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THE IMPACT OF TECHNOLOGICAL CHANGE ON THE FUNCTIONAL DISTRIBUTION OF INCOME

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PART 1

INTRODUCTION

CHAPTER 1

THE PROBLEM AND THE FRAMEWORK OF THE RESEARCH

Functional income distribution[1] in conjunction with the progress of a society has long been one of the major socio-economic problems.

The current trend of increasing automation seems to call forth not only an intellectual curiosity but also a real interest as regard to the nature and the consequence of technological changes on the output, the functional distribution, and the employment.

Such an interest is not a new one.
Even before the industrial revolution, governments and writers worried about the unfavorable consequence of automation on the labor share of the society's produce, and labour groups resisted against machineries.

The subject had been treated in more sophisticated ways by the Classical Economists, particularly by D. Ricardo and J.S. Mill.
However, the Classical Economics was not equipped with an uniform theory of income distribution.
Their theory of distribution was divided into the seperate theories of

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1 There are various ways of approaching to the subject of income distribution: Functional distribution focusses on the distribution of the produce among the owners of the production factors. The other approaches are Size, Personal, and Occupational distribution approaches. See T. Scitovsky, "A Survey of Some Theories of Income Distribution," The Behavior of Income Shares, National Bureau of Economic Research, New York, 1964, p.15.

the values of the three factors of production, i.e. capital, labor and land.

Ricardo's analysis of the effect of the society's progress on the distributive shares was based on his theory of rent, the Law of Diminishing Return, and the Malthusian population theory.
As the society progresses, the wage level tends to the minimum subsistance level, and the profit tends to the minimum level which does not provide any incentive to the firms neither to enter nor to depart from the industries, while the land owners alone experience the rising rents.[2] Thus, Ricardo presented us the possibility of the Classical Stationary State where the growth of population and capital accumulation would cease.
In his analysis, the technological progress was neglected, which was introduced by John S. Mill as a countervailing force against the tendency of the economy toward the stationary state.

Later, Ricardo in his newly added Chapter 31, ' On Machinery'[3] focussed on the influence of machinery on the interests of the different social classes.
He derived a conclusion that the opinion entertained by the laboring class that the employment of machinery was frequently detrimental to

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2 Absolute rents will rise in this case, and the relative rents will also rise when the degree of the diminishing return is strong enough.

3 The Principles of Political Economy, J.M. Dent and Sons, Ltd., London, 1957.

their interests was not founded on prejudice and error but was conformable to the correct principles of political economy.[4]
Nevertheless, in his analysis, the distinction between factor substitution within a given technology due to the consideration of the relative factor price and that due to change in technology was not clear. In any event, the conclusion that the mechanization might permanently reduce the relative share of labor to capital and possibly even the absolute amount of real wage bill provided a stimulus to the development of Marxian system.

Even though the significance of the problem was well recognized and discussed by the Classical Economists, the basis for a comprehensive and systematic analyses of functional distribution was founded by the Marginalists in terms of the Marginal Productivity Theory, which was essentially a static equilibrium theory of a firm behavior that could be readily applied to the analysis of the functional distribution of income.
The entrepreneurs, under perfect competition in the product and factor markets, would maximize their profits under tha constraint of the production function by deciding the optimum inputs at the levels where the factor prices are equal to the value marginal products of the input factors, the output being determined by the production function with the optimum inputs.

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[4] Ibid., p.267.

The theory, thus, bore a moral implication that under perfect competition the entrepreneurs' profit maximizing behavior would lead to the equilibrium where the productive agents would be remunerated at their marginal contributions to the production, ruling out the unsophisticated exploitation theses other than the Marxian.

The production function system then would be as follows:

(1.1) $Q = F(K,L)$: production function in general form

(1.2) $F_k = r/p$

(1.3) $F_l = w/p$

] : marginal productivity relations (Appendix A : 1)

Q: output
K: capital input
L: labor input
F_k: marginal productivity of capital
F_l: marginal productivity of labor
p: price of the product
r: price of capital(rental of capital)
w: price of labor(wage rate)

This postulates the situation where the input factors would be smoothly substituted to fulfill the marginal productivity relations. This type of production function, which is usually referred to as the Neoclassical Production Function, is supposed to have the well-known properties; positive marginal productivities (positive first partial derivatives of the production function with respect to capital and labor) and diminishing marginal rate of substitution (negative own second partial derivatives).[5]

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5 The cross second partial derivatives are positive.

After all, the marginal productivity theory was a static equilibrium theory in which the technology embodied in the production function was assumed to be constant.

The systematic organization and presentation of the marginal productivity theory of distribütion was done by J.R. Hicks, who added the element of the technological change to the theory.[6]
Hicks crystalized the concept of elasticity of substitution based on relative factor price or marginal rate of substitution.
He also called our attention to the nature of technological changes, and defined technological changes as neutral, capital biased, or labor biased depending on whether the marginal rate of substitution of capital for labour is constant, falls or rises at the constant capital-labour ratio.[7]
If the increase of the supply of one factor is relatively lower than the other's, the relative share of the former decreases, remains constant, or increases depending upon whether the elasticity of substitution is greater than, equal to, or smaller than unity.
If a technological change is biased to a certain factor, the relative share of that factor would increase, decrease or remain constant depending upon whether the elasticity of substitution is greater than, less

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6 The Theory of Wage, Macmillan and Co., London, 1964, Ch. VI and Section II, 3.

7 Ibid., pp.112-122.

than, or equal to unity.

Hicks also classified technological changes into autonomous and induced technological changes. The latter type is the result of a change in the relative factor price and or the elasticity of substitution, and the former type comprises all the noninduced technological changes. He expected that in practice all or nearly all induced inventions to be labor saving and the autonomous inventions to be random.[8]

Hicks contended that the decline in the real wage bill due to economic progress would be possible but extremely improbable, whereas it would be difficult to feel the same degree of optimism in the matter of the relative share.[9]

The implication might be that the change in the real wage bill can occur due to both income and substituion effects, whereas the change in the relative share occurs due to substitution effect. For instance, when sustitution of capital for labor takes place due to the relative cost consideration, the new process of production based on the lower cost of production would tend to lower the price of the product. Cetris paribus, the fall in the price of the industry's product would result in the increase in the demand for the product due to the conventional mechanism of substitution and income effects as illustrated in the indifference curve analysis. An increase in the demand for final product due to such a consideration as well as all the other reasons would raise the derived demand for labor as well as for capital.

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[8] Ibid., p.129.

[9] Ibid., p.130.

Thus, the real wage bill might not necessarily fall despite the initial substitution of capital for labor.
On the other hand, however, the relative share of labor to capital, cetris paribus, would fall due to the initial substitution of capital for labor.

Based on the Hicksian propositions and the Schumpetarian idea of discrete technological changes[10], Murray Brown and J.S. de Cani presented an ingenious way of estimating the impacts of the changes in relative factor price and technology[11] on the relative share of capital and labor.[12]
Their production function system is the same as the one discussed before , consisting of the production function and the marginal productivity relations of labor and capital. They, however, introduced some dynamic element in the marginal productivity relations, which is statistically similar to Koyck's distributed lag specification.
They estimated a number of the constant elasticity of substitution production function for the private domestic nonfarm economy of the United States from 1890 to 1960, by varying the subperiods which the

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10 Schumpeter regarded innovations to result in shifts in the production function and to be discontinous and periodic so as to induce business cycles.

11 Without distinction between induced and autonomous technological changes.

12 "Technological Change and the Distribution of Income," International Economic Review, 1963.

short-run production functions cover. Applying covariance analysis on these prodcution function estimates, they attempted to search for technological epochs within each of which the production function was stable.[13]

Once they broke down the whole period into a number of technological epochs, they tried to get the estimates of the contributing forces to the movements of the relative factor shares by the finite differencing method. Their estimating equations, in this case, were equivalent to the expansion path function with Koyck's distributed lag scheme.[14]
They found that the impact of the nonneutral technological change on the relative factor share was the same order of magnitude as the effect of changes in relative factor price.

It should be noted, however, that their analysis is based on the assumption that there exist clear-cut technological epochs. This assumption is very difficult to accept, particularly at the aggregate level of the economy.[15]

This study, based on the Marginal Productivity Theory of Distribution[16] and the idea of gradual technological changes

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13 Brown, Murray, <u>On the Theory and Measurement of Technological Change</u>, Cambridge University Press, 1965, pp. 199-201.

14 <u>Ibid.</u>, p. 191.

15 Prof. Brown pointed out to me that the neutral technological change was allowed to proceed gradually throughout the whole period.

16 There are alternative approaches to the analyses of the functional distribution of income. Notable ones are Nicholas Kaldorian

contrasted to the idea of discrete changes, will attempt to estimate the impact of technological change on the relative shares of the factors for the post-war period in the selected U.S. two-digit manufacturing industries.[17] The basis of the selection is the availability of the consistent set of data. For fifteen industries out of the twenty two-digit manufacturing industries, the consistent set of the data are avilable, while the rest of the industries do not have satisfactory data on the capital stock. The excluded five industries are as follows: Lumber and Wood Products, Petroleume and Related Industries, Transportation Equipment, Instruments and the Related Products, and Miscellaneous Manufacturing Industries. The list of the industries included in this study is shown in the Appendix B : 2.

When one attempts to perform an empirical estimation in this area as in others, the lack of satisfactory set of data emerges as a significant limiting factor. The more disaggregated the object of the study becomes, the more difficult to get the consistent set of data for all the relevant variables, whereas the marginal productivity theory demands for a high level of disaggregation.

Even though the two-digit classification of industries is not as much disaggregated as one would wish, it seems at the present this classification is the finest one for which one can get somewhat

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(continued from the previous page)
macro-economic approach and Michael Kalecki's mark-up theory approach, both of which need further refinements. For detailed discussion on these alternative approaches, read Scitovsky, op. cit.

17 The former type of technological change was advocated by Abbott P. Usher. For a comparative discussion on the Schumpetarian and Usherian technological changes, read Ch. 5 of Murray Brown, op. cit.

reasonable set of data.

Despite the difficiencies in the data as well as those in the researcher's imagination and research techniques, the importance of the topic demands for some estimation of the relevant parameters.

Throughout the whole study, a member of the Constant Elasticity of Substitution Production Function family will be used, which would be slightly modified to incorporate the factor augmenting technological changes.[18]

The specification in general form of factor augmening technological changes in a production function was well illustrated by R. Solow.[19]

Leaving aside the complications of the possible embodiment of technological change in capital goods and heterogeneity of labor, he represents technological change in a production function in general form as follows:

$$Q = F(K,L;\ t) \qquad (1.4)$$

where "t" represents the technology embedded in the production function.

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[18] Variable Elasticity of Substitution (VES) Production Function would be more general than the CES Production Function. However, the former has not been fully developed yet, and the elasticities of substitution would not likely show significant variations during two decades.

[19] " Developments in Production Theory," The Theory and Empirical Analyses of Production, National Bureau of Economic Research, 1967, pp. 28-9.

Consider that all the technological changes occur through the augmentation of either one or both of the production factors. Then, the specialized form of equation (1.4) is

$$(1.5)\quad Q = F[a(t)K,\ b(t)L]$$

a(t)K and b(t)L can be interpreted as inputs of capital and labour in "efficiency units," whereas K and L are inputs in "natural units". a(t) and b(t) are augmentation factors or efficiencies per natural units of capital and labor respectively. The former reflects capital innovations attributable to research and development, and the latter reflects the influences of increased education and training, and of improved health.

Hicksian neutrality criterion of technological change is equivalent to the following:[20]

a(t)/b(t)	is constant	:	neutral
a(t)/b(t)	is rising	:	capital biased
a(t)/b(t)	is falling	:	labor biased

Solow in his another article employed a more specialized form of the equation (1.5) assuming b(t) is unity.[21]

$$(1.6)\quad Q/L = F[a(t)K/L,\ 1]$$

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20 Harrod's neutrality condition is equivalent to requiring a(t) to be constant. Ibid., p.29.

21 "Capital, Labor and Income in Manufacturing," The Behavior of Income Shares, National Bureau of Economic Research, 1964.

Besides all the other numerous assumptions he made in order to estimate the technological change, the assumption of the unit b(t) is too restrictive, because it implies that all the technological changes occur through augmenation of capital while the efficiency of labor remains constant.

The specification of the augmentation factors in the production function which will be used in this study would be as follows:

$$(1.7)\ Q = (e^{g_1 t} K , e^{g_2 t} L)$$

This formulation is based on the assumption that the augmentation factors of capital and labor have exponential growth paths with different growth rates, g_1 and g_2 respectively.

The assumptions that all the technological changes are factor augmenting and that the factor augmenations follow exponential growth paths enable us to put aside the possible obstacle of the well-known Diamond-MacFadden Impossibility Theorem.

The impossibility theorem states that it is in fact impossible to measure either the bias in technology or the elasticity of substitution from a given time series of all observable market phenomena for a single economy which has a neoclassical production function, because these same time series could have been generated by an alternative production function with an arbitrary elasticity of substitution and arbitrary technological bias at the observed points of time. In the absence of technological change, however, one can measure the elasticity of substitution and in the absence of a change in capital-labor ratio one can determine the bias in the technology.[22]

In this study, however, even if there exist non-factor augmenting technological changes attributable to organizational improvements, they will have neutral effects on the expansion path functions and, thus, on the relative shares. (Appendix A : 4)

Equation (1.7) will become the original static production function in equation (1.1) if the augmentation parameters, g_1 and g_2 , are zero.

In the early part of the study, the two-factor production function will be used, and an attempt will be made to use a generalized three-factor Constant Elasticity of Substitution Production Function to examine the impact of factor augmenting technological changes on the distribution among capital owners and production workers in the production function with the three factors; capital, labor of production workers and labor of nonproduction workers.[23]

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22 Diamond, P.A., and D. McFadden, "Identification of the Elasticity of Substitution and the Bias of Technical Change: An Impossibility Theorem," 1965, unpublished. For the proof that the assumption of exponential factor augmenting technological change is sufficient for the identification, read Nerlove, Marc, "CES and Related Production Functions," The Theory and Empirical Analysis of Production, National Bureau of Economic Research, 1967, pp.92-100.

23 Production workers are all the nonsupervisory workers, and the nonproduction workers are workers other than the production workers.

The classification of laborers into the above two is expected to give us quite interesting result because this classification is similar to that of the laborers into the blue color workers and the white color workers, and also becuase the variation in the relative share of capital and production labor is more in line with the traditional concern.

PART 2

THE MODEL WITH TWO FACTORS OF PRODUCTION: CAPITAL AND LABOR

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CHAPTER 2

THEORETICAL SPECIFICATION OF THE MODEL

2.1 Long-run Static Equilibrium Model

The long-run equilibrium production function system, under perfect competition in all the relevant markets, is [1]

(2.1) $Q = A \left[d(e^{g_1 t} K)^{-s} + (1-d)(e^{g_2 t} L)^{-s} \right]^{-1/s}$

(2.2) $F_k = A^{-s} de^{-sg_1 t} K^{-1/\sigma} Q^{1/\sigma} = r/p$

(2.3) $F_l = A^{-s} (1-d) e^{-sg_2 t} L^{-1/\sigma} Q^{1/\sigma} = w/p$

where $\sigma = 1/(1+s)$

A: constant term (efficiency parameter)
d: distributive parameter
s: substitution parameter
t: technology expressed in terms of time
σ: elasticity of substitution
g_1 and g_2 : factor augmentation parameters as defined before

This production function is well-known Solow-Minhas-Arrow-Chenery Production Function with factor augmening technological changes, which has the properties of constant return to scale and constant elasticity of substitution.

The possibility of increasing returns to scale is put aside to eliminate the possibility of divergence of social marginal product from private marginal product. The case of the decreasing returns to scale can be examined in the following more general model:

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1 C.A. Knox Lovell also independently set up the same production function system in his paper presented at the annual meeting of the Econometric Society: Biased Technical Change and Factor Shares in U.S. Manufacturing, 1968, December.

The production function with homogeneity of "m" degree is as follows:

$$(2.4)\quad Q = A\left[d(e^{g_1 t} K)^{-s} + (1-d)(e^{g_2 t} L)^{-s} \right]^{-m/s}$$

where $m \leq 1$

As far as the marginal productivity ratio or cost minimization condition is concerned, on which the subsequent derivation of the estimating equation is based, it is not affected by the specification of the production function in either form, (2.1) or (2.4). (Appendix:A: 3)

The marginal productivity ratio is

$$(2.5)\quad F_k/F_l = [d/(1-d)]\, e^{s(g_2-g_1)t}\, (L/K)^{1/\sigma} = r/w$$

Logarithmic transformation of (2.5) is

$$(2.6)\quad \log r/w = \log d/(1-d) + s(g_2-g_1)t + 1/\sigma \log L/K$$

$$(2.7)\quad \log L/K = \sigma \log (1-d)/d + (\sigma-1)(g_2-g_1)t + \sigma \log r/w$$ [2]

The relative share of labor to capital, RS, is

$$(2.8)\quad RS = wL/rK$$

from (2.7) and (2.8),

$$(2.9)\quad \log RS = -\log r/w + \log L/K$$

$$= \sigma \log (1-d)/d + (\sigma-1)(g_2-g_1)t + (\sigma-1) \log (r/w)$$

In the equation (2.9), the coefficient of "t", i.e. $(\sigma-1)(g_2-g_1)$, measures the average annual percentage change in the relative shate of labour to capital attributable to the nonneutral technological changes.

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2 The performances of this static model in the empirical estimations were poorer than the more realistic estimating equations developed in the folling sections.

The term of (g_2-g_1) represents the degree of the bias in the technological change.

Technological changes can be termed as capital biased, labor biased, or neutral depending upon whether (g_2-g_1) is negative, positive or zero. And the technological change can be classified as capital using, labor using, or neutral depending upon whether $(\sigma-1)(g_2-g_1)$ is negative, positive, or zero. This means that capital biased technological change can be capital using, a labour using or a neutral technological change depending upon whether the elasticity of substitution is greater than, less than or equal to unity.
With the same logic, a labour biased technological change can be capital using, labor using, or neutral technological change depending upon whether the elasticity of substitution is less than, greater than, or equal to unity.

$(\sigma-1)(g_2-g_1)$ can be zero when σ is unity and / or (g_2-g_1) is zero. One implication of this is that the technological change would be neither labor saving nor capital saving in the Cobb-Douglas Production Function system regardless the bias in the technological changes, because it assumes the unit elasticity of substitution.

The coefficient of log (r/w), i.e. $(\sigma-1)$, would provide us with another criterion for the analysis of the relative share: the relative share of labour to capital will rise, fall, or remain constant, cetris paribus, when the relative price of labour rises, depending upon whether the elasticity of substitution is less than, greater than, or equal to unity.

The effect of the change in the relative factor price on the relative share can be decomposed into two offsetting effects, which will be arbitrarily termed as the "substitution effect" and the "accounting effect" or "valuation" effect. This means that a change in the relative factor price influences the relative share both indirectly through the substituion effect and directly through the accounting effect, the latter is termed as such because the change in the relative factor price changes the value of the input factors.[3]

The degree of substitution effect depends on the magnitude of the elasticity of substitution, and the degree of the accounting effect is, in principle, one hundred percent.

To express it differently, the substitution effect of the change in the relative price of capital to labor, i.e. r/w, on the relative share of labor to capital,i.e. wL/rK, is "100σ percent", and the accounting effect is "100 percent" to the opposite direction.

For example, if the elasticity of substitution were unity, the substitution effect of a fall in the relative price of capital to labor would work in the direction to reducing the relative share of labor to capital by 100 percent, whereas the accounting effect would raise the relative share by 100 percent, thus leaving the net effect as neutral.

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3 Such a distinction becomes clearer and necessary in the analysis of the dynamic models developed in the following sections, although the distinction does not seem to be necessary in the static model here.

That is, the relative share of labor to capital remains constant in this case.

In the same example, if the elasticity of substitution were less than unity, the substitution effect would lower the relative share of labor to capital by less than 100 percent, whereas the accounting effect would raise the relative share by 100 percent. Thus the net effect of the decline in the relative price of capital to labor in this case would be an increase of the relative share of labor to capital.

If the elasticity of substituion were greater than unity, the substitution effect of the fall in the relative price of capital to labor would reduce the relative share of labor to capital by greater than 100 percent, whereas the accounting effect raises the relative share by 100 percent. The net effect in this case would be a reduction of the relative share of labor to capital.

Therefore, cetris paribus, the relative share of labor to capital rises, falls, or remains constant depending upon whether the net effect of the fall in the relative price of capital to labor is positive, negative, or zero, i.e. whether

$$\sigma \frac{\partial \log r/w}{\partial t} - \frac{\partial \log r/w}{\partial t} = (\sigma - 1) \frac{\partial \log r/w}{\partial t} \gtrless 0$$

Now we turn to another aspect of the model.

The conventional practice of estimating elasticity of substitution from time series regression without holding technology constant would result in the overestimated elasticities of substitution, so that the change in the relative factor price alone would take the excessive weight in the explanation of the variations of the factor ratio and the relative factor share.[4]

In more technical term, the omission of the technology term from the estimating equation would be a specification error, which would result in specification bias in the estimated coefficient. (Appendix A : 4)

Now we can turn our attention to an estimation problem associated with the estimating equation (2.7).

The estimating equation (2.7) would face some difficulty in the estimation process if the two independant variables are highly correlated. The high intercorrelation would occur when the relative factor price has a strong exponential time trend. This means the case where the logarithmic value of the relative factor price has a linear time trend.

Such a colinearity problem is a common problem to almost all the estimations based on the time series data, even though the degree of the severity might be different.

In this study, therefore, the industries the relative factor price of which do not have the strong exponential time trend or at least the correlations between the logarithmic value of the relative factor price and the technology term are not too high are expected to give us relatively satisfactory estimates of the relevant coefficients.

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[4] This point has also been made by George Stigler in his monograph, Capital and Rates of Return in Manufacturing Industries, National Bureau of Economic Research, 1963, p.100.

On the other hand, the industries which are characterized by high intercorrelation between the logarithmic value of the relative factor price and the linear time trend term are expected to give us the estimates which should be very carefully examined.
Such a collinearity problem will also be present in the dynamic estimating equations which will be discussed in the next section.

If the multicollinearity is serious, the estimated parameters will have an unsatisfactorily low degree of precision expressed by the high standard errors of the estimates or very low values of "t" statistic. We can quote Professor Johnston's remark on such a case: [5]

> " If multicollinearity is serious, in the sense that estimated parameters have an unsatisfactorily low degree of precision, we are in the statistical position of not being able to make bricks without straw."

The remedy lies essentially in the acquisition, if possible, of new sample which will break the multicollinearity deadlock, or in obtaining some extraneous information. The extraneous information can be either a priori knowledge on or extraneous statistical estimates of the coefficients of certain independent variables.
A most common practice in the time series analyses, which will be used also in this study, is to get the cross-section estimates of the coefficients of certain explanatory variables and apply them to the time series regression to transform the relevant variables.

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5 Johnston, J., Econometric Methods, McGraw-Hill Book Co., New York, 1963, p.207.

2.2 Short-run Dynamic Model Based on the Adaptive Expectation Scheme: Model 1

The adjustments in the input decisions based on the changes in the relative factor price would not usually be instantaneous, partly becuase the entrepreneurs base their input decisions on the expected relative factor price which is formed by taking into account the history of the past relative factor prices, and also partly because there are some environmental factors which would cause certain lags in carrying out the input decisions.

Irving Hoch noted that entrepreneurs might be maximizing anticipated profit, which notion was, however, not fully developed.[6] Later, Zellner, Kmenta and Drèzè developed a model which explicitly took expectation and uncertainty into account.[7]

$$Q_t = A\, L_t^{\alpha_1}\, K_t^{\alpha_2}\, e^{u_{ot}}$$

$$\frac{\partial E(\pi)}{\partial L} = 0$$

u_o : random disturbance (e.g. weather, unpredictable variations in machine or labor performance)

$$\frac{\partial E(\pi)}{\partial K} = 0$$

$$E(\pi) = p^+ E(Q) - w^+ L - r^+ K$$

$E(\pi)$: expected profit

p^+, w^+, r^+ : expected prices of the product, labor and capital respectively

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6 Hoch, I.,"Simultaneous Equation Bias in the Context of the Cobb-Douglas Production Function," Econometrica, Oct., 1958, pp.566-78.

7 Zellner, A., J. Kmenta, and J. Drèzè,"Specification and Estimation of Cobb-Douglas Production Function Model," Econometrica, Oct., 1966, pp.784-795.

The profit maximizing conditions contain stochastic disturbance terms each of which has two sources of the disturbances.
One of them would be random managerial errors (u_1^* and u_2^*) originating in inertia, ignorance, etc.. Another source is the random difference between relaized prices from anticipated ones (u_1^+ and u_2^+).

$$\log (w^+/p^+)_t = \log(w/p)_t + u_{1t}^+$$
$$\log (r^+/p^+)_t = \log(r/p)_t + u_{2t}^+$$

Combining the two kinds of the error terms,

$u_{1t} = u_{1t}^* + u_{1t}^+$: disturbance term in labor input decision function
$u_{2t} = u_{2t}^* + u_{2t}^+$: disturbance term in capital input decision function

The above disturbance terms can be more explicitly specified in the input decision function by using the combined form of partial adjustment and adaptive expectation schemes.
The general formula for this is as follows:

Let $X = \log r/w$, $Y = \log F_k/F_1$

X_t^* : expected X for time t Y_t^* : desired Y for time t

(2.10) $Y_t - Y_{t-1} = \alpha(Y_t^* - Y_{t-1})$: partial adjustment scheme

(2.11) $X_t^* - X_{t-1}^* = \beta(X_{t-1} X_{t-1}^*)$: adaptive expectation scheme [8]

(2.12) $Y_t^* = X_t^*$

where

α : partial adjustment coefficient
β : adaptive expectation coefficient

$$0 < \begin{bmatrix}\alpha \\ \beta\end{bmatrix} \leq 1$$

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8 This scheme was originally used by Prof. P. Cagan in his "The Monetary Dynamics of Hyper-Inflations," in M. Friedman (ed.), Studies in the Quantity Theory of Money, University of Chicago Press, 1956.

As is well known, equation (2.10) indicates that the actual level of adjustment is a fraction of the desired level of adjustment, and equation (2.11) indicates that an adustment in expectation is based on the deviation of the last actual value from the previous expectation, where the adjustment value is a fraction of the deviation.

Equation (2.11) is equivalent to

$$(2.13) \quad X_t^* = \sum_{i=1}^{\infty} \beta(1-\beta)^{i-1} X_{t-i} \qquad \text{(Appendix A: 5)}$$

Thus, the expected value of X is a geometrically weighted average of all previous actual value of X, while the actual value of Y is a geometrically weighted average of all previous desired value of Y.

Equations (2.10), (2.11), (2.12) can be reduced to

$$(2.14) \quad Y_t = \alpha\beta X_{t-1} + [2-(\alpha+\beta)] Y_{t-1} - [1+\alpha\beta-(\alpha+\beta)] Y_{t-2}$$ [9]

Eventhough simplicity and easiness in the estimation procedure is an additional advantage of the geometrically weighted distributed lag scheme compared with other schemes, the estimation of equation (2.14) involves a number of difficulties; the biased estimates due to the lagged dependent variable as an explanatory variable and under-identification of each seperate parameter.[10]

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[9] Equation (2.14) becomes the adaptive expectation model when $\alpha=1$, and the partial adjustment model when $\beta=1$.

[10] Johnston, J., op. cit., p. 220.

In this study, the adaptive expectation scheme will be used to identify each relevant parameter by leaving out the partial adjustment possibility on the ground that the former is more significant than the latter.[11]

This can be done by assigning unit value to the partial adjustment coefficient. Then, the equation (2.14) becomes,

$$Y_t = \beta X_{t-1} + (1-\beta)\, Y_{t-1}$$

that is

$$(2.15) \quad \log(F_k/F_l) = \beta \log(r/w)_{t-1} + (1-\beta) \log(F_k/F_l)_{t-1}$$

Which is a short-run expansion path function based on the adaptive expectation hypothesis.

The assumption of unit partial adjustment coefficient, however, is too restrictive.

An alternative method, even though not quite satisfactory, is to specify the adaptive expectation scheme in the stochastic input decision function, letting the disturbance term in the function reflect the random partial adjustment effect. (c.f. equation (2.20))

On the other hand, in the ealier section the production function

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11 R. Waud found the statistical bias due to such a misspecification was not significant.
"Small Sample Bias due to Misspecification in the Partial Adjustment and Adaptive Expectation Models," Journal of American Statistical Association, 1966, pp. 1130-52.

was treated as a nonstochastic one, which will now be specified as a stochastic production function. The resulting stochastic production function system is as follows:

$$(2.16)\quad Q = A[\, d(e^{g_1 t} K)^{-s} + (1-d)(e^{g_2 t} L)^{-s}\,]^{-1/s}\, u_o$$

$$(2.17)\quad F_k = (Au_o)^{-s}\, d e^{-sg_1 t} K^{-1/\sigma} Q^{1/\sigma} = (r/p)^* \, u_1^*$$

$$(2.18)\quad F_l = (Au_o)^{-s}\, (1-d)\, e^{-sg_2 t} L^{-1/\sigma} Q^{1/\sigma} = (w/p)^* \, u_2^*$$

$$(2.19)\quad F_k/F_l = [d/(1-d)]\, e^{s(g_2-g_1)t} (L/K)^{1/\sigma} = (r/w)^*[u_1^*/u_2^*]$$

u_o, u_1^*, and u_2^* bear similar meanings to the corresponding disturbance terms of Zellner-Kmenta-Dreze, except that the disturbance terms here might or might not be random.[12]

Rewriting equation (2.19)

$$\log [d/(1-d)] + s(g_2-g_1)t + 1/\sigma \log(L/K)_t = \log(F_k/F_l)_t$$

$$= \log(r/w)^*_t + \log\,(u_1^*/u_2^*)_t$$

That is

$$(2.20)\quad Y_t = X_t^* + V_t$$

where $V_t = \log(u_1^*/u_2^*)_t$

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12 The problems associated with the randomness or nonradomness of the disturbance terms will be discussed at the later part of this section.

From (2.20) and (2.11),

$$X_t^* - X_{t-1}^* = (X_{t-1} - X_{t-1}^*)$$
$$Y_t = X_t^* + V_t$$

By combining these two equations,

$$Y_t = \beta X_{t-1} + (1-\beta)Y_{t-1} + [V_t - (1-\beta)\, V_{t-1}]$$

That is

$$\text{(2.21)} \qquad \log(F_k/F_l)_t = \log(r/w)_{t-1} + (1-\beta)\log(F_k/F_l)_{t-1} + \sigma[V_t - (1-\beta)\, V_{t-1}]$$

From equations (2.20) and (2.21),

$$\text{(2.22)} \quad \log(L/K)_t = [\,\beta\sigma\log(1-d)/d + (1-\beta)(\sigma-1)(g_2-g_1)] + \beta\sigma\log(r/w)_{t-1} + (1-\beta)\log(L/K)_{t-1} + \beta(\sigma-1)(g_2-g_1)\, t + \sigma[V_t - (1-\beta)\, V_{t-1}]$$

$$\text{(2.23)} \quad \log RS_t = [\,\beta\sigma\log(1-d)/d + (1-\beta)(\sigma-1)(g_2-g_1)] + \beta\sigma\log(r/w)_{t-1} - \log(r/w)_t + (1-\beta)\log(L/K)_{t-1} + \beta(\sigma-1)(g_2-g_1)\, t + \sigma[V_t - (1-\beta)\, V_{t-1}]$$

The interpretation of all the parameters except β and the disturbance terms would be the same as the case of equtions (2.7) and (2.9). Since the coefficients of the technology term, t , in both equations (2.22) and (2.23) is the same and the former is simpler than the latter, equation (2.22) will be used for the actual estimations.

In the estimating equation (2.22), we can derive the estimates of β, σ, and (g_2-g_1) from the estimated coefficients. σ represents the elasticity of substitution with respect to the change in the expected relative factor price, holding the technology constant.
$\beta(\sigma-1)(g_2-g_1)$ measures the average annual percentage change in the relative share of labor to capital attributable to the technological change, holding the relative factor price constant.
The estimated coefficient $\beta\sigma$ provides us with the estimate of the elasticity of substitution with respect to the change in the expected relative factor price when $\beta\sigma$ is divided by β, and $\beta\sigma$ measures the elasticity of substitution with respect to the change in the most recently observed relative factor price holding all the previously observed relative factor prices constant.
Since the adjustment coefficient, β, is positive and less than or equal to unity, the elasticity of substitution with respect to the change in the expected relative factor price is larger than or equal to that with repect to the change in the most recently observed relative factor price.

The exclusion of the current relative factor price, which is an expost value, from the right hand side of the equation is more in line with the ex-ante input decision making equation, and it also precludes a possible complication due to a simultaneous relation between the current input factor ratio and the current relative factor price. When the current relative factor price is included in the equation, the equation would contain two endogenous variables, i.e. $\log(L/K)_t$ and

$\log (r/w)_t$, because an adjustment in the input ratio based on the change in the relative factor price also can affect the current relative factor price. The exclusion of it would leave only the past relative factor prices, which are predetermined variables, on the right hand side of the equation, thus excluding the possibility of the simultaneous equation bias.

Someone might argue that the current relative factor price can be included in the computation of the current expected relative factor price on purely statistical grounds. Since we are considering such a time unit as one calendar year and using as the datum the average annual relative factor price, it might not be unreasonable to argue that the relative factor prices of the early parts of the period also enter the computational procedure of the expected relative factor price for the current time period.

According to this reasoning, the equation (2.13) becomes

$$(2.24) \quad X_t^* = \sum_{i=0}^{\infty} \beta(1-\beta)^i \; X_{t-i}$$

Let D be the delay or lag operator.

Then, $X_{t-i} = D^i X_t$, $(i=0,1,2,\dots,\infty)$.

Equation (2.24) becomes equivalent to

$$(2.25) \quad X_t^* = \sum_{i=0}^{\infty} \beta(1-\beta)^i D^i X_t = \frac{\beta X_t}{1-(1-\beta) D}$$

By rearranging this we obtain

$$(2.26) \quad X_t^* - X_{t-1}^* = \beta (X_t - X_{t-1}^*)$$

This is mathematically the same as the equation (2.11) except X_t term in the parenthesis of the equation. However, in such a formulation, the current X value is no longer of the same concept as all the past X values. Therefore, the inclusion of the current relative factor price, X_t , in the estimating equation is not advisable, and, thus, it will not be included in our estimating equation.

From the estimating equation (2.22), the changes in all the past observed relative factor prices represented by the changes in $\log(r/w)_{t-1}$ and $\log(L/K)_{t-1}$ will affect the expected relative factor price for the current period and thus the current relative share through the substitution effect.

On the other hand, the change in $\log(r/w)_t$ will influence the relative share to the opposite direction directly through the accounting effect.

The accounting effect associated with the change in the currently observed relative factor price would be reflected in the relative share for the current period, whereas the substitution effect of it will be reflected in the factor ratios or the relative share for the future periods.

Thus, there exists a difference in the timing of the two effects, and the relative share for the current period will be affected by the accounting effect associated with the change in the current relative factor price and also by the substitution effect associated with the change in the current expected relative factor price which is the geometrically weighted average of all the past observed relative factor prices.

The substitution effect associated with the change in the expected relative factor price, which is the concept of the substitution effect to be used for the quantification of the effect in the subsequent chapters of the empirical analyses, is

$$\beta\sigma\left[\log(r/w)_{t-1} - \log(r/w)_{t-2}\right] + (1-\beta)\left[\log(L/K)_{t-1} - \log(L/K)_{t-2}\right]$$

The accounting effect associated with the change in the observed relative factor price is

$$-\left[\log(r/w)_{t} - \log(r/w)_{t-1}\right]$$

The net effect can be shown in the following expression by adding these two effects:

$$\beta\sigma\left[\log(r/w)_{t-1} - \log(r/w)_{t-2}\right] + (1-\beta)\left[\log(L/K)_{t-1} - \log(L/K)_{t-2}\right] - \left[\log(r/w)_{t} - \log(r/w)_{t-1}\right]$$

The relative share of labor to capital will rise, fall, or remain constant, cetris paribus, depending upon whether the above net effect is positive, negative, or equal to zero.

Now, we can consider some econometric problems related with the estimating equation developed in this section.
The estimating equation (2.22) based on the adaptive expectation scheme carries some econometric problem due to the lagged dependent variable on the right hand side of the estimating equation.
When the residuals of the estimating equation are serially independent, we can obtain biased but consistent estimates of the coefficients.
However, when the residuals are autocorrelated, the estimates are biased and inconsistent.
Furthermore, we can not then use Durbin-Watson statistic to test the existence of autocorrelation, because the Durbin-Watson statistice itself in such a case is badly biased toward randomness.
Therefore, the application of the Ordinary Least Square method to such an estimating equation should be carefully considered.

This problem can be examined in the following way:

From equation (2.20),

$$Y_t = X_t^* + V_t \quad , \quad \text{where} \quad V_t = \log(u_1^* / u_2^*)_t$$

Assuming that the disturbance term V_t follows a first order autoregressive process,

$$V_t = a\, V_{t-1} + e_t$$

where "e" is a random disturbance term, and "a" is the first order autocorrelation coefficient.

In equation (2.22), the new disturbance term "U" after the transformation of the equation is

$$U_t = \sigma(V_t - (1-\beta)\, V_{t-1})$$

Since $(1-\beta)$ is positive, this relation indicates that the autoregressive transformation of the equation itself had introduced some degree of a negative autocorrelation.

Combining these two error relations,

$$U_t = \sigma[\, 1- (1-\beta)\, D\,]\, (1- aD)^{-1}\, e_t$$

where D is a lag operator.

When V_t is random,

$U_t = \sigma[\, 1- (1-\beta)\, D\,]\, e_t$; U_t is negatively autocorrelated.

When V_t are positively autocorrelated,[13]

If $(1-\beta) = a$, U_t is random.

If $(1-\beta) > a$, U_t is negatively autocorrelated.

If $(1-\beta) < a$, U_t is positively autocorrelated.

Thus, U_t might be random, but not quite likely.

Malinvaud points out that in spite of the imperfection revealed by the above analysis, the Ordinary Least Squares applied to the auto-regressive form is often the best method for estimating a model the coefficients of which have geometric or a Pascal Distribution.[14]

In this study, Durbin's two-step procedure will be used, whenever appropriate, to obtain consistent estimates, assuming that U_t has a first order autocorrelation[15] even though Malinvaud provides us with an excuse to apply the Ordinary Least Square method without any adjustment for an autocorrelation to autoregressive estimating equation with autocorrelated disturbance term.

Durbin's method is chosen among various methods of obtaining consistent estimates, because it is relatively simple and gives estimates that have asymptotically same mean vector and variance matrix as those obtained

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[13] Malinvaud, E., Statiscal Methods of Econometrics, Ch. 14, Rand McNally and Co., Chicago, 1966.

[14] Ibid., pp. 489-491.

[15] When we carefully examine the structure of the disturbance term, U, we are likely to find that it has an autocorrelation of higher order. Therefore, the application of the Durbin's method with the assumption of the first order autocorrelated disturbance term, U, should be interpreted as an approximation to the true model. This qualification is also applicable to the models developed in the subsequent sections.

by the Generalised Least Square method.[16]

When Durbin's two-step procedures are applied to the adaptive expectation model in this study, the first step is

$$(2.27)\ \log (L/K)_t = [\sigma\beta(1-\rho)\log(1-d)/d + (\sigma-1)(g_2-g_1)(1-\beta-\rho+2\beta\rho)] + \beta\sigma\log(r/w)_{t-1} - \rho\beta\sigma\log(r/w)_{t-2} + \beta(\sigma-1)(g_2-g_1)(1-\rho)\,t + (1-\beta+\rho)\log(L/K)_{t-1} - \rho(1-\beta)\log(L/K)_{t-2} + \sigma e_t$$

$$\hat{\rho} = -(-\widehat{\rho\beta\sigma}/\widehat{\beta\sigma}) \qquad , \quad \text{(Appendix A : 7)}$$

The second step is

$$(2.28)\ [\log(L/K)_t - \hat{\rho}\log(L/K)_{t-1}] = [\sigma\beta(1-\hat{\rho})\log(1-d)/d + (\sigma-1)(g_2-g_1)(1-\hat{\rho})(1-\beta)] + \beta\sigma[\log(r/w)_{t-1} - \hat{\rho}\log(r/w)_{t-2}] + \beta(\sigma-1)(g_2-g_1)[t - \hat{\rho}(t-1)] + (1-\beta)[\log(L/K)_{t-1} - \hat{\rho}\log(L/K)_{t-2}] + \sigma e_t$$

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16 Durbin, J., "Estimation of the Parameters in Time-series Regression Models," Journal of the Royal Statistical Society, Series B, 1960. Taylor-Wilson's Three-Pass Least Square method is an alternative, which, however, is valid only under the assumption that the explanatory variable is serially independent.
N. Liviatan's instrumental variable technique, which is tantamount to the Two-Stage Least Square method, is another alternative, which , however, is relatively inefficient.
Taylor, L., and T. Wilson, "Three-Pass Least Squares: A Method for Estimating Models with a Lagged Dependent Variable," Review of Economics and Statistics, 1964. Liviatan, N., "Consistent Estimation of Distributed Lags," International Economic Review, 1963.

For the industries, which have severe multicollinearity problem, the method of obtaining some extraneous information can be used as discussed in the previous section.

The cross-section estimates of the elasticities of substitution for the relevant industries would be applied to the time series estimating equation for the transformations of the involved variables.

As we can see from equation (2.28), however, the desired transformation can not be carried out without the knowledge of the adjustment coefficient , i.e. β , as well.

One way to solve this problem may be impose a priori restriction on the adjustment coefficient, e.g. $\beta = 1$.

An alternative way, which seems to be more reasonable, is an application of the Monte Carlo technique. Based on the basic condition that the value of the adjustment coefficient should be positive and less that or equal to unity, i.e. $0 < \beta \leq 1$, we can try the various values of the adjustment coefficient ranging from 0.1 through 0.9 to choose the value which gives us the highest coefficient of determination.

Then, the transformed equation of the equation (2.28) with which the Monte Carlo technique is going to be tried is as follows:

$$(2.29)\ \Big[\, (\log(L/K)_t - \hat{\rho}\log(L/K)_{t-1}) - \hat{\beta}\hat{\sigma}(\log(r/w)_{t-1} - \hat{\rho}\log(r/w)_{t-2}) - (1-\hat{\beta})(\log(L/K)_{t-1} - \hat{\rho}\log(L/K)_{t-2})\Big]$$

$$= \Big[\, \hat{\sigma}\hat{\beta}(1-\hat{\rho})\log(1-d)/d + (\hat{\sigma}-1)(g_2-g_1)(1-\hat{\rho})(1-\hat{\beta})\Big] + \hat{\beta}(\hat{\sigma}-1)(g_2-g_1)[t-\hat{\rho}(t-1)] + \hat{\sigma}\, e_t$$

$$\text{where}\quad \hat{\rho} = 0.1,\ 0.2, \ldots\ldots\ldots,\ 0.9$$

2.3 Short-run Dynamic Model Based on an Extension of the Adaptive Expectation Scheme: Model 2

In the adaptive expectation scheme, the expected relative factor price was the weighted average of all the past relative factor prices where the weights were geometrically declining. Therefore, this scheme assigns the heaviest weight to the most recent value and the weights decline geometrically as the time goes back. In practice, however, the heaviest weight might or might not fall on the most recent value, and the weight pattern might be more complicated than a monotonically declining weight such as the geometrically declining weight.

There are various schemes which accommodate more flexible weight patterns. Shirly Almon's distributed lag scheme based on the Lagrangian Interpolation Polynomial technique is quite promissing candidate for such a purpose.[17] Unfortunately, however, when Almon's lag scheme is applied to equation (2.7), the estimating equation faces under-identifiability. (Appendix A : 6)

Robert Solow's Pascal Distributed Lag scheme has a simple expression based on two parameters, and , yet, is a quite flexible in tracing various weighting patterns.[18]

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[17] "The Distributed Lag between Capital Appropriations and Expenditures," Econometrica, 1965.

[18] Solow, R., "On a Family of Lag Distributions," Econometrica, 1960.

Nevertheless, this scheme involves the estimation of the parameters in nonlinear form and the estimates are very sensitive to the specification error of the order of the scheme.

A distributed lag function which is more general and yet does not involve the estimation problem faced by the Solow's Pascal Distributed Lag scheme is Jorgenson's Rational Distributed Lag function.[19] The class of rational distributed lag function is defined by the condition that the sequence $[\beta_i]$ coefficients have a rational generating function. This class of distributed lag function has the properties that an arbitrary distributed lag function may be approximated to any desired degree of accuracy by a member of this class and that the number of parameters required for a satisfactory approximation is less than that required for an equally good approximation by a finite distributed lag function.

Consider the distributed lag function

$$(2.30)\quad Y_t = \sum_{i=1}^{\infty} \beta_i \; X_{t-i} + V_t$$

We impose the restrictions that the sequence $[\beta_i]$ corresponds to the probability distribution of a nonnegative, integer valued, random variable, so that:

$$\beta_i \geq 0 \qquad (i= 1,2,3,.......)$$

$$\sum_{i=1}^{\infty} \beta_i = 1$$

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[19] Jorgenson, Dale W., "Rational Distributed Lag Functions," Econometrica, 1966, pp.135-149.

Then, we may write

$$(2.31)\quad Y_t = \frac{A(D)}{B(D)} X_{t-1} + V_t$$

Where D is a lag operator.

$$A(D) = \quad a_1 + a_2 D + a_3 D^2 + \ldots..$$
$$B(D) = 1 + b_1 D + b_2 D^2 + b_3 D^3 + \ldots..$$

Equation (2.31) can be rewritten as

$$(2.32)\ B(D)\, Y_t = A(D)\, X_{t-1} + B(D)\, V_t$$ [20]

Which is

$$Y_t + b_1 Y_{t-1} + b_2 Y_{t-2} + \ldots + b_n Y_{t-n} = a_1 X_{t-1} + a_2 X_{t-2} + \ldots + a_n X_{t-n} + V_t + b_1 V_{t-1} + b_2 V_{t-2} + \ldots + b_n V_{t-n}$$

Equation (2.32) is called as the final form of a rational distributed lag function.

It can be shwwn that the adaptive expectation scheme is a well-defined and self-contained special case of the rational distributed lag function.

In equation (2.32), we take

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[20] Note that Jorgenson inserted the disturbance term in the final form of the rational distributed lag function, and simply assumed that the disturbance term had the conventional properties; randomness, finite variance and zero covariances.
This procedure seems to obsecure the nature of the disturbance term. When the disturbance term was included in the equation (2.31), the disturbance term in the final form (2.32) turned out to be a linear combination of the original disturbance term.

$$A(D) = 1 - \lambda$$

$$B(D) = 1 - \lambda D \quad , \qquad \text{where} \quad \lambda = 1 - \beta$$

Then,

$$(1-\lambda D)Y_t = (1-\lambda)X_{t-1} + (1-\lambda D)V_t$$

Rearranging the equation,

$$Y_t = \beta X_{t-1} + (1-\beta)Y_{t-1} + [V_t - (1-\beta)V_{t-1}]$$

Which is the familiar adaptive expectation model.

The more Y and X terms we put into the final equation, the more accurate approximation we might get.
However, we also have to consider some significant limiting factors in doing so. The inclusion of more Y and X terms in a time series estimating equation increases the possibility of multicollinearity, and decreases the degrees of freedom.
Therefore, it is doubtful that the experiments with higher order of lag structures than the following one, which is a simple extension of the adaptive expectation scheme, would be worthwhile.

Let $A(D) = a$

$$B(D) = 1 - b_1 D - b_2 D^2$$

When the above relations are applied to equation (2.32),

$$(2.33) \quad Y_t = a X_{t-1} + b_1 Y_{t-1} + b_2 Y_{t-2} + [V_t - b_1 V_{t-1} - b_2 V_{t-2}]$$

Which is a second order difference equation of Y.

Even though this is a simplest extension of the adaptive expectation scheme, this will give us a notable flexibility in the pattern of the distributed lag weights, and, thus, in the present study only this lag scheme will be tried, in addition to the adaptive expectation scheme, to estimate the relevant parameters.

In order for the equation (2.33) to have a non-negative lag distribution function for X,

$$Y_t = \frac{a}{1 - b_1 - b_2} \sum_{i=1}^{n} \beta_i \; X_{t-i} \quad , \text{ where } \beta_i \geq 0$$

b_1 and b_2 must satisfy the following conditions:

$$1 - b_1 - b_2 > 0 \; , \qquad 2 > b_1 > 0 \; , \qquad b_1^2 + 4b_2 \geq 0$$

The first two conditions imply

$$1 > b_2 > -1$$

In order for the equation to satisfy the other restriction, i.e. $\sum_{i=1}^{n} \beta_i = 1$, we will have the following relation:

$$\sum_{i=1}^{n} \beta_i = A(1)/B(1) = 1$$

Therefore $A(1) = B(1)$

That is $a = 1 - b_1 - b_2$

When this distributed lag function is introduced into (2.22), the expansion path function, the final estimating equation becomes

$$(2.34)\ \log(L/K)_t = [\sigma(1-b_1-b_2)\log(1-d)/d + (\sigma-1)(g_2-g_1)(b_1+2b_2)] + a\sigma\log(r/w)_{t-1} + (1-b_1-b_2)(\sigma-1)(g_2-g_1)t + b_1\log(L/K)_{t-1} + b_2\log(L/K)_{t-2} + \sigma(V_t - b_1V_{t-1} - b_2V_{t-2})$$

The equation is just identified.

This equation also contains the problem of autocorrelated disturbance term and lagged dependent variables on the right hand side of the estimating equation.

We can apply to this equation the same adjustment procedure as the one proposed for the adapitve expectation model at the expense of one degree of freedom.

The Durbin's first step applied to equation (2.34) is

$$(2.35)\ \log(L/K)_t = [\sigma(1-b_1-b_2)(1-\rho)\log(1-d)/d + (\sigma-1)(g_2-g_1)(b_1+2b_2+\rho-2b_1\rho-3b_2\rho)] + a\sigma\log(r/w)_{t-1} - \rho a\sigma\log(r/w)_{t-2} + (\sigma-1)(g_2-g_1)(1-b_1-b_2)(1-\rho)t + (b_1+\rho)\log(L/K)_{t-1} + (b_2-b_1\rho)\log(L/K)_{t-2} - b_2\rho\log(L/K)_{t-3} + \sigma e_t$$

The estimate of ρ can derived from the estimates of the coefficients of $\log(r/w)_{t-1}$ and $\log(r/w)_{t-2}$ by dividing the latter by the former and changing the sign of the resulting quotient.

$$\hat{\rho} = -(-\widehat{\rho a\sigma}/\widehat{a\sigma})$$

The second step is

$$(2.36)\quad [\log(L/K)_t - \hat{\rho}\log(L/K)_{t-1}] = [\sigma(1-b_1-b_2)(1-\hat{\rho})\log(1-d)/d + (\sigma-1)(g_2-g_1)(1-\hat{\rho})(b_1+2b_2)] + a\sigma[\log(r/w)_{t-1} - \hat{\rho}\log(r/w)_{t-2}] + (\sigma-1)(g_2-g_1)(1-b_1-b_2)[t-\hat{\rho}(t-1)] + b_1[\log(L/K)_{t-1} - \hat{\rho}\log(L/K)_{t-2}] + b_2[\log(L/K)_{t-2} - \hat{\rho}\log(L/K)_{t-3}] + \sigma e_t$$

β 's can be computed from the estimated coefficients, a, b_1, and b_2 in the following way:[21]

$$(2.37)\quad \beta_i = b_1\beta_{i-1} + b_2\beta_{i-2}$$

Where the initial condition is

$$\beta_1 = 1 - b_1 - b_2 = a$$

For the industries with severe multicollinearity, the same method of treatment as that used in the earlier section can be tried.

The final transformed estimating equation incorporating the cross-section estimates of the elasticities of substitution and the various values of the weight parameters, b_1 and b_2, would be as follows:

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[21] Griliches, Zvi, "Distributed Lag : A Survey," Econometrica, January, 1967, p.23.

$$(2.38)\quad [(\log(L/K)_t - \hat{\rho}\log(L/K)_{t-1}) - \hat{a}\hat{\rho}\,(\log(r/w)_{t-1} - \hat{\rho}\log(r/w)_{t-2}) - \hat{b}_1\,(\log(L/K)_{t-1} - \hat{\rho}\log(L/K)_{t-2}) - \hat{b}_2\,(\log(L/K)_{t-2} - \hat{\rho}\log(L/K)_{t-3})] = [\hat{\sigma}(1-b_1-b_2)(1-\hat{\rho})\log(1-d)/d + (\hat{\sigma}-1)(g_2-g_1)(1-\hat{\rho})(b_1+2b_2)] + (\hat{\sigma}-1)(g_2-g_1)(1-\hat{b}_1-\hat{b}_2)[t - \hat{\rho}(t-1)] + \hat{\sigma} e_t$$

Where $\hat{b}_1 = 0.1,\ 0.2, \ldots\ldots\ldots\ldots\ldots,\ 2.0$

$\hat{b}_2 = -0.9,\ -0.8, \ldots\ldots, 0.0, \ldots\ldots, 0.9$

The relative share relation from the equation (2.34) is

$$(2.39)\quad \log RS_t = \log(wL/rK)_t = [\sigma(1-b_1-b_2)\log(1-d)/d + (\sigma-1)(g_2-g_1)(b_1+2b_2)] + a\sigma\log(r/w)_{t-1} - \log(r/w)_t + b_1\log(L/K)_{t-1} + b_2\log(L/K)_{t-2} + (1-b_1-b_2)(\sigma-1)(g_2-g_1)\,t + \sigma(V_t - b_1V_{t-1} - b_2V_{t-2})$$

The substitution effect associated with the change in the expected relative factor price on the relative factor share is

$$a\sigma[\log(r/w)_{t-1} - \log(r/w)_{t-2}] + b_1[\log(L/K)_{t-1} - \log(L/K)_{t-2}] + b_2[\log(L/K)_{t-2} - \log(L/K)_{t-3}]$$

The accounting effect associated with the change in the observed relative factor price on the relative factor share is

$$- [\ \log(r/w)_t - \log(r/w)_{t-1}]$$

The net effect is

$$[\ a\sigma[\ \log(r/w)_{t-1} - \log(r/w)_{t-2}] + b_1 [\ \log(L/K)_{t-1} - \log(L/K)_{t-2}]$$
$$+ b_2[\ \log(L/K)_{t-2} - \log(L/K)_{t-3}]] - [\ \log(r/w)_t - \log(r/w)_{t-1}]$$

The relative share of labor to capital will increase, decrease, or remains constant depending upon the above net effect is positive, negative or equals to zero, cetris paribus.

2.4 Possible Complications Put Aside

2.4.1 Imperfect Competition

The neoclassical assumption of perfect competition in the product and factor markets seem to be very restrictive. It is tempting to examine the consequence of the relaxation of the assumption within the context of this study. The modified equilibrium marginal conditions are as follows:

(2.40) $F_k = r\ (\ 1 + 1/v_2\ )\ /\ p(\ 1 + 1/v_1)$

(2.41) $F_1 = w\ (\ 1 + 1/v_3\ )\ /\ p(\ 1 + 1/v_1)$

(2.42) $F_k/F_1 = r\ (\ 1 + 1/v_2\ )\ /\ w(\ 1 + 1/v_3) = (r/w)'$

where $(r/w)'$: relative marginal cost of capital to labour

v_i ; (i= 1, 2, 3) : the demand elasticity in the product market, the supply elasticity in the capital market, and the supply elasticity in the labor market

Equation (2.42) shows that the degree of competition in the product market does not have any direct influence on the marginal productivity ratio or the cost minimization condition.
Then, we can substitute the relative marginal factor cost for the relative factor price in the family of equation (2.7) to get the modified estimating equations.
If we can get some satisfactory extraneous estimates of the elasticities, we might be tempted to use it in this modified estimating equation. However, a satisfactory estimation of these elasticities itself is a very difficult task.

On the other hand, if the competitions in both factor markets have the same degree of imperfection, i.e. if the supply elasticities in both factor markets are the same, equation (2.42) becomes the original equation formulated under the assumption of perfect competition. The reality of the assumption that both factor market have equal degree of competition is an empirical question. On a priori ground, it seems that this result widens the applicable area of the estimating equation based on the assumption of perfect competition.

Another possible complication which was not explicitly taken into the model is the changes over time in the degree of competition in the factor markets. In most empirical studies this element has been put aside because of the difficulty in specifying and estimating it. This study can not be an exception to it at this stage.
On a priori ground, however, it seems that the bias in the estimation due to the misspecification is not going to be so significant as to discourage the estimation itself, particularly because the absolute degrees of competition in the factor markets might not have changed drastically, and , furthermore, the relative degree of competition in these markets might be more stable than the absolute one.
On the empirical ground, it is not unreasonable to get the impression that the absolute degree of competition in the labour market is quite drastically changing over time to the direction of the falling supply elasticity, i.e. increasing degree of imperfect competition, due to the rapid expansion of labour union in the recent decades.

However, there is a piece of information which does not support this view.
Leo Troy presents a set of data on the union membership as a percent of civilian labour force and of nonagricultural employees.[22]

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[22] Unfortunately no consistent time series on the union membership by industry is readily available.

As we can see from the Table 1,[23] the union membership as the percent of the nonagricultural employees has not really risen during the postwar period.

The percentage changes in this set of data are quite minor, and union membership as the percent of the nonagricultural employees had actually declined from 1957 on.

Table 1

Extent of Union Organization

Year	Union membership as % of nonagricultural employees	Year	Union membership as % of nonagricultural employees
1947	31.8	1955	31.7
1948	31.9	1956	31.3
1949	31.9	1957	31.4
1950	31.2	1958	30.4
1951	31.3	1959	28.9
1952	31.6	1960	28.6
1953	32.7	1961	27.9
1954	32.0	1962	26.7

23 Troy, Leo, "Trade Union Membership, 1897-1962," Review of Economics and Statistics, 1965, p. 94.

The variations in the figures of the union membership by industry might show some diverse patterns. Nevertheless, the movement of the union membership as the percent of the nonagricultural employees gives the researcher some degree of additional relief from the fear of misspecification.

2.4.2 Aggregation Bias

It is not easy to justify the application of the marginal productivity theory at the two-digit industry level even though this is a quite common practice and is still better than other numerous empirical studies which applied the theory to higher degree of aggregate segment of the economy. As mentioned before, the data limitation is the chief obstacle in scaling down to the finer detailed industry classification.

At the statistical level, Klein-Nataf condition[24] of aggregating micro-variables requires geometric averages of the micro-data, whereas most published data are arithmetic averages.[25] The bias due to such a misspecification is not clear, and H. Theil's formulation of measuring the aggregation bias[26] is not operational yet.[27]

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[24] Nataf, A., "Sur la Possibilité de construction de Certains Macremodeles," Econometrica, 1950, pp.232-244.

[25] See Appencix A : 8 .

[26] Theil, H., Linear Aggregation of Economic Relations, North-Holland Publishing Co., Amsterdam, 1954.

[27] See Appendix A : 9 .

CHAPTER 3

EMPIRICAL ESTIMATION OF THE PARAMETERS IN THE MODEL

3.1. Food and Kindred Products (SIC No. 20)

The estimating equation based on the adaptive expectation scheme, the Model 1, unadjusted for the autocorrelation is

$$(2.22)\quad \log(L/K)_t = 1.460 - \underset{(1.497)}{.072} \log(r/w)_{t-1} + \underset{(3.069)}{.634} \log(L/K)_{t-1} - \underset{(2.032)}{.010}\, t$$

$\bar{R}^2$ (adjusted coefficient of determination): .944

The numbers in the parentheses are the values of t statistic for the estimated parameters.

The elasticity of substitution with respect to the change in the expected relative factor price, i.e. σ, the adaptive expectation coefficient, i.e. β, and the degree of bias in the factor augmenting technological change, i.e. $(g_2 - g_1)$, can be derived from the estimated coefficients shown above.

σ	β	$(g_2 - g_1)$
-.198	.366	.023

The adaptive expectation coefficient satisfies the condition, $0 < \beta \leq 1$, and the degree of the bias in the technological change is around the expected magnitude.

However, the estimated elasticities of substitution with respect to the change in the observed and expected relative factor price, i.e. $\beta\sigma$ and σ, are negative, which can not be justified at the level of economic theory, because the negative elasticity of substitution indicates that more labor will be hired in place of capital when the relative price of labor rises, cetris paribus.

The alternative estimating equation based on the extended adaptive expectation model, the Model 2, unadjusted for the autocorrelation still produced the negative elasticities of substitution.

$$(2.34)\quad \log(L/K)_t = 1.442 - \underset{(.941)}{.050}\log(r/w)_{t-1} + \underset{(2.845)}{.856}\log(L/K)_{t-1} - \underset{(.762)}{.230}\log(L/K)_{t-2} - \underset{(1.885)}{.010}\,t$$

$$\bar{R}^2 : .935$$

The estimated coefficiant of the technology term is the same order of magnitude as that estimated in the Model 1.
Nevertheless, the negative elasticity of substitution precludes any further interpretation of the estimated results, although the estimated elasticity of substitution is not significantly different from zero at .05 level.

Even the Models 1 and 2 adjusted for the autocorrelation did not improve the situation.

To examine the case more closely, a regression was run using the conventional estimating equation, which does not contain the technology term and , thus, is subject to the specification error as discussed in the previous chapter.

$$\log(L/K)_t = 1.443 + \underset{(2.94)}{.32} \log(r/w)_{t-1}$$

$\bar{R}^2$: .34

This regression gave us the positive elasticity of substitution even though the estimated elasticity is biased due to specification error.

As soon as the technology term was included in the same equation, the estimated elasticity of substitution became negative.

$$\log(L/K)_t = -.012 - \underset{(1.59)}{.03} \log(r/w)_{t-1} - \underset{(8.71)}{.02}\, t$$

$\bar{R}^2$: .89

The dependence of $\log(L/K)_t$ on the explanatory variables expressed by $\bar{R}^2$ is drastically improved by including the technology term. However, the high intercorrelation between the two explanatory variables, $r = .7$, was responsible for the negative value for the elasticity of substitution.

The problem seems to be that of so called "harmful multicollinearity" which is generally defined as the cause of "wrong sign" or other symptoms of nonsense regressions.[1]

As a practical remedy, the extraneous information method with a Monte Carlo technique described at the end of the Section 2.2. was introduced into the Model 1 unadjusted for the autocorrelation.

$$\begin{aligned}[\log(L/K)_t - \hat{\beta}\hat{\sigma}\log(r/w)_{t-1} - (1-\hat{\beta})\log(L/K)_{t-1}] = \\ [\hat{\beta}\hat{\sigma}\log(1-d)/d + (1-\hat{\beta})(\hat{\sigma}-1)(g_2-g_1)] + \\ \hat{\beta}(\hat{\sigma}-1)(g_2-g_1)\, t + \hat{\sigma}\, e_t\end{aligned}$$

$$\hat{\beta} = 0.1,\ 0.2, \ldots\ldots\ldots\ldots, 0.9$$

Dhrymes' cross-section estimate of the elasticity of substitution from the interstate regression in 1957 was used as the extraneous information in our time series estimating equation, because his cross-section regression was based on the CES production function, and his estimated elasticities of substitution for the two-digit manufacturing industries are closest to the ones estimated in the estimating equations in this study.[2]

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1 Farrar, D.E. and R.G. Glauber, "Multicollinearity in Regression Analysis: The Problem Revisited," Review of Economics and Statistics, Feb., 1967, p.98.

2 Dhrymes, P.J., "Some Extensions and Tests for the CES Class of Production," Review of Economics and Statistics, Nov., 1965, pp.357-366. For summary table of the cross-section estimates of the elasticity of substitution obtained by various researcher, see Nerlove, Marc, op. cit. p.103.

Among the regressions with the varying values of the adaptive expectation coefficients from 0.1 to 0.9, the one with $\hat{\beta} = 0.9$ gave us the best fit in terms of the highest coefficient of determination.

$$[\log(L/K)_t - .504 \log(r/w)_{t-1} - .100 \log(L/K)_{t-1}]$$
$$= 1.026 - \underset{(.611)}{.003} \; t$$

R^2 : .161

The estimated coefficient of the explanatory variable t is not statistically significant, and the impact of the technological change on the labor-capital ratio and the relative share of labor to capital does not seem to be significant. Therefore, even though the estimated coefficient of the technology term is in the reasonable range, the interpretation of the parameters for this industry should be made with caution.

In this industry, a very heavy weight was assigned to the most recently observed relative factor price and the subsequent weights for all the previous relative factor prices declined very rapidly as time went back.

The influence of the relevant forces on the relative factor share for this industry is shown in the Table 2 . All the figures in the table are in percentage term.

TABLE 2

CONTRIBUTIONS OF THE RELEVANT FORCES TO THE CHANGES IN THE RELATIVE SHARE OF LABOR TO CAPITAL

YEAR	(1)	(2)	(3)	(4)	(5)	(6)
1948-49	-4.114	12.991	7.645	17.105	24.751	-0.300
1949-50	-5.319	14.541	-9.032	19.860	10.827	-0.300
1950-51	0.013	19.507	-10.541	19.493	8.952	-0.300
1951-52	0.065	-17.171	-9.823	-17.237	-27.061	-0.300
1952-53	4.456	3.492	8.694	-0.964	7.730	-0.300
1953-54	-3.847	10.254	0.931	14.101	15.033	-0.300
1954-55	-2.615	-17.551	-7.492	-14.936	-22.429	-0.300
1955-56	-1.565	27.498	7.266	29.063	36.330	-0.300
1956-57	-2.159	0.281	-14.804	2.440	-12.363	-0.300
1957-58	-3.870	-10.986	-1.446	-7.116	-8.563	-0.300
1958-59	-4.480	-5.823	3.199	-1.343	1.856	-0.300
1959-60	-0.372	10.001	0.228	10.373	10.602	-0.300
1960-61	-4.975	-29.432	-5.265	-24.457	-29.723	-0.300
1961-62	-3.725	-0.120	11.829	3.604	15.434	-0.300
1962-63	-3.130	24.447	-2.189	27.577	25.388	-0.300

(1) ANNUAL PERCENTAGE CHANGE IN LABOR-CAPITAL RATIO
(2) ANNUAL PERCENTAGE CHANGE IN THE RELATIVE SHARE OF LABOR TO CAPITAL
(3) SUBSTITUTION EFFECT OF THE CHANGE IN THE RELATIVE FACTOR PRICE
(4) ACCOUNTING EFFECT OF THE CHANGE IN THE RELATIVE FACTOR PRICE
(5) NET OF THE SUBSTITUTION AND ACCOUNTING EFFECTS
(6) EFFECT OF TECHNICAL CHANGE

From the column (2) of Table 2 , we can observe the relative share of labor to capital during the period, 1948-63, had generally risen, even though the labor-capital ratio in the column (1) had been generally declining with a few exceptional years in the early 1950's. Throughout the whole period, 1948-63, the relative share of labor to capital in this industry had increased by about 41.93 percent.
The column (3) of the table shows the substitution effect of the change in the expected relative price of capital to labor on the relative share of labor to capital, and the column (4) shows the accounting effect of the change in the observed relative factor price.
The column (5) presents the net of both the substitution and the accounting effect.

In general, the two effects had been working in the same direction with a few exceptional periods, and the net effect had been largely responsible for the variation of the relative share of labor to capital. Among the two effects, the accounting effect had been stronger than the substitution effect except five observations out of fifteen.

The annual influence of the technological change shown in the column (6) seems to have been rather minor compared to those of the change in the relative factor price.
However, by no means this indicates that the impact of the technological change had been insignificant in absolute sense, and that the impact of the technological change accumulated over time is insignificant relative to the impact of the changes in the relative factor price.

The absolute importance of the technological change can be shown in the following illustration:

Consider the year of 1954 which is in the middle of the period.

L: 3906.48 million manhours

W: 1.893 dollars

Then, the annual decrease of the labor-capital ratio by .3 percent is equivalent to a relative decrease of the labor input by about 11.72 million manhours a year, and , thus, a relative decrease of the labor share by about 23.43 million dollars per annum, which is significant.

The relative importance of the technological change over time can be acknowledged in view of the fact that the variation of the relative factor price shows alternations, whereas the technological changes are accumulative without alternations, thus, the accumulated impact of the latter can be as significant as that of the former.

The accumulated effect of the capital using technological changes would be a reduction of the relative share of labor to capital by about 4.5 percent.

3.2. Tobacco Manufactures (SIC No. 21)

The selected estimating equation among the alternative ones is the Model 1 adjusted for the autocorrelation.

The bases of the selection are the adjusted coefficient of determination and the precision of the estimation of the coefficients expressed by the standard errors of the estimates of the individual coefficients.

The estimated first order autocorrelation, $\hat{\rho}$, estimated by Durbin's method is .257.

$$(2.28)\ \left[\log(L/K)_t - .257\log(L/K)_{t-1}\right] = .688 + \underset{(1.487)}{.072}\left[\log(r/w)_{t-1} - .257\log(r/w)_{t-2}\right] + \underset{(4.946)}{.828}\left[\log(L/K)_{t-1} - .257\log(L/K)_{t-2}\right] - \underset{(1.923)}{.015}\left[t - .257(t-1)\right]$$

$\bar{R}^2$: .961

The coefficients of $\log(L/K)_{t-1}$ and t are significant at the .05 level, and that of $\log(r/w)_{t-1}$ is significant at the .10 level.[3]

The seperate parameters derived from the above estimates of the coefficients are

σ	β	(g_2-g_1)
.418	.172	.150

The low value of the adaptive expectation coefficient, β, indicates that the distributed lag weights do not decline rapidly as time goes back. This implies quite cautious and conservative

3 Probability of a deviation greater than t.

attitude to the changes in the relative factor price.

The value of .15 for (g_2-g_1) indicates that the technological change in this industry had been biased to labor, and that the annual exponential growth rate of the efficiency of labor was higher than that of capital by about 15 percentage points.
Since the elasticity of substitution is less than unity, the labor biased technological change, i.e. $(g_2-g_1)<0$, became capital using one, i.e. $\beta(\sigma-1)(g_2-g_1)>0$.

The contributions of the relevant forces to the variation of the relative share and the labor-capital ratio are shown in Table 3.

The column (2) of the table shows that the relative share of labor to capital in this industry had in general been declining with a some exceptional periods; 1949-50, 1953-56 and 1959-60.

In a majority of the cases, the substitution and accounting effects worked in the opposite direction, and the accounting efffect had generally been stronger than the substitution effect, and, thus, had been the dominant force among the ones affecting the variation of the relative share of labor to capital.

For the entire period, 1948-63, the relative share of labor to capital had declined by about 49.9 percent.

The technological change had also been capital using one, thus contributing to the declines of the relative share of labor to capital, and the average annual rate of decline in the relative share due to the capital using technological change is about 1.4 percent.

TABLE 3

CONTRIBUTIONS OF THE RELEVANT FORCES TO THE
CHANGES IN THE RELATIVE SHARE OF LABOR TO CAPITAL

YEAR	(1)	(2)	(3)	(4)	(5)	(6)
1948-49	-6.759	-20.720	-8.082	-13.960	-22.043	-1.450
1949-50	-0.753	20.961	-4.517	21.714	17.197	-1.450
1950-51	-2.795	-2.321	-1.136	0.474	-0.662	-1.450
1951-52	1.953	-2.298	-2.038	-4.252	-6.290	-1.450
1952-53	5.062	-22.525	1.530	-27.587	-26.056	-1.450
1953-54	0.067	9.859	4.418	9.791	14.210	-1.450
1954-55	-0.584	3.364	-0.217	3.949	3.731	-1.450
1955-56	-9.339	1.000	-0.530	10.339	9.808	-1.450
1956-57	-14.112	-9.315	-7.048	4.796	-2.251	-1.450
1957-58	-10.924	-8.475	-10.356	2.448	-7.907	-1.450
1958-59	-11.324	-10.495	-7.982	0.828	-7.154	-1.450
1959-60	-4.154	7.715	-8.227	11.870	3.642	-1.450
1960-61	-5.560	-6.756	-3.333	-1.196	-4.529	-1.450
1961-62	-6.764	-3.137	-3.996	3.626	-0.369	-1.450
1962-63	-10.665	-6.720	-4.999	3.944	-1.055	-1.450

(1) ANNUAL PERCENTAGE CHANGE IN LABOR-CAPITAL RATIO
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(3) SUBSTITUTION EFFECT OF THE CHANGE IN THE RELATIVE FACTOR PRICE
(4) ACCOUNTING EFFECT OF THE CHANGE IN THE RELATIVE FACTOR PRICE
(5) NET OF THE SUBSTITUTION AND ACCOUNTING EFFECTS
(6) EFFECT OF TECHNICAL CHANGE

In 1954, for example, the capital using technological change by 1.5 percent, cetris paribus, would be associated with the relative reduction in the labor input by about 2.57 million manhours a year, and, thus, with the relative reduction of the labor share by about 4.11 million dollars per annum.

The accumulated effect of the capital using technological change over the period of 1948 to 1963 would be a reduction of the relative share of labor to capital by 21.8 percent.

3.3. Textile Mill Products (SIC No. 22)

The selected estimating equation is Model 1 adjusted for the autocorrelation, which gave us the highest $\bar{R}^2$ and the lowest standard errors for the estimated coefficients.

$$(2.28)\quad [\log(L/K)_t + 1.0 \log(L/K)_{t-1}] = 1.518 + \underset{(2.944)}{.083}[\log(r/w)_{t-1} + 1.0 \log(r/w)_{t-2}] + \underset{(4.650)}{.474}[\log(L/K)_{t-1} + 1.0 \log(L/K)_{t-2}] + \underset{(1.747)}{.0032}[t + 1.0 (t-1)]$$

$\bar{R}^2$: .967

$\hat{\rho}$: -1.00

The estimated coefficients, $\beta\sigma$ and $(1-\beta)$, are statistically significant at the level of .05, and the estimated coefficient, $(\sigma-1)(g_2-g_1)$, is significant at the level of .10.

The seperate parameters derived from these estimates are

σ	β	(g_2-g_1)
.157	.526	-.0072

The elasticity of substitution with respect to the change in the expected relative factor price is .157 , and the technological change is a capital biased one in the sense that the exponential growth rate of the efficiency of capital is higher than that of labor, the difference of the growth rate being .72 percentage points per annum. Since the elasticity of substitution is less than unity, i.e. $(\sigma-1) < 0$, and the tebhnological change is capital biased, i.e. $(g_2-g_1) < 0$, the capital biased technological change in this industry is ultimately a labor using one, i.e. $\beta(\sigma-1)(g_2-g_1) > 0$.

The adaptive expectation coefficient of .526 indicates that about half of the weight in the formation of the price expectation is assigned to the most recently observed relative factor price and the other half to all the previous relative factor prices. The weights decline at the moderate rate as time goes back.

Table 4 presents the influence of the relevant factors on the variation of the relative share of labor to capital.

TABLE 4

CONTRIBUTIONS OF THE RELEVANT FORCES TO THE
CHANGES IN THE RELATIVE SHARE OF LABOR TO CAPITAL

YEAR	(1)	(2)	(3)	(4)	(5)	(6)
1948-49	-13.313	50.118	-12.320	63.432	51.111	0.319
1949-50	-5.326	25.910	-11.551	31.237	19.686	0.319
1950-51	-7.430	-48.486	-5.105	-41.056	-46.162	0.319
1951-52	-3.244	46.645	-0.131	49.889	49.758	0.319
1952-53	-1.867	35.379	-5.659	37.247	31.587	0.319
1953-54	-2.461	48.232	-3.961	50.693	46.732	0.319
1954-55	-8.799	-57.599	-5.354	-48.800	-54.155	0.319
1955-56	-3.992	0.512	-0.140	4.505	4.364	0.319
1956-57	4.517	19.815	-2.265	15.298	13.033	0.319
1957-58	3.711	-7.982	0.877	-11.693	-10.816	0.319
1958-59	-0.976	-21.932	2.725	-20.956	-18.230	0.319
1959-60	2.331	-5.994	1.267	-8.326	-7.059	0.319
1960-61	3.056	37.149	1.793	34.093	35.887	0.319
1961-62	-0.547	-27.288	-1.367	-26.741	-28.108	0.319
1962-63	-3.726	2.079	1.949	5.805	7.754	0.319

(1) ANNUAL PERCENTAGE CHANGE IN LABOR-CAPITAL RATIO
(2) ANNUAL PERCENTAGE CHANGE IN THE RELATIVE SHARE OF LABOR TO CAPITAL
(3) SUBSTITUTION EFFECT OF THE CHANGE IN THE RELATIVE FACTOR PRICE
(4) ACCOUNTING EFFECT OF THE CHANGE IN THE RELATIVE FACTOR PRICE
(5) NET OF THE SUBSTITUTION AND ACCOUNTING EFFECTS
(6) EFFECT OF TECHNICAL CHANGE

The labor-capital ratio had generally declined with some exceptional periods in the late 1950's and the early 1960's. However, the relative share of labor had generally risen, although there had been occasional declines.

In 5 observations among 15, both the substitution and the accounting effects had worked in the same direction, whereas they had worked in the opposite direction in the remaining cases and the accounting effect had been responsible for the generally rising relative share of labor to capital despite the general decline in the labor-capital ratio. For the period from 1948 to 1963, the relative share of labor to capital had increased by about 96.56 percent.

The annual average technological change of .32 percent had also contributed to pulling up the relative share of labor to capital. The labor using technological change by .32 percent per annum is equivalent to the case of a relative decline in the labor input by about 6.24 million manhours, and, thus, a relative decline in the labor share by about 9.99 million dollars per annum.

The accumulated effect of the labor using technological change over the entire period is an increase of the relative share by 4.80 percent.

3.4 Apparel and Related Industries (SIC No. 23)

All the estimating equations tried for this industry presented negative values for the elasticity of substitution, which occurred in the analysis of the Foods and Kindred Products Industry.

The same technique used for the analysis of the Food industry has been tried on the Model 1 adjusted for the autocorrelation, where Dhrymes' cross-section estimate of the elasticity of substitution was used as the extraneous information.
The estimating equation gave us the highest R^2 and the lowest standard errors of estimate when the value of .900 for the adaptive expectation coefficient was tried.

$$(2.29)\ [[\log(L/K)_t - .066\log(L/K)_{t-1}] - .477\,[\log(r/w)_{t-1} - .066\log(r/w)_{t-2}] - .100\,[\log(L/K)_{t-1} - .066\log(L/K)_{t-2}]] = .492 + \underset{(2.184)}{.013}\,[t - .066(t-1)]$$

R^2 : .504

$\hat{\sigma}$: .530

$\hat{\rho}$: .066

The estimated coefficient of the technology term is significant at the .05 level, and the magnitude of the coefficient is in the expected range.

The seperate parameters are

σ	β	(g_2-g_1)
.530	.900	.013

The high value for the adaptive expectation coefficient indicates that a very heavy weight is assigned to the most recently observed relative factor price and the subsequent weights for all the previous relative prices are declining very rapidly as time goes back.

The positive value of .013 for (g_2-g_1) indicates that the technological change in this industry had been on the average a labor biased one, where the annual exponantial growth rate of the efficiency of labor is higher than that of capital by 1.3 percentage points.

The contributions of the relevant factors to the variation of the relative share are presented in the Table 5 .

The labor-capital ratio had generally declined with a few exceptional periods, whereas the relaitve share of labor had not. In many instances, the substitution effect associated with falling labor-capital ratio was offset by the positive accounting effect on the relative share of labor to capital, e.g. 1949-50, 1953-54, 1958-59,...

For the period from 1948 to 1963, the relative share had risen in net by about 31.23 percent, and the accounting effect seems to have been the dominant factor influencing the relative factor share.

The technological change had also contributed to the increases of the relative share of labor to capital by 1.3 percent per annum, which is equivalent to the relative increase of the labor input, for instance in 1954, by about 28.23 million manhours and, thus, to the relative increase of the labor share by about 45.28 million dollars per annum, which is significant.

The effect of the technological change accumulated for the entire period would be an increase of the relative share of labor to capital by about 19.05 percent.

TABLE 5

CONTRIBUTIONS OF THE RELEVANT FORCES TO THE CHANGES IN THE RELATIVE SHARE OF LABOR TO CAPITAL

YEAR	(1)	(2)	(3)	(4)	(5)	(6)
1948-49	0.079	7.784	-19.566	7.704	-11.861	1.269
1949-50	-2.879	41.572	-3.667	44.451	40.783	1.269
1950-51	1.996	-50.289	-21.491	-52.286	-73.777	1.269
1951-52	-0.783	8.349	25.140	9.133	34.273	1.269
1952-53	6.335	20.834	-4.434	14.498	10.064	1.269
1953-54	-2.816	10.259	-6.282	13.076	6.793	1.269
1954-55	-4.111	-0.930	-6.518	3.180	-3.337	1.269
1955-56	0.327	-5.436	-1.928	-5.763	-7.692	1.269
1956-57	5.189	27.218	2.782	22.028	24.811	1.269
1957-58	-3.567	-25.702	-9.988	-22.134	-32.123	1.269
1958-59	-4.434	23.865	10.201	28.300	38.501	1.269
1959-60	-1.143	-20.595	-13.942	-19.452	-33.394	1.269
1960-61	-0.355	-11.715	9.164	-11.360	-2.195	1.269
1961-62	-4.674	5.765	5.383	10.439	15.823	1.269
1962-63	-1.040	0.342	-5.447	1.382	-4.064	1.269

(1) ANNUAL PERCENTAGE CHANGE IN LABOR-CAPITAL RATIO
(2) ANNUAL PERCENTAGE CHANGE IN THE RELATIVE SHARE OF LABOR TO CAPITAL
(3) SUBSTITUTION EFFECT OF THE CHANGE IN THE RELATIVE FACTOR PRICE
(4) ACCOUNTING EFFECT OF THE CHANGE IN THE RELATIVE FACTOR PRICE
(5) NET OF THE SUBSTITUTION AND ACCOUNTING EFFECTS
(6) EFFECT OF TECHNICAL CHANGE

3.5 Furnitures and Fixtures (SIC No. 25)

The estimating equation is Model 1 adjusted for the autocorrelation which shows the highest $\bar{R}^2$.

$$(2.28)\ \left[\log(L/K)_t + .092 \log(L/K)_{t-1}\right] = 1.799 + \underset{(3.481)}{.213}\left[\log(r/w)_{t-1} + .092 \log(r/w)_{t-2}\right] + \underset{(1.038)}{.173}\left[\log(L/K)_{t-1} + .092 \log(L/K)_{t-2}\right] - \underset{(2.664)}{.0103}\left[t + .092(t-1)\right]$$

$\bar{R}^2$: .932

$\hat{\rho}$: -.092

The estimated coefficients, $\beta\sigma$ and $\beta(\sigma-1)(g_2-g_1)$, are significant at the level of .05, and the coefficient, $(1-\beta)$, is significant at the level of .10.

The seperate parameters derived are

σ	β	(g_2-g_1)
.258	.827	.017

The high value of the adaptive expectation coefficient implies that the quite significant weight is assigned to the most recently observed relative factor price and the subsequent weights for all the previous relative factor prices rapidly decline as time goes back.

The value of .017 for (g_2-g_1) indicates that the technological change in this industry had been biased to labor in the sense that the annual exponential growth rate of the efficiency of labor had been higher than that of capital by 1.7 percentage points.

From Table 6 we can observe that there had been no distinctive downward trend in the movement of the labor-capital ratio, although there had been several occations where the ratio declined, and the relative share of labor to capital had shown rather random fluctuation. For the period from 1948 to 1963, the relative share had increased in net by about 21.47 percent.

The accounting effect of the change in the relative factor share again had been generally the dominant force influencing the direction of the variation of the relative share.

The capital using technological change in this industry had constantly worked in the direction to reducing the share of labor to capital by 1.03 percent per annum, which is equivalent to the relative decrease of the labor input, for instance in 1954, by about 7.33 million manhours, and, thus, to the relative decrease of the labor share by about 14.4 million dollars per annum.

The capital using technological change accumulated over the whole period under study had depressed the relative share of labor to capital by 15.45 percent.

TABLE 6

CONTRIBUTIONS OF THE RELEVANT FORCES TO THE CHANGES IN THE RELATIVE SHARE OF LABOR TO CAPITAL

YEAR	(1)	(2)	(3)	(4)	(5)	(6)
1948-49	4.733	-11.930	30.999	-16.664	14.335	-1.029
1949-50	-0.487	11.471	4.372	11.959	16.331	-1.029
1950-51	3.795	-20.990	-2.634	-24.785	-27.420	-1.029
1951-52	0.101	11.809	5.941	11.707	17.649	-1.029
1952-53	1.776	35.676	-2.478	33.899	31.420	-1.029
1953-54	-13.184	-8.583	-6.919	4.601	-2.318	-1.029
1954-55	-6.227	-16.348	-3.263	-10.121	-13.384	-1.029
1955-56	0.503	-4.890	1.079	-5.393	-4.314	-1.029
1956-57	1.732	15.209	1.237	13.476	14.714	-1.029
1957-58	0.636	31.664	-2.573	31.028	28.455	-1.029
1958-59	-11.558	-18.152	-6.505	-6.593	-13.099	-1.029
1959-60	-4.235	3.555	-0.595	7.790	7.195	-1.029
1960-61	3.296	-5.691	-2.394	-8.988	-11.382	-1.029
1961-62	1.812	-1.481	2.486	-3.293	-0.807	-1.029
1962-63	-5.647	0.152	1.015	5.799	6.815	-1.029

(1) ANNUAL PERCENTAGE CHANGE IN LABOR-CAPITAL RATIO
(2) ANNUAL PERCENTAGE CHANGE IN THE RELATIVE SHARE OF LABOR TO CAPITAL
(3) SUBSTITUTION EFFECT OF THE CHANGE IN THE RELATIVE FACTOR PRICE
(4) ACCOUNTING EFFECT OF THE CHANGE IN THE RELATIVE FACTOR PRICE
(5) NET OF THE SUBSTITUTION AND ACCOUNTING EFFECTS
(6) EFFECT OF TECHNICAL CHANGE

3.6. Paper and Allied Products (SIC No.26)

The estimating equation selected is Model 2 adjusted for the autocorrelation.

$$
\begin{aligned}
(2.36)\ \left[\log(L/K)_t - .202\log(L/K)_{t-1}\right] &= .157 + \underset{(3.396)}{.185}\left[\log(r/w)_{t-1}\right.\\
&\quad \left.- .202\log(r/w)_{t-2}\right] + \underset{(5.631)}{1.134}\left[\log(L/K)_{t-1}\right.\\
&\quad \left.- .202\log(L/K)_{t-2}\right] - \underset{(2.557)}{.543}\left[\log(L/K)_{t-2}\right.\\
&\quad \left.- .202\log(L/K)_{t-3}\right] + \underset{(\ .582)}{.005}\left[t - .202(t-1)\right]
\end{aligned}
$$

$\bar{R}^2$: .944

$\hat{\rho}$: .202

All the estimated coefficients except that of the technology term are significant at the level of .025, and the coefficient of the technology term is not significantly different from zero.

The seperate parameters are

σ	b_1	b_2	(g_2-g_1)
.451	1.134	-.543	-.021

The weight parameters of the price expectation, b_1 and b_2 , satisfy the given restrictions; $2 > b_1 > 0$, $1 > b_2 > -1$

The value of -.021 for (g_2-g_1) indicates that the technological change in this industry had been capital biased one, the degree of the bias being 2.1 percentage points.

The Table 7 shows the contributions of the relevant forces to the variation of the relative factor share.

The relative share of labor to capital had generally increased, whereas the labor-capital ratio had been steadily declining with two exceptional observations.

In seven instances, the strong accounting effect of the changes in the relative factor price had offset the depressing substitution effect, which had been largely responsible for the declining labor-capital ratio thus leaving the net effect on the relative share of labor to capital as increasing one.

The relative share had increased by 77.47 percents in net over the entire period under study.

The technological change in this industry had been labor using ones, and thus it had been contributing to the increases of the labor-capital ratio by .5 percent per annum, which is equivalent to the relative increase of the labor input, for instance in 1954, by .584 million manhours, and, thus, to the relative increase of the labor share by about 1.23 million dollars per annum.

Nevertheless, the above assessment of the effect of the technological change on the relative factor share should be interpreted with caution, because the assessment was based on the estimated coefficient of the technology term which is not significantly different from zero.

TABLE 7

CONTRIBUTIONS OF THE RELEVANT FORCES TO THE CHANGES IN THE RELATIVE SHARE OF LABOR TO CAPITAL

YEAR	(1)	(2)	(3)	(4)	(5)	(6)
1949-50	-7.119	-24.319	-0.208	-17.199	-17.408	0.479
1950-51	-4.569	-24.178	-6.461	-19.608	-26.070	0.479
1951-52	3.359	32.824	1.277	29.464	30.742	0.479
1952-53	-2.864	14.298	1.607	17.163	18.770	0.479
1953-54	-5.930	4.692	-8.883	10.623	1.739	0.479
1954-55	-10.424	-7.573	-8.462	2.850	-5.611	0.479
1955-56	-7.816	-12.753	-11.467	-4.937	-16.405	0.479
1956-57	-3.921	28.329	-4.044	32.251	28.206	0.479
1957-58	-0.947	14.832	-7.036	15.779	8.743	0.479
1958-59	-0.514	-10.521	-2.069	-10.007	-12.076	0.479
1959-60	0.672	8.989	1.662	8.316	9.979	0.479
1960-61	-0.794	9.520	-0.342	10.314	9.971	0.479
1961-62	-0.961	6.529	-3.347	7.491	4.144	0.479
1962-63	-1.604	17.087	-2.258	18.691	16.433	0.479

(1) ANNUAL PERCENTAGE CHANGE IN LABOR-CAPITAL RATIO
(2) ANNUAL PERCENTAGE CHANGE IN THE RELATIVE SHARE OF LABOR TO CAPITAL
(3) SUBSTITUTION EFFECT OF THE CHANGE IN THE RELATIVE FACTOR PRICE
(4) ACCOUNTING EFFECT OF THE CHANGE IN THE RELATIVE FACTOR PRICE
(5) NET OF THE SUBSTITUTION AND ACCOUNTING EFFECTS
(6) EFFECT OF TECHNICAL CHANGE

The labor using technological change accumulated over the entire period under study had contributed to an increase of the relative share of labor to capital by 7.20 percent.

3.7 Printing and Publishing (SIC No.27)

The selected estimating equation is the Model 2 unadjusted for the autocorrelation, which gives us the highest $\bar{R}^2$.

$$(2.34)\quad \log(L/K)_t = .811 + \underset{(1.938)}{.308}\ \log(r/w)_{t-1} + \underset{(7.337)}{1.250}\ \log(L/K)_{t-1}$$

$$- \underset{(4.031)}{.899}\ \log(L/K)_{t-1} + \underset{(\ .094)}{.0004}\ t$$

$\bar{R}^2$: .905

All the coefficients estimated except that of the technology term are sinificant at the level of .05. The coefficient of the technology term alone is not significantly different from aero.

The seperate parameters are

σ	a	b_1	b_2	(g_2-g_1)
.473	.649	1.250	-.899	-.0024

The weight parameters, b_1 and b_2 , satisfy the given restrictions: $0 < b_1 < 2$, $-1 < b_2 < 1$

The value of -.0024 for (g_2-g_1) indicates that the technological change had been capital biased one in the sense that the exponential growth rate of the efficiency of capital had been higher than that of labor by .24 percentage points per annum.

From Table 8 , we can observe that the relative share of labor had continously increased until 1954, then decreased for the later years of 1950's, and had increased again in the early 1960's except the period from 1961 to 1962, whereas the labor-capital ratio associated with the substitution effects of the changes in the relative factor price had declined except the early years of the 1950's.
The relative share shows a net increase of 24.99 percent for the entire period under study.

The increases of the relative share of labor to capital in spite of the decreases of the labor-capital raio could occur because of the positive accounting effect which had in many instances offset the depressing substitution effect , while the latter had been working in the direction to reducing the relative share.
The net effect of the changes in the relative factor price account for most of the variation of the relative share.

The influence of the technological change is not clear due to the unsatisfactorily low degree of precision in the estimation of the coefficient of the technology term in the estimating equation. When the estimated coefficient is accepted, the capital biased technological change associated with the elasticity of substitution less than unity would become a labor using technological change, which had contributed to the increases of the relative share of labor to capital by .04 percent per annum, which is equivalent to the relative increase

TABLE 8

CONTRIBUTIONS OF THE RELEVANT FORCES TO THE CHANGES IN THE RELATIVE SHARE OF LABOR TO CAPITAL

YEAR	(1)	(2)	(3)	(4)	(5)	(6)
1949-50	-3.189	5.501	-3.896	8.691	4.795	0.039
1950-51	3.312	7.721	-1.941	4.409	2.467	0.039
1951-52	5.245	1.480	6.784	-3.764	3.019	0.039
1952-53	2.811	1.602	6.528	-1.208	5.319	0.039
1953-54	-0.093	5.640	0.129	5.733	5.863	0.039
1954-55	-3.712	-10.821	-4.440	-7.108	-11.549	0.039
1955-56	-3.396	5.009	-3.639	8.405	4.766	0.039
1956-57	-1.927	-0.963	-4.653	0.963	-3.689	0.039
1957-58	0.942	21.850	-0.310	20.908	20.597	0.039
1958-59	-4.614	-14.160	-3.196	-9.545	-12.742	0.039
1959-60	-6.016	1.349	-5.256	7.365	2.109	0.039
1960-61	1.114	10.696	-7.693	9.581	1.887	0.039
1961-62	-3.954	-7.460	4.239	-3.506	0.733	0.039
1962-63	-3.807	7.805	-6.218	11.613	5.394	0.039

(1) ANNUAL PERCENTAGE CHANGE IN LABOR-CAPITAL RATIO
(2) ANNUAL PERCENTAGE CHANGE IN THE RELATIVE SHARE OF LABOR TO CAPITAL
(3) SUBSTITUTION EFFECT OF THE CHANGE IN THE RELATIVE FACTOR PRICE
(4) ACCOUNTING EFFECT OF THE CHANGE IN THE RELATIVE FACTOR PRICE
(5) NET OF THE SUBSTITUTION AND ACCOUNTING EFFECTS
(6) EFFECT OF TECHNICAL CHANGE

of the labor input by .05 million manhours per annum when the year of 1954 was used as an example, and, thus, to the relative increase of the labor share by about 1.49 million dollars per annum.

The accumulated effect of the labor using technological change over the period from 1948 to 1963 would raise the relative share of labor to capital by .6 percent.

3.8. Chemicals and Allied Products (SIC No. 28)

All the estimating equations tried produced negative values for the elasticity of substitution, which call forth the same method of treatment used in the Food and the Apparel industries.

The estimating equation, the Model 1 adjusted for the auto-correlation, with Dhrymes' cross-section estimate of the elasticity of substitution as the extraneous information is

$$(2.29)\quad [\log(L/K)_t + .561 \log(L/K)_{t-1}] - .270 [\log(r/w)_{t-1} + .561 \log(r/w)_{t-2}] - .100 [\log(L/K)_{t-1} + .561 \log(L/K)_{t-2}] = 1.628 - \underset{(2.188)}{.007} [t - .561 (t-1)]$$

$\bar{R}^2$: .505

$\hat{\sigma}$: .300 , $\hat{\rho}$: -.561 , $\hat{\lambda}$: .900

The estimated coefficient of the technology term is significant at the level of .025 and the magitude is in the expected range.

The seperate parammeters are

σ	β	(g_2-g_1)
.300	.900	.011

The high value for the adaptive expectation coefficient indicates that in the formation of the price expectation a very significant weight was assigned to the most recently observed relative factor price and the subsequent weights assigned to all the previous relative factor prices declined very rapidly as time went back.

The value of .011 for (g_2-g_1) indicates that the technological change in this industry had been on the average a labor biased one, where the annual expoenetial growth rate of the efficiency of labor had been higher than that of capital by 1.1 percentage points.

Although the labor-capital ratio had gnerally declined in this industry, the relative share of labor to capital had not declined.(Table 9) In fact, the relative share shows the increases in 11 observations out of 15, the net increase over the whole period being 31.95 percent.

The strong negative accounting effect of the changes in the relative factor price seem to be responsible for the decreases of the relative share of labor to capital for the four periods; 1948-49,1949-50, 1954-55, and 1958-59.
As a whole, the net effect of the changes in the relative factor price seem to be the dominant force affecting the direction of the fluctuations of the relative share of labor to capital.

TABLE 9

CONTRIBUTIONS OF THE RELEVANT FORCES TO THE CHANGES IN THE RELATIVE SHARE OF LABOR TO CAPITAL

YEAR	(1)	(2)	(3)	(4)	(5)	(6)
1948-49	-2.034	-5.645	5.127	-3.610	1.516	-0.680
1949-50	-7.485	-16.200	0.771	-8.715	-7.943	-0.680
1950-51	-0.638	1.493	1.604	2.131	3.736	-0.680
1951-52	-4.212	27.437	-0.639	31.650	31.010	-0.680
1952-53	-0.068	15.455	-8.966	15.524	6.557	-0.680
1953-54	3.679	1.458	-4.198	-2.221	-6.419	-0.680
1954-55	-11.617	-24.386	0.967	-12.768	-11.801	-0.680
1955-56	-3.051	18.461	2.285	21.512	23.798	-0.680
1956-57	-3.978	6.838	-6.113	10.817	4.703	-0.680
1957-58	4.680	9.852	-3.318	5.171	1.853	-0.680
1958-59	-3.804	-24.784	-0.928	-20.980	-21.908	-0.680
1959-60	1.458	15.805	5.284	14.347	19.631	-0.680
1960-61	0.237	3.131	-3.727	2.893	-0.833	-0.680
1961-62	-4.561	2.390	-0.757	6.952	6.194	-0.680
1962-63	-4.677	0.641	-2.333	5.319	2.985	-0.680

(1) ANNUAL PERCENTAGE CHANGE IN LABOR-CAPITAL RATIO
(2) ANNUAL PERCENTAGE CHANGE IN THE RELATIVE SHARE OF LABOR TO CAPITAL
(3) SUBSTITUTION EFFECT OF THE CHANGE IN THE RELATIVE FACTOR PRICE
(4) ACCOUNTING EFFECT OF THE CHANGE IN THE RELATIVE FACTOR PRICE
(5) NET OF THE SUBSTITUTION AND ACCOUNTING EFFECTS
(6) EFFECT OF TECHNICAL CHANGE

The labor biased technological change associated with the elasticity of substitution less than unity would be a capital using one. The capital using technological change had contributed to reducing the relative share of labor to capital by .7 percent per annum, which is equivalent to the relative reduction in the labor input, for instance in 1954, by about 11.18 million manhours a year, and to the relative reduction in the labor share by about 27.95 million dollars per annum, which is quite significant.

The accumulated effect of the capital using technological change over the entire period under study would be a reduction of the relative share of labor to capital by 10.2 percent.

3.9. Rubber and Miscellaneous Plastic Products (SIC No. 30)

The selected estimating equation is the Model 1 adjusted for the autocorrelation.

$$(2.28)\ \left[\log(L/K)_t - .063\log(L/K)_{t-1}\right] = 1.868 + \underset{(2.274)}{.103}\left[\log(r/w)_{t-1} - .063\log(r/w)_{t-2}\right] + \underset{(1.878)}{.385}\left[\log(L/K)_{t-1} - .063\log(r/w)_{t-2}\right] - \underset{(3.768)}{.015}\left[t - .063(t-1)\right]$$

$\bar{R}^2$: .901

$\hat{\rho}$: .063

All the estimated coefficients are significant at .05 level.

The seperate parameters are

σ	β	(g_2-g_1)
.168	.615	.030

The elasticity of substitution with respect to the change in the expected relative factor price is low in this industry.

The value of .615 for the adaptive expectation coefficient implies that about 62 percent of the weight, in the formation of the expected relative factor price, is assigned to the most recently observed relative factor price and the subsequent weights for all the previous relative factor prices decline at the moderate rate as time goes back.

The value of .030 for (g_2-g_1) indicates that the technological change had been labor biased one, where the annual exponential growth rate of the efficiency of labor had been higher than that of capital by 3 percentage points.

From Table 10 , we can see that the labor-capital ratio had been rising from 1948 to 1953 with one exceptional observation, i.e. 1949-50, and then it had been decling thereafter until 1963.
For the period from 1953 to 1963, the almost continous decline in the labor-capital can be explained by the almost continous negative substitution effect and the effect of the capital using technological change.

The relative share of labor to capital, on the other hand, had not followed the variation of the labor-capital ratio because of the

TABLE 10

CONTRIBUTIONS OF THE RELEVANT FORCES TO THE CHANGES IN THE RELATIVE SHARE OF LABOR TO CAPITAL

YEAR	(1)	(2)	(3)	(4)	(5)	(6)
1948-49	3.240	4.084	-1.707	0.844	-0.863	-1.519
1949-50	-3.343	6.518	1.161	9.861	11.023	-1.519
1950-51	7.013	-92.432	-2.305	-99.446	-101.752	-1.519
1951-52	5.206	15.074	12.965	9.868	22.833	-1.519
1952-53	2.505	23.367	0.987	20.862	21.849	-1.519
1953-54	-5.454	49.960	-1.187	55.414	54.227	-1.519
1954-55	-9.294	-20.097	-7.820	-10.802	-18.623	-1.519
1955-56	3.104	-25.081	-2.466	-28.185	-30.652	-1.519
1956-57	-5.457	22.442	4.105	27.899	32.005	-1.519
1957-58	-6.603	-0.999	-4.982	5.603	0.620	-1.519
1958-59	-4.364	-2.400	-3.122	1.963	-1.158	-1.519
1959-60	-1.953	0.146	-1.884	2.099	0.215	-1.519
1960-61	2.143	5.727	-0.969	3.584	2.614	-1.519
1961-62	-8.316	7.621	0.455	15.937	16.393	-1.519
1962-63	-6.390	5.611	-4.848	12.001	7.152	-1.519

(1) ANNUAL PERCENTAGE CHANGE IN LABOR-CAPITAL RATIO
(2) ANNUAL PERCENTAGE CHANGE IN THE RELATIVE SHARE OF LABOR TO CAPITAL
(3) SUBSTITUTION EFFECT OF THE CHANGE IN THE RELATIVE FACTOR PRICE
(4) ACCOUNTING EFFECT OF THE CHANGE IN THE RELATIVE FACTOR PRICE
(5) NET OF THE SUBSTITUTION AND ACCOUNTING EFFECTS
(6) EFFECT OF TECHNICAL CHANGE

intervening accounting effect of the changes in the observed relative factor price. Nevertheless, the relative share had decreased for the period as a whole by .46 percent.

The net effect of the changes in the relative factor price had still been dominant force influencing the direction of the movements of the relative factor share.

The labor biased technological change associated with the elasticity of substitution less than unity became a capital using one. The capital using technological change had contributed to the reduction of the relative share of labor to capital by 1.5 percent a year, which is equivalent to the relative reduction of the labor input , for instance in 1954, by about 10.21 million manhours, and to the relative reduction of the labor share by about 22.46 million dollars per annum, which is significant.

The effect of the capital using technological change accumulated for the entire period would be a reduction of the relative share of labor to capital by 22.80 percent.

3.10. Leather and Leather Products (SIC No. 31)

The selected estimating equation is the Model 2 unadjusted for the autocorrelation.

$$(2.34)\ \log(L/K)_t = 3.366 + \underset{(.046)}{.002} \log(r/w)_{t-1} + \underset{(1.671)}{.510} \log(L/K)_{t-1} - \underset{(1.748)}{.486} \log(L/K)_{t-2} - \underset{(2.959)}{.0252}\ t$$

$\bar{R}^2$: .909

All the estimated coefficients except that of log $(r/w)_{t-1}$ are significant at .10 level. The coefficient of $\log(r/w)_{t-1}$, the elasticity of substitution with respect to the change in the observed relative factor price, is not significantly different from zero.

The seperate parameters are

σ	a	b_1	b_2	(g_2-g_1)
.021	.619	.510	-.486	.024

All the weight parameters satisfy the given conditions, and the technological change in this industry had been a labor biased one , where the annual exponential growth rate of the efficiency of labor had been higher than that of capital by 2.4 percentage points.

The influences of the relevant forces on the relative factor share are shown in Table **11** .

The labor-capital ratio had gnerally declined with a few exceptional periods, whereas the changes in the relative sahre of labor to capital had **had** alternations around zero.
In many instances, the substitution effects, which had worked to lower the labor-capital ratio and the relative share of labor to capital, were overruled by the strong accounting effects, which had worked to raise

TABLE 11

CONTRIBUTIONS OF THE RELEVANT FORCES TO THE CHANGES IN THE RELATIVE SHARE OF LABOR TO CAPITAL

YEAR	(1)	(2)	(3)	(4)	(5)	(6)
1949-50	0.049	86.769	-20.258	86.720	66.461	-2.519
1950-51	0.725	-143.660	7.301	-144.385	-137.084	-2.519
1951-52	-0.813	38.910	1.241	39.723	40.965	-2.519
1952-53	2.482	22.501	-1.610	20.019	18.408	-2.519
1953-54	-3.630	-19.272	4.052	-15.642	-11.590	-2.519
1954-55	-3.637	45.706	-6.572	49.344	42.771	-2.519
1955-56	-6.797	-24.554	-3.701	-17.757	-21.459	-2.519
1956-57	-2.990	7.116	-8.299	10.106	1.807	-2.519
1957-58	0.918	40.136	-1.150	39.218	38.067	-2.519
1958-59	-3.003	-3.661	2.763	-0.658	2.104	-2.519
1959-60	-4.045	-52.522	-4.902	-48.477	-53.379	-2.519
1960-61	-2.760	76.360	-4.482	79.121	74.638	-2.519
1961-62	-4.214	-68.790	-2.235	-64.576	-66.812	-2.519
1962-63	-7.338	-18.652	-4.830	-11.314	-16.144	-2.519

(1) ANNUAL PERCENTAGE CHANGE IN LABOR-CAPITAL RATIO
(2) ANNUAL PERCENTAGE CHANGE IN THE RELATIVE SHARE OF LABOR TO CAPITAL
(3) SUBSTITUTION EFFECT OF THE CHANGE IN THE RELATIVE FACTOR PRICE
(4) ACCOUNTING EFFECT OF THE CHANGE IN THE RELATIVE FACTOR PRICE
(5) NET CF THE SUBSTITUTION AND ACCOUNTING EFFECTS
(6) EFFECT OF TECHNICAL CHANGE

the relative share of labor to capital, e.g. 1948-49, 1951-52, 1954-55, 1956-57, 1960-61.

The relative share of labor to capital shows a net increase of 35.58 percent for the whole period under study.

The labor biased technological change associated with the elasticity of substitution less than unity would be a capital using one, which had contributed to reducing the relative share of labor to capital by about 2.5 percent a year, which is equivalent to the relative reduction of labor input, for instance in 1954, by 17.94 million manhours, and, thus, to the relative reduction of the labor share by 28.70 million dollars per annum.

The capital using technological change accumulated over the entire period had depressed the relative share of labor to capital by about 37.8 percents.

3.11. Stone, Clay and Glass Products (SIC No. 32)

All the estimating equation tried yielded negative values for the elasticity of substitution, which again calls forth the same treatment used in the earlier industries with the same problem.

The selected estimating equation with Dhrymes' cross-section estimate of the elasticity of substitution as the extraneous information is the Model 1 adjusted for the autocorrelation.

$$(2.29)\ [[\log(L/K)_t - .017\log(L/K)_{t-1}] - .541[\log(r/w)_{t-1} - .100[\log(L/K)_{t-1} - .017\log(L/K)_{t-2}]]$$

$$= 1.892 - \underset{(3.621)}{.0195}[t - .017(t-1)]$$

$\bar{R}^2$: .696

$\hat{\rho}$: .017

The estimated coefficient is highly significant even at the level of .01 level, and its magnitude is in the expected range.

The seperate parameters are

σ	δ	(g_2-g_1)
.601	.900	.054

The high adaptive expectation coefficient implies that the input decisions in this industry depend on the most recently observed relative factor price, while the roles of all the previously observed relative factor prices in the input decision making process become progressively insignificant as time goes back.

The value of .054 for (g_2-g_1) indicates that the technological change in this industry had been biased to labor in the sense that the annual exponential growth rate of the efficiency of labor had been higher than that of capital by 5.4 percentage points.

From Table 12, we can observe that the labor-capital ratio had declined almost throughout the whole period, whereas the relative share of labor to capital had not followed the trend of the labor-capital ratio. The accounting effect of the changes in the relative factor price had been largely reponsible for the increases of the relative share of labor despite the declining labor-capital ratio associated with the substitution effect.
The relative share had increased in net by 10.48 percent for the whole period.

The labor biased technological change associated with the elasticity of substitution less than unity became the capital using technological change, which had contributed to the declines of the labor-capital ratio by 1.95 percent a year, which is equivalent to the relative reduction of labor input, for instance in 1954, by about 23.25 million dollars per annum.

The effect of the technological change accumulated over the entire period would be a reduction of the relative share of labor to capital by 29.25 percent.

3.12. Primary Metal Industries (SIC No. 33)

The selected estimating equation is the Model 1 unadjusted for the autocorrelation.

TABLE 12

CONTRIBUTIONS OF THE RELEVANT FORCES TO THE CHANGES IN THE RELATIVE SHARE OF LABOR TO CAPITAL

YEAR	(1)	(2)	(3)	(4)	(5)	(6)
1948-49	4.795	-11.747	1.804	-16.543	-14.738	-1.950
1949-50	-6.240	-26.216	9.429	-19.975	-10.546	-1.950
1950-51	-5.169	15.601	10.183	20.771	30.954	-1.950
1951-52	0.305	18.538	-11.754	18.232	6.478	-1.950
1952-53	-4.934	-5.599	-9.833	-0.664	-10.498	-1.950
1953-54	-4.658	-9.552	-0.133	-4.893	-5.027	-1.950
1954-55	-11.982	-21.360	2.181	-9.378	-7.196	-1.950
1955-56	-9.880	14.867	3.875	24.747	28.623	-1.950
1956-57	-9.348	12.202	-14.376	21.550	7.174	-1.950
1957-58	-0.072	3.353	-12.593	3.425	-9.168	-1.950
1958-59	-6.427	-11.029	-1.860	-4.602	-6.463	-1.950
1959-60	-1.319	24.420	1.847	25.739	27.587	-1.950
1960-61	0.586	4.095	-14.057	3.508	-10.548	-1.950
1961-62	-1.344	8.697	-1.839	10.042	8.202	-1.950
1962-63	-5.920	-5.790	-5.567	0.129	-5.437	-1.950

(1) ANNUAL PERCENTAGE CHANGE IN LABOR-CAPITAL RATIO
(2) ANNUAL PERCENTAGE CHANGE IN THE RELATIVE SHARE OF LABOR TO CAPITAL
(3) SUBSTITUTION EFFECT OF THE CHANGE IN THE RELATIVE FACTOR PRICE
(4) ACCOUNTING EFFECT OF THE CHANGE IN THE RELATIVE FACTOR PRICE
(5) NET OF THE SUBSTITUTION AND ACCOUNTING EFFECTS
(6) EFFECT OF TECHNICAL CHANGE

$$(2.22)\quad \log(L/K)_t = .612 + \underset{(1.511)}{.531}\ \log(r/w)_{t-1} + \underset{(\ .209)}{.058}\ \log(L/K)_{t-1} - \underset{(2.803)}{.025}\ t$$

$$\bar{R}^2 \ : \ .896$$

All the estimated coefficients except that of $\log(L/K)_{t-1}$ are significant at the .10 level.

The low value of the t statistic for the estimated coefficient of $\log(L/K)_{t-1}$ here implies that the contribution of all the relative factor prices observed prior to the most recent period is not significant.

The seperate parameters are

σ	β	(g_2-g_1)
.563	.942	.060

The value of .06 for (g_2-g_1) indicates that the technological change in this industry had been biased to labor in the sense that the annual exponential growth rate of the efficiency of labor had been higher than that of capital by 6 percentage points.

From Table 13, we can observe that about two third of the observations of the labor-capital ratio and the relative share of labor to capital show declines.

Prior to 1951, the strong accounting effect had worked in the direction to reducing the relative share of labor to capital in the face of the

TABLE 13

CONTRIBUTIONS OF THE RELEVANT FORCES TO THE
CHANGES IN THE RELATIVE SHARE OF LABOR TO CAPITAL

YEAR	(1)	(2)	(3)	(4)	(5)	(6)
1948-49	2.563	-21.134	2.111	-23.697	-21.586	-2.480
1949-50	-3.777	-18.608	12.718	-14.830	-2.111	-2.480
1950-51	-1.282	-13.462	7.646	-12.179	-4.533	-2.480
1951-52	-8.180	50.544	6.385	58.724	65.109	-2.480
1952-53	-10.827	-19.225	-31.621	-8.398	-40.019	-2.480
1953-54	6.077	33.646	3.826	27.569	31.396	-2.480
1954-55	-11.435	-36.780	-14.270	-25.344	-39.615	-2.480
1955-56	2.163	6.963	12.779	4.799	17.578	-2.480
1956-57	-8.107	-2.871	-2.419	5.236	2.816	-2.480
1957-58	3.540	34.336	-3.247	30.796	27.549	-2.480
1958-59	4.984	-16.724	-16.129	-21.709	-37.839	-2.480
1959-60	-10.678	20.524	11.804	31.202	43.007	-2.480
1960-61	-3.037	34.366	-17.169	37.403	20.234	-2.480
1961-62	-0.401	2.359	-20.015	2.761	-17.254	-2.480
1962-63	-3.655	-14.319	-1.487	-10.664	-12.151	-2.480

(1) ANNUAL PERCENTAGE CHANGE IN LABOR-CAPITAL RATIO
(2) ANNUAL PERCENTAGE CHANGE IN THE RELATIVE SHARE OF LABOR TO CAPITAL
(3) SUBSTITUTION EFFECT OF THE CHANGE IN THE RELATIVE FACTOR PRICE
(4) ACCOUNTING EFFECT OF THE CHANGE IN THE RELATIVE FACTOR PRICE
(5) NET OF THE SUBSTITUTION AND ACCOUNTING EFFECTS
(6) EFFECT OF TECHNICAL CHANGE

substitution effect which had worked in the direction to raising the relative share. Since the former effect had been stronger than the latter, the net effect of the changes in the relative factor price on the relative share of labor to capital had been negative.

During the later periods, the accounting effects had either joined the substitution effects in lowering the relative share or off-seting the substitution effects to raise the relative share.

The relative share of labor to capital had increased in net for the entire period by 39.61 percent.

The labor biased technological change associated with the elasticity of substitution less than unity became the capital using one, which had contributed to reducing the labor-capital ratio and the relative share of labor to capital by 2.48 percent a year, which is equivalent to the relative reduction of the labor input, for instance in 1954, by about 6.03 million manhours, and, thus, to the relative reduction of the labor share by about 15.08 million dollars per annum.

The effect of the capital using technological change accumulated over the entire period would be a reduction of the relative share of labor to capital by 37.2 percent.

3.13. Fabricated Metal Products (SIC No. 34)

The selected estimating equation is the Model 1 unadjusted for the autocorrelation.

$$(2.22)\quad \log(L/K)_t = 1.960 + \underset{(1.747)}{.110}\ \log(r/w)_{t-1} + \underset{(1.325)}{.358}\ \log(L/K)_{t-1} - \underset{(1.228)}{.017}\ t$$

$\bar{R}^2$: .975

All the estimated coefficients are statistically sinificant at the .15 level.

The seperate parameters are

σ	θ	$(g_2 - g_1)$
.171	.643	.0313

About 46 percent of the weight in the formation of the expected relative factor price is assigned to the most recently observed relative factor price, and the subsequent weights for all the previous relative factor prices decline at tha moderate rate as time goes back.

The technological change in this industry had been labor biased in the sense that the annual exponential growth rate of the efficiency of labor had been higher than that of capital by 3.13 percentage points.

The technological change associated with the elasticity of substitution less than unity became a capital using one.

Table 14 shows that the labor-capital ratio had declined throughout the whole period except two exceptional observations, whereas

TABLE 14

CONTRIBUTIONS OF THE RELEVANT FORCES TO THE
CHANGES IN THE RELATIVE SHARE OF LABOR TO CAPITAL

YEAR	(1)	(2)	(3)	(4)	(5)	(6)
1948-49	-0.430	13.945	-2.953	14.375	11.422	-1.670
1949-50	-5.190	-28.445	-1.733	-23.254	-24.988	-1.670
1950-51	-3.674	-0.322	0.699	3.352	4.052	-1.670
1951-52	2.316	34.089	-1.682	31.772	30.090	-1.670
1952-53	-6.185	19.486	-2.663	25.671	23.007	-1.670
1953-54	-2.731	-0.566	-5.032	2.165	-2.866	-1.670
1954-55	-6.060	0.748	-1.214	6.808	5.593	-1.670
1955-56	-5.888	3.782	-2.914	9.670	6.756	-1.670
1956-57	-10.374	0.677	-3.168	11.051	7.883	-1.670
1957-58	0.711	7.354	-4.923	6.643	1.719	-1.670
1958-59	-8.383	-0.525	-0.475	7.857	7.382	-1.670
1959-60	-1.132	24.125	-3.860	25.257	21.397	-1.670
1960-61	-3.928	-24.168	-3.180	-20.239	-23.420	-1.670
1961-62	-5.150	0.225	0.819	5.376	6.195	-1.670
1962-63	2.124	3.227	-2.431	1.103	-1.327	-1.670

(1) ANNUAL PERCENTAGE CHANGE IN LABOR-CAPITAL RATIO
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(3) SUBSTITUTION EFFECT OF THE CHANGE IN THE RELATIVE FACTOR PRICE
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the relative share of labor to capital had risen from 1954 to 1958 and had shown alternations during the remaining periods.

The substitution effects of the changes in the relative factor price along with the capital using effect of the technological change seem to have been responsible for the declines of the labor-capital ratio, whereas the strong accounting effects of the changes in the relative factor price had frequently offset the combined forces of the substitution effects and the effects of the capital using technological changes , thus raising the relative share of labor to capital in the face of declining labor-capital ratio.

The relative share of labor to capital had increased in net by 53.63 percent for the entire period.

The capital using technological change of 1.7 percent a year is equivalent to the relative annual reduction of the labor inputs, for incstance in 1954, by 38.59 million manhours, and, thus, to the relative annual reduction of the labor share by 88.55 million dollars.

The impact of the capital using technological change accumulated over the entire period would be a reduction of the relative share of labor to capital by 25.05 percent.

3.14 Machinery, Except Electrical (SIC No. 35)

The selected estimating equation is the Model 2 unadjusted for the autocorrelation.

$$(2.34)\quad \log(L/K)_t = 2.351 + \underset{(1.273)}{.113}\ \log(r/w)_{t-1} + \underset{(1.566)}{.481}\ \log(L/K)_{t-1} - \underset{(1.030)}{.373}\ \log(L/K)_{t-2} - \underset{(1.687)}{.013}\ t$$

$\bar{R}^2$: .872

All the estimated coefficients other than that of $\log(L/K)_{t-2}$ are significant at the .125 level, and the coefficient of $\log(L/K)_{t-2}$ is significant at the .175 level.

The seperate parameters are

σ	a	b_1	b_2	(g_2-g_1)
.126	.893	.481	-373	.016

The weight parameters satisfy the given restrictions, and the technological change in this industry had been labor biased one in the sense that the annual exponential growth rate of the efficiency of labor had been higher than that of capital by 1.6 percentage points.

Table 15 shows that the labor-capital ratio had generally declined, whereas the relative share of labor had not followed the movements of the labor-capital ratio.

About half of the declines in the relative share of labor to capital can be attributed to the negative accounting effect of the changes in the relative factor price supplemented by the effect of the capital using technological change, while the mild sustitution effect had

TABLE 15

CONTRIBUTIONS OF THE RELEVANT FORCES TO THE
CHANGES IN THE RELATIVE SHARE OF LABOR TO CAPITAL

YEAR	(1)	(2)	(3)	(4)	(5)	(6)
1948-49	0.898	-18.943	1.127	-19.842	-18.714	-1.259
1949-50	-0.499	-5.922	2.330	-5.423	-3.093	-1.259
1950-51	-2.563	-9.837	0.557	-7.274	-6.716	-1.259
1951-52	-3.383	5.934	0.543	9.318	9.861	-1.259
1952-53	-4.376	27.876	-1.412	32.252	30.839	-1.259
1953-54	-0.391	2.241	-4.102	2.632	-1.469	-1.259
1954-55	-6.533	15.520	-0.338	22.053	21.714	-1.259
1955-56	-8.015	-16.315	-3.185	-8.300	-11.485	-1.259
1956-57	1.851	12.494	0.072	10.643	10.715	-1.259
1957-58	4.195	19.993	-0.999	15.797	14.798	-1.259
1958-59	-8.006	-21.665	-1.327	-13.659	-14.986	-1.259
1959-60	3.013	16.314	0.677	13.301	13.978	-1.259
1960-61	1.457	-3.488	-1.173	-4.946	-6.119	-1.259
1961-62	-3.004	-11.882	0.713	-8.877	-8.164	-1.259
1962-63	-1.153	3.304	0.676	4.457	5.134	-1.259

(1) ANNUAL PERCENTAGE CHANGE IN LABOR-CAPITAL RATIO
(2) ANNUAL PERCENTAGE CHANGE IN THE RELATIVE SHARE OF LABOR TO CAPITAL
(3) SUBSTITUTION EFFECT OF THE CHANGE IN THE RELATIVE FACTOR PRICE
(4) ACCOUNTING EFFECT OF THE CHANGE IN THE RELATIVE FACTOR PRICE
(5) NET OF THE SUBSTITUTION AND ACCOUNTING EFFECTS
(6) EFFECT OF TECHNICAL CHANGE

tried to raise the relative share during 1940's, the early 1950's and 1960's.

The relative share of labor to capital had increased in net by 15.63 percent over the whole period.

The technological change had worked in the direction to reducing the relative share of labor to capital by 1.26 percent a year, which is equivalent to the relative reduction of the labor input, for instance in 1954, by 37.81 million manhours, and, thus, to the relative reduction of the labor share by about 90.74 million dollars per annum.

The effect of the capital using technological change accumulated over the entire period would be a reduction of the relative share of labor to capital by 18.90 percent.

3.15 Electrical Machinery (SIC) No. 36)

The selected estimating equation is Model 2 adjusted for the autocorrelation.

$$(2.36)\ \left[\log(L/K)_t - .147\log(L/K)_{t-1}\right] = 1.136 + \underset{(3.479)}{.220}\left[\log(r/w)_{t-1} - .147\log(r/w)_{t-2}\right] + \underset{(2.343)}{.160}\left[\log(L/K)_{t-1} - .147\log(L/K)_{t-2}\right] - \underset{(.097)}{.002}\left[\log(L/K)_{t-2} - .147\log(L/K)_{t-3}\right] - \underset{(1.034)}{.005}\left[t - .147(t-1)\right]$$

$\bar{R}^2$: .942

The estimated coefficients, $a\sigma$ and b_1 , are significant at the .025 level, that of $(1-b_1-b_2)(\sigma-1)(g_2-g_1)$ being at the .175 level. The estimated coefficient, b_2, is not significantly different from zero.

The seperate parameters are

σ	a	b_1	b_2	(g_2-g_1)
.261	.842	.160	-.002	.008

The weight parameters, a and b_1 as well as b_2 , satisfy the given restrictions, and the technological change in this industry had been biased to labor in the sense that the annual exponential growth rate of the efficiency of labor had been higher than that of capital by .8 percentage points.

Table 16, shows that the labor-capital ratio had generally declined with four exceptional observations, whereas the relative share of labor to capital had risen in half of the observations.
As in the other industries, the accounting effect of the changes in the relative factor price had been dominant forces in influencing the direction of the movements of the relative share of labor to capital.

The relative share of labor to capital had increased in net by 23.15 percent over the entire period.

TABLE 16

CONTRIBUTIONS OF THE RELEVANT FORCES TO THE CHANGES IN THE RELATIVE SHARE OF LABOR TO CAPITAL

YEAR	(1)	(2)	(3)	(4)	(5)	(6)
1949-50	-5.885	-27.249	7.783	-21.363	-13.579	-0.489
1950-51	13.735	15.235	0.101	1.500	1.602	-0.489
1951-52	-9.006	-6.511	10.417	2.495	12.912	-0.489
1952-53	-5.143	25.723	-7.617	30.867	23.249	-0.489
1953-54	5.074	4.359	-10.803	-0.715	-11.518	-0.489
1954-55	-5.686	25.862	4.134	31.549	35.683	-0.489
1955-56	-4.527	-0.423	-11.405	4.104	-7.300	-0.489
1956-57	5.258	-23.221	-4.432	-28.480	-32.912	-0.489
1957-58	8.667	9.377	10.393	0.709	11.102	-0.489
1958-59	-3.285	-9.303	6.609	-6.018	0.591	-0.489
1959-60	-2.872	30.477	-1.259	33.350	32.090	-0.489
1960-61	-2.335	2.612	-9.586	4.947	-4.638	-0.489
1961-62	-0.289	3.320	-2.910	3.609	0.699	-0.489
1962-63	-9.217	-0.025	-1.016	9.191	8.175	-0.489

(1) ANNUAL PERCENTAGE CHANGE IN LABOR-CAPITAL RATIO
(2) ANNUAL PERCENTAGE CHANGE IN THE RELATIVE SHARE OF LABOR TO CAPITAL
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(6) EFFECT OF TECHNICAL CHANGE

The substitution effect of the changes in the relative factor price and the effect of the capital using technological change had been responsible for the declines of the labor-capital ratio.

The technological change had worked in the direction to reducing the relative share of labor to capital by .5 pervent a year, which is equivalent to the relative reduction of the labor input, for instance in 1954, by 12.32 million manhours, and, thus, to the relative reduction of the labor share by 27.10 million dollars per annum.

The effect of the capital using technological change accumulated over the entire period is a reduction of the relative share of labor to capital by 7.35 percent.

PART 3

THE MODEL WITH THREE FACTORS OF PRODUCTION: CAPITAL, LABOR OF PRODUCTION WORKERS, AND LABOUR OF NONPRODUCTION WORKERS

CHAPTER 4

THEORETICAL SPECIFICATION OF THE MODEL

4.1 Long-run Static Equilibrium Model

Various alternative generalized Constant Elasticity of Substitution Production Functions have been presented by H. Uzawa and V. Mukerji as well as others.

Mukerji's function is based on the assumption that the ratios of the partial elasticities of factor substitutions are the same,[1] whereas Uzawa's first generalized function is based on the stronger assumption that all the partial elasticities are the same.[2]

Even though Mukerji's function is less restrictive than the Uzawa's first the latter will be used in this study because the application of the former to the estimating equation results in complications and underidentification of the important parameters.

When the factor augmenting technology terms with exponential growth pathe are specified in the Uzawa's first generalized three-factor Constant Elasticity of Substitution Production Function,

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1 "Production Functions with Constant Elasticities of Substitution," Review of Economic Studies, 1962, pp. 291-299.

2 "A Generalized SMAC Production Function with Constant Ratio of Elasticities of Substitution," Review of Economic Studies, 1963.

$$(4.1)\quad Q = A\left[d_1(e^{g_1 t} K)^{-s} + d_2(e^{g_2 t} L_1)^{-s} + d_3(e^{g_3 t} L_2)^{-s}\right]^{-1/s}$$

L_1 : Labor of production workers

L_2 : Labor of nonproduction workers

d_i : Distributive parameters; $i= 1,2,3$

$$(4.2)\quad F_k = d_1 A^{-s} e^{-sg_1 t} K^{-1/\sigma} Q^{1/\sigma}$$

$$(4.3)\quad F_{L_1} = d_2 A^{-s} e^{-sg_2 t} L_1^{-1/\sigma} Q^{1/\sigma}$$

$$(4.4)\quad F_{L_2} = d_3 A^{-s} e^{-sg_3 t} L_2^{-1/\sigma} Q^{1/\sigma}$$

$$(4.5)\quad F_k/F_{L_1} = (d_2/d_1)\, e^{s(g_2-g_1)t} (L_1/K)^{1/\sigma} = r/w_1$$

$$(4.6)\quad \log L_1/K = \sigma \log d_2/d_1 + (\sigma-1)(g_2-g_1)\, t + \sigma \log r/w_1$$

$$(4.7)\quad \log RS_1 = \sigma \log d_2/d_1 + (\sigma-1)(g_2-g_1)\, t + (\sigma-1) \log r/w_1$$

where $RS_1 = w_1L_1/rK$, $\sigma = 1/(1+s)$

The analysis can be extended to the relative shares of L_1 to L_2 and L_2 to K. The relations involving L_1 and L_2 are shown as equations (4.8) and (4.9), and that involving L_2 and K are shown in equation (4.10) and (4.11).

$$(4.8)\quad \log L_2/L_1 = \sigma \log d_3/d_2 + (\sigma-1)(g_3-g_2)\, t + \sigma \log w_1/w_2$$

$$(4.9)\quad \log RS_2 = \sigma \log d_3/d_2 + (\sigma-1)(g_3-g_2)\, t + (\sigma-1) \log w_1/w_2$$

where $RS_2 = w_2L_2/w_1L_1$

$$(4.10)\ \log L_2/K = \sigma \log d_3/d_1 + (\sigma-1)(g_3-g_1)\ t + \sigma \log r/w_2$$

$$(4.11)\ \log RS_3 = \sigma \log d_3/d_1 + (\sigma-1)(g_3-g_1)\ t + (\sigma-1)\log r/w_2$$

$$\text{where} \quad RS_3 = w_2L_2/rK$$

However, the extension raises a problem of explaining the more likely differences in the estimated elasticities of substitution from the estimating equation (4.6), (4.8), and (4.10), whereas the estimating equations are derived on the assumption of equal elasticities of substitution. This problem can be dealt with by using the Theil-Goldberger's Mixed Estimation method[3] or with a statistical a priori restriction on the elasticities of substitutions in equations (4.8) and (4.10). The restricted estimating equations, then, become (4.12) and (4.13).

$$(4.12)\ [\ \log L_2/L_1 - \hat{\sigma}\log w_1/w_2\ ] = \hat{\sigma}\log d_3/d_2 + (g_3-g_2)(\hat{\sigma}-1)\ t$$

$$(4.13)\ [\ \log L_2/K - \hat{\sigma}\log r/w_2\ ] = \hat{\sigma}\log d_3/d_1 + (g_3-g_1)(\hat{\sigma}-1)\ t$$

As a by-product, we can get seperate estimates of the factor augmenting parameters, i.e. g_1, g_2 and g_3, because the composit parameters , (g_2-g_1), (g_3-g_2) and (g_3-g_1) would be estimated from the estimating equations (4.6), (4.12) and (4.13).

However, the present study will limit its analysis to the

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[3] Theil, H. and A. Goldberger, "On Pure and Mixed Statistical Estimation in Economics," International Economic Review, 1960.

relation involving L_1 and K as an experiment.

4.2 Short-run Dynamic Model Based on the Adaptive Expectation Scheme: Model 1

The stochastic production function, marginal productivity relations and the expansion path functions are as follows:

$$(4.14)\quad Q = A[\, d_1(e^{g_1 t} K)^{-s} + d_2(e^{g_2 t} L)^{-s} + d_3(e^{g_3 t} L)^{-s}]^{-1/s} u_o$$

$$(4.15)\quad F_k = d_1 (Au_o)^{-s} \; e^{-sg_1 t} K^{-1/\sigma} Q^{1/\sigma} = (r/p)\, u_o^*$$

$$(4.16)\quad F_{l_1} = d_2 (Au_o)^{-s} \; e^{-sg_2 t} L_1^{-1/\sigma} Q^{1/\sigma} = (w_1/p) u_o^*$$

$$(4.17)\quad F_{l_2} = d_3 (Au_o)^{-s} \; e^{-sg_3 t} L_2^{-1/\sigma} Q^{1/\sigma} = (w_2/p) u_o^*$$

u_o, u_1^* , u_2^* , u_3^* are the disturbance terms in the above functions, and have the meaning as those in the case of the two-factor production function system.

$$(4.18)\quad F_k/F_{l_1} = d_1/d_2 \; e^{s(g_2-g_1)t} (L_1'/K)^{1/\sigma} = (r/w_1)(u_1^* / u_2^*)$$

$$(4.19)\quad F_{l_1}/F_{l_2} = d_2/d_3 \; e^{s(g_3-g_2)t} (L_2'/L_1)^{1/\sigma} = (w_1/w_2)(u_2^* / u_3^*)$$

$$(4.20)\quad F_k/F_{l_2} = d_1/d_3 \; e^{s(g_3-g_1)t} (L_2/K)^{1/\sigma} = (r/w_2)(u_1^* / u_3^*)$$

The short-run estimating equations based on the adaptive expectation model, then, are as follows:

$$(4.21)\quad \log(L_1/K)_t = [\beta\sigma \log d_2/d_1 + (1-\beta)(\sigma-1)(g_2-g_1)] + \beta\sigma \log (r/w_1)_{t-1} + \beta(\sigma-1)(g_2-g_1)\, t + (1-\beta) \log(L_1/K)_{t-1} + \sigma[V_{lt} - (1-\beta)V_{lt-1}]$$

$$(4.22)\ \log(L_2/L_1) = [\beta\sigma\log d_3/d_2 + (1-\beta)(\sigma-1)(g_3-g_2)] + \log(w_1/w_2)_{t-1} + \beta(\sigma-1)(g_3-g_2)\,t + (1-\beta)\log(L_2/L_1)_{t-1} + \sigma[V_{1t} - (1-\beta)V_{1t-1}]$$

$$(4.23)\ \log(L_2/K) = [\beta\sigma\log d_3/d_1 + (1-\beta)(\sigma-1)(g_3-g_1)] + \log(r/w_2)_{t-1} + \beta(\sigma-1)(g_3-g_1)\,t + (1-\beta)\log(L_2/K)_{t-1} + \sigma[V_{3t} - (1-\beta)V_{3t-1}]$$

Where $V_1 : u_1^*/u_2^*$ $V_2 : u_2^*/u_3^*$ $V_3 : u_1^*/u_3^*$

The expectation coefficient, β, is the same in the three estimating equation. The expectation coefficient is parameter of the behavioral pattern of the same decision making unit even if a variety of the different sets of the input combinations are being considered.

The restricted estimating equations for L_2/L_1 and L_1/K based on the estimated values of β and σ from equation (4.21) are as follows:

$$(4.24)\ [\log(L_2/L_1)_t - \hat{\beta}\hat{\sigma}\log(w_1/w_2)_{t-1} - (1-\hat{\beta})\log(L_2/L_1)_{t-1}] = [\hat{\beta}\hat{\sigma}\log d_3/d_2 + (1-\hat{\beta})(\hat{\sigma}-1)(g_3-g_2)] + \hat{\beta}(\hat{\sigma}-1)(g_3-g_2)\,t + [V_{2t} - (1-\hat{\beta})V_{2t-1}]$$

$$(4.25)\ [\log(L_2/K)_t - \hat{\beta}\hat{\sigma}\log(r/w_2)_{t-1} - (1-\hat{\beta})\log(L_2/K)_{t-1}] = [\hat{\beta}\hat{\sigma}\log d_3/d_1 + (1-\hat{\beta})(\hat{\sigma}-1)(g_3-g_1)] + \hat{\beta}(\hat{\sigma}-1)(g_3-g_1)\,t + [V_{3t} - (1-\hat{\beta})V_{3t-1}]$$

Where $\hat{\beta}$ and $\hat{\sigma}$ were estimated from the estimating equation (4.21).

The relative share relation from the equation (4.21) is

$$\log RS_{1t} = \log(w_1L_1/rK) = [\beta\sigma \log d_2/d_1 + (1-\beta)(\sigma - 1)(g_2-g_1)] + \beta\sigma \log(r/w_1)_{t-1} - \log(r/w_1)_t + (1-\beta)\log(L_1/K)_{t-1} + \sigma[V_{1t} - (1-\beta) V_{1t-1}]$$

The substitution effect associated with the expected relative factor price is

$$\beta\sigma[\log(r/w_1)_{t-1} - \log(r/w_1)_{t-2}] + (1-\beta)[\log(L_1/K)_{t-1} - \log(L_1/K)_{t-2}]$$

The accounting effect associated with the change in the observed relative factor price on the relative factor share is

$$-[\log(r/w_1)_t - \log(r/w_1)_{t-1}]$$

The net effect is

$$\beta\sigma[\log(r/w_1)_{t-1} - \log(r/w_1)_{t-2}] + (1-\beta)[\log(L_1/K)_{t-1} - \log(L_1/K)_{t-2}] - [\log(r/w_1)_t - \log(r/w_1)_{t-1}]$$

The relative share of production labour[4] to capital will increase, decrease, or remain constant depending upon whether the above net effect is positive, negative or zero, cetris paribus.

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4 Labour of production workers.

In the case where the adjustments for the bias and the inconsistency due to the autocorrelation and the lagged dependent variable on the right hand side of the estimating equation are needed, Durbin's two-step procedure will be used as before.

The first step under the same assumption as before on the nature of the disturbance term would be as follows:

$$(4.26)\ \log(L_1/K)_t = [\sigma\beta(1-\rho)\log d_2/d_1 + (\sigma-1)(g_2-g_1)(1-\beta-\rho+2\beta\rho)] + \beta\sigma\log(r/w_1)_{t-1} - \rho\beta\sigma\log(r/w_1)_{t-2} + \beta(\sigma-1)(g_2-g_1)(1-\rho)\ t + (1-\beta+\rho)\log(L_1/K)_{t-1} - \rho(1-\beta)\log(L_1/K)_{t-2} + \sigma e_t$$

The second step is

$$(4.27)\ [\log(L_1/K)_t - \hat{\rho}\log(L_1/K)_{t-1}] = [\sigma\beta(1-\hat{\rho})\log d_2/d_1 + (\sigma-1)(g_2-g_1)(1-\beta-\hat{\rho}+2\rho\beta)] + \beta\sigma[\log(r/w_1)_{t-1} - \hat{\rho}\log(r/w_1)_{t-2}] + \beta(\sigma-1)(g_2-g_1)[t-\hat{\rho}(t-1)] + (1-\beta)[\log(L_1/K)_{t-1} - \hat{\rho}\log(L_1/K)_{t-2}] + \sigma e_t$$

where $\hat{\rho}$ is from equation (4.26).

$$\hat{\rho} = -(-\widehat{\rho\beta\sigma}/\widehat{\beta\sigma})$$

For those industries which have severe multicollinearity problem, the same method as that used in the model with two factors of production may be tried if one can get a set of satisfactory cross-section estimate of the elasticity of substitution between capital and labor of production workers.

The transformed estimating equation is

$$(4.28)\ \Big[\ \big[\ \log(L_1/K)_t - \hat{\rho}\log(L_1/K)_{t-1}\big] - \hat{\beta}\hat{\sigma}\big[\ \log(r/w_1)_{t-1} - \hat{\rho}\log(r/w_1)_{t-2}\ \big] - (1-\hat{\beta})\big[\ \log(L_1/K)_{t-1} - \hat{\rho}\log(L_1/K)_{t-2}\ \big]\ \Big]$$
$$= \big[\hat{\beta}\hat{\sigma}(1-\hat{\rho})\log\ d_2\ /d_1 + (\hat{\sigma}-1)(g_2-g_1)(1-\hat{\rho})(1-\hat{\beta})\big] + \hat{\beta}(\hat{\sigma}-1)(g_2-g_1)\big[\ t-\hat{\rho}(t-1)\ \big] + \hat{\sigma}e_t$$

$$\hat{\beta} = 0.1, 0.2, \ldots\ldots, 0.9$$

4.3 Short-run Dynamic Model Based on an Extension of the Adaptive Expectation Scheme: Model 2

The simple but more generalized rational distributed lag function than the adaptive expectation scheme will also be tried here. The estimating equation based on this scheme is as follows:

$$(4.29)\ \log(L_1/K)_t = \big[\sigma(1-b_1-b_2)\log d_2/d_1 + (\sigma-1)(g_2-g_1)(b_1+2b_2)\big] + a\sigma\log(r/w_1)_{t-1} + (1-b_1-b_2)(\sigma-1)(g_2-g_1)\ t + b_1\log(L_1/K)_{t-1} + b_2\log(L_1/K)_{t-2} + \sigma(V_{1t} - b_1V_{1t-1} - b_2V_{1t-2})$$

a , b_1 and b_2 have the same meaning as those in the case of the two-factor pfoduction function system.

In the case where an adjustment is needed for the bias and inconsistency due to autocorrelation and the lagged dependent variables as the explanatory variables, Durbin's two-step technique will be used again.

Durbin's first step based on this distributed lag scheme is

$$(4.30)\ \log(L_1/K)_t = [\sigma(1-b_1-b_2)(1-\rho)\log d_2/d_1 + (\sigma-1)(g_2-g_1)(b_1+2b_2+\rho-2b_1\rho-3b_2\rho)] + a\sigma\log(r/w_1)_{t-1} - \rho a\sigma\log(r/w_1)_{t-2} + (\sigma-1)(g_2-g_1)(1-b_1-b_2)(1-\rho)\ t + (b_1+\rho)\log(L_1/K)_{t-1} + (b_2-b_1\rho)\log(L_1/K)_{t-2} - b_2\rho\log(L\ /K)_{t-3} + \sigma e_t$$

$$\hat{\rho} = -(-\rho\hat{a\sigma}/\hat{a\sigma})$$

The second step is

$$(4.31)[\log(L_1/K)_t - \hat{\rho}\log(L_1/K)_{t-1}] = [\sigma(1-b_1-b_2)(1-\hat{\rho})\log d_2/d_1 + (\sigma-1)(g_2-g_1)(1-\hat{\rho})(b_1+2b_2)] + a\sigma[\log(r/w_1)_{t-1} - \hat{\rho}\log(r/w_1)_{t-2}] + (\sigma-1)(g_2-g_1)(1-b_1-b_2)[t-\hat{\rho}(t-1)] + b_1[\log(L_1/K)_{t-1} - \hat{\rho}\log(L_1/K)_{t-2}] + b_2[\log(L_1/K)_{t-2} - \hat{\rho}(L_1/K)_{t-3}] + \sigma e_t$$

For the industries with severe multicollinearity, the same method of treatment as that used in the earlier section can be tried. The final transformed estimating equation incorporating the cross-section estimates of the elasticities of substitution and the various values of the weight parameters, b_1 and b_2, would be as follows:

$$(4.32)[\log[(L_1/K)_t - \hat{\rho}\log(L_1/K)_{t-1}] - \hat{a}\hat{\sigma}[\log(r/w_1)_{t-1} - \hat{\rho}\log(r/w_1)_{t-2}] - \hat{b}_1[\log(L_1/K)_{t-1} - \hat{\rho}\log(L_1/K)_{t-2}] - \hat{b}_2[\log(L_1/K)_{t-2} - \hat{\rho}\log(L_1/K)_{t-3}]] = [\hat{\sigma}(1-\hat{b}_1-\hat{b}_2)(1-\hat{\rho})\log d_2/d_1 + (\hat{\sigma}-1)(g_2-g_1)(1-\hat{\rho})(\hat{b}_1+2\hat{b}_2)] + (\hat{\sigma}-1)(g_2-g_1)(1-\hat{b}_1-\hat{b}_2)[t-\hat{\rho}(t-1)]$$

The relative share relation from the equation (4.29) is

$$\log RS_{1t} = \log(w_1L_1/rK)_t = \Big[\sigma(1-b_1-b_2)\log d_2/d_1 + (\sigma-1)(g_2-g_1)(b_1+2b_2)\Big] + a\sigma\log(r/w_1)_{t-1} - \log(r/w_1)_t + b_1\log(L_1/K)_{t-1} + b_2\log(L_1/K)_{t-2} + (1-b_1-b_2)(\sigma-1)(g_2-g_1)\,t + \sigma(V1_t - b_1V1_{t-1} - b_2V1_{t-2}\Big]$$

The substitution effect associated with the change in the expected relative facotr price on the relative factor share is

$$a\sigma\big[\log(r/w_1)_{t-1} - \log(r/w_1)_{t-2}\big] + b_1\big[\log(L_1/K)_{t-1} - \log(L_1/K)_{t-2}\big] + b_2\big[\log(L_1/K)_{t-2} - \log(L_1/K)_{t-3}\big]$$

The accounting effect associated with the observed relative factor price is

$$-\big[\log(r/w_1)_t - \log(r/w_1)_{t-1}\big]$$

The net effect is

$$a\sigma\big[\log(r/w_1)_{t-1} - \log(r/w_1)_{t-2}\big] + b_1\big[\log(L_1/K)_{t-1} - \log(L_1/K)_{t-2}\big] + b_2\big[\log(L_1/K)_{t-2} - \log(L_1/K)_{t-3}\big] - \big[\log(r/w_1)_t - \log(r/w_1)_{t-1}\big]$$

The relative share of production labor to capital will increase, decrease, or remain constant depending upon whether the above net effect is positive, negative, or zero, cetris paribus.

CHAPTER 5

EMPIRICAL ESTIMATION OF THE PARAMETERS IN THE MODEL

The following industries, which had shown the negative elasticities of the substitution in the previous estimations with the model for labor as a whole and capital, again showed the negative estimates of the elasticities: Food and Kindred Products, Apparel and Related Products, Chemicals and Allied Products, and Stone and Glass Products.

Since satisfactory extraneous estimates of the elasticities of substitution between capital and labor of production workers are not available, the mixed estimation method used in the previous model with two factors of production can not be used here.
Instead of attempting to estimate the parameters with unsatisfactory proxies for the extraneous estimates of the elasticities, these industries will be put aside.

When the satisfactory extraneous estimates of the elasticities are available, the mixed estimation method used in the model with two factors of production can be used.

5.1 Tobacco Manufactures (SIC No. 21)

The selected estimating equation is the Model 1 adjusted for the autocorrelation.

$$(4.27)\ \left[\log(L_1/K)_t - .191\log(L_1/K)_{t-1}\right] = 1.590 + \underset{(2.026)}{.360}\left[\log(r/w_1)_{t-1} - .191\log(r/w_1)_{t-2}\right] + \underset{(3.077)}{.597}\left[\log(L_1/K)_{t-1} - .191\log(L_1/K)_{t-2}\right] - \underset{(2.631)}{.028}\left[t - .191\,(t-1)\right]$$

$\bar{R}^2$: .973

$\hat{\rho}$: .191

All the estimated coefficients are significant at the .05 level. The seperate parameters are

σ	β	(g_2-g_1)
.891	.403	.064

The elasticity of substitution between capital and production labor is much higher than that between capital and labor as a whole.

The adaptive expectation coefficient of .403 , which is higher than that previously estimated for the case of capital and labor, implies that this industry had been more cautious in making input decision involving capital and labor as a whole than the case involving capital and labor of production worker,i.e. production labor.

The technological change had been biased to the production labor in the sense that the annual exponential growth rate of the efficiency of production labor had been higher than that of capital by about 6.42 percentage points.

When the labor biased technological change is associated with the elasticity of substitution lower than unity, it becomes a capital using one.

From Table 17 we can observe that the production labor-capital ratio, L_1/K, had declined almost throughout the whole period with two exceptional observations, and that the relative share of production labor and capital, w_1L_1/rK, had also declined with two exceptional observations. The degrees of declines are slightly higher than those associated with the cases involving labor-capital ratio, L/K, which is in line with our expectation.

After 1954 on, the strong substitution effect of the changes in the relative factor price enforced by the capital using technological changes had worked in the direction to reducing the relative share of production labor despite the accounting effect of the change in the relative factor price which had worked in the direction to raising the relative share.

Prior to 1953, the strong accounting effect this time had worked in the direction to reducing the relative share whereas the mild substitution effect showed alternations around zero.

The relative share of production labor to capital had declined in net over the entire period by 91.48 percent.

TABLE 17

CONTRIBUTIONS OF THE RELEVANT FORCES TO THE CHANGES IN THE RELATIVE SHARE OF LABOR TO CAPITAL

YEAR	(1)	(2)	(3)	(4)	(5)	(6)
1948-49	-7.105	-30.423	-0.376	-23.318	-23.694	-2.819
1949-50	-1.215	19.548	4.143	20.764	24.907	-2.819
1950-51	-2.714	-9.106	-8.189	-6.392	-14.581	-2.819
1951-52	2.032	-4.008	0.678	-6.041	-5.362	-2.819
1952-53	4.981	-24.826	3.384	-29.807	-26.423	-2.819
1953-54	-0.012	7.664	13.688	7.676	21.365	-2.819
1954-55	-0.666	4.309	-2.767	4.976	2.209	-2.819
1955-56	-11.708	-1.766	-2.187	9.942	7.754	-2.819
1956-57	-16.782	-13.958	-10.559	2.824	-7.735	-2.819
1957-58	-8.964	-6.903	-11.027	2.061	-8.965	-2.819
1958-59	-12.382	-12.464	-6.089	-0.082	-6.171	-2.819
1959-60	-4.294	-0.279	-7.357	4.015	-3.342	-2.819
1960-61	-5.998	-3.539	-4.005	2.459	-1.546	-2.819
1961-62	-8.021	-7.119	-4.463	0.902	-3.560	-2.819
1962-63	-11.006	-8.603	-5.110	2.403	-2.707	-2.819

(1) ANNUAL PERCENTAGE CHANGE IN LABOR-CAPITAL RATIO
(2) ANNUAL PERCENTAGE CHANGE IN THE RELATIVE SHARE OF LABOR TO CAPITAL
(3) SUBSTITUTION EFFECT OF THE CHANGE IN THE RELATIVE FACTOR PRICE
(4) ACCOUNTING EFFECT OF THE CHANGE IN THE RELATIVE FACTOR PRICE
(5) NET OF THE SUBSTITUTION AND ACCOUNTING EFFECTS
(6) EFFECT OF TECHNICAL CHANGE

The influences of capital using technological change had been steady and significant. The capital using technological chagne of 2.82 percent per annum is equivalent to the relative reduction of the production labor input, for instance in 1954, by 5.34 million manhours a year, and, thus, the relative reduction of the production labor share by about 8.65 million dollars per annum.

The capital using technological change accumulated over the entire period had depressed the relative share of production labor to capital by 42.30 percent.

5.2. Textile Mill Products (SIC No. 22)

The selected estimating equation is the Model 1 adjusted for the autocorrelation.

$$(4.27)\quad [\log(L_1/K)_t + 1.00 \log(L_1/K)_{t-1}] = 1.756 +$$

$$\underset{(3.302)}{.084}[\log(r/w_1)_{t-1} + 1.00 \log(r/w_1)_{t-2}] +$$

$$\underset{(4.757)}{.472}[\log(L_1/K)_{t-1} + 1.00 \log(L_1/K)_{t-2}] +$$

$$\underset{(.779)}{.002}[\,t + 1.00\,(t-1)\,]$$

R : .974

$\hat{\rho}$: - 1.00

The estimated coefficients except for that of the technology term are highly significant at the .005 level.
The coefficient of the technology term is significant only at .225 level which is not quite satisfactory.
Thus, it seems the influence of the technological change on the production labor-capital ratio and the relative share of production labor to capital had not been quite significant in this industry.

The seperate parameters are

σ	β	(g_2-g_1)
.159	.528	-.003

The estimated elasticity of substitution between production labor and capital is of almost same magnitude as the one between labor and capital.

The adaptive expectation coefficient is also of almost same magnitude as the one associated with input adjustment involving labor and capital, which assigns about a half weight to the most recently observed relative factor price and the subsequent weights on all the previous prices decline at the moderate rate as time goes back.

The technological chagne in this industry had been capital biased one in the sense that the exponential growth rate of the efficiency of production labor had been lower than that of capital by .3 percentage points per annum.

When the capital biased technological change is associated with the elasticity of substitution lower than unity, the technological change becomes a production labor using one.

From Table 18, we can observe that the production labor-capital ratio had generally declined with four exceptional observations, whereas the relative share of production labor to capital prior to 1954 had risen with one exceptional observation. These patterns of the variations of the production labor-capital ratio and the relative share of production labor to capital are the same as those of the labor-capital ratio and the relative share of labor to capital, except that the former ratio and relative share had shown slightly higher degree of decreases and slightly lower degree of increases than the latter ones.

The declines of the production labor-capital ratio from 1948 to 1957 had been associated with the depressing substitution effect of the changes in the relative factor price, while the depressing substitution effect had been mostly offset by the strong accounting effect, thus resulting in the increases of the relative share of production labor to capital.

From 1957 on, the strong accounting effect had worked in the direction to reducing the relative share, while the substitution effect had worked in the opposite direction to raise the relative share. The relative share of production labor to capital had increased in net over the entire period by 83.02 percent.

TABLE 18

CONTRIBUTIONS OF THE RELEVANT FORCES TO THE CHANGES IN THE RELATIVE SHARE OF LABOR TO CAPITAL

YEAR	(1)	(2)	(3)	(4)	(5)	(6)
1948-49	-14.140	46.735	-12.355	60.875	48.520	0.150
1949-50	-5.165	27.568	-11.773	32.733	20.959	0.150
1950-51	-7.974	-49.264	-5.179	-41.290	-46.469	0.150
1951-52	-3.576	45.462	-0.310	49.039	48.728	0.150
1952-53	-2.019	35.066	-5.793	37.085	31.292	0.150
1953-54	-3.182	45.654	-4.057	48.836	44.779	0.150
1954-55	-8.719	-56.709	-5.590	-47.989	-53.580	0.150
1955-56	-4.057	-0.114	-0.101	3.942	3.841	0.150
1956-57	4.031	18.597	-2.246	14.566	12.319	0.150
1957-58	3.284	-10.457	0.684	-13.741	-13.057	0.150
1958-59	-0.925	-21.433	2.701	-20.508	-17.806	0.150
1959-60	1.978	-7.316	1.279	-9.295	-8.015	0.150
1960-61	2.809	36.953	1.712	34.145	35.857	0.150
1961-62	-0.683	-28.084	-1.531	-27.401	-28.932	0.150
1962-63	-4.191	0.360	1.970	4.552	6.522	0.150

(1) ANNUAL PERCENTAGE CHANGE IN LABOR-CAPITAL RATIO
(2) ANNUAL PERCENTAGE CHANGE IN THE RELATIVE SHARE OF LABOR TO CAPITAL
(3) SUBSTITUTION EFFECT OF THE CHANGE IN THE RELATIVE FACTOR PRICE
(4) ACCOUNTING EFFECT OF THE CHANGE IN THE RELATIVE FACTOR PRICE
(5) NET OF THE SUBSTITUTION AND ACCOUNTING EFFECTS
(6) EFFECT OF TECHNICAL CHANGE

The production labor using technological change had worked in the direction to increasing the production labor-capital ratio and the relative share of production labor to capital by .15 percent per annum, which is equivalent to the relative increase of the production labor input, for instance in 1954, by 2.85 million manhours, and, thus, to the relative increase of production labor share by 3.87 million dollars per annum. The labor using technological change accumulated over the entire period had worked in the direction to raising the relative share of production labor to capital by 2.25 percent.

5.3. Furniture and Fixtures (SIC No. 25)

The selected estimating equation is the Model 1 adjusted for the autocorrelation.

$$(4.27)\ [\log(L_1/K)_t + .073\log(L_1/K)_{t-1}] = 1.877 + .217[\log(r/w_1)_{t-1} + .073\log(r/w_1)_{t-2}] \underset{(1.100)}{.187}[\log(L_1/K)_{t-1} + .073\log(L_1/K)_{t-2}] - \underset{(3.114)}{.014}[t + .073(t-1)]$$

$\bar{R}^2$: .946

$\hat{\rho}$: -.073

The coefficients of the relative factor price and the technology term are significant even at the .005 level, and the coefficient of the lagged dependent variable term is significant at the .15 level.

The seperate parameters are

σ	β	(g_2-g_1)
.266	.814	.023

The estimated elasticity of substitution is slightly higher and the adaptive expectation coefficient slightly lower than those involving the two factors of production, i.e. labor and capital.

The technological change had been biased to production labor in the sense that the annual exponential growth rate of the efficiency of production labor had been higher than that of capital by 2.3 percentage points. When the labor biased technological change was associated with the elasticity of substitution less than unity, it became a capital using one.

Table 19 shows that the production labor-capital ratio and the relative share of production labor to capital had had some irregular alternation of increases and decreases.
In general, those ratio and relative share showed the same patterns of the variations as those involving the labor and capital except that the degree of the increases of the former are lower and those of decreases higher than the latter.

The accounting effects of the changes in the relative factor price this time had not really worked to relieve the relative share of

TABLE 19

CONTRIBUTIONS OF THE RELEVANT FORCES TO THE
CHANGES IN THE RELATIVE SHARE OF LABOR TO CAPITAL

YEAR	(1)	(2)	(3)	(4)	(5)	(6)
1948-49	3.098	-17.151	31.439	-20.249	11.189	-1.370
1949-50	0.264	14.100	4.965	13.836	18.801	-1.370
1950-51	2.531	-23.160	-2.949	-25.692	-28.641	-1.370
1951-52	-0.504	11.270	6.039	11.775	17.814	-1.370
1952-53	1.696	36.266	-2.645	34.570	31.925	-1.370
1953-54	-14.594	-13.384	-7.175	1.210	-5.965	-1.370
1954-55	-6.072	-15.687	-2.983	-9.614	-12.598	-1.370
1955-56	0.150	-5.540	0.950	-5.691	-4.740	-1.370
1956-57	1.312	12.918	1.261	11.606	12.868	-1.370
1957-58	-0.402	30.112	-2.270	30.515	28.244	-1.370
1958-59	-10.896	-16.024	-6.687	-5.128	-11.815	-1.370
1959-60	-4.600	2.227	-0.920	6.827	5.906	-1.370
1960-61	2.735	-6.980	-2.337	-9.716	-12.053	-1.370
1961-62	2.167	-1.192	2.615	-3.359	-0.744	-1.370
1962-63	-5.436	0.034	1.132	5.471	6.604	-1.370

(1) ANNUAL PERCENTAGE CHANGE IN LABOR-CAPITAL RATIO
(2) ANNUAL PERCENTAGE CHANGE IN THE RELATIVE SHARE OF LABOR TO CAPITAL
(3) SUBSTITUTION EFFECT OF THE CHANGE IN THE RELATIVE FACTOR PRICE
(4) ACCOUNTING EFFECT OF THE CHANGE IN THE RELATIVE FACTOR PRICE
(5) NET OF THE SUBSTITUTION AND ACCOUNTING EFFECTS
(6) EFFECT OF TECHNICAL CHANGE

production labor to capital from declining due to the substitution effect.

The relative share of production labor to capital had increased in net over the entire period by 7.81 percent.

The capital using technological change had tried harder to reduce the relative share of production labor to capital than the case of the labor and capital. The reduction of the relative share of production labor to capital by 1.37 percen a year is equivalent to the relative reduction of production labor input by 8.22 million manhours , and , thus, to the relative reduction of the relative share by 13.12 million dollars per annum.

The capital using technological change accumulated over the entire period had depressed the relative share of production labor to capital by 20.55 percent.

5.4. Paper and Allied Products (SIC No. 26)

The selected estimating equation is the Model 2 unadjusted for the autocorrelation.

$$(4.29)\ \log(L_1/K)_t = 1.032 + \underset{(1.310)}{.090} \log(r/w_1)_{t-1} + \underset{(4.978)}{1.275} \log(L_1/K)_{t-1} - \underset{(2.220)}{.648} \log(L_1/K)_{t-2} - \underset{(\ .842)}{.008}\ t$$

$\bar{R}^2$: .970

All the estimated coefficients except that of the technology term are significant at the .125 level, and the coefficient of the technology term is significant at the .2 level.

The seperate parameters are

σ	a	b_1	b_2	(g_2-g_1)
.242	.373	1.275	-.648	.029

All the weight parameters satisfy the given restrictions on them, and the technological change in this industry had been labor biased, where the annual exponential growth rate of the efficiency of production labor had been higher than that of capital by 2.93 percentage points. Nevertheless, when the production labor biased technological change was associated with the elasticity of substitution less than the value of unity the technological change became a capital using one.

From Table 20 , we can observe that the production labor-capital ratio had declined throughout the entire period except one minor exceptional increase, whereas the relative share of production labor to capital had nine increases and five declines.

The movements of the factor ratio and the relative factor share show almost the same patterns as those shown in the case involving labor and capital except that the degree of the decreases of the factor ratio and the relative factor share of the former are slightly severer and that of the decreases slightly milder than that of the decreases and the increases of the latter.

TABLE 20

CONTRIBUTIONS OF THE RELEVANT FORCES TO THE
CHANGES IN THE RELATIVE SHARE OF LABOR TO CAPITAL

YEAR	(1)	(2)	(3)	(4)	(5)	(6)
1949-50	-7.049	-23.474	2.197	-16.424	-14.227	-0.829
1950-51	-5.325	-26.600	-9.049	-21.274	-30.323	-0.829
1951-52	1.705	30.198	-1.812	28.493	26.680	-0.829
1952-53	-3.226	14.269	3.538	17.496	21.034	-0.829
1953-54	-6.382	2.726	-7.707	9.108	1.401	-0.829
1954-55	-11.218	-7.170	-8.672	4.047	-4.624	-0.829
1955-56	-8.459	-13.531	-13.703	-5.071	-18.775	-0.829
1956-57	-4.705	27.796	-5.454	32.501	27.046	-0.829
1957-58	-1.851	13.779	-4.777	15.631	10.853	-0.829
1958-59	-0.623	-10.641	-1.244	-10.017	-11.261	-0.829
1959-60	-0.005	8.100	1.129	8.105	9.235	-0.829
1960-61	-1.210	8.187	-0.334	9.398	9.063	-0.829
1961-62	-1.442	4.274	-2.729	5.716	2.986	-0.829
1962-63	-2.253	16.527	-1.976	18.781	16.805	-0.829

(1) ANNUAL PERCENTAGE CHANGE IN LABOR-CAPITAL RATIO
(2) ANNUAL PERCENTAGE CHANGE IN THE RELATIVE SHARE OF LABOR TO CAPITAL
(3) SUBSTITUTION EFFECT OF THE CHANGE IN THE RELATIVE FACTOR PRICE
(4) ACCOUNTING EFFECT OF THE CHANGE IN THE RELATIVE FACTOR PRICE
(5) NET OF THE SUBSTITUTION AND ACCOUNTING EFFECTS
(6) EFFECT OF TECHNICAL CHANGE

The relative share shows a net increase of 44.44 percent over the period.

In general, the accounting effect had worked in the direction to offset the depressing substitution effect, thus resulting in the rising relative share of production labor to capital in the face of the declining production labor-capital ratio.

The technological change had contributed to reducing the relative share by about .83 percent a year, which is equivalent to the relative reduction of the production labor input, for instance in 1954, by 8.15 million manhours, and, thus, to the relative reduction of production labor share by 14.30 million dollars per annum.

The capital using technological change accumulated over the entire period had depressed the relative share of production labor to capital by 12.45 percent.

5.5. Printing and Publishing (SIC No. 27)

The selected estimating equation is the Model 2 adjusted for the autocorrelation.

$$(4.31)\quad [\log(L_1/K)_t + .279\log(L_1/K)_{t-1}] = 1.923 + \underset{(2.806)}{.409}[\log(r/w_1)_{t-1} + .279\log(r/w_1)_{t-2}] + \underset{(10.628)}{1.218}[\log(L_1/K)_{t-1} + .279\log(L_1/K)_{t-2}] - \underset{(6.183)}{.979}[\log(L_1/K)_{t-2} + .279\log(L_1/K)_{t-3}] - \underset{(.934)}{.002}[t - .279(t-1)]$$

R : .974

$\hat{\rho}$: -.279

All the estimated coefficients except that of the technology term are significant at the .01 level, and the coefficient of the technology term is significant at the .2 level.

The seperate parameters are

σ	a	b_1	b_2	(g_2-g_1)
.538	.760	1.218	-.979	.006

The elasticity of substitution in this case is greater than that involving the labor and capital.

The weight parameters satisfy the given restrictions, and they indicate that the more weights are assigned to the recently observed relative factor prices than the case involving labor and capital.

The technological change in this industry had been slightly biased to to production labor in the sense that the annual expoenetial growth rate of the efficiency of production labor had been higher than that of capital by .6 percentage points.
Nevertheless, the labor biased technological change associated with the elasticity of substitution lower than unity became capital using one.

Table 21 shows that the production labor-capital ratio had generally been declining except early 1950's, whereas the relative share of production labor to capital had experienced more frequent increases

TABLE 21

CONTRIBUTIONS OF THE RELEVANT FORCES TO THE
CHANGES IN THE RELATIVE SHARE OF LABOR TO CAPITAL

YEAR	(1)	(2)	(3)	(4)	(5)	(6)
1949-50	-3.043	4.145	-4.802	7.188	2.386	-0.209
1950-51	2.875	6.348	-0.200	3.472	3.271	-0.209
1951-52	4.679	1.283	6.069	-3.396	2.673	-0.209
1952-53	2.231	0.244	5.918	-1.987	3.931	-0.209
1953-54	-0.880	3.256	-0.264	4.136	3.871	-0.209
1954-55	-3.629	-10.209	-5.256	-6.580	-11.836	-0.209
1955-56	-2.757	4.510	-2.140	7.268	5.127	-0.209
1956-57	-2.138	-3.599	-3.751	-1.460	-5.211	-0.209
1957-58	0.534	21.287	-0.060	20.753	20.692	-0.209
1958-59	-4.323	-13.885	-5.561	-9.562	-15.123	-0.209
1959-60	-6.168	1.007	-3.393	7.175	3.782	-0.209
1960-61	0.857	10.015	-8.387	9.157	0.769	-0.209
1961-62	-4.424	-8.191	3.634	-3.767	-0.132	-0.209
1962-63	-4.975	5.457	-6.239	10.433	4.193	-0.209

(1) ANNUAL PERCENTAGE CHANGE IN LABOR-CAPITAL RATIO
(2) ANNUAL PERCENTAGE CHANGE IN THE RELATIVE SHARE OF LABOR TO CAPITAL
(3) SUBSTITUTION EFFECT OF THE CHANGE IN THE RELATIVE FACTOR PRICE
(4) ACCOUNTING EFFECT OF THE CHANGE IN THE RELATIVE FACTOR PRICE
(5) NET OF THE SUBSTITUTION AND ACCOUNTING EFFECTS
(6) EFFECT OF TECHNICAL CHANGE

than the decreases. The degrees of the decreases of the relative share had been slightly severer and of the increases slightly milder than those of the relative share of labor to capital.

The decreases of the production labor-capital ratio had been associated with the depressing substitution effect and the depressing effects of the capital using technological change, whereas the accounting effect had offset the depressing effects.

The relative share of production labor to capital had increased in net over the entire period by 24.99 percent.

The capital using technological change had constantly depressed the relative share of production labor to capital by .21 percent a year, which is equivalent to the relative reduction of the production labor input, for instance in 1954, by 2.21 million manhours, and, thus, to the relative reduction of the production labor share by 4.81 million dollars.

The capital using technological change accumulated over the entire period had depressed the relative share by 3.15 percent

5.6. Rubber and Miscellaneous Plastic Products (SIC No. 30)

The selected estimating equation is the Model 1 adjusted for the autocorrelation.

$$(4.27) \quad \left[\log(L_1/K)_t - .230 \log(L_1/K)_{t-1} \right] = 1.747 \ +$$

$$\underset{(1.500)}{.073}\left[\log(r/w_1)_{t-1} - .230\log(r/w_1)_{t-2}\right] +$$

$$\underset{(1.565)}{.384}\left[\log(L_1/K)_{t-1} - .230\log(L_1/K)_{t-2}\right] -$$

$$\underset{(3.187)}{.019}\left[t - .230\,(t-1)\right]$$

$\bar{R}^2$: .884

$\hat{\rho}$: .230

The estimated coefficients except that of the technology term are significant at the .10 level, and the coefficient of the technology term is significant at the .005 level.

The seperate parameters are

σ	β	(g_2-g_1)
.118	.616	.035

This estimated elasticity of substitution between production labor and capital is slightly lower than that between labor and capital.

The adaptive expectation coefficient is almost the same as the one for the input decision process involving labor and capital, and the technological change had been biased to production labor in the sense that the annual exponential growth rate of the efficiency of production labor had been higher than that of capital by 3.5 percentage points.

Nevertheless, when the labor biased technological change was associated with the elasticity of substitution less than unity, the technological change became capital using technological change.

Table 22 shows that the production labor-capital ratio had generally experienced increases until 1953 and decreases thereafter, whereas the relative share of production labor to capital had more frequent increases than decreases, although the magnitude of the single decrease from 1950 to 1951 was quite drastic.

The substitution effect enforced by the capital using technological change had been responsible for the decreases of the production labor-capital ratio.

In many instances, the accounting effect had offset the above depressing effects, although in some cases the accounting effect had also joined the above effects in depressing the relative share of production labor to capital, e.g. 1948-49, 1950-51, 1954-55, 1955-56.

The relative share of production labor to capital had decreased in net over the entire period by 12.21 percent.

The capital using technological change had depressed the relative share by 1.92 percent a year, which is equivalent to the relative reduction of the production labor share by about 19 million dollars per annum.

The capital using technological change accumulated over the entire period had depressed the relative share of production labor to capital by 28.80 percent.

TABLE 22

CONTRIBUTIONS OF THE RELEVANT FORCES TO THE CHANGES IN THE RELATIVE SHARE OF LABOR TO CAPITAL

YEAR	(1)	(2)	(3)	(4)	(5)	(6)
1948-49	1.709	1.224	-1.422	-0.484	-1.907	-1.919
1949-50	-1.886	9.560	0.691	11.447	12.138	-1.919
1950-51	6.849	-95.387	-1.555	-102.236	-103.791	-1.919
1951-52	3.944	15.514	10.050	11.570	21.620	-1.919
1952-53	2.375	23.560	0.673	21.184	21.858	-1.919
1953-54	-7.560	45.666	-0.626	53.226	52.599	-1.919
1954-55	-7.741	-14.884	-6.764	-7.143	-13.908	-1.919
1955-56	2.231	-29.498	-2.452	-31.729	-34.181	-1.919
1956-57	-6.340	19.893	3.160	26.234	29.394	-1.919
1957-58	-8.172	-5.871	-4.337	2.300	-2.036	-1.919
1958-59	-3.063	1.218	-3.302	4.281	0.979	-1.919
1959-60	-2.521	-1.651	-1.486	0.869	-0.616	-1.919
1960-61	1.483	6.577	-1.030	5.094	4.063	-1.919
1961-62	-7.400	7.866	0.199	15.266	15.466	-1.919
1962-63	-6.929	4.001	-3.948	10.931	6.982	-1.919

(1) ANNUAL PERCENTAGE CHANGE IN LABOR-CAPITAL RATIO
(2) ANNUAL PERCENTAGE CHANGE IN THE RELATIVE SHARE OF LABOR TO CAPITAL
(3) SUBSTITUTION EFFECT OF THE CHANGE IN THE RELATIVE FACTOR PRICE
(4) ACCOUNTING EFFECT OF THE CHANGE IN THE RELATIVE FACTOR PRICE
(5) NET OF THE SUBSTITUTION AND ACCOUNTING EFFECTS
(6) EFFECT OF TECHNICAL CHANGE

5.7. Leather and Leather Products (SIC No. 31)

The selected estimating equation is the Model 2 unadjusted for the autocorrelation.

$$(4.29)\quad \log(L_1/K)_t = 3.476 + \underset{(.145)}{.004}\ \log(r/w_1)_{t-1} + \underset{(1.677)}{.506}\ \log(L_1/K)_{t-1} - \underset{(1.800)}{.499}\ \log(L_1/K)_{t-2} - \underset{(3.015)}{.027}\ t$$

$$\bar{R}^2 : .917$$

The estimated coefficients except that of the relative factor price term are significant at the .10 level, and the coefficient of the relative factor price, i.e. the elasticity of substitution with respect to change in the observed relative factor price, is not significantly different from zero.

The seperate parameters are

σ	a	b_1	b_2	(g_2-g_1)
.004	.993	.506	-.499	.027

The weight parameters satisfy the given restrictions, and the technological change had been biased to production labor in the sense that the exponential grwwth rate of the efficiency of production labor had been higher than that of capital by 2.72 percentage points. When the production labor biased technological change was associated with the elasticity of substitution less than unity, the technological change turned out to be capital using one.

From Table 23, we can observe that the production labor-capital ratio had in general declined, whereas the relative share of production labor to capital had not followed the movements of the factor ratio. In general, the accounting effect had been the dominant one in influencing the direction of the variation of the relative share.

The relative share of production labor to capital had increased in net over the entire period by 26.29 percent.

The technological change had constantly depressed the relative share by 2.69 percent a year, which is equivalent to the relative reduction of the production labor share by about 23.27 million dollars.

The capital using technological change accumulated over the entire period had depressed the relative share of production labor to capital by 40.35 percent.

5.8. Fabricated Metal Products (SIC No. 34)

The selected estimating equation is the Model 1 unadjusted for the autocorrelation.

$$(4.21)\quad \log(L_1/K)_t = 1.257 + \underset{(2.124)}{.131}\log(r/w_1)_{t-1} + \underset{(2.120)}{.516}\log(L_1/K)_{t-1} - \underset{(.827)}{.011}\,t$$

$\bar{R}^2$: .982

TABLE 23

CONTRIBUTIONS OF THE RELEVANT FORCES TO THE
CHANGES IN THE RELATIVE SHARE OF LABOR TO CAPITAL

YEAR	(1)	(2)	(3)	(4)	(5)	(6)
1949-50	0.509	88.338	-20.026	87.828	67.802	-2.690
1950-51	0.574	-144.547	8.005	-145.122	-137.116	-2.690
1951-52	-0.984	39.970	1.198	40.954	42.152	-2.690
1952-53	2.373	22.645	-1.925	20.271	18.346	-2.690
1953-54	-4.133	-22.226	3.957	-18.092	-14.135	-2.690
1954-55	-3.544	45.768	-7.290	49.313	42.022	-2.690
1955-56	-6.892	-24.165	-3.438	-17.272	-20.710	-2.690
1956-57	-3.322	4.531	-8.464	7.853	-0.611	-2.690
1957-58	0.738	39.733	-1.558	38.995	37.436	-2.690
1958-59	-2.487	-3.767	2.601	-1.279	1.321	-2.690
1959-60	-5.005	-53.735	-4.082	-48.729	-52.812	-2.690
1960-61	-2.666	77.127	-6.040	79.794	73.754	-2.690
1961-62	-4.103	-69.100	-1.816	-64.996	-66.813	-2.690
1962-63	-7.466	-20.803	-4.537	-13.337	-17.874	-2.690

(1) ANNUAL PERCENTAGE CHANGE IN LABOR-CAPITAL RATIO
(2) ANNUAL PERCENTAGE CHANGE IN THE RELATIVE SHARE OF LABOR TO CAPITAL
(3) SUBSTITUTION EFFECT OF THE CHANGE IN THE RELATIVE FACTOR PRICE
(4) ACCOUNTING EFFECT OF THE CHANGE IN THE RELATIVE FACTOR PRICE
(5) NET OF THE SUBSTITUTION AND ACCOUNTING EFFECTS
(6) EFFECT OF TECHNICAL CHANGE

The estimated coefficients except that of the technology term are significant at the .05 level, and the coefficient of the technology term is significant only at the .225 level.

The seperate parameters are

σ	β	(g_2-g_1)
.270	.484	.031

The estimated elasticity of substitution between the production labor and capital is slightly higher than that between labor and capital, and the adaptive expectation coefficient is also slightly lower.

The technological change had been biased to labor in the sense that the annual exponential growht rate of the efficiency of production labor had been higher than that of capital by 3.1 percentage points.

From Table 24, we can observe that the production labor-capital ratio had declined thgoughout the whole period with two exceptional observations, whereas the relative share of production labor to capital had risen with four exceptional observations.

The decreases of the factor ratio can be explained by the substitution effect and the effect of the captial using technological changes. On the other hand, strong accounting effect had generally worked in the direction to raising the relative share of production labor to capital with three exceptional observations, and succeeded in overruling the depressing effects, i.e. substitution effect and

TABLE 24

CONTRIBUTIONS OF THE RELEVANT FORCES TO THE CHANGES IN THE RELATIVE SHARE OF LABOR TO CAPITAL

YEAR	(1)	(2)	(3)	(4)	(5)	(6)
1948-49	-2.374	11.257	-4.694	13.632	8.937	-1.100
1949-50	-3.182	-24.479	-3.007	-21.296	-24.303	-1.100
1950-51	-4.618	-0.954	1.140	3.664	4.804	-1.100
1951-52	0.867	29.977	-2.862	29.110	26.247	-1.100
1952-53	-5.786	21.471	-3.356	27.257	23.901	-1.100
1953-54	-4.627	-7.784	-6.549	-3.156	-9.706	-1.100
1954-55	-5.519	5.069	-1.976	10.589	8.612	-1.100
1955-56	-7.146	0.838	-4.232	7.984	3.752	-1.100
1956-57	-11.303	0.710	-4.732	12.014	7.282	-1.100
1957-58	-1.398	4.823	-7.405	6.222	-1.183	-1.100
1958-59	-7.369	0.943	-1.534	8.313	6.778	-1.100
1959-60	-1.621	23.708	-4.890	25.329	20.439	-1.100
1960-61	-5.072	-24.912	-4.147	-19.840	-23.987	-1.100
1961-62	-4.537	0.710	-0.025	5.247	5.222	-1.100
1962-63	2.166	3.638	-3.028	1.471	-1.556	-1.100

(1) ANNUAL PERCENTAGE CHANGE IN LABOR-CAPITAL RATIO
(2) ANNUAL PERCENTAGE CHANGE IN THE RELATIVE SHARE OF LABOR TO CAPITAL
(3) SUBSTITUTION EFFECT OF THE CHANGE IN THE RELATIVE FACTOR PRICE
(4) ACCOUNTING EFFECT OF THE CHANGE IN THE RELATIVE FACTOR PRICE
(5) NET OF THE SUBSTITUTION AND ACCOUNTING EFFECTS
(6) EFFECT OF TECHNICAL CHANGE

the effect of the capital using technological change .

The relative share of production labor to capital had increased in net over the entire period by 45.02 percent.

The technological change had constantly depressed the relative share by 1.1 percent a year, which is equivalent to the relative reduction of the production labor share, for instance in 1954, by about 36.34 million dollars.

The capital using technological change accumulated over the entire period had depressed the relative share of production labor to capital by 16.5 percent.

5.9. Machinery, Except Electrical (SIC No. 35)

The selected estimating equation is the Model 2 unadjusted for the autocorrelation.

$$(4.29)\quad \log(L_1/K)_t = 2.140 + \underset{(1.782)}{.128}\log(r/w_1)_{t-1} + \underset{(2.559)}{.752}\log(L_1/K)_{t-1} - \underset{(1.615)}{.499}\log(L_1/K)_{t-2} - \underset{(1.947)}{.016}\, t$$

$$\bar{R}^2 \; : \; .962$$

All the estimated coefficients except that of $\log(L_1/K)_{t-2}$ are significant at the .05 level, and that of $\log(L_1/K)_{t-2}$ is significant at the .10 level.

The seperate parameters are

σ	a	b_1	b_2	(g_2-g_1)
.171	.747	.752	-.499	.026

The weight parameters satisfy the given restrictions, and the estimated elasticity of substitution between the production labor and capital is slightly higher than the one between labor and capital.

The technological change had been biased to labor in the sense that the annual exponential growth rate of the efficiency of production labor had been higher than that of capital by 2.6 percentage points.

Table 25 indicates that the production labor-capital ratio had declined throughout the whole period with three exceptional observations. The changes in the relative share of production labor to capital showed unsystematic alternations around zero.
The patterns of the variations of the production labor-capital ratio and the relative share of production labor to capital are similar to those of the variations of the labor-capital ratio and the relative share of labor to capital except that the decreases of the former ratio and the relative share are slightly severer and the increaes slightly milder than the latter ratio and the relative share.

The relative share of production labor to capital had decreased in net over the entire period by 2.99 percent.

TABLE 25

CONTRIBUTIONS OF THE RELEVANT FORCES TO THE CHANGES IN THE RELATIVE SHARE OF LABOR TO CAPITAL

YEAR	(1)	(2)	(3)	(4)	(5)	(6)
1948-49	-1.870	-23.294	1.058	-21.424	-20.365	-1.619
1949-50	0.331	-2.909	2.259	-3.241	-0.981	-1.619
1950-51	-1.552	-9.288	0.497	-7.735	-7.238	-1.619
1951-52	-4.521	2.354	0.593	6.875	7.469	-1.619
1952-53	-5.164	26.703	-2.023	31.867	29.844	-1.619
1953-54	-3.476	-2.546	-5.375	0.929	-4.446	-1.619
1954-55	-6.452	17.858	-0.999	24.310	23.311	-1.619
1955-56	-8.147	-16.687	-4.737	-8.539	-13.277	-1.619
1956-57	-0.426	8.775	-0.975	9.202	8.226	-1.619
1957-58	0.432	13.561	-1.282	13.129	11.847	-1.619
1958-59	-6.116	-18.622	-1.565	-12.506	-14.072	-1.619
1959-60	2.043	15.058	0.045	13.015	13.061	-1.619
1960-61	-0.294	-5.680	-1.142	-5.385	-6.528	-1.619
1961-62	-2.000	-10.766	0.612	-8.765	-8.152	-1.619
1962-63	-1.534	2.495	0.611	4.029	4.641	-1.619

(1) ANNUAL PERCENTAGE CHANGE IN LABOR-CAPITAL RATIO
(2) ANNUAL PERCENTAGE CHANGE IN THE RELATIVE SHARE OF LABOR TO CAPITAL
(3) SUBSTITUTION EFFECT OF THE CHANGE IN THE RELATIVE FACTOR PRICE
(4) ACCOUNTING EFFECT OF THE CHANGE IN THE RELATIVE FACTOR PRICE
(5) NET OF THE SUBSTITUTION AND ACCOUNTING EFFECTS
(6) EFFECT OF TECHNICAL CHANGE

The capital using technological change had constantly depressed the relative share of production labor to capital by 1.62 percent a year, which is equivalent to the relative reduction of the production labor share, for instance in 1954, by about 70.82 million dollars per annum.

The capital using technological change accumulated over the entire period had depressed the relative share of production labor to capital by 24.30 percent.

5.10. Electrical Machinery (SIC No. 36)

The selected estimating equation is the Model 1 unadjusted for the autocorrelation.

$$(4.21)\quad \log(L_1/K)_t = 3.460 + \underset{(3.386)}{.199}\log(r/w_1)_{t-1} + \underset{(1.279)}{.363}\log(L_1/K)_{t-1} - \underset{(4.325)}{.017}\ t$$

$$\bar{R}^2 \ : \ .839$$

The estimated coefficients of the relative factor price and the technology term are significant even at the .005 level, and that of $\log(L_1/K)_{t-1}$ is significant at .125 level.

The seperate parameters are

σ	β	(g_2-g_1)
.312	.637	.040

The estimated elasticity of substitution between production labor and capital is slightly higher than that between labor and capital.

The technological change had been biased to labor of production workers, where the annual exponential growth rate of the efficiency of production labor had been higher than that of capital by about 4.0 percentage points.
The labor biased technological change associated with the elasticity of substitution less than unity became a capital using one.

From Table 26 we can observe that the production labor-capital ratio shows ten declines and five increases, whereas the relative share of production labor to capital eight declines and seven increases, the accumulated change being the decrease of the relative share by 5 percent.

The patterns of the movements of the production labor-capital ratio and the relative share of production labor to capital are similar to those of the labor-capital ratio and the relative share of labor to capital.

The relative share of production labor to capital had decreased in net over the entire period by 5.01 percent.
In many instances, strong accounting effect joined by the substitution effect of the changes in the relative factor price had depressed the relative share of production labor to capital.

TABLE 26

CONTRIBUTIONS OF THE RELEVANT FORCES TO THE CHANGES IN THE RELATIVE SHARE OF LABOR TO CAPITAL

YEAR	(1)	(2)	(3)	(4)	(5)	(6)
1948-49	-1.874	-31.746	-1.244	-29.871	-31.116	-1.739
1949-50	-1.027	-20.111	5.258	-19.084	-13.825	-1.739
1950-51	13.784	13.479	3.420	-0.304	3.115	-1.739
1951-52	-10.338	-9.139	5.065	1.199	6.264	-1.739
1952-53	-4.588	26.811	-3.992	31.400	27.408	-1.739
1953-54	1.226	-1.906	-7.908	-3.133	-11.042	-1.739
1954-55	-5.375	25.618	1.068	30.993	32.062	-1.739
1955-56	-5.554	-0.473	-8.113	5.080	-3.032	-1.739
1956-57	2.029	-28.264	-3.026	-30.293	-33.320	-1.739
1957-58	4.752	1.989	6.759	-2.762	3.996	-1.739
1958-59	-2.128	-8.583	2.275	-6.454	-4.179	-1.739
1959-60	-5.084	27.261	0.510	32.346	32.857	-1.739
1960-61	-4.466	0.601	-8.276	5.067	-3.209	-1.739
1961-62	0.620	3.282	-2.628	2.661	0.033	-1.739
1962-63	-10.014	-3.827	-0.303	6.187	5.883	-1.739

(1) ANNUAL PERCENTAGE CHANGE IN LABOR-CAPITAL RATIO
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(5) NET OF THE SUBSTITUTION AND ACCOUNTING EFFECTS
(6) EFFECT OF TECHNICAL CHANGE

The capital using technological change had constantly depressed the relative share by 1.74 percent a year, which is equivalent to the relative reduction of production labor share, for instance in 1954, by about 57.31 million dollars.

The capital using technological change accumulated over the entire period had depressed the relative share of production labor to capital by 26.10 percent.

PART 4

CONCLUSION

CHAPTER 6

SUMMARY OF FINDINGS AND CONCLUSION

The estimated elasticities of substitution were less than unity, indicating that the time series production functions were not the Cobb-Douglas Production Functions.

The estimated elasticities of substitution between production labor and capital were in general slightly higher than those between labor and capital, implying that the sensitivities in the input adjustments were slightly higher when the production labor and capital were involved than the case where labor and capital were involved.

In the model with the two factors of production, the technological changes had generally been biased to labor in the sense that the exponential growth rates of the efficiency of labor had been higher than those of the efficiency of capital.
The degrees of the biases, i.e. the differential exponential growth rates of the efficiencies of the two factors, ragned from .75 percentage points in the Food and Kindred Products Industry to 15 percentage points in the Tobacco Mannfactures Industry.

When those labor biased technological changes were associated with the elasticities of substitution less than unity, they worked as capital using technological changes.

The following four industries were the exceptions to the above: Textile Mill Products, Apparel and Related Products, Paper and Allied Products, and Printing and Publishing.
In these industries, the technological changes were biased to capital, indicating that the exponential growth rates of the efficiency of labor were lower than those of the efficiency of capital.
The degrees of the biases ranged from .11 percentage points in the Printing and Publishing Industry to 3 percentage points in the Apparel and Related Products Industry.
When these capital biased technological changes were associated with the elasticities of substitution less than unity, they worked as labor using technological changes.

In the model with the three factors of production, the technological changes had been biased to production labor with one exceptional industry, the Textile Mill Products.
In all the industries, except the Textile Mill Products Industry, the exponential growth rates of the efficiency of production labor had been higher than those of the efficiency of capital.
The degrees of the biases ranged from .59 percentage points in the Printing and Publishing Industry to 6.42 percentage points in the Tobacco Manufactures Industry.

When the production labor biased technological changes were associated with the elasticities of substitution less than unity, they worked as capital using technological changes.
In the Textile Mill Products Industry, the technological change had been a capital biased one, and it worked as production labor using technological changes when it was associated with the elasticity of substitution less thman unity.
The degree of the bias in the technological change was .33 percentage points.

The technological changes had influenced the factor ratios and the relative factor shares of the industries with varying degrees ranging from 1.49 million dollars in the Printing and Publishing Industry to 90.74 million dollars in the Machine Except Electrical Industry per annum when the figures for the year of 1954 were used for the illustrative purpose.

The labor-capital ratios as well as production labor-capital ratios had generally declined due to the capital using technological changes and the depressing substitution effects associated with the changes in the expected relative factor prices.

However, the relative shares of labor to capital in all the industries except the Tobacco Manufactures and the Rubber and Miscellaneous Plastic Products industries showed net increases over the entire

period under study despite the general declines in the labor-capital ratios, because the strong accounting effects of the changes in the relative factor prices had generally offset the effects of the capital using technological changes and the substitution effects.
Therefore, the results of this study do not support the view that the relative shares of labor to capital had been generally declining due to increasing degrees of the automations in the sense that the relatively more capital inputs had been used in place of labor inputs including the labor of the nonproduction workers as well as that of the production labor, although the results are in accordance with the general impression that the labor-capital ratios had generally been declining.
The analysis of the reasons why there had been strong positive accounting effects would be a part of the factor market analyses, and, thus, is beyond the scope of this study.

On the other hand, the relative shares of production labor to capital showed net decreases over the entire period in four industries among ten industries analyzed.
The industries which showed the decreases of the relative shares of production labor to capital were the Tobacco Manufacturers, the Rubber and Miscellaneous Plastic Products, the Machinery Except Electrical, and the Electrical Machinery industries.

Therefore, the results of this study partially support the view that the relative shares of production labor, which labor concept is closer to the classical concept of labor, to capital had declined.

One implication of the results of this study on the long-run constancy of the relative share of labor to capital at the level of the aggregate economy would be that since all the industries show diverse movements of the relative factor shares it may be possible that both the diverse intertemporal and interindustry variations of the movements of the relative factor shares cancel out, thus leaving the long-run relative factor share at the level of the aggregate economy as almost invariant.

The existance or nonexistence of the intertemporal and interindustry variations of the relations between the relative factor shares and the explanatory variables can be tested by applying the covariance analyses to the intertemporal and interindustry subgroups of the samples. Nevertheless, such tests presuppose that all the necessary data are available, which is not realistic presupposition at this moment.

This type of analysis can be extended to the other industries in the United States to increase the number of observations or extended to the industries in the other countries for the comparative analyses.

APPENDIX A

1.

(1) $Q = F(K,L)$: production function

(2) $\pi = pQ - (wL + rK)$: profit function

The problem of profit maximization constrained by the given production function can be solved by the familiar Lagrangian Multiplier method.

$$S = pQ - (wL + rK) - m(Q - F(K,L))$$

where S: Lagrangian function

m: Lagrangian multiplier

The necessary conditions for constrained profit maximization are as follows:

(3) $\partial S/\partial Q = p - m = 0$

(4) $\partial S/\partial K = -r + m F_k = 0$

(5) $\partial S/\partial L = -w + m F_l = 0$

(6) $\partial S/\partial m = Q - F(K,L) = 0$

From (3), (4), (5) and (6),

(7) $F_k = r/p$

(8) $F_l = w/p$

The sufficient condition for the maximization is

$$\begin{vmatrix} 0 & F_k & F_l \\ F_k & F_{kk} & F_{kl} \\ F_l & F_{kl} & F_{ll} \end{vmatrix} > 0$$

The postulate that the entrepreneurs try to maximize their profits at each instant of time might seem to be not quite realistic. A more realistic postulate might be that the entrepreneurs try to maximize their present discounted value of their current and future profits, net revenues.

The objective function is as follows:

$$(9) \quad \pi(t) = p(t)\, Q(t) - w(t)\, L(t) - r(t)\, K(t)$$

$$(10) \quad D = \int_0^{\infty} e^{-\int_0^t i(s)\, ds}\ \pi(t)\, dt$$

When the time rate of discount is a constant, equation (10) becomes

$$(11) \quad D = \int_0^{\infty} e^{-it}\ \pi(t)\, dt$$

The maximization of the present value of the profits subject to the constraint of the production function can be solved by the Lagrangian Multiplier method.

$$S = \int_0^{\infty} \left[e^{-it}\ (t) - m(t)\, (Q-F(K,L)) \right] dt = \int_0^{\infty} g(t)\, dt$$

where

$$g(t) = e^{-it}\ \pi(t) - m(t)(Q-F(K,L))$$

The necessary conditions for constrained profit maximization are as follows:

(12) $\partial g/\partial Q = e^{-it}\, p - m(t) = 0$

(13) $\partial g/\partial K = e^{-it}\, r - m(t)\, F_k = 0$

(14) $\partial g/\partial L = e^{-it}\, w - m(t)\, F_l = 0$

(15) $\partial g/\partial m = Q - F(K,L) = 0$

From these conditions,

(16) $F_k = r/p$

(17) $F_l = w/p$

(16) and (17) turn out to be the same as the static marginal conditions (7) and (8). However, the implication is different. The differece between these marginal productivity condions is that the conditions (16) and (17) hold at every point of time over the indefinite future whereas the conditions (7) and (8) hold only at a single point of time.[1]

Therefore, all the subsequent models in this paper would be consistent with the two different behavioral assumptions; maximization of profit and maximization of present value of the profits.

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1 Jorgenson, D., "The Theory of Investment Behavior," Determinants of Investment Behaviors, National Bureau of Economic Research, 1967, pp. 140-143.

2. (1) $Q = A(t) \left[d(e^{g_1 t} K)^{-s} + (1-d)(e^{g_2 t} L)^{-s} \right]^{-1/s}$

Let A(t) be a general expression for the nonfactor augmenting technological changes.

(2) $F_k = A^{-s}(t)\ d\ e^{-sg_1 t} K^{-1/\sigma} Q^{1/\sigma} = r/p$

(3) $F_l = A^{-s}(t)\ (1-d)\ e^{-sg_2 t} L^{-1/\sigma} Q^{1/\sigma} = w/p$

(4) $F_k/F_l = [d/(1-d)]\ e^{s(g_2-g_1)t} (L/K)^{1/\sigma} = r/w$

This is the same as the equation (2.5), which is based on the assumption that all the technological changes are factor augmenting.

3. Linear homogenous production function:

(1) $F_k = A^{-s}\ d\ e^{-sg_1 t} K^{-1/\sigma} Q^{1/\sigma}$

(2) $F_l = A^{-s}\ (1-d)\ e^{-sg_2 t} L^{-1/\sigma} Q^{1/\sigma}$

(3) $F_k/F_l = [d/(1-d)]\ e^{s(g_2-g_1)t} (L/K)^{1/\sigma}$

"m" degree homogenous production function:

(4) $F_k = A^{-s}\ d\ m\ e^{-sg_1 t} K^{-1/\sigma} Q^{1/\sigma}$

(5) $F_l = A^{-s}\ (1-d)\ m\ e^{-sg_2} L^{-1/\sigma} Q^{1/\sigma}$

(6) $F_k/F_l = [d/(1-d)]\ e^{s(g_2-g_1)t} (L/K)^{1/\sigma}$

Thus, the marginal productivity ratios or the cost minimization conditions based on the two types of production functions are the same.

4.

Let the correct estimating equation be

(1) $Y = X\beta + U$

where Y: $n \times 1$ column vector of the dependent variable

X: $n \times k$ matrix of the explanatory variables

β: $k \times 1$ column vector of the regression coefficients including intercept

U: $n \times 1$ column vector of the disturbance term

Now, let the researcher uses $\bar{X}$ instead of X, where $\bar{X}$ is the matrix of $n \times \bar{k}$, and k is larger than $\bar{k}$.

Then, the estimating equation with the specification error will be as follows:

(2) $Y = \bar{X}\,\bar{b} + \bar{U}$

where $\bar{b}$: $n \times 1$ column vector of the coefficients including the intercept

Following the Ordinary Least Square procedure,

(3) $\bar{b} = (\bar{X}'\bar{X})^{-1}\,\bar{X}'\,Y$

The expected value of $\bar{b}$ is

$$(4)\quad E[\,\bar{b}\,] = E\left[(\bar{X}'\bar{X})^{-1}\,\bar{X}'Y\right] = E\left[(\bar{X}'\bar{X})^{-1}\,X'(X\beta + U)\right]$$
$$= (\bar{X}'\bar{X})^{-1}\,\bar{X}'X\,\beta = P\,\beta$$

where P is an arbitrary notation:

$$P = (\bar{X}'\bar{X})^{-1}\,\bar{X}'X$$

We can see that P itself is a $\bar{k} \times k$ matrix of the coefficients including intercepts in the regressions of each variable in the X matrix on the $\bar{X}$ matrix.

Now, consider the correct matrix of the explanatory variables in our model without any distributed lag scheme be

$$X = \begin{bmatrix} 1 & x_{21} & x_{31} \\ 1 & x_{22} & x_{32} \\ 1 & x_{23} & x_{33} \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ 1 & x_{2n} & x_{3n} \end{bmatrix} \quad \text{where} \quad \left[\begin{array}{l} x_2 = \log(r/w)_{t-1} \\ x_3 = t \end{array} \right.$$

The frequently used matrix of the explanatory variables including the intercept which omits the technology term would be

$$\bar{X} = \begin{bmatrix} 1 & x_{21} \\ 1 & x_{22} \\ \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \\ 1 & x_{2n} \end{bmatrix}$$

Matrix P in this case would be

$$P = (\bar{X}'\bar{X})^{-1}\,\bar{X}'X = (\bar{X}'\bar{X})^{-1}\,\bar{X}'\,[\,X_1 \quad X_2 \quad X_3\,] = \begin{bmatrix} 1 & 0 & p_1 \\ 0 & 1 & p_2 \end{bmatrix}$$

where X_1, X_2, and X_3 are $n \times 1$ column vectors of the intercept and the explanatory variables, x_2 and x_3.

p_1 : intercept of the regression of x_3 , i.e. t , on $\bar{X}$

p_2 : coefficient of x_2 , i.e $\log(r/w)_{t-1}$, in the regression of x_3 , i.e. t , on $\bar{X}$.

From equation (4),

$$(5) \quad E[\bar{b}] = P\beta = \begin{bmatrix} 1 & 0 & p_1 \\ 0 & 0 & p_2 \end{bmatrix} \begin{bmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \end{bmatrix}$$

$$= \begin{bmatrix} \beta_1 \\ \beta_2 \end{bmatrix} + \begin{bmatrix} \beta_3 p_1 \\ \beta_3 p_2 \end{bmatrix}$$

Therefore, the specification biases of β_1 and β_2 are $\beta_3 p_1$ and $\beta_3 p_2$ respectively.

5.

From equation (2.13) in the text,

$$(2.13) \quad X_t^* = \sum_{i=1}^{\infty} \beta(1-\beta)^{i-1} X_{t-i} \quad , \quad 0 < \beta \leq 1$$

Let D be a delay or lag operator.

$$X_{t-i} = D^i X_t \quad , \quad (i = 1,2,\ldots\ldots,\infty)$$

The equation (2.13) can be rewritten as

$$X_t^* = \sum_{i=1}^{\infty} \beta(1-\beta)^{i-1} D^i X_t$$

$$= \beta DX_t + \beta(1-\beta)D^2X_t + \beta(1-\beta)^2 D^3 X_t + \ldots.$$

$$= \beta X_{t-1} / [1-(1-\beta)D]$$

$$X_t^* [1 - (1-\beta)D] = \beta X_{t-1}$$

$$X_t^* - (1-\beta)DX_t^* = \beta X_{t-1}$$

Since $DX_t^* = X_{t-1}^*$,

$$X_t^* - (1-\beta)X_{t-1}^* = \beta X_{t-1}$$

$$X_t^* - X_{t-1}^* = \beta (X_{t-1} - X_{t-1}^*) \tag{2.11}$$

Q.E.D.

6. Almon's scheme is not suitable for a straightforward econometric treatment. Yet, the scheme is quite flexible in tracing various distributed lag patterns, and does not involve the difficulties that Solow's scheme faces.

The conventional expression of the relation between X and Y is

(1) $Y_t = X_t^* + V_t$ — V_t : stochastic disturbance term

(2) $X_t^* = \sum_{i=0}^{n-1} \beta_i X_{t-i}$ — β_i : the distributed lag weights: values at $x = 0, 1, \ldots, n-1$, of a polynomial (x) of degree q+1, $q < n$.

(3) $\beta_i = \sum_{j=1}^{q} \phi_j(i)\, b_j$ — $\phi_j(i)$: values at $x=i$ of the Lagrangian interpolation polynomials; $i = 0, 1, \ldots, n-1$

Substituting equation (3) into (2),

$$X_t = \sum_{j=1}^{q} b_j \left[\sum_{i=0}^{n-1} \phi_j(i) \right] X_{t-i} \tag{4}$$

Once $\phi_j(i)$ are estimated by the Lagrangian interpolation, b_j can be estimated by the ordinary regression of Y_t on the q variables,

$[\sum_{i=0}^{n-1} \phi_j(i)X_{t-i}]$, j=i,2,.....,q.

Applying the estimated $\phi_j(i)$ and b_j values to equation (3), β_i will be estimated.

From equations (2,5), (1), and (2),

$$(5) \quad \log (L/K)_t = \sigma \log(1-d)/d + (\sigma-1)(g_2-g_1)\ t + \sigma \log(r/w)_t^* + V_t$$
$$= \sigma \log(1-d)/d + (\sigma-1)(g_2-g_1)\ t + \sigma \sum_{j=1}^{q} b_j \left[\sum_{i=0}^{n-1} {}_j(i) \right] \log(r/w)_{t-i} + V_t$$

The unknown parameters are σ, d, (g_2-g_1), and b_j, whereas the estimated terms are $\sigma \log (1-d)/d$, $(\sigma-1)(g_2-g_1)$, σb_j. Therefore, the parameters can not be identified.

7. Durbin's two-step procedure is as follows:

$$(1) \quad Y_t = X_{t-1} + (1-\beta)\ Y_{t-1} + U_t$$

Assuming the autocorrelation is of the first order,

$$(2) \quad U_t = \rho U_{t-1} + e_t$$

where "e" is a random stochastic term with a finite variance and zero covariances.

Combining equations (1) and (2), we can get Durbin's first step:

$$(3) \quad Y_t = \beta X_{t-1} - \beta\rho X_{t-2} + (1-\beta+\rho)Y_{t-1} - \rho(1-\beta)Y_{t-2} + e_t$$

The first step is to get the estimate of the first order auto-correlation coefficient, ρ, from the Ordinary Least Square regression of equation (1). ρ can be derived from the coefficients of X_{t-1} and X_{t-2} by dividing the coefficient of the latter by that of the former;[2] $\hat{\rho} = -(- \hat{\beta\rho} / \hat{\beta})$

Durbin's second step is to transform (1) into the following form using the estimated ρ and to get the estimates of the relevant parameters by running an Ordinary Least Square regression of equation (4).

$$(4) \quad [Y_t - \hat{\rho} Y_{t-1}] = \beta [X_{t-1} - \hat{\rho} X_{t-2}] + (1-\beta) [Y_{t-1} - \hat{\rho} Y_{t-2}] + e_t$$

8.

Klein noted that to get an aggregate production function and aggregate marginal productivity relations analogous to the micro production functions, weighted geometric means of the corresponding micro-vaiables were needed, where the weights would be proportinnal to the elasticities for each firm.[3]

Nataf went on to prove that for sensible aggregation the production function should be additively seperable.

Then, the output is equal to labour and capital components.

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2 Malinvaud, E., op. cit., p.469.

3 Klein, L. R., "Macroeconomics and the Theory of Rational Behavior," Econometrica, 1946, pp. 93-108.

The Constant Elasticity of Substitution Production Function satisfies this condition.

(1) $Q = A \left[d(e^{g_1 t} K)^{-s} + (1-d)(e^{g_2 t} L)^{-s} \right]^{-1/s}$

(2) $A^s Q^{-s} = d\, e^{-sg_1 t} K^{-s} + (1-d)\, e^{-sg_2 t} L^{-s}$

9. Walter rephrases Theil's formulation , taking the case of the Cobb-Douglas Production Function in the logarithmic form for an illustrative purpose.[4]

(1) $q_i = \alpha_i l_i + \beta_i k_i + a_i$ (i=1,2,.....,n)

This is a Cobb-Douglas Production Function for i th firm.

$q_i = \log Q_i$ $l_i = \log L_i$ $k_i = \log K_i$

Macro time series production function is

$q = \alpha l + \beta k + a + e$

First, examine the regression of the labour input of the i th firm on aggregate labour input and aggregate capital input for the

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4 Walters, A.A., "Production and Cost Functions: An Econometric Survey," Econometrica, 1963, pp. 10-11.

time series observation, i.e.,

$$(2) \quad l_i = B_{l_i} l + C_{l_i} k + D_{l_i} + U_{l_i}$$

Similarly the regression for capital is

$$(3) \quad k_i = B_{k_i} l + C_{k_i} k + D_{k_i} + U_{k_i}$$

The regression coefficients B and C describe the systematic movements of micro-variables as macro-quantities change. U's are the random disturbances with conventional characteristics.

Substituting these into the micro-equatinn,

$$q = \alpha l + \beta k + a$$

where

$$\alpha = \bar{\alpha} + n \left[\mathrm{cov}(\alpha_i\; B_{l_i}) + \mathrm{cov}(\beta_i\; B_{k_i}) \right]$$
$$\beta = \bar{\beta} + n \left[\mathrm{cov}(\alpha_i\; C_{l_i}) + \mathrm{cov}(\beta_i\; C_{k_i}) \right]$$
$$a = n \left[\bar{a} + \mathrm{cov}(\alpha_i\; D_{l_i}) + \mathrm{cov}(\beta_i\; D_{k_i}) \right]$$

The aggregation bias of the macro-parameters is measured by the covariance terms in these relations. However, these analyses are not operational, because a priori restrictions on the covariances can not easily be made.

10. (1) $K_t = K_t^{'} / p_{kt}$

K_t : proxy for the physical capital input adjusted for the capacity utilization at time t.

$K_t^{'}$: capital input in current value adjusted for the capacity utilization at time t.

p_{kt} : price deflator of capital goods at time t in index form (1954=100)

(2) $p_{kt} = (p_{kt}^{*} / p_{ko}^{*})\ 100$

(3) $p_{kt}^{*} = (p_{ko}^{*}/100)\ p_{kt}$

where p_{ko}^{*} : average price of capital goods at the base year

p_{kt}^{*} : average price of capital goods at t . (unknown)

(4) $K_t^{*} = (K_t^{'} / p_{kt}^{*})$

K_t^{*} : physical capital input adjusted for the capacity utilization at time t, which is unknown

From (1) and (2),

(5) $K_t = K_t^{'} / p_{kt} = (K_t^{'} / p_{kt}^{*})(p_{ko}^{*}/ 100) = K_t^{*}\ (p_{ko}^{*}/100)$

Thus, the proxy for the capital input is a product of the true capital input and constantterm, $(p_{ko}^{*}/ 100)$. The latter term measures the proportional bias in the proxy series.

However, such a bias will not affect the regression coefficients but the intercept term in the logarithmic estimating equation.

(6) $\log (L/K)_t = \log(L/K^*)_t - \log p^*_{k_o} / 100$.

Consider the simple expansion function based on the true capital input is

(7) $\log (L/K^*_t) = \sigma \log(1-d)/d + (\sigma - 1)(g_2 - g_1)\ t + \sigma \log(r/w)_{t-1}$

The estimating expansion function based on the proxy for the capital input is

(8) $\log (L/K)_t = [\sigma \log(1-d)/d - \log p^*_{k_o} / 100] + \sigma \log(r/w)_{t-1} + (\sigma - 1)(g_2 - g_1)\ t$

11. (1) $r^*_t = (pQ - D\ wL)_t \ /\ K^*_t$

where K^*_t : true capital input

K_t : proxy for K^*_t .

r^*_t : true compensation per unit of capital

r_t : proxy for r_t .

$p^*_{k_o}$: average price of capital goods at the base year

D_t : depreciation for time t.

Consider the simple expansion path function based on the true capital input and the capital compensation is

$$(4)\quad \log(L/K^*)_t = \sigma \log(1-d)/d + \sigma \log(r^*/w)_{t-1} + (\sigma-1)(g_2-g_1)\ t$$

The estimating expansion path function based on the proxies for K_t^* and r_t^* is

$$\begin{aligned}(5)\quad \log(L/K)_t &= \sigma \log(1-d)/d - \log 100/p_{k_o}^* + \sigma \log 100/p_{k_o}^* + \\ &\quad \log(r/w)_{t-1} + (\sigma-1)(g_2-g_1)\ t \\ &= \left[\sigma \log(1-d)/d + (\sigma-1) \log 100/p_{k_o}\right] + \sigma \log(r/w)_{t-1} \\ &\quad + (\sigma-1)(g_2-g_1)\ t\end{aligned}$$

Only the intercept term is different between the two equations.

APPENDIX B

1. Data

L: Labour input is measured by the annual total manhour series in the respective industry, which is computed by multiplying the average annual work hours of production workers to the number of employees on manufacturing payrolls.[5] In this process, the same movements of average hours are imputed to nonproduction workers.[6]

L_1: Annual total manhours of production workers, which are computed by multiplying the annual average work hours of production workers to the number of production workers.[7]

L_2: Annual total manhours of nonproduction workers can be derived by substracting L_1 from L.

K : Capital input is derived by multiplying capacity utilization rates to the estimated real net capital stocks. The real net capital stock series is one of the most difficult one to compile. In this study, Professor Murray Brown's unpublished estimates of the real net capital stocks based on the perpetual inventory method

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5 Source of the data: Handbook of Labor Statistics, 1968, Bulletin No. 1600, Tables 34 and 64, Bureau of Labor Statistics, Department of Labor, 1968.

6 Kendrick, J., Productivity Trends in the United States, National Bureau of Economic Research, 1961, p.50.

7 Bureau of Labor Statistics, op. cit., Table 36.

with the straight depreciation method are used as the real net capital stock series, and the Wharton Indexes of capacity utilizations are used for the derivation of the capital inputs by multiplying the former by the latter.

Depreciation assoicated with the derivation of the net capital stock includes both obsolescence and deteriaration which is inversely dependent on the economic lives of the assets.
For each industry a seperate economic life was estimated for structures and for the various equipment types with the assumption that all equipment of a specified type, industry, and vintage would retire at the same time.

In deriving the real net capital stock series from the net capital stock in historical price, price deflators with the base year of 1954 were developed for structure and for each of the equipment types.

The specific formulas used in the derivation of the capital stock figures are as follows:

$$(1) \quad K_j \ (t) = \sum_{v = t-\theta_j + 1}^{t} w_j \ (t-v) \ I_j \ (v)$$

K : net capital stock

K_g : gross capital stock

I : gross investmetn expenditures

θ_j : the economic life of the j th asset class

v : the vintage of the investment expenditure for the asset

For the gross stocks, the weight $w(t-v)$ is unity, and for the net stocks the weights are

$$(2)\quad w_j(t-v) = 1 - 1/2\theta_j - (t-v)/\theta_j$$
$$= 1 - (1/2)d_j - (t-v)\,d_j$$

From (1) and (2),

$$K_t = (1-(1/2)d)\,I_t + (1-(1/2)d-d)I_{t-1} + (1-(1/2)d-2d)I_{t-2} + \ldots + (1-(1/2)d-(t-v)d)I_{t-\theta+1}$$

$$-\quad K_{t-1} = (1-(1/2)d)I_{t-1} + (1-(1/2)d-d)I_{t-2} + \ldots + (1-(1/2)d-(t-1-v)d)I_{t-\theta+1}$$

$$K_t - K_{t-1} = (1-(1/2)d)I_t - d(I_{t-1} + I_{t-2} + \ldots\ldots + I_{t-\theta+1})$$
$$= (1-(1/2)d)I_t - d\,K^g_{t-1}$$

The subscript j was excluded in the above derivation just for the simplicity.

Thus,

$$(3)\quad K_t = (K_{t-1} + I_t) - d(K^g_{t-1} + (1/2)I_t)$$

The weight in (2) allowed for only half a year's depreciation for the current investment expenditure. When the full year's depreciation is allowed, the weight would be as follows:

$$(2)' \quad w_j(t-v) = 1 - (1+t-v)/\theta_j = 1 - (1+t-v)\, d_j$$

$$(3)' \quad K_t = (K_{t-1} + I_t) - d\, K_t^g$$

(3) is familiar straightforward relation between the net capital stock and gross capital stock based on the straight line depreciation method.

Further details related with the derivation of the depreciation and the capital stock can be found in the data appendix of Professor Brown's another paper.[8]

The real net capital stock series is derived from the net capital stock series in historical price by dividing the latter by the appropriate deflator of the investment expenditures.
The real net capital stock derived in this way is not strictly the physical net capital stock which is the desirable concept in the model. Thus, the capital input actually used in the empirical estimation is a proxy for the capital input adjusted for the capacity utilization. Nevertheless, this would not affect the regression coefficients in our estimating equation, but only the intercept term. (Appendix A ; 10)

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8 Brown, Murray, The Share of Corporate Profits in the Postwar Period, Staff Working Paper in Economics and Statistics, No.11, Department of Commerce, April, 1965, pp. 140-147.

The capacity utilization rate is another complicating factor. A relatively simple way of deriving the capacity utilization rates would be the Wharton School method, which derives the capacity utilization index by interpolating linearly the outputs of the adjacent cyclical peaks and computing the indexes of the actual output with respect to interpolated output, i.e. capacity output. (Chart 1 : (1))[9]

This method implicitly assumes that the capacity output grows at the constant rate from one cyclical peak to another, which is tantamount to imply that the growth rates of the capacity output are the same for the cyclical contraction (from peak to trough) and the expansion (from trough to peak). On a priori ground, this assumption does not seem to be right. The contraction and expansion phases might have to be treated differently.

During the contraction, we can think of two offestting forces in work ; downward pressure due to short-run cyclical effect and upward pressure due to long-run trend effect.
The capacity output will grow, decline or remain constant depending upon whether the former force is weaker than, stronger than or equal to the latter. The question of the relative significance of the forces is an empirical one.

During the expansion, on the other hand, we can imagine that the two forces are working together to raise the capacity output, which can produce an exponential growth in the capacity output.

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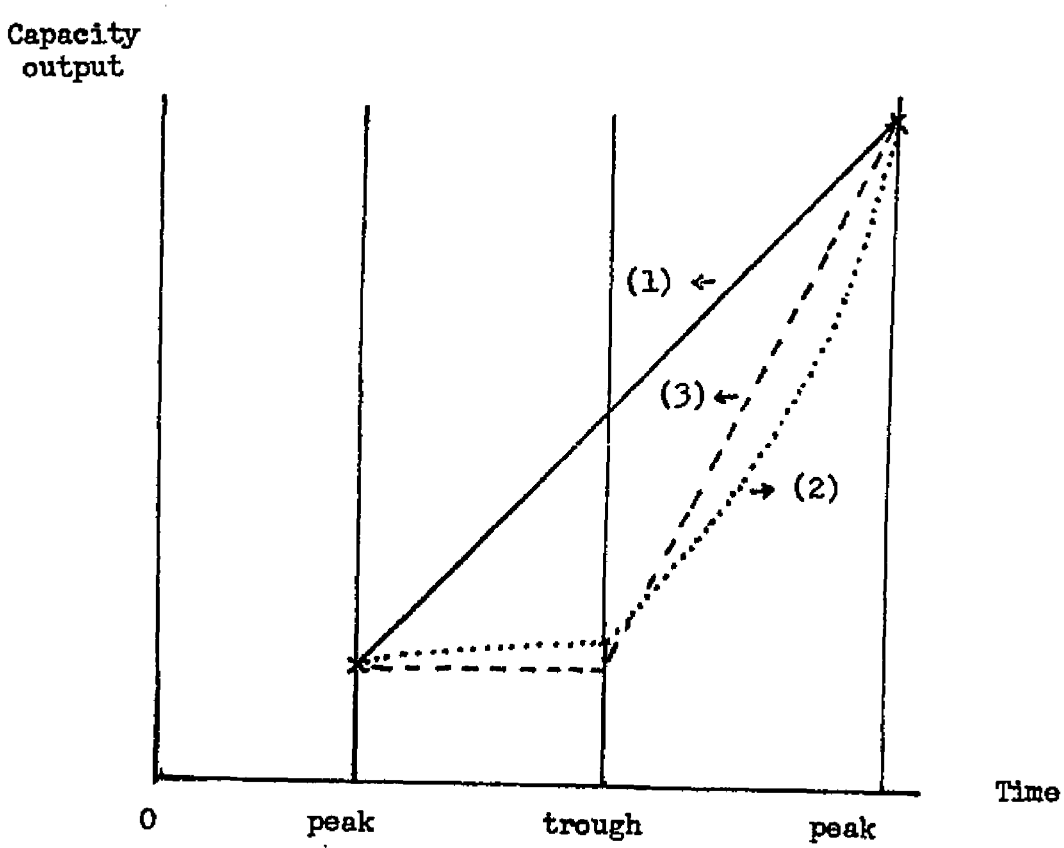
[9] Klein, L. and R. Summers, <u>The Wharton Index of Capacity Utilization</u>, University of Pennsylvania, Philadelphia, 1966.

Thus, we can postulate a variety of situations based on the above conjecture. On a priori ground, it seems that the growth rate of the capacity output during the contraction would be low and declining as the contraction proceeds, while that of the expansion would be high and increasing as the expansion proceeds. The hypothetical growth pattern, then, may be shown as Chart 1 : (2).

CHART 1

ALTERNATIVE MEASURES OF CAPACITY UTILIZATION



The simplest linear approximation to this pattern would be shown as Chart 1 :(3). If the growth pattern shown in Chart 1:(2) is the true one, then the Wharton School method generally overestimates the capacity output and, thus, underestimates the capacity utilization rates.

In this study, the Wharton School method will be used despite the short-coming, because this capacity utilization index series is readily available, and also because the two different measures of the utilization are expected to produce the empirical results which are not significantly different.

W : Average hourly earning series is derived by deviding the annual total employment compensation by the annual total manhours.[10] The complication due to the existence of the proprietors may arise. Nevertheless, such a possible complication is minimum in the manufacturing industries.[11]

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10 Source of the data on employment compensation: OBE, The National Income and Product Accounts of the United States, 1929-65; Statistical Tables, Table 6.1, Department of Commerce. For some industries, however, this statistical table gives us a slight inconsistency in the figures at the time of the cross-over from 1947 to 1948 due to the change in the industry classification.
The following work presents the adjusted series for this inconsistency: Gottsegen, J.J. and R.C. Ziemer, "Comparison of Federal Reserve and OBE Measures of Real Manufacturing Output, 1947-64, J.W. Kendrick(ed.), The Industrial Composition of Income and Product, National Bureau of Economic Research, 1968.

11 Schultze, C.L. and L. Weiner, "Introduction," The Behavior of Income Shares, National Bureau of Economic Research, 1964, p.7.

W_1 : BLS series on the average hourly earnings of production workers.[12]

W_2 : This series may be derived from the following identity relation:

$$w_2L_2 = (wL - w_1L_1)$$

$$w_2 = (wL - w_1L_1)/L_2$$

r : Price of capital series can be derived from the following identity relation:

$$pQ - D = wL + rK$$

$$r = (pQ - D - wL)/K$$

D: depreciation

As discussed before, when the proxy for capital is biased, the computed "r" series will also be biased inversely and lead to the bias in the relative factor price in the estimating equation.
"r" is again not strictly the price of capital but a proxy for it. Nevertheless, the regression coefficients will not be affected by such a bias. (Appendix A : 11)

This capital price or compensation is equivalent to the product of the rate of return to capital and the price of capital goods, and represents the sum of net interest payment and profit.[13]

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[13] Output (Gross Product Originating to the industry) and depreciation (Capital Consumption Allowance) series are from J.J. Gottsegen and R.C. Ziemer, op. cit.

[12] Bureau of Labor Statistics, op. cit., Table 80.

This method of deriving the capital price series has been well discussed by John W. Kendrick [14], and also been used by numerous other researchers.

This price of capital concept is different from an alternative price of capital concept, i.e. $r = p_k (i+d)$, in the respect that the former excludes the depreceation both from the input and output, whereas the latter includes it.
The former concept would be used in this study, because the model uses the capital input net of depreciation and the former is more suitable for the analysis of income distribution.

In the latter method, either the rate of return to capital or interest rate can be used for "i", and the share of capital computed using this price of capital concept would include depreciaition, which is not conventional in the analysis of the distribution of inoome.

Jorgenson's user cost concept is a generalization of the above concepts:

$$r = p_k \left[\frac{1-uv}{1-u} i + \frac{1-uw}{1-u} d \right]$$

where u, v, w are tax parameters.

In the empirical estimation, it is more common to exclude the tax parameters. With the zero tax parameters, Jorgenson's user cost concept becomes the second price of capital concept, i.e. $r = p_k (i+d)$.
When the depreciation is excluded both from the output and capital input, the second concept becomes the first one, i.e. $r = p_k \cdot i = [(pQ-D) - wL]/ K$.

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[14] Kendrick, John W., Productivity Trends in the United States, National Bureau of Economic Research, 1961, pp.112-114.

All the variants of the price of capital will be affected by the changes in the average price of capital goods, in the depreciation rate, and in the rate of return. This can be easily seen in the second and third variants of the price of capital, because they explicitly include both rate of return and the rate of depreciation as well as the price of the capital goods.

In the first concept of the price of capital,

$$r = \left[(pQ - D) - wL \right] / K = p_k i$$

A change in the average price of capital goods results in the direct change in the price of capital, and a change in the rate of return also brings about the same result.

A change in the depreciation rate results in the inverse change in the real net capital input, which in turn results in the inverse change in the price of capital, so that the net effect of the change in the depreciation rate on the price of capital is also direct.

2.

List of the Two-digit Manufacturing Industries Included in This Study

SIC code number	Industry Name
20	Food and Kindred Products
21	Tobacco Manufactures
22	Textile Mill Products
23	Apparel and Related Products
25	Furniture and Fixtures
26	Paper and Allied Products
27	Printing and Publishing
28	Chemicals and Allied Products
30	Rubber and Miscellaneous Plastic Products
31	Leather and Leather Products
32	Stone, Clay, and Glass Products
33	Primary Metal Industries
34	Fabricated Metal Products
35	Machinery, except Electrical
36	Electrical Machinery

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