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Mean and variance in the futures market

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City University of New York, 1992

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MEAN AND VARIANCE IN THE FUTURES MARKET

BY

JENN Y. SHIUE

A dissertation submitted to the Graduate faculty in
Economics in partial fulfillment of the requirements
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1992

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ABSTRACT

MEAN AND VARIANCE IN THE FUTURES MARKET

BY

JENN Y. SHIUE

ADVISER: PROFESSOR HARRY H. MARKOWITZ

This paper uses the marking to the market characteristic of futures contracts to explain both the normal backwardation theory and the contango theory under the assumption that the Capital Asset Pricing Model works day by day through the lives of futures contracts. We analyze ten consecutive 3-month T-bill futures contracts, starting March 14, 1985 through December 16, 1987. We provide not only evidence of the normal backwardation theory and evidence of the contango theory, but also evidence of both theories in a futures contract. From the efficient portfolio theory of hedging in futures market, the naive speculator is reimbursed in the market only for the amount of systematic risk or basis risk he takes. The large speculator would sacrifice some diversification risk and earn the highest expected abnormal return to compensate for an imperfectly diversified efficient portfolio.

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I. Introduction

Organized futures markets in financial securities were first established in the U.S. on October 20,1975 when the Chicago Board of Trade opened a futures market in Government National Mortgage Association (GNMA) 8% Pass-Through Certificates. This was followed in January 1976 by a 90 day Treasury Bill futures market on the International Monetary Market of the Chicago Mercantile Exchange. In terms of trading volume both were initially commercial successes and this led to the establishment, in 1977, of futures markets in Long Term U.S. Treasury Bonds and 91-day Commercial Paper, in 1978, of a market in one-year Treasury notes and New GNMA markets, in 1981 of futures market in Eurodollar Time Deposits, and in 1982 of futures markets in the S&P 500 Index and the Value Line Index.

The classic economic rational for futures markets is that they facilitate hedging - that they allow those who deal in a commodity to transfer the risk of price changes in that commodity to speculators more willing to bear such risks. Obviously it is possible to hedge by entering into forward contracts, but an organized futures market facilitates such transactions by providing a standardized contract and by substituting the trustworthiness of the exchange for that of the individual trader. However, there continues to be a theoretical and empirical debate over normal backwardation, the Keynes-Hicks argument from which the liquidity premium theory of the term structure was developed. Hick's argument

[See reference, pp 136-139] was that most hedgers of agricultural commodities maintain a long position in the cash market and a short position in the futures market, and that speculators will not step in and absorb risk until the futures price is sufficiently low so that the expected favorable price change will compensate for the risk. This theory, in effect, argues that speculators sell "insurance" to hedgers and that the market seems to be inefficient since the futures price is not an unbiased estimate of the subsequent spot price. Telser(1958,1960) and Cootner(1960) have both tested their interpretation of the theory of normal backwardation without using any standard pricing system, and have obtained conflicting results.

With the creation and maturation of futures markets, research interest naturally focuses on price behavior, market performance, and the market's fulfillment of its social role. In the interest rate futures market, one of the most important issues has been the question of market efficiency, and a number of studies on the efficiency of the interest rate futures market have appeared [See Branch(1978), Capozza and Cornell(1979), Lang and Rasche(1978), Poole(1978), Puglisi(1978), Rendleman and Carabini(1978), and Vignola and Dale(1979,1980)]. However, all of these studies have explored the efficiency of the Treasury-Bill futures contract by the use of the arbitrage-opportunity-method. Several studies test for market efficiency in various metal markets, for example aluminum, copper, lead, nickel, tin and zinc traded on the London

Metal Exchange, using both weak and semi-strong forms tests. Following Fama (1970), they employ either weak form tests of efficiency [See Cargill and Rausser (1975), Gross (1981), Kofi (1973), Solt and Swanson (1981), Tomek and Gray (1970)], or semi-strong form tests of efficiency [See Garcia et al. (1988), Gross (1983, 1988), Guta and Mayer (1981), Leuthold and Hartman (1979)]. The weak form test relies on the historical sequence of prices and involves regressing cash price at contract maturity on a previous futures price. If the intercept coefficient is zero and the slope is one, the market is regarded as efficient. However this test is criticized on the ground that the coefficient estimates are based on expost knowledge of the data that is not available to agents in the market. The semi-strong form efficiency test centers on whether futures prices fully reflect all publicly available information at the time of contracting. In this case, an economic model is employed to compare the forecast error of the model with that of futures price. However, results from this test are contradictory.

Recent developments in the theory of cointegration provide a new method for testing market efficiency [See Hakkio and Rush (1989), Shen and Wang (1990), Gravyer (1986), and Chowdhury (1991)]. The theory of cointegration demonstrates that, in an efficient market, the spot price and the futures price should be cointegrated and that cointegration between prices in two different markets implies inefficiency. However, the cointegration of spot price and futures price is a joint test of market efficiency and no risk premium. That is a test of theory of normal backwardation of futures contract. The cointegration between prices in two different markets is a test of the existence of a dominated market, which can be duplicated by other futures markets. Therefore, the rejection of cointegration between spot price and futures price is the rejection of no risk premium or the rejection of efficient market assumptions, and the rejection of cointegration between prices in different market is the rejection of existence of dominated market in the real world. However the outcomes of cointegration between two prices in different markets are mixed in the Chowdhury paper.

*1

Thus, the best way to test the efficiency of futures markets is to use a well-known ex ante model. The set of data for the model has to meet the empirical evidence of Goldenberg (1989) and Chowdhury (1991) that the futures price series is nonstationary, but the first difference of futures price series is stationary. After we modify the ex ante model to an ex post model we use it to calculate the basis risks of futures contracts from the ex post data set. If the market is efficient in semi-strong form, then the basis risks should be arbitrarily scattered. On the other hand, if a futures market is efficient then it is possible for us to use the model to interpret price behavior and market performance. Dusak (1973), and Carter, Rausser and Schmitz (1983) [hereafter represented by C-R-S] have examined the existence of a risk premium within the context of the Capital Asset Pricing Model (CAPM).

Breeden (1980) considers risk in the context of the intertemporal asset pricing model. This model derives the equilibrium expected excess returns on assets for more general economies than those considered by CAPM, because the model extends the financial market to contain the commodity market for the market portfolio.

Dusak argues that the Keynesian notion of a risk premium takes on a new interpretation. Namely, the risk premium required on a futures contract should depend on the extent to which the variations in prices are systematically related to variations in the return on the market portfolio. If the CAPM applies and if the risk of a futures contract is independent of the risk of changes of the market portfolio (i.e. no systematic risk), then investors will not have to be paid for that risk. The Keynesian insurance interpretation, on the other hand, identifies the risk of a futures asset solely with its own price variability (i.e. total variance of the price of futures contract). Dusak uses the CAPM to generalize the Keynesian formulation and tests the risk in the futures market for wheat, corn, and soybeans; she concludes that wheat, corn, and soybean futures contracts are not risky assets. C-R-S modifies the Dusak model to include the difference in taste or attitude toward risk among hedgers and speculators, and extends the market portfolio to include the nation's stock of agricultural and nonagricultural commodities. They show that, contrary to Dusak's results, holding futures contract is risky and that the generalized Keynesian theory of normal backwardation has some merit. Breeden concludes that some futures contracts have significant systematic risks that should result in risk premia in markets.

The major problems with Dusak's investigation are that it is based on a misspecified model and is restricted to a small set of commodities where systematic risk is most likely to be absent. The primary problems with the C-R-S model are that the inclusion of taste or attitude among investors is not a necessary condition since the utility function is canceled out when the capital market is in equilibrium, and that the returns of futures contracts, which they use for regression, are not the real returns of the contracts, because they do not calculate the certainty-equivalent investment of each contract. Finally, both of them use time series data to run a regression on a single period CAPM, and all contracts in a given commodity are pooled and a single regression is estimated. So both of them have model specification problems and the outcomes of both models are either under estimated or over estimated. The estimated risk premium, or estimated beta, of the sample period in both Dusak and the C-R-S papers is the weighted averages of the betas of futures contracts within their sample period respectively. The weights are proportional to the variances of each of the estimated betas.

The purpose of this paper is to use a multiperiod CAPM in the market factor model to interpret the Keynesian insurance concept, or the Markowitz mean-variance concept, of futures assets. Our paper focuses not only on the theoretical development of a multiperiod CAPM in the market factor model, but also on empirical evidence of insurance premia, or discounts, by using Treasury bills futures contracts' cash flows, or settlement price changes, under the assumption that the Capital Asset Pricing Model works through the lives of futures contracts. Our paper tries not only to explain the profit difference between large speculators and naive speculators, but also to show empirical evidence of normal backwardation theory and contango theory.

The remainder of the paper is organized as follows. After a description of the theoretical background in the next section, an ex-post model is established. The efficient portfolio theory of hedging in futures market is analyzed in section III. Section IV describes the methodology of our ex post mean-variance futures model on ten consecutive 3-month T-bills futures contracts. Fluctuation of these ten 3-month T-bills futures contracts is analyzed in Section V. The final section is the conclusion and some comments.

II. Theoretical Background.

II.1 Introduction.

The mean-variance portfolio model was originally suggested by Markowitz (1959) as a normative theory. The model only has content if there is some relationship between future returns and estimates of risk that can be made on the basis of current information. This was theoretically analyzed by Sharpe (1964) and Lintner (1965), and first used by Jensen (1968) to analyze mutual funds, and further extended to the Arbitrage Pricing Theory (APT) by Ross (1976). The normative theory shows an investor can minimize variance for different levels of expected return, subject to various constraints. The input for analysis is estimates of the means, variances and covariances of various securities, but does not assume that all investors hold these same beliefs. The Sharpe-Lintner CAPM requires the assumption that every investor acts according to mean and variance, and that all investors can borrow and/or lend at the same rate. These two strong assumptions the normative theory permits but does not require.

The market model of this paper is a special model of the normative theory, and also uses assumptions, required in both normative theory and CAPM. The market factor in the normative theory may not be an efficient portfolio, but the market portfolio in CAPM must be efficient. (See Markowitz "The Two Beta Trap").

Therefore, the market portfolio in our model may be an efficient portfolio. The capital market is assumed to be perfect in the sense that investors are price takers and there are no transactions costs. Distributions of one-period returns on all assets and portfolios are assumed to be normal. Investors are assumed to be risk averse and to behave as if they choose among portfolios on the basis of maximum expected utility. A perfect capital market, investor risk aversion, and a two-parameter (mean-variance) return distributions imply the efficient frontier portfolio: The optimal portfolio for any investor must be efficient in the sense that no other asset or portfolio with the same or higher expected return has lower dispersion of return. These assumptions were widely debated in the past two decades, and gradually accepted in academic textbooks. We are not interested in the discussion of assumptions of the market model, but in the application of the model itself.

The model simply states a linear relationship between the returns on any asset and a general market factor. That is, the returns on the jth asset as

$$(2.1) \qquad \tilde{R}_j = E[\tilde{R}_j] + b_j \times \tilde{\pi} + \tilde{e}_j \qquad j = 1,2...N$$

where the market factor $\tilde{\pi}$ is defined such that $E[\tilde{\pi}] = 0$, b_j is a constant, $\tilde{\pi}$ and $\tilde{e}_j$ are all normally distributed random variables, tildes ($\tilde{}$) are used to denote random variables, E[ ] represents the expected value, and N is the total number of assets in the market. The following assumptions are made regarding the residual terms $\tilde{e}_j$:

(2.1a) $E[\tilde{e}_j] = 0 \quad j=1,2......N$

(2.1b) $E[\tilde{e}_j \times \tilde{\pi}] = 0 \quad j=1,2.......N$

(2.1c) $$E[\tilde{e}_j \times \tilde{e}_i] = 0 \quad i \neq j \quad j=1,2.....N$$
$$= \sigma^2(\tilde{e}_j) \quad i = j$$

where σ^2 is the notation of variance.

There are at least three reasons for us to use the market factor to analyze futures contracts. First, if the futures price is the unbiased estimation of the future spot price, then the expected return, $E[\bar{R}_j]$, is the ratio of price difference between futures price and current spot price to the current spot price. The return $\bar{R}_j$ is determined by future spot price. Therefore, the basis risk of futures contract is determined in (2.1) by the market factor, constant term b_j and the residual random variable. The present value of volatility of intra-spreads through the life of futures contract is insurance premium or discount, which is the cash amount needed to guarantee the holder of futures contract to buy underlying commodity or financial asset on the maturity date for the futures price prevailing at the time the contract is initiated.

Second, marking to the market characteristics of futures contracts can be investigated through the data, which are taken at daily intervals and over which general price level and other macroeconomic changes are expected to be small by comparison to those market events affecting the prices of futures contracts. That means the market is efficient in the sense of semi-strong form efficiency: asset prices reflect all available public information, and the unexpected information is negligible in the market daily operation. The efficiency of market is close to the idea of Lucas's island world (1972). The only source to affect asset prices is the unexpected information, but the change of asset prices reflects all information available to the public, including those previously unexpected information. So the policy ineffectiveness of markets authorities may be the reason for the efficiency of the market factor model, because no policy can be adapted to the unexpected information as Lucas points out in his paper. Third, Bodie and Rosansky (1980) empirically test that common stock returns and commodity futures returns are negatively correlated, and this provides evidence for the inclusion of commodity assets in any efficient portfolio. We can include the Dow Jones Commodity Futures Index to the CAPM's market portfolio to organize the new market portfolio. Thus, the factor model may capture the impact of the change of the economy's consumption on the returns of assets through the change of the market factor. Finally, the major difference between the market factor model and the CAPM is the expected return of an asset. If the expected return of an asset in the market factor model is calculated according to the expected return formula in the CAPM world, then the market factor model is exactly the CAPM. That means the market factor model is an extension of the CAPM, and is more powerful for measuring the risk premium of asset in the real world. On the other hand, the market factor is exactly the APT model when all the factors of the APT model are collapsed into one factor.

The Capital Asset Pricing Model is indeed assumed in the analysis of this paper in the following few sections' derivations. Before we do further theoretical derivations, we would like to review some previous empirical tests of the Capital Asset Pricing Model. The earliest empirical tests of the CAPM were done by

Douglas (1968), whose results seem to refute that the systematic risk of a security, beta (ß), is a complete measure of the risk of the security in the efficient market portfolio. In annual and quarterly return data, there seem to be measures of risk, in addition to the beta, that contribute systematically to observed average returns. These results are inconsistent with the hypothesis that investors attempt to hold efficient portfolios. Miller and Scholes (1972) take both Douglas's statistical techniques and his use of annual and quarterly data. Using different methods and simulations, they show that Douglas's results could be expected even if the beta is a complete measure of the risk of a security in the market portfolio. Other tests of Friend and Blume (1970) and those of Black, Jensen and Scholes (1972) indicate that, at least in the period since 1940 to 1970, on average the risk free rate is underestimated. However, Fama and MacBeth (1973) test with monthly data and support the relationship between average return and risk for New York Stock Exchange common stocks, and also support that the market prices of securities fully reflect available information. We simply list three important empirical test papers, issued in the late 60's and early 70's, for reference, each of the papers represents respectively the rejection, inconclusion, and acceptance of the hypothesis that CAPM does work for the stock market as a measure of the risk of a security. We merely use part of the results of Markowitz, Fama, Jensen, Black, Sharp-Lintner,...etc. to investigate futures contracts and portfolios consisting of futures market holdings and spot market holdings.

II.2 Systematic risk in a single-period homogeneous market model.

Assume again that the capital market is perfect. In addition, suppose that from the information available without cost, all investors derive the same and correct assessment of the distribution of the future value of any asset or portfolio - homogeneous expectations. Finally, assume that short selling of any asset or all other assets is allowed. Then Black (1972) has shown that in a market equilibrium, the market portfolio is always efficient and defined by $w_j = \{ v_j / (\sum_{i=1}^{N} v_j) \}$, which is the fraction of the jth asset in the market portfolio. The returns on the market portfolio $\tilde{R}_m$ are given by

$$(2.2) \quad \tilde{R}_m = \sum_{j=1}^{N} w_j \times \tilde{R}_j$$

By direct substitution from (2.1) into (2.2), we get

$$(2.2a) \quad \tilde{R}_m = \sum_{j=1}^{N} w_j \times E[\tilde{R}_j] + \sum_{j=1}^{N} w_j \times b_j \times \tilde{\pi} + \sum_{j=1}^{N} w_j \times \tilde{e}_j$$

Without loss of generality, we can assume that $\sum_{j=1}^{N} w_j \times b_j = 1$[*2] and reduce equation (2.2a) to

$$(2.3) \quad \tilde{R}_m = E[\tilde{R}_m] + \tilde{\pi} + \sum_{j=1}^{N} w_j \times \tilde{e}_j$$

The Sharpe-Lintner Capital Asset Pricing Model, (CAPM) , indicates that the expected return on any asset is given by

$$(2.4) \quad E[\tilde{R}_j] = R_f + (E[\tilde{R}_m] - R_f) \times (\mathrm{Cov}(\tilde{R}_j , \tilde{R}_m)/ \sigma^2 (\tilde{R}_m))$$

Where R_f is the return on riskless asset. The measure of risk of any asset j in the CAPM relative to the risk of the market portfolio is $\mathrm{Cov}(\tilde{R}_j , \tilde{R}_m)/ \sigma^2 (\tilde{R}_m)$. By direct substitution from (2.1) and (2.3) into the definition of the covariance of asset j with respect to the market portfolio, we get

$$\mathrm{Cov}(\tilde{R}_j , \tilde{R}_m) = b_j \times \sigma^2 (\tilde{\pi}) + w_j \times \sigma^2 (\tilde{e}_j) \quad j=1,2...N$$

$$(2.4a) \quad \sigma^2 (\tilde{R}_m) = \sigma^2 (\tilde{\pi}) + \sum_{j=1}^{N} w_j^2 \times \sigma^2 (\tilde{e}_j)$$

Equation (2.4a) expresses the risk of an asset in the market factor model but measured by the CAPM system, and the relationship between the variance of the market portfolio and the variance of the market factor. It is obvious that the market factor is more efficient than the market portfolio of the CAPM, if both of them have the same return. Therefore, restating the results of the Capital Asset Pricing Model given in (2.4) in terms of the ex-post parameters of the market factor model, we have

$$(2.5)\quad E[\tilde{R}_j \mid E[\tilde{R}_m],\, b_j,\, \sigma^2(\tilde{\pi})] = R_f + (E[\tilde{R}_m] - R_f) \times \{ b_j \times \sigma^2(\tilde{\pi}) + w_j \times \sigma^2(\tilde{e}_j)\}/\, \sigma^2(\tilde{R}_m)$$

We define

$$(2.6)\quad \beta^{*} = \{ b_j \times \sigma^2(\tilde{\pi}) + w_j \times \sigma^2(\tilde{e}_j)\}/\, \sigma^2(\tilde{R}_m)$$

and

$$d_j = w_j \times \sigma^2(\tilde{e}_j)/\, \sigma^2(\tilde{R}_m)$$

where β^{*} is the measure of systematic risk of the market factor model. Substituting for $E[\tilde{R}_j]$ from (2.6) and (2.5) into (2.1), we have

$$\text{(2.7)} \quad \tilde{R}_j = E[\tilde{R}_j] + b_j \times \tilde{\pi} + \tilde{e}_j$$

$$= R_f \times (1 - \beta^*_j) + \beta^*_j \times E[\tilde{R}_m] + b_j \times \tilde{\pi} + \tilde{e}_j$$

Adding and subtracting $d_j \times \tilde{\pi}$ and $\beta^*_j \times \sum_{i=1}^{N} w_i \times \tilde{e}_i$ on the right hand side (RHS) of equation (2.7) gives

$$\tilde{R}_j = R_f \times (1 - \beta^*_j) + \beta^*_j \times E[\tilde{R}_m] + b_j \times \tilde{\pi} + d_j \times \tilde{\pi} + \beta^*_j \times \sum_{i=1}^{N} w_i$$

$$\times \tilde{e}_i - d_j \times \tilde{\pi} - \beta^*_j \times \sum_{i=1}^{N} w_i \times \tilde{e}_i + \tilde{e}_j$$

Noting that $\beta^*_j \approx b_j + d_j^{*3}$ (i.e. $\sigma^2(\tilde{R}_m) = \sigma^2(\tilde{\pi})$), using the definition of $\tilde{R}_m$ from (2.3) and simplifying, we get the ex post relationship.

$$\text{(2.8)} \quad \tilde{R}_j = R_f \times (1 - \beta^*_j) + \tilde{R}_m \times \beta^*_j - d_j \times \tilde{\pi} - \beta^*_j \times \sum_{i=1}^{N} w_i \times \tilde{e}_i$$

$$+ \ \tilde{e}_j$$

By assumption (2.1a), $E[\tilde{e}_j] = 0$. We see that to a very close approximation the conditional expected return on the jth asset in the market factor model is given by

$$(2.9) \quad E[\, \tilde{R}_j \mid R_m,\ \beta_j^* \,] \approx R_f \times (1 - \beta_j^*) \ + \ R_m \times \beta_j^*$$

Equation (2.9) gives us an expression for the expected return on asset j conditional on the expost realization of the return on the market portfolio and the systematic risk. Comparing the expected return of an asset in the market factor, equation (2.9), with that in the CAPM world, equation (2.4), one will find the difference between these two models. One is an ex-ante measure system, and the other is ex-post.

II.3 The Trading Interval Selection of Futures Contracts in CAPM (extension of unobservable trading period to observable one day period).

In deriving the results of the Sharpe-Lintner Capital Asset Pricing Model given in (2.4), it was assumed that all investors have identical length of interval. This implies that all trading in the market takes place only at the beginning and end of this period. This restriction on one trading period prohibits us from making a study of the cash flows generated by marking to the market over all holding periods of a futures contract. Thus, we have to extend the linear relationships (2.5) to as many trading periods as possible. We are going to prove in this section that the existence of a discrete one day interval is consistent with a world in which trading takes place almost continuously, as long as the return of riskless asset and the expected return of the market portfolio are expressed in terms of compounding interval. In considering the marking to the market character of futures contracts, we have to take the unit of time period as a one day interval. One does not expect that both interest rate and expected return of the market portfolio in one trading interval will fluctuate very much to violate the assumption expressed by the compounding interval within a one day duration. However, you can not anticipate that these two returns will be constant for a long time period. It seems reasonable to use this idea of infinitesimal returns of the riskless asset and infinitesimal expected returns of the market portfolio to measure systematic risks of 3 month T-bills futures contracts.

Looking at the Sharpe-Lintner capital market in terms of beta risks and the familiar expected return-risk relationship of equation (2.4), however, allows us to refer to available empirical works. Gonedes (1973), for example, presents evidence that the betas of individual firms are not literally constant through time, but on the other hand, they also do not usually change dramatically over five to ten year periods. Jensen (1970) also presents evidence that betas in the two ten-year periods 1945-54 and 1955-64 for fifty-six mutual funds seem to be approximately stationary over time. Here, we restrict our attention to a stationary beta over the life of futures contract, whose expiration period is usually less than three months. Non-stationary beta will be discussed and solved approximately in the methodology section of this paper. Under the assumption that CAPM works period by period, we usually do not have small trading period data for the empirical test. Therefore, we have to extend these unobservable small periods to an observable one day period CAPM, and show the necessary condition for these two CAPMs to have the same systematic risk for the same asset. We know that if the Capital Asset Pricing Model is valid for each small trading period, then the following holds in each small period for the jth asset:

$$(2.10) \quad E[\tilde{R}_j] = R_f \times (1 - \beta^*_j) + E[\tilde{R}_m] \times \beta^*_j$$

where $E[\tilde{R}_j] = E[\; P_j / P_j]$ is the expected rate of return for the jth asset in a small period, P_j is the initial price of the jth asset, and P_j is the change of price of the jth asset during the period. $E[\tilde{R}_m]$ and R_f are rates of return for the market portfolio and the riskless asset, and β^*_j is the systematic risk of the jth asset.

By equations (2.3) and (2.8), we know[*4]

$$\tilde{R}_j \approx R_f \times (1 - \beta^*_j) + \tilde{R}_m \times \beta^*_j + \tilde{e}_j \tag{2.12}$$

Now consider the daily rate of return $\tilde{R}_{nj}$ on the jth asset, where we assume one day has n consecutive trading periods, and n is a positive integer.

$$\begin{aligned} \tilde{R}_{nj} &= \left(\prod_{t=1}^{n} (1 + \tilde{R}_{j,t}) \right) - 1 \\ &= \left(\prod_{t=1}^{n} (1 + R_{f,t} \times (1 - \beta^*_j) + \tilde{R}_{m,t} \times \beta^*_j + \tilde{e}_{j,t}) \right) - 1 \end{aligned} \tag{2.13}$$

where $\tilde{R}_{j,t}$ is the rate of return of the jth asset in the t-th trading period of a day. As long as (1) $R_{f,t}$ and $E[\tilde{R}_{m,t}]$ are constant through successive n-period intervals[*5], (2) the $\tilde{R}_{m,t}$ and $\tilde{e}_{j,t}$ are independently distributed respectively through one day duration, and (3) the $\tilde{e}_{j,t}$ and $\tilde{R}_{m,t}$ are independent, it can be proved directly that the expected n-period return, $E[\tilde{R}_{nj}]$ is given by

$$(2.14) \qquad 1 + E[\tilde{R}_{nj}] = (1 + E[\tilde{R}_j])^{1/\Gamma}$$

$$= (1 + (1 - \beta^*_j) \times R_f + \beta^*_j \times E[\tilde{R}_m])^{1/\Gamma}$$

where $\Gamma = 1/n$. The assumption (2) is simply an extension of one period efficiency of the market factor to an n-period efficiency, and (3) is an extension of equation (2.1b). Both assumptions will be subject to the empirical test of efficiency of the market factor, and will be discussed in the empirical section of this paper.

Theoretically, these two assumptions are not assumptions, but are the characteristics of the multiperiod version of the CAPM. Fama (1977) examines the types of variability admissible under a stationary CAPM which assumes that the portfolio opportunity set is nonstochastic, and claims that the non-existence of these two assumptions is a consequence of the pricing model, originally derived by Bogue and Roll (1974), which results has been discussed by Merton (1973) and Long (1974). Bogue and Roll allow uncertainty in the parameters of the market opportunity set, but Fama points out that in a world where securities are priced according to the CAPM, relationships between uncertainty in the returns realized at time t-1 and the characteristics of the portfolio opportunity set are ruled out. Were such relationships to exist, they would provide initiative for investors to use their portfolio opportunities at time t-2 to hedge against uncertainty in portfolio opportunities at time t-1. This result is a pricing process different from the CAPM.

Solving (2.14) for $E[\tilde{R}_j]$, we have

$$(2.15)\quad E[\tilde{R}_j] = (1 + E[\tilde{R}_{nj}])^{\Gamma} - 1$$

$$= (1 - \beta_j^*) \times R_f + \beta_j^* \times E[\tilde{R}_m]$$

But now, what we need to complete the proof of consistency of the discrete one day model and continuous trading model is to express the infinitesimal returns of riskless asset, R_f, and the expected returns of the market portfolio $E[\tilde{R}_m]$ in terms of observable n-periods, or daily, return. Under the assumptions of constant expectations and independence through the one day period, we have

$$(2.16) \qquad R_f = (1 + R_{nf})^{\Gamma} - 1$$

$$E[\tilde{R}_m] = (1 + E[\tilde{R}_{nm}])^{\Gamma} - 1$$

Hence, rewriting (2.15) in terms of the potentially observable quantities given in (2.16), we have

$$(2.17) \qquad \{1 + E[\tilde{R}_{nj}]\}^{\Gamma} - 1 = (1 - \beta_j^*) \times \{(1 + R_{nf})^{\Gamma} - 1\} + \beta_j^* \times \{(1 + E[\tilde{R}_{nm}])^{\Gamma} - 1\}$$

Now this relationship still holds if we divide both side by Γ:

$$(2.18) \qquad \{(1 + E[\tilde{R}_{nj}])^{\Gamma} - 1\}/\Gamma = (1 - \beta_j^*) \times \{(1 + R_{nf})^{\Gamma} - 1\}/\Gamma + \beta_j^* \times \{(1 + E[\tilde{R}_{nm}])^{\Gamma} - 1\}/\Gamma$$

Define

$$E[\tilde{R}_j^*] = \{(1 + E[\tilde{R}_{nj}])^{\Gamma} - 1\}/\Gamma$$

$$R_f^* = \{(1 + R_{nf})^{\Gamma} - 1\}/\,\Gamma$$

$$E[\tilde{R}_m^*] = \{(1 + E[\tilde{R}_{nm}])^{\Gamma} - 1\}/\,\Gamma$$

We get

$$(2.19) \qquad E[\tilde{R}_j^*] = (1 - \beta_j^*) \times R_f^* + \beta_j^* \times E[\tilde{R}_m^*]$$

If we let the length of each small trading period go to zero, which is equivalent to Γ going to zero, and by L'Hospital's rule[*6], we have

$$(2.20) \qquad E[\tilde{R}_j^*] \approx \mathrm{Log}_e\,\{1 + E[\tilde{R}_{nj}]\} = \lim_{\Gamma \to 0} \{(1 + E[\tilde{R}_{nj}])^{\Gamma} - 1\}/\,\Gamma$$

Under the assumption of a perfect market, we consider the limit of equation (2.19) as the length of each trading time period goes to zero. We get

$$(2.21) \quad \lim_{n \to \infty} \{(1 + E[\tilde{R}_{nj}])^{1/n} - 1\}/(1/n) = \mathrm{Log}_e\,(1 + E[\tilde{R}_{nj}]) =$$

$$= (1 - \beta_j^*) \times \mathrm{Log}_e\,(1 + R_{nf}) + \beta_j^* \times \mathrm{Log}_e\,(1 + E[\tilde{R}_{nm}])$$

where Log_e denotes the natural logarithm. Substituting (2.20) into (2.21), we have

$$(2.22) \quad E[\tilde{R}_j^*] = (1 - \beta_j^*) \times R_f^* + \beta_j^* \times E[\tilde{R}_m^*]$$

Equation (2.22) states that as long as the market trading takes place almost continuously, we may use a natural logarithm form of equation (2.22) as a very good approximate to the discrete time model equation (2.18). This means the market factor model will hold for returns calculated over a one day period as long as the returns on the risk free asset and expected returns on market portfolio are constant in each compounding trading interval of a day, and market trading takes place almost continuously. Therefore, a one day interval does not cause problems if the futures market trades almost continuously as the security market does.

II.4 The measurement of systematic risk of futures contracts with T days' observations.

Recalling the specifications of the market factor model given in (2.1) and (2.1a), it was shown that

$$\beta_j^* = \mathrm{Cov}(\tilde{R}_j, \tilde{R}_m) / \sigma^2(\tilde{R}_m) = (b_j \times \sigma^2(\tilde{\pi}) + w_j \times \sigma^2(\tilde{e}_j)) / \sigma^2(\tilde{R}_m)$$

Since $\sigma^2(\tilde{R}_m) \approx \sigma^2(\tilde{\pi})$ as the number of assets in the market factor is large, the above equation can be rewritten as

$$\beta_j^* \approx b_j + w_j \times \sigma^2(\tilde{e}_j) / \sigma^2(\tilde{R}_m)$$

We note that β_j^* is derived within a single period model within which the relevant covariances and variances refer to the properties of the set of probability distributions on a one period return.

Now we have to consider the estimated systematic risk $\hat{\beta}_j$ derived from a sample of T daily returns observed over a 3-month time period under the assumption of stationary probability distributions of the sample observations. Let n be the number of trades in a day, k be the elapse of time between two trades, i.e. $k=1/n$, and T be the number of days of futures contract lasting before the expiration date. Thus, the total number of trading periods is $(T * n) = T/k$, assumed to be a positive integer. Then the estimated beta, $\hat{\beta}_j$, is calculated as

$$\text{(2.23)} \quad \hat{\beta}_j = \mathrm{Cov}(\tilde{R}_{Tj}, \tilde{R}_{Tm}) / \sigma^2(\tilde{R}_{Tm})$$

Note that $1 + \tilde{R}_{Tj}$ is given by

$$(2.24a) \qquad 1 + \tilde{R}_{Tj} = \prod_{i=1}^{T/k} (1 + \tilde{R}_{kj,i})$$

for integer T/k, where $\tilde{R}_{kj,i}$ is the return of the jth asset at the i-th trading period under the assumption that the CAPM works in a trading time duration k. Further, we know from basic algebra that

$$(2.24b) \qquad \lim_{k \to 0} \prod_{i=1}^{T/k} (1 + \tilde{R}_{kj,i}) = \exp\left(\sum_{i=1}^{T/k} \tilde{R}_{kj,i} \right)$$

where exp denotes the exponential function. Using (2.23), (2.24a), (2.24b) and the assumptions of stationary and serial independence, we take the limit of equation (2.23). We have

$$(2.25) \qquad \lim_{k \to 0} \hat{\beta}_j = \mathrm{Cov}(\tilde{R}_{kj}, \tilde{R}_{km}) / \sigma^2(\tilde{R}_{km})^{*7} = \beta^*_j$$

From equation (2.25), we find that as long as the sample data are transformed according to (2.20) into a natural logarithm form, the estimated systematic risk of $\hat{\beta}_j$ will be independent of the length of time over which the returns are calculated, when the trading time duration k is small. If the result is empirically true, then we may calculate the systematic risks of futures contracts on the base of the data which are taken from marking to the market.

Since the ordinary least square regression estimation is used to estimate systematic risk in the stable market factor model, the estimated results are unbiased and consistent, but not efficient. Lack of efficiency comes from the fact that both the market factor and the market portfolio are not observable in the real world, so the residual terms between the real market portfolio and the proxy market portfolio will be correlated with the residual terms of the equation (2.1). Thus, in light of this evidence, we get the estimated beta in (2.1) by the least-squares method, to be

$$(2.26) \quad b_j = \sum_{i=1}^{T} (\tilde{R}^*_{j,i} - \bar{R}^*_j) \times (\tilde{\pi}^*_i - \bar{\pi}^*) / (\sum_{i=1}^{T} (\tilde{\pi}^*_i - \bar{\pi}^*)^2)$$

where $\tilde{R}^*$ is the transformed natural logarithm of $\tilde{R}$, T is the total number of observations and the barred variables represent mean values of sample data. Note that we use the transformed returns in (2.26). This means we use the continuous time but discrete observations' model.

Thus all we need to complete the measurement of systematic risk is a measure of the market factor $\tilde{\pi}$. By the argument given in

(2.6) and (2.25), we have

$$(2.27) \qquad b_j \approx \beta^*_j \qquad \text{and} \qquad \lim_{k \to 0} \hat{\beta}_j = \beta^*_j$$

The first approximate equation is similar to the results of King (1966) and Blume (1968), which implies that $\sigma^2(\bar{e}_j)$ is roughly the same order of magnitude as $\sigma^2(\bar{\pi}) \approx \sigma^2(\bar{R}_m)$, and

$$\beta^*_j = b_j + w_j \times \{ \sigma^2(\bar{e}_j) / \sigma^2(\bar{R}_m) \} \approx b_j$$

as the weight w_j is small or the number of asset in the market portfolio is large. Since equation (2.27) is true and derived from theoretical analysis under the assumptions of the market factor for arbitrary T periods, the market factor should be highly correlated with the market portfolio. Therefore, replacing

$\bar{\pi}_i$ by $\bar{R}_{m,i}$ and $\bar{\pi}$ by $\bar{R}_m$ in (2.26) we have approximately

$$(2.28) \quad b_j \approx \sum_{i=1}^{T} (\bar{R}^*_{j,i} - \bar{R}^*_j) \times (\bar{R}^*_{m,i} - \bar{R}^*_m) \, / \, \{ \sum_{i=1}^{T} (\bar{R}^*_{m,i} - \bar{R}^*_m)^2 \}$$

$$\approx \beta^*_j$$

Equation (2.28) is measurable in the real world, because it is an ex post natural logarithm transformed model.

II.5 Systematic risk measured by futures contracts' marking to the market activity.

An individual who takes a long position in a futures contract agrees to buy a designated good or asset on the maturity date for the futures price prevailing at the time the contract is initiated. Hence, the futures price must also be equal to the spot price on the maturity date, and no money changes hands initially. Subsequently, however, as the futures price changes, the party in whose favor the price change occurred must immediately be paid the full amount of the change by the losing party. As a result, the payment required on the maturity date to buy the underlying commodity or asset is simply the spot price at that time. The difference between that amount and the initial futures price has to be paid (or received) in installments throughout the life of the contract. The equilibrium futures price must change daily over time. It must do so in such a way that the remaining stream of future payments described above always has an expected value of zero.

Day-to-day movements in financial asset prices, as reported in the Wall Street Journal have seemed too large to be realistically attributed to any objective new information. Sometimes, large movements in stock prices or commodity prices, for example the days around October 19 1987, have occurred only to be reversed in a few days immediately following. These episodes have instilled in investors a growing uncertainty about the future and about the future value of financial asset prices and commodity futures prices. There is a clear perception that volatility and the risk of holding financial assets and/or futures contracts has increased. Fluctuations in assets prices and/or futures contracts prices are sometime attributed not to changes in economic fundamentals or to changes in expectations about them, but to large, professionally-managed, speculative trading programs. This belief is reinforced by the knowledge that a larger share of financial asset trading and/or futures contracts trading is done by a relative few professionally-managed institutions and that these institutions often pursue computer-guided trading strategies that have them all selling or buying at the same time. This kind of advance technical analysis and computer program trading seems to cause assets' price volatility to be greater than or less than that which can be justified by any standard asset pricing models or by objective new information. In order to deny or justify this paradoxical concept held by most professional traders in the market, we turn to study cash flows generated by futures contracts and by portfolios consisting of cash market holdings and futures market holdings.

The cash flow method was first used in the study of futures by Cox-Ingersoll-Ross (1981) [hereafter represented by C-I-R] in a complete states economy. C-I-R introduces a forward contract as a quasi-futures contract, which is exactly the same as a regular futures contract, except that at the end of each period the person in whose favor the price change occurred is paid not the full amount of the change, but instead the present value that the full amount would have if it were paid on the maturity date. This means the quasi-futures contract allows the individual to arrange today to buy a specific type of good on the maturity date for a designated forward price. Here, we discuss the futures contract not in a complete states economy, but in a period by period Sharpe-Lintner Capital Asset Pricing Model, so we can not discount by risk free rate for each period as C-I-R does. Tracing the payments process backward through the life of the contract tells us that, in a capital market where each day's prices are determined according to the S-L model, the current market value of a future cash flow is the cost of insurance against price changes of the underlying commodity or asset.

Before further deriving any formula for the cash flows in the future through the life of futures contract, we have to ask whether the net present value method under uncertainty can be used to solve cash flows in any future period of any futures contract. This means we need some empirical references to support that the advent of futures markets have increased speculative activity, which does not destabilize spot markets or cash markets. The empirical studies of financial futures that have been done by Corgel and Gay (1984), Charles and Workman (1981), Figlewski (1981), Forewiss (1978), Moriarty and Tosini (1985), Seider (1981), and Simpson and Ireland (1982, 1985), suggest that the introduction of futures trading has not increased the volatility of cash prices. Edwards (1987) uses the day-to-day and intra-day price volatility of the stock market and of short-term debt instruments over an 18 year period from 1973 -1987 and concludes that the introduction of futures on these assets does not result in an increase in price volatility. No evidence was found which links futures trading to an increase in general market volatility. While there is some evidence of futures induced short-run volatility, such as that which occurs on futures contract expiration days, this does not carry over to a longer period time. Theoretically, we know that the positive net present value of a project activity undertaken by a firm will increase the value of a firm, and a zero net present value of a project activity does not affect the value of a firm. Similarly, a zero net present value of cash flows of futures contract will not affect the underlying cash market volatility. A positive net present value of cash flows of a contract will cause speculators to take long positions in futures contract and short positions in the cash market simultaneously. The results of arbitrage opportunities will bid down the value of the cash market. On the other hand, a negative net present value of futures contract will bid up the value of the cash market. The above-mentioned empirical results do support zero net present value of futures contract (i.e. no initial payment), so the futures prices' changes reflect only risk premia, or internal rate of return of contracts. The theoretical proof is beyond the scope of this paper.

Rewriting (2.1) in terms of period cash payments and dropping the subscript j, we have

$$(2.29)\quad \tilde{X}_t = E_{t-1}[\tilde{X}_t] + X_{t-1} \times b \times \tilde{\pi} + X_{t-1} \times \tilde{e}_t$$

$$= E_{t-1}[\tilde{X}_t] \times (1 + \tilde{\epsilon}_t) \qquad t = 1,2......T$$

where $\tilde{\epsilon}_t = \{X_{t-1} \times b \times \tilde{\pi} + X_{t-1} \times \tilde{e}_t\}/E_{t-1}[\tilde{X}_t]$, $\tilde{X}_t$ is cash[*8] flow at time t, $E_{t-1}[\tilde{X}_t]$ is the expected value of $\tilde{X}_t$ conditional on all information available at time t-1, $\tilde{\epsilon}_t$ is a random variable with expected value zero, and is called the expectations adjustment variable : the change in the expected value of cash payment, $\tilde{X}$, from time t-1 to t per unit of expected value of $\tilde{X}_t$ at time t, $E_{t-1}[\tilde{X}_t]$. The second equality in (2.29) holds only if the expected cash flows are bounded away from zero.

The interpretation of the expected value operator as conditioned on all available information is consistent with the world of the S-L model wherein information is costlessly available to all investors, who agree on its implication for probability distributions of future cash payment. In short, rational assessment of expectations requires that the expected value of the payment $\tilde{X}_t$ to be realized at the fixed time t evolves as a martingale. Equation (2.29) is a valuation of multiperiod cash payments through the life of futures contract in a world where prices are determined according to the S-L model of capital market equilibrium. Fama (1977) showed that (1) the risk adjustments in the discount rates arise because of uncertainties about reassessments through time of the expected value of marking to the market settlement price of futures contract, and the relationship's between these reassessments and the corresponding reassessments of the expected cash flows of the market; (2) the risk-adjusted discount rate $E[\tilde{R}_t]$ for any time t must be given and non-stochastic at all prior time. In other words, the contributions of the futures contract to the risk and expected value of market wealth at current time are uncertain, but the ratio of these uncertain contributions, i.e. beta, is always perfectly certain; and (3) the current market value of any future net cash flows at time zero, V_0, is the current expected value of the cash flows discounted at risk-adjusted discount rates for each day until the cash flows are realized.

The following equation is derived from (2.29)

$$(2.30) \qquad V_0 = \sum_{i=1}^{T} E_0[\,\tilde{X}_i\,] * \prod_{\tau=1}^{i} \{\, 1/(1 + E[\,\tilde{R}_{i,\tau}\,])\}$$

$$= E_0[\,\sum_{i=1}^{T} \tilde{X}_i * \prod_{\tau=1}^{i} \{\, 1/(1 + E[\,\tilde{R}_{i,\tau}\,])\}$$

where E_0 is the expectation at time zero, and the second equality is the result of (2) in Fama's paper and of the martingale process of $\tilde{X}_i$. Equation (2.30) describes the current market value of cash settlements of futures contract through its life, which is the cost of insurance against price changes of the underlying asset or commodity. This equation is similar to equation (46) in C-I-R (1981), which states that the futures price is the risk adjusted expected spot price at maturity in continuous time, continuous states economies. Their formula is not subject to empirical test, because we do not have continuous time data.

II.6 Ex post Mean-Variance Futures Model.

Equation (2.28) in section II.3 is the expression of estimated systematic risk, β^*, of a futures contract in continuous time CAPM. The systematic beta can be derived from the number, T, of observations of returns on the market portfolio and returns on the contract, if we transfer the observed returns to a natural logarithm form. For the sake of simplicity, we delete the subscript j in (2.28) and rewrite as

$$(2.31)\quad \beta^{*} \approx \sum_{i=1}^{T} (\tilde{R}^{*}_{i} - \bar{R}^{*}_{i}) \times (\tilde{R}^{*}_{m,i} - \bar{R}^{*}_{m}) / \left(\sum_{i=1}^{T} (\tilde{R}^{*}_{m,i} - \bar{R}^{*}_{m})^{2} \right)$$

$$\approx b$$

where

$$\bar{R}^{*}_{i} = \left(\sum_{i=1}^{T} \tilde{R}^{*}_{i} \right) / T \quad ; \quad \bar{R}^{*}_{m} = \left(\sum_{i=1}^{T} \tilde{R}^{*}_{m,i} \right) / T$$

$$\tilde{R}^{*}_{i} = \mathrm{Log}_{e} (1 + \tilde{R}_{i}) \quad ; \quad \tilde{R}^{*}_{m,i} = \mathrm{Log}_{e} (1 + \tilde{R}_{m,i})$$

$$\tilde{R}_{i} = X_{i} / (V_{0} + X_{1} + \dots\dots\dots + X_{i-1})$$

$$X_{i} = F(i) - F(i-1)$$

F(i) is the observable futures price at time i.

Unfortunately, returns on futures can not be observed directly from the futures market or spot market due to the fact that total cost of the insurance against price changes of the underlying asset or commodity is unknown through the life of the contract. Once the insurance cost, V_0, of a futures contract is given, the systematic risk is determined by (2.31). Therefore, we claim that the systematic risk is exclusively dependent on the insurance cost. However, equation (2.30) in section II.4 is used to calculate the insurance cost, but the equation itself has an errors-in-variables problem. First, the insurance cost, or certainty-equivalent-cost, at time zero, V_0, is based on an ex-ante expected value of future cash flows, but we can only use ex-post observations to calculate the cost. The difference between ex-ante expected value and ex-post realized value is a random variable with expected value zero. This martingale process implies that the changes of variables are unpredictable. The relationship between ex-ante and ex-post value of variable can be written as

$$(2.32) \quad \tilde{V}_0 = E[\tilde{V}_0] + \tilde{\epsilon}$$

Substituting (2.32) into (2.30), we have

$$(2.33) \quad \tilde{V}_0 = \sum_{i=1}^{T} \tilde{X}_i * \pi_{\tau=1}^{i} \left(1/(1 + E[\tilde{R}_{i,\tau}]) \right) + \tilde{\epsilon}$$

where $E[\tilde{\epsilon}] = 0$. Second, from equations (2.9) and (2.22), we know that the risk-adjusted discount rate for a continuous time model is $E[\tilde{R}_{i,\tau}] \approx (1 - \beta^*) \times R^*_{f,\tau} + \beta^* \times R^*_{m,\tau}$. For any day τ, it must be given and non-stochastic at all prior days i. This means the systematic risk of futures contract is given but unknown at time zero, because the systematic risk itself depends on the ex-ante cost, V_0 .

If we assume here that the first " ≈ " in (2.31) is " = ", then (2.31) can be written as

$$(2.34) \qquad \beta^* = \sum_{i=1}^{T} (\tilde{R}^*_{i} - \bar{R}^*_{i}) \times (\tilde{R}^*_{m,i} - \bar{R}^*_{m}) \Big/ \left(\sum_{i=1}^{T} (\tilde{R}^*_{m,i} - \bar{R}^*_{m})^2 \right)$$

Also, if in (2.33), we replace $\tilde{X}_i$ by the observable X_i and assume $\epsilon_i = 0$ (i.e. ex post is ex ante), we get

$$(2.35) \qquad V_0 = \sum_{i=1}^{T} X_i * \prod_{\tau=1}^{i} (1 / (1 + E[\tilde{R}_{i,\tau}]))$$

where replacing " ≈ " by " = " in the expression for $E[\tilde{R}_{i,\tau}]$

$$E[\tilde{R}_{i,\tau}] = (1 - \beta^*) \times R^*_{f,\tau} + \beta^* \times R^*_{m,\tau}$$

with $R^*_{f,\tau}$ and $R^*_{m,\tau}$ equal the observed (ex post) risk free rate and return on the market. Thus if the definitions of $\tilde{R}^*_i$,

..etc in (2.31) are substituted into (2.34) and definition of $E[\tilde{R}_{i,\tau}]$ into (2.35), then (2.34) and (2.35) become two equations in the two unknowns β^* and V_0.

Our ex post mean-variance continuous time but discrete observations' model consists of equations (2.34) and (2.35). We know there is a unique solution for the model, because β^* is constant (See section II. 1,2 and 3) and given (See section II.4) at the beginning of the time period. However, it may not be possible to find an exact solution, because these two equations are approximate equations. We have to use a trial and error method to solve simultaneously until the error term of beta lies in an allowable range. (we arbitrarily set it at value level 0.01)

III. Efficient portfolio theory of hedging in futures market.

III.1 Introduction.

In section II, we set up a model to calculate the risk and return of futures contract. Now we discuss the hedge theory of cash market holdings and futures market holdings in the CAPM world. We are not emphasizing how to construct an efficient portfolio, but on how to explain risk and return of the naive speculator (i.e. random selection buyer) and of the large speculator (i.e. financial institutions). Before going on with further discussion of the hedge theory, we introduce the general concept of futures contract.

In most academic discussions of futures contracts, a futures contract is described as an agreement between two investors to trade an asset or commodity at a future date at an agreed price. In practice futures contracts typically provide more flexibility for the short position to deliver the asset or commodity at maturity. They may allow some variation as to when, where, how much, and what will be delivered. Gay and Manaster (1986), Mac Donald and Hein (1989) describe these flexibilities as the time option, the location option, the quantity option, and the quality option. These flexibilities of the futures-cash portfolio have value and should be reflected in a reduced futures price, or a higher return on futures contract, and differentiate the portfolio from perfectly diversified efficient portfolios.*9

The timing option allows the short position in a futures contract to deliver the commodity or financial asset on any business day in the maturity month. If the asset or commodity could be rented profitably during the maturity month, then the short position will delay delivery to capture these profits. On the other hand, if the rental value is less than the cost of storing or maintaining the asset, then the short will tend to accelerate delivery in order to avoid these costs. However, the flexibility provided by quantity options will have no effect on futures prices. Settlement at delivery is always based on the par quantity specified by the contract. If less is delivered than the contracted amount, the cash received by the short position is reduced by the amount of the shortfall times the prevailing spot price. Similarly, if the quantity delivered is greater than the contracted amount, the long position is billed for the excess at the prevailing spot price.

The quality option allows the short position to satisfy the contract by delivering one of a variety of specified assets. Most futures contracts traded in the United States avoid the price pressures associated with inadequate supplies of specific varieties by allowing sellers to deliver any of several commodity grades. More particularly, contracts permit delivery of high quality varieties at a premium to the contract price and/or low quality grades at a discount from the contract price. For example, the Chicago Board of Trade T-bond futures contract calls for the delivery of $100,000 par value of 8% U.S. Treasury bonds, maturing no sooner than 15 years from delivery. Also, for callable bonds, the first call date must be at least 15 years after delivery. This means that a number of bonds with different coupon rates and maturities are deliverable against each contract. Since the short position holder selects the instruments to be delivered, it is reasonable to assume that he will deliver the bond that maximizes his wealth, and this gives rise to the concept of the bond that is "cheapest-to-deliver". Given different coupon rates and maturities on deliverable bonds, along with term structures of varying shapes, it is impossible to know in advance which bond will be best to deliver from the short's point of view.

Futures contracts on commodities specify that the delivery of the commodity may occur at more than one location. This provides the short position an opportunity to reduce the storage and transportation cost associated with making a delivery. This is important for agricultural commodities since in times of a plentiful harvest, the supply of grain might be high and storage space scarce. The location option therefore reduces the chance that the short position will be squeezed by locally expensive storage space. For example, the Chicago Board of Trade wheat contract permits delivery in Toledo at the normal commercial discount on wheat in that city relative to Chicago.

In addition to these options, there is another trading time problem between the futures market and spot market. For example, the nonsimultaneity of trading time between S&P 500 futures contracts and the S&P 500 index is itself exacerbated by the fact that the stock market closes at 4:00 PM EST while futures contracts trade for an additional fifteen minutes. Therefore, at the close of any trading day, the price of an index futures contract may reflect information in excess of that reflected in the value of the spot index.

The above-mentioned flexibilities of the futures-cash portfolio for the short position is a cost and uncertainty to the long position. It is impossible to reflect those options' risks merely by analyzing the systematic risk of portfolio, because those are the total risks of the futures-cash portfolio. Therefore, we can divide, as Eric Chang (1985) did, all investors into two classes: the naive speculators and the large speculators. The naive speculator, who was long when hedgers were net short and short when hedgers were net long, does not have enough money to investigate information concerning the uncertainty and flexibility of future-cash portfolios. This is the essence of the theory of normal backwardation when speculators are allowed to be either long or short. The large speculators, who possess some superior forecasting ability to sacrifice some diversification risk and earn the highest expected abnormal return to compensate for the portfolio to be imperfect diversified efficient, usually consist of professionally-managed institutions.

III.2 Efficiency and Portfolio Theory.

Traditional hedging theory emphasizes the risk avoidance potential of futures markets. Hedgers are envisioned as taking futures market positions equal in magnitude but of opposite sign to their position in the cash market. For instance, holders of an inventory of X units would protect themselves against the loss from a decline in the cash price by selling X futures of the same commodity or security. The theory argues that hedgers should always be completely hedged. However, Working's hypothesis (1953, 1962) indicated that hedgers would be completely hedged or unhedged. The application of portfolio theory allows Johnson (1960) and Stein (1961) to explain why hedgers would hold both hedged and unhedged commodity stocks. Cash market holdings are viewed as fixed and the discussion is about how much of the stock to hedge. Anderson and Danthine (1980 and 1981) have applied portfolio perspective to the problem of a firm whose output is not exactly the commodity traded in the futures markets. Britto (1984), and Anderson and Danthine (1983a and 1983b) have worked to extend the portfolio model of a single firm to the equilibrium resulting from the combined actions of many such firms.

Despite the spectacular growth in mid of 1980s of the portfolio theory of hedging, Jeffrey (1989) criticizes with examples that the portfolio theory of hedging examines inappropriate assets and misrepresents the risks associated with positions in the commodity markets. If we discuss the portfolio theory of hedging without considering the utility function of investor and firm, then the portfolio theory suggests a method for measuring the hedging effectiveness of a futures market, and also provides a method for measuring the costs of hedging as described in section II. The concept of an efficient portfolio in the Markowitz sense is one which provides maximum expected return for a given level of risk and minimum risk for a given level of expected return. It is important to note that the risk in the Markowitz' portfolio efficiency is the total risk of the portfolio which is like the Keynesian insurance interpretation of the risk of a futures asset as solely being its own price variability. Because of this definition of total risk, we have to describe portfolio return in terms of total relative risk of the portfolio with respect to the market portfolio risk.

According to the security market line in the derivation of Sharpe-Lintner Capital Asset Pricing Model, each efficient portfolio P will satisfy

$$(3.1)\quad E[\bar{R}_p \mid E[\bar{R}_m],\ \sigma(\bar{R}_p)/\sigma(\bar{R}_m)] = R_f + (E[\bar{R}_m] - R_f) \times \sigma(\bar{R}_p)/\sigma(\bar{R}_m)$$

where $\sigma(\bar{R}_p)/\sigma(\bar{R}_m)$ is the total relative risk of the portfolio P, R_f is the risk free rate, and $E[\bar{R}_m]$ is the expected return of the market portfolio. Recall that the results of the Capital Asset Pricing Model merely state the returns which should be expected on any asset given its level of systematic risk, and that the model also applies to any asset or portfolio. But equation (3.1) will be satisfied only by efficient portfolios. We know that the opportunity set for efficient portfolios is the set which would be determined by knowledge of only the parameters of each security and the parameters of the distribution on the market factor (we assume that security returns are normally distributed). Any short hedgers or long speculators in possession of information in the stock market, futures markets and/or cash markets which enables them to correctly form expectations on the market factor $\tilde{\pi}$, and disturbances $\tilde{e}_j$ of the jth asset which are non-zero, will be able to form portfolios which dominate the portfolio which lies in the security market line. Equation (3.1) presents the expected return on any efficient portfolio P conditional on the expected return on the market portfolio and the total risk of the portfolio.

In order to investigate the behavior of the hedger and speculator in the futures markets, we have to consider the derivation of an expression for the expected return on any efficient portfolio conditional on the realized returns on the market portfolio. Adding $\beta_p \times \tilde{\pi} + \tilde{e}_p$ to both sides of (3.1), we have

$$(3.2) \quad E[\, \tilde{R}_p \mid E[\tilde{R}_m],\ \sigma(\tilde{R}_p)/\sigma(\tilde{R}_m)\,] + \beta_p \times \tilde{\pi} + \tilde{e}_p = R_f + ($$

$$E[\tilde{R}_m] - R_f) \times \sigma(\tilde{R}_p)/\sigma(\tilde{R}_m) + \beta_p \times \tilde{\pi} + \tilde{e}_p$$

and since for all efficient portfolios as described in section II, the following expression is true.

$$(3.3) \quad E[\, \tilde{R}_p \mid E[\tilde{R}_m],\ \sigma(\tilde{R}_p)/\sigma(\tilde{R}_m) \,] + \beta_p \times \tilde{\pi} + \tilde{e}_p \approx \tilde{R}_p$$

Substituting (3.3) into (3.2), we can rewrite (3.2) as

$$(3.4) \quad \tilde{R}_p \approx R_f + (E[\tilde{R}_m] - R_f) \times \sigma(\tilde{R}_p)/\sigma(\tilde{R}_m) + \beta_p \times \tilde{\pi} + \tilde{e}_p$$

We can substitute market factor $\tilde{\pi}$ with $\tilde{R}_m - E[\tilde{R}_m]$ in (3.4), and arrive at

$$(3.5) \quad \tilde{R}_p \approx R_f + (E[\tilde{R}_m] - R_f) \times \sigma(\tilde{R}_p)/\sigma(\tilde{R}_m) + (\tilde{R}_m - E[\tilde{R}_m]) \times \beta_p + \tilde{e}_p$$

Note that

$$(3.6) \quad \sigma(\tilde{R}_p)/\sigma(\tilde{R}_m) = 1/\tau_p \times \beta_p \qquad \tau_p \neq 0$$

where τ_p is the correlation coefficient between returns on the portfolio P and the returns on the market portfolio. Using (3.6), adding and subtracting $\beta_p \times R_f$ on the RHS of (3.5) and rearranging, we have for the efficient portfolio

$$(3.7)\quad \tilde{R}_p \approx R_f + (\tilde{R}_m - R_f) \times \beta_p + (E[\tilde{R}_m] - R_f) \times \beta_p \times (1/\tau_p - 1) + \tilde{e}_p \qquad \tau_p \neq 0$$

Now, since $E[\tilde{e}_j] = 0$ for all $j = 1,2,....N$, then we have $E[\tilde{e}_p] = 0$, and

$$(3.8)\quad E[\tilde{R}_p \mid E[\tilde{R}_m], \tilde{R}_m, \beta_p, \sigma(\tilde{R}_p)/\sigma(\tilde{R}_m)] \approx R_f + (\tilde{R}_m - R_f) \times \beta_p + (E[\tilde{R}_m] - R_f) \times \beta_p \times (1/\tau_p - 1)$$

Equation (3.8) gives us the expected return on the efficient portfolio P conditional on the realized returns on the market portfolio, its systematic risk, and its total relative risk. But also we are left with a term involving $E[\tilde{R}_m]$ which indicates that we cannot find the expected return for the large speculator without taking into account the ex ante expected returns on the market portfolio. Note that the first two terms on the RHS of (3.8) tell us what the portfolio should earn given its level of systematic risk. However, if the portfolio is also to be efficient, its returns must be higher by an amount given by $(E[\tilde{R}_m] - R_f) \times$

$\beta_p \times (1/\tau_p - 1)$. Therefore, when the futures-cash portfolio is a perfectly diversified portfolio, whose total risk should be equal to its systematic risk, $\tau_p = 1$. Now the quantity $\beta_p \times (1/\tau_p - 1)$ $= \sigma(\tilde{R}_p)/\sigma(\tilde{R}_m) - \beta_p$ is just the increment of the portfolio's risk (measured in a relative sense), which is due to the lack of perfect diversification.

In the absence of transactions cost, a large speculator would never hold an imperfectly diversified portfolio unless he believed he could forecast future security prices or commodity prices to some extent. If he possessed some superior forecasting ability, it would be rational to sacrifice some diversification of risk and earn the highest expected abnormal return to compensate for an imperfect diversification of portfolio. So the term $\beta_p \times (1/\tau_p - 1)$ represents the incremental risk due to the lack of perfect diversification of futures-cash portfolio, and the term $E[\tilde{R}_m] - R_f$ is the expected premium per unit of risk possessed in the futures contracts. Thus, equation (3.8) explains the reason why the large speculator does actually forecast better than the naive speculator, and thus the naive speculator is reimbursed in the market only for the amount of systematic risk he has taken. This means that the naive speculator always chooses a perfectly diversified efficient portfolio and the large speculator chooses an imperfectly diversified efficient portfolio.

III.3 An Example of Negative Returns on a Futures-Cash Portfolio.

It is clear from equation (3.7) that the realized returns on all efficient portfolios will not scatter about the security market line, because the third term of RHS of the equation is not zero. To see the issues more clearly, we consider the following example in which the ex post returns of a hypothetical portfolio will be positive or negative depending on the relationships among realized and expected returns on the market portfolio and the risk free rate.

Let us consider our hypothetical portfolio with a level of systematic risk of 0.9 and managed by an institution which attempts to forecast the future prices of commodities or financial securities. In attempting to incorporate its forecasts into the portfolio, the institution is forced to accept additional diversifiable risk in the portfolio. We assume a total relative risk of $\sigma(\tilde{R}_p)/\sigma(\tilde{R}_m) = 4.5$ or $\tau_p = 0.2$. Now equation (3.7) indicates that in order for this portfolio to be imperfectly diversified efficient, the institution's forecasting efforts must increase the expected returns on the portfolio by an amount equal to 3.6 x $(E[\tilde{R}_m] - R_f)$, which is simply the amount of incremental risk in the portfolio multiplied by the ex ante price per unit of risk. For illustrative purposes, let us also assume that another institution can not forecast any better than naive speculators, and thus the institution is reimbursed in the market only for the amount of systematic risk it has taken $\beta_p^* = 0.9$. Let us also assume that the error terms $\tilde{e}_p$ are zero for all efficient portfolios, the risk free rate is 15%, the realized return on the market portfolio is 10%, and the expected return on the market portfolio is 12%. This is an economic environment in which the short run interest rate is higher than the return on the market portfolio, and the institution is also optimistic in the long run economy (i.e. $\tilde{R}_f > E[\tilde{R}_m] > R_m^*$).[10]

After plugging all data into (3.7), we have the negative realized return on the future-cash portfolio, $\tilde{R}_p = -0.3\%$. Therefore, the institution is able to take the profit from its forecasting power by engaging in a short position of the portfolio. This example shows the reason why there is profit or loss an efficient portfolio held through the hedging time period. The naive investor takes a long position, because the beta of the efficient portfolio, 0.9, seems profitable to him. Unfortunately, the imperfect diversification risk dominates the systematic risk of the efficient portfolio. Therefore, it may not be correct to confirm or deny the validity of the normal backwardation theory from a regression outcome without tracing back the economic history of the data collected for regression, and without knowing individual assessments about the market portfolio.

IV. Methodology

IV.1 Introduction.

Futures contracts on U.S. Treasury bills will be used to calculate approximate systematic risks and insurance premiums or discounts. There are four T-bills futures each year: March, June, September and December. The same methodology can be applied to any other financial asset or commodity. The futures contract as currently traded on The International Monetary Market Division (IMM) of the Chicago Mercantile Exchange (CME) calls for delivery of 91-day bills on the maturity date of a 3-month T-bill futures contract. For example, sale of the September contract obligates the seller to deliver $ 1 million face value of 91-day Treasury bills on the Thursday following the third Treasury bill auction of September. The T-bill futures prices are quoted according to the IMM index, which is a function of the discount yield. The IMM futures price indices should be converted to implied T-bill futures prices. Denoting the IMM index futures settlement price observed on day i as f(i), the conversion to implied T-bill futures price F(i) is made following the formula below

$$F(i) = 100 - [(100 - f(i)) \times (90/360)]$$

The daily cash flow from marking-to-market is calculated on the day i as F(i) - F(i-1). We use the IMM index futures settlement price f(i) for empirical calculation, because the return on futures are the same using either F(i) or f(i).

IV.2 Data description.

We will assume that the efficient market portfolio is an index composed of the S&P index of 500 common stocks and the Dow Jones commodity futures index, with each of the two components weighted by the fraction of each index in the market portfolio. Our market portfolio should be viewed as an approximation of the true market portfolio. There are two reasons to include the Dow Jones commodity futures index in the market portfolio. First, C-I-R (1981) theoretically shows that changes in futures prices, when combined with the market portfolio, will satisfy the CAPM in consumption form. Second, Bodie and Rosansky (1980) empirically find that common stock returns and commodity futures returns are negatively correlated, and this provides evidence for the inclusion of commodity assets in any efficient portfolio.

The daily IMM index futures settlement prices, from March 14, 1985 to December 16, 1987 of 3 month, 6 month and 9 month T-bills futures contracts are available from the Marvin M. Speiser Center for Research in Finance and Economics, Baruch College, CUNY. We have 11 futures contracts for each of 3-month, 6-month and 9-month T-bills, but two of the market portfolios' data are not available in one contract. Therefore, we can only calculate 10 futures contracts for each of 3-month, 6-month and 9-month T-bills. We use the weekly Federal Funds rate as the weekly risk free rate, and the daily risk free rate[*11] is derived from the weekly Federal Funds rate.

IV.3 General Approach.

Solving the ex post mean-variance futures model, equation (2.34) and (2.35), immediately presents an unavoidable errors-in-variables problem: the systematic risk β^* is expressed in terms of the observations of returns on the market portfolio and returns on the futures contract in equation (2.34), but the observations of returns on the futures contract depend not only on the settlement cash flows through the life of the futures contract but also on the systematic risk itself in equation (2.35).

The systematic risk of each futures contract is given for any time τ and is a real number at all prior days t, as expressed in the following equation.

$$(4.1) \qquad E[\bar{R}^{*}_{t,\tau}] = (1 - \beta^{*}) \times R^{*}_{f,\tau} + \beta^{*} \times R^{*}_{m,\tau}$$

We use a trial and error method to calculate beta to avoid the errors-in-variables problem. In this paper a real number for proxy systematic risk β^{*} is arbitrarily set at the beginning of the time period of each contract, and the risk adjusted discount rate in the continuous time model is calculated from equation (4.1) by the observations on returns of risk free rate and the market portfolio. Together with the T observations of settlement price of the futures contract, we obtain from equation (2.35) a certainty-equivalent-cost, or insurance premium/discount, of the futures contract. Once obtaining the insurance premium or discount, we can calculate the T implicit returns of the futures contract as follow

$$(4.2) \qquad \bar{R}_t = X_t / (V_0 + X_1 + \ldots.. + X_{t-1})^{*12} \qquad t= 1,2,\ldots\ldots T$$

and

$$(4.3) \qquad \bar{R}^{*}_t = \mathrm{Log}_e(1 + \bar{R}_t)$$

where X_t is the change of futures price, or settlement price, at time t. After plugging the set of T implicit returns and the T observations on the returns of market portfolio into (2.34), we obtain a new real number for systematic risk β^*. If the difference between the new derived beta and the proxy beta does not lie within the satisfactory range, set at value level 0.01, then we have to choose another new proxy beta and repeat the procedure again and again, until we find a approximate systematic risk β^* such that the difference of this proxy beta with the derived beta indeed lies within the satisfactory range. Through this kind of trial-and-error method, we can calculate simultaneously the approximate insurance premium/discount V_0, and the approximate systematic risk β^* of a futures contract.

In order to confirm that the approximate beta and insurance premium or discount is the actual systematic risk and the certainty-equivalent-cost of this futures contract, we have to use returns on futures contract, generated by computer program, and returns on the market portfolio to test the significance of the beta by running an ordinary least square (OLS) regression. If the beta of the OLS regression is exactly the same as the beta calculated by the ex post mean-variance futures model, and the t-test is also significant at the 5% level, then we can confirm the insurance premium or discount of the futures contract. If both constant term and beta of the OLS regression is also significant at the 5% level, then we confirm the correctness of our ex post mean-variance futures model.

In cases where the beta is not a constant through the life of a futures contract, we can still use the ex post mean-variance futures model to find the estimated beta of the futures contract. First, we have to use the model by trial-and-error method to divide the life of the futures contract into sub-periods. For example, if we can not get insurance premium/discount and approximate beta through the life of one contract, then we have to delete (from back to front) settlement prices of the contract day by day until the model finds the insurance premium/discount and beta. The second subperiod of the contract is searched by the same method but using the remaining settlement prices, or the settlement prices which are deleted in the first period, of the contract. If the contract has more than two subperiods, we continue to search the third, the fourth, ..., etc. Second, under the assumption that CAPM works through each subperiod, the OLS regression outcomes of each subperiod provide the beta and variance of the beta of each subperiod. Third, the estimated beta of the futures contract is the weighted average of the betas of subperiods, and the weights are proportional to the variances of each of the betas. (See section V)

IV.4 Empirical Results.

Tables I, II, and III report the outcomes of 3 month, 6 month, and 9 month T-bills futures contracts respectively, and the trading period of each table is from March 14, 1985 to December 16, 1987.

The first and the second columns list the current market trading periods and the expiration month and year of contracts. The third column is the total daily observations of each contract. The fourth and the fifth columns report the estimated beta and insurance premium or discount of each futures contract. Tables IV, V, and VI are the outcomes of ordinary least square (OLS) regressions of returns of futures, generated by our model, on returns of the market portfolio. The first and the second columns list the current market trading period and total observations of each contract. The third column reports the regression outcomes: r represents returns of futures contracts; R represents returns of the market portfolio; the coefficients of R are the estimated betas; constant terms of regression equations are the product of risk free rates by one minus betas, $r_f \times (1 - \beta^*)$; standard deviation of each estimated coefficient is below the coefficient, and the t-value of each estimated variable is within the parenthesis. The fourth and the fifth columns are R-square and F-value of the regression equation. The last column is Durbin-Watson.

The estimated betas in tables IV, V, and VI are the same as betas, calculated by our model, in Tables I, II, and III. Therefore, we are sure that the insurance premium or discount in Tables I, II, and III is the actual certainty-equivalent-cost of each futures contract. Besides the contracts, traded from December 19, 1985 to March 12 1986, we have 12 contracts out of a total of

27 contracts, which do not have a single beta and insurance premium/discount throughout the life of each contract. The detail distribution of these 12 contracts is as follows: three 3-month T-bill futures contracts expired in March, June, and December 1987; four 6-month T-bill futures contracts expired in September 1986, March and December 1987, and March 1988; five 9-month T-bill futures contracts expired in June, December 1986, June 1987, and March and June 1988. There are three different characteristics among those 12 contracts. The first is the 3-month T-bill futures expired in March 1987, which has three betas and insurance premiums or discounts through the life of the contract. The beta is changed from 6.8164 through -2.8092 to 3.3811, and the insurance premium/discount is changed from 19.4451 through -10.9812 to 1.6888. This means that there are two jumps in this contract. The second characteristic is the contracts traded in the time period, September 24 to December 16 1987. During this time period, there is a stock market crash, October 19, 1987. The betas of those contracts are changed from positive beta to positive beta, but the insurance premiums or discounts are changed from negative discount to positive premium, and the jump occurs on the same day for those three contracts. The third characteristic is the contracts whose beta is changed either from positive beta to negative beta or from negative beta to positive, and whose premium changes in the same direction as beta. This means there is only one jump in the contract and the jump occurs at different days for different contracts.

The contracts, traded from September 26 to December 18, 1985, are used to analyze the impact of the expiration day on the stability of beta of the next futures contract. We can not find a single beta and premium for each of these three contracts, and think that the first few days' settlement prices are affected by the precedent expiration day effect. Hence we take out the first settlement price of futures contract on 3-month T-bill, and use the model to simulate 58 out of a total of 59 observations of the contract. We obtain a single beta, -6.7902, and insurance discount, -24.216. On the other hand, we use the jump process of our model to simulate contracts on 6 month and 9 month T-bills. The betas are changed from -6.1172 to 7.0585 for 6-month T-bill futures, from -4.0178 to 5.1961 for 9-month T-bill futures, and the insurance premiums/discounts are changed from -18.8659 to 37.4774 for 6-month T-bill futures, from -4.8265 to 49.3259 for 9-month T-bill futures. These contracts provide the evidence of the expiration day effect on the price volatility of futures.

It is not easy to draw a conclusion from only 30 contracts. If there is summarization from Tables I, II, and III, we conclude that (1) the higher the absolute value of a beta is, the lower the absolute value of the insurance premium is for all contracts, which do not have a jump, on the same underlying T-bill. That means that the higher the risk of a futures contract you own, the higher the return you will receive; (2) the futures contracts, traded from December 18, 1986 to March 11, 1987, have different results on 6-

month and 9-month T-bills. These two contracts have higher risk but receive a lower return, because the 3-month futures has two jumps. This means the jump of beta may refute the theoretic assumption in finance: higher return is compensated by higher risk; (3) besides 3 contracts for sensitivity analysis, traded from September 26 to December 18 1985, we have 9 contracts, which present positive betas, are in favor of naive long hedgers, and 8 contracts, which present negative betas, and are in favor of naive short hedgers. This means 9 contracts provide evidences of the existence of the normal backwardation theory and 8 contracts show the contango theory; (4) we have 10 contracts, which present jumps of betas from positive to positive, or from positive to negative, or from negative to positive, but not from negative to negative, and show evidence of the existence of both the normal backwardation theory and the contango theory through the life of the futures contracts; (5) the contracts, traded from September 26 to December 18 1985, show the high sensitivity of the expiration day effect on the volatility of beta of futures contracts; (6) the contracts, traded from September 24 to December 16 1987 including October 19 1987, provide a case with three jumps occurring on the same day. This differs from the other jump examples. This is consistent with Merton's jump process model, but the evidence is too weak to be conclusive.

Tables IV, V, and VI report OLS regression outcomes of 3-month, 6-month and 9-month T-bills futures contracts on the market portfolio. F-values measure the correctness of model specification of equation (2.1), and t-values test the significance of betas, or basis risks, of futures contracts. R-squares measure the closeness of fit of the set of the sample data to the regression equations, and Durbin-Watson statistics test the serial independence of residual terms, $\tilde{e}_j$, of the equation (2.1). The Durbin-Watson value for most contracts is close to 2. This means that the assumption of serial independence of residual terms of the market factor model is not a big problem in this paper. The independence of residual terms of the market factor model implies that we only need one factor instead of many factors in a model to explain returns on futures contracts. The F-test is superfluous in a single factor model, because F-value is dominated by the value of R-square due to the equation $F = \{ (N - 2) \times \text{R-square} \} / (1 - \text{R-square})$, where N is the number of observation. This means that the F-test is only important to multiple factors model, like the APT model. There are 15 estimated betas which have significant t-values ($t > 2$) at the 5% level, but only 5 contracts which have both significant constant terms and betas at the 5% level.

Unfortunately, the highest R-square is 0.34 in all the empirical futures contracts. We use the certainty-equivalent-method to convert the time series data of futures contract into a set of cross section data, but it is impossible to do the same conversion of data for the market portfolio. Therefore, the goodness of fit test by OLS regression, R-square, is incorrect between the set of time series data and the set of cross section data. Therefore, we claim that the results of low R-squares and low F-values come from the problem of the statistical power of the OLS method, not from our mean-variance ex post model. By referring to R-squares and betas of the consumption oriented CAPM by Breeden, Gibbons and Litzenberger (1989), we find that both R-squares and betas of the spliced consumption, quarterly 1929-1982, and R-squares of financial assets of the maximum correlation consumption portfolio and the CRSP value-weigthed index, monthly 1926-1982, are close to our results. Fortunately, R-square is not important in our model, since our model is an ex post model, not an ex ante forecasting model.

The purpose of this paper is to provide empirical evidence as to whether normal backwardation or contango exist in a futures contract. This has been long debated both in theoretical and empirical papers, so the t-values of betas are more important. Almost half of our estimated betas have significant t-values, and 5 contracts even have significant constant terms and betas. The OLS results support that our ex post mean-variance futures model is able to measure systematic risk of futures contracts, and the 5 contracts prove the correctness of our model. The most important and unbelievable OLS result is that our model has the ability to measure the systematic risk of futures, even the occurrence of the stock market crash.

V. Fluctuation Analysis.

V.1 Introduction.

A T-bill futures contract has one of the three expiration periods of T-bills in our paper; 3 month, 6 month, and 9 month. A new 3-month T-bill futures is created immediately on the next business day for trade, after the current 3-month T-bill contract expires. Therefore, the 3-month T-bill futures close price, or settlement price, is available for every business day. The data observations on 3-month T-bill futures is similar to the data observations on any stock which is traded in the New York Stock Exchange. 6-month and 9-month T-bills futures contracts are traded like the 3-month T-bill futures, so we discuss fluctuation analysis only for the 3-month T-bill futures.

Our model calculates beta and insurance premium/discount of each contract which lasts only three months. We define the beta of 3-month contract as short run beta, or basis risk, and the beta of the whole sample time period, March 14, 1985 to December 16, 1987, as long run equilibrium beta. Now we turn to the question of what the long run equilibrium beta is and of whether the long run beta of futures contract is more risky than that of any stock.

We pool all observations of the ten consecutive 3-month T-bills futures contracts together to calculate the estimated long run equilibrium beta. The ten short run betas fluctuate highly from -6.2625 to 5.4657 (See table VII). The fluctuation of betas not only supports theoretical results, derived by Huang and Litzenberger (1988), that the beta of a financial asset depends on the previous period total value of the market portfolio, but also confirms our conclusion in section III that the argument of normal backwardation or contango theory is meaningless without describing the hedger's or speculator's expectation on the market portfolio.

V.2 Technique for long run equilibrium beta.

The returns on the t-th 3-month T-bill futures contract in the market factor model (2.1) is rewritten as

$$(5.1) \quad \tilde{R}_{j,t} = E[\tilde{R}_{j,t}] + b_t \times \tilde{\pi}_{j,t} + \tilde{e}_{j,t} \qquad \begin{array}{l} j = 1,2.....,N_t \\ t = 1,2....,10. \end{array}$$

where N_t is the N_t observations of the t-th contract, and t is the number of contracts. Under the assumption that CAPM works period by period, (5.1) can be rewritten by substituting

$\tilde{R}_{m,j,t} - E[\tilde{R}_{m,j,t}]$ for $\tilde{\pi}_{j,t}$

$$(5.2)\quad \tilde{R}_{j,t} = E[\tilde{R}_{j,t}] + b_t \times (\tilde{R}_{m,j,t} - E[\tilde{R}_{m,j,t}]) + \tilde{e}_{j,t}$$

$$= (1 - b_t) \times R_{f,j} + b_t \times \tilde{R}_{m,j,t} + \tilde{e}_{j,t}$$

where $(1 - b_t) \times R_{f,j} = E[\tilde{R}_{j,t}] - b_t \times E[\tilde{R}_{m,j,t}]$ is the expected return formula of an asset in CAPM. We will assume independence of all 3-month T-bill futures contracts. The empirical results (See table I) show that the betas of futures contracts are arbitrarily scattered, therefore the assumption is consistent with the efficiency of futures markets. The efficiency of futures markets can be tested by checking the set of time series data of betas. If the set of betas fits any ARIMA time series model, then we claim the market is not efficient. We can express the independence of contracts as

$$(5.3)\quad \mathrm{Cov}(\tilde{e}_{j,t} , \tilde{e}_{j,k}) = \sigma^2 (\tilde{e}_t) \quad \text{when } t = k$$

$$= 0 \quad \text{when } t \neq k$$

$$t,k = 1,2,.....,10.$$

In section II, we assume the constancy of infinitesimal return on the risk free rate and expected return on market portfolio within a short time period, so the first term of RHS of equation (5.2) is constant for each contract. Therefore, the variance-covariance of returns of futures contracts in the whole sample period, denoted by V, is a block diagonal matrix, and its inverse, V^{-1}, may be expressed as

$$(5.4) \qquad V^{-1} = \left| \begin{array}{llll} V_1^{-1}, & 0, & \ldots\ldots\ldots\ldots\ldots\ldots, & 0 \\ 0, & V_2^{-1}, & 0, \ldots\ldots\ldots\ldots\ldots, & 0 \\ . & . & & \\ . & & . \quad . & \\ 0, & \ldots\ldots\ldots\ldots\ldots\ldots\ldots, & 0, & V_{10}^{-1} \end{array} \right|$$

where V_t^{-1}, t = 1,2.....,10 denotes the inverse of the variance - covariance of the t-th 3-month T-bill futures contracts' returns. The variance-covariance of the t-th period 3-month T-bill futures contract is $V_t = \sigma_t^2 \times I_{t,N_t}$, where I_{t,N_t} is a $N_t \times N_t$ identity matrix. This can be regarded as returns on futures randomly selected N_t times from a normal distribution with variance σ_t^2. Because V is a block diagonal, separate generated least square (GLS) or ordinary least square (OLS) regression may be run cross-sectionally by using data from each futures contract. The estimated long run beta of the whole sample period is the weighted average of the 10 betas of the 3-month T-bills futures contracts, with the weights proportional to the variances of each of the estimated betas.

Formally, consider the sub-period t

$$(5.5) \qquad \tilde{R}^{*}_{t} = b_{t} \times \tilde{R}^{*}_{m,t} + \tilde{e}_{t} \qquad t = 1,2,\ldots\ldots\ldots,10^{*13}$$

where

$$\tilde{R}^{*}_{t} = \begin{vmatrix} \tilde{R}^{*}_{1,t} \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \vdots \\ \tilde{R}^{*}_{N_t,t} \end{vmatrix} \qquad \tilde{R}^{*}_{m,t} = \begin{vmatrix} 1 , \tilde{R}^{*}_{m,1,t} \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ 1 , \tilde{R}^{*}_{m,N_t,t} \end{vmatrix} \qquad \tilde{e}_{t} = \begin{vmatrix} \tilde{e}_{1,t} \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \vdots \\ \tilde{e}_{N_t,t} \end{vmatrix}$$

and $b_{t} = [\, b_{1,t} , b_{2,t} \,]$

where $b_{1,t}$ is the intercept of the regression equation ; $b_{2,t}$ is the slope or the estimated beta of the regression equation; and N_{t} is the number of observations in the sub-period t. The GLS or OLS estimator of b_{t}, denoted by $\hat{b}_{t(gls)}$, is

$$(5.6) \qquad \hat{b}_{t(gls)} = \hat{b}_{t(ols)} = (\tilde{R}^{*T}_{m,t} \times V^{-1}_{t} \times \tilde{R}^{*}_{m,t})^{-1} \times (\tilde{R}^{*T}_{m,t} \times V^{-1}_{t} \times$$

$$\tilde{R}^{*}_{t}) = (\tilde{R}^{*T}_{m,t} \times I_{N_t} \times \tilde{R}^{*}_{m,t})^{-1} \times (\tilde{R}^{*T}_{m,t} \times I_{N_t} \times \tilde{R}^{*}_{t})$$

where I_{N_t} is the $N_t \times N_t$ identity matrix, and $\tilde{R}^{*T}_{m,t}$ is the transpose of $\tilde{R}^{*}_{m,t}$. The estimated long run beta $\hat{b}$ can be verified to be

$$(5.7) \qquad \hat{b} = \sum_{t=1}^{10} w_t \times \hat{b}_{t(gls)}$$

where

$$w_t = \{ 1/var(\hat{b}_{t(gls)}) \} / \{ \sum_{t=1}^{10} (1/var(\hat{b}_{t(gls)})) \}$$

The variance of $\hat{b}$, denoted by Var($\hat{b}$), can be verified to be

$$(5.8) \qquad Var(\hat{b}) = \sum_{t=1}^{10} w_t^2 \times Var(\hat{b}_{t(gls)})$$

By substituting all variances of estimated betas of the 3-month T-bill futures contracts (Table VII) into (5.6), we obtain the estimated long run equilibrium beta of the whole sample period, 0.77191, which is the equilibrium risk of the ten sample futures contracts if the beta of the 3-month T-bill is regarded as zero in the CAPM world.

V.3 Comments and Conclusions

In order to incorporate the Dow Jones Commodity futures index into the S&P index of 500 common stocks to define a new market portfolio, we had to discuss in section II the multiperiod CAPM in the market factor model. However,the market factor model is the Capital Asset Pricing Model if the expected returns of assets are calculated by CAPM, and the number of securities is large. The assumptions of infinitesimal returns of the risk free asset and expected returns of the market portfolio of each short trading period is used to extend the single period CAPM to a multiperiod CAPM. The assumption of continuity of market trading is used to prove that the beta, systematic risk of financial asset or basis risk of futures contract, in the discrete time model is the beta in the continuous time model when we take the natural logarithm of returns of assets and the market portfolio. The cash flows method is used to calculate the certainty-equivalent-cost, or insurance premium or discount, of the futures contract. But the method can not be used to calculate any security traded in the New York Stock Exchange because we do not know when the security was initiated for trade and when it is going to be terminated. The cash flows method can also be used to convert time series returns of futures contracts to cross section returns, but can not be used to convert returns of the market portfolio. Therefore, the statistic results of ordinary least square regression are not powerful enough to support the significance of t-value, F-value, and R-square at the

5% level of all futures contracts in our empirical tests. Unless we can develop a new statistical method to measure basis risks of futures contracts between sets of time series data and cross section data, we can not say that the basis risks calculated by the ex post mean-variance futures model are inaccurate. Fortunately, the purpose of this paper is not to discuss the empirical weakness or strength of CAPM. Those who are interested in criticism of the CAPM equation are encouraged to read Roll (1978).

The classic debate on the futures contract focuses on the existence of normal backwardation or the contango theory. The debate comes from the fact that we do not have a way to measure returns on futures contracts. Even if the multiperiod CAPM works, we can only measure the approximate beta of T-bills futures contracts within an allowed error level. Our results show that not only do both theories exist in different futures contracts, but also exist in the same contract. The expiration date effect indeed has impact on the systematic risk, or basis risk, of the immediate following contract.

Our sample period includes the date of the stock market crash, October 19, 1987, and the empirical outcomes of these three contracts show that the jumps of contracts occurred on the same day, and that the jumps are totally different from those of other contracts. We do not know if the jump of these three contracts signals a jump of CAPM as described by Merton's jump process of continuous time CAPM, or just reflects the existence of both theories in the contracts. Fluctuations of short run betas only reflect basis risks of the contracts themselves, and the long run equilibrium beta is the systematic risk of futures contract in the CAPM world.

Finally, the naive speculator is reimbursed in the market only for the amount of systematic risk or basis risk he has taken, but large speculators who possess some superior forecasting ability, would be rational to sacrifice some diversification risk and earn the highest expected abnormal return to compensate for an imperfectly diversified efficient portfolio.

NOTES

*1 We use a cash flow technique to generate returns on futures contracts [See section IV.2]. The series of the first differences of futures close prices is the set at daily settlement prices, which is stationary. The return on a futures contract is the ratio of the settlement price to a constant term, instead of futures price. The constant term depends on the certainty-equivalent-cost. Therefore, the best model should be expressed in terms of returns on the futures contract. The stationarity of a series of returns of financial assets has been discussed by Fama.

*2 If $\sum_{j=1}^{n} w_j * b_j \neq 1$, then we can substitute $b_j^* = b_j / \{ \sum_{i=1}^{n} w_i * b_i \}$ into the market model.

*3 The last term on the RHS of (2.4a) can be approximately expressed as $\sum_{j=1}^{N} w_j^2 * \sigma^2(e_j) \approx N/N^2 * \bar{\sigma}^2(e) = 1/N * \bar{\sigma}^2(e)$ when N is large, where $\bar{\sigma}^2(e)$ is the average variance of the disturbance terms.

*4 Note that $\tilde{d}_j * \pi$ is trivially small since $\tilde{d}_j * \pi = \{ w_j * \sigma^2(\tilde{e}_j) \}/\sigma^2(\tilde{R}_m) * \pi \approx \tilde{w}_j * \pi$. If we know the market factor, then the term $\sum_{j=1}^{N} w_j x \tilde{e}_j$ is zero. Otherwise, we can only say that the term is small when N is large.

*5 We use n=1 in our model, so the daily interest rate and expected return of the market portfolio can be assumed to be constant through the life of a futures contract.
If we use trade by trade data of futures contracts, then k is the time interval between two trades.

*6 L'Hospital's rule:

$$\lim_{\Gamma \to 0} (x^{\Gamma} - 1)/ \Gamma = \lim_{\Gamma \to 0} \{ \delta(x^{\Gamma} - 1)/\delta(\Gamma) \}/\{\delta(\Gamma)/\delta(\Gamma)\}$$

$$= \lim_{\Gamma \to 0} x^{\Gamma} * \log_e x = \log_e x$$

where δ is the notation of partial derivative.

*7 $$\lim_{\Gamma \to 0} \hat{\beta}_j = \lim_{\Gamma \to 0} \mathrm{Cov}\{([\exp(\sum_{i=1}^{T/k} \tilde{R}_{k\,j,i})]^{\Gamma} - 1)/ \Gamma , ([\exp(\sum_{i=1}^{T/k} \tilde{R}_{k\,m,i})]^{\Gamma} - 1)/ \Gamma\} / \sigma^2\{([\exp(\sum_{i=1}^{T/k} \tilde{R}_{k\,m,i})]^{\Gamma} - 1)/ \Gamma \}$$

$$= \mathrm{Cov}\{ \lim_{\Gamma \to 0} ([\exp(\sum_{i=1}^{T/k} \tilde{R}_{k\,j,i})]^{\Gamma} - 1)/ \Gamma , \lim_{\Gamma \to 0} ([\exp(\sum_{i=1}^{T/k} \tilde{R}_{k\,m,i})]^{\Gamma} - 1)/ \Gamma \} / \sigma^2\{ \lim_{\Gamma \to 0} ([\exp(\sum_{i=1}^{T/k} \tilde{R}_{k\,m,i})]^{\Gamma} - 1)/ \Gamma \}$$

$$= \mathrm{Cov}\{ \tilde{R}_{k\,j} , \tilde{R}_{k\,m} \} / \sigma^2(\tilde{R}_{k\,m})$$

where $\Gamma = k/T$. The first equality comes from (2.21), (2.22), (2.24a), (2.24b), and the definition of (2.18); the second equality is due to the continuity of covariance and variance functions; the last equality is derived from L'Hospital's rule, and the compounding interval expression of returns, and series independence.

*8 $\tilde{X}_t$ is the difference between futures price at time t and t-1.

*9 Under the assumptions of no taxes, transaction costs, indivisibilities and other market imperfections, the perfectly diversified efficient portfolios are portfolios in the Markowitz' efficient portfolio frontier and those portfolios are perfectly correlated. The imperfectly diversified efficient portfolio may occur due to the fact that our market portfolio does not include the goods market.

*10 Equation (2.3), $\tilde{R}_m = E[\tilde{R}_m] + \tilde{\pi} + \sum_{j=1}^{N} w_j x \tilde{e}_j$, states the relationship between the ex post and ex ante returns on the market portfolio. Under the assumption $\tilde{e}_p = 0$, we can rewrite the equation as $\tilde{R}_m = E[\tilde{R}_m] + \tilde{\pi}$, where $E[\tilde{\pi}] = 0$. If the forecast returns on the market portfolio are below the realized returns, then the situation $\tilde{R}_f > E[\tilde{R}_m] > R_m$ is possible for a short time period, but not for a long run equilibrium CAPM. For example, you expect the economy to boom in the near future, so you take a long position in the futures market. Unfortunately, the economic data reports the economy suffering a bit of a recession in this quarter, so the negative return on your investment comes from a mistaken forecast.

*11 The reasons for us to use the higher Federal Funds rate as the risk free rate in our empirical test is that (1) Black, Jensen and Scholes (1972) show that the risk free rate is undervalued by using the T-bill as the risk free asset; (2) the available market one day or one week risk free rate is the Federal Funds rate, which is the overnight lending or borrowing rate between commercial banks. We do not expect that the one day rate will affect our empirical outcomes.

*12 The returns on T-bills futures contracts are called implicit returns because of the following two reasons. First, the certainty-equivalent-cost, V_0, is a function of the beta of the futures contract which is unknown. Second, the cost V_0 is the insurance premium or discount to guarantee the long position holder on the expiration date to pay the contract price the time the contract is initiated. If the futures contract close price goes up X dollar next day, the cost of the insurance premium should be increased to $V_0 + X$. Therefore, the return on the contract for a one day period is X/V_0, which is also a function of beta.

*13 The value of t is 14 for the 3-month T-bill futures contracts, since there are two contracts with one jump and one contract with two jumps.

Table I : 3-month T-bill futures trading from March 14, 1985 to December 16, 1987.

near contract trading period	maturity	obs.	basis risk (beta)	insurance premium
03/14/85 - 06/05/85	June '85	58	0.5379	174.61
06/06/85 - 09/25/85	Sept.'85	78	3.8614	45.067
09/26/85 - 12/18/85	Dec. '85	58	-6.7902	-24.216
12/19/85 - 03/12/86	Mar. '86	54	-	- *1
03/13/86 - 06/04/86	June '86	58	4.7090	19.579
06/05/86 - 09/24/86	Sept.'86	78	1.1848	126.999
09/25/86 - 12/17/86	Dec. '86	59	-2.3896	-48.058
12/18/86 - 03/11/87	Mar. '87	35 7 15	6.8164 -2.8092 3.3811	19.4451 -10.9812 1.6888
03/12/87 - 06/03/87	June '87	48 10	-7.2116 4.0063	-26.4927 36.1576
06/04/87 - 09/23/87	Sept.'87	78	-1.2599	-110.602
09/24/87 - 12/16/87	Dec. '87	18 41	1.5056 0.4536	-48.3079 152.089

*1 data errors occur during this period, so we can not calculate the beta and insurance premium.

Table II: 6-month T-bill futures trading from March 14, 1985 to December 16, 1987.

near contract trading period	maturity	obs.	basis risk (beta)	insurance premium
03/14/85 - 06/05/85	Sept.'85	58	0.72134	237.919
06/06/85 - 09/25/85	Dec. '85	78	-3.89200	-19.6373
09/26/85 - 12/18/85	Mar. '86	37 22	-6.11720 7.05850	-18.8659 37.4774
12/19/85 - 03/12/86	June '86	54	-	- *1
03/13/86 - 06/04/86	Sept.'86	55 *2 3	9.29950 - -	8.5732 - -
06/05/86 - 09/24/86	Dec. '86	78	1.5216	123.759
09/25/86 - 12/17/86	Mar. '87	13 46	19.1435 -4.07250	23.5687 -29.9814
12/18/86 - 03/11/87	June '87	57	-4.22200	-53.8540
03/12/87 - 06/03/87	Sept.'87	58	-2.96220	-98.9300
06/04/87 - 09/23/87	Dec. '87	52 26	0.24450 -3.09470	44.9960 -72.1656
09/24/87 - 12/16/87	Mar. '88	18 41	0.62483 0.15967	-94.1211 163.459

*2 there is a jump of beta during this contract period, and the second observation is only three. Therefore we skip the calculation.

Table III: 9-month T-bill futures trading from March 14, 1985 to December 16, 1987.

near contract trading period	maturity	obs.	basis risk (beta)	insurance premium
03/14/85 - 06/05/85	Dec. '85	58	0.6570	255.0354
06/06/85 - 09/25/85	Mar. '86	78	-3.5390	-21.7230
09/26/85 - 12/18/85	June '86	31 28	-4.0178 5.1961	-4.82650 49.32590
12/19/85 - 03/12/86	Sept.'86	54	-	- *1
03/13/86 - 06/04/86	Dec. '86	52 6	11.6567 -2.67960	16.14950 -51.4613
06/05/86 - 09/24/86	Mar. '87	78	1.48760	129.6610
09/25/86 - 12/17/86	June '87	19 40	-15.3147 2.92990	-20.5214 19.5522
12/18/86 - 03/11/87	Sept.'87	57	-3.82200	-47.759
03/12/87 - 06/03/87	Dec. '87	58	-2.70900	-142.159
06/04/87 - 09/23/87	Mar. '88	59 19	3.70530 -5.85920	25.5616 -43.4760
09/24/87 - 12/16/87	June '88	18 41	*3 0.63158 0.09080	-75.6688 152.1530

*3 It is surprising to have positive beta with negative insurance premium for all T-bills futures contracts around Oct. 19, 1987 of the stock market crash.

Table IV: OLS regression of rates of returns of 3-month T-bill futures on rates of returns of the market portfolio

near contract trading period	obs	regression equation	R-square	F-value	D-W
03/14/85 06/05/85	58	r = 0.0115 + 0.5425*R 0.0112 2.1978 (1.029) (0.247)	0.0011	0.061	1.8
06/06/85 09/25/85	78	r = 0.0100 + 3.8552*R 0.0186 3.2875 (0.540) (1.173)	0.018	1.375	1.8
09/26/85 12/18/85	58	r = 0.0239 - 6.2625*R 0.0166 2.5218 (1.449) (-2.484)	0.098	6.167	1.4
12/19/85 03/12/86	-	-	-	-	-
03/13/86 06/04/86	58	r = 0.0055 + 4.7129*R 0.0219 2.5455 (0.251) (1.852)	0.058	3.428	1.0
06/05/86 09/24/86	78	r = 0.0093 + 1.1973*R 0.0044 0.4392 (2.095) (2.726)	0.089	7.432	1.9
09/25/86 12/17/86	59	r = 0.0141 - 2.3876*R 0.0188 2.3706 (0.751) (-1.01)	0.018	1.015	2.0
12/18/86 03/11/87	35	r = -0.0179 + 6.779*R 0.0444 4.910 (-0.404) (1.381)	0.055	1.906	1.6
	7	r = 0.1062 - 2.7713*R 0.1388 12.589 (0.765) (-0.22)	0.009	0.049	1.4
	15	r = 0.0272 + 3.4069*R 0.1932 30.757 ( ) ( )	0.001	0.012	1.0
03/12/87 06/03/87	48	r = 0.0132 - 7.2167*R 0.0399 3.3322 (0.332) (-2.17)	0.093	4.691	1.5
	10	r = 0.0546 + 3.9613*R 0.0422 4.1952 (1.296) (0.944)	0.101	0.892	1.6

06/04/87 09/23/87	78	r = 0.0100 - 1.2546*R 0.0092 1.1163 (1.085) (-1.124)	0.016	1.263	1.3
09/24/87 12/16/87	18	r = 0.0827 + 1.6301*R 0.0389 0.7904 (2.127) (2.063)	0.210	4.254	1.3
	41	r = 0.0158 + 0.4855*R 0.0164 0.5911 (0.963) (0.821)	0.017	0.675	1.3

Note: r represents the logarithm of return of the futures contract. R represents the logarithm of return of the market portfolio.

Note: the numbers which are under the coefficients of regression equations are standard errors of those regressed coefficients. The numbers which are within the parentheses are t-values of those estimated coefficients.

Note: It is a surprise that the contract with positive beta but negative insurance premium, Sept. 24, 1987 to Dec. 16, 1987, fits the regression equation. That means our model is able to capture any big price change in the stock market.

Table V: OLS regression of rates of returns of 6-month T-bill futures on rates of returns of the market portfolio.

near contract trading period	obs	regression equation	R-square	F-value	D-W
03/14/85 06/05/85	58	r = 0.0114 + 0.7227 * R 0.0064 1.2500 (1.788) (0.578)	0.006	0.334	1.98
06/06/85 09/25/85	78	r = 0.0193 - 3.9089 * R 0.0267 4.7104 (0.072) (-0.83)	0.009	0.689	1.54
09/26/85 12/18/85	37	r = 0.0363 - 6.0907 * R 0.0393 6.3338 (0.925) (-0.96)	0.026	0.925	0.97
	22	r = 0.0165 + 7.0029 * R 0.0283 3.9666 (0.582) (1.765)	0.135	3.117	1.74
12/19/85 03/12/86	-	-	-	-	-
03/13/86 06/04/86	55	r = -0.002 + 9.3281 * R 0.0337 3.8667 (-0.06) (2.412)	0.1	5.82	1.52
	3	-	-	-	-
06/05/86 09/24/86	78	r = 0.0938 + 1.5335 * R 0.0051 0.5035 (1.840) (3.046)	0.11	9.28	2.1
09/25/86 12/17/86	13	r = 0.0469 + 19.3115 * R 0.1034 14.3005 (0.453) (1.3504)	0.14	1.83	2.8
	46	r = 0.0171 - 4.0925 * R 0.0161 - (1.066) (-2.066)	0.09	4.27	1.85
12/18/86 03/11/87	57	r = 0.0192 - 4.1987 * R 0.0137 1.5785 (1.397) (-2.66)	0.12	7.08	1.71
03/12/87 06/03/87	58	r = 0.0131 - 2.9645 * R 0.0066 0.5686 (1.980) (-5.213)	0.33	27.18	1.5

06/04/87 09/23/87	52	r = 0.0129 + 0.2626 * R 0.0465 7.1574 (0.277) (0.037)	0.003	0.001	2.07
	26	r = 0.0234 - 3.0883 * R 0.0097 0.8845 (2.410) (-3.49)	0.34	12.19	1.33
09/24/87 12/16/87	18	r = 0.0516 + 0.6680 * R 0.0298 0.6065 (1.729) (1.102)	0.07	1.22	1.42
	41	r = 0.0165 + 0.1900 * R 0.0143 0.5157 (1.154) (0.368)	0.004	0.14	1.52

Table VI: OLS regression of rates of returns of 9-month T-bill futures on rates of returns of the market portfolio.

near contract trading period	obs	regression equation	R-square	F-value	D-W
03/14/85 06/05/85	58	r = 0.0114 + 0.6577*R 0.0052 1.0098 (2.223) (0.651)	0.008	0.424	2.3
06/06/85 09/25/85	78	r = 0.0030 - 3.5542*R 0.0246 4.3402 (0.123) (-0.82)	0.009	0.671	1.5
09/26/85 12/18/85	31	r = 0.0485 - 4.0367*R 0.0835 15.073 (0.582) (-0.27)	0.005	0.072	0.9
	28	r = 0.0133 + 5.1496*R 0.0223 2.9564 (0.597) (1.742)	0.11	3.04	2.0
12/19/85 03/12/86	-	-	-	-	-
03/13/86 06/04/86	52	r = 0.0061 + 11.694*R 0.0430 4.8501 (0.141) (2.411)	0.11	5.82	1.8
	6	r = 0.1131 - 2.6507*R 0.0538 9.4549 (2.104) (-0.28)	0.02	0.08	2.8
06/05/86 09/24/86	78	r = 0.0093 + 1.4998*R 0.0056 0.5489 (1.677) (2.733)	0.09	7.47	2.3
09/25/86 12/17/86	19	r = 0.0325 - 15.257*R 0.0804 11.018 (0.405) (-1.39)	0.11	1.92	2.0
	40	r = 0.0136 + 2.9489*R 0.0178 2.1584 (0.768) (1.366)	0.05	1.87	1.7
12/18/86 03/11/87	57	r = 0.0191 - 3.8029*R 0.0128 1.4706 (1.494) (-2.59)	0.11	6.68	1.9

03/12/87 06/03/87	58	r = 0.0139 - 2.6036*R 0.0050 0.4326 (2.760) (-6.02)	0.393	36.23	1.6
06/04/87 09/23/87	59	r = 0.0035 + 3.7045*R 0.0335 4.8596 (0.103) (0.763)	0.01	0.58	2.2
	19	r = 0.0304 - 5.8411*R 0.0229 1.9813 (1.328) (-2.95)	0.34	8.69	1.8
09/24/87 12/16/87	18	r = 0.0517 + 0.6703*R 0.0322 0.6541 (1.609) (1.025)	0.06	1.1	1.6
	41	r = 0.0166 + 1.1242*R 0.0152 0.5482 (1.094) (0.227)	0.002	0.05	1.7

Note 1: Two contracts, Dec. 18, 1986 to June 3, 1987, fit the OLS regression. That means our model is able to measure the beta of futures contracts. Those two contracts provide examples of contango theory.

Note 2: The contract, June 5, 1986 to Sept. 24, 1986, fits the OLS regression. That contract provides an example of Normal Backwardation theory.

Note 3: The two contracts, Mar. 13, 1986 to June 4, 1986 and June 4 1987 to Sept. 23, 1987, provide examples of the existence of both normal backwardation and contango theories in a futures contract.

Note 4: It seems that our empirical results support the ex post mean-variance model for measuring beta of futures contracts, but there is statistical power problem that the OLS methodology is not power enough to capture fluctuations of all ten futures contracts.

Table VII: Long run beta of 3-month T-bill futures contract trading from March 14, 1985 to December 16, 1987.

near contract trading period	mat.	obs.	basis risk (beta)	comb. beta	stand. devia.	weight
03/14/85 06/05/85	June - 1985	58	0.5379	0.5379	2.1978	0.0181
06/06/85 09/25/85	Sept.- 1985	78	3.8614	3.8614	3.2875	0.0081
09/26/85 12/18/85	Dec. - 1985	58	-6.790	-6.790	2.5218	0.0138
12/19/85 03/12/86	Mar. - 1986	54	-	-	-	-
03/13/86 06/04/86	June - 1986	58	4.7090	4.7090	2.5455	0.0135
06/05/86 09/24/86	Sept.- 1986	78	1.1848	1.1848	0.4392	0.4537
09/25/86 12/17/86	Dec. - 1986	59	-2.389	-2.389	2.3706	0.0156
12/18/86 03/11/87	Mar. - 1987	35	6.8164	5.4657	4.9102	0.0036
		7	-2.809		12.589	0.0006
		15	3.3811		30.757	0.0001
03/12/87 06/03/87	June - 1987	48	-7.211	-2.879	3.3322	0.0079
		10	4.0063		4.1952	0.0050
06/04/87 09/23/87	Sept.- 1987	78	-1.259	-1.259	1.1163	0.0700
09/24/87 12/16/87	Dec. - 1987	18	1.5055	0.8964	0.7904	0.1400
		41	0.4536		0.5911	0.2501

Note: The long run beta is calculated by equation (5.7) and the value is 0.77191.

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