Order Number 9405575

Optimal government policies in a cash-in-advance environment

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City University of New York, 1993

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OPTIMAL GOVERNMENT POLICIES IN

A CASH-IN-ADVANCE ENVIRONMENT

by

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A dissertation submitted to the Graduate Faculty in Economics in partial fulfillment of the requirements for the degree of Doctor of Philosophy, The City University of New York.

1993

This manuscript has been read and accepted for the Graduate Faculty in Economics in satisfaction of the dissertation requirement for the degree of Doctor of Philosophy.

9/22/93

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Abstract

OPTIMAL GOVERNMENT POLICIES IN
CASH-IN-ADVANCE ENVIRONMENT
by
Spyridon Pipinis

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This thesis is concerned with the derivation of optimal monetary and fiscal policy in a typically employed monetary economy with optimizing agents and endogenous production. The demand for money is motivated by a cash-in-advance constraint which forces individuals to hold money for transaction purposes.

We derive the optimal monetary and fiscal policy by integrating the behavior of all three agents to one, in a dynamic competitive equilibrium context, and solve only the government's problem.

In addition to policy recommendations, the results show stochastic fluctuations in the total-output time path, representing periods of recessions and periods of expansions due to the stochastic component of the model economy. They also show that the level and direction of this time path depend on the particular fiscal and monetary policy that the government follows. Money has real effects in this setting.

Acknowledgements

First of all, I would like to thank the faculty members of the Department of Economics at the Graduate Center of the City University of New York for offering me the opportunity to a graduate study in Economics and for their support all these years.

Specially, I would like to thank my teacher and advisor professor Salih Neftci for his great influence in my economic way of thinking and for his encouragement that led me to conclude successfully my thesis work.

I would like also to thank the faculty members of the Department of Economics at Hunter College of the City University of New York for the collaboration in teaching matters, during the years I was holding a position there.

In addition, I would like to thank my cousin Ernest Daniels for his support and encouragement.

Above all, I would like to express my deep thanks to my fried George Tjionas for his help in developing the computer software, a necessity for the project's calculations.

September 3, 1993

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INTRODUCTION

Dynamic macroeconomics deals mainly with the determination of equilibrium time paths of the economy-wide variables of total output, per-capita consumption, employment, investments, and the price level. Moreover, it deals with the study of the inflation rate, the budget deficit, and the growth rate of the total output.

Understanding and controlling are the motivating forces for studying the behavior of the economic system and, to fulfil this aim, macroeconomics uses both abstract theory and empirical procedures.

In a continuous effort, this science tries to reveal that mechanism that generates the behavior of the actual economic system, and to explain the movements and the co-movements of the endogenously determined variables and their association with the exogenously determined ones. Therefore, it attempts to explain the fluctuations in the total production level by determining the reasons for expansions and recessions in the business cycle. In addition, by using the proper instruments, it seeks to control the performance of the macroeconomic magnitudes.

In the attempt to fulfil these purposes, there is a plethora of opinions and options and, consequently, controversies exist and questions are raised.

Do money and the form of money influence the real sector of the economy?

Should the government get involved in the operation of the economic system, and if so, how should monetary and fiscal policy be conducted?

Since its formulation as a separate branch of economics, macroeconomics has experienced a tremendous expansion in its development. This expansion is due partially to the development of new techniques in mathematics and the wide application of these techniques, traditional and new, in the field, and partially to the wide use of computers, and software and hardware innovations.

Of all these new mathematical techniques, control theory and dynamic programming played the most important role in the development of macroeconomics.

During recent years, emphasis has been given to models with agents optimizing their objective function. In this endeavor, the application of control theory and dynamic programming improved the ability of macroeconomics to approximate better the actual economic system. In addition, these techniques have made an important contribution in developing new orientations for the science.

Using the control-theory framework, macroeconomics tries to explain the way limited resources are allocated among competitive individuals in a dynamic sense. To fulfil this aim, time trajectories of the control variables are chosen in order to optimize the objective functional that describes the preferences of the society, taking under consideration the equations of motions, the feasibility of the control variable(s), and the initial values of the variables and parameters.

But the proper applications of these innovations need the proper environment. The massive use of computers with their hardware and software discoveries enables the macroeconomic researcher, with relatively low cost and little money, to test the behavior and performance of an economic model. In these tests the researcher can now determine the consequences of alternative government policies on the variables that characterize the performance of the economic system, and he/she can suggest policies that result in the most desirable outcome. Without this experimental environment provided by computers, the testing in reality of alternative policies would probably be prohibitive because of the political and economic costs.

This laboratory economic environment has generated a vast number of "computer type" or "artificial model" economies, and following traditions, we will employ in this thesis a model called Cash-In-Advance that recently has been used extensively in studying macroeconomic systems. The basic characteristic of this type of model is that the demand for money is motivated by a cash-in-advance constraint which forces individuals to hold money for transaction purposes. We will use this type of model not only to understand the operation of the economic system but also to make policy recommendations for the government.

More specifically, in this thesis, we investigate the way the optimal monetary and fiscal policy should be conducted in a dynamic monetary competitive equilibrium framework. We also compare the consequences of this optimal government policy on the time paths of the endogenously determined variables and on the corresponding welfare level with the consequences resulting from the application of an alternative monetary and fiscal policy, the so called the "sub-optimal" one.

To reach our destination, we use a one-sector growth model subject to stochastic fluctuations due to a technology shock in the supply side and due to fiscal and monetary policy shocks in the demand side.

We study a model economy that has no capital, no investments, no bond and stock markets, no international trade but includes money. We assume that all exchanges involve the use of fiat money which is introduced via a cash-in-advance or a Clower constraint.

Money is restricted to follow a particular flow. Nominal cash holdings that the individual carries from the previous period [t-1] finance consumption expenditure of the current period and money made through labor finances the purchase of the cash good in the next period [t+1]. The labor income of the current period cannot finance current consumption.

In addition, we assume that this model excludes Keynesian-type wage setting and allows the perfect-competition condition in the labor market to determine the wage rate endogenously.

We are dealing here with a perfect-foresight monetary economy where firms, households, and the government are involved in a dynamic differential game setting, in a discrete sense, where optimizing motivation is subject to the initial conditions and to the equations of motion and where the solution time paths of the endogenous variables result from the "cooperative" game interaction among these three agents.

We use mathematical techniques to integrate the three optimizing problems to one, and we use numerical methods technique to solve its final version.

More precisely, the work starts with formulation of the basic model to be used in the derivation of the optimal policy. This procedure involves two steps.

In the first step, we formulate the household's, firm's and the government's problems. We begin by assuming that this economy is populated with a fixed number of identical households living for ever, and we endow the household with one unit of time that can be spent either in supplying labor to firms or in leisure. The representative household faces consumption and labor-supply decisions. The objective is to maximize its utility function, subject to budget and cash-in-advance constraints. In this model formulation, the price level and the wage and tax rates are taken as given. We define also the cash and the credit good, something which will be very useful later in explaining the interaction between the nominal and the real variables of this economy. We introduce here the assumption of the indivisibility of labor and the existence of an employment lottery to ensure that the representative household works, with a given probability, a certain number of hours per period, as full time or not at all. We specify also the exact preference form, as well as the two types of constraints, the representative consumer faces.

Then, we proceed to the firm's problem. There exists a large number of identical firms which produce the single, non-storable output of this model economy. Each firm faces a labor demand-decision, and its objective is to maximize its profit function, subject to a technology constraint expressed by a constant return-to-scale production function that relates the total production of the output to the productivity factor and to the employment level.

In this artificial economy, there is a government which faces monetary and fiscal policy decisions. Its aim is to maximize the representative household's objective function, subject to the government's budget constraint which states that the exogenously given government expenditure time path must be financed partially by revenues from taxes on income and partially by revenues from seignorage.

In section II, we proceed to the second step of our demonstration. Here, we integrate the behavior of all three agents to one and develop the final version of our problem. This can be done by optimizing the representative household's and firm's objective functions, taking under consideration the constraints they face, and thereby developing the first-order conditions for each optimization. Before continuing into the final formulation of our problem, we develop the basic notions that characterize the dynamic competitive equilibrium the economy is assumed to operate under, as well as the notions concerning the government's cooperative behavior in synchronizing the private agents' actions. In the final formulation, we take under consideration the government's motivation, its budgetary constraints, the first-order conditions resulting from the optimization procedures of the other two agents, and the economy-wide assumption of market clearing. We choose to solve the government's problem as the final model because only the government has the ability to influence the relationship between the objectives of the individuals, on one hand, and the variables they take as given, on the other.

We employ the numerical-methods technique as the solution mechanism to this optimization problem, because the derivation of the control variable(s) may result in money-supply and total-production time paths which are not stationary.

In section III, we show our computational results. In the first part of this section, we solve the model by assuming that, for each period [t], the government specifies its spending level [G_t] as well as the tax-rate level [H_t] and has available and uses only one of its control instruments, monetary policy.

Our results show that different money-supply policies generate different employment, total output, and per-capita consumption paths, leading to different welfare levels. The government chooses as the optimal monetary policy the one that generates the highest value in the consumer's utility function.

Money has real effects in this economy. High levels of anticipating inflation, due to high growth rates of money expansion, force the individual consumer to substitute leisure for the taxable cash good, lowering then the employment time path, and thereby lowering the corresponding output and per-capita consumption levels.

We perform also another experiment in this setting by investigating the change of tax policy in the endogenous variables of the model, keeping monetary and government spending policies constant.

The data show that high tax rates result in lower employment and consequently lower output and consumption time paths. This lowering happens because high income-tax rates force consumers to substitute the taxable cash good with the nontaxable credit good, leading consequently to a lower employment time path.

In the second part of section III, the government specifies only the expenditure policy and is able to use both of its available instruments, the monetary and the tax rate policies. Our findings show that different combinations of monetary and fiscal policies result in different time paths of the economy-wide endogenous variables and, consequently, in different welfare levels. The government chooses that combination of fiscal and monetary policies that maximizes the representative consumer's objective function.

The important observation of this section is that the coordination of both, fiscal and monetary, policies leads to an improvement of the representative household's utility function, compared with the utility level that the individual consumer attains when the government has available and uses only one of its control instruments, monetary policy.

Finally, in the graph section, we provide graph illustrations to support our conclusions.

The purpose of this work is not to match our projections with real data but to make government policy recommendations in an agent-optimizing environment.

In addition to policy recommendation, the generated data show stochastic fluctuations in the total-output time path, representing periods of recession and periods of expansion due to the stochastic components of the model economy. They also show that the level and the direction of this time path depend on the particular fiscal and monetary policy that the government follows.

The association of the employment time path with the government's monetary policy implies a positive Phillips Curve relationship, where high levels of inflation are associated with a high levels of unemployment and where high levels of employment can be attained by low-inflation monetary policies.

Part of the results of this work is similar to the results in the work of others, for example, Carmichael (1989), Cooley and Hansen (1989), and Lucas (1986).

I. PROBLEM FORMULATION

This "computer type" monetary economy consists of three kinds of players: households, firms and the government.

It is an economy that excludes international trade and has no investments, no bond or stock markets and no capital of any type.

Time is used here in a discrete sense.

a) The household's problem

It is assumed that this economy is populated with a finite number [N] of identical households living for ever.

Also, it is assumed that there are two types of goods, the cash good and the credit good.

Each period [t] households are endowed with one unit of time that can be used for leisure or for supplying labor to firms as input for the production of the total output of the economy.

$$L_t = 1 - Z_t \qquad \text{(a)}$$

L_t : labor time supplied by the household at period [t]

Z_t : leisure enjoyed by the household at period [t].

Leisure is supposed to be the credit good in the sense that the household first enjoys it and then pays later.

Each period, the household faces two types of expenditure. The first expenditure is for the consumption good:

$$P_t \, C_t$$

P_t : price level at period [t]

C_t : consumption good at period [t].

Consumption good is the cash good in the sense that the household pays for it first and enjoys it later.

The second type of expenditure that the household faces is the tax on its labor income:

$$H_t \, W_t \, L_t$$

W_t : nominal wage rate at period [t]

H_t : tax rate at period [t].

The household starts its time period [t] with [m_{t-1}] cash holdings carried over from the previous period. These cash holdings are augmented during the current period only by the income of the household, the labor income, which is equal to the labor [L_t], supplied by the household, times the wage rate [W_t].

After using the cash balances from the previous period and the labor income of the current one to finance both types of expenditure, namely consumption and taxes, the household will end up at the end of the current period [t] with [m_t] nominal cash holdings to be available for use, the next period [t+1].

At this point we can formulate the first constraint that the household faces:

$$m_{t-1} + W_t L_t = P_t C_t + H_t W_t [1 - Z_t] + m_t$$

Then by substituting equation (a) into the above constraint, we get its final form:

$$m_{t-1} + W_t L_t = P_t C_t + H_t W_t L_t + m_t$$

In addition, we assume that the consumption good of the current period can be purchased only by the nominal cash holdings that the household carries over from the previous period. Following Lucas and Stokey (JME 1983), we can reason as follows.

During period [t], the representative household engages in two different activities: shopping and working. In its first activity, the household spends the whole period shopping, purchasing the cash good, and in its second one working, purchasing the other type of good, the credit good. There is no way that the two different activities of the representative household can combine during the current period [t], to allow current labor income to finance the current consumption expenditure.

Having this under consideration, we can formulate the second type of constraint that the representative household faces, the so called cash-in-advance or the Clower constraint:

$$P_t C_t = m_{t-1}$$

It is assumed that this constraint holds with equality and later we will develop the necessary and sufficient conditions to support it.

The household, therefore, spends all the nominal balances [m_{t-1}] on cash good replacing them with the labor income during the current period to satisfy the tax expenditure, and to create the nominal cash balances [m_t] to be available to finance the purchase of the cash good next period.

It is the cash-in-advance constraint that forces the economy to hold money with the purpose to be used as a medium of exchange. Money is needed to purchase the cash good and this transaction takes place before labor income becomes available to the consumer.

The representative household obtains utility by consuming both types of goods, the cash good [C_t] and the credit good [Z_t]. It is assumed here that for the current period [t] the household faces a logarithmic utility function with the following form:

$$U\ (C_t,\ Z_t) = [\ \log C_t + Q \log Z_t\ ] \qquad \text{where } Q > 0$$

The purpose of the household is to maximize the intertemporal version of the above time-invariant, logarithmic utility function which now becomes

$$U = \sum_{t=0}^{\infty} b^t\ (\log C_t + Q \log Z_t)$$

where [∞] stands for infinity and [b^t] is the discount factor that shows the weight the representative household puts on current over future consumption-leisure decisions. The fact that future utility levels get less and less weight, but a positive one, forces the discount factor to take values that are positive but less than unity.

Following Cooley and Hansen (AER 1989), we introduce here the assumption of indivisibility of labor and the existence of the employment lottery which allow some of the households to be unemployed. The assumption of the indivisibility of labor basically implies that fluctuation in employment is due to the fluctuation in the number of employed households and not to the fluctuation in the average work hours per period. To ensure that the solution of the household optimization problem, in the competitive-equilibrium framework, exists, it is necessary to convexify the consumption possibility set. This can be done by introducing into the model an employment lottery and the existence of a contract between the firm and the household that commits the household to work [L_t] hours per period with probability [a_t]. This contract can be traded between households and since all of them are identical, they all going to choose the same probability [a_t]. It is the assumption of the indivisibility of labor and the existence of the employment lottery that ensures that a fraction [a_t] of the households will work, as full-time, a given number of hours [L_0] and the remaining [$1-a_t$] fraction will be unemployed. The lottery determines which of the household is going to be employed and which not. We can, therefore, say that the employment level of the representative household during period [t] is given by the following equation:

$$L_t = a_t L_0$$

where

L_0 : fixed number of hours worked by the employed household.

Then, the representative household has an expected utility in period [t] given by the expression :

$$a_t [\log C_t + Q \log(1-L_0)] + (1 - a_t)[\log C_t + Q \log 1]$$

and after rearranging it becomes

$$U(C_t, L_t) = a_t \log C_t + a_t Q \log(1-L_0) + \log C_t - a_t \log C_t$$

or

$$U(C_t, L_t) = \log C_t + a_t Q \log(1-L_0)$$

by using the equation

$$L_t = a_t L_0$$

we get

$$U(C_t, L_t) = \left[\log C_t + \frac{Q \log(1-L_0) L_t}{L_0} \right]$$

and by setting

$$B = -Q\left[\frac{\log(1-L_0)}{L_0}\right]$$

we get

$$U(C_t, L_t) = [\log C_t - B L_t]$$

which is the same utility function used in Hansen (1985), Cooley and Hansen (1989), and Hansen and Sargent (1988).

As mentioned earlier, the purpose of the representative household is to maximize an intertemporal utility function which is equal to the summation of the discounted values of the utility the household enjoys at each period, subject to budget and cash-in-advance constraints.

In a mathematical frame, the household's problem takes the following form:

$$\max_{C_t,\ L_t} \quad U = \sum_{t=0}^{\infty} b^t \ [\log C_t - B L_t]$$

s.t.

1) $m_{t-1} + W_t L_t = P_t C_t + H_t W_t L_t + m_t$

2) $P_t C_t = m_{t-1}$

3) The non-negative constraints on C_t, L_t, P_t

[t] presents here a quarter time period.

The solution to the above optimization problem implies that given the time paths for the price level [P_t], the wage rate [W_t], the tax rate [H_t] and the initial value of the nominal cash holdings [m_{-1}] which the household takes as given, the representative consumer chooses time paths for [C_t] and [L_t] that maximize his/ her utility function subject to the above stated constraints.

□

b) The firm's problem

This economy is populated with a large number [s] of identical firms. Each firm operates under perfect competition in the product market in which it can sell as much output as it desires at the market price level [P_t] and under perfect competition in the labor market in which it can hire as much labor as it desires at the market nominal wage rate [W_t]. Firms use the labor supplied by the household [L_t], at a particular period [t], as input, to produce economy's output during the same period.

Each firm's objective is to maximize its profits which we define as

$$\Pi_t = P_t \, y_{it} - W_t \, L_{it}$$

by choosing the appropriate amount of labor to employ, subject to the technology constraint expressed by a constant return-to-scale production function of the following form:

$$y_{it} = A_t \, f_t \, L_{it}$$

where

[L_{it}] : labor demanded by the typical firm (i) from the representative household at period [t]

[y_{it}] : per capita output produced by the typical firm (i) at period [t]

$[A_t]$: temporary change in labor productivity at period [t] and

$[f_t]$: the realization of a permanent, exogenously given, technological shock at period [t] known to all agents.

It is assumed that all firms face the same production function and all produce the same, single output which is non-storable. Mathematically, the firm's problem is as follows:

$$\max_{L_{it}} \quad \Pi_t = P_t\, y_{it} - W_t\, L_{it}$$

s.t.

1) $y_{it} = A_t\, f_t\, L_{it}$

2) $[\, A_t\,]_{t=0}^{\infty}$

3) $[\, f_t\,]_{t=0}^{\infty}$

4) **taking $[P_t]$, $[W_t]$ as given.**

It is assumed also that the productivity shock $[f_t]$, behaves stochastically according to the first order autoregressive equation with drift.

so

$$f_t = p_0 + p_1\, f_{t-1} + e_t$$

where:

p_0 and p_1 are constants and

$[e_t]$ is a random variable following the normal distribution with zero mean and variance [v_{et}].

Variables [A_t] and [f_t] introduce the first type of uncertainty into the system in the form of productivity shocks to technology on the supply side of the economy.

The solution to the above optimization problem implies that given the time paths for the price level [P_t] and the wage rate [W_t], the firm chooses the time path for the demanded quantity of labor [L_{it}] with an aim to maximize the profit function subject to the above stated constraint.

□

c) The government's problem

There is a government in this economy, and policies that relate to macroeconomic issues have to be made.

The government has to decide on the total amount of government spending and the tax rates for every period (fiscal policy) and the central bank has to decide if it is necessary to increase or decrease the money base and at what rate.

The government also has to decide how to finance its spending. Fiscal and monetary policy are the two instruments the government has available to influence the behavior of the economic system and to manipulate its performance.

The fiscal policy of our economy consists of:

1) An expenditure-policy sequence:

$$[\, G_t \,]_{t=0}^{\infty}$$

2) A sequence of tax rates:

$$[\, H_t \,]_{t=0}^{\infty}$$

it is assumed here that the realization of the government expenditure $[G_t]$ as well as the realization of the tax rates $[H_t]$ do not enter into the utility function of the representative household and both realizations are known to all agents of the economy at period [t].

The total amount of tax revenues [T_t] that the government receives at the current period [t] is equal to the tax rate [H_t] times the total output of the economy [Y_t].

so

$$T_t = H_t\, Y_t$$

The government consumption path [G_t] is financed partially by taxes on the total output of the economy given by the above formula and partially by revenues from seignorage given, in real terms, by the expression

$$\textbf{Seignorage} = \frac{M_t - M_{t-1}}{P_t}$$

The revenues from seignorage are due to the government's ability (power) to issue fully unbacked currency and their purpose here is to finance the budget deficit.

The budget constraint then that the government faces takes the form:

$$G_t - T_t = \frac{M_t - M_{t-1}}{P_t}$$

where

M_t : money supply at period [t]

M_{t-1} : money supply at period [t-1].

In this problem, the determination of government spending and tax rate policies is independent of monetary policy but the reverse is not true. More precisely, it is assumed that the government manipulates the performance of the economic system by utilizing only one of its available instruments, monetary policy.

The advantage for dealing solely with the money supply is its ease of adjustment. Fiscal policy on the other hand has to face the hurtle of the legislature in increasing or decreasing both spending and tax rates. Beyond the legislature, there are also objective problems facing the government: for example neither the defence budget nor government employment can be cut drastically.

The purpose of the government in this model economy is to search for and apply that monetary policy, the so called the "optimal one", which will maximize the intertemporal utility function of the representative household (all agents of this economy are consumers) subject to its budget and previously specified spending and tax constraints.

Mathematically, the government's problem has the following form:

$$\max_{M_t} \quad U = \sum_{t=0}^{\infty} b^t \left[\log C_t - B\, L_t \right]$$

s.t.

1) $[\, G_t \,]_{t=0}^{\infty}$

2) $[\, H_t \,]_{t=0}^{\infty}$

3) $T_t = H_t Y_t$

4) $G_t - T_t = \dfrac{M_t - M_{t-1}}{P_t}$

5) initial value of [M_{-1}].

■□

II. MODEL'S SOLUTION

Having specified the household's, firm's and government's behavior, we proceed to the solution of our "artificial" monetary economy where these agents coexist and behave optimally.

As was stated earlier, households care about their welfare level. Facing time paths for price level [P_t], nominal wage rate [W_t] and tax rate [H_t], they aim to maximize their utility function by choosing time paths for the consumption level [C_t] and labor supply [L_t].

Firms, on the other hand, care about their respective profits. Facing the same time paths for [P_t] and [W_t], they aim to maximize them.

[P_t] and [W_t] are respectively the market values for the price level and the nominal wage rate that the product market and the labor market determine simultaneously.

Households and firms as well as our third agent, the government, participate in these markets by selling or buying output and/or labor. The government specifically buys output (the government expenditure) from the firms and finances this purchase partially by issuing totally unbacked currency.

The currency government issues influences the price level and the nominal wage rate, this economy experiences. Therefore, the government interferes in household's and firm's optimization problems by influencing the market value of price level and the nominal wage rate that both firm and household take as given.

Changes in the government's money supply policy generate changes in the price level and nominal wage rate time paths which consequently change the consumption and employment time paths, influencing finally the value of the intertemporal utility function of the household. So, the government's monetary policy influences, indirectly, the value of the utility function of the representative consumer.

These changes, combined with the assumption that markets clear (an assumption that will be presented later), imply that the solution mechanism of this monetary economy consists of a dynamic interaction among firms, households, and the government.

Departing from a pragmatic observation that governments exist and behave in a economic environment by getting involved in spending, collecting taxes, and issuing currency, we assign to the government of this particular economy an important role. More precisely, the government's role is to search for and apply the "optimal monetary policy" such that to generate consumption and leisure time paths that maximize the intertemporal utility function of the representative household.

With a cooperative behavior, our government synchronizes the private agents' behavior by influencing the price level and the nominal wage rate time paths, and by taking under consideration their motivations and restrictions as well as the objective conditions and specifications of our economy, so as to maximize consumers' welfare.

This altruistic behavior of the government is superior to others: the ones, for example, that minimize the unemployment rate, or stabilize the price level, or minimize the variance of the output.

In the remaining part of this section, we describe in detail the solution for the firm's, household's and the government's optimization problem in a competitive equilibrium framework.

□

a) Solution to the household's problem

The representative household maximizes the intertemporal and time invariant utility function:

$$U = \sum_{t=0}^{\infty} b^{t} \; [\; \log C_t - B\, L_t \;]$$

subject to the budget constraint:

$$m_{t-1} + W_t\, L_t = P_t\, C_t + H_t\, W_t\, L_t + m_t$$

and the cash-in-advance constraint:

$$P_t\, C_t = m_{t-1}$$

During period [t], the household chooses the consumption level [C_t] as well as the labor [L_t] that is willing to supply to the firms, taking as given the nominal wage rate, the price level, the tax rate, and the nominal cash holdings from the previous period [t-1].

To solve the household's optimization problem, we formulate the following Lagrangian function:

$$\Lambda_t = \sum_{t=0}^{\infty} \left(b^{t} \; [\; \log C_t - B\, L_t \;] \right) + k_{2t}\, [\; P_t C_t - m_{t-1} \;]$$

$$+ k_{1t}\, [\; m_{t-1} + W_t L_t - P_t C_t - H_t W_t L_t - m_t]$$

where:

$[k_{1t}]$ and $[k_{2t}]$ are the Lagrangian multipliers associated with the budget and the cash-in-advance constraints respectively. To derive the time paths for consumption level and labor supply that maximize the household's objective function subject to the preceding constraints, the partial derivatives of the Lagrangian function with respect to consumption, labor, multiplier $[k_{1t}]$ and multiplier $[k_{2t}]$ are derived and set equal to zero.

So, the first order conditions for the household's optimization problem become

for typical [t]

$$\frac{\partial \Lambda_t}{\partial C_t} = b^t \frac{1}{C_t} + k_{2t}P_t - k_{1t}P_t = 0 \qquad (1a)$$

$$\frac{\partial \Lambda_t}{\partial L_t} = - B\, b^t + k_{1t}W_t - k_{1t}H_tW_t = 0 \qquad (1b)$$

$$\frac{\partial \Lambda_t}{\partial k_{2t}} = P_tC_t - m_{t-1} = 0 \qquad (1c)$$

$$\frac{\partial \Lambda_t}{\partial k_{1t}} = m_{t-1} + W_tL_t - P_tC_t - H_tW_tL_t - m_t = 0 \qquad (1d)$$

from equation (1c) we get

$$P_t\, C_t = m_{t-1}$$

or

$$C_t = \frac{m_{t-1}}{P_t} \qquad (2c)$$

which means that knowing the nominal cash holdings that the household carries from the previous period and the price level at the current period, the representative household is able to determine its consumption level for period [t].

From equation (1b) we get

$$k_{1t}\, W_t - k_{1t}\, H_t\, W_t = B\, b^{t}$$

or

$$k_{1t}\, [\, W_t - H_t\, W_t\,] = B\, b^{t}$$

which determines the value of the multiplier [k_{1t}] as a function of [B, b, t, W_t, H_t] variables.

so

$$k_{1t} = \frac{B\, b^{t}}{W_t\, [1 - H_t]} \qquad (2b)$$

from equation (1a) we get

$$k_{2t}\, P_t = k_{1t}\, P_t - \frac{b^{t}}{C_t}$$

and by substituting (2b) and (2c) into the above equation we get

$$k_{2t}\, P_t = \frac{B\, b^{t}\, P_t}{W_t\, [1 - H_t]} - \frac{b^{t}\, P_t}{m_{t-1}}$$

now by dividing both sides by $[P_t]$ we get

$$k_{2t} = \frac{B\, b^t}{W_t\ [1 - H_t]} - \frac{b^t}{m_{t-1}}$$

or

$$k_{2t} = b^t \left(\frac{B}{W_t\ [1 - H_t]} - \frac{1}{m_{t-1}} \right) \qquad \text{(2a)}$$

which determine the value of the $[k_{2t}]$ multiplier as a function of $[B,\ b,\ t,\ W_t,\ H_t,\ m_{t-1}]$ variables.

Finally from (1d) we get

$$W_t\, L_t = P_t\, C_t + H_t\, W_t\, L_t + m_t - m_{t-1}$$

and by substituting equation (2c):

$$P_t\, C_t = m_{t-1}$$

we get

$$W_t\, L_t = H_t\, W_t\, L_t + m_t \qquad \text{(2d)}$$

which basically says that the labor income $[W_t\ L_t]$ of the consumer during period [t] is totally available to finance the tax expenditure $[H_t\ W_t\ L_t]$ and to create the nominal cash holdings $[m_t]$ to be available for use in the following period [t+1].

The other type of expenditure, the consumption one, is totally financed by the nominal cash holdings carried from the previous period [t-1]. This is supposed to happen due to the earlier stated assumption that the two activities of the household, namely working and shopping, do not intersect during the current period [t].

To summarize, equations (2a), (2b), (2c) and (2d) present the first order conditions for the representative household's optimization problem.

Another way to view equation (2d) is to rearrange it and solve it for [L_t]. By using simple algebra, we get

$$W_t L_t - H_t W_t L_t = m_t$$

or

$$L_t [W_t - H_t W_t] = m_t$$

which gives

$$L_t = \frac{m_t}{W_t [1 - H_t]} \qquad (3d)$$

equation (3d) shows the relationship between the current labor supply, the current nominal wage rate and the nominal cash holdings the consumer is willing to hold to be able to finance the consumption expenditure for the next period [t+1]. With fixed nominal wage and tax rates, this implies that a consumer's desire to have more nominal cash holdings available next period [t+1], can be fulfilled only if the representative household increases the amount of labor supplied during the current period [t].

Stated differently, it shows that the lower the nominal wage rate the more labor the representative household has to supply to be able to create the same amount of nominal cash holdings [m_t]. In other words, the consumer can give up leisure today (and produce more output today) with a purpose of ending up with a bigger amount of nominal cash holdings [m_t] and "via currency" to have more of the cash good tomorrow.

Viewing also equation (3d), we notice that the current price level [P_t] does not interfere with the household's decision on the current amount of the supplied labor. The current price level [P_t] was involved in the consumption expenditure financed by the nominal cash holdings of the previous period [t-1]. The only price level that influences the current labor supply decisions is the next period's price level [P_{t+1}] through the next period's cash-in-advance constraint:

$$P_{t+1}\, C_{t+1} = m_t$$

In other words equation (3d) shows an intertemporal relationship between labor supplied decisions at period [t] and the next period's price level.

□

b) Solution to the firm's problem

As was said earlier, the aim of the typical firm (i) is to maximize its profits at the current period [t] by choosing the appropriate amount of labor to employ, subject to the technology constraint and taking the current price level and nominal wage rate as given.

To derive the first order conditions for profit optimization, we substitute the production function into the profit function getting

$$\Pi_{it} = P_t A_t f_t L_{it} - W_t L_{it}$$

then, we take the first partial derivative of the profit function with respect to labor and set it equal to zero.

$$\frac{\partial(\Pi_{it})}{\partial L_{it}} = P_t A_t f_t - W_t = 0$$

or

$$P_t A_t f_t = W_t$$

and by dividing both sides by the price level [P_t], we get the first order condition for profits optimization as follows:

$$A_t \, f_t = \frac{W_t}{P_t} \qquad \text{(2e)}$$

which basically says that the firm maximizes profits where the marginal product of labor [MPL] at period [t] equals the real wage rate [W_t / P_t] at the same period.

Equation (2e), showing the first order conditions for profit optimization in the case of the typical firm (i) can be the equation presenting the first order condition for profit optimization for the whole industry, because:

1) all firms are identical
2) it shows a relationship between (A_t, f_t, W_t, P_t) variables which are all common to every firm.

The preceding assumptions about the existence of identical firms, the objectives they pursue, the perfect competition conditions they face in labor and product markets imply:

1) the total amount of the profit level is zero
2) it is possible to get the aggregate production function of the whole economy as follows:

The per-capita output of this economy is the summation of the per-capita output of all firms.

$$y_t = \sum_{i=1}^{s} y_{it}$$

by substituting the production function into the above equation we get

$$y_t = \sum_{i=1}^{s} (A_t \, f_t \, L_{it})$$

because the $[A_t]$ and $[f_t]$ variables are the same for all firms (they do not depend on [i]), we can take them out of the summation operator and have

$$y_t = A_t \, f_t \sum_{i=1}^{s} (L_{it})$$

and by using

$$L_t = \sum_{i=1}^{s} (L_{it})$$

we get

$$y_t = A_t \, f_t \, L_t$$

which relates the per-capita production of the economy to the productivity shocks and to the amount of labor demanded from the representative consumer.

Multiplying both sides of the above equation by the size of the population [N], we get the aggregate production function of the economy:

$$Y_t = N \, y_t$$

or

$$Y_t = N \, A_t \, f_t \, L_t$$

□

c) Solution to government's optimization problem and the dynamic competitive equilibrium framework

Before we solve for a competitive equilibrium, it is necessary to develop its concept.

We define as the competitive equilibrium for this economy:

1) a set of sequences for the price level

$$[\ P_t\ ]_{t=0}^{\infty}$$

and nominal wage rate

$$[\ W_t\ ]_{t=0}^{\infty}$$

where:

$$P_t = F_1\ (\ H_t,\ M_t,\ G_t\ )$$

$$W_t = F_2\ (\ H_t,\ M_t,\ G_t,\ f_t,\ A_t\ )$$

given the initial values for $[M_{-1}]$ and $[f_{-1}]$

2) a set of sequences for

$$[\ C_t,\ L_t\ ]_{t=0}^{\infty}$$

and

$$[\, M_t \,]_{t=0}^{\infty}$$

where:

$$C_t = F_3 \ (P_t, \ W_t \ )$$

$$L_t = F_4 \ (P_t, \ W_t \ )$$

such that

a) given the time paths for the price level and the nominal wage rate the typical firm and the representative household maximize their objective function subject to their constraints

b) the sequence of

$$[\, M_t \,]_{t=0}^{\infty}$$

maximizes the government's objective function subject to its fiscal specifications and its budgetary constraints.

c) money market clears :

$$M_t = N \, m_t$$

d) labor market clears

e) product market clears determining the price level $[P_t]$ which makes the total non-storable output produced by firms (total supply) to be totally consumed by households and by the government (total demand).

This integration of optimizing behavior of all agents under the previously specified frame of competitive equilibrium solves the economy's problem for an optimal allocation of the limited resources among competitive agents in a dynamic sense.

Our government though has a leading role in this dynamic setting because its monetary policy can influence the relationship between the private agents' objectives on one hand and the price level as well as the nominal wage rate time paths on the other, a possibility which is not available to the private agents.

Another factor that can interfere with this relationship by also influencing the time paths for the price level and the nominal wage rate is the productivity factor but this factor is not an endogenously determined variable but an exogenously specified one.

Next, we proceed for the solution of the economy's problem in a dynamic competitive equilibrium setting.

In this setting, we maximize the government's objective function by choosing the optimal monetary policy, taking under consideration its constraints as well as the first order conditions for optimization in the firm's and household's problems and the competitive equilibrium assumption that all markets clear.

Mathematically, the problem can be stated as follows:

$$\max_{M_t} \quad U = E \sum_{t=0}^{\infty} b^t \left[\log C_t - B\, L_t \right] \tag{1}$$

s.t.

$$[\, G_t \,]_{t=0}^{\infty} \tag{2}$$

$$[\, H_t \,]_{t=0}^{\infty} \tag{3}$$

$$T_t = H_t \, Y_t \tag{4}$$

$$G_t - T_t = \frac{M_t - M_{t-1}}{P_t} \tag{5}$$

$$Y_t = N \, A_t \, f_t \, L_t \tag{6}$$

$$f_t = p_0 + p_1 \, f_{t-1} + e_t \tag{7}$$

$$A_t \, f_t = \frac{W_t}{P_t} \tag{8}$$

$$C_t = \frac{m_{t-1}}{P_t} \tag{9}$$

$$k_{1t} = \frac{B \, b^t}{W_t \, [1 - H_t]} \tag{10}$$

$$k_{2t} = b^t \left(\frac{B}{W_t \, [1 - H_t]} - \frac{1}{m_{t-1}} \right) \tag{11}$$

$$W_t\, L_t = H_t\, W_t\, L_t + m_t \tag{12}$$

$$M_t = N\, m_t \tag{13}$$

$$M_{t-1} = N\, m_{t-1} \tag{14}$$

$$TC_t = N\, C_t \tag{16}$$

$$Y_t = TC_t + G_t \tag{15}$$

given the initial values for $[M_{-1}]$and $[f_{-1}]$

where:

TC_t : total consumption of period [t]

C_t : per-capita consumption at period [t]

E : the expectation operator.

The expectation operator is used in the government's problem because the optimization involves decisions based on realizations of random variables.

By substituting equation (6) into equation (4) we get

$$T_t = N\, H_t\, A_t\, f_t\, L_t \tag{4a}$$

by substituting equation (4a) into equation (5) we get

$$G_t - H_t\, N\, A_t\, f_t\, L_t = \frac{M_t - M_{t-1}}{P_t}$$

or

$$P_t\, (\, G_t - N\, H_t\, A_t\, f_t\, L_t\,) = M_t - M_{t-1} \tag{5a}$$

by substituting equation (8) into (4a) we get

$$T_t = N\, H_t\, \frac{W_t}{P_t}\, L_t \qquad \textbf{(4b)}$$

or

$$P_t\, T_t = N\, H_t\, W_t\, L_t \qquad \textbf{(4c)}$$

equation (4c) says that the total amount of taxes (in nominal terms), $[P_t\ T_t]$, the government collects is equal to the summation of the labor income taxes that all the individuals of this economy pay $[N\ H_t\ W_t\ L_t]$. This is due to the perfect competition condition that firms operate under that forces the total profit level to be equal to zero.

Using the profit function:

$$\Pi_t = P_t\, Y_t - N\, W_t\, L_t$$

and by putting it equal to zero we get

$$P_t\, Y_t = N\, W_t\, L_t$$

by multiplying both sides by $[H_t]$ we get

$$P_t\, H_t\, Y_t = N\, H_t\, W_t\, L_t$$

and by using equation (4)

$$P_t\, T_t = N\, H_t\, W_t\, L_t$$

by dividing equation (4c) by [N] we get

$$\frac{P_t\, T_t}{N} = H_t\, W_t\, L_t \qquad \textbf{(4d)}$$

by substituting equation (15) into equation (4) we get

$$T_t = H_t \; (\; TC_t + G_t) \tag{17}$$

by substituting equation (17) into (4d) we get

$$H_t \; W_t \; L_t = \frac{P_t}{N} \; H_t \; (\; TC_t + G_t \;) \tag{18}$$

by substituting equation (16) into equation (18) we get

$$H_t \; W_t \; L_t = \frac{P_t}{N} \; H_t \; (\; N \; C_t + G_t \;) \tag{18a}$$

or

$$H_t \; W_t \; L_t = P_t \; H_t \left(C_t + \frac{G_t}{N} \right) \tag{18b}$$

where $[G_t \; / \; N]$ stands for the per-capita government spending. By substituting equation (18b) into equation (12) we get

$$W_t \; L_t = P_t \; H_t \left(C_t + \frac{G_t}{N} \right) + m_t \tag{19}$$

by dividing both sides of equation (19) by the price level $[P_t]$ we get

$$\frac{W_t \; L_t}{P_t} = H_t \left(C_t + \frac{G_t}{N} \right) + \frac{m_t}{P_t} \tag{19a}$$

by substituting equation (8) into equation (19a) we get

$$A_t \; f_t \; L_t = H_t \left(C_t + \frac{G_t}{N} \right) + \frac{m_t}{P_t} \tag{20}$$

by dividing both sides of equation (20) by

$$A_t \; f_t$$

we get

$$L_t = \frac{1}{A_t\ f_t}\left[H_t \left(C_t + \frac{G_t}{N} \right) + \frac{m_t}{P_t} \right] \qquad \textbf{(20a)}$$

by substituting equation (14) into equation (9) we get

$$C_t = \frac{M_{t-1}}{N\ P_t} \qquad \textbf{(9a)}$$

by substituting equation (9a) into equation (20a) we get

$$L_t = \frac{1}{A_t\ f_t}\left[H_t \left(\frac{M_{t-1}}{N\ P_t} + \frac{G_t}{N} \right) + \frac{m_t}{P_t} \right] \qquad \textbf{(20b)}$$

by substituting equation (13) into equation (20b) we get

$$L_t = \frac{1}{A_t\ f_t}\left[H_t \left(\frac{M_{t-1}}{N\ P_t} + \frac{G_t}{N} \right) + \frac{M_t}{N\ P_t} \right] \qquad \textbf{(20c)}$$

equation (20c) determines the economy's employment path as a function of sequences of:

a) the productivity shocks

b) the price level

c) fiscal policy specifications

d) money supply policy

e) the constant population size [N]

by multiplying both sides of equation (20c) by [N] we get

$$N\ L_t = \frac{1}{A_t\ f_t}\left[H_t \left(\frac{M_{t-1}}{P_t} + G_t \right) + \frac{M_t}{P_t} \right] \qquad \textbf{(21)}$$

by substituting equation (21) into equation (5a) we get

$$P_t \left[G_t - H_t A_t f_t \frac{1}{A_t f_t} \left(H_t \left(\frac{M_{t-1}}{P_t} + G_t \right) + \frac{M_t}{P_t} \right) \right] = M_t - M_{t-1}$$

or

$$P_t \left[G_t - H_t \left(H_t \left(\frac{M_{t-1}}{P_t} + G_t \right) + \frac{M_t}{P_t} \right) \right] = M_t - M_{t-1}$$

or

$$P_t \left[G_t - H_t^2 \left(\frac{M_{t-1}}{P_t} + G_t \right) - H_t \left(\frac{M_t}{P_t} \right) \right] = M_t - M_{t-1}$$

or

$$P_t \left[G_t - H_t^2 \left(\frac{M_{t-1}}{P_t} \right) - H_t^2 G_t - H_t \left(\frac{M_t}{P_t} \right) \right] = M_t - M_{t-1}$$

or

$$P_t \left[G_t - H_t^2 G_t \right] - H_t^2 M_{t-1} - H_t M_t = M_t - M_{t-1}$$

and after rearranging we get

$$P_t \left[G_t - H_t^2 G_t \right] = M_t - M_{t-1} + H_t^2 M_{t-1} + H_t M_t$$

dividing both sides of the above equation by

$$\left[G_t - H_t^2 G_t \right]$$

we get

$$P_t = \frac{M_t - M_{t-1} + H_t^2 M_{t-1} + H_t M_t}{G_t - H_t^2 G_t}$$

or

$$P_t = \frac{M_t \left[1 + H_t \right] - M_{t-1} \left[1 - H_t^2 \right]}{G_t \left[1 - H_t^2 \right]} \qquad (23)$$

More formally, the economy's optimization problem has the following mathematical presentation:

$$\max_{M_t} \quad U = \mathbf{E} \sum_{t=0}^{\infty} b^t \left[\log C_t - B\, L_t \right] \qquad (1)$$

s.t.

$$\left[G_t \right]_{t=0}^{\infty} \qquad (2)$$

$$\left[H_t \right]_{t=0}^{\infty} \qquad (3)$$

$$C_t = \frac{M_{t-1}}{N\, P_t} \qquad (9a)$$

$$f_t = p_0 + p_1 f_{t-1} + e_t \qquad (7)$$

$$P_t = \frac{M_t \left[1 + H_t \right] - M_{t-1} \left[1 - H_t^2 \right]}{G_t \left[1 - H_t^2 \right]} \qquad (23)$$

$$L_t = \frac{1}{A_t\, f_t} \left[H_t \left(\frac{M_{t-1}}{N\, P_t} + \frac{G_t}{N} \right) + \frac{M_t}{N\, P_t} \right] \qquad (20c)$$

given the initial values for $[M_{-1}]$ and $[f_{-1}]$

Equation (23) shows that the current period's price level $[P_t]$ depends on:

1) Current period's government spending $[G_t]$, tax rate $[H_t]$,

money supply $[M_t]$ and

2) previous period's money quantity $[M_{t-1}]$.

It shows also the relationship between changes in money supply and changes in price level. To elaborate this relationship further, we simplify the government spending and tax rate paths by assigning to them a constant value of the form:

$$[\, G_t \,]_{t=0}^{\infty} = \overline{G}$$

$$[\, H_t \,]_{t=0}^{\infty} = \overline{H}$$

then equation (23) becomes

$$P_t = \frac{M_t - M_{t-1}\,(\,1 - \overline{H}\,)}{\overline{G}\,(\,1 - \overline{H}\,)} \qquad (23a)$$

and updating it for the next period [t+1] we get

$$P_{t+1} = \frac{M_{t+1} - M_t\,(\,1 - \overline{H}\,)}{\overline{G}\,(\,1 - \overline{H}\,)} \qquad (23b)$$

and by developing the difference in the price between the two following periods we get

$$P_{t+1} - P_t = \Delta P_t = \frac{M_{t+1} - M_t - (\,1 - \overline{H}\,)\,(\,M_t - M_{t-1}\,)}{\overline{G}\,(\,1 - \overline{H}\,)}$$

or

$$\Delta P_t = \frac{\Delta M_t - (\,1 - \overline{H}\,)\,\Delta M_{t-1}}{\overline{G}\,(\,1 - \overline{H}\,)} \qquad (23c)$$

by looking at equation (23c) and assuming that

$$\bar{G} > 0 \quad \text{and} \quad 1 > \bar{H} > 0$$

we can say that the economy experiences inflation

$$\Delta P_t > 0$$

$$\text{if} \quad \Delta M_t > (1 - \bar{H}) \Delta M_{t-1}$$

experiences deflation

$$\Delta P_t < 0$$

$$\text{if} \quad \Delta M_t < (1 - \bar{H}) \Delta M_{t-1}$$

and experiences a constant price level

$$\Delta P_t = 0$$

$$\text{if} \quad \Delta M_t = (1 - \bar{H}) \Delta M_{t-1}$$

Now, by dividing both sides of equation (23c) by $[P_{t-1}]$ we get the expression for the inflation rate of the form:

$$\pi_t = \frac{M_t - M_{t-1} - (1 - \bar{H})(M_{t-1} - M_{t-2})}{M_{t-1} - M_{t-2}(1 - \bar{H})}$$

or

$$\pi_t = \frac{\dfrac{M_t - M_{t-1}}{M_{t-1}} + \bar{H}}{1 - \dfrac{M_{t-2}(1 - \bar{H})}{M_{t-1}}} - 1$$

which shows the relationship between current inflation rate and the percentage change in money quantity.

III. COMPUTATIONAL RESULTS

a) optimal monetary policy

Our aim in the preceding problem is to maximize a non-linear objective function subject to a set of non-linear constraints. The two available methods for solving this type of model structure for the derivation of the optimal policy would be the numerical methods and the linear-quadratic approximation, the latter having the advantage of delivering an analytical solution to the problem and obtaining a linear decision optimal rule. In this particular case, however, the issue that the optimization involves decision on an optimal monetary policy rule which could be non-stationary, makes the use of linear quadratic approximation technique really unavailable. Consequently, the only other available approach for the derivation of the optimal monetary policy would be the numerical methods technique, having the disadvantage of lacking an analytical solution to the problem.

The first step for the solution to the problem is to substitute constraints (9a) and (20c) into the objective function (1) and create the final version of the government's problem with the following form:

$$\max U = E \sum_{t=0}^{\infty} b^t \left[\log \left(\frac{M_{t-1}}{N\, P_t} \right) - B \left(\frac{1}{A_t\, f_t\, N}\, H_t \left(\frac{M_{t-1}}{P_t} + G_t \right) + \frac{M_t}{P_t} \right) \right]$$

s.t.

$$[\, G_t \,]_{t=0}^{\infty} \qquad (2)$$

$$[\, H_t \,]_{t=0}^{\infty} \qquad (3)$$

$$f_t = p_0 + p_1\, f_{t-1} + e_t \qquad (7)$$

$$P_t = \frac{M_t \left[1 + H_t \right] - M_{t-1} \left[1 - H_t^2 \right]}{G_t \left[1 - H_t^2 \right]} \qquad (23)$$

Next the government expenditure and tax rate forms have to be specified. It is assumed that both of them behave according to a first order autoregressive process with drift of the following form:

$$G_t = g_0 + g_1\, G_{t-1} + n_t \qquad (2')$$

$$H_t = h_0 + h_1\, H_{t-1} + h_t \qquad (3')$$

with initial values: $[G_{-1}] = 2000$

and $[H_{-1}] = 0.25$

$[n_t]$ and $[h_t]$ are random variables following the normal distribution with particular mean and variance. The above equations, (2′) and (3′), introduce another type of uncertainty into the model, in the form of shocks to the fiscal behavior of the government.

The next task is to develop an "information bank" for money supply time paths generated by four types of money supply policies available to the monetary authorities.

* * *

- The first type of money supply policy is of the form :

$$M_t = (1 + m) M_{t-1}$$

This rule implies that the money quantity increases or decreases in a constant rate depending on the value of the (1 + m) parameter.

We assign to the (1 + m) parameter the following values:

[0.95, 0.96, 0.97, 0.98, 0.985, 0.9, 0.99, 1.0, 1.01, 1.02, 1.025, 1.03, 1.035, 1.04, 1.045, 1.05, 1.06, 1.07, 1.08, 1.09, 1.095, 1.10, 1.15, 1.20]

With this procedure it is possible to generate (24) money supply time paths representing the first money supply policy.

* * *

- The second type of money supply policy is of the form :

$$M_t = (M_{t-1})^{a2}$$

or

$$\log M_t = a2 \log M_{t-1}$$

which implies a constant elasticity of the current period's money quantity with respect to its previous period value. In other words, it assumes a one percent change in $[M_t]$ due to one percent change in $[M_{t-1}]$.

Parameter [a2] takes the following values:

[1.0001, 1.00015, 1.0002, 1.00025, 1.0003, 1.00035, 1.0004, 1.00045, 1.0005, 1.00055, 1.0006, 1.00065, 1.0007, 1.00075, 1.0008, 1.00085, 1.0009, 1.001, 1.002, 1.0025, 1.003, 1.0035, 1.004, 1.0045, 1.005, 1.0055, 1.006, 1.0065, 1.007, 1.0075]

giving another (30) money supply time paths representing the second type of money supply policy.

* * *

- The third type of money supply policy is of the form :

$$M_t = a3_t \, M_{t-1}$$

where:

1) the logarithm of the growth rate [$a3_t$] behaves stochastically according to a first order autoregressive scheme.

$$\log a3_t = b3 \log a3_{t-1} + theta3_t$$

and initial value: [$a3_{-1}$] = 1

2) [$theta3_t$] is a random variable following the normal distribution with zero mean and three different standard deviation values :

[0.0 , 0.001 , 0.003]

3) the parameter b3 takes the values :

[0.48, 0.5, 0.54, 0.58, 0.64, 0.68, 0.70, 0.74, 0.78, 0.82, 0.86, 0.90, 0.91, 0.92, 0.93, 0.94]

the combination of different values for [$theta3_t$] and for the

[b3] parameter gives (48) money supply time paths representing the third type of money supply policy.

* * *

- The fourth type of money supply policy is of the form :

$$M_t = (1 + a4_t) M_{t-1}$$

where:

1) the growth rate of money is not constant but behaves stochastically according to a first order autoregressive process

$$a4_t = b4\ a4_{t-1} + theta4_t$$

and initial value: $[a4_{-1}] = 0.1$

2) $[theta4_t]$ is a random variable following the normal distribution with zero mean and three different standard deviation values :

[0.0005 , 0.001 , 0.003]

3) the parameter [b4] takes the values :

[0.90, 0.91, 0.92, 0.93, 0.94, 0.95, 0.96, 0.97, 0.98, 0.99, 1.00]

The random components $[theta3_t]$ and $[theta4_t]$ introduce the last type of uncertainty into the economic system in the form of shocks in the monetary behavior of the government.

The combination of different values of [b4] and $[theta4_t]$ gives (33) money supply time paths representing the fourth money supply policy.

* * *

Adding all these generated time paths, representing the four money supply rules, we develop the "information bank" containing (135) time series.

An initial value: $[M_{-1}] = 2500$

is assumed for all of them.

How should the monetary policy be applied has been a major source of conflict in the macroeconomics and politics.

Some years ago, M. Friedman suggested that monetary authorities should "avoid sharp swings in policy" and follow a steady money supply growth rule. Friedman related the constant rate of money expansion with stable economic activity and estimated that this growth rate should be equal to four percent.

Friedman's four percent money growth rule was criticized by rational expectation theorists. In their paper "Rational Expectations and the Theory of Economic Policy", T. Sargent and N. Wallace criticized the four percent rule as being a "suboptimal" one because it is a rule with no feedback and does not take under consideration all the information available at time the monetary policy is conducted. Rational expectation economists go much further by suggesting that systematic monetary policy known to all agents has no real effect on the level of produced output. What has a real effect on the level of produced output is the unanticipated part of the monetary policy, the random component.

The monetary policy that "reacts to real shocks in just the right way" is going to take the form:

$$M_t = (1 + m) M_{t-1}$$

where:

$$m_t = k\, m_{t-1} + e_t$$

$[e_t]$ is an iid random variable with zero mean and $[v_{et}]$ variance.

Our purpose, here, is to put different types of monetary policy together, assign different values to their parameters, and derive as an optimal policy the one that maximizes the objective function of the representative household.

Before we proceed to the final step for the problem's solution, we have to develop the restrictions that this model economy must satisfy, such that the results make economic sense.

The first restriction is that the price level given by the equation (23) must be positive.

Of course the GNP level, the per-capita consumption level, the employment level, and the wage rate given by the following equations :

$$Y_t = A_t\, f_t\, N\, L_t$$

$$C_t = \frac{M_{t-1}}{N\, P_t}$$

$$W_t = A_t\, f_t\, P_t$$

$$L_t = \frac{1}{A_t\ f_t\ N}\left[H_t \left(\frac{M_{t-1}}{P_t} + G_t \right) + \frac{M_t}{P_t} \right]$$

must be positive.

By taking under consideration :

1) the fact that the size of the population [N], the level of the money base [M_t] and the level of the government spending [G_t] are always positive
2) the tax rate [H_t] is always positive and smaller than unity

and assuming :

1) that the parameter [A_t] takes always positive values
2) [e_t] follows the normal distribution with zero mean and standard deviation [0.001]
3) the parameters [p_0] and [p_1] take the values 5 and 0.95 respectively and
4) the initial value: [f_{-1}] = 1 ,

we can insist that the GNP level, the employment level and the wage rate are positive if the price level is positive.

So a positive price level together with the above specifications about the [e_t, p_0, p_1, N, M_t, G_t, H_t, A_t, f_t] variables/parameters ensures that the other endogenously determined variables will take positive values.

The second restriction is that the Cash-in-Advance constraint should hold with equality. A sufficient condition is that the Lagrange multiplier associated with this constraint must be positive.

so

$$k_{2t} = b^t \left(\frac{B}{W_t \; [1 - H_t]} - \frac{N}{M_{t-1}} \right) > 0$$

the above inequality determines the relationship between the last period's money supply value on the one hand, and [B, H_t, W_t, N] variable/parameter values on the other, such that the second restriction be satisfied.

Having specified all of the above, we can proceed to the final step for the model's solution. For the computations to be possible, the assumption that the time interval ranges from period zero to infinity has to be replaced by a more realistic one.

The time interval ranges from period zero to period 100, involving calculations for 101 periods.

Now the first of (135) money supply time paths, called $[M_t]^1$ is being taken.

To calculate the price level time path, we substitute into equation (23) the selected $[M_t]^1$ as well as the time paths for government spending and tax rates given by equations (2') and (3') respectively. The selected money supply time path must satisfy both of the above cited restrictions :

1) the corresponding price level path resulting from equation (23) must always have positive values and
2) the multiplier associated with the Cash-in-Advance constraint must always be positive too.

The money supply paths that satisfy both conditions are called "feasible", and they would be available for further elaboration. The ones that fail to satisfy at least one of the above conditions are rejected from our available money supply paths.

The time series for $[P_t, G_t, H_t]$ as well as the time series for the technology shock $[f_t]$ are substitute into the objective function to calculate the value of the expression:

$$\sum_{t=0}^{100} b^t \left[\log \left(\frac{M_{t-1}}{N\, P_t} \right) - B \left(\frac{1}{A_t\, f_t\, N}\, H_t \left(\frac{M_{t-1}}{P_t} + G_t \right) + \frac{M_t}{P_t} \right) \right]$$

The above procedure is repeated (50) times using the same feasible money supply time path $[M_t]^1$ and different time series for $[H_t, G_t, f_t]$, each one corresponding to the different realizations of the random variables $[h_t, n_t, e_t]$.

An average value (mean value) of these summations approximating the expected value

$$E \sum_{t=0}^{100} b^t \; [\ldots\ldots\ldots\ldots]$$

is calculated and this mean value is called $[EV]^1$ corresponding to money supply time path $[M_t]^1$.

We repeat the above procedure for all feasible money supply time paths, and consequently we calculate an equal amount of [EV] values.

The highest of these [EV] values is chosen and is called [Vmax].

Then the money supply time path that corresponds to this [Vmax] is checked and is called the optimal money supply rule.

Having determined the optimal monetary policy, we can go back to the original model and determine time series for the price level, the wage rate, the inflation rate, the employment rate, the GNP level, the per capita consumption level, and the deficit level that correspond to this optimal monetary policy from the following equations :

$$P_t^* = \frac{M_t^* \left[1 + H_t \right] - M_{t-1}^* \left[1 - H_t^2 \right]}{G_t \left[1 - H_t^2 \right]}$$

$$W_t^* = A_t \, f_t \, P_t^*$$

$$\pi_t^* = \frac{P_t^* - P_{t-1}^*}{P_{t-1}^*}$$

$$L_t^* = \frac{1}{A_t \, f_t \, N} \left[H_t \left(\frac{M_{t-1}^*}{P_t^*} + G_t \right) + \frac{M_t^*}{P_t^*} \right]$$

$$Y_t^* = A_t \, f_t \, N \, L_t^*$$

$$C_t^* = \frac{M_{t-1}^*}{N \, P_t^*}$$

$$D_t^* = G_t - H_t \, Y_t^*$$

To be able to deliver computational results we, also have to assign values for the other parameters of the model.

For our exercise we assume that :

$g_0 = 0.00$

$g_1 = 1.00$

$h_0 = 0.00$

$h_1 = 0.995$

$n_t \sim N(0, 0.001)$

$h_t \sim N(0, 0.001)$

$B = 2.86$

$[A_t] = 1$

and the population size $N = 100$.

Some of these selected parameters and initial values, for example (B, b, H_{-1}), are results taken from studies in available publications. Some other parameter/initial values though are selected in such a way as to allow the majority of money supply time paths generated by the four money supply rules, consisting the "information bank", to be feasible.

Our purpose here is to derive an optimal policy, and to make the best choice, we must have a sufficient number of available and feasible money supply paths from which to choose the optimal one.

Some other set of initial values and parameters may narrow our feasibility space, restricting in this way the ability for further elaboration.

Looking at the computer generated results for this monetary economy, we notice :

1) The highest calculated value for the welfare level using the first type of monetary policy of the form :

$$M_t = (1+m)\ M_{t-1}$$

is :

$$[\ EV\ ] = 32.802$$

which corresponds to the following money supply rule specification:

$$M_t = 1.2\ M_{t-1}$$

* * *

2) The highest calculated value for the welfare level using the second type of monetary policy of the form :

$$\log M_t = a2\ \log M_{t-1}$$

is:

$$[\ EV\ ] = 0.5439$$

which corresponds to the following money supply rule specification:

$$\log M_t = 1.0075\ \log M_{t-1}$$

* * *

3) The highest calculated values for the welfare level using the third type of monetary policy of the form :

$$M_t = a3_t\ M_{t-1}$$

where

$$\log a3_t = b3\ (\log a3_{t-1}) + theta3_t$$

is:

[EV] = - 0.528 for s.d of theta3 = 0.000

[EV] = - 0.183 .... = 0.001

[EV] = 0.672 .... = 0.003

which corresponds to the following money supply rule specification:

$$M_t = a3_t \, M_{t-1}$$

$$\log a3_t = 0.94 \, (\log a3_{t-1}) + theta3_t$$

* * *

4) The highest calculated value for the welfare level using the fourth type of monetary policy of the form :

$$M_t = (1+ a4_t) \, M_{t-1}$$

where

$$a4_t = b4 \, (a4_{t-1}) + theta4_t$$

is :

[EV] = 9.025 for s.d of theta4 = 0.0005

[EV] = 7.027 .... = 0.0010

[EV] = 5.875 .... = 0.0030

which corresponds to the following money supply rule specification:

$$M_t = (1+ a4_t) \, Mt-1$$

$$a4_t = a4_{t-1} + theta4_t$$

it is obvious from the above results that the highest values for the [EV] variable, corresponding to the four monetary policies, are the ones with the fastest money supply growth rate.

Comparing, we conclude that the preferable money supply policy is the one according to the first rule with a twenty percent money growth rate.

This policy gives the highest welfare level that the representative household can attain. The [Vmax] value is equal to 32.802.

In graphs [a, b, c, d, e, f, g and h], we present the plots for government spending [G_t], tax rate [H_t], GNP level [Y_t], per-capita consumption level [C_t], employment level [L_t], the price level [P_t], the deficit level [D_t] and the inflation rate [π_t] due to the application of the optimal monetary policy rule, shown in graph [m].

The work shows that this economy experiences a relatively constant inflation rate about the same level as the growth rate in money supply and a very "realistic" employment level fluctuating between [82] and [96] percent.

It also shows an increasing path of per-capita consumption and gross national product. We attribute the increasing behavior in these two variables partially to the productivity factor and partially to the decreasing tax rate policy the fiscal authorities follow.

A decreasing tax rate policy forces people away from leisure and consequently towards more labor which generates an increasing tendency in the total produced output and the corresponding per-capita consumption level.

On the other hand, an increasing tax rate policy, with the same government spending and money supply specifications, forces consumers to substitute the cash good with leisure and consequently providing less labor which leads to a decreasing total output and per-capita time paths.

In graphs [c', d' and e'], we present the plots for GNP level, per capita consumption level and the employment level that correspond to a tax policy shown in graph [b'] and having the form:

$$H_t = H_{t-1} + h_t$$

where in graphs [c", d" and e"], we show the plots for GNP level, the consumption level and the employment level that corresponds to a tax policy of the form:

$$H_t = 1.004\ H_{t-1} + h_t$$

shown in graph [b"].

In the following section, we would like to consider the consequences of an alternative monetary policy. Our government now does not apply the optimal monetary policy corresponding to a twenty percent growth in money supply but a suboptimal one of the form:

$$M_t = 1.05\ M_{t-1}$$

The results show that under the same fiscal specifications, the new monetary policy leads to a welfare level

$$[\ EV\ ] = -\ 12.443$$

which means that there is a welfare loss due to the alternative policy given by the difference

$$[\ VMAX\ ] - [\ EV\ ] = 45.245$$

Also, the employment time path is lower in the case of high growth rate in money supply, and consequently high inflation rate, than the case of lower growth money rate, and consequently lower inflation.

Money has real effects in this setting. In the case of high money growth rate, the anticipation of a twenty percent inflation forces consumers to move away from the cash good which is taxable with the inflation tax and consume more of the credit good, leisure.

This substitution generates the lower employment time path and, consequently, the higher welfare level.

On the other hand though, the anticipation of a inflation rate of five percent, as in the case of the suboptimal monetary policy, forces consumers to substitute leisure with the cash good generating a higher employment time path and consequently a lower welfare level.

Graphs [D, C, and E] show the time paths for per-capita consumption level, the total output level, and the employment level due to the application of the suboptimal monetary policy, shown in graph [M], and can be compared with graphs [d, c, and e] showing the results of the application of the optimal monetary policy.

□

b) optimal monetary and fiscal policy

This part describes the way fiscal and monetary policy should be conducted in the previously specified welfare maximizing setting.

In that section we had restricted the government to proceed in the optimization procedure by using only one of its available instruments, the monetary policy, assuming that the fiscal one was exogenously specified. Here though, we alter our setting by assuming that the government is able to use both of its available instruments, and examine if this availability results in a higher welfare level, higher than the one obtained by manipulating only one policy.

To proceed to the problem's solution, first, we will use exactly the same model as the one before with a slight modification in the choice of control variables. More precisely, here, the optimization in the objective function involves choices not only in the money supply time path, as in the previous case, but in fiscal policy as well.

Second we will develop the same "information bank" containing money supply time paths, representing the four types of the monetary policy.

Third, we will assume that the expenditure part of fiscal policy [G_t] is exogenously determined as in equation (2′) with the same initial value and the same stochastic behavior of the random component, where the other part of the fiscal policy, on the other hand, the tax rate policy [H_t], is endogenously determined as a result of the optimization procedure.

Fourth, we will develop another type of "information bank" containing different time trajectories for the other control variable, tax rates, that all follow the same stochastic autoregressive behavior as in the following equation :

$$H_t = h_0 + h_1 H_{t-1} + h_t$$

with an initial value: [H_{-1}] = 0.25

and [h_t] following the normal distribution with zero mean and (0.0001) standard deviation.

The suggested tax rate policies have the following parameter specifications:

#	h_0	h_1
1	0.000	0.800
2	0.000	0.900
3	0.000	0.995
4	0.000	0.999
5	0.000	1.000
6	0.000	1.004
7	0.150	0.000

#	h_0	h_1
8	0.200	0.000
9	0.300	0.000
10	0.350	0.000
11	0.035	0.990
12	0.035	0.900
13	0.500	-0.960

We select these parameter values in such a way as to create steady, increasing, and decreasing time paths for the tax rate policies.

As it was said in section [II], fiscal and monetary policy influences indirectly the consumer's objective function by altering the time paths for the price level and the wage rate that the consumer takes for granted as he/she chooses the consumption and employment paths.

Our problem in this setting is to choose that combination of money supply and tax rate time trajectories, which will maximize the representative household's objective function subject to equations (2'), (7) and (23).

The solution strategy is as follows:

The first of (135) money supply time paths, called $[M_t]^1$, is selected and substituted into equation (23) together with the government spending given by equation (2') and the first of [13] tax rate policy, called $[H_t]^1$, for the calculation of the price level time path. The selected government policies must satisfy of the previously cited restrictions:

1) that the price level path resulting from equation (23) must always experience positive values and

2) the multiplier associated with the Cash-in-Advance constraint to be always positive too.

The combinations of fiscal and monetary policies that satisfy both conditions are called "feasible", and they are available for further elaboration. The ones which do not meet at least one of the requirements are rejected from fiscal and monetary policy available space.

The selected feasible government policies $[M_t]^1$ and $[H_t]^1$ with the corresponding price level time path and the series for the productivity factor $[f_t]$ are substituted into the objective function to calculate the value of the expression:

$$\sum_{t=0}^{100} b^t \left[\log\left(\frac{M_{t-1}}{N\ P_t} \right) - B\left(\frac{1}{A_t\ f_t\ N} H_t \left(\frac{M_{t-1}}{P_t} + G_t \right) + \frac{M_t}{P_t} \right) \right]$$

The above procedure is repeated (50) times using the same feasible pair of monetary and fiscal policy and different time series for $[G_t]$ and $[f_t]$, each one corresponding to a different realization of the random variable $[n_t]$ and $[e_t]$.

An average value (mean value) of these summations approximating the expected value

$$E \sum_{t=0}^{100} b^t [\ldots\ldots\ldots\ldots\ldots]$$

is calculated and this mean value is called $[M\text{-}F\ EV]^{1\&1}$, corresponding to the money supply path $[M_t]^1$ (the monetary policy instrument) and to the tax rate path $[H_t]^1$ (the fiscal policy instrument).

We repeat the above procedure for all feasible money supply paths using the same feasible tax rate policy $[H_t]^1$ and consequently, we calculate an equal amount of $[\text{M-F EV}]^{j\&1}$ values, where the parameter [j] refers to the number of feasible money supply time paths.

The highest of these $[\text{M-F EV}]^{j\&1}$ values is chosen and is called $[\text{M-F Vmax}]^{j\&1}$.

We repeat the above procedure for all feasible tax rate policies and that leads to a calculation of an equal amount of $[\text{M-F Vmax}]^{j\&r}$ values where the parameter [r] refers to the number of the feasible tax rate time paths.

The highest of these values is chosen and is called:

$$[\text{M-F Vmax}]^*$$

Then, the combination of money supply and tax rate policies that correspond to this $[\text{M-F Vmax}]^*$ is chosen and is called the optimal monetary and fiscal policy.

Having defined the form of the optimal monetary and fiscal policy, we can determine the economy's time series for the wage rate, the inflation rate, the employment level, the output level, and the per-capita consumption level.

The computer generated results show that the highest calculated value for the welfare level

$$[\text{M-F Vmax}]^* = 48.110$$

which corresponds to a monetary policy of the following money supply rule

$$M_t = 1.2\ M_{t-1}$$

and a tax rate policy of the tenth rule, given by the formula:

$$H_t = 0.35 + 0\ H_{t-1} + h_t$$

Comparing the welfare level that this economy experiences in the case that the government has available and uses both of its instruments optimally with the welfare level that results from the application of the optimal monetary policy only, we notice an improvement in the welfare level in the case of the availability of both policies given by the difference

$$[M\text{-}F\ Vmax]^* - [Vmax] = 15.308$$

In graphs [1, 2, 3, 4, 5, 6], we present the plots for the GNP level, per-capita consumption level, employment level, price level, budget deficit, and the inflation rate resulting from the application of optimal monetary and fiscal policy, shown in graphs [7 and 8] respectively.

We would like also to consider the results of the application of an alternative policy. More precisely, this administration does not apply the optimal money supply policy which requires a twenty percent growth rate in the money base and the optimal fiscal one which requires a thirty five percent tax rate but a suboptimal one of the following form:

$$M_t = 1.07\ M_{t-1}$$

$$H_t = 0.035 + 0.9\ H_{t-1} + h_t$$

The results show that under the new government policy the consumer's welfare level is

$$[S\ F\text{-}M\ Vmax]^* = 24.529$$

which shows that the economy experiences a welfare loss due to the suboptimal policy given by the difference

$$[F\text{-}M\ Vmax]^* - [S\ F\text{-}M\ Vmax]^* = 23.581$$

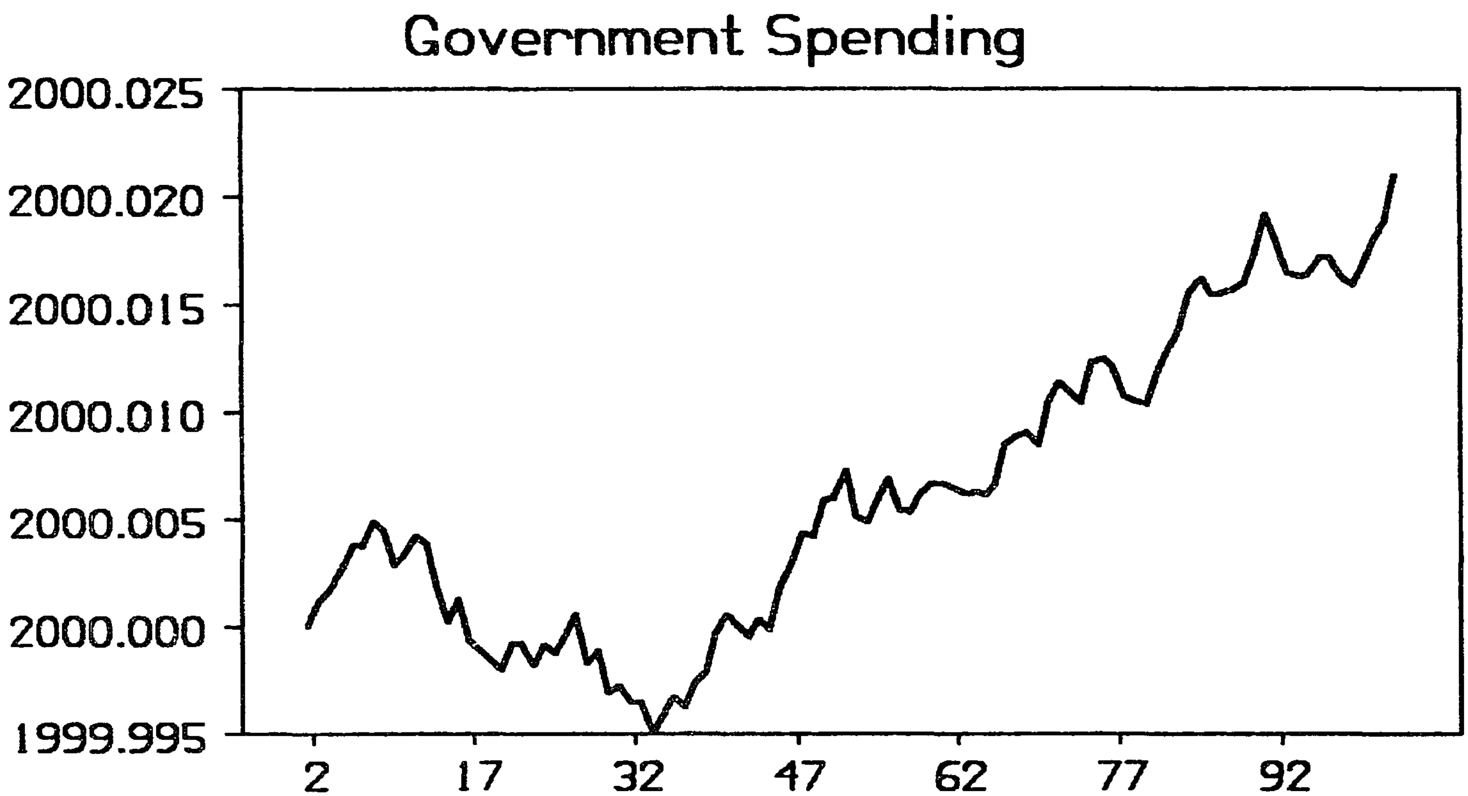
The application of the optimal policy generates a higher inflation rate, about twenty percent, which forces consumers to substitute the cash good with leisure and consequently the economy experiences a lower employment, output, and per capita consumption time paths. On the other hand though, the application of the suboptimal government policy leads to a decreasing inflation rate which creates higher employment, output, and consumption time paths.

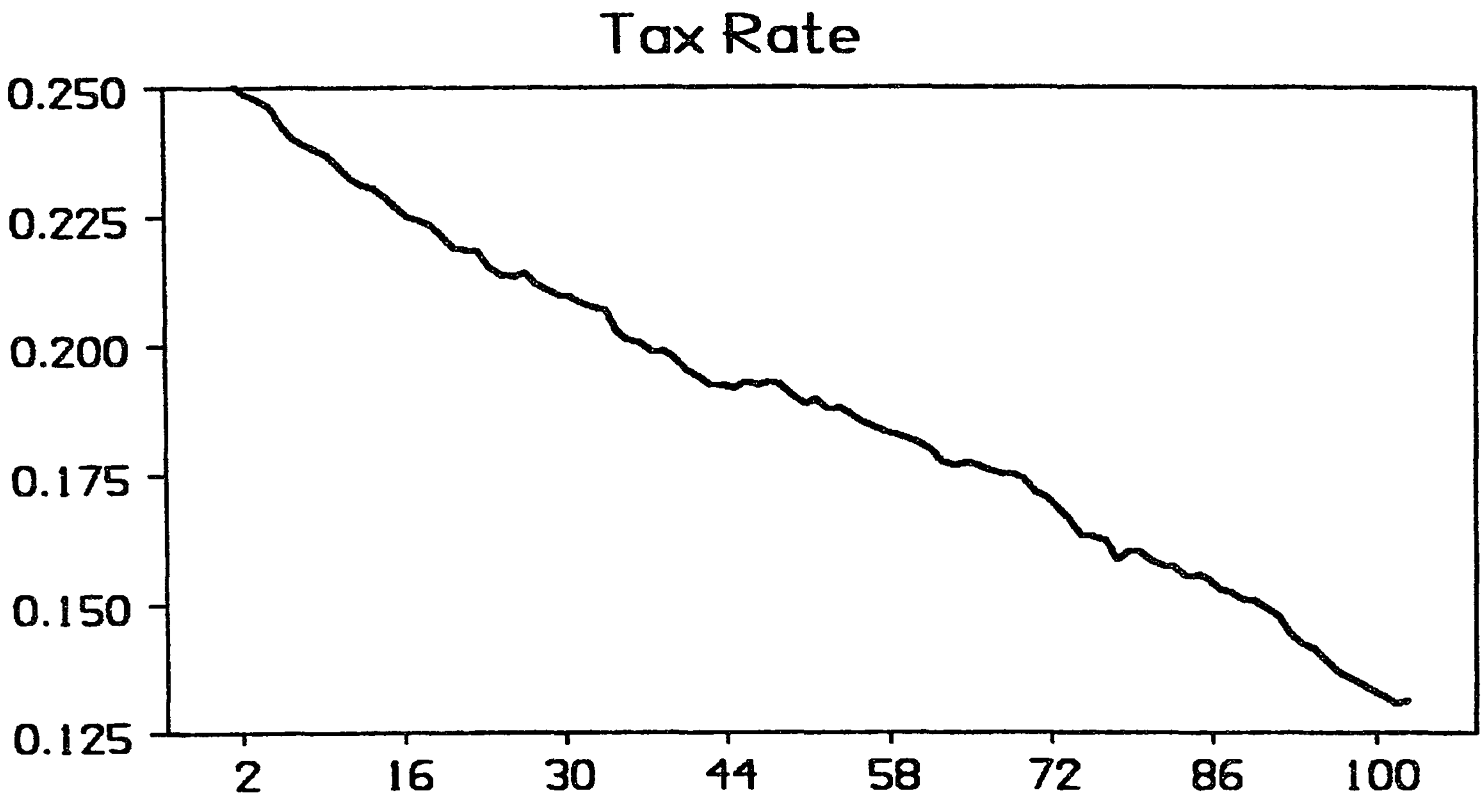
Graphs [1a, 2a, 3a, 4a and 6a] present the GNP level, per-capita consumption level, employment level, price level and the inflation rate due to the application of the suboptimal government policy, shown in graphs [7a and 8a] respectively.

■□

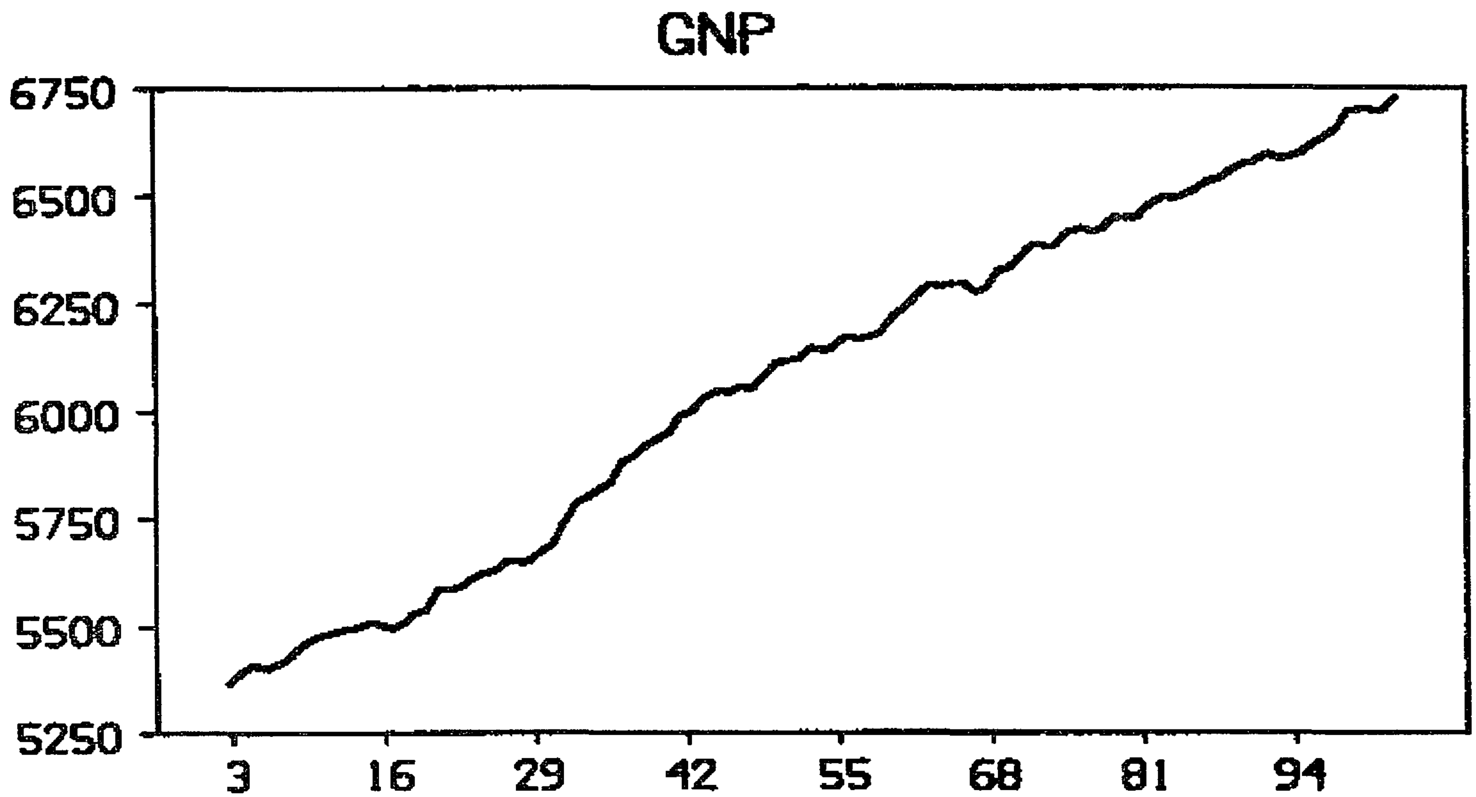
IV. GRAPHS

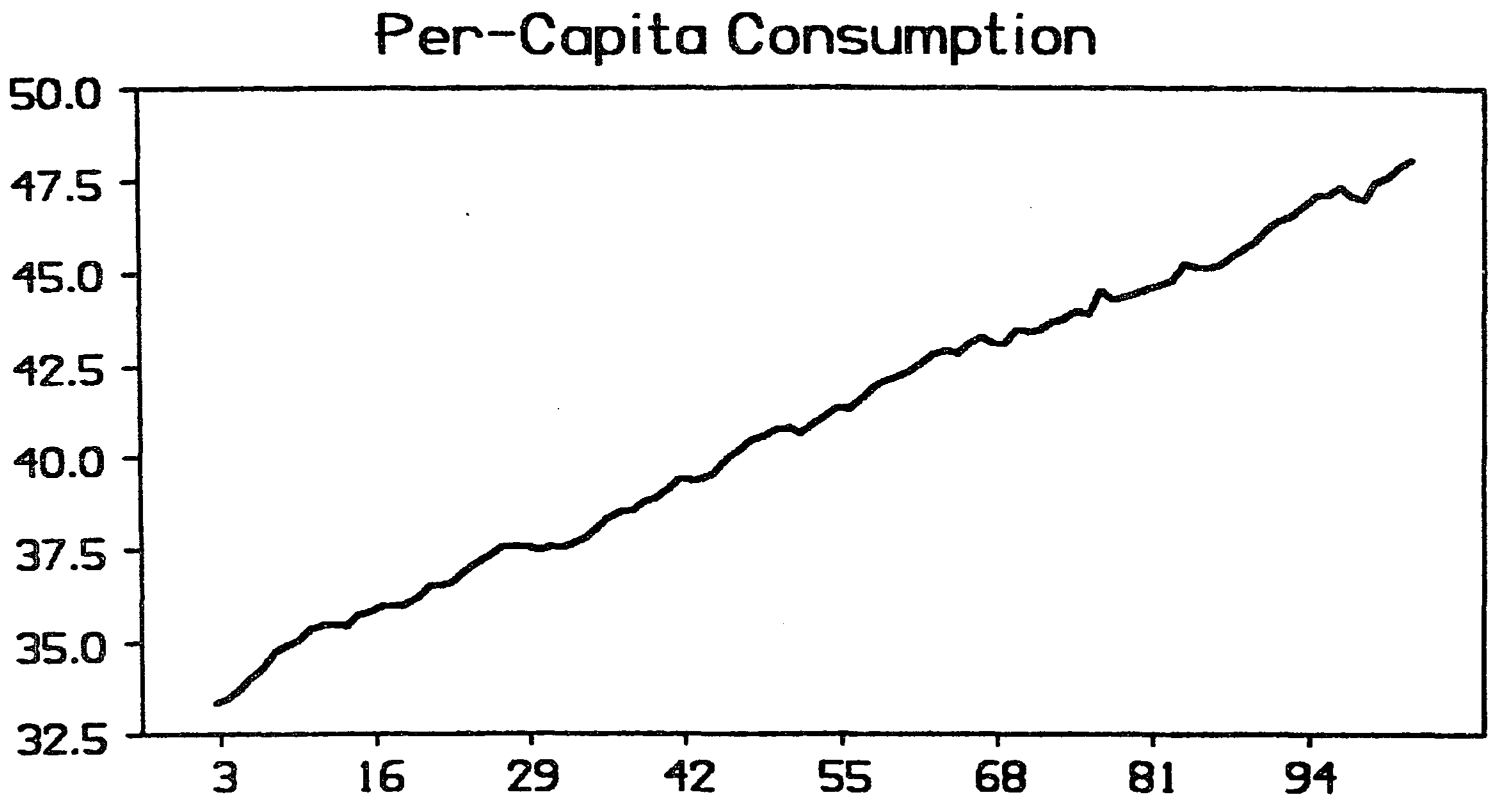
Graph a


Government Spending
2000.025
2000.020
2000.015
2000.010
2000.005
2000.000
1999.995
2
17
32
47
62
77
92


Tax Rate
0.250
0.225
0.200
0.175
0.150
0.125
2
16
30
44
58
72
86
100

Graph c

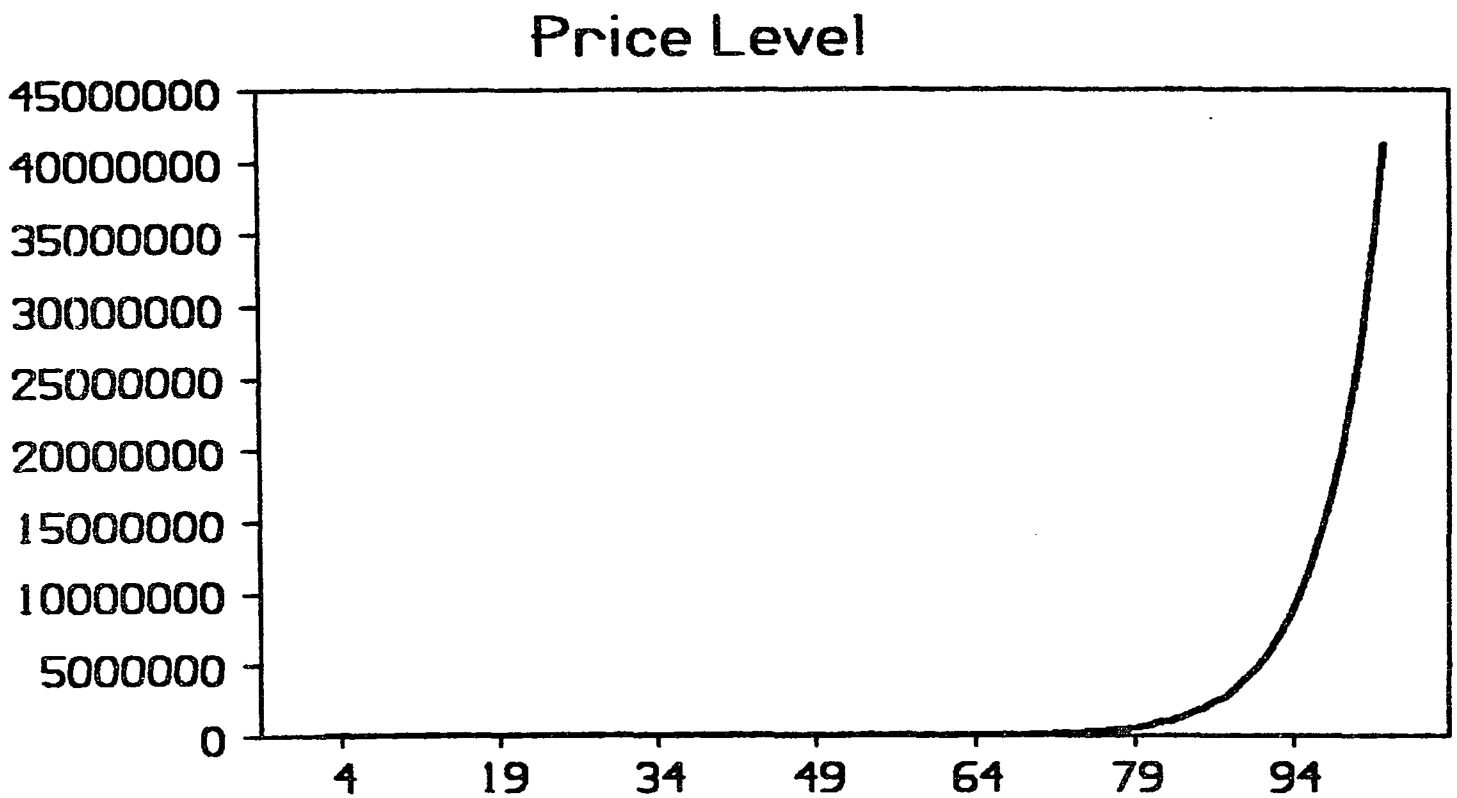



Per-Capita Consumption
50.0
47.5
45.0
42.5
40.0
37.5
35.0
32.5
3
16
29
42
55
68
81
94

Graph d

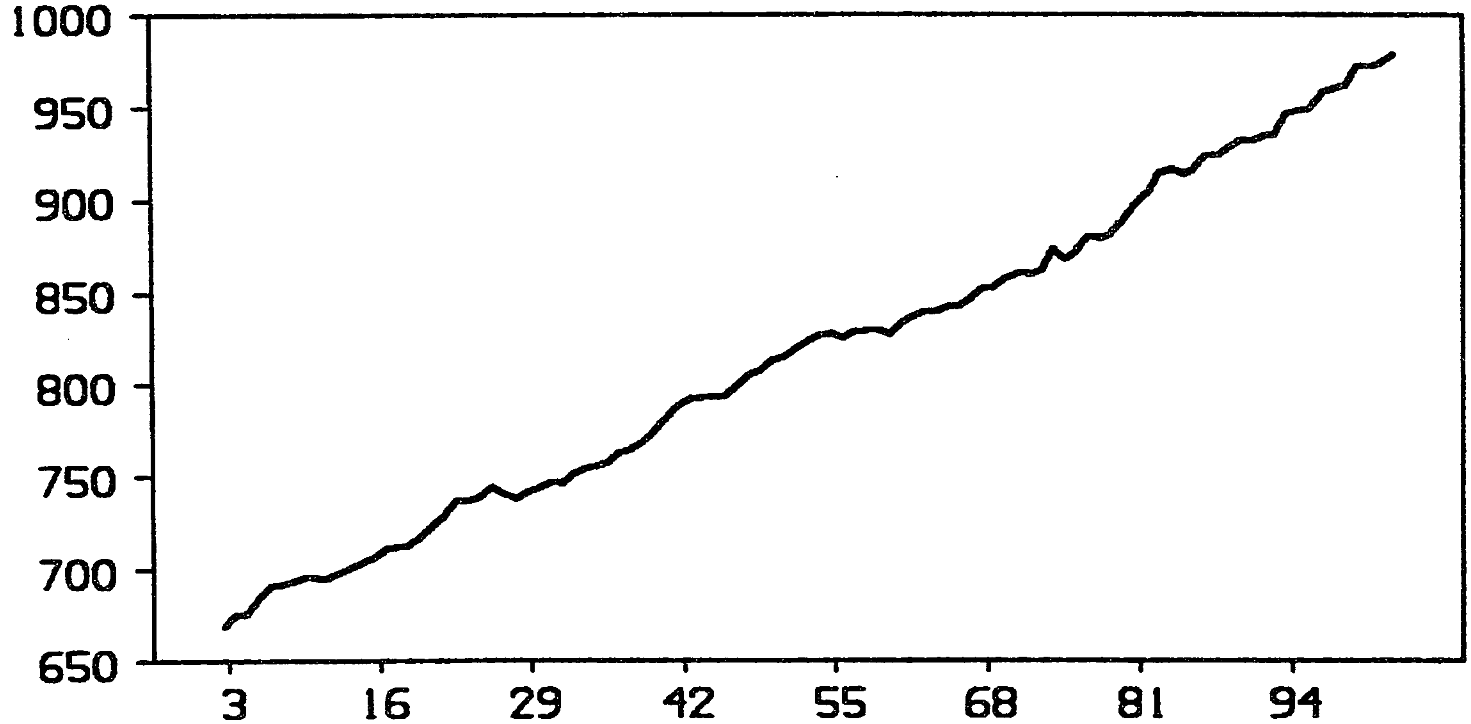
Graph e

Graph f

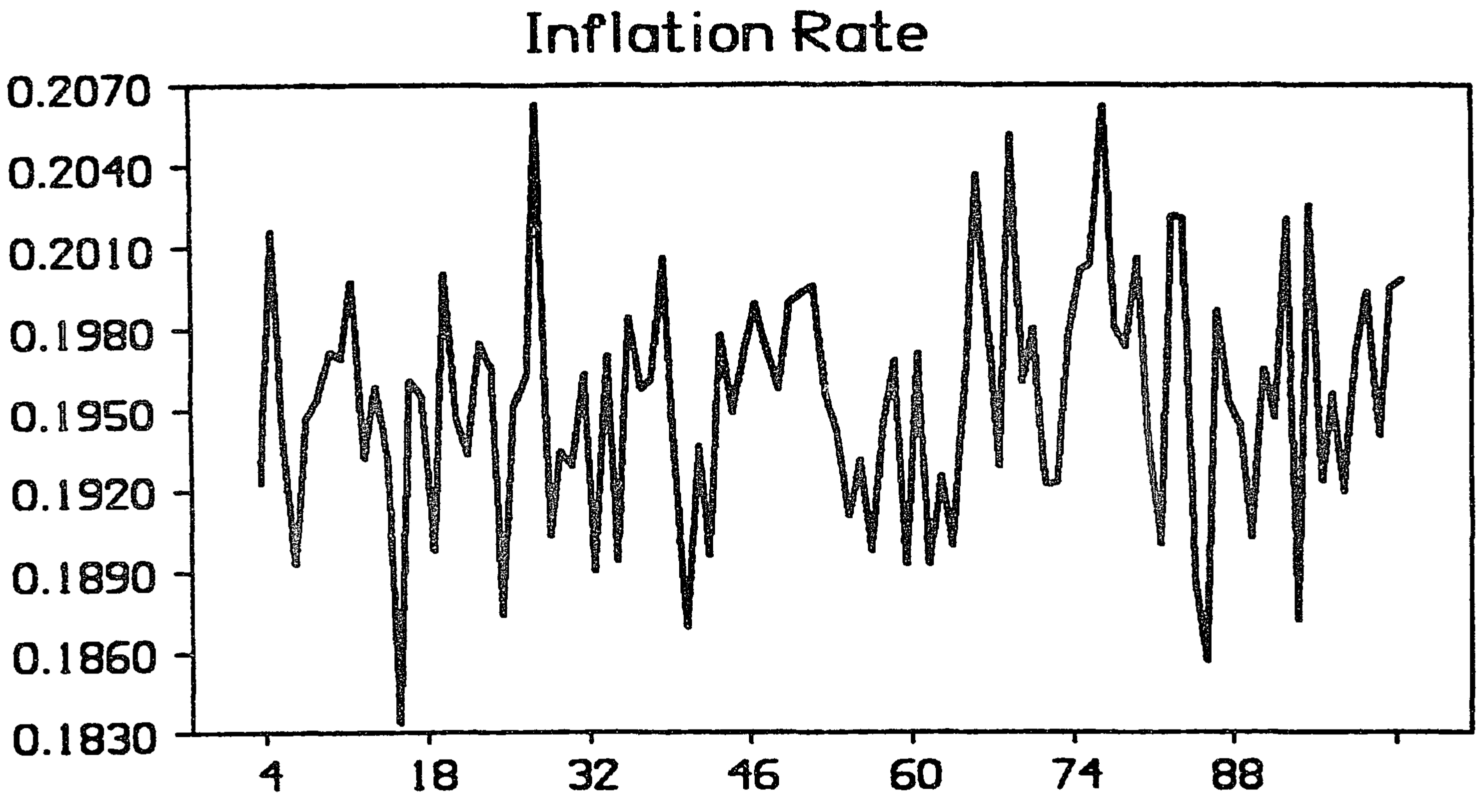

Price Level
45000000
40000000
35000000
30000000
25000000
20000000
15000000
10000000
5000000
0
4
19
34
49
64
79
94

Graph 9

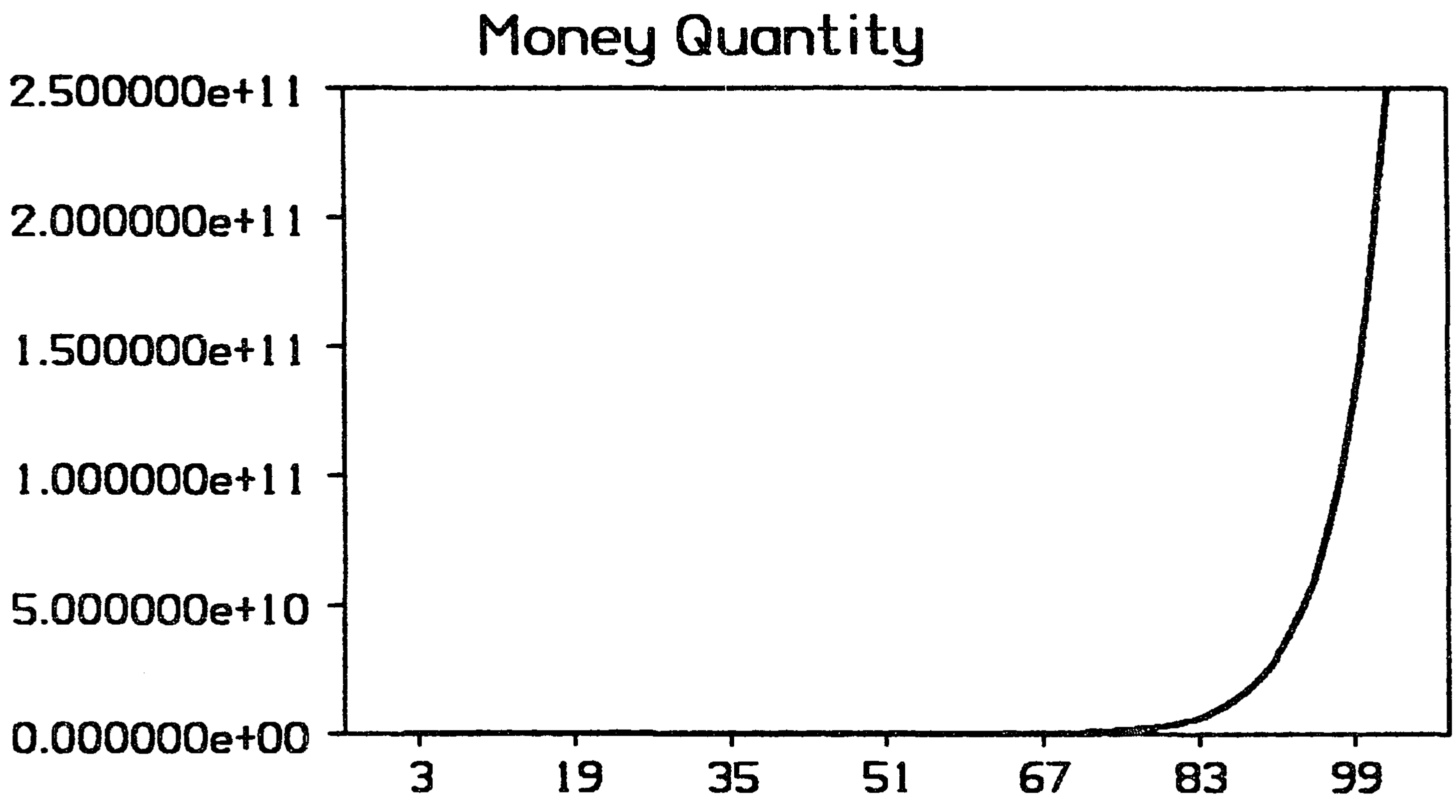
Budget Deficit



Graph h

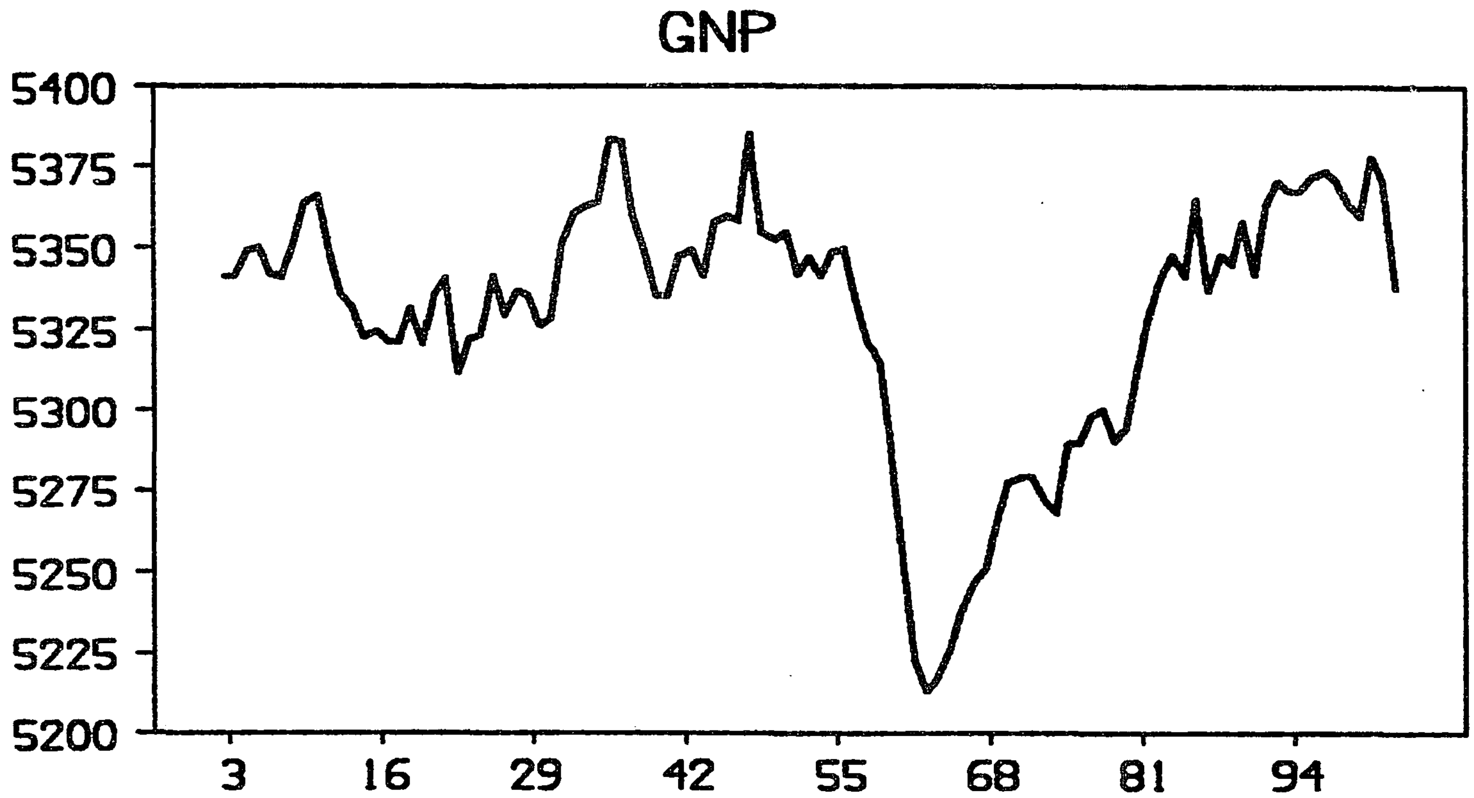

Inflation Rate
0.2070
0.2040
0.2010
0.1980
0.1950
0.1920
0.1890
0.1860
0.1830
4
18
32
46
60
74
88

Graph m


Money Quantity
2.500000e+11
2.000000e+11
1.500000e+11
1.000000e+11
5.000000e+10
0.000000e+00
3
19
35
51
67
83
99

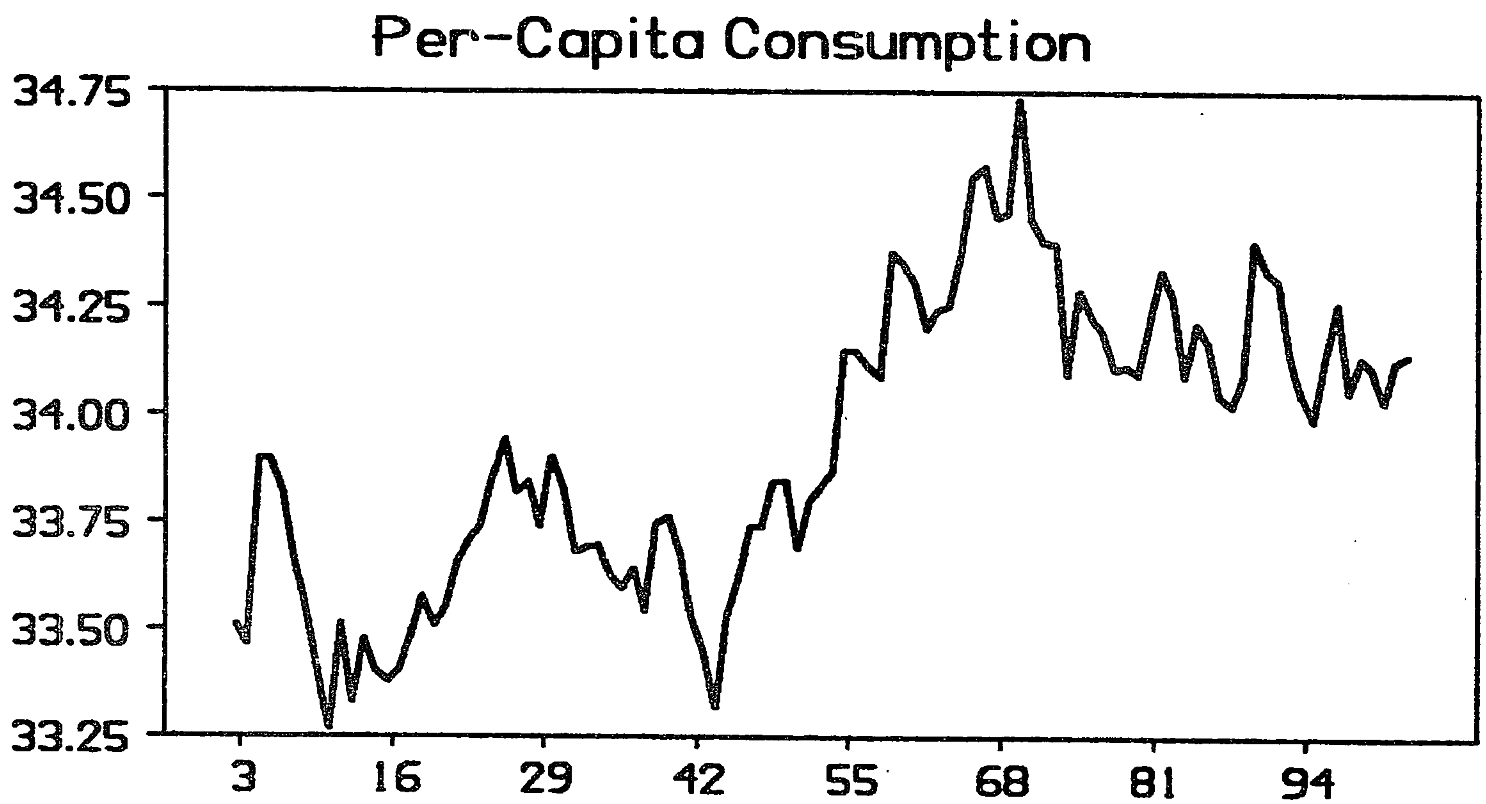
Graph b'


Tax Rate
0.2650
0.2625
0.2600
0.2575
0.2550
0.2525
0.2500
0.2475
0.2450
2
16
30
44
58
72
86
100

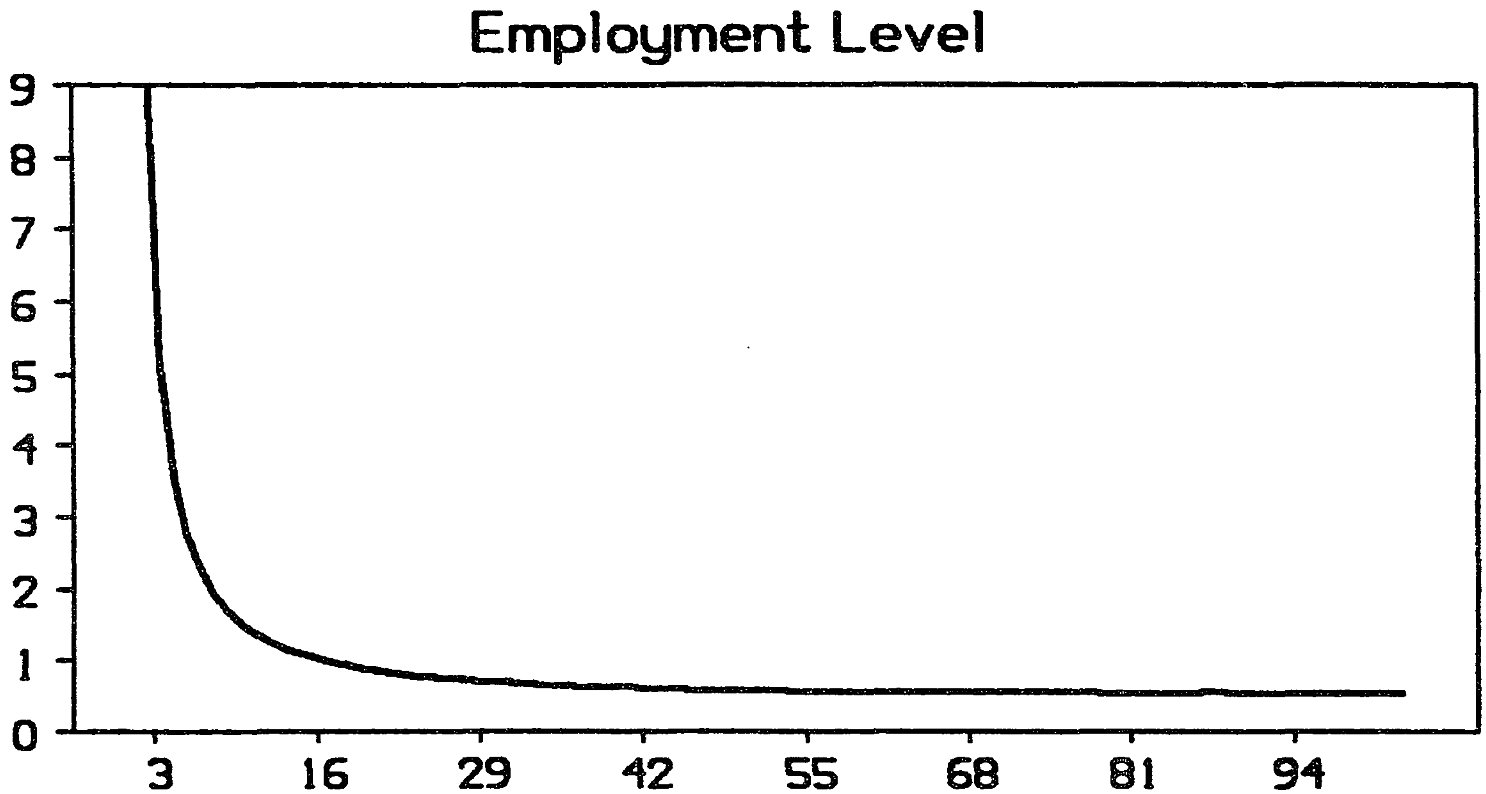

GNP
5400
5375
5350
5325
5300
5275
5250
5225
5200
3
16
29
42
55
68
81
94

Graph c'

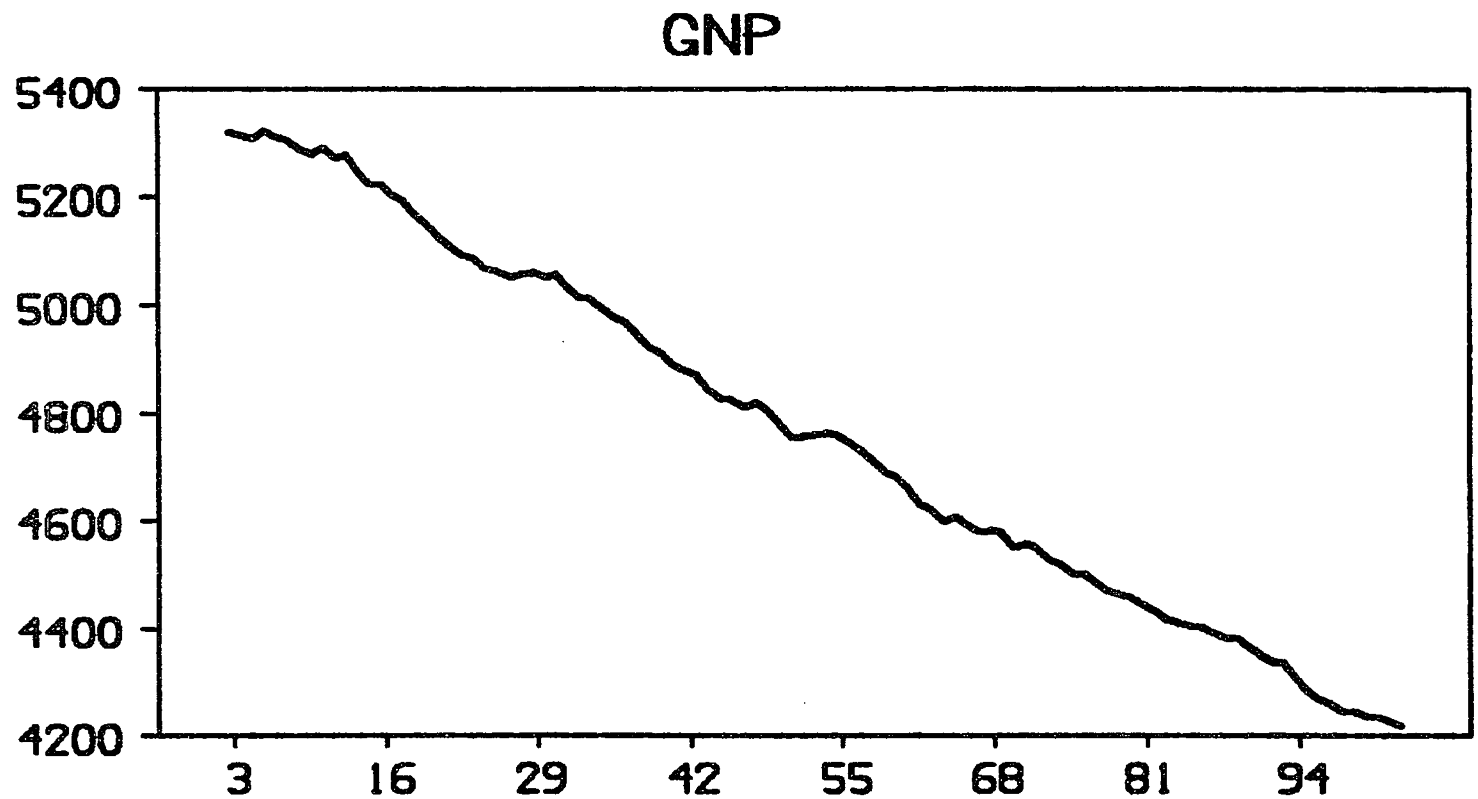
Graph d'


Per-Capita Consumption
34.75
34.50
34.25
34.00
33.75
33.50
33.25
3
16
29
42
55
68
81
94

Graph e'

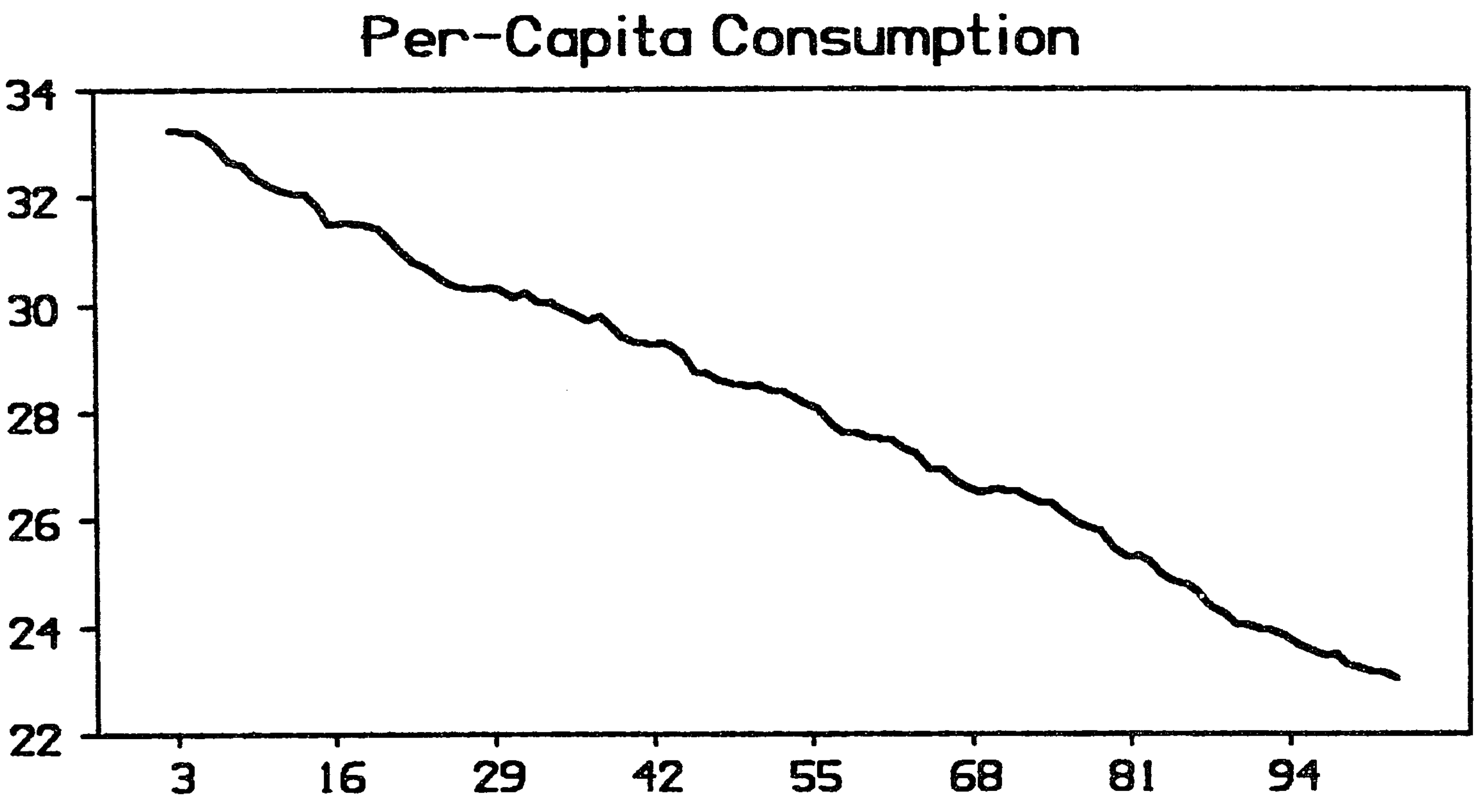


Graph b"

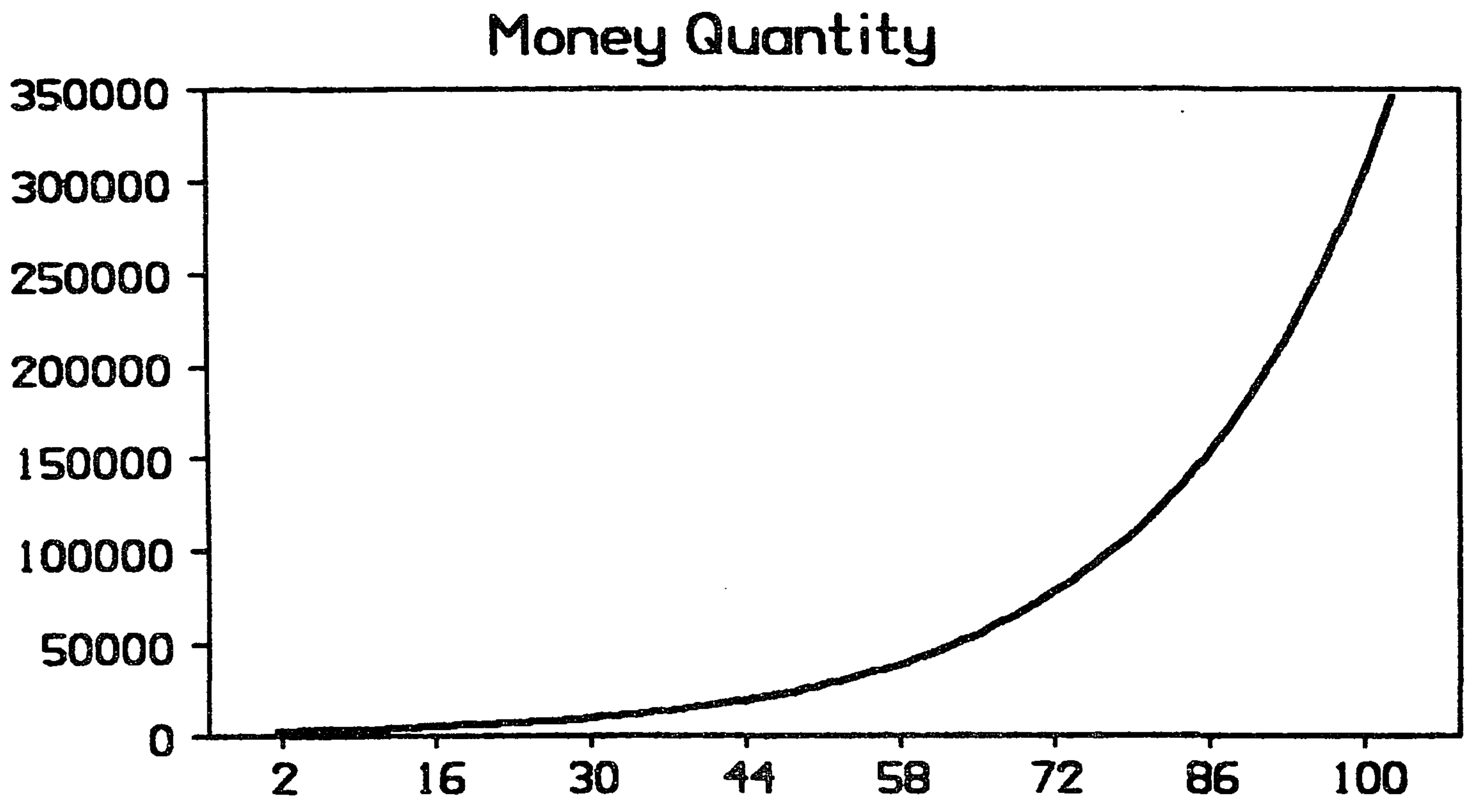


Graph c"

Graph d"

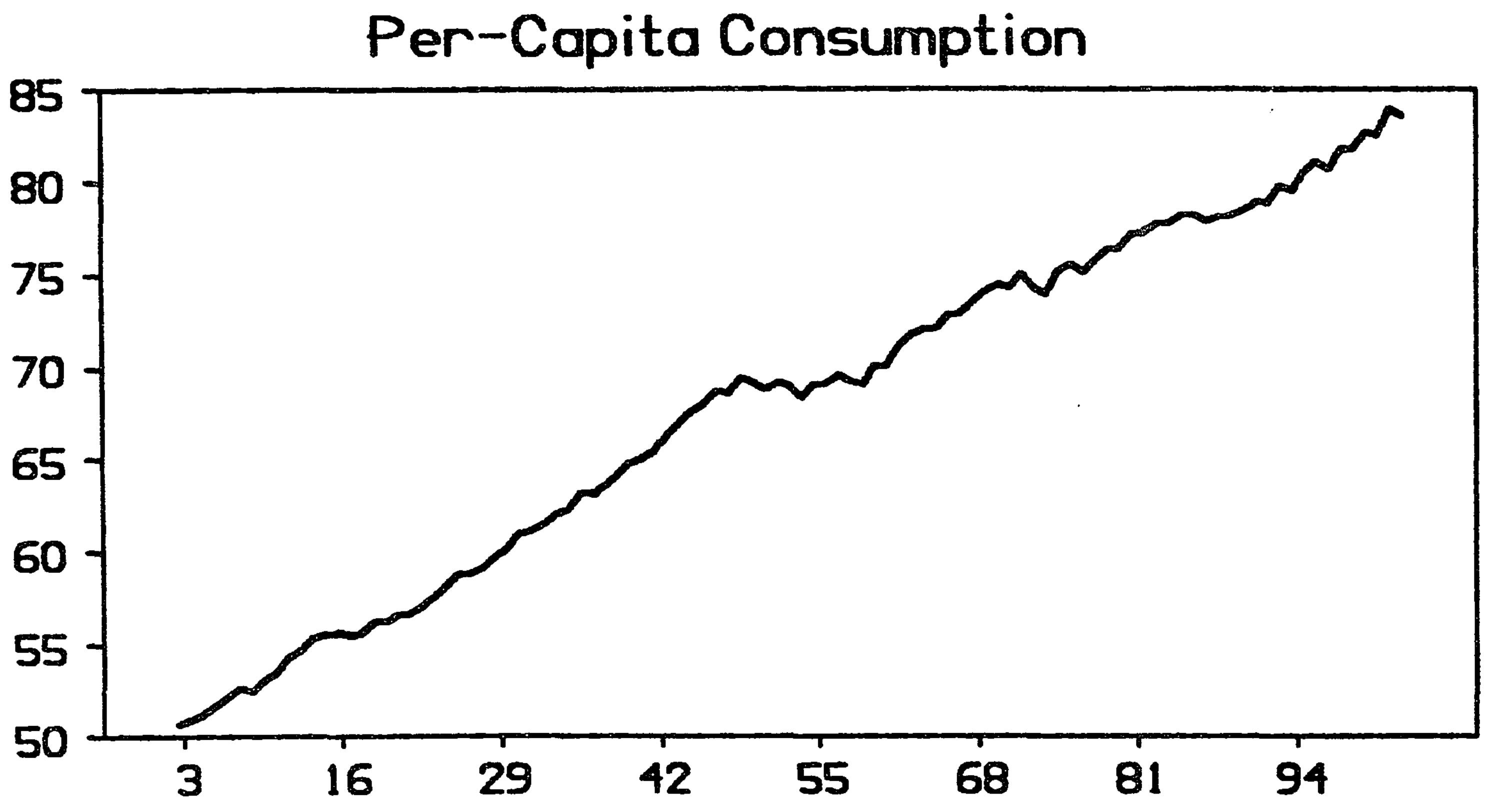


Graph e"


Money Quantity
350000
300000
250000
200000
150000
100000
50000
0
2
16
30
44
58
72
86
100

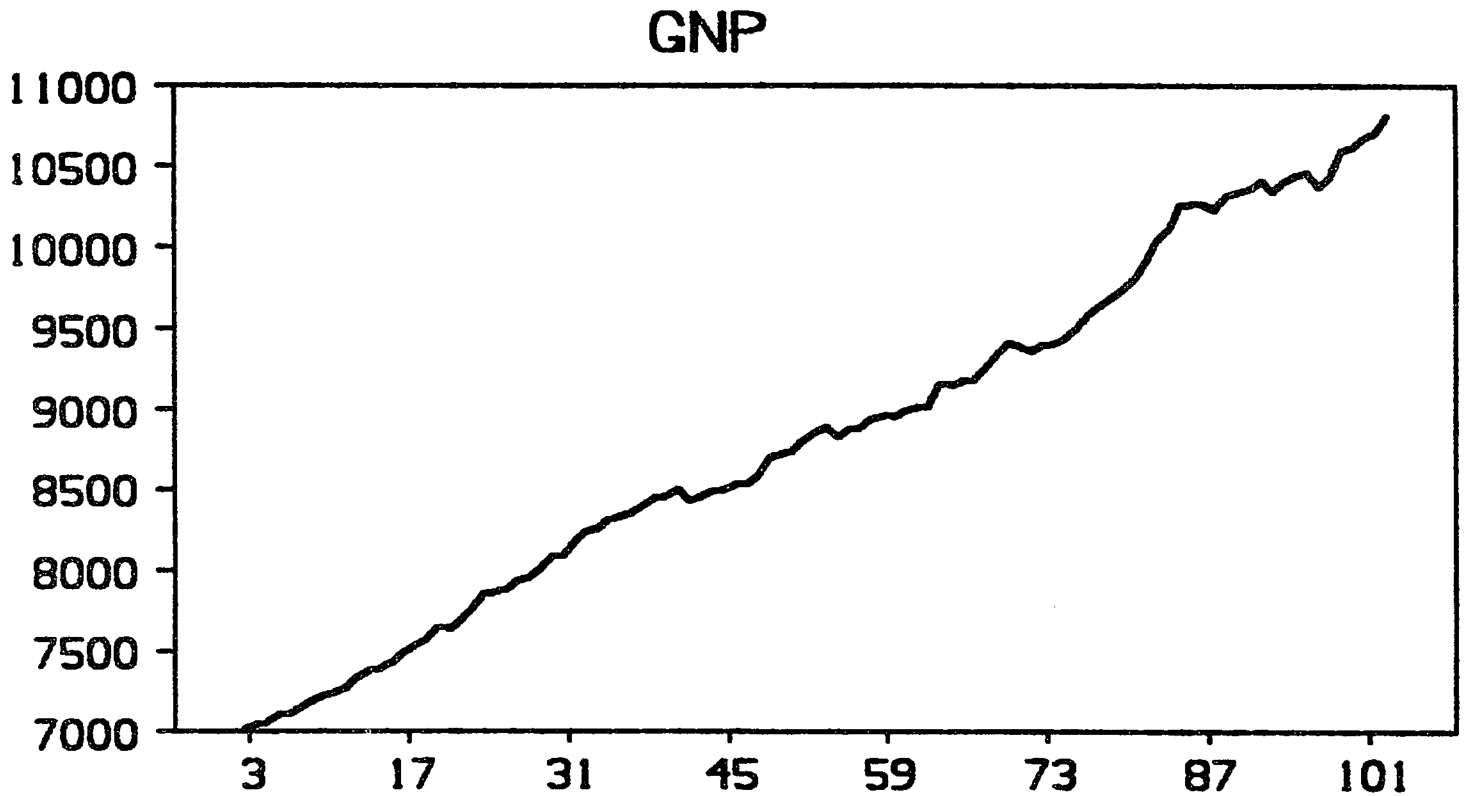
Graph M

Graph D



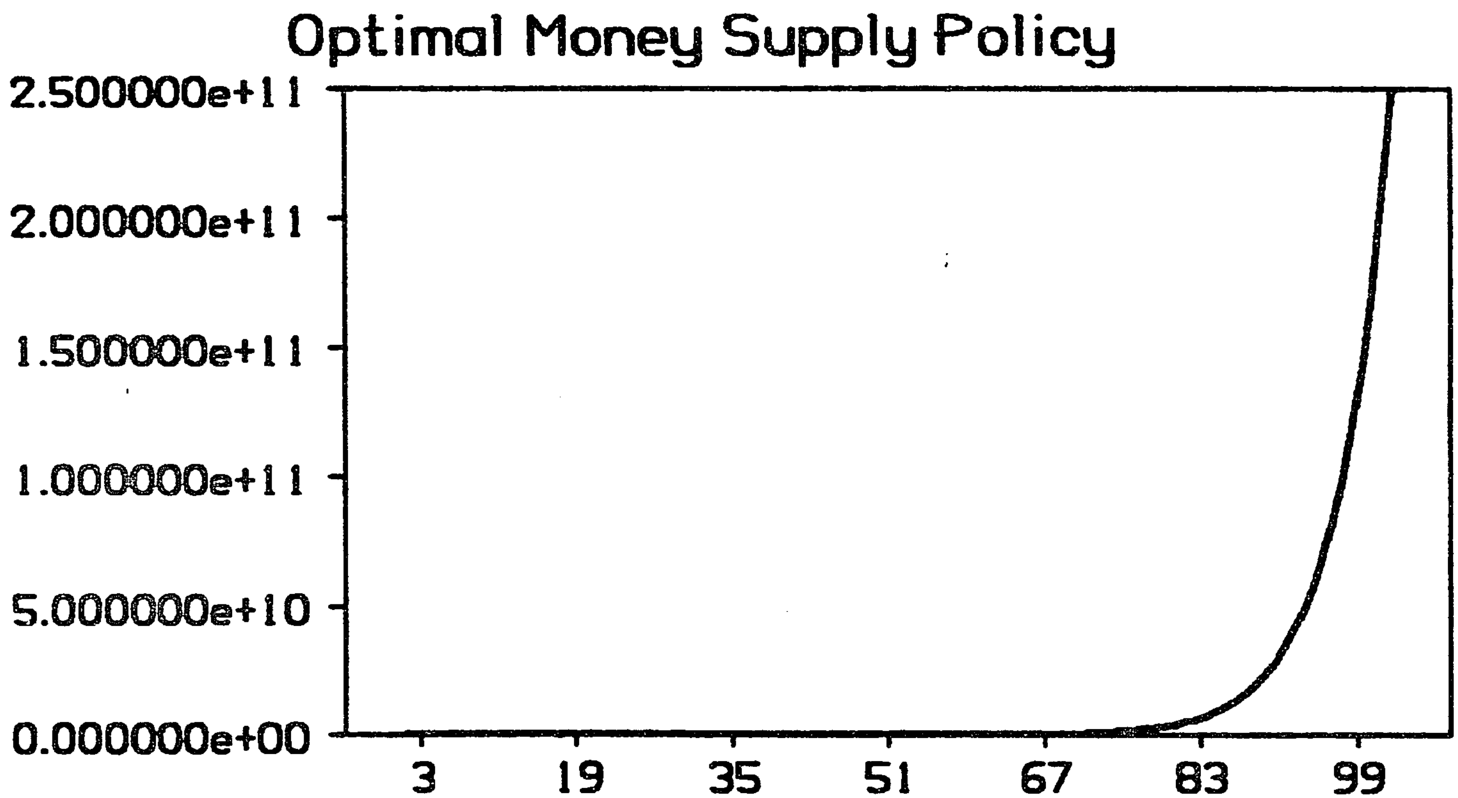
Per-Capita Consumption
85
80
75
70
65
60
55
50
3
16
29
42
55
68
81
94

Graph C


GNP
11000
10500
10000
9500
9000
8500
8000
7500
7000
3
17
31
45
59
73
87
101

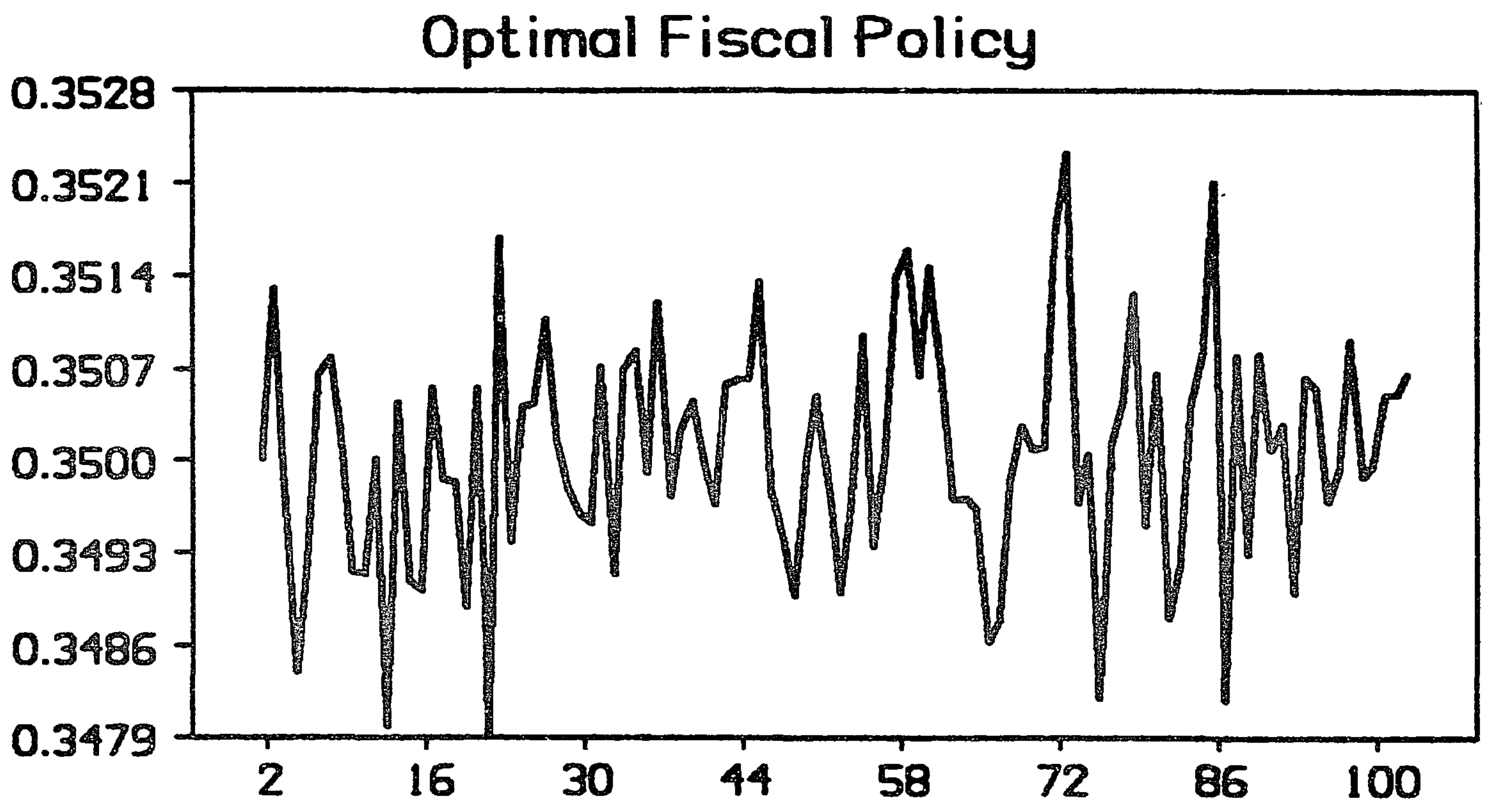
Graph E

Graph 7

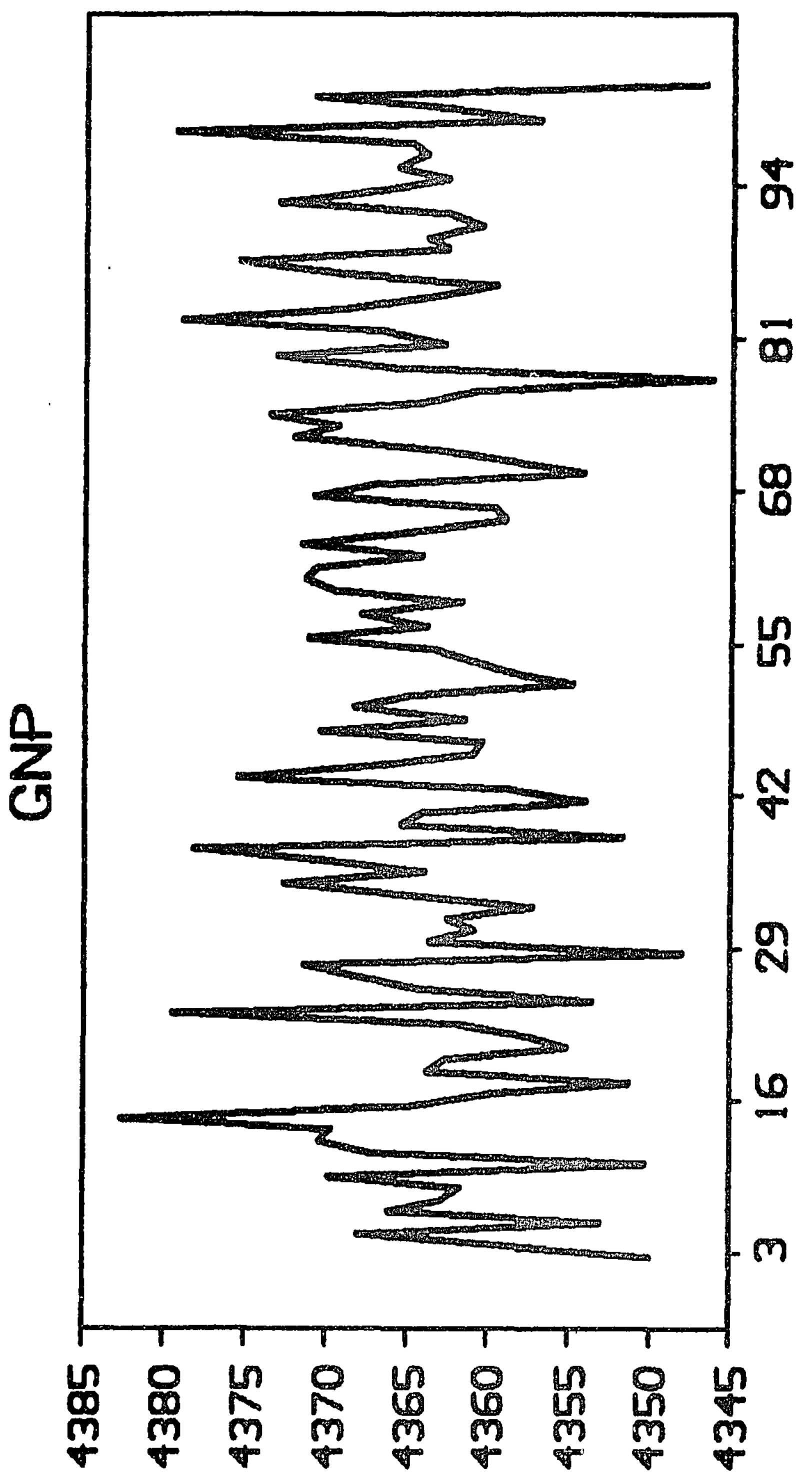


Optimal Money Supply Policy
2.500000e+11
2.000000e+11
1.500000e+11
1.000000e+11
5.000000e+10
0.000000e+00
3
19
35
51
67
83
99

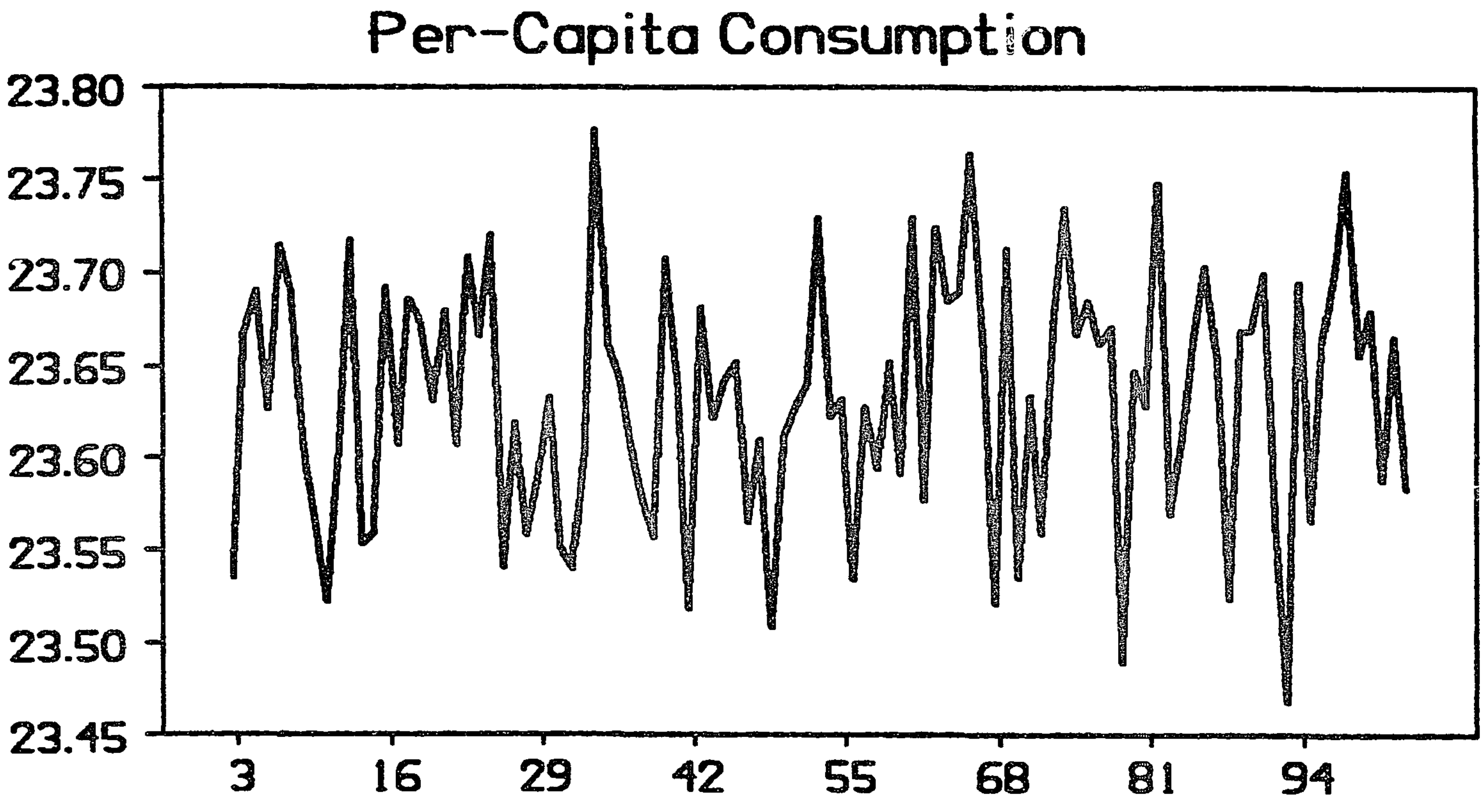
Graph 8


Optimal Fiscal Policy
0.3528
0.3521
0.3514
0.3507
0.3500
0.3493
0.3486
0.3479
2
16
30
44
58
72
86
100

Graph 1


GNP
4385
4380
4375
4370
4365
4360
4355
4350
4345
3
16
29
42
55
68
81
94

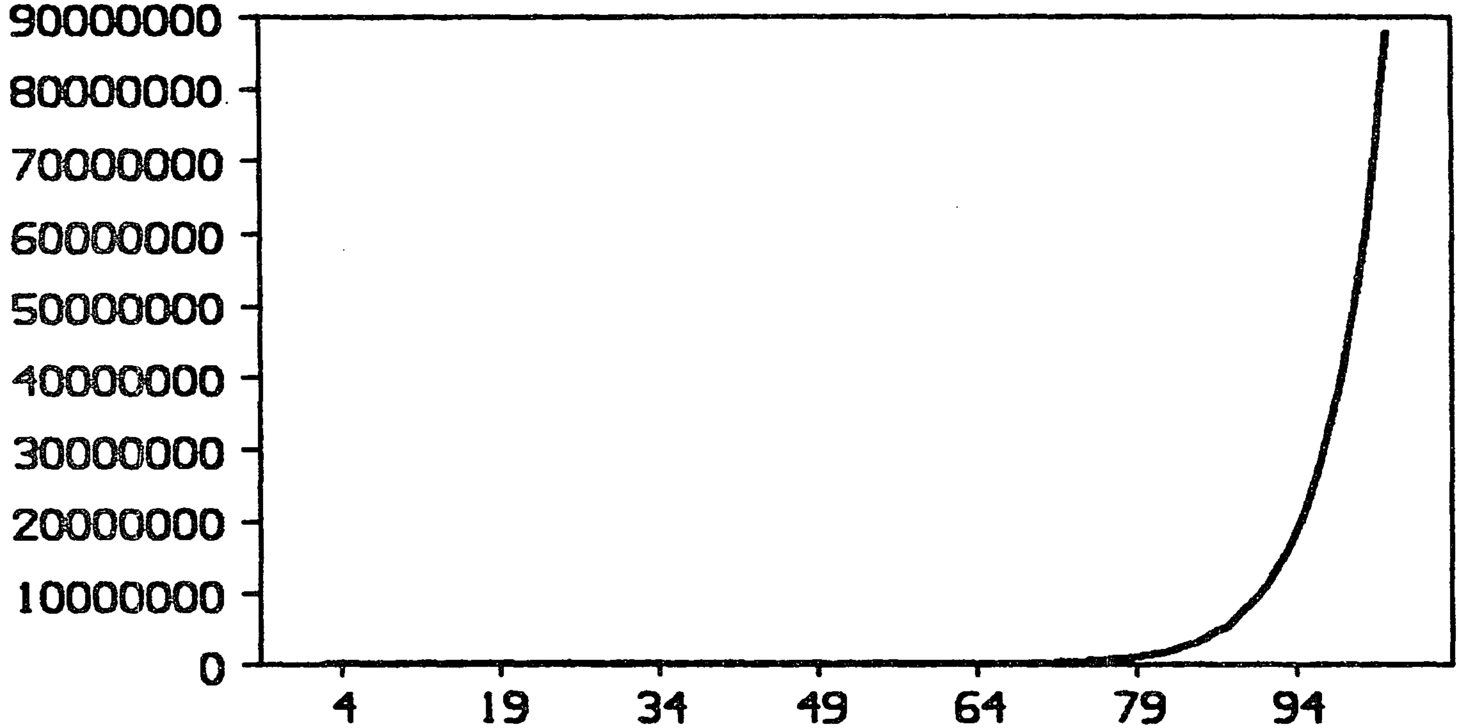
Graph 2


Per-Capita Consumption
23.80
23.75
23.70
23.65
23.60
23.55
23.50
23.45
3
16
29
42
55
68
81
94

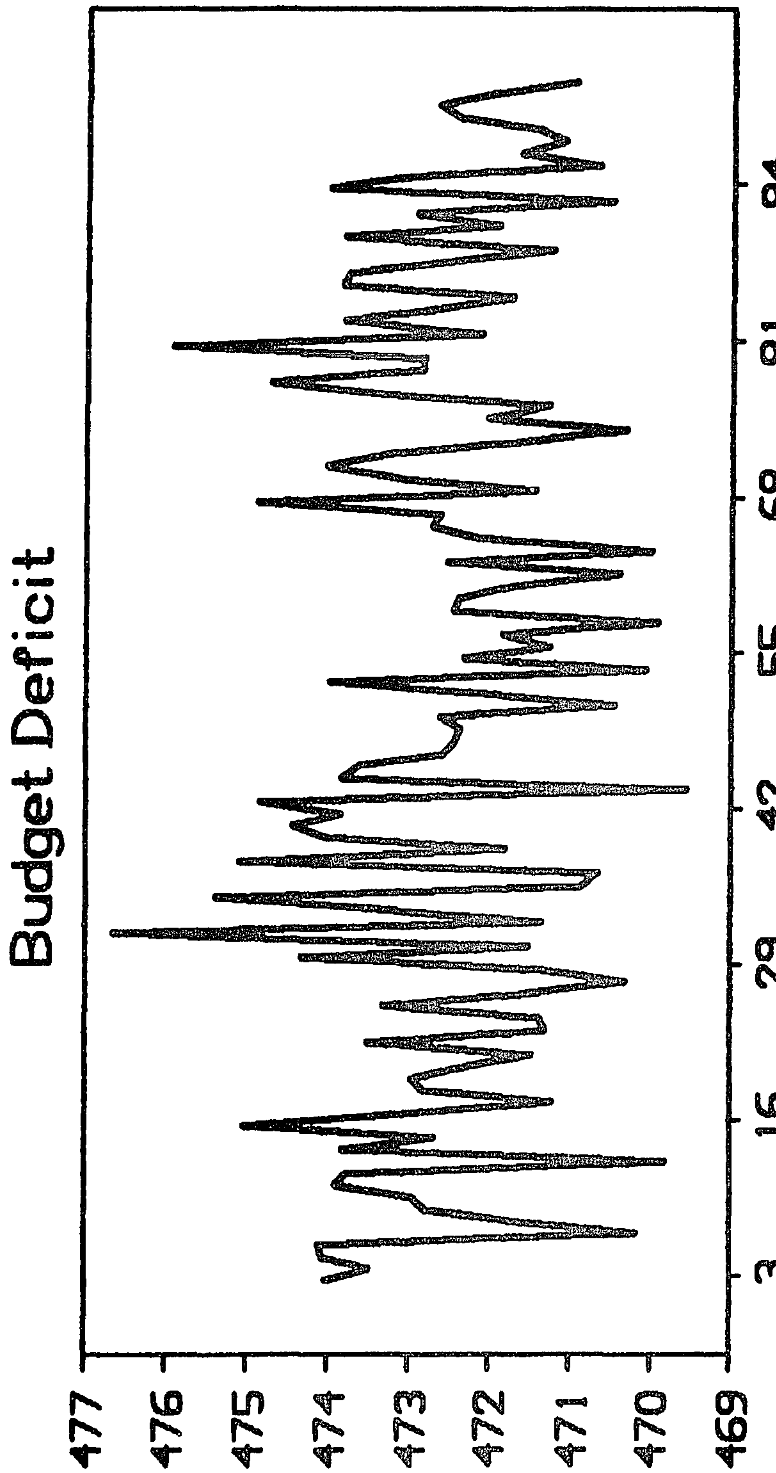
Graph 3

Graph 4

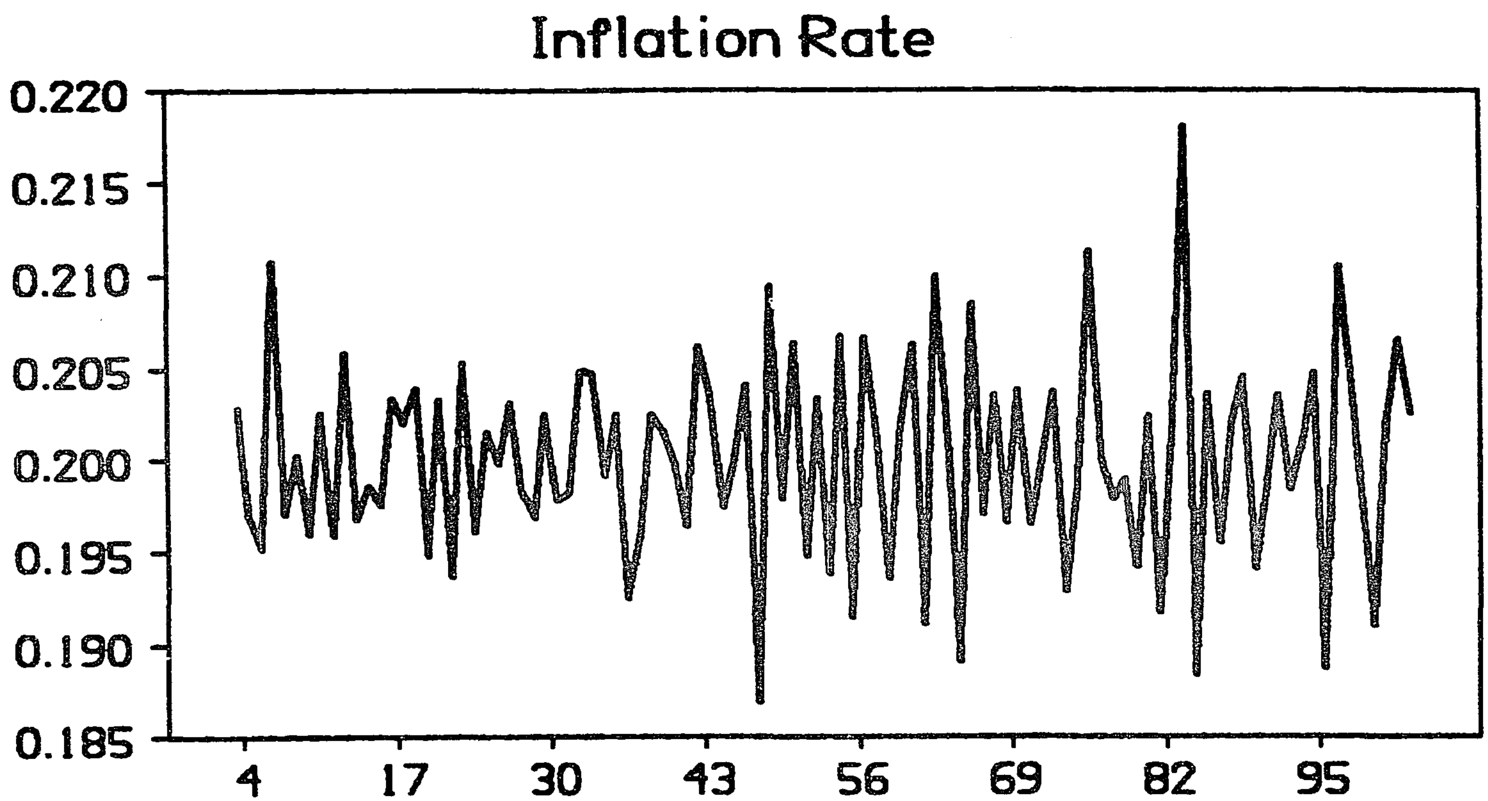
Price Level



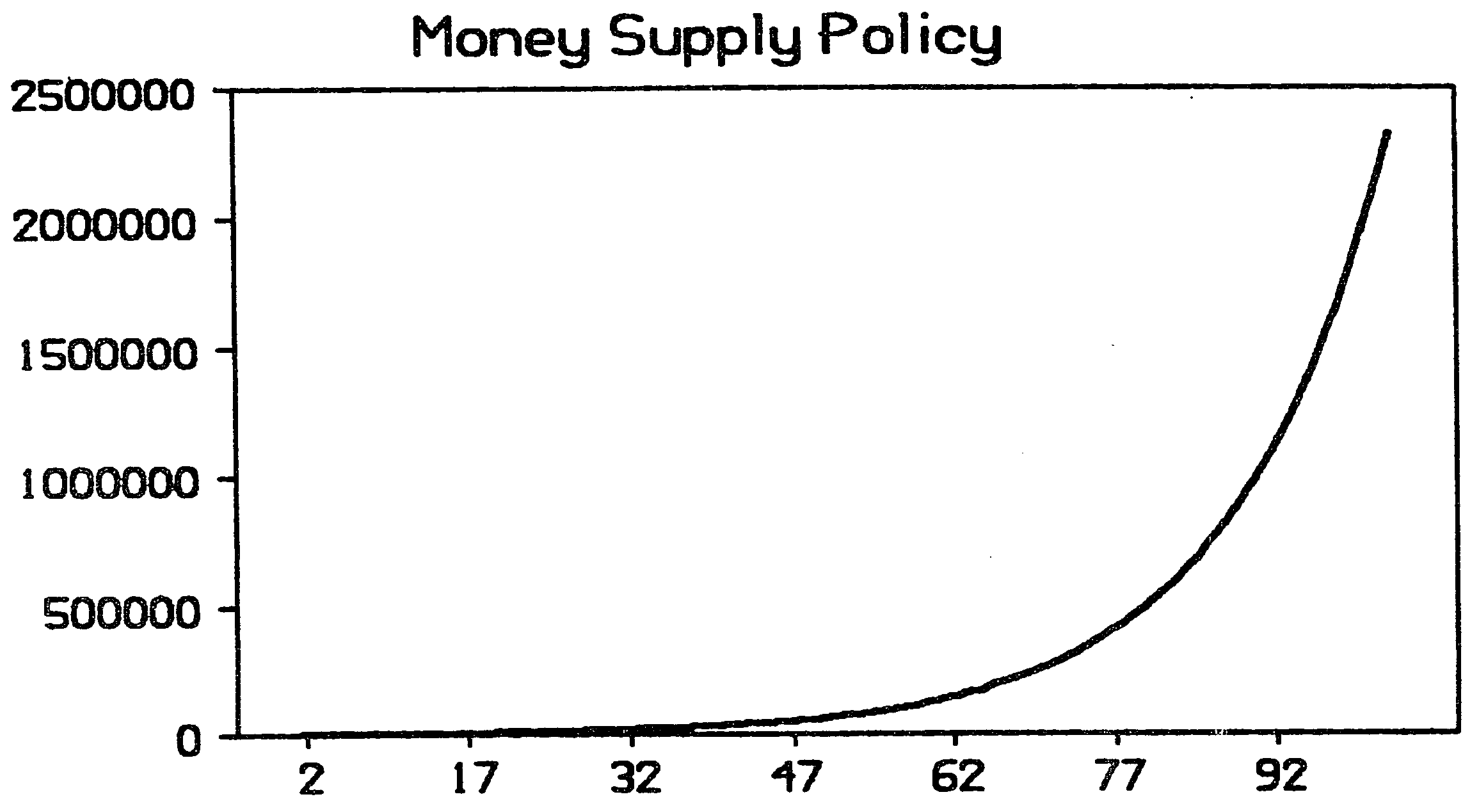
Graph 5


Budget Deficit
477
476
475
474
473
472
471
470
469
3
16
29
42
55
68
81
94

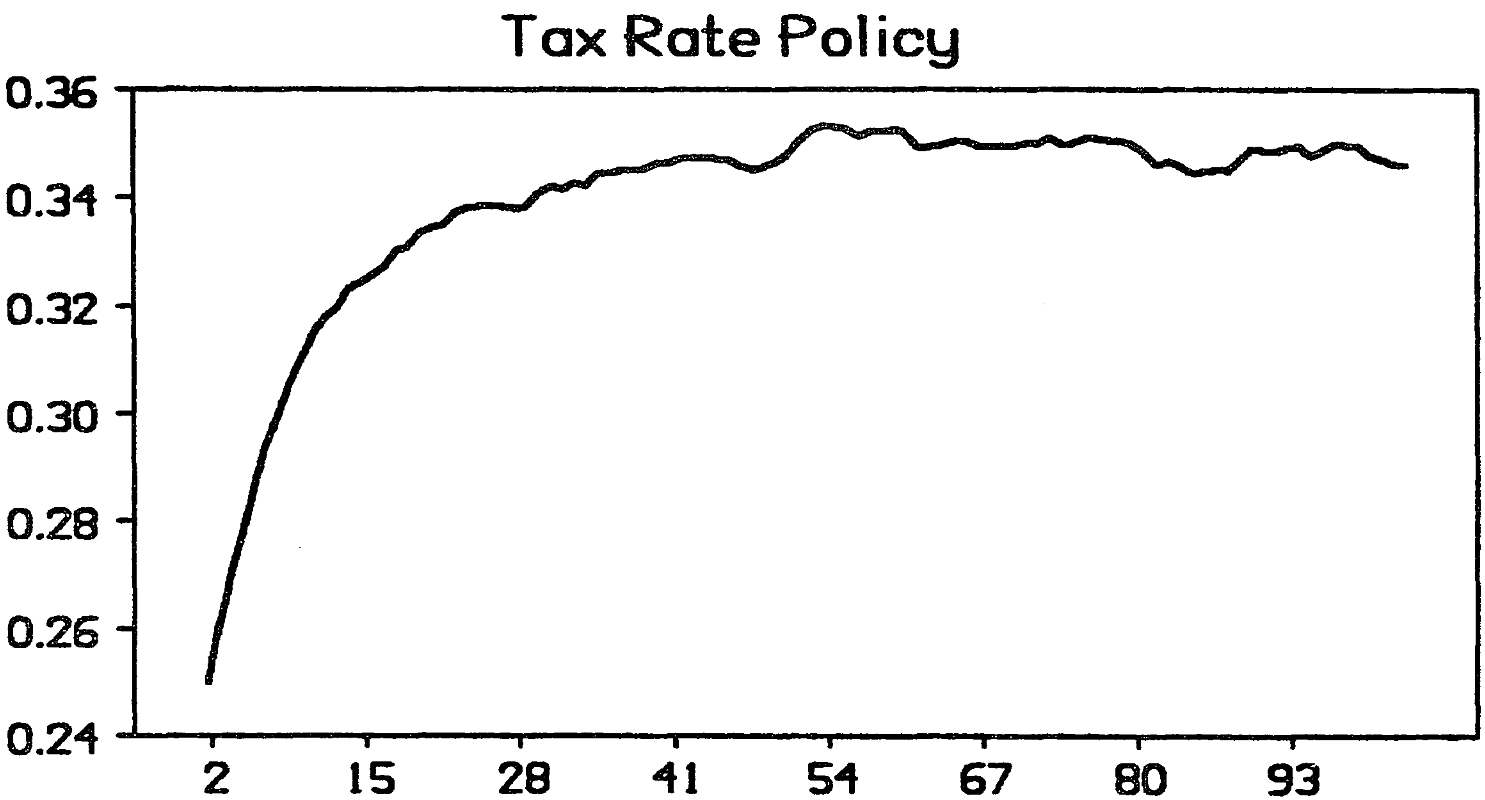
Graph 6


Inflation Rate
0.220
0.215
0.210
0.205
0.200
0.195
0.190
0.185
4
17
30
43
56
69
82
95

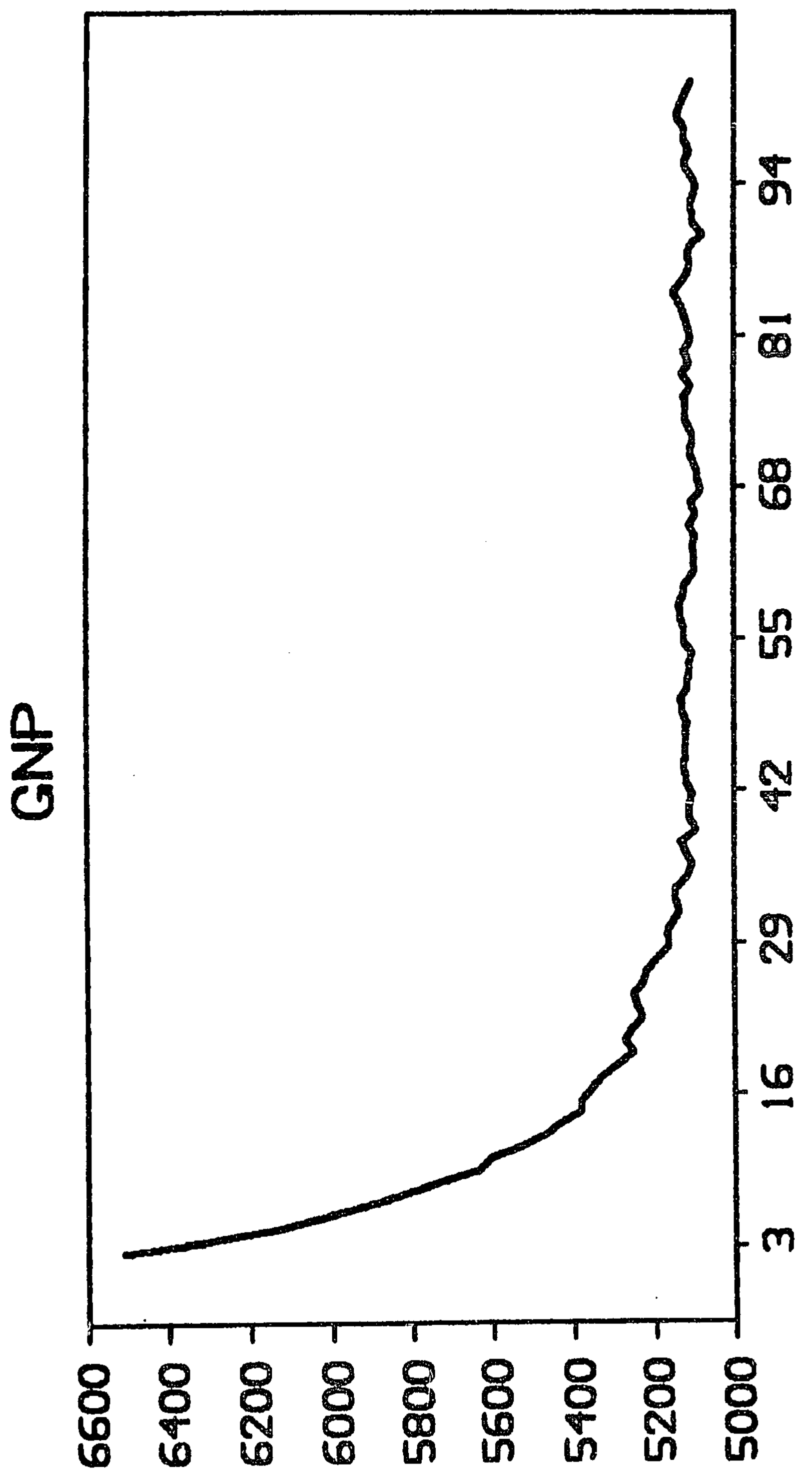
Graph 7a

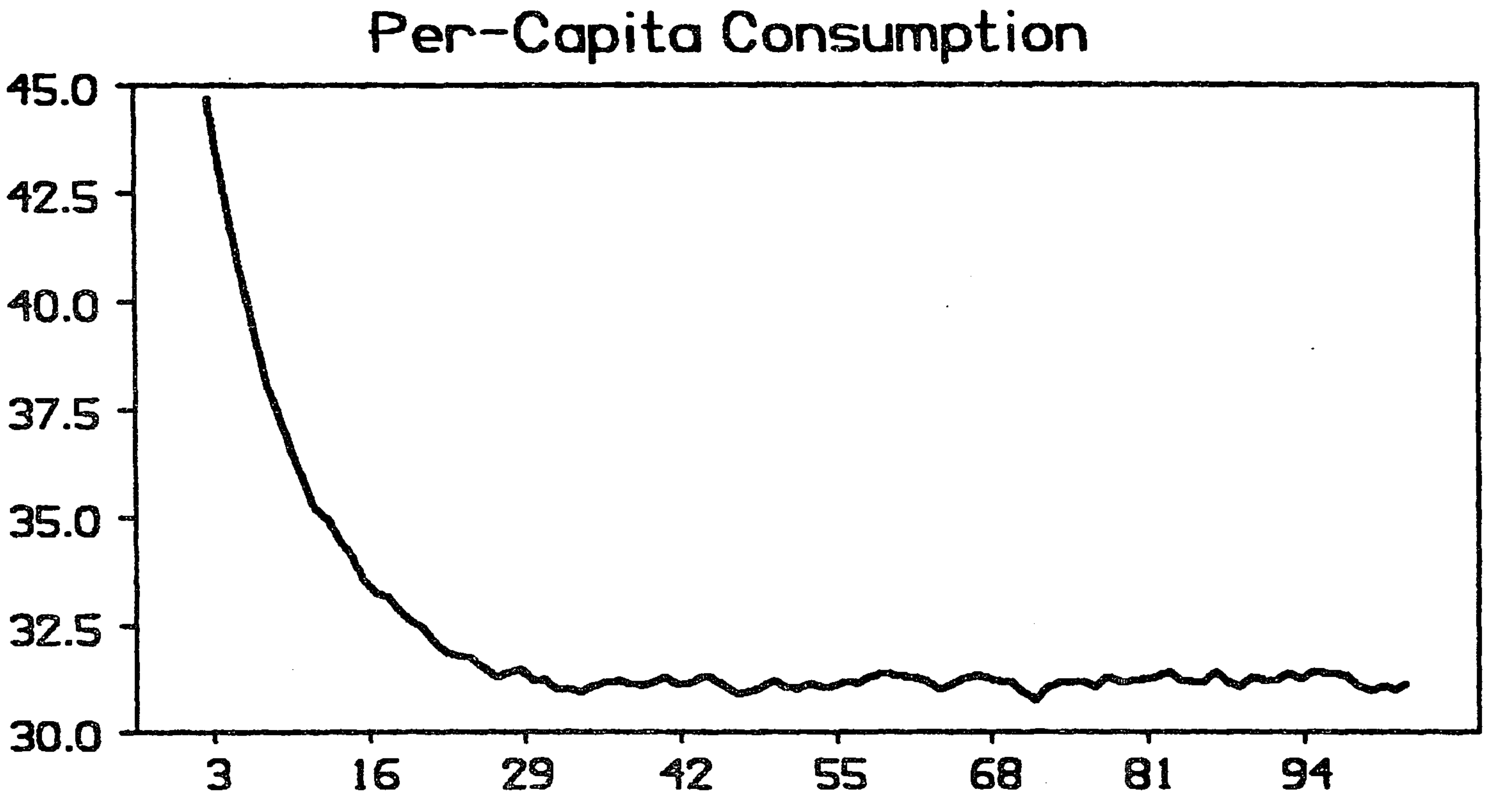

Money Supply Policy
2500000
2000000
1500000
1000000
500000
0
2
17
32
47
62
77
92

Graph 8a

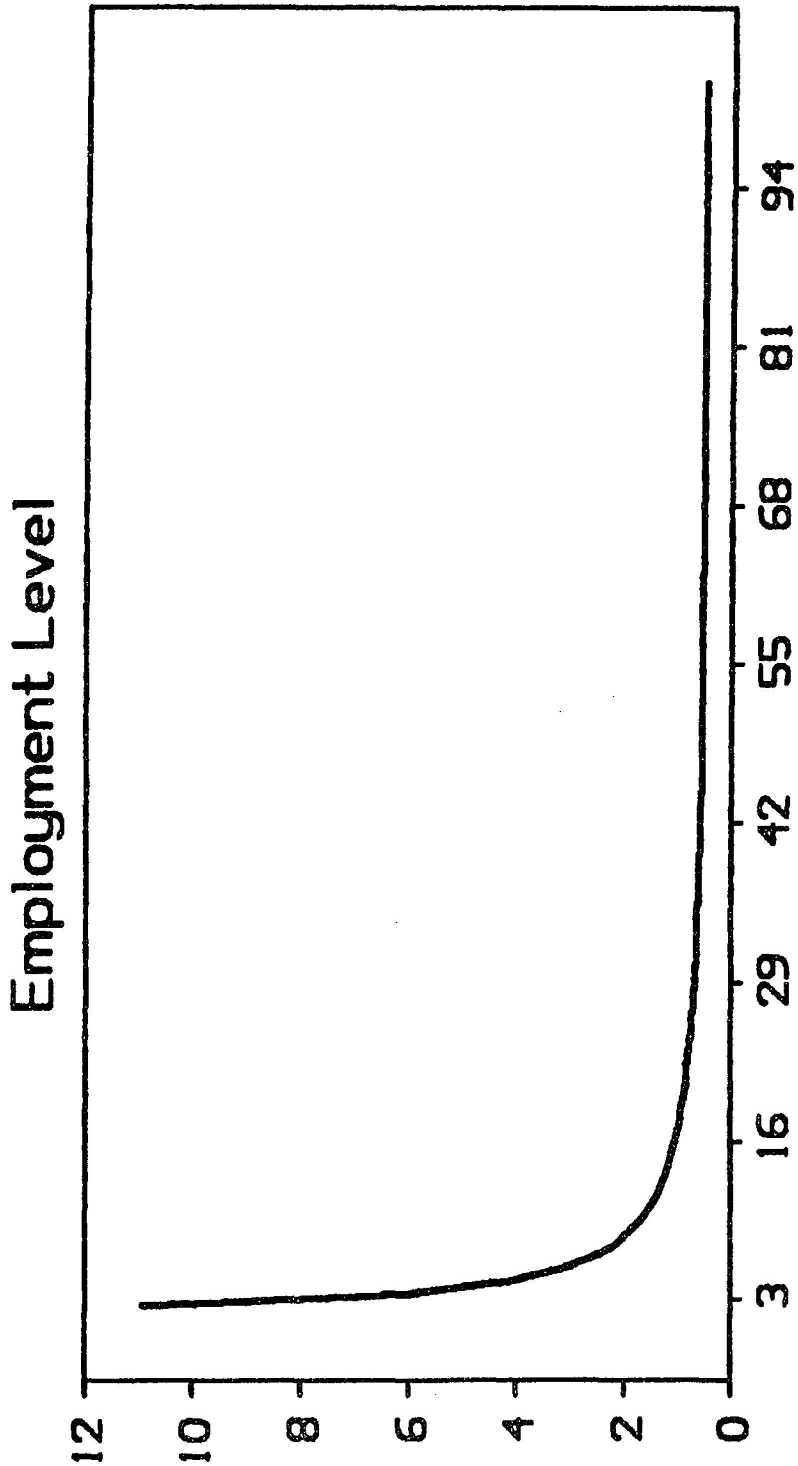


Graph 1a

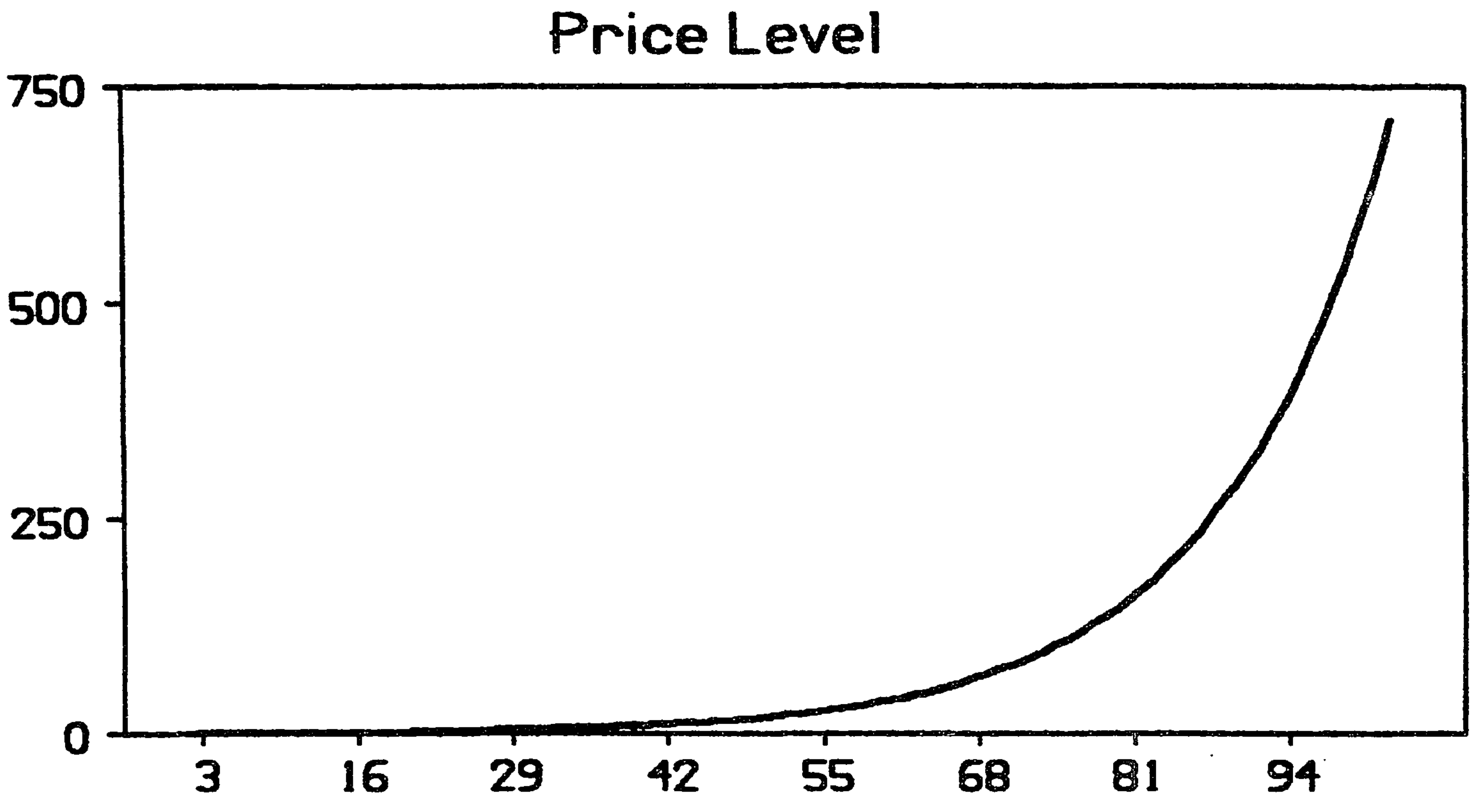

GNP
6600
6400
6200
6000
5800
5600
5400
5200
5000
3
16
29
42
55
68
81
94


Per-Capita Consumption
45.0
42.5
40.0
37.5
35.0
32.5
30.0
3
16
29
42
55
68
81
94

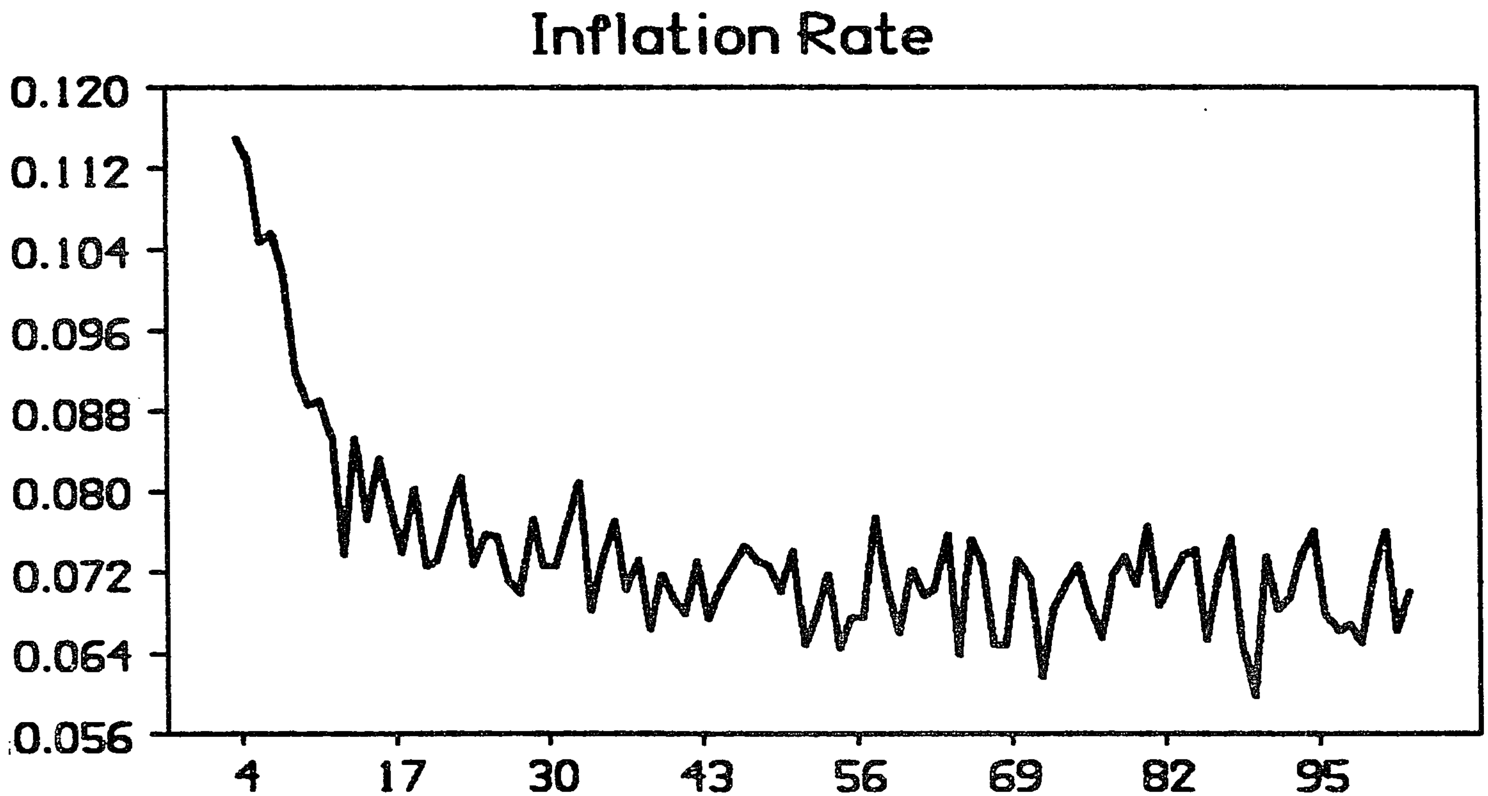
Graph 3a



Graph 4a



Graph 6a


Inflation Rate
0.120
0.112
0.104
0.096
0.088
0.080
0.072
0.064
0.056
4
17
30
43
56
69
82
95

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