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ECONOMIC ANALYSIS FOR PUBLIC DAY CARE CENTERS IN NEW  
YORK CITY

*City University of New York*

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ECONOMIC ANALYSIS FOR PUBLIC DAY CARE CENTERS IN NEW YORK CITY

by

SATYA CHUNDRU

A dissertation submitted to the Graduate Faculty in Economics  
in partial fulfillment of the requirements for the degree of  
Doctor of Philosophy, The City University of New York.

1980

This manuscript has been read and accepted for the Graduate Faculty in Economics in satisfaction of the dissertation requirement for the degree of Doctor of Philosophy.

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## ABSTRACT

## ECONOMIC ANALYSIS FOR PUBLIC DAY CARE CENTERS IN NEW YORK CITY

by

Satya Chundru

In this dissertation, an economic analysis of the Group Day Care Centers funded by New York City has been conducted. The specific issues investigated include the measurement of the center outputs, the comparison of costs and outputs of different centers, the factors affecting day care costs, the scale effects and the least cost center size, and the optimal inputmix.

Information about 185 of the 300 centers has been used in this study. Costliness, productivity and input efficiency have been proposed for use in evaluating the center performance. The cost curve is U-shaped and has a strong scale effect. The production function indicates that the current input ratios are not appropriate. In addition to the empirical analysis, a mathematical model demonstrating the scale economies has been developed.

Advisor: Professor Michael Grossman

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CHAPTER 1  
INTRODUCTION

New York City has been providing day care for children for a long time, and the cost of day care, for the most part is subsidized by public funds. The day care program has undergone rapid expansion since 1967, when the New York State and federal governments started sharing the costs of the public subsidy. The expansion was further propelled by the availability, since 1967, of publicly guaranteed financing<sup>1</sup> of the construction of new day care centers. In 1969, approximately 15,000 children were receiving publicly funded day care at any given time, and the corresponding figure for 1976 was 47,000 children, indicating a three-fold increase in a span of seven years. The growth was interrupted in 1976 due to limitations on day care funds resulting from the New York City fiscal crisis and the newly imposed ceiling<sup>2</sup> on federal subsidies for social services. The day care capacity changed little since then.

Most of the public funds for day care go for the operation of two types of day care programs: Family Day Care and Group Day Care. Family Day Care is a home-based form of care in which, the provider of services, who is usually a mother of young children herself, serves six or less

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1

New York State Housing Financing Agency mortgages financing, New York State Guaranteed Private Loans from New York State Department of Social Services, and Direct Lease Program of New York City.

2

Refers to the ceiling on Title XX funds.

children in her own home. In Group Day Care, the children who are at least seven in number (in actuality in much larger number) are cared for in a class room setting by trained teaching staff. Group Day Care is the more important of these two forms of care, in terms of the number of children served, and the expenditures. Forty thousand children are provided Group Day Care at an annual cost of approximately \$3,000 per child, while seven thousand are provided Family Day Care at a cost of \$1,600 per child per year (1977 figures).

In this study, the operation of Group Day Care Centers, of which there are three hundred in the city, are analysed. The discussion that follows concerns the Group Day Care Program only, except when specifically noted.

Most of the day care centers were originally sponsored by the neighborhood based community organizations, and later approved by the Agency for Child Development (ACD) for funding. ACD, a division of New York City Human Resources Administration is responsible for the management of day care programs. The centers are required to get a license from the New York City Department of Health before they can operate, and the license is usually granted after receiving a recommendation from ACD. The licensure requirement is to ensure that the centers meet certain standards with regard to space, safety, staff-children ratio, and staff qualifications.

The day care program had been troubled by serious problems until the recent past-- low capacity utilization rates in the presence of waiting lines to enroll in the centers, significant budget-overruns by the centers, longterm building leases with above-market rents, etc.

These were responsible, for the most part, for the escalation of costs from \$1,000 per child-year in 1969 to \$3,000 in 1975. The root causes had been the virtually unlimited availability of funds, the lack of performance monitoring of centers and an absence of penalties for poor management, which fostered unsound program practices, such as the following:<sup>3</sup>

1. The focus of ACD had been completely diverted (during 1967-74) from the management of centers to the starting of new centers, and there were enough funds to cover the expansion as well as the higher costs resulting from the neglect of the management of the centers. The management of the centers, including staff hiring, purchase of supplies, program content design and budget preparation had been left to the local neighborhood groups (center sponsors), virtually without any audits by ACD.

2. In its eagerness to acquire buildings for new centers, ACD initiated "The Direct Lease Program" whereby the agency signed longterm leases with rents far above the market prices. A more complete discussion of this issue is in Section 2.

3. In about 1975, ACD was forced to undertake steps to contain costs because of intense pressures from several sources: reduction in New York City funds for day care resulting from the fiscal crisis, imposition of a ceiling on Federal contributions, and city and state audits of day care center operations. Since then, ACD has made good

progress in introducing modern management methods in the operation, monitoring, and evaluation of the centers. It has increased considerably its role in the management of the centers, with a consequent reduction of the role played by the neighborhood groups strengthened its authority in determining the program contents of the centers and their budgets, and instituted management information systems.

Despite the large array of management activities undertaken by ACD, there are several aspects of day care operations that have not been analyzed intensively. One of these is the issue of the efficient production of day care services, and this is the focus of this study. Some of the questions that will be explored are: How do changes in the relative quantities of various inputs such as teachers, administrative staff, support staff, and building rent affect the output of a center? How to measure the output of a center? How to compare the costs and outputs of different day care centers? What factors influence day care costs? What are the effects of scale on day care costs?

## CHAPTER 2

## DAY CARE PROGRAM FEATURES AND DATA

2.1 Multi--product Output

The Group Day Care Centers provide two kinds<sup>1</sup> of day care: Pre-school-age Day Care and School-age Day Care. The pre-school program is for children of age 3-5 years, and the school-age program is for Children of age 5-12 years (who attend public schools). The pre-school program is run for a whole day, whereas the school-age program is operated for four hours a day (after the public school day is over) during the year when the public schools are open, and for a full day during the summer when the public schools are closed. Many centers offer both kinds of care, but some offer only one.

2.2 Unit of Output

Every center has a licensed capacity, expressed in children, which essentially indicate the "slots", or spaces for children contained in the center, meaning that the physical facilities were designed to accommodate the number of children indicated in the capacity. The capacity may be considered a measure of center output. The number of children enrolled in the center may differ from the capacity, and the enrollment (average enrollment in a year) is an alternative output measure. The third output measure is child-day (or, child-year)<sup>2</sup>, which unlike the

---

1

There is another kind, Day Care for Infants involving 2% of the centers and an equal percentage of the total expenditures, and this is not included in the study.

2

Child-years are obtained by dividing the child-days by the number of center working days in a year.

other two measures reflects the number of children actually present in the center.

The three measures--capacity, enrollment and child-day -- are further discussed in Chapter 4.

### 2.3 Product Prices

The public day care is provided to low income families only, most of whom do not pay any fee. Some, because their incomes have exceeded a certain level, pay fees scaled to their incomes. The fees paid are generally well below the day care production costs.

### 2.4 Center Inputs

The various resource inputs in a day care center can be classified into five broad groups:

#### Administrative Staff (A):

Every center has a Director, responsible for the overall management of the center, and a Bookkeeper responsible for maintaining the financial records. Large centers have additional administrative staff -- Assistant Director and Assistant Bookkeeper.

#### Teaching Staff (T):

The Teaching Staff in a center comprises the Teachers, Assistant Teachers and Teacher Aids. A Teacher directs the activities of a group of children in the center, supported by the staff in the other two titles.

#### Support Staff(S):

The Support Staff in a center includes Cooks, who prepare the meals and snacks served to the children, Helpers, who assist the cooks, in addition to performing other chores, and the

Custodians, who are responsible for the upkeep of the building. Incidentally, the food is supplied free by the United States Department of Agriculture.

Rent (R):

This category includes building rent, real estate taxes, and expenses for utilities.

Miscellaneous Costs (M):

This category includes bathroom supplies, toys, reading and writing materials, etc.

## 2.5 Day Care Quality

Day care is provided mostly to the children from families in which the parent is employed (in the case of two-parent families, both must be employed), or actively looking for work, or enrolled in a job-training program. The custodial aspect of any form of day care meets the objective of freeing the parent to enter the labor force. All the publicly funded day care programs in New York City, with the exception of the Group Day Care Program contains an educational component in addition to the custodial component, and therefore, has higher unit day care costs than other day care programs.

The three measures of day care output mentioned earlier--capacity, enrollment, and the child-days--apply to the custodial component only, and hence are not ideal measures of day care output. The educational component can probably be measured in terms of improvements in test scores attributable to day care. But day care centers do not administer any tests to measure educational (developmental) achievements. In fact, the Agency for Child Development has not, so far, demonstrated

the higher benefits of Group Day Care compared to Family Day Care or other forms of day care that justify its higher costs.

In this study, it is assumed that the opportunities for a significant trade-off between the custodial and educational components are very limited because of the strict enforcement of the licensure requirements of the centers, and the ongoing supervision of the centers by ACD. If meaningful quality measures are ever developed, their inclusion in the day care cost analyses will undoubtedly enhance the validity of the results. But in the absence of such measures, the possible existence of some differences in quality among centers is not a sufficient reason to give up attempts to measure and compare day care costs.

#### 2.6 Data Collection and Sample Size

Data concerning 185 of the 300 Group Day Care centers for the New York City fiscal year 7/1/1976-6/30/1977 have been collected. The data has been assembled from several documents in the individual records of the centers, kept at the Agency for Child Development. The data about the remaining centers is either incomplete or of doubtful validity. Since the data is not assembled and published by the Agency for Child Development, it is not responsible for any errors.

#### 2.7 Mean Center Costs and Weighted (overall) Unit Total Cost

3

Table 2.1 shows the average cost per center, by input and the total, and the weighted unit day care costs.

The mean total cost incurred by a center is \$252,509, with a coefficient of variation of 42.0, which may reflect the variation in the

---

3

Combined costs of all centers divided by the number of centers.

TABLE 2.1

Mean Center Costs and Weighted Unit Total Costs

Center Costs	Mean	Coefficient of Variation
Administration	\$33,636 (13.3%)	21.6
Director/Assistant Director	20,025	26.0
Bookkeeper/Assistant Bookkeeper	13,611	20.0
Teaching Staff	117,256 (46.4%)	34.2
Teacher	54,729	35.2
Assistant Teacher	45,423	34.9
Teacher Aide	17,104	37.4
Support Staff	32,310 (13.0%)	24.9
Cook	8,716	12.0
Helper	13,860	35.8
Custodian	9,735	31.7
Rent	57,203 (26.5%)	96.0
Miscellaneous	12,103 ( 4.9%)	43.5
Total Cost	252,509 (100%)	42.0

Weighted Unit Cost

Total Cost per Child-Year:	\$2,837
Total Cost per Unit Capacity:	2,444
Total Cost per Unit Enrollment:	2,948

center size. Of all the inputs, Teaching Staff takes up the largest share of Total cost-- 46.4%, followed by Rent-- 26.5%, and the lowest share goes to Miscellaneous Costs-- 4.9%. The shares of the Administrative Staff and the Support Staff are about the same-- 13.3% and 13%.

The coefficient of variation of Rent is extremely large -- 96%, which is twice as large as the coefficient of variation of the total cost per center. The coefficients of variation of other inputs are generally smaller than that of the total cost.

The system wide total cost per child-day, per unit capacity, and per unit enrollment are, respectively, \$2837, \$2444, and \$2948. These are obtained by dividing the mean total per center by the mean center size, (expressed in the three different output measures) listed in Section 2.10.

## 2.8 Unit Cost Variation Among Centers

Table 2.2 presents the unit costs--the unweighted mean of the unit costs in individual centers--by input and total.

For the child-day measure, the unit total cost is \$3011 with a coefficient of variation of 14.7% : The range of the unit costs, obtained from the complete distribution of total costs per child-year (Table 2.3), is \$2034 - 4254, indicating that the unit costs vary widely--by, as much as, 100%. The unit costs of the individual inputs also have large coefficients of variation--rent 66%, Administration 31%, Support Staff 24% etc.

TABLE 2.2

Unit Costs Variation Among Centers

Cost Type	Cost per Child-Year		Cost per Unit Capacity		Cost per Unit Enrollment	
	Mean <sup>1</sup>	C.V. <sup>2</sup>	Mean	C.V.	Mean	C.V.
Administration	\$441.26 (14.6%) <sup>3</sup>	31.1	425.46	31.5	368.63	32.4
Director/ Asst. Director	261.71	34.2	252.38	34.9	218.60	35.6
Bookkeeper/ Asst. Bookkeeper	179.55	30.6	173.08	30.5	150.03	31.6
Teaching Staff	1430.58 (47.5%)	16.3	1375.57	14.5	1191.3	16.0
Teacher	64.5	15.1	641.48	15.1	555.32	16.3
Asst. Teacher	533.5	16.3	533.44	16.3	462.30	18.3
Teacher Aide	200.7	27.7	200.70	27.7	173.68	28.4
Support Staff	412.78 (13.7%)	23.8	397.73	30.6	344.52	24.6
Cook	120.2	39.2	116.00	39.3	100.50	40.1
Helper	170.1	22.2	163.83	21.9	141.87	22.7
Custodian	122.4	24.8	117.90	24.5	102.1	25.4
Rent	580.2 (19.3%)	66.2	554.59	64.3	480.31	64.7
Miscellaneous	146.7 (4.9%)	30.4	141.40	30.2	122.08	30.0
Total	3011.55 (100.0%)	14.7	2894.72	12.6	2506.83	14.1
Total Excluding Rent	2431.34 (80.7%)	17.8	2340.13	16.8	2026.52	18.0

1: Unweighted mean of the center's average costs.

2: C.V.: Coefficient of Variation

3: The number in the parenthesis is the input share in the Total.

TABLE 2.3

Distribution of Total Cost Per Child-Year

Cost per Child-Year (dollars)	Number of Centers	Percent of Centers
2000-2399	17	9.2%
2400-2799	36	19.5%
2800-3199	74	40.0%
3200-3599	41	22.2%
3600-3999	13	7.0%
4000-4399	4	2.2%
	<hr/>	<hr/>
	185	100.0

Mean: 3012                      Range: 2034-4254

St. Dev: 444                    C.V. 14.7

## 2.9 Variables Affecting Unit Costs

In this section, the variables that might affect the unit day care costs are identified, and their sample values are presented. The discussion is limited to speculating the nature of the association with the unit costs, except in the case of the day care mix, for which some crude regression results are given.

### 2.9.1 Center Size

Table 2.4 provides data on the center sizes: For the child-years, the complete distribution is given, while for the other two measures, only the mean and the coefficient of variation are given. The child-years distribution shows that the centers are fairly distributed over a wide range--from 21-178 CY, and there is a significant representation of centers in each of the eight size-classes covering all the centers. The mean of the child-years, capacity and enrollment are, respectively, 86, 89 and 103, and the coefficient of variation is about the same for all the three--44.

Economic theory posits a size-cost relationship in the production operations (particularly, in manufacturing), and the investigation of this relationship (the scale effect) is usually the central part of cost analyses, as is the case in this study. The reasons for expecting a size-cost relationship in day care centers are explored in Chapter 4.

### 2.9.2 Day Caremix

Table 2.5 shows the day caremix distribution, measured by the child-days produced. Less than 1% of the centers produce School-age care whereas 58% produce exclusively Pre-school care and 41% produce both kinds. 98% of the centers produce Pre-school day care either

TABLE 2.4

Center Size Distribution

Child-Years	Number of Centers	Percent of Centers
20-39.9	11	5.94
40-59.9	57	30.81
60-79.9	28	15.13
80-99.9	29	15.67
100-119.9	24	12.97
120-139.9	16	8.64
140-159.9	11	5.94
160-179.9	9	4.86
	-----	-----
	185	100.0
	-----	-----

Mean: 85.67 CY      Range: 20.9-178.3

St. Dev: 37.13      C.V.: 43.4

	<u>Mean</u>	<u>C.V.</u>
Capacity:	89.01	43.8
Enrollment	103.31	44.4

TABLE 2.5

Day Caremix Distribution

Proportion of Pre-school Day Care (Child-Days)	Number of Centers	Percent of Centers	Cumulative Percentage
0	1	0.5%	0.5%
0.35-0.50	3	1.6%	2.1%
0.51-0.60	20	10.8%	12.9%
0.61-0.70	34	18.4%	31.3%
0.71-0.80	14	7.6%	38.9%
0.81-0.90	5	2.7%	41.6%
1	108	58.4%	100.0%
	-----	-----	
	185	100.0	
	-----	-----	

Mean: 0.850

Range: 0-1

St. Dev.: 0.19

C.V.: 22.8

exclusively or in greater part. The mean value of the day caremix proportion (proportion of Pre-school day care) is 0.850. Although the School-age program is much smaller (15% of the output), the number of centers producing it is large--42% of the centers. The mean of the day caremix is the same for the other output measures--0.850 and 0.855 for the capacity and enrollment measures, respectively.

Differences in day caremix could affect the day care costs. The Pre-school care could cost more because it is offered for a full day, while the School-age care is offered for half-a-day during most of the year, and it probably requires higher staff-child ratios--the children in the Pre-school care, being relatively young (3-5 yrs.) may require more attention and care than the children (5-12 yrs.) receiving the School-age care.

Table 2.6 presents the effects of day caremix on unit costs;  $\bar{R}^2$ , the square of the adjusted<sup>4</sup> multiple correlation coefficient provides a rough indication of the degree to which the day caremix affects the costs.

For the child-year measure, day caremix differences account for 19%<sup>5</sup> of the variation in unit total costs. Support Staff costs per child-year appear to be most sensitive to day caremix differences-- 44% of the variation is explained. Among the individual Support Staff components, Cook and Helper have relatively high  $\bar{R}^2$  values-- 0.47 and 0.35, respectively and the Custodian has very small  $\bar{R}^2$ -- 0.18. These are plausible since

---

4  
Adjusted for degrees of freedom.

5  
These are very preliminary results.

TABLE 2.6  
Effect of Day Caremix on Unit Costs

Cost Type	Effect of Day Caremix ( $\bar{R}^2$ )*		
	Child-year	Capacity	Enrollment
Administration	0.23	0.30	0.26
Director/ Asst. Director	0.18	0.24	0.21
Bookkeeper/ Asst. Bookkeeper	0.28	0.33	0.27
Teaching Staff	0.30	0.53	0.42
Teacher	0.28	0.57	0.41
Asst. Teacher	0.32	0.48	0.38
Teacher Aide	0.13	0.18	0.16
Support Staff	0.44	0.56	0.50
Cook	0.47	0.52	0.50
Helper	0.35	0.43	0.38
Custodian	0.18	0.29	0.23
Rent	0.03	0.02	0.02
Miscellaneous	0.12	0.13	0.11
Total	0.19	0.37	0.23
Total Non-rental Cost	0.36	0.57	0.49

\*  $\bar{R}^2$  is the square of the multiple correlation coefficient, adjusted for degrees of freedom.

the Pre-school program, being a full day operation, serves more meals and snacks than the School-age program, requiring more Cooks and Helpers, and the Custodian costs probably depend more or less on the center size. rather than on the day care mix.

Teaching Staff cost/child-year has the next highest  $\bar{R}^2$ -- 0.30, which is in accord with the earlier speculation (that the Pre-school children might require more care and supervision because of their relatively young age). Among the individual components, the Teacher and the Assistant Teacher have high  $\bar{R}^2$ , while the Teacher Aide has a very low  $\bar{R}^2$ , and the reasons are not clear without having a knowledge of day care operations.

Rent has the lowest  $\bar{R}^2$ --0.03, which is negligible, indicating that the day care mix has no influence on it. Apparently, the larger recreation space requirements of the School-age children are offset by the special space requirements of the Pre-School children, such as sleeping areas.

The  $\bar{R}^2$  value of the Miscellaneous costs is low--0.12, implying that these costs do not depend on the kind of day care.

The table also lists the  $\bar{R}^2$  values for the other output measures. Capacity has the highest  $\bar{R}^2$  value, followed by Enrollment. For the unit total cost, the  $\bar{R}^2$  for capacity, enrollment, and child-year are 37%, 23%, and 19% respectively. This might indicate that in the allocation of funds to the centers, the capacity and enrollment are given far more weight than the child-days.

### 2.9.3 Center Lease

As discussed earlier (Section 2.7 and Section 2.8), the rent costs (average rent per center as well as unit rent costs) vary widely. The major cause of variation is the lease covering the center building

rental, of which there are two kinds: the Direct Lease, and the Indirect Lease. Thirty six percent of the centers in the sample are of Direct Lease.

The following passage from (5) would provide information about the origin and the content of the Direct Lease.

Early in 1969 when the need for increased day care services reached the crisis stage, the Department of Social Services began exploring alternative methods to finance development of day care centers. The capital budget route was explored but under this process it was determined that it would take 3-5 years for any additional facilities to reach the construction stage. It was estimated that the time frame for construction or renovation of a day care facility via this route could be reduced to approximately 18 months. In view of the critical need for additional day care services, the Department of Social Services, in cooperation with the Department of Real Estate, embarked upon a program by which the city would lease day care facilities from a private owner through a direct lease (underline added) in the early months of 1969, on a "crash" basis. To date, 100 day care facilities have been completed under this program. An additional 72 are under construction.

New Direct lease sites have not been approved since December, 1970, when Administrator Jule M. Sugarman declared a moratorium on all new projects under the direct lease program in order that the feasibility of continuing this method of financing and constructing day care centers could be effectively studied.

Under the Direct Lease Program, the Agency for Child Development signed leases of 15 to 20 years duration with private developers at high rents (rent prices per unit space), three to five times as those in the centers not under the Direct Lease Program. Allegations of mismanagement and outright corruption were levelled against the Agency for Child Development and other New York City departments. The centers that are not covered by the Direct Lease are called the Indirect Lease, and

otherwise there is no significance attached to the name.

Table 2.7 lists some selected data by the lease, and for all the centers. The mean rent cost per unit capacity is \$972 in the Direct Lease, which is three times as large as that in the Indirect Lease--\$323. The mean rent cost per child-year is 3% higher than the mean rent cost per unit capacity in each lease category. The total cost per child year is \$2915 in the Indirect Lease, and \$3186 in the Direct Lease, and the difference is \$271 compared to the difference of \$649 between the two mean rent costs, implying that the non-rental costs are lower in the Direct Lease.

The Direct Lease Centers, on the average are 50% larger, 112 CY vs 71 CY.

#### 2.9.4 Capacity Utilization Rate

Capacity utilization rate (simply, utilization rate) is the proportion of child-days attended in a year to the child-days equivalent of the center's capacity. That is,

$$\text{Utilization Rate (UR)} = \frac{\text{Child-days attended in a year}}{\text{Capacity} \times \text{number of center working days in a year}^6}$$

---

<sup>6</sup>  
262 working days during 7/1/1976-6/30/1977.

TABLE 2.7

Selected Data by Type of Lease

	Indirect Lease		Direct Lease		All Centers	
	Mean	CV	Mean	CV	Mean	CV
Number of Centers in the Sample	119	-	66	-	135	-
<u>Unweighted Costs</u>						
Rent/Child-Year (\$)	333	39	1027	27	580	66
Rent/Unit Capacity (\$)	323	39	972	24	555	64
Non-Rental Cost/Child-Year(\$)	2582	16	2160	14	2431	18
Total Cost/Child-Year (\$)	2915	15	3186	13	3012	15
<u>Weighted Cost</u>						
Total Cost/Child-Year(\$)	2785	-	3131	-	2837	-
Size (CY)	70.9	42	112.2	31	85.7	43
Utilization Rate	0.973	7	0.96	8	0.966	7.2
Day Caremix (Prop CY)	0.9	21	0.72	23	0.85	23

The utilization rate of a center is an important variable in explaining the cost differences among centers because of its effect on unit costs, which is explained in Chapter 4. Table 2.8 shows the distribution of centers by the utilization rate. The range of values of the utilization rate is quite large: 0.8 - 1.15.

#### 2.9.5 Miscellaneous Variables

Three variables -- Turnover rates of Children, Enrollment--capacity Ratio and Age of a Center--, which may affect day care costs are grouped under Miscellaneous Variables, and are discussed below:

##### 2.9.5.1 Turnover of Children

Day care centers experience considerable turnover of children during the year. Children drop out of a center for a variety of reasons, such as, change of family residence out of the neighborhood, and the parent leaving the labor force. On occasion, a center may drop a child if he is excessively absent without a valid reason. In either case, new children are enrolled to take up the vacated slots.

In this study, the turnover rate of a center is defined as the monthly average of new enrollments in the center divided by the center's enrollment (average enrollment).

Table 2.9 shows the distribution of centers by the turnover rate. 90% of the centers have turnover rates under 9%, and 66% of the center have turnover rates under 6%. The mean is 4.7%, with a coefficient of variation of 55%.

High turnover rates may be associated with low utilization rates, and the rationale is as follows: When a child drops out, the center management may not be notified; the child simply stops attending and

TABLE 2.8

Distribution of Utilization Rates

Utilization Rate	Number of Centers	Percent of Centers
0.800-0.849	9	4.9
0.850-0.899	30	16.2
0.900-0.949	36	19.5
0.950-0.999	44	23.3
1.000-1.049	45	24.3
1.050-1.099	17	9.2
1.100-1.149	4	2.2
	—	—
	185	100.0
	—	—

Mean: 9.966

Range: 0.809-1.146

St. Dev.: 0.70

C.V. 7.3

TABLE 2.9

Distribution of Turnover Rates

Turnover Rate	Number of Centers	Percent of Centers
0-2.99%	41	22.2
3.00-5.99%	81	43.8
6.00-8.99%	44	23.8
9.00-1.99%	11	5.9
12.00-14.99%	8	4.3
	—	—
	185	100.0
	—	—

Mean: 4.7%                      Range: 0-13.9%

St. Dev.: 0.025                  C.V.: 54.5

the center initially treats it as a routine absence. Considerable time may elapse before the center finds out that the child dropped out, and initiates the procedures for enrolling a new child. The time lags involved will reduce the utilization rate.

But a lower turnover rate may not always mean that the center has a high utilization rate. A weak center management may be slow in reacting to poor attendance by a child, which would keep the computed turnover rate low, while the inadequate child attendance lowers the utilization rate. Changes in the utilization rate would affect the day care costs and this has already been discussed in Section 2.9.4. Turnover rates, however, could affect day care costs directly: Higher turn-over rates could mean increased administrative work (connected with the dropouts and the new enrollments), perhaps, resulting in higher Administrative Staff costs. Hence the turnover rate is an explanatory variable of day care costs.

#### 2.9.5.2. Enrollment-Capacity Ratio

Table 2.10 shows the distribution of the centers by the enrollment-capacity ratio (enrollment in a center divided by its capacity). Almost all centers (98%) have a ratio of greater than one; the mean is 1.16, and the range, 0.95-1.44. In the past, enrollments had been maintained at or below the capacity, which resulted in low utilization rates--because of absences, and turnover rates. To increase utilization rates, ACD now permits the centers to enroll up to 115% of capacity. The data, however, shows that about 50% of the centers have enrollments above this level, and some of these centers, perhaps, may suffer a drop in quality. Centers experiencing high absenteeism rates and large turnover rates are

TABLE 2.10

Enrollment-Capacity Distribution

Enrollment-capacity Ration	Number of Centers	Percent of Centers
0.900-0.999	4	2.2
1.000-1.099	36	19.5
1.100-1.199	87	47.0
1.200-1.299	51	27.6
1.300-1.399	5	2.7
1.400-1.499	2	1.1
	<hr/> 185 <hr/>	<hr/> 100.0 <hr/>

Mean: 1.159

Range: 0.953-1.443

St. Dev.: 0.074

C.V.: 6.3

likely to have relatively large enrollment-capacity ratios (if the centers do not wish to see a drop in the utilization rates). 2% of the centers have very low ratios (less than one), which could be due to lack of demand for day care, or due to poor management, and these centers will experience utilization rates of less than 100%.

The effect of the enrollment-capacity ratio on day care costs through its influence on the utilization rate would be accounted for by the inclusion of the utilization rate as an explanatory variable for the day costs (as discussed in Section 2.9.4). The enrollment-capacity ratio, however, could affect the day care costs directly -- higher ratios may be associated with higher Administrative Staff costs (hence, higher day care costs), if Administrative Staff costs increase with enrollment-- and therefore, the ratio should be considered as an explanatory variable of day care costs.

#### 2.9.5.3 Age of a Center

The age of a center refers to how long a center has been in operation. An older center may have more experienced staff, leading to increased efficiency in the center operations. On the other hand, older centers may have higher staff salaries (because of the possible existence of seniority based pay scales), contributing to higher costs of operation.<sup>7</sup> The next effect is not clear. Because of many errors in the age data collected, the age is not further considered in this study.

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7

Differences in pay scale could affect the input ratios.

## 2.10 Summary and Conclusions

Many Group Day Care Centers produce two products--Pre-School-age Day Care and School-age Day Care and this would make it difficult to assign separate cost for each. The problem is further compounded by the lack of market prices for these products-- the public day care is provided either free or at nominal prices; hence there is no market-price basis for aggregation of the outputs.

There are three basic units of output: The capacity of a center the enrollment in a center and the child-day (child-year), Before developing a framework for analyzing the day care costs, a choice must be made between the three.

Information about 185 of the 300 Group Day Care Centers in New York City has been collected. There is a substantial variation in the unit costs among centers: the mean is \$3011 per child-year, with a coefficient of variation of 14.7%, and a range of \$2000-4200. The basic inputs of a center and their shares of the total cost are: Administrative Staff (13%), Teaching Staff (46%), Support Staff (13%), Rent (27%) and the Miscellaneous costs (5%).

The unit cost variation among centers could be due to the influence of the following factors:

Center size: The center size has a range of 21-178 CY, with a mean of 86 CY and a coefficient of variation of 44.

Day caremix: Although the average proportion of Pre-school day care is 0.85, there is a substantial variation in the day caremix. Some centers produce exclusively Pre-school day care, while others produce both, and some others produce exclusively School-age Day Care. The input requirements are different for the two kinds of day care, and the former

is the costlier. Initial regression results indicate that 17% of the variation in costs is explained by the day care mix differences.

Day Care Center Lease: The rent cost (per unit capacity) in the Direct Lease category is three times as large as in the Indirect Lease category. Hence the lease status of a center could affect its costs.

Capacity Utilization Rate: Changes in the utilization rate of a center will affect the unit costs. There is widespread variation among the utilization rates of the centers.

Other variables that may affect day care costs are Child Turnover Rate and the Enrollment-capacity Ratio.

## CHAPTER 3

### COST AND PRODUCTIVITY MEASURES

#### 3.1 Introduction

Program management activities, such as, cost control, and budget allocations cannot be properly conducted without cost and productivity measures. Furthermore, decisions, such as expanding, contracting, or closing of a center cannot be objectively made without relying on these measures. A well-defined, and properly compiled set of measures would provide stimulus for local managers to improve the performance ratings of their centers through effective management.

A problem in developing the cost measures in day care is the difficulty of allocating various input costs between the two kinds of care. Moreover, there is no satisfactory method of aggregating, or valuing the diverse outputs of a day care center as there are no market prices for the output

In this chapter, first, the theoretical aspects of the cost and productivity measures, including the following are discussed:

- A statistical method of allocating costs between the two kinds of day care,
- Costliness, productivity and input efficiency indices.

Second, the empirical estimates of the costs of the different day care kinds, and the various indices, including an evaluation of the center performance based on these indices, are covered.

#### 3.2 Theoretical Issues

##### 3.2.1 Allocation of Costs between the Different Day Care Outputs

It is not possible to allocate the various input costs among the two outputs through the usual accounting methods. Nor, is there a satisfactory method of aggregating, or valuing the two outputs, as there are no market prices.

Described below is a statistical method of allocating costs between the two kinds of care:

Let  $C_1$  be systemwide average cost of Pre-school day care,  
 $C_2$  be systemwide average cost of School-age day care,  
 $n$  be total output of a center (physical output)  
 $n_1$  be Pre-school output of the center (physical output)  
 $n_2$  be School-age output of the center (physical output)  
 $P$  Proportion of Pre-school day care (in the total output)

Since  $n_1$  and  $n_2$  are known,  $n$  and  $P$  are also known:

$$n = n_1 + n_2, \quad P = n_1/n$$

Proportion of School-age day care is  $1-p$ , which is equal to  $n_2/n$

The objective is to find  $C_j$ ,  $J = 1, 2$  (or, simply  $C_1$  and  $C_2$ ).

The average cost of all day care in a center is the sum of the weighted and individual average costs of Pre-school care and School-age care; the weights are their respective proportions in the total output,

Thus,

$$(3.1) \quad C_n = C_1 (P) + C_2 (1 - P)^1$$

where,  $C_n$  is the average cost of all day care in a center.

Equation (3.1) can be rewritten as:

$$(3.2) \quad C_n = C_2 + (C_1 - C_2) P$$

If  $C_1$  and  $C_2$  are constant, equation (3.2) suggests a regression of  $C_n$  on  $P$ :

$$(3.3) \quad C_n = a_0 + a_1 P$$

where,

$a_0$  is the estimate of  $C_2$  ,

$a_1$  is the estimate of  $(C_1 - C_2)$  .

Then,  $(a_0 + a_1)$  is the estimate of  $C_1$  .

### 3.2.2 Costliness Index

As discussed in Chapter 2, a major cause of variation in unit costs among centers is the variation in day caremix. Ignoring other factors affecting costs, a measure for comparing costs among centers, after adjusting for day caremix differences, can be devised. Costliness

<sup>1</sup>

This relationship can be obtained in a different way:

Let  $T_n$  be total cost in a center,

$T_1$  be total cost of Pre-school care in the center,

$T_2$  be total cost of School-age day care in the center,

$n_1$  be the units of Pre-school care produced in the center,

$n_2$  be the units of School-age care produced in the center.

$$\text{Then, } T_n = T_1 + T_2 = C_1 \cdot n_1 + C_2 \cdot n_2$$

Dividing both sides by  $n = n_1 + n_2$ , the resulting equation is:

$$C_n = C_1 (P) + C_2 (1-P)$$

Index is such a measure, and is defined as:

$$C_i^* = \frac{\sum_j n_{ij} C_{ij}}{\sum_j n_{ij} C_{.j}}$$

(Which is simply, actual cost incurred by a center in producing a given output mix divided by the system-wide average cost of producing the same output mix).

Where,  $C_i^*$  : Costliness of Center i  
 $n_{ij}$  : Output of type j in Center i  
 $C_{ij}$  : Average Cost per unit of output of type j in Center i  
 $C_{.j}$  : System-wide average cost per unit of output of type j  
 $\sum_j n_{ij} C_{ij}$  is the total cost in Center i, which is directly available from the data.

Costliness Index, as defined above, is a Paache Index number, comparing a center's actual total cost with the hypothetical total cost arrived at by aggregating the different outputs of the center, after weighting them by the corresponding system-wide average costs ( $C_{.j}$ ). Thus, the costliness is a measure of the deviation of the actual cost of a center from the systemwide average cost of producing the same output.

The term costliness was first used by M. Feldstein (4), but the underlying concept is analagous to the overall efficiency proposed earlier by M. J. Farrell (3), which is the reciprocal of costliness.

### 3.2.3 Productivity Index:

A productivity index provides a different way of measuring a center's performance. Its construction requires examining the output a center

produces with a given set of inputs. In contrast, in the Costliness Index, the costs incurred by a center to produce a particular output are considered. Further comparisons between productivity and costliness are discussed in Section 3.3.2.

The productivity index<sup>2</sup> of a center is defined as the ratio of the actual output to the output expected from a center of average productivity using the same input quantities (as used by the center under consideration).

Let  $P_i^*$  be the productivity Index of a center,

$W_i$  be the actual output of the center (in dollars)

$\hat{W}_i$  be the output (in dollars) expected from a center of average productivity using the same input quantities.

$$(3.5) \quad \text{Then } P_i^* = W_i / \hat{W}_i$$

$W_i$  is approximated by weighting the different outputs of a center with the corresponding system average costs.

Thus,

$$W_i = n_{1i} C.1 + n_{2i} C.2$$

where,

$n_{1i}$  is Pre-school output in center  $i$  ,

$n_{2i}$  is School-age output in the center .

$\hat{W}_i$  is estimated from the center's production function:

---

2

The definition was, first, given by M. J. Farrell(3), under the name "Technical efficiency". M. Feldstein renamed it productivity index.

$$(3.6) \quad W_i = f(X_{i1}, X_{i2}, \dots, X_{ir}, V_i$$

where,

$X_{ir}$  are the quantities of inputs used by center  $i$ ,

$\hat{W}_i$  is the output of the center,

$V_i$  is the random error term.

All the inputs and outputs in equation (3.6) are expressed in dollars.

If a Cobb-Douglas production function is assumed to be the center's production function, then

$$(3.7) \quad \hat{W}_i = z (X_{ir}) V_i$$

The parameters of equation (3.8) are estimated by ordinary least squares. A more complete description of the production function, including the estimation of the parameters is given in Chapter 5.

The production function used in estimating the expected productivity is:

$$(3.8) \quad W_i = 0.812 A^{0.152} T^{0.600} S^{0.243} R^{0.072} M^{0.081}$$

where, A, T, S, R, and M are respectively, Administrative Staff costs, Teaching Staff costs, Support Staff costs, Rent costs and Miscellaneous Costs.

#### 3.2.4 Input Efficiency

Input efficiency deals with the issue of whether the center has chosen the right input mix-- for example, whether the center is spending too much for rent and too little for teachers. In comparison, productivity

deals with the issue of how well a center manages to extract from the given inputs -- whether the firm has chosen the right production function from among all those actually in use in the industry.

The input efficiency of a center is defined as the ratio of the costs incurred by a center of average input efficiency in producing the same output as produced by center  $i$  to the actual costs incurred by center  $i$ , assuming that both centers are using the right production function. Thus the input efficiency indicates the fraction of costs for which the output could be produced if the relative use of inputs is varied.

The overall efficiency (reciprocal of costliness) of a center depends on both the input efficiency and productivity of a center.

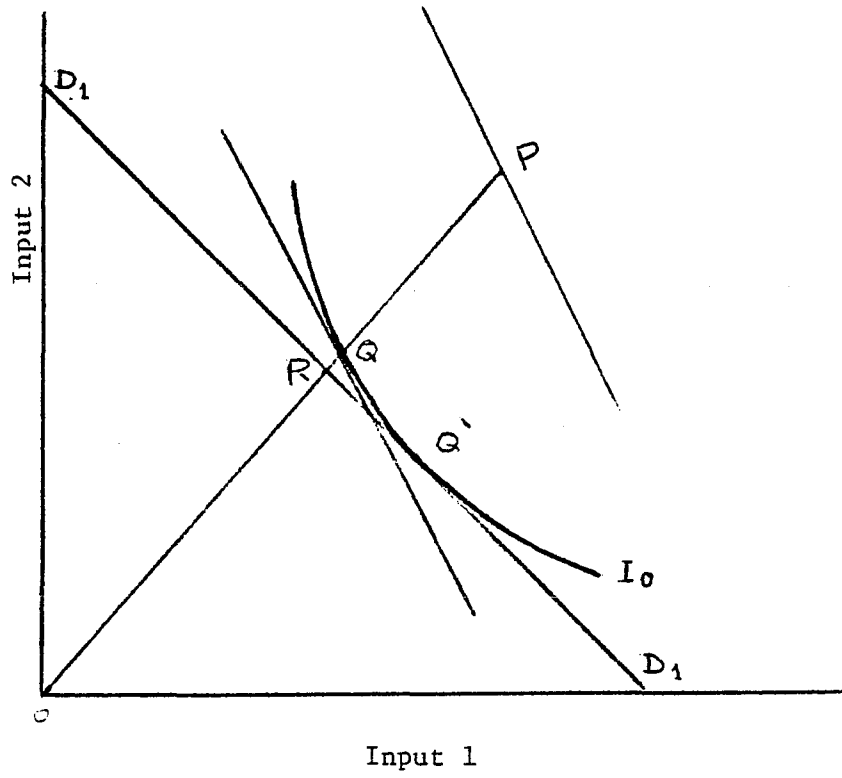
$$\begin{aligned}
 (3.9) \quad \text{overall efficiency} &= \text{input efficiency} \times \text{productivity, or} \\
 \text{input efficiency} &= \text{overall efficiency} / \text{productivity} \\
 &= 1 / (\text{costliness} \times \text{productivity})
 \end{aligned}$$

This relationship would be relied on in computing the input efficiencies of the centers in the study.

The inter-relationship among costliness, productivity, and input efficiency are clarified with the following diagrammatic exposition (given by M. Nerlove (9)):

A production process with constant returns to scale, a single output, and two factors of production is assumed. It is described by isoquant  $I_0$  in Figure 3.1. Let  $D_1$  be the price line. Then  $Q'$  represents the optimum input combination, and a firm of average productivity operates there. But any given firm may not be producing there. It may produce at  $P$ , meaning, the firm operates at an output level of  $I_0$ ,

FIGURE 3.1

Costliness, Productivity and Input Efficiency

with input amounts indicated by P. Thus the firm is using inputs of OP instead of OQ (at the same input ratio) to produce the same output. The distance OP relative to OQ measures the extent to which the same amount of output could be produced with fewer inputs used in the same proportion or the technical efficiency (productivity).

Producing at Q' is better than producing at Q. The distance OR relative to OQ measures the fraction of costs for which the output could be produced.  $OR/OQ$  can be called the input efficiency.

$$\begin{aligned} \text{Overall efficiency} &= OR/OP = OR/OQ \cdot OQ/OP \\ &= \text{Input Efficiency} \times \text{Productive Efficiency} \end{aligned}$$

Overall efficiency is also equal to the reciprocal of Costliness since  $OR/OP$  represents the ratio of average cost of producing output  $I_0$  divided by the actual cost of producing the same output, and costliness is the ratio of actual cost of producing  $I_0$  divided by the average cost of producing  $I_0$ .

$$\text{Thus, input efficiency} = 1/(\text{Costliness} \times \text{Productivity})$$

### 3.2.5 Relative Cost

The relative cost of a center is defined as the average cost of the center divided by the average cost of all centers. In this definition, no allowance is made for factors outside the control of the center management (for example, day care mix, center lease, and center size, etc.), and differences in these factors among the centers could be responsible for some of the observed cost differences among the centers. Hence, the indiscriminate use of relative cost in comparing the performance of the centers could be deceptive.

### 3.3 Empirical Results

#### 3.3.1 Cost Estimates for the Two Day Care Kinds

The following least squares regression of unit costs on the day care is used in estimating  $C_j$  (system-wide unit costs of type  $j$  day care).

$$(3.10) \quad AC_i = \sum_{j=1}^{k=2} C_j P_{ij} + u_i$$

where,

$AC_i$  : Unit cost in center  $i$

$P_{ij}$  : Proportion of type  $j$  output in center  $i$

$u_i$  : Random error term

In the regression, the day care mix is represented by the proportion of Pre-school day care ( $P$ ). The results of the regression are shown in Table 3.1. The  $\bar{R}^2$  values are low, which range from 0.187 for the child-year measure, to 0.299 for the capacity. The equations are highly significant (significant at 0.01% level).

The value of the intercept is 2,171 for the child-year measure, which according to the discussion in Section 3.2.1 represents  $C_2$  (system-wide unit cost of School-age care) in dollars. The coefficient of the day care mix is 989, which when added to  $C_2$  gives  $C_1$  (system-wide average cost of Pre-School care), which comes to 3,160 dollars.

Table 3.2 shows the estimates of  $C_j$  for the three output measures. For all the measures, the cost of School-age care is about two-thirds of the cost of Pre-school care, which appears reasonable in light of the differences between the two kinds (as discussed in Chapter 2).

TABLE 3.1

Regression of Average Cost on Day Caremix

	Average Cost per Child-Year	Average Cost per Unit Capacity	Average Cost per Unit Enrollment
Intercept	2171.3 (16.3)	2020.7 (20.0)	1693.9
Day Caremix	989.1 (6.4)	1027.0 (8.9)	950.3
R <sup>2</sup>	0.187	0.299	0.254

TABLE 3.2

Day Care System Average Costs (C.j)

Type of Care	Average Cost per Child-Year	Average Cost per Unit Capacity	Average Cost per Unit Enrollment
Per-School	\$3160	3040	2644
School-age	2171	2021	1694



FIGURE 3.2  
Scatter Diagrams of Cost and Efficiency Measures

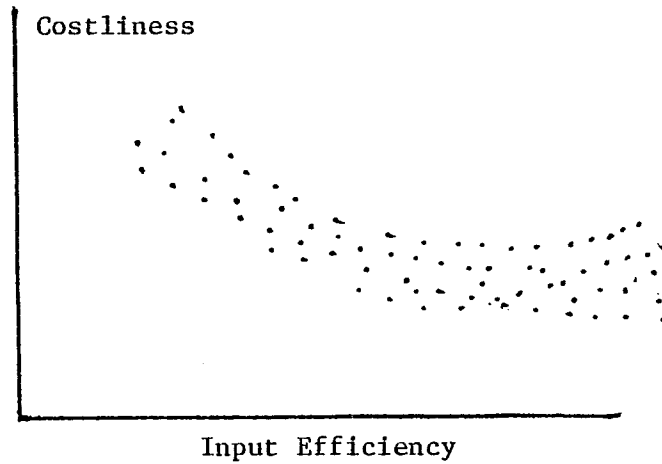
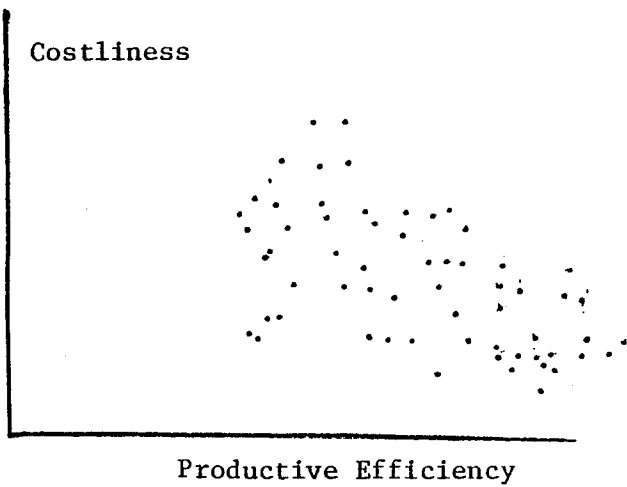
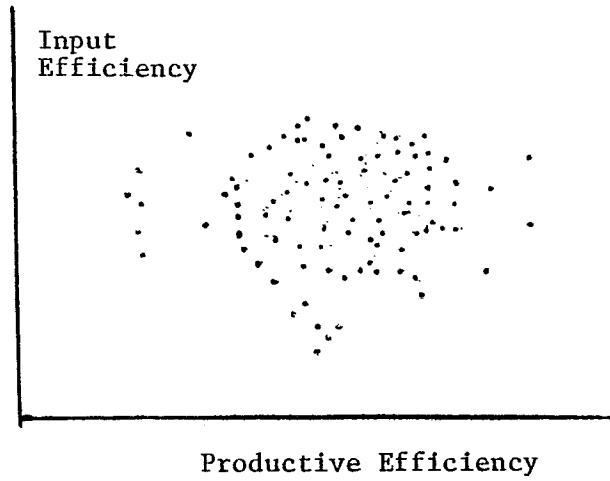
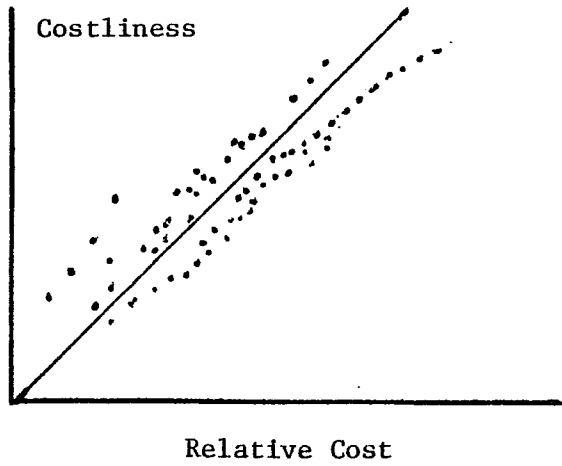


TABLE 3.3

A Comparison of the Relative Cost and Costliness of  
Twenty High and Low Relative Cost Centers

	<u>Low Relative Cost Centers</u>		<u>High Relative Cost Centers</u>	
	Rel. Cost	Costliness	Rel. Cost	Costliness
1.	0.6898	0.7331	1.2108	1.1297
2.	0.6907	0.7510	1.2193	1.1377
3.	0.7220	0.7931	1.2203	1.1386
4.	0.7273	0.8492	1.2237	1.1417
5.	0.7522	0.7444	1.2287	1.1464
6.	0.7556	1.0261	1.2310	1.2439
7.	0.7706	0.8111	1.2330	1.1504
8.	0.7724	0.7717	1.2453	1.1619
9.	0.7776	0.8077	1.2530	1.2350
10.	0.7799	0.8309	1.2535	1.1696
11.	0.7804	0.8320	1.2586	1.1743
12.	0.7816	0.8553	1.2745	1.1892
13.	0.7927	0.8330	1.2781	1.3489
14.	0.7990	0.8064	1.3222	1.2336
15.	0.8111	0.7455	1.3398	1.2500
16.	0.8120	0.8382	1.3459	1.2558
17.	0.8146	0.8821	1.3997	1.3060
18.	0.8148	0.8732	1.4404	1.3439
19.	0.8156	0.8476	1.4429	1.3463
20.	0.8241	0.8587	1.4431	1.3464

costliness exceeds  
relative cost in 17 cases.

costliness lags behind  
relative cost in 18 cases.

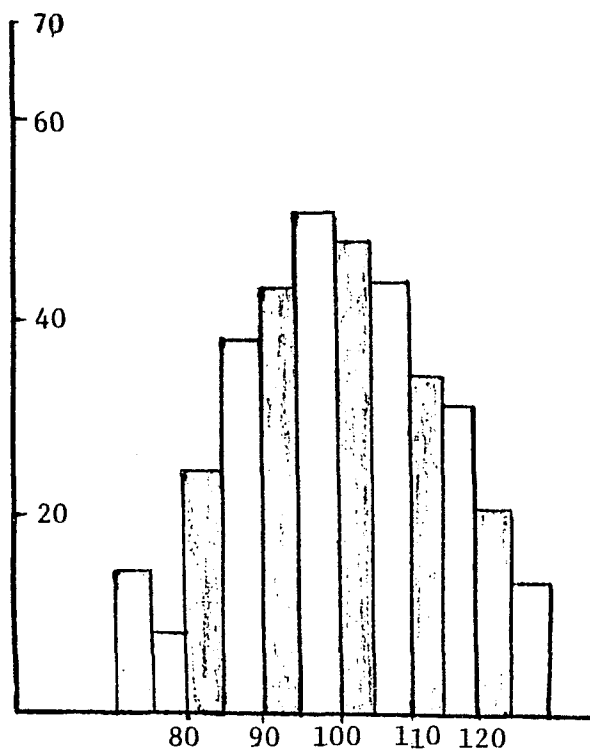
From the histograms of productivity and input efficiency of the centers (Figure 3.3), the mean of productivity is 1.007 with a standard deviation of 0.1182, and the mean input efficiency is 1.0225 with a standard deviation of 0.1238.

The plot of productivity vs input efficiency, shown in Figure 3.2, indicates that the relation between them is very weak -- the value of this simple correlation coefficient is 0.01. A higher correlation is expected since both indices are measures of a center's performance in using its budget to produce day care. A high correlation would indicate that good center management selects an efficient combination of inputs and extracts high productivity from them, while bad management selects an inefficient combination of inputs and uses them with low productivity. One reason for the observed low correlation may be that budget allocation decisions (input decisions) are not made by the same persons, nor in the same way as the decisions which affect the productivity of the inputs. Input combinations reflect administrative decisions, not only at the center level, but also at the central management level, as well as the inherited patterns of past allocations. The productivity with which resources are used is more likely determined by the administrative staff in an individual center. The low correlations may also be due to imperfections in the indices as measures of center performance; productivity and input efficiency may reflect not only management efficiency but also center size, center rent, insufficiency of enrollment, child attendance rates (to some extent) and turnover rates.

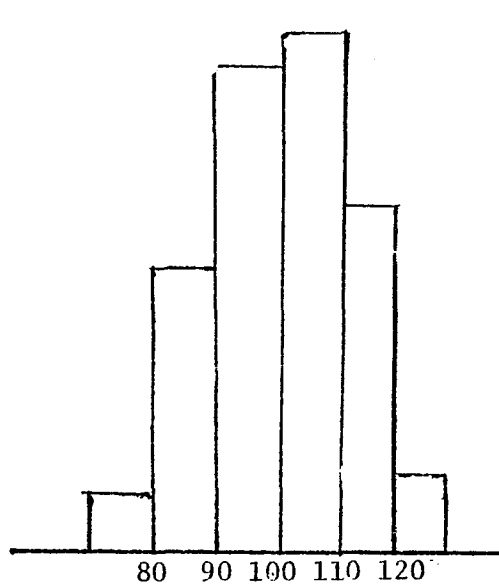
Costliness variation is a reflection of both the input efficiency differences and the productivity differences. This can be seen from

FIGURE 3.3  
Histograms of Cost and Efficiency Measures

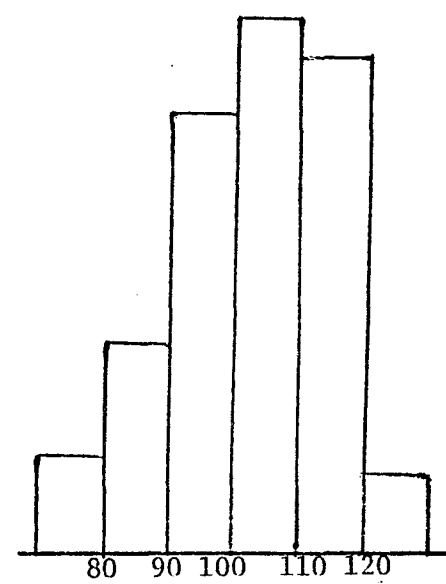
number of  
centers



□ Costliness  
mean: 0.99998 st.dev. 0.1335  
■ Relative Cost  
mean: 1.0215 st.dev. 0.1506



Productive Efficiency  
mean: 1.006 st.dev. 0.103



Input Efficiency  
mean: 1.021 st.dev: 0.110

Note: In all histograms, values of the measures above 120 and below 80 are not specifically designated.

the scatter diagrams in Figure 3.2 -- costlines vs input efficiency, and costliness vs productivity. Fifty five percent of the variation in costliness can be explained by a linear regression on input efficiency, while thirty eight percent can be explained by a regression on productivity. If high costlines values are to be lowered, the input efficiency and the productivity must be increased. The appropriateness of the current input proportions are discussed in Chapter 5.

### 3.4 Conclusions

A statistical method of allocating costs between the two kinds of day care has been used, and the resulting system-wide costs per child-year are, \$3,160 and \$2,171, respectively, for the Pre-school, and School-age day care kinds.

Costliness, productivity, and input efficiency indices have been proposed in place of the crude relative costs in evaluating the center performance. Crude relative costs, not only misrepresent the performance of individual centers, but also exaggerate the variation in center costs.

Costliness index could provide the day care management with a summary measure of a center's performance. The management's attention may be directed to centers that deviate substantially from the average.

The indices of productivity and input efficiency could be useful in identifying the cause of a center's high or low costliness value. If a center has high costliness, but average or above average productivity, its input ratios should be examined. By studying the centers in which productivity and input efficiency values are very high and very low, the day care management may better understand the factors influencing

day care costs. The results have shown that the costliness differences are due to differences in input efficiency as well as productivity differences, (meaning that the current input ratios are inappropriate).

The indices, however, should not be relied upon uncritically as measures of individual center's managerial efficiency. Factors outside the control of the center's management may affect costs: center size, center rent, low enrollments resulting from lack of demand, child attendance rates and child turnover rates.

## CHAPTER 4

4.1 Introduction

Central to the cost function estimation is the investigation of the size-cost relationship (the nature of scale economies) in the production of day care at the individual center level. Existence of significant economies or diseconomies of scale would greatly influence the size and the location of the centers. For example, if unit costs are found to be significantly lower in large centers, a tendency, whereby the existing small centers are consolidated into a much lower number of large centers and only large sizes are built when starting new centers, would emerge. There are, however, other factors influencing the size of a day care center, an example of which is the size of the catchment area served. As the center size increases, the area served by the center expands, increasing the average travel distance from a child's home to the center. Long travel distances would be a hardship on the parents, who drop their children in the center in the morning on the way to work, and pick them up in the evening on the way home.

In this chapter, first, the theoretical aspects underlying the day care cost curve are explored, which include the following:

- Discussion of the reasons for expecting center size to influence the day care costs, including the development of a mathematical model in support.

- Examination of alternative units for measuring center output, leading to the selection of a child-day as the measure of output.

- Review of the theory of long-run, focussing on the distinction between long run, and the short run cost curves and their relevance of the scale.

- Discussion of the regression fallacy problem in the estimation of the cost curves.

- Discussion of the choice of estimating the total cost function, or the average cost function.

Second, the empirical estimation of the cost curves is undertaken, which includes the following:

- Estimation of linear and quadratic average cost curves, with, and without the day care mix term.

- Estimation of separate cost curves by the lease category, after establishing the need. Some, or all of the parameters are allowed to vary between the categories.

- Estimation of size differentiated cost curves by lease and their comparison with the single cost curve of each lease category.

- Estimation of cost functions including the utilization rate and other variables -- the age, the turnover rate and the enrollment capacity ratio.

## 4.2 Theoretical Issues

### 4.2.1 The Scale Effect

There is no prior evidence concerning the economies of scale in day care centers, hence, it is imperative to examine the factors that give rise to scale economies, or diseconomies in day care centers.

There are several features favoring economies of scale:

1. Staff Indivisibilities: Staff such as Director, Bookkeeper, Custodian, Cook and Helper do not come in divisible units, and hence only large centers would be able to fully utilize them.

2. Physical facilities Indivisibilities and Costs: Day care centers, both large and small, are required to contain sleeping areas

(only for Pre-School) play areas, dining areas and kitchen facilities, which are indivisible, implying economies of scale. It is also likely that there are scale economies both in their construction and maintenance.

3. Fluctuations in Attendance Levels: Large centers will have relatively smaller fluctuations in child, and staff attendance levels (due to the law of large numbers). Hence the staff levels required to maintain a given staff-child ratio will be proportionately lower in large centers. A mathematical model developed in support, is shown in Section 4.2.1.1.

There are, however, as many features favourable to diseconomies of scale:

1. Travel Distances: Larger centers may experience higher absenteeism rates<sup>1</sup> of children because of the larger commuting distances.

2. "Personal Aspects": Small centers may be more personal-- they provide more opportunities for interaction among the staff and the children, therefore may be conducive to higher attendance levels of staff, and children. Additionally, the staff in small centers are more likely to be well acquainted with the parents (of the children), which, in addition to increasing the attendance levels of the children, as believed in ACD circles, could be an asset to the center when a child drops out. The center is likely to find out about it sooner, enabling it to enroll another child in the vacated slot promptly. Furthermore, the cohesive spirit prevailing in small centers, by reducing the unplanned (and, short-notice) staff absences, could moderate the fluctuations in staff absences.

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1

Diseconomy only when output is measured in child-days.

3. Managing and Communicating: As in other production enterprises, the problems of managing and communicating become increasingly difficult as the center size increases.

The overall scale effect - the relative distributions of the scale economies and diseconomies along the entire size range of the centers -- is unclear, and there is no basis for an a priori assumption about the shape of the cost curve. The answer should await the results of the empirical analysis.

#### 4.2.1.1 Fluctuations in Attendance Levels and Scale Economies--A Model

If the daily staff, and child censuses were free of fluctuations, the staff size (on payroll) required to maintain a given staff-child ratio in all days would exhibit constant returns to scale. The censuses, however, fluctuate widely even when the probability of an individual's absence in a day is constant in all days, because of the randomness of the absences. The staff size, containing an allowance for the fluctuations, exhibits economies of scale, since the intensity of the fluctuations in each census increases less rapidly than the size increase.

An earlier model, by B. Chiswick ( 2 ) demonstrates the scale economies in the presence of child absences (model interpreted in the context of day care), with an implicit assumption of no staff absences. The current model investigates the scale economies, in the presence of the staff absences, as well as, the child absences, and the resulting economies of scale would be higher.

Let  $\bar{y}_1^c$  be the mean daily child in a small center, and  $p_c$  be the probability of a child being absent in a day, which is assumed to be constant among all children in the day care system, and is independent.

The daily census may have a normal <sup>2</sup> distribution with mean  $\bar{y}_1^c$ , and standard deviation,

$$(4.1) \quad \sigma_1^c = \sqrt{\bar{y}_1^c \cdot p^c}$$

This indicates that, as the center size goes up, the standard deviation goes up only by the square root of the proportional increase in the center size.

Consider a large center, with mean daily child census  $\bar{y}_2^c$  equal to  $m\bar{y}_1^c$ .

$$(4.2) \quad \text{Then } \sigma_2^c = \sqrt{\bar{y}_2^c \cdot p^c} = \sqrt{m} \sqrt{\bar{y}_1^c \cdot p^c}$$

Let  $\bar{y}_1^s$  and  $\bar{y}_2^s$  be the mean staff censuses in the small, and large centers; and  $p^s$  be the probability of a staff being absent in a day, which is assumed to be constant throughout the system, and is independent.

Let  $r$  be the desired child-staff ratio.

The economies of scale are examined, first (case(1)), with the assumption that there are only child absences, and second (case(2)), with the assumption that there are both staff, and child absences.

case (1): The ray  $b^c a^s$  in the Figure 4.1 indicates the relation between the peak child census ( $2\sigma^c$  level) and the mean staff census, when the ratio  $r$  is maintained 97.5% of the time. Staff in the small center,

$$(4.3) \quad \bar{y}_1^s = r \left( \bar{y}_1^c + 2\sigma_1^c \right)$$

staff in the large center,

$$\bar{y}_2^s = r \left( \bar{y}_2^c + 2\sigma_2^c \right)$$

Substituting the value of  $\sigma_2^c$  indicated in equation (4.2)

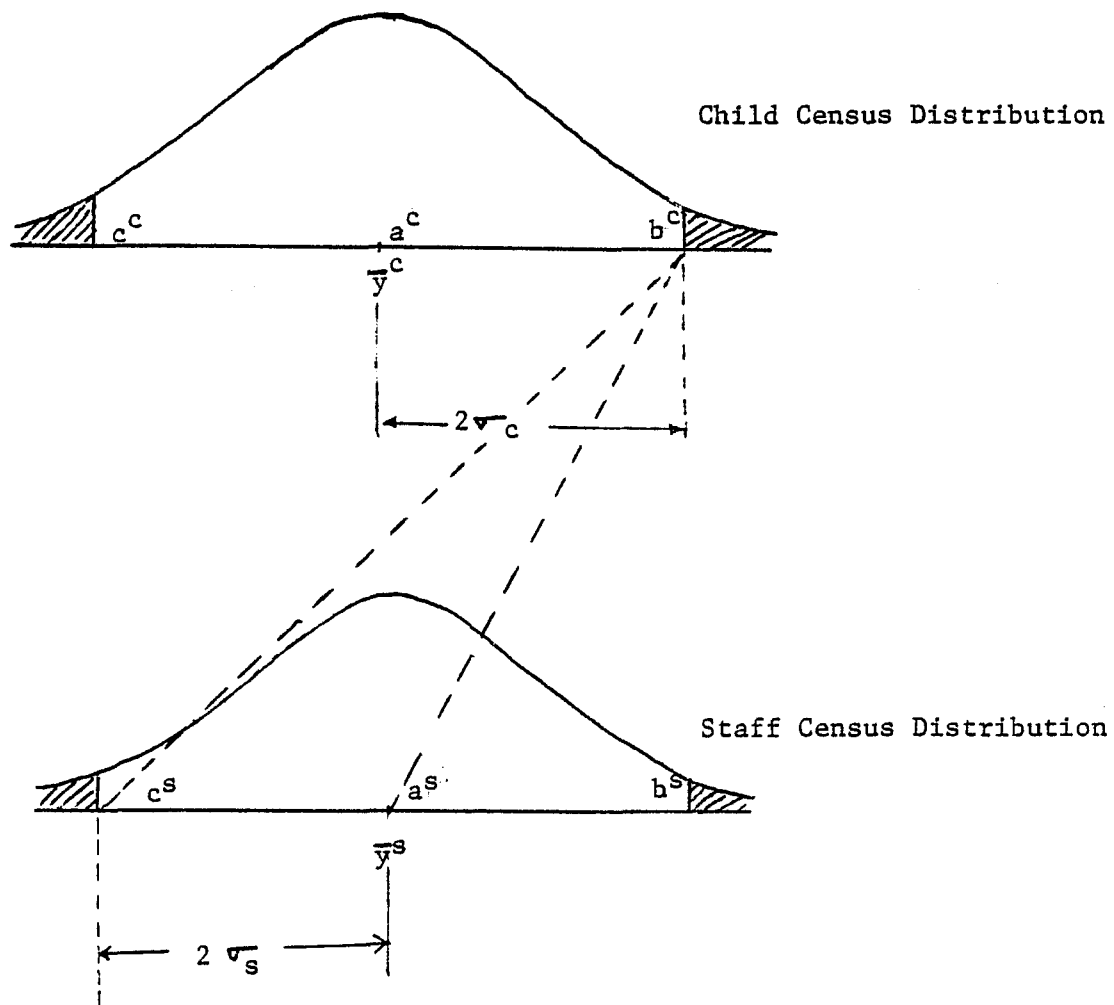
$$(4.4) \quad \bar{y}_2^s = r \left( m\bar{y}_1^c + 2\sqrt{m} \sigma_1^c \right)$$

---

<sup>2</sup>

The assumption of normality is not essential.

FIGURE 4.1

Distribution of Child, and Staff Censuses

The ratio of the staff in the two centers,

$$(4.5) \quad \frac{\bar{y}_2^s}{\bar{y}_1^s} = \frac{r (m \bar{y}_1^c + 2 \sqrt{m} \sqrt{v-1})}{r (\bar{y}_1^c + 2 \sqrt{v-1})} = K$$

K is clearly less than m, and hence there are economies of scale.

Case (2): The ray  $b^c c^s$  in Figure 4.1 indicates the relation between the peak child census ( $2\sqrt{v-1}$  level), and the minimum staff census ( $-2\sqrt{v-1}$  level).

In the small center

$$(4.6) \quad \bar{y}_1^s - 2\sqrt{v-1}^s = r (\bar{y}_1^c + 2\sqrt{v-1}^c), \text{ or}$$

$$\bar{y}_1^s - 2\sqrt{p^s \bar{y}_1^s} = r (\bar{y}_1^c + 2\sqrt{v-1}^c)$$

In the large center,

$$(4.7) \quad \bar{y}_2^s - 2\sqrt{v-1}^s = r (\bar{y}_2^c + 2\sqrt{v-1}^c), \text{ or}$$

$$\bar{y}_2^s - 2\sqrt{p^s \bar{y}_2^s} = r (\bar{y}_2^c + 2\sqrt{m} \sqrt{v-1}^c)$$

Dividing (4.7) by (4.6)

$$\frac{\bar{y}_2^s - 2\sqrt{p^s \bar{y}_2^s}}{\bar{y}_1^s - 2\sqrt{p^s \bar{y}_1^s}} = \frac{r (\bar{y}_2^c + 2\sqrt{m} \sqrt{v-1}^c)}{r (\bar{y}_1^c + 2\sqrt{v-1}^c)}$$

• K ( from equation (4.5) )

From this it can be easily shown that the ratio of the staffs<sup>3</sup>

$$\frac{\bar{y}_2^s}{\bar{y}_1^s} \text{ is less than } K, \text{ for } K > 1.$$

Thus, the economies of scale are greater in case (2) than in case (1).

#### 4.2.2 Choice of Output Measure

A choice must be made among the alternative output measures -- enrollment, capacity and child-day -- before cost, and production functions can be estimated.

Enrollment is the weakest of the three since it does not, by itself alone, indicate the work-load (or work-output) of a center. The work-load depends on the child attendance rates as well, meaning that the work-load depends on the number of children actually attending the center rather than on enrollment. Although the number actually attending may depend on the enrollment, there is no strict proportionality. According to sources in ACD, enrollment had been used as the output measure until a few years ago when ACD auditors uncovered that many centers contained "phantom" enrollees -- children who never attended, or stopped attending a long time ago -- and enrollees who attended sporadically, and the centers were very underutilized. Thus the enrollment as the output measure, by providing perverse incentives to enroll children who rarely attend, contributes to the underutilization of centers, in

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<sup>3</sup> Let  $\frac{x}{y} = z$ ; then  $\frac{z\bar{y} - \sqrt{z} \sqrt{y}}{\bar{y} - \sqrt{y}}$  is greater than  $z$ , since  $(z\bar{y} - \sqrt{z} \sqrt{y})$  is greater than  $y - \sqrt{y}$  for  $z > 1$ .

addition to not reflecting the actual work-load of a center.

The capacity of a center, essentially, indicates the number of "slots", or spaces, for children contained in the center, meaning that the physical facilities in the center were designed to accommodate the number of children indicated in the capacity. Therefore, it could be an appropriate physical measure of capital.

There has been varying interpretations of capacity over the years. At first, it had been interpreted as the upper limit of enrollment (apparently not realizing that even the peak daily census will be below enrollment). The consequent underutilization of the centers resulted in its current interpretation as the desired (or close to the desired) mean daily census of the center, thus permitting enrollments to exceed the capacity.

Obviously, the capacity cannot be a measure of output when interpreted as the upper limit of enrollment -- then the output would depend on enrollment. It could be a measure of output in its current interpretation as the desired mean census, if the centers were operating near full capacity (producing the desired mean census). Otherwise, the capacity measure of output will overestimate or underestimate the center work-loads. For example, if a center has a capacity of 100, with the same desired mean census, the capacity reflects the work-load (output) of the center only if the actual mean census is 100. If the mean census is less than 100, the capacity will overestimate the work-load. This is especially important in view of the wide variation in the utilization rates of the center (as discussed in Chapter 2). The capacity measure of output might even contribute to the under-utilization of the centers as the center managers will not have an incentive to strive to raise the

utilization rates, which do not raise the pre-determined (by capacity) center outputs. Additionally, the susceptibility of capacity to changing interpretations would render it unsuitable for the comparison of center outputs over time.

Child-days produced by a center in a year are equal to the average daily census multiplied by the number of center working days in a year (the same for all the centers). Since the daily census represents the work-load of a center, child-days reflect the work-load of the centers at all times, unlike enrollment and capacity, which reflect the work-load only under special conditions, as seen earlier. Unlike capacity, the child-day is not subject to varied interpretations, hence can be used for time series as well as cross-section comparisons of center outputs. The child-day, by positively linking the center output to the center utilization rate could encourage higher utilization rates. It may be argued, however, that the centers might lower the quality of output in their eagerness to increase the child-days. But it is rather a problem of day care standards definition and enforcement.

#### 4.2.3. Theory of Long Run Costs and the Measure of Size<sup>4</sup>

The long-run expansion path of a firm is the locus of points along which the marginal rate of technical substitution is constant, factor prices remaining constant. A line joining the points of tangency of parallel isocost curves and isoquants is the long-run expansion path, since the marginal rate of technical substitution is constant along the line. OP in Figure 4.2 represents a long-run expansion path.

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4

The contents of this section can be found in standard economics text books.

FIGURE 4.2

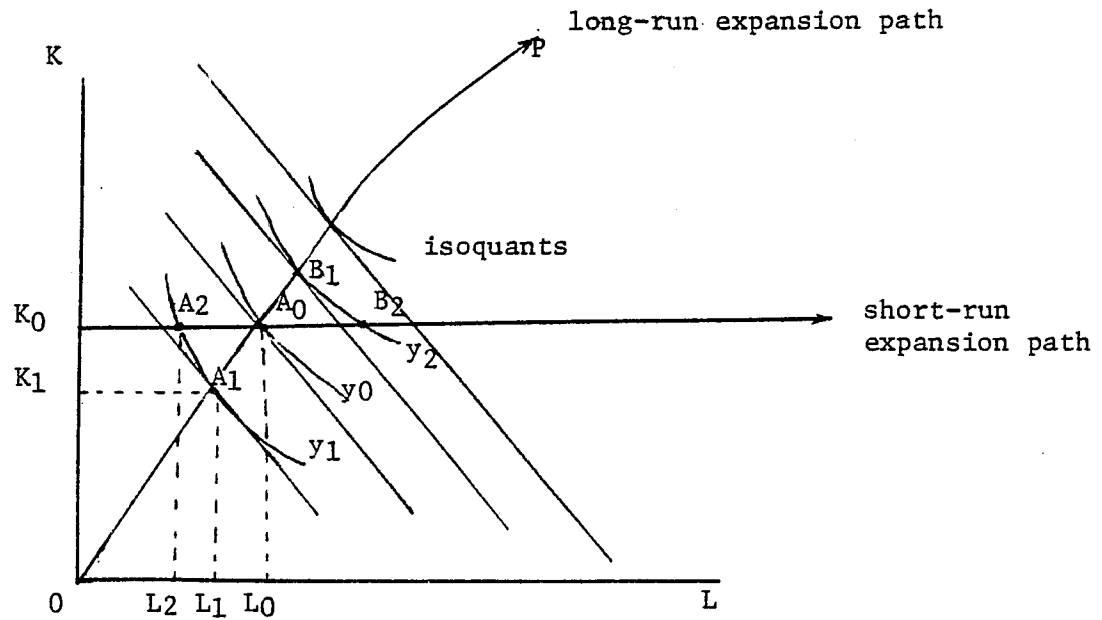
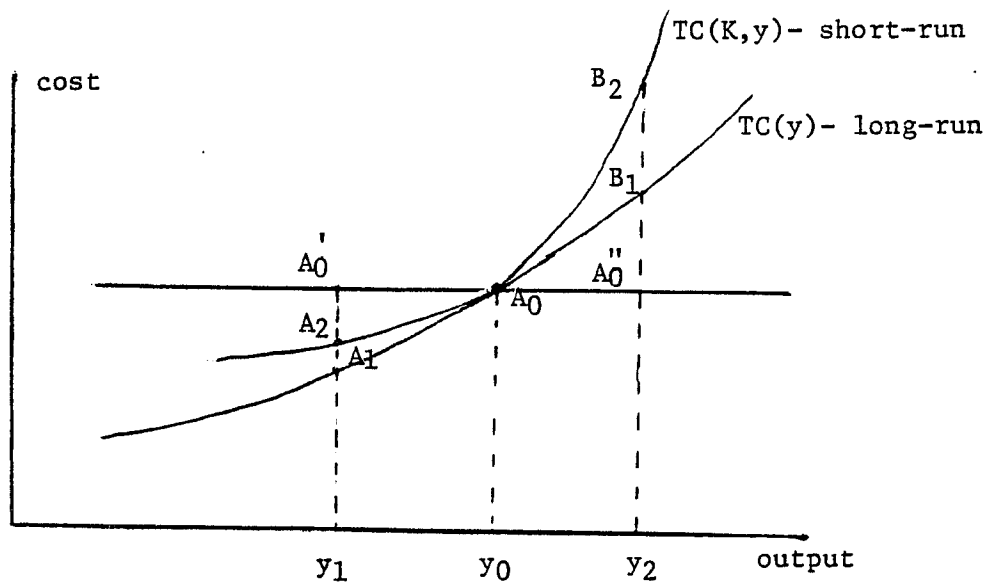
Expansion Path and Output Fluctuations

FIGURE 4.3

Total Cost Curves and Output Fluctuations

The combination of outputs ( $y$ ) and costs along expansion path, gives the long-run total cost curve (TC):

$$TC = f(y)$$

The short-run cost curve is defined along the short-run expansion path defined by the horizontal line at  $\bar{K}$  (in Figure 4.2), where the capital is fixed at this level, and the labor (the variable factor) is free to vary. The short-run cost curve given by the output and cost along such an alternative expansion path  $C_s(\bar{K}, y)$  must satisfy

$$C_s(\bar{K}, y) = C(y) \text{ at each } y.$$

Simply stated, the long-run and short-run costs of producing a given output are the same at the output level where the two expansion paths cross, while at other output levels the short-run costs are higher.

Economies of scale are measured by the effect of  $y$  on the long-run average cost curve.

$$\frac{\partial AC(y)}{\partial y} \begin{matrix} < 0 & \text{increasing returns} \\ = 0 & \text{constant returns} \\ > 0 & \text{decreasing returns} \end{matrix}$$

$\partial AC(y) / \partial K$ , when  $y$  held constant shows the effect of moving from one short-run cost curve to another, and hence does not measure scale effects. The value of  $K$  that minimizes the average cost (or, total cost) of a particular output gives a point on the long-run expansion path.

#### 4.2.4 Long-run Cost and Regression Fallacy

The meaning of regression fallacy in cost curve estimation, is essentially, that there is a difference between the observed and the true cost curves which is caused by short-run fluctuations in output and errors in measuring output, as explained below.

##### 4.2.4.1 Fluctuations in Utilization Rate:

If a center is not producing its long-run output at the time of observation (meaning that the capacity utilization rate differs from the

planned utilization rate, which is for simplicity of discussion assumed to be 100%), the observed cost-output data represents a point outside the long-run cost curve<sup>5</sup>. Nor does it represent a point on the short-run cost curve<sup>6</sup> except when the adjustment in the variable factor (Labor) is full. In the case when the adjustment in labor is less than full or none, the data represents a point outside of both the short-run and long-run cost curves, as explained below.

Additionally, the fluctuations in the utilization rate affect unit costs. In the case of a decrease in the utilization rate, the unit costs will go up regardless of the adjustment in labor, but the extent of the increase depends on the adjustment. The increase will be the largest in case of no adjustment, and the smallest with full adjustment. For example, when the enrollment in a center drops considerably and if the labor is not reduced, the unit costs will rise considerably (in proportion to the drop in child-days). If the labor is reduced somewhat, the rise in unit costs will be moderate. In the case of an increase in utilization rate (utilization rate exceeding 100%), the unit costs go up or down, depending on the adjustment in labor. In this case, the quality of output may also be affected. If there is full adjustment in labor (meaning that the labor is increased when there is an increase in child-days), the unit costs will go up (in the short-run additional output can only be produced with higher

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5

For additional discussion, see G. Botts (1).

6

As discussed earlier, the short cost curve is a schedule of minimum cost of producing different outputs, when the capital is fixed.

unit costs) and the quality of output remains unaffected. If there is no adjustment in labor, the unit costs will go down, but the quality will deteriorate. For partial adjustments of labor, the unit costs go up or down depending on the degree of adjustment, and there will be a corresponding decline in quality. The rationale behind these statements are described below.

The center shown in Figure 4.2 operates in the long-run at  $A_0$ , with output  $Y_0$  (planned output at full utilization of capacity) and inputs  $K_0$  and  $L_0$ . Due to a drop in demand, the center now produces a smaller output  $Y_1$ . If the drop in demand is permanent, the firm will move, in the long-run, to a new equilibrium at  $A_1$  (inputs  $K_1$  and  $L_1$ ), and will operate at the full utilization rate. In the short-run, regardless of whether the drop in demand is permanent or temporary, the center operates on the short-run expansion path, at  $K_0$ , since the capital cannot be adjusted in the short-run. If the center adjusts the labor fully, the center will achieve short-run equilibrium at  $A_2$ . If there is no adjustment in the labor (meaning that the center does not reduce labor when the output drops), the inputs in use will continue to be represented by  $A_0$ . Thus, for a lower output  $Y_1$ ,  $A_1$  represents long-run equilibrium,  $A_2$  represents short-run equilibrium, and  $A_0$  represents short-run operation involving no adjustment in labor.

Similarly, for an increase in the center output, the center will operate at  $B_1$  in the long-run (involving an increase in capital and labor). But in the short-run, the center operates at  $B_2$ , representing short-run equilibrium, if there is full-adjustment in labor. Otherwise, the center operates at  $A_0$  (involving no adjustment in labor) or in-between (if there is some adjustment in labor).

The total costs of production for the various factor adjustments are shown in the cost-output space shown in Figure 4.3. The arc  $A_1B_1$  represents the costs when both factors are fully adjusted (the long-run case), while the arc  $A_2B_2$  and the horizontal arrow through  $A_0$  represent, respectively, the short-run costs when the labor adjustment is full, or none. For a decrease in output to  $Y_1$ , the short-run cost of production is always above the long-run cost curve, and is in the interval  $A_2A_0^1$ ,  $A_2$  being the minimum cost in the short-run. In contrast, for an output increase, the short-run cost could be above or below the long-run curve and is in the interval  $B_2A''_0$ ,  $B_2$  being the maximum cost in the short-run (when the labor is fully adjusted). The apparent deviation from the theory of long-run and short-run costs is easily explained: Operating at  $B_1$  in the long-run, or  $B_2$  in the short-run does not entail any decline in the quality of the output, whereas the movement from  $B_2$  to  $A''_0$  in the short-run represents progressive decline in quality. The output produced at  $A''_0$  is of the least quality since the output ( $Y_2$ ) is produced with the same quantity of the variable factor used in producing a smaller output  $y_0$ .

7

According to the theory, the long-run cost represents the least cost, followed by the short-run equilibrium cost.

8

This is especially relevant to day care centers, and many other service industries, where quality is not reflected in the easily compilable output measures. In day care centers, the output measured by the child-days, or the enrollment can be increased by increasing the enrollment without an increase in the staff; in the outpatient clinics, the output measured by the number of patients seen by a doctor can be increased by rushing through the patients; in public transportation, the output measured by the number of passengers carried can be increased by overcrowding.

Given that the utilization rate changes affect unit costs, what would be the nature of the deviation of the estimated cost curve from the long-run cost curve? For the familiar Friedman case <sup>9</sup> -- negative output fluctuations occur in small firms, while positive fluctuations occur in large firms, and the adjustment in the variable factor is none -- the scale will be overstated in the case of a cost curve with increasing returns, and understated in the case of a cost curve with decreasing returns. However, this case is not applicable to day care centers as there is no association between size and the sign of the output fluctuation. It is hard to foresee the exact nature of the deviation as it depends on many factors, of which, there is little information. Some of these factors are, the signs of the output fluctuations and the magnitudes, their distribution along the center size range, the degree of adjustment in the variable factor in each of the centers, and the shape of the long-run curve. Compounding the problem is that the long-run output is not generally known in many industries. <sup>10</sup> For example, in day care, the capacity, a possible long-run output measure, has been interpreted differently from time to time (as discussed in Section 4.2.2).

Generally, there is no way of avoiding regression fallacy completely. Nonetheless it may be some what reduced by eliminating observations of very high (much higher than 100%) and very low utilization rates from the sample; by increasing the sample size when there is a concentration

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9

See M. Friedman (5).

10

A discussion of the problems of measuring the long-run output in electricity generation is contained in M. Nerlove (9).

of a particularly low or high utilization rate in one size group; and perhaps, by stratifying the sample by size groups and estimating individual cost curves -- the envelope of these cost curves is the long-run cost curve.<sup>11</sup>

#### 4.2.4.2 Measurement Errors

Measurement errors in output cause the estimated average cost curve to deviate from the long-run average cost curve. The bias always results in overstating the economies of scale (or understating the diseconomies),<sup>12</sup> as made clear in the following examples:

1. Linear Average cost curve:

Average cost  $\equiv \frac{TC}{Y} = a_0 + a_1 Y$ , where TC, and Y are total cost, and outputs,  $a_0$  and  $a_1$  are regression coefficients.

When Y is measured larger than its value, the ratio TC/Y goes down, while  $a_1 Y$  goes up in absolute value, creating a spurious negative correlation between them. For a declining cost curve,  $a_1$  is negative, and the spurious correlation will increase the absolute value of  $a_1$ , overstating the economies of scale. In the case of increasing costs, since  $a_1$  is positive, its value will be reduced, understating the diseconomies of scale.

The same results are obtained when Y is measured smaller than its value.

2. Quadratic cost curve:

$$AC \equiv \frac{TC}{Y} = a_0 + a_1 Y + a_2 Y^2, \quad a_1 < 0, \quad a_2 > 0$$

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11

G. Borts (1) estimated rail road cost functions by stratifying the sample.

12

See M. Grossman (7).

Measurement errors will increase the absolute value of  $a_1$ , while reducing  $a_2$ , thus increasing the slope of the curve, and the minimum cost size.

#### 4.2.5 Choice of Estimating Total Cost Function, or Average Cost Function

By estimating the cost function as an average cost function, the problem of multicollinearity is avoided and the problem of heteroscedasticity is reduced, as explained below.

Multicollinearity:

If a total cost function is estimated, the appropriate functional form in the presence of multiple outputs is

$$(4.8) \quad TC_i = a_0 + a_1 y_{i1} + a_2 y_{i2} + v_i$$

where  $TC_i$ ,  $y_{i1}$ , and  $y_{i2}$  are the total cost, and  $y_{i1}$  and  $y_{i2}$  are the different day care outputs produced in center  $i$ .

There is usually substantial multicollinearity between  $y_1$  and  $y_2$ . The problem, however, will not be present in an average cost function of the form.<sup>13</sup>

$$(4.9) \quad AC_i = b_0 + b_1 y_i + b_2 p_{i1} + u_i,$$

where,  $AC_i$ ,  $y_i$  and  $p_{i1}$  are the average cost, total output, and the proportion of type 1 day care in center  $i$ .<sup>14</sup>

13

This form was used by M. Feldstein (4) in the estimation of hospital costs. An alternative way to handle the multiple outputs is to aggregate them, using the weights  $C_j$  (from Chapter 3), according to M. Grossman (7).

14

If there are  $n$  day care types, only  $n-1$  proportions need to be used in the equation, to avoid over-determinacy.

Since the separate day care outputs are represented by the proportional composition, the multicollinearity is avoided.

Heteroscedasticity:

Let the variance of the stochastic disturbance term in the above total cost function,  $v_i$ , be  $k^2$ --  $\text{var}(v_i) = k^2$

Since the average cost in center  $i$  is obtained by dividing the total cost function by the output  $y_i$  (for simplicity, homogeneous output is assumed) in center  $i$ ,

$$\text{Var}(u_i) = \text{var}(v_i / y_i) = (1/y_i^2) \text{var}(v_i) = k^2 / y_i^2$$

where,  $u_i$  is the error term in the average cost function (equation 4.9). Thus the variance of the error term in the average cost function is a small fraction of the variance of the error term in the total cost function; hence the problem of heteroscedasticity will be smaller in the average cost function.

The relative superiority of the average cost function with respect to the multicollinearity and the heteroscedasticity problems outweighs its potential for containing measurement related regression fallacy (as discussed in Section 4.2.5); hence the cost curve is estimated as the average cost function in this study.

### 4.3 Empirical Results

#### 4.3.1 Cost Curve Without Day Caremix

In this study, the average cost (AC) is the total cost per child-year, and the center size is the center output in child-year (CY).

Regression of average cost on size produced the following linear,  
15  
and quadratic equations.

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15

Throughout the study, parentheses under the regression coefficients show the t-ratios.

$$(4.10) \quad AC = 3353 - 3.98 \text{ CY} \\ (t=43.1) \quad (4.8) \quad R^2 = 0.111$$

$$(4.11) \quad AC = 3841 - 15.77 \text{ CY} + 0.0599 \text{ CY}^2 \\ (21.2) \quad (3.8) \quad (2.9) \quad R^2 = 0.149$$

The equations, along with the relevant data are shown in columns (1) and (2) of Table 4.1. Since the day caremix is omitted, the equations may be biased (discussed in the next section), limiting their value. So only a limited evaluation is presented:

The large coefficient values, and the high t-ratios (small standard errors of the coefficients) suggest a strong size-cost relationship in the day care centers. The linear equation with a negative size coefficient, implies that the costs decline linearly with size, suggesting that economies of scale prevail in the entire size range. The quadratic equation, with a negative size coefficient and a positive size square coefficient, yields a U-shaped cost curve, meaning that economies of scale prevail up to a certain size only, and then diseconomies take over. Both equations are highly significant -- significant at 0.01% level, but the  $R^2$ s are low.

The quadratic curve reaches a minimum (since  $\partial AC / \partial \text{CY}^2 > 0$ ) at 132 CY. The expected average costs for different center sizes yielded by the equations are shown in Table 4.2, and they will be compared later with the cost estimate from the equations containing the day caremix team.

#### 4.3.2 Cost Functions with Day Caremix

The following linear and quadratic equations with the day caremix have been estimated (and shown in Table 4.1):

$$(4.12) \quad AC = 2338 - 0.85 \text{ CY} + 878 \text{ Prop} \\ (t=9) \quad (1.1) \quad (4.2) \quad R^2 = 0.189$$

TABLE 4.1

Single Cost Curve for All Centers

	(1)	(2)	(3)	(4)
Intercept	3353 (t=43.1)	3841 (21.0)	2338 (9.3)	2825 (9.2)
CY	-3.98 (4.8)	-15.77 (3.8)	-0.85 (1.1)	-11.84
CY <sup>2</sup>	-	0.0599 (2.9)	-	0.553
Prop	-	-	878 (4.2)	846 (4.2)
R <sup>2</sup>	0.1108	0.1492	0.1892	0.2217
SER	420	412	402	394
SER/AC	13.9%			
F-sig level	0.01%	0.01%	0.01%	0.01%
Minimum at	-	132 CY	-	107 CY
<sup>6</sup> ESSx10/DF	32.24/183	30.85/182	29.41/182	28.22/181
e at 86 CY	-0.113	-0.156	-0.024	-0.066

Notations:

SER: Standard Error of Regression

SER/AC: SER as a percent of average cost

F-sig level, or F-level: Significance level of F-statistic

Minimum at: The size at which the cost curve reaches the minimum

ESS/DF: Error Sum of Squares/Degrees of Freedom

e: elasticity of average cost curve with respect to size

$$(4.13) \quad AC = \underset{(9.2)}{2825} - \underset{(2.9)}{11.84} CY + \underset{(2.7)}{0.0553} CY^2 + \underset{(4.2)}{846} \text{ Prop} \quad R^2 = 0.222$$

Minimum at 107 CY

The specifications assume that the changes in the day care mix will shift the curves in parallel without affecting the shapes of the curves. The day care mix is represented by the proportion of Pre-school day care (Prop).

Since the equations are likely to be biased due to the omission of the day care center lease, as discussed in the next section, they have not been evaluated in detail. The coefficients of the day care mix in the two equations are large, and close -- 846 and 878, and have large t-ratios. Comparison with equations (4.10) and (4.11) reveals that the introduction of the day care mix has affected the slope coefficients considerably: In the linear form, the size coefficient has decreased by 75%, along with a drop in the significance level (from 1% to 44%). In the quadratic form, the coefficients of size and size square have decreased in value, by 25% and 8%, respectively, but their significance levels have not changed.<sup>16</sup> These suggest that the equations without the day care mix are biased and the bias is in the direction of exaggerating the scale economies. For example, the inclusion of the day care mix has reduced the elasticity of the quadratic curve at its mean size, from 0.16<sup>17</sup> to 0.06, and the minimum cost size from 132 CY to 107 (meaning that the

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Implies a shallower U-curve.

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The elasticity at the mean size is computed:

$$e_{AC, CY} = \frac{\partial AC}{\partial CY} \frac{CY}{AC}, \quad \partial AC / \partial CY \text{ is obtained as follows:}$$

$$AC = A + b_1 CY + b_2 CY^2 + \text{Prop (Quadratic curve)}$$

$$\partial AC / \partial CY = b_1 + 2 b_2 CY, \quad \overline{CY} \text{ used for } CY.$$

$$\text{For the linear curve } \partial AC / \partial CY = b_1$$

region of increasing returns is smaller).

The signs of the day caremix are, as expected, positive, implying that the average cost and the proportion of Pre-school day care (the more costly kind) move together. The higher elasticities of the cost curves without the day caremix indicate that the day caremix is correlated with the size, which is supported by the value of the observed simple correlation coefficient of 0.6 between the day caremix and the size.

The quadratic equation with the day caremix is obviously superior to the other equations: Its  $\bar{R}^2$  is the highest-0.22, while the standard error of regression (SER) is the smallest; its regression coefficients are large and statistically significant.

The average costs at different sizes yielded by each of the four equations are listed in Table 4.2. There are substantial differences among the estimate, therefore, the opportunity costs of not choosing the appropriate functional form are high. For example, in attempting to build a center with the least average cost within the size limits of 25-180CY, the day care management would have chosen a size of 180CY had it used one of the linear equations. According to the correct form, (the quadratic form with the day caremix), the average cost at 180CY is \$295 more than the true minimum cost (at 107CY).

TABLE 4.2

Initial Estimates of Average Costs by Size

	Center Size (CY)						
	25	50	100	107 (dollars)	132	150	180
Linear form							
Without Day Caremix	3254	3154	2955	2932	2827	2756	2637
With Day Caremix	3063	3041	2999	2993	2972	2957	2931
Quadratic form							
Without Day Caremix	3484	3202	2863	2840	2803	2824	2943
With Day Caremix	3283	3090	2913	2910	2945	3012	3205

Note: Costs computed at the average day caremix ( when day caremix is included in the cost function)

#### 4.3.3 Cost Functions by Lease Category

The focus of this section is to examine whether a single cost function for all the centers can properly represent the size cost relationships in the individual lease categories in view of their differences (as discussed in Chapter 2), and to estimate separate cost functions if needed.

The unweighted mean unit costs in the Direct, and the Indirect leases are, respectively, \$3186 and \$2915, yielding a difference of \$271, which increases to \$451, when adjusted for day care mix differences. This difference is approximately 15% of the unweighted mean unit cost of all centers. An initial inference from such a large cost differential is that a single cost curve would underestimate the costs in the Direct Lease and overestimate in the Indirect Lease. But when size differences between the categories are considered, the cost differential may be explained in terms of differences in scale within the framework of a single cost curve. Since the mean size in the Direct Lease is 112 CY compared to 71 CY in the Indirect Lease, the cost differential could have resulted from the diseconomies of scale exhibited by a single cost curve in the size region of the Direct Lease. The explanation, however, may not hold if there are considerable size overlap between the categories, with substantial cost differences between categories in the region of overlap -- scale differences cannot exist if sizes are the same. The standard deviations of size -- 34 and 30 in the Direct Lease and Indirect Lease respectively, -- indicate a considerable size overlap (slightly more than  
<sup>18</sup>  
50% of the Direct Lease Centers have a size in common with approximately

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The two standard deviations are assumed to be the same. The two means located apart by two standard deviations; and the normal curve shapes will yield the 50%.

the same percent of the other centers).

To examine the size overlap and the cost differences in detail, the centers are divided into eight size groups, differentiated by the lease status. For each size-lease cell, the number of observations, and the day caremix adjusted average costs (adjusted to the system-wide average proportion) are shown in Table 4.3. There is a complete size overlap: Every size class, except the lowest (20-40 CY), contains observations from both the categories, though the Indirect lease is dominant in the lower sizes as the Direct Lease is in the higher sizes. The day caremix adjusted average costs are higher in the Direct Lease in every size class, implying that a single cost curve cannot adequately describe the cost relations in the individual categories.

Column (3) shows the costs in each size class yielded by the single cost curve estimated earlier -- equation (4.13), at the mean of the size class and the system-wide day caremix. These costs are lower than the day caremix adjusted costs in the Direct lease, and higher than those in the Indirect lease, in every size class, and the errors are in the range 6-20%, supporting the earlier conclusion. Hence it is necessary to estimate separate cost curves for the two categories.

Three sets of lease differentiated cost curves are estimated with separate assumptions of whether the influence of the variables on costs is the same in both categories. The alternative assumptions are needed because very little is known about the day care operations. It is expected that the behavior of many of the factors affecting the slope parameters of the cost functions, such as, staff indivisibilities, random absences of staff and children, size of the catchment area, and the personal factors, would be invariant of the lease category. But it is not known how the

TABLE 4.3

Adjusted Unit Costs by Size

Size Class	(1)		(2)		(3)	
	Number of Centers	Indirect Lease Centers Day Caremix <sup>1</sup> Adjusted Unit Costs (unweighted)	Number of Centers	Direct Lease Centers Day Caremix Adjusted Unit Costs (unweighted)	Mean Size (CY)	Predicted <sup>2</sup> Unit Cost
20-40CY	11	\$3360	0	--	33.20	\$3212
40-60	51	3040	6	3272	53.3	3070
60-80	20	2784	8	3564	70.7	2983
80-100	19	2654	10	3076	90.0	2926
100-120	11	2651	13	3082	109.3	2911
120-140	2	2474	14	3308	128.9	2937
140-160	3	2472	8	3159	148.8	3006
160-180	2	2736	7	3231	170.7	3136

<sup>1</sup>Adjusted to day caremix proportion of 0.85(systemwide average)

<sup>2</sup>Predicted by equation(4.13) at the mean size of the class, and day caremix proportion of 0.85.

economies in building costs (or Rent Costs, which also affect the slope coefficients) differ between categories; so it is not possible to foresee if the slope coefficients differ between categories.

Set I assumes that all the coefficients, except the intercept are the same between categories, implying that the cost curves are identical in shape, but situated apart in parallel. The following dummy variable specification is used in estimating the curves:

$$(4.14) \quad AC = (b_0 + b_4)L + b_1 CY + b_2 CY^2 + b_3 Prop$$

$$L = 0/1 \text{ for Indirect Lease/Direct Lease}$$

The estimates are shown in Column (1), Table 4.4. All the coefficients are large, and statistically significant, both individually and collectively. Compared to the single curve, the  $R^2$  and the SER improved, respectively, by 250% (from 0.22 to 0.54) and 25% (from 394 to 305). The slope parameters have the signs leading to the U-shape, as in the single cost curve, but the magnitudes are larger. The size, and the size square coefficients have increased by 70% and 50%, respectively, indicating that the new curves are less shallow (more elastic, hence larger scale), and the region of increasing returns is wider, as seen from the data: The minima and the elasticities for the new and the old curves are, respectively, 137 CY and 107 CY, and -0.24 and -0.13.

Set II assumes that all the coefficients, except the day care mix coefficients change between the categories, meaning that their cost curves have different shapes, and the assumption is supported by the estimation results, as explained below:

TABLE 4.4

Average Cost Curves by Lease

	(1)	(2)	(3)	(4)	(5)	(6)
	Set 1	Set II	(A)	Set III (B)	(C) Indirect Lease	Direct Lease
Intercept	3295 (t=13.7)	3376 (13.7)	3307 (12.2)	3359 (13.9)	3307 (14.3)	3476 (4.9)
CY	-20.43 (6.2)	-22.80 (5.7)	-22.61 (5.6)	-22.23 (6.0)	-22.61 (6.6)	-10.20 (1.1)
CY <sup>2</sup>	.07453 (4.8)	.08442 (4.0)	.08456 (4.0)	.08143 (4.1)	.08465 (4.6)	.03064 (0.8)
Prop	701.0 (4.4)	733.5 (4.6)	784.2 (4.2)	725.8 (4.6)	794.2 (5.0)	534 (1.5)
Lease(L)	619.9 (11.0)	-147.8 (0.4)	51.6 (0.3)			
L.CY	-	14.19 (1.8)	12.4 (1.5)	11.83 (6.0)		
L.CY <sup>2</sup>	-	-0.05618 (1.5)	-0.05303 (1.4)	-0.04778 (3.0)		
L.Prop	-	-	-230.2 (0.6)	-		
R <sup>2</sup>	.5374	.5478	.5488	.5474	.6429	.2433
SER	305	304	304	302	8.9	11.7%
SER/AC	10.1%	10.1%	10.1%	10.1%		
Min at						
ILC's	137CY	135	134	136	134	137
DLC's	137	152	151	155	-	166
ESS/DFx10 <sup>6</sup>	16.8/180	16.40/178	16.36/177	16.41/179	7.78/115	8.58/62
F-sig <sub>α</sub> level	0.01%	0.01%	0.01%	0.01%	0.01%	0.01%
e at 86CY					-0.23	-0.14
Slope at 86CY						
ILC's	-7.61	-8.28	-8.08	-7.66	-0.08	-
DLC's	-7.61	-6.24	-4.93	-4.60	-	-4.93

The dummy variable specification of equation (4.14), with dummy coefficients added to the size and size square terms, has been used in the estimation, and the results are shown in the Column (2), Table 4.4.<sup>19</sup>

The differential coefficients are large, with fairly significant t-ratios (the differential coefficients of size and size square are significant at 94% and 89%, respectively), supporting the assumption of this set. Compared with Set I, the slope coefficients are 10% higher, implying that the Indirect lease curve is slightly steeper, without a change in the minimum (137 CY vs 135 CY). The Direct lease slope coefficients are about one-third as large as those of Indirect lease, indicating that the Direct lease curve is relatively shallower; the minimum cost size has increased to 152 CY. A more detailed analysis of the shapes of the curves are given in Set III.

Set III assumes that all the coefficients vary between categories, permitting alternative estimation approaches. The first is the dummy variable method, used earlier, by which a single equation containing the separate cost curves is estimated. The second is the direct estimation of the two cost curves separately. The two methods are referred henceforth as the Single Equation Estimation and the Two Equations Estimation, and their specifications are as shown:

Single Equation Estimation:

$$(4.15) \quad AC = (b_0 + b_4L) + (b_1 + b_5L)CY + (b_2 + b_6L)CY^2 + (b_3 + b_7L) Prop + u_i$$

L= 0/1 for Indirect Lease/Direct Lease

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The coefficients of the Direct lease curve are obtained by subtracting the differential coefficients from the corresponding regular coefficients of the Indirect Lease. The Direct lease coefficients are not separately shown in the tables.

Two Equations Estimation:

Indirect Lease:

$$(4.16) \quad AC = a_0 + a_1 CY + a_2 CY^2 + a_3 Prop + u_i$$

Direct Lease:

$$(4.17) \quad AC = b_0 + b_1 CY + b_2 CY^2 + b_3 Prop + v_i$$

Although both methods yield identical regression equations, their implications to the analysis of the cost differences are not the same. In the dummy variable method, the cost variation in the entire sample of centers (total variation) is investigated, whereas in the Two Equations Estimation, the variation within each category is investigated. In the latter method, the investigation is carried out without an indication of the proportion of the total variation being studied. Unlike the Two Equations Estimation, the dummy variable method provides a test of whether a coefficient is significantly different between the two categories, by providing the standard error of the differential coefficient.<sup>20</sup>

Since the study objective is to investigate the cost variation in the population, as well as, in the individual categories, both methods are used.

Set III, (A) in Table 4.4 shows the results of the dummy variable method. The differential coefficient of the day caremix has a value of minus 130, implying that the day caremix coefficient is smaller in the Direct Lease. But its t-ratio is very small, 0.6 (significant at 45% level).

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A Chow test can be used to test whether the two equations are different, but there is no test available for testing whether a single coefficient is different between the equations.

Compared with Set II, the regular coefficient of the day caremix is higher by 8%, while those of the size and the size square and the three t-ratios are the same. The differential coefficients of the size square and their t-ratios are lower by 10%. The t-ratio of the intercept is the same.

Since the t-ratios of the differential coefficients of the intercept, and the day caremix are very small, their exclusion from the equation might make the other differential coefficients significant by reducing their standard errors. Set III, (B) shows regressions obtained with the exclusion. The t-ratios of the differential coefficients of the size and the size square have increased considerably (making them significant at a level higher than 1%), followed by a slight decline in their values. The  $R^2$  has remained unchanged. Therefore the conclusion is that the influence of the day caremix is the same between the categories.

The results of the two equation estimation are shown in Set III, (C).  
 The t-ratios of all the coefficients are high in the ILC's<sup>21</sup> (significance levels higher than 1%), and small in the DLC's (significance levels ranging from 42% to 14%). The  $\bar{R}^2$  value in the Indirect Lease is 0.64, compared to 0.24 in the Direct Lease. Nonetheless, the Direct Lease as well as the Indirect Lease equations are statistically significant, while being different (confirmed by a Chow test).

Shown below are the final equations of the separate cost curves by lease:

Single Equation Estimation:

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ILC's: Indirect Lease Centers, DLC's: Direct Lease Centers.

$$(4.18) \quad AC = 3359 + (-22.23 + 11.36L)CY + (0.0814 - 0.0478L)CY^2 + 726 \text{ Prop}$$

L = 0/1 for ILC's/DLC's

$$R^2 = 0.55$$

Minimum at 136CY for ILC's and  
152CY for DLC's

Two Equations Estimation:

Indirect Lease:

$$(4.19) \quad AC = 3307 - 22.61 CY + 0.0847 CY^2 + 794 \text{ Prop}$$

$$R^2 = 0.643$$

Minimum is at 134 CY

Direct Lease:

$$(4.20) \quad AC = 3368 - 10.20 CY + 0.03064 CY^2 + 564 \text{ Prop}$$

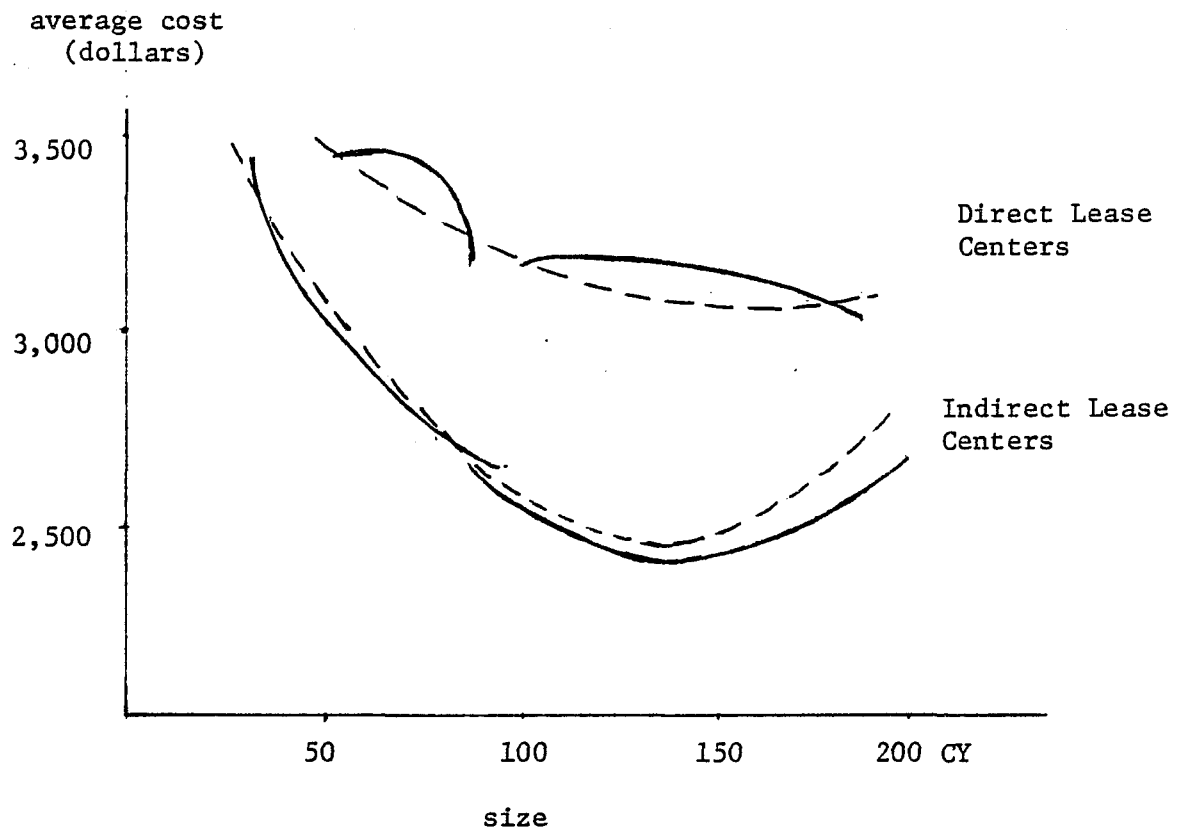
(t=1.1)            (0.8)            (1.5)

$$R^2 = 0.243$$

Minimum at 166 CY

Equations (4.18) and (4.19) have high  $R^2$  and high t-ratios for the coefficients, whereas equation (4.20) has a low  $R^2$  and small t-ratios (note significant at 5%). Both cost curves have a U-shape, but have different slopes, with the Direct Lease curve being the shallower, as seen from Figure 4.4. The elasticities measured at the mean size (86CY) are -0.13 for the Direct Lease Curve, and -0.24 for the other. The minimum cost size for the Indirect Lease is 135CY, which exceeds the size of 70% of the Indirect Lease Centers. For the Direct Lease the minimum

Figure 44

Single Cost Curves and Size Differentiated Cost Curves, by LeaseNotes:

Broken line curves represent the single cost curves.

Solid line curves represent the size differentiated cost curves.

The curves are estimated at the mean day care proportion of 0.85

cost size is in the range 155-166, and there are only 5% of the Direct Lease Centers above this range, meaning that economies of scale exist almost throughout their size range.

The minimum average cost in the Indirect Lease Centers is \$2473, which is about 8% less than the observed average (the costs in both the lease categories are adjusted to the mean day care mix of all centers). The estimated average costs at sizes 50 CY, 90 CY, 180 CY are, respectively, \$3062, \$2633 and \$3656 which are higher than the minimum by, 24%, 6.4% and 7% respectively. The minimum cost in the Direct Lease is \$3000, which is about 6.6% less than the observed average cost. The estimated average costs at sizes 50 CY, 90 CY and 180 CY, are respectively \$3414, 3177, and 3004, which are higher than the minimum by, 14%, 6%, and 0%, respectively.

The cost curves are shown in Figure 4.5, ( page 94 ) which will be compared later with the cost curves containing the utilization rate.

#### 4.3.4 Cost Functions by Size and Lease

It may be an oversimplification to assume that the cost-size relationship can be represented by a single equation over the entire size range. By dividing centers into size groups and estimating a separate cost function for each group, better estimates of cost curves can be obtained. Such differentiated cost functions are useful when the policy issue is to choose an optimal center within certain size limits, perhaps, necessitated by neighborhood demand for daycare, and the availability of funds. As an example, if the issue is to select an optimum center under 90 CY, the cost function within the range of 90-300 CY is less relevant.

Moreover, the size differentiated cost curves, perhaps free of

regression fallacy, when juxtaposed with the single cost curve, could reveal the extent of regression fallacy in the single cost curve.

For this purpose, the centers are divided into two groups: small, and large centers with sizes above, and below 89 CY,<sup>23</sup> respectively. A separate equation estimated for each group is shown in Table 4.5.

The small size group in the Indirect Lease has a U-shaped cost curve, reaching a minimum at 86.5 CY, which is almost the upper limit of the group. The large size group also has a U-curve, which reaches a minimum at 142 CY, which is close to the minimum cost size of the single cost curve for the Indirect Lease (equation (4.19)), 134 CY. The plots shown in Figure 4.4 reveal that the single cost curve approximates the size-differentiated cost curves very well. The slopes at 58 CY (mean size of the small centers in the Indirect Lease) are -12.76 and 12.79 for the small size curve, and the single cost curve respectively; and the slopes at 111 CY (mean size of the large centers in the Indirect Lease) are -4.4, and -3.90, respectively, for the large size curve and the single curve.

Additionally, the reduction in the error sum of squares with the two curves compared with one curve is very small, 3.4% (from  $7.78 \times 10^6$  to  $7.51 \times 10^6$ ). Hence it can be concluded that a single quadratic curve can very well represent the size-cost relationship for the entire size range of the Indirect Lease Centers. If the policy problem is to choose a small center with an upper size limit of, say under 90 CY, the choice

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The plan was to divide the centers at 86 CY, the mean of all centers, but a key punch error resulted in the division at 89 CY, and this will not affect the conclusions of this section.

TABLE 4.5

Cost Curves by Size and Lease

	Indirect Lease		Direct Lease	
	Small Centers	Large Centers	Small Centers	Large Centers
Intercept	3531 (t=8.8)	3482 (3.1)	448 (0.1)	2092 (1.2)
CY	-37.12 (3.1)	-20.58 (1.2)	81.97 (0.6)	8.31 (0.3)
CY <sup>2</sup>	0.2149 (2.2)	0.0726 (1.1)	-0.6466 (0.7)	-0.0348 (0.4)
Prop	955 (4.5)	638 (2.0)	581 (0.5)	730 (1.8)
R <sup>2</sup> SER	0.440 276	0.262 188	0.075 502	0.113 331
Min (Max) at	87CY	142	(64)	(120)
Sample Size	91	28	18	48
ESSx10 <sup>6</sup>	6.66	0.85	3.53	4.83
F-sig. level	0.01%	6%	77%	15%
	<u>Means Data</u>			
CY	57.8	111.0	68.5	28.6
Prop	0.96	0.69	0.94	0.10
Average Cost	\$3065	2427	3476	3078
Utilization Rate	0.969	0.985	0.949	0.957

should be the center with a size equal to the upper limit. If the problem is to choose a large center with a lower size limit, say, above 90 CY, the choice should be the center with size equal to minimum cost size of the single cost curve.

In the Direct Lease, the equations of the size groups shown in Table 4.5, have very large slope coefficients with signs signifying cost curves of the shape of an inverted U, in contrast to the U-shape of the single cost curve in the Direct Lease (equation (4.20)). The maxima are reached at 64 CY, and 124 CY in the small, and large center groups. The statistical characteristics of these equations are extremely poor.<sup>24</sup> The t-ratios of most of the coefficients are in the range 0.3-0.4, and none has a coefficient higher than 1.8. The equations are significant at low confidence levels only: 23% and 85% for the small size, and large size curves; respectively.

From the plots in Figure 4.4, a general trend of declining average costs almost throughout the size range of the Direct Lease centers is clearly visible, despite the inverted U-shape costs of the individual size groups. The maxima exhibited by a sharp surge in the average costs<sup>25</sup> in the average costs resulting from a strong localized disturbance in each group. The shape of the single cost curve, however has not been affected by the disturbances, and hence captured the general trend well, although its  $R^2$  and t-values have been influenced.

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24

A higher upper size limit for the small size would have produced a larger sample than the current 18 observations. Similar results have been obtained with a larger sample (not shown in the study).

25

The disturbance is explored in the next section.

Moreover, the reduction in the error sum of squares achieved by the separate cost curves is negligible -- 2.5% (from  $8.56 \times 10^6$  to  $8.36 \times 10^6$ ). Therefore the conclusion is that a single cost curve would better explain the nature of scale economies in the Direct Lease Centers than size differentiated curves would.

#### 4.3.5 Cost Curve with Utilization Rate

As discussed in Chapter 2, the utilization rate (UR) varies widely among the centers: mean -- 0.966, coefficient of variation -- 7%, range -- 0.81-1.15. Although the coefficient of variation of 7% may not appear very large, it can effect several times larger differences in the costs. For example, a coefficient of variation of 7% in the utilization rate of a center can create a difference of as much as 28% in the average cost of the center.

The large fluctuations in the utilization rate could introduce considerable regression fallacy in the estimation of the cost curves, as described in Section 4.2.4. For example, the maximums exhibited by the size-differentiated cost curves in the Direct Lease could be due to the bias resulting from a concentration of unusually low utilization rates in a narrow size band in each of the two size groups, as discussed:

Table 4.6 shows the utilization rates by size class and lease when the centers are divided into 8 size classes. In the Direct Lease, the general shape of the size-utilization relationship is of U-shape when size classes 60-80 CY and 120-140 CY are excluded. The curves starts at

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26

28% represents two standard deviations above and below the mean utilization rate of 100%. It is assumed that the factor levels do not change with utilization rates.

TABLE 4.6

Utilization Rates by Center Size

Size Class	Indirect Lease			Direct Lease		
	Number of Centers	Utilization Rate		Number of Centers	Utilization Rate	
		Mean	Range		Mean	Range
20-40CY	11	0.98	0.85-1.13	0	--	---
40-60	51	0.96	0.82-1.05	6	0.97	0.90-1.04
60-80	20	1.00	0.86-1.11	8	0.91	0.81-1.02
80-100	19	0.96	0.84-1.11	10	0.96	0.85-1.12
100-120	11	0.98	0.89-1.08	13	0.95	0.83-1.08
120-140	2	1.06	1.05-1.07	14	0.92	0.83-1.07
140-160	3	1.07	0.97-1.04	8	0.97	0.85-1.07
160-180	2	0.98	0.97-1.00	7	0.96	0.85-1.02

a value of 0.97 in the 40-60 CY size class, more or less gradually moves to 0.95 in the 100-120 CY class, and reaches a value of 0.96 in the end class. The utilization rates in the excluded size classes are unusually low -- 0.91 and 0.92, which pushed the average costs in the two classes very high, creating the local maxima observed at sizes 64 CY and 120 CY.

The U-shaped size-utilization relationship, by raising the average costs in the middle sizes relative to the outer sizes would render a U-shaped long-run cost curve shallower -- lower elasticities and possibly a higher minimum cost size. This will partly explain why the cost curve of the Direct Lease is less elastic (than the Indirect Lease curves), with declining average costs in almost the entire size region.

In the Indirect Lease Centers, the utilization rates are generally higher in the top four size classes. The resulting bias would make the upward sloping portion of the U-shape cost curve for the all the Indirect Lease centers less steep.

Inclusion of the utilization rate in the cost equation (partialing it out) would reduce the effects of its fluctuations, and its systematic relationship with size. But this would not altogether remove the regression fallacy, as explained below:

The observed utilization rates may not truly measure the fluctuations in the longrun output since the desired longrun utilization rate may not be one hundred percent of capacity, and it may vary from center to center -- larger centers may have higher desired utilization rates since they have proportionately smaller fluctuations in staff, and child censuses (as seen in Section 4.2.1), requiring smaller safety margins (to cope with situations involving unusually low staff attendance and high student attendance). Then, by holding the observed utilization rates constant, the real utilization rates (of which there is no data available) are varied, creating a bias.

Uniformity of output fluctuations does not guarantee bias-free estimates since the bias depends as well on the adjustment in the variable factor (discussed in Section 4.2.5) and the shape of the longrun curve. For the simple case of no adjustment in the variable factor, bias-free estimates are obtained only in the case of constant returns to scale when output variations are held constant. In the case of a cost curve containing either increasing returns or decreasing returns, uniform output fluctuations along the entire size range would bias the cost curve, and the bias would reinforce the scale.

The following are the equations obtained when the utilization rate is included in the regressions.

Single Equations Estimation:<sup>27</sup>

$$(4.21) \quad AC = 11058 + (-20.66 + 10.00L) CY + (.0847 - .0559L) + CY^2 \\ + 881 \text{ Prop} - 13649 UR + 5604 UR^2 \\ (t=1.8) \\ L=0/1 \text{ for ILC's / DLC's} \\ R^2 = 0.7415; \\ \text{Minimum at: } 126 \text{ CY, ILC's, and} \\ 135 \text{ CY, DLC's}$$

Two Equations Estimation:

Indirect Lease:

$$(4.22) \quad AC = 10,067 - 21.34 CY + 0.0846 CY^2 + 884 \text{ Prop} - 11785 UR + \\ (1.7) \\ 4769 UR^2 \\ (1.0) \\ R^2 = 0.7893; \text{ Minimum is at } 126 \text{ CY}$$

Direct Lease:

$$(4.23) \quad AC = 10,290 - 7.85 CY + 0.0283 CY^2 + 922 \text{ Prop} - 12,095 UR \\ (1.9) \quad (1.1) \quad (1.0) \quad (3.0) \quad (1.1) \\ + 4613 UR^2 \\ (1.3) \\ R^2 = 0.5822; \text{ Minimum is at } 139 \text{ CY}$$

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27.

t-ratios above 2.0 are not shown

The differential coefficients of the intercept, the day caremix, the utilization rate, and its square term are not included in equation (4.21), since they are both small and insignificant (t-ratios of less than 0.6), as shown in Column (7), Table 4.7, which shows alternative specifications. How small these are can also be seen from equations (4.23 and (4.22) ) though indirectly: Each of their regular coefficients has approximately the same value in the two equations, meaning that the differential coefficients are small.

There is almost no difference in the predictive power between the specifications containing the linear, and the quadratic utilization terms. They have the same SER value and almost the same  $R^2$  value. Nonetheless, the quadratic form <sup>28</sup> has been chosen since the quadratic term is large and significant at 11% level.

By comparing the equations in this section with equations (4.18), (4.19) and (4.20), the effects of including the utilization rate in the cost curve specification can be assessed:

Although all the equations registered substantial increases in the  $R^2$  values, the increase in the Direct lease is huge -- 141%, compared to 17% in the Indirect lease and 36% in the dummy variable equation; this is not surprising in view of the severity of the fluctuations in the utilization rate of the Direct lease. The SER's have declined by 24-30% in all the equations.

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The optimal capacity for producing a given output, using the quadratic form can be derived as follows:

$$AC = a_1 Y + a_2 Y^2 + a_3 UR + a_4 UR^2$$

$$a_1 Y + a_2 Y^2 + a_3 \frac{Y}{cap} + a_4 \left( \frac{Y^2}{cap} \right)$$

$$\frac{\partial AC}{\partial C} = a_3 cap^{-2} - 2 a_4 Y cap^{-3}$$

Equating it to zero and simplifying

$$Cap^* = \frac{2 a_4 Y}{a_3}$$

TABLE 4.7

## Average Cost Curves with Utilization Rate

	(1)		(2)		(3)	(4)	(5)	(6)	(7)	(8)	(9)
	Indirect Lease	Direct Lease	Indirect Lease	Direct Lease							
Intercept	5555.0 (t=17.8)	6022 (9.3)	10067 (3.0)	10290 (1.9)	5719 (19.4)	5698 (19.1)	5554 (15.5)	10186 (3.5)	11058 (3.9)	10980 (3.8)	10054 (2.6)
CY	-21.18 (7.9)	-6.78 (1.0)	-21.34 (8.0)	-7.85 (1.1)	-21.67 (7.2)	-21.62 (7.2)	-21.18 (6.9)	-21.32 (7.0)	-20.66 (7.3)	-20.99 (7.2)	-21.31 (7.0)
CY <sup>2</sup>	0.0833 (5.9)	0.0236 (0.83)	0.0846 (6.0)	0.0283 (1.0)	0.0864 (5.4)	0.0852 (5.3)	0.0833 (5.1)	0.0847 (5.3)	0.0817 (5.5)	0.0837 (5.3)	0.0847 (5.2)
Prop	886 (7.1)	940 (3.2)	884 (7.1)	922 (3.0)	906 (7.4)	864 (6.2)	886.3 (6.2)	894 (7.3)	881 (7.3)	906 (6.7)	894 (7.3)
UR	-2492 (8.8)	-3267 (7.0)	-11785 (1.7)	-12096 (1.1)	-2661 (9.8)	-2599 (9.0)	-2492 (7.7)	-12052 (2.0)	-13649 (2.3)	-13513 (2.2)	-11781 (1.5)
UR <sup>2</sup>			4769 (1.3)	4613 (0.8)	-	-	-	4905 (1.6)	5604 (1.8)	5535 (1.8)	4766 (1.2)
Lease(L)					-	-	467.0 (0.7)	-	-	-	309 (.05)
L. CY					16.85 (3.3)	17.11 (3.3)	14.40 (2.3)	13.24 (2.2)	10.00 (7.0)	11.37 (3.2)	13.18 (2.2)
L. CY <sup>2</sup>					-.0717 (2.9)	-.0703 (2.9)	-.0597 (2.1)	-.0559 (2.0)	-.0422 (3.7)	-.0494 (2.4)	-.0556 (2.0)
L. Prop					-	153.5 (0.6)	53.4 (0.2)	-	-	-73.1 (0.4)	-
L. UR					-366 (1.4)	-536.5 (1.4)	-775.2 (1.5)	245 (0.3)	-	-	-392 (.03)
L. UR <sup>2</sup>								-438 (0.7)	-	-	-109 (.01)
R <sup>2</sup>	0.7880	0.5780	0.7913	0.5822	0.7393	0.7399	0.7407	0.7439	0.7415	0.7418	0.7439
SER	201	280	201	281	231	232	232	231	230	231	231
Min at (CY)											
ILC's	127CY	144	126	139	125	127	127	126	126		126
DLC's					164	164	141	140	135		140
ESSx10 <sup>6</sup> /DF	$\frac{4.62}{114}$	$\frac{4.79}{61}$	$\frac{4.54}{113}$	$\frac{4.73}{60}$	$\frac{9.45}{177}$	$\frac{9.43}{176}$	$\frac{9.40}{184}$	$\frac{9.37}{195}$	$\frac{9.37}{177}$	$\frac{9.34}{176}$	$\frac{9.29}{174}$

The day caremix coefficients in both categories have increased in value to about 900, and the difference in their values has narrowed to 4%, from 33%. The coefficient in the dummy variable equation has increased by 20% from 726 to 884, which indicates that the day caremix and the utilization may be positively correlated.

The slope coefficients in the Indirect lease have changed very little. The size coefficient has increased by 5%, while the size square coefficient remained the same, which implies that the cost curve is slightly less shallow, and the minimum cost size is smaller (127 CY vs 134). In the Direct lease, the size and its square term have declined, respectively, by 23% and 7%; the changes in the cost curve are of the same kind as in the Indirect lease, but are larger. The minimum cost size has moved from 166 CY to 139 CY (but, as seen from Table 4.7, the 139 CY value is not stable and the minimum varies between 135 and 164 CY, depending on the specification).

The cost estimates by center size are shown in Table 4.8, and the cost curve plots are in Figure 4.5.

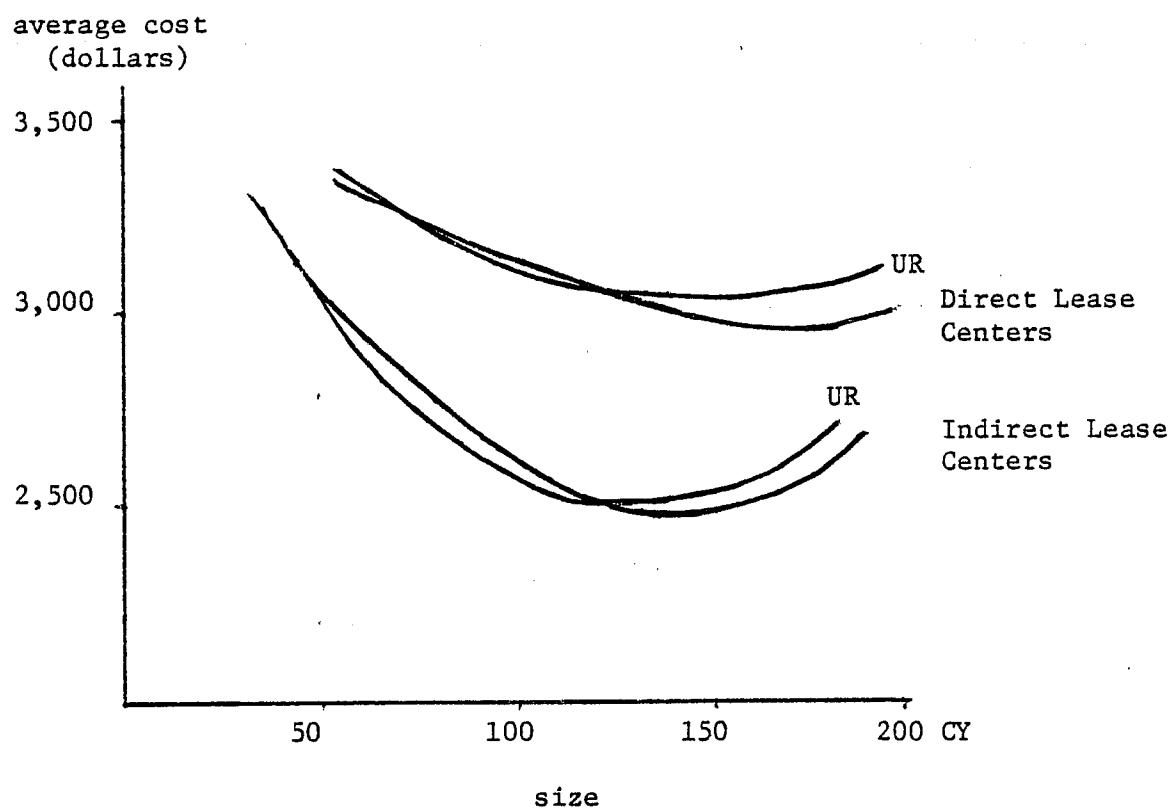
Comparison of the cost curves with, and without the utilization rate show that the partialing out of the utilization rate has made the upward sloping portion of the cost curves of both the lease categories steeper. In the case of the Direct lease, the minimum cost has increased by 2.5% (from \$3000 to \$3070). The cost curve is still very shallow, and the costs are more or less uniform between 120-160 CY. When the size deviates by 50 CY from the minimum cost size of 139 CY, the costs go up by less than 2%.

TABLE 4. 8

Final Estimates of Average Costs by Size

Center Size (CY)	Cost Curves without Utilization Rate		Cost Curves with Utilization Rate	
	Indirect Lease Centers	Direct Lease Centers	Indirect Lease Centers	Direct Lease Centers
50	\$3062	3414	2976	3296
90	2633	3177	2595	3140
125			2500	
135	2473			3074
164		3000		
180	2656	3004	2721	3120

FIGURE 4.5  
Cost Curves With, and Without Utilization Rate



Notes:

Curves marked by UR contain the utilization rate.

In the Indirect Lease, the partialing out of the utilization rate has not brought any significant changes, except for the above mentioned increase in the slope of upward portion of the curve, and a slight decline in the slope of the downward sloping portion, which have not significantly altered the size-cost relationship.

#### 4.3.6 Cost Curve with Miscellaneous Variables

The Miscellaneous Variables -- the Turnover rate and the Enrollment-Capacity Ratio --, and their effects on costs have been discussed in Chapter 2. To recall, these variables, in addition to affecting the utilization rates (and hence, affect the costs, indirectly) may affect the center costs directly: the administrative costs may increase with increases in Turnover Rates, and Enrollment- Capacity Ratios.

Table 4.9 presents the estimates containing these variables. The regression coefficients of the two Miscellaneous Variables are small: The coefficients of the Turnover Rate are 55, and 23 in DLC's and ILC's, respectively. Their impacts on cost differences are clearly limited since the mean value of the Turnover Rate is only 0.03. The coefficients of the Enrollment-capacity Ration are 15 and 23 in DLC's and ILC's, respectively, and the mean value of the variable is 1.1; hence the variable is inconsequential to explaining the cost differences. The t-ratios of the coefficients are extremely small, ranging from 0.2 to 0.5.

Comparison with equations (4.22) and (4.23) indicates that the addition of Miscellaneous Variables has neither affected the coefficients

TABLE 4.9

Average Cost Curves with Miscellaneous Variables

	Indirect Lease	Direct Lease
Intercept	9850	9901
CY	-21.57	-7.36
CY <sup>2</sup>	0.0856	0.0265
Prop	881	930
UR	-11300	-11800
UR <sup>2</sup>	4500	4000
Age	0.7 (0.4)	8.1 (1.2)
Turnover	23.1 (0.3)	55.3 (0.6)
Enrollment- capacity Ratio	10.53 (0.02)	15.0 (0.7)
R <sup>2</sup>	0.7917	0.5931
Min cost size	126CY	139CY
ESSx10 <sup>6</sup> /DF	4.54/110	4.61/65

nor the  $R^2$ s. For example, the size coefficients in the Indirect, and Direct lease center equations with, and without the Miscellaneous Variables, are, respectively, -21.34, -7.85, and -21.29, -7.79. The size square coefficients, listed in the same order are: 0.0846, 0.0283, and 0.0831, 0.0281. The  $R^2$ s, listed in the same order are: 0.7913, 0.5822, and 0.7916, 0.5829.

Therefore, the conclusion is that the Miscellaneous Variables do not affect the day care costs (directly).

#### 4.4 Summary and Conclusions

In the theoretical discussion, first, the factors giving rise to scale economies, or diseconomies have been explored. The case for expecting increasing returns to scale in the staff size because of fluctuations in the staff, and child attendance levels has been demonstrated with a mathematical model. The theoretical discussion, however, has not clarified the nature of the overall economies of scale (the overall size-cost relationship). Second, alternative output measures--child-day, capacity and enrollments have been examined, leading to the selection of the child-day as the day care center output measure. The theory of long run and short run costs has been reviewed, leading to the selection of the output to represent center size. The choice of estimating the cost function as an average cost function, or a total cost function has been discussed, and the average cost function has been chosen to reduce the problems of multicollinearity and heteroscedasticity. The nature of the regression fallacy in the estimation of cost functions resulting from fluctuations in the output, and errors in measuring the output have been discussed. The results of the estimation indicate that the center size, day care mix and center lease are important in explaining the day care cost differences among centers. The cost curve for the Indirect Lease Centers has explained 64% of the variation in costs, while the cost curve for the Direct Lease Centers has explained 24% of the variation in cost. A single equation for all the centers, containing the lease status in the dummy variable form has explained 55% of the total variation (variation in costs over the entire sample of centers).

Although the cost curves of both the lease categories are U-shaped, their slopes are different. The cost curve of the ILC's is less shallow.

The elasticities are -0.25 and -0.13 for the ILC's and DLC's, respectively, at the mean size of all centers (86 CY). In the ILC's, which have a mean size of 71 CY and a range of 20-180 CY, the cost curve reaches a minimum at 135 CY. The day caremix adjusted minimum cost is \$2375 which is 8% less than the observed average cost. The estimated costs increase by 8% when the center size moves away from the minimum cost size by 50 CY. In the DLC's, the cost curve reaches minimum at 166 CY, compared to their mean size of 112 CY, and the size range of 40-180 CY, indicating that the costs decline in almost the entire range of DLC's. The day caremix adjusted minimum cost is \$3000, which is less than the observed average cost by 6%. The estimated costs are higher by 5% at sizes 50 CY above and below the minimum cost size.

A single cost curve in each lease category can explain the size-cost relationship very well. Size differentiated cost curves in the Indirect Lease almost coincide with the single cost curve, whereas in the Direct Lease, the size differentiated curves, reflecting the influence of the highly erratic utilization rates, exhibited local maxima.

There has been no significant change in the cost curve of ILC's when the utilization rate is partialled out of the cost equations. However, the cost curve of the Direct Lease has turned less shallow -- the upward sloping portion of the curve has turned steeper. The minimum cost size has moved to 139 CY. But the costs are more or less the same between 120-160 CY. The percentage of cost variation explained in the ILC's and DLC's have increased to 79% and 58%, respectively. The total variation explained has increased to 74%.

The miscellaneous Variables -- Turnover Rate and Enrollment-Capacity ratio -- have no effect on costs. The results of this chapter would affect the costliness ratios of Chapter 3. The costliness ratios should include, in addition to day caremix, the size and the lease, and perhaps, the utilization rate of a center (to the extent, the utilization rate is outside the influence of the center management).

## CHAPTER 5

ESTIMATION OF PRODUCTION FUNCTION5.1 Introduction

The production function approach is useful for examining the scale effects, optimum input proportions and the productive efficiency: In examining the scale effects, the production function analysis is an alternative to the cost curve analysis of Chapter 4.

Furthermore, from a production function, the input proportions can be analysed to see if the production unit is achieving maximum output for a given cost. If the returns to a factor are found to be increasing, it may be desirable to increase the usage of that factor.

Lastly, the production function can be used for evaluating the productive efficiency of individual production units (the application to the day care centers has been discussed in Chapter 3).

In this chapter, first, the theoretical aspects of the production function are examined, covering the following topics:

- Relationship between production function and scale.
- Relationship between the long run average cost curve and technical economies of scale.
- Specification of the production function for day care centers.
- Framework for examining the appropriateness of the observed input ratios.

Second, the empirical estimation of the production functions are undertaken, involving:

- A single production function (Cobb-Douglas type) for all centers.

- Separate production functions for the two lease categories.
- Size differentiated production functions by lease.
- Production functions with the utilization rate.

## 5.2 Theoretical Issues

### 5.2.1 Production Function and Scale

In a production process of the type  $Q=f(K,L)$ , there are said to be increasing, constant, or decreasing returns to scale, when the ratio  $Z$ , shown in equation (5.1), is greater, equal to, or less than  $\lambda$ , where  $\lambda$  is a constant with a value greater than one.

$$(5.1) \quad Z = \frac{F(\lambda K, \lambda L)}{F(K, L)}$$

This measure of returns to scale is called technical economies of scale, since it depends only on the production function, which is a technological relationship between the inputs and outputs. The technical economies of scale are defined at any point on the production function, but only on a line through the origin (meaning that the factor ratios are held constant). Therefore, if the long-run expansion path is not a straight line through the origin, input variations on this path will not be equi-proportional for all factors, and the technical description of economies of scale will not apply to the full range of outputs along the long-run expansion path. Only, if the long-run expansion path is a line through the origin, will the technical description of economies of scale be meaningful for the potential levels of output of the production unit.

The above discussion is made more clear with the aid of Figure 5.1.  $K$  and  $L$  are inputs,  $I_0$  and  $I_1$  are isoquants, and  $OPQ$  is the long-run expansion path. If we consider scale effects between  $P$  and  $Q$  which lie

FIGURE 5.1

Production Function and Scale

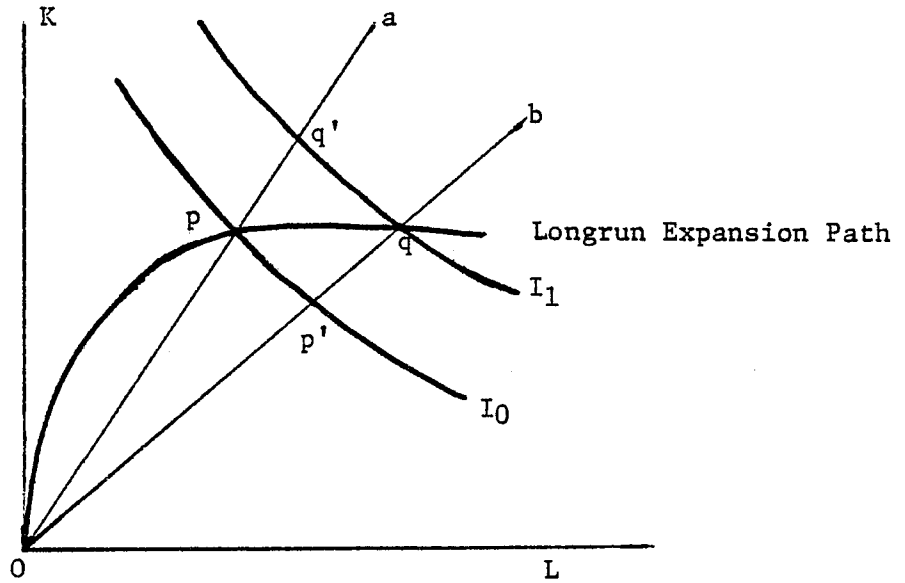
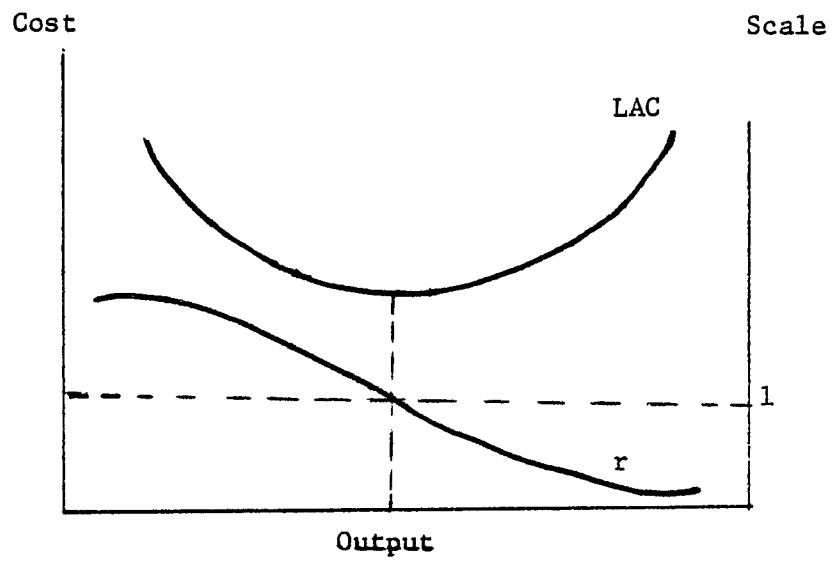


FIGURE 5.2

Average Cost and Scale



on the longrun expansion path, then the technical economies of scale are applicable between P and Q' on Oa, and between P' and Q on ob.

Although the subject of technical economies of scale can be found in any standard economics text book, a particularly clear presentation is made by M. Galatin (6).

### 5.2.2 Average Cost Curve and Technical Economies of Scale

When factor prices remain constant, the average cost curve decreases or increases according as there are increasing or decreasing returns to scale. This can be algebraically shown as (the derivation can be found in standard economic text books):

$$(5.2) \quad e^1 = 1/r - 1$$

where,

r: Scale parameter

e: Elasticity of average cost with respect to output

Then,  $e \begin{matrix} > \\ < \end{matrix} 0$  according as  $r \begin{matrix} \geq \\ < \end{matrix} 1$

This is illustrated in Figure 5.2. The scale parameter has a value greater than one for the downward sloping portion of the longrun cost curve, and less than one for the upward sloping portion of the longrun cost curve.

### 5.2.3 Specification of the Production Function:

The fixed coefficient production function is initially considered:

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The technical economies of scale may differ from the scale parameter of the cost curve, if the cost function is influenced by factors additional to those included in the production function.

$$(5.3) \quad Q = \min \left( \frac{X_1}{a_1}, \frac{X_2}{a_2}, \frac{X_3}{a_3}, \dots, \frac{X_k}{k} \right)$$

where,  $Q$ ,  $X_i$ , and  $a_i$ , are the output, and input coefficients.

This form implies no substitution among the inputs, but it is likely that there is some substitution among day care center inputs. For example, centers located in newer buildings may need smaller quantities of janitorial services and guard services than those in older buildings. Centers with better kitchen equipment may need fewer cooks and helpers. Centers with more safety features may need fewer teacher assistants and helpers. There may also be some substitution between teaching staff and administrative staff, especially at the lower level titles. Hence, the fixed proportion production function is not further considered.

The Cobb-Douglas form may be a good choice because of its input substitutability, and ease of estimation. The basic form employed is:

$$(5.4) \quad Y_i = Z X_{1i}^{a_1} X_{2i}^{a_2} \dots X_{ki}^{a_k} e^{u_i}$$

where,

$Z$  : efficiency parameter

$Y_i$ : Output in child-days in center  $i$

$X_{ji}$ ,  $J = 1, K$  are the inputs used in center  $i$

$a_j$ ,  $j=1, k$  : input coefficients

$u_i$ : error term

The basis form is modified to allow for day care mix differences among centers, and the resulting alternative forms are:

$$(5.5) \quad Y_i = Z X_{1i}^{a_1} X_{2i}^{a_2} \dots X_{ki}^{a_k} e^{p_i + u_i}$$

$$(5.6) \quad W_i = Z \begin{matrix} a_1 & a_2 & & a_k & u_i \\ X_{1i} & X_{2i} & \dots & X_{ki} & e \end{matrix}$$

where,

$P_i$  : day care mix proportion

$W_i$  : output in dollars of center  $i$ , which is the weighted sum of the different day care outputs in a center, the weights being the system-wide average costs ( $C_j$ ) of different kinds of day care (from Chapter 2).

The production function is estimated in the double logarithmic form, and the regression coefficients of the inputs provide estimates of their output elasticities.

The scale parameter  $r$  is the sum of the output elasticities --

$$(5.7) \quad r = \sum_i a_i$$

The standard error of the scale parameter is obtained as follows:

$$(5.8) \quad \sigma_r = \sum_i \text{var } a_i + \sum_i \sum_{j \neq i} \text{cov} (a_i, a_j)$$

A test for the significance of the economies of scale is derived as follows:

The null hypothesis is that there are no scale economies or diseconomies, i.e.

$$H_0: r = 1$$

$$(5.9) \quad \text{The t-ratio} = \frac{(r-1)}{\sigma_r}$$

If the computed t-statistic is less than the critical t-value, the null hypothesis of constant returns to scale is accepted.<sup>2</sup>

Before concluding the section, a point to note is that the Cobb-Douglas function assumes a unitary elasticity of substitution between any pair of inputs, which is a limitation in its application to day care centers. The CES function allows differing elasticities of substitution between input pairs, but it is difficult to estimate.

#### 5.2.4 Optimal Input Proportions

The familiar condition for cost minimization is that the marginal rate of substitution between two inputs is equal to their price ratio. For a production function  $y = f(X_1, X_2)$  the optimality condition is:

$$(5.10) \quad \frac{MP_1}{MP_2} = \frac{P_{x1}}{P_{x2}}$$

Where,  $MP_1$ ,  $MP_2$  are the marginal products of the inputs  $X_1$  and  $X_2$ .  $P_{x1}$ ,  $P_{x2}$  are the prices of the inputs.

For the Cobb-Douglas function, it is generally known:

$$MP_1 = a_1 \frac{y}{X_1}, \quad MP_2 = a_2 \frac{y}{X_2}$$

Substituting for  $MP_1$  and  $MP_2$  in equation (5.10)

$$\frac{a_1 \frac{y}{X_1}}{a_2 \frac{y}{X_2}} = \frac{P_{x1}}{P_{x2}}, \quad \text{or}$$

$$(5.11) \quad \frac{a_1}{a_2} = \frac{X_1 P_{x1}}{X_2 P_{x2}}$$

<sup>2</sup>

In the estimations, only t-values are shown in the parentheses under the scale.

$X_1 P_{x1}$  and  $X_2 P_{x2}$  are the expenditures on inputs  $X_1$  and  $X_2$ .

In the analyses of the next section, the ratios of the output elasticities are termed optimal ratios. The expenditures ratios are termed the observed ratios, and are computed from the mean expenditures of the individual inputs.

### 5.3 Empirical Results

#### 5.3.1 Production Function for All Centers

Table 5.1 presents the estimates of the production function for the two output measures -- the physical output<sup>3</sup> and the dollar-output. The coefficient of the day caremix proportion is large, and highly significant, suggesting that, had the production function (with the physical output) been estimated without the day caremix, the output elasticities would have been biased. The sign of the day caremix, as expected, is negative, implying that the output produced by a given input quantities decreases as the proportion of Pre-school day caremix (the more costly kind) increases -- since the value of a negative number raised to the power  $e$  decreases as the number increases.

The  $\bar{R}^2$  values of the equations are high -- 0.95 and 0.935, the standard errors of the regression are small, and the equations are significant at 0.01% level.

The scale parameter has a value of 1.078 for the production function (CY), with the increasing returns being significant at 10% significance

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The production function with the physical output, and the production function with the dollar output, are referred, respectively, as the production function (CY), and the production function (\$).

TABLE 5.1

Single Production Function for all Centers

	ln CY (Physical Output)	ln W (Dollar Output)
ln A	0.068 (t=1.0)	0.152 (2.3)
ln T	0.593 (10.3)	0.600 (10.0)
ln S	0.267 (3.5)	0.243 (2.9)
ln R	0.061 (4.7)	0.072 (5.5)
ln M	0.069 (2.4)	0.081 (2.7)
Prop	-0.518 (10.7)	-
e <sup>intercept</sup>	0.0009 (13.1)	0.81 (0.4)
R <sup>2</sup>	0.950	0.935
SER	0.099	0.102
F-sig.level	0.01%	0.01%
ESS/DF	1.767/178	1.903/179
Scale (r)	1.078 (1.5)	1.148 (1.8)

level (meaning that the null hypothesis of constant returns to scale is rejected), and a value of 1.148 for the production function (\$), with the increasing returns being significant at 5% level, which indicates that there are slight economies of scale on the average. These scale values are close to the scale of 1.071 implied by the elasticity of the single cost curve for all centers (Chapter 4), 0.666 at the mean size (based on the relationship  $e = 1/r - 1$ ).

The relative order of the output elasticities is the same in both the equations. Rent has the lowest output elasticity while the Teaching Staff has the lowest. All the output elasticities, have approximately the same values between the equations, with the exception of the Administrative Staff output elasticity, whose value is higher by 70% in the dollar output equation.

The  $\bar{R}^2$  values of the two equations are high and are almost the same -- 0.95 and 0.935 and both equations are significant at 0.01%. All the output elasticities, are significant at 1% level, with the exception of the Administrative Staff output elasticity in the physical output production function, which is significant at 30% only.

The observed and the optimal input ratios are shown in column (1), Table 5.2. As mentioned in Section 5.2.4, the observed input ratios are the ratios of the mean expenditures of the individual inputs (shown in Appendix ) and the optimal input ratios are the ratios of the estimated output elasticities. Since all the output elasticities in the dollar-output production function are all highly significant, they have been used in the computation of the optimal ratios. Compared to the optimum ratios, the observed ratios of  $\frac{A}{T}$ ,  $\frac{T}{M}$ , and  $\frac{S}{M}$  are close to the optimum, while those of  $\frac{A}{S}$ ,  $\frac{A}{M}$ , and  $\frac{R}{M}$  are higher, and those of  $\frac{A}{R}$ ,  $\frac{T}{R}$ ,  $\frac{S}{R}$  are

TABLE 5.2

Observed and Optimal Input Ratios

Input Ratio	(1)		(2)		(3)		(4) (with utilization rate)		(5)	
	All centers		Indirect Lease		Direct Lease		Indirect Lease Observed	Direct Lease Optimal	Observed	Optimal
	Observed	Optimal	Observed	Optimal	Observed	Optimal				
A/T	0.29	0.25	0.31	0.17	0.26	0.15	0.23	0.12		
A/S	1.04	0.63	1.08	0.34	0.98	0.48	0.56	0.43		
A/R	0.59	2.11	1.32	0.65	0.32	-	1.23	2.67		
A/M	2.78	1.88	3.11	1.12	2.39	2.62	2.76	1.88		
T/S	3.63	2.47	3.52	1.08	3.79	3.44	2.40	3.98		
T/R	2.05	8.33	4.28	3.84	1.23	-	5.29	25.00		
T/M	9.69	7.41	10.10	6.60	9.20	17.7	11.91	17.40		
S/R	0.56	4.37	1.22	1.03	0.32	-	2.20	6.22		
S/M	2.67	3.0	2.87	3.33	2.43	5.49	4.95	4.37		
R/M	4.73	0.89	2.36	1.72	7.5	-	2.25	0.70		

lower, indicating that too much is being spent on Rent and not enough on the Support Staff and the Miscellaneous Costs.

The adjustments in the factor levels necessary to equalize the observed and the optimal input ratios would depend on the current shares as well as the extent of the divergence between the optimal, and the observed ratios

### 5.3.2 Production Functions by Lease Category

As seen in Chapter 2, there are significant unit cost differences between the two lease categories: In the DLC's the rent cost per child-year is 321% higher, the non-rental cost per child-year is 18% lower, and the total cost per child-year (after adjusting for day care mix differences), is 8% higher. Some or all of the differences are incompatible with the assumption of a single production function for both the categories:

For exposition, let it be assumed that there are only two inputs -- capital, representing the building (the capital cost being the rent), and other inputs (whose cost is referred as non-rental cost); and the capital<sup>4</sup> price is the same for both categories. If both categories had the same production function, the rent/non-rental cost ratio and the unit total cost would be the same in both categories. Since this is not the case with the observed data, the assumption of a single production for all the centers is untenable:

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Capital price is the rent per unit space or per unit capacity.

However as discussed in Chapter 2, the capital price<sup>5</sup> facing the DLC's is higher. The higher capital price in the case of a single production function would have led to the substitution of other inputs for capital in DLC's (ignoring the zero elasticity of substitution case), leading to a higher unit non-rental costs in the DLC's. As this has not happened, according to the observed data, the conclusion is that the production processes in the two categories must be different.

Table 5.3 shows the production functions estimated by lease, for both the physical output and the dollar output measures. The latter measure has yielded better results -- it yielded fewer negative output elasticities, the t-ratios of the coefficients are larger -- although it has the same  $R^2$  value: Hence only its production functions are further discussed below:

The estimation results support the need for separate production functions for the two lease categories. Compared to the coefficients of the single production function (\$), in ILC's the coefficients of the production function are larger, whereas in DLC's, some coefficients are larger (Teaching Staff, and Support Staff, by 30%), and some are smaller (Administrative Staff, by 25% and Miscellaneous costs, by 50%), and the rent coefficient is negative.

The negative rent coefficient suggests that either the Cobb-Douglas form is inappropriate or an important variable is omitted, and this is discussed in Section 5.3. A Chow test confirms that the two production

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Input prices, except capital price, is assumed to be the same for all centers. Capital price is assumed to be the same in each lease category.

TABLE 5.3

Production Functions by Lease

	Indirect Lease		Direct Lease	
	ln CY (Physical Output)	ln W (Dollar Output)	ln CY (Physical Output)	ln W (Dollar Output)
ln A	-0.0121 (t=0.1)	0.1029 (1.0)	0.0652 (0.7)	0.1111 (1.4)
ln T	0.6228 (7.8)	0.6042 (7.4)	0.7545 (6.4)	0.7523 (6.5)
ln S	0.3213 (2.6)	0.3047 (2.4)	0.3348 (3.0)	0.2329 (2.1)
ln R	0.1431 (4.8)	0.1575 (5.2)	-0.0479 (1.1)	-0.0159 (0.4)
ln M	0.0832 (2.1)	0.0915 (2.2)	0.0286 (0.6)	0.0424 (0.9)
Prop	-0.5120 (8.5)	-	-0.5603	-
e <sup>intercept</sup>	0.0012 (8.7)	0.9117 (0.1)	0.0004 (10.4)	0.4623 (1.1)
R <sup>2</sup>	0.9341	0.9160	0.9418	0.9163
SER	0.1028	0.106	0.086	0.09
SER/Dv	2.5%	0.9%	1.8%	0.7%
F-sig level	0.01%	0.01%	0.01%	0.01%
ESS/DF	1.18/112	1.27/113	0.43/59	0.49/60
Scale	1.1704 (0.9)	1.2608 (1.0)	1.07 (0.8)	1.1128 (0.3)

functions are different at 95% confidence level. The reduction in the error sum of squares resulting from the separate functions is about 10%. The  $R^2$ s are high--0.916 for both the ILC's and for the DLC's.

The scale parameter has a value of 1.26 in ILC's compared to 1.110 in DLC's, and this agrees with the results of the cost curve analysis (Chapter 4) -- the cost curve of the ILC's is steeper.

The observed, and optimal input ratios are show in columns (2) and (3) of Table 5.2. In the DLC's, the optimal ratios involving rent are not shown since its output elasticity is negative. In the DLC's, in comparison to the optimum, the observed ratios of  $\frac{A}{T}$  and  $\frac{A}{S}$  are too high,  $\frac{T}{M}$  and  $\frac{S}{M}$  are somewhat low, and  $\frac{T}{S}$  is close to the optimum. This implies that too much is being spent on Administrative Staff and too little on Miscellaneous Costs. If output elasticity for rent is taken to be small, ignoring the negative sign, it will indicate that too much is being spent on rent at the expense of other inputs. In ILC's, in comparison with the optimal ratios, the observed  $A/T$ ,  $A/S$ ,  $A/R$ ,  $T/S$ ,  $T/M$  are higher,  $S/R$  is slightly lower, and  $T/R$  is close, indicating that too much on Administrative Staff, and too little on Support Staff and slightly lower amounts than needed on Miscellaneous Costs, are being spent.

### 5.3.3 Production Function by Size Group and Lease

The implicit assumption behind a single production function for all center sizes is that the scale economies are uniform throughout the size range. However, the U-shaped cost curves in the preceding chapter indicate that the production relationships may not have the same scale in the entire

size region<sup>6</sup> -- the production relationships exhibit increasing returns in the lower sizes and decreasing returns in the higher sizes. Then an estimated single production function for all sizes would underestimate the scale in the lower sizes, while overestimating in the higher sizes.<sup>7</sup> Therefore, it is necessary to estimate size-differentiated production functions.

The centers in each lease category are classified into two size groups -- above and below the mean size, and separate production functions are estimated. If the results provide evidence of size related scale, the analysis may be repeated using several size-groups to obtain a more accurate estimate of the size-scale relationship.<sup>8</sup>

The estimates, shown in Table 5.4, suggest that the scale indeed is different between groups. In ILC's, the scale is 1.33 in the smaller size group, and 0.88 in the larger group, compared with the scale of 1.26 in the single function (preceding section). In DLC's, the scale is 1.23 in the smaller size group, and 0.87 in the larger size group, compared with 1.11 in the single function. The reduction in the error sum of squares resulting from the size-differentiated function is considerable: 20% in ILC's and 30% in DLC's.

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In view of the earlier result (Section 5.3.1) that the technical scale economies are close to the economies of scale implied by the cost curve.

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An examination of the residual plots could reveal this.

8

The sample size of 66 observations in DLC's would not allow more than three groups.

TABLE 5.4

Size-differentiated Production Functions by Lease

(Dollar Output)

	Indirect Lease Centers		Direct Lease Centers	
	small centers (<71 CY)	large centers (≥71 CY)	small centers (<112 CY)	large centers (≥112 CY)
ln A	0.2193 (t=1.2)	0.0986 (0.7)	0.3245 (1.5)	0.0976 (1.2)
ln T	0.6387 (5.6)	0.5484 (4.2)	0.7547 (4.6)	0.3922 (2.7)
ln S	0.2599 (1.3)	0.0093 (0.1)	0.1779 (1.1)	0.3842 (2.6)
ln R	0.1533 (3.3)	0.1472 (3.4)	-0.0633 (1.0)	-0.0355 (0.4)
ln M	0.0600 (1.1)	0.0726 (1.0)	0.0379 (0.7)	0.0656 (1.0)
e <sup>intercept</sup>	3.6256 (1.2)	15.86 (2.2)	0.0921 (1.2)	16.56 (3.0)
R <sup>2</sup>	0.8312	0.7870	0.8619	0.8082
SER	0.096	0.099	0.095	0.065
F-sig level	0.01%	0.01%	0.01%	0.01%
ESS/DF	0.62/67	0.39/40	0.22/25	0.12/29
Scale	1.3312 (1.7)	0.8761 (0.6)	1.2317 (0.90)	0.874 (0.80)

In the Indirect Lease, the output elasticities are more or less consistent between groups -- the values are higher in the smaller size group, in proportion to the scale difference, and the ratios have not changed. But in the DLC's, the values of the output elasticities are erratic between the groups -- the ratios of output elasticities are different. Compared with the smaller size group, the output elasticities of Administrative Staff, and Teaching Staff are 50% smaller, and the output elasticities of Support Staff and Miscellaneous Costs are larger by about 100%, whereas the scale value is 30% smaller.

The rent coefficient is negative in the two groups of DLC's, as in the single function.

#### 5.3.4 Production Function with Utilization Rate

The large variation in the center utilization rates could cause regression fallacy in the estimation of the production function as in the cost function, biasing the regression coefficients (output elasticities in case of the production function), and hence the scale. Since the utilization rate fluctuations are very erratic in DLC's, their production function estimates are likely to be affected. This could be the reason for the negative output elasticities of Rent (in all the DLC production functions) and the erratic output elasticity ratios in the size differentiated production functions of DLC's.

In general, as in the cost curve estimation, the regression fallacy can not be fully avoided, and any purported corrective measure would introduce a new bias. Discussed below are some corrective measures.

1. The capacity of a center actually utilized may be substituted for rent in estimating the production function. For example, if the capacity of a center is 200 slots, and the utilization rate is 80%, then 160 slots

are used in the production function estimation; if the utilization rate is 110%, then 220 slots are used. This would alleviate the problem of invariability of capital when output fluctuates. A major short-coming in this approach is that it is against the theory of short-run: in the short run, the capital is fixed and the variable factors are adjusted (as discussed in Chapter 4). Another problem is that it assumes that the desired utilization rate is 100% for all the centers.

2. By stratifying centers into several groups according to the utilization rates, separate production functions may be estimated for each group. The near homogeneity of utilization rates in each group would reduce the regression fallacy. However, the current sample size, especially in the Direct Lease, precludes its use: Since the utilization rates are in the range 0.80 - 1.10, a minimum of three utilization rate stratifications may be needed to assure homogeneity. When these groups are differentiated by two size classifications, a total of six groups would be needed, but there are only 66 observations in the Direct Lease,

3. The utilization rate may be partialled out by explicitly introducing it in the production function as shown:

$$(5.12) \quad Y_i = Z \times \begin{matrix} a_1 \\ 1_i \end{matrix} \times \begin{matrix} a_2 \\ 2_i \end{matrix} \times \begin{matrix} a_3 \\ 3_i \end{matrix} \times \begin{matrix} a_4 \\ 4_i \end{matrix} \times \begin{matrix} a_5 \\ 5_i \end{matrix} e^{P_i + UR_i + u_i}$$

The specification assumes that the changes in the utilization rate are neutral -- meaning that the factor ratios are unaffected by the changes in the utilization rate. But the factor ratios remain constant only if there are no adjustments in the variable factors when output fluctuates. Despite this limitation, because of the ease of estimation, this approach has been tried.

Table 5.5 shows the estimates with the utilization rate, by lease. The coefficient of utilization rate in each leases is highly significant. Although the table shows estimates for both the physical output and the dollar output, the discussion is confined to the latter only. The output elasticity of rent has now the correct sign -- positive. Compared with the estimates by lease without the utilization rate, the t-ratios of several coefficients have increased. The output elasticity of rent, now, has the correct sign -- positive. The increases in the predictive power of the equations have been remarkable, although not readily apparent from the 3% increases in  $R^2$  values: the reduction in the error sum of squares is 60% in DLC's, and 50% in the ILC's; the reduction in the standard error of regression is 30% in DLC's and 25% in ILC's. Similar improvements have resulted in the estimates for the production function (CY) -- now it has a positive Administrative Staff output elasticity in ILC's, and positive rent output elasticity in DLC's.

The increases in scale are small: less than 2% in ILC's and 3% in DLC's. In ILC's, the output elasticities have not changed, the exception being the Administrative Staff elasticity, which has increased by 50%. The optimal ratios, shown in Column (4) of Table 5.2, indicate that, the earlier (Section 5.3.2) conclusion concerning Support Staff is unchanged, but the inferences concerning the Administrative Staff and the Miscellaneous Staff have changed: substantial increase in the Support Staff is required as suggested earlier, the decrease required in the Administrative Staff is much smaller than that suggested earlier, and the Miscellaneous Staff requires a slight decrease instead of a slight increase. In DLC's, although the rent coefficient has not the correct sign, its value is tiny -- 0.03. The output elasticity ratios,

TABLE 5.5

Production Function with Utilization Rate, by Lease

	Indirect Lease Centers		Direct Lease Centers	
	ln CY (Physical Output)	ln W (Dollar Output)	ln CY (Physical Output)	ln W (Dollar Output)
ln A	0.0474 (0.6)	0.1540 (1.9)	0.0387 (0.7)	0.0842 (1.4)
ln T	0.6923 (11.8)	0.6647 (10.9)	0.8218 (10.8)	0.7826 (10.5)
ln S	0.2963 (3.3)	0.2764 (3.0)	0.2346 (8.2)	0.1964 (2.0)
ln R	0.1147 (5.2)	0.1257 (5.6)	0.0035 (0.1)	0.0316 (1.1)
ln M	0.0494 (1.7)	0.0558 (1.8)	0.0040 (0.1)	0.0449 (0.5)
Prop	-0.5006 (11.4)	-	-0.5245 (9.8)	-
UR	1.06 (9.9)	1.07 (9.6)	0.8782 (9.3)	0.9043 (8.8)
e <sup>intercept</sup>	0.00025 (14.1)	0.2043 (2.9)	0.0002 (18.0)	0.2567 (2.8)
R <sup>2</sup>	0.9650	0.9537	0.9766	0.9639
SER	0.075	0.079	0.054	0.060
F sig level	0.01%	0.01%	0.01%	0.01%
ESS/DF	0.63/111	0.70/112	0.17/58	0.21/59
Scale	1.209 <sup>*</sup> (1.8)	1.277 (1.7)	1.103 (0.8)	1.14 (1.2)

shown in Column (5) of Table 5.2, basically confirms the earlier Section 5.3.2) inferences: a major reduction in rent, a decrease (smaller than suggested earlier) in Administrative Staff, and an increase in Miscellaneous Costs are needed.

Table 5.6 shows the estimate with the utilization rate, by size and lease. Compared to the estimates without the utilization rate (Table 5.4), the reduction in the error sum of squares are large: 50% in each of the size groups of ILC's and 75% in each of the size groups of DLC's. The negative rent coefficients in the DLC groups have turned positive and the coefficients have become less erratic. In both the lease categories, the t-ratios have increased. However, the scale values have not changed.

#### 5.4 Summary and Conclusions

The definition of scale economies in the production function context, and the relationship between the average cost curve and the technical economies of scale have been reviewed. The Cobb-Douglas function has been selected for the day care centers, and the basic form has been modified to allow for day caremix differences.

A single production function for all the centers has been rejected in favor of a separate production function for each lease category, which has been supported by the results of the estimation. The scale values obtained are: 1.26 in ILC's and 1.10 in DLC's, which indicate that there are slight economies of scale in both lease categories, and the economies of scale are somewhat higher in the ILC's.

Size-differentiated cost curves in each lease category has explained the production relations better than the single cost function

TABLE 5.6

Size-differentiated Production Functions with Utilization Rate, by Lease

(Dollar Output)

	Indirect Lease Centers		Direct Lease Centers	
	small centers ( $<71$ CY)	large centers ( $>71$ CY)	small centers ( $<112$ CY)	large centers ( $>112$ CY)
ln A	0.2016 (t=1.5)	0.1134 (1.1)	0.2872 (2.3)	0.0972 (1.9)
ln T	0.6979 (8.4)	0.5524 (5.9)	0.7442 (8.1)	0.5795 (6.1)
ln S	0.3044 (2.3)	0.0359 (0.3)	0.1743 (1.8)	0.1937 (3.8)
ln R	0.1045 (3.0)	0.1119 (2.8)	0.0130 (0.4)	0.0199 (0.6)
ln M	0.0401 (1.0)	0.0238 (0.5)	0.0274 (0.9)	0.0354 (1.0)
UR	0.9936 (7.2)	0.9115 (4.8)	0.90 (6.7)	0.73 (6.6)
$e^{\text{intercept}}$	0.5610 (0.8)	1.7192 (0.5)	0.0341 (2.0)	3.6969 (2.1)
$R^2$	0.9097	0.874	0.96	0.93
SER	0.0714	0.0787	0.0519	0.0412
F-sig level	0.01%	0.01%	0.01%	0.01%
ESS	0.33/66	0.237/39	0.06/24	0.04/28
Scale	1.3485 (2.0)	0.8378 (1.9)	1.2440 (1.0)	0.9257 (1.6)

in each category. The small size group (less than the mean size) in ILC's has a scale of 1.33, compared to 0.88 in the large size group. In DLC's, the scale in the small size group is 1.23 compared to 0.87 in the large size group. These results suggest that the scale is not uniform in the entire size range, the small sizes have increasing returns, and the large sizes have decreasing returns, agreeing with the results of Chapter 4.

In DLC's, the single production function and the size-differentiated production functions have negative rent coefficient, and the ratios of the output elasticities are not consistent between the size groups. These problems could be due to the large fluctuations in output in DLC's. Production functions estimated with the utilization rate have been free of these problems.

There is a substantial divergence between the observed and the optimal input ratios, agreeing with a conclusion of Chapter 3 that the current input ratios are inappropriate (the costliness differences are in greater part due to input efficiency differences). In DLC's too much on Rent, and somewhat higher (than the optimal) amounts on Administrative Staff, and somewhat lower amounts on Miscellaneous Costs are being spent. In the ILC's, too little on the Support Staff, somewhat higher amounts on the Administrative Staff, and the Miscellaneous Costs are being spent.

## CHAPTER 6

SUMMARY

In this study, an economic analysis of the Group Day Care Centers operated by New York City has been conducted. The specific issues dealt with include: How to measure the output of a center? How to compare the costs and outputs of different centers? What factors influence day care costs? What are the scale effects? How do changes in the relative quantities of inputs affect the center output? Since detailed summaries are presented at the end of each chapter, only a brief summary of the results of the analysis is presented below.

Many centers produce two outputs -- Pre-school-age day care and School-age day care. Differences in the day care mix among centers and the lack of market prices for the outputs make it difficult to compare costs among centers.

Information about 185 of the 300 Group Day Care Centers has been used in the study. The unweighted average cost of day care is \$3011 per-child-year, with a coefficient of variation of 14.7%.

Costliness, productivity and input efficiency have been proposed for use in evaluating the center performance. The costliness index would provide the day care management with a summary measure of a center's performance. The indices of productivity and input efficiency could be useful in identifying the cause of a center's high or low costliness value.

Results of the cost function estimation indicate that separate cost functions by center lease are necessary to explain day care cost differences. In both lease categories (Direct and Indirect) the day care mix

and the center size affect the day care costs. Additionally, in the Direct Lease, the capacity utilization rate affects the cost curve shape.

The cost curves of both the lease categories are U-shaped. The cost curve of the Indirect Lease Centers is less shallow, reaches a minimum of 135 CY (compared to the mean center size of 71 CY). The day caremix adjusted minimum cost is \$2475 per child-year, which is 8% less than the observed average cost in the Indirect Lease Centers. The estimated costs increase by about 8% when the center size differs from the minimum cost size by 50 CY. In the Direct Lease Centers, the cost curve reaches a minimum at 139 CY (compared to the mean size of 112 CY). The day caremix adjusted minimum cost is \$3000 per child-year, which is 6% less than the observed average cost. The estimated costs are 5% higher for center sizes differing from the minimum cost size by 50 CY. The cost functions have explained about 79% of the cost variation in the Indirect Lease Centers and 58% of the cost variation in the Direct Lease Centers.

The results of the production function estimation indicate that the current input ratios are not appropriate. In the Direct Lease Centers, too much money is being spent on Rent Costs and slightly more than the optimal amounts are being spent on Administrative Staff. In the Indirect Lease Centers, too little on Support Staff and somewhat more than the optimal amounts on the Administrative Staff and the Miscellaneous Costs are being spent.

## APPENDIX

Mean Input Expenditures

Input	All centers	Indirect Lease centers	Direct Lease centers
Administrative Staff(A)	\$33636	31637	37241
Teaching Staff(T)	117256	102711	143482
Support Staff(S)	32310	29212	37895
Rent(R)	57202	24019	117033
Miscellaneous cost(M)	12103	10165	15597

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