70-1108

PETITO, Joseph Michael, 1942-
CHIRAL DYNAMICS WITHOUT A_1.

The City University of New York, Ph.D., 1969
Physics, nuclear

University Microfilms, Inc., Ann Arbor, Michigan

CHIRAL DYNAMICS WITHOUT A_1

by

JOSEPH M. PETITO

A dissertation submitted to the Graduate Faculty in Physics in partial fulfillment of the requirements for the degree of Doctor of Philosophy, The City University of New York.

1969

This manuscript has been read and accepted for the Graduate Faculty in Physics in satisfaction of the dissertation requirement for the degree of Doctor of Philosophy.

May 21, 1969
date

Chairman of Examining Committee

May 21, 1969
date

Executive Officer

Supervisory Committee

Professor M.A.B. Beg, Rockefeller University
Professor R.M. Lea, City College
Professor R. P. Saxena, City College

Abstract

CHIRAL DYNAMICS WITHOUT A_1

by

Joseph M. Petito

Advisor: Professor Ngee Pong Chang

The nonlinear realization method of Weinberg is used to construct a chiral invariant Lagrangian consisting of π, N, N*(1236) and ρ but not A_1. Chiral invariance can be, of course, maintained without invoking the A_1 meson. In view of the experimental status of A_1, it may be worthwhile studying such a model. The tree diagram technique is used, with an off mass-shell N* propagator, to calculate pion nucleon S wave scattering lengths, isobar production parameters and single pion photoproduction differential cross sections. A comparison with the current algebra method is made.

ACKNOWLEDGMENTS

I would like to thank Professor N. P. Chang for suggesting this problem and for his constant advice and encouragement, without which this thesis would not have been possible.

Also, I am grateful to Professors M.E. Arons and R. P. Saxena for many helpful discussions.

Finally, I would like to thank Rosella Petito who did a wonderful job in typing this manuscript.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

CHAPTER I

INTRODUCTION

In the study of the properties of strong interactions among hadrons (pions, nucleons, etc.), curiously enough, the structure of the weak interactions, in which these hadrons also participate, plays a very central role. This role is embodied in the Gell-Mann postulates of current algebra and the partial conservation of axial vector current (P.C.A.C.) hypothesis, which we briefly review.[1]

From the Fermi theory, it is known that the weak interactions can be described in terms of an effective Lagrangian

$$\mathcal{L}_{eff} = \frac{G_F}{\sqrt{2}} J_\lambda^{*} J_\lambda \tag{1.1}$$

where $G_F = 10^{-5}/m^2_{proton}$ is the Fermi coupling constand and J_λ is the current which consists of leptonic and hadronic parts

$$J_\lambda = J_\lambda^{\ell} + J_\lambda^{h} \tag{1.2}$$

The leptonic current is

$$J_\lambda^{\ell} = \bar{\nu}_\mu i \gamma_\lambda (1-\gamma_5)\mu + \bar{\nu}_e i \gamma_\lambda (1-\gamma_5) e \tag{1.3}$$

where $\mu(e)$ and $\nu_\mu(\nu_e)$ are the fields associated with the muon (electron) and its neutrino. When the effective Lagrangian (1.1) is used to lowest order in G_F, it describes very well both the leptonic and non-leptonic decays of the hadrons. If the leptonic decays of the hadrons are studied, a measure of the matrix elements of the hadron current, J_λ^h, between initial and final hadron states, can, in principle, be obtained.

The hadron current can be decomposed into several pieces: (i) V_λ, vector current, (ii) A_λ, the axial vector current. These currents can in turn be further subdivided into parts labelled by the internal quantum numbers they carry, viz. (a) $V_\lambda^{(\Delta S=0)}$, the strangeness conserving vector current, which transforms as an isovector under isospin rotations of the initial and find hadron states, (b) $V_\lambda^{(|\Delta S|=1)}$, the strangeness changing vector current, which transforms as an $I = 1/2$ spinor under isospin rotations. Similarly for the axial vector current.

These currents transform in a simple way under the operations of a symmetry group which relabels the hadron internal quantum numbers. Thus, under an SU_3 transformation, the vector and axial vector currents seemto transform as members of an octet. The success of the Cabbibo theory of weak interactions[2] in fitting with experiments serves to confirm this.

The current algebra hypothesis of Gell-Mann,[3] with far-reaching consequences, is to fix the scale of the hadron currents by imposing the equal time commutation rules

$$\begin{aligned}
[V_4^i(\vec{x},t), V_4^j(\vec{y},t)] &= -f^{ijk} V_4^k(\vec{x},t)\,\delta(\vec{x}-\vec{y}) \\
[V_4^i(\vec{x},t), A_4^j(\vec{y},t)] &= -f^{ijk} A_4^k(\vec{x},t)\,\delta(\vec{x}-\vec{y}) \qquad (1.4) \\
[A_4^i(\vec{x},t), A_4^j(\vec{y},t)] &= -f^{ijk} V_4^k(\vec{x},t)\,\delta(\vec{x}-\vec{y})
\end{aligned}$$

With these commutation rules to fix the magnitudes of the matrix elements of the hadron currents, the magnitudes of the weak decay are given by (1.1). The most important point is that the commutation rules involve hadron dynamics only and therefore the current algebra hypothesis places restrictions on hadron dynamics.

Evaluation of these commutation relations leads to sum rules which, in principle, can be tested by experiment since the matrix elements of the hadron currents are measurable in weak interaction physics. However, this is not a very practical procedure. In order to obtain useful physical information from these sum rules, the P.C.A.C. hypothesis and infinite momentum limit are introduced.

The P.C.A.C. hypothesis is

$$\partial_\mu \vec{A}_\mu = c\,\vec{\pi} \tag{1.5}$$

where $\vec{A}_\mu$ is the strangeness conserving axial vector current and is the pion field. Note that the divergence of $\vec{A}_\mu$ cannot be zero because the pion has mass and the pion decays weakly. To the extent that m_π is small, $\vec{A}_\mu$ is almost conserved.

The usefulness of the P.C.A.C. condition lies in the relation of the sum rules of (1.4) to the sum rules containing zero pion mass cross sections on hadron targets. The extrapolation from these cross sections to the physical cross sections is expected to be smooth and since m_π^2 is small in hadron physics, the sum rules, now involving measurable quantities, can be confronted with experiment. This was first done by Adler and Weisberger and the succes of current algebra has since been well established in many other areas of low energy hadron dynamics.

The method of chiral dynamics arose from attempts by Weinberg, Schwinger and others[4,5] to place the results of the current algebra and P.C.A.C. procedures in a Lagrangian context. A field theoretic model in which the currents satisfy a $SU_2 \times SU_2$ algebra and the axial vector current satisfies the P.C.A.C. condition is the so called σ model.[6,7]

The Lagrangian in the σ model is

$$\begin{aligned} \mathcal{L} = & -\bar{N}\left[\gamma\cdot\partial - g(\sigma + i\vec{\tau}\cdot\vec{\pi}\gamma_5)\right]N \\ & -\tfrac{1}{2}\left[(\partial_\mu\vec{\pi})^2 + (\partial_\mu\sigma)^2\right] - \frac{m_\pi^2}{2}\left[\vec{\pi}^2+\sigma^2\right] \qquad (1.6) \\ & -\tfrac{1}{2}\left[m_\sigma^2 - m_\pi^2\right]\frac{g^2}{4m_N^2}\left[\vec{\pi}^2+\sigma^2-\frac{m_N^2}{g^2}\right] - \left(\frac{m_N}{g}\right)m_\pi^2\,\sigma \end{aligned}$$

and the P.C.A.C. condition comes from a variation of this Lagrangian under the chiral gauge transformations

$$\begin{aligned} N &\rightarrow \left(1 + i\frac{\vec{\tau}}{2}\cdot\vec{\omega}\gamma_5\right)N \\ \vec{\pi} &\rightarrow \vec{\pi} - \vec{\omega}\,\sigma \qquad (1.7) \\ \sigma &\rightarrow \sigma + \vec{\omega}\cdot\vec{\pi} \end{aligned}$$

The axial vector currents associated with these chiral gauge transformations form an $SU_2 \times SU_2$ algebra with the vector currents associated with the isospin gauge transformations.

In the σ model, chirality is treated as a linear symmetry and the pion is assigned to the four dimensional representation of $SU_2 \times SU_2$ along with an isoscalar, scalar field, the σ meson . Since the σ meson does not seem to appear in nature, it is desirable to find a model where the P.C.A.C. and current algebra relations still hold, but not requiring a σ meson.

Within the framework of the σ model, Weinberg and others[4,8,9] have accomplished a nonlinear realization of chiral symmetry which exhibits all the features of current algebra and P.C.A.C. They transform away the σ meson field by rewriting the chiral and isospin invariant condition

$$\sigma^2 + \vec{\pi}^2 = C^2 \tag{1.8}$$

as a field equation for σ , C being a c-number constant. At the same time, a transformation dependent upon the pion field is performed on the nucleon so that the πNN vertex is pseudovector. In order to reproduce the current algebra results, the Lagrangian, constructed as indicated, is used in lowest order perturbation theory in conjunction with the tree diagram technique. Only those Feynman diagrams, given by lowest order perturbation theory, having no internal loops are considered. It is for this reason that the Lagrangian is called the phenomenological chiral Lagrangian.

It is the purpose of this work to extend the considerations of phenomenological Lagrangians to include the N*, spin 3/2, resonances.[10,11] We rely heavily on the technique developed by Weinberg for a general construction of chiral invariant Lagrangians.[12] This technique is reviewed in the next chapter.

In constructing the chiral Lagrangian, we have omitted the A_1 meson. The doubtful existence of A_1 meson has long been a controversy. While the outcome of this controversy is by no means clear, we entertain here the possibility that A_1 meson may not exist.

It may be surprising, but a chiral invariant Lagrangian can be constructed without the presence of A_1.

This chiral Lagrangian is given in Chapter III. Applications to $\pi N \rightarrow \pi N$, $\pi N \rightarrow \pi N^*$, $\gamma N \rightarrow \pi N$ are discussed in detail in Chapters IV, VI, and VII.

FOOTNOTES AND REFERENCES
FOR CHAPTER I

1. For a thorough discussion of this subject see S. L. Adler and R. F. Dashen, Current Algebras, W. A. Benjamin, Inc., New York (1968). Also, J. Bernstein, Elementary Particles and Their Currents, W. H. Freeman and Co., San Francisco (1968).
2. N. Cabbibo, Phys. Rev. Letters **10**, 531 (1963).
3. M. Gell-Mann, Physics **1**, 63 (1964).
4. S. Weinberg, Phys. Rev. Letters **18**, 188 (1967).
5. J. Schwinger, Phys. Letters **24B**, 473 (1967).
6. J. Schwinger, Ann. Phys. (N.Y.) **2**, 407 (1957); F. Gursey, Nuovo Cimento **16**, 230 (1960); Ann. Phys. (N.Y.) **12**, 91 (1961).
7. M. Gell-Mann and M. Levy, Nuovo Cimento **16**, 705 (1960).
8. J. Wess and B. Zumino, Phys. Rev. **163**, 1727 (1967).
9. B. W. Lee and H. T. Nieh, Phys. Rev. **166**, 1507 (1968).
10. H. W. Huang, Phys. Rev. **174**, 1799 (1968).
11. R. D. Peccie, Phys. Rev. **176**, 1812 (1968).
12. S. Weinberg, Phys. Rev. **166**, 1568 (1968).

CHAPTER II

NONLINEAR REALIZATION METHOD

TRANSFORMATION LAWS

One procedure for constructing general chiral Lagrangians was described by Weinberg.[1] The construction is generally called the nonlinear realization method. In this method, a chiral SU_2 transformation carries the pion field, which has isospin one, into a nonlinear function of itself. This same chiral transformation induces a linear isospin gauge transformation on any other field, ψ except that the gauge depends on the pseudoscalar pion field. The rules for constructing chiral invariant Lagrangians are then simply to write down isospin invariant Lagrangians, except that where derivatives are involved, they are to be replaced by the covariant derivatives, $D_\mu \psi$. $D_\mu \psi$ transforms under chiral transformations in the same way as ψ. In chiral invariant Lagrangians, the pion field appears only in $D_\mu \psi$ and in its own covariant derivative $D_\mu \vec{\pi}$.

There are other methods of constructing chiral Lagrangians that have been discussed in the literature.[2-5] We have chosen to emphasize the method of Weinberg because of its simplicity and far-reaching generality. Since our construction rests so heavily on it, we present a review of the technique in this chapter. We shall also summarize in a table the transformation laws of all the fields that we shall need.

The generators of $SU_2 \times SU_2$ will be denoted by Q^i (isospin generator), $Q_5{}^i$ (chiral generator) with $i = 1,2,3$ and have the commutation relations

$$
\begin{aligned}
[Q^i, Q^j] &= i\,\varepsilon^{ijk} Q^k \\
[Q^i, Q_5{}^j] &= i\,\varepsilon^{ijk} Q_5{}^k \\
[Q_5{}^i, Q_5{}^j] &= i\,\varepsilon^{ijk} Q_5{}^k
\end{aligned}
\tag{2.1}
$$

First, consider the pion field π^j which is intrinsic to all chiral transformations of the realizations of $SU_2 \times SU_2$. Under isospin transformations Q^i, π^j being an isovector, we have

$$[Q^i, \pi^j] = i\,\varepsilon^{ijk}\pi^k \tag{2.2}$$

and under $Q_5{}^i$ we write

$$[Q_5{}^i, \pi^j] = -i f^{ij}(\vec{\pi}) \tag{2.3}$$

where $f^{ij}(\vec{\pi})$ is an arbitrary function of $\vec{\pi}$. From the Jacobi identity

$$\left[Q^i, [Q_5{}^j, \pi^k]\right] + \left[[Q^i, \pi^k], Q_5{}^j\right] + \left[\pi^k, [Q^i, Q_5{}^k]\right] = 0 \tag{2.4}$$

we find

$$[Q^i, f^{jk}(\vec{\pi})] = i\,\varepsilon^{ijl} f^{lk}(\vec{\pi}) + i\,\varepsilon^{ikl} f^{jl}(\vec{\pi}) \tag{2.5}$$

or $f^{ij}(\vec{\pi})$ is an isotopic tensor and combining this with the fact that $f^{ij}(\vec{\pi})$ has positive parity (under parity $Q_5{}^i \longrightarrow -Q_5{}^i$, $\pi^i \rightarrow -\pi^j$), $f^{ij}(\vec{\pi})$ can be written as

$$f^{ij}(\vec{\pi}) = \delta^{ij} f(\vec{\pi}^2) + \pi^i \pi^j g(\vec{\pi}^2) \tag{2.6}$$

It is assumed that $f^{ij}(\vec{\pi})$ does not contain derivatives of the pion field. Next, from the Jacobi identity

$$\left[Q_5^i,[Q_5^j,\pi^k]\right]+\left[[Q_5^i,\pi^k],Q_5^j\right]+\left[\pi^k,[Q_5^i,Q_5^j]\right]=0 \tag{2.7}$$

follows

$$\left[Q_5^i, f^{jk}(\vec{\pi})\right]-\left[Q_5^j, f^{ik}(\vec{\pi})\right]=-i\left[\delta^{ik}\pi^j-\delta^{jk}\pi^i\right] \tag{2.8}$$

or, since
$$\delta f^{jk}(\vec{\pi})=i\,\omega^i\left[Q_5^i, f^{jk}(\vec{\pi})\right]=\omega^i f^{il}(\vec{\pi})\frac{\partial}{\partial\pi^l}f^{jk}(\vec{\pi}) \tag{2.9}$$

ω = infinitesimal chiral parameter

we find

$$\frac{\partial f^{jk}(\vec{\pi})}{\partial\pi^l}f^{il}(\vec{\pi})-\frac{\partial f^{ik}(\vec{\pi})}{\partial\pi^l}f^{jl}(\vec{\pi})=\delta^{ik}\pi^j-\delta^{jk}\pi^i \tag{2.10}$$

and a condition between $f(\vec{\pi}^2)$ and $g(\vec{\pi}^2)$ is obtained

$$g(\vec{\pi}^2)=\frac{1+2f(\vec{\pi}^2)\,f'(\vec{\pi}^2)}{f(\vec{\pi}^2)-2\vec{\pi}^2 f'(\vec{\pi}^2)} \tag{2.11}$$

The transformation (2.3) with $g(\vec{\pi}^2)$ given in (2.11) can be shown to be unique up to a redefinition of the pion filed. Following Weinberg, we pick

$$f(\vec{\pi}^2)=(f_\pi/2)\left(1-f_\pi^{-2}\,\vec{\pi}^2\right) \tag{2.12}$$

where f_π is the pion decay constant. Thus

$$g(\vec{\pi}^2)=f_\pi^{-1} \tag{2.13}$$

and

$$\left[Q_5^i,\pi^j\right]=(f_\pi/2)\left[\left(1-f_\pi^{-2}\vec{\pi}^2\right)\delta^{ij}+2f_\pi^{-2}\pi^i\pi^j\right] \tag{2.14}$$

Now that the pion field transformation are known, we can use them to determine the transformation laws of other fields.

Consider an arbitrary field ψ which transforms under Q^i as

$$\left[Q^i,\psi\right]=-t^i\psi \tag{2.15}$$

where t^i is the appropriate isospin matrix associated with ψ (for the nucleon field N, $t^i=\tau^i/2$). Under Q_5^i, assume that ψ has the following transformation law linear in ψ

$$[Q_5^i, \psi] = \vartheta^{ij}(\vec{\pi})\, t^j \psi \qquad (2.16)$$

If the Jacobi identities (2.4), (2.7), with ψ substituted for π^k, are used, $\vartheta^{ij}(\vec{\pi})$ is determined to be

$$\vartheta^{ij}(\vec{\pi}) = \varepsilon^{ijk}\pi^k \vartheta(\vec{\pi}^2) \qquad (2.17)$$

$$\vartheta(\vec{\pi}^2) = \left[f(\vec{\pi}^2) + \sqrt{f(\vec{\pi}^2) + \vec{\pi}^2}\right]^{-1}$$

and with the form (2.12) for $f(\vec{\pi}^2)$

$$\vartheta(\vec{\pi}^2) = f_\pi^{-1}$$

$$[Q_5^i, \psi] = f_\pi^{-1}\varepsilon^{ijk} t^j \pi^k \psi \qquad (2.18)$$

A covariant derivative of the pion field can be defined such that

$$D_\mu \pi^i = d^{ij}(\vec{\pi})\,\partial_\mu \pi^j \qquad (2.19)$$

$$[Q^i, D_\mu \pi^j] = i\,\varepsilon^{ijk} D_\mu \pi^k$$

$$[Q_5^i, D_\mu \pi^j] = -i\,\vartheta^{il}(\vec{\pi})\,\varepsilon^{ljk} D_\mu \pi^k$$

$d^{ij}(\vec{\pi})$ can be determined by using the Jacobi identity process as before. It turns out that

$$d^{ij}(\vec{\pi}) = \left[1 + f_\pi^{-2}\vec{\pi}^2\right]^{-1} \qquad (2.20)$$

or

$$D_\mu \pi^i = \left[1 + f_\pi^{-2} \vec{\pi}^2\right]^{-1} \tag{2.21}$$

It is necessary to introduce a covariant derivative of the pion field because chiral (Q_5) transformation of the pion field is nonlinear and it is only possible to couple π's and ψ's to form invariants through the covariant derivatives. Pseudovector coupling is, therefore, built into this model, e.g., nucleon and pions are combined in the following manner in a Lagrangian

$$\bar{N} i \gamma_5 \gamma_\mu \vec{\tau} N \cdot D_\mu \vec{\pi} \tag{2.22}$$

This would complete the construction of chiral Lagrangians if only pions and nucleons exist in nature. To extend the construction to other higher spin resonances, it is easy to generalize the discussion above.

The vector mesons may be introduced into the analysis by defining a covariant derivative of the general field ψ

$$\mathcal{D}_\mu \psi = (\partial_\mu - i f_\rho \vec{t} \cdot \vec{\rho}_\mu) \psi \tag{2.23}$$

and requiring

$$\begin{aligned} \left[Q^i, \mathcal{D}_\mu \psi\right] &= -t^i \mathcal{D}_\mu \psi \\ \left[Q_5^i, \mathcal{D}_\mu \psi\right] &= v^{ij}(\vec{\pi}) t^j \mathcal{D}_\mu \psi \end{aligned} \tag{2.24}$$

Since we know the transformation laws of ψ, this imposes the following transformation law on the field ρ_μ^i

$$\left[Q_5^i, \rho_\mu^j\right] = -i\vartheta^{il}(\vec{\pi})\,\varepsilon^{ljk}\rho_\mu^k - i f_\rho^{-1}\partial_\mu \vartheta^{ij}(\vec{\pi}) \tag{2.25}$$

$\vartheta^{ij}(\vec{\pi})$ is given earlier in (2.17), (2.18). A covariant curl $V_{\mu\nu}^i$ of the ρ field can be defined

$$V_{\mu\nu}^i = \partial_\mu \rho_\nu^i - \partial_\nu \rho_\mu^i + f_\rho \varepsilon^{ijk}\rho_\mu^j \rho_\nu^k \tag{2.26}$$

such that

$$\left[Q_5^i, V_{\mu\nu}^j\right] = -i\vartheta^{il}(\vec{\pi})\,\varepsilon^{ljk} V_{\mu\nu}^k \tag{2.27}$$

Note that since the ρ field has a nonlinear chiral transformation law, it is not possible to write a chiral invariant ρ mass term involving the ρ field alone in a Lagrangian. However, a new quantity ϕ_μ^i, which has the same transformation law as the field ψ, viz.

$\left[Q_5^i, \phi_\mu^j\right] = -i\vartheta^{il}(\vec{\pi})\,\varepsilon^{ljk}\phi_\mu^k$ can be constructed out of the $\vec{\rho}_\mu$ and $\vec{\pi}$ fields and is given as follows

$$\phi_\mu^i = \rho_\mu^i + 2 f_\rho^{-1} f_\pi^{-2}\left[1 + f_\pi^{-2}\vec{\pi}^2\right]^{-1}\varepsilon^{ijk}\pi^j \partial_\mu \pi^k \tag{2.28}$$

The ρ mass term is then

$$-\frac{m_\rho^2}{2}\vec{\phi}_\mu^{\,2} \equiv -\frac{m_\rho^2}{2}\left[\vec{\rho}_\mu + 2 f_\rho^{-1} f_\pi^{-2}\left(1 + f_\pi^{-2}\vec{\pi}^2\right)^{-1}\vec{\pi}\times\partial_\mu\vec{\pi}\right]^2$$

$$= -\frac{m_\rho^2}{2}\vec{\rho}_\mu^{\,2} - 2 m_\rho^2 f_\rho^{-1} f_\pi^{-2}\vec{\rho}_\mu\cdot\vec{\pi}\times\partial_\mu\vec{\pi}$$

$$+ \begin{pmatrix}\text{higher order} \\ \text{in the pion field}\end{pmatrix} \tag{2.29}$$

Now, if ψ is identified with the nucleon field and ρ_μ is indentified with the ρ meson field, we see that the ρ NN and $\rho\pi\pi$ coupling constants are not equal (ρ_μ is not coupled to the pion like an ordinary gauge field) but if the condition that ρ is universally coupled to pions and nucleons[6] is imposed, then they are equal and

$$f_\rho^2 = 2 m_\rho^2 f_\pi^{-2} \tag{2.30}$$

which is the Kawarabayashi, Suzuki, Riazuddin and Fayyazuddin relation.[7-9]

Finally we state how the N*(1236) field transforms. It is assumed that the N* field transforms linearly under chiral transformations (similar to the ψ transformation) . Let $N_{\mu j}$ be the positive parity nucleon isobar with IJ = 3/2, 3/2 where μ and j are the space time and isotopic vector indices respectively. There is an implied isospin index, i.e., $N_{\mu j} = (N_{\mu j})_1^\alpha$ $\alpha = 1, 2$ (Appendix A). In the absence of interactions, this object satisfies the subsidiary conditions:

$$\begin{aligned} \gamma_\mu N_{\mu j} &= 0 \\ \partial_\mu N_{\mu j} &= 0 \end{aligned} \tag{2.31}$$

and

$$\tau^j N_{\mu j} = 0 \tag{2.32}$$

Under chiral transformations we have

$$\left[Q_5^i, N_{\mu j}\right] = \vartheta^{il}(\vec{\pi})\left[(\tau^l/2)\delta^{jk} - i\,\varepsilon^{ljk}\right] N_{\mu k} \tag{2.33}$$

The results of this chapter are summarized in Table I.

The N_μ field we use here does not satisfy the Dirac equation

$$(\gamma\cdot\partial + m_{N^*}) N_\mu = 0 \tag{2.34}$$

In the context of Lagrangian field theory, the N_μ field does not appear in the free Lagrangian. Thus, we do not consider the N_μ field to correspond to a fundamental particle.

In order to obtain an understanding of our use of the N_μ field, let us look at the following intuitive picture. Consider π N scattering in the s-channel. The success of current algebra and chiral dynamics seems to indicate that the amplitude (q, q' = pion momenta, s = $-p^2$)

$$\overline{N}\left[S(q, q'; p) - \text{NUCLEON S-CHANNEL POLE TERM}\right] N \tag{2.35}$$

is approximated at low energies by

$$q_\mu q'_\nu \overline{N} \Delta_{\mu\nu} N \tag{2.36}$$

where $\Delta_{\mu\nu}(p)$ is no longer a sensitive function of q, q'. In what follows, we ignore q, q'. The effective propagator is given by

$$\Delta_{\mu\nu}(p) = \int \frac{dm^2}{p^2+m^2}\left\{\sigma_1(m^2)\, S^{\frac{3}{2}}_{\mu\nu}(p) + \sigma_2(m^2)\, S^{\frac{1}{2}+}_{\mu\nu}(p) + \sigma_3(m^2)\, S^{\frac{1}{2}-}_{\mu\nu}(p)\right\} \tag{2.36}$$

where $S^{\frac{3}{2}}_{\mu\nu}(p)$, $S^{\frac{1}{2}+}_{\mu\nu}(p)$, $S^{\frac{1}{2}-}_{\mu\nu}(p)$ are the J = 3/2, 1/2(+parity), 1/2(-parity) spin projection operators respectively. The spectral functions $\sigma_i(m^2)$ contain the correct s-channel unitarity and σ_1 contributes to the J = 3/2 partial wave amplitude, σ_2, σ_3 contribute to the J = 1/2, $\pm$ parity partial wave amplitudes respectively.

Experimentally a resonance (N*) is observed in the πN system at 1238 MeV. The usual procedure would be to assume that the propagator $\Delta_{\mu\nu}$ is approximated well by the N* and use the Rarita-Schwinger[10] propagator for $\Delta_{\mu\nu}$.

In the Rarita-Schwinger formalism, each of the spectral functions σ_1, σ_2, σ_3 are assumed to have a $\delta(m^2 - m_{N^*}{}^2)$ such that the propagator $\Delta_{\mu\nu}$ contains only $J = 3/2$ on the N* mass shell $(p^2 = -m_{N^*}{}^2)$. However, this is an unrealistic assumption about dynamics.

A more realistic approach would be to assume that the $J = 3/2$ partial wave amplitude is saturated in the region of 1238 MeV. by the N* resonance, i.e., σ_1 has a simple pole at $m^2 = m_{N^*}^2 - im_{N^*}\Gamma$ while the $J = 1/2^{\pm}$ spectral functions do not have poles at $m_{N^*}{}^2$ on the second Riemann sheet. If we saturate the propagator in this manner, in the zero width limit, σ_1 has a $\delta(m^2 - m_{N^*}{}^2)$ and the propagator $\Delta_{\mu\nu}$ contains only $J = 3/2$. It is in this manner that we use the N* field.

Of course, $\sigma_1(m^2)$ has the correct analyticity properties, but in the δ-function approximation we lose this analyticity aspect of σ_1. However, in a complete theory, this analyticity and unitarity could be restored by using a full $\sigma_1(m^2)$. The singularities due to the kinematic projection operation $S^{\frac{3}{2}}_{\mu\nu}$ we thus consider spurious.

TABLE I

Summary of the infinitesimal $SU_2 \times SU_2$ transformation properties of the quantities cited in the text.

ISOSPIN	CHIRAL
$\vec{\pi}$ = PION FIELD	
$\delta\vec{\pi} = \vec{\omega}\times\vec{\pi}$	$\delta\vec{\pi} = \frac{f_\pi}{2}\left\{\vec{\omega}(1-f_\pi^{-2}\vec{\pi}^2)+2f_\pi^{-2}\vec{\pi}\,\vec{\omega}\cdot\vec{\pi}\right\}$
$D_\mu\vec{\pi}$ = COVARIANT DERIVATIVE OF THE PION $= [1+f_\pi^{-2}\vec{\pi}^2]^{-1}\partial_\mu\vec{\pi}$	
$\delta D_\mu\vec{\pi} = \vec{\omega}\times D_\mu\vec{\pi}$	$\delta D_\mu\vec{\pi} = f_\pi^{-1}(\vec{\omega}\times\vec{\pi})\times D_\mu\vec{\pi}$
$\vec{\rho}_\mu$ = RHO MESON FIELD	
$\delta\vec{\rho}_\mu = \vec{\omega}\times\vec{\rho}_\mu$	$\delta\vec{\rho}_\mu = f_\pi^{-1}\left\{(\vec{\omega}\times\vec{\pi})\times\vec{\rho}_\mu - f_\rho^{-1}\vec{\omega}\times\partial_\mu\vec{\pi}\right\}$
$\vec{\phi}_\mu \equiv \vec{\rho}_\mu + \frac{f_\rho}{m_\rho^2}[1+f_\pi^{-2}\vec{\pi}^2]^{-1}\vec{\pi}\times\partial_\mu\vec{\pi}$	
$\delta\vec{\phi}_\mu = \vec{\omega}\times\vec{\phi}_\mu$	$\delta\vec{\phi}_\mu = f_\pi^{-1}(\vec{\omega}\times\vec{\pi})\times\vec{\phi}_\mu$

$\vec{V}_{\mu\nu}$ = COVARIANT CURL OF $\vec{\rho}_\mu$	
$= \partial_\mu \vec{\rho}_\nu - \partial_\nu \vec{\rho}_\mu + f_\rho \vec{\rho}_\mu \times \vec{\rho}_\nu$	
$\delta \vec{V}_{\mu\nu} = \vec{\omega} \times \vec{V}_{\mu\nu}$	$\delta \vec{V}_{\mu\nu} = f_\pi^{-1} (\vec{\varpi} \times \vec{\pi}) \times \vec{V}_{\mu\nu}$
N = NUCLEON FIELD	
$\delta N = -i \vec{\omega} \cdot \frac{\vec{\tau}}{2} N$	$\delta N = i f_\pi^{-1} \vec{\varpi} \cdot \frac{\vec{\tau}}{2} \times \vec{\pi} N$
$N_{\mu j} = N^*(1236)$ FIELD	
$\delta N_{\mu j} = \left[-i \vec{\omega} \cdot \frac{\vec{\tau}}{2} \delta_{jk} + \omega_l \epsilon_{jlk} \right] N_{\mu k}$	$\delta N_{\mu j} = f_\pi^{-1} \left[i \vec{\varpi} \cdot \frac{\vec{\tau}}{2} \times \vec{\pi} \delta_{jk} + (\vec{\varpi} \times \vec{\pi})_l \epsilon_{jlk} \right] N_{\mu k}$

Here $\vec{\omega}$ and $\vec{\varpi}$ are the infinitesimal parameters associated with the generators Q and Q_5 respectively.

FOOTNOTES AND REFERENCES

FOR CHAPTER II

1. S. Weinberg, Phys. Rev. 166, 1568 (1968).
2. J. Wess and B. Zumino, Phys. Rev. 163, 1727 (1967).
3. P. Chang and F. Gursey, Phys. Rev. 164, 1752 (1967).
4. B. W. Lee and H. T. Nieh, Phys. Rev. 166, 1507 (1967).
5. See, S. Weinberg, Ref. 1 for a complete list of references to previous literature.
6. J. J. Sakurai, Ann. Phys. (N. Y.) 11, 1 (1960).
7. K. Kawarabayshi and M. Suzuki, Phys. Rev. Letters 16, 225 (1966).
8. Riazuddin and Fayyazuddin, Phys. Rev. 147, 1071 (1966).
9. J. J. Sakurai, Phys. Rev. Letters 17, 552 (1966).
10. W. Rarita and J. Schwinger, Phys. Rev. 60, 61 (1941).

CHAPTER III

A. CHIRAL DYNAMICS WITHOUT A_1

In the conventional analyses of chiral symmetric theories, i.e., those analyses where chirality is treated as a linear symmetry like isospin, both vector and axial vector guage fields must be introduced simultaneously in order to preserve chiral invariance.[1,2] Of course, the introduction of these fields in a phenomenological model requires the indentification of some known particle with them. On the level of $SU_2 \times SU_2$, the vector fields are associated with the well known rho mesons. As far as the axial vector fields are concerned the experimental situation is not quite clear as to the existence of axial vector mesons with which the axial vector fields can be associated.[3] The most likely candidate for an axial vector meson is the A_1 but its existence is in doubt.[4,5] It is, therefore, questionable if chiral Lagrangians should contain the A_1 as a fundamental field.

Note that in the discussion of the chiral transformations of the various fields in the nonlinear realization method, especially the vector or rho field, it was not necessary to introduce an axial vector field to preserve chiral symmetry. In fact, it is possible to introduce vector fields independent of the axial vector fields without destroying the chiral symmetry. We make the simple observation that there can be chiral invariance in a Lagrangian theory <u>without</u> the A_1 field.

Hence, we will consider a chiral invariant phenomenological

Lagrangian model that would apply to reactions involving pions, nucleons, nucleon isobars, ρ mesons, but no A_1 mesons. This model will be used to determine the pion nucleon S wave scattering lengths, partly as a check, nucleon isobar S wave production parameters and single pion photoproduction threshold cross sections, these parameters being most accessible to experimental analysis. The experimental fit is found to be quite good. (Even if we were to include the A_1 meson, its contribution is found to be negligible.)

B. PHENOMENOLOGICAL LAGRANGIAN

In this section we write down the Lagrangian that will be used in our applications. We have followed several overall requirements: (i) We demand chiral invariance (and, of course, isotopic invariance). The nonchiral-invariant part of the Lagrangian, following Weinberg,[6] is assumed to involve only the pion fields and will not affect the reactions to be considered. (ii) We construct the Lagrangian out of nucleons, nucleon isobars N*(1236), pions and ρ mesons. Note first that our discussion is at the level of $SU_2 \times SU_2$ and not that of $SU_3 \times SU_3$. More importantly, we consider reactions involving π, N, N*, γ and at the phenomenological level the Lagrangian should involve these fields (γ comes in through vector dominance). The most important fact about our requirements is that we do not consider the A_1 meson in our Lagrangian.

The experimental status of the A_1 meson is not too clear[4, 5] and it is perhaps relevant theoretically to consider phenomenological theories where the A_1 is not present. We make the simple observation that theories without the A_1 meson can be nevertheless manifestly chiral invariant.

Therefore, based upon the two principle requirements, we have the following:[7]

$$
\begin{aligned}
\mathcal{L} = & -\bar{N}(\gamma\cdot\partial + m_N)N \\
& -\tfrac{1}{2} D_\mu \vec{\pi}\cdot D_\mu \vec{\pi} - \tfrac{1}{4}\vec{V}_{\mu\nu}\cdot\vec{V}_{\mu\nu} - \tfrac{1}{2} m_\rho^2 \vec{\phi}_\mu\cdot\vec{\phi}_\mu \\
& -(g/2m_N)\bar{N} i\gamma_\mu\gamma_5 \vec{\tau} N\cdot D_\mu\vec{\pi} \\
& -(g_{N^*N\pi}/m_\pi) i\bar{N}_{\lambda\gamma}\, g_{\lambda\mu} N D_\mu \pi^\gamma + (g_{N^*N\pi}/m_\pi) i\bar{N} N_{\mu\gamma} D_\mu \pi^\gamma \\
& -(g_{N^*N^*\pi}/2m_{N^*})\bar{N}_{\lambda\gamma}\, g_{\lambda\mu}\, i\gamma_\nu\gamma_5 \vec{\tau} N_{\mu\gamma}\cdot D_\mu\vec{\pi} \\
& + f_\rho \bar{N} i\gamma_\mu \frac{\vec{\tau}}{2} N\cdot\vec{\rho}_\mu \\
& -(f_{\rho N^*N}/m_\pi)\bar{N}_{\lambda\gamma}\, g_{\lambda\mu}\, i\gamma_\nu\gamma_5 N V_{\mu\nu}^\gamma
\end{aligned}
\qquad (3.1)
$$

where the notation and chiral transformations of Chapter II have been utilized. (The relative phases of the coupling terms are those of Sakita and Wali.)[8]

With this Lagrangian at our disposal, we then use the tree diagram technique to perform the calculations for any given process (we consider those graphs given by lowest order perturbation theory where no integration over internal momenta is required or retain only those graphs which have the structure of trees).

C. VECTOR AND AXIAL VECTOR CURRENTS

An interesting aspect of our model, in connection with our deleting A_1 from the Lagrangian, is that the field current identities[9] no longer follow, e.g., in the usual field current identities the axial vector current, apart from a $c\,\partial_\mu\pi$ term, is directly proportional to the A_1 field. This is not so in our model because we have no A_1; but, if the currents are defined in the manner of Gell-Mann-Levy,[10] the currents obey the same $SU_2 \times SU_2$ algebra as before.

The vector current and axial vector current are given by the Gell-Mann-Levy equations

$$
\begin{aligned}
V_\mu^i(x) &= \frac{\delta \mathcal{L}}{\delta\, \partial_\mu \omega^i(x)} \\
A_\mu^i(x) &= \frac{\delta \mathcal{L}}{\delta\, \partial_\mu \omega'^i(x)}
\end{aligned}
\tag{3.2}
$$

and

$$
\begin{aligned}
\partial_\mu V_\mu^i(x) &= \frac{\delta \mathcal{L}}{\delta \omega^i(x)} \\
\partial_\mu A_\mu^i(x) &= \frac{\delta \mathcal{L}}{\delta \omega'^i(x)}
\end{aligned}
\tag{3.3}
$$

where the coordinate dependent form of the transformations in Table I are to be used. In our model these equations are given as (we ignore the N* field for simplicity):

$$\vec{V}_\mu = (m_\rho^2/f_\rho)\vec{\phi}_\mu - f_\rho[1 + f_\pi^{-2}\vec{\pi}^2]^{-1}\vec{\pi}\times(\vec{\phi}_\mu \times \vec{\pi})$$
$$- [1 + f_\pi^{-2}\vec{\pi}^2]^{-2}\vec{\pi}\times\partial_\mu\vec{\pi}$$
$$+ (g/2m_N)[1 + f_\pi^{-2}\vec{\pi}^2]^{-1}\bar{N}i\gamma_\mu\gamma_5\vec{\tau}N\times\vec{\pi} \tag{3.4}$$

$$\vec{A}_\mu = -(f_\pi/2)[1 + f_\pi^{-2}\vec{\pi}^2]^{-2}[(1 - f_\pi^{-2}\vec{\pi}^2)\partial_\mu\vec{\pi} + 2f_\pi^{-2}\vec{\pi}\,\vec{\pi}\cdot\partial_\mu\vec{\pi}]$$
$$+ (f_\rho f_\pi/m_\rho^2)[1 + f_\pi^{-2}\vec{\pi}^2]^{-1}\vec{\pi}\times\vec{\phi}_\mu$$
$$- (g f_\pi/2m_N)\bar{N}i\gamma_\mu\gamma_5\frac{\vec{\tau}}{2}N[1 - f_\pi^{-2}\vec{\pi}^2][1 + f_\pi^{-2}\vec{\pi}^2]^{-1}$$
$$- (g/2m_N f_\pi)\bar{N}i\gamma_\mu\gamma_5\vec{\tau}N\cdot\vec{\pi}\,\vec{\pi}[1 + f_\pi^{-2}\vec{\pi}^2]^{-1} \tag{3.5}$$

From these equations it follows that under isospin transformations

$$\delta\vec{V}_\mu = \vec{\omega}\times\vec{V}_\mu \qquad \delta\vec{A}_\mu = \vec{\omega}\times\vec{A}_\mu \tag{3.6}$$

and under chiral transformations

$$\delta\vec{V}_\mu = \vec{a}\times\vec{A}_\mu \qquad \delta\vec{A}_\mu = \vec{a}\times\vec{V}_\mu \tag{3.7}$$

The transformation laws (3. 6), (3. 7) are equivalent to

$$\left[V_4^i(\vec{x}), V_4^j(\vec{y})\right] = -\varepsilon^{ijk} V_4^k(\vec{x})\,\delta(\vec{x}-\vec{y})$$

$$\left[V_4^i(\vec{x}), A_4^j(\vec{y})\right] = -\varepsilon^{ijk} A_4^k(\vec{x})\,\delta(\vec{x}-\vec{y})$$

$$\left[A_4^i(\vec{x}), A_4^j(\vec{y})\right] = -\varepsilon^{ijk} V_4^k(\vec{x})\,\delta(\vec{x}-\vec{y}) \qquad (3.8)$$

In other words, in a chiral invariant world without the A_1 meson, the currents still satisfy the $SU_2 \times SU_2$ algebra. The axial vector current propagator, in such a world, will not be saturated by a simple 1^+ meson state as can be seen from equation (3. 5).

D. SPIN 3/2 PROJECTION OPERATOR

If the Lagrangian (3.1) is used to determine the pion nucleon S wave scattering lengths, it is found that when the standard N* projection operator

$$\sum_{s=-3/2}^{3/2} N_\mu(p,s)\bar{N}_\lambda(p,s)\gamma_{\lambda\nu} =$$

$$\frac{(-i\gamma\cdot p+m_{N^*})}{2p_0}\left[\delta_{\mu\nu}+\frac{2}{3}\frac{p_\mu p_\nu}{m_{N^*}^2}-\frac{i}{3}\frac{p_\mu\gamma_\nu}{m_{N^*}}+\frac{i}{3}\frac{\gamma_\mu p_\nu}{m_{N^*}}-\frac{1}{3}\gamma_\mu\gamma_\nu\right] \tag{3.9}$$

is employed in the calculation, the experimental fit is not good unless large contact terms (S wave $\bar{N}N\pi\pi$ term in a phenomenological Lagrangian) are postulated ad hoc.[11] These contact terms correspond to the usual σ meson terms of the current algebra. Note, that it might be possible to adjust the coupling constants in (3.1) to give the correct scattering lengths; but in our case these contants are to be independently fixed by experiment.

Also, the high energy behavior of the pion nucleon elastic scattering matrix elements so found is bad. By arranging the contact term (isospin even), the isotopic even amplitude can have good behavior at infinity but the isotopic odd amplitude would still be badly divergent.[12] This prompts us to consider an alternative approach. A new "off mass" shell N* projection operator is proposed and the results obtained using this projection operator are in good agreement with experiment.

We pick an N* projection operator which is a projection operator for spin 3/2 both on and off the mass shell and is given as follows

$$\frac{\Lambda_{\mu\nu}(p)}{2p_0} \equiv \sum_{s=-3/2}^{3/2} N_\mu(p,s)\bar{N}_\lambda(p,s)\, g_{\lambda\nu} =$$

$$\frac{(-i\gamma\cdot p+\sqrt{-p^2})}{2p_0}\left[\delta_{\mu\nu}+\frac{2}{3}\frac{p_\mu p_\nu}{-p^2}-\frac{i}{3}\frac{p_\mu\gamma_\nu}{\sqrt{-p^2}}+\frac{i}{3}\frac{\gamma_\mu p_\nu}{\sqrt{-p^2}}-\frac{1}{3}\gamma_\mu\gamma_\nu\right] \tag{3.10}$$

which was constructed by taking the subsidiary conditions $\partial_\mu N_\mu = 0$, $\gamma_\mu N_\mu = 0$, as operator identities. The usual spin 3/2 projection operator is obtained by putting $p^2 = -m_{N^*}{}^2$ or when $\Lambda_{\mu\nu}(p)$ is on the N* mass-shell. This freedom, in picking a projection operator, is allowed because of the ambiguity in quantizing the N* field, e. g., in Q. E. D. one has the freedom of taking $\partial_\mu a_\mu = 0$ either as a matrix element or operator identity. Also with this choice of N* projection operator a pole and branch cut are introduced at $p^2 = 0$ but since the tree diagram method is only an approximation scheme in lowest order perturbation theory and does not contain unitarity, it is possible that these singularities are cancelled by higher order terms. In any case this is an assumption upon which we proceed.

E. COUPLING CONSTANTS

It is assumed that the coupling constants given in the Lagrangian (3.1) are determined experimentally. The renormalized pion nucleon coupling is g where $g^2/4\pi = 15$ and the universal vector meson coupling constant is f_ρ where $f_\rho^2/4\pi = 2.4$. Also, $g_{N^*N\pi} = 2.13$ which is based on a N* width $\Gamma_{N^*} = 120$ MeV. and $g_{N^*N^*\pi} = (9/5)g$ which can be determined from U(6,6) or superconvergence arguments.[8,13] The ρNN^* coupling constant is determined from the dominant γNN^* coupling constant C_3 by employing vector dominance in the usual manner. Briefly, this is done as follows: Gourdin and Salin have given a phenomenological form for the γNN^* vertex[14,15] which involves three possible couplings, characterized by the constants C_3, C_4, C_5

$$
\begin{aligned}
\mathcal{L} = & -e\, C_3/m_\pi\, \bar{N}_\lambda\, g_{\lambda\mu}\, i\gamma_\nu \gamma_5 N F_{\mu\nu} \\
& -e\, C_4/m_\pi\, \partial_\nu \bar{N}_\lambda\, g_{\lambda\mu}\, \gamma_5 N F_{\mu\nu} \\
& +e\, C_5/m_\pi\, \bar{N}_\lambda\, g_{\lambda\mu}\, \gamma_5 \partial_\nu N F_{\mu\nu}
\end{aligned}
\tag{3.11}
$$

It is then found by experimental fit to single pion photoproduction in the region of the N* that C_3 is the dominant coupling and C_4, $C_5 \approx 0$. Then by using vector dominance,[16] we find $f_{\rho NN^*} = C_3 f_\rho$ where $C_3 = .37$. We remark that the Gourdin and Salin analysis was done with the N* propagator $\Lambda_{\mu\nu}(p)$ on the mass-shell, i.e., $p^2 = -m_{N*}{}^2$. If the propagator (3.10), which is not on the mass-shell, is used, we find that the results of Gourdin and Salin are reproduced because the data was fitted in the region of the N* where $p^2 \approx -m_{N*}{}^2$. We have also investigated the effect on this analysis by the inclusion of the A_1 and found it to be negligible.

The relative phases of the coupling terms are chosen by comparison with the relative phases as would be given by a U(6,6) invariant theory.[8] Purely experimental considerations could not, of course, yield phase information on the coupling constants.

FOOTNOTES AND REFERENCES

FOR CHAPTER III

1. J. Wess and B. Zumino, Phys. Rev. 163, 1727 (1967).

2. B. W. Lee and H. T. Nieh, Phys. Rev. 166, 1507 (1968).

3. B. French, Rapporteur's Talk, Proceedings of the 14th International Conference on High Energy Physics (Cern, Geneva, 1968), p. 91.

4. A partial list of recent references on A_1 is: S. U. Chung, O. I. Dahl, J. Kirz, D. H. Miller, Phys. Rev. 165, 1491 (1967).
   A. W. Key, J. D. Prentice, N. R. Stenberg, E. West, T. S. Yoon, W. A. Cooper, W. Manner, L. Vovvodic, W. D. Walker, Phys. Rev. 166, 1430 (1968).
   Aachen-Berlin-Cern Collaboration, Nuc. Phys. B4, 501 (1968).
   For further references on A_1 see: N. Barash-Schmidt, A. Barbaro-Galtieri, L. R. Price, A. H. Rosenfeld, P. Soping, C. G. Wohl, M. Roos, G. Conforto, Rev. Mod. Phys. 41, 109 (1969).

5. Recently, T. Ferbel, et. al. (Rochester Group), have seen a large A_1 peak in the reaction $K^+p \longrightarrow K^o p\, \pi^+ \pi^+ \pi^-$
   Report given at A. P. S. Meeting , Washington, D.C. (1969).

6. S. Weinberg, Phys. Rev. 166, 1568 (1968).

7. We have used the Pauli Metric, i.e., $X_\mu = (\vec{x}, it)$ and the four momentum of a particle of mass m is given by $p_\mu = (\vec{p}, ip_o)$, $p^2 = p_\mu p_\mu = -m^2$.

8. B. Sakita and K. C. Wali, Phys. Rev. 139, B1355 (1965).

9. T. D. Lee, S. Weinberg and B. Zumino, Phys. Rev. Letters 18, 1029 (1967). N. M. Kroll, T. D. Lee and B. Zumino, Phys. Rev. 157, 1376 (1967).

10. M. Gell-Mann and M. Levy, Nuovo Cimento 16, 705 (1960).

11. B. D. Roy, I. R. Lapidus and M. J. Tausner, Phys. Rev. 177, 2529 (1969), have introduced a scalar meson with a coupling constant adjusted to give the correct S wave scattering lengths. At threshold, this is equivalent to the introduction of a contact term.

12. According to S. Weinberg, Phys. Rev. 177, 2604 (1969), the behavior should be no worse than that indicated by Regge theory.

13. H.F. Jones and M. D. Scadron, Nuovo Cimento 48, 545 (1967).

14. M. Gourdin and Ph. Salin, Nuovo Cimento 27, 193 (1963).

15. M. Gourdin and Ph. Salin, Nuovo Cimento 27, 310 (1963).

16. L. Stodolsky, Phys. Rev. 134, B1099 (1964).

CHAPTER IV

PION NUCLEON S WAVE SCATTERING LENGTHS

Let the amplitude for the process

$$N(p) + \pi^i(q) \longrightarrow N(p') + \pi^j(k) \tag{4.1}$$

where p(p') and q(k) are the four momenta of the incident (final) nucleon and pion respectively, be given by[1]

$$\langle \pi^j(k) N(p') | S | \pi^i(q) N(p) \rangle$$

$$= \delta_{fi} + (2\pi)^4 \delta(p' + k - p - q) \sqrt{\frac{m_N^2}{p_0' p_0} \frac{1}{4 q_0 k_0}}\, T^{ji}$$

$$T^{ji} = \bar{N}(p') \left[A^{ji} + \tfrac{1}{2} i \gamma \cdot (q + k) B^{ji} \right] N(p) \tag{4.2}$$

and the isospin amplitudes are

$$T^{ji} = T^+ \delta^{ji} + T^- \tfrac{1}{2} [\tau^j, \tau^i]$$

$$T^+ = \tfrac{1}{3} (T^{1/2} + 2 T^{3/2}) \tag{4.3}$$

$$T^- = \tfrac{1}{3} (T^{1/2} - T^{3/2})$$

The normalization is such that the differential cross section is given by

$$\frac{d\sigma}{d\Omega} = \left(\frac{m_N}{4\pi W} \right)^2 \sum_{\text{spins}} |T|^2 \tag{4.4}$$

A further decomposition into two component helicity amplitude form can be given for T as

$$T_{\lambda_2 \lambda_1} = \chi^{\dagger}(\hat{p}_2' \lambda_2)\left[f_1 + \vec{\sigma}\cdot\hat{p}_2\,\vec{\sigma}\cdot\hat{p}_1\, f_2\right]\left(\frac{4\pi W}{m_N}\right)$$

λ_2, λ_1 = helicity of the final and initial nucleons

$$f_1 = \frac{E+m_N}{8\pi W}\left[A - (W - m_N)B\right]$$

$$f_2 = -\frac{E-m_N}{8\pi W}\left[A + (W + m_N)B\right] \tag{4. 5}$$

E= nucleon energy in the c. m. frame

In terms of the usual partial wave amplitudes

$$f_1 + \cos\theta f_2 = \sum_{\ell=0}^{\infty}\left[(\ell+1)f_{\ell+} + \ell f_{\ell-}\right]P_\ell(\cos\theta)$$

$$f_2 = \sum_{\ell=0}^{\infty}\left[f_{\ell-} - f_{\ell+}\right]P_\ell(\cos\theta) \tag{4. 6}$$

θ = c. m. scattering angle

The amplitudes $T^{\pm}$, A, B, f_1, f_2 are all invariant functions of the Mandelstam variables $s = -(p+q)^2$, $t = -(q-k)^2$, $u = -(p-k)^2$ and the

f_ℓ are functions of $s = w^2$ alone. Finally the scattering lengths are

$$a_\ell = \left.\frac{f_\ell}{(\vec{q}^{\,2})^\ell}\right|_{\vec{q}^{\,2}=0} \tag{4.7}$$

Our main interest here is in the S wave (ℓ=0) scattering lengths given by

$$a_S^\pm = \frac{1}{4\pi}\,\frac{1}{1+\frac{m_\pi}{m_N}}\left[A^\pm - m_\pi B^\pm\right]_{\vec{q}^{\,2}=0} \tag{4.8}$$

Referring to the Lagrangian (3.1), it is easily seen that the processes which contribute to elastic π N scattering are N and N*, s and u channel poles, and a ρ meson t channel pole, Fig. 1. The contribution of these processes to the amplitude T^{ji} are calculated using the tree diagram method and given as follows:

$$T_N^{ji} = \frac{g^2}{4\pi}\bar{N}(p')\left[\frac{i\gamma\cdot k\,\gamma_5[-i\gamma\cdot(p+q)+m_N]\,i\gamma\cdot q\,\gamma_5\,P_N^{ji}}{m_N^2 - s}\right.$$

$$\left. + \frac{i\gamma\cdot q\,\gamma_5[-i\gamma\cdot(p-k)+m_N]\,i\gamma\cdot k\,\gamma_5\,P_N^{ij}}{m_N^2 - u}\right]N(p)$$

$$P_N^{ji} = \delta^{ji} + \tfrac{1}{2}[\tau^j, \tau^i]$$

$$T_{N^*}^{ji} = \frac{g_{N^*N\pi}^2}{m_\pi^2} \bar{N}(p') \left[\frac{k_\mu q_\nu \Lambda_{\mu\nu}(p+q) P_{N^*}^{ji}}{m_{N^*}^2 - s} \right.$$

$$\left. + \frac{q_\mu k_\nu \Lambda_{\mu\nu}(p-k) P_{N^*}^{ij}}{m_{N^*}^2 - u} \right] N(p)$$

$$P_{N^*}^{ji} = \frac{1}{3}\left[2\delta^{ji} - \frac{1}{2}[\tau^j, \tau^i] \right]$$

$$T_\rho^{ji} = -\frac{f_\rho^2}{m_\rho^2 - t} \bar{N}(p') \frac{1}{2} i\gamma\cdot(q+k) \frac{[\tau^j, \tau^i]}{2} N(p) \qquad (4.9)$$

where the subscripts indicate the appropriate pole diagram. In terms of the invariant amplitudes (4. 2)

$$A_N^+ = \frac{g^2}{m_N} \qquad B_N^+ = -g^2\left[\frac{1}{m_N^2 - s} - \frac{1}{m_N^2 - u} \right]$$

$$A_N^- = 0 \qquad B_N^- = \frac{g^2}{2m_N^2} - g^2\left[\frac{1}{m_N^2 - s} + \frac{1}{m_N^2 - u} \right]$$

$$A^{\pm}_{N^*} = \begin{pmatrix} 2/3 \\ -1/3 \end{pmatrix} \frac{g_{N^*N\pi}}{m_\pi^2} \left\{ \left[(m_N + \sqrt{s}) \left(\frac{t}{2} - m_\pi^2 + \frac{[s - m_N^2 + m_\pi^2]^2}{6s} + \frac{s - m_N^2}{3} \right) \right. \right.$$

$$\left. - \frac{(s - m_N^2 - m_\pi^2)(s - m_N^2 + m_\pi^2)}{6\sqrt{s}} \right] \frac{1}{m_{N^*}^2 - s}$$

$$\left. \pm \, (s \to u, t \to t) \right\}.$$

$$B^{\pm}_{N^*} = - \begin{pmatrix} 2/3 \\ -1/3 \end{pmatrix} \frac{g_{N^*N\pi}}{m_\pi^2} \left\{ \left[\frac{t}{2} - \frac{2}{3} m_\pi^2 + \frac{[s - m_N^2 + m_\pi^2]^2}{6s} \right. \right.$$

$$\left. + \frac{m_N (s - m_N^2 + m_\pi^2)}{3\sqrt{s}} - \frac{2}{3} m_N (m_N + \sqrt{s}) \right] \frac{1}{m_{N^*}^2 - s}$$

$$\left. \mp \, (s \to u, t \to t) \right\}$$

$$A^+_\rho = A^-_\rho = B^+_\rho = 0$$

$$B^-_\rho = - \frac{f_\rho^2}{m_\rho^2 - t} \tag{4.10}$$

From the expression (4.8) for the S wave scattering lengths

$$a^+_N = - \frac{g^2}{4\pi} \frac{1}{1 + \frac{m_\pi}{m_N}} \, 2 \left(\frac{m_\pi}{2 m_N} \right)^3 \frac{1}{1 - \frac{m_\pi^2}{4 m_N^2}} \, \frac{1}{m_\pi}$$

$$a_N^- = \frac{g^2}{4\pi} \frac{1}{1+\frac{m_\pi}{m_N}} 2\left(\frac{m_\pi}{2m_N}\right)^4 \frac{1}{1-\frac{m_\pi^2}{4m_N^2}} \frac{1}{m_\pi}$$

$$a_{N^*}^{\pm} = 0$$

$$a_\rho^+ = 0$$

$$a_\rho^- = \frac{f_\rho^2}{4\pi}\left(\frac{m_\pi}{m_\rho}\right)^2 \frac{1}{1+\frac{m_\pi}{m_N}} \frac{1}{m_\pi} \tag{4.11}$$

and the scattering lengths $a_s^{\pm}$ are just the sum of the appropriate terms and the result is

$$\begin{aligned} a^+ &\simeq -0.01\, m_\pi^{-1} \\ a^- &\simeq 0.08\, m_\pi^{-1} \end{aligned} \tag{4.12}$$

which is in good agreement with the experimentally determined values[2]

$$\begin{aligned} a^+ &\simeq -0.009\, m_\pi^{-1} \\ a^- &\simeq 0.093\, m_\pi^{-1} \end{aligned} \tag{4.13}$$

Observe that the ρ meson term a_ρ^- is the dominant contribution (about ten times as large as the nucleon term) and is therefore consistent with the usual ρ dominance assumption. This would not be so if the usual on mass-shell N* propagator were used, since the N* pole term would give a large contribution to a^+. It is then necessary to make assumptions about including other particles, whose existence is doubtful, in our Lagrangian, e.g., an IJ = 0 meson, and fix their couplings in such a way that their contribution cancels the large N* value. We avoid this difficulty by our choice of N* propagator and including only known particles in the Lagrangian. Furthermore, it should be noted that this selection of N* propagator introduces a technical problem. Consider the expression (4.10) for the invariant amplitudes $A(s,t,u)$ and $B(s,t,u)$. We find that they have poles and branch cuts at $s, u = 0$ which is in contradiction to what is known about the analytic properties of these amplitudes. These spurious poles and cuts do not cause much difficulty, since the tree diagram method is just an approximation scheme used in lowest order perturbation theory and it is possible that higher order terms will cancel these singularities. Also, we are not too disturbed by these singularities since the poles are at $s, u = 0$ which is far from the physical region and the branch cuts, which are of the square root type, can be chosen away from the physical region in the s plane.

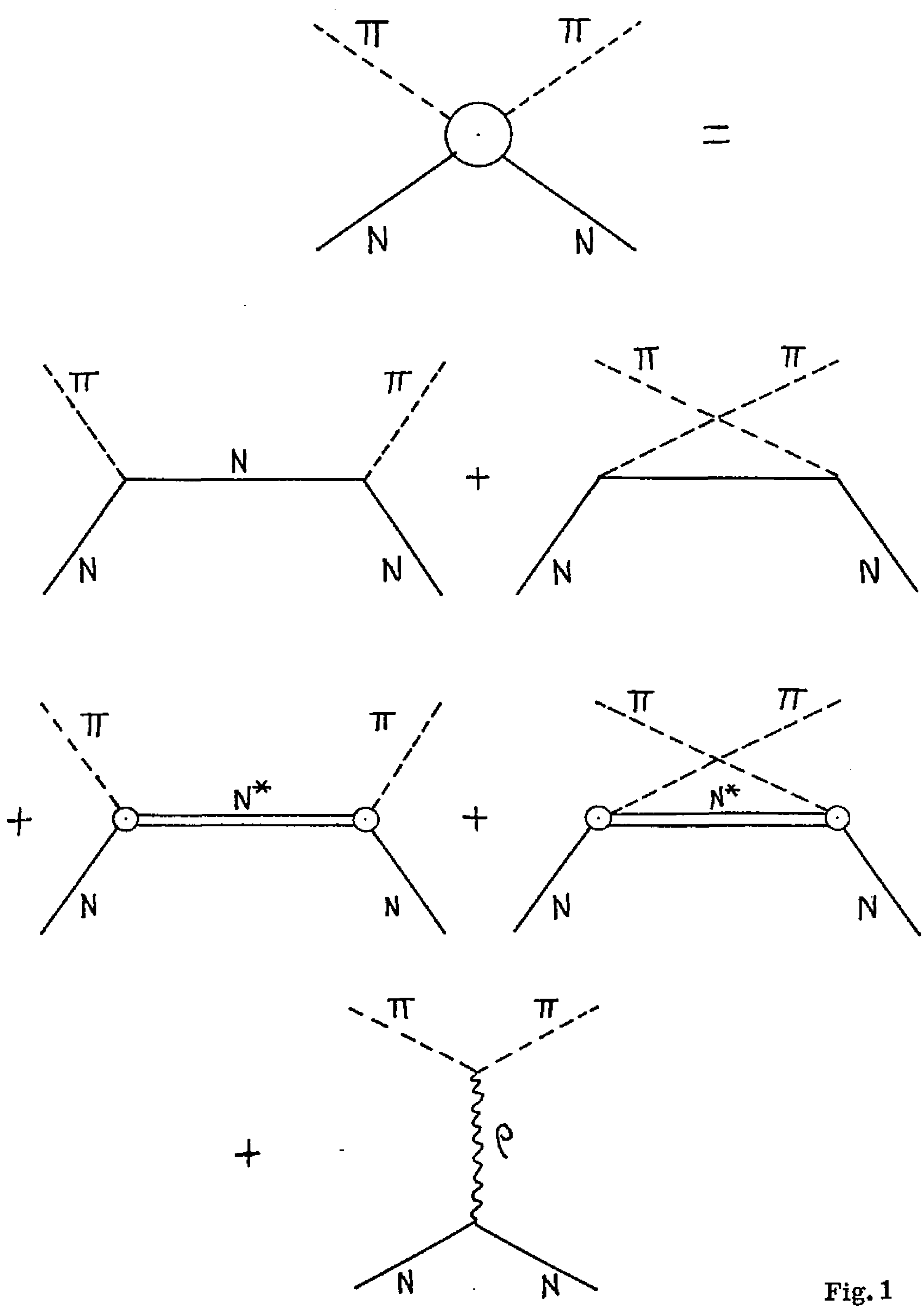

Fig. 1

FOOTNOTES AND REFERENCES FOR CHAPTER IV

1. We have followed the notation of G. F. Chew, M. L. Goldberger, F. E. Low, Y. Nambu, Phys. Rev. 106, 1337 (1957).

2. See K. Raman, Phys. Rev. 164, 1736 (1967) for references to experimental results.

CHAPTER V

CURRENT ALGEBRA

A COMPARISON WITH THE PHENOMENOLOGICAL LAGRANGIAN METHOD

Consider the pion nucleon scattering process

$$N(p) + \pi^i(q) \rightarrow N(p') + \pi^j(k) \tag{5.1}$$

and let the amplitude T^{ji} be given by (4.2). By using the Lehmann, Symanzik, Zimmerman (LSZ) formalism and contracting the final pion

$$\sqrt{\frac{m_N^2}{p_0' p_0}} \frac{1}{\sqrt{2q_0}} T^{ji} = (k^2 + m_\pi^2) \, {}_{in}\langle N(p') | \pi^j(0) | N(p) \pi^i(q) \rangle_{in} \tag{5.2}$$

With this expression it is possible to test the P.C.A.C. hypothesis[1] $\partial_\mu A_\mu^i(x) = m_\pi^2 f_\pi \pi^i(x)$, where $A_\mu(x)$ is the axial vector current and f_π is the pion decay constant (f_π = 190 MeV.). Inserting this relation in the expression (5.2), we find

$$\sqrt{\frac{m_N^2}{p_0' p_0}} \frac{1}{\sqrt{2q_0}} T^{ji} = i (k^2 + m_\pi^2) \frac{k_\mu}{m_\pi^2 f_\pi} \, {}_{in}\langle N(p') | A_\mu^j(0) | N(p) \pi^i(q) \rangle_{in} \tag{5.3}$$

and the usual procedure is to let $q^2 = -m_\pi^2$, $k^2 = 0$ and evaluate this expression by a method devised by Adler.[2] If we let $\nu = -\frac{p \cdot q}{m_\pi}$ then

(5. 3) will give only terms of zeroth order in ν but in order to calculate the pion nucleon S wave scattering lengths first order in ν, which is the main contribution, viz., the ρ meson term in (4. 11), is needed. This requires us to proceed to a further, more complicated expression about which additional assumptions will have to be made.

Contracting out the initial pion in (5. 2)

$$\sqrt{\frac{m_N^2}{p_0' p_0}}\, T^{ji} = (q^2+m_\pi^2)(k^2+m_\pi^2)\, i \int d^4x\, e^{-ik\cdot x} \langle N(p')|T(\pi^j(x)\pi^i(0))|N(p)\rangle \tag{5.4}$$

and substituting the P. C. A. C. expression for both pions

$$\sqrt{\frac{m_N^2}{p_0' p_0}}\, T^{ji} = \frac{(q^2+m_\pi^2)(k^2+m_\pi^2)}{m_\pi^4 f_\pi^2}\, i \int d^4x\, e^{-ik\cdot x} \langle N(p')|T(\partial_\mu A_\mu^i(x)\partial_\nu A_\nu^j(0))|N(p)\rangle \tag{5.5}$$

Applying the usual Ward-Takahashi technique we obtain

$$\begin{aligned}
\sqrt{\frac{m_N^2}{p_0' p_0}}\, T^{ji}
&= \frac{(q^2+m_\pi^2)(k^2+m_\pi^2)}{m_\pi^4 f_\pi^2} \Big\{ i \int d^4x\, e^{-ik\cdot x} k_\mu q_\nu \langle N(p')|T(A_\mu^j(x)A_\nu^i(0))|N(p)\rangle \\
&\quad + i q_\nu \int d^4x\, e^{-ik\cdot x}\, \delta(x_0) \langle N(p')|[A_4^j(x), A_4^i(0)]|N(p)\rangle \\
&\quad - m_\pi^2 f_\pi \int d^4x\, e^{-ik\cdot x}\, \delta(x_0) \langle N(p')|[A_4^i(0), \pi^j(x)]|N(p)\rangle \Big\}
\end{aligned} \tag{5.6}$$

For the first commutator in (5.6) we insert the current algebra relationship

$$\delta(x_0)\left[A_4^j(x), A_\nu^i(0)\right] = -\,\varepsilon^{jik}\,V_\nu^k(0)\;\delta(x) + S.T. \qquad (5.7a)$$

where S.T. is the Schwinger term[3] which is symmetric in ij. It is customary to ignore this term since it only serves to make the T-product covariant and is assumed not to affect the results. The last commutator in (5.6) is symmetric in ij and is assumed to be

$$\delta(x_0)\left[A_4^i(0), \pi^j(0)\right] = \delta^{ij}\,\sigma\,\delta(x) \qquad (5.7b)$$

which is the usual σ meson term. The result is

$$\sqrt{\frac{m_N^2}{p_0' p_0}}\; T^{ji}$$

$$= \frac{(q^2+m_\pi^2)(k^2+m_\pi^2)}{m_\pi^4 f_\pi^2}\Big\{ i\int d^4x\, e^{-ik\cdot x} k_\mu q_\nu \langle N(p')|\,T(A_\mu^j(x)\,A_\nu^i(0))\,|N(p)\rangle$$

$$+\, i\varepsilon^{jik} q_\nu \langle N(p')|\,V_\nu^k(0)\,|N(p)\rangle$$

$$-\, \delta^{ji} m_\pi^2 f_\pi \langle N(p')\,|\sigma|\,N(p)\rangle \Big\} \qquad (5.8)$$

and to evaluate this expression, we let q, k $\longrightarrow$ 0. This procedure gives off mass-shell quantities which must be extrapolated to on shell values by some suitable method.[4]

Calculation of the pion nucleon scattering lengths has been done by Raman[4, 5] and the reader is referred to his paper for details. A brief description of his results for the S wave scattering lengths will be given here and a comparison will be made with those given in Chapter IV obtained by using the phenomenological Lagrangian method.

Consider the T-product in (5.8) which has contributions from N* and N pole terms. Following Raman, it is found the N pole gives the values

$$a_N^+ = -0.0105\ m_\pi^{-1} \qquad a_N^- = 0.0008\ m_\pi^{-1} \tag{5.9}$$

to be compared with those given in (4.11)

$$a_N^+ = -0.010\ m_\pi^{-1} \qquad a_N^- = 0.0008\ m_\pi^{-1} \tag{5.10}$$

For the N* pole, we have the values

$$a_{N*}^+ = -0.06\ m_\pi^{-1} \qquad a_{N*}^- = 0.001\ m_\pi^{-1} \tag{5.11}$$

which are different from the values given in (4.11), which are $a_{N*}^{\pm} = 0$. Also, note that the value of the a_{N*}^+ given by the current algebra method is about ten times larger than the experimental value for the total $a^+ = -0.009\ m_\pi^{-1}$. This difficulty can be removed by assuming that the σ meson term in (5.8) is adjusted in such a way that it gives the experimental result for a^+ when combined with the N and N* contributions.[6] The σ meson term is fixed by this procedure and the consistency can be checked by looking at the P wave scattering lengths.[5] Finally, the vector current term in (5.8) gives

$$a_V^- = \frac{1}{4\pi}\frac{2m_\pi^2}{f_\pi^2}\frac{1}{1+\frac{m_\pi}{m_N}}\frac{1}{m_\pi}$$
$$a_V^+ = 0 \tag{5.12}$$

and using the K.S.F.R. relation , (2.30), we find this is identical with the ρ meson contribution in (4.11). We note that the ρ meson (vector current) term is the dominant contribution to the S wave scattering lengths in both methods.

FOOTNOTES AND REFERENCES FOR CHAPTER V

1. For a review and further details of current algebra and P.C.A.C. calculations, see S. L. Adler and R. F. Dashen, Current Algebras, W. A. Benjamin, Inc., New York (1968).
2. S. L. Adler, Phys. Rev.
3. J. Schwinger, Phys. Rev. Letters 3, 296 (1959).
4. K. Raman and E. C. G. Sudarshan, Phys. Rev. 154, 1499 (1967).
5. K. Raman, Phys. Rev. 159, 1501 (1967); Phys. Rev. 164, 1736 (1967).
6. This procedure (introducing an I = J=0 meson) has been instituted in the phenomenological Lagrangian method by B. D. Roy, I. R. Lapidus, and M. J. Tausner, Phys. Rev. 177, 2529 (1969).

CHAPTER VI

A. $\pi N \to \pi N^*$ S WAVE THRESHOLD PRODUCTION PARAMETERS

The tree diagram method can be easily applied to the production process

$$\pi(q) + N(p) \longrightarrow \pi(k) + N^*(p') \tag{6.1}$$

where N* is produced in a relative s-state. Of course, this method will give N* in any state, but the data is not sufficient to give information on states other than s-states accurately.

Let the matrix element for this process be given by

$$\langle N^*(p')\, \pi(k) \,|\, S \,|\, N(p)\, \pi(q) \rangle = \delta_{fi} + i\,(2\pi)^4\, \delta(p' + k - p - q) \sqrt{\frac{m_{N^*} m_N}{p_0'\, p_0} \frac{1}{4 q_0 k_0}}\; T \tag{6.2}$$

and in terms of the invariant amplitudes

$$T = i\, \bar{N}_\nu(p')\, g_{\nu\mu} \left[A q_\mu + B k_\mu + i \gamma \cdot k \,(C q_\mu + D k_\mu) \right] \gamma_5 N(p) \tag{6.3}$$

where A, B, C, D are invariant functions of s, t, u and multiplication by an overall Clebsch-Gordan coefficient for specific charge states in understood. The value of T at threshold, which is what we are interested in, is given by the following simple two component form:

$$T\Big|_{\vec{p}',\vec{k}=0} = \frac{i\chi_{\gamma}^{\dagger}(s')\vec{\sigma}\cdot\vec{p}\, g_{\gamma}\chi(s)}{\sqrt{2m_N(p_0+m_N)}}\left[A-(m_{N^*}+m_N+m_\pi)B\right]\Big|_{\vec{p}',\vec{k}=0}$$

$$\equiv \frac{i\chi_{\gamma}^{\dagger}(s')\vec{\sigma}\cdot\vec{p}\, g_{\gamma}\chi(s)\ F}{\sqrt{2m_N(p_0+m_N)}} \tag{6.4}$$

$$F = \left[A-(m_{N^*}+m_N+m_\pi)B\right]_{\vec{p}',\vec{k}=0}$$

and the total cross-section at threshold is

$$\left(\frac{q}{p'}\right)\sigma\Big|_{\vec{p}'=0} = \frac{g^4}{12\pi}\,\frac{m_{N^*}}{(m_{N^*}+m_\pi)^2}\,\frac{|F|^2}{p_0+m_N} \tag{6.5}$$

There are five processes that may contribute $\pi N \rightarrow \pi N^*$, Fig. 2. These are the s and u-channel N and N* poles and the t-channel ρ pole. Both the N and N* s-channel poles, Fig. 2(a) and 2(c), will not, however, contribute to isobar production in a final state S wave. This can be seen by the following arguments: (i) The s-channel N pole, Fig. 2(a), does not enter since conservation of angular momentum at the πNN* vertex will permit only relative P waves or higher. (ii) The s-channel N* pole, Fig. 2(c), is forbidden by parity conservation at the πN*N* vertex. If the final πN* is in a relative S wave, then its parity is negative and the parity of the intermediate state is positive, thus, violating parity conservation.

From the Lagrangian (3.1) of Chapter III, we find that these diagrams give

$$T_N = \frac{g_{N^*N\pi}}{m_\pi}\frac{g}{2m_N}\, i\,\bar{N}_\lambda(p')\, g_{\lambda\sigma}\, q_\sigma \left[-1 + \frac{2m_N(m_N+m_{N^*})}{m_N^2-u} + \frac{2m_N}{m_N^2-u}\, i\gamma\cdot q\right]\gamma_5 N(p)$$

$$T_{N^*} = \frac{g_{N^*N^*\pi}}{2m_{N^*}}\frac{g_{N^*N\pi}}{m_\pi}\, i\bar{N}_\lambda(p')\, g_{\lambda\sigma}\Bigg\{$$

$$q_\sigma\left(-\frac{2}{3}\frac{(p-k)\cdot k}{u} + \frac{\sqrt{u}-m_N}{3\sqrt{u}} + \frac{2}{3}\frac{(m_{N^*}+\sqrt{u})}{\sqrt{u}}\frac{(p-k)\cdot k}{m_{N^*}^2-u} - \frac{2}{3}\frac{(m_{N^*}+\sqrt{u})(m_{N^*}+m_N)}{m_{N^*}^2-u}\right)$$

$$+ k_\sigma$$

$$+ i\gamma\cdot q\, q_\sigma\left(-\frac{2}{3}\frac{(p-k)\cdot k}{u}\frac{(m_{N^*}+\sqrt{u})}{m_{N^*}^2-u} - \frac{1}{3\sqrt{u}} + \frac{1}{3}\frac{(m_{N^*}+\sqrt{u})}{\sqrt{u}}\frac{(m_{N^*}-m_N)}{m_{N^*}^2-u} - \frac{2}{3}\frac{(m_{N^*}+\sqrt{u})}{m_{N^*}^2-u}\right)$$

$$+ i\gamma\cdot q\, k_\sigma\left(\frac{m_{N^*}+\sqrt{u}}{m_{N^*}^2-u}\right)\Bigg\}\gamma_5 N(p)$$

$$T_\rho = \frac{2 f_{\rho N^* N} f_\rho}{m_\pi} i \bar{N}_\lambda(p') g_{\lambda\nu} \left(- \frac{m_{N^*}+m_N}{m_\rho^2 - t} g_\nu - \frac{1}{m_\rho^2 - t} q_\nu i \gamma \cdot g + \frac{1}{m_\rho^2 - t} k_\nu i \gamma \cdot g \right) \gamma_5 N(p) \tag{6.6}$$

where, as usual, the subscripts indicate the appropriate pole. At threshold these yield the following results:

$$F_N = -\frac{g}{2m_N} \frac{g_{N^*N\pi}}{m_\pi} \alpha_N \left[1 + \frac{m_N}{p_0 - \frac{m_\pi}{2}} \right]$$

$$F_{N^*} = \frac{g_{N^*N\pi}\, g_{N^*N^*\pi}}{3m_\pi} \alpha_{N^*} \frac{m_{N^*}+\sqrt{u}}{2m_{N^*}} \frac{1}{\sqrt{u}} \left[1 + \frac{3\, m_\pi}{m_{N^*}+\sqrt{u}} - \frac{(m_{N^*}+m_N+m_\pi)(m_{N^*}-m_N+2m_\pi)}{m_{N^*}^2 - u} \right]$$

$$F_\rho = 2 f_{\rho N^* N} f_\rho \alpha_\rho \frac{1}{m_\rho^2 - (m_{N^*} - m_N)^2} \tag{6.7}$$

and the α's are the relevant Clebsch-Gordan coefficients. The full threshold amplitude F is given by

$$F = F_N + F_{N^*} + F_\rho \tag{6.8}$$

The isotopic spin content of isobar production is fairly simple. Conservation of isospin allows production $I = 1/2,\ 3/2$ channels, i.e., there will be only two independent amplitudes for this process. For convenience of calculation, we pick the processes $\pi^+ p \to \pi^+ N^{*+}$ and $\pi^- p \to \pi^- N^{*+}$ and list the pertinent Clebsch-Gordan coefficients in Table II. We can decompose the two amplitudes $F(\pi^+ p \to \pi^+ N^{*+})$ and $F(\pi^- p \to \pi^- N^{*+})$ into isotopic spin invariant amplitudes for the $I = 1/2,\ 3/2$ channels as follows:

$$\begin{aligned} F(\pi^+ p \to \pi^+ N^{*+}) &= -\sqrt{2/5}\, F_{3/2} \\ F(\pi^- p \to \pi^- N^{*+}) &= \tfrac{2}{3}\sqrt{2/5}\, F_{3/2} - \tfrac{1}{3} F_{1/2} \end{aligned} \tag{6.9}$$

where the subscripts represent the isotopic spin. In order to compare with experiment, we use the notation of Olsson and Yodh[1] and define production parameters in terms of the F's by mulitplication with an appropriate scale factor. If the S wave production parameters are defined as a_{2I} then

$$a_{2I} = \beta F_I \tag{6.10}$$

where

$$\beta = m_\pi / 500 \tag{6.11}$$

Combining the results of (6.7), (6.9), Table II, we find

$$\begin{aligned} a_3 &\approx 0.0168\, F \\ a_1 &\approx 0.022\, F \end{aligned} \tag{6.12}$$

and the results are in fermis.

B. COMPARISON WITH EXPERIMENT

The experimental analysis of single pion production data, in reactions $\pi N \to \pi\pi N$ above the N* threshold, by Olsson and Yodh,[1] will provide the basis for comparison of the theoretical predictions from the previous section. Some discussion will be required because of ambiguities involved in extracting data on the final πN* states from the observed $\pi\pi$N states.

Olsson and Yodh have been able to fit single pion production data near the N* threshold by assuming that the final $\pi\pi$N state proceeds predominantly through a πN* state in a relative S wave. This requires the initial πN state to be either D_{13} or D_{33} where the notation is L_{2I2J}

L = relative orbital angular momentum of the initial state

J = total angular momentum of the initial state

I = isotopic spin of the initial state

The predictions for the S wave isobar production parameters from the analysis of Olsson and Yodh are

$$a_3 = 0.0175 \pm 0.0008\ F$$

$$\frac{a_1}{a_3} = 3.4 \pm .3 \tag{6.13}$$

or

$$a_1 = 0.059 \pm 0.005\ F \tag{6.14}$$

where the errors are statistical. There are, also, model dependent errors (S wave πN^* final state), determined by Olsson,[2] in the ratio a_1/a_3. His estimate of the error in a_1/a_3 is ± 1 thus introducing further uncertainty in the production parameter a_1. Recently, Morgan has analyzed isobar production in the $I = 1/2$ channel.[3,4] He finds that there are possible effects on the D_{13} amplitude from dipion ($I = J = 0$ being strongest) resonating final states; whereas Olsson and Yodh completely neglect this. Morgan assumes that the D_{13} amplitude is a combination of πN^* and ∇N (Olsson and Yodh consider only πN^*) which would reduce a_1 by a factor of .57. Futhermore, according to Morgan, the Olsson and Yodh estimate of the D_{13} partial wave cross-section is in disagreement with the known experimental result. Instead of the Olsson and Yodh value of 8.5 mb, it should be 4mb reducing a_1 by a factor of $\sqrt{4/8.5} \sim .8$. Combining this with the other factor, we find a_1 should be reduced by about .4 from the Olsson and Yodh value. The "adjusted" value of a_1 is then

$$a_1 \sim 0.022 \text{ F} \tag{6.15}$$

It should be noted that Morgan retains the Olsson Yodh result for the $I = 3/2$ channel. We can now compare the theoretical predictions given in (6.12) with those in (6.13) and (6.15) and we see the agreement is quite good.

C. COMPARISON WITH CURRENT ALGEBRA

The use of current algebra in the usual way, i.e., $q = k$, $q \rightarrow 0$ for calculating isobar production, involves certain technical ambiguities and thus there is no real comparison with current algebra. To the extent possible, we discuss here briefly the difficulties in the current algebra approach.

For instance, consider the expression (5.8) with the final nucleon state $N(p')$ replaced by the isobar state $N^*(p')$ everywhere. There is an immediate difficulty if we set $q = k$, $q \rightarrow 0$. If energy and momentum are to be conserved, this would require $m_N = m_{N^*}$ for the usual application of current algebra or the baryons are extrapolated off the mass shell. Let us consider for a moment the condition $m_N = m_{N^*}$. In applying the reduction technique, then, as usual, we encounter the σ term which cannot now be ignored since it could be isotopic spin two, thus connecting N^* and N and we would have to estimate it somehow. The vector current term does not contribute here since it vanishes in the limit $k = q$, $q \rightarrow 0$, $m_N = m_{N^*}$. A parity and angular momentum analysis of isobar production in a final S state, tells us that the initial πN must be in a D wave and this presents a further difficulty if q, the initial pion momentum, is allowed to go to zero.

An alternative to the current algebra approach is to take one of the pions off the mass shell (in the case of isobar production the natural one would be the final pion) and restrict the other to be on the mass shell. This will provide a test of the P.C.A.C. hypothesis. An expression similar to (5.3) is obtained in the limit $q^2 = -m_\pi^2$, $k \to 0$ or

$$T = i(q^2+m_\pi^2)\,\frac{k_\mu}{f_\pi}\,\langle N^*(p')|\,A_\mu(0)\,|\,\pi(q)N(p)\rangle\Big|_{\substack{k\to 0\\ q^2=-m_\pi^2}} \tag{6.16}$$

Arons has calculated the S wave isobar production parameters using the Adler technique to evaluate the right hand side (R.H.S.) of (6.16). Briefly, only terms on the R.H.S. which have poles as $k \to 0$ will contribute. These terms are just nucleon and N* poles coupled to the external nucleon and N* respectively through the axial vector current A_μ . The results obtained for the production parameters are similar to ours.

Of course another approach would be to keep both pions on the mass shell ($q^2 = -m_\pi^2$, $k^2 = -m_\pi^2$) but all this does is to give the usual L.S.Z. formula for the process $\pi N \to \pi N^*$ which we are not interested in. It should be kept in mind that with any of these procedures, an additional assumption must be made as to how one extrapolates from off the mass shell to on mass shell quantities.

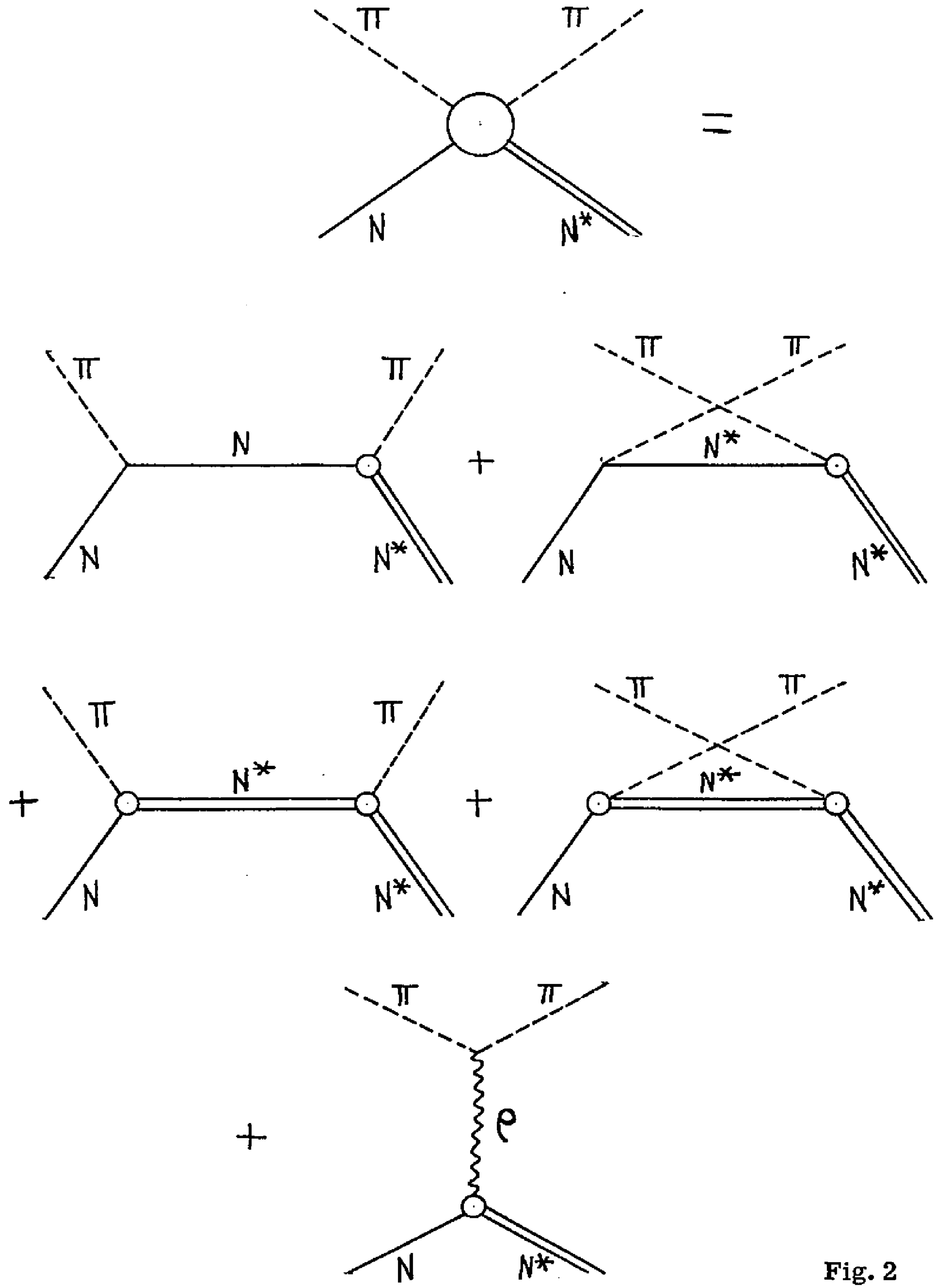

Fig. 2

TABLE II

COEFFICIENT / REACTION	α_N	α_{N^*}	α_ρ
$\pi^+ p \to \pi^+ N^{*+}$	$-\sqrt{\frac{2}{3}}$	$\sqrt{\frac{8}{27}}$	$\sqrt{\frac{2}{3}}$
$\pi^- p \to \pi^- N^{*+}$	0	$-\sqrt{\frac{2}{3}}$	$\sqrt{\frac{2}{3}}$

Table II. Clebsch-Gordan coefficients appropriate to each pole term for given initial and final charge states in isobar production.

FOOTNOTES AND REFERENCES

FOR CHAPTER VI

1. M. G. Olsson and G. B. Yodh, Phys. Rev. 145, 130 (1966).

2. M. G. Olsson, Phys. Rev. Letters 15, 710, 768 E (1965).

3. D. Morgan, Phys. Rev. 166, 1731 (1968).

4. For further details see M. E. Arons, Phys. Rev. 175, 1905 (1968).

CHAPTER VII

SINGLE PION PHOTOPRODUCTION

The Kroll-Ruderman theorem[1] gives the threshold pion photoproduction amplitude to lowest order in m_π/m_N and to all orders in strong interactions in terms of the Born approximation alone. If this amplitude is used to calculate the threshold differential cross-section, it is found to be in disagreement with experiment, indicating that higher order corrections in m_π/m_N are not negligible. The tree diagrams can be evaluated without taking the m_π/m_N limit and can be expected to agree better with experiment than the Kroll-Ruderman theorem. Of course, in the limit of $m_\pi/m_N \to 0$, the tree diagrams reproduce the Kroll-Ruderman results.

Let the invariant amplitude for the process

$$\gamma(q) + N(p) \rightarrow \pi(k) + N(p') \tag{7.1}$$

be given by

$$T = \frac{e}{m_N} \bar{N}(p') \left[A \frac{i\gamma_\mu \gamma_\nu}{2} + B P_\mu \gamma_\nu + C k_\mu \gamma_\nu + D i P_\mu k_\nu \right] \gamma_5 N(p) F_{\mu\nu}(q)$$

$$P_\mu = p'_\mu + p_\mu \tag{7.2}$$

$$F_{\mu\nu}(q) = q_\mu \varepsilon_\nu(q) - q_\nu \varepsilon_\mu(q)$$

At threshold, the differential cross-section is

$$\frac{q}{k}\frac{d\sigma}{d\Omega}\bigg|_{\vec{k}=0} = \frac{e^2}{4\pi}\frac{1}{\left(1+\frac{m_\pi}{m_N}\right)^3}\frac{\left(1+\frac{m_\pi}{2m_N}\right)^2}{m_N^{\,2}}\frac{\left|m_\pi A - P\cdot q\, B - k\cdot q\, C\right|^2}{4\pi} \tag{7.3}$$

Consider the process

$$\gamma p \longrightarrow \pi^+ n \tag{7.4}$$

In the presence of electromagnetism, the interaction part of the Lagrangian (3.1) for this process can be written as

$$\begin{aligned}\mathcal{L}_{EM} = {} & e\,\bar{p}\, i\gamma_\mu p A_\mu - \frac{g}{\sqrt{2}\, m_N}\,\bar{n}\, i\gamma_\mu\gamma_5\, p\, \partial_\mu \pi^- \\ & - i\frac{e\, g}{\sqrt{2}\, m_N}\,\bar{n}\, i\gamma_\mu\gamma_5\, p A_\mu \pi^- + ie(\pi^+\partial_\mu\pi^- - \pi^-\partial_\mu\pi^+)A_\mu \\ & - \frac{e\, C_3}{m_\pi}\,\bar{N}^{*+}_\lambda\, g_{\lambda\mu}\, i\gamma_\nu\gamma_5\, p(\partial_\mu A_\nu - \partial_\nu A_\mu) - \frac{1}{\sqrt{3}}\frac{g_{N^*N\pi}}{m_\pi}\, i\bar{n} N^+_\mu\, \partial_\mu \pi^- \\ & + \frac{f_\rho}{\sqrt{2}}\,\bar{n}\, i\gamma_\mu p\, \rho^-_\mu + \frac{e f_{\rho\pi\gamma}}{m_\pi}\, i\epsilon_{\mu\nu\lambda\sigma}\,\partial_\mu \rho^+_\nu \partial_\lambda \pi^- A_\sigma \\ & + \left(\begin{matrix}\text{HIGHER ORDER}\\ \text{IN THE PION}\end{matrix}\right)\end{aligned} \tag{7.5}$$

where we have added a $\rho\pi\gamma$ interaction term. The $\rho\pi\gamma$ coupling constant is determined from the ω° decay rate into $\pi^\circ\gamma$ by SU_3 arguments and based on a width

$$\Gamma(\omega^\circ \to \pi^\circ\gamma) = 1.15\ \mathrm{MeV}. \tag{7.6}$$

we obtain

$$\frac{1}{3} f_{\omega^\circ\pi\gamma} = f_{\rho\pi\gamma} = .1375 \tag{7.7}$$

A similar Lagrangian can be written for the processes

$$\begin{aligned} \gamma n &\to \pi^- p \\ \gamma p &\to \pi^\circ p \end{aligned} \tag{7.8}$$

The process $\gamma p \to \pi^\circ p$ requires the addition of $\omega\pi\gamma$ and ωNN interaction terms.

$$\mathcal{L}_\omega = \frac{f_\rho}{2} \bar{p} i \gamma_\mu p \, \omega^\circ_\mu + \frac{e\, 3 f_{\rho\pi\gamma}}{m_\pi} i \, \epsilon_{\mu\nu\lambda\sigma} \partial_\mu \omega^\circ_\nu \partial_\lambda \pi^\circ A_\sigma \tag{7.9}$$

Tree diagrams, which contribute to the three photoproduction processes, are given in Figs. 3, 4, 5.[2] At threshold, the N* intermediate states do not contribute to any of the three processes because of our choice of N* propagator. The ρ and ω t-channel diagrams gives negligible contributions to all three processes and thus the nucleon, pion pole diagrams and the contact term give the dominant contribution to all processes. The results for charged pion photoproduction at threshold are

$$\begin{aligned} \frac{q}{k} \frac{d\sigma}{d\Omega}(\gamma p \to \pi^+ n) &= \frac{e^2}{4\pi} \frac{g^2}{4\pi} \frac{1}{2m_N^2} \frac{1}{\left(1 + \frac{m_\pi}{m_N}\right)^3} \\ &= 15 \ \mu \ \text{BARNS/STER.} \end{aligned} \tag{7.10}$$

$$\begin{aligned} R = \frac{\frac{d\sigma}{d\Omega}(\gamma n \to p\pi^-)}{\frac{d\sigma}{d\Omega}(\gamma p \to n\pi^+)} &= 1 + \frac{m_\pi}{m_N} \\ &= 1.15 \end{aligned} \tag{7.11}$$

to be compared with the experimental values[3]

$$\frac{q}{k}\frac{d\sigma}{d\Omega}(\gamma p \to \pi^+ n) = (15.6 \pm 0.5)\ \mu\ \text{barns/ster.} \tag{7.12}$$

$$R = 1.265 \pm 0.075$$

For neutral pion photoproduction

$$\frac{q}{k}\frac{d\sigma}{d\Omega}(\gamma p \to \pi^0 p) = \frac{e^2}{4\pi}\,\frac{g^2}{4\pi}\,\frac{1}{2m_N^2}\,\frac{1}{1+\frac{m_\pi}{m_N}}\,\frac{m_\pi^2}{2m_N^2} \tag{7.13}$$

$$= 0.24\ \mu\ \text{barns/ster}$$

which is about half of the experimental value.

We note that a current algebra calculation of single pion photoproduction, using the P.C.A.C. condition with electromagnetic interaction, gives exactly the same results.[4] Futhermore, if the on mass-shell N* propagator is used, the charged pion photoproduction results remain unchanged and the neutral pion differential cross-section is increased somewhat but not enough to explain the discrepancy with the experimental data.

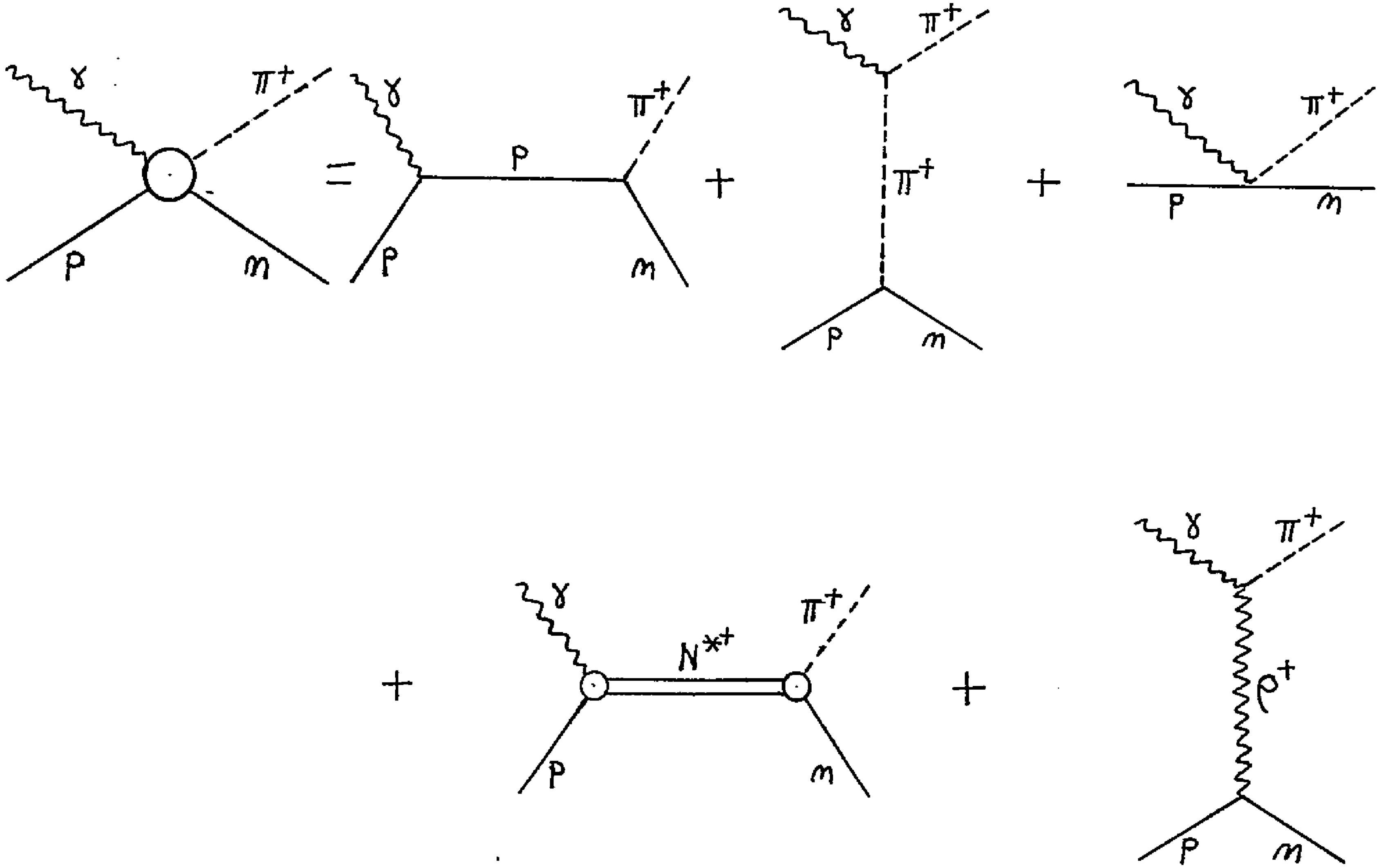

Fig. 3

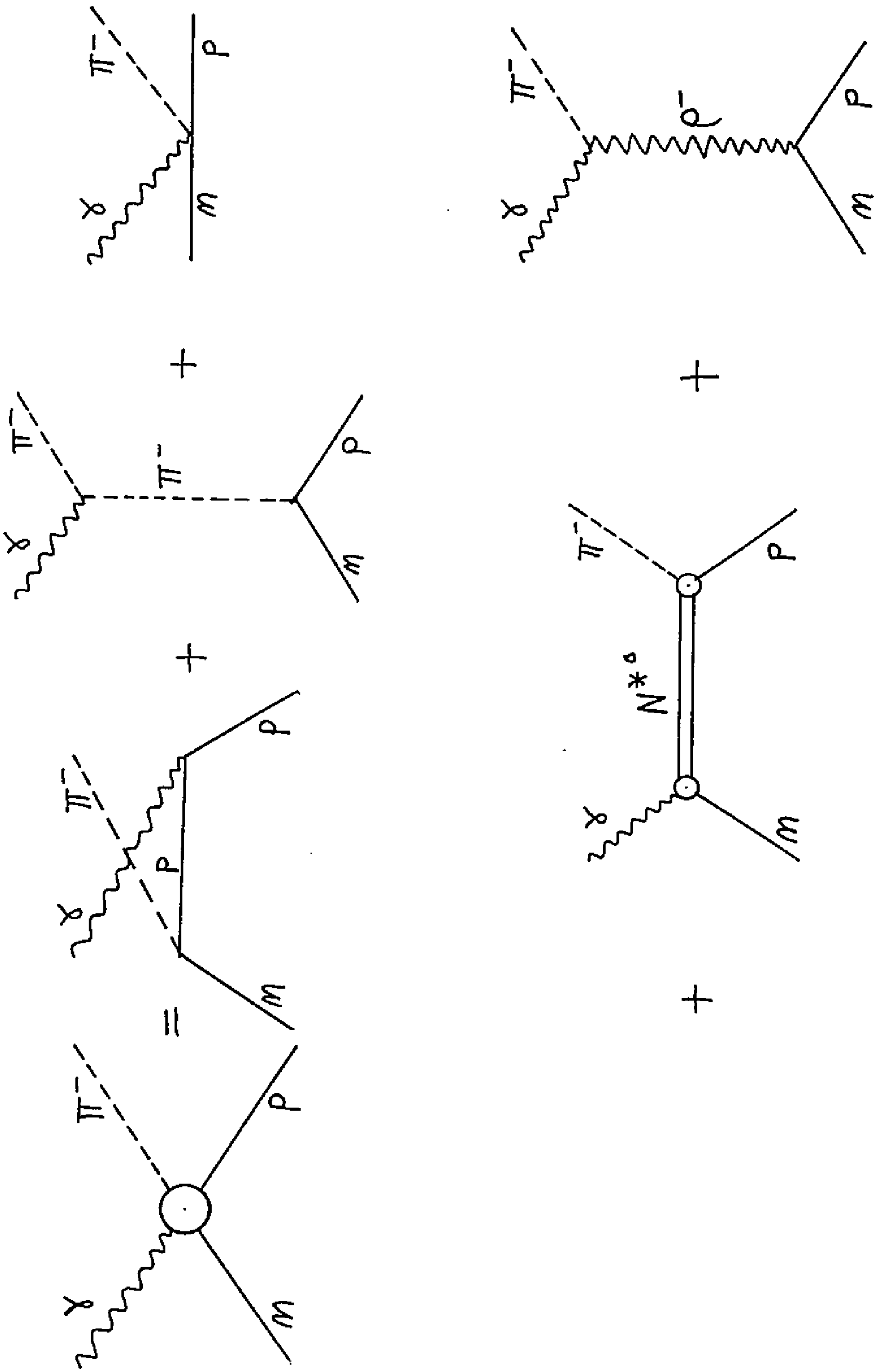

Fig. 4

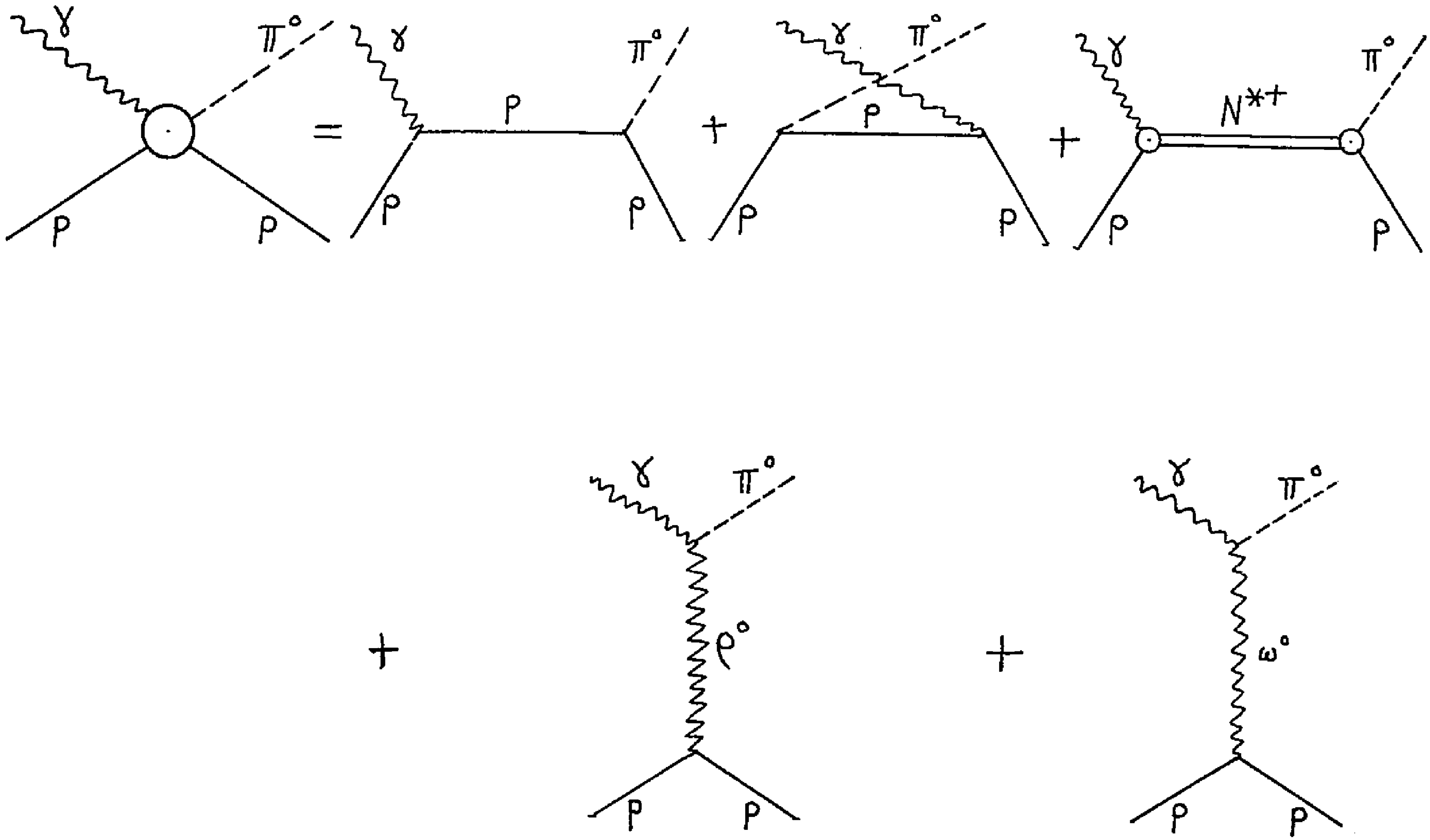

Fig. 5

FOOTNOTES AND REFERENCES

FOR CHAPTER VII

1. N. M. Kroll and M. A. Ruderman, Phys. Rev. 93, 233 (1954).
2. The crossed N* pole diagrams give small contribution and have been neglected. See Ph. Dennery, Phys. Rev. 124, 2000 (1961).
3. References to the experimental results are given by G. W. Gaffney, Phys. Rev. 161, 1599 (1967).

APPENDIX A

For convenience, we indicate here the method used to determine the isospin content of $N_{\mu j}$. In what follows, the space-time index μ is suppressed and the isospinor index $\alpha = 1,2,$ is written explicitly. If Q^ℓ, $\ell = 1,2,3,$ are the isospin generators, then

$$\left[Q^\ell, N_j^\alpha\right] = -\left(\frac{\tau^\ell}{2}\right)^\alpha{}_\beta N_j^\beta + (T^\ell)^{mj} N_m^\alpha \tag{A.1}$$

where $(T^\ell)^{mj} = -i\epsilon^{\ell m j}$. Using this relation and the subsidiary condition

$$\tau^j N_j^\alpha = 0 \tag{A.2}$$

we find

$$N_1^1 = N^{*0}/\sqrt{6} - N^{*++}/\sqrt{2}$$

$$N_2^1 = i\left[N^{*0}/\sqrt{6} + N^{*++}/\sqrt{2}\right]$$

$$N_1^2 = N^{*-}/\sqrt{2} - N^{*+}/\sqrt{6}$$

$$N_2^2 = i\left[N^{*-}/\sqrt{2} + N^{*+}/\sqrt{6}\right]$$

$$N_3^1 = \sqrt{2/3}\, N^{*+}$$

$$N_3^2 = \sqrt{2/3}\, N^{*0}$$

The convention that all operators destroy when operating foward is used.