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SEISMIC ANALYSIS OF STRUCTURES EMBEDDED IN SATURATED SOILS

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SEISMIC ANALYSIS OF STRUCTURES EMBEDDED IN SATURATED SOILS

by

Richard Hai Lung

A dissertation submitted to the Graduate Faculty in Engineering in partial fulfillment of the requirements for the degree of Doctor of Philosophy, The City University of New York.

1980

This manuscript has been read and accepted for the Graduate Faculty in Engineering in satisfaction of the dissertation requirement for the degree of Doctor of Philosophy.

12 Dec 1980 — Carl J Costantino

date — Chairman of Examining Committee

December 22, 1980

date — Executive Officer

Prof. D.H. Cheng
Prof. C.J. Costantino
Prof. D. Goldfarb
Prof. C.A. Miller
Supervisory Committee

The City University of New York

ACKNOWLEDGEMENTS

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1.0 INTRODUCTION

Interaction between structure and surrounding soil is of considerable interest in soil mechanics and a variety of geotechnical situations. Earthquake analysis of structures built on dry or saturated soils requires special consideration due to the interaction between structure and soil. When one analyzes the response of a structure due to earthquake ground motion, one must recognize the additional dynamic forces and modifications in the dynamic properties cause by the soil surrounding the structure.

The main objective of this thesis research is to determine the influence of pore water on the dynamic response of structure subjected to earthquake ground motion. To date, extensive studies have been made for earthquake analysis of structures built on single-phased non-saturated soil. However, no such study has been made for earthquake analysis of structures built on two-phased saturated soil.

For any media, numerical approximation techniques are used widely since analytical solutions are available only for the simplest problems. These numerical methods are based on either finite difference(9) or finite element methods.(11,15,16) However, systems with complex geometry and boundary conditions limit the use of the finite difference method. The finite element method on the other hand is very effective in analyzing the response of continua of complex geometry to dynamic excitation. The finite element method is used in conjunction with variational principles herein to study the coupled response of structures embedded in saturated soils and subjected to earthquake motions.

The finite element idealization of the saturated soil system replaces the actual continuum with a finite number of discrete elements, where the geometry of the elements is defined by a set of nodal points of the system. Shape functions for the unknown field variables are assumed within each element. The field variables within each element are then defined completely in terms of the values at the nodal points.

The standard finite element methods, in principle, could deal with multiple-dimensional situations. However, the cost of solutions increase greatly with each dimension added and could overtax the capabilities of the available computer. Only two-dimensional plane problems are considered herein.

For actual problems, the structure is typically idealized by simple lumped mass system connected by equivalent beams, this structure resting on or in the saturated soil. The structure is considered to be a free-free system for analysis purposes and the interaction forces developed between the soil and the structure act as forcing functions to this free-free system. The interaction forces are proportional to the difference between the actual motion of the ground and the ground motion that would develop without the structure (criteria earthquake motion). The constants of proportionality are then the soil compliance coefficients. The objective of this research is to find these compliance coefficients for the two-dimensional case, for saturated soils, for which no solutions currently exist.

2.0 TECHNICAL DISCUSSION

In Appendix A, the equations of motion of both soil and pore water are presented. These equations prescribe the propagation of stress waves in a porous elastic solid containing a compressible viscous fluid. When inertia terms are taken out of these equations, one of them will reduce to Darcy's law of seepage. These equations are valid for the relative low frequency range where the assumption of Poiseuille flow is valid. The assumption is valid only below a cutoff frequency which we denote by f_t, and which depends on the kinematic viscosity of the fluid, ν, and the size of the pores,(5) d (or the average diameter of the pores), as follows.

$$f_t = \pi \nu / 4\ d^2$$

For average(7) pore sizes of 0.1 mm or less (sand or finer soils), a cutoff frequency of about 100 cps is computed. For most interaction problems, upper bounds of approximately 30 cps are of interest, implying that the laminar assumption is valid.

2.1 FINITE ELEMENT ANALYSIS

The problems of applied physics encountered in engineering can be specified in one of two ways. In the first, differential equations governing the behavior of a typical, infinitesimal, region are given. In the second, a variational (extremum) principle valid over the whole region is postulated and the correct solution is the one minimizing a 'functional' which is defined by suitable integration of the unknown quantities over the whole domain.

For a single-phased elastic continuum, the second method mentioned above can be used. The finite element method uses the process of minimizing the total potential energy of the system in terms of a prescribed displacement field. First, the continuum is separated by imaginary lines or surfaces into a number of finite elements, which are assumed to be interconnected at a discrete number of nodal points situated on their vertices. The displacements of these nodal points will be the basic unknown parameters of the problem. A set of functions is chosen to define uniquely the state of displacement within each 'element' in terms of its nodal displacements. This process is the well-known Ritz procedure. The displacement functions now define uniquely the state of strain within an element in terms of the nodal displacements. These strains, together with any initial strains and the constitutive properties of the material define the state of stress throughout the element and, hence, also on its boundaries. By the virtual work method, the finite element equilibrium equations are obtained for the whole media. A finite element discretization of a general elastic media is presented in Appendix B. For a single-phased (or dry) soil, the procedure outlined in Appendix B is generally used.

For the two-phased soil/water system, differential equations governing the behavior of a typical, infinitesimal, region are defined in Appendix A. The differential equations must be transformed into functional form before the finite element method could be used. The Galerkin process of weighted residuals(21) is used to transform the differential equations into functional form. In Appendix C, these equations are discretized using the finite element method and the resulting equations developed. Similar results have been obtained by Ghaboussi and Wilson(10) using a method suggested by Sandhu and Pister.(19,20) The method involves bilinear mapping and utilization of symmetry of convolution. The advantage of the Galerkin process is the ease and directness of obtaining the boundary terms.

Appendix C presents the finite element discretization of the equations of motion for two-dimensional saturated elastic porous media for steady state solutions.

Appendix D presents the two-dimensional isoparametric elements to be used in the finite element analysis of the soil/water media. It also presents the method of evaluating certain relevant element matrices and the numerical integration scheme by Gauss quadrature.

2.2 BOUNDARY CONDITIONS

When the finite element method is used to solve the equations of motion of the soil and pore water, an infinite half-space is idealized into a finite space with appropriate boundary conditions. In the infinite space problem, the wave propagation thru the half space will allow the energy to radiate away from the site. In the finite computational space, however, appropriate boundary conditions must be specified so as to allow this energy to be transmitted away from the zone of interest, rather than artificially reflecting back into the zone by the boundary conditions.

In Appendix F, proper transmitting boundary conditions for dry soil and for steady state case are presented. These boundary conditions work perfectly for the one-dimensional case and have been found to work reasonably well for the two-dimensional case.

It is shown, in Appendix E, that analytical solutions could be obtained for steady state one-dimensional saturated elastic porous media. It is also shown that normal stress and water pressure could be expressed in terms of bulk displacment and relative water displacement for an infinite rod. Results of Appendix E are used in Appendix F to formulate the transmitting boundary conditions for two-phased soil/water problem. It is also shown in Appendix F that these transmitting boundary conditions adequately simulate the transmission of energy for the one-dimensional problem.

2.3 UTILIZATION OF SOIL-STRUCTURE INTERACTION COEFFICIENTS

Once the soil-structure interaction coefficients are obtained, they can be used in general earthquake analysis of structures embedded in soil. The procedure of how to utilize these coefficients are outlined in Appendix G. Since this post processing is not the main objective of this thesis research, we will limit ourself with only the general presentation of Appendix G.

2.4 COMPUTER PROGRAM

For small structural problems, computer programs can be written such that only core memory is used. Programming for these small problems which can be handled without using too much back-up storage space is usually fairly straightforward. However, for problems involving many equations and matrices with wide bandwidths, a very large core memory is required for the ordinary band solver subroutines. In many cases such core requirements far exceed the capacity of the existing computer.

In Appendix H, a simple and straightforward finite element computer program procedure capable of handling very large practical problems by the use of random access devices to link the back-up storage of the computer is presented.

The two very important features of this program procedure are (1) it does not store any zero submatrices within the band and (2) during the solution of equations all operations dealing with zero submatrices within the band are skipped automatically.

3.0 NUMERICAL RESULTS

A series of 36 two-dimensional finite element runs were made to compute the frequency dependent soil-structure interaction coefficients. For all the computer runs, soil and water properties listed on Table M-1 are used. These properties which are obtained from Appendix K are essentially those of sand.

A variation of parameter study was then made in an attempt to ascertain the impact of the various parameters on the computed responses. The variations considered are as follows.

(a) Four different mesh sizes are used as shown in Figures 1, 2, 4, and 5 with the location of lateral and bottom boundaries varied.

(b) Two different element sizes are used as shown in Figures 2 and 3.

(c) For model shown on Figure 1, six different levels of soil saturation are used.

- Model 1 (Fully Saturated): soil is fully saturated to depth = 0.
- Model 1A: soil is saturated to depth of = -½a, where 'a' is the half width of the footing.
- Model 1B: soil is saturated to depth = -a.
- Model 1F: soil is saturated to depth = -1½a.

- Model 1C: soil is saturated to depth = -2a.
- Model 1 (Dry): dry soil.

(d) For Model 1 (Fully Saturated), shown in Figure 1, three different coefficients of permeability are used.

For all the problem runs, no hysteretic soil damping was used. Although the computer code was written to include the effects of soil damping, it was felt that the undamped results were of particular interest.

For all the computed results, the following boundary conditions were used for the finite element mesh:

(a) The right and bottom sides have transmitting boundaries.

(b) The left sides have vertical soil and vertical relative water movements restricted for horizontal and rocking displacements of the footings, as shown in Figure 6, and have horizontal soil and horizontal relative water movements restricted for vertical displacements of the footings.

(c) The top sides are stress free everywhere except right under the footing where the displacements are prescribed.

The results from computer runs are tabulated on Tables M-3 thru M-14. The frequencies and the compliance coefficients have been non-dimensionalized according to Table M-2. The results are also plotted on Plots N-1 thru N-72.

3.1 MESH SIZES

By comparing the compliance coefficients of Model 2 (Fully Saturated), tabulated on Table M-11 and plotted on Plots N-17, N-18, N-41, N-42, N-65, N-66, and those of Model 4 (Fully Saturated), tabulated on Table M-13 and plotted on Plots N-21, N-22, N-45, N-46, N-69, N-70, the following conclusions can be made. When Model 2 mesh has been widened laterally,

(a) the results improve considerably for the horizontal case,

(b) the results improve slightly for the rocking case, and

(c) the results improve slightly for the vertical case.

By comparing the compliance coefficients of Model 2 (Fully Saturated) and those of Model 5 (Fully Saturated), tabulated on Table M-14 and plotted on Plots N-23, N-24, N-47, N-48, N-71, N-72, the following conclusions can be made. When Model 2 mesh has been deepened vertically,

(a) the results improve considerably for the horizontal case,

(b) the results improve a fair amount for the rocking case, and

(c) the results improve a fair amount for the vertical case.

By comparing the compliance coefficients of Model 4 (Fully Saturated) and those of Model 1 (Fully Saturated), tabulated on Table M-3 and plotted on Plots N-1, N-2, N-25, N-26, N-49, N-50, the following conclusions can be made. When Model 4 mesh has been deepened vertically,

(a) the results improve slightly for the horizontal case,

(b) the results improve slightly for the rocking case, and

(c) the results improve slightly for the vertical case.

By comparing the compliance coefficients of Model 5 (Fully Saturated) and those of Model 1 (Fully Saturated), the following conclusions can be made. When Model 5 mesh has been widened laterally,

(a) the results improve slightly for the horizontal case,

(b) the results improve slightly for the rocking case, and

(c) the results improve slightly for the vertical case.

Since only slight improvements are obtained by deepening the mesh of Model 4 to that of Model 1 or by widening the mesh of Model 5 to that of Model 1, it can be concluded that Model 1 mesh size will yield reasonable compliance coefficients and further widening or deepening of the mesh will result with very insignificant improvements.

3.2 ELEMENT SIZES

By comparing the compliance coefficients of Model 2 (Fully Saturated) and those of Model 3 (Fully Saturated), tabulated on M-12 and plotted on Plots N-19, N-20, N-43, N-44, N-67, N-68, the following conclusions can be made. When the element sizes of Model 2 are reduced to one quarter sizes,

(a) the results improve slightly for the horizontal case,

(b) the results improve a fair amount for the rocking case, and

(c) the results improve slightly for the vertical case.

The improvements due to element size change are smaller than the improvements due to mesh size change. Since element size change adds to computer time requirements significantly, only the element size shown on Model 1, 2, 4, and 5 were used.

3.3 DEGREE OF SATURATION

Comparisons were made between the results from Model 1 (Dry), tabulated on Table M-10 and plotted on Plots N-15, N-16, N-39, N-40, N-63, N-64, and the analytical results from Luco, Reference 13. Since the results are very close, we know that the element size and mesh size used in Model 1 are acceptable.

By comparing the compliance coefficients of Model 1 (Dry) and those of Model 1 (Fully Saturated), the following conclusions can be made.

(a) For the horizontal case, the average real part of the compliance coefficient of the saturated soil is about 50 % higher than that of dry soil, and the imaginary part of saturated soil is about 50 % higher than that of dry soil for all frequencies.

(b) For the rocking case, the real part of the compliance coefficient of the saturated soil is higher than that of dry soil for low frequencies and changes sign for high frequencies. The imaginary part of the compliance coefficient of the saturated soil is more than twice that of dry soil for all frequencies.

(c) For the vertical case, the real part of the compliance coefficient of the saturated soil is slightly higher than that of dry soil for very low frequencies and changes sign for all other higher frequencies.

The imaginary part of the compliance coefficient of the saturated soil is more than five times that of dry soil for all frequencies.

By comparing the compliance coefficients of Model 1 (Fully Saturated), Model 1A (tabulated on Table M-6 and plotted on Plots N-7, N-8, N-31, N-32, N-55, N-56), Model 1B (tabulated on Table M-7 and plotted on Plots N-9, N-10, N-33, N-34, N-57, N-58), Model 1F (tabulated on Table M-8 and plotted on Plots N-11, N-12, N-35, N-36, N-59, N-60), Model 1C (tabulated on Table M-9 and plotted on Plots N-13, N-14, N-37, N-38, N-61, N-62), and Model 1 (Dry), the following conclusions can be made.

(a) For the horizontal case, as the depth of the ground water table increases the effect of water decreases. When the soil is dry to a depth equal to the width of the footing ($d_f = 2a$), the soil can be considered as dry soil.

(b) For the rocking case, as the depth of the ground water table increases the effect of water decreases. When the soil is dry to a depth equal to the width of the footing, the soil can be considered as dry soil.

(c) For the vertical case, the effect of water is present for all depth of saturation.

As previously stated, for the case of a fully saturated soil, the stiffness coefficients for both the rocking and vertical motion cases become negative at the higher frequency range. Although the real parts of the rocking and vertical compliance coefficients become negative at these higher frequencies, the imaginary parts are always positive and the magnitudes of the imaginary parts of the saturated soil are considerably higher than those of the dry soil.

These results are compared with the analytic results for the case of an incompressible single phase elastic solid presented in Ref. 14. As may be noted from Figures 7 thru 12, the character of the compliance coefficients are similar to the incompressible case, although different in magnitude. This indicates that the effect of the pore water is to make the soil behave essentially as an incompressible solid, with greater amount of damping caused by the additional dissipation caused by the permeability effect between the solid and water phases.

3.4 EFFECT OF COEFFICIENT OF PERMEABILITY

For Model 1 (Fully Saturated), three different coefficients of permeability (k_T= 0.1 cm/sec, 0.01 cm/sec, and 0.0001 cm/sec) are used. The compliance coefficients with various coefficients of permeability are tabulated on Table M-3 thru M-5 and are plotted on Plots N-1 thru N-6, N-25 thru N-30, N-49 thru N-54. By comparing those compliance coefficients, the following conclusions can be made.

When the coefficient of permeability decreases from k_T = 0.1 cm/sec to k_T = 0.01 cm/sec,

(a) for the horizontal case and for all frequencies, the real part of the compliance coefficient increases by approximately 10 %, with the imaginary part of the compliance coefficient increasing slightly (about 2 %);

(b) for the rocking case and for all frequencies, the absolute value of the real part of the compliance coefficient increases very slightly and the imaginary part of the compliance coefficient increases slightly;

(c) for the vertical case and for all frequencies, the real part of the compliance coefficient has negligible change and the imaginary part of the compliance coefficient increases slightly.

When the coefficient of permeability decreases from k_z = 0.01 cm/sec to k_z = 0.0001 cm/sec,

(a) for the horizontal case and for all frequencies, the real part of the compliance coefficient increases slightly and the imaginary part of the compliance coefficient has negligible change;

(b) for the rocking case and for all frequencies, the real part and imaginary part of the compliance coefficient have negligible changes;

(c) for the vertical case and for all frequencies, the real part and imaginary part of the compliance coefficient have negligible changes.

When the fully saturated soil with coefficient of permeability of 0.1 cm/sec or less is excited by the rocking or vertical motion of the footing, the pore water is essentially trapped by the soil. For rocking and vertical cases, there is no need to find the compliance coefficients for the fully saturated soil with coefficient of permeability lower than 0.1 cm/sec, because we can use the compliance coefficients obtained for the saturated soil with coefficient of permeability of 0.1 cm/sec. However, under the horizontal motion of the footing, the pore water becomes trapped by the soil only when the coefficient of permeability falls below 0.01 cm/sec. For horizontal case, there is no need to find the compliance coefficients for the fully saturated soil with coefficient of permeability lower than 0.01 cm/sec, because we can used the compliance coefficients obtained for the saturated soil with coefficient of permeability of 0.01 cm/sec.

Since pore water is essentially trapped by the soil for saturated soil with coefficient of permeability of 0.01 cm/sec or less, the question arises as to the ability to predict the saturated response of the soil from the previously known dry solution by simply modifying some of the parameters involved, as follows.

(a) Replace $\frac{E(1-\nu)}{(1+\nu)(1-2\nu)}$ of the dry soil by $\frac{E(1-\nu)}{(1+\nu)(1-2\nu)}+M$, where E is the Young's modulus, ν is the Poisson's ratio and M is the bulk modulus of fluid.

(b) Replace ρ_d by ρ, where ρ_d is the mass density of the dry soil and ρ is the mass density of the bulk of fluid-solid.

(c) Reduce ν such that G (shear modulus) will be unchanged from that of the dry soil.

After various trials, no correlation factors could be found to predict the saturated response of soil of low permeability from the previously known dry solution.

4.0 CONCLUSION

The influence of pore water on the dynamic response of structure subjected to earthquake ground motion is quite substantial.

For the fully saturated horizontal case, pore water increases the values of the real parts and the imaginary parts of the compliance coefficients by approximately 50 %. For the fully saturated rocking case, pore water increases the imaginary parts of the compliance coefficients by a factor of two and changes the signs (directions) of the real parts of the compliance coefficients above certain frequencies. For the fully saturated vertical case, pore water increases the imaginary parts of the compliance coefficients by a factor of five and changes the signs (directions) of the real parts of the compliance coefficients above certain frequencies.

For the horizontal and rocking cases, as the depth of the ground water increases the effect of water decreases as expected. When the soil is dry to a depth equal to the width of the building, the soil can be considered as dry soil. However, for the vertical case, the effect of water is felt even when the depth of the ground water increases substantially.

For fully saturated soil under the excitation of the footing, the pore water can be considered essentially to be trapped by the soil, when the coefficient of permeability is below a certain cutoff value. The cutoff value of coefficient of permeability for the horizontal motion of the footing is lower than those of the rocking and vertical motions of the footing.

Although no computer run has been obtained to study the effect of the coefficient of permeability on partially saturated soil, we can expect that the coefficient of permeability to have greater effect on partially saturated soil than on fully saturated soil.

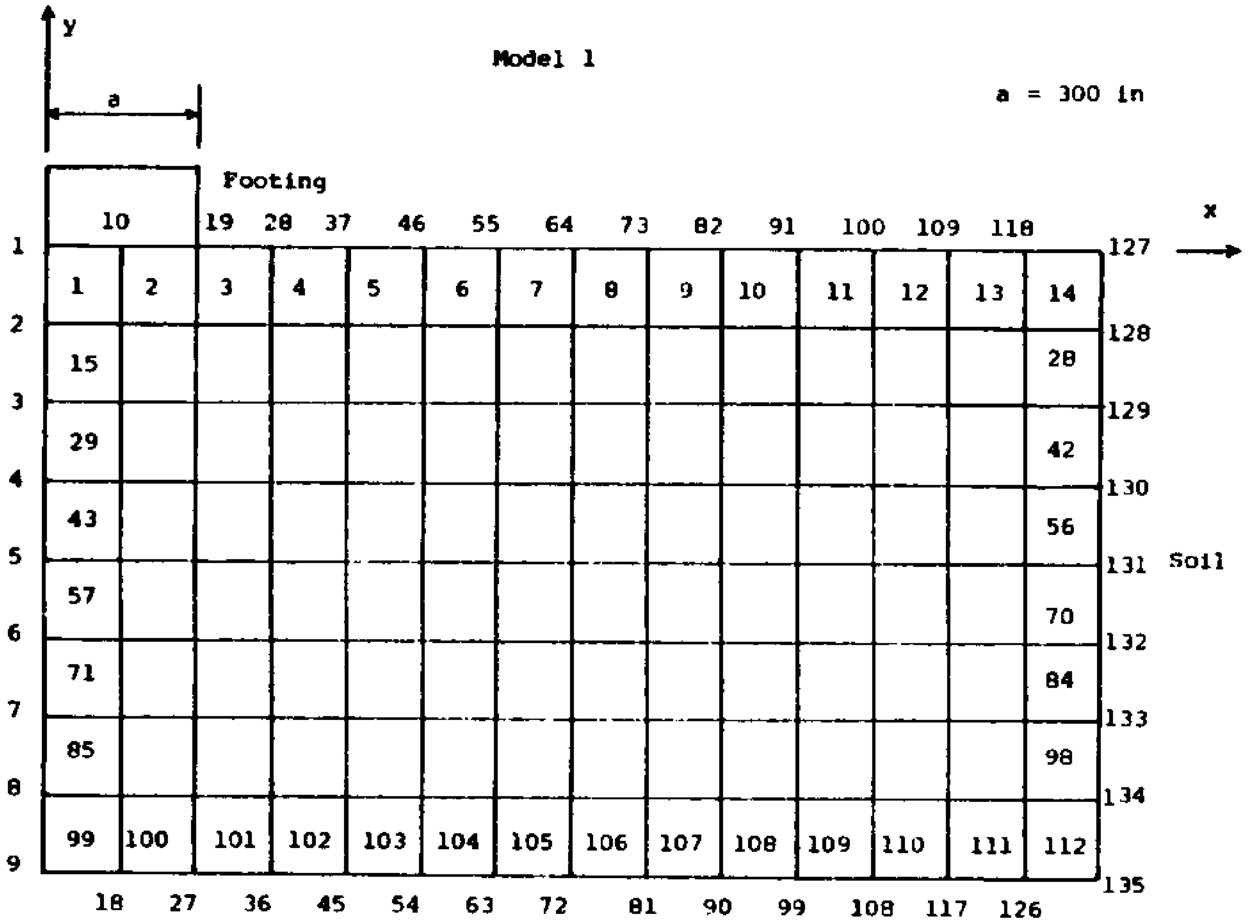
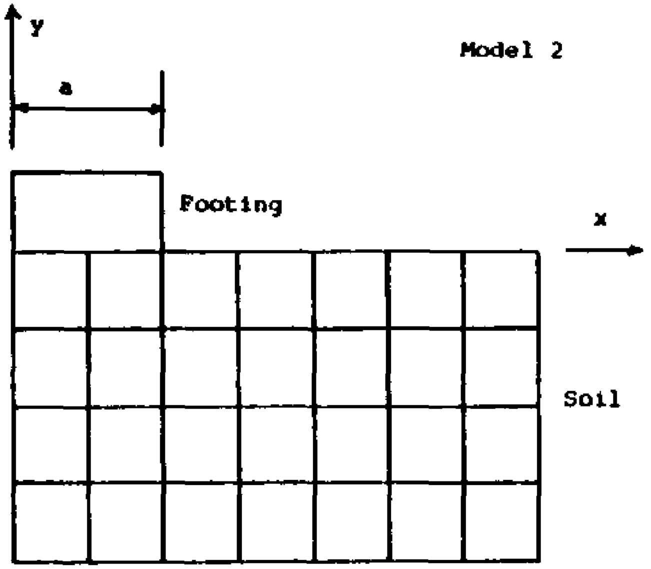

y
Model 1
a
a = 300 in
Footing
10
19 28 37 46 55 64 73 82 91 100 109 118
127
x
1 2 3 4 5 6 7 8 9
1 2 3 4 5 6 7 8 9 10 11 12 13 14
15 29 43 57 71 85
28 42 56 70 84 98
99 100 101 102 103 104 105 106 107 108 109 110 111 112
128 129 130 131 132 133 134 135
Soil
18 27 36 45 54 63 72 81 90 99 108 117 126

Figure 1



Model 2

a = 300 in

Figure 2

Model 3

a = 300 in

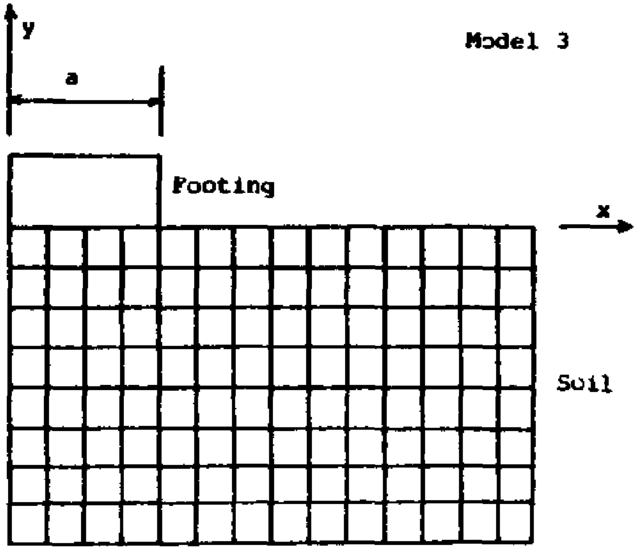


Figure 3

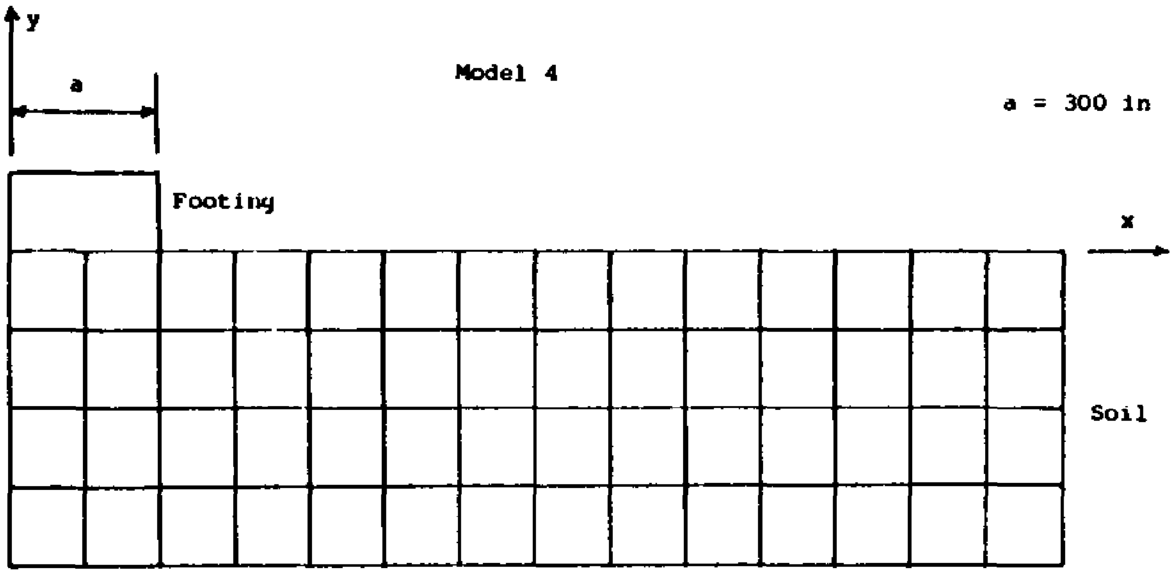


Figure 4

Model 5

a = 300 in

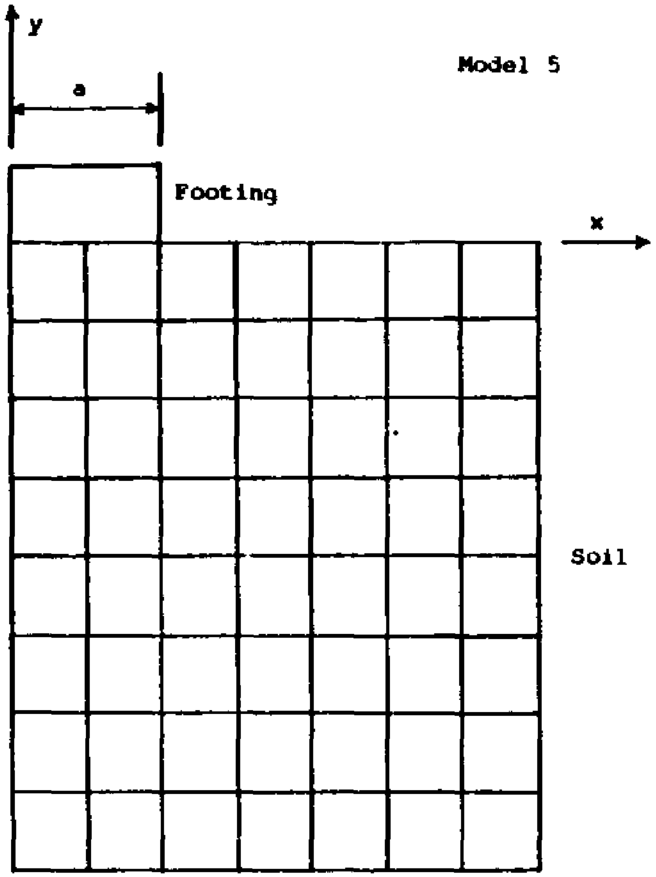
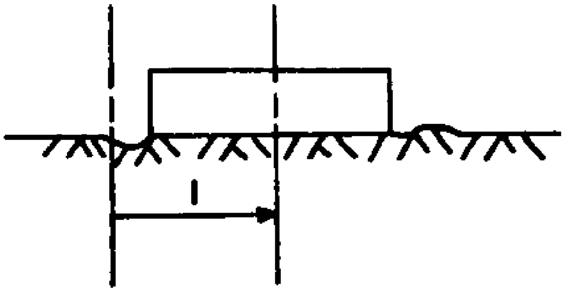
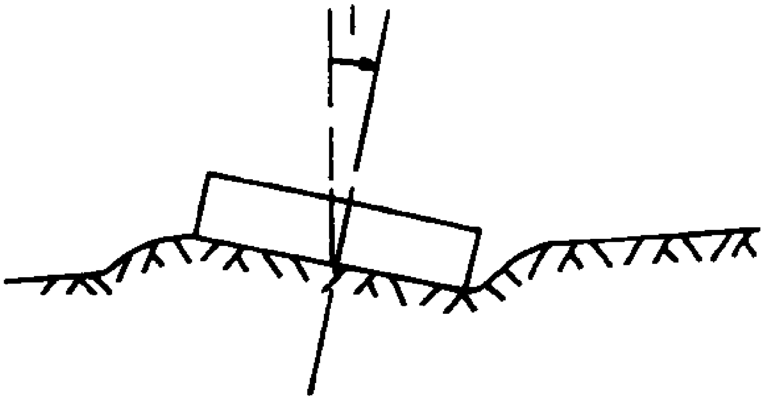


Figure 5



HORIZONTAL DISPLACEMENT

VERTICAL DISPLACEMENT



ROCKING DISPLACEMENT

Figure 6

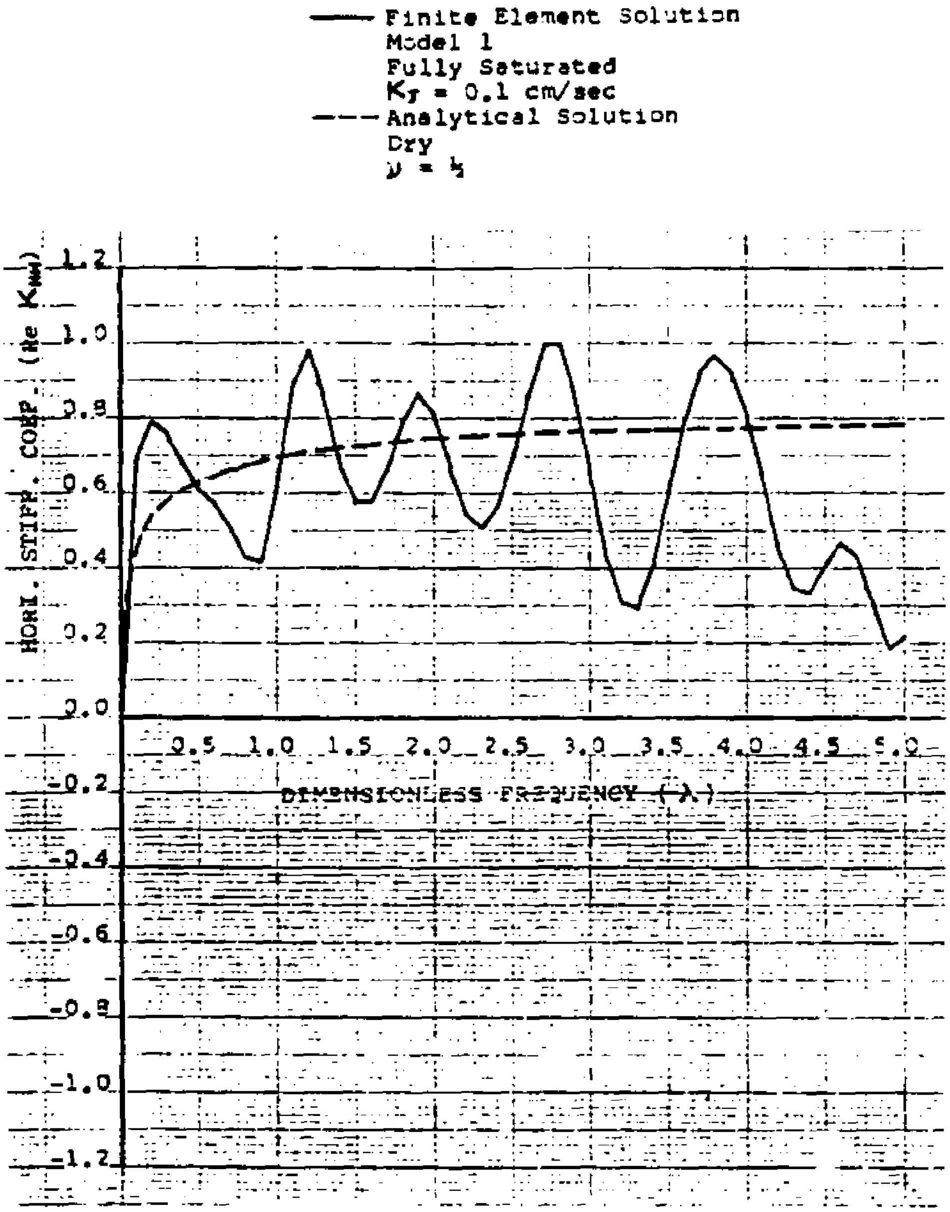

Finite Element Solution
Model 1
Fully Saturated
K_f = 0.1 cm/sec
Analytical Solution
Dry
$\nu = \frac{1}{2}$
HORI. STIFF. COEF. (Re K_{HH})
1.2
1.0
0.8
0.6
0.4
0.2
0.0
-0.2
-0.4
-0.6
-0.8
-1.0
-1.2
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
5.0
DIMENSIONLESS FREQUENCY (λ)

Figure 7

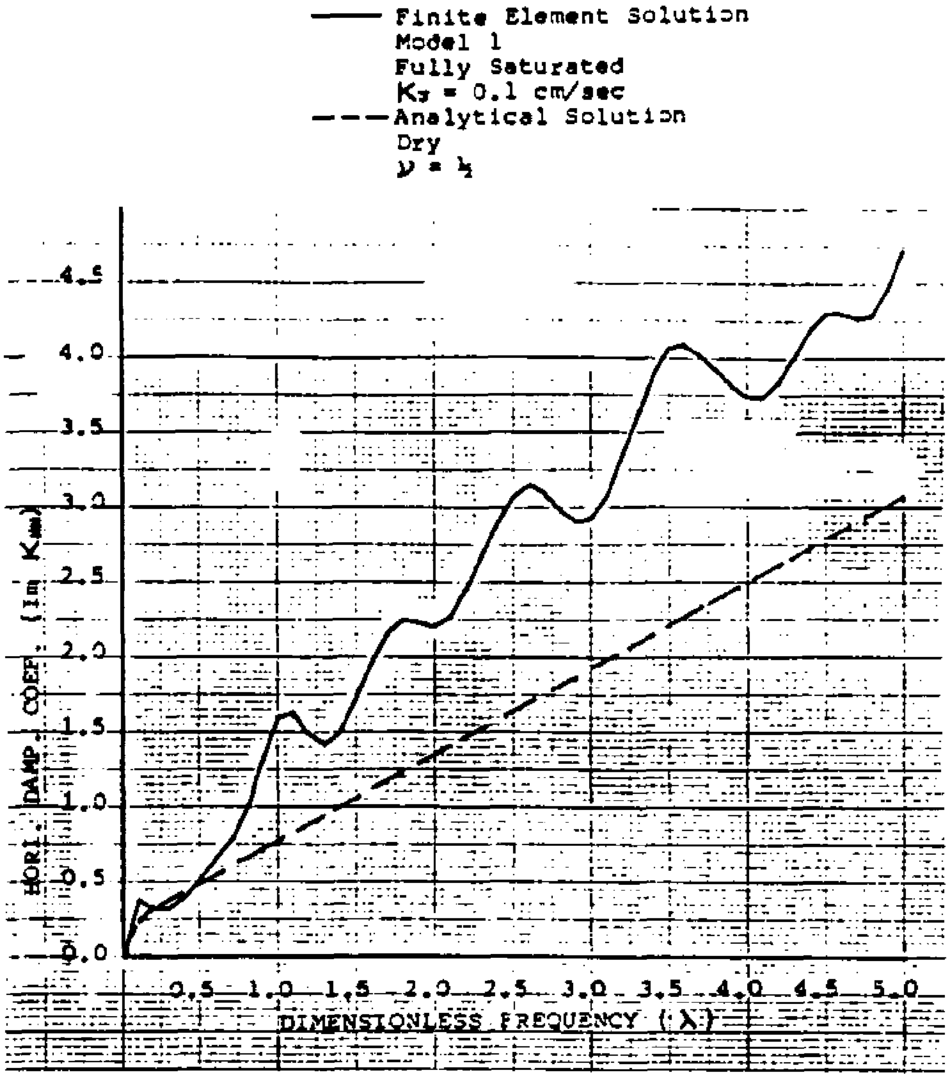
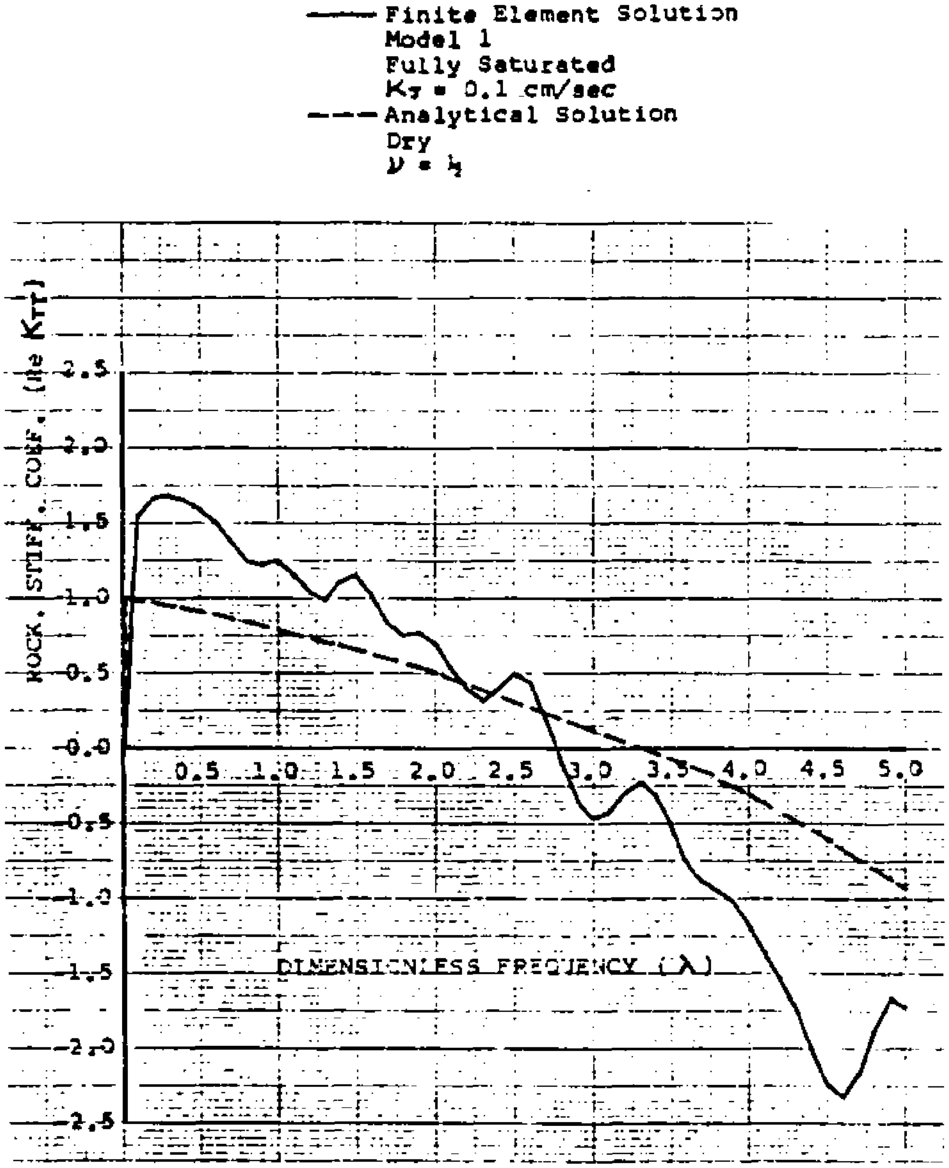

Finite Element Solution
Model 1
Fully Saturated
Ks = 0.1 cm/sec
Analytical Solution
Dry
ν = ½
4.5
4.0
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.0
HORI. DAMP. COEF. (Im Khh)
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
5.0
DIMENSIONLESS FREQUENCY (λ)

Figure 8



Finite Element Solution
Model 1
Fully Saturated
K_T = 0.1 cm/sec
Analytical Solution
Dry
$\nu = \frac{1}{2}$
ROCK. STIFF. COEF. (Re K_{TT})
2.5
2.0
1.5
1.0
0.5
0.0
-0.5
-1.0
-1.5
-2.0
-2.5
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
5.0
DIMENSIONLESS FREQUENCY (λ)

Figure 9

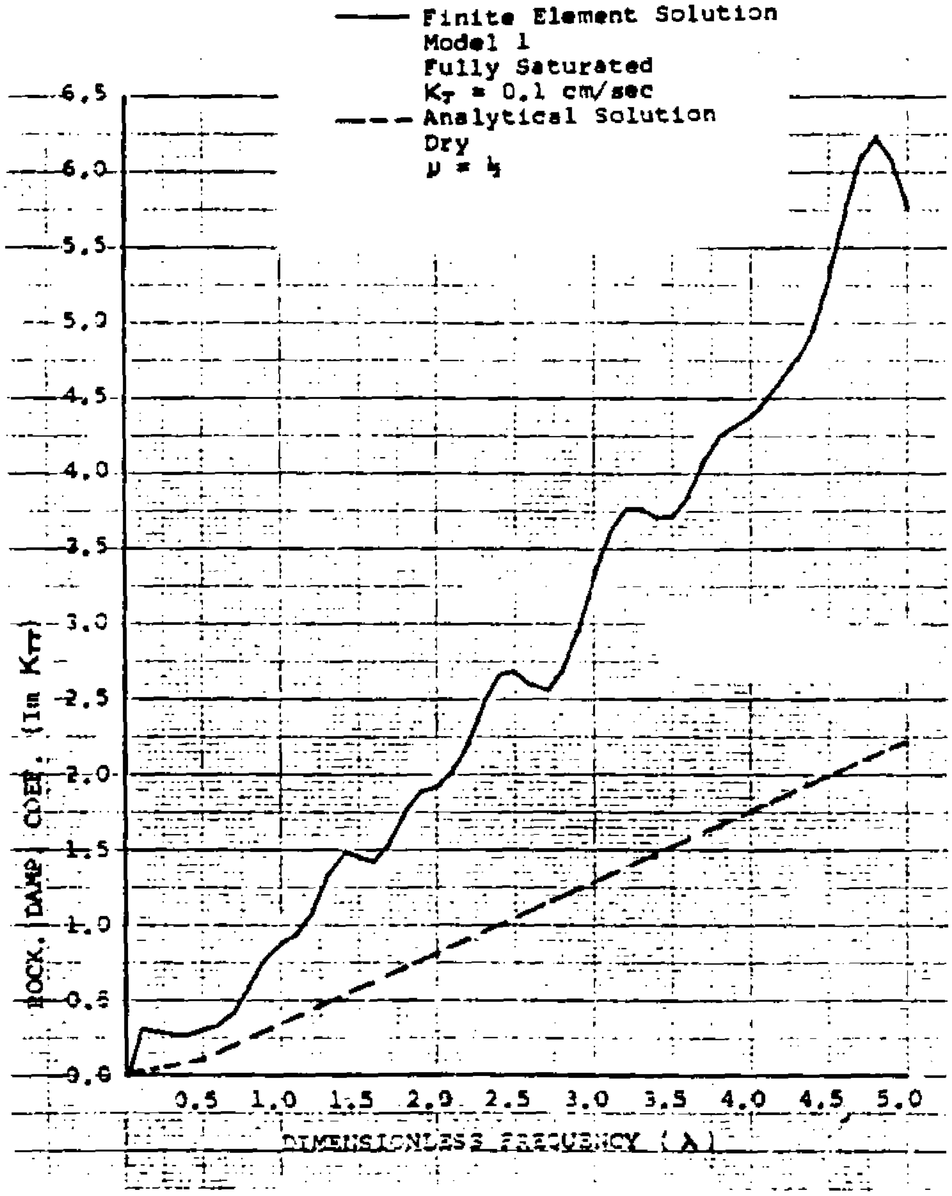

Finite Element Solution
Model 1
Fully Saturated
K_T = 0.1 cm/sec
Analytical Solution
Dry
μ = ¼
ROCK. DAMP. COEF. (Im K_{TT})
6.5
6.0
5.5
5.0
4.5
4.0
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
5.0
DIMENSIONLESS FREQUENCY (λ)

Figure 10

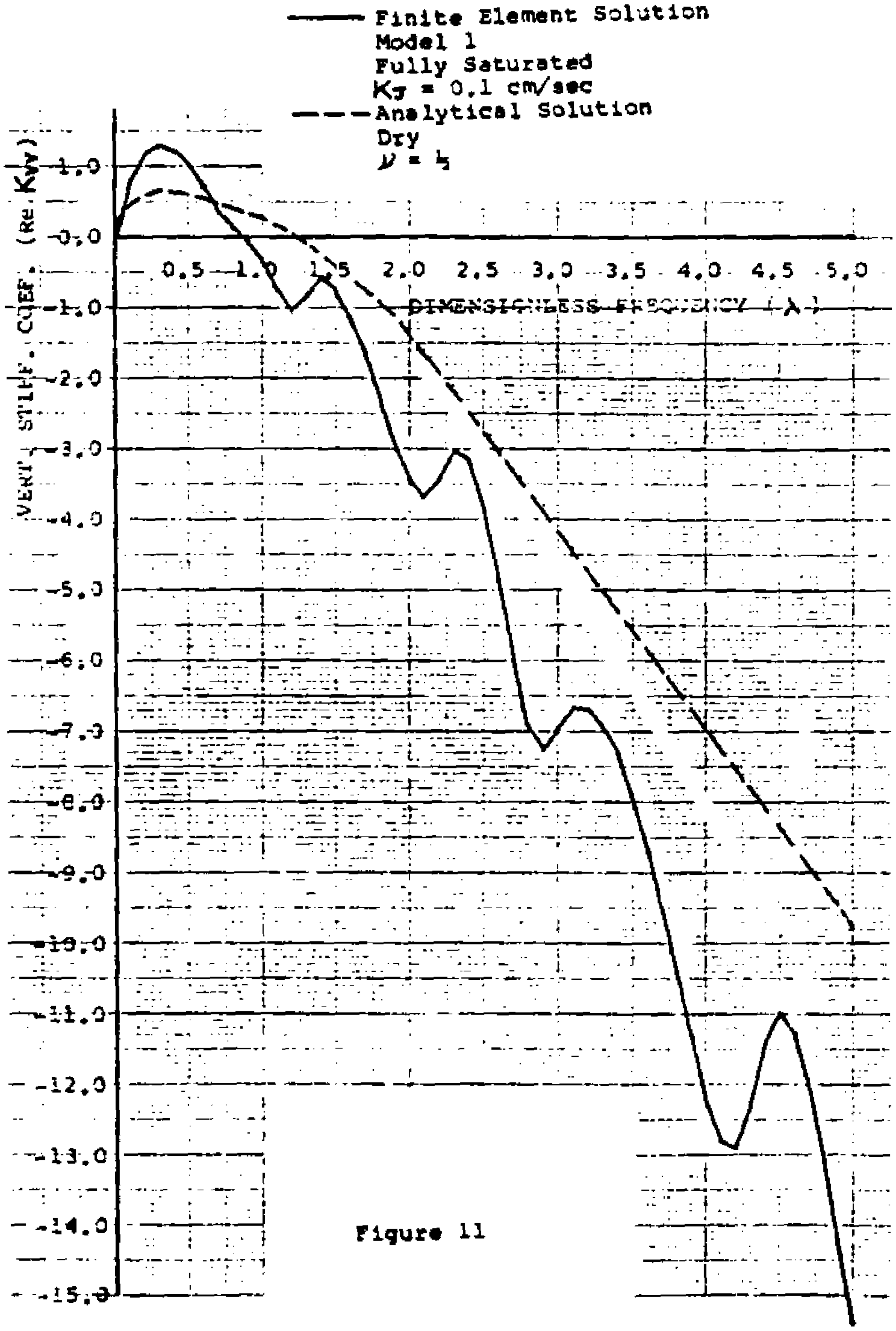
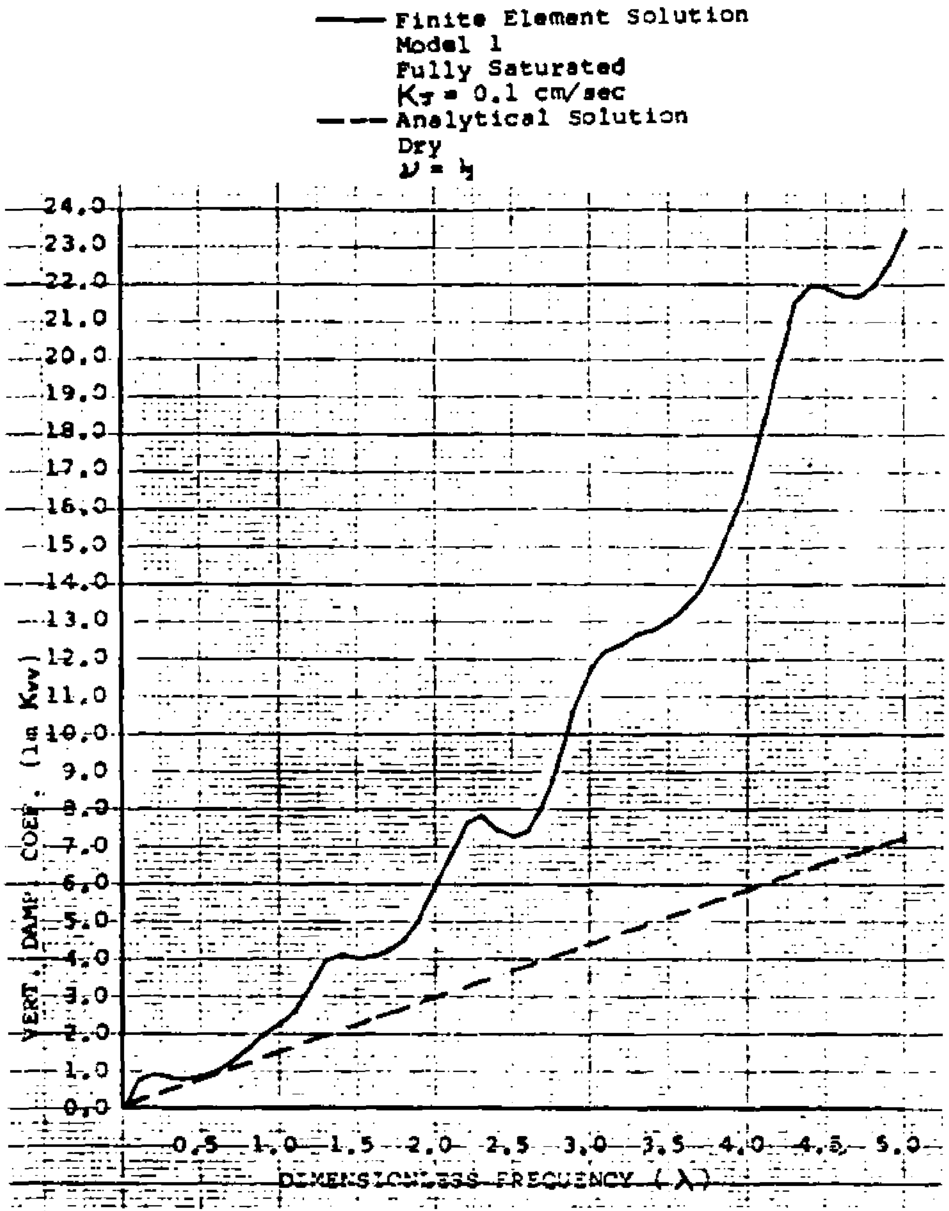

Finite Element Solution
Model 1
Fully Saturated
$K_{\bar{z}}$ = 0.1 cm/sec
Analytical Solution
Dry
$\nu = \frac{1}{3}$
VERT. STIFF. COEF. (Re. K_{vv})
1.0
-0.0
-1.0
-2.0
-3.0
-4.0
-5.0
-6.0
-7.0
-8.0
-9.0
-10.0
-11.0
-12.0
-13.0
-14.0
-15.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
5.0
DIMENSIONLESS FREQUENCY (λ)

Figure 11



Finite Element Solution
Model 1
Fully Saturated
K_T = 0.1 cm/sec
Analytical Solution
Dry
$\nu = \frac{1}{4}$
24.0
23.0
22.0
21.0
20.0
19.0
18.0
17.0
16.0
15.0
14.0
13.0
12.0
11.0
10.0
9.0
8.0
7.0
6.0
5.0
4.0
3.0
2.0
1.0
0.0
VERT. DAMP. COEF. (in Kvv)
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
5.0
DIMENSIONLESS FREQUENCY (λ)

Figure 12

APPENDIX A

THE EQUATIONS OF MOTION FOR SATURATED ELASTIC POROUS MEDIA

APPENDIX A

THE EQUATIONS OF MOTION FOR SATURATED ELASTIC POROUS MEDIA

Denoting by u_x, u_y, u_z the components of the displacement of the soil and assuming the strain to be small, the values of the strain components are (2,3,4)

$$
\begin{aligned}
e_{xx} &= \frac{\partial u_x}{\partial x} & e_{xy} &= \frac{\partial u_y}{\partial x} + \frac{\partial u_x}{\partial y} \\
e_{yy} &= \frac{\partial u_y}{\partial y} & e_{xz} &= \frac{\partial u_z}{\partial x} + \frac{\partial u_x}{\partial z} \\
e_{zz} &= \frac{\partial u_z}{\partial z} & e_{yz} &= \frac{\partial u_z}{\partial y} + \frac{\partial u_y}{\partial z}
\end{aligned}
\tag{1)A}
$$

Assume the soil to have isotropic properties and neglect water pressure, we have the well-known stress-strain relationships

$$
\begin{aligned}
e_{xx} &= \frac{\sigma_{xx}}{E} - \frac{\nu}{E}\left(\sigma_{yy} + \sigma_{zz}\right) & e_{xy} &= \frac{\sigma_{xy}}{G} \\
e_{yy} &= \frac{\sigma_{yy}}{E} - \frac{\nu}{E}\left(\sigma_{xx} + \sigma_{zz}\right) & e_{xz} &= \frac{\sigma_{xz}}{G} \\
e_{zz} &= \frac{\sigma_{zz}}{E} - \frac{\nu}{E}\left(\sigma_{xx} + \sigma_{yy}\right) & e_{yz} &= \frac{\sigma_{yz}}{G}
\end{aligned}
\tag{2)A}
$$

In these relations, the constants E, G, ν are, respectively, Young's modulus, the shear modulus and Poisson's ratio for the solid skeleton. The shear modulus can be expressed as

$$G = \frac{E}{2(1+\nu)} \qquad (3)A$$

Solving Eq.(2)A with respect to stresses we find

$$\{\sigma\} = [D]\{e\} \qquad (4)A$$

where

$$\{\sigma\} = \begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{zz} \\ \sigma_{xy} \\ \sigma_{xz} \\ \sigma_{yz} \end{Bmatrix} \qquad \{e\} = \begin{Bmatrix} e_{xx} \\ e_{yy} \\ e_{zz} \\ e_{xy} \\ e_{xz} \\ e_{yz} \end{Bmatrix} \qquad (5)A$$

and

$$[D] = \frac{E(1-\nu)}{(1+\nu)(1-2\nu)} \begin{bmatrix} 1 & \frac{\nu}{1-\nu} & \frac{\nu}{1-\nu} & 0 & 0 & 0 \\ \frac{\nu}{1-\nu} & 1 & \frac{\nu}{1-\nu} & 0 & 0 & 0 \\ \frac{\nu}{1-\nu} & \frac{\nu}{1-\nu} & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1-2\nu}{2(1-\nu)} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1-2\nu}{2(1-\nu)} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1-2\nu}{2(1-\nu)} \end{bmatrix} \qquad (6)A$$

If water pressure is also considered, the bulk stress tensor must be considered. The components of the bulk stress tensor are τ_{ij} where i, j = x,y,z. Let us first consider a unit volume of bulk material. The components of the average fluid-displacement vector are U_x , U_y , U_z . These components are defined in such a way that the volume of fluid displaced through unit areas normal to the x,y,z directions are fU_x, fU_y, fU_z , respectively, where f denotes the porosity. The flow of the fluid relative to the solid but measured in terms of volume per unit area of the bulk medium is

$$\begin{aligned} w_x &= f(U_x - u_x) \\ w_y &= f(U_y - u_y) \\ w_z &= f(U_z - u_z) \end{aligned} \tag{7)A}$$

The fluid volumetric strain is obtained by Biot(5,6) as

$$\zeta = -\left(\frac{\partial w_x}{\partial x} + \frac{\partial w_y}{\partial y} + \frac{\partial w_z}{\partial z}\right) \tag{8)A}$$

Biot has shown that the water pressure is as following

$$p = -\alpha M(e_{xx}+e_{yy}+e_{zz})+M\zeta \tag{9)A}$$

where α is the compressibility of the solid particles and M is the compressibility of the fluid.

The bulk stress tensor is

$$
\begin{aligned}
\tau_{xx} &= \sigma_{xx} - \alpha p_f \\
\tau_{yy} &= \sigma_{yy} - \alpha p_f \\
\tau_{zz} &= \sigma_{zz} - \alpha p_f \\
\tau_{xy} &= \sigma_{xy} \\
\tau_{xz} &= \sigma_{xz} \\
\tau_{yz} &= \sigma_{yz}
\end{aligned}
\qquad (10)\text{A}
$$

Substituting Eq.(9)A into Eq.(10)A and using Eq.(4)A, we have

$$\{\tau\} = \{[D] + \alpha^2 M [\bar{D}]\}\{e\} + \alpha M [\delta]\{w\} \qquad (11)\text{A}$$

where

$$\{\tau\} = \begin{Bmatrix} \tau_{xx} \\ \tau_{yy} \\ \tau_{zz} \\ \tau_{xy} \\ \tau_{xz} \\ \tau_{yz} \end{Bmatrix} \qquad \{w\} = \begin{Bmatrix} W_x \\ W_y \\ W_z \end{Bmatrix} \qquad (12)\text{A}$$

and

$$[\bar{D}] = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \qquad [\delta] = \begin{bmatrix} \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad (13)\text{A}$$

The kinetic energy of a unit volum of bulk material in low frequency range is (5,6)

$$T = \frac{1}{2}\rho(\dot{u}_x^2 + \dot{u}_y^2 + \dot{u}_z^2) + \rho_f(\dot{u}_x\dot{w}_x + \dot{u}_y\dot{w}_y + \dot{u}_z\dot{w}_z) + \frac{1}{2}m(\dot{w}_x^2 + \dot{w}_y^2 + \dot{w}_z^2) \qquad (14)A$$

where

$m = \frac{\rho_f}{f}$

$f =$ porosity

$\rho =$ total mass of bulk material per unit volume

$\rho_f =$ mass density of the fluid.

The dissipation energy of a unit volume of bulk material is (5,6)

$$2D = \frac{\eta}{k}(\dot{w}_x^2 + \dot{w}_y^2 + \dot{w}_z^2) \qquad (15)A$$

where

$k =$ the permeability of soil

$\eta =$ the viscosity of the fluid

If we now look at the forces applied to a unit volume of bulk material and consider u_x, u_y, u_z, w_x, w_y, and w_z as generalized coordinates, we can apply Lagrange's equations.

$$\frac{d}{dt}\left(\frac{\partial T}{\partial u_i}\right) - \frac{\partial T}{\partial u_i} = Q_i \qquad (16)A$$

where

$$i = x, y, z$$

We have

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{u}_x}\right) = \frac{d}{dt}\left(\rho \dot{u}_x + \rho_f \dot{w}_x\right) = \rho \ddot{u}_x + \rho_f \ddot{w}_x$$

$$\frac{\partial T}{\partial u_x} = 0$$

$$Q_x = \frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + \frac{\partial \tau_{xz}}{\partial z} + \rho g_x \qquad \text{,etc.}$$

But $g_x = g_y = 0 \quad , \quad g_z \neq 0 \quad .$

Therefore

$$\begin{aligned} \frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + \frac{\partial \tau_{xz}}{\partial z} &= \rho \ddot{u}_x + \rho_f \ddot{w}_x \\ \frac{\partial \tau_{yx}}{\partial x} + \frac{\partial \tau_{yy}}{\partial y} + \frac{\partial \tau_{yz}}{\partial z} &= \rho \ddot{u}_y + \rho_f \ddot{w}_y \\ \frac{\partial \tau_{zx}}{\partial x} + \frac{\partial \tau_{zy}}{\partial y} + \frac{\partial \tau_{zz}}{\partial z} + \rho g_z &= \rho \ddot{u}_z + \rho_f \ddot{w}_z \end{aligned} \qquad (17)\text{A}$$

Similary

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{w}_i}\right) + \frac{\partial D}{\partial \dot{w}_i} = -\frac{\partial p}{\partial i} + \rho_f g_i \qquad (18)\text{A}$$

Therefore

$$\begin{aligned} -\frac{\partial p}{\partial x} &= \rho_f \ddot{u}_x + \frac{1}{f}\rho_f \ddot{w}_x + \frac{\eta}{k}\dot{w}_x \\ -\frac{\partial p}{\partial y} &= \rho_f \ddot{u}_y + \frac{1}{f}\rho_f \ddot{w}_y + \frac{\eta}{k}\dot{w}_y \\ -\frac{\partial p}{\partial z} + \rho_f g_z &= \rho_f \ddot{u}_z + \frac{1}{f}\rho_f \ddot{w}_z + \frac{\eta}{k}\dot{w}_z \end{aligned} \qquad (19)\text{A}$$

APPENDIX B

FINITE ELEMENT DISCRETIZATION OF A GENERAL ELASTIC MEDIA

APPENDIX B

FINITE ELEMENT DISCRETIZATION OF A GENERAL ELASTIC MEDIA

Static Cases

Elastic media could be represented as an assemblage of finite elements interconnected at a discrete number of nodal points. The displacements of these nodal points will be the basic unknown parameters of the problem. A set of functions is chosen to define uniquely the state of displacement within each 'finite element' in terms of its nodal displacements. Let the displacements at any point within the element, $\bar{a}$, be defined as a column vector

$$\{f(x,y,z)\}^{\bar{a}} = \begin{Bmatrix} u_x \\ u_y \\ u_z \end{Bmatrix}^{\bar{a}} = [N]^{\bar{a}} \begin{Bmatrix} \bar{u}_i \\ \bar{u}_j \\ \bar{u}_k \\ \bar{u}_l \\ \vdots \end{Bmatrix} \tag{1)B}$$

where

$$\bar{u}_r = \begin{Bmatrix} u_x \\ u_y \\ u_z \end{Bmatrix}_r$$

and

$$[N]^{\bar{a}} = [IN_i, IN_j, IN_k, IN_l, \cdots\cdots\cdots]^{\bar{a}}$$

I is a three by three identity matrix and N_i's are the shape functions. The shape function matrix has size 3 by $3q$, where q is the number of nodes in each element.

The strains at any point can be determined from the relationship between strains and displacements.

$$\{e\}^{\bar{a}} = \begin{Bmatrix} e_{xx} \\ e_{yy} \\ e_{zz} \\ e_{xy} \\ e_{xz} \\ e_{yz} \end{Bmatrix}^{\bar{a}} = \begin{bmatrix} \frac{\partial}{\partial x} & 0 & 0 \\ 0 & \frac{\partial}{\partial y} & 0 \\ 0 & 0 & \frac{\partial}{\partial z} \\ \frac{\partial}{\partial y} & \frac{\partial}{\partial x} & 0 \\ \frac{\partial}{\partial z} & 0 & \frac{\partial}{\partial x} \\ 0 & \frac{\partial}{\partial z} & \frac{\partial}{\partial y} \end{bmatrix} \begin{Bmatrix} u_x \\ u_y \\ u_z \end{Bmatrix}^{\bar{a}} \qquad (2)B$$

Combine Eq.(1)B with Eq.(2)B, we now have

$$\{e\}^{\bar{a}} = [N']^{\bar{a}} \begin{Bmatrix} \bar{u}_i \\ \bar{u}_j \\ \bar{u}_k \\ \bar{u}_l \\ \vdots \end{Bmatrix} \qquad (3)B$$

where

$$[N']^{\bar{a}} = \begin{bmatrix} \frac{\partial N_i}{\partial x} & 0 & 0 & \frac{\partial N_j}{\partial x} & 0 & 0 & \cdot & \cdot & \cdot & \cdot \\ 0 & \frac{\partial N_i}{\partial y} & 0 & 0 & \frac{\partial N_j}{\partial y} & 0 & \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & \frac{\partial N_i}{\partial z} & 0 & 0 & \frac{\partial N_j}{\partial z} & \cdot & \cdot & \cdot & \cdot \\ \frac{\partial N_i}{\partial y} & \frac{\partial N_i}{\partial x} & 0 & \frac{\partial N_j}{\partial y} & \frac{\partial N_j}{\partial x} & 0 & \cdot & \cdot & \cdot & \cdot \\ \frac{\partial N_i}{\partial z} & 0 & \frac{\partial N_i}{\partial x} & \frac{\partial N_j}{\partial z} & 0 & \frac{\partial N_j}{\partial x} & \cdot & \cdot & \cdot & \cdot \\ 0 & \frac{\partial N_i}{\partial z} & \frac{\partial N_i}{\partial y} & 0 & \frac{\partial N_j}{\partial z} & \frac{\partial N_j}{\partial y} & \cdot & \cdot & \cdot & \cdot \end{bmatrix}^{\bar{a}}$$

For isotropic material, we have the well-known stress-strain relationships

$$
\begin{aligned}
e_{xx} &= \frac{\sigma_{xx}}{E} - \frac{\nu}{E}(\sigma_{yy}+\sigma_{zz}) & e_{xy} &= \frac{\sigma_{xy}}{G} \\
e_{yy} &= \frac{\sigma_{yy}}{E} - \frac{\nu}{E}(\sigma_{xx}+\sigma_{zz}) & e_{xz} &= \frac{\sigma_{xz}}{G} \\
e_{zz} &= \frac{\sigma_{zz}}{E} - \frac{\nu}{E}(\sigma_{xx}+\sigma_{yy}) & e_{yz} &= \frac{\sigma_{yz}}{G}
\end{aligned}
\tag{4)B}
$$

The constants E , G , ν are, respectively, Young's modulus, the shear modulus and Poisson's ratio.

Solving Eq.(4)B with respect to stresses, we find

$$\{\sigma\} = [D]\{e\} \tag{5)B}$$

where

$$\{\sigma\} = \begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{zz} \\ \sigma_{xy} \\ \sigma_{xz} \\ \sigma_{yz} \end{Bmatrix}$$

and

$$[D] = \frac{E(1-\nu)}{(1-\nu)(1-2\nu)} \begin{bmatrix} 1 & \frac{\nu}{1-\nu} & \frac{\nu}{1-\nu} & 0 & 0 & 0 \\ \frac{\nu}{1-\nu} & 1 & \frac{\nu}{1-\nu} & 0 & 0 & 0 \\ \frac{\nu}{1-\nu} & \frac{\nu}{1-\nu} & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1-2\nu}{2(1-\nu)} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1-2\nu}{2(1-\nu)} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1-2\nu}{2(1-\nu)} \end{bmatrix}$$

Combine Eq.(3)B and Eq.(5)B, we now have

$$\{\sigma\}^{\bar{a}} = [D]\,[N']^{\bar{a}} \begin{Bmatrix} \bar{u}_i \\ \bar{u}_j \\ \bar{u}_k \\ \bar{u}_l \\ \vdots \end{Bmatrix} \qquad (6)B$$

Let

$$\{F\}^{\bar{a}} = \begin{Bmatrix} \bar{F}_i \\ \bar{F}_j \\ \bar{F}_k \\ \bar{F}_l \\ \vdots \end{Bmatrix} \qquad (7)B$$

where

$$\bar{F}_r = \begin{Bmatrix} F_x \\ F_y \\ F_z \end{Bmatrix}_r$$

define the nodal forces which are equivalent statically to the internal stresses and the distributed loads on the element.

Let

$$\{b\}^{\bar{a}} = \rho \begin{Bmatrix} 0 \\ 0 \\ g_z \end{Bmatrix}^{\bar{a}} \qquad (8)B$$

be the 'body force' components.

Let us have a virtual displacement $\delta \left[\bar{u}_i \;\; \bar{u}_j \;\; \bar{u}_k \;\; \bar{u}_l \cdots\right]^T$ at the nodes. With the help of Eq.(1)B and (3)B, we have

$$\delta\{\bar{f}\}^{\bar{a}} = [N]^{\bar{a}}\,\delta \begin{Bmatrix} \bar{u}_i \\ \bar{u}_j \\ \bar{u}_k \\ \bar{u}_l \\ \vdots \end{Bmatrix} \tag{9-a)B}$$

and

$$\delta\{e\}^{\bar{a}} = [N']^{\bar{a}}\,\delta \begin{Bmatrix} \bar{u}_i \\ \bar{u}_j \\ \bar{u}_k \\ \bar{u}_l \\ \vdots \end{Bmatrix} \tag{9-b)B}$$

The virtual work done by the external nodal forces is equal to the sum of the products of the individual force components and the corresponding displacements,

$$\delta\left[\bar{u}_i \;\; \bar{u}_j \;\; \bar{u}_k \;\; \bar{u}_l \cdots\cdots\cdots\right]\{F\}^{\bar{a}}$$

Similarly, the virtual work per unit volume done by the stresses and 'body force' is

$$\begin{aligned} &\delta\{e\}^{\bar{a}T}\{\sigma\}^{\bar{a}} - \delta\{\bar{f}\}^{\bar{a}T}\{b\}^{\bar{a}} \\ &= \delta\left[\bar{u}_i \;\; \bar{u}_j \;\; \bar{u}_k \;\; \bar{u}_l \cdots\cdots\cdots\right]\left\{[N']^{\bar{a}T}\{\sigma\}^{\bar{a}} - [N]^{\bar{a}T}\{b\}^{\bar{a}}\right\} \end{aligned}$$

The external work is equal to the internal virtual work. Therefore, we have

$$\delta\left[\bar{u}_i \ \ \bar{u}_j \ \ \bar{u}_k \ \ \bar{u}_l \cdots\cdots\right]\{F\}^{\bar{a}}$$
$$= \delta\left[\bar{u}_i \ \ \bar{u}_j \ \ \bar{u}_k \ \ \bar{u}_l \cdots\cdots\right] \int_{V^{\bar{a}}} \left\{ [N']^{\bar{a}T}\{\sigma\}^{\bar{a}} - [N]^{\bar{a}T}\{b\}^{\bar{a}} \right\} dv^{\bar{a}}$$

For non-trivial $\delta\left[\bar{u}_i \ \ \bar{u}_j \ \ \bar{u}_k \ \ \bar{u}_l \cdots\cdots\right]$, we have

$$\{F\}^{\bar{a}} = \int_{V^{\bar{a}}} \left\{ [N']^{\bar{a}T}\{\sigma\}^{\bar{a}} - [N]^{\bar{a}T}\{b\}^{\bar{a}} \right\} dv^{\bar{a}} \qquad (10)B$$

With the help of Eq.(6)B, we now have

$$\{F\}^{\bar{a}} = [K]^{\bar{a}} \begin{Bmatrix} \bar{u}_i \\ \bar{u}_j \\ \bar{u}_k \\ \bar{u}_l \\ \vdots \end{Bmatrix} - \int_{V^{\bar{a}}} [N]^{\bar{a}T}\{b\}^{\bar{a}} dv^{\bar{a}} \qquad (11)B$$

where

$$[K]^{\bar{a}} = \int_{V^{\bar{a}}} [N']^{\bar{a}T}[D][N']^{\bar{a}} dv^{\bar{a}} \qquad (12)B$$

In general, external concentrated forces may exist at the nodes:

$$\{R\} = \begin{Bmatrix} \bar{R}_i \\ \bar{R}_j \\ \bar{R}_k \\ \bar{R}_l \\ \vdots \end{Bmatrix}$$

where

$$\bar{R}_r = \begin{Bmatrix} R_x \\ R_y \\ R_z \end{Bmatrix}_r$$

They will be added to the consideration of equilibrium at the nodes.

Now let us look at the elements near the boundary. If, at the boundary, displacements are specified, no problem arises. If the boundary is subject to a distributed external load, $\{\sigma\}^{\bar{b}}$ per unit area, a loading term on the nodes of the element which has a boundary face will now have to be added:

$$\{F\}^{\bar{b}} = -\int_{S^{\bar{b}}} [N]^{*T} [\bar{n}] \{\sigma\}^{\bar{b}} ds^{\bar{b}}$$

where

$$[\bar{n}] = \begin{bmatrix} l_x & 0 & 0 & l_y & l_z & 0 \\ 0 & l_y & 0 & l_x & 0 & l_z \\ 0 & 0 & l_z & 0 & l_x & l_y \end{bmatrix}$$

The preceding analysis for individual element may be applied directly to the whole continuum. The virtual work principle will be applied to the whole structure. Inter-element forces no longer needed to be considered.

Let

$$\{R\} = \begin{Bmatrix} \bar{R}_1 \\ \bar{R}_2 \\ \vdots \\ \bar{R}_Q \end{Bmatrix}$$

where

$$\bar{R}_r = \begin{Bmatrix} R_x \\ R_y \\ R_z \end{Bmatrix}_r$$

be the external concentrated forces. Q is the number of total nodal points.

The external virtual work during any virtual displacements of all nodes $\delta\left[u_1 \ u_2 \cdots\cdots\cdots u_Q\right]^T$ becomes

$$\delta\left[\bar{u}_1 \, \bar{u}_2 \cdots \bar{u}_Q\right] \begin{Bmatrix} \bar{R}_1 \\ \bar{R}_2 \\ \vdots \\ \bar{R}_Q \end{Bmatrix} + \sum_{\bar{a}=1}^{N} \int_{V^{\bar{a}}} \delta\left[\bar{u}_i \, \bar{u}_j \, \bar{u}_k \, \bar{u}_l \cdots\right]^{\bar{a}} [N]^{\bar{a}T} \{b\}^{\bar{a}} \, dv^{\bar{a}}$$

$$+ \sum_{\bar{b}=1}^{\bar{N}} \int_{S^{\bar{b}}} \delta\left[\bar{u}_i \, \bar{u}_j \, \bar{u}_k \, \bar{u}_l \cdots\right]^{a} [N]^{\bar{a}T} \left[\bar{\bar{n}}\right] \{\sigma\}^{\bar{b}} \, ds^{\bar{b}}$$

where N is the number of total elements and $\bar{N}$ is the number of elements at the boundaries. The above could be arranged and let the summation stands for addmix. We then have

$$\delta\left\{\begin{matrix}\bar{u}_1\\ \bar{u}_2\\ \vdots\\ \bar{u}_q\end{matrix}\right\}^T\left\{\left\{\begin{matrix}\bar{R}_1\\ \bar{R}_2\\ \vdots\\ \bar{R}_q\end{matrix}\right\}+\sum_{\bar{a}=1}^{N}\int_{V^{\bar{a}}}[N]^{\bar{a}T}\{b\}^{\bar{a}}dv^{\bar{a}}+\sum_{\bar{b}=1}^{\bar{N}}\int_{S^{\bar{b}}}[N]^{\bar{b}T}[\bar{n}]\{\sigma\}^{\bar{b}}ds^{\bar{b}}\right\}$$

The internal virtual work during any virtual displacement of all nodes $\delta\left[\bar{u}_1\ \bar{u}_2\cdots\cdots\cdots\bar{u}_q\right]^T$ is

$$\sum_{\bar{a}=1}^{N}\int_{V^{\bar{a}}}\delta\left\{\begin{matrix}\bar{u}_i\\ \bar{u}_j\\ \bar{u}_k\\ \bar{u}_l\\ \vdots\end{matrix}\right\}^T[N']^{\bar{a}T}[D][N']^{\bar{a}}\left\{\begin{matrix}\bar{u}_m\\ \bar{u}_n\\ \bar{u}_o\\ \bar{u}_p\\ \vdots\end{matrix}\right\}dv^{\bar{a}}$$

Again, the above could be rearranged and let the summation stands for addmix. We have

$$\delta\left\{\begin{matrix}\bar{u}_1\\ \bar{u}_2\\ \vdots\\ \bar{u}_q\end{matrix}\right\}^T\sum_{\bar{a}=1}^{N}\int_{V^{\bar{a}}}[N']^{\bar{a}T}[D][N]^{\bar{a}}dv^{\bar{a}}\left\{\begin{matrix}\bar{u}_1\\ \bar{u}_2\\ \vdots\\ \bar{u}_q\end{matrix}\right\}$$

The external virtual work is equal to the internal virtual work and, for non-trivial virtual displacement, we have

$$[K]\begin{Bmatrix}\bar{u}_1\\ \bar{u}_2\\ \vdots\\ \bar{u}_n\end{Bmatrix} = [G] + \begin{Bmatrix}\bar{R}_1\\ \bar{R}_2\\ \vdots\\ \bar{R}_n\end{Bmatrix}$$

$$+\sum_{\bar{b}=1}^{\bar{N}}\int_{S^{\bar{b}}}[N]^{\bar{b}T}[\bar{n}]\,\{\sigma\}^{\bar{b}}ds^{\bar{b}} \qquad (13)\text{B}$$

where

$$[K] = \sum_{\bar{a}=1}^{N}\int_{V^{\bar{a}}}[N']^{\bar{a}T}[D]\,[N']^{\bar{a}}\,dv^{\bar{a}}$$

$$(14)\text{B}$$

$$[G] = \sum_{\bar{a}=1}^{N}\int_{V^{\bar{a}}}[N]^{\bar{a}T}\{b\}^{\bar{a}}dv^{\bar{a}}$$

Dynamic Cases

When a structure is under dynamic loads, the displacements will vary with time. We must consider two sets of additional forces.

The first is the inertia force. By the well-known d'Alembert principle, we have the static equivalent,

$$-\rho \frac{\partial^2}{\partial t^2}\{\bar{f}(x,y,z,t)\}$$

$\{\bar{f}\}$ is the generalized displacement and it is a function of space and time coordinates.

The second force is due to resistances opposing the motion. Usually it is a linear, viscous type and the static equivalent,

$$-\mu \frac{\partial}{\partial t}\{\bar{f}(x,y,z,t)\}$$

Add the two additional forces to Eq.(13)B and recognize that the forcing functions and displacements are functions of both space and time. We have

$$[K]\begin{Bmatrix}\bar{u}_1\\ \bar{u}_2\\ \vdots\\ \bar{u}_n\end{Bmatrix}+\sum_{\bar{a}=1}^{N}\int_{V^{\bar{a}}}[N]^{\bar{a}T}\rho\{\ddot{\bar{f}}\}dv^{\bar{a}}+\sum_{\bar{a}=1}^{N}\int_{V^{\bar{a}}}[N]^{\bar{a}T}\mu\{\dot{\bar{f}}\}dv^{\bar{a}}$$

$$=[G]+\begin{Bmatrix}\bar{R}_1\\ \bar{R}_2\\ \vdots\\ \bar{R}_n\end{Bmatrix}+\sum_{\bar{b}=1}^{N}\int_{S^{\bar{b}}}[N]^{\bar{b}T}[\bar{\bar{n}}]\{\sigma\}^{\bar{b}}ds^{\bar{b}} \qquad (15)B$$

By Eq.(1)B, Eq.(15)B becomes

$$[M]\begin{Bmatrix}\ddot{\bar{u}}_1\\ \ddot{\bar{u}}_2\\ \vdots\\ \ddot{\bar{u}}_a\end{Bmatrix}+[C]\begin{Bmatrix}\dot{\bar{u}}_1\\ \dot{\bar{u}}_2\\ \vdots\\ \dot{\bar{u}}_a\end{Bmatrix}+[K]\begin{Bmatrix}\bar{u}_1\\ \bar{u}_2\\ \vdots\\ \bar{u}_a\end{Bmatrix}$$

$$=[G]+\begin{Bmatrix}\bar{R}_1\\ \bar{R}_2\\ \vdots\\ \bar{R}_a\end{Bmatrix}+\sum_{\bar{b}=1}^{\bar{N}}\int_{S^b}[N]^{\bar{b}T}[\bar{\bar{n}}]\{\sigma\}^{\bar{b}}ds^{\bar{b}} \tag{16)B}$$

where

$$[M]=\sum_{\bar{a}=1}^{N}\int_{V^{\bar{a}}}[N]^{\bar{a}T}\rho[N]^{\bar{a}}dv^{\bar{a}}$$

$$[C]=\sum_{\bar{a}=1}^{N}\int_{V^{\bar{a}}}[N]^{\bar{a}T}\mu[N]^{\bar{a}}dv^{\bar{a}} \tag{17)B}$$

The summation stands for addmix.

The mass matrix we are using is called 'consistent mass matrix'. In place of 'consistent mass matrix', 'lumped mass matrix' is often used. 'Lumped mass matrix' is always a diagonal matrix even if no actual concentrated masses existed. This approach is often taken although the results are less accurate.

The damping matrix, $[C]$ is called 'consistent damping matrix'. However, there are various alternative damping matrices. The selection depends on the problem.

One alternative damping matrix is proportional damping matrix. Proportional damping matrix is a matrix which is proportional to the stiffness matrix,

$$[C]=\beta[K]$$

where β is a constant. However, in this method the damping coefficient, η_i , is proportional to the natural frequency, ω_i . This will cause the high frequency components of the solution to be damped out.

Another alternative damping matrix is that of modal damping,

$$[C]=a[M]+b[K]$$

This method could be carried out to higher order type,

$$[C]=\sum_{j=1}^{J} e_j [M]\left\{[K]^{-1}[M]\right\}^{j}+\sum_{j=1}^{J} d_j [K]\left\{[M]^{-1}[K]\right\}^{j}$$

where J is selected by judgment and is less than the total number of the natural frequency of the structure.

APPENDIX C

FINITE ELEMENT DISCRETIZATION OF THE EQUATIONS OF MOTION FOR TWO-DIMENSIONAL SATURATED ELASTIC POROUS MEDIA

APPENDIX C

FINITE ELEMENT DISCRETIZATION OF THE EQUATIONS OF MOTION FOR TWO-DIMENSIONAL SATURATED ELASTIC POROUS MEDIA

The equations of motion for saturated elastic porous media, Eq.(17)A and Eq.(19)A, can easily be reduced into two-dimensional case. Eq.(17)A reduces to the following,

$$\begin{aligned} \frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} &= \rho \ddot{u}_x + \rho_f \ddot{w}_x \\ \frac{\partial \tau_{yx}}{\partial x} + \frac{\partial \tau_{yy}}{\partial y} &= \rho \ddot{u}_y + \rho_f \ddot{w}_y \end{aligned} \tag{1)C}$$

Eq. (19)A reduces to the following,

$$\begin{aligned} -\frac{\partial p}{\partial x} &= \rho_f \ddot{u}_x + \frac{1}{f} \rho_f \ddot{w}_x + \frac{\eta}{k} \dot{w}_x \\ -\frac{\partial p}{\partial y} &= \rho_f \ddot{u}_y + \frac{1}{f} \rho_f \ddot{w}_y + \frac{\eta}{k} \dot{w}_y \end{aligned} \tag{2)C}$$

The body force (gravitational force) is taken out of Eq.(1)C and Eq.(2)C, because its effect could easily be solved statically. From herein, all displacements and forces are values in excess of values due to gravitational force.

For two-dimensional plane strain, the strain-displacement relationships are

$$\{e\} = \begin{Bmatrix} e_{xx} \\ e_{yy} \\ e_{zz} \end{Bmatrix} = \begin{Bmatrix} \frac{\partial u_x}{\partial x} \\ \frac{\partial u_y}{\partial y} \\ \frac{\partial u_x}{\partial y} + \frac{\partial u_y}{\partial x} \end{Bmatrix} \tag{3)C}$$

and

$$e_{zz} = e_{xz} = e_{yz} = 0 \tag{4)C}$$

The stress-strain relationships are

$$\{\sigma\} = \begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{Bmatrix} = E_c \begin{bmatrix} 1 & \frac{\nu}{1-\nu} & 0 \\ \frac{\nu}{1-\nu} & 1 & 0 \\ 0 & 0 & \frac{1-2\nu}{2(1-\nu)} \end{bmatrix} \{e\} \tag{5)C}$$

where

$$E_c = \frac{(1-\nu)\,E}{(1-\nu)(1-2\nu)} \tag{6)C}$$

From Eq.(9)A, the water pressure in two-dimensional case reduces to

$$p = -\alpha M(e_{xx}+e_{yy}) + M\zeta \tag{7)C}$$

where

$$\zeta = -\left(\frac{\partial w_x}{\partial x} + \frac{\partial w_y}{\partial y}\right) \tag{8)C}$$

In matrix form, we have

$$p_f = -\alpha M \begin{bmatrix} \frac{\partial}{\partial x} & \frac{\partial}{\partial y} \end{bmatrix} \begin{Bmatrix} u_x \\ u_y \end{Bmatrix} - M \begin{bmatrix} \frac{\partial}{\partial x} & \frac{\partial}{\partial y} \end{bmatrix} \begin{Bmatrix} W_x \\ W_y \end{Bmatrix} \tag*{(9)C}$$

The bulk stress tensor is

$$\{\tau\} = \begin{Bmatrix} \tau_{xx} \\ \tau_{yy} \\ \tau_{xy} \end{Bmatrix} = \begin{Bmatrix} \sigma_{xx} - \alpha p_f \\ \sigma_{yy} - \alpha p_f \\ \sigma_{xy} \end{Bmatrix} \tag*{(10)C}$$

Combining Eq.(3)C, Eq.(5)C and Eq.(7)C, we have

$$\{\tau\} = E_c [D_0][D_1] \begin{Bmatrix} u_x \\ u_y \end{Bmatrix} + \alpha^2 M [D_2] \begin{Bmatrix} u_x \\ u_y \end{Bmatrix} + \alpha M [D_2] \begin{Bmatrix} W_x \\ W_y \end{Bmatrix} \tag*{(11)C}$$

where

$$[D_0] = \begin{bmatrix} 1 & \frac{\nu}{1-\nu} & 0 \\ \frac{\nu}{1-\nu} & 1 & 0 \\ 0 & 0 & \frac{1-2\nu}{2(1-\nu)} \end{bmatrix}$$

$$[D_1] = \begin{bmatrix} \frac{\partial}{\partial x} & 0 \\ 0 & \frac{\partial}{\partial y} \\ \frac{\partial}{\partial y} & \frac{\partial}{\partial x} \end{bmatrix} \tag*{(12)C}$$

$$[D_2] = \begin{bmatrix} \frac{\partial}{\partial x} & \frac{\partial}{\partial y} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} \\ 0 & 0 \end{bmatrix}$$

Add hysteretic damping ratios and we have

$$\{\tau\} = E_c[D_0][D_1]\begin{Bmatrix} u_x \\ u_y \end{Bmatrix} + E_c[D_3][D_1]\frac{\partial}{\partial t}\begin{Bmatrix} u_x \\ u_y \end{Bmatrix} + \alpha^2 M[D_2]\begin{Bmatrix} u_x \\ u_y \end{Bmatrix} + \alpha M[D_2]\begin{Bmatrix} W_x \\ W_y \end{Bmatrix} \tag{13)C}$$

where

$$[D_3] = \begin{bmatrix} \lambda_c & \frac{\lambda_c \nu}{1-\nu} & 0 \\ \frac{\lambda_c \nu}{1-\nu} & \lambda_c & 0 \\ 0 & 0 & \frac{\lambda_s(1-2\nu)}{2(1-\nu)} \end{bmatrix} \tag{14)C}$$

λ_c is the compressive hysteretic damping ratio and λ_s is the shearing hysteretic damping ratio.

For steady state solutions, let

$$\begin{Bmatrix} u_x \\ u_y \end{Bmatrix} = \begin{Bmatrix} U_x \\ U_y \end{Bmatrix} e^{i\Omega t} \qquad \begin{Bmatrix} w_x \\ w_y \end{Bmatrix} = \begin{Bmatrix} W_x \\ W_y \end{Bmatrix} e^{i\Omega t} \tag{15}$$

Also, let

$$\begin{aligned} \theta &= \frac{\Omega x}{V_c} \\ \phi &= \frac{\Omega y}{V_c} \\ V_c &= \sqrt{\frac{E_c + \alpha^2 M}{\rho}} \\ N &= \frac{\rho_f}{\rho} \\ K &= \frac{M}{E_c + \alpha^2 M} \end{aligned} \tag{16)C}$$

With aid of Eq.(15)C and Eq.(16)C, Eq.(9)C becomes

$$\beta e^{-i\Omega t} = -\alpha M \frac{\Omega}{V_c}\begin{bmatrix}\frac{\partial}{\partial\theta} & \frac{\partial}{\partial\phi}\end{bmatrix}\begin{Bmatrix}U_x\\U_y\end{Bmatrix} - M\frac{\Omega}{V_c}\begin{bmatrix}\frac{\partial}{\partial\theta} & \frac{\partial}{\partial\phi}\end{bmatrix}\begin{Bmatrix}W_x\\W_y\end{Bmatrix} \quad (17)C$$

and Eq.(13)C becomes

$$\{\tau\}e^{-i\Omega t} = E_c\frac{\Omega}{V_c}[D_0][D_4]\begin{Bmatrix}U_x\\U_y\end{Bmatrix} + iE_c\frac{\Omega^2}{V_c}[D_3][D_4]\begin{Bmatrix}U_x\\U_y\end{Bmatrix}$$
$$+\alpha^2 M\frac{\Omega}{V_c}[D_5]\begin{Bmatrix}U_x\\U_y\end{Bmatrix} + \alpha M\frac{\Omega}{V_c}[D_5]\begin{Bmatrix}W_x\\W_y\end{Bmatrix} \quad (18)C$$

where

$$[D_4] = \begin{bmatrix}\frac{\partial}{\partial\theta} & 0\\ 0 & \frac{\partial}{\partial\phi}\\ \frac{\partial}{\partial\phi} & \frac{\partial}{\partial\theta}\end{bmatrix}$$

$$[D_5] = \begin{bmatrix}\frac{\partial}{\partial\theta} & \frac{\partial}{\partial\phi}\\ \frac{\partial}{\partial\theta} & \frac{\partial}{\partial\phi}\\ 0 & 0\end{bmatrix} = \begin{bmatrix}1 & 1 & 0\\ 1 & 1 & 0\\ 0 & 0 & 0\end{bmatrix}[D_4] \quad (19)C$$

With aid of Eq.(18)C and dividing each term by $\rho\Omega^2$, Eq.(1)C can be written in matrix form as following,

$$[D_4]^T\left\langle \frac{E_c}{\rho V_c^2}[D_0][D_4]\begin{Bmatrix}U_x\\U_y\end{Bmatrix} + \frac{i\Omega E_c}{\rho V_c^2}[D_3][D_4]\begin{Bmatrix}U_x\\U_y\end{Bmatrix}\right.$$
$$\left. +\alpha^2 K[D_5]\begin{Bmatrix}U_x\\U_y\end{Bmatrix} + \alpha K[D_5]\begin{Bmatrix}W_x\\W_y\end{Bmatrix}\right\rangle + \begin{Bmatrix}U_x\\U_y\end{Bmatrix} + N\begin{Bmatrix}W_x\\W_y\end{Bmatrix} = 0 \quad (20)C$$

With aid of Eq.(17)C and dividing each term by $\rho\Omega^2$, Eq.(2)C can be written in matrix form as following,

$$\begin{Bmatrix}\frac{\partial}{\partial\theta}\\ \frac{\partial}{\partial\phi}\end{Bmatrix}\left\langle \alpha K\left[\frac{\partial}{\partial\theta}\quad \frac{\partial}{\partial\phi}\right]\begin{Bmatrix}U_x\\ U_y\end{Bmatrix}+K\left[\frac{\partial}{\partial\theta}\quad \frac{\partial}{\partial\phi}\right]\begin{Bmatrix}W_x\\ W_y\end{Bmatrix}\right\rangle$$

$$+N\begin{Bmatrix}U_x\\ U_y\end{Bmatrix}+\frac{N}{f}\begin{Bmatrix}W_x\\ W_y\end{Bmatrix}-\frac{i\eta}{\rho k\Omega}\begin{Bmatrix}W_x\\ W_y\end{Bmatrix}=0 \tag{21)C}$$

Start finite element discretization of Eq.(20)C and Eq.(21)C. From Eq.(9)D and Eq.(11)D, the bulk displacements within each finite element, $\bar{a}$, can be expressed as

$$\begin{Bmatrix}U_x\\ U_y\end{Bmatrix}_{\bar{a}}=[A]_{\bar{a}}\begin{Bmatrix}U_{x_i}\\ \vdots\\ U_{x_l}\\ U_{y_i}\\ \vdots\\ U_{y_l}\end{Bmatrix} \tag{22)C}$$

where

$$[A]_{\bar{a}}=\begin{bmatrix}[N]_{\bar{a}} & [0]\\ [0] & [N]_{\bar{a}}\end{bmatrix}$$

$$[N]_{\bar{a}}=\left[N_i\quad N_j\quad N_k\quad N_l\right] \tag{23)C}$$

$N_m =$ shape function $\qquad m=i,j,k,l$

Similarly, the relative fluid displacements can be expressed as

$$\begin{Bmatrix} W_x \\ W_y \end{Bmatrix}_{\bar{a}} = [B]_{\bar{a}} \begin{Bmatrix} W_{x_i} \\ \vdots \\ W_{x_Q} \\ W_{y_i} \\ \vdots \\ W_{y_Q} \end{Bmatrix} \quad \text{and} \quad [B]_{\bar{a}} = [A]_{\bar{a}} \qquad (24)C$$

Discretize Eq.(20)C, then premultiply each term by $[A]^T_{\bar{a}}$, addmix and itegrate with respect to the whole area S, such that

$$\sum_{\bar{a}=1}^{L} \int_{S^{\bar{a}}} [A]^T_{\bar{a}} [A]_{\bar{a}} ds \begin{Bmatrix} U_{x_1} \\ \vdots \\ U_{x_Q} \\ U_{y_1} \\ \vdots \\ U_{y_Q} \end{Bmatrix} + N \sum_{\bar{a}=1}^{L} \int_{S^a} [A]^T_a [B]_a \, ds \begin{Bmatrix} W_{x_1} \\ \vdots \\ W_{x_Q} \\ W_{y_1} \\ \vdots \\ W_{y_Q} \end{Bmatrix}$$

$$+ \sum_{\bar{a}=1}^{L} \int_{S^{\bar{a}}} [A]^T_{\bar{a}} [D_4]^T \left\langle \frac{E_s}{\rho V_c^2} [D_1][D_4][A]_{\bar{a}} + \frac{i\Omega E_s}{\rho V_c^2} [D_1][D_4][A]_{\bar{a}} + \alpha^2 K [D_5][A]_{\bar{a}} \right\rangle ds \begin{Bmatrix} U_{x_1} \\ \vdots \\ U_{x_Q} \\ U_{y_1} \\ \vdots \\ U_{y_Q} \end{Bmatrix}$$

$$+ \sum_{\bar{a}=1}^{L} \int_{S^{\bar{a}}} [A]^T_{\bar{a}} [D_4]^T \alpha K \, [D_5][B]_{\bar{a}} ds \begin{Bmatrix} W_{x_1} \\ \vdots \\ W_{x_Q} \\ W_{y_1} \\ \vdots \\ W_{y_Q} \end{Bmatrix} = 0 \qquad (25)C$$

Note that the summation stands for addmix. L is number of elements and Q is number of total nodal points.

Let us look at the last term of Eq.(25)C.

$$\underline{\underline{I}} = \alpha K \sum_{\bar{a}=1}^{L} \int_{S^{\bar{a}}} [A]^{T}_{\bar{a}} [D_t]^{T} \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} [B']^{T}_{\bar{a}} \, ds \begin{Bmatrix} W_{x_1} \\ \vdots \\ W_{x_n} \\ W_{y_1} \\ \vdots \\ W_{y_n} \end{Bmatrix} \quad (26)C$$

where

$$\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} [B']_{\bar{a}} = [D_s][B]_{\bar{a}}$$

$$= \begin{bmatrix} \frac{\partial}{\partial\theta} & \frac{\partial}{\partial\phi} \\ \frac{\partial}{\partial\theta} & \frac{\partial}{\partial\phi} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} N_i & N_j & N_k & N_l & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & N_i & N_j & N_k & N_l \end{bmatrix}_{\bar{a}}$$

$$= \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \begin{bmatrix} \frac{\partial N_i}{\partial\theta} & \frac{\partial N_j}{\partial\theta} & \frac{\partial N_k}{\partial\theta} & \frac{\partial N_l}{\partial\theta} & \frac{\partial N_i}{\partial\phi} & \frac{\partial N_j}{\partial\phi} & \frac{\partial N_k}{\partial\phi} & \frac{\partial N_l}{\partial\phi} \end{bmatrix}_{\bar{a}} \quad (27)C$$

Expand Eq.(26)C

$$\underline{\underline{I}} =$$

$$\alpha K \sum_{\bar{a}=1}^{L} \int_{S^{\bar{a}}} \begin{bmatrix} N_i & 0 \\ N_j & 0 \\ N_k & 0 \\ N_l & 0 \\ 0 & N_i \\ 0 & N_j \\ 0 & N_k \\ 0 & N_l \end{bmatrix}_{\bar{a}} \begin{bmatrix} \frac{\partial}{\partial\theta} & 0 & \frac{\partial}{\partial\phi} \\ 0 & \frac{\partial}{\partial\phi} & \frac{\partial}{\partial\theta} \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \begin{bmatrix} \frac{\partial N_i}{\partial\theta} & \frac{\partial N_j}{\partial\theta} & \frac{\partial N_k}{\partial\theta} & \frac{\partial N_l}{\partial\theta} & \frac{\partial N_i}{\partial\phi} & \frac{\partial N_j}{\partial\phi} & \frac{\partial N_k}{\partial\phi} & \frac{\partial N_l}{\partial\phi} \end{bmatrix}_{\bar{a}} ds \begin{Bmatrix} W^{T}_{x_1} \\ \vdots \\ W^{T}_{x_n} \\ W^{T}_{y_1} \\ \vdots \\ W^{T}_{y_n} \end{Bmatrix}$$

$$
= \alpha K \sum_{e=1}^{L} \int_{S^e}
\begin{bmatrix}
N_i \frac{\partial^2 N_i}{\partial\theta\partial\theta} & N_i \frac{\partial^2 N_j}{\partial\theta\partial\theta} & N_i \frac{\partial^2 N_k}{\partial\theta\partial\theta} & N_i \frac{\partial^2 N_l}{\partial\theta\partial\theta} & N_i \frac{\partial^2 N_i}{\partial\theta\partial\phi} & N_i \frac{\partial^2 N_j}{\partial\theta\partial\phi} & N_i \frac{\partial^2 N_k}{\partial\theta\partial\phi} & N_i \frac{\partial^2 N_l}{\partial\theta\partial\phi} \\
N_j \frac{\partial^2 N_i}{\partial\theta\partial\theta} & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
N_k \frac{\partial^2 N_i}{\partial\theta\partial\theta} & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
N_l \frac{\partial^2 N_i}{\partial\theta\partial\theta} & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
N_i \frac{\partial^2 N_i}{\partial\phi\partial\theta} & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
N_j \frac{\partial^2 N_i}{\partial\phi\partial\theta} & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
N_k \frac{\partial^2 N_i}{\partial\phi\partial\theta} & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
N_l \frac{\partial^2 N_i}{\partial\phi\partial\theta} & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot
\end{bmatrix}_e dS
\begin{Bmatrix} W_{x_1} \\ \vdots \\ W_{x_n} \\ W_{y_1} \\ \vdots \\ W_{y_n} \end{Bmatrix}
\tag{(28)C}
$$

Consider the first row of Eq.(28)C, and use integration by parts.

$$
\begin{aligned}
&\int_{S^e} \left[N_i \frac{\partial^2 N_i}{\partial\theta\partial\theta} \; N_i \frac{\partial^2 N_j}{\partial\theta\partial\theta} \; N_i \frac{\partial^2 N_k}{\partial\theta\partial\theta} \; N_i \frac{\partial^2 N_l}{\partial\theta\partial\theta} \; N_i \frac{\partial^2 N_i}{\partial\theta\partial\phi} \; N_i \frac{\partial^2 N_j}{\partial\theta\partial\phi} \; N_i \frac{\partial^2 N_k}{\partial\theta\partial\phi} \; N_i \frac{\partial^2 N_l}{\partial\theta\partial\phi} \right]_e d\theta\, d\phi \\
&= \int_{\phi} \left[N_i \frac{\partial N_i}{\partial\theta} \; N_i \frac{\partial N_j}{\partial\theta} \; N_i \frac{\partial N_k}{\partial\theta} \; N_i \frac{\partial N_l}{\partial\theta} \; N_i \frac{\partial N_i}{\partial\phi} \; N_i \frac{\partial N_j}{\partial\phi} \; N_i \frac{\partial N_k}{\partial\phi} \; N_i \frac{\partial N_l}{\partial\phi} \right]_{\theta_1}^{\theta_2} d\phi \\
&- \int_{S^e} \left[\frac{\partial N_i}{\partial\theta}\frac{\partial N_i}{\partial\theta} \; \frac{\partial N_i}{\partial\theta}\frac{\partial N_j}{\partial\theta} \; \frac{\partial N_i}{\partial\theta}\frac{\partial N_k}{\partial\theta} \; \frac{\partial N_i}{\partial\theta}\frac{\partial N_l}{\partial\theta} \; \frac{\partial N_i}{\partial\theta}\frac{\partial N_i}{\partial\phi} \; \frac{\partial N_i}{\partial\theta}\frac{\partial N_j}{\partial\phi} \; \frac{\partial N_i}{\partial\theta}\frac{\partial N_k}{\partial\phi} \; \frac{\partial N_i}{\partial\theta}\frac{\partial N_l}{\partial\phi} \right]_e dS \\
&= \oint_{L^e} \left[N_i \frac{\partial N_i}{\partial\theta} \; N_i \frac{\partial N_j}{\partial\theta} \; N_i \frac{\partial N_k}{\partial\theta} \; N_i \frac{\partial N_l}{\partial\theta} \; N_i \frac{\partial N_i}{\partial\phi} \; N_i \frac{\partial N_j}{\partial\phi} \; N_i \frac{\partial N_k}{\partial\phi} \; N_i \frac{\partial N_l}{\partial\phi} \right] n_\theta \, dl \\
&- \int_{S^e} \left[\frac{\partial N_i}{\partial\theta}\frac{\partial N_i}{\partial\theta} \; \frac{\partial N_i}{\partial\theta}\frac{\partial N_j}{\partial\theta} \; \frac{\partial N_i}{\partial\theta}\frac{\partial N_k}{\partial\theta} \; \frac{\partial N_i}{\partial\theta}\frac{\partial N_l}{\partial\theta} \; \frac{\partial N_i}{\partial\theta}\frac{\partial N_i}{\partial\phi} \; \frac{\partial N_i}{\partial\theta}\frac{\partial N_j}{\partial\phi} \; \frac{\partial N_i}{\partial\theta}\frac{\partial N_k}{\partial\phi} \; \frac{\partial N_i}{\partial\theta}\frac{\partial N_l}{\partial\phi} \right]_e dS
\end{aligned}
$$

in which n_θ is the direction cosine of the outward normal in the θ-direction, and integral $L_{\bar{a}}$ is taken over the whole boundary of each element.

Carry out similar procedure for all rows. We now have

$$\underline{\mathrm{I}} = \alpha K \sum_{\bar{a}=1}^{L} \oint_{L_{\bar{a}}} [A]^T_{\bar{a}} [\bar{n}] [D_5] [B]_{\bar{a}} \, dl \begin{Bmatrix} W_{x_1} \\ \vdots \\ W_{x_n} \\ W_{y_1} \\ \vdots \\ W_{y_n} \end{Bmatrix}$$

$$- \alpha K \sum_{\bar{a}=1}^{L} \int_{S_{\bar{a}}} [B']^T_{\bar{a}} [B']_{\bar{a}} \, ds \begin{Bmatrix} W_{x_1} \\ \vdots \\ W_{x_n} \\ W_{y_1} \\ \vdots \\ W_{y_n} \end{Bmatrix} \qquad (29)C$$

where

$$[\bar{n}] = \begin{bmatrix} n_\theta & 0 & n_\phi \\ 0 & n_\phi & n_\theta \end{bmatrix}$$

Now look at the second last term of Eq.(25)C.

$$\underline{\mathrm{II}} =$$

$$\sum_{\bar{a}=1}^{L} \int_{S_{\bar{a}}} [A]^T_{\bar{a}} [D_4]^T \left\langle \frac{E_c}{\rho C_1^2} [D_1][D_4][A]_{\bar{a}} + \frac{(1+\nu)E_c}{\rho C_2^2} [D_3][D_4][A]_{\bar{a}} + \alpha^2 K [D_4][A]_{\bar{a}} \right\rangle ds \begin{Bmatrix} U_{x_1} \\ \vdots \\ U_{x_n} \\ U_{y_1} \\ \vdots \\ U_{y_n} \end{Bmatrix}$$

Integrate by parts.

$$\mathrm{I} =$$

$$\sum_{\bar{a}=1}^{L} \oint_{L_{\bar{a}}} [A]_{\bar{a}}^{T} [\bar{n}] \left\langle \frac{E_c}{\rho V_c^2} [D_1][D_4][A]_{\bar{a}} + \frac{i\Omega E_c}{\rho V_c^2} [D_2][D_4][A]_{\bar{a}} + \alpha^2 K [D_3][A]_{\bar{a}} \right\rangle dl \begin{Bmatrix} U_{x_1} \\ \vdots \\ U_{x_n} \\ U_{y_1} \\ \vdots \\ U_{y_n} \end{Bmatrix}$$

$$- \sum_{\bar{a}=1}^{L} \int_{S_{\bar{a}}} [A']_{\bar{a}}^{T} \left\langle \frac{E_c}{\rho V_c^2} [D_1] + \frac{i\Omega E_c}{\rho V_c^2} [D_2] + \alpha^2 K \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \right\rangle [A']_{\bar{a}} \, ds \begin{Bmatrix} U_{x_1} \\ \vdots \\ U_{x_n} \\ U_{y_1} \\ \vdots \\ U_{y_n} \end{Bmatrix}$$

(30)C

where

$$[A']_{\bar{a}} = [D_4][A]_{\bar{a}}$$

$$= \begin{bmatrix} \frac{\partial N_i}{\partial \theta} & \frac{\partial N_j}{\partial \theta} & \frac{\partial N_k}{\partial \theta} & \frac{\partial N_l}{\partial \theta} & 0 & 0 & 0 & 0 \\ & & & & \frac{\partial N_i}{\partial \phi} & \frac{\partial N_j}{\partial \phi} & \frac{\partial N_k}{\partial \phi} & \frac{\partial N_l}{\partial \phi} \\ \frac{\partial N_i}{\partial \phi} & \frac{\partial N_j}{\partial \phi} & \frac{\partial N_k}{\partial \phi} & \frac{\partial N_l}{\partial \phi} & \frac{\partial N_i}{\partial \theta} & \frac{\partial N_j}{\partial \theta} & \frac{\partial N_k}{\partial \theta} & \frac{\partial N_l}{\partial \theta} \end{bmatrix}_{\bar{a}} \qquad (31)C$$

Combine Eq.(29)C and Eq.(30)C.

$$\underline{\overline{\mathrm{I}}} + \underline{\overline{\mathrm{II}}} =$$

$$\sum_{\bar{a}=1}^{L} \oint_{L_{\bar{a}}} [A]_{\bar{a}}^{T} [\bar{n}] \left\langle \left\{ \frac{E_c}{\rho V_c^2} [D_1][D_4] + \frac{i\Omega E_c}{\rho V_c^2} [D_2][D_4] + \alpha^2 K [D_3] \right\} \begin{Bmatrix} U_x \\ U_y \end{Bmatrix}_{\bar{a}} + \alpha K [D_3] \begin{Bmatrix} W_x \\ W_y \end{Bmatrix}_{\bar{a}} \right\rangle dl$$

$$- \sum_{\bar{a}=1}^{L} \int_{S_{\bar{a}}} [A']_{\bar{a}}^{T} \left\langle \frac{E_c}{\rho V_c^2} [D_1] + \frac{i\Omega E_c}{\rho V_c^2} [D_2] + \alpha^2 K \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \right\rangle [A']_{\bar{a}} \, ds \begin{Bmatrix} U_{x_1} \\ \vdots \\ U_{x_n} \\ U_{y_1} \\ \vdots \\ U_{y_n} \end{Bmatrix}$$

$$- \alpha K \sum_{\bar{a}=1}^{L} \int_{S_{\bar{a}}} [B']_{\bar{a}}^{T} [B']_{\bar{a}} \, ds \begin{Bmatrix} W_{x_1} \\ \vdots \\ W_{x_n} \\ W_{y_1} \\ \vdots \\ W_{y_n} \end{Bmatrix}$$

With help of Eq.(18)C, the above becomes

$$\underline{I} + \underline{II} = \frac{1}{\rho \Omega V_c e^{i\Omega t}} \sum_{\bar{a}=1}^{L} \oint_{L^{\bar{a}}} [A]_{\bar{a}}^{T} [\bar{n}] \begin{Bmatrix} \tau_{xx} \\ \tau_{yy} \\ \tau_{xy} \end{Bmatrix} dl$$

$$- \sum_{\bar{a}=1}^{L} \int_{S\bar{a}} [A']_{\bar{a}}^{T} \left\langle \frac{E_c}{\rho V_c^2} [D_0] + \frac{i\Omega E_c}{\rho V_c^2} [D_s] + \alpha^2 K \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \right\rangle [A']_{\bar{a}} ds \begin{Bmatrix} U_{x_1} \\ \vdots \\ U_{x_n} \\ U_{y_1} \\ \vdots \\ U_{y_n} \end{Bmatrix}$$

$$- \alpha K \sum_{\bar{a}=1}^{L} \int_{S\bar{a}} [B']_{\bar{a}}^{T} [B']_{\bar{a}} ds \begin{Bmatrix} W_{x_1} \\ \vdots \\ W_{x_n} \\ W_{y_1} \\ \vdots \\ W_{y_n} \end{Bmatrix} \qquad (32)C$$

Let

$$[M] = \sum_{\bar{a}=1}^{L} \int_{S\bar{a}} [A]_{\bar{a}}^{T} [A]_{\bar{a}} ds = \sum_{\bar{a}=1}^{L} \int_{S\bar{a}} [A]_{\bar{a}}^{T} [B]_{\bar{a}} ds$$

$$[E] = \sum_{\bar{a}=1}^{L} \int_{S\bar{a}} [B']_{\bar{a}}^{T} [B']_{\bar{a}} ds \qquad (33)C$$

$$[K] = \sum_{\bar{a}=1}^{L} \int_{S\bar{a}} [A']_{\bar{a}}^{T} \left\langle \frac{E_c}{\rho V_c^2} [D_0] + \frac{i\Omega E_c}{\rho V_c^2} [D_s] + \alpha^2 K \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \right\rangle [A']_{\bar{a}} ds$$

Now Eq.(25)C can be expressed as

$$[M] \begin{Bmatrix} U_{x_1} \\ \vdots \\ U_{x_n} \\ U_{y_1} \\ \vdots \\ U_{y_n} \end{Bmatrix} + N [M] \begin{Bmatrix} W_{x_1} \\ \vdots \\ W_{x_n} \\ W_{y_1} \\ \vdots \\ W_{y_n} \end{Bmatrix} - [K] \begin{Bmatrix} U_{x_1} \\ \vdots \\ U_{x_n} \\ U_{y_1} \\ \vdots \\ U_{y_n} \end{Bmatrix} - \alpha K [E] \begin{Bmatrix} W_{x_1} \\ \vdots \\ W_{x_n} \\ W_{y_1} \\ \vdots \\ W_{y_n} \end{Bmatrix}$$

$$+ \frac{1}{\rho \Omega V_c e^{i\Omega t}} \sum_{\bar{a}=1}^{L} \oint_{L\bar{a}} [A]_{\bar{a}}^{T} [\bar{n}] \begin{Bmatrix} \tau_{xx} \\ \tau_{yy} \\ \tau_{xy} \end{Bmatrix} dl = 0 \qquad (34)C$$

Discretize Eq.(21)C, then premultiply each term by $[A]^T_{\bar{a}}$, addmix and integrate with respect to the whole area S, such that

$$N\sum_{\bar{a}=1}^{L}\int_{S_{\bar{a}}}[A]^T_{\bar{a}}[A]_{\bar{a}}\,ds\begin{Bmatrix}U_{x_1}\\ \vdots\\ U_{x_e}\\ U_{y_1}\\ \vdots\\ U_{y_e}\end{Bmatrix}+\left(\frac{N}{f}-\frac{i\eta}{\rho k\Omega}\right)\sum_{\bar{a}=1}^{L}[A]^T_{\bar{a}}[B]_{\bar{a}}\,ds\begin{Bmatrix}W_{x_1}\\ \vdots\\ W_{x_e}\\ W_{y_1}\\ \vdots\\ W_{y_e}\end{Bmatrix}$$

$$+\sum_{\bar{a}=1}^{L}\int_{S_{\bar{a}}}[A]^T_{\bar{a}}\begin{Bmatrix}\frac{\partial}{\partial\theta}\\ \frac{\partial}{\partial\phi}\end{Bmatrix}\alpha K\begin{bmatrix}\frac{\partial}{\partial\theta} & \frac{\partial}{\partial\phi}\end{bmatrix}[A]_{\bar{a}}\,ds\begin{Bmatrix}U_{x_1}\\ \vdots\\ U_{x_e}\\ U_{y_1}\\ \vdots\\ U_{y_e}\end{Bmatrix}$$

$$+\sum_{\bar{a}=1}^{L}\int_{S_{\bar{a}}}[A]^T_{\bar{a}}\begin{Bmatrix}\frac{\partial}{\partial\theta}\\ \frac{\partial}{\partial\phi}\end{Bmatrix}K\begin{bmatrix}\frac{\partial}{\partial\theta} & \frac{\partial}{\partial\phi}\end{bmatrix}[B]_{\bar{a}}\,ds\begin{Bmatrix}W_{x_1}\\ \vdots\\ W_{x_e}\\ W_{y_1}\\ \vdots\\ W_{y_e}\end{Bmatrix}=0 \qquad (35)C$$

Let us look at the last term of Eq.(35)C.

$$\underline{\underline{III}}=\sum_{\bar{a}=1}^{L}\int_{S_{\bar{a}}}[A]^T_{\bar{a}}\begin{Bmatrix}\frac{\partial}{\partial\theta}\\ \frac{\partial}{\partial\phi}\end{Bmatrix}K\begin{bmatrix}\frac{\partial}{\partial\theta} & \frac{\partial}{\partial\phi}\end{bmatrix}[B]_{\bar{a}}\,ds\begin{Bmatrix}W_{x_1}\\ \vdots\\ W_{x_e}\\ W_{y_1}\\ \vdots\\ W_{y_e}\end{Bmatrix}$$

$$=\sum_{\bar{a}=1}^{L}\int_{S_{\bar{a}}}\begin{bmatrix}N_i & 0\\ N_j & 0\\ N_k & 0\\ N_l & 0\\ 0 & N_i\\ 0 & N_j\\ 0 & N_k\\ 0 & N_l\end{bmatrix}_{\bar{a}}\begin{Bmatrix}\frac{\partial}{\partial\theta}\\ \frac{\partial}{\partial\phi}\end{Bmatrix}K\begin{bmatrix}\frac{\partial}{\partial\theta} & \frac{\partial}{\partial\phi}\end{bmatrix}\begin{bmatrix}N_i & N_j & N_k & N_l & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & N_i & N_j & N_k & N_l\end{bmatrix}_{\bar{a}}ds\begin{Bmatrix}W_{x_1}\\ \vdots\\ W_{x_e}\\ W_{y_1}\\ \vdots\\ W_{y_e}\end{Bmatrix}$$

$$\underline{\overline{III}} =$$

$$\sum_{\bar{a}=1}^{L}\int_{S_{\bar{a}}} K \begin{bmatrix} N_i\frac{\partial^2 N_i}{\partial\theta\partial\theta} & N_i\frac{\partial^2 N_i}{\partial\theta\partial\theta} & N_i\frac{\partial^2 N_i}{\partial\theta\partial\theta} & N_i\frac{\partial^2 N_i}{\partial\theta\partial\theta} & N_i\frac{\partial^2 N_i}{\partial\theta\partial\phi} & N_i\frac{\partial^2 N_i}{\partial\theta\partial\phi} & N_i\frac{\partial^2 N_i}{\partial\theta\partial\phi} & N_i\frac{\partial^2 N_i}{\partial\theta\partial\phi} \\ N_i\frac{\partial^2 N_i}{\partial\theta\partial\theta} & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ N_i\frac{\partial^2 N_i}{\partial\theta\partial\theta} & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ N_i\frac{\partial^2 N_i}{\partial\theta\partial\theta} & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ N_i\frac{\partial^2 N_i}{\partial\theta\partial\theta} & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ N_i\frac{\partial^2 N_i}{\partial\theta\partial\theta} & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ N_i\frac{\partial^2 N_i}{\partial\theta\partial\theta} & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ N_i\frac{\partial^2 N_i}{\partial\phi\partial\theta} & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \end{bmatrix}_{\bar{a}} ds \begin{Bmatrix} W_{x_1} \\ \vdots \\ W_{x_e} \\ W_{y_1} \\ \vdots \\ W_{y_e} \end{Bmatrix}$$

Integrate by parts.

$$\underline{\overline{III}} = \sum_{\bar{a}=1}^{L}\oint_{L_{\bar{a}}}[A]_{\bar{a}}^{T}\begin{Bmatrix} n_\theta \\ n_\phi \end{Bmatrix} K\begin{bmatrix}\frac{\partial}{\partial\theta} & \frac{\partial}{\partial\phi}\end{bmatrix}[B]_{\bar{a}}\,dl \begin{Bmatrix} W_{x_1} \\ \vdots \\ W_{x_e} \\ W_{y_1} \\ \vdots \\ W_{y_e} \end{Bmatrix} - K\sum_{\bar{a}=1}^{L}\int_{S_{\bar{a}}}[B']_{\bar{a}}^{T}[B']_{\bar{a}}\,ds \begin{Bmatrix} W_{x_1} \\ \vdots \\ W_{x_e} \\ W_{y_1} \\ \vdots \\ W_{y_e} \end{Bmatrix}$$

$$= \sum_{\bar{a}=1}^{L}\oint_{L_{\bar{a}}}[A]_{\bar{a}}^{T}\begin{Bmatrix} n_\theta \\ n_\phi \end{Bmatrix} K\begin{bmatrix}\frac{\partial}{\partial\theta} & \frac{\partial}{\partial\phi}\end{bmatrix}[B]_{\bar{a}}\,dl \begin{Bmatrix} W_{x_1} \\ \vdots \\ W_{x_e} \\ W_{y_1} \\ \vdots \\ W_{y_e} \end{Bmatrix} - K[E]\begin{Bmatrix} W_{x_1} \\ \vdots \\ W_{x_e} \\ W_{y_1} \\ \vdots \\ W_{y_e} \end{Bmatrix} \quad (36)C$$

Similarly, the second last term of Eq.(35)C is to be integrated by parts and we have

$$\underline{\overline{IV}} = \sum_{\bar{a}=1}^{L}\int_{S_{\bar{a}}}[A]_{\bar{a}}^{T}\begin{Bmatrix} \frac{\partial}{\partial\theta} \\ \frac{\partial}{\partial\phi} \end{Bmatrix}\alpha K\begin{bmatrix}\frac{\partial}{\partial\theta} & \frac{\partial}{\partial\phi}\end{bmatrix}[A]_{\bar{a}}\,ds \begin{Bmatrix} U_{x_1} \\ \vdots \\ U_{x_e} \\ U_{y_1} \\ \vdots \\ U_{y_e} \end{Bmatrix}$$

$$\underline{\overline{\text{IV}}} =$$

$$\sum_{\bar{a}=1}^{L} \oint_{L_{\bar{a}}} [A]_{\bar{a}}^{T} \begin{Bmatrix} n_\theta \\ n_\phi \end{Bmatrix} \alpha K \left[\frac{\partial}{\partial\theta} \quad \frac{\partial}{\partial\phi} \right] [A]_{\bar{a}} \, dl \begin{Bmatrix} U_{x_1} \\ \vdots \\ U_{x_e} \\ U_{y_1} \\ \vdots \\ U_{y_e} \end{Bmatrix} - \alpha K [E] \begin{Bmatrix} U_{x_1} \\ \vdots \\ U_{x_e} \\ U_{y_1} \\ \vdots \\ U_{y_e} \end{Bmatrix}$$

(37)C

Combine Eq.(36)C and Eq.(37)C.

$$\underline{\overline{\text{III}}} + \underline{\overline{\text{IV}}} =$$

$$\sum_{\bar{a}=1}^{L} \oint_{L_{\bar{a}}} [A]_{\bar{a}}^{T} \begin{Bmatrix} n_\theta \\ n_\phi \end{Bmatrix} \left\langle \alpha K \left[\frac{\partial}{\partial\theta} \quad \frac{\partial}{\partial\phi} \right] \begin{Bmatrix} U_x \\ U_y \end{Bmatrix}_{\bar{a}} + K \left[\frac{\partial}{\partial\theta} \quad \frac{\partial}{\partial\phi} \right] \begin{Bmatrix} W_x \\ W_y \end{Bmatrix}_{\bar{a}} \right\rangle dl$$

$$-\alpha K [E] \begin{Bmatrix} U_{x_1} \\ \vdots \\ U_{x_e} \\ U_{y_1} \\ \vdots \\ U_{y_e} \end{Bmatrix} - K [E] \begin{Bmatrix} W_{x_1} \\ \vdots \\ W_{x_e} \\ W_{y_1} \\ \vdots \\ W_{y_e} \end{Bmatrix}$$

With help of Eq.(17)C, the above becomes

$$\underline{\overline{\text{III}}} + \underline{\overline{\text{IV}}} = - \frac{1}{\rho \Omega V_\infty e^{i\Omega t}} \sum_{\bar{a}=1}^{L} \oint_{L_{a}} [A]_{\bar{a}}^{T} \begin{Bmatrix} n_\theta \\ n_\phi \end{Bmatrix} p_f \, dl$$

$$-\alpha K [E] \begin{Bmatrix} U_{x_1} \\ \vdots \\ U_{x_e} \\ U_{y_1} \\ \vdots \\ U_{y_e} \end{Bmatrix} - K [E] \begin{Bmatrix} W_{x_1} \\ \vdots \\ W_{x_e} \\ W_{y_1} \\ \vdots \\ W_{y_e} \end{Bmatrix} \qquad (38)C$$

Now Eq.(35)C can be expressed as

$$N[M]\begin{Bmatrix} U_{x_1} \\ \vdots \\ U_{x_e} \\ U_{y_1} \\ \vdots \\ U_{y_e} \end{Bmatrix} + \left(\frac{N}{f} - \frac{i\eta}{\rho k \Omega}\right)[M]\begin{Bmatrix} W_{x_1} \\ \vdots \\ W_{x_e} \\ W_{y_1} \\ \vdots \\ W_{y_e} \end{Bmatrix} - \alpha K[E]\begin{Bmatrix} U_{x_1} \\ \vdots \\ U_{x_e} \\ U_{y_1} \\ \vdots \\ U_{y_e} \end{Bmatrix}$$

$$-K[E]\begin{Bmatrix} W_{x_1} \\ \vdots \\ W_{x_e} \\ W_{y_1} \\ \vdots \\ W_{y_e} \end{Bmatrix} - \frac{1}{\rho \Omega V_c e^{i\Omega t}} \sum_{\bar{a}=1}^{L} \oint_{L_{\bar{a}}} [A]_{\bar{a}}^{T} \begin{Bmatrix} n_\theta \\ n_\phi \end{Bmatrix} p_f \, dl = 0 \tag{39)C}$$

Let

$$\begin{Bmatrix} \widetilde{U} \\ \widetilde{W} \end{Bmatrix} = \begin{Bmatrix} U_{x_1} \\ \vdots \\ U_{x_e} \\ U_{y_1} \\ \vdots \\ U_{y_e} \\ W_{x_1} \\ \vdots \\ W_{x_e} \\ W_{y_1} \\ \vdots \\ W_{y_e} \end{Bmatrix} \tag{40)C}$$

Eq.(34)C and Eq.(39)C can be combined and expressed in matrix form.

$$\rho V_c^2 \begin{bmatrix} [K]-[M] & \alpha K[E]-N[M] \\ \alpha K[E]-N[M] & K[E]-\left(\frac{N}{f}-\frac{i\eta}{\rho k \Omega}\right)[M] \end{bmatrix} \begin{Bmatrix} \widetilde{U} \\ \widetilde{W} \end{Bmatrix}$$

$$= \frac{V_c}{\Omega e^{i\Omega t}} \begin{Bmatrix} \sum_{\bar{a}=1}^{L} \oint_{L_{\bar{a}}} [A]_{\bar{a}}^{T} [\bar{n}] \begin{Bmatrix} \tau_{xx} \\ \tau_{yy} \\ \tau_{xy} \end{Bmatrix} dl \\ -\sum_{\bar{a}=1}^{L} \oint_{L_{\bar{a}}} [A]_{\bar{a}}^{T} \begin{Bmatrix} n_\theta \\ n_\phi \end{Bmatrix} p_f \, dl \end{Bmatrix} \tag{41)C}$$

APPENDIX D

TWO-DIMENSIONAL ISOPARAMETRIC ELEMENTS

APPENDIX D

TWO-DIMENSIONAL ISOPARAMETRIC ELEMENTS

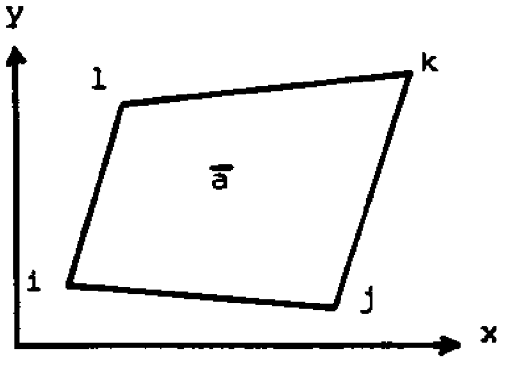


Figure D-1

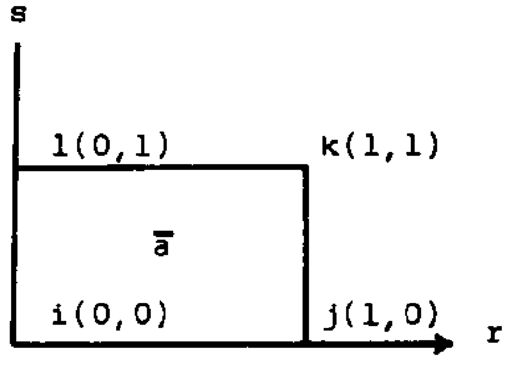


Figure D-2

Figure D-1 shows a typical quadrilateral finite element to be considered, with node i, j, k, l numbered in an anti-clockwise order. The element is shown in Cartesian coordinates, x and y.

The element can also be shown in its local coordinates, r and s, as in Figure D-2. A most convenient method of establishing the coordinate transformations is to use the shape function.(21)

TRANSFORMATION MATRIX

We can write that

$$x = \alpha_1 + \alpha_2 r + \alpha_3 s + \alpha_4 rs \qquad (1)D$$

and we know that

$$\begin{aligned} X_i &= \alpha_1 \\ X_j &= \alpha_1 + \alpha_2 \\ X_k &= \alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 \\ X_l &= \alpha_1 + \alpha_3 \end{aligned} \qquad (2)D$$

Solving Eq.(2)D, we obtain

$$\begin{aligned} \alpha_1 &= X_i \\ \alpha_2 &= X_j - X_i \\ \alpha_3 &= X_l - X_i \\ \alpha_4 &= X_k + X_i - X_j - X_l \end{aligned} \qquad (3)D$$

and

$$X = \begin{bmatrix} T_i & T_j & T_k & T_l \end{bmatrix} \begin{Bmatrix} X_i \\ X_j \\ X_k \\ X_l \end{Bmatrix} \qquad (4)D$$

where

$$
\begin{aligned}
T_i &= (1-r)(1-s) \\
T_j &= r(1-s) \\
T_k &= rs \\
T_l &= (1-r)s
\end{aligned}
\tag{5)D}
$$

Similarly, we have

$$y = \begin{bmatrix} T_i & T_j & T_k & T_l \end{bmatrix} \begin{Bmatrix} y_i \\ y_j \\ y_k \\ y_l \end{Bmatrix} \tag{6)D}$$

DISPLACEMENT SHAPE FUNCTION MATRIX

The displacement of each node have two components

$$\{\bar{u}_i\} = \begin{Bmatrix} u_{x_i} \\ u_{y_i} \end{Bmatrix} \tag{7)D}$$

The displacements within an element can be uniquely defined by the shape functions in its local coordinates,

$$u_x = \beta_1 + \beta_2 r + \beta_3 s + \beta_4 rs \tag{8)D}$$

and

$$u_x = \begin{bmatrix} N_i & N_j & N_k & N_l \end{bmatrix} \begin{Bmatrix} u_{x_i} \\ u_{x_j} \\ u_{x_k} \\ u_{x_l} \end{Bmatrix} \qquad (9)D$$

where

$$\begin{aligned} N_i &= (1-r)(1-s) \\ N_j &= r(1-s) \\ N_k &= rs \\ N_l &= (1-r)s \end{aligned} \qquad (10)D$$

Similarly, we have

$$u_y = \begin{bmatrix} N_i & N_j & N_k & N_l \end{bmatrix} \begin{Bmatrix} u_{y_i} \\ u_{y_j} \\ u_{y_k} \\ u_{y_l} \end{Bmatrix} \qquad (11)D$$

Clearly, we have selected the transformation matrix to be equal to the displacement shape function matrix,

$$[T] = [N] \qquad (12)D$$

where

$$[T] = [T_i \ T_j \ T_k \ T_l] \qquad (13)D$$

and

$$[N] = [N_i \ N_j \ N_k \ N_l] \qquad (14)D$$

When the shape functions defining geometry (transformation matrix) and function (displacement shape function matrix) are the same, the element will be called isoparametric.

EVALUATION OF ELEMENT MATRICES

To perform finite element analysis the matrices defining element properties have to be found. These will be of the form

$$\int_A [Q] \, dA \qquad (15)D$$

in which the expression $[Q]$ depends on $[N]$ or its derivatives with respect to global coordinates.

To evaluate these matrices, two transformations are needed. As $[N]$ is defined in terms of local coordinates, r and s, it is necessary to express the global derivatives in terms of local derivatives. The element of area over which the integration has to be carried out needs to be expressed in terms of the local coordinates with an appropriate change of limits of integration.

By usual rules of partial differentiation, we have

$$\begin{Bmatrix} \dfrac{\partial N_m}{\partial r} \\ \dfrac{\partial N_m}{\partial s} \end{Bmatrix} = [J] \begin{Bmatrix} \dfrac{\partial N_m}{\partial x} \\ \dfrac{\partial N_m}{\partial y} \end{Bmatrix} \qquad (16)D$$

where

$$[J] = \begin{bmatrix} \dfrac{\partial x}{\partial r} & \dfrac{\partial y}{\partial r} \\ \dfrac{\partial x}{\partial s} & \dfrac{\partial y}{\partial s} \end{bmatrix} \qquad (17)D$$

Combining Eqs.(4)D, (5)D, (6)D and (17)D, we have

$$[J] = \begin{bmatrix} (-x_i+x_j)+(x_i-x_j+x_k-x_l)s & (-y_i+y_j)+(y_i-y_j+y_k-y_l)s \\ (-x_i+x_l)+(x_i-x_j-x_k-x_l)r & (-y_i+y_l)+(y_i-y_j+y_k-y_l)r \end{bmatrix}$$

(18)D

To find the global derivatives we invert $[J]$ and we have

$$\begin{Bmatrix} \dfrac{\partial N_m}{\partial x} \\ \dfrac{\partial N_m}{\partial y} \end{Bmatrix} = \left[J \right]^{-1} \begin{Bmatrix} \dfrac{\partial N_m}{\partial r} \\ \dfrac{\partial N_m}{\partial s} \end{Bmatrix} \tag{19)D}$$

where

$$\left[J \right]^{-1} = \frac{\begin{bmatrix} (-y_i+y_l)+(y_i-y_j+y_k-y_l)r & -(-y_i+y_j)-(y_i-y_j+y_k-y_l)s \\ -(-x_i+x_l)-(x_i-x_j+x_k-x_l)r & (-x_i+x_j)+(x_i-x_j+x_k-x_l)s \end{bmatrix}}{\det\left[J \right]} \tag{20)D}$$

and

$$\begin{aligned} \det\left[J \right] = & \left[x_i(y_j-y_l)+x_j(y_l-y_i)+x_l(y_i-y_j) \right] \\ & + \left[(x_j-x_i)(y_k-y_l)+(x_k-x_l)(y_i-y_j) \right] r \\ & + \left[(x_j-x_k)(y_i-y_l)+(x_i-x_l)(y_k-y_j) \right] s \end{aligned} \tag{21)D}$$

From vector analysis, we can easily show that

$$dA = dx\,dy = \det\left[J \right] dr\,ds \tag{22)D}$$

We can now modify Eq.(15)D into

$$\int_0^1\!\!\int_0^1 \left[\tilde{Q}(r,s) \right] \det\left[J \right] dr\,ds = \int_0^1\!\!\int_0^1 f(r,s)\,dr\,ds \tag{23)D}$$

in which $[\bar{Q}(r,s)]$ is the transform of function $[Q]$ in the local coordinates.

NUMERICAL INTEGRATION

One of the most effective methods of evaluating integral is to use Gauss quadrature. The most obvious way of obtaining the integral of Eq.(23)D is to first evaluate the inner integral while keeping s constant, such that

$$\int_0^1 f(r,s)\,dr = g(s) \tag{24)D}$$

Four integrating points will be used and we have

$$\int_0^1 f(r,s)\,dr = \sum_{n=1}^{4} H_n f(r_n,s) \tag{25)D}$$

where

$$\begin{aligned} H_1 &= 0.1739274227 & r_1 &= 0.0694318442 \\ H_2 &= 0.3260725775 & r_2 &= 0.3300094782 \\ H_3 &= H_2 & r_3 &= 0.6699905220 \\ H_4 &= H_1 & r_4 &= 0.9305681560 \end{aligned} \tag{26)D}$$

The integration of Eq.(23)D can now be taken in s-direction, such that

$$\int_0^1 \int_0^1 f(r,s)\,dr\,ds = \sum_{m=1}^{4} \sum_{n=1}^{4} G_m H_n f(r_n, s_m) \qquad (27)D$$

where

$$G_i = H_i$$

$$s_i = r_i \qquad (28)D$$

$$i = 1,\ 2,\ 3,\ 4$$

APPENDIX E

ANALYTICAL SOLUTION FOR ONE-DIMENSIONAL SATURATED ELASTIC POROUS MEDIA

APPENDIX E

ANALYTICAL SOLUTION FOR ONE-DIMENSIONAL SATURATED ELASTIC POROUS MEDIA

The equations of motion for saturated elastic porous media, Eq.(17)A and Eq.(19)A will reduce to one-dimensional case in the following forms

$$\frac{\partial \tau_{xx}}{\partial x} = \rho \ddot{u}_x + \rho_f \ddot{w}_x \qquad (1)E$$

and

$$-\frac{\partial p}{\partial x} = \rho_f \ddot{u}_x + \frac{1}{f} \rho_f \ddot{w}_x + \frac{\eta}{k} \dot{w}_x \qquad (2)E$$

For one-dimensional wave, the strain-displacement relationships are

$$e_{xx} = \frac{\partial u_x}{\partial x} \qquad (3)E$$

and

$$e_{yy} = e_{zz} = 0 \qquad (4)E$$

The stress-strain relationships are

$$\sigma_{xx} = E_c e_{xx} \qquad (5)E$$

where

$$E_c = \frac{(1-\nu)E}{(1+\nu)(1-2\nu)} \qquad (6)E$$

From Eq.(9)A, the water pressure in one-dimensional wave reduce to

$$p = -\alpha M e_{xx} + M\zeta \qquad (7)E$$

where

$$\zeta = -\frac{\partial w_x}{\partial x} \qquad (8)E$$

The bulk stress tensor is

$$\tau_{xx} = \sigma_{xx} - \alpha p \qquad (9)E$$

Combining Eq.(3)E, Eq.(5)E and Eq.(7)E, we have

$$\tau_{xx} = E_c \frac{\partial u_x}{\partial x} + \alpha^2 M \frac{\partial u_x}{\partial x} + \alpha M \frac{\partial w_x}{\partial x} \qquad (10)E$$

Add hysteretic damping ratio and we have

$$\tau_{xx} = E_c \left[\frac{\partial u_x}{\partial x} + \lambda_c \frac{\partial}{\partial t}\left(\frac{\partial u_x}{\partial x}\right)\right] + \alpha^2 M \frac{\partial u_x}{\partial x} + \alpha M \frac{\partial w_x}{\partial x} \qquad (11)E$$

For steady state solutions, let

$$
\begin{aligned}
u_x &= U_x\, e^{i\Omega t} \\
w_x &= W_x\, e^{i\Omega t}
\end{aligned}
\tag{12)E}
$$

Also let

$$
\begin{aligned}
\theta &= \frac{\Omega x}{V_c} \\
V_c &= \sqrt{\frac{E_c+\alpha^2 M}{\rho}} \\
N &= \frac{\beta}{\rho} \\
K &= \frac{M}{E_c+\alpha^2 M}
\end{aligned}
\tag{13)E}
$$

With aid of Eq.(12)E and Eq.(13)E, Eq.(7)E becomes

$$\beta e^{-i\Omega t} = -\alpha M \frac{\Omega}{V_c} \frac{\partial U_x}{\partial \theta} - M \frac{\Omega}{V_c} \frac{\partial W_x}{\partial \theta} \tag{14)E}$$

and Eq.(11)E becomes

$$
\begin{aligned}
\tau_{xx} e^{-i\Omega t} = E_c \frac{\Omega}{V_c}\left(1 + i\Omega\lambda_c\right)\frac{\partial U_x}{\partial \theta} + \alpha^2 M \frac{\Omega}{V_c}\frac{\partial U_x}{\partial \theta} \\
+ \alpha M \frac{\Omega}{V_c}\frac{\partial W_x}{\partial \theta}
\end{aligned}
\tag{15)E}
$$

With aid of Eq.(14)E and Eq.(15)E, Eq.(1)E and Eq.(2)E can be expressed as

$$
\begin{aligned}
\frac{\partial^2 U_x}{\partial \theta^2} + 2\mu i \frac{\partial^2 U_x}{\partial \theta^2} + \alpha K \frac{\partial^2 W_x}{\partial \theta^2} &= -U_x - N W_x \\
\alpha K \frac{\partial^2 U_x}{\partial \theta^2} + K \frac{\partial^2 W_x}{\partial \theta^2} &= -N U_x - \frac{N}{f} W_x + \frac{i\eta}{\rho k \Omega} W_x
\end{aligned}
\tag{16)E}
$$

where

$$2\mu = \frac{E_c \Omega \lambda_c}{\rho V_c^2} \tag{17)E}$$

The simultaneous linear differential equations in Eq.(16)E can be solved by method of operator.(12) Rewrite Eq.(16)E in operational form

$$\begin{aligned} \left[(1+2\mu i)D^2+1\right]U_x+(\alpha K D^2+N)W_x &= 0 \\ (\alpha K D^2+N)U_x+\left(K D^2+\frac{N}{f}-\frac{i\eta}{\rho k \Omega}\right)W_x &= 0 \end{aligned} \tag{18)E}$$

The operator is

$$\Delta = \begin{vmatrix} (1+2\mu i)D^2+1 & \alpha K D^2+N \\ \alpha K D^2+N & K D^2+\frac{N}{f}-\frac{i\eta}{\rho k \Omega} \end{vmatrix} \tag{19)E}$$

$$\Delta = A D^4 + B D^2 + C$$

where

$$\begin{aligned} A &= K-\alpha^2 K^2+2\mu K i \\ B &= \left(\frac{N}{f}+K-2\alpha K N+\frac{2\mu\eta}{\rho k \Omega}\right)+i\left(\frac{2\mu N}{f}-\frac{\eta}{\rho k \Omega}\right) \\ C &= \frac{N}{f}-N^2-\frac{i\eta}{\rho k \Omega} \end{aligned}$$

We now have

$$(AD^4+BD^2+C)U_x = 0$$

$$(AD^4+BD^2+C)W_x = 0$$

We notice that the characteristic equation for both U_x and W_x is obtained by formally replacing D by r in the expression

$$\Delta = 0$$

$$Ar^4 + Br^2 + C = 0 \tag{20)E}$$

Solving for r , we have

$$r = \pm R_1, \pm R_2 \tag{21)E}$$

where

$$R_1 = \sqrt{\frac{-B+\sqrt{B^2-4AC}}{2A}}$$

$$R_2 = \sqrt{\frac{-B-\sqrt{B^2-4AC}}{2A}}$$

$$R_e R_1 > 0$$

$$R_e R_2 > 0$$

Hence we obtain

$$U_x = a_1 e^{R_1\theta} + a_2 e^{-R_1\theta} + a_3 e^{R_2\theta} + a_4 e^{-R_2\theta}$$

$$W_x = b_1 e^{R_1\theta} + b_2 e^{-R_1\theta} + b_3 e^{R_2\theta} + b_4 e^{-R_2\theta} \tag{22)E}$$

Insert Eq.(22)E into Eq.(16)E, we obtain the following relationships,

$$b_1 = -\frac{(1+2\mu i)R_1^2+1}{\alpha K R_1^2+N}\, a_1$$

$$b_2 = -\frac{(1+2\mu i)R_1^2+1}{\alpha K R_1^2+N}\, a_2$$

$$b_3 = -\frac{(1+2\mu i)R_2^2+1}{\alpha K R_2^2+N}\, a_3$$

$$b_4 = -\frac{(1+2\mu i)R_2^2+1}{\alpha K R_2^2+N}\, a_4$$

We now have

$$U_x = a_1 e^{R_1\theta} + a_2 e^{-R_1\theta} + a_3 e^{R_2\theta} + a_4 e^{-R_2\theta}$$

(23)E

$$W_x = -\frac{(1+2\mu i)R_1^2+1}{\alpha K R_1^2+N}(a_1 e^{R_1\theta} + a_2 e^{-R_1\theta}) - \frac{(1+2\mu i)R_2^2+1}{\alpha K R_2^2+N}(a_3 e^{R_2\theta} + a_4 e^{-R_2\theta})$$

$$W_x = -\frac{(1+2\mu i)R_1^2+1}{\alpha K R_1^2+N}(a_1 e^{R_1\theta} + a_2 e^{-R_1\theta}) - \frac{(1+2\mu i)R_2^2+1}{\alpha K R_2^2+N}(a_3 e^{R_2\theta} + a_4 e^{-R_2\theta})$$

With Appropriate boundary conditions, coefficients a_1, a_2, a_3 and a_4 can be obtained.

INFINITE ROD

Typical boundary conditions for the infinite rod are the following,

$$\begin{aligned} &U_x(0) = 1 \\ &U_x(\infty) \text{ is finite} \\ &W_x(0) = 0 \\ &W_x(\infty) \text{ is finite} \end{aligned} \tag{24)E$$

Since $U_x(\infty)$ and $W_x(\infty)$ are finite, coefficients a_1 and a_3 in Eq.(23)E must be zero. Therefore, we have

$$\begin{aligned} U_x &= a_2 e^{-R_1\theta} + a_4 e^{-R_2\theta} \\ W_x &= -\frac{(1+2\mu i)R_1^2+1}{\alpha K R_1^2+N} a_2 e^{-R_1\theta} - \frac{(1+2\mu i)R_2^2+1}{\alpha K R_2^2+N} a_4 e^{-R_2\theta} \end{aligned} \tag{25)E$$

Since $U_x(0)=1$ and $W_x(0)=0$, we have

$$a_2 + a_4 = 1$$

$$-\frac{(1+2\mu i)R_1^2+1}{\alpha K R_1^2+N} a_2 - \frac{(1+2\mu i)R_2^2+1}{\alpha K R_2^2+N} a_4 = 0$$

Solving the above two simultaneous equations, we obtain

$$a_2 = \frac{(\alpha K R_1^2+N)\left[(1+2\mu i)R_2^2+1\right]}{\left[\alpha K - N(1+2\mu i)\right](R_1^2-R_2^2)}$$

$$a_4 = \frac{(\alpha K R_2^2+N)\left[(1+2\mu i)R_1^2+1\right]}{\left[\alpha K - N(1+2\mu i)\right](R_1^2-R_2^2)}$$

Therefore

$$U_x = \frac{(\alpha K R_1^2+N)\left[(1+2\mu_i)R_2^2+1\right]}{\left[\alpha K-N(1+2\mu_i)\right](R_1^2-R_2^2)} e^{-R_1\theta} - \frac{(\alpha K R_2^2+N)\left[(1+2\mu_i)R_1^2+1\right]}{\left[\alpha K-N(1+2\mu_i)\right](R_1^2-R_2^2)} e^{-R_2\theta} \qquad (26)E$$

$$W_x = -\frac{\left[(1+2\mu_i)R_1^2+1\right]\left[(1+2\mu_i)R_2^2+1\right]}{\left[\alpha K-N(1+2\mu_i)\right](R_1^2-R_2^2)}\left(e^{-R_1\theta}-e^{-R_2\theta}\right)$$

Combining Eq.(15)E and Eq.(26)E, we have

$$\frac{\tau_{xx}}{e^{j\Omega t}} = \frac{V_c\Omega\rho(1+2\mu_i)}{\left[\alpha K-N(1+2\mu_i)\right](R_1^2-R_2^2)}\Big\{-R_1(\alpha K R_1^2+N)\left[(1+2\mu_i)R_2^2+1\right] e^{-R_1\theta}$$

$$+R_2(\alpha K R_2^2+N)\left[(1+2\mu_i)R_1^2+1\right] e^{-R_2\theta}\Big\}$$

$$-\frac{\alpha K V_c\Omega\rho\left[(1+2\mu_i)R_1^2+1\right]\left[(1+2\mu_i)R_2^2+1\right]}{\left[\alpha K-N(1+2\mu_i)\right](R_1^2-R_2^2)}\left(-R_1e^{-R_1\theta}+R_2e^{-R_2\theta}\right) \qquad (27)E$$

Similarly, combining Eq.(14)E and Eq.(26)E, we have

$$\frac{p_x}{e^{j\Omega t}} = \frac{V_c\Omega\rho\alpha K}{\left[\alpha K-N(1+2\mu_i)\right](R_1^2-R_2^2)}\Big\{R_1(\alpha K R_1^2+N)\left[(1+2\mu_i)R_2^2+1\right] e^{-R_1\theta}$$

$$-R_2(\alpha K R_2^2+N)\left[(1+2\mu_i)R_1^2+1\right] e^{-R_2\theta}\Big\}$$

$$+\frac{V_c\Omega\rho K\left[(1+2\mu_i)R_1^2+1\right]\left[(1+2\mu_i)R_2^2+1\right]}{\left[\alpha K-N(1+2\mu_i)\right](R_1^2-R_2^2)}\left(R_1e^{-R_1\theta}-R_2e^{-R_2\theta}\right) \qquad (28)E$$

It will be very convenient if we can express stresses, τ_{xx} and p_x, in terms of displacements, U_x and W_x, such that

$$\tau_{rx} = C_1 U_x + C_2 W_x$$

(29)E

$$f_r = C_3 U_x + C_4 W_x$$

The above conditions are achievable for an infinite rod. Solving for C_1, C_2, C_3, and C_4 from Eq.(26)E, Eq.(27)E, Eq.(28)E and Eq.(29)E, we have

$$C_1 = \frac{V_0 \Omega_0 \rho \left[1-(1+2\mu i)R_1 R_2\right]}{R_1 + R_2}$$

$$C_2 = -\frac{V_0 \Omega_0 \rho (\alpha K R_1 R_2 - N)}{R_1 + R_2}$$

$$C_3 = \frac{V_0 \Omega_0 \rho K \left[(1+2\mu i)R_1^2+1\right]\left[(1+2\mu i)R_2^2+1\right]}{\left[\alpha K - N(1+2\mu i)\right](R_1^2 - R_2^2)}$$

(30)E

$$\times \left\{ \frac{\alpha R_1(\alpha K R_1^2 + N)}{(1+2\mu i)R_1^2+1} - \frac{\alpha R_2(\alpha K R_2^2 + N)}{(1+2\mu i)R_2^2+1} - (R_1 - R_2) \right\}$$

$$C_4 = \frac{V_0 \Omega_0 \rho K (\alpha K R_1^2 + N)(\alpha K R_2^2 + N)}{\left[\alpha K - N(1+2\mu i)\right](R_1^2 - R_2^2)}$$

$$\times \left\{ - \frac{R_1\left[(1+2\mu i)R_1^2+1\right]}{\alpha K R_1^2 + N} + \frac{R_2\left[(1+2\mu i)R_2^2+1\right]}{\alpha K R_2^2 + N} + \alpha(R_1 - R_2) \right\}$$

Now we can express stresses, τ_{rx} and f_r, in terms of displacements, U_x and W_x, such that

$$\tau_{xx}=\frac{V_c\Omega\rho\left[1-(1+2\mu_i)R_1R_2\right]}{R_1+R_2}u_x-\frac{V_c\Omega\rho(\alpha KR_1R_2-N)}{R_1+R_2}w_x$$

$$p_3=\frac{V_c\Omega\rho K\left[(1+2\mu_i)R_1^2+1\right]\left[(1+2\mu_i)R_2^2+1\right]}{\left[\alpha K-N(1+2\mu_i)\right](R_1^2-R_2^2)}$$

$$\times\left\{\frac{\alpha R_1(\alpha KR_1^2+N)}{(1+2\mu_i)R_1^2+1}-\frac{\alpha R_2(\alpha KR_2^2+N)}{(1+2\mu_i)R_2^2+1}-(R_1-R_2)\right\}u_x$$

$$+\frac{V_c\Omega\rho K(\alpha KR_1^2+N)(\alpha KR_2^2+N)}{\left[\alpha K-N(1+2\mu_i)\right](R_1^2-R_2^2)}$$

$$\times\left\{-\frac{R_1\left[(1+2\mu_i)R_1^2+1\right]}{\alpha KR_1^2+N}+\frac{R_2\left[(1+2\mu_i)R_2^2+1\right]}{\alpha KR_2^2+N}+\alpha(R_1-R_2)\right\}w_x$$

APPENDIX F

TRANSMITTING BOUNDARY CONDITIONS

APPENDIX F

TRANSMITTING BOUNDARY CONDITIONS

When the finite element method is used to solve the equations of motion of the soil and pore water, an infinite half-space is idealized into a finite space with appropriate boundary conditions. In the infinite space problem, the wave propagation thru the half space will allow the energy to radiate away from the site. In the finite computational space, however, appropriate boundary conditions must be specified so as to allow this energy to be transmitted away from the zone of interest, rather than artificially reflecting back into the zone by the boundary conditions. To date, transmitting boundary conditions have been used for the single-phased non-saturated soil problem. However, no such development has been used for the two-phased soil/water problem.

The most promising way to incorporate the transmitting boundary conditions in the single-phased non-saturated soil is to express them by conditions (1,16,18)

$$\sigma = \rho V_c \dot{u}_c \qquad (1)F$$

$$\tau = \rho V_s \dot{u}_s \qquad (2)F$$

in which σ and τ are the boundary normal and shear stresses, respectively; $\dot{u}_c$ and $\dot{u}_s$ are the corresponding normal and tangential velocities, respectively; ρ is the mass density of the solid; V_s and V_c are the shear wave and compressive wave velocities. The above transmitting boundary formulation is called the viscous boundary, and has been used extensively.

Now let us find how the condition of Eq.(1)F could be obtained for steady state wave propagation. The wave equation for one-dimensional dry soil is the following

$$\frac{E(1-\nu)}{(1+\nu)(1-2\nu)} \frac{\partial^2 u_c}{\partial x^2} = \rho \ddot{u}_c \qquad (3)F$$

where E and ν are, respectively, Young's modulus and Poisson's ratio.

For steady state solutions, let

$$u_c = U_c e^{i\Omega t} \qquad (4)F$$

With help of Eq.(4)F, Eq.(3)F becomes

$$\frac{V_c^2}{\Omega^2} \frac{\partial^2 U_c}{\partial x^2} = -U_c \qquad (5)F$$

where

$$V_c^2 = \frac{E(1-\nu)}{(1+\nu)(1-2\nu)\rho}$$

The solution of Eq.(5)F is

$$U_c = a_1 e^{\left(\frac{\Omega x}{V_c}\right)i} + a_2 e^{-\left(\frac{\Omega x}{V_c}\right)i} \qquad (6)F$$

The problem of interest is an infinite rod, where

$$U_c(0) = c \qquad (7)F$$

and

$$U_c(\infty) = \text{finite} \qquad (8)F$$

Apply the above conditions, we have

$$U_c = c\, e^{-\left(\frac{\Omega x}{V_c}\right)i} \qquad (9)F$$

The normal stress, σ , can be expressed in term of displacement as

$$\sigma = \frac{E(1-\nu)}{(1+\nu)(1-2\nu)} \frac{\partial u_c}{\partial x} \qquad (10)F$$

With help of Eq.(4)F and Eq.(9)F, Eq.(10)F becomes

$$\sigma = -\rho V_c \Omega i u_c \qquad (11)F$$

Since $\dot{u}_c = i\Omega u_c$, the stress can be expressed as

$$\sigma = -\rho V_c \dot{u}_c \qquad (12)F$$

Notice that now the normal stress can be expressed in term of the normal velocity. The transmitting boundary takes the negative of the normal stress for the one-dimensional infinite rod problem and is applied to the artificial boundary, imposed when the infinite space is cut into a finite space.

For steady state solutions, it is more convenient to express the normal stress in term of the normal displacement as shown in Eq.(11)F.

If a hysteretic damping ratio, λ , is included in the wave equation, Eq.(3)F becomes

$$\frac{E(1-\nu)}{(1+\nu)(1-2\nu)}\left[\frac{\partial^2 u_c}{\partial x^2} + \lambda \frac{\partial}{\partial t}\left(\frac{\partial^2 u_c}{\partial x^2}\right)\right] = \rho \ddot{u} \qquad (13)F$$

Upon solving Eq.(13)F, the normal stress can be expressed as

$$\sigma = -\rho V_c \Omega i (1+2\mu i)^{1/2} u_c \qquad (14)F$$

where

$$2\mu = \Omega \lambda$$

Similarly, for shear wave, we have

$$\tau = -\rho V_s \Omega i (1+2\mu i)^{1/2} u_s \qquad (15)F$$

where

$$V_s^2 = \frac{G}{\rho}$$

Essentially, we have

$$\sigma = a_1 u_c \qquad (16)F$$

$$\tau = b_1 u_s \qquad (17)F$$

The values of a_1 and b_1 are used to modify the master stiffness matrix of the finite element models to simulate the transmitting boundary conditions for dry soil.

Now let us outline the procedure of incorporating the transmitting boundary conditions for the two-phased soil/ water problem. The equations of motion for saturated elastic porous media for one-dimensional case are shown in Appendix E. For an infinite rod with hysteretic soil damping and considering the steady state solution, the bulk stress, τ_{xx}, and water pressure, π_f , can be expressed in terms of the displacement of soil, u_x , and the relative water displacement, w_x , from Eq.(29)E as

$$\tau_{xx} = C_1 U_x + C_2 W_x$$

$$p_f = C_3 U_x + C_4 W_x \tag{18)F}$$

The coefficients C_1, C_2, C_3 and C_4 are shown in Eq.(30)E. The values of these coefficients are used to modify the master stiffness matrix of the finite element models to simulate the transmitting boundary conditions for two-phased saturated soil.

The steady state normalized non-dimensional equations of motion for saturated elastic porous media for the one-dimensional case is given by Eq.(16)E as

$$\frac{\partial^2 U_x}{\partial \theta^2} + 2\mu i \frac{\partial^2 U_x}{\partial \theta^2} + \alpha K \frac{\partial^2 W_x}{\partial \theta^2} = -U_x - N W_x$$

$$\alpha K \frac{\partial^2 U_x}{\partial \theta^2} + K \frac{\partial^2 W_x}{\partial \theta^2} = -N U_x - \frac{N}{f} W_x + \frac{i\eta}{\rho k \Omega} W_x \tag{19)F}$$

where θ, N and K are non-dimensional parameters defined in Eq.(13)E.

The finite element discretization of Eq.(19)F begins by letting the bulk displacements within each finite element, $\bar{a}$, be expressed as

$$U_{x\bar{a}} = [N]_{\bar{a}} \begin{Bmatrix} U_i \\ U_j \end{Bmatrix} \tag{20)F}$$

where $[N]_a$ is the shape function matrix, as described in Appendix C.

Similarly

$$W_{xa} = [N]_a \begin{Bmatrix} W_i \\ W_j \end{Bmatrix}$$

The procedure for the one-dimensional case is similar to that for the two-dimensional case, which is presented in Appendix C. After discretization, Eq.(19)F becomes

$$\begin{bmatrix} -[M]+(1+2\mu i)[K] & -N[M]+\alpha K[K] \\ -N[M]+\alpha K[K] & \left(\frac{\eta i}{\rho k \Omega}-\frac{N}{f}\right)[M]+K[K] \end{bmatrix} \begin{Bmatrix} U_1 \\ \vdots \\ U_a \\ W_1 \\ \vdots \\ W_a \end{Bmatrix} \tag{21)F}$$

$$= \frac{1}{V_c \Omega \rho e^{i\Omega t}} \begin{Bmatrix} -\tau_{xx_1} \\ 0 \\ \vdots \\ 0 \\ \tau_{xx_a} \\ p_1 \\ 0 \\ \vdots \\ 0 \\ -p_a \end{Bmatrix}$$

where

$$[M] = \sum_{a=1}^{L} \int_{\theta_a} [N]_a^T [N]_a \, d\theta$$

$$[K] = \sum_{\ell=1}^{L} \int_{\theta_\ell} [N']^T_\ell \, [N']_\ell \, d\theta$$

$$[N'] = \frac{\partial [N]}{\partial \theta}$$

Note that the summation stands for addmix. L is number of elements and Q is number of total nodal points.

Let the transmitting boundary to be located at node Q. The master stiffness matrix of Eq.(21)F must be modified and τ_{xx_Q} and f_{θ_Q} are set equal to zero. The new master stiffness matrix is formed as

$$[R] = \begin{bmatrix} -[M]+(1+2\mu i)[K] & -N[M]+\alpha K[K] \\ -N[M]+\alpha K[K] & \left(\frac{2i}{\rho\alpha\Omega}-\frac{N}{f}\right)[M]+K[K] \end{bmatrix}$$

$$-\frac{1}{V_c \Omega \rho e^{i\Omega t}} \begin{array}{ll} \left[\begin{array}{ccccc} \circ & \cdots & \circ & \cdots & \circ \\ \vdots & & \vdots & & \vdots \\ \circ & \cdots & C_1 & \cdots & C_2 \\ \vdots & & \vdots & & \vdots \\ \circ & \cdots & -C_3 & \cdots & -C_4 \end{array}\right] & \begin{array}{l} \\ \\ \leftarrow Q\text{th row} \\ \\ \leftarrow (2Q)\text{th row} \end{array} \\ \quad Q\text{th column} \uparrow \qquad\qquad \uparrow (2Q)\text{th column} & \end{array}$$

(22)F

From Eq.(22)F, we have a set of matrix equations to simulate the infinite rod problem and they can be written as

$$[R]\begin{Bmatrix} U_1 \\ \vdots \\ U_\bullet \\ W_1 \\ \vdots \\ W_\bullet \end{Bmatrix} = \frac{1}{V_c \Omega \rho e^{i\Omega t}} \begin{Bmatrix} -\tau_{xx_1} \\ 0 \\ \vdots \\ 0 \\ \tau_{f_1} \\ 0 \\ \vdots \\ 0 \end{Bmatrix}$$

Once the appropriate boundary conditions are specified at node 1 (the free end of the rod), the above equations can be solved easily by computer. As an example, the simple rod of Figure F-1 was used to generate comparison solutions.

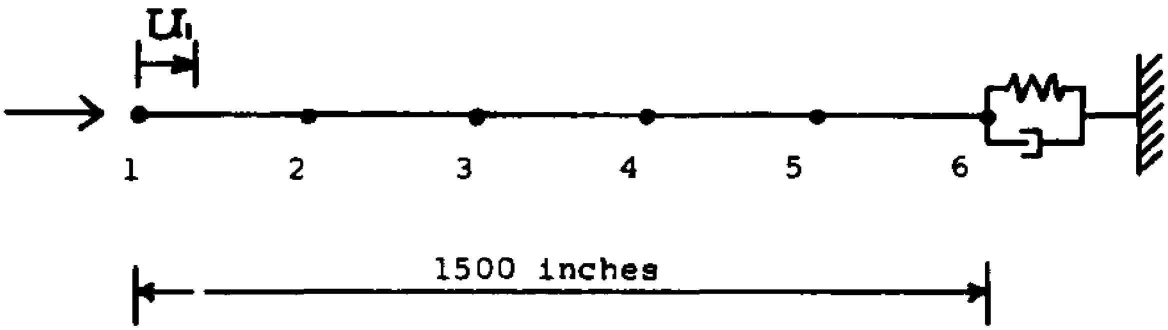


Figure F-1

For this problem the following parameters were used.

$E_c = \frac{E(1-\nu)}{(1+\nu)(1-2\nu)}$
$= 14{,}000$ psi
$\alpha = 1$
$M = 293{,}900$ psi
$\rho = 0.002324$ slug/in^3
$\rho_f = 0.0003925$ slug/in^3

$f = 0.35$
$\frac{k}{\gamma} = 0.009096$ in^3-sec/slug
($k_T = 0.01$ cm/sec)
$V_c = 39{,}823$ in/sec
$\lambda = 0$

(See Appendix K for detail parameter definitions.)

Linear shape functions were used in the elements and the boundary conditions $U_1 = 1$ and $W_1 = 0$ were chosen. These boundary conditions represent a impervious boundary at the left end of the rod (node 1). Analytic and finite element solutions are tabulated in Table F-1 and Table F-2 for comparison purposes. As may be noted, the results of the analytic solution are very close to those of the finite element solution for a frequency range of 0.5 rad/sec to 64.0 rad/sec. Differences occur at a frequency of 128.0 rad/sec. The cause for the differences does not come from the transmitting boundary conditions, rather, it arises from the fact that the element size chosen (300-inch long element) is too large for the high frequency being transmitted. At the high frequency, we are trying to simulate sine or cosine behavior with only 6 or less points in one period. Since the differences are caused by the mesh sizes and not by the transmitting boundary conditions, we can conclude that the transmitting boundary conditions adequately simulate the transmission of energy for the one-dimensional problem. This formulation is used to attempt to simulate the two-dimensional transmitting boundary conditions.

Table F-1

(Analytic Solution)

FREQUENCY (RAD/SEC)	BULK DISPLACEMENT (INCHES)	WATER DISPLACEMENT (INCHES)
0.5	(.100E 1,-.188E-1)	(.156E-6, .829E-5)
1.0	(.999E 0,-.376E-1)	(.624E-6, .166E-4)
2.0	(.997E 0,-.752E-1)	(.250E-5, .331E-4)
4.0	(.989E 0,-.150E 0)	(.995E-5, .656E-4)
8.0	(.955E 0,-.297E 0)	(.394E-4, .127E-3)
16.0	(.824E 0,-.566E 0)	(.150E-3, .219E-3)
32.0	(.358E 0,-.933E 0)	(.496E-3, .190E-3)
64.0	(-.742E 0,-.668E 0)	(.709E-3,-.789E-3)
128.0	(.104E 0, .991E 0)	(-.210E-2, .223E-3)

FREQUENCY (RAD/SEC)	BULK STRESS (PSI)	WATER PRESSURE (PSI)
0.5	(-.726E-1,-.386E 1)	(.693E-1, .368E 1)
1.0	(-.290E 0,-.772E 1)	(.277E 0, .737E 1)
2.0	(-.116E 1,-.154E 2)	(.111E 1, .147E 2)
4.0	(-.463E 1,-.305E 2)	(.442E 1, .292E 2)
8.0	(-.183E 2,-.590E 2)	(.175E 2, .563E 2)
16.0	(-.700E 2,-.102E 3)	(.668E 2, .972E 2)
32.0	(-.231E 3,-.886E 2)	(.220E 3, .846E 2)
64.0	(-.331E 3, .367E 3)	(.316E 3,-.350E 3)
128.0	(.979E 3,-.102E 3)	(-.935E 3, .972E 2)

Table F-2

(Finite Element Solution)

FREQUENCY (RAD/SEC)	BULK DISPLACEMENT (INCHES)	WATER DISPLACEMENT (INCHES)
0.5	(.100E 1,-.188E-1)	(.138E-6, .828E-5)
1.0	(.999E 0,-.376E-1)	(.586E-6, .166E-4)
2.0	(.997E 0,-.752E-1)	(.244E-5, .332E-4)
4.0	(.989E 0,-.150E 0)	(.989E-5, .658E-4)
8.0	(.955E 0,-.297E 0)	(.393E-4, .127E-3)
16.0	(.824E 0,-.566E 0)	(.150E-3, .219E-3)
32.0	(.359E 0,-.931E 0)	(.494E-3, .192E-3)
64.0	(-.720E 0,-.685E 0)	(.727E-3,-.762E-3)
128.0	(-.584E-1, .956E 0)	(-.203E-2,-.117E-3)

FREQUENCY (RAD/SEC)	BULK STRESS (PSI)	WATER PRESSURE (PSI)
0.5	(-.654E-1,-.386E 1)	(.624E-1, .368E 1)
1.0	(-.261E 0,-.772E 1)	(.250E 0, .737E 1)
2.0	(-.105E 1,-.154E 2)	(.998E 0, .147E 2)
4.0	(-.417E 1,-.306E 2)	(.398E 1, .292E 2)
8.0	(-.165E 2,-.595E 2)	(.158E 2, .568E 2)
16.0	(-.637E 2,-.106E 3)	(.608E 2, .101E 3)
32.0	(-.217E 3,-.115E 3)	(.207E 3, .110E 3)
64.0	(-.409E 3, .264E 3)	(.390E 3,-.252E 3)
128.0	(.794E 3, .445E 3)	(-.758E 3,-.425E 3)

APPENDIX G

TYPICAL EARTHQUAKE ANALYSIS OF STRUCTURES EMBEDDED IN SOIL

APPENDIX G

TYPICAL EARTHQUAKE ANALYSIS OF STRUCTURES EMBEDDED IN SOIL

Earthquake analysis of structures built on soil involves equations of motion of structure, equations of motion of soil and its entrapped water, and interaction forces between the structure and the soil. Since structural response to earthquake is primarily due to the first few modes of vibration, only the low frequency components of the structural motion need to be considered.

A structure of interest built on soil is shown in Figure G-1. Since the primary purpose of this research is to investigate soil behavior and since extensive studies are already being made on the structure, a simple equivalent structural model will be used herein. In fact, such simplified models are often used in earthquake analysis of actual facilities. The structure is idealized into the nodal configuration such as shown in Figure G-2.

The base of the structure is allowed to have horizontal motion, vertical motion and rocking. For simplicity, let us consider only the horizontal motion and rocking of the base.

Consider the structure as a free-free system. Each node has two degrees of freedom, the horizontal deflection and rotation as shown in Figure G-3. Let the direction of the earthquake motion coincide with x-direction. The motion in y-direction is considered zero herein. Let u_i

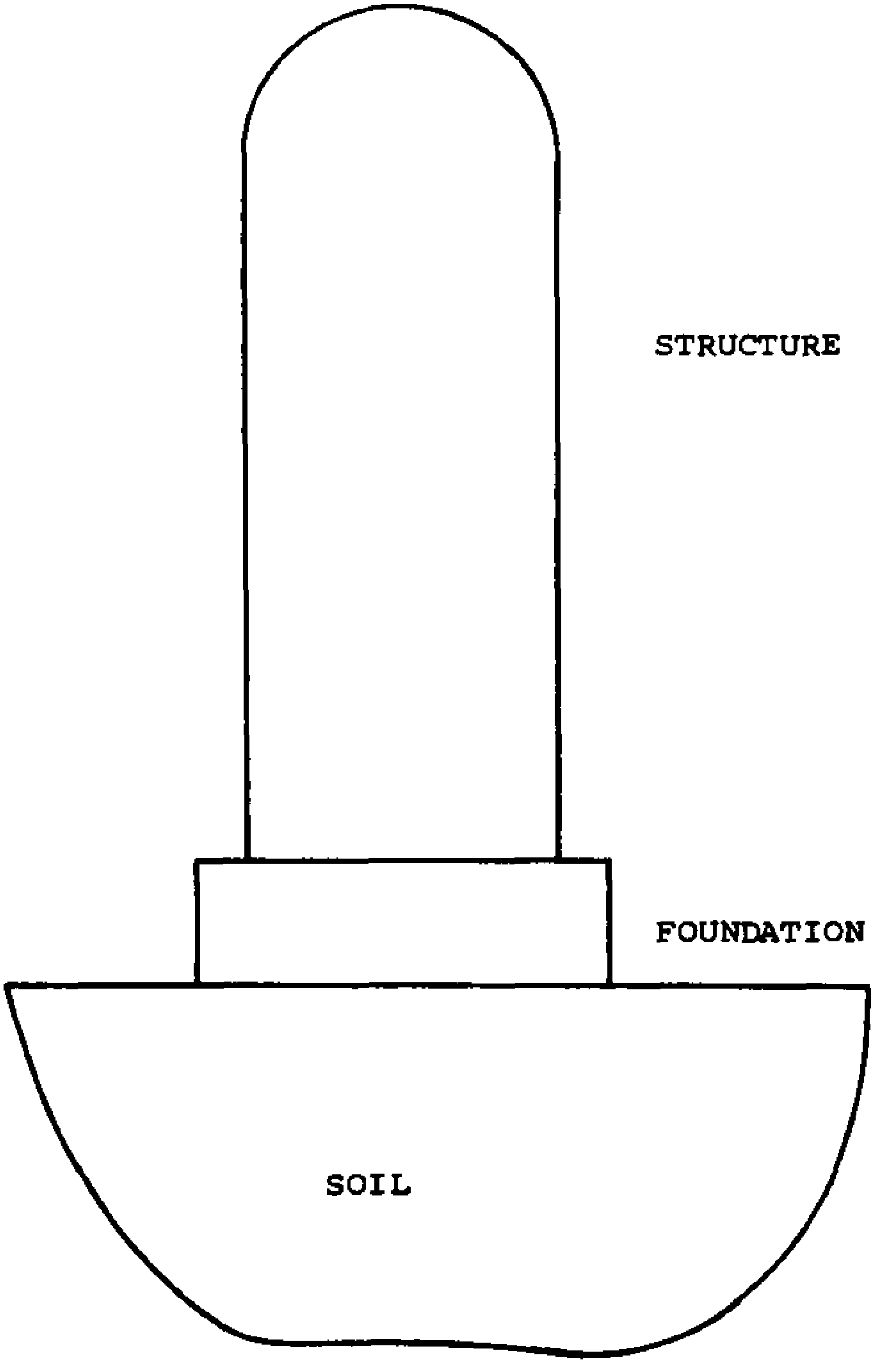


Figure G-1

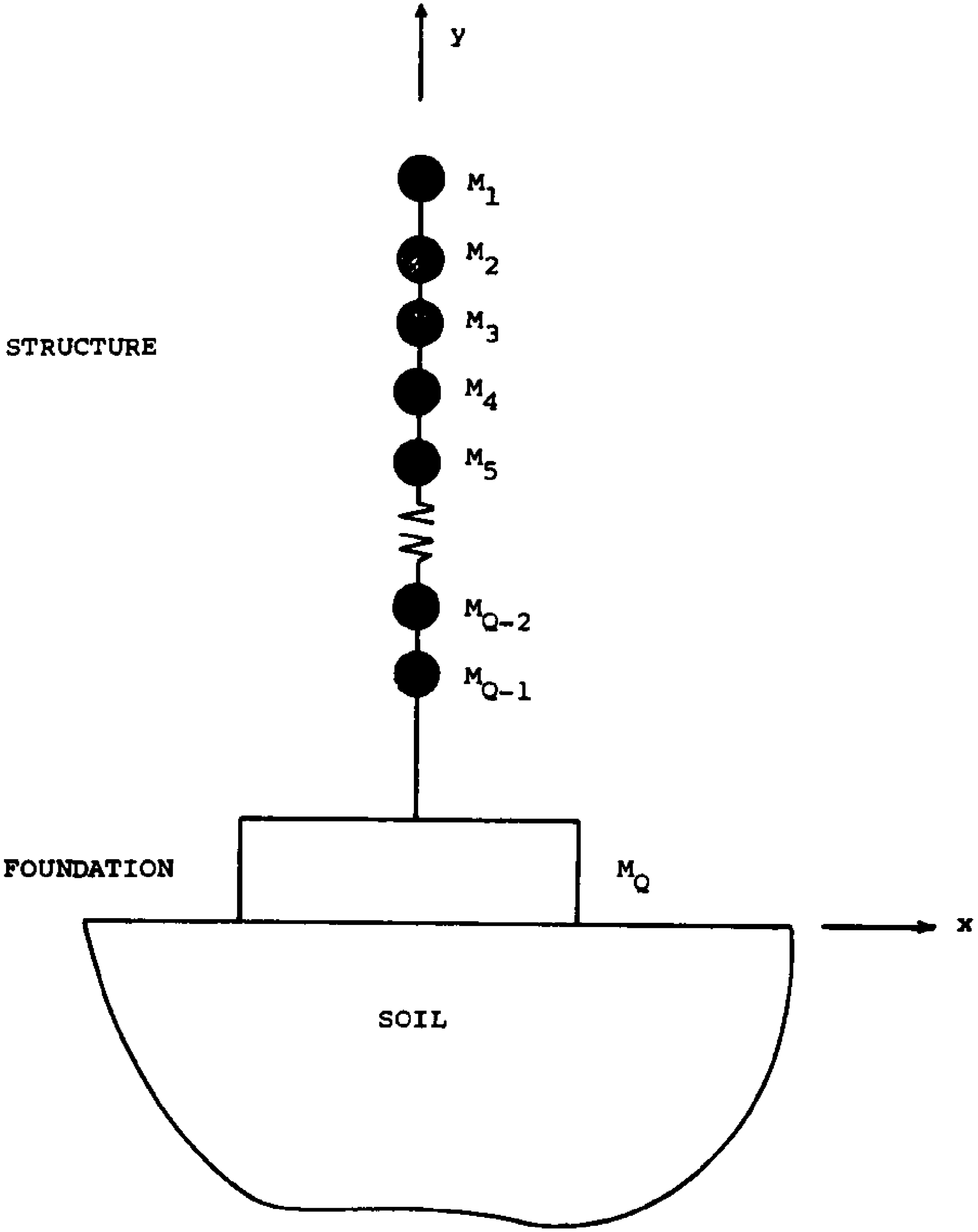


Figure G-2

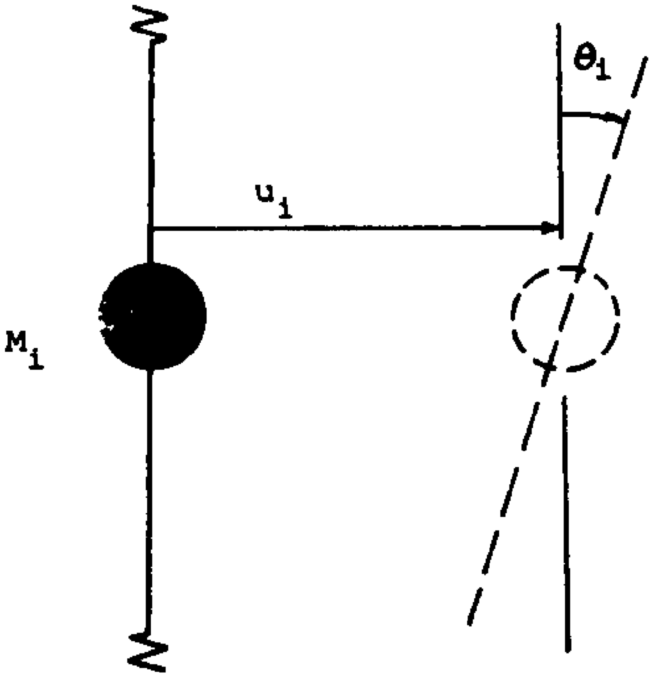


Figure G-3

be the horizontal displacement and θ_i be the rotation of the 'i'th node. $\overline{\underline{\Theta}}$ is the rigid body rotation of the center of gravity of the structural mass. The equations of motion are

$$[M]\begin{Bmatrix}\ddot{\bar{\bar{u}}}\\ \\ \ddot{\bar{\bar{\theta}}}\end{Bmatrix}+[K]\begin{Bmatrix}\bar{\bar{u}}\\ \\ \bar{\bar{\theta}}\end{Bmatrix}=\begin{Bmatrix}F_x\\ \\ C_\theta\end{Bmatrix} \tag*{(1)G}$$

where

$$\begin{Bmatrix}\bar{\bar{u}}\\ \\ \bar{\bar{\theta}}\end{Bmatrix}=\begin{Bmatrix}u_1\\ u_2\\ \vdots\\ u_Q\\ \theta_1\\ \theta_2\\ \vdots\\ \theta_Q\end{Bmatrix}$$

$[M]$ is the lumped mass matrix, $[K]$ is the stiffness matrix, and $\{F_x\}$ and $\{C_\theta\}$ are the load vectors. In the above formulation the damping terms are neglected and they will be considered later.

Let

$$\left\{\begin{matrix}\bar{\bar{u}}\\ \bar{\bar{\theta}}\end{matrix}\right\} = u_{c.g.}\left\{\begin{array}{c}1\\1\\ \vdots\\1\\ \hline 0\\0\\ \vdots\\0\end{array}\right\} + \underline{\bar{\theta}}\left\{\begin{array}{c}t_1\\t_2\\ \vdots\\t_a\\ \hline 1\\ \vdots\\1\end{array}\right\} + \left\{\begin{matrix}\bar{\bar{u}}^f\\ \bar{\bar{\theta}}^f\end{matrix}\right\} \qquad (2)G$$

where t_i is the vertical distance between 'i'th node and the center of gravity. $\{\bar{\bar{u}}^f\}$ are the flexible displacements and $\{\bar{\bar{\theta}}^f\}$ are the flexible rotations. However Eq.(2)G could be expressed in modal terms (solutions of free vibration).

$$\left\{\begin{matrix}\bar{\bar{u}}\\ \bar{\bar{\theta}}\end{matrix}\right\} =$$

$$u_{c.g.}\left\{\begin{array}{c}1\\1\\ \vdots\\1\\ \hline 0\\0\\ \vdots\\0\end{array}\right\} + \underline{\bar{\theta}}\left\{\begin{array}{c}t_1\\t_2\\ \vdots\\t_a\\ \hline 1\\ \vdots\\1\end{array}\right\} + \sum_{j=1}^{2a-2} \alpha_j \{\bar{\bar{r}}^f\}_j \qquad (3)G$$

where $\{\bar{\bar{r}}^f\}_j$ is the 'j'th modal displacement vector.

Let

$$[Z] = \begin{bmatrix} 1 & z_1 & & & \\ 1 & z_2 & & & \\ \vdots & \vdots & & & \\ 1 & z_Q & \{\bar{\bar{r}}^f\}_1 & \cdots & \{\bar{\bar{r}}^f\}_{2Q-2} \\ 0 & 1 & & & \\ 0 & 1 & & & \\ \vdots & \vdots & & & \\ 0 & 1 & & & \end{bmatrix}$$

Then Eq.(3)G becomes

$$\begin{Bmatrix} \bar{\bar{u}} \\ \bar{\bar{\theta}} \end{Bmatrix} = [Z]\{q\} \tag{4)G}$$

where

$$\{q\} = \begin{Bmatrix} u_{C.G.} \\ \bar{\Theta} \\ \alpha_1 \\ \alpha_2 \\ \vdots \\ \alpha_{2Q-2} \end{Bmatrix}$$

Premultiplying Eq.(1)G by $[Z]^T$ and utilizing Eq.(4)G the equations of motion become

$$[Z]^T[M][Z]\{\ddot{q}\}+[Z]^T[K][Z]\{q\} =[Z]^T\begin{Bmatrix} F_x \\ C_\theta \end{Bmatrix}$$

$$\begin{bmatrix} M_T & & & & \\ & I_{c.g.} & & & \\ & & \overline{M}_1 & & \\ & & & \ddots & \\ & & & & \overline{M}_{2n-2} \end{bmatrix}\{\ddot{q}\} + \begin{bmatrix} 0 & & & & \\ & 0 & & & \\ & & \omega_1^2\overline{M}_1 & & \\ & & & \ddots & \\ & & & & \omega_{2n-2}^2\overline{M}_{2n-2} \end{bmatrix}\{q\}$$

$$=\{Q(t)\} \qquad (5)G$$

where

M_T = total mass of the system

$I_{c.g.}$ = rotary inertia about c.g.

$\overline{M}_i$ = 'i'th modal mass

ω_i = 'i'th modal frequency

$$\{Q(t)\} = [Z]^T\begin{Bmatrix} F_x \\ C_\theta \end{Bmatrix}$$

For the forced vibration problem, we will consider modal damping in the form:

$$\begin{bmatrix} M_T & & & & \\ & I_{c.G.} & & & \\ & & \overline{M}_1 & & \\ & & & \ddots & \\ & & & & \overline{M}_{2Q-2} \end{bmatrix} \{\ddot{q}\} + \begin{bmatrix} 0 & & & & \\ & 0 & & & \\ & & 2\beta_1\omega_1 & & \\ & & & \ddots & \\ & & & & 2\beta_{2Q-2}\,\omega_{2Q-2} \end{bmatrix} \{\dot{q}\}$$

$$\begin{bmatrix} 0 & & & & \\ & 0 & & & \\ & & \omega_1^2\,\overline{M}_1 & & \\ & & & \ddots & \\ & & & & \omega_{2Q-2}^2\,\overline{M}_{2Q-2} \end{bmatrix} \{q\} = \{Q(t)\} \qquad (6)G$$

where β_i is the 'i'th modal damping coefficient. Once the interaction forces are specified, the load vectors $\{F_x\}$ and $\{C_\theta\}$ can be determined and the forced vibration solution obtained.

If there is no building present, the criteria earthquake motion of the ground surface would be

$$u_g = \sum_{j=1}^{NN} \tilde{u}_g^{\Omega_j} = \sum_{j=1}^{NN} X_j\, e^{i\Omega_j t} \qquad (7)G$$

where X_j is the 'j'th complex amplitude and Ω_j is the 'j'th circular frequency. NN is usually from 500 to 1000. Each X_j is the Fourier coefficient of the actual transient earthquake motion.

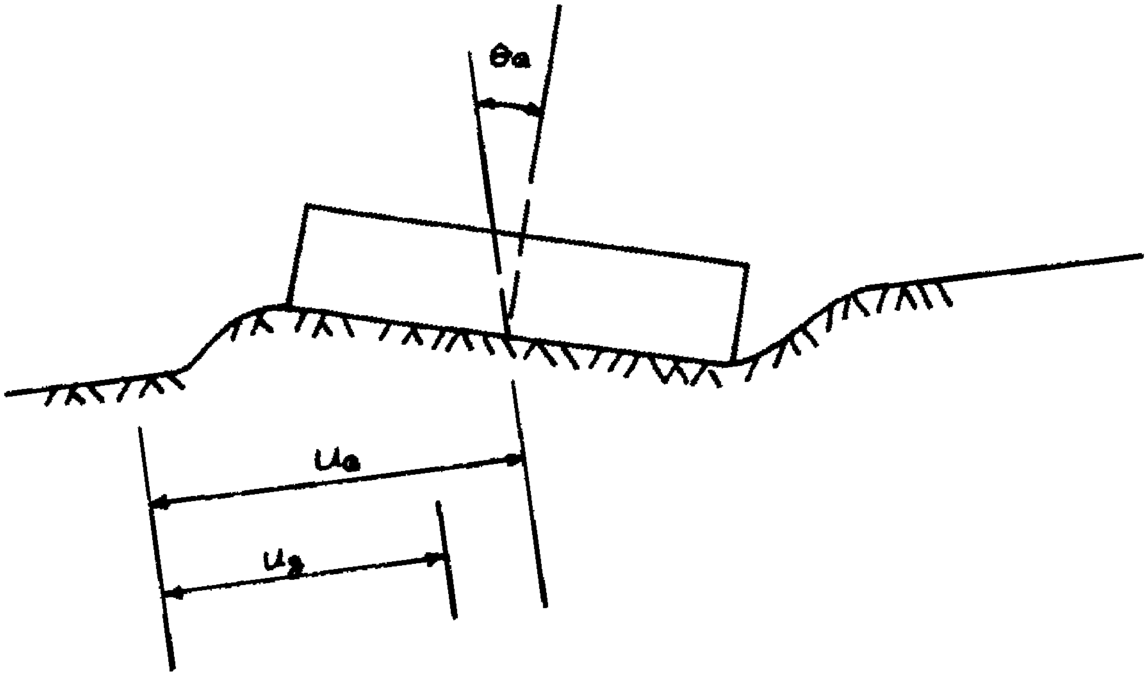


Figure G-4

The displacement of the base as cause by the incident wave is

$$u_a = \sum_{j=1}^{NN} \tilde{u}_a^{\Omega_i} = \sum_{j=1}^{NN} A_j \mathrm{X}_j e^{i\Omega_j t} \tag{8)G}$$

where A_j is the 'j'th complex magnification factor for the horizontal displacement of the base mass, as shown in Figure G-4.

The angle of rotation of the base as cause by the incident wave is

$$\theta_a = \sum_{j=1}^{NN} \tilde{\theta}_a^{\Omega_j} = \sum_{j=1}^{NN} B_j \mathrm{X}_j e^{i\Omega_j t} \tag{9)G}$$

in which B_j is the 'j'th complex magnification factor.

For each Ω_j, we have

$$\{q\} = \{\tilde{q}\}_j e^{i\Omega_j t} \tag{10)G}$$

The interaction forces are,

$$\{F_x\} = \begin{Bmatrix} 0 \\ \vdots \\ 0 \\ F_a \end{Bmatrix}$$

and

$$\{C_o\} = \left\{ \begin{matrix} 0 \\ \vdots \\ 0 \\ C_a \end{matrix} \right\}$$

where

$$F_a = \sum_{j=1}^{NN} (k_H^{\Omega_j} + i\Omega_j c_H^{\Omega_j})(\tilde{u}_a^{\Omega_j} - \tilde{u}_j^{\Omega_j})$$

$$C_a = \sum_{j=1}^{NN} (k_\theta^{\Omega_j} + i\Omega_j C_\theta^{\Omega_j})\,\tilde{\theta}_a^{\Omega_j}$$

One of the primary purposes of this research is to find $(k_H^{\Omega_j} + i\Omega_j C_H^{\Omega_j})$ and $(k_\theta^{\Omega_j} + i\Omega_j C_\theta^{\Omega_j})$. These are the compliance coefficients of the saturated soil at each respective frequency. Once these coefficients are known, we could solve Eq.(6)G as function of each frequency. The final results are then the summations of solutions at each frequency.

APPENDIX H

FINITE ELEMENT SOLUTION PROGRAM FOR LARGE STRUCTURES

APPENDIX H

FINITE ELEMENT SOLUTION PROGRAM FOR LARGE STRUCTURES

For small structural problems, computer programs can be written such that only core memory is used. Programming for these small problems which can be handled without using too much back-up storage space is usually fairly straightforward. However, for problems involving many equations and matrices with wide bandwidths, a very large core memory is required for the ordinary band solver subroutines. In many cases such core requirements far exceed the capacity of the existing computers. The more sophisticated solution programs such as frontal solutions went a long way in minimizing storage requirements and reducing execution time. However, they still require a fair amount of core and therefore are unsuitable for very large structures.

A simple and straightforward finite element computer program procedure capable of handling very large practical problems by the use of random access devices to link the back-up storage of the computer is used herein.(8) This computer program procedure is used for assembling and solving widely banded and sparse finite element matrix equations. The two very important features of this program procedure are: (1) it does not store any zero submatrices within the band and (2) during the solution of equations all operations

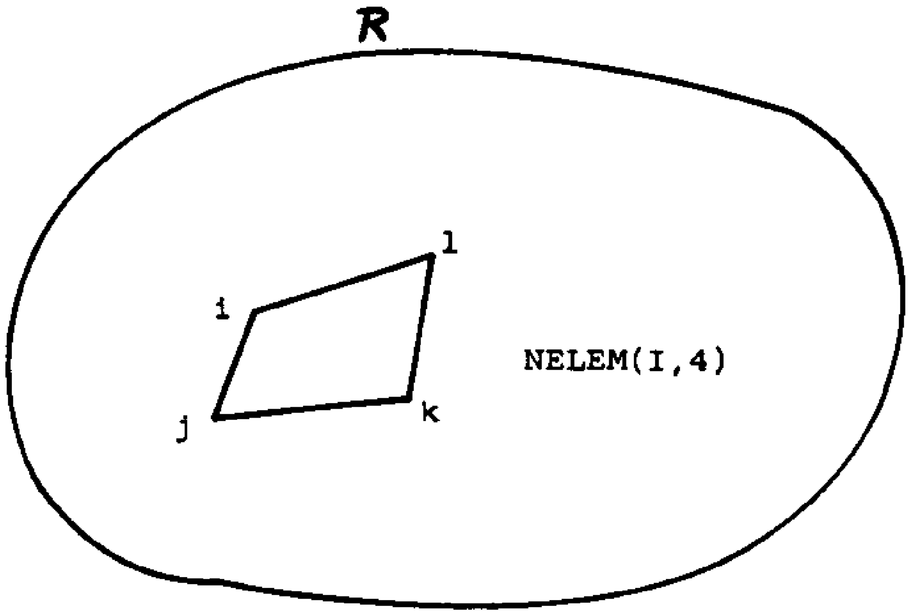


Figure H-1

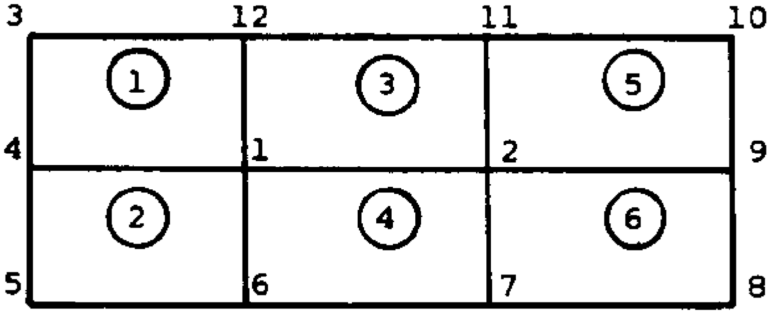


Figure H-2

(Note that the numbering system is not chosen to minimize the bandwidths, but it is chosen only for example purposes.)

dealing with zero submatrices within the band are skipped automatically.

Let $\mathcal{R}$, Figure H-1, be the region subjected to a number of loadings $\{P\}$ and with certain specified boundary conditions. The finite element idealization of region $\mathcal{R}$ results in NNODES number of nodes and NEMENT number of elements. For our analysis, only quadrilateral elements are used. A matrix NELEM(I,4), in which I varies from 1 to NEMENT, is formed to specify element connectivity. Each node in any element has four degrees-of-freedoms. They are two degrees-of-freedoms, U_x and U_y , for the soil displacements and two degrees-of-freedoms, W_x and W_y, for the flow of the fluid relative to the soil. The whole degrees-of-freedoms are stored in a vector CSPD(I), in which I varies from 1 to 4 x NNODES. They are stored in the following order, (U_{x_1}, U_{y_1} , W_{x_1}, W_{y_1} , U_{x_2} , U_{y_2}, W_{x_2} , W_{y_2} ,........, U_{x_I} , U_{y_I} , W_{x_I} , W_{y_I} ,.............).

An example is considered for illustration of methodology. Figure H-2 shows a soil region, with loadings and with certain specified boundary conditions. The soil is idealized by dividing it into NEMENT (=6) number of quadrilateral elements with NNODES (=12) number of nodes and the element numbers are shown inside the small circles. A stiffness matrix for the soil can be easily assembled. One important information needed in solution scheme is the effective bandwidth. The bandwidths of the stiffness matrix after

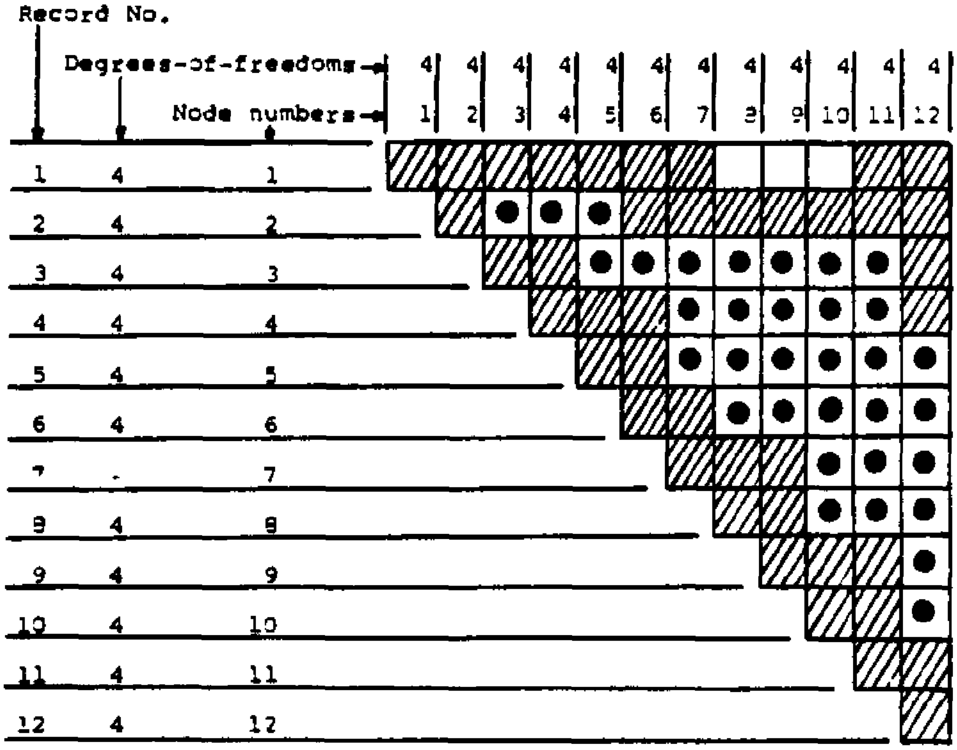

Record No.
Degrees-of-freedoms
Node numbers
4 4 4 4 4 4 4 4 4 4 4 4
1 2 3 4 5 6 7 8 9 10 11 12
1 4 1
2 4 2
3 4 3
4 4 4
5 4 5
6 4 6
7 7
8 4 8
9 4 9
10 4 10
11 4 11
12 4 12

Figure H-3

assembly and the effective bandwidths (during reduction) are not necessarily the same in all cases.

Figure H-3 shows the master stiffness matrix for soil region of Figure H-2. The shaded areas are filled during stiffness matrix assembly. The blocks with shaded small circles are filled up by non-zero terms in the process of reduction by the Gauss elimination techneque. From Figure H-3 it can be seen that the effective bandwidth at node 6 is 28 although the last five blocks contain zeros before reduction. This is because these five blocks will be filled up by non-zero terms in the process of reduction by the Gauss elimination technique when using node 2 as the pivotal node.

Consider the Jth node of the soil region at which certain elements are connected. From matrix NELEM, we can find out the difference between J and the highest number in each of the elements including J node as connecting node. The highest among all these differences is store in a vector MMDL as MMDL(J) in which J varies from 1 to NNODES. However MMDL(J) should satisfy the following condition

$$\mathrm{MMDL}(J) \geq \mathrm{MMDL}(J-1)-1 \qquad (1)\mathrm{H}$$

where

$J = 2,3,4,\ldots\ldots\ldots \mathrm{NNODES}$

Also MMDL(J) = total number of small blocks representing one node in the upper triangular part of the stiffness equations of the Jth node minus 1 (see Figure H-3). The effective bandwidth of Jth node is 4 x (MMDL(J) + 1).

In many practical problems there may be many zero submatrices inside the band and, as it is logical not to store them. To do this it is necessary to calculate the total number of non-zero matrix coefficients inside the band, and this requires the introduction of another vector NLOG whose Ith coefficient contains the total number of non-zero blocks above the Ith diagonal block plus 1. In the example NLOG vector is (1,2,3,4,5,6,7,7,8,9,11,12), and can be automatically calculated using NELEM. MMDL and NLOG together will identify all the zero blocks.

Consider now the same example of soil region shown in Figure H-2. The upper diagonal terms of the master stiffness matrix (see Figure H-3) and the load matrix are divided into NNODES number of rectangular blocks, and the blocks are numbered 1,2,........,I,......,NNODES. The Ith block contains an upper diagonal portion of the master stiffness equations of the Ith node and is stored in the ramdom access file no. 14 of the disc as the Ith record. The total number of non-zero matrix coefficients in the Ith record is NBND(I), which is created from MMDL and NLOG and which varies from node to node, and it is therefore convenient to store the rectangular matrix as one-dimensional vector A in order to save storage space in the disc.

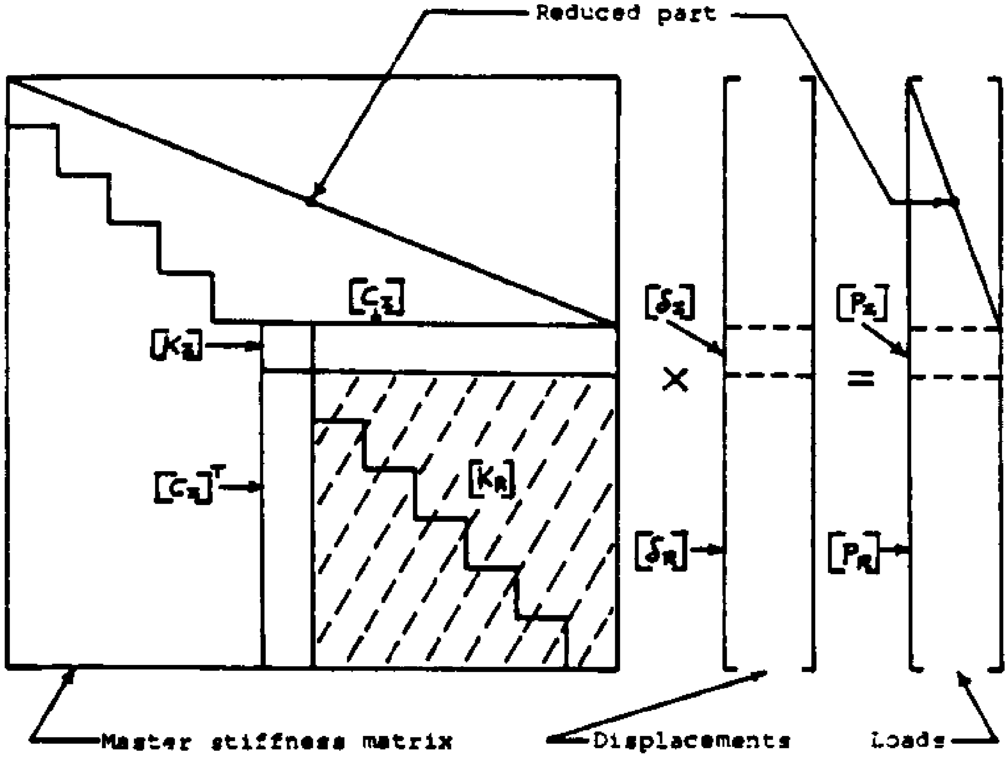

Reduced part
$[C_I]$
$[K_I]$
$[C_I]^T$
$[K_R]$
$[\delta_I]$
$[\delta_R]$
×
=
$[P_I]$
$[P_R]$
Master stiffness matrix
Displacements
Loads

Figure H-4

BOUNDARY CONDITIONS

Boundary conditions are incorporated by letting the leading diagonal terms to be ones and corresponding rows and columns to be zeros. The load vector is modified to include the effects of the given specified displacements.

REDUCTION OF MASTER STIFFNESS MATRIX

Consider the stage where the stiffness equations of (I - 1) nodes have already been reduced. The form of the master stiffness, to be reduced further, is shown in Figure H-4. The expression for reduction of the Ith node equations is as follows:

$$[\overline{K_R}] = [K_R] - [C_I]^T [K_I]^{-1} [C_I] \qquad (2)H$$

and

$$[\overline{P_R}] = [P_R] - [C_I]^T [K_I]^{-1} [P_I] \qquad (3)H$$

where $[\overline{K_R}]$ is the reduced form of the stiffness matrix $[K_R]$, and $[\overline{P_R}]$ is the reduced form of load matrix $[P_R]$.

Let $[C_I]$ be divided into MMDL(I) small submatrices:

$$[C_I] = \left[[C_I]^*_1, [C_I]^*_2, \cdots\cdots, [C_I]^*_{MMDL(I)} \right] \qquad (4)H$$

Similarly $[\bar{K}_R]$ can also be divided into (MMDL(I) x MMDL(I)) submatrices as follows:

$$[\bar{K}_R] = \begin{bmatrix} [\bar{K}_R]^*_{I,1} & \cdots\cdots\cdots & [\bar{K}_R]^*_{I,MMDL(I)} \\ \vdots & & \vdots \\ [\bar{K}_R]^*_{MMDL(I),1} & \cdots\cdots & [\bar{K}_R]^*_{MMDL(I),MMDL(I)} \end{bmatrix} \qquad (5)H$$

Finally $[\bar{P}_R]$ is divided into MMDL(I) submatrices:

$$[\bar{P}_R] = \begin{bmatrix} [\bar{P}_R]^*_1 \\ \vdots \\ [\bar{P}_R]^*_{MMDL(I)} \end{bmatrix} \qquad (6)H$$

In submatrix form we can rewrite Equations (2)H and (3)H as:

$$[\bar{K}_R]^*_{ij} = [K_R]^*_{ij} - [C_I]^{*T}_i [K_I]^{-1} [C_I]^*_j \qquad (7)H$$

and

$$[\bar{P}_R]^*_i = [P_R]^*_i - [C_I]^{*T}_i [K_I]^{-1} [P_I] \qquad (8)H$$

Submatrices $[K_R]^*_{ij}$ and $[P_R]^*_i$ are obtained by dividing $[K_R]$ and $[P_R]$ in similar way as $[\overline{K_R}]$ and $[\overline{P_R}]$ are divided. Now, if $[C_I]^*_i$ or $[C_I]^*_j$ happens to be a null matrix, then the above reduction is not performed and thereby a considerable amount of computer time is saved. In the program this is tested by the vector NLOG.

BACK SUBSTITUTION

The stiffness equations of the Ith node take the following form after reduction:

$$[K_I][\delta_I]+[C_I][\delta_R]=[P_I] \qquad (9)H$$

or

$$[K_I][\delta_I]+\sum_{i=1}^{MMDL(I)}[C_I]^*_i[\delta_R]^*_i=[P_I] \qquad (10)H$$

where $[\delta_R]^*_i$ is similar to $[P_R]^*_i$.

At this stage $[\delta_R]$ is known, hence $[\delta_I]$ can be obtained by using the following equation:

$$[\delta_I]=[K_I]^{-1}[P_I]-[K_I]^{-1}\sum_{i=1}^{MMDL(I)}[C_I]^*_i[\delta_R]^*_i \qquad (11)H$$

During the process of reduction $[K_I]^{-1}$, $[C_I]$ matrices are stored in the random access file no.14. Here also the multiplication $[C_I]^*_i[\delta_R]^*_i$ is skipped if $[C_I]^*_i$ is null.

APPENDIX I

PROGRAM LISTING

APPENDIX I

PROGRAM LISTING

```
      DIMENSION NELEM(300,4),ENODES(300,2),MMDL(300),NLOG(300),NBND(300)
      DIMENSION MATYPE(300),NTBC(300,4),NUC(300),MUC(300),SPD(1200)
      DIMENSION STP(1200),OMST(50),MPLOT(50,10),MDIR(10),MLENG(10)
      DIMENSION ISIGN(80),NDCC(10)
      DOUBLE PRECISION SNODES(300,2),PAR(12)
      COMPLEX CKVV,CKHH,CKTT,CKHT,CKTH
      COMPLEX*16 CSTP(1200),CSPD(1200)
      DEFINE FILE 14( 600,1600,U,INT)
  100 FORMAT(8I10)
      NNNN=300
      N444=1200
      CALL READDD(ISIGN,NEMENT,NNODES,NELEM,MATYPE,ENODES,MMDL,
     1 NLOG,NBND,NUC,MUC,JUW,SPD,NTBC,NTRAN,JTBC,STP,NPLOT,MDIR,
     2 MLENG,MPLOT,NDCCP,NCC1,NCC2,NCC3,NPSUM1,NPSUM2,NDCC,ADIM,NNNN,
     3 N444,NODECC)
      READ(5,100)NCASE
      DO 1000 IC=1,NCASE
      CALL PARAM(PAR,AL1,CF1,DF,OMS)
      CALL DMOM(NOAMP,DINDAM,DINDAS,NOM,OMST)
      DO 1001 KOM=1,NOAMP
      CALL WTPAR(PAR,AL1,CF1,DF,OMS)
      DO 1002 KOM=1,NOM
      OM=OMST(KOM)
      PAR(7)=OM
      DO 401 I=1,NNODES
      DO 401 J=1,2
  401 SNODES(I,J) = ENODES(I,J)*OMST(KOM)/PAR(12)
      PAR(11)=2.*PAR(11)/OM
      OMS=2.*OMS/OM
      CALL STMATR(NNODES,NEMENT,SNODES,NELEM,NBND,PAR,NLOG,MATYPE,DF,
     1OMS,NNNN)
      CALL STMODI(NNODES,NBND,NLOG,MMDL,NUC,MUC,JUW,SPD,STP,CSTP,NTBC,
     1 NTRAN,JTBC,PAR,MATYPE,SNODES,DF,OMS,NNNN,N444,CSPD)
      CALL SOLUTN( NNODES,NBND,NLOG,MMDL,CSPD,CSTP,NNNN,N444)
      CALL PRINU(NNODES,PAR(7),CSPD,N444)
      CALL PRINF(NNODES,PAR(7),CSTP,N444)
      IF(NDCCP.LE.0)GO TO 53
      VS2=PAR(10)*(1.-2.*PAR(9)) / (2.*(1.-PAR(9))*(PAR(8)-DF))
      VS=SQRT(VS2)
      OMDL=OM*ADIM/VS
      CALL COMCCF(NODECC,NCC1,NCC2,NCC3,NPSUM1,NPSUM2,NDCC,ADIM,
     1 ENODES,CSTP,CSPD,NNNN,N444,PAR,CKVV,CKHH,CKTT,CKHT,CKTH)
      CALL WRITEC(NCC1,NCC2,NCC3,OMDL,CKHH,CKVV,CKTT,CKHT,CKTH)
   53 IF(NPLOT.LE.0)GO TO 1002
```

```
      CALL PLOTUW(NPLOT,MLENG,MDIR,MPLOT,CSPD,CSTP,N444)
      PAR(11)=DM*PAR(11)/2.
      DMS=DM*DMS/2.
 1002 CONTINUE
      DMS=DMS+DINDAS
 1001 PAR(11)=PAR(11)+DINDAM
 1000 CONTINUE
      STOP
      END
      SUBROUTINE STMODI(NNODES,NBNO,NLOG,MMOL,NUC,MUC,JUM,SPD,STP,CSTP,
     1 NTBC,NTRAN,JTBC,PAR,MATYPE,SNODES,DF,DMS,NNNN,N444,CSPD)
      COMPLEX*16 A( 400),CSTP(N444),B( 400),CSPD(N444)
      DIMENSION MATYPE(NNNN)
      DIMENSION NUC(NNNN),MUC(NNNN),SPD(N444),STP(N444),NTBC(NNNN,4)
      DIMENSION NBNO(NNNN),NLOG(NNNN),MMOL(NNNN),NSPACE(25)
      DOUBLE PRECISION PAR(12),SNODES(NNNN,2),XLL,DABS
      COMPLEX*16 C1,C2,D1,D2,C1D,TC1,TC2,PD1,PD2,TC1D,SH,TSH
      N4=NNODES*4
      DO 2 I=1,N4
      CSPD(I)=0.
    2 CSTP(I)=STP(I)
      IF(NTRAN.LE.0)GO TO 40
C          ADD IN TRANSMITTING BOUNDARY
      CALL TRANBC(C1,C2,D1,D2,C1D,SH,PAR,DF,DMS)
      DO 44 I=1,JTBC
      FIND(14'NTBC(I,2)+NNODES)
      J1=NBNO(NTBC(I,2))
      J2=NBNO(NTBC(I,3))
      IF(IABS(NTBC(I,4)).EQ.1)GO TO 41
      IF(IABS(NTBC(I,4)).EQ.2)GO TO 42
   41 XLL=DABS(SNODES(NTBC(I,2),2)-SNODES(NTBC(I,3),2))
      ID=1
      IS=6
      GO TO 45
   42 XLL=DABS(SNODES(NTBC(I,2),1)-SNODES(NTBC(I,3),1))
      ID=6
      IS=1
   45 SSIGN=-1.
      J99=NTBC(I,1)
      IF(MATYPE(J99).LE.0)GO TO 43
      TC1D=SSIGN*C1D*XLL/3.
      TSH=SSIGN*SH*XLL/3.
      READ(14'NTBC(I,2)+NNODES)(A(K),K=1,J1)
      A(ID)=A(ID)-TC1D*(PAR(9)-DF)*PAR(12)**2
      A(IS)=A(IS)-TSH*(PAR(8)-DF)*PAR(12)**2
      IF(NTBC(I,3)-NTBC(I,2))50,50,51
   51 MEST=NTBC(I,3)-NTBC(I,2)
      JJ1=NTBC(I,2)
      JJ2=NTBC(I,3)
      DO 52 JJ=JJ1,JJ2
```

```
      JN1=JJ-NTBC(I,2)+1
      IF(NLOG(JJ).LT.JN1)MEST=MEST-1
   52 CONTINUE
      L1=MEST*16
      A(L1+ID)=A(L1+ID)-TC1D*(PAR(8)-DF)*PAR(12)**2/2.
      A(L1+IS)=A(L1+IS)-TSH*(PAR(8)-DF)*PAR(12)**2/2.
   50 WRITE(14'NTBC(I,2) +NNODES)(A(K),K=1,J1)
      READ(14'NTBC(I,3)+NNODES)(A(K),K=1,J2)
      A(ID)=A(ID)-TC1D*(PAR(8)-DF)*PAR(12)**2
      A(IS)=A(IS)-TSH*(PAR(8)-DF)*PAR(12)**2
      IF(NTBC(I,2)-NTBC(I,3))60,60,61
   61 MEST=NTBC(I,2)-NTBC(I,3)
      JJ1=NTBC(I,3)
      JJ2=NTBC(I,2)
      DO 62 JJ=JJ1,JJ2
      JN1=JJ-NTBC(I,3)+1
      IF(NLOG(JJ).LT.JN1)MEST=MEST-1
   62 CONTINUE
      L1=MEST*16
      A(L1+ID)=A(L1+ID)-TC1D*(PAR(8)-DF)*PAR(12)**2/2.
      A(L1+IS)=A(L1+IS)-TSH*(PAR(8)-DF)*PAR(12)**2/2.
   60 WRITE(14'NTBC(I,3) +NNODES)(A(K),K=1,J2)
      GO TO 44
   43 TC1=SSIGN*C1*XLL/3.
      TC2=SSIGN*C2*XLL/3.
      PD1=-SSIGN*D1*XLL/3.
      PD2=-SSIGN*D2*XLL/3.
      TSH=SSIGN*SH*XLL/3.
      READ(14'NTBC(I,2)+NNODES)(A(K),K=1,J1)
      A(ID)=A(ID)-TC1*PAR(8)*PAR(12)**2
      A(ID+8)=A(ID+8)-TC2*PAR(8)*PAR(12)**2
      A(ID+2)=A(ID+2)-PD1*PAR(8)*PAR(12)**2
      A(ID+10)=A(ID+10)-PD2*PAR(8)*PAR(12)**2
      A(IS)=A(IS)-TSH*(PAR(8)-DF)*PAR(12)**2
      IF(NTBC(I,3)-NTBC(I,2))70,70,71
   71 MEST=NTBC(I,3)-NTBC(I,2)
      JJ1=NTBC(I,2)
      JJ2=NTBC(I,3)
      DO 72 JJ=JJ1,JJ2
      JN1=JJ-NTBC(I,2)+1
      IF(NLOG(JJ).LT.JN1)MEST=MEST-1
   72 CONTINUE
      L1=MEST*16
      A(L1+ID)=A(L1+ID)-TC1*PAR(8)*PAR(12)**2/2.
      A(L1+ID+8)=A(L1+ID+8)-TC2*PAR(8)*PAR(12)**2/2.
      A(L1+ID+2)=A(L1+ID+2)-PD1*PAR(8)*PAR(12)**2/2.
      A(L1+ID+10)=A(L1+ID+10)-PD2*PAR(8)*PAR(12)**2/2.
      A(L1+IS)=A(L1+IS)-TSH*(PAR(8)-DF)*PAR(12)**2/2.
   70 WRITE(14'NTBC(I,2)+NNODES)(A(K),K=1,J1)
      READ(14'NTBC(I,3)+NNODES)(A(K),K=1,J2)
```

```
      A(ID)=A(ID)-TC1*PAR(8)*PAR(12)**2
      A(ID+8)=A(ID+8)-TC2*PAR(8)*PAR(12)**2
      A(ID+2)=A(ID+2)-PD1*PAR(8)*PAR(12)**2
      A(ID+10)=A(ID+10)-PD2*PAR(8)*PAR(12)**2
      A(IS)=A(IS)-TSH*(PAR(8)-DF)*PAR(12)**2
      IF(NTBC(I,2)-NTBC(I,3))80,80,81
   81 MEST=NTBC(I,2)-NTBC(I,3)
      JJ1=NTBC(I,3)
      JJ2=NTBC(I,2)
      DO 82 JJ=JJ1,JJ2
      JN1=JJ-NTBC(I,3)+1
      IF(NLOG(JJ).LT.JN1)MEST=MEST-1
   82 CONTINUE
      L1=MEST*16
      A(L1+ID)=A(L1+ID)-TC1*PAR(8)*PAR(12)**2/2.
      A(L1+ID+8)=A(L1+ID+8)-TC2*PAR(8)*PAR(12)**2/2.
      A(L1+ID+2)=A(L1+ID+2)-PD1*PAR(8)*PAR(12)**2/2.
      A(L1+ID+10)=A(L1+ID+10)-PD2*PAR(8)*PAR(12)**2/2.
      A(L1+IS)=A(L1+IS)-TSH*(PAR(8)-DF)*PAR(12)**2/2.
   80 WRITE(14'NTBC(I,3)+NNODES)(A(K),K=1,J2)
   44 CONTINUE
   40 NIN=NNODES-1
      DO 3 I=1,NIN
      J=NBND(I)
      M4=J/4
      READ(14'I+NNODES)(A(K),K=1,J)
      DO 4 K=1,J
    4 B(K)=A(K)
      CALL ALOCAL(I,MMOL,NLOG,NSPACE,IJ,NNNN)
      DO 5 K=1,JUW
      IF(I.NE.NUC(K))GO TO 5
      DO 6 KK=1,M4
    6 A((KK-1)*4+MUC(K))=0.
      DO 7 KK=1,4
    7 A((MUC(K)-1)*4+KK)=0.
      A((MUC(K)-1)*4+MUC(K))=1.
      IF(SPD(K).EQ.0.)GO TO 5
      DO 8 KK=1,4
      IF(KK.EQ.MUC(K))GO TO 8
      N=(I-1)*4+KK
      CSTP(N)=CSTP(N)-B((MUC(K)-1)*4+KK)   *SPD(K)
    8 CONTINUE
      DO 30 L=1,IJ
      DO 30 LL=1,4
      NN=(NSPACE(L)-1)*4+LL
   30 CSTP(NN)=CSTP(NN)-B(L*15+(LL-1)*4+MUC(K))*SPD(K)
    5 CONTINUE
      DO 9 K=1,IJ
      DO 9 L=1,JUW
      IF(NSPACE(K).NE.NUC(L))GO TO 9
```

```
      DO 10 KK=1,4
   10 A(K*16+(MUC(L)-1)*4+KK)=0.
      IF(SPD(L).EQ.0.)GO TO 9
      DO 11 KK=1,4
      N=(I-1)*4+KK
   11 CSTP(N)=CSTP(N)-B(K*16+(MUC(L)-1)*4+KK)   *SPD(L)
    9 CONTINUE
    3 WRITE(14'1+NNODES)(A(K),K=1,J)
      J=NBND(NNODES)
      M4=J/4
      READ(14'2*NNODES)(A(K),K=1,J)
      DO 12 K=1,J
   12 B(K)=A(K)
      DO 13 K=1,JUW
      IF(NNODES.NE.NUC(K))GO TO 13
      DO 14 KK=1,M4
   14 A((KK-1)*4+MUC(K))=0.
      DO 15 KK=1,4
   15 A((MUC(K)-1)*4+KK)=0.
      A((MUC(K)-1)*4+MUC(K))=1.
      IF(SPD(K).EQ.0.)GO TO 13
      DO 16 KK=1,4
      IF(KK.EQ.MUC(K))GO TO 16
      N=(NNODES-1)*4+KK
      CSTP(N)=CSTP(N)-B((MUC(K)-1)*4+KK)*SPD(K)
   15 CONTINUE
   13 CONTINUE
      WRITE(14'2*NNODES)(A(K),K=1,J)
      DO 20 I=1,JUW
      N=(NUC(I)-1)*4+MUC(I)
   20 CSTP(N)=SPD(I)
      RETURN
      END
      SUBROUTINE AINV(A,N,N2,D,L,M)
      DIMENSION        L(N),M(N)
      COMPLEX*16 A(N2),D,BIGA,HOLD
      COMPLEX AB,BB
C
C     SEARCH FOR LARGEST ELEMENT
C
      D=1.0
      NK=-N
      DO 80 K=1,N
      NK=NK+N
      L(K)=K
      M(K)=K
      KK=NK+K
      BIGA=A(KK)
      DO 20 J=K,N
      IZ=N*(J-1)
```

```
      DO 20 I=K,N
      IJ=IZ+I
10    AB=BIGA
      ABR=REAL(AB)
      ABI=AIMAG(AB)
      ABA=ABR**2+ABI**2
      BB=A(IJ)
      BBR=REAL(BB)
      BBI=AIMAG(BB)
      BBA=BBR**2+BBI**2
      IF(ABA-BBA)15,20,20
15    BIGA=A(IJ)
      L(K)=I
      M(K)=J
20    CONTINUE
C
C     ITERCHANGE ROWS
C
      J=L(K)
      IF(J-K) 35,35,25
25    KI=K-N
      DO 30 I=1,N
      KI=KI+N
      HOLD=-A(KI)
      JI=KI-K+J
      A(KI)=A(JI)
30    A(JI)=HOLD
C
C     INTERCHANGE COLUMNS
C
35    I=M(K)
      IF(I-K) 45,45,38
38    JP=N*(I-1)
      DO 40 J=1,N
      JK=NK+J
      JI=JP+J
      HOLD=-A(JK)
      A(JK)=A(JI)
40    A(JI)=HOLD
C
C     DIVIDE COLUMN BY MINUS PIVOT(VALUE OF
C     PIVOT ELEMENT IS CONTAINED IN BIGA)
45    AB=BIGA
      ABR=REAL(AB)
      ABI=AIMAG(AB)
      ABA=ABR**2+ABI**2
      IF(ABA)48,46,48
46    D=0.0
      RETURN
48    DO 55 I=1,N
```

```
      IF(I-K) 50,55,50
50    IK=NK+I
      A(IK)=A(IK)/(-BIGA)
55    CONTINUE
C
C     REDUCE MATRIX
C
      DO 65 I=1,N
      IK=NK+I
      HOLD=A(IK)
      IJ=I-N
      DO 65 J=1,N
      IJ=IJ+N
      IF(I-K) 60,65,60
60    IF(J-K) 62,65,62
62    KJ=IJ-I+K
      A(IJ)=HOLD*A(KJ)+A(IJ)
65    CONTINUE
C
C     DIVIDE ROW BY PIVOT
C
      KJ=K-N
      DO 75 J=1,N
      KJ=KJ+N
      IF(J-K) 70,75,70
70    A(KJ)=A(KJ)/BIGA
75    CONTINUE
C
C     PRODUCT OF PIVOTS
C
      D=D*BIGA
C
C     REPLACE PIVOT BY RECIPROCAL
C
      A(KK)=1.0/BIGA
80    CONTINUE
C
C     FINAL ROW AND COLUMN INTERCHANGE
C
      K=N
 100  K=(K-1)
      IF(K) 150,150,105
 105  I=L(K)
      IF(I-K) 120,120,108
 108  JQ=N*(K-1)
      JR=N*(I-1)
      DO 110 J=1,N
      JK=JQ+J
      HOLD=A(JK)
      JI=JR+J
```

```
      A(JK)=-A(JI)
110   A(JI)=HOLD
120   J=M(K)
      IF(J-K) 100,100,125
125   KI=K-N
      DO 130 I=1,N
      KI=KI+N
      HOLD=A(KI)
      JI=KI-K+J
      A(KI)=-A(JI)
130   A(JI)=HOLD
      GO TO 100
150   RETURN
      END
      SUBROUTINE SOLUTN(NNODES,NBND,NLOG,MMDL,CSPD,CSTP,NNNN,N444)
      COMPLEX*16 CSTP(N444),CSPD(N444),D,E(400)
      DIMENSION NBND(NNNN),NLOG(NNNN),MMDL(NNNN)
      COMPLEX*16 AII(16),A( 400),B( 400),C( 400) ,BII(16),CII(16)
      DIMENSION L(4),M(4),NSPACE(25)
      NIN=NNODES-1
      DO 1 I=1,NIN
      J=NBND(I)
      READ(14'I+NNODES)(A(K),K=1,J)
      DO 2 K=1,16
    2 AII(K)=A(K)
      CALL AINV(AII,4,16,D,L,M)
      DO 30 K=1,16
   30 A(K)=AII(K)
      WRITE(14'I+NNODES)(A(K),K=1,J)
      MK=(J-16)/16
      MN=J-16
      MK4=(J-16)/4
      DO 22 K=1,4
   22 BII(K)=CSTP((I-1)*4+K)
      CALL AMULTI(AII,BII,4,4,1,16,4,4,CII)
      DO 23 K=1,MN
   23 C(K)=A(16+K)
      CALL ATRANP(C,4,MK4,4*MK4,B)
      CALL AMULTI(B,CII,MK4,4,1,4*MK4,4,MK4,C)
      CALL ALOCAL(I,MMDL,NLOG,NSPACE,IJ,NNNN)
      DO 45 KK=1,MK
      DO 45 LL=1,4
   45 CSTP((NSPACE(KK)-1)*4+LL)=CSTP((NSPACE(KK)-1)*4+LL)-
     1 C((KK-1)*4+LL)
      DO 3 JB=1,MN
    3 B(JB)=A(JB+16)
      CALL AMULTI(AII,B,4,4,MK4,16,4*MK4,4*MK4,C)
      CALL ALOCAL(I,MMDL,NLOG,NSPACE,IJ,NNNN)
      DO 6 K=1,IJ
      FIND(14'NSPACE(K)+NNODES)
```

```
      DO 7 KK=1,16
    7 BII(KK)=A(KK+K*16)
      CALL ATRANP(BII,4,4,16,AII)
      CALL AMULTI(AII,C,4,4,MK4,16,4*MK4,4 *MK4,B)
      NNN=NBND(NSPACE(K))
      READ(14'NSPACE(K)+NNODES) (E(LL),LL=1,NNN)
      DO 8 KK=K,IJ
      LS=NSPACE(K)
      LF=NSPACE(KK)
      MEST=-1
      DO 9 LL=LS,LF
      JNI=LL-LS+1
      MEST=MEST+1
      IF(NLOG(LL).LT.JNI)GO TO 9
    9 CONTINUE
      DO 10 M10=1,16
   10 E(MEST*16 +M10)= E(MEST*16 +M10)-B((KK-1)*16+ M10)
    8 CONTINUE
      WRITE(14'NSPACE(K)+NNODES)(E(LL),LL=1,NNN)
    5 CONTINUE
    1 CONTINUE
      J=NBND(NNODES)
      READ(14'2*NNODES)(A(I),I=1,J)
      CALL AINV(A,4,16,D,L,M)
      DO 50 K1=1,4
      DO 50 K2=1,4
   50 CSPD((NNODES-1)*4+K1)=CSPD((NNODES-1)*4+K1)+A((K1-1)+(K2-1)*4+1)
     1 *CSTP((NNODES-1)*4+K2)
      DO 51 I=2,NNODES
      J=NNODES+1-I
      JJ=NBND(J)
      READ(14'J+NNODES)(A(K),K=1,JJ)
      IJJ=J-1
      FIND(14'IJJ+NNODES)
      DO 52 K1=1,4
      DO 52 K2=1,4
   52 CSPD((J-1)*4+K1)=CSPD((J-1)*4+K1)+A((K1 -1)+(K2-1)*4+1)
     1 *CSTP((J-1)*4+K2)
      MK=(JJ-16)/16
      MN=JJ-16
      MK4=(JJ-16)/4
      DO 31 K=1,16
   31 AII(K)=A(K)
      DO 32 K=1,MN
   32 B(K)=A(K+16)
      CALL AMULTI(AII,B,4,4,MK4,16,4*MK4,4*MK4,C)
      CALL ALOCAL(J,MMOL,NLOG,NSPACE,IJ,NNNN)
      DO 56 KK=1,MK
      DO 56 LL=1,4
      DO 56 NN=1,4
```

```
55    CSPD((J-1)*4+LL)=CSPD((J-1)*4+LL)- C((KK-1)*16+(LL-1)+(NN-1)*4+1)
     1 *CSPD((NSPACE(KK)-1)*4+NN)
51    CONTINUE
      N4=4*NNODES
      DO 60 I=1,N4
60    CSTP(I)=0.
      DO 61 I=1,NIN
      J=NBND(I)
      READ(14'I)(A(K),K=1,J)
      FIND(14'I+1)
      CALL ALOCAL(I,MMOL,NLOG,NSPACE,IJ,NNNN)
      DO 62 K=1,16
62    AII(K)=A(K)
      DO 63 K=1,4
63    BII(K)=CSPD((I-1)*4+K)
      CALL AMULTI(AII,BII,4,4,1,16,4,4,CII)
      DO 64 K=1,4
64    CSTP((I-1)*4+K)=CSTP((I-1)*4+K)+CII(K)
      DO 65 K=1,IJ
      DO 66 KK=1,16
66    AII(KK)=A(K*16+KK)
      DO 67 KK=1,4
67    BII(KK)=CSPD((NSPACE(K)-1)*4+KK)
      CALL AMULTI(AII,BII,4,4,1,16,4,4,CII)
      DO 68 KK=1,4
68    CSTP((I-1)*4+KK)=CSTP((I-1)*4+KK)+CII(KK)
      CALL ATRANP(AII,4,4,16,CII)
      DO 69 KK=1,4
69    BII(KK)=CSPD((I-1)*4+KK)
      CALL AMULTI(CII,BII,4,4,1,16,4,4,AII)
      DO 70 KK=1,4
70    CSTP((NSPACE(K)-1)*4+KK)=CSTP((NSPACE(K)-1)*4+KK)+AII(KK)
65    CONTINUE
61    CONTINUE
      J=NBND(NNODES)
      READ(14'NNODES)(A(K),K=1,J)
      DO 71 K=1,16
71    AII(K)=A(K)
      DO 72 K=1,4
72    BII(K)=CSPD(NIN*4+K)
      CALL AMULTI(AII,BII,4,4,1,16,4,4,CII)
      DO 73 K=1,4
73    CSTP(NIN*4+K) = CSTP(NIN*4+K)+CII(K)
      RETURN
      END
      SUBROUTINE TRANBC(C1,C2,D1,D2,C1D,SH,PAR,DF,DMS)
      DOUBLE PRECISION PAR(12),Z1,Z2,Z3,Z4
      COMPLEX*16 C1, C2, D1, D2, C1D,SH
      COMPLEX*16 E4, DCMPLX,E1,E2,E3,CZ1,CZ2,CZ3,CZ6,CZ7,CDSQRT
      COMPLEX Q
```

```
      DO 10 IORY=1,2
      IF(IORY.EQ.1)GO TO 11
      Z1=1.
      Z2=PAR(7)*PAR(11)
      E4=DCMPLX(Z1,Z2)
      E4=CDSQRT(E4)
      Z3=0.
      E3=DCMPLX(Z3,-Z1)
      VP2=PAR(10)/(PAR(8)-DF)
      VP=SQRT(VP2)
      C10=E3*E4*VP/PAR(12)
      Z2=PAR(7)*DMS
      E4=DCMPLX(Z1,Z2)
      E4=CDSQRT(E4)
      VS2=VP2*(1.-2.*PAR(9))/(2.*(1.-PAR(9)))
      VS=SQRT(VS2)
      SM=E3*E4*VS/PAR(12)
      GO TO 10
   11 Z1=1.
      Z2=(PAR(10)*PAR(11)*PAR(7))/(PAR(0)*PAR(12)**2)
      E4=DCMPLX(Z1,Z2)
      Z3=PAR(2)-(PAR(3)*PAR(2))**2
      Z4=PAR(2)*Z2
      E1=DCMPLX(Z3,Z4)
      Z3=PAR(1)/PAR(4)+PAR(2)-2.*PAR(3)*PAR(2)*PAR(1)
     1 +(Z2*PAR(5))/(PAR(8)*PAR(6)*PAR(7))
      Z4=(Z2*PAR(1))/PAR(4)-PAR(5)/(PAR(8)*PAR(6)*PAR(7))
      E2=DCMPLX(Z3,Z4)
      Z3=PAR(1)/PAR(4)-PAR(1)**2
      Z4=PAR(5)/(PAR(8)*PAR(6)*PAR(7))
      E3=DCMPLX(Z3,-Z4)
      CZ1=E2**2-4.*E1*E3
      CZ1=CDSQRT(CZ1)
      CZ2=(-E2+CZ1)/(2.*E1)
      CZ3=(-E2-CZ1)/(2.*E1)
      CZ6=CDSQRT(CZ2)
      Q=CZ6
      IF(REAL(Q).GT.0)GO TO 1
      CZ6=-CZ6
    1 CZ7=CDSQRT(CZ3)
      Q=CZ7
      IF(REAL(Q).GT.0)GO TO 2
      CZ7=-CZ7
    2 CONTINUE
      C1= -(E4*CZ6*CZ7-1.)/(CZ6+CZ7)
      C2= -(PAR(3)*PAR(2)*CZ6*CZ7-PAR(1))/(CZ6+CZ7)
      D1= ((PAR(2)*(E4*CZ6**2+1.)*(E4*CZ7**2+1.))
     1/ ((PAR(3)*PAR(2)- PAR(1)*E4)*(CZ6**2-CZ7**2)))
     2*((PAR(3)*CZ6*(PAR(3)*PAR(2)*CZ6**2+PAR(1)))/
     3(E4*CZ6**2+1.    )-(PAR(3)*CZ7*(PAR(3)*PAR(2)*CZ7**2 +PAR(1)))
```

```
     4/(E4*CZ7**2+1.) -CZ6+CZ7)
      U2= ((PAR(2)*(PAR(3)*PAR(2)*CZ6**2+PAR(1))*(PAR(3)*PAR(2)
     1*CZ7**2+PAR(1)))/((PAR(3)*PAR(2)-PAR(1)*E4)
     2*(CZ6**2 -CZ7**2)))*((-CZ6*(E4*CZ6**2+1.))/
     3(PAR(3)*PAR(2)*CZ6**2+PAR(1)) +(CZ7*(E4*CZ7**2+1.))/
     4(PAR(3)*PAR(2)*CZ7**2+PAR(1)) + PAR(3)*(CZ6-CZ7))
  10 CONTINUE
      RETURN
      END
      SUBROUTINE CSPOIN(NNODES,NUC,MUC,JJW,SPD,CSPD,NNNN,N444)
      DIMENSION NUC(NNNN),MUC(NNNN),SPD(N444)
      COMPLEX*16 CSPD(N444)
      N4=4*NNODES
      DO 1 I=1,N4
    1 CSPD(I)=0.
      RETURN
      END
      SUBROUTINE ALOCAL(J,MMOL,NLOG,NSPACE,IJ,NNNN)
      DIMENSION MMOL(NNNN),NLOG(NNNN),NSPACE(25)
      IS=J+1
      IP=J+MMOL(J)
      IJ=0
      DO 55 KK=IS,IP
      JN1=KK-IS+1
      IF(NLOG(KK).LT.JN1)GO TO 55
      NSPACE(IJ+1)=KK
      IJ=IJ+1
  55 CONTINUE
      RETURN
      END
      SUBROUTINE COORD(NNODES,ENODES,NNNN)
      DIMENSION ENODES(NNNN,2)
    1 FORMAT(8I10)
    2 FORMAT('1', ' NODE POINTS',/,6X,'NODE',5X,'X(IN)',5X,'Y(IN)',/ )
    3 FORMAT(I10,2F10.2)
    4 FORMAT(8F10.0)
      DO 310 I=1,NNODES
      DO 310 J=1,2
 310 ENODES(I,J)=0.
 303 READ(5,1)L,NR,NI,NR2,NI2,NSTOP
      READ(5,4)ENODES(L,1),ENODES(L,2),X1,Y1,X2,Y2
      IF(NR.LE.0)GO TO 301
      DO 302 K=2,NR
      ENODES(L+NI*(K-1),1) =  ENODES(L+NI*(K-2),1)+X1
 302 ENODES(L+NI*(K-1),2) =  ENCODES(L+NI*(K-2),2)+Y1
      IF(NR2.LE.0)GO TO 301
      DO 306 M=2,NR2
      DO 306 K=1,NR
      ENODES(L+NI*(K-1)+NI2*(M-1),1)=ENODES(L+NI*(K-1)+NI2*(M-2),1)+X2
 306 ENODES(L+NI*(K-1)+NI2*(M-1),2)=ENODES(L+NI*(K-1)+NI2*(M-2),2)+Y2
```

```
301 IF(NSTOP.GT.0)GO TO 300
    GO TO 303
303 WRITE(6,2)
    DO 304 I=1,NNODES
304 WRITE(6,3)   I,ENODES(I,1),ENODES(I,2)
    RETURN
    END
    SUBROUTINE PRINF(NNODES,F,CSTP,N444)
    COMPLEX*16 CSTP(N444)
    DOUBLE PRECISION F
  1 FORMAT('1',' FREQUENCY= ',F12.6,/,6X,'NODE',20X,'TX(I)',20X,
   1 'TY(I)',20X,'FX(I)',20X,'FY(I)',/)
  2 FORMAT(I10,4(3X,(2E11.4)))
    N4=4*NNODES
    DO 100 I=1,N4
100 CSTP(I)=CSTP(I)/12.
    FG=F
    WRITE(6,1)FG
    DO 10 I=1,NNODES
    K=(I-1)*4
 10 WRITE(6,2)I,(CSTP(K+J),J=1,4)
    RETURN
    END
    SUBROUTINE SPDISP(NUC,MUC,JUW,SPD,NNNN,N444)
    DIMENSION NUC(NNNN),MUC(NNNN),SPD(N444)
 11 FORMAT(2I10,F10.0,3I10)
 12 FORMAT('1',' SPECIFIED DISPLACEMENT',/,
   1 9X,'I',4X,'NUC(I)',4X,'MUC(I)',4X,'SPD(I)',/)
 13 FORMAT(3I10,F10.2)
    J=1
351 READ(5,11)NUM,MUM,UMN,NSTUP,NREP,NINC
    MUC(J)=MUM
    NUC(J)=NUM
    SPD(J)=UMN
    IF(NREP.LE.0)GO TO 360
    DO 361 M=2,NREP
    MUC(J+M-1)=MUM
    NUC(J+M-1)=NUM+(M-1)*NINC
361 SPD(J+M-1)=UMN
    J=J+NREP
    GO TO 362
360 J=J+1
362 IF(NSTUP.LE.0)GO TO 351
    JUW=J-1
    WRITE(6,12)
    DO 600 I=1,JUW
600 WRITE(6,13)I,NUC(I),MUC(I),SPD(I)
    RETURN
    END
    SUBROUTINE ANELEM(NEMENT,NELEM,MATYPE,NNNN)
```

```
      DIMENSION NELEM(NNNN,4),MATYPE(NNNN)
    1 FORMAT(16I5)
    2 FORMAT('1', ' ELEMENT CONNECTIVITY',/,
     1 3X,'ELEMENT',4X,'NODE I', 4X,'NODE J', 4X, 'NODE K', 4X ,'NODE L',
     2 1X,'MATYPE(I)',/)
    3 FORMAT(8I10)
      I=1
  202 READ(5,1)   (NELEM(I,J),J=1,4),NREP,NINC,NREP2,NINC2,MTP
      MATYPE(I)=MTP
      IF(NREP.LE.0)GO TO 200
      DO 201 K=2,NREP
      MATYPE(I+K-1)=MTP
      DO 201 J=1,4
  201 NELEM(I+K-1,J) = NELEM(I+K-2,J)+NINC
      IF(NREP2.LE.0)GO TO 203
      DO 204 M=2,NREP2
      DO 204 K=1,NREP
      MATYPE(I+K-1+NREP*(M-1))=MTP
      DO 204 J=1,4
  204 NELEM( I+K-1+NREP*(M-1),J)= NELEM(  I+K-1+NREP*(M-2),J) +NINC2
  203 NR2=1
      IF( NREP2.GT.0) NR2=NREP2
      I=I+NR2*NREP
      GO TO 205
  200 I=I+1
  205 IF(I.LE.NEMENT)GO TO 202
      WRITE(6,2)
      DO 206 I=1,NEMENT
  206 WRITE(6,3)   I,(NELEM(I,J),J=1,4),MATYPE(I)
      RETURN
      END
      SUBROUTINE EMMOL(NNODES,NEMENT,NELEM,MMOL,NNNN)
      DIMENSION NELEM(NNNN,4),MMOL(NNNN)
      DIMENSION MDIF(4,4)
      DO 1 I=1,NNODES
    1 MMOL(I)=0
      DO 2 I=1,NEMENT
      DO 2 J=1,4
      DO 3 K=1,4
    3 MDIF(J,K)=NELEM(I,K)-NELEM(I,J)
      MT=MAX0(MDIF(J,1),MDIF(J,2),MDIF(J,3),MDIF(J,4))
      IDX=NELEM(I,J)
    2 MMOL(NELEM(I,J))=MAX0(MT,MMOL(IDX))
      DO 4 I=2,NNODES
      M=MMOL(I-1)-1
      IF(MMOL(I).LT.M)MMOL(I)=M
    4 CONTINUE
      RETURN
      END
      SUBROUTINE BANCA(NNODES,NEMENT,NELEM,NLOG,NNNN)
```

```
      DIMENSION NELEM(NNNN,4),NLOG(NNNN)
      DIMENSION MDIF(4,4)
      DO 1 I=1,NNODES
    1 NLOG(I)=1
      DO 2 I=1,NEMENT
      DO 3 J=1,4
      DO 3 K=1,4
    3 MDIF(J,K)=NELEM(I,K)-NELEM(I,J)
      DO 4 K=1,4
      MT=1+MAX0(MDIF(1,K),MDIF(2,K),MDIF(3,K),MDIF(4,K))
    4 NLOG(NELEM(I,K))=MAX0(MT,NLOG(NELEM(I,K)))
    2 CONTINUE
      RETURN
      END
      SUBROUTINE ANBND(NNODES,MMOL,NLOG,NBND,NNNN)
      DIMENSION MMOL(NNNN),NLOG(NNNN),NBND(NNNN)
      DO 1 I=1,NNODES
    1 NBND(I)=0
      DO 2 I=1,NNODES
      M=MMOL(I)+1
      DO 3 J=1,M
      IF(NLOG(I+J-1).GE.J)NBND(I)=NBND(I)+1
    3 CONTINUE
    2 NBND(I)=NBND(I)*16
      RETURN
      END
      SUBROUTINE SPFORC(NNODES,STP,N444)
      DIMENSION STP(N444)
    1 FORMAT(8I10)
    2 FORMAT(2I10,F10.0,3I10)
      N4=4*NNODES
      DO 370 I=1,N4
  370 STP(I)=0.
      READ(5,1)NSCASE
      IF(NSCASE.LE.0)GO TO 371
  373 READ(5,2)NUM,MUM,TPNM,NSTUP,NREP,NINC
      I=(NUM-1)*4+MUM
      STP(I)=TPNM
      IF(NREP.LE.0)GO TO 372
      DO 374 M=2,NREP
      I=(NUM-1+(M-1)*NINC)*4+MUM
  374 STP(I)=TPNM
  372 IF(NSTUP.LE.0)GO TO 373
  371 CONTINUE
      DO 100 I=1,N4
  100 STP(I)=12.*STP(I)
      RETURN
      END
      SUBROUTINE BCTRAN(NTBC,NTRAN,JTBC,NNNN)
      DIMENSION NTBC(NNNN,4)
```

```
  1 FORMAT(8I10)
  2 FORMAT('1',' TRANSMITTING BOUNDARY',/,4X,'NODE I',4X,'NODE J',
   1 1X,'DIRECTION',/)
  3 FORMAT('1',' NO TRANSMITTING BOUNDARY')
    READ(5,1)NTRAN
    JTBC=0
    IF(NTRAN.LE.0)GO TO 100
    J=1
 10 READ(5,1)NELM,NOD1,NOD2,NDIR,NREP,NINE,NINC,NSTOP
    NTBC(J,1)=NELM
    NTBC(J,2)=NOD1
    NTBC(J,3)=NOD2
    NTBC(J,4)=NDIR
    IF(NREP.LE.0)GO TO 20
    DO 40 M=2,NREP
    NTBC(J+M-1,1)=NELM+(M-1)*NINE
    NTBC(J+M-1,2)=NOD1+(M-1)*NINC
    NTBC(J+M-1,3)=NOD2+(M-1)*NINC
 40 NTBC(J+M-1,4)=NDIR
    J=J+NREP
    GO TO 30
 20 J=J+1
 30 IF(NSTOP.LE.0)GO TO 10
    JTBC=J-1
    WRITE(6,2)
    DO 60 I=1,JTBC
 60 WRITE(6,1)(NTBC(I,K+1),K=1,3)
    GO TO 70
100 WRITE(6,3)
 70 CONTINUE
    RETURN
    END
    SUBROUTINE READDD(ISIGN,NEMENT,NNODES,NELEM,MATYPE,ENODES,MMDL,
   1 NLOG,NBND,NUC,MUC,JUW,SPD,NTBC,NTRAN,JTBC,STP,NPLOT,MDIR,
   2 MLENG,MPLOT,NDCCP,NCC1,NCC2,NCC3,NPSUM1,NPSUM2,NDCC,ADIM,NNNN,
   3 N444,NODECC)
    DIMENSION MDIR(10),MLENG(10),MPLOT(50,10),NOCC(10),ISIGN(80)
    DIMENSION NELEM(NNNN,4),MATYPE(NNNN),ENODES(NNNN,2)
    DIMENSION MMDL(NNNN),NLOG(NNNN),NBND(NNNN),STP(N444)
    DIMENSION NUC(NNNN),MUC(NNNN),SPD(N444),NTBC(NNNN,4)
100 FORMAT(8I10)
111 FORMAT(8F10.0)
115 FORMAT(80A1)
116 FORMAT('1',///////////)
117 FORMAT(25X,80A1)
C
C          READ TITLE
C          -----CARD TITLE-----
C
    READ(5,115)(ISIGN(I),I=1,80)
```

```
      WRITE(6,116)
      DO 300 J=1,5
  300 WRITE(6,117)(ISIGN(I),I=1,80)
C
C          READ NUMBER OF ELEMENT AND NUMBER OF NODES
C          -----CARD A-----
C
      READ(5,100)NEMENT,NNODES
C
C          READ AND GENERATE ELEMENT CONNECTIVITY
C          -----CARD B-----
C
      CALL ANELEM(NEMENT,NELEM,MATYPE,NNNN)
C
C          READ AND GENERATE NODAL COORDINATES
C          -----CARD C1 & C2-----
C
      CALL COORD(NNODES,ENODES,NNNN)
C
C          GENERATE EFFECTIVE HALF BANDWIDTH BLOCK VECTOR
C
      CALL EMMDL(NNODES,NEMENT,NELEM,MMDL,NNNN)
C
C          GENERATE VECTOR SHOWING TOTAL NUMBER OF NON-ZERO BLOCKS
C          ABOVE THE I-TH DIAGONAL BLOCK PLUS ONE
C
      CALL BANCA(NNODES,NEMENT,NELEM,NLOG,NNNN)
C
C          VECTOR SHOWING TOTAL NUMBER OF NON-ZERO
C          MATRIX COEFFICIENTS AT NODE I
C
      CALL ANBNO(NNODES,MMDL,NLOG,NBAD,NNNN)
C
C          READ SPECIFIED NODAL DISPLACEMENT
C          -----CARD D-----
C
      CALL SPDISP(NUC,MUC,JJW,SPD,NNNN,N444)
C
C          READ TRANSMITTING BOUNDARY
C          -----CARD E1 & E2-----
C
      CALL BCTRAN(NTBC,NTRAN,JTBC,NNNN)
C
C          READ SPECIFIED NODAL FORCE
C          -----CARD F1 & F2-----
C
      CALL SPFORC(NNODES,STP,N444)
C
C          PLOT OPTION
C          -----CARD G1 & G2-----
```

```
C
      CALL PLOTOP(NPLOT,MDIR,MLENG,MPLOT)
C
C          READ DIMENSIONLESS COMPLIANCE COEFFICIENT PARAMETERS
C          -----CARD H1,H2,H3, $ H4-----
C
      READ(5,100)NDCCP
      IF(NDCCP.LE.0)GO TO 50
      READ(5,100)NODECC,NCC1,NCC2,NCC3,NPSUM1,NPSUM2
      READ(5,100)(NDCC(LN),LN=1,NODECC)
      READ(5,111)ADIM
   50 RETURN
      END
      SUBROUTINE DMOM(NDAMP,DINDAM,DINDAS,NOM,OMST)
      DIMENSION OMST(50)
  100 FORMAT(8I10)
  111 FORMAT(8F10.0)
  114 FORMAT(I10,2F10.0)
C          READ FREQUENCIES TO BE ANALIZED AND NUMBER OF DAMPING
C          -----CARD I4,I5 $ I6-----
      READ(5,114)NDAMP,DINDAM,DINDAS
      READ(5,100)NOM
      READ(5,111)(OMST(I),I=1,NOM)
      RETURN
      END
      SUBROUTINE PARAM(PAR,AL1,CF1,DF,OMS)
      DOUBLE PRECISION PAR(12)
    1 FORMAT(8F10.0)
      READ(5,1)   AL1,CS,CF1,DS,DF,PO,VI,PE
      READ(5,1)   OM,POIS,OMS
      AL=12.*AL1
      CF=12.*CF1
      AK=CF/(AL+CS**2*CF)
      VC2=(AL+CS**2*CF)/DS
      VC=SQRT(VC2)
      AN=DF/DS
      PAR(1)=AN
      PAR(2)=AK
      PAR(3)=CS
      PAR(4)=PO
      PAR(5)=VI
      PAR(6)=PE
      PAR(7)=0.
      PAR(8)=DS
      PAR(9)=POIS
      PAR(10)=AL
      PAR(11)=OM
      PAR(12)=VC
      RETURN
      END
```

```
      SUBROUTINE WTPAR(PAR,AL1,CF1,DF,DMS)
      DOUBLE PRECISION PAR(12)
  112 FORMAT('1', 'AL= ELASTIC COMPLIANCE CONSTANT= ',E13.6,/,' CS= MATE
     1RIAL CONSTANT REPRESENTING THE COMPRESSIBILITY OF THE SOLID PARTIC
     2LES= ',E13.6,/,' CF= BULK MODULUS OF FLUID= ',E13.6,/, ' DS= MASS
     3DENSITY OF BULK OF FLUID-SOLID= ',E13.6,/,' DF= MASS DENSITY OF FL
     4ID= ',E13.6,/,' PO= POROSITY= ',E13.6,/,' VI= VISCOSITY OF THE FLU
     5ID= ',E13.6)
  113 FORMAT(' PE= COEFFICIENT OF PERMEABILITY= ',E13.6,/,' DM= MATERIAL
     1 DAMPING COMPRESSIVE= ',E13.6,/,' DMS= MATERIAL DAMPING SHEARING=
     2',E13.6)
      WRITE(6,112)AL1,PAR(3),CF1,PAR(9),DF,PAR(4),PAR(5)
      WRITE(6,113)PAR(6),PAR(11),DMS
      RETURN
      END
      SUBROUTINE  STMATR(NNODES,NEMENT,SNODES,NELEM,NBND,PAR,NLOG,MATYPE,
     1 DF,DMS,NNNN)
      DIMENSION NELEM(NNNN,4),NBND(NNNN),NLOG(NNNN),MATYPE(NNNN)
      DOUBLE PRECISION SNODES(NNNN,2),PAR(12),APAR8,APAR3
      COMPLEX DQ
      COMPLEX*16 A( 400),COM,DCMPLX,ESTU(8,8),B(16,16)
      DOUBLE PRECISION TH(4),PH(4),EMAS(4,4),ESTW(8,8),C1,C2
      DO 1 I=1,NNODES
      J=NBND(I)
      DO 2 K=1,J
    2 A(K)=0.
    1 WRITE(14'I)(A(K),K=1,J)
      DO 3 I=1,NEMENT
      IF(MATYPE(I).EQ.0)GO TO 20
      APAR8=PAR(8)
      PAR(8)=PAR(8)-DF
      APAR3=PAR(3)
      PAR(3)=0.
      GO TO 30
   20 C1=PAR(1)/PAR(4)
      C2=PAR(5)/(PAR(3)*PAR(7)*PAR(6))
      COM=DCMPLX(C1,-C2)
   30 DO 4 K=1,4
      TH(K)=SNODES( NELEM(I,K),1)
    4 PH(K)=SNODES( NELEM(I,K),2)
      CALL EMASS(TH,PH,EMAS)
      CALL ESTIFW(TH,PH,ESTW)
      CALL ESTIFU(TH,PH,PAR,ESTU,DMS)
      DO 5 M=1,16
      DO 5 N=1,16
    5 B(M,N)=0.
      IF(MATYPE(I).EQ.1)GO TO 31
      DO 6 M=1,4
      DO 6 N=1,4
      M1=(M-1)*4
```

```
      NI=(N-1)*4
      B(MI+1,NI+1)=B(MI+1,NI+1)+EMAS(M,N)-ESTU(M,N)
      B(MI+1,NI+2)=B(MI+1,NI+2)-ESTU(M,N+4)
      B(MI+1,NI+3)=B(MI+1,NI+3)+EMAS(M,N)*PAR(1)-ESTW(M,N)*PAR(3)*PAR(2)
      B(MI+1,NI+4)=B(MI+1,NI+4)-ESTW(M,N+4)*PAR(3)*PAR(2)
      B(MI+2,NI+1)=B(MI+2,NI+1)-ESTU(M+4,N)
      B(MI+2,NI+2)=B(MI+2,NI+2)+EMAS(M,N)-ESTU(M+4,N+4)
      B(MI+2,NI+3)=B(MI+2,NI+3)-ESTW(M+4,N)*PAR(3)*PAR(2)
      B(MI+2,NI+4)=B(MI+2,NI+4)+EMAS(M,N)*PAR(1)-ESTW(M+4,N+4)*PAR(3)
     1 *PAR(2)
      B(MI+3,NI+1)=B(MI+3,NI+1)+EMAS(M,N)*PAR(1)-ESTW(M,N)*PAR(3)*PAR(2)
      B(MI+3,NI+2)=B(MI+3,NI+2)-ESTW(M,N+4)*PAR(3)*PAR(2)
      B(MI+3,NI+3)=B(MI+3,NI+3)+EMAS(M,N)*COM-ESTW(M,N)*PAR(2)
      B(MI+3,NI+4)=B(MI+3,NI+4)-ESTW(M,N+4)*PAR(2)
      B(MI+4,NI+1)=B(MI+4,NI+1)-ESTW(M+4,N)*PAR(3)*PAR(2)
      B(MI+4,NI+2)=B(MI+4,NI+2)+EMAS(M,N)*PAR(1)-ESTW(M+4,N+4)*PAR(3)
     1 *PAR(2)
      B(MI+4,NI+3)=B(MI+4,NI+3)-ESTW(M+4,N)*PAR(2)
    5 B(MI+4,NI+4)=B(MI+4,NI+4)+EMAS(M,N)*COM-ESTW(M+4,N+4)*PAR(2)
      GO TO 40
   31 DO 32 M=1,4
      DO 32 N=1,4
      MI=(M-1)*4
      NI=(N-1)*4
      B(MI+1,NI+1)=B(MI+1,NI+1)+EMAS(M,N)-ESTU(M,N)
      B(MI+1,NI+2)=B(MI+1,NI+2)-ESTU(M,N+4)
      B(MI+2,NI+1)=B(MI+2,NI+1)-ESTU(M+4,N)
   32 B(MI+2,NI+2)=B(MI+2,NI+2)+EMAS(M,N)-ESTU(M+4,N+4)
   40 DO 33 M=1,16
      DO 33 N=1,16
   33 B(M,N)=B(M,N)*PAR(3)*PAR(12)**2
      DO 7 M=1,4
      JB=NBND(NELEM(I,M))
      READ(14'NELEM(I,M))(A(K),K=1,JB)
      DO 10 N=1,4
      IF(NELEM(I,N)-NELEM(I,M))10,8,9
    8 DO 11 N1=1,4
      DO 11 M1=1,4
      L1=(N1-1)*4+M1
   11 A(L1)=B((M-1)*4+M1,(N-1)*4+N1)+A(L1)
      GO TO 10
    9 MEST=NELEM(I,N)-NELEM(I,M)
      JJ1=NELEM(I,M)
      JJ2=NELEM(I,N)
      DO 12 JJ=JJ1,JJ2
      JN1=JJ-NELEM(I,M)+1
      IF(NLOG(JJ).LT.JN1)MEST=MEST-1
   12 CONTINUE
      DO 13 N1=1,4
      DO 13 M1=1,4
```

```
      L1=MEST*16+(N1-1)*4+M1
   13 A(L1)=B((M-1)*4+M1,(N-1)*4+N1)+A(L1)
   10 CONTINUE
    7 WRITE(14'NELEM(I,M))(A(K),K=1,JB)
      IF(MATYPE(I).EQ.0)GO TO 3
      PAR(3)=APAR3
      PAR(8)=APAR8
    3 CONTINUE
      DO 200 I=1,NNODES
      J=NBND(I)
      READ(14'I)(A(K),K=1,J)
      DO 100 IK=1,4
      KI=(IK-1)*5+1
      QQ=A(KI)
      IF(REAL(QQ).NE.0)GO TO 100
      IF(AIMAG(QQ).EQ.0)A(KI)=1.
  100 CONTINUE
  200 WRITE(14'I+NNODES)(A(K),K=1,J)
      RETURN
      END
      SUBROUTINE UWPLOT(NORDER,NDF,X,N)
      INTEGER TITLE(8)/'  UX','  UY','  WX','  WY','  TX','  TY',
     1 '  PX','  PY'/
      INTEGER DOT/'.'/,BLANK/' '/,ESS/'X'/
      DIMENSION X(50),ISIGN(101),Y(50),NORDER(50)
    1 FORMAT(I5,E15.5,5X,101A1)
    2 FORMAT(25X,101A1)
    3 FORMAT('1','NODE',11X,A4,/)
      WRITE(6,3)TITLE(NDF)
      M=1
      IF(N.LE.25)M=2
      IF(N.LE.16)M=3
      IF(N.LE.12)M=4
      IF(N.LC. 8)M=6
      IF(N.LE. 6)M=8
      DO 10 I=1,101
   10 ISIGN(I)=BLANK
      ISIGN(51)=DOT
      BIGA=X(1)
      DO 20 I=2,N
      IF(X(I).GT.BIGA)BIGA=X(I)
   20 CONTINUE
      SMALLA=X(1)
      DO 30 I=2,N
      IF(X(I).LT.SMALLA)SMALLA=X(I)
   30 CONTINUE
      XMAX=ABS(BIGA)
      IF(ABS(SMALLA).GT.XMAX)XMAX=ABS(SMALLA)
      IF(XMAX.LE.10.**(-10))GO TO 40
      DO 50 I=1,N
```

```
50 Y((I-1)*M+1)=X(I)/XMAX
   IF(M.LE.1)GO TO 60
   N1=N-1
   DO 70 I=1,N1
   DELTA=(Y(I*M+1)-Y((I-1)*M+1))/M
   DO 70 J=2,M
70 Y((I-1)*M+J)=Y((I-1)*M+1)+(J-1)*DELTA
60 NM=N+(N-1)*(M-1)
   GO TO 61
40 NM=N+(N-1)*(M-1)
   DO 41 I=1,NM
41 Y(I)=0.
61 K=1
   KK=1
   DO 80 J=1,NM
   IS=50.*Y(J)+51.1
   ISIGN(IS)=ESS
   IF(J.EQ.K)GO TO 81
   WRITE(6,2)(ISIGN(JJ),JJ=1,101)
   GO TO 82
81 WRITE(6,1)NORDER(KK),X(KK),(ISIGN(JJ),JJ=1,101)
   K=K+M
   KK=KK+1
82 ISIGN(IS)=BLANK
   ISIGN(51)=DOT
80 CONTINUE
   RETURN
   END
   SUBROUTINE CONCOF(NODECC,NCC1,NCC2,NCC3,NPSUM1,NPSUM2,NOCC,ADIM,
  1 ENODES,CSTP,CSPD,NNNN,N444,PAR,CKVV,CKHH,CKTT,CKHT,CKTH)
   DIMENSION NOCC(10),ENODES(NNNN,2)
   DOUBLE PRECISION PAR(12)
   COMPLEX TXOMLS,TYOMLS,TANMLS,Q,CKVV,CKHH,CKTT,CKHT,CKTH,Q2
   COMPLEX*16 CSTP(N444),CSPD(N444)
   G=PAR(10)*(1.-2.*PAR(9))/(2.*(1.-PAR(9)))
   G=G/12.
   PI=3.141593
   TXOMLS=0.
   TYOMLS=0.
   TANMLS=0.
   CKVV=0.
   CKHH=0.
   CKTT=0.
   CKHT=0.
   CKTH=0.
   IF(NCC2.LE.0)GO TO 51
   DO 52 I=1,NODECC
52 TYOMLS=TYOMLS-CSTP((NOCC(I)-1)*4+2)*2.
   Q=CSPD((NPSUM1-1)*4+2)
   CKVV=TYOMLS/(REAL(Q)*PI*G)
```

```
 51 IF((NCC1+NCC3).LE.0)GO TO 60
    DO 54 I=1,NODECC
    K=NDCC(I)
    TXOMLS=TXOMLS-CSTP((K-1)*4+1)*2.
54  TANMLS=TANMLS+CSTP((K-1)*4+2)*(ENODES(K,1)-ENODES(NPSUM1,1))*2.
    IF(NCC1.LE.0)GO TO 55
    Q=CSPD((NPSUM1-1)*4+1)
    CKHH=TXOMLS/(REAL(Q)*PI*G)
    CKHT=TANMLS/(ADIM    *REAL(Q)*PI*G)
 55 IF(NCC3.LE.0)GO TO 60
    Q=CSPD((NPSUM1-1)*4+1)
    Q2=CSPD((NPSUM1-1)*4+2)
    DH1=REAL(Q)
    DV1=REAL(Q2)
    Q=CSPD((NPSUM2-1)*4+1)
    Q2=CSPD((NPSUM2-1)*4+2)
    DH2=REAL(Q)
    DV2=REAL(Q2)
    AX=ENODES(NPSUM2,1)-ENODES(NPSUM1,1)
    AY=ENODES(NPSUM2,2)-ENODES(NPSUM1,2)
    BX=AX+DH2-DH1
    BY=AY+DV2-DV1
    COSANG=(AX*BX+AY*BY)/(SQRT(AX**2+AY**2)*SQRT(BX**2+BY**2))
    RAD=ARCOS(COSANG)
    CKTT=TANMLS/(ADIM**2*RAD*PI*G)
    CKTH=TXOMLS/(ADIM*RAD*PI*G)
 60 RETURN
    END
    SUBROUTINE DETJJ(TH,PH,DETJ,Q)
    DOUBLE PRECISION A,B,C,TH(4),PH(4),Q(4),DETJ(4,4)
    A=TH(1)*(PH(2)-PH(4))+TH(2)*(PH(4)-PH(1))+TH(4)*(PH(1)-PH(2))
    B=(TH(2)-TH(1))*(PH(3)-PH(4))+(TH(3)-TH(4))*(PH(1)-PH(2))
    C=(TH(2)-TH(3))*(PH(1)-PH(4))+(TH(1)-TH(4))*(PH(3)-PH(2))
    Q(1)=6.943184E-2
    Q(2)=3.300095E-1
    Q(3)=6.699905E-1
    Q(4)=9.305602E-1
    DO 1 I=1,4
    DO 1 J=1,4
  1 DETJ(I,J)=A+B*Q(I)+C*Q(J)
    RETURN
    END
    SUBROUTINE PLOTUM(NPLOT,MLENG,MDIR,MPLOT,CSPD,CSTP,N444)
    DIMENSION X(50),NORDER(50)
    DIMENSION MLENG(10),MDIR(10),MPLOT(50,10)
    COMPLEX QQ
    COMPLEX*16 CSPD(N444),CSTP(N444)
    DO 40 K=1,NPLOT
    NL=MLENG(K)
    DO 50 KK=1,NL
```

```
      NORDER(KK)=MPLOT(KK,K)
      IF(MDIR(K).EQ.1)QQ    =CSPD((MPLOT(KK,K)-1)*4+1)
      IF(MDIR(K).EQ.2)QQ    =CSPD((MPLOT(KK,K)-1)*4+2)
      IF(MDIR(K).EQ.3)QQ    =CSPD((MPLOT(KK,K)-1)*4+3)
      IF(MDIR(K).EQ.4)QQ    =CSPD((MPLOT(KK,K)-1)*4+4)
      IF(MDIR(K).EQ.5)QQ    =CSTP((MPLOT(KK,K)-1)*4+1)
      IF(MDIR(K).EQ.6)QQ    =CSTP((MPLOT(KK,K)-1)*4+2)
      IF(MDIR(K).EQ.7)QQ    =CSTP((MPLOT(KK,K)-1)*4+3)
      IF(MDIR(K).EQ.8)QQ    =CSTP((MPLOT(KK,K)-1)*4+4)
   50 X(KK)=REAL(QQ)
      NDF=MDIR(K)
      N=MLENG(K)
   40 CALL UWPLOT(NORDER,NDF,X,N)
      RETURN
      END
      SUBROUTINE PRINU(NNODES,F,CSPD,N444)
      COMPLEX*16 CSPD(N444)
      DOUBLE PRECISION F
    1 FORMAT('1',' FREQUENCY= ',F12.6,/,6X,'NODE', 20X,'UX(I)',20X,
     1 'UY(I)',20X,'WX(I)',20X,'WY(I)',/)
    2 FORMAT(I10,4(3X,(2E11.4)))
      FG=F
      WRITE(6,1)FG
      DO 10 I=1,NNODES
      K=(I-1)*4
   10 WRITE(6,2)I,(CSPD(K+J),J=1,4)
      RETURN
      END
      SUBROUTINE GAUSSQ(F,E)
      DOUBLE PRECISION F(4,4),E
      AG1=1.739274E-1
      AG2=3.260726E-1
      E= AG1*(AG1*F(1,1)+AG2*F(1,2)+AG2*F(1,3)+AG1*F(1,4))
     1  +AG2*(AG1*F(2,1)+AG2*F(2,2)+AG2*F(2,3)+AG1*F(2,4))
     2  +AG2*(AG1*F(3,1)+AG2*F(3,2)+AG2*F(3,3)+AG1*F(3,4))
     3  +AG1*(AG1*F(4,1)+AG2*F(4,2)+AG2*F(4,3)+AG1*F(4,4))
      RETURN
      END
      SUBROUTINE ESTIFU(TH,PH,PAR,ESTU,OMS)
      DOUBLE PRECISION  TH(4),PH(4),Q(4),DETJ(4,4),AS(8,4,4),F(4,4)
      DOUBLE PRECISION ETU(3,8),PAR(12),E,G2,Z1,Z2
      COMPLEX*16 G1,ESTU(8,8),H1,H2,H3,E4,DCMPLX,E5
      CALL DETJJ(TH,PH,DETJ,Q)
      CALL OSHAPE(TH,PH,AS,Q)
      Z1=1.
      Z2=PAR(7)*PAR(11)
      E4= DCMPLX(Z1,Z2)
      G1=E4*PAR(10)/(PAR(8)*PAR(12)**2)
      G2=(PAR(3)**2)*PAR(2)
      H1=G1+G2
```

```
      H2=(PAR(9)/(1.-PAR(9))) *G1+G2
      H3=(1.-2.*PAR(9))* G1/(2.*(1.-PAR(9)))
      Z2=DMS*PAR(7)
      E5=DCMPLX(Z1,Z2)
      H3=H3*E5/E4
      IK=1
      DO 1 K=1,8
      DO 9 L=IK,8
      DO 2 I=1,4
      DO 2 J=1,4
    2 F(I,J)=(AS(K,I,J)*AS(L,I,J))/DETJ(I,J)
      CALL GAUSSQ(F,E)
    9 ETU(K,L)=E
    1 IK=IK+1
      K=8
      L=0
      DO 12 I=1,7
      DO 15 J=2,K
   15 ETU(L+J,I)=ETU(I,L+J)
      K=K-1
   12 L=L+1
      DO 3 I=1,4
      DO 3 J=1,4
      ESTU(I,J)=H1*ETJ(I,J)+H3*ETU(I+4,J+4)
      ESTU(I,J+4)=H2*ETJ(I,J+4)+H3*ETU(I+4,J)
      ESTU(I+4,J)=H2*ETU(I+4,J)+H3*ETU(I,J+4)
    3 ESTU(I+4,J+4)=H1*ETU(I+4,J+4)+H3*ETU(I,J)
      RETURN
      END
      SUBROUTINE ESTIFW(TH,PH,ESTW)
      DOUBLE PRECISION TH(4),PH(4),ESTW(8,8),Q(4),DETJ(4,4)
      DOUBLE PRECISION AS(8,4,4),F(4,4),E
      CALL DETJJ(TH,PH,DETJ,Q)
      CALL DSHAPE(TH,PH,AS,Q)
      IK=1
      DO 2 K=1,8
      DO 9 L=IK,8
      DO 3 I=1,4
      DO 3 J=1,4
    3 F(I,J)=(AS(K,I,J)*AS(L,I,J))/DETJ(I,J)
      CALL GAUSSQ(F,E)
    9 ESTW(K,L)=E
    2 IK=IK+1
      K=8
      L=0
      DO 12 I=1,7
      DO 15 J=2,K
   15 ESTW(L+J,I)=ESTW(I,L+J)
      K=K-1
   12 L=L+1
```

```
      RETURN
      END
      SUBROUTINE EMASS(TH,PH,EMAS)
      DOUBLE PRECISION TH(4),PH(4),EMAS(4,4),Q(4),DETJ(4,4)
      DOUBLE PRECISION SHAPE(4,4,4),F(4,4),E
      CALL DETJJ(TH,PH,DETJ,Q)
      CALL ESHAPE(Q,SHAPE)
      IK=1
      DO 3 K=1,4
      DO 9 L=IK,4
      DO 4 I=1,4
      DO 4 J=1,4
    4 F(I,J)=SHAPE(K,I,J)*SHAPE(L,I,J)*DETJ(I,J)
      CALL GAUSSQ(F,E)
    9 EMAS(K,L)=E
    3 IK=IK+1
      K=4
      L=0
      DO 12 I=1,3
      DO 15 J=2,K
   15 EMAS(L+J,I)=EMAS(I,L+J)
      K=K-1
   12 L=L+1
      RETURN
      END
      SUBROUTINE ATRANP(A,M,N,MN,B)
      COMPLEX*16 A(MN),B(MN)
C          MATRIX A IS M BY N
C          MATRIX B IS TRANSPOSE OF A
C          MATRIX B IS N BY M
      DO 1 I=1,N
      DO 1 J=1,M
    1 B((J-1)*N+I)=A((I-1)*M+J)
      RETURN
      END
      SUBROUTINE WRITEC(NCC1,NCC2,NCC3,OMDL,CKHH,CKVV,CKTT,CKHT,CKTH)
      COMPLEX CKVV,CKHH,CKTT,CKHT,CKTH
  118 FORMAT(///)
  119 FORMAT(1X,'DIMENSIONLESS FREQUENCY = ',E10.3,/,21X,'CKVV',21X,
     1 'CKHH',21X,'CKTT',21X,'CKHT',21X,'CKTH')
  120 FORMAT(30X,(2E10.3),30X,(2E10.3))
  121 FORMAT(5X,(2E10.3))
  122 FORMAT(55X,(2E10.3),30X,(2E10.3))
      WRITE(6,118)
      IF(NCC1.LE.0)GO TO 51
      WRITE(6,119)OMDL
      WRITE(6,120)CKHH,CKHT
   51 IF(NCC2.LE.0)GO TO 52
      WRITE(6,119)OMDL
      WRITE(6,121)CKVV
```

```
   52 IF(NCC3.LE.0)GO TO 53
      WRITE(6,119)OMOL
      WRITE(6,122)CKTT,CKTH
   53 RETURN
      END
      SUBROUTINE PLOTOP(NPLOT,MDIR,MLENG,MPLOT)
      DIMENSION MDIR(10),MLENG(10),MPLOT(50,10)
  100 FORMAT(8I10)
      READ(5,100)NPLOT
      IF(NPLOT.LE.0)GO TO 500
      DO 501 IP=1,NPLOT
      READ(5,100)L,NREP,NINC,NDIR
      MDIR(IP)=NDIR
      MLENG(IP)=NREP
      MPLOT(1,IP)=L
      NREP1=NREP-1
      DO 501 KK=1,NREP1
  501 MPLOT(KK+1,IP)=MPLOT(KK,IP)+NINC
  500 RETURN
      END
      SUBROUTINE DSHAPE(TH,PH,AS,Q)
      DOUBLE PRECISION B1,B2,B3,A1,A2,A3, TH(4),PH(4),Q(4),AS(8,4,4)
      B1=-PH(1)+PH(4)
      B2=PH(1)-PH(2)+PH(3)-PH(4)
      B3=-PH(1)+PH(2)
      A1=-TH(1)+TH(4)
      A2=TH(1)-TH(2)+TH(3)-TH(4)
      A3=-TH(1)+TH(2)
      DO 1 I=1,4
      DO 1 J=1,4
      AS(1,I,J)= (B1+B2*Q(I))*(Q(J)-1.)-(B3+B2*Q(J))*(Q(I)-1.)
      AS(2,I,J)= (B1+B2*Q(I))*(1.-Q(J))-(B3+B2*Q(J))*(-Q(I)   )
      AS(3,I,J)= (B1+B2*Q(I))*Q(J)      -(B3+B2*Q(J))*Q(I)
      AS(4,I,J)= (B1+B2*Q(I))*(-Q(J)   )-(B3+B2*Q(J))*(1.-Q(I))
      AS(5,I,J)=-(A1+A2*Q(I))*(Q(J)-1.)+(A3+A2*Q(J))*(Q(I)-1.)
      AS(6,I,J)=-(A1+A2*Q(I))*(1.-Q(J))+(A3+A2*Q(J))*(-Q(I)   )
      AS(7,I,J)=-(A1+A2*Q(I))*Q(J)      +(A3+A2*Q(J))*Q(I)
    1 AS(8,I,J)=-(A1+A2*Q(I))*( -Q(J)) +(A3+A2*Q(J))*(1.-Q(I))
      RETURN
      END
      SUBROUTINE AMULTI(A,B,M,N,L,MN,NL,ML,C)
      COMPLEX*16 A(MN),B(NL),C(ML)
C          MATRIX A IS M BY N
C          MATRIX B IS N BY L
C          MATRIX C IS A TIMES B
      DO 2 I=1,ML
    2 C(I)=0.
      DO 1 I=1,L
      DO 1 J=1,M
      DO 1 K=1,N
```

```
1 C(( I-1)*M+J)=C((I-1)*M+J)+A(J+(K-1)*M)*B((I-1)*N+K)
  RETURN
  END
  SUBROUTINE ESHAPE(Q,SHAPE)
  DOUBLE PRECISION Q(4),SHAPE(4,4,4)
  DO 2 I=1,4
  DO 2 J=1,4
  SHAPE(1,I,J)=(1.-Q(I))*(1.-Q(J))
  SHAPE(2,I,J)=Q(I)*(1.-Q(J))
  SHAPE(3,I,J)=Q(I)*Q(J)
2 SHAPE(4,I,J)=(1.-Q(I))*Q(J)
  RETURN
  END
```

APPENDIX J

USER'S MANUAL FOR PROGRAM

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All input dimensions should be either dimensionless or in combinations of inches, pound-forces, slugs, seconds, and radians. All integers are right adjusted to the spaces provided. All real numbers are free to occupy the provided spaces without being either right adjusted or left adjusted.

Displacement outputs will be in inches. Force outputs will be in pound-forces. Moment outputs will be in inch-pound-forces. The compliance coefficients will be dimensionless.

Card Title: Read Title

FORMAT(80A1) one card

80
TITLE

TITLE = array of letters or number (up to 80 spaces) to identify the problem to be solved.

Card A: Read Number of Elements and Number of Nodes

FORMAT(8I10) one card

10	20	
NEMENT	NNODES	

NEMENT = total number of elements.

NNODES = total number of nodes.

Card B: Read Element Connectivity

FORMAT(16I5) as many cards as necessary

5	10	15	20	25	30
NELEM(I,1)	NELEM(I,2)	NELEM(I,3)	NELEM(I,4)	NREP	NINC

35	40	45	
NREP2	NINC2	MTP	

NELEM(I,1) = first node of element 'I'. 'I' is one at the start of first B Card.

NELEM(I,2) = second node of element 'I'.

NELEM(I,3) = third node of element 'I'.

NELEM(I,4)	=	fourth node of element 'I'.
NREP	=	total number of elements by first level generation, including the original element. It is blank or zero or one, if no first level generation is required.
NINC	=	increment added to each node number of the subsequent element by first level generation. It is blank, if no first level generation is required.
NREP2	=	total number of sets of elements by second level generation, including the original set of elements. It is blank or zero or one, if no second level generation is required.
NINC2	=	increment added to each node number of subsequent set of elements by second level generation. It is blank, if no second level generation is

required.

MTP = type of soil. If the soil is saturated, it is blank or zero. If the soil is unsaturated, it is one. The type is assigned to all elements (or element) generated by each B Card.

<u>Card C1 & C2: Read Nodal Coordinates</u>

FORMAT(8I10) set of two cards, as many sets

FORMAT(8F10.0) as necessary

10	20	30	40	50	60	
L	NR	NI	NR2	NI2	NSTOP	

10	20	30	40	50	60	
ENODES (L,1)	ENODES (L,2)	X1	Y1	X2	Y2	

L = node number.

NR = total number of nodes by first level generation, including the original node. It is blank or zero or one, if no first level generation is required.

NI = increment added to each subsequent node number by first level generation. It is blank, if no first level generation is required.

NR2 = total number of sets of nodes by second level generation, including the original set of nodes. It is blank or zero or one, if no second level generation is required.

NI2 = increment added to each node number of subsequent set of nodes by second level generation. It is blank, if no second level generation is required.

NSTOP = one, if the card set is the last set of C Cards. It is blank or zero, if it is otherwise.

ENODES(L,1) = x-coordinate of node 'L'. Dimension is in inches.

ENODES(L,2) = y-coordinate of node 'L'. Dimension is in inches.

X1 = increment added to each subsequent nodal x-coordinate by first level generation.

Dimension is in inches.

Y1 = increment added to each subsequent nodal y-coordinate by first level generation. Dimension is in inches.

X2 = increment added to each nodal x-coordinate of subsequent set of nodes by second level generation. Dimension is in inches.

Y2 = increment added to each nodal y-coordinate of subsequent set of nodes by second level generation. Dimension is in inches.

Card D: Read Specified Nodal Displacements

FORMAT(2I10,F10.0,3I10)

as many cards as necessary

10	20	30	40	50	60	
NUM	MUM	UMN	NSTUP	NREP	NINC	

NUM = node number.

MUM = specified displacement direction. Ux is one. Uy is two. Wx is three. Wy is four.

UMN = magnitude of specified displacement in inches.

NSTUP = one, if the card is the last card of D Cards. It is blank or zero, if it is otherwise.

NREP = total number of nodes with same specified displacement by first level generation.

NINC = increment added to each subsequent node number with same specified displacement by first level generation. Dimension is in inches.

<u>Card E1: Read If There Is Transmitting Boundary</u>

FORMAT(8I10) one card

10	
NTRAN	

NTRAN = one, if there is transmitting boundary. It is blank or zero, if there is no transmitting boundary and there should be no E2 Card.

Card E2: Read Transmitting Boundary

FORMAT(8I10) as many cards as necessary

10	20	30	40	50	60	70	80
NELM	NOD1	NOD2	NDIR	NREP	NINE	NINC	NSTOP

NELM = element 'J'.

NOD1 = first node connecting the transmitting boundary side of the element 'J'.

NOD2 = second node connecting the transmitting boundary side of the element 'J'.

NDIR = one, if it has positive x-face. It is two, if it has positive y-face. Negative value will be for negative face.

NREP = total number of elements with same transmitting boundary by first level generation, including the original element. It is blank or zero or one, if no first level generation is required.

NINE = increment added to each subsequent element number with same transmitting boundary by first level generation.

NINC = increment added to each node number connecting the transmitting boundary side of the subsequent element by first level generation. It is blank, if no first level generation is required.

NSTOP = one, if the card is the last of E2 Cards. It is blank or zero, if it is otherwise.

Card F1: Read If There Is Specified Nodal Forces

FORMAT(8I10) one card

10	
NSCASE	

NSCASE = one, if there is specified nodal force. It is blank or zero, if there is no specified nodal force and there should be no F2 Cards.

Card F2: Read Specified Nodal Forces

FORMAT(2I10,F10.0,3I10)

as many cards as necessary

10	20	30	40	50	60	
NUM	MUM	TRNM	NSTUP	NREP	NINC	

NUM = node number.

MUM = specified force direction. Tx is one. Ty is two. Px is three. Py is four.

TRNM = magnitude of specified force in pounds.

NSTUP = one, if the card is the last of F2 Cards. It is blank or zero, if it is otherwise.

NREP = total number of nodes with same specified force by first level generation.

NINC = increment added to each subsequent node number with same specified force by first level generation.

Card G1: Read If Plot Option Is Needed

FORMAT(8I10) one card

10	
NPLOT	

NPLOT = number of plots. It is blank or zero, if no plot is needed and there should be no G2 Card.

Card G2: Read Plot Option Parameters

FORMAT(8I10) as many cards as specified by the integer value of NPLOT on G1 Card

10	20	30	40	
L	NREP	NINC	NDIR	

L = node number.

NREP = total number of nodes to be plotted in one plot by first level generation.

NINC = increment added to each subsequent node number by first level generation.

NDIR = displacement or force direction to be plotted. Ux is one. Uy is two. Wx is three. Wy is four. Tx is five. Ty is six. Px is seven. Py is eight.

Card H1: Read If Compliance Coefficients Are Needed

FORMAT(8I10)   one card

10	
NDCCP	

NDCCP = one, if compliance coefficients are to be generated. It is blank or zero, if no compliance coefficient is needed and there should be no H2, H3, and H4 Cards.

Card H2, H3 & H4: Read Compliance Coefficient Parameters

FORMAT(8I10)   one card for H2 Card, as many
FORMAT(8I10)   cards as necessary for H3 Card,
FORMAT(8F10.0)   and one card for H4 Card

10	20	30	40	50	60	
NODECC	NCC1	NCC2	NCC3	NPSUM1	NPSUM2	

10	20							
NDCC (1)	NDCC (2)			NDCC (I)			NDCC (NODECC)	

10	
ADIM	

NODECC = number of nodes whose forces and/or moments are to be summed.

NCC1 = one, if forces in x-direction are to be summed. It is blank or zero if it is otherwise.

NCC2 = one, if forces in y-direction are to be summed. It is blank or zero is it is otherwise.

NCC3 = one, if moments are to be summed. It is blank or zero if it is otherwise.

NPSUM1 = node point, where the forces and/or moments are summed.

NPSUM2 = node point used to determine angular rotation.

NDCC(I) = node whose force and/or moment are to be summed.

ADIM = radius or half-length of footing used to non-dimensionize the frequency and the compliance coefficients.

<u>Card I1: Read Number of Cases</u>

FORMAT(8I10) one card

10	
NCASE	

NCASE = number of cases to be analyzed. Each case will have a set of I2, I3, I4, I5, and I6 Cards.

Card I2, I3, I4, I5 & I6

1) Card I2 & I3: Read Material Properties

FORMAT(8F10.0) two cards for each set

FORMAT(8F10.0)

10	20	30	40	50	60	70	80
AL1	CS	CF1	DS	DF	PO	VI	PE

10	20	30	
DM	POIS	DMS	

AL1 = elastic compliance constant. Dimension is in psi.

CS = material constant representing the compressibility of the solid. It is dimensionless.

CF1 = bulk modulus of the fluid. Dimension is in psi.

DS = mass density of the bulk of the fluid-solid. Dimension is in slug./in.^{3} .

DF = mass density of the fluid. Dimension is in slug./in.^{3} .

PO = porosity. It is dimensionless.

VI = viscosity of the fluid. Let it be dimensionless and let it to be one.

PE = coefficient of permeability. Dimension is in in./sec. .

DM = material compressive damping. It is dimensionless.

POIS = Poisson's ratio. It is dimensionless.

DMS = material shearing damping. It is dimensionless.

2) Card I4: Read Number of Material Damping To Be Considered

FORMAT(I10,2F10.0) one card for each set

10	20	30	
NDAMP	DINDAM	DINDAS	

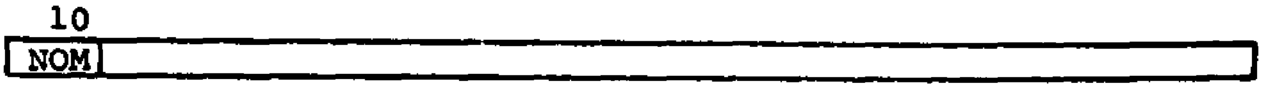
NDAMP = number of material damping to be considered. If it is two or greater, the values of DM and DMS of I3 Card will be altered.

DINDAM = increment added to the previous value of DM. It is dimensionless.

DINDAS = increment added to the previous value of DMS. It is dimensionless.

3) <u>Card I5: Read Number of Frequencies To Be Considered</u>

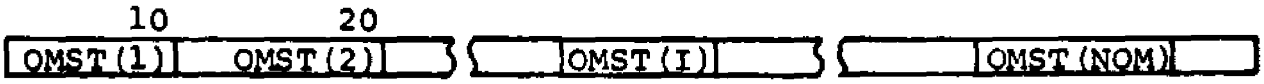
FORMAT(8I10) one card for each set



NOM = number of frequencies to be considered.

4) <u>Card I6: Read Frequencies To Be Considered</u>

FORMAT(8F10.0) as many cards as necessary for each set



OMST(I) = 'I'th frequency to be analyzed.

APPENDIX K

PROGRAM INPUT DATA AND SOURCES

APPENDIX K

PROGRAM INPUT DATA AND SOURCES

The computer program, listed in Appendix I and used for solving soil problems of interest, requires a set of relevant soil and water properties as inputs. These input parameters, along with corresponding computer symbols, are listed and defined on Table K-1.

Table K-1

INPUT PARAMETER	COMPUTER SYMBOL	DEFINITION
$\frac{E(1-\nu)}{(1+\nu)(1-2\nu)}$	AL	elastic compliance constant
α	CS	material constant representing the compressibility of the solid particles; equal one for saturated soil
M	CF	bulk modulus of fluid
ρ	DS	mass density of bulk of fluid-solid
ρ_f	DF	mass density of fluid
f	PO	porosity
η	VI	viscosity of the fluid
k	PE	coefficient of permeability

Elastic Compliance Constant

Instead of obtaining the individual values of E and ν, use the compressive modulus, M_ν, which is equivalent to $\frac{E(1-\nu)}{(1+\nu)(1-2\nu)}$. Range of values for M_ν are obtained from Reference 15.

Table K-2

SOIL MATERIAL	M_ν	
	kg/cm^2	psi
Sand, dense	500 to 1000	7112 to 14223
Clay, medium hard	80 to 150	1138 to 2134
Clay, plastic	5 to 40	71 to 569

Material Constant Representing the Compressibility of the Solid Particles

Use α equals one for saturated soil.

Bulk Modulus of Fluid

$$M = 42.324 \times 10^6 \text{ lb/ft}^2$$
$$= 293.917 \times 10^3 \text{ psi}$$

Mass Density of Bulk of Fluid-solid

Mass density of bulk of fluid-solid is related to porosity of soil, relative density of dry soil and water density as following

$$\rho = (1-f)G\delta_w + f\delta_w$$

Use

$$G = 2.65$$

$$\delta_w = \frac{62.4}{g} \text{ lb/ft}^3 = 0.00112146 \text{ slug/in}^3$$

Table K-3

SOIL MATERIAL	f	ρ (slug/in^3)
Sand, dense	0.35	0.0023243
Clay, medium hard	0.40	0.0022317
Clay, plastic	0.50	0.0020466

Mass Density of Fluid

Mass density of fluid is related to porosity of soil and water density as following

$$\rho_f = f\delta_w$$

Table K-4

SOIL MATERIAL	f	ρ_f (slug/in^3)
Sand, dense	0.35	0.0003925
Clay, medium hard	0.40	0.0004486
Clay, plastic	0.50	0.0005607

Porosity

For values of porosity see Table K-3 or Table K-4.

Viscosity of the Fluid and Coefficient of Permeability

Biot's definition of coefficient of permeability (2,3,4) is different from Jumikis's definition. (13) They are related in the following way,

$$\left(\frac{k}{\eta}\right)_{Biot} = \frac{k_{Jumikis}}{\gamma_w}$$

where γ_w is specific weight of water.

Table K-5

SOIL MATERIAL	$k_{Jumikis}$ (cm/sec)	$\left(\frac{k}{\eta}\right)_{Biot}$ (in^3-sec/slug)
Sand, dense	10^{-3} to 10^{-1}	$9.0854 \times (10^{-4}$ to $10^{-2})$
Clay, medium hard	10^{-6} to 10^{-5}	$9.0854 \times (10^{-7}$ to $10^{-6})$
Clay, plastic	10^{-6}	9.0854×10^{-7}

APPENDIX L

SAMPLE PROGRAM INPUT DATA

APPENDIX L

SAMPLE PROGRAM INPUT DATA

Program input data for the horizontal case of Model 1 (Fully Saturated) are listed on Table L-1.

```
CARD   12345678901234567890123456789012345678901234567890123456789012345678901234567890
TITLE  HORI. STIFFNESS, 0% SOIL DAMPING, TRANSMITTING BOUNDARIES, SAT, 135 NODES,A=300
A             112       135
B          1    2   11   10    8    1   14    9
C1                  1         9         1        15         9         1
C2         0.        0.        0.      -150.      150.       0.
D                   1         1    10.                   3         9
D                   1         2     0.                   3         9
D                   1         4     0.                   3         9
D                   2         2     0.                   8         1
D                   2         4     0.            1         8         1
E1                  1
E2                  8         9        18        -2        14         8         9
E2                105       127       128         1         8         1         1         1
F1                  1
F2                 28         2    0.                     12         9
F2                 28         4    0.             1       12         9
G1                  0
H1                  1
H2                  3         1                             1        19
H3                  1        10        19
H4     300.
I1                  1
I2       14000.         1.    293900. 0.0023243 0.0003925      0.35        1. 0.090854
I3                 0.      0.25        0.
I4                  1      0.05      0.05
I5                 50
I6         1.7949    3.5894    5.3840    7.1787    8.9734   10.7681   12.5628   14.3574
I6        16.1521   17.9468   19.7415   21.5362   23.3308   25.1255   26.9202   28.7149
I6        30.5096   32.3042   34.0989   35.8936   37.6883   39.4830   41.2776   43.0723
I6        44.8670   46.6617   48.4564   50.0457   52.0457   53.8404   55.6351   57.4298
I6        59.2244   61.0191   62.8138   64.6085   66.4032   68.1978   69.9925   71.7872
I6        73.5819   75.3766   77.1712   78.9659   80.7606   82.5553   84.3500   86.1446
I6        87.9393   89.7340
```

Table L-1

APPENDIX M

TABLES

Table M-1

(Parameters Used for Computer Runs)

PARAMETER	VALUE
$\frac{E(1-\nu)}{(1+\nu)(1-2\nu)}$	14,000 psi
α	1
M	293,900 psi
ρ	0.002324 slug/in^3
ρ_f	0.0003925 slug/in^3
f	0.35
k/η k_J	9.0854 x (10^{-5} to 10^{-2}) in^3-sec/slug 10^{-4} to 10^{-1} cm/sec
V_c	39,823 in/sec
λ	0

(See Appendix K for detail parameter definitions.)

Table M-2

(Definition of Dimensionless Parameters)

PARAMETER	DIMENSIONED VARIABLE	DIMENSIONLESS VARIABLE
Frequency	Ω (1/sec)	$\lambda = a\Omega / V_s$
Compliance Coefficients:		
Horizontal	F_H / Δ_H (psf)	$K_{HH} = F_H / (\pi G \Delta_H)$
Vertical	F_V / Δ_V (psf)	$K_{VV} = F_V / (\pi G \Delta_V)$
Rocking	M / ϕ (lbs)	$K_{TT} = M / (\pi G a^2 \phi)$

where

Ω = frequency,

F_H = horizontal force acting on the footing due to Δ_H, horizontal displacement,

F_V = vertical force acting on the footing due to Δ_V, vertical displacement,

M = moment acting on the footing due to ϕ, rocking displacement,

a = half width of the footing.

Table M-3

MODEL 1 (FULLY SATURATED,KJ=0.1 CM/SEC)

FREQUENCY	KHH (HORIZONTAL)	KVV (VERTICAL)	KTT (ROCKING)
0.1	(.694E 0, .385E 0)	(.781E 0, .797E 0)	(.155E 1, .304E 0)
0.2	(.794E 0, .315E 0)	(.118E 1, .873E 0)	(.167E 1, .282E 0)
0.3	(.762E 0, .314E 0)	(.129E 1, .835E 0)	(.168E 1, .269E 0)
0.4	(.682E 0, .384E 0)	(.122E 1, .817E 0)	(.165E 1, .277E 0)
0.5	(.615E 0, .516E 0)	(.102E 1, .859E 0)	(.159E 1, .302E 0)
0.6	(.577E 0, .650E 0)	(.719E 0, .994E 0)	(.150E 1, .343E 0)
0.7	(.515E 0, .785E 0)	(.383E 0, .126E 1)	(.137E 1, .426E 0)
0.8	(.429E 0, .986E 0)	(.110E 0, .163E 1)	(.125E 1, .582E 0)
0.9	(.417E 0, .129E 1)	(-.562E-1, .198E 1)	(.122E 1, .772E 0)
1.0	(.610E 0, .159E 1)	(-.307E 0, .222E 1)	(.124E 1, .874E 0)
1.1	(.893E 0, .163E 1)	(-.737E 0, .261E 1)	(.116E 1, .929E 0)
1.2	(.981E 0, .149E 1)	(-.103E 1, .333E 1)	(.103E 1, .108E 1)
1.3	(.860E 0, .142E 1)	(-.823E 0, .399E 1)	(.994E 0, .133E 1)
1.4	(.679E 0, .151E 1)	(-.556E 0, .407E 1)	(.111E 1, .148E 1)
1.5	(.577E 0, .173E 1)	(-.758E 0, .390E 1)	(.116E 1, .145E 1)
1.6	(.591E 0, .196E 1)	(-.116E 1, .408E 1)	(.102E 1, .142E 1)
1.7	(.667E 0, .215E 1)	(-.163E 1, .423E 1)	(.828E 0, .155E 1)
1.8	(.797E 0, .225E 1)	(-.229E 1, .451E 1)	(.755E 0, .177E 1)
1.9	(.867E 0, .223E 1)	(-.296E 1, .510E 1)	(.771E 0, .188E 1)
2.0	(.806E 0, .220E 1)	(-.344E 1, .588E 1)	(.696E 0, .192E 1)
2.1	(.666E 0, .227E 1)	(-.369E 1, .679E 1)	(.546E 0, .202E 1)
2.2	(.547E 0, .245E 1)	(-.347E 1, .763E 1)	(.397E 0, .220E 1)
2.3	(.511E 0, .268E 1)	(-.302E 1, .781E 1)	(.324E 0, .245E 1)
2.4	(.562E 0, .291E 1)	(-.312E 1, .746E 1)	(.401E 0, .266E 1)
2.5	(.693E 0, .308E 1)	(-.383E 1, .727E 1)	(.457E 0, .268E 1)
2.6	(.869E 0, .316E 1)	(-.484E 1, .744E 1)	(.430E 0, .259E 1)
2.7	(.998E 0, .310E 1)	(-.596E 1, .805E 1)	(.193E 0, .256E 1)
2.8	(.100E 1, .298E 1)	(-.690E 1, .918E 1)	(-.109E 0, .269E 1)
2.9	(.870E 0, .290E 1)	(-.727E 1, .106E 2)	(-.354E 0, .297E 1)
3.0	(.659E 0, .292E 1)	(-.699E 1, .117E 2)	(-.461E 0, .331E 1)
3.1	(.444E 0, .308E 1)	(-.667E 1, .122E 2)	(-.416E 0, .361E 1)
3.2	(.308E 0, .334E 1)	(-.671E 1, .124E 2)	(-.294E 0, .377E 1)
3.3	(.296E 0, .364E 1)	(-.694E 1, .127E 2)	(-.227E 0, .377E 1)
3.4	(.410E 0, .390E 1)	(-.728E 1, .128E 2)	(-.304E 0, .371E 1)
3.5	(.603E 0, .406E 1)	(-.788E 1, .130E 2)	(-.515E 0, .372E 1)
3.6	(.797E 0, .409E 1)	(-.868E 1, .134E 2)	(-.752E 0, .386E 1)
3.7	(.925E 0, .402E 1)	(-.953E 1, .139E 2)	(-.897E 0, .406E 1)
3.8	(.966E 0, .392E 1)	(-.104E 2, .147E 2)	(-.944E 0, .425E 1)
3.9	(.925E 0, .382E 1)	(-.113E 2, .156E 2)	(-.102E 1, .432E 1)
4.0	(.813E 0, .375E 1)	(-.122E 2, .167E 2)	(-.118E 1, .438E 1)
4.1	(.646E 0, .374E 1)	(-.128E 2, .183E 2)	(-.136E 1, .450E 1)
4.2	(.468E 0, .383E 1)	(-.129E 2, .200E 2)	(-.153E 1, .463E 1)
4.3	(.349E 0, .400E 1)	(-.123E 2, .215E 2)	(-.175E 1, .476E 1)
4.4	(.338E 0, .419E 1)	(-.114E 2, .220E 2)	(-.202E 1, .497E 1)
4.5	(.410E 0, .430E 1)	(-.110E 2, .219E 2)	(-.225E 1, .531E 1)
4.6	(.469E 0, .431E 1)	(-.113E 2, .217E 2)	(-.232E 1, .573E 1)
4.7	(.430E 0, .427E 1)	(-.121E 2, .217E 2)	(-.217E 1, .610E 1)
4.8	(.301E 0, .430E 1)	(-.130E 2, .220E 2)	(-.188E 1, .624E 1)
4.9	(.187E 0, .447E 1)	(-.142E 2, .225E 2)	(-.167E 1, .608E 1)
5.0	(.217E 0, .472E 1)	(-.154E 2, .234E 2)	(-.172E 1, .577E 1)

Table M-4

MODEL 1 (FULLY SATURATED,KJ=0.01 CM/SEC)

FREQUENCY	KHH(HORIZONTAL)	KVV(VERTICAL)	KTT(ROCKING)
0.1	(.754E 0, .416E 0)	(.805E 0, .808E 0)	(.168E 1, .224E 0)
0.2	(.859E 0, .321E 0)	(.121E 1, .879E 0)	(.175E 1, .211E 0)
0.3	(.814E 0, .314E 0)	(.132E 1, .838E 0)	(.174E 1, .212E 0)
0.4	(.722E 0, .390E 0)	(.124E 1, .819E 0)	(.169E 1, .235E 0)
0.5	(.649E 0, .534E 0)	(.103E 1, .864E 0)	(.162E 1, .273E 0)
0.6	(.611E 0, .679E 0)	(.718E 0, .101E 1)	(.153E 1, .324E 0)
0.7	(.544E 0, .821E 0)	(.373E 0, .128E 1)	(.140E 1, .417E 0)
0.8	(.450E 0, .104E 1)	(.987E-1, .167E 1)	(.128E 1, .585E 0)
0.9	(.432E 0, .137E 1)	(-.573E-1, .203E 1)	(.126E 1, .782E 0)
1.0	(.651E 0, .171E 1)	(-.310E 0, .226E 1)	(.128E 1, .877E 0)
1.1	(.100E 1, .176E 1)	(-.767E 0, .264E 1)	(.119E 1, .928E 0)
1.2	(.113E 1, .157E 1)	(-.110E 1, .338E 1)	(.104E 1, .108E 1)
1.3	(.988E 0, .145E 1)	(-.895E 0, .409E 1)	(.100E 1, .135E 1)
1.4	(.772E 0, .154E 1)	(-.607E 0, .418E 1)	(.113E 1, .152E 1)
1.5	(.647E 0, .177E 1)	(-.823E 0, .409E 1)	(.119E 1, .147E 1)
1.6	(.650E 0, .203E 1)	(-.125E 1, .423E 1)	(.103E 1, .144E 1)
1.7	(.750E 0, .223E 1)	(-.169E 1, .441E 1)	(.822E 0, .158E 1)
1.8	(.902E 0, .233E 1)	(-.233E 1, .470E 1)	(.749E 0, .183E 1)
1.9	(.982E 0, .229E 1)	(-.298E 1, .530E 1)	(.782E 0, .195E 1)
2.0	(.914E 0, .225E 1)	(-.346E 1, .609E 1)	(.714E 0, .198E 1)
2.1	(.759E 0, .231E 1)	(-.372E 1, .701E 1)	(.554E 0, .207E 1)
2.2	(.623E 0, .250E 1)	(-.348E 1, .789E 1)	(.385E 0, .225E 1)
2.3	(.577E 0, .275E 1)	(-.299E 1, .807E 1)	(.297E 0, .253E 1)
2.4	(.629E 0, .300E 1)	(-.307E 1, .771E 1)	(.380E 0, .276E 1)
2.5	(.776E 0, .320E 1)	(-.377E 1, .752E 1)	(.500E 0, .279E 1)
2.6	(.981E 0, .327E 1)	(-.476E 1, .767E 1)	(.445E 0, .269E 1)
2.7	(.114E 1, .321E 1)	(-.588E 1, .825E 1)	(.202E 0, .265E 1)
2.8	(.116E 1, .305E 1)	(-.686E 1, .936E 1)	(-.114E 0, .278E 1)
2.9	(.101E 1, .294E 1)	(-.727E 1, .108E 2)	(-.372E 0, .306E 1)
3.0	(.768E 0, .295E 1)	(-.701E 1, .120E 2)	(-.485E 0, .342E 1)
3.1	(.527E 0, .311E 1)	(-.669E 1, .125E 2)	(-.440E 0, .374E 1)
3.2	(.372E 0, .339E 1)	(-.674E 1, .127E 2)	(-.314E 0, .391E 1)
3.3	(.353E 0, .371E 1)	(-.695E 1, .130E 2)	(-.242E 0, .391E 1)
3.4	(.471E 0, .400E 1)	(-.725E 1, .132E 2)	(-.318E 0, .386E 1)
3.5	(.678E 0, .417E 1)	(-.779E 1, .134E 2)	(-.540E 0, .387E 1)
3.6	(.892E 0, .421E 1)	(-.853E 1, .138E 2)	(-.791E 0, .403E 1)
3.7	(.104E 1, .415E 1)	(-.931E 1, .143E 2)	(-.931E 0, .428E 1)
3.8	(.110E 1, .404E 1)	(-.102E 2, .149E 2)	(-.941E 0, .447E 1)
3.9	(.108E 1, .392E 1)	(-.111E 2, .157E 2)	(-.971E 0, .453E 1)
4.0	(.968E 0, .382E 1)	(-.121E 2, .169E 2)	(-.112E 1, .456E 1)
4.1	(.787E 0, .379E 1)	(-.128E 2, .184E 2)	(-.132E 1, .465E 1)
4.2	(.585E 0, .386E 1)	(-.130E 2, .202E 2)	(-.152E 1, .476E 1)
4.3	(.442E 0, .403E 1)	(-.123E 2, .218E 2)	(-.177E 1, .489E 1)
4.4	(.413E 0, .423E 1)	(-.114E 2, .224E 2)	(-.207E 1, .511E 1)
4.5	(.475E 0, .436E 1)	(-.110E 2, .222E 2)	(-.233E 1, .548E 1)
4.6	(.526E 0, .438E 1)	(-.113E 2, .220E 2)	(-.241E 1, .596E 1)
4.7	(.480E 0, .435E 1)	(-.120E 2, .221E 2)	(-.224E 1, .640E 1)
4.8	(.347E 0, .440E 1)	(-.130E 2, .224E 2)	(-.188E 1, .657E 1)
4.9	(.235E 0, .460E 1)	(-.140E 2, .229E 2)	(-.160E 1, .640E 1)
5.0	(.288E 0, .488E 1)	(-.153E 2, .237E 2)	(-.161E 1, .603E 1)

Table M-5

MODEL 1 (FULLY SATURATED,KJ=0.0001 CM/SEC)

FREQUENCY	KHH(HORIZONTAL)	KVV(VERTICAL)	KTT(ROCKING)
0.1	(.800E 0, .414E 0)	(.808E 0, .816E 0)	(.171E 1, .191E 0)
0.2	(.884E 0, .306E 0)	(.122E 1, .887E 0)	(.176E 1, .190E 0)
0.3	(.830E 0, .303E 0)	(.133E 1, .842E 0)	(.175E 1, .197E 0)
0.4	(.734E 0, .385E 0)	(.125E 1, .821E 0)	(.170E 1, .224E 0)
0.5	(.662E 0, .534E 0)	(.103E 1, .866E 0)	(.163E 1, .265E 0)
0.6	(.624E 0, .680E 0)	(.724E 0, .101E 1)	(.154E 1, .319E 0)
0.7	(.557E 0, .825E 0)	(.378E 0, .129E 1)	(.141E 1, .414E 0)
0.8	(.462E 0, .104E 1)	(.105E 0, .169E 1)	(.129E 1, .584E 0)
0.9	(.446E 0, .139E 1)	(-.477E-1, .204E 1)	(.127E 1, .781E 0)
1.0	(.673E 0, .173E 1)	(-.301E 0, .226E 1)	(.129E 1, .875E 0)
1.1	(.104E 1, .177E 1)	(-.763E 0, .265E 1)	(.120E 1, .927E 0)
1.2	(.116E 1, .157E 1)	(-.110E 1, .339E 1)	(.105E 1, .109E 1)
1.3	(.101E 1, .145E 1)	(-.893E 0, .411E 1)	(.101E 1, .135E 1)
1.4	(.791E 0, .154E 1)	(-.599E 0, .421E 1)	(.114E 1, .152E 1)
1.5	(.664E 0, .177E 1)	(-.814E 0, .411E 1)	(.120E 1, .148E 1)
1.6	(.668E 0, .203E 1)	(-.124E 1, .426E 1)	(.104E 1, .145E 1)
1.7	(.771E 0, .224E 1)	(-.167E 1, .445E 1)	(.829E 0, .154E 1)
1.8	(.928E 0, .233E 1)	(-.231E 1, .473E 1)	(.757E 0, .184E 1)
1.9	(.101E 1, .229E 1)	(-.296E 1, .533E 1)	(.794E 0, .196E 1)
2.0	(.936E 0, .225E 1)	(-.344E 1, .611E 1)	(.726E 0, .199E 1)
2.1	(.778E 0, .231E 1)	(-.370E 1, .704E 1)	(.563E 0, .208E 1)
2.2	(.640E 0, .250E 1)	(-.346E 1, .792E 1)	(.392E 0, .226E 1)
2.3	(.593E 0, .275E 1)	(-.296E 1, .810E 1)	(.303E 0, .254E 1)
2.4	(.647E 0, .301E 1)	(-.304E 1, .774E 1)	(.390E 0, .278E 1)
2.5	(.798E 0, .321E 1)	(-.374E 1, .754E 1)	(.514E 0, .281E 1)
2.6	(.101E 1, .329E 1)	(-.472E 1, .768E 1)	(.460E 0, .270E 1)
2.7	(.117E 1, .321E 1)	(-.586E 1, .826E 1)	(.215E 0, .266E 1)
2.8	(.119E 1, .305E 1)	(-.684E 1, .937E 1)	(-.104E 0, .279E 1)
2.9	(.104E 1, .293E 1)	(-.726E 1, .108E 2)	(-.363E 0, .308E 1)
3.0	(.788E 0, .294E 1)	(-.700E 1, .120E 2)	(-.476E 0, .344E 1)
3.1	(.542E 0, .311E 1)	(-.668E 1, .125E 2)	(-.431E 0, .376E 1)
3.2	(.386E 0, .339E 1)	(-.672E 1, .127E 2)	(-.303E 0, .392E 1)
3.3	(.367E 0, .372E 1)	(-.693E 1, .130E 2)	(-.231E 0, .393E 1)
3.4	(.486E 0, .401E 1)	(-.723E 1, .132E 2)	(-.308E 0, .387E 1)
3.5	(.696E 0, .419E 1)	(-.776E 1, .134E 2)	(-.531E 0, .388E 1)
3.6	(.914E 0, .422E 1)	(-.849E 1, .138E 2)	(-.782E 0, .405E 1)
3.7	(.107E 1, .416E 1)	(-.927E 1, .143E 2)	(-.920E 0, .431E 1)
3.8	(.113E 1, .405E 1)	(-.101E 2, .149E 2)	(-.923E 0, .450E 1)
3.9	(.111E 1, .392E 1)	(-.111E 2, .157E 2)	(-.948E 0, .455E 1)
4.0	(.997E 0, .382E 1)	(-.121E 2, .168E 2)	(-.110E 1, .457E 1)
4.1	(.811E 0, .378E 1)	(-.128E 2, .184E 2)	(-.130E 1, .466E 1)
4.2	(.604E 0, .386E 1)	(-.130E 2, .203E 2)	(-.151E 1, .477E 1)
4.3	(.457E 0, .403E 1)	(-.123E 2, .218E 2)	(-.176E 1, .490E 1)
4.4	(.426E 0, .423E 1)	(-.114E 2, .224E 2)	(-.206E 1, .513E 1)
4.5	(.486E 0, .436E 1)	(-.110E 2, .223E 2)	(-.232E 1, .550E 1)
4.6	(.537E 0, .438E 1)	(-.113E 2, .220E 2)	(-.241E 1, .599E 1)
4.7	(.491E 0, .436E 1)	(-.120E 2, .221E 2)	(-.224E 1, .644E 1)
4.8	(.358E 0, .441E 1)	(-.129E 2, .224E 2)	(-.187E 1, .661E 1)
4.9	(.248E 0, .461E 1)	(-.140E 2, .229E 2)	(-.158E 1, .643E 1)
5.0	(.306E 0, .490E 1)	(-.152E 2, .237E 2)	(-.159E 1, .605E 1)

Table M-6

MODEL 1A (PARTIALLY SATURATED UP TO DEPTH OF -1/2 A,KJ=0.1 CM/SEC)

FREQUENCY	KHH(HORIZONTAL)	KVV(VERTICAL)	KTT(ROCKING)
0.1	(.619E 0, .283E 0)	(.724E 0, .568E 0)	(.981E 0, .106E 0)
0.2	(.674E 0, .218E 0)	(.988E 0. .555E 0)	(.102E 1, .105E 0)
0.3	(.644E 0, .225E 0)	(.105E 1, .520E 0)	(.102E 1, .109E 0)
0.4	(.593E 0, .293E 0)	(.101E 1, .515E 0)	(.101E 1, .122E 0)
0.5	(.572E 0, .402E 0)	(.883E 0, .565E 0)	(.990E 0, .140E 0)
0.6	(.579E 0, .497E 0)	(.698E 0, .701E 0)	(.961E 0, .159E 0)
0.7	(.565E 0, .590E 0)	(.507E 0, .974E 0)	(.912E 0, .194E 0)
0.8	(.554E 0, .737E 0)	(.438E 0, .138E 1)	(.865E 0, .269E 0)
0.9	(.645E 0, .924E 0)	(.547E 0, .171E 1)	(.866E 0, .362E 0)
1.0	(.850E 0, .979E 0)	(.597E 0, .192E 1)	(.890E 0, .404E 0)
1.1	(.951E 0, .865E 0)	(.587E 0, .227E 1)	(.866E 0, .434E 0)
1.2	(.875E 0, .774E 0)	(.809E 0, .268E 1)	(.830E 0, .513E 0)
1.3	(.735E 0, .821E 0)	(.115E 1, .282E 1)	(.849E 0, .615E 0)
1.4	(.671E 0, .985E 0)	(.125E 1, .282E 1)	(.903E 0, .662E 0)
1.5	(.727E 0, .114E 1)	(.123E 1, .297E 1)	(.912E 0, .666E 0)
1.6	(.818E 0, .121E 1)	(.123E 1, .318E 1)	(.872E 0, .704E 0)
1.7	(.902E 0, .123E 1)	(.116E 1, .355E 1)	(.858E 0, .796E 0)
1.8	(.956E 0, .120E 1)	(.136E 1, .426E 1)	(.903E 0, .864E 0)
1.9	(.941E 0, .116E 1)	(.213E 1, .475E 1)	(.916E 0, .874E 0)
2.0	(.866E 0, .115E 1)	(.299E 1, .467E 1)	(.874E 0, .920E 0)
2.1	(.793E 0, .123E 1)	(.352E 1, .418E 1)	(.859E 0, .103E 0)
2.2	(.783E 0, .134E 1)	(.354E 1, .369E 1)	(.902E 0, .114E 1)
2.3	(.837E 0, .141E 1)	(.328E 1, .360E 1)	(.101E 1, .121E 1)
2.4	(.907E 0, .143E 1)	(.325E 1, .393E 1)	(.111E 1, .119E 1)
2.5	(.955E 0, .139E 1)	(.371E 1, .424E 1)	(.111E 1, .112E 1)
2.6	(.943E 0, .131E 1)	(.438E 1, .406E 1)	(.102E 1, .111E 1)
2.7	(.858E 0, .124E 1)	(.473E 1, .351E 1)	(.935E 0, .121E 1)
2.8	(.718E 0, .123E 1)	(.470E 1, .300E 1)	(.940E 0, .138E 1)
2.9	(.566E 0, .129E 1)	(.451E 1, .270E 1)	(.107E 1, .152E 1)
3.0	(.454E 0, .142E 1)	(.433E 1, .261E 1)	(.124E 1, .154E 1)
3.1	(.425E 0, .160E 1)	(.428E 1, .259E 1)	(.136E 1, .145E 1)
3.2	(.491E 0, .174E 1)	(.432E 1, .253E 1)	(.135E 1, .134E 1)
3.3	(.617E 0, .180E 1)	(.438E 1, .240E 1)	(.127E 1, .130E 1)
3.4	(.732E 0, .175E 1)	(.439E 1, .218E 1)	(.119E 1, .134E 1)
3.5	(.768E 0, .161E 1)	(.432E 1, .194E 1)	(.117E 1, .145E 1)
3.6	(.688E 0, .145E 1)	(.416E 1, .175E 1)	(.123E 1, .155E 1)
3.7	(.511E 0, .135E 1)	(.397E 1, .163E 1)	(.133E 1, .160E 1)
3.8	(.289E 0, .135E 1)	(.380E 1, .155E 1)	(.145E 1, .158E 1)
3.9	(.839E-1, .143E 1)	(.363E 1, .149E 1)	(.152E 1, .150E 1)
4.0	(-.727E-1, .158E 1)	(.346E 1, .146E 1)	(.153E 1, .141E 1)
4.1	(-.170E 0, .176E 1)	(.331E 1, .145E 1)	(.149E 1, .136E 1)
4.2	(-.189E 0, .195E 1)	(.319E 1, .146E 1)	(.145E 1, .135E 1)
4.3	(-.112E 0, .211E 1)	(.310E 1, .146E 1)	(.145E 1, .135E 1)
4.4	(.252E-1, .216E 1)	(.301E 1, .142E 1)	(.144E 1, .133E 1)
4.5	(.130E 0, .209E 1)	(.291E 1, .136E 1)	(.143E 1, .131E 1)
4.6	(.131E 0, .195E 1)	(.277E 1, .130E 1)	(.140E 1, .131E 1)
4.7	(.256E-1, .183E 1)	(.262E 1, .126E 1)	(.139E 1, .132E 1)
4.8	(-.146E 0, .177E 1)	(.246E 1, .123E 1)	(.142E 1, .133E 1)
4.9	(-.339E 0, .178E 1)	(.231E 1, .122E 1)	(.146E 1, .129E 1)
5.0	(-.522E 0, .185E 1)	(.215E 1, .121E 1)	(.147E 1, .122E 1)

Table M-7

MODEL 1B (PARTIALLY SATURATED UP TO DEPTH OF -A,KJ=0.1 CM/SEC)

FREQUENCY	KHH (HORIZONTAL)	KVV (VERTICAL)	KTT (ROCKING)
0.1	(.573E 0, .259E 0)	(.715E 0, .495E 0)	(.911E 0, .675E-1)
0.2	(.619E 0, .199E 0)	(.931E 0, .448E 0)	(.928E 0, .651E-1)
0.3	(.592E 0, .210E 0)	(.972E 0, .411E 0)	(.922E 0, .700E-1)
0.4	(.553E 0, .276E 0)	(.928E 0, .411E 0)	(.908E 0, .834E-1)
0.5	(.548E 0, .366E 0)	(.822E 0, .463E 0)	(.892E 0, .100E 0)
0.6	(.557E 0, .433E 0)	(.673E 0, .602E 0)	(.865E 0, .115E 0)
0.7	(.537E 0, .500E 0)	(.544E 0, .885E 0)	(.819E 0, .145E 0)
0.8	(.521E 0, .616E 0)	(.617E 0, .127E 1)	(.776E 0, .213E 0)
0.9	(.588E 0, .753E 0)	(.887E 0, .145E 1)	(.785E 0, .300E 0)
1.0	(.721E 0, .774E 0)	(.962E 0, .146E 1)	(.820E 0, .326E 0)
1.1	(.750E 0, .690E 0)	(.963E 0, .166E 1)	(.802E 0, .334E 0)
1.2	(.652E 0, .648E 0)	(.118E 1, .187E 1)	(.766E 0, .384E 0)
1.3	(.526E 0, .725E 0)	(.143E 1, .186E 1)	(.770E 0, .458E 0)
1.4	(.485E 0, .876E 0)	(.150E 1, .179E 1)	(.807E 0, .494E 0)
1.5	(.525E 0, .989E 0)	(.152E 1, .183E 1)	(.811E 0, .493E 0)
1.6	(.578E 0, .104E 1)	(.160E 1, .189E 1)	(.779E 0, .512E 0)
1.7	(.612E 0, .104E 1)	(.174E 1, .195E 1)	(.761E 0, .569E 0)
1.8	(.593E 0, .101E 1)	(.197E 1, .189E 1)	(.781E 0, .608E 0)
1.9	(.498E 0, .993E 0)	(.209E 1, .165E 1)	(.779E 0, .614E 0)
2.0	(.369E 0, .106E 1)	(.203E 1, .140E 1)	(.756E 0, .639E 0)
2.1	(.293E 0, .120E 1)	(.186E 1, .124E 1)	(.745E 0, .675E 0)
2.2	(.300E 0, .131E 1)	(.166E 1, .118E 1)	(.744E 0, .709E 0)
2.3	(.312E 0, .135E 1)	(.151E 1, .120E 1)	(.757E 0, .723E 0)
2.4	(.269E 0, .135E 1)	(.141E 1, .119E 1)	(.743E 0, .705E 0)
2.5	(.158E 0, .137E 1)	(.127E 1, .116E 1)	(.681E 0, .710E 0)
2.6	(.572E-2, .146E 1)	(.110E 1, .115E 1)	(.636E 0, .769E 0)
2.7	(-.132E 0, .163E 1)	(.937E 0, .116E 1)	(.647E 0, .812E 0)
2.8	(-.210E 0, .186E 1)	(.759E 0, .118E 1)	(.648E 0, .809E 0)
2.9	(-.182E 0, .211E 1)	(.583E 0, .122E 1)	(.615E 0, .792E 0)
3.0	(-.570E-1, .227E 1)	(.405E 0, .126E 1)	(.546E 0, .788E 0)
3.1	(-.378E-1, .230E 1)	(.195E 0, .131E 1)	(.466E 0, .833E 0)
3.2	(.128E-2, .229E 1)	(-.346E-1, .140E 1)	(.439E 0, .897E 0)
3.3	(-.124E 0, .240E 1)	(-.239E 0, .155E 1)	(.428E 0, .908E 0)
3.4	(-.189E 0, .265E 1)	(-.377E 0, .172E 1)	(.372E 0, .904E 0)
3.5	(-.880E-1, .294E 1)	(-.418E 0, .185E 1)	(.290E 0, .921E 0)
3.6	(.149E 0, .312E 1)	(-.456E 0, .181E 1)	(.195E 0, .970E 0)
3.7	(.383E 0, .312E 1)	(-.681E 0, .168E 1)	(.126E 0, .107E 1)
3.8	(.501E 0, .303E 1)	(-.107E 1, .165E 1)	(.138E 0, .116E 1)
3.9	(.491E 0, .296E 1)	(-.148E 1, .176E 1)	(.172E 0, .115E 1)
4.0	(.423E 0, .299E 1)	(-.188E 1, .198E 1)	(.113E 0, .108E 1)
4.1	(.392E 0, .314E 1)	(-.224E 1, .231E 1)	(-.361E-1, .105E 1)
4.2	(.485E 0, .334E 1)	(-.244E 1, .273E 1)	(-.224E 0, .110E 1)
4.3	(.735E 0, .349E 1)	(-.241E 1, .305E 1)	(-.410E 0, .123E 1)
4.4	(.106E 1, .346E 1)	(-.236E 1, .310E 1)	(-.524E 0, .146E 1)
4.5	(.132E 1, .324E 1)	(-.251E 1, .297E 1)	(-.506E 0, .169E 1)
4.6	(.142E 1, .294E 1)	(-.288E 1, .286E 1)	(-.386E 0, .182E 1)
4.7	(.136E 1, .268E 1)	(-.342E 1, .282E 1)	(-.272E 0, .181E 1)
4.8	(.123E 1, .250E 1)	(-.410E 1, .291E 1)	(-.260E 0, .171E 1)
4.9	(.109E 1, .242E 1)	(-.489E 1, .320E 1)	(-.385E 0, .159E 1)
5.0	(.995E 0, .239E 1)	(-.569E 1, .373E 1)	(-.617E 0, .156E 1)

Table M-8

MODEL 1F (PARTIALLY SATURATED UP TO DEPTH OF -1 1/2 A,KJ=0.1 CM/SEC)

FREQUENCY	KHH(HORIZONTAL)	KVV(VERTICAL)	KTT(ROCKING)
0.1	(.539E 0, .254E 0)	(.706E 0, .445E 0)	(.884E 0, .555E-1)
0.2	(.584E 0, .198E 0)	(.888E 0, .382E 0)	(.896E 0, .531E-1)
0.3	(.561E 0, .213E 0)	(.913E 0, .347E 0)	(.888E 0, .580E-1)
0.4	(.534E 0, .274E 0)	(.869E 0, .351E 0)	(.873E 0, .703E-1)
0.5	(.532E 0, .347E 0)	(.775E 0, .404E 0)	(.855E 0, .841E-1)
0.6	(.530E 0, .396E 0)	(.654E 0, .537E 0)	(.825E 0, .981E-1)
0.7	(.495E 0, .454E 0)	(.582E 0, .796E 0)	(.778E 0, .128E 0)
0.8	(.460E 0, .560E 0)	(.724E 0, .108E 1)	(.734E 0, .194E 0)
0.9	(.494E 0, .688E 0)	(.945E 0, .111E 1)	(.745E 0, .278E 0)
1.0	(.580E 0, .728E 0)	(.949E 0, .107E 1)	(.780E 0, .296E 0)
1.1	(.588E 0, .687E 0)	(.939E 0, .118E 1)	(.759E 0, .293E 0)
1.2	(.489E 0, .680E 0)	(.107E 1, .125E 1)	(.714E 0, .329E 0)
1.3	(.366E 0, .781E 0)	(.114E 1, .117E 1)	(.699E 0, .393E 0)
1.4	(.323E 0, .953E 0)	(.109E 1, .111E 1)	(.714E 0, .430E 0)
1.5	(.365E 0, .109E 1)	(.101E 1, .111E 1)	(.705E 0, .435E 0)
1.6	(.425E 0, .117E 1)	(.960E 0, .111E 1)	(.663E 0, .455E 0)
1.7	(.453E 0, .118E 1)	(.887E 0, .108E 1)	(.630E 0, .508E 0)
1.8	(.402E 0, .118E 1)	(.761E 0, .104E 1)	(.626E 0, .553E 0)
1.9	(.291E 0, .126E 1)	(.581E 0, .102E 1)	(.619E 0, .573E 0)
2.0	(.242E 0, .144E 1)	(.369E 0, .105E 1)	(.597E 0, .579E 0)
2.1	(.304E 0, .157E 1)	(.144E 0, .112E 1)	(.539E 0, .579E 0)
2.2	(.333E 0, .160E 1)	(-.123E 0, .122E 1)	(.446E 0, .619E 0)
2.3	(.252E 0, .165E 1)	(-.407E 0, .144E 1)	(.374E 0, .718E 0)
2.4	(.155E 0, .186E 1)	(-.573E 0, .177E 1)	(.370E 0, .824E 0)
2.5	(.265E 0, .214E 1)	(-.553E 0, .203E 1)	(.394E 0, .868E 0)
2.6	(.501E 0, .225E 1)	(-.510E 0, .210E 1)	(.380E 0, .868E 0)
2.7	(.702E 0, .220E 1)	(-.638E 0, .198E 1)	(.301E 0, .868E 0)
2.8	(.774E 0, .206E 1)	(-.109E 1, .194E 1)	(.182E 0, .933E 0)
2.9	(.689E 0, .195E 1)	(-.165E 1, .219E 1)	(.103E 0, .108E 1)
3.0	(.553E 0, .199E 1)	(-.210E 1, .272E 1)	(.108E 0, .120E 1)
3.1	(.512E 0, .213E 1)	(-.232E 1, .340E 1)	(.961E-1, .127E 1)
3.2	(.574E 0, .225E 1)	(-.222E 1, .400E 1)	(.661E-1, .135E 1)
3.3	(.700E 0, .229E 1)	(-.203E 1, .422E 1)	(.390E-1, .144E 1)
3.4	(.821E 0, .222E 1)	(-.223E 1, .418E 1)	(.244E-1, .154E 1)
3.5	(.858E 0, .207E 1)	(-.290E 1, .439E 1)	(-.656E-2, .163E 1)
3.6	(.772E 0, .190E 1)	(-.380E 1, .515E 1)	(-.259E-1, .175E 1)
3.7	(.578E 0, .178E 1)	(-.457E 1, .673E 1)	(-.176E-1, .184E 1)
3.8	(.329E 0, .178E 1)	(-.432E 1, .915E 1)	(-.993E-1, .193E 1)
3.9	(.851E-1, .189E 1)	(-.232E 1, .110E 2)	(-.171E 0, .220E 1)
4.0	(-.971E-1, .210E 1)	(-.256E 1, .109E 2)	(-.227E-2, .252E 1)
4.1	(-.163E 0, .237E 1)	(.308E 0, .105E 2)	(.312E 0, .259E 1)
4.2	(-.827E-1, .261E 1)	(.443E 0, .113E 2)	(.443E 0, .250E 1)
4.3	(.765E-1, .271E 1)	(.223E 1, .131E 2)	(.457E 0, .247E 1)
4.4	(.181E 0, .269E 1)	(.610E 1, .128E 2)	(.419E 0, .249E 1)
4.5	(.179E 0, .262E 1)	(.818E 1, .972E 1)	(.356E 0, .266E 1)
4.6	(.899E-1, .261E 1)	(.792E 1, .714E 1)	(.486E 0, .300E 1)
4.7	(-.135E-1, .269E 1)	(.710E 1, .580E 1)	(.986E 0, .319E 1)
4.8	(-.424E-1, .281E 1)	(.650E 1, .512E 1)	(.150E 1, .289E 1)
4.9	(-.462E-1, .289E 1)	(.620E 1, .460E 1)	(.159E 1, .238E 1)
5.0	(-.730E-1, .299E 1)	(.599E 1, .394E 1)	(.138E 1, .204E 1)

Table M-9

MODEL 1C (PARTIALLY SATURATED UP TO DEPTH OF -2A,KJ=0.1 CM/SEC)

FREQUENCY	KHH(HORIZONTAL)	KVV(VERTICAL)	KYY(ROCKING)
0.1	(.508E 0, .259E 0)	(.698E 0, .406E 0)	(.873E 0, .505E-1)
0.2	(.559E 0, .207E 0)	(.851E 0, .335E 0)	(.883E 0, .478E-1)
0.3	(.542E 0, .223E 0)	(.865E 0, .303E 0)	(.874E 0, .521E-1)
0.4	(.523E 0, .278E 0)	(.820E 0, .309E 0)	(.857E 0, .628E-1)
0.5	(.519E 0, .336E 0)	(.735E 0, .359E 0)	(.836E 0, .754E-1)
0.6	(.501E 0, .378E 0)	(.635E 0, .477E 0)	(.802E 0, .904E-1)
0.7	(.450E 0, .437E 0)	(.589E 0, .681E 0)	(.754E 0, .121E 0)
0.8	(.399E 0, .548E 0)	(.695E 0, .866E 0)	(.709E 0, .187E 0)
0.9	(.412E 0, .689E 0)	(.815E 0, .867E 0)	(.716E 0, .270E 0)
1.0	(.460E 0, .761E 0)	(.793E 0, .830E 0)	(.749E 0, .290E 0)
1.1	(.488E 0, .758E 0)	(.736E 0, .846E 0)	(.728E 0, .284E 0)
1.2	(.403E 0, .788E 0)	(.660E 0, .845E 0)	(.677E 0, .311E 0)
1.3	(.303E 0, .924E 0)	(.508E 0, .874E 0)	(.641E 0, .365E 0)
1.4	(.298E 0, .113E 1)	(.343E 0, .994E 0)	(.620E 0, .415E 0)
1.5	(.410E 0, .130E 1)	(.249E 0, .117E 1)	(.595E 0, .466E 0)
1.6	(.546E 0, .134E 1)	(.229E 0, .129E 1)	(.589E 0, .521E 0)
1.7	(.607E 0, .130E 1)	(.186E 0, .130E 1)	(.580E 0, .547E 0)
1.8	(.573E 0, .126E 1)	(-.190E-1, .127E 1)	(.546E 0, .579E 0)
1.9	(.473E 0, .130E 1)	(-.349E 0, .137E 1)	(.512E 0, .616E 0)
2.0	(.425E 0, .142E 1)	(-.698E 0, .164E 1)	(.459E 0, .653E 0)
2.1	(.432E 0, .154E 1)	(-.991E 0, .207E 1)	(.389E 0, .722E 0)
2.2	(.471E 0, .165E 1)	(-.105E 1, .266E 1)	(.336E 0, .829E 0)
2.3	(.560E 0, .169E 1)	(-.793E 0, .307E 1)	(.324E 0, .958E 0)
2.4	(.588E 0, .167E 1)	(-.616E 0, .307E 1)	(.369E 0, .107E 1)
2.5	(.556E 0, .167E 1)	(-.871E 0, .299E 1)	(.439E 0, .112E 1)
2.6	(.523E 0, .168E 1)	(-.147E 1, .327E 1)	(.460E 0, .109E 1)
2.7	(.467E 0, .169E 1)	(-.208E 1, .413E 1)	(.388E 0, .108E 1)
2.8	(.362E 0, .173E 1)	(-.221E 1, .565E 1)	(.270E 0, .116E 1)
2.9	(.256E 0, .185E 1)	(-.107E 1, .730E 1)	(.183E 0, .133E 1)
3.0	(.223E 0, .203E 1)	(.913E 0, .752E 1)	(.222E 0, .158E 1)
3.1	(.286E 0, .218E 1)	(.189E 1, .665E 1)	(.440E 0, .174E 1)
3.2	(.351E 0, .222E 1)	(.201E 1, .621E 1)	(.652E 0, .168E 1)
3.3	(.314E 0, .225E 1)	(.232E 1, .638E 1)	(.687E 0, .153E 1)
3.4	(.296E 0, .240E 1)	(.330E 1, .633E 1)	(.585E 0, .147E 1)
3.5	(.417E 0, .254E 1)	(.414E 1, .540E 1)	(.476E 0, .155E 1)
3.6	(.560E 0, .254E 1)	(.421E 1, .433E 1)	(.483E 0, .168E 1)
3.7	(.645E 0, .246E 1)	(.397E 1, .359E 1)	(.560E 0, .176E 1)
3.8	(.619E 0, .236E 1)	(.367E 1, .298E 1)	(.660E 0, .177E 1)
3.9	(.513E 0, .234E 1)	(.325E 1, .246E 1)	(.721E 0, .171E 1)
4.0	(.439E 0, .242E 1)	(.272E 1, .207E 1)	(.715E 0, .164E 1)
4.1	(.448E 0, .250E 1)	(.214E 1, .183E 1)	(.651E 0, .159E 1)
4.2	(.507E 0, .254E 1)	(.156E 1, .172E 1)	(.584E 0, .161E 1)
4.3	(.551E 0, .249E 1)	(.982E 0, .172E 1)	(.566E 0, .163E 1)
4.4	(.493E 0, .243E 1)	(.451E 0, .184E 1)	(.531E 0, .160E 1)
4.5	(.443E 0, .249E 1)	(.727E-2, .203E 1)	(.434E 0, .161E 1)
4.6	(.543E 0, .251E 1)	(-.357E 0, .224E 1)	(.368E 0, .169E 1)
4.7	(.606E 0, .231E 1)	(-.647E 0, .239E 1)	(.398E 0, .176E 1)
4.8	(.432E 0, .209E 1)	(-.101E 1, .242E 1)	(.449E 0, .168E 1)
4.9	(.128E 0, .202E 1)	(-.156E 1, .251E 1)	(.341E 0, .154E 1)
5.0	(-.175E 0, .211E 1)	(-.218E 1, .282E 1)	(.937E-1, .149E 1)

Table M-10

```
MODEL 1   (DRY)

FREQUENCY      KHH(HORIZONTAL)          KVV(VERTICAL)            KTT(ROCKING)
---------      ---------------          -------------            ------------
   0.1      ( .322E 0, .344E 0)   ( .204E 0, .354E 0)   ( .830E 0, .645E-1)
   0.2      ( .479E 0, .323E 0)   ( .394E 0, .521E 0)   ( .847E 0, .746E-1)
   0.3      ( .508E 0, .316E 0)   ( .532E 0, .586E 0)   ( .851E 0, .815E-1)
   0.4      ( .494E 0, .343E 0)   ( .603E 0, .615E 0)   ( .843E 0, .882E-1)
   0.5      ( .466E 0, .404E 0)   ( .622E 0, .641E 0)   ( .824E 0, .980E-1)
   0.6      ( .447E 0, .498E 0)   ( .598E 0, .681E 0)   ( .795E 0, .113E 0)
   0.7      ( .461E 0, .605E 0)   ( .544E 0, .752E 0)   ( .756E 0, .138E 0)
   0.8      ( .505E 0, .689E 0)   ( .488E 0, .865E 0)   ( .711E 0, .182E 0)
   0.9      ( .547E 0, .735E 0)   ( .472E 0, .100E 1)   ( .680E 0, .253E 0)
   1.0      ( .563E 0, .760E 0)   ( .477E 0, .108E 1)   ( .695E 0, .322E 0)
   1.1      ( .547E 0, .782E 0)   ( .380E 0, .113E 1)   ( .711E 0, .328E 0)
   1.2      ( .500E 0, .823E 0)   ( .214E 0, .134E 1)   ( .655E 0, .328E 0)
   1.3      ( .440E 0, .905E 0)   ( .238E 0, .168E 1)   ( .569E 0, .407E 0)
   1.4      ( .404E 0, .103E 1)   ( .453E 0, .185E 1)   ( .574E 0, .557E 0)
   1.5      ( .437E 0, .117E 1)   ( .543E 0, .180E 1)   ( .684E 0, .601E 0)
   1.6      ( .511E 0, .126E 1)   ( .425E 0, .182E 1)   ( .695E 0, .536E 0)
   1.7      ( .569E 0, .128E 1)   ( .352E 0, .202E 1)   ( .600E 0, .527E 0)
   1.8      ( .578E 0, .128E 1)   ( .399E 0, .218E 1)   ( .511E 0, .623E 0)
   1.9      ( .549E 0, .129E 1)   ( .441E 0, .227E 1)   ( .522E 0, .750E 0)
   2.0      ( .503E 0, .133E 1)   ( .438E 0, .233E 1)   ( .581E 0, .795E 0)
   2.1      ( .440E 0, .139E 1)   ( .375E 0, .240E 1)   ( .593E 0, .782E 0)
   2.2      ( .387E 0, .148E 1)   ( .384E 0, .254E 1)   ( .537E 0, .776E 0)
   2.3      ( .368E 0, .160E 1)   ( .239E 0, .274E 1)   ( .450E 0, .839E 0)
   2.4      ( .371E 0, .170E 1)   ( .267E 0, .294E 1)   ( .417E 0, .960E 0)
   2.5      ( .397E 0, .181E 1)   ( .333E 0, .308E 1)   ( .455E 0, .106E 1)
   2.6      ( .459E 0, .189E 1)   ( .376E 0, .317E 1)   ( .508E 0, .109E 1)
   2.7      ( .525E 0, .192E 1)   ( .372E 0, .325E 1)   ( .505E 0, .107E 1)
   2.8      ( .556E 0, .190E 1)   ( .358E 0, .337E 1)   ( .435E 0, .108E 1)
   2.9      ( .530E 0, .187E 1)   ( .362E 0, .350E 1)   ( .367E 0, .117E 1)
   3.0      ( .451E 0, .187E 1)   ( .380E 0, .363E 1)   ( .369E 0, .129E 1)
   3.1      ( .352E 0, .192E 1)   ( .410E 0, .374E 1)   ( .416E 0, .136E 1)
   3.2      ( .265E 0, .203E 1)   ( .446E 0, .383E 1)   ( .450E 0, .138E 1)
   3.3      ( .216E 0, .218E 1)   ( .469E 0, .390E 1)   ( .443E 0, .137E 1)
   3.4      ( .234E 0, .235E 1)   ( .470E 0, .396E 1)   ( .364E 0, .139E 1)
   3.5      ( .332E 0, .249E 1)   ( .448E 0, .402E 1)   ( .318E 0, .147E 1)
   3.6      ( .476E 0, .254E 1)   ( .400E 0, .410E 1)   ( .313E 0, .158E 1)
   3.7      ( .595E 0, .249E 1)   ( .343E 0, .420E 1)   ( .345E 0, .165E 1)
   3.8      ( .642E 0, .238E 1)   ( .306E 0, .433E 1)   ( .371E 0, .170E 1)
   3.9      ( .580E 0, .226E 1)   ( .283E 0, .445E 1)   ( .395E 0, .173E 1)
   4.0      ( .436E 0, .220E 1)   ( .261E 0, .458E 1)   ( .402E 0, .174E 1)
   4.1      ( .262E 0, .223E 1)   ( .247E 0, .471E 1)   ( .368E 0, .175E 1)
   4.2      ( .104E 0, .234E 1)   ( .235E 0, .484E 1)   ( .328E 0, .180E 1)
   4.3      (-.829E-3, .253E 1)   ( .222E 0, .499E 1)   ( .305E 0, .186E 1)
   4.4      (-.130E-1, .277E 1)   ( .251E 0, .517E 1)   ( .286E 0, .193E 1)
   4.5      ( .104E 0, .300E 1)   ( .352E 0, .532E 1)   ( .286E 0, .201E 1)
   4.6      ( .346E 0, .314E 1)   ( .472E 0, .540E 1)   ( .322E 0, .208E 1)
   4.7      ( .643E 0, .310E 1)   ( .574E 0, .542E 1)   ( .374E 0, .212E 1)
   4.8      ( .861E 0, .286E 1)   ( .640E 0, .538E 1)   ( .397E 0, .211E 1)
   4.9      ( .877E 0, .253E 1)   ( .626E 0, .531E 1)   ( .392E 0, .211E 1)
   5.0      ( .696E 0, .226E 1)   ( .528E 0, .547E 1)   ( .367E 0, .211E 1)
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Table M-11

MODEL 2 (FULLY SATURATED, KJ=0.1 CM/SEC)

FREQUENCY	KHH(HORIZONTAL)	KVV(VERTICAL)	KII(ROCKING)
0.1	(.470E 0, .612E 0)	(.466E 0, .847E 0)	(.123E 1, .522E 0)
0.2	(.631E 0, .672E 0)	(.788E 0, .137E 1)	(.145E 1, .612E 0)
0.3	(.100E 1, .640E 0)	(.113E 1, .176E 1)	(.160E 1, .667E 0)
0.4	(.107E 1, .612E 0)	(.144E 1, .201E 1)	(.169E 1, .697E 0)
0.5	(.108E 1, .595E 0)	(.170E 1, .216E 1)	(.175E 1, .718E 0)
0.6	(.106E 1, .607E 0)	(.187E 1, .224E 1)	(.176E 1, .738E 0)
0.7	(.101E 1, .640E 0)	(.196E 1, .227E 1)	(.179E 1, .762E 0)
0.8	(.943E 0, .699E 0)	(.197E 1, .227E 1)	(.177E 1, .792E 0)
0.9	(.875E 0, .790E 0)	(.189E 1, .226E 1)	(.175E 1, .827E 0)
1.0	(.822E 0, .901E 0)	(.173E 1, .226E 1)	(.173E 1, .863E 0)
1.1	(.790E 0, .102E 1)	(.149E 1, .228E 1)	(.169E 1, .893E 0)
1.2	(.772E 0, .112E 1)	(.119E 1, .233E 1)	(.164E 1, .915E 0)
1.3	(.749E 0, .120E 1)	(.824E 0, .241E 1)	(.157E 1, .932E 0)
1.4	(.707E 0, .127E 1)	(.399E 0, .254E 1)	(.147E 1, .953E 0)
1.5	(.639E 0, .134E 1)	(-.776E-1, .273E 1)	(.134E 1, .984E 0)
1.6	(.545E 0, .142E 1)	(-.595E 0, .299E 1)	(.119E 1, .103E 1)
1.7	(.431E 0, .152E 1)	(-.114E 1, .333E 1)	(.103E 1, .111E 1)
1.8	(.305E 0, .166E 1)	(-.163E 1, .375E 1)	(.845E 0, .121E 1)
1.9	(.176E 0, .184E 1)	(-.221E 1, .428E 1)	(.656E 0, .135E 1)
2.0	(.569E-1, .205E 1)	(-.266E 1, .488E 1)	(.472E 0, .153E 1)
2.1	(-.355E-1, .231E 1)	(-.305E 1, .555E 1)	(.308E 0, .175E 1)
2.2	(-.823E-1, .261E 1)	(-.330E 1, .623E 1)	(.186E 0, .201E 1)
2.3	(-.652E-1, .293E 1)	(-.342E 1, .685E 1)	(.129E 0, .229E 1)
2.4	(.276E-1, .324E 1)	(-.346E 1, .733E 1)	(.144E 0, .255E 1)
2.5	(.194E 0, .351E 1)	(-.351E 1, .765E 1)	(.218E 0, .275E 1)
2.6	(.414E 0, .372E 1)	(-.365E 1, .782E 1)	(.310E 0, .286E 1)
2.7	(.656E 0, .384E 1)	(-.395E 1, .792E 1)	(.376E 0, .289E 1)
2.8	(.682E 0, .387E 1)	(-.442E 1, .800E 1)	(.380E 0, .287E 1)
2.9	(.106E 1, .384E 1)	(-.505E 1, .813E 1)	(.314E 0, .283E 1)
3.0	(.119E 1, .376E 1)	(-.581E 1, .835E 1)	(.182E 0, .279E 1)
3.1	(.125E 1, .367E 1)	(-.667E 1, .869E 1)	(-.201E-2, .276E 1)
3.2	(.127E 1, .358E 1)	(-.760E 1, .917E 1)	(-.222E 0, .281E 1)
3.3	(.124E 1, .350E 1)	(-.857E 1, .981E 1)	(-.463E 0, .286E 1)
3.4	(.117E 1, .344E 1)	(-.955E 1, .106E 2)	(-.713E 0, .298E 1)
3.5	(.108E 1, .341E 1)	(-.105E 2, .116E 2)	(-.964E 0, .312E 1)
3.6	(.978E 0, .339E 1)	(-.113E 2, .127E 2)	(-.121E 1, .329E 1)
3.7	(.863E 0, .340E 1)	(-.121E 2, .140E 2)	(-.145E 1, .346E 1)
3.8	(.744E 0, .343E 1)	(-.127E 2, .154E 2)	(-.167E 1, .369E 1)
3.9	(.624E 0, .346E 1)	(-.131E 2, .168E 2)	(-.189E 1, .392E 1)
4.0	(.506E 0, .355E 1)	(-.133E 2, .183E 2)	(-.209E 1, .417E 1)
4.1	(.390E 0, .363E 1)	(-.133E 2, .197E 2)	(-.226E 1, .444E 1)
4.2	(.279E 0, .372E 1)	(-.131E 2, .210E 2)	(-.243E 1, .472E 1)
4.3	(.174E 0, .383E 1)	(-.127E 2, .223E 2)	(-.256E 1, .499E 1)
4.4	(.790E-1, .396E 1)	(-.123E 2, .233E 2)	(-.268E 1, .526E 1)
4.5	(-.863E-3, .410E 1)	(-.118E 2, .241E 2)	(-.278E 1, .550E 1)
4.6	(-.600E-1, .426E 1)	(-.112E 2, .247E 2)	(-.288E 1, .574E 1)
4.7	(-.932E-1, .442E 1)	(-.108E 2, .250E 2)	(-.299E 1, .596E 1)
4.8	(-.966E-1, .458E 1)	(-.105E 2, .252E 2)	(-.311E 1, .618E 1)
4.9	(-.677E-1, .473E 1)	(-.104E 2, .253E 2)	(-.324E 1, .642E 1)
5.0	(-.650E-2, .487E 1)	(-.105E 2, .254E 2)	(-.337E 1, .668E 1)

Table M-12

MODEL 3 (FULLY SATURATED, KJ=0.1 CM/SEC)

FREQUENCY	KHH(HORIZONTAL)	KVV(VERTICAL)	KTT(ROCKING)
0.1	(.492E 0, .533E 0)	(.449E 0, .798E 0)	(.101E 1, .373E 0)
0.2	(.786E 0, .531E 0)	(.811E 0, .125E 1)	(.118E 1, .420E 0)
0.3	(.907E 0, .488E 0)	(.116E 1, .152E 1)	(.128E 1, .435E 0)
0.4	(.951E 0, .459E 0)	(.144E 1, .166E 1)	(.134E 1, .436E 0)
0.5	(.953E 0, .446E 0)	(.164E 1, .172E 1)	(.136E 1, .437E 0)
0.6	(.927E 0, .452E 0)	(.174E 1, .174E 1)	(.136E 1, .442E 0)
0.7	(.880E 0, .477E 0)	(.176E 1, .173E 1)	(.134E 1, .455E 0)
0.8	(.822E 0, .524E 0)	(.171E 1, .172E 1)	(.131E 1, .475E 0)
0.9	(.761E 0, .591E 0)	(.158E 1, .172E 1)	(.128E 1, .502E 0)
1.0	(.705E 0, .675E 0)	(.140E 1, .174E 1)	(.123E 1, .532E 0)
1.1	(.658E 0, .765E 0)	(.116E 1, .179E 1)	(.118E 1, .562E 0)
1.2	(.613E 0, .855E 0)	(.868E 0, .187E 1)	(.112E 1, .591E 0)
1.3	(.564E 0, .944E 0)	(.540E 0, .199E 1)	(.104E 1, .625E 0)
1.4	(.502E 0, .104E 1)	(.183E 0, .215E 1)	(.947E 0, .671E 0)
1.5	(.429E 0, .115E 1)	(-.193E 0, .236E 1)	(.841E 0, .735E 0)
1.6	(.347E 0, .128E 1)	(-.572E 0, .261E 1)	(.731E 0, .820E 0)
1.7	(.269E 0, .145E 1)	(-.942E 0, .291E 1)	(.626E 0, .930E 0)
1.8	(.208E 0, .165E 1)	(-.129E 1, .321E 1)	(.538E 0, .106E 1)
1.9	(.186E 0, .188E 1)	(-.163E 1, .351E 1)	(.477E 0, .120E 1)
2.0	(.223E 0, .214E 1)	(-.198E 1, .378E 1)	(.444E 0, .132E 1)
2.1	(.333E 0, .237E 1)	(-.239E 1, .403E 1)	(.427E 0, .142E 1)
2.2	(.505E 0, .255E 1)	(-.288E 1, .428E 1)	(.401E 0, .148E 1)
2.3	(.700E 0, .265E 1)	(-.348E 1, .460E 1)	(.347E 0, .152E 1)
2.4	(.867E 0, .266E 1)	(-.416E 1, .503E 1)	(.253E 0, .156E 1)
2.5	(.972E 0, .261E 1)	(-.487E 1, .564E 1)	(.122E 0, .161E 1)
2.6	(.101E 1, .256E 1)	(-.553E 1, .644E 1)	(-.338E-1, .169E 1)
2.7	(.982E 0, .252E 1)	(-.605E 1, .743E 1)	(-.202E 0, .182E 1)
2.8	(.924E 0, .252E 1)	(-.634E 1, .854E 1)	(-.368E 0, .199E 1)
2.9	(.852E 0, .255E 1)	(-.632E 1, .965E 1)	(-.511E 0, .221E 1)
3.0	(.777E 0, .262E 1)	(-.599E 1, .106E 2)	(-.598E 0, .249E 1)
3.1	(.708E 0, .271E 1)	(-.546E 1, .112E 2)	(-.598E 0, .277E 1)
3.2	(.651E 0, .283E 1)	(-.490E 1, .115E 2)	(-.506E 0, .301E 1)
3.3	(.618E 0, .297E 1)	(-.452E 1, .113E 2)	(-.369E 0, .315E 1)
3.4	(.617E 0, .311E 1)	(-.449E 1, .110E 2)	(-.249E 0, .320E 1)
3.5	(.651E 0, .325E 1)	(-.484E 1, .106E 2)	(-.185E 0, .319E 1)
3.6	(.712E 0, .336E 1)	(-.551E 1, .103E 2)	(-.175E 0, .316E 1)
3.7	(.780E 0, .343E 1)	(-.640E 1, .102E 2)	(-.201E 0, .313E 1)
3.8	(.834E 0, .347E 1)	(-.741E 1, .104E 2)	(-.252E 0, .309E 1)
3.9	(.858E 0, .348E 1)	(-.847E 1, .108E 2)	(-.341E 0, .303E 1)
4.0	(.851E 0, .349E 1)	(-.955E 1, .113E 2)	(-.485E 0, .297E 1)
4.1	(.817E 0, .351E 1)	(-.106E 2, .121E 2)	(-.693E 0, .293E 1)
4.2	(.765E 0, .354E 1)	(-.116E 2, .131E 2)	(-.953E 0, .293E 1)
4.3	(.706E 0, .360E 1)	(-.125E 2, .143E 2)	(-.124E 1, .301E 1)
4.4	(.647E 0, .367E 1)	(-.131E 2, .156E 2)	(-.153E 1, .315E 1)
4.5	(.590E 0, .376E 1)	(-.135E 2, .171E 2)	(-.179E 1, .335E 1)
4.6	(.536E 0, .385E 1)	(-.135E 2, .185E 2)	(-.202E 1, .360E 1)
4.7	(.484E 0, .395E 1)	(-.131E 2, .197E 2)	(-.220E 1, .389E 1)
4.8	(.437E 0, .407E 1)	(-.127E 2, .206E 2)	(-.232E 1, .421E 1)
4.9	(.400E 0, .420E 1)	(-.122E 2, .210E 2)	(-.236E 1, .453E 1)
5.0	(.377E 0, .435E 1)	(-.120E 2, .211E 2)	(-.232E 1, .483E 1)

Table M-13

MODEL 4 (FULLY SATURATED, KJ=0.1 CM/SEC)

FREQUENCY	KHH(HORIZONTAL)	KVV(VERTICAL)	KTT(ROCKING)
0.1	(.499E 0, .404E 0)	(.546E 0, .879E 0)	(.124E 1, .524E 0)
0.2	(.620E 0, .422E 0)	(.948E 0, .125E 1)	(.148E 1, .589E 0)
0.3	(.633E 0, .486E 0)	(.123E 1, .146E 1)	(.162E 1, .613E 0)
0.4	(.615E 0, .594E 0)	(.140E 1, .158E 1)	(.170E 1, .623E 0)
0.5	(.600E 0, .744E 0)	(.145E 1, .167E 1)	(.173E 1, .630E 0)
0.6	(.615E 0, .921E 0)	(.140E 1, .176E 1)	(.173E 1, .642E 0)
0.7	(.678E 0, .110E 1)	(.126E 1, .190E 1)	(.170E 1, .664E 0)
0.8	(.776E 0, .123E 1)	(.107E 1, .212E 1)	(.165E 1, .703E 0)
0.9	(.885E 0, .132E 1)	(.896E 0, .243E 1)	(.156E 1, .764E 0)
1.0	(.965E 0, .135E 1)	(.614E 0, .273E 1)	(.151E 1, .850E 0)
1.1	(.598E 0, .137E 1)	(.835E 0, .304E 1)	(.147E 1, .952E 0)
1.2	(.583E 0, .138E 1)	(.809E 0, .312E 1)	(.144E 1, .104E 1)
1.3	(.543E 0, .143E 1)	(.589E 0, .314E 1)	(.141E 1, .108E 1)
1.4	(.899E 0, .150E 1)	(.199E 0, .325E 1)	(.133E 1, .111E 1)
1.5	(.869E 0, .156E 1)	(-.242E 0, .353E 1)	(.120E 1, .118E 1)
1.6	(.825E 0, .166E 1)	(-.607E 0, .395E 1)	(.106E 1, .131E 1)
1.7	(.737E 0, .174E 1)	(-.825E 0, .437E 1)	(.973E 0, .147E 1)
1.8	(.734E 0, .181E 1)	(-.976E 0, .466E 1)	(.948E 0, .162E 1)
1.9	(.655E 0, .189E 1)	(-.124E 1, .482E 1)	(.933E 0, .170E 1)
2.0	(.550E 0, .199E 1)	(-.166E 1, .500E 1)	(.857E 0, .173E 1)
2.1	(.434E 0, .214E 1)	(-.221E 1, .533E 1)	(.715E 0, .179E 1)
2.2	(.340E 0, .235E 1)	(-.269E 1, .560E 1)	(.554E 0, .192E 1)
2.3	(.301E 0, .259E 1)	(-.304E 1, .630E 1)	(.435E 0, .210E 1)
2.4	(.333E 0, .284E 1)	(-.333E 1, .668E 1)	(.384E 0, .228E 1)
2.5	(.427E 0, .304E 1)	(-.371E 1, .694E 1)	(.363E 0, .238E 1)
2.6	(.551E 0, .317E 1)	(-.428E 1, .721E 1)	(.297E 0, .243E 1)
2.7	(.663E 0, .324E 1)	(-.499E 1, .764E 1)	(.158E 0, .248E 1)
2.8	(.729E 0, .326E 1)	(-.572E 1, .829E 1)	(-.297E-1, .258E 1)
2.9	(.736E 0, .327E 1)	(-.630E 1, .911E 1)	(-.214E 0, .274E 1)
3.0	(.691E 0, .330E 1)	(-.668E 1, .998E 1)	(-.352E 0, .295E 1)
3.1	(.623E 0, .338E 1)	(-.691E 1, .107E 2)	(-.431E 0, .314E 1)
3.2	(.573E 0, .351E 1)	(-.712E 1, .114E 2)	(-.464E 0, .329E 1)
3.3	(.571E 0, .368E 1)	(-.740E 1, .119E 2)	(-.561E 0, .339E 1)
3.4	(.620E 0, .380E 1)	(-.773E 1, .125E 2)	(-.687E 0, .348E 1)
3.5	(.699E 0, .389E 1)	(-.805E 1, .130E 2)	(-.848E 0, .361E 1)
3.6	(.730E 0, .394E 1)	(-.836E 1, .135E 2)	(-.101E 1, .377E 1)
3.7	(.642E 0, .394E 1)	(-.873E 1, .139E 2)	(-.113E 1, .397E 1)
3.8	(.872E 0, .392E 1)	(-.926E 1, .143E 2)	(-.121E 1, .415E 1)
3.9	(.864E 0, .388E 1)	(-.100E 2, .147E 2)	(-.127E 1, .429E 1)
4.0	(.820E 0, .385E 1)	(-.109E 2, .153E 2)	(-.134E 1, .437E 1)
4.1	(.745E 0, .382E 1)	(-.118E 2, .162E 2)	(-.146E 1, .442E 1)
4.2	(.644E 0, .382E 1)	(-.127E 2, .174E 2)	(-.165E 1, .446E 1)
4.3	(.528E 0, .384E 1)	(-.134E 2, .188E 2)	(-.191E 1, .453E 1)
4.4	(.398E 0, .385E 1)	(-.137E 2, .204E 2)	(-.221E 1, .467E 1)
4.5	(.271E 0, .398E 1)	(-.135E 2, .220E 2)	(-.252E 1, .490E 1)
4.6	(.157E 0, .410E 1)	(-.130E 2, .233E 2)	(-.278E 1, .520E 1)
4.7	(.672E-1, .425E 1)	(-.123E 2, .240E 2)	(-.299E 1, .555E 1)
4.8	(.140E-1, .442E 1)	(-.117E 2, .243E 2)	(-.312E 1, .591E 1)
4.9	(.910E-2, .460E 1)	(-.114E 2, .242E 2)	(-.316E 1, .627E 1)
5.0	(.569E-1, .476E 1)	(-.117E 2, .239E 2)	(-.317E 1, .658E 1)

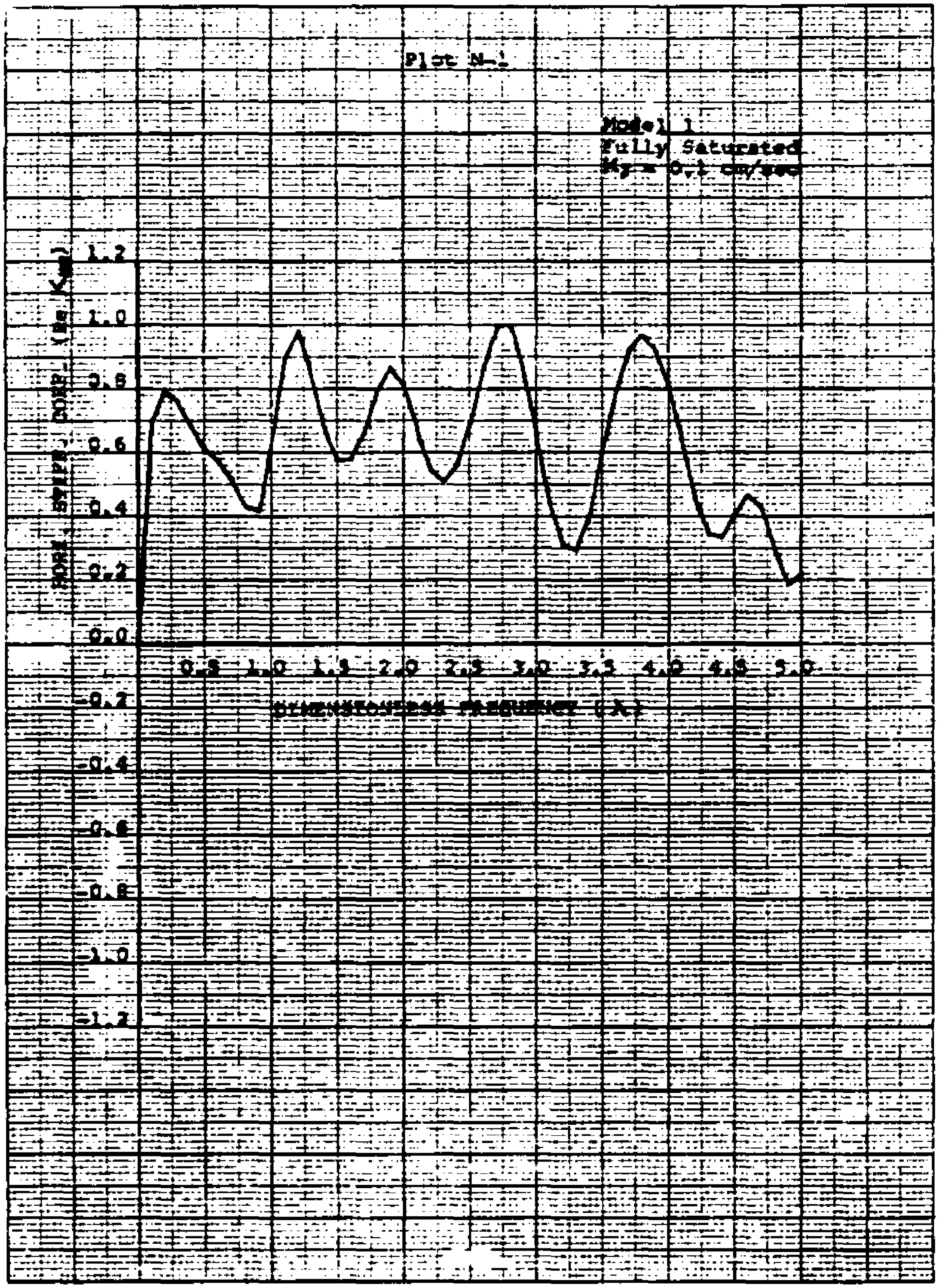
Table M-14

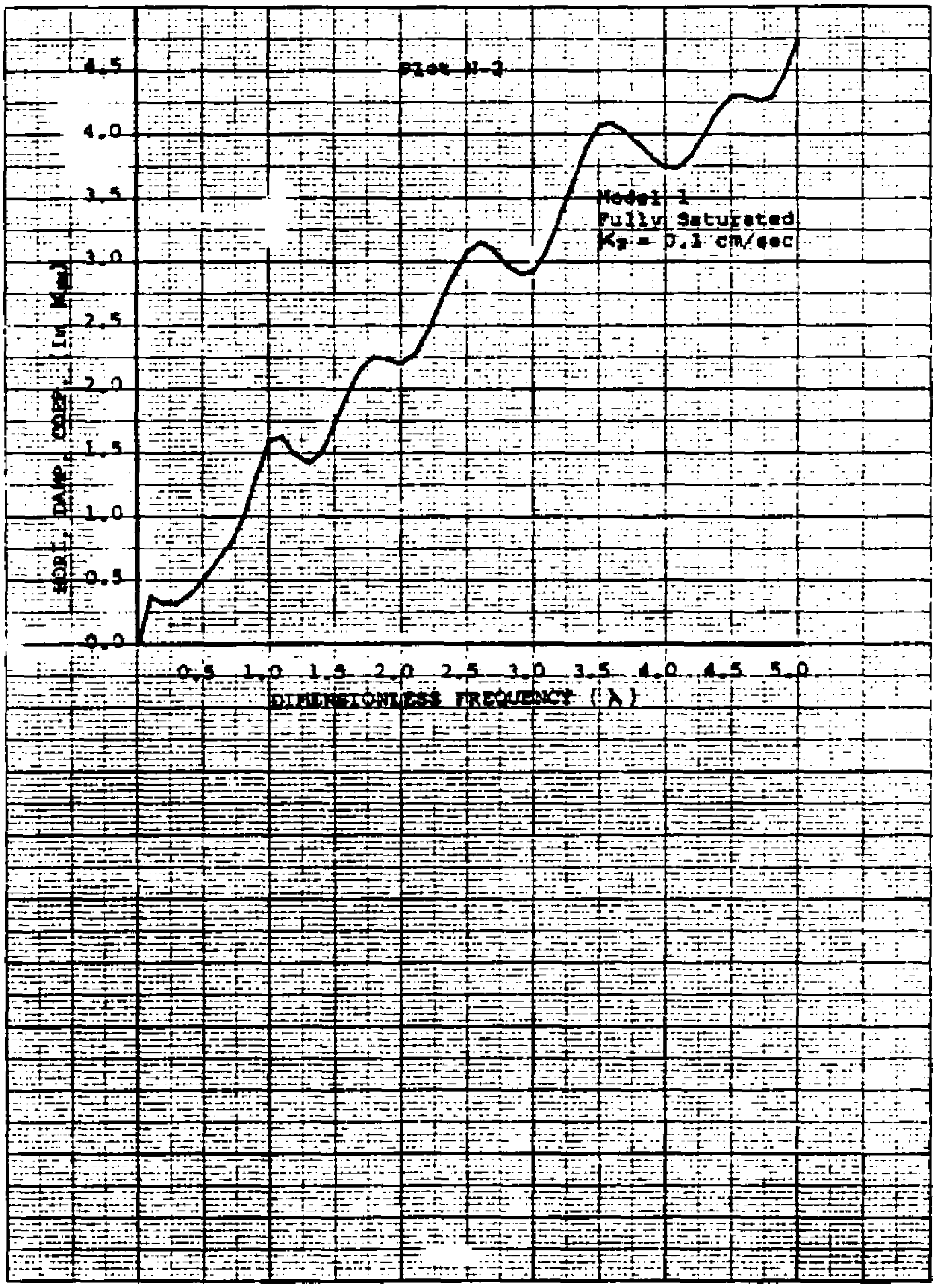
```
MODEL 5 (FULLY SATURATED, KJ=0.1 CM/SEC)

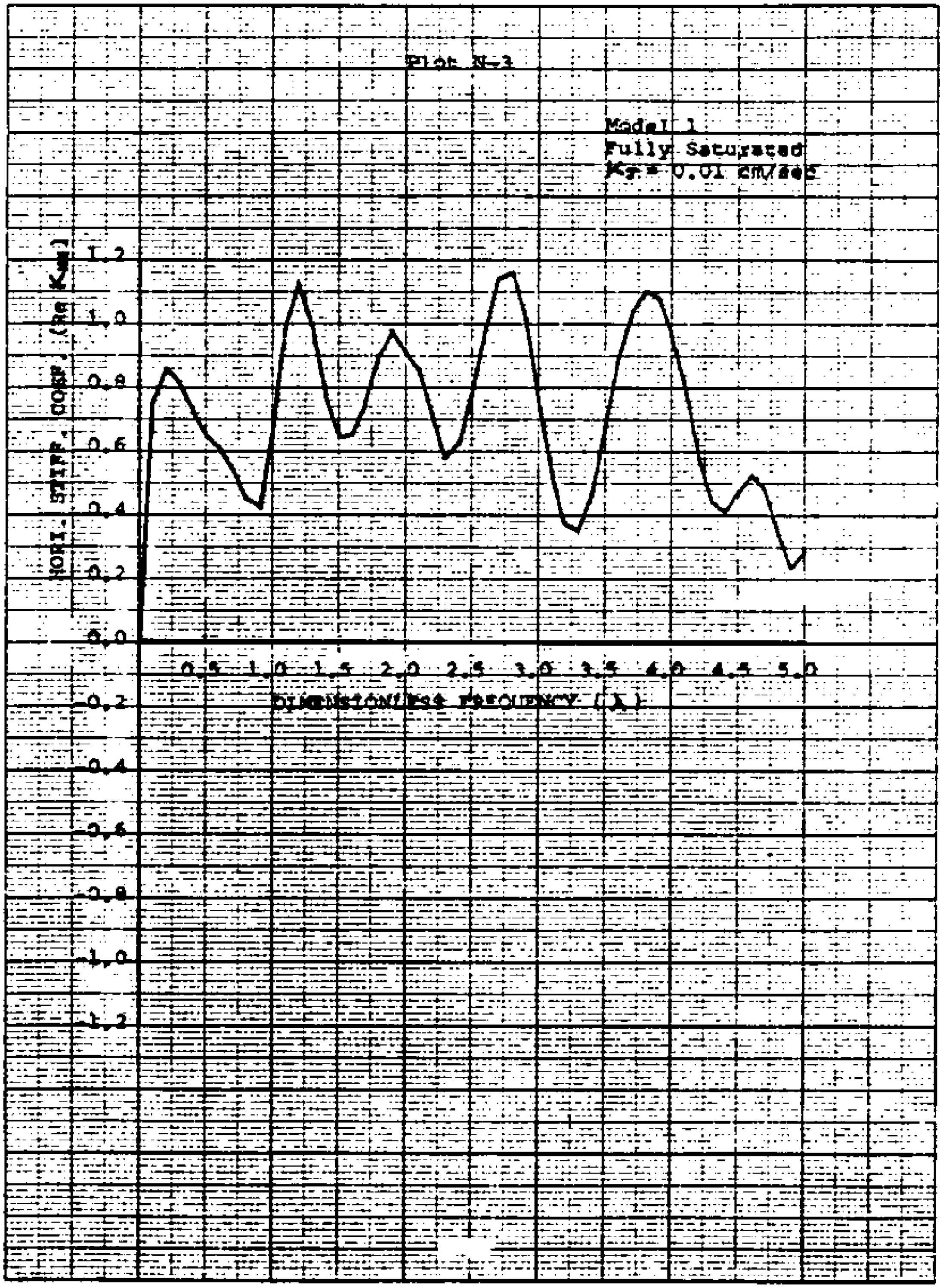
FREQUENCY      KHH(HORIZONTAL)          KVV(VERTICAL)            KTT(ROCKING)
---------     ----------------         -------------            ------------
   0.1      ( .573E 0, .598E 0)   ( .651E 0, .815E 0)   ( .150E 1, .367E 0)
   0.2      ( .913E 0, .596E 0)   ( .984E 0, .114E 1)   ( .164E 1, .343E 0)
   0.3      ( .105E 1, .538E 0)   ( .126E 1, .134E 1)   ( .166E 1, .327E 0)
   0.4      ( .109E 1, .496E 0)   ( .144E 1, .144E 1)   ( .163E 1, .339E 0)
   0.5      ( .107E 1, .474E 0)   ( .152E 1, .149E 1)   ( .158E 1, .390E 0)
   0.6      ( .103E 1, .468E 0)   ( .151E 1, .152E 1)   ( .149E 1, .484E 0)
   0.7      ( .942E 0, .481E 0)   ( .140E 1, .155E 1)   ( .144E 1, .615E 0)
   0.8      ( .825E 0, .519E 0)   ( .122E 1, .160E 1)   ( .144E 1, .742E 0)
   0.9      ( .677E 0, .595E 0)   ( .956E 0, .168E 1)   ( .145E 1, .825E 0)
   1.0      ( .504E 0, .731E 0)   ( .635E 0, .182E 1)   ( .144E 1, .866E 0)
   1.1      ( .334E 0, .954E 0)   ( .266E 0, .203E 1)   ( .139E 1, .897E 0)
   1.2      ( .239E 0, .128E 1)   (-.123E 0, .234E 1)   ( .129E 1, .943E 0)
   1.3      ( .322E 0, .163E 1)   (-.484E 0, .271E 1)   ( .117E 1, .102E 1)
   1.4      ( .550E 0, .183E 1)   (-.750E 0, .317E 1)   ( .104E 1, .114E 1)
   1.5      ( .715E 0, .181E 1)   (-.673E 0, .359E 1)   ( .917E 0, .131E 1)
   1.6      ( .705E 0, .174E 1)   (-.917E 0, .384E 1)   ( .641E 0, .152E 1)
   1.7      ( .564E 0, .174E 1)   (-.107E 1, .388E 1)   ( .834E 0, .174E 1)
   1.8      ( .367E 0, .166E 1)   (-.147E 1, .366E 1)   ( .695E 0, .190E 1)
   1.9      ( .197E 0, .212E 1)   (-.207E 1, .393E 1)   ( .978E 0, .197E 1)
   2.0      ( .149E 0, .250E 1)   (-.279E 1, .415E 1)   ( .102E 1, .196E 1)
   2.1      ( .305E 0, .266E 1)   (-.357E 1, .452E 1)   ( .577E 0, .190E 1)
   2.2      ( .610E 0, .305E 1)   (-.437E 1, .505E 1)   ( .646E 0, .183E 1)
   2.3      ( .873E 0, .301E 1)   (-.515E 1, .575E 1)   ( .634E 0, .180E 1)
   2.4      ( .955E 0, .286E 1)   (-.583E 1, .663E 1)   ( .362E 0, .183E 1)
   2.5      ( .866E 0, .275E 1)   (-.634E 1, .762E 1)   ( .581E-1, .193E 1)
   2.6      ( .688E 0, .275E 1)   (-.663E 1, .862E 1)   (-.251E 0, .212E 1)
   2.7      ( .498E 0, .288E 1)   (-.678E 1, .954E 1)   (-.541E 0, .239E 1)
   2.8      ( .365E 0, .312E 1)   (-.682E 1, .104E 2)   (-.787E 0, .274E 1)
   2.9      ( .342E 0, .341E 1)   (-.673E 1, .111E 2)   (-.954E 0, .317E 1)
   3.0      ( .448E 0, .366E 1)   (-.656E 1, .116E 2)   (-.996E 0, .364E 1)
   3.1      ( .649E 0, .386E 1)   (-.651E 1, .118E 2)   (-.872E 0, .409E 1)
   3.2      ( .867E 0, .390E 1)   (-.673E 1, .118E 2)   (-.595E 0, .442E 1)
   3.3      ( .102E 1, .383E 1)   (-.726E 1, .116E 2)   (-.264E 0, .453E 1)
   3.4      ( .106E 1, .372E 1)   (-.805E 1, .119E 2)   (-.159E-1, .445E 1)
   3.5      ( .101E 1, .362E 1)   (-.905E 1, .122E 2)   ( .639E-1, .429E 1)
   3.6      ( .885E 0, .358E 1)   (-.102E 2, .126E 2)   ( .543E-1, .410E 1)
   3.7      ( .744E 0, .361E 1)   (-.114E 2, .136E 2)   (-.732E-1, .393E 1)
   3.8      ( .618E 0, .370E 1)   (-.127E 2, .149E 2)   (-.291E 0, .377E 1)
   3.9      ( .532E 0, .384E 1)   (-.136E 2, .166E 2)   (-.596E 0, .367E 1)
   4.0      ( .500E 0, .399E 1)   (-.142E 2, .187E 2)   (-.971E 0, .362E 1)
   4.1      ( .519E 0, .413E 1)   (-.140E 2, .208E 2)   (-.140E 1, .366E 1)
   4.2      ( .573E 0, .422E 1)   (-.132E 2, .226E 2)   (-.185E 1, .379E 1)
   4.3      ( .637E 0, .427E 1)   (-.119E 2, .236E 2)   (-.232E 1, .401E 1)
   4.4      ( .690E 0, .427E 1)   (-.106E 2, .238E 2)   (-.278E 1, .433E 1)
   4.5      ( .715E 0, .424E 1)   (-.103E 2, .235E 2)   (-.320E 1, .477E 1)
   4.6      ( .706E 0, .420E 1)   (-.105E 2, .230E 2)   (-.355E 1, .532E 1)
   4.7      ( .661E 0, .414E 1)   (-.113E 2, .229E 2)   (-.377E 1, .597E 1)
   4.8      ( .580E 0, .409E 1)   (-.123E 2, .231E 2)   (-.382E 1, .668E 1)
   4.9      ( .465E 0, .408E 1)   (-.133E 2, .230E 2)   (-.365E 1, .737E 1)
   5.0      ( .325E 0, .406E 1)   (-.143E 2, .248E 2)   (-.327E 1, .795E 1)
```

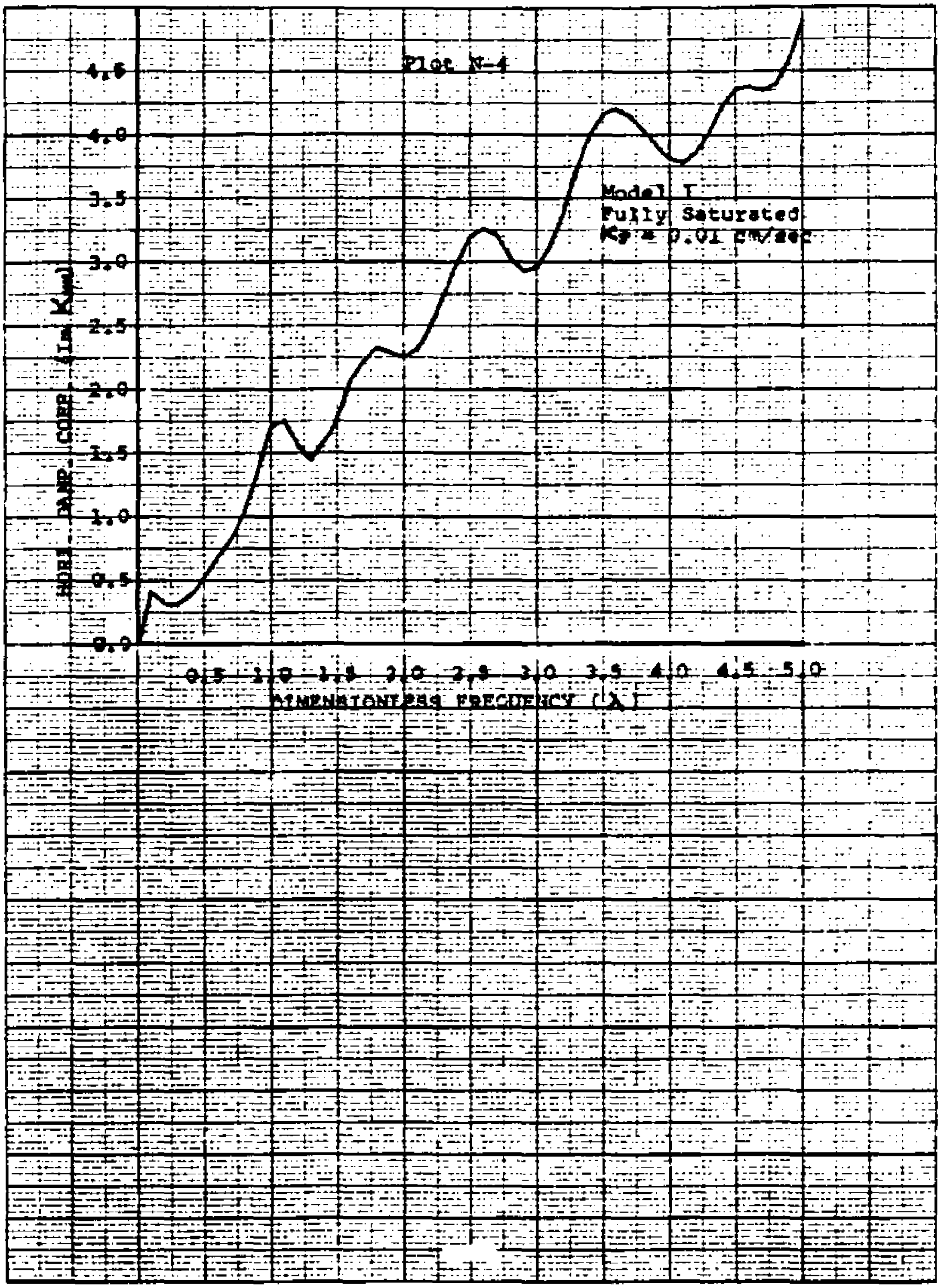
APPENDIX N

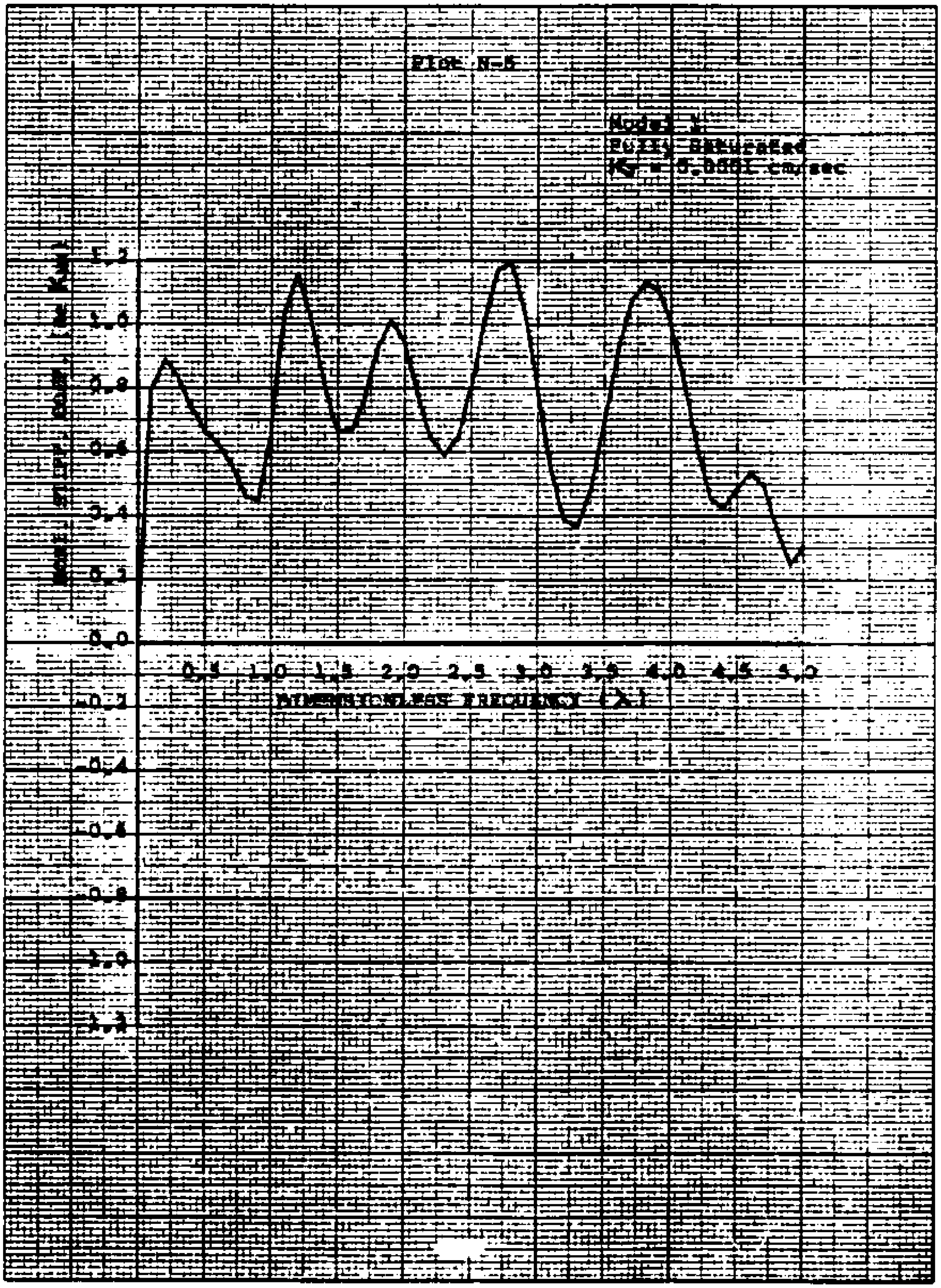
PLOTS


Plot N-1
Model 1
Fully Saturated
Ky = 0.1 cm/sec
HORZ. STIFF. COEF. (Re K_HH)
1.2
1.0
0.8
0.6
0.4
0.2
0.0
-0.2
-0.4
-0.6
-0.8
-1.0
-1.2
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
5.0
DIMENSIONLESS FREQUENCY (λ)


Plot H-2
Model 1
Fully Saturated
Kp = 0.1 cm/sec
HORI. DAMP. COEF. (Im Kmm)
4.5
4.0
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
5.0
DIMENSIONLESS FREQUENCY (λ)

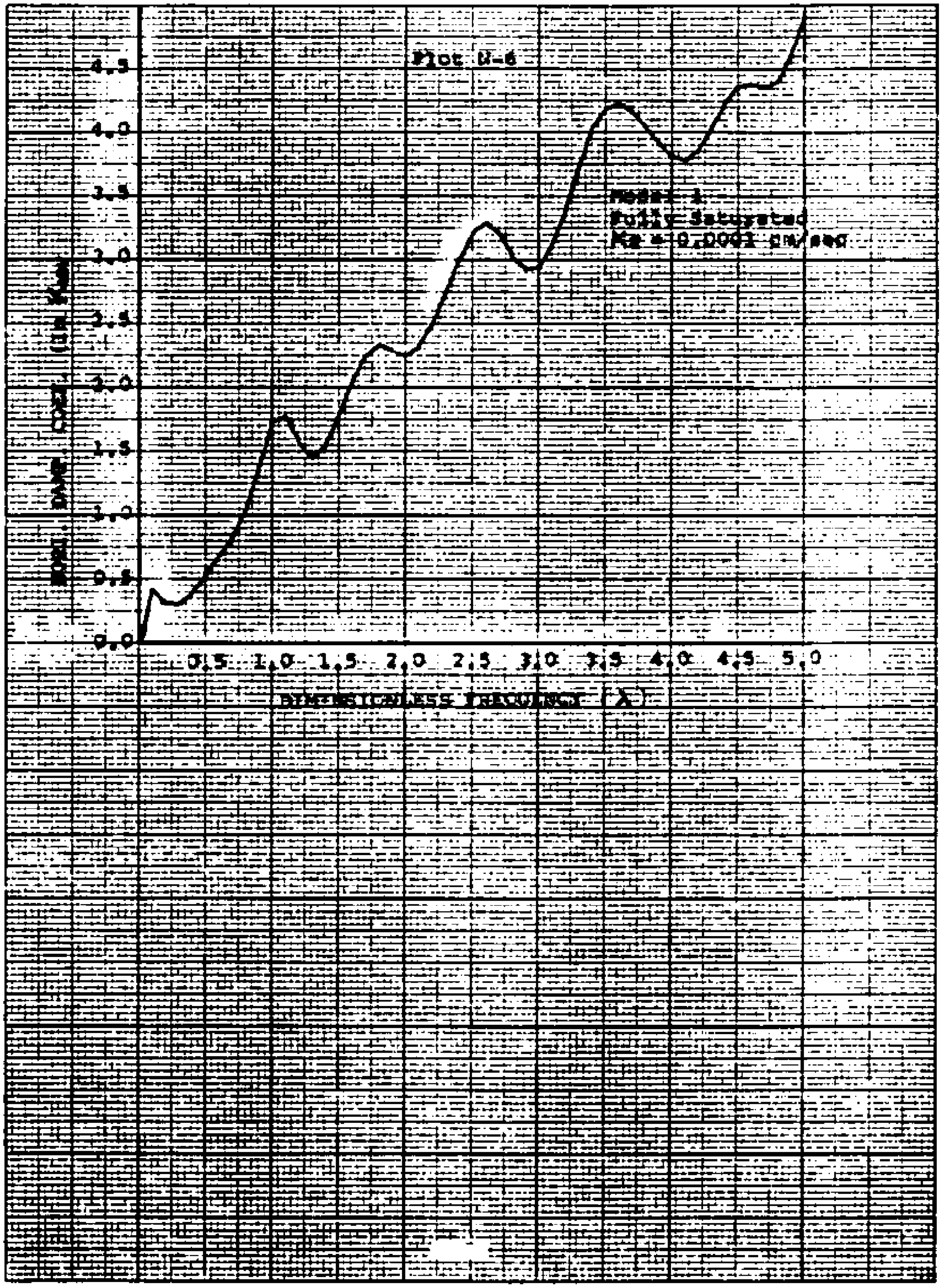

Plot N-3
Model 1
Fully Saturated
K_z = 0.01 cm/sec
HORI. STIFF. COEF. (Re K_{HH})
1.2
1.0
0.8
0.6
0.4
0.2
0.0
-0.2
-0.4
-0.6
-0.8
-1.0
-1.2
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
5.0
DIMENSIONLESS FREQUENCY (λ)

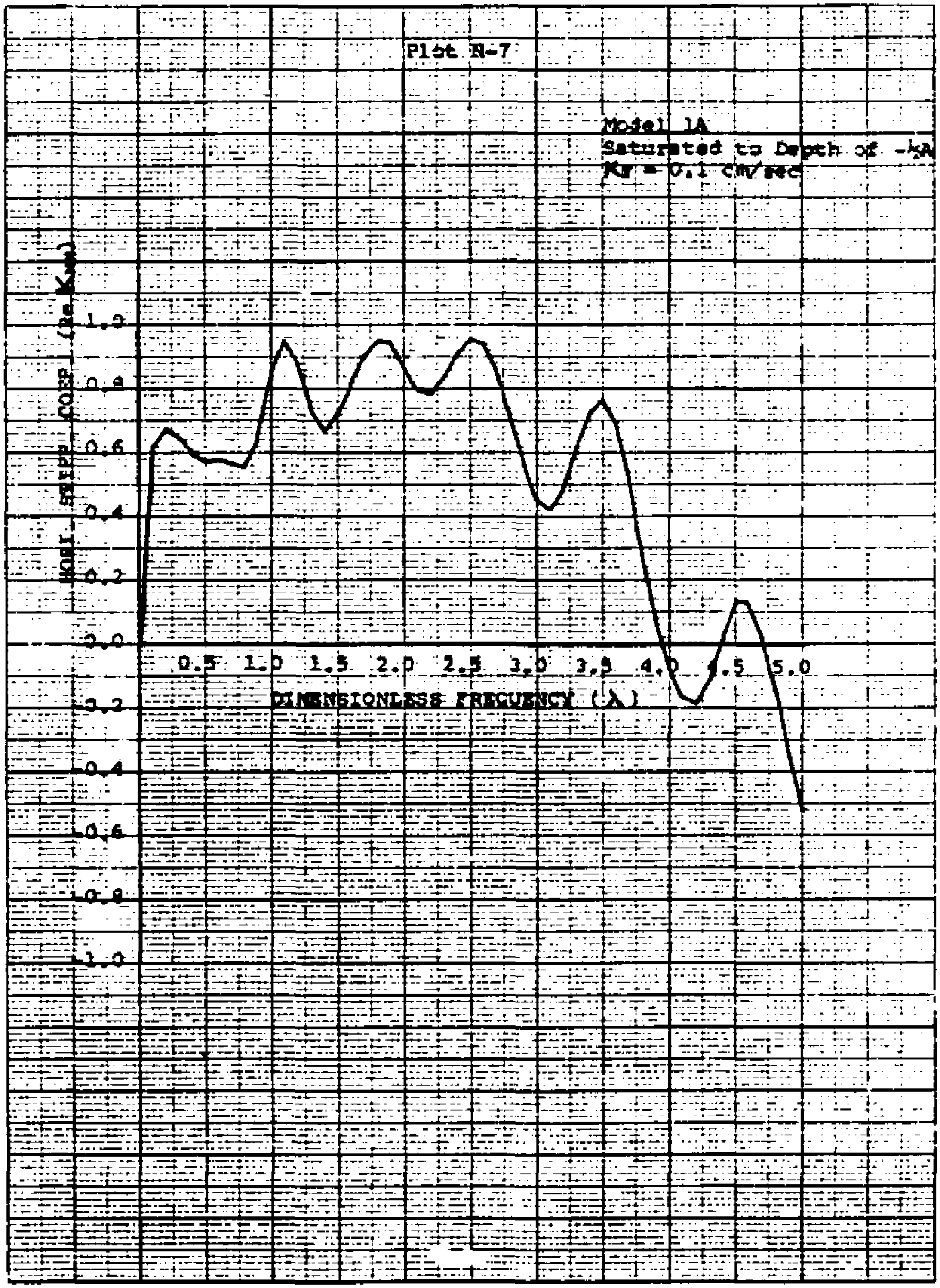

Plot N-4
Model I
Fully Saturated
Ks = 0.01 cm/sec
HORZ. DAMP. CORR. (Im K)
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
5.0
DIMENSIONLESS FREQUENCY (λ)

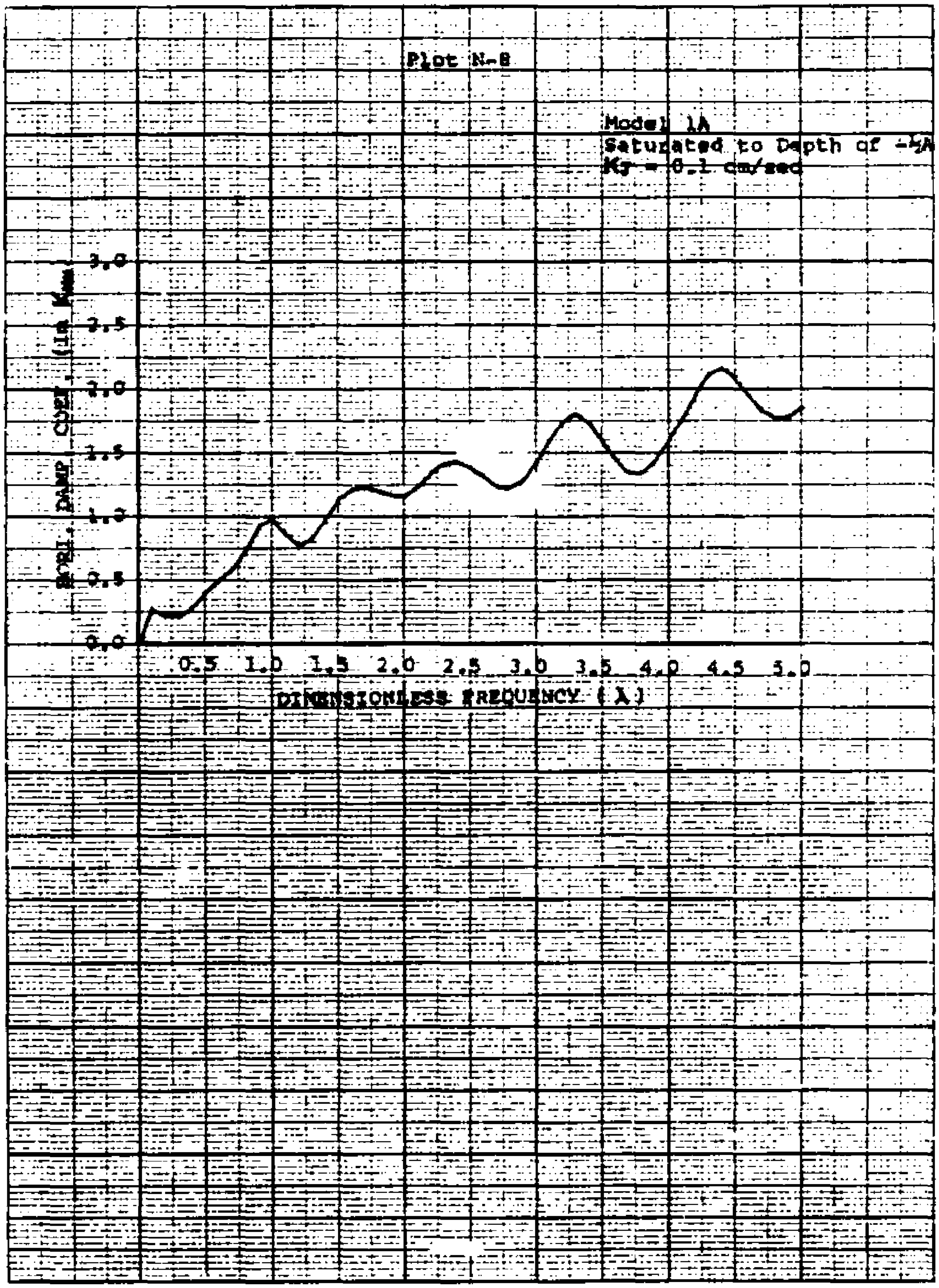


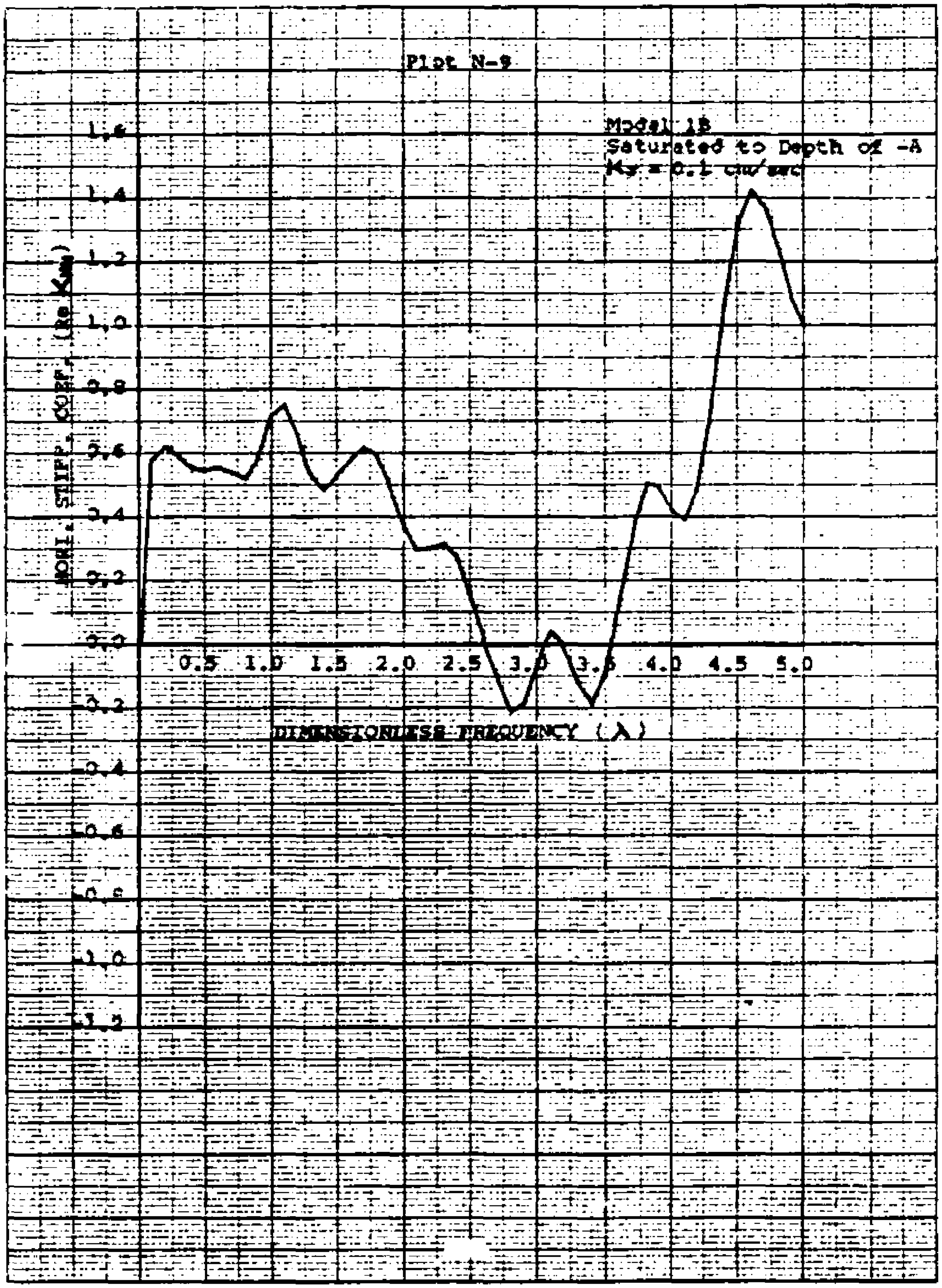
Plot H-5

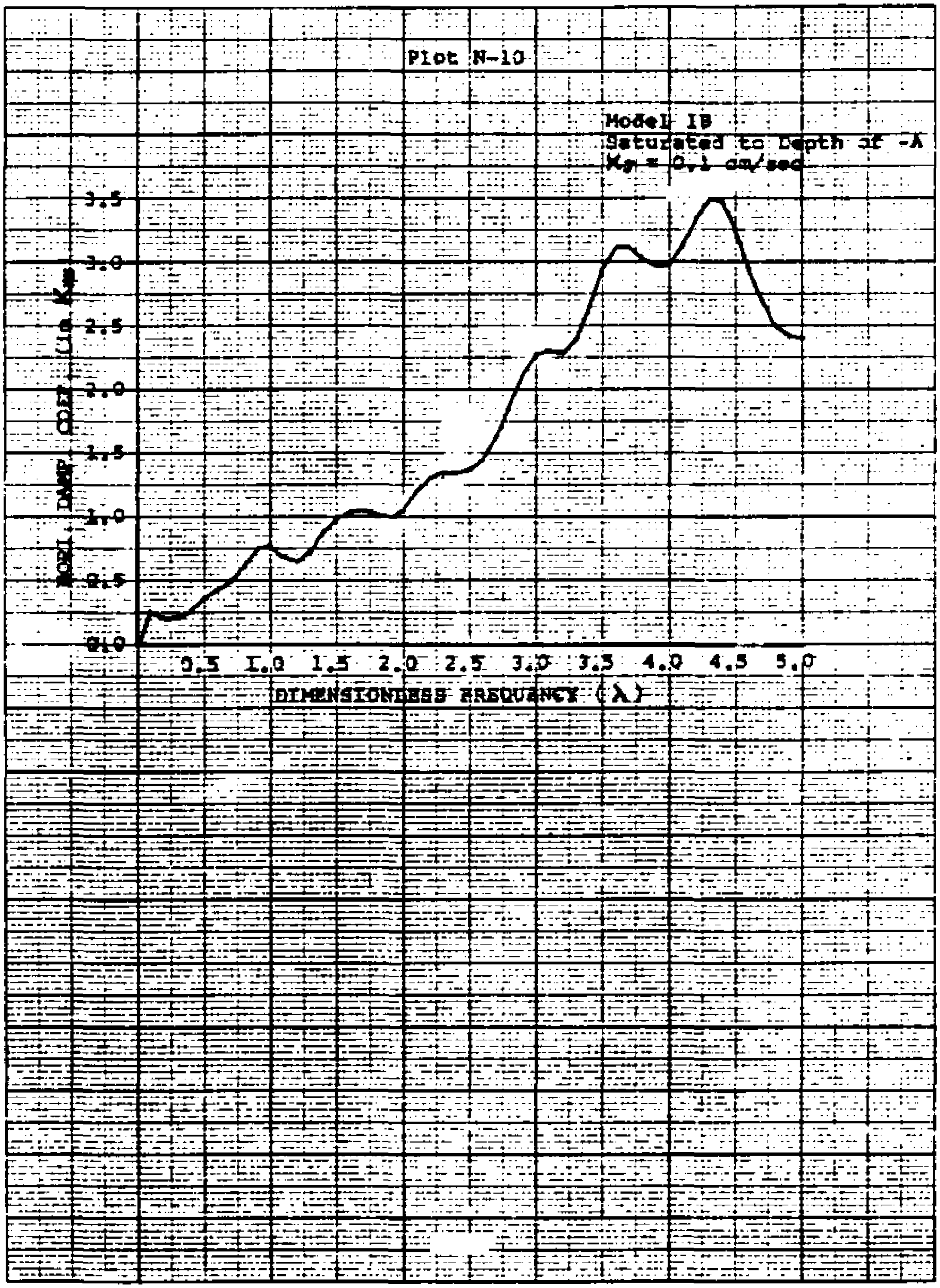
Model 1
Fully Saturated
K_f = 0.0001 cm/sec
HORZ. STIFF. COEF. (Re Kuu)
1.2
1.0
0.8
0.6
0.4
0.2
0.0
-0.2
-0.4
-0.6
-0.8
-1.0
-1.2
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
5.0
DIMENSIONLESS FREQUENCY (λ)

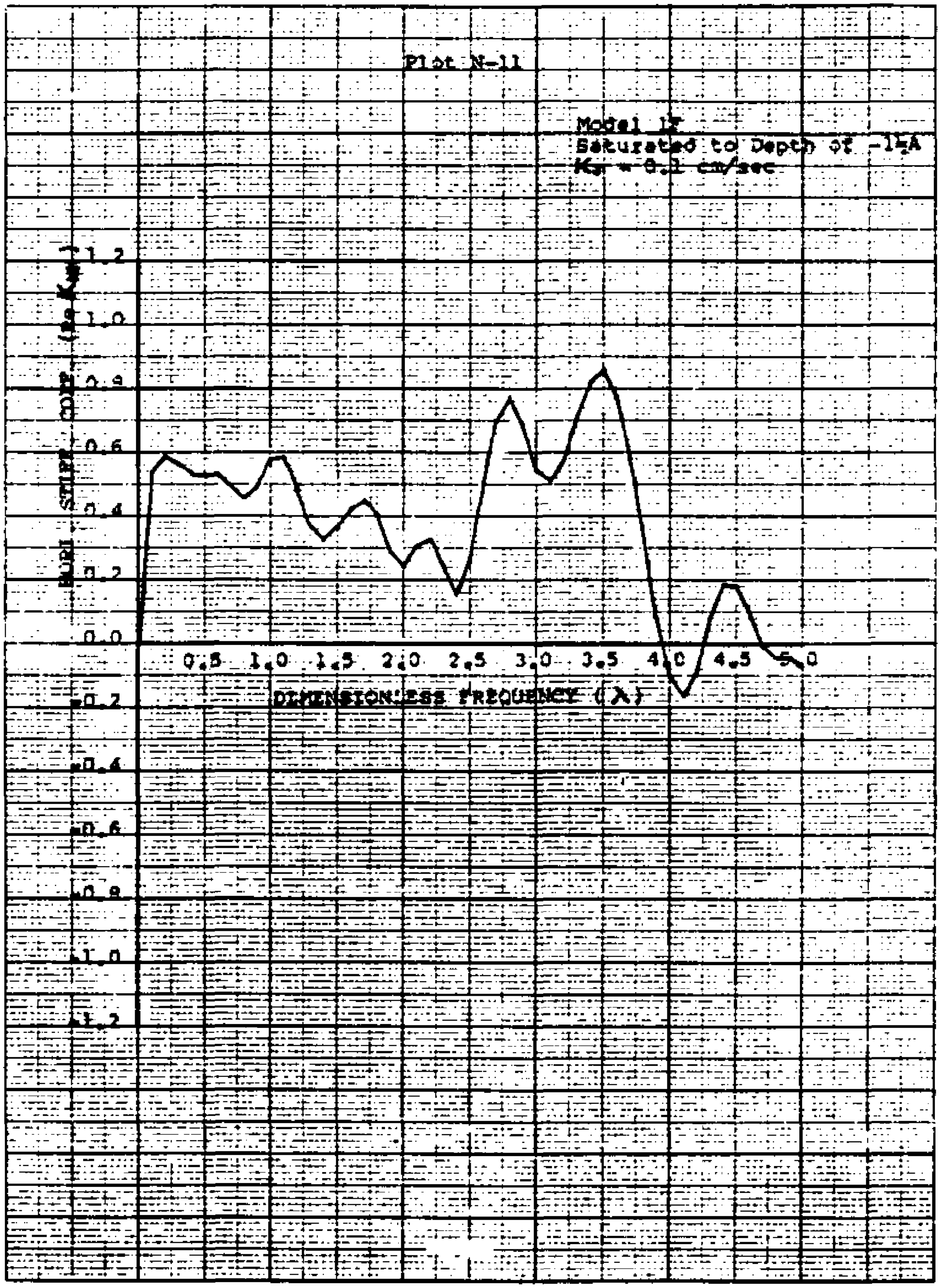

Plot U-6
Model 1
Fully Saturated
Kz = 0.0001 cm/sec
HORZ. DAMP. COEF. (in Khh)
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
5.0
DIMENSIONLESS FREQUENCY (λ)

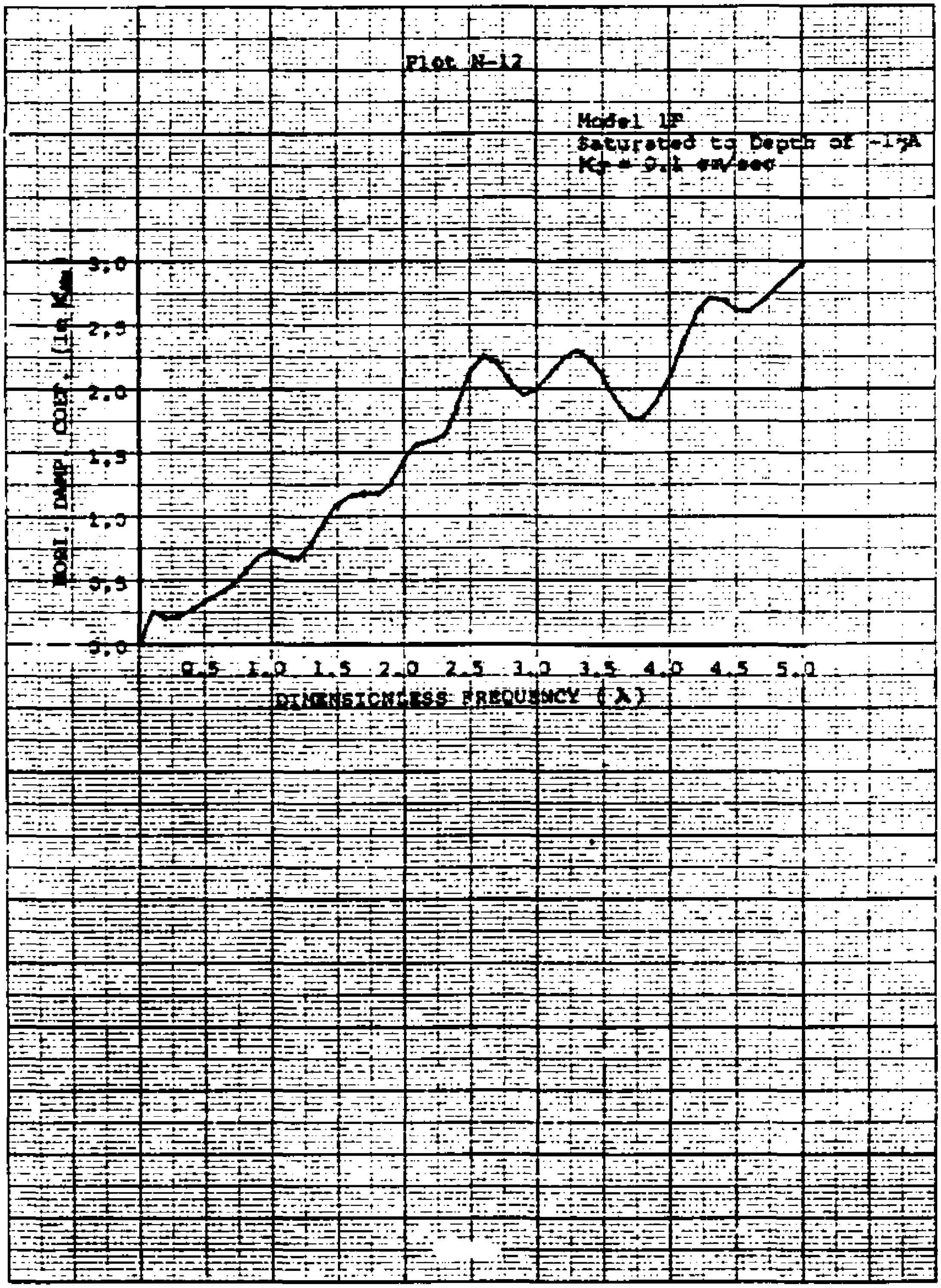

Plot N-7
Model 1A
Saturated to Depth of -½A
Kr = 0.1 cm/sec
HORI. STIFF. COEF. (Re K_HH)
1.0
0.8
0.6
0.4
0.2
0.0
-0.2
-0.4
-0.6
-0.8
-1.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
5.0
DIMENSIONLESS FREQUENCY (λ)

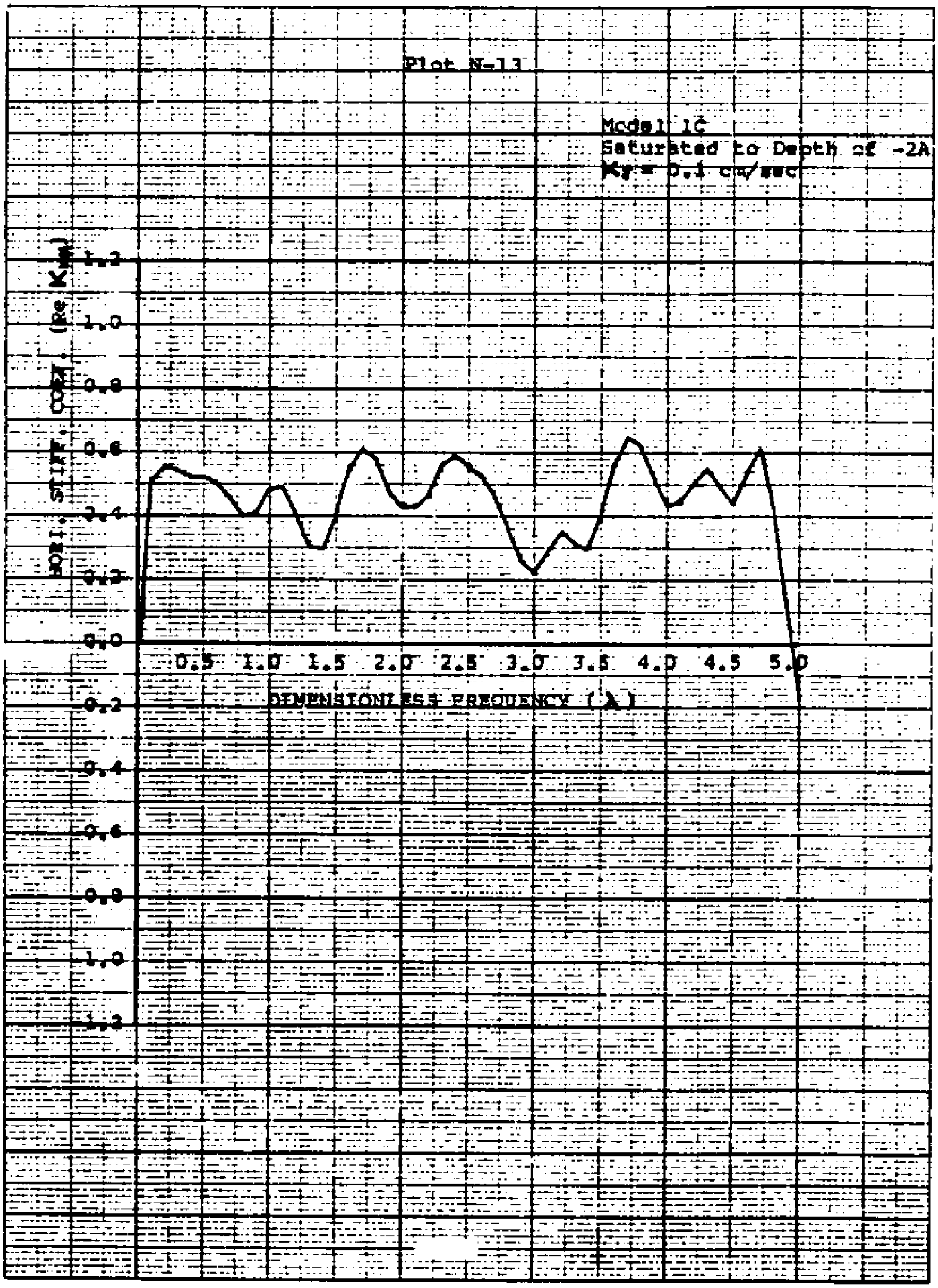

Plot N-8
Model 1A
Saturated to Depth of -½λ
Kf = 0.1 cm/sec
HORI. DAMP. COEF. (in K
0.0
0.5
1.0
1.5
2.0
2.5
3.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
5.0
DIMENSIONLESS FREQUENCY (λ)

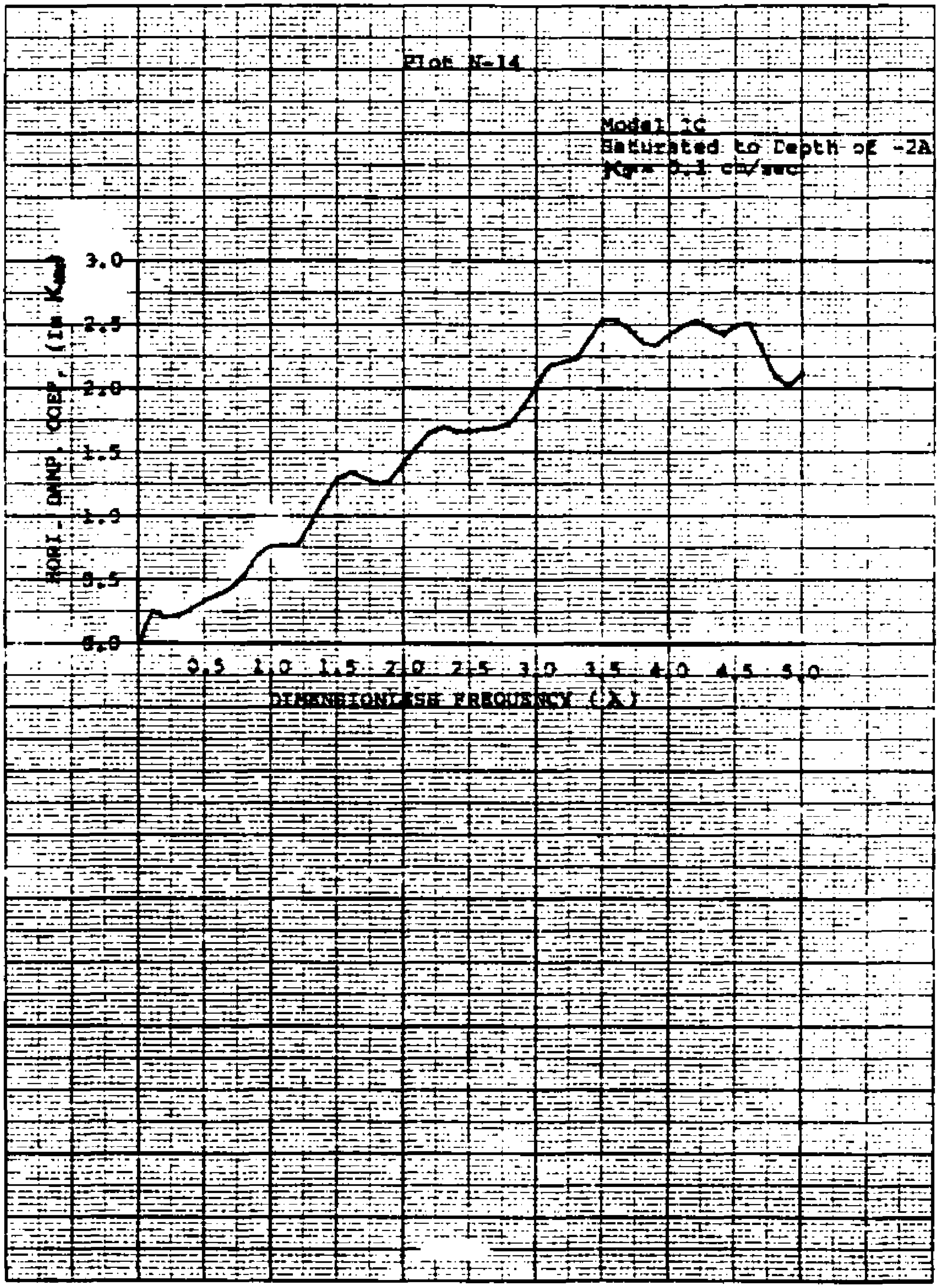

Plot N-9
Model 1B
Saturated to Depth of -A
Ky = 0.1 cm/sec
HORI. STIFF. COEF. (Re K_HH)
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
-0.2
-0.4
-0.6
-0.8
-1.0
-1.2
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
5.0
DIMENSIONLESS FREQUENCY (λ)

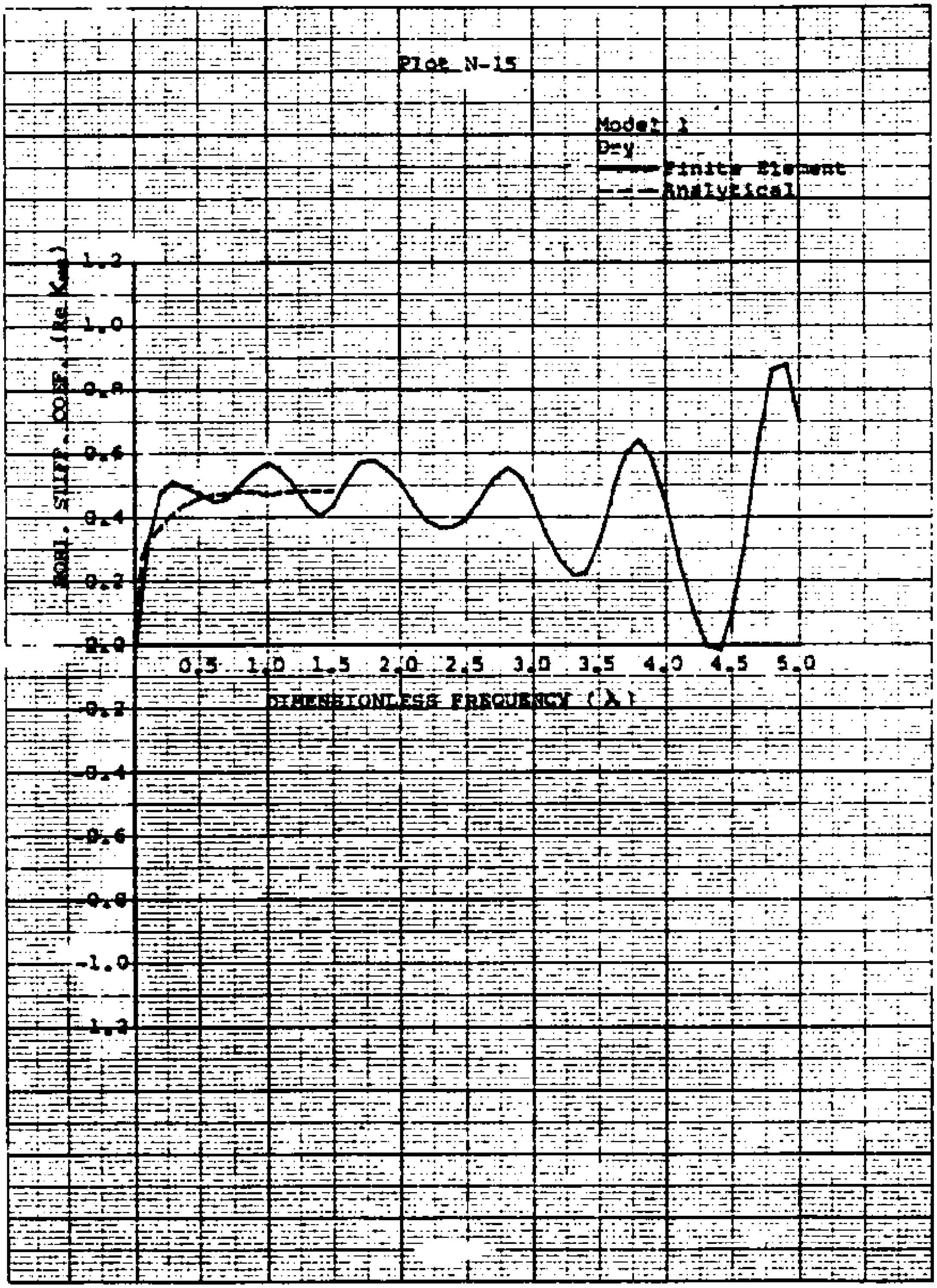

Plot N-10
Model IB
Saturated to Depth of -λ
K_s = 0.1 cm/sec
HORI. DAMP. COEF. (in K_{m})
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
5.0
DIMENSIONLESS FREQUANCY (λ)

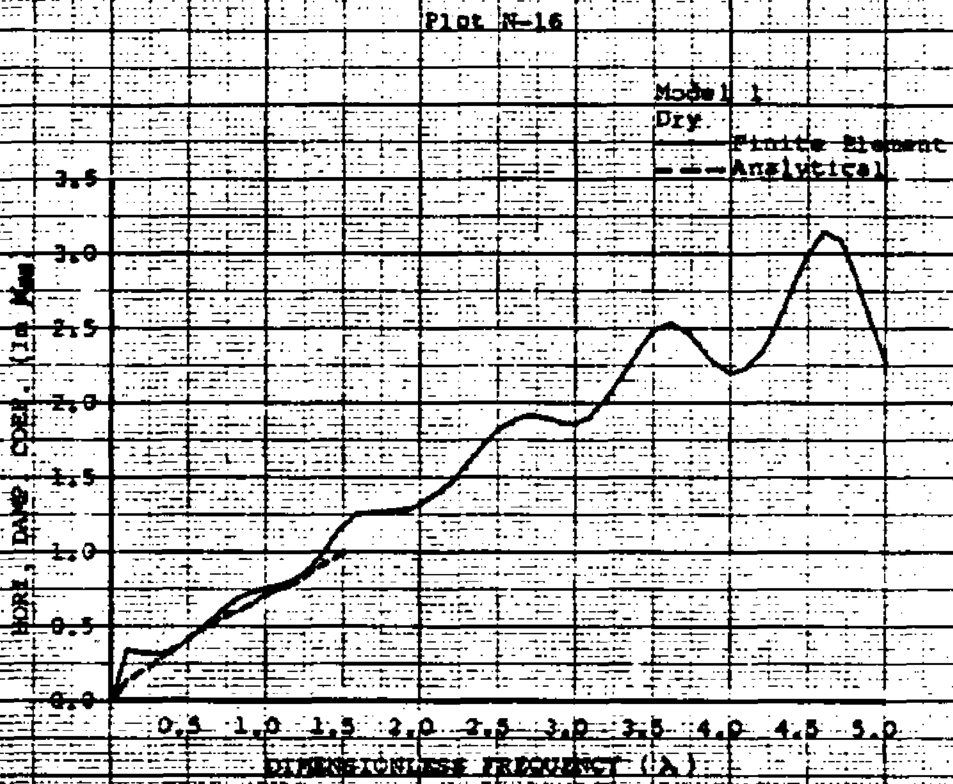

Plot N-11
Model 1F
Saturated to Depth of -1½A
K_s = 0.1 cm/sec
BURI. STIFF. COEF. (Re K_{uu})
1.2
1.0
0.8
0.6
0.4
0.2
0.0
-0.2
-0.4
-0.6
-0.8
-1.0
-1.2
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
5.0
DIMENSIONLESS FREQUENCY (λ)


Plot N-12
Model 1F
Saturated to Depth of -1½λ
K_z = 0.1 cm/sec
HORI. DAMP. COEF. (in K_{hh})
0.0
0.5
1.0
1.5
2.0
2.5
3.0
0.5 1.0 1.5 2.0 2.5 3.0 3.5 4.0 4.5 5.0
DIMENSIONLESS FREQUENCY (λ)

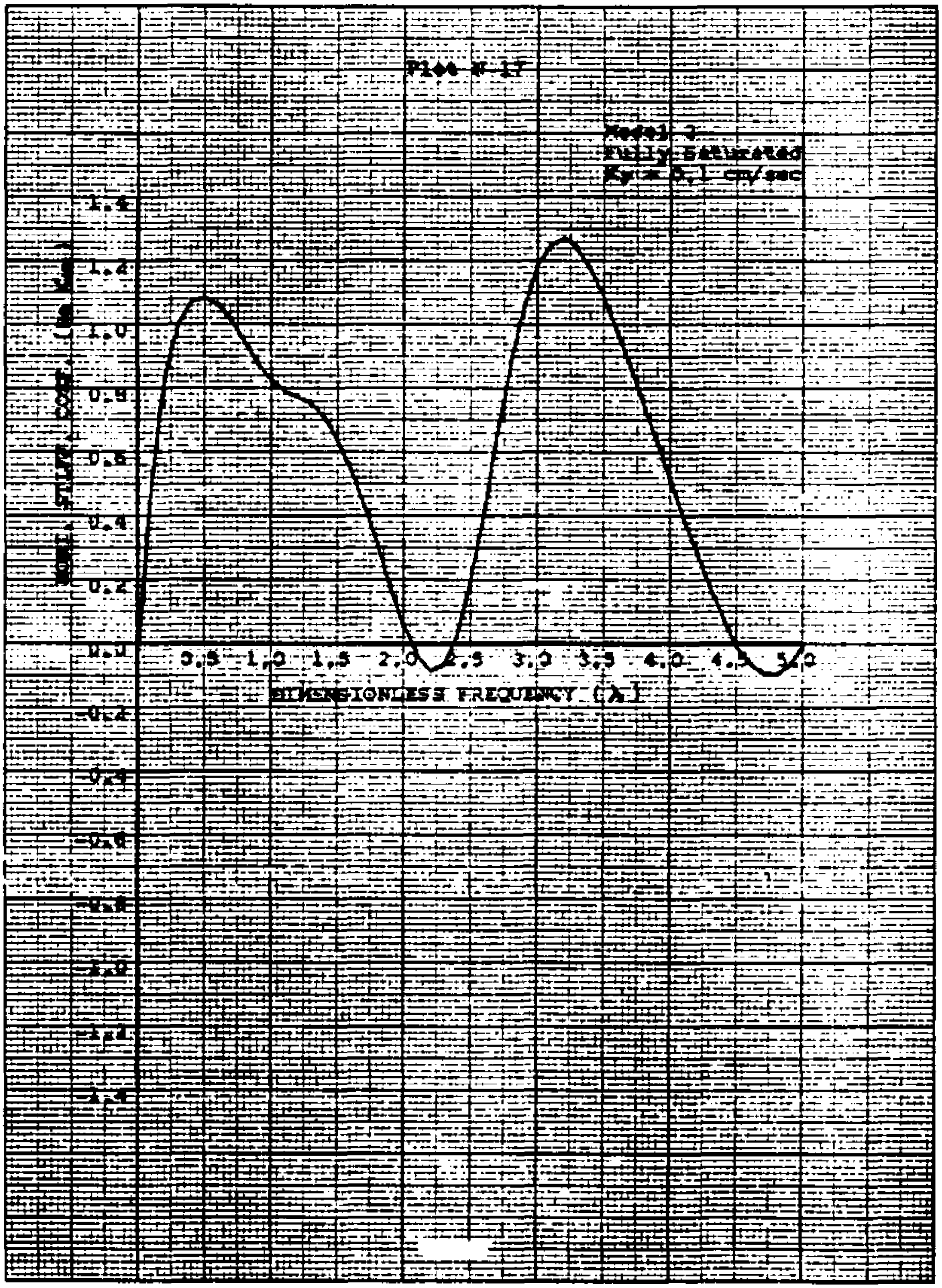

Plot N-13
Model 1C
Saturated to Depth of -2A
K_y = 0.1 cm/sec
HORI. STIFF. COEF. (Re K_{HH})
1.2
1.0
0.8
0.6
0.4
0.2
0.0
0.2
0.4
0.6
0.8
1.0
1.2
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
5.0
DIMENSIONLESS FREQUENCY (λ)

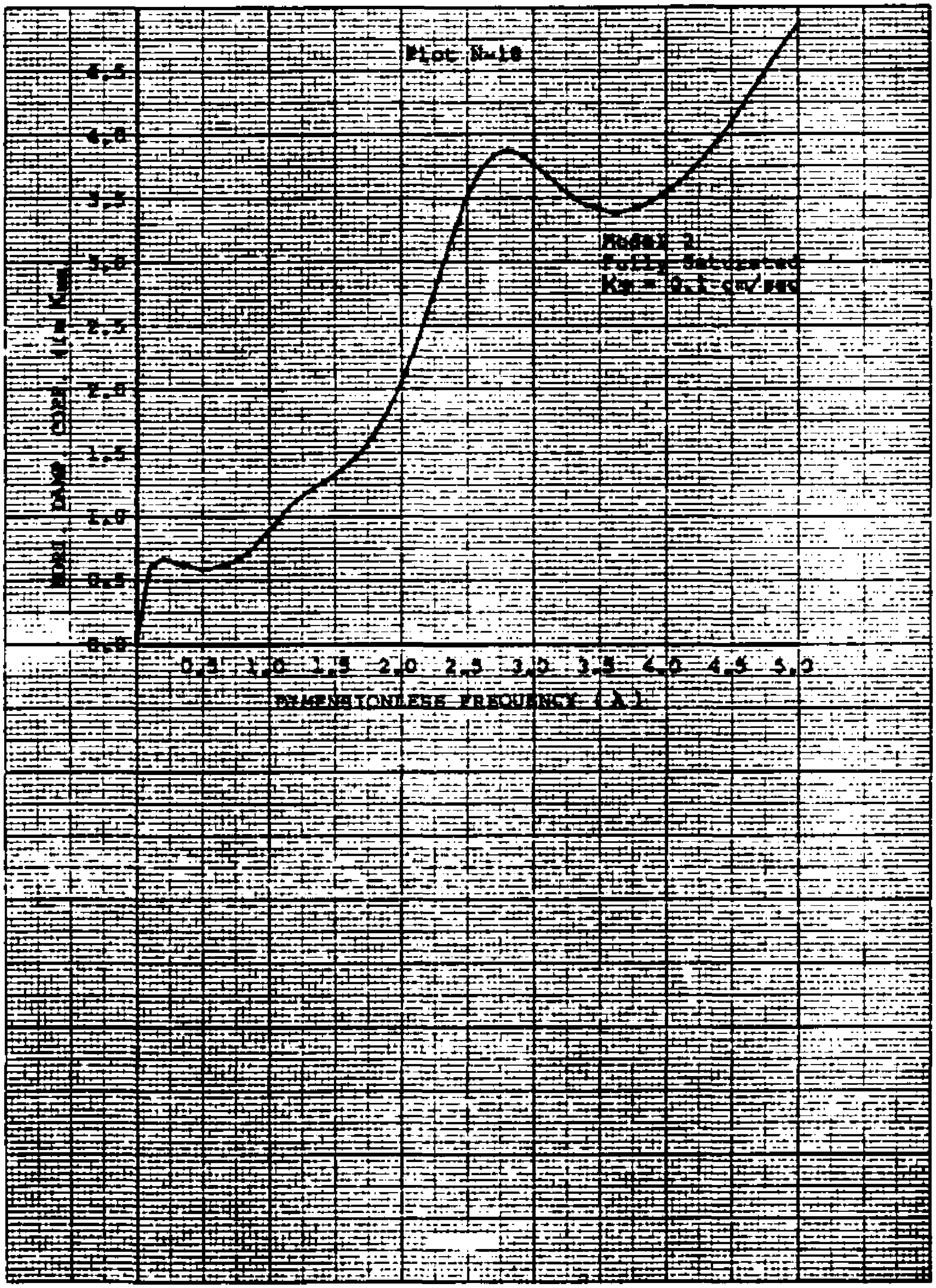

Plot N-14
Model IC
Saturated to Depth of -2A
K = 0.1 cm/sec
HORI. DAMP. COEF. (In K_{mm})
0.0
0.5
1.0
1.5
2.0
2.5
3.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
5.0
DIMENSIONLESS FREQUENCY (λ)

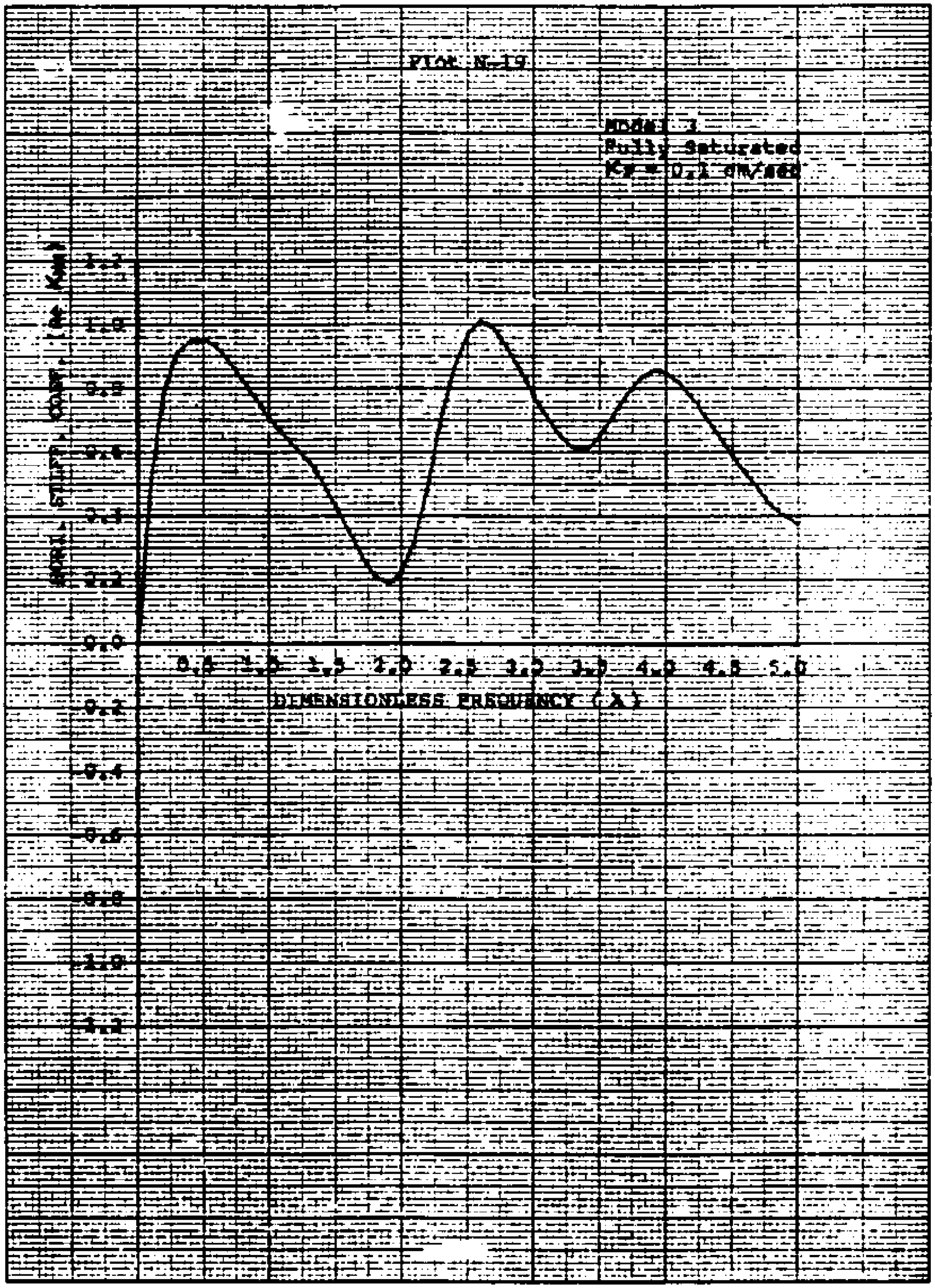

Plot N-15
Model 1
Dry
Finite Element
Analytical
HORI. STIFF. COEF. (Re K_HH)
1.2
1.0
0.8
0.6
0.4
0.2
0.0
-0.2
-0.4
-0.6
-0.8
-1.0
-1.2
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
5.0
DIMENSIONLESS FREQUENCY (λ)


Plot N-16
Model 1
Dry
Finite Element
Analytical
HORZ. DAMP. COEF. (in Mm)
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
5.0
DIMENSIONLESS FREQUENCY (λ)

K•E 20 X 20 TO THE INCH • 7 X 10 INCHES KEUFFEL & ESSER CO. MADE IN U.S.A. 46 1242




Plot N-18
HORI. DAMP. COEF. (K = Khh)
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
5.0
DIMENSIONLESS FREQUENCY (λ)
Model 2
Fully Saturated
Kw = 0.1 cm/sec


PLOT N-19
Model 3
Fully Saturated
Kw = 0.1 cm/sec
HORI. STIFF. COEF. (Re Khh)
1.2
1.0
0.8
0.6
0.4
0.2
0.0
-0.2
-0.4
-0.6
-0.8
-1.0
-1.2
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
5.0
DIMENSIONLESS FREQUENCY (λ)

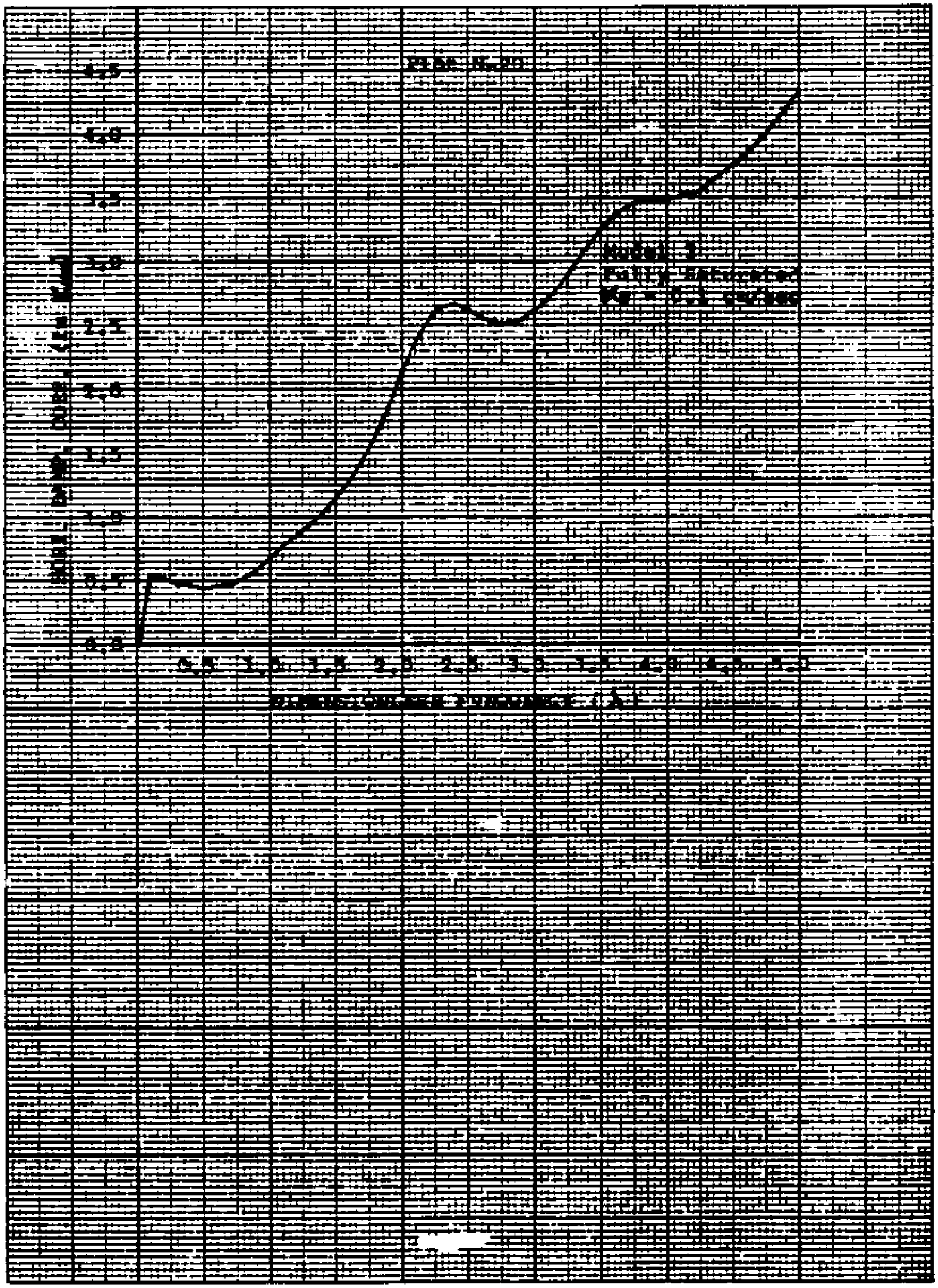
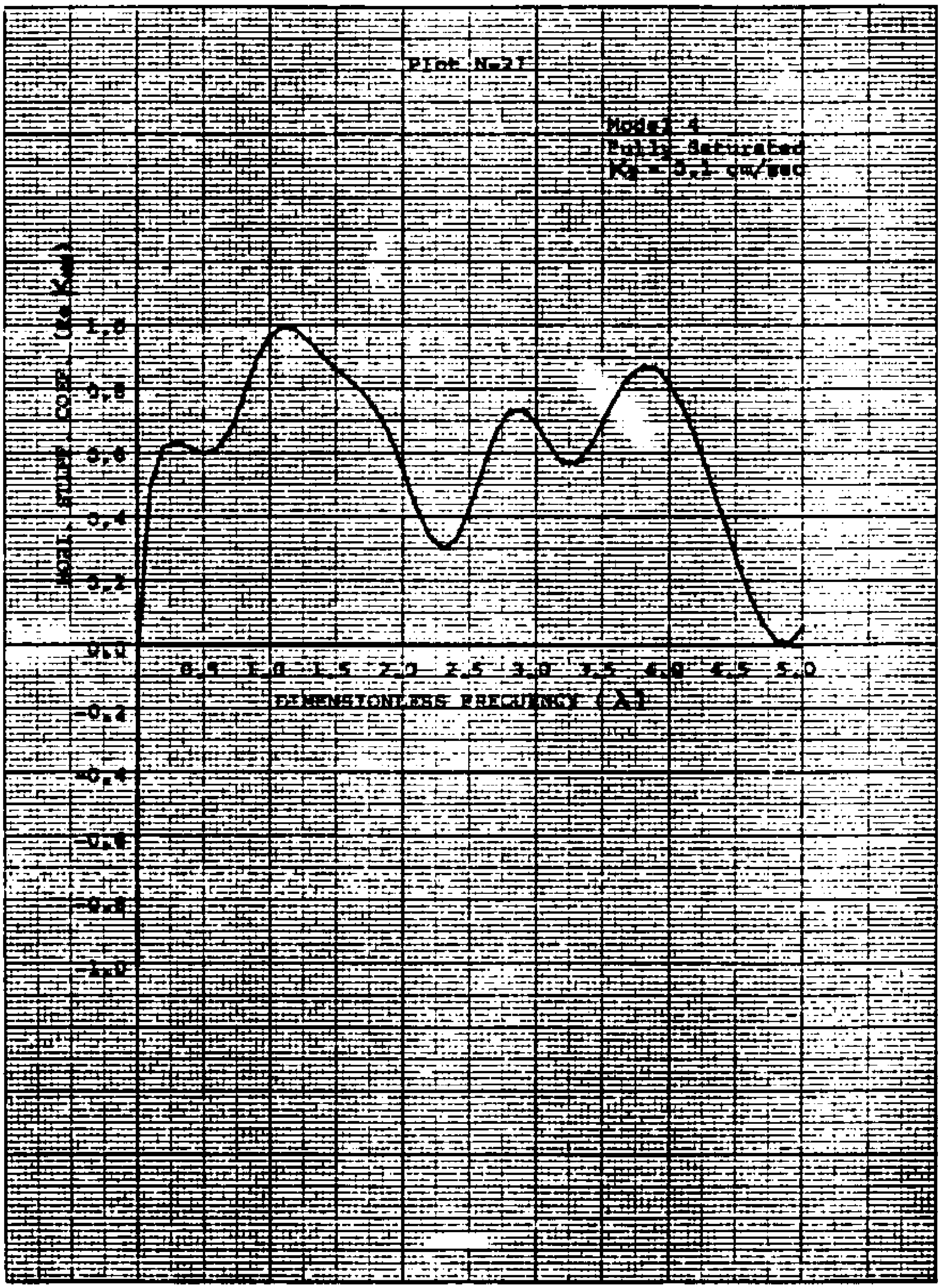


FIG. B.20
HORZ. DAMP. COEF.
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
5.0
DIMENSIONLESS FREQUENCY (A)
Model 1
Fully Saturated
Kr = 0.1 cm/sec


Plot N=27
Model 4
Fully Saturated
K_s = 3.1 cm/sec
NORM. STIFF. COEF. (Re Kmm)
1.0
0.8
0.6
0.4
0.2
0.0
-0.2
-0.4
-0.6
-0.8
-1.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
5.0
DIMENSIONLESS FREQUENCY (λ)

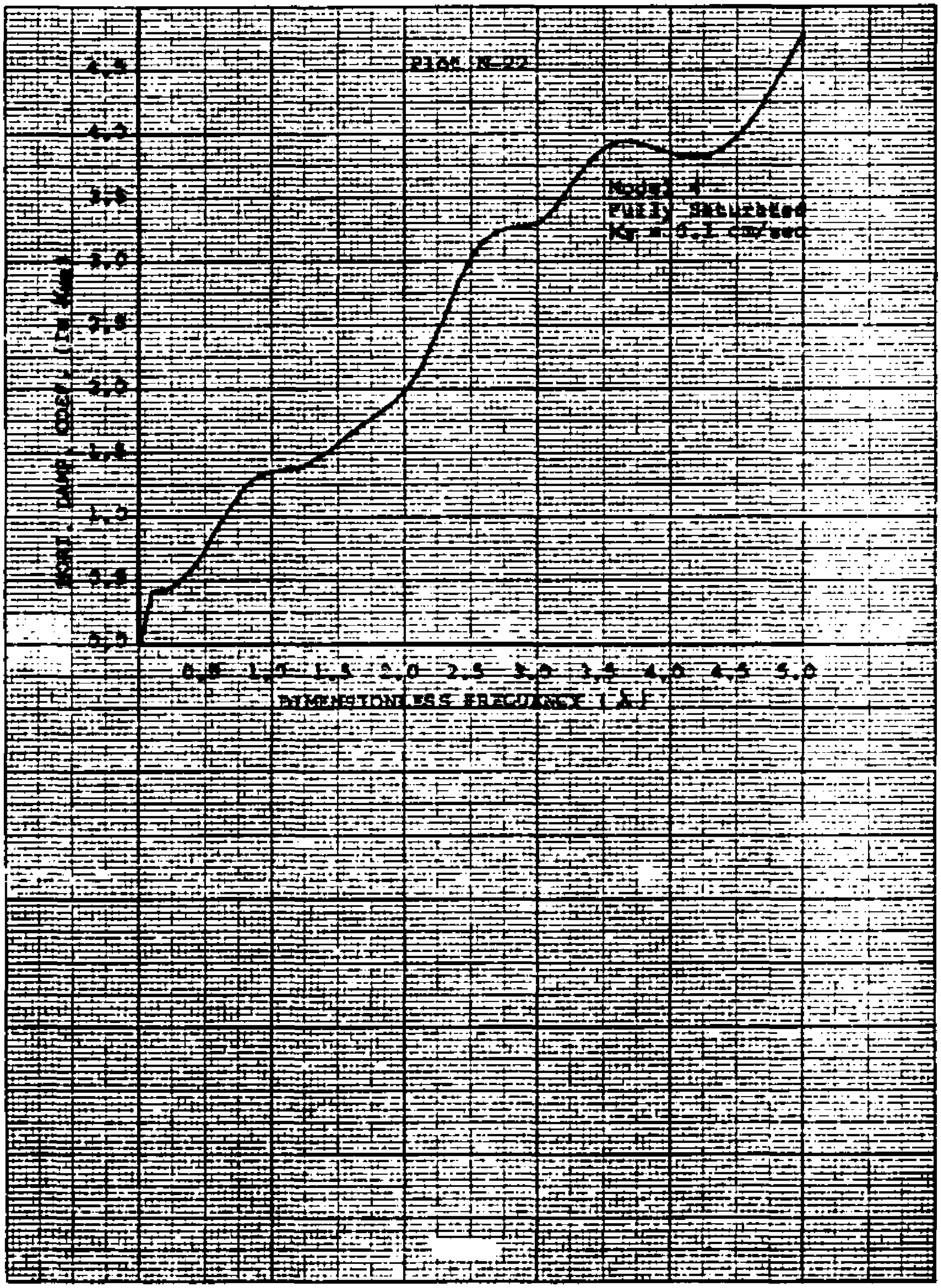
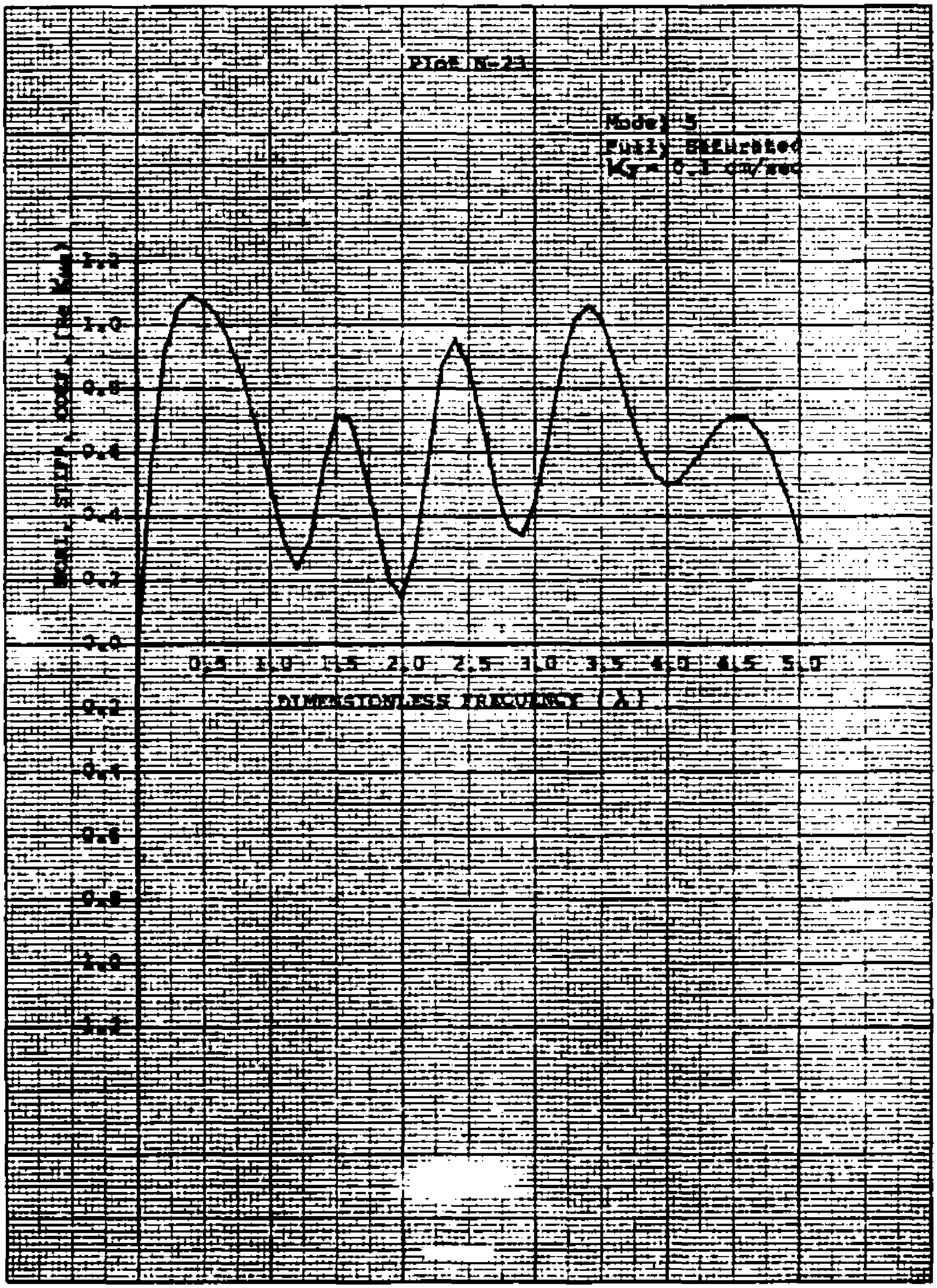
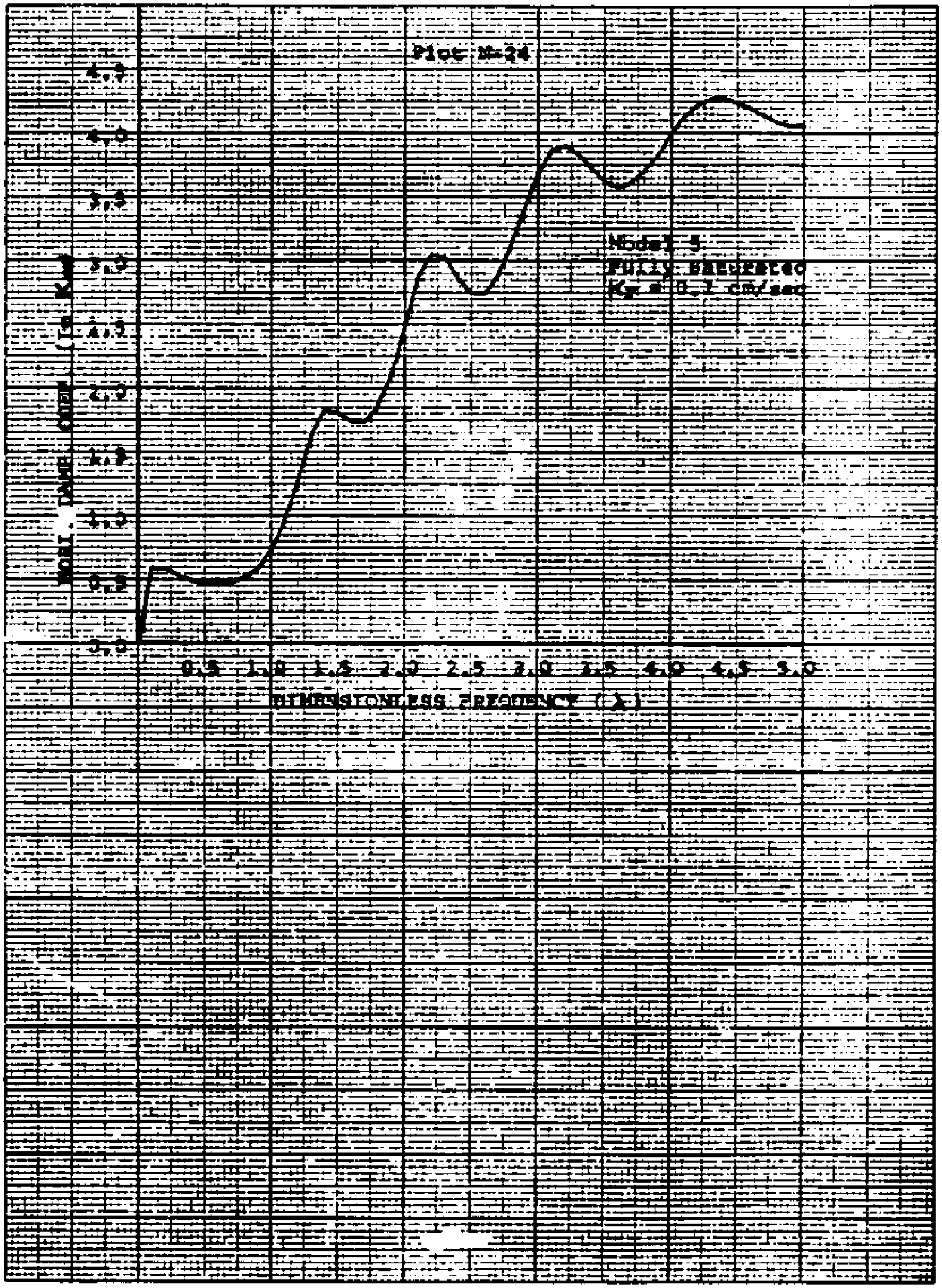


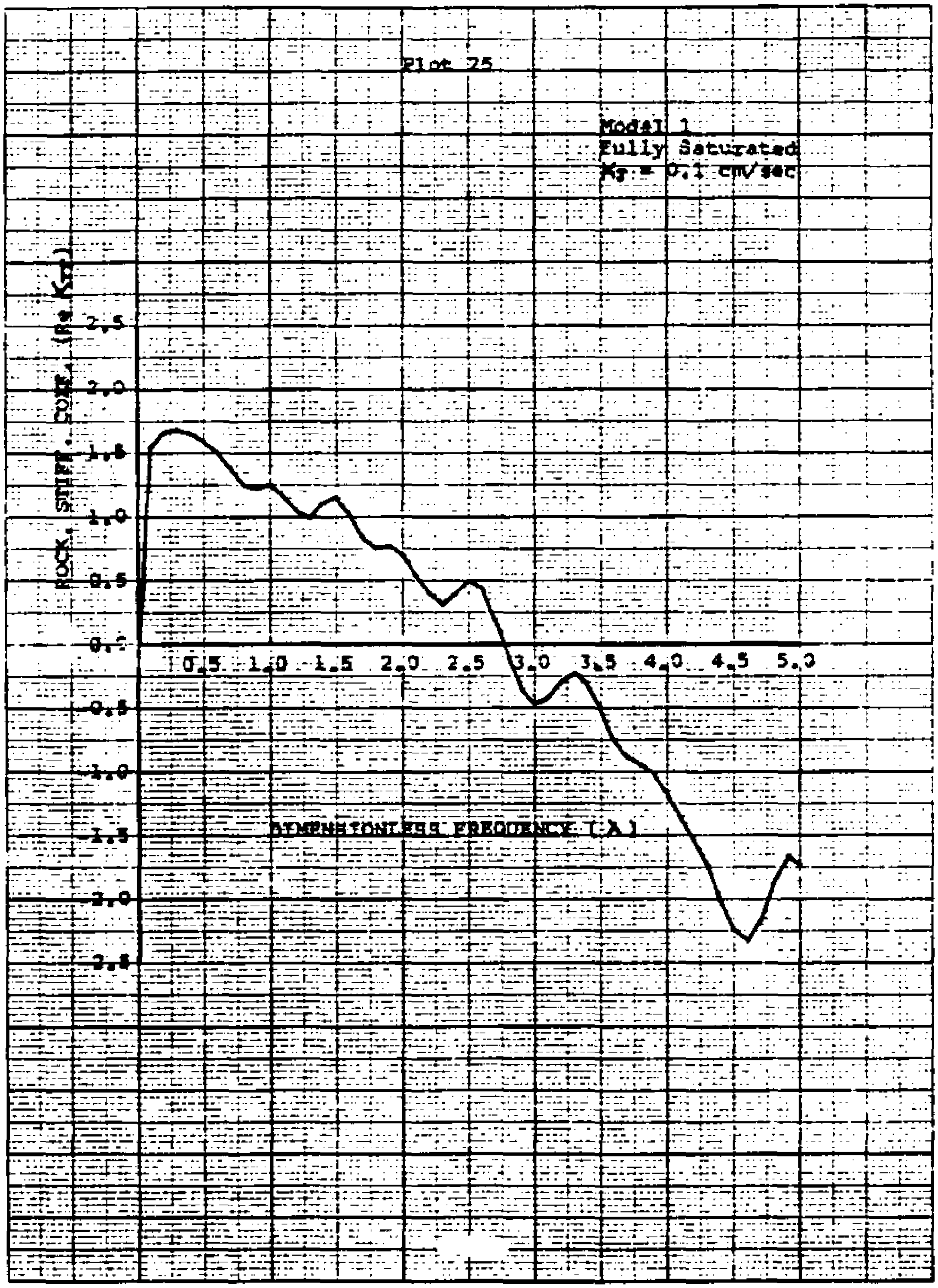
FIG. N-22
HORI. DAMP. COEF. (in Kam)
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
0.5 1.0 1.5 2.0 2.5 3.0 3.5 4.0 4.5 5.0
DIMENSIONLESS FREQUENCY (λ)
Model 4
Fully Saturated
Kr = 0.1 cm/sec

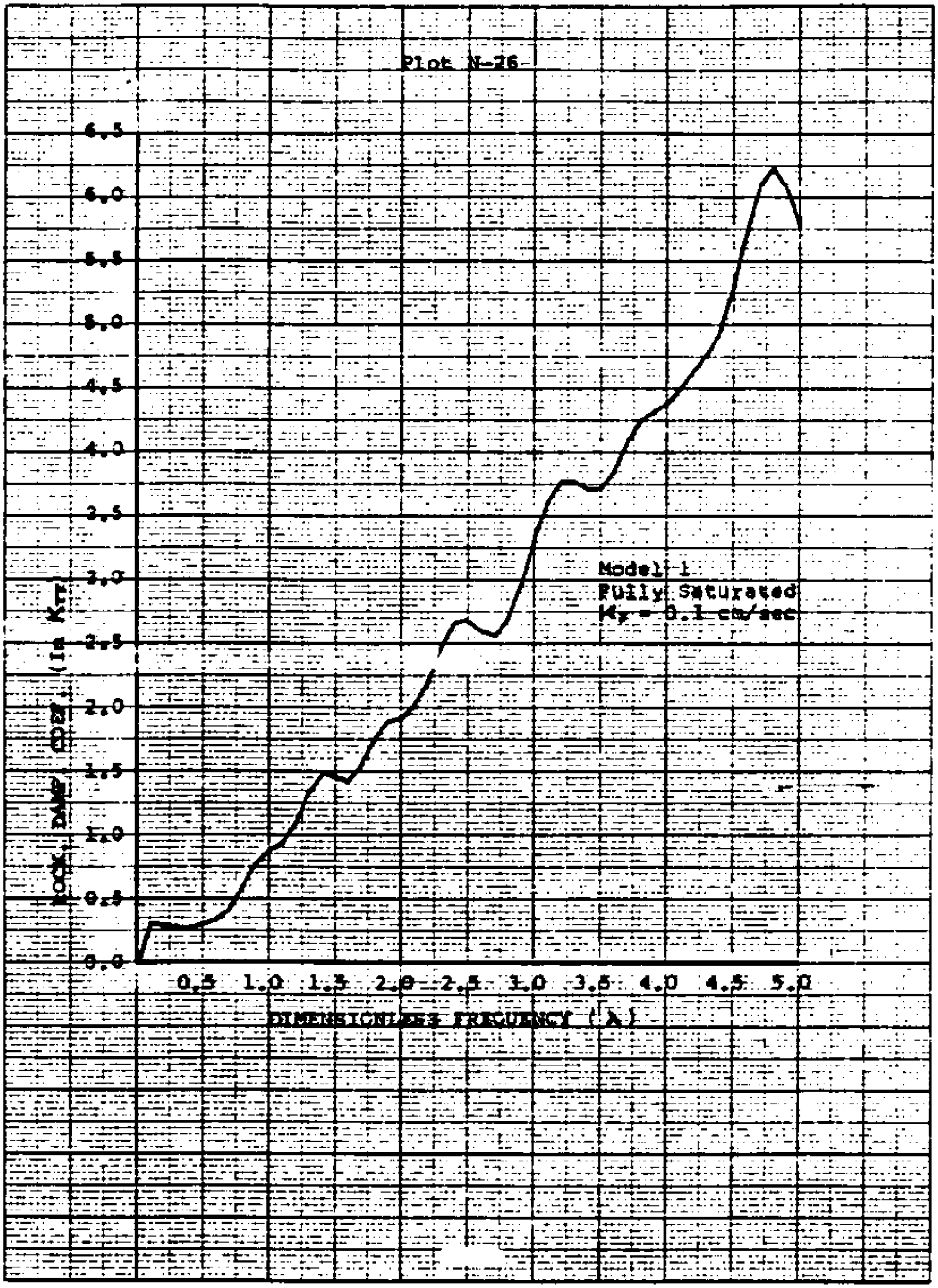

Plot N-23
Model 5
Fully Saturated
Kf = 0.3 cm/sec
NORL. STIFF. COEF. (Re Kmm)
1.2
1.0
0.8
0.6
0.4
0.2
0.0
0.2
0.4
0.6
0.8
1.0
1.2
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
5.0
DIMENSIONLESS FREQUENCY (λ)

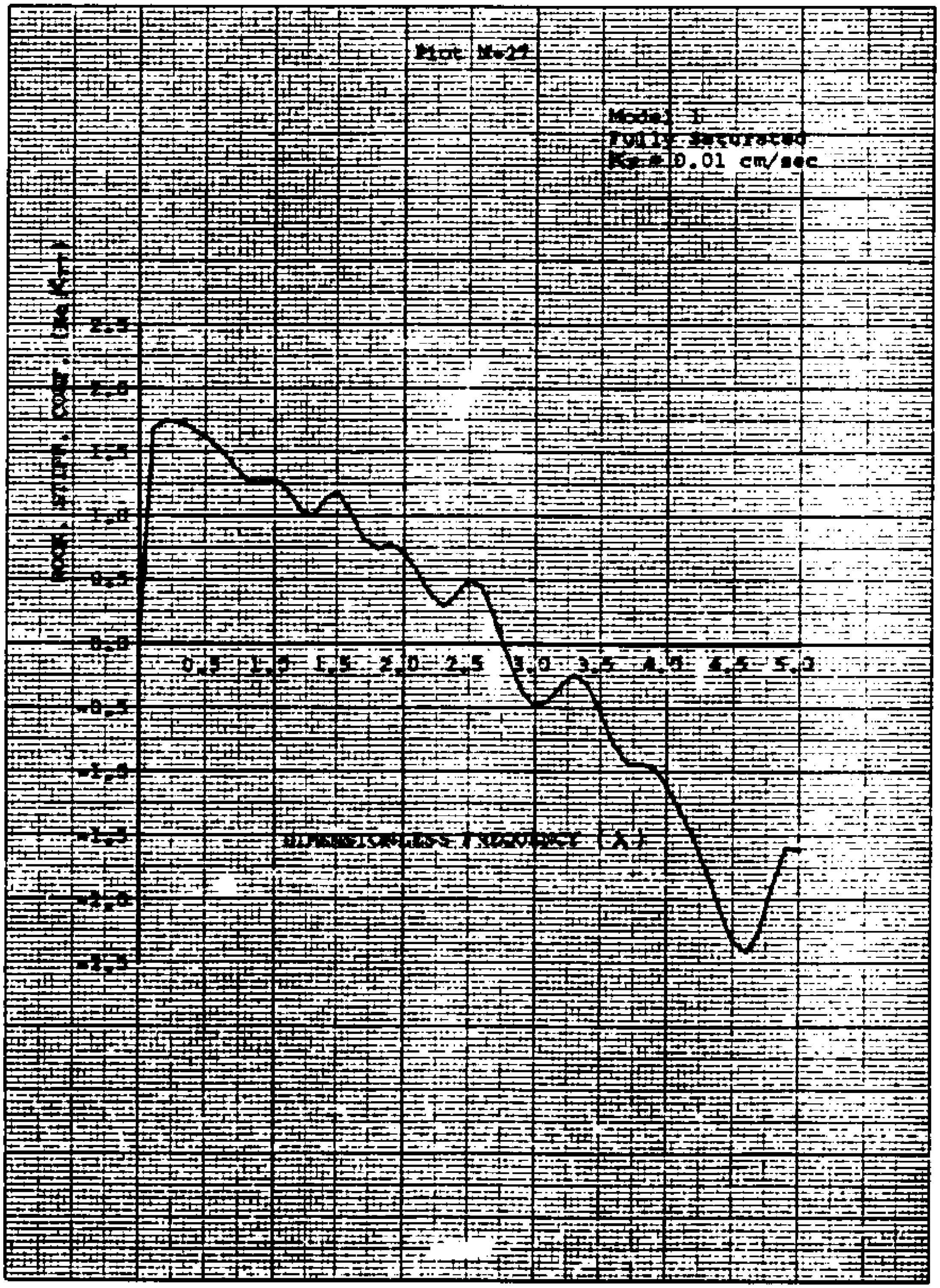
K+E [illegible]

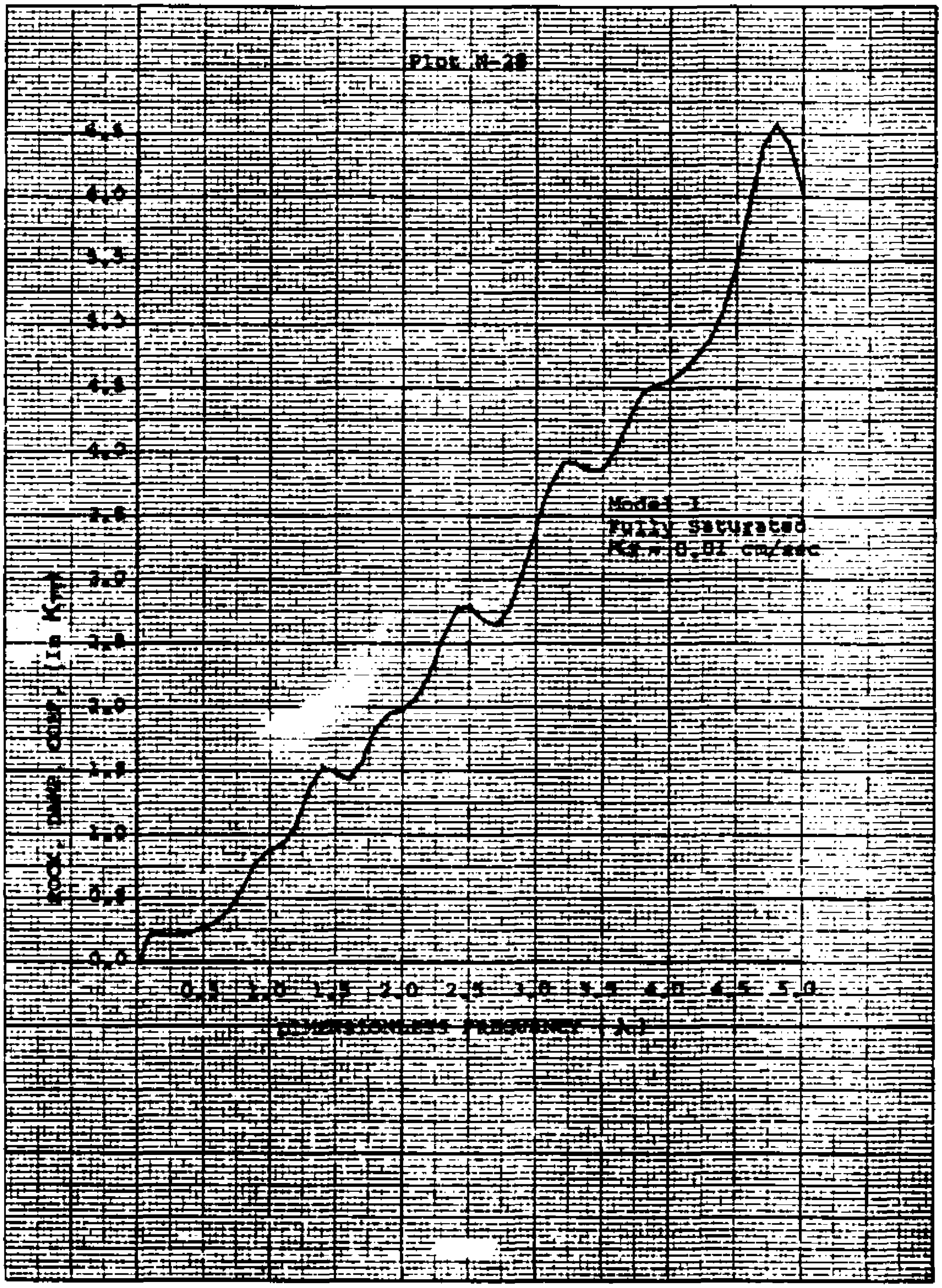
46 1242

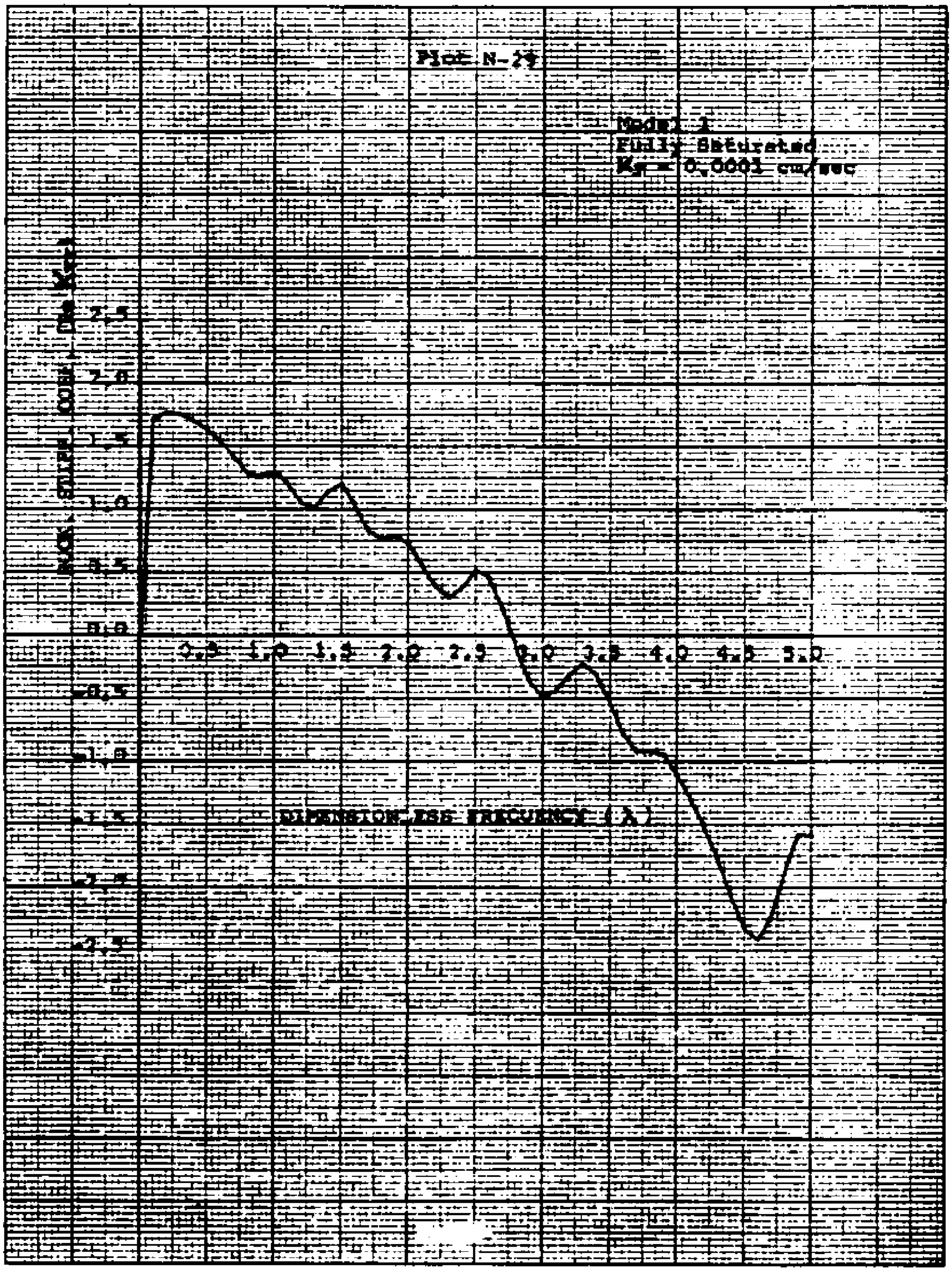

Plot M-24
HORI. DAMP. COEF. (in Kh*)
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
0.5 1.0 1.5 2.0 2.5 3.0 3.5 4.0 4.5 5.0
DIMENSIONLESS FREQUENCY (λ)
Model 5
Fully saturated
Kr = 0.1 cm/sec


Plot 25
Model 1
Fully Saturated
K_f = 0.1 cm/sec
ROCK. STIFF. COEF. (Re K_{rr})
2.5
2.0
1.5
1.0
0.5
0.0
-0.5
-1.0
-1.5
-2.0
-2.5
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
5.0
DIMENSIONLESS FREQUENCY (λ)


Plot N-26
ROCK. DAMP. COEF. (In K_{TT})
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
5.0
5.5
6.0
6.5
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
5.0
DIMENSIONLESS FREQUENCY (λ)
Model 1
Fully Saturated
K_y = 0.1 cm/sec


Plot M-27
Model I
Fully saturated
K_s = 0.01 cm/sec
ROCK. STIFF. COEF. (Re K_{rr})
2.5
2.0
1.5
1.0
0.5
0.0
-0.5
-1.0
-1.5
-2.0
-2.5
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
5.0
DIMENSIONLESS FREQUENCY (λ)


Plot M-28
ROCK. DAMP. COEF. (In K_m)
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
5.0
5.5
6.0
6.5
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
5.0
DIMENSIONLESS FREQUENCY (λ)
Model 1
Fully Saturated
Kd = 0.01 cm/sec



Plot N-29
Model 1
Fully Saturated
Kf = 0.0001 cm/sec
ROCK. STIFF. COEF. (Re Kr)
2.5
2.0
1.5
1.0
0.5
0.0
-0.5
-1.0
-1.5
-2.0
-2.5
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
5.0
DIMENSIONLESS FREQUENCY (λ)

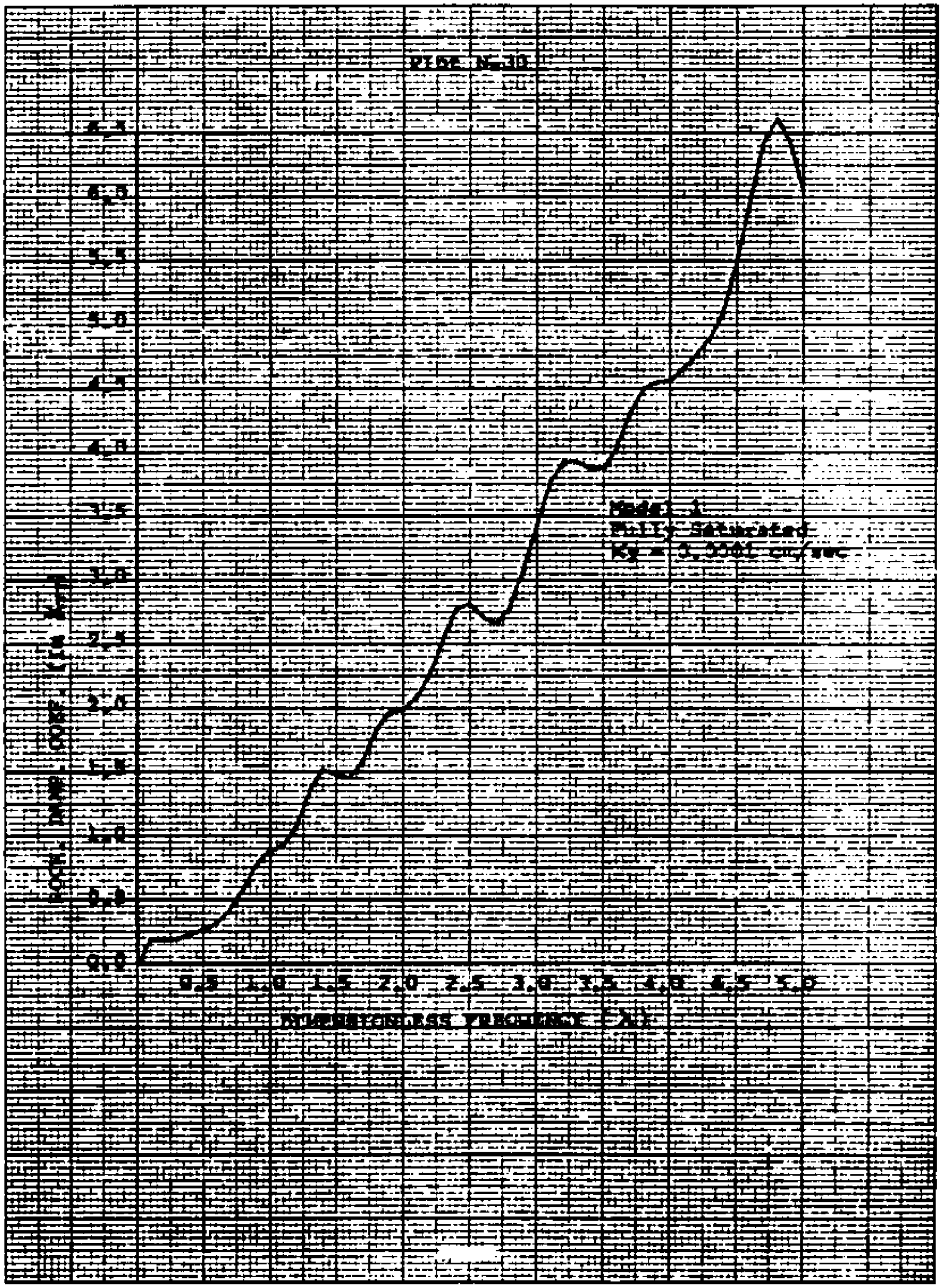
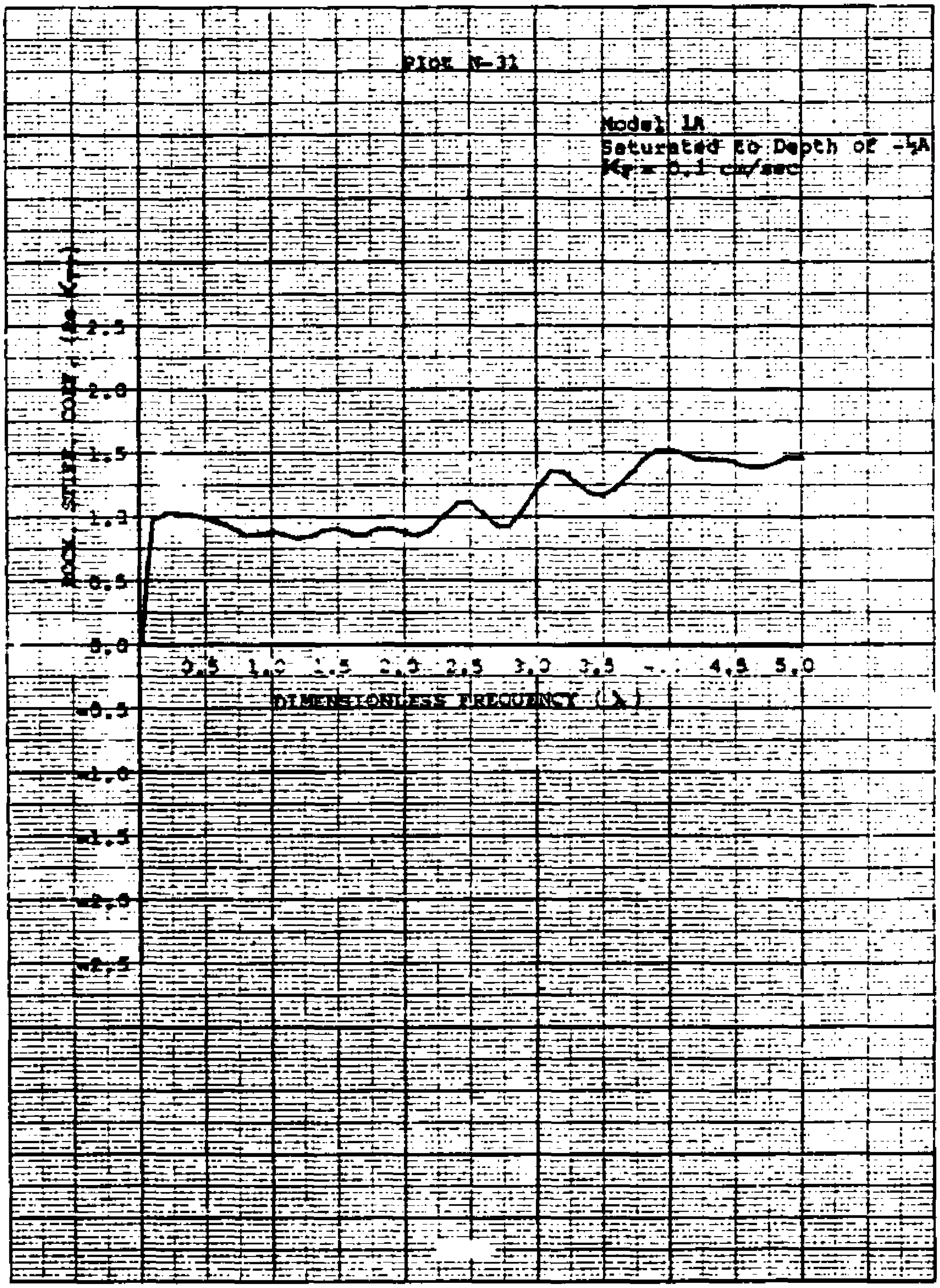
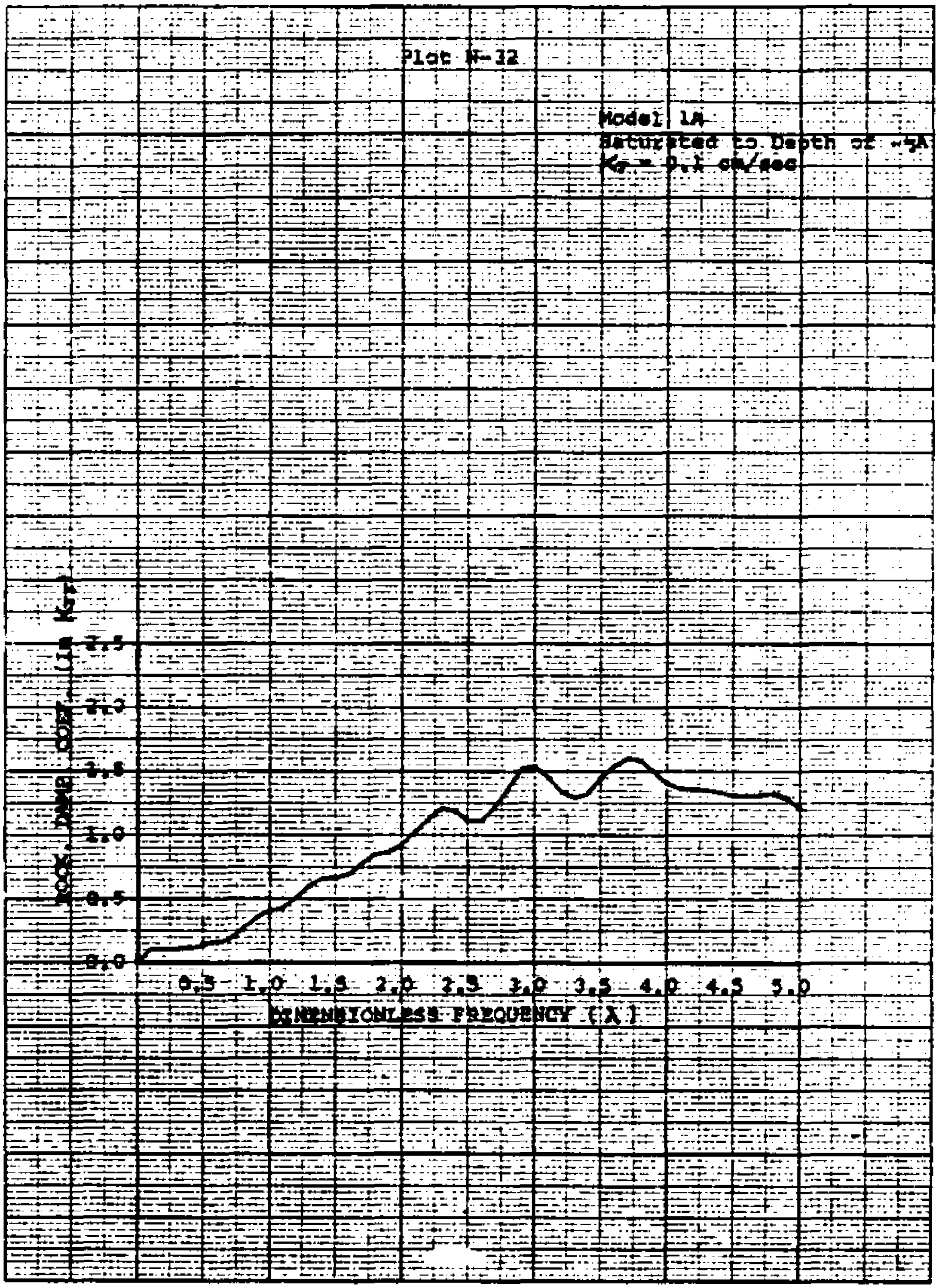
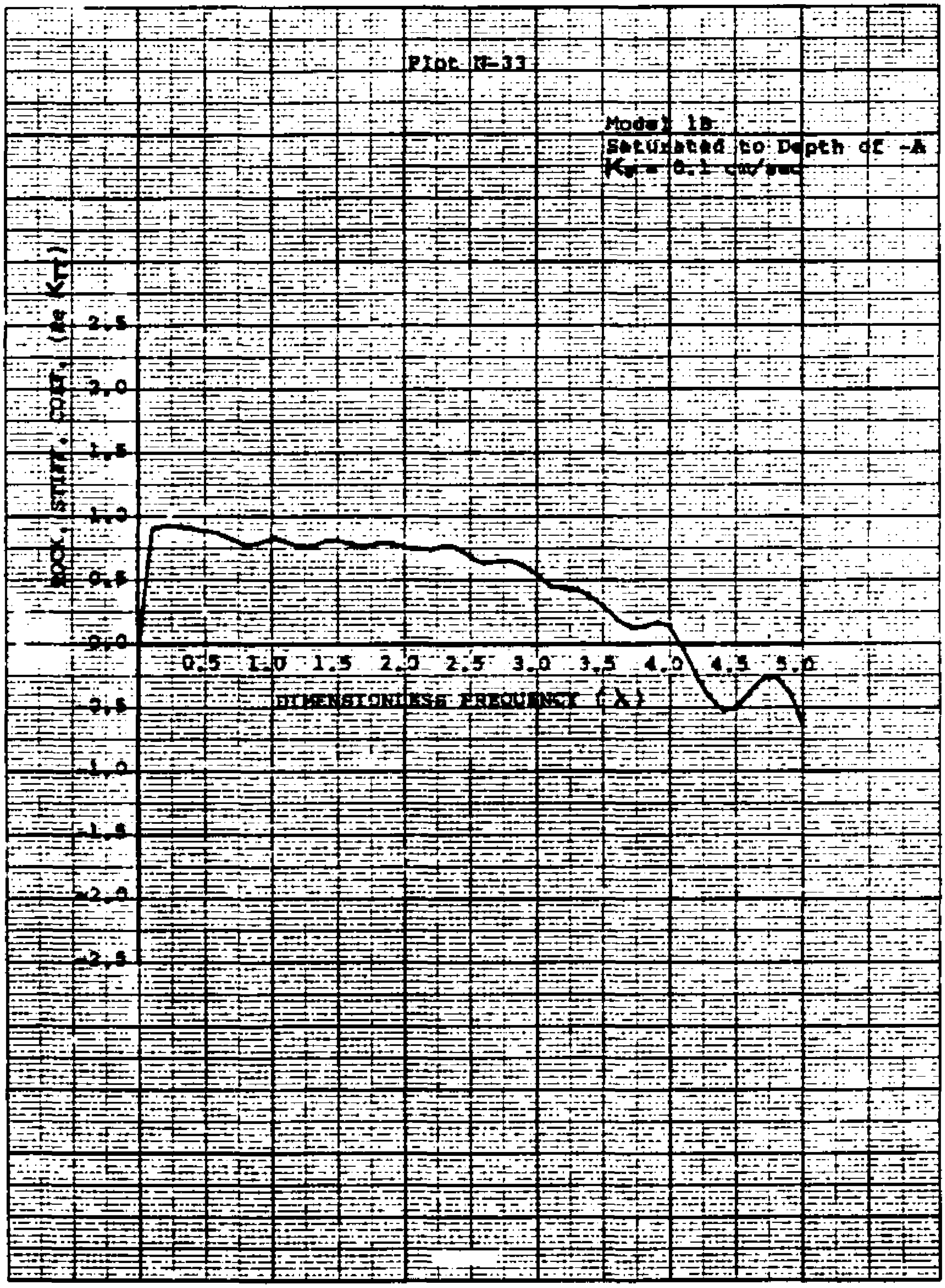
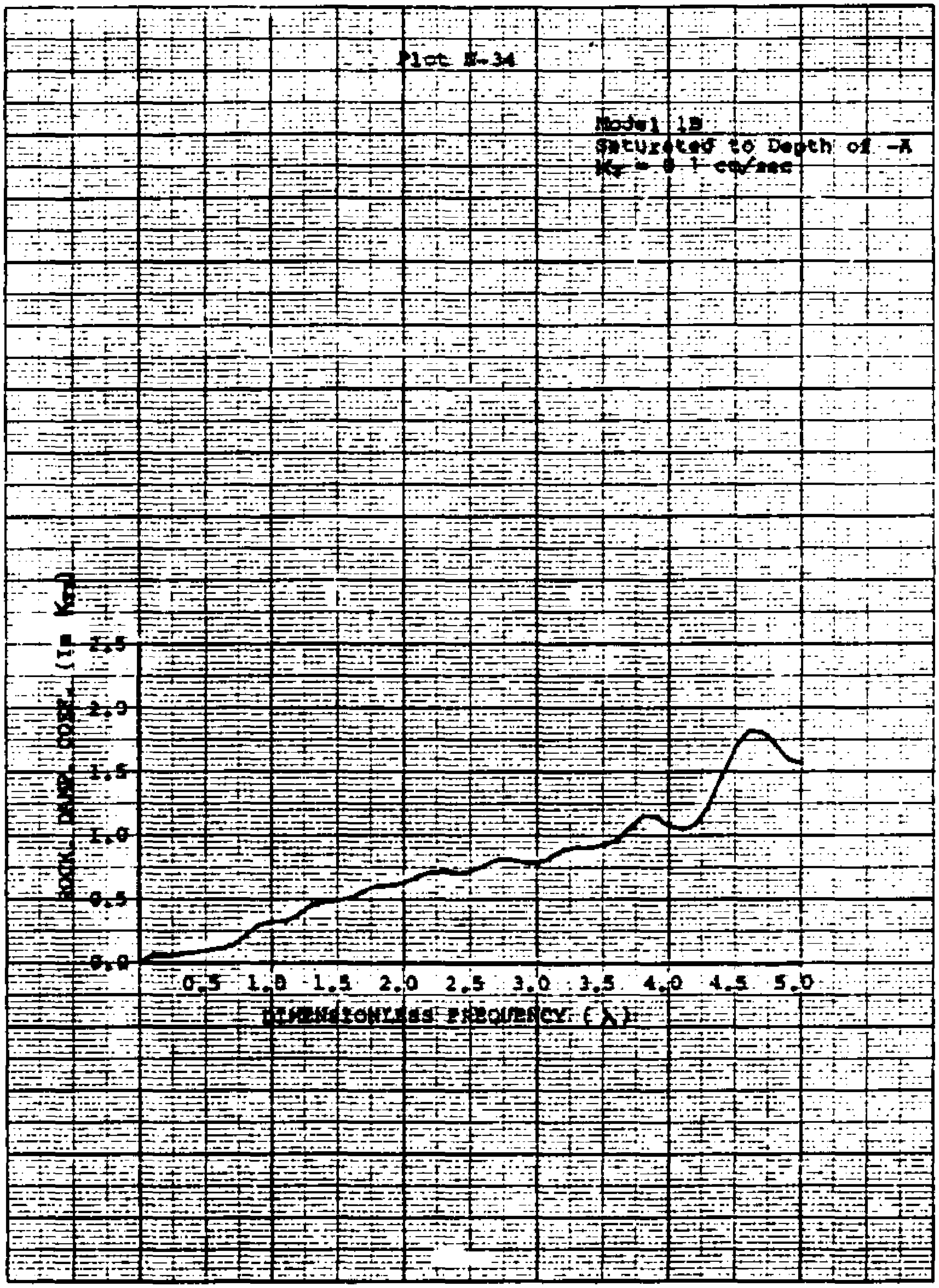

PILE No.10
ROCK. DAMP. COEF. (in K_r)
6.5
6.0
5.5
5.0
4.5
4.0
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.0
0.5 1.0 1.5 2.0 2.5 3.0 3.5 4.0 4.5 5.0
DIMENSIONLESS FREQUENCY (A)
Model 1
Fully Saturated
Ky = 3.3301 cm/sec

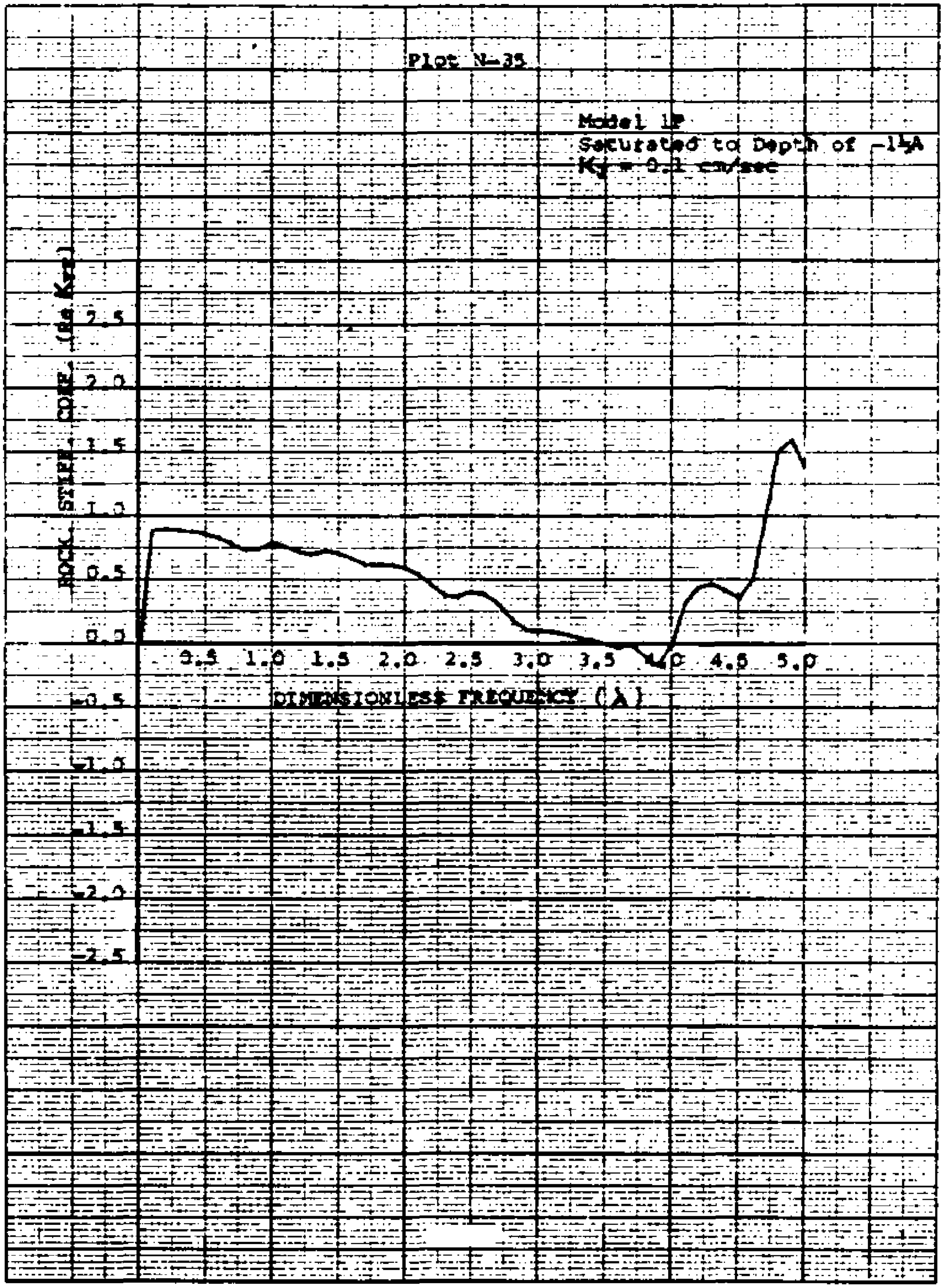
FIG. N-31

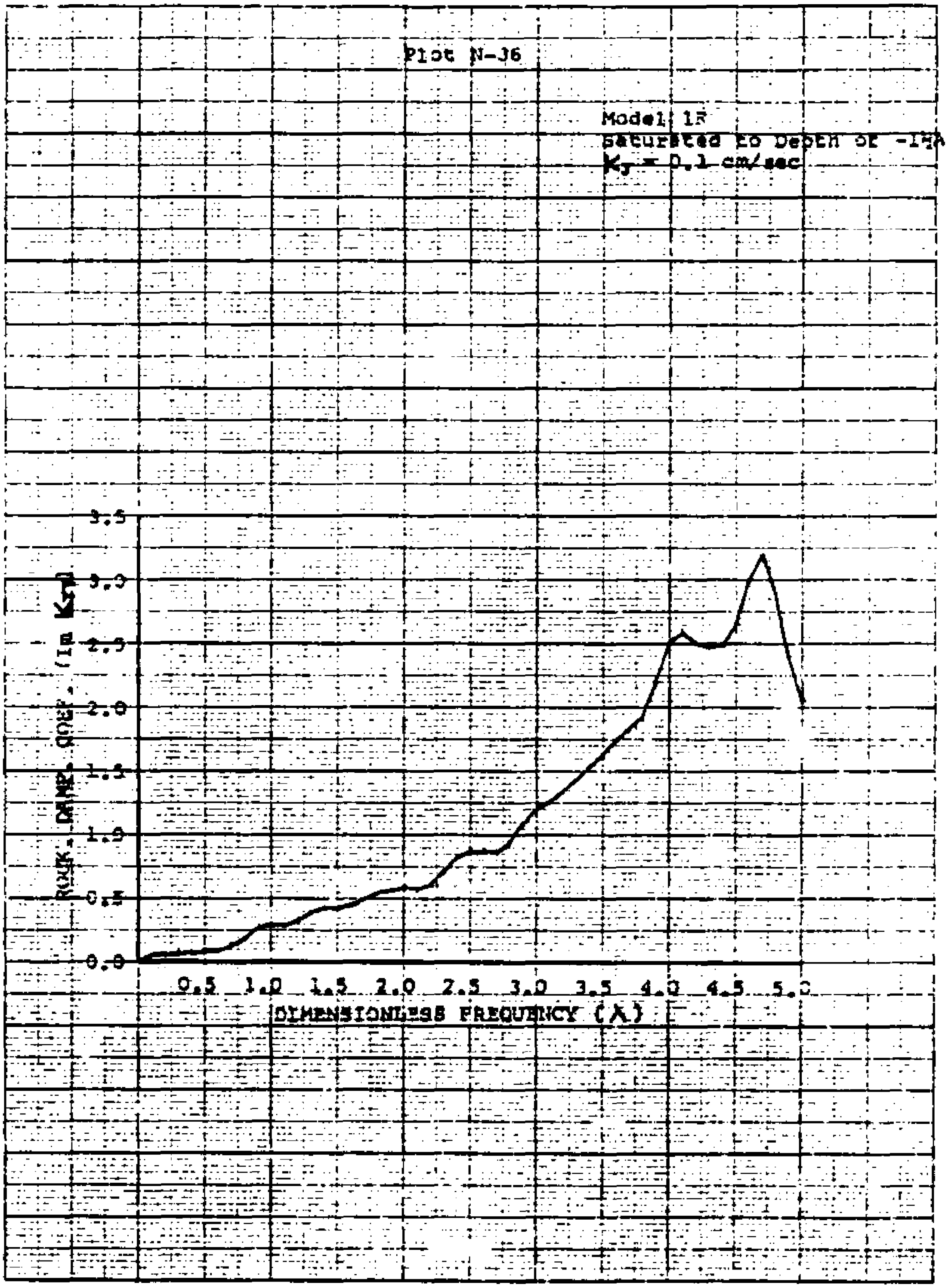

Model 1A
Saturated to Depth of -¼A
K_f = 0.1 cm/sec
ROCK. STIFF. COEF. (Re K_{TT})
2.5
2.0
1.5
1.0
0.5
0.0
-0.5
-1.0
-1.5
-2.0
-2.5
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
5.0
DIMENSIONLESS FREQUENCY (λ)

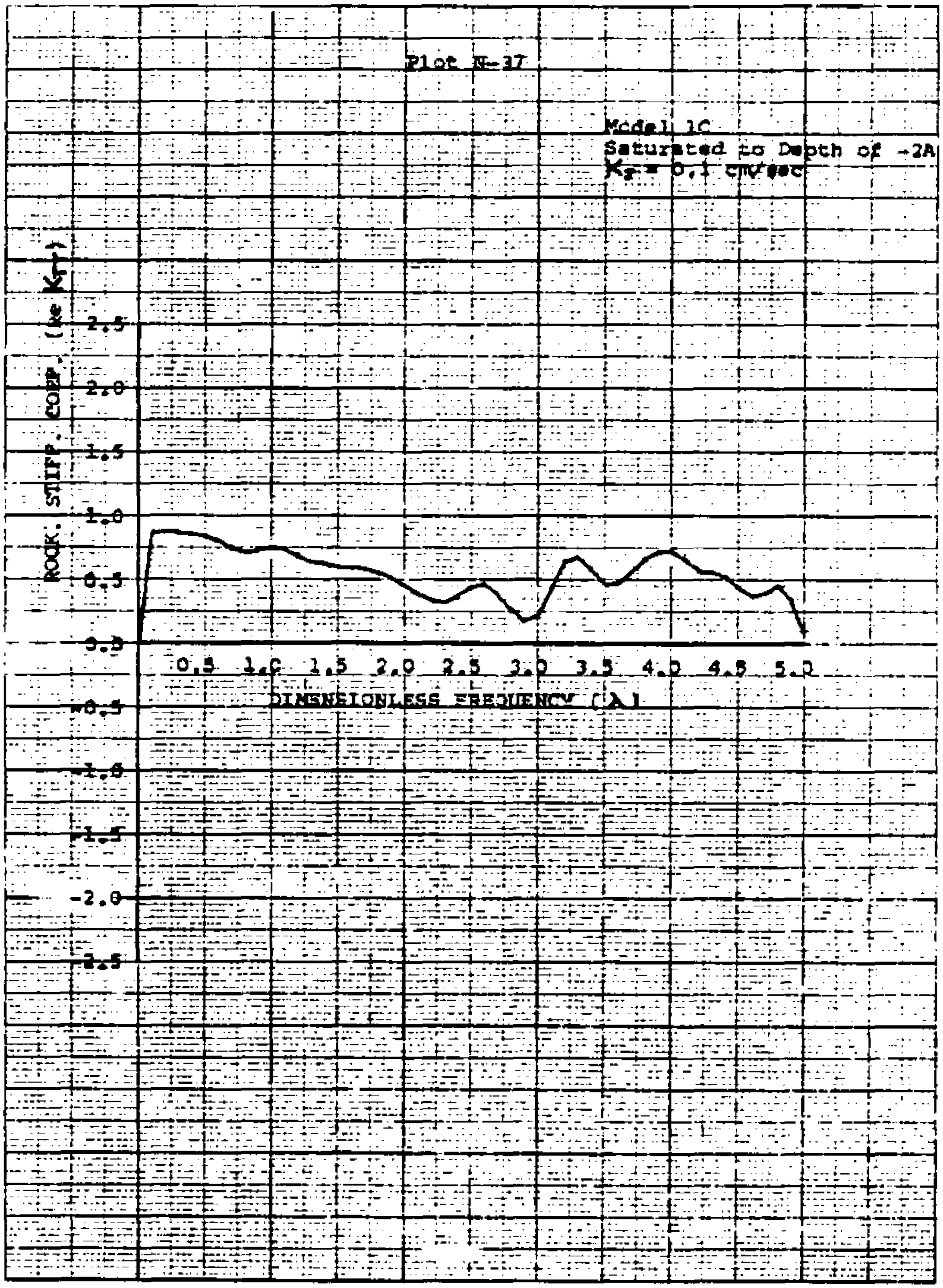

Plot N-32
Model 1A
Saturated to Depth of ~.5A
K_σ = 0.1 cm/sec
ROCK. DAMP COEF. (in K_ST)
0.0
0.5
1.0
1.5
2.0
2.5
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
5.0
DIMENSIONLESS FREQUENCY (λ)


Plot IV-33
Model 1B
Saturated to Depth of -A
K_s = 0.1 cm/sec
ROCK. STIFF. COEF. (Re K_{TT})
2.5
2.0
1.5
1.0
0.5
0.0
-0.5
-1.0
-1.5
-2.0
-2.5
0.5 1.0 1.5 2.0 2.5 3.0 3.5 4.0 4.5 5.0
DIMENSIONLESS FREQUENCY (λ)


Plot B-34
Model 1B
Saturated to Depth of -λ
Kr = 0.1 cm/sec
ROCK. DAMP. COEF. (Im Krr)
0.0
0.5
1.0
1.5
2.0
2.5
0.5 1.0 1.5 2.0 2.5 3.0 3.5 4.0 4.5 5.0
DIMENSIONLESS FREQUENCY (λ)

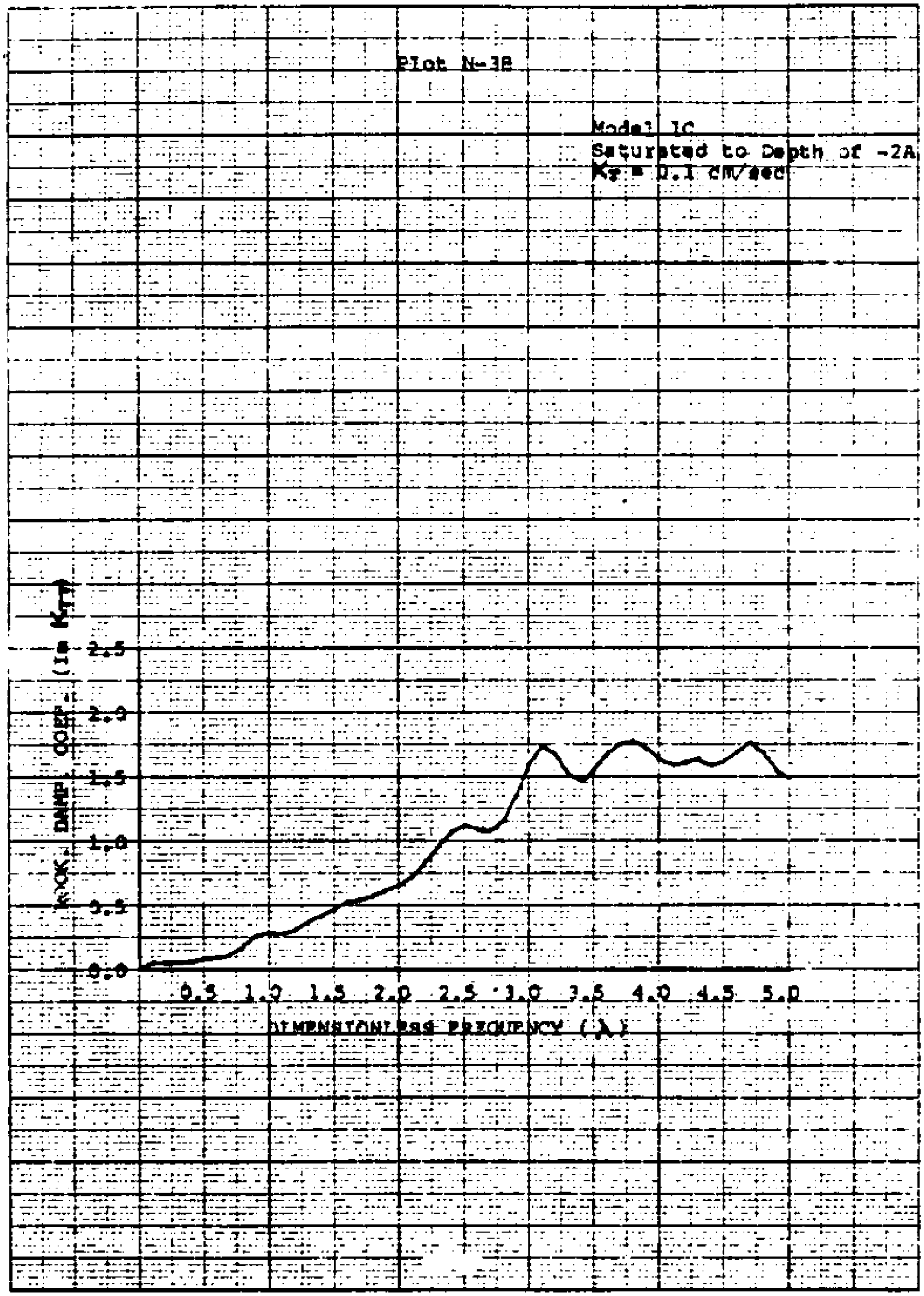

Plot N-35
Model 1P
Saturated to Depth of 1½A
Kf = 0.1 cm/sec
ROCK. STIFF. COEF. (Re Krr)
2.5
2.0
1.5
1.0
0.5
0.0
-0.5
-1.0
-1.5
-2.0
-2.5
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
5.0
DIMENSIONLESS FREQUENCY (λ)


Plot N-36
Model 1F
Saturated to Depth of -1½A
K_J = 0.1 cm/sec
ROCK. DAMP. COEF. (Im K_r)
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
5.0
DIMENSIONLESS FREQUENCY (λ)



Plot N-37

Model 1C
Saturated to Depth of -2A
K_s = 0.1 cm/sec
ROCK. STIFF. COEF. (re K_{rr})
2.5
2.0
1.5
1.0
0.5
0.0
-0.5
-1.0
-1.5
-2.0
-2.5
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
5.0
DIMENSIONLESS FREQUENCY (λ)


Plot N-3B
Model IC
Saturated to Depth of -2A
K_z = 0.1 cm/sec
ROCK. DAMP. COEF. (Im K_{TT})
0.0
0.5
1.0
1.5
2.0
2.5
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
5.0
DIMENSIONLESS FREQUENCY (λ)

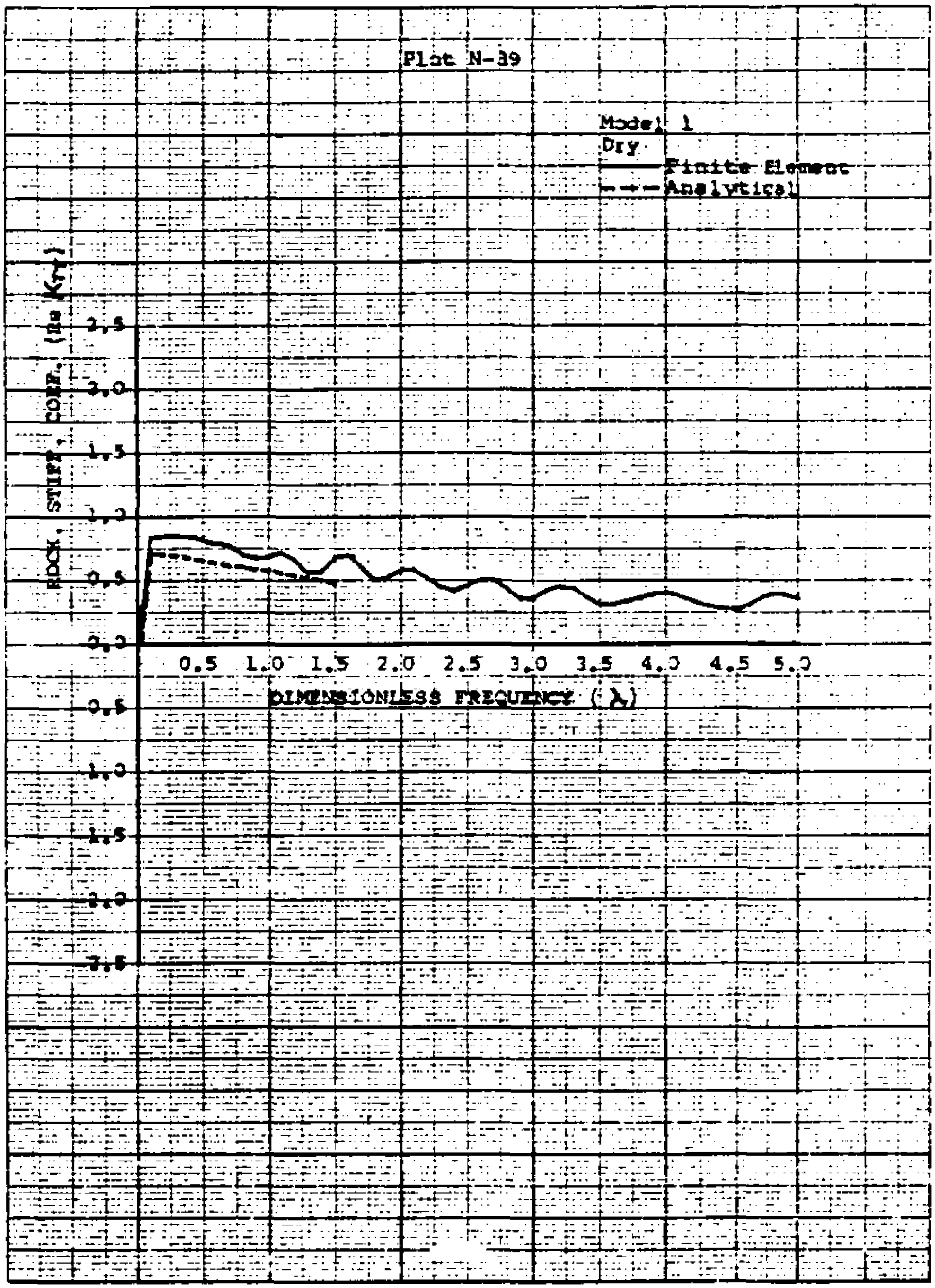
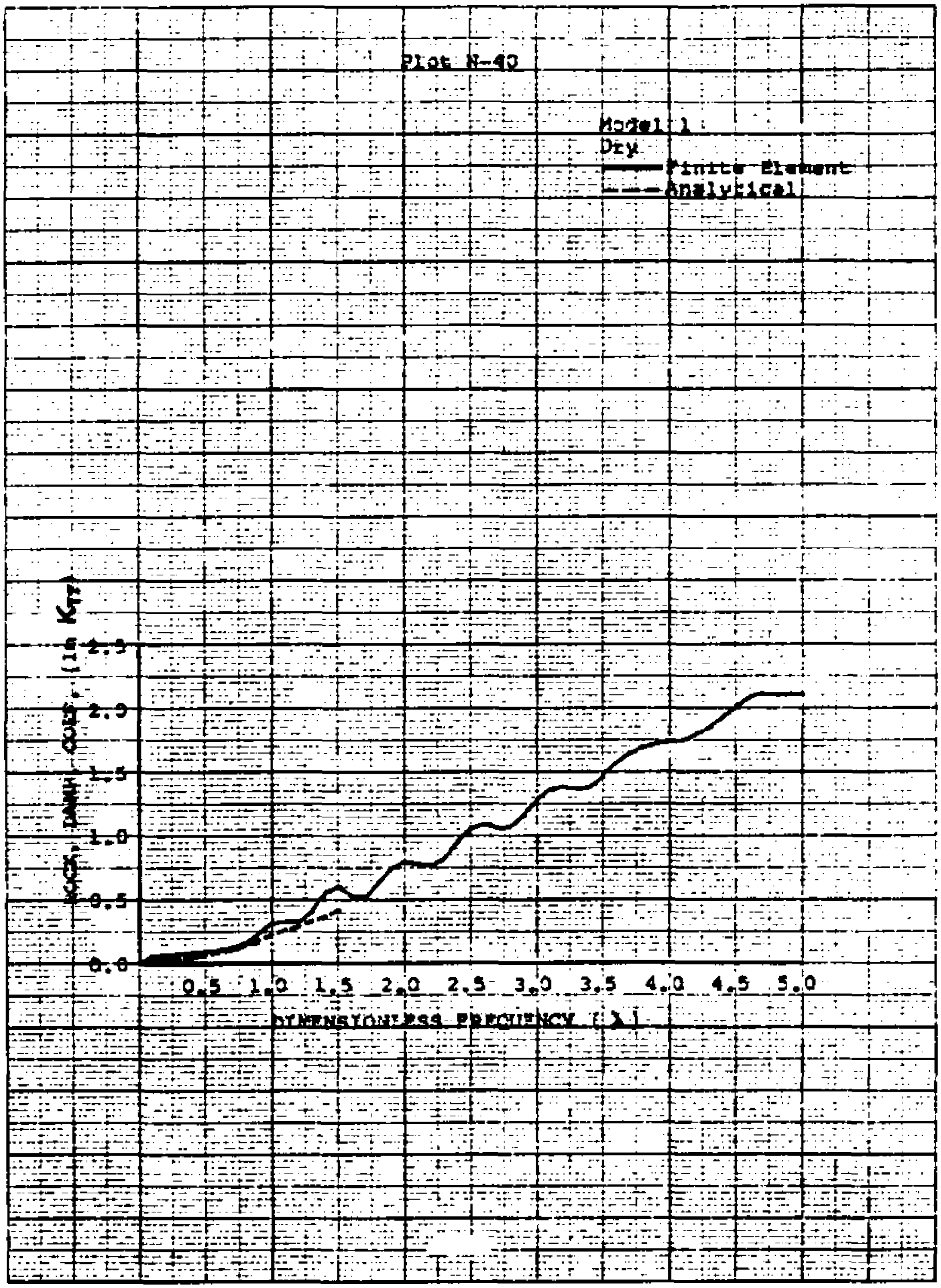
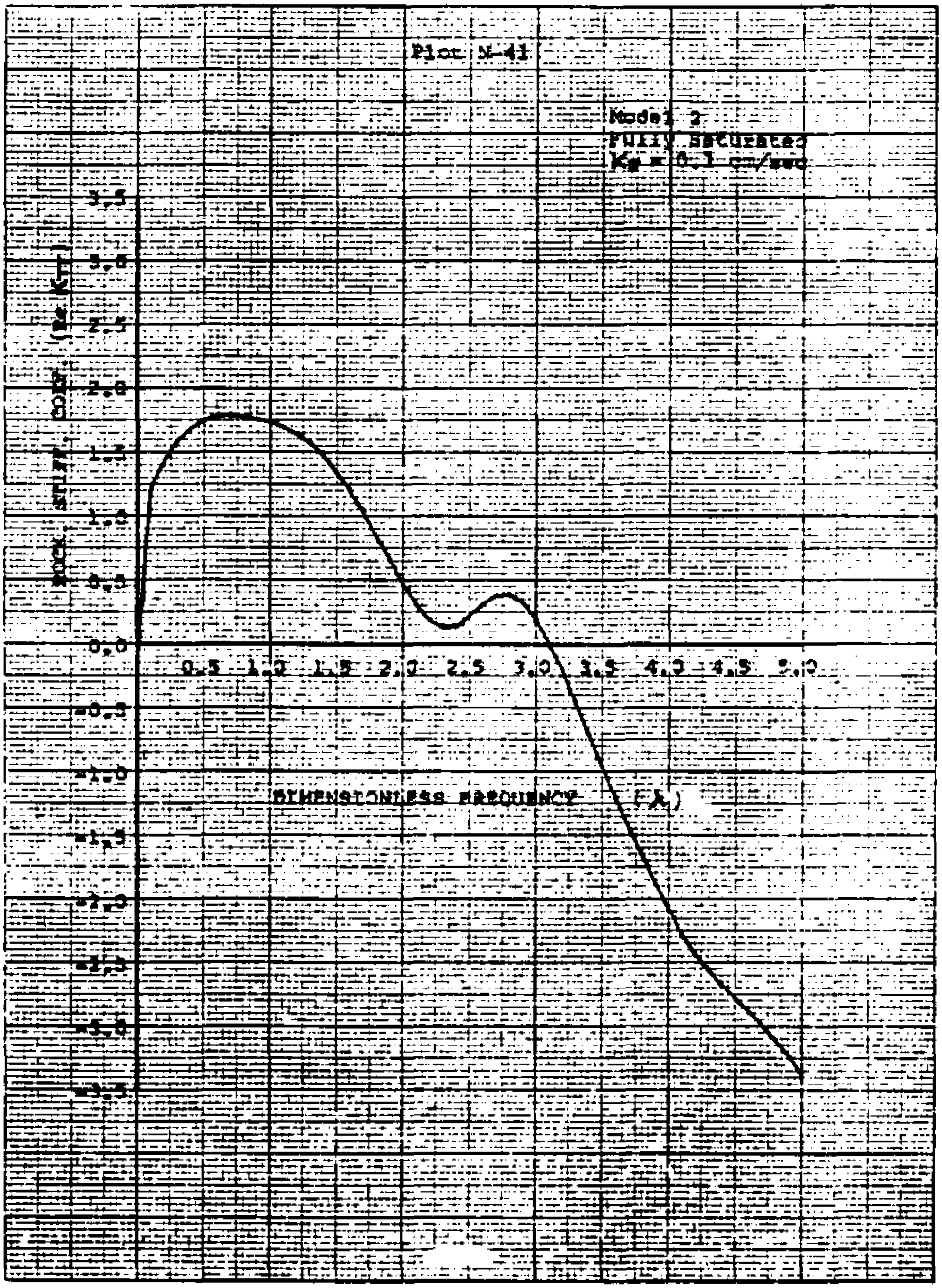
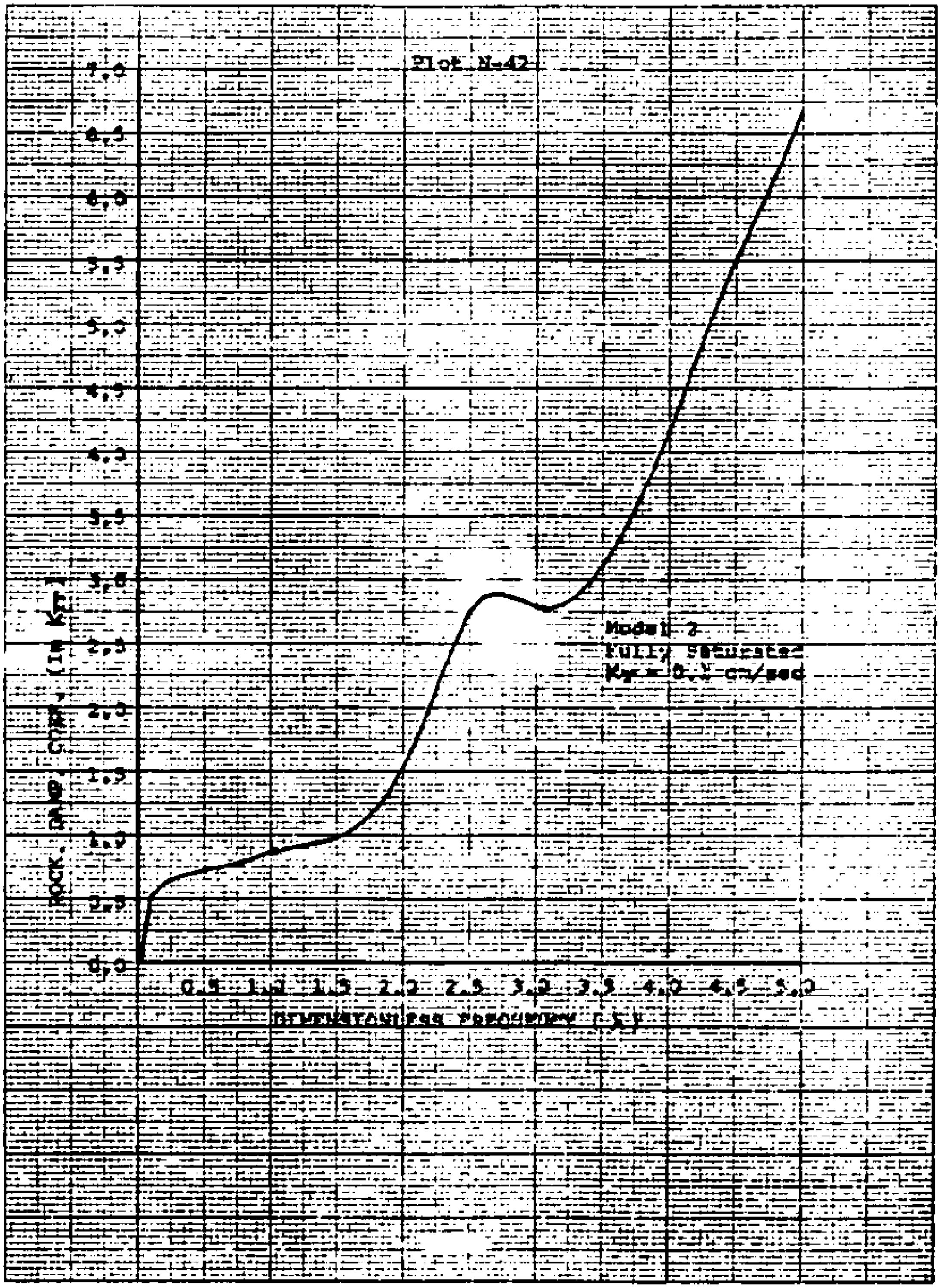


Plate N-39
Model 1
Dry
Finite Element
Analytical
ROCK. STIFF. COEF. (Re K_{rr})
2.5
2.0
1.5
1.0
0.5
0.0
0.5
1.0
1.5
2.0
2.5
0.5 1.0 1.5 2.0 2.5 3.0 3.5 4.0 4.5 5.0
DIMENSIONLESS FREQUENCY (λ)


Plot N-4D
Model 1
Dry
Finite Element
Analytical
ROCK. DAMP. COEF. (in K_{TT})
0.0
0.5
1.0
1.5
2.0
2.5
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
5.0
DIMENSIONLESS FREQUENCY (λ)


Plot M-41
Model 2
Fully Saturated
K_s = 0.1 cm/sec
ROCK. STIFF. COEF. (Re K_{rr})
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.0
-0.5
-1.0
-1.5
-2.0
-2.5
-3.0
-3.5
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
5.0
DIMENSIONLESS FREQUENCY (λ)

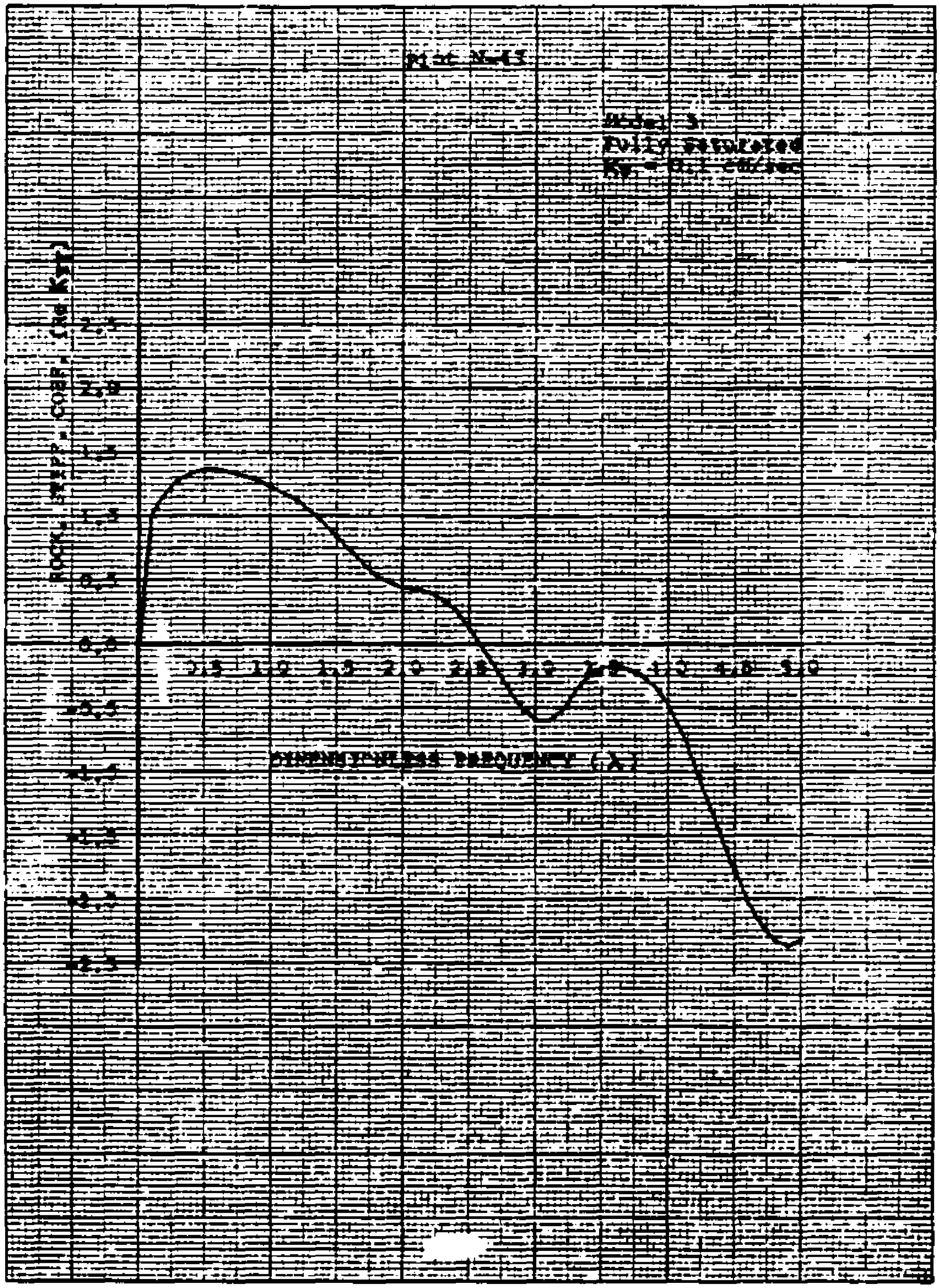
K+E 46 1242

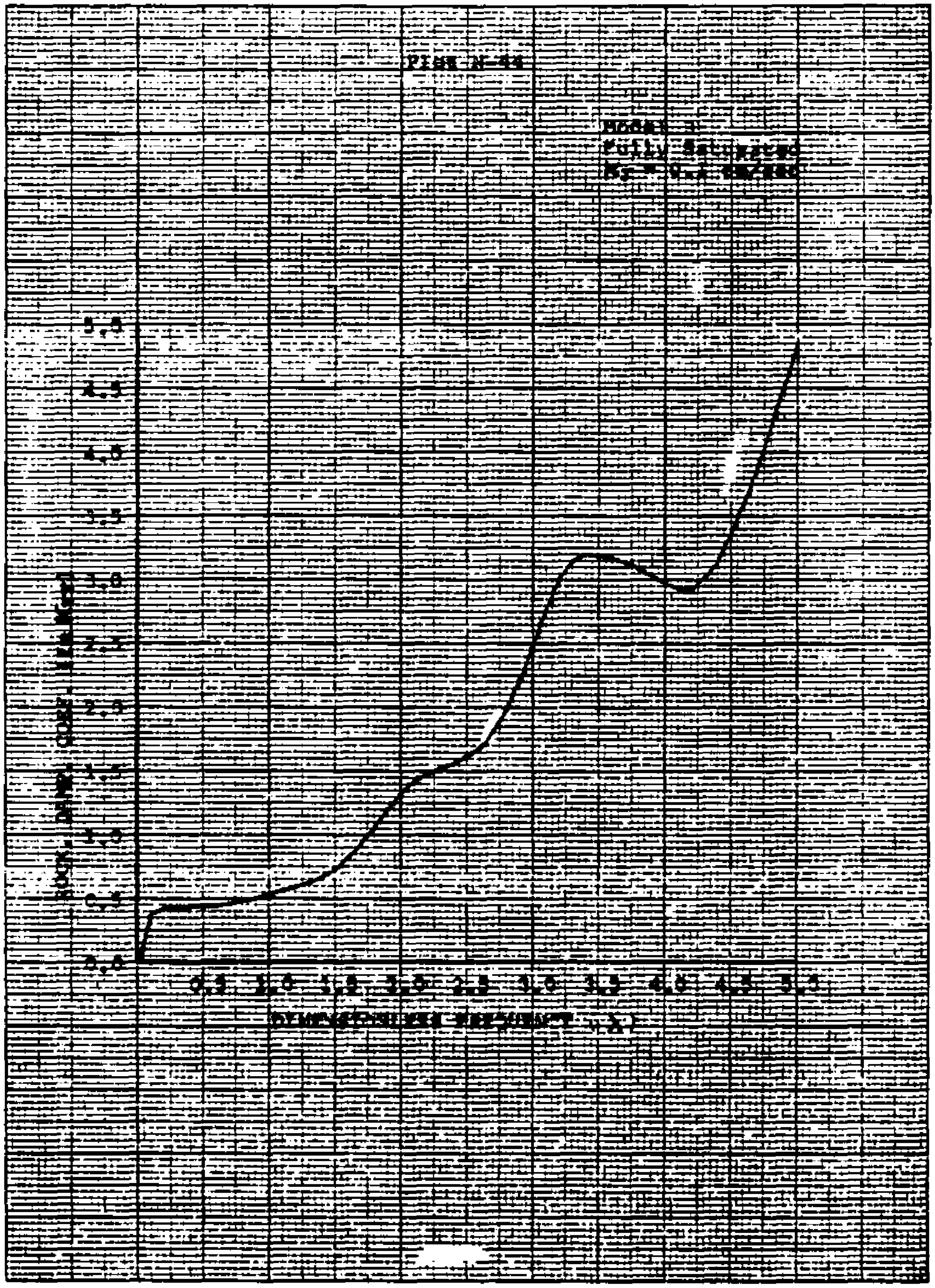


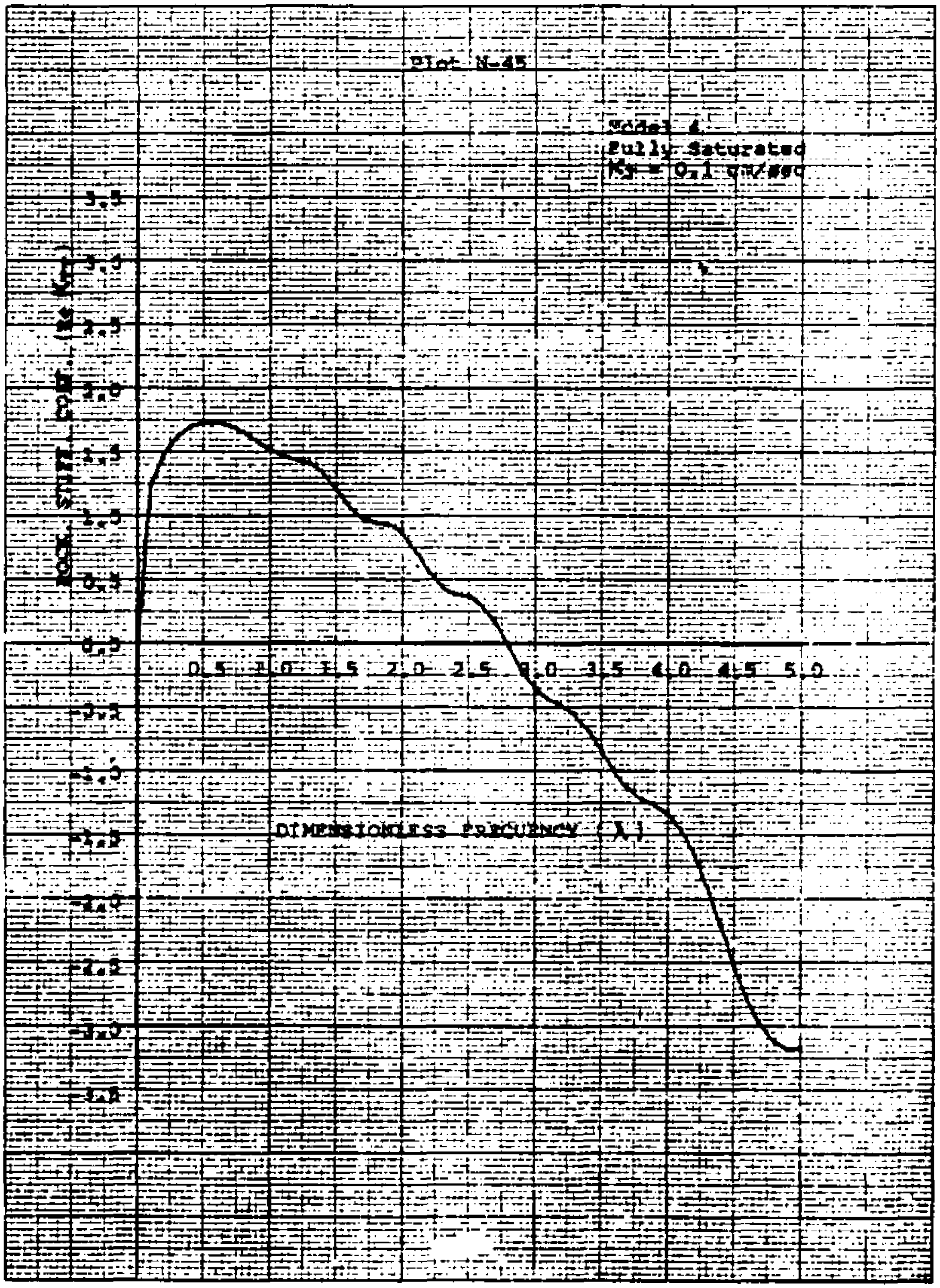
Plot N-42
ROCK. DAMP. COEF. (In K_{rr})
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
5.0
5.5
6.0
6.5
7.0
0.5 1.0 1.5 2.0 2.5 3.0 3.5 4.0 4.5 5.0
DIMENSIONLESS FREQUENCY (a_0)
Model 2
Fully Saturated
K_y = 0.1 cm/sec

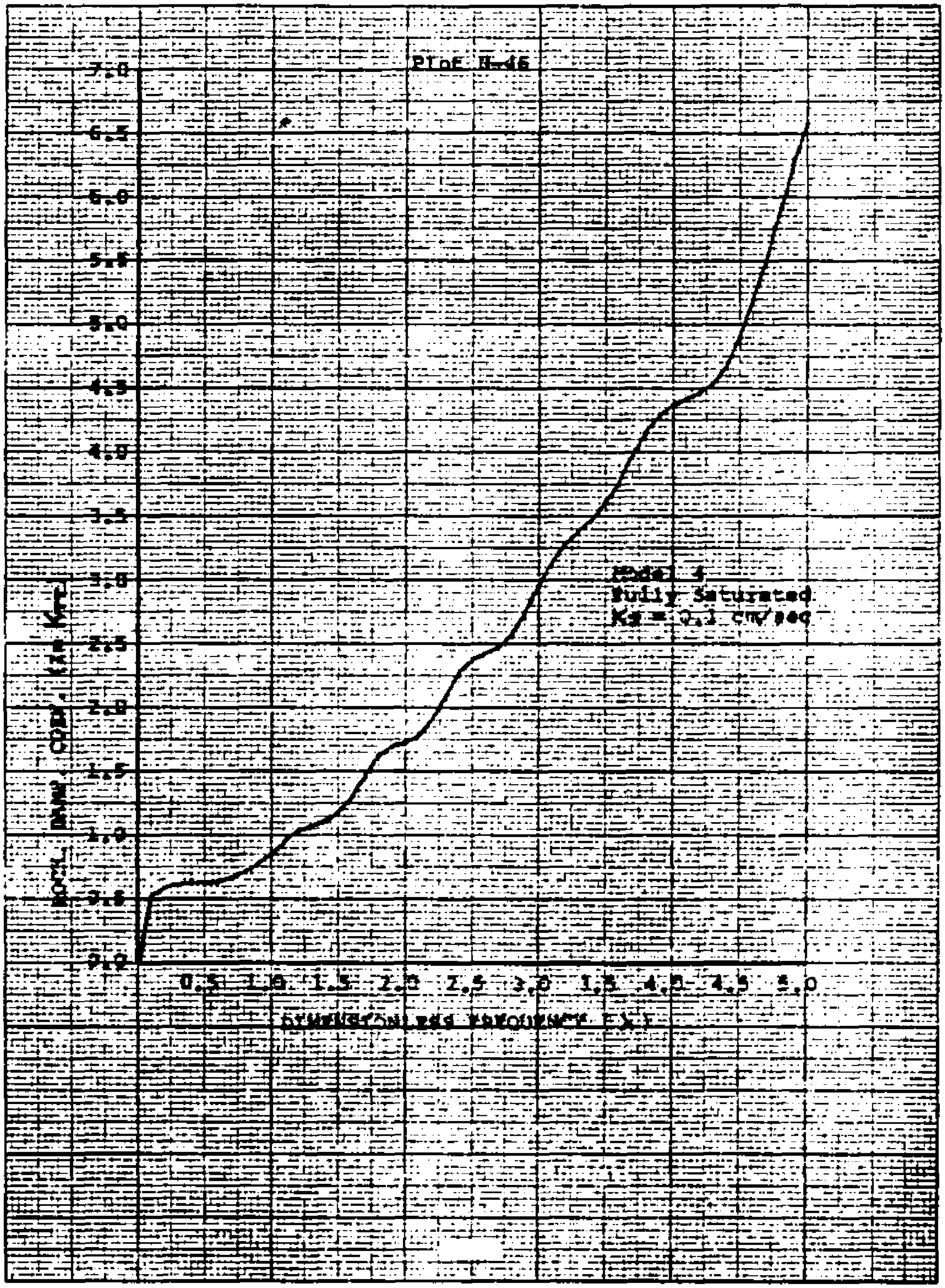
K-E 20 X 20 TO THE INCH • 7 X 10 INCHES KEUFFEL & ESSER CO.

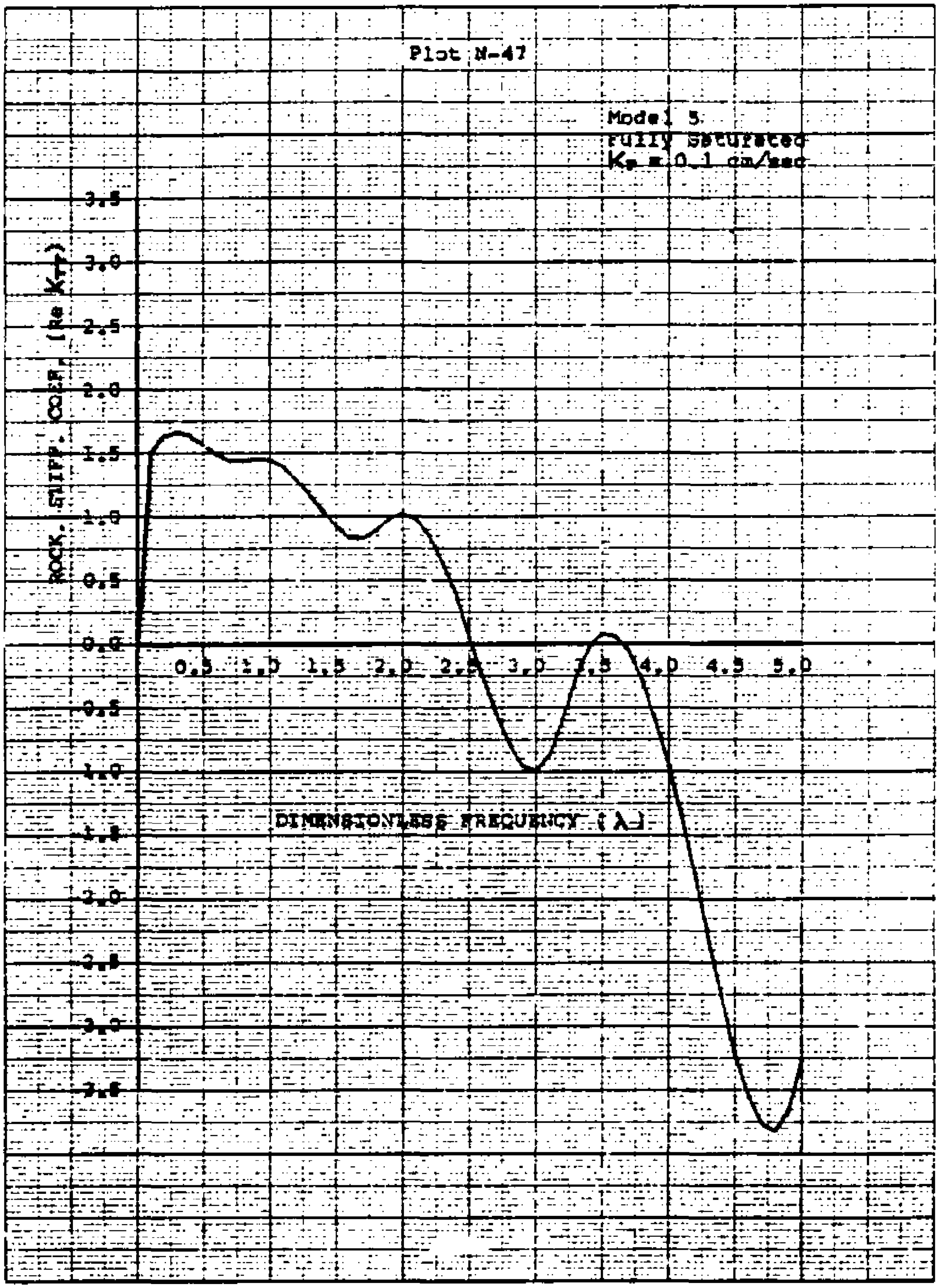
46 1242

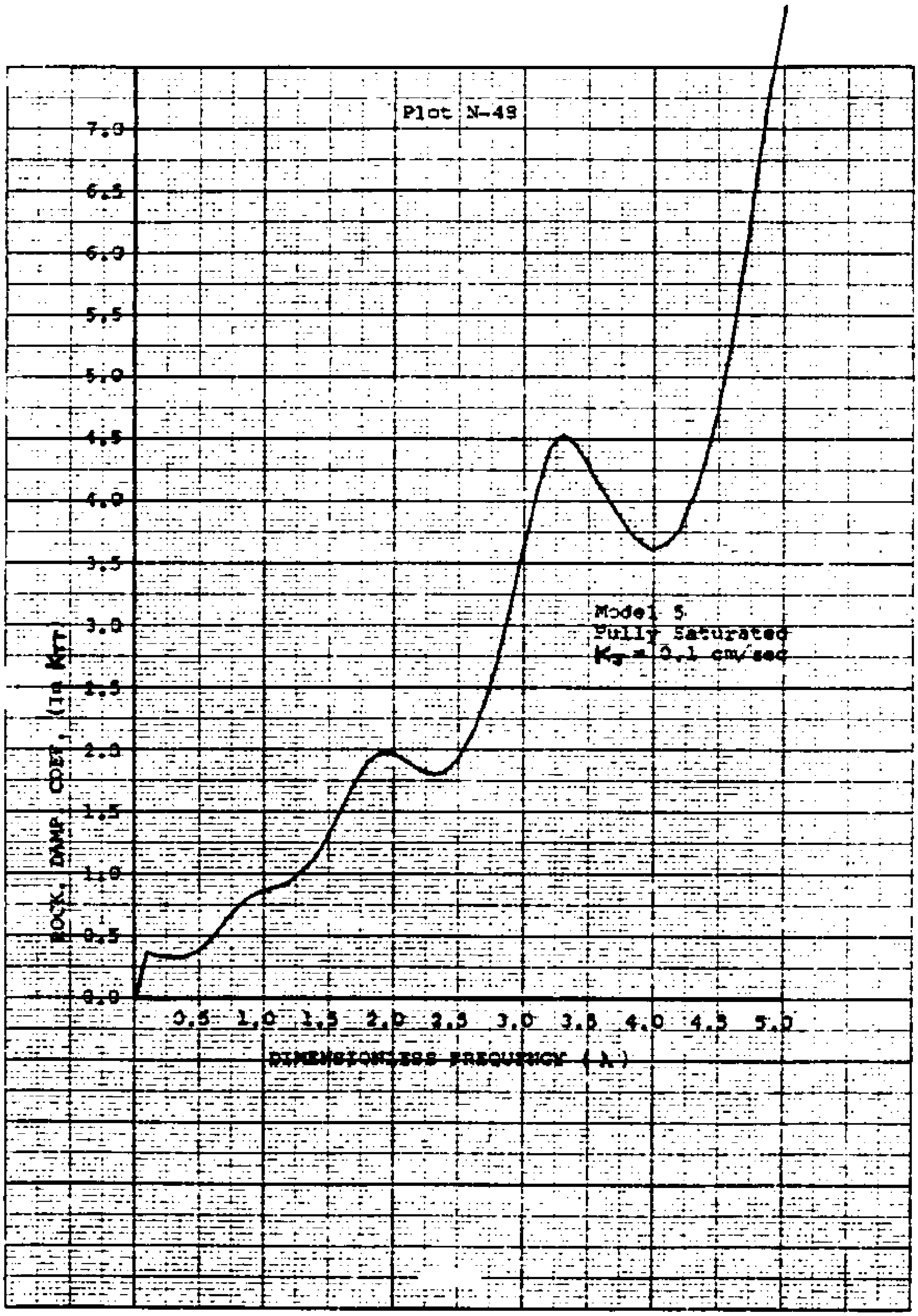


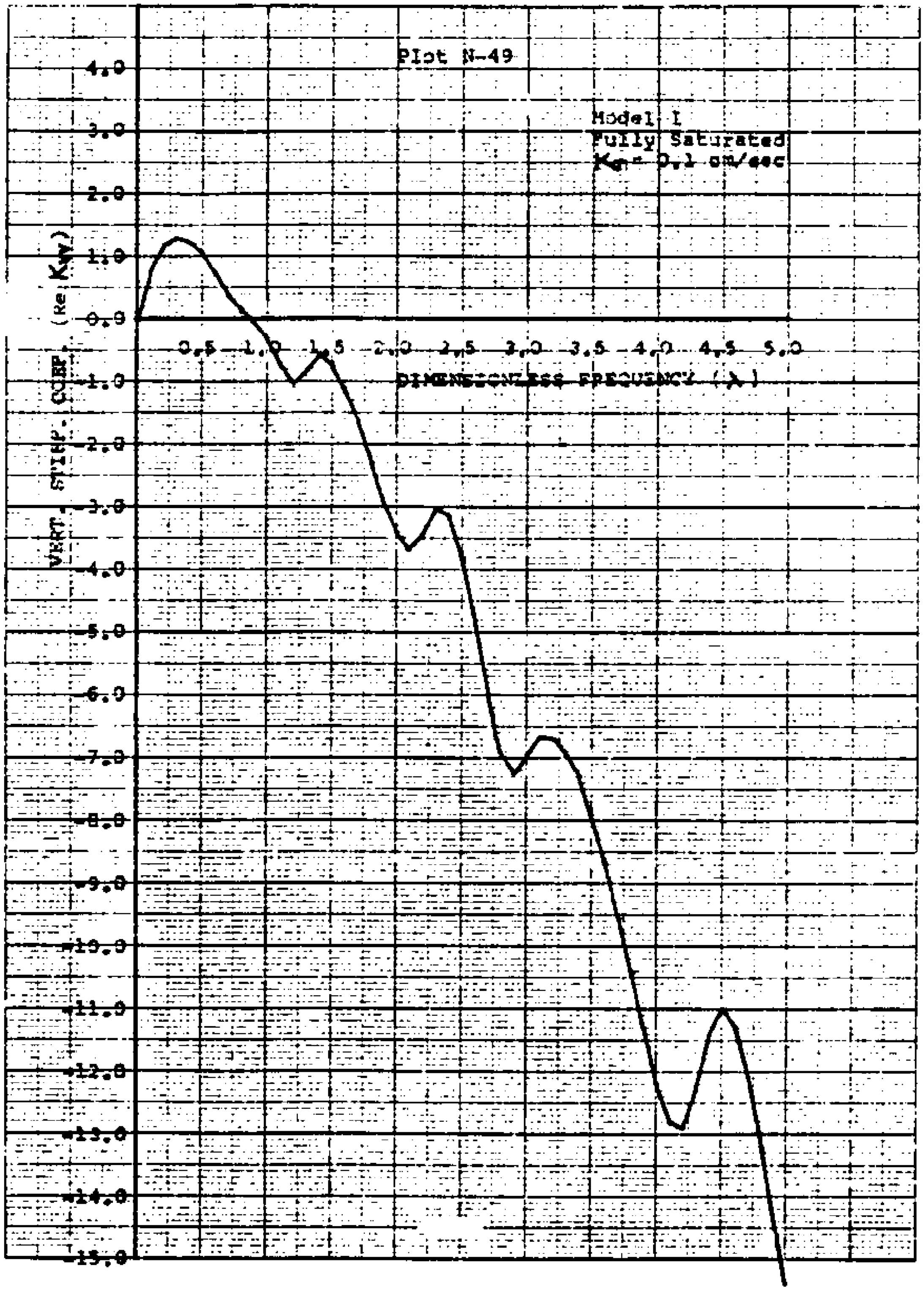

PIER N-44
Model 3:
Fully Saturated
ROCK. DAMPL COEF.
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
5.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
5.0
DIMENSIONLESS FREQUENCY

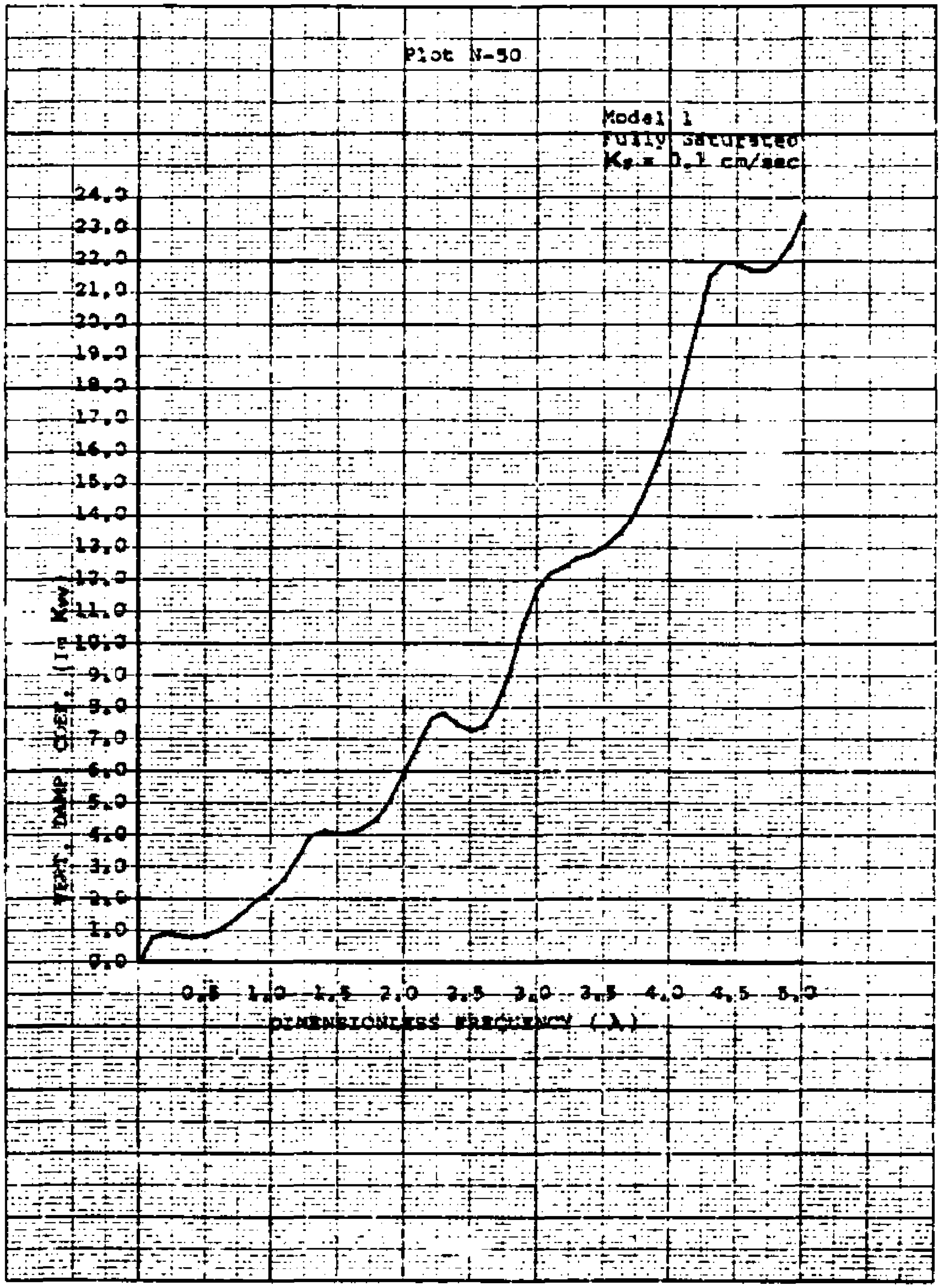

Plot N-45
Model 4
Fully Saturated
Ks = 0.1 cm/sec
ROCK. STIFF. COEF. (Re Krr)
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.0
-0.5
-1.0
-1.5
-2.0
-2.5
-3.0
-3.5
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
5.0
DIMENSIONLESS FREQUENCY (λ)

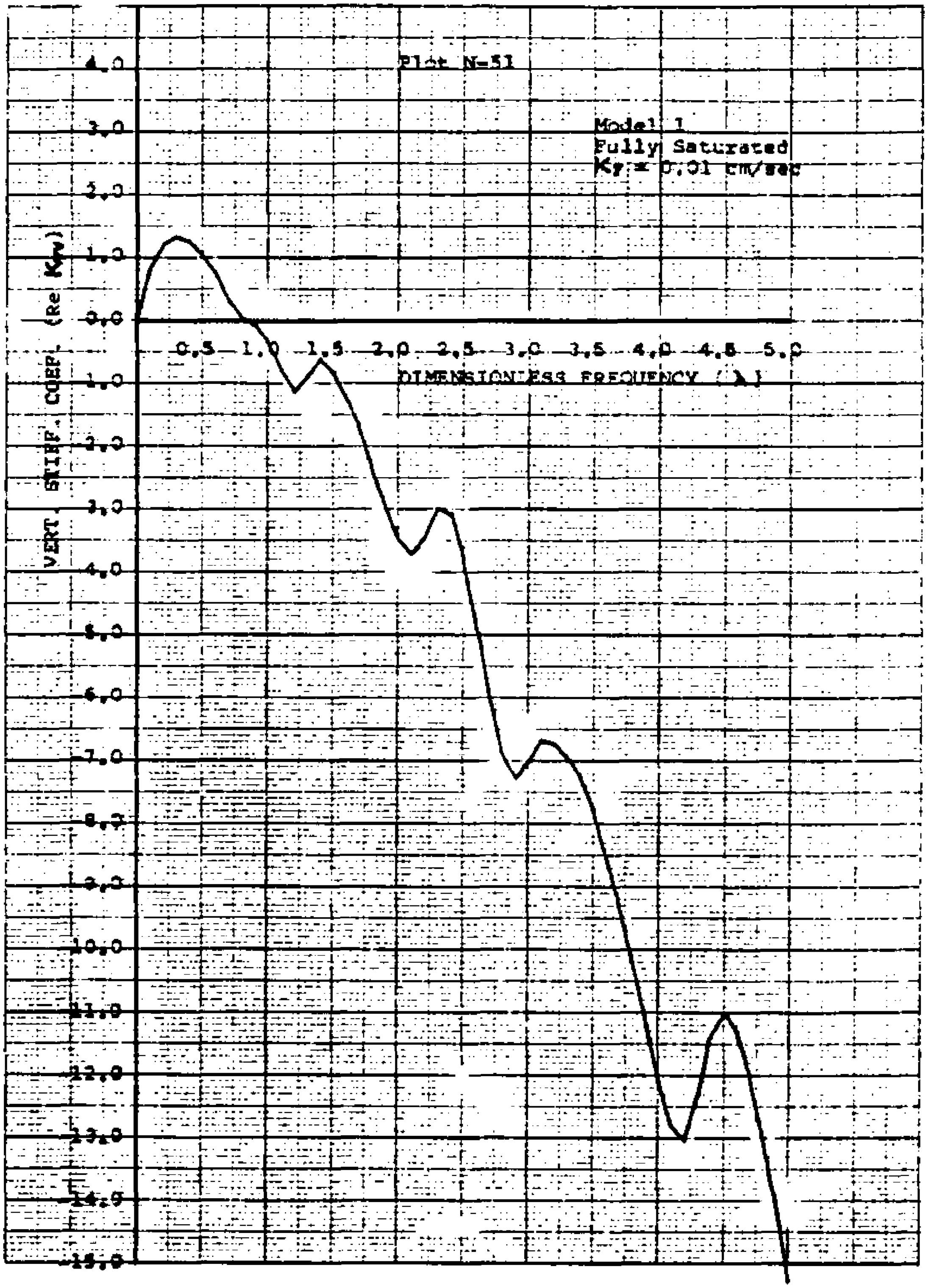

PLOT B-46
7.0
6.5
6.0
5.5
5.0
4.5
4.0
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.0
ROCK. DAMP. COEF. (Kr= Krr)
Model 4
Fully Saturated
Ks = 0.1 cm/sec
0.5
1.0
1.5
2.0
2.5
3.0
1.5
4.0
4.5
5.0
DIMENSIONLESS FREQUENCY

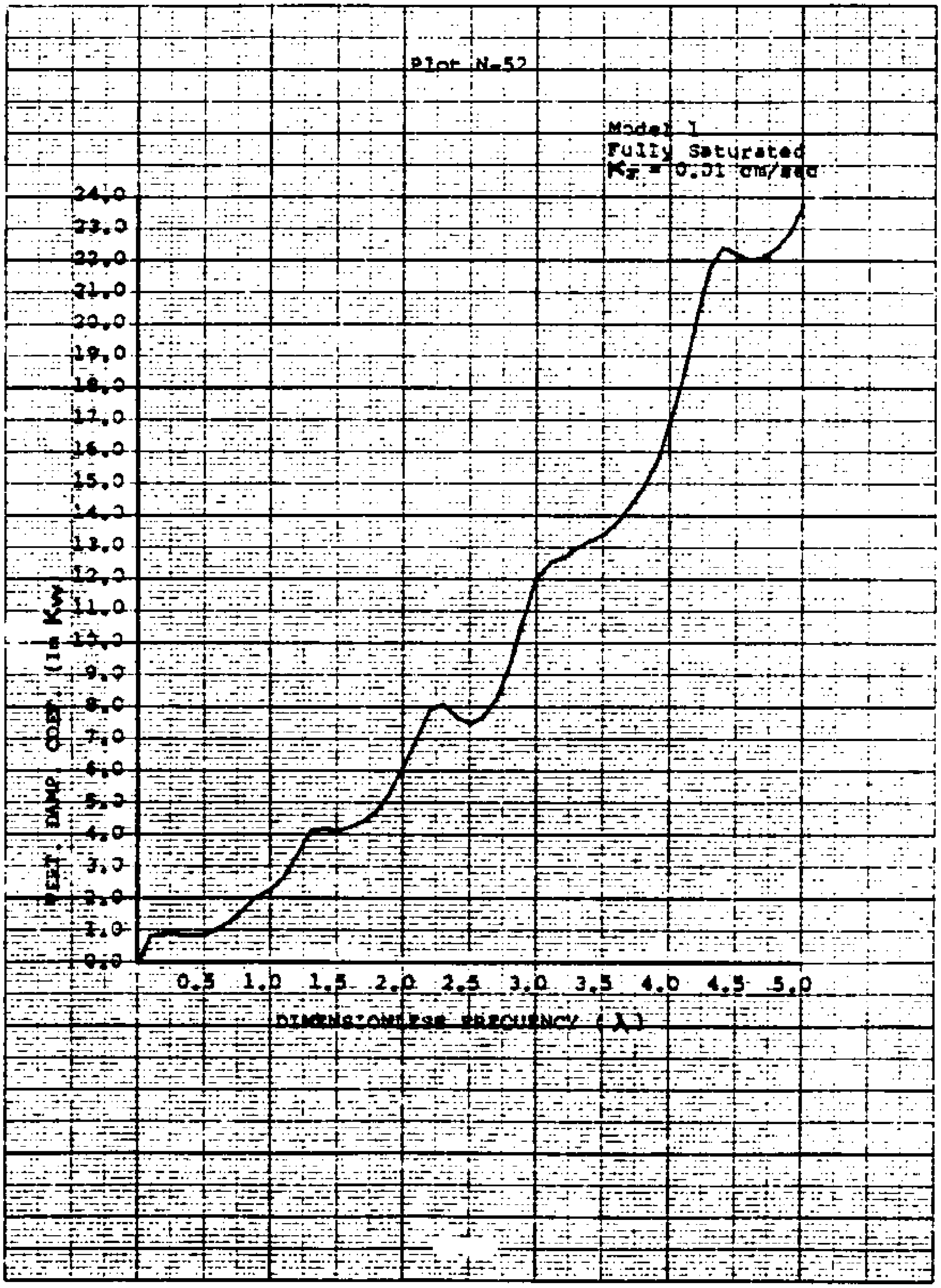

Plot N-47
Model 5
Fully Saturated
K_p = 0.1 cm/sec
ROCK. STIFF. COEF. (Re K_{TT})
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
5.0
DIMENSIONLESS FREQUENCY (λ)

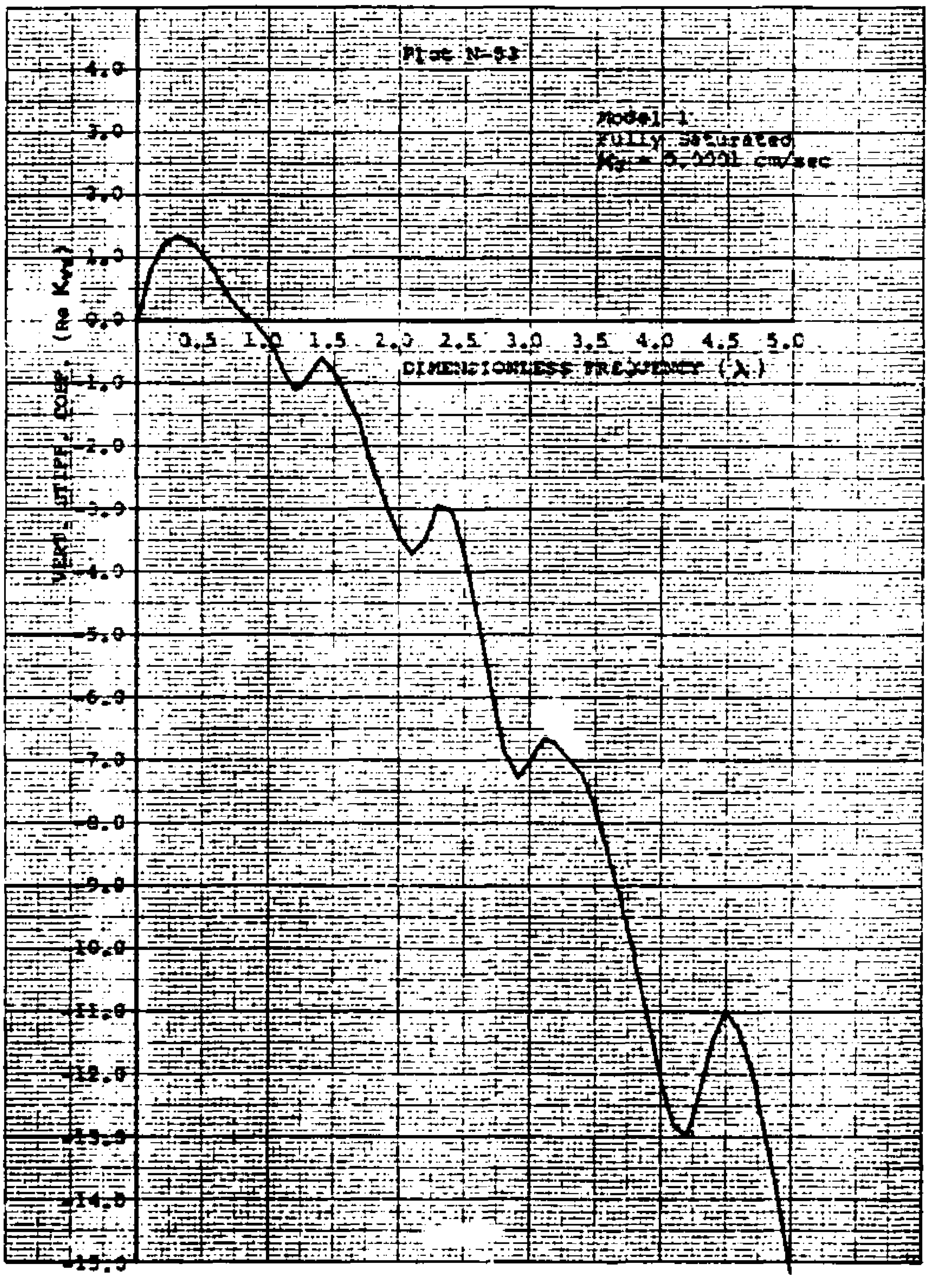

Plot N-49
Model 5
Fully Saturated
K_s = 0.1 cm/sec
ROCK. DAMP. COEF. (In K_{TT})
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
5.0
5.5
6.0
6.5
7.0
0.5 1.0 1.5 2.0 2.5 3.0 3.5 4.0 4.5 5.0
DIMENSIONLESS FREQUENCY (λ)

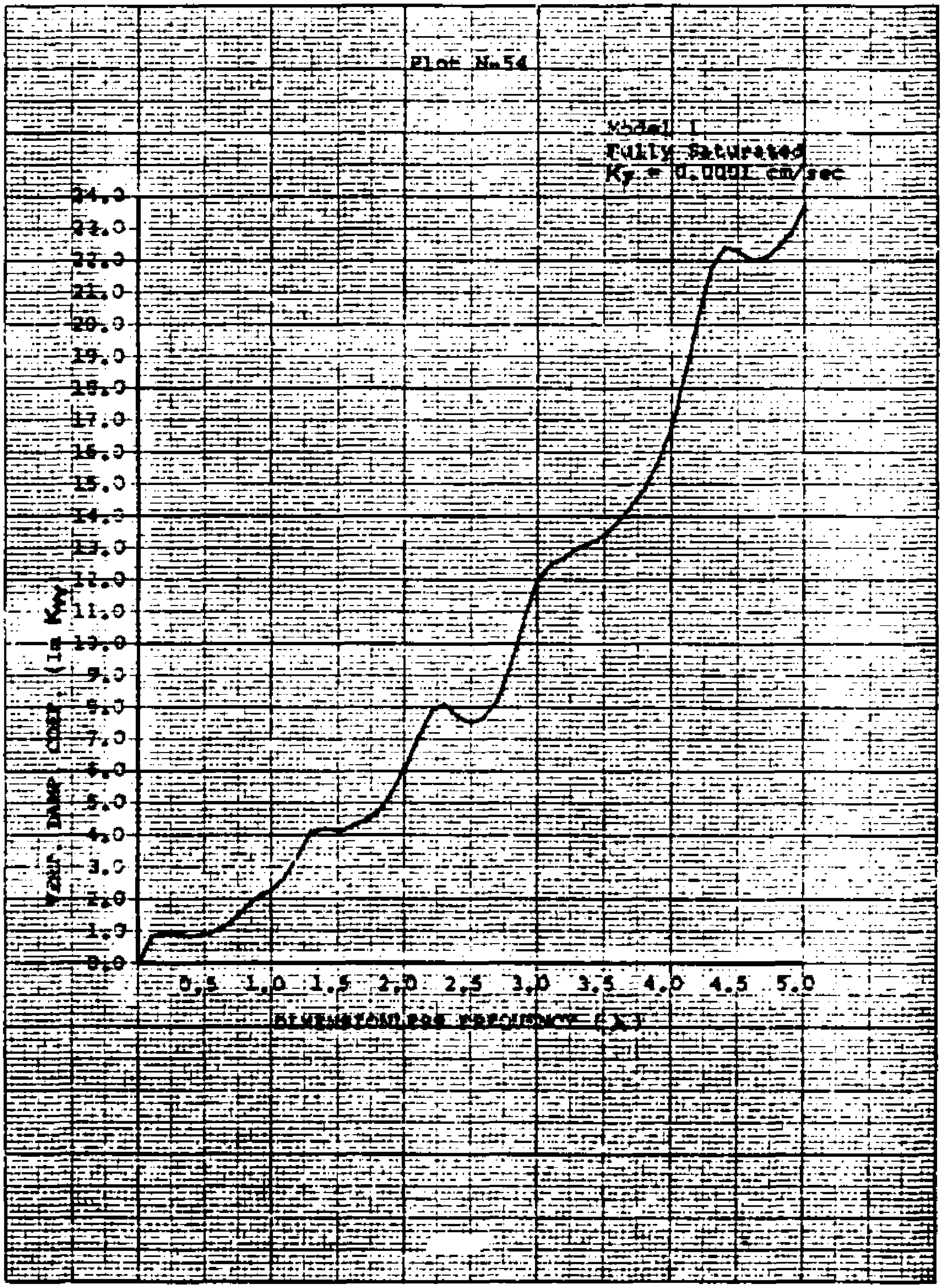

Plot N-49
Model I
Fully Saturated
K_{m} = 0.1 cm/sec
VERT. STIFF. COEF. (Re K_{vv})
4.0
3.0
2.0
1.0
0.0
-1.0
-2.0
-3.0
-4.0
-5.0
-6.0
-7.0
-8.0
-9.0
-10.0
-11.0
-12.0
-13.0
-14.0
-15.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
5.0
DIMENSIONLESS FREQUENCY (λ)

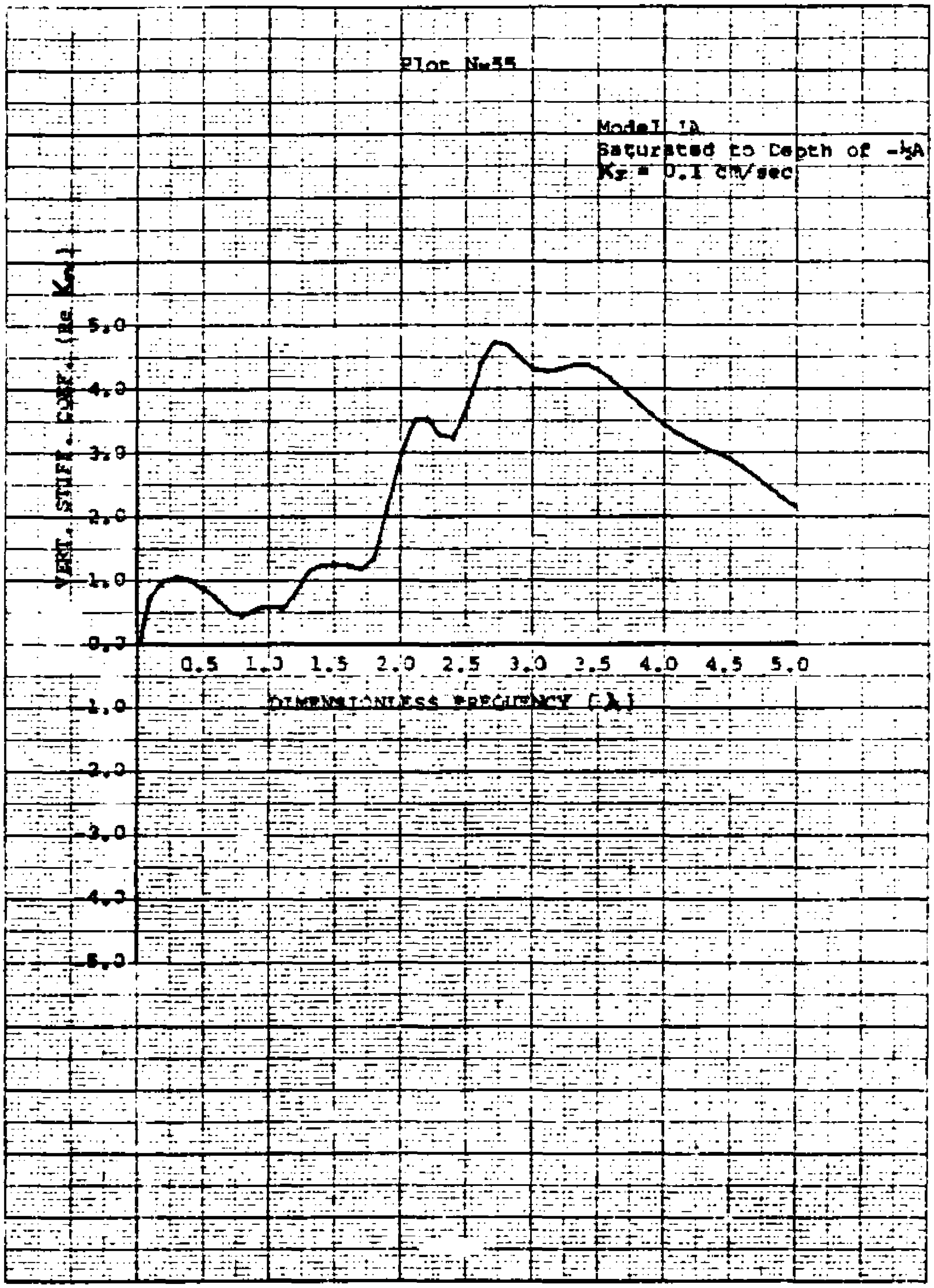

Plot N-50
Model 1
Fully saturated
K$_s$ = 0.1 cm/sec
VERT. DAMP. COEF. (Im Kvv)
0.0
1.0
2.0
3.0
4.0
5.0
6.0
7.0
8.0
9.0
10.0
11.0
12.0
13.0
14.0
15.0
16.0
17.0
18.0
19.0
20.0
21.0
22.0
23.0
24.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
5.0
DIMENSIONLESS FREQUENCY (λ)

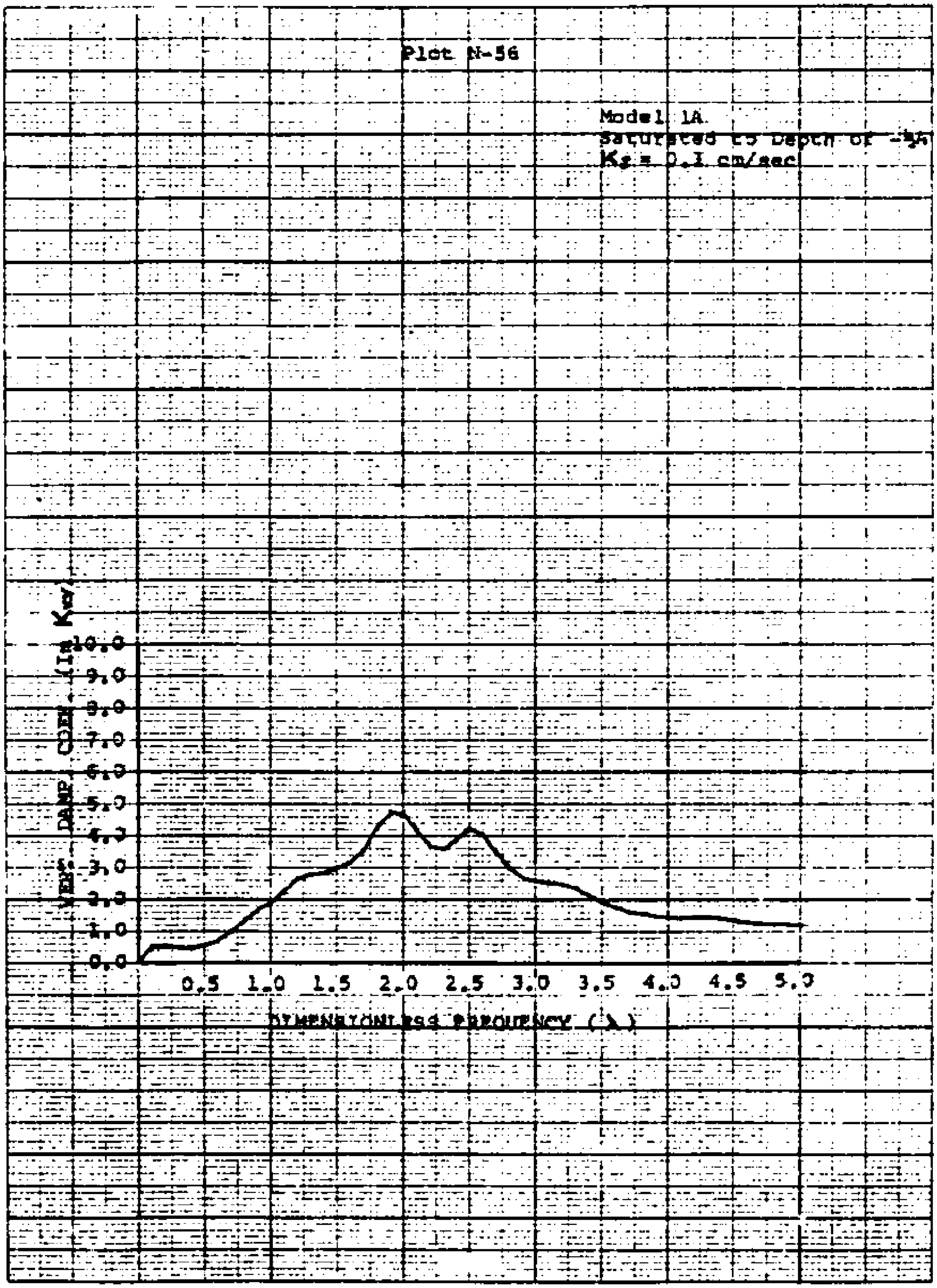

Plot N-51
Model 1
Fully Saturated
K_y = 0.01 cm/sec
VERT. STIFF. COEF. (Re K_{vv})
4.0
3.0
2.0
1.0
0.0
1.0
2.0
3.0
4.0
5.0
6.0
7.0
8.0
9.0
10.0
11.0
12.0
13.0
14.0
15.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
5.0
DIMENSIONLESS FREQUENCY (λ)

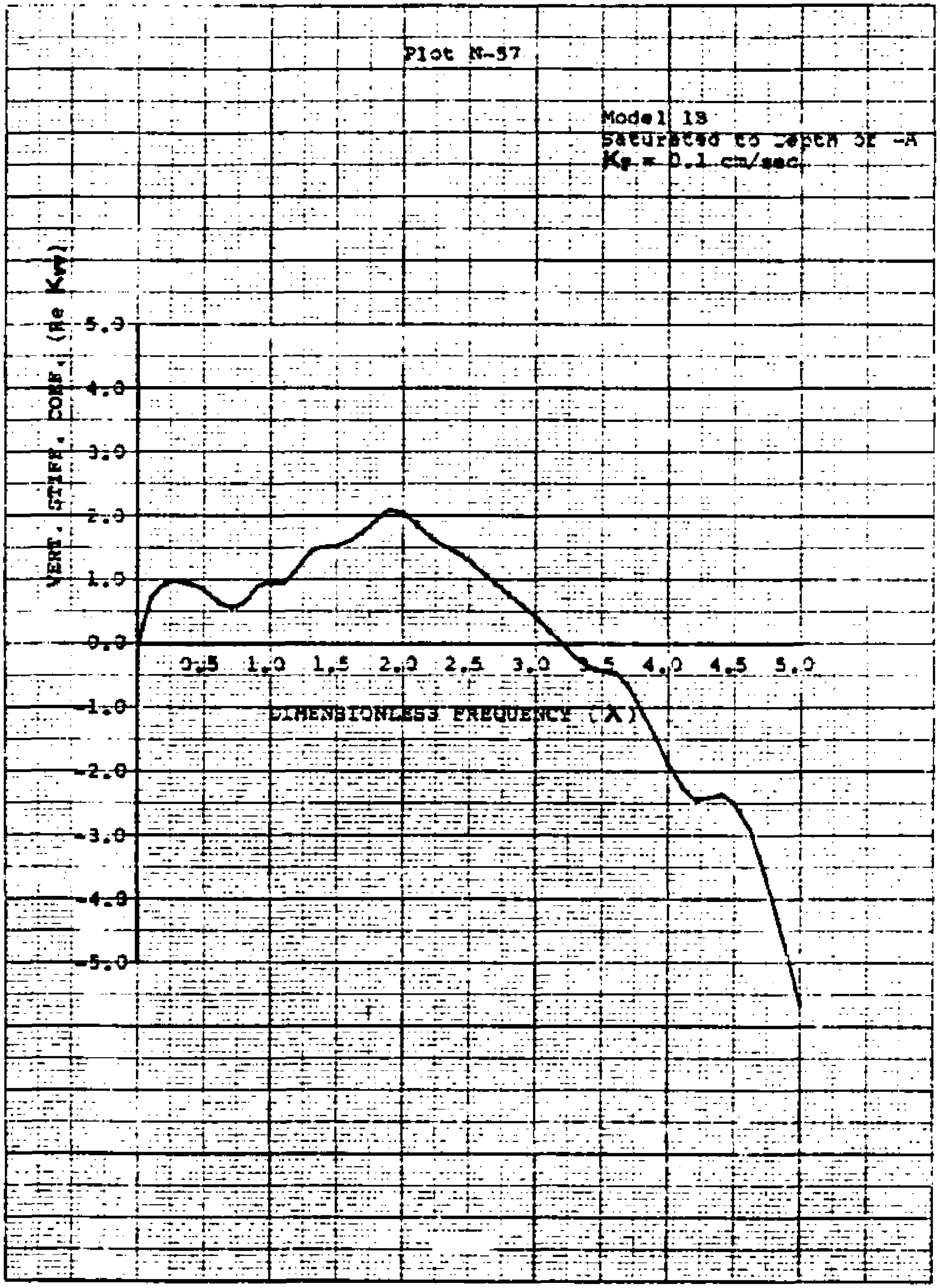

Plot N-52
Model 1
Fully Saturated
K_z = 0.01 cm/sec
VERT. DAMP. COEF. (Im K_{vv})
0.0
1.0
2.0
3.0
4.0
5.0
6.0
7.0
8.0
9.0
10.0
11.0
12.0
13.0
14.0
15.0
16.0
17.0
18.0
19.0
20.0
21.0
22.0
23.0
24.0
0.5 1.0 1.5 2.0 2.5 3.0 3.5 4.0 4.5 5.0
DIMENSIONLESS FREQUENCY (λ)

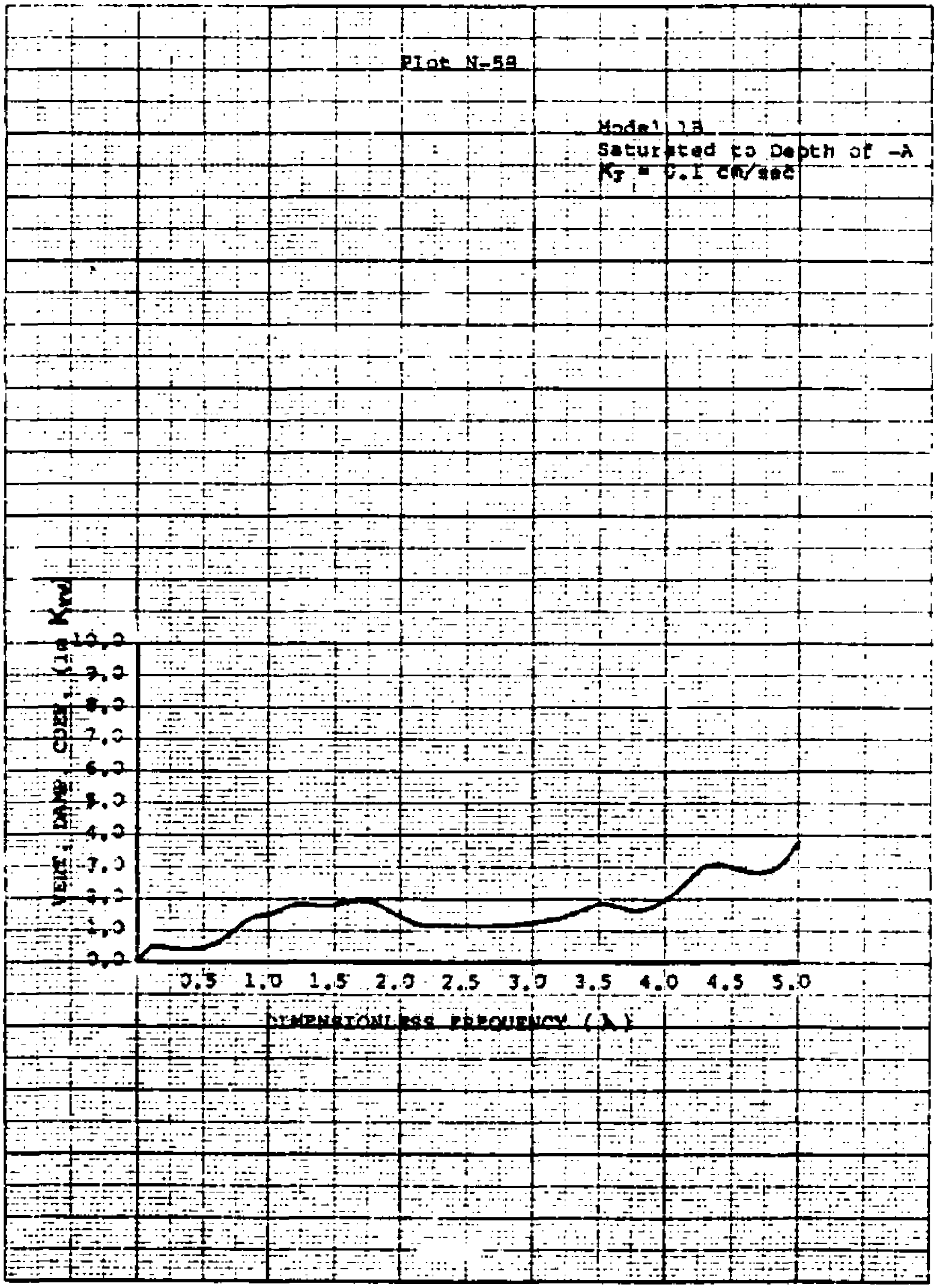

Plot N-53
Model-1
Fully Saturated
K_z = 0.0001 cm/sec
VERT. STIFF. COEF. (Re K_{vv})
4.0
3.0
2.0
1.0
0.0
-1.0
-2.0
-3.0
-4.0
-5.0
-6.0
-7.0
-8.0
-9.0
-10.0
-11.0
-12.0
-13.0
-14.0
-15.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
5.0
DIMENSIONLESS FREQUENCY (λ)

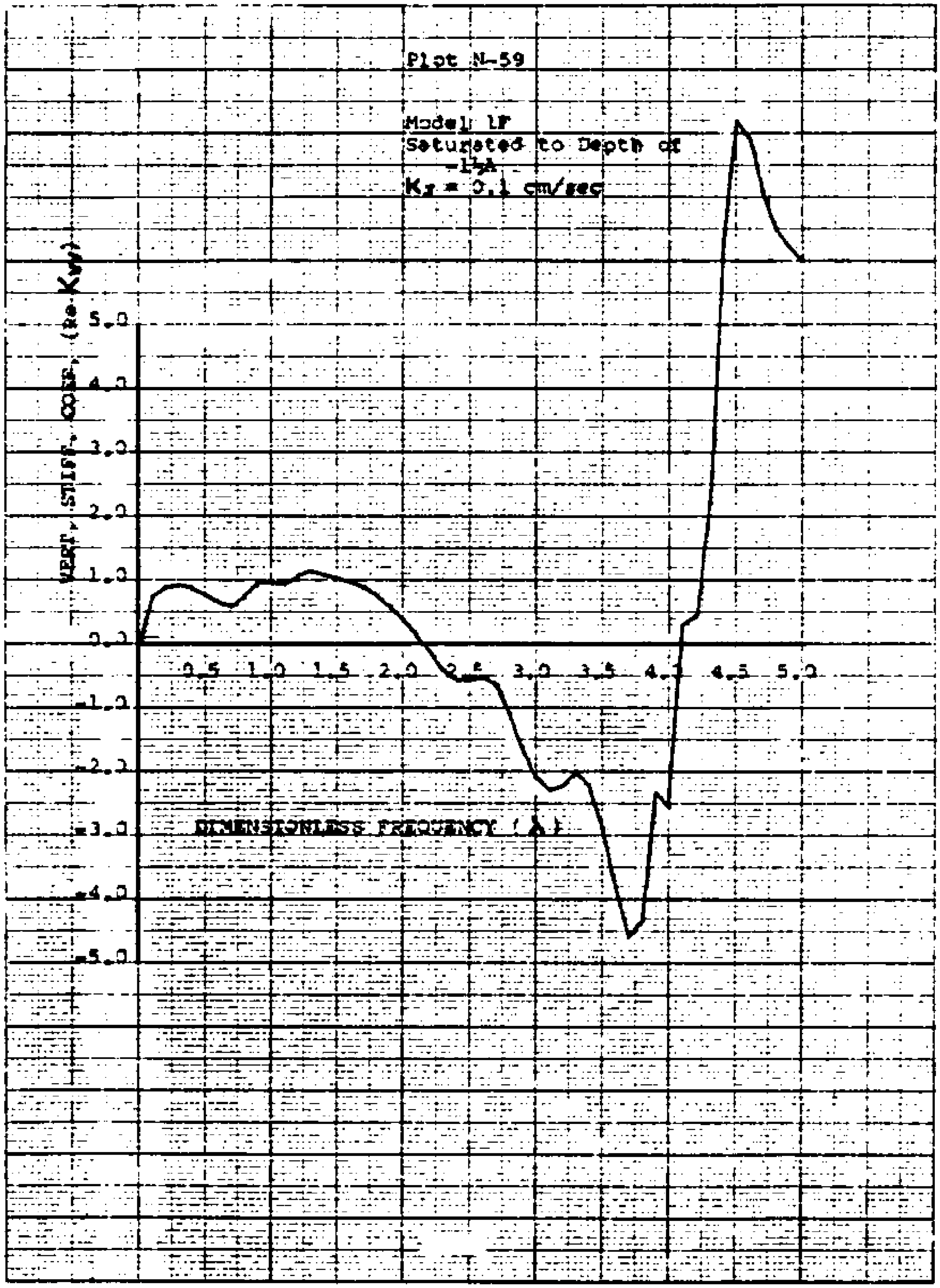

Plot No.54
Model I
Fully Saturated
K_y = 0.0001 cm/sec
24.0
23.0
22.0
21.0
20.0
19.0
18.0
17.0
16.0
15.0
14.0
13.0
12.0
11.0
10.0
9.0
8.0
7.0
6.0
5.0
4.0
3.0
2.0
1.0
0.0
VERT. DAMP. COEF. (in K_{vy})
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
5.0
DIMENSIONLESS FREQUENCY (λ)
K+E
46 1242

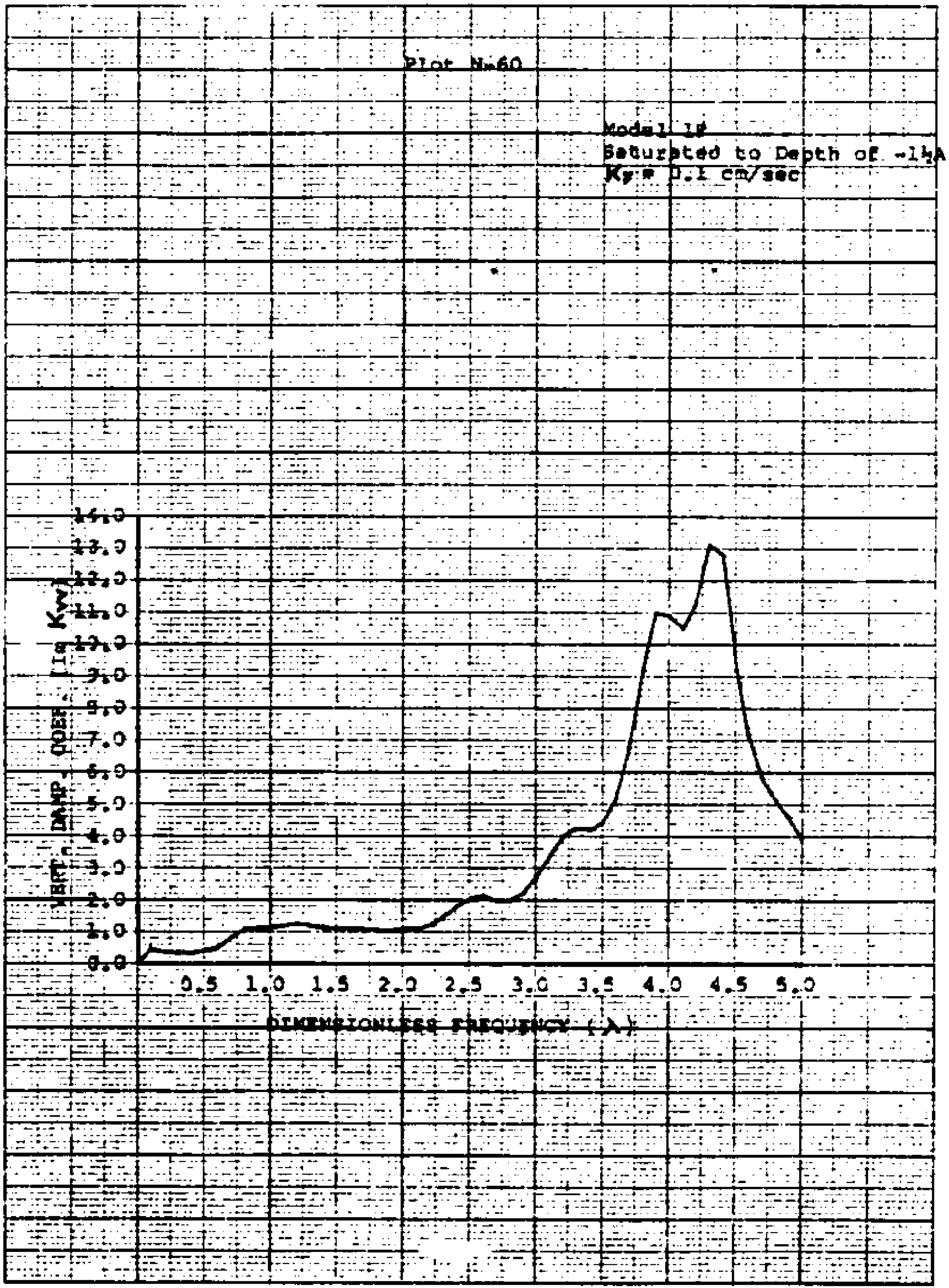

Plot No.55
Model 1A
Saturated to Depth of -½A
Kz = 0.1 cm/sec
VERT. STIFF. COEF. (Re Kv)
5.0
4.0
3.0
2.0
1.0
0.0
1.0
2.0
3.0
4.0
5.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
5.0
DIMENSIONLESS FREQUENCY (λ)

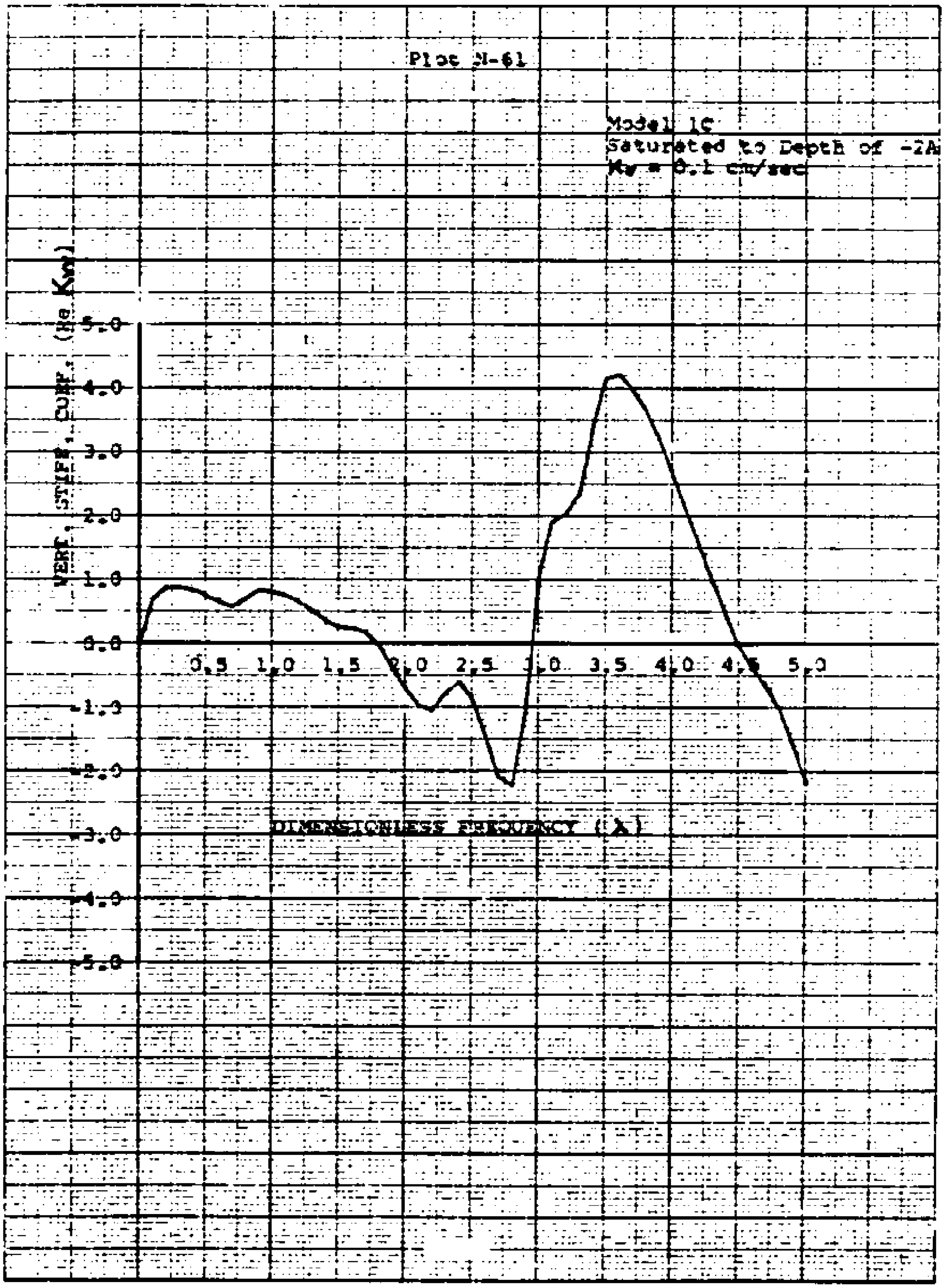

Plot N-56
Model 1A
Saturated to Depth of -½A
K_s = 0.1 cm/sec
VERT. DAMP. COEF. (in K_{wv})
10.0
9.0
8.0
7.0
6.0
5.0
4.0
3.0
2.0
1.0
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
5.0
DIMENSIONLESS FREQUENCY (λ)

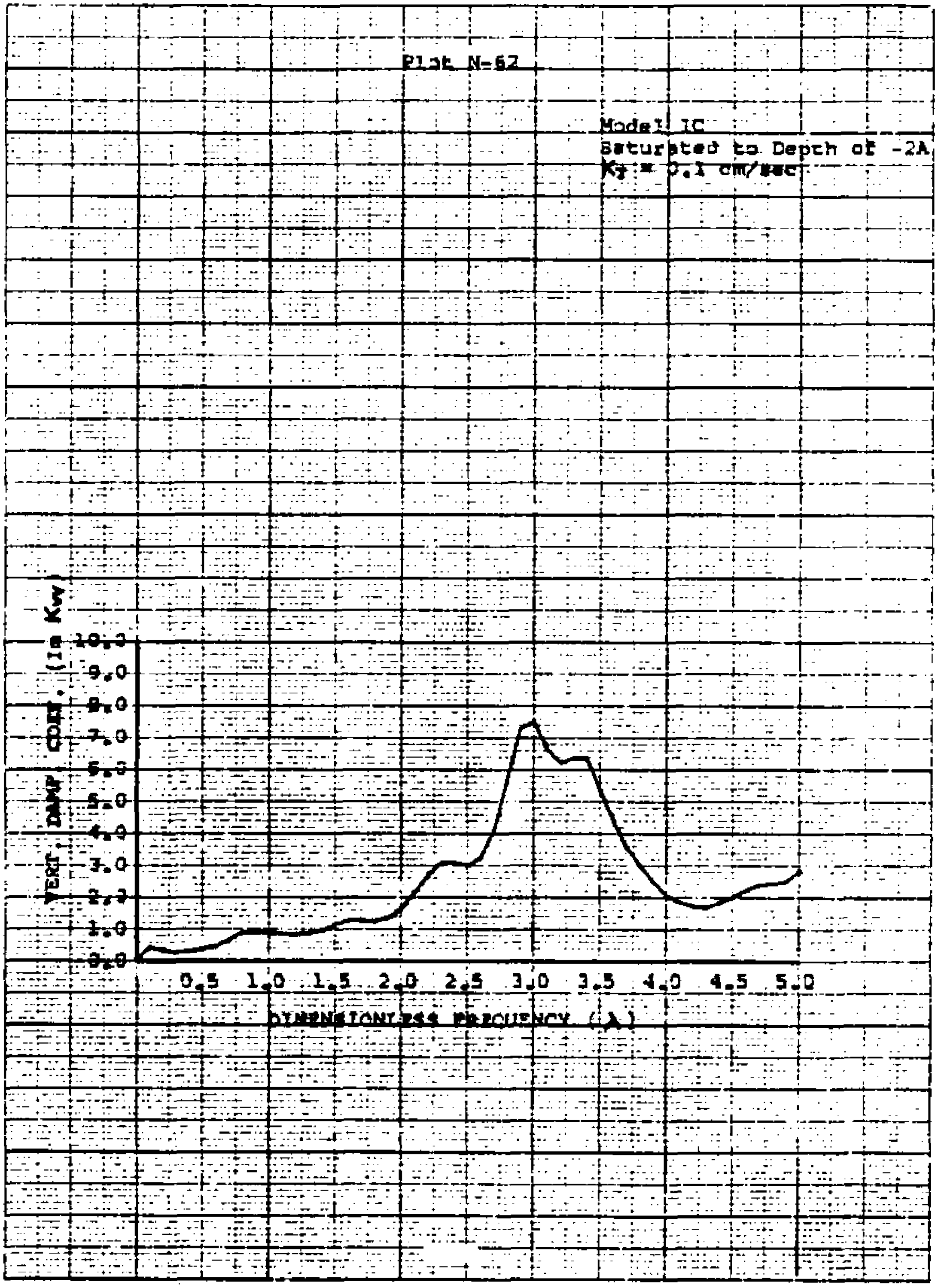

Plot M-57
Model 1B
Saturated to Depth of -A
K_s = 0.1 cm/sec.
VERT. STIFF. COEF. (Re K_{vv})
5.0
4.0
3.0
2.0
1.0
0.0
-1.0
-2.0
-3.0
-4.0
-5.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
5.0
DIMENSIONLESS FREQUENCY (λ)

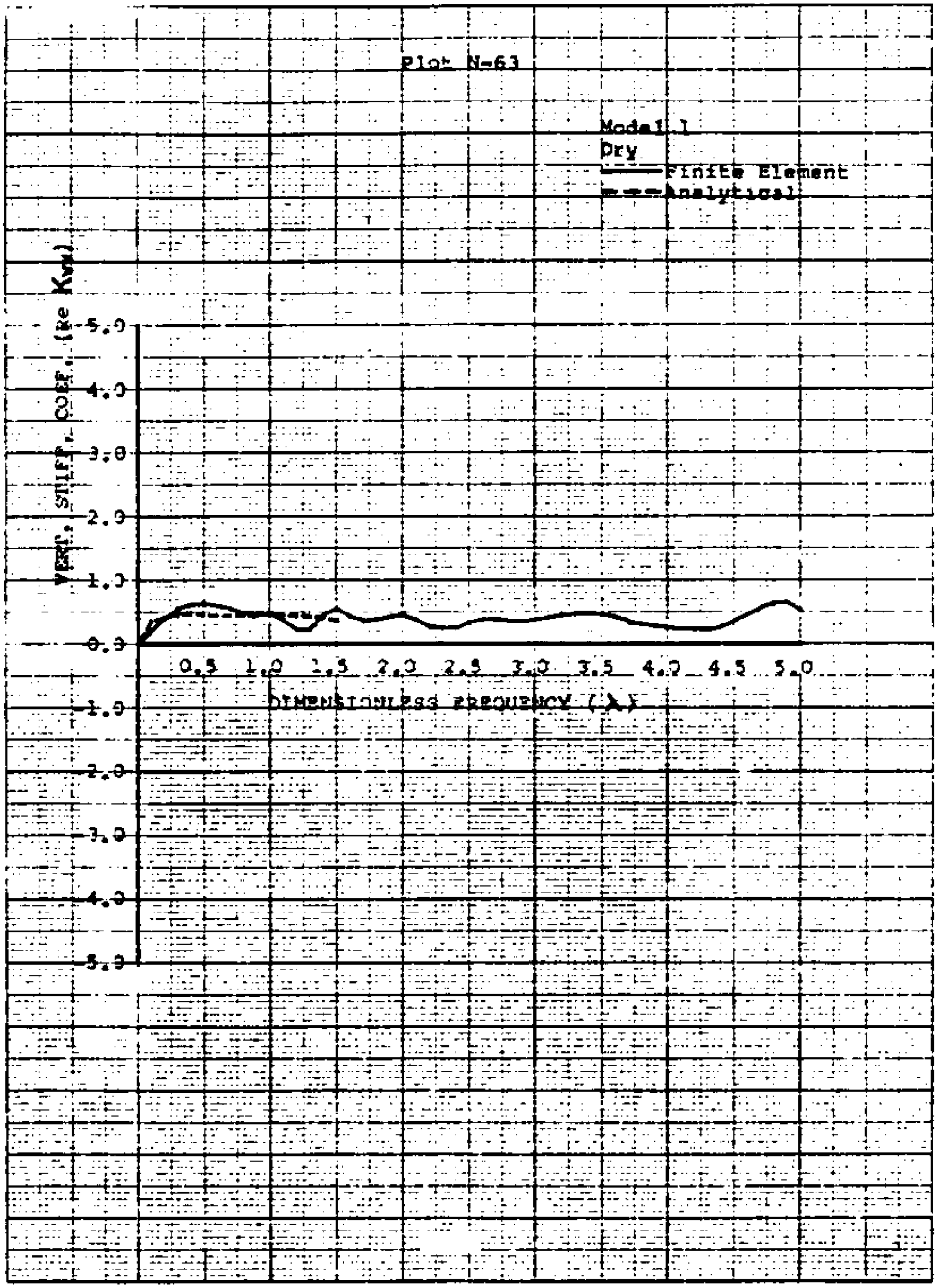

Plot N-59
Model 1B
Saturated to Depth of -A
K_T = 0.1 cm/sec
VERT. DAMP. COEF. (in K_{vd})
0.0
1.0
2.0
3.0
4.0
5.0
6.0
7.0
8.0
9.0
10.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
5.0
DIMENSIONLESS FREQUENCY (λ)

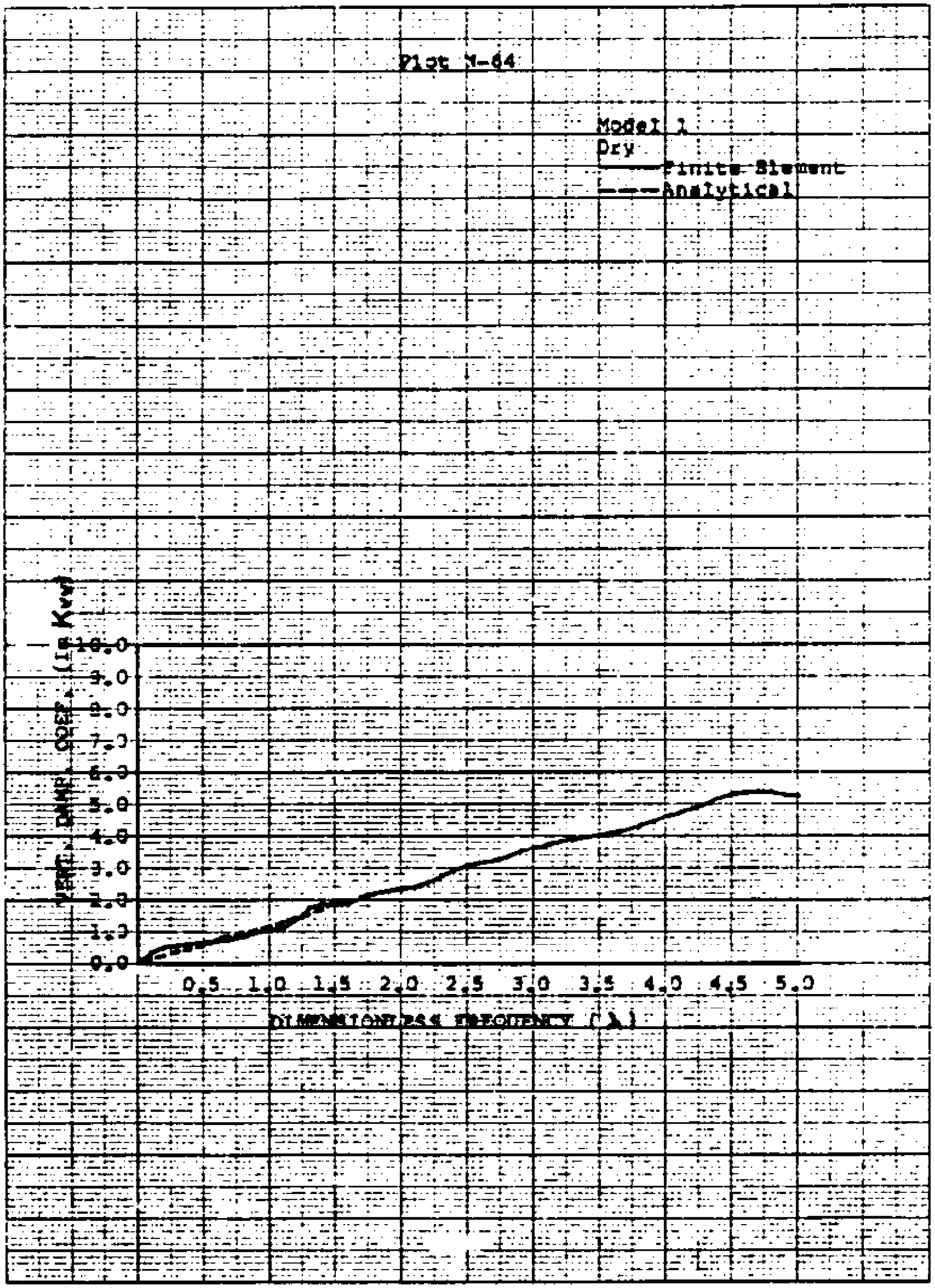

Plot N-59
Model 1F
Saturated to Depth of
-1½λ
K_z = 0.1 cm/sec
VERT. STIFF. COEF. (Re Kvv)
5.0
4.0
3.0
2.0
1.0
0.0
-1.0
-2.0
-3.0
-4.0
-5.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
5.0
DIMENSIONLESS FREQUENCY (λ)

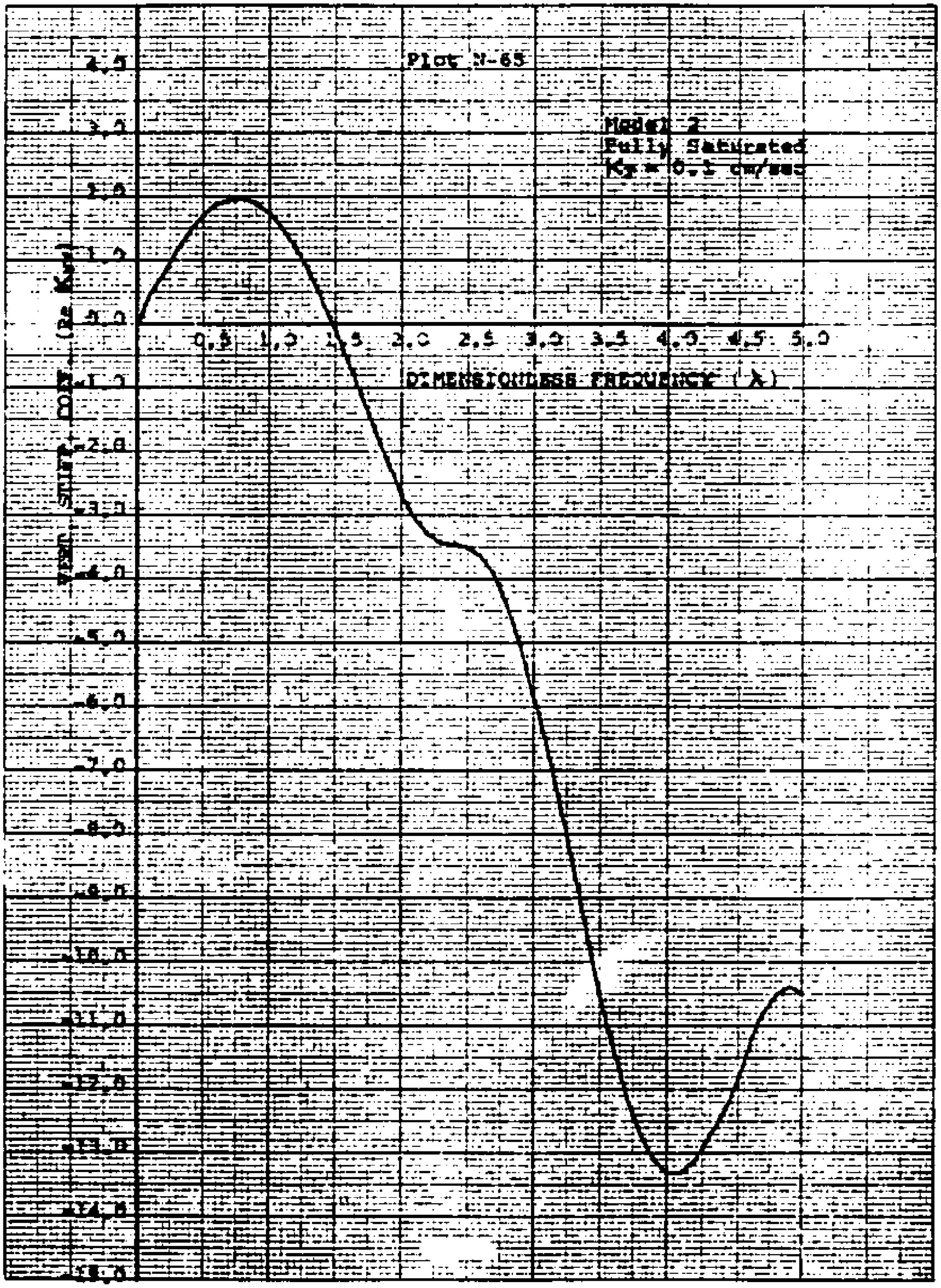

Plot N-60
Model 1#
Saturated to Depth of -1¼A
Ky = 0.1 cm/sec
VERT. DAMP. COEF. [in Kv]
14.0
13.0
12.0
11.0
10.0
9.0
8.0
7.0
6.0
5.0
4.0
3.0
2.0
1.0
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
5.0
DIMENSIONLESS FREQUENCY (λ)

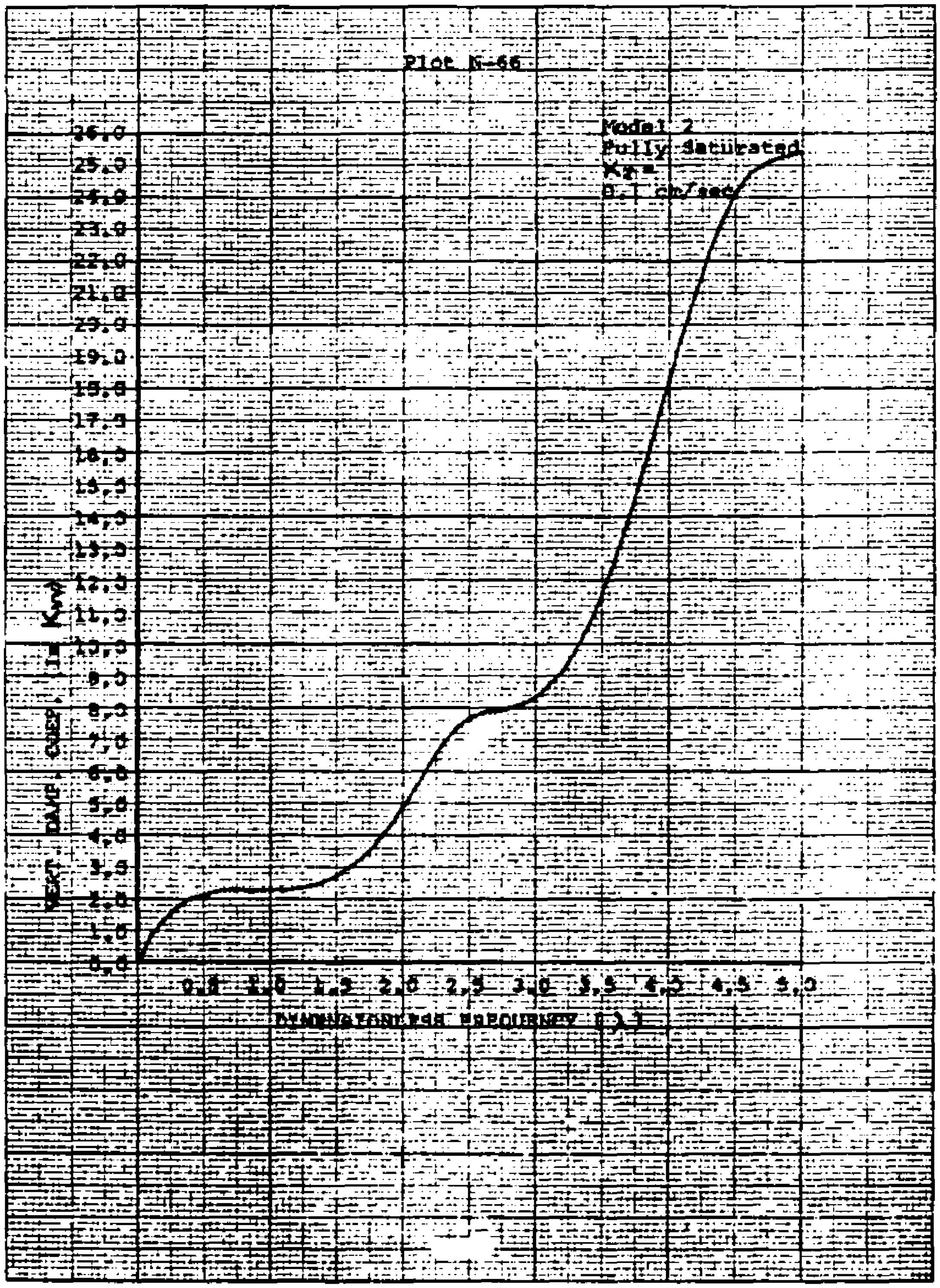

Plot N-61
Model 1C
Saturated to Depth of -2A
Kv = 0.1 cm/sec
VERT. STIFF. COEF. (Re Kvv)
5.0
4.0
3.0
2.0
1.0
0.0
-1.0
-2.0
-3.0
-4.0
-5.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
5.0
DIMENSIONLESS FREQUENCY (λ)

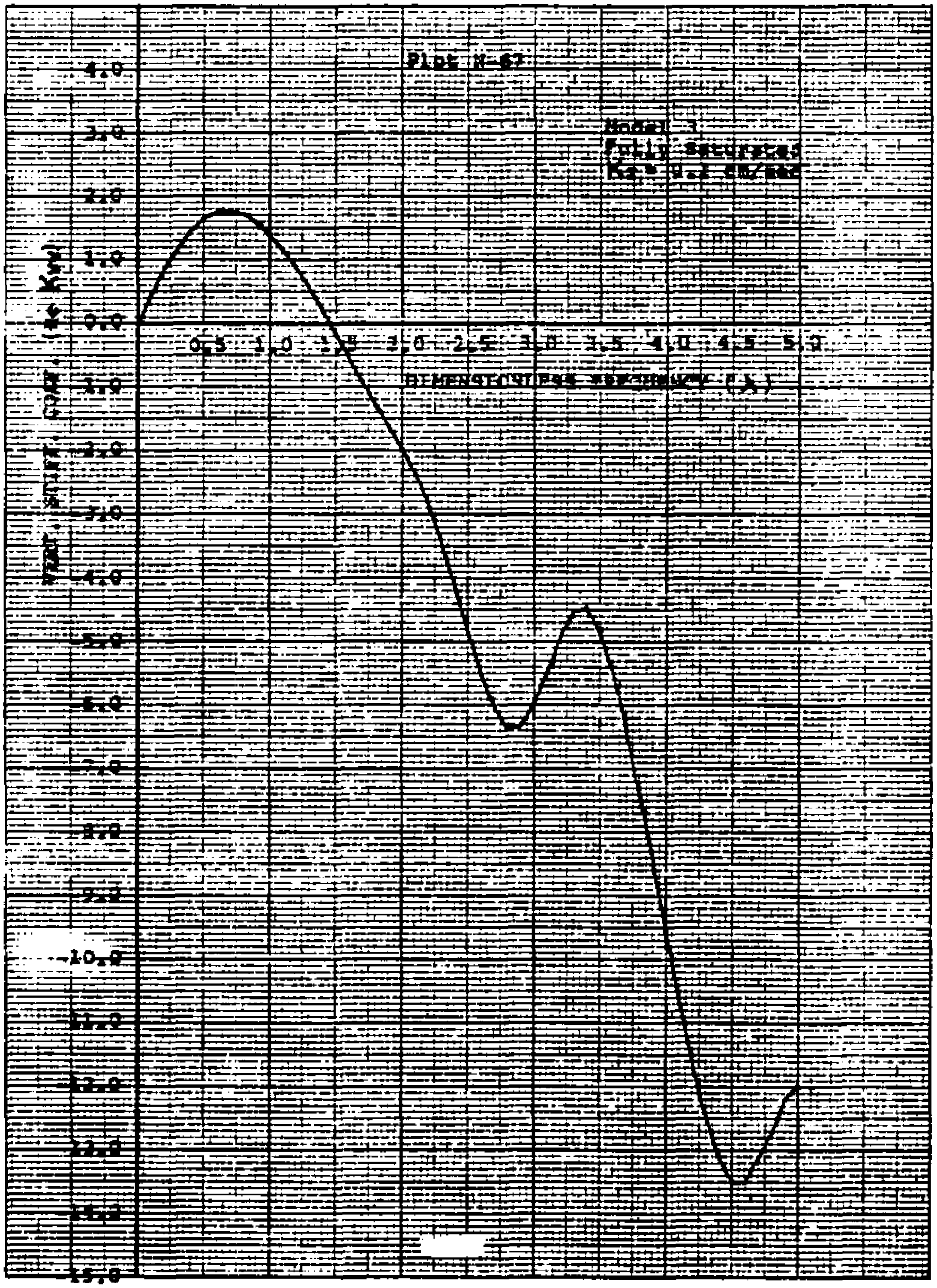

Plot N-62
Model IC
Saturated to Depth of -2A
K_3 = 0.1 cm/sec
VERT. DAMP. COEF., (Im K_{VV})
10.0
9.0
8.0
7.0
6.0
5.0
4.0
3.0
2.0
1.0
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
5.0
DIMENSIONLESS FREQUENCY (A)

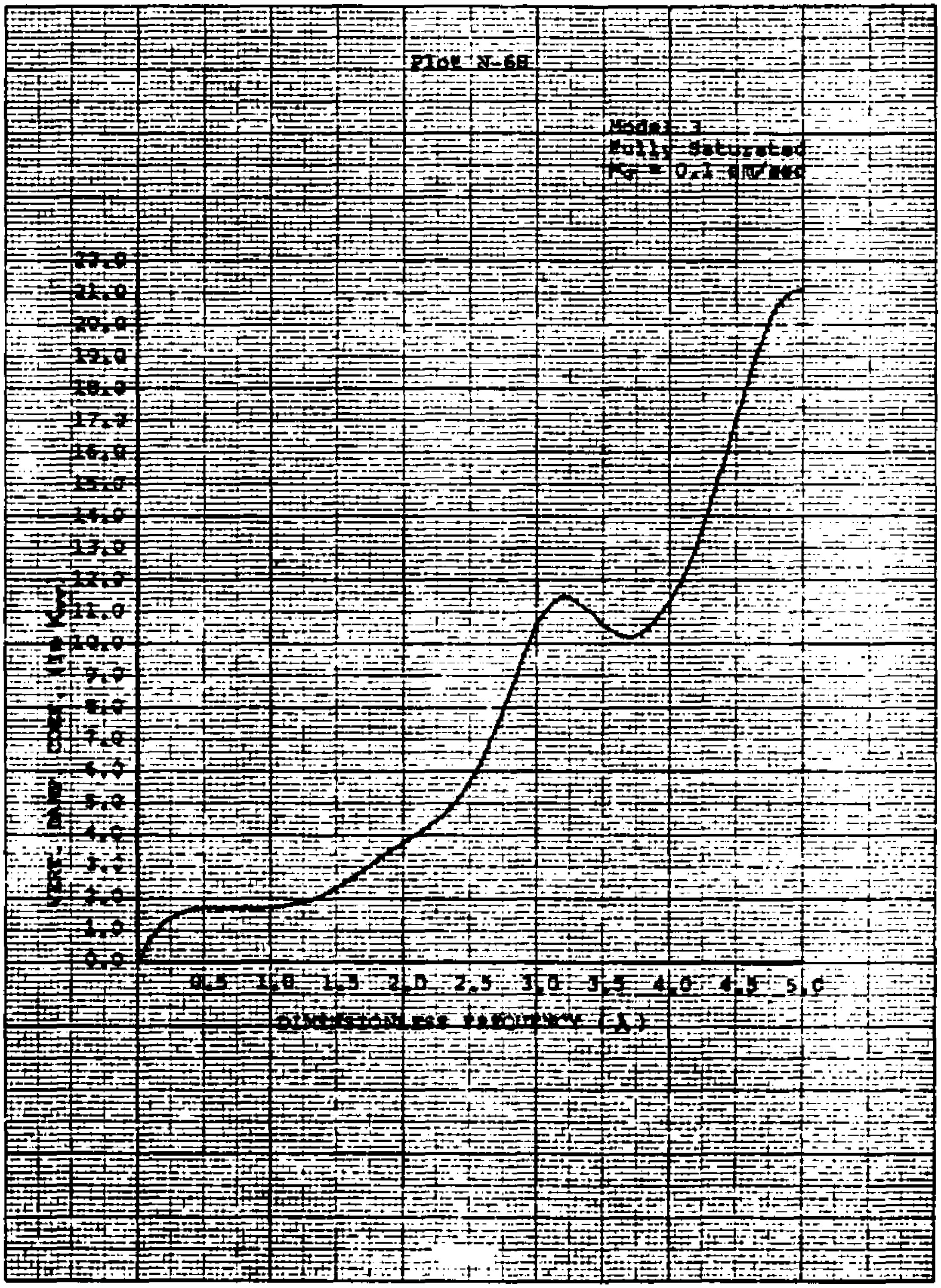

Plot N-63
Model 1
Dry
Finite Element
Analytical
VERT. STIFF. COEF. (Re Kvv)
5.0
4.0
3.0
2.0
1.0
0.0
-1.0
-2.0
-3.0
-4.0
-5.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
5.0
DIMENSIONLESS FREQUENCY (λ)


Plot N-64
Model 1
Dry
Finite Element
Analytical
VERT. DAMP. COEF. (in Kvv)
10.0
9.0
8.0
7.0
6.0
5.0
4.0
3.0
2.0
1.0
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
5.0
DIMENSIONLESS FREQUENCY (λ)


Plot N-65
Model 2
Fully Saturated
K_z = 0.1 cm/sec
VERT. STIFF. COEF. (Re K_{vv})
DIMENSIONLESS FREQUENCY (λ)
4.0
3.0
2.0
1.0
0.0
-1.0
-2.0
-3.0
-4.0
-5.0
-6.0
-7.0
-8.0
-9.0
-10.0
-11.0
-12.0
-13.0
-14.0
-15.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
5.0


Plot K-66
Model 2
Fully Saturated
K = 0.1 cm/sec
VERT. DAMP. COEF. (1= Kvv)
0.0
1.0
2.0
3.0
4.0
5.0
6.0
7.0
8.0
9.0
10.0
11.0
12.0
13.0
14.0
15.0
16.0
17.0
18.0
19.0
20.0
21.0
22.0
23.0
24.0
25.0
26.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
5.0
DIMENSIONLESS FREQUENCY (λ)


Plot N-67
Model 3
Fully Saturated
Kz= 0.1 cm/sec
VERT. STIFF. COEF. (Re Kvv)
4.0
3.0
2.0
1.0
0.0
-1.0
-2.0
-3.0
-4.0
-5.0
-6.0
-7.0
-8.0
-9.0
-10.0
-11.0
-12.0
-13.0
-14.0
-15.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
5.0
DIMENSIONLESS FREQUENCY (λ)


Plot N-6B
Model 1
Fully Saturated
K_T = 0.1 cm/sec
22.0
21.0
20.0
19.0
18.0
17.0
16.0
15.0
14.0
13.0
12.0
11.0
10.0
9.0
8.0
7.0
6.0
5.0
4.0
3.0
2.0
1.0
0.0
VERT. DAMP. COEF. [illegible]
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
5.0
DIMENSIONLESS FREQUENCY (λ)

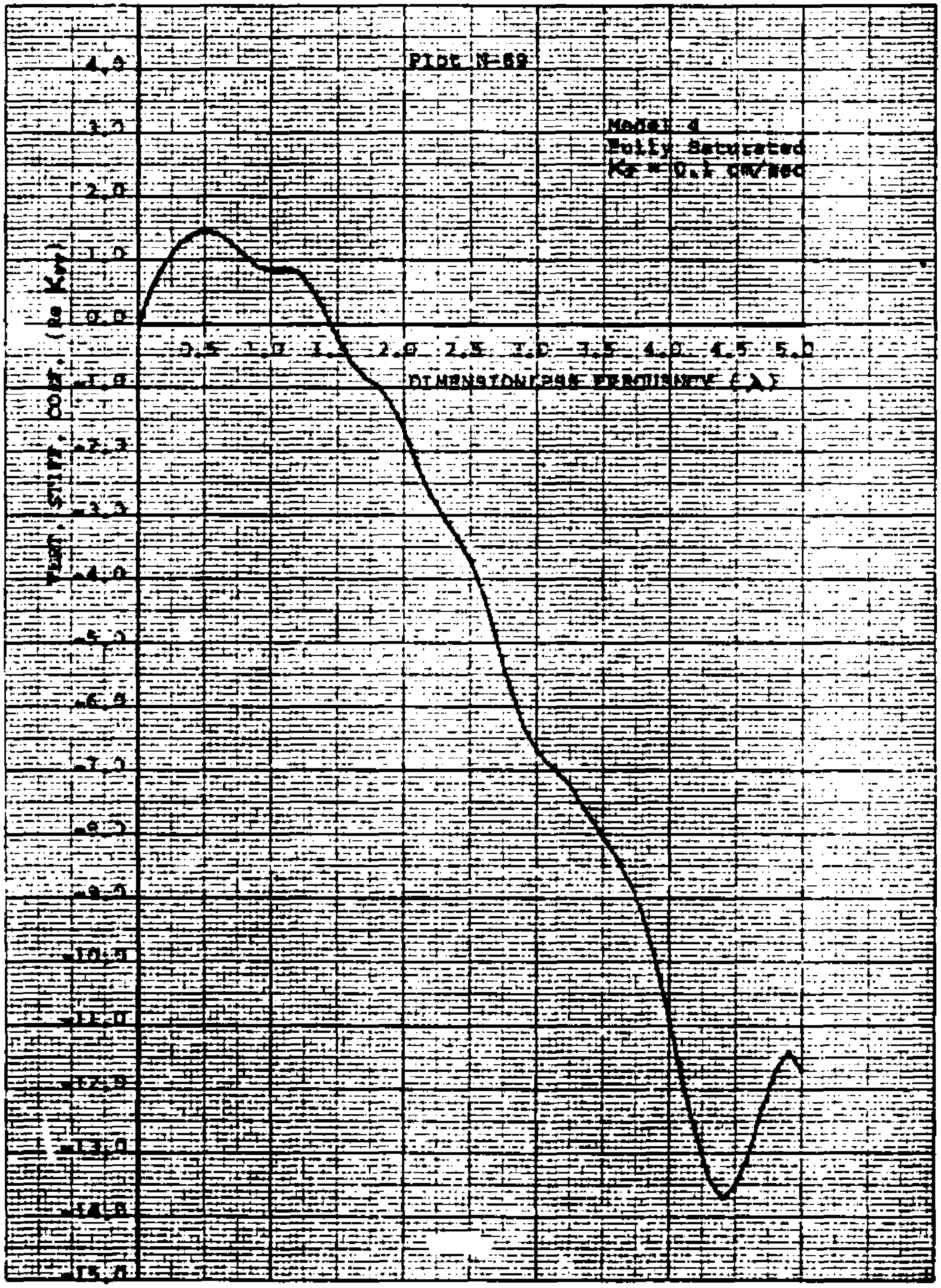
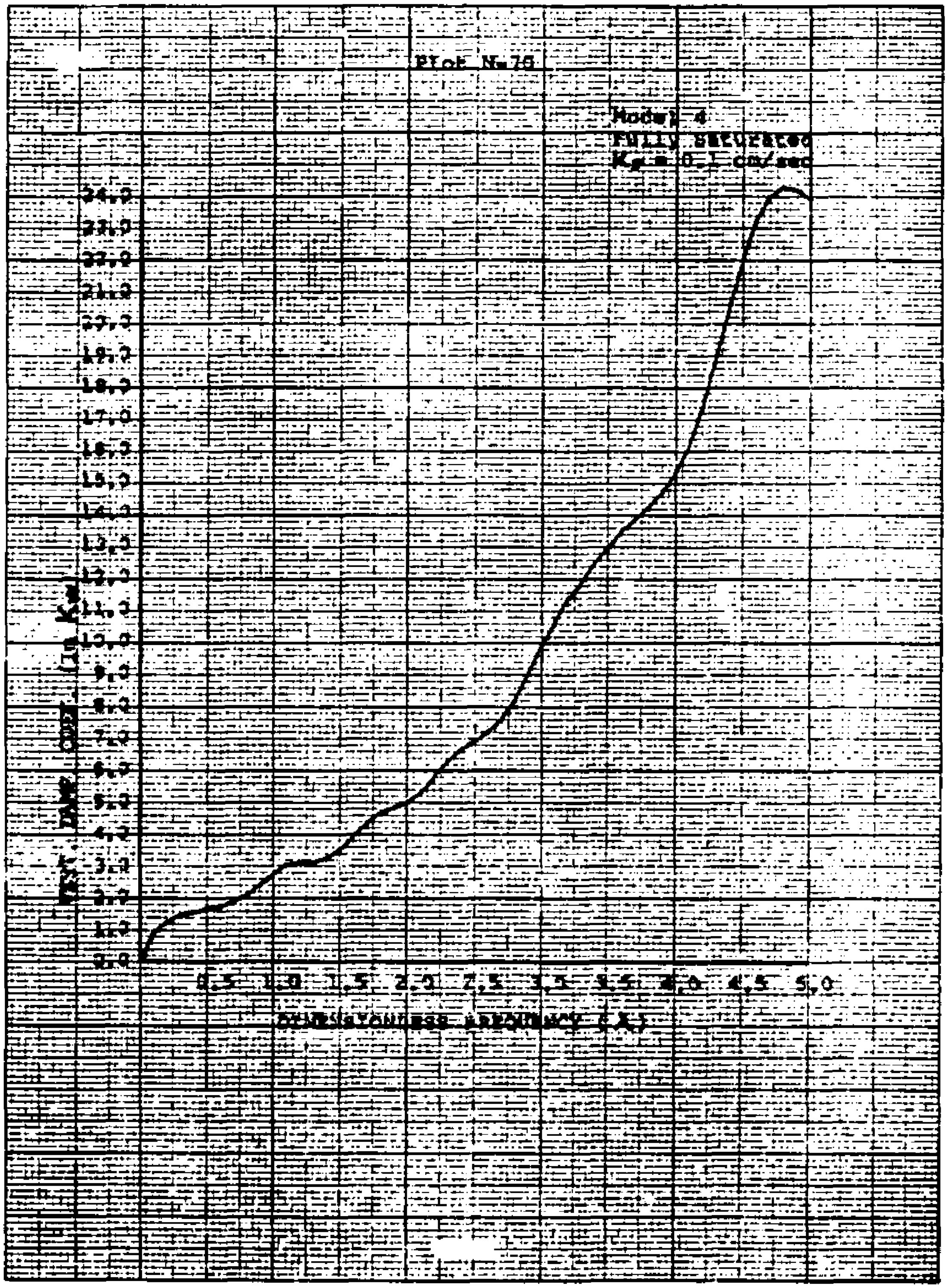
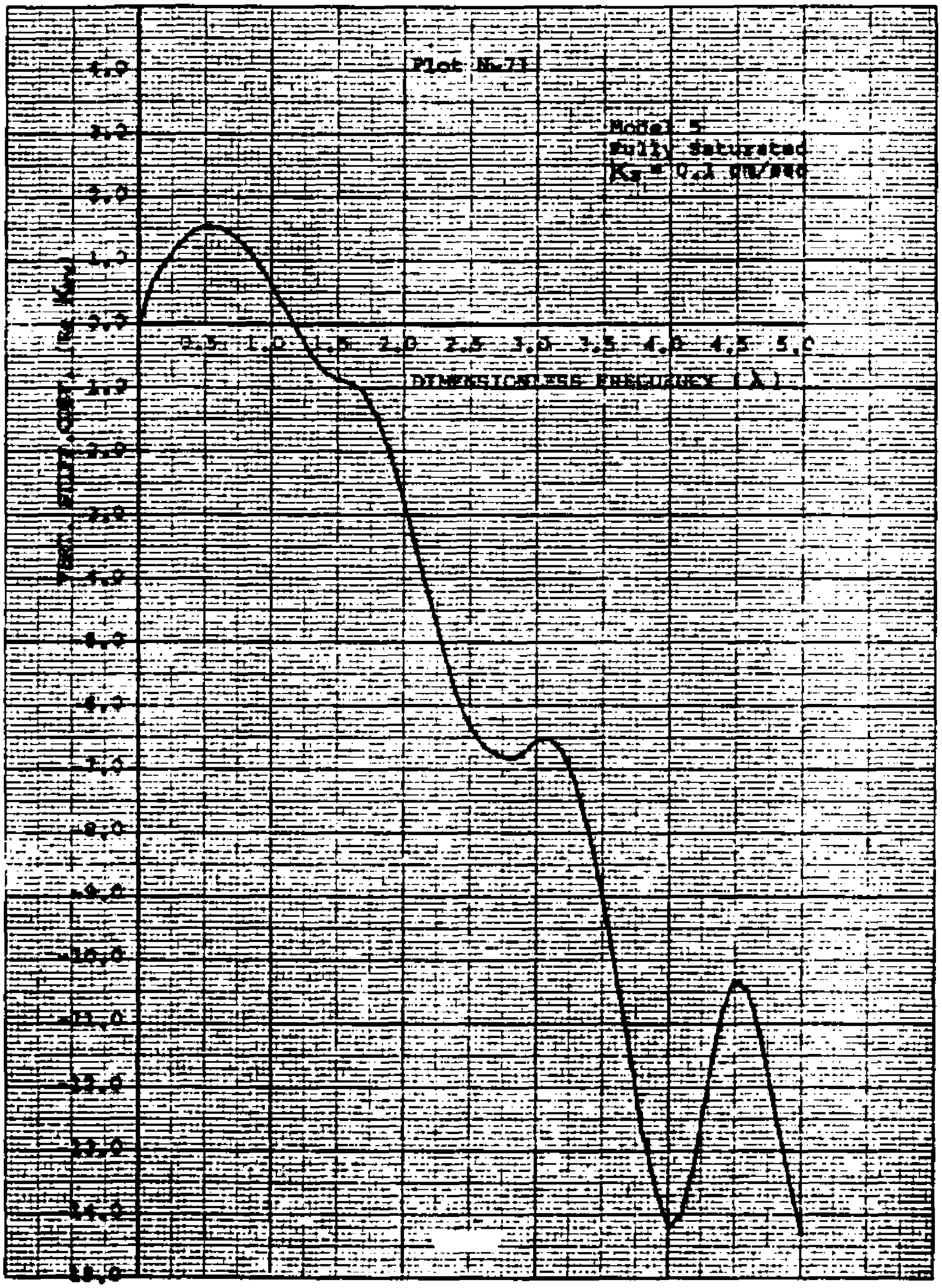

Model 4
Fully Saturated
Kz = 0.1 cm/sec
VERT. STIFF. COEF. (Re K_{vv})
4.0
3.0
2.0
1.0
0.0
-1.0
-2.0
-3.0
-4.0
-5.0
-6.0
-7.0
-8.0
-9.0
-10.0
-11.0
-12.0
-13.0
-14.0
-15.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
5.0
DIMENSIONLESS FREQUENCY (λ)

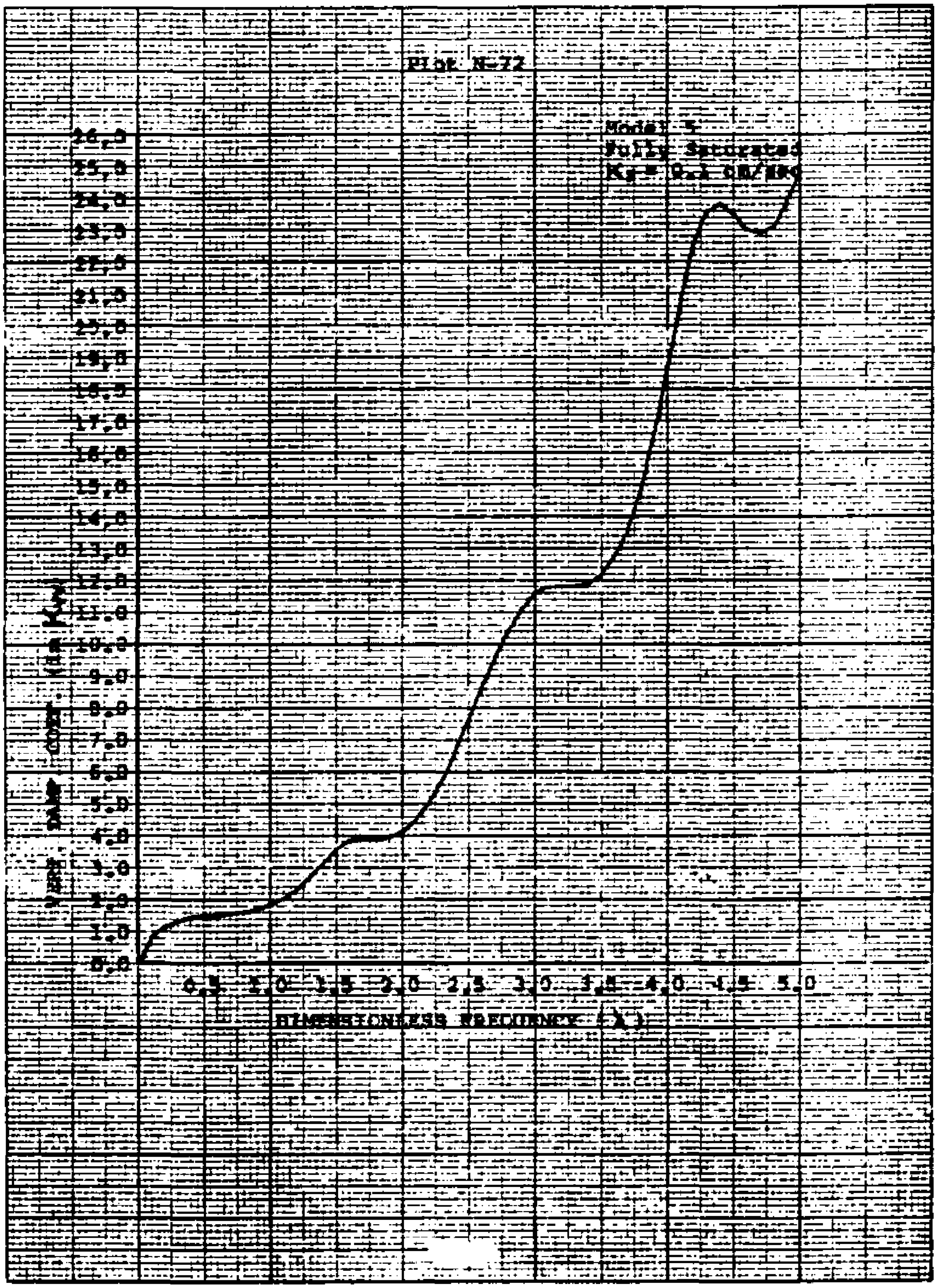
FIG. N-89


Plot No.76
Model 4
Fully saturated
K_s = 0.1 cm/sec
VERT. DAMP. COEF. (In K_w)
0.0
1.0
2.0
3.0
4.0
5.0
6.0
7.0
8.0
9.0
10.0
11.0
12.0
13.0
14.0
15.0
16.0
17.0
18.0
19.0
20.0
21.0
22.0
23.0
24.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
5.0
DIMENSIONLESS FREQUENCY (λ)


Plot No.71
Model 5
Fully Saturated
K_s = 0.1 cm/sec
VERT. STIFF. COEF. (Re. Kvv)
4.0
3.0
2.0
1.0
0.0
-1.0
-2.0
-3.0
-4.0
-5.0
-6.0
-7.0
-8.0
-9.0
-10.0
-11.0
-12.0
-13.0
-14.0
-15.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
5.0
DIMENSIONLESS FREQUENCY (λ)

K+E 20 X 20 TO THE INCH 7 X 10 INCHES KEUFFEL & ESSER CO. MADE IN U.S.A.

46 1242



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