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A study of Federal Reserve discount policy as a point process

Dieringer, Stephanie Louise, Ph.D.

City University of New York, 1988

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A STUDY OF FEDERAL RESERVE DISCOUNT POLICY
AS A POINT PROCESS

by

STEPHANIE L. DIERINGER

A dissertation submitted to the Graduate Faculty in
Economics in partial fulfillment of the requirements for
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1988

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Abstract

A STUDY OF FEDERAL RESERVE DISCOUNT POLICY
AS A POINT PROCESS

by

Stephanie L. Dieringer

Adviser: Professor Salih N. Neftci

This thesis models Federal Reserve discount policy as a stochastic process where realizations consist of a series of point events; the events are occurrences of discount rate changes. Using the timing of discount rate changes we examine two hypotheses regarding the policy motivations behind discount rate changes.

The first hypothesis studied is that the discount rate is an independent tool used for macrostabilization. The second hypothesis is that discount rate changes are technical adjustments instituted in response to conditions outside the monetary authorities control, for instance, changes in other market interest rates.

Our empirical work suggests that the alternative hypothesis is consistent with the history of the discount process. Thus we present a model of Federal Reserve behavior in which discount rate changes are motivated by significant changes in market interest rates.

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To David - Thanks for the tea and sympathy.

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Chapter I

Introduction

Generally the approach taken to examine discount rate policy has been to test the relationships between the discount rate and specific macroeconomic variables. By determining how, and indeed, if the discount rate acts in relation to these variables researchers have attempted to draw conclusions about Federal Reserve discount policy. This thesis differs in that it uses characteristics of the stochastic process of discount rate changes to examine the behavior of the Federal Reserve.

The discount mechanism is employed to achieve a variety of goals. They include:

1. To serve as a tool of monetary policy.
2. To serve as a safety valve.
3. To meet seasonal fluctuations in deposits and loans or seasonal reserve drains.
4. To serve as "lender of last resort" to individual banks or member banks as a whole.
5. To serve as "lender of last resort" to nonmember banks and nonbank financial institutions.
6. To encourage Federal Reserve membership and arrest rising defections from the system.¹

In essence, the Federal Reserve may direct discount policy in one of two ways. Rate changes may be implemented to influence economic activity with the intention of

¹ Carson [1970] p 161.

achieving macroeconomic stabilization (in accordance with the first goal); or to achieve maximum effectiveness of the discount mechanism (in accordance with the remaining goals).²

The discount rate process can be thought of as a series of rate changes occurring randomly over time. That is, it is a family of random variables which evolves over time or a stochastic process. Since different policy motivations have different implications for the nature, or evolution, of the discount rate process we can use the characteristics of the observed process to examine discount rate policy.

We investigate the sample path properties of the timing of discount rate changes. If discount policy is discretionary and plays an independent role in macroeconomic stabilization then a discount rate change would represent a signal of change in current monetary policy and be made in consideration of the state of economic activity or the business cycle.

By definition real output has cyclical frequencies and movements are serially correlated over the course of the business cycle. If the Federal Reserve was following a policy in which they reacted to these movements discount

² It should be noted that in some instances a rate change may not be directed towards one goal exclusively but, in fact, may be aimed at both. Smirlock and Yawitz [1985] make this point and discuss various methods which can be used for classifying such changes.

rate changes would be related in a similar fashion. That is, the timing of rate changes would have a discernible serially correlated pattern.

If, however, discount policy is carried out with regard to the "financial goals" of the discount mechanism, then a discount rate change would represent a reaction to forces that are largely independent of system policy. The rate change would be then made to assure that the discount mechanism functions effectively in line with current policy.

Changes in the discount rate would be merely technical adjustments designed to keep the discount rate in line with other short term market rates of interest. We could interpret the behavior of the Federal Reserve as following a policy of maintaining a rate for some undetermined period of time and then randomly switching policy at a point in time. The timing of such "on and off" behavior would generate a series of adjustments which would behave as a Poisson process.³

The purpose of this thesis is twofold: first, to present techniques for inference in Federal Reserve discount rate policy, and second, to propose a model which is consistent with the observed characteristics of the discount rate process.

Chapter II contains a selective survey of articles

³ Leadbetter et. al. [1983]

which were drawn upon in this thesis. It is divided into two parts. The first part focuses on three studies related to the discount rate and discount rate policy. First we present Froyen [1975] which is an empirical investigation of the determinants of Federal Reserve discount rate policy. His work uses a reaction function to explain Federal Reserve behavior.

The second section focuses on the role of the discount rate as an active tool of monetary policy. In this instance the Federal Reserve uses discount policy in an attempt to stabilize the economy. Their ability to influence the future path of the economy is dependent upon the existence of "announcement effects."

If economic agents perceive rate changes as signals of the future course of monetary policy they may act in response to that anticipated policy change. More specifically, announcement effects are said to exist if discount rate changes affect economic activity by causing a revision of agents' expectations.

Studies on the existence of announcement effects include Thornton [1986], Smirlock and Yawitz [1985], Lombra and Torto [1977], and Waud [1970]. Lombra and Torto is chosen as an illustration of various methodologies which have been used to test for announcement effects.

The third section, which is a discussion of work by Frost and Sargent [1970], deals with the relationship

between the discount rate and market interest rates as it pertains to member bank borrowing behavior. Its inclusion allows us to gain some insight into the conditions under which banks borrow at the discount window and thus, consider how the Federal Reserve may make decisions regarding the implementation of discount rate policy.

The second part of the chapter presents the simplest form of problems of cash-balance management. Baumol [1952] is credited with the first work in this area, therefore we concentrate on his model.

Baumol's model is derived using an inventory-theoretic approach. Since we employ this approach in our work with Federal Reserve discount rate policy it is worthwhile to outline the underlying assumptions and the basic methodology.

Chapter III begins our study of the Federal Reserve's use of discount policy. It addresses our first hypothesis that the discount rate is used as an active tool for macroeconomic stabilization. In this section we make some preliminary remarks about 1) the nature of the data and 2) selecting an appropriate time series which can be used to test our hypothesis. An empirical examination of the timing of discount rate changes is conducted which yields little evidence in support of this hypothesis.

In Chapter IV we use results from the theory of stochastic processes to pursue our alternative hypothesis.

We present a framework for the selection of a data series which is consistent with the nature of the process and can be used to make inferences regarding the implementation of discount rate changes by the Federal Reserve.

The second objective of this thesis, to propose a model of Federal Reserve behavior, is incorporated in Chapter V.

The model assumes that the Federal Reserve seeks to maintain an appropriate relationship between the discount rate and other forms of short-term reserve adjustment. Implicit in this assumption is the notion that the System acts as a cost-minimizer in instituting discount rate adjustments. Such a model produces a series of discount rate changes which is consistent with our earlier empirical results.

Finally, in Chapter VI we summarize our findings and offer some concluding remarks.

Chapter II

Literature Survey

This chapter is concerned with presenting material relevant to this thesis. The first section reviews methodology which has been used to examine discount rate policy and is comprised of three reviews.

Froyen's [1975] paper is an example of how researchers have attempted to study discount rate changes in relation to economic fluctuations. The Federal Reserve's behavior with respect to discount rate changes is examined with a reaction function which relates the discount rate to lagged levels of interest rates, and deviations of macroeconomic goal variables from their target values. While such work makes no attempt to separate the two goals of discount policy, that of financial versus stabilization, it does offer some information on the relationship of the discount rate to macroeconomic variables pertinent to these goals.

The second article presented, Lombra and Torto [1977], focuses on the relationship between the discount rate and the Federal funds rate. By employing various tests to determine the direction of causation between changes in the discount rate and the Federal funds rate the authors attempt to determine if Federal Reserve discount policy leads or lags market behavior. Thus, their work implicitly examines the issue of whether Federal Reserve discount policy is directed towards financial or stabilization

goals.

The third article deals with the costs of reserve adjustment and member bank borrowing. Frost and Sargent [1970] empirically test the relationship between the costs of borrowing funds from alternative sources available to banks and the levels of borrowings to determine which is the primary source outside of the discount window that member banks use.

It is useful for our thesis in that it presents a model of the determinants of member bank borrowing which incorporates the relative costs of borrowing funds. Therefore it permits an examination of how member banks respond to changes in the discount rate as it is manifested in spreads between the discount rate and other sources of short-term reserve adjustment.

In addition, by knowing the conditions under which member banks borrow the study can be used to hypothesize how the Federal Reserve uses discount policy. For instance, if the Federal Reserve was attempting to maintain an appropriate relationship between the discount rate and other forms of short-term reserve adjustment, knowing the primary alternative source of funds outside of the discount window would give the Federal Reserve has information as to which interest rate is most relevant to "follow."

The second section of the chapter outlines a methodology often used in problems of cash-balance

management. Since our model of Federal Reserve behavior adopts this "inventory theoretic" approach we present the main ideas and assumptions underlying the cash-balance management model.

A. Literature Relating to Discount Rate Policy

i.

Froyen [1975] is an examination of the role that financial and stabilization goals play in Federal Reserve discount rate setting policy.

Discount rate policy is assumed to be characterized by the following equation:

$$r_d = a_0 + \sum_{i=1}^n b_i (r_s)_{t-i} + \sum_{i=1}^n c_i (B-B^*)_{t-i} \\ + \sum_{i=1}^n d_i (U-U^*)_{t-i} + \sum_{i=1}^n e_i (P-P^*)_{t-i},$$

where,

r_d - discount rate,

r_s - representative short-term interest rate or index of short-term rates;

if $s = b$ - three month treasury bill rate for new issues,

$s = f$ - federal funds rate,

$s = i$ - simple average of r_b , r_f and the (3-6) month commercial paper rate,

$(B-B^*)$ - deviation of balance of payments from target surplus,

$(U-U^*)$ - deviation of unemployment rate from target rate,

$(P-P^*)$ - deviation of inflation rate from target rate,

a_0, b_i, c_i, d_i, e_i - parameters such that,

$$\sum_{i=1}^n b_i \geq 0, \quad \sum_{i=1}^n c_i \leq 0, \quad \sum_{i=1}^n d_i \leq 0, \quad \sum_{i=1}^n e_i \geq 0.$$

Lagged levels of interest rates enter the function to examine the role of financial goals in discount policy. These goals are best served by maintaining an appropriate relationship between the discount rate and other forms of short-term reserve adjustment. This prevents an incentive for banks to use the discount window as anything other than that for which it was designed - a temporary source of funds for meeting seasonal or unexpected reserve drains.

Macroeconomic goal variables enter the reaction function to examine the role of stabilization goals in discount policy. If the discount mechanism is used as an active tool of monetary policy then movements in the discount rate would be related to deviations of goal variables from desired target levels.

The coefficients of the reaction function are estimated using monthly observations for the period 1955:1-1972:12, and for the two subperiods 1955:1-1968:7 and 1968:8-1972:12. Froyen chooses the starting point of 1955 in consideration of a "reformulation of discount rate policy following a dormant period at the discount window in the 1930s and an unsettled policy response to revived bank borrowing in the post-war period."⁴

⁴ Froyen [1975] p.196.

From the mid 1930s through the mid 1940s the volume of excess reserves was continuously so large that the lending facilities of the Federal Reserve banks were rarely used. In fact, for the entire period 1934 to 1943 member bank borrowing averaged less than one-tenth of one percent of the total member bank reserves.⁵ During World War II The Federal Reserve kept the banks amply supplied with reserves through open market operations, and although member banks resumed the practice of borrowing in the years following World War II it has been suggested that discount rate changes did not assume the importance they once had.⁶

The break at 1968 is made to account for a structural shift thought to have taken place in the late 1960s. July 1968 is chosen since at that time there was an "reappraisal" of the Federal Reserve discount mechanism which suggested several changes in the administration of the discount window. The expected effect of these changes was more frequent changes in the discount rate.⁷

Froyen's results for all equations are presented in Table I. The significance of the coefficients on the (lagged) interest rate variable(s) in all equations indicates a consistent response of the discount rate to movements in short-term market rates. Froyen interprets

⁵ Smith [1963] p.49.

⁶ Simmons [1952] p.18.

⁷ Froyen [1975] p. 196.

Table I

Estimates of a Discount Rate Reaction Function

Time Period	a ₀	r _b	r _f	r _i	(B-B*)	(U-U*)	(P-P*)	Lag
1955:1 - 1972:12	1.31** (.275)	.643** (.066)	---	---	-.088* (.050)	.009 (.078)	.072* (.043)	9
1955:1 - 1968:6	.916** (.284)	.692** (.077)	---	---	-.379* (.166)	.036 (.074)	.096** (.039)	8
1968:7 - 1972:12	3.85** (.405)	---	.142* (.060)	---	-.092** (.022)	-.458** (.128)	.260 (.225)	10
1968:7 - 1972:12	4.28** (.317)	---	---	.198** (.060)	-.090** (.017)	-.584** (.095)	.060 (.152)	10

The table gives the time period for each estimate, the lag length, and regression coefficients with their standard errors in parentheses beneath them. Regression coefficients and standard errors refer to the sum of the distributed lag coefficients. A (*) or (**) above the coefficient denotes statistical significance at the .05 and .01 levels of significance, respectively, using a one-tailed test.

Source: R. Froyen, The Determinants of Federal Reserve Discount Rate Policy

this as evidence of the policymakers efforts to meet financial goals in forming discount policy.

Evidence of the macrostabilization goals is mixed with regard to the response of the discount rate to different target variables. While the balance of payments target is significant in both periods, there is a notable difference in the effect of the inflation rate target and the unemployment target. In the earlier subperiod the inflation target is significant whereas the unemployment target is not, and in the later subperiod the significance of the variables is reversed. Thus, Froyen concludes that while the stabilization goal was a significant influence in discount rate setting policy, the target variables under consideration may have varied at different times.

Looking at the distributed lag coefficients for the first two monthly values of each of the variables permits an examination of the response time of the discount rate with respect to the two goals. These results are summarized in Table II.

It is interesting to note that both for the period as a whole and the two subperiods the only significant coefficients are those of the interest rate variable. Since the other target variables were significant (at some point) in the first set of regressions this suggests that the response to the stabilization variables occurred with a longer lag.

Table II

Current Quarter Distributed Lag Weights

a. 1955:1 - 1972:12			
r_b	(B-B*)	(U-U*)	(P-P*)
$b_1 = .117^{**}$ (.042)	$c_1 = -.039$ (.038)	$d_1 = -.066$ (.053)	$e_1 = .010^*$ (.005)
$b_2 = .143^{**}$ (.028)	$c_2 = -.010$ (.024)	$d_2 = -.047$ (.039)	$e_2 = .011$ (.006)
b. 1955:1 - 1968:6			
r_b	(B-B*)	(U-U*)	(P-P*)
$b_1 = .134^*$ (.062)	$c_1 = -.079$ (.119)	$d_1 = -.046$ (.062)	$e_1 = .007$ (.005)
$b_2 = .127^{**}$ (.045)	$c_2 = .067$ (.093)	$d_2 = -.052$ (.049)	$e_2 = .008$ (.007)
c. 1968:7 - 1972:12			
r_i	(B-B*)	(U-U*)	(P-P*)
$b_1 = .143^*$ (.058)	$c_1 = -.0004$ (.0012)	$d_1 = .025$ (.100)	$e_1 = .023^*$ (.011)
$b_2 = .081^*$ (.038)	$c_2 = -.001$ (.001)	$d_2 = -.049$ (.086)	$e_2 = .013$ (0.15)

Figures in parentheses beneath the regression coefficients are standard errors. A (*) and (**) denote statistical significance at the .05 and .01 levels using a two tailed t test.

Source: R. Froyen, The Determinants of Federal Reserve Discount Rate Policy

Thus, Froyen's results suggest that the discount rate is affected (within the quarter) by movements in short-term market interest rates, and therefore econometric models which assume an exogenous discount rate may be improperly specified.

ii.

Lombra and Torto [1977] focuses on the question of discount rate exogeneity by testing for the existence of "announcement effects" of discount rate changes.

"Announcement effects" refer to the ability of the Federal Reserve to influence economic activity by changing the discount rate. Essentially the argument is that if discount rate changes contain new information regarding the future intentions of the monetary authorities they will influence economic activity by altering the profit expectations of economic agents.

Then the possibility of announcement effects could be ruled out if movements in the discount rate are not exogenous. Therefore Lombra and Torto test the hypothesis that the variance in the discount rate can be explained by the spread between the discount rate and the Federal funds rate, or by the level of member bank borrowing.

Two regressions were run in which the dependent variable was the discount rate. In the first equation the independent variable was the Federal funds rate minus the

discount rate lagged over four periods, and in the second equation the independent variable was the quantity of borrowing at the discount window lagged over six periods.⁸

Their results show that in each case movements in the independent variable explained over 95% of the variance in the discount rate. Thus, they conclude that discount rate movements are endogenous and do not contain any information that is not already embedded in the movements of these financial variables.

To check their findings the authors use a test for unidirectional causality applied by Sims [1972]. The test is based on the idea that if causality runs one way from current and past values of an exogenous variable to a given endogenous variable then in a regression of the endogenous variable on past, current, and future values of the exogenous variable, the future values of the exogenous variable should have zero coefficients.⁹

The application of the test is done in two stages. In the first the change in the Federal funds rate is regressed on past, present, and future changes in the discount rate. If the coefficient on future changes in the discount rate is significantly different from zero the direction of causation is not from the discount rate to the Federal

⁸ Because the quantity of borrowing is not independent of the spread between the rates the authors cannot include both variables as independent variables in the same equation.

⁹ Sims [1972] p. 541.

funds rate. Their test indicates that it is statistically significant.

The second stage is a check of the direction of causality implied by the results of the first regression. In this stage the change in the discount rate is regressed on past, present, and future changes in the Federal funds rate. A nonsignificant future coefficient on the on the Federal funds rate confirms that the direction of causality is from the Federal funds rate to the discount rate.

iii.

Frost and Sargent [1970] focuses on identifying the cost variable which most appropriately reflects the relative costs which affect aggregate borrowing from the Federal Reserve System.

A bank has the ability to obtain funds from different sources. It can borrow from the Federal Reserve, borrow in the Federal funds market or sell Treasury bills. The bank's decision as to the source of funds will depend upon the costs associated with each option. Frost and Sargent argue that the relevant cost is not simply the spread between the market rate and the discount rate. For each method of borrowing there is in addition to the interest expense incurred an adjustment cost. This adjustment cost consists of a lump sum cost and a variable cost. The lump sum cost reflects administrative expenses, i.e. the time

management spends on handling the transaction. The variable cost reflects service charges which vary with the amount of funds borrowed. They assume that the expected length of the holding period is known, therefore the total adjustment cost can be stated in the same dimension as the interest rate. This is referred to as the average adjustment cost.

Then the total costs of borrowing funds are:

1. $RDR_t + \tau_{Di} + \alpha_i$ for the discount window
 where RDR_t is the discount rate in period t , τ_{Di} is the average adjustment cost to the i^{th} bank associated with borrowing from the Federal Reserve, and α_i is a measure of the bank's reluctance to have its banking practices subject to surveillance by the authorities.¹⁰

2. $FFR_t + \tau_{Fi}$ for the federal funds market
 where FFR_t is the Federal funds rate in period t , and τ_{Fi} is the average adjustment cost to the i^{th} bank associated with borrowing in the Federal funds market.

3. $TBR_t + \tau_{Ti}$ for selling Treasury bills
 where TBR_t is the Treasury bill rate in period t , and τ_{Ti} is the average adjustment cost to the i^{th} bank associated with selling bills.

A cost advantage to borrowing from the Federal Reserve will exist if

$$FFR_t - RDR_t + \phi_{Fi} > 0, \quad (\phi_{Fi} = \tau_{Fi} - \tau_{Di} - \alpha_i) \quad \text{and}$$

$$TBR_t - RDR_t + \phi_{Ti} > 0, \quad (\phi_{Ti} = \tau_{Ti} - \tau_{Di} - \alpha_i).$$

¹⁰ In practice this "surveillance cost" is a subjective estimate which varies across banks. However, for simplicity their model assumes it is constant.

The expected level of aggregate borrowing (if selling bills is a cheaper means of generating reserves than is borrowing Federal funds) can be expressed as

$$E(B_t) = g(TBR_t - RDR_t + \phi_{T,max})$$

$$g' > 0 \text{ for } \phi_{T,max} > RDR_t - TBR_t > \phi_{T,min}$$

$$g' = 0 \text{ otherwise}$$

where

E is the mathematical expectation operator and, the parameters $\phi_{T,max}$ and $\phi_{T,min}$ are the maximum, and minimum respectively, adjustment cost differential faced by any bank.¹¹

The regression of borrowing on the cost-advantage variable is characterized by a break where the cost-advantage variable is zero. Above this point, borrowing on the average respond positively to increases in the cost advantage; below it there is no response.¹²

To test the specification of the cost advantage variables Frost and Sargent resort to cross-spectral techniques. The cross spectral results for 1) member bank borrowing and the cost advantage (FFR) and 2) member bank borrowing and the cost advantage (TBR) are given in Table

¹¹ The specification for the instance when borrowing Federal funds is cheaper than selling Treasury bills is similarly defined.

¹² That is, if $TBR_t - RDR_t + \phi_{T,max} < 0$ no bank borrows. As $TBR_t - RDR_t + \phi_{T,max}$ increases above zero, more and more banks borrow. Frost and Sargent [1970] pp. 62-63.

III. Also included in the table are cross spectral results for 3) member bank borrowing and the (simple) spread between the Federal funds rate and the discount rate and 4) member bank borrowing and the (simple) spread between the Treasury bill rate and the discount rate.

The coherence is somewhat analogous to the R^2 of correlation analysis. It is a measure of the proportion of one series' variance at a frequency explained by the component of similar frequency in the other series. Both the average coherence (over 60-, 30-, and 20-month frequencies) and the coherence at 30 months are higher when the cost advantage variable for the Federal funds rate is used. Thus, they conclude that the primary alternative to borrowing from the Federal Reserve is the Federal Funds market.

Table III also shows higher coherences for the cost advantage variables as compared to the simple spreads. This improvement in explanatory power suggests that banks' borrowing behavior may be better modeled by assuming that banks take into account other costs besides simple interest costs.

Table III
Cross-Spectral Results At the 30-Month Frequency

	Average Coherence*	Coherence
New York Reserve City Banks:		
Cost Advantage (FFR)	.7725	.8842
FFR - RDR	.6134	.8056
Cost Advantage (TBR)	.5651	.6739
TBR - RDR	.5620	.6624
Reserve City Banks:		
Cost Advantage (FFR)	.8580	.9258
FFR - RDR	.7566	.8435
Cost Advantage (TBR)	.6634	.7235
TBR - RDR	.6598	.7046
Country Banks:		
Cost Advantage (FFR)	.8797	.9163
FFR - RDR	.8341	.8799
Cost Advantage (TBR)	.7230	.7071
TBR - RDR	.7071	.7943

*Average coherence over 60-, 30-, and 20-month frequencies.

Source: Frost, Peter A. and Thomas J. Sargent, Money-Market Rates, the Discount Rate, and Borrowing from the Federal Reserve.

B. An Optimal Adjustment Model

Baumol's [1952] work on cash-balance management can be considered part of a larger class of optimal adjustment problems. The solution to such problems can be found by applying inventory theory analysis. In that our model of Federal Reserve behavior is derived using a similar approach it is useful to present Baumol's main assumptions and methodology.¹³

The assumptions of Baumol's model are as follows. An individual's income is received in lump sums at regularly spaced intervals. Between intervals expenses occur and are paid gradually at a steady rate. The funds held to meet these expenditures can be kept in one of two forms: 1) interest bearing savings deposits and 2) noninterest bearing cash.

The cost of holding savings deposits is the "brokerage" or transaction fee associated with making a transfer to cash. This fee can be thought of as consisting of direct costs (i.e., the bank service charge) and opportunity costs (i.e., the time spent making an trip to the bank.) The cost of holding cash is the interest forgone.

¹³ Our point of departure is the nature of the adjustment variable. While Baumol's model assumes a deterministic adjustment variable (the cash balance); our model assumes a stochastic adjustment variable (the relationship between the discount rate and short-term market rates of interest).

The optimal cash management policy will require balancing the cost of making frequent transfers against the cost of holding cash. Specifically, if

τ is the transaction cost,

m is the rate (in dollars, per day) at which expenditures are made, and

ν is rate (per dollar per day) of interest paid on savings deposits;

the optimal policy calls for a transfer of

$$M^* = [(2\tau m)/\nu] \text{ dollars every}$$

$$L^* = [(2\tau)/(m\nu)] \text{ days,}$$

implying an average cash balance of

$$\bar{M}^* = ((\tau m)/2\nu).$$

Baumol's model, which incorporates the idea of costs associated with holding idle cash, has frequently served as the foundation for other researchers work. Eppen and Fama's [1968] and Miller and Orr's [1966] articles dealing with a firm's adjustment of cash-balance are predicated upon Baumol's work.

In a broader context while the papers of Barro [1972] and Frost [1971] differ in the area of application, Barro deals with a monopolistic firm's adjustment of price and Frost with a bank's adjustment of excess reserves, both share Baumol's "inventory theoretic" approach.

Chapter III

The Discount Rate As A Tool For Stabilization

Our examination of the Federal Reserve's use of discount policy begins with the hypothesis that the discount rate is used as an active tool for macroeconomic stabilization. This implies that discount changes would occur in relation to changes in the economic time series which characterize the business cycle.

The first step in testing this hypothesis is to determine an appropriate time series which 1) reflects the use of discount policy by the Federal Reserve, and 2) can be used for comparison with a "typical" business cycle series.

To choose an appropriate series for empirical tests we must take into account the following. Changes in the discount rate are discrete increments, typically of one-fourth or one-half percent, which occur (instantaneously) at a moment in time. The discount rate process may then be thought of as consisting of a series of periods where the rate remains constant for some random length of time and then "jumps" to a new level. The use of such a step function on the "left" hand side of a regression equation may end up biasing the resulting tests.¹⁴ We therefore reduce the continuous record of the discount rate process to a set of observable coordinates which reflect the

¹⁴ See, for instance, Neftci [1983] pp. 2-6.

occurrence of a "discrete change."

The stochastic process $\{T_k\}$ is a series of interoccurrence times. It is a collection of random variables which measure the time (in days) between successive discount rate changes.¹⁵

Figure I shows how a typical value of $\{T_k\}$ is determined. On November 16, 1914 the discount rate was 6%. The discount rate was lowered to 5.5% on December 18, 1914. Therefore T_1 , which represents the first occurrence of a discount rate change, is equal to 32. Similarly, the discount rate was changed on December 23, 1914 (to 5%), therefore T_2 takes on the value 5.

Figure II charts the series $\{T_k\}$ from 1914 to 1986. Again, each observation denotes the number of days between (successive) discount rate changes. On the horizontal axis the years referenced are periods in which the administration of the discount window is thought to have differed. Due to the relative inactivity of the discount window in the period prior to 1955, Figure III is presented which covers 1955 - 1986. This gives a (somewhat) clearer picture of $\{T_k\}$ for the latter period.

¹⁵The data is taken from Federal Reserve Bank Discount Rates on Loans to Member Banks under Sections 13 and 13a tables which are published in (consecutive) issues of the Board of Governors' Banking and Monetary Statistics and Annual Statistical Digest. It should also be noted that the rate chosen is that of the Federal Reserve Bank of New York. For a study on the behavior of Federal Reserve district banks' discount rate changes with respect to each other see Brimmer [1972].

Representation of Interoccurrence Times

November 16, 1914 - February 18, 1915

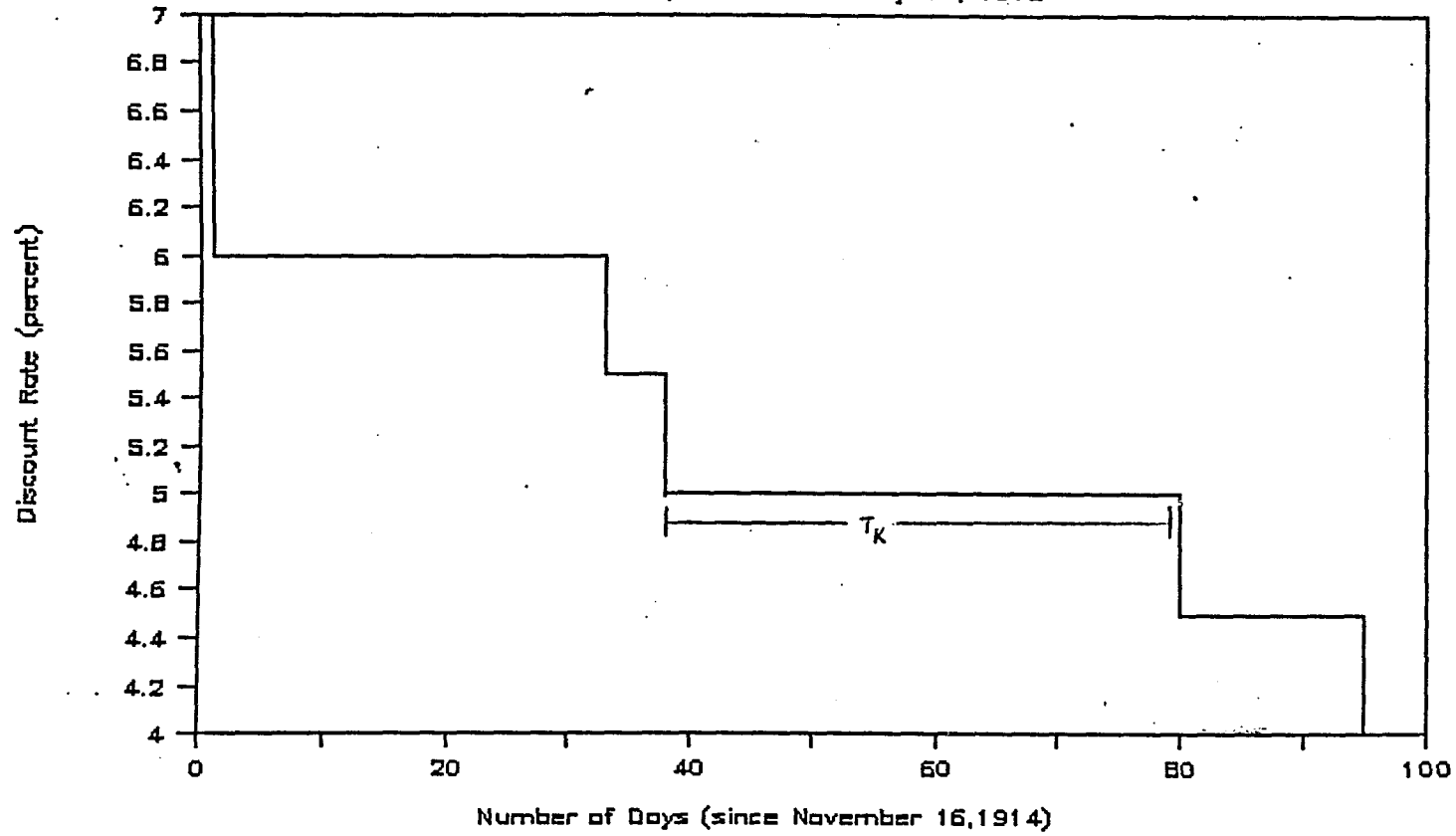
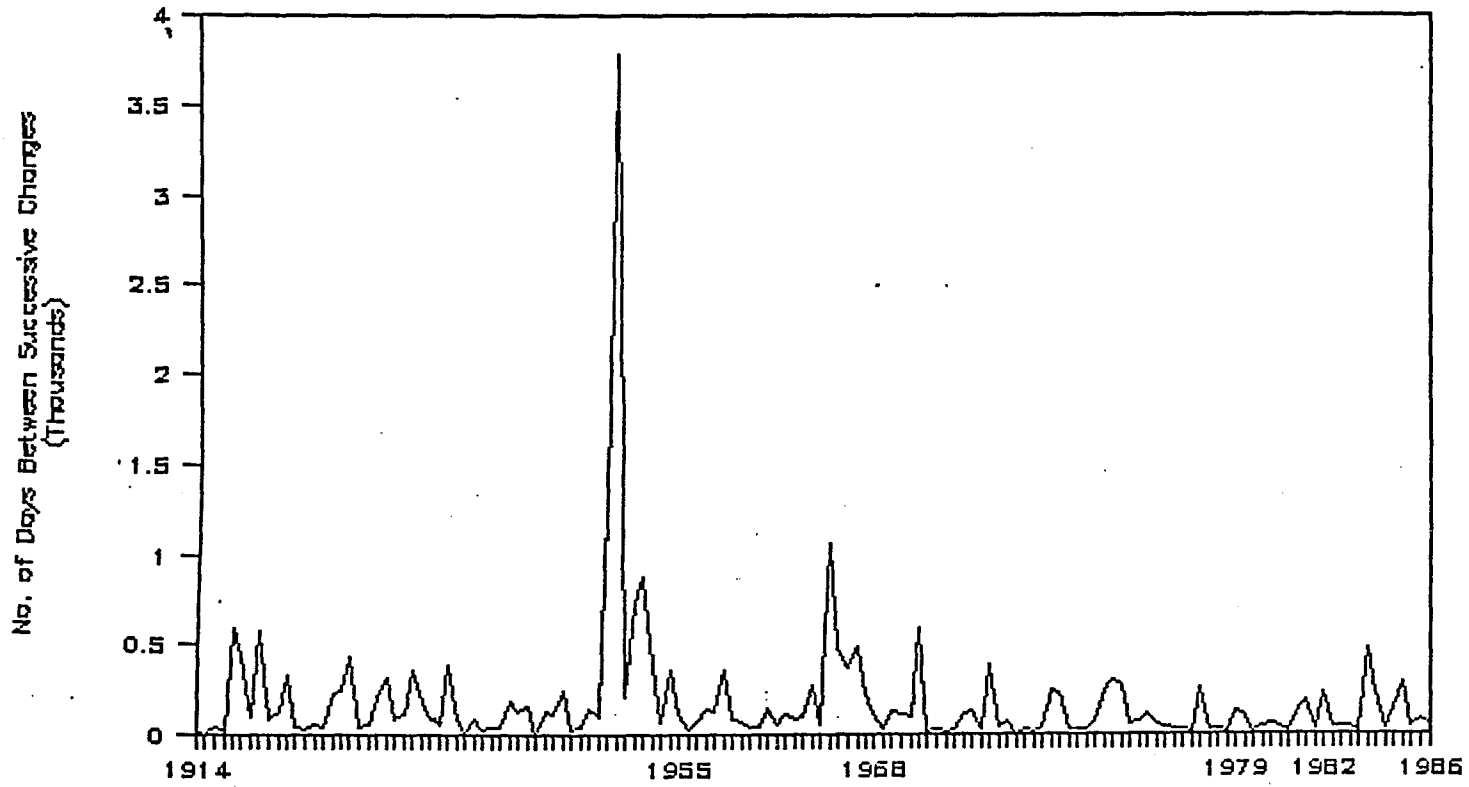


Figure I
Representation of T_K

Interoccurrence Times 1914 - 1986



Values of T_k for 1914 - 1986

Figure II

Interoccurrence Times 1955 - 1986

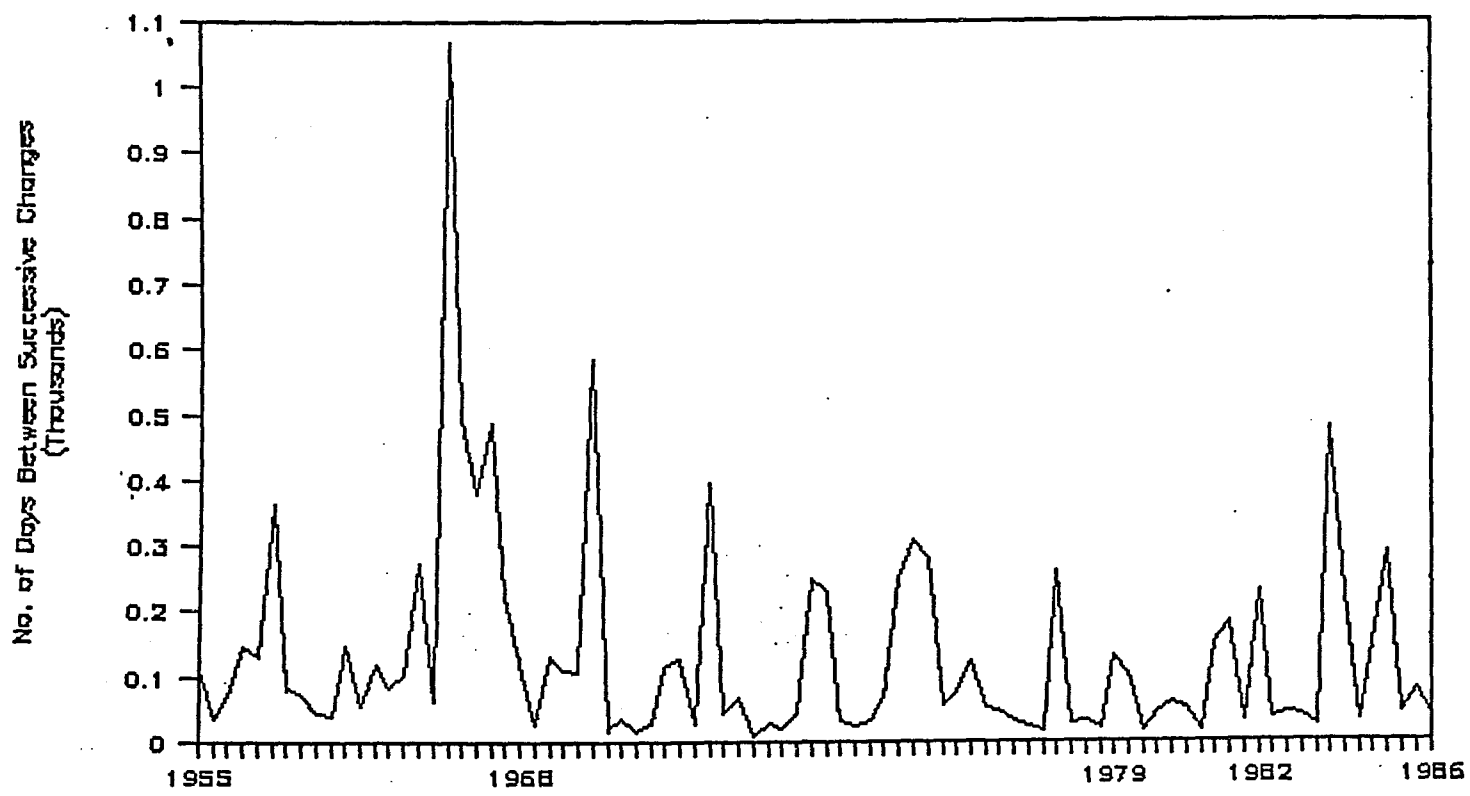


Figure III
Values of TK for 1955 - 1986

$\{T_K\}$ is a time series that is unusual in some sense. As defined, the series has an index which is not a measure of conventional time. Rather, it reflects the occurrence of an particular event. The observation itself is the amount of time between events. Thus, $\{T_K\}$ is a representation of intervals of time over time.

We assume that the series of observations on the time between discount rate changes can be approximated by

$$T_K = \beta_0 + \sum_{s=1}^n \beta_s T_{K-s} + \epsilon_K \quad (1)$$

where ϵ_K is assumed to be a white noise process and satisfies the following

$$E[\epsilon_K] = 0 \text{ for all } K,$$

$$E[\epsilon_K^2] = \sigma^2 \text{ for all } K,$$

$$E[\epsilon_K \epsilon_{K-s}] = 0 \text{ all } K \text{ and all } s \neq 0.$$

The use of such a representation can be justified in the sense that any linearly indeterministic covariance stationary process, obeying certain root conditions, admits an autoregressive representation.¹⁶

Before proceeding, a caveat is in order. If the time series is a non-linear stochastic process then current methods of VARs will fail to capture all of the useful

¹⁶For a definition of root conditions and autoregressive representations see, for instance, Sargent [1979] pp. 262 ff.

information available in the information set observed at time t . Linear time series analysis won't yield forecasts which are optimal in the sense of mean square error. This is clear when one considers the following.

$\{\epsilon_K\}$ is the white noise process which is the building block for the indeterministic part of T_K . It can be interpreted as the sequence of innovations or one-step-ahead linear least square forecasting errors in predicting T_K as a linear function of $\{T_{K-1}, T_{K-2}, \dots\}$. The key here is *linear*. That is, the representation in (1) will, in general, give the best forecasts if the conditional expectations of the random variables are linear. Since T_K is derived from a 0 - 1 type process the expectation (conditional on T_{K-1}) will not be a linear function of the observed data. Therefore (1) may fail to yield the best forecast.

The expression in (1) is estimated for the period 1914-1986 and the four subperiods; 1914-1955, 1955-1986, 1914-1968 and 1968-1986.¹⁷ We initially specify a lag length of 8. Subsequent regressions are run with 4 lags.

If we assume that the monetary authorities are using the discount mechanism as a tool for macroeconomic stabilization then rate changes would be instituted in response to movements in the underlying "real" events. By

¹⁷ We choose these breakpoints in accordance with the previous studies of discount rate policy.

definition real output has cyclical frequencies and undergoes movements which are serially correlated over the course of the business cycle. Then if the Fed was using discount policy as an "active" tool, adjusting the discount rate as a reaction to movements in real output, a series which reflected the timing of rate changes would have a (discernible) serially correlated pattern.

The reasoning is as follows. Real output varies rhythmically over a business cycle. A substantial proportion of the series' overall variance occurs around turning points. If the Federal Reserve was adjusting the discount rate in reaction to changes in real output then we would expect more frequent changes during these periods. (T_k) would be characterized by a sequence of small values. Between turning points we would expect less frequent discount rate adjustments. (T_k) would then be characterized by a sequence of large values. Therefore the series (T_k) would have a serially correlated pattern.

The results shown in Tables IV to VII indicate that in all periods there is no significant coefficient on any lag beyond the first. And, in fact, the coefficient on the first lag is significant in only two of the subperiods, 1914-1986 and 1914-1968.

A series of Chow tests examines whether there were any structural shifts which took place between various time periods. For the subperiods 1914-1955 and 1955-1986 the

null hypothesis that the coefficients of the estimated regression are stable is accepted; for the subperiods 1914-1968 and 1968-1986 it is rejected.¹⁸

The F-statistic for testing the significance of the block of lags in the regression equation is significant only for the period 1914-1986. Thus it appears that the past history of T_K makes no contribution to explaining the occurrence of a discount rate change.

This contention is also supported by the low R^2 s of the regression equations. The R^2 for any subperiod does not exceed 0.12; therefore the proportion of the variance in T_K which can be attributed to past values of T_K is almost negligible.

Tables IV to VII also give the marginal significance levels for the Ljung-Box Q-statistics. The Q-statistic provides a test for general serial correlation of residuals. All equations have significance levels for which the hypothesis of no serial correlation cannot be rejected.

Figures IV - VIII are representations of the (estimated) residuals for various subperiods. Each is generated using an AR(8). That is, each set of residuals is from a regression run with 8 lags. These plots indicate that there are no significant patterns which can

¹⁸ Note that this latter rejection is consistent with Froyen's [1975] contention of a shift in discount rate policy during the late 1960s.

be detected.

The low R^2 s, the insignificance of lagged values of T_K , and the lack of serial correlation of residuals suggest that the series is random. Thus there is little evidence for us to conclude that the timing of discount rate changes is related to the business cycle. The hypothesis that the Federal Reserve uses discount policy to influence business conditions, with the specific intent of stabilizing the economy, must be rejected.

Table IV
Correlation Properties of T_K

$T_K = t_K - t_{K-1}$ where t_K = time period of K'th
discount rate change

$$\log T_K = \sum_{s=1}^n \beta_s \log T_{K-s} + \epsilon_K$$

Variable	Period of Estimation		
	1914-1986	1914-1955	1955-1986
log T_{K-1}	.2532**	.2631	.1870
log T_{K-2}	.0611	.0003	.0916
log T_{K-3}	.0708	.0888	.0351
log T_{K-4}	-.0102	.0980	-.0864
log T_{K-5}	.0148	-.0205	.0402
log T_{K-6}	.0077	.0128	-.0173
log T_{K-7}	.0837	-.0017	.1696
log T_{K-8}	.0192	-.0069	.0730
Constant	2.2460**	2.7446	2.1911*
Typical Standard Error	.0910	.1650	.1220
R^2	.11	.11	.11
D.W.	1.98	1.90	1.97
Q-statistic (significance level)	.86	.98	.99
F-statistic ($\beta_s=0, s=1, \dots, n$)	1.853**	.5571	1.036
F-statistic that $\{\beta_s\}$ are "stable" across two periods = 1.22			

* denotes significance at 5% level

** denotes significance at 1% level

Table V
Correlation Properties of T_K

$T_K = t_K - t_{K-1}$ where $t_K =$ time period of K 'th
discount rate change

$$\log T_K = \sum_{s=1}^n \beta_s \log T_{K-s} + \epsilon_K$$

Variable	Period of Estimation		
	1914-1986	1914-1955	1955-1986
log T_{K-1}	.2492**	.2230	.2131
log T_{K-2}	.0671	-.0143	.1065
log T_{K-3}	.0581	.0580	.0207
log T_{K-4}	-.0434	-.01800	-.0893
Constant	3.0593**	3.7328**	3.2544**
Typical Standard Error	.0870	.1440	.1150
R^2	.08	.07	.06
D.W.	1.91	1.79	1.99
Q-statistic (significance level)	.91	.99	.96
F-statistic ($\beta_s=0, s=1, \dots, n$)	3.037**	.6844	1.491
F-statistic that $\{\beta_s\}$ are "stable" across two periods = 1.66			

* denotes significance at 5% level
** denotes significance at 1% level

Table VI

Correlation Properties of T_K

$T_K = t_K - t_{K-1}$ where t_K = time period of K'th
discount rate change

$$\log T_K = \sum_{s=1}^n \beta_s \log T_{K-s} + \epsilon_K$$

Variable	Period of Estimation	
	1914-1968	1968-1986
$\log T_{K-1}$.2696**	.0730
$\log T_{K-2}$.0482	-.0402
$\log T_{K-3}$.1032	-.0672
$\log T_{K-4}$	-.0002	-.0754
$\log T_{K-5}$	-.0013	-.0760
$\log T_{K-6}$	-.0429	-.0198
$\log T_{K-7}$.0120	.0601
$\log T_{K-8}$	-.0701	.1382
Constant	3.3077**	4.1660*
Typical Standard Error	.1300	.1400
R^2	.12	.07
D.W.	1.97	1.95
Q-statistic (significance level)	.42	.97
F-statistic ($\beta_s=0, s=1, \dots, n$)	.9855	.4236
F-statistic that $\{\beta_s\}$ are "stable" across two periods - 2.70**		

* denotes significance at 5% level

** denotes significance at 1% level

Table VII
Correlation Properties of T_K

$T_K = t_K - t_{K-1}$ where $t_K =$ time period of K 'th
discount rate change

$$\log T_K = \sum_{s=1}^n \beta_s \log T_{K-s} + \epsilon_K$$

Variable	Period of Estimation	
	1914-1968	1968-1986
log T_{K-1}	.2402**	.0797
log T_{K-2}	.0399	-.0389
log T_{K-3}	.0675	-.0675
log T_{K-4}	-.0754	-.1572
Constant	3.5962**	4.9105**
Typical Standard Error	.1190	.1330
R^2	.08	.04
D.W.	1.85	2.02
Q-statistic (significance level)	.52	.96
F-statistic ($\beta_s=0, s=1, \dots, n$)	1.422	.5803
F-statistic that $\{\beta_s\}$ are "stable" across two periods =	3.97**	

* denotes significance at 5% level

** denotes significance at 1% level

Residuals 1914 - 1986
(AR(B))

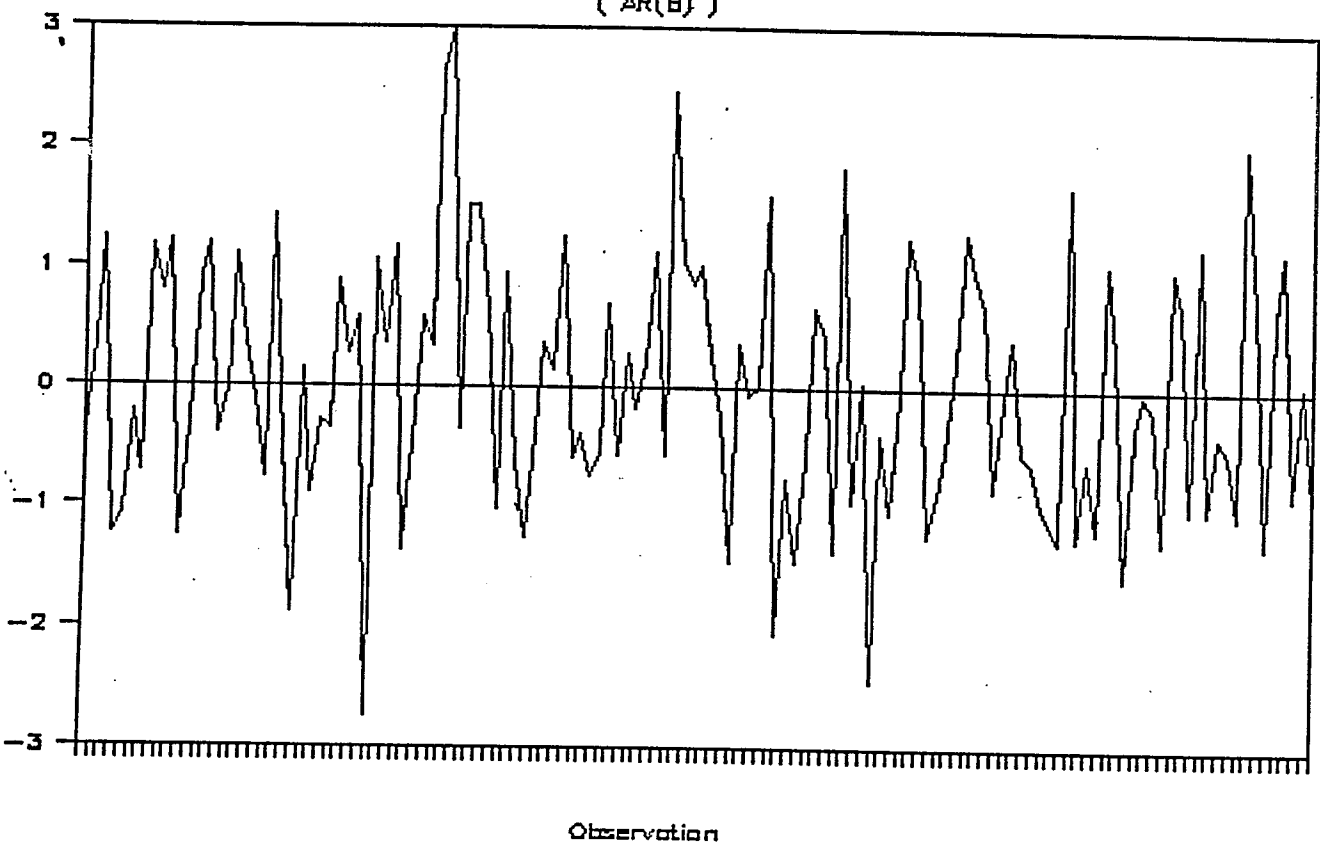


Figure IV
Estimated Residuals 1914 - 1986
 $\log TK - \sum_{s=1}^n \beta_s \log TK-s + \epsilon_K$

Figure V

Estimated Residuals 1914 - 1955

$$\log T_K = \sum_{s=1}^n \beta_s \log T_{K-s} + \epsilon_K$$

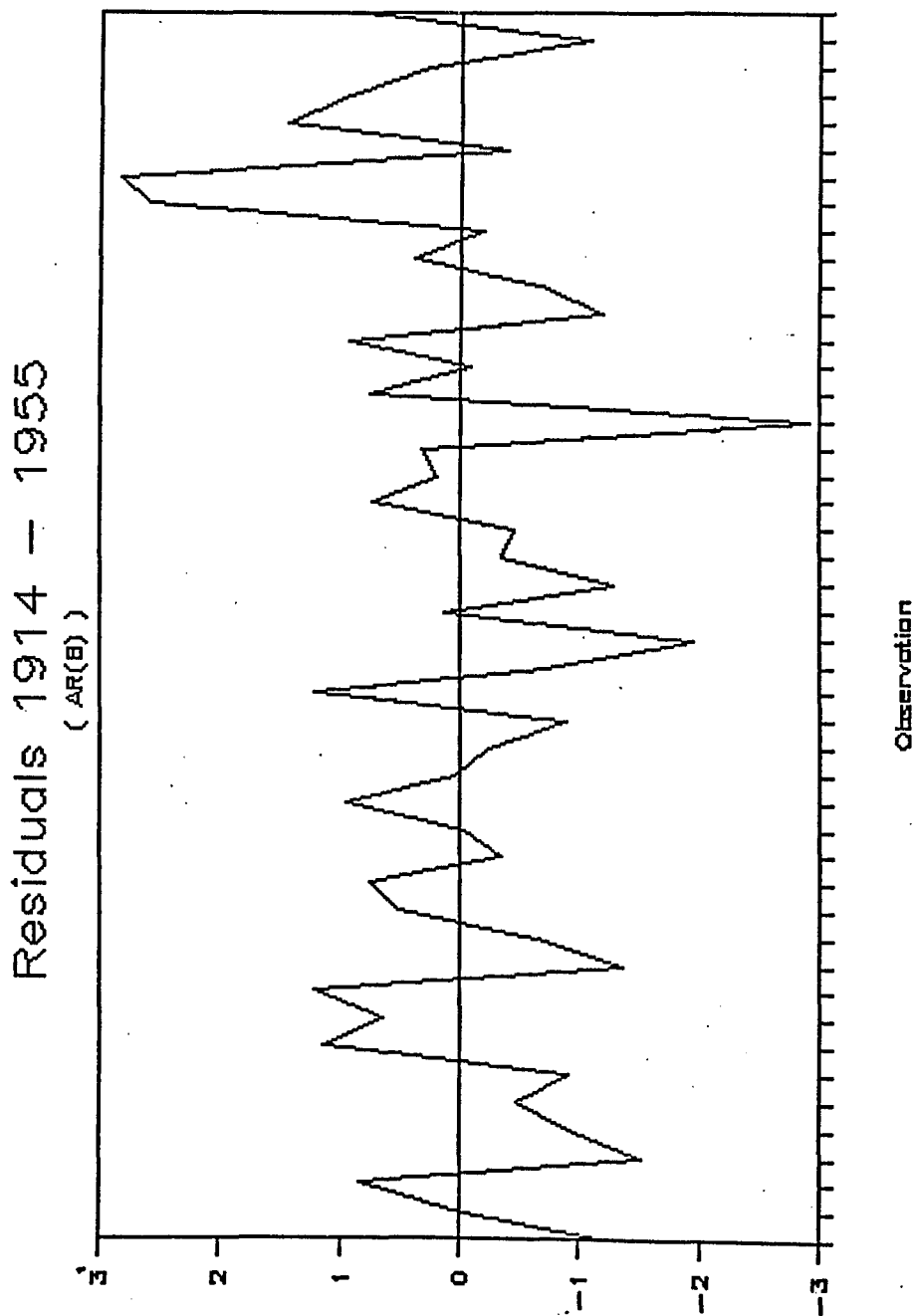
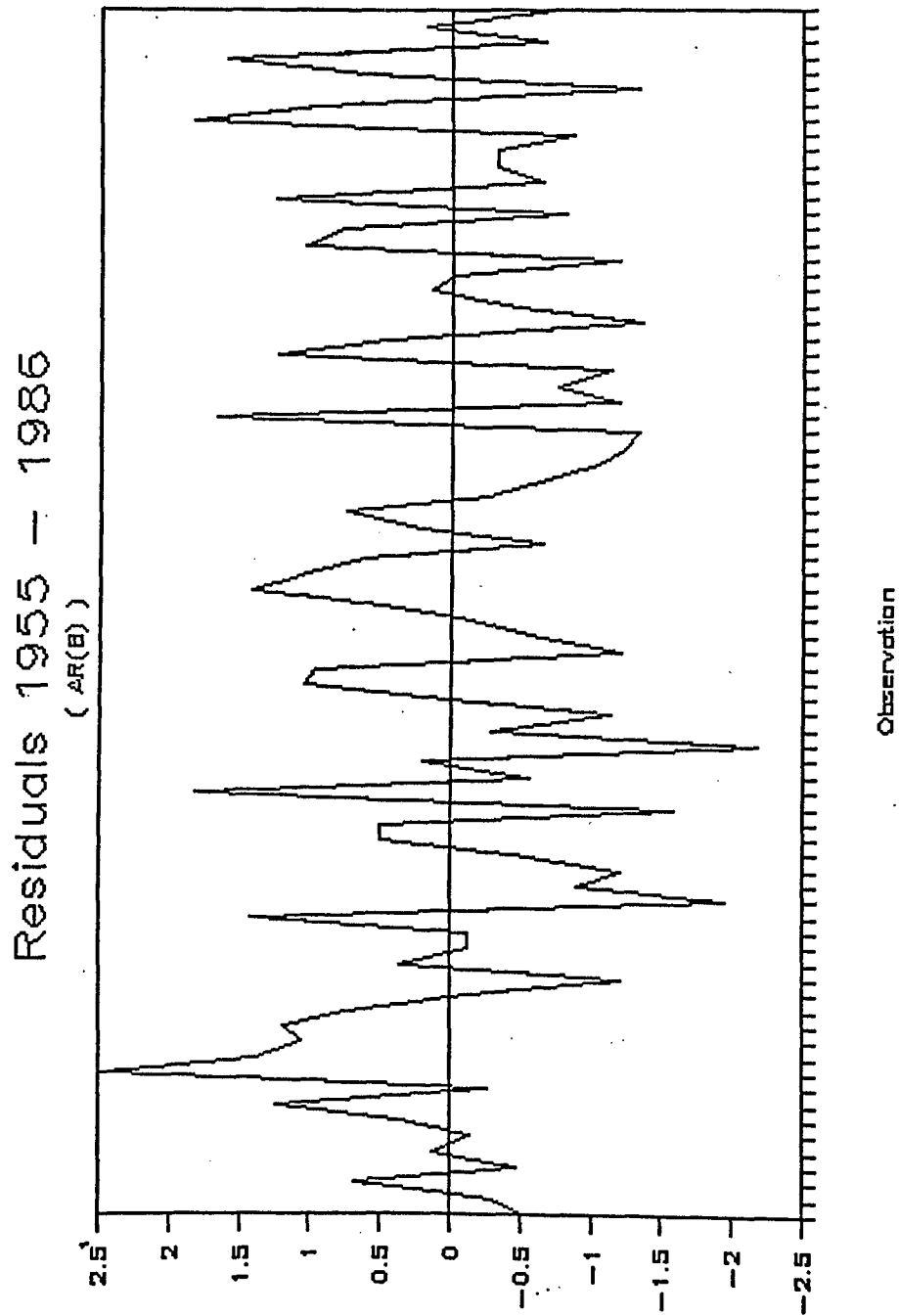


Figure VI

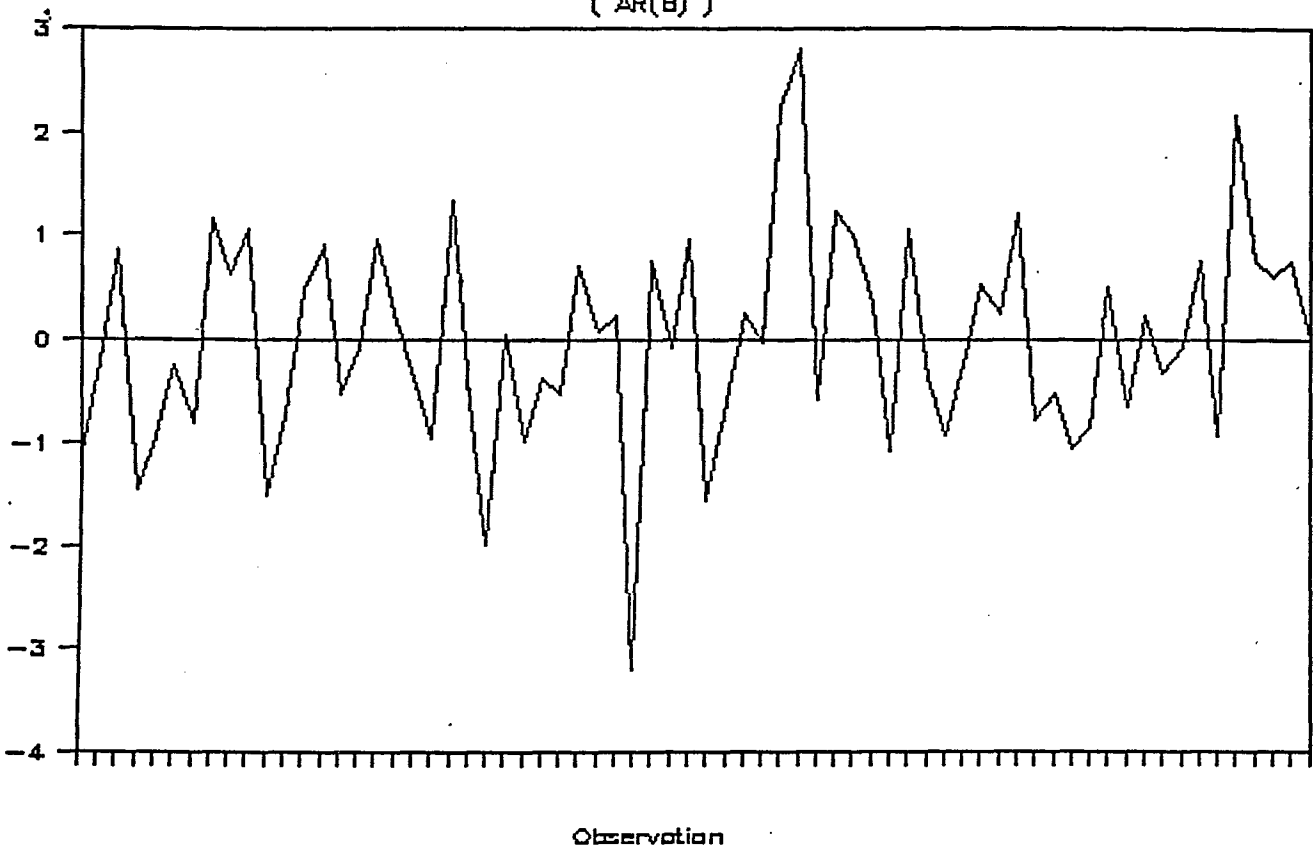
Estimated Residuals 1955 - 1986

$$\log T_K = \sum_{s=1}^n \beta_s \log T_{K-s} + \epsilon_K$$



Residuals 1914 - 1968

(AR(B))



Estimated Residuals 1914 - 1968

$$\log T_k = \sum_{s=1}^n \beta_s \log T_{k-s} + \epsilon_k$$

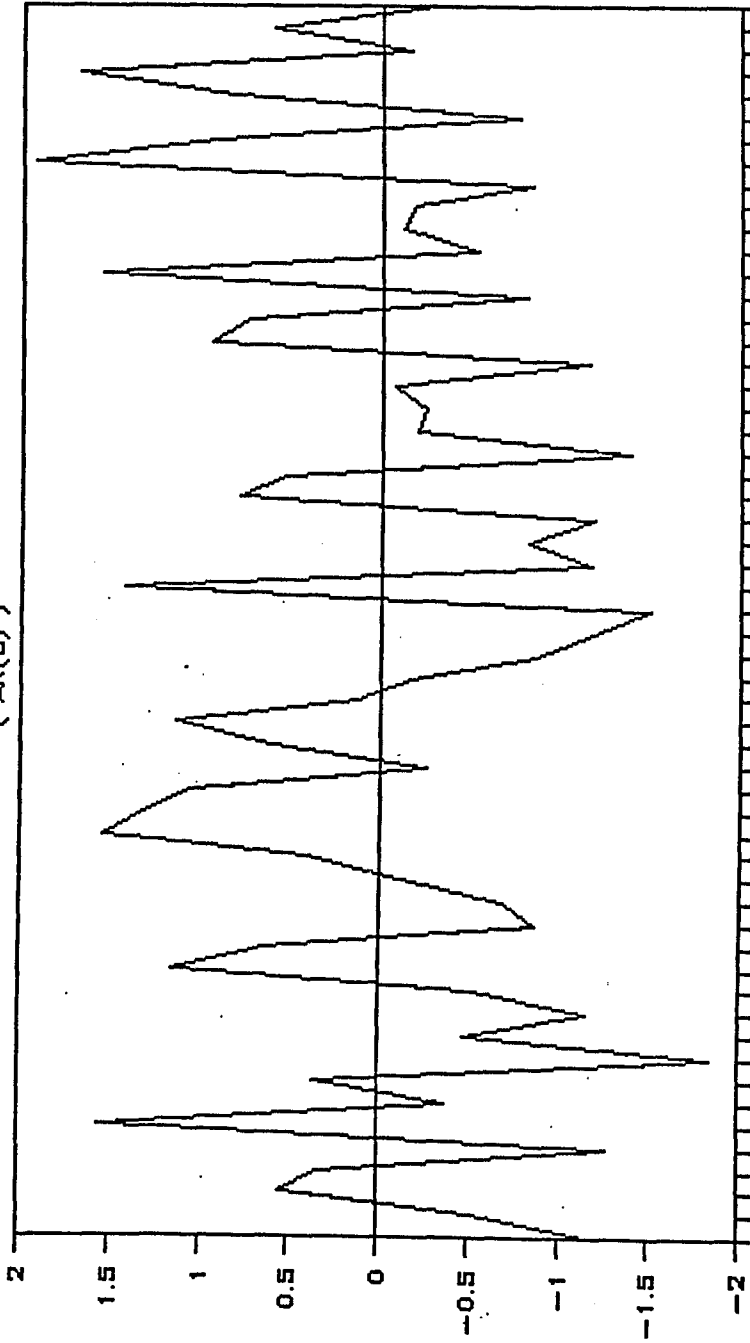
Figure VII

Figure VIII

Estimated Residuals 1968 - 1986

$$\log T_K = \sum_{s=1}^n \beta_s \log T_{K-s} + \epsilon_K$$

Residuals 1968 - 1986
(AR(B))



Observation

Chapter IV

Discount Rate Changes With Respect To Market Rate Changes

Our alternative hypothesis suggests that the Federal Reserve follows a policy designed to keep the discount rate in line with other short term market rates of interest. The theory of efficient markets contends that interest rates are a Martingale process; changes occur randomly and cannot be predicted. Then adjustments in the discount rate would also be random. Furthermore, suppose that the Fed does not respond to every change in the market rate but waits until the change in the market rate is large enough to justify an adjustment. That is, the Fed reacts only when the spread between the discount rate and the market rate exceeds some "critical" level. A policy such as this, in which rate adjustments are based upon "level crossings", would generate a series whose distribution would be Poisson.

A Poisson process may be thought of as a stochastic process where:

- 1) The time of reoccurrence of the event is independent of past occurrences.
- 2) The distribution of the number of events in any interval of time depends only on the length of time for which we observe the process.
- 3) The probability of more than one event occurring in a small (time) interval is negligible in comparison with the probability of one event occurring in that interval.

Then we can pursue our alternative hypothesis by investigating the sample path properties of the series of discount rate changes to determine if the process can be similarly characterized.

We begin by interpreting the sequence of discount rate changes as a stochastic process. This process can then be defined as a counting process.

Let $\{N(t), t \in T\}$ be a stochastic process with index set $T = \{t: t \geq 0\}$ and where the state space is defined to be the number of discount rate changes that have occurred after time 0 but not later than time t for each $t \in T$.

$\{N(t), t \geq 0\}$ constitutes a counting process provided that:

- 1) $N(0) = 0$
- 2) $N(t)$ assumes only nonnegative integer values,
- 3) $s < t$ implies that $N(s) < N(t)$, and
- 4) $N(t) - N(s)$ is the number of events that have occurred after s but not later than t , that is, in the time interval $(s, t]$.¹⁹

$\{N(t), t \geq 0\}$ can then be defined as a Poisson process using the following:

¹⁹ Allen [1978] p 115.

A counting process $\{N(t), t \geq 0\}$ is a Poisson process with parameter λ if and only if the following conditions hold,

- 1) $N(t+h) - N(t)$ is independent of $N(s)$, $s < t$, for $h > 0$,
- 2) $N(t+h) - N(t)$ has the same distribution as $N(h)$ for $h > 0$,
- 3) $P[N(h) = 1] = \lambda h + o(h)$, and
- 4) $P[N(h) > 1] = o(h)$.

(The quantity $o(h)$ is such that $o(h)/h \rightarrow 0$ as $h \rightarrow 0$.)²⁰

This definition yields the following proposition.

Proposition I.

Let $\{N(t), t \geq 0\}$ be a Poisson process with parameter λ .

If the process is observed over the fixed interval $(0, T_0)$ and

$$0 < t_1 < t_2 < t_3 < \dots < t_N$$

denote the epochs at which events occur, then the interoccurrence times

$$T_K = t_K - t_{K-1}, K = 1, 2, \dots, N \text{ (with}$$

$t_0=0$) are independent identically distributed as negative exponential with parameter λ .²¹

What is important for our work is the fact that the

²⁰ Bickel and Kjell [1977] p 466.

²¹ Basawa and Rao [1980] pp. 98-99 and also Allen [1978] pp. 119-120 which contains the proof.

converse of Proposition I is true.²² That is, if the interoccurrence times $\{T_K\}$ are independent identically distributed exponential random variables with parameter λ then $\{N(t), t \geq 0\}$ is a Poisson process. Thus, we can proceed with our inquiry by testing the series $\{T_K\}$ to determine if these observations are independent identically distributed exponential random variables.²³

We first estimate the parameter of the process, λ . Assuming $t_0=0$ the process is continually observed until N_0 events have occurred. These events occur at the epochs $0 < t_1 < t_2 \cdots < t_{N_0}$. If $\{T^K\}$, with $T_K = t_k - t_{k-1}$, $k = 1, 2, \dots, N_0$, are independent identically distributed exponential random variables, each with parameter λ , then the likelihood function is just the joint density function of the realization and is given by

$$L = \lambda^{N_0} \exp(-\lambda \sum_{k=1}^{N_0} T_K). \quad 24$$

The maximum likelihood estimate of λ is given by

$$\hat{\lambda} = \frac{N_0}{T}.$$

²² For a proof the reader is referred to Chung [1974] pp. 200-202.

²³ The reader will note that this is the same series which we used in our previous work.

²⁴ The density function for an exponential random variable with parameter λ is $f(x) = \lambda \exp(-\lambda x)$ for $x > 0$.

Using λ we can apply a goodness-of-fit test to determine if the $\{T_k\}$ are exponentially distributed. We use the Kolmogorov-Smirnov test.

The Kolmogorov-Smirnov is a non-parametric method used to test the hypothesis that a given population is determined by a specific random variable.

To do the test the sample is arranged in ascending order, $x_1 \leq x_2 \leq \dots \leq x_n$. We define a sample or empirical distribution function $S_n(\cdot)$ by $S_n(x) = j/n$, where j is the number of sample values $\leq x$. Then $S_n(\cdot)$ is a step function which is zero for x less than the smallest x_j , has a jump of $1/n$ at each x_j , and is 1 for x greater than or equal to the largest x_j .

If the distribution function from which our sample was drawn is really $F(\cdot)$, then this empirical distribution function S_n should be an approximation to $F(\cdot)$. A comparison can be made by considering the distance of S_n from the hypothesized distribution function $F(\cdot)$. The Kolmogorov-Smirnov test uses the statistic

$$D = \max_x | F(x) - S_n(x) |.$$

to make that comparison.

Since our hypothesized distribution is continuous it is necessary to compute two differences to find the maximum absolute deviation between $F(x)$ and $S_n(x)$. That is, the test statistic becomes

$$D = \max_j D_j$$

where

$$D_j = \max(| F(x_j) - S_n(x_j) |, | F(x_j) - S_n(x_{j-1}) |)$$

and $F(\cdot)$ is assumed to be exponential with parameter λ .

If the value of D exceeds the critical value then there is enough evidence to reject the hypothesis that the observations are from an exponential population.

The critical values are taken from Lilliefors [1969]. This work recognizes that since the parameter of the process is not specified, but rather estimated from the data, the standard tables for the Kolmogorov-Smirnov test are not valid. However he notes that in David and Johnson [1948]

"...it is shown that if the parameters estimated are parameters of scale and location, and the estimators satisfy certain general conditions, then when one applies the probability integral transformation, the joint distribution of the transformed variables will not depend on the true parameter values. The distribution will, in general, depend upon the functional form of the distribution of the original variables." ²⁵

And, thus a table for use with the Kolmogorov-Smirnov statistic for that particular distribution can be constructed.²⁶

²⁵ Lilliefors [1969] p. 387.

²⁶ Using the standard tables (as opposed to Lilliefors' table) would give extremely conservative results in that the actual significance level would be much lower. The critical value from his table for a .05 significance level is approximately equivalent to the

For the period 1914-1986 the D statistic is calculated to be .1499. The critical value (at a 1% level of significance) is .1056. Therefore we must reject the hypothesis that over the entire history of the process the interoccurrence times are exponentially distributed.

It is important to note that Kolmogorov-Smirnov test statistic was calculated using the (hypothesized) exponential distribution function $F(\cdot)$ where the parameter λ was estimated using the full sample. It may be that the rejection of the null hypothesis can be attributed to a misspecification of the parameter of the process and not of the distribution. This point demands clarification.

The parameter λ of the exponential distribution can be interpreted as the rate of occurrence of events. In our case, the rate of occurrence of discount rate changes. The estimation of λ for the Kolmogorov-Smirnov test was done using the full sample; therefore, it implicitly assumed that the rate was constant over the entire process. However, studies have indicated that the frequency of discount rate changes has varied over time.

Initially we specified two breakpoints, 1955 and 1968, in which the administration of discount rate policy was said to have differed. At this time we add a breakpoint at October 1979. This is chosen in view of the inception of

critical value for a .20 significance level in a standard table.

new operating procedures by the Federal Reserve. Instead of pursuing money growth targets through an adjustable federal funds rate peg the Fed switched to a nonborrowed reserve operating procedure.²⁷ Essentially the FOMC specified that the growth in nonborrowed reserves was to be controlled in a manner consistent with the target money growth subject to a federal funds rate tolerance range.²⁸

Table VIII provides the estimated frequency of discount rate changes for the different subperiods. The table shows that λ has varied considerably across sample periods, and in fact, appears to have increased over time with the most noticeable change taking place after 1968.²⁹

The apparent difference in the rates of occurrence suggests that one way the (entire) discount rate process may be better identified is as a collection of (several) separate regimes, each with its own rate of occurrence.

For this reason a series of Kolmogorov-Smirnov tests is run which investigates the possibility of separate

²⁷ While we acknowledge that there was a further change in these operating procedures in October 1982 when the Fed switched from a nonborrowed reserve to a borrowed reserve operating procedure, we do not specify a breakpoint here since the observations are limited (there were only eight discount rate changes during this period.)

²⁸ Poole [1982] p. 576.

²⁹ Froyen [1975] makes this point noting that "there were almost three times as many discount rate changes in the five years following the (Federal Reserve's) reappraisal (of the discount mechanism in 1968) as in the five years preceding it.", p.196.

Table VIII

Estimated Frequencies of Discount Rate Changes
For Various Subperiods

Sample Period	Number of Observations	Average Number of Days Between Discount Rate Changes	Frequency of Occurrence $\hat{\lambda}$
1914 - 1986	140	189	.0053
1914 - 1955	54	269	.0037
1955 - 1968	22	209	.0048
1968 - 1979	41	105	.0095
1979 - 1982	13	85	.0118
1982 - 1986	10	140	.0071
1955 - 1986	86	133	.0075
1968 - 1986	64	106	.0094
1979 - 1986	23	109	.0092

regimes each characterized by exponentially distributed interoccurrence times. Table IX gives the D statistics for all sample periods. Again, critical values are taken from the table due to Lilliefors [1969].

The hypothesis that the distribution is exponential can be rejected for any subperiod which begins with 1914. That is, in addition to the full sample period, the subperiod 1914-1955.

Figure IX shows the empirical distribution function for the period 1914-1955 in relation to the hypothesized exponential distribution function. $F(x)$, the hypothesized distribution function, is the step function "cutting" across the figure and $S_n(x)$, the empirical distribution function, is represented by the second line. That the hypothesis is rejected for this subperiod can be seen by noting the position $S_n(x)$ occupies in relation to $F(x)$.

Thus, it appears that our model is inappropriate when the period consisting of the years 1914-1955 is included.

There is however, not evidence to reject the hypothesis of an exponential distribution for the years 1955-1986. This can be seen by examining the position of $F(x)$ relative to $S_n(x)$ calculated using this subperiod. Figure X gives a representation of the two functions. Clearly, the difference between the two curves is not as pronounced as in the case of 1914-1955. Therefore we may assume that over this period the process of discount rate

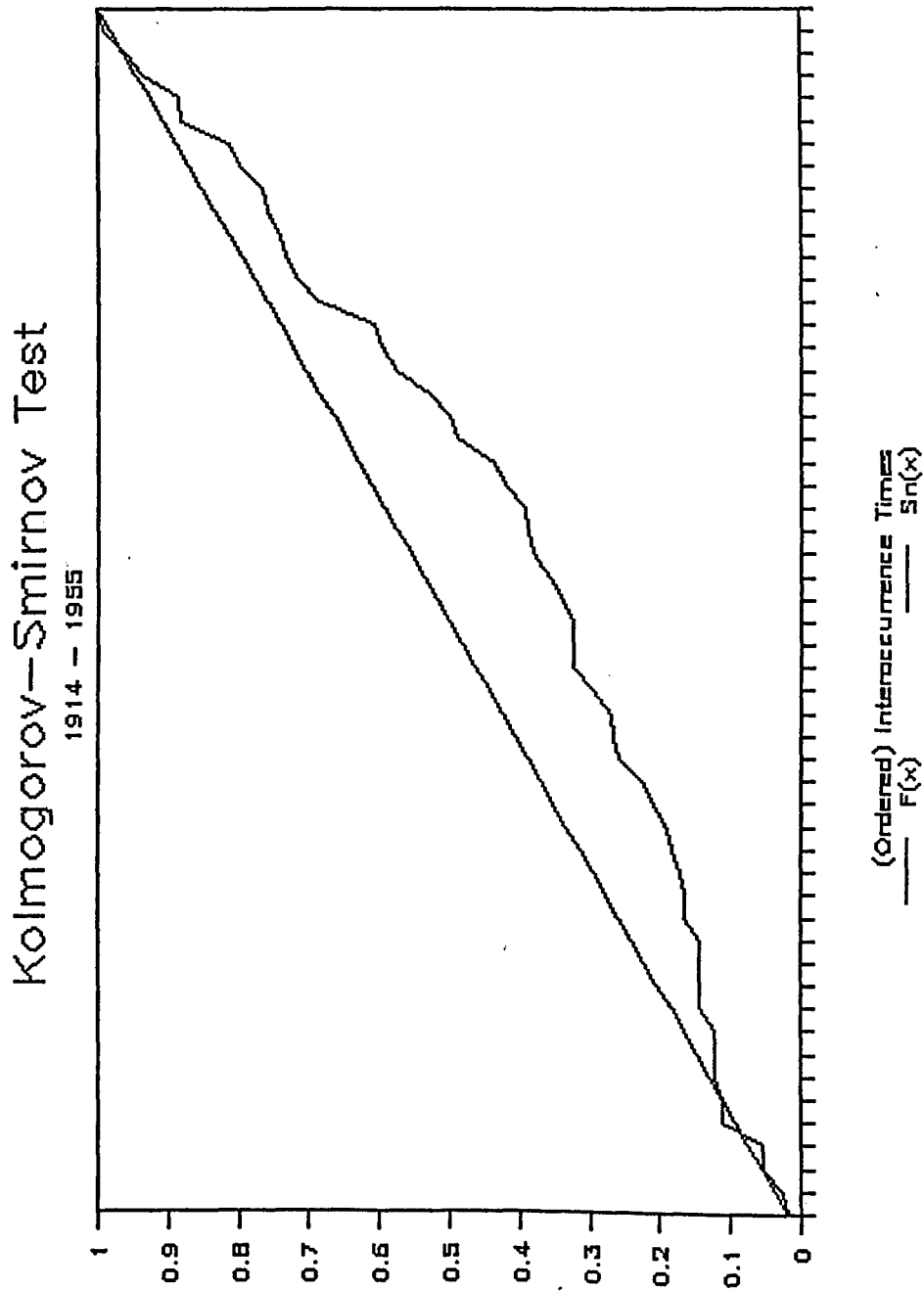
Table IX

Kolmogorov-Smirnov Test Statistics

Sample Period	D statistic	Critical Value	
		$\alpha = .05$	$\alpha = .01$
1914 - 1986	.1499	.0896	.1056
1914 - 1955	.1980	.1442	.1701
1955 - 1968	.1770	.2100	.2470
1968 - 1979	.1556	.1655	.1952
1979 - 1982	.1130	.2870	.3380
1982 - 1986	.2260	.3250	.3800
1955 - 1986	.1134	.1143	.1348
1968 - 1986	.1390	.1325	.1563
1979 - 1986	.1470	.2210	.2606

Critical values are for use with the Kolmogorov-Smirnov statistic when testing that a set of observations is from an exponential distribution where the population mean is not specified but estimated from the sample.

Figure IX
Empirical Versus Hypothesized Distribution Function
1914 - 1955



Kolmogorov-Smirnov Test

1955 - 1986

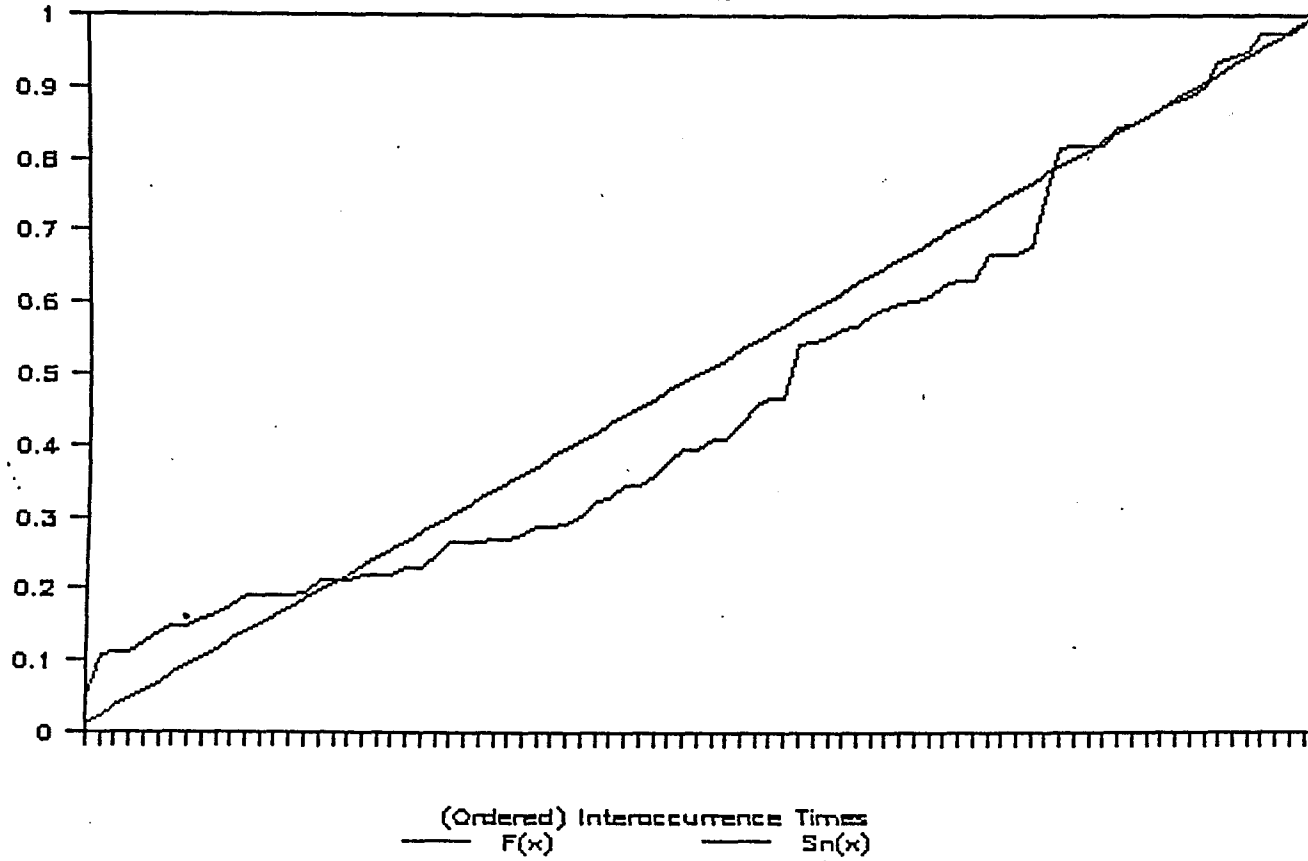


Figure X
Empirical Versus Hypothesized Distribution Function
1955 - 1986

changes is Poisson.

The fact that we must reject the hypothesis of a Poisson distribution for the subperiod 1914-1955 and not for the subperiod 1955-1986 indicates that a decisive shift in the use of the discount mechanism may have occurred after 1955.

While we can consider the subperiod 1955-1986 as a period in which the Federal Reserve implemented discount rate changes randomly in response, perhaps, to movements in interest rates, a further examination yields some interesting points.

There was not enough evidence to reject the null hypothesis for the subperiod 1955-1986 however, the D-statistic is very close to the critical value. In contrast, the difference for each of the subperiods 1955-1968, 1968-1979, 1979-1982, 1982-1986 is relatively large. The stronger evidence for (four) individual exponential distributions within the subperiod 1955-1986 suggests that a better model for the discount rate process may be one in which the timing of discount rate changes is a function of the subperiod or regime in which it takes place. That is, while the time of reoccurrence of a discount rate change may be independent of past occurrences it may be dependent upon the moment in time.

This point can be addressed if we consider the second condition of the definition of the Poisson process, the

property of identical distributions or stationarity of increments. The process is said to have stationary increments if the distribution of the number of events in any interval depends only on the length of the time for which we observe the process and not on the moment in time. We can however, model a more generalized process by dropping this assumption.

Specifically, if we replace the assumption of stationarity with

$$\lim_{h \rightarrow 0} \frac{1 - \text{Prob}[N(t+h) - N(t)]}{h} = \lambda(t)$$

the resulting process will have a rate of occurrence of events which is a function of time.

This process is called a non-homogeneous Poisson process; $\lambda(t)$ is called the intensity function of the process. The Poisson process we initially assumed, one which had stationary increments, is called a homogenous Poisson process. It is characterized by an intensity function of the form $\lambda(t) = \lambda$, or a constant rate of occurrence.

The non-homogeneous or time-dependent Poisson process is characterized by interoccurrence intervals, $\{T_k\}$, which are independent random variables with densities

$$f_{T_K}(t_K) = \lambda(t_K) \exp \left[- \int_{t_{k-1}}^{t_k} \lambda(s) ds \right]$$

where, once again, t_K represents the time of the rate change. The correct specification for the functional form of $\lambda(t)$ would allow us to make the transition from (possibly four) separate homogeneous Poisson processes to one non-homogeneous Poisson process.

We choose an exponential form for the intensity function or

$$\lambda(t) = \eta e^{-\theta t}$$

with

$$\eta > 0,$$

and

$$\theta \geq 0.$$

In choosing this functional form we have assumed a (simple) trend in the rate of occurrence of events. More importantly, this specification affords a check on the stationarity of the process. A test of the hypothesis $H:\theta = 0$ is equivalent to testing that the Poisson process is homogeneous.

Suppose we observe $\{N(t)\}$ continuously over a fixed interval $(0, \tau_0)$ and n_0 denotes the number of events occurring by time τ_0 . The Neyman-Pearson Lemma for testing $H:\theta = 0$ against $K:\theta > 0$ gives the (best) critical region

$\sum_{i=1}^{n_0} t_i \geq \text{constant}$. Under H , $\sum_{i=1}^{n_0} t_i$ is the sum of independent

uniform $[0, \tau_0]$ random variables, and hence the statistic

$$Z = \frac{\sum_{i=1}^{n_0} t_i - \frac{1}{2} n_0 \tau_0}{\tau_0 \sqrt{(n_0 / 12)}}$$

is approximately standard normal for large n_0 .³⁰

We calculate the relevant Z statistic for the time period 1914-1986. Its value of 2.1 leads to a rejection of the hypothesis that the process observed over the entire history is homogeneous.

To calculate the parameters of the intensity function we follow the suggestion of Basawa and Rao [1970] and regard the parameter η as a nuisance parameter. This allows us to find an estimate for θ .

In order to calculate θ we again observe $N(t)$ continuously over the fixed interval $(0, \tau_0)$ and let n represent the number of discount rate changes that have occurred in that interval. We consider the conditional likelihood function given the number of observations is equal to n_0 . The likelihood function of a random sample from a truncated (at τ_0) negative exponential distribution

³⁰ Basawa and Rao [1980] p. 104.

would then be

$$L^* = \frac{\theta^{n_0} e^{-\theta \sum_{r=1}^{n_0} t_r}}{(1 - e^{-\theta \tau_0})^{n_0}} \quad \text{for } \theta > 0$$

$$1 / \tau_0^{n_0} \quad \text{for } \theta = 0.$$

The likelihood equation for estimating θ is given by

$$\frac{1 - \tau_0 e^{-\theta \tau_0}}{\theta (1 - e^{-\theta \tau_0})^{n_0}} = \frac{1}{n_0} \sum_{r=1}^{n_0} t_r$$

In accordance with this equation θ is estimated to be (approximately) -0.0000694. The presence of a negative parameter means that the intensity function is increasing over time.

In terms of our process an increasing intensity function implies that implies a shorter time between discount rate changes. This is consistent with the studies which have maintained that beginning with 1955 the discount rate was adjusted more frequently.

Chapter V

A Model of Federal Reserve Behavior

In this chapter we present a model of Federal Reserve discount rate policy which is consistent with the process being Poisson.³¹

It is often suggested that in order to keep the effect of its monetary policy unchanged the Federal Reserve adjusts the discount rate in response to movements in the market interest rate. We assume that:

- 1) there is a "target" spread between the market interest rate and the discount rate which the monetary authorities seek to maintain, and
- 2) there is a cost associated with not maintaining that spread.

We shall refer to this cost as an "out-of-equilibrium" cost and define it as $(S_t - S^*)$.

If the discount rate could be adjusted instantaneously and at zero cost then it is clear that the Federal Reserve would never incur out-of-equilibrium costs. As market rates varied the policymakers would continually adjust the discount rate so that $S_t = S^*$. However, if some cost is incurred in adjusting the discount rate, then these should be weighed against the cost of being out of equilibrium.

³¹ For simplicity we assume a stationary, or homogeneous, Poisson process. The "problem" of non-homogeneity can be dealt with by a suitable transformation of the time scale to remove time dependencies.

Discount rate adjustments typically involve administrative costs. These costs reflect labor or management expense, i.e. the time that the Federal Reserve must spend in meeting and implementing discount rate adjustments. Generally they are lump sum and are independent of the size or direction of the change.

The presence of administrative costs gives rise to problems which can best be solved by adopting a policy of "(S,s)" form.

In essence the Federal Reserve chooses ceiling and floor "thresholds" between which the spread is allowed to fluctuate.³² Once either of these thresholds is hit an adjustment is made. This adjustment restores the spread to its optimal value.

The thresholds are chosen so as to minimize the expected cost of adjusting the discount rate. Therefore the first step is formulating an expression which reflects this cost.

We define $E[Y]$ as the expected number of discount rate adjustments over some time interval T ; γ as the cost of making a discount rate adjustment; and $E[S_t]$ as the expected spread. Then the expected cost per unit time is:

$$E[C] = \gamma \frac{E[Y]}{T} + E[S_t - S^*]$$

³² Because the deviation in the spread will be symmetric the optimal thresholds will also be symmetric.

Next we seek an equivalent expression for the expected cost in terms of the thresholds. To find an expression for the expected number of discount rate adjustments we appeal to the properties of the Poisson process.

If $\{N(t), t \geq 0\}$ is a Poisson process with rate $\lambda > 0$ then the random variable Y describing the number of discount rate changes in any time interval of length $t > 0$ has a Poisson distribution with parameter λt . Or,

$$P\{Y=k\} = e^{-\lambda t} \frac{(\lambda t)^k}{k!}, \quad k = 0, 1, 2, \dots$$

Then the average number of rate changes occurring in any time interval of length t is λt , and the average number of rate changes occurring per unit time is $\lambda t/t = \lambda$.

To find an equivalent expression in terms of the thresholds we begin by noting that if λ is the average number of rate changes (per unit time) then $1/\lambda$ is the mean waiting time between discount rate adjustments.³³

Recall that in order for a rate change to have occurred the spread must have reached a critical level. The assumption that the lengths of time between rate changes (and thus, between level crossings) are ruled by statistically independent exponential random variables

³³ This conclusion can also be reached by referring back to our earlier work which contends that the interoccurrence times are exponentially distributed with parameter λ . The mean of such a distribution is $1/\lambda$.

suggests that the spread on successive days is determined by independent random events. Because the discount rate is "fixed" between level crossings, these random events must be attributable to the market rate process. The efficient markets model contends that interest rates follow a random walk, therefore we assume that the process which rules the variation of the spread is also a random walk. Specifically we assume that

$$S_t = S_{t-1} + \xi_t,$$

ξ_t serially independent.

We model $(\xi_t)_{t \geq 0}$ as a sequence of independent Bernoulli variables with

$$P(\xi_t = +1) = p,$$

$$P(\xi_t = -1) = q,$$

$$p + q = 1.$$

where $P(\xi_t = +1)$ is the probability that the interest rate will increase by 1 unit in the interval $[t-1, t]$; and $P(\xi_t = -1)$ is the probability that the interest rate will decrease by 1 unit in the interval $[t-1, t]$.

If we define $S_0 = 0$ and $S_t = \xi_1 + \dots + \xi_t$, $1 \leq t \leq n$ then the sequence S_0, S_1, \dots, S_t can be interpreted as the path of the random motion of the spread starting at zero.³⁴ Let x be an integer such that $-h \leq x \leq h$ and for $0 \leq k \leq n$ let $S_k^x = x + S_k$. We define a stopping time,

³⁴ This is equivalent to saying that we are in equilibrium or that $S_0 = S^*$.

$$\tau_t^x = \min \{0 \leq L \leq t: S_L^x = -h \text{ or } h\}$$

where

$$\tau_t^x = t \quad \text{if} \quad -h < S_L^x < h \text{ for all } 0 \leq L \leq t.$$

The stopping time can then be interpreted as the event that the spread, starting at time zero at the point x , leaves the interval $(-h, h)$; and the expectation of the stopping time is the mean duration of the random walk or $1/\lambda$.

Let $m_k(x) = E\tau_k^x$ be the expectation of the stopping time τ_k^x , $k \leq n$. For x belonging to $(-h, h)$

$$\begin{aligned} E \tau_k^x &= \sum_{1 \leq L \leq k} L P(\tau_k^x = L) \\ &= \sum_{1 \leq L \leq k} L [pP(\tau_k^x = L | \xi_1 = 1) + qP(\tau_k^x = L | \xi_1 = -1)] \\ &= \sum_{1 \leq L \leq k} L [pP(\tau_{k-1}^{x+1} = L - 1) + qP(\tau_{k-1}^{x-1} = L - 1)] \\ &= \sum_{0 \leq L \leq k-1} (L+1) [pP(\tau_{k-1}^{x+1} = L) + qP(\tau_{k-1}^{x-1} = L)] \\ &= pm_{k-1}(x+1) + qm_{k-1}(x-1) \\ &+ \sum_{0 \leq L \leq k-1} [pP(\tau_{k-1}^{x+1} = L) + qP(\tau_{k-1}^{x-1} = L)] \\ &= pm_{k-1}(x+1) + qm_{k-1}(x-1) + 1. \end{aligned}$$

Thus, for x belonging to $(-h, h)$ and $0 \leq k \leq n$, the functions $m_k(x)$ satisfy the recurrent relations

$$m_k(x) = 1 + pm_{k-1}(x+1) + qm_{k-1}(x-1),$$

with $m_0(x) = 0$. With these equations, and given the boundary conditions

$$m_k(-h) = m_k(h) = 0,$$

we can successively find $m_1(x), \dots, m_n(x)$.

Assuming equal probability for increases or decreases in the spread or $p = q = \frac{1}{2}$, the general solution has the form

$$m(x) = a + bx - x^2,$$

and since $m(-h) = m(h) = 0$ we have

$$m(x) = (-h - x)(x - h).$$

And

$$m(0) = h^2.35$$

This expectation is in terms of steps therefore it is necessary to multiply h^2 by the amount of time per step in

³⁵ Shirayayev pp. 88-89.

order to arrive at the expected amount of time between threshold crossings. If δ represents the amount of time per step the expected amount of time between threshold crossings is δh^2 and λ , the expected number of discount rate changes per unit time, equals

$$\frac{1}{\delta h^2}.$$

The variance of the Bernoulli process involved in the symmetric random walk, or equivalently, the variability of the spread, may be derived as:

$$\sigma_t^2 = \frac{t}{\delta}, \text{ where } t \text{ denotes the total elapsed time since the start (from the origin) at time zero.}$$

The "daily" variance is:

$$\sigma^2 = \sigma_t^2 (t = 1) = \frac{1}{\delta} .36$$

The expected adjustment cost per unit can then be written as

$$\frac{\gamma \sigma^2}{h^2} .$$

Next we require an expression for the expected out of equilibrium cost in terms of the thresholds. This is just the mean of the steady-state distribution of deviations in the spread.

We assume the deviation of the spread from the target spread varies from an initial value, u_0 , to a final value, u_1 . Setting $u_0 = 0$ and $u_1 = u$ we define this deviation as

u. The probability that the spread deviates by u units can be found by deriving the density function of u.

The difference equation

$$f(u) = \frac{1}{2} [f(u+1) + f(u-1)] \quad (-h+1 \leq u \leq h-1; u = 0)$$

with boundary conditions

$$f(h) = f(-h) = 0$$

$$f(0) = \frac{1}{2} [f(1) + f(-1) + f(h-1) + f(-h+1)]$$

and the density condition

$$\sum_{u=-h}^h f(u) = 1$$

implies that

$$f(u) = \begin{cases} \frac{1}{h} - \frac{u}{h^2} & (0 \leq u \leq h) \\ \frac{1}{h} + \frac{u}{h^2} & (-h \leq u \leq 0).^{37} \end{cases}$$

The mean of this distribution is

$$E[u] = \frac{h}{3}.$$

With this expression we can now state the total expected cost per time as

³⁷ The solution is derived in the appendix.

$$E[C] = \frac{\gamma\sigma^2}{h^2} + \frac{h}{3} .$$

The value of h which minimizes this expression is

$$h^* = (3/2 \gamma\sigma^2)^{1/3} .$$

According to this solution the optimal spread between the discount rate and the market rate will be a function of the administrative costs of making a rate adjustment, and the observed variability of the daily spread.

There is a positive relationship between the variability of the spread and the rate of occurrence of discount rate changes. The magnitude of this relationship will depend upon two "offsetting" forces. For a given critical threshold, or equivalently an optimal spread, an increase in the variance of the observed spread implies an increased frequency of discount rate changes. However this effect will be partially offset by the positive relationship between the threshold and the variance.

The latter effect is the result of the Fed widening the interval or increasing the (absolute) value of the thresholds. This occurs because the increased variability of the spread causes the Fed to readjust h^* . In accordance with the above equation the new optimal value of h is greater. Given the symmetry of the thresholds, $-h = h$, the new adjusted interval is wider and therefore we would expect (relatively) fewer discount rate adjustments.

It is interesting to look at these implications of the

optimal policy for the subperiods 1968 - 1979 and 1979-1982.

In October 1979 the Federal Reserve switched to a nonborrowed reserve operating procedure as opposed to the earlier adjustable federal funds rate peg system. The Open Market Desk at the Federal Reserve Bank of New York was instructed by the Federal Open Market Committee to control the growth in nonborrowed reserves in a way calculated to hit or be consistent with the target money growth, subject to a federal funds rate tolerance range that has typically been five percentage points wide.³⁸ The allowance of a greater range for the federal funds rate implies an increased variability for the spread.

Recall that the variance of the spread is dependent upon the difference between a market rate of interest and the discount rate. A fixed discount rate then implied that the total variability could be attributed to the market rate process. In this case, the Federal funds rate process. Since the Federal Reserve no longer targeted the Federal funds rate and instead allowed it to vary over a wider range its variability increased. The result of this would then be an increase in the observed daily variance of the spread.

The positive relationship between the spread variance and the frequency of discount rate changes is borne out by

³⁸ Poole [1982] p. 577.

the data. In effect, the increased volatility of the Federal funds rate (and therefore the spread) served to cause more frequent discount rate adjustments. The average number of days between discount rate changes in the subperiod 1968 - 1979 was 105; the rate of occurrence was .0095. In contrast, the average number of days between discount rate changes in the subperiod 1979 - 1982 was 85; the rate of occurrence was .0118. Thus, there was an increase in the frequency of discount rate changes.

Chapter VI

Summary and Conclusions

In this thesis we examined two possible hypotheses with regard to the interpretation of Federal Reserve discount rate policy. The methodology used for studying the discount rate process differed from more conventional techniques in order to take into account the special nature of the data.

The mechanics of discount policy imply a different type of behavior; one which puts restrictions on the data. Therefore observations on discount rate changes cannot be accurately represented as a typical time series.

This thesis characterized discount policy as an "on/off" process where individual observations represented the occurrence of an event. The event was a change in the discount rate. By using the sample path properties of the timing of discount rate changes to determine the probabilistic law which the timing obeyed, we had a model which was narrower and better suited to investigate the two interpretations of Federal Reserve discount policy.

The first hypothesis studied was that the discount rate plays an independent role in monetary policy and is used to achieve a goal of macroeconomic stabilization. In accordance with this type of policy discount rate changes would be interpreted as "active" signals of tightening or easing of monetary policy made in consideration of the

state of the economy.

The response of the monetary authorities to movements in real output could be represented by a specific pattern in the timing of discount rate changes. More specifically, a series reflecting the timing of discount rate changes, in which the observations represent the number of days between successive discount rate changes, would be characterized by sequences of "small" values (representing more frequent changes around turning points) alternating with sequences of "large" values (representing the relative inactivity during periods between turning points). Thus, the series would have a discernible serially correlated pattern.

In studying the pattern of the timing of discount rate changes this thesis found little evidence which was consistent with this hypothesis.

The alternative hypothesis suggested was that Federal Reserve directs discount policy towards financial goals. Rate changes are employed to achieve maximum effectiveness of the discount mechanism and thus ensure that the discount window serves exclusively as a source of funds for temporary or unexpected reserve drains.

It would be necessary for the Federal Reserve to maintain an appropriate relationship between the discount rate and the (short-term) market interest rate. No incentive for member banks to use the discount window to obtain funds to either circumvent current monetary

objectives or exploit profit opportunities offered by significant cost differentials between the two rates could exist. Since the market rate is a Martingale process and changes cannot be predicted, a policy of responding to movements in interest rates would produce a random pattern in the timing of discount rate changes. Moreover if the Federal Reserve had not responded to every movement in the market rate but waited until the difference was "large" enough to warrant such a change in the discount rate, then the realizations of the events of discount rate changes would be distributed as Poisson.

To pursue this hypothesis we tested the timing of discount rate changes to see if the intervals between rate changes were independent, identically distributed exponential random variables. This property would imply that the discount rate process was Poisson. Our results indicated that a Poisson distribution could be rejected for the period 1914-1955, but not for the period 1955-1986.

We concluded that there was some evidence to suggest that the Federal Reserve had changed the discount rate sporadically, and that the pattern in the timing of changes was consistent with what would be expected if the authorities had followed changes in other market interest rates.

Additionally, we used results from the theory of stochastic processes to suggest an extension of the model

which might better explain the period 1955-1986. By allowing the frequency of discount rate changes to be a function of time, this extension could take into account the shifts in the administration of the discount window which are said to have occurred.

And finally we presented a model which was consistent with the observed characteristics of the discount rate process. The random pattern of discount rate changes can be justified theoretically with a model which assumes that a fixed transaction cost accompanies every discount rate change and is weighed against the cost of not making an adjustment. These cost considerations cause the Fed to react only to significant changes in the relationship between the discount rate and market interest rate smaller changes not being worthwhile.

Appendix

The system

$$f(u) = \frac{1}{2} [f(u+1) + f(u-1)] \quad (-h+1 \leq u \leq h-1; u \neq 0)$$

$$f(h) = f(-h) = 0$$

$$f(0) = \frac{1}{2} [f(1) + f(-1) + f(h-1) + f(-h+1)]$$

$$\sum_{u=-h}^h f(u) = 1$$

can be solved by considering

$$f(u) = A + B|u|$$

with

$$f(1) = A + B$$

$$f(-1) = A + B$$

$$f(h-1) = A + Bh - B$$

$$f(-h+1) = A + Bh - B$$

$$f(0) = \frac{1}{2} [2(A + B) + 2(A + Bh - B)]$$

$$= 2A + Bh$$

$$f(0) = A$$

$$2A + Bh = A$$

$$B = \frac{-A}{h}$$

$$[f(-h) + f(h)] + [f(-h+1) + f(h-1)] + \dots + f(0) = 1$$

$$2(A + Bh) + 2[A + B(h-1)] + \dots + 2(A + B) + A = 1$$

$$2 \cdot h \cdot A + 2B(h + (h-1) + \dots + 1) + A = 1$$

$$(2h + 1)A + 2B \cdot \frac{h(h+1)}{2} = 1$$

$$(2h+1)A - \frac{A}{h} \cdot h(h+1) = 1$$

$$A(2h + 1 - h - 1) = 1$$

$$A = \frac{1}{h}$$

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