SPECIFICATION OF THE ERROR COVARIANCE STRUCTURE FOR LINEAR MIXED EFFECTS MODELS WITH AUTOREGRESSIVE CHARACTERISTICS: A SIMULATION STUDY

by

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Abstract

Specification of the Error Covariance Structure for Linear Mixed Effects Models with Autoregressive Characteristics: A simulation study

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This study examines the effects of specifying different error covariance structures on linear mixed models with autoregressive characteristics. Computer simulations were used to generate data varying magnitudes of autocorrelations, sample size, and series lengths. The data were fitted with error covariance structure specifications of compound symmetry, identity, autoregressive lag-1, Toeplitz, and unstructured. The effectiveness of using information criteria to correctly identify the error covariance structures was investigated and the impact of error covariance structure specification on estimates of fixed effects and tests of fixed effects were examined. In addition, a statistical power analysis of detecting the AR(1) autocorrelation parameter was conducted. Results provide recommendations on which information criteria to used for data with autoregressive characteristics, demonstrate how misspecifying the error covariance structure impact tests of fixed effects, and the data conditions necessary to accurately detect the AR(1) autocorrelation parameter.

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Chapter I

Introduction

Longitudinal studies have long held the interest of educational researchers because they allow researchers to answer important questions: Does reading ability improve over time when students are given a reading intervention? What is the rate of language acquisition during the first three years of childhood? These questions involve the measurement of change or improvement over a period of time. Selecting the best method to analyze change has been a topic essential to educational research (Harris, 1963; Collins & Sayer, 2001; Singer & Willett, 2003). With the advent of advanced statistical software packages and the development of new statistical techniques, researchers have been given the ability to examine complex and intricate patterns in their data with increased ease and sophistication.

One method that has been developed is the linear mixed effects model, which is also known as the multilevel model, hierarchical linear model, variance component model, random coefficient model, or growth model. The major difference among these models is mostly a matter of conceptualization and not statistical computation, except when it comes to the selection of alternative error covariance structures. Models statistically developed via the linear mixed effects methodology allow for additional specification of alternative error covariance structures when compared to models conceptualized via the multilevel and hierarchical linear methodologies. For this paper, we would refer to this group of models collectively as the linear mixed effects model.

Bryk and Raudenbush (1987) listed several advantages the linear mixed effects model has when compared to traditional statistical methods such as ANOVA, repeated-measures ANOVA, and MANOVA. First, the number of observations and the spacing of time between measurements can vary across individuals. Second, modeling at the occasion level allows for the examination of individual growth curves which conceptually fits the way that individuals change over time, varying by rate and magnitude of growth. Third, linear mixed effects models allow for the addition of systematic factors to examine the effects of school characteristics or social groupings on individual change. Fourth, covariances between repeated observations can be examined and modeled. A fifth advantage, noted by Hox (2002), surrounds the simplicity of including time-constant or time-varying predictors into the model, which would allow for the examination of individual development and mean group development. Given the development of software programs that allow for the analysis of longitudinal data and the advantages linear mixed models have over traditional statistical methods, educational researchers are increasingly using linear mixed effects models (Singer & Willett, 2003).

A review of recent educational studies using linear mixed effects models for longitudinal data include: measuring the effectiveness of a preschool swimming program on improving student's swimming abilities (Zhu & Erbaugh, 1997); investigating the effects of intense intervention on the cognitive performance of children of low income families (Burchinal, Campbell, Bryant, Wasik, & Ramey, 1997); whether acquiring a GED, receiving postsecondary education, or given job training resulted in increased wages over time (Murnane, Willett, & Boudett, 1999); measuring the relationship between the quality of child care and early cognitive and language development (Burchinal, Roberts, Riggins, Zeisel, Neebe, & Bryant, 2000); modeling growth curves for children in a reading intervention program (Plewis, 2000); comparing the math competencies of students with math difficulties to students without math difficulties across academic terms (Jordan, Kaplan, & Hanich, 2002); and exploring longitudinal changes in behavior of students with emotional and behavioral problems in a school-based intervention program (Hussey & Guo, 2003). These studies have all utilized the linear mixed effect model to examine changes or improvement and to adjust for missing and varying time-points. However, few peer reviewed journal articles considered specifying alternative error covariance structures. All but one used the "standard" error covariance structure, where the error terms were assumed to be independent and normally distributed with a mean of zero and a constant variance, to model their data. The exception was Murnane, Willett, and Boudett (1999) who specified an unstructured error covariance structure. Here error terms are unrestricted and take on the values that the data demand. Finding that most of these studies did not fit alternative error covariance structures was unexpected given that it was a standard option in all the popular software packages for linear mixed effects models (Kreft, de Keeuw, & van der Leede, 1995), and that it is widely-thought that the error covariance structure of educational measurements collected over time would have autoregressive characteristics (Lunneborg & Lunneborg, 1970; Humphreys, 1985; Rogosa & Willett, 1985; Ackerman, 1988; Fitzmaurice, Laird, & Ware, 2004).

After reviewing the above studies it was found that none of them examined or identified the conditions necessary to correctly specify error covariance structures. Therefore, the purpose of this dissertation is to 1) examine data with autoregressive errors and to determine which error covariance structure provides the best fit and unbiased estimates and whether the correct structure can be detected (power), 2) approximate data constraints that are realistic and common to educational research, such as small sample sizes and limited length of observations, to examine their effects on model statistics, and 3) establish guidelines and recommendations on how to fit data with autoregressive errors.

Chapter II

Literature Review

A. The Linear Mixed Effects Model

Linear mixed effects models are conceptualized to measure a combination of population characteristics that are assumed to be shared by all individuals, and subject-specific effects, which are unique to a particular individual. Population characteristics are referred to as fixed effects and subject-specific characteristics are referred to as random effects. The term mixed is used to indicate that the model includes both fixed and random effects.

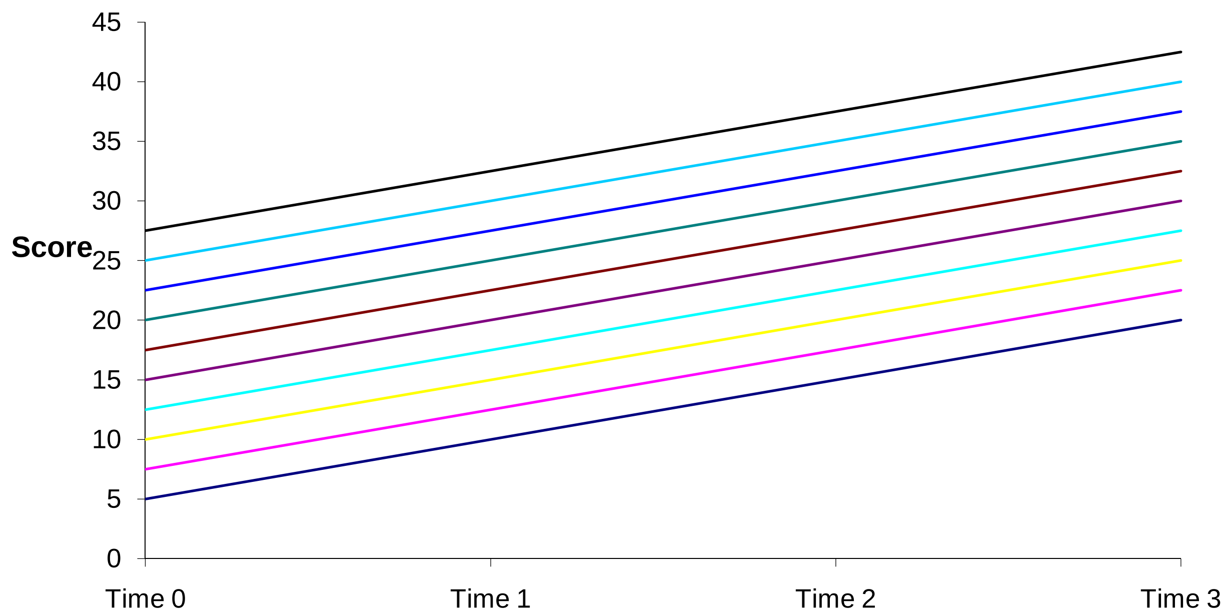
Mixed models can also be expressed and conceptualized as a linear model with hierarchical structure or multiple levels. A basic hierarchical (multilevel) linear model can be formulated as equations at two separate levels in which the first level, representing repeated measures over time within individuals, is nested within a second level that contains systemic factors that might affect individual change. Both linear mixed effect and hierarchical linear models are similar, and in most cases the ideas behind the models are easier to understand when presented in the multilevel conceptualization inherent to hierarchical linear models. However, using the hierarchical linear model formulation involves certain constraints to modeling that the linear mixed effects model does not. The purpose of this section is to present the formulation of the linear mixed effects model and the hierarchical linear model and then compare them in terms of statistical notation used.

The most basic linear mixed effects model is a random intercept model. In this model it is assumed that each individual has a different intercept but a constant slope; the equation is expressed as

$$Y_{ij} = X'_{ij}\beta + b_i + e_i$$

where Y_{ij} is the observed outcome for the i^{th} individual on j^{th} occasion, β is the fixed effect intercept and describes patterns of changes over time for the model in relationship to X_{ij}, the covariate, b_i is the random intercept effect that describes how the trend over time for the i^{th} individual deviates from the population, and e_{ij} is the error term. Both the b_i and e_{ij} are assumed to be random, independent, and normally distributed with a mean of zero, and variances Var (b_i) = σ_b^2 and Var (e_{ij}) = σ^2. Figure 1 shows a model with random intercepts and constant slopes.

Figure 1. Random Intercepts Constant Slopes Model

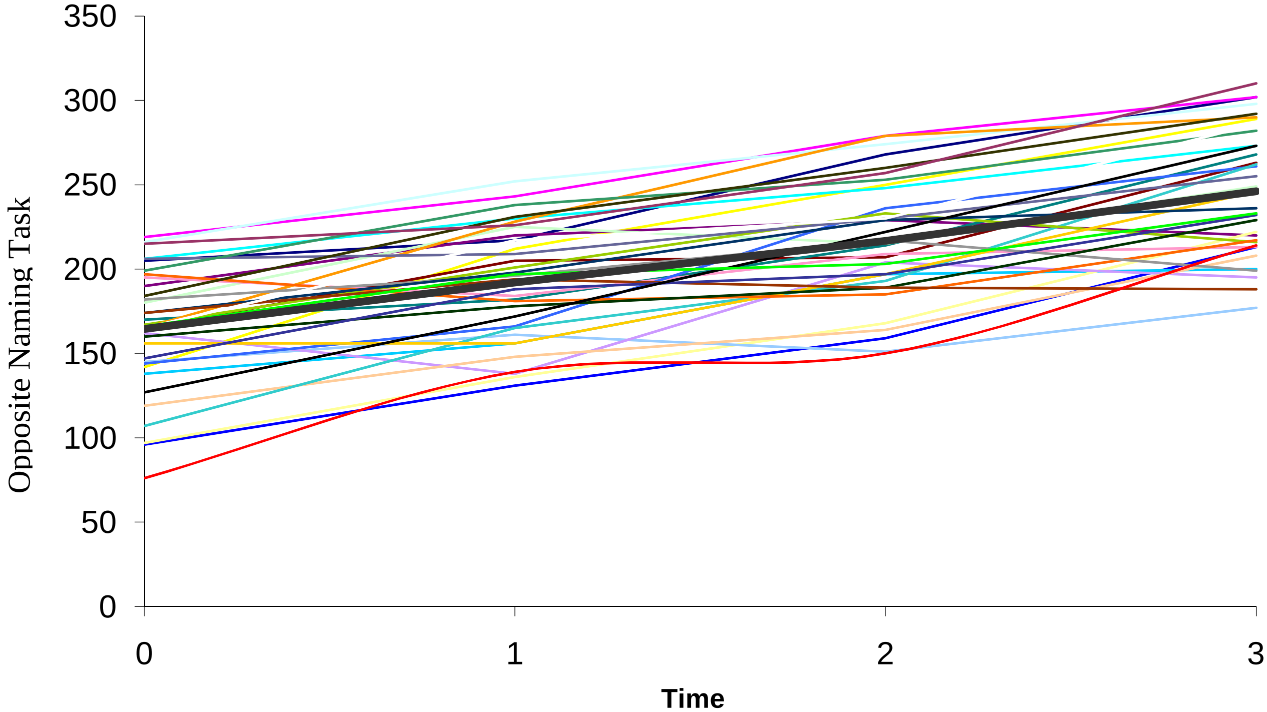


The individual mean, which is the mean observed outcome, given time, for any individual is $X'_{ij}\beta + b_i$ and the marginal mean, which is the mean observed outcome averaging across all individuals in the population is, $\beta + b_i$. The linear mixed effects model with random intercepts and constant slopes is represented in matrix form for an individual (subscript *i*) observed at four time points

$$\begin{bmatrix} Y_{i0} \\ Y_{i1} \\ Y_{i2} \\ Y_{i3} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 2 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} \beta_0 \\ \beta_1 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} [b_i] + \begin{bmatrix} e_0 \\ e_1 \\ e_2 \\ e_3 \end{bmatrix},$$

β_0 represents the fixed effect for the intercept, β_1 represents the fixed effect for the slope, and b_i is the random effect that allows the intercepts to vary. An individual's mean growth curve over time can be described as the combination of the fixed effects and b_i, a random effect. In this model $\beta_0 + b_i$ represents the intercept for the i^{th} individual and will vary randomly by individuals; b_i is assumed have a mean of zero and represents the deviation of the individual's intercept from the population intercept. This concept can be illustrated by Figure 2 which is created using data from Willett (1988) on 35 individuals measured over four occasions on an opposite-naming task.

Figure 2. Mean Profile of Opposite Naming Task



The bold line represents the marginal mean of the observed outcome across time, while all the other lines represent the conditional means of observed outcome scores across time. One can see that the conditional means deviate from the marginal mean and therefore their conditional mean outcome scores across time will also deviate from marginal mean of the observed outcome across time. Individuals who have a higher conditional mean than the marginal mean will have a positive b_i; individuals who have a lower conditional mean than the marginal mean will have a negative b_i. The marginal mean of the expected observed outcome, Y_{ij}, is $X'_{ij}\beta$. The marginal covariance of Y_{ij} in a model with random intercepts is

$$\begin{aligned} \text{Var}(Y_{ij}) &= \text{Var}(X'_{ij}\beta + b_i + e_{ij}) \\ &= \text{Var}(b_i + e_{ij}) \\ &= \text{Var}(b_i) + \text{Var}(e_{ij}) \\ &= \sigma_b^2 + \sigma^2. \end{aligned}$$

The marginal covariance between any two outcome scores for the same person is

$$\begin{aligned}\mathrm{Cov}\,(Y_{ij}, Y_{ik}) &= \mathrm{Cov}\,(X'_{ij}\beta + b_i + e_{ij}, X'_{ik}\beta + b_i + e_{ik}) \\ &= \mathrm{Cov}\,(b_i + e_{ij}, b_i + e_{ik}) \\ &= \mathrm{Cov}\,(b_i, b_i) \\ &= \mathrm{Var}\,(b_i) \\ &= \sigma_b^2.\end{aligned}$$

Therefore, the marginal covariance matrix of the repeated observations will follow a compound symmetry pattern

$$Cov(Y_i) = \begin{bmatrix} \sigma^2+\sigma_b^2 & \sigma_b^2 & \sigma_b^2 & \cdots & \sigma_b^2 \\ \sigma_b^2 & \sigma^2+\sigma_b^2 & \sigma_b^2 & \cdots & \sigma_b^2 \\ \sigma_b^2 & \sigma_b^2 & \sigma^2+\sigma_b^2 & \cdots & \sigma_b^2 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \sigma_b^2 & \sigma_b^2 & \sigma_b^2 & \cdots & \sigma^2+\sigma_b^2 \end{bmatrix}$$

and the correlation between any two outcome scores is

$$Corr(Y_{ij}, Y_{ik}) = \frac{\sigma_b^2}{\sigma_b^2 + \sigma^2}.$$

The correlation between the repeated outcome scores is induced by the random effect, b_i, which is an important aspect of linear mixed effect model. However, the covariance matrix structure generated by this random intercept model is not appropriate for most longitudinal data. General concepts from the random intercept model can be extended to a model with random intercepts and random slopes, the standard model used to examine multilevel growth and change.

To fully understand the advantages of linear mixed effects model we would have to extend the random intercept model by allowing slopes to vary randomly, which can be expressed as

$$Y_i = X_i\beta + Z_i b_i + e_i,$$

where Y_i is a (n_i x 1) vector of outcomes for person i, X_i is a (n_i x p) matrix of covariates, β is a (p x 1) vector of fixed effects, Z_i is a (n_i x p) matrix of covariates, b_i are a vector of random effects, and e_i is a (n_i x 1) vector of residuals. Consider individual i who is measured four times on the opposite-naming task where the model can be expressed in matrix notation as

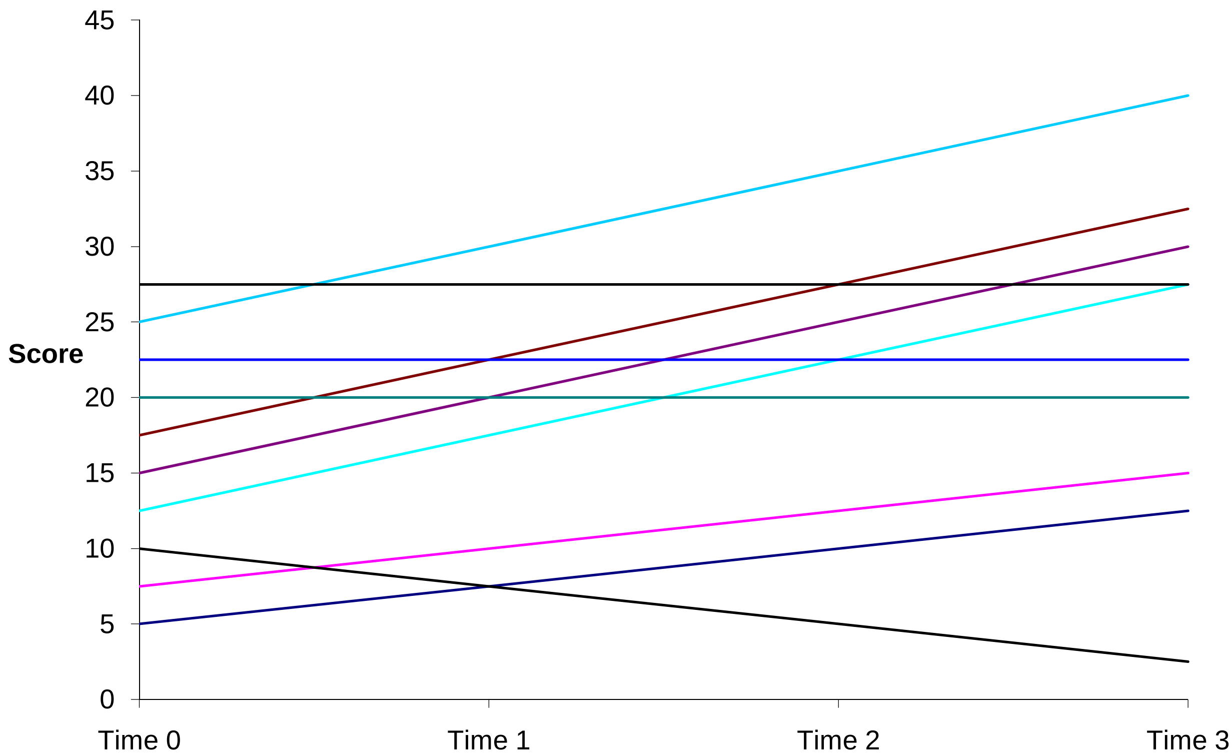
$$\begin{bmatrix} Y_{i0} \\ Y_{i1} \\ Y_{i2} \\ Y_{i3} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 2 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} \beta_0 \\ \beta_1 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 2 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} b_{i0} \\ b_{i1} \end{bmatrix} + \begin{bmatrix} e_0 \\ e_1 \\ e_2 \\ e_3 \end{bmatrix}$$

The matrix notation can be expanded to include individuals with different numbers of observations. For three individuals with four repeated observation, the model can be expressed as

$$\begin{bmatrix} Y_{10} \\ Y_{11} \\ Y_{12} \\ Y_{13} \\ Y_{20} \\ Y_{21} \\ Y_{22} \\ Y_{23} \\ Y_{30} \\ Y_{31} \\ Y_{32} \\ Y_{33} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 2 \\ 1 & 3 \\ 1 & 0 \\ 1 & 1 \\ 1 & 2 \\ 1 & 3 \\ 1 & 0 \\ 1 & 1 \\ 1 & 2 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} \beta_0 \\ \beta_1 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 2 & 0 & 0 & 0 & 0 \\ 1 & 3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 2 & 0 & 0 \\ 0 & 0 & 1 & 3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 & 1 & 3 \end{bmatrix} \begin{bmatrix} b_{10} \\ b_{11} \\ b_{20} \\ b_{21} \\ b_{30} \\ b_{31} \end{bmatrix} + \begin{bmatrix} e_{10} \\ e_{11} \\ e_{12} \\ e_{13} \\ e_{20} \\ e_{21} \\ e_{22} \\ e_{23} \\ e_{30} \\ e_{31} \\ e_{32} \\ e_{33} \end{bmatrix}.$$

In this example, the columns of Z_i is a partitioned matrix linking the vector of random effects, b_i, to the observed scores, Y_i and is a subset of the columns of X_i. The regression parameters, β, vary according to the column of X_i in Z_i, with random effects, b_i, assumed to be normally distributed with a mean of zero with a covariance matrix G. The vector of residuals, e_i, is assumed to be independent of the random effects with a multivariate normal distribution, mean of zero, and covariance matrix R_i. In a random coefficient model where both intercepts and slopes vary randomly, R_i is a diagonal matrix, $\sigma^2 I$. This matrix will be examined in detail in later sections. A graphical representation of a random intercepts and random slopes model is shown in Figure 3.

Figure 3. Random Intercepts Random Slopes Model



The conditional or individual specific mean of the outcome given the random effects is

$$E(Y_i | b_i) = X_i\beta + Z_i b_i,$$

and the marginal or population mean of Y_i is

$$\begin{aligned} E(Y_i) &= \mu_{\text{i}} \\ &= E\{E(Y_i | b_i)\} \\ &= E(X_i\beta + Z_i b_i) \\ &= X_i\beta + \text{Z}_{\text{i}}E(\text{b}_{\text{i}}) \\ &= X_i\beta. \end{aligned}$$

The covariance and variances of the random coefficient model can be expressed as a 2 x 2 matrix G, where Var (b_{1i}) = g_{11}, Var (b_{2i}) = g_{22}, and Cov (b_{1i}, b_{2i}) = g_{12},

$$Cov(b_i) = G = \begin{pmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{pmatrix}.$$

Assuming that R_i is a diagonal matrix, $\sigma^2 I$, then

$$\begin{aligned} \text{Var}(Y_{ij}) &= \text{Var}(\text{X'}_{\text{ij}}\beta + Z'_{ij}b_i + e_{ij}) \\ &= \text{Var}(Z'_{ij}b_i + e_{ij}) \\ &= \text{Var}(b_{1i} + b_{2i}t_{ij} + e_{ij}) \\ &= \text{Var}(b_{1i}) + 2t_{ij}\,\text{Cov}(b_{1i}, b_{2i}) + t^2_{ij}\,\text{Var}(b_{2i}) + \text{Var}(e_{ij}) \\ &= \text{g}_{11} + 2t_{ij}\text{g}_{12} + t^2_{ij}\,\text{g}_{22} + \sigma^2 \end{aligned}$$

and the covariance between any pair of observations for a given individual is

$$\text{Cov}(Y_{ij}, Y_{ik}) = \text{g}_{11} + (t_{ij} + t_{ik})\,\text{g}_{12} + t_{ij}\,t_{ik}\,\text{g}_{22},$$

showing that the covariance matrix, Cov (Y_i), is expressed as a function of time, t_{ij}.

A linear mixed effects model can be alternatively formulated as having multiple levels or stages. Hence, models that are formulated in such a way are often referred to as multilevel models or hierarchical linear models. A hierarchical linear model for longitudinal data can be conceptualized at two levels (Bryk & Raudenbush, 1987). The level-1 component represents individual change over time where each individual has his or her own unique growth curve. The level-2 component represents systematic factors that might affect change. In this way, hierarchical modeling can be used to examine repeated measures or observations that are nested within individuals. Using hierarchical notation consistent with Singer and Willett (2003) and Raudenbush and Bryk (2002), the level-1 equation of a hierarchical linear model with random intercept and slopes and no covariates is expressed as

$$Y_{ij} = \pi_{0i} + \pi_{1i}\text{Time}_{ij} + e_{ij}$$

where Y_{ij}, is the observed measure for individual i on occasion j expressed as a linear function of Time, in which π_{0i} represents the intercept and π_{1i} represents the slope. In matrix form the model for individual i is

$$\begin{bmatrix} Y_{i1} \\ Y_{i2} \\ Y_{i3} \\ \vdots \\ Y_{ij^{th}} \end{bmatrix} = \begin{bmatrix} 1 & t_1 \\ 1 & t_2 \\ 1 & t_3 \\ \vdots & \vdots \\ 1 & t_i \end{bmatrix} \begin{bmatrix} \pi_{0i} \\ \pi_{1i} \end{bmatrix} + \begin{bmatrix} e_{i1} \\ e_{i2} \\ e_{i3} \\ \vdots \\ e_{ij^{th}} \end{bmatrix}$$

The vector of errors, e_{ij}, is assumed to be mutually independent, across time and individuals, and normally distributed with a mean of zero and common variance σ^2_ε,

$$e_{ij} \sim N(0, \sigma^2_\varepsilon).$$

To formulate the second level of this model, it is assumed that the individual-specific effects, π_{0i} and π_{1i}, are random with unknown means and covariances. The individual-specific effects are modeled by the level-2 equations

$$\pi_{0i} = \beta_{00} + u_{0i},$$

$$\pi_{1i} = \beta_{10} + u_{1i}.$$

β_{00} represents the mean intercept, β_{10} represents the mean growth rate, and both μ_{0j} and μ_{1j} are random effects. The level-2 equations can be expressed

in matrices and vectors as

$$\begin{bmatrix}\pi_{0i}\\ \pi_{1i}\end{bmatrix} = \begin{bmatrix}\beta_{00}\\ \beta_{10}\end{bmatrix}[1] + \begin{bmatrix}u_{0i}\\ u_{1i}\end{bmatrix}$$

It is assumed that the level-2 residuals, u_{0i} and u_{1i}, are independent and normally distributed with means of zero, variances τ_{00}, and τ_{11} and covariance τ_{01},

$$\begin{bmatrix}u_{0i}\\ u_{1i}\end{bmatrix} \sim N\left[\begin{pmatrix}0\\ 0\end{pmatrix}, \begin{pmatrix}\tau_{00} & \tau_{01}\\ \tau_{10} & \tau_{11}\end{pmatrix}\right].$$

The above equation contains the level-2 covariance matrix which, except for the notation, is the same as the *G* matrix in mixed model notation.

$$\begin{pmatrix}\tau_{00} & \tau_{01}\\ \tau_{10} & \tau_{11}\end{pmatrix},$$

which describes the variances and covariances of the level-2 residuals. The combination of the level-1 residual variance, σ_{ε}^{2}, and the level-2 covariance matrix is widely referred to as the variance components of a model.

The two levels of the hierarchical linear model can be expressed as a composite equation by combining the fixed effects with the variance components,

$$Y_{ij} = \beta_{00} + \beta_{10}\text{Time}_{ij} + (u_{0i} + u_{1i}\text{Time}_{j} + e_{ij}),$$

which is the equivalent to the linear mixed effects equation,

$$Y_i = X_i\beta + Z_i b_i + e_i.$$

The induced covariance structure of a linear mixed effects model is referred to in the literature by different names; examples include the random coefficients structure

(Wolfinger, 1993), the random effects structure (Raudenbush & Bryk, 2002), and the variance components structure (Littell, Milliken, Stroup, Wolfinger, & Schabenberger, 2006). In a linear mixed effects model, the covariance of the outcome score, Y_i, can be separated conditionally, given b_i,

$$Cov\ (Y_i\ |b_i) = \mathrm{Cov}\ (e_i) = R_i,$$

and marginally, averaging over the a distribution of random effects, b_i,

$$\begin{aligned} \mathrm{Cov}\ (Y_i) &= \mathrm{Cov}\ (Z_i b_i) + \mathrm{Cov}\ (e_i) \\ &= Z_i \mathrm{Cov}(b_i) Z_i' + \mathrm{Cov}\ (e_i) \\ &= Z_i G Z_i' + R_i. \end{aligned}$$

where$R_{i,}$ in most cases is assumed to be an identity matrix, $\sigma^2 I$, where given random effects b_i, errors are independently distributed with a common variance of σ^2. The overall covariance structure of a standard linear mixed effects model then can be represented as

$$Cov\ (Y_i) = Z_i G Z_i' + \sigma^2 I$$

and in vector and matrix form

$$Cov(Y_i) = \begin{pmatrix} 10 \\ 11 \\ 12 \\ \vdots 3 \\ 1 \vdots \end{pmatrix} \begin{pmatrix} \tau_{00}\tau_{01} \\ \tau_{10}\tau_{11} \end{pmatrix} \begin{pmatrix} 1 & 1 & 1 & \cdots & 1 \\ 0 & 1 & 2 & 3 & \dots \end{pmatrix} + \sigma^2 \begin{pmatrix} 1 & 0 & 0 & \dots & 0 \\ 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \end{pmatrix}.$$

This induced covariance structure has several characteristics that make it robust to certain data conditions when compared to other covariance structures. First, because this covariance structure allows for covariances to be expressed as a function of time, designs where each individual has different measurement intervals can be accommodated.

Second, the assumption of homogeneity of variance over time that is required by many other covariance structures does not apply to the random effects covariance structure since it allows for the increase or decrease of the variance and covariance as a function of time. Lastly, the number of covariance parameters that need to be estimated remains the same regardless of the number of observed occasions, saving valuable degrees of freedom when sample size is small. The main focus of this dissertation is to examine the covariance structure of the error matrix, R_i, which will be referred to as the error covariance matrix.

B. The Error Covariance Matrix

In order to better understand the error covariance structure of a linear mixed effects model, I will first examine the error matrix of an OLS regression and then apply the underlying similarity to the error covariance matrix of a linear mixed effects model. First, consider an OLS regression on the opposite-task-naming data provided by Willett (1988) where the outcome variable is measured on four occasions, and is regressed on the main effect of Time,

$$Y_{ij} = \beta_{00} + \beta_{10}\ \text{Time}_{ij} + e_{ij},$$

with the assumption that all errors, e_{ij}, are independent and normally distributed, with a mean of zero and homoscedastic variance. The error structure and its distributional assumptions for this example can illustrated in matrix notation as

$$
\begin{bmatrix} e_{11} \\ e_{12} \\ e_{13} \\ e_{14} \\ e_{21} \\ e_{22} \\ e_{23} \\ e_{24} \\ \vdots \\ e_{n1} \\ e_{n2} \\ e_{n3} \\ e_{n4} \end{bmatrix} \sim N \left(\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \vdots \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix}
\sigma_r^2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 \\
0 & \sigma_r^2 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 \\
0 & 0 & \sigma_r^2 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \sigma_r^2 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \sigma_r^2 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \sigma_r^2 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \sigma_r^2 & 0 & \cdots & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \sigma_r^2 & \cdots & 0 & 0 & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & \sigma_r^2 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & \sigma_r^2 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & \sigma_r^2 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & \sigma_r^2
\end{bmatrix} \right).
$$

This matrix represents the residuals in a typical OLS regression. The first matrix contains a set of errors, e_{nj}, for each individual on each observation, where e_{11} is the residual for the first person on the first observation and e_{24} is the residual for the second person on the fourth observation and onwards to the residual for the n^{th} person on the fourth observation. The vector of residuals is distributed normally, shown symbolically as $\sim N$. The second matrix represents the expected means of the residuals in this case the vector consist of only zeros, to fulfill the assumption that the population mean for each residual is zero. The final matrix contains the variances and covariances of all residuals, also known as the error covariance matrix. Note that all elements within the matrix except for the diagonal are zero. These zeros represent the assumption that residuals are independent and do not covary with each other. In the diagonal are population variances, σ^2_{r}, which are identical, fulfill the assumption of homoscedasticity for residuals. This

OLS regression error matrix is unsuitable for longitudinal data because errors within individuals are expected to be correlated and heteroscedastic over time (Singer, 1998). A more general error matrix for longitudinal data, accounting for correlations among observations within individuals, is

$$\begin{bmatrix} e_{11} \\ e_{12} \\ e_{13} \\ e_{14} \\ e_{21} \\ e_{22} \\ e_{23} \\ e_{24} \\ \vdots \\ e_{n1} \\ e_{n2} \\ e_{n3} \\ e_{n4} \end{bmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \vdots \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} & \sigma_{14} & 0 & 0 & 0 & 0 & \vdots & 0 & 0 & 0 & 0 \\ \sigma_{21} & \sigma_{22} & \sigma_{23} & \sigma_{24} & 0 & 0 & 0 & 0 & \vdots & 0 & 0 & 0 & 0 \\ \sigma_{31} & \sigma_{32} & \sigma_{33} & \sigma_{34} & 0 & 0 & 0 & 0 & \vdots & 0 & 0 & 0 & 0 \\ \sigma_{41} & \sigma_{42} & \sigma_{43} & \sigma_{44} & 0 & 0 & 0 & 0 & \vdots & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \sigma_{11} & \sigma_{12} & \sigma_{13} & \sigma_{14} & \vdots & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \sigma_{21} & \sigma_{22} & \sigma_{23} & \sigma_{24} & \vdots & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \sigma_{31} & \sigma_{32} & \sigma_{33} & \sigma_{34} & \vdots & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \sigma_{41} & \sigma_{42} & \sigma_{43} & \sigma_{44} & \vdots & 0 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \vdots & \sigma_{11} & \sigma_{12} & \sigma_{13} & \sigma_{14} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \vdots & \sigma_{21} & \sigma_{22} & \sigma_{23} & \sigma_{24} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \vdots & \sigma_{31} & \sigma_{32} & \sigma_{33} & \sigma_{34} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \vdots & \sigma_{41} & \sigma_{42} & \sigma_{43} & \sigma_{44} \end{bmatrix} \right)$$

Note that the residuals and expected means remain the same as those of the OLS equation but the error covariance matrix is different. The error covariance matrix of longitudinal data has a block diagonal structure, where each block represents the observations within an individual. It is these block structures that represent the covariance and variances of residuals within individuals. The zeros surrounding the blocks signify that residuals between different individuals are independent and do not covary. Expressed in more parsimonious terms

$$e \sim N\left(0, \begin{bmatrix} R_i & 0 & 0 & \cdots & 0 \\ 0 & R_i & 0 & \cdots & 0 \\ 0 & 0 & R_i & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & R_i \end{bmatrix}\right),$$

where the submatrix for each individual can be expressed as

$$R_i = \Sigma_i = \begin{bmatrix} \sigma_{11}^2 & \sigma_{12} & \sigma_{13} & \sigma_{14} \\ \sigma_{21} & \sigma_{22}^2 & \sigma_{23} & \sigma_{24} \\ \sigma_{31} & \sigma_{32} & \sigma_{33}^2 & \sigma_{34} \\ \sigma_{41} & \sigma_{42} & \sigma_{24} & \sigma_{44}^2 \end{bmatrix}.$$

This error covariance matrix contains four error variances along the diagonal and six covariances taking the form of an unstructured error covariance matrix. This is the most general case considered here. However, the error covariance matrix can also be specified to take on different structures that lie between the OLS pattern and the completely unstructured covariance matrix.

The modeling approach that is chosen to fit the data makes certain assumptions about the structure and patterns of error variances and covariances. I will examine several structures for the error covariance matrix in detail.

C. The Standard Error Covariance Structure

The standard linear mixed effects model with an underlying covariance structure is given by the equation

$$\text{Cov}(Y_i) = Z_i G Z_i' + R_i.$$

The introduction of random effects, b_i, induces correlation among the components of Y_i. Hence, Cov (Y_i) accounts for the correlations among the repeated observations within the same individuals. The standard error covariance structure allows for the explicit analysis of the between-subject (G) and within-subject (R_i) sources of variability. The error term, $R_\mathrm{i} = \sum_i = \sigma^2 I$, is a scaled identity matrix that is expressed as

$$\sum_i = \begin{bmatrix} \sigma^2 & 0 & 0 & 0 \\ 0 & \sigma^2 & 0 & 0 \\ 0 & 0 & \sigma^2 & 0 \\ 0 & 0 & 0 & \sigma^2 \end{bmatrix},$$

a diagonal matrix, where all pairwise correlations are equal to zero. Other covariance error structures take into account all sources of variability, therefore the sources of between-subject and within-subject variability cannot be explicitly modeled.

While this identity (ID) error covariance structure is simple, several features make it particularly useful when it is adopted as part of the covariance structure. First, this error covariance structure allows for individuals with missing observation scores. Second, the number of covariance parameters that need to be estimated stay the same regardless of number of repeated observations. Third, the assumption of homogeneity of variance is relaxed, because the variances may increase or decrease as a function of time.

D. The Unstructured Error Covariance Structure

When no assumptions are made about the error variances and covariances among the repeated observations, the error covariance matrix takes on an unrestricted or unstructured form, where Cov (Y_i) = Σ_i ,

$$\Sigma_i = \begin{bmatrix} \sigma_1^2 & \sigma_{12} & \sigma_{13} & \sigma_{14} \\ \sigma_{21} & \sigma_2^2 & \sigma_{23} & \sigma_{24} \\ \sigma_{31} & \sigma_{32} & \sigma_3^2 & \sigma_{34} \\ \sigma_{41} & \sigma_{42} & \sigma_{43} & \sigma_4^2 \end{bmatrix}.$$

As mentioned previously, fitting a model with an unstructurederror covariance structure (which I will abbreviate as (UN)) usually provides the best model fit. However, there is a problem, because as the covariances can take on any value as needed, the number of covariance parameters that need to be estimated may grow large. The covariance parameters that need to be estimated grow as the number of observations per person increases. The number of variance and covariance parameters is

$$\frac{n(n+1)}{2}$$

where n is the number of repeated observations. Given a hypothetical case where there are 12 observation occasions, the number of covariance parameters that need to be estimated is 78. If the sample size of the study is small relative to the number of covariance parameters then estimates of the fixed effects will be unstable and frequently the estimation procedure will not converge (Singer & Willett, 2003).

E. Alternative Error Covariance Structures

Ware (1985) was the first to consider that the level-1 errors in a linear mixed effects model might be correlated, and suggested that alternative error covariance structures should be considered. A comprehensive list of error covariance structures that can used to model linear mixed effects model is provided by Wolfinger (1993). For the purposes of this paper, we will consider alternative error covariance structures that are considered appropriate for educational data, which are compound symmetry (CS), autoregressive lag-1 (AR(1)), and Toeplitz (TP) (Singer & Willett, 2003; Hox, 2002).

An error covariance structure that assumes that covariances and correlations between all observations are equal is called compound symmetry, a structure that is often assumed in a univariate repeated measures ANOVA. Under compound symmetry, the variance is constant across occasion and Corr (Y_{ij} , Y_{ik}) = ρ for all j and k. That is,

$$Cov\ (e_{ij}) = \sigma^2 \begin{bmatrix} 1 & \sigma_1^2 & \sigma_1^2 & \sigma_1^2 \\ \sigma_1^2 & 1 & \sigma_1^2 & \sigma_1^2 \\ \sigma_1^2 & \sigma_1^2 & 1 & \sigma_1^2 \\ \sigma_1^2 & \sigma_1^2 & \sigma_1^2 & 1 \end{bmatrix},$$

the error covariance structure matrix can be expressed as:

$$\Sigma_i = \begin{bmatrix} \sigma^2 & \sigma_1^2 & \sigma_1^2 & \sigma_1^2 \\ \sigma_1^2 & \sigma^2 & \sigma_1^2 & \sigma_1^2 \\ \sigma_1^2 & \sigma_1^2 & \sigma^2 & \sigma_1^2 \\ \sigma_1^2 & \sigma_1^2 & \sigma_1^2 & \sigma^2 \end{bmatrix}$$

.

This matrix expresses compound symmetry in which all variances are equal and all covariances are equal, meaning that residual errors are independent and have constant variance over time. It assumes that the correlation between repeated observations is the same regardless of the amount of time separating observations. Given the nature of longitudinal data, where observations measured closer in time are more highly correlated than observations measured further apart, this assumption is restrictive and unrealistic. However, the compound symmetry structure is parsimonious, requiring only two parameters regardless of the number of observation occasions.

The autoregressive lag-1 (AR(1)) error covariance structure assumes that observations that are closer together in time have a higher correlation than observations that are further apart. In the AR(1) error covariance structure, the autoregressive process is considered to be first-order because the error for an observation depends on the immediately preceding error. An autoregressive process that is second-order would have dependence on the preceding two errors.

The variance for the autoregressive model is assumed to be constant across occasions and Corr (Y_{ij}, Y_{ij+k}) = ρ^k for all j and k. That is,

$$Cov\ (Y_i) = \sigma^2 \begin{bmatrix} 1 & \rho & \rho^2 & \rho^3 \\ \rho & 1 & \rho & \rho^2 \\ \rho^2 & \rho & 1 & \rho \\ \rho^3 & \rho^2 & \rho & 1 \end{bmatrix}.$$

The AR(1) error covariance structure is represented as not just decreasing correlations over, time but in a specific pattern:

$$\Sigma_i = \begin{bmatrix} \sigma^2 & \sigma^2\rho & \sigma^2\rho^2 & \sigma^2\rho^3 \\ \sigma^2\rho & \sigma^2 & \sigma^2\rho & \sigma^2\rho^2 \\ \sigma^2\rho^2 & \sigma^2\rho & \sigma^2 & \sigma^2\rho \\ \sigma^2\rho^3 & \sigma^2\rho^2 & \sigma^2\rho & \sigma^2 \end{bmatrix}.$$

This error covariance structure requires only two parameters, regardless of the number of observation occasions.

An error covariance structure called the Toeplitz structure, shown in Jennrich and Schluter (1986), assumes that each time lag has a unique autocorrelation, where all observations that are one time point apart will share an autocorrelation, whereas all observations that are two time points apart will share another autocorrelation, and so on. The Toeplitz model contains k-1 autocorrelation parameters compared to a single autocorrelation parameter estimated by an autoregressive structure.

The variance for the Toeplitz model is assumed to be constant across occasions and Corr (Y_{ij} , Y_{ij+k}) = ρ_k for all j and k. That is,

$$Cov\ (Y_i) = \sigma^2 \begin{bmatrix} 1 & \rho_1 & \rho_2 & \rho_3 \\ \rho_1 & 1 & \rho_1 & \rho_2 \\ \rho_2 & \rho_1 & 1 & \rho_1 \\ \rho_3 & \rho_2 & \rho_1 & 1 \end{bmatrix}.$$

The Toeplitz error covariance matrix structure is:

$$\sum_i = \begin{bmatrix} \sigma^2 & \sigma^2\rho_1 & \sigma^2\rho_2 & \sigma^2\rho_3 \\ \sigma^2\rho_1 & \sigma^2 & \sigma^2\rho_1 & \sigma^2\rho_2 \\ \sigma^2\rho_2 & \sigma^2\rho_1 & \sigma^2 & \sigma^2\rho_1 \\ \sigma^2\rho_3 & \sigma^2\rho_2 & \sigma^2\rho_1 & \sigma^2 \end{bmatrix}.$$

Other covariance structures can be specified such as moving average (MA) and autoregressive integrated moving average (ARIMA). However, for the purpose of fitting educational data using autoregressive error covariance structures the AR(1) and Toeplitz structures appear to be the most logical, as they are more appropriate for the very short time series usually found in such datasets.

F. Model Specification and the Error Covariance Structure

The following section provides a review of the literature on how choosing certain covariance structures affects parameter estimates and inferences for linear mixed effects models. Although the overall covariance structure is represented by equation

$$\text{Cov}(Y_i) = Z_i G Z_i' + \Sigma_i$$

in practice the modeling of the covariance structure is done through the random effects matrix, G, and the error covariance matrix, Σ_i, or more often only through Σ_i (Littell, Pendergast, & Natarajan, 2000).

There are three general approaches to modeling the covariance structure (Fitzmaurice et al., 2004): The first approach is to impose structure on the error covariance matrix by using random effects. This approach is similar to the repeated measures ANOVA design, where a single individual-specific random effect is included to account for the correlation among observations. The result of adding this random effect to every observation for each individual is that the correlation among the repeated observations will be positive, thus imposing a general structure on the covariances. However, by just adding one random effect into the model, the correlation and variance would remain constant (i.e., not varying as a function of time), resulting in a compound symmetry structure. For the correlation and variance among repeated observations to vary would require that an additional random effect be added, which would allow both intercepts and slopes to vary randomly across individuals, creating an error covariance structure that is implicit in a standard linear mixed effects model.

The second approach is to impose no error covariance structure, allowing the variances and covariances to take on any values necessary. This approach is also known as an unrestricted covariance structure and usually produces the best model fit compared to other covariance structures (Singer & Willett, 2003). However, because the covariances can take on any value, the number of covariance parameters can get very large and, when sample size is small, estimates of the fixed effects may not be stable nor estimable (Fitzmaurice et al., 2004).

The third approach is to use error covariance structures originally developed for time series models. While the purpose of time series models is different from that of longitudinal data analysis, they are similar in that both are concerned with examining repeated observations that are correlated. Most of these error covariance structures assume that repeated observations taken closer together are more correlated than repeated observations taken further apart and the remaining error covariance structures assume that the correlation between observations is a function of amount of time that separate observations (Wolfinger, 1993).

McLean, Sanders, and Stroup (1991) showed that when the covariance parameters, *G* and $\sum_i$, of a linear mixed effects model are known, best linear unbiased estimates of the fixed effects and best linear unbiased predictors of the random effects in a linear model can be obtained,

$$C = \begin{bmatrix} X'\hat{\Sigma}^{-1}X & X'\hat{\Sigma}^{-1}Z \\ Z'\hat{\Sigma}^{-1}X & Z'\hat{\Sigma}^{-1}Z + \hat{G}^{-1} \end{bmatrix}$$

where *C* is the covariance matrix of known *G* and $\sum_i$. However, when the covariance parameters are not known, unbiased estimates of the fixed and random effects can only be obtained if the error structures are specified correctly and estimated by using either maximum likelihood estimation or restricted maximum likelihood estimation through solving mixed model equations (Henderson, 1984):

$$\begin{bmatrix} X^{'}\hat{\Sigma}^{-1}X & X^{'}\hat{\Sigma}^{-1}Z \\ Z^{'}\hat{\Sigma}^{-1}X & Z^{'}\hat{\Sigma}^{-1}Z+\hat{G}^{-1} \end{bmatrix}\begin{bmatrix} \hat{\beta} \\ \hat{b}_i \end{bmatrix}=\begin{bmatrix} X^{'}\hat{\Sigma}^{-1}Y \\ Z^{'}\hat{\Sigma}^{-1}Y \end{bmatrix}$$

$$\begin{bmatrix} \hat{\beta} \\ \hat{b}_i \end{bmatrix}=\begin{bmatrix} (X'\hat{V}^{-1}X)X'\hat{V}^{-1}Y \\ GZ'\hat{V}^{-1}(Y-X\hat{\beta}) \end{bmatrix}$$

where $\hat{C}$ is the estimated covariance matrix of an unknown G and $\sum_i$,

$$\hat{C}=\begin{bmatrix} X'\hat{\Sigma}^{-1}X & X'\hat{\Sigma}^{-1}Z \\ Z'\hat{\Sigma}^{-1}X & Z'\hat{\Sigma}^{-1}Z+\hat{G}^{-1} \end{bmatrix}$$

$$\hat{C}=\begin{bmatrix} X^{'}\hat{\Sigma}^{-1}Y \\ Z^{'}\hat{\Sigma}^{-1}Y \end{bmatrix}.$$

G. Information Criteria

The ability to specify different error covariance structures has prompted the use of several information criteria to decide which covariance structure provides the "best" fit for the data (Fitzmarurice et al., 2004; Littell et al., 2006). This study will use four information criteria to select the "best" fitting error covariance structure for given data set. The first is the Akaike Information Criterion (Akaike, 1974),

AIC = 2(number of parameters) - 2(maximized log-likelihood)

$$=2k-2\ln(L),$$

where k represents the number of covariance parameters in the statistical model, and L is the maximized value of the likelihood function. Modeling linear mixed effect models, the purpose of the AIC is to provide a score-based on the best combination of model parsimony and fit of a covariance structure, in which specifying a complex covariance structure that requires the estimation of additional covariance parameters gets penalized.

Covariance structures that result in a lower AIC are considered better than covariance structures that result in a higher AIC. In this way, the AIC can be used to compare models with the same fixed effects but with different covariance structures. In addition, a small sample bias correction to the AIC is available (Hurvich & Tsai, 1989). The method is called the Akaike Information Criterion Correction (AICC); it is interpreted in the same way as AIC, and is used when n = 20 or less. However, studies have not been conducted on the efficiency of using AICC on larger samples or for selecting the correct covariance structure. This study will examine the performance of AICC compared to the other indices.

The third method is the Schwartz (1978) Bayesian Information Criterion (BIC),

$$\text{BIC} = 2(\text{maximized log-likelihood}) + \log N^{*}(\text{number of parameters})$$

$$= 2(\hat{l} + \log\sqrt{N^{*}}\ c),$$

where N is the number of subjects and c is the product of the number of between-subjects and within-subjects factors. The BIC is based on the Bayesian approach where the model with the highest posterior probability is considered the optimum model. As with the AIC, a model with a lower BIC is considered better than models with a higher BIC. Also like the AIC, the BIC penalizes additional covariance parameters estimated. While both AIC and BIC provide a method to evaluate the fit of covariance structures on a particular data set, they do not always lead to the correct selection of the covariance structure. Keselman, Algina, Kowalchuk, and Wolfinger (1998) found that, on average, the AIC selected the correct covariance structure 47 percent of the time and BIC selected the correct covariance structure 35 percent of the time. The relatively poor accuracy of the BIC for selecting the correct covariance structure makes it of limited use when modeling the covariance structure. This perception is further reinforced by Fitzmaurice et al. (2004) who recommend against using the BIC when selecting covariance structures because of the high risk of selecting covariance structures that are too simple for the data. In addition, it was found that Type I error rates can be biased even when the covariance structure is correctly selected by the AIC or BIC (Keselman, Algina, Kowalchuk, & Wolfinger, 1999).

The fourth information criterion is Bozdogan's Criterion (Bozdogan, 1987),

$$\text{CAIC} = 2(\text{maximized log-likelihood}) + \log N^*(\text{number of parameters} + 1)$$

$$= 2l+s(\ln N+1)$$

A thorough review of the linear mixed model literature has not produced research that examined the performance of the CAIC on selecting error covariance structure. This study will add to the literature by comparing the CAIC to other information criteria.

Hamaker, Hattum, Kuiper, and Hoijtink (2011) provide some guidelines and qualifications in using information criteria to select for the correct or best model within the context of linear mixed effect models. The first consideration is to ensure that information criteria should not be used to compare models that have difference inferential emphasis, for example, comparing the AIC of a marginal model to the AIC of a conditional model. Second, it is important for the researcher to decide on whether inferences using information criteria are made to the clusters or the population. It is suggested that the conditional AIC is appropriate to use when making inferences to observations from the same cluster, the marginal AIC should be used to make inferences to observations from different clusters, and the BIC should be used for making inferences to observations from different populations. Third, the information criteria obtained by different statistical software packages have different inferential focus. The AIC obtained from SPSS has a population focus whereas the AIC obtained from WinBUGs has a cluster focus. Finally, there is an inherent difference between the AIC and BIC, where the former selects the "best" model as a function of sample size and the latter selects model based on the posterior probability and does not depend on sample size. This consideration needs to be accounted for in simulation studies, such as, whether the true model is included in the set of fitted models and the sample sizes that are being considered.

H. Effects of Misspecification of the Error Covariance Structure

As a statistical rule of thumb, Van Belle (2002) states that the not taking in account correlated errors will inflate Type I error rates, if the correlation is positive, and deflate Type I error rates, if the correlation is negative. In addition, several studies have examined the effects of misspecifying covariance structures on parameter estimates, standard errors, and significance values. Verbeke and Molenberghs (2000) showed that, in order to obtain valid inferences for the parameter estimates of the fixed effects, selecting an appropriate covariance structure is necessary since,

$$\hat{\beta} = (X^{'}\hat{V}^{-1}X)^{-}X^{'}\hat{V}^{-1}Y$$

and the variance-covariance matrix of $\hat{\beta}$ is

$$\text{var}(\hat{\beta}) = (X^{'}\hat{V}^{-1}X)^{-}(X^{'}\hat{V}^{-1}\,\text{var}(Y)\hat{V}^{-1}X)\;(X^{'}\hat{V}^{-1}X)^{-}$$

$$= (X^{'}\hat{V}^{-1}X)^{-},$$

which assumes the marginal covariance matrix to be Cov (Y_i) = $Z_i G Z_i^{'} + \sum_i$. Hence, fixed effects estimates based on the above equation of the variance-covariance matrix of $\hat{\beta}$ will not be robust if the covariance structure of the model is not correctly specified. A potential remedy to misspecification of the covariance structure is to use robust sandwich estimators for $\text{var}(\hat{\beta})$ proposed by Liang and Zeger (1986) to provide valid inferences for the fixed effects. However, because robust inference only provides valid outcomes based on strict assumptions related to missing data (Verbeke & Molenberghs, 2000), where the pattern of missing data must fulfill the condition that they are missing completely at random, the only reliable method to ensure valid inferences for estimates of the fixed effects is to select an appropriate covariance structure for the model. On the other hand, Altham (1984) found that restrictive specification leads to faulty inferences about the covariance structure and over-specification, where there are too many parameters in the covariance structure, leads to biased estimates and too-small standard errors.

Littell, Pendergast, and Natarajan (2000) conducted a study that showed specification of different covariance structures will affect parameter estimates, *F*-test values, and standard errors. A longitudinal data set comparing the effects of two drugs (A and B) on respiratory ability (FEV1) for 24 patients was used. Patients were randomly assigned into the three treatment groups. A baseline measurement was recorded prior to the administration of the drugs and then additional measurements were taken hourly for eight hours. The model used was as follows:

$$Y_{ij} = \beta_1 + \beta_2 Hour_{ij} + \beta_3 Drug_{ij} + \beta_4 Drug_i \text{x} Hour_{ij} + e_{ij}$$

For individual *i* on occasion *j*, it is assumed that respiratory score, Y_{ij}, is a linear function of *Hour*, and *Drug* is the treatment group to which an individual was assigned. Five different error covariance structures were fitted to the data: ID, CS, AR(1), TP, and UN. Results of the values for *F* tests for fixed effects were compared across error covariance structures.

Error Covariance Structure	β_1Intercept	β*2Hour*	β_3*Drug*	β_4*DrugxHour*
ID	490.76	9.20	46.50	1.69
CS	76.42	38.86	7.24	7.11
AR(1)	90.39	7.39	8.40	2.46
TP	76.31	13.75	7.30	3.82
UN	92.58	13.72	7.25	4.06

The values for the *F* tests for fixed effects differ substantially across error covariance structures. This shows that modeling the error covariance structure potentially has a large impact on tests for fixed effects. Standard errors for within-subject comparisons across time obtained from the five error covariance structure were also examined.

Parameter	ID	CS	AR (1)	TP	UN
hour1 – hour2 drug A	0.15	0.07	0.06	0.06	0.05
hour1 – hour3 drug A	0.15	0.07	0.08	0.06	0.05
hour1 – hour4 drug A	0.15	0.07	0.09	0.07	0.07
hour1 – hour5 drug A	0.15	0.07	0.10	0.08	0.08
hour1 – hour6 drug A	0.15	0.07	0.11	0.08	0.08
hour1 – hour7 drug A	0.15	0.07	0.12	0.09	0.10
hour1 – hour8 drug A	0.15	0.07	0.12	0.10	0.09

It was found that each of the five error covariance structures produced standard errors that were different. Littell, Pendergast, and Natarajan (2000) concluded that it is important to model the error covariance structure even when the estimates of fixed effects do not depend on the covariance structure, because tests of significance may depend on the covariance structure.

Using a data set containing 35 individuals measured over four occasions on an inventory that assesses performance on a timed "opposites naming" cognitive task, Singer and Willett (2006) examined the effects of error covariance specification on the estimates of fix effects, standard errors, and p-values. Each individual completed a standardized instrument that assessed baseline cognitive skills during the initial occasion, then scores from an opposite's naming task across four occasions, spaced exactly one week apart, were obtained. Hypotheses for this study are 1) opposites-naming task scores will improve due to practice, not cognitive improvement and 2) individuals with stronger cognitive skills will see greater improvement in opposites-naming skills over time. A general model was specified to examine these hypotheses:

$$Y_{ij} = \beta_1 + \beta_2 Time_{ij} + \beta_3(Cog_i - \overline{Cog})_{ij} + \beta_4(Cog_i - \overline{Cog})_i \text{x} Time_{ij} + e_{ij}$$

For individual *i* on occasion *j*, it is assumed that opposites-naming score, Y_{ij}, is a linear function of *Time*, and *Cog* is the baseline cognitive skill score. In examining the data, Singer and Willett (2006) specified three different error covariance structures: ID, TP, and UN. Results are shown in the table below.

	ID		TP		UN	
Fixed Effects	Parameter Estimate	Standard Error	Parameter Estimate	Standard Error	Parameter Estimate	Standard Error
β_1Intercept	164.37***	6.21	165.10***	5.92	165.83***	5.95
β_2Time	26.96***	1.99	26.90**	1.94	26.58***	1.93
$\beta_3(Cog_i - \overline{Cog})$	-0.11	0.50	-0.00	0.48	-0.07	0.48
$\beta_4(Cog_i - \overline{Cog})\text{x}Time$	0.43**	0.16	0.44**	0.16	0.46**	0.16

**p* < .05; ***p* < .01; ****p* < .001

The results showed that specifying different error covariance structures affect the precision of the parameter estimates and will in turn affect hypothesis testing and confidence interval construction. Standard errors generated by the TP and UN error covariance structures are smaller than those generated by the ID error covariance structure. While the differences in precision are small among the three models, it is specific to this data set. Singer and Willett (2006) suggested that differences in precision based on the specification of the error covariance structure might be greater and have a significant impact on other data sets.

To examine the effects of misspecification of the error covariance structure on estimates of fixed effects and variance components Ferron, Dailey, and Yi (2002) conducted a simulation study. Using a model where,

$$Y_{ij} = \beta_1 + \beta_2\text{Time}_{ij} + \beta_3\text{Covariate}_{ij} + \beta_4\text{Covariate}_i\text{xTime}_{ij} + e_i.$$

and setting β_1, β_2, β_3, and β_4 to 0, data were generated for a AR (1) model with autocorrelations of ρ = .3 and ρ = .6, series lengths of 3, 4 ,6, 8 and12, and sample sizes of 30, 100, 500. A total of 30 conditions were obtained, 10,000 data sets were simulated for each condition. Parameter values for the variance components,σ_0^2, σ_1^2, and σ^2 were set to 1 and σ_{01} was set to 0. The error covariance structure is incorrectly specified as $\sum_i = \sigma^2 I$, ID. It was found that estimates of fixed effects are unbiased across all conditions. However, it was found that both variance for both intercepts, σ_0^2, and slopes, σ_1^2, were inflated across conditions, the greater the autocorrelation the larger the inflation, sample size and series length had little or no effect. Estimates of σ^2 were also inflated, where a higher autocorrelation and shorter series length increased inflation. Covariance between errors in the intercepts and errors in the slopes,σ_{01}, were found to be negative, the bias becoming greater when there was a higher autocorrelation and shorter series length. The misspecification of the error covariance structure makes interpretation regarding the amount of random variance in the intercepts and slopes, the correlation in errors in the intercepts and slopes, and the variability in individual growth curves problematic.

Chi and Reinsel (1989) showed that the appropriate specification of the random effects and the error covariance structure leads to better prediction of future observation scores,

$$\hat{Y}_i^* = X_i^* \hat{\beta} + Z_i^* \hat{b}_i + e(\varepsilon_{(2)i}^* \mid Y_i),$$

where $\hat{Y}_i^*$ is a predicted future observation score for the i^{th} individual, to be taken at time t_i^*, X_i^* are fixed effects, Z_i^* are random effects, and $\varepsilon_{(2)i}^*$ is an error at time t_i^*. The addition of an autoregressive parameter, ρ, to the parameter error component

$$e(\varepsilon_{(2)i}^* \mid Y_i) = \rho^{(t_i^* - t_{i,n_i})} \left[Y_i - X_i \hat{\alpha} - Z_i \hat{b}_i \right]_{n_i},$$

may improve prediction because it draws on the correlation between the observed score to be predicted and the last observed score.

I. Additional Considerations for Modeling

There are other studies that did not directly study the effect that model specification has on the covariance structure or the error covariance structure but do provide additional factors for consideration when preparing data for analysis. Centering and transforming variables might change the covariance structure of the model in some situations (VanLeeuwen, 1997). Failure to adjust for measurement error will result in fitting an error covariance structure that contains measurement errors leading to biased estimates of fixed effects and variance components (Woodhouse, Yang, Goldstein, Rasbash, 1996). Both centering variables and adjusting for measurement error are common practices in the fields of education and psychology.

Chapter III

Methods

A. Rationale for the Current Study

After a review of the literature it was found that the current research has not provided adequate guidelines for specifying an appropriate covariance structure. First, studies that examined model specification of covariance error structures and its effect on the estimates of model parameters are often based on real data sets (Singer & Willett, 2003; Littell et al., 2000), which may contain idiosyncrasies not generalizable to other data sets. The study that used simulated data (Ferron et al., 2002) only considered the misspecification of error covariance structure where $\sum_i = \sigma^2 I$, which raises the question of whether estimates and the standard errors would remain unbiased when more complex error covariance structures are selected for a model. Second, current research does not address which error covariance structure provides the best fit and least biased estimates when modeling data with autoregressive errors, and under what circumstances the correct structure can be detected (power). Finally, there is a lack of studies that provide practical suggestions with regard to length of observations or sample size in relations to the specification of the error covariance structure. How does the length of observation or sample size effect model estimates? One purpose of this study is to consider the above questions with simulated data where $\sum_i$ = AR(1).

B. Overview of Procedures

This section will outline the methodology used for this study. The study uses a linear mixed effects model where repeated observation scores are a linear function of Time and Group, a bivariate predictor variable dummy coded (0,1),

$$Y_{ij} = \beta_1 + \beta_2 \text{Time}_{ij} + \beta_3 \text{Group}_{ij} + \beta_4 \text{Group}_i \text{xTime}_{ij} + b_{1i} + b_{2i} \text{Time}_{ij} + e_i,$$

where the parameter values of β_1, β_2, β_3, and β_4 were all set to equal 1, errors were generated to be normally distributed N(0, 1), and values for the variance components, σ_0^2, σ_1^2, and σ^2 were set to 1 and σ_{01} was set to 0. SPSS Matrix (SPSS, 2004) was used to produce values used to modify the error covariance structure into varying magnitudes of AR(1) errors correlations, specifically .0, .1, .3, .5 and .7. Data sets with sample sizes of 30, 50, 100, 250, and 500 with series length of 3, 4, 6, and 8 were generated. A total of 100 scenarios were possible given the combination of magnitude of AR(1) errors, sample sizes, and series lengths. A SPSS macro was used to simulate 1,000 samples for each scenario for a total of 100,000 datasets. Each sample was analyzed five times using the linear mixed model procedure in SPSS: a different specification for the error covariance structure was used each time a sample was analyzed. The specifications for the error covariance structure are CS, ID, AR(1), TP, and UN. Syntax and programming were written to ensure model results and fit statistics were comparable following the guidelines created by SPSS (2005) and Hamaker, Hattum, Kuiper, and Hoijtink (2011). The rationale for using the aforementioned error covariance structures is that 1) the CS error covariance structure is the most restrictive and least complex specification, duplicating results from repeated measures ANOVAs, 2) the ID error covariance structure is considered the "standard" structure, 3) the AR(1) specification is the correct error covariance structure for the data generated, 4) the TP error covariance structure provides the most general structure that would account for autoregressive errors among repeated observations, and 5) an UN error covariance structure is the least restrictive and most complex specification, approximating results from MANOVAs.

Of the 100,000 datasets simulated, 1,274 datasets produced non-convergent results (see Appendix A), replacement datasets were generated in these cases to ensure only results from convergent datasets are used. Results generated by each analysis were compared. First, fit statistics (AIC, AICC, BIC, and CAIC) were used to examine the proportion of times a particular error covariance structure was identified as providing the best fit for the data. This provides evidence about which error covariance structure best fits the data given sample conditions. Second, mean fixed effects estimates (β_1, β_2, β_3, and β_4) were compared across the five error covariance structures to see whether unbiased estimates were obtainable under different conditions. Third, mean p-values of the fixed effects estimates were evaluated across the five error covariance structures to determine whether there is a Type I error cost under certain model scenarios. Fourth, the proportion of times the AR(1) error covariance structure was significantly detected when the error covariance structure was correctly specified was reported to provide information about model conditions needed to estimate the AR(1) error parameter.

Chapter IV

Results

For this study a series of computer simulations were conducted to examine the effectiveness of using different information criteria in identifying linear mixed models with an AR(1) error covariance structure, to determine whether estimates of fixed effects are biased and the type I error cost when the error covariance structure is misspecified and finally, to determine the conditions necessary to detect the autocorrelation (ρ) parameter.

A. Identifying the error covariance structure using information criteria

The first goal was to examine the effectiveness of using AIC, AICC, BIC, and CAIC information criteria to correctly identify models with an AR(1) error covariance structure. Each data condition was analyzed using CS, ID, AR(1), TP, and UN error covariance structures. To examine how models are usually selected, the aforementioned information criteria were used to determine which covariance structure provided the best fit for the various data conditions. Previous studies (Singer, 2003) have shown that for data without autoregressive error an ID error covariance structure should provide the best fit. Therefore, we expect data conditions with an AR(1) error covariance structure of 0, no autocorrelation, to be correctly identified by fit statistics generated by the ID error covariance structure the vast majority of the time. Similarly, we expect that data conditions generated with an AR(1) error covariance structure should be correctly identified by fit statistics as having an AR(1) error covariance structure. The purpose is to examine the effects of the magnitude of the AR(1) autocorrelation, sample size, and series length on identification accuracy.

For models generated with an AR(1) error covariance structure with p = 0, no autocorrelation, the AIC correctly identified 82.1%, AICC correctly identified 83.3%, BIC correctly identified 98.0%, and CAIC correctly identified 93.6% as being best fitted by the ID error covariance structure. The data presented in Figures 4 through 6 (see Appendix A for data) illustrate identification accuracy for models generated with an AR(1) error covariance structure. It is apparent that as the magnitude of the AR(1) error, sample size, and series length increase so does the identification accuracy. It is also apparent that both AIC and AICC outperform BIC and CAIC in identification accuracy with an AR(1) error covariance structure across all conditions. The difference in performance between the AIC and AICC is negligible and the BIC outperforms the CAIC by a small margin.

In examining the main effects of the magnitude of the AR(1) error, sample size, and series length on identification accuracy, it was found that series length had the most prominent effect. Using the AIC, identification accuracy increases from 16.9% with a series length of 3 to 90.6% with a series length of 8, and similar large increases are seen with the other information criteria. Another noticeable effect is the sharp increase in the identification accuracy from an AR(1) error of .1 to an AR(1) error of .3. For example using the AIC, changing from an AR(1) error of .1 to an AR(1) error of .3, increases identification accuracy from 37.8% to 65.1%. Changes in the magnitude of AR(1) from .3 to .5 and from .5 to .7 resulted in a comparatively smaller increase in identification accuracy. The relationship between sample size and identification accuracy is steady, with a 7% to 10% increase in percent correctly identified each time sample size increases.

Figure 4: Percentage of AR(1) models correctly identified by magnitudes of AR(1) autocorrelation

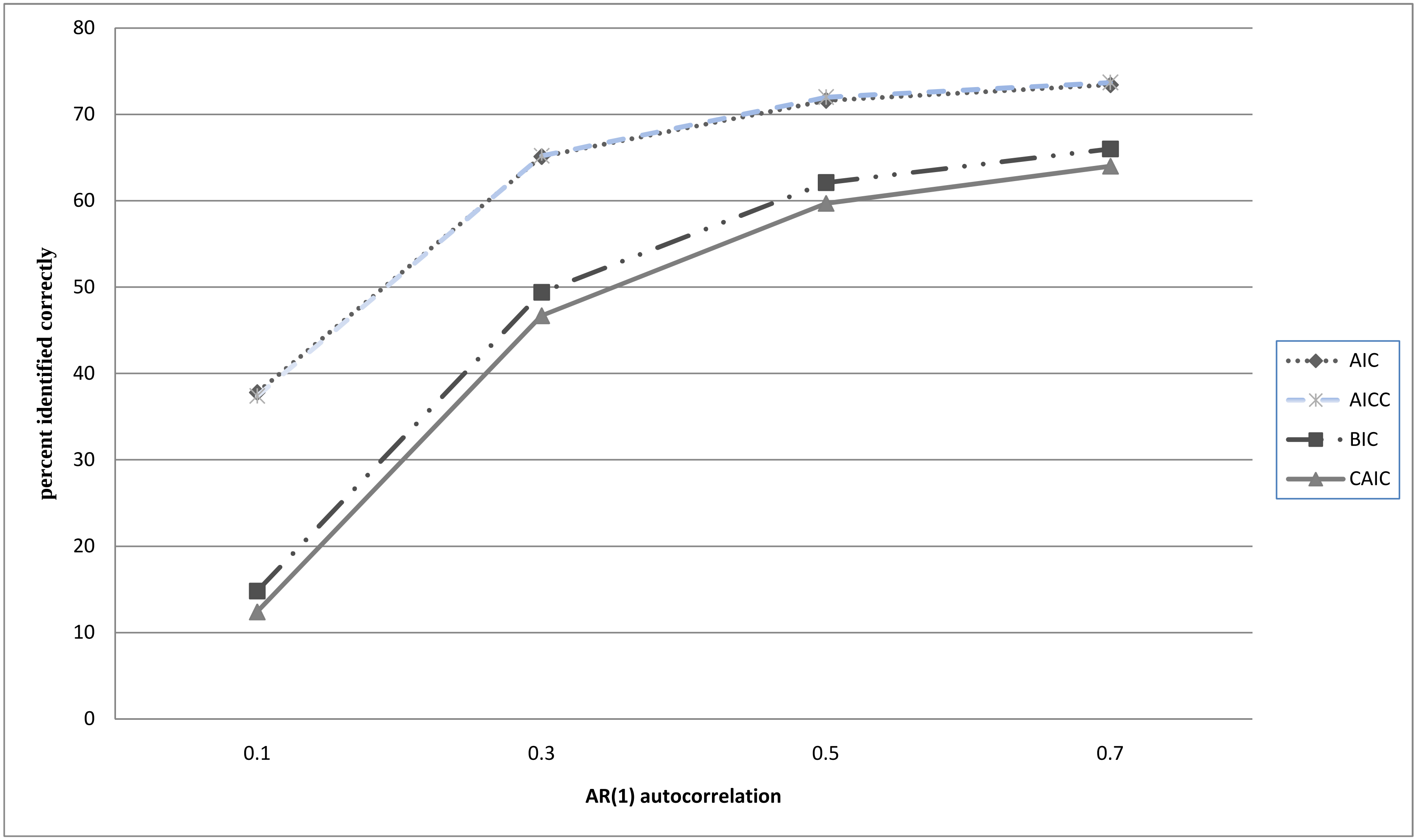


Figure 5: Percentage of AR(1) models correctly identified by varying series lengths

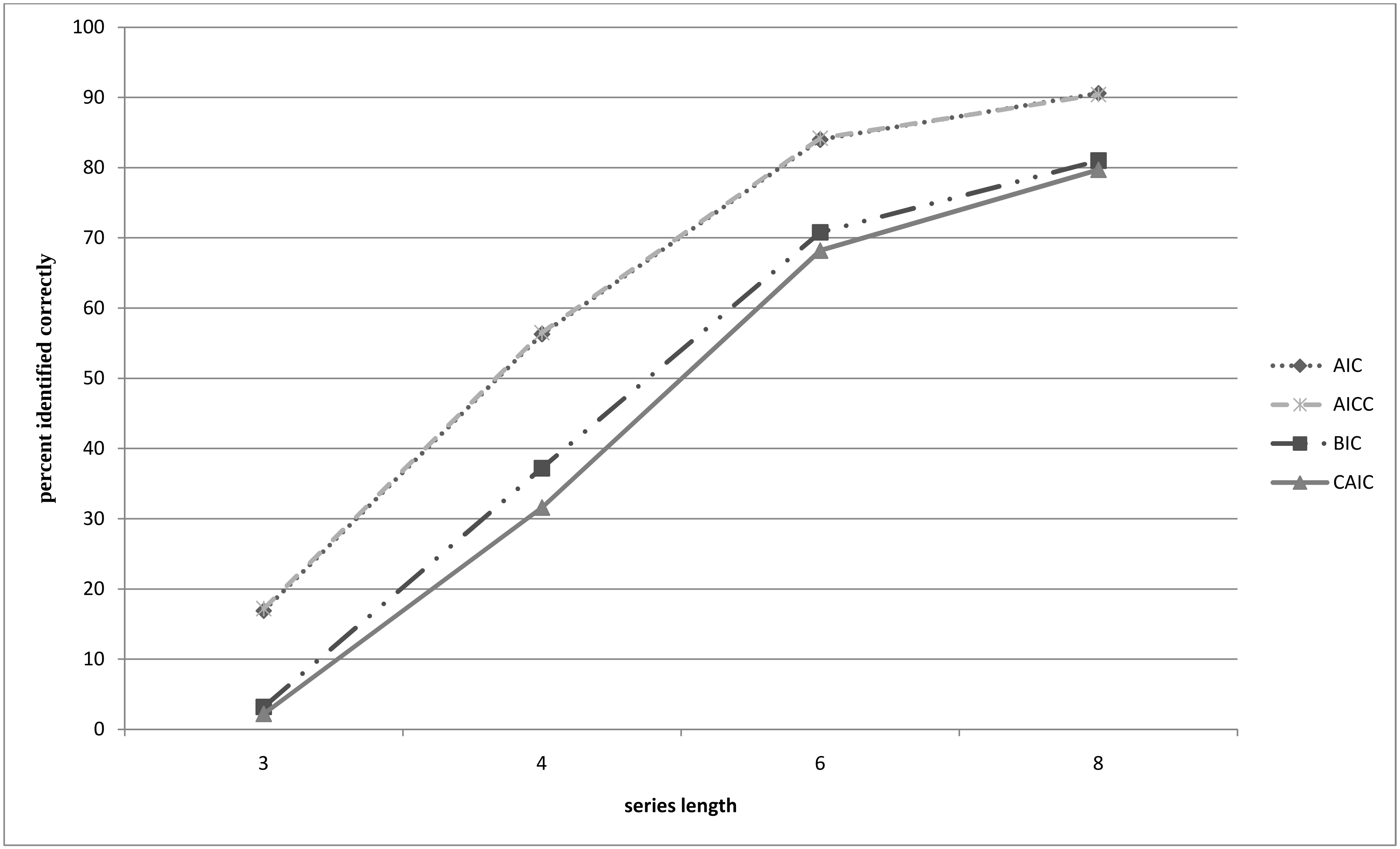
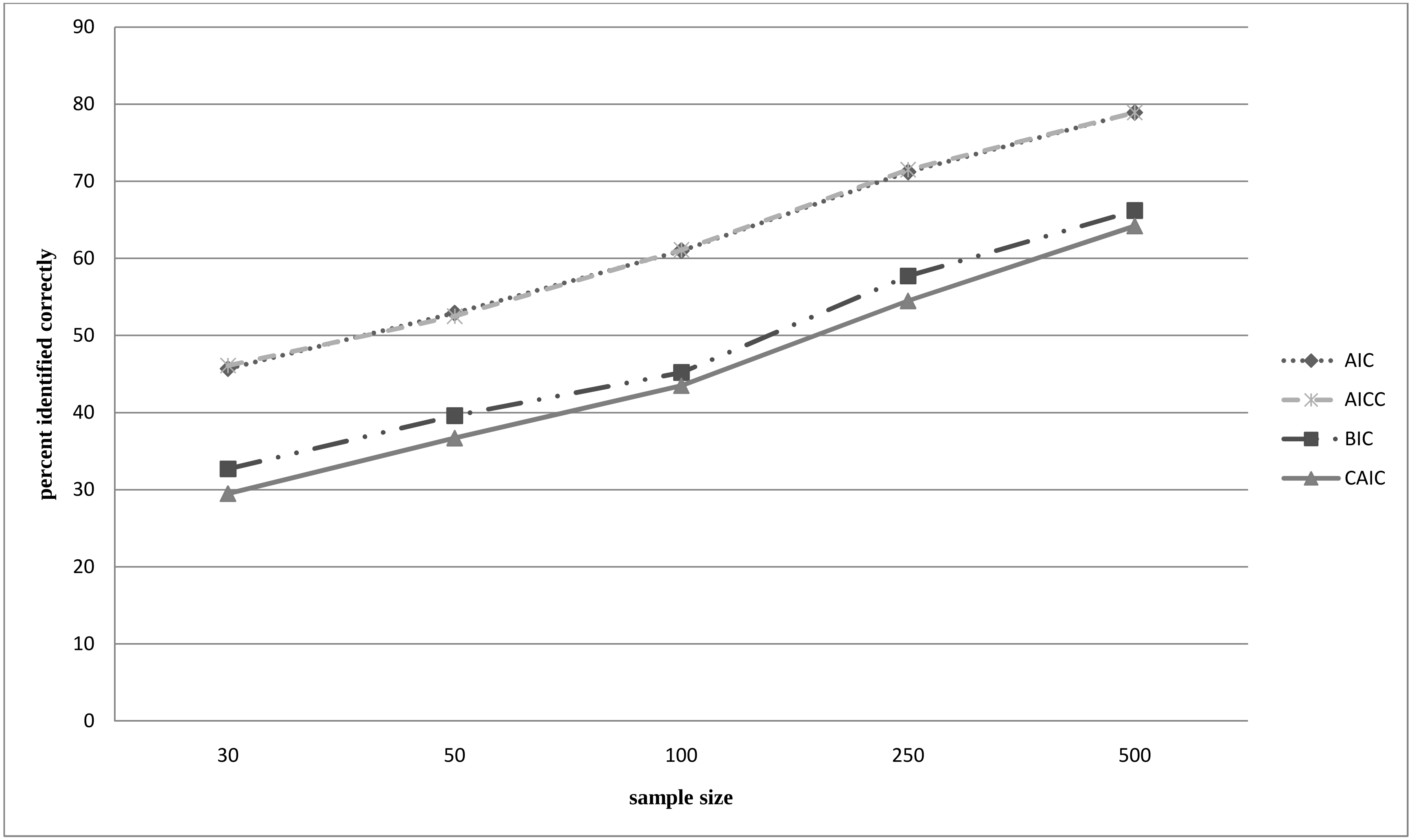


Figure 6: Percentage of AR(1) models correctly identified by varying sample sizes



To examine the interaction effects that magnitude of the AR(1) error, sample size, and series length have on identification accuracy and how the different error covariance structures fitted the data we will examine results provided by the AIC, as it outperformed the other indices in correctly identifying data with AR(1) error covariance structure. Tables 1 through 5 show detailed data on the models identified as best fitting by the AIC across covariance structures and data conditions (data for AICC, BIC, and CAIC are in Appendix B).

Table 1 shows that the AIC correctly identifies the identity error covariance structure as the correct fit for models generated were $\boldsymbol{\rho} = 0$, without autocorrelation, 82.1% of the time. However, the AIC also misidentified these models as having an AR(1) error structure 13.4% of the time. Tables 2 through 5 examine the identification accuracy of data with an AR(1) error covariance structure, show an interaction between sample size and series length. Sample size has a larger impact on identification accuracy when the series length is shorter. For example, in the case where $\boldsymbol{\rho} = .3$ and series length is 3, changing the sample size from 30 to 500 results in an increase in identification accuracy from 1.1% to 36.0%. However, when the series length is 8 (holding $\boldsymbol{\rho} = .3$) changing the sample size from 30 to 500 increased the percent of correct identifications from 91.3% to 99.9%, a smaller gain than the previous case. A three-way interaction is notable: When we examine the case where $\boldsymbol{\rho} = .1$ and series length is 3, changing the sample size from 30 to 500 results in a change in identification accuracy from 2.6% to 18.1%, whereas holding $\boldsymbol{\rho} = .1$, a series length of 8 and changing the sample size from 30 to 500 increased the percent of identification accuracy from 35.1% to 99.7%, a much larger change. Thus, a change in the magnitude of the AR(1) error along with sample size and series length interact to affect identification accuracy.

In examining how the other error covariance structures fitted the data it was found that models fitted with the CS and TP error covariance structures were identified in a small percentage of cases as the “correct” model in data conditions where sample size is small (30 or 50) and series length short (3 or 4). The percentage of time that models with an UN error covariance structure were mistakenly identified as correct increased slightly as the magnitude of AR(1) error increases, and decreases when sample size and series length increase. The incorrect identification of the ID error covariance structure as the preferred model decreased as the magnitude of the AR(1) error, sample size, and series length increase.

Tables 1 through 5 also presents, in bold, the data conditions in which the AR(1) error can be detected with an AR(1) error covariance structure with a high probability of success (the percent of correctly identified is 90.0% or more). Although, these results are for the AIC, patterns of main and interactions effects on identification accuracy for AIC, BIC, and CAIC are similar and can be reviewed in Appendix B.

Table 1. Identification accuracy using AIC across covariance error structure for ***ρ*** = .0 by sample size and series length

Akaike's Information Criterion (AIC)							
Conditions			**CS**	**ID**	**AR1**	**TP**	**UN**
ρ	**N**	**Length**	**% identified as best fit**	**% identified as best fit**	**% identified as best fit**	**% identified as best fit**	**% identified as best fit**
0	30	3	1.7	78.4	3.5	2.9	13.5
0	50	3	0.0	82.5	5.4	0.1	12.0
0	100	3	0.0	81.3	7.1	0.0	11.6
0	250	3	0.0	78.7	13.6	0.0	7.7
0	500	3	0.0	78.6	12.8	0.0	8.6
0	30	4	0.0	80.4	13.4	0.1	6.1
0	50	4	0.0	81.0	12.0	0.0	7.0
0	100	4	0.0	78.8	15.6	0.0	5.6
0	250	4	0.0	80.3	16.5	0.0	3.2
0	500	4	0.0	81.8	14.1	0.0	4.1
0	30	6	0.0	82.6	15.7	0.0	1.7
0	50	6	0.0	82.6	16.8	0.0	0.6
0	100	6	0.0	82.4	16.8	0.0	0.8
0	250	6	0.0	84.3	15.2	0.0	0.5
0	500	6	0.0	84.6	15.0	0.0	0.4
0	30	8	0.0	84.7	15.0	0.0	0.3
0	50	8	0.0	85.3	14.2	0.0	0.5
0	100	8	0.0	83.9	15.9	0.0	0.2
0	250	8	0.0	85.5	14.4	0.0	0.1
0	500	8	0.0	85.6	14.3	0.0	0.1
Total			**< 0.1**	**82.1**	**13.4**	**< 0.1**	**4.2**

Table 2. Identification accuracy using AIC across covariance error structure for ρ = .1 by sample size and series length

Akaike's Information Criterion (AIC)							
Conditions			**CS**	**ID**	**AR1**	**TP**	**UN**
ρ	**N**	**Length**	**% identified as best fit**	**% identified as best fit**	**% identified as best fit**	**% identified as best fit**	**% identified as best fit**
0.1	30	3	0.3	80.7	2.6	3.6	12.8
0.1	50	3	0.0	83.6	4.1	0.4	11.9
0.1	100	3	0.0	78.9	9.4	0.0	11.7
0.1	250	3	0.0	74.9	10.4	0.0	10.4
0.1	500	3	0.0	71.8	18.1	0.0	10.1
0.1	30	4	0.0	78.5	13.5	0.0	8.0
0.1	50	4	0.0	76.5	17.2	0.0	6.3
0.1	100	4	0.0	74.4	19.6	0.0	6.0
0.1	250	4	0.0	64.2	29.3	0.0	6.5
0.1	500	4	0.0	50.3	43.8	0.0	5.9
0.1	30	6	0.0	74.3	24.5	0.0	1.2
0.1	50	6	0.0	68.1	31.2	0.0	0.7
0.1	100	6	0.0	56.6	42.1	0.0	1.3
0.1	250	6	0.0	28.4	70.5	0.0	1.1
0.1	500	6	0.0	8.1	91.3	0.0	0.6
0.1	30	8	0.0	64.3	35.1	0.0	0.6
0.1	50	8	0.0	57.4	42.6	0.0	0.0
0.1	100	8	0.0	35.7	64.0	0.0	0.0
0.1	250	8	0.0	13.5	86.5	0.0	0.0
0.1	500	8	0.0	0.3	99.7	0.0	0.0
Total			**< 0.1**	**57.0**	**37.8**	**0.2**	**4.8**

Table 3. Identification accuracy using AIC across covariance error structure for ρ = .3 vary sample size and series length

Akaike's Information Criterion (AIC)							
Conditions			**CS**	**ID**	**AR1**	**TP**	**UN**
ρ	**N**	**Length**	**% identified as best fit**	**% identified as best fit**	**% identified as best fit**	**% identified as best fit**	**% identified as best fit**
0.3	30	3	0.3	79.8	1.1	3.6	15.2
0.3	50	3	0.0	78.2	4.3	0.8	16.7
0.3	100	3	0.0	72.0	13.8	0.0	14.2
0.3	250	3	0.0	65.6	24.9	0.0	9.5
0.3	500	3	0.0	53.1	36.0	0.0	10.9
0.3	30	4	0.0	63.9	25.9	0.2	10.0
0.3	50	4	0.0	39.6	51.2	0.0	9.2
0.3	100	4	0.0	46.2	47.0	0.0	6.8
0.3	250	4	0.0	18.3	73.6	0.0	8.1
0.3	500	4	0.0	2.0	89.1	0.0	8.9
0.3	30	6	0.0	28.6	69.5	0.0	1.9
0.3	50	6	0.0	14.2	83.6	0.0	2.2
0.3	100	6	0.0	2.4	96.9	0.0	0.7
0.3	250	6	0.0	0.1	98.5	0.0	1.4
0.3	500	6	0.0	0.0	98.7	0.0	1.3
0.3	30	8	0.0	9.2	90.3	0.0	0.5
0.3	50	8	0.0	1.7	97.7	0.0	0.6
0.3	100	8	0.0	0.1	99.9	0.0	0.0
0.3	250	8	0.0	0.1	99.9	0.0	0.0
0.3	500	8	0.0	0.1	99.9	0.0	0.0
Total			**< 0.1**	**28.8**	**65.1**	**0.2**	**5.9**

Table 4. Identification accuracy using AIC across covariance error structure for ρ = .5 by sample size and series length

Akaike's Information Criterion (AIC)								
Conditions			**CS**	**ID**	**AR1**	**TP**	**UN**
ρ	**N**	**Length**	**% identified as best fit**	**% identified as best fit**	**% identified as best fit**	**% identified as best fit**	**% identified as best fit**
0.5	30	3	0.0	80.4	0.4	4.4	14.8
0.5	50	3	0.0	77.3	5.6	0.3	16.8
0.5	100	3	0.0	67.4	16.5	0.0	16.1
0.5	250	3	0.0	52.4	33.1	0.0	14.4
0.5	500	3	0.0	32.9	51.1	0.0	16.0
0.5	30	4	0.0	51.6	38.3	0.1	10.0
0.5	50	4	0.0	38.8	52.0	0.0	9.2
0.5	100	4	0.0	20.0	71.7	0.0	8.3
0.5	250	4	0.0	2.0	90.9	0.0	7.1
0.5	500	4	0.0	0.0	92.6	0.0	7.4
0.5	30	6	0.0	8.3	88.6	0.0	3.1
0.5	50	6	0.0	1.5	96.2	0.0	2.3
0.5	100	6	0.0	0.0	99.0	0.0	1.0
0.5	250	6	0.0	0.0	99.0	0.0	1.0
0.5	500	6	0.0	0.0	98.6	0.0	1.4
0.5	30	8	0.0	0.3	99.1	0.0	0.6
0.5	50	8	0.0	0.0	99.7	0.0	0.3
0.5	100	8	0.0	0.0	99.9	0.0	0.1
0.5	250	8	0.0	0.0	99.9	0.0	0.1
0.5	500	8	0.0	0.0	99.9	0.0	0.1
Total			**0.0**	**21.6**	**71.6**	**0.2**	**6.5**

Table 5. Identification accuracy using AIC across covariance error structure for ρ = .7 by sample size and series length

Akaike's Information Criterion (AIC)							
Conditions			CS	ID	AR1	TP	UN
ρ	N	Length	% identified as best fit	% identified as best fit	% identified as best fit	% identified as best fit	% identified as best fit
0.7	30	3	0.0	78.3	0.9	2.7	18.1
0.7	50	3	0.0	75.8	3.6	0.1	20.5
0.7	100	3	0.0	65.5	15.5	0.0	19.0
0.7	250	3	0.0	48.8	32.6	0.0	18.6
0.7	500	3	0.0	27.3	54.1	0.0	18.6
0.7	30	4	0.0	42.2	47.8	0.0	10.0
0.7	50	4	0.0	32.7	59.9	0.0	7.4
0.7	100	4	0.0	9.8	82.2	0.0	7.9
0.7	250	4	0.0	0.3	90.4	0.0	9.3
0.7	500	4	0.0	0.1	90.6	0.0	9.3
0.7	30	6	0.0	2.3	95.5	0.0	2.2
0.7	50	6	0.0	0.3	97.5	0.0	2.2
0.7	100	6	0.0	0.0	99.3	0.0	0.7
0.7	250	6	0.0	0.0	99.2	0.0	0.8
0.7	500	6	0.0	0.0	99.6	0.0	0.4
0.7	30	8	0.0	0.0	99.1	0.0	0.9
0.7	50	8	0.0	0.0	99.5	0.0	0.5
0.7	100	8	0.0	0.0	99.9	0.0	0.1
0.7	250	8	0.0	0.0	100.0	0.0	0.0
0.7	500	8	0.0	0.0	99.9	0.0	0.1
Total			**0.0**	**19.2**	**73.4**	**0.1**	**7.3**

B. Impact of fitting different error covariance structures on estimated fixed effects

Data were simulated with values of β_1, β_2, β_3, and β_4 are set to equal 1.00. Examining these results allows determination of whether misspecifying the error covariance will bias estimated fix effects. The mean effects estimates for β_1, β_2, β_3, and β_4 were very close to the true parameter of 1.00; across all conditions the estimates ranged between .97 and 1.03. Thus, as expected, there does not seem to be any bias in the estimates of fixed effects due to misspecification of the error covariance structure.

C. Impact of fitting error covariance structures on tests of fixed effects

Effects of fitting each of the five error covariance structures on type I error rates were examined, where H_0: $\beta_1 = \beta_2 = \beta_3 = \beta_4 = 1$. Tables 6 through 11 show the mean type I error rates of parameter estimates across assumed error covariance structures, magnitude of the AR(1) error, sample size, and series length (p1, p2, p3, and p4 are type I error rates for the fixed effects estimates β_1, β_2, β_3, and β_4, respectively). Mean type I error rates that are greater than .05 are in bold.

It is apparent that mean type I error rates for estimated parameters fitted with the ID, AR(1), and UN error covariance structures are nearly identical across conditions. For these three error covariance structures, when sample size is 30 or 50, mean type I error rates for p2, p4, and in some cases p3, were above .05. Once sample size is 100 or larger, mean type I error rates were below .05; this was consistent across magnitudes of AR(1) error and series lengths.

Results from data fitted with the CS error covariance structure showed that as series length increases mean type I error rates for p1 and p2 also increase. For example, for p1 where the magnitude of AR(1) is 0, sample size is 100, and series length is 3, the mean p-value is below .01; as series length increases to 8, the mean type I error rates for p1 becomes .09. In the same conditions the mean type I error rates for p2 changes from .03 to .23 as series length increases from 3 to 8, respectively. This relationship remains consistent regardless of the magnitude of the AR(1) error. On the other hand, as series length increases the mean type I error rates of p4 (the interaction) decreases. For example, when the magnitude of AR(1) is .0, sample size is 30, and series length is 3, the mean type I error rate for p4 is .08, as series length increases to 8, the mean type I error rate becomes .01. This relationship also remains consistent regardless of the magnitude of the AR(1) error.

Data fitted with a TP error covariance structure has the most instances where mean type I error rates were above .05. Across conditions, as series length increases the instances where p1 and p2 had mean type I error rates above .05 also increases. The reverse occurs for p4; as series length increases the mean type I error rates decreases.

While the effects of sample size and series length on type I error rates varied by error covariance structure, the effect of the magnitude of the AR(1) error on type I error rates are consistent: As the magnitude of the AR(1) error increases the instances were mean type I error rates are above .05 also increased. For example, when AR(1) error is .1, in conditions fitted with the ID error covariance structure there were ten instances where mean type I error rates where above .05, as AR(1) error increased to .7, in conditions fitted with the ID error covariance structure there were 15 instances where mean type I error rates where above .05. This pattern holds for the other four error covariance structures.

Table 6. Mean type I error rates across error covariance structures for ρ = .0 by sample size and series length

Conditions			**CS**				**ID**				**AR1**				**TP**				**UN**			
ρ	**N**	**Length**	**p1**	**p2**	**p3**	**p4**	**p1**	**p2**	**p3**	**p4**	**p1**	**p2**	**p3**	**p4**	**p1**	**p2**	**p3**	**p4**	**p1**	**p2**	**p3**	**p4**
0.0	30	3	**0.09**	**0.24**	0.02	**0.08**	0.05	**0.16**	0.03	**0.12**	0.05	**0.16**	0.04	**0.12**	**0.11**	**0.25**	0.03	**0.11**	0.05	**0.16**	0.03	**0.12**
0.0	50	3	0.03	**0.13**	0.00	0.03	0.01	**0.07**	0.01	0.05	0.01	**0.07**	0.01	0.05	0.04	**0.15**	0.01	0.04	0.01	**0.07**	0.01	0.05
0.0	100	3	0.00	0.03	0.00	0.00	0.00	0.01	0.00	0.00	0.00	0.01	0.00	0.00	0.00	0.04	0.00	0.00	0.00	0.01	0.00	0.00
0.0	250	3	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.0	500	3	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.0	30	4	**0.14**	**0.27**	0.01	0.04	0.05	**0.14**	0.02	**0.09**	0.05	**0.14**	0.02	**0.09**	**0.19**	**0.32**	0.01	**0.07**	0.05	**0.13**	0.02	**0.09**
0.0	50	4	0.05	**0.16**	0.00	0.01	0.01	0.05	0.00	0.03	0.01	0.05	0.00	0.03	**0.08**	**0.21**	0.00	0.02	0.01	0.05	0.00	0.02
0.0	100	4	0.01	0.04	0.00	0.00	0.00	0.01	0.00	0.00	0.00	0.01	0.00	0.00	0.02	**0.07**	0.00	0.00	0.00	0.01	0.00	0.00
0.0	250	4	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00
0.0	500	4	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.0	30	6	**0.24**	**0.39**	0.00	0.02	0.04	**0.11**	0.01	**0.07**	0.04	**0.11**	0.01	**0.07**	**0.34**	**0.48**	0.01	0.04	0.04	**0.11**	0.01	**0.06**
0.0	50	6	**0.13**	**0.29**	0.00	0.00	0.01	0.05	0.00	0.02	0.01	0.05	0.00	0.02	**0.23**	**0.39**	0.00	0.01	0.01	0.05	0.00	0.02
0.0	100	6	0.03	**0.13**	0.00	0.00	0.00	0.01	0.00	0.00	0.00	0.01	0.00	0.00	**0.10**	**0.23**	0.00	0.00	0.00	0.01	0.00	0.00
0.0	250	6	0.00	0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	**0.06**	0.00	0.00	0.00	0.00	0.00	0.00
0.0	500	6	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00
0.0	30	8	**0.35**	**0.51**	0.00	0.01	0.03	**0.11**	0.01	**0.06**	0.03	**0.11**	0.01	**0.06**	**0.46**	**0.56**	0.01	0.03	0.03	**0.12**	0.01	**0.06**
0.0	50	8	**0.23**	**0.39**	0.00	0.00	0.00	0.04	0.00	0.02	0.00	0.04	0.00	0.02	**0.37**	**0.50**	0.00	0.01	0.00	0.04	0.00	0.02
0.0	100	8	**0.09**	**0.23**	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	**0.21**	**0.38**	0.00	0.00	0.00	0.00	0.00	0.00
0.0	250	8	0.01	**0.06**	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.05	**0.17**	0.00	0.00	0.00	0.00	0.00	0.00
0.0	500	8	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.05	0.00	0.00	0.00	0.00	0.00	0.00

Table 7. Mean type I error rates across error covariance structures for ρ = .1 by sample size and series length

Conditions			**CS**				**ID**				**AR1**				**TP**				**UN**			
ρ	**N**	**Length**	**p1**	**p2**	**p3**	**p4**	**p1**	**p2**	**p3**	**p4**	**p1**	**p2**	**p3**	**p4**	**p1**	**p2**	**p3**	**p4**	**p1**	**p2**	**p3**	**p4**
0.1	30	3	**0.09**	**0.22**	0.02	**0.07**	0.05	**0.15**	0.03	**0.11**	0.05	**0.15**	0.04	**0.12**	0.11	**0.25**	0.03	**0.10**	0.05	**0.14**	0.03	**0.11**
0.1	50	3	0.03	**0.13**	0.00	0.02	0.01	**0.07**	0.00	0.04	0.01	**0.08**	0.01	0.05	0.04	**0.15**	0.00	0.04	0.01	**0.07**	0.00	0.04
0.1	100	3	0.00	0.03	0.00	0.00	0.00	0.01	0.00	0.01	0.00	0.01	0.00	0.01	0.00	0.04	0.00	0.00	0.00	0.01	0.00	0.01
0.1	250	3	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.1	500	3	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.1	30	4	**0.13**	**0.28**	0.00	0.04	0.05	**0.14**	0.02	**0.09**	0.05	**0.14**	0.02	**0.09**	**0.18**	**0.33**	0.01	**0.07**	0.04	**0.14**	0.02	**0.08**
0.1	50	4	**0.06**	**0.18**	0.00	0.01	0.01	**0.07**	0.00	0.03	0.01	**0.07**	0.00	0.03	**0.09**	**0.23**	0.00	0.02	0.01	**0.07**	0.00	0.03
0.1	100	4	0.01	0.05	0.00	0.00	0.00	0.01	0.00	0.00	0.00	0.01	0.00	0.00	0.02	**0.08**	0.00	0.00	0.00	0.01	0.00	0.00
0.1	250	4	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00
0.1	500	4	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.1	30	6	**0.24**	**0.40**	0.00	0.01	0.03	**0.13**	0.01	**0.07**	0.04	**0.13**	0.02	**0.07**	**0.34**	**0.48**	0.01	0.05	0.03	**0.12**	0.01	**0.07**
0.1	50	6	**0.13**	**0.29**	0.00	0.01	0.01	0.05	0.00	0.03	0.01	0.05	0.00	0.03	**0.23**	**0.40**	0.00	0.02	0.01	**0.06**	0.00	0.03
0.1	100	6	0.03	**0.12**	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	**0.09**	**0.22**	0.00	0.00	0.00	0.00	0.00	0.00
0.1	250	6	0.00	0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	**0.06**	0.00	0.00	0.00	0.00	0.00	0.00
0.1	500	6	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00
0.1	30	8	**0.35**	**0.50**	0.00	0.02	0.03	**0.11**	0.01	**0.07**	0.03	**0.11**	0.01	**0.07**	**0.45**	**0.57**	0.00	0.05	0.03	**0.11**	0.01	**0.07**
0.1	50	8	**0.23**	**0.39**	0.00	0.00	0.00	0.04	0.00	0.01	0.00	0.04	0.00	0.01	**0.37**	**0.51**	0.00	0.01	0.00	0.04	0.00	0.01
0.1	100	8	**0.09**	**0.22**	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	**0.23**	**0.37**	0.00	0.00	0.00	0.00	0.00	0.00
0.1	250	8	0.03	**0.10**	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	**0.09**	**0.22**	0.00	0.00	0.00	0.00	0.00	0.00
0.1	500	8	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.05	0.00	0.00	0.00	0.00	0.00	0.00

Table 8. Mean type I error rates across error covariance structures for ρ = .3 by sample size and series length

Conditions			**CS**				**ID**				**AR1**				**TP**				**UN**			
ρ	**N**	**Length**	**p1**	**p2**	**p3**	**p4**	**p1**	**p2**	**p3**	**p4**	**p1**	**p2**	**p3**	**p4**	**p1**	**p2**	**p3**	**p4**	**p1**	**p2**	**p3**	**p4**
0.3	30	3	**0.10**	**0.23**	0.02	**0.07**	**0.06**	**0.16**	0.03	**0.11**	**0.07**	**0.16**	0.04	**0.12**	**0.12**	**0.25**	0.03	**0.10**	**0.06**	**0.16**	0.03	**0.12**
0.3	50	3	0.04	**0.14**	0.00	0.02	0.02	**0.08**	0.01	0.05	0.02	**0.09**	0.01	0.05	0.05	**0.16**	0.00	0.04	0.02	**0.08**	0.01	0.05
0.3	100	3	0.00	0.03	0.00	0.00	0.00	0.01	0.00	0.00	0.00	0.01	0.00	0.00	0.00	0.04	0.00	0.00	0.00	0.01	0.00	0.00
0.3	250	3	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.3	500	3	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.3	30	4	**0.14**	**0.29**	0.01	0.03	0.05	**0.16**	0.02	**0.08**	0.05	**0.16**	0.02	**0.08**	**0.19**	**0.34**	0.01	**0.06**	0.05	**0.15**	0.02	**0.08**
0.3	50	4	0.06	**0.18**	0.00	0.01	0.02	**0.07**	0.00	0.03	0.02	**0.08**	0.00	0.03	**0.09**	**0.24**	0.00	0.02	0.02	**0.08**	0.00	0.03
0.3	100	4	0.01	0.06	0.00	0.00	0.00	0.01	0.00	0.00	0.00	0.02	0.00	0.00	0.02	**0.09**	0.00	0.00	0.00	0.01	0.00	0.00
0.3	250	4	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00
0.3	500	4	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.3	30	6	**0.24**	**0.41**	0.00	0.01	0.05	**0.14**	0.02	**0.07**	0.05	**0.15**	0.02	**0.07**	**0.33**	**0.48**	0.01	0.05	0.04	**0.13**	0.01	**0.07**
0.3	50	6	**0.14**	**0.27**	0.00	0.00	0.01	**0.06**	0.00	0.02	0.01	**0.06**	0.00	0.02	**0.24**	**0.39**	0.00	0.01	0.01	0.05	0.00	0.02
0.3	100	6	0.03	**0.13**	0.00	0.00	0.00	0.01	0.00	0.00	0.00	0.01	0.00	0.00	**0.09**	**0.23**	0.00	0.00	0.00	0.01	0.00	0.00
0.3	250	6	0.00	0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	**0.06**	0.00	0.00	0.00	0.00	0.00	0.00
0.3	500	6	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00
0.3	30	8	**0.34**	**0.52**	0.00	0.01	0.03	**0.13**	0.01	**0.07**	0.04	**0.14**	0.01	**0.07**	**0.45**	**0.56**	0.00	0.04	0.04	**0.13**	0.01	**0.06**
0.3	50	8	**0.23**	**0.40**	0.00	0.00	0.01	**0.06**	0.00	0.02	0.01	0.05	0.00	0.02	**0.38**	**0.50**	0.00	0.01	0.01	0.05	0.00	0.02
0.3	100	8	**0.09**	**0.23**	0.00	0.00	0.00	0.01	0.00	0.00	0.00	0.01	0.00	0.00	**0.21**	**0.37**	0.00	0.00	0.00	0.01	0.00	0.00
0.3	250	8	0.01	**0.06**	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.05	**0.16**	0.00	0.00	0.00	0.00	0.00	0.00
0.3	500	8	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.05	0.00	0.00	0.00	0.00	0.00	0.00

Table 9. Mean type I error rates across error covariance structures for ρ = .5 by sample size and series length

Conditions			**CS**				**ID**				**AR1**				**TP**				**UN**			
ρ	**N**	**Length**	**p1**	**p2**	**p3**	**p4**	**p1**	**p2**	**p3**	**p4**	**p1**	**p2**	**p3**	**p4**	**p1**	**p2**	**p3**	**p4**	**p1**	**p2**	**p3**	**p4**
0.5	30	3	**0.09**	**0.24**	0.01	**0.06**	0.05	**0.17**	0.03	**0.10**	**0.06**	**0.17**	0.03	**0.10**	**0.11**	**0.27**	0.02	**0.09**	0.05	**0.16**	0.03	**0.11**
0.5	50	3	0.04	**0.13**	0.00	0.02	0.02	**0.08**	0.00	0.04	0.03	**0.09**	0.01	0.04	0.05	**0.16**	0.00	0.03	0.02	**0.08**	0.00	0.04
0.5	100	3	0.00	0.04	0.00	0.00	0.00	0.02	0.00	0.00	0.01	0.02	0.00	0.01	0.00	0.05	0.00	0.00	0.00	0.02	0.00	0.00
0.5	250	3	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.5	500	3	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.5	30	4	**0.15**	**0.29**	0.00	0.03	**0.06**	**0.16**	0.02	**0.08**	**0.06**	**0.16**	0.02	**0.08**	**0.20**	**0.35**	0.01	**0.06**	**0.06**	**0.15**	0.02	**0.08**
0.5	50	4	**0.06**	**0.18**	0.00	0.01	0.01	**0.08**	0.00	0.02	0.02	**0.08**	0.01	0.03	**0.10**	**0.23**	0.00	0.02	0.01	**0.07**	0.00	0.02
0.5	100	4	0.01	**0.06**	0.00	0.00	0.00	0.01	0.00	0.00	0.00	0.01	0.00	0.00	0.02	**0.09**	0.00	0.00	0.00	0.01	0.00	0.00
0.5	250	4	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00
0.5	500	4	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.5	30	6	**0.25**	**0.40**	0.00	0.01	**0.06**	**0.15**	0.01	**0.07**	**0.06**	**0.16**	0.01	**0.07**	**0.34**	**0.48**	0.00	0.04	0.05	**0.15**	0.01	**0.06**
0.5	50	6	**0.14**	**0.29**	0.00	0.00	0.02	**0.07**	0.00	0.02	0.02	**0.07**	0.00	0.02	**0.24**	**0.39**	0.00	0.01	0.02	**0.07**	0.00	0.02
0.5	100	6	0.04	**0.14**	0.00	0.00	0.00	0.01	0.00	0.00	0.00	0.01	0.00	0.00	0.10	**0.24**	0.00	0.00	0.00	0.01	0.00	0.00
0.5	250	6	0.00	0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.07	0.00	0.00	0.00	0.00	0.00	0.00
0.5	500	6	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00
0.5	30	8	**0.35**	**0.51**	0.00	0.01	0.05	**0.14**	0.01	**0.06**	**0.06**	**0.15**	0.02	**0.07**	**0.46**	**0.56**	0.01	0.03	0.05	**0.13**	0.01	**0.06**
0.5	50	8	**0.23**	**0.40**	0.00	0.00	0.01	**0.07**	0.00	0.02	0.01	**0.07**	0.00	0.02	**0.38**	**0.50**	0.00	0.01	0.01	**0.06**	0.00	0.02
0.5	100	8	**0.09**	**0.23**	0.00	0.00	0.00	0.01	0.00	0.00	0.00	0.01	0.00	0.00	**0.22**	**0.38**	0.00	0.00	0.00	0.01	0.00	0.00
0.5	250	8	0.01	**0.06**	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.05	**0.17**	0.00	0.00	0.00	0.00	0.00	0.00
0.5	500	8	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.05	0.00	0.00	0.00	0.00	0.00	0.00

Table 10. Mean type I error rates across error covariance structures for ρ = .7 by sample size and series length

Conditions			**CS**				**ID**				**AR1**				**TP**				**UN**			
ρ	**N**	**Length**	**p1**	**p2**	**p3**	**p4**	**p1**	**p2**	**p3**	**p4**	**p1**	**p2**	**p3**	**p4**	**p1**	**p2**	**p3**	**p4**	**p1**	**p2**	**p3**	**p4**
0.7	30	3	**0.11**	**0.24**	0.01	0.04	**0.07**	**0.17**	0.02	**0.09**	**0.07**	**0.18**	0.02	**0.09**	**0.13**	**0.27**	0.01	**0.07**	**0.06**	**0.17**	0.02	**0.09**
0.7	50	3	0.04	**0.12**	0.00	0.01	0.02	**0.07**	0.00	0.03	0.03	**0.08**	0.01	0.04	**0.06**	**0.15**	0.00	0.02	0.02	**0.07**	0.00	0.03
0.7	100	3	0.00	0.03	0.00	0.00	0.00	0.01	0.00	0.00	0.01	0.02	0.01	0.01	0.01	0.05	0.00	0.00	0.00	0.01	0.00	0.00
0.7	250	3	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.7	500	3	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.7	30	4	**0.14**	**0.30**	0.00	0.03	0.05	**0.17**	0.02	**0.07**	**0.06**	**0.17**	0.02	**0.08**	**0.19**	**0.35**	0.01	0.05	0.05	**0.16**	0.02	**0.07**
0.7	50	4	0.07	**0.18**	0.00	0.01	0.02	**0.09**	0.00	0.03	0.03	**0.09**	0.01	0.03	**0.11**	**0.25**	0.00	**0.02**	0.02	**0.09**	0.00	0.03
0.7	100	4	0.01	**0.06**	0.00	0.00	0.00	0.02	0.00	0.00	0.01	0.02	0.01	0.01	0.02	**0.11**	0.00	0.00	0.00	0.02	0.00	0.00
0.7	250	4	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00
0.7	500	4	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.7	30	6	**0.25**	**0.43**	0.00	0.01	**0.06**	**0.18**	0.01	**0.08**	**0.06**	**0.18**	0.02	**0.08**	**0.34**	**0.50**	0.01	0.05	**0.06**	**0.16**	0.01	**0.07**
0.7	50	6	**0.14**	**0.30**	0.00	0.00	0.02	**0.08**	0.00	0.02	0.03	**0.09**	0.01	0.03	**0.25**	**0.40**	0.00	0.01	0.01	**0.08**	0.00	0.02
0.7	100	6	0.04	**0.14**	0.00	0.00	0.00	0.01	0.00	0.00	0.01	0.02	0.00	0.01	**0.11**	**0.25**	0.00	0.00	0.00	0.01	0.00	0.00
0.7	250	6	0.00	0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	**0.07**	0.00	0.00	0.00	0.00	0.00	0.00
0.7	500	6	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00
0.7	30	8	**0.37**	**0.52**	0.00	0.01	**0.06**	**0.18**	0.01	**0.07**	**0.07**	**0.18**	0.02	**0.07**	**0.48**	**0.57**	0.00	0.04	**0.06**	**0.15**	0.01	**0.07**
0.7	50	8	**0.23**	**0.41**	0.00	0.00	0.01	**0.09**	0.00	0.02	0.02	**0.08**	0.01	0.02	**0.38**	**0.50**	0.00	0.01	0.01	**0.08**	0.00	0.02
0.7	100	8	**0.10**	**0.24**	0.00	0.00	0.00	0.02	0.00	0.00	0.00	0.02	0.00	0.00	**0.23**	**0.37**	0.00	0.00	0.00	0.01	0.00	0.00
0.7	250	8	0.01	**0.06**	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	**0.06**	**0.17**	0.00	0.00	0.00	0.00	0.00	0.00
0.7	500	8	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	**0.06**	0.00	0.00	0.00	0.00	0.00	0.00

D. <u>Statistical power of detecting the AR(1) autocorrelation parameter</u>

Statistical power is represented by the proportion of Wald tests where the estimated $\boldsymbol{\rho}$ parameter was found to be statistically significant, where $p < .05$. The data presented in Figures 7 through 9 (see Appendix C for data) illustrate the main effects that magnitude of AR(1) $\boldsymbol{\rho}$, sample size, and series length have on the statistical power of detecting the AR(1) autocorrelation. In examining these figures, it is apparent that as magnitude of the AR(1) $\boldsymbol{\rho}$, sample size, and series length increase so does the statistical power to detect the AR(1) autocorrelation. The magnitude of the AR(1) $\boldsymbol{\rho}$ had the most prominent effect. The statistical power for detecting the AR(1) autocorrelation increases from .28, when $\boldsymbol{\rho} = .1$, .79, when $\boldsymbol{\rho} = .7$. The main effect of series length on statistical power is also prominent, with statistical power increasing from .34 to .86 as series length increases from 3 to 8. Changes in the sample size from 30 to 500 resulted in a comparatively smaller increase in statistical power, from .47 to .78.

Figures 10 through 12 (see Appendix D for data) illustrate the interaction effects that magnitude of AR(1) $\boldsymbol{\rho}$, sample size, and series length have on the statistical power of detecting the AR(1) autocorrelation. Caution is needed when interpreting these figures, as power is a probability, so the limit is 1.0 and lines must converge near 1.0. Figure 10 shows that there is little or no interaction between the magnitude of the AR(1) $\boldsymbol{\rho}$ and sample size. The lines are relatively parallel, with an intersection between the lines where sample size is 30 or 50 and AR (1) autocorrelation is .1. Figure 11 shows some interaction between sample size and series length. The lines for series length 3 and series length 4 intersect, while the lines for series lengths 6 and 8 remain parallel. Figure 12 shows little or no interaction between series length and magnitude of the AR(1) $\boldsymbol{\rho}$, with lines for AR(1) $\boldsymbol{\rho}$ .1 through .7 running parallel as a group.

Table 11 shows the statistical power of detecting the AR(1) autocorrelation across magnitude of the AR(1) $\boldsymbol{\rho}$, sample size, and series length. This table show data conditions that are necessary in order to detect the AR(1) autocorrelation. Results where statistical power is .90 or greater are bolded. The general finding is that in order to successfully detect the AR(1) autocorrelation larger samples with longer series lengths are needed, especially when magnitude of the AR(1) parameter is suspected to be small or moderate. For example, $\boldsymbol{\rho}$ is .1 there is only one data condition where statistical power is above .90, when sample size is 500 and series length is 8. In addition, there are no data conditions with a series length of 3 where statistical power is above .90. The highest statistical power obtained with a series length of 3 is .65, when the magnitude of $\boldsymbol{\rho}$ is .7 and sample size is 500.

Figure 7. Statistical power to detect the AR(1) autocorrelation by magnitude

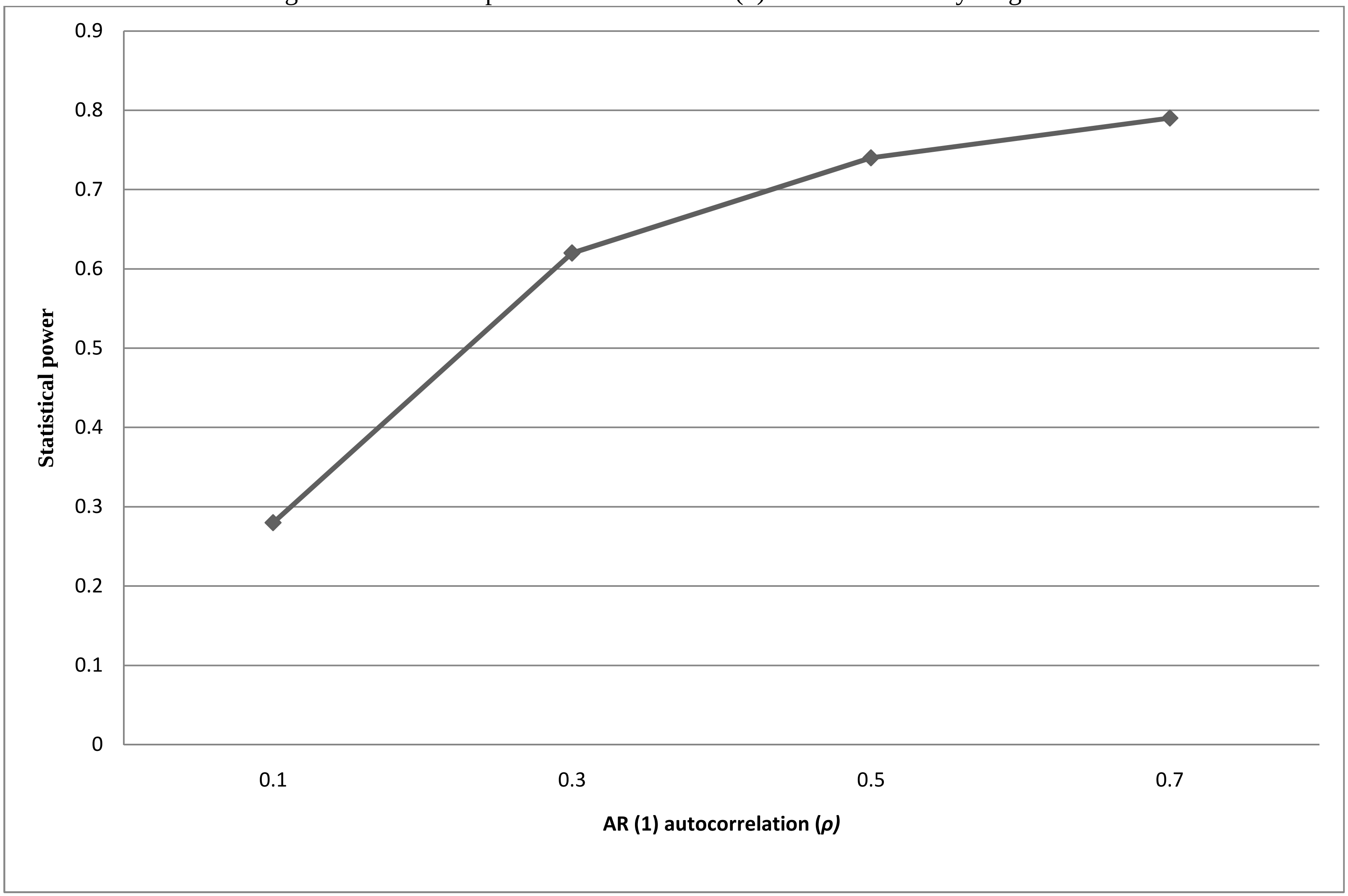


Figure 8. Statistical power to detect the AR(1) autocorrelation by sample size

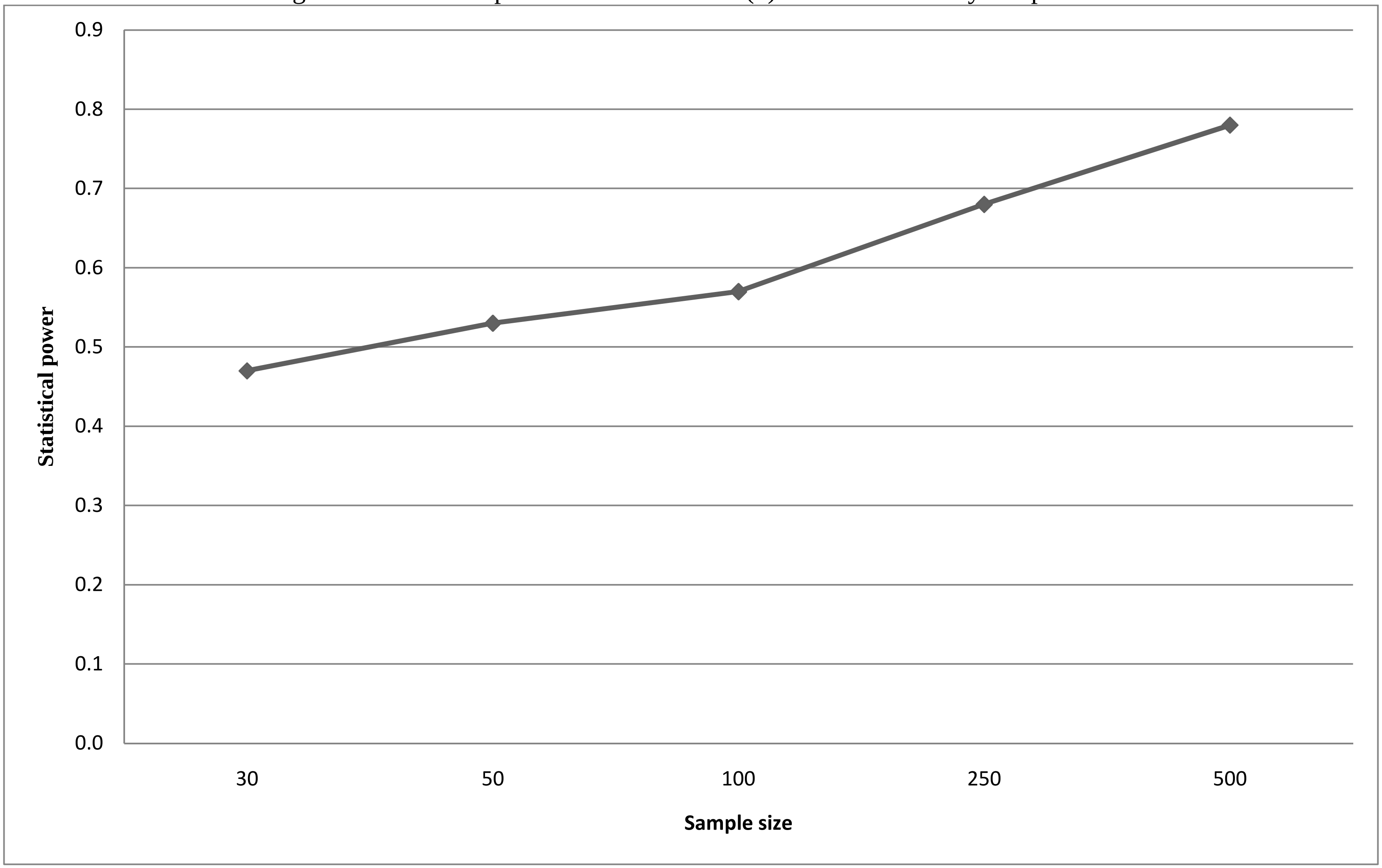


Figure 9. Statistical power to detect the AR(1) autocorrelation by series length

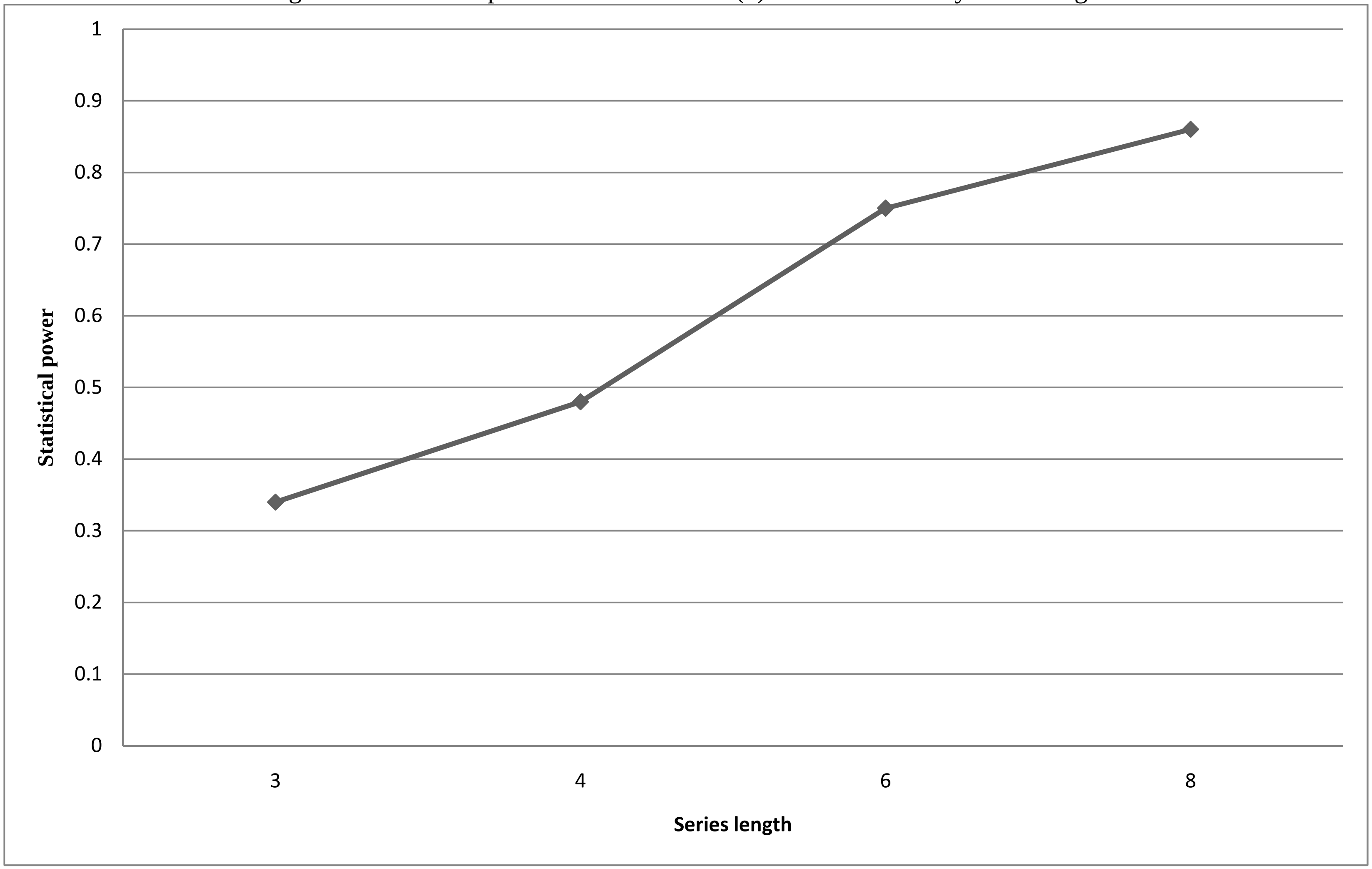


Figure 10. Effect of magnitude of AR(1) autocorrelation on statistical power and sample size

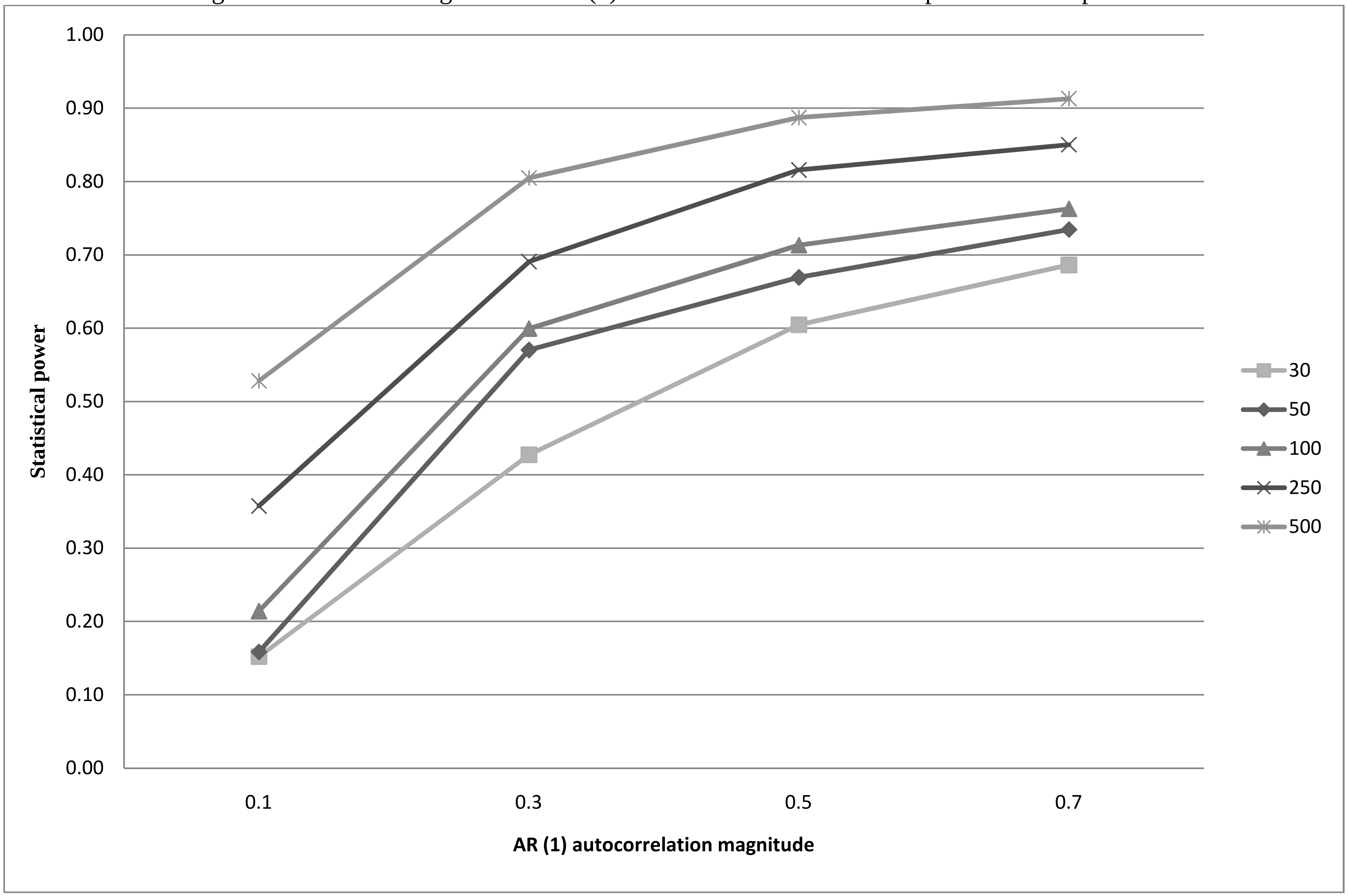


Figure 11. Effect of sample size on statistical power by series length

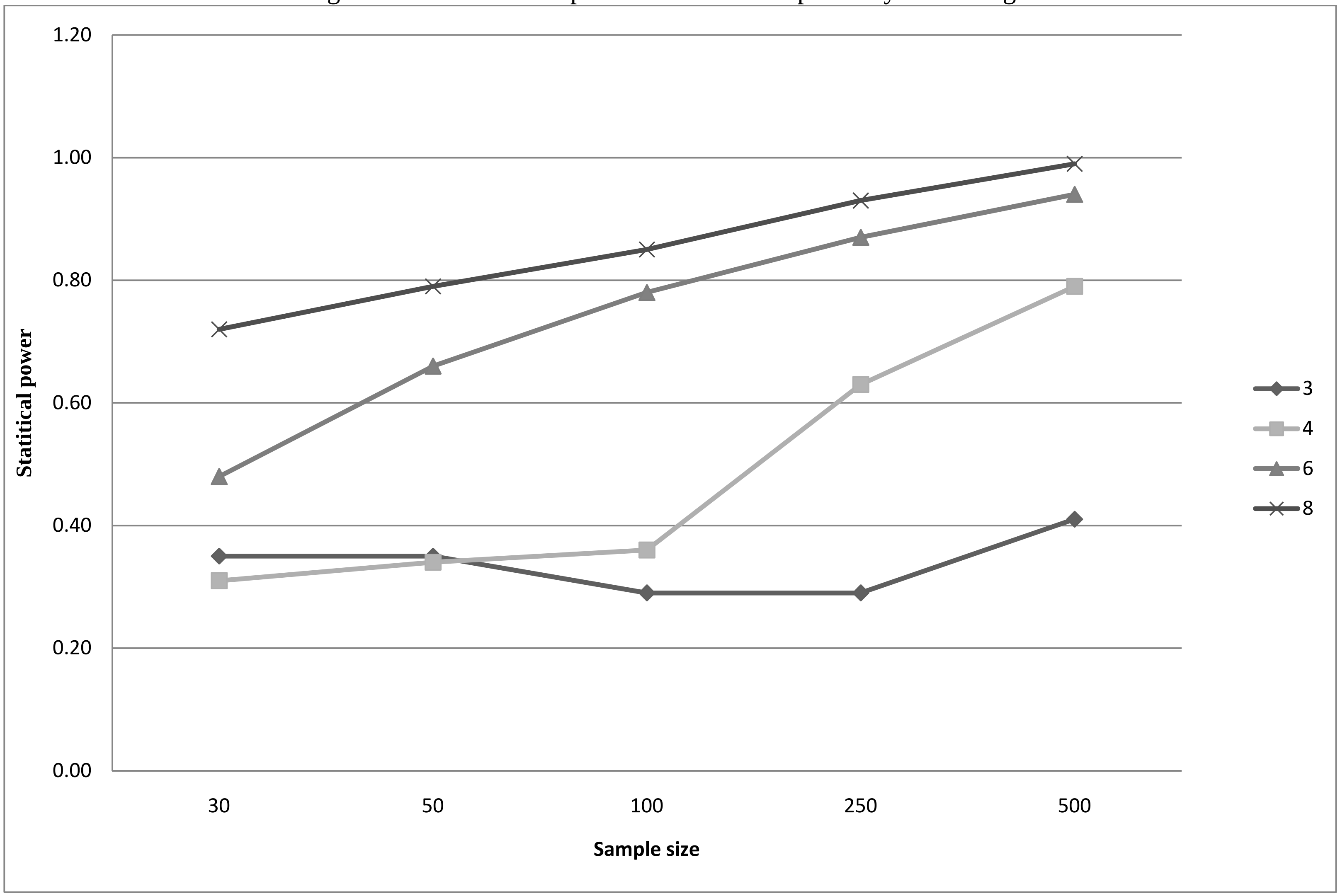


Figure 12. Effect of series length on statistical power by AR(1) autocorrelation

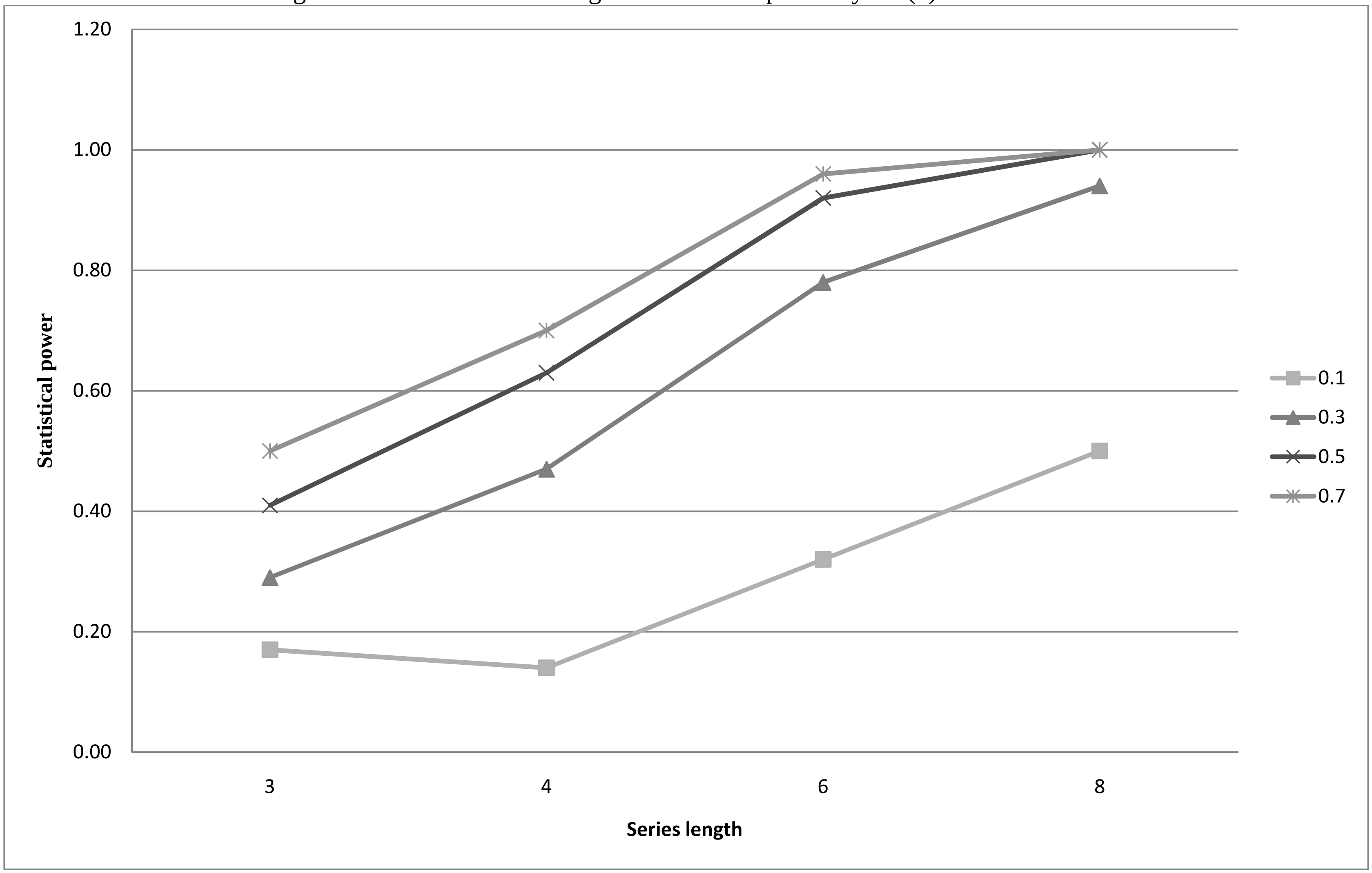


Table 11. Statistic power of detecting the AR(1) autocorrelation parameter across conditions

Conditions		Magnitude of AR(1)				
N	**Length**	**0**	**0.1**	**0.3**	**0.5**	**0.7**
30	3	0.20	0.23	0.33	0.37	0.48
30	4	0.12	0.15	0.25	0.39	0.45
30	6	0.05	0.07	0.37	0.69	0.82
30	8	0.05	0.16	0.76	**0.98**	**1.00**
50	3	0.15	0.22	0.32	0.40	0.47
50	4	0.08	0.11	0.39	0.36	0.49
50	6	0.05	0.09	0.64	**0.92**	**0.98**
50	8	0.05	0.22	**0.93**	**1.00**	**1.00**
100	3	0.12	0.15	0.25	0.35	0.43
100	4	0.08	0.08	0.25	0.51	0.62
100	6	0.05	0.21	**0.90**	**1.00**	**1.00**
100	8	0.06	0.42	**1.00**	**1.00**	**1.00**
250	3	0.04	0.11	0.22	0.37	0.45
250	4	0.07	0.12	0.54	**0.90**	**0.95**
250	6	0.04	0.48	**1.00**	**1.00**	**1.00**
250	8	0.06	0.72	**1.00**	**1.00**	**1.00**
500	3	0.04	0.11	0.32	0.55	0.65
500	4	0.04	0.24	**0.91**	**1.00**	**1.00**
500	6	0.06	0.78	**1.00**	**1.00**	**1.00**
500	8	0.05	**0.98**	**1.00**	**1.00**	**1.00**

Chapter V

Summary and Discussion

For this study a series of computer simulations were conducted to 1) examine the effectiveness of using different information criteria in identifying linear mixed models with an AR(1) error covariance structure, 2) to find out whether estimates of fixed effects are biased and the type I error cost when the error covariance structure is misspecified and finally, 3) to determine the conditions necessary to detect the autocorrelation (ρ) parameter with reasonably large power.

A. <u>Identifying the AR(1) error covariance structure using information criteria</u>

It was found that both the AIC and AICC, with very little difference between the two, outperformed the BIC and CAIC in identifying data with an AR(1) error covariance structure. However, it should be noted that both the AIC and AICC have a higher rate of false positives in identifying data with no autocorrelation as having an AR(1) error covariance structure when there is none, compared to the BIC and CAIC.

The factor that had the greatest influence on identification accuracy is series length, where identification accuracy increases as series length increases. Using the AIC, accuracy ranged from 16.9% with a series length of 3 to 90.6% with a series length of 8. The second most influential factor is magnitude of the AR(1) autocorrelation; when magnitude increases so does accuracy. Using the AIC, accuracy ranged from 37.8% with an AR(1) autocorrelation of .1 to 73.4% with an AR(1) $\boldsymbol{\rho}$ of .7. Sample size also affects identification accuracy. Using the AIC, accuracy ranged from 45.7% with a sample size of 30 to 78.9% with a sample size of 500. Two-way and three-way interactions among the factors are also present.

B. Impact of fitting different error covariance structures on estimated fixed effects

There was no observable bias in estimating the fixed estimates using any of the five error covariance structures. Mean estimated fixed effects (β_1, β_2, β_3, and β_4) varied between .97 and 1.03, very close to the true parameter value of 1.00. The magnitude of the AR(1) autocorrelation, sample size, and series length had no impact on the mean estimated fixed effects.

C. Impact of fitting error covariance structures on tests of fixed effects

Type I error rates for tests of fixed effects from data fitted with ID, AR(1), and UN error covariance structures are nearly identical across all conditions. For these three error covariance structures, once sample size is 100 or larger mean type I error rates for the estimated fixed parameters are below .05, across magnitudes of AR(1) error and series lengths. It is unexpected that misspecification of data with the ID error covariance structure did not inflate type I error, as expected based on Van Belle (2002), this finding warrants additional examination to whether it is due to statistical properties specific to the ID error covariance structure.

On the other hand, type I error rates obtained from data fitted with the CS and TP error covariance structures showed some interesting patterns. Mean type I error rates for p1 and p2 were inflated as series length increased, regardless of the magnitude of the AR(1) error and sample size. At the same time mean type I error rates for p3 and p4 decreases as series length increases.

While the effects of sample size and series length on type I error rates varied by error covariance structure, the effect of the magnitude of the AR(1) autocorrelation on type I error rates are consistent. It was found that as the magnitude of the AR(1) increases the instances where mean type I error rates are inflated above .05 also increases. This pattern holds across all five error covariance structures.

D. **Statistical power for detecting the AR(1) autocorrelation parameter**

It was found that magnitude of AR(1) autocorrelation, sample size, and series length have an effect on the statistical power of detecting the AR(1) autocorrelation: As each factor increases so does statistical power. The magnitude of AR(1) autocorrelation had the greatest impact on statistical power. Series length had the second largest impact on statistical power, followed by sample size. A graphical examination of two–way interactions found little or no interaction effects on statistical power among the three factors.

Overall, when the magnitude of the AR(1) is .1, small, it is very difficult to detect the autocorrelation parameter; both large sample size and long series length is needed. When series length is 3 or 4, the statistical power for detecting the autocorrelation parameter is less than .5. The same is true when sample size is 100 or below.

E. **Recommendations**

In considering which information criteria to use for data with autoregessive characteristics, AIC and AICC provide the highest accuracy in selecting the correct error covariance structure, keeping in mind that sample size, series length, and magnitude of the AR(1) autocorrelation play a role in accuracy. Accuracy decreases drastically when sample size is small, below 100 and when series length is short, 3 or 4 observations. The development of more accurate indices for small sample and short series data conditions, if possible, are needed. This then leads to the next question: What will happen to the estimated fixed effects and test of significance for these estimates when the wrong error covariance structure is selected? This study found that there is no impact on the estimated fixed effects for misspecifying the error covariance structure, which confirms the findings of a number of previous studies (Ferron et al., 2002 and Singer and Willet, 2003). On the other hand, the effect of misspecification on type I error rates is important. Mean type I error rates differ greatly when the error covariance structure for data with autocorrelation characteristic is misspecified as having an ID or UN structure, a common practice. However, if the error covariance structure is misspecified as either having a CS or TP structure then type I error rates will be inflated, particularly for p1 and p2, corresponding to the tests of significance for β_1 and β_2. These results should give caution to researchers not to carelessly under-specify or over-specify the error covariance structure when modeling.

Given that there is no practical type I error cost for specifying ID, AR(1), or UN error covariance structures, what are some additional considerations for choosing the optimal error covariance structure when modeling? Ideally, researchers should be making decisions about specifying the error covariance structure based on careful consideration of the nature of the phenomenon being examined or results from previous studies. It is unrealistic to assume that longitudinal data will have an ID error covariance structure, which assumes no relation between residuals of pair of observations once random effects have been modeled. When presuming an ID error covariance structure there is a potential loss of information about the true relation between residuals of repeated observations in the case of data with autoregressive characteristics, an estimate of the $\boldsymbol{\rho}$ parameter. Leaving the error variance structure as unstructured also results in a loss of information, not just a potential estimate of $\boldsymbol{\rho}$, but also the magnitude of the variation of random intercepts and slopes that can be used to calculate the pseudo R^2 (Singer and Willet, 2003).

However, if one is unsure of the true nature of the error covariance structure underlying the data or unable to specifying an autoregressive error covariance structure due to model constraints, it might be appropriate to use robust standard errors to avoid bias to the standard error estimates. Several statistical packages are able to compute robust standard errors, including HLM, SAS PROC MIXED and Stata. Semiparametric methods such as the partially linear model, which are primarily used in econometrics, can also be used to provide accurate approximations and adjusted standard errors estimates for models that have autoregressive errors.

It is recommended that more studies, especially when sample size is large and series length is greater than four repeated observation, to consider fitting error structures with autoregressive characteristics. This will provide the field with information about the nature of the phenomenon being studied and literature to guide future research on specifying an optimal error covariance structure to use for modeling.

F. Future Research

This study focused on data with autoregressive characteristics, specifically AR(1), without missing data, with a balanced design, no level-two predictors, and normally distributed error distributions. These are not typical conditions researchers encounter when working with “real” data. In order to examine the generalizability of the current findings, further examination across different types of models and conditions are needed. For example, models with multiple predictors across multiple levels, missing data, an unbalanced design, and nonnormality of errors.

In addition, data models with no autoregressive characteristics, or different autoregressive characteristics (i.e., ARMA or Toeplitz), can be generated and examined. Finally, further studies can be conducted on the specification of error covariance structures not used in this study, which can have different effects on parameter estimates and type I error rates.

Appendix A

Percentage of non-convergence datasets across conditions

Conditions		Autocorrelation Magnitude				
N	**Length**	**0**	**0.1**	**0.3**	**0.5**	**0.7**
30	3	14.9%	15.0%	12.6%	15.2%	12.4%
30	4	0.0%	0.0%	0.0%	0.0%	0.0%
30	6	0.0%	0.0%	0.0%	0.0%	0.0%
30	8	0.0%	0.0%	0.0%	0.0%	0.0%
50	3	9.5%	7.9%	8.6%	7.9%	7.1%
50	4	0.0%	0.0%	0.0%	0.0%	0.0%
50	6	0.0%	0.0%	0.0%	0.0%	0.0%
50	8	0.0%	0.0%	0.0%	0.0%	0.0%
100	3	3.7%	4.2%	2.1%	3.0%	2.3%
100	4	0.0%	0.0%	0.0%	0.0%	0.0%
100	6	0.0%	0.0%	0.0%	0.0%	0.0%
100	8	0.0%	0.0%	0.0%	0.0%	0.0%
250	3	0.3%	0.3%	0.1%	0.1%	0.2%
250	4	0.0%	0.0%	0.0%	0.0%	0.0%
250	6	0.0%	0.0%	0.0%	0.0%	0.0%
250	8	0.0%	0.0%	0.0%	0.0%	0.0%
500	3	0.0%	0.0%	0.0%	0.0%	0.0%
500	4	0.0%	0.0%	0.0%	0.0%	0.0%
500	6	0.0%	0.0%	0.0%	0.0%	0.0%
500	8	0.0%	0.0%	0.0%	0.0%	0.0%

Appendix B

Percentage of AR(1) models correctly identified by information criteria varying size of autocorrelation

Information Criteria	Autocorrelation			
	0.1	**0.3**	**0.5**	**0.7**
AIC	37.8	65.1	71.6	73.4
AICC	37.4	65.2	72.0	73.7
BIC	14.8	49.4	62.1	66
CAIC	12.4	46.7	59.7	64

Percentage of AR(1) models correctly identified by information criteria varying sample size

Information Criteria	Sample Size				
	30	**50**	**100**	**250**	500
AIC	45.7	52.9	61.0	71.2	78.9
AICC	46.1	52.5	61.1	71.5	78.9
BIC	32.7	39.6	45.2	57.7	66.2
CAIC	29.5	36.7	43.5	54.5	64.2

Percentage of AR(1) models correctly identified by information criteria varying series length

Information Criteria	Series Length			
	3	4	6	8
AIC	16.9	56.3	84.0	90.6
AICC	17.2	56.5	84.2	90.4
BIC	3.2	37.2	70.8	81.0
CAIC	2.2	31.6	68.2	79.7

Appendix C

Identification accuracy using AICC across covariance error structure for ***ρ*** = .0 by sample size and series length

Hurvich and Tsai's Information Criterion (AICC)								
Conditions			**CS**	**ID**	**AR1**	**TP**	**UN**
ρ	**N**	**Length**	**% identified as best fit**	**% identified as best fit**	**% identified as best fit**	**% identified as best fit**	**% identified as best fit**
0	30	3	1.9	81.6	2.8	3.4	10.3
0	50	3	0.0	84.9	4.9	0.1	10.1
0	100	3	0.0	83.1	6.6	0.0	10.3
0	250	3	0.0	79.9	13.1	0.0	7.6
0	500	3	0.0	78.7	12.8	0.0	8.5
0	30	4	0.0	84.6	12.4	0.1	2.9
0	50	4	0.0	83.5	11.8	0.0	4.7
0	100	4	0.0	79.8	15.2	0.0	5.0
0	250	4	0.0	80.5	16.4	0.0	3.1
0	500	4	0.0	81.8	14.1	0.0	4.1
0	30	6	0.0	84.8	14.7	0.0	0.5
0	50	6	0.0	83.9	16.1	0.0	0.0
0	100	6	0.0	83.0	16.4	0.0	0.0
0	250	6	0.0	84.5	15.2	0.0	0.3
0	500	6	0.0	84.7	15.0	0.0	0.3
0	30	8	0.0	85.7	14.3	0.0	0.0
0	50	8	0.0	86.0	14.0	0.0	0.0
0	100	8	0.0	84.2	15.6	0.0	0.2
0	250	8	0.0	85.5	14.4	0.0	0.1
0	500	8	0.0	85.6	14.3	0.0	0.1
Total			**0.1**	**83.3**	**13.0**	**0.2**	**3.4**

Identification accuracy using AICC across covariance error structure for $\boldsymbol{\rho}$ = .1 by sample size and series length

Hurvich and Tsai's Information Criterion (AICC)							
Conditions			**CS**	**ID**	**AR1**	**TP**	**UN**
$\boldsymbol{\rho}$	**N**	**Length**	**% identified as best fit**	**% identified as best fit**	**% identified as best fit**	**% identified as best fit**	**% identified as best fit**
0.1	30	3	0.6	83.4	2.1	3.8	10.1
0.1	50	3	0.0	85.8	4.0	0.4	9.8
0.1	100	3	0.0	79.2	9.4	0.0	11.4
0.1	250	3	0.0	75.2	14.6	0.0	10.2
0.1	500	3	0.0	72.1	17.8	0.0	10.1
0.1	30	4	0.0	82.3	13.1	0.0	4.6
0.1	50	4	0.0	78.4	17.0	0.0	4.6
0.1	100	4	0.0	75.3	19.3	0.0	5.4
0.1	250	4	0.0	64.5	29.2	0.0	6.3
0.1	500	4	0.0	50.5	43.9	0.0	5.6
0.1	30	6	0.0	76.1	23.4	0.0	0.5
0.1	50	6	0.0	69.5	30.1	0.0	0.4
0.1	100	6	0.0	57.5	41.9	0.0	0.6
0.1	250	6	0.0	28.6	70.5	0.0	0.9
0.1	500	6	0.0	8.4	91.1	0.0	0.5
0.1	30	8	0.0	65.9	34.1	0.0	0.0
0.1	50	8	0.0	63.8	36.1	0.0	0.1
0.1	100	8	0.0	35.7	64.0	0.0	0.0
0.1	250	8	0.0	13.7	86.3	0.0	0.0
0.1	500	8	0.0	0.3	99.7	0.0	0.0
Total			**< 0.1**	**58.3**	**37.4**	**0.2**	**4.1**

Identification accuracy using AICC across covariance error structure for ρ = .3 vary sample size and series length

Hurvich and Tsai's Information Criterion (AICC)							
Conditions			**CS**	**ID**	**AR1**	**TP**	**UN**
ρ	**N**	**Length**	**% identified as best fit**	**% identified as best fit**	**% identified as best fit**	**% identified as best fit**	**% identified as best fit**
0.3	30	3	0.3	82.6	1.2	4.8	11.1
0.3	50	3	0.0	80.8	4.1	0.8	14.3
0.3	100	3	0.0	73.2	13.2	0.0	13.6
0.3	250	3	0.0	65.8	25.1	0.0	9.1
0.3	500	3	0.0	53.4	35.8	0.0	10.8
0.3	30	4	0.0	67.8	26.2	0.3	5.7
0.3	50	4	0.0	41.6	51.6	0.0	6.8
0.3	100	4	0.0	47.6	47.2	0.0	5.2
0.3	250	4	0.0	18.5	73.8	0.0	7.7
0.3	500	4	0.0	2.0	89.4	0.0	8.6
0.3	30	6	0.0	30.8	69.0	0.0	0.2
0.3	50	6	0.0	14.9	84.0	0.0	1.1
0.3	100	6	0.0	2.4	97.1	0.0	0.5
0.3	250	6	0.0	0.1	98.9	0.0	1.0
0.3	500	6	0.0	0.0	98.7	0.0	1.3
0.3	30	8	0.0	9.8	90.2	0.0	0.0
0.3	50	8	0.0	1.8	98.1	0.0	0.1
0.3	100	8	0.0	0.1	99.9	0.0	0.0
0.3	250	8	0.0	0.0	100.0	0.0	0.0
0.3	500	8	0.0	0.1	99.9	0.0	0.0
Total			**< 0.1**	**29.7**	**65.2**	**0.295**	**4.9**

Identification accuracy using AICC across covariance error structure for ρ = .5 by sample size and series length

Hurvich and Tsai's Information Criterion (AICC)							
Conditions			**CS**	**ID**	**AR1**	**TP**	**UN**
ρ	**N**	**Length**	**% identified as best fit**	**% identified as best fit**	**% identified as best fit**	**% identified as best fit**	**% identified as best fit**
0.5	30	3	0.0	83.6	5.2	5.2	10.8
0.5	50	3	0.0	80.5	4.5	0.3	14.7
0.5	100	3	0.0	68.8	16.2	0.0	15.0
0.5	250	3	0.0	53.0	33.0	0.0	14.0
0.5	500	3	0.0	33.2	50.9	0.0	15.9
0.5	30	4	0.0	55.7	38.2	0.1	6.0
0.5	50	4	0.0	41.6	51.8	0.0	6.6
0.5	100	4	0.0	20.6	72.3	0.0	7.1
0.5	250	4	0.0	2.0	91.3	0.0	6.7
0.5	500	4	0.0	0.0	92.8	0.0	7.2
0.5	30	6	0.0	9.3	90.4	0.0	0.3
0.5	50	6	0.0	1.5	97.7	0.0	0.8
0.5	100	6	0.0	0.0	99.3	0.0	0.7
0.5	250	6	0.0	0.0	99.0	0.0	1.0
0.5	500	6	0.0	0.0	98.7	0.0	1.3
0.5	30	8	0.0	0.3	99.7	0.0	0.3
0.5	50	8	0.0	0.0	100.0	0.0	0.0
0.5	100	8	0.0	0.0	99.9	0.0	0.1
0.5	250	8	0.0	0.0	99.9	0.0	0.1
0.5	500	8	0.0	0.0	99.9	0.0	0.1
Total			**0.0**	**22.5**	**72.0**	**0.3**	**5.4**

Identification accuracy using AICC across covariance error structure for ρ = .7 by sample size and series length

Hurvich and Tsai's Information Criterion (AICC)							
Conditions			**CS**	**ID**	**AR1**	**TP**	**UN**
ρ	**N**	**Length**	**% identified as best fit**	**% identified as best fit**	**% identified as best fit**	**% identified as best fit**	**% identified as best fit**
0.7	30	3	0.0	82.0	0.6	3.2	14.2
0.7	50	3	0.0	78.6	3.1	0.2	18.1
0.7	100	3	0.0	66.7	15.7	0.0	17.6
0.7	250	3	0.0	49.3	32.6	0.0	18.1
0.7	500	3	0.0	27.3	54.1	0.0	18.6
0.7	30	4	0.0	45.5	47.9	0.0	6.6
0.7	50	4	0.0	32.7	59.9	0.0	7.4
0.7	100	4	0.0	10.1	83.1	0.0	6.8
0.7	250	4	0.0	0.3	91.0	0.0	8.7
0.7	500	4	0.0	0.1	90.9	0.0	9.0
0.7	30	6	0.0	2.4	97.1	0.0	0.5
0.7	50	6	0.0	0.3	98.7	0.0	1.0
0.7	100	6	0.0	0.0	99.8	0.0	0.2
0.7	250	6	0.0	0.0	99.4	0.0	0.6
0.7	500	6	0.0	0.0	99.6	0.0	0.4
0.7	30	8	0.0	0.0	99.9	0.0	0.1
0.7	50	8	0.0	0.0	100.0	0.0	0.0
0.7	100	8	0.0	0.0	100.0	0.0	0.0
0.7	250	8	0.0	0.0	100.0	0.0	0.0
0.7	500	8	0.0	0.0	99.9	0.0	0.1
Total			**0.0**	**19.8**	**73.7**	**0.2**	**6.4**

Identification accuracy using BIC across covariance error structure for ρ = .0 by sample size and series length

BIC (Schwarz's Bayesian Criterion)							
Conditions			**CS**	**ID**	**AR1**	**TP**	**UN**
ρ	**N**	**Length**	**% identified as best fit**	**% identified as best fit**	**% identified as best fit**	**% identified as best fit**	**% identified as best fit**
0	30	3	8.2	82.1	0.4	8.3	1.0
0	50	3	0.6	97.7	0.2	1.2	0.3
0	100	3	0.0	99.2	0.6	0.0	0.2
0	250	3	0.0	99.1	0.6	0.0	0.3
0	500	3	0.0	99.4	0.6	0.0	0.0
0	30	4	0.1	98.3	1.5	0.1	0.0
0	50	4	0.0	98.8	1.2	0.0	0.0
0	100	4	0.0	98.4	1.6	0.0	0.0
0	250	4	0.0	98.5	1.5	0.0	0.0
0	500	4	0.0	99.5	0.5	0.0	0.0
0	30	6	0.0	98.3	1.7	0.0	0.0
0	50	6	0.0	98.5	1.5	0.0	0.0
0	100	6	0.0	98.7	1.3	0.0	0.0
0	250	6	0.0	99.6	0.4	0.0	0.0
0	500	6	0.0	99.4	0.6	0.0	0.0
0	30	8	0.0	98.9	1.1	0.0	0.0
0	50	8	0.0	98.5	1.5	0.0	0.0
0	100	8	0.0	98.8	1.2	0.0	0.0
0	250	8	0.0	99.0	1.0	0.0	0.0
0	500	8	0.0	99.8	0.2	0.0	0.0
Total			**0.4**	**98.0**	**1.0**	**0.5**	**0.1**

Identification accuracy using BIC across covariance error structure for $\boldsymbol{\rho}$ = .1 by sample size and series length

BIC (Schwarz's Bayesian Criterion)							
Conditions			**CS**	**ID**	**AR1**	**TP**	**UN**
ρ	**N**	**Length**	**% identified as best fit**	**% identified as best fit**	**% identified as best fit**	**% identified as best fit**	**% identified as best fit**
0.1	30	3	4.4	84.8	0.7	8.8	1.3
0.1	50	3	0.7	96.0	0.1	2.7	0.5
0.1	100	3	0.0	98.9	0.4	0.3	0.4
0.1	250	3	0.0	99.4	0.5	0.0	0.1
0.1	500	3	0.0	99.0	1.0	0.0	0.0
0.1	30	4	0.0	98.2	1.8	0.0	0.0
0.1	50	4	0.0	98.3	1.7	0.0	0.0
0.1	100	4	0.0	96.5	3.5	0.0	0.0
0.1	250	4	0.0	95.6	4.4	0.0	0.0
0.1	500	4	0.0	92.7	7.3	0.0	0.0
0.1	30	6	0.0	94.4	5.6	0.0	0.0
0.1	50	6	0.0	93.3	6.7	0.0	0.0
0.1	100	6	0.0	89.7	10.3	0.0	0.0
0.1	250	6	0.0	77.8	22.2	0.0	0.0
0.1	500	6	0.0	51.1	48.9	0.0	0.0
0.1	30	8	0.0	90.0	10.0	0.0	0.0
0.1	50	8	0.0	88.1	11.9	0.0	0.0
0.1	100	8	0.0	79.5	20.5	0.0	0.0
0.1	250	8	0.0	48.7	51.3	0.0	0.0
0.1	500	8	0.0	13.0	87.0	0.0	0.0
Total			**0.3**	**84.3**	**14.8**	**0.6**	**0.1**

Identification accuracy using BIC across covariance error structure for ρ = .3 vary sample size and series length

BIC (Schwarz's Bayesian Criterion)								
Conditions			**CS**	**ID**	**AR1**	**TP**	**UN**
ρ	**N**	**Length**	**% identified as best fit**	**% identified as best fit**	**% identified as best fit**	**% identified as best fit**	**% identified as best fit**
0.3	30	3	1.2	84.4	0.2	12.9	1.3
0.3	50	3	0.1	96.1	0.2	2.5	1.1
0.3	100	3	0.0	99.1	0.4	0.0	0.5
0.3	250	3	0.0	98.0	2.0	0.0	0.0
0.3	500	3	0.0	93.8	6.0	0.0	0.2
0.3	30	4	0.1	93.9	5.4	0.6	0.0
0.3	50	4	0.0	74.8	25.2	0.0	0.0
0.3	100	4	0.0	83.1	16.9	0.0	0.0
0.3	250	4	0.0	60.9	39.1	0.0	0.0
0.3	500	4	0.0	25.6	74.4	0.0	0.0
0.3	30	6	0.0	61.8	38.2	0.0	0.0
0.3	50	6	0.0	44.4	55.6	0.0	0.0
0.3	100	6	0.0	19.5	80.5	0.0	0.0
0.3	250	6	0.0	0.3	99.7	0.0	0.0
0.3	500	6	0.0	0.0	100.0	0.0	0.0
0.3	30	8	0.0	31.9	68.1	0.0	0.0
0.3	50	8	0.0	12.9	87.1	0.0	0.0
0.3	100	8	0.0	0.5	88.5	0.0	0.0
0.3	250	8	0.0	0.0	100.0	0.0	0.0
0.3	500	8	0.0	0.0	100.0	0.0	0.0
Total			**0.1**	**49.1**	**49.4**	**0.8**	**0.2**

Identification accuracy using BIC across covariance error structure for ρ = .5 by sample size and series length

BIC (Schwarz's Bayesian Criterion)							
Conditions			**CS**	**ID**	**AR1**	**TP**	**UN**
ρ	**N**	**Length**	**% identified as best fit**	**% identified as best fit**	**% identified as best fit**	**% identified as best fit**	**% identified as best fit**
0.5	30	3	0.0	88.8	0.4	9.6	1.2
0.5	50	3	0.0	95.5	0.0	3.1	1.4
0.5	100	3	0.0	98.6	0.5	0.2	0.7
0.5	250	3	0.0	92.8	6.7	0.0	0.5
0.5	500	3	0.0	82.9	16.5	0.0	0.6
0.5	30	4	0.0	84.3	15.3	0.3	0.1
0.5	50	4	0.0	77.7	22.3	0.0	0.0
0.5	100	4	0.0	56.9	43.1	0.0	0.0
0.5	250	4	0.0	19.2	80.8	0.0	0.0
0.5	500	4	0.0	1.5	98.5	0.0	0.0
0.5	30	6	0.0	27.5	72.5	0.0	0.0
0.5	50	6	0.0	10.9	89.1	0.0	0.0
0.5	100	6	0.0	0.1	99.9	0.0	0.0
0.5	250	6	0.0	0.0	100.0	0.0	0.0
0.5	500	6	0.0	0.0	100.0	0.0	0.0
0.5	30	8	0.0	3.2	96.8	0.0	0.0
0.5	50	8	0.0	0.1	99.9	0.0	0.0
0.5	100	8	0.0	0.0	100.0	0.0	0.0
0.5	250	8	0.0	0.0	100.0	0.0	0.0
0.5	500	8	0.0	0.0	100.0	0.0	0.0
Total			**0.0**	**37.0**	**62.1**	**0.7**	**0.2**

Identification accuracy using BIC across covariance error structure for ρ = .7 by sample size and series length

BIC (Schwarz's Bayesian Criterion)							
Conditions			**CS**	**ID**	**AR1**	**TP**	**UN**
ρ	**N**	**Length**	**% identified as best fit**	**% identified as best fit**	**% identified as best fit**	**% identified as best fit**	**% identified as best fit**
0.7	30	3	0.0	88.7	0.6	8.9	1.8
0.7	50	3	0.0	97.2	1.8	1.8	0.9
0.7	100	3	0.0	98.8	0.0	0.0	1.2
0.7	250	3	0.0	91.6	7.1	0.0	1.3
0.7	500	3	0.0	79.5	19.8	0.0	0.7
0.7	30	4	0.0	79.6	20.0	0.3	0.1
0.7	50	4	0.0	66.6	33.4	0.0	0.0
0.7	100	4	0.0	41.6	58.4	0.0	0.0
0.7	250	4	0.0	6.5	93.5	0.0	0.0
0.7	500	4	0.0	0.5	99.5	0.0	0.0
0.7	30	6	0.0	11.1	88.9	0.0	0.0
0.7	50	6	0.0	1.9	98.1	0.0	0.0
0.7	100	6	0.0	0.0	100.0	0.0	0.0
0.7	250	6	0.0	0.0	100.0	0.0	0.0
0.7	500	6	0.0	0.0	100.0	0.0	0.0
0.7	30	8	0.0	0.2	99.8	0.0	0.0
0.7	50	8	0.0	0.0	100.0	0.0	0.0
0.7	100	8	0.0	0.0	100.0	0.0	0.0
0.7	250	8	0.0	0.0	100.0	0.0	0.0
0.7	500	8	0.0	0.0	100.0	0.0	0.0
Total			**0.0**	**33.2**	**66.0**	**0.6**	**0.3**

Identification accuracy using CAIC across covariance error structure for ρ = .0 by sample size and series length

CAIC (Bozdogan's Criterion)							
Conditions			**CS**	**ID**	**AR1**	**TP**	**UN**
ρ	**N**	**Length**	**% identified as best fit**	**% identified as best fit**	**% identified as best fit**	**% identified as best fit**	**% identified as best fit**
0	30	3	13.7	76.5	0.2	9.3	0.3
0	50	3	1.3	96.7	0.1	1.8	0.1
0	100	3	0.0	99.6	0.2	0.0	0.2
0	250	3	0.0	99.8	0.2	0.0	0.0
0	500	3	0.0	99.7	0.3	0.0	0.0
0	30	4	0.2	98.5	1.2	0.1	0.0
0	50	4	0.0	99.5	0.5	0.0	0.0
0	100	4	0.0	99.1	0.9	0.0	0.0
0	250	4	0.0	99.2	0.8	0.0	0.0
0	500	4	0.0	99.8	0.2	0.0	0.0
0	30	6	0.0	99.4	0.6	0.0	0.0
0	50	6	0.0	99.0	1.0	0.0	0.0
0	100	6	0.0	99.4	0.6	0.0	0.0
0	250	6	0.0	99.8	0.2	0.0	0.0
0	500	6	0.0	99.7	0.3	0.0	0.0
0	30	8	0.0	99.2	0.8	0.0	0.0
0	50	8	0.0	99.4	0.6	0.0	0.0
0	100	8	0.0	99.4	0.6	0.0	0.0
0	250	8	0.0	99.6	0.4	0.0	0.0
0	500	8	0.0	99.9	0.1	0.0	0.0
Total			**0.8**	**93.6**	**0.5**	**0.6**	**0.0**

Identification accuracy using CAIC across covariance error structure for ρ = .1 by sample size and series length

CAIC (Bozdogan's Criterion)								
Conditions			**CS**	**ID**	**AR1**	**TP**	**UN**
ρ	**N**	**Length**	**% identified as best fit**	**% identified as best fit**	**% identified as best fit**	**% identified as best fit**	**% identified as best fit**
0.1	30	3	8.1	80.6	0.5	10.3	0.5
0.1	50	3	0.7	96.0	0.1	2.7	0.5
0.1	100	3	0.0	99.4	0.3	0.3	0.0
0.1	250	3	0.0	99.8	0.1	0.0	0.1
0.1	500	3	0.0	99.5	0.5	0.0	0.0
0.1	30	4	0.0	99.3	0.7	0.0	0.0
0.1	50	4	0.0	98.9	1.1	0.0	0.0
0.1	100	4	0.0	98.2	1.8	0.0	0.0
0.1	250	4	0.0	96.9	3.1	0.0	0.0
0.1	500	4	0.0	95.2	4.8	0.0	0.0
0.1	30	6	0.0	96.6	3.4	0.0	0.0
0.1	50	6	0.0	95.5	4.5	0.0	0.0
0.1	100	6	0.0	92.5	7.5	0.0	0.0
0.1	250	6	0.0	82.5	17.5	0.0	0.0
0.1	500	6	0.0	56.4	43.6	0.0	0.0
0.1	30	8	0.0	93.1	6.9	0.0	0.0
0.1	50	8	0.0	91.6	8.4	0.0	0.0
0.1	100	8	0.0	84.4	15.6	0.0	0.0
0.1	250	8	0.0	56.2	43.8	0.0	0.0
0.1	500	8	0.0	17.1	82.9	0.0	0.0
Total			**0.4**	**86.5**	**12.4**	**0.7**	**0.1**

Identification accuracy using CAIC across covariance error structure for ρ = .3 vary sample size and series length

CAIC (Bozdogan's Criterion)							
Conditions			CS	ID	AR1	TP	UN
ρ	N	Length	% identified as best fit	% identified as best fit	% identified as best fit	% identified as best fit	% identified as best fit
0.3	30	3	2.2	80.5	0.2	16.7	0.4
0.3	50	3	0.1	95.8	0.2	3.3	0.6
0.3	100	3	0.0	99.4	0.3	0.1	0.2
0.3	250	3	0.0	98.8	1.2	0.0	0.0
0.3	500	3	0.0	96.0	3.9	0.0	0.1
0.3	30	4	0.1	96.8	2.4	0.7	0.0
0.3	50	4	0.0	82.4	17.6	0.0	0.0
0.3	100	4	0.0	88.8	11.2	0.0	0.0
0.3	250	4	0.0	67.7	32.3	0.0	0.0
0.3	500	4	0.0	32.2	67.8	0.0	0.0
0.3	30	6	0.0	69.0	31.0	0.0	0.0
0.3	50	6	0.0	50.6	49.4	0.0	0.0
0.3	100	6	0.0	25.5	74.5	0.0	0.0
0.3	250	6	0.0	0.5	99.5	0.0	0.0
0.3	500	6	0.0	0.0	100.0	0.0	0.0
0.3	30	8	0.0	39.7	60.3	0.0	0.0
0.3	50	8	0.0	17.3	82.7	0.0	0.0
0.3	100	8	0.0	0.7	99.3	0.0	0.0
0.3	250	8	0.0	0.0	100.0	0.0	0.0
0.3	500	8	0.0	0.0	100.0	0.0	0.0
Total			**0.1**	**52.1**	**46.7**	**1.0**	**0.1**

Identification accuracy using CAIC across covariance error structure for ρ = .5 by sample size and series length

CAIC (Bozdogan's Criterion)								
Conditions			**CS**	**ID**	**AR1**	**TP**	**UN**
ρ	**N**	**Length**	**% identified as best fit**	**% identified as best fit**	**% identified as best fit**	**% identified as best fit**	**% identified as best fit**
0.5	30	3	0.4	87.2	0.4	11.6	0.4
0.5	50	3	0.0	94.9	0.0	4.7	0.4
0.5	100	3	0.0	98.6	0.1	0.4	0.3
0.5	250	3	0.0	96.1	3.7	0.0	0.2
0.5	500	3	0.0	87.7	12.0	0.0	0.3
0.5	30	4	0.0	91.3	8.3	0.4	0.0
0.5	50	4	0.0	84.4	15.6	0.0	0.0
0.5	100	4	0.0	64.0	36.0	0.0	0.0
0.5	250	4	0.0	25.7	74.3	0.0	0.0
0.5	500	4	0.0	2.3	97.7	0.0	0.0
0.5	30	6	0.0	35.1	64.9	0.0	0.0
0.5	50	6	0.0	14.6	85.4	0.0	0.0
0.5	100	6	0.0	0.1	99.9	0.0	0.0
0.5	250	6	0.0	0.0	100.0	0.0	0.0
0.5	500	6	0.0	0.0	100.0	0.0	0.0
0.5	30	8	0.0	4.6	95.4	0.0	0.0
0.5	50	8	0.0	0.1	99.9	0.0	0.0
0.5	100	8	0.0	0.0	100.0	0.0	0.0
0.5	250	8	0.0	0.0	100.0	0.0	0.0
0.5	500	8	0.0	0.0	100.0	0.0	0.0
Total			**<0.1**	**39.3**	**59.7**	**0.9**	**0.1**

Identification accuracy using CAIC across covariance error structure for $\boldsymbol{\rho}$ = .7 by sample size and series length

CAIC (Bozdogan's Criterion)							
Conditions			**CS**	**ID**	**AR1**	**TP**	**UN**
$\boldsymbol{\rho}$	**N**	**Length**	**% identified as best fit**	**% identified as best fit**	**% identified as best fit**	**% identified as best fit**	**% identified as best fit**
0.7	30	3	0.0	87.1	0.6	11.9	0.4
0.7	50	3	0.0	96.8	0.1	2.8	0.3
0.7	100	3	0.0	99.2	0.0	0.2	0.6
0.7	250	3	0.0	94.5	4.8	0.0	0.7
0.7	500	3	0.0	84.8	14.9	0.0	0.3
0.7	30	4	0.0	87.5	11.7	0.8	0.0
0.7	50	4	0.0	74.9	25.1	0.0	0.0
0.7	100	4	0.0	50.7	49.3	0.0	0.0
0.7	250	4	0.0	8.6	91.4	0.0	0.0
0.7	500	4	0.0	0.5	99.5	0.0	0.0
0.7	30	6	0.0	14.4	85.6	0.0	0.0
0.7	50	6	0.0	2.7	97.3	0.0	0.0
0.7	100	6	0.0	0.0	100.0	0.0	0.0
0.7	250	6	0.0	0.0	100.0	0.0	0.0
0.7	500	6	0.0	0.0	100.0	0.0	0.0
0.7	30	8	0.0	0.4	99.6	0.0	0.0
0.7	50	8	0.0	0.0	100.0	0.0	0.0
0.7	100	8	0.0	0.0	100.0	0.0	0.0
0.7	250	8	0.0	0.0	100.0	0.0	0.0
0.7	500	8	0.0	0.0	100.0	0.0	0.0
Total			**0.0**	**18.8**	**64.0**	**0.8**	**0.1**

Appendix D

Statistical power of detecting the AR(1) autocorrelation parameter by size of the AR(1) autocorrelation

	Size of Autocorrelation				
	0	0.1	0.3	0.5	0.7
Statistical Power	0.07	0.28	0.62	0.74	0.79

Statistical power of detecting the AR(1) autocorrelation parameter by sample size

	Sample Size				
	30	50	100	250	500
Statistical Power	0.40	0.44	0.47	0.55	0.64

Statistical power of detecting the AR(1) autocorrelation parameter by series length

	Series Length			
	3	4	6	8
Statistical Power	0.29	0.4	0.61	0.7

Appendix E

Statistical power of detecting the AR(1) autocorrelation parameter by size of the AR(1) autocorrelation and sample size

	Size of Autocorrelation				
Sample Size	0	0.1	0.3	0.5	0.7
30	0.11	0.15	0.43	0.60	0.69
50	0.09	0.16	0.57	0.67	0.73
100	0.08	0.21	0.60	0.71	0.76
250	0.05	0.36	0.69	0.82	0.85
500	0.05	0.53	0.81	0.89	0.91

Statistical power of detecting the AR(1) autocorrelation by sample size and series length

	Sample Size				
Series Length	30	50	100	250	500
3	0.32	0.31	0.26	0.24	0.33
4	0.27	0.29	0.31	0.51	0.64
6	0.40	0.54	0.63	0.70	0.77
8	0.59	0.64	0.70	0.76	0.81

Statistical power of detecting the AR(1) autocorrelation by series length and size of the AR(1) autocorrelation

Size of AR(1)	Series Length			
Autocorrelation	3	4	6	8
0	0.11	0.08	0.05	0.05
0.1	0.17	0.14	0.32	0.50
0.3	0.29	0.47	0.78	0.94
0.5	0.41	0.63	0.92	1.00
0.7	0.50	0.70	0.96	1.00

Appendix F

```
***Marco to generate dataset***.
***OMS used to gather parameter estimates and significance test***.
***Model: y=b0 + b1+ e1, where b1 is a bivariate variable***.
***betas and errors are normal with mean = 0 and SD = 1***.
***Data generated is transposed variables to cases***.
***Mixed models with covariance structures: CS, ID, TP, UN, AR1***.
***This example is used to simulate 100 samples where AR(1) = .3 and n = 100***.

OMS
 /SELECT TABLES
 /IF COMMANDS = ["Mixed"]
    SUBTYPES = ["Model Dimension" "Parameter Estimates" "Information Criteria" ]
 /DESTINATION FORMAT = SAV OUTFILE = "C:\Documents and Settings\jjung\My
Documents\Dissertation\mixedoutput.sav"
 /COLUMNS SEQUENCE = [RALL CALL LALL].

DEFINE Linearmixed_bootstrap (samples=!TOKENS (1))
!Do  !Other=1 !To !samples
Set seed random.
input program.
 loop #i=1 to 100.
compute b0=rv.normal(1,1).
compute b1=rv.normal(1,1).
compute e1=rv.normal(0,1).
compute e2=rv.normal(0,1).
compute e3=rv.normal(0,1).
compute e4=rv.normal(0,1).
Compute x1= rv.normal(0,1).
COMPUTE x2 = RV.UNIFORM(0,1) .
COMPUTE group = 0 .
IF (x2 > .50) group = 1 .
IF (group=0) b1 = b1 + 1.
IF (group=0) b0 = b0 + 1.
compute r2=.3*e1 + .954*e2.
compute r3=.09*e1 + .286*e2 + .954*e3.
compute r4=.027*e1 + .086*e2 + .286*e3 + .954*e4.
compute y1 = b0 + b1*(0)+e1.
compute y2 = b0 + b1*(1)+r2.
compute y3 = b0 + b1*(2)+r3.
compute y4 = b0 + b1*(3)+r4.
end case.
end loop.
 end file.
end input program.
VARSTOCASES /ID = id
 /MAKE trans1 FROM y1 y2 y3 y4
 /KEEP =  group
 /INDEX = Index1(4)
 /NULL = KEEP.
COMPUTE wave = index1 .
EXECUTE .
```

```
COMPUTE score = trans1 .
EXECUTE .
RECODE
  wave
  (1=0)  (2=1)  (3=2)  (4=3)  INTO  time .
EXECUTE .

MIXED
  score  by group with time
  /CRITERIA = CIN(95) MXITER(10000) MXSTEP(5) SCORING(1) SINGULAR(0.000000000001)
HCONVERGE(0, ABSOLUTE) LCONVERGE(0, ABSOLUTE)
  PCONVERGE(0.000001, ABSOLUTE)
  /FIXED = group time group*time  | SSTYPE(3)
  /METHOD = REML
  /PRINT =  CORB COVB G LMATRIX R SOLUTION TESTCOV
 /REPEATED = wave | SUBJECT(id) COVTYPE(CS) .


MIXED
score  by group with time
  /CRITERIA = CIN(95) MXITER(10000) MXSTEP(5) SCORING(1) SINGULAR(0.000000000001)
HCONVERGE(0, ABSOLUTE) LCONVERGE(0, ABSOLUTE)
  PCONVERGE(0.000001, ABSOLUTE)
  /FIXED = group time group*time  | SSTYPE(3)
  /RANDOM INTERCEPT time  | SUBJECT(id) COVTYPE(UN)
  /METHOD = REML
  /PRINT =  CORB COVB G LMATRIX R SOLUTION TESTCOV  .

MIXED
  score  by group with time
  /CRITERIA = CIN(95) MXITER(10000) MXSTEP(5) SCORING(1) SINGULAR(0.000000000001)
HCONVERGE(0, ABSOLUTE) LCONVERGE(0, ABSOLUTE)
  PCONVERGE(0.000001, ABSOLUTE)
  /FIXED = group time group*time | SSTYPE(3)
  /RANDOM INTERCEPT time  | SUBJECT(id) COVTYPE(UN)
  /METHOD = REML
  /PRINT =  CORB COVB G LMATRIX R SOLUTION TESTCOV
 /REPEATED = wave | SUBJECT(id) COVTYPE(ar1) .

MIXED
  score  by group with time
  /CRITERIA = CIN(95) MXITER(10000) MXSTEP(5) SCORING(1) SINGULAR(0.000000000001)
HCONVERGE(0, ABSOLUTE) LCONVERGE(0, ABSOLUTE)
  PCONVERGE(0.000001, ABSOLUTE)
  /FIXED = group time group*time | SSTYPE(3)
  /METHOD = REML
  /PRINT =  CORB COVB G LMATRIX R SOLUTION TESTCOV
 /REPEATED = wave | SUBJECT(id) COVTYPE(TP) .

MIXED
  score  by group with time
  /CRITERIA = CIN(95) MXITER(10000) MXSTEP(5) SCORING(1) SINGULAR(0.000000000001)
HCONVERGE(0, ABSOLUTE) LCONVERGE(0, ABSOLUTE)
  PCONVERGE(0.000001, ABSOLUTE)
  /FIXED = group time group*time | SSTYPE(3)
  /METHOD = REML
```

```
  /PRINT =  CORB COVB G LMATRIX R SOLUTION TESTCOV
 /REPEATED = wave | SUBJECT(id) COVTYPE(UN) .

!DOEND
!ENDDEFINE.

Linearmixed_bootstrap
  samples=100.
omsend.
```

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