ENDURANCE AND MULTILOCATION

by

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Abstract

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Material objects exist at different times. Endurance theory is the view that they are wholly present at each of the times at which they exist—or, that they are located at multiple regions of spacetime. In this dissertation, I argue that endurance theory is coherent by explaining how cases of multilocation (whether in space or in spacetime) are possible. My goals are twofold. The first is to show that there is nothing incoherent, both metaphysically and formally, in cases of multilocation and, thereby, in endurance theory. After having introduced temporal and regional variants of classical extensional mereology together with some principles about the location of objects in space, I show how our reluctance to admit cases of multilocation can be resisted by responding to an argument to the effect that they are incoherent. I then defend the view that endurance is multilocation in spacetime against rival characterizations. And, in the Appendix to the Dissertation, I develop formal theories of location in which objects can be located at several regions of space (or spacetime).

The second goal is to explain how the possibility of multilocation arises. I claim that it is possible for material objects to be located at several disjoint regions of space (or spacetime) because their haecceities, or the properties they have of being themselves, can be instantiated at these several regions. I offer an analysis of haecceities that allows us to give necessary and/or sufficient conditions for their instantiation. It is these conditions that constitute an explanation of the possibility of multilocation. I end the dissertation by showing that my analysis of haecceities, and of how they could come to be instantiated at distinct places, solves other issues in the metaphysics of persistence and, specifically, issues regarding the coincidence of material objects.

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Introduction

My future self, 20 years my elder, sent "his" watch back in time. It is sitting next to the watch I now have on my otherwise empty desk. Apparently, I will not acquire a new watch in the next 20 years, since his is exactly the same watch as I now have, despite the fact that it looks older and lacks the second hand. I am puzzled. The very same material object, the watch, is located at two different places on my desk, and is composed, at these different places, of different parts. How could that be? It seems that the description of the case is itself incoherent, or (if it is coherent) that the case is not possible.

This dissertation is an attempt to articulate the view that it is *possible* for material objects to be located at several places at once (or at different times) while having different parts at these places (or times)—it is not concerned with time travel, an issue hotly debated in the philosophy of physics. Why am I interested in defending such a view? The reason is that cases where an object is at different places at once and has different parts at each of these places, can be used to defend a certain theory about the *persistence* of material objects. Typically, we think that there are objects, like watches, that exist at more than one time. When an object exists at more than one time, we say that it *persists* in time. And there are *two* popular theories that attempt to give an account of the fact that material objects persist in time.

The one I am interested in is *endurance theory*. Its claim is that

> objects exist at different times and are wholly present (or completely there) at each of these times.

The whole dissertation is my attempt to make sense of this claim. And I do so in the following way. I understand this claim to mean that objects are located at several regions of spacetime, while having different parts at these regions. I take it that *that* phenomenon, i.e., that objects are located at several regions of spacetime, is a special case of the following possible phenomenon of multilocation.

> Material objects are located at more than one region of space (at once or at different times).

Endurance theory is a *special case* of multilocation. So the watch's being located at different regions of spacetime (that correspond to different times) is (just like) the realization of the possibility of its being located at two places at once, such as the two places on my desk. So it is by making sense of the latter possibility that I make sense of endurance theory.

So the dissertation attempts to make coherent and possible cases in which material objects are located at several places at once. It does not do more than that. In particular, it does not show that cases of multilocation are plausible or likely. And so it does not show that endurance theory itself is plausible. Although I aim to make endurance theory (or cases of multilocation) coherent, I will not argue that endurance theory is the best theory of persistence on the market. Similarly, I will not be concerned with showing that the possibility of multilocation for material objects has any use in our investigations of material objects (other than making endurance theory coherent and possible, that is). In fact, I will not argue that some philosophical problems, other than the ones posed by endurance theory, can be solved by appealing to multilocation. I believe that there are such problems, but I do not defend this belief here.

The other popular theory of the persistence of material objects is *perdurance theory*. It claims that objects persist in time by being temporally extended, and by having temporal parts at each of the times at which they exist. Perdurance theorists think that objects are extended in time in much the same way that they are extended in space. My guitar is extended in space, and it has parts at each of the regions of space where it is located. Perdurance theory says that my guitar is also extended in space, that it has some sort of temporal "length," and that it has parts (temporal parts) at each of the times at which it is. These temporal parts of the guitar are, so to speak, "slices" of the temporally extended guitar that are perpendicular to its temporal "length."

I do not defend perdurance theory. As we will see, it is already very well defended in the literature. But I do talk about it. The reason is that it is helpful to keep it in mind when discussing endurance theory, or cases of multilocation. For it helps to see how both theories, i.e., endurance and perdurance theory, differ on specific issues, and to see what is assumed by each. The sole purpose of writing about perdurance theory in this dissertation is to make endurance theory, or cases of multilocation, clearer.

The dissertation is an essay on the ontology of endurance theory. And the part of perdurance theory with which I will be concerned is its ontology. Sometimes, philosophers mean to talk about something more than ontological theories when they talk about perdurance and endurance theory. And so the ontologies of these theories are sometimes distinguished from the rest of the theory: the ontology of endurance theory is sometimes called "three-dimensionalism," while that of perdurance theory is sometimes called "four-dimensionalism." That is so because endurance theory entails that persisting objects in our world are three-dimensional, i.e., extended in three spatial dimensions only, while perdurance theory entails that they are extended in four dimensions (the three spatial ones, in addition to the temporal one). But in the dissertation, any occurrence of "perdurance theory" or of "endurance theory" is meant to designate the ontologies of either one of the theories. It may happen that I will use "four-dimensionalism" or "three-dimensionalism" instead of "perdurance theory" or "endurance theory." But whichever expression I use, I do not mean to talk about anything other than the ontologies of each theory of persistence.

The dissertation is composed of 6 chapters, and one appendix. Here is how the dissertation is organized. In the first three chapters, I defend the view that the best characterization of endurance theory is in terms of location in spacetime. And so my discussion is mainly about persistence. To that end, I introduce mereological and locative relations in Chapter 1. More specifically, I give the axioms and (important) theorems of classical extensional mereology, and I introduce some principles about the location of objects in space (or in spacetime). I make clear that endurance theory should *reject* the principle called "Functionality," which claims that the relevant locative relation, the one that is of use in the debate over the persistence of material objects, is a function. In the Appendix to the Dissertation, I develop theories of location that lack Functionality, and I contrast them with theories that are already on the market. In my view, this shows that Functionality is dispensable in formal theories of location. I finish Chapter 1 by introducing perdurance theory. As I said, this will be useful for the rest of the discussion of endurance theory.

In Chapter 2, I adopt a variant of classical extensional mereology in which the part-whole relation, i.e., the relation holding between a part and a whole, is a ternary relation that holds between parts, wholes, and regions of space (or spacetime). I stress that what motivates my adoption of that mereological theory is my rejection of Functionality. I end the chapter by responding to an argument to the effect that the resulting metaphysical view of material objects is incoherent.

As I said above, I take it that endurance theory is the view that objects are located at multiple regions of spacetime, which is just a special case of the view that they can be located at distinct regions of space. But not everybody would characterize the theory in such a way. Some rather think that endurance theory can be formulated in mereological terms. In Chapter 3, I show that this is not the case, and I further defend my version of endurance theory against arguments given in the literature.

Chapters 4, 5, and 6 explain how it is possible for the same object to be located at several regions of space. I steer away from the persistence of material objects. By explaining mutlilocation in space, however, I offer an explanation of endurance theory. In Chapter 4, I introduce the notion of an object's haecceity. An object *o*'s haecceity is the property it has *of being identical to itself*—or, more precisely, *of being identical to o*. Chapter 4 aims at offering an analysis of haecceities that would allow us to formulate sufficient and/or necessary conditions for their *instantiation*. It does not aim at doing more than that. I introduce haecceities because I explain, in Chapter 5, multilocation in terms of them. There, I claim that it is possible for a material object to be located at more than one region of space (or spacetime) because its haecceity can be instantiated at more than one region of space (or spacetime). In Chapter 6, I discuss a problem that plagues endurance theory and cases of multilocation, namely, the problem of the coincidence of material objects. I will then offer a solution to it.

Chapter 1

Parthood and Location

1. Introduction

In order to discuss endurance theory, it will be useful to have some idea of the ontology of perdurance theory. According to the latter, objects persist by being temporally extended, and by having temporal parts at all times at which they are present. In this chapter, I first introduce the axioms (and some theorems) of classical extensional mereology (CEM). I suppose that CEM is a component of perdurance theory, and that a suitable variant of it (introduced in Chapter 2) is a component of endurance theory. Second, I discuss some principles regarding the location of objects in space. Finally, I introduce four-dimensionalism, i.e., the ontology of perdurance theory.

2. Classical Mereology

In this section, I aim to introduce CEM's axioms, along with the theorems relevant to my discussion of persistence. I will discuss other related issues as well, but only to illustrate CEM further. I will proceed by first introducing the relations which different formulations of CEM take as basic, and then talk about the extensionality of parthood. The domain of classical mereology is closed under several operations. Thus, I will secondly introduce the operations that will be relevant to my discussion of persistence.

2.1 Parthood

Mereology is a theory of parthood, i.e., of the relation that holds between two objects whenever one is part of another. We intuitively think of the part-whole relation as being non-reflexive, anti-symmetric, and transitive. That is to say, we do not generally think of objects as being part of themselves (non-reflexivity); nor do we tend to think of two objects as being both parts of each other (anti-symmetry). Finally, if an object *a* is part of another object *b*, which is part of yet another object *c*, then we are tempted to say that *a* is part of *c* as well (transitivity).

More formally, the relation of parthood just introduced is that of *proper parthood* (PP). It satisfies the following conditions:[1]

(PP0) $\sim \mathrm{PP}xx$ (Non-reflexivity)
(PP1) $\mathrm{PP}xy \rightarrow \sim \mathrm{PP}yx$ (Anti-symmetry)
(PP2) $(\mathrm{PP}xy \wedge \mathrm{PP}yz) \rightarrow \mathrm{PP}xz$ (Transitivity)

One could bring a modification to our intuitive understanding of parthood and introduce a relation similar to PP except that it is reflexive, i.e., that it allows an object to be a part of itself. Such a relation of parthood (P) satisfies the following conditions:

(P0) $\mathrm{P}xx$ (Reflexivity)
(P1) $(\mathrm{P}xy \wedge \mathrm{P}yx) \rightarrow y = x$ (Weak Anti-symmetry)
(P2) $(\mathrm{P}xy \wedge \mathrm{P}yz) \rightarrow \mathrm{P}xz$ (Transitivity)

It is worth noting that P can be defined in terms of PP and vice versa in the following way:

(D1) $\mathrm{P}xy =_{\mathrm{df}} (\mathrm{PP}xy \vee x = y)$ (P in terms of PP)
(D2) $\mathrm{PP}xy =_{\mathrm{df}} (\mathrm{P}xy \wedge x \neq y)$ (PP in terms of P)

(D1) says that an object is part of another just in case either the former is a proper part of the latter, or they are both the same object. (D2) claims that an object is a proper part of

[1] I take the following principles from (Varzi 2007b) and (Simons 1987), and borrow the notation from (Varzi 2007b). Accordingly, I omit the primary universal (first-order) quantifiers in order to lighten up the presentation.

[2] (LM) and (LII) are the objects of controversies when the identity predicate is given a diachronic reading. For it seems that the same object at different times could have different parts or different properties. In the

another just in case the former is part of the latter, and they are distinct (i.e., not identical). Neither of these relations is thus more primitive than the other.

Another relation worth introducing is that of overlap (O). Two objects overlap whenever they share a part in common. Thus, overlap can be defined in terms of the parthood relation.

(O) $Oxy =_{df} (\exists z)(Pzx \wedge Pzy)$ (Overlap)

However, parthood (P) can also be defined in terms of overlap (O).

(O1) $Pxy =_{df} (\forall z)(Ozx \rightarrow Ozy)$ (P in terms of O)

One thing is part of another just in case everything that overlaps the former also overlaps the latter. Unlike the parthood relation (P), the relation of overlap (O) is symmetric. Thus,

$$Oxy \leftrightarrow Oyx.$$

Finally, we can say that two objects are *disjoint* (D) just in case they do not overlap.

(D) $Dxy =_{df} \sim Oxy$

We can therefore define overlap (O), similarly, in terms of disjointness (D). Disjointness, like overlap, is symmetric.

Because they can all be defined in terms of one another, any of the relations of parthood, proper parthood, overlap, and disjointness can be used as primitive in a mereological system. The choice of one over the other is thus purely arbitrary. I will use (P).

2.2 Extensionality

When mereology is employed as a theory of material objects, the issue arises as to whether criteria of identity of objects can be given only in terms of their parts. The principle of *extensionality of parthood* states that

(E) $[(\exists y)(\mathrm{PP}ya) \wedge (\exists y)(\mathrm{PP}yb)] \rightarrow [(\forall x)(\mathrm{PP}xa \leftrightarrow \mathrm{PP}xb) \rightarrow a = b]$

Here is how I read it:

> It is sufficient for composite objects, i.e., objects with proper parts, to have the same proper parts for them to be identical.

We would also think that it is necessary that objects have the same proper parts for them to be identical.

(LM) $x = y \rightarrow (\forall z)(\mathrm{PP}zx \leftrightarrow \mathrm{PP}zy)$

(Note that we do not need to restrict (LM) to composite objects.) (LM) reads in the following way:

> Identical objects share all of their proper parts.

(LM) seems correct, for if there is one object that is a proper part of x but not of y, then x and y are not identical. Equivalently, it is sufficient for objects to be identical for them to have the same proper parts. (LM) is an instance of Leibniz's Indiscernability of Identicals (II).

(II) $(\forall P)\ [x = y \rightarrow (Px \leftrightarrow Py)]$

(II) claims that

> Identical objects share all of their properties.

And both principles, i.e., (LM) and (II), find a motivation in the thought that identical objects should be alike in all respects.[2]

The principle of *mereological extensionality* (ME) follows from both (E) and (LM). It claims that it is necessary and sufficient for identical objects to have the same proper parts.

(ME) $[(\exists y)(PPya) \wedge (\exists y)(PPyb)] \rightarrow [(\forall x)\ (PPxa \leftrightarrow PPxb) \leftrightarrow a = b]$

I will put (ME) aside for the moment and focus on (E).

2.2.1 *Extensionality of Parthood*

We can understand how (E) comes about in a mereological system by analyzing the principles of supplementation. Our colloquial use of "part" suggests that a whole cannot have only one proper part. For then, what would be the difference between the whole and the proper part? A mereological system should thus include a principle of supplementation, i.e., a principle according to which other proper parts always supplement a proper part of a whole. More formally:

(WS) $PPxy \rightarrow (\exists z)\ (PPzy \wedge \sim Ozx)$. (Weak Supplementation)

(I follow common usage in employing the notion of overlap rather than disjointness.) Here is how we can read (WS):

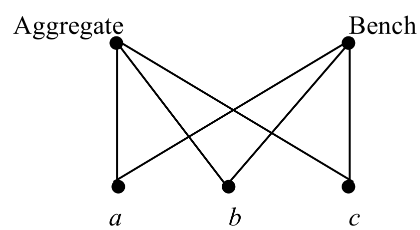
> If an object is part of another, then there is something that is a proper part of the latter but that does not overlap the former.

(WS) is the principle of *weak supplementation*.

---

[2] (LM) and (LII) are the objects of controversies when the identity predicate is given a diachronic reading. For it seems that the same object at different times could have different parts or different properties. In the next chapter, I will analyze the diachronic reading of the identity predicate with regard to (LM). (LII) will be discussed in Chapter 4.

(WS) is an axiom of CEM, and it contributes to the fact that the latter mereological system is extensional. For (WS) prevents wholes from being *distinct* from the totality of their parts. But it is not sufficient for (E). The latter indeed prevents distinct objects from having the same proper parts. For ease of exposition, imagine a bench made of three rectangular wood pieces *a*, *b*, and *c*, where *a* and *b* are the legs, and *c* is the seat. (WS) entails that neither the bench nor the aggregate of *a*, *b*, and *c* have parts distinct from *a*, *b*, and *c* together. But it leaves open the possibility that the bench and the aggregate are distinct. Refer to Model 1[3] below for clarity. In this model, *a*, *b*, and *c* are all proper parts of both the bench and the aggregate, and so both the bench and the aggregate have proper parts that are disjoint from *a*, *b*, or *c* (but not from all three). So (WS) holds. Because *a*, *b*, and *c* are the only parts of the two distinct objects, (E) does not hold.



Model 1

A way to exclude that the aggregate and the bench are distinct is to show that they are both parts of each other, and to use (P1) to conclude that they are identical. We could do so upon introducing the principle of *strong supplementation*.

[3] Here is how we can read this figure: The dots represent things (material objects in our case), and the lines represent the relationship between these things. In this case, the lines represent *proper parthood*, and the dots at the lower level are proper parts of the dots at the higher level.

(S) $(\forall z)(Pzx \rightarrow Ozy) \rightarrow Pxy$ (Strong Supplementation)

Here is how I read (S):

> If it is the case that every part of object *x* overlaps another object *y*, then *x* is part of *y*.

Model 1 is not compatible with (S) and (P0—P2). For it follows from (S) that the aggregate is part of the bench (since all of the aggregate's parts overlap those of the bench), and that the bench is part of the aggregate (since all of the bench's parts overlap the aggregate). With (P1), we conclude that the aggregate and the bench are identical.

Although it was better suited to our purpose, (S) is often informally introduced in the literature in the following way. If two objects are not part of each other, then one of them has at least one part disjoint from the other.

(S') $\sim Pxy \rightarrow (\exists z)(Pzx \wedge \sim Ozy)$

We can see from (S') that it is really a supplementation principle, whereas it is less obvious in the case of (S). It goes without saying, however, that (S) and (S') are equivalent; indeed, they are contrapositives of one another.

Many philosophers (e.g. Judith Jarvis Thomson (Thomson 1998) and Peter Simons (Simons 1987, p. 180)) would recommend rejecting (P1) when material objects are at issue. The point is precisely to vindicate the idea that distinct material objects can be part of each other. One reason for claiming that the bench and the aggregate are distinct stems from (II) above: the aggregate and the bench do not have the same historical or modal properties. Indeed, the aggregate was created well before the bench was, and it would not survive the replacement of any part (whereas the bench would). In Chapter 6, I will analyze cases like the bench and the aggregate, and I will show why they do not make (P1) false. At this point, let us only note that mereological systems that do not include (P1) can be made to contain (S), and yet fail to be extensional. In any case, we can conclude that (P0 – P2) and (S) are jointly sufficient for (E).

2.3 Fusion and Extensionality

I have been analyzing principles related to the extensionality of parthood, as well as the properties of the part-whole relation. Here, I will focus on some operations of CEM.

2.3.1 *Fusion and Product*

The domain of CEM is closed under several operations. In this section, I focus on the operations of fusion, product, and least upper bound. For any set of entities one already recognizes, these operations can be used to generate other entities on the basis of the former.

The *fusion* of any two objects *a* and *b* is an object such that anything that overlaps it overlaps either *a* or *b*, and vice versa.

$$x + y =_{\mathrm{df}} i(z)(\forall w)(\mathrm{O}wz \leftrightarrow (\mathrm{O}wx \vee \mathrm{O}wy))$$ [4]

(In section 2.4 below, I slightly change the definition of the operation of fusion for reasons that are indicated there.) A least upper bound *c* of objects *a* and *b* is such that it is part of anything of which *a* and *b* are parts, and vice versa. More formally,

$$x +' y =_{\mathrm{df}} i(z)(\forall w)(\mathrm{P}zw \leftrightarrow (\mathrm{P}xw \wedge \mathrm{P}yw)).$$

[4] When defining the operations of fusion, product, and least upper bound, I use an operator for definite description. This use is warranted in an extensional system such as CEM, since there is no more than one object that can be the fusion, the product, or the least upper bound of two further objects. We will see that in more detail below.

(I follow the notation of (Simons 1987, p. 32) in using " + " for the fusion of two objects, and " +′ " for the least upper bound of any two objects.) If objects *a* and *b* have a fusion, it follows that they also have a least upper bound (indeed, that the fusion *is* the least upper bound). But there could be restrictions on fusion such that objects with a least upper bound do not have a fusion. Simons (Simons 1987, p. 32-33) gives the following example: two disjoint intervals on the real line have a least upper bound, namely, the smallest interval that includes both of them as parts. But since no interval can be scattered on the real line, it does not follow that our initial two intervals have a sum.

The product of two overlapping objects *a* and *b* ($a \times b$) is an object the parts of which are all parts of both *a* and *b*, and of which all the parts common to *a* and *b* are also parts. More formally,

$$x \times y =_{\mathrm{df}} \iota(z)(\forall w)(\mathrm{P}wz \leftrightarrow (\mathrm{P}wx \wedge \mathrm{P}wy))$$

There is a formal similarity between the principle for product and that for least upper bound. It reflects the fact that the least upper bound of any objects is the smallest object that contains them as a part, while the product of any objects is the largest object that is part of all of them.

What has been said above about the fusion, least upper bound, and product of two objects can be generalized to cases that involve an infinity of objects. I will set aside the operation of least upper bound, and focus from now on only on fusion and product. The fusion (∑) of the objects that are F is defined in the following way:

(F) $\sum x(\mathrm{F}x) =_{\mathrm{df}} \iota(z)\,(\forall y)(\mathrm{O}yz \leftrightarrow (\exists w)(\mathrm{F}w \wedge \mathrm{O}yw))$ Fusion

Here is how I read (F):

> The fusion of everything that is an F is the object such that anything that overlaps *it* overlaps at least one of the Fs, and such that anything that overlaps an F, overlaps it.

Typically, (F) is meant to assert the existence of an object that has all the Fs as parts, and that has no part that is disjoint from the Fs. Hence, we can formulate (F) with the part-whole relation in order to formalize the informal reading I just gave to it.

(F') $\sum' x(\mathrm{F}x) =_{\mathrm{df}} i(z)((\forall y)(\mathrm{F}y \rightarrow \mathrm{P}yz) \wedge (\forall y)(\mathrm{P}yz \leftrightarrow (\exists w)(\mathrm{F}w \wedge \mathrm{O}yw)))$

Here is how I read (F'):

> The fusion of the Fs is the object such that all of the Fs are part of it, and any part of it overlaps at least one of the Fs (and vice versa).

Paul Hovda (Hovda 2009) discusses different axiomatizations of CEM in a clear and elegant way. He claims, as is expected, that (F) and (F') are equivalent once we have selected the axioms and definitions that yield CEM. But there are important differences between (F) and (F') such that axiomatizing CEM using one or the other will result in a slightly different mereological system. I will discuss one of these differences presently. Discussing Hovda's conclusion, however, would lead me too far afield.

Before turning to the differences between (F) and (F'), and to the axiomatization of CEM, let us note that the operation for the product (∏) of objects that are F is defined in the following way:

(Π) $\prod x(\mathrm{F}x) =_{\mathrm{df}} i(z)\ (\forall y)(\mathrm{P}yz \leftrightarrow (\forall w)(\mathrm{F}w \rightarrow \mathrm{P}yw))$ Product

Here is how it should be read:

> The product of everything that is F is the object all of whose parts are parts of all of the Fs.

Let us now turn to CEM itself.

2.4 Classical Extensional Mereology (CEM)

Classical Extensional Mereology (CEM) is the mereological theory on which my discussion of the persistence of material objects, as well as their location in space, is based. There are many ways to axiomatize it, and I will settle on a set of axioms below. For now, let us focus on a set of axioms that does not yield CEM, contrary to what is assumed in the literature. (See (Casati and Varzi 1999; Simons 1987)). The set of axioms I have in mind is composed of (P0), (P1), (P2), (WS) (given again below), and the schema (FE) for fusion existence. (FE) says that if there are objects that are F, then there is a fusion of them.

(FE) $\exists x Fx \rightarrow \exists z \forall y (Oyz \leftrightarrow \exists x\, (Fx \wedge Oyx))$

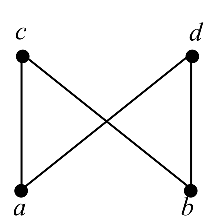
(where the predicate F can be replaced by any predicate whatever).

(P0)	Pxx	(Reflexivity)
(P1)	$Pxy \wedge Pyx \rightarrow y = x$	(Weak Anti-symmetry)
(P2)	$(Pxy \wedge Pyz) \rightarrow Pxz$	(Transitivity)
(WS)	$PPxy \rightarrow (\exists z)\, (PPzy \wedge \sim Ozx)$.	(Weak Supplementation)

I assume, as is commonly accepted in the literature, that CEM (as well as all the mereological systems I discuss) also contains the axioms of first order predicate calculus with identity.

While discussing the case of the bench and the aggregate above, we saw that (P0—P2) together with (WS) was not sufficient to ensure extensionality (E). We needed, in addition to (WS) (or instead of (WS)), the principle (S) of strong supplementation. Now, we can derive (S) from (P0—P2) together with a principle asserting the existence of a product (Π) of any overlapping entity. The proof is given by Simons (Simons 1987, p. 31), and takes the following form. Assume two composite objects *a* and *b* such that *a* is not part of *b* (~P*ab*). Then, *a* and *b* overlap, or they do not. In the latter case, it is clear that *a*'s parts do not overlap *b*. And hence, *a* has at least one part (if only an improper part) that does not overlap *b*. In the first case, however, it follows from (Π) that *a* and *b* have a product—here, we should not suppose that the product is *unique*; otherwise, we assume what we want to show. Call the product of *a* and *b* '*a* × *b*'. From the assumption that *a* is not part of *b* but that they overlap, it follows that *a* × *b* is a proper part of *a* and is also a part of *b*. In that case, we can conclude, with the help of (WS), that *a* has at least one part that does not overlap *a* × *b*. Therefore, *a* has at least one part that overlaps no parts of *b*. But (S) precisely claims that if *a* is not part of *b*, then *a* has at least one part that does not overlap *b*. (S) is thus derivable from (P0—P2), (WS), and (Π). And, as we have seen, (S) together with the axioms P(0) – P(2) implies (E).

Simons thinks that the principle that asserts the existence of a product of overlapping objects is also a part of CEM, since he thinks of the product of objects as the sum of their common parts (and, hence, that the principle for product follows, among others, from (FE) above). Unfortunately, the set of axioms given above, i.e., (P0—P1), (WS), and (FE), is not sufficient for the principle for product. To see this, consider Model 2:



Model 2

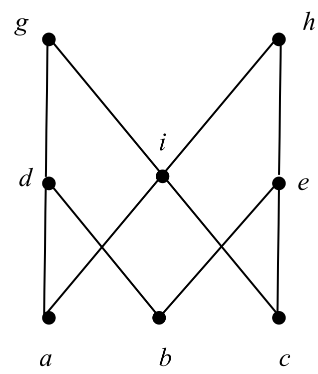
Model 2 is similar to Model 1 in that we have two distinct objects, *c* and *d*, composed of the same proper parts. We already know that Model 2 is compatible with (P0—P2) and (WS). As Carsten Pontow (Pontow 2004) and Paul Hovda (Hovda 2009) show, Model 2 is compatible with (FE). Here is how Pontow shows it. Consider the domain *D* of the model which comprises *a*, *b*, *c*, and *d*. We have to show that all the subsets of *D* have a fusion, the existence of which is postulated by (FE). So, take *B*, a subset of *D*, and consider it empty. In that case, (FE) holds trivially, since there are no Fs to fuse. Any *B* that is a subset of *D* and that contains only one element will have a fusion, namely that element itself. Now, the subset *B* = {*a*,*b*} also has a fusion. For there is an object, say *c*, such that anything that overlaps it overlaps *a* or *b*, and such that anything that overlaps *a* or *b* overlaps *it*. The same would hold true of *d*. Finally, both *c* and *d* are fusions of the subset B = {*a*, *b*, *c*, *d*}. For anything that overlaps either *c* or *d* overlaps every member of *B*. *c* is such a thing, and so is *d*. So, (FE) is compatible with Model 2. And yet, (Π) is not: *c* and *d* do *not* have a product. For the product of *c* and *d* would be the object such that anything that is part of *both c* and *d* is part of it (and vice versa). Neither *c* nor *d* can qualify as their product. Indeed, suppose that *c* is the product of *c* and *d*. (The thought here is that *c* is the fusion of *a* and *b*, and that *a* and *b* are also parts of *d*). Then, *c* would have to be a part of *d*, which it is not. The same holds for *d*. There is nothing in our set of axioms that makes it so that *c* and *d* are part of each other.

So, the set of axioms (P0—P2), (WS), and (FE), is not sufficient for the principle for product. *A fortiori*, it is not sufficient for (E). We can indeed see that (E) is not a theorem of the set of axioms under discussion just by examining Model 2. The latter presents us with *two* fusions of *a* and *b*, i.e., two distinct objects with the same proper parts. Perhaps one could salvage the widespread thought that our set of axioms yields CEM by adopting the following principle of the *uniqueness of fusions*:

> (Uniqueness) For any class of objects, there is at most one fusion of the members of that class.

(Uniqueness) would clearly rule out Model 2, and it is sufficient for (E). For take a model in which (E) does not hold (such as Model 2). It is clear that (uniqueness) does not hold in that model either, since the latter would allow two distinct objects to have the same proper parts. Because these objects qualify as the sum of their proper parts, it follows that (uniqueness) is false (see (Varzi 2008, p. 109)). By contraposition, therefore, (uniqueness) implies (E).

But why not just add (E) to the set of axioms under consideration? Because a mereological system that has (E) does not necessarily contain (uniqueness). Refer to Model 3, which is adapted from a model given in (Varzi 2008, p. 109). In this model, both *g* and *h* are fusions of *a*, *b*, and *c*, and yet they are distinct by (E). *g* (but not *h*) has *d* as a part, and *h* (but not *g*) has *e* as a part. Model 3 is also a model of our set of axioms (P0—P2), (WS), and (FE). Because Model 3 is similar to Model 1, we know that it is compatible with (P0—P2) and (WS). We can see that (WS) holds since nothing has proper parts *only*. It remains to show that it is also compatible with (FE).



Model 3

Take the domain *D* in Model 3. We need to show that all of its subsets *B* have a fusion. If *B* is the empty set, then (FE) holds vacuously. There is simply nothing to fuse. And if *B* is a singleton, then its member *is* its fusion. Any subset *B* of which *g* is a member will be fused by *g*. For *g* overlaps everything in Model 3. The same holds for any *B* that contains *h*; *h* will fuse *B* since it overlaps everything. (So *D* itself has two fusions, *g* and *h*.) We are left with the following subsets: $B_1 = \{a, b, d\}$, $B_2 = \{e, b, c\}$, $B_3 = \{a, b, c\}$, $B_4 = \{d, e\}$, $B_5 = \{d, i\}$, $B_6 = \{i, e\}$, $B_7 = \{a, b\}$, $B_8 = \{b, c\}$, $B_9 = \{a, c\}$. *d* fuses B_1, while *e* fuses B_2. For, indeed, anything that overlaps *d* overlaps *a* or *b* (and vice versa), while anything that overlaps *b* or *c* also overlaps *e* (and vice versa). B_4, B_5, and B_6 all have the same two fusions, namely *g* and *h*. For the latter objects overlap everything, and are overlapped by anything that overlap either a) *d* or *e*; or b) *d* or *i*; or c) *i* or *e*. As for B_7, B_8, and B_9, they are fused by *d*, *e*, and *i*, respectively. Hence, (FE) is compatible with Model 3, and so is (E).

So (E) does not imply (uniqueness). And, in any case, adding either (E) or (uniqueness) to our set of axioms would be a rather *ad hoc* way to make it so that our axioms yield CEM. Fortunately, there is a simpler solution. Instead of taking (FE) as our axiom-schema of fusion existence, we could take the following one (FE'):

(FE') $(\exists x)(Fx) \rightarrow (\exists z)((\forall y)(Fy \rightarrow Pyz) \wedge (\forall y)(Pyz \leftrightarrow (\exists w)(Fw \wedge Oyw))))$[5] (Fusion Existence)

Here is how we can read (FE'):

> If there are objects that are F, then there is an object that contains all of them as parts and that is such that any part of it overlaps at least one of the Fs, and anything that overlaps at least one of the Fs is part of it.

Note that Model 3 is ruled out by (FE') together with (WS) and (P0—P2). For, in model 3, objects *a*, *b*, and *c* (the Fs, let us suppose) are all parts of both *g* and *h*. But *g* and *h* overlap everything. And yet, not everything that overlaps any of *a*, *b*, or *c* is part of *g* (or of *h*). In compliance with (FE'), we would get that anything that overlaps either *a*, *b*, or *c* is part of both *g* and *h*. Thus, *g* would be part of *h*, and *h* part of *g*. From (P1), it would follow that $g = h$.

As Hovda (Hovda 2009, p. 66) notes, that we can obtain (uniqueness) from (FE') (or F') is a strong result. I make use of this insight to offer the axiomatization of CEM which I adopt. I take CEM to be composed of the following axioms:

(P0) Pxx (Reflexivity)
(P1) $Pxy \wedge Pyx \rightarrow y = x$ (Weak Anti-symmetry)
(P2) $(Pxy \wedge Pyz) \rightarrow Pxz$ (Transitivity)
(WS) $PPxy \rightarrow (\exists z)\ (PPzy \wedge \sim Ozx)$. (Weak Supplementation)
(FE') $(\exists x)(Fx) \rightarrow (\exists z)((\forall y)(Fy \rightarrow Pyz) \wedge (\forall y)(Pyz \leftrightarrow (\exists w)(Fw \wedge Oyw))))$ (Fusion Existence)

(We find in (Hovda 2009, p. 65-66) the suggestion that the aforementioned axioms yield CEM.) It would have been possible to hold on to (FE), and replace (WS) by (S), without losing the spirit of CEM. However, some (e.g., Simons (Simons 1987, p. 26)), take (WS)

[5] For coherence, I replaced the operator for definite description '*ι*' in (F') by an existential quantifier.

to be an intuitive truth about proper parthood (and not (S)), and I find that (FE') captures better what we think of when we think of a fusion of objects.[6]

I aim to introduce the perdurance theorist's ontology, namely, four-dimensionalism, since it will be helpful in our discussion of endurance theory. To that end, it is appropriate to introduce, in addition to CEM, the relations holding between objects and space. For it is in terms of the location of a persisting object that one can cash out the fact that it is temporally extended. I now turn to these relations.

3. Location

3.1 Location and Occupation

An object can be said to occupy a certain region of space or to be located at a region.[7] There is no consensus in the literature on whether and how the relations of occupation and of location are distinct. Some (e.g. (Parsons 2008)) even take them to be the same relation. In the course of the dissertation, however, I will construe them as two different families of relations, because I am interested in defending the view that objects can be located (in a way soon to be specified) at multiple disjoint regions of space, without being

[6] See (Hovda 2009) for other axiomatization of CEM. Due to space constraints, I cannot discuss them here.

[7] From here on, I make the following two assumptions: 1) I assume substantivalism about any space I may appeal to in the course of the dissertation. I assume that space is a particular along other particular objects such as cars, cups, teeth, and teddy bears. I further suppose that space can be arbitrarily divided into regions, which themselves can be decomposed into subregions. A region *r*'s subregion is a part of that region, while *r*'s superregion is any region that has *r* as a part. *r* itself is both an improper subregion and an improper superregion of itself. I will refer to parts of regions and subregions interchangeably, as they are synonymous. I suppose that space has atomic parts, namely points of space. 2) When I talk about spacetime, I assume a non-relativistic four-dimensional spacetime ultimately composed of primitive spacetime points (a Galilean spacetime would be perfect). Both the relations of simultaneity and of directionality are well defined; that is to say, for any two points, there is an absolute (i.e., not relative to any scheme of reference) answer to the question as to whether they are simultaneous. Similarly, for any two points, there is an absolute answer to the question as to which one is earlier (or later) than the other. These assumptions about space are taken from (Sattig 2006, p. 21-22).

so located at the fusion of these regions. Yet, I want to use the notion of occupation in order to say that these objects occupy the latter fusion. I will also suppose that the *locative properties* of an object are the relations of occupation and location between that object and regions of space. I start by defining the notion of occupation and then turn to that of location. The notion of location is the most important one.

3.1.1 *Occupation*

Take an object *o* and a region *r*, and let *o exactly occupy* (OC) *r* whenever *o* fills up *r* and no other superregion of *r*. I do not intend to impose peculiar conditions on this very informal gloss of exact occupation. The idea is simply that a region all parts of which contain an object, and of which no other superregion is so filled up by an object, is a region that the object exactly occupies. I take it that objects exactly occupy only regions, and thus that

$$\mathrm{OC}or \rightarrow \mathrm{R}r$$

where the predicate R is that for region. I will define the region predicate below in section 3.1.2, where I will make clear that, in my view, whether or not something is a region depends on whether or not that thing is *located* at itself. Taking OC as a primitive, we may move on to define other occupation relations. I will focus here on the notion of generic occupation and on pervasive occupation.

An object *o generically occupies* any region that overlaps the region it exactly occupies. More formally,

$$\text{(OG)} \quad \mathrm{OG}or =_{\mathrm{df}} (\exists s)(\mathrm{OC}os \wedge \mathrm{O}rs \wedge \mathrm{R}r) \qquad \text{(Generic Occupation)}$$

Because the fusion of a region *r* (exactly occupied by an object) and of anything whatsoever overlaps *r*, we need to specify that *o* can generically occupy only regions—hence, the presence of "R*r*" in (OG). The idea behind OG is that an object generically occupies any region that is not completely free of it. Thus, if this informal gloss on the notion of generic occupation is clearer than the gloss on the notion of exact occupation, we can take generic occupation (OG) as our primitive and define OC in terms of it.

(OC) $\mathrm{OC}or =_{\mathrm{df}} (\mathrm{R}r \rightarrow (\forall s)\,(\mathrm{P}sr \leftrightarrow \mathrm{OG}os))$ (Exact Occupation)

o exactly occupies *r* just in case it generically occupies all of region *r*'s parts, and all of the latter are generically occupied by *o*.

An object *o pervasively occupies* a region *r* just in case *r* is a part of the region *s* it exactly occupies.

(OP) $\mathrm{OP}or =_{\mathrm{df}} (\exists s)(\mathrm{R}s \wedge \mathrm{OC}os \wedge \mathrm{P}rs)$ (Pervasive Occupation)

The idea behind (OP) is, simply, that for any object that exactly occupies a certain region, that object also occupies some parts of that region. Whenever I talk about an object *occupying* a region, I have exact occupation in mind (unless otherwise specified). I will use exact occupation as the primitive occupation relation, but one can also take generic occupation as primitive, and that will not change anything I say about the relations holding between an object and a region.

3.1.2 *Location*

Let us suppose that an object *o* is *exactly located* L at a region *r* if and only if *o* generically occupies *r* and *o* and *r* have the same shape.[8] I informally define exact location in terms of the notion of generic occupation, but I would be happy to take exact location as primitive in the discussion to follow. I take it that an object's persistence is a matter of its being located in special ways in spacetime, and *not* a matter of its occupying spacetime. This is not to say that there is no relation to be drawn between the notions of occupation and location. I will come back to the relations holding between occupation and location in section 3.1.4.

Roberto Casati and Achille Varzi (Casati and Varzi 1999), Varzi (Varzi 2007b), and Josh Parsons (Parsons 2007) offer theories of location. The following assumption of functionality is endorsed by them all:

$$(\mathrm{L}oy \wedge \mathrm{L}oz) \rightarrow y = z \qquad \text{(Functionality)}$$

This says that a single object cannot be exactly located at more than one region, and ensures that L is a function. In my view, Functionality is an important principle in the debate over the persistence of material objects. For,

> as it turns out, *perdurance theory requires Functionality to hold, while endurance theory requires that it should not.*

Indeed, endurance theory involves the rejection of Functionality. I will say more about that in the next chapter. Another assumption, explicitly made by Casati and Varzi (Casati and Varzi 1999, p. 121), is that of conditional reflexivity.

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[8] I shall use the expression "exact location" (or its cognates) to talk either about the relation between an object and region(s) of space, or about the regions of space where an object is exactly located.

$Lxy \rightarrow Lyy$ (Conditional Reflexivity)

This says that if there is an object located at a region *r*, then *r* is located at itself. I take it that regions, in addition to objects, are located somewhere.[9]

As will become clear in the next chapter, I am interested in locative systems in which Functionality does *not* hold. However, I think that something in the spirit of Functionality holds for regions, for if we allow regions to have locative properties, and – in particular – if we allow them to be exactly located at themselves, it should be the case that regions cannot be exactly located at more than one region. In order to express such a thing, let us first define the region predicate R, and let us make use of the fact that regions are exactly located at themselves in order to do so (as in (Casati and Varzi 1999, p. 123)).

$Rx =_{df} Lxx$ (Region Predicate)

Then, let us stipulate that if *x* is exactly located at *y* and both *x* and *y* are regions, then *x* and *y* are identical.

$(Lxy \wedge Rx \wedge Ry) \rightarrow x = y$ (Regional Uniqueness)

It thus follows that regions are solely located at themselves (and not also at other regions) without needing it to be the case that exact location is a function.

Two more stipulations are needed about our region predicate. First, let us stipulate that if something is a region, then any part of it is also a region.

$(Rx \wedge Pyx) \rightarrow Ry$ (Regional Dissection)[10]

[9] This may not be widely shared. For instance, Simons (Simons 1987, p. 179) defines the predicate "exist at" in such a way that it cannot be true only of times. So, one could follow Simons's idea and introduce a relation of location such that regions cannot be located anywhere. When introducing Conditional Reflexivity, Parsons does not mind our thinking of regions as the things that are located at themselves.

Furthermore, let us say that the fusion of regions is also a region.

$$((\exists x)(Fx) \wedge (\forall y)(Fy \rightarrow Ry)) \rightarrow R(\sum x(Fx)) \qquad \text{(Regional Additivity)}$$

I will make little use of Regional Additivity, but I thought it appropriate to introduce it anyway.

Given either Functionality *or* Regional Uniqueness above, the following two properties of exact location L follow.

$$(Lxy \wedge Lyx) \rightarrow x = y \qquad \text{(Weak anti-symmetry)}$$
$$(Lxy \wedge Lyz) \rightarrow Lxz \qquad \text{(Transitivity)}$$

That completes my introduction of exact location.

3.1.3 *Generic and Pervasive Location (and Occupation)*

We can define generic and pervasive location similar to the way we defined generic and pervasive occupation. Informally, we can claim that an object *o* is generically located at any region that is not completely free of *o*. Parsons's (Parsons 2007, p. 4) cashes out the informal gloss thus:

$$L_g or =_{df} (\exists s)(Los \wedge Osr).$$[11]

An issue immediately arises with (L_g). It follows from Conditional Reflexivity above and from the definition given to the region predicate R that the exact location of an object is a region. Our present definition of generic location does not ensure that objects are

[10] As Casati and Varzi (Casati and Varzi 1999, p. 124) note, Regional Dissection follows from Conditional Reflexivity and the definition of entire location (whole location in (Casati and Varzi 1999)) that I shall introduce below.

[11] Casati and Varzi (Casati and Varzi 1999, p. 120) offer the following definition of generic location in terms of exact location (rewritten using the notation I employ): $L_g xy =_{df} (\exists z)(\exists w)(Pzx \wedge Pwy \wedge Lzw)$. In other words, *x* is generically located at *y* just in case a part of *x* is exactly located at a part of *y*. Should *x* be an extended simple, it would follow that it would be generically located at *y* only if its exact location is a part of *y*. Surely, though, *y* is not completely free of *x* even when the exact location of *x* merely overlaps *y*. These are Parsons's reasons for offering his definition of generic location. I agree with him.

generically located *only* at regions. From what has been said, r could be the mereological fusion of a part of the region s and my scarf. We want it to be the case, though, that objects can be generically located only at regions. I make that explicit in a definition of generic location. Thus,

(L$_g$) $\quad \mathrm{L_g}or =_{\mathrm{df}} (\exists s)(\mathrm{R}r \wedge \mathrm{L}os \wedge \mathrm{O}sr)$. (Generic Location)

I turn now to discussing a locative system with Generic Location as a primitive.

We could take generic location as our primitive and define exact location in terms of it. The following is given by Parsons

(L) $\quad \mathrm{L}or =_{\mathrm{df}} (\forall s)(\mathrm{O}sr \leftrightarrow \mathrm{L_g}os)$.

(L) claims that o is exactly located at anything r such that it is both necessary and sufficient that r should overlap all of the regions at which o is generically located. Here again, nothing ensures in (L) that objects are exactly located at *regions*. For the regions at which o is generically located are overlapped by objects that are not regions. So, a correct definition of exact location in terms of generic location would be the following:

(L') $\quad \mathrm{L}or =_{\mathrm{df}} (\forall s)((\mathrm{R}s \wedge \mathrm{O}sr) \leftrightarrow \mathrm{L_g}os)$.

But (L') obviously does not hold if Functionality above is false, i.e., if our object o is exactly located at multiple disjoint regions of space. The right-to-left reading of the right-hand side of the identity sign fails. For it is not true, of one of the exact locations r of such an object, that it is overlapped by *all* of the regions at which the object is generically located. The problem is that there are regions not in the vicinity of r at which the object is generically located.

The correct definition of exact location in terms of generic location is as follows:

(L'') $\mathrm{L}or =_{\mathrm{df}} (\forall s)((\mathrm{R}s \wedge \mathrm{O}sr) \rightarrow \mathrm{L_g}os) \wedge [(\forall s)((\mathrm{PP}rs \wedge \mathrm{SC}s) \rightarrow \sim (\forall t)((\mathrm{R}t \wedge \mathrm{O}ts) \rightarrow \mathrm{L_g}ot))]$

Here is how I read (L'):

> An object *o* is exactly located at a region *r* just in case *o* is generically located at all the regions that overlap *r*, and for any *self-connected* proper superregion *s* of *r*, it is false that *o* is generically located at all of the regions that overlap *s*.

In (L''), I introduce a topological notion, namely that of self-connection. A region (or an object) is *self-connected* just in case it is not scattered. The idea behind (L'') is simple. I suppose that an object's exact location is a region that is self-connected, and that it is not connected to any of the object's other exact locations. That is to say, an object's exact locations are separated by regions of space (if only points of space). So, in (L''), we claim that an object's exact location is any region *r* such that *o* is generically located at any region that overlaps *r*. More crucially, we ensure that *r* is not connected to any other of the object's exact locations by making sure that all of the proper self-connected supperregions of *r* contain a subregion at which the object is *not* generically located.

Obviously, (L'') comes somewhat unannounced at this stage in the dissertation. I give a justification for it, together with the topological assumptions on which it is based, in the Appendix to the Dissertation. There, I tackle an obvious problem with (L''), namely, that of the exact locations of scattered objects (such as bikinis). I also show that locative systems with L or $\mathrm{L_g}$ as primitive are equivalent, whether or not Functionality holds. I make clear, finally, that (L'') is *also* a good definition of L in terms of $\mathrm{L_g}$ in a system in which Functionality holds. Let us now end our discussion of locative relations.

An object *o* is *pervasively located* at any parts of the region at which it is exactly located. We may express this in the following way:

(L_p) $L_p or =_{df} (\exists s)(Los \wedge Prs)$ Pervasive location

Here again, the idea is the same as that for pervasive occupation, namely, that an object entertains a locative relation with any parts of the region at which it is exactly located. Note that we do not need to stipulate that *r* is a region, since it follows from Regional Part, Conditional Reflexivity, and the Region predicate that it is.

The informal gloss I gave to both the notion of generic occupation and that of generic location is the same. This suggests that the two notions are equivalent.

$$OGor \leftrightarrow L_g or$$

If Functionality is true, it follows that exact location and exact occupation are equivalent. For *the* region at which the object is exactly located is also *the* region it exactly occupies.

$$OCor \leftrightarrow Lor$$

This last equivalence fails in the absence of Functionality. It still remains the case that an object exactly located at only one region of space exactly occupies that region. But objects that are exactly located at several disjoint regions do not exactly occupy these regions. To see this, suppose *o* is exactly located at disjoint *r* and *s*. Then *o* exactly occupies neither *r* nor *s*, for in each case there is a superregion other than the region at which it is exactly located that *o* fills up. In particular, *o* fills up the fusion of *r* and *s*. So, it is not the case that *o* exactly occupies the regions at which it is exactly located. The converse is also true; for if *o* is exactly located at region *r* and *s*, then it follows that it exactly occupies the fusion *t* of *r* and *s* without being exactly located at *t*. In the absence of Functionality, therefore, there are neither necessary nor sufficient conditions between exact location and exact occupation.

I will not analyze the equivalences of the other locative notions with the other occupation relations, since most of the discussion to follow will be about location.

3.1.4 *Whole and Entire Location*

Objects are completely located at some regions of space. An analysis of the persistence of objects brings with it its share of questions about the regions at which an object can be said to be completely located. I will postpone my discussion of persistence until later in the dissertation. In this section, I will analyze how the idea that an object is completely located at a region should be understood.

There are two routes we can follow to understand this idea. The first is to suppose that an object *o* is completely located at a region *r* just in case *o* is exactly located at one of *r*'s subregions. This way of cashing out the complete location is Parsons's (Parsons 2007) and Varzi's (Varzi 2007b) notion of entire location. More formally,

(L_e) $L_e or =_{df} (\exists y)(Pyr \wedge Loy)$. (Entire Location)

Obviously, whenever P in Entire Location is read as improper parthood, entire location (L_e) is exact location (L).[12] In contrast, the second route consists in cashing out an object's complete location in terms of the location of its parts. One could say that an object *o* is completely located at a region *r* just in case *o*'s parts are exactly located at subregions of *r*, or rather just in case *o*'s parts are generically located at *r*. More formally,

(L_w) $L_w or =_{df} (\forall y)(Pyo \rightarrow L_g yr)$ (Whole Location)

[12] Casati and Varzi (Casati and Varzi 1999, p. 120) define in exactly the same way what they call "whole location." As will become clear presently, I follow the notation of Parsons (Parsons 2007) and Varzi (Varzi 2007b).

(I follow Parsons in labelling this latter mode of location 'Whole Location.' Whole Location and Whole Presence – to be discussed in the third chapter – are not the same notion.) As Parsons (Parsons 2007, p. 212) points out, it seems intuitive to suppose that Whole Location and Entire Location are equivalent, or that

$L_w or \leftrightarrow L_e or$ (Equivalence (L_w—L_e))

For it seems to be the case that any region at which I am entirely located is a region at which all of my parts are generically located, and vice versa.

Parsons argues that Equivalence (L_{WL}—L_E) is not true. Take an extended simple. Because it is extended, we may suppose that the region *r* at which it is exactly located contains subregions. Given the way we defined Whole Location, it turns out that our extended simple is wholly located at each of *r*'s subregions. For the only (improper) part it has is generically located at each of these regions. Entire location is therefore not necessary for whole location. As will become clear in the second chapter, Parsons gladly accepts this consequence. However, any region at which an object (simple or otherwise) is entirely located is also a region at which it is wholly located. Thus,

$L_e or \rightarrow L_w or.$

The last implication holds simply because an object's parts are all generically located at the region it exactly occupies (and any superregion thereof). And if that is so, it follows that an object is wholly located wherever it is exactly located. In Chapter 3, I will argue that whole presence, a central notion of endurance theory, has to be understood in terms of entire location, and not in terms of whole location.

The relations of location and occupation introduced in this section allow us to formulate the multifarious ways an object is in space or spacetime. I now turn to theses often discussed in the literature in conjunction with both CEM and locative relations.

3.1.5 *CEM, Decomposition, and Arbitrary Parts*

Here are the *doctrines of arbitrary undetached part* (DAUP) and *arbitrary undetached temporal part* (DATP) that are often discussed in the literature.

(DAUP) for every material object *o*, if *r* is a region of space at which *o* is exactly located at time *t*, and if sub-*r* is any occupiable subregion of *r*, there exists a material object that is exactly located at sub-*r* at *t* and that is part of *o*. (see (van Inwagen 1981, p. 75))[13]

(DATP) For every persisting object *o*, if *I* is the interval of time through which *o* persists, if *r* is a region at which *o* is exactly located, and if sub-*I* is any interval included in *I*, there exists an object that is exactly located at the product of sub-*I* and *r*, which is a part of *o* and which, for every moment *t* that falls within sub-*I*, has at *t* exactly the same momentary properties that *o* has.

The formulation I give of DATP is significantly different from that of van Inwagen, for I am concerned with formulating the doctrine in terms of the locative relations introduced above.

[13] van Inwagen formulates it thus: "For every material object M, if R is the region of space occupied by M at time *t*, and if sub-R is any occupiable sub-region of R whatever, there exists a material object that occupies the region sub-R at *t*." I reformulated the principle in order to conform it to my terminology, and made it consistent with the view that objects can be exactly located at multiple regions of space. Moreover, if we read what van Inwagen says with my notion of occupation in mind, we miss the intended idea behind DAUP. Further, the doctrine as formulated by van Inwagen does not succeed in cashing out the intended idea behind DAUP, despite the fact that it may well be true. For DAUP claims that any object exactly located at a subregion of the region of space at which another object M is exactly located *is part of M*. As stated, however, the doctrine would be made true by any object that is located at sub-regions of the region at which M is exactly located, whether or not it is a part of M. van Inwagen (p.75) acknowledges that the statement he gives of DAUP, "though it is imperfect in some respects, at least captures the *generality* of the doctrine [he] mean[s] to denote by the name." (Italics in the original) So, what I have just said should not be understood as a criticism of his formulation.

Some take endurance theory to be a denial of DATP. In Chapter 3, I will evaluate whether this is a good characterization. For the moment, I turn to four-dimensionalism, i.e., the ontology of perdurance theory.

4. Four-Dimensionalism

I will end this expository chapter by introducing the ontology of perdurance theory in terms of the part-whole and locative relations introduced above.

4.1 Some Definitions and Three Assumptions

Perdurance theory says that objects are temporally extended and have temporal parts at each of the times at which they exist. A temporal part (TP) is defined in the following way (taken from Theodore Sider (Sider 2001, p. 60)):

> (TP) *x* is a temporal part of *y* at *t* $=_{df}$ (1) *x* exists at, but only at, *t*; (2) *x* is part of *y*; and (3) *x* overlaps every part of *y* that exists at *t*.

When "*t*" stands for an instant, (TP) yields the definition of the notion of an *instantaneous* temporal part. A temporal part at a time *t* of a temporally extended object is thus the largest part of the object that exists at *t*.

A similar notion could be defined in terms of regions. Instead of talking of an object's largest part that exists at a time *t*, one could talk about an object's part that is exactly located at a region *r*. Such a part would be the object's *regional part at region r*. A more formal definition of a regional part is as follows:

> (RP) *x* is a regional part of *y* at *r* $=_{df}$ (1) *x* is exactly located at *r*, (2) *x* is part of *y*; and (3) *x* overlaps everything that is part of *y* and that is generically located at *r*.

A definition of the notion of temporal parts can then be given in terms of the notion of a regional part.

(TP_R) *x* is a temporal part of *y* at *t* $=_{df}$ Either a) *x* is a regional part of *y* at the region *r* that is the product of the hyperplane of simultaneity *h* that corresponds to *t* and the region at which *y* is exactly located, or b) *x* is the fusion of any such regional parts.

A hyperplane *h* of simultaneity for a spacetime *s* with N dimensions is a (N-1) dimensional subregion such that all of the latter's parts are simultaneous, and such that there is no subregion of *s* that is not part of *h* and that is simultaneous with all of *h*'s parts. More precisely,

> *h* is a hyperplane of simultaneity of space *s* $=_{df}$ (1) *h* is part of *s*; (2) all of *h*'s parts are simultaneous; and (3) there is no region *r* such that: a) *h* is a proper part of *r*, b) there is a region *z* such that *z* is part of *r* but not of *h*, and *z* is simultaneous with all of *h*'s parts.

(A hyperplane of simultaneity is a slice of spacetime.) Each time corresponds to a specific hyperplane in a non-relativistic spacetime. And so I use the phrase "hyperplane of simultaneity that corresponds to a time *t*" in order to talk about the relevant region of space. (TP_R) defines the notion of a temporal part as an object's largest instantaneous regional part, or the fusion of any such regional parts. (TPR)'s clause b) is there since many four-dimensionalists are satisfied with the idea that objects may have temporal parts that last longer than an instant or temporal parts that are scattered.

A perdurance theorist accepts the following:

1- CEM
2- DATP
3- Persisting objects are exactly located at four-dimensional regions of spacetime.
4- Functionality

It indeed follows from (TP_R), (RP), and the four assumptions above, that a persisting object has temporal parts at each time at which it exists. Take a persisting object *o* exactly located at a four-dimensional region *r* of spacetime. This region *r* has many subregions. In particular, there is, for each hyperplanes h_i of simultaneity, a three-dimensional region r_i that is the product of h_i and r. It follows that each r_i is a subregion of r. From DATP, it follows that o has a part exactly located at each of the r_is. Because r is a four-dimensional region and the region at which o is exactly located, o also is a four-dimensional object. Thus, the parts of o exactly located at each of the r_is are distinct regional parts of o at the r_is. Each regional part of this type is exactly located at a single r_is. Hence, because the r_is are the cross-section of r and the hyperplanes h_i, it follows (from TP_R) that each of o's parts just discussed is a temporal part of o.

5. Conclusion

I have been concerned with introducing four-dimensionalism in this chapter. To that end, I introduced CEM and some locative principles. In this dissertation, I am interested in defending the view that objects can be exactly located at multiple disjoint regions of space, and – more to the point – that persisting objects are exactly located at multiple disjoint regions of spacetime. In other words, I defend the view that Functionality is false. In the next chapter, I analyze variants of CEM that can be used in accounts of the view that objects can be located at several disjoint regions of space.

Chapter 2

Parthood, Time, and Space

1. Introduction

In the last chapter, I introduced the ontology of perdurance theory. In so doing, I introduced the axioms of classical mereology, discussed some of its theorems, and introduced the relations that hold between material objects and regions of space. In this dissertation, I plan to show that endurance theory offers a coherent metaphysical picture of the persistence of material objects. According to endurance theory, persisting objects are not temporally extended, and do not have temporal parts. In this chapter, I motivate and adopt a ternary part-whole relation whose *relata* are objects and regions of spacetime. I first examine ternary part-whole relations that hold between objects and times, before turning to those that hold between objects and regions of space (or spacetime). I end the chapter by responding to an argument according to which endurance theory, as I construe it, is metaphysically inconsistent.

2. Regional Mereologies: Parthood and Regions

The problem of mereological change is the endurance theorist's main motivation for taking into account a ternary part-whole relation, whose *relata* are two objects and a time. I further contend that a similar problem motivates a ternary part-whole relation whose *relata* are two objects and a region of space. I introduce both problems, as well as both types of ternary relations, below. Finally, I claim that an account of the persistence of material objects requires that we adopt as primitive a part-whole relation that holds between objects and regions.

2.1 The Problem of Mereological Change

Objects change parts over time. Cars often get their bumpers replaced, bicycles get new wheels, and humans lose their teeth. An object *o* that changes its parts from time t_1 to t_2 is such that it is present at both t_1 and t_2, and has some part *p* at t_1 that it does not have at t_2. The problem of mereological change can be formulated in at least two ways.[14] First, one can claim that the contradiction

(c) *o* has proper part *p* and does not have proper part *p*

follows from the commonsensical claim

(a) *o* at t_1 is identical to *o* at t_2, which is identical to *o*

and the change report

(b) *o* at t_1 has proper part *p* and *o* at t_2 does not have proper part *p*.

Our commonsensical view of identity over time together with the fact that objects change their proper parts over time seem contradictory.

The second way one can formulate the problem of change also relies on our commonsensical view about identity over time. It consists in denying the common view about identity, i.e. to conclude to the claim that

(c') *o* at t_1 is *not* identical to *o* at t_2

[14] The problem of mereological change is closely related to, if not a special case of, the more general problem of change (also called the problem of temporary intrinsics). Suppose an object has a property P at t_1 but not at t_2. Leibniz Indiscernability of Identicals (introduced in Chapter 1, section 2.2) claims that

(LII) $(\forall P)\ [x = y \rightarrow (Px \leftrightarrow Py)]$.

It follows from (LII) that our object at t_1 is not identical to our object at t_2. See (Lewis 1986), (Haslanger 1989), (Johnston 1987), (Merrick 1994) for a discussion of the more general problem of change.

from the change report

(b’) o at t_1 has proper part p and o at t_2 does not have proper part p

together with (LM) introduced in the last chapter (section 2.2)

(a’) $x = y \rightarrow (\forall z)(\text{PP}zx \leftrightarrow \text{PP}zy)$.

(c’) follows from both (b’) and (LM)’s contrapositive.

In both of its formulations, the problem of change has a metaphysical as well as a semantic aspect. The semantic aspect involves the problem of finding an appropriate semantic for temporal predications, such as “o has p at t_1” or “o at t_1 has proper part p,” such that there is no conflict between our change reports and our commonsensical view about identity over time (see (Sattig 2006, p. 34)). I mention the semantic problem only to leave it aside. The metaphysical problem of change arises out of the adoption of the binary part-whole relation introduced in the last chapter, together with the view that objects change their parts over time. The problem arises because it is assumed that truth-conditions of change reports (i.e., (b) and (b’) above), in both formulations of the problem, must involve a binary part-whole relation. So, (b) and (b’) are true, it is thought, if and only if

> o present at t_1 has proper part p and o present at t_2 does not have proper part p.

An obvious solution to the metaphysical problem of mereological change, in both of its formulations, requires giving truth-conditions of change reports in terms of a ternary part-whole relation that holds between objects and times.

2.2 Parthood and Time

Change reports have truth-conditions that involve a ternary part-whole relation. That is to say, it is true to claim that *o* has *p* at t_1 and not at t_2 if and only if *o* stands in a ternary part-whole relation holding between it, *p*, and t_1, but not in a similar relation holding between it, *p*, and t_2. It is clear that once change reports are understood in such a way, then no contradiction follows from them and our commonsensical view of diachronic identity. For there is nothing contradictory in saying that *o* has a part at one time, but not at another. Similarly, the conclusion of the second formulation of the problem of change does not follow from the change reports and (LM). For (LM) itself needs to be reformulated in terms of a three-place parthood predicate that denotes the ternary part-whole relation objects instantiate with times (otherwise, (LM) is not about the same part-whole relation as our change reports). Thus,

(LM') $a = b \rightarrow (\forall t)(\forall z)(PP_t zx \leftrightarrow PP_t zy)$.

('$PP_t xy$' is short for '*x* is a proper part of *y* at *t*'. The variable *t* is thus bound by the appropriate universal quantifier.)[15] (LM') claims that identical objects share all of their proper parts at all times at which they exist. Our initial worry about the possibility of mereological change is thus deflated. (LM') is silent on objects at distinct times; it merely claims that distinct objects are such that there is at least one time at which they have different proper parts.

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[15] I follow Simons (Simons 1987, p. 179) in expressing the ternary part-whole relation with the variable "*t*" as a subscript. I prefer it to "PP(*x*,*y*,*t*)" simply because it makes clearer that we are only claiming that *x* is part of *y* (i.e., that we are not making any claim as to the parts of *t*).

In face of this revision of (LM), the axioms and theorems introduced in the previous chapter must similarly be relativized to times. Thus, the axioms (P0), (P1), and (P2), as well as the definitions (D2), (O), (O1) and (D) should be translated in the following way:

(P0') $(\forall t)\ P_t xx$
(P2') $(\forall t)\ [(P_t xy \wedge P_t yz) \rightarrow P_t xz]$
(D2') $(\forall t)\ [PP_t xy \leftrightarrow (P_t xy \wedge \sim P_t yx)]$
(O') $(\forall t)\ [O_t xy \leftrightarrow (\exists z)\ (P_t zx \wedge P_t zy)]$
(O1') $(\forall t)\ [P_t xy \leftrightarrow (\forall z)(O_t zx \rightarrow O_t zy)]$
(D') $(\forall t)\ (D_t xy \leftrightarrow \sim O_t xy)$

Note here that (D2') makes no use of the identity relation. For an object *a* could be part of *b* at one time *t*, and have no part disjoint from *b* at *t*. And yet *a* and *b* may fail to be identical (they could be disjoint at other times).

What about (P1)? (P1) is the axiom of anti-symmetry, and its counterpart in a temporal mereology would read in the following way:

(P1') $(\forall t)\ [(P_t xy \wedge P_t yx) \rightarrow y = x]$.

I assume, like most metaphysicians, that no sense is attached to a time-relative identity predicate or to questions asking when objects *a* and *b* are identical.[16] Thus, (P1') claims that it is sufficient for two objects to be part of each other at all times for them to be identical. A temporal version of classical mereology should include (P1').

Some philosophers might contend that two objects coincide at all times at which they exist without being identical. For instance, one could claim that a statue and the clay it is made of could coincide at all times, but still be distinct since there are worlds in

[16] André Gallois (Gallois 1998) defends the view that identity can be occasional, i.e., relative to times. Allan Gibbard (Gibbard 1975) defends the view that it is contingent.

which the clay exists, but not the statue. In their view, a temporal mereology suited for the persistence of material objects should have a principle of coincidence (CC).

(CC) $(\forall t)\ (a <>_t b \leftrightarrow (P_t ab \wedge P_t ba))$.[17]

In (CC), "$<>_t$" stands for "coincide at t". The coincidence of distinct objects is metaphysically controversial, and I will come back to it in Chapter 6.

The principles of supplementation, i.e., both (WS) and (S), are translated thus:

(WS') $(\forall t)\ [PP_t ab \rightarrow (\exists x)\ (PP_t xb \wedge \sim O_t xa)]$
(S') $(\forall t)\ [(\sim P_t ab \wedge E_t a \wedge E_t b) \rightarrow (\exists x)\ (PP_t xa \wedge \sim O_t xb)]$

The predicate "E_t" in (S') is short for "exists at t", which should not be conflated with the existential quantifier. I will call this predicate the predicate of existence. Many philosophers define it in mereological terms. For instance, Thomson (Thomson 1983, p. 152) defines "x exists at t" with the notion of discreteness, where "x exists at t" means that it is not the case that everything is discrete from x at t (i.e., there are things that overlap x at t). As will become clear below, I will use the predicate of generic location (L_g) introduced in Chapter 1 (section 3.1.3) instead of the predicate of existence in order to convey the commonsensical idea that objects exist at a time. I will show in the next chapter, Chapter 3, that locative relations cannot be defined in mereological terms.

Finally, the temporal analogue of fusion existence (FE') that I take to be an axiom of classical mereology (CEM) is the following:

(FE_t') $(\forall t)[(\exists x)(F_t x) \rightarrow (\exists z)((\forall y)(F_t y \rightarrow P_t yz) \wedge (\forall y)(P_t yz \leftrightarrow (\exists w)(F_t w \wedge O_t yw))))]$

Here is how I read (FE_t'):

[17] I have taken the principle from (Simons 1987, p. 180), but substituted his existential quantifier with a universal quantifier.

> At all times at which there are Fs, there is an object that has all those Fs as parts at each of these times and that is such that anything which overlaps an F at any time is part of it at that time, and anything that is part of it at any time overlaps at least one of the Fs at that time.

There are many questions that may be asked about cross-temporal fusion, but I will not analyze them here. See (Simons 1987, p. 183-187) for an interesting discussion.

The temporal analogue of CEM has the axioms (P0'—P2'), (WS'), and (FE_t'), given again below.

(P0') $(\forall t)\ \mathrm{P}_t xx$
(P1') $(\forall t)\ [(\mathrm{P}_t xy \wedge \mathrm{P}_t yx) \rightarrow y = x]$.
(P2') $(\forall t)\ [(\mathrm{P}_t xy \wedge \mathrm{P}_t yz) \rightarrow \mathrm{P}_t xz]$
(WS') $(\forall t)\ [\mathrm{PP}_t ab \rightarrow (\exists x)\ (\mathrm{PP}_t xb \wedge \sim \mathrm{O}_t xa)]$
(FE_t') $(\forall t)[(\exists x)(\mathrm{F}_t x) \rightarrow (\exists z)((\forall y)(\mathrm{F}_t y \rightarrow \mathrm{P}_t yz) \wedge (\forall y)(\mathrm{P}_t yz \leftrightarrow (\exists w)(\mathrm{F}_t w \wedge \mathrm{O}_t yw))))]$

2.2.1 *An Ambiguity in Temporal Talk about Parthood*

In adopting a temporal mereology, several questions about parthood and times come to mind. In particular, we may wonder *when* it is the case that "*p* is part of *o*" is true. By asking this question, we may be asking one of the following two questions. We may be wondering

> *of* what time it is true to say that *p* is a part of *o*.

or

> *at* what time(s) it is true to say that *p* is a part of *o*

The following case will make the above distinction clear. Suppose an instantaneous object *o* that exists at (and, of course, only at) *t*, and that counts the instantaneous *p* among its parts. For someone who adopts an atemporal part-whole relation, there is no question about the times *of* which the part-whole relation holds. For there is *no* time of which the relation holds. But suppose we take parthood to be a ternary relation holding between objects and time. Then, we can give either of the following truth-conditions to "*p* is a part of *o*." First, we may claim that

(1) $(\forall t)$ ("*p* is part of *o*" is true at *t* if and only if $P_t po$.)

"*p* is part of *o*" is true at the times figuring among the *relata* of the part-whole relation. (1) constitutes an answer to our question *of* what time it is true to say that *p* is part *o*. But we may as well claim that

(2) $(\forall t)$ ("*p* is part of *o*" is true at *t* if and only if $(\exists t')$ $P_{t'} po$.)

In that case, we are claiming that "*p* is part of *o*" is true at all times provided there be a time that figure in the relata of the part-whole relation. (2) constitutes an answer to our question *at* what time it is true to say that *p* is part of *o*.

Perhaps another example may help us clarify this. Suppose *o* persists from t_1 to t_2, and has *p* as a part at t_1 and not at t_2. When is "*p* is part of *o*" true? Typically, one would ask such a question in order to know which times instantiate the part-whole relation holding between it, *p*, and *o*. And if this is so, then one wonders *of* what time(s) it is true to say that *p* is part of *o*. In our case, t_1 figures in the *relata* of such a part-whole relation. But suppose we correctly answer the question by claiming that there is a part-whole relation instantiating t_1, *p*, and *o*. It would be trivial, yet meaningful, to claim that it is true at *all times* that a relation instantiates t_1, *p*, and *o*.

The distinction I just introduced will become relevant in the next chapter where I will analyze recent accounts of the notion of whole presence. It will become apparent that some accounts of whole presence fail because they do not take into account the foregoing distinction. I turn now to my preferred part-whole relation, namely, a ternary relation holding between objects and regions.

3. Parthood and Regions

Regional mereologies take as primitive a ternary part-whole relation holding between objects and regions of space. When the relevant space is spacetime, it can be seen that temporal mereologies are special cases of regional mereologies. For there is a hyperplane of simultaneity that corresponds to any of the times at which parthood can be relativized. Recall that a hyperplane of simultaneity is a "slice" of spacetime all parts of which are simultaneous. (I gave a formal definition of the notion of a hyperplane of simultaneity in Chapter 1 (section 4.1).) I motivate regional mereologies in the same way as I do temporal mereologies.

3.1 A Regional Analogue of the Problem of Mereological Change

An analogue of the problem of mereological change could be stated with reference to regions instead of times. An object can have a part at some regions of space that it does not have at other regions. More precisely, an object *o* exactly located at *r* could have part *p*, and not have part *p* when exactly located at *r'*. As it was the case above, two formulations of the regional analogue of the problem of mereological change can be given as follows. First, it appears that the contradiction

(c) *o* has proper part *p* and *o* does not have proper part *p*

follows from our "change" report

(a) *o* at *r* has proper part *p* and *o* at *r'* does not have proper part *p* and an identity statement

(b) *o* at *r* is strictly identical to *o* at *r'* (which is identical to *o*).

"Change" reports like (a) together with the identity statement (b) seems to lead to a contradiction.

Second, one can instead claim that the denial of an identity statement follows both from our "change" report and (LM). Indeed,

(a) *o* at *r* has proper part *p* and *o* at *r'* does not have proper part *p*.
(b') $x = y \rightarrow (\forall z)(\text{PP}zx \leftrightarrow \text{PP}zy)$.
(c') Therefore, *o* at *r* is not identical to *o* at *r'*.

Here, (c') contradicts the identity statement that *o* exactly located at *r* is identical to *o* exactly located at *r'*.

A few comments about these two arguments are in order and concern the identity statement (b). If (b) is true, it follows that what is exactly located at *r* is also exactly located at *r'*. Thus, it follows, from the assumption that *r* and *r'* are disjoint, that one object can be exactly located at two disjoint regions of space. This result contradicts the principle of Functionality introduced in the last chapter, and to which I will return in the next section. (The principle is fully discussed in the Appendix to the Dissertation, where I show that it is dispensable in formal theories of location.) More important is the fact that I took (b) to be our commonsensical conception of identity over regions. Is that right? In the original formulation of the problem of mereological change, the conclusion "*o* at t_1 is not identical to *o* at t_2" is the denial of our common conception of diachronic identity. In the case of the regional problem of "change," it seems that we lack a commonsensical conception of an object's identity across regions. In fact, we tend to think that an object *cannot* be exactly located at two disjoint regions at any one time. I will argue below that we should revise this intuition about exact location. Yet, we have enough to motivate the introduction of a regional part-whole relation. For the same commonsensical view about identity that allowed us to express the problem of mereological change can be of use now in expressing its regional analogue. For we can stipulate, for now, that the disjoint region *r* and *r'* at which *o* is exactly located belong to two different hyperplanes of simultaneity. Under this stipulation, both formulations of the analogue to the problem of change show that a problem arises out of our common conception of the identity of material objects together with regional change reports.

As it was the case above, the metaphysical regional analogue of the problem of mereological change is due to the adoption of a binary part-whole relation in conjunction with the claim that objects can be exactly located at more than one region of spacetime. An obvious solution available to the supporters of the view that objects can be exactly located at more than one disjoint region of space requires adopting a ternary part-whole relation that holds between objects and regions of space. If such a ternary part-whole relation is on board, then the contradictory conclusion (c) does not follow from the premises (a) and (b). For either (c) is not contradictory anymore when formulated with a three place parthood predicate, or else it has nothing to do with the part-whole relations involved in an analysis of premises (a) and (b). Similarly, the counterintuitive conclusion (c') does not follow from both (a) and (b') if a ternary part-whole relation is adopted. For (b') also needs to be formulated in such a way that objects with distinct proper parts at all of the regions at which they are exactly located are distinct. That is,

$(\mathrm{LM_r})\quad (x = y \rightarrow (\forall r)(\forall z)(\mathrm{PP}_r zx \leftrightarrow \mathrm{PP}_r zy))$

(where '$PP_r xy$' is short for 'x is a proper part at r of y'). From this, however, the conclusion (c') does not follow.

Confronted with such a ternary part-whole relation, CEM's definitions and principles need to be revised thus:

($P0_r$) $(\forall r)\ P_r xx$
($P1_r$) $(\forall r)\ [(P_r xy \wedge P_r yx) \rightarrow y = x]$.
($P2_r$) $(\forall r)\ [(P_r xy \wedge P_r yz) \rightarrow P_r xz]$
($D2_r$) $(\forall r)\ [PP_r xy \leftrightarrow (P_r xy \wedge \sim P_r yx)]$
(O_r) $(\forall r)[O_r xy \leftrightarrow (\exists z)(P_r zx \wedge P_r zy)]$
($O1_r$) $(\forall r)\ [P_r xy \leftrightarrow (\forall z)(O_r zx \rightarrow O_r zy)]$
(D_r) $(\forall r)\ (D_r xy \leftrightarrow \sim O_r xy)$
(WS_r) $(\forall r)\ [PP_r ab \rightarrow (\exists x)\ (PP_r xb \wedge \sim O_r xa)]$
(S_r) $(\forall r)\ [(\sim P_r ab \wedge L_g ar \wedge L_g br) \rightarrow (\exists x)\ (PP_r xa \wedge \sim O_r xb)]$

As it was the case for temporal mereology, ($P1_r$) is here also questionable. Some may want to claim that distinct objects are part of each other at the region(s) at which they are exactly located. The principle of coincidence, if any, would read as follows:

(CC_r) $(\forall r)\ (a <>_r b \leftrightarrow (P_r ab \wedge P_r ba))$.

As I said above, I discuss the coincidence of material objects in Chapter 6 and show that ($P1_r$) can be kept. From here on, I will sometimes talk about the regional analogue of CEM as if it was CEM itself.

3.2 Parts at a region

In the context of temporal mereological systems, it is necessary that both an object *o* and one of its parts *p* exist at a time *t* in order for *p* to be part of *o* at *t*. How should this necessary condition be expressed in the context of a regional mereological system? There are two issues to tackle here. First, times, as they are ordinarily conceived, are hyperplanes of simultaneity of spacetime, i.e., the largest three-dimensional regions of spacetime all points of which are simultaneous. In relativizing parthood to regions of spacetime, we are in no way restricted to hyperplanes. For we certainly want it to be meaningful to claim that an object has a part at a region that is not a hyperplane. So, which regions are the ones at which an object can be said to have a part?[18] Second, the very predicate of existing at a time needs to be replaced by a more precise locative notion. For it is quite imprecise to claim merely that an object is at a hyperplane of simultaneity. One would also want a specification of where, more precisely, the object is located.

Let us start with the first issue. To simplify the discussion, let us suppose that an object *o* is exactly located at a region *r* of space with proper parts r_1 and r_2. And let us also suppose that *o* has proper parts p_1 and p_2 exactly located at r_1 and r_2, respectively. We ask: at what region(s) is p_1 part of *o*? In tackling this issue, Hud Hudson (Hudson 2001, p. 67) concludes that the largest region at which an object is part of another is the region the latter exactly occupies, while the smallest region is the one the former exactly occupies. Hudson defines his exact occupation in much the same way I define my exact location. (See (Hudson 2001, p. 63).) So, in order to avoid any further confusion in what follows, I will substitute Hudson's exact occupation with my exact location. One good consequence of such a substitution is that many principles to be listed below will be true independent of whether or not Functionality holds.

[18] There are some philosophers (Balashov 2008; Gibson and Pooley 2006; Sattig 2006) who would be happy with relativizing parthood to hyperplanes of simultaneity (in absolute or relativistic spacetime). Maureen Donnelly (Donnelly 2010) discusses different regional parthood relations that are found in the literature. She discusses the formal properties of Hudson's regional parthood relation that I discuss below.

In our case, if Hudson is right about the regions at which an object can have a part, then no proper subregions of r_1 will do. For suppose r_1* a proper subregion of r_1. Hudson would claim that what is exactly located at r_1* is consistent with p_1 and *o* overlapping at r_1* without their being part of each other at any region whatsoever.[19] So, the ternary part-whole relation holds between objects and a region *r only if* what is exactly located at *r* is *not* consistent with there being a mere overlap at *r* of the objects of concern without their being part of each other at any region whatsoever. The smallest region that can figure in the necessary condition expressed in the last sentence is the region at which p_1 is exactly located. It can indeed be said of r_1 that p_1 is part of *o* since both p_1 and *o* are generically located at r_1, and what there is at r_1 is not consistent with p_1 *not* being a part of *o* at any region whatsoever.

Hudson claims that the largest region at which an object can be said to be a part of another one is the region at which the latter is exactly located. In my case, it would be *a* region at which the object is exactly located. Why make such a supposition? Hudson does it because of the way he defines his notions of exact occupation and whole presence (see (Hudson 2001, p. 63-67), on which more in Chapter 3, section 2.2.1). But should *I* make such a supposition? After all, if I am exactly located at a proper subregion of the region at which my apartment is exactly located, then it seems true to say that my arm is part of me at the region at which my apartment is exactly located. Despite the apparent correctness of such a claim, I have no reason to accept that an object can be part of a whole at a

[19] "…we cannot determine anything with respect to parthood regarding two objects, if all we know is that they are both present at [a region]; … that relation is consistent with a simple case of overlap at [that region] in which neither of the overlapping continuants is a part of the other at any region whatsoever." In the body of the text, I stripped Hudson's assertion of its epistemic flavor. Parthood is not a matter of our epistemic access to the world.

superregion "larger" than the whole's exact location (i.e., at a proper superregion of the whole's exact location). For I adopted a ternary parthood relation holding between objects and regions because I was confronted with an analogue to the problem of mereological change, namely, the fact that objects can have different parts at different regions. I succeeded in constructing the analogue to the problem of change by supposing that Functionality is false, and thus by allowing objects to be exactly located at more than one disjoint region. Now, suppose an object *o* is exactly located at several regions the sum of which is *r*. To claim that *o* has a part at *r* would not solve the analogue of the problem of change, and thus would undermine the very motivation I had for introducing the ternary part-whole relation.

However, the largest region at which an object can be said to be a part of another is, in fact, the part's exact location. Or so I will stipulate. I have two main motivations for such a stipulation. The first is that if we accept the fact that *p* is part of *o* at any superregion of *p*'s exact location, it is so because it makes sense to say that *p* is part of *o* at *p*'s exact location. So adding other regions would unnecessarily complicate our regional mereology. For we would always have to keep in mind that the variable *r* in "*p* is part of *o* at *r*" admits of many values, if *p* is not identical to *o*. Secondly, I will defend a certain view of material objects in Chapters 4, 5, and 6 according to which *p* cannot be part of *o* at any other (super)regions of *p*'s exact location. So, just like Hudson, I constrain the regions at which objects can have parts in order to make my theory consistent. It should be noted that there is nothing wrong with doing such a thing. For once we have determined that *p* is part of *o* at (at least) *p*'s exact location, it is a matter of stipulation if we want to claim that *p* is also part of *o* at other regions. Beyond the fact that *p* is part of *o* at *p*'s exact location, the world does not inform us on the region of which it can be said that *p* is part of *o*.[20]

Let us turn to the second issue mentioned above. In temporal mereological systems, an object exists at a time *t* only if *t* is not completely free of the object. The times of which the predicate of existence is true are not restricted to the interval of times that comprise all of the objects, nor to the times that comprise at least one of the object's parts. In particular, a temporally extended simple can exist at a time *t* included in the interval of time through which it is extended. My goal is to keep this characteristic of the predicate of existence in regional mereology, and I choose to substitute the predicate for one denoting the relation of generic location. I find the choice appealing since there is no further requirement imposed on the object for it to exist or to be present at a region. Hudson (Hudson 2001, p. 63), however, offers another definition of being present at a region. He defines "*x* is present at *s*" as "*x* has a part at *s*." (Judith Jarvis Thomson (Thomson 1983) does the same, although she uses disjointness instead of parthood.) His definition (in terms of parthood) and mine (in locative terms) are not equivalent. According to Hudson's definition, a world whose mereological atoms are extended would be a world in which the simples are not present or do not exist at proper subregions of their exact locations. This is counterintuitive, and I prefer to avoid this consequence by defining the presence or existence predicate in terms of generic location.

3.2.1 *An ambiguity in Regional Talk about Parthood*

[20] Incidentally, some would not even take p's exact location as a region that figure in the relata of the part-whole relation. Sattig (Sattig 2006) takes hyperplanes of simultaneity as the appropriate regions, and others would take the whole of space (e.g., (Crisp and Smith 2005, p. 333)).

We saw above that there are two ways to cash out the claim that "*p* is a part of *o*" is true at a time *t*. The claim that "*p* is a part of *o*" is true at a region *r* is similarly ambiguous. As was the case above, we could cash it out in either one of the following ways:

(3) $(\forall r)$ ("*p* is a part of *o*" is true at *r* if and only if $P_r po$)
(4) $(\forall r)$ ("*p* is a part of *o*" is true at *r*) if and only if $(\exists r')\ P_{r'} po$

(3) claims that "*p* is part of *o*" is true at the region that figures in the *relata* of the ternary regional part-whole relation, while (4) claims that "*p* is part of *o*" is true at all regions just in case there is a region that figures in the *relata* of the regional part-whole relation.

3.3 Further Principles

A principle, one mereological operator, and a doctrine deserve some discussion. These are: the operator for unrestricted fusion, a variant on the principle of transitivity, and a doctrine of undetached temporal parts more general than the two doctrines introduced in Chapter 1 (section 3.1.5).

The operator for unrestricted fusion in regional mereology is similar to the one in temporal mereology. More precisely, the fusion *x* of everything that is an F can be defined as follows:

$$(F_r') \quad \sum(F_r x) = (iz)(\forall r)((\forall y)(F_r y \rightarrow P_r yz) \wedge (\forall y)(P_r yz \leftrightarrow (\exists w)(F_r w \wedge O_r yw))))].$$

As it was the case for (F_r'), "$F_r x$" is short for "*x* is F at *r*." Here is how I read (Fr'):

> The fusion of the things that are F at any region *r* is the object such that anything that is an F at any region is part of it there, and such that every part of it at any region overlaps there at least one of the things that is an F there, and anything that is an F at any region overlaps there at least one of its parts there.

I assume that objects at different regions can make different predicates true.

The regional principle of fusion existence can be formulated with the help of (F_r'). Here it is:

(FE_r') $(\forall r)[(\exists x)(F_r x) \rightarrow (\exists z)((\forall y)(F_r y \rightarrow P_r yz) \wedge (\forall y)(P_r yz \leftrightarrow (\exists w)(F_r w \wedge O_r yw))))]$

(FE_r') reads in the following way.

> For all regions, if there are things that are F at any of these regions, then there is an object that has them as parts at any of these regions.

The variant on the principle of transitivity that deserves attention is given by Hudson (Hudson 2001, p. 68). Take the principle of transitivity ($P2_r$). It says that for a given region *r*, if an object *x* is part at *r* of *y*, while *y* is part at *r* of *z*, then *x* is part at *r* of *z*. But it seems true that for any three regions *t, s,* and *r*, where *s* is a proper subregion of *r*, and *t* a proper subregion of *s*, if *x* is part at *t* of *y*, and *y* is a part at *s* of *z*, then *x* is part of *z* at *t*. Refer to Figure 1 for clarity.

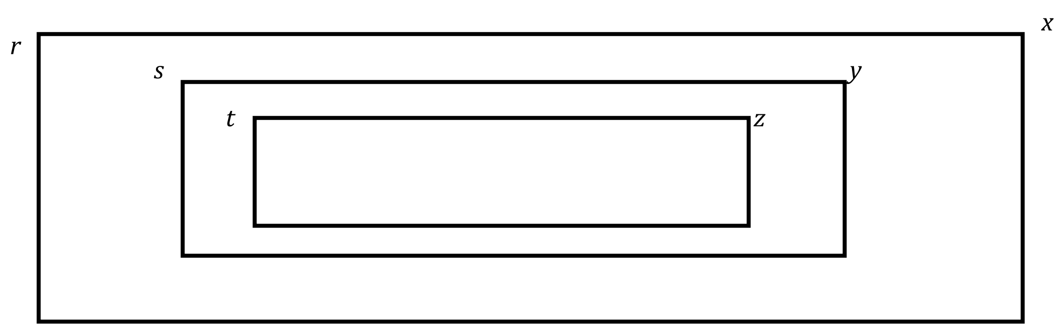

Figure 1: Extended Transitivity

Following Hudson, we can formulate the more general principle of extended transitivity (ET) in the following way:

$$\text{(ET)} \quad (\forall r)(\forall s)(\forall t)((P_t ts \wedge P_s sr \wedge P_t zy \wedge P_s yx) \rightarrow (P_t zx))$$[21]

I also think (ET) should be adopted in an account of the persistence of material objects. I will have more to say about that in Chapter 6.

DAUP and DATP of Chapter 1 can receive a simpler, more general formulation (given by Parsons (Parsons 2007, p. 10)) in terms of the locative terminology I introduced in the last chapter and the regional part-whole relation (Parsons uses a binary part-whole relation). One could cash out the intended idea behind both DAUP and DATP in the following way:

$$\text{(DAP)} \; L_p xr \rightarrow (\exists y)(P_r yx \wedge Lyr)$$[22]

Here is how (DAP) reads.

> If an object is pervasively located at a region *r*, then there is a part of that object at that region that is exactly located at that region.

4. "Change" and Ternary Part-Whole Relations

Does the introduction of the regional part-whole relation represent a substantial addition to an account of the persistence of material objects? The pressure to adopt such a ternary relation comes from the problem of mereological change. Change reports, such as

(5) *o* at t_1 has proper part *p* and *o* at t_2 does not have proper part *p*; and
(6) *o* exactly located at *r* has *p* as a proper part, and *o* exactly located at *r'* does not have proper part *p*

do not square well with the view that it is the very same object that can exist at two different times, or is exactly located at two different regions. I have shown above that

[21] (ET) is a formal rendering of Hudson's informal definition, with the appropriate modifications that reflect the fact that *p* is part of an object at *p*'s exact location only. For Hudson's definition, see (Hudson 2001, p. 68).

[22] This is Parsons's principle of arbitrary partition, though, unlike him, I employ a ternary part-whole relation.

conceptual problems having to do with these change reports together with our commonsensical conception of identity are eschewed if one adopts a ternary part-whole relation. Four-dimensionalists, however, adopt another strategy.

4.1 Four-Dimensionalism and Change

Four-dimensionalists refuse to see the ternary part-whole relation as primitive. Instead, they understand the phrase "*o* has *p* as a part at t_1" to be given truth conditions in terms of the notion of a temporal part and of the *atemporal* part-whole relation. They will offer the following truth conditions for (5):

(5') an object *o*'s temporal part at t_1 has the temporal part at t_1 of *p* as *part* and *o*'s temporal part at t_2 does not have any temporal part at t_2 of *p* as *part*.

The same applies to regional change reports, which are given the following truth-conditions:

(6') *o*'s regional part at *r* has a regional part of *p* at *r* as a part and *o*'s regional part at *r'* does not have a regional part of *p* at *r'* as a part.

Recall that the notions of a temporal part at a time *t* and of a regional part at a region *r* are defined in terms of a two-place part-whole relation (see Chapter 1, section 4.1).

(TP) *x* is a temporal part of *y* at $t =_{df}$ (1) *x* exists at, but only at, *t*; (2) *x* is part of *y*; and (3) *x* overlaps every part of *y* that exists at *t*.

(RP) *x* is a regional part of *y* at $r =_{df}$ (1) *x* is exactly located at *r*, (2) *x* is part of *y*; and (3) *x* overlaps everything that is part of *y* and that is generically located at *r*.

Perdurance theorists are therefore giving up what I called the commonsensical view of identity, and claim that, strictly speaking, what is present at times *t* and *t'*, or what is exactly located at regions *r* and *r'*, are distinct parts of the same extended objects, i.e., temporal parts or regional parts of the same object, the exact location of which is the fusion of the exact location of all of its parts.

4.2 “Change” and Part at a Region

The perdurance theorist’s solution to the problem of change consists in taking a binary part-whole relation as primitive, and understanding change reports in terms of this relation and of the notion of a temporal part. For the solution to succeed in making true the change reports we ordinarily deem to be true, it must be wedded to a certain ontological view of persisting objects wherein the latter are extended in spacetime. Of course, I defend a view of persistence according to which persistence is not spatiotemporal extension, and the perdurance theorist’s solution is unavailable to me. Indeed, we shall see in the next chapter that objects wholly present at different times are exactly located at multiple regions of spacetime.

Regardless of what is available to me, adopting a regional part-whole relation allows us to express many more possibilities about the location of objects than does adopting a binary part-whole relation or a temporal one. The problem with adopting a binary part-whole relation is that it is biased towards a specific view of material objects. For it precludes the expression of some possibilities that are *prima facie* coherent, such as that in which an object is exactly located at two disjoint regions. The ternary regional part-whole relation, however, is not wedded to any picture of material objects. Or, at least, it does not prevent us from formulating four-dimensionalism. It is perfectly possible to express the notion of a temporal part in terms of the ternary part-whole relations.

(TP_T) *x* is a temporal part of *y* at *t* =df (1) *x* is part of *y* at *t*; (2) *x* exists at, but only at *t*; and (3) *x* overlaps at *t* every part of *y* at *t*. (see (Sider 2001, p. 59))

We arrive at the definition of the notion of a temporal part in terms of the regional part-whole relation by offering a definition of a regional part.

(RP_R) *x* is a regional part of *y* at *r* $=_{df}$ (1) *x* is exactly located at *r*, (2) *x* is part of *y* at *r*; and (3) *x* overlaps at some subregions of *r* everything that is part of *y* at *r*.[23]

The resulting definition of the notion of a temporal part in terms of a regional part can then stay the same as before, i.e.,

(TP_R) *x* is a temporal part of *y* at *t* $=_{df}$ (1) *x* is a regional part of *y* at the region *r* that is the product of the exact location of *y* and of the hyperplane *h* of simultaneity (corresponding to *t*).

So even if a ternary part-whole relation is taken as primitive, four-dimensionalism can still be expressed.

I have introduced the regional analogue to the problem of mereological change by stipulating that the regions at which an object *o* is exactly located belong to different hyperplanes of simultaneity. But there is nothing beyond common sense that can force us to accept such a stipulation. In fact, it is *prima facie* possible that an object is exactly located at two different regions of space (at a given time), and that it has different proper parts at each of these regions. Take again the regional "change" report

(6) *o* exactly located at *r* has *p* as a proper part and *o* exactly located at *r'* does not have proper part *p*

in which it is assumed that *r* and *r'* belong to the same hyperplane of simultaneity. Someone who only has a temporal part-whole relation available cannot express (5) without running into the contradiction

(7) *o* has proper part *p* at t_1 and *o* does not have proper part *p* at t_1

[23] The definition of a regional part I just gave is the natural alternative to the one couched in terms of the binary part-whole relation (that I gave in Chapter 1, section 4.1). Hudson (Hudson 2001, p. 65) gives the same definition to what he calls "spacetime part." He wants his spacetime parts to be the analogue to the four-dimensionalist's instantaneous temporal parts. I think Hudson loses an important aspect of the definition of a temporal part, namely, the idea that such a part has to exist at, and only at, a *time*. There are cross-sections of the exact location of a persisting object such that the object has what Hudson calls a spacetime part at that region without its being a temporal part of the object, since the part runs along more than one hyperplane of simultaneity.

(t_1 corresponds to the hyperplane of simultaneity of which *r* and *r'* are disjoint subregions). The problem is that the times that figure in the *relata* of the temporal part-whole relation correspond to regions much larger than the ones at which *o* is exactly located. Because a temporal part-whole relation makes it difficult for us to express these cases, the regional part-whole relation is preferable.

5. Regional Mereologies and Functionality: A Coherent Metaphysical Picture?

I have been arguing that adopting a regional part-whole relation allows us to express cases we could not have expressed otherwise. The cases in question are all cases in which the principle of Functionality fails. Functionality is introduced in the last chapter, and claims that an object can be exactly located only at one region, or

$$(\mathrm{L}ox \wedge \mathrm{L}oy) \rightarrow x = y.$$

Perhaps, cases of exact location at multiple regions ("cases of multilocation," from now on) are simply impossible. If this were the case, then our motivation for adopting the regional part-whole relation would be ill-guided. The latter would indeed appeal to cases that are not possible anyway. It is thus appropriate to dispel such a worry before moving on to offer an account of persistence in terms of exact location at multiple regions.

Whether or not Functionality holds is a matter of dispute. The battle may be seen as fought on different grounds. First, one may claim that a denial of Functionality is incoherent or undesirable in a formal theory of location. Second, one may claim that the debate over persistence has nothing to do with Functionality. Third, one may claim that, although there are formal theories of location in which Functionality does not hold, it leaves us with an incoherent metaphysical picture of material objects. In this section, I will address only the third and final ground. Of course, the whole dissertation is devoted to cases of multilocation. What I want to address here is an argument to the effect that cases of multilocation involving material objects are simply incoherent. In Chapter 3, I will show that endurance theory involves a denial of Functionality, thus addressing the second worry one may have about denying Functionality. As for the first worry, I discuss it in the appendix to the dissertation. There, I show what the axioms and theorems of theories of location in which Functionality does not figure are. I also show that theories with distinct primitives and that do not include Functionality are nevertheless equivalent.

5.1 Barker and Dowe's Arguments

Stephen Barker and Phil Dowe (Barker and Dowe 2003) offer an argument to the effect that the notion of multilocation and, more specifically, accounts of endurance in terms of multilocation are incoherent. Their understanding of the notion of multilocation is not completely clear. They talk as though an enduring object is wholly located at several three-dimensional regions of spacetime, but fail to define their notion of whole location. Their arguments suggest, however, that they have in mind what I would call exact location. And, in any case, since I am mainly interested in the coherence of the claim that enduring objects are exactly located at multiple regions of spacetime, I will interpret their argument as if they had in mind the notion of exact location. In what follows, I analyze two of Barker and Dowe's arguments. They offer a third one meant to analyze endurance theory against the backdrop of presentism, i.e., the view that only what is present exists. Since I am not concerned with the latter, I shall leave this third argument aside.

5.1.1 *First Argument*

The first argument runs as follows. Suppose an object *o* is multilocated throughout a four-dimensional spatiotemporal region *R*. We are supposing that *o* is exactly located at multiple three-dimensional subregions *r* the sum of which is *R*. Let us call '*or*' what is exactly located at each of the *rs*, and 'F(*or*)' the fusion of what is exactly located at each of the *rs*. Barker and Dowe (Barker and Dowe 2003, p. 109) suppose the following part/whole location principle (WLP):

> (WLP) If an entity *W* and a space-time region *R* are such that for some division of *R* into sub-regions *r*, *W* has a part *p* located[24] at each *r*, then *W* is located at *R* and has zero- or non-zero temporal extent according to the dimension of *R* itself. (I modified their formulation slightly so that it may conform to my terminology.)[25]

Barker and Dowe argue that a contradiction follows from what has just been said. Here is their argument.

1- All of the *ors* are identical to each other. (Multilocation)
2- Each *or* is an entity of zero temporal extent.
3- Therefore, F(*or*) is also of zero temporal extent. (The fusion of an object with itself indeed is nothing else than the object in question.) (from 1 and 2)
4- But F(*or*) has parts at every subregion of R that have no temporal extent.
5- Therefore, F(*or*) has non-zero temporal extent. (from 4, and WLP)
6- Therefore, F(*or*) both has zero and non-zero temporal extent. A contradiction. (from 3 and 6)

As Barker and Dowe (p. 109) note, an obvious reply available to the proponent of multilocation would be to claim that WLP is right only if the parts located at each subregion *r* of *R* are distinct from each other. Multilocation entails that *o* has an improper

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[24] I intentionally leave "located" ambiguous. And I do so in order to be as charitable as possible to Barker and Dowe.

[25] Here is their formulation. "If an entity *W* and a space-time region *R* are such that for some division of *R* into sub-regions *r*, *W* has a part *p* located at each subregion *r*, then *W* is located at *R* and a 3 or 4D entity according to the dimension of *R* itself." In the body of the text, I make no mention of the dimension of the entities concerned.

part at each *rs*, but this hardly makes WLP true. Since *o* is exactly located at each of these *rs*, it follows that the *ors* are identical to one another, and thus that F(*or*) is only an entity that has zero temporal extent. Contrary to Barker and Dowe, I think that this response is right. Before analyzing what they have to say about it, let us note that appealing to WLP is a methodological mistake. WLP is true only if the notion of exact location is a function (i.e., it satisfies Functionality). Only in that case can we claim that what is exactly located at each of these *rs* are objects that are distinct from one another. But the appropriate notion of exact location under scrutiny, the one that would allow us to claim that objects are exactly located at multiple regions of spacetime, is precisely *not* a function. There is thus no reason why an advocate of multilocation should accept WLP. Furthermore, it comes as no surprise that they are able to derive a contradiction if WLP entails that exact location is a function, while the notion of multilocation entails that it is not.

How do Barker and Dowe support WLP? They argue that F(*or*) has non-zero temporal extent. To that end, they suppose another entity exactly located at a region *R** adjacent to *R*. For simplicity's sake, suppose that *R** share *R*'s time, but not its space. Barker and Dowe suppose, in addition, that there is a perduring object F(*e*) that has proper temporal parts at each of the three-dimensional slices of *R**. By all standards, F(*e*) is an entity that has non-zero temporal extent. Because, they claim, there is a one-to-one correspondence between parts of F(*e*) and parts of F(*or*), it follows that F(*e*) and F(*or*) have the same temporal extent. Since the former has non-zero temporal extent, so does the latter.[26]

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[26] McDaniel (McDaniel 2003) responds to Barker and Dowe's argument. Interestingly, he sees their argument as an instance of the problem of specifying the shape of an object exactly located (my

What seems to be doing all the work in Barker and Dowe's argument is this alleged one-to-one correspondence between F(*e*)'s and F(*or*)'s parts. To a certain extent, there is something right in their intuitions about this one-to-one correspondence. For it serves to count the objects exactly located at several disjoint regions. In cases of multilocation, it seems that several intuitions we have about multilocated objects are correctly accounted for by assuming that an object exactly located at two disjoint regions should be seen as two objects. In any case, it is clear that Barker and Dowe's one-to-one correspondence between F(*e*)'s and F(*or*)'s parts cannot do the work they intend it to do. For if the one-to-one correspondence is meant to lead us to the conclusion that F(*or*)'s parts are, like F(*e*)'s parts, *distinct from each other*, it follows that the correspondence relation undermines the premise according to which *o* is multilocated, in the relevant sense, in spacetime. In other words, an advocate of multilocation would simply deny that there is a *one-to-one* correspondence between F(*e*)'s and F(*or*)'s parts. For the latter "parts" are in fact one and the same object. If there is any correspondence between F(*e*)'s and F(*or*)'s parts, it is a *many-one* correspondence. In Barker and Dowe's argument, the contradiction is derived not because something is incoherent in the notion of

terminology) at several disjoint regions of spacetime. For he claims that an object's shape is analyzed solely in terms of its geometrical, metrical, and topological properties. He distinguishes between an object's intrinsic and extrinsic shape. Arguably, the shape of *o* is intrinsic just in case an analysis of *o*'s exemplification of its geometrical, metrical, and topological properties needs to involve nothing more than properties *o* has in virtue of being composed of its parts. *o*'s shape would then be extrinsic just in case such an analysis needs to involve properties of other objects, notably, those of the region(s) of spacetime to which *o* is related. McDaniel then claims that *o*'s *extrinsic* shape is that of *R*, while it has the same *intrinsic* shape as each of the *rs* at which it is exactly located. Presumably, the extrinsic shape of an object is the shape of the region it exactly occupies, i.e., occupies throughout its lifetime, and of no smaller regions. Otherwise, *o*'s extrinsic shape would be that of any subregions of *R*, and McDaniel's solution – as it stands – would not be particularly illuminating. Barker and Dowe (Barker and Dowe 2005) offer the (rather uncharitable) and contradictory interpretation of McDaniel's argument according to which *o* has the extrinsic shape of *R* and of any of the *rs*. The notion of intrinsic shape is fairly controversial, and a solution tying endurance theory to the latter should be adopted only as a last resort. Despite McDaniel's attempt at defusing Barker and Dowe's charge against the notion of multilocation, I think an adequate answer to their argument shows it is invalid.

multilocation, but because the alleged one-to-one correspondence implies that multilocation does not occur. An advocate of multilocation will simply deny that there is such a one-to-one correspondence, and no contradiction will follow from multilocation.

At some point in their article, Barker and Dowe (Barker and Dowe 2003, p. 109) claim that the objection to WLP "is untenable. By [their] lights, there is an entity *or* located at each *r* in *R*, and so consequently, *R* is, as a region, *filled up*" (their italics), They reiterate this point in (Barker and Dowe 2005, p. 70), claiming that "[i]n terms of occupation there is no difference between this situation [i.e., one in which something exactly occupies a four-dimensional region by being exactly located at disjoint non-extended 'slices' of that region] and that where each occupier of a 4-D space is non-identical to every other occupier. The fact of identity does not affect the matter of occupation and the whole space being occupied." Because an object has the same shape as the regions at which it is exactly located, their claim suggests that they think "*x* occupies (fills up) region *r*" entails "*x* is exactly located at *r*" (in my sense of "exact location"). I have shown, in Chapter 1, that no such entailment needs to hold in the absence of Functionality. I turn now to the second argument.

5.1.2 *Second Argument*

Another version of the paradox relies on weaker mereological and locative assumptions, and runs as follows. Suppose our object *o* is exactly located at each of the three-dimensional slices *r* of a four-dimensional region *R*. There are events whose occurrence is necessarily connected to *o*'s persistence. One such event could simply be *o*'s life, which would in that case be exactly located at *R*. Another such event could be *o*'s motion from one place to another. For simplicity's sake, take *o*'s life. In a Humean spirit, Barker and Dowe endorse the *principle of independence* according to which there cannot be a necessary connection (metaphysical or logical) between two disjoint *entities*.[27] They thus conclude that *o* and its life are not disjoint entities. Because *o* and its life are not identical, and since it is counterintuitive to contend that something's life is part of that thing, they interpret this conclusion as entailing that *o* is a part of its life. Finally, they (Barker and Dowe 2003, p. 111) assume the following location principle:

Location a whole is located where its parts are.

It follows from Location, they think, that a whole with a multilocated part is itself multi-located.[28]

The paradox can thus be stated as follows. Let L(*o*) be *o*'s life, and suppose we divide L(*o*) into two halves $L(o)_1$ and $L(o)_2$. Both halves are exactly located at regions R_1 and R_2 (whose sum is *R*) , respectively.

1- $L(o)_1$ and $L(o)_2$ are exactly located at regions R_1 and R_2, respectively. (Intuitive conception of events)
2- *o* is part of $L(o)_1$ and $L(o)_2$ (from independence).
3- *o* is exactly located at multiple distinct regions *rs* (*o* is multilocated).
4- Some of the *rs* are not subregions of R_1, and some of the *rs* are not subregions of R_2.

[27] They say (Barker and Dowe 2003, p. 110) "distinct entities." But they go on to conclude that "if there is a necessary connection between *o*'s persisting and L(*o*)'s existing [where L(*o*) is *o*'s life], *o* and L(*o*) can't be distinct things. That means … that *o* is part of L(*o*); L(*o*) is somehow constituted out of *o* and possibly other entities." I take it that by "distinct," they have "disjoint" in mind, for a proper part is distinct from (i.e., not identical to) the whole. Their use of the rather cumbersome "non-identical" further substantiates my interpretation of their "distinct."

[28] See (Barker and Dowe 2003, p. 111): "… this reasoning invokes the principle that a thing is located where its parts are and in particular, a thing with a multi-located part is itself multi-located." What follows the "in particular" is actually not an instance of the mentioned principle, as I will illustrate shortly. Perhaps the fact that they take what follows "in particular" to be an instance of the principle suggests that Location in the text is not the principle they have in mind. I doubt this, however, since they obviously mean to appeal to an uncontroversial principle about the location of the whole and its parts. And Location is precisely this principle.

5- Therefore, $L(o)_1$ and $L(o)_2$ have a part (o) located at regions that are disjoint from either *R1* or *R2*. (from 2 and 3)
6- Therefore, $L(o)_1$ and $L(o)_2$ are located at regions that are disjoint from R_1 or from R_2 (from Location and 5).
7- Therefore, $L(o)_1$ and $L(o)_2$ are not exactly located at regions R_1 and R_2, respectively (from Exact Location, chapter 1).[29]
8- Therefore, $L(o)_1$ and $L(o)_2$ are both exactly located and not exactly located at R_1 and at R_2, respectively (from 1 and 7). A contradiction.

In Barker and Dowe's view, the contradiction shows that the notion of multilocation is incoherent.

Many aspects of the argument can be put into question. One could indeed question whether one's endorsing the principle of independence really commits one to claim that *o* is part of L(*o*). Or one could question the very existence of L(*o*). But I will focus on Location, and see what kind of principle Location is if it is meant to do the work Barker and Dowe want it to do. Principles like Location abound in the literature. For example, a similar principle is judged by Ted Sider (Sider 2007, p. 52) to be uncontroversial, and, in his view, it merely claims that "[t]he locations of a thing's parts are automatically reflected in the thing's location." (p. 75) So it remains to see whether Barker and Dowe's use of Location is innocent.

Location itself seems to be a rendering of two distinct uncontroversial ideas. The first is that

(Idea 1) the region at which the parts are exactly located are subregions of the region at which the whole is exactly located.

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[29] Barker and Dowe claim that events $L(o)_1$ and $L(o)_2$ are bounded by regions R_1 and R_2, respectively. But they don't seem to have in mind anything else than the fact that $L(o)_1$ and $L(o)_2$ are exactly located at, respectively, R_1 and R_2. See (Barker and Dowe 2003, p. 111): "$L(o)_1$ and $L(o)_2$ are entities bounded by distinct regions R_1 and R_2. Qua event-like entities they occur or are located at R_1 and R_2 respectively, and nowhere else." Here, I interpret their talk of location in terms of my talk of exact location. I thus omit any mention of the notion of boundary from my rendering of their paradox.

The regions of space at which my arms are, respectively, exactly located are subregions of the region of space at which I am exactly located. Another intuitive idea of which Location could be a rendering states that

> (Idea 2) a whole is not generically located wherever its parts are not generically located.

I cannot be generically located at the region where Paris is exactly located since no parts of me are in Paris. Neither of these two uncontroversial ideas entails that all of the locative properties of a whole are locative properties of its parts. In particular, a whole with some multilocated parts is not also obviously multilocated, contrary to what Barker and Dowe claim.[30] For there may be other parts, ones that do not enjoy multilocation, that enter into the mereological composition of the whole. And even a whole all parts of which are multilocated is not necessarily also multilocated. Suppose, indeed, that distinct *a*, *b*, and *c* together compose a whole *o*. Suppose *a* is exactly located at r_1 and r_2, that *b* is exactly located at r_2 and r_3, while *c* is exactly located at r_3 and r_4. Then, *o* is exactly located at the region *r* that is the fusion of r_1, r_2, r_3, and r_4, and at no regions other than *r*. If Barker and Dowe need to claim that a whole inherits all of its locative properties from its parts, then Location is not up to the job.

I have been careful in giving a rendering of their argument that does not revolve around $L(o)_1$ and $L(o)_2$ being multilocated. In the rendering above, we need only draw from Location the sole consequence that $L(o)_1$ and $L(o)_2$ are partly located at regions disjoint from the regions at which we initially took them to be exactly located. Here, it seems that this consequence is vindicated by the first of the intuitive ideas of which

[30] See (Barker and Dowe 2003, p. 111) where they claim that it is a particular case of Location that "… a thing with a multilocated part is itself multilocated." And below: "So L(O)1 and L(O)2 are … singly located, but [also] multi-located."

Location could be a rendering, but not the second. For it comes as no surprise that $L(o)_1$ (say) is partly located at a region disjoint from the one we take it to be located at (i.e., at regions disjoint from R_1) if it is exactly located wherever its parts are exactly located. But, then, Barker and Dowe are guilty of neglecting Location early on in their argument, while taking it into consideration at later stages. It indeed seems that it should follow from Location and the fact that the multilocated *o* is part of $L(o)_1$, that the latter is exactly located at a region of which the regions at which *o* is exactly located are subregions, contrary to what the first premise says.[31] So do they have independent support for the first premise?

Some time after presenting their argument, they claim (Barker and Dowe 2003, p. 111) that denying the first premise would amount to supposing it possible that an event could be exactly located at multiple disjoint regions. They go on to claim that "this looks highly implausible since the whole point of event talk depends upon events being singly located." Assuming this to be true, a necessary condition on eventhood is that

Event something is an event only if it is exactly located at no more than one region.

To suppose, as Barker and Dowe do, that a denial of the first premise amounts to denying Event is a mistake. I take it that they are led to this conclusion because they think Location entails that an event with a multilocated part is itself multilocated. But, as we saw, no such conclusion follows.

Someone who considers the notion of multilocation or the metaphysical picture with which a denial of Functionality leaves us coherent can, on the other hand, reconcile

[31] And this is contrary to what they (Barker and Dowe 2003, p. 111) later claim: that an answer to their argument that denies the first premise is misguided.

Location and Event. I take it that there is no problem, real or apparent, with this position so long as we understand Location according to the second of the uncontroversial ideas. As for the first idea of which Location is a rendering, it only requires that the region at which an object is exactly located be the fusion of the region at which its parts are exactly located. So $L(o)_1$ is exactly located not at R_1, but at the fusion of the regions at which its parts are exactly located, which include as subregions all the regions *rs* at which *o* is exactly located. Here, it does not follow that $L(o)_1$ is exactly located at R_1, and both the uncontroversial ideas behind Location and Event are satisfied. There is no contradiction, and nothing is incoherent with the notion of multilocation.

The above objection shows Barker and Dowe's argument defective. Yet, one may insist that although coherent, the notion of multilocation is counterintuitive, and an account of endurance theory in terms of multilocation is highly implausible. For suppose a multilocated *o* is part of $L(o)_1$. It follows from my response to the argument that $L(o)_1$ is exactly located at *R*, some suberegions of which are the regions at which *o* is exactly located at the end of its life. It turns out that $L(o)_1$ is not the first half of *o*'s life, contrary to the way we informally introduced $L(o)_1$. Moreover, it follows from what I said that the regions at which $L(o)_1$ and $L(o)_2$ are exactly located overlap. Because we informally introduced $L(o)_1$ and $L(o)_2$ as the first and second halves of *o*'s life, respectively, this consequence strikes me as odd. If these indeed follow from endurance theory, then it seems that we cannot hold on to Barker and Dowe's claim that *o* is part of its life, or circumscribe the first and second half of *o*'s life. Is there a way to capture our intuition that $L(o)_1$ and $L(o)_2$ are the first and second halves of *o*'s life that would be consistent with endurance theory (or multilocation)?

Helen Beebee and Michael Rush (Beebee and Rush 2003) point out that Barker and Dowe's paradox does not arise if we think, as an endurance theorist would, of parthood as a ternary relation holding between two objects, and a region.[32] While there is no paradox to tackle anymore, the same strategy could be used to eschew the odd consequences of the last paragraph. Suppose, for simplicity's sake, that there are six regions *rs* at which *o* is exactly located throughout *R*, and assume the first three are locations of *o* in the first half of its life, while the last three are locations of *o* in the second half. With a ternary regional part-whole relation, we can say that *o* is part of $L(o)_1$ at the first three regions at which it is exactly located, but not at the last three. We avoid in that way the implausible consequences mentioned in the last paragraph.

Conclusion

In this chapter, I have been concerned with introducing and adopting a ternary part-whole relation holding between objects and regions of space. Part of my motivation for adopting such a part-whole relation relied on cases where objects are exactly located at multiple disjoint regions of space (i.e., cases in which Functionality fails). I thus examined an argument to the effect that these cases are incoherent. In the next chapter, I will argue that endurance theory can only be characterized as the view that objects are exactly located at multiple disjoint regions of spacetime.

[32] Barker and Dowe (Barker and Dowe 2005, p. 73-74) give an unconvincing answer to Beebee and Rush. They claim that L(O) has distinct temporal parts, all of which have O as a part. By construing temporal parts as facts, they conclude that each temporal part is multilocated if O is. For the fact {O, x} has O and something else, x, as constituents. By relativizing parthood and fusion to times, each fact has the form

O instantiate-at-t1 the property of being-fused-with-x.

O thus appears as a (timeless) constituent of the fact in question. Their argument succeeds because of the way they chose to express themselves, and not because there is anything inconsistent in what Beebee and Rush say.

Chapter 3

Wholly Present Objects

1. Introduction

The version of endurance theory I discuss entails that continuants are wholly present at each of the times at which they are present. Continuants are persisting objects like tables, humans, cars, etc., that do not *occur*. Objects that occur are called occurrents, and count events, states, and processes among them. In this chapter, I give a characterization of whole presence in terms of the notion of exact location (or, as we will see, entire location), and I distinguish wholly present objects from other kinds of objects recently discussed in the literature.

I am not the first to think of whole presence as location in spacetime. Cody Gilmore (Gilmore 2006, 2007), Thomas Sattig (Sattig 2006), and John Hawthorne (J. Hawthorne 2006, 2008) also think a proper characterization of whole presence should be couched in terms of location.[33] Sattig and Gilmore do not say much about their notion of exact location; all they include is the informal characterization I also gave to my exact location, i.e., that an object is exactly located at a region that has the same size and shape. But it appears that they take this notion to be basic. The theories of location that I develop in the appendix show that one could also take generic location as basic in giving an

[33] The Sattig-Gilmore conception is further discussed by Yuri Balashov in (Balashov 2008), Ian Gibson and Oliver Pooley (Gibson and Pooley 2006), Mark Moyer (Moyer 2009), and Maya Eddon (Eddon 2010).

account of whole presence. As a result, it could be the case that their notion of exact location and mine differ. I will not analyze this issue here.

In fact, I will have little to say about the Sattig-Gilmore characterization of whole presence here, since my goal is to show that there are no good mereological characterizations of whole presence. I now turn to this subject.

2. Whole Presence

In this section, I analyze different characterizations of whole presence. I start by explaining four conditions a correct characterization should fulfill. Then, I analyze characterizations couched in terms of parthood, and move on to those given in locative terms.

2.1 Conditions of Adequacy

Informally, an object is wholly present at a time or at a region just in case it is not extended to other disjoint times or regions. This informal rendering of whole presence is typically cashed out in mereological terms. It is commonly thought that

> (1) an object is wholly present at a time if and only if all of its parts are present at that time.

With (1) in mind, Ted Sider (Sider 1997, p. 209-213; 2001, p. 63-66) argues that the notion of whole presence cannot be defined correctly whenever the part-whole relation is ternary (see Chapter 2, section 2.2, for a discussion of the temporal ternary part-whole relation). The problem is one of specifying the relevant times that figure in the *relata* of the ternary part-whole relation. Two suggestions come to mind: the relevant time is either the time *t* at which the object is said to be wholly present, or it is the interval of time *I* through which the object persists. (Because any interval of time in-between these two would appear to be arbitrarily chosen, I will not consider any suggestions other than these two.) If the former, then there is nothing more behind the notion of whole presence than the triviality according to which

> (WP1) an object is wholly present at *t* if and only if all the parts it has at *t* are present at *t*.

If the latter, however, then the proposal amounts to claiming that

> (WP2) an object is wholly present at *t* if and only if all the parts the object *ever* has (or has at *I*) are present at *t*.

While there is nothing incoherent with either of these proposals, neither is particularly useful for an endurance theorist. (WP1) fails to highlight what is distinctive of the endurance theorist's views of continuants. In fact, it trivializes the notion of whole presence. Any continuant, whether it endures or perdures, will be deemed wholly present at any time at which it has a part. Intuitively, however, it is not the case that a perduring object that has a proper temporal part at *t* is wholly present at *t*. On the other hand, (WP2) would have it that endurance theory entails mereological constantism, i.e., the view that persisting objects do not lose nor gain parts over time.[34] Many endurance theorists, as Sider (Sider 2001, p. 64) notes, would rather have their favored thesis not imply mereological constantism.

Our brief analysis of (WP2) and (WP1) allows us to give the following two conditions of adequacy that we would like a characterization of whole presence to fulfill.

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[34] I adopt from Kris McDaniel (McDaniel 2004, p. 143) the expression "mereological constantism." The literature, e.g., Sider (Sider 2001, p. 64), often uses "mereological essentialism" to refer to the same thesis. But it is clear that (WP2) does not entail mereological essentialism, namely, the view that it is impossible for an object to be composed of parts other than the ones it actually has (at all times at which it exists). See (R. Chisholm 1976; R. M. Chisholm 1971, 1973; van Cleve 1986) for discussions of mereological essentialism.

(C1) Any characterization of whole presence should allow the endurance theorist to claim that an object that changes its parts over time is wholly present at each of the times at which it exists.
(C2) No characterization should trivialize the notion of whole presence.

To these two conditions, I add the following:

(C3) No characterization of whole presence can bar endurance theorists from expressing what it is for an object *not* to be wholly present at a time.
(C4) A characterization of whole presence should allow the endurance theorist to answer the following question: "for a certain persisting object *o*, and any time *t* (or any region *r*), which are the times (or regions) at which *o* is wholly present?"

It is necessary that an adequate characterization of whole presence meet all of the above conditions.

Condition (C3) is motivated as follows. The view that (at least some) objects are temporally extended and have temporal parts should certainly be taken to be *coherent* whether or not endurance theorists think such objects possible. And no such object is wholly present at any time strictly included in the interval of time through which it persists (or is wholly present at subregions of the region at which it is exactly located). As a matter of fact, many endurance theorists are willing to contend that occurents are just objects of this sort. Without the third condition, endurance theorists could be barred from expressing a view that *should not* be inconsistent with their favored notion, and one that may turn out to be consistent with their preferred ontology. Hence, the third condition is appropriately imposed on any adequate characterization of whole presence.

Some have suggested that an endurance theorist should employ two metaphysically primitive part-whole relations, a binary and a ternary one. (Katherine Hawley (Hawley 2001, p. 29) holds the view, while it is left open as a possibility by, e.g., Sider (Sider 2001, p. 57).) On their view, endurance theorists need the ternary part-whole relation to describe correctly enduring objects, i.e., objects wholly present at each of the times at which they exist, while they need the binary part-whole relation to characterize perduring objects, or those that are temporally extended and have temporal parts. This view could be seen as threatening (C3) above. For why would an endurance theorist want to express what it is for an object *not* to be wholly present given that the notion of whole presence (characterized in terms of the ternary part-whole relation) does not even apply to temporally extended objects?

(Here again, the use of both part-whole relations is not dependent on whether or not the endurance theorist accepts that some objects perdure. Accounts of persistence according to which some objects perdure should not be made inconsistent with the use of some notions.)

The view just described strikes me as unappealing. An object that is temporally extended cannot be deemed wholly present at any time strictly included in its duration (or any proper subregion of its exact location). So, an endurance theorist should be able to claim that there are times at which such an object is *not* wholly present. But if perduring and enduring objects are described with different mereological primitives, it turns out that a perduring object's *not* being wholly present at a time or region is unrelated to an enduring object's *being* wholly present at a time or region. Yet, temporally extended objects, when (or where) they are not wholly present, should be lacking precisely what a wholly present object would have. It is better, then, to suppose that both objects are characterized with the same primitive.

For the same reasons, the notion of whole presence an endurance theorist should be interested in characterizing is one that would apply to enduring and perduring objects.

Any notion of whole presence that is only applicable to enduring objects would fail to capture the difference between the two types of objects. And so, (C3) had better hold.

A characterization of an object's whole presence is bound to appeal to some characteristics of the object, a description of which can eventually be couched in terms of the endurance theorist's metaphysical primitive. For instance, (WP1) and (WP2) above, as well as all of the characterizations I criticize below, appeal to the parts an object has at a time (or, for some characterizations, at a region). Condition (C4) states that a characterization, together with a specification of the characteristics to which it appeals, should be informative about the time(s) or region(s) at which a certain object is wholly present.

Let us suppose that a non-trivial characterization of whole presence states that an object *o* is wholly present at a time *t* (or region *r*) if and only if *o* has some characteristics *X* at *t* (such as, say, having at *t* all the parts it ever has—by 'non-trivial,' I merely mean that *being wholly present* is not among these characteristics *X*). And suppose we specify, for a certain time *t*, the characteristics *X* of the object. An adequate characterization of whole presence should entail, together with this specification, whether or not the object is *wholly present* at that time *t*. But singling out the object's relevant characteristics should not require that we already can tell the times or regions at which it is wholly present (nor the time or region to which applies any notion in the conceptual vicinity of that of whole presence). For, otherwise, our characterization of whole presence would appeal to characteristics of objects that would not be specifiable without an idea of what that characterization is meant to illuminate. And our characterization would therefore fail to be illuminating.

Though never discussed, such a condition is important. Leaving the debate over persistence aside for a short while, suppose I set myself to offer a characterization of what the notion of a reciprocal in mathematics is (my example is inspired, though completely different in its form and objective, from Peter van Inwagen (van Inwagen 1990b, p. 41)). A good characterization could be thought to be the following: "*x* is a reciprocal of *y* if and only if *x* is the result of dividing 1 by *y*." The characterization not only specifies what "*x* is a reciprocal of *y*" means, but it allows us to know, for any description of a number I could substitute for *y*, what its reciprocal is. For I can give whatever description I choose to substitute for *y* (such as "5," or "the number that is the result of adding 1 to 4," or "the whole number before 6") without relying on what the characterization is meant to capture, i.e., the number's reciprocal. Suppose now that a correct description of the number I wish to substitute for *y* requires that I have available the number's reciprocal. In that case, the characterization would be quite uninformative as to what is the reciprocal of a specific number. The characterization would thus be useless to anyone desiring to master the notion of a reciprocal, for it would offer no clue as to how it should be applied. We would indeed have to be able to tell what a specific number's reciprocal is in order to apply the notion to it. In the previous example, the characterization of a reciprocal plays the role we expect from a characterization of whole presence, and the specification of the number the role of a specification of an object's relevant characteristics (such as, say, its having parts). I thus take it that condition (C4) is well motivated.

With (C1) – (C4) now stated, let us analyze characterizations of whole presence couched in mereological terms. I will show that mereological characterizations fail any one of these conditions of adequacy. I will first focus on characterizations that do not appeal to locative relations. The most promising ones fail condition (C3). The most promising characterization that appeals to locative relations characterize the latter in mereological terms. And so, I will argue, it fails condition (C4). Finally, I will examine another characterization that takes locative relations as primitive. But, I will show, it fails condition (C2). I stop discussing a characterization whenever I show that it fails one of the conditions (even, that is, if it fails other conditions as well).

2.2 Whole Presence and Parthood

Sider understands whole presence along the lines of (1) above, i.e., that an object is wholly present whenever all of its parts are present. (2) below is another way to understand this notion. It relies on the idea that perduring objects, unlike enduring ones, have, for any time *t* strictly included in the interval of times throughout which they persist, parts at *t* that are present at other times.

> (2) An object wholly present at a time has then no parts at other (disjoint) times.[35]

Objections similar to the ones formulated by Sider are not looming if one understands the notion of whole presence in terms of (2).

An example of a characterization in lines with (2) above is suggested by Sider (Sider 2001, p. 64) who analyzes the claim that the ontology of endurance theory is the denial of the possibility of proper temporal parts. Thus, one may think under such a view that

[35] For this strategy to work, we should not suppose at the outset that "*p* is part of *o* at *t*" entails that *p* exists at *t*. (WP4) and (WP5) below makes such a thing clear. I see this as a drawback of both characterization, and of the strategy.

(WP3) an object is wholly present at *t* if and only if it is present at *t* but does not have a proper temporal part then.

(WP3) rightly allows an object with *proper* temporal parts to be wholly present *at* the interval *I* of time through which it persists; for it then has no proper temporal part, only an improper one.

Sider argues against (WP3) on the following grounds.[36] He claims that a flat lump of clay shaped at *t* into the form of a statue and immediately flattened would be a proper temporal part of the lump at *t*. In his view, the statue would exist at and only at *t*, and it would overlap every part of the lump at *t*. From the temporal principle of strong supplementation (S') (introduced in Chapter 2 (section 2.2) and given below),

(S') $(\forall t)\, [(\sim P_t ab \wedge E_t a \wedge E_t b) \rightarrow (\exists x)\, (PP_t xa \wedge \sim O_t xb)]$

and the definition (TP_T) of a temporal part in terms of temporal parthood (introduced in Chapter 2 (section 4.2) and given again below),

(TP_T) *x* is a temporal part of *y* at *t* =df (1) *x* is part of *y* at *t*; (2) *x* exists at, but only at *t*; and (3) *x* overlaps at *t* every part of *y* at *t*. (see (Sider 2001, p. 59))

it follows, in Sider's view, that the statue counts as a proper temporal part of the lump. Sider reasons as follows. The statue and the lump have the same subatomic particles as proper parts, and so every part of the lump at *t* overlaps the statue at *t*. Hence, from (S'), it follows that the statue is part of the lump at *t*. The statue, by hypothesis, exists only at *t* and overlaps every part of the lump at *t*. So Sider concludes that all of (TP_T)'s clauses are satisfied.

[36] In what follows, I assume that Sider thinks that three-dimensionalism is defined by the notion of whole presence, when he analyzes the claim that three-dimensionalism is the denial of the possibility of proper temporal parts. My assumption seems justified in light of Sider's project laid out in the few pages surrounding the one from which I quote. Should it be mistaken, though, then what I say should not be understood as a criticism of his point.

But the reasoning is *wrong*. Take the temporal principle of weak supplementation (WS') (introduced in Chapter 2 (section 2.2) and given again below).

(WS') $(\forall t)\ [PP_t ab \rightarrow (\exists x)\ (PP_t xb \wedge \sim O_t xa)]$

Because the lump is supposed to be "all there" at *t* (i.e., the lump has, at *t*, no parts at other times), there is no proper part of the lump at *t* that does not overlap the statue at *t*. So the statue is not a proper temporal part of the lump at *t*.[37]

It is clear, therefore, that Sider's reasons for rejecting (WP3) are not cogent. And, in fact, it seems that (WP3) highlights a distinctive characteristic of wholly present objects—the latter do not have proper temporal parts. But though it may be true, (WP3) is not philosophically illuminating—a definition of whole presence should specify the manner in which an object exists at a time, and not merely what it does not have at a time. A more positive characterization is what I am looking for. I take it that (WP3) is inadequate, regardless of whether it meets all of the conditions of adequacy.

Another characterization that is in line with (2) is strongly suggested by Hawley (Hawley 2001, p. 27),[38] and runs as follow.

(WP4) an object is wholly present at *t* if and only if it does not have at *t* parts that are not then present.[39]

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[37] See (McKinnon 2002, p. 305) for a similar objection to Sider's argument considered in the context of presentism. I think the objection is good regardless of whether presentism is true. See also (Crisp and Smith 2005, p. 334-335) for the conclusion that either the statue is an improper part of the lump, or neither is part of the other.

[38] She claims (p.27) that "[she] characterized endurance theorists as believing that persisting things are wholly present whenever they exist—that, at any given time, no persisting object has parts which are not then present."

[39] I am concerned in this section with analyzing characterizations of whole presence couched in terms of the notion of parthood. (WP4) contains the presence or existence predicate, and it is not clear whether that predicate is mereological. I indeed contend that one should understand the presence predicate in terms of my notion of generic location (cf. Chap. 2, section 2.2). Many philosophers understand it in terms of the parthood relation, and claim that to be present at a time is to have a part at that time (cf. Chap. 2, section 2.2). So the following formulation of (WP4) should be the one analyzed in this section.

The idea behind this is that for an object to be wholly present at a time, it must not *at that time* have parts present at other times. (WP4) appears not to count objects with proper temporal parts as being wholly present. For such objects have at a time t (strictly included in the interval of time through which they persist) parts, temporal parts among others, that are not present at t. Otherwise, they would not have a *proper* temporal part at t. So, the second condition of adequacy, (C2), is satisfied; (WP4) does not trivialize whole presence. Similarly, it does not entail mereological constantism, since it does not prevent objects wholly present at a time to have different parts at other times. It solely prevents them from having, at the time at which they are wholly present, parts present at other times. Hence, condition (C1) is satisfied.

Unfortunately, (WP4) trades on the ambiguity of "having a part at a time" (see Chapter 2, section 2.2.1), and fails condition (C3). To see why, suppose a *perduring* object o composed of temporal parts p_1 and p_2 respectively present at disjoint times t_1 and t_2, the fusion of which is the interval I through which o persists. Since o perdures, it is *not* wholly present at t_1 (say). Thus, (WP4) entails that, at t_1, o has a part—namely p_2—that is present at t_2 (i.e., not present at t_1). How should we understand the last claim?

Recall from the last chapter that there is an ambiguity in temporal talk about parthood (once a ternary part-whole relation is on board). Then, we disambiguated temporal talk about parthood with the following.

(3) $(\forall t)$ ("p is part of o" is true at t if and only if $P_t po$)

(WP4*) x is wholly present at t if and only if x does not have parts at t that have no parts at t.
(WP4*) presents us with a very strange way of understanding the notion of whole presence. An object is wholly present at t just in case it has no parts at t that do not have parts at t; and an object is *not* wholly present at t just in case it has no parts at t (i.e., does not exist at t), or else it has parts at t which, in turn, have no parts at t. Because of that, I will overlook (WP4*) in the body of the text.

(4) $(\forall t)$ ("p is part of o" is true at t if and only if $(\exists t')$ $P_{t'}po$)

(4) gives an adequate rendering of the claim that o has p_2 as a part at t_1, given that the part-whole relation holds between o, p_2, and t_2. In the case at hand, we are claiming that "p_2 is part of o" is true *at* t_1 given that there is a time, namely t_2, such that it is true *of* it that p_2 is part of o. It follows that there is *nothing* significant about t_1 such that we understand why o's having a part *at* it makes of o a four-dimensional object. Whether or not o is wholly present at t_1 (say), it is true at t_1—in fact, it is true at all times—that p_2 stands in the part-whole relation to o and t_2. We would adequately capture the fact that o has p_2 as a part at t_1 by claiming that it is true *of* t_1 that o has part p_2 at t_2. But no ternary part-whole relation can offer a rendering of the last claim, for it cannot instantiate more objects than the part, the whole, and a time. So (WP4) fails condition (C3).

Another characterization that fails (C3) is offered by Kristie Miller (Miller 2005). This characterization is close, in appearance, to (WP4)—indeed, it is unclear whether Miller offers an essentially distinct characterization at all. She suggests one could have an *adverbialist* account of the temporal part-whole relation. Since this account leads her to express her characterization of whole presence in a different way than (WP4), perhaps, it may be thought, her characterization would fare better with condition (C3).

Adverbialism is a view found in accounts of property instantiation that claims that times modify the manner an object instantiates a certain property. The *instantiation* of a property by an object is, on this view, a relation holding between the object, the property, and the time. So when we say that the car is blue at some time t, we are claiming that the car stands in the ternary instantiation relation to the property of being blue, and the time t; we are claiming, in other words, that the car is blue in a *t*ly manner. And it remains true at all times that the car is blue in a *t*ly way; at all times, the car instantiates all the properties it ever has, but in a different manner (corresponding to the different times at which these properties are exemplified). I will come back to adverbialism about properties in Chapter 5.

Miller suggests that we could conceive of the part-whole relation in an adverbialist way.[40] Her suggestion amounts to claiming, perhaps wrongly, that times modify the manner in which objects have parts. So, instead of claiming that our object *o* introduced above has parts *a* and *b* at *t*, she claims that it has parts *a* and *b* in a *t*ly way. Regardless of whether *o* is wholly present at *t* or *t'*, it is true at all times that *o* has parts *a* and *b* *t*ly—it is true, in particular, that, at *t'*, *o* has parts *a* and *b* *t*ly. She then argues that the expression "*p* is part of *o simpliciter*" expresses different propositions depending on the times at which it is uttered. For any time *t*, "*p* is part of *o simpliciter*" is true at *t* if and only if *p* is part of *o* *t*ly. She finally offers the following gloss of whole presence:

(WP5) An object is wholly present at *t* if and only if all of its parts *simpliciter* are present at *t*.

(WP5) has it that an object is wholly present at *t* if and only if all the parts the object has *t*ly are present at *t*. Thus, a four-dimensional object—not wholly present at *t*—would be such that some of the parts it has *t*ly are not present at *t*. That is what follows from (WP5) and her characterization of having a part simpliciter.

Indeed, Miller seems to have this conception of four-dimensional objects in mind. She (p. 25) claims that an object "not composed of non-present parts" has no temporal

[40] See the way she expresses herself on page 326, for example: "Just as we began with the idea that there is, for the endurantist, some metaphysically basic sense of having a property in a temporally modified way..., so too we can see that there is a metaphysically basic sense of having a part *in a temporally modified way*" (my emphasis).

extension. So an object with temporal extension, on her view, would be composed of non-present parts. How could that be expressed in terms of her notion of parts *simpliciter*?

Recall our object *o* composed of *a* and *b* at *t* and of *c* and *d* at *t'*, and bear in mind that there are no other times. Suppose *o* is not wholly present at *t*. Intuitively, Miller would like to claim that at *t*, *o* has part *c* present at *t'* (i.e., not present at *t*), and contend that this claim would be false were *o* wholly present at *t*. But the desired claim cannot translate into "at *t*, *o* has part *c* *t'*ly", since that would be true even if *o* was wholly present at *t*—it is true at all times that *o* has part *c* *t'*ly, regardless of *o*'s whole presence at *t*. "At *t*, *o* has part *c* *t'*ly" fails to express that *o* has, at *t*, a non-present part.

Could the desired claim be that at *t*, *o* has part *c* *t*ly? The example we are working with is constructed in such a way that *c* is present at *t'* only. And so we could claim that perduring *o* has *c* at *t*, while enduring *o* does not. Yet the claim would be false of enduring *o* just because of the peculiarity of the case, and not because it is about a distinctive characteristic of wholly present objects. Suppose indeed *c* is present at *t*, and that it is then part of another object *o' that is* wholly present at *t*. Then, "at *t*, *o'* has part *c* *t*ly" is true, despite the fact that *c* is present at *t*. The conclusion I draw from all this is that "at *t*, *o* has part *c* *t*ly" fails to capture that *o* has at *t* a non-present part. And that is what we intended, with Miller, to capture.

And so (WP5) fails to meet condition (C3). There is no way to express that *o* has, at *t*, a non-present part *c* with a claim that really captures that fact. Therefore, (WP5) does not allow endurance theorists to say what it is for an object *not* to be wholly present at a time.

It does not provide them with claims that can capture a characteristic distinctive of objects that are not wholly present at a time.

While discussing Miller's view, I supposed that, say, "at *t*, *c* is part of *o t'*ly" entails that *o* stands in the ternary part-whole relation to *c* and *t*. And it is for that reason that I am led to conclude that (WP5) is plagued with the same problem as (WP4); in both cases, our ternary part-whole relation does not allow us to express the fact that *o* has, at *t*, a then non-present part. One could contend, perhaps, that Miller's adverbialist account of the having of a part at a time is meant precisely to deny this supposed entailment. Just as in the case of property instantiations, one could claim that it is not only the part-whole relation that holds between objects and times, but also *the having of a part* relation. (That position is not held by anybody in the literature.) On that view, the ternary part-whole relation I introduced in the last chapter would hold between a part *p*, an object *o*, and a time *t*.

(5) $P_t po$

That part-whole relation would, in turn, figure among the *relata* of a ternary *having of* relation H, whose other *relata* would be *o* and another time *t*'.

(6) $H[o, t', P_t po]$

(6) would be interpreted as expressing the view that *o* instantiates at *t'* the ternary part-whole relation holding between *p*, *o*, and another time *t*. It may then be thought that (6) gives an acceptable gloss of the view that *o* has at *t'* a part at another time *t*—i.e., that *o* has at *t'* a then non-present part *p*. A perduring object that has (distinct) temporal parts at *t* and at *t'* would be one of which (6) is true. An enduring object that changes parts from *t* to *t'* would not make (6) true.

I believe that (6) is coherent, though the predicate H cannot denote the *having of a part* relation as we commonly use it. Usually, when we claim that *o* has *p* as a part at a time, we do not mean to suggest that there is something beyond standing in the ternary part-whole relation that constitutes *o*'s instantiating the part-whole relation at a time. So we suppose it true that *o*'s having *p* as a part is just *p*'s being part of *o*, such that the having of relation is identical to the part-whole relation. Yet (6) entails that instantiating the part-whole relation (i.e., having a part) and standing in the part-whole relation are two different things; it even entails that *o* can have *p* at a time *t* without it being the case that there is a ternary part-whole relation holding between *o*, *p*, and *t*. The *having of* relation is thus not even equivalent to the mereological ternary part-whole relation.

Furthermore, (6) presents us with a distinction between perduring and enduring objects in terms of *the having of* relation, and not in terms of the parthood relation. (6) is thus not the intended analysis of whole presence. We set ourselves to express the idea that the main difference between an object that is, and one that is not, wholly present at *t* is that the latter, but not the former, is at *t composed* of non-present parts. If the *having of* relation is not the part-whole relation, then it turns out that endurance theorists need to appeal to relations other than parthood in order to express what it is for an object to be wholly present at a time. Hence, adopting (6) as the correct analysis of "*o* has at a time a then non-present part" amounts to admitting that no characterizations of whole presence can be couched solely in mereological terms. Although I contend that relations other than the part-whole relation should be brought into the picture while characterizing whole presence, there are relations that fit the bill much better than the having of relation (understood as in (6)). And so I will put (6) aside in my discussion of whole presence.

In sum, I analyzed some attempts at defining the notion of whole presence in terms of the notion of parthood. I concluded that these attempts ultimately fail to allow the endurance theorist to express what it is for an object *not* to be wholly present at a time, i.e., that they all fail condition (C3). Perhaps a better strategy would be to take into consideration the object's location in spacetime. I now turn to characterizations that put an emphasis on the object's location.

2.2 Whole Presence and Location

Our difficulty in saying what it is for an object not to be wholly present at a time arose because we wanted to cash out the notion of extension in terms of parthood at a time. Perhaps a better strategy would be to appeal to locative relations holding between objects and regions of spacetime. In this section, I analyze attempts at defining the notion of whole presence that appeals to locative relations, be they further defined in mereological terms (as with Hudson) or not (as with Parsons). I will argue that the first fails to be informative as to the regions or times at which the object is wholly present (and so fails condition C(4)), while the second is simply the regional counterpart of (WP2), and fails for exactly the same reasons. Let us analyze them in more detail.

2.2.1 *Hudson's Whole Presence*

Hudson (Hudson 2001, p. 63-64) understands the notion of whole presence in terms of his notion of exact occupation, and makes use of the regional part-whole relation (that I discussed in Chapter 2, section 3.2) to define the latter. In order to facilitate the discussion of Hudson's thesis, I will suppose that his exact occupation is my exact location, and employ "exact location" when talking about his exact occupation. My exact location and his exact occupation are similar, for Hudson claims – as do I – that an object exactly occupies a region of spacetime it completely and exactly fits (Hudson 2001, p. 63). Despite this similarity, I will be careful not to assume too much of Hudson's exact occupation, and I will present his own definition of the notion (but label it "exact location"). Here are his definitions:

> "*x* is wholly present at region $R =_{df} x$ [is exactly located at] some subregion of *R*." (p. 64)
>
> "*x* [is exactly located at] region *R* of spacetime $=_{df}$ (i) *x* has a part at *R*, (ii) there is no region of spacetime *R** such that *R** has *R* as a proper subregion, while *x* has a part at *R**, and (iii) for every subregion *R'* of *R*, *x* has a part at *R*'." (p. 63)

So Hudson's characterization of whole presence is

> (WP6) *x* is wholly present at *r* if and only if there is a subregion *r'* of *r* such that (i) *x* has a part at *r'*, (ii) there is no region of spacetime *r** such that *r** has *r'* as a proper subregion, while *x* has a part at *r**, and (iii) for every subregion r_s' of *r'*, *x* has a part at r_s'.[41]

Recall from the last chapter (chapter 2, section 3.2) that Hudson contends that the part-whole relation holds true, among other things, of any region that is both a subregion of the whole's exact location, and a superregion of the part's exact location. So, the smallest region at which an object has a part is the region at which the part is exactly located, while the largest region is the whole's exact location. Thus, clause (ii) of the definition of exact location ensures that the region the object exactly fits is no bigger than the region at which it is exactly located. Clause (iii), on the other hand, ensures that the region the object exactly fits is no smaller than the region at which it is exactly located. Incidentally,

[41] A similar definition could be couched in terms of the temporal part-whole relation. Indeed, we could claim that

(WP6*) *x* is wholly present at *t* if and only if there is a time *t'* strictly included in *t* such that i) *x* has a part at *t'*, ii) there is no interval of time *t** that strictly includes *t'* while *x* has a part at *t**, and iii) for any time t_s' included in the time *t'*, *x* has a part at t_s'.

In effect, my reasons for rejecting (WP6) count against (WP6*) as well. So I will not analyze (WP6*) separately.

the third clause is the doctrine of arbitrary undetached parts (DAP introduced in Chapter 2, section 3.3). Finally, (WP6) allows for an object to be wholly present at a region "bigger" than the one it exactly fits, but not "smaller."[42] So, (WP6) has it that an object is wholly present at a region *r* just in case a subregion of *r* is the largest region at which the object has a part.

At this point, one may think a problem arises for Hudson. He characterizes an object's exact location in terms of the regions at which it has (or does not have) parts, but then circumscribes the regions that figure in the *relata* of the part-whole relation by appealing to the part's and the whole's *exact location*. There is something circular in Hudson's characterization of an object's exact location: the notion of exact location figures, one way or another, in its own characterization. But it is not easy to see what is the source of the circularity in Hudon's characterization.

The circularity is not related to our understanding of an object's exact location, nor is it a circularity that would show up in a full-fledged analysis of the characterization. Indeed, it seems to me that we can understand locutions such as "*x* is part of *y* at region *r*" without having an idea of the exact location of the objects in question. Furthermore, Hudson's characterization of exact location only appeals to the regions at which the object has parts. And neither the notion of regions, nor the ternary part-whole relation, needs to be analyzed in terms of an object's exact location (recall that the part-whole

[42] Hudson's definition is reminiscent of the notion of entire location introduced in Chapter 1 (section 3.1.6), except that exact location is here defined in terms of parthood. His definition, like that of entire location, does not account for Parsons's intuition that extended simples are wholly located at each subregion of the region at which they are exactly located (see Chapter 1, section 3.1.6). Indeed, (WP6) has it that an object is wholly located at superregions of the region at which it is exactly located. None of the proper subregions of the region at which it is exactly located can be such a region. Furthermore, an extended simple cannot even be exactly located anywhere on Hudson's view, since it fails to satisfy the third clause.

relation is here taken as primitive).[43] So, while it may be true that Hudson appeals to an object's exact location in order to say which regions should figure in the *relata* of the parthood relation, it remains that—on Hudson's view—for a region to be an object's exact location it must be the largest region at which it has a part.

The circularity in Hudson's characterization becomes apparent when we focus on the way he circumscribes the regions at which objects have parts. To see this, first note that there is nothing intuitive about the regions (or even the times) at which objects can be said to have parts—although the point is often missed in the literature.[44] Of course, there may be many restrictions that endurance theorists need to take into consideration. For instance, there should not be any region (or time) of which it would be true to say that an object has *and* does not have a part. The point of adopting a ternary part-whole relation is precisely to avoid giving contradictory truth-conditions to our everyday change reports. Also, there may be restrictions that arise out of one's own theory of material objects. Hudson, for example, justifies the choice of the whole's exact location as a (the) largest region at which it has a part by claiming that, without making it, one would not be able to give his definition of exact location. Beyond that, however, one is free to choose whichever times or regions one wants to see in the *relata* of the part-whole relation.

So when Hudson says that the largest region at which an object has a part is the latter's exact location, he is not being particularly helpful. We should understand his claim, according to his characterization of exact location, in terms of the largest region at

[43] Here, I put aside the fact that regions *are* the objects that are exactly located at themselves. See the Appendix. As far as I know, Hudson does not hold this view.

[44] See (Donnelly 2010) for a thorough study of the different views expressed in the literature. It is clear that different authors think differently with regard to the regions at which objects have parts, and for no apparent metaphysical reasons.

which the object has a part. But he appeals precisely to the object's exact location in order to specify the largest region at which it has a part. When attempting to capture the largest region at which the object has a part, it does not help to be told that *that* region is the largest one at which the object has a part. Hudson's characterization of exact location, therefore, does not allow him to specify such a region in an illuminating way. In order to proceed with my analysis of (WP6), I will put aside this problem in Hudson's characterization. We will see that other problems are looming.

(WP6) meets condition (C3); it allows endurance theorists to express what it is for an object to be wholly present, or not to be wholly present, at a region. An object is *not* wholly present at a region *r* (at which it is nonetheless present) just in case *r* is a proper subregion of the region at which the object is exactly located, i.e., just in case there is a superregion of *r* distinct from *r* at which the object has a part. (WP6) succeeds to the task because it allows endurance theorists to account for an object's extension in terms of the extension of regions. Further, it also meets condition (C2), since it is not trivial. Some objects present at a region *r* are not wholly present there in view of (WP6). Finally, it meets condition (C1); it allows endurance theorists to claim that an object that changes its parts over time is wholly present at each of the times at which it exists – though there is no indication in what Hudson (Hudson 2001) says that he is willingly leaving this possibility open. Suppose that *o* is exactly located at the disjoint regions r_1, r_2, and r_3. In view of (WP6), such a possibility is realized just in case *o* has a part at r_1, r_2, and at r_3, at no superregion of either, and at all subregions of each. Nothing in what Hudson says about exact location prevents this. Furthermore, nothing in (WP6) prevents *o* from having distinct parts at r_1, r_2, and r_3. So (C1) is met.

(WP6) also allows an object *o* exactly located at disjoint regions r_1, r_2, and r_3, to be wholly present at a superregion of the fusion $r_1 + r_2 + r_3$. Suppose indeed that r_1 and r_2 fuse into the region r_n, while r_1, r_2, and r_3 fuse into the region *r* (*r* has r_n as a proper part, of course). (WP6) entails that *o* is wholly present at each of the regions r_1, r_2, and r_3, as well as the regions r_n and *r*. This last consequence of (WP6) is actually intuitive; the regions at which *o* is wholly present need not be the only ones at which it is exactly located. For suppose we wonder, of a region, how much of the object is present there. It follows from the fact that *o* is exactly located at r_1, r_2, and r_3, that each is a region that contains the whole of the object. But it is also true of r_n and of *r* that they, too, contain the whole of the object, even though they contain it many times over, so to speak. It is perfectly acceptable to have a view of persistence that enables perdurance and endurance theorists to agree on the fact that the region a persisting object comes to occupy during its lifetime is a region at which the object is wholly present. Perdurance theorists claim that it is *the* only region, while endurance theorists think there are many more.

So far so good for (WP6). What about condition (C4), namely, that an adequate characterization of whole presence should be informative about the time(s) or region(s) at which an object is wholly present? Does (WP6) allow us to answer the question

(Q) for a certain object *o*, and any region *r*, which is (are) the region(s) at which *o* is wholly present?

Presumably, a tenant of (WP6) would answer by pointing out that the regions at which *o* is wholly present are superregions of the region at which *it* is exactly located. But since an object's exact location is explicated in terms of the more primitive ternary part-whole relation, (WP6) would in effect force one to answer question (Q) by singling out the largest regions at which *o* has a part. But would the answer be at all informative?

An attempt to answer this question will unveil the fact that (WP6) fails condition (C4). It thus conveys no information about a specific object's whole presence. Recall that condition (C4) says that a characterization of whole presence is informative provided one can specify the characteristics of the objects to which it appeals without already being able to tell where or when the object is wholly present. In the case of (WP6), this means that one should be able to give a specification of the largest region(s) at which an object has a part, and, moreover, to do so in terms of the ternary regional part-whole relation, but without already being able to tell that *that* region is the object's exact location (that is, a region at which it is wholly present). As far as (WP6) is concerned, as I argue below, this cannot be done. (WP6) turns out, therefore, to be uninformative.

To see this, take an object *o* that has only one exact location *r*. How could we single out *r* in terms of the ternary part-whole relation as *o*'s exact location, without relying on the fact that *o* is exactly located there? Clearly, in (WP6)'s spirit, the largest region at which *o* has a part is the region at which it has an improper part. And it would seem that we have there the desired specification, namely, that

r is the region at which *o* has an improper part.[45]

Under this supposition, the characteristics to which (WP6) appeals in order to cash out whole presence would be the object's improper part. But to answer question (Q) above, that specification is not particularly useful. For we still need to *stipulate* what the regions at which an object *o* has parts are. To be precise, in order to get from the specification under consideration an answer to question (Q), we need to stipulate that the largest region

[45] The specifications I consider are formulated so as to put an emphasis on the regions at which objects have parts. But it is the parts objects have at a region, and not the regions themselves, that are the objects' relevant characteristics.

at which *o* has a part is *its* exact location. But if that is so, it turns out that (WP6) is uninformative; we would get an object's exact location or whole presence from it only if we can already tell what the regions at which it is exactly located and wholly present are. The desired specification, therefore, cannot be that *r* is a region at which *o* has an improper part.

In the argument just given, the fact that we need to stipulate what the regions at which objects have parts are is doing a lot of work. When I say that the regions at which objects have parts are a matter of stipulation, I do not mean to suggest that it is a mind-dependent matter. Nor do I mean to suggest that we legislate reality by stipulating which regions should figure in the *relata* of the part-whole relation. In fact, I think the world tells us something about where objects have parts, because it tells us where they are located. But what this "something" *is* is very minimal. Someone who adopts a ternary part-whole relation can specify many regions where it would seem appropriate to say that objects have parts there. And, indeed, many philosophers have different views on the topic.[46] In any case, suppose that we first decide what the regions at which an object has parts are, and that we claim that facts about exact location supervene on facts about parthood at a region. It seems that we could arrive at an intuitively correct rendering of the notion of exact location (in terms of parthood) *only if* the regions at which an object has parts are appropriately circumscribed (i.e., that they give us the object's exact location). I am urging that we cannot circumscribe such regions without relying on the object's exact location.

[46] Here again, see the discussion in (Donnelly 2010).

(WP6), as I said above, is in line with the idea that an object is wholly present wherever all of its parts are present. Given that our object *o* is the mereological fusion of all of its parts, *o*'s exact location *is the region at which all of its parts fuse into it*. Hence, it may be thought that *that* is a specification that would allow (WP6)'s tenants to give an adequate answer to question (Q). The specification under consideration here is (to repeat),

r is the region at which all of *o*'s parts fuse into it.

Unfortunately, that is not an appropriate specification. (In order not to embark on considerations that space would not allow me to treat correctly, let us assume that the largest region at which an object has a part is its exact location. The point of the discussion to follow is that even with that assumption, there is still a problem with the specification under discussion here.)

In Chapter 2 (section 3.3), I introduced the regional fusion operator

$$(F_r') \quad \sum(F_r x) = (iz)(\forall r)((\forall y)(F_r y \rightarrow P_r yz) \wedge (\forall y)(P_r yz \leftrightarrow (\exists w)(F_r w \wedge O_r yw))))].$$

Here is how I read it, then:

> The fusion of all the things that are F at any region *r* is *the object* such that all the things that are F at any region are parts of it at these regions, and anything that overlaps at any region something that is an F at that region is a part of it at that region, and any part of it at any region overlaps there something that is an F there.

Thus, for any objects and any region, the fusion of the objects has all of them as parts at the relevant regions. But suppose, as it should be, that

our object *o* is the fusion of all of the objects that are part of it.

In terms of the regional part-whole relation, the last supposition should be rephrased thus:

o is the fusion at *r* of all of the objects that are part of it at *r*.

But clearly, the last claim is inadequate. Recall that *o* is exactly located at a region *r*, and suppose that *r'* is a proper subregion of *r*. It is not true that *o* is the fusion at *r'* of all of the objects that are part of it at *r'*, though it follows from the axiom of fusion existence that *there is* a fusion of all of *o*'s parts at *r'*.

We palliate this problem by appealing to the following uncontroversial principle.

> For any *x*s and for any region *r*, the *x*s fuse into an object *o* at *r* if and only if *o* is exactly located at *r*.

The last principle is uncontroversial in the case of material objects. For it is uncontroversial to claim that a whole is located wherever its parts are (recall our discussion of similar principles in Chapter 2, section 5.1.2). Equally uncontroversial is the claim that an object's parts are exactly located at subregions of the object's exact location. There cannot be subregions of the whole's exact location at which the whole has no parts. Conversely, a region at which a whole has a part is a subregion of its exact location.

Does (WP6) allow us, in an informative way, to answer our initial question, namely, for a certain object *o* and any time *t* or region *r*, which are the time *t* or region *r* at which *o* is wholly present? It follows from the above considerations that we cannot rely on the parts an object has at one region or another in order to supply the region at which the object is exactly located, or the region at which it is wholly present. For we need to rely on the object's exact location in order to be able to specify the parts that fuse into it at that region. So, a specification of the parts that fuse into an object at a region already supplies the region at which the object is exactly located. It already supplies, in other words, an answer to (Q). And (WP6) played no role in our answering it.

The conclusion is that (WP6) fails condition (C4). It is not informative with respect to the region(s) at which an object is exactly located, and it conveys no information as to the region(s) at which it is wholly present. I now turn to a characterization that has locative relations as primitive.

2.2.2 *Parsons's Whole Presence*

Parsons (Parsons 2007, p. 218) claims that endurance theory should be defined in terms of the notion of whole location.

$(L_w) \quad L_w or =_{df} (\forall y)(Pyo \rightarrow L_g yr)$

(I introduced (Lw) in Chapter 1, section 3.1.5.) Recall that an object is generically located at a region just in case the region is not free of the object.

Here is how (L_w) reads.

> *o* is wholly located at *r* just in case (by definition) all of its parts are generically located at *r*.

I take it that, for Parsons, whole location is an appropriate rendering of the endurance theorist's whole presence. Nothing really hinges on that assumption. Parsons thus contends that an object is wholly present at a region whenever all of its parts are present at that region, and he adopts the first understanding of the notion of whole presence above, i.e., (1). Surprisingly, contrary to most endurance theorists, he apparently does not understand (1) as a way to cash out an object's extension (or its lack thereof) in mereological terms. In Parsons' view, an enduring object is exactly located at a four-dimensional region of spacetime and an extended simple. The sole (improper) part of the object is generically located at every subregion of the region at which the object is exactly located, and thus the object is wholly present at all such subregions. In Parsons's view, the main difference between enduring and perduring objects is not their temporal extension, but rather the fact that the former lack proper temporal parts. In choosing the notion of whole location as a rendering of whole presence, Parsons makes the acceptance or rejection of DATP[47] the main difference between perdurance and endurance theory. I will come back to Parsons's characterization of endurance theory in the next section.

Parsons's characterization is inadequate if we suppose that enduring objects are exactly located at multiple disjoint regions of spacetime. For it appeals to a binary part-whole relation, whereas a ternary regional part-whole relation is needed in that context. Parsons's use of the binary part-whole relation is justified given that he supposes that objects are exactly located at only one spatiotemporal region. The problem of mereological change is thus no motivation for him to adopt a ternary part-whole relation. In any case, let us rephrase Parsons's characterization in the following way:

(WP7) *o* is wholly present at *r* if and only if all of *o*'s parts at *r* are present at *r*.

It should be clear that none of the criticism I will formulate against (WP7) should bother Parsons, given that he employs a binary part-whole relation in giving his characterization. (I will argue against Parsons's account in section 3 of the present chapter).

(WP7) meets condition (C1), since nothing in it prevents objects from having distinct parts at each of the regions at which they are wholly present. It fails condition (C2), however. (WP7) is indeed the regional counterpart of (WP1), and it trivializes the

[47] DATP is the Doctrine of Arbitrary Undetached Temporal Parts that I introduce in Chapter 1, section 3.1.6. To repeat, it claims that

> for every persisting object *o*, if I is the interval of time through which *o* persist, if *r* is a region at which *o* is exactly located, and if sub-I is any interval included in I, there exists an object that is exactly located at the product of sub-I and *r*, which is a part of *o* and which, for every moment *t* that falls within sub-I, has at *t* exactly the same momentary properties that *o* has.

notion of whole presence. It turns out that any object is wholly present at any region at which it is present, regardless of whether our object is extended to other disjoint regions.

We seem to be in a quandary. Characterizations of whole presence formulated in terms of the object's having or lacking at a time a part present at another time prevent endurance theorists from expressing what it is for an object not to be wholly present at a time. A characterization that appeals to locative notions but that further defines them in mereological terms fails, as I said above, to be informative as to the regions or times at which the object is wholly present. Finally, appealing to locative relations as primitive is fruitless if, in addition, we attempt to cash out whole presence in terms of the location of all of an object's parts. What would it take for a characterization of whole presence to fulfill all four conditions of adequacy?

2.3 An Adequate Characterization

(WP6) comes close to be an acceptable characterization of whole presence, since it correctly allowed endurance theorists to account for an object's extension in terms of the extension of its exact location. It fails, however, because it appeals to an object's mereological characteristics in attempting to define exact location. So, a better strategy would be to characterize an object's whole presence in terms of its exact location, but also to take the latter as primitive, i.e., as not being further defined in terms of the mereological characteristics of the objects.

I suggest that the notion of *entire location* (see Chapter 1, section 3.1.5) offers a better characterization of whole presence.

(L_e) $L_e or =_{df} (\exists y)(Pyr \wedge Loy)$

(Recall that I read (L_e) in the following way.

> *o* is entirely located at *r* just in case (by definition) it is exactly located at a subregion of *r*.)

I suggest that

> an object is wholly present at a region *r* if and only if it is entirely located at *r*; or

> (WP) an object is wholly present at a region *r* if and only if it is exactly located at one of *r*'s subregions.

An object is thus wholly present at a region just in case it is entirely located at that region. (WP) entails that an extended simple is not wholly present at any of the proper subregions of the region at which it is exactly located. It thus fails to accommodate Parsons's intuition about the whole presence of an extended simple. Unlike Hudson, however, it does allow an extended simple to be wholly present at any superregion of the region at which it is exactly located.

It should be noted that Roberto Casati and Achille Varzi (Casati and Varzi 1999, p. 120) define their notion of whole location the way I defined (WP). They are not, however, endurance theorists, and are not even concerned with the persistence of material objects. Clearly, though, their notion of whole location is the best rendering of that of whole presence, especially in theories of persistence that suppose that persisting objects are exactly located at multiple disjoint regions of spacetime. As I noted above, the Sattig-Gilmore conception of endurance theory also seems to take (WP) as a correct characterization of whole presence.

(WP) meets all four conditions of adequacy. First, it allows endurance theorists to claim that an object that changes its parts throughout its existence is wholly present at several distinct regions of spacetime (or several distinct times). It is so because (WP) makes no mention of an object's parts. Second, it is not trivial; it is not true that every object is wholly present at every region at which it is present. Third, an object not wholly present at a region *r* (at which it is nevertheless present) is such that it is exactly located at one of *r*'s superregions. Fourth, because (WP) ties whole presence to exact location, it is informative as to the region at which an object is wholly present.

A few comments about (WP)'s fulfilling condition (C4) are in order. Above, I said that (WP6) fails the fourth condition because a specification of an object's mereological composition at a region already entails the region at which the object is exactly located. Cannot the same be said about (WP)? After all, a specification of the object's relevant characteristics, namely, its exact location, already entails (rather trivially) its exact location. Still, the situations are dissimilar in that, in (WP6), an object's mereological composition at a region is meant to illuminate its exact location. (WP6) fails because we already need the object's exact location in order to provide its mereological composition at a region. In (WP), however, there is nothing that supplying the object's exact location is meant to illuminate. (WP) only tells us that the region at which the object is exactly located, or any superregion thereof, is the region at which the object is wholly present. In other words, the relevant characteristic of the objects, the one that should be appealed to in a characterization of whole presence, is their exact location. That is what (WP) says.

I conclude, therefore, that (WP) is an adequate characterization of whole presence. But is (WP) interesting in an account of the persistence of material objects? I turn now to this very question.

3. Whole Presence and Persistence

If (WP) is the correct characterization of whole presence, then endurance theory claims that persisting objects are exactly located at multiple disjoint regions of spacetime. Is that an adequate way to characterize endurance theory? And what kind of objects are persisting objects in its view?

3.1 Whole Presence and Endurance

Parsons objects (Parsons 2007, 2008) to anyone's using, as I do, the notion of exact location in characterizing whole presence. For he thinks that endurance theory is a denial of DATP (see section 3.1.5 of the first chapter), and not of Functionality.[48] Recall that he thinks enduring objects are temporally extended simples, i.e., objects with no proper parts that are exactly located at (temporally) extended regions of spacetime. In his view, then, the sole difference between an enduring and a perduring object is that the latter has temporal parts, but not the former. An odd, benign consequence of Parsons's account can immediately be observed: the notion of whole presence plays no role in his account of the persistence of material objects. The latter persist because they are temporally extended. Most endurance theorists hold, however, that enduring object persist *by* being wholly present at each of the times at which they exist.

Parsons (Parsons 2007) gives one main reason why he thinks characterizing endurance theory as a denial of Functionality is inadequate: by accounting for endurance theory in terms of exact location at multiple disjoint regions of spacetime, one fails to

[48] Parsons (Parsons 2007, p. 211) introduces what he calls the doctrine of arbitrary partition (DAP) that I introduced in Chapter 2 (section 3.3). In talking about endurance theory, however, he refers to a temporal version of arbitrary partition (e.g., see p. 219). I interpret him as though he had DATP in mind.

capture the distinctive characteristics of the theory. Speaking of an account of endurance theory that has it deny Functionality, he states that

> … what unifies endurantists, on any interpretation, is their opposition to temporal parts. But the truth or falsity of Functionality has nothing to do with whether objects have temporal parts. If a thing could be exactly located at two disjoint times, *t* and *t'*, it could do so and have a part exactly located at *t* but not *t'*, and another part exactly located at *t'* and not *t*. (Parsons 2007, p. 219)

In Parsons's view, endurance theorists disagree with the claim that continuants have temporal parts, and a denial of Functionality has no bearing on whether objects have temporal parts.

There is a simple reason why I do not think it true that a denial of Functionality has *no* bearing on whether objects have temporal parts. Suppose Functionality is false, and that *o* is exactly located at r_1 and r_2 (which correspond to times t_1 and t_2). We know that the fusion of *o* at r_1 and of *o* at r_2 is *o* itself (as opposed to an object composed of two proper temporal parts), and that *o* is not exactly located at the fusion *r* of r_1 and r_2. There is thus no unique region at which *o* is exactly located. Yet, it follows from the definition of the notion of a temporal part that something *x* is a temporal part of something else at a time just in case it is not also present at another time. As a result, *o* is not a temporal part of itself, since it is present at t_1 and t_2.[49] And *o* has proper temporal parts neither at r_1 nor

[49] To be fair, Parsons's definition of the notion of a temporal part slightly differs from the one introduced in the last chapter and borrowed from Sider. Parsons (Parsons 2007, p. 16) defines a temporal part as follows:

x's temporal part at $r =_{df} (\sigma y)\ (Pyx \wedge WLyr)$

(The operator σ is a description-like operator for mereological fusion.) Here is how Parsons's definition should be read:

> *x* has a temporal part at *r* just in case (by definition) there is one unique object that is part of *x* and that is wholly located at *r*.

Because he thinks enduring objects are uniquely located at a four-dimensional region of spacetime, his definition of the notion of a temporal part allows him to claim that enduring objects are improper temporal parts of themselves. Could the same thing be said of an object exactly located at multiple regions of spacetime? It could not, since for any region at which the object is exactly located, the object has parts at other regions (and thus fails to be wholly located – in Parsons's sense – at the region in question).

at r_2 (or t_1 and t_2), since it is not exactly located at a region that has r_1 and r_2 as proper parts. That *o* has no temporal parts follows from a denial of Functionality.

Perhaps, however, Parsons merely wants to claim that an object exactly located at different disjoint regions could have distinct temporal parts at these regions, as suggested in the quotation above. Presumably, Parsons has the following in mind. The two distinct parts exactly located at *t* and *t'*, respectively, are mereological fusions of the parts our persisting object *o* has at these regions. (I take it that "*t*" and "*t*'" are names of regions.) They count as temporal parts of *o* because they are each located (wholly and exactly) at, and only at, the regions in question. Parsons thus presents us with a situation where the following are all true:

a) *o* is exactly located at both t_1 and t_2.
b) distinct temporal parts are exactly located at t_1 and t_2.
c) *o* is identical with neither of the temporal parts at t_1 and t_2.
d) the temporal parts at t_1 and t_2 are part of *o*.

For a) through d) to be true, it has to be the case either that the temporal versions of the principle of weak supplementation fails, or else that the temporal version of the axiom of weak anti-symmetry fails (see Chapter 2, section 2.2). For at either t_1 or t_2, there is no part that *o* has and that the relevant temporal part lacks. Our object *o* would thus have, at t_1 or t_2, temporal parts that have exactly the same parts as *o*. And so weak supplementation would fail. Should we decide to hold on to weak supplementation, we would have to conclude that *o*'s temporal part at t_1 and *o* are part of each other there, without *o* and the temporal part being identical (so as to make c) true) (*mutatis mutandis* for t_2). Both views would have us maintain that *o* coincides with its temporal parts at both t_1 and t_2. For someone who rejects either weak supplementation or the axiom of weak anti-symmetry, a rejection of Functionality does not entail that an object lacks temporal parts. But I accept CEM, and so that argument does not work for me. Is Parsons right, then, to claim that endurance theory is better characterized by a rejection of DATP?

Parsons's case is unnecessarily complicated. By itself, DATP does not entail anything about an object's temporal parts. For DATP to entail that an object has temporal parts, it is necessary that the object be exactly located at a temporally extended region of spacetime. Similarly, a rejection of DATP will entail that objects do not have temporal parts only if the latter are exactly located at temporally extended regions of spacetime. After all, an instantaneous object lacks proper temporal parts, regardless of whether we regard Functionality or DATP as true. But an endurance theorist may very well account for the *persistence* of material objects by pointing out that they persist by being exactly located at different three-dimensional regions (i.e., by a rejection of Functionality). Such a philosopher would also claim that enduring objects lack temporal extension and temporal parts. But she would not need to reject DATP, for the enduring object lacks temporal parts simply because it is not temporally extended. Hence, it is true to claim that what unifies endurance theorists is their opposition to temporal parts, but it does not support the view that endurance theory is better characterized as a rejection of DATP. For their rejection of temporal parts does not necessarily come from a rejection of DATP. I conclude, therefore, that it is perfectly acceptable to characterize endurance theory as a rejection of Functionality, as acceptable, that is, as characterizing it as a rejection of DATP.[50]

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[50] Parsons illustrates his point by taking Cody Gilmore's assertion about locations as examples. Gilmore takes a notion close to our exact location as primitive, and Parsons claims that he ends up making "the question of "perdurantism" [i.e., perdurance theory] vs. "endurantism" [i.e., endurance theory] independent

3.2 Multilocated Objects

What kind of objects does endurance theory *cum* (WP) leave us with? Objects exactly located at multiple regions of spacetime are to be contrasted with other objects, the possibility of the existence of which is often expressed with the help of locative notions. Hudson (Hudson 2005, p. 100-106) and Kris McDaniel (McDaniel 2007, p. 133-134) write about spanners and multi-locators (the name "spanners" for the kind of object I am about to describe is due to McDaniel). In my terminology, a spanner is

> an object *o* that is exactly located at (only) one non-point-sized region, *r*, while there is no proper subregion of *r* at which *o* is generically located (i.e., *o* does not bear any of the location relations to *r*'s proper subregions).[51]

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of whether objects are divisible into arbitrary temporal parts" (p. 220). Regardless of the peculiarities of Gilmore's theory, the argument just given works for Gilmore also.

[51] Here are Hudson's and McDaniel's characterizations of what a spanner is.

> "[a spanner] is a material object that is wholly and entirely located at exactly one non-point-sized region, *r*, and there is no proper subregion of *r*, *r**, such that any part of *x* is located at *r**." (Hudson 2005, p. 101)
>
> "[a spanner] bears the occupation relation to exactly one extended spatiotemporal region, without bearing the *location* relation to any proper part of that extended region. Spanners… uniquely occupy a single extended region of spacetime" (McDaniel 2007, p. 134) (my emphasis).

McDaniel takes the relation of occupation to be primitive, and he describes it similarly to the way I defined the notion of exact location in the last chapter. To be precise, he claims (p. 132-133) that a) "if an object *x* occupies a region of spacetime *r*, then, typically, every part of *x* occupies some part of *r*;" b) "if *x* occupies a region of spacetime *r*, it does not follow that *x* occupies a proper sub-region of *r*;" and c) "if an object occupies two disjoint regions, it does not follow that the object occupies the fusion of these regions. (When an object occupies two disjoint regions, I say that the object enjoys multi-location.)" Note that McDaniel's relation of occupation does not satisfy something that would look like Functionality. Finally, his location relation seems to be a generic relation that could be any relation holding between objects and regions.

For the record, here is how Hudson (Hudson 2005, p. 99) defines the notion of whole location and of entire location:

> *x* is entirely located at $r =_{df} x$ is located at *r*, and there is no region of spacetime disjoint from *r* at which *x* is located.
>
> *x* is wholly located at $r =_{df} x$ is located at *r*, and there is no proper part of *x* not located at *r*.

Hudson understands his primitive location relation "in such a way that the object completely fills any region at which it is located (as opposed to 'located within', which suggests that the region might be vastly bigger than the object contained somewhere or other in the depths of its interior)" (p. 99). More on it below.

Parsons's enduring objects are not spanners. For these enduring objects are wholly located (in the way the notion is defined in the last chapter) at every subregion of the region *r* at which they are exactly located. My multilocated objects, which enjoy exact locations at multiple regions of spacetime, are not spanners either, since they are exactly located at more than one non-point-sized region of spacetime.

I claim that enduring objects are multilocated in spacetime in the sense of being exactly located at multiple disjoint regions of spacetime without being exactly located at the fusion of these regions. McDaniel (McDaniel, 2007) and Hudson (Hudson, 2005) talk about multilocated objects. For McDaniel, an object is multilocated if and only if it is exactly located (in my terminology) at at least two distinct regions *r* and *r**.[52] He (McDaniel, 2007, p. 134) goes on to claim that multilocators are "extended in virtue of covering an extended region," where an object covers a region just in case that region is the union of the regions at which it is exactly located. Though McDaniel explicitly claims that multilocators are not exactly located (in my terminology) at the fusion of the regions at which they are exactly located, he counts Parsons's enduring objects as multilocators. If so, then his definition of the notion of a multilocator is different both from Hudson's and mine. Moreover, if we take McDaniel at his word, we find that he leaves no room for a distinction between location and occupation. For he talks throughout of multilocators as *occupying* several disjoint regions, without occupying the fusion of these regions. But if a simple is exactly located at regions r_1 and r_2, then it seems that the fusion *r* of r_1 and r_2 is occupied by our simple (though, of course, it does not follow that the simple is exactly

[52] McDaniel (McDaniel 2007, p. 133) defines multi-location in the following way: "'*x* is multi-located' $=_{df.}$ There are regions *R* and *R'* such that (i) *R* is not identical with *R'* and (ii) *x occupies R* and *x* occupies *R'*" (my emphasis).

located at *r*). All this is probably only a matter of terminology, and I have now merely expressed my own preference for mine. No conceptual issues really revolve around McDaniel's choice as to how to express himself.

Hudson (Hudson 2005, p. 103) claims that a multilocator is H-located (i.e., located in Hudson's sense – Parsons (Parsons 2008) coins the term) at more than one region, and is not H-located at the fusion of these regions.[53] Hudson informally characterizes his location relation in such a way that objects are H-located at any of the regions they fill up (where an object is H-located at a region whenever it completely fills up that region, as opposed to a proper part of that region. (See (Hudson 2005, p. 98-99)). In an attempt to clarify H-location, Parsons (Parsons 2008, p. 432) admits that his best take on H-location is that it "is sufficient but not necessary for pervasive location." Recall that, in Parsons's and my own view, an object is pervasively located at a region just in case the latter is a subregion of the object's exact location. A multilocator (in Hudson's sense) could therefore be an object H-located at r_1 and at disjoint r_2, not be H-located at $r_1 + r_2$, and yet be exactly located at a superregion of $r_1 + r_2$ (proper or improper). If that is correct, then Hudson's multilocators are not my multilocated objects.

I initially thought that H-location was exact location, and that Hudson was denying that Functionality is a conceptual truth (as I do). But when he defines (Hudson 2005, p. 99) his notion of whole location, he claims that an object wholly located at *r* is such that it, as well as all of its proper parts, are H-located at a region *r*. On my rendering of exact location, no objects can be exactly located at the same region as one of its proper

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[53] His definition: " '*x* multiply locates' $=_{df}$ (i) *x* is a material object that is located at more than one region, and (ii) *x* is not located at the fusion of the regions at which *x* is located" (p. 103).

parts. In fact, only my generic location would allow an object to be wholly located at a region in Hudson's sense. Yet, my generic location does not conform to the informal characterization of H-location. In addition, since Hudson grants that his multilocated objects are not H-located at the fusion *r* of the regions at which they are H-located, he admits that they do not fill *r* up. I claim that my multilocated objects fill up the fusion of the regions at which they are exactly located (though, without being exactly located there). In other words, they occupy the regions at which they are exactly located, in addition to the fusion of these regions. So here, again, Hudson's multilocators are not my multilocated objects.

The point of my discussion of Hudson's multilocators was to make sure that my characterization of my multilocated objects is clear.

6. Conclusion

In this chapter, I defended the view that the best characterization of whole presence is one according to which an object is wholly present at a region if and only if it is exactly located at a subregion of that region. Since endurance theory claims that objects—continuants, more specifically—are wholly present at each of the times at which they exist, it entails that persisting objects are exactly located at several disjoint regions. I thus took the time to distinguish these persisting objects from other kinds of objects discussed in the literature.

Chapter 4

Haecceities and Universals

1. Introduction

So far, I have argued both that there is nothing incoherent in supposing that objects can be exactly located at multiple regions of spacetime (see Chapter 2 and the Appendix), *and* that the notion of whole presence, at the heart of endurance theory, can only be cashed out in locative terms. In effect, endurance theory implies that objects are exactly located at multiple disjoint regions of spacetime. Now, I see the latter fact as a special case of a more general one, namely, that objects *can* be exactly located at more than one region of three-dimensional space. Or, if one wishes, that they can be exactly located at more than one disjoint region of spacetime, even in the same hyperplane of simultaneity. In this chapter and the next, I seek (and find) an explanation of the possibility for material objects to be exactly located at more than one disjoint region of three-dimensional space (or, as I will often say, I seek an explanation of the possibility of *multilocation*). I focus on three-dimensional space instead of hyperplanes of simultaneity for ease of presentation. I do not think that there is a restriction on the spaces in which objects can be multilocated, nor am I interested here in investigating the issue.

In the next chapter, I will offer a theory of material objects that makes it apparent that it is possible for them to be multilocated in space. I will then also make clear what kind of explanation of multilocation I am after. Here, however, I introduce and discuss some notions that will play a major role in the next chapter.

2. Introducing Haecceities

2.1 Haecceities and Persistence

I have characterized endurance and perdurance theory partly in terms of a persisting object's exact location in spacetime. I gave the characterization of perdurance theory in Chapter 1 (section 4), whereas the characterization of endurance theory was given in Chapter 3 (section 2.3). In an attempt to explain the possibility of multilocation, it will be useful to digress from these characterizations.

Let us say that the property an object has *of being identical with itself*[54] is the object's *haecceity*. So *o*'s haecceity would be the property

being identical to o (or, equivalently, the property $x = o$),

and *not* the property shared by every objects *of being identical to some object*.[55] Obviously, *o*'s haecceity cannot be shared with any object that is not identical to *o*. Here, I follow Robert Merrihew Adams's (Adams 1979, p. 6) terminology. For the moment, I am not supposing anything else about *o*'s haecceity.[56]

My motivation for appealing to haecceities is that they will play a central role in the explanation of multilocation that I offer in Chapter 5. Roughly, I will then defend the view that it is possible for objects to be exactly located at multiple disjoint regions of

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[54] For clarity here and in the rest of the dissertation, I italicize the expressions that stand for properties.

[55] See (Rosenkrantz 1993) for a slightly different characterization. He (p. 3) tentatively characterizes a haecceity in the following way: "F is a haecceity $=_{df}$ $(\exists x)$(F is the property of being identical to *x*)."

[56] One may think of haecceities as individual essences of objects, for they are unsharable and unique to their bearer. Many philosophers (see, e.g., (Mackie 2006, p. 20)) who discuss the topic of individual essences think that making haecceities individual essences trivialize the latter notion. But they would trivialize the notion of individual essence if they were taken as primitive. I will not have anything to say about individual essences, since discussing the issue would be a little bit off the topic. I will come back to the issue as to whether haecceities are primitive.

space because their haecceities can be instantiated at these multiple regions. I will not attempt to make this explanation clearer in the present chapter. I postpone any discussion of multilocation until Chapter 5.

Endurance and perdurance theory differ with respect to the distribution of a persisting object's haecceity in spacetime. The former theory entails that it is instantiated at multiple disjoint regions of spacetime, and the latter that it is instantiated at no more than one region. I regard this difference in distribution of haecceities over spacetime as a simple and uncontroversial consequence of the disagreement between the theories over the exact locations of objects in spacetime.[57]

Despite the fact that the disagreement over the distribution of haecceities is innocuous, it is generally thought that the debate over the persistence of material objects is not one about haecceities.[58] Few indeed regard the problem of specifying persistence conditions for objects of a certain kind as problems about identity strictly speaking,

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[57] I recently became aware of the work of David Velleman and Thomas Hofweber (Hofweber and Velleman 2011) and Hofweber (Hofweber 2009) who also think of the debate over persistence (and probably multilocation) as a matter of the identity of the object. The account they give is, of course, very different from the one I will give. Here is what I take to be the best account they give (from Hofweber (Hofweber 2009, p. 307)).

> "An object is wholly present at a time *t* iff the property of being *o* is local to *t*."

In Hofweber's view, a property *P* is local to a time or region *R* "iff something in *R* has *P* and that thing would still have *P* as long as *R* were the same, even if things were different outside of *R*." There is little I have to say about their account, except for two things. First, I cannot help noting that Hofweber appeals to an object's haecceity in his characterization of whole presence. That is removed in (Hofweber and Velleman 2011). Second, both Velleman and Hofweber think that the characterization of whole presence in terms of an object's identity is a new way of thinking about persistence, i.e., it is not related to the normal characterizations of perdurance and endurance theory. In my view, the characterizations in terms of an object's identity (indeed, in terms of its haecceity) is simply a consequence of the usual characterizations.

[58] Sider (Sider 1999, p. 924), for instance, asks us to "[n]otice that in the familiar debates involving 'identity over time,' the haecceities of objects involved are irrelevant. When we discuss a case of amnesia, we want to know whether the person at the beginning of the thought experiment survives; it is irrelevant whether that person is *Frank* or *Joe*." And later: "[i]t is possible to [ask]: what are the conditions under which Frank exists at times t_1 and t_2? Here we ask not only about what I am calling persistence, but also about what conditions would have to be satisfied to have Frank present, and that is a question about haecceity."

although the expressions "diachronic identity" or "identity over time" are often used as labels for this problem. Typically, a specification of persistence conditions for objects of a kind will appeal to, among other things, causal relations that hold between different "stages" of a persisting object, where a "stage" is nothing more than an object at a certain time (I think of "stages" as represented by ordered pairs the members of which are an object and a time at which it exists, such as "$\{o,t\}$"). In effect, then, the problem of specifying persistence conditions for material objects (of some kinds) is orthogonal to the problem of formulating a coherent picture of the ontology of endurance theory. For endorsing either endurance or perdurance theory does not amount to specifying an object's persistence conditions. Indeed, some specifications of an object's persistence conditions are available to both theorists.[59] So it may well be that problems of diachronic identity or identity over time are misnamed. Yet that does not show that either endurance or perdurance theory cannot appeal to haecceities.

In fact, I think that the confusion between the two different problems is often to be found on the side of perdurance theorists. David Lewis, for instance, rightly claims (Lewis 1986, p. 193) that it is not the fact that a certain problem is formulated with the identity predicate that makes it a problem about identity. He goes on to claim, however, that "it is a good question … whether there could be a time traveler who meets his younger self…" (p. 193), but that is still not a question about identity. It strikes me as apparent that Lewis's claim is ambiguous: whether a time traveler could meet his

[59] See, e.g., (Hirsch 1982) for an account of the persistence conditions of material objects (especially the first part). Although Eli Hirsch formulates his account by referring to objects' stages, he does not obviously commit himself to a certain metaphysical theory of persistence. In fact, he seems to think that the debate over the metaphysics of persistence is a moot issue (p.149-156). His notion of a stage could be compatible with endurance theory, and so could his account.

younger self involves many questions about causation, persistence conditions, the nature of spacetime, etc. But it also invites questions about the location of objects in three-dimensional space. How could the time traveler be exactly located at two places at once? For the perdurance theorist, the time traveler and his younger self are distinct entities, i.e., distinct proper parts of the same temporal parts.[60] So none of the problems posed by time travel are about identity. For the endurance theorist, however, the time traveler and his younger self are, strictly speaking, identical. It is the same object that is exactly located at two disjoint regions of space.

The point can be generalized. It is not the case, for the perdurance theorist, that an object's haecceity is instantiated at several disjoint regions of spacetime. And so it becomes easy to claim that questions about the persistence of material objects are not questions about identity. For the endurance theorist, however, an object's persistence is a matter of *its* being exactly located at several disjoint regions of spacetime; it is a matter of the same haecceity's being instantiated at these several disjoints regions. And, to that extent, it *is* a question about identity. How is it possible for what is exactly located at one region and what is exactly located at another to be identical? It is this question endurance theorists need to answer.

2.2 Haecceities and Universals

I think of (most) properties as universals. Universals, it is generally agreed, are *repeatable* entities. That is to say, they can be instantiated at multiple places. There are

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[60] Temporal parts, as I characterized them in the first chapter (section 4.1) and as they are generally characterized, are individuated by times. The time-traveler and his younger self are present at the same time. So it follows that they are both (proper) parts of the same temporal part.

other characteristics that have come to be recognized as the mark of universality. For instance, a universal can in principle be instantiated by an infinite number of particulars. It is possible, indeed, for an infinite number of things to instantiate *redness*. From such characteristics, David Armstrong concludes that haecceities "lack a necessary mark of universals, the logical possibility that the class of particulars which have this property be an infinite class" (Armstrong 1978a, p. 93). As far as I can see, there are two lines of arguments against considering haecceities as universals hidden behind Armstrong's conclusion.

The first line has it that, as a consequence of the fact that haecceities cannot be instantiated by an infinite number of individuals, they cannot be instantiated at more than one place, and, therefore, cannot be universals. Here, it is contended that haecceities are not universals in virtue of the fact that they cannot be instantiated at more than one place. But this line of argument can hardly constitute a good reason for me to deny that haecceities are universals: both universals and particulars, in my view, can be at many places at once. Disregarding the differences between location at a region and instantiation at a region, I cannot regard multiple instantiation as a *defining* characteristic of universals. For they are not the only things that can be at multiple places. And even if I were to insist that there is a relevant difference between instantiation and location, i.e., a difference that could be used in an attempt to salvage the fact that only universals can be *instantiated* at multiple regions (and that haecceities are universals), it would not do given my project. I indeed wish to defend the view that objects are exactly located at multiple regions of space, and the claim I defend in this chapter and the next is that they can be so located because their haecceities can be instantiated at multiple regions. I would therefore end up explaining multilocation with the notion of multi-instantiation. But because it would come too close to taking multilocation as basic, such an explanation would not be very illuminating.

The other line of argument takes Armstrong to the letter, and merely consists in the claim that haecceities, unlike universals, are unsharable entities. Haecceities cannot be universals since they cannot possibly be instantiated by an infinite number of particulars, regardless of whether they are capable of multiple instantiations in space. It is hard to evaluate this line of argumentation correctly. For once one allows that particulars can be exactly located at multiple places, one is free to claim that counting objects in space is not, contrary to what is usually supposed, a matter of counting non-identical objects. One could claim, for instance, that counting is related not to the objects, but to the regions at which objects are exactly located. So, if I were to see an older woman talking to a younger woman, I would count them as two women talking to each other given that there are two regions at which the woman is exactly located, even if I came to realize that they are in fact the same woman: an older time-travelling woman that meets her younger self.[61] There could therefore be a sense in which "many" objects could instantiate the same haecceity. Yet there is no sense to be attached to the view that haecceities can be instantiated by *distinct* objects. And universals can be so instantiated. So I will grant that haecceities do not "bear the necessary mark of universals," and yet

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[61] This whole issue about counting is actually an important one once we allow an object to be exactly located at multiple disjoint regions of space. But I will not be able to touch upon it in this dissertation. The view that we do not always count by identity has been held by David Lewis (Lewis 1976), in another context. And John Hawthorne and J.A. Cover (J. O. L. Hawthorne and Cover 1998) suggest that we employ it precisely for multilocation. They do not go as far as claiming, as I am inclined to, that we count by regions in cases of multilocation. At any rate, the view that we do no count by the relation of identity, though it is controversial, should not be dismissed at the outset.

claim that they can be of use in a good explanation for the possibility of multilocation for material objects *because of the characteristics they share with universals*. It remains to see what these are.

2.2.1 *What is Common To Haecceities and Universals?*

Universals are repeatable entities: they can be instantiated at different regions of space. How should we understand that very possibility? We should first note that not everybody who embraces universals thinks that they are located in space. With respect to universals, a Platonician thinks that universals are transcendent entities that are located nowhere, and the existence of which is entirely independent of the particulars that “participate” in them. It is rather the Aristotelian conception of universals, what is called *immanent realism*, that has them located in space. Under such a view, universals are *instantiated* by particulars, and are located where the latter are; they are *in* the particulars that instantiate them. Immanent universals have the following two locative properties. First, they can be wholly present, or completely present, at different regions of space. Second, several universals can be located at the same region of space. (See (Oliver 1996, p. 25).) The way universals are wholly present at distinct regions of space is different from the way material objects are, as I conceive it, wholly present at different places. This will become clear in Chapter 5, where I develop a theory of material objects. It will then be apparent that I do not rely on this characteristic of universals in explaining multilocation. And issues about the co-location of universals will be taken up in Chapter 5 and 6. For now, let us see how we can make sense of the claim that universals are “located” in space.

Typically, a universal is instantiated at a specific region in virtue of the fact that an object, which is exactly located (to use my terminology) at that specific region, instantiates it. Universals are multi-instantiated in space, it is usually thought, in virtue of being instantiated by distinct objects that are themselves exactly located at distinct regions of that space. A simple way to characterize a universal's being instantiated at a region or another, then, is to claim that its location is a characteristic it has in virtue of being instantiated by an object that is *itself* located in space. In other words, a universal is located at a place because an object that instantiates it is located at that place. The idea is L.A. Paul's (Paul 2002, p. 583-585), and I find it a very compelling one.[62] Universals, under the view suggested here, get their location from that of the objects.[63] I will consider this simple and rather broad claim about the location of universals to be part and parcel of the immanent realist view.

Let us come back to haecceities. Are they also the kind of things that are located in space in virtue of being instantiated by located material objects? I think they are, and that *that* is what is common to both haecceities and universals. Although I do not think of haecceities as universals, they are properties. And we can explain the location of any properties in space simply by generalizing the explanation of the location of universals given above. For universals are a specific kind of properties, and nothing in the

[62] She claims, however, that universals have themselves no location, but acquire one by being instantiated by an object. I do not think that she means to say something different than what I just said.

[63] Alex Oliver (Oliver 1996, p. 26) tries to make sense of the claim that immanent universals are *in* their instances. He claims that one obvious way to do so is to suppose that particulars are bundles of universals, and that a universal is "in" the particular just in case it is part of the bundle. I will have more to say about bundle theories of particular below and, especially, in Chapter 5. I just want to point out that I am discussing a slightly different point. I am not yet interested in specifying the way universals are in particulars, but rather how it is that they are located in space.

explanation of their location appeals to their being *universals*.[64] So if we are ready to grant that universals get their location from objects, then we should likewise accept the same thing for haecceities. Still, one could point out that I blur the order of explanation I had wished to maintain: multilocation is possible in virtue of a haecceity's ability to be instantiated at multiple regions. In granting that a haecceity's location is the exact location of an object that instantiates it, I seem to suggest that it is, in fact, the reverse order of explanation that holds. But the worry is too hastily formulated. I have said nothing so far that looks like an explanation of multilocation. As will become clear in Chapter 5, the important characteristic of haecceities in an explanation of multilocation is the one I claim they share with universals: that their location in space is that of the objects that instantiate them.

When I say that universals and haecceities inherit a location from the objects that instantiate them, I do not mean to suggest that universals and haecceities exist outside of space—unless, of course, the objects that instantiate them are not located in space (such as, say, the property of *being a prime number* instantiated by 2). Although I will not discuss the issue at great length, I am tempted to subscribe to the following principle of the existence of properties:

> A property (of a material object) exists if and only if it is instantiated by an object that is exactly located at at least one region of space.

Such a principle of existence is in accordance with the immanent conception of universals, which is plagued with the problem of uninstantiated universals. It is indeed

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[64] To be sure, universals and other properties like haecceities differ with respect to their location in space. For instance, a universal that is located at more than one region *can* inherit its many locations from *distinct* objects, whereas haecceities would be multilocated in space in a different fashion (that remains to be specified). But these differing characteristics of universals and haecceities were not appealed to in the explanation of the location of universals.

argued that the immanent conception of universals fails because it cannot accommodate uninstantiated universals. For reasons of space, I will not investigate the problem of uninstantiated universals here.

One final concern needs to be addressed. Many philosophers talk of haecceities as properties of objects. But are they really such properties? Armstrong (Armstrong 1978b, p. 11) argues that they are not.[65] He gives two reasons for this view. First, he contends that properties of objects are discovered by scientific investigation. The fact that it is *a priori* knowledge that an object is identical with itself indicates, in his view, that haecceities are not properties. Second, properties of objects should endow them with causal powers. But the fact that haecceities do not seem to endow objects with causal powers indicates, as a consequence, that they are not properties. I am not convinced by Armstrong's reasons. I am ready to accept that the knowledge of the existence of some properties is *a priori*, and I do not find this incompatible with claiming that scientific investigation informs us on what properties there are. Furthermore, I find the issue as to whether haecceities endow objects with causal powers rather moot. Obviously, the scope of this dissertation does not allow me to investigate either of these reasons further. Instead, I will endorse in section 4 the motivation that leads philosophers to recognize haecceities as properties of objects—a motivation that, incidentally, Armstrong (Armstrong 1978a, p. 91-97) himself endorses. The motivation is a counterexample to Leibniz's Identity of Indiscernibles. But more on that in section 4.

[65] In the passage to which I am referring, Armstrong is not really concerned with haecceities, but rather with the alleged property of *being identical with itself*, which is *shared* by all objects. In his view, we have seen, haecceities are not universals. And he thinks all properties are universals. I bring up this passage because it would constitute further reasons to deny that haecceities are properties of objects, should one recognize that there *are* properties other than universals.

2.2.2 *What is Not Common to Haecceities and Universals*

Universals are qualitative properties of objects, whereas haecceities are regarded as non-qualitative properties.[66] The distinction between qualitative and non-qualitative properties is hard to pin down. But it is often expressed in the following rough way: qualitative properties can be specified with descriptions in a language that contains no proper names, whereas describing non-qualitative properties requires the use of proper names. (See (Adams 1979; Lewis 1986; O'Leary-Hawthorne and Cover 1997).) Hence, the property of *being blue* can be specified without the use of a proper name, but not the property of *being the brother of Chloe*. The former would be qualitative, while the latter would not. Under that characterization of non-qualitative properties, haecceities are non-qualitative since their specification involves a proper name for the object of which they are haecceities ("JD" in the specification of my haecceities, i.e., in "$x = JD$").

This way of marking the difference between qualitative and non-qualitative properties is meant to characterize our intuitive grasp of the distinction. It is tied to a certain view of proper names. Should "Chloe" be a descriptive name, i.e., should it apply to its bearer in virtue of descriptions that could be formulated in a language that contains no proper names, then *being the brother of Chloe* would be a qualitative property. I have no knock-down argument against this semantic criterion for qualitative and non-

[66] I will simply assume that universals are qualitative properties. Some philosophers write as though universals could be non-qualitative. For instance, Oliver (Oliver 1996, p. 26) writes about the possibility of introducing "non-qualitative universals into the bundles" of properties that capture a particular, and he even goes so far as to claim that haecceities are "universals such as the properties of being identical to particular *a*." On the other hand, O'Leary-Hawthorne (O'Leary-Hawthorne 1995, p. 191) claims that "'universal' is the term of art that most safely excludes haecceities from its instances." He does not say whether there are non-qualitative universals. These differences in expression show that the terminology is not really rigid. So I allow myself to stipulate that universals are qualitative properties. The stipulation changes nothing about the point that I want to make.

qualitative properties. What, in my view, speaks against it is that it leaves open the possibility that a property could be non-qualitative, and yet be reducible to (or supervene on) qualitative properties only. For it *could be* that the semantics of proper names treats them as rigid designators, i.e., as terms with the same referent in all possible worlds, and yet that objects are nothing more than bundles of (perhaps essential) qualitative properties.

A more illuminating approach is to claim that whether or not a property is qualitative depends on whether it can be reduced (or whether it supervenes on, emerges from, is equivalent to, etc.) qualitative properties. Such a metaphysical "partial-criterion" is more telling as to whether or not a (non-primitive) property is qualitative because it focuses on what makes objects instantiate the property in question. If a certain property *P* can be reduced (or supervene on, is equivalent to, etc…) *qualitative* properties *Q* (where *Q* could be a conjunctive property), then the object has *P* just because (or just in case) it has *Q*. There would be absolutely no reason to suppose, then, that *P* is non-qualitative. On the other hand, it suffices that the conjunctive property *Q* has only one conjunct that is a non-qualitative property for *P* to be non-qualitative. For the object has *P* partly because (or partly in case) it has a non-qualitative property. So the "partial-criterion" for qualitative/non-qualitative property I will adopt is the following:

> (C) A property *P* is qualitative if and only if *all* the properties to which it is reduced (on which it supervenes, from which it emerges, to which it is equivalent, etc.) are qualitative properties. Otherwise, *P* is non-qualitative.

(C)'s scope is somewhat limited, or at least narrower than the semantic criterion. For the latter allows us to decide whether or not a *primitive* property is qualitative. As a result, (C) serves only to tell whether a certain non-primitive property is qualitative, and is *not* a characterization of what it is for a property to be qualitative—nor is it, strictly speaking, a criterion (hence my calling it a "partial-criterion"). On the contrary, it relies on a previous (perhaps intuitive) understanding of what qualitative properties are. (C) will come up when discussing whether or not haecceities are non-qualitative.[67]

3. An 'Analysis' of Haecceities

The explanation of multilocation that I propose in Chapter 5 has it that objects can be exactly located at multiple disjoint regions of space in virtue of the fact that their haecceities can be instantiated at these regions. But what exactly are these haecceities? In this section, I offer an "analysis" of them in terms of universals. In the following section, I discuss the main motivation for recognizing haecceities as properties of objects. I leave to the next chapter, Chapter 5, an account of multilocation.

I claim that an object's haecceity *h* is "analyzed" in terms of a group *B* of universals. *h* is *not* a universal, and the relation holding between it and B is *not* identity. For it could be that the conjunction of universals in *B* retains the characteristics of universals. And in any case, we will see below that I think of universals, though not haecceities, as qualitative properties. I will argue in the next chapter that, nevertheless, *B* allows us to offer necessary and sufficient conditions for the *instantiation* of *h*, and constitutes, *in that sense*, an adequate "analysis" of *h*. (And that is why I put "analysis" in quotation marks. I will drop them in what follows.) After all, as far as an explanation of

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[67] Adams (Adams 1979, p. 7-9) suggests the following for basic qualitative properties. He claims that it is sufficient for a property *P* to be qualitative that a) it is not a haecceity, nor equivalent to one (provided, I should add, the haecceity is not itself qualitative); b) it is not a property of being related to an individual or to its haecceity (with the same proviso as above); and c) it is not related to a set whose members are individuals, or that has members whose members are individuals, or that has members whose members are sets whose members contain individuals, etc. He claims that *P* is qualitative also if it can be constructed (with the help of logical operations) out of qualitative property.

multilocation is concerned, I am more interested in explaining the fact that *h* is instantiated at multiple regions than in specifying the nature of haecceities in general.

The analysis of haecceities I offer consists in the following simple claim:

A haecceity is uniquely captured by a group *B* of universals.

In order to understand the proposed analysis, it will be useful to introduce in very broad outline the bundle theory of material objects. I model the analysis of haecceities on this theory, and so it will be discussed alongside the proposed analysis. For this purpose, I assume that the bundle theory is true. I will discuss it further in Chapter 5.

3.1 Bundle Theories and Haecceities

Typically, the bundle theory of material objects is given the following gloss:

(A) Objects are identical to bundles of co-instantiated properties.[68]

Never mind for the moment the identity relation holding between the object and its bundle, nor the co-instantiation relation. I will come back to the issues they raise in Chapter 5 (section 3). I prefer not to go into too much detail in characterizing what bundles of properties are. It suffices for now to say that I think of them as aggregates of properties. In the next chapter, it will become clear that I take bundles to be mereological fusions of properties (or universals). But I will not discuss this point in the present chapter. Instead, let us note that (A) entails that a material object is *uniquely* captured by

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[68] See (van Cleve 1985, p. 102-104) for the same claim minus the identity relation. He holds that the bundle theory eliminates particulars, and does not make any claim about the identity of particulars to bundles. See also (Rodriguez-Pereyra 2004) for a version of the theory according to which material objects are constituted of (but not identical to) bundles of universals.

a bundle of properties, in the sense that the material object is nothing more than the bundle itself.[69]

3.1.1 *Haecceities Uniquely Captured by Universals*

In the context of the bundle of universals theory of material objects, bundles of universals *capture* objects just in case the objects are identical to them. In the context of my analysis of haecceity, that is *not* what "capture" means. For haecceities are not identical to the groups of universals that capture them. Instead, what I mean when I say that a group *captures* a haecceity *h* is nothing more than the fact that the universals in the group allow us to derive necessary and sufficient conditions for the instantiation of *h*. The fact that this "capture" relation is vague is not really an issue. As I said, here I am interested in the *instantiation* of haecceities, not primarily in their nature.

When I say that haecceities are *uniquely* captured by universals, I mean to claim the conjunction of the following two claims, which are *desiderata* of the proposed analysis of haecceities. (The conjunction explains the meaning I attach to "uniquely," not to "capture.")

(a) There is no other group *B'* of universals that captures *h*.

(b) There is no other haecceity *h'* that is captured by the group *B* of universals.

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[69] Versions of the bundle of universals theory of material objects have been offered by Bertrand Russell (Russell 1962 (1940)), John O'Leary Hawthorne (O'Leary-Hawthorne 1995), and Hawthorne and J.A. Cover (J. O. L. Hawthorne and Cover 1998). Gonzalo Rodriguez-Pereyra (Rodriguez-Pereyra 2004) has also examined such theories. Some bundle theorists (see (Casullo 1988; J. O. L. Hawthorne and Cover 1998)) are also perdurance theorists. In their case, it is the temporal parts of objects that are uniquely captured by bundles of properties, and not the persisting objects itself. In any case, it is this characteristic of the bundle theory, i.e., that material objects are *uniquely captured* by bundles of properties, that I aim to keep in my analysis of haecceities.

There is an obvious parallel between the bundle theory of material objects and my analysis of haecceities. (A) claims that material objects are identical to bundles of universals. From the identity relation and the uniqueness of bundles of universals, it results that no two bundles can capture the same material objects, and that the same bundle cannot capture more than one object. Indeed, since there could be no *distinct* bundles of the very same universals, an object that is identical to that bundle is necessarily identical to it (because of the necessity of identity). Because identity is transitive, the object cannot also be identical to a distinct bundle. So an object cannot be captured by more than one bundle. Similarly, and for the same reasons, there could be no two distinct material objects that are identical to the same bundle. Incidentally, that is precisely what Leibniz's Identity of Indiscernibles (LII) claims.

(LII) $(\forall x)(\forall y)(\forall P)((Px \leftrightarrow Py) \rightarrow x = y)$[70]

Objects that instantiate exactly the same universals are identical. Each of these consequences of (A) corresponds, respectively, to (a) and (b) above.

Different explanations must be given of (a) and of (b), and each involves a version of (LII) to some degree. The principle is controversial, but let us put that aside for the moment. I will discuss the issues it raises in section 4 of this chapter. My explanation of why (a) holds appeals to the bundle theory of material objects, and it appeals indirectly to (LII) (since the theory implies it).

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[70]The fact that the bundle theory entails (LII) is recognized by many. See, for instance, (van Cleve 1985). Gonzalo Rodriguez-Pereyra (Rodriguez-Pereyra 2004) rejects it, however. Note also that (LII) is the converse of (II) introduced in the first chapter (section 2.2).

Here is my explanation of (a). Suppose the bundle theory of material objects and take the variable *P* in (LII) to range over relativized properties. That is to say, the properties (LII) is about are the properties in which figure the regions at which *they* are instantiated.[71] Now, suppose we have an object *o* that instantiates the universals *D* at r_1 and *C* at r_2. The bundle theory has *o* identical to the bundle [*D* at r_1; *C* at r_2].[72] Abstract the universals from the regions in the latter bundle. You get the group *B* of universals [*D*, *C*]. *B* captures the property of being identical to *o*, i.e., *o*'s haecceity *h*, in a way that remains to be specified. But note that there is no other group that can be associated with *h*. Of course, the explanation really supports (a), if we suppose that the resulting group includes all the universals *o* instantiates at all regions at which it is located, as well as all the universals it *could* instantiate. The universals in *B*, I stress, are not relativized to regions or worlds. In any case, (a) has been shown to follow from the bundle theory, as well as some operation of abstraction that I will not discuss in this dissertation.

An explanation of why (b) holds is more controversial. (b) claims that a group of universals that captures a haecceity captures no more than one haecceity. Suppose that an object *o* instantiates many universals throughout its actual and possible careers. Take all of these universals, and call "*Z*" the group they compose. And let us make the following plausible assumption (PA):

> (PA) No objects distinct from *o* could come to instantiate exactly and only all the universals in *Z*.

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[71] I take 'relativized properties' to be neutral among the many different views of property instantiation at a region. I will come back to two such views in Chapter 5.

[72] Here and in the rest of the dissertation, I use brackets "[" and "]" in order to refer to bundles or groups or subgroups of properties or of universals.

No objects other than *o* instantiate exactly and only the same universals as *o throughout its actual and possible careers*. Once this assumption is on board, it results that a group of universals that captures a haecceity *h* captures no other haecceity. To take the example of group *B* above, it follows from the plausible assumption (PA) that it is in relation to *h* only. The explanation of why (b) holds is of course more convincing when we realize that the group of universals that captures the haecceity of a material object would contain many more universals than *B* does.

(PA) is reminiscent of Leibniz's take on his Identity of Indiscernibles. He (Leibniz 1991) claims that an individual substance contains everything that happened or will happen to it.[73] Leibniz makes clear that his principle is about all the properties an object comes to instantiate throughout its life. The difference between the latter and (LII) above is that (LII), or so I take it, ranges over all the universals the object instantiates in its actual *and possible* careers. For Leibniz, individuals are bound to one world only, whereas I presuppose that objects exist in more than one world. I will have more to say about this supposition at the very end of this chapter. (PA) is justified by an interpretation of (LII) that entails that two objects could *not* come to instantiate the same universals throughout their actual and possible lives, i.e., regardless of the places or worlds where they instantiate it. In section 4 of this chapter, I will consider the relation between (LII)

[73] See (Leibniz 1991, p. 8): "…we can say that the nature of an individual substance or of a complete being is to have a notion so complete that it is sufficient to contain and to allow us to deduce from it all the predicates of the subject to which this notion is attributed." And later: "God, seeing in Alexander's individual notion or haecceity, sees in it at the same time the basis and reason for all the predicates which can be said truly of him, for example, that he vanquished Darius and Porus… Thus…we can say that from all time in Alexander's soul there are vestiges of everything that has happened to him and marks of everything that will happen to him…" It goes without saying that, in my view, God is out of the picture.

and typical counterexamples voiced against Leibniz's Identity of Indiscernibles in more detail.

Two points need to be addressed in order to substantiate (a) and (b) further. First, a group *B* that contains all the universals an object ever has, and could have had, contains incompatible universals. So, a group of *incompatible* universals uniquely captures a haecceity. Is that a problematic consequence of the proposed analysis of haecceities? The second point is that I supposed (LII) to be true. (LII) is far from being obvious; some variant of it is generally taken to be false. I turn now to the first issue. I will discuss the second one in section 4 of the present chapter.

3.1.2 *Incompatible Properties*

B contains universals that are not relativized to the regions of space where they are instantiated. In consequence, *B* contains incompatible universals even though the object, the haecceity of which is uniquely captured by *B*, never instantiates incompatible universals at the same region. Should *o* be blue at *r* and red at *r'*, *B* would contain the incompatible properties of *being red* and of *being blue*. I should make clear that *B* would not be, in my view, inconsistent. I reject "negative" properties, i.e., properties such as *not being red* and *not being blue*. So, one cannot claim that the property of *being red* is somehow related to that of *not being blue*, and that the latter is included in *B*. Given the scope of this dissertation, I will give no argument against negative properties.[74] I will simply stipulate that there are none. My motivation, of course, is to make it so that groups

[74] See (Armstrong 1978b, p. 23-29) for such arguments.

like *B* are not inconsistent. Nevertheless, one could question the adequacy of an analysis that appeals to incompatible properties.

The charge has some validity, but fails to undermine, in my view, the adequacy of the analysis. For regardless of why *P* and *Q* are incompatible, the important point to note is that their incompatibility is not relevant given the proposed analysis of haecceities. For we are interested in an analysis of *h* insofar as it gives us necessary and sufficient conditions for the instantiation of *h*. A bundle's containing the incompatible *P* and *Q* would prevent that to happen only if it would entail that *h*'s instantiation at a region *r* involves the instantiation of both *P* and *Q at r*. We will see that *that* is not the case in Chapter 5, where I give an account of property instantiation. So the fact that the bundle contains incompatible universals is not problematic after all.

For similar reasons, the fact that *B* contains incompatible universals is not a problem for the intended interpretation of (LII)—that is, (LII) construed as ranging over all the universals an objects instantiate throughout its actual and possible careers. (I supported (PA) above by relying on that interpretation of (LII).) *Prima facie*, (LII) claims that two objects that have exactly and only the same universals are identical. Because no objects instantiate incompatible universals, it would seem that the intended interpretation of (LII) is vacuously true. Here again, the problem arises because we are supposing that objects cannot instantiate incompatible properties *simpliciter*, whereas we should only be assuming that they cannot instantiate them at one region or another. Once we contend that objects instantiate universals *at one region or another*, the notion of instantiating one property *simpliciter* is up for grabs. I will simply stipulate that an object instantiates a property *simpliciter* just in case it instantiates the property at one region or another. My motivation for this stipulation is simply to make sure that (LII), under the intended interpretation, is not vacuously true. I should note that my stipulation on having a property *simpliciter* is not typical. Most philosophers, and notably perdurance theorists, think that an object has a property *simpliciter* just in case it enters in a binary instantiation relation with a property.

The second issue that I wanted to discuss is my reliance on (LII) in my analysis of haecceities. I now turn to it.

4. Haecceities and (LII): Black's World

The explanations given above of why (a) and (b) hold rely on the truth of (LII), either directly or indirectly. What grounds do I have to suppose that (LII) is true? I will focus on the interpretation I gave to the principle in the explanation of (b). I then supposed it impossible that distinct objects could instantiate exactly and only the same universals throughout their actual and possible careers.

Typically, (LII) is understood as the claim that there can be no distinct indiscernibles within one world. Counterexamples to Leibniz's principle are therefore directed against the version of the principle, call it "$(LII)_{bw}$," that ranges over the universals objects instantiate in one world.[75] The most famous counterexample to $(LII)_{bw}$ is given by Max Black (Black 1952, p. 156ff.). He supposes that there is a world in which there are two distinct but qualitatively indiscernible spheres: both spheres instantiate exactly the same *qualitative* properties. Black (Black 1952, p. 163) concludes that such a world is possible simply because we cannot derive a contradiction from the supposition

[75] This is not to say that the objects must exist only in one world. I do not make that assumption, and I will come back to it below.

that the spheres are qualitatively indiscernible. It is clear from what Black says that he does not aim to prove that the latter supposition is true. He merely wants to show that the supposition is logically possible, i.e., that it does not imply a contradiction. In recent literature, however, it is generally agreed that Black had to make the extra step to prove that his world is possible, and not simply to assume it is so. (See (Hacking 1975), on which more below.) Within that perspective, we need reasons to think that Black's spheres are really distinct. We now take it that a criterion for the distinctness of the spheres is given by the fact that they are *spatially separated*. (See (Adams 1979; Diekemper 2009; Hacking 1975).) Black's world, if possible, is a counterexample to $(LII)_{bw}$.

In addition, it is taken to be the main motivation for accepting *primitive*, *non-qualitative* haecceities. (See (Adams 1979; Diekemper 2009; Swinburne 1995).)[76] To see this, suppose that all of the spheres' properties are universals. Worlds like those of Black are symmetrical, and so any relational properties one sphere may have is also had by the other. The spheres will therefore share all the same (relational and nonrelational) universals. Their qualitative indiscernibility fails to secure their identity, and in that sense $(LII)_{bw}$ fails.[77] But since the spheres are distinct, their haecceities are not reducible to universals, i.e., to any of the spheres' qualitative properties. Their haecceities are therefore non-qualitative properties (by (C) above). Because the spheres have, by hypothesis, no other properties than qualitative ones (save their haecceities, of course),

[76] But see (Rosenkrantz 1993) for another motivation, namely, that they best explain the diversity of particulars.

[77] We should keep in mind that I suppose $(LII)_{bw}$ to range over universals. Haecceities are not universals, and so $(LII)_{bw}$ does not range over them. If it did, $(LII)_{bw}$ would be a trivial principle. For it would only claim that objects that are identical are identical. And this is not a metaphysically substantial claim. (See (Rodriguez-Pereyra 2006), on which more below, for a discussion of trivializing properties.)

their haecceities cannot be reduced to other, non-qualitative properties of the spheres. They are, therefore, primitive properties of the spheres. And since the metaphysical structure of objects is the same in all possible worlds (or so I assume following (Adams 1979, p. 13)), haecceities are primitive, non-qualitative properties of *any* material objects (actual or possible).

What is the relation between (LII) and $(\text{LII})_{\text{bw}}$? It is clear that (LII) does not entail $(\text{LII})_{\text{bw}}$, since it is consistent with (LII) that some objects could be indiscernible in some worlds (though not in *all* worlds). However, if $(\text{LII})_{\text{bw}}$ is true, then we have some reasons to think that (LII) is true also. Or, at least, we can have these reasons if we suppose that objects can exist in more than one world. When I say that I assume that an object can exist in more than one possible world, I am assuming that the relation between an object in one world and this same object in another world is that of strict identity. As I said above, I will come back to this assumption at the very end of the present chapter. Assuming that objects exist in more than one world allows us to take $(\text{LII})_{\text{bw}}$ to be true in all worlds in which a certain object *o* exists. This, in turn, undermines any motivation for the view that, though there are no indiscernible in any world, there could be another object that comes to instantiate exactly and only the same universals as *o* instantiates throughout its actual and possible careers.[78] As a result, $(\text{LII})_{\text{bw}}$ constitutes a good

[78] (LII), taken to range over all the universals objects come to instantiate throughout their actual and possible lives, rules out the following case. Suppose worlds *w* and *w'*. In both of these, there are two balls, one red and one blue. Call *w*'s blue ball "bb_w" and its red ball "br_w," and suppose that bb_w = *w'*'s red ball and that br_w = *w'*'s blue ball. Finally, let us say that there are no other worlds. Then, (LII_{bw}) is true since, by hypothesis, neither *w* nor *w'* contain distinct indiscernibles. And yet, both balls instantiate the same universals throughout their respective possible and actual lives. So (LII) is false. Nevertheless, I still think that the view that (LII_{bw}) is true in all worlds gives credence to the view that (LII) is true. For anybody who thinks that there are no distinct indiscernibles in any world whatsoever (anyone who takes (LII_{bw}) to be true) would be in a rather weak position to argue that there could be distinct objects that instantiate exactly and only the same universals throughout their actual and possible lives.

motivation for accepting (LII). And a counterexample to $(LII)_{bw}$ would not necessarily make (LII) false, but it would undermine a very good motivation and support for it.

I claimed in section 2.2.2 above that haecceities are generally taken to be non-qualitative properties, and I will give an argument for that view below. I should now say that I do not take them to be primitive properties of objects. One may think that, because a haecceity is uniquely captured by a group of universals, it is *not* a primitive property. But I aim only to offer necessary and sufficient conditions for the instantiation of a haecceity. It is clear, then, that the *instantiation* of a haecceity is *not* primitive. Although it is true that the primitiveness of a property *H* is not obviously decided by an analysis that shows that the instantiation of *H* is not primitive, I will nevertheless argue below that haecceities are equivalent (or reducible) to some other, non-qualitative properties of the spheres. So while non-qualitative, haecceities are *not* primitive.

I have made myself face some sort of dilemma. On the one hand, I offer my "analysis" of haecceities on the basis that (LII) is true. A good motivation and support for this is $(LII)_{bw}$, but I cannot argue for the view that haecceities are properties of objects without accepting Black's world. The latter, in turn, is a counterexample to $(LII)_{bw}$. And so it undermines the very motivation I have to accept (LII). But the dilemma is not really a dead end. As I said above, it is consistent with accepting (LII) that $(LII)_{bw}$ be false. But it leaves (LII) unsupported. My way out of this dilemma is the following argument that I will articulate in the remainder of this chapter. I will show that I ought to accept Black's world as a counterexample to $(LII)_{bw}$. In so doing, however, I will endorse the best argument (or motivation) there is for recognizing haecceities. To the extent that (LII) does not serve to establish the fact that haecceities are properties, I am free to appeal to it in order to say what they are. For (LII) is not put to use in order to elucidate what properties objects have. It serves to analyze one such property. And there lies my motivation for relying on (LII): without it, I would not be able to say anything interesting about haecceities.

Let us examine how both sides of the dilemma can be made to represent true alternatives. The goal of the resulting discussion will be to demonstrate that we have good grounds for accepting (LII) in the context of an "analysis" of haecceities even though we reject $(\text{LII})_{bw}$.

4.1 Horns of the Dilemma

Black's world uncovers a substantial metaphysical truth provided that it is meant as a counterexample to a *non-trivial* $(\text{LII})_{bw}$. Joseph Diekemper (Diekemper 2009, p. 258) suggests that $(\text{LII})_{bw}$ becomes trivial as soon as it ranges over non-qualitative properties. Of course, the non-qualitative properties he has in mind are the haecceities of the objects $(\text{LII})_{bw}$ is about. And as far as these haecceities are concerned, he is right in his conclusion about $(\text{LII})_{bw}$; it is trivial to claim that all objects that have the property of *being identical to o* are identical (and identical to *o*). But note that $(\text{LII})_{bw}$ is trivial (at least) when it merely claims that objects that are identical are identical.[79] So any non-qualitative properties that can be analyzed in terms of the haecceities of the objects instantiating them will trivialize $(\text{LII})_{bw}$. This is mirrored in the way we must think of Black's world. For the latter to be a good counterexample to $(\text{LII})_{bw}$, it should not be the

[79] See (Rodriguez-Pereyra 2006) for a discussion of which properties trivialize (LII). His criterion is the following (p.219): a property is trivializing just in case "differing with respect to it is or may be differing numerically." The idea is that concluding to the non-identity of two objects by singling out a trivializing property amounts, in effect, to appealing to the non-identity in order to conclude to it.

case that the only criterion we have to tell the spheres apart is that they have different haecceities. For that is the conclusion we want to reach, and not something we should assume. So, any property that cannot be analyzed in any other way than by appealing to the haecceities of the spheres should not be part of Black's world.

A property the analysis of which appeals to another object's haecceity, a property that will likely be relational, may be acceptable, as long as it cannot be analyzed in terms of the haecceity of the initial object. Rodriguez-Pereyra (Rodriguez-Pereyra 2006, p. 212) gives the examples of properties such as *thinking about a*. The possession of the property *thinking about a* does not secure that the object is, say, *identical to b*, since the property could be shared by many individuals. Should we discover that instantiating *thinking about a* is sufficient for securing the identity of objects, we would make a substantial metaphysical discovery! So $(LII)_{bw}$ is not necessarily made trivial by ranging over non-qualitative properties. It results that a counterexample to it can introduce other non-qualitative properties of objects in order to secure the distinctness of the spheres in Black's world.

This observation is reminiscent of one made by Ian Hacking. He (Hacking 1975) argues that Black's world constitutes a counterexample to $(LII)_{bw}$ only if its construction or description does not already presuppose that $(LII)_{bw}$ is false.[80] In order to construct an appropriate counterexample to $(LII)_{bw}$, one must *argue* that there is a world with two indiscernible individuals, and not simply suppose that such a world is possible. Otherwise, Black's world would represent a *petitio* against $(LII)_{bw}$. Hacking is not interested in arguing against or in favor of $(LII)_{bw}$; he only makes a point about what

[80] Compare with (Black 1952, p. 163).

would really constitute good counterexamples to it. In particular, he is concerned about spatial dispersion being a sufficient condition for the distinctness of the spheres. Obviously, the condition really is sufficient only if we are able to specify the spatial dispersion without relying on the distinctness of the sphere. In other words, we need to show that there is something in Black's world other than the spheres that guarantees that the spheres are spatially dispersed. Otherwise, we would already suppose that $(LII)_{bw}$ is false in constructing Black's world.

Hacking's point relocates the debate about the possibility of Black's world. In order to secure that spatial dispersion allows us to tell the spheres apart, we seek to identify something that ensures that Black's is a world *distinct* from similar worlds that are not counterexamples to $(LII)_{bw}$, and yet would make the description of Black's world true. For a start, let us note that adopting substantivalism about space, as I do, is the first step in making sure that spatial separation is a criterion for the distinctness of the spheres. (See (Hacking 1975, p. 251).)[81] For, then, we can suppose that the locations of the spheres are distinct without having already supposed anything about the distinctness of the spheres.[82] Yet there are worlds in which there is a substantival space and that can be described as worlds in which a sphere is at some distance from a sphere, but in which spatial dispersion does not make the "spheres" distinct. Suppose a world *w'* in which space is a cylinder, on which a sphere is exactly located in such a way that it is spatially separated from itself. (I will say of such a space that it forms a loop, because one can

[81] In what follows, I am overlooking the possibility of multilocation. I merely want to come up with an appropriate counterexample to $(LII)_{bw}$, one that would motivate my appeal to haecceities.

[82] See (Adams 1979, p. 15-16) for the claim that relationalism about space, i.e., the view that space is a relation between objects, could accommodate the spatial dispersion argument; we could simply suppose that objects instantiate primitive spatiotemporal relational properties. I will not pursue this avenue here.

travel from an object to itself through a straight line.) Such a world satisfies the description Black gives to his world *w* (minus the claim that there are two spheres), but it contains only one sphere. Spatial dispersion, therefore, does not ensure in *w'* the distinctness of anything. So, in order to secure the existence of *w*, we need to suppose that *w* and *w'* are distinct worlds. We should suppose that, unlike *w'*, *w* has no looping space. The property of *having a space that is not a loop* is a property of Black's world that distinguishes it from other worlds, and that ensures that spatial dispersion indicates that the spheres are distinct.

Or, rather, it ensures it if we make the further assumption that space is not an object identical to a bundle of universals. Without this supposition, Hacking is wrong to claim that the assumption of substantivalism about space allows us to distinguish worlds such as Black's from worlds that are not counterexamples to $(LII)_{bw}$. John O'Leary-Hawthorne and J.A. Cover (J. O. L. Hawthorne and Cover 1998) argue, precisely, that one can have a bundle of universals theory (BTU) that can account for substantival space. Their argument is a rejoinder to Hawthorne's (O'Leary-Hawthorne 1995) attempt to salvage $(LII)_{bw}$ from putative counterexamples. Hawthorne recognizes that BTU entails $(LII)_{bw}$, but he argues that BTU allows one to interpret Black's world as a world in which there is only one sphere that is spatially separated from itself. This is so because bundles of universals can be instantiated at more than one place at once. So, as a result, BTU allows objects to be exactly located (my terminology) at more than one place at once. That universals are capable of multi-instantiation also allows BTU to accommodate substantivalism about space. According to the theory, space is identical to the bundle composed of *pointhood*, *instanthood,* and of the relations holding between them. The universal *pointhood* stands in the *five-meters-from* relation, say, to itself. Black's world is thus a world in which there is *a* bundle *B* of universals that is identical to one sphere, whose universals stand in the co-instantiation relation to the universal *pointhood*. (Wrongly put, the bundle is co-instantiated at *pointhood* twice over.) And there are no distinct indiscernibles. In other words, when BTU is taken on board, one cannot distinguish Black's world from other worlds that contain only one sphere, even if all worlds contain a non-looping space as an object.

So a counterexample to $(LII)_{bw}$ is a world in which space is an object that is not, in turn, identical to a bundle of universals. On the conception of space I am assuming, there are *distinct* regions of space at which objects are located. And space is not reduced to a bundle of universals. Besides, the thesis of multilocation is interesting provided it claims that objects can be exactly located at regions of space that are *really* distinct. It is much less interesting, and a different issue anyway, how one could account for the multilocation of objects in space on a conception of particulars that have all regions of space identical. I ought, therefore, to recognize Black's world as a counterexample to $(LII)_{bw}$.[83]

Taking Black's world as one that has a substantival space amounts to introducing non-qualitative properties (or, rather, non-qualitative relations) of the spheres that would also secure their distinctness. For each sphere bears a relation to space, and these relations are differentiating characteristics of the spheres. The relations that I have in mind are the (co-)instantiation relations that I discuss in Chapter 5 (section 3.2), i.e. the relations of instantiation indexed to regions of space. The spheres' haecceities are, in

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[83] See (Hawley 2009) for a nice discussion of defenses of $(LII)_{bw}$.

turn, reducible (or equivalent) to the properties the spheres instantiate—which include this instantiation-at-a-region relation. (This claim is an oversimplification, since it overlooks the fact that the spheres exist also in other worlds and instantiate other properties in these. I will have more to say about that below.) Since the instantiation relation is non-qualitative, it follows from (C) above that haecceities are also non-qualitative. It may seem strange to say that the instantiation relation is a property of the spheres. But I assume a bundle theory of material objects. Accordingly, objects are identical to bundles of co-instantiated properties, and so the instantiation relation is in the bundle to which the object is identical. (In Chapter 5, I discuss the foregoing claim about bundles of properties.)

A similar idea can be found in (Rodriguez-Pereyra 2004) where Rodriguez-Pereyra claims that material objects are identical, not to bundles of universals, but to instances of such bundles. He suggests that instances are spatiotemporally individuated, but he does not explain this topic at great length. It is clear, however, that instances of bundles of universals are distinct from such bundles. And so, in his view, objects are *not* identical to bundles.[84] (See (Rodriguez-Pereyra 2004, p. 72) on this point.) But, at least, he takes the instantiation of the universals forming the bundle seriously.

[84] In fact, Rodriguez-Pereyra is concerned with the bundle of universals theory of material objects (BTU). In his view, BTU is not committed to $(LII)_{bw}$. He contends that an advocate of BTU accounts for Black's world by claiming that the same bundle has two distinct instances. Since this way of accounting for Black's world makes BTU true and $(LII)_{bw}$ false, it follows that BTU does not entail $(LII)_{bw}$ after all. Despite the fact that objects are identical to instances of bundles, and not to the bundles themselves, Rodriguez-Pereyra still claims that BTU is the correct theory of objects. He takes BTU to imply simply that objects are *constituted* by bundles of universals, and not that they are *identical* to them. But, then, it comes as no surprise that BTU does not entail $(LII)_{bw}$. There is simply no identity claim in BTU. When the bundle theory is taken to imply $(LII)_{bw}$, it is because it claims that objects are *identical* to bundles of properties.

4.1.1 *Interlude: Black's World as a Motivation for Haecceities?*

I said above that Black's world is a motivation or an argument that supports the view that haecceities are properties of objects. Typically, however, it is thought that Black's world allows this because it shows that haecceities are non-qualitative, *primitive* properties of objects. Considerations about Black's world show that haecceities, so conceived, are indispensable. That is not something I have shown. On the contrary, my discussion of Black's world concluded that haecceities are non-qualitative, *non-primitive* properties. But if they are not primitive, are they really indispensable? More generally, given the way I think about haecceities, can I also use Black's world as a motivation for the latter?

I think that I can, and my argument for that view is given in two steps. The first consists recognizing that Black's world is a motivation for accepting haecceities, and to use it to analyze what kind of properties haecceities are. That is what I have just done. The second step consists in showing that haecceities do some metaphysical work in Black's world, despite the fact that they are not primitive. So, recall that I concluded that the non-qualitative properties of the spheres (other than their haecceities) are the (co)-instantiation relations indexed to the region of space where the spheres are. These, I contended, are differentiating characteristics of the spheres, i.e., ones that ensure that the spheres are distinct. But could it be something that constitutes the *individuality* of the spheres, i.e., a characteristic in virtue of which the spheres are the individuals that they are? The view that would have this question answered in the affirmative would be very strange. For once we recognize that the individuality of the spheres is not qualitative (i.e., that haecceities are non-qualitative, or that there are other objects with the same qualitative properties), we cannot suppose that it is the *instantiation* of qualitative properties that confers individuality to objects. For, as we have just recognized, it is not in virtue of instantiating the same qualitative properties that objects are identical. We cannot suppose, either, that it is the *instantiation-at-a-region* that confers individuality to objects. For it would be very strange to suppose that being at a region is what makes an object the object that it is. In other words, it could have been at other regions. I conclude, therefore, that haecceities, though non-primitive, are not dispensable properties of objects.

4.2 Taking Stock

Let us take stock of the above. It is typical to take Black's world as confronting us with a numerical difference that is not captured by any properties that objects instantiate, qualitative or not. That is what is meant when it is said that Black's world presents us with a difference *solo numero*. This typical take on the world is the starting point for arguments in favor of non-qualitative, primitive haecceities. But Black's world would remain a counterexample to $(LII)_{bw}$ even if other non-qualitative properties of objects were introduced in it. In fact, once we suppose a substantival space that is not reducible to any universals, *there are* non-qualitative properties that account for the distinctness of the spheres. It results that a counterexample to $(LII)_{bw}$, i.e., Leibniz's Identity of Indiscernibles that ranges over qualitative properties which objects instantiate within a world, brings into the picture a non-qualitative property as a differentiating property of the spheres. It is allowed to rely on these properties since the properties do not trivialize a version of (LII) that ranges over them. Finally, they secure the fact that the haecceities of the spheres are non-qualitative.

4.3 A Dilemma?

So I recognize the possibility of Black's world, namely, a world in which there are two qualitatively indiscernible, spatially separated spheres, and in which space is a particular that is not, in turn, reducible to any universals. The apparent dilemma I mentioned above can now be formulated more precisely.

The first horn is

> accepting $(LII)_{bw}$ (and so (LII));
> offering the proposed analysis of haecceities;
> but claiming that haecceities are qualitative.

The second is

> rejecting $(LII)_{bw}$;
> claiming that haecceities are non-qualitative;
> and leaving (LII), as well as the proposed analysis of haecceities, unsupported.

I think that the dilemma is only apparent.

Black's world is a counterexample to $(LII)_{bw}$. But we should note that $(LII)_{bw}$ finds a counterexample in Black's world because it fails to range over all of the spheres' properties. Both spheres are discernible if we take into account their non-qualitative properties. So I can accept that Black's world is a counterexample to $(LII)_{bw}$, without necessarily committing myself to the view that the only differentiating properties of objects are their haecceities. The non-qualitative properties of the spheres I claimed were differentiating characteristics are the *instantiation-at-a-region* relations. If $(LII)_{bw}$ ranges over all properties of the spheres, and still fails to establish their identity because of the instantiation relation, then $(LII)_{bw}$ is simply not a good principle about the identity of objects. And, indeed, why should it be, since it is not a principle about the instantiation of properties? So, a working principle that respect the spirit of $(LII)_{bw}$ takes these instantiation relations into question. Granted, there are many ways to do so, some of which consist in reformulating $(LII)_{bw}$ so that it also quantifies on the regions where properties are instantiated. (R-LII) below is just such a principle. Or one could take $(LII)_{bw}$ to range over qualitative and non-qualitative properties of objects.

So I grant that $(LII)_{bw}$ is false. And I cannot rely on it to support (LII). But here is the way out of the dilemma. The reasons why $(LII)_{bw}$ fails do not apply to and are *not valid reasons* for (LII) to fail. This can be seen in two ways. First, as I have already noted, it is simply not the case that I can take the failure of $(LII)_{bw}$ in some worlds to be generalizable to its failure in all worlds—a failure that would perhaps give us reasons to say that (LII) is false. Indeed, (LII) does not imply $(LII)_{bw}$. Second, and more convincingly, the failure of $(LII)_{bw}$ is a motivation, and even an argument, for recognizing haecceities as properties of objects. I have said that $(LII)_{bw}$ fails because it is not a principle about property instantiation at one region or another, and I interpreted it as failing to range over all the properties of the object. (LII) is also not a principle about property instantiation, and it does not need to be. For it is meant to supply a group of properties that would, precisely, give the conditions under which haecceities enter into the co-instantiation relation with universals. The need for such an account is by itself a good motivation to appeal to (LII). In other words, the proposed analysis should satisfy the two desiderata (a) and (b) mentioned above if it is to say anything informative about haecceities. (LII) allows us to have (b), despite the fact that $(LII)_{bw}$ fails. And so my appeal to (LII) is correctly motivated.

Perhaps a problem derivative from the dilemma concerns the following. There is a tension between the proposed analysis of haecceities and the fact that the latter are non-qualitative and non-primitive properties of objects. For I claimed that, in Black's world, the haecceities of the spheres are necessarily equivalent to both the qualitative and non-qualitative properties of the spheres. If that is so, however, then the spheres' haecceities could be reduced to these properties of the spheres. And yet, the proposed analysis captures haecceities by appealing to properties that are situated beyond Black's world. To be exact, it entails that the very same haecceities could be instantiated with a bunch of *other* universals. If the reductive story I just gave about the haecceities of the spheres in Black's world is to hold water, the very same haecceity would have to be captured by distinct groups of universals. And that is prevented by the proposed analysis; no haecceities can be captured by distinct groups of universals.

In fact, I am tempted to think that something that looks like a reductive analysis of haecceities can be given with the following variant of (LII).

(R-LII) $(\forall x)(\forall y)(\forall r)(\forall P)((P_r x \leftrightarrow P_r y) \rightarrow x = y)$

Here is how I read (R-LII):

All objects that instantiate the same universals at the same regions are identical.

(I leave aside possible lives of objects for simplicity of expression, but I will take them into account in my argument below). The group of appropriately relativized properties is, I am tempted to admit, necessarily equivalent to the haecceity of the object. (And the group of universals that result from abstracting them from the regions or worlds where they are instantiated captures the haecceity.) In Black's world, I was not supposing that the spheres were exactly located at other regions of space. And so, there was only one region to which the properties were relativized. Now, I did say that I thought of the spheres as existing in other worlds as well. But if that were the case, then their haecceities in Black's world would not, contrary to what I suggested, be reducible or equivalent to qualitative and non-qualitative properties the spheres have *in that world*. (The earlier claim that haecceities would be reducible to the properties of the spheres in Black's world was an oversimplification, made to simplify my account of haecceities.) Yet, they are still non-primitive since conjunctions of (non-qualitative) properties (given by (R-LII) appropriately reformulated so as to take into account other possible worlds) are necessarily equivalent to them. The properties are the ones relativized to regions and worlds, and not the universals of the groups that capture haecceities. The relevant properties are, for that reason, non-qualitative.

Before moving on to the next chapter, two precisions should be made. Take (R-LII) above. We could imagine counterexamples to it, for instance a "Black" world in which there are two indiscernible spheres exactly located at the very same region. However, an important issue that such a world would raise is that of the co-location not only of the same kind of objects, but of *indiscernible* objects. Such cases, at least for mundane objects like tables, cars, theater tickets, and teddy bears, are rather implausible. I will analyze cases of co-location in more detail in Chapter 6.

Yet it is a debatable subject whether counterexamples to (R-LII) could be found at the microlevel. Bosons, or so our current physics tell us, would appear to be the kind of "particles" that could be co-located, and it would seem that they could be indistinguishable as far as their qualitative properties (mass, spin, etc.) are concerned.

The issue is at least taken at face value by Jonathan Schaffer (Schaffer 2001, p. 255-256). Let us just note here that whether or not bosons can be co-located in *spacetime* throughout their career is not a pivotal issue for my analysis. For a case of co-location that would jeopardize it would be one in which indiscernible objects are co-located at all regions of space in all possible worlds. My analysis rules out those cases because they present us with a single group of universals that capture more than one haecceity; the objects share all the universals they come to instantiate throughout their actual and possible careers.

The same concern arises in Black's world, with $(LII)_{bw}$. For the spheres to have different haecceities (as they should, since they are distinct), the groups of universals that capture their haecceities must be distinct. Now, suppose that the spheres exist *only* in Black's world. Then, the same group of universals captures distinct haecceities, and the proposed analysis fails (or, at least, has its scope reduced). A way out is to deny that objects can exist only in one world. I already acknowledged that I suppose strict transworld identity, and I am inclined to think of the spheres as existing in other worlds. What comes out of this observation is that the proposed analysis of haecceities presupposes strict transworld identity.[85] Although I do not want to discuss the issue here, let us just note that strict transworld identity is a special case of multilocation: it is being exactly located at subregions of the space of each of these worlds. So, if I succeed in explaining multilocation adequately, i.e., in a non-circular way, then my analysis of

[85] See (O'Leary-Hawthorne and Cover 1997) for an explanation of why recognizing haecceities presupposes strict transworld identity. In their view, it is doubtful that objects, such as the spheres in Black's world, that exist only in one world have haecceities. For if there are no distinct *de re* proposition true of them (i.e. if the relation holding between them in Black's world and them in other worlds is not strict identity), then there is no sense we can attach to the claim that one of the sphere is the sphere that *it* is. If they are right, then my 'analysis' of haecceities would be in accordance with their conclusion.

haecceities must rely on a phenomenon that is explainable, though not accepted by many. (See (Leibniz, 1991; Lewis, 1986; O'Leary-Hawthorne & Cover, 1997).)

5. Conclusion

In this chapter, I offered an analysis of haecceities. The analysis claims that haecceities are captured by groups of universals. The main aim of the analysis is, in effect, to offer necessary and sufficient conditions for the instantiation of haecceities. With these conditions, an explanation of multilocation becomes possible, since it suffices to state what they are in order to explain multilocation. I turn to that issue in the next chapter.

Chapter 5

Multilocation

1. Introduction

In Chapter 4, I announced my strategy for explaining the fact that it is possible for material objects to be exactly located at several disjoint regions of spacetime (i.e., the fact that it is possible for material objects to be multilocated in spacetime). The strategy consists in the claim that objects can be multilocated because their haecceities can be instantiated at multiple disjoint regions of spacetime. I here proceed to make clear what kind of explanation I am after, and why multilocation is possible. The goal is to end up with a theory of material objects that makes it apparent that multilocation is a possibility for them. Here again, I shall speak mainly of space rather than spacetime.

2. Two kinds of explanation

How should we attempt to explain the possibility of multilocation? Broadly stated, there are two kinds of explanation one could give of that possibility. One could suppose that an object is exactly located at specific disjoint regions r_1, r_2, r_3,… and attempt to dissolve the (hopefully apparent) problems such a supposition poses. Typically, accounts of endurance theory aim to do just that. They are concerned with explaining an object's difference in properties at one time (or region) and another. Accordingly, they rely on the regions where the object is exactly located, and they relativize properties to these.[86] They succeed in explaining an object's differences in properties because they adequately show how it is possible, coherent, or plausible that objects change their properties over time. Similarly, an explanation of multilocation could be concerned with an object's differences in properties from one region to another, and explain these by appealing to properties relativized to regions of space.

These accounts do *not* intend to give an explanation of the possibility that an object exactly located at *any* region is identical to one that is exactly located at *any* other region. So, the second kind of explanation one could give of the possibility of multilocation would be to offer necessary and/or sufficient conditions for multilocation—or for the instantiation of an object's haecceity. Such an account would presumably appeal to properties instantiated at some region or another, but they would not appeal to the regions where the object is exactly located. In fact, as suggested in the first sentence, they would specify necessary and/or sufficient conditions that should be met at any region whatsoever.

More precisely, there are no free variables for regions of space in an explanation of the second kind unlike in that of the first. Here are more concrete examples of the two kinds of explanation. Suppose that a certain baseball is white at a time *t* (or region *r*), but red at a future time *t'* (or region *r'*). The baseball changed its color property from *t* or *r* to *t'* or *r'*. An explanation of multilocation of the first kind would simply explain how it is possible for the baseball to instantiate both the property of being white and the property

[86] Here and in what follows, the notion of a property that it relativized to a region, time, or world is meant to be neutral among the many ways one could think of an object's having a property at a time. Below, in section 3.1, I will discuss two such ways.

of being red. The problem posed by the instantiation of properties would be motivated by the realization that objects cannot instantiate incompatible properties *simpliciter*. Presumably, the explanation would relativize properties to times. It would then conclude that it is possible for the baseball to be white at *t* or *r* and red at *t'* or *r'*, because the baseball instantiates *whiteness at t or r*, and it instantiates *redness at t' or r'*. Nothing else needs to be said in order to explain the differences in the properties of the same object.

The second kind of explanation would, in addition, attempt to specify necessary and/or sufficient conditions for the fact that it is the *baseball* that is exactly located at *r* and at *r'* (or wholly present both at *t* and at *t'*). These conditions could appeal to nothing more than the baseball's properties (presumably, to many more properties than its color properties). But the explanation would aim to specify these conditions for any regions of space where the baseball could be exactly located (or any times at which it could be wholly present). Most notably, such an explanation would not contain free variables for regions in stating the aforementioned necessary and sufficient conditions.

In this chapter, I seek an explanation of the second kind. Though I am concerned with differences in properties from one region to another, I also intend to explain, through necessary and/or sufficient conditions that contain no free variables for regions, an object's being exactly located at disjoint regions of space. I will formulate my intention in the form of a criterion for an explanation of the possibility of multilocation.

(R) No free variable for regions figure in the explanation of multilocation.

(R) will be important below.

Let me mention one last issue before we move on. I already said in Chapter 4 (section 2.2) how I wish to explain the possibility of multilocation. On my view, an object can be exactly located at disjoint regions of space because its haecceity can be instantiated at these regions. Stated thus, the proposed explanation of multilocation seems obviously circular. It seems that I am attempting to explain multilocation in terms of multi-instantiation. But, clearly, were the explanation left at that, I would have explained almost nothing. I would have explained multilocation in terms of a closely related notion, that of multi-instantiation. These two notions are obviously too closely related for one to serve as an explanation for the other. My explanation of multilocation avoids this problem.

Before offering an explanation of the possibility of multilocation, it will be useful to leave haecceities aside for a moment and discuss material objects.

3. Bundle Theories of Material Objects (Again)

In Chapter 4 (section 3.1), I introduced in very broad outline the bundle theory of material objects. I then said that it consists in the following simple claim:

(A) A material object is identical to a bundle of co-instantiated properties.

In this section, I will make clear what I take bundles of co-instantiated properties to be.

The bundle theory is a rival to two other accounts of material objects, namely, the bare-particular theory and the substratum theory. The former claims that nothing can explain the fact that objects are particulars. It is commonly supposed that there is a propertyless substance that “supports” properties. The substratum theory is similar in supposing that properties are instantiated by something, i.e., a substance, that we cannot describe, because it is propertyless. Both of these theories have a lot in common, but I prefer to distinguish them in view of the fact that some authors claim that we can be acquainted with bare particulars (see (Allaire B. 1963 (1976))) and not with a substratum ((Locke 1689 (2004), p. Book II, Chap. 23), (Armstrong 1978a, p. 105)), and because other philosophers claim that there is a benign version of the substratum theory (see (Denkel 1996)), of which the bundle theory could be an instance (there is no benign version of the bare particular thesis).

No explanation of the possibility of multilocation is forthcoming should I take either the substratum or the bare particular thesis on board. For both theories could account for the latter possibility only by claiming that the bare particular or the substratum is itself exactly located at multiple disjoint regions of space. And this is hardly an illuminating account of the possibility of multilocation. The bundle theory, on the other hand, could have the locative properties of objects tied to the instantiation of the bundles of properties. It offers, therefore, the prospect of explaining the possibility of multilocation by appealing to the instantiation of the properties of the bundle. Although I will not give arguments in favor of the bundle theory of material objects, I will nevertheless offer a bundle theory in what follows.

In an attempt to clarify what bundles of properties are on my view, let us consider three problems typically thought to afflict the traditional bundle theory, i.e., (A) above (see (van Cleve 1985)).[87]

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[87] James van Cleve identifies in fact six problems with the bundle theory. The first three, which I omitted in the body of the text, are problems plaguing what he calls an unsophisticated bundle theory, i.e., a theory (call it "C") that claims that objects are identical to bundles of properties but do not require the latter to be co-instantiated. They are: d) C wrongly allows any bundle or set of properties to be an object (the bundle

(a) (A) precludes things from changing their properties.

(b) (A) makes all of an object's properties essential to it.

(c) (A) entails the Identity of Indiscernibles.

Problems (a) and (b) invite us to take a stand on property instantiation and on the relation of co-instantiation. I discussed in the last chapter the Identity of Indiscernibles. I said in section 4.3 that there was nothing problematic with recognizing such a principle that quantifies over regions of space, in addition to quantifying over properties and objects. The bundle theory I will develop entails that unproblematic principle.

3.1 Bundle Theories and Property Instantiation

Problems (a) and (b) can both be seen to originate in the fact that the relation of identity is necessary and atemporal. For if *o* is identical to a bundle *B* of co-instantiated properties, it follows from the identity relation that it is so identical at all times and in all worlds in which it exists. Hence, objects cannot change their properties, nor could they have had properties different from the ones they actually have.

Problem (a) can be solved by including in *B* all the properties the object ever has, but by relativizing the properties to the regions or times at which the object instantiates them. In that case, an object changes from being blue at *r* (or *t*) to being red at *r'* (or *t'*) just in case *B* contains the relativized property of *being blue at r (or t)*, as well as the relativized property of *being red at r' (or t')*. At all times or regions, the object instantiates all the relativized properties it ever has. Similarly, we can solve (b) by including in *B* all the properties the object actually has, as well as the ones it could have

constituted of the properties of being an alligator and being purple is, intuitively speaking, not an object); e) C entails that objects are eternal and necessary beings, since properties are eternal and necessary; f) C wrongly analyzes the relation of instantiation.

had. Here, the properties are relativized to worlds. In that case, all the properties in *B* are essential to the object, but in an innocuous way. The object's being red could be said to be contingent just in case *B* contains color properties that are relativized to distinct worlds, in addition to the property of *being red in the actual world*. The object instantiates in all worlds *all* the relativized properties that are in *B*. The solutions to both problems do not have *B* contain incompatible properties. The properties of being completely red and completely yellow are incompatible only if they are indexed to the same region, time, or world, but are perfectly compatible when indexed to distinct regions, times, or worlds. The solution entails that B should contain properties that are relativized either to regions of spacetime, or to worlds.[88] How should we think of these relativized properties?

One way to proceed would be to *index* the properties in question to times, regions and/or worlds. This way of understanding relativized properties is known as *indexicalism* (cf. (van Inwagen 1990a, p. 113)). It makes the region and/or world a constituent of the property. So if an object *o* is red at a region *r* and yellow at a disjoint region *r'*, a bundle *B*, to which *o* is identical, includes the properties of *being-red-at-r* and *being-yellow-at-r'*. Indexicalism also works for properties that are relativized to worlds. For instance, an object that is actually red and possibly yellow is identical to a bundle *B* that contains *red-at-w* and *yellow-at-w'* (where I assume that *w* is the actual world). This strategy still claims that all of the properties included in the bundle are co-instantiated: at any one region at which the object is exactly located (or in any one world in which it is), *all* of the

[88] Note that the strategy is disputed by van Cleve (van Cleve 1985, p. 100-101). van Cleve focused on world-indexed properties, but the strategy is similar. Yet the claim that objects are bundles of relativized properties is fairly common.

properties of the bundle are co-instantiated. So indexicalism leaves us with the following bundle theory of material objects. An object *o* that is square and blue at r_1 and round and red at r_2, but one that could have been square and red, is identical to the following bundle

B = [*square-at-*r_1, *blue-at-*r_1, *round-at-*r_2, *red-at-*r_2, *square-at-w*, *red-at-w*]

(where "w" is a constant for a non-actual world). All of the properties of the bundle are instantiated wherever the object is.[89]

Another way to proceed would be to suppose that the instantiation relation is itself indexed to a time. (Peter van Inwagen (van Inwagen 1990a, p. 113) would also be content with this view.)[90] This view is called '*Adverbialism*' (cf. (Haslanger 1989; Johnston 1987)). So an object is blue at a region *r* just in case it has-at-*r*, or instantiates-at-*r*, the property of being blue. The view is called 'adverbialism' because the region, time, or world to which properties are relativized modifies the manner properties are instantiated. And so, it is common to express an object *o*'s instantiating the property of being blue at *r* by claiming that *o* is blue in a *r*ly way. Similarly, an object that is only possibly black is

[89] In his (Benovsky 2008, p. 188), Jiri Benovsky claims that time-indexed properties are tropes on the grounds that properties are, for the indexicalist, time-bound. (*Mutatis mutandis*, of course, for region-indexed properties.) But while I agree that time- or region-indexed properties are bound to a time or a region, there is still a sense in which they can be instantiated at other regions. Indexicalists do not relativize the instantiation relation to a region or time. An object that is yellow at *r*, therefore, instantiates (*simpliciter*) the property of being *yellow-at-r*. And it does so wherever it is exactly located. Furthermore, there is no reason why, in principle, several objects should not instantiate the property of being *yellow-at-r*. Of course, there may be issues about co-location of material objects, but whether or not a property can be instantiated by many objects cannot be a function of whether or not the co-location of objects is possible.

[90] While describing his view, he (p. 113) writes, "[w]hen we say that Descartes was hungry at t_1, we are saying either (take your pick) that this object bore the relation *having* to the time-indexed property *hunger-at-*t_1, or else that it bore the time-indexed relation *having-at-*t_1 to hunger."

black in a *w*ly manner (where *w* is not the actual world). I alluded to this conception of property instantiation in Chapter 3 (section 2.2)[91]

Adverbialism poses an apparent difficulty for the bundle theorist who is also an endurance theorist (or an advocate of multilocation). For take an object *o* that is square and blue at r_1 and round and red at r_2, but that could have been square and red. The object is apparently identical to more than one distinct bundle. Here are the bundles.

B_1 = [*square, blue, instantiation-at-*r_1]
B_2 = [*round, red, instantiation-at-*r_2]
B_3 = [*square, red, instantiation-at-w*]

(where "*w*" is a constant for a non-actual world). Since B_1, B_2, and B_3 are distinct, and since the identity relation is transitive, such a situation is impossible. A perdurance theorist could be an adverbialist, since on her view B_1 and B_2 would be regional parts of the same extended objects. These would therefore be identical to distinct objects. As for B_3, it would be identical to a *counterpart* of *o*, where an object's counterpart is another distinct object that exists in another possible world. (See (Lewis, 1986) for an account of counterpart theory as a theory of metaphysical possibilities.).

In any case, the bundle theory of material objects that I will offer, i.e., the one that makes the possibility of multilocation apparent, is adverbialist. Thus, contrary to what is commonly supposed (see, e.g., (Benovsky 2008, p. 189)), adverbialism about property

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[91] The names 'indexicalism' and 'adverbialism' are used differently by different philosophers. As we saw, Peter van Inwagen would be happy to index either the property or the instantiation relation to a region (or time). And he is recognized as an advocate of indexicalism. On the other hand, indexing the instantiation relation to regions or times is taken to be adverbialism by Hawley (Hawley 2001, p. 21) and is suggested by Benovsky (Benovsky 2008, p. 189), who refers to Johnston (Johnston 1987). Sally Haslanger (Haslanger 1989) is typically taken to be an adverbialist, but she does not suppose that the instantiation relation is relativized to times. Rather, it is the obtaining of a fact that an object instantiates a property that is relativized to a time (or region). In any case, there are many views one can have, but I will take the instantiation relation to be indexed to regions or times.

instantiation is compatible with the conjunction of both endurance theory (or multilocation) and the bundle theory. The theory will be articulated in section 3.3 below.

3.2 Bundle Theory and the Co-Instantiation Relation

Let us turn to the co-instantiation relation. It is also sometimes called the "compresence relation." What is its nature? Some take it to be a primitive relation (see (Russell 1962 (1940))), while others analyze it in terms of other relations. Most notably, Jonathan Schaffer (Schaffer 2001) analyzes it in terms of the co-location relation, while L.A. Paul (Paul 2002, 2006) and Kris McDaniel (McDaniel 2001) analyze it in terms of mereological relations.[92] Schaffer's bundle theory states that material objects are identical to bundles of co-located *tropes*, where tropes are properties that are also particulars. They do not enjoy multilocation. In Schaffer's view, tropes are co-located just in case they occupy the same regions of space. Paul and McDaniel are concerned with the problem of coincidence, with which I will be concerned in Chapter 6, and claim that the co-instantiation relation is, at bottom, the mereological part-whole relation. Like Schaffer, McDaniel thinks of properties as tropes. On the other hand, as I have noted in Chapter 4, Paul and I think of them as immanent universals.

Schaffer thinks that the co-instantiation relation is a co-location relation, where tropes are "co-instantiated" wherever they are co-located. (For the record, tropes are not instantiated since they are particulars.) I recognize locative relations as primitive. So it may be thought that I could follow Schaffer in thinking of the co-instantiation relation in locative terms as well, but – this time – universals would be co-located. But adopting this

[92] Some, e.g., (J. O. L. Hawthorne and Cover 1998), even contend that the claim that ordinary objects are mereological fusions of properties is the typical gloss of the bundle theory.

analysis of the co-instantiation relation would be doomed to fail. Indeed, I seek to explain the possibility of multilocation for material objects in terms of the possibility that their haecceities can be instantiated at multiple disjoint regions. The instantiation of a haecceity by an object, given the view under consideration here, would be nothing more than its being co-located with the object's other properties. So an object's multilocation would be, under such a view, a matter of its haecceity's ability to be located at multiple regions of space. But multilocation, be that of material objects or of haecceities, is what I aim to explain! Hence, my explanation of multilocation would be circular.

As I have made clear in Chapter 4 (section 2.2.1), I follow Paul in thinking of properties as immanent universals (or, at least, of most of them). Like Paul, I think that universals enter into mereological relations in order to form bundles. The mereological relation is what "binds" the properties together. Unlike Paul (Paul 2006, p. 631) and McDaniel, however, I take mereological relations to be *ternary*. And unlike Paul, I contend that the mereological relations that hold between properties are the same as the ones that hold between spatiotemporal parts of objects. (The difference between Paul's mereology and mine are important, but they are best illustrated when talking about the coincidence of material objects. And so I will come back to them in Chapter 6, section 3.) They are the ones I introduced in the second chapter. It is also my contention that universals acquire a location by entering into these relations.

How should we understand these last contentions? I do not suppose that there is a difference of kind between the instantiation and the co-instantiation relation. So, on my view, instantiation itself is understood as the fusion of some properties with other properties. *That* universals can enter into mereological relations I take to be uncontroversial. We know, after all, that mereology can be used to analyze relations that hold between abstract entities—David Lewis (Lewis 1990) shows that set-inclusion can be given a mereological characterization (although sets are abstract particulars and not universals). But that the mereological relations in question are the same as the ones holding between spatiotemporal parts of objects perhaps require some explaining. On my view, the latter mereological relations are the ternary part-whole relations introduced in Chapter 2, and they define the operation of fusion. What needs explaining here is that universals enter into *a ternary regional* mereological relation and acquire a location in so doing.

Typically, as I said in Chapter 4 (section 2.2.1), an advocate of immanent universals contends that the latter acquire a location in virtue of the fact that the objects that instantiate them have a location. Paul (Paul 2006, p. 632) has it that universals acquire a location by being fused with locative properties, i.e., with the relational properties of having a location. I want my immanent universals to acquire a location by being fused-at-a-region. I will defend my view by showing that it is a plausible analysis alongside Paul's. And, though short of stressing the point, I will even claim that my analysis has some advantages over Paul's.

So both Paul and I want to give an interpretation of the immanent realist's claim that

(D) universals acquire a location by being instantiated by objects that have a location.

And should there be an object *o* exactly located at *r* that instantiates the universal *P*, we would both recognize the following as true:

(E) *o* instantiates *P* at *r*.

An adverbialist thinks of the instantiation relation as a region-indexed relation. Paul would account for the truth of (E) by claiming that *P* is fused with, among other things, relational locative properties. The fusion of all properties, including *P*, composes the object. Therefore, (D) is given the following interpretation: *P* has a location because it is part of a bundle of properties that include some relational (locative) properties. On the face of it, the account is quite complex. It involves the part-whole relation and relational (locative) properties that are *not P*. So, whereas the adverbialist merely wants to claim that the instantiation relation is relativized to a region, Paul would have her say that, in fact, whatever plays the role of the instantiation relation is a mereological relation and a relational property (presumably analyzed in terms of another non-mereological relation).

In my view, the truth of (E) is accounted for by the fact that a ternary part-whole relation holds between *P*, the region *r*, and the rest of the bundle. This is what it is for the properties in the bundle to fuse at a region. And so I interpret (D) as the claim that *P* is fused-at-*r* with the object's other properties. In truth, on my view, *P* acquires a location by entering into a region-indexed relation. But since the object is nothing more than the fusion at a region of universals, it still remains true that *P* acquires a location by being instantiated by an object that has a location. What we should note, however, is that I do not distort the adverbialist claim that the instantiation relation is relativized to regions or times. The instantiation relation is accounted for in terms of the ternary part-whole relation with which we are already familiar.

In effect, both my view and Paul's are coherent. Though I think that my view has certain advantages over Paul's, the crucial point of this discussion was to make the claim that universals enter into region-indexed part-whole relations more amenable to our intuition. My intention here has been simply to show that there is a contender to Paul's analysis, and one that remains within the spirit of an adverbialist approach to property instantiation. I will assume the truth of it in what follows.

3.3 An Adverbialist Bundle Theory of Material Objects

I said above that an adverbialist conception of property instantiation poses an apparent threat to the bundle theorist who is also an endurance theorist (or a friend of multilocation). The reason is that adverbialism has objects identical to more than one distinct bundle, which is apparently impossible. In fact, however, the objection was too hastily formulated. For, recall from Chapter 2 that it is perfectly consistent to claim that an object that changes its parts over time can still remain the same object, provided that we employ a ternary part-whole relation. That was indeed the conclusion we reached when evaluating the problem of mereological change in Chapter 2. But the proposed bundle theory is a mereological theory about material objects. And an object's changing its properties from one region to the next is just an object's changing its parts from one region to another.

Let us be more precise. Suppose an object *o* that is square and blue at *r*, but red and round at *r'*. On my view, *o* is identical to the fusion at *r* of *blueness*, *squareness*, and *o*'s haecceity h_o, and the fusion at *r'* of *redness*, *roundness*, and h_o.

(1) *fr*(blueness, squareness, h_o); and

(2) *fr'*(redness, roundness, h_o)

(where "f_r(blueness, squareness, h_o)" should be read as "the fusion at r of blueness, squareness, and h_o"). Despite having distinct parts, the "two" fusions are identical and identical to o. They are simply a manifestation of the fact that o changes its properties from r to r', while remaining selfsame.

I have not yet said anything about the instantiation of haecceities. I will devote the rest of the chapter to this subject. For the moment, I would like to dispel two worries one may have about what I have just said. First, I claimed that (1) and (2) are fusions of different things at different regions. And so, it seems, that they should be distinct, and—therefore—not identical to o. Such a reaction, however, fails to take into consideration that we are working with ternary mereological relations. Recall from Chapter 2 Leibniz's (LM_r) (the mereological instance of the Indiscernibility of Identicals), given again below.

(LM_r) $(x = y \rightarrow (\forall r)(\forall z)(PP_r zx \leftrightarrow PP_r zy))$

It says that identical objects share all of their proper parts at all regions at which they have them. In order to conclude from (1) and (2) that the fusions are not identical, we would need to show that there is a time or region at which both fusions have distinct parts. But since the fusions are tied to distinct regions, that is not something that we can conclude.

The second worry is the following. On my view, material objects are fusions of properties. But material objects are particulars, whereas most of the properties that are fused at a region are universals. What confers particularity to material objects, then? I can only give a brief response to this worry. The bundle of universals theory of objects attempts to reduce material objects to universals. So on *that* view, it could be argued that material objects are *not* particulars. My bundle theory, however, claims that material objects are fusions of universals and haecceities at specific regions. Haecceities are not universals. Although they are properties, they are non-qualitative. And so they *could* be used in order to account for the particularity of material objects. Or I could appeal to the particularity of the regions of space in order to account for the particularity of the objects (other than regions) that are located in it. Material objects would then be fusions of properties, but made into particulars by the regions at which the properties are fused. If none of these solutions end up working, I would have to claim that what exists are properties, and that material objects are reducible to them. I am happy with any of these consequences.

In sum, universals fuse together at a region of space. They form bundles of universals in just such a way, and material objects are identical to these fusions of universals. Furthermore, it is by fusing at a region that universals acquire a location in space. Since the operation of fusion is defined in terms of the ternary part-whole relation, at least on my view, universals enter into the part-whole relations that hold between them, the fusion (or bundle) of universals and a region of space. On this basis, we can formulate necessary and sufficient conditions for the instantiation of haecceities, and arrive at an explanation of multilocation. I explain all this in the next section.

4 Multilocation: The General Picture

In this section, I give the general picture of an explanation of the possibility of multilocation. In the next section, I will fill in the details.

Let us start with a simple case of multilocation: an object *o* is exactly located both at region r_1 and at region r_2 of space. It is completely blue and round at r_1 and completely red and cubical at r_2. o's haecceity h_o is uniquely captured by (and not identical to) the group B containing the following properties: *blueness*, *redness*, *roundness*, and *cubicness*. Not all properties need to be instantiated at a region in order for h_o to be instantiated there. Indeed, our group B includes incompatible properties: o cannot be blue and red, nor can it be cubical and round, at the same region. Instead, the instantiation of sub-groups S of B at some regions bring about the instantiation of h_o at these regions. Of course, for reasons just noted, not *every* sub-group of B brings about the instantiation of h_o. Only maximally consistent sub-groups of B do, where a maximally consistent subgroup is one that includes all compatible properties of B. Put differently, a maximally consistent subgroup of B includes compatible properties of B, to which another of B's properties cannot be added on pain of inconsistency. In our case, the sub-group S_1 = [*blueness*, *roundness*] and S_2 = [*redness*, *cubicness*] both bring about the instantiation of h_o at their respective region.

What brings about the instantiation of h_o at a region is the fusion-at-a-region of the universals in the appropriate sub-group. In other words, the fusion-at-a-region of a sub-group is *sufficient* (though not necessary) for the instantiation of h_o. More formally, and according to our example,

(1) $f_{r1}(S_1) \rightarrow f_{r1}(h_o, S_1)$; and

(2) $f_{r2}(S_2) \rightarrow f_{r2}(h_o, S_2)$,

where "$f_{r1}(S_1)$" should be read as the sentence "the universals in S_1 together fuse-at-r_1," while "$f_{r1}(h_o, S_1)$" should be read as "h_o fuse-at-r_1 with the universals in S_1"—it is not the same reading as above. (And similarly for "$f_{r2}(S_2)$" and "$f_{r2}(h_o, S_2)$.") We can of course generalize on r_1 and r_2 in (1) and (2) to obtain:

(1’) $(\forall r)\ (f_r(S_1) \rightarrow f_r(h_o, S_1))$; and

(2’) $(\forall r)\ (f_r(S_2) \rightarrow f_r(h_o, S_2))$.

Hence, at any region at which the universals in S_1 or in S_2 are fused, h_o is fused with them.

Of course, the operation of fusion is unrestricted on my view. So the operation of fusion is *not* restricted by the sufficient conditions just given. The latter are rather an indication of the fact that h_o is compatible with the universals in S_1 (or S_2). I will have more to say about this issue in section 5.1.1 below.

If haecceities and universals inherit their locations from the objects that instantiate them, how can they be of use in explaining the possibility of multilocation for material objects? In answering this question, the first thing to note is that I seek an explanation of the fact that objects can be exactly located at multiple regions of space, and *not* of the fact that they can be exactly located in space. The fact that objects are in space is not controversial, and does not need explaining, or so I shall suppose given the scope of this dissertation.

The haecceity *h* of an object inherit its location from the object that instantiate it (just like the redness of the blood and the rose inherits its locations from the latter). That is the view about the location of properties that I defended in Chapter 4. The inheritance of location should be understood in the following way. It is when *h* is fused-at-a-region that it acquires a location, and its being fused with the object’s other properties (i.e., the ones it has at that region) constitutes its inheriting its location from that object. What, in effect, determines that it is fused at a certain region *r* instead of another is the fusion at that region of a sub-group *S* of the group *B* of universals that uniquely captures *h*. The fusion-at-*r* of *S* is indeed, I have said, sufficient for the fusion-at-*r* of *h* with *S*. In effect, it is the principles expressing the different sufficient conditions for the instantiation at a region of *h* (like (1') and (2') above) that explains why *h* is instantiated (or can be instantiated) at different disjoint regions. By accounting in this way for the instantiation of *h* at distinct regions, we also account for the identity of what is *exactly located* at these regions (since *h* is the property the object has of being identical to itself). We obtain, therefore, an explanation of (the possibility of) multilocation. Or, at least, that is the idea.

The explanation of multilocation I am suggesting is in accordance with (R) above. For it specifies sufficient conditions for the instantiation of a haecceity for any region whatsoever, and not the ones at which an object is exactly located. In other words, it does not aim merely to explain away the apparent difficulty posed by the differences in properties of the same objects at distinct regions. Rather, it gives conditions, for any region of space, that are sufficient for the instantiation of a haecceity; it explains how it is possible that what is exactly located at one place is identical to what is exactly located at another place. I turn now to specifying in more detail the relation holding between an object's exact location and the instantiation of its haecceity.

4.2 Primacy of Location

In Chapter 3 (section 2), I defended the view that no mereological characterizations of whole presence are forthcoming. Instead, an object's whole presence at a region consists in its being exactly located at a sub-region (proper or improper) of that region. The relation of exact location is taken as primitive, and is *not*—in particular—characterized in mereological terms. On the other hand, the general picture here presented takes properties to be parts of objects, and claims that it is sufficient that a bunch of properties fuse at a region in order for a haecceity to be instantiated at that region. According to the general picture, what makes it so that a certain object is at a region, or so it seems, is the fusing of properties at that region. There is an apparent inconsistency between the claim of Chapter 3 and the general picture. In Chapter 3, I said that an object's exact location could not be explained in mereological terms. Here, I seem to claim that the mereological composition of an object is sufficient for its being exactly located at a region.

The problem arises because I have not yet said something about the regions of space that figure in the *relata* of the part-whole relation holding between universals. In Chapter 2 (section 3.2), I claimed that an object *p* is part of a whole at *p*'s exact location. Since the operation of fusion-at-a-region is defined in terms of the ternary part-whole relation, the region at which distinct parts can be said to fuse together is the resulting object's exact location. The fusion is indeed an improper part of the object. That is a claim I made in Chapter 3 (section 2.2.1). Similarly, there is no reason why the region at which universals fuse together should be different from the object's exact location. And so the region at which universals fuse together is the object's exact location. But note that the regions that figure in *part-whole* relations holding between universals are the objects' exact locations. Universals inherit their location from the object that instantiate them. So when we claim that redness is part of the rose at *r*, *r* can only be the rose's exact location.

The fact that we need to rely on the object's exact location in order to specify the regions at which the universals can be instantiated shows that the inconsistency introduced above is only apparent. We cannot hope to analyze the relation of exact location in terms of the instantiation relation, because we need to rely on the object's exact location in order to determine the region at which universals can be instantiated. So the general picture does not give us sufficient conditions for an object's exact locations in terms of the instantiation of its haecceity, i.e., in terms of the fusion-at-a-region of its haecceity with universals. Our locative relation remains, therefore, a primitive.

Perhaps one could see here a problem about the adequacy of the general picture. For one could argue as follows. Although the relation of exact location is needed while specifying the regions at which properties can be instantiated (or fused), it remains that what explains multilocation is the instantiation (appropriately understood) of haecceities at multiple regions. But I have just said that we can adequately specify the regions at which a haecceity can be instantiated only by relying on the relation of exact location. The crucial point, so the argument goes, is that specifying the regions at which the haecceity is instantiated involves the exact location of the object that is *itself* involved in the haecceity in question. That is to say, the general picture offers an explanation of the multilocation of an object that, in turn, presupposes that *that* object is exactly located at the multiple regions where its haecceity is to be found. The general picture, therefore, is circular and can hardly constitute an explanation of multilocation.

This charge of circularity misses the point. To see this, let us first focus on properties that are not haecceities. The general picture entails that blueness is instantiated at a region r_1 provided that r_1 is the (or an) exact location of a blue object. It also entails that blueness is instantiated at a region r_2 provided that the latter region is the (or an) exact location of a blue object. Nowhere need it be mentioned that the blue object located at r_1 and the one located at r_2 are identical (or not). So a reply to the aforementioned charge can be given at this point. The general picture gives sufficient conditions for the instantiation of (let us say) a haecceity *h* at a region. Of course, the instantiation of *h* cannot figure among the sufficient conditions for its own instantiation, on pain of circularity. Instead, the sufficient conditions appeal to the instantiation of universals the instantiation of which does not presuppose anything about the identity of the object that has them. Specifying the region at which the universals are instantiated, therefore, does not presuppose anything about the identity of the object located there with objects located elsewhere. Put differently, the haecceity *h* inherits (so to speak) the region at which it is instantiated from the instantiation of the appropriate sub-group of universals. The specification of the region at which *h* is instantiated, therefore, does not involve the instantiation of *h* at other regions.

5. Haecceities and Multilocation

I illustrated above the general picture I wish to defend. It is time now to fill in the details. The resulting theory will show how material objects can be exactly located at multiple regions of space (and spacetime).

Let us recall that an object's haecceity h_o is uniquely captured by a group *B* of (perhaps incompatible) universals. The fusion-at-a-region of a maximally consistent sub-group *S* of *B* is sufficient for the instantiation there of h_o. Which sub-group *S* of *B* is sufficient for the instantiation of h_o? The only requirement formulated so far is that *S* be maximally consistent. But there are many sub-bundles of *B* that satisfy this requirement and yet are not sufficient for the instantiation of h_o.

5.1 Sub-Bundles and Haecceities

Two types of problems arise here concerning sub-groups of universals. The first can be seen in the following way. Suppose a group *B* of universals uniquely captures a haecceity *h*. I mentioned earlier that not every sub-group of *B* (that is, the instantiation of their universals) is sufficient for the instantiation of *h*. A red ball at r_1 that turns into a blue cube at r_2 is not a red cube at r_1, even though *redness* and *cubicness* are compatible universals (at r_1) that belong to *B*. In other words, the sub-group *S* that is [*redness*, *cubicness*], although a sub-group of *B*, is not sufficient for the instantiation of our object's haecceity. Hence, if the proposed explanation of multilocation is to work at all, there must be a way to restrict sub-groups so that only the appropriate ones figure in sufficient conditions for the instantiation of *h*. The second problem can be seen if we suppose, in addition to *B*, another group *B'* of universals that uniquely captures another haecceity *h'*. It appears that *B'* cannot share a sub-group *S* with *B*, for in that case *S* would be sufficient for the instantiation of both *h* and *h'*. I find this counterintuitive: it seems that two objects could have, at distinct regions, the same properties regardless of the properties they have at other regions. And in any case, I want that possibility left open, whether or not it is an intuitive one. (Black's world would realize such a possibility.) Whether it turns out impossible should not depend on a theory of haecceities, but on an analysis of possibilities.

Both problems demand that we find some sort of restrictions on the sub-groups of a group of universals. But in looking for such restrictions we are *not* looking for restrictions on the operation of fusion. It is the part-whole relation that I introduced in the second chapter that "binds" universals together at a region, and the axioms and definitions of the mereological system introduced *then* hold *here* as well. I do not wish to make use of two mereological systems in my account of material objects. And regardless of the mereological system I wish to use, restrictions on the operation of fusion that would result in the correct restrictions on sub-groups would be rather *ad hoc*. Why should it be in the nature of the operation of fusion that *redness* and *cubicness* be not fused at a certain region? Similarly, the impossibility of fusing incompatible properties at a region does *not* arise from the nature of the operation of fusion.

The point is apparently worth noting, since Paul (Paul 2006, p. 655) suggests that restrictions on the operation of fusion are necessary in order to prevent the fusion of incompatible properties.[93] In my view, it is the fact that the mereological system includes the axioms of the predicate calculus with the identity predicate that prevents the fusion of incompatible properties. To see this, suppose a view of properties that recognizes negative properties, i.e., properties such as *not standing up*. (That view of properties is not mine.) Then, it is clear that an object cannot instantiate at the same region both the property of *standing up* and the property of *not standing up*. In view of the theory of property instantiation that I am defending, both properties cannot fuse together at the same region *r*. Does that impossibility arise because of some restriction on the operation of fusion? I doubt it. A statement expressing the fusion of contradictory properties would itself be contradictory, for it would appeal to the predicates denoting each property. So

[93] In the footnote 25 of her article, she contemplates the fact that imposing restrictions on the operation of fusion involves countenancing that composition is vague. The point is due to David Lewis (Lewis 1986), and is further developed by Ted Sider (Sider 2001). She admits that she can grant that spatiotemporal composition is unrestricted in order to avoid claiming that spatiotemporal composition is vague, but still maintain that the fusion of properties is restricted. For, she says, “[t]here are no vague intuitions about the (relevant) qualitative fusions [i.e., fusions of properties]: for example, fusions of contradictory properties and the like [such as incompatible properties] are clearly not possible.”

the impossibility of fusing contradictory properties arises because of the fact that mereology contains the axioms of predicate calculus. Mereology would entail a contradiction, if we were to allow fusion of contradictory properties.

The same holds for the fusion of incompatible properties at a region. Typically, one judges of the incompatibility of (say) two properties by noting that they cannot both be instantiated at the same region. Now, it is hard to evaluate what, from a metaphysical point of view, gives rise to such incompatibility. On the one hand, we could claim that properties *P* and *Q* are incompatible *because* they cannot both be instantiated at the same region by the same object. This metaphysical "explanation" of the incompatibility would seem to follow our judgment about *P* and *Q*'s incompatibility. On the other hand, it may well be that *P* and *Q* cannot both be instantiated at the same region by the same object because they are incompatible—I have a preference for the latter way of conceiving of the incompatibility of properties. In both cases, the incompatibility of the properties indicates that there is a certain inconsistency related to the predicates that denote them. For although there is no negative property to which that of *being blue* is related, the predicate that denotes the latter property, i.e., "… is blue," is surely related to the predicate "… is not red." The relation between the two predicates, whatever it may be, makes it so that the sentence "*o* is blue" entails "*o* is not red." So on pain of contradiction, one cannot suppose that incompatible properties are instantiated by the same object. For that reason, our mereological system cannot entail that incompatible properties fuse at the same region. For it would then entail a contradiction, since it contains the axioms of the calculus of predicates.

Simply put, what prevents the fusion of incompatible properties is whatever prevents them from being instantiated at the same region by the same object, not mereology.

I will treat both problems mentioned above as formal problems of the theory I am advocating. In other words, I will be happy with a solution that modifies the formal aspect of the theory, whether or not it is commonsensical.

5.1.1 *First Problem. Elimination of Inappropriate Sub-bundles: Some Precisions on the Sufficiency Condition*

The only criterion I have imposed so far on sub-groups is that they be maximally consistent. Eliminating inappropriate sub-groups does not require imposing any more conditions on them. To see this, suppose a group *B*, composed of universals *A*, *C*, *T*, and *R*, that uniquely captures *h*. *A* and *T* are incompatible, and so are *C* and *R*. I want the instantiation of subgroups [*A*,*C*] and [*T*,*R*] to be sufficient for the instantiation of *h*, but not [*A*,*R*] and [*T*,*C*]. Since the latter two sub-groups contain compatible properties, I cannot rely on their being incompatible in order to exclude them. But I can claim that their fusing with *h* is itself incompatible. For, under the theory I am proposing,

(3) $f_r([A,C]) \rightarrow f_r([A,C],h)$.

Since the object of which *h* is the haecceity is *A* and *C* at *r*, the consequent expresses the fact that compatible properties *h*, *A*, *C*, are fused together at *r*. But the following conditional

(4) $f_r([A,R]) \rightarrow f_r([A,R],h)$

is false, since its consequent expresses the fact that *incompatible* properties, namely *A*, *R*, and *h*, are fused together at *r* on the assumption that *A* and *R* fuse at *r*. That is impossible. And it is so because *h* is the haecceity of an object that is never, or nowhere, *A* and *R*. Since it is possible for *A* and *R* to be instantiated at *r*, it results that (4) is false. And that is the result we wanted.

Let us examine this in more detail. Take again our object *o*. Here are the two sufficient conditions for the instantiation of its haecceity *h*.

(5) $(\forall r)\ (f_r([A,C]) \rightarrow f_r([A,C],h))$; and

(6) $(\forall r)\ (fr([T,R]) \rightarrow fr([T,R],h))$.

The two sufficient conditions disjoin to form a necessary condition for the instantiation of *h*.

(7) $(\forall r)\ (fr(S,h) \rightarrow (f_r([A,C]) \vee fr([T,R])))$

(where "*S*" is a schematic letter for *any* maximally consistent sub-group of *B*). Of course, it follows from (5), (6), and (7) that

(8) $(\forall r)\ (fr(S,h) \leftrightarrow (f_r([A,C]) \vee fr([T,R])))$

My argument to the effect that *h* is incompatible with [*A*,*R*] is the claim that [*A*,*R*] does not figure among the disjuncts that are *both* necessary *and* sufficient for the instantiation of *h* at a region.

Why is that a reason to think that *h* is incompatible with [*A*,*R*]? Suppose we want to claim that [*A*,*R*] fuses at a region with *h*. Since (8) is a schema, where "S" can be substituted for any sub-group of *B*, then we would have to claim that

(9) $(\forall r)\ (fr([A,R],h) \leftrightarrow (f_r([A,C]) \vee fr([T,R])))$.

Now, (9) is *not* inconsistent. But it is a very bizarre claim. Read from left to right, (9) says that it is sufficient that [*A*,*R*] be instantiated at a region with *h* in order for either [*A*,*C*] or [*T*,*R*] to be instantiated there. (9) would not be inconsistent since it would not entail that [A,R] fuses with either [A,C] or [T,R], which is what would be required for it to be inconsistent. Rather, (9) would specify a sufficient condition for the instantiation of sub-groups in terms of another sub-group that is incompatible with anyone of them. And *that* is odd. Read from right to left, (9) says that it is sufficient for either [*A*,*C*] or [*T*,*R*] to be instantiated at a region for [*A*,*R*] and *h* to be instantiated there. Here again, it is a bizarre claim (though a consistent one). The instantiation of a sub-group is sufficient for the instantiation of another sub-group that is incompatible with it. Clearly, therefore, the explanation of multilocation proposed here would have to rule out claims such as (9), while still making it clear why *h* and [*A*,*R*] are incompatible.

We are looking for appropriate restrictions on maximally consistent sub-groups of *B* that would allow us to select only those that are sufficient for the instantiation of *h*. I contend that nothing other than maximal consistence is such a restriction. The schematic (8) allows, without resulting in an inconsistent claim, that "S" be substituted with any maximally consistent sub-group of *B*. However, by restraining the maximally consistent sub-groups that can be substituted for "S" in (8), we arrive at the sought-for selection of the maximally consistent sub-groups of *B*. The idea here is to use (8) as a principle selecting the appropriate sub-groups among *B*, by putting restrictions on what can be substituted for "*S*." The restriction on the substitution in (8) is simple: whenever we substitute "*S*" with a maximally consistent subgroup, the resulting claim together with the sufficient conditions for the instantiation of *h* should not give us any new information about the instantiation of properties at a region. It is clear, then, that (9) is ruled out. For (9) precisely contains new information about the instantiation of *h*, namely, that *h* is instantiated with a sub-group that figures nowhere in the sufficient conditions for its own instantiation.

In other words, (8) succeeds in selecting the subgroups that are compatible with *h* provided that the subgroup substituted for "*S*" is any one of (8)'s disjuncts. One may worry that the restriction on (8) is circular. And one may articulate this worry in two ways. First, the restriction on substitution in (8) relies on the sufficient conditions delivered by the proposed explanation of multilocation. So it seems that the restriction merely restates that the sub-groups that are sufficient for the instantiation of *h* are sufficient for it. And so it would appear to be a poor principle for selecting the appropriate maximally consistent sub-groups of *B*. It would merely be the claim that a maximally consistent sub-group is not sufficient for the instantiation of *h* because it is not among the maximally consistent sub-groups that *are* sufficient for the instantiation of *h*. Articulated in such a way, the worry is misplaced. We arrived at the schematic (8) above by disjoining the sufficient conditions expressed in (5) and (6). So (8) had better not give any new information about the instantiation of *h*. Should it do so, we would have no justification or motivation to cling to it.

The second way to state the worry is the following. When we claim that (8) does not provide any new information as to the instantiation of *h*, we fail to give the *requirements* that sub-groups should meet in order to be sufficient for the instantiation of *h*. The worry arises because, it is thought, the hope was initially to specify such requirements. But that was never the hope! The sufficient conditions are not *conditions* that maximally consistent sub-groups of universals should *satisfy* in order for them to bring about *h*. It is not as though we were given a group of universals, and were asked to select the appropriate subgroups that would correctly represent an object at one region or another. That is in no way part of an explanation of multilocation. And indeed, how could it be? Selecting the sub-groups of universals that correctly represent an object is a task for empirical sciences and theories of possibilities. It is not a task an explanation of the possibility of multilocation should fulfill. As far as multilocation is concerned, we should take it for granted that the world tells us which universals are instantiated by objects at different regions, so that it is not incumbent on us to select those. The sufficient conditions expressed, for instance, by (5) and (6) *say only that the instantiation of h is compatible with that of a sub-group of universals*.

5.1.2 *Second Problem. Uniqueness of Sufficient Conditions*

A solution to the second problem requires that a sub-group should figure in the sufficient condition of only *one* haecceity. The solution to the first problem still does not ensure such uniqueness. Working with the same example, we could introduce another bundle *B'* that contains *A*, *C*, *H*, *P*, and that uniquely captures *h'*. Here, [*A*,*C*] is a sub-bundle of both *B* and *B'*. So the proposed explanation would entail the following:

(10) $f_r([A,C]) \rightarrow f_r([A,C],h)$

(11) $f_r([A,C]) \rightarrow f_r([A,C],h')$

(10) and (11) entails that two objects, of which *h* and *h'* are the respective haecceities, are exactly located at the same region of space. Since no account is given of the possibility of co-location by these conditionals, there does not need to be a problem with such a

possibility. I will be concerned with co-location in the next chapter. Let us put it aside for now.

The problem is rather this. The theory entails the universalizations of (3) and (4), respectively. It entails

(12) $(\forall r)((f_r([A,C]) \rightarrow f_r([A,C],h))$

(13) $(\forall r)((f_r([A,C]) \rightarrow f_r([A,C],h'))$.

And it follows from (12) and (13) that wherever *A* and *C* are fused at a region, they are also fused with *h* and *h'*, respectively. So at any region where we find *A* and *C* fused, we also find *h* and *h'*. But it could very well be that *h* and *h'* are *not* co-located at the region at which *A* and *C* are fused. An object *o*, whose haecceity is *h*, could be *A* and *C* at *r*, while *o'*, whose haecceity is *h'*, is *A* and *C* at *r'*. And yet *o* and *o'* could *fail* to be co-located at any region. Or, more simply, only *o* could be exactly located at *r*, and not *o'*. These cases are ruled out by the theory.

In order to avoid a situation like that described above, we need to make sure that a sufficient condition for the instantiation of a haecceity is unique, i.e., is not a sufficient condition for the instantiation of another haecceity. By hypothesis, *A* and *C* could be fused with either *h* or *h'*. This goes to show that it is not enough of a sufficient condition for the instantiation of a haecceity that universals such as *A* and *C* be instantiated at a region. On the face of it, this is rather obvious. For an object's haecceity is the property it has to be identical with itself. In giving a sufficient condition for *h*, I am, in effect, specifying a criterion for the fact that it is a specific object that is exactly located at a region. But the identity of an object is, at least partly, determined by the properties it instantiates at all of the regions at which it is exactly located. That is a consequence of

Leibniz's Identity of Indiscernibles that was discussed in Chapter 4, and that is appropriately relativized to regions of space. For recall from section 4.3 of Chapter 4 the principle R(-LII), given again below.

(R-LII) $(\forall x)(\forall y)(\forall r)(\forall P)((P_r x \leftrightarrow P_r y) \rightarrow x = y)$

(R-LII), as I said, is an adequate principle. It claims that any objects that share all of their properties at all regions at which they are exactly located are identical. So the identity of objects is determined by the properties they instantiate at all regions (and also worlds) at which they are located. The project of giving sufficient conditions for the instantiation of haecceities by focusing on the properties instantiated at only one region is, therefore, doomed to fail. We need to appeal, then, to how the object is at other regions.

We find ourselves in some sort of a dilemma. I claimed at the beginning of this chapter that I seek a theory of material objects that would offer an explanation of multilocation in which no free variables for regions would figure (condition (R) of section 2 above). And yet, it seems that I am appealing to how the object is at all regions at which it is exactly located in order to determine its identity. But the dilemma is not real. I already took into consideration the properties of the object when specifying the bundle of universals that captures its haecceity. The explanation of multilocation still satisfies the requirement laid out at the beginning of this chapter, since once we have the bundle, no free variables for regions figure in our explanation of the instantiation of the haecceity.

I think the correct way to make a sufficient condition unique to a certain haecceity *h* is to make use of the fact that *h* is uniquely captured by a group *B* of universals. There is no mention in *B* of any region at which an object has a specific property, and yet a mention of *B* in the sufficient condition would render it unique. What I have in mind is to modify the sufficient condition we have been working with so far in the following way (where, again, *A* and *C* are compatible properties that an object, whose haecceity is *h*, has at a region *r*).

(14) $f_r([A,C],[A,C < B]) \rightarrow f_r([A,C],[A,C < B], h)$

where "*A*,*C* < *B*" should be read as "*A*,*C* are properties that belong to the group *B*." I understand it to be a relational property of the sub-group [*A*,*C*], which is, in turn, meant to be analyzed in terms of a relation that holds between the sub-group and the group. I do not wish to take a stand on whether or not groups of universals are set. If they are, then "<" is the set-inclusion relation of set theory.[94] If not, it is simply the "sub-bundle" relation, whatever it is. I am happy, given the scope of this dissertation, with working with an informal understanding of "<."

In adopting (14), I steer away from the idea that the instantiation of a sub-group of *B* is sufficient for the instantiation of the haecceity *h* uniquely captured by *B*. It is rather the instantiation of the sub-group, together with the relational property that the sub-group is a sub-group of *B*, that is sufficient for the instantiation of *h*. (14) captures the fact that there is much more to a sufficient condition of the instantiation of the haecceity of a specific object than the properties the object instantiates at that exact location. Yet, it does not mention the regions at which the properties are had by the object. No free variable for regions is used in (14)'s antecedent.

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[94] It is not set-membership because the relational properties it partly analyzes are properties of sub-groups (or sub-sets), and not of universals. When I say "whether or not groups are sets," I mean to say whether or not we should think of what I have been calling "groups" as sets. I am aware that the notion of a group is not that of a set. There are no empty groups, and no group contains only one member.

One may worry that (14)'s antecedent claims, among other things, that *A* and *C* belong to *B at a region*, since they appear within the scope of the operation of fusion-at-a-region. But we cannot make sense of the claim that something is a sub-group of a group at one region, and not at another. Nor, similarly, could we say that a set is a sub-set of a set *at a region*. The worry arises out of a misreading of (14)'s antecedent, however. The latter takes the atemporal "sub-bundle" relation (or, if one wishes, set-inclusion relation), and claims that the relational property formed out of them is instantiated at a region. Granted, one may feel uneasy claiming that such a relational property is instantiated at one region or another. The worry would be real, however, should the sub-bundle relation be indexed to a region. And *that* is not the case here.

Another worry may be thought to arise. We generalized on (10) and (11) above in order to illustrate the problem I am attempting to solve here. But it seems that we should, similarly, generalize on (14) in order to obtain

(15) $(\forall r)(fr([A,C],[A,C < B]) \rightarrow fr([A,C],[A,C < B], h))$.

Should *h'* be uniquely captured by *B'* that also has [*A*,*C*] as a sub-bundle, we would be led, by parity of reasoning, to accept

(16) $(\forall r)(fr([A,C],[A,C < B']) \rightarrow fr([A,C],[A,C < B'], h'))$.

And, it would seem, we are back to square one, the very problem I set out to resolve.

The worry would miss the point. Though (15) and (16) are both true, there is no guarantee that their antecedent is true. The initial problem arose precisely out of the supposition that an object is both *A* and *C* at a certain region, while another is also *A* and *C* at another region. But I am not claiming that wherever *A* and *C* are fused together, they are also fused with either [*A*,*C* < *B*] or [*A*,*C* < *B’*]. I only claim that their being so fused is sufficient for the instantiation of *h*, or *h’*. Of course, there is nothing that prevents all of these properties, i.e., *A*,*C*, [*A*,*C* < *B*], and [*A*,*C* < *B’*], from being fused together. But the object to which that fusion is identical is neither *h* nor *h’*.

6. Conclusion

The goal of this chapter was to offer an explanation of the possibility of multilocation for material objects. My strategy was to claim that material objects can be exactly located at more than one disjoint region of space in virtue of the fact that their haecceities can be instantiated at more than one region. The bulk of the chapter was devoted to an account of the instantiation of haecceity. In the next chapter, I will analyze the issue of the coincidence of material objects.

Chapter 6

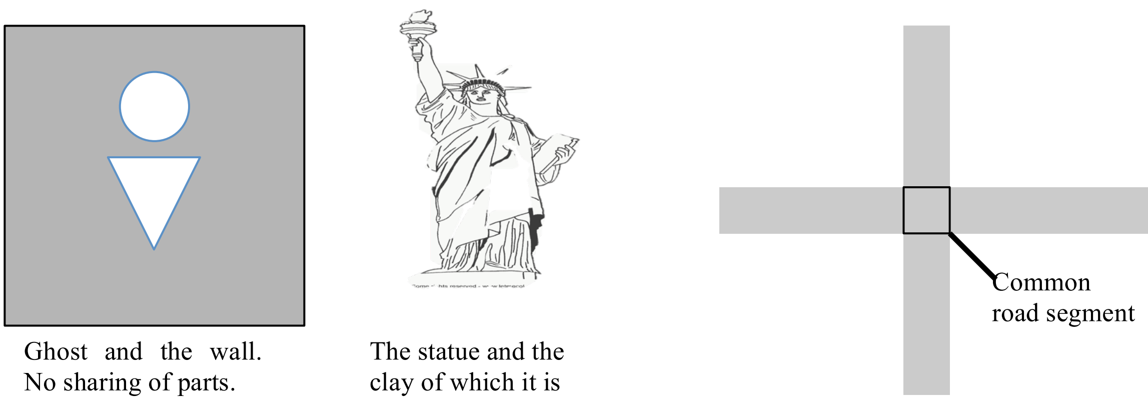
Coincidence

1. Introduction

In the last chapter, I argued that multilocation is possible for material objects given the fact that the instantiation at a region of some properties, conceived as universals, is sufficient for the instantiation at that region of a material object's haecceity. In this chapter, I will be concerned with the problem of coincidence to which endurance theorists and friends of multilocation must attend. I will end the chapter (and the dissertation) by solving some problems brought about by the solution to the problem of coincidence I will offer.

2. Coincidence

Material objects coincide (or are co-located) at a region provided that they are both located, generically or exactly, at a region of space (or spacetime). (Recall from Chapter 1 (Section 3.1.3) that an object is generically located at a region provided the region is not free of it.) A ghost passing through a wall coincides with the wall, since this ghost and a proper part of the wall are exactly located at the same region of space. Two roads coincide at an intersection by sharing there a common road-segment; both roads are generically located at the region where the intersection is. A statue and the clay of which it is made coincide at a region, since they are both generically (indeed, exactly) located there. To be sure, the coincidence of the statue and the clay is unlike that of the ghost and the wall; the ghost interpenetrates the wall without overlapping it, while, *prima facie*, we would be inclined to say that the statue and the clay overlap. In the case of the statue and the clay, it is their overlapping at a region at which they are both exactly located that creates a problem (as we will see). The coincidence of the roads is unproblematic, for it is analyzed away as a case of mere overlap. Figure 2 is an illustration of the three cases of coincidence just introduced. I will not discuss cases like the ghost and the wall.



Ghost and the wall. No sharing of parts.

The statue and the clay of which it is made. Problematic case of overlap.

Coincidence of Roads. Unproblematic case of overlap

Figure 2: Cases of Coincidence

Whether or not there are coincident objects like the statue and the clay is a philosophical issue. We are inclined, by common sense, to think that coincident objects are impossible—or, perhaps, implausible, if we think that the coincidence of the ghost and the wall is vindicated by common sense. John Locke (Locke 1689 (2004), p. 261) wrote this commonsensical inclination into his theory of objects, when he claimed that objects of the same kind cannot be located at the same region of space at the same time. And the view that objects of the same kind cannot coincide is often referred to as Locke's thesis. Presumably, common sense vindicates Locke's thesis because, confronted with the apparent coincidence of two objects at a region, such as the statue and the clay of which it is made, we are *prima facie* tempted to say that there is only one object that is exactly located at that region—we see, after all, only one object.[95] Nevertheless, I will defend the view that there *are* coincident objects, but that they do not pose any problem when cases of coincidence are correctly understood—that is, no philosophical problem beyond the rejection of our pre-theoretical intuition about coincidence.

In this section, I expound some cases of coincidence and explain *what* makes them apparently problematic. Many cases of coincidence are given in the literature, but I will here focus only on two.

2.1 Case 1: The Statue and the Clay

The first, call it "Case 1," is the famous case introduced by Allan Gibbard (Gibbard 1975) of the statue and the lump of clay, to which I alluded earlier. A statue *s* is made of a lump of clay *l*, and both *s* and *l* are exactly located at a region *r*. *s* and *l* differ in their persistence conditions, for the statue, unlike the lump, cannot survive being squashed. It results, from Leibniz's Indiscernibility of Identicals appropriately relativized to regions (II_r) (given below), that *s* and *l* are distinct. Since both are exactly located at the same region, they coincide. Moreover, they are composed of exactly the same constituents; they have the same physical micro-structure. We could say, in order to distinguish cases

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[95] Note here that if we are right in counting only one object at the region at which the coincidence occurs, we are not necessarily led to admit that there are no coincidents. For, as I briefly alluded in Chapter 4 (section 2.2), cases of multilocation already create a problem for counting—they suggest that we do not count by identity. And I speculated that counting objects in space might be related to the regions at which they are exactly located, and not to their identity. An advocate of multilocation could therefore accept that there is only one object at a region *r*, and yet claim that non-identical objects coincide at *r*. Another approach that distinguishes counting from identity is discussed in (Rea 1998, p. 320-322).

where coincidents are composed of the same atoms from cases where there is no overlap, that *s* and *l mereologically coincide*.

(Here is, for the record, the principle (II_r).

(II_r) $(\forall P)(\forall r)\ (x = y \rightarrow (P_r x \leftrightarrow P_r y))$

I reformulated it so as to take regions of space into consideration. Here is how I read (II_r):

> For any property and any region, identical objects share all of their properties at all of the regions at which they have them.

In short, identical objects share all of their properties everywhere. In the argument above, I appealed to (II_r)'s contrapositive.)

There are other ways in which *s* and *l* differ. Presumably, they both have different historical properties at *r*. For, at *r*, *s* has the property of being destroyed in the year 2015 (let us say), but not *l*. Similarly, *l* (let us say) existed well before *s* came into existence. These differences in historical properties find corollaries in cases of multilocation. For suppose that *l*, but not *s*, is also exactly located at *r'*. Then *s* and *l* differ at *r*, even if we grant (pace their modal properties) that they are otherwise indiscernible at *r*. For *l*, but not *s*, has properties at *r'*. Finally, it has been noted by Kit Fine (Fine 2003) that *s* and *l* can differ in what I will call their evaluative properties. For instance, the statue has at *r* the property of *being well made*, though not the lump. But I will focus on modal properties only. The reason for doing so will become clear shortly.

2.2 Massive Coincidence

The other case I want to consider is best understood by analyzing an argument formulated by Achille Varzi (Varzi 2005, 2007a) against some versions of endurance theory, among which mine figures. Varzi claims that a promiscuous endurance theorist, i.e., one that accepts the temporal version of the principle of unrestricted fusion (see Chapter 2, section 2.2), runs the risk of countenancing, in her ontology, the coincidence of an uncountable multitude of material objects. To see this, consider again the temporal version of the principle of (unrestricted) fusion existence (FE_t').

(FE_t') $(\forall t)[(\exists x)(F_t x) \rightarrow (\exists z)((\forall y)(F_t y \rightarrow P_t yz) \wedge (\forall y)(P_t yz \leftrightarrow (\exists w)(F_t w \wedge O_t yw))))]$

Recall that (FE_t') is a schema, and it is read like this:

> The fusion of all of the things that are *F* at any time *t* is an object that has all of these as parts and that has no other parts.

Now, suppose a function *f* whose domain is a set of times and whose image is a set of objects existing at these times. If *a* and *b* both exist at time *t* while *c* and *d* exist at *t'*, then $f(t) = \{a,b\}$ and $f(t') = \{c,d\}$.[96] For proponents of unrestricted fusion, *f* selects an object *o* that exists only at *t* and *t'*; it is composed of *a* and *b* at *t*, and of *c* and *d* at *t'*. Suppose now another function, *f**, such that $f^*(t) = \{a,b\}$ and $f^*(t') = \{e,g\}$. Then, *f** selects an object *o** that also exists only at *t* and at *t'*, that is also composed at *t* of *a* and *b*, but that is composed at *t'* of *e* and *g*. *o* and *o** coincide at *t*, but not at *t'*.

Perhaps the coincidence of both fusions at *t* can be explained away as a case of mere overlap, just like the roads in Figure 2. In that case the fusion $a + b$ at *t* is exactly located at *r* (at *t*) and is a part of both *o* and *o**. It would, therefore, accommodate Locke's thesis. Indeed, the thought goes, there is no more than one object that is exactly located at *r* (at *t*), and that object is part of both *o* and *o**. In fact, that strategy is precisely the one adopted by the perdurance theorist. Should we think of *o* and *o** as being

[96] This way of expressing issues about cross-temporal fusion is due to Ted Sider. See, e.g., (Sider 2001, p. 133ff.) I use it because Varzi does.

composed of temporal parts, then *a* + *b* would, at *t*, be a temporal part of both *o* and *o**. The latter two objects would coincide in exactly the same way that roads coincide at their intersection: no more than one object (the temporal part) is at *t*, and that object is a proper part of both *o* and *o**. In that case, the coincidence of *o* and *o** would not be problematic.

But, as Varzi points out, such a way of thinking of the coincidence of *o* and *o** at *t* is not available to the endurance theorist. For the latter, persisting objects endure, i.e., they are wholly present whenever they exist. So *o*, say, is wholly present at *t* and is also wholly present at *t'*, from which it follows that the relation holding between *o* at *t* and *o* at *t'* is strict identity. The same goes, *mutatis mutandis*, for *o**. Suppose the endurance theorist really wants to claim that *a* + *b* is a common part of *o* and *o** in the same way that roads have a common road-segment at the region where their intersection is—just as the perdurance theorist would say. Because *o* and *o** are wholly present at both *t* and *t'* and since they do not overlap at *t'*, the endurance theorist who desires to eschew any commitment to coincident objects is *prima facie* committed to the following option: she has to claim that the relation of identity is indexed to times. (See (Varzi 2007a, p. 184).)

To see this, take the temporal principle of unrestricted fusion existence (FE_t') and the definition (O1') from Chapter 2 (section 2.2), which is given again below.

(O1') $(\forall t)\,(P_t xy \leftrightarrow (\forall z)(O_t zx \rightarrow O_t zy))$

Recall that (O1') claims that

> if everything that overlaps *x* overlaps at a time *y*, then *x* is part of *y* at that time (and vice versa).

Together with a reluctance to recognize coincident objects at *t*, (FE_t') and (O1') lead to the view that *a* + *b, o*, and *o** are all identical at *t*.[97] Since the latter two objects are distinct, it results that identity is relative to times. Indeed, it is the only way we can accommodate a) that *a* + *b* is a common part of both *o* and *o**, none of which has its parts disjoint from *a* + *b* (not even improper parts); b) that *o* and *o** are both wholly present at *t* and *t'*; and c) that no objects coincide. On this view, *o* and *o** are identical at *t*, but not at *t'*. Of course, the thesis of occasional identity is controversial and unattractive. And I will not defend it.[98] (See (Gallois 1998) for a defense of it.)

Varzi contends that the endurance theorist can avoid occasional identity at the cost of a massive coincidence of material objects. Take again *o* and *o** that are composed of the same parts at *t*, but not at *t'*. And recall that they are both wholly present at *t* and *t'*. *o* and *o** are not identical. And although *o*, *o**, and *a* + *b* are composed of the same parts, i.e., *a* and *b*, there is not one single object in virtue of which we could explain away the

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[97] *a* and *b* exist at *t*, and they are both part at *t* of *o* and of *o**. We are supposing that neither *o* nor *o** has a part disjoint at *t* from either *a* or *b*, so that *a* + *b* is a common part at *t* of *o* and *o**. From the temporally modified principle of unrestricted fusion existence, we get that anything that is a part of *a* or of *b* at *t* is a part of the fusion *a* + *b* at *t,* and that *a* + *b* has no other part at *t*. Everything that overlaps *a* + *b* overlaps *o* at *t*, and vice versa. (*Mutatis mutandis* for *o**.) From (O1') we get that *a* + *b* is a part at *t* of *o*, and *o* is a part at *t* of *a* + *b*. Similarly, *a* + *b* is a part at *t* of *o**, and *o** is a part at *t* of *a* + *b*. We also have that whatever overlaps *o* at *t* overlaps *o** at *t*, and vice versa. And from (O1'), it follows that *o* is a part at *t* of *o'*, and *o'* is a part at *t* of *o*. The strategy I am now analyzing seeks to avoid coincidence. At *t*, we avoid coincidence if we claim that all of *o*, *o**, and *a* + *b* are identical. Since none of them are identical at *t'*, the desired result is obtained by the view that identity is occasional.

[98] Another option available to the endurance theorist who desires to explain coincidence away is mereological constantism, i.e., the view that objects neither gain nor lose parts. (See (R. Chisholm 1976; R. M. Chisholm 1971, 1973) for a defense of mereological essentialism, which entails mereological constantism.) (See, again, (Varzi 2007a, p. 185).) I have encountered the expression "mereological constantism" in (McDaniel 2004, p. 143), but people generally use "mereological essentialism" instead. Since modalities are not mentioned anywhere in the body of the text, I prefer the first expression. According to this view, it is neither *o* nor *o** that persists through time; it is rather the object that is composed, at all times, of (say) *a* and *b*. *o* and *o** are fictional entities, or logical constructions, and are not really entities that populate the world. They are made up of a succession of persisting objects. So *o* is the succession of the fusion *a* + *b*, followed by *c* + *d*. Since, for any persisting object, all the parts it ever has are present whenever the object is present, it is wholly present whenever it exists. (Recall WP2 from section 2.1 of Chapter 3.) As I did in Chapter 3, I will now put mereological constantism aside.

coincidence of *o* and *o** at *t* in the way the perdurance theorist would. In such a case, *o*, *o**, and $a + b$ would be distinct fusions of *a* and *b*. That represents a departure from Classical Extensional Mereology (CEM), the temporal version of which I introduced in Chapter 2 (section 2.2). Our objects *o* and *o** mereologically coincide at *t* in a problematic way. For they are both composed there of exactly the same parts. And Varzi contends that the endurance theorist that accepts the coincidence of *o* and *o** would be caught with a *massive* case of coincidence, since there are countless functions f_i the value of which is identical at *t*, i.e., $f_i(\mathrm{t}) = \{a,b\}$, but which differ at some (or all) other times.

Pace Varzi, cases of massive coincidence do not result from the endurance theorist's adoption of (FE_t'), nor do they result from the adoption of trans-regional unrestricted fusion by friends of multilocation—a principle introduced in Chapter 2, section 3.3 (and given again in the following section).[99] The reason is that an endurance theorist, just like a friend of multilocation, does not need to claim that every persisting object is an enduring object, or that every object composed of parts that are exactly

[99] In his (Varzi 2007a, p. 182-184), Varzi suggests that it is temporally unrestricted fusion that creates a problem for the endurance theorist (see especially page 184), but it is, in fact, the recourse to that mereological principle in an attempt to explain vagueness as a case of semantic indeterminacy. In a nutshell, perdurance theory is particularly well suited to explain vagueness as a case of semantic indeterminacy. For suppose that an object perdures, and say that there is a young cat in front of us that we name "Tibble" at *t*. Suppose, finally, that we place Tibble, at the same time *t*, in a machine that destroys one of its atoms every second. When will Tibble go out of existence? It is in fact a vague matter, for we know that Tibble enters the machine alive, and is destroyed when the machine finishes its routine. But we are at pain to specify the number of atoms, the loss of which constitutes the death of the cat. And yet, it would seem odd to claim that it is the cat itself that is vague. In fact, perdurance theory entails that it is not the cat that is vague. For objects, according to perdurance theory, are fusions of temporal parts, and fusions are not vague objects. The vagueness comes instead from the fact that we fail to select *the* fusion of temporal parts that would be the referent of "Tibble." In other words, there are many fusions of temporal parts, some of which include more temporal parts than others, that are equally good candidates for the referent of "Tibble." Of course, all of these fusions coincide at *t*, for there is at *t* one temporal part that is a proper part of all of them. But the coincidence in that case is akin to the coincidence of the roads that I briefly discussed above. The endurance theorist cannot offer the same kind of solution, for, and that *is* Varzi's argument, she would then be committed to cases of coincidence. All of the equally good candidates would be wholly present at *t*, and – therefore – coincide there.

located at no more than one region needs be multilocated. Actually, the case is clearer if we focus on trans-regional unrestricted fusion. For suppose two distinct objects o_1 and o_2 each exactly located at no more than one region of space, say the disjoint regions r_1 and r_2. The friend of multilocation is free to claim that there is a fusion *o* of o_1 and o_2. But she is also, of course, under no obligation to make the further claim that *o* is exactly located at r_1 and at r_2. I can say that there is a fusion of the computer and the lamp (both of which are on my desk), but it would be quite unnatural to say that I have just acknowledged that there is an object exactly located at two disjoint regions of space. Similarly, an endurance theorist can say that there is an object composed, at instant *t*, of the Eiffel tower and, at instant *t'*, of the head of a goat without, in addition, granting that this object should endure through time. She may agree with the perdurance theorist that the object perdures, and explain away the coincidence, at *t*, of that object with the Eiffel tower just as the perdurance theorist would—as a case of unproblematic coincidence.

So not all functions *f* that select a persisting object need to select an enduring one.[100] As a result, the endurance theorist, or friend of multilocation, does not face *massive* coincidence.

2.3.1 *Case 2: Multilocation and (non-massive) Coincidence*

Although this response is available to the endurance theorist, it re-introduces the problem of coincidence in another way. (For simplicity, in what follows I will focus on cases of coincidence at regions instead of times.) To see this, suppose that we have an object *o*

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[100] Of course, this does not solve the problem of vagueness. As we saw, the endurance theorist would still be committed to the view that all of the functions selecting equally good candidates for the referent of "Tibble" select an object that is wholly present at *t*. It still results a case of coincidence that cannot be explained in the perdurance theorist's way. But the endurance theorist does not face *massive* coincidence.

exactly located at regions r_1, r_2, and r_3, the sum of which is the region r. And suppose, furthermore, that our object is composed of a and b at r_1, of c and d at r_2, and of e and g at r_3, as is depicted in Figure 3. Consider now the regional principle of unrestricted fusion introduced in Chapter 2.

(FE$_r$') $(\forall r)((\exists x)(F_r x) \rightarrow (\exists z)((\forall y)(F_r y \rightarrow P_r yz) \wedge (\forall y)(P_r yz \leftrightarrow (\exists w)(F_r w \wedge O_r yw)))))$

(FE$_r$') is a schema, and I read it like this:

> The fusion of things that are F at any region is an object that has all of these Fs as parts and no parts disjoint from any of the Fs at any region.

It follows from (FE$_r$') (in CEM) that there is a unique object that is the fusion at r_1 of a and b. Since o is composed at r_1 of a and b only, it follows that everything that overlaps $a + b$ at r_1 overlaps o (at r_1), and vice versa. From Chapter 2's (O1$_r$) (see section 3.1),

(O1$_r$) $(\forall r)\,[P_r xy \leftrightarrow (\forall z)(O_r zx \rightarrow O_r zy)]$,

it follows that o is a part at r_1 of $a + b$, and that $a + b$ is a part at r_1 of o. By parity of reasoning, the same holds for o and $c + d$ at r_2, and for o and $e + g$ at r_3.

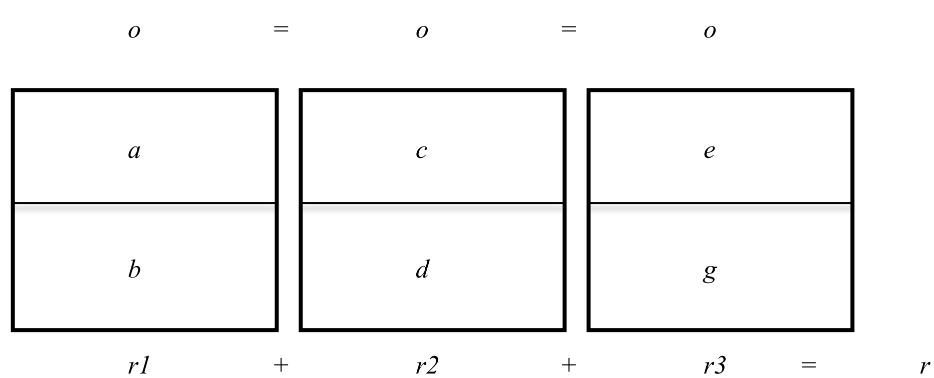

Figure 3: Case of Coincidence 2

Now, call "mf" the fusion of a, b, c, d, e, and g at r (= $r_1 + r_2 + r_3$). That mf exists is a consequence of (FE_r'). To be sure, *mf* is exactly located at *r*, while *o* is not. Therefore, what is exactly located at *r*, i.e., *mf*, is *not* the fusion at *r* of *o* with itself. In chapter 2, we indeed saw that the region at which an object can have a part is the part's exact location. Since the fusion (at any region) of an object with itself is the object itself, the region at which an object can be fused with itself is (one of) its exact location(s). And *r* is not one of *o*'s exact locations. It is equally clear that *mf* cannot be said to be composed of *o* at r_1, r_2, or r_3. Indeed, we know already that *mf* is not the fusion of *o* with itself. And so *mf* is not composed at *r* of *o*. But, then, it cannot be composed at r_1 of *o* either. For if that were the case, we could say that *mf* is composed of *o* at r_1, r_2, and r_3 (unless we impose arbitrary restrictions on where objects can compose other objects). So *o* is nowhere a part of *mf*. But it follows from the description of the case that *o* is composed at r_1 (say) of *a* and *b*. There are therefore two objects composed at r_1 of *a* and *b*, namely a part of *mf* and *o*. The same goes for r_2 and r_3. And here again, the uniqueness of composition is in trouble (even though (II_r) does not enter into the picture).

In Classical Extensional Mereology (CEM) the operation of fusion is unrestricted, and so a failure of the uniqueness of unrestricted fusion amounts to a failure of extensionality. Or, at least, this is so according to the axiomatization of (CEM) I adopted in Chapter 1 (section 2.4). Should there be more than one fusion of the same objects, some axioms would have to fail so as to let go extensionality. And the same is true for the regional counterpart of CEM (R-CEM). Since I suppose the latter, the case of coincidence presented above really is a case that puts the extensionality of parthood in trouble. For at r_1, r_2, and r_3, we are confronted with distinct objects that have exactly the same proper parts.

So I have argued that it results from (FE_r'), from the view that objects can be exactly located at multiple disjoint regions of space or spacetime, and from the view that they can have different parts at these regions, that there is a coincidence of material objects. That was Case 2. I also introduced another case to illustrate coincidence: that of the statue and the clay (Case 1).

2.4 Hope for a Solution

What exactly coincides in Cases 1 and 2? In case 2, the object that is the fusion of *a* and *b* (say) is distinct from *o*. (See Figure 3) And yet, both *o* and the fusion are composed at r_1 of *a* and *b*. The *sorts* to which the fusion and *o* respectively belong are distinct; *o* and the fusion have different kinds of persistence conditions.[101] The object *o*, though not the fusion, could have had different parts at r_1. And *o*, though not the fusion, could lose a part (let us say). Case 1 is similar in that the piece of clay and the statue have different kinds of persistence conditions (and so fall under different sorts), and yet are composed of exactly the same objects.[102] Here again, the lump of clay, though not the statue, could

[101] Do objects differ in their modal properties because they are of different sorts, or is it the other way around? Never mind. I use differences in modal properties as a criterion for differences of sorts, and not as indicating any metaphysical priority of one kind of properties over another.

[102] In fact, the situation is much more complex than that. First, it could well be that neither the lump nor the statue is composed of mereological atoms, since it is possible that there are no such things (i.e., that the statue and the lump are gunk). Dean Zimmerman (Zimmerman 1995) gives a very nice analysis of the problems encountered by certain theories of constitution with the possibility of "gunky" composition. Second, even if there *are* mereological simples, it is not straightforward that the lump and the statue overlap. Varzi (Varzi 2008) shows that the mereological parts of the statue and those of the lump are not necessarily identical. We already know that, *prima facie*, cases such as the statue and the lump present us with coincident objects with distinct *improper* parts. But the reasoning that led us to accept that view can be applied to any parts of either the lump or the statue. Take the right half of the statue. Since it does not share the same properties as the right half of the lump, it is distinct from the latter. And so the right half of the statue and that of the lump are also distinct coincident objects. By parity of reasoning, we could be led to conclude that the statue's and the lump's atomic parts are distinct coincidents, unless we want to make the arbitrary claim that some parts are "too small" to be the subject of the argument. The coincidence of the atomic parts would be akin to that of the ghost and the wall, where two non-overlapping objects are exactly

have had a radically different shape. And the statue, though not the lump, could lose some of its parts.

In my discussion of coincidence below, I will appeal only to modal properties of objects and the relevant instance of (II_r) in order to conclude that there is a coincidence, in both cases, of objects of the same kind.[103]

2.4.3 *Two Aspects of the Problem of Coincidence*

From inspecting Cases 1 and 2, we can see that there are two aspects to the problem of coincidence. In both of theses cases, coincident objects are composed of the same atomic constituents. And yet, according to R-CEM, there can be no distinct objects composed of exactly the same parts. I shall call this aspect of the problem of coincidence "the mereological aspect." Since I adhere to R-CEM, a solution to that aspect of the problem will either have to show that coincident objects are not distinct after all, or else that they are, contrary to appearance, composed of different parts.

The second aspect is commonly referred to as the supervenience aspect. It can be formulated thus: how can distinct objects that are exactly located at a region *r* and share

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located at the same region of space. Because the latter case of coincidence is more problematic than the one I set out to solve, it is no solution to it.

[103] Why do I focus on modal properties instead of regional ones, i.e., the ones that coincident objects instantiate at some region or another? For one thing, we could suppose, for instance, that the lump of clay and the statue are coincident at every region at which they are exactly located. And so focusing on modal properties allows us to take that special case into account. But there is a more philosophical reason for doing so. Focusing on modal properties may demand a solution to the problem of coincidence that will be distinct from the one we would be led to, were we to focus on regional properties. For an object's locative profile, i.e., where it is exactly located, is most plausibly taken as basic (at least, by me). It is much less plausible to do so with the object's modal profile, i.e., with the modal properties it instantiates (though I am inclined to take them as basic, too). So focusing on modal properties allows us to uncover an aspect of the problem that would not necessarily arise should we focus only on regional properties. Unfortunately, it uncovers an aspect of the problem that, as we will see, I will not be able to discuss. See (Fine 2008, p. 104-105) for the suggestion that the times at which an object is present could be a brute fact, but not its instantiating modal properties.

their qualitative (physical) properties differ, at *r*, in their modal (or historical or evaluative) properties? The supervenience aspect is actually not confined to the problem of coincidence. For any two objects that are indiscernible with respect to their qualitative (physical) properties but differ in their historical, evaluative, or modal properties pose exactly the same problem, whether they coincide or not. Kit Fine (Fine 2008, p. 106-107) goes even farther and claims that, in fact, the supervenience aspect of the problem demands an explanation of how modal properties arise out of non-modal ones, and not simply an account of the modal difference between objects—though, of course, the explanation should also clarify the difference in modal properties between coincident objects. These considerations cast doubt on whether the second aspect of the problem of coincidence really is *an aspect* of the problem. Still, an advocate of coincidence would be forced to explain how objects indiscernible in their non-modal properties differ in their modal ones.[104]

In what follows, I use what has been developed in the last two chapters to solve the mereological aspect of the problem of coincidence. The main focus will be on Case 1. The latter can be simply and elegantly presented. The solution of the problem of coincidence for Case 1 carries over to Case 2. Or, rather, it works for Case 2 provided some further issues are put to rest. I will unfortunately leave out the supervenience aspect of the problem, since discussing it would lead me too far afield.

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[104] See (Olson 2001) for a nice discussion of the different attempts either to dismiss the supervenience aspect of the problem, or to solve it.

3. A Solution to the Problem of Coincidence

In the last two chapters, I was concerned with explaining the possibility of multilocation for material objects. In so doing, I offered a bundle theory of material objects that says that objects are identical to fusions, at distinct regions, of universals and haecceities. (See Chapter 5, section 3.3.) I then said nothing about modal properties which objects instantiate at a region of space. Modal properties of an object are properties that they instantiate in the actual world in virtue of instantiating universals (at least, on my view of properties) in possible worlds that they do not instantiate in the actual world. So a red ball instantiates in the actual world the modal property of *being possibly blue* in virtue of instantiating in a non-actual world the universal *being blue*. I did not offer an account of modal property. I did not suppose, in particular, that an object exactly located at a region *r*, say, could instantiate there a property whose analysis appeals to other worlds (or even other regions). So, strictly speaking on my view, a bundle of universals to which an object *o* is identical, i.e., that contains *o*'s haecceity, does not contain any modal properties. But there are two things my account could say about the fact that an object could have been different. The first would be to say that an object could have been different simply because there is a bundle of properties in this world that contains the same haecceity as another bundle of properties in another world. Here, I would not need to appeal to modal properties. The second one would be to appeal to the sub-group relation I introduced in Chapter 5 (section 5.1.2). I then said that a bundle of properties at a region that is identical to a material objects contains, in addition to all of the properties an object has at that region and its haecceity, the relational property the sub-group has of being a sub-group of the group of universals that captures the haecceity of the object.

Modal properties, then, could be given an analysis in terms of this relational property. I favor the second approach, but I will not develop it here.

Now, it would be quite difficult to treat the problem of coincidence by appealing to either of these two ways of accounting for how an object could have been different. Instead, and for reasons of simplicity *only*, I will suppose that modal properties are also parts of the bundles of properties to which objects are identical. To that end, I will take them to be also primitive properties of the object.[105] All of this is meant to simplify the discussion to follow.

Take the statue and the lump of clay again, both of which are exactly located at r and have exactly the same spatial parts. The statue and the lump differ with respect to their modal properties, but share their qualitative properties (or any non-modal ones, like *being five meters from the bar*). Under the theory of material objects I am suggesting, both objects are the fusions of their respective properties. So suppose the statue and the lump share their non-modal properties A, B, C, and D. And suppose that the statue has modal property M (only), while the lump has modal property N (only). We end up with two distinct fusions of properties, namely

(1) $f_r(A, B, C, D, M, h_s)$; and

(2) $f_r(A, B, C, D, N, h_l)$.

[105] See (Olson 2001) and (Bennet 2004) for suggestions to the effect that taking modal properties or kind-membership as basic is an avenue for advocates of coincidence. The sort of solution I am giving to the problem of coincidence does not need to take modal properties as basic. Paul (Paul 2006, p. 640ff) gives a nice account of the supervenience of modal properties on non-modal ones. She claims that modal properties supervene on physical properties of the object together with the object's relational properties of being *de re* represented in a certain way. The latter properties are relational since they are analyzed by a similarity relation that holds between the object and its representations in possible worlds. I cannot avail myself of such a treatment, given that my "analysis" of haecceities presupposes strict transworld identity (see Chapter 4, section 4.3). In any case, I prefer to leave the issue aside.

“h_s” and “h_l” stand for the haecceity of the statue and that of the lump, respectively. (One could think of *M* and *N* as being properties instantiated by the subgroup [*A*,*B*,*C*,*D*] of the group B_s that captures h_s (in the case of *M*), or of the group B_l that captures h_l (in the case of *N*). As I said, I will not do so for reasons of simplicity.) Both fusions overlap at *r*, since they both have *A*, *B*, *C*, and *D* as parts. And yet each is distinct since it has parts that are not shared by the other fusion. The mereological aspect of the problem is therefore solved. For I have shown that, under the view of material objects I am defending, it is *not* the case that the statue and the lump share all of their parts. The same solution can be offered for Case 2.

The solution I recommend for the mereological aspect of the problem of coincidence is not new. L.A. Paul (Paul 2002, 2006) and Kris McDaniel (McDaniel 2001), for instance, give similar solutions. But here is an issue that my solution faces but Paul’s does not. (I do not engage with McDaniel’s argument since he is concerned with tropes, as I said in Chapter 5). Paul takes two part-whole relations as basic—one for properties, the other for spatiotemporal parts of objects—whereas I take only the ternary part-whole relation introduced in Chapter 2 as basic. But let us focus on Paul’s adoption of two kinds of primitive part-whole relations. She (Paul 2006, p. 650ff) claims that the property mereology underlies the spatiotemporal mereology. What she means is that physical objects, such as particles, are reducible to fusions of qualitative (physical) properties. But the spatiotemporal part-whole relation can be used to (spatiotemporally) fuse these fusions of properties into bigger material objects. And, again, these spatiotemporal fusions are nothing more than fusions of properties. In Paul’s view, a property mereology underlies the spatiotemporal one simply because material objects are *reducible* to fusions of properties, and *not* because only the property part-whole relation could be used at the fundamental level. For, clearly, the qualitative fusions of particles (understood as being themselves fusions of properties) *cannot* be the spatiotemporal fusions of these particles. The spatiotemporal fusion is likely to have properties that neither of its particles has (and vice versa). By taking two part-whole relations as primitive, Paul is able to claim that the (spatiotemporal) fusion of fusions of properties does not necessarily instantiate the properties that are fused.

The issue just discussed is crucial, so let us use an example to illustrate it. Suppose an object *o* is in fact nothing other than a fusion of properties *A* and *B*. When Paul says that the property mereology underlies the spatiotemporal mereology, she only means to say that *o* is reducible to the (property) fusion of *A* and *B*. Suppose now another object *o** that is reducible to the (property) fusion of properties *D* and *C*. A spatiotemporal mereology would have it that there is a fusion, call it "*z*," of *o* and *o**. It is clear, however, that z could instantiate none of *A*, *B*, *C*, or *D*. And so *z*, though the spatiotemporal fusion of *o* and *o**, is not the fusion of properties *A*, *B*, *C*, and *D*. So, on Paul's view, a spatiotemporal mereology is *not* reducible to a property one. And all she means when she says that the property mereology underlies the spatiotemporal one, is that objects are reducible to fusions of properties. That Paul is able to claim that *z* is not the fusion of *A*, *B*, *C*, and *D*, while claiming that objects are identical to fusions of properties, is an advantage of her theory. For the latter allows her to avoid a problem that has to do with the properties of fusions of spatiotemporal objects. It remains to see whether I can avoid this problem, too.

I fail to see any distinction between a spatiotemporal and a property mereology. On my view, objects are fusions of properties, and the mereological axioms and principle of regional mereology (introduced in Chapter 2) hold whether or not we are dealing with properties or spatiotemporal parts of objects.[106] But then, how am I to deal with the problem Paul averts? I now turn to this and other related problems.

3.3 Case 2 and the Proposed Theory of Objects

Recall Figure 3 above. And take the region r_1 at which *o* and the fusion at r_1 of *a* and *b* coincide. R-CEM pushes us to claim that *o* is the fusion at r_1 of *a* and *b*. The solution to the mereological aspect of the problem of coincidence given above shows that it does not conflict with R-CEM. The solution entails that the fusion *f* at r_1 of *a* and *b* is, in fact, the fusion at r_1 of *f*'s properties, whereas *o* is the fusion at r_1 of *its* properties. The two fusions overlap, but do not share all of their parts.

Two difficulties arise from this solution: the first I mentioned when discussing Paul's adoption of two primitive mereological relations, and the second is that my view forces me to recognize many objects that we do not typically recognize.

3.3.1 *The First Difficulty: Transitivity of Parthood*

Take Figure 3 again. It is clear that objects *a* and *b* have distinct properties. They have distinct haecceities, and do not share their locative properties. But let us also suppose that

[106] Although I will not be able to discuss the issue here, I do not necessarily suppose that properties are more fundamental than objects. Indeed, the exact location of objects is needed in order to specify the regions at which properties fuse together to form them. And I am tempted to think that this prevents me from thinking of properties, or fusions thereof, as more fundamental than objects. But that is an issue I leave for another time.

they differ even more than that. Let us say that *a* has properties *A* and *B*, while *b* has properties *C* and *D*. Finally, let us grant that *a* is exactly located at the region r_{1a}, while *b* is at r_{1b} (the fusion of r_{1a} and r_{1b} is r_1). According to the theory on offer, we end up with two objects:

(3) $f_{r1a}(A, B, h_a)$; while

(4) $f_{r1b}(C, D, h_b)$.

What we would ordinarily say is the fusion at r_1 of *a* and *b* is not the fusion at r_1 of f_{r1a} and f_{r1b}. The fusion of *a* and *b* may instantiate none of the properties instantiated by either *a* or *b*. Furthermore, and quite simply, it is inconsistent to claim that h_a and h_b fuse together, while maintaining that *a* and *b* are distinct. For h_a and h_b are, respectively, the properties of *being identical to a* and of *being identical to b*. And objects that instantiate both of these haecceities would be identical to *a* and to *b*. Since *a* and *b* are distinct, and because the identity relation is transitive, we cannot have such a fusion on pain of incoherence. So there simply is no fusion of f_{r1a} and f_{r1b}.

Suppose now that the fusion *f* of *a* and *b*, as we would ordinarily say, has properties *E*, *G*, and *N*. Then, we have the following fusion: $f_{r1}(E, G, N, h_f)$. Now, we would ordinarily say that *a* is part of *f*. But none of *a*'s properties are properties of *f*. And yet, or so it seems, because the ternary part-whole relation introduced in Chapter 2 is transitive, we seem to be committed to making the false claim that *a*'s properties are parts of $f_{r1}(E, G, N, h_f)$. Clearly the claim is wrong for many objects: my hand's mass is not *my* mass.

Lewis (Lewis 1986, p. 65) makes clear that this problem arises from the transitivity of parthood. In his view, "[i]t cannot be said…that a universal is instantiated by just anything that has it as a part. For one thing, the relation of part to whole is transitive; so if a universal of charge is part of a particle which is part of an atom, then the universal in turn is part of the atom; but it is the particle, not the atom, which instantiates the universal." We could just accept, Lewis suggests, that a whole has as parts all of the universals its spatial parts instantiate, but we would then have to relinquish the view that an object's having a universal as a part is just its instantiating it; we would have to claim that the part-whole relation is not an explication of the instantiation relation. Interestingly, Lewis suggests in the passage quoted above that there is only one part-whole relation that is primitive and not two, as Paul would have it.

I want to claim that the very same part-whole relation holds between an object and its spatiotemporal parts, as well as between an object's properties. In view of the fact that I analyze material objects as fusions of their properties, what is true about the part-whole relation holding between spatiotemporal parts of objects should be true about the part-whole relation holding between the properties of the spatiotemporal parts and wholes. And yet, I cannot claim that any property that the parts possess is also a property of the whole. Rejecting the transitivity of parthood would be a rather drastic solution, and would constitute a departure from R-CEM.

Here is, schematically, the problem with which we are now dealing. *a* is part of *f*, and both *a* and *f* are the fusions (at their respective exact location) of their respective properties. So the claim that

(5) *a* is part of *f*

should be understood as the claim that

(6) $f_{r1a}(A, B, h_a)$ is part of $f_{r1}(E, G, N, h_f)$.

But $f_{r1a}(A, B, h_a)$ is not part of $f_{r1}(E, G, N, h_f)$ in the intended sense because the properties that are part of $f_{r1a}(A, B, h_a)$ (in the intended sense) are not part of $f_{r1}(E, G, N, h_f)$ (in the same intended sense).

Now, following Lewis's lead in stating the problem, I have overlooked the important fact that parthood also holds of regions of space. Taking the appropriate ternary part-whole relation into consideration, several solutions to the problem come to mind. I will focus here only on two, the second of which is the one I advocate. Recall from Chapter 2 (section 3.3) the mereological principle of Extended Transitivity (ET), given again below.

(ET) $(\forall r)(\forall s)(\forall t)((\mathrm{P}_t ts \wedge \mathrm{P}_s sr \wedge \mathrm{P}_t zy \wedge \mathrm{P}_s yx) \rightarrow (\mathrm{P}_t zx))$

(ET) claims that if z is a part of y at region t, while y is a part of x at region s, and if t is a subregion of s and s a subregion of r, then z is part of x at t. And it would of course follow from (ET) that any part of z is a part of x at some subregion of t. Refer to Figure 3 for clarity.

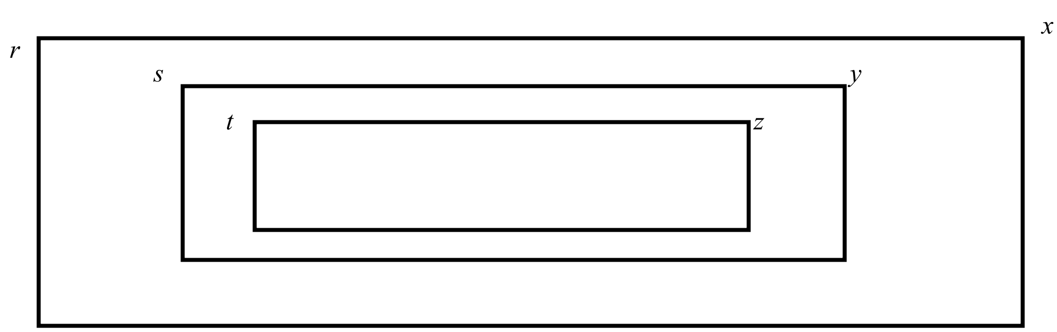

Figure 4: Extended Transitivity

It may be suggested that the problem surfaces only if one thinks that (ET) holds. For suppose, to change the example slightly, that a is exactly located at s while f is exactly located at r. (See Figure 4) So we end up with $f_s(A, B, h_a)$ at s, and with $f_r(E, G, N, h_f)$ at r. We want to say that

(6') $f_s(A, B, h_a)$ is a part at s of $f_r(E, G, N, h_f)$.

$f_s(A, B, h_a)$ is the fusion at s of properties A, B, and h_a, while $f_r(E, G, N, h_f)$ is the fusion at r of properties E, G, N, and h_f. It follows from (6') that $f_s(A, B, h_a)$'s parts, say A and B, are also part at s of $f_r(E, G, N, h_f)$, *only if* (ET) holds. So rejecting it blocks the undesired consequence of (6'). And our problem is solved. That is the first solution I wanted to introduce.

Note that (ET) is not an axiom of R-CEM. It was introduced in Chapter 2 because it is an intuitive principle for spatiotemporal objects. So rejecting it would not constitute a departure from R-CEM. The axiom of transitivity that is a component of R-CEM is Chapter 2's ($P2_r$). Here it is.

($P2_r$) $(\forall r)\,((P_rxy \wedge P_ryz) \rightarrow P_rxz)$

It claims that for any given region r, any part of y at r, where y is part at r of z, is a part at r of z. In R-CEM, there are no specific requirements on the regions at which objects can have a part. It is clear, however, that we prevent ($P2_r$) from implying (ET) if we suppose that all of x, y, and z are exactly located at the same region. Any fusion of properties at a region will have as part any fusions of any set of its properties. For instance, $f_s(A,B)$ is a part at s of $f_s(A,B,h_a)$.

The second solution accepts (ET), and solves the problem by pointing out that, in fact, it rests on a confusion. Lewis's rendering of the problem is convincing because he fails to take into consideration a ternary part-whole relation.[107] In fact, when proper attention is given to the regions at which objects have parts, we realize that no problem arises from accepting (ET). For recall that object *a* is exactly located at *s* while *f* is exactly located at region *r*. (See Figure 4.) I want to claim that *a* is part at *s* of *f*, since *s* is *a*'s exact location. *a* is not part at *r* of *f*, since *a* is not exactly located there. So (6') above is true. Yet it does not entail that *A* and *B*, say, are properties of $f_r(E, G, N, h_f)$. For them to be so, they would have to be parts at *r* of $f_r(E, G, N, h_f)$. And that is *not* what (6') claims. So $f_r(E, G, N, h_f)$ does not have the properties *A* and *B*. It is rather the *s*-part of $f_r(E, G, N, h_f)$ that is *A* and *B*, and that has *A* and *B* as a part at *s*. Similarly, whatever is a part at the subregion *t* of *s* of $f_s(A, B, h_a)$ (see Figure 4), will be a part at *t* of $f_r(E, G, N, h_f)$. Here, again, that does not imply that the properties that make up the *t*-part of $f_r(E, G, N, h_f)$ are parts at *r* of $f_r(E, G, N, h_f)$.

3.3.2 *Many Fusions and Odd Objects*

The second problem that deserves attention is the following. Take Figure 3 again, and focus once more on the region r_1. The fusion *f* of *a* and *b* at r_1 is, let us continue to suppose, the fusion at r_1 of *E*, *G*, *N*, and h_f. Let us suppose again $f_{ra1}(A, B, h_a)$ and $f_{rb1}(C, D, h_b)$. I have already noted that there is no fusion of *A*, *B*, *C*, *D*, *E*, *G*, *N*, h_f, h_a, and h_b,

[107] There are other claims Lewis makes about his modal realism and endurance theory that result from his failure to take ternary part-whole relation into consideration. For instance, in giving his argument against strict transworld identity, he (Lewis 1986, p. 202) claims that such a view about transworld identity would entail an overlap of worlds. For the same object would be part of both worlds, and, as a result, the worlds would overlap. He goes on to claim: "Our question of overlap of worlds parallels the this-worldly problem of identity through time… Endurance involves overlap: the content of two different times has the enduring thing as a common part." It is clear that this last assertion is a mistake. An endurance theorist is likely to take a ternary part-whole relation as primitive. But, then, there is no time at which an enduring thing is a common part of (the content of) distinct times. And so the contents of the times do not overlap. I believe that the same sort of strategy could be appealed to in order to undermine Lewis's argument against the overlap of worlds. I will of course not analyze the issue here.

since that fusion would be inconsistent. In Chapter 5 (section 5.1.1), I indicated that there is no fusion of any of an object's haecceity with properties it does not instantiate (at any region or world). So there could not be a fusion of h_a, say, with *C* and *D*, or with *E* and *G*.

Because fusion is unrestricted, one may think that there is a fusion of *A*, *B*, *C*, *D*, *E*, *G*, *N*, and of any of these properties, in addition to the ones corresponding to the objects we ordinarily recognize. But such a thought would be mistaken, since it would again fail to take into account the ternary part-whole relation introduced in Chapter 2. As we just saw, because *A* and *B* are fused (with h_a) at r_{a1} and *C* and *D* are fused (with h_b) at r_{a2}, there is no fusion at r_1 of *A*, *B*, *C*, and *D*. None of the initial fusions are exactly located at r_1. There is, however, a fusion at r_{a1} of *A* and *B*, and a fusion at r_{a2} of *C* and *D*. And these fusions are parts (at their respective region) of $f_{ra1}(A, B, h_a)$ and of $f_{rb1}(C, D, h_b)$, respectively. And I am ready to recognize these fusions. In fact, they would resemble David Armstrong's abstract particulars. (See (Armstrong 1978a, p. 121).) Armstrong gives the following example of a colored cube. He claims that the cube could be stripped of its visible properties. That would result in its having only its tactual properties. Similarly, we can imagine the cube devoid of its tactual properties. That, in turn, would yield a visible cube. Both the visual and the tactual cubes are parts, in Armstrong's view, of the initial colored cube. The odd fusions I acknowledge would be like Armstrong's visual and tactual cubes; they could be proper parts of the object we readily acknowledge, and be composed of the latter's properties.

Perhaps one could think that recognizing these odd fusions leads to a problem for my view of material objects. For, after all, these odd fusions would also have haecceities. And for reasons given above, these fusions could not be conceived as parts of the objects we recognize. For their haecceities, it would be thought, could not be part of the fusions of the material objects we are ready to recognize; otherwise, it would result in incoherence. There are many ways out of this quandary. For reasons of space, I will not analyze these, however. Instead, I will just mention my preferred solution. It is true that the odd fusions would have haecceities that, presumably, would be inconsistent with the haecceities of the object of which they are parts. But recall from Chapter 5 (section 5.1.1) that the sufficient condition for the instantiation of a haecceity is only one that indicates that a certain haecceity is consistent with a sub-group of properties. It is *not* a sufficient condition for the existence of the fusion of the properties in a sub-group with a haecceity. So I can safely claim that the odd fusions that are parts of material objects (like the tactual cube's being a part of the cube) are the fusions of properties alone, and not the fusions of the latter with haecceities.

4. Conclusion

In this chapter, I used the apparatus developed in the preceding chapters to solve an important aspect of the problem of the coincidence of material objects. I recognize that many distinct material objects can coincide at a region, i.e., that they can all be exactly located at the same region, without their being composed of exactly the same parts. Indeed, these coincident objects are the fusions at the region of coincidence of distinct properties. This solution, already suggested in the literature, brought about problems for my view of material objects. I ended the chapter by solving them.

Conclusion

The dissertation is an attempt to make sense of endurance theory, i.e., to show that it is a coherent and even possible view of the persistence of material objects. It did so by assuming that endurance theory is a special case of a more general phenomenon, namely, that it is possible for material objects to be exactly located at more than one disjoint region of space (or spacetime). In Chapter 1, I introduced classical extensional mereology, some principles about location, and the ontology of perdurance theory. I then suggested that Functionality, a principle that claims that an object cannot be exactly located at more than one place, is dispensable. In the Appendix to the Dissertation, I showed that it was formally dispensable, i.e., that formal theories of location can do without it. In Chapter 2, I introduced the temporal and regional variants of classical extensional mereology, and responded to an argument claiming that endurance theory is metaphysically incoherent. In Chapter 3, I showed that an object's whole presence can only be characterized in terms of exact location, and I argued that endurance theory really amounts to a rejection of Functionality.

In Chapters 4 and 5, I gave an explanation of the possibility of multilocation for material objects. The idea was to claim that it is possible for material objects to be exactly located at several disjoint regions of space (or spacetime) because their haecceities can be instantiated at these several regions. Chapter 4 introduced the notion of a haecceity, which is the unsharable property an object has of being identical to itself. And it gave an analysis of haecceities that allows us to give necessary and/or sufficient conditions for their instantiation. In Chapter 5, I explained how these conditions could be drawn from the analysis of haecceities, and showed that they explain the possibility of multilocation for material objects. I there adopted a property mereology in order to give my version of the bundle theory of material objects. In Chapter 6, I was concerned with the problem of the coincidence of material objects. I concluded that two objects could in fact coincide at a region of space, without posing any threat to the regional variant of classical extensional mereology.

Much remains to be done in order to strengthen the explanation of multilocation I offered. Specifically, I rested content with an analysis of haecceities that allows us to derive necessary and/or sufficient conditions for their instantiation. But I have not clearly articulated what haecceities are, beyond saying that they are non-qualitative, non-primitive properties of objects. I think that the explanation of multilocation I offered can be strengthened if I were to say more about what haecceities are, and about their relation to the groups of properties that capture them. But that is a project for another time.

Appendix to the Dissertation

(Other) Theories of Location

In Chapters 1 and 2, I claimed that there were theories of location in which Functionality does not hold, i.e., theories that do not take the relation of exact location to be a function. Josh Parsons (Parsons 2007) contends that Functionality is needed to show that locative systems in which exact location is the primitive and those in which generic location is the primitive are equivalent. An appropriate response consists in developing theories of location in which Functionality does not hold. Here, I will first give an overview of theories of location with Functionality, and show the role the latter principle plays in them. Then, I will introduce theories of location in which Functionality does not hold.

1. Theories of Location with Functionality

All of the theories of location to be given below contain the axioms (and theorems) of Classical Extensional Mereology (CEM). Here is a list of CEM's axioms and of its main definitions.[108]

Axioms

(P0) $\mathrm{P}xx$ (Reflexivity)

(P1) $\mathrm{P}xy \wedge \mathrm{P}yx \rightarrow y = x$ (Weak Anti-symmetry)

(P2) $(\mathrm{P}xy \wedge \mathrm{P}yz) \rightarrow \mathrm{P}xz$ (Transitivity)

(WS) $\mathrm{PP}xy \rightarrow (\exists z)(\mathrm{PP}zy \wedge \sim \mathrm{O}zx)$. (Weak Supplementation)

(FE') $(\exists x)(\mathrm{F}x) \rightarrow (\exists z)((\forall y)(\mathrm{F}y \rightarrow \mathrm{P}yz) \wedge (\forall y)(\mathrm{P}yz \leftrightarrow (\exists w)(\mathrm{F}w \wedge \mathrm{O}yw))))$ (Fusion Existence)

[108] In what follows, I adopt the following convention. I use letters "*x*," "*y*," and "*z*" as variables for objects that are not regions, and "*r*," "*s*," "*t*," and "*u*" as variables for regions. It will sometimes be the case that I will use "*o*" as a constant for an object, and "*r*" as one for a region. As it was the case in the dissertation, I leave out the universal quantifiers that have the largest scope, for clarity.

Definitions

(D2)	$PPxy =_{df} (Pxy \wedge x \neq y)$	(PP in terms of P)
(O)	$Oxy =_{df} (\exists z)(Pzx \wedge Pzy)$	(Overlap)
(O1)	$Pxy =_{df} (\forall z)(Ozx \rightarrow Ozy)$	(P in terms of O)

I should make clear at the outset that, although Parsons shows the equivalence of theories of location with distinct primitives, he does not endorse CEM. So none of his theories of location contain CEM. More precisely, Parsons rejects (FE'). Nevertheless, we do not lose the spirit of his derivations and conclusions about Functionality by adopting CEM. Note also that I here take the part-whole relation to be binary, and not ternary. This has the advantage of immensely simplifying the arguments below. It also makes clear that a denial of Functionality does not prevent one from employing binary mereological relations in a theory of location.

1.1 CEML

Let us call "CEML" the theory of location that extends CEM with the following axioms:

$(Lor \wedge Los) \rightarrow r = s$	(Functionality)
$Lxr \rightarrow Lrr$	(Conditional Reflexivity)[109]
$(Rr \wedge Psr) \rightarrow Rs$	(Regional Dissection)
$(\forall t)[((Rt \wedge Otr) \leftrightarrow (Rt \wedge Ots)) \leftrightarrow r = s] \rightarrow (Rr \wedge Rs)$	(Region Identity)

Recall that "L" denotes the relation of exact location, and that an object is exactly located at a region provided they both have the same size and shape. (Conditional Reflexivity) ensures that objects are *exactly located* only at regions. For regions are the only things that are located at themselves (as I stipulated in Chapter 1 (sections 3.1.2) following

[109] Parsons (Parsons 2007, p. 224) does not have strong intuitions about conditional reflexivity, and sees no problem with it (nor with its denial).

Roberto Casati and Achille Varzi). (Regional Dissection) makes sure that parts of regions are also regions, while (Functionality) says that objects have no more than one exact location. Finally, "R" is the region predicate.

(Region Identity) is a simple axiom. It is meant to mimic the following theorem of CEM, one that is derived from (O1) and (P1).

(A) $(\forall z)((\mathrm{O}zr \leftrightarrow \mathrm{O}zs) \leftrightarrow r = s)$

Here is how I read (A):

> Objects are identical just in case everything that overlaps one does so just in case it overlaps the other.

Of course, (A) is true if *r* and *s* are regions of space. But we could restrict the things that overlap *r* and *s* and *still* ensure that the latter be identical. In particular, *r* and *s* are identical if and only if every region that overlaps the one overlaps the other, and vice versa. In other words, we would like to claim something like this:

(A') $(\forall t)[((\mathrm{R}t \wedge \mathrm{O}tr) \leftrightarrow (\mathrm{R}t \wedge \mathrm{O}ts) \leftrightarrow r = s]$

The problem, however, is that (A') is clearly false. For there is no mention that *r* and *s* are *regions*. And it is just not true that two objects are identical just in case every region that overlaps the one does so if and only if it overlaps the other. Take, for instance, the fusion *f* of *r* and the sun, and the fusion *f** of *r* and the moon. Every region that overlaps *f* overlaps *f**, and vice versa. And yet they are distinct. It is thus necessary that *r* and *s* be regions for (A') to hold.

(A'') $(\forall t)[((\mathrm{R}t \wedge \mathrm{O}tr) \leftrightarrow (\mathrm{R}t \wedge \mathrm{O}ts)) \leftrightarrow r = s] \rightarrow (\mathrm{R}r \wedge \mathrm{R}s)$

Of course, for any two regions, if all regions that overlap one do so just in case they overlap the other, they are identical. So

$$(\forall t)[((\mathrm{R}t \wedge \mathrm{O}tr) \leftrightarrow (\mathrm{R}t \wedge \mathrm{O}ts)) \leftrightarrow r = s] \leftrightarrow (\mathrm{R}r \wedge \mathrm{R}s) \qquad \text{(Region Identity)}$$

Here is how I read (Region Identity):

> Any regions *r* and *s*, which are such that every region that overlaps the one does so just in case it overlaps the other, are identical. And identical regions are such that every region that overlaps the one does so just in case it overlaps the other.

(Region Identity) is used in some derivations below.

As we saw in Chapter 1 (sections 3.1.2 and 3.1.3), the following can all be defined in CEML.

$$\mathrm{L_g}or =_{\mathrm{df}} (\exists s)(\mathrm{R}r \wedge \mathrm{L}os \wedge \mathrm{O}sr). \qquad \text{(Generic Location)}$$
$$\mathrm{R}r =_{\mathrm{df}} \mathrm{L}rr \qquad \text{(Region Predicate)}$$
$$\mathrm{L_p}or =_{\mathrm{df}} (\exists s)(\mathrm{L}os \wedge \mathrm{P}rs) \qquad \text{(Pervasive location)}$$
$$\mathrm{L_e}or =_{\mathrm{df}} (\exists s)(\mathrm{P}sr \wedge \mathrm{L}os). \qquad \text{(Entire Location)}$$
$$\mathrm{L_w}or =_{\mathrm{df}} (\forall y)(\mathrm{P}yo \rightarrow \mathrm{L_g}yr) \qquad \text{(Whole Location)}$$

Recall that:

1. "$\mathrm{L_g}$" denotes the relation of generic location, and that an object is *generically located* at a region provided that the latter is not free of *it*;
2. "R" is the region predicate;
3. "$\mathrm{L_p}$" denotes the relation of pervasive location, and that an object is pervasively located at a region provided that the latter is a part of the object's exact location;
4. "$\mathrm{L_e}$" denotes the relation of entire location, and that an object is entirely located at a region provided that it is exactly located at a subregion of the latter;
5. "$\mathrm{L_w}$" denotes the relation of whole location, and that an object is wholly located at a region provided that all of its parts are generically located at that region.

   The following is an interesting theorem of CEML.

$$(\exists r)\mathrm{L_g}or \rightarrow (\exists s)\mathrm{L}os \qquad \text{(Exactness)}$$

(Exactness) makes sure that objects have an exact location, if they are in space at all. It follows directly from the definition (Generic Location), as Parsons (Parsons 2007, p. 222) claims.

The above completes my introduction of CEML. Let us move on to a theory of location in which Functionality holds and which takes generic location (L_g) as a primitive.

1.2 CMG

Let us call "CMG" the locative theory that extends CEM with the following axioms:

$(\forall s)(L_g os \rightarrow (\exists t)(Lot \wedge Ost))$. (Exactness*)
$(\forall x)(L_g xr \rightarrow L_g rr)$ (Conditional Reflexivity—CMG)
$(\forall t)[((Rt \wedge Otr) \leftrightarrow (Rt \wedge Ots)) \leftrightarrow r = s] \leftrightarrow (Rr \wedge Rs)$ (Region Identity)

Some of CMG's important definitions are given below.

$Rr =_{df} L_g rr$. (Region Predicate—CMG)
$Lor =_{df} (\forall s)((Rs \wedge Osr) \leftrightarrow L_g os)$ (Exact Location)

Before explaining CMG's definitions, a few words on the axioms are in order.

Parsons (Parsons 2007, p. 205) proposes (Exactness) in order to capture the fact that an object generically located somewhere has an exact location. It is clear, however, that he wants to avoid an object's having a lone generic location, as depicted in Figure 1. The latter represents a model consistent with (Exactness), but one that we want to rule out.

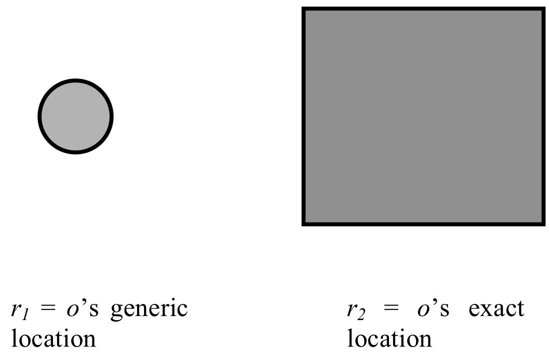

Figure 5: Generic Location

An object is generically located at r_1, and exactly located at r_2. So (Exactness) holds. (Exactness*) comes to the rescue by ruling out the model of Figure 1, since r_1 does not overlap r_2. I make it an axiom of CMG. For the record, CEML's (Exactness) follows from (Exactness*)[110]. As for (Conditional Reflexivity—CMG), it makes sure that objects are generically located only at regions.

(Exact Location) is the definition of the relation of exact location in CMG. Here is how I read it.

> An object is exactly located at *r* just in case (by definition) every region that overlaps *r* is a region at which the object is generically located, and vice versa.

The region predicate of CMG is not the same as CEML's, but is equivalent to it. (Though I will not show such a thing, for reason of space.)

That completes my introduction of CMG. I will presently show that CMG and

[110] (Exactness*) is also a theorem of CEML. Recall CEML's definition of L_g, i.e. $L_g or = (\exists s)(Rr \wedge Los \wedge Osr)$. It follows from $L_g or$ that there is an exact location of *o* that overlaps *r*. That is what (Exactness*) says. I did not introduce it above because I prefer to stay as close as possible to what Parsons says.

CEML are equivalent. Before doing so, let us note an interesting issue related to (Exactness) in CMG.[111] Suppose that space is composed of three *atomic, but extended* regions r_1, r_2, and r_3. And suppose furthermore that an object *o* "outstretches" r_2, without "outstretching" r_1 and r_3. (See Figure 6.) Then, *o* is generically located at each region of r_1, r_2, and r_3, given that none of these regions is free of *o*. It would follow from (Exact Location) that *o* is exactly located at the fusion *f* of r_1, r_2, and r_3. Indeed, *f* overlaps all of the regions at which *o* is generically located, and all of *o*'s generic locations overlap *f* (though *o* does not fit the whole of r_1 and r_3, both of these regions are simples, and their respective sole improper parts overlap *o*'s generic location). This does not square with the informal rendering of exact location, according to which an object is exactly located at a region that has the same shape and size as the object. Worse still, the model of Figure 6 is not a model of CEML, given that the latter's primitive fails to capture *o*'s mode of location at at least r_1 and r_3 (it would fail to capture *o*'s mode of location at all regions in cases in which *o* does not have a proper part exactly located at r_2). If the model of Figure 6 is at all coherent, CMG and CEML are not equivalent.[112]

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[111] What I am about to say resembles greatly a counterexample to (Exactness) that Parsons considers in (Parsons 2007, p. 207). The point I want to make is not that there is a counterexample to (Exactness), but that (Exact location) may fail. This, in turn, undermines the equivalence of CMG and CEML (Parsons contends that locative theories close to CMG and CEML are equivalent).

[112] I suppose that Arbitrary Partition is neither a theorem nor an axiom of CMG. Here is the Doctrine of Arbitrary Partition, introduced in Chapter 2 (section 3.3).

(DAP) $L_p xr \rightarrow (\exists y)(Pyx \wedge Lyr)$

But note that the counterexample to the equivalence of CMG and CEML just given would equally hold should both theories contain (DAP). For the latter does not prevent regions from being extended simples. The point that I want to emphasize, though, is that objects could, as it were, be "smaller" than the regions of space, whether or not the latter are extended simples.

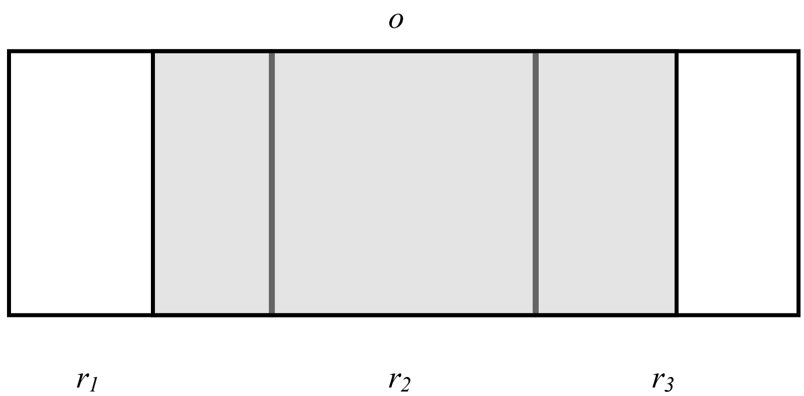

Figure 6: Model of CMG

Fortunately, it is not really important for my argument whether the model of Figure 6 is coherent. For my aim is to show that removing Functionality from either CEML or CMG does not jeopardize the equivalence of the resulting systems. Since Parsons contends that theories close to CEML and CMG are equivalent, I will suppose his contention correct and put aside the counterexample I just gave. I should note, however, that the counterexample under discussion here is equally problematic for Parsons. On pain of making CEML and CMG non-equivalent, therefore, let us suppose that the counterexample to Figure 6 is not coherent.

1.3 Equivalence of CMG and CEML

In this section, I show that CMG and CEML are equivalent theories of location. Following Parsons, I do so in three steps. First, I take CEML as our theory of location, and I show that I can derive CMG's definition of exact location in it. The derivations require Functionality to hold in order to go through. Second, I show that CMG contains Functionality as a theorem. Third, I show that we can derive CEML's definition of generic location in CMG. All derivations are *due* to Parsons, but I modified them slightly in order to accommodate the fact that CEM is a component of both CMG and CEML.

1.3.1 *(Exact Location) in CEML*

Let us start by showing that one can derive CMG's definition of L in CEML. That is, we want to show that

$$\mathrm{L}or \leftrightarrow (\forall s)((\mathrm{R}s \wedge \mathrm{O}rs) \leftrightarrow \mathrm{L_g}os)$$

holds in CEML. This derivation is due to Parsons, but has been slightly modified to accommodate the definition of exact location in CMG. (Recall from Chapter 1 that Parsons does not offer the same definition of exact location in terms of generic location, since he does not need to stipulate that objects are exactly located only at regions.) I have broken down the derivation into four parts. Note that Functionality is used in the first and third part of the derivation.

In the first part, we show that

$$\mathrm{L}or \rightarrow (\forall s)((\mathrm{R}s \wedge \mathrm{O}sr) \leftrightarrow \mathrm{L_g}os)$$

holds in CEML.

1.	$\mathrm{L}or$	Assumption
2.	$(\mathrm{R}s \wedge \mathrm{O}sr)$	Assumption
3.	$\mathrm{R}s \wedge \mathrm{L}or \wedge \mathrm{O}sr$	∧-Introduction(1,2)
4.	$(\exists t)(\mathrm{R}s \wedge \mathrm{L}ot \wedge \mathrm{O}st)$	∃-Introduction(3)
5.	$\mathrm{L_g}os$	(Generic Location)
6.	$(\mathrm{R}s \wedge \mathrm{O}sr) \rightarrow \mathrm{L_g}os$	→-Introduction(2,5)
7.	$\mathrm{L_g}os$	Assumption
8.	$(\exists t)(\mathrm{R}s \wedge \mathrm{L}ot \wedge \mathrm{O}ts)$	(Generic Location)
9.	$\mathrm{R}s \wedge \mathrm{L}ou \wedge \mathrm{O}us$	∃-Elimination(8)
10.	$\mathrm{L}or \wedge \mathrm{L}ou$	∧-Introduction(1,9), ∧-Elimination(9)

11.	$(\mathrm{L}or \wedge \mathrm{L}ou) \rightarrow r = u$	**Functionality Axiom**
12.	$r = u$	→-Elimination(10,11)
13.	$\mathrm{O}us$	∧-Elimination(9)
14.	$\mathrm{O}rs$	Substitution(12,13)
15.	$\mathrm{R}s \wedge \mathrm{O}sr$	∧-Elimination(9), ∧-Introduction(9,14)
16.	$\mathrm{L_g}os \rightarrow (\mathrm{R}s \wedge \mathrm{O}sr)$	→-Introduction(7,15)
17.	$(\mathrm{R}s \wedge \mathrm{O}sr) \leftrightarrow \mathrm{L_g}os$	↔-Introduction(6,16)
18.	$(\forall s)((\mathrm{R}s \wedge \mathrm{O}sr) \leftrightarrow \mathrm{L_g}os)$	∀-Introduction(17)
19.	$\mathrm{L}or \rightarrow (\forall s)((\mathrm{R}s \wedge \mathrm{O}sr) \leftrightarrow \mathrm{L_g}os)$	→-Introduction(1,18)

Let us now show that

$$(\forall s)((\mathrm{R}s \wedge \mathrm{O}sr) \leftrightarrow \mathrm{L_g}os) \rightarrow \mathrm{L}or$$

holds in CEML. The trick is to show that $(\forall s)((\mathrm{R}s \wedge \mathrm{O}sr) \leftrightarrow \mathrm{L_g}os)$ entails that *o* has an exact location *u* that overlaps everything overlapped by *r* (and vice versa). (See (Parsons 2007, p. 230-231).) The second part of the derivation concludes that *o* is exactly located at this region *u*. It supposes that there are regions.

1.	$(\exists r)\mathrm{R}r$	Assumption
2.	$\mathrm{R}r$	∃-Elimination(1)
3.	$(\forall s)((\mathrm{R}s \wedge \mathrm{O}sr) \leftrightarrow \mathrm{L_g}os)$	Assumption
4.	$(\forall s)((\mathrm{R}s \wedge \mathrm{O}sr) \leftrightarrow (\exists t)(\mathrm{L}ot \wedge \mathrm{O}st \wedge \mathrm{R}s))$	Generic Location(3)
5.	$(\mathrm{R}r \wedge \mathrm{O}rr) \leftrightarrow (\exists t)(\mathrm{L}ot \wedge \mathrm{O}st \wedge \mathrm{R}s)$	∀-Elimination(4)
6.	$\mathrm{O}rr$	(P0) and (O)
7.	$\mathrm{R}r \wedge \mathrm{O}rr$	∧-Introduction(2,6)
8.	$(\exists t)(\mathrm{L}ot \wedge \mathrm{O}st \wedge \mathrm{R}s)$	→-Elimination(5,7)
9.	$\mathrm{L}ou \wedge \mathrm{O}su \wedge \mathrm{R}s$	∃-Elimination(8)
10.	$\mathrm{L}ou$	∧-Elimination(9)

Having found an exact location *u* for *o*, we aim to show that *u* overlaps the same regions as *r* does (and vice versa) in order to show that *u* and *r* are identical. The derivation to be given appeals to (Region Identity), and not to CEM's theorem to the effect that identical objects overlap the same things. Parsons's own derivation appeals to the latter theorem.

11.	$\mathrm{R}b \wedge \mathrm{O}ub$	Assumption
12.	$\mathrm{R}b \wedge \mathrm{L}ou \wedge \mathrm{O}ub$	∧-Introduction(10,11)

13.	$(\exists t)(\mathrm{R}b \wedge \mathrm{L}ot \wedge \mathrm{O}tb)$	∃-Introduction(12)
14.	$\mathrm{L}_g ob$	(Generic Location)
15.	$(\mathrm{R}b \wedge \mathrm{O}br) \leftrightarrow \mathrm{L}_g ob$	∀-Elimination(3)
16.	$(\mathrm{R}b \wedge \mathrm{O}br)$	→-Elimination(14,15)
17.	$(\mathrm{R}b \wedge \mathrm{O}ub) \rightarrow (\mathrm{R}b \wedge \mathrm{O}br)$	→-Introduction(11,16)
18.	$\mathrm{R}b \wedge \mathrm{O}br$	Assumption
19.	$(\mathrm{R}b \wedge \mathrm{O}br) \leftrightarrow \mathrm{L}_g ob$	∀-Elimination(3)
20.	$\mathrm{L}_g ob$	→-Elimination(18,19)
21.	$(\exists t)(\mathrm{R}b \wedge \mathrm{L}ot \wedge \mathrm{O}tb)$	(Generic Location)
22.	$\mathrm{R}b \wedge \mathrm{L}ot \wedge \mathrm{O}tb$	∃-Elimination(21)
23.	$\mathrm{L}ot \wedge \mathrm{L}ou$	∧-Elimination(22),∧-Introduction(10,22)
24.	$(\mathrm{L}ot \wedge \mathrm{L}ou) \rightarrow t = u$	**Functionality Axiom**
25.	$t = u$	→-Elimination(23,24)
26.	$\mathrm{O}tb$	∧-Elimination
27.	$\mathrm{O}ub$	Substitution(25,26)
28.	$\mathrm{R}b \wedge \mathrm{O}ub$	∧-Elimination(18),∧-Introduction(18,27)
29.	$(\mathrm{R}b \wedge \mathrm{O}br) \rightarrow (\mathrm{R}b \wedge \mathrm{O}ub)$	→-Introduction(18,28)
30.	$(\mathrm{R}b \wedge \mathrm{O}br) \leftrightarrow (\mathrm{R}b \wedge \mathrm{O}ub)$	↔-Introduction(17,29)
31.	$(\forall t)((\mathrm{R}t \wedge \mathrm{O}tr) \leftrightarrow (\mathrm{R}t \wedge \mathrm{O}tu))$	∀-Introduction(30)
32.	$\mathrm{R}u$	Conditional Reflexivity(23), (Region Predicate)
33.	$\mathrm{R}r \wedge \mathrm{R}u$	∧-Introduction(2,32)
34.	$(\forall t)[((\mathrm{R}t \wedge \mathrm{O}tr) \leftrightarrow (\mathrm{R}t \wedge \mathrm{O}tu)) \leftrightarrow (r = u)] \leftrightarrow (\mathrm{R}r \wedge \mathrm{R}u)$	(Region Identity)
35.	$(\forall t)((\mathrm{R}t \wedge \mathrm{O}tr) \leftrightarrow (\mathrm{R}t \wedge \mathrm{O}tu)) \leftrightarrow (r = u)$	→-Elimination(33,34)
36.	$r = u$	→Elimination(31,35)
37.	$\mathrm{L}or$	Substitution(10,36)

In CEML, therefore, Functionality is required to derive the equivalence

$$\mathrm{L}or \leftrightarrow (\forall s)((\mathrm{R}s \wedge \mathrm{O}sr) \rightarrow \mathrm{L}_g os)$$

1.3.2 *Functionality Is a Theorem of CMG*

The derivation I will present in this section is an adaptation of Parsons's own derivation (Parsons 2007, p. 28). It is also meant to stay within the spirit of Parsons's derivation, even though it appeals to principles Parsons does not appeal to. Finally, it shows that Functionality is a theorem of CMG. We start by supposing that an object *o* is exactly located at regions *r* and *s*, and we aim to show that *r* and *s* are identical.

1.	$\mathrm{L}or \wedge \mathrm{L}os$	Assumption

2. $(\forall z)((Rz \wedge Ozr) \leftrightarrow L_g oz)$ (Exact Location)(1)
3. $(\forall z)((Rz \wedge Ozs) \leftrightarrow L_g oz)$ (Exact Location)(2)
4. $(\forall z)((Rz \wedge Ozr) \leftrightarrow (Rz \wedge Ozs))$ $\leftrightarrow$-(2,3)
5. $(\forall z)[((Rz \wedge Ozr) \leftrightarrow (Rz \wedge Ozs)) \leftrightarrow r = s] \leftrightarrow (Rs \wedge Rr)$ (Region Identity)
6. $Rs \wedge Rr$ (Region Predicate)[113]
7. $(\forall z)[((Rz \wedge Ozr) \leftrightarrow (Rz \wedge Ozs)) \leftrightarrow r = s]$ $\rightarrow$-Elimination(5,6)
8. $r = s$ $\rightarrow$-Elimination(4,7)

The derivation just given shows that Functionality is a theorem of CMG. It parallels exactly Parsons's own derivation, with the appropriate modifications made necessary by CEM.

1.3.3 *(Generic Location) in CMG*

I have just shown where Functionality was necessary in order to show the equivalence of CMG and CEML. And I showed that CMG implies Functionality. Here I will show that CEML's definition of Lg and CMG's primitive Lg are equivalent in CMG. That is, I show that

$$L_g or \leftrightarrow (\exists s)(Rr \wedge Los \wedge Osr)$$

is derivable in CMG. None of these derivations appeals to Functionality. But I thought I would introduce them anyway.

First, let us start with the left to right reading of the biconditional. Here is how we can derive it in CMG.

1. $L_g or$ Assumption
2. $(\forall s)(L_g os \rightarrow (\exists t)(Lot \wedge Ost))$ (Exactness*)
3. $L_g or \rightarrow (\exists t)(Lot \wedge Ort)$ $\forall$-Elimination(2)

[113] Here, I took a shortcut. In CMG, the region predicate is defined in terms of (L_g). Nevertheless, it follows from line 1 that *r* and *s* are regions. For line 1 implies L*rr* and L*ss*. But a region at which *r* (or *s*) is exactly located is also a region at which it is generically located. For the exact location of something is never free of that thing. It follows, therefore, that *r* and *s* are, respectively, generically located at themselves. (Note also that it is a fact about the location of regions that a region is never free of itself.)

4. $(\exists t)(\mathrm{L}ot \wedge \mathrm{O}rt)$ →-Elimination(1,3)
5. $\mathrm{L}os \wedge \mathrm{O}rs$ ∃-Elimination(4)
6. $(\forall u)((\mathrm{R}u \wedge \mathrm{O}us) \leftrightarrow \mathrm{L_g}ou)$ (Exact Location)
7. $(\mathrm{R}r \wedge \mathrm{O}rs) \leftrightarrow \mathrm{L_g}or$ ∀-Elimination(6)
8. $\mathrm{R}r \wedge \mathrm{O}rs$ →-Elimination(1,7)
9. $\mathrm{R}r \wedge \mathrm{L}os \wedge \mathrm{O}sr$ ∧-Introduction(5,8)
10. $(\exists s)(\mathrm{R}r \wedge \mathrm{L}os \wedge \mathrm{O}sr)$ ∃-Introduction(9)

Now, I show how to derive the right to left reading of the biconditional above.

1. $(\exists s)(\mathrm{R}r \wedge \mathrm{L}os \wedge \mathrm{O}sr)$ Assumption
2. $\mathrm{R}r \wedge \mathrm{L}os \wedge \mathrm{O}sr$ ∃-Elimination(1)
3. $\mathrm{L}os$ ∧-Elimination(2)
4. $(\forall u)((\mathrm{R}u \wedge \mathrm{O}us) \leftrightarrow \mathrm{L_g}ou)$ (Exact Location)
5. $(\mathrm{R}r \wedge \mathrm{O}rs) \leftrightarrow \mathrm{L_g}or$ ∀-Elimination(4)
6. $\mathrm{R}r \wedge \mathrm{O}rs$ ∧-Elimination(2)
7. $\mathrm{L_g}or$ →-Elimination(5,6)

From these two derivations, it results that

$$\mathrm{L_g}or \leftrightarrow (\exists s)(\mathrm{R}r \wedge \mathrm{L}os \wedge \mathrm{O}sr)$$

holds in CMG.

2. Theories of Location Without Functionality

Parsons's claim that we cannot forgo Functionality in theories of location rests on the assumption that Functionality is required to show the equivalence between theories with different primitives. An adequate answer to his claim consists in showing that *there are* theories of location without Functionality *with* different primitives that are nevertheless equivalent. That there are such theories, I take it, shows that, from a purely formal point of view, Functionality is not needed.

2.1 CEML *Minus* Functionality: CEML*

Let us start with CEML, and let us remove the axiom of Functionality from it. I call "CEML*" the resulting theory of location. Here are CEML*'s axioms.

$Lxr \rightarrow Lrr$ (Conditional Reflexivity)
$(Rx \wedge Pyx) \rightarrow Ry$ (Regional Dissection)
$(\forall z)[((Rz \wedge Ozr) \leftrightarrow (Rz \wedge Ozs)) \leftrightarrow (r = s)] \leftrightarrow (Rr \wedge Rs)$ (Region Identity)
$(Lor \wedge Los \wedge Ors) \rightarrow r = s$ (Preciseness)

The definitions of CEML* are exactly the same as the ones of CEML. To repeat:

$L_g or =_{df} (\exists s)(Rr \wedge Los \wedge Osr)$. (Generic Location)
$Rx =_{df} Lxx$ (Region Predicate)
$L_p or =_{df} (\exists s)(Los \wedge Prs)$ (Pervasive location)
$L_e or =_{df} (\exists y)(Pyr \wedge Loy)$. (Entire Location)
$L_w or =_{df} (\forall y)(Pyo \rightarrow L_g yr)$ (Whole Location)

Let us discuss CEML*'s axioms. I have already introduced the first three axioms, and will not discuss them here. However, (Regional Uniqueness), introduced in Chapter 1 (section 3.1.2), follows from (Region Identity) with the assumption that a region exactly located at another overlaps it. (For reasons of space, I will unfortunately not show this.) (Regional Uniqueness) is *not* Functionality, since it is restricted to regions of space. As for (Preciseness), it makes sure that objects exactly located at distinct regions are exactly located at *disjoint* regions.[114] I simply assume that it is an axiom of CEML*.

That completes my introduction of CEML*. Let us turn to CMG*, i.e., CMG minus Functionality.

2.2 CMG *Minus* Functionality: CMG*

Functionality was a theorem of CMG. So CMG* must have axioms and definitions from which Functionality does not follow. Moreover, CMG* has to be equivalent to CEML*.

Before introducing CMG*'s axioms and theorems, it is important that I discuss CMG*'s definition of exact location which I introduced in Chapter 1 (section 3.1.3).

[114] Hud Hudson (Hudson 2001) would not be happy with such a principle. For it is apparent from his Partist View that he rejects Functionality. But he thinks that the same object is exactly located (my terminology) at overlapping regions of space.

There, I said that the correct definition of exact location in a theory that lacks Functionality (such as CMG*) is the following:

$$\mathrm{L}or =_{\mathrm{df}} (\forall s)((\mathrm{R}s \wedge \mathrm{O}sr) \rightarrow \mathrm{L_g}os) \wedge (\forall s)[((\mathrm{PP}rs \wedge \mathrm{SC}s) \rightarrow \sim (\forall t)((\mathrm{R}t \wedge \mathrm{O}ts) \rightarrow \mathrm{L_g}ot))] \qquad \text{(Exact Location*)}$$

The idea behind (Exact Location*) is that an object's exact location is a self-connected region such that all of its proper superregions have subregions that are free of the object (i.e., at which the object is not generically located). The notion of self-connection is a topological notion, and (Exact Location*) itself requires some assumptions about space. In order to understand (Exact Location*) correctly, see how (Exact Location) fails in CMG*.

2.2.1 *(Exact Location) in CMG**

Recall CMG's (Exact Location).

$$\mathrm{L}or =_{\mathrm{df}} (\forall s)((\mathrm{R}s \wedge \mathrm{O}sr) \leftrightarrow \mathrm{L_g}os) \qquad \text{(Exact Location)}$$

As I pointed out in Chapter 1 (section 3.1.3), (Exact Location) is false in theories that lack Functionality. For it fails to express adequate conditions that a region *r* should satisfy in order to be one of the object's exact locations. Suppose, indeed, that an object *o* is exactly located at both *r* and *s*. Then it is clear that all of the regions that overlap either *r* or *s* are regions at which *o* is generically located. Yet the converse does not hold. For there are regions that overlap all of *o*'s generic locations that do not figure among *o*'s exact locations. For instance, the fusion *r* + *s* overlaps all of *o*'s generic locations, and yet *o* is not exactly located at *r* + *s*.

2.2.1.1 Failure of (Exact Location) in CMG*

In his (Varzi 2007b, p. 74), Achille Varzi points out that the relation of exact location is equivalent to the conjunction of entire location and pervasive location (what he calls ubiquitous location). Both definitions were given in Chapter 1 (section 3.1.2 and 3.1.5) and above. To repeat,

$L_p or =_{df} (\exists s)(Los \wedge Prs)$. (Pervasive location)
$L_e or =_{df} (\exists y)(Pyr \wedge Loy)$ (Entire Location)

Varzi's point is thus that

(B) $Lor \leftrightarrow (L_p or \wedge L_e or)$.

An object's exact location is *the* region at which it is *both* pervasively *and* entirely located. Since we are working with a theory that takes (L_g) as a primitive, we need to translate L_p and L_e in terms of L_g. We could do so by using CMG's (Exact Location). But our discussion of (Exact Location*) will be clearer if we follow Varzi in using the following (with some minor modifications to make sure that objects are pervasively and entirely located only at regions).

(C) $L_p or =_{df} (L_g or \wedge (\forall t)((Otr \wedge Rt) \rightarrow L_g ot))$
(D) $L_e or =_{df} (L_g or \wedge (\forall t)(L_g ot \rightarrow Otr))$

Here is how I read the above.

> An object o is *pervasively located* (L_p) at a region *r* just in case (by definition) it is generically located at *r*, and any region that overlaps *r* is a region at which *o* is generically located.
>
> An object *o* is *entirely located* (L_e) at a region *r* just in case (by definition) *o* is generically located at *r*, and everything at which *o* is generically located overlaps *r*.

Note, finally, that we need to specify in (C) that what overlaps *o*'s generic location is a region, but that we do not need to do so in (D), since it follows from the fact that *o* is generically located at something that *that* something is a region (in CMG, at least).

Now that we have (C) and (D), which one of these fails in CMG*? I have just said above that it is false, in CMG*, that everything that overlaps all of the regions at which an object *o* is generically located is one of *o*'s exact locations. The converse, I said, holds: every region that overlaps one of *o*'s exact locations is one of *o*'s generic locations. In CMG*, therefore, (B) does not hold because (D) is false. To see this, suppose a region *r* where *o* is entirely located. For simplicity, take *r* to be one of *o*'s exact locations. Then, as I have said above, there are some regions disjoint from *r* at which *o* is generically located. So (B) fails. Now, one could take that to mean that an object's exact location in theories in which Functionality does not hold is not a region at which the object is *both* pervasively *and* entirely located. But according to my characterization of (WP) (see Chapter 3 (section 2.3)), an object is wholly present wherever it is entirely located. And an object is entirely located at any superregion of its exact location (including its exact location). So, on my view, an object's exact location is *a* region at which it is both pervasively and entirely located, even when Functionality does not hold. What comes out of this discussion is that one should define entire location in a different way than it is defined in (B) in theories in which Functionality does not hold.

Note that (C) minus *its* first conjunct is the first conjunct of (Exact Location*)—that is, (C) holds in theories in which Functionality does not hold. (The first conjunct of (C) could be included in (Exact Location*), but it would be redundant.) The second conjunct of (Exact Location*) cannot be the appropriate definition of entire location in CMG*. For if we define an object *o*'s entire location in CMG* as a region *r* such that any proper (self-connected) superregion of *r* is such that some of its subregions are free of *o*, then we need to add the proviso that the object *o* is generically located at all of *r*'s subregions. (I will have more to say below about the notion of self-connection, and why it is present in (Exact Location*). Recall that a self-connected region is not scattered.) Otherwise, we could simply have a region *r'* at which *o* is only generically located (*r'* has subregions free of *o*), and *r'* would count as *o*'s entire location. But if we add the proviso according to which *o* is generically located at all of *r*'s subregions, then we end up with (Exact Location*), i.e. *o*'s exact location. And we fail to define exact location in CMG* as a region at which the object is both entirely and pervasively located.

So when I say that an object *o*'s exact location in CMG* is a region at which it is both entirely and pervasively located, it is because I define entire location in terms of (Exact Location*) (which is further defined in terms of L_g). Entire location does not help in capturing *o*'s exact location in CMG*, despite the fact that it is true in CMG* that any one of *o*'s exact locations is a region at which it is both pervasively and entirely located.

So let us further explain how we arrive at (Exact Location*).

2.2.1.2 Bad Attempts at Defining Exact Location in CMG*

We are attempting to impose adequate conditions on a region *r* for it to be one of an object's exact locations. The difficulty is that we need to couch these conditions in terms of generic location L_g. And yet we cannot say that all of an object's generic locations overlap one of its many exact locations. In order to define exact location in terms of (L_g), we need to pay attention only to the generic locations that are in the vicinity of one of the object's exact locations. And so one may think that to require that *o*'s generic locations be themselves generically located at *r*, for any *r* that is one of *o*'s exact locations, would do the trick.

More formally, one may think that the following would work as a definition of exact location:

(E) $\mathrm{L}or =_{\mathrm{df}} (\forall s)((\mathrm{R}s \wedge \mathrm{L_g}sr) \rightarrow (\mathrm{O}sr \leftrightarrow \mathrm{L_g}os))$

But it does not. One of the problems with (E) is that we end up with quite an uninformative criterion for a region to be an object's exact location when we read the biconditional from right to left. For

(F) $(\forall s)((\mathrm{R}s \wedge \mathrm{L_g}sr) \rightarrow (\mathrm{L_g}os \rightarrow \mathrm{O}sr))$

claims that any region *s* that is generically located at *r* is such that, if *o* is generically located at *s*, then *s* overlaps *r*. But it already follows from the fact that *s* is generically located at *r* that it overlaps *r* (it is a trivial fact about the location of regions). And (F) is only a consequence of that triviality. Even if it were not trivial, it would be hard to see how *o*'s being generically located at *s* would play any role in *s*'s overlapping of *r* (as it should), given that *s* is generically located at *r* anyway.

The main problem with (E), however, is that it is easy to find a counterexample to it. To see this, consider the model in Figure 7, where *o* is exactly located at regions r_1 and r_2. The region *r*, represented by the dotted lines, is *not* a region at which *o* is exactly located. But it would follow from (E) that it is. Indeed, all of the regions that are generically located at *r* are such that *o* is generically located there just in case they overlap *r*. Obviously, (E) is not a good definition of exact location in CMG*. However, some very plausible assumptions about space, as well as some plausible topological assumptions about an object's exact location, will lead us to the correct definition.

Figure 7: Counterexample to (E)

2.2.2 *A Correct Definition of Exact Location CMG**

2.2.2.1 Assumptions about Space and Exact Location

Let us start by supposing that space is dense. That is to say, for any non-point-sized region of space, there is an infinite number of points making it up. We may define density, following Varzi (Varzi 2007b, p. 77), thus:

$$(\mathrm{R}r \wedge \mathrm{R}s \wedge \mathrm{PP}rs) \rightarrow (\exists t)(\mathrm{PP}ts \wedge \mathrm{PP}rt) \qquad \text{(Density)}$$

Here is how I read (Density):

> If r is a proper part of s and both r and s are regions, then there is a region such that it is a proper part of s and of which r is a proper part.

(Density) does not require that we specify that t is a region, since it follows from (Region Dissection) that it is.

Now, let us say that an object is exactly located at r_1 and also at r_2. These regions are disjoint. And I will stipulate, with (Density) in mind, that there are regions of space (an infinity of points of space) that separate them. ((Location Assumption) below is a formal rendering of that stipulation.) We can use that stipulation, therefore, to define an object's exact location in CMG*. For note that, if there are regions that lie in-between r_1 and r_2, then the model in Figure 7 is ruled out by our definition of exact location. For it is not the case that all of *r*'s subregions are regions at which *o* is generically located (where *r* is a region that is not one of *o*'s exact locations). But the model in Figure 8 is *not* ruled out.

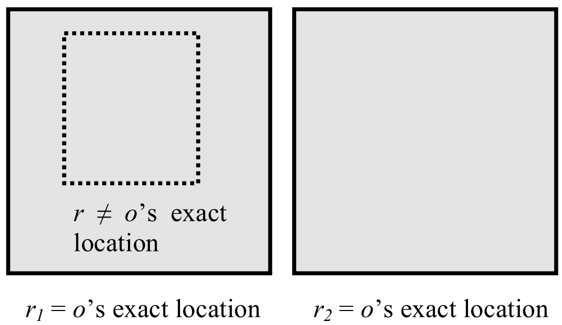

Figure 8: A Model not Ruled Out

For any region *s* that is generically located at *r* is such that *o* is generically located there just in case *s* overlaps *r*. In order to rule out the model in Figure 8, we need to make sure that an object's exact location is not a proper part of another one of the object's exact locations.

How can we specify this requisite with L_g? Intuitively, and given (Density), we would like to say that an object's exact location is such that its (proper) superregions have regions at which *o* is *not* generically located. As it stands, CMG* does not allow us to claim such a thing. For *there are* proper superregions of r_1 (say), all of whose subregions are generic locations of *o*. Take, for instance, the fusion of r_1 and r_2. Instead, we need to claim that the appropriate superregions are self-connected.

The notion of *self-connection* (SC) is a topological notion. Intuitively, entities that are self-connected are neither scattered, nor do they contain holes. In our case, a self-connected superregion of a region *r* is a region that is not scattered. So the fusion of r_1 and r_2 in Figure 8, though a superregion of r_1, is *not* a self-connected superregion of r_1. It is clear that we cannot define SC in mereological terms. For although mereology is the theory of parts and wholes, it has very little to say about wholes (it lacks a whole predicate, so to speak). I believe that the notion of self-connection could be captured within a theory of location.[115] But for reasons of space, I will not discuss this issue. Instead, I will take SC as basic, alongside P and L_g (in CMG*).

So we arrive at an object *o*'s exact location *r* by claiming that any region that overlaps *r* is a region at which *o* is generically located (exactly what (C) above claims), and any self-connected proper superregion *s* of *r* is such that it has subregions at which *o* is *not* generically located. That is precisely what (Exact Location*) claims.

---

[115] Carola Eschenbach and Wolfgang Heydrich (Eschenbach and Heydrich 1995) claim that a simple distinction between regions of space and parts of space that are not regions is enough to define a topology for space. Suppose that we think of points as parts of space that are not regions. Then, we can say that two regions are *connected* (C) just in case they overlap. And we can say that they are *externally connected* just in case they overlap, but have no regions as common parts. Closer to our concern, we could define a self-connected region as a region such that dividing it in two regions leaves us with connected *regions* (or overlapping *regions*) (see (Casati and Varzi 1999, p. 57)). Eschenbach and Heydrich suggest that we should take a region predicate as basic. But our theories of location allow us to define such a region predicate. So, if they are right, it seems that a topology for space could be constructed out of theories of location. Of course, it would come with counterintuitive consequences. First, because they would not be regions of space, points of space would be exactly located nowhere, and would be the location of nothing. Point-size objects (if any) would not have an exact location. These objects, as well as spatial points, would only enjoy a generic location. So they would constitute a counterexample to any claimed equivalence between CEML* and CMG*.

In order to arrive at (Exact Location*), I stipulated, with (Density) in mind, that the exact locations of an object are always "separated" by an infinite number of points of space. Here is the stipulation, more formally.

$$(\mathrm{L}or \wedge \mathrm{R}s \wedge \mathrm{PP}rs \wedge \mathrm{SC}s) \rightarrow (\exists z)(\mathrm{PP}zs \wedge \sim \mathrm{L_g}oz) \qquad \text{(Location Assumption)}$$

Here is how I read it.

> If an object is exactly located at a region *r*, and if *r* is a proper part of a self-connected region *s*, then there is a region that is also a proper part of *s* that is free of *o*.

(Location Assumption) is an axiom of CMG*.

2.2.2.2 Problems with (Exact Location*) and Interesting Consequences

An immediate problem arises with (Exact Location*). It implies that an object's exact location is also a self-connected region. Otherwise, self-connected superregions of an object's exact location will have subregions that are free of the object. But scattered objects exist, and they have exact locations (since they are generically located somewhere (recall (Exactness*)). A bikini is an example. Although it is true that a self-connected superregion of the exact location of a bikini will have subregions at which the bikini is not generically located, that will also be true of some subregions of the bikini's exact location. Suppose a scattered object like the one in Figure 9 (my (in)ability to draw does not allow me to draw a bikini).

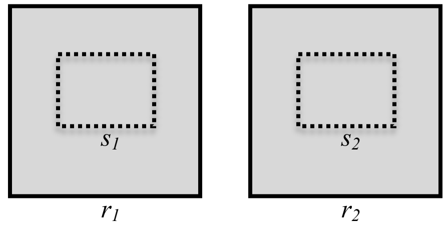

Figure 9: A Scattered Object

In Figure 9, the gray areas represent two parts of a scattered object, and not an object exactly located at two disjoint regions. Take the fusion f of s_1 and s_2, both of which are subregions of the scattered object's exact location. Self-connected superregions of f have subregions at which the object is not generically located, and every region that overlaps f is a region where the scattered object is generically located. And yet, f is not the scattered object's exact location.

An obvious solution to this problem consists in looking for the self-connected parts of scattered objects (what is exactly located at r_1 and at r_2, in Figure 9), and fusing their exact locations in order to obtain the scattered object's exact location. I will overlook this difficulty in what follows. Discussing it would lead me too far afield.

An interesting thing follows from what I have just said. Suppose that a self-connected region r is *not* an object o's exact location, but that it is the case that any region that overlaps r is one of o's generic locations. Then, it is clear that r is a part of one of o's exact locations. Indeed, it cannot be a superregion of one of o's exact locations, since either the latter have subregions at which o is not generically located, or else they are scattered regions. This will be an important part of our demonstration below, where I will show the equivalence of Lor and its definition in CMG*.

2.2.3 *Equivalence of CMG* and CEML**

Let us go back to CMG*. Here are its axioms.

$(\forall s)(L_g os \rightarrow (\exists t)(Lot \wedge Ost))$. (Exactness*)

$(Lor \wedge Los \wedge Ors) \rightarrow r = s$ (Preciseness)

$(\exists x)L_g xr \rightarrow L_g rr$ (Conditional Reflexivity—CMG)

$(\forall z)[((Rz \wedge Ozr) \leftrightarrow (Rz \wedge Ozs)) \leftrightarrow (r = s)] \leftrightarrow (Rr \wedge Rs)$ (Region Identity)

$(Rx \wedge Pyx) \rightarrow Ry$ (Regional Dissection)

$(Lor \wedge Rs \wedge PPrs \wedge SCs) \rightarrow (\exists z)(PPzs \wedge \sim L_g oz)$ (Location Assumption)

And here are some important definitions.

$Lor =_{df} (\forall s)((Rs \wedge Osr) \rightarrow L_g os) \wedge (\forall s)((PPrs \wedge SCs) \rightarrow \sim(\forall t)(Rt \wedge Ots \rightarrow L_g ot))$

(Exact Location*)

$Rr =_{df} L_g rr$ (Region)

I start by showing that one can derive the equivalence

$$Lor \leftrightarrow [(\forall s)((Rs \wedge Osr) \rightarrow L_g os) \wedge (\forall s)((PPrs \wedge SCs) \rightarrow \sim(\forall t)(Rt \wedge Ots \rightarrow L_g ot))]$$

in CMG*.

<u>2.2.3.1 (Exact Location*) in CEML*</u>

Let us start by deriving the following implication

$$Lor \rightarrow [(\forall s)((Rs \wedge Osr) \rightarrow L_g os) \wedge (\forall s)((Rs \wedge PPrs \wedge SCs) \rightarrow \sim(\forall t)(Rt \wedge Ots \rightarrow L_g ot))]$$

in CEML*.

1.	Lor	Assumption
2.	$Rs \wedge Osr$	Assumption
3.	$Rs \wedge Lor \wedge Osr$	$\wedge$-Introduction(1,2)
4.	$(\exists t)(Rs \wedge Lot \wedge Ost)$	$\exists$-Introduction(3)
5.	$(Rs \wedge Osr) \rightarrow (\exists t)(Rs \wedge Lot \wedge Ost)$	$\rightarrow$-Introduction(2-4)

6. $(\forall s)((Rs \wedge Osr) \rightarrow (\exists t)(Rs \wedge Lot \wedge Ost))$ ∀-Introduction(5)
7. $(\forall s)((Rs \wedge Osr) \rightarrow L_g os)$ (Generic Location)
8. $Rs \wedge PPrs \wedge SCs$ Assumption
9. $(Lor \wedge Rs \wedge PPrs \wedge SCs) \rightarrow (\exists z)(PPzs \wedge \sim L_g oz)$ (Location Assumption)
10. $Lor \wedge Rs \wedge PPrs \wedge SCs$ ∧-Introduction(1,8)
11. $(\exists z)(PPzs \wedge \sim L_g oz)$ →-Elimination(9,10)
12. $PPbs \wedge \sim L_g ob$ ∃-Elimination(11)
13. Rb (Regional Dissection)
14. Obs (O) and (12)
15. $Rb \wedge Obs \wedge \sim L_g ob$ ∧-Elimination(12), ∧-Introduction(12,13,14)
16. $(\exists t)(Rt \wedge Ots \wedge \sim L_g ot)$ ∃-Introduction(15)
17. $\sim (\forall t)((Rt \wedge Ots) \rightarrow L_g ot)$ Equivalence(16)
18. $(Rs \wedge PPrs \wedge SCs) \rightarrow \sim (\forall t)((Rt \wedge Ots) \rightarrow L_g ot)$ →-Introduction(8,17)
19. $(\forall s)((Rs \wedge PPrs \wedge SCs) \rightarrow \sim (\forall t)((Rt \wedge Ots) \rightarrow L_g ot))$ ∀-Introduction(18)
20. $(\forall s)((Rs \wedge Osr) \rightarrow L_g os) \wedge (\forall s)((Rs \wedge PPrs \wedge SCs) \rightarrow \sim (\forall t)((Rt \wedge Ots) \rightarrow L_g ot))$ ∧-Introduction(7,19)
21. $Lor \rightarrow (\forall s)((Rs \wedge Osr) \rightarrow L_g os) \wedge (\forall s)((Rs \wedge PPrs \wedge SCs) \rightarrow \sim (\forall t)((Rt \wedge Ots) \rightarrow L_g ot))$ →-Introduction(1,20)

In the derivation above, no use was made of Functionality.

I show that one can derive

$$[(\forall s)((Rs \wedge Osr) \rightarrow L_g os) \wedge (\forall s)((Rs \wedge PPrs \wedge SCs) \rightarrow \sim (\forall t)(Rt \wedge Ots \rightarrow L_g ot))] \rightarrow Lor$$

in CEML*. The derivation is a *reductio*. In order for it to work, I will use the universalization of (Exact Location*) that I gave above. So, instead of starting directly with the constant *r*, I will treat *r* as a variable. The derivation supposes that there are self-connected regions.

1. $(\exists r)(Rr \wedge SCr)$ Premise
2. $(\forall r)\ [(\forall s)((Rs \wedge Osr) \rightarrow L_g os) \wedge (\forall s)((Rs \wedge PPrs \wedge SCs) \rightarrow \sim (\forall t)(Rt \wedge Ots \rightarrow L_g ot))]$ Assumption
3. $Rr \wedge SCr$ ∃-Elimination(1)
4. $\sim Lor$ Assumption (for *reductio*)
5. $(\forall s)((Rs \wedge Osr) \rightarrow L_g os) \wedge (\forall s)((Rs \wedge PPrs \wedge SCs) \rightarrow \sim (\forall t)(Rt \wedge Ots \rightarrow L_g ot))$ ∀-Elimination(2)
6. $(Rr \wedge Orr) \rightarrow L_g or$ ∧-Elimination(5),∀-Elimination(5)

7.	$\mathrm{R}r \wedge \mathrm{O}rr$	$\wedge$-Introduction(3, CEM's Theorem)
8.	$\mathrm{L_g}or$	$\rightarrow$-Elimination(6)
9.	$(\exists t)(\mathrm{R}r \wedge \mathrm{L}ot \wedge \mathrm{O}tr)$	(Generic Location)
10.	$\mathrm{R}r \wedge \mathrm{L}ou \wedge \mathrm{O}ur$	$\exists$-Elimination(9)
11.	$\mathrm{L}ou \wedge \sim \mathrm{L}or$	$\wedge$-Elimination(10),$\wedge$-Introduction(4,10)
12.	$u \neq r$	From (11)[116]
13.	$(\forall s)((\mathrm{R}s \wedge \mathrm{O}su) \rightarrow \mathrm{L_g}os) \wedge (\forall s)((\mathrm{R}s \wedge \mathrm{PP}us \wedge \mathrm{SC}s) \rightarrow \sim (\forall t)(\mathrm{R}t \wedge \mathrm{O}ts \rightarrow \mathrm{L_g}ot))$	$\forall$-Elimination(2)
14.	$(\forall s)((\mathrm{R}s \wedge \mathrm{O}su) \rightarrow \mathrm{L_g}os)$	$\wedge$-Elimination(13)
15.	$(\forall s)((\mathrm{R}s \wedge \mathrm{PP}rs \wedge \mathrm{SC}s) \rightarrow \sim (\forall t)((\mathrm{R}t \wedge \mathrm{O}ts) \rightarrow \mathrm{L_g}ot)$	$\wedge$-Elimination(5)
16.	$(\mathrm{R}u \wedge \mathrm{PP}ru \wedge \mathrm{SC}u) \rightarrow \sim (\forall t)((\mathrm{R}t \wedge \mathrm{O}ut) \rightarrow \mathrm{L_g}ot)$	$\forall$-Elimination(15)
17.	$(\mathrm{R}u \wedge \mathrm{P}ru \wedge u \neq r \wedge \mathrm{SC}u) \rightarrow \sim (\forall t)((\mathrm{R}t \wedge \mathrm{O}ut) \rightarrow \mathrm{L_g}ot)$	(PP)
18.	$\mathrm{R}u$	(Conditional Reflexivity), (Region Predicate)(11)
19.	$\mathrm{SC}u$	Assumption about Exact Location
20.	$(\forall s)((\mathrm{R}s \wedge \mathrm{O}sr) \rightarrow \mathrm{L_g}os)$	$\wedge$-Elimination(5)
21.	$\mathrm{P}ru$	From (3,10,11,20)[117]
22.	$\mathrm{R}u \wedge \mathrm{P}ru \wedge u \neq r \wedge \mathrm{SC}u$	$\wedge$-Introduction(12,18,19,21)
23.	$\sim (\forall t)((\mathrm{R}t \wedge \mathrm{O}ut) \rightarrow \mathrm{L_g}ot)$	$\rightarrow$-Elimination(17,22)
24.	$(\forall s)((\mathrm{R}s \wedge \mathrm{O}su) \rightarrow \mathrm{L_g}os) \wedge \sim (\forall t)((\mathrm{R}t \wedge \mathrm{O}ut) \rightarrow \mathrm{L_g}ot)$	$\wedge$-Introduction(14,23)
25.	$\sim\sim \mathrm{L}or$	From 4 and 24
26.	$\mathrm{L}or$	Double Negation

2.2.3.2 (Generic Location) in CMG*

I now turn to show that the following equivalence

$$\mathrm{L_g}or \leftrightarrow (\exists t)(\mathrm{R}r \wedge \mathrm{L}ot \wedge \mathrm{O}tr)$$

is derivable in CMG*. I start by deriving

$$(\exists t)(\mathrm{R}r \wedge \mathrm{L}ot \wedge \mathrm{O}tr) \rightarrow \mathrm{L_g}or.$$

1.	$(\exists t)(\mathrm{R}r \wedge \mathrm{L}ot \wedge \mathrm{O}tr)$	Assumption

[116] I took a shortcut here. In fact, it follows from Leibniz's Indiscernability of Identicals that regions *u* and *r* are not identical if one is an exact location of *o*, and not the other.

[117] I took a shortcut here. From 20, it follows that any region that overlaps *r* is one of *o*'s generic locations. And I stipulated that *r* was self-connected. It therefore follows that *r* is a part of one of *o*'s exact locations. That was indeed the interesting consequence observed above. Because *u* overlaps *r* (line 10), and none of an object's exact locations overlap (cf. (Preciseness)), I conclude that *r* is a part of *u*.

2. $\mathrm{R}r \wedge \mathrm{L}ou \wedge \mathrm{O}ur$ ∃-Elimination(1)
3. $\mathrm{R}r \wedge (\forall s)((\mathrm{R}s \wedge \mathrm{O}su) \rightarrow \mathrm{L_g}os) \wedge (\forall s)((\mathrm{R}s \wedge \mathrm{PP}us \wedge \mathrm{SC}s) \rightarrow \sim (\forall t)(\mathrm{R}t \wedge \mathrm{O}ts \rightarrow \mathrm{L_g}ot)) \wedge \mathrm{O}ur$ (Exact Location*)
4. $(\forall s)((\mathrm{R}s \wedge \mathrm{O}su) \rightarrow \mathrm{L_g}os)$ ∧-Elimination(3)
5. $(\mathrm{R}r \wedge \mathrm{O}ru) \rightarrow \mathrm{L_g}or$ ∀-Elimination(4)
6. $\mathrm{R}r \wedge \mathrm{O}ur$ ∧-Elimination(2)
7. $\mathrm{L_g}or$ →-Elimination(5,6)

Next, I will show that

$$\mathrm{L_g}or \rightarrow (\exists t)(\mathrm{R}r \wedge \mathrm{L}ot \wedge \mathrm{O}tr)$$

is derivable in CMG*. I will make use of (Exactness*) in order to do so.

1. $\mathrm{L_g}or$ Assumption
2. $(\forall s)(\mathrm{L_g}os \rightarrow (\exists t)(\mathrm{L}ot \wedge \mathrm{O}st))$ (Exactness*)
3. $\mathrm{L_g}or \rightarrow (\exists t)(\mathrm{L}ot \wedge \mathrm{O}rt)$ ∀-Elimination(2)
4. $(\exists t)(\mathrm{L}ot \wedge \mathrm{O}rt)$ →-Elimination(2,3)
5. $\mathrm{L}ou \wedge \mathrm{O}ru$ ∃-Elimination(4)
6. $\mathrm{R}r$ (Conditional Reflexivity),(Region Predicate)
7. $\mathrm{R}r \wedge \mathrm{L}ou \wedge \mathrm{O}ru$ ∧-Introduction(5,6)
8. $(\exists t)(\mathrm{R}r \wedge \mathrm{L}ot \wedge \mathrm{O}rt)$ ∃-Introduction(7)

3. Conclusion

In this appendix, I was concerned with showing that there are theories of location in which Functionality does not hold, but which are nevertheless equivalent. In presenting the theories in which Functionality does hold, I was careful not to depart too much from the axiomatization to be found in (Parsons 2007) and (Varzi 2007b). However, the definition of exact location in CMG*, i.e., (Exact Location*), would also work quite well in theories in which Functionality holds and which take ($\mathrm{L_g}$) as basic. Of course, we would then have to add (Functionality) as an axiom, just as it is done in CEML. This shows, again, that Functionality is in no way necessary in theories of location.

Bibliography of Cited Works

Adams, R. M. (1979). "Primitive Thisness and Primitive Identity." *The Journal of Philosophy, 76*(1), 5-26.

Allaire B., E. (1963 (1976)). "Bare Particulars." In *Universals and Particulars: Readings in Ontology* (pp. 248-254): University of Notre Dame.

Armstrong, D. M. (1978a). *Nominalism and Realism: Universals and Scientific Realism* (Vol. 1). Cambridge: Cambridge University Press.

Armstrong, D. M. (1978b). *A Theory of Universals: Universals and Scientific Realism* (Vol. 2). Cambridge: Cambridge University Press.

Balashov, Y. (2008). "Persistence and Multilocation in Spacetime." In D. Dieks (Ed.), *The Ontology of Spacetime* (Vol. 2). Elsevier: Philosophy and Foundations of Physics Series.

Barker, S., & Dowe, P. (2003). Paradoxes of Multi-Location. *Analysis, 63*(2), 106-114.

Barker, S., & Dowe, P. (2005). Endurance is Paradoxical. *Analysis, 65*(1), 69-74.

Beebee, H., & Rush, M. (2003). Non-Paradoxical Multi-Location. *Analysis, 63*(4), 311-317.

Bennet, K. (2004). "Spatio-Temporal Coincidence and the Grounding Problem." *Philosophical Studies, 118*(3), 339-371.

Benovsky, J. (2008). "The bundle theory and the substratum theory: deadly enemies or twin brothers?" *Philosophical Studies, 141*, 175-190.

Black, M. (1952). "The Identity of Indiscernibles." *Mind, 61*(242), 153-164.

Casati, R., & Varzi, A. C. (1999). *Parts and places : the structures of spatial representation*. Cambridge, Mass.: MIT Press.

Casullo, A. (1988). "A Fourth Version of the Bundle Theory." *Philosophical Studies, 54*(1), 125-139.

Chisholm, R. (1976). "The Persistence of Persons." In Kim, Jaegwon and Sosa, Ernest (Ed.), *Metaphysics: An Anthology* (1999), Oxford: Blackwell.

Chisholm, R. M. (1971). "Problems of Identity." In M. Munitz (Ed.), *Identity and Individuation* (pp. 3-30). New York: NYU Press.

Chisholm, R. M. (1973). "Parts as Essential to Their Wholes." *The Review of*

Metaphysics, 26, 581-603.

Crisp, T. M., & Smith, D. P. (2005). “"Wholly Present" Defined.” *Philosophy and Phenomenological Research, 71*, 318-344.

Denkel, A. (1996). *Object and Property*. Cambridge: Cambridge University Press.

Diekemper, J. (2009). “Thisness and Events.” *The Journal of Philosophy, 106*(5), 255-276.

Donnelly, M. (2010). “Parthood and Multi-Location.” *Oxford Studies in Metaphysics*, vol. 5. Oxford: Oxford University Press.

Eddon, M. (2010). “Why Four-Dimensionalism Explains Coincidence.” *Australasian Journal of Philosophy, 88*(4), 721-728.

Eschenbach, C., & Heydrich, W. (1995). “Classical mereology and restricted domains.” *International Journal of Human-Computer Studies, 43*, 723-740.

Fine, K. (2003). “The Non-Identity of a Material Thing and Its Matter.” *Mind, 112*, 195-234.

Fine, K. (2008). “Coincidence and Form.” *Proceedings of the Aristotelian Society, Supplementary Volume 82*, 101-117.

Gallois, A. (1998). *Occasions of identity : a study in the metaphysics of persistence, change, and sameness*. Oxford: Oxford University Press.

Gibbard, A. (1975). “Contingent Identity.” *Journal of Philosophical Logic, 4*.

Gibson, I., & Pooley, O. (2006). “Relativistic Persistence.” *Philosophical Perspectives, 20*(Metaphysics), 157-198.

Gilmore, C. (2006). “Where in the Relativistic World are We?” *Philosophical Perspectives, 20*, 199-236.

Gilmore, C. (2007). “Time Travel, Coinciding Objects, and Persistence.” In D. Zimmerman (Ed.), *Oxford Studies in Metaphysics* (Vol. 3). Oxford: Clarendon Press.

Hacking, I. (1975). “The Identity of Indiscernibles.” *The Journal of Philosophy, 72*(9), 249-256.

Haslanger, S. (1989). “Endurance and Temporary Intrinsics.” *Analysis, 49*, 119-125.

Hawley, K. (2001). *How things persist*. Oxford: Oxford University Press.

Hawley, K. (2009). “Identity and Indiscernibility.” *Mind, 118*, 101-119.

Hawthorne, J. (2006). "Three-Dimensionalism." In *Metaphysical Essays* (pp. 85-109). Oxford: Clarendon Press.

Hawthorne, J. (2008). "Three-Dimensionalism vs. Four-Dimensionalism." In T. Sider, J. Hawthorne, & D. Zimmerman (Eds.), *Contemporary Debates in Metaphysics*. Oxford: Blackwell Publisher.

Hawthorne, J. O. L., & Cover, J. A. (1998). "A World of Universals." *Philosophical Studies: An International Journal for Philosophy in the Analytic Tradition, 91*(3), 205-219.

Hirsch, E. (1982). *The Concept of Identity*. Oxford: Oxford University Press.

Hofweber, T. (2009). "The Meta-Problem of Change." *Noûs, 43*(2), 286-314.

Hofweber, T., & Velleman, D. (2011). "How to Endure." *The Philosophical Quarterly, 61*(242), 37-57.

Hovda, P. (2009). "What is Classical Mereology?" *Journal of Philosophical Logic, 38*, 55-82.

Hudson, H. (2001). *A Materialist Metaphysics of the Human Person*. Ithaca: Cornell University Press.

Hudson, H. (2005). *The Metaphysics of Hyperspace*. Oxford: Oxford University Press.

Johnston, M. (1987). "Is There a Problem about Persistence?" In S. Haslanger, & R. M. Kurtz (Eds.), *Persistence: Contemporary Readings* (pp. 241-265). Cambridge: MIT Press.

Leibniz, G. W. (1991). *Discourse on Metaphysics and Other Essays*. Indianapolis: Hackett Publishing Company.

Leonard, H. S., & Goodman, N. (1940). "The Calculus of Individuals and Its Uses." *The Journal of Symbolic Logic, 5*(2), 45-55.

Lewis, D. (1976). "Survival and Identity." In A. O. Rorty (Ed.), *The Identities of Persons* (pp. 17-40). Berkeley: University of California Press.

Lewis, D. (1986). *On the plurality of worlds*. Oxford, UK ; New York, NY, USA: B. Blackwell.

Lewis, D. (1990). *Parts of classes*. Oxford, UK ; Cambridge, Mass., USA: B. Blackwell.

Locke, J. (1689 (2004)). *An Essay Concerning Human Understanding*. New York: Barnes and Noble Books.

Mackie, P. (2006). *How Things Might Have Been*. Oxford: Clarendon Press.

McDaniel, K. (2001). “Tropes and Ordinary Physical Objects.” *Philosophical Studies, 104*(3), 269-290.

McDaniel, K. (2003). “No Paradox of Multi-Location.” *Analysis, 63*(4), 309-311.

McDaniel, K. (2004). “Modal Realism With Overlap.” *Australasian Journal of Philosophy, 82*(1), 137-152.

McDaniel, K. (2007). “Extended Simples.” *Philosophical Studies, 133*, 131-141.

McKinnon, N. (2002). “The Endurance/Perdurance Distinction.” *Australasian Journal of Philosophy, 80*(3), 288-306.

Merrick, T. (1994). “Endurance and Indiscernibility.” *The Journal of Philosophy, 91*(4), 165-184.

Miller, K. (2005). “A New Definition of Endurance.” *Theoria, 4*, 309-332.

Moyer, M. (2009). “Does Four-Dimensionalism Explain Coincidence?” *Australasian Journal of Philosophy, 87*(3), 479-488.

O'Leary-Hawthorne, J. (1995). “The Bundle Theory of Substance and the Identity of Indiscernibles.” *Analysis, 55*, 191-196.

O'Leary-Hawthorne, J., & Cover, J. A. (1997). “Framing the Thisness Issue.” *Australasian Journal of Philosophy, 75*(1), 102-108.

Oliver, A. (1996). “The Metaphysics of Properties.” *Mind, 105*(417), 1-80.

Olson, E. T. (2001). “Material Coincidence and The Indiscernibility Problem.” *The Philosophical Quarterly, 51*(204), 337-335.

Parsons, J. (2007). “Theories of Location.” In *Oxford Studies in Metaphysics* (pp. 201-232). Oxford: Oxford University Press.

Parsons, J. (2008). “Hudson on Location.” *Philosophy and Phenomenological Research, 76*(2), 427-435.

Paul, L. A. (2002). “Logical Parts.” *Noûs, 36*(4), 578-596.

Paul, L. A. (2006). “Coincidence as Overlap.” *Noûs, 40*(4), 623-659.

Pontow, C. (2004). “A Note of the Axiomatics of Theories in Parthood.” *Data and Knowledge Engineering, 50*, 195-213.

Rea, M. (1998). “Sameness Without Identity: An Aristotelian Solution to the Problem of Material Constitution.” *Ratio, 11*(3), 316-328.

Rodriguez-Pereyra, G. (2004). “The Bundle Theory is compatible with distinct but indiscernible particulars." *Analysis, 64*(1), 72-81.

Rodriguez-Pereyra, G. (2006). "How Not to Trivialize the Identity of Indiscernibles." In P. F. Strawson, & A. Chakrabarti (Eds.), *Universals, Concepts and Qualities: New Essays on the Meaning of Predicates* (pp. 205-223). Aldershot: Ashgate Publishing Limited.

Rosenkrantz, G. S. (1993). *Haecceity: An Ontological Essay* (Philosophical Studies Series). Norwell: Kluwer Academic Publisher.

Russell, B. (1962 (1940)). *An Enquiry into Meaning and Truth*. Baltimore, Md: Penguin Books.

Sattig, T. (2006). *The Language and Reality of Time*. Oxford: Oxford University Press.

Schaffer, J. (2001). "The Individuation of Tropes." *Australasian Journal of Philosophy, 79*(2), 247-257.

Sider, T. (1997). "Four-Dimensionalism." *Philosophical Review, 106*, 197-231.

Sider, T. (1999). "Global Supervenience and Identity across Times and Worlds." *Philosophy and Phenomenological Research, 59*, 913-937.

Sider, T. (2001). *Four-dimensionalism : an ontology of persistence and time*. Oxford: Oxford University Press.

Sider, T. (2007). "Parthood." *Philosophical Review, 116*, 51-91.

Simons, P. M. (1987). *Parts : a study in ontology*. Oxford: Oxford University Press.

Swinburne, R. (1995). "Thisness." *Australasian Journal of Philosophy, 73*(3), 389-400.

Thomson, J. J. (1983). "Parthood and Identity Across Time." *The Journal of Philosophy, 80*(4), 201-220.

Thomson, J. J. (1998). "The Statue and the Clay." *Noûs, 32*, 149-173.

van Cleve, J. (1985). "Three Versions of the Bundle Theory." *Philosophical Studies, 47*(1), 95-107.

van Cleve, J. (1986). "Mereological Essentialism, Mereological Conjunctivism, and Identity Through Time." *Midwest Studies in Philosophy, 11*(1), 141-156.

van Inwagen, P. (1981). "The Doctrine of Arbitrary Undetached Parts." In *Ontology, Identity, and Modality: Essays in Metaphysics*, Cambridge: Cambridge University Press.

van Inwagen, P. (1990a). "Four-Dimensional Objects." In *Ontology, Identity, and Modality: Essays in Metaphysics*, Cambridge: Cambridge University Press.

van Inwagen, P. (1990b). *Material Beings*. Ithaca: Cornell University Press.

Varzi, A. C. (2005). “Change, Temporal Parts, and the Argument from Vagueness.” *Dialectica, 59*(4), 485-498.

Varzi, A. C. (2007a). “Promiscuous Endurantism and Diachronic Vagueness.” *American Philosophical Quartely, 44*, 181-189.

Varzi, A. C. (2007b). “Spatial Reasoning and Ontology: Parts, Wholes, and Locations.” In M. Aiello, I. Pratt-Hartmann, & J. v. van Benthem (Eds.), *The Logic of Space* (pp. 1-99). Dordrecht: Kluwer Academic Publishers.

Varzi, A. C. (2008). “The Extensionality of Parthood and Composition.” *The Philosophical Quarterly, 58*(230), 108-133.

Zimmerman, D. (1995). “Theories of Masses and Problems of Constitution.” *The Philosophical Review, 104*(1), 53-110.