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A DECISION THEORY APPROACH TO THE PROBLEM OF FAIR SELECTION

by

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INTRODUCTION

Tests for screening of applicants for admission and employment to educational institutions and businesses, have been used extensively. With the advent of the equal right's movement, the question of constructing a fair or unbiased selection procedure has become one of great concern for many institutions. The purpose of this study is to view the question of how one constructs a fair selection model from the viewpoint of decision theory. In this dissertation, a general procedure for constructing a "fair" and "optimal" selection procedure is developed and will be referred to as the Expected Utility Model.

The proposed procedure consists of the assignment of utilities to each selection outcome and the derivation of acceptance scores that maximize the expected utility of selection. In a typical selection situation where the question of "fairness" occurs, one can consider applicants as either "minority" or "majority" group members. For each applicant group, one can consider four outcomes of selection, selecting or rejecting potentially successful applicants and selecting or rejecting potentially unsuccessful applicants. The utility of a particular outcome represents a measure of the "preference" of this outcome to the decision-maker (e.g., the institution). Utilities can be very subjective. In other words, different decision-makers (e.g., the minority group, the majority group, the institution) may assign different utility to a given outcome. Once utilities have been assigned to the outcomes of selection, the decision-maker can derive acceptance scores (i.e., the scores that applicants must achieve to be selected) that maximize the expected utility of the selection process.

In terms of this conceptualization, the fairness of a selection procedure is judged on the basis of the utilities that the decision-maker has assigned to the outcomes of selection. If one agrees with the assignment of utilities, he will consider the selection procedure as fair. On the other hand, if one disagrees with the assignment, he may claim that the selection procedure is unfair. Competing goals of test users introduce disagreement on how these utilities should be assigned. For example, suppose a decision-maker considers the selection of potentially successful minority applicants as more important than the selection of potentially successful majority applicants. In this instance, the decision-maker would assign a greater utility value to the selection of potentially successful minority applicants as compared to the selection of potentially successful majority applicants. If the selection procedure based on this assignment of utilities is followed, the minority group would probably consider the selection procedure as fair while the majority group would probably consider it to be unfair. Therefore, a universal procedure for fair selection can never exist. The best a decision-maker can do is to develop an optimal procedure that maximizes the expected utility of selection after utilities are determined in accordance with his goals. The fairness of a selection procedure is a function of utilities which the decision-maker has assigned to the outcomes of selection.

It is important to realize that a very basic difference exists between the Expected Utility Model and previously proposed models of fair selection (Cleary, 1968; Einhorn and Bass, 1971; Thorndike, 1971; Cole, 1973; Darlington, 1971). The Expected Utility Model derives a selection procedure which is "consistent" with a set of utilities that have been explicitly assigned to the outcomes of selection by the decision-maker (e.g., the institution). The previously proposed models, unlike the Expected Utility Model, do not provide for such an explicit assignment of utilities to the outcomes of selection. Rather, these models derive a set of acceptance scores which satisfy a certain definition of "fairness". Let us now consider the previously proposed models of "fair" selection.

The frequently accepted definition of test fairness requires that two populations of individuals have equal test score means. This definition rests on the assumption that the groups do not differ on the test scores being measured and that any group differences in the test scores are due to bias in the measuring instrument. This assumption of equivalence is frequently part of the argument for so-called culture-free or culture-fair tests. The assumption is judged to be unrealistic as it is certain that almost any variable will show some differences between almost any two specified subpopulations. The equal mean definition also overlooks the fact that the criterion variable is not culture free.

As stated by Thorndike (1971):

> If one acknowledges the differences in average test performance may exist between population A and B, then a judgement on test-fairness must rest on the inferences that are made from the test rather than on a comparision of mean scores in the two populations. One must then focus attention on fair use of test scores, rather than the scores themselves (p.63).

Due to these obvious shortcomings, the equal mean model will not be discussed in this dissertation.

Five other models for "fair use of test scores" have been proposed by Cleary (1968), Einhorn and Bass (1971), Thorndike (1971), Cole (1973) and Darlington (1971). For all of these models, it is assumed that applicants comprise more than one subgroup (e.g., the minority group and the majority group) and only a certain number of institutional openings are available. Each of these models recommends a specific definition of fairness and the selection procedure which satisfies its definition. Again, it should be remembered that these models do not require a decision-maker to state his "utility function" explicitly. In the Regression Model proposed by Cleary (1968), test bias is used to refer to cases in which the criterion score predicted from the common regression line is consistently too high or too low for members of a subgroup. This definition implies a fair selection procedure where separate regression equations are computed for subpopulations and a certain number of applicants having the highest predicted criterion score are selected. The Employer's Model proposed by Einhorn and Bass (1971) considers a selection procedure as fair when the conditional probability of success for the individuals scoring at the acceptance scores is a constant for a given selection problem. In Thorndike's Model (1971), a selection procedure is fair only if the success ratio (i.e., the proportion of potentially successful minority applicants to the proportion of potentially successful majority applicants) equals the acceptance ratio (i.e., the proportion of minority applicants accepted to the proportion of majority applicants accepted). Cole's (1973) definition of test fairness (which will be referred to as the Equal Opportunity Model), requires that a potentially successful applicant should have the same probability for being accepted whether he is a minority or majority group member. The model proposed by Darlington (1971), on the other hand, recommends that the decision-maker (e.g., the institution) must first decide a subjective value in selecting members of the minority group. If it is valuable to obtain minority members, one might think that a minority member's score of y-w (w is greater than zero) on the criterion is as desirable as a score of y on the criterion variable for majority members. The choice of w represents the special value that a decision-maker places in selecting members of the minority group. By using this dependent variable, the applicants having the highest predicted performance are selected. It is interesting to know that for a wide class of selection situations, the selection procedure implied by one model is not necessarily the same as the selection procedure derived from the other models (Darlington, 1971). These five models will be carefully reviewed and evaluated in a later chapter.

In the present study, it will be shown that although these models basically differ from the Expected Utility Model, they still can be viewed in a certain sense in terms of the Expected Utility Model. More specifically, for a given selection problem, if one of these models is employed, the decision-maker can be described as acting as if he possessed a certain utility function. For example, if the Equal Opportunity Model is applied to the selection situations where the minority group has lower mean values than the majority group on the predictor and/or criterion variables, the decision-maker can be considered as if he is placing a greater utility on the selection of potentially successful minority applicants than the selection of potentially successful majority applicants. Using the Expected Utility Model, the utility structures implied by these models can be identified. Each definition of "fair" selection implies the maximization of a different utility structure, although the assignment of preferences to outcomes is not presented in the explicit manner of the Expected Utility Model. The Regression Model and the Employer's Model will be shown to imply the same utility structure for the two subpopulations. In other words, accepting an unsuccessful applicant and rejecting a successful applicant from the minority group are as "costly" as from the majority group. The Darlington's Model also has the same implicit utility structure for the two subpopualtions if the minority group is allowed to have a lower minimal performance level deemed satisfactory than the majority group. The Equal Opportunity Model and Thorndike's Model will be shown to implicitly assign a greater utility to the correctly classified members and/or a greater disutility to the misclassified members of the group which has a poorer predictive validity and a lower mean performance than the other group. Detailed analyses of utility structures for these five models have been performed and will be presented in a later chapter. These analyses will demonstrate the basic difficulties associated with the models which do not assign utilities openly. When utilities are not assigned explicitly, the implicit utilities of selection will depend on data situations and occasionally will be incompatible with the decision-maker's goals. These problems will never occur when the Expected Utility Model is applied.

The manner in which the Expected Utility Model leads one to select minority and majority group members varies as a function of the validity of the test (i.e., the correlation between the predictor variable and the criterion variable), the selection ratio (i.e., the number of institutional openings divided by the total number of applicants), the distribution of the criterion scores (i.e., the mean and the standard deviation of the criterion scores), and the assigned utility structure. In a later section of the dissertation, the proportions of applicants accepted for the minority group and the majority group are computed and will be presented for a variety of data situations and utility structures. These results will be used to demonstrate how the assignment of utilities affects the selection procedure (i.e., the proportions of applicants accepted for the minority group and the majority group) for a given set of data. The effect of the selection ratio, the validity of the test, and the distribution of the criterion scores on selection procedures will also be examined and discussed.

In addition, the Expected Utility Model will be compared with the five previously mentioned fair models in terms of the proportions of applicants accepted for the minority group and the majority group. The six models will also be evaluated on the criteria of: a) the average criterion performance of applicants accepted; b) the success rate of accepted applicants (i.e., the number of potentially successful applicants accepted divided by the total number of applicants accepted); c) the rate of selection errors (i.e., the total number of potentially unsuccessful applicants accepted and potentially successful applicants rejected divided by the total number of applicants); and d) the expected loss of selection. In these comparisons, the validity of the test, the selection ratio, and the distribution of the criterion scores will be varied in selection situations.

The final data analyses consist of a detailed study of real life selection situations. In these analyses, the selection procedure (i.e., the acceptance scores for the minority and majority groups) will first be identified from the proportions of the minority and majority applicants accepted; then using the Expected Utility Model, the implicit utilities associated with the selection procedure of the institution will be identified. These implicit utilities will clearly demonstrate the basic problems associated with the use of selection models (i.e., if they be one of the previously mentioned models for any other procedures) that do not state utilities explicitly.

The real life data were drawn from the 1970 Open Admission Program at the City University of New York. Under the Open Admission Program, the City University of New York has commitment to accept all high school graduates regardless of their academic achievement. However, a selection process is believed to be performed by the central admissions office for every senior college since the number of applicants was far greater than the number of institutional openings. In this study, the selection procedures of four of the senior colleges (e.g., City College, Hunter College, Brooklyn College and Queens College) have been carefully evaluated. The data of each senior college have been first analyzed by sex subgrouping and then by race subgrouping. In these analyses, the Open Admission test scores (e.g., Stanford High School Reading Test and Stanford Arithmetic Test) and the high school grade averages (e.g., the averages of English, Mathematics, Science and Social Studies), were considered to be predictor variables, and the first year college grade point average was considered to be the criterion variable. It must be emphasized that the predictor and criterion variables actually used by the central admissions office were not known, and therefore, the selection procedures studied might only be hypothetical.

In summary, this dissertation consists of the following chapters:

(1) Review of literature: The five previously proposed "fair" models will be reviewed and evaluated.

(2) Expected Utility Model: A method for computing the subgroup acceptance scores that maximize the expected utility of selection will be described.

(3) Viewing "fair" selection models in terms of the Expected Utility Model: It will be shown that each definition of "fair" models implies the maximization of a different utility structure and the implicit utilities of the model may not be compatible with the decision-maker's goals.

(4) The proportions of minority and majority applicants accepted by applying the Expected Utility Model: The effect of utility structures and data situations on the proportions of minority and majority applicants accepted will be examined.

(5) Comparison of the Expected Utility Model and the previously proposed fair models: The Expected Utility Model and the five "fair" models will be compared in terms of the proportions of applicants accepted for the minority group and the majority group, the average criterion performance and the success rate of accepted applicants, the rate of selection errors and the expected loss of selection.

(6) Analyses of the selection process for real life data: It will be shown that there are problems associated with the selection procedures in real life situations when the institution can not state the utilities of outcomes explicitly.

2. REVIEW OF LITERATURE

The definitions of five "fair" models proposed by Cleary (1968), Einhorn and Bass (1971), Thorndike (1971), Cole (1973) and Darlington (1971), and their corresponding "fair" selection procedures will be described and compared. Finally, these models will be evaluated on the criterion of practical implication. It will be shown that some of these models are inconsistent and incompatible with the decision-maker's goals.

Description of the Selection Situation

For all of these five models, it is assumed that applicants comprise k subpopulations (π_i, i=1, 2,----------k). In each of the subpopulations, test scores (X's) and a criterion score (Y) have some joint distribution and can be assumed to be multivariate normally distributed. A test score is any information which is gathered from interviews, letters of recommendation, transcripts, standardized tests or questionnaires. The criterion measure is assumed to be a relevant and reliable measure of performance for applicants, and can be predicted from test scores with a certain degree of accuracy. The applicants who can achieve the minimum satisfactory performance (Y_s) on the criterion measure are considered to be successful. Each of k subpopulations can be represented by a certain number of applicants (N_i). Only a certain number of institutional openings (N_0) are available. In other words, the selection ratio (SR, i.e., the proportion of applicants accepted) is less than one in the selection situations studied.

The selection of applicants is accomplished through the use of test scores. The definition of any "fair" model is a rule for making "fair" use of test scores. The problem of "fair" selection is to choose acceptance scores (C_i) on the test, so that N_0 applicants are selected and the definition of a particular "fair" model is satisfied. Applicants with a test score above the acceptance score for their subpopulation will be selected, and applicants with a test score below the acceptance score for their subpopulation will be rejected. In the case of multiple test scores, the acceptance scores can be calculated on the linear combination of test scores which is developed by the least square method. For simplicity, only the case of a single test score is considered in the following sections.

Regression Model

According to Cleary (1968):

> A test is biased for members of a subgroup of the population if, in the prediction of a criterion for which test was designed, consistent non-zero errors of prediction are made for members of the subgroup. In other words, the test is biased if the criterion score predicted from the common regression line is consistently too high or too low for members of the subgroup. With this definition of bias, there may be a connotation of "unfair" particularly if the use of the test produces a prediction that is too low (p.115).

Anastasi (1968) gave a similar definition: "Test bias refers to overprediction or underprediction of criterion measure." These definitions imply that a selection procedure is fair when separate regression equations are computed for each subpopulation and the N_0 applicants having the highest predicted criterion score are selected. In other words, first the predicted criterion score of each applicant is obtained by using the regression equation of his subpopulation. The applicants are then ordered according to their values of predicted criterion scores, and the N_0 applicants having the highest predicted criterion score are selected.

In the Regression Model, the acceptance score for each subgroup be determined to meet the restriction of the selection ratio and the following condition.

$$\hat{Y} = a_1 + b_1C_1 = a_2 + b_2C_2 = \text{--------------} = a_k + b_kC_k \qquad (2.1)$$

where

$\hat{Y}$ = the predicted criterion score for the individuals scoring at the acceptance score C_i

a_i = the intercept of the regression equation for group i

b_i = the slope of the regression equation for group i

C_i = the acceptance score for group i

The Regression Model can be shown to possess a certain optimal property. Cochran (1951) has shown that the expected criterion score of the applicants accepted will be a maximum when the selection follows the Regression Model and is based upon the population regression equations. In other words, the average criterion performance of applicants accepted from the Regression Model will always be higher than the average criterion performance of applicants accepted from other procedures. The Regression Model can be considered as "fair" to the institution in the sense that it provides the institution with a procedure of selecting applicants such that the average criterion score is a maximum. It also can be considered as "fair" to individual members of each group since the criterion performance is not systematically over or under-predicted for members of any group.

The illustrations of the Regression Model are shown in Figure 1. Figure 2 and Figure 3 (Adapted from Einhorn and Bass, 1971). In Figure 1, the regression lines are identical in two subpopulations. The use of the common regression equation for selecting applicants would not underpredict or overpredict the criterion score for any member of either subpopulation. Figure 2 shows the situation in that two subpopulations have the regression lines with the same slope but different intercepts. If the combined regression line (π_c) were used to predict criterion scores for all applicants, then the criterion scores for members of subpopulation π_2 would be consistently underpredicted. Therefore, the test would be considered as "unfair" to subpopulation π_2 according to the definition of the Regression Model. Figure 3 illustrates the situation in that two subpopulations have different slopes and different intercepts. Let us consider the problems associated with the use of the combined regression line for predicting criterion measures. For the individuals scoring at point X_H, criterion scores for members of subpopulation π_2 would be underpredicted, while for the individuals scoring at point X_L, criterion scores for members of subpopulation π_1 would be underpredicted. The test would be considered as "unfair" unless separate regression lines were used for two subpopulations.

The Regression Model is a widely used model of test bias within the predictive context. It has been followed by investigators (Bower, 1970; Cleary, 1968; Temp, 1971) in the empirical studies of bias in the use of tests in college admissions. Also, it has been basic

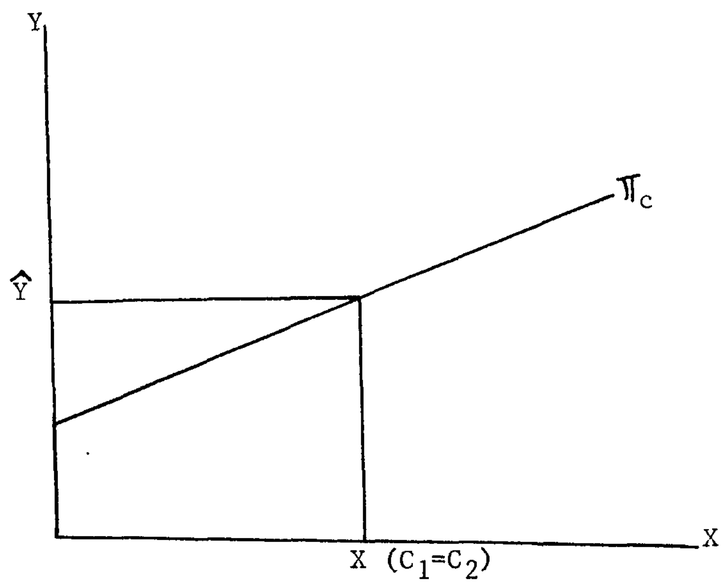

Figure 1. An illustration of test bias as defined by the Regression Model in the situation of common regression equation.

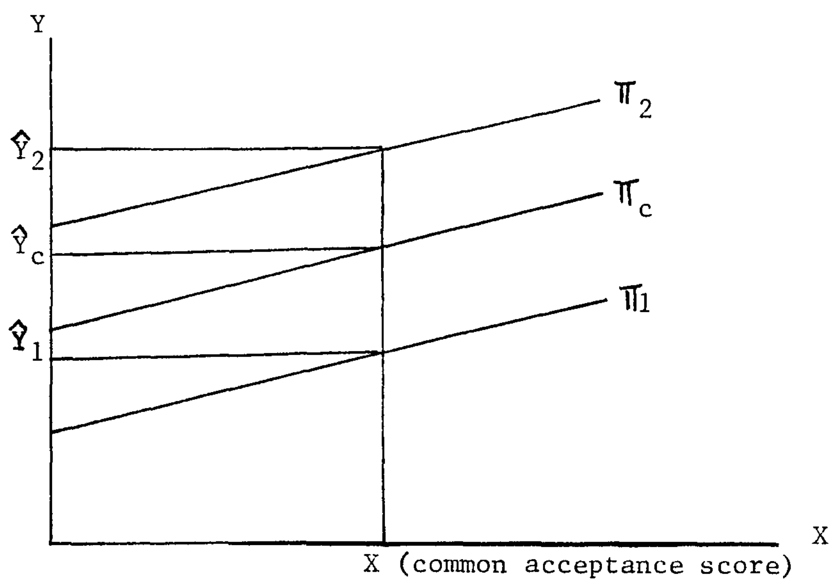

Figure 2. An illustration of test bias as defined by the Regression Model in the situation of parallel regression lines.

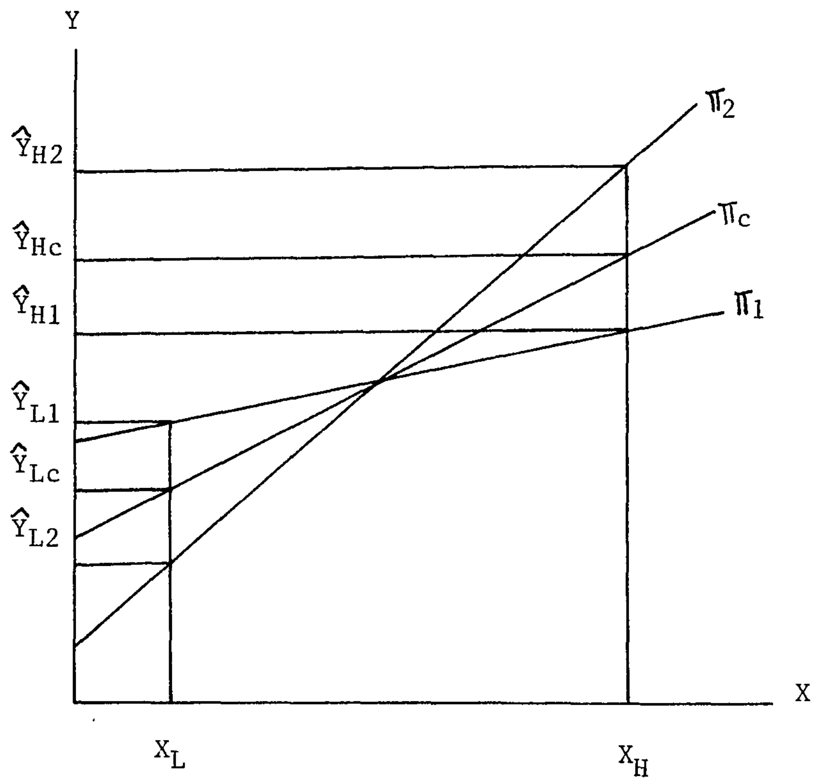

Figure 3. An illustration of test bias as defined by the Regression Model in the situation of different regression lines.

in the conceptualizations and discussions of test bias that may be found in Anastasi (1968), Bartlett and O'Leary (1969), Einhorn and Bass (1971), Guion (1966), Linn (1973), Linn and Wert (1971), and Schmidt and Hunter (1974).

Employer's Model

Guion (1966) stated that:

> Unfair discrimination exists when persons with equal probabilities of success on the job have unequal probabilities of being hired for the job (p.26).

This definition was implemented in the Employer's Model proposed by Einhorn and Bass (1971). The objective of this model is not only to accept those persons who were predicted to be above a certain minimum point on the criterion, but also to make these predictions with as high a degree of confidence as possible. This means that it is necessary for the decision-maker to take into account not only an individual's predicted criterion score, but also the probability with which this score will fall below or beyond some minimum performance level deemed satisfactory by the institution (Y_s). In order to avoid "test bias", the acceptance scores are chosen so that the conditional probability of success for the individuals scoring at acceptance scores is the same for each group.

In the Employer's Model, the test acceptance scores C_i be determined so that

$$\text{Prob}(Y \geq Y_s \mid X = C_1, \pi_1) = \text{Prob}(Y \geq Y_s \mid X = C_2, \pi_2)$$

$$= \cdots\cdots = \text{Prob}(Y \geq Y_s \mid X = C_k, \pi_k) \qquad (2.2)$$

For example, suppose that the applicants of an institution can be subdivided into two subpopulations, π_1 and π_2. The institution is willing to hire all applicants with at least 85% chance of success as gauged by the test scores used. Then the acceptance scores are chosen at the points at which the minimum satisfactory performance on the criterion (Y_S) is approximately one standard error of estimate ($\sigma_{y.x}$) below the predicted criterion ($\hat{Y}$) associated with the acceptance scores. In other words, the following condition should be met.

$$\text{Prob}(\hat{Y} \geq Y_s \mid X = C_1, \pi_1) = \text{Prob}(\hat{Y} \geq Y_s \mid X = C_2, \pi_2) = .85$$

Einhorn and Bass (1971) stated that this procedure is non-discriminatory since there is no over or underprediction of an individual criterion score. In addition, the probabilities of success for the persons selected from different groups are necessarily the same, since the standard errors of estimate have been taken into account in making selection decisions.

Figure 4 and Figure 5 illustrate the Employer's Model compared with the Regression Model. In Figure 4, two subpopulations have an identical regression line but differ in their standard errors of estimate. The standard error of estimate for subpopulation π_2 is smaller than the standard error of estimate for subpopulation π_1. The shaded portion of the distribution represents the probability of scoring above the minimum satisfactory criterion score (Y_S) for point X on the test. In this situation, for the applicants scoring at point X on the test, the probability of being successful is greater for the applicants of subpopulation π_2 than for the applicants of subpopulation π_1. The selection procedure derived from the Regression Model would be considered "unfair" to subpopulation π_2 according to the definition of fairness given by the Employer's Model. In Figure 5, two subpopu-

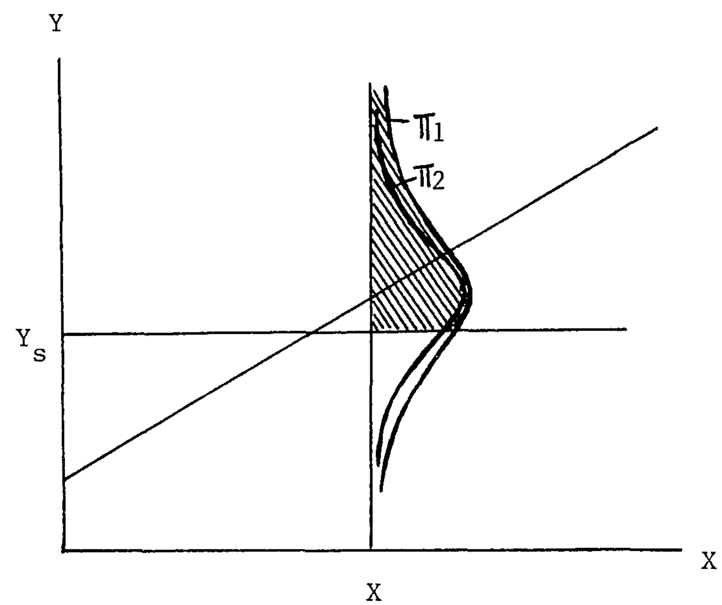

Figure 4. An illustration of test bias as defined by the Employer's Model in the situation of the same regression line but different standard errors of estimate.

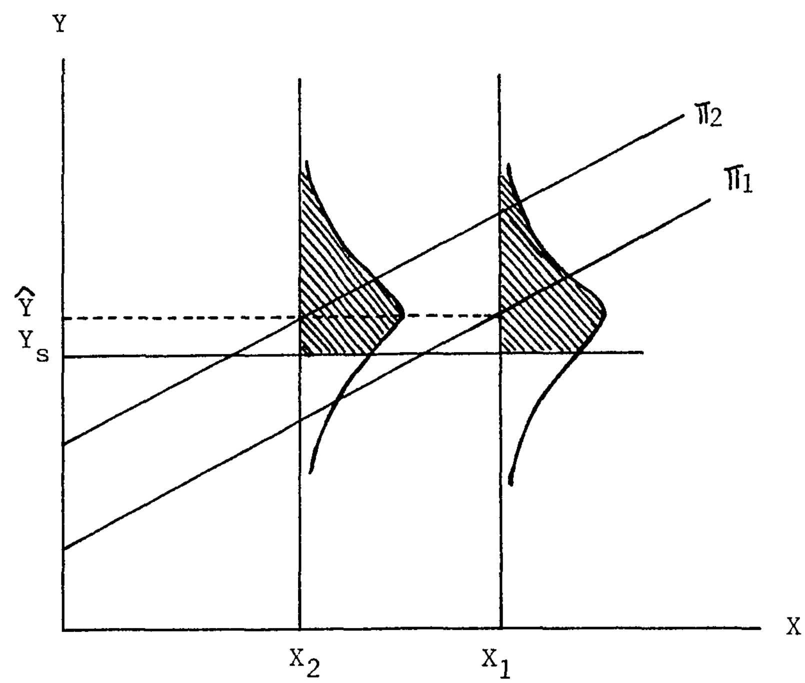

Figure 5. An illustration of test bias as defined by the Employer's Model in the situation of parallel regression lines and the same standard error of estimate.

lations have an identical standard error of estimate and parallel regression equations. According to the Regression Model, the acceptance scores X_1 and X_2 were derived. The probability of being successful for the applicants of subpopulation π_1 scoring at point X_1 equals the probability of being successful for the applicants of subpopulation π_2 scoring at point X_2. In other words, the selection procedure derived from the Regression Model also satisfied the definition of fairness given by the Employer's Model in the data situations where two subpopulations have the same standard error of estimate and parallel regression equations. This statement is also true when two subpopulations have an identical standard error of estimate and different slopes. In general, if subpopulations have an identical standard error of estimate, then the selection procedures proposed by the Regression Model and the Employer's Model are exactly the same.

As with the Regression Model, the Employer's Model also possesses an optimal property. The Employer's Model is equivalent to a selection procedure that minimizes the rate of selection errors (i.e., the total number of potentially unsuccessful applicants accepted and potentially successful applicants rejected divided by the total number of applicants). The proof of this statement is given in a later chapter.

In the Employer's Model as well as the Regression Model, poor prediction (i.e., low correlation between test scores and a criterion score) in one group decreases the chances of acceptance of members of that group. When the prediction is poor in one group, the standard error of estimate is larger for that group as compared to the other group. Consequently, a higher acceptance score for that group is required to maintain the same conditional probability of success for the individuals scoring at the acceptance score.

Thorndike's Model

Thorndike (1971) clearly demonstrated that when two groups have equal regression equation but differ appreciably in mean test scores, then using the selection procedure implied by the Regression Model,

> which is "fair" to individual members of the group scoring lower on the test, is "unfair" to the lower scoring group as a whole in the sense that the proportion qualified on the test will be smaller, relative to the higher scoring group, than the proportion that will reach any specified level of criterion performance (p. 63).

Thorndike proposed a model which requires that the success ratio (i.e., the proportion of potentially successful minority applicants to the proportion of potentially successful majority applicants) equals the acceptance ratio (i.e., the proportion of minority applicants accepted to the proportion of majority applicants accepted). In a fair selection procedure

> the qualifying scores on a test should be set at levels that will qualify applicants in the two groups in proportion to the fraction of the two groups reaching a specified level of criterion performance (p. 63).

In terms of probability statements, the acceptance scores should be chosen so that the following condition is satisfied.

$$\text{Prob}(X \geq C_1 \mid \pi_1)/\text{Prob}(Y \geq Y_s \mid \pi_1) = \text{Prob}(X \geq C_2 \mid \pi_2)/\text{Prob}(Y \geq Y_s \mid \pi_2)$$

$$= \text{----------------} = \text{Prob}(X \geq C_k \mid \pi_k)/\text{Prob}(Y \geq Y_s \mid \pi_k) \qquad (2.3)$$

For example, if in Group 1, 40% of the members are successful, and in Group 2, 80% of the members are successful, then the proportion of Group 1 members accepted to those accepted from Group 2 should be 1:2.

Thorndike's formulation suggests that we should be looking at the implications for the proportions of students admitted as well as regression lines. This model tries to eliminate the inequality of over-selecting members from the group with better prediction, even though a substantial proportion of other group members could succeed if selected.

Illustrations of Thorndike's Model are shown in Figure 6, Figure 7 and Figure 8 (Adapted from Thorndike, 1971). In Figure 6, two subpopulations have an identical regression line. For any given test score, the predicted criterion score for an individual is the same without regard to the group of which he is a member. According to the Regression Model, the same acceptance score on the test would be derived for both subpopulations. Since the overlapping of two subpopulations is much greater on the criterion score than on the test score, the same acceptance score for both groups would result in a larger percentage of subpopulation π_1 which would achieve the minimum satisfactory score than the percentage which would be accepted on the basis of the test. In other words, the selection procedure derived from the Regression Model would be considered as "unfair" to population π_1 according to the definition of fairness given by Thorndike's Model when two populations have an identical regression line. In Figure 7, two subpopulations have parallel regression lines and the same distribution of criterion scores. The same selection procedure would be considered "fair" by both the Regression Model and Thorndike's Model in this data situation. In Figure 8, the regression lines are parallel and the difference

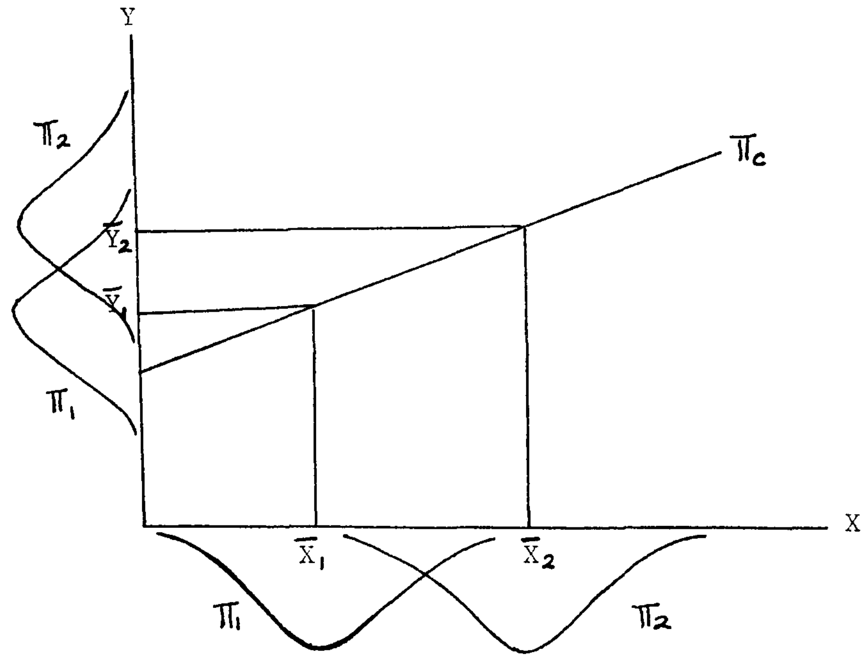

Figure 6. An illustration of test bias as defined by Thorndike's Model in the situation of common regression line and different mean differences on test and criterion scores.

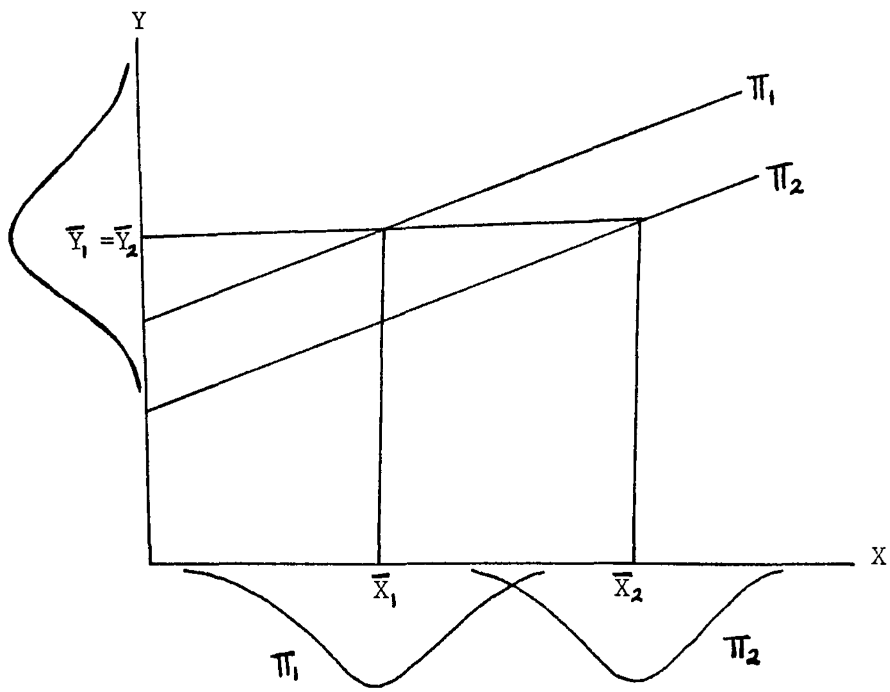

Figure 7. An illustration of test bias as defined by Thorndike's Model in the situation of parallel regression lines and an identical distribution of criterion scores.

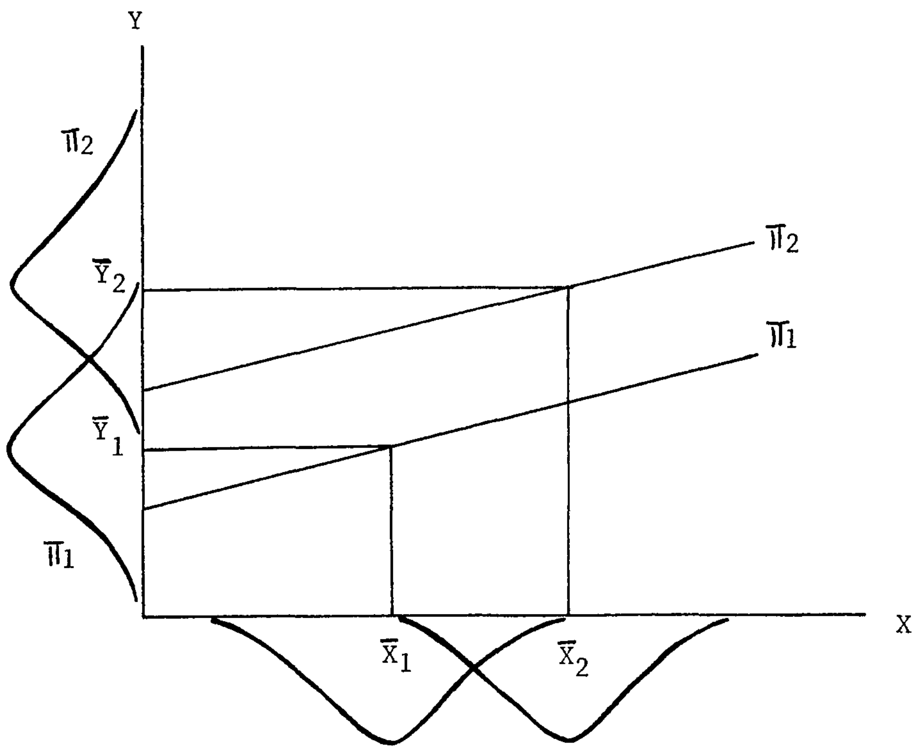

Figure 8. An illustration of test bias as defined by Thorndike's Model in the situation of parallel regression lines and the same mean difference on test and criterion scores.

between means on the test equals the difference between means on the criterion. In this situation, the selection procedure derived from Thorndike's Model would accept applicants who have lower predicted criterion scores from subpopulation π_1 than the applicants accepted from subpopulation π_2. In other words, the selection procedure derived from Thorndike's Model would not satisfy the definition of "fairness" given by the Regression Model. This would be true whenever the test mean of subpopulation π_1 is lower than the test mean of subpopulation π_2 and the correlation between test and criterion is less than perfect. The extent of the discrepancy between these two models will increase in direct proportion to the difference between two test means and will be negatively related to the correlation between test and criterion score (Thorndike, 1971).

Equal Opportunity Model

The principle of the Equal Opportunity Model is that as a group, the people who can achieve a satisfactory criterion score (i.e., Y_s) should have the same probability of being selected whether they are minority or majority group members. Cole (1973) claimed that

> Under each of the previous models discussed, it may happen that the chance of selection of a potentially successful applicant in Group A is different from the chance of selection of such an applicant in Group B. Thus, two applicants, both of whom could succeed (achieve a criterion score above a criterion pass point) if selected, may have different chances of selection because of their group membership. Under the Equal Opportunity Model, this type of unfairness is eliminated.

In this model, one should determine the acceptance scores (C_i)

so that the conditional probability of being selected, given that an applicant is capable of successful performance is equal for each group. In terms of probability statements, the Equal Opportunity Model defines that a selection procedure is fair if

$$\text{Prob}(X \geq C_1 \mid Y \geq Y_S, \pi_1) = \text{Prob}(X \geq C_2 \mid Y \geq Y_S, \pi_2)$$

$$= \text{---------------} = \text{Prob}(X \geq C_K \mid Y \geq Y_S, \pi_K) \qquad (2.4)$$

Following Linn (1973) we can summarize and compare the above two definitions of "fair" selection. Figure 9 is an illustration of a hypothetical bivariate distribution of criterion and test scores. The cases that fall in Region I of Figure 9, have test scores below the acceptance point (would be rejected), and would have satisfactory performance on the criterion if accepted. They are "false negatives". False positives, those with test scores above the acceptance point but unsatisfactory criterion performance, are in Region IV. The cases in Regions I and IV are considered as errors of selection, while the cases in Regions II and III are considered as "correct" classification.

In terms of Regions in Figure 9, the Equal Opportunity Model focuses on the ratio of the number of cases in Region II to the number of cases in Regions I and II combined. If this ratio is same for the subpopulations that are being compared, then the procedure would be considered as "fair" according to the definition given by the Equal Opportunity Model. On the other hand, the definition of "fairness" given by Thorndike's Model would be satisfied whenever the ratio of the number of cases in Regions II and IV combined to the number of cases in Regions I and II combined, is identical for each group.

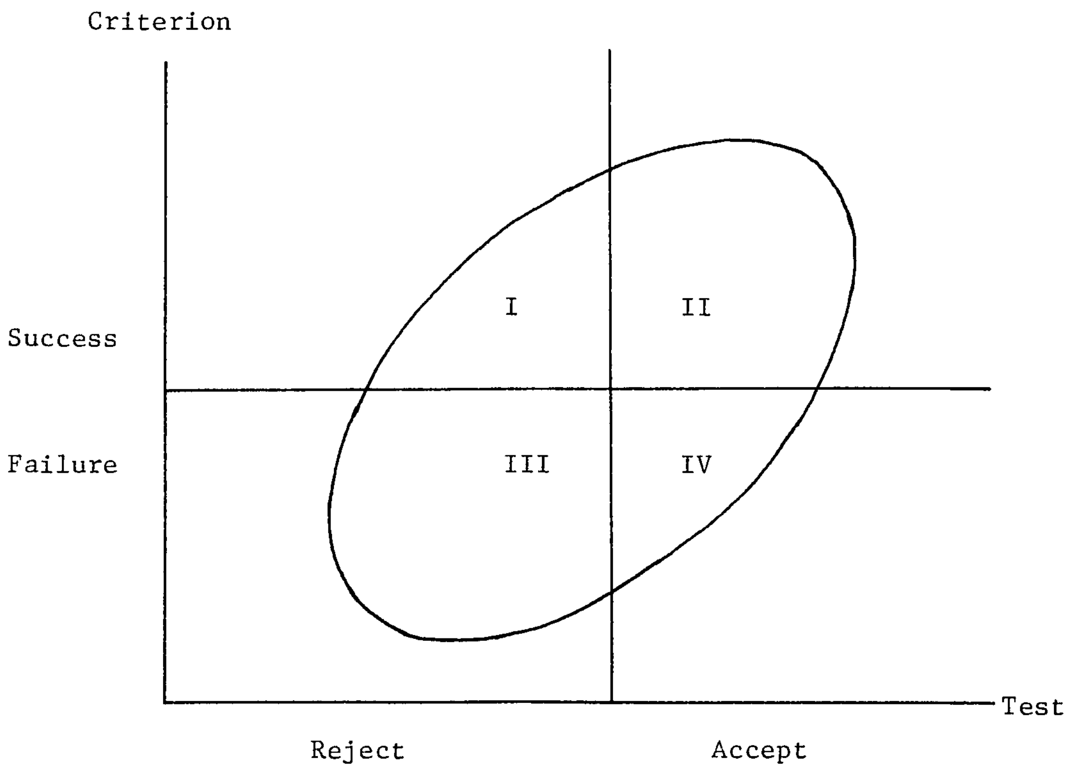

Figure 9. A bivariate distribution of criterion and test scores.

Darlington's Model

Darlington (1971) evaluated the four definitions of "fairness" (i.e., the Regression Model, Thorndike's Model, the Equal Opportunity Model and the Equal Test Mean Model). He concluded that these four definitions yield contradictory results except in the case of perfect validity (i.e., the correlation between the predictor and criterion variables is one) or in the case of equal subpopulation mean performance on the criterion. Darlington (1971) argued that "fairness" can be achieved only by a kind of combination of the Regression Model and the type of value judgement made in selecting some group members. Therefore, he suggests that instead of predicting the criterion variable, Y, a variable Y-wg be defined where w is established by a subjective value judgement, and g denotes an individual's group membership. The variable g has value zero for minority group members, and one for majority group members. He urges that:

> the term "cultural fairness" be replaced in public discussion by the concept of "culture optimality". The question of whether a test is culturally optimum can be divided in two: a subjective, policy-level question concerning the optimum balance between criterion performance and cultural factors (operationalized---------as the optimum value of w), and a purely empirical question concerning the test's correlation with the culture-modified variable (Y-wg) and whether that correlation can be raised (p. 79-80).

According to Darlington, each institution must first decide if there is any special value in the selection of members of some culture group. If so, then one accepts some difference between criterion scores which will yield equally desirable candidates from different groups. One might think that a minority member's score Y-w (w is greater than zero) on a criterion is as desirable as a score of Y on the criterion for majority members if it is valuable to obtain minority group members. The choice of w represent the special value that an institution places in selecting members of the minority group.

Under Darlington's Model, the dependent variable is Y-wg. The data from two groups should be combined, and a prediction equation for Y-wg is computed using both the predictor variable X and the group membership variable g (i.e., $\widehat{Y\text{-}wg} = a + b_1X + b_2g$). Then by using this dependent variable, the applicants having the highest predicted performance are selected. When w is set equal to zero (i.e., there is no reason to favor one culture group) and two groups have an identical regression line, Darlington's Model reduces to the Regression Model.

The Problems in Using These Five "Fair" Models

In using the Regression Model, the criterion score of each applicant is not over or under predicted and the average criterion performance for accepted applicants is maximized. The Employer's Model leads to a selection process which minimizes the rate of selection errors. Thorndike's Model and the Equal Opportunity are the selection models which tend to compensate minority applicants. However, Petersen and Novick (1974) examined these models in terms of "converse model" and "wrong conditional probability". They found that Thorndike's Model and the Equal Opportunity Model are internally contradictory and based on the wrong conditional probability. They also found that these two models do not work properly in selection situations where Japanese or Chinese-Americans are minority members and white Americans are majority members. In this situation, both Thorndike's Model and the Equal Opportunity Model will give a lower predictor acceptance score to the majority group instead of the minority group. These two models make access more difficult rather than easier for a Japanese or Chinese-American minority group. Darlington's Model is the revised form of the Regression Model. In this model, the public desire for compensating the minority group is carried out by adding a constant to the criterion measure of a minority applicant.

The major problem in using these models is the lack of a clear rule for choosing one model among them in a selection situation. Each of these "fair" models recommends a specific definition of "fairness" and the selection procedure which satisfies this definition. The selection procedure of one model may be in conflict with the selection procedure of another model. The conflict among models created a considerable confusion to test users. Unforunately, there is no advice in the literature on how to make a judgement to accept or reject a selection model. Additionally, these models are too specific. A decision-maker may have a goal which can not be accomplished by using one of these models. Therefore, a general and flexible model for constructing a "fair" selection procedure is needed. In the next chapter, a general model will be developed in the framework of decision theory. This general model will provide a method for constructing a selection procedure which will be compatible with a decision-maker's goal and also will maximize the expected utility of selection. In addition, the model will provide a method for identifying implicit utilities associated with a selection procedure and then a decision-maker will be able to judge whether the selection procedure is "fair" or "unfair" based on the identified implicit utilities.

3. EXPECTED UTILITY MODEL

In this chapter, the Expected Utility Model will be derived. The basic assumptions underlying this derivation are: 1) the decision-maker can represent his preferences of selection outcomes explicitly; 2) the decision-maker can only accept a certain proportion of applicants. Only the case of a single predictor variable is considered in the following sections, although the results are identical for multiple predictors when the within-group multiple regression equation is used as a single predictor. For simplicity, the solution is derived only if the predictor and criterion variables have a bivariate normal distribution in each subpopulation. However, the solution can be obtained even if the predictor and criterion variables are not bivariate normally distributed, and this solution may differ from the solution derived in the following section.

Outcomes of Selection

Assume that the population of applicants consists of k subpopulations (π_i, i=1,2,3,------k). A predictor variable X and a criterion variable Y have possibly different bivariate normal distribution for each subpopulation. In the ith subpopulation, an applicant will be accepted if his test score is greater than or equal to an acceptance point C_i on the predictor variable X. Additionally, for all subpopulations, an applicant is defined to be successful if his performance on the criterion variable is greater than or equal to the minimum successful point Y_s. To simplify the derivation of the selection procedure, first transform the test score (X), acceptance scores (C_i), the criterion score (Y), and the minimum successful score (Y_s) into their corresponding standardized values (Z_x, C_i^*, Z_y, and Y_{si}^*). In other words, let

$$Z_x = (X - \mu_{xi}) / \sigma_{xi}$$
$$C_i^* = (C_i - \mu_{xi}) / \sigma_{xi}$$
$$Z_y = (Y - \mu_{yi}) / \sigma_{yi}$$
$$Y_{si}^* = (Y_s - \mu_{yi}) / \sigma_{yi}$$

where

μ_{xi} = the mean of the test score for ith group

σ_{xi} = the standard deviation of the test score for ith group

μ_{yi} = the mean of the criterion score for ith group

σ_{yi} = the standard deviation of the criterion score for ith group

Given Y_{si}^* and C_i^*, four outcomes of the selection procedure can be identified in each subpopulation. The problem to be considered is represented schematically in Figure 10. The following are outcomes defined in terms of whether or not $Z_x \geqq C_i^*$ and $Z_y \geqq Y_{si}^*$.

O_{1i}: $Z_x < C_i^*$, $Z_y \geqq Y_{si}^*$ The set of all ith group members who are not selected and would have been successful if selected.

O_{2i}: $Z_x \geqq C_i^*$, $Z_y \geqq Y_{si}^*$ The set of all ith group members who are selected and successful.

O_{3i}: $Z_x < C_i^*$, $Z_y < Y_{si}^*$ The set of all ith group members who are not selected and would not have been successful if selected.

O_{4i}: $Z_x \geqq C_i^*$, $Z_y < Y_{si}^*$ The set of all ith group members who are selected and not successful.

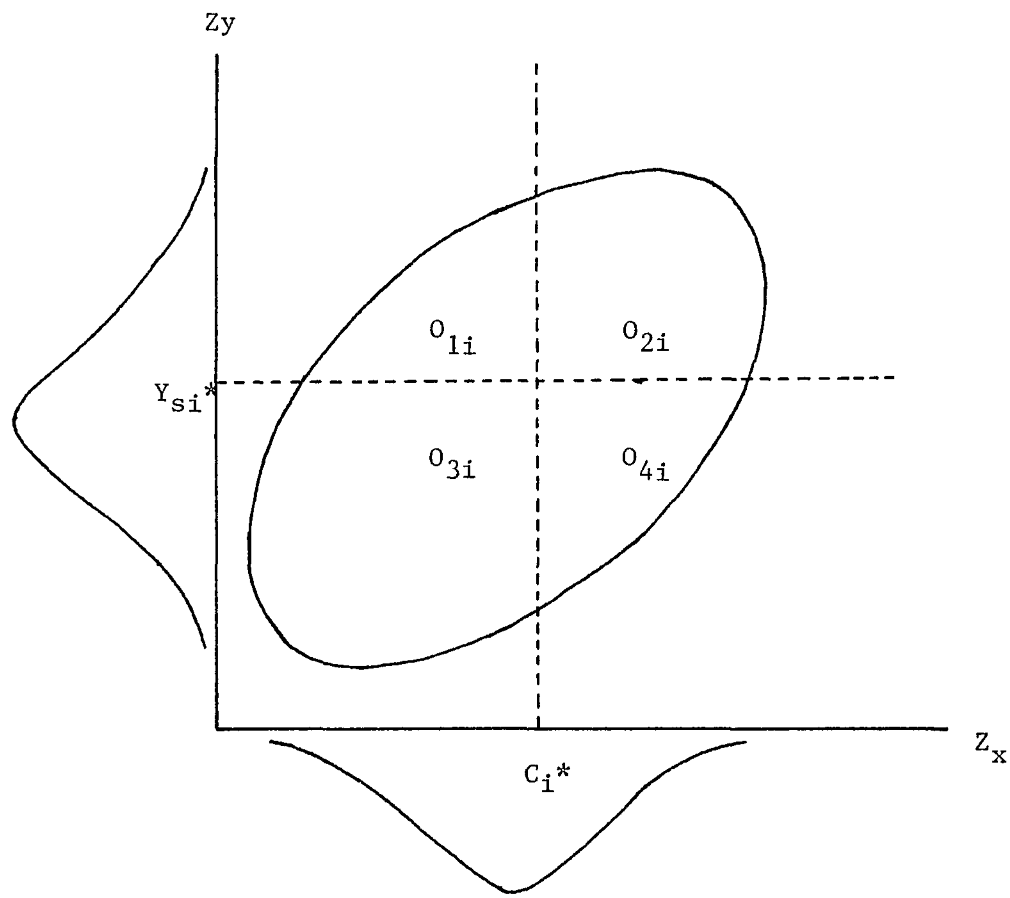

Figure 10. Pictorial representation of the problem.

Since there are k subpopulations and four outcomes for each subpopulation, k x 4 outcomes can be identified for a selection problem.

Utilities of Outcomes

Given these outcomes, the institution designates $U(O_{ji})$ be the corresponding utility of the outcome O_{ji}. In the educational setting, various factors might be considered to determine $U(O_{ji})$: 1) the cost of the program; 2) the benefit that the institution will obtain from accepting an individual of the ith subpopulation; 3) the contribution that a successful individual of the ith subpopulation can make to the institution; and 4) the contribution that a successful individual of the ith subpopulation can make to his own subpopulation. For example, the determination of $U(O_{1i})$ and $U(O_{2i})$ might depend upon the above four factors, while the determination of $U(O_{3i})$ and $U(O_{4i})$ might depend only upon the first two factors. In general, from the institution's viewpoint, rejecting a potentially unsuccessful applicant is preferable to accepting a potentially successful applicant. This implies that the utility $U(O_{3i})$ associated with outcome O_{3i} is greater than the utility $U(O_{2i})$ associated with outcome O_{2i}. From the applicant's point of view, the situation may be reversed and $U(O_{2i}) > U(O_{3i})$. Also, from the institution's viewpoint, rejecting a potentially successful applicant may be less "costly" than accepting a potentially unsuccessful applicant (i.e., $U(O_{1i}) > U(O_{4i})$). Again, from the applicant's point of view, the situation may be reversed and $U(O_{4i}) > U(O_{1i})$.

In theory, utility judgements might be obtained by asking the decision-maker to state how many utility units each outcome is worth on his personal value scale. These judgements are, however, quite difficult to make, and if the judgements are obtained repeatedly, there is most likely variation in the evaluation of an outcome and inconsistency in the information about different outcomes. The most common procedure to improve the estimation of values is to employ some method of psychophysical scaling. These methods give a satisfactory degree of reliability, although it is often necessary to supress certain inconsistencies in the data by describing them as "error" (Coombs, 1953). It would be desirable if the utility structure for each subpopulation is determined, in the educational setting by a faculty and student committee, and in the business setting by a joint personnel and management committee. It should be noted that it is not necessary for the decision-maker to specify a different utility function for each subpopulation, but the decision-maker is likely to want to do so.

Expected Utility of Selection

The process of selection can be viewed as a series of separate decisions, each of which involves one applicant. Let q_i be the proportion of applicants who are members of π_i and $Prob(O_{ji})$ be the probability of outcome O_{ji} occuring. The expected utility E(U) over all applicants can be computed as follows:

$$E(U)=\sum_{i=1}^{k} q_i \sum_{j=1}^{4} U(O_{ji})\ Prob(O_{ji}) \qquad (3.1)$$

Under the assumption of a bivariate normal distribution for the predictor and criterion variables, the expected utility E(U) of the selection procedure can be expressed as:

$$E(U) = \sum_{i=1}^{K} \frac{U(O_{1i})\ q_i}{2\pi\sqrt{1-\rho_i^2}} \int_{Y_{si}^*}^{\infty} \int_{-\infty}^{C_i^*} e^{-\frac{(Z_x^2 - 2\rho_i Z_x Z_y + Z_y^2)}{2(1-\rho_i^2)}}\, dZ_x dZ_y$$

$$+ \sum_{i=1}^{K} \frac{U(O_{2i})\ q_i}{2\pi\sqrt{1-\rho_i^2}} \int_{Y_{si}^*}^{\infty} \int_{C_i^*}^{\infty} e^{-\frac{(Z_x^2 - 2\rho_i Z_x Z_y + Z_y^2)}{2(1-\rho_i^2)}}\, dZ_x dZ_y$$

$$+ \sum_{i=1}^{K} \frac{U(O_{3i})\ q_i}{2\pi\sqrt{1-\rho_i^2}} \int_{-\infty}^{Y_{si}^*} \int_{-\infty}^{C_i^*} e^{-\frac{(Z_x^2 - 2\rho_i Z_x Z_y + Z_y^2)}{2(1-\rho_i^2)}}\, dZ_x dZ_y$$

$$+ \sum_{i=1}^{K} \frac{U(O_{4i})\ q_i}{2\pi\sqrt{1-\rho_i^2}} \int_{-\infty}^{Y_{si}^*} \int_{C_i^*}^{\infty} e^{-\frac{(Z_x^2 - 2\rho_i Z_x Z_y + Z_y^2)}{2(1-\rho_i^2)}}\, dZ_x dZ_y \qquad (3.2)$$

where

ρ_i = the pearson-product correlation coefficient between X and Y for the ith subpopulation.

Derivation of the "Optimal" Selection Procedure

In most of the situations, the decision-maker can only select a certain fixed proportion of total applicants (i.e., the number of "openings" is limited). This fixed proportion is referred to as the selection ratio (SR) which can be expressed as:

$$SR = \sum_{i=1}^{K} q_i \text{Prob}(\ Z_x \geq C_i^* \mid \pi_i\ )$$

$$= \sum_{i=1}^{K} \frac{q_i}{2\pi\sqrt{1-\rho_i^2}} \int_{Y_{si}^*}^{\infty} \int_{C_i^*}^{\infty} e^{-\frac{(Z_x^2 - 2\rho_i Z_x Z_y + Z_y^2)}{2(1-\rho_i^2)}}\, dZ_x dZ_y$$

$$+ \sum_{i=1}^{K} \frac{q_i}{2\pi\sqrt{1-\rho_i^2}} \int_{-\infty}^{Y_{si}^*} \int_{C_i^*}^{\infty} e^{-\frac{(Z_x^2 - 2\rho_i Z_x Z_y + Z_y^2)}{2(1-\rho_i^2)}}\, dZ_x dZ_y \qquad (3.3)$$

The problem of deriving the "optimal" selection procedure is to find the values of C_i^* in order to maximize the expected value as defined in (3.2) under the restriction of (3.3). Introducing the SR constraint by a LaGrange multiplier and setting the partial derivative of E(U) with respect to C_i^* to zero, (3.2) becomes:

$$0 = U(O_{1i})\, A_i \int_{Y_{si}^*}^{\infty} e^{-B_i}\, dZ_y - U(O_{2i})\, A_i \int_{Y_{si}^*}^{\infty} e^{-B_i}\, dZ_y$$
$$+ U(O_{3i})\, A_i \int_{-\infty}^{Y_{si}^*} e^{-B_i}\, dZ_y - U(O_{4i})\, A_i \int_{-\infty}^{Y_{si}^*} e^{-B_i}\, dZ_y$$
$$+ \lambda \left(A_i \int_{Y_{si}^*}^{\infty} e^{-B_i}\, dZ_y + A_i \int_{-\infty}^{Y_{si}^*} e^{-B_i}\, dZ_y \right)$$
$$i = 1, 2, \text{-----------------} k \qquad (3.4)$$

where

$$A_i = \frac{q_i\, e^{-C_i^*/2}}{2\pi\sqrt{1-\rho_i^2}}$$

$$B_i = \frac{(Z_y - \rho_i C_i^*)^2}{2(1-\rho_i^2)}$$

Define P_i as

$$P_i = \text{Prob}(Z_y \geq Y_{si}^* \mid Z_x = C_i^*)$$
$$= \frac{1}{\sqrt{2\pi(1-\rho_i^2)}} \int_{Y_{si}^*}^{\infty} e^{-\frac{(Z_y-\rho_i C_i^*)^2}{2(1-\rho_i^2)}}\, dZ_y \qquad (3.5)$$

Equation (3.4) can be simplified as

$$-\lambda = P_i\left[U(O_{1i})-U(O_{2i})\right] + (1-P_i)\left[U(O_{3i}) - U(O_{4i})\right]$$

It can be shown that the values of C_i^* that maximize E(U) should satisfy the constraint of the selection ratio (SR) and the following equation:

$$P_1 \left[U(O_{11})-U(O_{21})-U(O_{31})+U(O_{41})\right] + \left[U(O_{31})-U(O_{41})\right]$$
$$= P_2 \left[U(O_{12})-U(O_{22})-U(O_{32})+U(O_{42})\right] + \left[U(O_{32})-U(O_{42})\right]$$
$$= P_3 \left[U(O_{13})-U(O_{23})-U(O_{33})+U(O_{43})\right] + \left[U(O_{33})-U(O_{43})\right]$$
$$\cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot$$
$$= P_k \left[U(O_{1k})-U(O_{2k})-U(O_{3k})+U(O_{4k})\right] + \left[U(O_{3k})-U(O_{4k})\right] \qquad (3.6)$$

Using an iterative procedure one can find the values of C_i^* that satisfy both equation (3.6) and the selection ratio (3.3), and thus maximize the expected utility of selection (3.2). To begin an iterative procedure, the convenient initial value of P_1 is

$$P_1 = \left[U(O_{31})-U(O_{41})\right] / \left\{\left[U(O_{31})-U(O_{41})\right] - \left[U(O_{11})-U(O_{21})\right]\right\}$$

All the other values of P_i $(i \neq 1)$ can be found from equation (3.6). In order to find the values of C_i^*, one should utilize a standardized normal table and obtain the standardized values associated with the proportions P_i. Designate these standardized scores as Z_i. The values of Z_i equal the following:

$$Z_i = \frac{Y_{si}^* - \rho_i C_i^*}{\sqrt{1 - \rho_i^2}} \qquad (3.7)$$

Solving the C_i^*, it is found that

$$C_i^* = (Y_{si}^* - Z_i \sqrt{1 - \rho_i^2}\,)/\rho_i \qquad (3.8)$$

The values of acceptance scores computed from the equation (3.8) are then used to calculate the total proportion of applicants selected (TPS). The value of P_1 should be increased if the computed proportion (TPS) is greater than the selection ratio (SR). The values of P_1 should be decreased if the computed proportion (TPS) is less than the selection ratio. Then, the values of P_i and C_i^* are again computed from equations (3.6) and (3.8) respectively. The same procedure should be repeated until the equation (3.3) is satisfied.

Computer Program of the Expected Utility Model

For implementing the Expected Utility Model, a computer program has been developed and will be presented in Appendix I. The iterative procedure of this computer program follows the method presented in the last section. For simplicity, only the cases of two subpopulations and one predictor variable are considered in the program.

The basic inputs of the computer program are distributions of predictor and criterion scores (i.e., mean and standard deviation), the proportion of the ith subpopulation applicants and preferences for four outcomes of selection for two subpopulations, and the selection ratio. To obtain the distributions of predictor and criterion scores, institutions must be able to provide the predictor scores for all applicants, and the predictor and criterion scores for the applicants who had been accepted for the previous year. In addition, institutions must be able to classify each applicant as to his subpopulation membership. From the accepted applicants, the correlation between the predictor variable and the criterion variable, and the means and standard deviations of the predictor and criterion variables can be calculated for each subpopulation. Then, applying the method of range adjustment suggested by Gulliksen (1967), the correlation between the predictor variable and the criterion variable, and the mean and standard deviation of the criterion variable for total applicants can be estimated. However, it must be emphasized that the accuracy of estimations is very poor if the selection ratio is small (e.g., SR= 0.1). The proportion of the ith subpopulation applicants and the selection ratio can be estimated from the total number of applicants, the number of applicants from the ith subpopulation, and the total number of applicants that can be accepted during a given time period.

The formal outputs of the program are standardized acceptance scores (C_i^*) on the predictor variable that divide a particular subpopulation into subgroups of those selected and not-selected, and the proportions of applicants accepted for two subpopulations. In addition, the expected utility of the derived selection procedure, the average criterion performance of accepted applicants, the rate of selection errors and the success rate of accepted applicants, are provided. The detailed procedure for computing these outputs will be presented in Appendix I.

4. VIEWING FAIR SELECTION MODELS IN TERMS OF THE EXPECTED UTILITY MODEL

In this chapter, we will examine the underlying implicit utilities of the five "fair" selection models proposed by Cleary (1968), Einhorn and Bass (1971), Thorndike (1971), Cole (1973) and Darlington (1971). As was noted in the introduction section, one can demonstrate that each of these five definitions of "fair" selection models implies the maximization of some utility function. Unlike the Expected Utility Model, these utilities are not assigned directly by the decision-maker,but indirectly as a function of data situations. By examining the acceptance scores derived from the given definition as well as means and standard deviations of the observed data, one can find the set of utilities that are implied by the selection procedure.

For simplicity, only the most practical situations of two subgroups with the restriction of a selection ratio will be considered in the following analyses.

The Employer's Model

According to the Expected Utility Model, the "optimal" standardized acceptance scores (i.e., C_1^* and C_2^*) should be chosen to meet the following two equations.

$$\begin{aligned} & P_1 [U(O_{11})-U(O_{21})-U(O_{31})+U(O_{41})] + [U(O_{31})-U(O_{41})] \\ = \; & P_2 [U(O_{12})-U(O_{22})-U(O_{32})+U(O_{42})] + [U(O_{32})-U(O_{42})] \qquad (4.1) \end{aligned}$$

$$SR = q_1 Prob(Z_x \geq C_1^* | \pi_1) + q_2 Prob(Z_x \geq C_2^* | \pi_2) \qquad (4.2)$$

where

$$P_i = Prob(Z_y \geq Y_{si}^* | Z_x = C_i^* , \pi_i)$$

Suppose that two subpopulations have same utility structure. In other words, if

$$U(O_{11}) - U(O_{21}) = U(O_{12}) - U(O_{22}) \text{ and } U(O_{31}) - U(O_{41}) = U(O_{32}) - U(O_{42})$$

Then equation (4.1) can be simplified as

$$P_1 = P_2 \qquad (4.3)$$

Equation (4.1) shows when one assigns the same utility structure to both groups, the resulting "optimal" acceptance scores (C_1^* and C_2^*) that maximize the expected utility are those which produce the equality of conditional probabilities given in equation (4.3). We also know that these acceptance scores satisfy the definition of "fairness" given by the Employer's Model. Since equation (4.3) is the special case of equation (4.1) and equation (4.3) is the definition of "fairness" given by the Employer's Model, it can be concluded that the Employer's Model is the special case of the Expected Utility Model. In other words, under the situations where the same utility structure is assigned to both subpopulations, the Employer's Model leads to the selection procedure that maximizes the expected utility of selection. In the special cases of same utility structure where only selection errors are considered (i.e., $U(O_{11}) = U(O_{41}) = U(O_{12}) = U(O_{42}) = -1$ and $U(O_{21}) = U(O_{31}) = U(O_{22}) = U(O_{32}) = 0$), the selection procedure derived from the Employer's Model is a procedure that minimizes the rate of selection errors.

The Regression Model and Darlington's Model

In the following analysis it will be shown that the Regression Model and Darlington's Model, as with the Employer's Model, can also be viewed in terms of the Expected Utility Model with the same utility structure assigned to two subpopulations. The ten hypothetical data situations which were originally given by Cole (1972) are considered in this section. These data situations are described in Table 1 in terms of the standardized minimum success point (Y_s*), the correlation between the predictor and criterion variables (ρ_{xy}), the standard deviation of the predictor variable (σ_x) and the standard deviation of the criterion variable (σ_y) within each subgroup.

For all these data situations, the common assumptions are: 1) only 20% of total applicants can be accepted (i.e., SR= 0.2); 2) the population of applicants comprises two subpopulations, the minority group (Group 1) and the majority group (Group 2); 3) 20% of applicants are minority group members and 80% of applicants are majority group members (i.e., q_1= 0.2 and q_2= 0.8). In Case A, the regression lines are parallel with the minority regression line falling above the majority regression line. In Case B, the regression lines are identical, but the minority group has a lower proportion of potentially successful applicants than the majority group (i.e., $Y_{s1}^* > Y_{s2}^*$). In Case C, two groups differ in the slope of the regression lines with the minority slope smaller. In Case D, the minority regression line

Table 1
Ten Hypothetical Data Situations

	Minority Group				Majority Group			
Situation	σ_x	ρ_{xy}	σ_y	Y_s^*	σ_x	ρ_{xy}	σ_y	Y_s^*
A	1.0	0.5	1.0	0.0	1.0	0.5	1.0	0.0
B	1.0	0.5	1.0	0.5	1.0	0.5	1.0	0.0
C	1.0	0.2	1.0	0.0	1.0	0.7	1.0	0.0
D	1.0	0.4	1.0	1.0	1.0	0.6	1.0	0.0
E	1.0	0.6	1.0	1.0	1.0	0.4	1.0	0.0
A'	1.0	0.5	1.0	0.0	1.0	0.5	1.0	0.0
B'	1.0	0.5	1.0	0.0	1.0	0.5	1.0	0.5
C'	1.0	0.7	1.0	0.0	1.0	0.2	1.0	0.0
D'	1.0	0.6	1.0	0.0	1.0	0.4	1.0	1.0
E'	1.0	0.4	1.0	0.0	1.0	0.6	1.0	1.0

Note. The symbols σ_x, ρ_{xy}, σ_y and Y_s^* represent the standard deviation of the predictor variable, the correlation between the predictor and criterion variables, the standard deviation of the criterion variable, and the standardized minimal success score respectively.

is with smaller slope and intercept than the majority group. In Case E, the minority regression line is with larger slope and smaller intercept than the majority group. The last five cases (i.e., A', B', C', D', and E') are reversals of minority and majority data situations in the five above mentioned cases. Under these data situations, Cole (1972) computed the acceptance scores (C_1 and C_2) for each of the five "fair" selection models. For comparing the Regression Model and Darlington's Model with the Employer's Model, the conditional probability of success (P_1) for the individuals scoring at C_1 in the minority group, the conditional probability of success (P_2) for the individuals scoring at C_2 in the majority group, and the ratio of two conditional probabilities (P_1/P_2) are computed for three models. The values of P_1/P_2 are computed under the assumption that 0. is the minimum successful point for minority group members and majority group members. In addition, the adjusted conditional probability of success (P_1*) for the minority group members scoring at C_1 and the adjusted ratio (P_1*/P_2) are computed for Darlington's Model. The values of P_1*/P_2 are calculated under the assumption that 0. is the minimum successful point for the majority group members, while -0.5 is the minimum successful point for the minority group. The results are presented in Table 2.

It is seen that P_1/P_2 is one in all situations for the Employer's Model. The values of P_1/P_2 for the Regression Model are very close to one in all cases. This indicates that the acceptance scores derived from the Regression Model are approximately the same as those from the Employer's Model. Thus when the Regression Model is viewed in

Table 2

Results of Employer's, Regression and Darlington's Models

Case	Employer's Model					Regression Model					Darlington's Model						
	C_1^*	C_2^*	P_1	P_2	P_1/P_2	C_1^*	C_2^*	P_1	P_2	P_1/P_2	C_1^*	C_2^*	P_1	P_2	P_1/P_2	P_1^*	P_1^*/P_2
A	0.84	0.84	0.69	0.69	1.0	0.84	0.84	0.69	0.69	1.0	0.10	1.10	0.52	0.74	0.71	0.74	1.0
B	1.71	0.71	0.66	0.66	1.0	1.71	0.71	0.66	0.66	1.0	0.84	0.84	0.46	0.69	0.68	0.69	1.0
C	3.24	0.67	0.75	0.75	1.0	2.38	0.68	0.69	0.75	0.92	0.22	1.05	0.52	0.85	0.61	0.71	0.84
D	3.66	0.67	0.69	0.69	1.0	3.51	0.67	0.67	0.70	0.96	1.61	0.72	0.35	0.71	0.50	0.56	0.80
E	2.07	0.69	0.62	0.62	1.0	2.13	0.69	0.64	0.62	1.03	1.84	0.70	0.55	0.62	0.89	0.77	1.25
A'	0.84	0.84	0.69	0.69	1.0	0.84	0.84	0.69	0.69	1.0	0.10	1.10	0.52	0.74	0.71	0.74	1.0
B'	0.10	1.10	0.52	0.52	1.0	0.10	1.10	0.52	0.52	1.0	-0.55	1.45	0.38	0.60	0.62	0.60	1.0
C'	0.22	1.06	0.59	0.59	1.0	0.29	1.02	0.61	0.58	1.05	-0.34	1.33	0.37	0.61	0.61	0.64	1.06
D'	-0.59	1.48	0.33	0.33	1.0	-0.65	1.52	0.31	0.34	0.93	-1.38	2.03	0.15	0.42	0.36	0.34	0.81
E'	-0.46	1.40	0.42	0.42	1.0	-0.43	1.38	0.43	0.42	1.02	-0.97	1.73	0.34	0.52	0.65	0.55	1.06

Note. For Tables 2-4, the symbols C_1*, P_1 and P_1* represent the standardized acceptance score, the conditional prabability of success at the acceptance score, and the adjusted conditional probability of success at the acceptance score for the minority group respectively. The symbols C_2* and P_2 represent the standardized acceptance score and the conditional probability of success at the acceptance score for the majority group.

terms of the Expected Utility Model, its implicit utility structure is quite similar to that of the Employer's Model. Therefore, one may conclude that the Regression Model as well as the Employer's Model are both "optimal" (i.e., maximizes the expected utility) and "fair" selection procedures in those cases when one can defend the assignment of same utility structure to both subgroups.

From Table 2, it is seen that in Darlington's Model, the values of P_1/P_2 are always below one. In other words, the minority group has lower conditional probability of success at the acceptance point than the majority group. However, the adjusted values of $P_1{*}/P_2$ are all one for the cases with equal slope (A, B, A' and B'). In the cases with different slopes, the values of $P_1{*}/P_2$ are closer to one than the values of P_1/P_2, but they are not as close to one as the values of P_1/P_2 in the Regression Model. These results are due to the fact that the acceptance scores of Darlington's Model were derived from the pool regression equation (i.e., the regression equation which is computed by using all applicants of two subgroups), while the acceptance scores of the Regression Model and the Employer's Model were derived from the subgroup regression equations. If the acceptance scores of Darlington's Model are computed from the subgroup regression equations, the values of $P_1{*}/P_2$ will be quite similar to those of the Regression Model. In order to test this hypothesis, the results of the new Darlington's Model in which the subgroup regression equations instead of the pool regression equation are used to derive acceptance scores, are computed and presented in Table 3.

Table 3 shows that the values of P_1/P_2 are still always below

Table 3
Results of New Darlington's Model

Situation	C_1^*	C_2^*	P_1	P_2	P_1/P_2	P_1^*	P_1^*/P_2
A	0.10	1.10	0.52	0.74	0.71	0.74	1.0
B	0.84	0.84	0.46	0.69	0.68	0.69	1.0
C	0.64	0.90	0.54	0.81	0.67	0.74	0.91
D	2.30	0.70	0.46	0.70	0.66	0.68	0.97
E	1.37	0.81	0.41	0.64	0.65	0.66	1.03
A'	0.10	1.10	0.52	0.74	0.71	0.74	1.0
B'	-0.55	1.45	0.38	0.60	0.62	0.60	1.0
C'	-0.33	1.33	0.37	0.60	0.61	0.61	1.06
D'	-1.22	1.91	0.18	0.40	0.45	0.38	0.96
E'	-1.06	1.80	0.33	0.54	0.60	0.53	0.99

Note. See Note in Table 2.

one. However, the values of P_1*/P_2 are very close to one in all cases. In addition, for every case, the value of P_1*/P_2 is approximately the same as the value of P_1/P_2 for the Regression Model. This indicates that by using different success criterion scores (i.e., 0. for the majority group and -0.5 for the minority group) and subgroup regression equations, the acceptance scores derived from the Darlington's Model almost satisfy the definition of "fairness" (i.e., $P_1 = P_2$) given by the Employer's Model. Using the same reasoning as for the Regression Model, the new Darlington's Model can be considered as the Expected Utility Model with an identical utility structure assigned to subgroups.

Thorndike's Model and the Equal Opportunity Model

In studying the implicit utility structure of Thorndike's Model and the Equal Opportunity Model, let us first consider whether or not the acceptance scores of these two models satisfy the definition of "fairness" given by the Employer's Model. The conditional probabilities of success at the acceptance scores for two subpopulations (P_1 and P_2) and the ratio of these two conditional probabilities (P_1/P_2) are computed and presented in Table 4 for Thorndike's and the Equal Opportunity Models under Cole's ten hypothetical data situations. Table 4 shows that the values of P_1/P_2 range from 0.46 to 1.99 for Thorndike's Model and the values of P_1/P_2 range from 0.35 to 2.95 for the Equal Opportunity Model. This indicates that the acceptance scores of these two models cannot consistently satisfy the definition of the Employer's Model (i.e., $P_1/P_2 = 1.0$). In other words, Thorndike's

Table 4

Results of Thorndike's and Equal Opportunity Models

	Thorndike's Model					Equal Opportunity Model				
Situation	C_1^*	C_2^*	P_1	P_2	P_1/P_2	C_1^*	C_2^*	P_1	P_2	P_1/P_2
A	0.84	0.84	0.68	0.68	1.0	0.84	0.84	0.68	0.68	1.0
B	1.11	0.78	0.52	0.67	0.78	0.98	0.81	0.49	0.68	0.73
C	0.84	0.84	0.57	0.80	0.71	0.60	0.91	0.55	0.82	0.67
D	1.45	0.73	0.32	0.71	0.46	0.97	0.81	0.25	0.73	0.35
E	1.45	0.73	0.43	0.59	0.74	1.30	0.76	0.39	0.63	0.62
A'	0.84	0.84	0.69	0.69	1.0	0.84	0.84	0.69	0.69	1.0
B'	0.56	0.92	0.63	0.48	1.30	0.71	0.88	0.66	0.47	1.39
C'	0.84	0.84	0.80	0.57	1.40	1.08	0.79	0.86	0.56	1.52
D'	0.15	1.12	0.54	0.27	1.99	0.72	0.87	0.70	0.24	2.95
E'	0.15	1.12	0.53	0.34	1.54	0.40	0.98	0.57	0.30	1.87

Note. See Note in Table 2.

Model and the Equal Opportunity Model cannot be viewed in terms of the Expected Utility Model with an identical implicit utility structure assigned to two subgroups for all data situations. The variation of P_1/P_2 indicates that the implicit utility structure of these two models varies as a function of data situations. In other words, the utilities implied by the selection procedures of these two models in one data situation may differ from the utilities implied by the selection procedures of these two models in another data situation. The data situation determines the implicit utilities of these two models.

One can identify the actual implicit utility structure associated with the value of P_1/P_2 in the following manner. Let LS_1 be the utility difference between accepting and rejecting a potentially successful applicant for the minority group (i.e., $LS_1 = U(O_{21})-U(O_{11})$), and LS_2 be the corresponding utility difference for the majority group (i.e., $LS_2 = U(O_{22})-U(O_{12})$). On the other hand, let LUS_1 be the utility difference between rejecting and accepting a potentially unsuccessful applicant for the minority group (i.e., $LUS_1 = U(O_{31})-U(O_{41})$), and LUS_2 be the corresponding utility difference for the majority group (i.e., $LUS_2 = U(O_{32})-U(O_{42})$). In terms of loss notations, these utility differences can be summaried as follows:

LS_1 = the loss resulting from rejecting (instead of accepting) a potentially successful minority applicant.

LS_2 = the loss resulting from rejecting (instead of accepting) a potentially successful majority applicant.

LUS_1 = the loss resulting from accepting (instead of rejecting) a potentially unsuccessful minority applicant.

LUS_2 = the loss resulting from accepting (instead of rejecting) a potentially unsuccessful majority applicant.

In terms of these loss notations, the optimal acceptance scores derived from the Expected Utility Model must satisfy the following equation.

$$P_1(LS_1 + LUS_1) - LUS_1 = P_2(LS_2 + LUS_2) - LUS_2 \qquad (4.4)$$

where

$$P_i = \text{Prob}(Z_y \geq Y_{si}^* \mid Z_x = C_i^*, \pi_i)$$

To simplify equation (4.4), LUS_1 can be expressed in terms of LUS_2. For h greater than zero, let $LUS_1 = h(LUS_2)$. For example, if the loss resulting from accepting a potentially unsuccessful applicant from the minority group is only one half as "costly" as the corresponding loss for the majority group, one would set $h = 0.5$ and $LUS_1 = 0.5(LUS_2)$. In many practical situations, it is reasonable to assume that h is less than or equal to one. For instance, when the institution wants to recruit minority group members, the loss resulting from accepting a potentially unsuccessful applicant for the minority group is most likely smaller than the corresponding loss for the majority group (i.e., $LUS_1 = h(LUS_2) < LUS_2$). Using "h" notation, equation (4.4) can be simplified and rewritten as

$$LS_1 = R_1(LS_2) + R_2(h-R_3)(LUS_2) \qquad (4.5)$$

where

$$R_1 = P_2/P_1$$

$$R_2 = (1-P_1)/P_1$$

$$R_3 = (1-P_2)/(1-P_1)$$

Equation (4.5) indicates that the values of P_1/P_2 can imply the relationship between the losses associated with the misclassification of a potentially successful applicant for the minority group and the majority group (i.e., LS_1 and LS_2) if the ratio of the two losses associated with the misclassification of a potentially unsuccessful applicant (i.e., the value of h) is known. In other words, suppose the institution adopts a selection procedure in which C_1 and C_2 are the acceptance scores for the minority and majority groups. Using these acceptance scores, the minimal level of successful performance (Y_s) and the subgroup regression equations, one can compute the conditional probabilities of success (P_1 and P_2) for the individuals scoring at the acceptance scores (C_1 and C_2) and the ratio of these two conditional probabilities. However, the loss structure implied by the selection procedure is unknown. In this situation, one can work backwards and use equation (4.5) to obtain a relationship between LS_1, LS_2 and LUS_2 given a certain value of h. In other words, the observed value of P_1/P_2 can be used to identify the loss structure implied by the selection procedure which has been adopted.

Let us consider an example of identification of an implicit loss structure by using equation (4.5). Suppose the loss resulting from accepting a potentially unsuccessful applicant from the minority group is smaller than or equal to the corresponding loss for the majority group (i.e., $0 < h \leq 1.0$). If $P_1/P_2 > 1.0$, then $(1-P_2)/(1-P_1) = R_3 > 1.0$. Since h is less than or equal to one, $(h-R_3) < 0$. Therefore, one can conclude that if $P_1/P_2 > 1.0$, then $LS_1 < (P_2/P_1)(LS_2)$ (i.e., the loss associated with the misclassification of a potentially successful minority applicant is smaller than the loss associated with the misclassification of a potentially successful majority applicant). When $P_1/P_2 = 1.0$, $LS_1 \leqq LS_2$ with the equal sign only holding for an h value of 1.0. In other words, an observed P_1/P_2 value of 1.0 implies a loss structure such that the two groups have an identical loss structure or the majority group has a greater loss associated with the misclassification of a potentially successful applicant than the minority group. Finally, when $P_1/P_2 < 1.0$, then $LS_1 > (P_2/P_1)(LS_2)$ given that the value of h is greater than the value of $(1-P_2)/(1-P_1)$. In other words, when the observed value of P_1/P_2 is less than one and the value of h is greater than the ratio of the conditional probability of being unsuccessful at the majority acceptance score to the corresponding conditional probability at the minority acceptance score, the minority group has a greater loss associated with the misclassification of a potentially successful applicant than the majority group. For a value of h greater than zero and less than or equal to one (i.e., $LUS_1 \leqq LUS_2$), the loss structure implied by the observed value of P_1/P_2 can be summaried in Table 5. The loss structure under the assignment of h greater than 1.0 will not be discussed, although they can also be identified by using equation (4.5).

When Thorndike's Model and the Equal Opportunity Model are viewed in terms of the Expected Utility Model, the values of P_1/P_2 can be used to identify the loss structure associated with the acceptance scores derived from these two models. Under the assumption that the loss resulting from accepting a potentially unsuccessful applicant from the minority group is less than or equal to the corresponding loss for the majority group, the value of P_1/P_2 were computed and

Table 5
The Implicit Loss Structure Associated with An Observed Value of P_1/P_2 under A Certain Assigned Value of h

Assigned value of h	Observed value of P_1/P_2	Implicit relationship between LS_1 and LS_2	Meaning of implicit loss structure
$h = 1.0$	$P_1/P_2 < 1.0$	$LS_1 > (P_2/P_1)(LS_2)$	LS_1 exceeds LS_2 by the factor of P_2/P_1
$h = 1.0$	$P_1/P_2 = 1.0$	$LS_1 = LS_2$	LS_1 is as "costly" as LS_2
$h = 1.0$	$P_1/P_2 > 1.0$	$LS_1 < (P_2/P_1)(LS_2)$	LS_1 is less "costly" than LS_2 by the fraction of P_2/P_1
$1.0 > h > (1-P_2)/(1-P_1)$	$P_1/P_2 < 1.0$	$LS_1 > (P_2/P_1)(LS_2)$	LS_1 exceeds LS_2 by the factor of P_2/P_1
$1.0 > h > 0.$	$P_1/P_2 = 1.0$	$LS_1 < LS_2$	LS_1 is less "costly" than LS_2
$1.0 > h > 0.$	$P_1/P_2 > 1.0$	$LS_1 < (P_2/P_1)(LS_2)$	LS_1 is less "costly" than LS_2 by the fraction of P_2/P_1

presented in Table 4. These values imply loss structures for the Thorndike's and Equal Opportunity Models in Cole's ten hypothetical data situations. Table 4 shows that when two groups have an identical slope and an identical proportion of potentially successful applicants (i.e., the cases A and A'), the value of P_1/P_2 is found to be one. Using Table 5 in the cases of A and A', Thorndike's Model and the Equal Opportunity Model can be viewed as implying either an identical loss structure for the two groups or a greater loss assigned to the misclassified potentially successful applicants for the majority group than for the minority group.

The value of P_1/P_2 is found to be less than one in the cases B, C, D and E (i.e., the data situations that the minority group has a lower validity and/or a smaller proportion of potentially successful applicants than the majority group). This implies that in any of these cases, if either Thorndike's Model or the Equal Opportunity Model is applied, the decision-maker can be described as if he implicitly assigns a greater loss associated with the misclassification of a potentially successful applicant for the minority group than for the majority group given h value greater than $(1-P_2)/(1-P_1)$. In order to have a greater loss associated with the misclassification of a potentially successful applicant for one group, the decision-maker has to assign a greater utility to the correctly classified members (i.e., O_{2i}) and/or a greater disutility to the misclassified members (i.e., O_{1i}) for that group.

On the other hand, for the cases B', C', D' and E' (i.e., the cases where the minority group has a higher validity and/or mean performance), the value of P_1/P_2 is found to be greater than one. Using Table 5 this indicates that these two models can be viewed in terms of the Expected Utility Model having a loss structure where the minority group has a smaller loss resulting from rejecting a potentially successful applicant than the majority group in these cases.

In summary, Thorndike's Model and the Equal Opportunity Model can be viewed in terms of the Expected Utility Model with a certain utility structure which varies as a function of data situations. When two groups have parallel regression lines and an identical distribution of criterion scores, these two models may be viewed in terms of the Expected Utility Model with the same utility structure implicitly assigned to two groups. For other data situations, Thorndike's Model and the Equal Opportunity Model can be viewed in terms of the Expected Utility Model with a greater utility assigned to the correctly classified members and/or a greater disutility to the misclassified members for the group which has a lower validity and mean performance than the other group.

5. THE PROPORTIONS OF MINORITY AND MAJORITY APPLICANTS ACCEPTED BY APPLYING THE EXPECTED UTILITY MODEL

It is important to consider the proportions of minority and majority applicants that will be accepted in various situations if the selection procedure is performed by applying the Expected Utility Model. In this chapter, 1458 hypothetical selection situations were constructed. For each hypothetical data situation, the proportions of minority applicants and majority applicants accepted were computed. The entire set of results is presented in Appendix II. These 1458 hypothetical data situations were constructed by varying the values of the minority test validity (ρ_1) and the majority test validity (ρ_2). These coefficients were set as 0.2, 0.4 and 0.6. Further, the proportions of potentially successful applicants for the minority group and the majority group were varied and set as 0.31, 0.50 and 0.69. These proportions correspond to the standardized minimal success scores of 0.5, 0.0 and -0.5 respectively. In other words, the standardized minimal success scores for two groups (Y_{s1}^* and Y_{s2}^*) were set as 0.5, 0.0 and -0.5 in the study. The value of selection ratio (SR) was also varied at values of 0.2, 0.4 and 0.6. In all these data situations, it was assumed that 30% of applicants (i.e., q_1 = 0.3) are minority group members. The final variable to be varied was the utility structure. Six hypothetical utility assignments were considered in this chapter. The analyses of 1458 hypothetical selection situations can be used to demonstrate how the assignment of utilities, the value of selection ratio, the validity of the test score and the standardized minimal success score affect the proportions of minority and majority applicants accepted. Let us first consider the effect of utility structures.

Utility Structures

In all situations it is always possible to set the utility of the most desirable outcome equal to +1.0 and the disutility of the least desirable outcome to -1.0. Then it is true that $2.0 \geq [U(O_{2i}) - U(O_{1i})] \geq 0.$ and $2.0 \geq [U(O_{3i}) - U(O_{4i})] \geq 0.$ In terms of loss notations, it must be true that LS_1 and LS_2 (i.e., the losses resulting from the misclassification of a potentially successful applicant for the minority and majority groups) and LUS_1 and LUS_2 (i.e., the losses resulting from the misclassification of a potentially unsuccessful applicant for the minority and majority groups) are in the range of 0.0 to 2.0 (i.e., $2.0 \geq LS_i \geq 0.$ and $2.0 \geq LUS_i \geq 0.$). To illustrate how a utility structure affects the proportions of applicants accepted for the minority and majority groups, six different hypothetical utility structures are considered in the following analyses. Rather than presenting six different utility structures, it is sufficient to present them in terms of loss structures. These six hypothetical utility structures can be described as follows:

(1) Minority: $LS_1 = 1.0$ and $LUS_1 = 1.0$
Majority: $LS_2 = 1.0$ and $LUS_2 = 1.0$
$LS_1/LS_2 = 1.0$ and $LUS_1/LUS_2 = 1.0$

(2) Minority: $LS_1 = 1.5$ and $LUS_1 = 1.0$

Majority: $LS_2 = 1.0$ and $LUS_2 = 1.0$

$LS_1/LS_2 = 1.5$ and $LUS_1/LUS_2 = 1.0$

(3) Minority: $LS_1 = 2.0$ and $LUS_1 = 1.0$

Majority: $LS_2 = 1.0$ and $LUS_2 = 1.0$

$LS_1/LS_2 = 2.0$ and $LUS_1/LUS_2 = 1.0$

(4) Minority: $LS_1 = 1.0$ and $LUS_1 = 0.5$

Majority: $LS_2 = 1.0$ and $LUS_2 = 1.0$

$LS_1/LS_2 = 1.0$ and $LUS_1/LUS_2 = 0.5$

(5) Minority: $LS_1 = 1.5$ and $LUS_1 = 0.5$

Majority: $LS_2 = 1.0$ and $LUS_2 = 1.0$

$LS_1/LS_2 = 1.5$ and $LUS_1/LUS_2 = 0.5$

(6) Minority: $LS_1 = 2.0$ and $LUS_1 = 0.5$

Majority: $LS_2 = 1.0$ and $LUS_2 = 1.0$

$LS_1/LS_2 = 2.0$ and $LUS_1/LUS_2 = 0.5$

The first three utility structures (i.e., the utility structures (1), (2) and (3)) represent the viewpoints that the misclassification of a potentially unsuccessful minority applicant is as "important" as the misclassification of a potentially unsuccessful majority applicant (i.e., $LUS_1/LUS_2 = h = 1.0$). However, these utility structures differ in the assignment of losses associated with the misclassification of a successful applicant for two groups. Under the assignment of utility structure (1), rejecting a potentially successful minority applicant is as "costly" as rejecting a potentially successful majority applicant (i.e., $LS_1/LS_2 = 1.0$). The utility structure (1) is equivalent to the utility structure which will lead to minimize the rate of selection errors. On the other hand, under the assignment of utility structure (2) or (3), the misclassification of a potentially successful minority applicant is considered as 1.5 times or twice as "important" as the misclassification of a potentially successful majority applicant. Unlike the utility structures (1), (2) and (3), the utility structures (4), (5) and (6) are the utility assignments where the loss resulting from the misclassification of a potentially unsuccessful applicant is related to the applicant's group membership. In the last three utility structures, the loss resulting from accepting a potentially unsuccessful minority applicant is only half as "costly" as the corresponding loss for the majority group (i.e., $LUS_1/LUS_2 = h = 0.5$). As with the first three utility structures, these utility structures also differ with each other in the assignment of losses associated with the misclassification of a potentially successful applicant for two groups. The misclassification of a potentially successful minority applicant is the same, 1.5 times and twice as "important" as the misclassification of a potentially successful majority applicant under the utility structures (4), (5) and (6), respectively.

For a given utility structure and a given value of selection ratio, eighty-one hypothetical data situations were considered by varying the values of the test validity (i.e., ρ_1 and ρ_2) and the standardized minimal success score (i.e., Y_{s1}^* and Y_{s2}^*) for two

groups. Since there are six utility structures and three different values of SR, eighteen such set of eighty-one hypothetical data situations can be considered. Within each data set of size eighty-one, the number of situations for which the Expected Utility Model produced a result "favoring' the minority group" (i.e., the minority group had a greater proportion of applicants accepted than the majority group) was computed and presented in Table 6.

Table 6 shows that in terms of frequency of "favoring the minority group", utility structure (3) is the highest and followed in order by utility structure (2) and utility structure (1). Among the last three utility structures, utility structure (6) is the highest and followed in order by utility structure (5) and utility structure (4). These results indicate that across the eighty-one data situations, if the value of LS_1 (i.e., the loss resulting from rejecting a potentially successful minority applicant) is increased, the chance that the optimal selection procedure favors the minority group will be increased. Table 6 also shows that in terms of frequency of "favoring the minority group", utility structure (4) is greater than utility structure (1); utility structure (5) is greater than utility structure (2); and utility structure (6) is greater than utility structure (3). Thus, as one reduces the value of LUS_1 (i.e., the loss resulting from accepting a potentially unsuccessful minority applicant), the Expected Utility Model has a better chance of "favoring the minority group". In summary, the results in Table 6 indicate that there is a direct relationship between the Expected Utility Model "favoring the minority group" and the value of LS_1 (or the ratio of LS_1 to LS_2), but there

Table 6

The Number of Situations Favoring the Minority Group for Each Utility Structure and Each Value of Selection Ratio

	Utility Structure					
Situation	(1)	(2)	(3)	(4)	(5)	(6)
SR = 0.2	36(44.4%)	57(70.4%)	67(82.7%)	52(64.2%)	66(81.5%)	74(91.4%)
SR = 0.4	35(43.2%)	54(66.7%)	62(76.5%)	53(65.4%)	70(86.4%)	72(88.9%)
SR = 0.6	35(43.4%)	53(65.4%)	61(75.3%)	57(70.4%)	70(86.4%)	72(88.9%)

Note. Each entry was based on eighty-one hypothetical data situations.

is an inverse relationship between the Expected Utility Model "favoring the minority group" and the value of LUS_1 (or the ratio of LUS_1 to LUS_2). In utility structure (1) where the two groups were assigned an identical utility structure (i.e., $LS_1/LS_2 = 1.0$ and $LUS_1/LUS_2 = 1.0$), the optimal selection procedure favors the minority group almost as often as it favors the majority group over the eighty-one data situations. It should be noted that the proportion of minority applicants accepted is equivalent to the proportion of majority applicants accepted in some situations. On the other hand, in utility structure (6) where the minority group has a larger value of LS_1 and a smaller value of LUS_1 than the majority group (i.e., $LS_1/LS_2 = 2.0$ and $LUS_1/LUS_2 = 0.5$), it is seen in Table 6 that the optimal selection procedure will favor the minority group almost nine out of ten times.

Another method for understanding how utility structures affect the degree of "favoring the minority group" is to compare differences between the proportions of applicants accepted for two groups. The average difference across the eighty-one data situations was computed and presented in Table 7 for all utility structures and three values of selection ratio. Table 7 shows that an increase in the value of LS_1 (or the ratio of LS_1 to LS_2) and a decrease in the value of LUS_1 (or the ratio of LUS_1 to LUS_2) lead to an increase in the degree of "favoring the minority group". Under most utility structures, the degree of "favoring the minority group" is the largest when the value of SR is 0.4.

Table 7
The Mean Difference between the Proportion of
Applicants Accepted for Two Subgroups

	Utility Structure					
Situation	(1)	(2)	(3)	(4)	(5)	(6)
SR = 0.2	0.071	0.244	0.360	0.196	0.371	0.477
SR = 0.4	0.037	0.233	0.371	0.263	0.443	0.554
SR = 0.6	-0.037	0.120	0.226	0.206	0.323	0.394

Note. Each entry is the mean of eighty-one differences between the proportions of applicants accepted for two groups. The difference is always taken as the proportion of minority applicants accepted minus the proportion of majority applicants accepted.

Selection Ratio

To demonstrate the effect of the selection ratio constraint on the proportions of applicants accepted for two groups, the value of selection ratio was varied in the study. Three different values of SR (i.e., SR = 0.2, SR = 0.4 and SR = 0.6) were considered in each data situation. The average proportion of applicants accepted (across the eighty-one data situations) was computed and presented in Table 8 for two subgroups under each assignment of utility structures and a given value of selection ratio. Table 8 clearly shows that when the value of SR is increased from 0.2 to 0.4 to 0.6, the proportion of minority applicants accepted and the proportion of majority applicants accepted are also increased for all utility structures. However, the rate of change is much sharper for the majority group. For example, under utility structure (6), the proportion of minority applicants accepted is increased about 1.5 times, while the proportion of majority applicants accepted is increased about 4.1 times when the value of SR is increased from 0.2 to 0.4. We can simply conclude that the value of selection ratio has a direct relationship with the proportion of applicants accepted for each subgroup.

Standardized Minimal Success Scores

The standardized minimal success scores were varied at values of 0.5, 0.0 and -0.5 such as corresponding to 0.31, 0.50 and 0.69 of potentially successful applicants in subpopulations. These proportions have been found in the empirical studies (Cleary, 1968; Bowers, 1970; Temp, 1971; and Petersen, 1974). The average proportion of minority applicants accepted (across twenty-seven data situations) was computed

Table 8
Average Proportions of Applicants Accepted for Two
Subgroups under Different Situations

Situation	Utility Structure (1)	(2)	(3)	(4)	(5)	(6)
Minority Group						
SR = 0.2	0.249	0.371	0.452	0.337	0.460	0.534
SR = 0.4	0.426	0.563	0.660	0.584	0.710	0.788
SR = 0.6	0.574	0.684	0.758	0.744	0.826	0.876
Majority Group						
SR = 0.2	0.178	0.127	0.092	0.141	0.089	0.057
SR = 0.4	0.389	0.330	0.289	0.321	0.267	0.234
SR = 0.6	0.611	0.564	0.532	0.538	0.503	0.482

presented in Table 9 for each value of Y_{s1}^*. The twenty-seven data situations were produced by varying the values of ρ_1, ρ_2 and Y_{s2}^*. Similarly, the average proportion of minority applicants accepted for each value of Y_{s2}^* was computed and presented in Table 10.

Table 9 shows that when the value of Y_{s1}^* is decreased, the average proportion of minority applicants accepted is increased for every value of selection ratio and every assignment of utility structures. In other words, a smaller value of Y_{s1}^* leads the optimal selection procedure to accept a greater proportion of minority applicants. It is also noted in Table 9 that the increasing rate of the proportion of minority applicants accepted with the decreasing of Y_{s1}^* is much sharper when the value of SR is 0.2 and the assignment of utilities is (1). On the other hand, Table 10 shows that if the value of Y_{s2}^* is decreased, the proportion of minority applicants accepted is also decreased. By applying the Expected Utility Model, the proportion of minority applicants accepted is directly related to the standardized minimal success score of the majority group. However, the decreasing rate of the proportion of minority applicants accepted is much sharper when the selection situation is with a smaller value of SR and under the assignment of utility structure (1).

Validity of the Test

The effect of the test validity will be demonstrated by varying the value of the test validity for the minority group (ρ_1) and the Majority group (ρ_2). Since previous studies (Seashore, 1962; Lavin, 1965; Stanley, 1967; Thomas and Stanley, 1969; Kallingall, 1971; Temp, 1971; Merritt, 1972; and Wright and Bean, 1974) have shown that the test

Table 9

Average Proportions of Minority Applicants Accepted in Terms of the Standardized Minimal Success Score for the Minority Group

Situation	Utility Structure (1)	(2)	(3)	(4)	(5)	(6)
SR = 0.2						
$Y_{s1}^{*} = 0.5$	0.084	0.153	0.226	0.185	0.273	0.357
0.0	0.239	0.372	0.489	0.334	0.476	0.579
-0.5	0.424	0.587	0.642	0.490	0.632	0.667
SR = 0.4						
$Y_{s1}^{*} = 0.5$	0.178	0.272	0.364	0.372	0.479	0.575
0.0	0.417	0.573	0.698	0.585	0.736	0.832
-0.5	0.685	0.843	0.918	0.795	0.916	0.959
SR = 0.6						
$Y_{s1}^{*} = 0.5$	0.315	0.419	0.511	0.564	0.656	0.732
0.0	0.583	0.715	0.807	0.757	0.858	0.914
-0.5	0.822	0.918	0.955	0.910	0.965	0.981

Table 10
Average Proportions of Minority Applicants Accepted in Terms of the Standardized Minimal Success Score for the Majority Group

Situation	Utility Structure (1)	(2)	(3)	(4)	(5)	(6)
SR = 0.2						
Y^*_{s2} = 0.5	0.423	0.509	0.563	0.540	0.595	0.623
0.0	0.236	0.363	0.453	0.331	0.464	0.541
-0.5	0.089	0.240	0.341	0.138	0.322	0.439
SR = 0.4						
Y^*_{s2} = 0.5	0.689	0.777	0.833	0.884	0.921	0.942
0.0	0.414	0.559	0.661	0.596	0.728	0.805
-0.5	0.176	0.352	0.487	0.273	0.481	0.618
SR = 0.6						
Y^*_{s2} = 0.5	0.824	0.878	0.911	0.980	0.986	0.989
0.0	0.586	0.698	0.772	0.792	0.865	0.905
-0.5	0.311	0.476	0.590	0.460	0.629	0.732

validity is usually in the range of 0.2 to 0.6, the value of ρ_1 and ρ_2 were set as 0.2, 0.4 and 0.6. The average proportion of minority applicants accepted (across twenty-seven data situations) was computed and presented in Table 11 for each value of ρ_1. These twenty-seven data situations were constructed by varying the values of Y_{s1}^*, Y_{s2}^* and ρ_2. Similarly, the average proportion of minority applicants accepted was computed and presented in Table 12 for each value of ρ_2.

It can be seen in Table 11 that in those cases where SR = 0.2, the average proportion of minority applicants accepted is increased when the test validity of the minority group (ρ_1) is increased. In those cases where SR = 0.4 or SR = 0.6 (except the case where SR = 0.4 and utility structure is (1)), less minority applicants (more majority applicants) will be accepted when the test validity of the minority group (ρ_1) is increased. Table 12 shows that when the test validity of the majority group (ρ_2) is increased, the average proportion of minority applicants accepted (across the twenty-seven data situations) will be decreased in those cases where SR = 0.2 or 0.4, but will be increased in those cases where SR = 0.6. AS with the effect of the standardized minimal success scores, the rate of change for the proportion of minority applicants accepted as a function of test validities is much sharper when the value of SR is smaller and the underlying utility structure does not favor any subgroup.

Table 11
Average Proportions of Minority Applicants Accepted in Terms of the Test Validity for the Minority Group

	Utility Structure					
Situation	(1)	(2)	(3)	(4)	(5)	(6)
SR = 0.2						
ρ_1 = 0.2	0.199	0.331	0.426	0.300	0.437	0.523
0.4	0.248	0.373	0.456	0.337	0.463	0.538
0.6	0.300	0.409	0.475	0.373	0.481	0.542
SR = 0.4						
ρ_1 = 0.2	0.411	0.577	0.696	0.594	0.745	0.836
0.4	0.426	0.563	0.662	0.586	0.713	0.793
0.6	0.442	0.548	0.622	0.573	0.672	0.736
SR = 0.6						
ρ_1 = 0.2	0.589	0.724	0.810	0.770	0.866	0.921
0.4	0.574	0.686	0.761	0.747	0.831	0.881
0.6	0.558	0.643	0.702	0.714	0.782	0.825

Table 12
Average Proportions of Minority Applicants Accepted in Terms of the Test Validity for the Majority Group

Situation	Utility Structure (1)	(2)	(3)	(4)	(5)	(6)
SR = 0.2						
ρ_2 = 0.2	0.328	0.444	0.514	0.438	0.538	0.590
0.4	0.247	0.372	0.459	0.337	0.465	0.537
0.6	0.172	0.296	0.384	0.235	0.378	0.476
SR = 0.4						
ρ_2 = 0.2	0.463	0.603	0.696	0.644	0.762	0.831
0.4	0.427	0.565	0.662	0.587	0.714	0.792
0.6	0.390	0.521	0.622	0.523	0.654	0.742
SR = 0.6						
ρ_2 = 0.2	0.537	0.662	0.744	0.724	0.818	0.873
0.4	0.573	0.684	0.758	0.745	0.828	0.877
0.6	0.610	0.706	0.771	0.762	0.833	0.877

Summary

The underlying utility structure of the Expected Utility Model strongly affects the proportions of minority and majority applicants accepted. It was found that there is a direct relationship between the degree to which the Expected Utility Model "favors the minority group" (i.e., accepts a greater proportion of minority applicants than majority applicants) and the value of LS_1 (or the ratio of LS_1 to LS_2), but there is an inverse relationship between the degree to which the Expected Utility Model "favors the mincrity group" and the value of LUS_1 (or the ratio of LUS_1 to LUS_2). These results imply that given a selection situation, an increase in the value of LS_1 (i.e., the loss resulting from rejecting a potentially successful minority applicant) and/or a decrease in the value of LUS_1 (i.e., the loss resulting from accepting a potentially unsuccessful minority applicant) can lead the Expected Utility Model to favor the minority group to a certain degree.

The data situation of a selection problem also has some effect on the proportions of minority and majority applicants accepted when the Expected Utility Model is applied. The degree to which the Expected Utility Model "favors the minority group" is greater in situations having a smaller value of Y_{s1}^* and/or a greater value of Y_{s2}^*. In most situations, the degree to which the Expected Utility Model "favors the minority group" is the highest when the value of SR is 0.4. Additionally, the variable SR also played as a moderate variable for test validities. For the situations where the value of

SR is 0.2 or 0.4, the degree to which the Expected Utility Model "favors the minority group" is directly related to the minority test validity, but is inversely related to the majority test validity. For the situations where the value of SR is 0.6, the relationship is just reversed. In other words, for the situations where the value of SR is 0.6, the degree to which the Expected Utility Model "favors" the minority group" is greater in situations having a lower minority test validity and/or a higher majority test validity, However, the effect of the data situation on the proportions of minority and majority applicants accepted will be dominated by the effect of the underlying utility structure when the underlying utility structure strongly favors the minority group.

6. COMPARISON OF THE EXPECTED UTILITY MODEL WITH THE REGRESSION, THE EMPLOYER'S, THORNDIKE'S, THE EQUAL OPPORTUNITY AND DARLINGTON'S MODELS

In this chapter, the Expected Utility Model will be compared with the five previously proposed fair models (Cleary, 1968; Einhorn and Bass, 1971; Thorndike, 1971; Cole, 1973; and Darlington, 1971) in eighty-one hypothetical data situations. To recruit minority applicants, the federal government often gives an institution some financial support when the institution accepts minority applicants. Consequently, for recruiting minority applicants, an institution may consider the acceptance of a potentially unsuccessful minority applicant not to be "costly" as the acceptance of a potentially unsuccessful majority applicant (i.e., $LUS_1/LUS_2 < 1.0$), but the rejection of a potentially successful minority applicant to be as "important" as the rejection of a potentially successful majority applicant (i.e., $LS_1/LS_2 = 1.0$). Therefore, only the Expected Utility Model with the assignment of utility structure (4) will be considered in the following analyses. In utility structure (4), the loss resulting from rejecting a potentially successful minority applicant is the same as the corresponding loss for the majority group (i.e., $LS_1/LS_2 = 1.0$), while the loss resulting from accepting a potentially unsuccessful minority applicant is only half as "costly" as the corresponding loss for the majority group (i.e., $LUS_1/LUS_2 = 0.5$). The eighty-one hypothetical data situations were produced by varying the values of the standardized minimal success scores (i.e., Y_{s1}^* and Y_{s2}^*), the test validity of the minority group (i.e., ρ_1) and selection ratio (SR). In all these data situations, it was assumed that the test validity of the majority group (i.e., ρ_2) is 0.4 and 30% of applicants are minority members (i.e., $q_1 = 0.3$). First, the six selection models will be compared in terms of the proportions of minority and majority applicants accepted. Then, for evaluating the consequence of the selection procedures, these six models will also be compared on average criterion performance of accepted applicants, the rate of selection errors, the success rate of accepted applicants and the expected loss of selection.

The Proportions of Minority and Majority Applicants Accepted

The proportions of minority and majority applicants accepted were computed for each of the six selection models under each of the eighty-one hypothetical data situations. The entire set of results is presented in Appendix III. It is important to consider whether a selection procedure favors the minority group (i.e., the minority group has a greater proportion of applicants accepted than the majority group) or not. For a given value of the minority test validity (ρ_1) and a given value of selection ratio (SR), the number of situations (across the nine data situations with the values of Y_{s1}^* and Y_{s2}^* varied) "favoring the minority group" was computed and presented in Table 13 for each selection model.

Table 13 shows that for the Expected Utility Model (EXPT) with utility structure (4) and Thorndike's Model (THOR), the number of situations "favoring the minority group" does not tend to be related

Table 13
The Number of Situations Favoring the Minority Group under Each Value of Minority Test Validity and Selection Ratio for the Six Models

Situation	Model					
	EXPT	REGR	EMPL	THOR	EQOP	DARL
ρ_1 = 0.2 and ρ_2 = 0.4						
SR = 0.2	5 (55.6%)	3 (33.3%)	3 (33.3%)	3 (33.3%)	8 (88.9%)	6 (66.7%)
0.4	6 (66.7%)	3 (33.3%)	3 (33.3%)	3 (33.3%)	8 (88.9%)	6 (66.7%)
0.6	7 (77.8%)	6 (66.7%)	6 (66.7%)	3 (33.3%)	8 (88.9%)	8 (88.9%)
ρ_1 = 0.4 and ρ_2 = 0.4						
SR = 0.2	6 (66.7%)	3 (33.3%)	3 (33.3%)	3 (33.3%)	3 (33.3%)	6 (66.7%)
0.4	6 (66.7%)	3 (33.3%)	3 (33.3%)	3 (33.3%)	3 (33.3%)	6 (66.7%)
0.6	6 (66.7%)	3 (33.3%)	3 (33.3%)	3 (33.3%)	3 (33.3%)	6 (66.7%)
ρ_1 = 0.6 and ρ_2 = 0.4						
SR = 0.2	6 (66.7%)	6 (66.7%)	6 (66.7%)	3 (33.3%)	3 (33.3%)	8 (88.9%)
0.4	6 (66.7%)	6 (66.7%)	5 (55.6%)	3 (33.3%)	2 (22.2%)	8 (88.9%)
0.6	6 (66.7%)	3 (33.3%)	2 (22.2%)	3 (33.3%)	1 (11.1%)	6 (66.7%)

Note. For Tables 13-17, each entry was based on nine hypothetical data sitautions. The abbreviations EXPT, REGR, THOR, EQOP and DARL are for the Expected Utility Model, the Regression Model, the Employer's Model, Thorndike's Model and the Equal Opportunity Model. The symbols ρ_1, ρ_2 and SR represent the minority test validity, the majority test validity and the selection ratio respectively.

to the minority test validity (or the relationship between ρ_1 and ρ_2) and the value of selection ratio. By applying the Expected Utility Model, the selection procedure favors the minority group about six out of nine times for almost every value of ρ_1 and every value of SR. Whether the Expected Utility Model favors the minority group depends only upon the underlying utility structure and the relationship of Y_{s1}^* to Y_{s2}^*. Under the nine hypothetical data situations, the Expected Utility Model with utility structure (4) favors the minority group except in the situations where the minority group has a higher standardized minimal success score than the majority group (i.e., the minority group has a smaller proportion of potentially successful applicants than the majority group). However, the results of the last chapter imply that adjusting the utility structure (i.e., increasing the ratio of LS_1 to LS_2 and/or decreasing the ratio of LUS_1 to LUS_2) can lead the Expected Utility Model to favor the minority group even in the situations where the minority group has a smaller proportion of potentially successful applicants than the majority group. By using Thorndike's Model, the minority group was favored three out of nine times. It can be seen in Appendix III that Thorndike's Model favors the minority group only when the minority group has a lower value of standardized minimal success score than the majority group. These results indicate that Thorndike's Model favoring the minority group completely depend upon the relationship of Y_{s1}^* to Y_{s2}^*.

It is also shown in Table 13 that for the Regression Model (REGR), the Employer's Model (EMPL), the Equal Opportunity Model

(EQOP) and Darlington's Model (DARL), the number of situations "favoring the minority group" is related to the minority test validity. By applying the Regression Model or the Employer's Model or Darlington's Model, the number of situations "favoring the minority group" and the minority test validity (ρ_1) are directly related to each other when the value of SR is 0.2 or 0.4, but inversely related to each other when the value of SR is 0.6. However, Darlington's Model has a greater number of situations "favoring the minority group" than the Regression Model and the Employer's Model in every data situation. The Regression Model has almost the same number of situations "favoring the minority group" as the Employer's Model (which is the Expected Utility Model with an identical utility structure assigned to two groups) in every data situation. Under the Equal Opportunity Model, the number of situations "favoring the minority group" is decreased when the minority test validity is increased. This implies that by applying the Equal Opportunity Model, the minority group was likely favored when it has a lower test validity than the majority group, but it was likely disfavored when it has a higher test validity than the majority group.

Overall, Darlington's Model is the highest followed by the Expected Utility Model in terms of frequency "favoring the minority group" across the eighty-one hypothetical data situations. The ratio of Y_{s1}^* to Y_{s2}^* affects the degree to which the selection procedure "favors the minority group" for all models. The value of SR and the value of ρ_1 (or the ratio of ρ_1 to ρ_2) have no effect on whether the selection procedure favors the minority group by applying the

Expected Utility Model or Thorndike's Model. However, these two variables, SR and ρ_1 have effect on whether the selection procedure favors the minority group when any of the other four models is applied. Especially for the Equal Opportunity Model, the minority test validity (or the ratio of ρ_1 to ρ_2) can almost determine if the selection procedure favors the minority group or not. In summary, the Expected Utility Model is the only selection model for which the degree to which the selection procedure favors the minority group is not completely determined by the underlying data situation. Given a selection situation, the Expected Utility Model always can be applied in order to favor the minority group to a certain degree by adjusting the underlying utility structure.

Average Criterion Performance of Accepted Applicants

One can compare the Expected Utility Model with the other selection models in terms of the average criterion performance of accepted applicants. Clearly, a high average criterion performance for accepted applicants is desirable. For example, suppose students are accepted into college on the basis of their high school averages. One would hope that this group of selected students will achieve a high grade point average in the first year of college. For each of eighty-one hypothetical data situations, the average criterion performance of accepted applicants was computed by following Gross's formula (1973). The entire set of results is presented in Table G of Appendix IV. For a given value of the minority test validity and a given selection ratio, the mean for average criterion performance of accepted applicants (across nine data situations) was computed and presented in Table 14 for each of these models. Each set of nine data situations was produced by varying the values of standardized minimal success scores for the minority group and the majority group (i.e., Y_{s1}^{*} and Y_{s2}^{*}).

Table 14 shows that for every selection model, the average criterion performance of accepted applicants is directly related to the minority test validity, but inversely related to the value of selection ratio. In other words, when the value of ρ_1 is increased and/or the value of SR is decreased, the average criterion performance of accepted applicants is increased. This relationship holds for every one of the six selection models. Additionally, it also can be seen in Table G that the average criterion performance of accepted applicants is inversely related to the values of the standardized minimal success scores for the minority and majority groups. In other words, the average criterion performance of accepted applicants is increased when the values of Y_{s1}^{*} and/or Y_{s2}^{*} are decreased. Comparing these selection models in terms of average criterion performance of accepted applicants, the Regression Model is always the highest, and followed in order by the Employer's Model and the Expected Utility Model. The Employer's Model which is the Expected Utility Model with an identical utility structure assigned to two groups, has either the same or almost the same average criterion performance of accepted applicants as the Regression Model in every data situation.
The Equal Opportunity Model and Darlington's Model take turns

Table 14

The Mean for Average Criterion Performance of Accepted Applicants under Each Value of Minority Test Validity and Selection Ratio for the Six Models

Situation	Model					
	EXPT	REGR	EMPL	THOR	EQOP	DARL
$\rho_1 = 0.2$ and $\rho_2 = 0.4$						
SR = 0.2	0.5760	0.5992	0.5990	0.5257	0.4866	0.5291
0.4	0.3999	0.4256	0.4256	0.3771	0.3389	0.3753
0.6	0.2660	0.2839	0.2836	0.2651	0.2281	0.2658
$\rho_1 = 0.4$ and $\rho_2 = 0.4$						
SR = 0.2	0.6432	0.6630	0.6630	0.6090	0.5827	0.5993
0.4	0.4377	0.4597	0.4597	0.4320	0.4028	0.4158
0.6	0.2879	0.3068	0.3068	0.2976	0.2689	0.2808
$\rho_1 = 0.6$ and $\rho_2 = 0.4$						
SR = 0.2	0.7220	0.7416	0.7404	0.6922	0.6629	0.6850
0.4	0.4854	0.5030	0.5026	0.4874	0.4603	0.4651
0.6	0.3188	0.3353	0.3351	0.3301	0.3098	0.3124

Note. See Note in Table 13.

being the lowest in these data situations. However, in terms of the overall average (across the eighty-one data situations), the Equal Opportunity Model is the lowest followed in order by Darlington's Model and Thorndike's Model. The differences between two models in Table 14 are not great, since those values are the means (across the nine situations with the values of Y_{s1}^* and Y_{s2}^* varied) for average criterion scores of accepted applicants. The difference between two average criterion scores can be large in some situations. Let us consider a situation which might occur often and in which the minority group has a lower validity (ρ_1) and a smaller proportion of potentially successful applicants than the majority group (i.e., the situation where $\rho_1 = 0.2$, $\rho_2 = 0.4$, $Y_{s1}^* = 0.5$ and $Y_{s2}^* = -0.5$) under the SR value of 0.2. It can be seen in Table G of Appendix IV that in this situation, the average criterion score of accepted applicants for the Expected Utility Model is 0.976, while the average criterion score of the Equal Opportunity Model is only 0.691. The average criterion scores of accepted applicants for the Regression Model, the Employer's Model, Darlington's Model and Thorndike's Model are 0.976, 0.976, 0.976 and 0.830 respectively.

The Rate of Selection Errors

The six selection models can also be compared in terms of the rate of selection errors. The rate of selection errors is defined to be the sum of the proportion of potentially successful applicants rejected and the proportion of potentially unsuccessful applicants accepted. For each of the eighty-one hypothetical data situations, the rate of selection errors was computed and presented in Table H of Appendix IV. For a given value of selection ratio and a given value of the minority test validity (ρ_1), the average rate of selection errors (across the nine data situations with the values of Y_{s1}^* and Y_{s2}^* varied) was computed and presented in Table 15 for the six selection models.

It can be seen in Table 15 that in terms of the average rate of selection errors, the Employer's Model (which is the Expected Utility Model with an identical utility structure assigned to both groups) is the lowest and followed in order by the Regression Model and the Expected Utility Model with utility structure (4). As with the evaluation in terms of the average criterion performance for accepted applicants, the Equal Opportunity Model is the poorest among the six selection models if they are compared in terms of the average rate of selection errors across the eighty-one data situations. Also as with the results in Table 14 the differences between models in Table 15 are small. However, the difference between two rates of selection errors can be large. Let us consider a situation which might occur often. In the situation where the minority group has a lower validity and a smaller proportion of potentially successful applicants (i.e., the situation where ρ_1 = 0.2, ρ_2 = 0.4, Y_{s1}^* = 0.5 and Y_{s2}^* = -0.5) under the SR value of 0.4, it can be seen in Table H that the Equal Opportunity Model has 8% more applicants misclassified than the Expected Utility Model. The rates of selection errors for the six models are 0.3429, 0.3429, 0.3430, 0.3445, 0.3800 and 0.4212 for

Table 15
Average Rate of Selection Errors under Each Value of Minority Test Validity and Selection Ratio for the Six Models

Situation	EXPT	REGR	EMPL	THOR	EQOP	DARL
$\rho_1 = 0.2$ and $\rho_2 = 0.4$						
SR = 0.2	0.4143	0.4105	0.4105	0.4219	0.4280	0.4216
0.4	0.3779	0.3700	0.3699	0.3858	0.3982	0.3856
0.6	0.3778	0.3699	0.3699	0.3792	0.3971	0.3779
$\rho_1 = 0.4$ and $\rho_2 = 0.4$						
SR = 0.2	0.4041	0.4009	0.4009	0.4094	0.4136	0.4106
0.4	0.3654	0.3593	0.3593	0.3685	0.3784	0.3735
0.6	0.3678	0.3593	0.3593	0.3639	0.3782	0.3718
$\rho_1 = 0.6$ and $\rho_2 = 0.4$						
SR = 0.2	0.3903	0.3880	0.3878	0.3967	0.4018	0.3955
0.4	0.3504	0.3444	0.3443	0.3497	0.3592	0.3570
0.6	0.3530	0.3444	0.3443	0.3472	0.3573	0.3568

Note. See Note in Table 13.

the Employer's Model, the Expected Utility Model, the Regression Model, Darlington's Model, Thorndike's Model and the Equal Opportunity Model, respectively. Table 15 also shows that for each selection model and each value of selection ratio, the rate of selection errors is decreased when the minority test validity is increased. For a given selection model and a given value of the minority test validity, the rate of selection errors is the highest when the value of SR is 0.2, while the rate of selection errors is identical for the SR values of 0.4 and 0.6.

The Success Rate of Accepted Applicants

The six selection models were additionally compared in terms of the success rate of accepted applicants. The success rate of accepted applicants was computed as the ratio of the proportion of potentially successful applicants accepted to the proportion of applicants accepted. For each of the eighty-one data situations, the success rate of accepted applicants was computed and presented in Table I of Appendix IV for the six selection models. For a given value of the minority test validity and a given value of selection ratio, the average success rate of accepted applicants (across the nine data situations with the values of Y_{s1}^* and Y_{s2}^* varied) is presented in Table 16 for these six models.

First, it is noted in Table 16 that for every selection model, the average success rate of accepted applicants is inversely related to the value of SR, but is directly related to the minority test validity (ρ_1). It also can be seen in Table I that for all six selection models, the success rate of accepted applicants is inversely

Table 16
Average Success Rate of Accepted Applicants under Each Value of Minority Test Validity and Selection Ratio for the Six Models

Situation	Model					
	EXPT	REGR	EMPL	THOR	EQOP	DARL
$\rho_1 = 0.2$ and $\rho_2 = 0.4$						
SR = 0.2	0.7142	0.7238	0.7239	0.6955	0.6801	0.6955
0.4	0.6526	0.6626	0.6626	0.6429	0.6275	0.6393
0.6	0.6018	0.6083	0.6083	0.6006	0.5857	0.5813
$\rho_1 = 0.4$ and $\rho_2 = 0.4$						
SR = 0.2	0.7398	0.7477	0.7478	0.7266	0.7162	0.7234
0.4	0.6669	0.6759	0.6759	0.6645	0.6522	0.6581
0.6	0.6102	0.6173	0.6173	0.6134	0.6014	0.6069
$\rho_1 = 0.6$ and $\rho_2 = 0.4$						
SR = 0.2	0.7742	0.7800	0.7804	0.7582	0.7457	0.7612
0.4	0.6870	0.6945	0.6946	0.6881	0.6762	0.6788
0.6	0.6225	0.6297	0.6297	0.6273	0.6192	0.6193

Note. See Note in Table 13.

related to the standardized minimal success scores for the minority and majority groups. In other words, the success rate of accepted applicants is increased when the value of Y_{s1}^* and/or the value of Y_{s2}^* are decreased. In every data situation, the success rate of accepted applicants for the Employer's Model is always the highest among these six models. The Regression Model has the same success rate of accepted applicants as the Employer's Model in most of these eighty-one data situations. Comparing these six models in terms of the average success rate of accepted applicants (across the eighty-one data situations), the Employer's Model is the highest and followed in order by the Regression Model, the Expected Utility Model with utility structure (4), Thorndike's Model, Darlington's Model and the Equal Opportunity Model. Although the differences between two average success rates in Table 16 are not great, the difference between two success rates of accepted applicants can be large in some situations. Especially, in a situation where the minority group has a lower validity and smaller proportion of potentially successful applicants than the majority group (i.e., the situation in which $\rho_1 = 0.2$, $\rho_2 = 0.4$, $Y_{s1}^* = 0.5$ and $Y_{s2}^* = -0.5$), the difference between the success rates for the Expected Utility Model and the Equal Opportunity Model is significant. In this data situation, the success rate of the Expected Utility Model is 0.8508, while the success rate of the Equal Opportunity Model is 0.7395 under the SR value of 0.2. The success rates for the other models are 0.8508, 0.8508, 0.8508 and 0.7936 for the Employer's Model, the Regression Model, Darlington's Model and Thorndike's Model, respectively.

The Expected Loss of Selection

Finally, the six selection models are compared in terms of the expected loss of selection. The expected loss of selection was computed under the assignment of utility structure (4). It was assumed that the loss resulting from accepting a potentially unsuccessful minority applicant (i.e., LUS_1) is 0.5 unit, while the loss resulting from accepting a potentially unsuccessful majority applicant is 1.0 unit. Further, it was assumed that the loss resulting from rejecting a potentially successful minority or majority applicant (i.e., LS_1 or LS_2) is 1.0 unit. Additionally, it was assumed that the loss is zero for the applicants who were correctly classified (i.e., the potentially successful applicants accepted and the potentially unsuccessful applicants rejected). For each of the eighty-one hypothetical data situations, the expected unit loss of selection was computed and presented in Table J of Appendix IV for each selection model. The average expected loss of selection (across the nine data situations with the values of Y_{s1}^* and Y_{s2}^* varied) is presented in Table 17 for each value of selection ratio and the minority test validity under each selection model.

Table 17 shows that the expected loss of selection is decreased when the value of selection ratio and/or the value of the minority test validity are increased. It can be seen in Table J that for every selection model, the expected loss of selection is inversely related to the values of Y_{s1}^* and Y_{s2}^* under the SR values of 0.2 and 0.4, but is directly related to the values of Y_{s1}^* and Y_{s2}^* under the SR value of 0.6. This inverse relationship of the expected loss with

Table 17
Average Expected Loss of Selection under Each Value of Minority Test Validity and Selection Ratio for the Six Models

Situation		Model					
		EXPT	REGR	EMPL	THOR	EQOP	DARL
$\rho_1 = 0.2$ and $\rho_2 = 0.4$							
	SR = 0.2	0.3977	0.4010	0.4012	0.4106	0.4145	0.4000
	0.4	0.3395	0.3468	0.3467	0.3611	0.3695	0.3415
	0.6	0.3251	0.3339	0.3334	0.3397	0.3528	0.3262
$\rho_1 = 0.4$ and $\rho_2 = 0.4$							
	SR = 0.2	0.3885	0.3912	0.3912	0.4012	0.4052	0.3904
	0.4	0.3322	0.3383	0.3383	0.3482	0.3578	0.3339
	0.6	0.3193	0.3272	0.3273	0.3293	0.3427	0.3205
$\rho_1 = 0.6$ and $\rho_2 = 0.4$							
	SR = 0.2	0.3769	0.3796	0.3790	0.3917	0.3977	0.3783
	0.4	0.3219	0.3268	0.3268	0.3340	0.3465	0.3233
	0.6	0.3112	0.3175	0.3183	0.3157	0.3316	0.3123

Note. The expected loss of selection was computed when $LUS_1 = 0.5$ and $LUS_2 = LS_1 = LS_2 = 1.0$.

SR, Y_{s1}^* and Y_{s2}^* (under the SR values of 0.2 and 0.4) is due to the studied selection situations in which the proportion of potentially successful applicants is much greater than the proportion of applicants which can be accepted (i.e., the selection ratio). In those selection situations, a lot of potentially successful applicants were rejected. However, the inverse relationship between the expected loss and the minority test validity is not obvious. An increase in the minority test validity leads to a decrease in the expected loss of selection.

Comparing the six selection models in terms of the expected loss of selection, it is found that in every data situation, the Expected Utility Model with utility structure (4) is always the lowest among the six models. The order of the other five models varies with data situations. For the overall average expected loss of selection (across the eighty-one data situations), the Expected Utility Model with utility structure (4) is the lowest, followed in order by Darlington's Model, the Regression Model, the Employer's Model, Thorndike's Model and the Equal Opportunity Model. As with the results in previous sections, the difference between two expected losses in some situations is greater than the difference between two average expected losses in Table 17. To understand the difference between the expected losses for two models, let us consider the data situation where the minority group has ρ_1 Of 0.2 and Y_{s1}^* Of 0.5 and the majority group has ρ_2 of 0.4 and Y_{s2}^* of -0.5. In this situation, it can be seen in Table J that the expected loss of the Expected Utility Model with utility structure (4) is 0.3429, and the expected loss of the Equal Opportunity Model is 0.3847 under the SR value of 0.4. The expected losses for the other models are 0.3429, 0.3430, 0.3434 and 0.3610 for the Employer's Model, the Regression Model, Darlington's Model and Thorndike's Model, respectively. Suppose a selection process is applied to some training program with 2500 applicants and the cost of training a person is $10,000. In this situation, the loss resulting from accepting a potentially unsuccessful majority applicant (LUS_2) may be $10,000. Under utility structure (4), LS_2 and LS_1 may also be $10,000, but LUS_1 may be $5,000. If the institution applies the Expected Utility Model instead of the Equal Opportunity Model to this selection situation, over one million dollars (i.e., $1,045,000) may be saved.

7. ANALYSES OF THE SELECTION PROCESS FOR REAL LIFE DATA

In this chapter some real life selection procedures will be examined and evaluated in terms of their implicit utility structures. The real life data were drawn from the 1970 Open Admission Program at the City University of New York. Under the Open Admission Program, the City University of New York accepted all high school graduates regardless of their academic achievement. Although every applicant was admitted to some branch of the University, not everyone was admitted to a four-years' senior college. Some applicants were admitted to junior colleges or community colleges. Therefore, there was a selection process performed in every senior college. In this chapter, we attempt to identify the selection procedures in four of the senior colleges (i.e., City College, Hunter College, Brooklyn College and Queens College) and analyze these procedures in terms of their implicit utilities.

With the advent of the equal right's movement, it is important to consider the "fairness" of the admission programs for women applicants and non-white applicants. Therefore, the admission data of every senior college was first analyzed by sex subgrouping, and then by race subgrouping. In the sex subgrouping analyses,women applicants were considered as minority members, and male applicants as majority members. In the race subgrouping analyses, applicants who had attended a high school with less than 10% white students were considered as minority members, and applicants who had attended a high school with more than 90% white students were considered as majority members. Those high schools which could not be grouped into one of the above two categories were not analyzed.

In these analyses, it was assumed that: 1) the admission committee of every senior college desired to select applicants who would achieve a satisfactory first year grade point average; 2) the central admissions office selected applicants based on their performance on the Open Admission Tests (i.e., Stanford High School Reading Test and Stanford Arithmetic Test) and the high school grade averages (i.e., the averages of English, Mathematics, Science and Social Studies). In other words, it was assumed that the first year grade point average was considered to be the criterion variable, and the Open Admission Test scores and the high school grade averages were considered to be predictor variables. It should be emphasized that if other criterion and predictor variables were used in the selection procedure, the selection procedures which were studied should be considered to be only hypothetical.

In each analysis, the selection procedure (i.e., the acceptance scores for the minority and majority groups) was identified from proportions of minority and majority applicants accepted. Then, using the Expected Utility Model, the implicit utilities associated with the institutional selection procedure were identified. Additionally, the institutional selection procedure was compared with the selection procedures of the five previously mentioned fair models in terms of acceptance scores, the average criterion performance of accepted students, the rate of selection errors and the success rate of accepted students. These results are used to demonstrate the basic problems associated with the use of selection models that do not state utilities explicitly. Let us first consider the sex subgrouping analysis.

Sex Subgrouping

In these analyses, the first year college grade point average (GPA) was the criterion variable; the Open Admission Test Mathematic Score (OATM), the Open Admission Test Reading Score (OATR), and High School Averages on Enalish (ENGL), Mathematics (MATH), Science (SCIE), and Social Studies (SOCI) were predictor variables. Since every college might have its own unique grading system, the grade point average of a student in one college might not be comparable to the grade point average of a student in another college. Also, the High School Average of a graduate from one high school might not be comparable to the High School Average of a graduate from another high school. Therefore, the analyses are only meaningful when they were performed separately for each high school and each senior college. It should be noted that the data were never pooled for high schools or for colleges. For each of the four senior colleges (i.e., City College, Hunter College, Brooklyn College and Queens College), only the admission data of high schools which had a sufficient number of applicants to that college were analyzed. A total of twelve high schools were considered in the study.

In the first part of the analyses, the descriptive statistics of a selection situation (i.e., means and standard deviations of the criterion Y and the linear least square composite of predictor variables X, and the correlation between these two variables) were obtained. For each high school, female applicants were assigned to the minority group (i.e., Group 1), and male applicants were assigned to the majority group (i.e., Group 2). Based on the applicants who had been admitted to the senior college of their first choice, the regression equation was developed for each subgroup by using OATM, OATR, ENGL, MATH, SCIE and SOCI as predictor variables and GPA as the criterion variable. The entire set of regression coefficients for the twelve high schools is presented in Appendix V. The results show that in general, the regression equations for the female groups are different from the regression equations for the male groups.

For each applicant, a linear composite (X) was computed by using the regression coefficients of his/her subgroup equation. This linear composite was defined as the admission index (AI). The means and standard deviations of AI (X) and GPA (Y), and the correlation coefficient between AI and GPA were calculated for the accepted applicants of each subgroup. The entire set of statistics for accepted applicants is presented in Table L of Appendix V. Since these statistics were computed on selected groups, they were corrected by following Gulliksen's formulas (1967). These formulas are:

$$R_{xy} = S_x\, r_{xy} \Big/ \sqrt{S_x^2 r_{xy}^2 + s_x^2 - s_x^2 r_{xy}^2} \qquad (7.1)$$

$$S_y = s_y \sqrt{1 - r_{xy}^2 + r_{xy}^2 \left(S_x^2 / s_x^2 \right)} \qquad (7.2)$$

$$\overline{Y} = \overline{y} - R_{xy} \left(\left(\overline{x} - \overline{X} \right) / S_x \right) S_y \qquad (7.3)$$

where

$\overline{x}$ = the computed mean of AI for accepted applicants

s_x = the computed standard deviation of AI for accepted applicants

$\bar{y}$ = the computed mean of GPA for accepted applicants

s_y = the computed standard deviation of GPA for accepted applicants

r_{xy} = the computed correlation coefficient between AI and GPA for accepted applicants

$\bar{X}$ = the computed mean of AI for total applicants

S_x = the computed standard deviation of AI for total applicants

$\bar{Y}$ = the estimated mean of GPA for total applicants

S_y = the estimated standard deviation of GPA for total applicants

R_{xy} = the estimated correlation coefficient between AI and GPA for total applicants

The entire set of corrected statistics is presented in Table 18. These statistics represent the distribution of AI and GPA under the assumption that all applicants to a college were admitted to that college. The first letter of a school name represents the college to which the high school graduates applied. The last three letters of a school name represent the public high school number. For example, A33 are the applicants to College A who had graduated from High School 33. It can be seen in Table 18 that for every high school, the grade point average of the female group is always higher than the grade point average of the male group. Only in four of the twelve schools, female groups have a higher correlation between AI and GPA than the male groups.

Table 18
Description of Female and Male Applicants for Each School

School	Group	N	$\overline{X}$	S_x	$\overline{Y}$	S_y	R_{xy}
A33	Female	45	3.0901	0.3410	3.0843	0.5809	0.5868
	Male	102	2.7068	0.5590	2.6941	0.9058	0.6167
A76	Female	51	2.4564	0.6393	2.4538	0.9066	0.7052
	Male	51	2.1618	0.3566	2.1591	0.7433	0.4796
B115	Female	50	2.9025	0.2584	2.8976	0.4539	0.5695
	Male	22	2.7064	0.2056	2.7005	0.5412	0.3795
B199	Female	35	2.4883	0.4333	2.4890	0.5832	0.7434
	Male	23	1.9018	1.3542	1.9018	1.4268	0.9493
C1	Female	90	2.6225	0.4555	2.6316	0.7393	0.6170
	Male	114	2.4700	0.5974	2.4733	0.8883	0.6731
C38	Female	133	2.5700	0.6421	2.5650	0.8241	0.7781
	Male	106	2.4645	0.3700	2.4532	0.7001	0.5276
C62	Female	93	2.6475	0.3937	2.6511	0.6651	0.5924
	Male	101	2.4718	0.4581	2.4719	0.7690	0.5960
C103	Female	107	2.5046	0.4324	2.5015	0.7802	0.5542
	Male	98	2.2811	0.4629	2.2669	0.7161	0.6450
D18	Female	95	2.8369	0.2892	2.8369	0.5073	0.5702
	Male	90	2.5763	0.5375	2.5662	0.6797	0.7902
D70	Female	82	2.9262	0.1017	2.9284	0.4087	0.2495
	Male	91	2.4097	0.6183	2.4185	0.7273	0.8515
D112	Female	84	2.6653	0.3641	2.6703	0.5247	0.6951
	Male	95	2.4823	0.3859	2.4899	0.6235	0.6201
D138	Female	84	2.8327	0.2130	2.8389	0.4410	0.4843
	Male	69	2.4772	0.3538	2.4796	0.5145	0.6880

In the second part of the analyses, the acceptance scores of AI (C_1 and C_2), and the ratio of P_1/P_2 (i.e., the ratio of the conditional probability being successful for minority applicants scoring at C_1 to the conditional probability being successful for majority applicants scoring at C_2) were computed and presented in Table 19. For each college and each high school, the proportions of minority and majority applicants accepted were identified by dividing the numbers of minority and majority members who had selected that college as their first choice by the numbers of minority and majority members who were admitted to that college. For each school, the acceptance scores for the minority and majority groups were computed by using the proportions of minority and majority applicants accepted and the descriptive statistics of two groups. Then, using the computed acceptance scores and the descriptive statistics, the ratio of P_1/P_2 was computed for each school. In these analyses, the grade point average of 2.0 was considered as the minimum success performance.

It can be seen in Table 19 that for most schools (except C1, C38, C62 and C103), the institution accepted a greater proportion of female applicants than male applicants. Since the female groups have a higher mean of AI than the male groups, the female groups still have a higher acceptance score than the male groups for ten of the twelve schools (except A76 and C38). The values of P_1/P_2 vary with the schools. Only two schools, A76 and C38 have a value of P_1/P_2 less than 1.0. The ten other schools have a value of P_1/P_2 greater than 1.0. Using the results in Table 5, the implicit utilities associated with every institutional selection procedure can be iden-

Table 19
Acceptance Scores, Proportions of Applicants Accepted and the Ratio of P_1/P_2 for Sex Subgrouping

School	C_1	C_2	PS_1	PS_2	P_1/P_2
A33	2.6305	1.9511	0.9511	0.9118	1.9491
A76	1.6298	1.7721	0.9020	0.8628	0.7761
B115	2.6455	2.5530	0.8400	0.7727	1.1094
B199	2.0773	1.2091	0.8286	0.6957	14.8557
C1	2.1165	1.7499	0.8667	0.8860	1.6562
C38	1.3628	1.7590	0.9699	0.9717	0.3210
C62	2.2404	1.8256	0.8495	0.9208	1.7378
C103	1.9997	1.7188	0.8785	0.8878	1.6855
D18	2.7520	2.3775	0.6633	0.6444	1.1797
D70	2.8743	2.2813	0.6951	0.5824	1.2707
D112	2.3918	2.2623	0.7738	0.7158	1.2032
D138	2.6366	2.3250	0.8214	0.6667	1.1759

Note. The symbols C_1 and PS_1 represent the acceptance score and the proportion of applicants accepted for the female group. The C_2 and PS_2 are for the acceptance score and the proportion of applicants accepted for the male group.

tified. Under the assumption that LUS_1 (i.e., the loss resulting from accepting a female unsuccessful applicant) is less than or equal to LUS_2 (i.e., the loss resulting from accepting a male unsuccessful applicant), the City University implicitly assigned a greater value to LS_1 (i.e., the loss resulting from rejecting a potentially successful female applicant) than LS_2 (i.e., the loss resulting from rejecting a potentially successful male applicant) for the applicants from high schools A76 and C38. For the applicants from the other high schools, the University seemed to implicitly assign a smaller value to LS_1 than LS_2 when LUS_1 is less than or equal to LUS_2. However, it seems to be impractical that the central admissions office would assign a different utility structure to the female and male groups from one high school, than the utility structure assigned to the female and male groups from another high school. Therefore, the inconsistency of these underlying utilities may only be the basic problem associated with the use of selection models that do not state utilities explicitly.

The third part of the analyses compares the institutional selection procedure with the five previously proposed fair models (i.e., the Regression Model, the Employer's Model, Thorndike's Model, the Equal Opportunity Model and Darlington's Model). The five "fair" models were applied to the selection of applicants in the twelve schools. Given a selection situation, the acceptance scores of AI, the proportions of applicants accepted for the minority and majority groups, the estimated grade point average of accepted applicants, the estimated rate of selection errors, the success rate of accepted applicants and the ratio of P_1/P_2 were computed and presented in Table M of Appendix V for each selection model. In the application of Darlington's Model, the subgroup regression equations were used and the grade point average of 2.0 was considered as the minimum success point for the female and male groups. Therefore, the results for Darlington's Model were the same as those for the Regression Model. It can be seen in Table M that the institutional selection procedure is most similar to the selection procedure for the Equal Opportunity Model in eight schools, and most similar to the selection procedure for Thorndike's Model in the other three schools (i.e., B115, C38 and D138). Table M also shows that the Regression Model is always the highest in terms of the average grade point average of accepted applicants (GS), and the Employer's Model is always the lowest in terms of the estimated rate of selection errors (RSE) and the highest in terms of the success rate of accepted applicants (SRA). On the other hand, the institutional selection procedure is the lowest on GS and SRA, and the highest on RSE in eight of the twelve schools. These results imply that if the institution desires to obtain a high grade point average and high success rate for accepted applicants, and a low rate of selection errors, the selection procedure should be modified.

Race Subgrouping

In these analyses, the applicants were grouped depending upon the high school from which they had graduated, since there was no race information available for each individual applicant. The applicants were placed in the minority group (i.e., the Non-White group) if they had graduated from the high school with less than 10% white students. The applicants were considered as majority applicants (i.e., members of the White group) if they had graduated from the high school with more than 90% white students. The first year college grade point average (GPA) was considered as the criterion variable. The Open Admission Test Mathematic Score (OATM) and the Open Admission Test Reading Score (OATR) were used as predictor variables. Since the grading system of one high school might be different from the grading system of another high school, the High School Averages were not used in these analyses. The analyses were performed separately for each of the four senior colleges.

The regression coefficients of predictor variables (i.e., OATM and OATR) for predicting GPA were computed and presented in Table N for every subgroup of the four senior colleges. It can be seen in Table N that the regression equations of the Non-White groups are in general different from the regression equations of the White groups. By using these subgroup regression equations, the least square linear composite of predictor variables (AI) was computed for each applicant. The means and standard deviations of AI (X) and GPA (Y), and the correlation coefficient between these two variables were computed and presented in Table O for the accepted applicants of each subgroup. Then, using Gulliksen's correction formulas (1967), the descriptive statistics for the total applicants in each subgroup were estimated and presented in Table 20. It can be seen in Table 20 that the means of AI and GPA for the Non-White group are lower than those

Table 20
Description of Non-White and White Applicants

College	Group	N	$\overline{X}$	S_x	$\overline{Y}$	S_y	R_{xy}
A	Non-White	241	1.7422	0.0791	1.7437	0.8921	0.0891
	White	48	2.4881	0.5930	2.4863	0.9759	0.6072
B	Non-White	259	2.0386	0.2364	2.0385	0.9140	0.2586
	White	47	2.7218	0.5200	2.7249	0.7097	0.7336
C	Non-White	178	2.1480	0.3400	2.1460	0.8199	0.4139
	White	338	2.8928	0.2366	2.8914	0.6993	0.3373
D	Non-White	55	2.1661	0.1734	2.1653	0.6993	0.2475
	White	79	3.0377	0.2572	3.0366	0.5540	0.4639

Note. The applicants who had graduated from the high school with less than 10% white students were considered as non-white applicants. The applicants who had graduated from the high school with more than 90% white students were considered as white applicants.

for the White group in every senior college. In three senior colleges (i.e., College A, College B and College D) the Non-White groups have a lower validity (i.e., the correlation between AI and GPA) than the White groups. In College C, the Non-White group has a slightly higher validity than the White group.

The selection procedures of the four senior colleges were identified from the proportions of Non-White and White applicants accepted. The acceptance scores of AI (C_1 and C_2), the proportions of applicants accepted (PS_1 and PS_2) and the ratio of P_1/P_2 (i.e., the ratio of the conditional probability being successful for individuals scoring at C_1 to the conditional probability being successful for individuals scoring at C_2) are presented in Table 21. In the computation of the ratio of P_1/P_2, the grade point average of 2.0 was considered as the minimum success point. Table 21 shows that the acceptance score for the Non-White group is lower than the acceptance score for the White group in every college. In College A and College C, the White groups have a greater proportion of applicants accepted than the Non-White groups. In College B and College D, the Non-White groups have a greater proportion of applicants accepted than the White groups. However, the value of P_1/P_2 is always less than 1.0 for each college. By using the results of Table 5 the implicit utilities associated with these institutional selection procedures can be identified. Under the assumption that LUS_1 is less than or equal to LUS_2, the admission procedure of every senior college can be described _as if_ it assigned a greater loss to the misclassification of a potentially successful Non-White applicant than a potentially

Table 21
Acceptance Scores, Proportions of Applicants Accepted
and the Ratio of P_1/P_2 for Race Subgrouping

College	C_1	C_2	PS_1	PS_2	P_1/P_2
A	1.6681	1.9145	0.8257	0.8333	0.7791
B	1.9076	2.4776	0.7104	0.6809	0.5453
C	1.7981	2.5613	0.8483	0.9201	0.4893
D	1.9962	2.9927	0.8364	0.5696	0.5085

Note. The symbols C_1 and C_2 represent acceptance scores for the non-white and white groups. The symbols PS_1 and PS_2 represent proportions of applicants accepted for the non-white and white groups.

successful White applicant. However, as with the results of the sex subgrouping analyses, the values of P_1/P_2 are different for each college. In other words, the implicit utilities associated with the institutional selection procedure vary across colleges.

Finally, the five "fair" selection models were applied to the selection of applicants in the four senior colleges. The entire set of results is presented in Table P of Appendix VI. For Darlington's Model, the grade point average of 1.5 was considered as the minimum success point for Non-White applicants, while the grade point average of 2.0 was considered as the minimum success point for White applicants. In the selection situations of College A and College B, no acceptance scores can be found in order to meet the definition of Thorndike's Model. It can be seen in Table P that in all of these four senior colleges, the institutional selection procedures are most similar to the selection procedure for the Equal Opportunity Model. As with the findings from the sex subgrouping analyses, these institutional selection procedures could be modified if the senior colleges desire a high grade point average and success rate of accepted applicants and a low rate of selection errors for the selection process.

8. SUMMARY AND CONCLUSIONS

In this study, the Expected Utility Model which is a general procedure for constructing a "fair" and "optimal" selection procedure has been developed. Given that utilities have been explicitly assigned to the outcomes of selection, the Expected Utility Model can lead to a selection process which maximizes the expected utility of selection. In terms of the conceptualization of this model, the fairness of a selection procedure is judged on the basis of the utilities that the decision-maker has assigned to the outcomes of selection. If one agrees with the assignment of utilities, he will consider the selection procedure as "fair". On the other hand, if one disagrees with the assignment of utilities, he may claim that the selection procedure is unfair.

The advantages of this model are: a) it requires the decision-maker to state utilities for each subpopulation explicitly and openly; b) it can be used to identify the implicit utilities associated with a selection procedure which does not state utilities explicitly and; c) it provides a certain degree of compensation that the decision-maker thinks the minority members should have.

In situations where the decision-maker can assign utilities explicitly to selection outcomes, not only does the Expected Utility Model lead to a selection process which maximizes the expected utility of selection; but also the explicit statement of utilities gives all interested parties (e.g., the institution, the minority group) a basis for judging the "fairness" of selection. Due to competition among applicants, a selection procedure will probably never be considered as "fair" by everyone. However, by applying the Expected Utility Model, the discussion and evaluation of a selection procedure can be open and participated in by all interested parties. Further, the justification of utilities can be made in order to satisfy the majority of interested parties.

When the decision-maker applies a selection model (e.g., the Equal Opportunity Model) that does not state utilities explicitly, the Expected Utility Model can be used to identify the utility structure implied by this selection procedure. The present study has demonstrated in general how an implicit utility structure can be identified. The implicit utilities were identified for the five previously proposed "fair" models by using hypothetical data. In addition, implicit utilities were identified for real life data. The Regression Model and the Employer's Model were shown to implicitly assign an identical utility structure to the minority and majority groups. Under the condition that the minority group is allowed to have a lower minimal success level than the majority group, Darlington's Model was shown to have an identical implicit utility structure for two subpopulations. The Equal Opportunity Model and Thorndike's Model were shown in general to implicitly assign a greater loss to a misclassified potentially successful applicant and/or a smaller loss to a misclassified potentially unsuccessful applicant from the subgroup which has a poorer predictive validity and a smaller proportion of potentially successful applicants than the other group.

Let us consider how the Expected Utility Model provides a certain degree of compensation to the minority group. The study has shown that the underlying utility structure of the Expected Utility Model strongly affects the proportions of minority and majority applicants accepted. There is a direct relationship between the degree to which the Expected Utility Model selects minority applicants and the ratio of LS_1 to LS_2 (i.e., the ratio of the assigned loss to a misclassified potentially successful minority applicant to the assigned loss to a misclassified potentially successful majority applicant). On the other hand, it was also found that there is an inverse relationship between the degree to which the Expected Utility Model selects minority applicants and the ratio of LUS_1 to LUS_2 (i.e., the ratio of the assigned loss to a misclassified potentially unsuccessful minority applicant to the assigned loss to a misclassified potentially unsuccessful majority applicant). Although the characteristics of data for a given selection problem has an effect on the selection procedure, the Expected Utility Model is strongly affected by the assigned utility structure. In other words, the utility structure tends to dominate the characteristic of data. For example, suppose a decision-maker wants to emphasize the acceptance of minority applicants. By increasing the assigned loss to a misclassified potentially successful minority applicant and/or decreasing the assigned loss to a misclassified potentially unsuccessful minority applicant, the decision-maker would be able to admit a greater proportion of minority members than majority members.

However, the characteristics of data can still affect the degree to which the Expected Utility Model selects minority applicants. In using the Expected Utility Model, the proportion of minority applicants accepted is increased when the proportion of potentially successful minority applicants is increased and/or the proportion of potentially successful majority applicants is decreased. It was also found across a large set of data situations that, in general, the difference between the proportions of minority and majority applicants accepted, is highest when the selection ratio is 0.4 as compared to situations where the selection ratio is 0.2 or 0.6. In addition, the selection ratio was found to operate as a moderator variable for the test validities. When the selection ratio is 0.2 or 0.4, the proportion of minority applicants accepted by the Expected Utility Model is increased if the minority test validity is increased and/or the majority test validity is decreased. Under the selection ratio of 0.6, the proportion of minority applicants accepted is inversely related to the minority test validity but directly related to the majority test validity.

The conceptualization of the five previously proposed "fair" selection models (i.e., the Regression Model, the Employer's Model, Thorndike's Model, the Equal Opportunity Model and Darlington's Model) basically differ from that of the Expected Utility Model. Each of the former models recommends a specific definition of "fairness" and the selection procedure which satisfies this definition. All of these models do not require a decision-maker to state his/her utility structure explicitly. Although the study has shown that the implicit utilities associated with these models can be identified, their utility structures (especially the utility structures for Thorndike's Model and the Equal Opportunity Model) depend upon the characteristics of data for a given selection problem. Any of these models may be judged as "fair" for one population of applicants, but may be judged as "unfair" for another population of applicants only because these two populations of applicants have different distributions of predictor and criterion variables. These results also imply that the degree to which the selection procedure for any of these models selects minority applicants, is completely determined by the characteristics of data (e.g., whether the minority group has a greater proportion of potentially successful applicants than the majority group). In situations where the minority group has a smaller proportion of potentially successful applicants and/or a lower test validity than the majority group, Thorndike's Model and the Equal Opportunity Model tend to select more minority applicants than the Expected Utility Model with an identical utility structure assigned to two groups. On the other hand, in situations where the minority group has a greater proportion of potentially successful applicants and/or a higher test validity than the majority group, these two models tend to select less minority applicants than the Expected Utility Model with an identical utility structure assigned to two groups. Those results can be seen in the analyses of hypothetical data and the sex subgrouping analyses for real life data. Suppose a decision-maker wants to compensate the minority group in the second type of situations.

The selection process will be very likely incompatible with his/her goal if either Thorndike's Model or the Equal Opportunity Model is applied.

In addition to the "fairness" of a selection procedure, it is also important to consider how a selection procedure "performs" on traditional criteria other than fairness (e.g., the average criterion performance for accepted applicants). The six selection models were compared in terms of the average criterion performance for accepted applicants, the rate of selection errors, the success rate of accepted applicants and the expected loss of selection in eighty-one hypothetical data situations. In general, the Equal Opportunity Model was found to be the poorest among the six models in terms of the overall average performance (across eighty-one data situations) on these four criteria. Especially in the most common situation where the minority group has a lower test validity and a smaller proportion of potentially successful applicants, the Equal Opportunity Model was found to be much poorer than the other models in terms of these four criteria.

Due to the flexibility and advantages of the model, the Expected Utility Model is recommended for selection situations. This should not be taken to mean that a decision-maker should always implement the Expected Utility Model but never consider any of the other models. Rather, in situations where a decision-maker can state utilities explicitly, the Expected Utility Model may be directly applied in order to maximize the expected utility of selection. In situations where a dceision-maker cannot explicitly assign utilities to selection outcomes, a study of implicit utilities associated with the past selection procedure or various selection procedures is suggested.

It also should be noted that the Expected Utility Model has its limitations. The model is based on the distribution of predictor and criterion variables for total applicants. In many practical situations, a criterion measure is only available for selected applicants but not rejected applicants. Thus, the statistics for total applicants must be estimated by using range correction formulas. Unforunately, these range correction formulas do not provide precise estimations in situations where a small proportion of applicants (e.g., 10% of applicants) were selected. This problem also exists for all other models of fair selection. The second limitation of the model is the assumption that the available criterion score is a relevant and reliable measure of performance for applicants in each subpopulation. Clearly, this assumption would not be reasonable in many situations. Again, this limitation also applies to other models of fair selection.

APPENDIX I. COMPUTER PROGRAM OF THE EXPECTED UTILITY MODEL

This program is written in FORTRAN IV. It consists of a main program and five subroutines, BINOR, NDTR, NDTRI, EXEC and OUTC.

Main Program

The main program is for handling the input and output information. The inputs of the program are: 1) the selection ratio and the minimum success point on the criterion; 2) the proportion of the first subpopulation applicants, the mean of the criterion variable, the standard deviation of the criterion variable and the correlation between the predictor and criterion variables for subpopulation 1; 3) the utilities of outcomes for subpopulation 1, $U(O_{11})$, $U(O_{21})$, $U(O_{31})$ and $U(O_{41})$; 4) the proportion of the second subpopulation applicants, the mean of the criterion variable, the standard deviation of the criterion variable and the correlation between the predictor and criterion variables for subpopulation 2; 5) the utilities of outcomes for subpopulation 2, $U(O_{12})$, $U(O_{22})$, $U(O_{32})$ and $U(O_{42})$. The outputs of the program are: 1) standardized acceptance scores on the predictor variable for two subpopulations; 2) proportions of applicants accepted for two subpopulations; 3) total utility of the selection procedure; 4) the estimated average performance of admitted applicants; 5) the estimated rate of selection errors; 6) the estimated success rate of accepted applicants.

BINOR

The subroutine BINOR is for computing proportions of bivariate normal distribution given pearson-product moment correlation coefficient and standardized normal deviates of two variables. The proportions of bivariate normal distribution are approximated by Kelley's formula (1949). The smallest proportion among outcomes can be computed by the formula:

$$\text{The smallest proportion} = q_x q_y + Z_x Z_y [\rho + C Y_S \rho^2 / 2! + (C^2 - 1)(Y_S^2 - 1) \rho^3 / 3! + (C^3 - 3C)(Y_S^3 - 3Y_S) \rho^4 / 4! + \text{----}] \quad (I.1)$$

where

q_x= the smallest X marginal when the distribution is divided at C

q_y= the smallest Y marginal when the distribution is divided at Y_S

Z_x= the ordinate of the normal curve at C

Z_y= the ordinate of the normal curve at Y_S

NDTR

This subroutine is directly adopted from the IBM scientific subroutines. The subroutine computes $Y= P(x)= \text{Prob}(X \leq x)$, where X is a random variable distributed normally with mean zero and variance one. The following approximation is used:

$$P(x) = 1 - f(x) \sum_{i=1}^{5} a_i w^i ; \; x \geq 0$$

where

$$w = 1/(1 + px)$$
$$f(x) = \exp(-x^2/2)/\sqrt{2\pi}$$
$$P = 0.2316419$$
$$a_1 = 0.3193815$$
$$a_2 = -0.3565638$$
$$a_3 = 1.781478$$
$$a_4 = -1.821256$$
$$a_5 = 1.330274$$

<u>NDTRI</u>

This subroutine is adopted from the IBM scientific subroutines. The routine computes the standardized value of x associated with a certain cumulative proportion p (i.e., p= Prob($X \leqq x$)). The following approximation is used:

$$x = w - \sum_{i=0}^{2} a_i w^i / \sum_{i=0}^{3} b_i w^i$$

where:

$$w = \sqrt{\ln(1/p^2)} \quad (0 < p \leq .5)$$
$$a_0 = 2.515517$$
$$a_1 = 0.802853$$
$$a_2 = 0.010328$$
$$b_1 = 1.432788$$
$$b_2 = 0.189269$$
$$b_3 = 0.001308$$

EXEC

The subroutine EXEC computes the standardized acceptance scores for two subpopulations by following the definition of the Expected Utility Model. The iterative procedure of the routine follows the method that has been presented in chapter 3. The iterative computational procedure stops when the difference between the total proportion of applicants accepted (TPR) and the selection ratio is less than or equal to 0.00001, or when the number of iteration exceed 2000. From the derived acceptance scores and the normal distribution table, the proportions of applicants accepted can be computed for two subpopulations.

OUTC

This subroutine computes the average criterion performance of accepted applicants, the rate of selection errors and the success rate of accepted applicants. From Gross (1973), the average value of the dependent variable Y within the accepted group for each subpopulation is

$$G_{si} = E\ (Y \mid X \geq C_i)$$
$$= 1\ /\ \text{Prob}\ (X \geq C_i)\ \{ \sigma_{yi}\ \rho_i\ N(C_i^*)$$
$$+ [\mu_{yi} - \mu_{yp}]\ [1 - F(C_i^*)] \} \qquad (I.2)$$

where

$$C_i^* = (C_i - \mu_{yi} + \mu_{yp})\ /\ (\beta_i' \Sigma_{xx}^{(i)} \beta_i)^{1/2} \qquad (I.3)$$

$N(C_i^*)$= the unit normal density evaluated at the point C_i^*

$F(C_i^*)$= the unit normal distribution function

ρ_i= the multiple correlation in the ith subpopulation

C_i^*= the standardized acceptance score for the ith subpopulation

μ_{yi}= the mean of the criterion variable for the ith subpopulation

μ_{yp}= the mean of the criterion variable for the combined population

σ_{yi}= the standard deviation of Y for the subpopulation i

The average performance of total accepted applicants is

$$G_s = 1/SR \sum_{i=1}^{k} q_i \text{ Prob}(X \geq C_i \mid \pi_i)\, G_{si} + 1/SR \sum_{i=1}^{k} q_i \{ \sigma_{yi} \rho_i N(C_i^*) + \mu_{yi} [1 - F(C_i^*)] \} \quad \text{(I.4)}$$

Using the subroutine BINOR , the rate of selection errors can be computed for each subpopulation. The rate of selection errors for the ith subpopulation e_i is

$$e_i = \text{Prob}(O_{1i}) + \text{Prob}(O_{4i}) = \text{Prob}(Z_x < C_i^*, Z_y \geq Y_{si}^* \mid \pi_i) + \text{Prob}(Z_x \geq C_i^*, Z_y < Y_{si}^* \mid \pi_i) \quad \text{(I.5)}$$

The rate of selection errors for the total population is

$$e = q_1 e_1 + q_2 e_2 \quad \text{(I.6)}$$

Using the subroutine BINOR, the success rate of accepted applicants can be computed. The success rate of accepted applicants for the ith subpopulation is

$$s_i = \mathrm{Prob}(O_{2i}) / \ \mathrm{Prob}(O_{2i}) + \mathrm{Prob}(O_{4i})$$
$$= \mathrm{Prob}(Z_x \geq C_i^*, \ Z_y \geq Y_{si}^* \mid \pi_i) / \mathrm{Prob}(\ Z_x \geq C_i^* \mid \pi_i) \qquad \text{(I.7)}$$

The success rate of accepted applicants for the total population is

$$s = \{ q_1 \ \mathrm{Prob}(Z_x \geq C_1^* \mid \pi_1) \ s_1 + q_2 \ \mathrm{Prob}(Z_x \geq C_2^* \mid \pi_2) \ s_2 \} / SR$$
$$= q_1 \ \mathrm{Prob}(Z_x \geq C_1^*, \ Z_y \geq Y_{s1}^* \mid \pi_1) + q_2 \ \mathrm{Prob}(Z_x \geq C_2^*, \ Z_y \geq Y_{s2}^* \mid \pi_2) \qquad \text{(I.8)}$$

The following are lists of the source program, sample data and sample outputs.

List of the Source Program

```
C THIS PROGRAM IS FOR COMPUTING THE STANDARD CUTOFF SCORES AND THE
C UTILITY FOR THE EXPECTED UTILITY MODEL
      DIMENSION U(2,4)
C READ IN THE SELECTION RATIO,MINIMUM SUCCESS POINT ON CRITERION SCOR
   50 READ (5,1) SR,SUPT
    1 FORMAT (6F6.2)
      READ (5,1) Q,YM1,YSD1,R1
      READ (5,1) (U(1,J),J=1,4)
      READ (5,1) Q2,YM2,YSD2,R2
      READ (5,1) (U(2,J),J=1,4)
      YS1=SUPT-YM1
      YS1=YS1/YSD1
      YS2=SUPT-YM2
      YS2=YS2/YSD2
      U1=U(1,2)-U(1,1)
      U2=U(1,3)-U(1,4)
      U3=U(2,2)-U(2,1)
      U4=U(2,3)-U(2,4)
      WRITE (6,21) SR,SUPT
   21 FORMAT (1H0,///,1X,'SR=',F6.4,',MINIMUM SUCCESS POINT=',F8.4)
      WRITE (6,22) Q,YM1,YSD1,R1,(U(1,J),J=1,4)
   22 FORMAT (1X,'Q1=',F6.4,',YM1=',F8.4,',YSD1=',F8.4,',R1=',F6.4 ,
     1 ',UTILITIES=',4F6.2)
      WRITE (6,23) Q2,YM2,YSD2,R2,(U(2,J),J=1,4)
   23 FORMAT (1X,'Q2=',F6.4,',YM2=',F8.4,',YSD2=',F8.4,',R2=',F6.4 ,
     1 ',UTILITIES=',4F6.2)
      CALL EXEC(U1,U2,U3,U4,SR,Q,R1,R2,YS1,YS2,C1,C2,P1,P2)
      WRITE (6,2) C1,P1
    2 FORMAT (     1X,'STANDARD CUTOFF OF GROUP 1 =',F8.4,
     1 ',PROPORTION THAT SHOULD BE SELECTED=',F8.5)
      WRITE (6,4) C2,P2
    4 FORMAT (1X,'STANDARD CUTOFF OF GROUP2 =',F8.4,
     1 ',PROPORTION THAT SHOULD BE SELECTED=',F8.5)
C COMPUTE THE TOTAL UTILITY
      CALL BINOR(C1,YS1,R1,Q11,Q21,Q31,Q41)
      CALL BINOR(C2,YS2,R2,Q12,Q22,Q32,Q42)
      UT1=Q11*U(1,1)+Q21*U(1,2)+Q31*U(1,3)+Q41*U(1,4)
      UT2=Q12*U(2,1)+Q22*U(2,2)+Q32*U(2,3)+Q42*U(2,4)
      UTT=Q*UT1+(1-Q)*UT2
      WRITE (6,3) UTT
    3 FORMAT (1X,'TOTAL UTILITY OF THE SELECTION PROCEDURE=',F10.4)
      CALL OUTC(SUPT,YSD1,YSD2,SR,Q,R1,R2,YS1,YS2,C1,C2,GS,PMC,SRC,
     1 PR1,PR2,PR3,THMR,FOMR)
      WRITE (6,7) GS
    7 FORMAT (1X,'THE ESTIMATED AVERAGE PERFORMANCE OF ACCEPTED ',
     1 'APPLICANTS=',F10.4)
      WRITE (6,5) PMC
    5 FORMAT (1X,'THE ESTIMATED RATE OF SELECTION ERRORS=',
     1 F7.4)
      WRITE (6,6) SRC
    6 FORMAT (1X,'THE ESTIMATED SUCCESS RATE OF ACCEPTED APPLICANTS=
     1 F7.4)
```

```
      GO TO 50
  100 CONTINUE
      CALL EXIT
      END
      SUBROUTINE BINOR(Z1,Z2,C,O1,O2,O3,O4)
C  Z1= STANDARD CUTOFF ON PREDICTOR
C  Z2= STANDARD CUTOFF ON CRITERION
C  C= PEARSON-PRODUCT MOMENT CORRELATION BETWEEN PREDICTOR AND CRITERION
C  O1,O2,O3,AND O4= THE FOUR PROPORTIONS OF THE BIVARIATE NORMAL
C  DISTRIBUTION
      DOUBLE PRECISION X31,X32,X312,X322
      X1=ABS(Z1)
      X2=ABS(Z2)
      CALL NDTR(X1,P1,D1)
      CALL NDTR(X2,P2,D2)
      P1=1-P1
      P2=1-P2
      IF ((Z1 .GE. 0.) .AND. (Z2 .GE. 0.)) R=C
      IF ((Z1 .GE. 0.) .AND. (Z2 .LT. 0.)) R=-C
      IF ((Z1 .LT. 0.) .AND. (Z2 .GE. 0.)) R=-C
      IF ((Z1 .LT. 0.) .AND. (Z2 .LT. 0.)) R=C
      XC1=1.0*R
      XC2=(X1*X2/2.0)*(R**2)
      XC3=((X1*X1-1.0)*(X2*X2-1.0)/6.0)*(R**3)
      X31=DBLE(X1)
      X32=DBLE(X2)
      X312=X31**2
      X322=X32**2
      XC4=(SNGL(X31*(X312-3.0D0)*X32*(X322-3.0D0)/24.0D0))*(R**4)
      XC5=(SNGL((X312*(X312-6.0D0)+3.0D0)*(X322*(X322-6.0D0)+3.0D0)
     1 /120.0D0))*(R**5)
      XC6=(SNGL(X31*(X312*(X312-10.0D0)+15.0D0)*X32*(X322*(X322-10.0D0)
     1 +15.0D0)/720.0D0))*(R**6)
      XC7=(SNGL((((X312-15.0D0)*X312+45.0D0)*X312-15.0D0)*(((X322-
     1 15.0D0)*X322+45.0D0)*X322-15.0D0)/5040.0D0))*(R**7)
      S=XC1+XC2+XC3+XC4+XC5+XC6+XC7
      S=S*D1*D2
      IF ((Z1 .GE. 0.) .AND. (Z2 .GE. 0.)) GO TO 10
      IF ((Z1 .GE. 0.) .AND. (Z2 .LT. 0.)) GO TO 20
      IF ((Z1 .LT. 0.) .AND. (Z2 .GE. 0.)) GO TO 30
      IF ((Z1 .LT. 0.) .AND. (Z2 .LT. 0.)) GO TO 40
   10 O2=S+P1*P2
      O1=P2-O2
      O3=1-P1-P2+O2
      O4=P1-O2
      GO TO 50
   20 O4=S+P1*P2
      O3=P2-O4
      O1=1-P1-P2+O4
      O2=P1-O4
      GO TO 50
   30 O1=S+P1*P2
```

```
      O2=P2-O1
      O4=1-P1-P2+O1
      O3=P1-O1
      GO TO 50
   40 O3=S+P1*P2
      O4=P2-O3
      O2=1-P1-P2+O3
      O1=P1-O3
   50 RETURN
      END
      SUBROUTINE NDTR(X,P,D)
      AX=ABS(X)
      T=1.0/(1.0+.2316419*AX)
      D=0.3989423*EXP(-X*X/2.0)
      P=1.0-D*T*((((1.330274*T-1.821256)*T+1.781478)*T-0.3565638)*T
     1 +0.3193815)
      IF (X) 1,2,2
    1 P=1.0-P
    2 RETURN
      END
      SUBROUTINE NDTRI(P,X,D,IE)
      IE=0
      IF (P) 1,4,2
    1 IE=-1
      GO TO 12
    2 IF (P-1.0) 7,6,1
    4 X=-.999999E+74
    5 D=0.0
      GO TO 12
    6 X=0.999999E+74
      GO TO 5
    7 D=P
      IF (D-0.5) 9,9,8
    8 D=1.0-D
    9 T2=ALOG(1.0/(D*D))
      T=SQRT(T2)
      X=T-(2.515517+0.802853*T+0.010328*T2)/(1.0+1.432788*T
     1 +0.189269*T2+0.001308*T*T2)
      IF (P-0.5) 10,10,11
   10 X=-X
   11 D=0.3989423*EXP(-X*X/2.0)
   12 RETURN
      END
```

```
      SUBROUTINE EXEC(U1,U2,U3,U4,SR,Q,R1,R2,YS1,YS2,C1,C2,P1,P2)
C U1  U(021)-U(011)
C U2  U(031)-U(041)
C U3  U(022)-U(012)
C U4  U(032)-U(042)
C SR  SELECTION RATIO
C Q  PROPORTION OF APPLICANTS IN THE MINORITY GROUP
C R1  MULTIPLE CORRELATION FOR THE MINORITY GROUP
C R2  MULTIPLE CORRELATION FOR THE MAJORITY GROUP
C YS1  STANDARD SUCCESS POINT FOR THE MINORITY GROUP
C YS2  STANDARD SUCCESS POINT FOR THE MAJORITY GROUP
C C1  STANDARD CUTOFF POINT FOR THE MINORITY GROUP
C C2  STANDARD CUTOFF POINT FOR THE MAJORITY GROUP
C P1  PERCENTAGE OF MINORITY STUDENTS WILL BE SELECTED
C P2  PERCENTAGE OF MAJORITY STUDENTS WILL BE SELECTED
      ND=0
      IS=-1
      PP1=U2/(U2+U1)
   50 PP2=-PP1*(U1+U2)+U2
      PP2=(U4-PP2)/(U3+U4)
      PP1=1-PP1
      PP2=1-PP2
      CALL NDTRI(PP1,Z1,D1,IE)
      C1=YS1-Z1*(SQRT(1-R1*R1))
      C1=C1/R1
      CALL NDTRI(PP2,Z2,D2,IE)
      C2=YS2-Z2*(SQRT(1-R2*R2))
      C2=C2/R2
      PP1=1-PP1
      PP2=1-PP2
      CALL NDTR(C1,P1,D1)
      CALL NDTR(C2,P2,D2)
      P1=1-P1
      P2=1-P2
      TSR=Q*P1+(1-Q)*P2
      ND=ND+1
      IF (ND .GT. 2000) GO TO 100
      IF (ND .EQ. 1) TSRP=TSR
      IF ((TSR .GT. SR) .AND. (TSRP .LT. SR)) IS=IS-1
      IF ((TSR .LT. SR) .AND. (TSRP .GT. SR)) IS=IS-1
      IF ((TSR-SR) .GT. 0.00001)PP1=PP1+10.0**IS
      IF ((TSR-SR) .LT. -0.00001)PP1=PP1-10.0**IS
      IF (ABS(TSR-SR) .LE. 0.00001)GO TO 100
      TSRP=TSR
      GO TO 50
  100 RETURN
      END
```

```
      SUBROUTINE OUTC(YP,SY1,SY2,SR,Q,R1,R2,YS1,YS2,C1,C2,GS,PMC,
     1 SRS,PR1,PR2,PR3,THMR,EOMR)
C YP MINIMUM PASSING POINT ON THE CRITERION VARIABLE
C SY1 S.D. OF THE CRITERION VARIABLE FOR THE MINORITY
C SY2 S.D. OF THE CRITERION VARIABLE FOR THE MAJORITY GROUP
C SR  SELECTION RATIO
C Q   PROPORTION OF APPLICANTS IN THE MINORITY GROUP
C R1  MULTIPLE CORRELATION FOR THE MINORITY GROUP
C R2  MULTIPLE CORRELATION FOR THE MAJORITY GROUP
C YS1  STANDARD SUCCESS POINT FOR THE MINORITY GROUP
C YS2  STANDARD SUCCESS POINT FOR THE MAJORITY GROUP
C C1   STANDARD CUTOFF POINT FOR THE MINORITY GROUP
C C2   STANDARD CUTOFF POINT FOR THE MAJORITY GROUP
C GS AVERAGE CRITERION PERFORMANCE OF SELECTEES
C PMC THE PROPORTION OF MISCLASSIFICATION
C SRC THE SUCCESS RATE OF SELECTEES
C PR1 P1/P2 PI=PROB(Y .GE. YSI/X=CI)
C PR2 P1*/P2 * COMPUTED BY USING THE SUCCESS POINT WHICH IS 0.5 SY2
C     LOWER THAN THE SUCCESS POINT OF GROUP 2
C PR3 (1-P1)/(1-P2)
C THMR PROB(X .GE. C1)/PROB(X .GE. C2)
C EOMR PROB(X .GE. C1 .AND. Y .GE. YS1)/PROB(X .GE. C2 AND Y .GE. YS2)
C COMPUTE THE MEAN OF CRITERION FOR EACH GROUP
      YM1=YP-YS1*SY1
      YM2=YP-YS2*SY2
C COMPUTE THE RESULTS OF EVALUATION
      CALL NDTR(C1,PP1,D1)
      CALL NDTR(C2,PP2,D2)
      CALL BINOR(C1,YS1,R1,O11,O21,O31,O41)
      CALL BINOR(C2,YS2,R2,O12,O22,O32,O42)
      GS=Q*SY1*R1*D1+(1-Q)*SY2*R2*D2
      GS=GS+Q*YM1*(1-PP1)+(1-Q)*YM2*(1-PP2)
      GS=GS/SR
      PMC=Q*(O11+O41)+(1-Q)*(O12+O42)
      SRS=Q*O21+(1-Q)*O22
      SRS=SRS/SR
      IF ((1-PP2) .EQ. 0.) THMR=9999.0
      IF ((1-PP2) .NE. 0.) THMR=(1-PP1)/(1-PP2)
      IF (O22 .EQ. 0.) EOMR=9999.0
      IF (O22 .NE. 0.) EOMR=O21/O22
      Z1=YS1-R1*C1
      Z1=Z1/SQRT(1.0-R1*R1)
      Z2=YS2-R2*C2
      Z2=Z2/SQRT(1.0-R2*R2)
      Z3=YS1-0.5-R1*C1
      Z3=Z3/SQRT(1.0-R1*R1)
      CALL NDTR(Z1,P1,F1)
      CALL NDTR(Z2,P2,F2)
      CALL NDTR(Z3,P3,F3)
      IF ((1-P2) .EQ. 0.) PR1=9999.0
      IF ((1-P2) .NE. 0.) PR1=(1-P1)/(1-P2)
      IF ((1-P2) .EQ. 0.) PR2=9999.0
```

```
      IF ((1-P2) .NE. 0.) PR2=(1-P3)/(1-P2)
      IF (P2 .EQ. 0.) PR3=9999.0
      IF (P2 .NE. 0.) PR3=P1/P2
      RETURN
      END
```

Sample Data

```
 0.20  0.00
 0.20  0.00  1.00  0.5
 0.00  1.00  0.00 -1.0
 0.80  0.00  1.00  0.50
 0.00  1.00  0.00 -1.0
 0.20  0.00
 0.20 -0.50  1.00  0.50
 0.00  1.00  0.00 -1.0
 0.80  0.00  1.00  0.50
 0.00  1.00  0.00 -1.0
 0.20  0.00
 0.20  0.00  1.00  0.20
-0.50  0.50  0.50 -0.50
 0.80  0.00  1.00  0.70
-0.50  0.50  0.50 -0.50
```

```
SR=0.2000,MINIMUM SUCCESS POINT=  0.0
Q1=0.2000,YM1=  0.0    ,YSD1=  1.0000,R1=0.5000,UTILITIES=  0.0   1.00  0.0  -1.00
Q2=0.8000,YM2=  0.0    ,YSD2=  1.0000,R2=0.5000,UTILITIES=  0.0   1.00  0.0  -1.00
STANDARD CUTOFF OF GROUP 1 =  0.8416,PROPORTION THAT SHOULD BE SELECTED= 0.20000
STANDARD CUTOFF OF GROUP2 =  0.8416,PROPORTION THAT SHOULD BE SELECTED= 0.20000
TOTAL UTILITY OF THE SELECTION PROCEDURE=    0.1129
THE ESTIMATED AVERAGE PERFORMANCE OF ACCEPTED APPLICANTS=    0.6999
THE ESTIMATED RATE OF SELECTION ERRORS= 0.3871
THE ESTIMATED SUCCESS RATE OF ACCEPTED APPLICANTS= 0.7822
```

```
SR=0.2000,MINIMUM SUCCESS POINT=  0.0
Q1=0.2000,YM1= -0.5000,YSD1=  1.0000,R1=0.5000,UTILITIES=  0.0   1.00  0.0  -1.00
Q2=0.8000,YM2=  0.0    ,YSD2=  1.0000,R2=0.5000,UTILITIES=  0.0   1.00  0.0  -1.00
STANDARD CUTOFF OF GROUP 1 =  1.7092,PROPORTION THAT SHOULD BE SELECTED= 0.04370
STANDARD CUTOFF OF GROUP2 =  0.7092,PROPORTION THAT SHOULD BE SELECTED= 0.23909
TOTAL UTILITY OF THE SELECTION PROCEDURE=    0.1052
THE ESTIMATED AVERAGE PERFORMANCE OF ACCEPTED APPLICANTS=    0.6449
THE ESTIMATED RATE OF SELECTION ERRORS= 0.3565
THE ESTIMATED SUCCESS RATE OF ACCEPTED APPLICANTS= 0.7630
```

```
SR=0.2000,MINIMUM SUCCESS POINT=  0.0
Q1=0.2000,YM1=  0.0    ,YSD1=  1.0000,R1=0.2000,UTILITIES= -0.50  0.50  0.50 -0.50
Q2=0.8000,YM2=  0.0    ,YSD2=  1.0000,R2=0.7000,UTILITIES= -0.50  0.50  0.50 -0.50
STANDARD CUTOFF OF GROUP 1 =  3.2411,PROPORTION THAT SHOULD BE SELECTED= 0.00060
STANDARD CUTOFF OF GROUP2 =  0.6749,PROPORTION THAT SHOULD BE SELECTED= 0.24986
TOTAL UTILITY OF THE SELECTION PROCEDURE=    0.1486
THE ESTIMATED AVERAGE PERFORMANCE OF ACCEPTED APPLICANTS=    0.8899
THE ESTIMATED RATE OF SELECTION ERRORS= 0.3514
THE ESTIMATED SUCCESS RATE OF ACCEPTED APPLICANTS= 0.8715
```

APPENDIX II. PROPORTIONS OF APPLICANTS ACCEPTED FOR THE EXPECTED UTILITY MODEL

The proportions of applicants accepted for the minority and majority groups are presented for the Expected Utility Model in various hypothetical data situations. The following symbols and abbreviations are used.

SR: The selection ratio (i.e., the proportion of applicants accepted).

LS_1: The assigned loss to a misclassified potentially successful applicant from the minority group.

LUS_1: The assigned loss to a misclassified potentially unsuccessful applicant from the minority group.

LS_2: The assigned loss to a misclassified potentially successful applicant from the majority group.

LUS_2: The assigned loss to a misclassified potentially unsuccessful applicant from the majority group.

Y_{s1}^*: The standardized minimal success score for the minority group.

Y_{s2}^*: The standardized minimal success score for the majority group.

ρ_1: The test validity for the minority group.

ρ_2: The test validity for the majority group.

Table A
Proportions of Applicants Accepted for the Expected Utility Model under the SR Value of 0.2

Utility Structure (1)
Minority: LS_1 = 1.0 and LUS_1 = 1.0, Majority: LS_2 = 1.0 and LUS_2 = 1.0

		Y_{s1}^* = 0.5			Y_{s1}^* = 0.0			Y_{s1}^* = -0.5		
ρ_2	Y_{s2}^*	0.5	0.0	-0.5	0.5	0.0	-0.5	0.5	0.0	-0.5
ρ_1 = 0.2										
0.2	Minority	.200	.001	.000	.631	.200	.001	.667	.631	.200
	Majority	.200	.285	.286	.015	.200	.285	.000	.015	.200
0.4	Minority	.096	.000	.000	.446	.077	.000	.638	.424	.061
	Majority	.245	.286	.286	.095	.253	.286	.012	.104	.260
0.6	Minority	.043	.000	.000	.288	.016	.000	.512	.227	.004
	Majority	.267	.286	.286	.162	.279	.286	.066	.188	.284
ρ_1 = 0.4										
0.2	Minority	.293	.060	.004	.605	.315	.070	.667	.618	.337
	Majority	.160	.260	.284	.027	.151	.256	.000	.021	.141
0.4	Minority	.200	.032	.001	.460	.200	.032	.631	.460	.200
	Majority	.200	.272	.285	.089	.200	.272	.015	.089	.200
0.6	Minority	.135	.012	.000	.338	.111	.008	.520	.308	.088
	Majority	.228	.280	.286	.141	.238	.282	.063	.154	.248
ρ_1 = 0.6										
0.2	Minority	.329	.149	.046	.578	.377	.183	.667	.614	.425
	Majority	.145	.222	.266	.038	.124	.207	.000	.023	.104
0.4	Minority	.258	.107	.028	.470	.284	.124	.627	.496	.311
	Majority	.175	.240	.274	.084	.164	.232	.017	.073	.152
0.6	Minority	.200	.068	.012	.375	.200	.068	.534	.375	.200
	Majority	.200	.256	.280	.125	.200	.256	.057	.125	.200

Table A - Continued

Utility Structure (2)

Minority: LS_1 = 1.5 and LUS_1 = 1.0, Majority: LS_2 = 1.0 and LUS_2 = 1.0

		Y_{s1}^* = 0.5			Y_{s1}^* = 0.0			Y_{s1}^* = -0.5		
ρ_2	Y_{s2}^*	0.5	0.0	-0.5	0.5	0.0	-0.5	0.5	0.0	-0.5
	ρ_1 = 0.2									
0.2	Minority	.420	.039	.000	.666	.514	.118	.667	.667	.641
	Majority	.106	.269	.286	.000	.065	.235	.000	.000	.011
0.4	Minority	.251	.012	.000	.590	.315	.046	.666	.638	.473
	Majority	.178	.281	.286	.033	.151	.266	.000	.012	.083
0.6	Minority	.147	.002	.000	.444	.166	.010	.630	.507	.296
	Majority	.223	.285	.286	.095	.215	.282	.016	.069	.159
	ρ_1 = 0.4									
0.2	Minority	.428	.166	.038	.663	.509	.266	.667	.667	.621
	Majority	.102	.215	.269	.002	.068	.172	.000	.000	.019
0.4	Minority	.313	.108	.022	.572	.377	.184	.665	.626	.503
	Majority	.151	.239	.276	.041	.124	.207	.001	.017	.070
0.6	Minority	.228	.063	.009	.457	.264	.113	.622	.520	.390
	Majority	.188	.259	.282	.090	.172	.237	.019	.063	.118
	ρ_1 = 0.6									
0.2	Minority	.418	.243	.124	.645	.506	.349	.667	.666	.615
	Majority	.107	.182	.233	.009	.069	.136	.000	.000	.022
0.4	Minority	.340	.190	.094	.556	.415	.282	.663	.621	.533
	Majority	.140	.204	.245	.048	.108	.165	.001	.019	.057
0.6	Minority	.274	.142	.066	.467	.330	.219	.619	.540	.459
	Majority	.168	.225	.258	.086	.144	.192	.020	.054	.089

Table A - Continued

Utility Structure (3)

Minority: LS_1 = 2.0 and LUS_1 = 1.0, Majority: LS_2 = 1.0 and LUS_2 = 1.0

		Y_{s1}^* = 0.5			Y_{s1}^* = 0.0			Y_{s1}^* = -0.5		
ρ_2	Y_{s2}^*	0.5	0.0	-0.5	0.5	0.0	-0.5	0.5	0.0	-0.5
ρ_1 = 0.2										
0.2	Minority	.571	.169	.009	.667	.655	.413	.667	.667	.667
	Majority	.041	.213	.282	.000	.005	.109	.000	.000	.000
0.4	Minority	.389	.082	.003	.651	.515	.257	.667	.667	.667
	Majority	.119	.251	.285	.007	.065	.176	.000	.000	.000
0.6	Minority	.257	.030	.000	.554	.352	.138	.667	.666	.464
	Majority	.176	.273	.286	.048	.135	.227	.000	.000	.087
ρ_1 = 0.4										
0.2	Minority	.527	.279	.110	.667	.626	.450	.667	.667	.667
	Majority	.060	.166	.239	.000	.018	.093	.000	.000	.000
0.4	Minority	.405	.200	.076	.634	.510	.353	.667	.667	.662
	Majority	.112	.200	.253	.014	.067	.134	.000	.000	.002
0.6	Minority	.309	.135	.046	.543	.399	.268	.665	.652	.464
	Majority	.153	.228	.266	.053	.115	.171	.001	.006	.087
ρ_1 = 0.6										
0.2	Minority	.485	.322	.201	.665	.591	.469	.667	.667	.667
	Majority	.078	.148	.200	.001	.032	.085	.000	.000	.000
0.4	Minority	.404	.264	.165	.611	.507	.404	.667	.666	.650
	Majority	.113	.173	.215	.024	.068	.113	.000	.000	.007
0.6	Minority	.334	.209	.130	.533	.429	.346	.662	.641	.464
	Majority	.142	.196	.230	.057	.102	.137	.002	.011	.087

Table A - Continued

Utility Structure (4)

Minority: $LS_1 = 1.0$ and $LUS_1 = 0.5$, Majority: $LS_2 = 1.0$ and $LUS_2 = 1.0$

		$Y_{s1}^* = 0.5$			$Y_{s1}^* = 0.0$			$Y_{s1}^* = -0.5$		
ρ_2	Y_{s2}^*	0.5	0.0	-0.5	0.5	0.0	-0.5	0.0	0.0	-0.5
$\rho_1 = 0.2$										
0.2	Minority	.619	.088	.000	.666	.518	.030	.667	.665	.435
	Majority	.021	.248	.286	.000	.064	.273	.000	.001	.099
0.4	Minority	.401	.017	.000	.598	.272	.003	.660	.551	.186
	Majority	.114	.278	.286	.030	.169	.285	.003	.049	.206
0.6	Minority	.235	.001	.000	.437	.104	.000	.579	.343	.032
	Majority	.185	.285	.098	.241	.286	.038	.139	.272	
$\rho_1 = 0.4$										
0.2	Minority	.588	.206	.017	.666	.513	.164	.667	.664	.477
	Majority	.034	.197	.278	.000	.066	.215	.000	.001	.081
0.4	Minority	.410	.110	.005	.585	.336	.079	.658	.553	.296
	Majority	.110	.239	.283	.035	.142	.252	.004	.049	.159
0.6	Minority	.277	.047	.001	.448	.196	.022	.577	.388	.142
	Majority	.167	.266	.285	.094	.202	.276	.039	.119	.225
$\rho_1 = 0.6$										
0.2	Minority	.545	.268	.086	.662	.509	.268	.667	.661	.515
	Majority	.052	.171	.249	.002	.068	.171	.000	.002	.065
0.4	Minority	.407	.184	.051	.571	.379	.179	.655	.562	.378
	Majority	.111	.207	.264	.041	.123	.209	.005	.045	.124
0.6	Minority	.304	.116	.022	.457	.265	.099	.580	.432	.245
	Majority	.155	.236	.276	.090	.172	.243	.037	.101	.181

Table A - Continued

Utility Structure (5)

Minority: LS_1 = 1.5 and LUS_1 = 0.5, Majority: LS_2 = 1.0 and LUS_2 = 1.0

		Y_{s1}^* = 0.5			Y_{s1}^* = 0.0			Y_{s1}^* = -0.5		
ρ_2 \ Y_{s2}^*		0.5	0.0	-0.5	0.5	0.0	-0.5	0.5	0.0	-0.5
ρ_1 = 0.2										
0.2	Minority	.661	.333	.010	.667	.661	.387	.667	.667	.667
	Majority	.002	.143	.281	.000	.003	.120	.000	.000	.000
0.4	Minority	.523	.152	.002	.655	.517	.198	.667	.665	.617
	Majority	.062	.221	.285	.005	.064	.201	.000	.001	.021
0.6	Minority	.354	.049	.000	.559	.328	.071	.658	.604	.468
	Majority	.134	.265	.286	.046	.145	.255	.004	.027	.085
ρ_1 = 0.4										
0.2	Minority	.646	.373	.112	.667	.645	.432	.667	.667	.666
	Majority	.009	.126	.238	.000	.009	.101	.000	.000	.000
0.4	Minority	.505	.243	.065	.645	.512	.304	.667	.661	.603
	Majority	.069	.182	.258	.009	.066	.155	.000	.002	.027
0.6	Minority	.373	.146	.030	.550	.374	.198	.653	.596	.500
	Majority	.126	.223	.273	.050	.125	.201	.006	.030	.072
ρ_1 = 0.6										
0.2	Minority	.606	.382	.200	.667	.618	.456	.667	.667	.664
	Majority	.026	.122	.200	.000	.021	.090	.000	.000	.001
0.4	Minority	.481	.290	.150	.629	.509	.367	.667	.657	.602
	Majority	.080	.162	.221	.016	.067	.129	.000	.004	.028
0.6	Minority	.379	.211	.104	.541	.407	.287	.649	.597	.531
	Majority	.123	.195	.241	.054	.111	.163	.008	.030	.058

Table A - Continued

Utility Structure (6)

Minority: $LS_1 = 2.0$ and $LUS_1 = 0.5$, Majority: $LS_2 = 1.0$ and $LUS_2 = 1.0$

		$Y_{s1}^* = 0.5$			$Y_{s1}^* = 0.0$			$Y_{s1}^* = -0.5$		
ρ_2	Y_{s2}^*	0.5	0.0	-0.5	0.5	0.0	-0.5	0.5	0.0	-0.5
$\rho_1 = 0.2$										
0.2	Minority	.666	.527	.114	.667	.667	.643	.667	.667	.667
	Majority	.000	.060	.237	.000	.000	.010	.000	.000	.000
0.4	Minority	.597	.317	.045	.666	.639	.487	.667	.667	.667
	Majority	.030	.150	.266	.000	.012	.077	.000	.000	.000
0.6	Minority	.449	.163	.011	.631	.514	.325	.667	.667	.667
	Majority	.093	.216	.281	.015	.065	.146	.000	.000	.000
$\rho_1 = 0.4$										
0.2	Minority	.664	.497	.241	.667	.667	.605	.667	.667	.667
	Majority	.001	.073	.182	.000	.000	.027	.000	.000	.000
0.4	Minority	.570	.358	.167	.664	.616	.491	.667	.667	.667
	Majority	.042	.132	.214	.001	.022	.075	.000	.000	.000
0.6	Minority	.447	.246	.105	.615	.510	.391	.667	.667	.667
	Majority	.094	.180	.241	.022	.067	.118	.000	.000	.000
$\rho_1 = 0.6$										
0.2	Minority	.640	.465	.298	.667	.662	.572	.667	.667	.667
	Majority	.011	.086	.158	.000	.002	.040	.000	.000	.000
0.4	Minority	.533	.371	.241	.657	.591	.494	.667	.667	.667
	Majority	.057	.127	.183	.004	.033	.074	.000	.000	.000
0.6	Minority	.436	.290	.189	.597	.507	.428	.667	.667	.667
	Majority	.099	.161	.205	.030	.068	.102	.000	.000	.000

Table B
Proportions of Applicants Accepted for the Expected Utility Model under the SR Value of 0.4

Utility Structure (1)
Minority: LS_1 = 1.0 and LUS_1 = 1.0, Majority: LS_2 = 1.0 and LUS_2 = 1.0

		Y_{s1}^* = 0.5			Y_{s1}^* = 0.0			Y_{s1}^* = -0.5		
ρ_2 \ Y_{s2}^*		0.5	0.0	-0.5	0.5	0.0	-0.5	0.5	0.0	-0.5
ρ_1 = 0.2										
0.2	Minority	.400	.010	.000	.939	.400	.010	.999	.939	.400
	Majority	.400	.567	.571	.169	.400	.567	.143	.169	.400
0.4	Minority	.377	.016	.000	.835	.342	.010	.998	.810	.309
	Majority	.410	.565	.571	.214	.425	.567	.144	.224	.439
0.6	Minority	.383	.026	.000	.739	.299	.008	.968	.665	.220
	Majority	.407	.560	.571	.255	.443	.568	.156	.286	.477
ρ_1 = 0.4										
0.2	Minority	.417	.111	.010	.795	.444	.125	.974	.814	.470
	Majority	.393	.524	.567	.231	.381	.518	.154	.223	.370
0.4	Minority	.400	.116	.010	.728	.400	.116	.939	.728	.400
	Majority	.400	.522	.567	.260	.400	.522	.169	.260	.400
0.6	Minority	.400	.122	.008	.672	.360	.096	.883	.635	.320
	Majority	.400	.519	.568	.283	.417	.530	.193	.299	.434
ρ_1 = 0.6										
0.2	Minority	.411	.200	.069	.697	.465	.241	.898	.743	.519
	Majority	.395	.486	.542	.273	.372	.468	.186	.253	.349
0.4	Minority	.400	.197	.067	.657	.433	.223	.859	.687	.466
	Majority	.400	.487	.543	.290	.386	.476	.203	.277	.372
0.6	Minority	.400	.194	.060	.625	.400	.194	.816	.625	.400
	Majority	.400	.488	.546	.304	.400	.488	.222	.304	.400

Table B - Continued

Utility Structure (2)

Minority: LS_1 = 1.5 and LUS_1 = 1.0, Majority: LS_2 = 1.0 and LUS_2 = 1.0

		Y_{s1}^* = 0.5			Y_{s1}^* = 0.0			Y_{s1}^* = -0.5		
ρ_2	Y_{s2}^*	0.5	0.0	-0.5	0.5	0.0	-0.5	0.5	0.0	-0.5
ρ_1 = 0.2										
0.2	Minority	.658	.113	.001	.993	.754	.230	.999	.998	.888
	Majority	.290	.523	.571	.146	.248	.473	.143	.144	.191
0.4	Minority	.576	.120	.001	.956	.632	.195	.999	.979	.765
	Majority	.325	.520	.571	.162	.300	.488	.143	.152	.243
0.6	Minority	.533	.128	.001	.878	.528	.150	.999	.899	.617
	Majority	.343	.516	.571	.195	.345	.507	.143	.186	.307
ρ_1 = 0.4										
0.2	Minority	.562	.241	.062	.890	.646	.349	.992	.930	.760
	Majority	.330	.468	.545	.190	.295	.422	.146	.173	.246
0.4	Minority	.525	.233	.061	.833	.582	.313	.978	.876	.685
	Majority	.347	.472	.545	.214	.322	.437	.152	.196	.278
0.6	Minority	.504	.224	.054	.776	.519	.266	.949	.802	.596
	Majority	.356	.475	.548	.239	.349	.458	.165	.228	.316
ρ_1 = 0.6										
0.2	Minority	.501	.299	.154	.775	.592	.406	.937	.840	.702
	Majority	.357	.443	.505	.239	.318	.397	.170	.212	.270
0.4	Minority	.482	.288	.148	.736	.554	.378	.909	.796	.656
	Majority	.365	.448	.508	.256	.334	.410	.182	.230	.290
0.6	Minority	.472	.277	.135	.700	.513	.339	.875	.744	.597
	Majority	.369	.453	.514	.271	.352	.426	.196	.253	.315

Table B - Continued

Utility Structure (3)
Minority: LS_1 = 2.0 and LUS_1 = 1.0, Majority: LS_2 = 1.0 and LUS_2 = 1.0

		Y_{s1}^* = 0.5			Y_{s1}^* = 0.0			Y_{s1}^* = -0.5		
ρ_2 \ Y_{s2}^*		0.5	0.0	-0.5	0.5	0.0	-0.5	0.5	0.0	-0.5
ρ_1 = 0.2										
0.2	Minority	.824	.308	.025	.999	.926	.575	.999	.999	.991
	Majority	.218	.440	.561	.143	.175	.325	.143	.143	.147
0.4	Minority	.725	.280	.025	.992	.829	.479	.999	.999	.963
	Majority	.261	.451	.561	.146	.216	.366	.143	.143	.159
0.6	Minority	.654	.257	.021	.956	.714	.379	.999	.987	.894
	Majority	.291	.461	.563	.162	.265	.409	.143	.148	.188
ρ_1 = 0.4										
0.2	Minority	.668	.368	.150	.938	.773	.540	.887	.971	.891
	Majority	.285	.414	.507	.169	.240	.340	.144	.155	.190
0.4	Minority	.620	.344	.142	.896	.710	.488	.991	.943	.842
	Majority	.306	.424	.511	.187	.267	.362	.147	.167	.211
0.6	Minority	.586	.320	.126	.845	.641	.426	.977	.896	.777
	Majority	.320	.434	.518	.209	.297	.389	.153	.187	.238
ρ_1 = 0.6										
0.2	Minority	.569	.380	.235	.825	.677	.524	.957	.892	.800
	Majority	.328	.409	.471	.218	.281	.347	.161	.189	.229
0.4	Minority	.545	.363	.224	.789	.639	.492	.938	.859	.764
	Majority	.338	.416	.475	.233	.298	.360	.170	.203	.244
0.6	Minority	.529	.345	.206	.754	.597	.452	.911	.817	.718
	Majority	.345	.423	.483	.248	.316	.378	.181	.221	.264

Table B - Continued

Utility Structure (4)

Minority: $LS_1 = 1.0$ and $LUS_1 = 0.5$, Majority: $LS_2 = 1.0$ and $LUS_2 = 1.0$

		$Y_{s1}^* = 0.5$			$Y_{s1}^* = 0.0$			$Y_{s1}^* = -0.5$		
ρ_2	Y_{s2}^*	0.5	0.0	-0.5	0.5	0.0	-0.5	0.5	0.0	-0.5
$\rho_1 = 0.2$										
0.2	Minority	.980	.283	.000	.999	.839	.133	.999	.999	.721
	Majority	.151	.450	.571	.143	.212	.515	.143	.143	.262
0.4	Minority	.853	.251	.000	.997	.669	.112	.999	.963	.540
	Majority	.206	.464	.571	.144	.285	.523	.143	.159	.340
0.6	Minority	.734	.228	.000	.933	.534	.079	.999	.822	.379
	Majority	.257	.474	.571	.171	.342	.538	.143	.219	.409
$\rho_1 = 0.4$										
0.2	Minority	.859	.346	.043	.983	.716	.272	.999	.946	.656
	Majority	.203	.423	.553	.150	.265	.455	.143	.166	.290
0.4	Minority	.749	.312	.042	.928	.615	.234	.994	.866	.547
	Majority	.240	.438	.553	.174	.308	.471	.145	.200	.337
0.6	Minority	.672	.283	.035	.849	.525	.185	.966	.758	.431
	Majority	.284	.450	.557	.208	.346	.492	.158	.247	.387
$\rho_1 = 0.6$										
0.2	Minority	.734	.365	.128	.905	.643	.352	.981	.862	.636
	Majority	.257	.415	.517	.183	.296	.421	.151	.202	.299
0.4	Minority	.660	.339	.121	.839	.580	.315	.950	.796	.564
	Majority	.288	.426	.520	.212	.323	.436	.164	.230	.330
0.6	Minority	.612	.314	.105	.776	.519	.267	.905	.718	.478
	Majority	.309	.437	.526	.239	.349	.457	.184	.264	.367

Table B - Continued

Utility Structure (5)
Minority: LS_1 = 1.5 and LUS_1 = 0.5, Majority: LS_2 = 1.0 and LUS_2 = 1.0

		Y_{s1}^* = 0.5			Y_{s1}^* = 0.0			Y_{s1}^* = -0.5		
ρ_2	Y_{s2}^*	0.5	0.0	-0.5	0.0	0.0	-0.5	0.5	0.0	-0.5
ρ_1 = 0.2										
0.2	Minority	.998	.591	.044	.999	.982	.602	.999	.999	.990
	Majority	.144	.318	.553	.143	.150	.314	.143	.143	.147
0.4	Minority	.945	.477	.042	.999	.892	.473	.999	.999	.933
	Majority	.166	.367	.553	.143	.189	.369	.143	.143	.172
0.6	Minority	.836	.395	.032	.991	.752	.352	.999	.983	.802
	Majority	.213	.402	.558	.147	.249	.421	.143	.150	.228
ρ_1 = 0.4										
0.2	Minority	.919	.527	.174	.994	.861	.556	.999	.987	.889
	Majority	.177	.345	.497	.145	.202	.333	.143	.148	.190
0.4	Minority	.827	.464	.159	.969	.774	.484	.999	.958	.816
	Majority	.217	.373	.503	.156	.240	.364	.143	.161	.222
0.6	Minority	.747	.408	.134	.914	.680	.402	.991	.897	.720
	Majority	.251	.396	.514	.180	.280	.399	.147	.187	.263
ρ_1 = 0.6										
0.2	Minority	.791	.480	.251	.937	.752	.535	.990	.924	.801
	Majority	.232	.365	.464	.170	.249	.342	.147	.175	.228
0.4	Minority	.720	.443	.234	.885	.692	.489	.972	.881	.748
	Majority	.263	.382	.471	.192	.275	.362	.155	.194	.251
0.6	Minority	.668	.406	.207	.831	.630	.433	.942	.824	.682
	Majority	.285	.397	.483	.215	.302	.386	.168	.218	.279

Table B - Continued

Utility Structure (6)

Minority: $LS_1 = 2.0$ and $LUS_1 = 0.5$, Majority: $LS_2 = 1.0$ and $LUS_2 = 1.0$

		$Y_{s1}^* = 0.5$			$Y_{s1}^* = 0.0$			$Y_{s1}^* = -0.5$		
ρ_2	Y_{s2}^*	0.5	0.0	-0.5	0.5	0.0	-0.5	0.5	0.0	-0.5
$\rho_1 = 0.2$										
0.2	Minority	.999	.797	.228	.999	.999	.884	.999	.999	.999
	Majority	.143	.230	.474	.143	.143	.193	.143	.143	.143
0.4	Minority	.985	.657	.193	.999	.979	.764	.999	.999	.998
	Majority	.149	.290	.489	.143	.152	.244	.143	.143	.144
0.6	Minority	.907	.541	.148	.999	.897	.622	.999	.999	.981
	Majority	.183	.340	.508	.143	.187	.305	.143	.143	.151
$\rho_1 = 0.4$										
0.2	Minority	.951	.655	.321	.998	.929	.734	.999	.996	.962
	Majority	.164	.291	.434	.144	.173	.257	.143	.145	.159
0.4	Minority	.878	.579	.288	.985	.866	.661	.999	.986	.927
	Majority	.195	.323	.448	.149	.200	.288	.143	.149	.174
0.6	Minority	.803	.509	.245	.951	.785	.576	.997	.958	.875
	Majority	.227	.353	.467	.164	.235	.324	.144	.161	.197
$\rho_1 = 0.6$										
0.2	Minority	.829	.563	.351	.954	.816	.650	.994	.953	.877
	Majority	.216	.330	.421	.162	.222	.293	.146	.163	.196
0.4	Minority	.763	.520	.327	.913	.763	.605	.982	.924	.840
	Majority	.245	.349	.431	.180	.244	.312	.150	.175	.211
0.6	Minority	.710	.477	.294	.867	.706	.551	.962	.883	.794
	Majority	.264	.367	.445	.200	.269	.335	.159	.193	.231

Table C
Proportions of Applicants Accepted for the Expected Utility Model under the SR Value of 0.6

Utility Structure (1)
Minority: LS_1 = 1.0 and LUS_1 = 1.0, Majority: LS_2 = 1.0 and LUS_2 = 1.0

		Y_{s1}^* = 0.5			Y_{s1}^* = 0.0			Y_{s1}^* = -0.5		
ρ_2	Y_{s2}^*	0.5	0.0	-0.5	0.5	0.0	-0.5	0.5	0.0	-0.5
ρ_1 = 0.2										
0.2	Minority	.600	.061	.000	.990	.600	.061	1.000	.990	.600
	Majority	.600	.831	.857	.433	.600	.831	.429	.433	.600
0.4	Minority	.691	.190	.002	.990	.658	.165	1.000	.984	.623
	Majority	.561	.776	.856	.433	.575	.786	.429	.435	.590
0.6	Minority	.780	.335	.032	.992	.701	.261	1.000	.974	.617
	Majority	.523	.714	.844	.432	.557	.745	.429	.440	.593
ρ_1 = 0.4										
0.2	Minority	.530	.186	.026	.875	.556	.205	.990	.889	.583
	Majority	.630	.777	.846	.482	.619	.769	.433	.476	.607
0.4	Minority	.600	.272	.061	.884	.600	.272	.990	.884	.600
	Majority	.600	.740	.831	.478	.600	.740	.433	.478	.600
0.6	Minority	.680	.365	.117	.904	.640	.328	.992	.878	.600
	Majority	.566	.701	.807	.470	.583	.717	.432	.481	.600
ρ_1 = 0.6										
0.2	Minority	.481	.257	.102	.759	.535	.303	.931	.800	.589
	Majority	.651	.747	.813	.532	.628	.727	.458	.514	.605
0.4	Minority	.534	.313	.141	.777	.567	.343	.933	.803	.600
	Majority	.628	.723	.797	.524	.614	.710	.457	.513	.600
0.6	Minority	.600	.375	.184	.806	.600	.375	.940	.806	.600
	Majority	.600	.696	.778	.512	.600	.696	.454	.512	.600

Table C - Continued

Utility Structure (2) Minority: LS_1 = 1.5 and LUS_1 = 1.0, Majority: LS_2 = 1.0 and LUS_2 = 1.0										
		Y_{s1}^* = 0.5			Y_{s1}^* = 0.0			Y_{s1}^* = -0.5		
ρ_2	Y_{s2}^*	0.5	0.0	-0.5	0.5	0.0	-0.5	0.5	0.0	-0.5
ρ_1 = 0.2										
0.2	Minority	.819	.245	.008	.999	.884	.375	1.000	1.000	.957
	Majority	.506	.752	.854	.429	.478	.696	.429	.429	.447
0.4	Minority	.847	.372	.043	.999	.876	.443	1.000	1.000	.936
	Majority	.494	.698	.839	.429	.482	.667	.429	.429	.456
0.6	Minority	.887	.481	.120	1.000	.869	.479	1.000	.999	.896
	Majority	.477	.651	.806	.429	.485	.652	.429	.429	.473
ρ_1 = 0.4										
0.2	Minority	.662	.328	.102	.939	.733	.431	.997	.962	.824
	Majority	.574	.717	.813	.455	.543	.672	.430	.445	.504
0.4	Minority	.704	.398	.153	.940	.745	.466	.997	.957	.812
	Majority	.555	.687	.792	.454	.538	.658	.430	.447	.509
0.6	Minority	.758	.468	.208	.949	.757	.486	.997	.951	.789
	Majority	.532	.657	.768	.451	.533	.649	.430	.450	.519
ρ_1 = 0.6										
0.2	Minority	.565	.354	.192	.823	.649	.458	.958	.877	.746
	Majority	.615	.705	.775	.504	.579	.661	.447	.481	.537
0.4	Minority	.604	.398	.227	.833	.665	.478	.958	.875	.743
	Majority	.598	.686	.760	.500	.572	.652	.446	.482	.539
0.6	Minority	.657	.447	.262	.852	.682	.490	.962	.872	.731
	Majority	.576	.665	.745	.492	.565	.647	.445	.483	.544

Table C - Continued

Utility Structure (3)
Minority: LS_1 = 2.0 and LUS_1 = 1.0, Majority: LS_2 = 1.0 and LUS_2 = 1.0

		Y_{s1}^{*} = 0.5			Y_{s1}^{*} = 0.0			Y_{s1}^{*} = -0.5		
ρ_2	Y_{s2}^{*}	0.5	0.0	-0.5	0.5	0.0	-0.5	0.5	0.0	-0.5
ρ_1 = 0.2										
0.2	Minority	.931	.464	.071	1.000	.977	.702	1.000	1.000	.998
	Majority	.458	.658	.827	.429	.438	.556	.429	.429	.430
0.4	Minority	.935	.545	.148	1.000	.970	.701	1.000	1.000	.996
	Majority	.456	.624	.794	.429	.441	.557	.429	.429	.430
0.6	Minority	.950	.610	.230	1.000	.960	.681	1.000	1.000	.989
	Majority	.450	.596	.759	.429	.446	.565	.429	.429	.433
ρ_1 = 0.4										
0.2	Minority	.753	.453	.203	.968	.838	.609	.999	.985	.924
	Majority	.534	.663	.770	.442	.498	.596	.429	.435	.461
0.4	Minority	.779	.503	.252	.968	.837	.618	.999	.983	.914
	Majority	.523	.641	.749	.442	.498	.592	.429	.436	.466
0.6	Minority	.816	.553	.297	.972	.837	.616	.999	.979	.894
	Majority	.508	.620	.730	.441	.499	.593	.429	.437	.474
ρ_1 = 0.6										
0.2	Minority	.627	.433	.273	.864	.725	.567	.972	.918	.830
	Majority	.588	.672	.740	.487	.547	.614	.441	.464	.501
0.4	Minority	.657	.467	.301	.870	.733	.576	.972	.915	.824
	Majority	.575	.657	.728	.484	.543	.610	.441	.465	.504
0.6	Minority	.700	.506	.328	.883	.741	.578	.974	.911	.811
	Majority	.557	.640	.716	.479	.539	.609	.440	.467	.510

Table C - Continued

Utility Structure (4)

Minority: LS_1 = 1.0 and LUS_1 = 0.5, Majority: LS_2 = 1.0 and LUS_2 = 1.0

		Y_{s1}^* = 0.5			Y_{s1}^* = 0.0			Y_{s1}^* = -0.5		
ρ_2 \ Y_{s2}^*		0.5	0.0	-0.5	0.5	0.0	-0.5	0.5	0.0	-0.5
ρ_1 = 0.2										
0.2	Minority	1.000	.537	.009	1.000	.971	.325	1.000	1.000	.896
	Majority	.429	.627	.853	.429	.441	.718	.429	.429	.473
0.4	Minority	1.000	.628	.083	1.000	.956	.426	1.000	1.000	.857
	Majority	.429	.588	.821	.429	.448	.674	.429	.429	.490
0.6	Minority	1.000	.688	.198	1.000	.936	.475	1.000	1.000	.798
	Majority	.429	.562	.772	.429	.456	.654	.429	.429	.515
ρ_1 = 0.4										
0.2	Minority	.969	.498	.101	.999	.837	.396	1.000	.983	.772
	Majority	.442	.644	.814	.429	.498	.687	.429	.436	.526
0.4	Minority	.969	.570	.178	.999	.837	.451	1.000	.978	.760
	Majority	.442	.613	.781	.429	.499	.664	.429	.438	.531
0.6	Minority	.980	.629	.254	.999	.836	.481	1.000	.970	.732
	Majority	.437	.588	.748	.429	.499	.651	.429	.442	.544
ρ_1 = 0.6										
0.2	Minority	.860	.463	.186	.965	.732	.433	.996	.915	.713
	Majority	.489	.659	.778	.443	.544	.672	.430	.465	.552
0.4	Minority	.879	.520	.239	.966	.744	.466	.995	.910	.710
	Majority	.480	.634	.755	.443	.538	.657	.431	.467	.553
0.6	Minority	.913	.575	.289	.974	.757	.486	.996	.903	.696
	Majority	.466	.611	.733	.440	.533	.649	.430	.470	.559

Table C - Continued

Utility Structure (5)

Minority: LS_1 = 1.5 and LUS_1 = 0.5, Majority: LS_2 = 1.0 and LUS_2 = 1.0

		Y_{s1}^* = 0.5			Y_{s1}^* = 0.0			Y_{s1}^* = -0.5		
ρ_2 \ Y_{s2}^*		0.5	0.0	-0.5	0.5	0.0	-0.5	0.5	0.0	-0.5
ρ_1 = 0.2										
0.2	Minority	1.000	.801	.145	1.000	.999	.769	1.000	1.000	.999
	Majority	.429	.514	.795	.429	.429	.528	.429	.429	.429
0.4	Minority	1.000	.809	.261	1.000	.998	.753	1.000	1.000	.997
	Majority	.429	.510	.745	.429	.430	.534	.429	.429	.430
0.6	Minority	1.000	.815	.345	1.000	.994	.719	1.000	1.000	.987
	Majority	.429	.508	.709	.429	.431	.549	.429	.429	.434
ρ_1 = 0.4										
0.2	Minority	.986	.658	.258	1.000	.931	.654	1.000	.996	.936
	Majority	.435	.575	.746	.429	.458	.577	.429	.430	.456
0.4	Minority	.985	.690	.322	1.000	.923	.659	1.000	.995	.921
	Majority	.435	.561	.719	.429	.462	.575	.429	.431	.461
0.6	Minority	.990	.718	.372	1.000	.914	.650	1.000	.993	.894
	Majority	.433	.550	.698	.429	.466	.579	.429	.432	.474
ρ_1 = 0.6										
0.2	Minority	.894	.567	.308	.978	.817	.597	.998	.954	.845
	Majority	.474	.614	.725	.438	.507	.601	.430	.448	.495
0.4	Minority	.905	.602	.347	.978	.819	.607	.997	.950	.835
	Majority	.469	.599	.708	.438	.506	.597	.430	.450	.499
0.6	Minority	.930	.638	.379	.983	.820	.606	.998	.944	.816
	Majority	.459	.584	.695	.436	.506	.598	.429	.453	.507

Table C - Continued

Utility Structure (6)
Minority: LS_1 = 2.0 and LUS_1 = 0.5, Majority: LS_2 = 1.0 and LUS_2 = 1.0

		Y_{s1}^* = 0.5			Y_{s1}^* = 0.0			Y_{s1}^* = -0.5		
ρ_2 \ Y_{s2}^*		0.5	0.0	-0.5	0.5	0.0	-0.5	0.5	0.0	-0.5
ρ_1 = 0.2										
0.2	Minority	1.000	.933	.382	1.000	1.000	.956	1.000	1.000	1.000
	Majority	.429	.457	.693	.429	.429	.447	.429	.429	.429
0.4	Minority	1.000	.919	.453	1.000	1.000	.934	1.000	1.000	1.000
	Majority	.429	.463	.663	.429	.429	.457	.429	.429	.429
0.6	Minority	1.000	.905	.489	1.000	1.000	.893	1.000	1.000	1.000
	Majority	.429	.469	.648	.429	.429	.474	.429	.429	.429
ρ_1 = 0.4										
0.2	Minority	.993	.764	.408	1.000	.968	.804	1.000	.999	.979
	Majority	.432	.530	.682	.429	.442	.513	.429	.429	.437
0.4	Minority	.992	.774	.448	1.000	.962	.793	1.000	.999	.973
	Majority	.432	.525	.665	.429	.445	.517	.429	.429	.440
0.6	Minority	.995	.784	.473	1.000	.954	.770	1.000	.998	.960
	Majority	.431	.521	.654	.429	.448	.527	.429	.430	.446
ρ_1 = 0.6										
0.2	Minority	.915	.639	.405	.984	.867	.699	.999	.972	.905
	Majority	.465	.583	.684	.435	.486	.558	.429	.441	.469
0.4	Minority	.922	.662	.432	.984	.864	.698	.998	.969	.896
	Majority	.462	.574	.672	.435	.487	.558	.429	.442	.473
0.6	Minority	.941	.685	.451	.987	.862	.689	.999	.964	.881
	Majority	.454	.563	.664	.434	.488	.562	.429	.444	.480

APPENDIX III. PROPORTIONS OF APPLICANTS ACCEPTED FOR THE SIX MODELS

The proportions of minority and majority applicants accepted are presented for the Expected Utility Model with utility structure (4), the Regression Model, the Employer's Model, Thorndike's Model, the Equal Opportunity Model and Darlington's Model under various hypothetical data situations. The following symbols and abbreviations are used.

ρ_1: The test validity for the minority group.

ρ_2: The test validity for the majority group.

SR: The selection ratio.

q_1: The proportion of applicants from the minority group.

Y_{s1}^*: The standardized minimal success score for the minority group.

Y_{s2}^*: The standardized minimal success score for the majority group.

EXPT: The Expected Utility Model with utility structure (4).

REGR: The Regression Model.

EMPL: The Employer's Model.

THOR: Thorndike's Model.

EQOP: The Equal Opportunity Model.

DARL: Darlington's Model.

Table D

Proportions of Applicants Accepted in the Situations of $\rho_1 = 0.2$, $\rho_2 = 0.4$ and $q_1 = 0.3$ for Each Value of SR and the Six Models

SR = 0.2

Y^*_{s2} \ Model		EXPT	REGR	EMPL	THOR	EQOP	DARL
$Y^*_{s1} = 0.5$							
0.5	Minority	.4007	.0871	.0958	.2000	.2413	.4472
	Majority	.1140	.2484	.2447	.2000	.1823	.0941
0.0	Minority	.0173	.0001	.0001	.1394	.2134	.0871
	Majority	.2783	.2857	.2857	.2260	.1943	.2484
-0.5	Minority	.0000	.0000	.0000	.1070	.1914	.0001
	Majority	.2857	.2857	.2857	.2398	.2037	.2957
$Y^*_{s1} = 0.0$							
0.5	Minority	.5978	.4472	.4461	.2732	.2552	.6427
	Majority	.0295	.0941	.0945	.1686	.1763	.0102
0.0	Minority	.2718	.0871	.0772	.2000	.2269	.4472
	Majority	.1693	.2484	.2526	.2000	.1885	.0941
-0.5	Minority	.0027	.0001	.0001	.1577	.2045	.0871
	Majority	.2346	.2857	.2857	.2181	.1981	.2484
$Y^*_{s1} = -0.5$							
0.5	Minority	.6605	.6421	.6384	.3266	.2669	.6662
	Majority	.0027	.0102	.0121	.1457	.1713	.0002
0.0	Minority	.5514	.4472	.4240	.2481	.2382	.6427
	Majority	.0494	.0941	.1040	.1794	.1836	.0102
-0.5	Minority	.1858	.0871	.0606	.2000	.2155	.4472
	Majority	.2061	.2484	.2598	.2000	.1934	.0941

Table D - Continued

SR = 0.4							
Y_{s2}^* \ Model		EXPT	REGR	EMPL	THOR	EQOP	DARL
$Y_{s1}^* = 0.5$							
0.5	Minority	.8533	.3476	.3767	.4000	.4636	.8260
	Majority	.2057	.4225	.4100	.4000	.3728	.2174
0.0	Minority	.2507	.0149	.0157	.2789	.4245	.3476
	Majority	.4640	.5650	.5647	.4519	.3895	.4225
-0.5	Minority	.0003	.0000	.0000	.2141	.3922	.0149
	Majority	.5713	.5711	.5714	.4797	.4033	.5650
$Y_{s1}^* = 0.0$							
0.5	Minority	.9967	.8260	.8351	.5465	.4822	.9980
	Majority	.1443	.2174	.2135	.3372	.3648	.1437
0.0	Minority	.6692	.3476	.3424	.4000	.4430	.8260
	Majority	.2846	.4225	.4247	.4000	.3816	.2174
-0.5	Minority	.1121	.0149	.0104	.3155	.4106	.3476
	Majority	.5234	.5650	.5670	.4362	.3954	.4225
$Y_{s1}^* = -0.5$							
0.5	Minority	.9999	.9980	.9981	.6532	.4749	.9999
	Majority	.1429	.1437	.1437	.2915	.3581	.1429
0.0	Minority	.9627	.8260	.8103	.4962	.4587	.9980
	Majority	.1589	.2174	.2242	.3588	.3749	.1437
-0.5	Minority	.5400	.3476	.3086	.4000	.4262	.8260
	Majority	.3400	.4225	.4392	.4000	.3888	.2174

Table D - Continued

SR = 0.6							
Y^*_{s2} \ Model		EXPT	REGR	EMPL	THOR	EQOP	DARL
$Y^*_{s1} = 0.5$							
0.5	Minority	.9999	.6524	.6914	.6000	.6686	.9851
	Majority	.4286	.5776	.5608	.6000	.5706	.4350
0.0	Minority	.6285	.1740	.1897	.4183	.6298	.6524
	Majority	.5878	.7826	.7758	.6779	.5872	.5776
-0.5	Minority	.0834	.0020	.0019	.3211	.5963	.1740
	Majority	.8214	.8563	.8564	.7196	.6016	.7826
$Y^*_{s1} = 0.0$							
0.5	Minority	.9999	.9851	.9896	.8197	.6864	.9999
	Majority	.4286	.4350	.4330	.5058	.5630	.4275
0.0	Minority	.9557	.6524	.6576	.6000	.6481	.9851
	Majority	.4476	.5776	.5753	.6000	.5794	.4350
-0.5	Minority	.4265	.1740	.1649	.4732	.6149	.6524
	Majority	.6744	.7826	.7865	.6544	.5936	.5776
$Y^*_{s1} = -0.5$							
0.5	Minority	.9999	.9999	.9999	.9798	.7013	.9999
	Majority	.4286	.4275	.4286	.4372	.5566	.4286
0.0	Minority	.9999	.9851	.9843	.7443	.6634	.9999
	Majority	.4286	.4350	.4353	.5382	.5728	.4275
-0.5	Minority	.8574	.6524	.6233	.6000	.6305	.9851
	Majority	.4897	.5776	.5900	.6000	.5869	.4350

Table E

Proportions of Applicants Accepted in the Situations of $\rho_1 = 0.4$, $\rho_2 = 0.4$ and $q_1 = 0.3$ for Each Value of SR and the Six Models

SR = 0.2

Y^*_{s2} \ Model		EXPT	REGR	EMPL	THOR	EQOP	DARL
$Y^*_{s1} = 0.5$							
0.5	Minority	.4097	.2000	.2000	.2000	.2000	.4601
	Majority	.1101	.2000	.2000	.2000	.2000	.0885
0.0	Minority	.1099	.0317	.0317	.1394	.1745	.2000
	Majority	.2386	.2721	.2721	.2260	.2109	.2000
-0.5	Minority	.0053	.0011	.0011	.1070	.1547	.0317
	Majority	.2835	.2853	.2853	.2398	.2194	.2721
$Y^*_{s1} = 0.0$							
0.5	Minority	.5850	.4601	.4602	.2732	.2268	.6312
	Majority	.0350	.0885	.0885	.1686	.1885	.0152
0.0	Minority	.3357	.2000	.2000	.2000	.2000	.4601
	Majority	.1419	.2000	.2000	.2000	.2000	.0885
-0.5	Minority	.0785	.0317	.0317	.1577	.1790	.2000
	Majority	.2521	.2721	.2721	.2181	.2090	.2000
$Y^*_{s1} = -0.5$							
0.5	Minority	.6579	.6312	.6312	.3266	.2496	.6656
	Majority	.0038	.0152	.0152	.1457	.1788	.0005
0.0	Minority	.5530	.4601	.4602	.2481	.2219	.6312
	Majority	.0487	.0885	.0885	.1794	.1906	.0152
-0.5	Minority	.2956	.2000	.2000	.2000	.2000	.4601
	Majority	.1591	.2000	.2000	.2000	.2000	.0885

Table E - Continued

SR = 0.4							
Model / Y^*_{s2}		EXPT	REGR	EMPL	THOR	EQOP	DARL
Y^*_{s1} = 0.5							
0.5	Minority	.7493	.3999	.4000	.4000	.4000	.7276
	Majority	.2503	.4001	.4000	.4000	.4000	.2596
0.0	Minority	.3122	.1160	.1160	.2789	.3628	.3999
	Majority	.4376	.5217	.5217	.4519	.4160	.4001
-0.5	Minority	.0421	.0099	.0099	.2141	.3324	.1160
	Majority	.5534	.5672	.5672	.4797	.4290	.5217
Y^*_{s1} = 0.0							
0.5	Minority	.9283	.7276	.7276	.5465	.4379	.9385
	Majority	.1736	.2596	.2596	.3372	.3838	.1692
0.0	Minority	.6150	.3999	.4000	.4000	.4000	.7276
	Majority	.3079	.4001	.4000	.4000	.4000	.2596
-0.5	Minority	.2342	.1160	.1160	.3155	.3689	.3999
	Majority	.4710	.5217	.5217	.4362	.4134	.4001
Y^*_{s1} = -0.5							
0.5	Minority	.9943	.9385	.9385	.6532	.4700	.9964
	Majority	.1453	.1692	.1692	.2915	.3700	.1444
0.0	Minority	.8664	.7276	.7276	.4962	.4317	.9385
	Majority	.2001	.2596	.2596	.3588	.3864	.1692
-0.5	Minority	.5470	.3999	.4000	.4000	.4000	.7276
	Majority	.3370	.4001	.4000	.4000	.4000	.2596

Table E - Continued

SR = 0.6							
Y^*_{s2} \ Model		EXPT	REGR	EMPL	THOR	EQOP	DARL
$Y^*_{s1} = 0.5$							
0.5	Minority	.9694	.6000	.6000	.6000	.6000	.8840
	Majority	.4417	.6000	.6000	.6000	.6000	.4783
0.0	Minority	.5699	.2724	.2724	.4183	.5611	.6000
	Majority	.6129	.7404	.7404	.6779	.6167	.6000
-0.5	Minority	.1782	.0615	.0615	.3211	.5279	.2724
	Majority	.7808	.8308	.8308	.7200	.6309	.7404
$Y^*_{s1} = 0.0$							
0.5	Minority	.9988	.8840	.8841	.8197	.6384	.9901
	Majority	.4291	.4783	.4783	.5058	.5835	.4328
0.0	Minority	.8366	.6000	.6000	.6000	.6000	.8840
	Majority	.4986	.6000	.6000	.6000	.6000	.4783
-0.5	Minority	.4511	.2724	.2724	.4732	.5670	.6000
	Majority	.6638	.7404	.7404	.6544	.6141	.6000
$Y^*_{s1} = -0.5$							
0.5	Minority	.9999	.9901	.9901	.9798	.6706	.9998
	Majority	.4286	.4328	.4328	.4372	.5697	.4287
0.0	Minority	.9779	.8840	.8841	.7443	.6327	.9901
	Majority	.4381	.4783	.4783	.5382	.5860	.4328
-0.5	Minority	.7599	.6000	.6000	.6000	.6000	.8840
	Majority	.5315	.6000	.6000	.6000	.6000	.4783

Table F

Proportions of Applicants Accepted in the Situations of $\rho_1 = 0.6$, $\rho_2 = 0.4$ and $q_1 = 0.3$ for Each Value of SR and the Six Models

SR = 0.2

Y^*_{s2} \ Model		EXPT	REGR	EMPL	THOR	EQOP	DARL
$Y^*_{s1} = 0.5$							
0.5	Minority	.4074	.2643	.2580	.2000	.1635	.4678
	Majority	.1111	.1724	.1751	.2000	.2157	.0852
0.0	Minority	.1845	.0975	.1067	.1394	.1414	.2643
	Majority	.2067	.2439	.2400	.2260	.2251	.1724
-0.5	Minority	.0513	.0197	.0280	.1070	.1246	.0975
	Majority	.2638	.2773	.2737	.2398	.2323	.2439
$Y^*_{s1} = 0.0$							
0.5	Minority	.5712	.4678	.4704	.2732	.2021	.6191
	Majority	.0409	.0852	.0841	.1686	.1991	.0204
0.0	Minority	.3791	.2643	.2845	.2000	.1775	.4678
	Majority	.1233	.1724	.1638	.2000	.2096	.0825
-0.5	Minority	.1795	.0975	.1245	.1577	.1585	.2643
	Majority	.2088	.2439	.2324	.2181	.2178	.1724
$Y^*_{s1} = -0.5$							
0.5	Minority	.6555	.6191	.6274	.3266	.2355	.6645
	Majority	.0048	.0204	.0168	.1457	.1848	.0009
0.0	Minority	.5622	.4678	.4955	.2481	.2091	.6191
	Majority	.0448	.0852	.0733	.1794	.1961	.0204
-0.5	Minority	.3784	.2643	.3115	.2000	.1883	.4678
	Majority	.1236	.1724	.1522	.2000	.2050	.0852

Table F - Continued

SR = 0.4							
Model / Y_{s2}^*		EXPT	REGR	EMPL	THOR	EQOP	DARL
$Y_{s1}^* = 0.5$							
0.5	Minority	.6603	.4254	.4000	.4000	.3345	6738
	Majority	.2884	.3891	.4000	.4000	.4281	.2827
0.0	Minority	.3391	.1964	.1970	.2789	.3011	.4254
	Majority	.4261	.4873	.4870	.4519	.4424	.3891
-0.5	Minority	.1209	.0563	.0670	.2141	.2741	.1964
	Majority	.5196	.5473	.5427	.4799	.4540	.4873
$Y_{s1}^* = 0.0$							
0.5	Minority	.8385	.6738	.6572	.5465	.3923	.8657
	Majority	.2121	.2827	.2898	.3372	.4033	.2004
0.0	Minority	.5802	.4254	.4330	.4000	.3571	.6738
	Majority	.3228	.3891	.3859	.4000	.4184	.2827
-0.5	Minority	.3149	.1964	.2229	.3155	.3283	.4254
	Majority	.4365	.4873	.4759	.4362	.4308	.3891
$Y_{s1}^* = -0.5$							
0.5	Minority	.9505	.8657	.8594	.6532	.4421	.9663
	Majority	.1641	.2004	.2031	.2915	.3820	.1573
0.0	Minority	.7959	.6738	.6875	.4962	.4055	.8657
	Majority	.2303	.2827	.2768	.3588	.3976	.2004
-0.5	Minority	.5640	.4254	.4663	.4000	.3754	.6738
	Majority	.3297	.3891	.3716	.4000	.4106	.2827

Table F - Continued

Y_{s2}^* \ Model		SR = 0.6 EXPT	REGR	EMPL	THOR	EQOP	DARL
$Y_{s1}^* = 0.5$							
0.5	Minority	.8789	.5746	.5338	.6000	.5188	.8037
	Majority	.4805	.6109	.6284	.6000	.6348	.5127
0.0	Minority	.5199	.3262	.3126	.4183	.4822	.5746
	Majority	.6343	.7174	.7232	.6779	.6505	.6109
-0.5	Minority	.2394	.1343	.1406	.3211	.4514	.3262
	Majority	.7545	.7996	.7969	.7196	.6637	.7174
$Y_{s1}^* = 0.0$							
0.5	Minority	.9659	.8037	.7771	.8197	.5816	.9438
	Majority	.4432	.5127	.5241	.5058	.6079	.4527
0.0	Minority	.7443	.5746	.5670	.6000	.5447	.8037
	Majority	.5382	.6109	.6141	.6000	.6237	.5127
-0.5	Minority	.4663	.3262	.3428	.4732	.5133	.5746
	Majority	.6573	.7174	.7103	.6544	.6372	.6109
$Y_{s1}^* = -0.5$							
0.5	Minority	.9954	.9438	.9330	.9798	.6347	.9915
	Majority	.4306	.4527	.4573	.4372	.5851	.4322
0.0	Minority	.9095	.8037	.8030	.7443	.5980	.9438
	Majority	.4674	.5127	.5130	.5382	.6009	.4527
-0.5	Minority	.7103	.5746	.6000	.6000	.5663	.8037
	Majority	.5527	.6109	.6000	.6000	.6145	.5127

APPENDIX IV. THE PERFORMANCE ON TRADITIONAL CRITERIA FOR THE SIX MODELS

The performance on four traditional criteria are presented for the six selection models in various hypothetical data situations. Table G, Table H, Table I and Table J present average criterion performance of accepted applicants, the rate of selection errors, the success rate of accepted applicants and the expected loss of selection respectively for the six models. The following symbols are used.

SR: The selection ratio.

q_1: The proportion of applicants from the minority group.

Y_{s1}*: The standardized minimal success score for the minority group.

Y_{s2}*: The standardized minimal success score for the majority group.

ρ_1: The test validity for the minority group.

ρ_2: The test validity for the majority group.

EXPT: The Expected Utility Model with utility structure (4).

REGR: The Regression Model.

EMPL: The Employer's Model.

THOR: Thorndike's Model.

EQOP: The Equal Opportunity Model.

DARL: Darlington's Model.

Table G
Average Criterion Performance of Accepted Applicants in Different Situations for the Six Models

	SR = 0.2 and q_1 = 0.3								
	Y_{s1}^* = 0.5			Y_{s1}^* = 0.0			Y_{s1}^* = -0.5		
Model \ Y_{s2}^*	0.5	0.0	-0.5	0.5	0.0	-0.5	0.5	0.0	-0.5
ρ_1=0.2, ρ_2=0.4									
EXPT	-0.114	0.470	0.976	0.158	0.453	0.975	0.612	0.675	0.979
REGR	-0.009	0.476	0.976	0.189	0.491	0.976	0.614	0.689	0.991
EMPL	-0.009	0.476	0.976	0.189	0.491	0.976	0.614	0.688	0.990
THOR	-0.024	0.383	0.830	0.157	0.476	0.867	0.418	0.648	0.976
EQOP	-0.036	0.312	0.691	0.150	0.469	0.822	0.355	0.644	0.972
DARL	-0.147	0.426	0.976	0.132	0.354	0.926	0.609	0.632	0.854
ρ_1=0.4, ρ_2=0.4									
EXPT	-0.003	0.464	0.975	0.281	0.535	0.976	0.723	0.793	1.047
REGR	0.060	0.484	0.976	0.308	0.560	0.984	0.727	0.808	1.060
EMPL	0.060	0.484	0.976	0.308	0.560	0.984	0.727	0.808	1.060
THOR	0.060	0.450	0.885	0.257	0.560	0.939	0.527	0.743	1.060
EQOP	0.060	0.428	0.825	0.229	0.560	0.925	0.431	0.726	1.060
DARL	-0.037	0.410	0.960	0.253	0.463	0.910	0.719	0.753	0.963
ρ_1=0.6 , ρ_2=0.4									
EXPT	0.115	0.501	0.975	0.405	0.628	1.003	0.834	0.909	1.128
REGR	0.152	0.521	0.982	0.427	0.652	1.021	0.840	0.927	1.152
EMPL	0.152	0.521	0.981	0.427	0.651	1.019	0.840	0.925	1.148
THOR	0.144	0.516	0.941	0.357	0.644	1.011	0.635	0.838	1.144
EQOP	0.132	0.516	0.926	0.296	0.637	1.011	0.504	0.803	1.141
DARL	0.076	0.454	0.948	0.376	0.576	0.954	0.829	0.876	1.076

Table G - Continued

	SR = 0.4 and q_1 = 0.3								
	Y_{s1}^* = 0.5			Y_{s1}^* = 0.0			Y_{s1}^* = -0.5		
Model \ Ys_2^*	0.5	0.0	-0.5	0.5	0.0	-0.5	0.5	0.0	-0.5
ρ_1=0.2, ρ_2=0.4									
EXPT	-0.266	0.232	0.775	0.034	0.292	0.765	0.408	0.543	0.816
REGR	-0.171	0.276	0.775	0.054	0.329	0.776	0.408	0.554	0.829
EMPL	-0.171	0.276	0.775	0.054	0.329	0.776	0.408	0.554	0.829
THOR	-0.172	0.223	0.662	0.020	0.328	0.711	0.285	0.508	0.829
EQOP	-0.176	0.168	0.535	0.004	0.326	0.674	0.195	0.497	0.827
DARL	-0.256	0.199	0.770	0.034	0.244	0.699	0.410	0.534	0.744
ρ_1=0.4, ρ_2=0.4									
EXPT	-0.182	0.265	0.772	0.069	0.361	0.783	0.410	0.586	0.875
REGR	-0.114	0.294	0.776	0.099	0.386	0.794	0.417	0.599	0.886
EMPL	-0.114	0.294	0.776	0.099	0.386	0.794	0.417	0.599	0.886
THOR	-0.114	0.273	0.706	0.079	0.386	0.764	0.341	0.567	0.886
EQOP	-0.114	0.250	0.635	0.050	0.386	0.748	0.236	0.548	0.886
DARL	-0.173	0.236	0.750	0.065	0.327	0.736	0.410	0.565	0.826
ρ_1=0.6, ρ_2=0.4									
EXPT	-0.096	0.312	0.779	0.128	0.427	0.817	0.432	0.639	0.931
REGR	-0.056	0.330	0.786	0.152	0.445	0.830	0.443	0.652	0.945
EMPL	-0.056	0.330	0.786	0.151	0.445	0.829	0.443	0.651	0.944
THOR	-0.056	0.324	0.750	0.139	0.444	0.818	0.396	0.627	0.945
EQOP	-0.062	0.320	0.722	0.091	0.441	0.815	0.276	0.597	0.943
DARL	-0.101	0.285	0.756	0.118	0.399	0.785	0.427	0.618	0.899

Table G - Continued

	SR = 0.6 and q_1 = 0.3								
	Y_{s1}^* = 0.5			Y_{s1}^* = 0.0			Y_{s1}^* = -0.5		
Model \ Ys_2^*	0.5	0.0	-0.5	0.5	0.0	-0.5	0.5	0.0	-0.5
ρ_1 = 0.2, ρ_2 = 0.4									
EXPT	-0.317	0.062	0.596	-0.067	0.194	0.601	0.183	0.433	0.709
REGR	-0.280	0.119	0.605	-0.066	0.220	0.619	0.184	0.434	0.720
EMPL	-0.281	0.119	0.605	-0.066	0.220	0.619	0.183	0.434	0.719
THOR	-0.281	0.102	0.532	-0.083	0.220	0.594	0.179	0.404	0.719
EQOP	-0.280	0.062	0.421	-0.109	0.220	0.565	0.070	0.385	0.719
DARL	-0.313	0.057	0.576	-0.066	0.187	0.557	0.274	0.433	0.687
ρ_1 = 0.4, ρ_2 = 0.4									
EXPT	-0.302	0.115	0.601	-0.066	0.236	0.637	0.183	0.439	0.748
REGR	-0.242	0.150	0.611	-0.054	0.258	0.650	0.184	0.446	0.758
EMPL	-0.242	0.150	0.611	-0.054	0.258	0.650	0.184	0.446	0.758
THOR	-0.242	0.141	0.568	-0.056	0.258	0.633	0.183	0.436	0.757
EQOP	-0.242	0.117	0.492	-0.083	0.258	0.615	0.091	0.415	0.757
DARL	-0.275	0.108	0.581	-0.064	0.225	0.608	0.183	0.436	0.725
ρ_1 = 0.6, ρ_2 = 0.4									
EXPT	-0.254	0.165	0.620	-0.052	0.282	0.674	0.185	0.462	0.787
REGR	-0.203	0.184	0.629	-0.030	0.297	0.684	0.191	0.470	0.796
EMPL	-0.204	0.184	0.629	-0.030	0.296	0.684	0.191	0.470	0.796
THOR	-0.204	0.180	0.604	-0.030	0.296	0.673	0.188	0.468	0.796
EQOP	-0.205	0.172	0.563	-0.058	0.296	0.666	0.112	0.446	0.796
DARL	-0.231	0.153	0.603	-0.045	0.269	0.653	0.186	0.455	0.769

Table H
The Rate of Selection Errors in Different Situations for the Six Models

SR = 0.2 and q_1 = 0.3									
	$Y_{s1}^{*} = 0.5$			$Y_{s1}^{*} = 0.0$			$Y_{s1}^{*} = -0.5$		
Model \ Ys_2^{*}	0.5	0.0	-0.5	0.5	0.0	-0.5	0.5	0.0	-0.5
$\rho_1 = 0.2, \rho_2 = 0.4$									
EXPT	.3260	.3662	.4363	.3403	.4276	.4938	.3306	.4555	.5526
REGR	.3102	.3652	.4363	.3350	.4207	.4937	.3302	.4527	.5502
EMPL	.3102	.3652	.4363	.3350	.4207	.4937	.3302	.4527	.5501
THOR	.3122	.3808	.4592	.3403	.4236	.5104	.3599	.4573	.5531
EQOP	.3140	.3924	.4808	.3415	.4249	.5172	.3698	.4578	.5537
DARL	.3311	.3736	.4363	.3447	.4443	.5015	.3310	.4621	.5702
$\rho_1 = 0.4, \rho_2 = 0.4$									
EXPT	.3093	.3673	.4363	.3198	.4144	.4939	.3134	.4379	.5444
REGR	.2986	.3640	.4362	.3151	.4100	.4928	.3128	.4358	.5428
EMPL	.2986	.3640	.4362	.3151	.4100	.4928	.3128	.4358	.5428
THOR	.2986	.3698	.4500	.3239	.4100	.4989	.3454	.4452	.5428
EQOP	.2986	.3735	.4594	.3288	.4100	.5008	.3607	.4476	.5428
DARL	.3150	.3766	.4386	.3245	.4260	.5030	.3140	.4436	.5544
$\rho_1 = 0.6, \rho_2 = 0.4$									
EXPT	.2896	.3599	.4356	.2971	.3969	.4883	.2941	.4190	.5324
REGR	.2820	.3570	.4352	.2931	.3941	.4876	.2935	.4177	.5319
EMPL	.2819	.3570	.4351	.2131	.3939	.4873	.2934	.4174	.5315
THOR	.2832	.3575	.4399	.3066	.3962	.4876	.3313	.4343	.5340
EQOP	.2853	.3576	.4420	.3179	.3977	.4877	.3534	.4397	.5346
DARL	.2969	.3682	.4389	.3023	.4049	.4937	.2948	.4230	.5370

Table H - Continued

	SR = 0.4 and q_1 = 0.3								
	Y_{s1}^* = 0.5			Y_{s1}^* = 0.0			Y_{s1}^* = -0.5		
Model \ Ys_2^*	0.5	0.0	-0.5	0.5	0.0	-0.5	0.5	0.0	-0.5
ρ_1 = 0.2, ρ_2 = 0.4									
EXPT	.3910	.3661	.3429	.3545	.4053	.4040	.2974	.3891	.4512
REGR	.3653	.3521	.3430	.3486	.3924	.4002	.2974	.3848	.4459
EMPL	.3652	.3521	.3429	.3486	.3924	.4002	.2974	.3848	.4458
THOR	.3653	.3690	.3800	.3607	.3928	.4221	.3378	.3980	.4465
EQOP	.3661	.3866	.4212	.3663	.3936	.4341	.3674	.4011	.4471
DARL	.3880	.3736	.3445	.3546	.4213	.4260	.2966	.3922	.4735
ρ_1 = 0.4, ρ_2 = 0.4									
EXPT	.3685	.3559	.3438	.3341	.3822	.3986	.2967	.3746	.4338
REGR	.3477	.3459	.3426	.3340	.3734	.3951	.2944	.3700	.4304
EMPL	.3477	.3459	.3426	.3340	.3734	.3951	.2944	.3700	.4304
THOR	.3477	.3530	.3659	.3408	.3734	.4044	.3207	.3805	.4304
EQOP	.3477	.3612	.3896	.3509	.3734	.4097	.3560	.3868	.4304
DARL	.3659	.3657	.3510	.3452	.3941	.4133	.2969	.3816	.4479
ρ_1 = 0.6, ρ_2 = 0.4									
EXPT	.3447	.3395	.3405	.3254	.3581	.3855	.2892	.3554	.4153
REGR	.3284	.3325	.3389	.3153	.3519	.3832	.2848	.3516	.4133
EMPL	.3282	.3325	.3388	.3152	.3519	.3830	.2847	.3516	.4130
THOR	.3282	.3349	.3499	.3189	.3522	.3855	.3015	.3619	.4139
EQOP	.3294	.3364	.3593	.3354	.3535	.3862	.3448	.3732	.4147
DARL	.3463	.3496	.3476	.3288	.3684	.3949	.2912	.3626	.4235

Table H - Continued

	SR = 0.6 and q_1 = 0.3								
	Y_{s1}^* = 0.5			Y_{s1}^* = 0.0			Y_{s1}^* = -0.5		
Model \ Ys_2^*	0.5	0.0	-0.5	0.5	0.0	-0.5	0.5	0.0	-0.5
ρ_1 = 0.2, ρ_2 = 0.4									
EXPT	.4578	.4097	.3023	.4003	.4047	.3586	.3429	.3524	.3716
REGR	.4459	.3848	.2974	.4002	.3924	.3486	.3426	.3521	.3653
EMPL	.4458	.3848	.2974	.4002	.3924	.3486	.3429	.3521	.3652
THOR	.4465	.3920	.3335	.4088	.3928	.3623	.3452	.3666	.3653
EQOP	.4458	.4098	.3878	.4219	.3924	.3765	.3987	.3754	.3653
DARL	.4563	.4123	.3121	.4001	.4078	.3810	.2966	.3525	.3822
ρ_1 = 0.4, ρ_2 = 0.4									
EXPT	.4531	.3870	.2998	.4002	.3847	.3406	.3429	.3496	.3524
REGR	.4304	.3700	.2944	.3951	.3734	.3340	.3426	.3459	.3477
EMPL	.4304	.3700	.2944	.3951	.3734	.3340	.3426	.3459	.3477
THOR	.4304	.3743	.3167	.3962	.3734	.3423	.3428	.3512	.3477
EQOP	.4304	.3861	.3555	.4089	.3734	.3514	.3890	.3617	.3477
DARL	.4434	.3905	.3099	.3992	.3900	.3554	.3429	.3509	.3636
ρ_1 = 0.6, ρ_2 = 0.4									
EXPT	.4379	.3636	.2890	.3949	.3609	.3197	.3422	.3373	.3316
REGR	.4133	.3516	.2848	.3832	.3519	.3153	.3389	.3325	.3284
EMPL	.4130	.3516	.2847	.3830	.3519	.3152	.3388	.3325	.3282
THOR	.4139	.3548	.2978	.3835	.3522	.3202	.3406	.3338	.3282
EQOP	.4130	.3597	.3201	.3940	.3520	.3237	.3778	.3465	.3285
DARL	.4284	.3704	.2985	.3921	.3681	.3307	.3417	.3413	.3402

Table I
The Success Rate of Accepted Applicants in Different Situations for the Six Models

SR = 0.2 and $q_1 = 0.3$

	$Y_{s1}^* = 0.5$			$Y_{s1}^* = 0.0$			$Y_{s1}^* = -0.5$		
Model \ Ys_2^*	0.5	0.0	-0.5	0.5	0.0	-0.5	0.5	0.0	-0.5

$\rho_1 = 0.2$, $\rho_2 = 0.4$

Model	0.5	0.0	-0.5	0.5	0.0	-0.5	0.5	0.0	-0.5
EXPT	.4563	.6908	.8508	.5642	.6810	.8506	.7320	.7548	.8473
REGR	.4958	.6934	.8508	.5774	.6983	.8508	.7329	.7618	.8530
EMPL	.4959	.6935	.8508	.5774	.6984	.8508	.7331	.7619	.8534
THOR	.4910	.6546	.7936	.5642	.6912	.8092	.6588	.7505	.8461
EQOP	.4864	.6255	.7395	.5614	.6879	.7922	.6341	.7492	.8444
DARL	.4436	.6722	.8508	.5530	.6393	.8314	.7309	.7382	.8032

$\rho_1 = 0.4$, $\rho_2 = 0.4$

Model	0.5	0.0	-0.5	0.5	0.0	-0.5	0.5	0.0	-0.5
EXPT	.4981	.6882	.8506	.6155	.7140	.8504	.7750	.7989	.8677
REGR	.5249	.6965	.8509	.6272	.7250	.8529	.7763	.8042	.8716
EMPL	.5249	.6966	.8509	.6272	.7250	.8530	.7765	.8041	.8716
THOR	.5250	.6821	.8166	.6052	.7251	.8379	.6951	.7807	.8718
EQOP	.5250	.6729	.7930	.5932	.7251	.8331	.6568	.7747	.8717
DARL	.4839	.6650	.8450	.6035	.6851	.8277	.7734	.7845	.8427

$\rho_1 = 0.6$, $\rho_2 = 0.4$

Model	0.5	0.0	-0.5	0.5	0.0	-0.5	0.5	0.0	-0.5
EXPT	.5474	.7066	.8525	.6722	.7579	.8643	.8233	.8462	.8976
REGR	.5665	.7139	.8535	.6822	.7649	.8661	.8247	.8495	.8990
EMPL	.5665	.7140	.8537	.6822	.7652	.8668	.8250	.8500	.9000
THOR	.5635	.7127	.8418	.6486	.7595	.8661	.7303	.8080	.8937
EQOP	.5582	.7126	.8366	.6204	.7560	.8660	.6751	.7945	.8923
DARL	.5292	.6860	.8441	.6592	.7377	.8508	.8214	.8361	.8863

Table I - Continued

	SR = 0.4 and q_1 = 0.3								
	Y_{s1}^* = 0.5			Y_{s1}^* = 0.0			Y_{s1}^* = -0.5		
Model \ Ys_2^*	0.5	0.0	-0.5	0.5	0.0	-0.5	0.5	0.0	-0.5
ρ_1 = 0.2, ρ_2 = 0.4									
EXPT	.3969	.5956	.7921	.5144	.6183	.7876	.6575	.7104	.8004
REGR	.4291	.6130	.7918	.5218	.6346	.7923	.6575	.7158	.8071
EMPL	.4292	.6130	.7921	.5218	.6346	.7923	.6575	.7158	.8072
THOR	.4292	.5922	.7458	.5067	.6343	.7651	.6070	.6995	.8064
EQOP	.4283	.5702	.6944	.4998	.6332	.7501	.5702	.6956	.8057
DARL	.4007	.5824	.7901	.5142	.5983	.7601	.6289	.7066	.7725
ρ_1 = 0.4, ρ_2 = 0.4									
EXPT	.4251	.6083	.7910	.5274	.6472	.7943	.6585	.7285	.8221
REGR	.4511	.6208	.7925	.5400	.6583	.7987	.6612	.7343	.8264
EMPL	.4511	.6208	.7925	.5400	.6583	.7987	.6612	.7343	.8263
THOR	.4512	.6121	.7635	.5316	.6585	.7872	.6285	.7213	.8266
EQOP	.4512	.6019	.7339	.5191	.6585	.7806	.5844	.7134	.8266
DARL	.4282	.5962	.7820	.5259	.6324	.7760	.6582	.7198	.8044
ρ_1 = 0.6 , ρ_2 = 0.4									
EXPT	.4548	.6289	.7951	.5508	.6774	.8107	.6677	.7525	.8452
REGR	.4753	.6376	.7971	.5633	.6852	.8136	.6733	.7572	.8477
EMPL	.4755	.6376	.7973	.5634	.6851	.8138	.6733	.7574	.8481
THOR	.4756	.6347	.7835	.5590	.6850	.8109	.6525	.7446	.8472
EQOP	.4741	.6329	.7718	.5385	.6833	.8099	.5985	.7305	.8462
DARL	.4527	.6163	.7863	.5465	.6644	.7990	.6653	.7435	.8349

Table I - Continued

SR = 0.6 and q_1 = 0.3									
	Y_{s1}^* = 0.5			Y_{s1}^* = 0.0			Y_{s1}^* = -0.5		
Ys_2^* / Model	0.5	0.0	-0.5	0.5	0.0	-0.5	0.5	0.0	-0.5
ρ_1 = 0.2, ρ_2 = 0.4									
EXPT	.3756	.5274	.7286	.4714	.5794	.7295	.5671	.6709	.7665
REGR	.3855	.5481	.7327	.4715	.5897	.7379	.5667	.6711	.7717
EMPL	.3857	.5482	.7327	.4715	.5897	.7379	.5671	.6711	.7719
THOR	.3849	.5421	.7026	.4643	.5892	.7264	.5653	.6589	.7717
EQOP	.3856	.5272	.6572	.4533	.5896	.7145	.5205	.6516	.7717
DARL	.3769	.5252	.7205	.4710	.5768	.7108	.4222	.6702	.7577
ρ_1 = 0.4, ρ_2 = 0.4									
EXPT	.3795	.5463	.7307	.4715	.5961	.7446	.5671	.6732	.7826
REGR	.3984	.5605	.7351	.4758	.6055	.7501	.5674	.6763	.7864
EMPL	.3984	.5605	.7351	.4758	.6055	.7500	.5674	.6763	.7865
THOR	.3984	.5568	.7165	.4748	.6054	.7431	.5673	.6718	.7863
EQOP	.3984	.5470	.6841	.4641	.6054	.7354	.5286	.6630	.7863
DARL	.3876	.5434	.7223	.4723	.5917	.7322	.5671	.6721	.7732
ρ_1 = 0.6, ρ_2 = 0.4									
EXPT	.3922	.5658	.7397	.4759	.6159	.7620	.5677	.6835	.7999
REGR	.4126	.5758	.7432	.4857	.6234	.7657	.5705	.6874	.8026
EMPL	.4129	.5758	.7432	.4858	.6234	.7657	.5705	.6874	.8027
THOR	.4121	.5731	.7323	.4854	.6230	.7614	.5691	.6863	.8026
EQOP	.4128	.5689	.7173	.4765	.6232	.7585	.5379	.6757	.8023
DARL	.4001	.5601	.7318	.4782	.6099	.7528	.5681	.6801	.7927

Table J

The Expected Loss of Selection in Different Situations for the Six Models

SR = 0.2 and q_1 = 0.3									
	Y_{s1}^* = 0.5			Y_{s1}^* = 0.0			Y_{s1}^* = -0.5		
Model \ Y_{s2}^*	0.5	0.0	-0.5	0.5	0.0	-0.5	0.5	0.0	-0.5
ρ_1 = 0.2, ρ_2 = 0.4									
EXPT	.2886	.3649	.4363	.3001	.4112	.4937	.3040	.4342	.5467
REGR	.3030	.3652	.4363	.3062	.4160	.4937	.3045	.4362	.5478
EMPL	.3022	.3652	.4363	.3062	.4166	.4937	.3047	.4372	.5485
THOR	.2945	.3688	.4501	.3238	.4119	.5015	.3486	.4491	.5467
EQOP	.2924	.3735	.4640	.3262	.4115	.5053	.3608	.4500	.5467
DARL	.2890	.3664	.4363	.3010	.4155	.4968	.3041	.4364	.5537
ρ_1 = 0.4, ρ_2 = 0.4									
EXPT	.2754	.3603	.4361	.2855	.3982	.4914	.2912	.4209	.5377
REGR	.2843	.3624	.4362	.2904	.4018	.4921	.2920	.4230	.5390
EMPL	.2843	.3624	.4362	.2904	.4017	.4920	.2920	.4230	.5390
THOR	.2843	.3605	.4433	.3115	.4017	.4928	.3377	.4400	.5390
EQOP	.2843	.3614	.4490	.3190	.4017	.4937	.3555	.4432	.5390
DARL	.2759	.3623	.4370	.2864	.4012	.4947	.2914	.4228	.5416
ρ_1 = 0.6, ρ_2 = 0.4									
EXPT	.2608	.3507	.4341	.2694	.3830	.4843	.2768	.4064	.5268
REGR	.2664	.3533	.4348	.2733	.3864	.4861	.2782	.4090	.5291
EMPL	.2669	.3528	.4345	.2731	.3853	.4851	.2777	.4078	.5276
THOR	.2728	.3513	.4357	.2985	.3914	.4844	.3271	.4318	.5324
EQOP	.2775	.3518	.4367	.3130	.3937	.4844	.3512	.4379	.5331
DARL	.2618	.3527	.4353	.2704	.3851	.4861	.2770	.4077	.5284

Table J - Continued

SR = 0.4 and q_1 = 0.3									
	Y_{s1}^* = 0.5			Y_{s1}^* = 0.0			Y_{s1}^* = -0.5		
Model \ Y_{s2}^*	0.5	0.0	-0.5	0.5	0.0	-0.5	0.5	0.0	-0.5
ρ_1 = 0.2, ρ_2 = 0.4									
EXPT	.3048	.3436	.3429	.2798	.3595	.3978	.2511	.3455	.4304
REGR	.3332	.3510	.3430	.2897	.3707	.3995	.2513	.3494	.4336
EMPL	.3302	.3510	.3429	.2889	.3711	.3997	.2513	.3503	.4351
THOR	.3280	.3437	.3610	.3245	.3674	.4027	.3116	.3792	.4321
EQOP	.3222	.3468	.3847	.3349	.3651	.4080	.3485	.3841	.4315
DARL	.3049	.3446	.3434	.2798	.3625	.4043	.2503	.3460	.4381
ρ_1 = 0.4, ρ_2 = 0.4									
EXPT	.2971	.3315	.3416	.2776	.3455	.3884	.2510	.3396	.4172
REGR	.3147	.3385	.3422	.2875	.3529	.3910	.2539	.3439	.4200
EMPL	.3147	.3385	.3422	.2875	.3529	.3910	.2539	.3439	.4200
THOR	.3148	.3317	.3504	.3095	.3529	.3895	.2987	.3661	.4200
EQOP	.3148	.3319	.3633	.3277	.3529	.3913	.3428	.3752	.4200
DARL	.2971	.3327	.3435	.2776	.3476	.3928	.2510	.3411	.4218
ρ_1 = 0.6, ρ_2 = 0.4									
EXPT	.2873	.3173	.3355	.2712	.3296	.3753	.2490	.3291	.4027
REGR	.2977	.3224	.3372	.2784	.3350	.3785	.2528	.3333	.4062
EMPL	.3001	.3223	.3367	.2799	.3345	.3772	.2533	.3324	.4045
THOR	.3002	.3182	.3384	.2931	.3370	.3753	.2843	.3521	.4076
EQOP	.3077	.3176	.3429	.3206	.3409	.3753	.3371	.3670	.4091
DARL	.2874	.3189	.3375	.2714	.3316	.3780	.2492	.3306	.4052

Table J - Continued

SR = 0.6 and q_1 = 0.3									
	Y_{s1}^* = 0.5			Y_{s1}^* = 0.0			Y_{s1}^* = -0.5		
Model \ Y_{s2}^*	0.5	0.0	-0.5	0.5	0.0	-0.5	0.5	0.0	-0.5
ρ_1 = 0.2, ρ_2 = 0.4									
EXPT	.3540	.3484	.2954	.3253	.3342	.3313	.2966	.3062	.3345
REGR	.3821	.3696	.2973	.3268	.3479	.3386	.2963	.3070	.3391
EMPL	.3777	.3681	.2973	.3263	.3475	.3392	.2966	.3070	.3405
THOR	.3884	.3528	.3041	.3505	.3524	.3316	.3004	.3357	.3417
EQOP	.3802	.3484	.3301	.3747	.3482	.3350	.3700	.3486	.3402
DARL	.3545	.3485	.2968	.3251	.3344	.3365	.2966	.3062	.3370
ρ_1 = 0.4, ρ_2 = 0.4									
EXPT	.3537	.3362	.2874	.3253	.3279	.3165	.2966	.3056	.3244
REGR	.3763	.3493	.2910	.3334	.3379	.3216	.2974	.3096	.3285
EMPL	.3763	.3493	.2910	.3334	.3379	.3217	.2974	.3096	.3285
THOR	.3762	.3395	.2915	.3410	.3379	.3266	.2986	.3242	.3285
EQOP	.3762	.3362	.3093	.3702	.3379	.3186	.3661	.3409	.3285
DARL	.3553	.3364	.2892	.3255	.3283	.3199	.2966	.3057	.3273
ρ_1 = 0.6, ρ_2 = 0.4									
EXPT	.3517	.3228	.2755	.3248	.3171	.3000	.2966	.3013	.3110
REGR	.3663	.3307	.2789	.3330	.3239	.3045	.2994	.3056	.3153
EMPL	.3707	.3318	.2785	.3357	.3245	.3035	.3004	.3056	.3138
THOR	.3639	.3249	.2773	.3315	.3220	.3000	.2969	.3111	.3139
EQOP	.3724	.3231	.2868	.3654	.3264	.3005	.3617	.3322	.3158
DARL	.3525	.3234	.2775	.3250	.3179	.3026	.2966	.3018	.3133

APPENDIX V. THE STATISTICS FOR REAL LIFE DATA

The statistics are presented for sex subgroups and race subgroups. Table K, L and M are the statistics for sex subgroups. Table N, O and P are the statistics for race subgroups. The following symbols are used.

OATM: The Open Admission Test Mathematic Score.

OATR: The Open Admission Test Reading Score.

ENGL: The high school average on English.

MATH: The high school average on Mathematics.

SCIE: The high school average on Science.

SOCI: The high school average on Social Studies.

CONST: The intercept of the regression equation for predicting the grade point average of the first year college.

C_1: The acceptance score for the minority group.

C_2: The acceptance score for the majority group.

PS_1: The proportion of applicants accepted for the minority group.

PS_2: The proportion of applicants accepted for the majority group.

GS: The average criterion performance for accepted applicants.

RSE: The rate of selection errors.

SRA: The success rate of accepted applicants.

P_1/P_2: The ratio of the conditional probability being successful at C_1 to the conditional probability being successful at C_2.

INST: The institutional selection procedure.

Regression Coefficients for Sex Subgroups

Group		OATM	OATR	ENGL	MATH	SCIE	SOCI	CONST
A33	F	-0.0041	0.0121	0.0387	0.0253	-0.0196	0.0226	-3.1576
	M	0.0107	-0.0033	0.0371	0.0373	-0.0015	0.0529	-8.3096
A76	F	-0.0122	0.0092	0.0133	0.0176	0.0018	0.0165	-1.5074
	M	0.0088	-0.0004	0.0330	0.0167	-0.0089	-0.0018	-1.0832
B115	F	-0.0180	-0.0024	0.0347	0.0148	0.0257	0.0023	-2.8388
	M	-0.0035	0.0085	0.0473	-0.0139	0.0072	-0.0096	-0.2378
B199	F	-0.0058	0.0225	0.0107	0.0001	0.0045	0.0376	-2.6185
	M	0.0286	-0.0493	-0.0210	-0.0208	0.0564	0.1126	-7.2236
C1	F	-0.0262	0.0148	-0.0183	0.0205	0.0248	0.0439	-3.0302
	M	-0.0194	0.0035	0.0063	0.0045	0.0332	0.0650	-6.4161
C38	F	0.0139	0.0051	0.0426	0.0079	0.0353	-0.0242	-3.4509
	M	0.0010	-0.0075	0.0349	0.0108	0.0244	0.0019	-3.4338
C62	F	-0.0115	0.0166	-0.0055	0.0408	-0.0217	0.0201	-0.3696
	M	0.0022	0.0070	0.0161	-0.0006	0.0176	0.0320	-3.3766
C103	F	-0.0197	0.0123	0.0181	0.0242	0.0007	0.0218	-2.9268
	M	0.0085	-0.0042	0.0006	0.0132	-0.0008	0.0580	-3.8665
D18	F	0.0008	0.0029	0.0612	-0.0050	0.0004	0.0115	-3.2337
	M	-0.0373	0.0196	0.0600	0.0205	0.0192	-0.0109	-4.5338
D70	F	-0.0042	-0.0051	0.0007	0.0073	0.0014	0.0118	1.5425
	M	-0.0224	0.0041	0.0503	0.0121	0.0341	0.0087	-5.7260
D112	F	-0.0063	-0.0127	0.0489	0.0339	0.0031	-0.0035	-3.4706
	M	-0.0381	0.0073	0.0033	0.0351	-0.0017	0.0376	-2.6056
D138	F	-0.0086	0.0059	0.0122	0.0143	-0.0139	0.0171	0.3855
	M	-0.0249	0.0100	-0.0258	0.0141	-0.0007	0.0536	-0.5843

The Means, Standard Deviations and Correlations of Admitted Students for Sex Subgroups

Group		n	$\bar{x}$	sx	$\bar{y}$	sy	rxy
A33	F	41	3.1424	0.3049	3.1366	0.5605	0.5438
	M	93	2.7954	0.4889	2.7826	0.8644	0.5652
A76	F	46	2.5708	0.4055	2.5682	0.7600	0.5336
	M	44	2.2049	0.3606	2.2022	0.7452	0.4837
B115	F	42	2.9680	0.2191	2.9633	0.4327	0.5065
	M	17	2.7187	0.2115	2.7128	0.5435	0.3888
B199	F	29	2.5675	0.3711	2.5682	0.5385	0.6895
	M	16	2.2800	0.6530	2.2801	0.7924	0.8242
C1	F	78	2.7300	0.3801	2.7393	0.6953	0.5475
	M	101	2.5849	0.5024	2.5883	0.8273	0.6078
C38	F	129	2.6343	0.4342	2.6292	0.6753	0.6421
	M	103	2.4904	0.3370	2.4791	0.6833	0.4924
C62	F	79	2.7283	0.3405	2.7320	0.6350	0.5366
	M	93	2.5396	0.4012	2.5397	0.7365	0.5450
C103	F	94	2.5845	0.3520	2.5814	0.7387	0.4765
	M	87	2.3725	0.3677	2.3581	0.6588	0.5569
D18	F	63	2.9669	0.2161	2.9669	0.4695	0.4604
	M	58	2.8352	0.3577	2.8249	0.5489	0.6512
D70	F	57	2.9566	0.1016	2.9589	0.4087	0.2493
	M	53	2.8043	0.3790	2.8138	0.5381	0.7055
D112	F	65	2.7847	0.2936	2.7899	0.4783	0.6149
	M	68	2.6369	0.3034	2.6448	0.5759	0.5278
D138	F	69	2.8777	0.1885	2.8840	0.4296	0.4399
	M	46	2.6291	0.2483	2.6316	0.4485	0.5539

The Statistics for A33, A76, B115, B199, C1, C38, C62, C103, D18 D70, D112 and D138

Model	C_1	C_2	PS_1	PS_2	GS	RSE	SRA	P_1/P_2
A33								
INST	2.6305	1.9511	0.9111	0.9118	2.9000	0.1613	0.8705	1.9491
REGR	2.0702	2.0702	0.9999	0.8726	2.9139	0.1434	0.8803	1.0000
EMPL	2.0429	2.0691	0.9989	0.8730	2.9139	0.1434	0.8803	1.0000
THOR	2.0702	2.0702	0.9999	0.8726	2.9139	0.1434	0.8803	1.0000
EQOP	2.0702	2.0702	0.9999	0.8726	2.9139	0.1434	0.8803	1.0000
DARL	2.0702	2.0702	0.9999	0.8726	2.9139	0.1434	0.8803	1.0000
A76								
INST	1.6298	1.7721	0.9019	0.8627	2.4168	0.2972	0.6932	0.7761
REGR	1.7236	1.7234	0.8742	0.8905	2.4179	0.2962	0.6938	0.9932
EMPL	1.7261	1.7218	0.8734	0.8914	2.4179	0.2962	0.6938	1.0000
THOR	1.3637	1.8508	0.9563	0.8085	2.4074	0.3060	0.6882	0.3924
EQOP	1.7553	1.7025	0.8636	0.9011	2.4177	0.2964	0.6937	1.0849
DARL	1.7236	1.7234	0.8742	0.8905	2.4179	0.2962	0.6938	0.9932
B115								
INST	2.6455	2.5530	0.8400	0.7727	2.9173	0.1773	0.9736	1.1094
REGR	2.6035	2.6045	0.8764	0.6900	2.9188	0.1733	0.9760	1.0697
EMPL	2.5068	2.6796	0.9372	0.5519	2.9145	0.1706	0.9777	1.0000
THOR	2.6468	2.5509	0.8388	0.7754	2.9171	0.1775	0.9735	1.1108
EQOP	2.6631	2.5248	0.8229	0.8115	2.9156	0.1797	0.9721	1.1308
DARL	2.6035	2.6045	0.8764	0.6900	2.9188	0.1733	0.9760	1.0697

Table M - Continued

Model	C_1	C_2	PS_1	PS_2	GS	RSE	SRA	P_1/P_2
B199								
INST	2.0773	1.2091	0.8286	0.6957	2.6081	0.1798	0.8156	14.8557
REGR	1.8139	1.8141	0.9402	0.5258	2.6469	0.1499	0.8349	0.9337
EMPL	1.8247	1.7986	0.9372	0.5304	2.6468	0.1497	0.8351	1.0000
THOR	1.8617	1.7404	0.9259	0.5476	2.6461	0.1494	0.8353	1.2853
EQOP	1.9389	1.5919	0.8976	0.5907	2.6409	0.1524	0.8333	2.4135
DARL	1.8139	1.8141	0.9402	0.5258	2.6469	0.1499	0.8349	0.9337
C1								
INST	2.1165	1.7499	0.8667	0.8860	2.6652	0.2096	0.8064	1.6562
REGR	1.9041	1.9093	0.9426	0.8260	2.6726	0.2018	0.8108	0.9847
EMPL	1.9112	1.9060	0.9408	0.8274	2.6725	0.2018	0.8108	1.0000
THOR	1.9006	1.9110	0.9435	0.8253	2.6725	0.2018	0.8108	0.9773
EQOP	2.0416	1.8231	0.8989	0.8606	2.6700	0.2044	0.8094	1.3507
DARL	1.9041	1.9093	0.9426	0.8260	2.6726	0.2018	0.8108	0.9847
C38								
INST	1.3628	1.7590	0.9699	0.9717	2.5514	0.2305	0.7666	0.3210
REGR	1.5050	1.5112	0.9514	0.9950	2.5538	0.2274	0.7682	0.8350
EMPL	1.5152	1.4486	0.9498	0.9970	2.5537	0.2273	0.7683	1.0000
THOR	1.2798	1.8083	0.9777	0.9619	2.5494	0.2329	0.7654	0.2210
EQOP	1.5292	1.0884	0.9475	0.9999	2.5535	0.2274	0.7682	2.9553
DARL	1.5050	1.5112	0.9514	0.9950	2.5538	0.2274	0.7682	0.8350

Table M - Continued

Model	C_1	C_2	PS_1	PS_2	GS	RSE	SRE	P_1/P_2
C62								
INST	2.2404	1.8256	0.8495	0.9208	2.6437	0.2050	0.8249	1.7378
REGR	2.0236	2.0269	0.9435	0.8343	2.6546	0.1915	0.8325	1.0051
EMPL	2.0211	2.0282	0.9442	0.8336	2.6546	0.1915	0.8325	1.0000
THOR	2.0028	2.0367	0.9492	0.8289	2.6545	0.1915	0.8325	0.9635
EQOP	2.1442	1.9452	0.8994	0.8748	2.6521	0.1947	0.8307	1.3099
DARL	2.0236	2.0269	0.9435	0.8343	2.6546	0.1915	0.8325	1.0051
C103								
INST	1.9997	1.7188	0.8785	0.8878	2.4881	0.2558	0.7485	1.6855
REGR	1.8412	1.8514	0.9375	0.8234	2.4932	0.2490	0.7524	1.0467
EMPL	1.8227	1.8613	0.9426	0.8178	2.4931	0.2489	0.7524	1.0000
THOR	1.8310	1.8570	0.9403	0.8202	2.4932	0.2489	0.7524	1.0204
EQOP	1.9422	1.7795	0.9033	0.8607	2.4915	0.2515	0.7509	1.3820
DARL	1.8412	1.8514	0.9375	0.8234	2.4932	0.2490	0.7524	1.0467
D18								
INST	2.7520	2.3775	0.6632	0.6444	2.9382	0.2618	0.9695	1.1797
REGR	2.5820	2.5920	0.8110	0.4884	2.9592	0.2499	0.9787	0.9999
EMPL	2.5821	2.5918	0.8109	0.4885	2.9593	0.2499	0.9787	1.0000
THOR	2.6773	2.4467	0.7096	0.5954	2.9490	0.2556	0.9742	1.1116
EQOP	2.6837	2.4355	0.7019	0.6035	2.9475	0.2565	0.9736	1.1218
DARL	2.5820	2.5920	0.8110	0.4884	2.9592	0.2499	0.9787	0.9999

Table M - Continued

Model	C_1	C_2	PS_1	PS_2	GS	RSE	SRA	P_1/P_2
D70								
INST	2.8743	2.2813	0.6951	0.5824	2.9088	0.2485	0.9696	1.2707
REGR	2.5480	2.7199	0.9999	0.3078	2.9791	0.2204	0.9920	0.9437
EMPL	2.7206	2.6816	0.9784	0.3272	2.9793	0.2201	0.9922	1.0000
THOR	2.8599	2.3488	0.7430	0.5393	2.9284	0.2391	0.9771	1.1932
EQOP	2.8638	2.3311	0.7304	0.5507	2.9235	0.2413	0.9753	1.2115
DARL	2.5480	2.7199	0.9999	0.3078	2.9791	0.2204	0.9920	0.9437
D112								
INST	2.3918	2.2623	0.7738	0.7158	2.7407	0.2075	0.9243	1.2032
REGR	2.3189	2.3160	0.8293	0.6667	2.7432	0.2021	0.9280	1.0786
EMPL	2.2684	2.3464	0.8621	0.6377	2.7424	0.2012	0.9286	1.0000
THOR	2.3625	2.2856	0.7973	0.6951	2.7422	0.2047	0.9262	1.1508
EQOP	2.4058	2.2504	0.7621	0.7262	2.7396	0.2091	0.9232	1.2296
DARL	2.3189	2.3160	0.8293	0.6667	2.7432	0.2021	0.9280	1.0786
D138								
INST	2.6366	2.3250	0.8214	0.6667	2.8129	0.2035	0.9667	1.1759
REGR	2.4814	2.4687	0.9504	0.5096	2.8297	0.1926	0.9740	1.0000
EMPL	2.4814	2.4687	0.9504	0.5096	2.8297	0.1926	0.9740	1.0000
THOR	2.6483	2.3073	0.8067	0.6848	2.8093	0.2059	0.9651	1.1989
EQOP	2.6632	2.2830	0.7870	0.7086	2.8040	0.2096	0.9626	1.2327
DARL	2.4814	2.4687	0.9504	0.5096	2.8297	0.1926	0.9740	1.0000

Table N
Regression Coefficients for Race Subgroups

Group		OATM	OHTR	CONST
A	Non-White	0.0042	0.0049	1.4944
	White	0.0567	0.0140	-0.0139
B	Non-White	0.0114	0.0170	1.2348
	White	0.0057	0.0474	0.3674
C	Non-White	0.0076	0.0255	1.0031
	White	0.0120	0.0265	1.1266
D	Non-White	0.0030	0.0159	1.5522
	White	0.0265	0.0125	1.5250

Note. The applicants who had graduated from the high school with less than 10% white students were considered as non-white applicants. The applicants who had graduated from the high school with more than 90% white students were considered as white applicants.

Table O
The Means, Standard Deviations and Correlations of Admitted Students for Race Subgroups

	Group	n	$\bar{x}$	s_x	$\bar{y}$	s_y	r_{xy}
A	Non-White	199	1.7505	0.0779	1.7520	0.8920	0.0878
	White	40	2.5407	0.5749	2.5389	0.9650	0.5953
B	Non-White	184	2.0749	0.2370	2.0748	0.9142	0.2592
	White	32	2.8177	0.4243	2.8209	0.6427	0.6610
C	Non-White	151	2.1915	0.3319	2.1894	0.8166	0.4057
	White	311	2.9195	0.2155	2.9180	0.6925	0.3102
D	Non-White	46	2.1684	0.1699	2.1676	0.6984	0.2428
	White	45	3.1611	0.1734	3.1599	0.5205	0.3329

Table P
The Statistics for the Four Senior Colleges

Model	C_1	C_2	PS_1	PS_2	GS	RSE	SRA	P_1/P_2
College A								
INST	1.6681	1.9145	0.8257	0.8333	1.9183	0.5050	0.4592	0.7791
REGR	1.6729	1.6752	0.8095	0.9148	1.9204	0.5037	0.4599	1.0586
EMPL	1.6722	1.7161	0.8117	0.9035	1.9204	0.5037	0.4599	1.0000
THOR	******	******	******	******	******	******	*******	******
EQOP	1.6628	2.0889	0.8424	0.7497	1.9126	0.5091	0.4566	0.6471
DARL	1.6600	2.1625	0.8506	0.7085	1.9088	0.5120	0.4549	0.6036
College B								
INST	1.9076	2.4776	0.7104	0.6809	2.2772	0.3913	0.6247	0.5453
REGR	1.9384	1.9362	0.6644	0.9346	2.2955	0.3736	0.6372	1.0513
EMPL	1.9374	1.9636	0.6657	0.9276	2.2958	0.3736	0.6373	1.0000
THOR	******	******	******	******	******	******	*******	******
EQOP	1.9086	2.4654	0.7089	0.6892	2.2783	0.3903	0.6254	0.5500
DARL	1.1931	2.4104	0.7024	0.7254	2.2825	0.3863	0.6282	0.5730

Note. The symbol ****** indicates that there is no solution which satisfies the definition of Thorndike's Model.

Table P - Continued

Model	C_1	C_2	PS_1	PS_2	GS	RSE	SRA	P_1/P_2
College C								
INST	1.7981	2.5613	0.8483	0.9201	2.7042	0.2209	0.8154	0.4893
REGR	1.9730	1.9698	0.6968	0.9995	2.7369	0.1883	0.8335	1.0042
EMPL	1.9729	1.9731	0.6968	0.9995	2.7370	0.1883	0.8335	1.0000
THOR	1.9729	2.0129	0.6968	0.9999	2.7369	0.1883	0.8335	0.9528
EQOP	1.7780	2.5710	0.8617	0.9131	2.7001	0.2247	0.8132	0.4739
DARL	1.9269	2.4252	0.7423	0.9759	0.7301	0.1953	0.8296	0.6211
College D								
INST	1.9962	2.9927	0.8364	0.5696	2.7093	0.4011	0.8047	0.5085
REGR	2.2993	2.2988	0.2211	0.9980	2.9527	0.2222	0.9365	0.9201
EMPL	2.3005	2.2174	0.2192	0.9993	2.9526	0.2221	0.9366	1.0000
THOR	2.1685	2.8141	0.4945	0.8077	2.8789	0.2845	0.8906	0.6284
EQOP	2.0989	2.9039	0.6509	0.6987	2.8097	0.3336	0.8544	0.5766
DARL	2.2212	2.7212	0.3751	0.8908	2.9207	0.2523	0.9144	0.6755

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