

**SEISMIC RESPONSE OF PILES AND PILE-SUPPORTED BRIDGE
PIERS EVALUATED THROUGH CASE HISTORIES**

by

KONSTANTINOS (COSTIS) SYNGROS

A dissertation submitted to the Graduate Faculty in Civil Engineering in partial fulfillment of the requirements for the degree of Doctor of Philosophy, The City University of New York

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ABSTRACT**SEISMIC RESPONSE OF PILES AND PILE-SUPPORTED BRIDGE PIERS
EVALUATED THROUGH CASE HISTORIES**

by

KONSTANTINOS SYNGROSAdvisor: *Professor George Mylonakis*

Numerical solutions are presented for the static, dynamic, and seismic response of single piles, pile groups, and pile-supported structures in homogeneous, inhomogeneous, and layered soil deposits. The seismic analysis considers both the kinematic and inertial interaction between the foundation and the superstructure in the frequency and time domains.

The proposed methods make use of dynamic finite-element formulations equipped with pertinent absorbing boundaries, spatial Fourier expansions, and other special analysis techniques. The role of soil inhomogeneity is investigated considering the presence of: (1) pre-existing inhomogeneity (in the vertical direction) arising from soil layering and overburden; (2) stress-induced inhomogeneity (in both horizontal and vertical directions) due to pile installation and loading. It is shown that ignoring soil inhomogeneity may substantially underestimate pile head stiffness and overestimate damping. In addition, modeling an inhomogeneous deposit as a homogeneous layer providing the same static stiffness to the pile, may grossly overestimate radiation

damping, leading to un-conservative predictions of dynamic response. The accuracy of the proposed theoretical solutions is demonstrated by comparing their predictions with the measured response of actual piles and pile groups.

Two case histories involving pile-supported bridges are presented: The first bridge, Ohba Ohashi, instrumented by the Institute of Technology of Shimizu Corporation, has been subjected to several moderate earthquake motions. Successful comparisons between predicted and measured response are presented. The role of valley effects, non-uniform seismic excitation, and structural connections on seismic response is discussed.

The second case study refers to the Fukae Section (a 630m segment) of the elevated Hanshin Expressway, which collapsed during the 1995 Kobe earthquake. Analytical and recorded evidence suggest a detrimental role of soil in the collapse. It is shown that: (1) the soil modified the incoming seismic waves such that the resulting ground surface motion became very severe for the particular bridge. (2) the presence of compliant soil in the foundation resulted to an increase in natural period of the bridge which moved the system to a region of stronger response; (3) the compliance of the foundation increased the participation of the fundamental mode, inducing stronger response. Implications of the above effects in design are discussed.

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CHAPTER 1

INTRODUCTION

1.1 The soil-structure interaction (SSI) problem

A common issue in the analysis and design of bridges is the effect of soil-structure interaction (SSI). The problem, as the name suggests, deals with predicting the response of a structure taking into consideration the compliance of the foundation. During earthquake shaking, soil deforms under the influence of the incident seismic wave and carries dynamically with it the foundation and the supported structure. In turn, the induced motion of the superstructure generates inertial forces, which result in dynamic stresses at the base that are transmitted into the supporting soil. Thus, superstructure-induced deformations develop in the soil while additional waves emanate from the soil-foundation interface. In response, foundation and superstructure undergo further dynamic displacements, which generate further inertia forces, and so on.

The above phenomena occur simultaneously. There are two major approaches of solving this complicated problem. The first is a direct one-step method where the whole soil-structure system is discretized formulation (usually by means of finite or boundary elements) and subjected to seismic loading. The second approach is the substructure-method where the SSI problem is split into subproblems. In the realm of the second approach, it is convenient (both conceptually and computationally) to separate the SSI problem into two successive problems, referred to as “kinematic interaction” and “inertial interaction”. Kinematic interaction requires evaluating the free-field site response and the

foundation response in the absence of the superstructure (often referred to as “foundation scattering”). Inertial interaction requires evaluating the foundation stiffness (termed foundation “impedance”), and the foundation-structure interaction (FSI). It is pointed out that the substructure approach is applicable only to linear problems. These problems typically involve strains in the soil not exceeding 0.1%.

The first step in SSI analysis is to determine the variation of the free-field motion within the ground regions, which interact with the foundation. This constitutes the free-field site response problem. Next, the kinematic response problem is concerned with the interaction of the foundation with the surrounding soil (driven by the free-field site response), which results in the foundation input motion. Following is the foundation stiffness problem, which provides the soil-foundation interface boundary flexibilities or compliances. Lastly, the inertial interaction problem involves determining the structural response of the bridge as affected by the soil-foundation boundary flexibilities and the foundation input motion. These four subproblems are solved in that order to evaluate the effects of SSI on the seismic demand on the superstructure and they form the state-of-the-art of analyzing a bridge under seismic loading. Extensive information on the subject can be found in publications by Luco (1982), Wolf (1985), Gazetas (1983), Pender (1993), Gazetas & Mylonakis (1998), Penzien & Tsang (1999), and Mylonakis et al. (2003).

The main concern of this study is determining the inertial response of a bridge pier on a pile foundation, due to earthquake loading. To analyze the problem, two major approaches are currently employed: (1) the “elastodynamic” method developed and practiced in the nuclear power industry for large mat-type foundations and (2) the so-called p-y method developed and practiced in the offshore oil industry for pile

foundations. Regarding the other part of inertial response, the FSI problem is a classical structural analysis problem, in which the foundation can be represented with springs and dashpots attached to the foundation-structure interface. In the case of seismic excitation the structure is subjected to the foundation input motion. Many methods exist for the evaluation of the response of the superstructure, varying from simple stick models to complicated finite element models. In the following sections, the available methods for treating the foundation impedance problem are presented.

1.2 The “Elastodynamic” method

This method is based on the well-established elastodynamic theory of wave propagation in a linear elastic or viscoelastic soil medium. The fundamental element of this method is the constitutive relation between an applied harmonic point load and the resulting dynamic response displacements within the medium called the dynamic Green’s functions. Since these functions apply only to cases where strains are proportional to stresses, they are valid only for linear SSI problems. This method was developed for the needs of the nuclear industry where the foundations are essentially rigid, and is valid for the rigid-like bridge-foundations such as spread footings, caissons and large-diameter shafts. By adopting an equivalent linearization procedure, the method has been extended to accommodate soil nonlinearities (Tseng & Hadjian, 1991).

1.3 The empirical “p-y” method (foundation impedance problem)

This method was originally developed for the evaluation of pile-foundation response to lateral loads (Matlock & Reese, 1961; Matlock, 1970; Reese et al., 1974) of

offshore structures. It characterizes the lateral soil resistance per unit pile length, p , as a function of the lateral pile displacement, y . The p - y relation is generally developed on the basis of an empirical curve, which reflects the nonlinear resistance of the soil in the vicinity of the pile at a specified depth. Based on the same concept, the method has been extended to treat axial soil-pile interaction in terms of the soil reaction along the pile shaft, t , per unit pile length as a nonlinear function of the corresponding axial soil displacement, z (t - z curve). The soil resistance at the pile tip, Q , to a tip soil displacement, d , is treated in an analogous way, resulting in the so-called Q - d curve. By utilizing a pertinent set of p - y , t - z and Q - d curves, the response of a pile subjected to general three-dimensional loadings applied at the pile head can be solved using the beam on Winkler foundation theory.

Since this method depends primarily on soil strength and stiffness parameters and does not incorporate soil inertia and damping, it is, strictly speaking, only applicable for capacity evaluations (under pseudo-static conditions) of slender-pile foundations and is not suitable for seismic demand evaluations. However, as in the elastodynamic method, this method is also extended to demand analysis using pertinent modifications (Nogami et al 1992; Boulanger et al. 1999).

1.4 Superstructure analysis (FSI problem)

The superstructure (bridge pier and deck) can be modeled using standard finite element approaches capable of performing linear or non-linear analyses in the time domain. However, when the shape of the pier is simple enough to be approximated by a stick model, a variety of methods can be used, employing the theory of the single-degree-

of-freedom (SDOF) oscillator. These methods (which are widely used in modern performance-based design) aim at capturing the non-linear response of the superstructure in a simple cost-effective manner. They can be roughly classified into three main groups: (i) linearization techniques in which the actual inelastic system is replaced by an equivalent elastic system; (ii) simplified relations between structural strength and inelastic displacements ($R-\mu$ relations) and (iii) capacity-spectrum methods superimposing seismic spectra and force-displacement (“push-over”) diagrams of the structure.

1.5 Research Objectives

This study is primarily concerned with the foundation-impedance and FSI problems, which constitute the inertial interaction problem. For the solution of the first problem, a finite-element (F.E.) formulation is proposed which falls in the realm of “elastodynamic” methods (chapters 1 and 2). This formulation makes use of the axisymmetry of a single cylindrical pile, to substantially reduce the computational effort. Using the results of the F.E. formulation, impedance curves for single piles are derived, as well as simplified expressions that take into account the effects of soil inhomogeneity. Special attention is paid to the case of simultaneous vertical and radial inhomogeneity in the medium, which come forth as a result of overburden, pile installation, and head loading. Specifically, while overburden tends to make soil stiffer with depth, pile installation and subsequent loading tend to “soften” the soil in the vicinity of the pile, generating inhomogeneity in the horizontal (radial) direction. The effects of vertical and radial inhomogeneity are explored both separately and combined, and found to reduce

both stiffness and damping at the pile. In addition, using the F.E. solutions of displacements and stresses along the pile shaft, equivalent Winkler springs along the pile are determined, to be used in simplified Beam-on-Dynamic-Winkler-Foundation (BDWF) models. A simplified method to make use of the single pile solutions to account for group effects is also presented.

The results of this research are used to compare predictions of the dynamic response of real foundations against field measurements. The study includes two unique case studies of pile-supported bridges on soft ground. The first study involves Ohba Ohashi, an instrumented bridge in Japan founded on extremely soft clay of high-plasticity, which has been subjected to several seismic motions of moderate intensity.

The second case study deals with the collapse of Hanshin Expressway in the 1995 Kobe earthquake. The objective of the case study is to use the proposed analysis tools for the evaluation of the foundation stiffness and to propose and explore simple techniques to estimate inelastic seismic demand on bridge piers. The analyses shed light on the role of soil-structure interaction in the collapse.

1.6 Organization

In Chapter 1 a brief introduction to soil-structure-interaction problems is presented.

Chapter 2 deals with the first part of *inertial response*, the “foundation impedance” problem. A F.E. formulation is presented for the axisymmetric problem of a cylindrical pile subjected to axially symmetric loading. The pile and the surrounding soil are treated as solids of revolution. This formulation incorporates: infinite elements (for

the static case), special absorbing boundaries (for the dynamic case) as well as a sophisticated integration scheme to treat singularities encountered in the derivation of element stiffness matrices. The analysis is performed in the frequency domain (steady-state response) and includes the special case of static loading (zero frequency). A parametric investigation is performed to determine the effect of simultaneous vertical and radial soil inhomogeneity in the dynamic impedance of an axially loaded pile. The formulation is then extended to determine the dynamic impedance of a pile-group foundation to vertical loading. Simplified expressions for estimating the axial dynamic impedance of a single pile are provided.

In Chapter 3, a F.E. formulation is developed to analyze a pile subjected to horizontal and moment head loading. These two loadings are examined separately using Fourier series expansions. It is shown that for the case of a horizontal pile-head force and moment, only the first term of the expansion is necessary. The corresponding “infinite” elements and absorbing boundaries presented in Chapter 2 are modified to account for the extra tangential degree of freedom resulting from axisymmetric loading. Simplified closed form expressions are presented for estimating the pile-head dynamic impedance as well as for Winkler springs along the pile shaft for horizontal and moment head loading.

In Chapter 4 the F.E. formulation is used to assess the response of actual single piles and pile groups under vertical and horizontal harmonic excitation. Simplified procedures, which make use of the derived closed form expressions, are proposed and theoretical results (pile impedance and damping) are checked against actual measurements.

In Chapter 5, the performance of Ohba bridge (Ohba Ohashi) is examined. The response of the bridge was recorded by the Institute of Technology of Shimizu Corporation during several earthquakes having PGA's at ground level up to approximately 0.11g. The soil and the superstructure behaved linearly during these earthquakes, which makes the available measurements suitable for evaluating the proposed elastodynamic analysis tools.

In Chapter 6, the collapse of a section of the elevated Hansin expressway in Kobe is studied. In this case, both soil and superstructure behave nonlinearly. For the case of the soil, an iterative linear procedure is carried out to compute the free-field response. The F.E. formulations of chapters 2 and 3 are used to evaluate the foundation impedances. For the FSI problem, nonlinear inelastic analyses are performed to determine the ductility demand imposed to the bridge pier.

Chapter 7 contains conclusions and recommendations for future research.

CHAPTER 2

ELASTODYNAMIC ANALYSIS OF AXIALLY-LOADED PILES IN INHOMOGENEOUS SOIL

2.1 INTRODUCTION

The problem of determining the elastic settlement of a pile has been investigated extensively over the past forty years, starting from the simplest case of static loading and homogeneous soil, and progressing to the more complicated cases of dynamic loading, inhomogeneous soil and group action (Poulos & Davis 1980, Banerjee & Sen 1987, Novak 1991, Fleming et al 1992, Pender 1993, Gazetas & Mylonakis 1998, Randolph 2003). Most early studies considered the static case to determine pile head settlement for foundation design. More recently, the dynamic case has been considered for designing piles to support vibrating machines and/or resist earthquake and blast loads. In most studies, the supporting soil has been treated as either homogeneous or layered, with constant material properties in each layer (Novak 1974, Kaynia & Kausel 1982). In a more realistic case it should be considered combined vertical and radial inhomogeneity, to account for the weakening of the soil around a pile due to installation and loading effects. Only a handful of studies on the effect of radial inhomogeneity on pile response are available today. Most of these studies adopt simplified plane-strain models which cannot capture adequately salient features of the problem such as static stiffness and cutoff frequency (Novak & Sheta 1980; Veletsos & Dotson, 1986; Poulos 1988; Han & Sabin 1995; Michaelides et al. 1998). No systematic investigations of the effect of

combined vertical and horizontal soil inhomogeneity on dynamic pile response are available.

The available solutions indicate that radiation damping in the soil generally decreases as a result of soil inhomogeneity, leading to increased response at the pile head for a given dynamic load. These effects are expected to be stronger under combined vertical and horizontal inhomogeneity, having potentially significant design implications (Poulos 1988, Novak 1991).

2.1.1 Available methods of solution

Several methods are presently available for the analysis of the dynamic response of piles. These can be roughly classified into three groups: Winkler methods, Finite Element and Boundary element methods. Simplified analytical models based on the theory of a beam on elastic foundation (Winkler models) are appealing because they can easily incorporate variation of soil properties with depth and require smaller computational effort than rigorous numerical formulations (Novak 1974; O'Rourke & Dobry 1978; Nogami, 1983; Veletsos & Dotson 1986; Nogami & Konagai 1987, Mylonakis & Gazetas 1998, Guo 2000). Their main drawback lies in the assessment of the modulus of the Winkler springs (modulus of subgrade reaction) and the difficulty in handling radial soil inhomogeneity and group effects.

Boundary element methods are essentially semi-analytic formulations based on fundamental solutions (Green's functions) for the displacement field due to point loads acting in the interior of an unbounded elastic or viscoelastic solid (Kaynia & Kausel

1982; Banerjee & Sen 1987; Poulos 1988; El-Sharnouby & Novak 1990; El-Marsafawi et al. 1992). These methods have the advantage of satisfying the governing elastodynamic equations at all points within the domain and the radiation condition at infinity, while they require discretization only along the boundaries. This leads to a relatively small number of elements. On the other hand, these methods encounter difficulties in handling inhomogeneous media because of the lack of pertinent fundamental solutions. For more details on these methods, the reader is referred to the books by Wolf (1985), Manolis and Beskos (1988) and Banerjee (1995).

Owing to the aforementioned difficulties, the finite-element technique has become increasingly popular over the years for analyzing dynamic pile response (Blaney et al. 1976; Wolf & Von Arx 1978; Angelides & Roesset 1980; Wolf et al. 1981; Waas & Hartmann 1984; Zhang & Kimura 2002). The method has been criticized for certain difficulties when dealing with unbounded media (especially in three dimensions). These include: large mesh size and associated computational effort, “box” effects due to wave reflections at the boundaries of the mesh, numerical difficulties when dealing with combinations of very soft and very stiff materials. Theoretical and technological advances over the past two decades have overcome these problems making finite elements an appealing tool to Geotechnical engineers for analyzing the dynamic response of inhomogeneous media in two and three dimensions.

To alleviate the aforementioned problems, the mesh size is kept reasonably small by exploiting the advantages of axisymmetric analysis (spatial Fourier expansions) and using variable element size. In addition, special infinite elements and transmitting boundaries are utilized to treat box effects, while novel numerical techniques are

employed to address singularities in element stiffness matrices close to the axis of symmetry.

2.2 PROBLEM DEFINITION AND MODEL DEVELOPMENT

In this chapter, a finite element solution is proposed for analyzing the response of an axially loaded single pile in inhomogeneous viscoelastic soil. To this end, a finite element computer code named K-PAX is developed. Special attention is paid to modeling soil inhomogeneity, to properly capture its effect on dynamic pile response. The scope of the study is threefold: (a) to examine the effect of the combined vertical and radial soil inhomogeneity on dynamic pile response; (b) to compare the dynamic impedance (stiffness and damping) of piles embedded in different soil profiles; (c) to derive coefficients of subgrade reaction to be used in the calibration of Winkler models;

The system considered is depicted in Figure 2.1. A solid cylindrical pile embedded in a soil layer over rigid or elastic bedrock, subjected to an axial harmonic head load. For the simulation of the pile and the surrounding soil, quadratic isoparametric triangular elements are used in axisymmetric mode. Use of solid elements instead of beam elements allows for Poisson effects and associated radial displacements in the pile to be captured. In addition, linear variation of Young's modulus within the elements is considered to more accurately represent soil inhomogeneity.

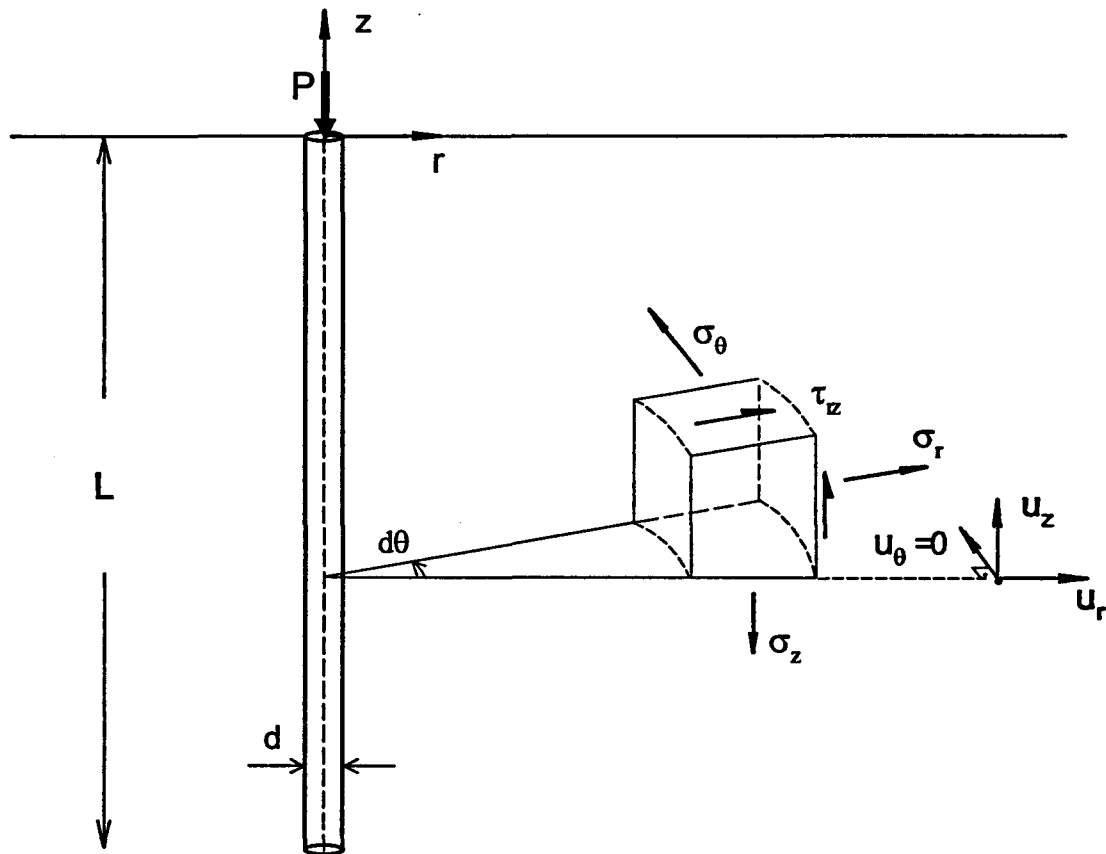


FIGURE 2.1: Axially-loaded single pile. Model geometry and notation for stresses and displacements.

2.2.1 Finite Element Mesh

The mesh is designed to be finer close to the pile (where large stress gradients are anticipated) and becomes coarser towards the boundaries (Figure 2.2). The change in element size is done accordingly.

With reference to the size of the mesh, tall meshes ($H/L > 2$) were found to produce results that are in close agreement with rigorous boundary element solutions and are generally recommended (Figure 2.2). However, in the case of very slender piles ($L/d > 80$) this approach may result in an excessive number of elements.

To balance computational effort and accuracy, the optimum mesh height, H , was found to be 1.25 pile lengths for slender piles ($L/d > 60$) and 2.5 pile lengths for short piles ($L/d < 20$). The aspect ratio of the mesh, H/S (Figure 2.2) was taken equal to 1 in all cases. To maximize accuracy, the aspect ratio of the triangular elements is kept very close to unity close to the pile and less than 2 elsewhere.

2.2.2 Isoparametric finite element formulation

The axial problem is axisymmetric, thus each node has only two degrees of freedom: a radial displacement u , and a vertical displacement w (Figure 2.1).

In cylindrical coordinates, the stress-strain relationships of a linearly elastic isotropic solid are:

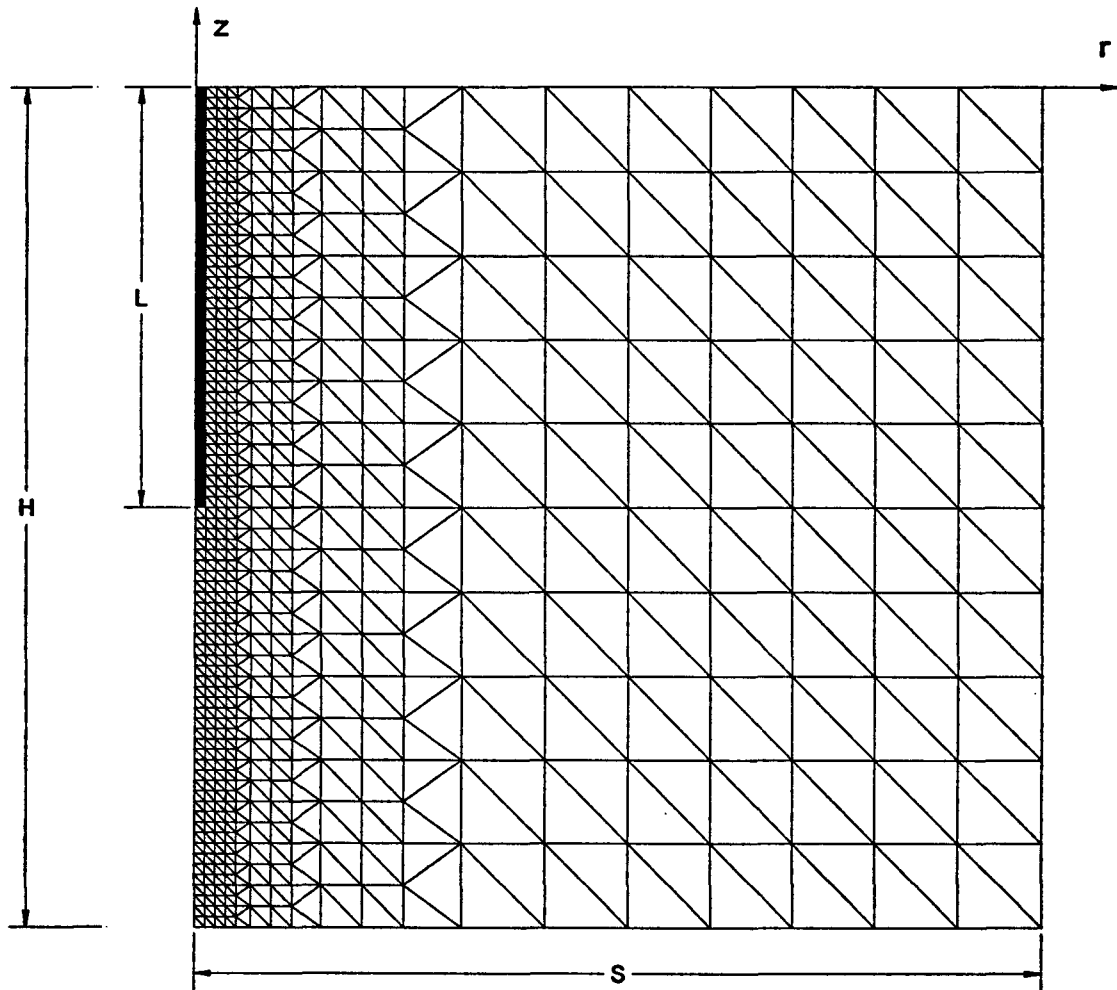


FIGURE 2.2: Typical F.E. mesh used in this study

$$\begin{aligned}
\sigma_{rr} &= \lambda \frac{1-\nu}{\nu} \varepsilon_{rr} + \lambda \varepsilon_{\theta\theta} + \lambda \varepsilon_{zz} \\
\sigma_{\theta\theta} &= \lambda \varepsilon_{rr} + \lambda \frac{1-\nu}{\nu} \varepsilon_{\theta\theta} + \lambda \varepsilon_{zz} \\
\sigma_{zz} &= \lambda \varepsilon_{rr} + \lambda \varepsilon_{\theta\theta} + \lambda \frac{1-\nu}{\nu} \varepsilon_{zz} \\
\tau_{rz} &= \lambda \frac{1-2\nu}{2\nu} \gamma_{rz} \\
\tau_{r\theta} &= 0 \\
\tau_{z\theta} &= 0
\end{aligned} \tag{2.1}$$

where λ denotes the first Lamé constant and ν the Poisson's ratio of the material.

The corresponding axisymmetric strain – displacement relationships, omitting the anti-plane terms, are:

$$\begin{aligned}
\varepsilon_{rr} &= \frac{\partial u}{\partial r} \\
\varepsilon_{\theta\theta} &= \frac{u}{r} \\
\varepsilon_{zz} &= \frac{\partial w}{\partial z} \\
\gamma_{rz} &= \frac{\partial u}{\partial z} + \frac{\partial w}{\partial r} \\
\gamma_{r\theta} &= 0 \\
\gamma_{z\theta} &= 0
\end{aligned} \tag{2.2}$$

where r and z denote the radial and vertical coordinates respectively [Note that the sign convention in Eqns. (2.2) is in accordance with elasticity notation].

Expressed in matrix form, the constitutive relationship in (2.1) is:

$$\boldsymbol{\sigma} = \mathbf{D} \boldsymbol{\varepsilon} \tag{2.3}$$

Where σ is a vector of element stresses, D is the constitutive (stress-strain) matrix and ϵ is a vector containing the element strains.

Equation (2.2) can be put in matrix form as (Appendix A)

$$\epsilon = S u \quad (2.4)$$

where S is a strain-compatibility matrix relating strains to stresses and u is a vector containing the element displacements.

The displacements within the element are related to the nodal displacements through an interpolation matrix N_{QT} according to

$$u = N_{QT} U \quad (2.5)$$

where U contains the element's nodal displacements, and the subscript QT refers to the quadratic triangular elements used in this study. In eq. (2.5), N_{QT} interpolates the displacement variables throughout the element. While, N_{QT} can be written with its elements being explicit functions of the model's spatial variables, r , z and θ , it is frequently more convenient to express the relationship between (r, z, θ) and u parametrically using computational coordinates (ξ, η, ρ) . Thus

$$u = N_{QT}(\xi, \eta, \theta) U \Rightarrow \begin{bmatrix} r \\ z \\ \theta \end{bmatrix} = N_{QT} R \quad (2.6)$$

where R is a vector of the elements nodal coordinates. Substituting equation (2.5) into (2.4) one obtains the expression

$$\epsilon = (S N_{QT}) U = B U \quad (2.7)$$

where $\mathbf{B} = \mathbf{S} \mathbf{N}_{QT}$ is the strain-displacement matrix.

Upon changing variables so as to integrate over the volume using computational coordinates, the corresponding element stiffness matrix is given by

$$\mathbf{K}_i = \Delta\theta \iint \mathbf{B}^T \mathbf{D} \mathbf{B} r |J| d\xi d\eta \quad (2.8)$$

In eq. 2.8, r is the radial coordinate, J is the Jacobian matrix relating the coordinates of model space to computational space (Appendix A) and $\Delta\theta$ is the angle of the element ‘slice’ in the axisymmetric mode (Figure 2.1). Note that for $\Delta\theta = 2\pi$, the element becomes toroidal. Parameters ξ and η are coordinates in computational space and are used for the numerical evaluation of the integral (Appendix A).

Based on the above notation, the element mass matrix can be written as (Bathe, 1996)

$$\mathbf{M}_i = \Delta\theta \iint \rho \mathbf{N}_{QT}^T \mathbf{N}_{QT} r |J| d\xi d\eta \quad (2.9)$$

where ρ is mass density.

The vertical motion of the pile head is modeled by constraining the corresponding nodes to experience a common displacement. This assumption is realistic since the axial loads carried by the pile are usually transferred from a rigid cap.

The derivation of the equation of motion can be found in textbooks (Bathe 1996) and need not be repeated here. At the structural level, the equation of motion is

$$\mathbf{M} \ddot{\mathbf{U}} + \mathbf{C} \dot{\mathbf{U}} + \mathbf{K} \mathbf{U} = \mathbf{P} \quad (2.10)$$

where $\mathbf{M}$ is the assembled global mass matrix, $\mathbf{K}$ the global stiffness matrix, $\mathbf{C}$ the global damping matrix, $\mathbf{P}$ the global load vector and $\mathbf{U}$ is the global nodal displacement vector.

The element load vector is obtained as

$$\mathbf{P}_i = \Delta\theta \int \mathbf{N}^T \mathbf{p} \, dA \quad (2.11)$$

where $\mathbf{p}$ stands for distributed forces over the element surface.

In this study the harmonic steady-state problem is examined, leading to solutions that are functions of the excitation frequency. Results from such an analysis can be used for solving problems in the time domain by means of the discrete Fourier transform (Veletsos & Ventura, 1986).

For harmonic steady-state oscillations, the loading-vector is

$$\mathbf{P} = e^{i\omega t} \mathbf{P}_0 \quad (2.12)$$

where $\mathbf{P}_0$ is (usually) a real-valued vector. To account for material damping it is convenient to introduce the complex Young's modulus

$$E^* = E (1 + 2 i \beta) \quad (2.13)$$

where β is the dimensionless hysteretic material damping coefficient of the soil (Kramer, 1995). The advantages of this representation over viscous damping are well known and need not be repeated here.

To solve the steady-state problem, we consider a solution of the form

$$\mathbf{U} = e^{i\omega t} \mathbf{U}_0 \quad (2.14)$$

where U_0 is a complex-valued displacement vector encompassing both the amplitude of the response as well as the phase difference between load and displacement. Substituting eq. (2.14) in eq. (2.10) and considering constant material damping in the domain, one obtains the well known solution

$$U_0 = [K(1 + 2i\beta) - \omega^2 M]^{-1} P_0 \quad (2.15)$$

which, naturally is independent of time t .

In K-PAX, the solution to equation (2.15) is obtained using standard routines available in MATLAB.

2.2.3 Variable modulus within elements

Since the effect of soil inhomogeneity is a main objective of this study, the variation of elasticity modulus needs to be described as accurately as possible. To achieve this, linear variation in Young's modulus within the elements is considered by linearly interpolating the values of the modulus at the corner nodes. The use of variable modulus is desirable to minimize wave reflections due to discontinuities in elastic modulus at the interfaces between neighboring elements. As soil density varies spatially much less than stiffness, it is assumed to remain constant within the elements.

Considering a complex modulus of elasticity yields

$$E = (1 + 2i\beta) N_{\text{Linear}} \begin{bmatrix} E_1 \\ E_2 \\ E_3 \end{bmatrix} \quad (2.16)$$

where E_1 , E_2 and E_3 denote the values of E at the corners of the element; N_{linear} an interpolation matrix containing the linear shape functions

$$N_{\text{Linear}} = [\xi \quad \eta \quad 1-\xi-\eta] \quad (2.17)$$

which are used in the computation of stiffness matrix K in eq. (2.8).

2.2.4 Transmitting boundaries

Dynamic finite-element solutions require special care at the boundaries, since the outgoing waves tend to reflect at the boundaries and get trapped inside the domain, generating spurious resonances known as “box effect”. This is a major drawback of the F.E. technique for the simulation of propagating waves in unbounded media. There is an extensive literature on pertinent “silent” boundaries developed to mitigate this problem (Lysmer & Kuhlemeyer 1969; Wolf 1986; Kausel, 1988; Kellezi, 2000). In K-PAX, the transmitting boundary developed by Kellezi (2000) is adopted. In this formulation, appropriate springs and dashpots, expressed through pertinent stiffness and damping matrices, are attached to each node of the boundaries as shown in Figure 2.3. The formulation is as follows:

The global equation of motion can be decomposed as

$$M \ddot{U} + (C + C_{\infty}) \dot{U} + (K + K_{\infty}) U = P \quad (2.18)$$

where K and C denote the familiar stiffness and damping arising from the contribution of the model domain, and K_{∞} and C_{∞} those arising from the transmitting boundary.

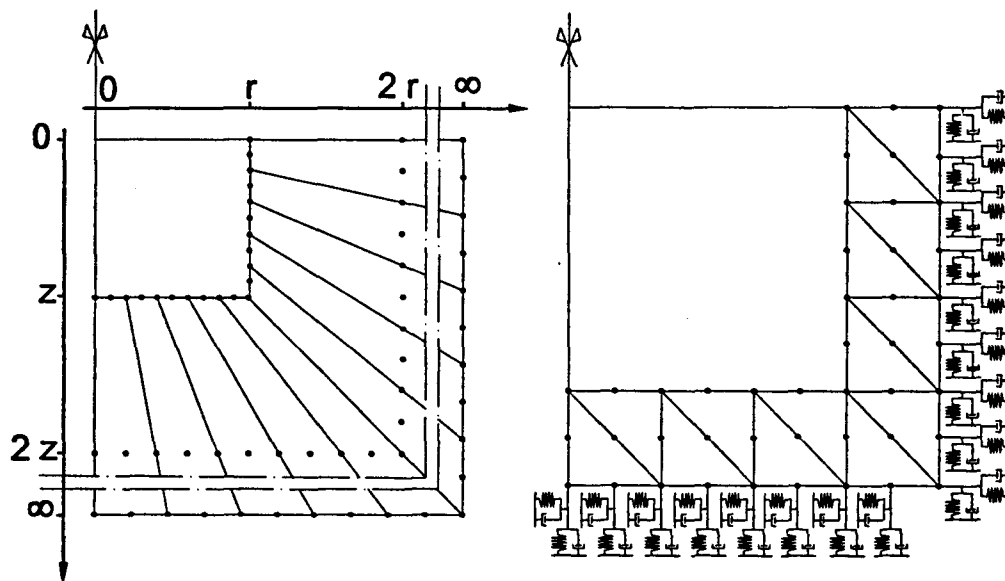


FIGURE 2.3: Simulation of the unbounded medium using infinite elements (static case); (b) Simulation of the infinite medium using 1-D “silent” elements at the edges of the mesh (dynamic case)

The consistent stiffness matrix at the boundary surface is assembled from the stiffness contribution of the elements at the boundary, each of which is given by:

$$\mathbf{K}^e = \int_{\Gamma_\infty} \mathbf{N}_Q^T \mathbf{D}_K \mathbf{N}_Q d\Gamma \quad (2.19)$$

where Γ_∞ is the surface of the model's outer boundary, $\mathbf{D}_K$ is the constitutive stiffness matrix and $\mathbf{N}_Q$ is a displacement interpolation matrix.

The damping matrix is obtained in a similar manner:

$$\mathbf{C}^e = \int_{\Gamma_\infty} \mathbf{N}_Q^T \mathbf{D}_C \mathbf{N}_Q d\Gamma \quad (2.20)$$

where $\mathbf{D}_C$ is the constitutive damping matrix.

The following constitutive matrices are derived for the case of axisymmetric analysis (Appendix A):

$$\mathbf{D}_K = \frac{\rho V_p^2}{r} (\bar{\mathbf{n}} \cdot \bar{\mathbf{r}}) (\bar{\mathbf{n}} \cdot \bar{\mathbf{n}}^T) + \frac{\rho V_s^2}{r} (\bar{\mathbf{n}} \cdot \bar{\mathbf{r}}) \{ [\mathbf{I}] - (\bar{\mathbf{n}} \cdot \bar{\mathbf{n}}^T) \} \quad (2.21)$$

$$\mathbf{D}_C = \rho V_p (\bar{\mathbf{n}} \cdot \bar{\mathbf{r}}) (\bar{\mathbf{n}} \cdot \bar{\mathbf{n}}^T) + \rho V_s (\bar{\mathbf{n}} \cdot \bar{\mathbf{r}}) \{ [\mathbf{I}] - (\bar{\mathbf{n}} \cdot \bar{\mathbf{n}}^T) \} \quad (2.22)$$

where

V_p and V_s = velocities of P and S waves in the soil, respectively

ρ = soil mass density

$\bar{\mathbf{n}}$ = unit vector normal to the boundary

$\bar{\mathbf{r}}$ = normalized wave direction vector

r = radial coordinate

$[I]$ = identity matrix.

The above formulation is designed to absorb body i.e., P and S waves. The formulation to absorb surface waves is very similar and was found to produce results which are very close to those produced by eq. (2.21) and (2.22). For simplicity, the body waves formulation was used in K-PAX. Note that no special mass matrices are needed in the boundaries, since the effect of soil inertia is incorporated as radiation damping in the damping matrix.

2.2.5 Infinite elements

For low frequency body waves, or static analysis, Kellezi's (2000) transmitting boundaries poorly represent the medium outside the mesh. For the static case, special infinite elements are implemented following Marques and Owen (1984) and Cook et al. (1989). At the boundaries, 9-noded quadratic quadrilateral elements of infinite extent are used to model the transmitting boundary (Figure 2.3). This is achieved by using two sets of shape functions: (i) standard quadratic shape functions to represent the field variable (displacement) and (ii) "growth" shape functions to incorporate the element's infinite geometry. The "growth" shape functions increase without limit as the local coordinate approaches one. Infinite elements of this kind are called "mapped" and are formed based on the location of the boundary nodes with respect to a "pole". In K-PAX the pole is taken at the origin of the coordinate system (as done in Kellezi's formulation).

2.2.6 Integration scheme in the Axisymmetric Model Domain

Concerning the integration of the stiffness matrix, there are difficulties associated with the axisymmetric geometry. To eliminate the problem, consider the stiffness matrix written as:

$$\mathbf{K}_i = \Delta\theta \iint \mathbf{B}^T \mathbf{D} \mathbf{B} r |J| d\xi d\eta = \Delta\theta \iint \left(\frac{1}{r} \mathbf{B}_0 + \mathbf{B}_1 \right)^T \mathbf{D} \left(\frac{1}{r} \mathbf{B}_0 + \mathbf{B}_1 \right) r |J| d\xi d\eta$$

Using straightforward algebraic manipulations, $\mathbf{K}_i$ can be cast in the form (Appendix A)

$$\mathbf{K}_i = \Delta\theta \iint \left[\frac{1}{r} \mathbf{B}_0^T \mathbf{D} \mathbf{B}_0 + \mathbf{B}_0^T \mathbf{D} \mathbf{B}_1 + \mathbf{B}_1^T \mathbf{D} \mathbf{B}_0 + r \mathbf{B}_1^T \mathbf{D} \mathbf{B}_1 \right] |J| d\xi d\eta \quad (2.23)$$

In equation (2.23), $\mathbf{B}_0$ contains linear combinations of the polynomial shape functions and $\mathbf{B}_1$ contains derivatives of the shape functions with respect to ξ and η . The presence of radial coordinate r in the denominator of the first term in eq. (2.23) leads to integration errors if Gauss-Legendre (G-L) quadrature stencils (sets of abscissas and weights) are used, as is commonly done. This is because the denominator becomes zero along the axis of symmetry ($r = 0$) and thus the integral becomes singular. In the case where the limits of integration are far away from the singular point, no difficulty arises. However, for elements near the axis of symmetry, the singular point lies near the limits of integration and special care is needed to integrate accurately the stiffness matrix. A tedious solution is to use many points from the G-L stencil and evaluate the integral repeatedly before deciding on the appropriate number of points. Following Price (2003), a more rational technique is used in K-PAX that allows apriori determination of a set of “exact” integration points and weights.

While the rest of the terms inside the integral are linear combinations of polynomials and can be exactly integrated using the G-L stencils, the first term in eq. (2.23) contains a singularity so that a different stencil is needed. Price (2003) has developed a stencil called a Rational-Gauss (R-G) stencil for the exact evaluation of these terms, as long as the denominator is a linear function of no more than one of the variables of integration. In the case of 6-noded triangular elements and 9-noded quadrilateral elements, the numerical integration can be performed by using the R-G stencil with respect to the singular direction, and the G-L stencil with respect to the other direction. For the triangular elements this is done by orienting the element without loss of generality, in the model space (r,z) to have one axis parallel to the axis of symmetry. The derivation of the integration abscissas and weights can be found in the work of Price (2003).

With reference to the order of integration, the orders of the associated matrices are:

$$\begin{aligned}
 \mathbf{B}_0 &= O(N) = O(\xi^2) \\
 \mathbf{B}_1 &= O(\xi) \\
 \mathbf{D} &= O(\xi) \\
 r &= O(\xi)
 \end{aligned}
 \tag{2.24}$$

From equations (2.23) and (2.24) it is evident that the maximum order in the integral's numerator is five, corresponding to the singular term $\frac{1}{r} \mathbf{B}_0^T \mathbf{D} \mathbf{B}_0$.

If we were to use the standard G-L stencil we would require more than 7 points to accurately evaluate the integral for the triangular element and we would need to evaluate

it repeatedly before determining the number of points to be used. On the other hand, using the R-G stencil we need 3 points in the normal G-L direction and 3 in the singular one, thus needing a total of 9 points to integrate the stiffness matrix exactly. Similarly, for the infinite elements using the rational Gauss stencil we require a set of 4 rational-Gauss points in the singular direction and 2 Gauss-Legendre in the other direction, which gives a total of 8 points.

2.3 FORMULATION OF RADIAL INHOMOGENEITY

2.3.1 Treatment of radial soil inhomogeneity in previous studies

In their pioneering study, Novak and Sheta (1980) modeled the radial soil inhomogeneity around a pile with a toroidal inner zone of reduced elastic properties (“near-field”), and an outer zone (“far-field”) of constant properties using an axisymmetric plane-strain model. This formulation introduces a discontinuity in soil stiffness at the boundary of the two zones, thus making the results sensitive to both frequency and ring radius. To prevent wave reflections at the discontinuity, Sheta and Novak considered the inner zone to be massless. In later studies, Veletsos and Dotson (1986) and Kim (1987) introduced a smooth transition of soil properties to the far-field, by using concentric soil rings of increasing elastic properties. The mass of the ring is also considered. More recently, Han & Sabin (1995), Michaelides et al. (1998) and El Naggar (2000) considered a continuous variation of elastic soil properties with radial distance from the pile and derived solutions for the plane-strain axisymmetric problem.

With reference to stress-induced inhomogeneity, it is assumed that, before loading the pile, soil is laterally homogeneous, implying that installation effects have

“dissipated”. Considering a static shear stress τ_0 acting at the pile-soil interface, the variation of shear stress with radial distance from the pile is given to a good approximation by the solution of Randolph and Wroth (1978):

$$\tau(r) = \tau_0 \left(\frac{d}{2r} \right) \quad (2.25)$$

Let's also consider the hyperbolic shear stress - shear strain relation (Kuwabara 1991)

$$\tau = \frac{\gamma}{\frac{1}{G_0} + \frac{\gamma}{\tau_f}} \quad (2.26)$$

where γ = shear strain; G_0 = initial (low strain) shear modulus; τ_f = shear stress at failure.

The variation in secant shear modulus $G(r) = \tau(r) / \gamma(r)$, with radial distance r is derived from equations (2.25) and (2.26) as

$$\frac{G(r)}{G_\infty} = 1 - \Lambda \left(\frac{d}{2r} \right) \quad (2.27)$$

where G_∞ = shear modulus at infinity; the factor $\Lambda = \tau_0 / \tau_f$ expresses the level of applied shear stress at the soil-pile interface. In deriving the above equation, it was implicitly assumed that G_∞ is identical to G_0 , as shear stresses diminish with r .

Using experimental data for a variety of soils and accounting for dynamic effects, Michaelides et al. (1998) derived a modified formula for the variation of soil shear modulus around the pile

$$\frac{G(r)}{G_{\infty}} = 1 - \left\{ \Lambda \frac{d}{2r} F(a_r) \right\}^{0.72} \quad (2.28)$$

In the above equation, $F(a_r)$ is a dimensionless coefficient given by

$$F(a_r) \approx \begin{cases} 1 & , \text{ if } a_r < 1 \\ a_r^{0.57} & , \text{ if } a_r > 1 \end{cases} \quad (2.29)$$

where a_r is the well known dimensionless frequency

$$a_r = \omega r \sqrt{\frac{\rho}{G(r)}} \quad (2.30)$$

Λ is an empirical loading intensity factor accounting for the reduction in shear modulus near the pile due to loading and is a function of the physical properties of the soil. Λ is given by

$$\Lambda = 2700 \frac{\tau_{c0}}{f_s} \frac{10^{-1.4 (I_p/\lambda)}}{G_{\infty} / \alpha S_u} \quad (2.31)$$

where λ is a function of the soil plasticity index I_p

$$\lambda = 0.002(I_p)^2 + 0.25(I_p) + 60 \quad (2.32)$$

τ_{c0} is the amplitude of the imposed cyclic shear stress at the pile-soil interface, and f_s is the frictional capacity of the pile-soil interface expressed as a fraction of the undrained shear strength S_u

$$f_s = \alpha S_u \quad (2.33)$$

Coefficient α varies approximately from 0.4 to 1 and can be obtained from standard geotechnical literature (Bowles, 1996).

As the above formulation is cumbersome for the purposes of this study, the following simplified expression is adopted:

$$\frac{G(r)}{G_{\infty}} = \frac{E(r)}{E_{\infty}} \approx 1 - \left\{ A \frac{d}{2r} \right\}^{3/4} \quad (2.34)$$

in which $A = A(\gamma, \omega)$ is a “load intensity factor” or a “radial inhomogeneity factor”, encompassing the combined effects of soil plasticity, inertia, and interface effects. The similarity between equations (2.27) and (2.34) is evident. Unless specifically stated otherwise, equation (2.34) will be used to describe radial soil inhomogeneity in this study. Figure 2.4 shows typical diagrams obtained from equations (2.34) and (2.27) computed for different values of the radial inhomogeneity factor A . It is observed that the shear modulus varies most rapidly in the immediate vicinity of the pile, while at larger radial distances ($r/d \geq 5$) tends asymptotically to the far-field value. For the needs of the parametric investigations presented in this paper, it is assumed that parameter A attains the distinct values 0.1, 0.5 and 0.8 corresponding to low, medium and high stress levels, respectively. As evident in Figure 2.4, these three values correspond to soil moduli at the pile-soil interface equal to 80, 40 and 15 percent of the far-field modulus, respectively.

The four different profiles used in this study are depicted in Figure 2.5:

- A homogeneous soil profile characterized by a constant soil modulus E_s

$$E_s(r, z) = E_s \quad (2.35)$$

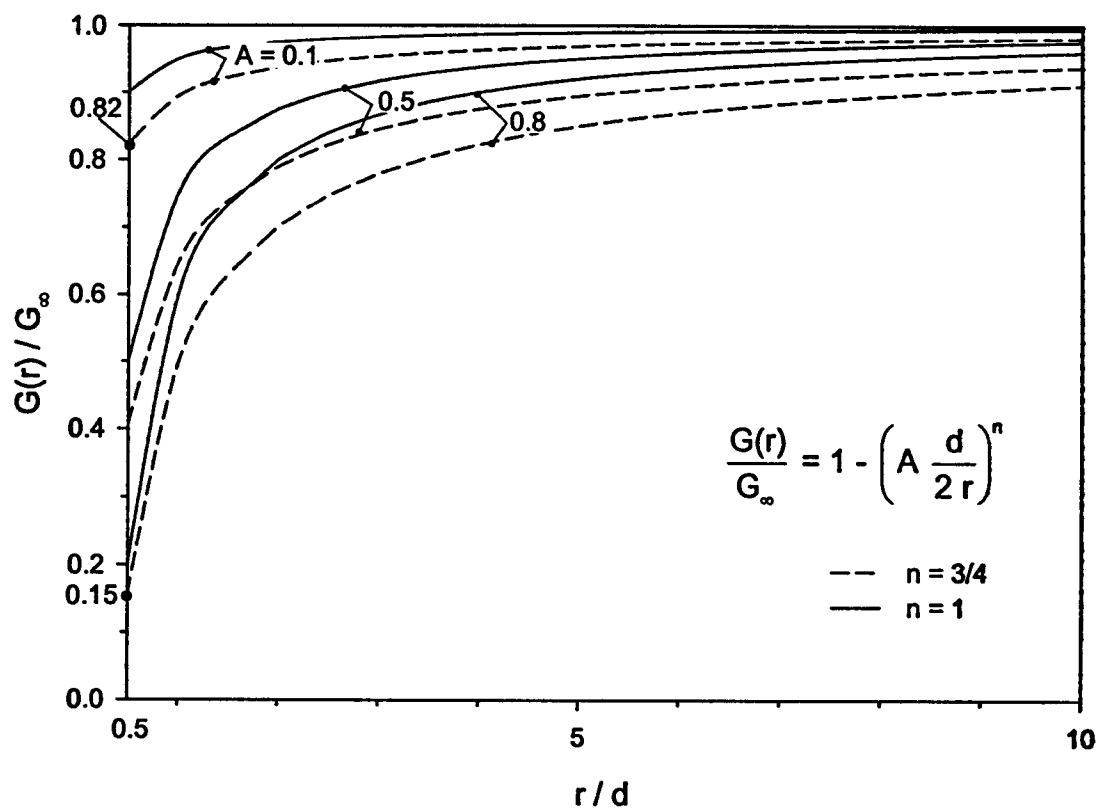


FIGURE 2.4: Variation of soil shear modulus with radial distance from the pile for various strain levels

- A soil profile whose modulus of elasticity increases proportionally with depth z up to a depth $z = L$, and remains constant for $z > L$

$$E_s(r,z) = E_s(z) = \begin{cases} E_{sl} \frac{z}{L} & , z \leq L \\ E_{sl} & , z \geq L \end{cases} \quad (2.36)$$

- A soil profile whose elastic modulus is constant with depth but varies with radial distance from the pile, starting from a near field value E_{s0} and tending asymptotically to a far-field value $E_{s\infty}$. The rate that the soil modulus changes with radial distance is given from eq. (2.34). As in the case of the linear soil profile, soil modulus is constant underneath the pile tip and is equal to $E_{s\infty}$.

$$E_s(r,z) = E_s(r) = \begin{cases} E_{s\infty} \left[1 - \left(\frac{A d}{2r} \right)^{3/4} \right] & , z \leq L, r \geq d/2 \\ E_{s\infty} & , z \geq L \end{cases} \quad (2.37)$$

- A doubly inhomogeneous soil profile in which soil modulus varies linearly with depth (up to a depth $z = L$) and with radial distance from the pile according to eq. (2.34). This is essentially a combination of the two previous cases and is characterized by the soil modulus $E_{sL\infty}$ taken at a depth equal to the pile length ($z = L$) and infinite radial from the pile. Underneath the pile tip, soil has a constant Young's modulus equal to $E_{sL\infty}$.

$$E_s(r,z) = \begin{cases} E_{sL\infty} \frac{z}{L} \left[1 - \left(\frac{A d}{2r} \right)^{3/4} \right] & , z \leq L, r \geq d/2 \\ E_{sL\infty} & , z \geq L \end{cases} \quad (2.38)$$

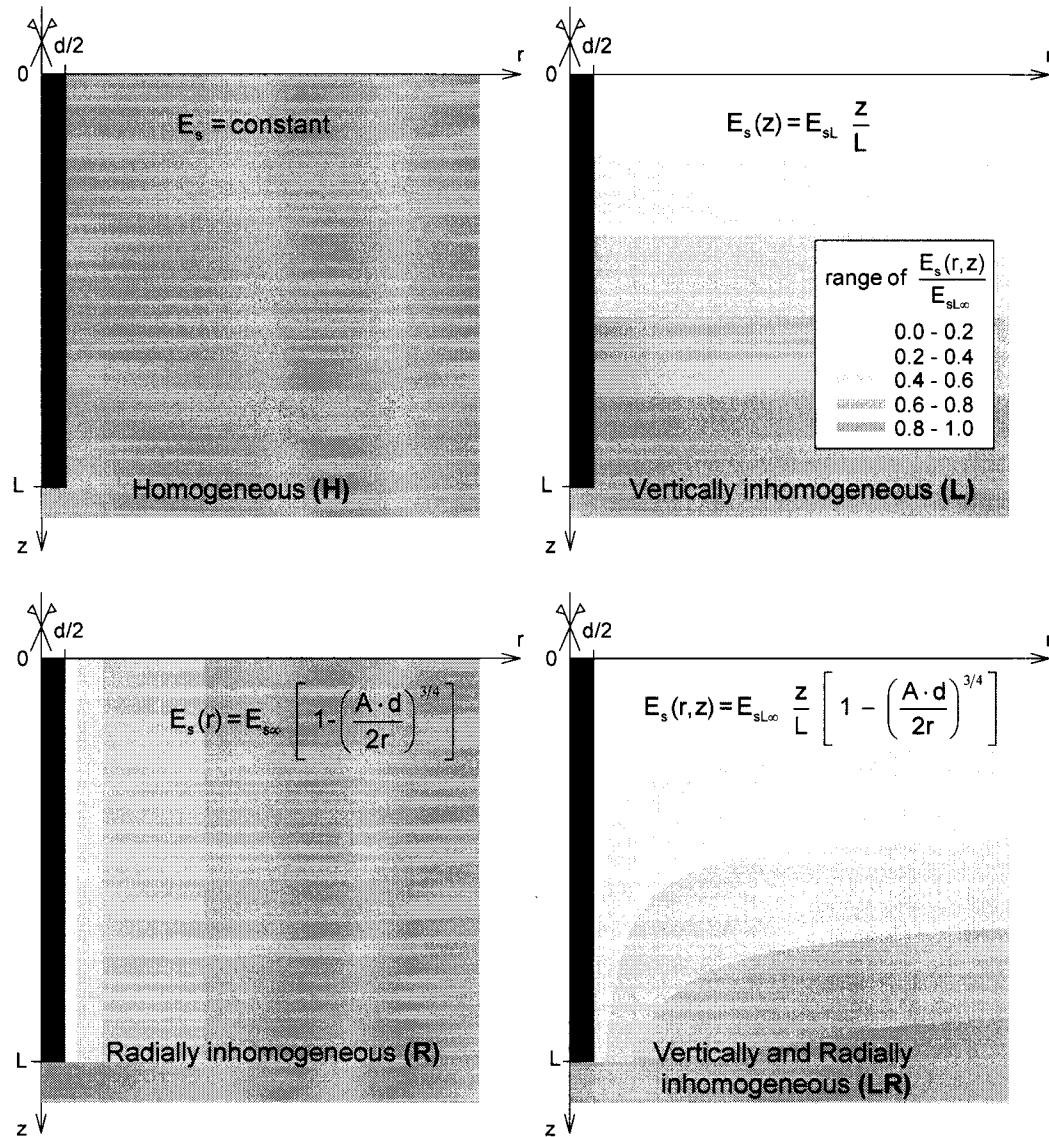


FIGURE 2.5: Soil profiles considered; [E_s = constant soil modulus (H); E_{sL} = soil modulus at pile tip (L); $E_{s\infty}$ = soil modulus at infinite radial distance from pile (R); $E_{sL\infty}$ = soil modulus at depth of pile tip and infinite radial distance from pile (LR).]

2.3.2 Pile-soil parameters used in the analysis

The following main parameters are used in the study:

- Pile slenderness ratio (L/d). Three distinct values are considered: 20 (corresponding to short piles), 40 (moderately long piles) and 60 (long piles).
- Pile to soil stiffness contrast (E_p / E_{sL}). For the case of a vertically inhomogeneous soil profile, the value of soil modulus at the pile tip E_{sL} is used to describe the soil stiffness. For a radially inhomogeneous profile the far-field modulus $E_{s\infty}$ is used. For the doubly inhomogeneous profile, the far field value at depth $z = L$, $E_{sL\infty}$, is used. E_p / E_s values considered range from (Homogeneous soil): 100 corresponding to the case of soft pile/stiff soil to 10,000 for stiff pile/soft soil.
- Poisson's ratio (ν). Unless specifically otherwise stated, the value of $\nu_s = 0.4$ is assumed for the soil and $\nu_p = 0.25$ was assumed for the pile.
- Pile-soil mass density contrast (ρ_p / ρ_s). The ratio of $\rho_p / \rho_s = 2.0$ is used in all cases.
- Material damping ratio β . A value of $\beta_s = 0.05$ is used for the soil and $\beta_p = 0.01$ is used for the pile.

The objective of this chapter is to: (1) validate the results of K-PAX by comparing with published results; (2) investigate the effects of soil inhomogeneity on

- static pile stiffness
- static Winkler springs
- dynamic pile stiffness and damping
- dynamic Winkler spring and dashpots

2.4 RESULTS OF FINITE ELEMENT SOLUTIONS

2.4.1 Comparisons with static results

K-PAX has been thoroughly tested under static conditions for floating and end-bearing piles in homogeneous and inhomogeneous soil. The infinite elements described in section 2.2.5 were employed in the analyses. Tables 2.1 through 2.4 compare results from K-PAX and the study of El-Marsafawi et al. (1992) for a variety of pile-soil configurations. In their study, Marsafawi et al. used the boundary element method which is considered very accurate, especially in the case of a homogeneous halfspace. The maximum difference in pile head stiffness was found to be less than 3%.

TABLE 2.1: Comparison of normalized pile-head stiffness of end-bearing piles, in homogeneous soil layer.

L / d	E_p / E_s	$K_z d / E_s$		difference (%)
		Marsafawi et al. (1992)	FEM	
20	100	7.94	7.98	0.6
	300	16.30	16.34	0.3
	1000	44.00	44.09	0.2
40	100	7.00	6.92	1.1
	300	12.30	12.13	1.4
	1000	27.00	26.75	0.9

TABLE 2.2: Comparison of normalized pile-head stiffness of floating piles in homogeneous halfspace.

L / d	E_p / E_s	$K_z d / E_s$		difference (%)
		Marsafawi et al. (1992)	FEM	
20	30	3.91	3.96	1.4
	100	6.21	6.26	0.8
	300	8.64	8.69	0.6
	1000	10.50	10.54	0.4
	3000	11.20	11.20	0.0
	10000	11.50	11.50	0.0
40	100	6.44	6.47	0.5
	300	10.10	10.02	0.8
	1000	14.60	14.52	0.5
60	100	6.47	6.48	0.2
	300	10.40	10.26	1.3
	1000	16.50	16.07	2.6

TABLE 2.3: Comparison of normalized pile-head stiffness of floating piles, embedded in a linear soil profile.

L / d	E_p / E_{sL}	$K_z d / E_{sL}$		difference (%)
		Marsafawi et al. (1992)	FEM	
20	100	3.81	3.81	0.0
	300	5.59	5.60	0.2
	1000	6.84	6.86	0.3
40	50	2.2	2.18	0.9
	150	4.2	4.17	0.7
	500	7.5	7.37	1.7

TABLE 2.4: Comparison of normalized pile-head stiffness of floating piles, embedded in a parabolic soil profile.

L / d	E_p / E_{sL}	$K_z d / E_{sL}$		difference (%)
		Marsafawi et al. (1992)	FEM	
20	30	2.62	2.60	0.76
	100	4.63	4.63	0.00
	300	6.69	6.72	0.45
	1000	8.18	8.26	0.98
	3000	8.77	8.87	1.14
	10000	9.00	9.11	1.22

In Figure 2.6, results from the present study are compared against results from four different studies. For piles in homogeneous soil, it is evident that pile-head stiffness increases with increasing pile length. The effect is more pronounced in soft soil profiles. The comparisons are satisfactory for all combinations of pile lengths, Poisson's ratios and pile-soil stiffnesses.

The opposite trends are observed for end-bearing piles shown in Figure 2.7: the longer the pile, the smaller the pile-head stiffness. It can also be seen that this drop is more pronounced in the case of soft soil. Again, results from K-PAX compare favorably with results from other studies.

In Figure 2.8, results by El-Sharnouby and Novak (1990) (boundary element formulation) are compared with the results from the present study, for a pile embedded in three different soil profiles, as function of pile-soil stiffness contrast (E_p / E_{sL}). It is shown that the pile stiffness is greater in the case of a homogeneous soil profile than in a parabolic or a linear soil profile of equal Young's modulus at the pile tip.

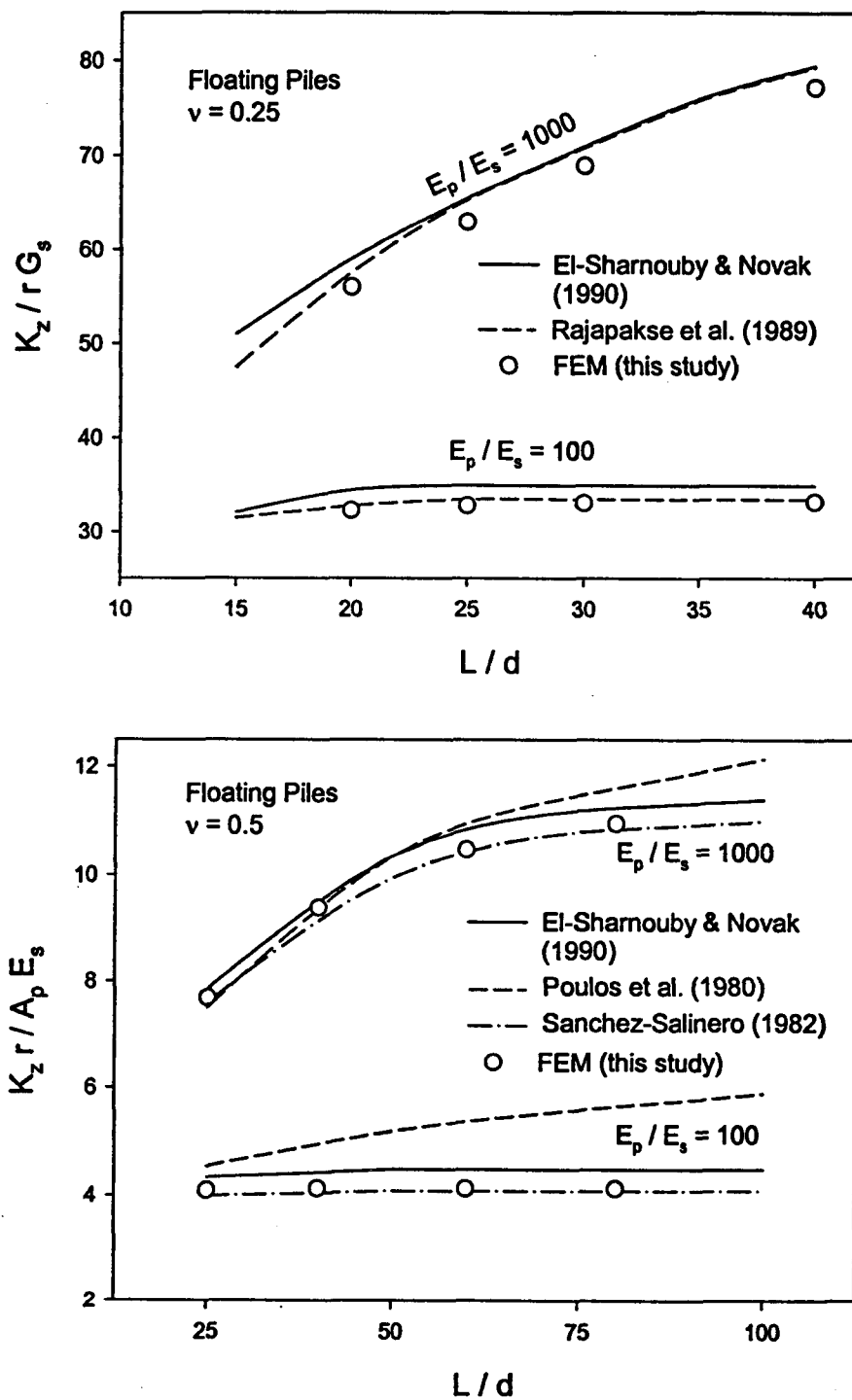


FIGURE 2.6: Normalized static stiffness of floating piles in homogeneous soil.

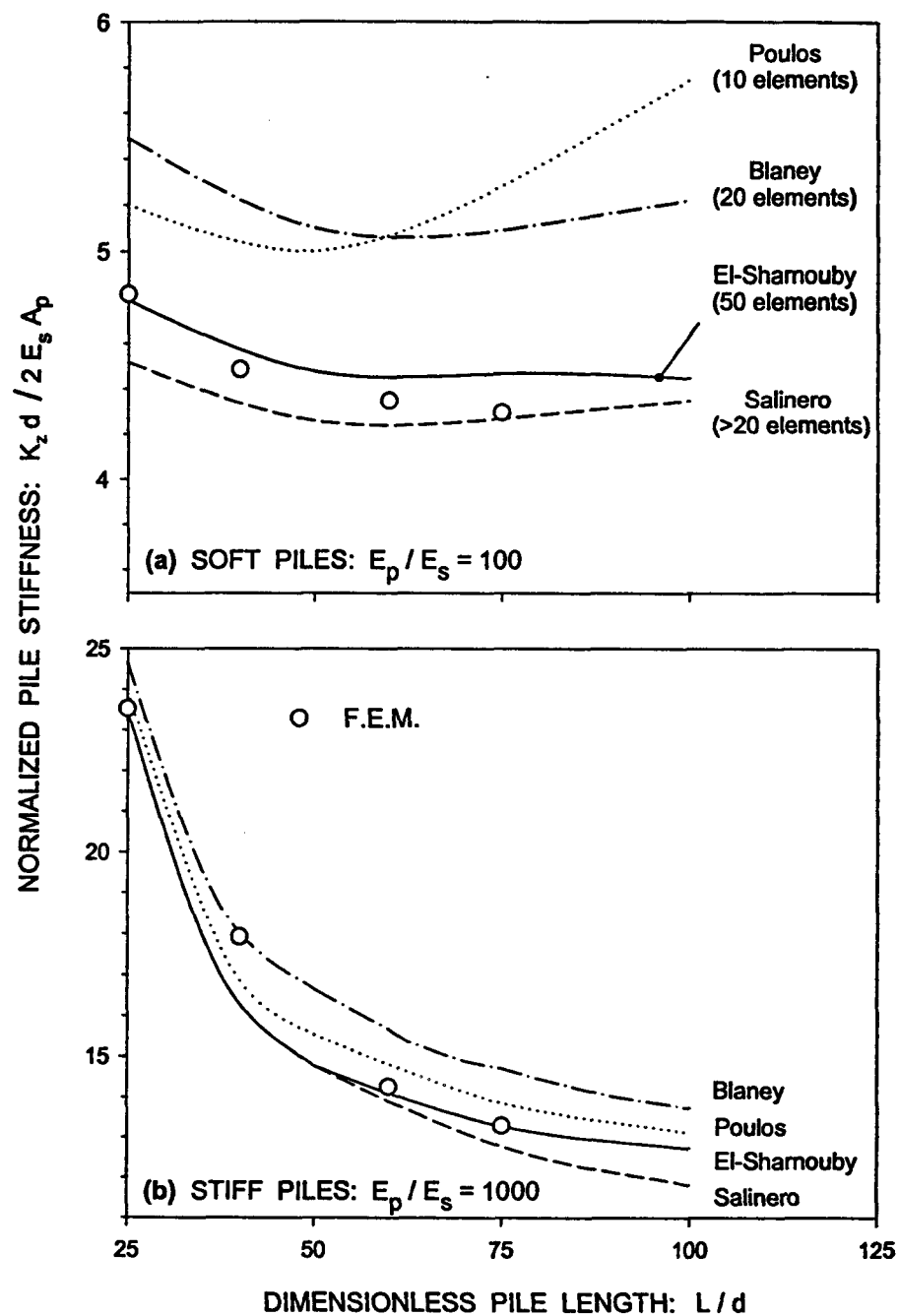


FIGURE 2.7: Normalized stiffness of end-bearing piles in a homogeneous soil stratum.

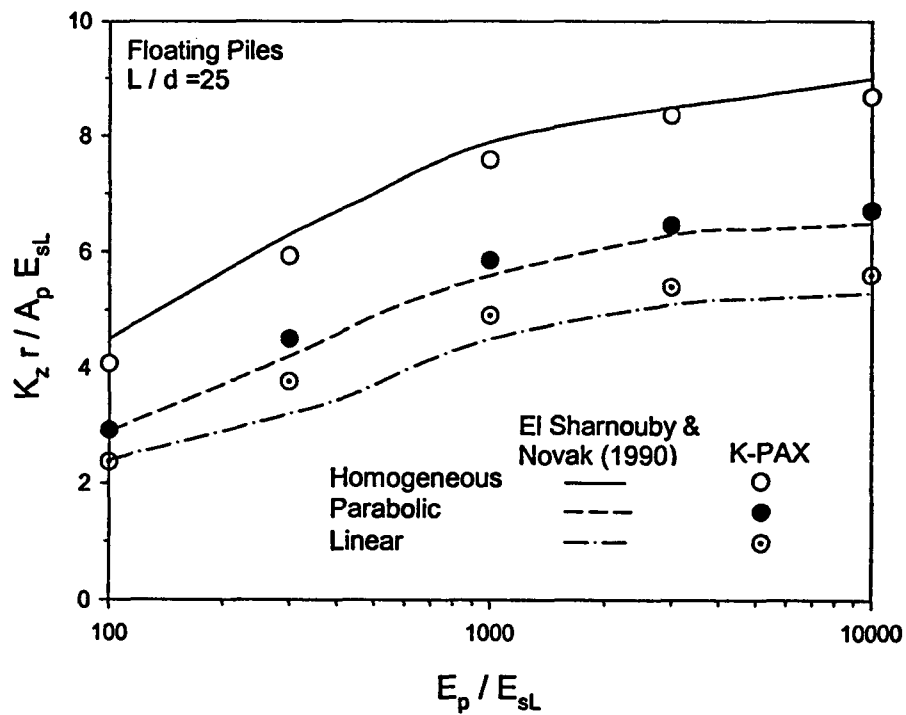


FIGURE 2.8: Normalized stiffness of floating piles in different soil profiles. Comparison of the proposed solution with results from El-Sharnouby & Novak (1990); $\nu = 0.5$.

This is expected since the linear and the parabolic soil profiles attain their maximum modulus value at the pile tip, whereas the homogeneous soil profile has the same (maximum) modulus throughout the pile length. It is noted that although the dimensionless stiffness increases as the soil becomes softer, (which is counter-intuitive), the actual pile-head stiffness increases. This is due to the fact that the pile-head stiffness is normalized with the soil modulus, which increases as the soil gets stiffer, therefore decreasing the normalized stiffness value.

Another informative plot is provided in Figure 2.9 for a pile embedded in a homogeneous soil layer (E_s) underlined by a homogeneous stratum (E_b). As the base stratum becomes progressively stiffer, the floating pile tends to become end-bearing. It is apparent that by increasing the top layer soil stiffness, the pile head flexibility decreases, as was expected. It can be seen that for all combinations of pile-soil stiffnesses and pile lengths, the results of the two studies are very close to each other.

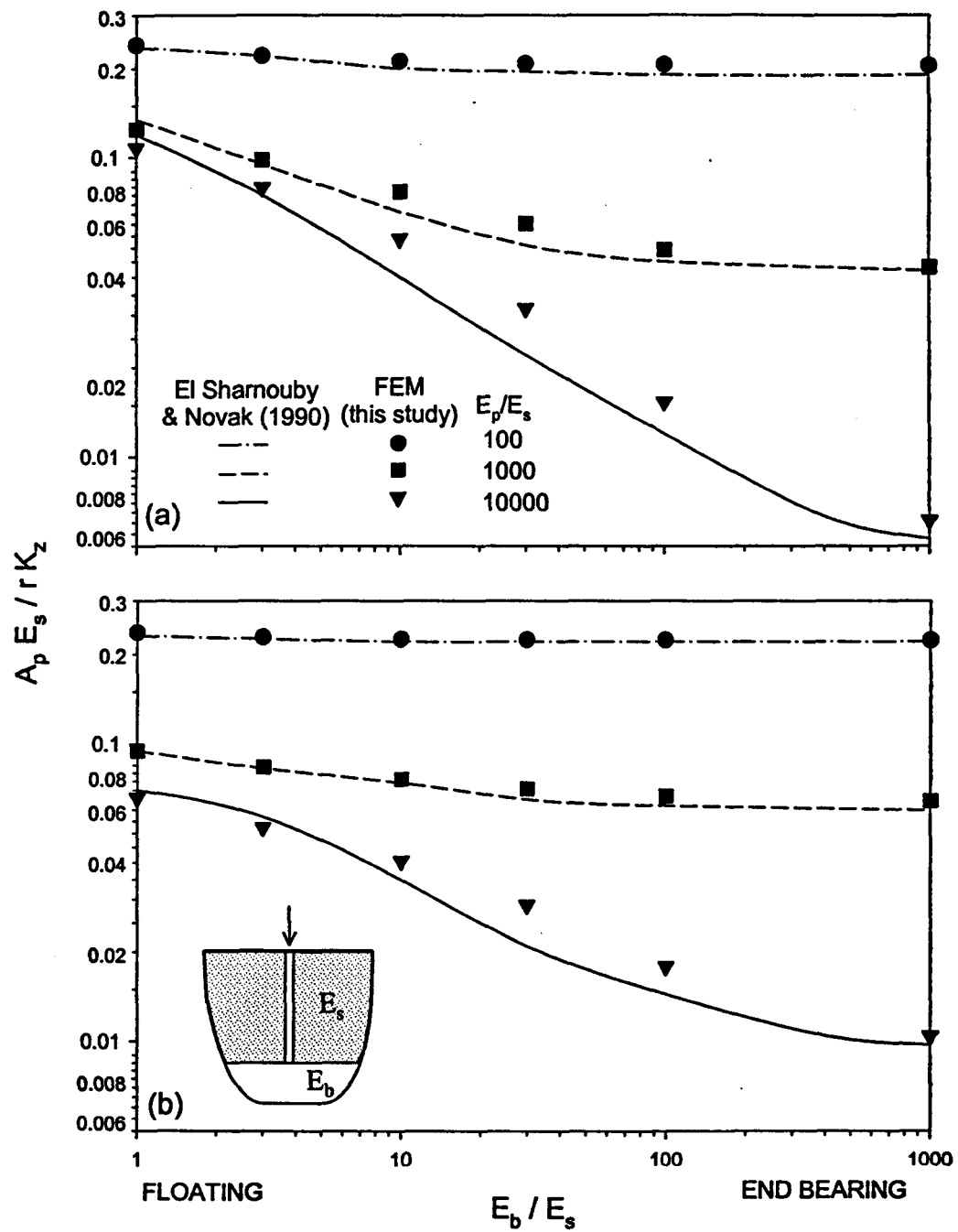


FIGURE 2.9: Dimensionless flexibility of an end bearing pile in a homogeneous soil layer: (a) $L/d = 25$, (b) $L/d = 50$; $\nu = 0.25$

2.4.2 Effect of radial inhomogeneity

The effect of radial inhomogeneity was investigated for the pile-soil configurations examined previously. Tables 2.5 and 2.6 show the reduction in pile head stiffness for different levels of radial inhomogeneity. It is evident that for high levels of radial soil inhomogeneity ($A = 0.8$), which could be encountered, say, in soft clay, static pile stiffness drops by an appreciable 30 percent. This is interesting considering that the present study is limited to small deformations, because even in small deformations the pile-head settlement can be significantly underestimated. This also may explain the practical recommendations in the analysis of single piles, to use the soil modulus of the soil material near the pile (O'Neill et al. 1977), because using the high soil modulus far from the pile will lead to unconservative settlements.

TABLE 2.5: Comparison of normalized pile-head stiffness of floating piles, embedded in a vertically homogeneous soil profile for different levels of radial inhomogeneity.

L/d	$E_p/E_{s,\infty}$	$A = 0$	$A = 0.1$		$A = 0.5$		$A = 0.8$	
		$K_z d/E_s$	$K_z d/E_{s,\infty}$	(%)	$K_z d/E_{s,\infty}$	(%)	$K_z d/E_{s,\infty}$	(%)
20	100	6.26	5.99	-4.3	5.24	-16.3	4.50	-28.1
	300	8.69	8.29	-4.6	7.13	-18.0	5.99	-31.1
	1000	10.54	9.99	-5.2	8.42	-20.1	6.93	-34.3
40	100	6.47	6.21	-4.0	5.49	-15.1	4.82	-25.5
	300	10.02	9.64	-3.8	8.18	-18.4	7.58	-24.4
	1000	14.52	13.91	-4.2	12.11	-16.6	10.26	-29.3
60	100	6.48	6.22	-4.0	5.50	-15.1	4.83	-25.5
	300	10.26	9.90	-3.5	8.87	-13.5	7.85	-23.5
	1000	16.07	15.49	-3.6	13.77	-14.3	11.97	-25.5

TABLE 2.6: Comparison of normalized pile-head stiffness of floating piles, embedded in a vertically inhomogeneous soil profile for different levels of radial inhomogeneity

L / d	$E_p / E_{sL\infty}$	A = 0	A = 0.1		A = 0.5		A = 0.8	
		$K_z d / E_{sL}$	$K_z d / E_{sL\infty}$	(%)	$K_z d / E_{sL\infty}$	(%)	$K_z d / E_{sL\infty}$	(%)
20	100	3.81	3.66	-3.9	3.24	-15.0	2.81	-26.2
	300	5.60	5.34	-4.6	4.57	-18.4	3.84	-31.4
	1000	6.86	6.48	-5.5	5.41	-21.1	4.44	-35.3
40	50	2.18	2.11	-3.2	1.92	-11.9	1.74	-20.2
	150	4.17	4.04	-3.1	3.66	-12.2	3.27	-21.6
	500	7.37	7.08	-3.9	6.22	-15.6	5.35	-27.4

2.4.2.1 Regression analyses

Performing a non-linear regression analysis with the stiffness results obtained from K-PAX for a variety of piles and soil profiles, analytical expressions were obtained for the pile-head stiffness:

- In the case of a radially inhomogeneous profile ($0 < A < 1$), pile-head stiffness is given as ($R^2 = 0.985$):

$$K_z = 1.8 E_{s\infty} d \left(\frac{L}{d} \right)^{2/3} (1 - 0.4A^{0.73}) \left(\frac{E_p}{E_{s\infty}} \right)^{-0.5 (1 - 0.17A^{0.63})} \left(\frac{L}{d} \right)^{0.7} \left(\frac{E_p}{E_{s\infty}} \right)^{-0.7} \quad (2.39)$$

- In the case of a combined vertically and radially inhomogeneous profile ($0 < A < 1$), pile-head stiffness K_z is obtained as ($R^2 = 0.984$):

$$K_z = 1.8 E_{sL\infty} d \left(\frac{L}{d} \right)^{0.55} (1 - 0.34A^{1.2}) \left(\frac{E_p}{E_{sL\infty}} \right)^{-0.5} \left(\frac{L}{d} \right)^{0.6} \left(\frac{E_p}{E_{sL\infty}} \right)^{-0.6} \quad (2.40)$$

2.4.3 Static Winkler springs

All previous graphs deal with pile-head stiffness. This way one can substitute the whole foundation with a single vertical spring at the location of the pile head.

It is of interest to investigate how soil stiffness varies along the pile shaft. As mentioned in the introduction, a simple method for analyzing a pile is by substituting the soil with equivalent Winkler springs and solving the corresponding rod problem. Although this method is perceived as being always approximate, use of exact Winkler springs (i.e. ratio of shear tractions over displacements) will yield an exact answer. In this section we examine the variation with depth of the modulus of Winkler springs for a pile embedded in inhomogeneous soil.

The governing equation of a rod on Winkler springs is:

$$E_p A_p \frac{d^2 w}{dz^2} - k_z(z) w = 0 \quad (2.41)$$

where $w = w(z)$ is the vertical pile displacement and k_z is the modulus (stiffness) of the distributed Winkler springs.

The Winkler modulus can be expressed as function of the Young's modulus of the soil material

$$k_z(z) = \delta(z) E_s(z) \quad (2.42)$$

where $\delta(z)$ is a dimensionless parameter.

It is known that the actual value of δ varies with depth even for the case of a homogeneous halfspace. Various attempts have been made to derive average values of δ

for the simple case of a homogeneous halfspace. These values are determined so that eq. (2.41) yields correct settlement at the pile head (but not necessarily along the shaft).

- Randolph and Wroth (1978) proposed the following expression for δ :

$$\delta = \frac{2\pi}{\ln \left(\frac{2 r_m}{d} \right)} \quad (2.43)$$

where, r_m is an empirical length parameter, termed “magical radius”, given by

$$r_m \approx 2.5 L (1 - \nu) \quad (2.44)$$

- O’Rourke and Dobry (1978) proposed an expression that is a function of the pile length (L) and relative soil-pile stiffness (E_p / E_s)

$$\delta = 1 + 5.5 \left(\frac{L}{d} \right)^{-0.74} + 0.6 \left(\frac{L}{d} \right)^{1.3} \left(\frac{E_p}{E_s} \right)^{-1.04} \quad (2.45)$$

- For a homogeneous layer, Roesset (1980) proposed $\delta = 0.6$.

In Figure 2.10 the variation of δ with depth is shown for four different soil profiles and three cases of pile-soil relative stiffness. To evaluate δ using the finite element method the following procedure was followed:

- a) Vertical displacements (w) were determined for several points along the pile shaft
- b) Shear stresses (τ_{xz}) at these points were evaluated. It was found that evaluating the stresses at the pile elements yields more accurate results than evaluating them at the soil elements or averaging stresses from pile and soil elements.

- c) Shear stresses were integrated over the corresponding pile surface area to provide the resisting forces per pile length $p(z) = \pi d \tau_z(z)$.
- d) Finally, $\delta(z)$ was calculated by normalizing the distributed spring stiffnesses with the soil modulus of elasticity at the corresponding depth $[p(z) E_s(z)/ w(z)]$. This normalization was found to be advantageous over the normalization with the soil modulus at a single predetermined depth because it yields values of $\delta(z)$ that are approximately constant with depth.

The values of δ have been verified using a F.E. formulation with one dimensional, 2-noded beam elements along with the distributed Winkler springs evaluated from K-PAX. The maximum differences in the pile head displacements were of the order of 3%.

The results show the following (Figure 2.10):

- For the case of a homogeneous halfspace (H), δ varies with depth but the variation is not strong. It can also be seen that Roesset's proposed value of 0.6 is a very good approximation.
- In the case of a radially inhomogeneous soil layer (R) with $A = 0.8$, δ varies with depth much less than in the homogeneous halfspace, and it tends to be independent of soil-pile relative stiffness. It is also noted that in both vertically homogeneous profiles (H and R), δ starts from a very small value at the pile head and attains a peak value within a depth of less than one pile diameter from the head (boundary layer behavior). This trend will be discussed in greater detail later on.

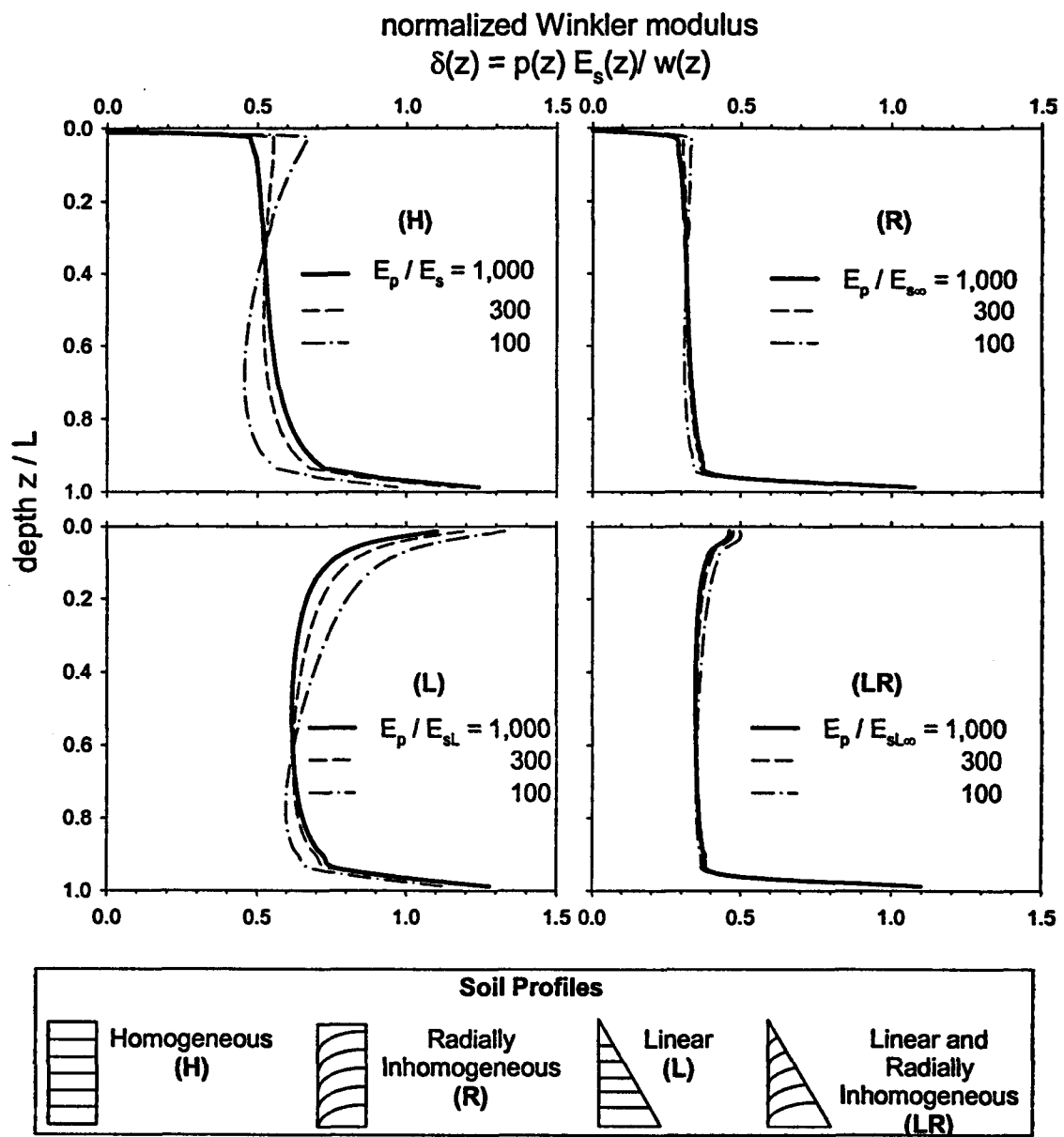


FIGURE 2.10: Normalized Winkler moduli for piles in four different soil profiles with the same properties at the far field; $L / d = 20$.

- In the linear profile (L) δ tends to infinity at the soil surface (where E_s is zero) but quickly becomes approximately constant with depth reaching a value of the order of 0.7-0.8.
- The above trend becomes stronger in the case of the doubly inhomogeneous profile (LR). The normalized Winkler modulus is equal to approximately 0.4 throughout the pile length. This modulus is normalized with the far field soil modulus ($r = \infty$) at the respective depth.
- δ becomes infinite at the pile tip in all soil profiles. This singularity has been discussed by Pak & Ji 1993.

From the plots in Figure 2.10, it can be seen that δ attains different values for different soil profiles with the same properties at the far field and under the pile tip. This is expected given the differences in material properties with depth and the different pile head stiffnesses.

It is instructive to explore the variation of Winkler modulus with depth for different soil profiles having matched pile-head stiffness. The elastic properties (E_p / E_s) of such profiles can be found with a trial-and-error procedure. In Figure 2.11 a comparison is made between 4 different profiles with matched pile head stiffness ($L / d = 20$):

- A homogeneous soil profile (H) with $E_p / E_s = 1,000$
- A linear soil profile (L) with $E_p / E_{sL} = 630$
- A radial soil profile (R) with $E_p / E_{s\infty} = 650$
- A linear-radial soil profile (LR) with $E_p / E_{sL\infty} = 380$

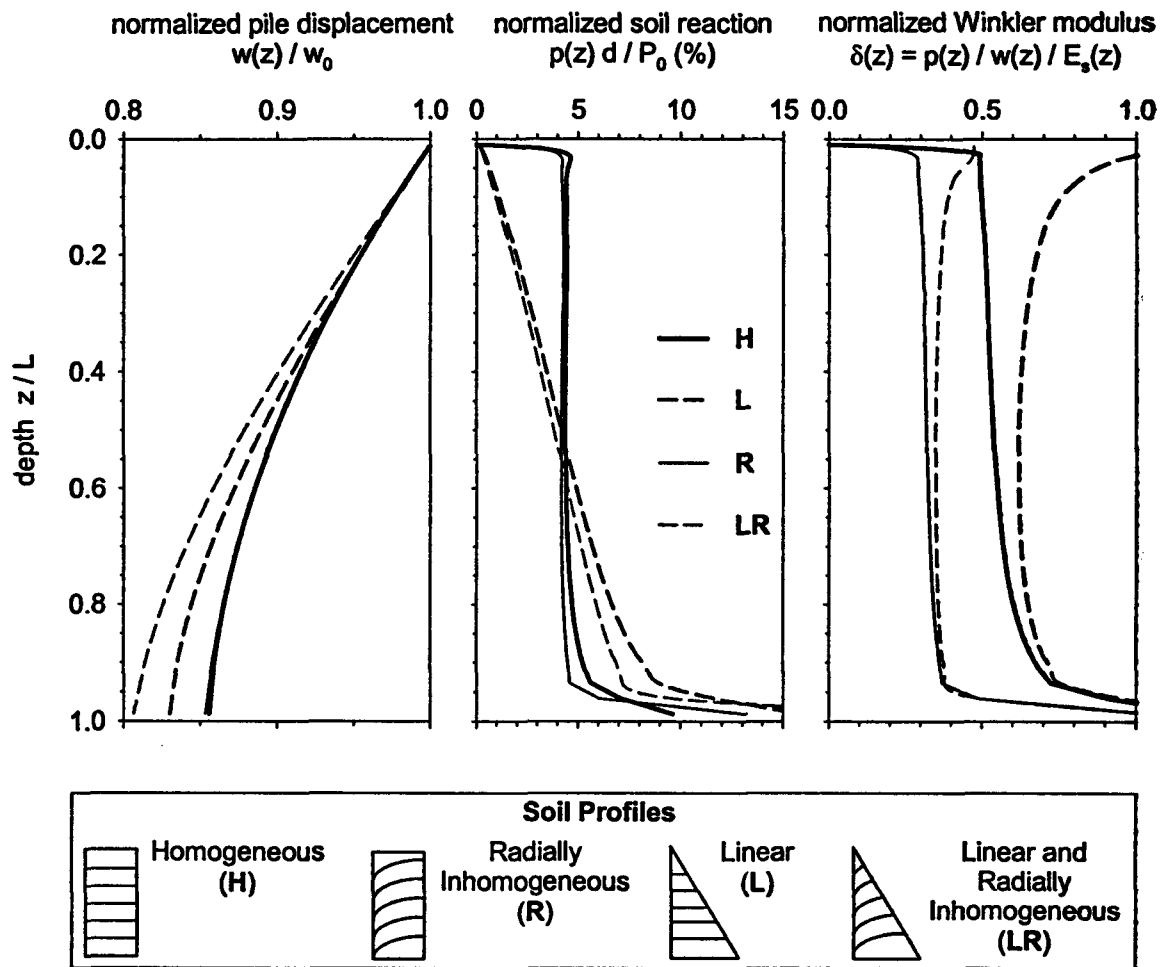


FIGURE 2.11: Normalized pile displacement, soil reaction and normalized Winkler modulus for piles of equal vertical stiffness in different soil profiles; $L/d = 20$.

The following trends are observed:

- In the homogeneous (H) and radially inhomogeneous (R) profile, pile displacements attenuate with depth practically in the same manner. This is understood since both profiles are vertically homogeneous, thus equal pile head displacements generate the same displacement profile with depth. On the other hand, displacements in the linear (L) and the linear-radial (LR) inhomogeneous profile die out faster than in the inhomogeneous case. This can also be explained, given that soil modulus increases with depth, providing stronger reactions.
- For the case of the normalized soil reaction (load transfer) with depth, the vertically inhomogeneous soil profiles (L and LR) start with zero reaction at the pile top which increases linearly with depth. In the case of vertically homogeneous soil profiles, soil reaction starts at zero and attains a maximum value within a depth of less than one diameter (this trend was also observed in Figure 2.10 for the normalized Winkler modulus). This is a boundary layer (or edge layer) phenomenon which can be explained considering that $p(z)$ has to be simultaneously zero and maximum at the pile top. Indeed, considering the vertical equilibrium of an infinitesimal pile segment, one obtains

$$\frac{\partial P(z)}{\partial z} = \pi d \tau_{zr} \quad (2.46)$$

Since the load $p(z)$ is maximum at the pile head and the corresponding shear traction has to be zero

$$\tau_{zr} = 0 \Big|_{z=0} \quad (2.47)$$

At the same time, however, it is intuitively evident the peak soil reaction will develop near the point of application of the load (where displacement is maximum). Therefore the soil reaction has to “jump” to a local maximum value over a very short distance generating the aforementioned boundary layer effect. The reason for not observing the same trend in a linear soil profile is because soil modulus is very small near the surface, therefore unable to mobilize soil reaction.

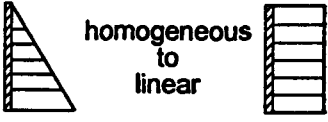
- Another interesting trend is that the soil reaction (p) is, for all practical purposes, equal for the two vertically homogeneous soil profiles (H and R) and the two vertically inhomogeneous soil profiles (L and LR). This means that the introduction of soil radial inhomogeneity doesn't affect the stresses at the pile-soil interface! On, the other hand, it does affect displacements and thereby the Winkler modulus.

2.4.4 Equivalent soil profiles

In the comparison of Figure 2.11, four different profiles were used, all having different properties in the far field but common pile-head stiffness. This kind of comparison will be used repeatedly in this study and will shed light to many aspects of the dynamic pile response. In the remainder of this study, the soil profiles providing the same (static) pile-head stiffness will be referred to as “equivalent”.

Performing a series of non-linear regression analyses using the results from K-PAX, expressions were found that relate the properties of equivalent soil profiles. These expressions are summarized in Table 2.7; they will be used extensively in the following sections.

Table 2.7: Pile-soil relative stiffness for soil profiles that have the same pile-head vertical stiffness

correlation	
$\left(\frac{E_p}{E_{sL}} \right)_L = 4.1 \left(\frac{L}{d} \right)^{-0.85} \left(\frac{E_p}{E_s} \right)_H^{0.9 \left(\frac{L}{d} \right)^{0.065}}$	
$\left(\frac{E_p}{E_{s\infty}} \right)_R = (1-0.45A) \left(\frac{E_p}{E_s} \right)_H$	
$\left(\frac{E_p}{E_{sL\infty}} \right)_{L-R} = (1-0.47A) \left(\frac{E_p}{E_{sL}} \right)_L$	

2.4.5 Comparisons with dynamic results

In order to validate the dynamic results of K-PAX, comparisons were made with other studies, mainly the boundary element results of El-Marsafawi et al. (1992).

It is known that in the realm of dynamic steady-state analysis, displacements become complex quantities with the imaginary part accounting for the phase difference with the applied load. Accordingly, pile-head stiffness can be represented by a complex

number $\mathcal{K}_z$ composed of a real and an imaginary part. This stiffness can be represented in the alternative forms:

$$\mathcal{K}_z = \text{Re}(\mathcal{K}_z) + i \text{Im}(\mathcal{K}_z) = K_z + i \omega C_z = K_z (1 + 2 i \beta) \quad (2.48)$$

where K_z is the real part of the complex stiffness, ω is the frequency of the force oscillations, C_z is the pile-head damping and β is the equivalent hysteretic damping ratio. For more details, the reader is referred to Gazetas (1991).

In Figure 2.12 the real and imaginary part of the pile-head stiffness are plotted against the dimensionless excitation frequency $a_0 = \omega d/V_s$. These plots refer to two sets of normalized pile length (L / d) and pile-soil relative stiffness (E_p / E_s) for a pile embedded in a homogeneous soil layer. It is observed that at low frequencies ($a_0 < 0.1$) oscillations appeared in the pile-head stiffness, a characteristic of incomplete wave absorption at the boundaries (box effect). This is anticipated as the absorbing boundary ceases to be acceptable at low frequencies. Following the recommendations by Novak (1991), for low frequencies ($a_0 < 0.1$), the static pile-head stiffness (obtained using infinite elements) is adopted.

The results from the present solution follow closely those of El-Marsafawi. It is obvious from Figure 2.12 that the real part of the pile-head stiffness slowly increases with frequency and this increase is stronger in soft soils. The imaginary part of the pile-head stiffness ($C_z \omega$) increases approximately in a linear manner with dimensionless frequency (a_0) thus rendering damping (C_z) approximately frequency independent.

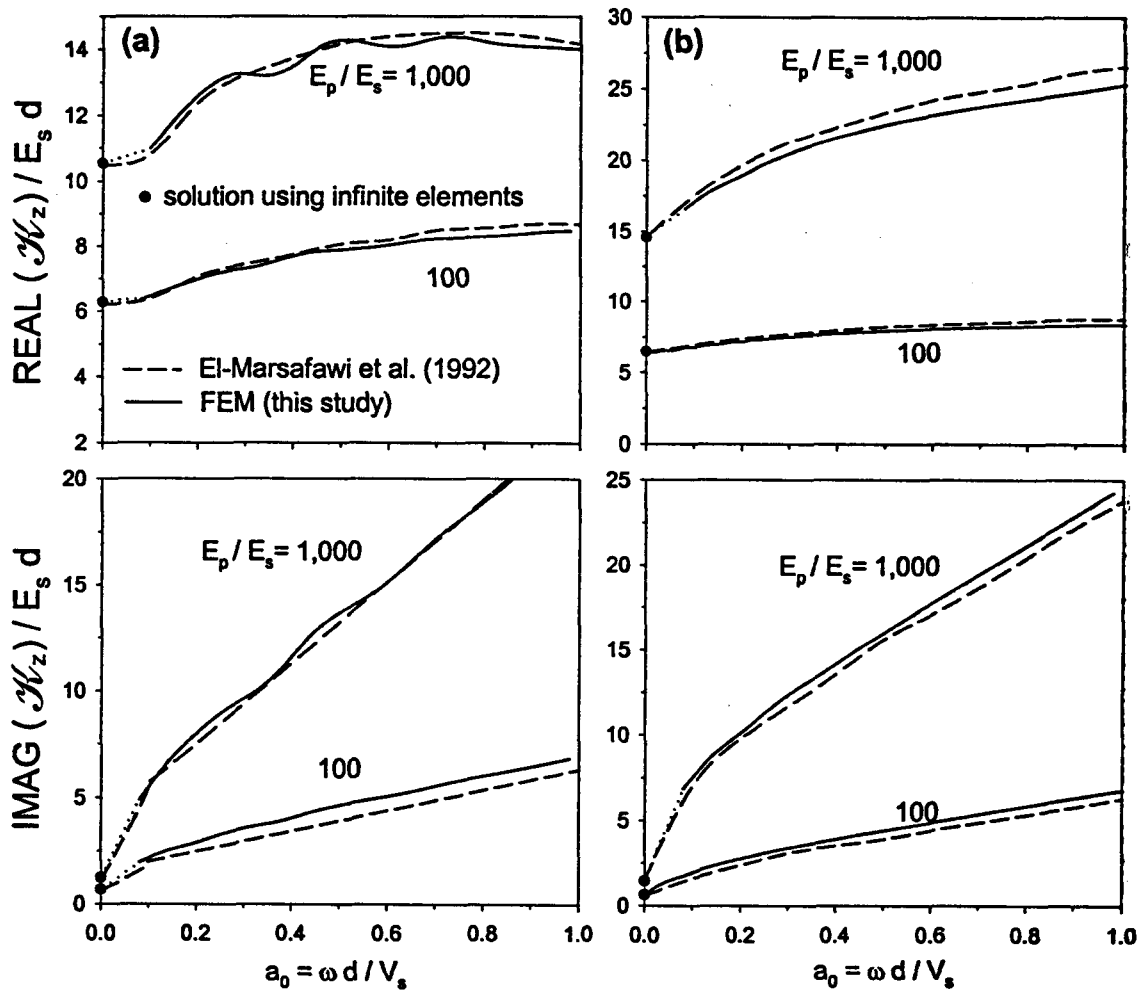


FIGURE 2.12: Normalized dynamic stiffness of a floating pile in homogeneous halfspace (profile H): (a) $L/d = 20$; (b) $L/d = 40$

Figure 2.13 presents a similar comparison, but for a pile in a linear soil profile. The results of Marsafawi et al. are limited to $a_0 < 0.5$, with the explanation that after this frequency, the results from the B.E.M. did not converge. It can be seen that up to that frequency the two solutions agree closely. Unlike the case of the homogeneous soil profile, the real part of the pile-head stiffness seems to be dropping at high frequencies.

Figures 2.14 and 2.15 deal with a single pile being embedded in a soil profile composed by two homogeneous soil layers. The first soil layer ends at the pile tip, and the second soil layer is located underneath the pile tip (bearing stratum). In Figure 2.14, the bearing stratum is 4 times stiffer than the overlying soil layer, whereas in Figure 2.15 the bearing stratum is rigid (end-bearing pile). For both soil profiles, the results of the two solutions are in close agreement.

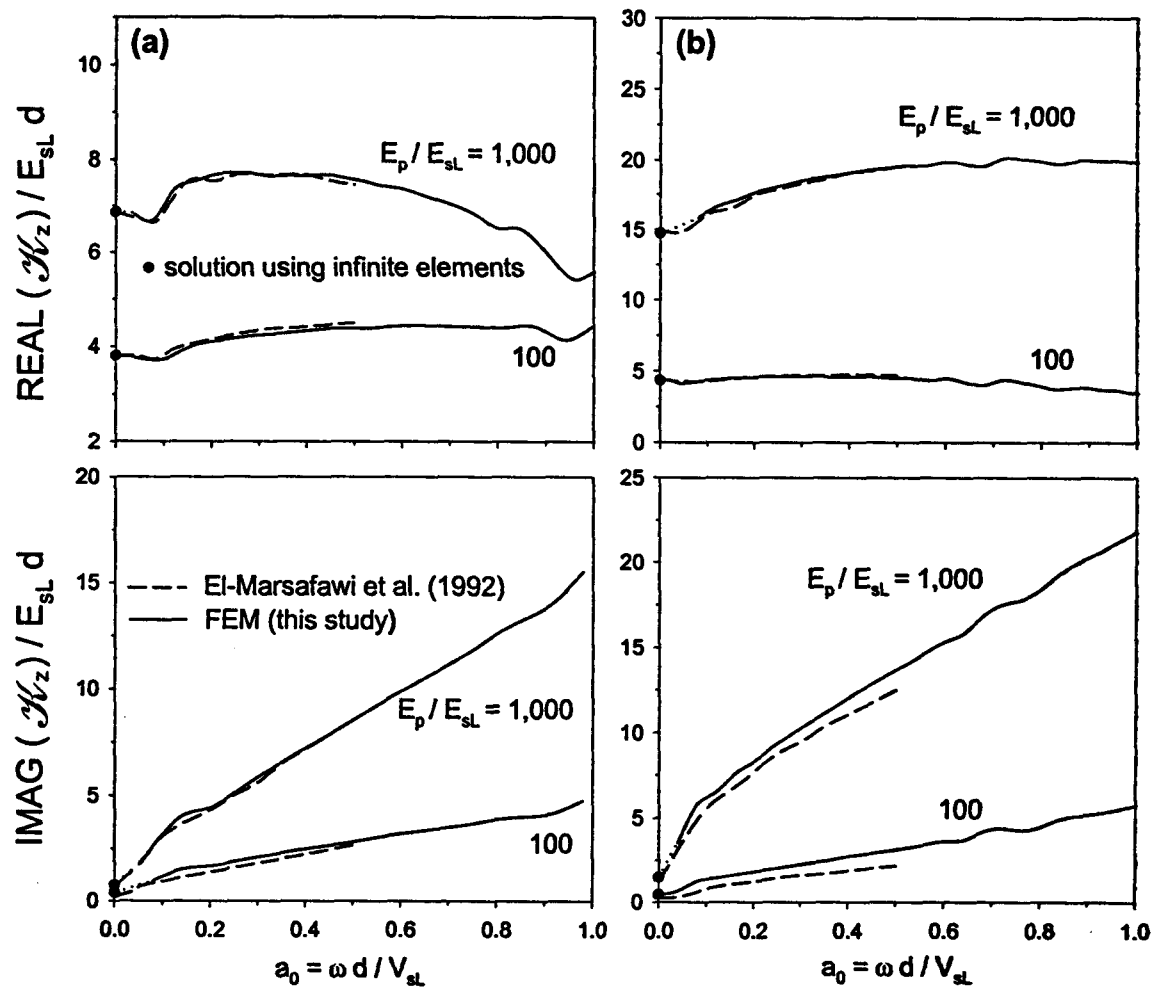


FIGURE 2.13: Normalized dynamic stiffness of a floating single pile in linear soil profile (profile L): (a) $L/d = 20$; (b) $L/d = 40$

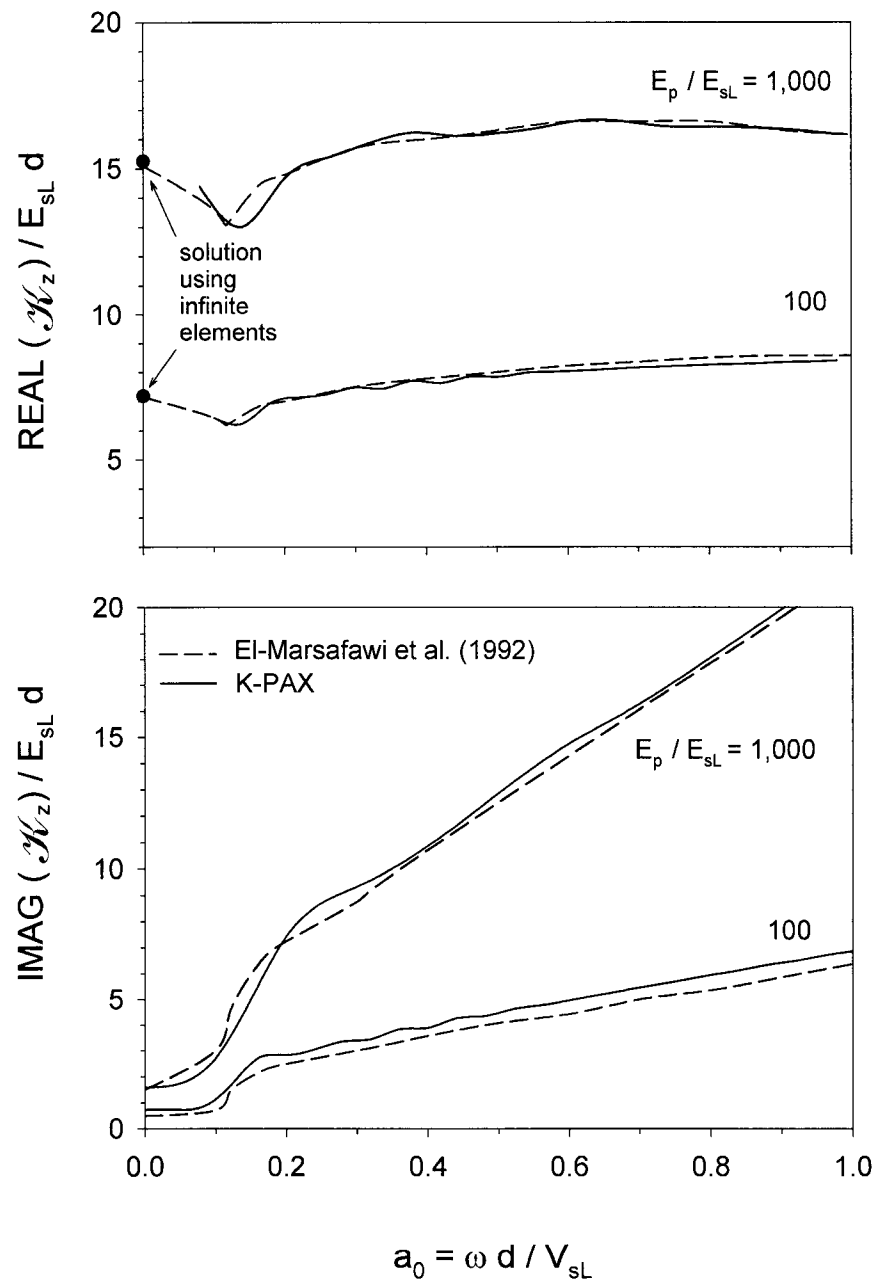


FIGURE 2.14: Normalized vertical dynamic stiffness of a single pile resting on a stiffer stratum; ($L / d = 20$, $E_b / E_{sl} = 4$)

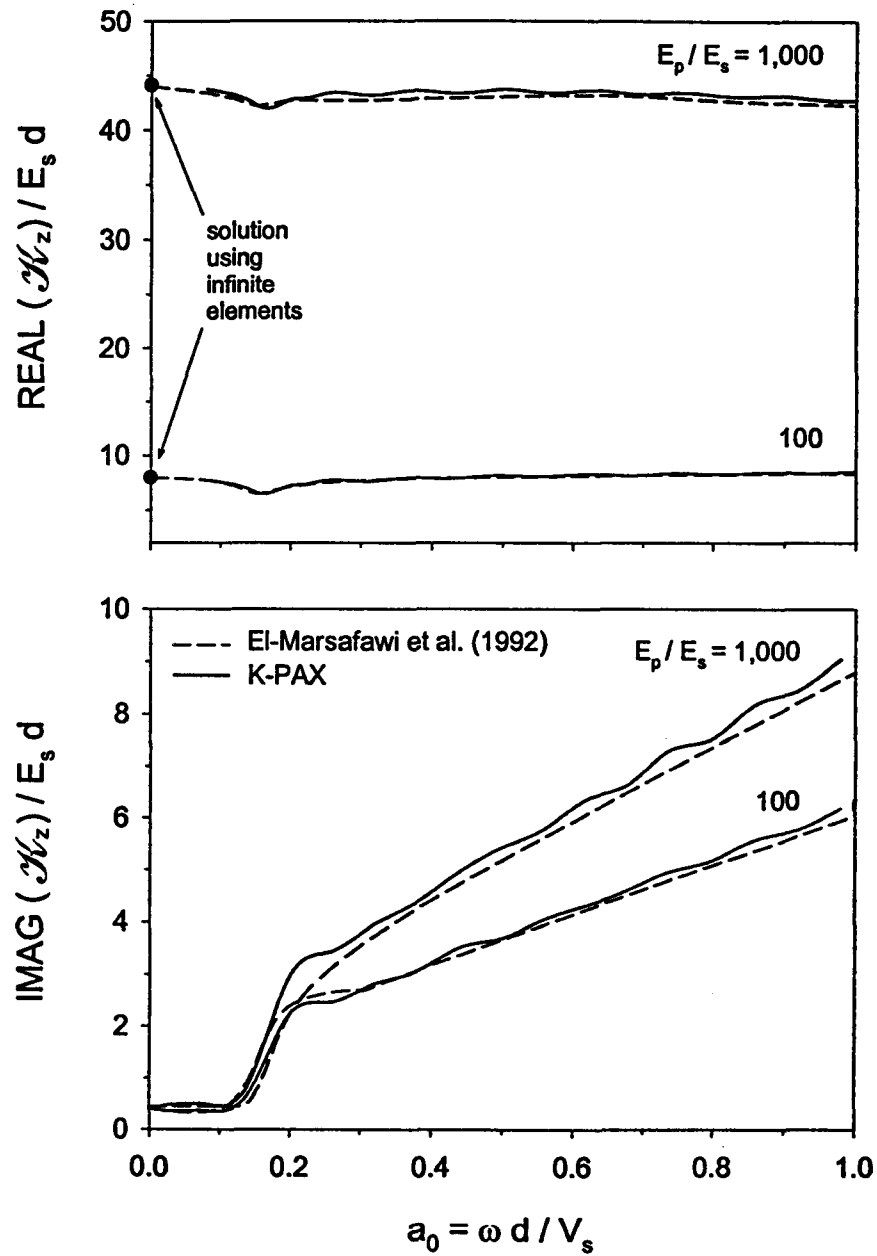


FIGURE 2.15: Normalized vertical dynamic stiffness of an end bearing single pile; ($L / d = 20$)

2.4.6 Dynamic case: effect of radial inhomogeneity

In this section, the effect of radial inhomogeneity on dynamic pile-head stiffness is explored. In Figure 2.16, the real and imaginary part of the complex stiffness ($\mathcal{K}_z$) are plotted against the dimensionless frequency a_0 for the case of vertically homogeneous soil profiles. The case of $A = 0$ corresponds to a homogeneous halfspace with $E_p / E_s = 1,000$. The cases where $A = 0.1, 0.5$ and 0.8 correspond to soil profiles with low, medium and high levels of radial inhomogeneity, respectively. All soil profiles have the same elastic properties at the far field ($r = \infty$) and underneath the pile tip. It can be seen that the introduction of radial inhomogeneity causes a significant drop in dynamic stiffness (as was expected) but also causes a change in the slope of the dynamic stiffness curves. The same trend appears for all pile lengths ($L / d = 20$ and 40). An analogous drop exists for the imaginary part of the impedance.

In Figure 2.17 a vertically inhomogeneous soil profiles with a linear variation of Young's modulus with depth is examined. The case of $A = 0$ corresponds to a radially homogeneous medium with $E_p / E_{sL} = 1,000$. With $A > 0$, the profile gradually becomes radially inhomogeneous, having the same elastic properties with the linear soil profile at the far field ($r = \infty$) and underneath the pile tip. It can be seen that, like in Figure 2.16, the introduction of radial inhomogeneity produces a drop in dynamic stiffness and a change in the inclination of the curves. In the case of strong radial inhomogeneity ($A = 0.8$), and for high frequencies, the real part of the dynamic stiffness becomes negative. This is not surprising, given the soft soil in the vicinity of the pile and the increasing participation of mass term $-\omega^2$ in the real part of the impedance.

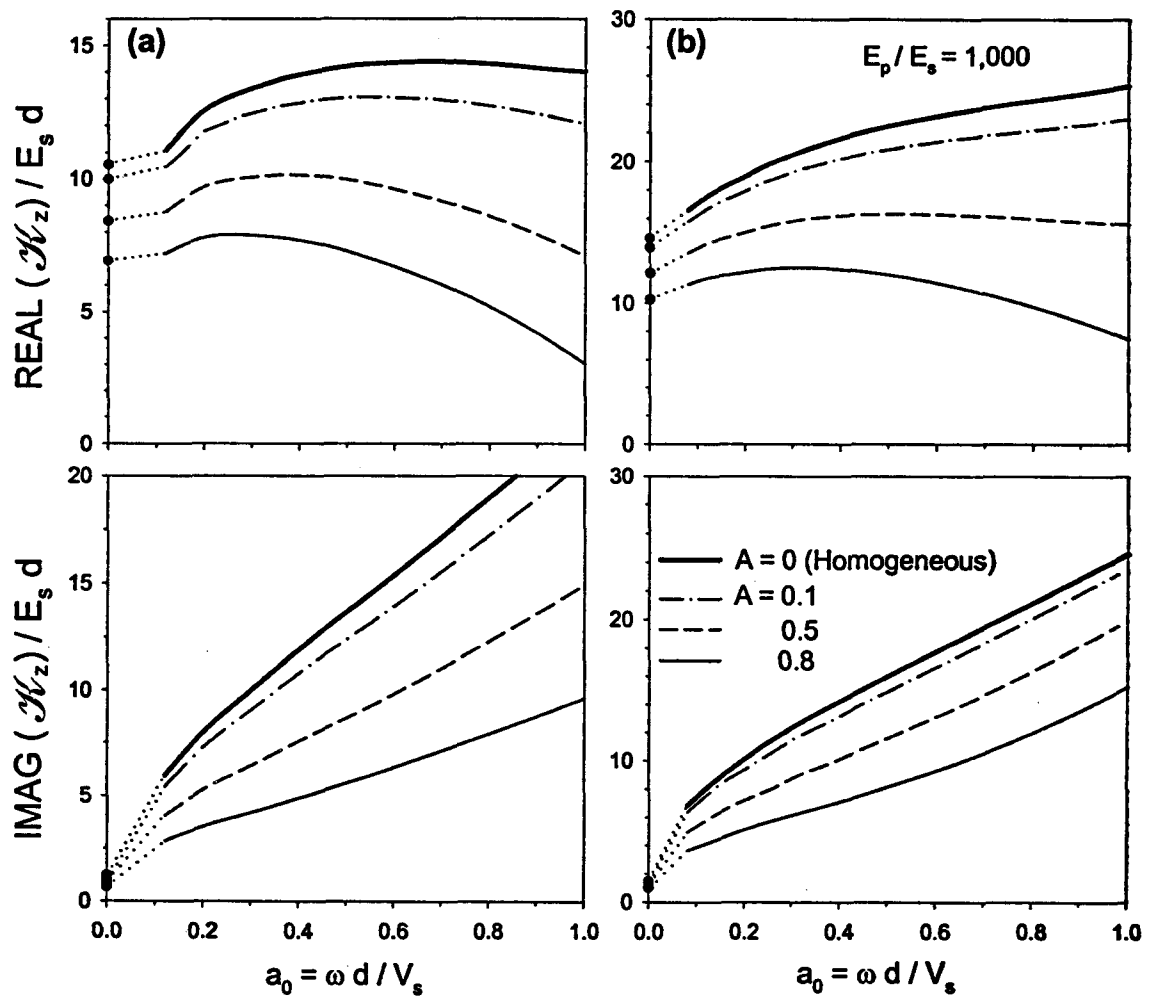


FIGURE 2.16: Normalized dynamic stiffness of a floating pile in a radially inhomogeneous halfspace (profile R): (a) $L/d = 20$; (b) $L/d = 40$

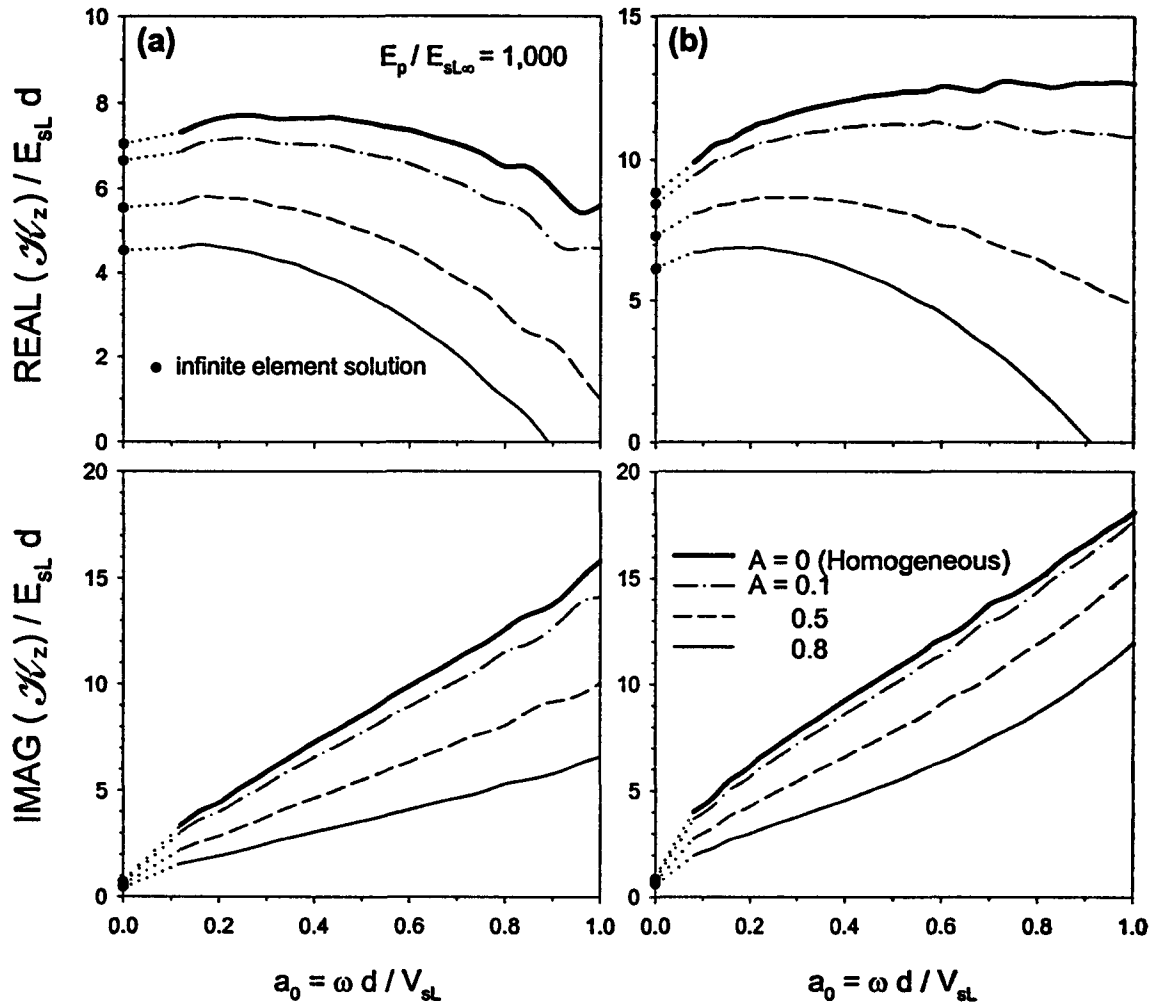


FIGURE 2.17: Normalized dynamic stiffness of a floating pile in a linear soil profile with radial inhomogeneity (profile LR): (a) $L/d = 20$; (b) $L/d = 40$

2.4.7 Effect of radial inhomogeneity on damping

As was mentioned above, damping in the medium consists of material and radiation damping.

Energy dissipation in unbounded media arises from two main: Material damping, where energy is consumed mainly through hysteretic action (e.g. friction between soil particles) and radiation damping, where energy is radiated to infinity as waves are emitted from the foundation and propagate in the unbounded medium. The latter effect has no counter part in media of finite size.

From Eq. (2.48), the damping ratio apparent at the pile head can be written as

$$\beta = \frac{\text{Im}(\mathcal{K}_z)}{2 \cdot \text{Re}(\mathcal{K}_z)} = \beta_{(\text{radiation})} + \beta_{(\text{hysteresis})} \quad (2.49)$$

where $\beta_{(\text{radiation})}$ is the radiation damping ratio and $\beta_{(\text{hysteresis})}$ is the material damping ratio.

The effect of soil inhomogeneity in damping has not been thoroughly explored. Intuition, suggests that the more inhomogeneous a soil medium, the stronger the reflections and distortions of the waves emanating from the source and, thereby, the less the energy radiated to infinity.

This effect was first identified in the pioneering work of Reissner (1936), and has been studied extensively over the years. Gazetas (1980) and Gazetas & Dobry (1984) attribute the reduction in radiated energy “to 1-D reflections and 2-D and 3-D upward curving of the waves rays”. In the second of these studies radiation damping was observed with the use of an approximate, but physically sound, plane strain model.

To elucidate the above effects, we plot results from the study of Veletsos and Dotson (1987), (Figure 2.18a), in the form of damping ratio β_z at the head of a pile embedded in a soil layer consisting of a weak inner zone (modulus E_i) and a stiffer outer zone (modulus E_o). The case of $E_i = E_o$ corresponds to a homogeneous halfspace. It can be seen that by increasing E_o while keeping E_i constant, the pile damping ratio decreases significantly at all dimensionless frequencies a_0 .

Referring to the study of Veletsos and Dotson (1987), Figure 2.18b depicts the damping ratio of a unit soil slice with a circular inclusion as a function of dimensionless frequency a_0 . In this case, soil modulus varies with the power m at a radial distance r . Obviously, for $m = 0$, the slice is homogeneous. The drop in radiation damping at all frequencies with increasing m is evident.

Using the results of El-Marsafawi et al. (1992), pile damping ratio in vertical oscillations is plotted against dimensionless frequency for three different soil profiles having the same properties underneath the pile tip (Figure 2.19). This time, the interpretation of the results is not so straightforward: at low frequencies radiation damping seems to be slightly less in the inhomogeneous profiles compared to the homogeneous halfspace, whereas for higher frequencies the trend reverses. The trends of Figures 2.18 and 2.19 will be explained in greater detail later on.

In Figure 2.20(a and b) pile damping ratio is plotted for the cases of a homogeneous halfspace and a linear soil profile under different pile-soil relative stiffnesses. Note that frequencies are not the same in graphs a, b as the normalization of the frequency factor a_0 is made with different shear wave velocities in each profile.

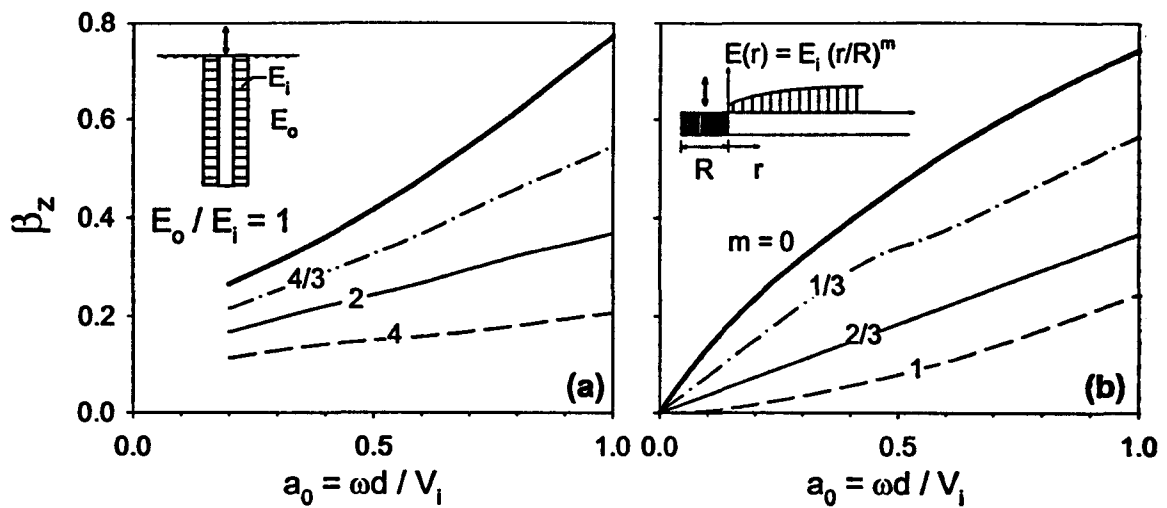


FIGURE 2.18: Damping ratio for a: (a) floating pile ($L/d = 20$, $E_p/E_i = 960$) in a homogeneous halfspace surrounded by a weak soil zone; (b) a rigid pile slice in radially inhomogeneous soil. Note the drop in damping with increasing levels of inhomogeneity (after Veletsos & Dotson 1987)

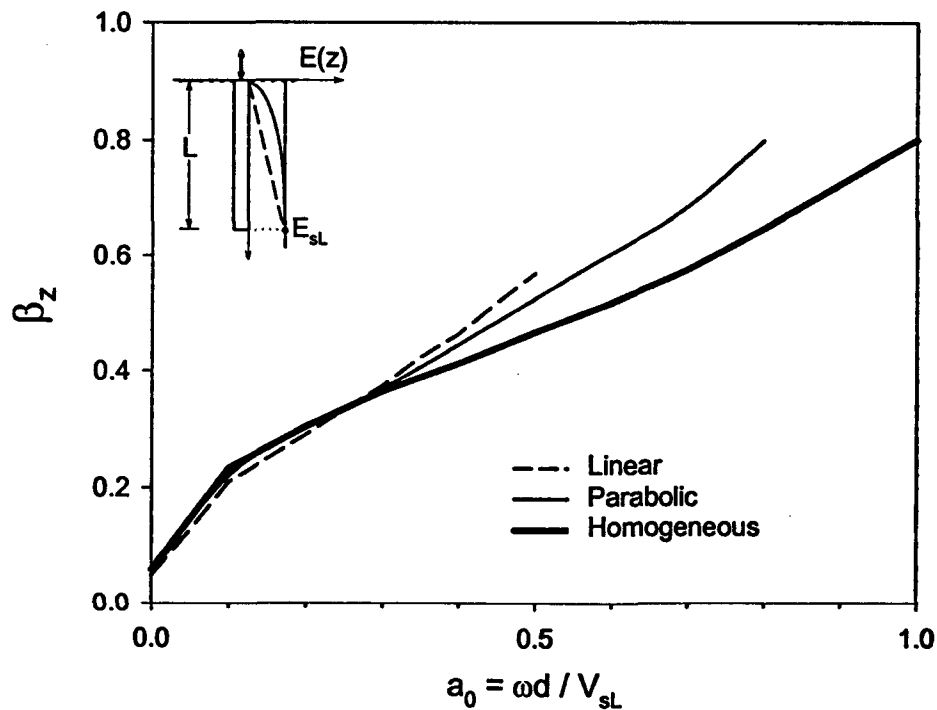


FIGURE 2.19: Equivalent damping ratio at pile head of a floating pile embedded in three different soil profiles having the same elastic modulus at the pile tip; $E_p/E_{sL} = 1000$, ($L/d = 20$) (after El-Marsafawi et al 1992)

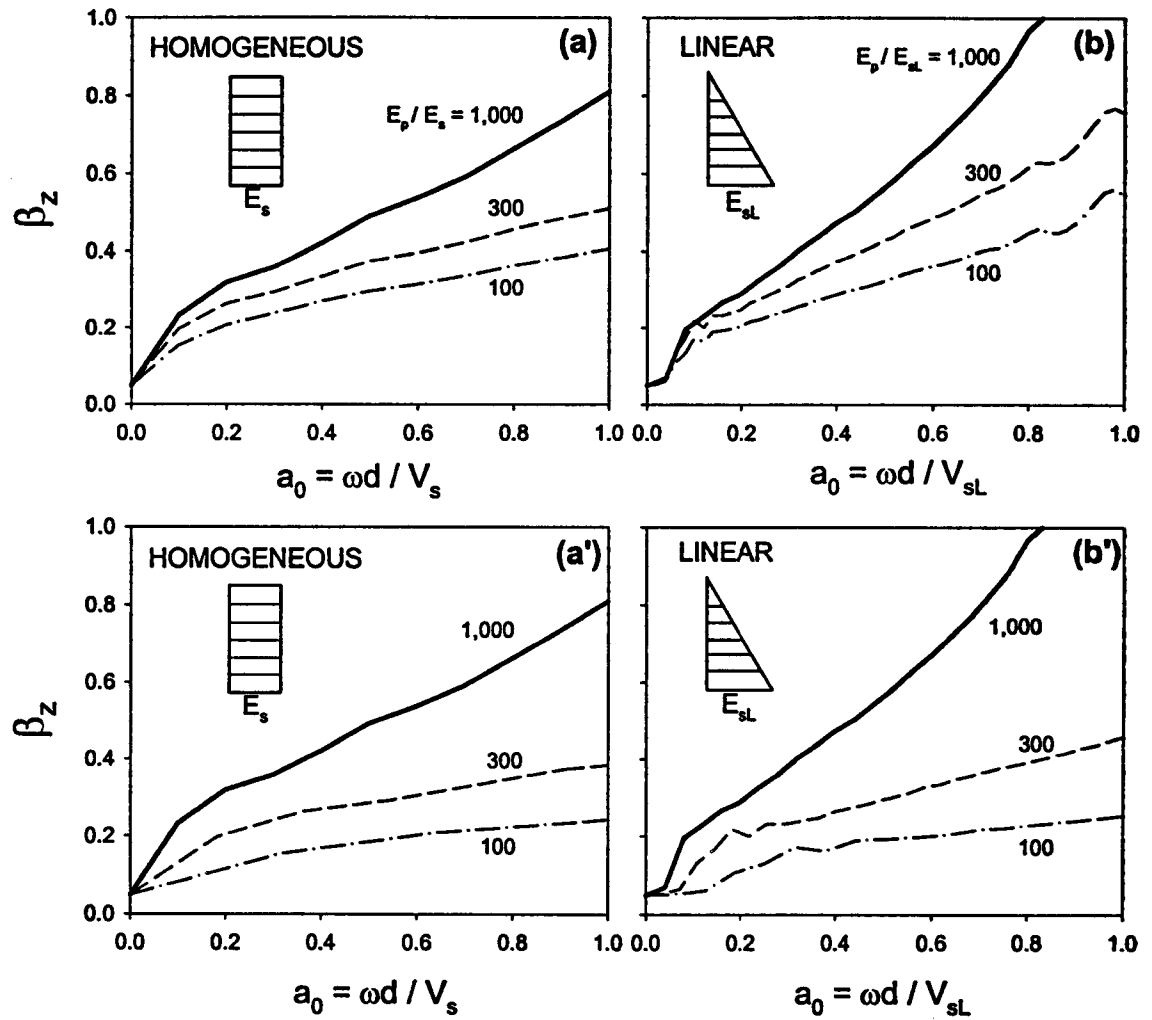


FIGURE 2.20: Equivalent damping ratio of a floating pile in a homogeneous and a linear soil profile for different pile-soil relative stiffnesses. In plots (a) and (b) a_0 in each curve is normalized with different shear wave velocities (V_s). In plots (a') and (b'), this is done with a common V_s of $E_p / 1000$

However, in plots (a') and (b'), the normalization of the dimensionless frequency a_0 is made with a common shear wave velocity, that of the softest soil profile ($E_s = E_p / 1000$). This way, equal values of a_0 correspond to equal exciting frequencies. This normalization is more appropriate since the damping ratios compared should refer to the same harmonic loading, or in other words, to the same frequency of excitation. It can be seen that for all frequencies and soil types, the softer the soil the higher the damping ratio. This effect is even more pronounced in plots a', b', associated with the consistent frequency axis (ω). Evidently, soft soils radiate more energy than stiff soils.

In Figure 2.21 damping ratio is plotted for radially inhomogeneous soil profiles having the same elastic properties at the far field as well as beneath the pile tip. In this case, a different trend appears. In low to moderate frequencies, the radially inhomogeneous soil profiles ($A > 0$), radiate less energy than the homogeneous profile ($A=0$). On the other hand, after a certain frequency the trend reverses, with the radially inhomogeneous soil profiles radiating more energy than the homogeneous halfspace. Two effects are deemed responsible for this behavior: The radially inhomogeneous soil profiles are softer than the homogeneous halfspace (soil moduli are equal only at $r \rightarrow \infty$), and as was shown in Figure 2.20 due to their lower E_p/E_s values they should radiate more in the whole range of frequencies (stiffness contrast effect). On the other hand, due to the increased inhomogeneity they tend to radiate less (wave transmission effect). The combination of these two counteracting effects yields the trend shown in Figure 2.21.

In order to separate the two counteracting effects, "equivalent" soil profiles (having the same static pile-head stiffness) are introduced, as shown in Figure 2.22. All cases shown refer to vertically homogeneous soil profiles. Dimensionless frequency a_0 is

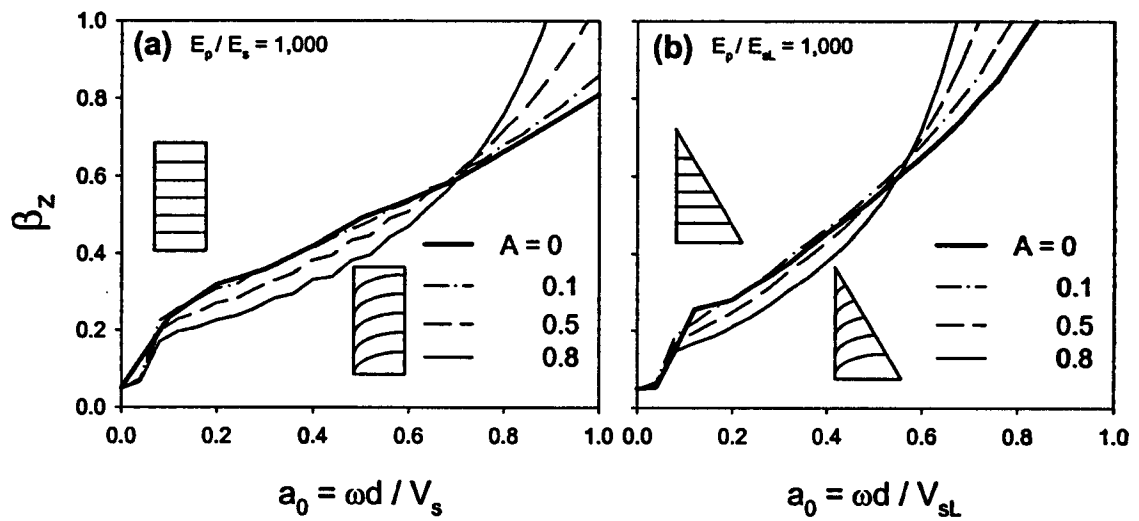


FIGURE 2.21: Damping ratio for a pile embedded in a radially inhomogeneous profile: (a) constant modulus with depth; (b) modulus increasing proportionally with depth; $L / d = 20$

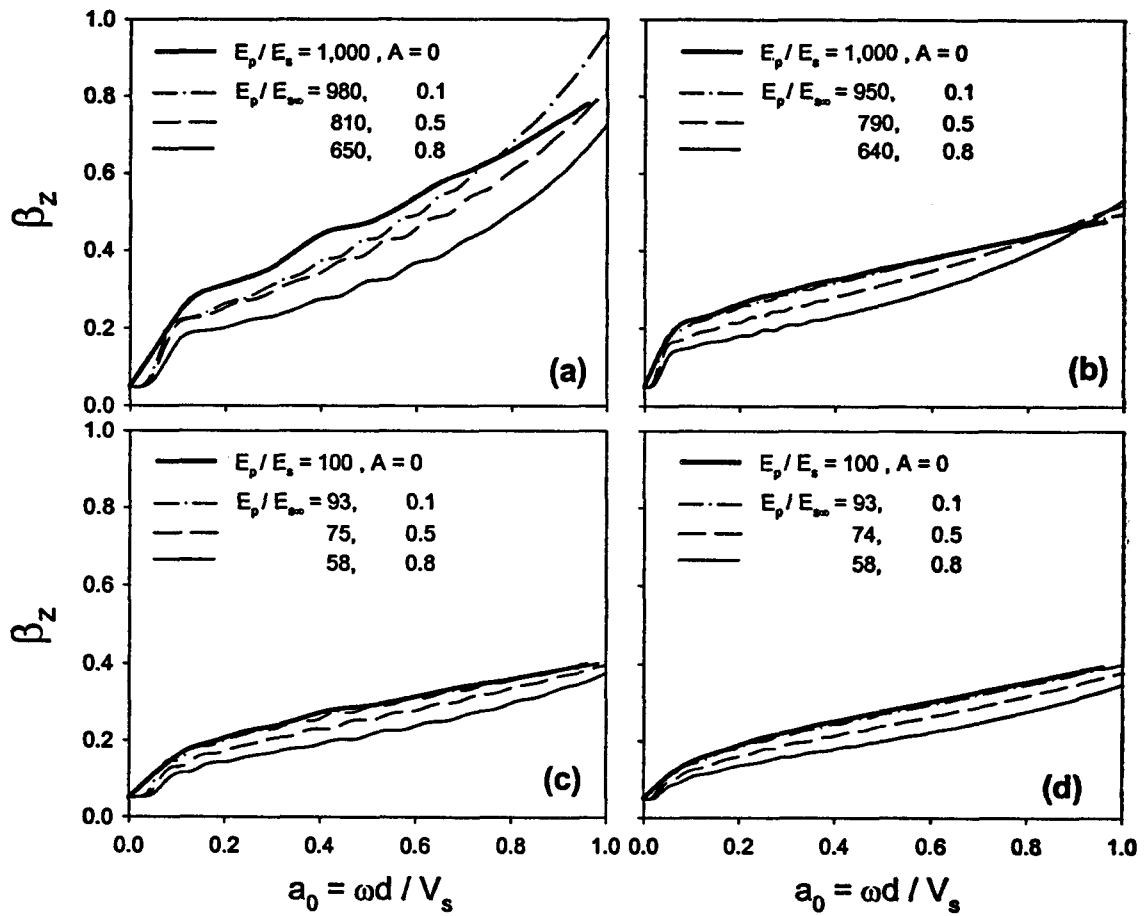


FIGURE 2.22: Damping ratio of a pile in radially inhomogeneous soil. All piles have equal static stiffness; cases (a) and (c): $L/d = 20$; cases (b) and (d): $L/d = 40$.

normalized with the shear wave velocity V_s of the homogeneous halfspace. In the light of these results, it becomes evident that inhomogeneous soil profiles radiate less than equivalent homogeneous profiles for a wide range of frequencies. It is noteworthy that the radially inhomogeneous soil profiles exhibit a cut-off frequency which, understandably, is absent from the homogeneous profiles.

Similar trends are apparent in Figure 2.23 for a set of vertically inhomogeneous profiles. In this plot, dimensionless frequency is normalized with the shear wave velocity corresponding to the soil modulus underneath the pile tip of the linear soil profile with no radial inhomogeneity (V_{sL}). Evidently, the stronger the radial inhomogeneity, the lower the radiation damping. For the case of a linear profile, a cut-off frequency is observed, that increases with radial inhomogeneity. This increase seems to be unimportant for all practical purposes.

Based on the above observations it becomes easier to explain the trends in Figures 2.18 and 2.19. In Figure 2.18, the damping ratio for the inhomogeneous soil profiles is much lower than the homogeneous halfspace because: (a) soil inhomogeneity impedes the transmission of wave energy away from the pile (wave transmission effect) (b) the inhomogeneous soil profiles are stiffer than the homogeneous halfspace, and thereby emit waves forcing the pile to deform “out-of-phase” along its shaft thus radiate less energy to infinity (emission effect). On, the other hand, in Figure 2.19 the inhomogeneous soil profiles (linear and parabolic) are softer than the homogeneous halfspace, thus radiation damping follows the same trends as in Figure 2.21.

In Figure 2.24, four statically “equivalent” soil profiles are considered to examine their differences in pile damping ratio. The properties of these profiles are the same as in

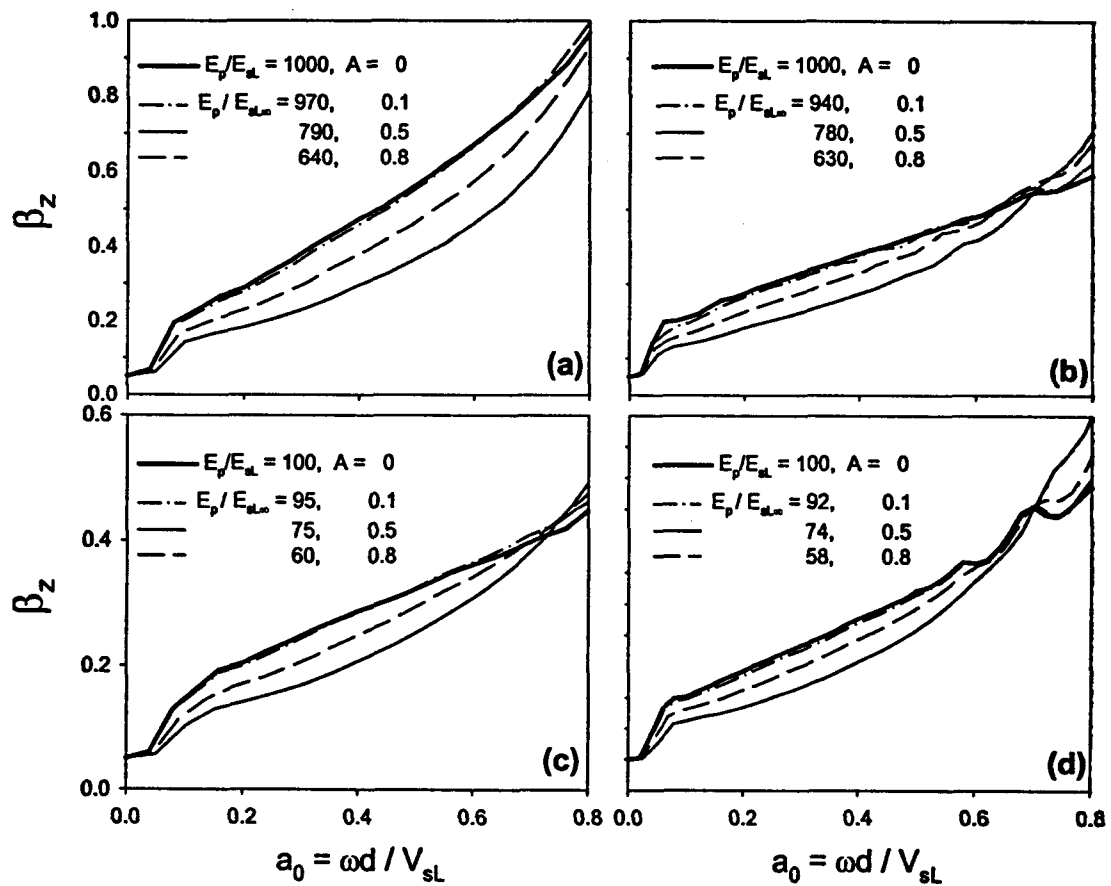


FIGURE 2.23: Damping ratio of a pile in linear and radially inhomogeneous soil profile. All piles have equal static stiffness; cases (a) and (c): $L / d = 20$; cases (b) and (d): $L / d = 40$

Figure 2.11. a_0 is normalized with the shear wave velocity of the homogeneous soil profile ($E_s = 1,000/ E_p$). The role of soil inhomogeneity in reducing radiation damping becomes evident in this plot. It is noteworthy that an increased inhomogeneity is associated with an increased cut-off frequency. This yields substantial differences in radiation damping at low frequencies, compared to the homogeneous halfspace.

Figure 2.24 raises some important issues. Since a real soil profile is never homogeneous, radiation damping is typically much less than the damping estimated by considering a homogeneous halfspace. For example, at a dimensionless frequency of 0.2, taking out the 5% material damping of the graph, the homogeneous halfspace generates radiation damping of 27% whereas the doubly inhomogeneous soil layer only 11%. The importance of these differences in design is evident.

A common approximation in static analysis, is to substitute the actual profile with an equivalent homogeneous profile to calculate pile settlement. To be on the “safe side” one may assume a reduced soil modulus, which would lead to increased pile settlement. However, it has already been shown that both assumptions yield increased radiation damping- an unconservative consequence. In other words, applying these pile-head dashpots in a dynamic analysis of the superstructure, will result to smaller response, thus rendering the analysis unconservative. Therefore, assumptions that result to a safe static analysis of a pile may not be safe in a dynamic analysis.

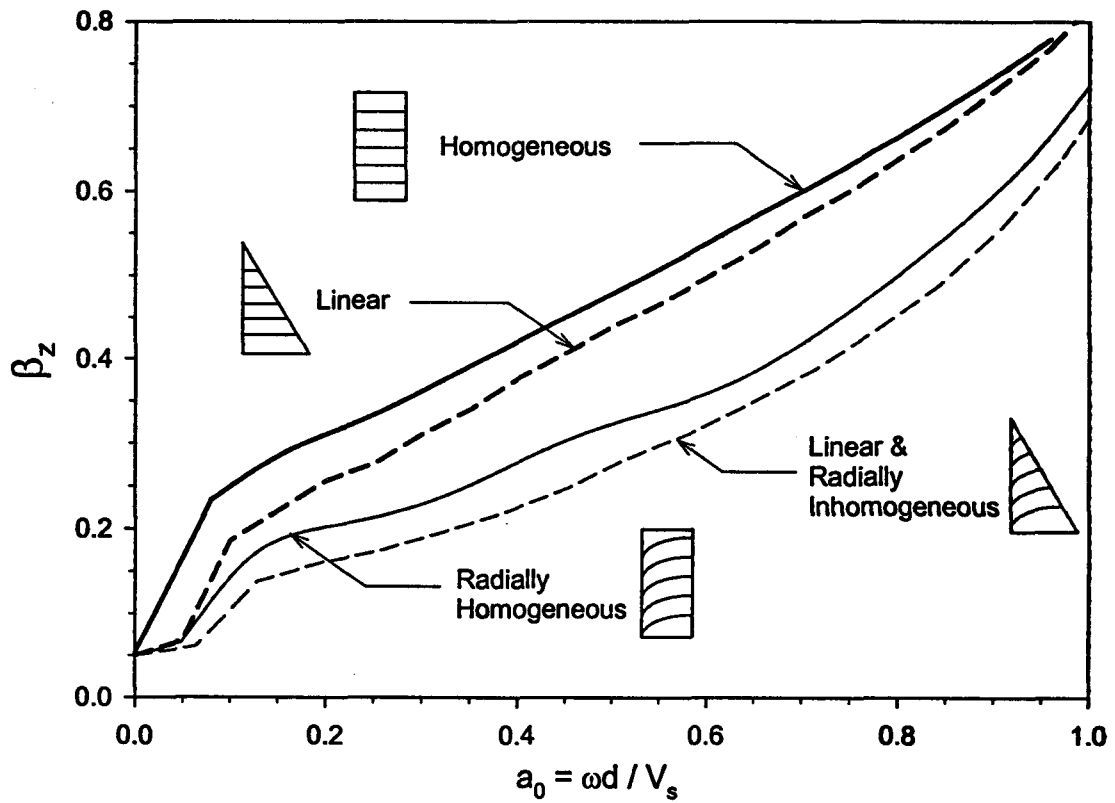


FIGURE 2.24: Equivalent damping ratio of a pile in four different soil profiles. All piles have equal static stiffness. Note the decrease in damping with increasing soil inhomogeneity; $L / d = 20$

2.4.8 Equivalent Winkler springs along the pile shaft

It is of interest to investigate how the stiffness and damping of the soil vary along the pile shaft. To this end, the complex-valued Winkler impedance k_z is defined as

$$k_z = \frac{p(z)}{w(z)} = k_z + i \omega c_z \quad (2.50)$$

where $p(z)$ is the soil reaction per unit pile length at depth z ; $w(z)$ is the pile settlement at depth z , k_z is the real part of the impedance (stiffness per unit pile length) and c_z is the imaginary part (damping per unit pile length). The latter parameter incorporates both material and radiation damping. It is noted that for a complete representation of the soil medium, a complex-valued impedance at the pile tip is required to account for the stiffness of the bearing material:

$$k_z^{(b)} = k_z^{(b)} + i \omega c_z^{(b)} \quad (2.51)$$

As in the previous case, the damping ratio of the Winkler springs is defined as

$$\beta = \frac{\text{Im}(k_z)}{2 \cdot \text{Re}(k_z)} \quad (2.52)$$

Figure 2.25 depicts four different soil profiles having the same properties at the far field and under the tip. The normalized real and imaginary part of k_z at each depth are plotted for five selected dimensionless frequencies. The following trends can be observed:

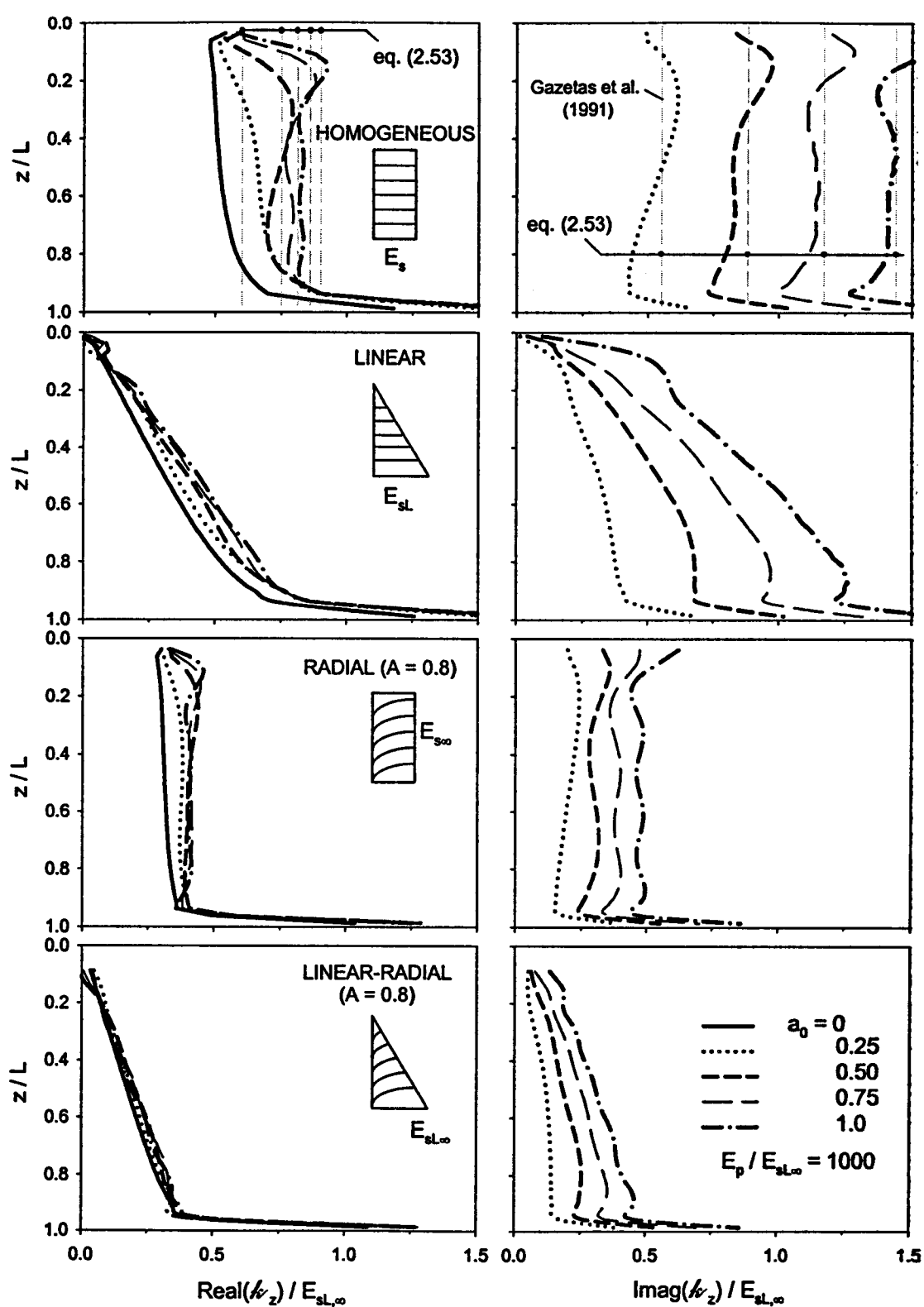


FIGURE 2.25: Real and imaginary part of Winkler impedance along the shaft of a floating pile in different soil profiles, for selected frequencies; $L / d = 20$

- The values of the Winkler springs become smaller as the soil becomes inhomogeneous. This can be easily explained since, for matched soil modulus under the pile tip, the more inhomogeneous the soil profile the softer it becomes¹. It is also interesting to observe that the real part of the Winkler springs tends to become frequency independent with increased inhomogeneity in the profile. This can be clearly seen in the plot of the linear-radial (doubly inhomogeneous) profile, where the curves corresponding to different frequencies fall on top of each other.
- The approximate (constant with depth) Winkler springs of Gazetas et al (1992) are plotted for the case of a homogeneous halfspace:

$$k_z = 0.6 E_s \left(1 + \frac{1}{2} \sqrt{a_0} \right) + i \left(1.2 a_0^{-1/4} \pi d \rho_s V_s + 1.2 \beta_{\text{hysteresis}} E_s \left(1 + \frac{1}{2} \sqrt{a_0} \right) \right) \quad (2.53)$$

The above springs are in agreement with the results from the present analysis for all frequencies.

- The imaginary part of the Winkler springs seems to follow the trends of the real part, exhibiting an additional dependence of frequency. It is obvious that the information obtained from this representation is limited.

Figure 2.26 elucidates the behavior of Winkler springs for two different, statically equivalent, soil profiles. The real parts of the normalized displacements, soil reactions and Winkler moduli along the pile shaft are plotted for various frequencies for a homogeneous halfspace and a linear soil profile. The excitation frequencies used in these plots are normalized with the shear wave velocity of the homogeneous profile. The pile-soil relative stiffness ratio for the homogeneous halfspace is equal to $E_p / E_s = 1000$,

¹ δ is proportional to a negative power of E_p/E_s as, for instance, showed by Dobry and O'Rourke (1978)

whereas for the linear profile it is equal to $E_p / E_{sL} = 630$. The following trends are noteworthy:

- a) The displacements attenuate faster with increasing frequency. This is reasonable since the contribution of pile inertia (which increases with frequency) causes an increased phase difference between excitation and response. The rate of attenuation is very similar in the two profiles.
- b) Increasing the frequency of excitation, soil reactions tend to increase near the surface and decrease close to the base. This indicates that the soil along the pile shaft is contributing more to soil reaction.
- c) As a consequence of the above trends, Winkler springs tend to remain approximately constant with depth and increase with increasing frequency. It can also be observed that normalizing the real part of the Winkler springs with the soil modulus at each respective depth produces a curve almost constant with depth in both profiles. Furthermore, the values of the Winkler springs for the two profiles are now very close to each other, at the same frequency (which is not evident in Figure 2.25).

To gain further insight, normalized displacements, soil reactions and Winkler moduli are plotted in Figure 2.27 for four different but statically equivalent soil profiles at two excitation frequencies. As in the previous Figure, a common shear wave velocity (of the homogeneous soil profile) was used to define a_0 .

The following trends are observed:

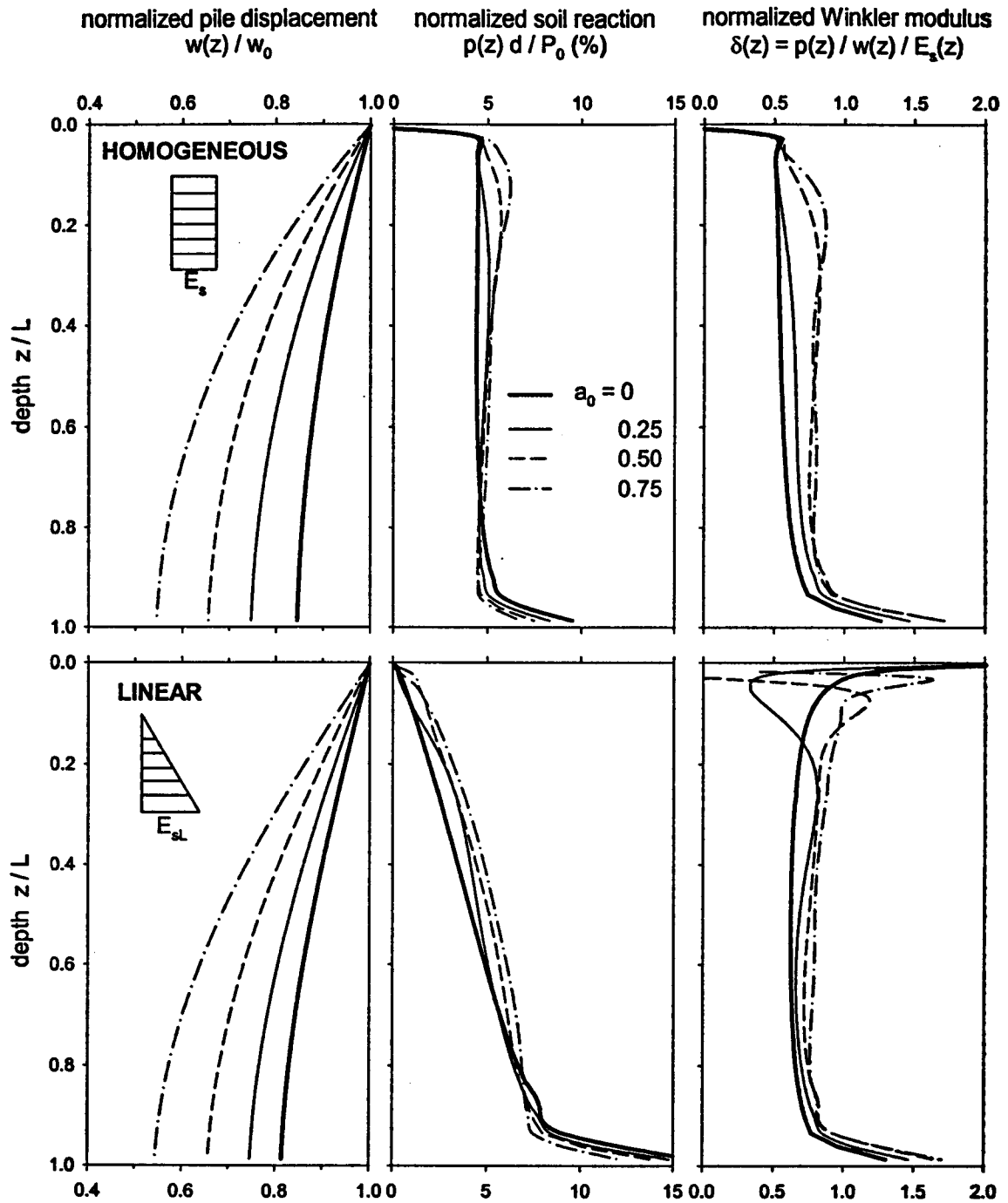


FIGURE 2.26: Real parts of normalized pile displacement, soil reaction per pile length and normalized Winkler modulus for different excitation frequencies, for piles of equal stiffness in two different soil profiles. ($L/d = 20$, $E_p/E_s = 1,000$, $E_p/E_{sL} = 630$)

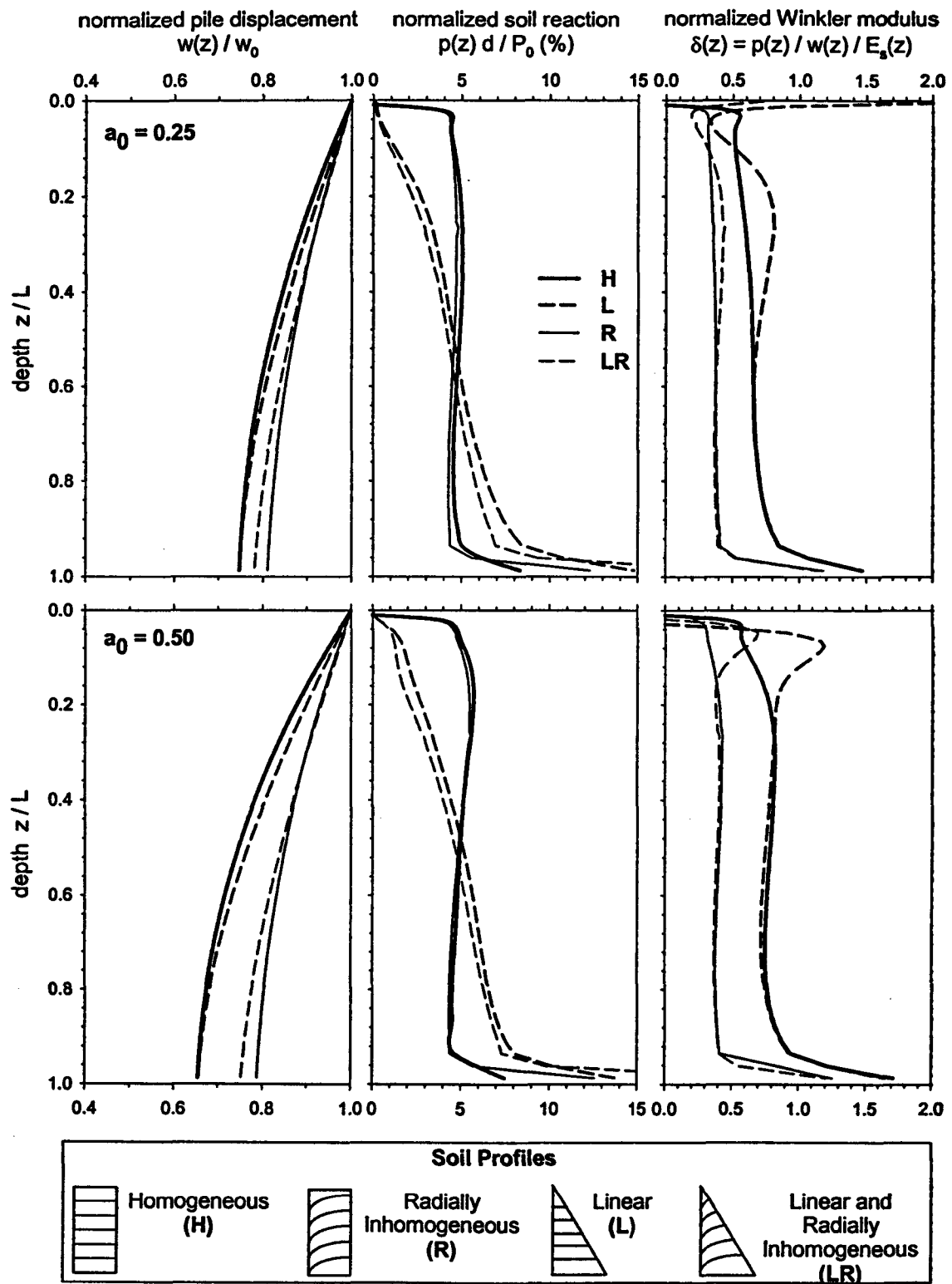


FIGURE 2.27: Real parts of normalized displacement, soil reaction and normalized Winkler modulus at two different excitation frequencies, for piles of equal stiffness in four different soil profiles; $L/d = 20$; $E_p/E_s = 1000$ (H), $E_p/E_{sL} = 630$ (L), $E_p/E_{s\infty} = 650$ (R), $E_p/E_{sL\infty} = 380$ (LR)

- The displacements for the linear and the homogeneous profile are very close as are those for the radial and the linear-radial profile. This is because in the radially inhomogeneous profiles the soil modulus near the pile is smaller, thus a bigger portion of the head load is carried by the base.
- Soil reactions for the vertically similar soil profiles (H and R; L and LR) are very close to each other in both frequencies. The same trend was observed as in the static case.
- Normalizing the Winkler modulus of the linear soil profiles with the soil modulus at the corresponding depth yields the following interesting pattern: The normalized Winkler moduli are very close for the radially similar profiles (H and L, R and LR). This implies that contrary to soil reaction, Winkler modulus is controlled by the near-pile soil properties.

The above trends are interesting but incomplete, as the behavior of the imaginary part is missing. To better understand the influence of inhomogeneity on the imaginary part of the Winkler modulus, damping ratios are plotted in Figure 2.28 for four statically equivalent soil profiles (same as those used in Figure 2.27).

The following are worthy of note:

- a) The real part of the Winkler modulus follows the patterns discussed in Figures 2.26 and 2.27.
- b) The damping ratio is less informative than that apparent at the pile-head, as it exhibits stronger variations, both with depth and with frequency. As a general trend, comparing the H and R profiles shows that damping ratio drops with increasing levels of inhomogeneity. This effect is also visible between the L and LR profiles.

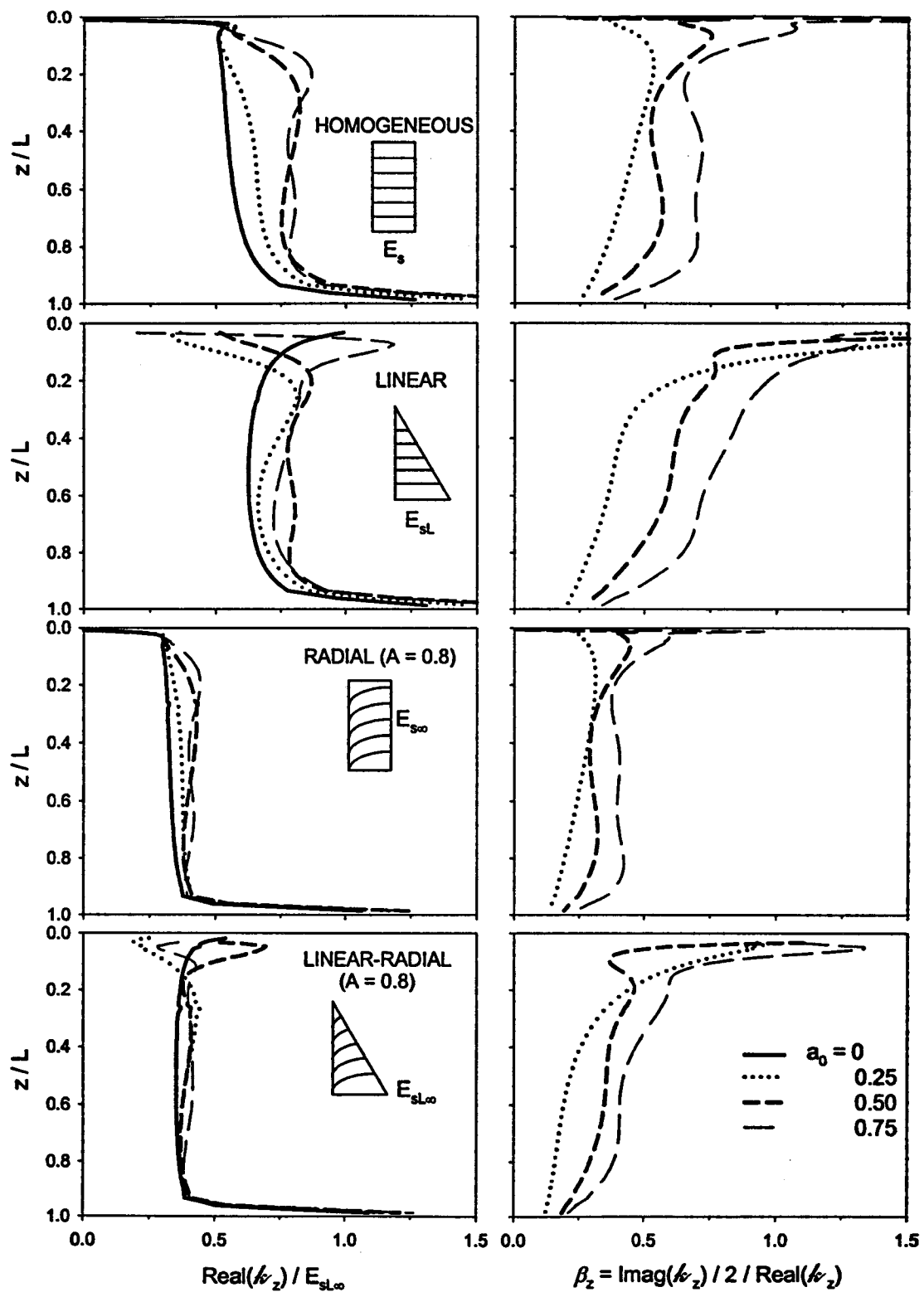


FIGURE 2.28: Real and imaginary parts of Winkler springs along the pile shaft of a pile in soil profiles of equal static stiffnesses, for different frequencies.

- c) The damping ratio, seems to be slightly higher for the linear than for the homogeneous profile. This trend is surprising, as it contradicts the previous results. The trend can be explained by recalling that the real part of the Winkler modulus (which is in the denominator of the damping ratio) becomes zero at the surface in the L profile, thereby increasing β_z . Furthermore, to achieve equal static pile stiffness, the soil modulus at the tip is greater in the linear soil profiles than in the homogeneous profiles. This tends to lower the values of β_z close to the base of the L profile, as shown in Figure 2.28.

2.5 TREATMENT OF PILE GROUP EFFECTS

The pile group problem is generally non axisymmetric and, thereby, unsuitable for the analytical solutions presented above. However, two approximate techniques can be applied to analyze the problem with the current tools. The first is to consider axisymmetric pile groups in the form of concentric rings of densely spaced piles (Tyson & Kausel, 1983). The second method is to employ the superposition approach, which is based on interaction factors (Appendix D). For both techniques K-PAX can be used to provide impedance curves for pile groups in inhomogeneous soil. With reference to the second method, the following three consecutive steps are followed: (1) The impedance of a single pile is determined using a pertinent analytical method such as K-PAX. (2) The interaction factors are determined between all possible pile pairs in the group by calculating the displacements at the pile head and at the corresponding distances from the pile, and (3) the interaction factors are combined to determine the impedance in the whole group (Appendix D).

The interaction factor α between two piles is defined based on the displacement of a pile carrying no load at its head (“receiver” pile), subjected to vibrations transmitted through the ground by a neighboring (“source”) pile which is loaded with an axial harmonic head load. The interaction factor then is defined as the displacement amplitude atop the receiver pile, normalized by the corresponding displacement amplitude of the source pile. Mylonakis and Gazetas (1998) showed that this interaction factor can be written as the product of two coefficients $\alpha = \psi \cdot \zeta$; the first coefficient, $\psi = \psi(r, \omega)$, denotes the induced free-field displacement and is a function of the distance between the two piles and the exciting frequency. Essentially, ψ is the ratio of the settlement of a point in the soil at a distance $r = r_0$ from the source pile and at a specified depth (say the middle of the pile $z = L/2$) over the settlement of a point in the perimeter of the pile, at the same depth

$$\psi = \psi(r_0, \omega) \approx \frac{U(r=r_0, z=L/2)}{U(r=d/2, z=L/2)} \quad (2.54)$$

which is valid for piles with slenderness ratio L/d up to approximately 30. The second coefficient, ζ , stands for the effect of diffraction of the arriving wave field due to the rigidity of the pile and the interaction between pile and surrounding soil. Analytical expressions for both coefficients exist, based on Beams on dynamic Winkler foundation models (Mylonakis, 1995; Mylonakis & Gazetas, 1998). The F.E. formulation described above can be used to derive rigorously both coefficients, by determining the displacement field expressed by ψ , and impose it on a “receiver” pile. This method is used extensively in Chapter 4 to evaluate experimental results for pile groups.

2.6 MAIN CONCLUSIONS

The main conclusions of this chapter are:

- (1) Accounting for radial soil inhomogeneity may increase elastic settlement by 50% or so.
- (2) A boundary layer phenomenon appears in the soil profiles with considerable soil stiffness at the surface. This is because of the boundary conditions, maximum axial force at the pile top combined with the maximum soil reaction slightly under the head.
- (3) Soil reaction along the pile shaft remains constant in equivalent profiles (same pile head stiffness) with the same vertical variation in modulus. This is valid at all frequencies.
- (4) Radiation damping in equivalent soil profiles (equal pile head stiffness) is significantly reduced with increasing inhomogeneity. Radiation damping in a homogeneous halfspace can be twice as high as damping in (a more realistic) inhomogeneous soil.
- (5) Reducing soil stiffness- a conservative assumption in a static analysis- may increase radiation damping, thereby resulting to unconservative results in a dynamic analysis of a pile.

CHAPTER 3

ELASTODYNAMIC ANALYSIS OF Laterally-Loaded PILES IN INHOMOGENEOUS SOIL

3.1 INTRODUCTION

In the past 25 years, a variety of theoretical methods have been developed for the dynamic analysis of piles under dynamic lateral loads (Wolf & Von Arx, 1978; Kuhlemeyer 1979; Kaynia & Kausel 1982; Sanchez-Salineró 1982; 1983; Sen et al 1985; El-Sharnouby & Novak 1986; El-Marsafawi et al 1992). Two important families of methods use finite and boundary elements which treat the soil as an unbounded continuum, and are used extensively in research.

Simplified solutions have also been developed. Most of these use the Beam-on-Winkler-Foundation (BWF) model in which the soil reaction to lateral pile movement, is represented by independent soil springs and dashpots. For static loads, these springs are calibrated using field load tests leading to semi-empirical non-linear “p-y” curves (Matlock & Reese 1960; Reese 1986). For dynamic loads, theoretically-calculated soil springs and dashpots have been proposed based on two-dimensional (Novak 1974; Novak et al 1978; Veletsos & Dotson 1986) and three-dimensional wave propagation theory (Nogami & Novak 1977; Mylonakis 2001b). Furthermore, spring values have been obtained by matching results from Winkler and from more rigorous solutions (Blaney et al 1976; Kagawa & Kraft 1980; Dobry et al 1982; Mylonakis 2001a).

As in the case of vertical loading, most of the published work addresses only homogeneous soil (i.e., halfspace or layer over rock). Results for inhomogeneous soil are limited, usually obtained by means of simplified Winkler models (e.g. Novak & Aboul-Ella 1978). In very few cases double-soil inhomogeneity, arising from pile installation and load-induced stresses, has been considered (Michaelides et al 1998). It is known that radial inhomogeneity results to reduced stiffness and radiation damping in the system, therefore increasing the dynamic displacements of the pile.

Like in Chapter 1, this chapter focuses in the foundation impedance problem of the overall SSI problem. More specifically, the purpose of this chapter is to:

- Provide formulas to estimate the pile-head static stiffness for different pile-head conditions and soil profiles including the effect of radial inhomogeneity.
- Provide formulas to estimate the soil moduli of radially inhomogeneous soil profiles that have matched pile-head static swaying stiffness with radially homogeneous profiles. This is necessary for comparing the pile response in different soil profiles.
- Provide relations for the normalized Winkler modulus (δ) to be used in conjunction with the versatile Winkler method.
- Compare existing formulas for the static active length of a free head pile and propose new ones for fixed head piles.
- Propose a new definition for the static pile active length.
- Demonstrate the way different soil profiles resist the statically loaded pile and the effect of radial inhomogeneity.

- Propose a new way to estimate the variation of the normalized Winkler modulus with depth.
- Provide formulas to estimate the damping at the pile-head for different pile-head conditions and soil profiles including the effect of radial inhomogeneity.
- Demonstrate the significance of soil inhomogeneity in the amount of radiation damping apparent at the pile head and propose ways to employ it in the dynamic analysis of piles.
- Demonstrate the way different soil profiles resist the dynamically loaded pile and the effect of radial inhomogeneity.

3.2 PROBLEM DEFINITION

The system considered is depicted in Figure 3.1: a solid cylindrical pile embedded in a soil layer, subjected to a horizontal head load or head moment. Unlike axial loading, there are two different boundary conditions at the pile head (Figure 3.2). The first case is when the pile top is restrained against rotation. This happens when a thick cap connects the heads of the piles in a pile group. In the second case, the pile top is free to rotate as it displaces horizontally. Likewise, for moment loading, the pile top displaces horizontally while rotating.

3.2.1 Finite Element Mesh

The finite-element mesh is done in such a way that the discretization is finer close to the pile head, where a large concentration of stresses is expected, and gets coarser

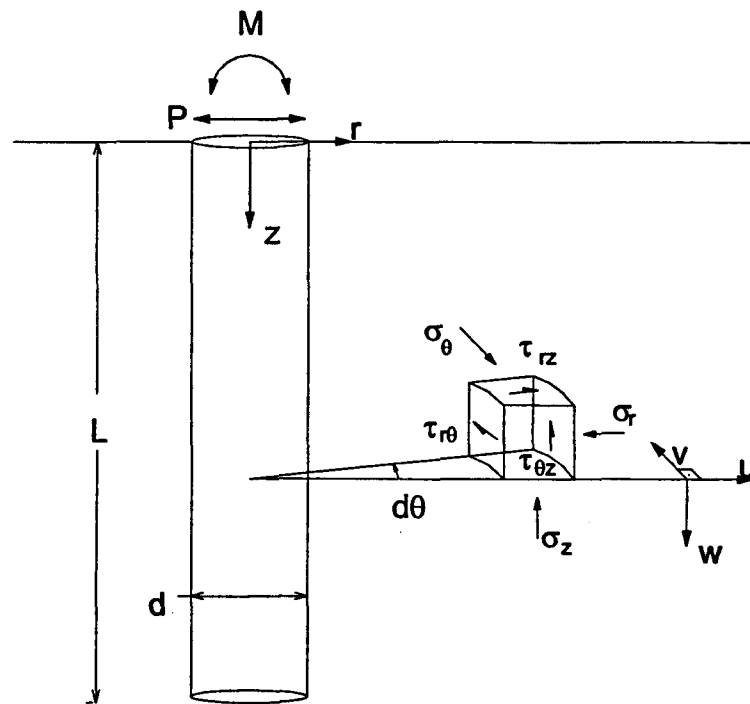


FIGURE 3.1: Problem considered

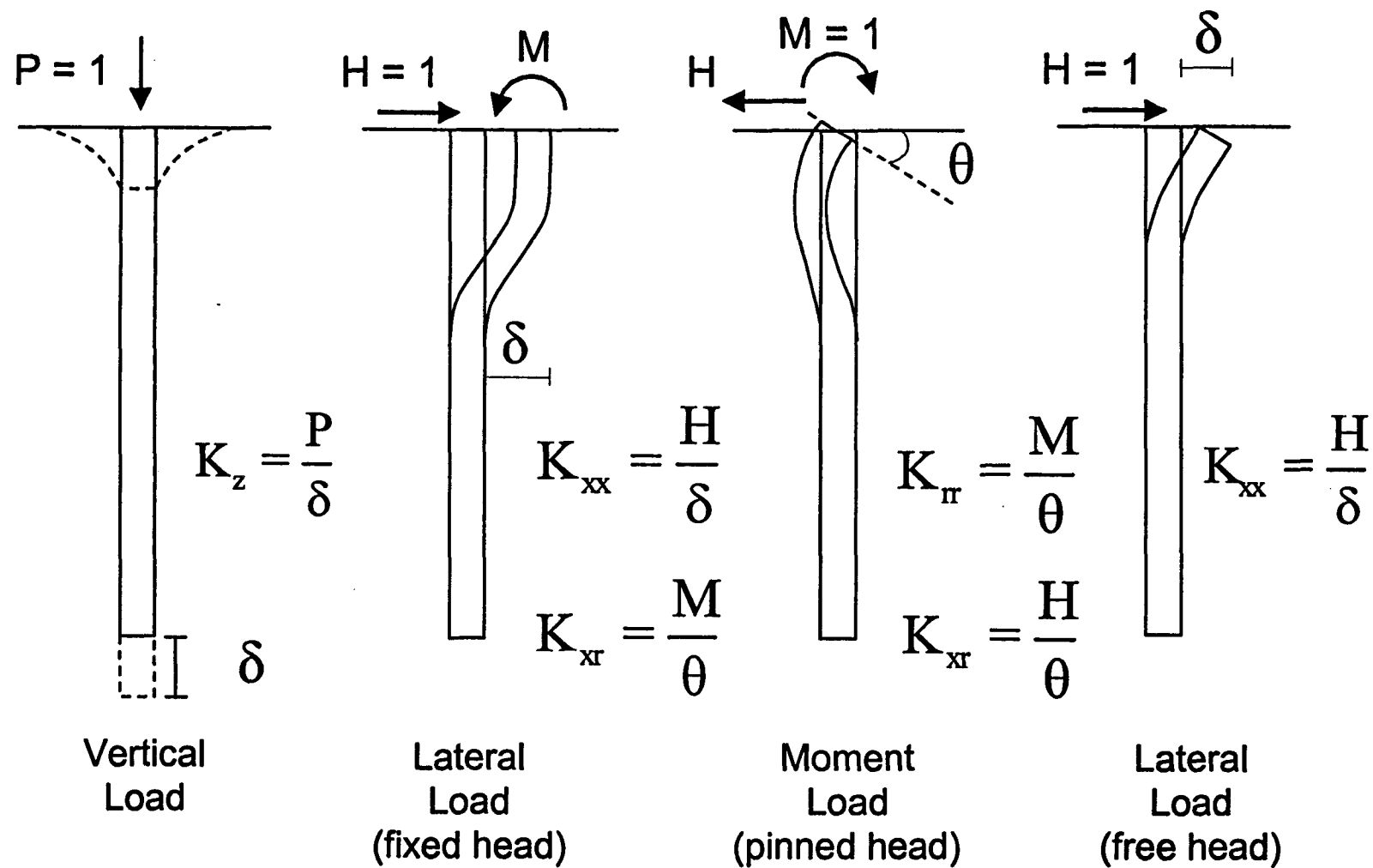


FIGURE 3.2: Different pile-head loadings examined in this study

towards the tip and away from the pile like in the axial problem (Figure 2.2). In addition, the width of the mesh is kept equal to the mesh height, as done in the axial problem.

Although in the construction of the mesh, some similarities with the axial problem exist, there are also major differences. In the lateral problem, stresses in the soil are concentrated near the pile top and become very small below a certain depth. This is known as the “active pile length” and is typically of the order of 10 to 15 pile diameters (Randolph, 1981) which means that in the case of long piles (e.g. $L/d > 40$) the pile impedance is independent of pile length and tip conditions. Exploiting this phenomenon, long floating piles can be analyzed considering a single length (e.g. $L/d = 40$), and a shallow mesh ($H/L \approx 1$).

3.2.2 Isoparametric formulation

The problem of a laterally loaded pile involves non symmetric loading meaning that the response depends on the angle θ and the tangential displacement v is no longer zero. For such loads the problem can be decomposed into a number of uncoupled two-dimensional problems, by expanding loading and displacements in Fourier series. The three-dimensional nature of the problem is preserved because of the three degrees of freedom associated with each node. Due to the orthogonality of the Fourier series, each term of the loading produces a displacement in the same Fourier mode. In general, this

formulations. Note that the axisymmetric problem is a special case of this analysis, since it corresponds to the constant term (zeroeth mode) of the Fourier series.

In the general case, the displacements $\{u, v, w\}$ and the loads $\{P_r, P_\theta, P_z\}$ are taken as functions of the angular coordinate θ and can be expressed in Fourier series by (Cook et al 1989):

$$\begin{aligned} u &= \sum_{n=0}^{\infty} [\bar{u}_s^n \cos(n\theta) + \bar{u}_a^n \sin(n\theta)] \quad , \quad P_r = \sum_{n=0}^{\infty} [\bar{P}_{r,s}^n \cos(n\theta) + \bar{P}_{r,a}^n \sin(n\theta)] \\ v &= \sum_{n=0}^{\infty} [-\bar{v}_s^n \sin(n\theta) + \bar{v}_a^n \cos(n\theta)] \quad , \quad P_\theta = \sum_{n=0}^{\infty} [-\bar{P}_{\theta,s}^n \sin(n\theta) + \bar{P}_{\theta,a}^n \cos(n\theta)] \quad (3.1) \\ w &= \sum_{n=0}^{\infty} [\bar{w}_s^n \cos(n\theta) + \bar{w}_a^n \sin(n\theta)] \quad , \quad P_z = \sum_{n=0}^{\infty} [\bar{P}_{z,s}^n \cos(n\theta) + \bar{P}_{z,a}^n \sin(n\theta)] \end{aligned}$$

where the mode amplitudes are denoted with an overbar and the subscripts "s" and "a" correspond to components that are, symmetric and antisymmetric respectively with respect to the circumferential direction.

The reason for using the sine expansion for the symmetric terms of v and P_θ is that it produces the same stiffness matrix whether we consider symmetric or antisymmetric components.

Zienkiewicz (1977) points out that nodal loads represent forces per unit length of circumference. Smith and Griffiths (1988) give the expansion referring to symmetric terms as

$$\begin{aligned} P_r &= \frac{1}{2} \bar{P}_r^0 + \bar{P}_r^1 \cos(\theta) + \bar{P}_r^2 \cos(2\theta) + \dots \\ P_\theta &= \bar{P}_\theta^1 \sin(\theta) + \bar{P}_\theta^2 \sin(2\theta) + \dots \\ P_z &= \frac{1}{2} \bar{P}_z^0 + \bar{P}_z^1 \cos(\theta) + \bar{P}_z^2 \cos(2\theta) + \dots \end{aligned} \quad (3.2)$$

and state that the above loads represent forces per radian in the radial, tangential and vertical directions, respectively. This implies that a radial load of unit amplitude ($P_r = 1$) in the first symmetric harmonic ($n = 1$) results in a net thrust in the $\theta = 0^\circ$ direction of $\bar{P}_r = \pi$. Thus the load considered in the analysis, should have an amplitude of $1/\pi$ to produce a unit horizontal load.

It is noted that the above expressions can also be written using complex notation as

$$\begin{Bmatrix} u \\ v \\ w \end{Bmatrix} = \sum_{n=-\infty}^{\infty} e^{in\theta} \begin{Bmatrix} \bar{u}_n \\ i \bar{v}_n \\ \bar{w}_n \end{Bmatrix} \quad (3.3)$$

which is not adopted herein to avoid confusion with complex fourier expansions over time.

3.2.2.1 Stiffness matrix:

The stress-strain relationships can be expressed in a matrix form as:

$$\sigma = D \varepsilon \quad (3.4)$$

where, σ and ε are vectors holding the element's stresses and strains respectively

$$\sigma = [\sigma_{rr} \quad \sigma_{\theta\theta} \quad \sigma_{zz} \quad \tau_{rz} \quad \tau_{r\theta} \quad \tau_{z\theta}]^T \quad (3.5)$$

$$\varepsilon = [\varepsilon_{rr} \quad \varepsilon_{\theta\theta} \quad \varepsilon_{zz} \quad \gamma_{rz} \quad \gamma_{r\theta} \quad \gamma_{z\theta}]^T \quad (3.6)$$

D is given by

$$\mathbf{D} = \begin{bmatrix} \lambda+2\mu & \lambda & \lambda & 0 \\ \lambda & \lambda+2\mu & \lambda & 0 \\ \lambda & \lambda & \lambda+2\mu & 0 \\ 0 & 0 & 0 & \mu \\ 0 & 0 & 0 & \mu \\ 0 & 0 & 0 & \mu \end{bmatrix} \quad (3.7)$$

where λ and μ are the complex Lamé's constants

$$\lambda = \frac{\nu E^*}{(1+\nu)(1-2\nu)} \quad (3.8)$$

$$\mu = G = \frac{E^*}{2(1+\nu)} \quad (3.9)$$

where ν is the Poisson's ratio of the material and E^* is the familiar complex modulus of linear viscoelasticity

$$E^* = E(1 + i2\beta) \quad (3.10)$$

The corresponding strain-displacement can be written in a matrix form as:

$$\boldsymbol{\varepsilon} = \mathbf{S} \mathbf{u} \quad (3.11)$$

where $\mathbf{S}$ is the familiar strain-compatibility matrix.

$$\mathbf{S} = \begin{bmatrix} \frac{\partial}{\partial r} & 0 & 0 \\ \frac{1}{r} & \frac{1}{r} \frac{\partial}{\partial \theta} & 0 \\ 0 & 0 & \frac{\partial}{\partial z} \\ \frac{\partial}{\partial z} & 0 & \frac{\partial}{\partial r} \\ \frac{1}{r} \frac{\partial}{\partial \theta} & \left(\frac{\partial}{\partial r} - \frac{1}{r} \right) & 0 \\ 0 & \frac{\partial}{\partial z} & \frac{1}{r} \frac{\partial}{\partial \theta} \end{bmatrix} \quad (3.12)$$

The Fourier expansion of the displacements can be written in matrix form as

$$\mathbf{u} = \begin{bmatrix} u \\ v \\ w \end{bmatrix} = \sum_{n=0}^n \mathbf{R}_n \bar{\mathbf{u}}_n \quad (3.13)$$

where $\mathbf{R}_n$ is

$$\mathbf{R}_n = \begin{bmatrix} \cos(n\theta) & 0 & 0 \\ 0 & -\sin(n\theta) & 0 \\ 0 & 0 & \cos(n\theta) \end{bmatrix} \quad (3.14)$$

for the symmetric terms and

$$\mathbf{R}_n = \begin{bmatrix} \sin(n\theta) & 0 & 0 \\ 0 & \cos(n\theta) & 0 \\ 0 & 0 & \sin(n\theta) \end{bmatrix} \quad (3.15)$$

for the antisymmetric ones.

One can relate the displacement amplitudes $\bar{\mathbf{u}}_n$ of any point within the element to the nodal displacement amplitudes $\bar{\mathbf{U}}^n$ by using an appropriate interpolation matrix $\mathbf{N}$. Accordingly,

$$\bar{\mathbf{u}}_n = \mathbf{N} \bar{\mathbf{U}}^n \quad (3.16)$$

In this study, quadratic shape functions are used to interpolate the element's displacements. In the spirit of eq. (3.3) we can rewrite eq. (3.16) as

$$\mathbf{u} = \begin{bmatrix} u \\ v \\ w \end{bmatrix} = \sum_{n=0}^{\infty} (\mathbf{R}_n \mathbf{N} \bar{\mathbf{U}}^n) \quad (3.17)$$

In the above equation, $\bar{\mathbf{U}}^n$ is a matrix containing the nodal displacement amplitudes

$$\bar{\mathbf{U}}^n = [\bar{U}_1^n \quad \bar{V}_1^n \quad \bar{W}_1^n \mid \bar{U}_2^n \quad \bar{V}_2^n \quad \bar{W}_2^n \mid \dots]^T \quad (3.18)$$

where $\bar{U}_1^n$, $\bar{V}_1^n$, $\bar{W}_1^n$ etc. are, respectively, the radial, tangential and vertical displacement amplitudes of nodes 1, 2 etc. (in the element level) for the n-th harmonic. Note that all these amplitudes are independent of the coordinate θ .

Substituting (3.17) into (3.11) one obtains for the nth circumferential mode

$$\boldsymbol{\varepsilon}_n = \mathbf{S} \mathbf{u}_n = \mathbf{S} (\mathbf{R}_n \mathbf{N} \bar{\mathbf{U}}^n)$$

which can be rewritten as

$$(\mathbf{S} \mathbf{R}_n) \mathbf{N} \bar{\mathbf{U}}^n = (\mathbf{R}'_n \mathbf{S}_n) \mathbf{N} \bar{\mathbf{U}}^n = \mathbf{R}'_n (\mathbf{S}_n \mathbf{N}) \bar{\mathbf{U}}^n = \mathbf{R}'_n \mathbf{B}_n \bar{\mathbf{U}}^n \quad (3.19)$$

where $\mathbf{B}_n$ is the strain-displacement matrix for the n-th mode and $\mathbf{R}'_n$ is a 6 by 6 transformation matrix.

For symmetric loading, matrices B_n and R'_n are determined from the following expressions:

$$\begin{aligned}
 S R_n &= \left[\frac{1}{r} S_0 + S_1 + \frac{1}{r} S_2 \frac{\partial}{\partial \theta} \right] \begin{bmatrix} \cos(n\theta) & 0 & 0 \\ 0 & -\sin(n\theta) & 0 \\ 0 & 0 & \cos(n\theta) \end{bmatrix} = \\
 &= \left[\frac{1}{r} \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} \frac{\partial}{\partial r} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \frac{\partial}{\partial z} \\ \frac{\partial}{\partial z} & 0 & \frac{\partial}{\partial r} \\ 0 & \frac{\partial}{\partial r} & 0 \\ 0 & \frac{\partial}{\partial z} & 0 \end{bmatrix} + \frac{1}{r} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \frac{\partial}{\partial \theta} \right] \begin{bmatrix} \cos(n\theta) & 0 & 0 \\ 0 & -\sin(n\theta) & 0 \\ 0 & 0 & \cos(n\theta) \end{bmatrix} = \\
 &= S'_n \begin{bmatrix} \cos(n\theta) & 0 & 0 \\ 0 & -\sin(n\theta) & 0 \\ 0 & 0 & \cos(n\theta) \end{bmatrix} = \\
 &= \begin{bmatrix} \cos(n\theta) & 0 & 0 & 0 & 0 & 0 \\ 0 & \cos(n\theta) & 0 & 0 & 0 & 0 \\ 0 & 0 & \cos(n\theta) & 0 & 0 & 0 \\ 0 & 0 & 0 & \cos(n\theta) & 0 & 0 \\ 0 & 0 & 0 & 0 & -\sin(n\theta) & 0 \\ 0 & 0 & 0 & 0 & 0 & -\sin(n\theta) \end{bmatrix} S'_n = R'_n S'_n
 \end{aligned}$$

(3.20)

$$\text{where } \mathbf{S}'_n = \begin{bmatrix} \frac{\partial}{\partial r} & 0 & 0 \\ \frac{1}{r} & 0 & 0 \\ 0 & 0 & \frac{\partial}{\partial z} \\ \frac{\partial}{\partial z} & 0 & \frac{\partial}{\partial r} \\ 0 & \frac{\partial}{\partial r} - \frac{1}{r} & 0 \\ 0 & \frac{\partial}{\partial z} & 0 \end{bmatrix} + \frac{n}{r} \begin{bmatrix} 0 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (3.21)$$

Substituting (3.18) and (3.21) into (3.11)

$$\boldsymbol{\varepsilon} = \sum_{n=0}^{\infty} \begin{bmatrix} \cos(n\theta) \mathbf{I}_4 & 0 \\ 0 & -\sin(n\theta) \mathbf{I}_2 \end{bmatrix} (\mathbf{S}'_n \mathbf{N}) \bar{\mathbf{U}}^n =$$

where $\mathbf{I}_4$ and $\mathbf{I}_2$ are the identity matrices of order 4 and 2 respectively.

Thus:

$$\boldsymbol{\varepsilon} = \begin{bmatrix} \begin{bmatrix} \mathbf{I}_4 & 0 \\ 0 & 0 \end{bmatrix} \mathbf{B}_0 & \begin{bmatrix} \cos(\theta) \mathbf{I}_4 & 0 \\ 0 & -\sin(\theta) \mathbf{I}_2 \end{bmatrix} \mathbf{B}_1 \\ \begin{bmatrix} \cos(2\theta) \mathbf{I}_4 & 0 \\ 0 & -\sin(2\theta) \mathbf{I}_2 \end{bmatrix} \mathbf{B}_2 & \cdots \end{bmatrix} \begin{bmatrix} \bar{\mathbf{U}}^0 \\ \bar{\mathbf{U}}^1 \\ \bar{\mathbf{U}}^2 \\ \vdots \end{bmatrix} = \mathbf{B} \bar{\mathbf{U}} \quad (3.22)$$

which is the well known strain-displacement formulation, and in which the $\mathbf{B}_n$ represents the strain displacement relations for the n^{th} mode.

The element stiffness matrix in its general form is defined as

$$\mathbf{K} = \int_V \mathbf{B}^T \mathbf{D} \mathbf{B} dV$$

Substituting $\mathbf{B}$ from eq.(3.22) and carrying out the calculations yields:

$$\mathbf{K} = \begin{bmatrix} \mathbf{K}_{00} & \mathbf{K}_{01} & & \\ \mathbf{K}_{10} & \mathbf{K}_{11} & & \\ & & \ddots & \\ & & & \mathbf{K}_{mm} \end{bmatrix}$$

the indices correspond to the harmonics m and n . It will be shown that for m different than n the stiffness matrix is zero, which means that the harmonics are uncoupled.

Indeed, carrying out the matrix multiplication one obtains for the symmetric case:

$$\mathbf{K}_{mn} = \int_V \mathbf{B}_m^T \begin{bmatrix} \cos(m\theta) \mathbf{I}_4 & 0 \\ 0 & -\sin(m\theta) \mathbf{I}_2 \end{bmatrix} \mathbf{D} \begin{bmatrix} \cos(n\theta) \mathbf{I}_4 & 0 \\ 0 & -\sin(n\theta) \mathbf{I}_2 \end{bmatrix} \mathbf{B}_n dV$$

$$\mathbf{K}_{mn} = \int_{A_{rz}} \mathbf{B}_m^T \left\{ \int_{-\pi}^{\pi} \begin{bmatrix} \cos(m\theta) \cos(n\theta) \mathbf{I}_4 & 0 \\ 0 & \sin(m\theta) \sin(n\theta) \mathbf{I}_2 \end{bmatrix} d\theta \right\} \mathbf{D} \mathbf{B}_n dA_{rz}$$

where dA_{rz} indicates integration over the element area within the plane of the model.

Because of the special nature of the transform matrix $\mathbf{R}'_n$, the matrix multiplication can be rearranged so that:

$$\Rightarrow \mathbf{K}_{mn} = \begin{cases} 2\pi \int_{A_{rz}} \mathbf{B}_0^T \begin{bmatrix} \mathbf{I}_4 & 0 \\ 0 & 0 \end{bmatrix} \mathbf{D} \mathbf{B}_0 dA_{rz} , & m = n = 0 \\ \pi \int_{A_{rz}} \mathbf{B}_n^T \mathbf{D} \mathbf{B}_n dA_{rz} , & m = n = 1, 2, .. \\ 0 & , m \neq n \end{cases} \quad (3.23)$$

Repeating equations (3.21) through (3.23) one obtains the similar expressions for the antisymmetric case:

$$\mathbf{K}_{mm} = \begin{cases} 2\pi \int_{A_{rz}} \mathbf{B}_0^T \begin{bmatrix} 0 & 0 \\ 0 & \mathbf{I}_2 \end{bmatrix} \mathbf{D} \mathbf{B}_0 dA_{rz} , m = n = 0 \\ \pi \int_{A_{rz}} \mathbf{B}_n^T \mathbf{D} \mathbf{B}_n dA_{rz} , m = n = 1, 2, .. \\ 0 , m \neq n \end{cases} \quad (3.24)$$

which differ from the one in eq. (3.23) only in the zeroeth harmonic.

3.2.2.2 Load vector:

The element load vector due to tractions on the surface of the element is obtained through the virtual work expression:

$$\delta \mathbf{U}^T \mathbf{P} = \int_S \delta \mathbf{U}^T \mathbf{T} dS \quad (3.25)$$

$$\text{thus } \mathbf{P} = \sum_n \int_S (\mathbf{N}^T \mathbf{R}_n^T) (\mathbf{T}) dS \quad (3.26)$$

where $\mathbf{R}_n$ is given in eq. (3.14) for the symmetric terms and in eq. (3.15) for the antisymmetric terms. In the case where the surface tractions ($\mathbf{T}$) can be expressed as a Fourier series, $\mathbf{T}$ can be written as

$$\mathbf{T} = \sum_{m=0}^{\infty} (\mathbf{R}_m \bar{\mathbf{P}}_m) \quad (3.27)$$

Plugging equation (3.27) into (3.26), one obtains, in the case of the symmetric terms

$$\mathbf{P} = \sum_n \int_A (\mathbf{N}^T \mathbf{R}_n^T) (\mathbf{T}) dA = \sum_m \sum_n \int_A (\mathbf{N}^T \mathbf{R}_n^T) (\mathbf{R}_m \bar{\mathbf{P}}_m) dA \Rightarrow$$

$$\Rightarrow \mathbf{P}_{mm} = \int_s \mathbf{N}^T \begin{bmatrix} \cos(n\theta) \cos(m\theta) & 0 & 0 \\ 0 & \sin(n\theta) \sin(m\theta) & 0 \\ 0 & 0 & \cos(n\theta) \cos(m\theta) \end{bmatrix} \bar{\mathbf{P}}_m dS \quad (3.28)$$

where $\bar{\mathbf{P}}_m$ is the load amplitude vector corresponding to the m^{th} harmonic.

As the interpolation matrix and the load amplitude vector are independent of θ , integration with respect to the angular coordinate yields:

$$\mathbf{P}_{mm} = \int_{\Xi} \mathbf{N}^T \left\{ \int_{-\pi}^{\pi} \begin{bmatrix} \cos(n\theta) \cos(m\theta) & 0 & 0 \\ 0 & \sin(n\theta) \sin(m\theta) & 0 \\ 0 & 0 & \cos(n\theta) \cos(m\theta) \end{bmatrix} d\theta \right\} \bar{\mathbf{P}}_m d\Xi \quad (3.29)$$

where Ξ is the projection of the traction-loaded surface onto the plane of the model.

Knowing that

$$\int_{-\pi}^{\pi} \cos(n\theta) \cos(m\theta) d\theta = \int_{-\pi}^{\pi} \sin(n\theta) \sin(m\theta) d\theta = 0, \text{ for } m \neq n$$

eq. (3.29) can be reduced to

$$\mathbf{P}_n = \begin{cases} 2\pi \int_{\Xi} \mathbf{N}^T \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \bar{\mathbf{P}}_n d\Xi, & n=0 \\ \pi \int_{\Xi} \mathbf{N}^T \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \bar{\mathbf{P}}_n d\Xi, & n=1, 2, .. \end{cases} \quad (3.30)$$

where $\mathbf{P}_n$ is the nodal load vector corresponding to the n^{th} harmonic. Similarly, for the antisymmetric terms, one obtains

$$\mathbf{P}_n = \begin{cases} 2\pi \int_{\Xi} \mathbf{N}^T \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \bar{\mathbf{P}}_n d\Xi, n=0 \\ \pi \int_{\Xi} \mathbf{N}^T \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \bar{\mathbf{P}}_n d\Xi, n=1, 2, \dots \end{cases} \quad (3.31)$$

3.2.2.2.1 Horizontal unit load distributed over the pilehead:

In the case of horizontal loading, the horizontal unit load over the pile-head area is equal to

$$\mathbf{P}_x = \frac{\text{unit load}}{\text{Area}} = \frac{1}{\pi(R_o^2 - R_i^2)} \quad : \text{units [F / L}^2 \text{]} \quad (3.32)$$

where R_o and R_i are the outer and inner radii of a hollow cylindrical pile. Decomposing this load into a radial and a tangential component yields the following force per surface area:

$$\mathbf{T} = \{P_r \quad P_\theta \quad P_z\}^T = \{P_x \cos\theta \quad -P_x \sin\theta \quad 0\} \quad (3.33)$$

which describes the loading in the model domain. From inspection, one may notice that the radial and tangential components are already expressed in Fourier series. The radial component is the first term of a cosine expansion and the tangential component is the first term of the sine expansion. It follows that $\mathbf{T}$ can be written as:

$$\mathbf{T} = \sum_n \mathbf{R}_n \bar{\mathbf{P}}_n \quad (3.34)$$

where $\mathbf{R}_n$ is taken from eq. (3.14) for symmetric terms and $\bar{\mathbf{P}}_n$ is equal to

$$\bar{P}_n = \begin{cases} [P_x & P_x & 0]^T, n=1 \\ [0 & 0 & 0]^T, n=0, 2, 3, .. \end{cases} \quad (3.35)$$

which implies that only the first harmonic needs be analyzed.

Following is the derivation of the consistent nodal load vector for a quadratic triangle that is loaded over its surface. With reference to Figure 3.3 the inner node (3) has a distance R_1 from the axis of symmetry and the outer node (2) has a distance R_2 . Using eq. (3.31) and (3.35) yields

$$\Rightarrow P = \pi P_x (R_2 - R_1) \left[\begin{array}{c|cc} 0_{1 \times 3} & \frac{R_2}{6} & -\frac{R_2}{6} & 0 \\ \hline \frac{R_1}{6} & -\frac{R_1}{6} & 0 & 0_{1 \times 3} \end{array} \left| \begin{array}{cc} \frac{R_1 + R_2}{3} & -\frac{R_1 + R_2}{3} \\ 0 & 0_{1 \times 3} \end{array} \right|^T \right] \quad (3.36)$$

In the case of the horizontal load being applied as a nodal ring load (e.g., to node 3), eq. (3.36) yields:

$$P = P_{n-1} = \pi (R_2^2 - R_1^2) P_x [0_{1 \times 3} \mid 1 \quad 1 \quad 0 \mid 0_{1 \times 12}]^T \quad (3.37)$$

3.2.2.2.2 Vertical unit load distributed over the pile-head:

In this case the force per surface vector is

$$T = \{P_r \quad P_\theta \quad P_z\}^T = \{0 \quad 0 \quad P_z\} \quad (3.38)$$

where

$$P_z = \frac{\text{unit load}}{\text{Area}} = \frac{1}{\pi (R_o^2 - R_i^2)} : \text{units [F / L}^2 \text{]} \quad (3.39)$$

Expanding this load in cosine series, it follows that only the constant term (zeroeth harmonic) survives. Thus T can be written in the form of eq. (3.34) with R_n taken from eq. (3.14) for symmetric terms and $\bar{P}_n$ as

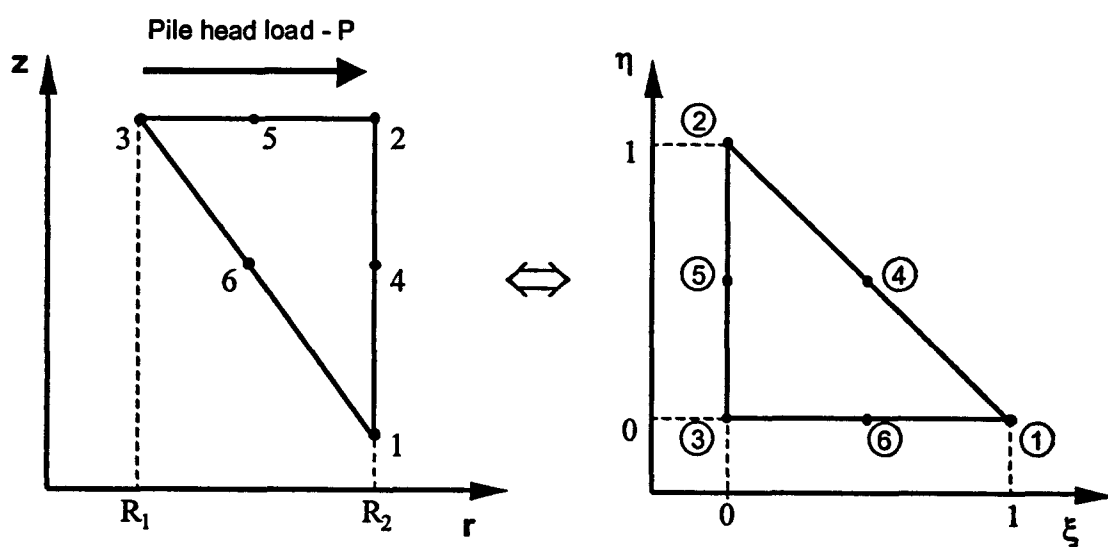


FIGURE 3.3: Representation of a triangular quadratic element at the pile-head, in actual and computational space.

$$\bar{\mathbf{P}}_n = \begin{cases} [0 & 0 & P_z]^T & \text{for } n=0 \\ [0 & 0 & 0]^T & \text{for } n=1, 2, 3, \dots \end{cases} \quad (3.40)$$

Plugging in eq. (3.30) yields

$$\mathbf{P} = \mathbf{P}_{n=0} = \int_{\Xi} \mathbf{N}^T \begin{bmatrix} 2\pi & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 2\pi \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ P_z \end{bmatrix} d\Xi \quad (3.41)$$

In the case of the load being distributed over the element, as with the horizontal load mentioned before, one obtains

$$\mathbf{P} = \mathbf{P}_{n=0} = 2\pi P_z (R_2 - R_1) \begin{bmatrix} \mathbf{0}_{1 \times 3} & 0 & 0 & \frac{R_2}{6} \\ 0 & 0 & \frac{R_1}{6} & \mathbf{0}_{1 \times 3} \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ \frac{R_1 + R_2}{3} & \mathbf{0}_{1 \times 3} \end{bmatrix}^T \quad (3.42)$$

and for the case of a nodal ring load, the load vector is

$$\mathbf{P} = \mathbf{P}_{n=0} = \pi (R_2^2 - R_1^2) P_z [\mathbf{0}_{3 \times 1} \mid 0 \quad 0 \quad 1 \mid \mathbf{0}_{1 \times 3}]^T \quad (3.43)$$

3.2.2.2.3 Unit Torsional Moment distributed over the pile-head area:

This loading condition is very similar to the vertical loading (constant load) with the difference that now, the antisymmetric terms are used.

$$\mathbf{T} = \{P_r \quad P_\theta \quad P_z\}^T = \{0 \quad P_\theta \quad 0\} \quad (3.44)$$

where

$$P_\theta = \frac{\text{unit load}}{\text{Area}} = \frac{1}{\pi (R_o^2 - R_i^2)} \quad : \text{units [F / L}^2 \text{]} \quad (3.45)$$

and $\mathbf{T}$ can be written as

$$\mathbf{T} = \sum_n \mathbf{R}_n \bar{\mathbf{P}}_n$$

where R_n is given from eq. (3.15) and $\bar{P}_n$ is equal to

$$\bar{P}_n = \begin{cases} [0 \ P_0 \ 0]^T & \text{for } n = 0 \\ [0 \ 0 \ 0]^T & \text{for } n = 1, 2, 3, \dots \end{cases} \quad (3.46)$$

Plugging in eq. (3.31) yields

$$\mathbf{P} = \mathbf{P}_{n=0} = \int_{\Xi} \mathbf{N}^T \begin{bmatrix} 0 & 0 & 0 \\ 0 & 2\pi & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ P_0 \\ 0 \end{bmatrix} d\Xi \quad (3.47)$$

In the case of the load being distributed over the element ($\xi = 0$):

$$\mathbf{P} = \mathbf{P}_{n=0} = 2\pi P_0 (R_2 - R_1) \begin{bmatrix} \mathbf{0}_{1 \times 3} & \left| 0 \ \frac{R_2}{6} \ 0 \right| \\ \left| 0 \ \frac{R_1}{6} \ 0 \right| & \mathbf{0}_{1 \times 3} \end{bmatrix} \begin{bmatrix} 0 \\ \frac{R_1 + R_2}{3} \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ \mathbf{0}_{1 \times 3} \end{bmatrix}^T \quad (3.48)$$

This distributed over the element load yields a torsional moment (M_T) about the axis of symmetry, equal to

$$M_T = \frac{\pi}{3} (R_2^3 - R_1^3) P_0 \quad (3.49)$$

This implies, for a unit torsional moment about the axis of symmetry, the tangential load amplitude P_0 should be set equal to

$$P_0 = \frac{3 \text{ (Unit Moment)}}{\pi (R_2^3 - R_1^3)} \quad (3.50)$$

In the case of a nodal ring load, using eq. (3.47) similarly one obtains

$$\mathbf{P} = \mathbf{P}_{n=0} = \pi (R_2^2 - R_1^2) P_0 \begin{bmatrix} \mathbf{0}_{3 \times 1} & | & 0 & 1 & 0 & | & \mathbf{0}_{12 \times 1} \end{bmatrix}^T \quad (3.51)$$

This nodal ring load yields a torsional moment (M_T) about the axis of symmetry, equal to

$$M_T = \pi R_2 (R_2^2 - R_1^2) P_0 \quad (3.52)$$

Which implies, that for a unit torsional moment about the axis of symmetry, the tangential load amplitude P_θ should be set equal to

$$P_\theta = \frac{\text{Unit Moment}}{\pi R_2 (R_2^2 - R_1^2)} \quad (3.53)$$

3.2.2.2.4 Unit Moment loading distributed over the pile-head area:

For the case of a moment M_0 applied to the pile head, the force per surface unit area can be obtained from strength-of-materials theory as

$$P_z = \frac{M_0}{I} x \quad (3.54)$$

where the Cartesian coordinate x can be expressed as $x = r \cos\theta$ and I is the cross-sectional moment of inertia of the pile. For a cylindrical cross-section,

$$I = \frac{\pi (R_2^4 - R_1^4)}{4} \quad (3.55)$$

so, eq. (3.54) becomes

$$P_z = \frac{M_0 r \cos\theta}{I} \quad (3.56)$$

Thus, the surface load vector becomes

$$\mathbf{T} = \{P_r \quad P_\theta \quad P_z\}^T = \{0 \quad 0 \quad P_z\} \quad (3.57)$$

and it can be expressed as a Fourier series, using the symmetric terms as follows

$$\mathbf{T} = \sum_n \mathbf{R}_n \bar{\mathbf{P}}_n$$

where $\mathbf{R}_n$ is given from eq. (3.14) and $\bar{\mathbf{P}}_n$ is equal to

$$\bar{\mathbf{P}}_n = \begin{cases} \begin{bmatrix} 0 & 0 & \frac{M_0 r}{I} \end{bmatrix}^T & \text{for } n = 1 \\ \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}^T & \text{for } n = 0, 2, 3, \dots \end{cases} \quad (3.58)$$

Substituting to eq. (3.30) yields

$$\mathbf{P} = \mathbf{P}_{n=1} = \int_r \mathbf{N}^T \begin{bmatrix} \pi & 0 & 0 \\ 0 & \pi & 0 \\ 0 & 0 & \pi \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ \frac{M_0 r}{I} \end{bmatrix} r \, dr \quad (3.59)$$

where $M_0 = 1 \text{ kNm}$.

In the case of the load being distributed over the element, one obtains

$$\mathbf{P} = \mathbf{P}_{n=0} = \pi \frac{M_0}{I} (R_2 - R_1) \left[\begin{array}{c|cc} \mathbf{0}_{3 \times 1} & 0 & 0 & -\frac{R_1^2}{60} + \frac{R_1 R_2}{30} + \frac{3R_2^2}{20} \\ \hline 0 & 0 & \frac{3R_1^2}{20} + \frac{R_1 R_2}{30} - \frac{R_2^2}{60} & \mathbf{0}_{3 \times 1} \end{array} \right] \quad (3.60)$$

Similarly, in the case of a nodal ring load, using eq. (3.59) one obtains

$$\mathbf{P} = \mathbf{P}_{n=0} = \frac{\pi}{3} (R_2^3 - R_1^3) \frac{M_0}{I} [\mathbf{0}_{3 \times 1} \mid 0 \quad 0 \quad 1 \mid \mathbf{0}_{12 \times 1}]^T \quad (3.61)$$

As a final remark, the following comments can be made on the similarities between the solutions of the axial and the lateral problem:

- The axial load used in Chapter 2 is the zero mode (constant term) of the Fourier series.
- The equations for the dynamic problem remain the same, with the stiffness, mass, and load matrices being bigger in size as they include the terms associated with tangential

displacement. Accordingly, equations 2.7, 2.12 – 2.14 can be used without modification for the lateral problem.

- Formulation of the infinite elements, the matrices **S**, **B** and **D** remain the same, as do the shape functions. The only difference is that the displacement interpolation matrix **N_{QQ}** is a 3 by 18 matrix (instead of a 2 by 18 for the axial problem). So the calculation of the infinite elements is straightforward.
- Since the boundary equations remain unchanged, the only difference in the derivation of the transmitting boundaries is that the constitutive matrices are 3x3 instead of 2x2. In the Kellezi formulation, the tangential and circumferential directions are treated identically. Thus the stiffness and damping matrices at the boundaries can be readily evaluated (see Appendix B).
- The order of integration is ξ^4 for the numerator with a singularity in the denominator, so the same number of points and weights as in the axisymmetric case is going to be used for evaluating the integrals.

3.3 COMPARISON WITH EXISTING SOLUTIONS AND INTRODUCTION OF NEW SOLUTIONS

3.3.1 Static response - Comparison with other studies

Like in Chapter 2, it is important to validate K-PAX against published results. A comprehensive parametric study was published by Marsafawi et al (1992) using a rigorous boundary element method. In their report a large number of piles was analyzed using different loading and soil conditions. Their results are considered very precise in

the case of a homogeneous soil medium and are considered approximate for the case of a linear or a parabolic soil profile. It can be seen from Tables 3.1 and 3.2 that the results from the present study follow closely those of Marsafawi et al. These results are presented for a variety of soil profiles, relative pile-soil stiffnesses and normalized pile lengths. In the case of a fixed head pile, it can be seen that the results are very close to each other, with the maximum difference being of the order of 3%. On the other hand, the results for the free head pile have bigger differences, of the order of 9%. It should be noted, that although the results from K-PAX arise from actually analyzing a free head pile, the results from Marsafawi et al. are calculated based on the relation that connects the swaying, rocking and cross swaying –rocking stiffness terms of a fixed head pile, with the swaying stiffness of a free head pile. It can easily be proved that the relation connecting all the terms mentioned above is:

$$K_{xx, free} = K_{xx, fixed} - \frac{K_{xr}^2}{K_{rr}} \quad (3.62)$$

In theory, the results from the equation above and the analysis of a free head pile should be identical. On the other hand, due to numerical errors, the cross swaying-rocking stiffness term obtained from a fixed head pile while being pushed with a horizontal head load differs from the one of a pinned head pile with a moment applied on top. This phenomenon is the reason for the observed difference in the free head swaying stiffness. In the present study, the cross term used was the one obtained from the fixed head pile with a horizontal load being applied on the top.

After performing a series of runs, the following relations were obtained through curve fitting to provide the swaying static stiffness of a pile due to different pile head restraints and for different soil profiles:

→ Homogeneous halfspace – Fixed Head

$$\frac{K_{xx}}{E_s d} = 1.24 \left(\frac{E_p}{E_s} \right)^{0.18} \quad \text{or} \quad 0.98 \left(\frac{E_p}{E_s} \right)^{0.21} \quad (3.63)$$

Comparing with the formula of Gazetas (1991),

$$\frac{K_{xx}}{E_s d} = 1.0 \left(\frac{E_p}{E_s} \right)^{0.21} \quad (3.64)$$

the two solutions are almost identical.

→ Homogeneous halfspace – Free Head

$$\frac{K_{xx, \text{free}}}{E_s d} = 0.72 \left(\frac{E_p}{E_s} \right)^{0.18} \quad (3.65)$$

Comparing with eq. (3.63) for fixed head, the pile head stiffness is 1.7 times smaller, for all pile-soil relative stiffnesses (E_p/E_s).

→ Linear soil profile – Fixed Head

$$\frac{K_{xx}}{E_{sd} d} = 0.65 \left(\frac{E_p}{E_{sd}} \right)^{0.35} \quad (3.66)$$

where E_{sd} is the soil modulus evaluated at a depth equal to one pile diameter ($z=d$).

Comparing, again, with Gazetas (1991), where $\frac{K_{xx}}{E_{sd} d} = 0.60 \left(\frac{E_p}{E_{sd}} \right)^{0.35}$, the two

solutions are very close to each other.

→ Linear soil profile – Free Head

$$\frac{K_{xx}}{E_{sd} d} = 0.28 \left(\frac{E_p}{E_{sd}} \right)^{0.35} \quad (3.67)$$

This time, the pile head stiffness is 2.3 times smaller than the corresponding for fixed head pile (eq. (3.66)).

In the case of the rocking and cross swaying-rocking component, the results from regression analysis are:

○ Homogeneous soil profile

$$K_{\pi} = E_s d^3 0.15 \left(\frac{E_p}{E_s} \right)^{0.75} \quad (3.68)$$

and

$$K_{\kappa} = E_s d^2 0.21 \left(\frac{E_p}{E_s} \right)^{0.5} \quad (3.69)$$

○ Linear soil profile

$$K_{\pi} = E_{sd} d^3 0.13 \left(\frac{E_p}{E_{sd}} \right)^{0.8} \quad (3.70)$$

and

$$K_{\kappa} = E_{sd} d^2 0.17 \left(\frac{E_p}{E_{sd}} \right)^{0.6} \quad (3.71)$$

TABLE 3.1: Comparison of results with El-Marsafawi et al., 1992. The values reported refer to a dimensionless static swaying stiffness ($K_{xx} / E_s / d$) for the case of piles embedded in a homogeneous halfspace for different pile-soil relative stiffness and head constraints.

FLOATING PILE							
L/d	E_p/E_s	Fixed Head			Free Head		
		this study	Marsafawi	deviation %	this study	Marsafawi	deviation %
10	100	2.81	2.83	0.6	1.87	1.72	8.9
	300	3.46	3.41	1.4	2.18	2.04	6.8
	1000	4.31	4.21	2.5	2.62	2.48	5.4
20	100	2.80	2.83	1.0	1.87	1.72	8.5
	300	3.44	3.42	0.6	2.17	2.04	6.3
	1000	4.32	4.24	1.9	2.61	2.48	5.1
40	100	2.79	2.83	1.5	1.86	1.72	8.2
	300	3.42	3.42	0.1	2.16	2.04	6.0
	1000	4.29	4.24	1.2	2.59	2.48	4.6

TABLE 3.2: Comparison of results with El-Marsafawi et al. (1992) for static pile-head stiffness; $L/d = 20$

Soil Profile	E_p/E_s	$K_{xx}/E_s(z=20d)/d$		$K_{yy}/E_s(z=20d)/d^3$		$K_{xz}/E_s(z=20d)/d^2$	
		this study	Marsafawi	this study	Marsafawi	this study	Marsafawi
HOMOGENEOUS	100	2.80	2.83	4.97	5.18	2.17	2.40
	300	3.44	3.42	11.22	11.50	3.79	3.97
	1000	4.32	4.24	27.27	27.40	6.85	6.95
LINEAR	100	0.463	0.464	3.117	3.200	0.903	0.926
	300	0.676	0.676	7.449	7.550	1.697	1.720
	1000	1.028	1.030	19.266	19.400	3.381	3.410
PARABOLIC	100	0.97	1.01	3.76	3.94	1.35	1.42
	300	1.32	1.36	8.79	9.03	2.43	2.51
	1000	1.84	1.88	22.17	22.50	4.60	4.69

Another comparison was made with published solutions. The formulas of Gazetas (1991) and Randolph (1981) were obtained by curve fitting finite-element solutions. The results from Mylonakis & Roubas (2001) were obtained using the Winkler theory, utilizing results from finite elements for the value of the normalized Winkler modulus (δ). This is depicted in Figure 3.4. It can be seen that all results are close to each other. It should be noted that the curve from Winkler theory is very close to the results of the finite-element solutions and illustrates the accuracy of the method when appropriate values of normalized Winkler moduli (δ) are used.

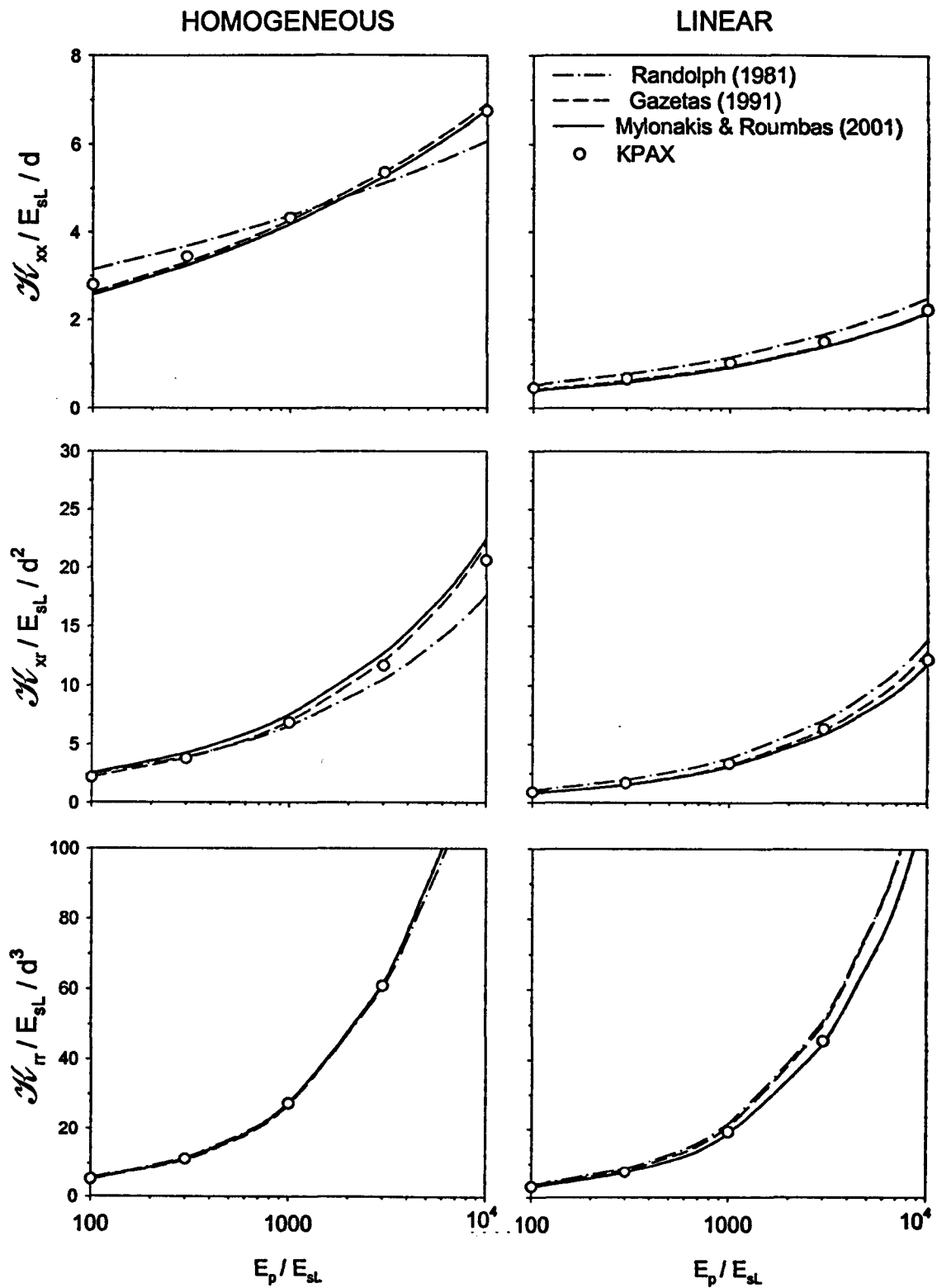


FIGURE 3.4: Normalized horizontal (swaying), rocking and cross term (swaying-rocking) dimensionless stiffness of a single pile in different soil profiles.

3.3.2 Effect of radial inhomogeneity

The softening (or hardening) of soil around a pile is expected to modify the pile's response. The present study deals with the effects of soil softening around the pile and addresses the questions of how big is the reduction in the pile head stiffness due to radial inhomogeneity and how is this reduction related to the degree of radial inhomogeneity (amount of softening), the pile-soil relative stiffness and the length of the pile.

Tables 3.3 and 3.4 show the swaying static stiffness of piles with different pile-soil relative stiffnesses (E_p/E_s) and different soil profiles. As was expected, the greater the degree of radial inhomogeneity (A) the greater is the drop in pile static stiffness. Depending on the soil properties, the drop in the static stiffness can reach a very appreciable 30%.

TABLE 3.3: Pile-head normalized static stiffness for various soil profiles; ($L/d=20$, fixed-head conditions)

Soil Profile	E_p/E_s	$K_{xx}/E_s(z=20d)/d$				$K_{\theta\theta}/E_s(z=20d)/d^3$				$K_{\alpha\alpha}/E_s(z=20d)/d^2$			
		A=0	0.1	0.5	0.8	A=0	0.1	0.5	0.8	A=0	0.1	0.5	0.8
HOMOGENEOUS	100	2.80	2.61	2.11	1.71	4.97	4.81	4.45	4.15	2.17	2.10	1.89	1.71
	300	3.44	3.22	2.64	2.18	11.2	10.9	10.2	9.55	3.79	3.67	3.31	3.00
	1000	4.32	4.07	3.38	2.84	27.3	26.6	25.0	23.6	6.85	6.63	6.01	5.47
LINEAR	100	0.46	0.44	0.36	0.31	3.12	3.04	2.89	2.74	0.90	0.87	0.78	0.70
	300	0.68	0.64	0.54	0.46	7.45	7.29	6.93	6.59	1.70	1.64	1.48	1.34
	1000	1.03	0.98	0.83	0.71	19.3	18.9	18.0	17.1	3.38	3.27	2.98	2.70

**TABLE 3.4: Pile-head normalized swaying stiffness for various soil profiles.
($L/d=20$, free-head conditions)**

Soil Profile	$E_p/E_s(z=20d)$	$K_{xx, free}/E_s(z=20d)/d$			
		$A = 0$	0.1	0.5	0.8
HOMOGENEOUS	100	1.87	1.71	1.33	1.02
	300	2.17	2.00	1.59	1.25
	1000	2.61	2.42	1.96	1.58
LINEAR	100	0.23	0.20	0.16	0.13
	300	0.35	0.28	0.23	0.19
	1000	0.58	0.44	0.35	0.29

Performing a regression analysis, a relation was found between the static stiffness for radially homogeneous medium and the static stiffness of a radially inhomogeneous medium (see Tables 3.5 and 3.6). Accounting for the effect of the relative pile-soil stiffness yields the following formula for different pile head constraints and stiffness terms:

□ Fixed Head – Homogeneous halfspace

$$K_{xx}(A) = K_{xx}(A=0) [1-0.52 A] \left(\frac{E_p}{E_s} \right)^{0.017 A} \quad (3.72)$$

or, for simplicity, neglecting the effect of the relative pile-soil stiffness yields

$$K_{xx}(A) = K_{xx}(A=0) [1-0.46 A] \quad (3.73)$$

□ Free head – Homogeneous halfspace

$$K_{xx}(A) = K_{xx}(A=0) [1-0.6 A] \left(\frac{E_p}{E_s} \right)^{0.018 A} \quad (3.74)$$

or neglecting the effect of the relative pile-soil stiffness yields

$$K_{xx}(A) = K_{xx}(A=0) [1-0.53 A] \quad (3.75)$$

Similarly,

- Fixed Head – Linear soil profile

$$K_{xx}(A) = K_{xx}(A=0) [1-0.47 A] \left(\frac{E_p}{E_{sd}} \right)^{0.011 A} \quad (3.76)$$

or

$$K_{xx}(A) = K_{xx}(A=0) [1-0.41 A] \quad (3.77)$$

- Free Head – Linear soil profile

$$K_{xx}(A) = K_{xx}(A=0) [1-0.03 A] \left(\frac{E_p}{E_{sd}} \right)^{-0.093 A} \quad (3.78)$$

or

$$K_{xx}(A) = K_{xx}(A=0) [1-0.63 A] \quad (3.79)$$

For the rocking and cross-term swaying rocking components, the results are

- Homogeneous soil profile

$$K_{rr}(A) = K_{rr}(A=0) [1 - 0.16A] \quad (3.80)$$

and

$$K_{rx}(A) = K_{rx}(A=0) [1 - 0.25A] \quad (3.81)$$

- Linear soil profile

$$K_{rr}(A) = K_{rr}(A=0) [1 - 0.18 A^{3.5}] \quad (3.82)$$

and

$$K_{rx}(A) = K_{rx}(A=0) [1 - 0.25A] \quad (3.83)$$

TABLE 3.5(a): Dynamic stiffness coefficients for a fixed head pile under harmonic horizontal head load (vertically homogeneous soil)

Soil Profile	Homogeneous (A=0) 	Radially Inhomogeneous (0<A<1) 
Dimensionless cutoff frequency	$a_{0,\infty \text{ cut-off}} \approx 1.6 (H/d)^{-1.0}$	
Swaying stiffness coefficient	$K_{xx}(A) = 1.24 E_{s,\infty} d \left(\frac{E_p}{E_{s,\infty}} \right)^{0.18+0.017A} (1-0.52A)$	
Swaying damping ratio	$\beta_{xx} = \begin{cases} 0.8 \beta_m + 0.19 a_{0,\infty} (1+0.44A) \left(\frac{E_p}{E_{s,\infty}} \right)^{0.18-0.1A} & \text{for } a_{0,\infty \text{ cut-off}} < a_{0,\infty} < 1 \\ 0.8 \beta_m, & \text{for } a_{0,\infty} \leq a_{0,\infty \text{ cut-off}} \end{cases}$	
Rocking stiffness coefficient	$K_{rr}(A) = E_s d^3 0.15 \left(\frac{E_p}{E_s} \right)^{0.75} (1-0.16A)$	
Rocking damping ratio	$\beta_{rr} = \begin{cases} 0.4 \beta_m + 0.1 \left(a_{0,\infty} + 0.013 \left(\frac{E_p}{E_{s,\infty}} \right)^{0.4} (1-0.23A) \right) & \text{for } a_{0,\infty \text{ cut-off}} < a_{0,\infty} < 1 \\ 0.4 \beta_m & \text{for } a_{0,\infty} \leq a_{0,\infty \text{ cut-off}} \end{cases}$	
Cross Swaying-Rocking stiffness coefficient	$K_{xr}(A) = E_s d^2 0.21 \left(\frac{E_p}{E_s} \right)^{0.5} (1-0.25A)$	
Cross Swaying-Rocking damping ratio	$\beta_{xr} = \begin{cases} 0.6 \beta_m + 0.3 \left(a_{0,\infty} + 0.03 \left(\frac{E_p}{E_{s,\infty}} \right)^{0.27} (1-0.64A) \right) & \text{for } a_{0,\infty \text{ cut-off}} < a_{0,\infty} < 1 \\ 0.6 \beta_m & \text{for } a_{0,\infty} \leq a_{0,\infty \text{ cut-off}} \end{cases}$	

TABLE 3.5(b): Dynamic stiffness coefficients for a fixed head pile under harmonic horizontal head load (vertically inhomogeneous soil)

Soil Profile	Linear ($A=0$) 	Linear and Radially Inhomogeneous ($0 < A < 1$) 
Dimensionless cutoff frequency	$a_{0d,\infty \text{ cut-off}} \approx 1.2 (H/d)^{-0.5}$	
Swaying stiffness coefficient	$K_{xx}(A) = 0.65 E_{sd,\infty} d \left(\frac{E_p}{E_{sd,\infty}} \right)^{0.35+0.011A} (1-0.47A)$	
Swaying damping ratio	$\beta_{xx} = \begin{cases} 0.7 \beta_m + 0.3 a_{0d,\infty} (1+0.77A) \left(\frac{E_p}{E_{sd,\infty}} \right)^{-0.09A} & \text{for } a_{0d,\infty \text{ cut-off}} < a_{0d,\infty} < 1 \\ 0.7 \beta_m, & \text{for } a_{0d,\infty} \leq a_{0d,\infty \text{ cut-off}} \end{cases}$	
Rocking stiffness coefficient	$K_{rr}(A) = E_{sd} d^3 0.13 \left(\frac{E_p}{E_{sd}} \right)^{0.8} (1-0.18A^{3.5})$	
Rocking damping ratio	$\beta_{rr} = \begin{cases} 0.8 \beta_m + 0.055 a_{0d,\infty} \left(\frac{E_p}{E_{sd,\infty}} \right)^{-0.02} (1-0.2A) & \text{for } a_{0,\infty \text{ cut-off}} < a_{0d,\infty} < 1 \\ 0.35 \beta_m & \text{for } a_{0d,\infty} \leq a_{0,\infty \text{ cut-off}} \end{cases}$	
Cross Swaying-Rocking stiffness coefficient	$K_{xr}(A) = E_{sd} d^2 0.17 \left(\frac{E_p}{E_{sd}} \right)^{0.6} (1-0.25A)$	
Cross Swaying-Rocking damping ratio	$\beta_{xr} = \begin{cases} 0.55 \beta_m + 0.12 \left(a_{0d,\infty} + 0.52 \left(\frac{E_p}{E_{sd,\infty}} \right)^{-0.04} (1-0.72A) \right) & \text{for } a_{0d,\infty \text{ cut-off}} < a_{0d,\infty} < 1 \\ 0.55 \beta_m & \text{for } a_{0d,\infty} \leq a_{0d,\infty \text{ cut-off}} \end{cases}$	

TABLE 3.6(a): Dynamic stiffness coefficients for a free head pile under harmonic horizontal head load (vertically homogeneous soil)

Soil Profile	Homogeneous (A=0) 	Radially Inhomogeneous (0<A<1) 
Dimensionless cut-off frequency	$a_{0,\infty \text{ cut-off}} \approx 1.6 (H/d)^{-1.0}$	
Swaying stiffness coefficient	$K_{xx}(A) = 0.72 E_{s,\infty} d \left(\frac{E_p}{E_{s,\infty}} \right)^{0.18+0.018A} (1-0.6A)$	
Swaying damping ratio	$\beta_{xx} = \begin{cases} 0.8 \beta_m + 0.17 \left(a_{0,\infty} \left(\frac{E_p}{E_{s,\infty}} \right)^{0.15} - 0.7 \left(\frac{E_p}{E_{s,\infty}} \right)^{-0.7} (1-0.6A) \right) & \text{for } a_{0,\infty \text{ cut-off}} < a_{0,\infty} < 1 \\ 0.8 \beta_m, & \text{for } a_{0,\infty} \leq a_{0,\infty \text{ cut-off}} \end{cases}$	

TABLE 3.6(b): Dynamic stiffness coefficients for a free head pile under harmonic horizontal head load (vertically inhomogeneous soil)

Soil Profile	Linear (A=0) 	Linear and Radially Inhomogeneous (0<A<1) 
Dimensionless cut-off frequency	$a_{0d,\infty \text{ cut-off}} \approx 1.2 (H/d)^{-0.5}$	
Swaying stiffness coefficient	$K_{xx}(A) = 0.28 E_{sd,\infty} d \left(\frac{E_p}{E_{sd,\infty}} \right)^{0.35-0.093A} (1-0.03A)$	
Swaying damping ratio	$\beta_{xx} = \begin{cases} 0.7 \beta_m + 0.7 \left(\frac{E_p}{E_{sd,\infty}} \right)^{-0.07} (1-0.07A) (a_{0d,\infty} - a_{0d,\infty \text{ cut-off}})^{2.7} \left(\frac{E_p}{E_{sd,\infty}} \right)^{-0.15} (1-0.17A) & \text{for } a_{0d,\infty \text{ cut-off}} < a_{0d,\infty} < 1 \\ 0.7 \beta_m, & \text{for } a_{0d,\infty} \leq a_{0d,\infty \text{ cut-off}} \end{cases}$	

3.3.3 Normalized Winkler modulus δ

3.3.3.1 Fixed Head pile in a Homogeneous Halfspace

Using the theory of a Beam-on-Winkler-Foundation (BWF), the following differential equation describes the static equilibrium of an infinitesimal piece of the pile:

$$E_p I_p \frac{d^4 u(z)}{dz^4} + k_x u(z) = 0 \quad (3.84)$$

the solution of which is

$$u(z) = e^{\lambda z} (A \sin(\lambda z) + B \cos(\lambda z)) + e^{-\lambda z} (C \sin(\lambda z) + D \cos(\lambda z)) \quad (3.85)$$

Imposing the boundary conditions for an infinitely long ($L > L_a$) floating pile, one finds that the pile-head stiffness matrix is

$$[K] = \begin{bmatrix} 4 E I \lambda^3 & 2 E I \lambda^2 \\ 2 E I \lambda^2 & 2 E I \lambda \end{bmatrix} \quad (3.86)$$

where λ is given as

$$\lambda = \left[\frac{\delta E_s}{4 E_p I_p} \right]^{1/4} \quad (3.87)$$

and δ is the normalized Winkler modulus.

For the value of δ , Roesset (1980) suggested an approximate constant value of 1.2. Later, Dobry et al (1982) suggested the following expression, which encompasses the effect of the soil-pile relative stiffness:

$$\delta = 1.67 \left(\frac{E_p}{E_s} \right)^{-0.053} \quad (3.88)$$

Given the advantages of the BWF method, it is important to find a representative expression of δ for different pile head restraints and different soil profiles. In the present

study, results from K-PAX are used in conjunction with Winkler theory to derive accurate moduli δ .

In the case of a fixed head pile embedded in a homogeneous halfspace, the swaying stiffness of an infinitely long pile (or a pile with a length greater than L_a) is given as

$$K_{xx} = 4 E_p I_p \lambda^3 \quad (3.89)$$

In combination with eq. (3.87), one may solve for δ and obtain

$$\delta = \left(\frac{16}{\pi} \right)^{1/3} \left(\frac{K_{xx}}{E_s d} \right)^{4/3} \left(\frac{E_p}{E_s} \right)^{-1/3} \approx 1.72 \left(\frac{K_{xx}}{E_s d} \right)^{4/3} \left(\frac{E_p}{E_s} \right)^{-1/3} \quad (3.90)$$

Using K-PAX to find the swaying stiffness K_{xx} , δ is plotted against E_p/E_s in Figure 3.5. After performing a regression analysis it was found that δ can be expressed as a function of E_p/E_s as

$$\delta \approx 2.0 \left(\frac{E_p}{E_s} \right)^{-0.075} \quad (3.91)$$

A similar expression can be obtained by substituting the static pile-head stiffness from eq. (3.63) (which was found using regression analysis) into eq. (3.90)

$$\delta \approx 2.3 \left(\frac{E_p}{E_s} \right)^{-0.09} \quad (3.92)$$

but it was found that eq. (3.91) is more accurate.

The results of Gazetas & Dobry (1984) are also plotted in Figure 3.5 and they seem to be in general agreement with the results of the present study.

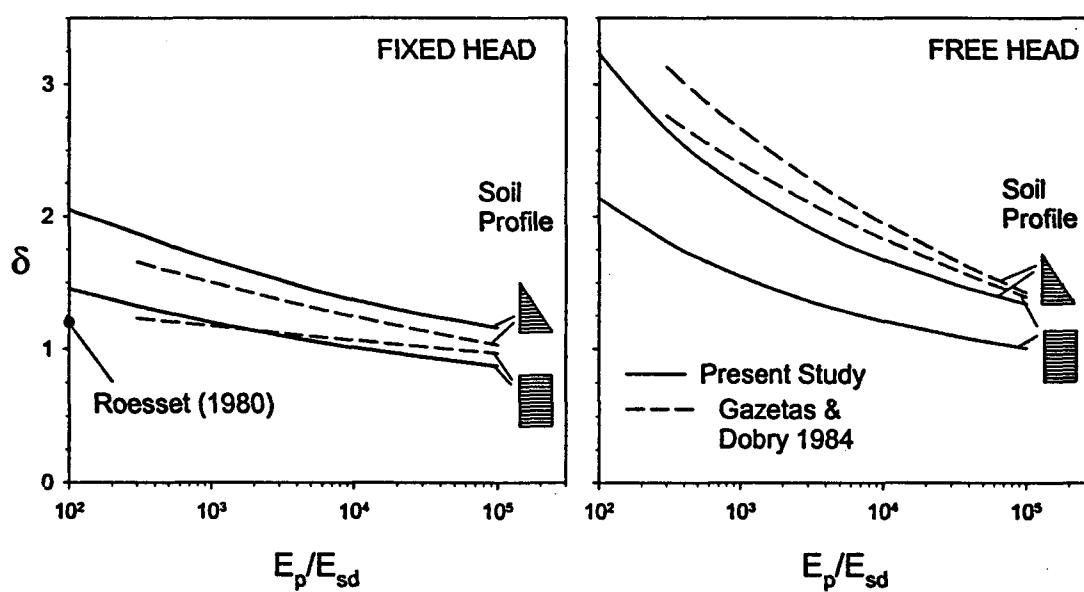


FIGURE 3.5: Normalized Winkler modulus versus relative pile-soil stiffness, for different pile head constraints and soil profiles.

3.3.3.2 Free Head pile in a Homogeneous Halfspace

In a similar way, δ can be found for the case of a free head pile embedded in a homogeneous halfspace. It can be easily proved that the swaying stiffness is

$$K_{xx, \text{free}} = 2 E_p I_p \lambda^3 \quad (3.93)$$

Substituting in eq. (3.112) one obtains

$$\delta = \left(\frac{256}{\pi} \right)^{1/3} \left(\frac{K_{xx, \text{free}}}{E_s d} \right)^{4/3} \left(\frac{E_p}{E_s} \right)^{-1/3} \approx 4.33 \left(\frac{K_{xx, \text{free}}}{E_s d} \right)^{4/3} \left(\frac{E_p}{E_s} \right)^{-1/3} \quad (3.94)$$

Similarly, using K-PAX to evaluate $K_{xx, \text{free}}$, the resulting δ is plotted against E_p/E_s in Figure 3.5.

Performing a regression analysis, it was found that

$$\delta \approx 3.5 \left(\frac{E_p}{E_s} \right)^{-0.11} \quad (3.95)$$

Comparing with Gazetas & Dobry (1984), their results seem to slightly under-predict the value of δ .

3.3.3.3 Fixed Head pile in a linear soil profile

In the case of a linear soil profile, Mylonakis and Roumbas (2001) have expressed the pile head stiffnesses as

$$K_{xx} = E_p I_p \mu^3 + \frac{3 \delta E_{sd} [(1-\alpha) + 2 \alpha \mu d]}{8 \mu^2 d} \quad (3.96)$$

$$K_{xr} = E_p I_p \mu^2 + \frac{\delta E_{sd} [3(1-\alpha) + 4 \alpha \mu d]}{16 \mu^3 d} \quad (3.97)$$

and

$$K_{\pi} = \frac{3}{2} E_p I_p \mu + \frac{\delta E_{sd} [(1-\alpha) + 2\alpha \mu d]}{8 \mu^4 d} \quad (3.98)$$

where in the case of zero stiffness at the top of the soil, parameter α is equal to zero. In this case, μ is given as

$$\begin{aligned} \mu &= \left[\frac{\delta E_{sd}}{4 E_p I_p} \right]^{1/4} \frac{4}{5} \left(\frac{L_a}{d} \right)^{-1} \frac{\left[(1-\alpha) \frac{L_a}{d} + \alpha \right]^{5/4} - \alpha^{5/4}}{1-\alpha} \Rightarrow \\ \Rightarrow \mu &= \left(\frac{64}{\pi} \right)^{1/4} \frac{4^{3/4}}{5} \frac{1}{d} \left[\delta \frac{E_{sd}}{E_p} \frac{L_a}{d} \right]^{1/4} \Rightarrow \\ \Rightarrow \mu &\approx 1.2 \frac{1}{d} \left[\delta \frac{E_{sd}}{E_p} \frac{L_a}{d} \right]^{1/4} \end{aligned} \quad (3.99)$$

Plugging into eq. (3.96) yields

$$\frac{K_{xx}}{E_{sd} d} = 0.085 \left(\frac{E_p}{E_{sd}} \right)^{1/4} \left(\frac{L_a}{d} \right)^{3/4} \delta^{3/4} + 0.26 \left(\frac{E_p}{E_{sd}} \right)^{2/4} \left(\frac{L_a}{d} \right)^{-2/4} \delta^{2/4} \quad (3.100)$$

Using K-PAX to evaluate the swaying stiffness K_{xx} and the active pile length L_a , in conjunction with eq. (3.100) one can determine the value of δ numerically. The resulting δ is plotted against E_p/E_{sd} in Figure 3.5. Performing a regression analysis, it was found that δ can be expressed as a function of E_p/E_{sd} as

$$\delta \approx 3.0 \left(\frac{E_p}{E_{sd}} \right)^{-0.08} \quad (3.101)$$

In this case, Gazetas & Dobry seem to over-predict the value of δ .

3.3.3.4 Free Head pile in a linear soil profile

Similarly, in the case of a free-head pile in a linear soil profile, the swaying stiffness can be found from the known formula

$$K_{xx, \text{free}} = K_{xx, \text{fixed}} - \frac{K_{xr}^2}{K_{rr}} \quad (3.102)$$

Plugging eq. (3.99) into eq. (3.97) and (3.98) yields

$$\frac{K_{rr}}{E_{sd} d^3} = 0.088 \left(\frac{E_p}{E_{sd}} \right)^{3/4} \left(\frac{L_a}{d} \right)^{1/4} \delta^{1/4} + 0.05 \left(\frac{E_p}{E_{sd}} \right) \left(\frac{L_a}{d} \right)^{-1} \quad (3.103)$$

and

$$\frac{K_{xr}}{E_{sd} d^2} = 0.071 \left(\frac{E_p}{E_{sd}} \right)^{2/4} \left(\frac{L_a}{d} \right)^{2/4} \delta^{2/4} + 0.11 \left(\frac{E_p}{E_{sd}} \right)^{3/4} \left(\frac{L_a}{d} \right)^{-3/4} \delta^{1/4} \quad (3.104)$$

Plugging eq. (3.100), (3.103) and (3.104) into (3.102) and using K-PAX to evaluate $K_{xx, \text{free}}$ and the active pile length L_a , δ can be numerically evaluated. The resulting δ is plotted against E_p/E_{sd} in Figure 3.5. Performing a regression analysis, it was found that δ can be expressed as a function of E_p/E_{sd} as

$$\delta \approx 5.8 \left(\frac{E_p}{E_{sd}} \right)^{-0.13} \quad (3.105)$$

In this case, Gazetas & Dobry seem to slightly over-predict the value of δ .

3.3.4 Active pile length

It is known that the response of a pile being loaded laterally becomes independent of the pile length after a certain depth (Randolph 1981). The depth (or pile length) at which this phenomenon occurs is called active (or critical) pile length (L_a). According to Kuhlemeyer (1979) who studied free head piles, this corresponds to the length at the

point where a fixed tip pile and a pinned tip pile have the same pile head swaying stiffness. This length is a function of the pile-soil relative stiffness and for static conditions is equal to (Kuhlemeyer 1979):

$$\frac{L_a}{d} \approx 1.7 \left(\frac{E_p}{E_s} \right)^{0.25} \quad (3.106)$$

Similarly, Randolph (1981) and Gazetas (1991) defined the active length as the depth below which pile deflexions are less than one-thousandth of the top deflexion and came up with the following formulas respectively (assuming a Poisson's ratio of 0.4):

$$\frac{L_a}{d} \approx 1.25 \left(\frac{E_p}{E_s} \right)^{2/7} \quad (3.107)$$

and

$$\frac{L_a}{d} \approx 2.0 \left(\frac{E_p}{E_s} \right)^{0.25} \quad (3.108)$$

The formulas above correspond to a free-head pile embedded in a homogeneous halfspace. In the case of a linear soil profile, the above formulas change to

$$\frac{L_a}{d} \approx 1.38 \left(\frac{E_p}{E_s} \right)^{2/9} \quad (3.109)$$

and

$$\frac{L_a}{d} \approx 2.0 \left(\frac{E_p}{E_s} \right)^{0.20} \quad (3.110)$$

All the formulas above were obtained through curve fitting the results of finite element analyses. With the help of the Winkler theory for a beam resting on an elastic foundation, approximate active lengths can be obtained for the case of free and fixed

head piles embedded in a homogeneous halfspace. Mylonakis (1995) demonstrated this solution for the case of a fixed head pile.

In the case of a floating pile resting in a homogeneous halfspace, the stiffness matrix of the pile head can be written as (Mylonakis, 1995)

$$[K] = \begin{bmatrix} 4 E I \lambda^3 \frac{\sin(2 \lambda L) + \sinh(2 \lambda L)}{2 + \cos(2 \lambda L) + \cosh(2 \lambda L)} & 2 E I \lambda^2 \frac{-\cos(2 \lambda L) + \cosh(2 \lambda L)}{2 + \cos(2 \lambda L) + \cosh(2 \lambda L)} \\ 2 E I \lambda^2 \frac{-\cos(2 \lambda L) + \cosh(2 \lambda L)}{2 + \cos(2 \lambda L) + \cosh(2 \lambda L)} & 2 E I \lambda \frac{-\sin(2 \lambda L) + \sinh(2 \lambda L)}{2 + \cos(2 \lambda L) + \cosh(2 \lambda L)} \end{bmatrix} \quad (3.111)$$

where λ is given as

$$\lambda = \left[\frac{\delta E_s}{4 E_p I_p} \right]^{1/4} \quad (3.112)$$

and δ is the normalized Winkler modulus.

Plotting the variation of the swaying stiffness K_{xx} for fixed head conditions (K_{11}), with respect to λL one may find that for $\lambda L \approx 2.5$ the swaying stiffness of the pile “matches” the one of an infinite pile ($L = L_a$). Plugging in eq. (3.112) one obtains

$$L_a/d = L \lambda \left(\frac{\pi}{16 \delta} \right)^{1/4} \left(\frac{E_p}{E_s} \right)^{1/4} \quad (3.113)$$

Using the expression for the normalized Winkler modulus δ that was derived in the present study (eq. (3.91)) leads to the following expression for the active length of a fixed head pile embedded in a homogeneous halfspace:

$$L_a/d \approx 1.4 \left(\frac{E_p}{E_s} \right)^{0.27} \quad (3.114)$$

One may use a similar procedure to find the active length of a free-head pile. The flexibility matrix is found by inverting the stiffness matrix (eq. (3.111))

$$[F] = \begin{bmatrix} \frac{-\sin(2\lambda L) + \sinh(2\lambda L)}{2EI\lambda^3(-2 + \cos(2\lambda L) + \cosh(2\lambda L))} & \frac{\cos(2\lambda L) - \cosh(2\lambda L)}{2EI\lambda^2(-2 + \cos(2\lambda L) + \cosh(2\lambda L))} \\ \frac{\cos(2\lambda L) - \cosh(2\lambda L)}{2EI\lambda^2(-2 + \cos(2\lambda L) + \cosh(2\lambda L))} & \frac{\sin(2\lambda L) + \sinh(2\lambda L)}{EI\lambda(-2 + \cos(2\lambda L) + \cosh(2\lambda L))} \end{bmatrix} \quad (3.115)$$

The swaying pile-head stiffness for a free head pile can be easily found by inverting the 1st term of the flexibility matrix:

$$K_{xx, free} = 2EI\lambda^3 \frac{(-2 + \cos(2\lambda L) + \cosh(2\lambda L))}{-\sin(2\lambda L) + \sinh(2\lambda L)} \quad (3.116)$$

Plotting the variation of $K_{xx, free}$ with respect to λL one may find that for $\lambda L \approx 3.2$ the swaying stiffness of the pile matches the one of an infinite pile ($L = L_a$). Plugging in eq. (3.113) and using the results for delta found in the present study (eq. (3.95)) one obtains

$$L_a/d \approx 1.6 \left(\frac{E_p}{E_s} \right)^{0.28} \quad (3.117)$$

In addition, using the results from K-PAX, two new formulas are derived using the same definition for the active length. In the case of a fixed head pile in a homogeneous halfspace, the active length is given by

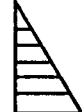
$$L_a/d \approx 2.4 \left(\frac{E_p}{E_s} \right)^{0.25} \quad (3.118)$$

and in the case of a fixed head pile in a linear soil profile, the active length is given by

$$L_a/d \approx 2.5 \left(\frac{E_p}{E_s} \right)^{0.20} \quad (3.119)$$

The powers are selected to match those of Gazetas in the corresponding free-head cases. Comparing these formulas it can be seen that the active length of fixed head piles is approximately 25% more than the one of the free-head piles (see Table 3.7).

TABLE 3.7: Active length and normalized Winkler moduli for various soil profiles and pile-head constraints

Soil Profile	Homogeneous (A=0) 		Linear (A=0) 	
	Pile-head conditions	Fixed Head	Free Head	Fixed Head
Active length L_a	$2.4 d \left(\frac{E_p}{E_{s, \infty}} \right)^{0.25}$	$2.0 d \left(\frac{E_p}{E_{s, \infty}} \right)^{0.25}$	$2.5 d \left(\frac{E_p}{E_{sd, \infty}} \right)^{0.20}$	$2.0 d \left(\frac{E_p}{E_{sd, \infty}} \right)^{0.20}$
Normalized Winkler Modulus δ (constant)	$2.0 \left(\frac{E_p}{E_s} \right)^{-0.075}$	$3.5 \left(\frac{E_p}{E_s} \right)^{-0.11}$	$3.0 \left(\frac{E_p}{E_{sd}} \right)^{-0.08}$	$5.8 \left(\frac{E_p}{E_{sd}} \right)^{-0.13}$
Normalized Winkler Modulus $\delta(z)$	$2.0 \frac{L_a - z}{L_a}$	$2.0 \frac{L_a - z}{L_a}$	$3.75 \frac{L_a - z}{L_a}$	$4.0 \frac{L_a - z}{L_a}$

3.3.5 Static p-y curves

One of the objectives of this study was to examine static and dynamic p-y curves valid for small deformations. These curves form the initial slope of the non-linear p-y curves that are valid for large deformations. Using K-PAX to find the soil reactions and corresponding displacements at the soil-pile interface the corresponding springs were calculated. The soil reactions were estimated by evaluating the normal stresses at the soil pile interface and integrating over the pile surface. In Figure 3.6 the normalized pile displacements, soil reactions and normalized Winkler moduli are plotted versus the depth from the soil surface. The plots are for piles with different pile head constraints and for different soil profiles. These soil profiles are equivalent, meaning that the piles resting in them have the same swaying stiffness at the pile head. The following trends are noted:

- Comparing the homogeneous (H) and the linear profile (L), the displacements die out much faster in the case of the linear profile. In the case of a radially homogeneous (R) and a doubly inhomogeneous soil profile (LR), the displacements attenuate faster than the corresponding homogeneous (H) and linear (L) profiles. These trends can be explained because the more inhomogeneous the soil layer (L, LR), the weaker it is at the surface. On the other hand, to match the static stiffness at the pile head of the homogeneous soil layer (H), the soil modulus must increase very fast with depth, thus

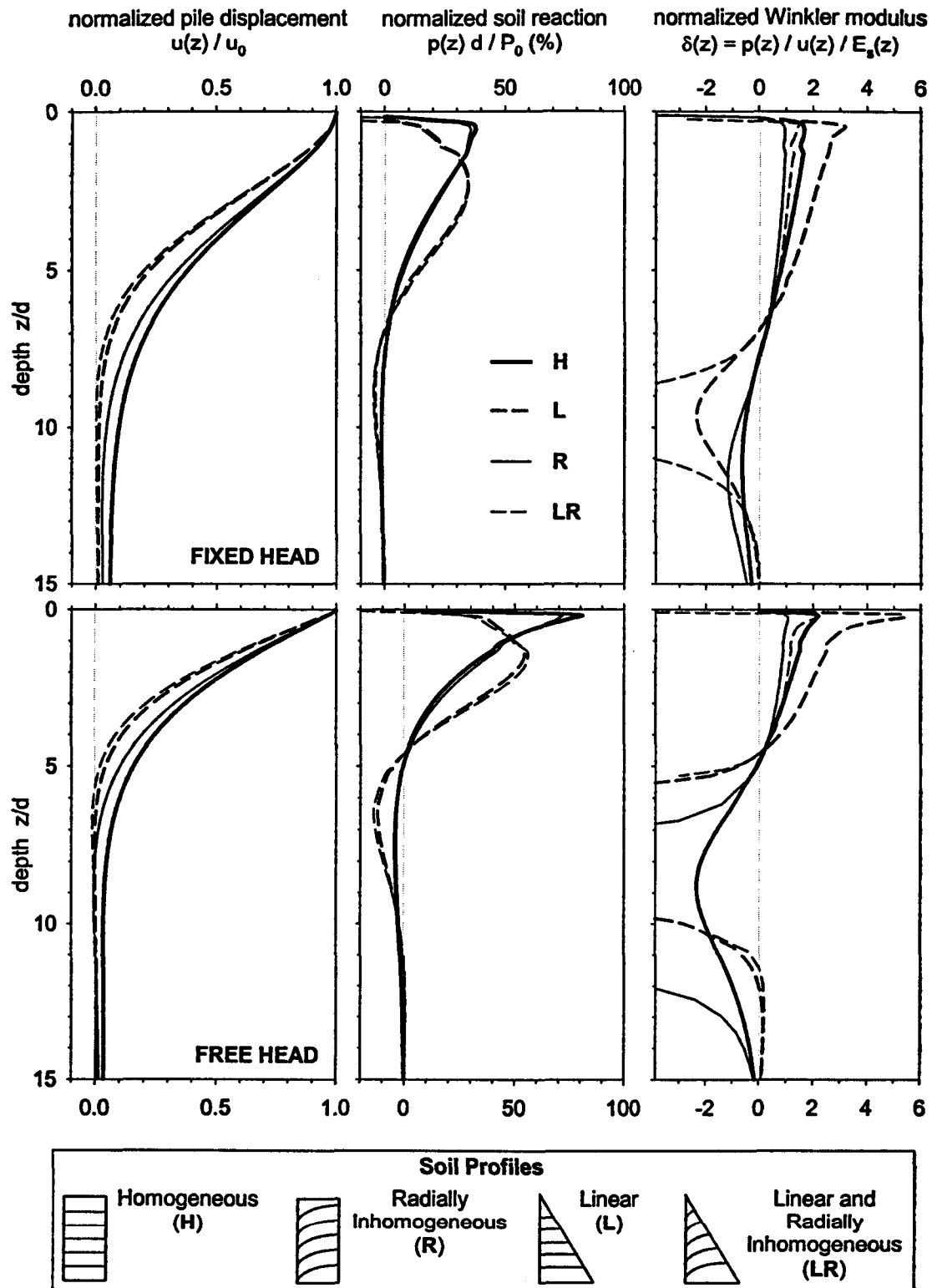


FIGURE 3.6: Deflected shape, soil reaction and normalized Winkler modulus for piles with different head constraints and of equal swaying stiffness in four different soil profiles.

the displacements die out faster with depth for the more inhomogeneous media. The same trend follows for both, fixed and free head piles.

- The soil reaction for the homogeneous halfspace (H) attains its peak earlier than the linear profile (L), which is reasonable since at the top the linear profile has very small modulus of elasticity, thus making it unable to produce a high reaction.
- On the other hand, it is observed that the homogeneous profile (H) has the same soil reactions with the radially inhomogeneous soil profile (R) and the linear (L) with the linear-radial (LR). This result is counter-intuitive since one may have expected that the displacements would be matched and the reactions would be different.
- Plotting the variation of the normalized Winkler modulus with depth it can be seen, for the case of a fixed head pile, that at a specific depth δ becomes infinite for the case of the (LR) soil profile. This trend is even more obvious for the case of the free head pile. This is due to the zeroing of the displacement at the same depth. For free head piles, this happens for all profiles with the exception of the homogeneous halfspace.
- It is also apparent that δ takes negative values after a certain depth. It will be shown that this depth is below the active pile length and thus plays no important role in the pile response.

Another plot was made (Figure 3.7) for the variation of the normalized Winkler modulus (δ) with depth. This plot was made for different pile-soil relative stiffnesses (E_p/E_s), two different soil profiles, both radially homogeneous, with the addition that the piles in the homogeneous and the linear profile have matched static stiffnesses (are “equivalent”). The following very interesting trends are observed:

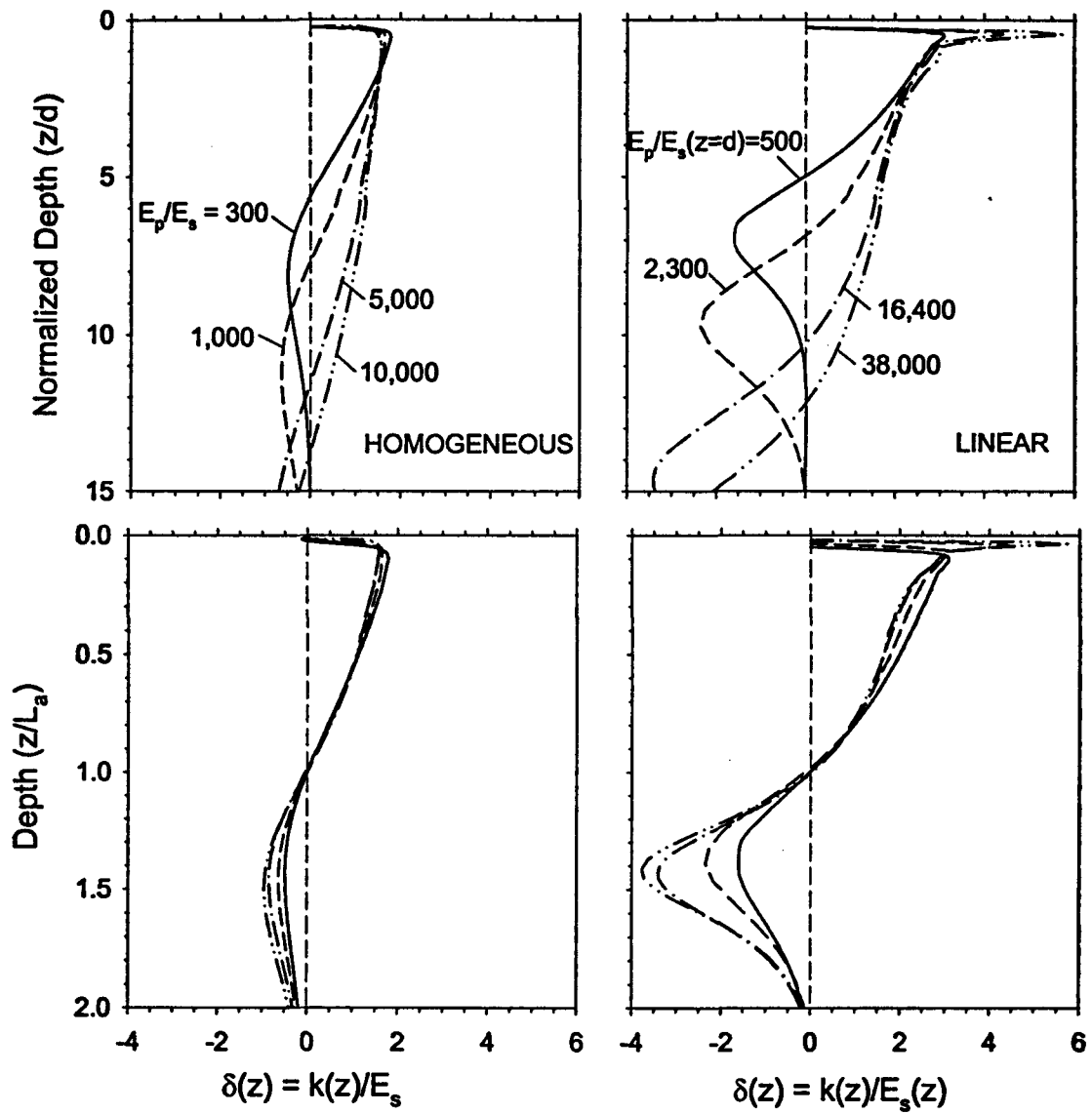


FIGURE 3.7: Variation of coefficient δ with respect to depth for various pile-soil stiffness contrasts for fixed head piles.

- In the case of the homogeneous soil profile, for all cases, δ achieves the value of approximately 2 (two) very near the soil surface and drops linearly until it becomes zero at a certain length. This length is very close (within two pile diameters) to the pile active length.
- In the case of a linear soil profile there seems to be a bigger variation in the peak of δ near the soil surface, but the trend of almost linear drop with depth remains.
- Normalizing the depth with the pile active length it becomes obvious that the variation of delta with depth can be approximated with a linear equation of the form:

$$\delta(z) = \delta_{\max} \cdot \frac{L_a - z}{L_a} \quad (3.120)$$

- The same trends are found in Figure 3.8 where again δ is plotted against depth for a free head pile.
- In the case of a homogeneous half space, $\delta_{\max}$ can be taken equal to approximately 2.0 for both free and fixed head. In the case of a linear soil profile the values of 3.75 and 4 can be taken for fixed and free head piles respectively.

A similar plot was made in Figures 3.9 and 3.10 where this time the profiles used were a radially inhomogeneous (R) and a linear with radial inhomogeneity (LR). It can be seen that they follow the same trends as the homogeneous and linear soil profiles, meaning that δ decreases linearly with depth. It can be seen that the value of δ near the surface ($\delta_{\max}$) is now different for both fixed and free head piles compared with the values of the radially homogeneous profiles (H and L). This becomes more obvious in Figure 3.11 where δ is plotted against depth for soil profiles with different degrees of radial inhomogeneity (A). It should be noted that in each plot, the piles have matched

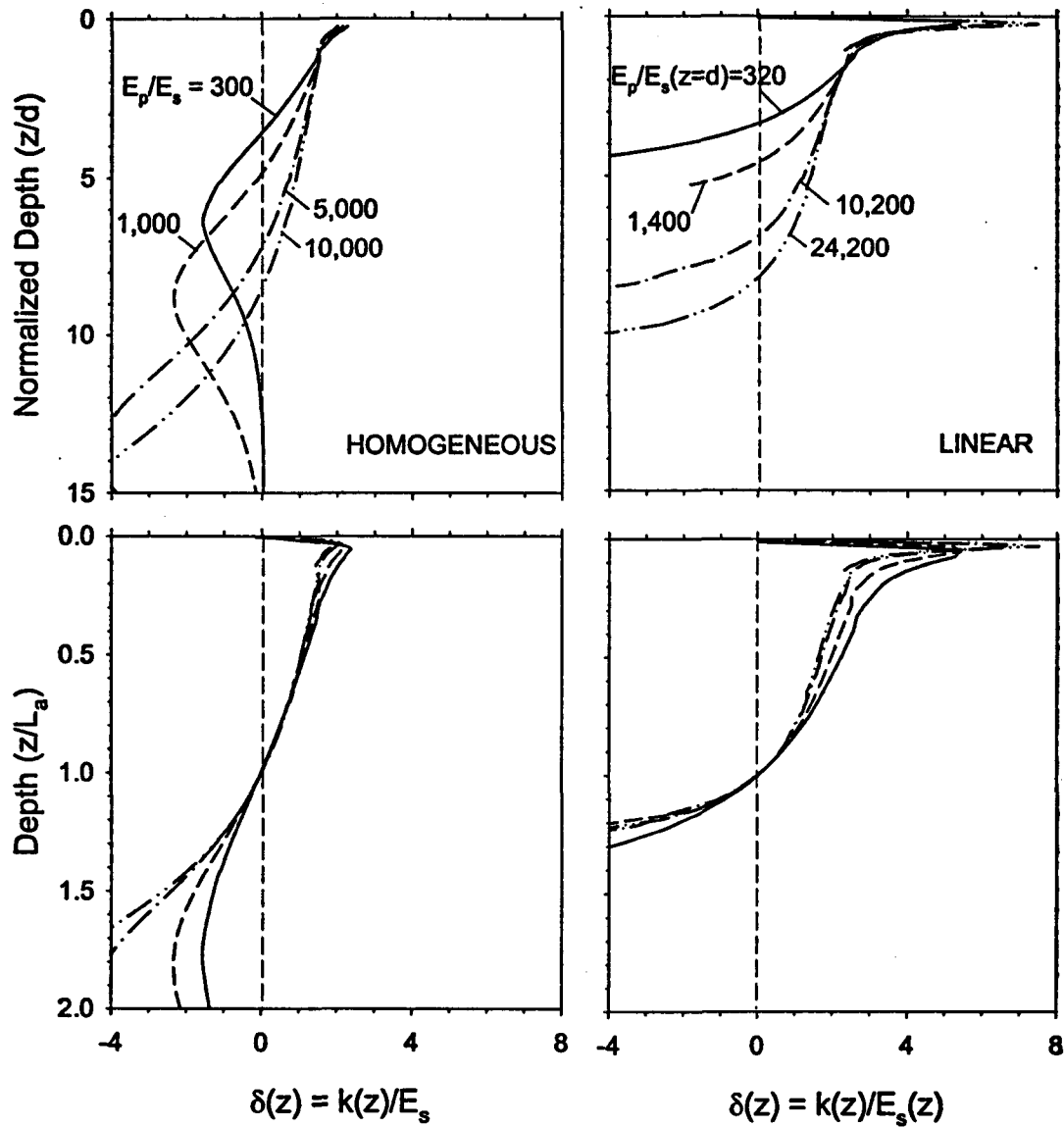


FIGURE 3.8: Variation of coefficient δ with respect to depth for various pile-soil stiffness contrasts for free head piles.

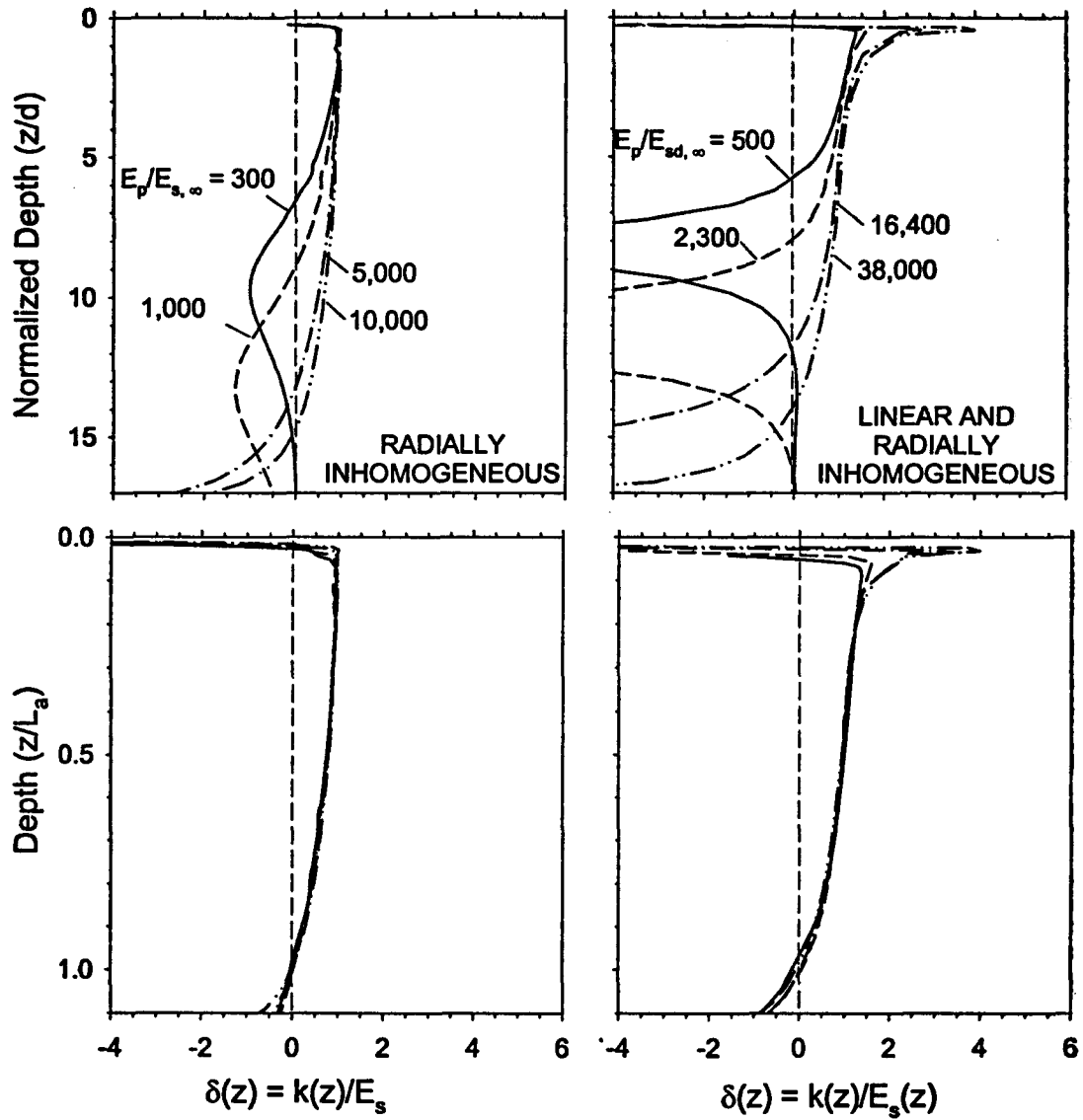


FIGURE 3.9: Variation of coefficient δ with respect to depth for various pile-soil stiffness contrasts for fixed head piles; ($A = 0.8$)

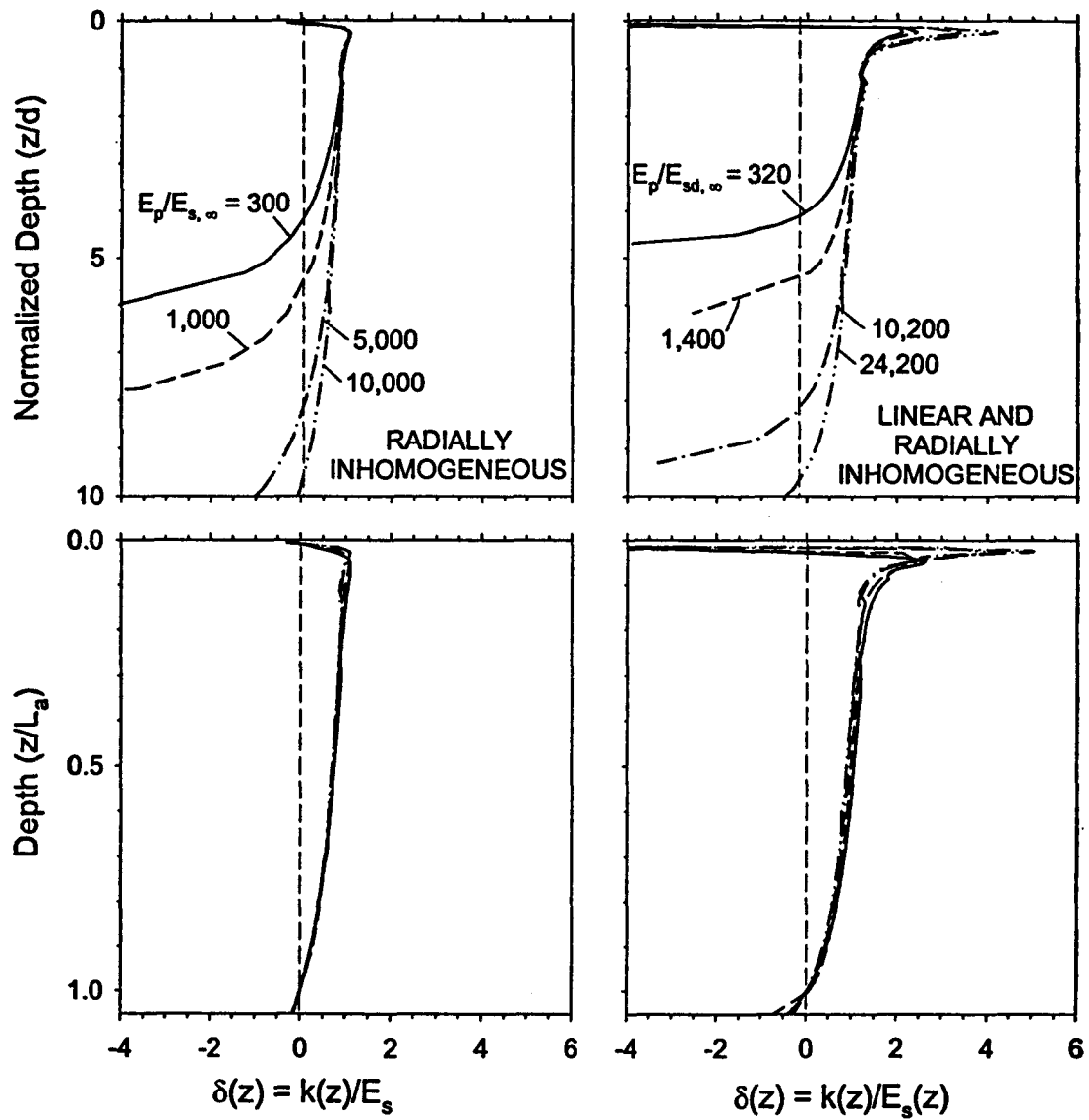


FIGURE 3.10: Variation of coefficient δ with respect to depth for various pile-soil stiffness contrasts for free head piles; ($A = 0.8$)

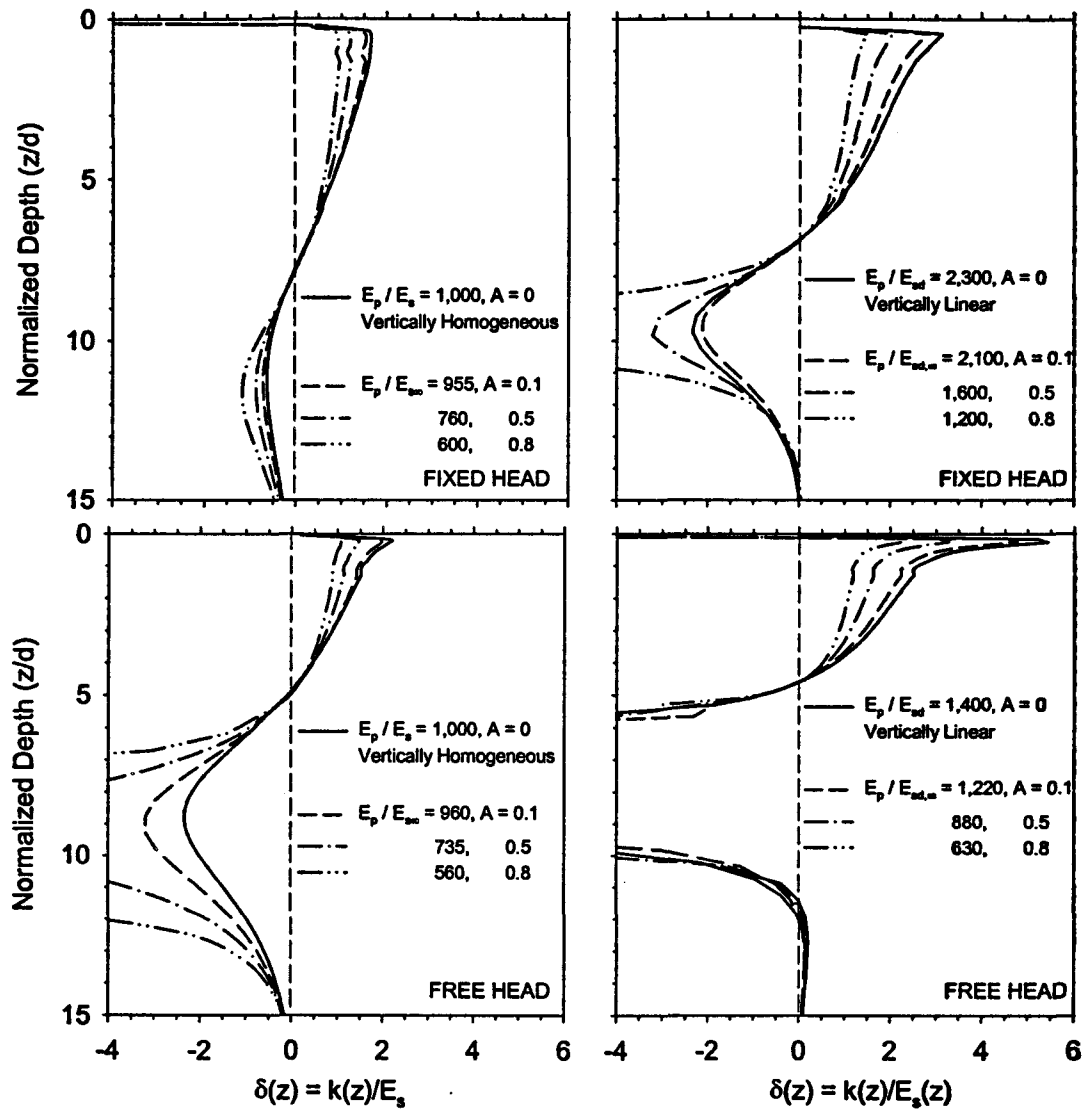


FIGURE 3.11: Variation of coefficient δ with respect to depth for various types of vertical and radial inhomogeneity and different head restraints. Piles with the same head restraints have matched pile-head swaying stiffness.

static stiffness. In these plots it is apparent that the value of $\delta_{\max}$ is a function of A and for the cases of fixed and free head piles in a radially inhomogeneous soil profile, it can be approximated as

$$\delta_{\max}(A) = \delta_{\max, \text{HOM}} (1 - 0.63 A) \quad (3.121)$$

where $\delta_{\max, \text{HOM}} = 2.0$

For the case of a linear soil profile with radial inhomogeneity, the reduction in $\delta_{\max}$ is very similar to the one for the radially inhomogeneous soil, so

$$\delta_{\max}(A) = \delta_{\max, \text{LIN}} (1 - 0.63 A) \quad (3.122)$$

where $\delta_{\max, \text{LIN}} = 3.75$ for the case of a fixed head pile and $\delta_{\max, \text{LIN}} = 4.0$ for the free head piles.

An also very interesting trend that is apparent from Figure 3.11 is that the soil reaction for soil profiles with different degree of radial inhomogeneity but with matched pile-head stiffness becomes zero at the same depth. This happens for all cases. In the next section it will be shown how this relates to the pile active length and how it can be used.

3.3.6 Active length – new definition

All the definitions of active length are somewhat arbitrary since they assume that the displacement at a depth of L_a should be equal to a certain (small) percentage of the displacement at the pile top. In this section a new definition is proposed that makes use of the mechanics involved in the pile response.

It is observed from all plots at the previous section that at a certain depth, the soil reaction adjacent to the pile becomes zero. This implies that the soil reaction becomes negative after this depth, thus the soil medium experiences tension. Although actual soil

can't sustain tension, in the absence of overburden pressure the elastic soil medium will experience tension due to the so-called arching behavior of the soil. To better illustrate this, after a certain depth the pile tends to separate from the soil because it is much stiffer and tends to deform less. On the other hand, the compatibility of displacements condition needs to be met for the neighboring soil and pile elements. Thus the soil appears to be pulling the pile (or vice versa) leading to a negative soil reaction. In reality there is an overburden pressure and the end result is a reduced compressive soil reaction.

Plotting the normalized swaying stiffness versus the normalized pile length (Figure 3.12) and superimposing the values of published formulas for the active length, one may reach the conclusion that all expressions are fairly accurate. Note that the piles embedded in a homogeneous and a linear soil profile have matched static stiffness. The following trends are observed:

The point of zero reaction is proportional to the relative pile-soil stiffness (E_p / E_s) for all pile head and soil conditions, and in all cases the swaying stiffness of a pile that has a length equal to the depth of zero soil reaction is above 80% of the one of an infinitely long pile. The accuracy becomes substantially greater if a length of two diameters is added to the depth of zero soil reaction. Therefore, a new definition is proposed: The pile active length is approximately equal to the depth of zero soil reaction plus two pile diameters. It can be seen from Figure 3.12 that the active length according to the new definition is fairly accurate.

Another conclusion that can be drawn by inspection is that for the same pile-head static stiffness, the active length of a pile embedded in a homogeneous halfspace is somewhat greater (of the order of 1 to 2 pile diameters) than that of a pile in an

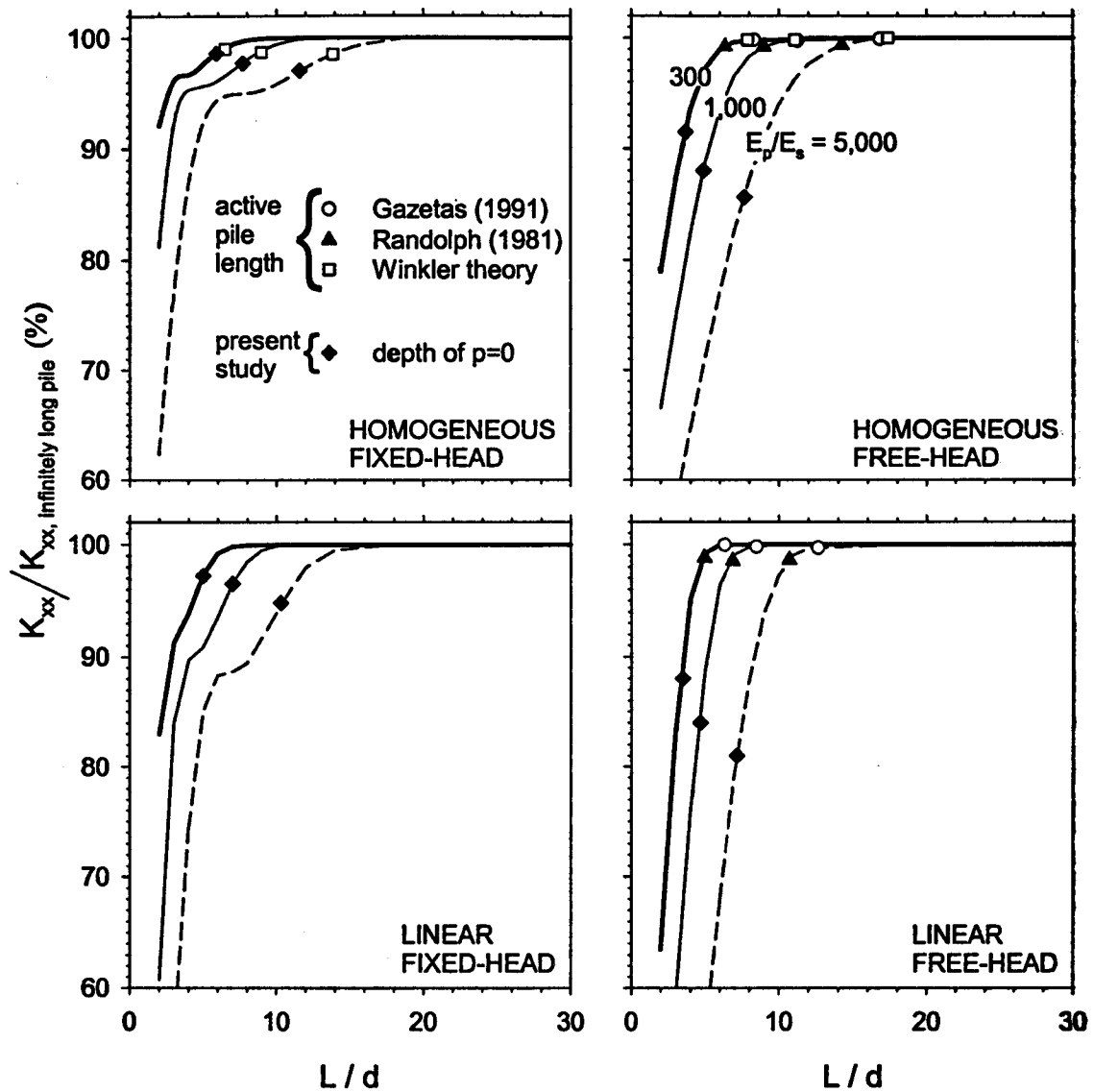


FIGURE 3.12: Variation of swaying stiffness with pile length for different pile-head restraints and soil profiles. The depth of zero soil reaction and active pile length are indicated. Long piles ($L > L_a$) in homogeneous and linear soil have equal swaying stiffnesses (Table 3.8).

“equivalent” soil profile. This could be seen also from figure 3.6 where the displacements with depth of a pile were dying out faster in the linear profile than in the homogeneous. For the case of radial inhomogeneity one can reach similar conclusions. Although it was shown from Figure 3.6 that the displacements die out faster for the profiles with radial inhomogeneity, in Figure 3.11 it is clear that the soil reactions become zero at the same depth. This suggests that the active length of radially inhomogeneous profiles is approximately equal to that of “equivalent” (matched pile head stiffness) radially homogeneous soil profiles.

3.3.7 Equivalent soil profiles

It becomes clear that finding the properties of different kinds of soil profiles with matched pile-head stiffnesses is important. To find these properties (mainly the pile-soil relative stiffness ratio, E_p / E_s) so far was done by a trial-and-error procedure. Now, using regression analysis new formulas can be obtained to predict the “equivalent” properties for different soil profiles. Equating the stiffnesses of the piles embedded in two different soil profiles, yields the following relations:

- For fixed head conditions, knowing the relative pile-soil stiffness for a pile embedded in a homogeneous halfspace enables us to find the elastic properties of an equivalent linear soil profile were the pile swaying stiffness of the two piles are equal.

Performing a regression analysis, the following formula was derived:

$$\left(\frac{E_p}{E_{sd}} \right)_{LIN} = 0.38 \left(\frac{E_p}{E_s} \right)_{HOM}^{1.25} \quad (3.123)$$

- Similarly, for free head piles

$$\left(\frac{E_p}{E_{sd}} \right)_{LIN} = 0.24 \left(\frac{E_p}{E_s} \right)_{HOM}^{1.25} \quad (3.124)$$

As in the case of finding an equivalent linear soil profile with the same swaying static stiffness of a pile embedded in a homogeneous halfspace, it is useful to find the equivalent radially inhomogeneous soil profile with a radially homogeneous soil profile.

In the case of a homogeneous halfspace, the pile-head stiffnesses from eqs. (3.63) and (3.73) are rewritten as:

$$K_{xx, HOM, A=0} = 1.24 E_{s, HOM, A=0} d \left(\frac{E_p}{E_{s, HOM, A=0}} \right)^{0.18} \quad (3.125)$$

for the case of a fixed head pile in a homogeneous halfspace
and

$$K_{xx, A}(A) = K_{xx, A}(A=0) [1 - 0.46 A] = 1.24 E_{s\infty, A} d \left(\frac{E_p}{E_{s\infty, A}} \right)^{0.18} [1 - 0.46 A] \quad (3.126)$$

for the case of a fixed head pile in a radially inhomogeneous soil profile.

Equating the two pile head stiffnesses and solving with respect to the unknown elastic modulus at the far field of the radially inhomogeneous soil profile, one obtains

$$\left(\frac{E_p}{E_{s\infty}} \right)_A = \left(\frac{E_p}{E_s} \right)_{HOM, A=0} [1 - 0.46 A]^{1.22} \quad (3.127)$$

The above equation will yield the corresponding $E_p / E_{s\infty}$ of a radially inhomogeneous soil profile given the E_p / E_s of a homogeneous profile and these two profiles will have the same static swaying pile-head stiffness.

Similarly in the case of a free head pile

$$\left(\frac{E_p}{E_{so}} \right)_A = \left(\frac{E_p}{E_s} \right)_{\text{HOM}, A=0} [1 - 0.53 A]^{1.22} \quad (3.128)$$

For the case of matching a vertically inhomogeneous soil profile (linear) with a doubly inhomogeneous profile (linear and radial) the corresponding relations for fixed head and free head piles are:

$$\left(\frac{E_p}{E_{sd, \infty}} \right)_A = \left(\frac{E_p}{E_{sd}} \right)_{\text{LIN}, A=0} [1 - 0.41 A]^{1.54} \quad (3.129)$$

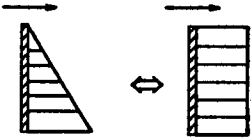
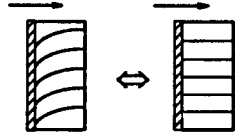
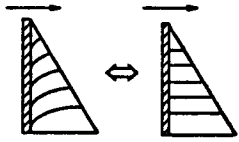
and

$$\left(\frac{E_p}{E_{sd, \infty}} \right)_A = \left(\frac{E_p}{E_{sd}} \right)_{\text{LIN}, A=0} [1 - 0.63 A]^{1.54} \quad (3.130)$$

respectively.

The above results are summarized in Table 3.8.

**TABLE 3.8: Relative pile-soil moduli for equivalent soil profiles
(matched static swaying pile-head stiffness)**

Pile-head fixity conditions	Pile soil relative stiffness for equivalent soil profiles	Equivalent soil profiles: Different soil moduli - same static swaying stiffness
Fixed Head	$\left(\frac{E_p}{E_{sd}}\right)_L = 0.38 \left(\frac{E_p}{E_s}\right)_H^{1.25}$	
Free Head	$\left(\frac{E_p}{E_{sd}}\right)_L = 0.24 \left(\frac{E_p}{E_s}\right)_H^{1.25}$	
Fixed Head	$\left(\frac{E_p}{E_{s,\infty}}\right)_R = (1-0.46A)^{1.22} \left(\frac{E_p}{E_s}\right)_H$	
Free Head	$\left(\frac{E_p}{E_{s,\infty}}\right)_R = (1-0.53A)^{1.22} \left(\frac{E_p}{E_s}\right)_H$	
Fixed Head	$\left(\frac{E_p}{E_{sd,\infty}}\right)_{L-R} = (1-0.41A)^{1.54} \left(\frac{E_p}{E_{sd}}\right)_L$	
Free Head	$\left(\frac{E_p}{E_{sd,\infty}}\right)_{L-R} = (1-0.63A)^{1.54} \left(\frac{E_p}{E_{sd}}\right)_L$	
Soil Profiles  Homogeneous (H) A=0  Radially Inhomogeneous (R) 0<A<1  Linear (L) A=0  Linear and Radially Inhomogeneous (LR) 0<A<1		

Example (static case)

Assume a concrete pile ($E_p = 3 \cdot 10^7$ KPa) with a length of 20m, and diameter of 1m ($L/d=20$). This pile is embedded in a homogeneous halfspace with an elasticity modulus of $E_s = 30,000$ KPa ($E_p / E_s = 1000$). The question is, what is the swaying static stiffness at the pile-head and what are the “equivalent” soil profiles for a) a fixed-head and b) a free-head pile?

- a) Using eq. (3.118) (fixed-head) the active length can be found to be equal to 13.5 m, less than the pile’s length, thus this is a “flexible” (or “long”) pile. Similarly, for the free-head pile, using eq. (3.108) the active length is 11.25 m, also less than the pile length.
- b) The dimensionless stiffness $K_{xx} / E_s / d$ is given by eq. (3.63) to be 4.3 for the fixed-head and 2.5 for the free-head using eq. (3.65). Thus, the pile-head stiffnesses are approximately equal to 129,000 KN/m and 74,900 KN/m for the fixed and free-head pile respectively.
- c) For the case of the fixed-head pile, a hypothetical equivalent linear soil profile (the one that would have a pile-head stiffness equal to 129,000 KN/m if the concrete pile was embedded in it) will have a soil modulus approximately equal to 14,000 KPa at a depth of one diameter, using eq. (3.123), ($E_p / E_{sd} = 2,140$). Similarly, in the case of a free-head pile, using eq. (3.124) one obtains $E_{sd} = 26,700$ KPa ($E_p / E_{sd} = 1,125$).
- d) To find an equivalent radially inhomogeneous soil profile with a inhomogeneity factor (A) equal to 0.8, using eq. (3.127) yields a soil modulus at the far field equal to $E_{s,\infty} = 52,500$ KPa ($E_p / E_{s,\infty} = 571$) for the case of a fixed head pile and 58,800 KPa ($E_p / E_{s,\infty} = 510$) for the case of a free-head pile using eq.(3.128). Comparing with the

elasticity modulus of the radially homogeneous soil profile (30,000 KPa) it is only natural that the soil modulus in the far field is greater, so that the two profiles can have the same pile-head stiffness.

- e) To find an equivalent linear soil profile with radial inhomogeneity ($A=0.8$) for a fixed-head pile, using eq. (3.129) with the ratio $E_p / E_{sd} = 2,140$ found earlier, yields $E_{sd,\infty} = 25,900$ KPa ($E_p / E_{sd,\infty} = 1,158$) which is the soil elasticity modulus corresponding to a depth of one diameter at the far-field. Similarly for the free-head pile, using eq. (3.130) yields $E_{sd,\infty} = 78,600$ KPa ($E_p / E_{sd,\infty} = 382$). It is obvious how much stiffer the doubly inhomogeneous soil profile has to be in the far field, in order to have the same pile-head stiffness.

3.3.8 Dynamic response - Comparison with other studies

Again, the comprehensive parametric results of Marsafawi et al. are used to validate the dynamic results of K-PAX. It can be seen from Figure 3.13 that in all soil profiles and for all stiffness coefficients the results are in close agreement. In the case of a homogeneous halfspace, it can be observed that the dynamic pile head stiffness is approximately constant and equal to the static stiffness. On the other hand the dashpot coefficient increases linearly with frequency. These arguments are also valid for the case of a linear soil profile as it can be seen from Figure 3.14. For a small range of low frequencies it was observed that the transmitting boundaries used in K-PAX by Kellezi were producing some oscillations in the dynamic stiffness. On the other hand, the infinite elements by Marques & Owen, implemented in K-PAX were producing very accurate results for the static case. Since it is obvious that for low frequencies the real part of the

pile response is practically constant, the static results of K-PAX were kept. For the case of the imaginary part, the transmitting boundaries were producing excellent results for low and high frequencies as can be seen from the Figures 3.13 and 3.14 and thus these results were kept for the whole range of frequencies.

3.3.9 Radial Inhomogeneity - Dynamic case:

In Figures 3.15 and 3.16 the stiffness terms of the pile head are plotted for the soil profiles that have the same properties in the far field. It can be clearly seen that the stronger the radial inhomogeneity the lower the stiffness coefficients, as it was observed for the static case. One may also observe a progressive change in the inclination of the impedance curves as the inhomogeneity factor (A) increases. This becomes even more obvious in Figure 3.17. This trend becomes even stronger in the case of a linear soil profile where for high frequencies the pile head stiffness becomes negative. This is not surprising as it is also predicted from the Winkler theory for very high frequencies.

It should be noted that in these figures, the soil profiles have the same properties in the far field. These plots are of interest to see what are the effects of radial soil inhomogeneity if neglected completely. Usually engineers take into account the soil softening near the pile and intentionally lower the elastic properties of the soil to come up with an equivalent homogeneous soil. In such a case, Figures 3.15 and 3.16 don't apply. A more representative plot would be one that would compare the pile embedded in a homogeneous soil profile and radially inhomogeneous soil profiles with matched pile head static stiffnesses. The implications are the following: If one selects an equivalent

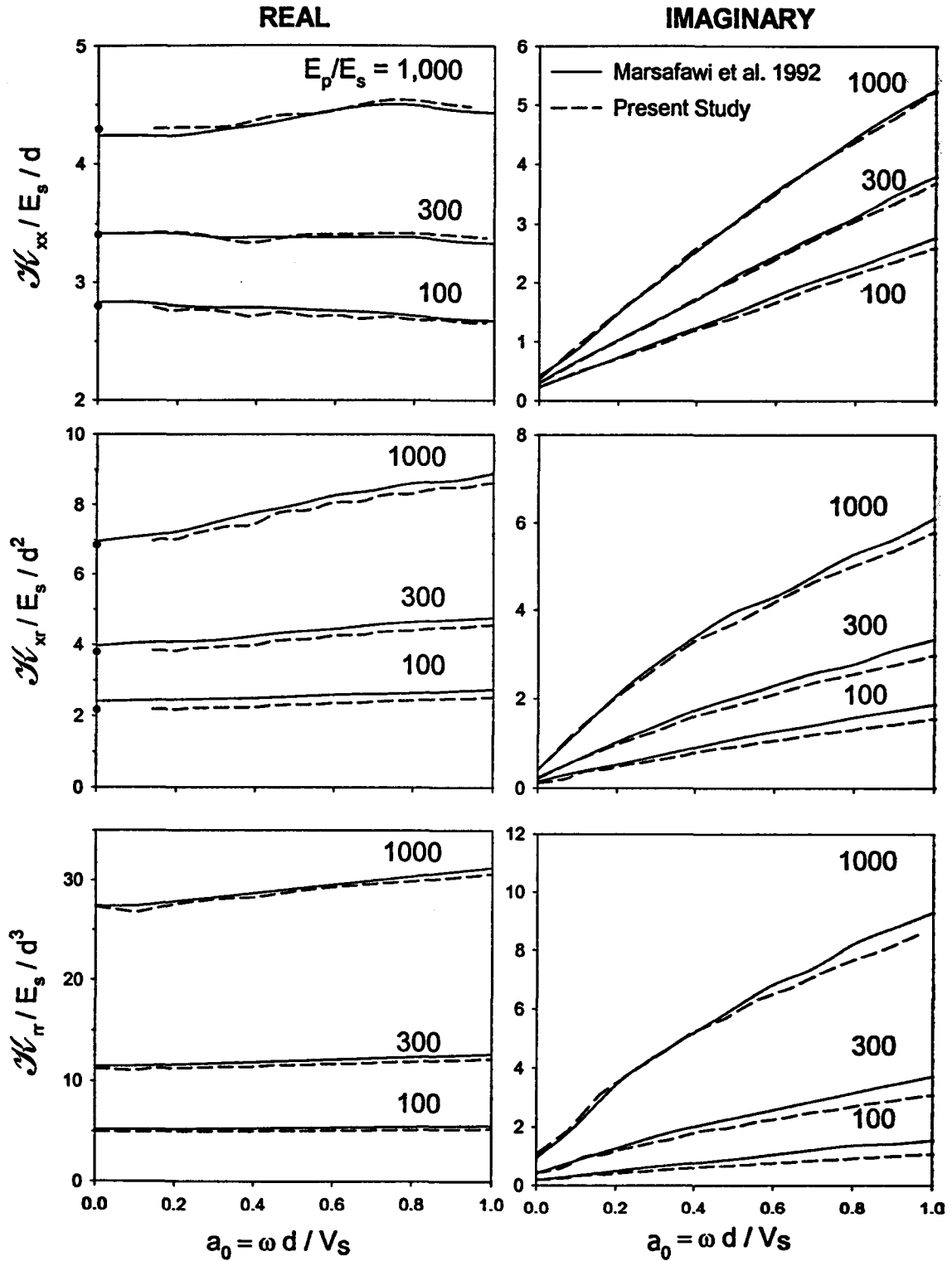


FIGURE 3.13: Normalized horizontal (swaying), rocking and cross term (swaying-rocking) dimensionless stiffness of a single pile in homogeneous Halfspace; ($L/d=20$, $\rho_p/\rho_s=1.25$, $\beta_s=0.05$, $\nu=0.4$)

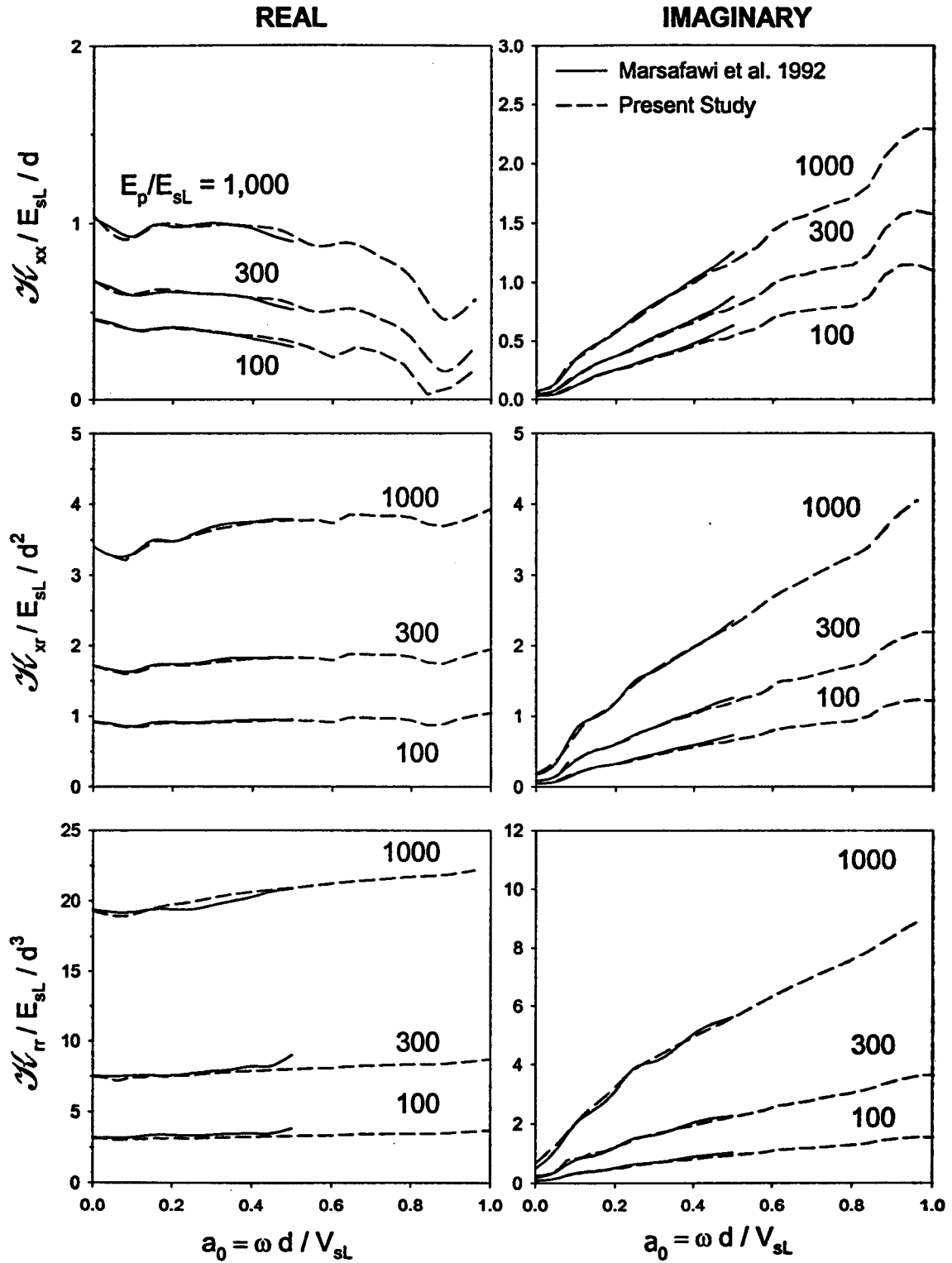


FIGURE 3.14: Normalized horizontal (swaying), rocking and cross term (swaying-rocking) dimensionless stiffness of a single pile in linear soil profile (Gibson soil); ($L/d=20$, $\rho_p/\rho_s=1.25$, $\beta_s=0.05$, $\nu=0.4$)

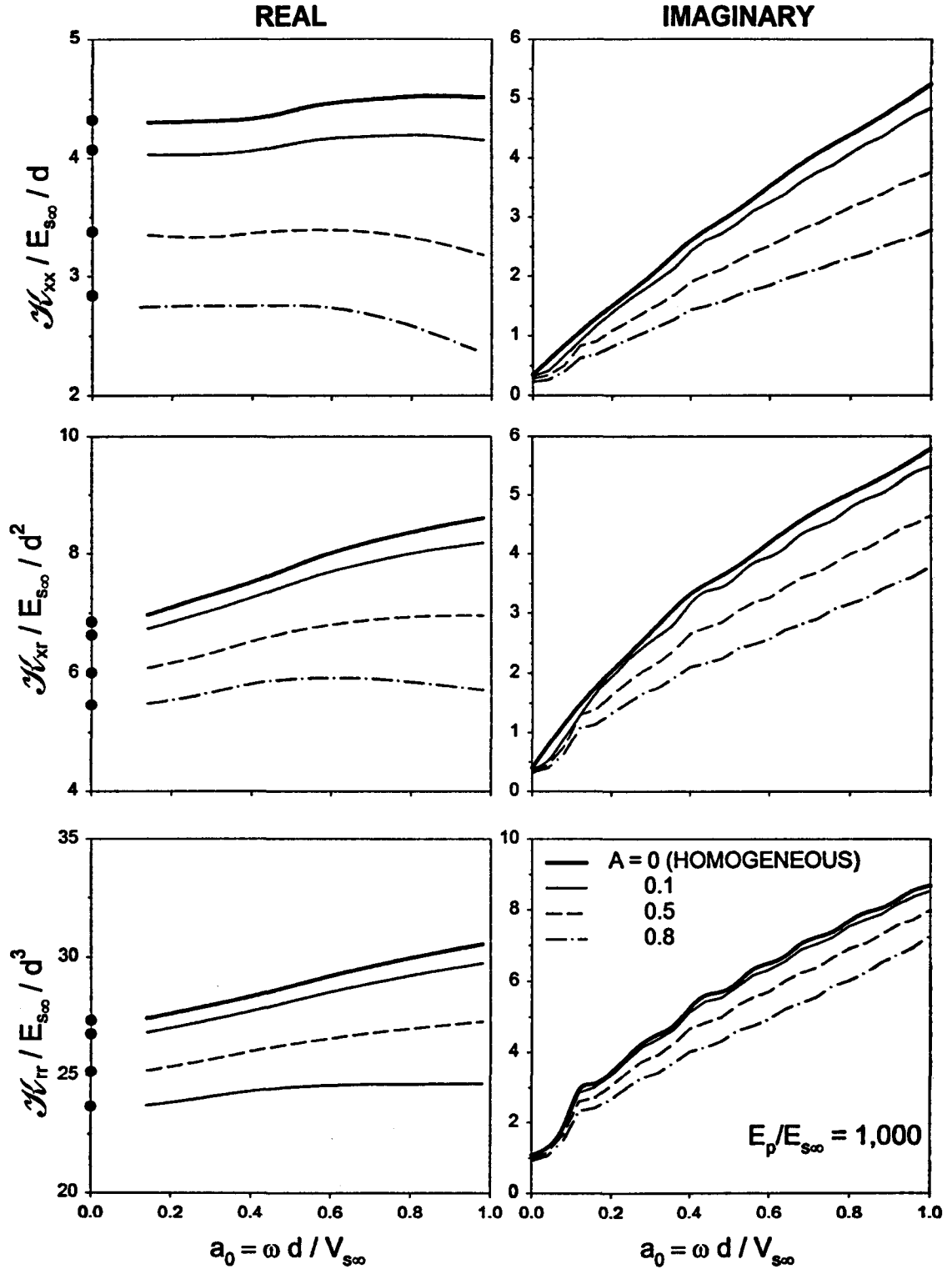


FIGURE 3.15: Normalized horizontal (swaying), cross term (swaying-rocking) and rocking dynamic stiffness of a floating single pile in radially inhomogeneous halfspace; ($L/d=20$, $\rho_p/\rho_s=1.25$, $\beta_s=0.05$, $\nu=0.4$)

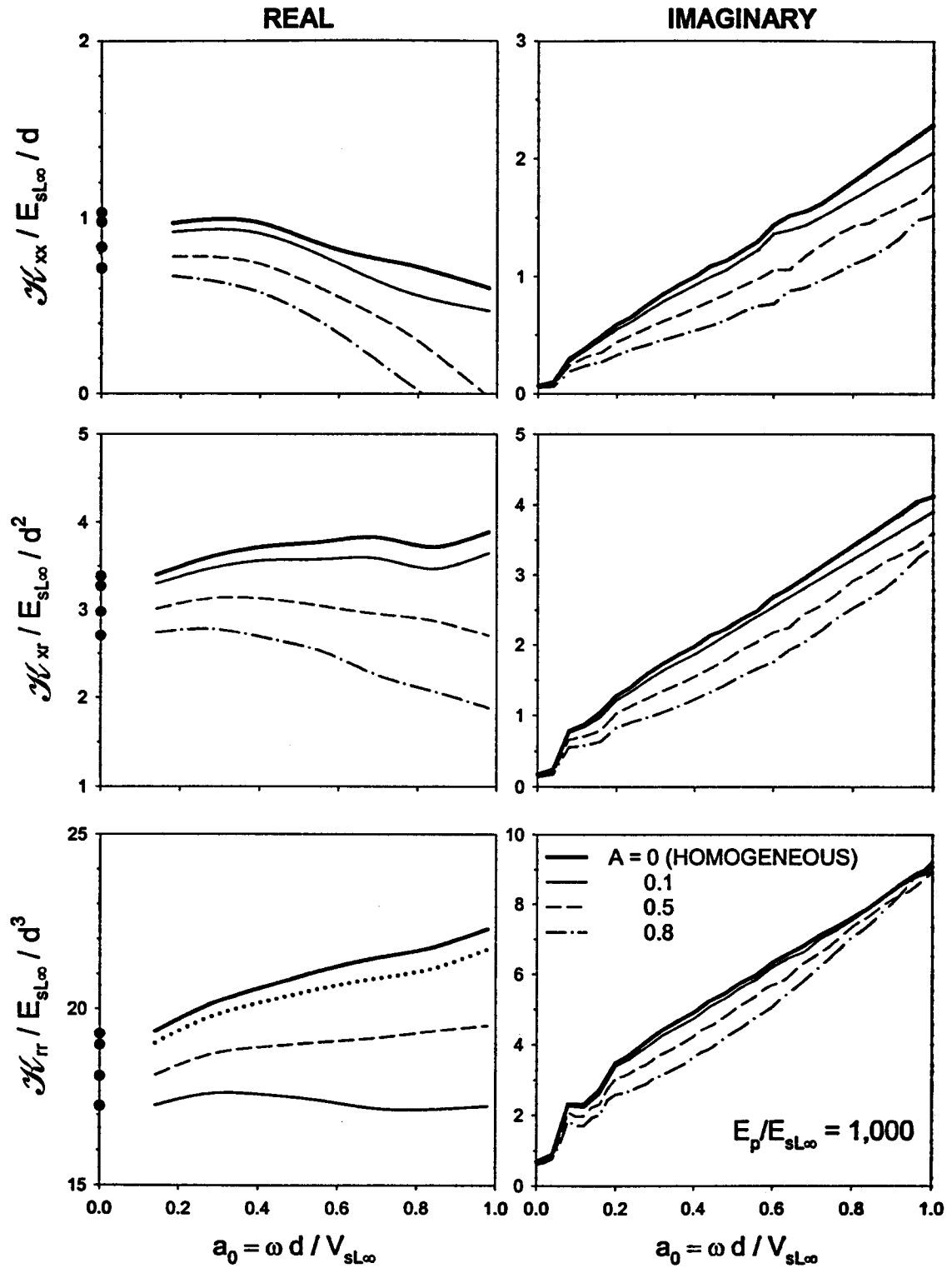


FIGURE 3.16: Normalized horizontal (swaying), cross term (swaying-rocking) and rocking dynamic stiffness of a floating single pile in a linear soil profile with radial inhomogeneity; ($L/d=20$, $\rho_p/\rho_s=1.25$, $\beta_s=0.05$, $\nu=0.4$)

homogeneous soil profile that will encompass the effect of radial inhomogeneity, the result in a dynamic analysis will lead to an overestimation of pile head stiffness and damping. This can be seen more clearly in figure 3.18 where the damping ratio apparent at the pile head is plotted against dimensionless frequency. Observing Figure 3.18 a trend is clear: the stronger the radial inhomogeneity the larger the difference in the damping ratio between the “actual” soil profile and the equivalent homogeneous. This phenomenon is discussed in greater length in the next section where the effect of vertical inhomogeneity is included.

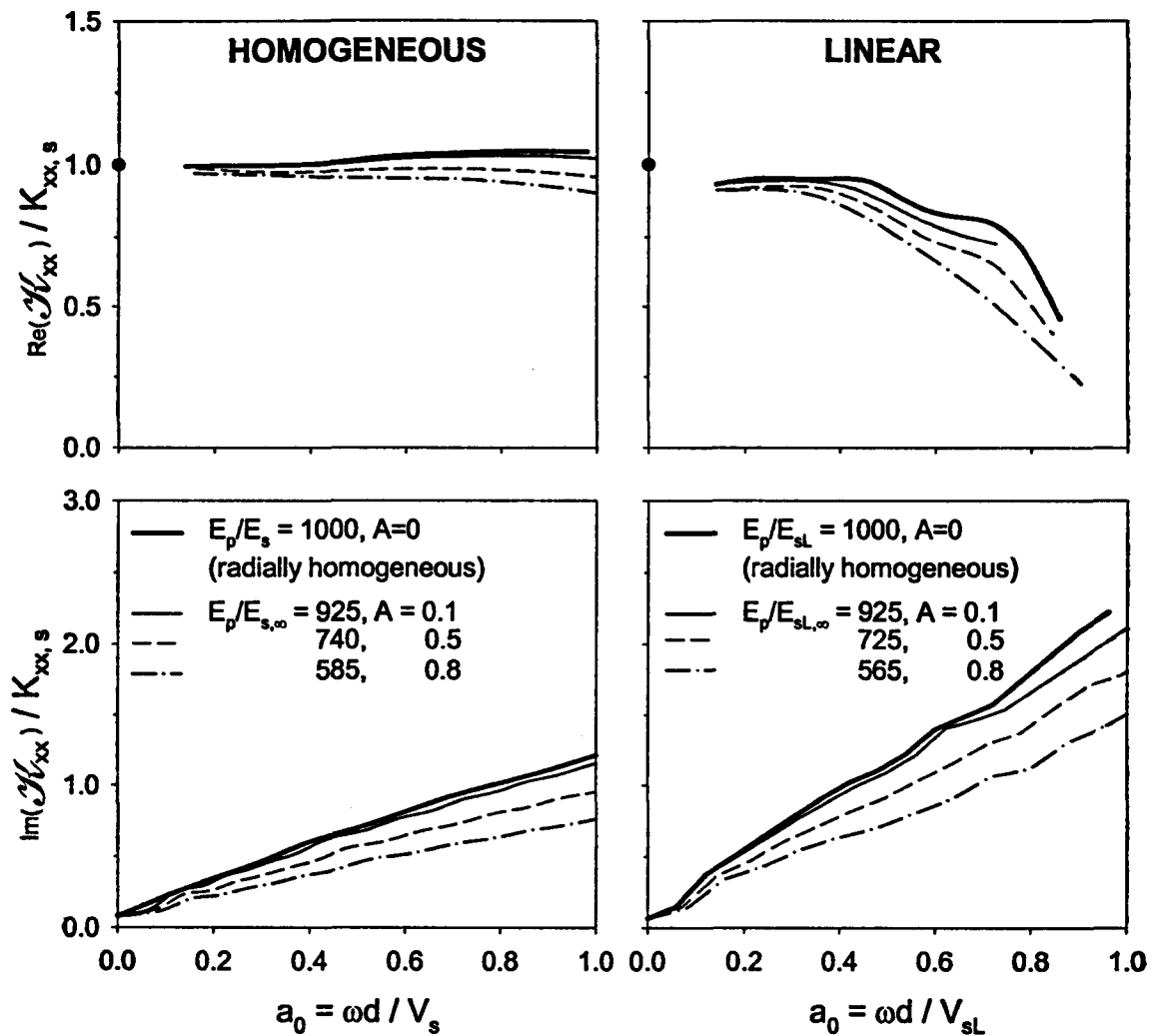


FIGURE 3.17: Normalized real and imaginary part of swaying pile head stiffness for fixed head conditions. In each of the two different soil profiles, the piles have matched swaying static stiffness. ($L/d = 20$)

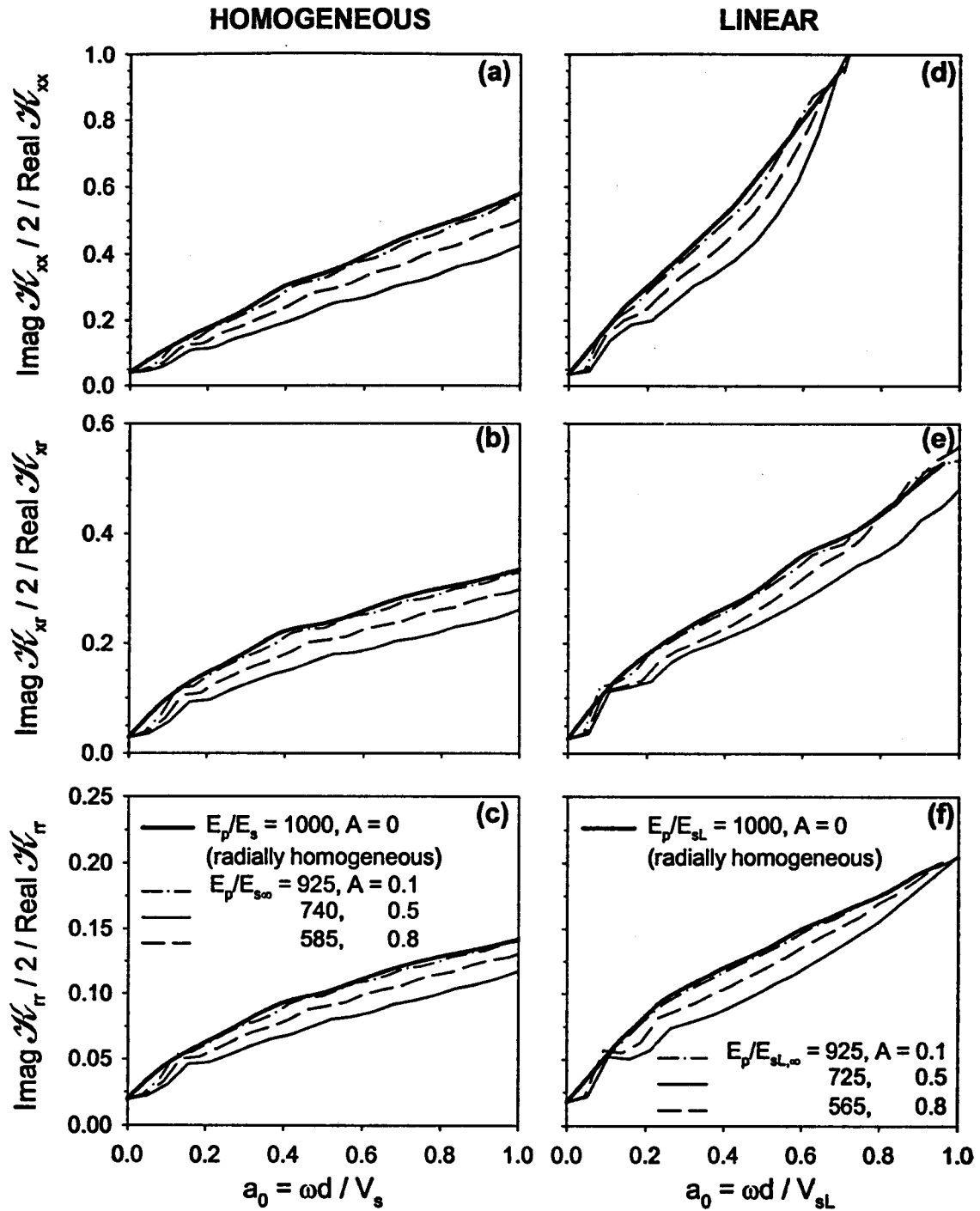


FIGURE 3.18: Equivalent damping ratio at pile head for a pile embedded in a vertically homogeneous (a through c) and a vertically linear soil profile (d through f) with ($A > 0$) and without ($A = 0$) radial inhomogeneity. All piles (a through c and d through f) have equal static swaying stiffness;

3.3.10 Radiation damping of “equivalent” soil profiles

The high values of radiation damping of embedded structures has been one of the arguments of engineers to avoid performing an analysis with soil-structure interaction. It will be shown that the actual radiation damping apparent at the head of a pile embedded in a realistic soil medium is significantly less than the damping of the same pile embedded in a homogeneous halfspace as is commonly assumed.

In Figures 3.19 and 3.20 the damping ratio (material and radiation) apparent at the pile head is plotted against dimensionless frequency.

In the first plot, all soils have the same elastic modulus in the far field. This plot would be useful to demonstrate the difference in damping that the radial inhomogeneity creates if neglected completely. So, in the case one takes only into account the properties of the far field and completely neglects the soil properties near the pile (which would be considered wrong), comparing the curves of a radially homogeneous and radially inhomogeneous profile shows a small drop in the radiation damping for low frequencies and an increase for higher frequencies.

The second plot shows the radiation damping for different soil profiles. The difference with the first plot is that these soil profiles are now “equivalent” in the sense that pile head stiffness will be the same in all profiles. It is obvious from this plot that the more inhomogeneous the soil medium, the smaller the radiation damping. This is intuitively correct, since the more inhomogeneous the medium the more the refractions and the reflections thus making it harder for the wave energy to dissipate, which in turn translates to reduced radiation damping. It should be noted that for the second plot, the dimensionless frequency ($a_0 = \omega d / V_s$) is based on the shear wave velocity (V_s) of the

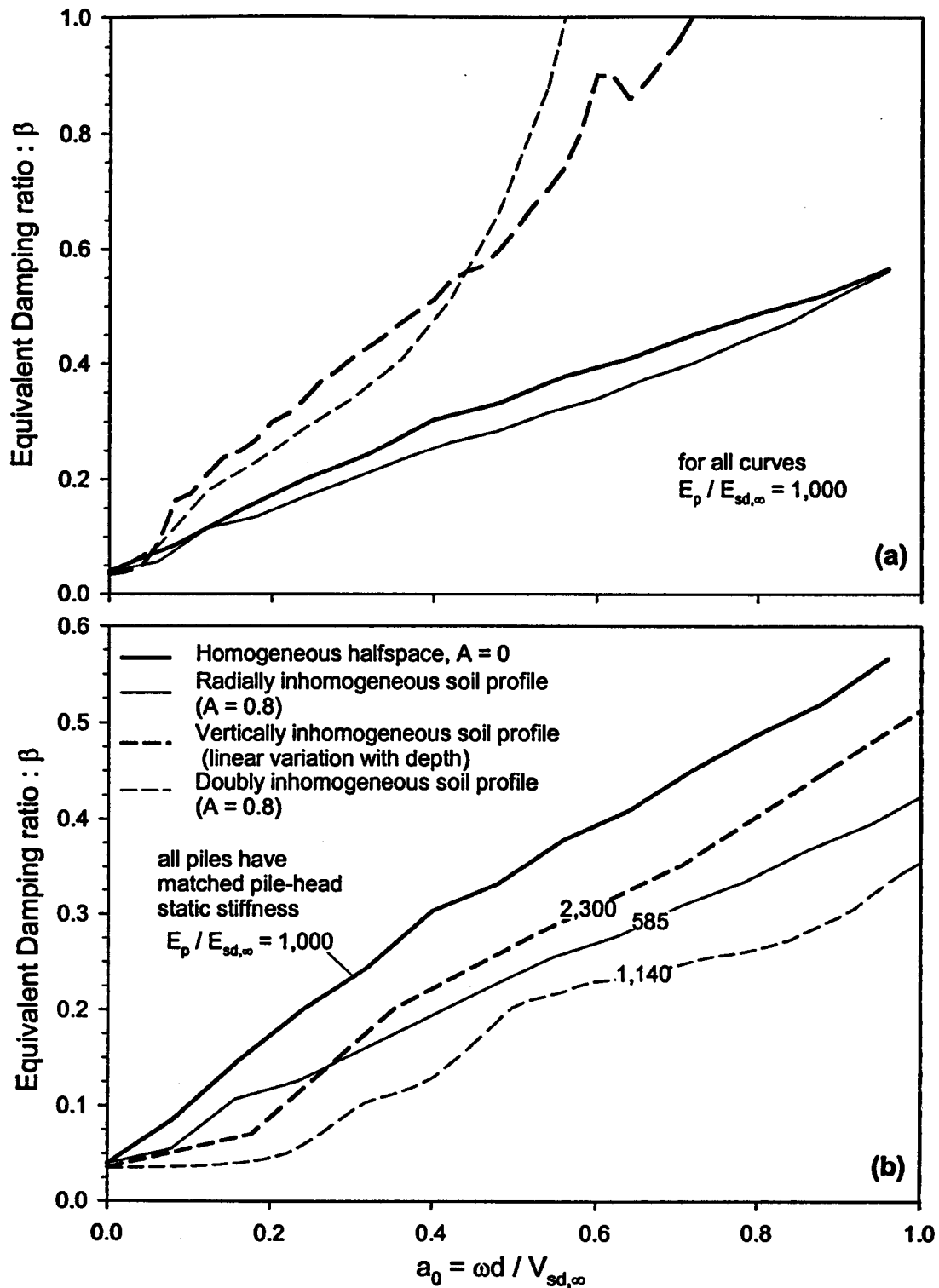


FIGURE 3.19: Equivalent damping ratio for swaying mode at pile head for different soil profiles. (a) All piles have same soil properties at the far field ($E_p / E_{sd,\infty} = 1000$) (b) All piles have matched pile-head static stiffness.

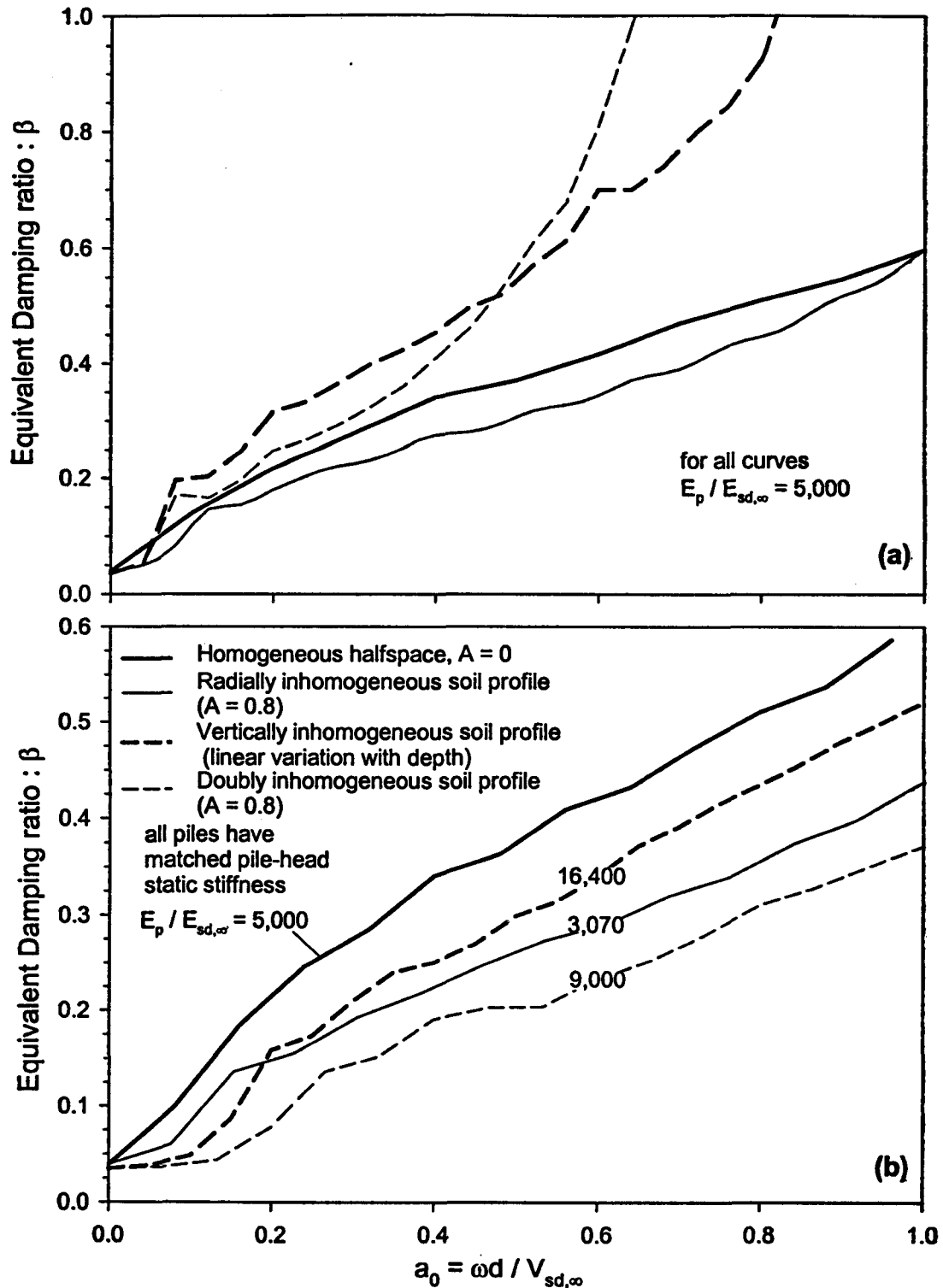


FIGURE 3.20: Equivalent damping ratio for swaying mode at pile head for different soil profiles. (a) All piles have same soil properties at the far field ($E_p / E_{sd,\infty} = 5000$) (b) All piles have matched pile-head static stiffness.

homogeneous halfspace ($E_p / E_s = 1,000$ and $5,000$ for Figures 3.19 and 3.20 respectively).

The implications of these graphs are important:

- a) It is understood that real soil profiles are hardly ever homogeneous. The assumption of homogeneity, although suitable for static analysis, leads to an overestimation in radiation damping thus making the analysis (or design) unconservative.
- b) Radiation damping plays an integral role in soil-structure-interaction analyses. The greater the radiation damping the lower the amplitude of response of the superstructure. Oversimplifications like the assumption of homogeneity or the assumption of a weaker soil with the justification of erring on the safe side lead to increased radiation damping. This leads to the following conclusion: The assumption of weaker soil leads to increased displacements for static analysis and therefore making the analysis more conservative. On the other hand, the same assumption yields greater radiation damping for dynamic analyses, thus making the analysis unsafe.

After performing a regression analysis, the following expressions for the damping ratio apparent at the pile head were found:

- For the case of a vertically homogeneous soil profile and a fixed head pile, the damping ratio apparent at the pile head (horizontal – swaying mode) can be approximated as:

$$\beta_{xx} = 0.8 \beta_m + 0.19 a_0 (1 + 0.44 A) \left(\frac{E_p}{E_{s,\infty}} \right)^{0.18 - 0.1A} \quad (3.131)$$

for $a_0 > a_{0, \text{CUTOFF}}$

where

$$a_{0, \text{CUTOFF}} = \frac{\pi}{2} \left(\frac{H}{d} \right)^{-1} \quad (3.132)$$

and H describes the depth at which the rigid bedrock is located.

This dimensionless cut-off frequency is derived from the well known 1st period of a homogeneous soil layer of depth H resting on top of a rigid bedrock. This period comes from the well-known formula

$$T = 4H / V_s \quad (3.133)$$

It was observed that the change in the cut-off frequency due to radial inhomogeneity is negligible, so for all practical purposes it is taken equal to the one for the radially homogeneous layer.

In the case of a halfspace, the cut-off frequency becomes zero, which means that radiation damping starts at zero frequency.

Also,

$$\beta_{xx} = 0.8 \beta_m \text{ for } a_0 \leq a_{0, \text{CUTOFF}}$$

It should be noted that for A equal to 0, eq. (3.131) describes the damping ratio of a homogeneous soil profile.

For the rocking mode it follows:

$$\beta_r = 0.4 \beta_m + 0.1 \left(a_0 + 0.013 \left(\frac{E_p}{E_{s, \infty}} \right)^{0.4} (1 - 0.23 A) \right) \quad (3.134)$$

for $a_0 > a_{0, \text{CUTOFF}}$ and

$$\beta_r = 0.4 \beta_m \text{ for } a_0 \leq a_{0, \text{CUTOFF}}$$

Similarly, for the cross swaying-rocking mode:

$$\beta_{xr} = 0.6 \beta_m + 0.3 \left(a_0 + 0.03 \left(\frac{E_p}{E_{s,\infty}} \right)^{0.27} (1 - 0.64 A) \right) \quad (3.135)$$

for $a_0 > a_{0, \text{CUTOFF}}$ and

$$\beta_{xr} = 0.6 \beta_m \text{ for } a_0 \leq a_{0, \text{CUTOFF}}$$

- For the case of a vertically linear soil profile and a fixed head pile, the damping ratio apparent at the pile head (horizontal – swaying mode) can be approximated as:

$$\beta_{xx} = 0.7 \beta_m + 0.3 a_{0,d} (1 + 0.77 A) \left(\frac{E_p}{E_{sd,\infty}} \right)^{-0.09A} \quad (3.136)$$

for $a_0 > a_{0, \text{CUTOFF}}$ and

where

$$a_{0, \text{CUTOFF}} \approx 1.2 \left(\frac{H}{d} \right)^{-1/2} \quad (3.137)$$

and H describes the depth at which the rigid bedrock is located.

This dimensionless cut-off frequency was derived from the cut-off frequency suggested by Gazetas (1991)

$$f = 0.19 V_{sH} / H \quad (3.138)$$

Like in the case of the vertically homogeneous soil layer, no significant change was observed in the cut-off frequency with the introduction of radial inhomogeneity. Thus, the dimensionless cut-off frequency is taken as the one for a radially homogeneous soil profile.

$$\beta_{xx} = 0.7 \beta_m \text{ for } a_0 \leq a_{0, \text{CUTOFF}}$$

Similarly for the rocking mode:

$$\beta_{\pi} = 0.8 \beta_m + 0.055 a_{0d} (1 - 0.20 A) \left(\frac{E_p}{E_{sd, \infty}} \right)^{-0.02} \quad (3.139)$$

for $a_0 > a_{0, \text{CUTOFF}}$ and

$$\beta_{\pi} = 0.35 \beta_m \text{ for } a_0 \leq a_{0, \text{CUTOFF}}$$

and for the cross swaying-rocking mode:

$$\beta_{xr} = 0.55 \beta_m + 0.12 \left(a_0 + 0.52 \left(\frac{E_p}{E_{sd, \infty}} \right)^{-0.04} (1 - 0.72 A) \right) \quad (3.140)$$

for $a_0 > a_{0, \text{CUTOFF}}$ and

$$\beta_{xr} = 0.55 \beta_m \text{ for } a_0 \leq a_{0, \text{CUTOFF}}$$

- For the case of a vertically homogeneous soil profile and a free head pile, the damping ratio apparent at the pile head (horizontal – swaying mode) can be approximated as:

$$\beta_{xx} = 0.8 \beta_m + 0.17 \left(a_{0d} \left(\frac{E_p}{E_{s, \infty}} \right)^{0.15} - 0.7 \left(\frac{E_p}{E_{s, \infty}} \right)^{-0.7} (1 - 0.6 A) \right) \quad (3.141)$$

for $a_0 > a_{0, \text{CUTOFF}}$ and

$$\beta_{xx} = 0.8 \beta_m \text{ for } a_0 \leq a_{0, \text{CUTOFF}}$$

- For the case of a vertically linear soil profile and a free head pile, the damping ratio apparent at the pile head (horizontal – swaying mode) can be approximated as:

$$\beta_{xx} = 0.7 \beta_m + 0.7 \left(\frac{E_p}{E_{sd, \infty}} \right)^{-0.07} (1 - 0.07 A) (a_{0d} - a_{0, \text{CUTOFF}})^{2.7} \left(\frac{E_p}{E_{sd, \infty}} \right)^{-0.15} (1 - 0.17 A) \quad (3.142)$$

for $a_0 > a_{0, \text{CUTOFF}}$ and

$$\beta_{xx} = 0.7 \beta_m \text{ for } a_0 \leq a_{0, \text{CUTOFF}}$$

All the above formulas are concentrated in Tables 3.5 and 3.6.

In Figure 3.21 the swaying damping ratio of a fixed head pile for four “equivalent” soil profiles is plotted against dimensionless frequency. In the first plot, the rigid bedrock is placed at infinity and at the second plot it is placed a quarter of the pile length beneath the pile tip. The pile and the profiles used have the same properties with those used in the previous example:

- A homogeneous soil profile with $E_p / E_s = 1,000$. The dimensionless cut-off frequency is zero for the case of the halfspace and for the case where $H/L = 1.25$ it is equal to (eq. (3.132)) $a_{0, \text{CUTOFF}} = 0.063$. It should be noted, that in the horizontal axis, the dimensionless frequency a_0 is normalized with the shear wave velocity (V_s) of the homogeneous halfspace ($a_0 = \omega d / V_s$). It can be easily found that this velocity is equal to $V_s = 73.2 \text{ m/s}$ ($\rho_s = 2.0 \text{ Mg/m}^3$, $\nu_s = 0.4$). The corresponding cut-off cyclic frequency (ω) is equal to 4.6 rad/s . The expression for the swaying damping ratio is eq.(3.131) with $A=0$.
- A linear soil profile with $E_p / E_{sd} = 2,140$. The dimensionless cut-off frequency in this case ($H/L=1.25$) is equal to (eq. (3.137)) $a_{0, \text{CUTOFF}} = 0.24$. This dimensionless frequency is normalized with the shear wave velocity at a depth of one diameter, which is equal to $V_{sd} = 50 \text{ m/s}$. The corresponding cut-off cyclic frequency (ω) is equal to 12 rad/s . For plotting the damping ratio of this linear profile along with the one for the homogeneous halfspace a common normalization must be used for the dimensionless frequency a_0 . It can be easily proved that the way the normalized frequency for the homogeneous halfspace ($a_{0, \text{HOM}}$) and the one for the linear soil profile ($a_{0, \text{LIN}}$) are connected through the following expression:

$$a_{0, \text{LIN}} = a_{0, \text{HOM}} \sqrt{\frac{\left(\frac{E_p}{E_{sd}} \right)_{\text{LIN}}}{\left(\frac{E_p}{E_s} \right)_{\text{HOM}}}} \quad (3.143)$$

In fact, eq. (3.143) can be generalized to connect the dimensionless frequencies of any kind of soil profile (X) with the one from the homogeneous halfspace:

$$a_{0, X} = a_{0, \text{HOM}} \sqrt{\frac{\left(\frac{E_p}{E_s} \right)_X}{\left(\frac{E_p}{E_s} \right)_{\text{HOM}}}} \quad (3.144)$$

So, the dimensionless cut-off frequency of the linear soil profile, normalized with the shear wave velocity of the homogeneous halfspace is equal to:

$$0.24 = a_{0, \text{CUTOFF-HOM}} \sqrt{\frac{(2,140)}{(1,000)}} \Rightarrow a_{0, \text{CUTOFF-HOM}} \approx 0.16$$

Alternatively, one can, easily, come up with the same answer by following the definition of the dimensionless frequency. Thus,

$$a_{0, \text{CUTOFF-HOM}} = \frac{\omega d}{V_{s, \text{HOM}}} = \frac{12 \cdot 1}{73.2} \approx 0.16$$

Plugging in eq. (3.143) into eq. (3.136) with $A = 0$ yields the expression for the curve shown in figure 3.21.

$$\beta_{xx} = 0.7 \beta_m + 0.3 a_{0, \text{HOM}} \sqrt{\frac{\left(\frac{E_p}{E_{sd}} \right)_{\text{LIN}}}{\left(\frac{E_p}{E_s} \right)_{\text{HOM}}}} \quad (3.145)$$

- A radially inhomogeneous soil profile with $A = 0.8$ and $E_p / E_{s, \infty} = 571$. In this case, the dimensionless cut-off frequency is approximately the same with the one for the

homogeneous halfspace. Thus, $a_{0, \text{CUTOFF}} = 0.063$. This dimensionless frequency is normalized with the shear wave velocity at the far field, which is equal to $V_{s, \infty} = 96.9$ m /s. The corresponding cut-off cyclic frequency (ω) is equal to 6.1 rad /s. Once again, the normalization of the cut-off frequency has to be done with the shear wave velocity of the homogeneous halfspace. Thus, either following eq. (3.144) or the definition of the dimensionless frequency, the result is $a_{0, \text{CUTOFF}} = 0.083$.

Plugging in eq. (3.144) into eq. (3.131) with $A = 0.8$ yields the expression for the curve shown in Figure 3.21.

$$\beta_{xx} = 0.8 \beta_m + 0.19 a_{0, \text{HOM}} \sqrt{\left(\frac{\left(\frac{E_p}{E_{s, \infty}} \right)_{\text{RAD}}}{\left(\frac{E_p}{E_s} \right)_{\text{HOM}}} \right)} (1 + 0.44 A) \left(\frac{E_p}{E_{s, \infty}} \right)^{0.18 - 0.1A} \quad (3.146)$$

- A linear soil profile that is also radially inhomogeneous with $A = 0.8$ and $E_p / E_{sd, \infty} = 1,158$. In this case, the dimensionless cut-off frequency is approximately the same with the one for the linear soil profile. Thus, $a_{0, \text{CUTOFF}} = 0.24$. This dimensionless frequency is normalized with the shear wave velocity at the far field, at a depth of one diameter, which is equal to $V_{sd, \infty} = 68$ m /s. The corresponding cut-off cyclic frequency (ω) is equal to 16.3 rad /s. Once again, the normalization of the cut-off frequency has to be done with the shear wave velocity of the homogeneous halfspace. Thus, either following eq. (3.144) or the definition of the dimensionless frequency, the result is $a_{0, \text{CUTOFF}} = 0.22$.

Plugging in eq. (3.144) into eq. (3.136) with $A = 0.8$ yields the expression for the curve shown in figure 3.21.

$$\beta_{xx} = 0.7 \beta_m + 0.3 a_{0,HOM} \sqrt{\left(\frac{\left(\frac{E_p}{E_{sd,\infty}} \right)_{LIN-RAD}}{\left(\frac{E_p}{E_s} \right)_{HOM}} \right)} (1 + 0.77 A) \left(\frac{E_p}{E_{sd,\infty}} \right)^{-0.09A} \quad (3.147)$$

The plots in Figure 3.21 show the same trends observed in the plots with the actual results but also one additional trend is shown and clarified. That is the effect of the cut-off frequency. The more inhomogeneous the soil profile is, the bigger the cut-off frequency becomes (for soil profiles with matched static stiffness). This makes the difference in the radiation damping much more pronounced! As an example, let's assume that the exciting period is 0.5 sec, a reasonable value in the realm of seismic excitations, and the bedrock is placed relatively shallow ($H/L = 1.25$). The corresponding dimensionless frequency for the homogeneous halfspace is equal to $a_0 = 0.17$. The damping ratio due to radiation, apparent at the pile head for swaying mode will be:

- Homogeneous halfspace (eq. (3.131), $A=0$, $\beta_m = 0$): $\beta_{xx} = 0.10$ (0.11 using K-PAX)
- Linear soil profile (eq.(3.145), $A=0$, $\beta_m = 0$) : $\beta_{xx} = 0.06$ (0.035 using K-PAX)
- Radially inhomogeneous soil profile (eq.(3.146), $A=0$, $\beta_m = 0$): $\beta_{xx} = 0.06$ (0.07 using K-PAX)
- Doubly inhomogeneous soil profile ($a_0 < a_{0,CUTOFF}$, $\beta_m = 0$) : $\beta_{xx} = 0$!!! (0.007 using K-PAX)

These results show clearly that the radiation damping predicted for the homogeneous halfspace is far greater than more realistic values.

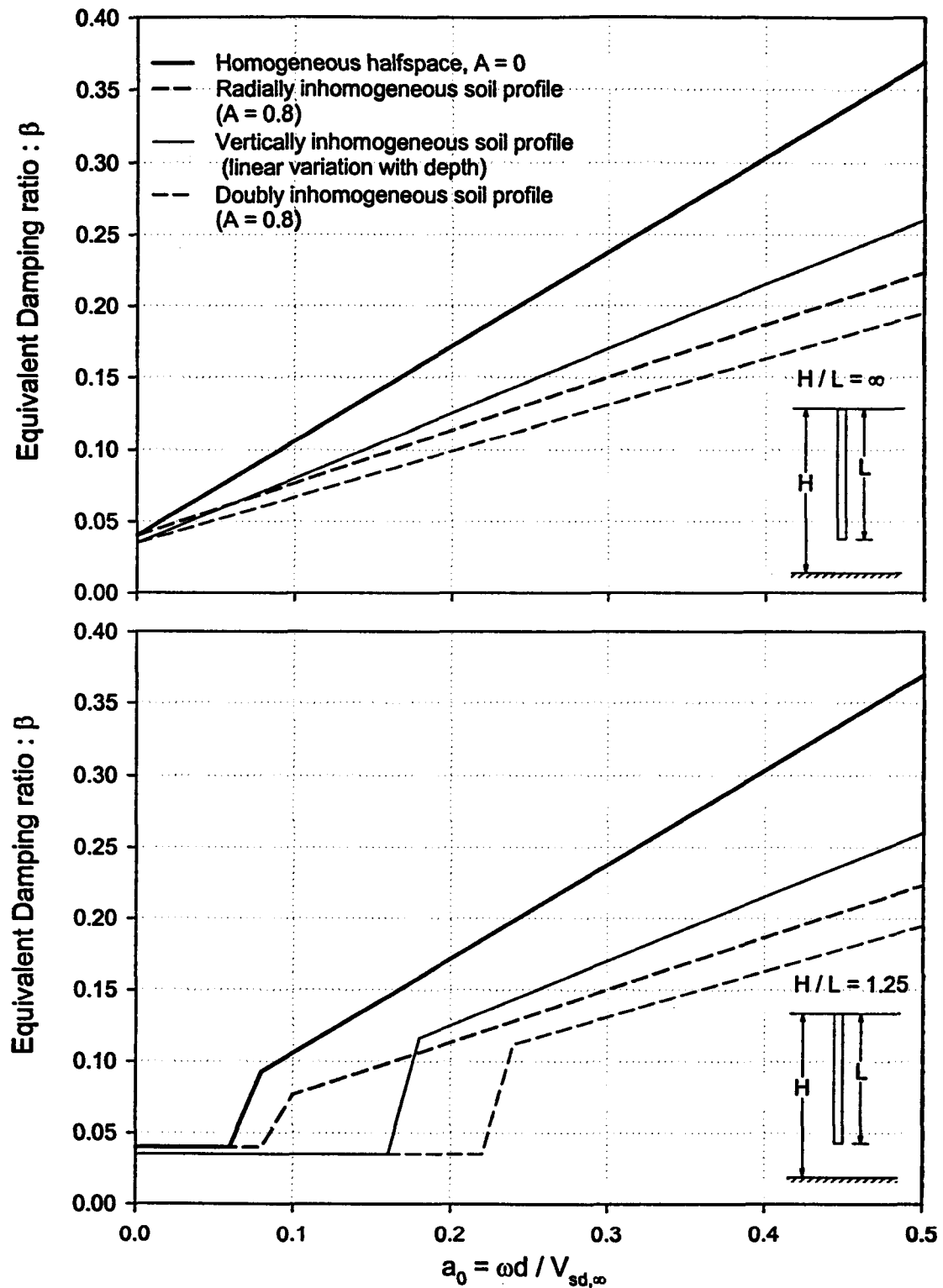


FIGURE 3.21: Equivalent damping ratio for swaying mode at pile head for "equivalent" soil profiles using the regression analysis expressions. Two different depths for the bedrock are chosen.

3.3.11 Dynamic p-y curves

The difference of dynamic p-y curves with static is that now in addition to the force and displacement, the frequency of the excitation is involved. The real part of the Winkler modulus is examined first. Plotting the real parts of the deflection (Figures 3.22 and 3.23), the soil reaction and the normalized Winkler modulus, the following trends can be observed for both fixed head and free head piles:

- Now there are negative displacements for both homogeneous and linear soil profile and as the exciting frequency increases the displacements become negative in shallower depths.
- The soil reaction is almost the same for the static and the dynamic case. This is observed for both soil profiles, both for fixed and free head piles.
- The normalized Winkler moduli follow closely each other up to a point and then, although the static modulus becomes negative, the dynamic moduli remain positive. This trend means that the new definition of the active length for the static case may not be applied in the dynamic case, since the zero soil reaction comes much later.

In figure 3.24, the real parts of the deflection, soil reaction and $\delta(z)$ are plotted for four different soil profiles with matched static stiffnesses, for a single frequency and for fixed and free head piles. It can be seen that unlike in Figure 3.6, which applies for the static case, the displacements now seem to be smaller for the radially inhomogeneous soil profiles. The soil reactions continue to be very close to each other and the Winkler modulus doesn't become infinite at the points of zero displacement because now the reactions and displacements are complex numbers.

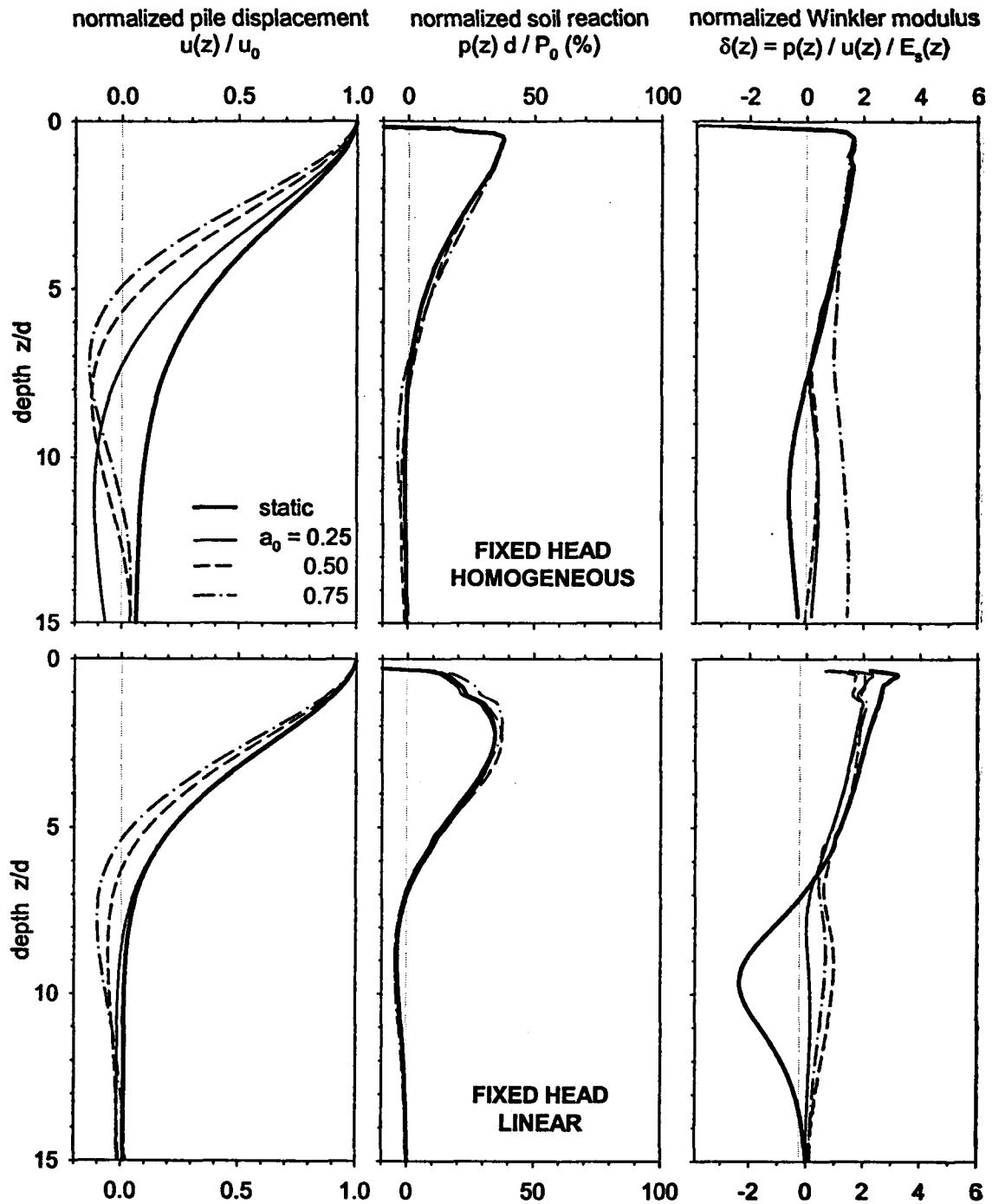


FIGURE 3.22: Deflected shape, soil reaction and normalized Winkler modulus for different excitation frequencies, for fixed-head piles of equal swaying stiffness. ($E_p/E_s=1000$, $E_p/E_{sd}=2300$)

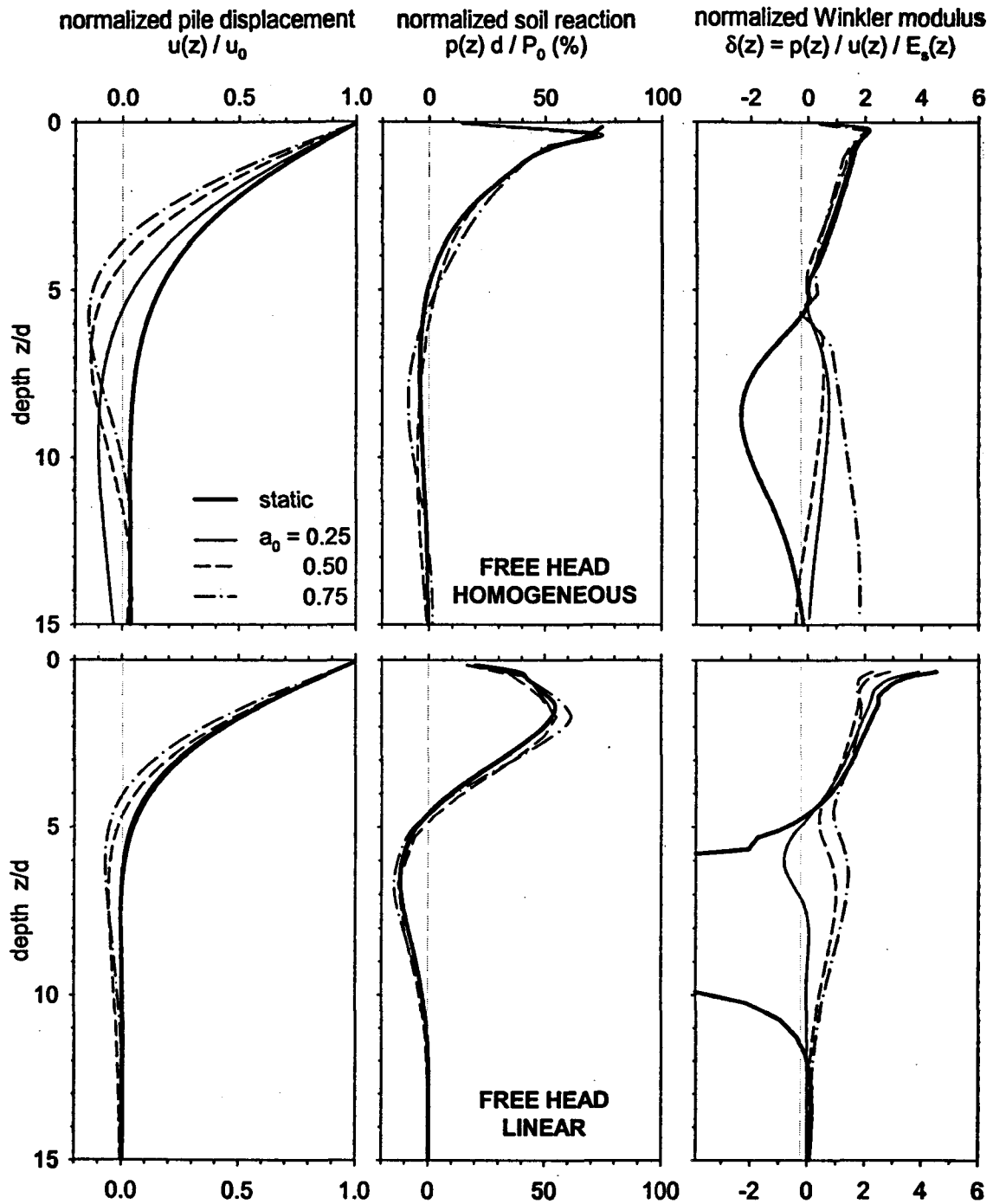


FIGURE 3.23: Real parts of deflected shape, soil reaction and normalized Winkler modulus for different excitation frequencies, for free-head piles of equal swaying stiffness in two different soil profiles. ($E_p/E_s = 1000$, $E_p/E_{sd} = 1400$)

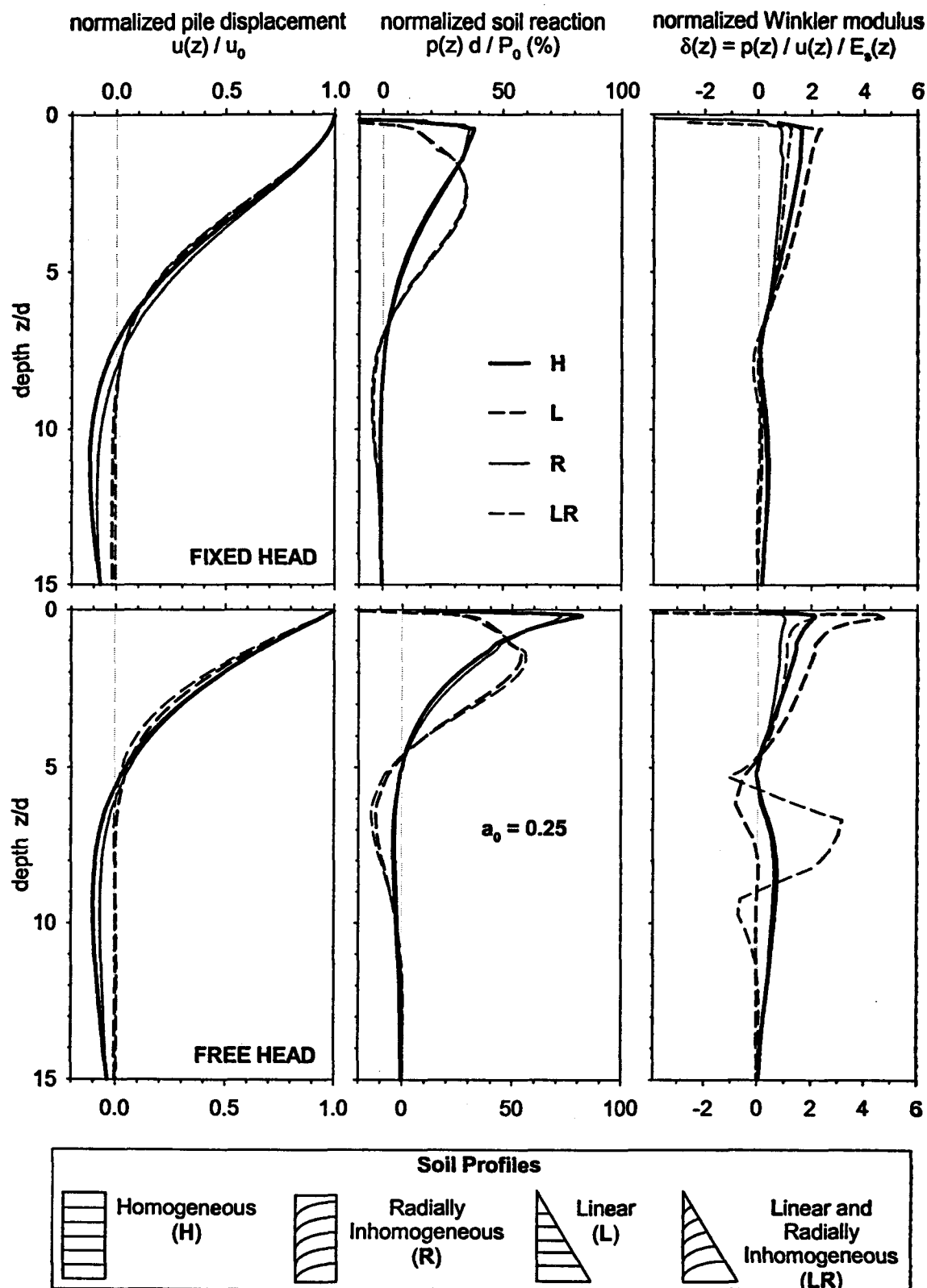


FIGURE 3.24: Deflected shape, soil reaction and normalized Winkler modulus for piles with different head constraints and of equal swaying stiffness in four different soil profiles.

In the case of the imaginary part of the Winkler modulus (dashpots), things are not straightforward. Different normalizations were tried in order to achieve the most meaningful representation (Figures 3.25 and 3.26). It can be seen that for both fixed and free head piles, the damping ratio oscillates strongly with depth, and in the case of free-head piles becomes infinity at some points. On the other hand, it should be noted that the dashpot values that control the pile response are located in the uppermost part of the pile. This can be seen from the known energy-conservation expression

$$C = \int_0^{L_p} (c_r + c_m) Y_s^2(z) dz \quad (3.148)$$

where C is the dashpot value at the head of a pile with distributed dashpots c_r (radiation damping) and c_m (material damping) along its shaft, and $Y_s(z)$ is the normalized static deflected shape. It is obvious that the higher the depth the smaller the contribution of the distributed dashpots. Observing the quick rate of drop in the pile displacements with depth (Figures 3.6, 3.22 and 3.23), it can be seen that for all practical purposes minimum 85% of the dashpot value C is attained within the first 5 diameters. These dashpot values are approximately constant in the first 5 diameters of a linear soil profile and increase linearly in the case of a homogeneous soil profile. Normalizing the imaginary part with the elastic soil modulus and the dimensionless frequency yields results that oscillate less rapidly but may be less informative. The variations are still strong and there is no obvious way to model the imaginary part. Of interest is the fact that even in the case of a homogeneous profile the fluctuations are strong.

Gazetas and Dobry (1984) proposed a relation for constant dashpots along the pile shaft, adding to the work of Berger et al. 1977, Novak et al. 1978 and Roesset 1980.

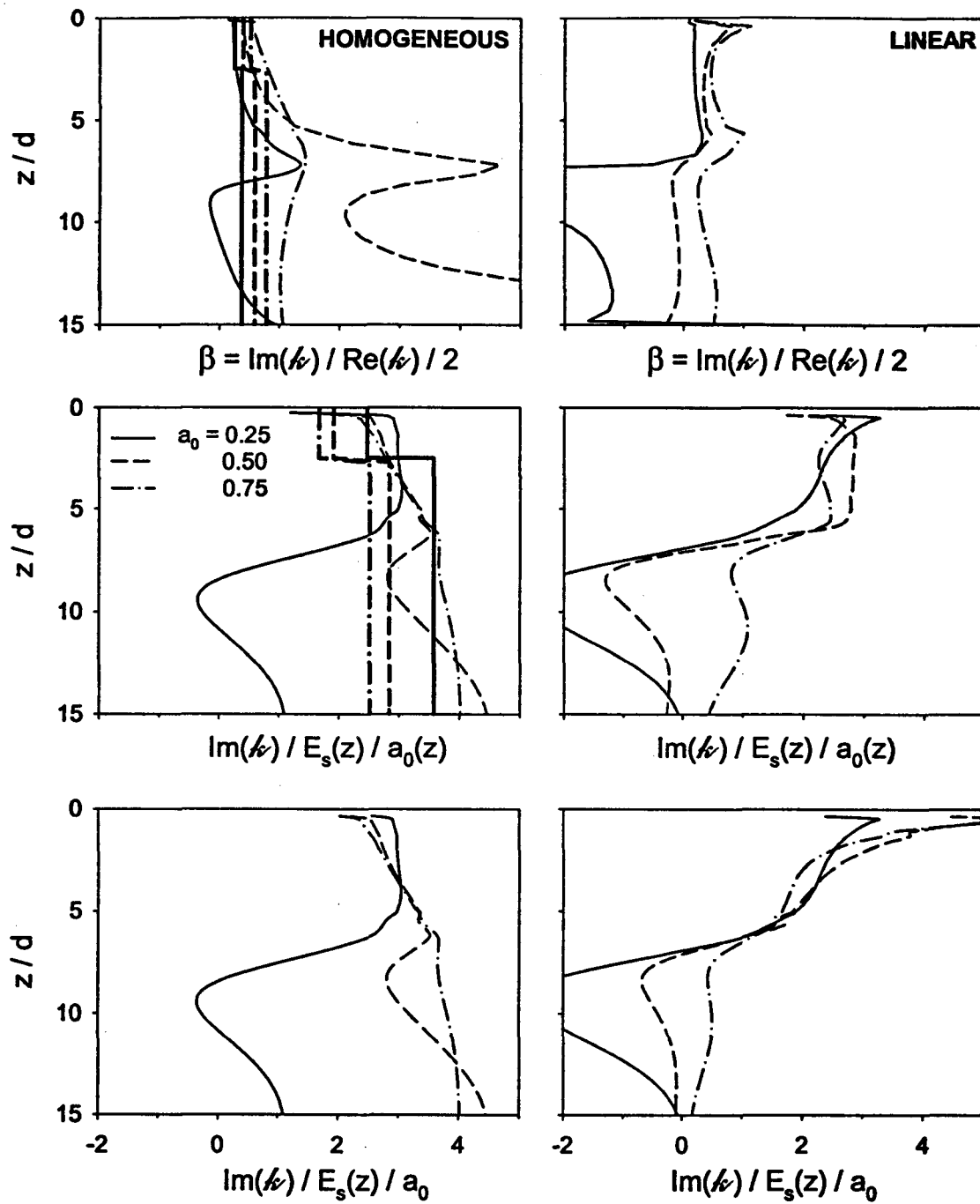


FIGURE 3.25: Various normalizations of the imaginary part of the Winkler springs along the pile shaft of a fixed-head pile embedded in different soil profiles and for selected excitation frequencies; $E_p / E_s = 1000$, $E_p / E_{sd} = 2300$

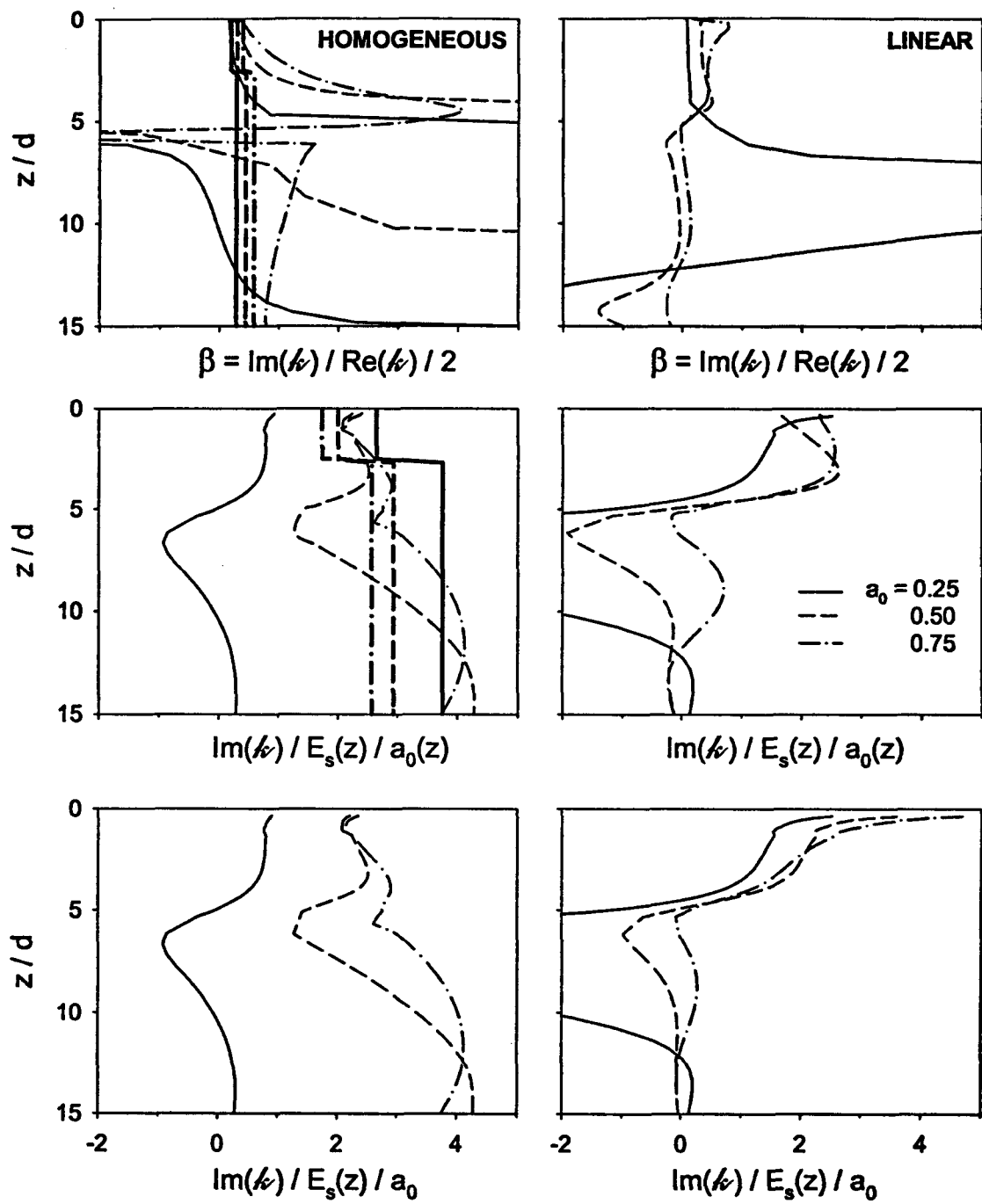


FIGURE 3.26: Various normalizations of the imaginary part of the Winkler springs along the pile shaft of a free-head pile embedded in different soil profiles and for selected excitation frequencies; $E_p / E_s = 1000$, $E_p / E_{sd} = 1400$

Gazetas and Dobry proposed the following formula for the distributed radiation damping dashpot:

$$c_r = \alpha_r \frac{E_s a_0^{3/4}}{\omega} \quad (3.149)$$

where the coefficient α_r is taken as

$$\alpha_r = \begin{cases} \left(\frac{\pi^3}{2(1+\nu)^4} \right)^{1/4} \approx 1.41, & z \leq 2.5d \\ \left(1 + \left(\frac{3.4}{\pi(1-\nu)} \right)^{5/4} \right) \left(\frac{\pi^3}{32(1+\nu)^4} \right)^{1/4} \approx 2.19, & z > 2.5d \end{cases} \quad (3.150)$$

This expression was based on a rigid, vertical, infinitely long circular inclusion, which means that the dashpot is strictly a function of the soil modulus (E_s) and not of the relative pile-soil stiffness (E_p / E_s).

To plot these values in Figures 3.25 and 3.26 two different normalizations were selected. The first is plotting the damping ratio along the pile shaft. This is given as:

$$\beta = \frac{\text{Im}(k)}{2 \text{Re}(k)} \quad (3.151)$$

where

$$\text{Im}(k) = c \omega \quad (3.152)$$

and

$$\text{Re}(k) = \delta E_s \quad (3.153)$$

The total dashpot is formed by the radiation and the material damping dashpot. The material damping dashpot is given as

$$c_m = \frac{2 \operatorname{Re}(k) \beta_m}{\omega} \quad (3.154)$$

Same for the radiation damping

$$c_r = \frac{2 \operatorname{Re}(k) \beta_r}{\omega} \quad (3.155)$$

and

$$c = c_r + c_m \quad (3.156)$$

Plugging eq. (3.154) and (3.155) into (3.156) and then into (3.151) along with (3.153) yields

$$\beta = \beta_m + \frac{c_r \omega}{2 \delta E_s} \quad (3.157)$$

For the value of δ the expressions derived in the present study will be used. Thus, for a fixed head pile in homogeneous soil eq. (3.91) will be used, and for a free-head pile eq.(3.95) will be used.

Plugging in the value of the radiation dashpot, along with the value of δ from the present study for a fixed head pile, one obtains:

$$\beta = \beta_m + \frac{\alpha_r a_0^{3/4}}{4 \left(\frac{E_p}{E_s} \right)^{-0.075}} \Rightarrow \beta = \begin{cases} \beta_m + 0.35 \left(\frac{E_p}{E_s} \right)^{0.075} a_0^{3/4}, & z \leq 2.5d \\ \beta_m + 0.54 \left(\frac{E_p}{E_s} \right)^{0.075} a_0^{3/4}, & z > 2.5d \end{cases} \quad (3.158)$$

Similarly, for free head piles,

$$\beta = \beta_m + \frac{\alpha_r a_0^{3/4}}{7 \left(\frac{E_p}{E_s} \right)^{-0.11}} \Rightarrow \beta = \begin{cases} \beta_m + 0.20 \left(\frac{E_p}{E_s} \right)^{0.11} a_0^{3/4}, & z \leq 2.5d \\ \beta_m + 0.31 \left(\frac{E_p}{E_s} \right)^{0.11} a_0^{3/4}, & z > 2.5d \end{cases} \quad (3.159)$$

These values are plotted in Figures 3.25 and 3.26 and they seem to be in general agreement with the present study's results.

Another way of normalizing the dashpots is by dividing the Winkler modulus imaginary part with the soil modulus E_s and the dimensionless frequency a_0 . This yields

$$\frac{\text{Im}(k)}{E_s a_0} = \alpha_r a_0^{-1/4} + 2 \delta \beta_m a_0^{-1} \quad (3.160)$$

and substituting the expressions for α_r and δ , one obtains:

for fixed head pile

$$\frac{\text{Im}(k)}{E_s a_0} = \begin{cases} 1.41 a_0^{-1/4} + 4 \left(\frac{E_p}{E_s} \right)^{-0.075} \beta_m a_0^{-1}, & z \leq 2.5d \\ 2.19 a_0^{-1/4} + 4 \left(\frac{E_p}{E_s} \right)^{-0.075} \beta_m a_0^{-1}, & z > 2.5d \end{cases} \quad (3.161)$$

and for free head

$$\frac{\text{Im}(k)}{E_s a_0} = \begin{cases} 1.41 a_0^{-1/4} + 7 \left(\frac{E_p}{E_s} \right)^{-0.11} \beta_m a_0^{-1}, & z \leq 2.5d \\ 2.19 a_0^{-1/4} + 7 \left(\frac{E_p}{E_s} \right)^{-0.11} \beta_m a_0^{-1}, & z > 2.5d \end{cases} \quad (3.162)$$

It can be clearly seen that even though the variation of the Winkler dashpots with depth is very strong, only the top few diameters actually define the pile's response.

In Figures 3.27 and 3.28, the real part of the Winkler springs and the Winkler damping ratios are plotted as they vary within the first 5 diameters. Figure 3.27 depicts these

variations for four different, but equivalent, soil profiles, for three different frequencies for the case for a fixed head pile. The following trends can be identified:

- It can be seen that the pile response in the profiles with radial inhomogeneity has the same variation but with smaller amplitudes with the response in the profiles without radial inhomogeneity. This trend is apparent for all frequencies.
- The real part of the Winkler springs has a maximum value near the top, as was seen in the static case, and drops linearly with depth. As the frequency increases, the real part tends to become constant for the case of the vertically homogeneous soil profiles.
- Observing the variation of the damping ratio, it increases linearly with depth for low frequencies, but in high frequencies it the damping ratio seems to oscillate rapidly, making things unclear. The only clear trend, as in the case of the real part, is that the pile response in the soil profiles with radial inhomogeneity has the same variation with depth, with smaller amplitudes like the pile response in the radially homogeneous soil profiles. This is a clear indication that the pile response due to radial inhomogeneity can be reproduced from the pile response in a radially homogeneous medium by adjusting the amplitudes.
- The trends observed for the case of a fixed-head pile also appear in the response of a free-head pile (Figure 3.28).

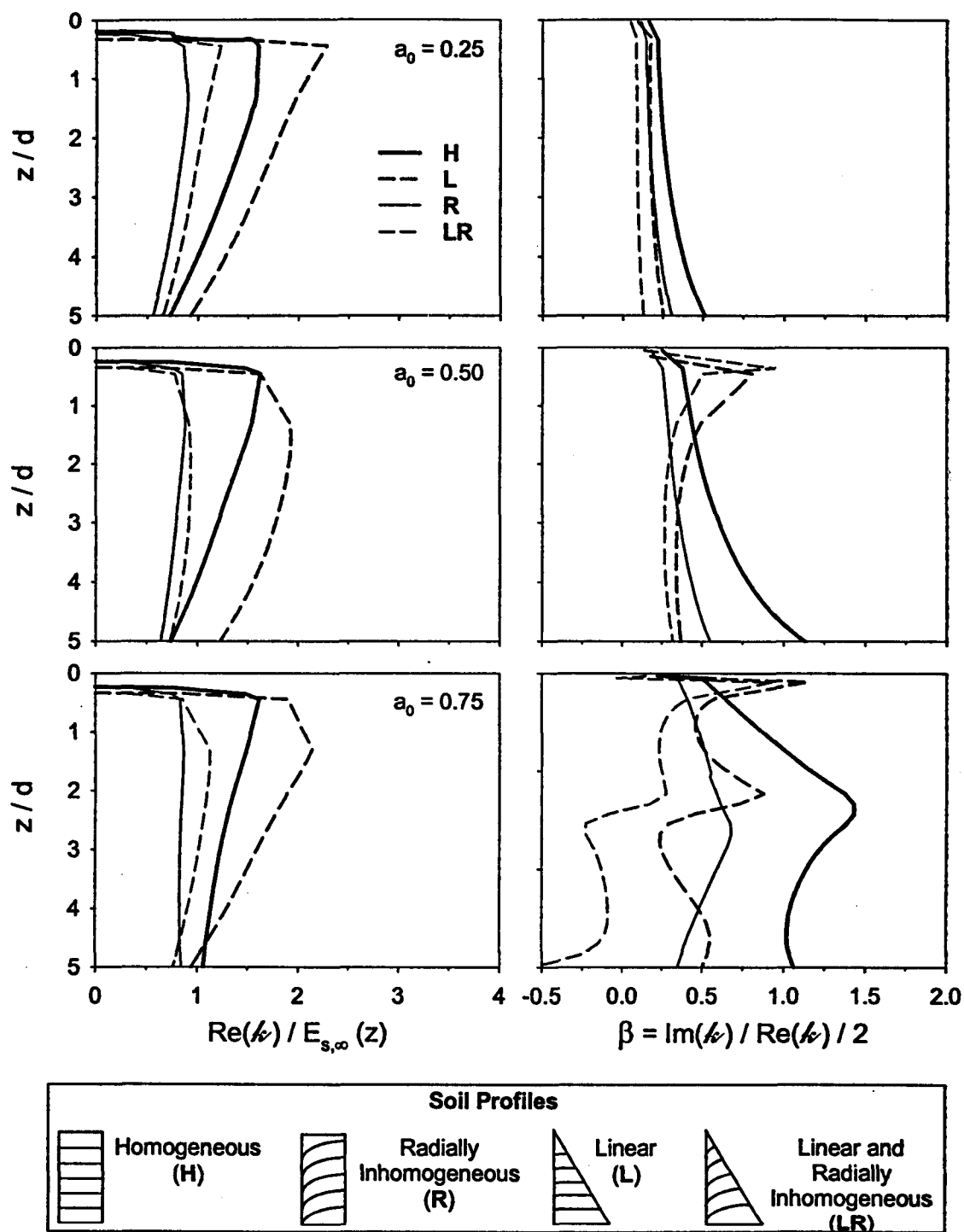


FIGURE 3.27: Normalized real part and damping ratio of the Winkler springs along the pile shaft of fixed-head piles embedded in 4 different soil profiles and for selected excitation frequencies. All piles have matched swaying stiffness.

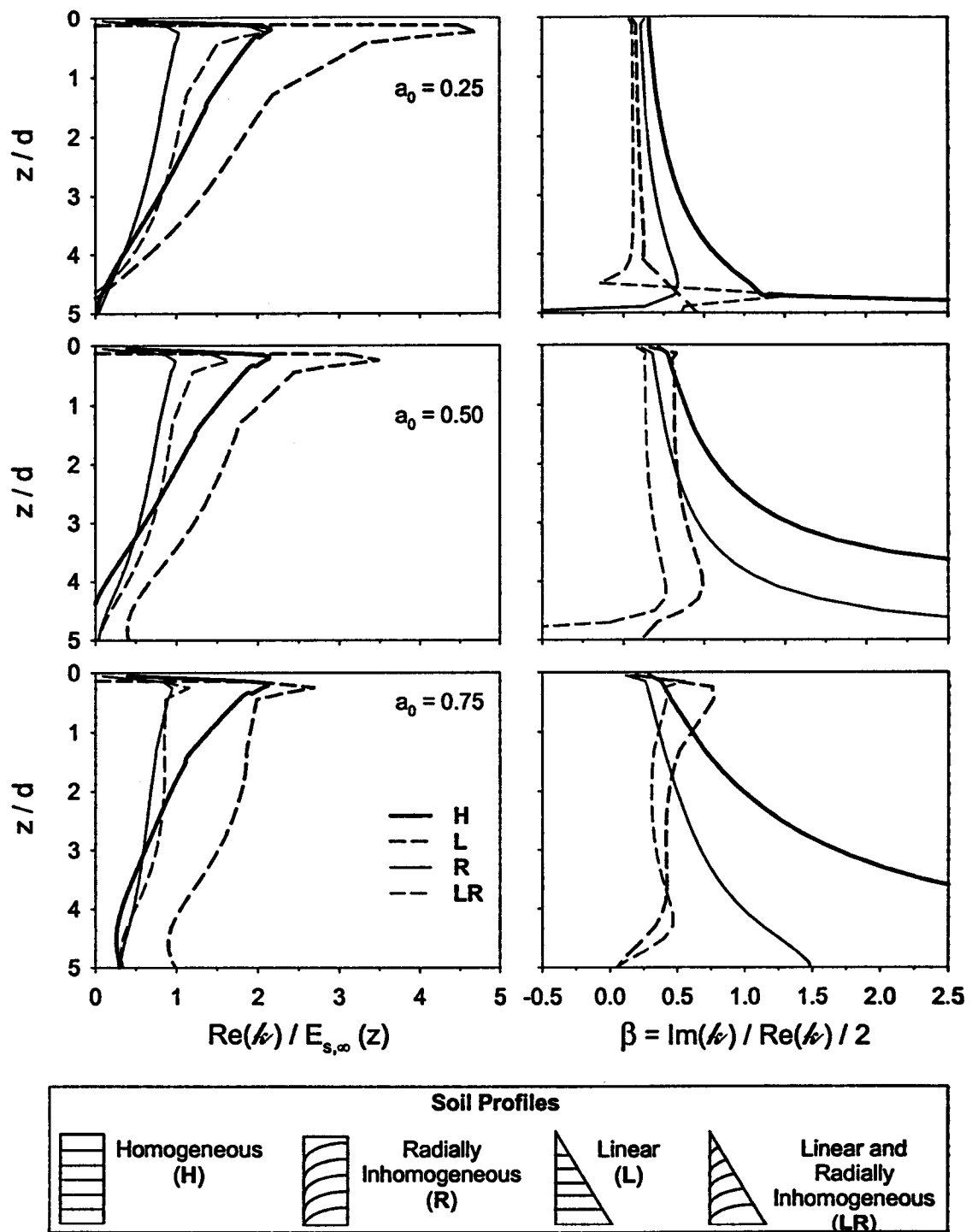


FIGURE 3.28: Normalized real part and damping ratio of the Winkler springs along the pile shaft of free-head piles embedded in 4 different soil profiles and for selected excitation frequencies. All piles have matched swaying stiffness.

3.4 PILE GROUP

As in Chapter 2, a pile group can be analyzed with the present method by either considering concentric rings of densely-spaced piles (axisymmetric group), or using the superposition method. The first method is straightforward and needs no further explanation. With reference to the second method, the impedance of the group is obtained by: (i) determining the three impedances (i.e., swaying, rocking and cross-swaying-rocking) of a single pile; (ii) determining the interaction factors between any two piles in the group; and (iii) combining these interaction factors to determine the impedance of the whole group.

In the lateral problem, translation and rocking are coupled so the interaction factor is expressed by a 2x2 complex matrix

$$[\alpha] = [\alpha(s, \theta)] = \begin{bmatrix} \alpha_{uP} & \alpha_{uM} \\ \alpha_{\phi P} & \alpha_{\phi M} \end{bmatrix} \quad (3.163)$$

where α_{uP} = swaying interaction factor; $\alpha_{\phi M}$ = rocking interaction factor; α_{uM} and $\alpha_{\phi P}$ = cross-swaying-rocking factors; s = axis to axis distance between the piles; and θ = "aperture" angle between the direction of loading and the line connecting the pile centers on a horizontal plane.

Following Mylonakis and Gazetas (1999), the interaction factor can be cast in the following form:

$$\begin{bmatrix} \alpha_{uP} & \alpha_{uM} \\ \alpha_{\phi P} & \alpha_{\phi M} \end{bmatrix} = \psi(s, \theta, \omega) [\zeta] \quad (3.164)$$

where $\psi = \psi(s, \theta, \omega)$ denotes the attenuation function of the horizontal soil displacement with radial distance from the pile and direction of loading. This function is defined as (Kaynia & Kausel, 1982)

$$\psi(s, \theta, \omega) \equiv \frac{U(s, \theta, z)}{U(d/2, \theta, z)} \approx \psi(s, 0, \omega) \cos^2 \theta + \psi(s, \pi/2, \omega) \sin^2 \theta \quad (3.165)$$

The second term, ζ , accounts for the effect of diffraction of the wave field due to the rigidity of the receiver pile and the interaction between pile and soil. ζ is expressed by the following 2×2 matrix (Appendix C).

$$[\zeta] = \begin{bmatrix} \zeta_{uP} & \zeta_{uM} \\ \zeta_{\phi P} & \zeta_{\phi M} \end{bmatrix} \quad (3.166)$$

Analytical expressions for both terms have been developed using analytical elastodynamic solutions (e.g., Mylonakis, 1995; Mylonakis & Gazetas, 1999). The F.E. formulation described above can be used to determine the above coefficients in a rigorous manner.

CHAPTER 4

VALIDATION OF F.E. MODEL USING EXPERIMENTAL RESULTS

4.1 INTRODUCTION

The objective of this chapter is to compare the numerical results obtained from the F.E. analysis against published experimental results. For this purpose, seven well-documented experiments on single piles and pile groups are selected to be numerically simulated using K-PAX. This chapter combines the results from Chapters 2 and 3 and demonstrates how to solve the foundation impedance problem, which is a crucial step in solving the complete SSI problem. In the first part of the chapter, three tests on single piles are analyzed. It is found that accounting for radial inhomogeneity in the soil yields a pile response which is closer to the actual response. In the second part, the response of four pile groups is examined. Analytical results are found to agree satisfactorily with measurements.

4.2 VERTICAL RESPONSE OF MODEL STEEL PILE (EL-MARSAFAWI ET AL 1992)

El-Marsafawi et al (1992) investigated the response of a single model steel pile by attaching a rigid mass on the pile head and exciting the system harmonically. The objective was to find the resonant frequency and determine the variation of the response with frequency. At the same site, the authors also constructed a pile group and used the

same equipment to document its response to harmonic loading. The response of this group is analyzed later in this chapter.

4.2.1 Test pile foundation and site

The piles used in this experiment (single pile and pile group) were made of hollow steel pipes with an outer diameter of 0.1016 m and a wall thickness of 0.00635 m. The embedded length of the piles was 2.75 m corresponding to a slenderness ratio $L / d = 27$. The pile cap used for single pile had a mass of 0.941 Mg; for the pile group the mass was 1.874 Mg.

The experiment took place at the campus of the University of Western Ontario; the soil at the site consists of a layered stratum of silty fine sand with a gravel seam and silty till. The stratigraphy and material properties of the profile are shown in Figure 4.1. It should be mentioned that the shear wave velocities were measured using the cross-hole technique and the steady-state vibration technique.

4.2.2 Test Description

The pile-mass system was excited vertically using a mechanical oscillator that generates forces proportional to the square of the excitation frequency (see eq. 4.1). The oscillator was driven by means of a flexible shaft and motor equipped with a Kopp variator which yields stable operating speeds between 6 and 62 Hz. To get the displacement measurements, four velocity transducers were used, with each pair mounted vertically at equal distances from the foundation center. The measured displacements

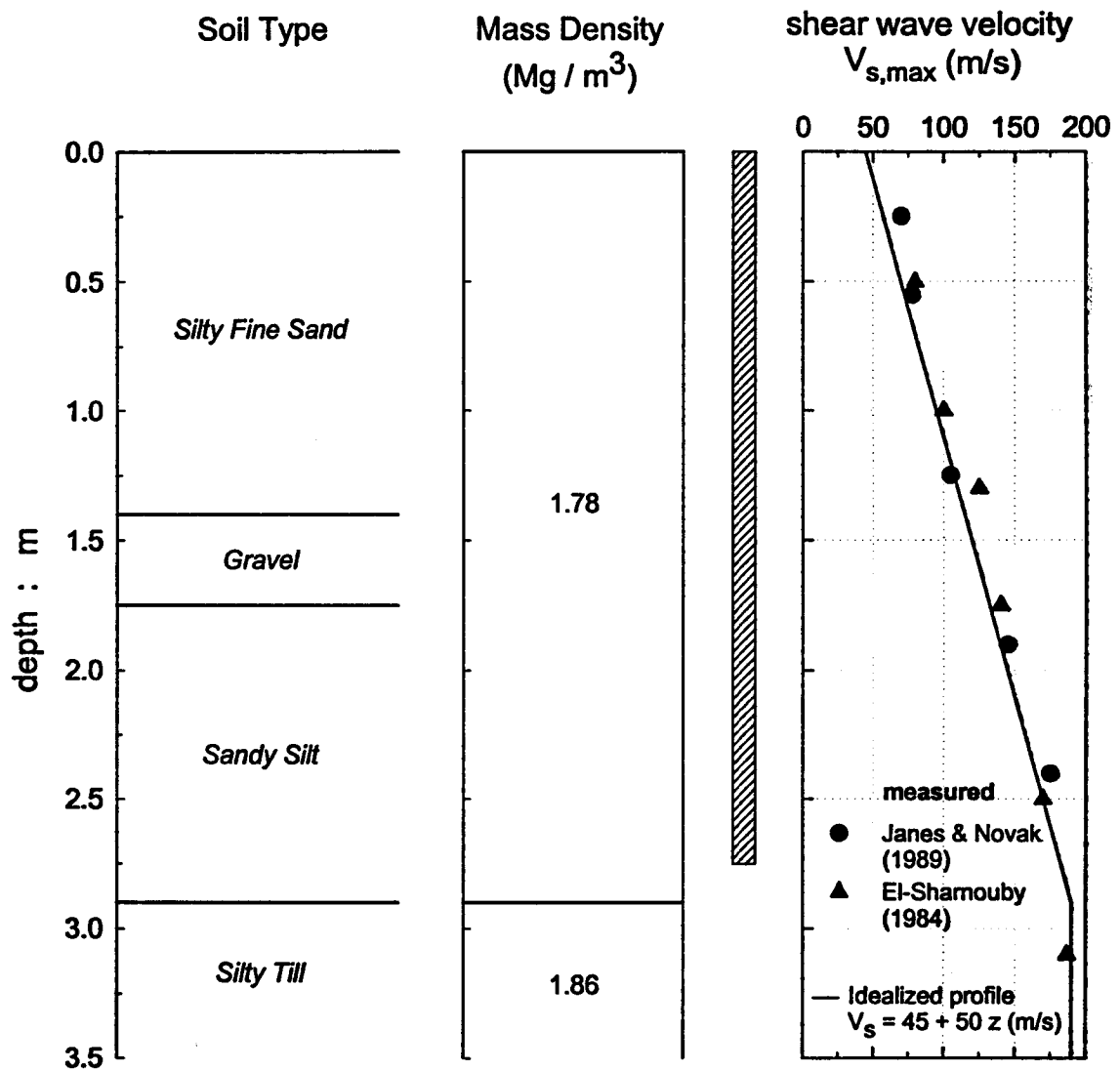


FIGURE 4.1: Soil properties at University of Western Ontario test site (El-Marsafawi et al 1992)

were then averaged to eliminate the rocking component. For more details, the reader is referred to El-Marsafawi et al (1992).

4.2.3 Representation of results

The excitation force of the system is described by

$$F(t) = (m_e e) \omega^2 \cos(\omega t) \quad (4.1)$$

in which the quantity $m_e e$ represents the excitation intensity, with m_e being the accelerating mass and e the eccentricity.

It is understood that in a linear elastic system the stiffness is not affected by the magnitude of the applied force. In reality, however, the excitation amplitude affects stiffness, thus experimental results using different force amplitudes are not identical. The measured displacement (u_z) is normalized with the pile cap mass (m_{cap}) and the “excitation intensity” $m_e e$:

$$\mathcal{D}_z = \left(\frac{m_{cap}}{m_e e} \right) u_z \quad (4.2)$$

In the realm of elastodynamic theory, the normalized amplitude $\mathcal{D}_z$ is insensitive to excitation intensity.

4.2.4 Analysis of single pile vertical response

4.2.4.1 Rigorous analysis (F.E.M.)

Figure 4.2 shows the impedance curves for the single pile as calculated using K-PAX. In some of the analyses it is necessary to assume that there is gap opening near the pile head (Guin 1997). As an approximate solution, instead of setting soil modulus equal to zero along the gap region, it was assumed that the soil modulus starts from zero at the soil surface and varies linearly with depth (Gibson soil). This way it is possible to make use of the analytical tools developed for the “linear” soil profile (Chapter 2). To account for radial soil inhomogeneity, a value of 0.8 was assigned to the inhomogeneity factor A . This choice was made empirically, based on the reduced near-pile soil modulus adopted in other studies (El-Marsafawi et al 1992, Han 1997, Guin 1997). As will be shown in the subsequent sections, the selected value of 0.8 yields satisfactory comparisons for a wide variety of soil profiles and load amplitudes.

The vertical displacement of the cap (u_z) can be calculated using the following well-known relation:

$$u_z(\omega) = \frac{F(\omega)}{\left[\left(\text{Re}(\mathcal{H}_z) - m_{\text{cap}} \omega^2 \right) + i \text{Im}(\mathcal{H}_z) \right]} \quad (4.3)$$

in which $F(\omega) = (m_e e) \omega^2$ denotes the amplitude of the applied force and $\mathcal{H}_z$ the complex dynamic impedance (stiffness and damping) of the pile.

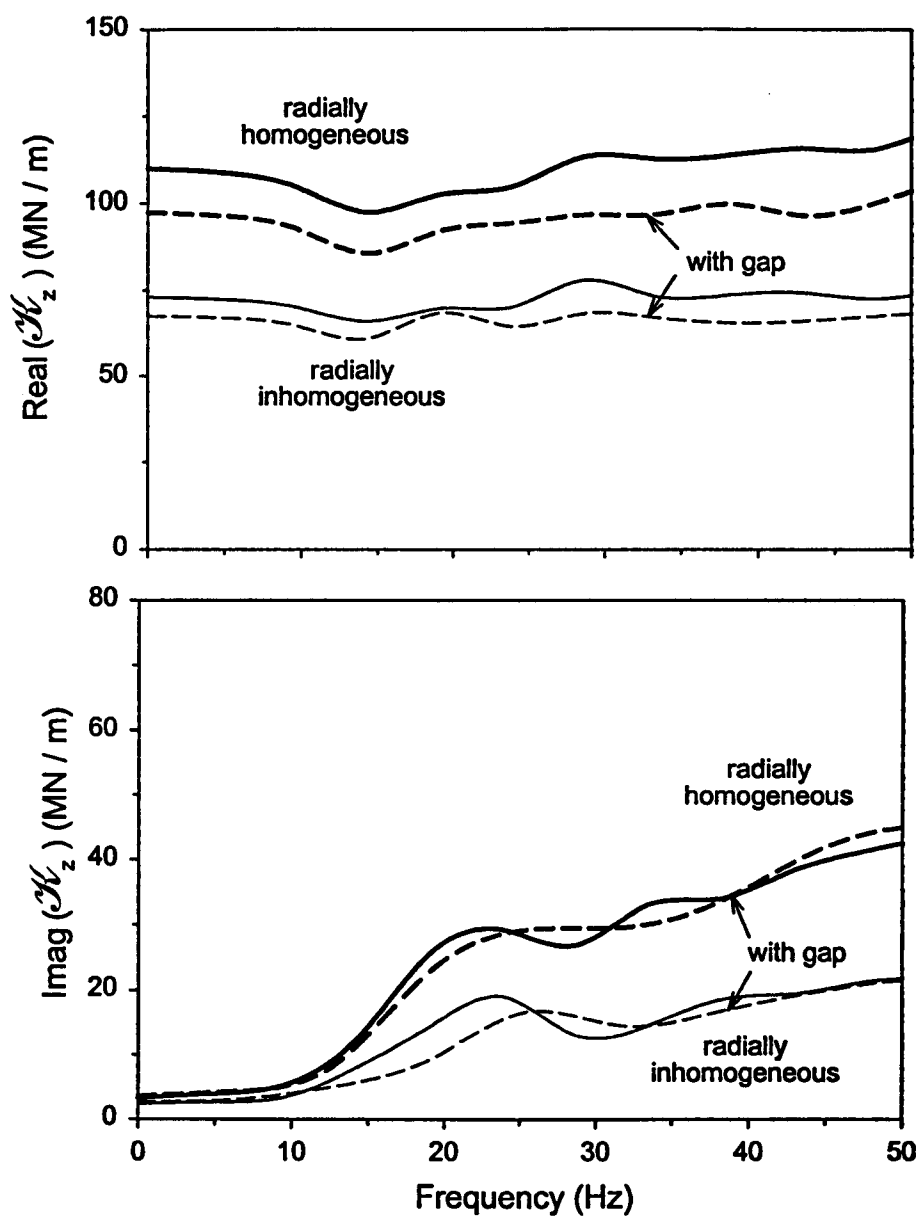


FIGURE 4.2: Real and imaginary parts of pile head stiffness for the tested single steel pipe (El-Marsafawi et al 1992).

Figure 4.3 refers to the adjusted soil profile. It is obvious that the measured response has a resonant frequency lower than that predicted for the radially homogeneous profile and also a much higher amplitude. On the other hand, considering radial inhomogeneity, the resonant frequency and amplitude are closer to the observed. It can also be seen that the analysis overestimates damping by as much as 100% despite the small material damping considered ($\beta_s = 3.5\%$). These findings are in agreement with those by Novak and co-workers (El-Marsafawi et al, 1992).

4.2.4.2 Simplified analysis (Analytical formulas)

In this section, a simplified solution is attempted using the analytical formulas developed in Chapter 2. The soil profile is considered to be “linear” (Gibson soil), with the soil shear modulus being zero at the surface and increasing linearly with depth up to 2.9 m. Below that depth, the shear modulus is assumed to be constant. In addition, the actual hollow pile is substituted with an “equivalent” solid pile since the analytical formulations pertain to solid piles. The soil-pile parameters used in this analysis are summarized in Table 4.1.

The modulus of elasticity and mass modulus of the substitute solid pile having the same diameter and axial rigidity ($E_p A$) as the hollow pile, were determined at 4.69×10^7 kpa and $1,842 \text{ Mg/m}^3$ respectively. The corresponding pile-soil elasticity ratio and pile length to diameter ratio are $E_p / E_{sL} = 295$ and $L / d = 27$.

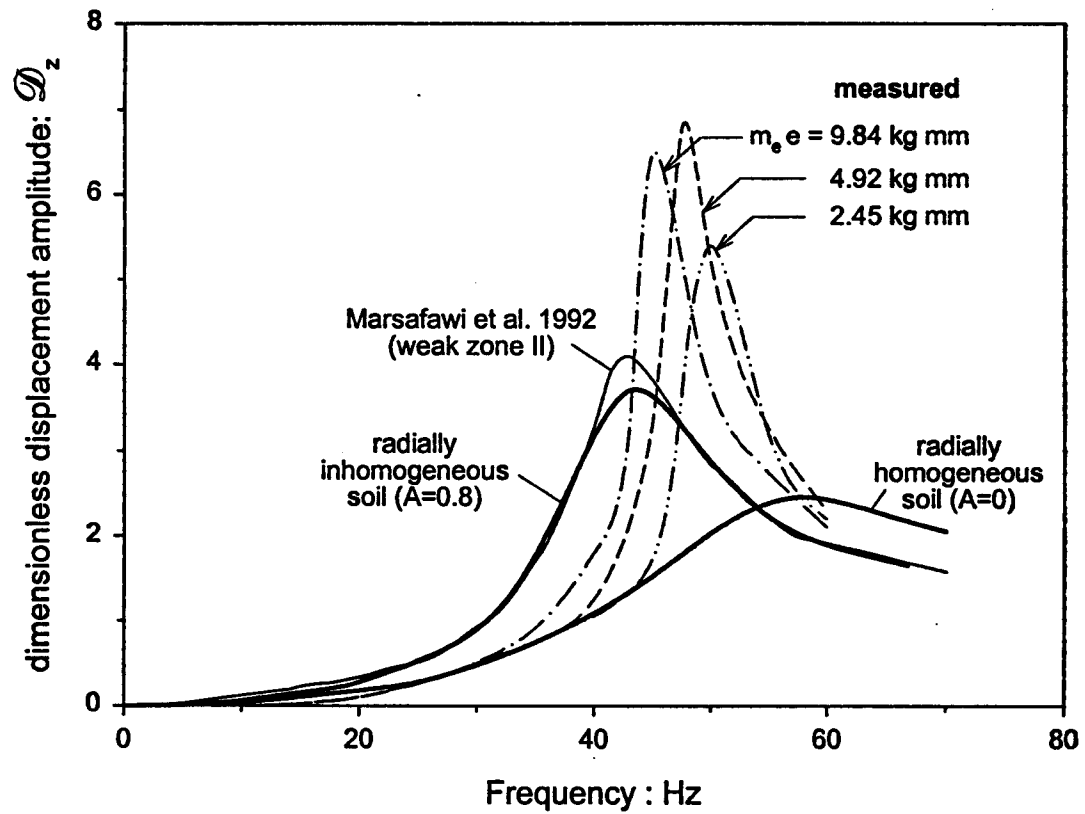


FIGURE 4.3: Comparison of measured and predicted vertical response of a single steel pipe (El-Marsafawi et al 1992). Note the overprediction of natural frequency assuming radially homogeneous soil and the improved prediction assuming radial inhomogeneity. The overprediction of damping in both cases is evident.

TABLE 4.1: Soil, pile and SSI parameters used in the analysis (vertical response of model steel pipe)

SOIL	PILE	SSI PARAMETERS
$V_{s1} = 45 + 50 z \text{ m/s}$	$E_p = 2.0 \cdot 10^8 \text{ kPa}$	$E_{sL} = 158,900 \text{ kPa}$
$V_{s2} = 190 \text{ m/s}$	$L = 2.75 \text{ m}$	$\bar{E}_p = 4.7 \cdot 10^7 \text{ kPa}$
$\rho_{s1} = 1.78 \text{ Mg/m}^3$	$d_o = 0.1016 \text{ m}$	$\bar{\rho}_p = 1.84 \text{ Mg/m}^3$
$\rho_{s2} = 1.86 \text{ Mg/m}^3$	$t = 0.00635 \text{ m}$	$\frac{\bar{E}_p}{E_{sL}} = 295$
$\beta_s = 3.5\%$	$\rho_p = 7.86 \text{ Mg/m}^3$	$\frac{L}{d} = 27.5$
$\nu_s = 0.3$	$\beta_p = 1\%$	
	$\nu_p = 0.25$	

Using equation 2.40, the static stiffness at the pile head is found to be 90,460 kN / m, 8% lower than the estimate from the finite element solution. Accounting for radial soil inhomogeneity by using an inhomogeneity factor $A=0.8$ results to a reduced vertical static stiffness of 66,930 kN / m. The latter value differs by 1% from the more rigorous prediction of K-PAX.

The real part of the complex dynamic stiffness of the pile doesn't vary significantly with frequency, as observed in Figure 4.2. Thus, it was taken to be approximately equal to the static stiffness at all frequencies. The imaginary part of the complex dynamic stiffness can be calculated as (Gazetas 1991):

$$C_z = \frac{2}{3} a_0^{1/3} \rho V_{sL} \pi d L r_d \quad (4.4)$$

where

$$r_d = 1 - \exp \left[-2 \left(\frac{E_p}{E_{sL}} \right) \left(\frac{L}{d} \right)^2 \right] \quad (4.5)$$

and

$$a_0 = \frac{\omega d}{V_{sL}} \quad (4.6)$$

is the corresponding dimensionless frequency. V_{sL} is the shear wave velocity of the soil at the elevation of the pile tip (L). Carrying out the calculations one obtains

$$C_z = 1297 \omega^{-1/3} \text{ kN s /m} \quad (4.7)$$

The dynamic impedance of the pile is thus expressed as:

$$\mathcal{H}_z(\omega) = 90,460 + i 1297 \omega^{2/3} \text{ KN/m} \quad (4.8)$$

To account for radial inhomogeneity in the soil, the previous expression is modified as

$$\mathcal{H}_z(\omega) = 66,930 + i \alpha 1297 \omega^{2/3} \text{ KN/m} \quad (4.9)$$

where α is a dimensionless factor accounting for the reduction in radiation damping due to inhomogeneity. The results above are concentrated in Table 4.2.

TABLE 4.2: Static and dynamic pile-head stiffness for the single steel pile

SOIL TYPE	Analysis Procedure	Pile stiffness (kN/m)	Equation
RADIALLY HOMOGENEOUS A = 0	K-PAX	$K_{z,static} = 98,800$	-
	ANALYTICAL	$K_{z,static} = 90,500$	2.40
		$\mathcal{H}_z(\omega) = 90,500 + (1,300 \omega^{2/3}) i$	4.8
RADIALLY INHOMOGENEOUS A = 0.8	K-PAX	$K_{z,static} = 66,100$	-
	ANALYTICAL	$K_{z,static} = 66,900$	2.40
		$\mathcal{H}_z(\omega) = 66,900 + (1,300 \alpha \omega^{2/3}) i$	4.9

From Figure 4.4, it appears that analytical expressions overestimate damping. Using a trial-and-error procedure it was determined that the estimated damping should be decreased 1.75 times in the case of radially homogeneous soil. For radially inhomogeneous soil, damping should be decreased 3 times. The bigger reduction was expected since the dashpot used corresponds to a radially homogeneous profile, which yields higher values (as was discussed in Chapter 2). It should be noted that the stiffness calculated using the analytical formulas is 10 percent less than the one calculated with finite elements. This suggests that the assumed soil modulus is somewhat lower than the modulus that would yield the correct (F.E.M.) pile-head stiffness.

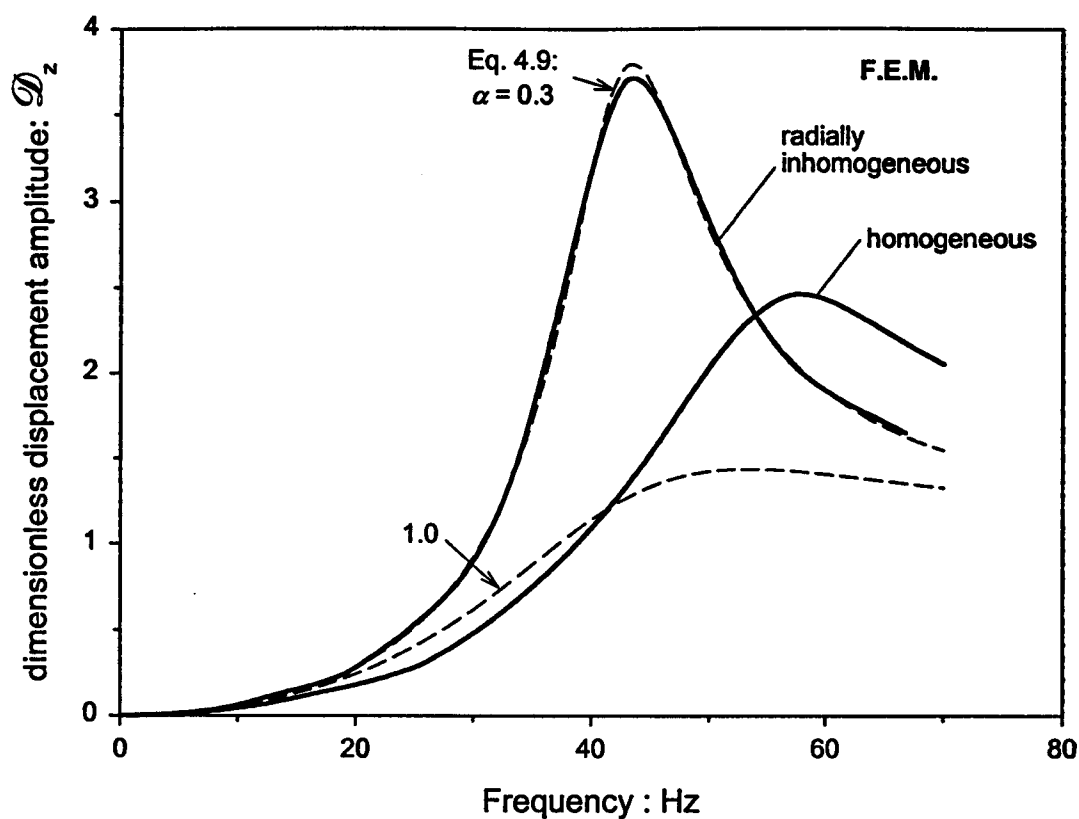


FIGURE 4.4: Comparison of predicted vertical response of a single steel pipe using finite elements and analytical relations.

As demonstrated in Chapter 2, softer soils tend to radiate more energy, thus having higher radiation damping. This is one of the reasons for the increased damping observed. It will be shown later that the opposite occurs in Figure 4.4 if soil stiffness is overestimated.

4.3 HORIZONTAL RESPONSE OF A FULL-SCALE CONCRETE PILE (EL-MARSAFAWI ET AL 1992)

El-Marsafawi et al (1992) also investigated the response of a single concrete pile by attaching a rigid mass on the top of the pile and exciting it harmonically to determine the natural frequency and damping. The response of a corresponding pile group is analyzed later in this chapter.

4.3.1 Test pile foundation and site

The piles used were cast-in-place reinforced concrete piles with a diameter of 0.32 m and a length of 7.5 m ($L / d = 23.4$). The pile cap had a mass of 0.86 Mg (single pile), including the mass of the exciter, and 5.12 Mg (pile group).

The experiment took place at the pile research site at the Institute of Engineering Mechanics, Harbin, China. The soil at the site consisted of a relatively homogeneous sandy clay with yellow and brown coloring. The properties of the profile are shown in Figure 4.5.

4.3.2 Test Description

An exciter with two counter-rotating eccentric masses was fixed in the cap to generate a horizontal harmonic excitation. The exciter produced forces proportional to the square of the excitation frequency. The horizontal displacements were measured using two pickups mounted horizontally at the foundation surface. The rotations about x-axis were measured using two pickups mounted vertically on the axis of symmetry, y (Figure 4.13).

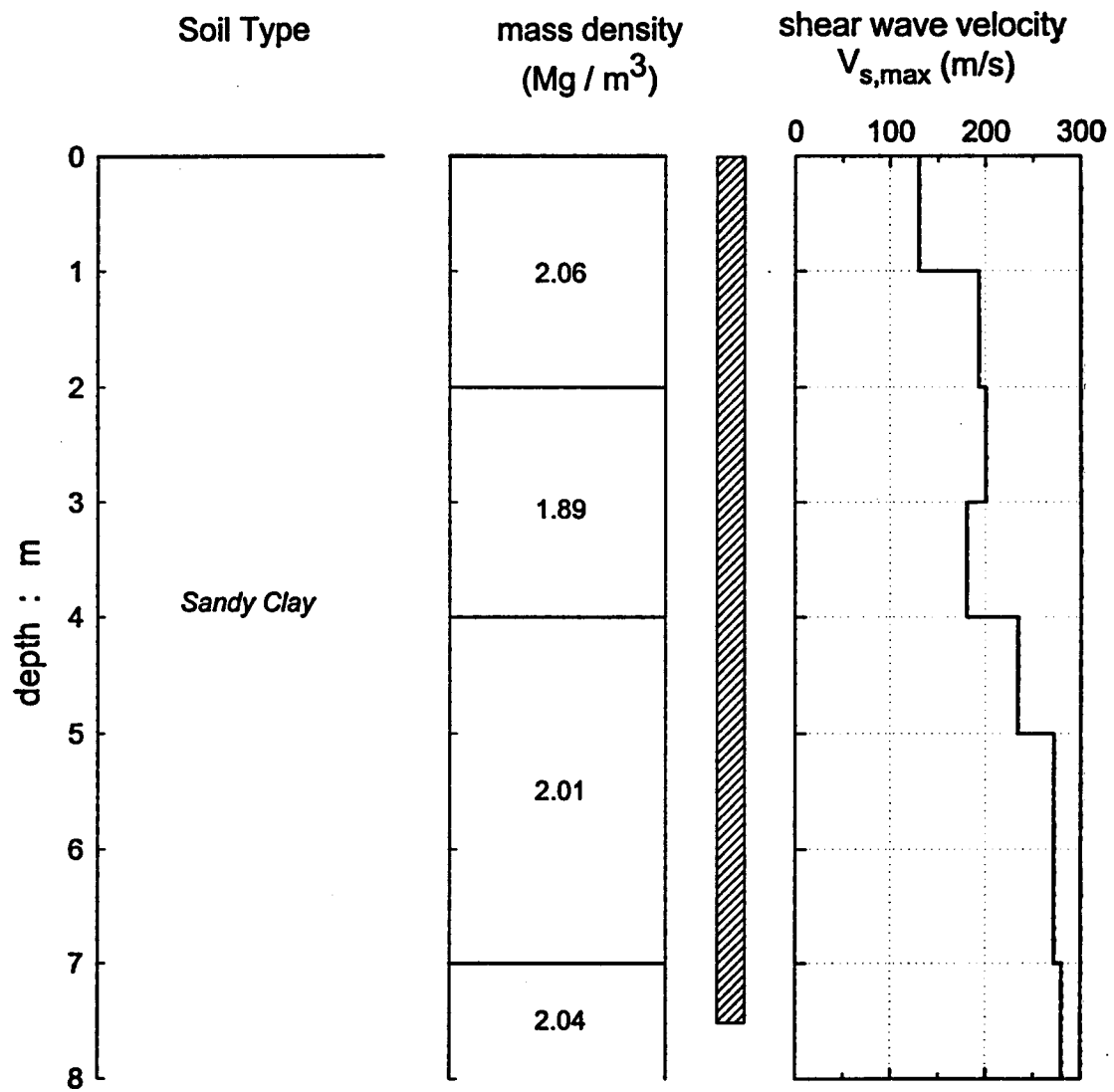


FIGURE 4.5: Soil properties at Harbin test site, China (El-Marsafawi et al 1992)

4.3.3 Representation of results

The excitation of the system is described by equation (4.1). The measured horizontal displacement is normalized with the pile cap mass and the excitation intensity $\mathcal{D}_x$ (eq. 4.2).

The measured rotation (φ) is normalized with the mass moment of inertia, I_o , of the pile cap-exciter system, the “excitation intensity” ($m_e e$) and the moment arm, Z_e , of the horizontal load above the center of gravity:

$$\mathcal{D}_r = \left[\frac{I_o}{(m_e e Z_e)} \right] \varphi \quad (4.10)$$

where $I_o = 0.095 \text{ Mg m}^2$, $Z_e = 0.3 \text{ m}$ (El-Marsafawi et al 1992).

4.3.4 Analysis of single pile

4.3.4.1 Rigorous analysis (F.E.M.)

Figure 4.6 shows the impedance curves for the single pile as calculated using K-PAX. Following Guin (1997) in some of the analyses it was assumed that a gap exists near the pile head extending 0.2 m below the surface. As in the previous simulation, this was accounted for, approximately, by reducing the Young’s modulus of the soil near the surface. To account for radial inhomogeneity, a value of A equal to 0.8 was adopted.

The horizontal displacement and rotation of the cap at the ground surface level can be determined by solving the following system of algebraic equations:

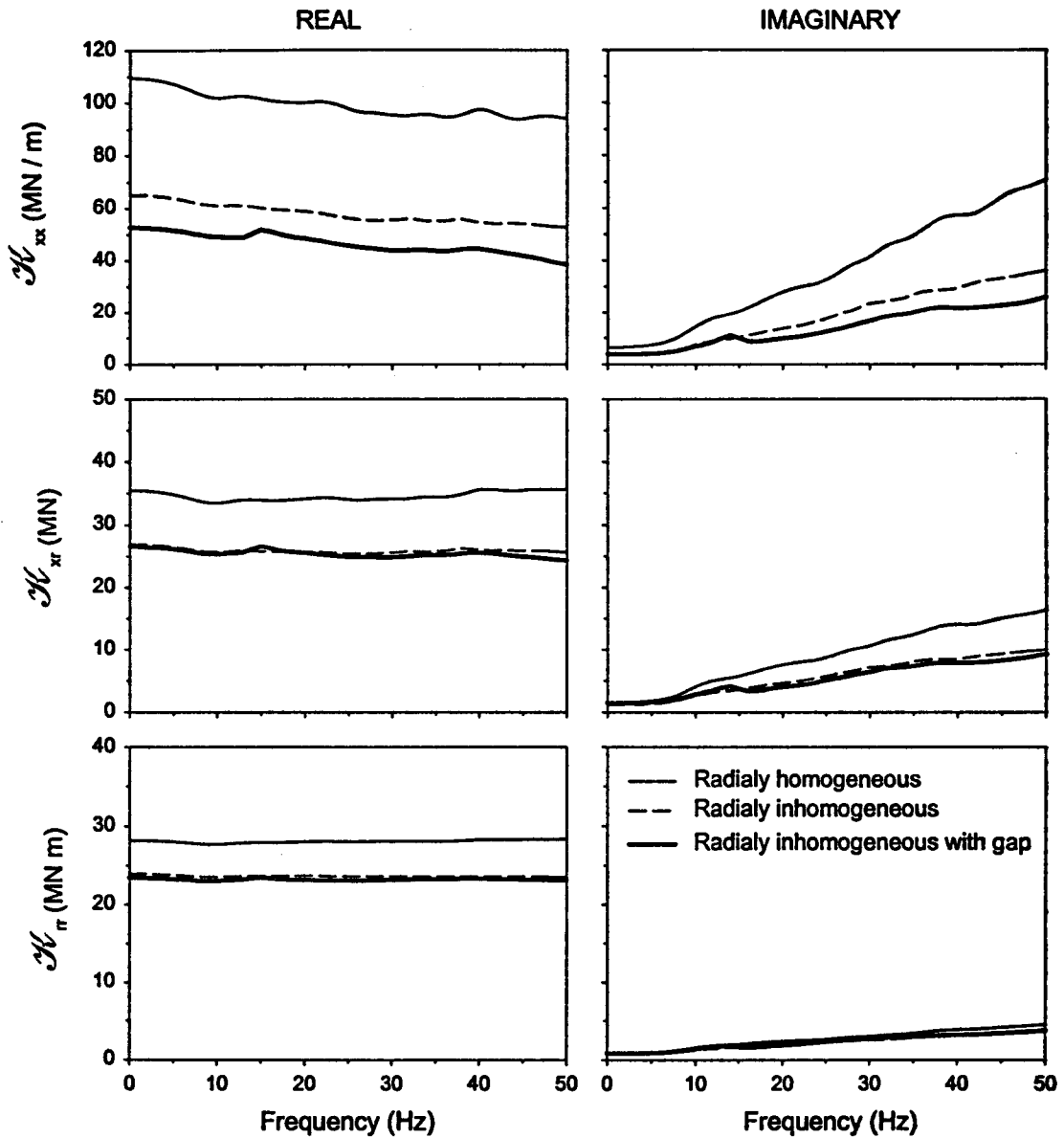


FIGURE 4.6: Real and imaginary part of pile head stiffness coefficients for the tested single concrete pile at the Harbin site. Note the effect of radial soil inhomogeneity in swaying stiffness K_{xx} .

$$F(\omega) \begin{bmatrix} 1 \\ H \end{bmatrix} = \begin{bmatrix} \mathcal{H}_{xx} - m_{\text{cap}} \omega^2 & \mathcal{H}_{xr} - m_{\text{cap}} y_{\text{GC}} \omega^2 \\ \mathcal{H}_{xr} - m_{\text{cap}} y_{\text{GC}} \omega^2 & \mathcal{H}_{rr} - (I_o + m_{\text{cap}} y_{\text{GC}}^2) \omega^2 \end{bmatrix} \begin{bmatrix} u(\omega) \\ \phi(\omega) \end{bmatrix} \quad (4.11)$$

in which y_{GC} denotes the elevation of the center of gravity of the cap. The displacement at the pile cap, u_{TOP} , can then be calculated as

$$u_{\text{TOP}}(\omega) = u(\omega) + \phi(\omega) y_{\text{TOP}} \quad (4.12)$$

The dimensionless displacement amplitude is depicted in Figure 4.7. In this case the amplitude of resonant response is replicated relatively well using a radially homogeneous profile ($A = 0$). This implies that radiation damping in the soil is properly modeled. On the other hand, resonant frequency (and thereby stiffness) is significantly overestimated. This frequency is estimated much better with a radially inhomogeneous soil profile as clearly shown in Figure 4.7. The prediction is further improved if a gap between pile and soil is considered.

4.3.4.2 Simplified analysis (Analytical formulas)

For this profile, the solutions for the dynamic response of a pile in homogeneous halfspace will be used. This requires a representative “average” soil modulus over the active pile length. It should be mentioned that averaging the soil moduli of the soil layers over the pile length is not a representative soil modulus since pile response is mostly affected by the soil reaction close to the surface. A more rational approach is to consider the weighted average:

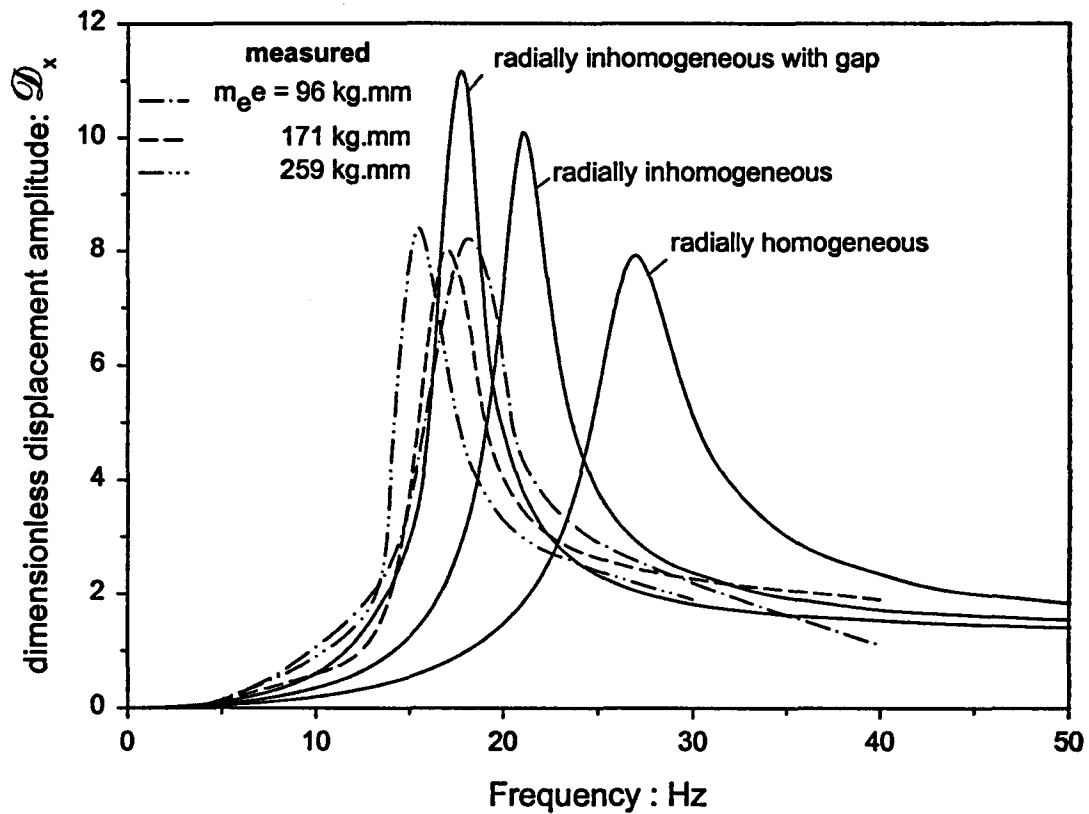


FIGURE 4.7: Comparison of measured and predicted horizontal response of a single concrete pile (El-Marsafawi et al 1992). Note the overprediction of natural frequency for a radially homogeneous soil and the improved prediction considering radial inhomogeneity.

$$\bar{E}_s = \frac{\sum_{i=1}^N \left(E_{si} \int_{z_1}^{z_2} \psi^2(z) dz \right)}{\int_0^L \psi^2(z) dz} \quad (4.13)$$

in which $\psi(z)$ denotes the normalized shape (shape function) of the deflected pile and E_{si} the soil modulus in layer i . In the case of a long pile in homogeneous halfspace with a horizontal head-load under no rotation, the shape function is (Mylonakis and Gazetas 1999)

$$\psi(z) = e^{-\lambda z} (\sin(\lambda z) + \cos(\lambda z)) \quad (4.14)$$

and

$$\lambda = \left[\frac{\delta \bar{E}_s}{4 E_p I_p} \right]^{1/4} \quad (4.15)$$

denotes the well-known wave number (Chapter 3).

From Winkler theory, the swaying, cross swaying-rocking and rocking stiffness coefficients of a pile in a homogeneous halfspace are given by the simple expressions:

$$\mathcal{K}_{xx} = 4 E_p I_p \lambda^3 \quad (4.16a)$$

$$\mathcal{K}_{xr} = 2 E_p I_p \lambda^2 \quad (4.16b)$$

$$\mathcal{K}_{rr} = 2 E_p I_p \lambda \quad (4.16c)$$

This procedure requires an initial estimate for the equivalent soil modulus but it converges really fast and is not sensitive to the initial value of $\bar{E}_s$. An initial value equal to the modulus of the first soil layer is adopted in the analyses.

Following the procedure outlined above, the equivalent soil modulus was estimated at $E_s = 94,800$ kpa, leading to a pile-soil stiffness contrast $E_p/\bar{E}_s = 207$.

Results for the stiffness coefficients are summarized in Table 4.3 and Figure 4.8. It can be seen that the amplitude of the response computed using the analytical formulas, is lower than that obtained with finite elements in the radially homogeneous profile and higher in the radially inhomogeneous profile. This indicates that the analytical formulas overestimate radiation damping in homogeneous soil and underestimate it in radially inhomogeneous soil. Overall, the analytical formulas yield satisfactory results.

TABLE 4.3: Static and dynamic pile-head stiffness coefficients for the single concrete pile

Soil Modeling	Analysis method	Pile stiffness	Equation
RADIALLY HOMOGENEOUS A = 0	K-PAX	$K_{xx,static} = 110,000 \text{ kN/m}$	-
		$K_{xr,static} = 37,100 \text{ kN}$	-
		$K_{rr,static} = 28,700 \text{ kN m}$	-
	ANALYTICAL	$K_{xx,static} = 98,000 \text{ kN/m}$	3.63
		$K_{xr,static} = 29,300 \text{ kN}$	3.69
		$K_{rr,static} = 25,430 \text{ kN m}$	3.68
		$\beta_{xx}(\omega) = 0.028 + 0.001176 \omega$	3.131
		$\beta_{xr}(\omega) = 0.06 + 0.00071 \omega$	3.135
		$\beta_{rr}(\omega) = 0.021 + 0.00024 \omega$	3.134
RADIALLY INHOMOGENEOUS A = 0.8	K-PAX	$K_{xx,static} = 66,900 \text{ kN/m}$	-
		$K_{xr,static} = 27,600 \text{ kN}$	-
		$K_{rr,static} = 24,300 \text{ kN m}$	-
	ANALYTICAL	$K_{xx,static} = 62,100 \text{ kN/m}$	3.72
		$K_{xr,static} = 22,200 \text{ kN}$	3.81
		$K_{rr,static} = 23,500 \text{ kN m}$	3.80
		$\beta_{xx}(\omega) = 0.06 + 0.0018 \omega$	3.131
		$\beta_{xr}(\omega) = 0.04 + 0.0005 \omega$	3.135
		$\beta_{rr}(\omega) = 0.08 + 0.0014 \omega$	3.134

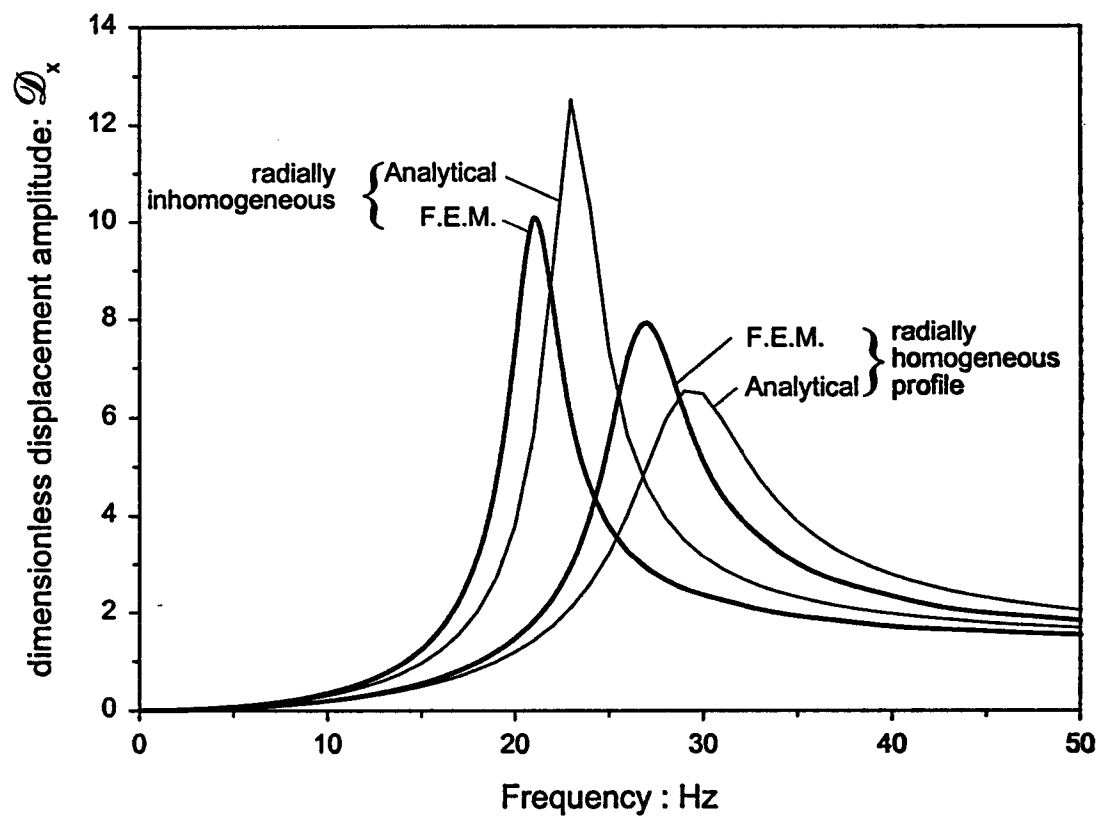


FIGURE 4.8: Comparison of predictions of horizontal response of a single concrete pile using finite elements (K-PAX) and analytical formulas.

4.4 VERTICAL RESPONSE OF A FULL-SCALE STEEL PIPE

Blaney et al (1987) investigated the response of a full-scale single pipe pile and a 3x3 pile group, driven in overconsolidated clay. The objective was to compare how well existing analysis tools simulated the pile response to dynamic loads and to compare the response of a single pile to that of a pile group.

4.4.1 Test pile foundation and site

The pipes used had a length of 13.4 m, an outer diameter of 0.273 m and a wall thickness of 0.0093 m. The pile length to diameter ratio is equal to $L / d = 49$ making this a long ($L > L_a$) pile. The soil profile consisted of Pleistocene clays that have been overconsolidated by desiccation. Geophysical logs indicated the presence of layers of silt and sand below the pile toes, but no rock to a depth of at least 250 ft (76.3 m). Detailed geotechnical descriptions of the site soils are given elsewhere (O'Neill et al 1982). Details on the soil profile are given in Figure 4.9. It should be mentioned that the vibration tests were performed 4 years after the piles were installed and tested statically. It is certain that in these 4 years, the bond of the pile with soil and the soil itself must have changed appreciably (strengthened, due to thixotropy, consolidation, adhesion etc.). This adds uncertainty in the simulations.

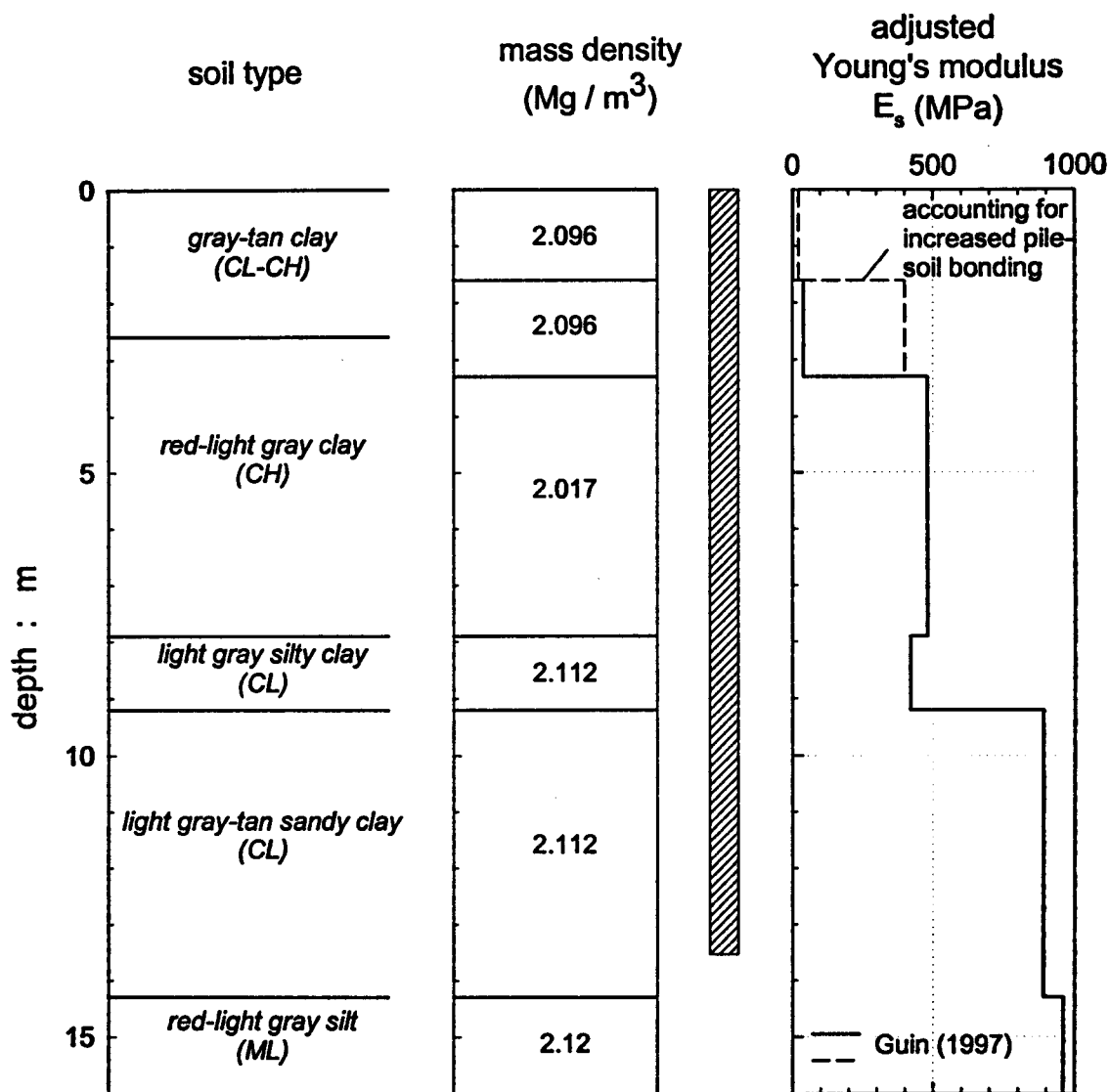


FIGURE 4.9: Soil properties at the University of Houston test site (Blaney et al 1987)

4.4.2 Test Description

The pile-mass system was excited vertically using a USAE Waterways Experiment Station (WES). This actuator displaced reaction masses sinusoidally on a vertical linear path to affect nearly harmonic loading. Load cells were placed between the vibrator and the top of each cap. Continuous monitoring of the load functions permitted the mass of the vibrator to be excluded from the analytical computations. The pile cap masses for the single pile and the pile group are respectively 5.637 Mg and 25.42 Mg. The amplitude of the applied load (F) was constant for all frequencies, with a value of 17.8 kN and 71.2 kN for the single pile and the pile group respectively.

4.4.3 Analysis of single pile vertical response

4.4.3.1 Rigorous analysis (F.E.M.)

Figure 4.10 depicts the impedance curves for the single pile as calculated using K-PAX. Following Guin (1997), in some of the analyses a stiffer soil profile (see discussion in Section 4.4.1) was used. To account for radial inhomogeneity, an inhomogeneity factor $A = 0.8$ was utilized. The pile-soil properties used in this analysis are summarized in Table 4.4. In Figure 4.11 the experimental results are compared with the analytical results from K-PAX in terms of a flexibility coefficient ($\mathcal{F}$), where

$$\mathcal{F} = \frac{u_z}{F} / \text{number of piles} \quad (4.17)$$

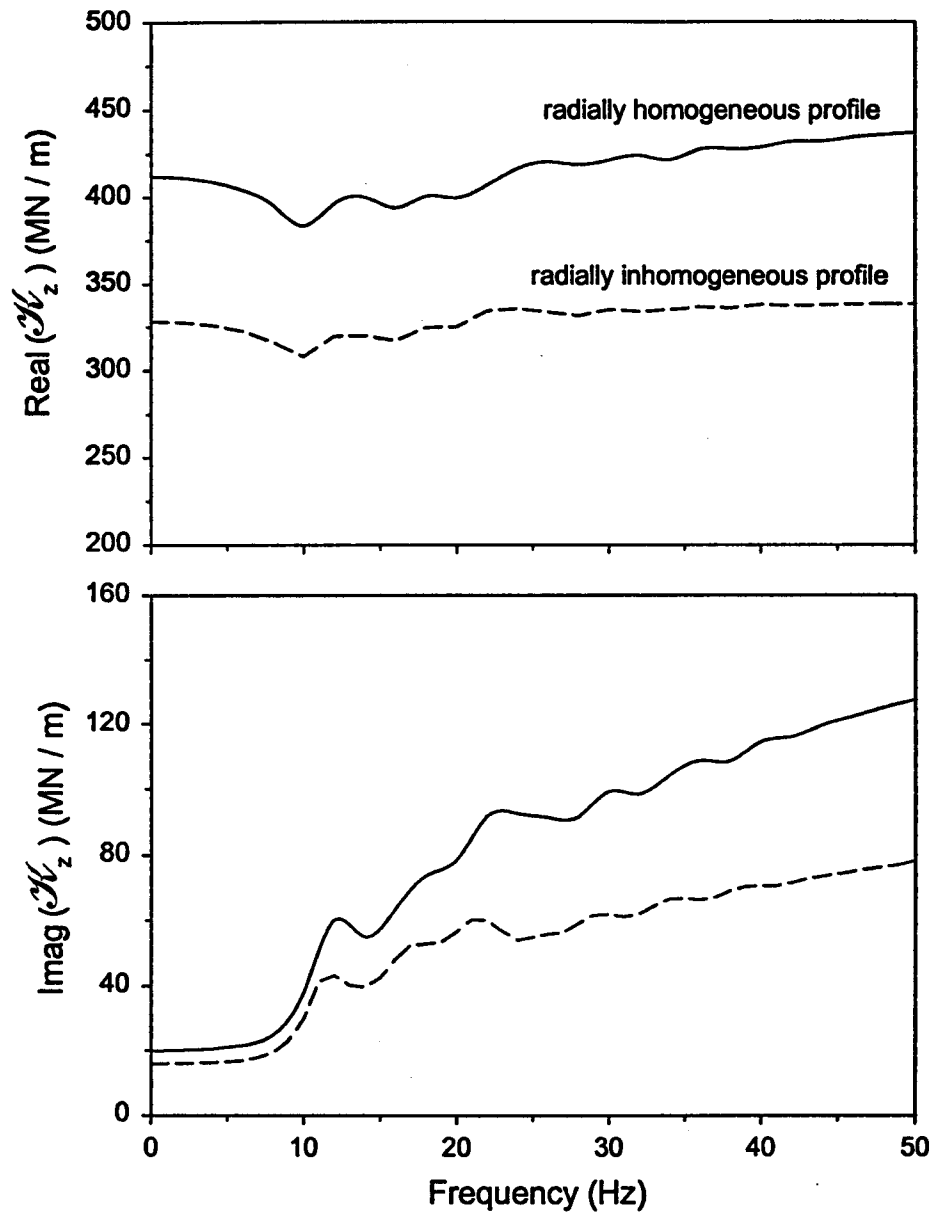


FIGURE 4.10: Real and imaginary part of pile head stiffness for the tested single steel pipe (Blaney et al 1987).

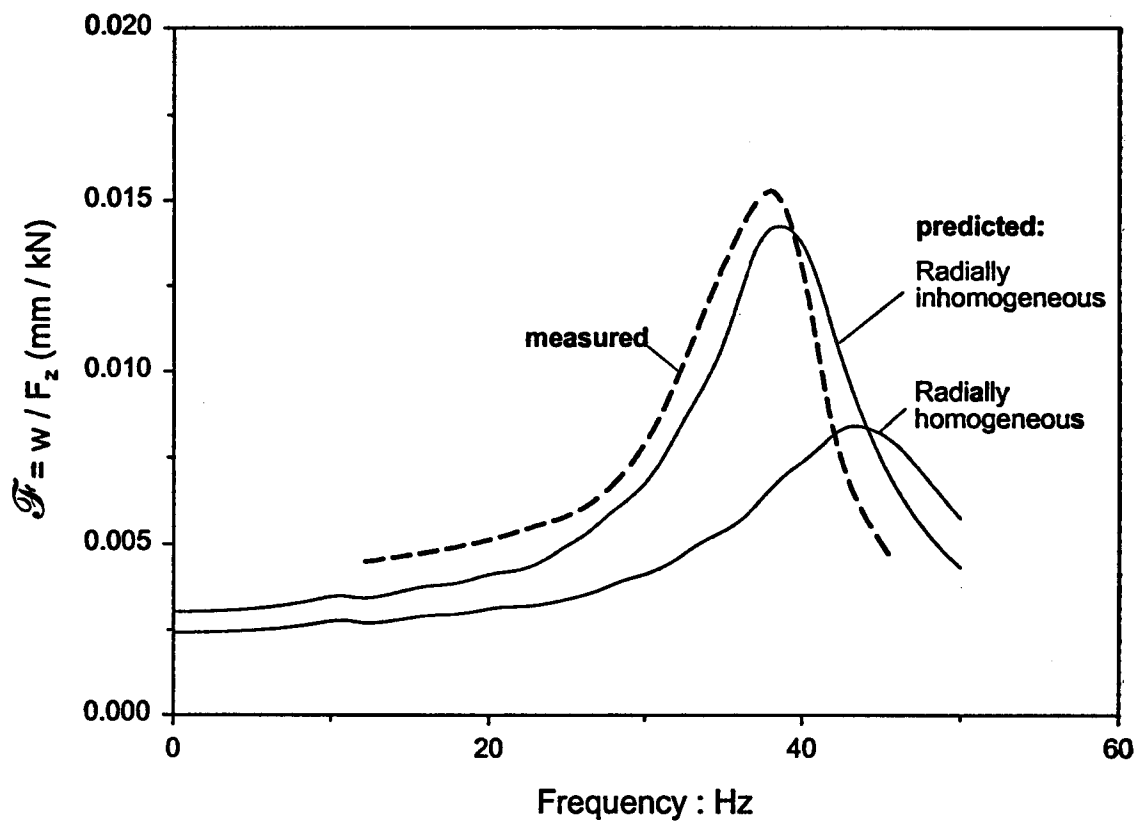


FIGURE 4.11: Comparison of measured vertical response of a single steel pipe (Blaney et al 1987) against analytical predictions. Note the overprediction of natural frequency and damping assuming radially homogeneous soil, and the improved prediction considering radial inhomogeneity in the soil.

and u_z is the vertical displacement and F is the excitation force. The agreement between recorded and predicted response is satisfactory.

4.4.3.2 Simplified analysis (Analytical formulas)

The soil profile consists of layers of clay and as in the previous case an equivalent homogeneous soil profile is going to be used. The soil modulus of the substitute homogeneous halfspace will be a weighted average of the moduli of the soil layers. This average modulus depends on the shape function of the vertically loaded pile, which can be assumed to be a linear function of depth:

$$\psi(z) = 1 - \frac{z}{L} \quad (4.18)$$

where L is the length of the pile.

- a) Using eq. (4.18) along with equation (4.13), one finds the equivalent soil modulus to be $\bar{E}_s = 325,000$ kpa. Thus, the pile to soil modulus of elasticity ratio is $E_p / \bar{E}_s = 81$.

The pile-soil parameters used in the analysis are summarized in Table 4.4.

- b) Using equation 2.39 the static stiffness at the pile head is found to be 455,900 kN / m, 10% higher than the estimate from the finite element solution. Accounting for radial soil inhomogeneity through an inhomogeneity factor $A = 0.8$ results to a reduced vertical static stiffness of 326,600 kN / m. The latter value differs only by 0.6% from the more rigorous prediction of K-PAX.

TABLE 4.4: Soil, pile and SSI parameters used in the analysis (vertical response of steel pipe)

SOIL			PILE	SSI PARAMETERS
V_s (m/s)	ρ_s (Mg/m ³)	v_s, β_s		
$V_{s,1} = 59$	$\rho_{s,1} = 2.096$	$v_{s,1-5} = 0.49$	$E_p = 2.0 \cdot 10^8$ kPa	$\bar{E}_s = 0.325 \cdot 10^6$ kPa
$V_{s,2} = 253$	$\rho_{s,2} = 2.096$	$v_{s,6} = 0.45$	$L = 13.4$ m	$\bar{E}_p = 2.63 \cdot 10^7$ kPa
$V_{s,3} = 283$	$\rho_{s,3} = 2.017$	$\beta_s = 5\%$	$d_o = 0.273$ m	$\bar{\rho}_p = 1.03$ Mg/m ³
$V_{s,4} = 258$	$\rho_{s,4} = 2.112$		$t = 0.0093$ m	$\frac{\bar{E}_p}{E_{st}} = 81$
$V_{s,5} = 376$	$\rho_{s,5} = 2.112$		$\rho_p = 7.86$ Mg/m ³	$\frac{L}{d} = 49$
$V_{s,6} = 395$	$\rho_{s,6} = 2.112$		$\beta_p = 1\%$	
			$v_p = 0.25$	

- c) The real part of the complex dynamic impedance is again assumed to remain constant with frequency. The imaginary part can be calculated from the formulas of Gazetas (1991):

$$C_z = a_0^{1/5} \rho V_s \pi d L r_d \quad (4.19)$$

where

$$r_d = 1 - \exp \left[- \left(\frac{E_p}{E_s} \right) \left(\frac{L}{d} \right)^2 \right] \quad (4.20)$$

Carrying out the calculations the following expression for the dashpot coefficient yields

$$C_z = 697 \omega^{-1/5} \text{ kN s /m} \quad (4.21)$$

The dynamic impedance of the pile is expressed as:

$$\mathcal{H}_z(\omega) = 455,900 + i 697 \omega^{4/5} \text{ kN/m} \quad (4.22)$$

To account for radial inhomogeneity in the soil, the previous expression is modified as

$$\mathcal{H}_z(\omega) = 326,600 + i \alpha 697 \omega^{4/5} \text{ kN/m} \quad (4.23)$$

where α is a dimensionless factor accounting for the reduction in radiation damping due to inhomogeneity.

The results for the dynamic stiffness of the pile are summarized in Table 4.5; the response is plotted in Figure 4.12 in terms of the flexibility coefficient in equation (4.17) and is compared with the rigorous results from K-PAX. It appears that the response curves from the simplified analysis are somewhat higher than those predicted by the finite elements showing that damping is underestimated. Recalling the first experiment, one will notice the opposite trend with damping being overestimated. The explanation for the contradicting results may be found in the properties of the soil profile. In the first experiment, the static stiffness of the pile was about 10% lower than the one calculated with finite elements, thus the assumed equivalent soil profile was softer. In this

experiment, the pile-head static stiffness is 10% higher, thus the assumed equivalent profile is stiffer. As mentioned in Chapter 2, soft soil profiles tend to radiate more energy than stiff profiles, hence the first profile overestimates damping and the second underestimates it! Using a trial and error procedure it was found that, to match the F.E. numerical results, radiation damping should be increased by a factor (α) of 1.6 and 1.25 for the cases of the radially homogeneous and the radially inhomogeneous soil profiles respectively.

TABLE 4.5: Static and dynamic pile-head stiffness for single steel pipe

Soil Modeling	Analysis method	Pile stiffness	Equation
RADIALLY HOMOGENEOUS $A = 0$	K-PAX	$K_{z,static} = 407,000$	-
	ANALYTICAL	$K_{z,static} = 455,900$	2.39
		$\mathcal{H}_z(\omega) = 455,900 + (700 \omega^{4/5}) i$	4.22
RADIALLY INHOMOGENEOUS $A = 0.8$	K-PAX	$K_{z,static} = 328,200$	-
	ANALYTICAL	$K_{z,static} = 326,600$	2.39
		$\mathcal{H}_z(\omega) = 326,600 + (700 \alpha \omega^{4/5}) i$	4.23

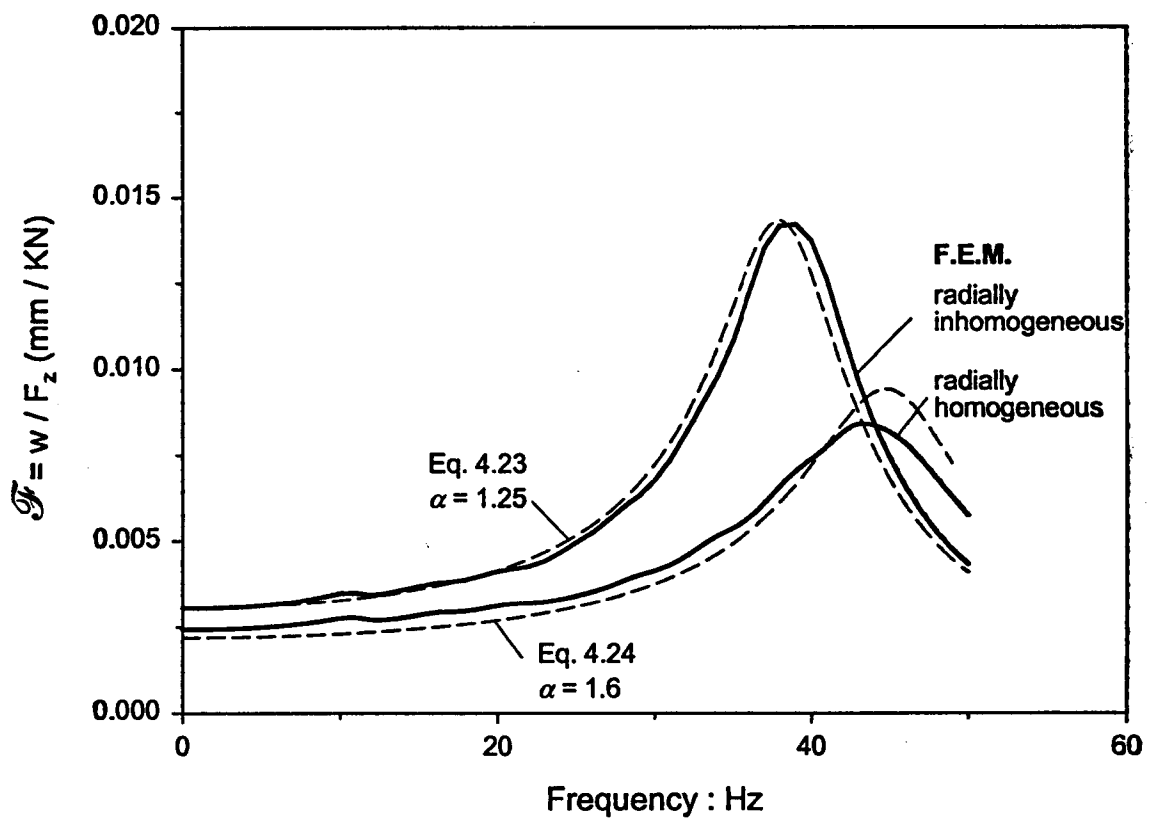


FIGURE 4.12: Comparison of predicted vertical response of a single steel pipe using finite elements and analytical formulas.

4.5 Analysis of pile groups

In this section the results of the single pile analysis are used to predict the response of pile groups. The horizontal and vertical response of three different pile groups is examined and compared with experimental results. The pile groups examined were part of the three experiments presented in the previous sections. Using the F.E. formulation displacement attenuation functions (ψ factors) are determined and used to compute pile-to-pile interaction factors. Two solutions for pile groups are compared: (a) conventional solutions neglecting radial inhomogeneity (b) modified solutions taking into account radial inhomogeneity. The latter utilize interaction factors that are calculated using an average soil modulus between the two interacting piles (Poulos 1988). It is shown that the modified approach yields superior predictions.

4.5.1 Horizontal and Vertical response of a model pile group (El-Marsafawi et al 1992)

El-Marsafawi et al (1992) analyzed a group of 6 model steel piles; a plan view of the group is shown in Figure 4.13. The experimental equipment was identical to the one used for the analysis of the single pile. Owing to the characteristics of the testing equipment, the mass of the pile cap was 1.874 Mg for the vertical excitation, 1.89 Mg and 1.13 Mg for the horizontal excitation at low and high displacement amplitudes respectively. In the case of horizontal excitation and low displacement amplitudes, the force acted at a height $H = 0.86$ m above ground level, the center of gravity was at

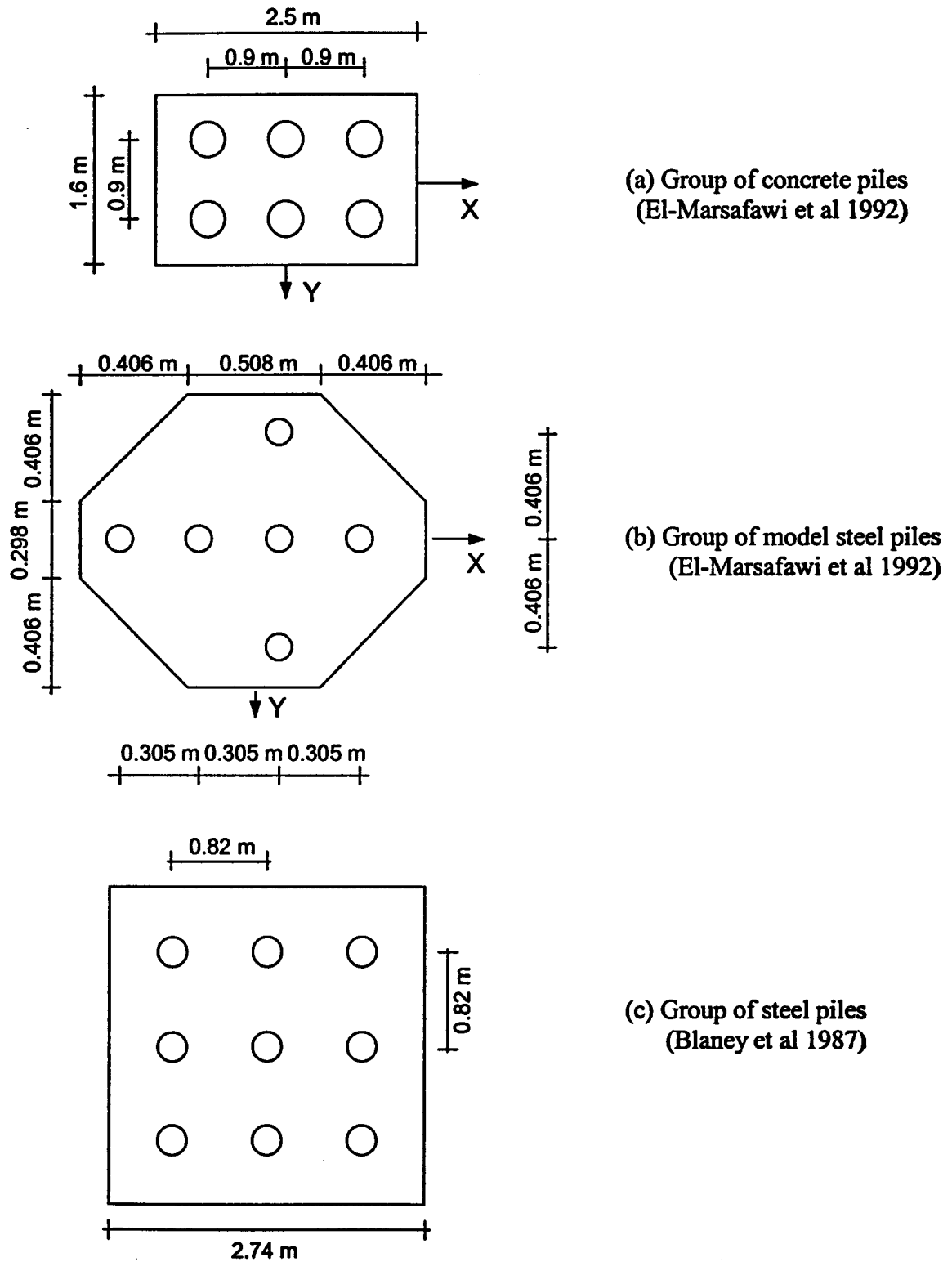


FIGURE 4.13: Configurations of the pile groups examined in this study

$y_{GC} = 0.45$ m, and the response was measured at $y_{TOP} = 0.35$ m. The mass moment of inertia was given to be 0.164 Mg m^2 and the height of the horizontal excitation above the center of gravity Z_c was 0.41 m. In the case of horizontal excitation and high displacement amplitudes, the properties mentioned above change to $H = 0.64$ m, $y_{GC} = 0.34$ m, $y_{TOP} = 0.35$ m, $I_o = 0.107 \text{ Mg m}^2$ and $Z_c = 0.3$ m. Figure 4.14 shows the dimensionless amplitude ($\mathcal{D}_z$) of the response of the pile group in vertical excitation (eq. (4.2)). Comparing predicted and measured response, it is seen that the predicted response curves do not follow closely the measured response curve. Nevertheless, the response using a radially inhomogeneous soil profile is closer to the measured ones.

In the case of horizontal excitation, Figure 4.15 shows dimensionless displacement amplitudes of swaying and rocking response ($\mathcal{D}_x$ and $\mathcal{D}_r$) at low displacement amplitudes. Comparing predicted and measured response, it is seen that the predicted response curves predict accurately the resonant frequency but are overdamped. For high displacement amplitudes (Figure 4.16), the predicted response is much closer to the recorded. In both cases, accounting for radial inhomogeneity yields improved predictions.

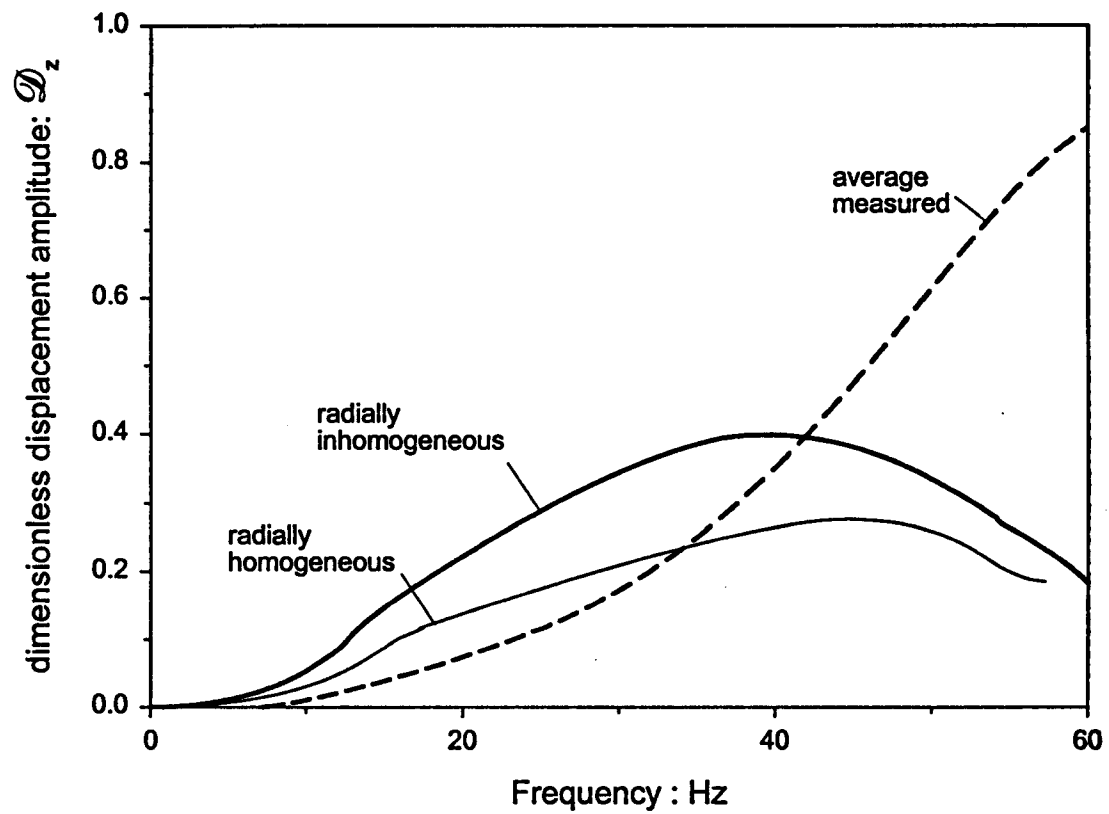


FIGURE 4.14: Comparison of measured vertical response of a steel pile group (El-Marsafawi et al 1992) with analytical results. Note the overprediction of natural frequency assuming radially homogeneous soil and the improved prediction assuming radial inhomogeneity.

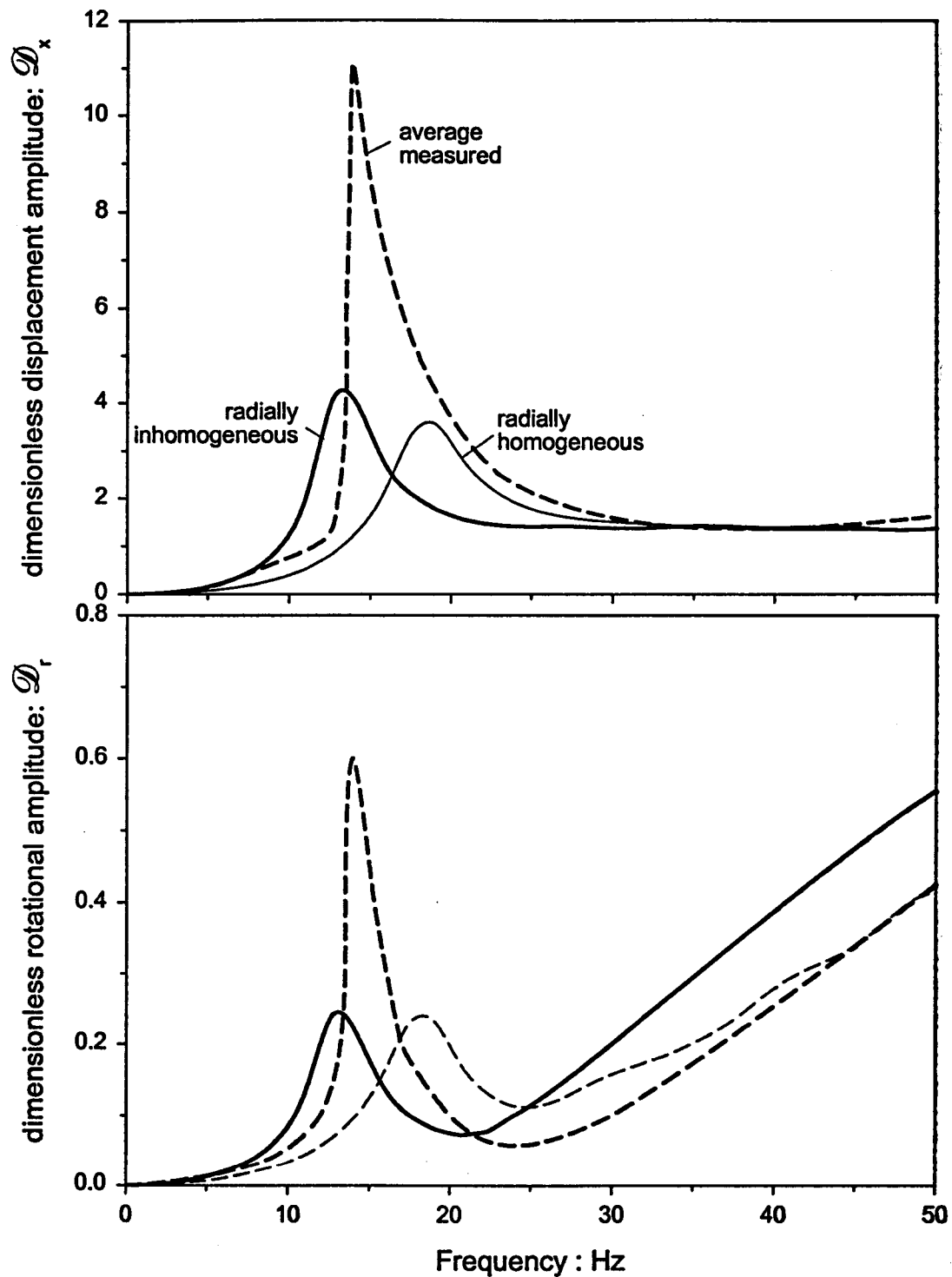


FIGURE 4.15: Comparison of the measured horizontal response of a steel pile group (low amplitudes- El-Marsafawi et al 1992) with analytical results. Note the overprediction of natural frequency assuming radially homogeneous soil and the improved prediction assuming radial inhomogeneity.

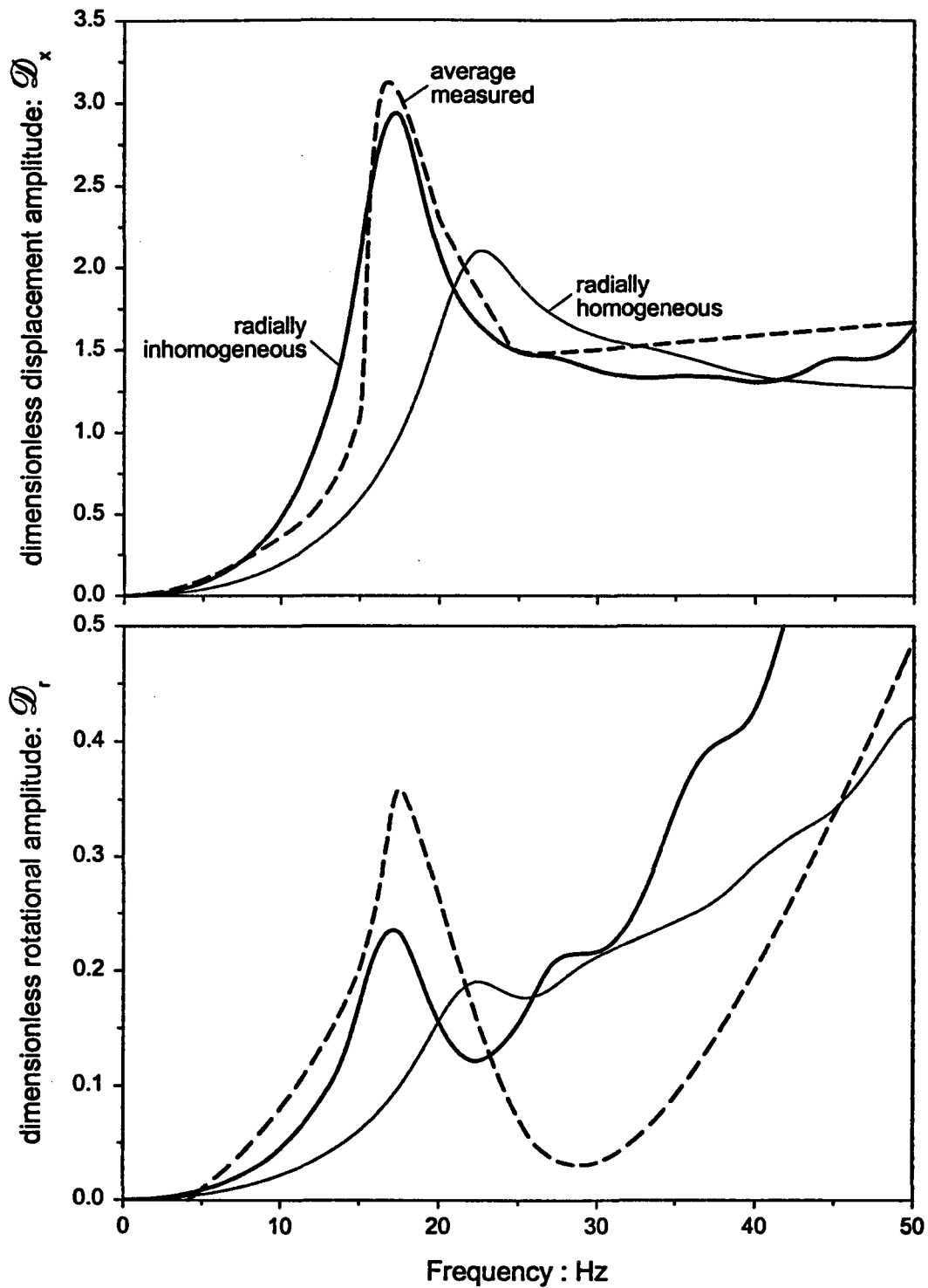


FIGURE 4.16: Comparison of the measured horizontal response of a steel pile group (high amplitudes - El-Marsafawi et al 1992) with analytical results. Note the overprediction of natural frequency assuming radially homogeneous soil and the improved prediction assuming radial inhomogeneity.

4.5.2 Horizontal response of full-scale concrete pile group (El-Marsafawi et al 1992)

El-Marsafawi et al also analyzed a full-scale pile group consisting of 6 concrete piles (see Section 4.3). A plan view of this pile group is shown in Figure 4.13. The measured and predicted responses are compared graphically in Figure 4.17. The predicted response curve is close to the recorded curve, especially in the case where radial inhomogeneity is accounted for. As in previous cases, incorporation of radial inhomogeneity yields improved results.

4.5.3 Vertical response of full-scale steel pile group (Blaney et al 1987)

Blaney et al measured the response of a group of nine full scale-steel pipes in clay. A plan view of the group is shown in Figure 4.13. The measured response curve is plotted in Figure 4.18 along with the predicted response curve in terms of flexibility ($\mathcal{F}$). Evidently, the calculated response curve considering radial soil inhomogeneity yields improved results compared to traditional theory assuming homogeneous soil.

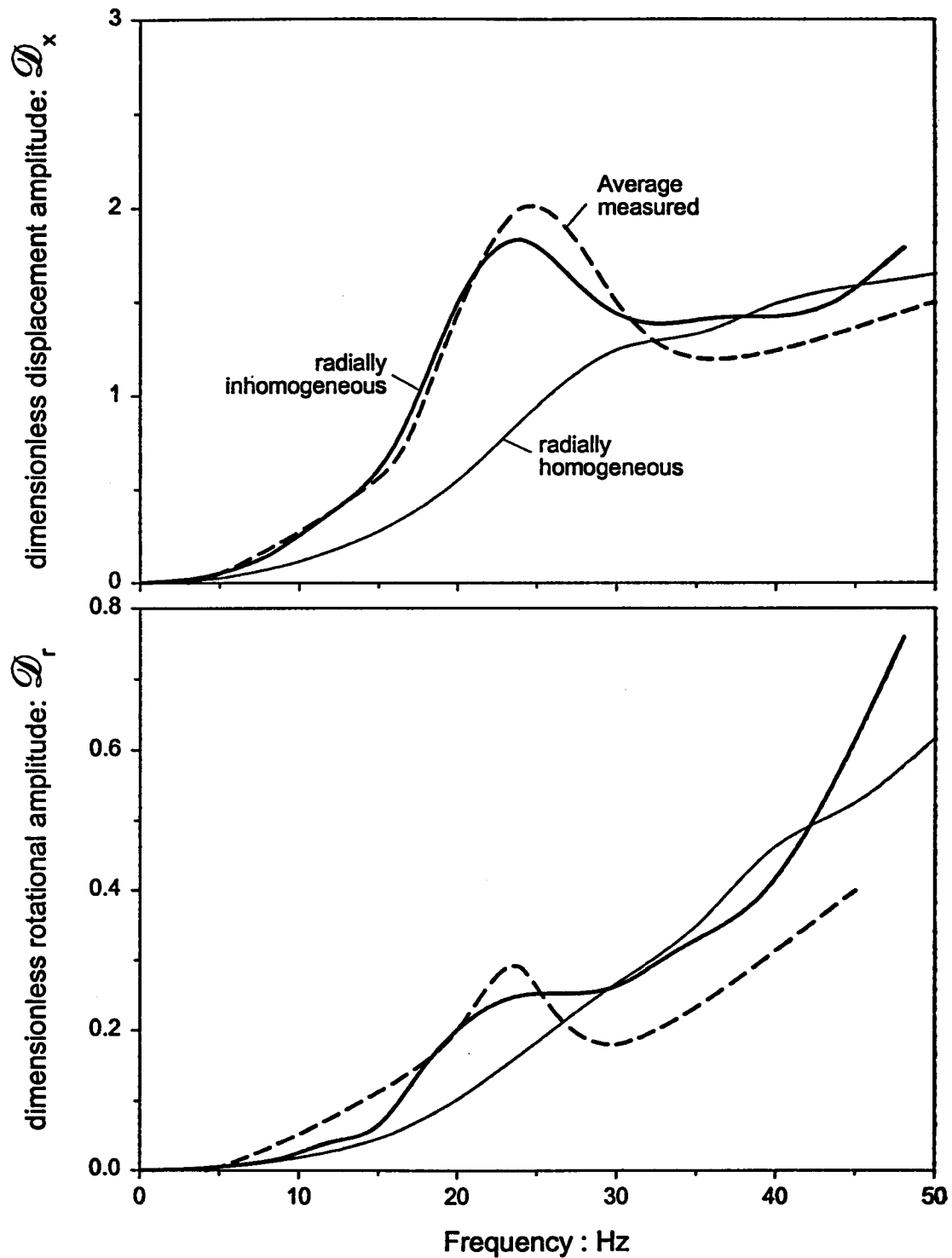


FIGURE 4.17: Comparison of the measured horizontal response of a concrete pile group (El-Marsafawi et al 1992) with analytical results. Note the overprediction of natural frequency assuming radially homogeneous soil and the improved prediction considering radial inhomogeneity.

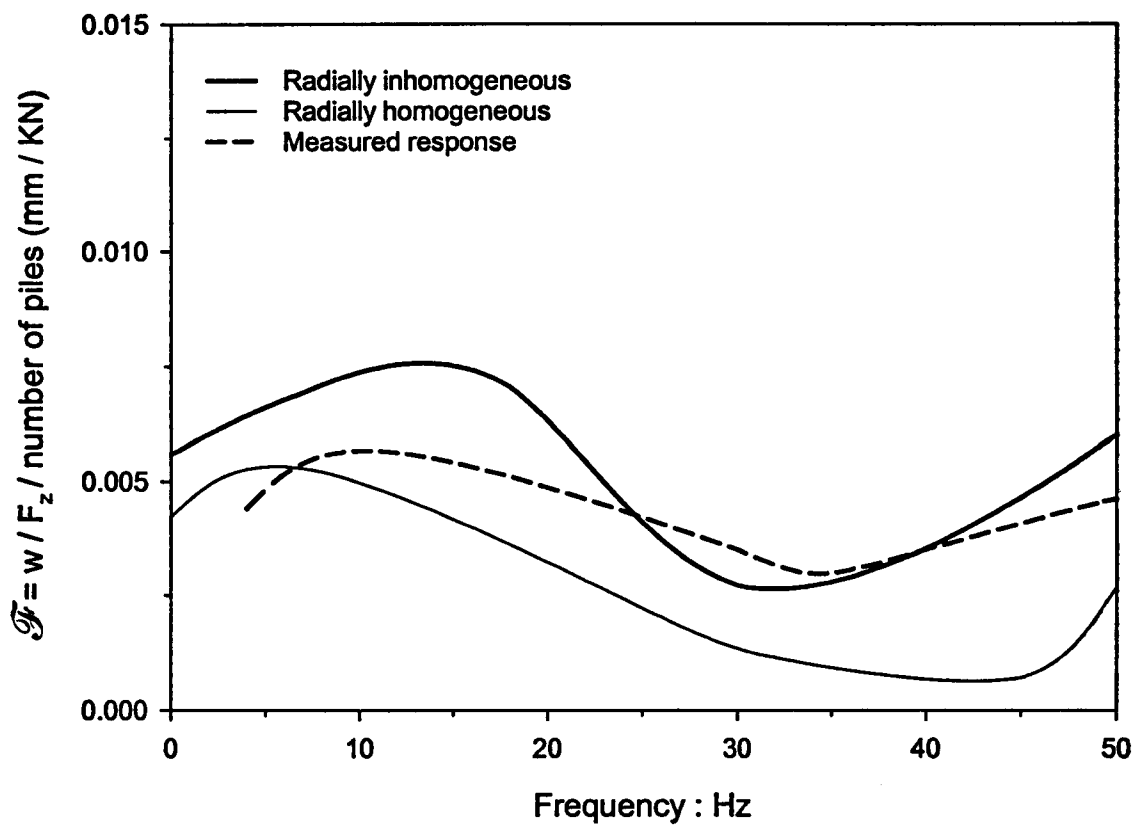


FIGURE 4.18: Comparison of the measured and computed vertical response of a steel pile group in the University of Houston site (Blaney et al 1987). Note the improved predictions considering radial inhomogeneity.

4.6 Conclusions

It was shown that the pile response is predicted more accurately when a radially inhomogeneous profile is considered. More specifically:

- (1) The traditional assumption of radially homogeneous soil overpredicts both radiation damping and stiffness of single piles in all modes of loading. The same trend is apparent in the case of pile groups.
- (2) The formulas derived in Chapter 2 for vertical pile response tend to overpredict radiation damping, but predict satisfactorily static stiffness.
- (3) The formulas derived in Chapter 3 for horizontal pile response seem to underpredict radiation damping.
- (4) An inhomogeneity factor $A = 0.8$ yields satisfactory results for all pile configurations, soil profiles, and oscillation modes examined.

CHAPTER 5

CASE STUDY I: SEISMIC RESPONSE OF OHBA OHASHI BRIDGE

5.1 Introduction

The objective of this chapter is to investigate the seismic response of an actual bridge pier founded on a group of 64 steel piles in extremely soft clay ($N_{SPT} = 0$), which has been instrumented specifically for studying soil-pile-superstructure interaction. Several earthquake motions have been recorded by the Institute of Technology of Shimizu Corporation, with peak accelerations in the free field as high as 0.11g. The theoretical developments presented earlier for the solution of the foundation impedance problem are utilized herein in an attempt to reproduce the field measurements. The relatively low levels of peak shear strains in the soil ($\gamma < 10^{-4}$) and the high plasticity index of the clay justifies the use of elastodynamic methods. The effects of soil amplification, kinematic interaction, foundation compliance, and superstructure inertia on seismic response are considered. Group effects are treated using the superposition method in conjunction with wave-propagation-based interaction factors between the piles. It is shown that records and predictions compare reasonably well for the response of the pile cap and bridge deck. Implications of valley effects, multiple-support excitation and structural connections on seismic response are discussed. The study complements and extends the seminal studies by Tazoh, Gazetas and co-workers.

5.2 System Description and Seismic Observations

Ohba Ohashi is a 10-span steel road bridge located in Fujisawa City, Kanagawa prefecture near Tokyo (Figure 5.1). The 600m-long, 16m-wide structure is supported on eleven piers founded on pile groups in extremely soft clay. Four piers (P5 to P8) are connected with a 143m-long continuous girder. The girder is attached with a fixed-shoe connection to Pier 6 and with movable bearings to the other piers. Pier 6 is supported by a group of 64 piles (8 rows by 8 columns), 32 of which are vertical and 32 raked at an angle of 10° along the longitudinal direction of the structure. The piles are 22m-long, 0.6m-wide steel pipes having wall thickness of 9mm (12mm for raked piles).

Pier 6 is a two-column steel frame oriented perpendicular to the plane of the bridge. The columns are about 7.4m tall, having square cross section of 2.4m x 2.4m and steel thickness of approximately 4cm. The weight carried by each column is approximately 5000 KN, which accounts for the mass of the deck and the columns. For in-plane oscillations, the columns can be considered moment free at the top. In the out-of-plane direction, however, significant bending moments may develop at the column heads due to frame action. Structural properties are summarized in Table 5.1.

The bridge crosses a valley along the back marsh of a stream running between piers P6 and P7 (Figure 5.1). Borehole data obtained close to Pier 6 reveal a 22m-thick soft alluvial stratum of humus and silt with practically zero SPT blow counts over the whole thickness of the profile (Figure 5.2). The water table is located 1m below ground surface. Shear wave velocities measured using geophysical methods range from 40 to 65 m/s. As reported by Fan and Gazetas (1992), even after improvement shear wave velocities close to the surface did not exceed 100 m/s. According to the same authors, the

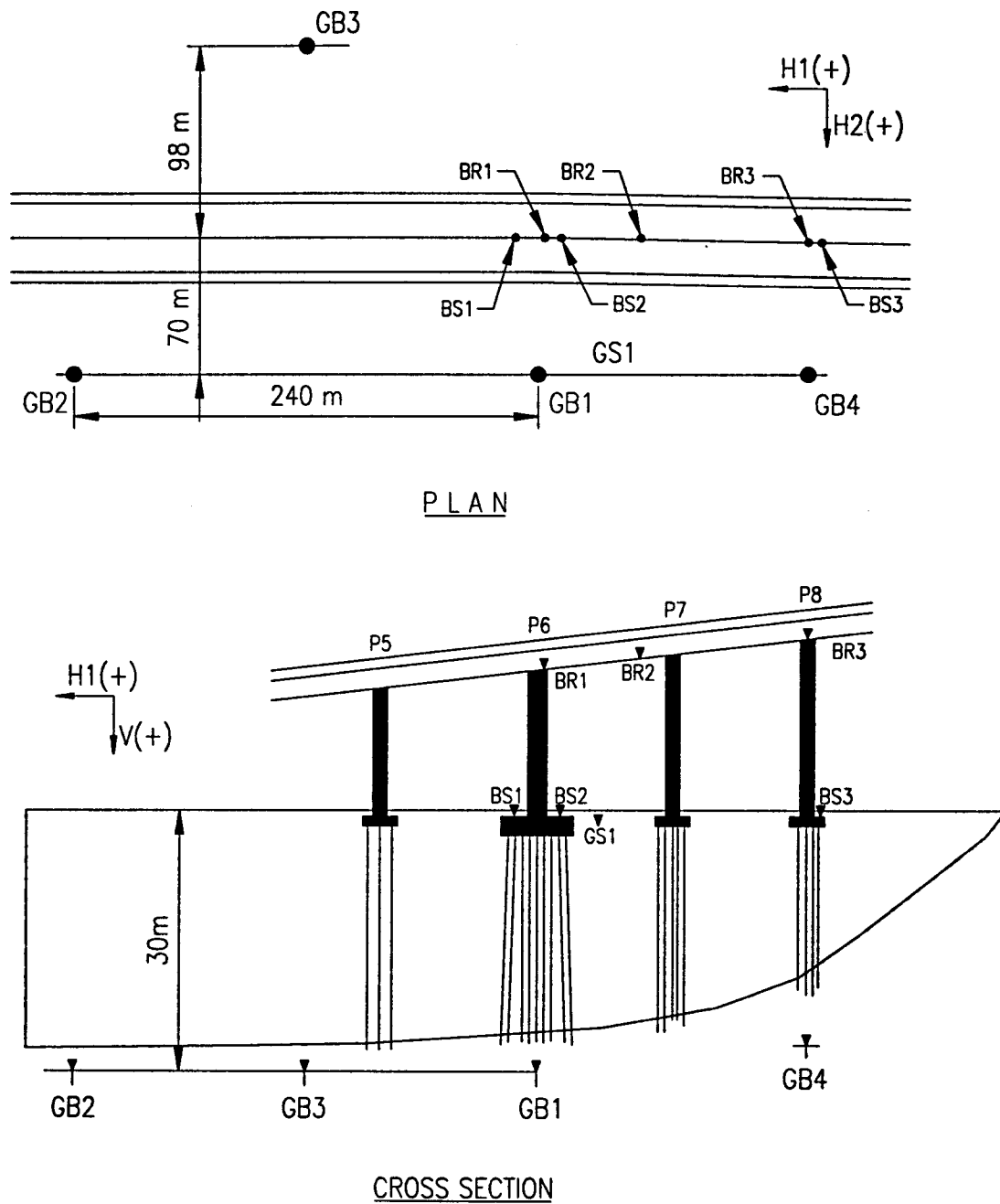


FIGURE 5.1: Plan view and cross section of Ohba bridge. Note the locations of accelerometers on the bridge and in the soil. (After Tazoh et al, 1989)

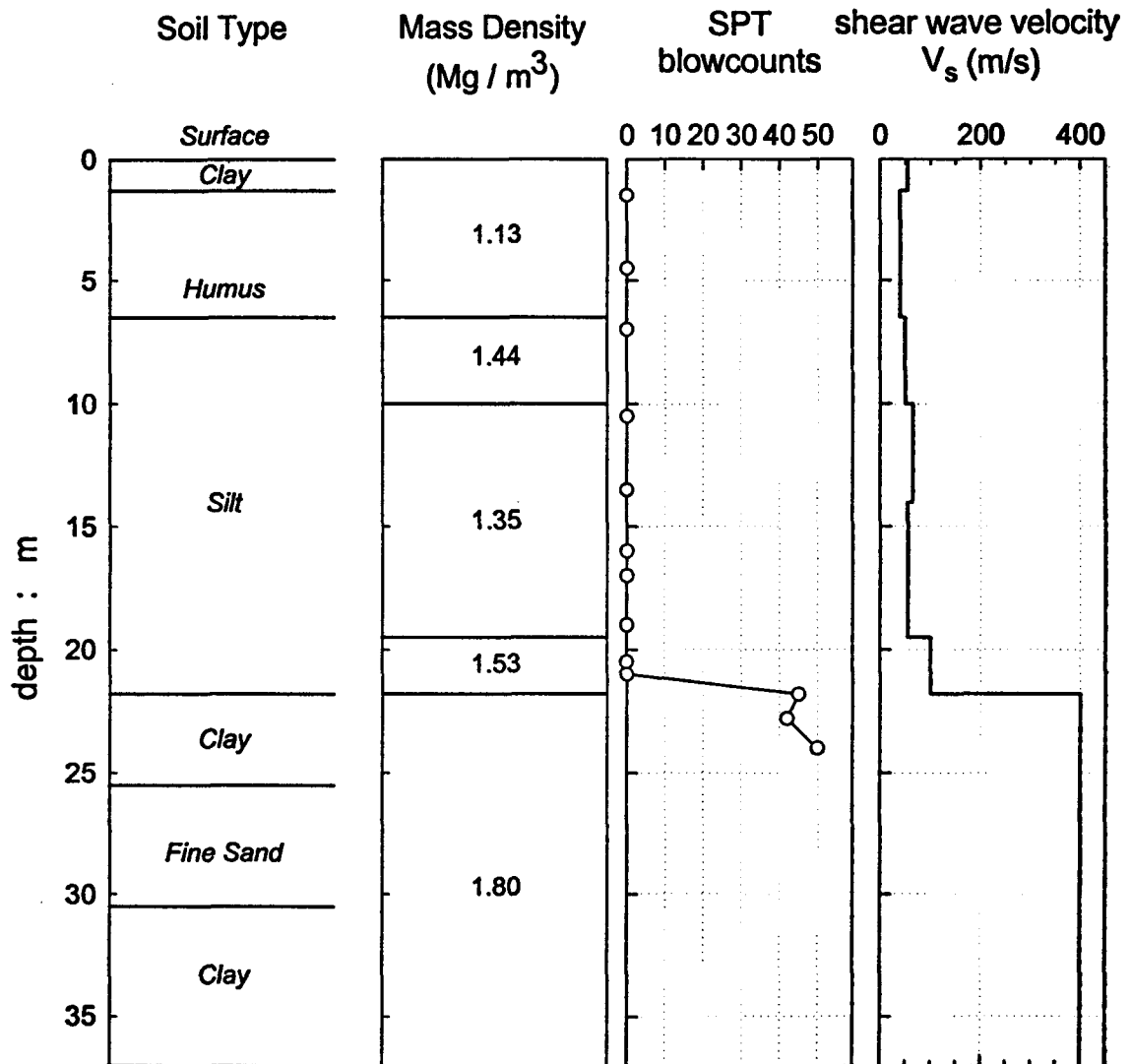


FIGURE 5.2: Soil profile and properties at Pier 6 (after Tazoh et al., 1989)

Table 5.1: Structural properties considered

	Tazoh et al (1989)	This study
	(bearings free to slide)	(bearings fixed against sliding)
m_{deck} (Mg)	2,192	700 [†]
m_{cap} (Mg)	801	801
$J_{\text{deck}}^{(i)}$ (Mg m ²)	–	420,000
$J_{\text{deck}}^{(o)}$ (Mg m ²)	–	11,500
J_{cap} (Mg m ²)	12,502	12,502
h_c (m)	–	8.7 m
E (kPa)	–	2.0×10^8
$K_c^{(i)}$ (kN/m) [*]	346,000 ^{††}	900,000 ^{**}
$K_c^{(o)}$ (kN/m) [*]	–	3,000,000 ^{**}
ζ (%)	0.02	0.02

^{*} superscripts (i) and (o) refer to in-plane and out-of-plane oscillations, respectively

^{**} estimated based on column wall thickness of 4cm

[†] encompassing tributary mass of pier 6 approximately 1/3 of deck mass

^{††} backcalculated based on an observed natural period of 0.45s

water content of the material is over 100%, reaching values as high as 250% at certain locations. This suggests a very elastic material with stress-strain and damping-strain properties similar to those encountered in Mexico City. The underlying stratum is a dilluvial deposit of clay and fine sand having shear wave velocity of about 400 m/s. As the bridge crosses a valley with steep edges, soil thickness varies under the piers giving rise to multi-dimensional waves in the deposit (even under uniform rock excitation) and, thereby, non-uniform seismic excitation on the piers. Key geotechnical properties are summarized in Table 5.2.

Table 5.2: Soil and foundation properties

Depth	Soil [†]	Pile
0 – 7 m	Humus $N_{SPT} = 0$	$E_p = 2.0 \times 10^8 \text{ kPa}$ $\rho_p = 7.8 \text{ Mg/m}^3$
7 – 22 m	Silt and Clay $N_{SPT} = 0$ $w > 100\%$	$L = 22 \text{ m}$ $d = 0.6 \text{ m}$ $t_{\text{vertical}} = 0.09 \text{ m}$
> 22 m	Clay and Sand $N_{SPT} > 50$	$t_{\text{raked}} = 0.12 \text{ m}$

[†] hysteretic damping of 3% and Poisson's ratio of 0.49 were considered for all layers

Comprehensive instrumentation of the bridge was undertaken by the Institute of Technology of Shimizu Corporation (Tazoh et al 1984, 1989, 1993). A total of eleven three-channel accelerometers were installed (Figures 5.1, 5.3), four of which were placed on the bearing stratum (GB1 to GB4), one on the free-field surface (GS1), three on pile caps (BS1 to BS3), and three on piers (BR1 to BR3). In addition, thirty-two strain gauges (SA1 to SA4, SB1 to SB4) were installed at four elevations along two piles under Pier 6, for the purpose of measuring longitudinal (axial & bending) strains. The first pile (pile A) is located close to the center of the group plan, the second (pile B) close to the edge (Figures 5.3, 5.4). The accelerometers were oriented so that one horizontal axis coincides with the longitudinal axis of the bridge (H1 direction), the other with the perpendicular axis (H2 direction).

Seismic observations were carried out by the Institute of Technology of Shimizu Corporation, Japan, using the installed instruments. Over the period from April 1981 to

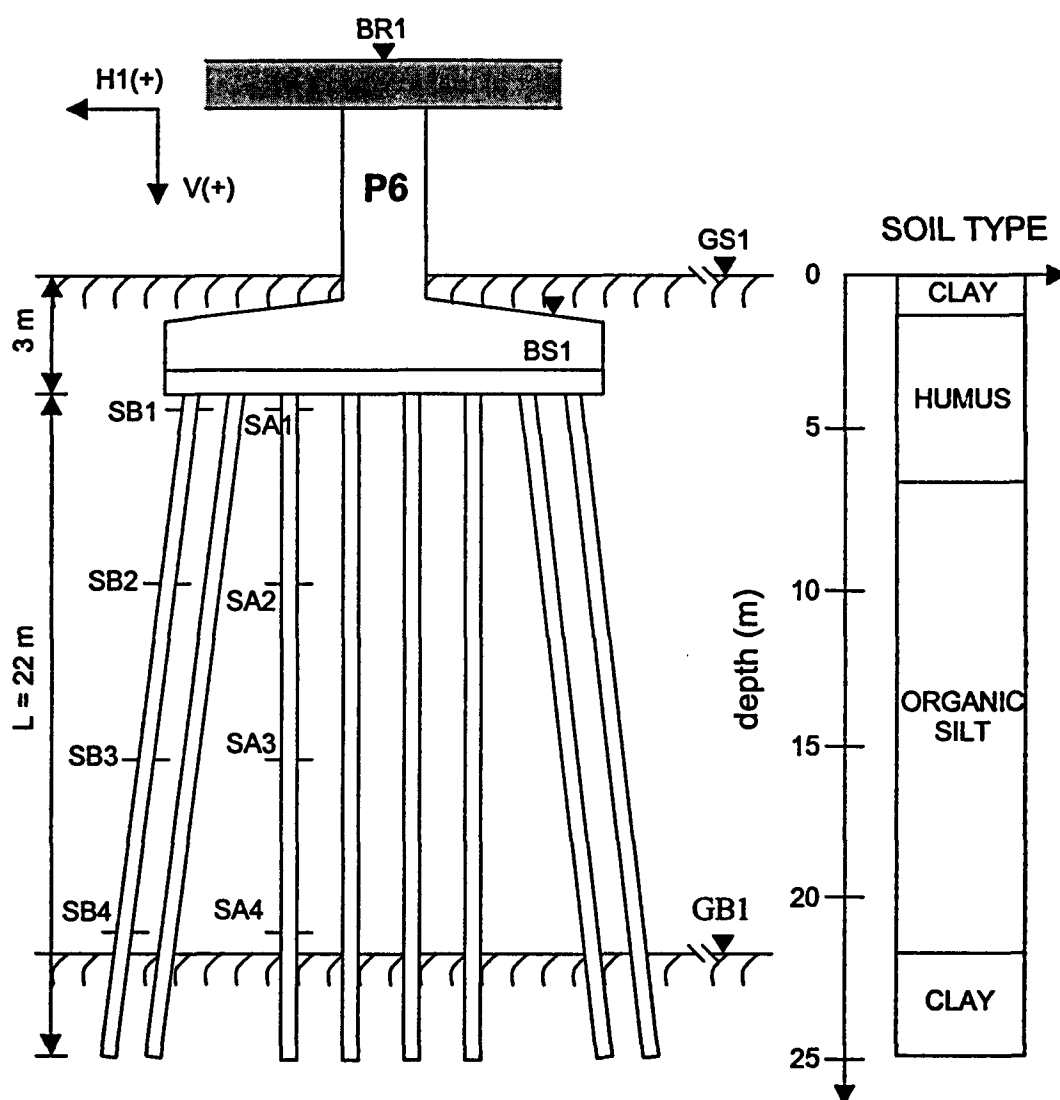


FIGURE 5.3: Cross section at pier 6.

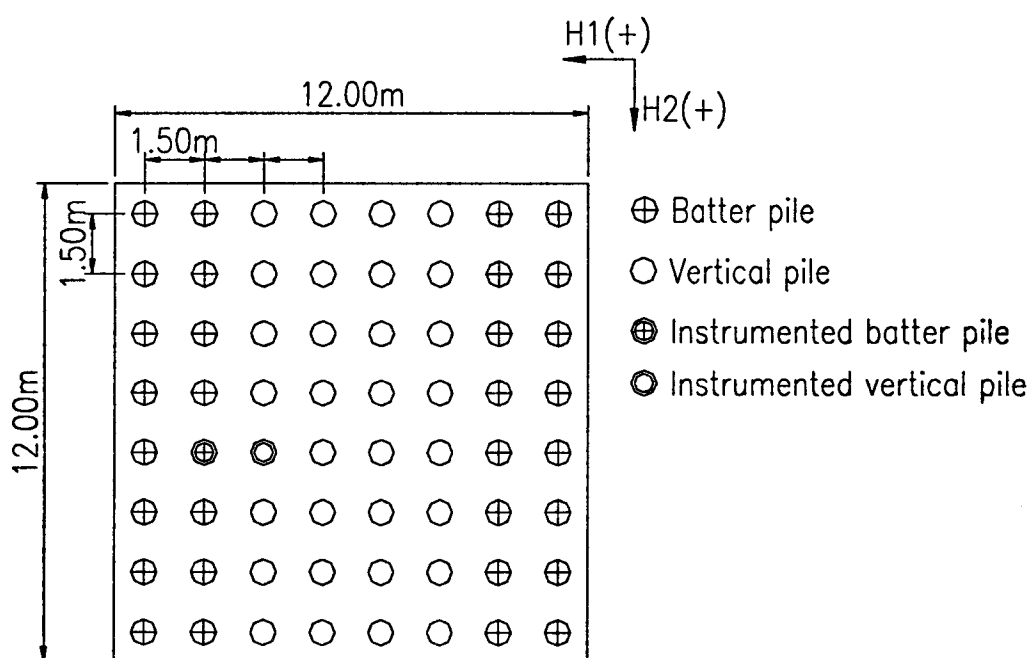


FIGURE 5.4: Pile group configuration (pier 6)

Table 5.3: List of observed earthquakes with peak accelerations in the free field above 0.01g.

Event #	Date	M _{JMA}	Epicentral	Focal	PGA (g%)
			Distance (km)	Depth (km)	
4	2/21/82	6.7	263	0	1.02
7	7/23/82	7.0	238	10	1.85
11	2/27/83	6.0	81	70	3.13
12 [†]	8/8/83	6.0	42	20	11.36
13	2/14/84	5.4	38	20	2.55
14	3/6/84	7.9	666	460	3.6

[†] event considered

April 1985, a total of fourteen earthquakes were recorded (Table 5.3). The most important event (# 12) occurred on August 8, 1983, with JMA magnitude 6, epicentral distance from the site 42km and focal depth 20km. The maximum acceleration at the surface (GS1-H1) was about 0.11g. The work reported herein is based exclusively on records from this earthquake.

5.3 Analysis

Based on the procedures described earlier, the analysis involves the following steps:

- (1) determining the free-field soil response
- (2) analyzing the kinematic response of the pile group
- (3) determining the dynamic impedance (stiffness and damping) of the pile group
- (4) analyzing the inertial response of the foundation-superstructure system

Each step is discussed below.

5.3.1 Free-Field Response

A successful analysis of free-field soil response is an essential step in SSI analyses, to: (a) estimate the input motions to be imposed to the structure; (b) assess the level of nonlinearity in the soil; (c) assess the values of the soil parameters needed in the subsequent soil-pile-superstructure interaction analyses.

In Ohba Ohashi, free-field response has been recorded by five instruments, four at bedrock level (GB1 to GB4) and one near the soil surface close to pier 6 (GS1, see Figure 5.1). Acceleration histories at bedrock (GB1), ground surface (GS1), pile cap (BS1) and bridge deck (BR1) are depicted in Figure 5.5 for both H1 and H2 directions.

Corresponding spectra are given in Figures 5.6 to 5.8. The following trends are noteworthy:

- Rock motions in directions H1 and H2 are similar both in amplitude and frequency content beyond about 1s. The same holds for the ground surface motions. This indicates a similar filtering of seismic waves through the soil in the two directions.
- Rock motions are significantly amplified at the soil surface in both directions. Peak accelerations are amplified approximately 3.4 times in the H1 direction (parallel to bridge axis) and 2.4 times in the H2 direction. This is an indication of multi-dimensional wave propagation, as explained below.

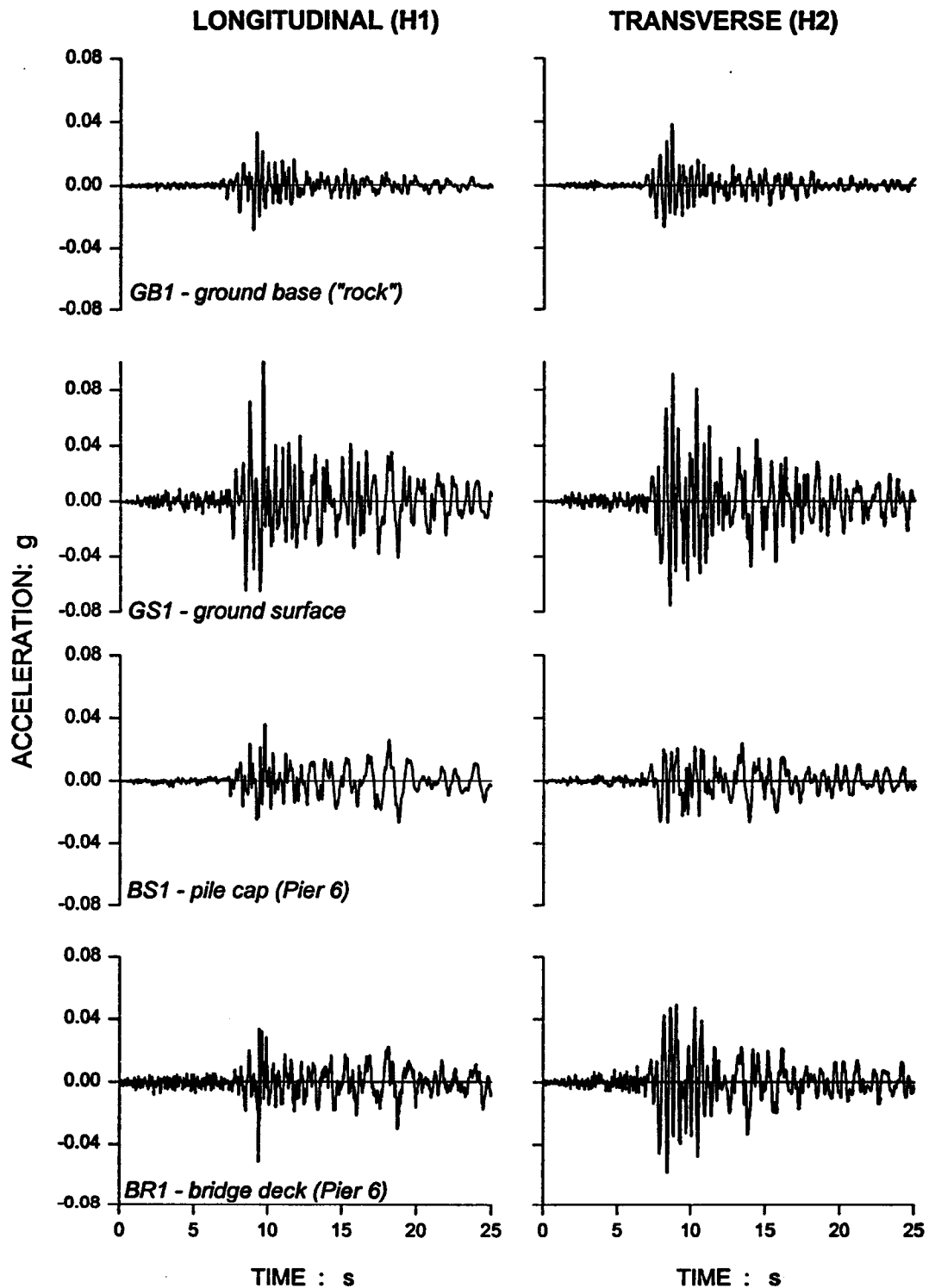


FIGURE 5.5: Selected acceleration recordings from earthquake No. 12. Note the deamplification of motion at pile cap and deck compared to the ground surface.

- The above trends become clearer in Figure 5.7, in which ratios of response spectra between surface and rock are provided for the two recording directions. It is evident that the fundamental natural period of the deposit lies in the range of 1.2 to 1.4s in both directions. Corresponding amplifications exceed the astonishing value of 6, which indicates very small damping in the medium. While the amplification patterns in the two directions are similar beyond approximately 1s, at smaller frequencies amplification is stronger in the H1 than in the H2 direction. This suggests the existence of a multi-dimensional wave field at short wavelengths, which is probably associated with the presence of the valley.
- Significant amplification is observed in the vertical component of the motion (Figures 5.7, 5.8). In this case, the fundamental period seems to be about 0.45s, which suggests an effective wave velocity in vertical oscillations of approximately 3 times the shear wave velocity. This velocity is compatible with that of P waves propagating at a steep angle in an imperfectly saturated soft soil medium.

With reference to analytical modeling, attempts to reproduce the motion at GS1 using one-dimensional wave propagation based on the rock motion at GB1, substantially under-predict the actual surface motion (Gazetas et al 1993). Note that this discrepancy cannot be attributed to high levels of material damping resulting from the use of standard ξ - γ curves for ordinary soils (Figure 5.9). Gazetas and his co-workers showed that values of damping as low as 0.02%, compatible with the imposed level of strain and the large plasticity index of the clays in Ohba Ohashi (up to $I_p = 250\%$) under-predict the peak surface motion by over 300%. Gazetas et al (1993) attribute the above discrepancy to

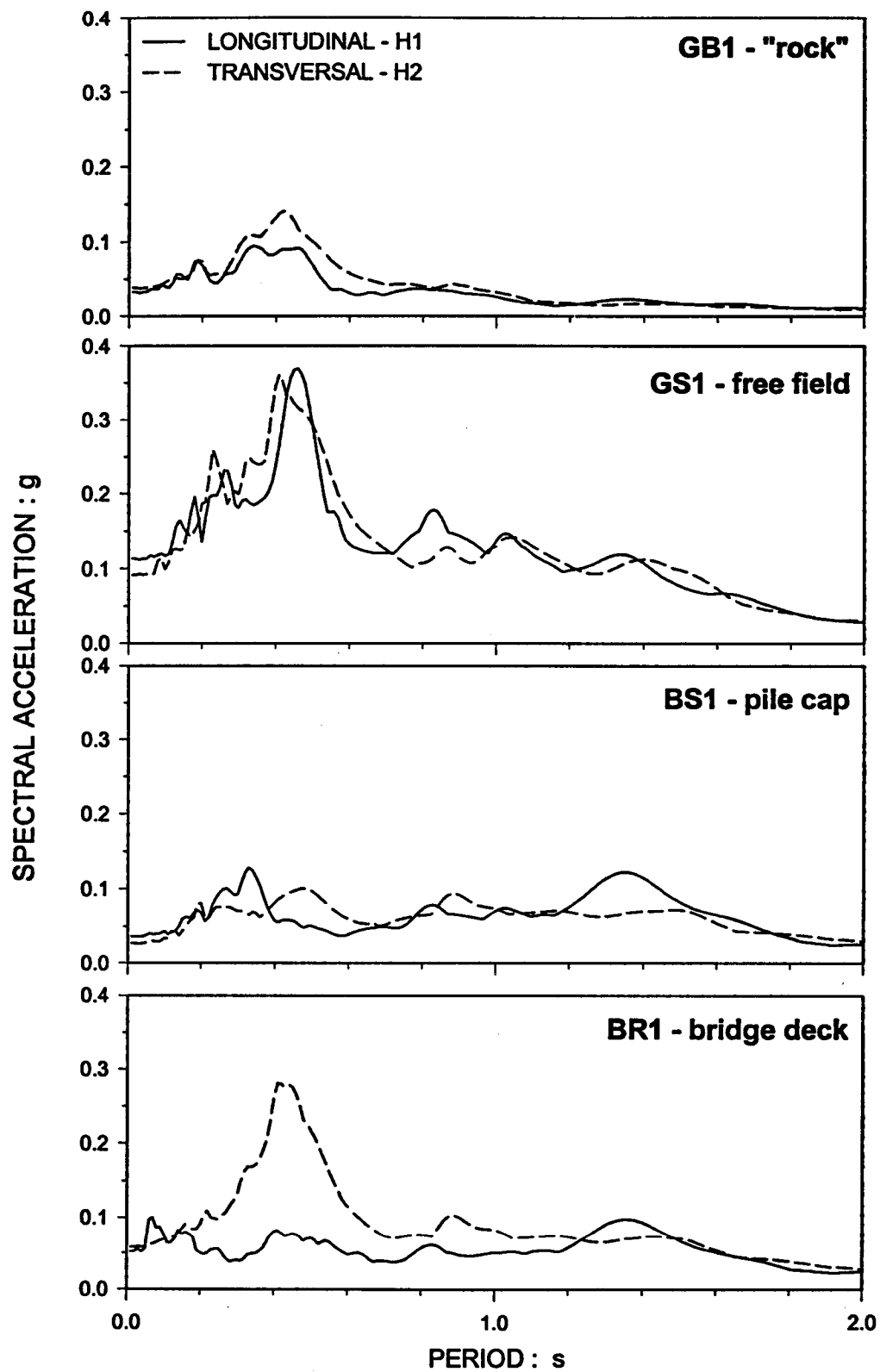


FIGURE 5.6: Elastic response spectra of recorded motions; $\zeta = 5\%$

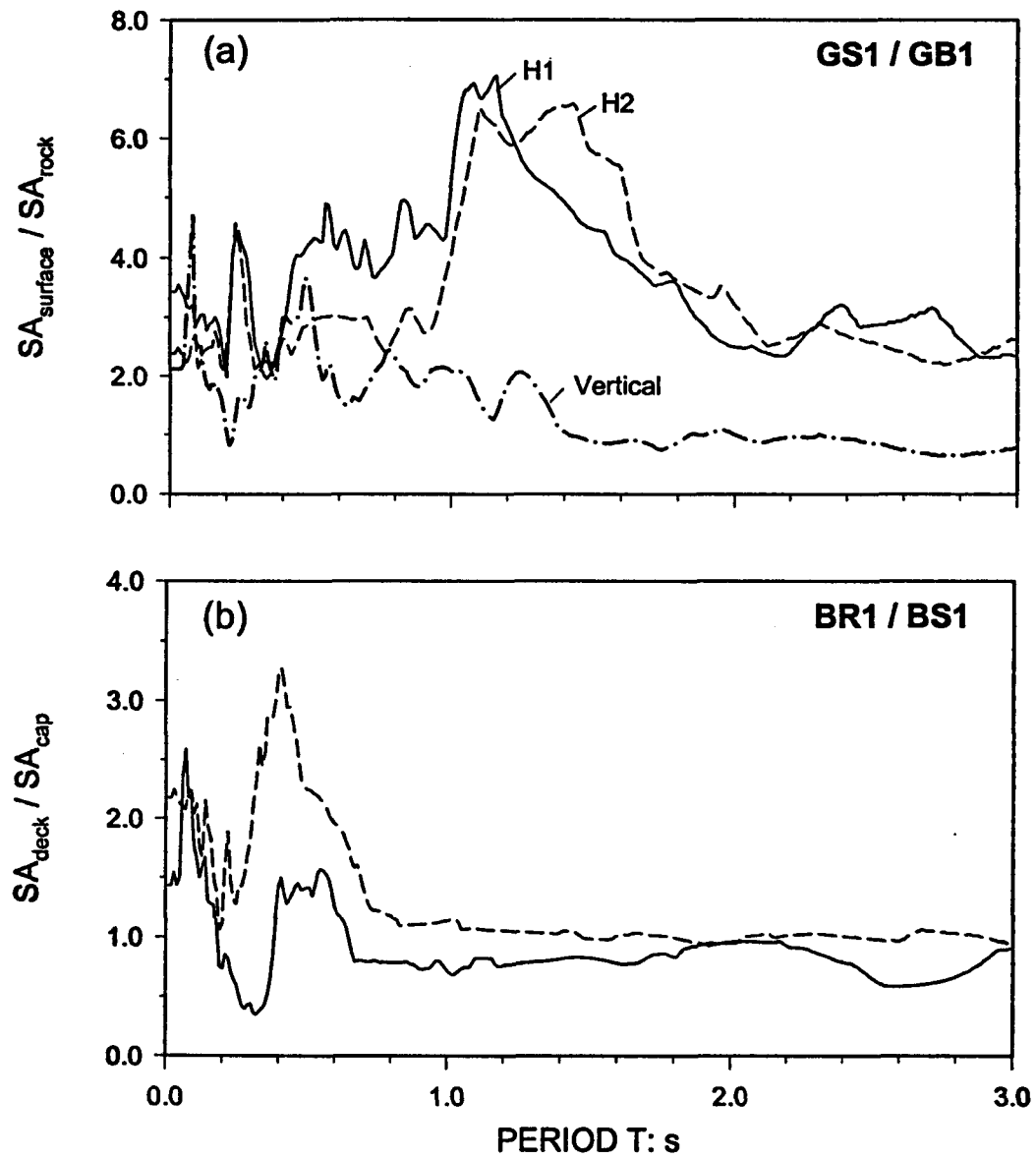


FIGURE 5.7: Elastic response spectra ratio of the recorded motions: (a) free field over rock motion; (b) deck over pile cap motion; $\zeta = 5\%$

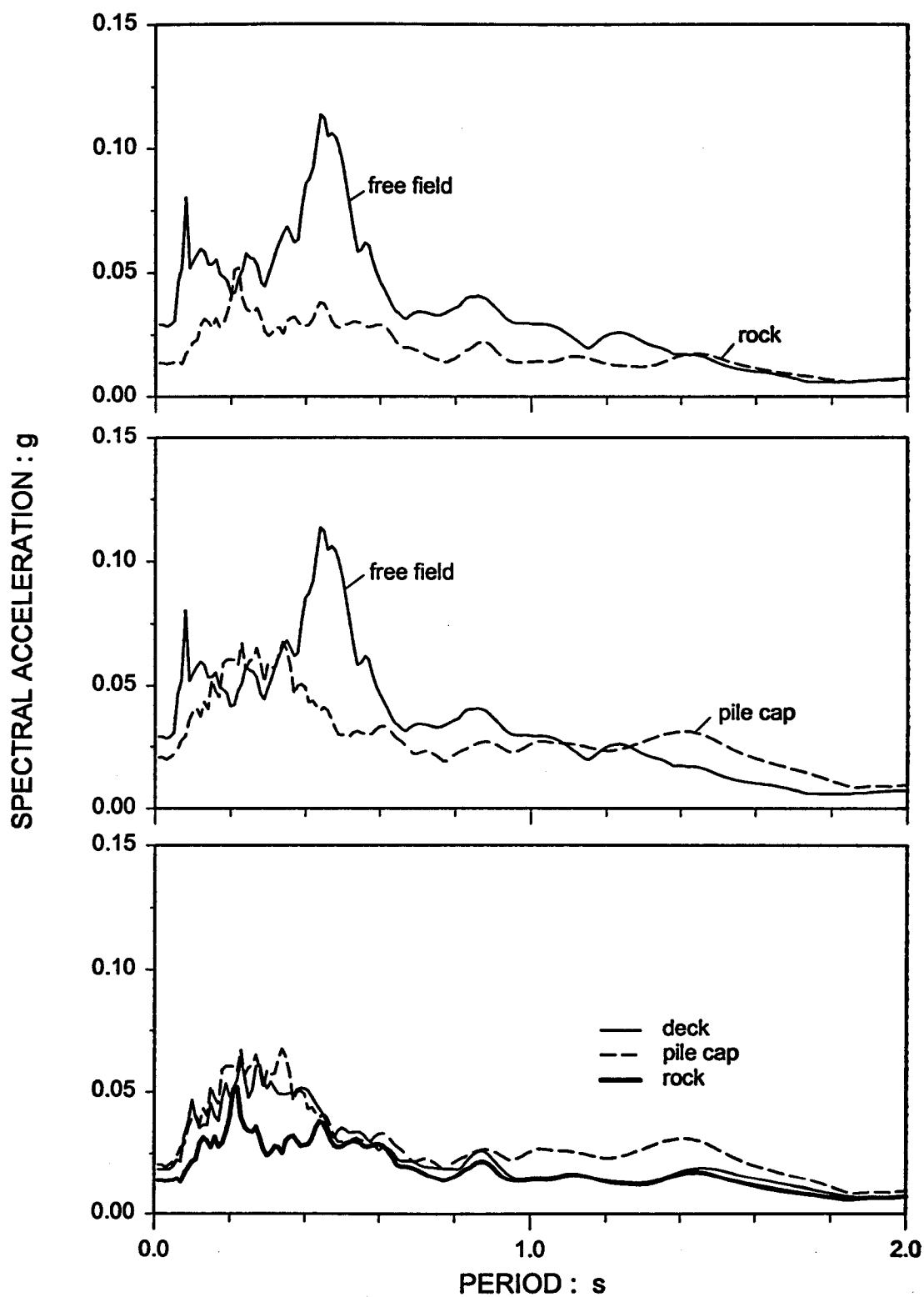


FIGURE 5.8: Elastic response spectra of recorded motions along the vertical direction; $\zeta = 5\%$

geometric factors associated with the 3-D shape of the valley. Indeed, a longitudinal cross section of the ground, depicted in Figure 5.1, reveals that the base of the soft soil deposit is not horizontal (as implicitly assumed in 1-D analyses), but dips at an angle of about 14 degrees close to pier 6. Evidently, this is a relatively narrow alluvial valley, one edge of which has a relatively steep slope.

Empirical and theoretical evidence compiled in recent years indicate that earthquake ground motions at the surface of such valleys are stronger and longer than the motions predicted by 1-D wave propagation theories (Sanchez-Sesma et al 1989). Several wave-propagation phenomena akin to 3-D geometry have been recognized as producing the following deleterious effects:

- (a) wave “focusing” which tends to amplify the motion primarily near the center of the valley;
- (b) “trapping” of obliquely-incident body waves which amplifies the motion experienced near the edges of the valley (Papageorgiou & Kim 1991).

One can qualitatively argue that some of the above-mentioned phenomena must have taken place in the alluvial valley of Ohba-Hashi. On the other hand, a rigorous analysis of the 3-D response of this valley is a formidable task that goes beyond the objectives of this work. Attempts by the Author to capture the phenomenon by adjusting the material properties of the soil, failed. Similar findings have been reported by Gazetas et al (1993). The remainder of this chapter will deal with the prediction of the response of the superstructure (pile cap and deck), not with the response of the foundation that is controlled

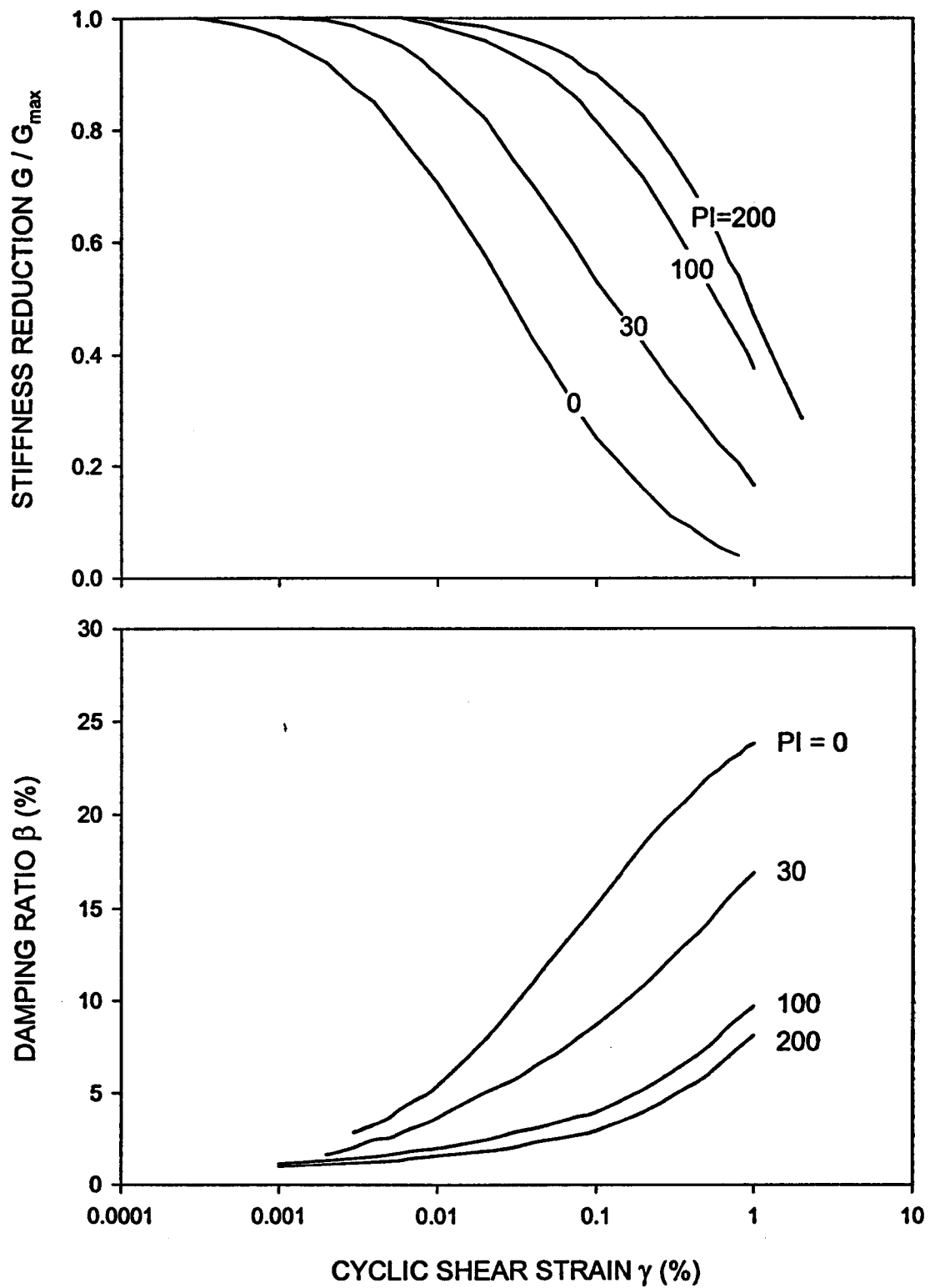


FIGURE 5.9: Relations between $G/G_{\max}$ and β versus γ curves and soil plasticity for normally consolidated and overconsolidated soil (Vucetic & Dobry, 1991).

by the free field motion. It will be shown that although the free field motion is not replicated, the predicted pile cap motion is very close to the recorded one, which provides confidence for a subsequent analysis of the super-structure.

5.3.2 Kinematic Response

From Figures 5.5 and 5.6, the following observations can be made regarding the motion of the pile cap:

- The cap motion in the longitudinal and transversal directions differ both in amplitude and (especially) frequency content (deck motions differ even more). Specifically the motion along direction H1 is stronger both at short and long periods, especially close to the fundamental natural period of the deposit ($\sim 1.2\text{s}$). These discrepancies are anticipated given the different stiffness of the pile group due to the raking of the piles in the longitudinal direction.
- The cap motion is much smaller (by a factor of 2 or so) than the free field motion. This indicates strong interaction between soil and foundation during shaking. A possible is that the soft soil amplifies the rock motion, yet cannot force the stiff foundation to comply. This could be true in cases of rigid foundations under high frequency excitation but can't fully explain the very weak cap motion in this case. Material nonlinearity in the soil cannot also be blamed, given the small intensity of excitation and the high plasticity of Ohba clay. Therefore this reason alone is not sufficient to explain this observation.

- Another possibility is that inertial interaction is responsible for the small motion of the cap. Indeed a flexible foundation tends to move less than the surrounding soil in certain frequencies associated with the natural frequencies of the superstructure (Mylonakis et al 1997, 2004). Analyses did not confirm this scenario.

To gain further insight on the problem, Figure 5.10 depicts acceleration spectra of the recorded motions on rock (GB1), free field (GS1) and pile cap (BS1) along the longitudinal and transverse directions. Three distinct frequency ranges can be identified: (1) long periods (long seismic wavelengths), in which the cap tends to follow the free field motion. This effect is apparent in both H1 and H2 directions. (2) short periods in which cap and rock spectra converge – well separated from the free field spectrum. This suggests that the foundation behaves as a stiff body, connected to rock, which is insensitive to the motion of the soil. (3) intermediate frequencies at which a transition behavior is observed, indicative of interaction between soil and foundation.

To explain the small response of the cap alternative scenarios should be considered. An appealing hypothesis is that the continuous girder connecting piers 5 to 8 reduces the response of the system by averaging the (different) input motions at the bases of the piers. Note that piers 5, 7 and 8 are connected to the deck with sliding bearings. In earlier investigations of the response of the bridge, it was invariably assumed that the bearings were free to slide. However, given that the deck accelerations in the longitudinal direction do not exceed 0.03g, it seems unlikely that the movable bearings would slide. To analyze this case, structural properties should be modified as shown in table 5.1.

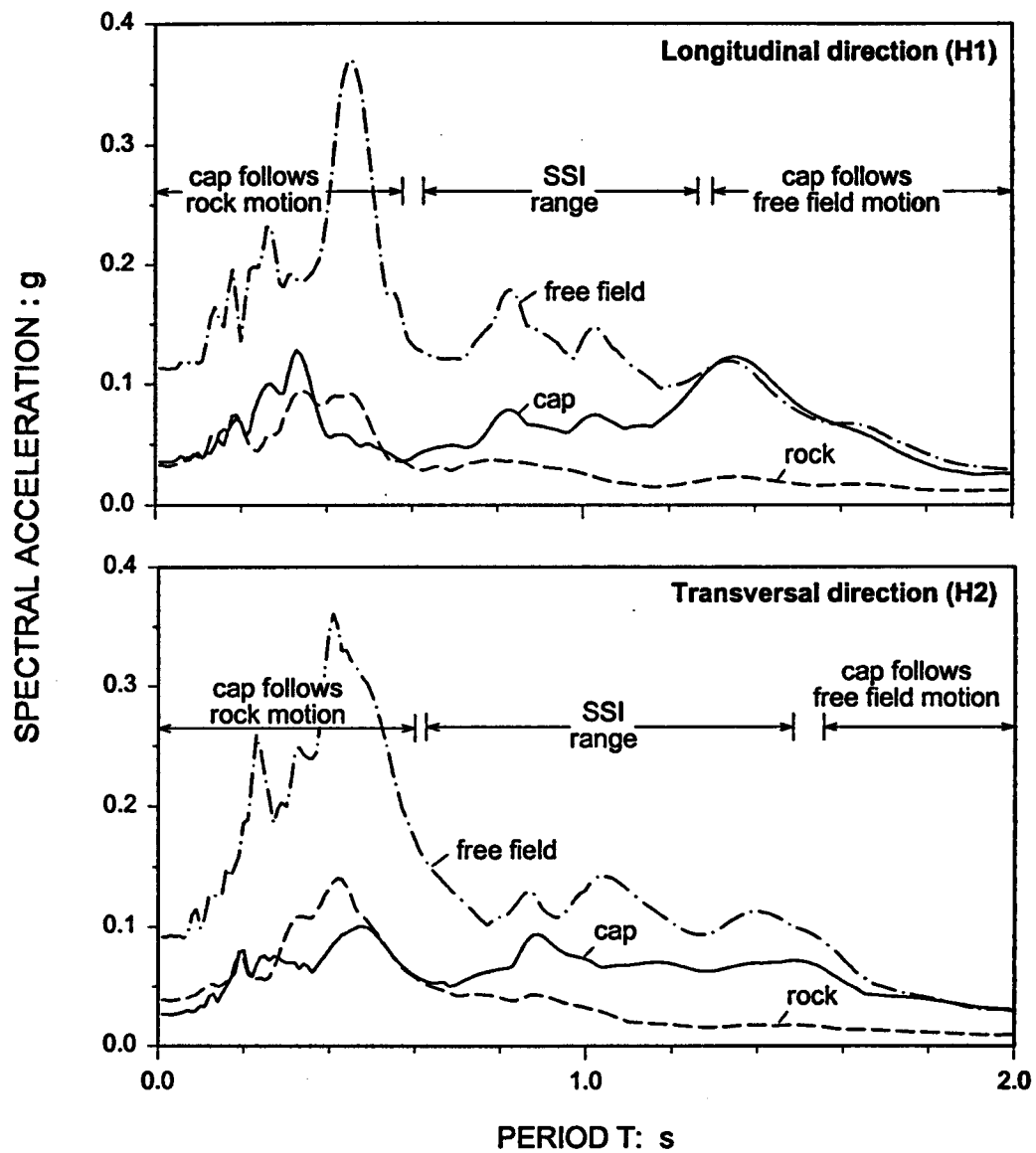


FIGURE 5.10: Comparison of elastic response spectra of recorded motions; $\zeta = 5\%$

In Figure 5.12 the recorded pile cap and bridge deck motions are plotted. Using the recorded motions and a multiply supported simple frame model (Figure 5.11), the foundation input motion (FIM) was back-calculated. This FIM is taking into account the effect of the nearby piers. Using this motion as input to a single pier model (Fig. 5.11) the response of the pile cap and the deck was obtained (Figure 5.12). Evidently, had pier 6 responded as an independent cantilever (as opposed to a part of a frame), cap and deck motions would be similar to the results of the SPIAB analysis (Figure 5.12). This result supports the view that the girder did not in fact slide but stayed connected to the piers and, thereby, suppressed the differential foundation input motion. To account for the influence of the nearby piers a pertinent transfer function ("filter") was introduced to modify the foundation input motion. The filter is essentially a deamplification function used in the Fourier domain to modify the F.I.M. of the single pier model. This transfer function is plotted in Figure 5.13. It can be seen that in most frequencies the ordinates of the transfer function lie below 1.0, suggesting a reduction in FIM due to differential seismic excitation at the base of the piers. Using the above transfer function, the effective foundation input motion was adjusted and used for a complete seismic soil-pile-foundation-structure interaction analysis.

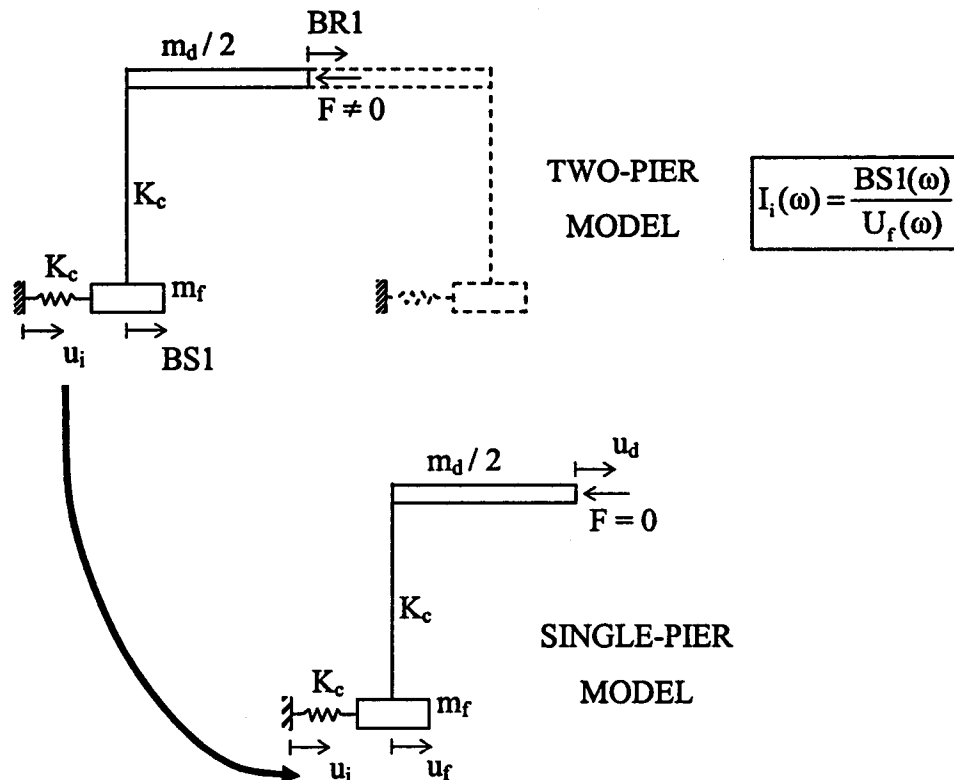


FIGURE 5.11: Procedure for determining non – uniform excitation index I_i . BR1, BS1 = measured deck and footing cap response; u_i = back – calculated input motion; u_d , u_f = computed responses of single – pier model; F = axial deck force in non – uniformly excited two – pier system.

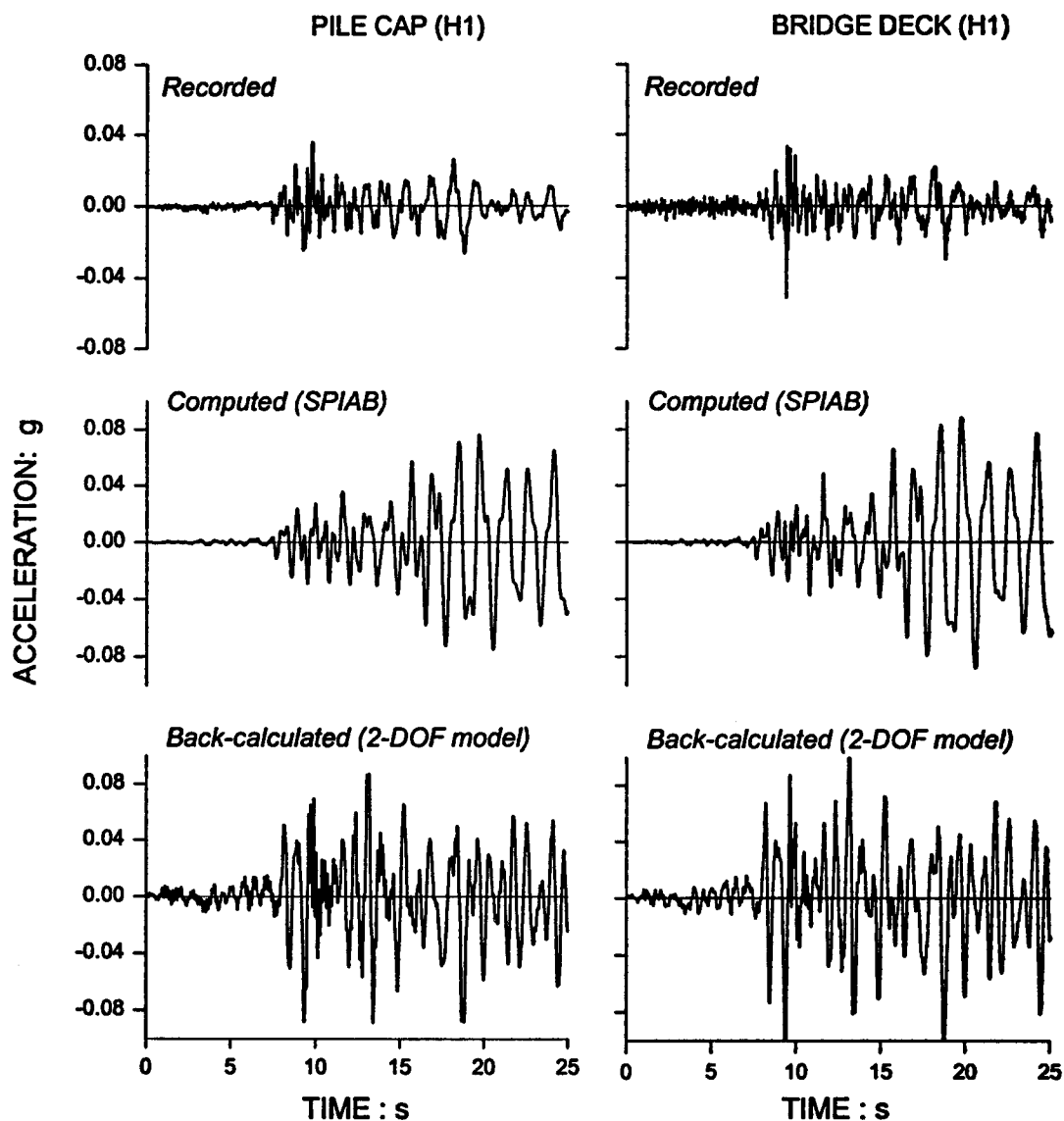


FIGURE 5.12: Recorded and predicted acceleration time histories at the pile cap and the bridge deck at pier 6. Note the overestimated response by ignoring the effect of nearby piers.

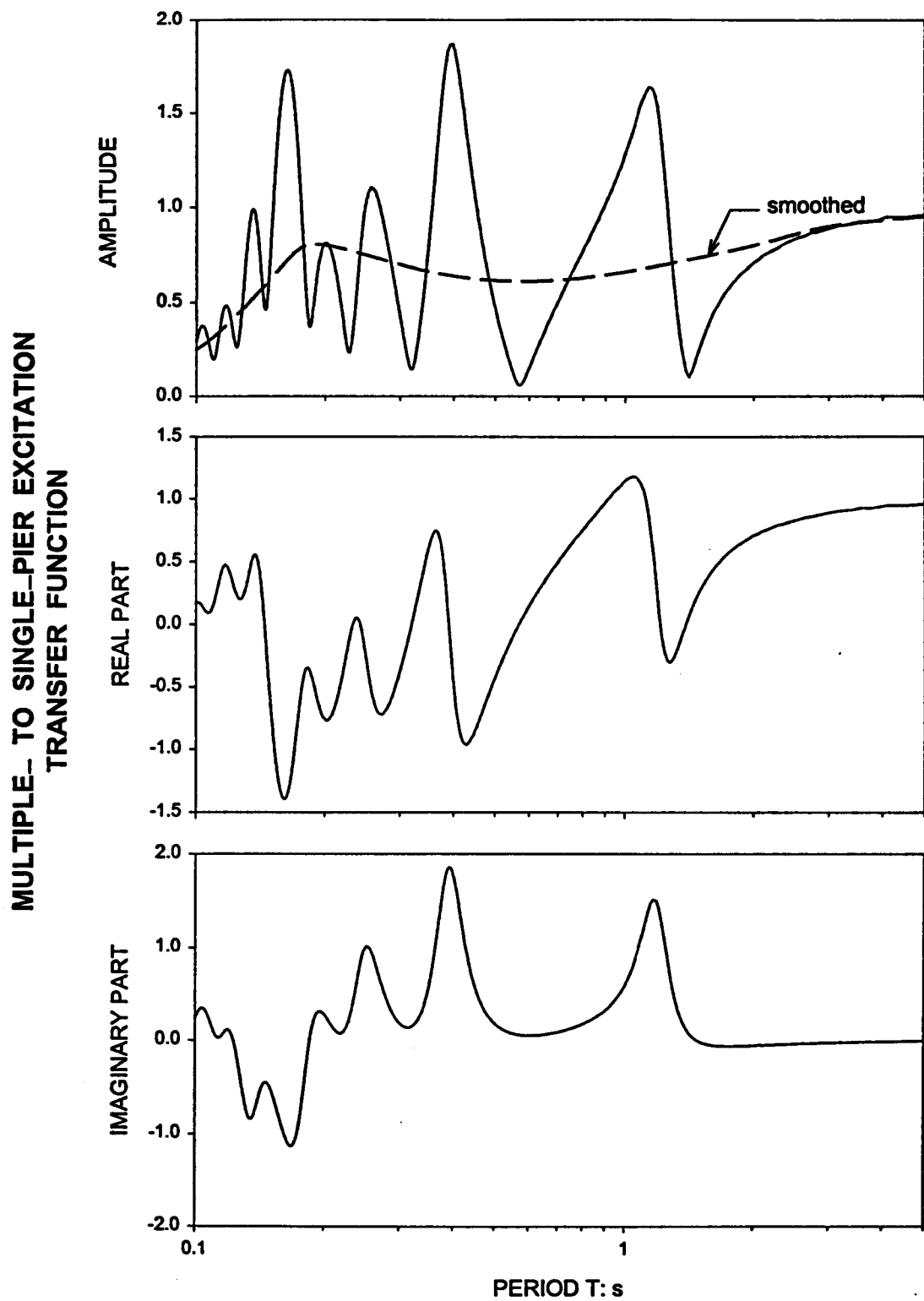


FIGURE 5.13: Transfer function relating kinematic accelerations (effective input motions) for multiple and single support excitation of the system.

5.3.3 Dynamic Impedance of Pile Foundation

In the analysis, the batter piles were treated as vertical elements with higher bending stiffness ($E_p I$) in the longitudinal direction of the bridge. For the 8x8 pile group, pile spacing of 1.5m was used between batter and vertical piles in the transversal direction. Following Poulos and Davis (1980), a higher spacing of 2.5m between batter and vertical piles was considered in the longitudinal direction.

The dynamic stiffness and damping functions (dynamic impedance) of a single pile in the pile group were determined using the finite-element formulation K-PAX, as described in Chapters 2 and 3. Frequencies ranging from 0 to 10 Hz were analyzed. Figure 5.14 shows the stiffness coefficients of a single vertical pile using two different damping ratios for the soil material. It can be seen that the results are very close to those obtained by Fan (1992) using a boundary-element type solution (Kaynia and Kausel 1982). For simplicity and given the low levels of imposed strains, a conservative value of 0.2 was selected for the inhomogeneity coefficient A . As expected, the real and imaginary parts of the impedances decrease with the introduction of radial inhomogeneity. Figures 5.15 and 5.16 display the stiffness coefficients of a batter pile (treated as a single pile) along directions H1 and H2. Note that raking is along the H1 direction, which explains the corresponding increase in swaying stiffness.

Figure 5.17 shows the attenuation of ground displacement (function ψ) around a vertical pile. These attenuation functions are the base of the superposition method, which

is used to determine the stiffness and damping of a pile group. The method is well known and need not be repeated here (For a brief description see Appendices C and D).

Figures 5.18 and 5.19 show the dynamic stiffness coefficients of the pile group. As expected, the behavior is strongly frequency dependent. Comparing with the solutions by Fan (1992) and Mylonakis (1995), it can be seen that the results are in satisfactory agreement. The real parts of the vertical, rocking and cross swaying rocking stiffness differ from method to method, but the imaginary parts are similar. This difference is due to the differences in the interaction factors employed by each method. The differences are pronounced because of:

- the small pile spacing ($s/d = 2.5$)
- the large number of piles
- the small differences in the interaction factors
- differences in pile group geometry to account for batter piles

The stiffness coefficient that influences the group response the most is the swaying stiffness for which all analyses are in close agreement.

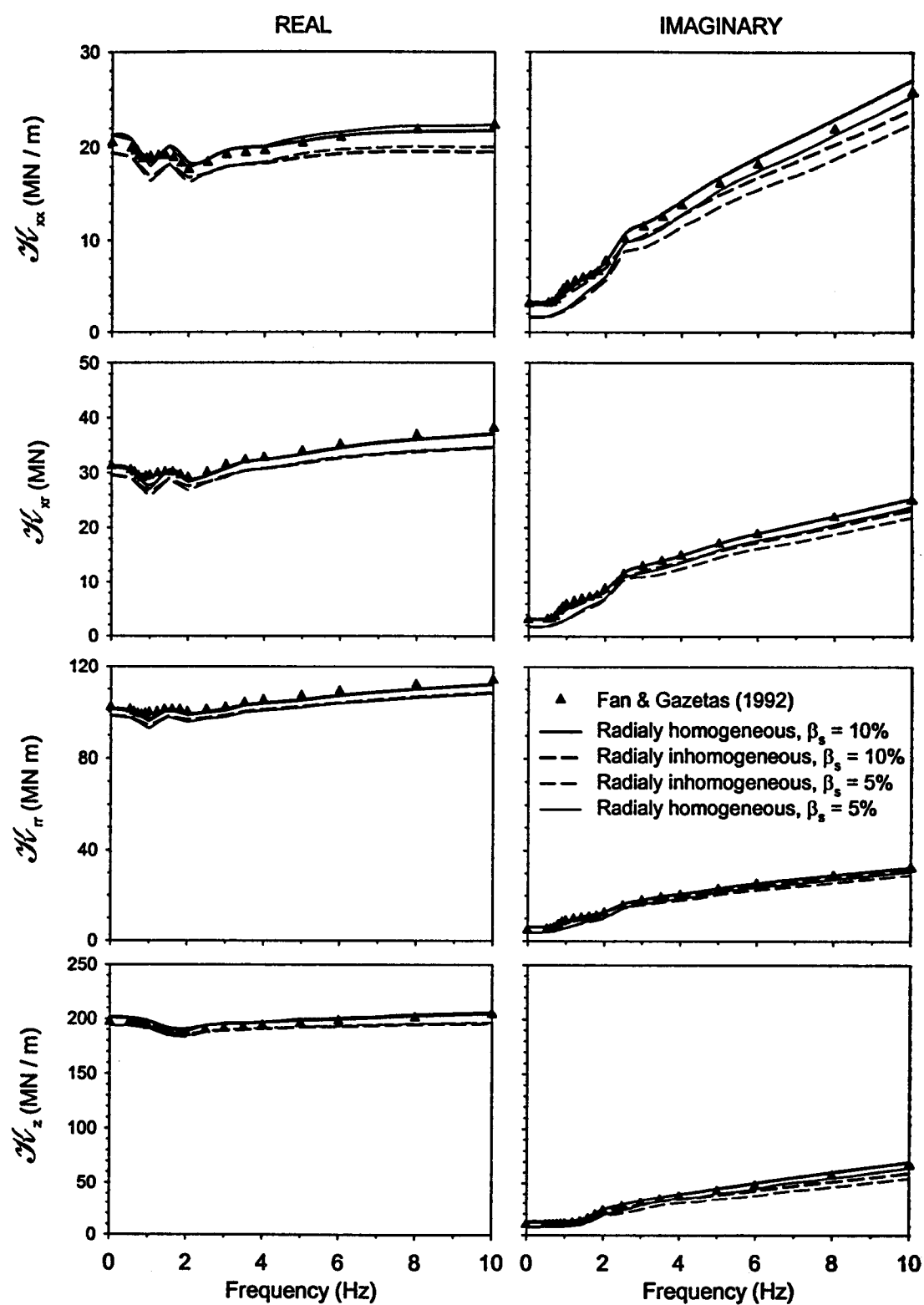


FIGURE 5.14: Real and imaginary parts of pile stiffness for a single vertical pile of pier 6.

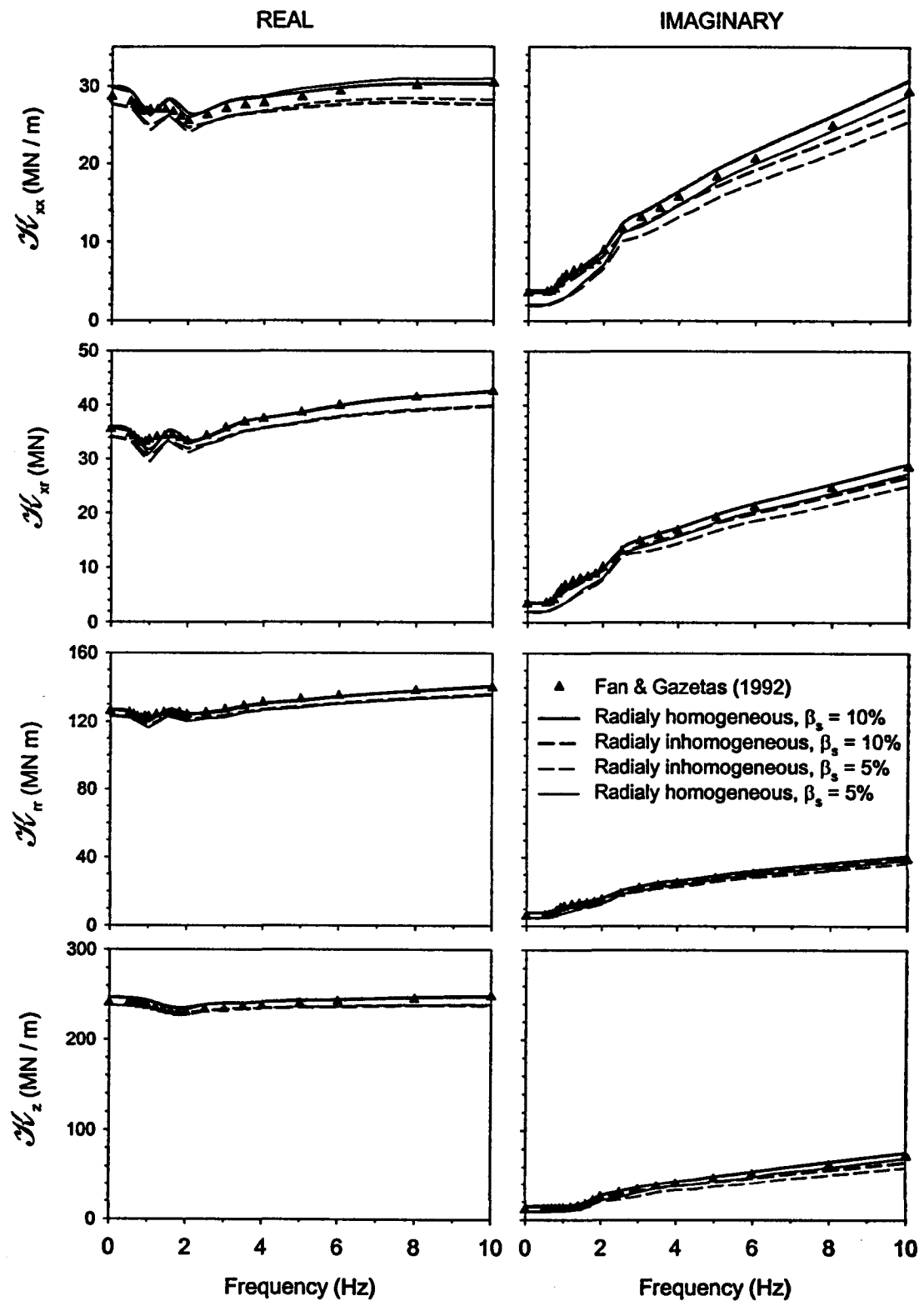


FIGURE 5.15: Real and imaginary parts of pile stiffness for a single batter pile of pier 6 in the longitudinal direction (H1). [Coefficients refer to the global reference system]

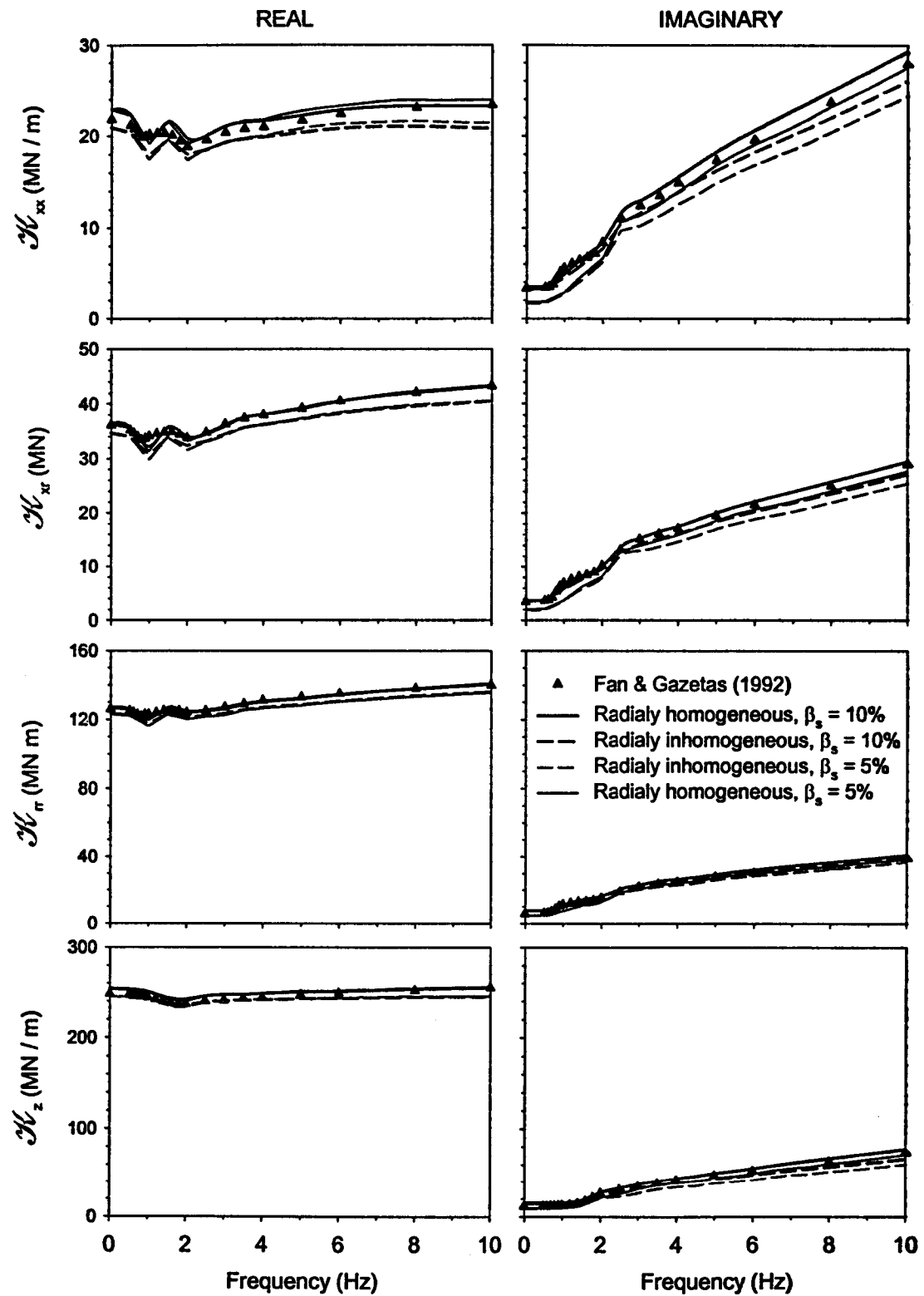


FIGURE 5.16: Real and imaginary parts of pile stiffness for a single batter pile of pier 6 in the transversal direction (H2). [Coefficients refer to the global reference system]

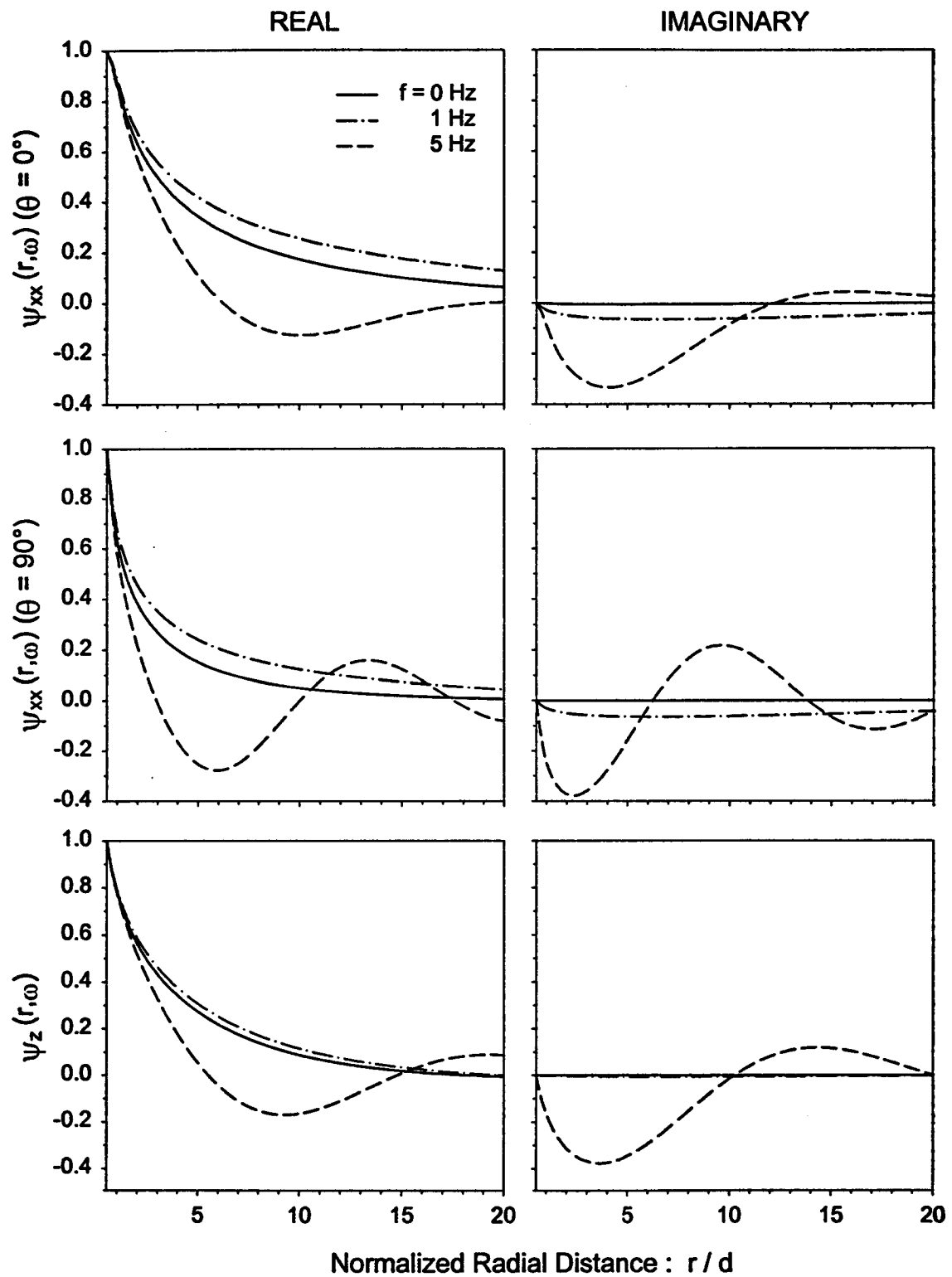


FIGURE 5.17: Attenuation functions of surface displacement around a single vertical pile, for different frequencies.

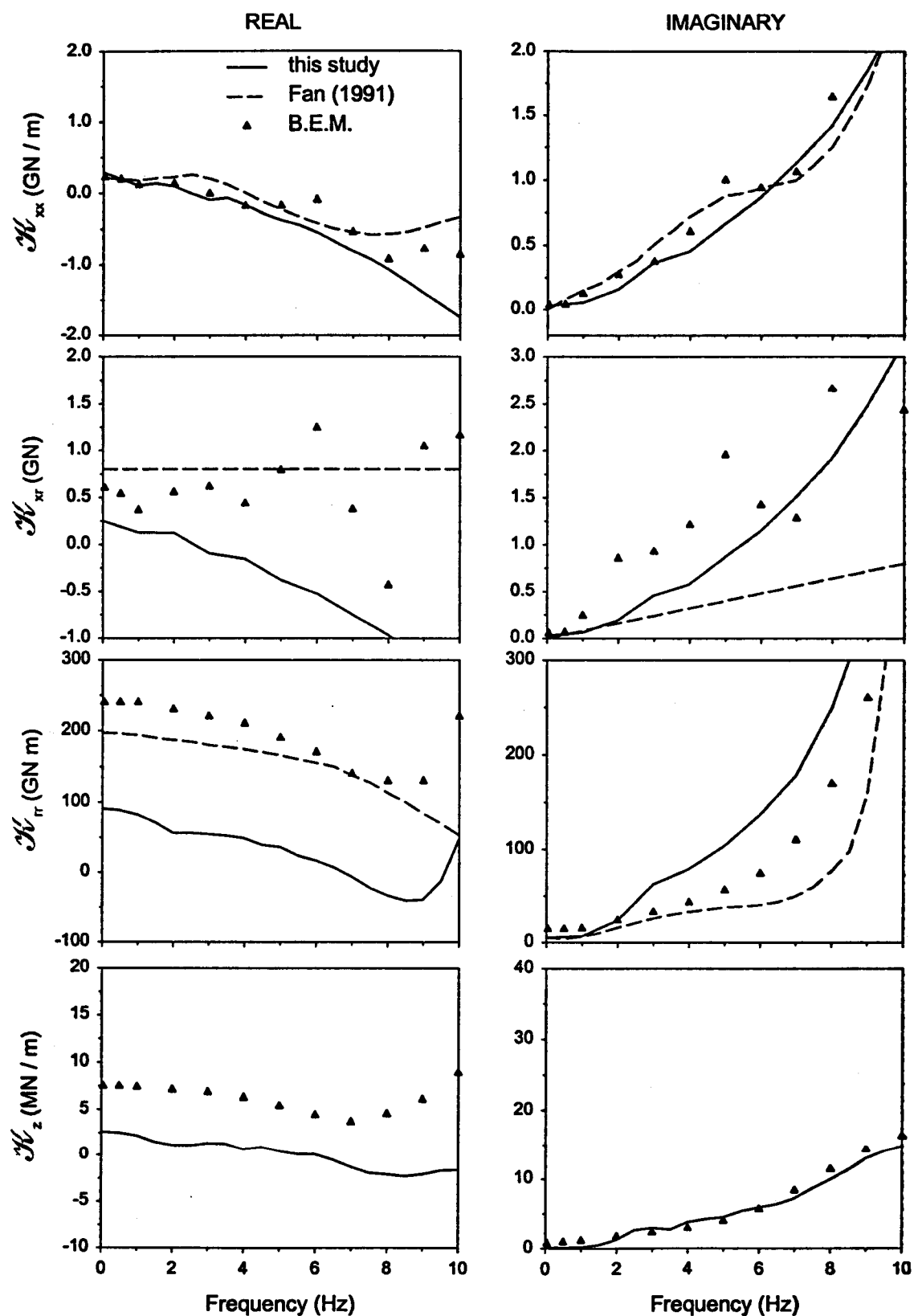


FIGURE 5.18: Dynamic impedance of pile group in the longitudinal direction (H1)

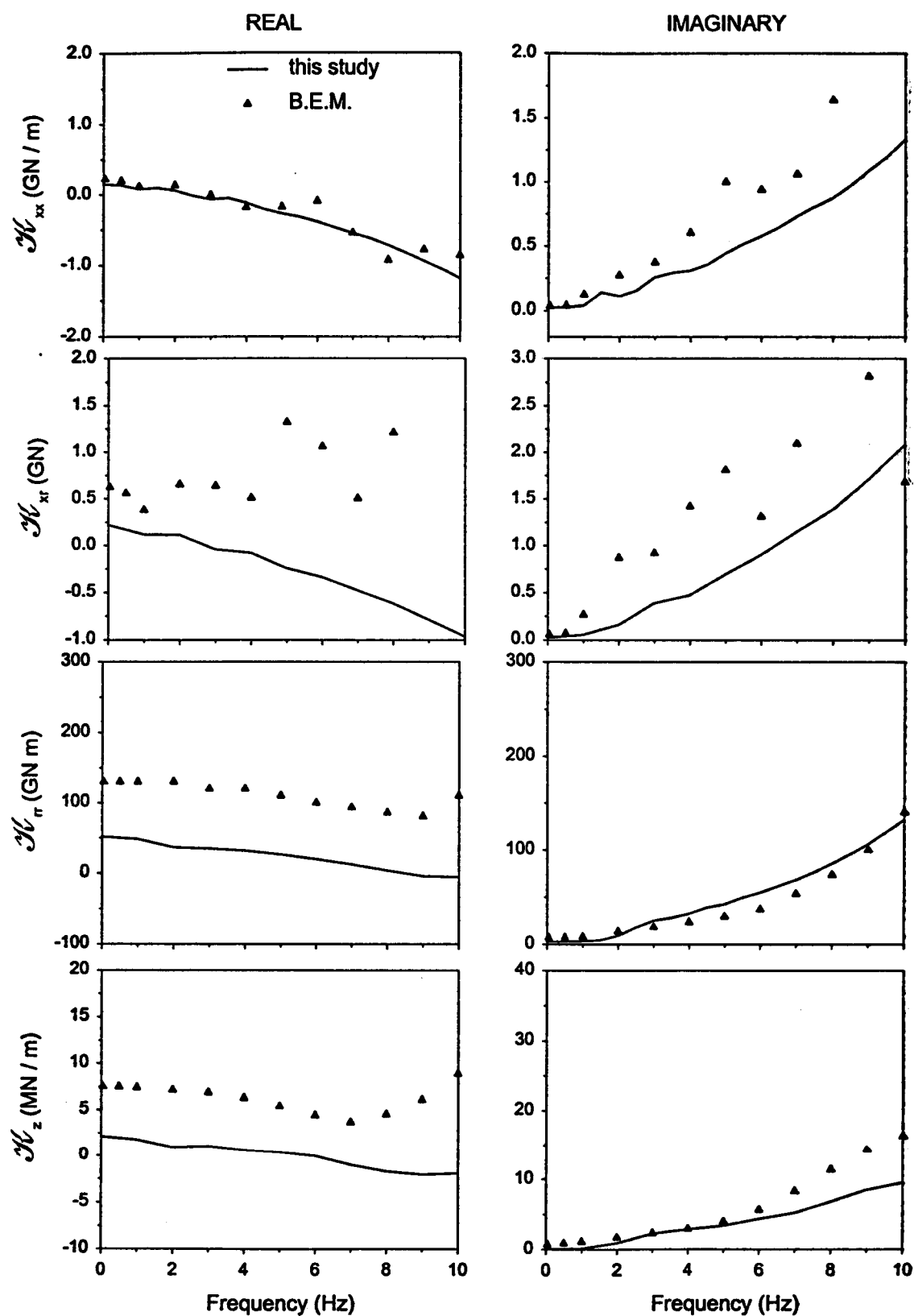


FIGURE 5.19: Dynamic impedance of pile group in the transversal direction (H2).

5.3.4 Complete Seismic Analysis of the Soil-Pile-Foundation-Bridge System

Having determined the two components (translation U_k and rotation ϕ_k) of the kinematic response of the foundation (i.e., the 'Foundation Input Motion), and (ii) the pile-group dynamic impedances $[\mathcal{H}_{ij}]$, the response of the system, namely the displacement and rotation of the pier top (deck) and bottom (pile cap), can be computed from (Fig. 5.20)

$$\begin{Bmatrix} U_s \\ \phi_s \\ U_b \\ \phi_b \end{Bmatrix} = ([K] - \omega^2 [m])^{-1} \begin{Bmatrix} 0 \\ 0 \\ [\mathcal{H}_{ij}] \begin{Bmatrix} U_k \\ \phi_k \end{Bmatrix} \end{Bmatrix} \quad (5.1)$$

where

$$[K] = \begin{bmatrix} [K_{ss}] & [K_{sb}] \\ [K_{bs}] & [K_{bb}] + [\mathcal{H}_{ij}] \end{bmatrix} \quad (5.2)$$

and

$$[m] = \begin{bmatrix} [m_{ss}] & [0] \\ [0] & [m_{bb}] \end{bmatrix} \quad (5.3)$$

in which $[K_{ss}]$, $[K_{sb}]$, $[K_{bs}]$, and $[K_{bb}]$ denote the stiffness submatrices corresponding to the superstructure (s) and the base (b) $[m_{ss}]$, $[m_{bb}]$ are the mass submatrices of the superstructure and the base, respectively.

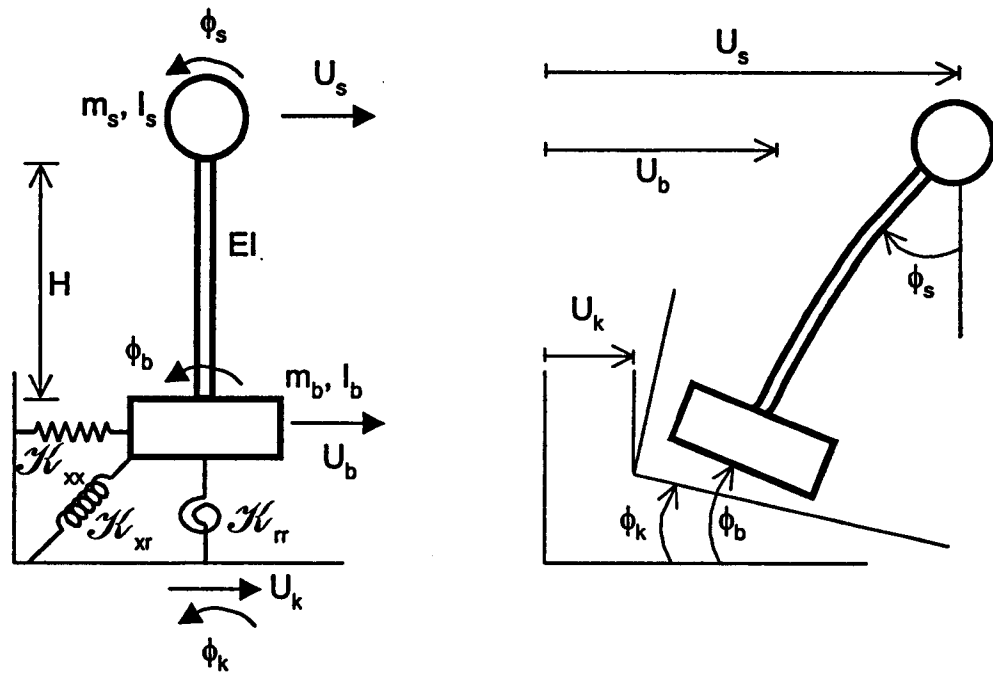


FIGURE 5.20: The model for the analysis of soil-structure interaction: (1) model parameters and notation; (2) sketch of response in terms of absolute displacements and rotations.

The properties of the pile cap and the pier are shown in Table 5.2, for both horizontal directions. The pier is considered free to rotate along the H1 direction but is restricted from rotating along the H2 rotation.

Using the computer code SPIAB (Mylonakis, 1995) the response of the cap and the deck was computed. In Figures 5.21 and 5.22 the predicted time histories at the cap and the deck are compared with the recorded. It can be seen that recordings and predictions are in very good agreement. It can also be seen that the high frequency content of the records is not replicated. This is evident in the longitudinal direction (H1).

In Figure 5.23 the acceleration spectra are compared to show more clearly the similarities and differences of the recorded and predicted response for a wide range of frequencies. The responses of the pile cap and the deck are very close to the recorded for both directions. The relatively small differences can be attributed to the simplified stick model employed for the pier.

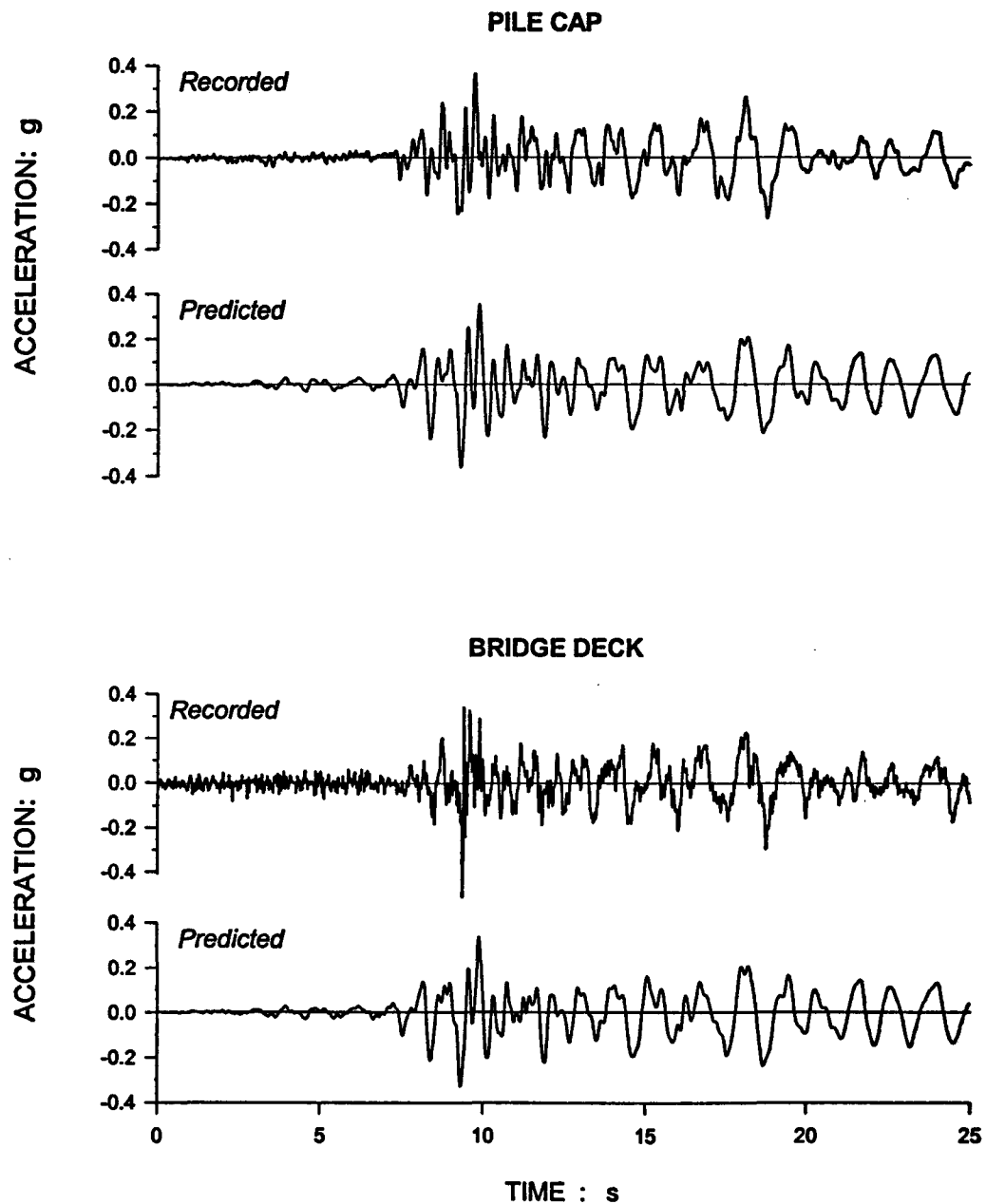


FIGURE 5.21: Predicted and recorded acceleration time histories at pile cap and bridge deck in the longitudinal (H1) direction. Note the presence of high frequencies in the deck recording.

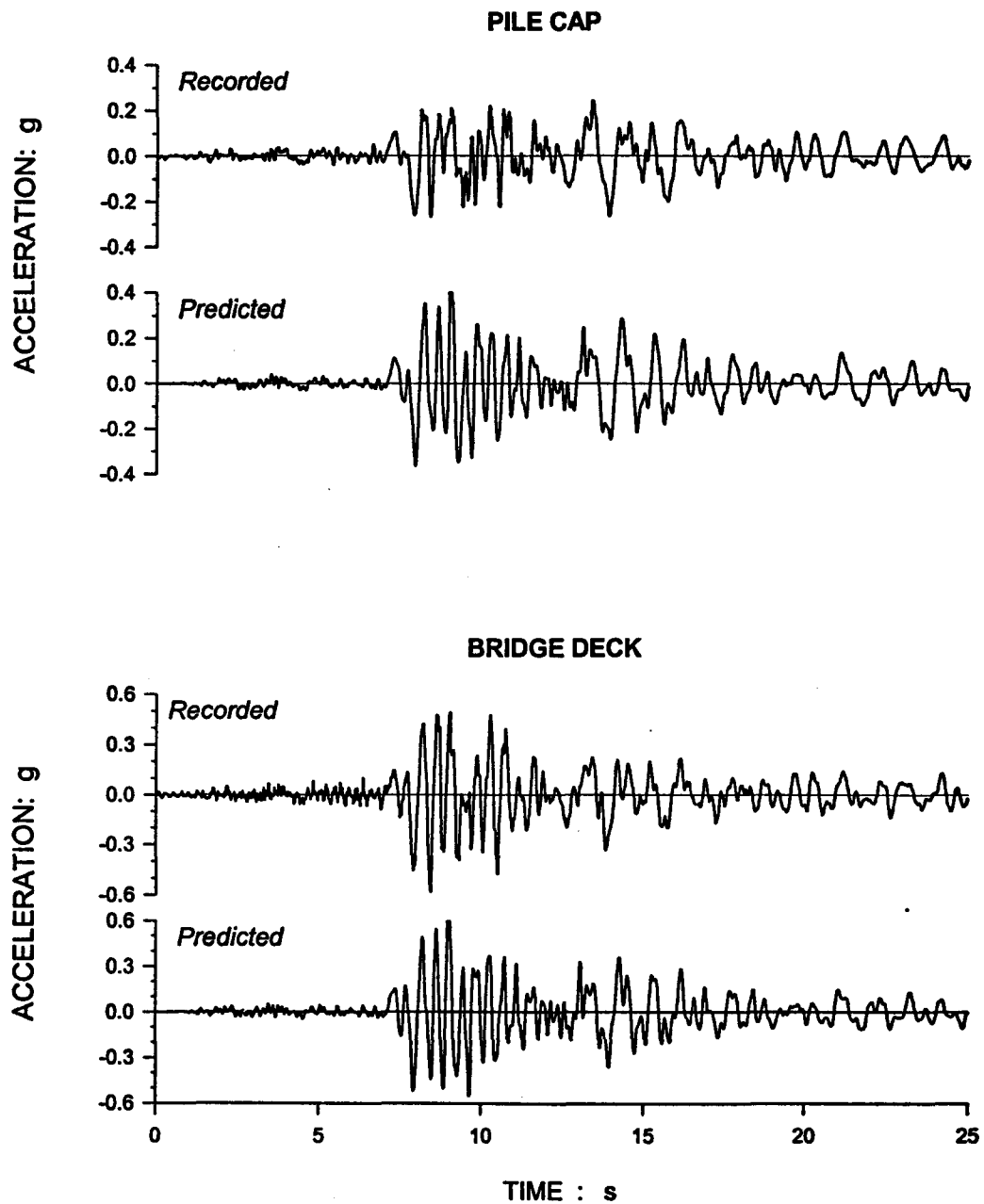


FIGURE 5.22: Predicted and recorded acceleration time histories at pile cap and bridge deck in the transversal (H2) direction.

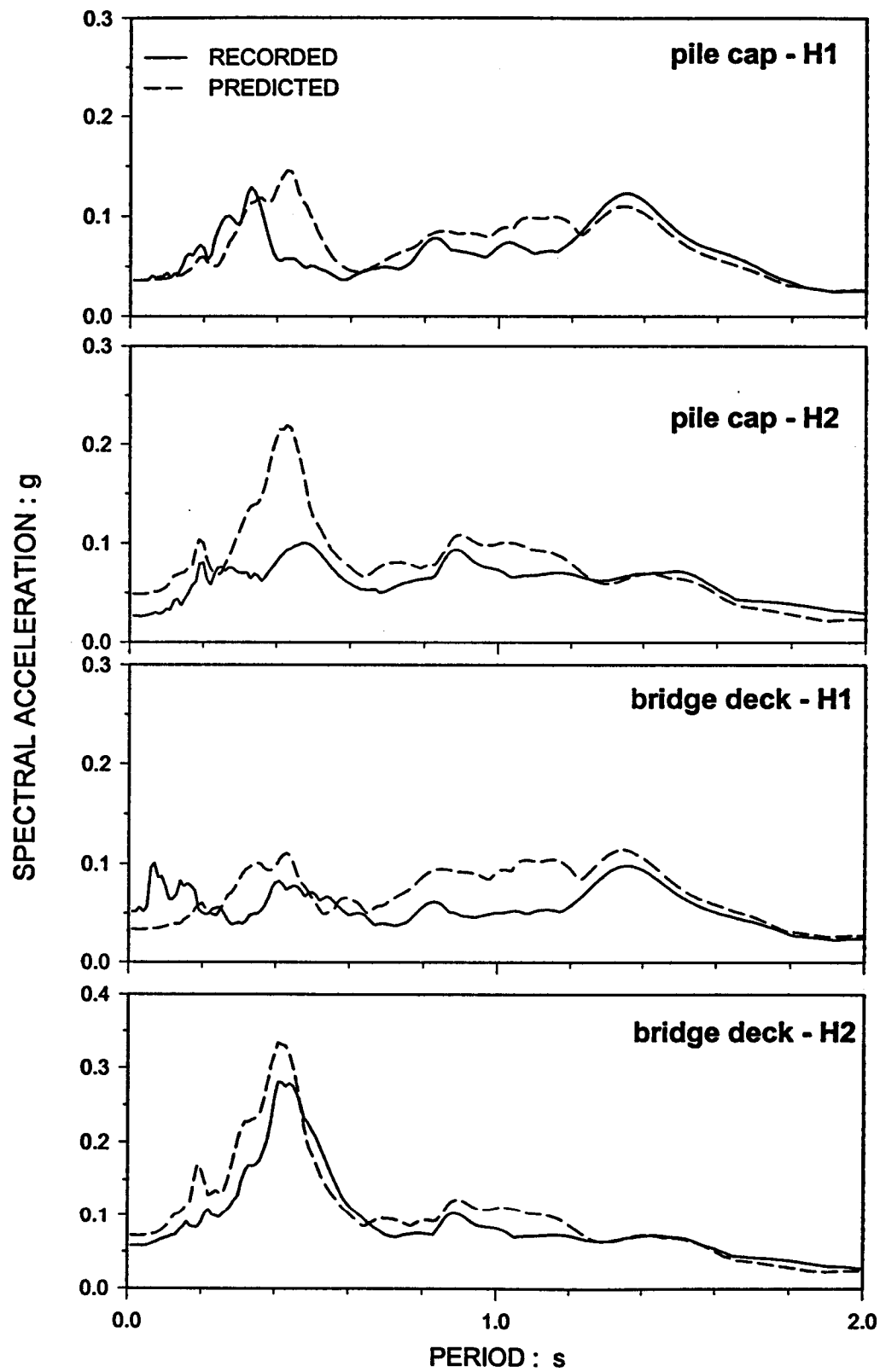


FIGURE 5.23: Elastic response spectra of recorded and predicted motions; $\zeta = 5\%$

CHAPTER 6

CASE STUDY II: THE ROLE OF SOIL AND SOIL-STRUCTURE INTERACTION IN THE COLLAPSE OF HANSHIN EXPRESSWAY, KOBE, 1995

6.1 SUMMARY

The collapse of a 630m segment (Fukae section) of the elevated Hanshin Expressway during the 1995 Kobe earthquake is investigated. The earthquake has, from a geotechnical viewpoint, been associated with extensive liquefactions, lateral soil spreading, and damage to waterfront structures. A complete soil- structure interaction (SSI) analysis of the bridge is performed accounting for non-linearities in the soil and the superstructure and evidence is presented that SSI in non-liquefied ground played a *detrimental* role in the seismic performance of this major structure. The findings of the previous chapters are implemented in the analysis and new simplified analysis methods are presented for evaluating the seismic response of bridge piers. The bridge consisted of single circular concrete columns monolithically connected to a concrete deck, founded on groups of 17 piles in moderately soft sand and gravel. There were 18 spans in total, all of which suffered a spectacular pier failure and transverse overturning. Several factors associated with poor structural design have already been identified. The scope of this chapter is to extend the previous studies by investigating the role of soil in the collapse. The following issues are examined: (1) seismological and geotechnical information pertaining to ground motions at the site; (2) free-field soil response; (3) response of soil-foundation-superstructure system; (4) evaluation of results against field data and earlier studies that did not consider SSI. Results indicate that the role of soil in the collapse was

triple: First, it modified the bedrock motion so that the frequency content of the resulting surface ground motion became disadvantageous for the particular structure. *Second*, the compliance of soil and foundation altered the vibrational characteristics of the bridge and moved it to a region of stronger response. *Third*, the compliance of the foundation increased the participation of the fundamental mode, inducing stronger response. It is shown that the increase in inelastic seismic demand in the piers may have exceeded 100% in comparison with piers fixed at the base. These conclusions contradict the widespread view of an always-beneficial role of seismic SSI.

6.2 INTRODUCTION

The Hyogo-Ken Nanbu earthquake struck the city of Kobe on Tuesday January 17, 1995, exactly one year after the severe earthquake in Northridge, California. It is the first major earthquake to hit a modern city with a high concentration of population and urban facilities. The event had a magnitude $M_s = 6.8$ ($M_w = 6.9$; $M_{JMA} = 7.2$) and occurred at a shallow depth right under the city; it resulted in the worst earthquake-related disaster in Japan since the Kanto earthquake of 1923: 5,500 lives were lost, 35,000 people were injured and over 150,000 buildings collapsed or were damaged beyond repair. The port of the city, which was of considerable importance to the Japanese economy, was destroyed almost completely. The overall economic loss has been estimated at U.S. \$100 billion (Kimura, 1996). Major reports on engineering aspects of the earthquake have been published by Akai et al. (1995), EERI (1995), Werner and Dickenson (1995), NIST (1996), JGS (1996; 1998).

The earthquake came as a surprise, not only because it hit a relatively *aseismic* (by Japanese standards) geographic region, but primarily because of the severe ground motions that were recorded – much stronger than in any previous earthquake in Japan.

In the devastation caused by the earthquake, the collapse and transverse overturning of the 630m section of Hanshin Expressway at Fukae was perhaps the most spectacular failure and became the “trademark” of the disaster in the press (Figure 6.1). The bridge was part of elevated Route 3 that runs parallel to the shoreline. Completed in 1969, it consisted of single circular columns, 3.1 to 3.3m in diameter and approximately 12m in height, monolithically-connected to a concrete deck and founded on groups of 17 piles. The main geometric and material characteristics of the bridge are depicted in Figures 6.2a,b.

Detailed investigations of the seismic performance of the superstructure are available (Seible et al. 1995, Park 1996, Kawashima and Unjoh 1997, Anastasopoulos 1999, Sun et al. 2000, Abe et al. 2000, Sun et al. 2003). In these studies, several factors associated with poor structural design have been identified including: (1) inadequate transverse reinforcement in the piers; (2) inadequate anchorage of longitudinal reinforcement; (3) use of non-conservative (elastic) methods for determining design shear forces in the columns. Notwithstanding the importance of these findings, there is evidence presented in this chapter that local soil conditions and dynamic interaction between foundation and superstructure further aggravated its inelastic behavior, thereby contributing to the collapse.

An additional concern comes from the fact that SSI has been traditionally considered as invariably *beneficial* for seismic response. Apparently, this perception

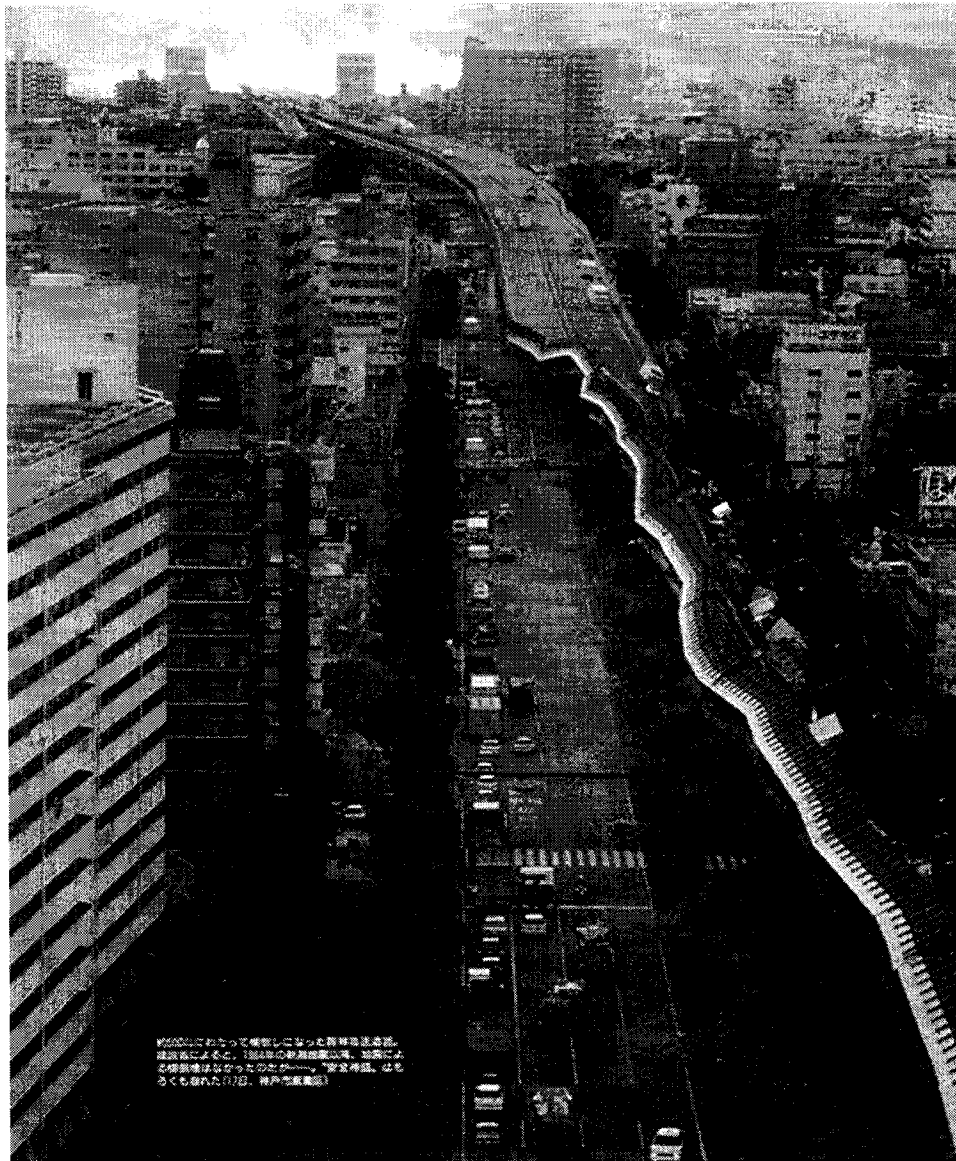


FIGURE 6.1: The collapsed Fukae Bridge of Hanshin Expressway (courtesy of NISEE, University of California, Berkeley)

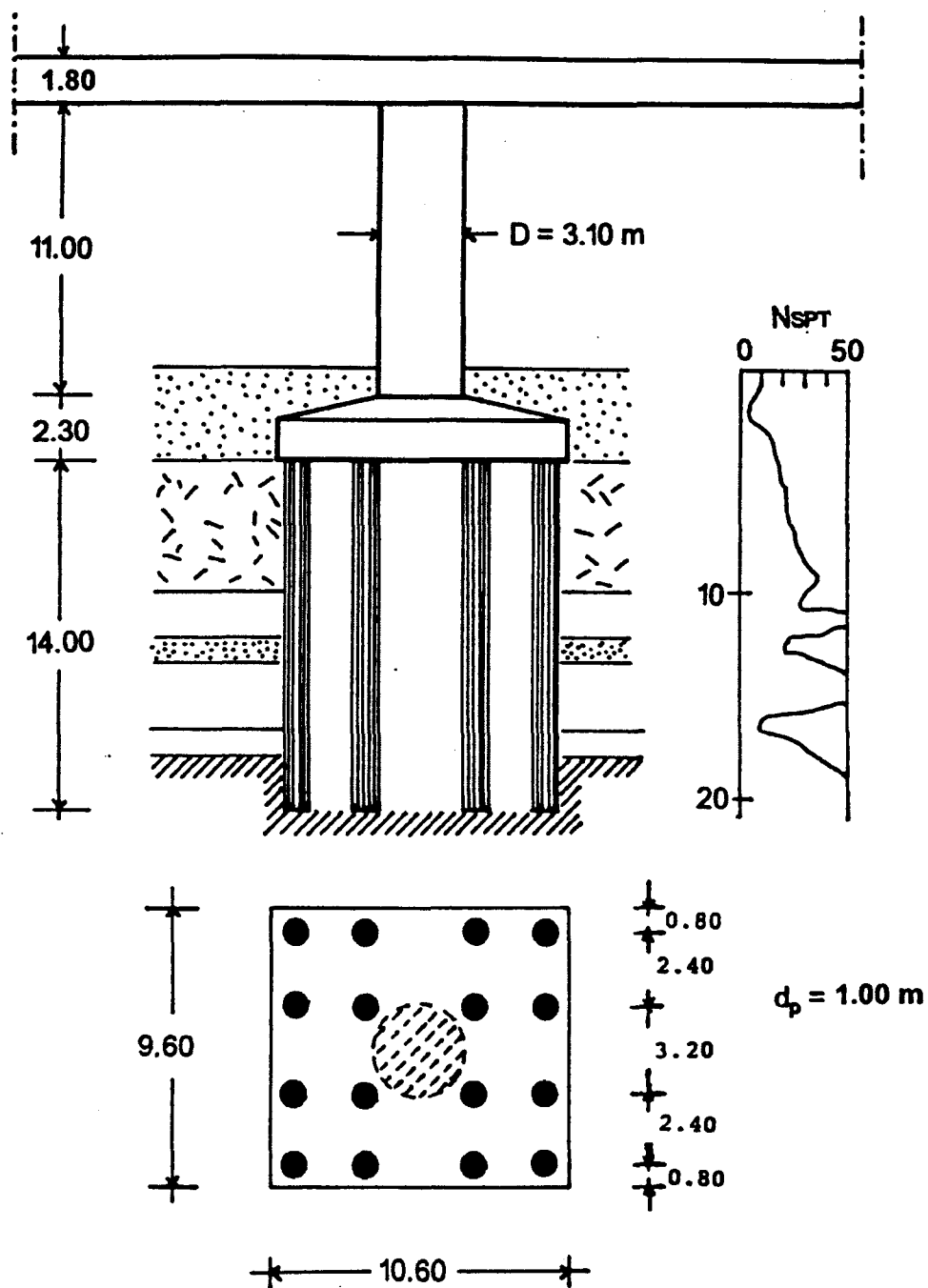


FIGURE 6.2(a): Geometric characteristics of a typical collapsed pier in Fukae section.

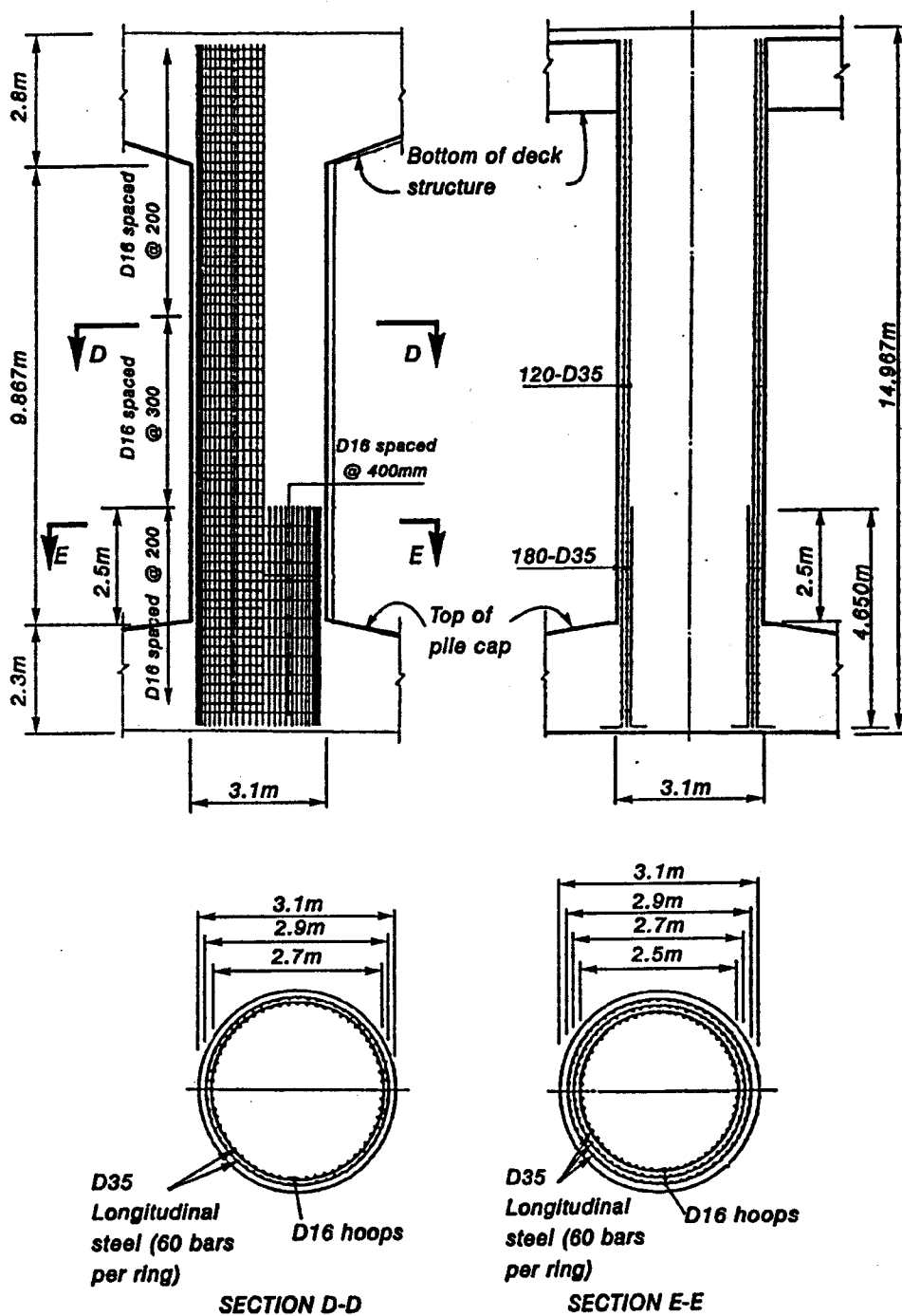


FIGURE 6.2(b): Pier reinforcement and structural details (Iwasaki et al 1995)

stems from oversimplifications in the nature of seismic demand adopted in seismic code provisions. The most important of these simplifications, with reference to SSI, are (Mylonakis and Gazetas 2000): (1) design acceleration spectra that decrease monotonically with increasing structural period; (2) response modification coefficients (i.e., “behaviour” factors used to derive seismic forces) which are either period independent, or increase with increasing structural period; (3) foundation damping derived assuming homogeneous halfspace conditions for the soil, which tends to overpredict the overall effective damping (see Chapters 2 and 3); (4) kinematic response coefficients for spread footings indicating that foundation response is smaller than free-field soil motion. The above perception is reinforced by the lack of a sufficient number of structural failures associated with SSI.

This apparently beneficial role of SSI, although realistic in many cases, has been essentially turned into a dogma. Thus, practicing engineers usually avoid the complication of accounting for SSI, as a conservative simplification that supposedly leads to improved safety margins. Results presented in this study are in contrast with this perception. It is worth mentioning that detrimental effects of SSI on seismic response have been pointed out in the past (Bielak 1978, Resendiz and Roesset 1987, Celebi 1998, Takewaki 1998, Mylonakis and Gazetas 2000). However, these studies have not yet received much attention by code writers and engineers.

The work reported in this chapter involves: (1) discussion of seismological and geotechnical information pertaining to the bridge site; (2) analysis of free-field response; (3) analysis of response of foundation-superstructure system; (4) evaluation of results through comparisons with earlier studies that did not consider SSI.

6.3 THE FIRST ROLE OF SOIL: INFLUENCE ON GROUND MOTIONS

Kobe and the nearby towns of Asiya, Nisinomiya, and Amagasaki are built along the shoreline in the form of an elongated rectangle with length of about 30 km and width 2 to 3 km. Soil in the region consists primarily of holocene alluvial deposits of sand and gravel, underlain by stiffer neogenes and soft rock (Figure 6.3). The thickness of the holocene alluvium increases to the southeast, from about 10m in uptown Kobe (e.g., Motoyama), to 20m in downtown (e.g., Fukiai, Takatori), to over 40m in Port Island. The granitic bedrock that outcrops in the mountain region bordering the city to the north, dips steeply in the northwest-southwest region; in the shoreline it lies at a depth of about 1 to 1.5 km (Iwasaki et al. 1995, Kawase 1996, Tokimatsu et al. 1996). Figures 6.4a and 6.4b show an approximate geologic plan and a cross section of the region, including the locations of strong motion accelerometers.

The mainshock was recorded in several strong motion instruments. A number of their records were of unusually high intensity, with peak ground accelerations (PGA) and peak ground velocities (PGV) in excess of 0.8g and 100cm/s, respectively. PGA's in non-liquefied ground were over 0.5g throughout Kobe, Asiya, and Nishinomiya. Peak accelerations above 0.4g were recorded at 17 sites, while at least in three locations (JMA, Fukiai, and Motoyama stations) PGA reached the astounding 0.80g (Table 6.1). A comprehensive catalog has been published by Fukushima et al (2000). Analytical studies have confirmed the above observations of the rupture and the basin response (Figure 6.5).

The attenuation of peak ground acceleration with distance from source is examined in Figure 6.6, with reference to rock and stiff soil sites. Two different distance variables are used: *epicentral distance* (Figure 6.6a) and *distance from fault* (Fig. 6.6b).

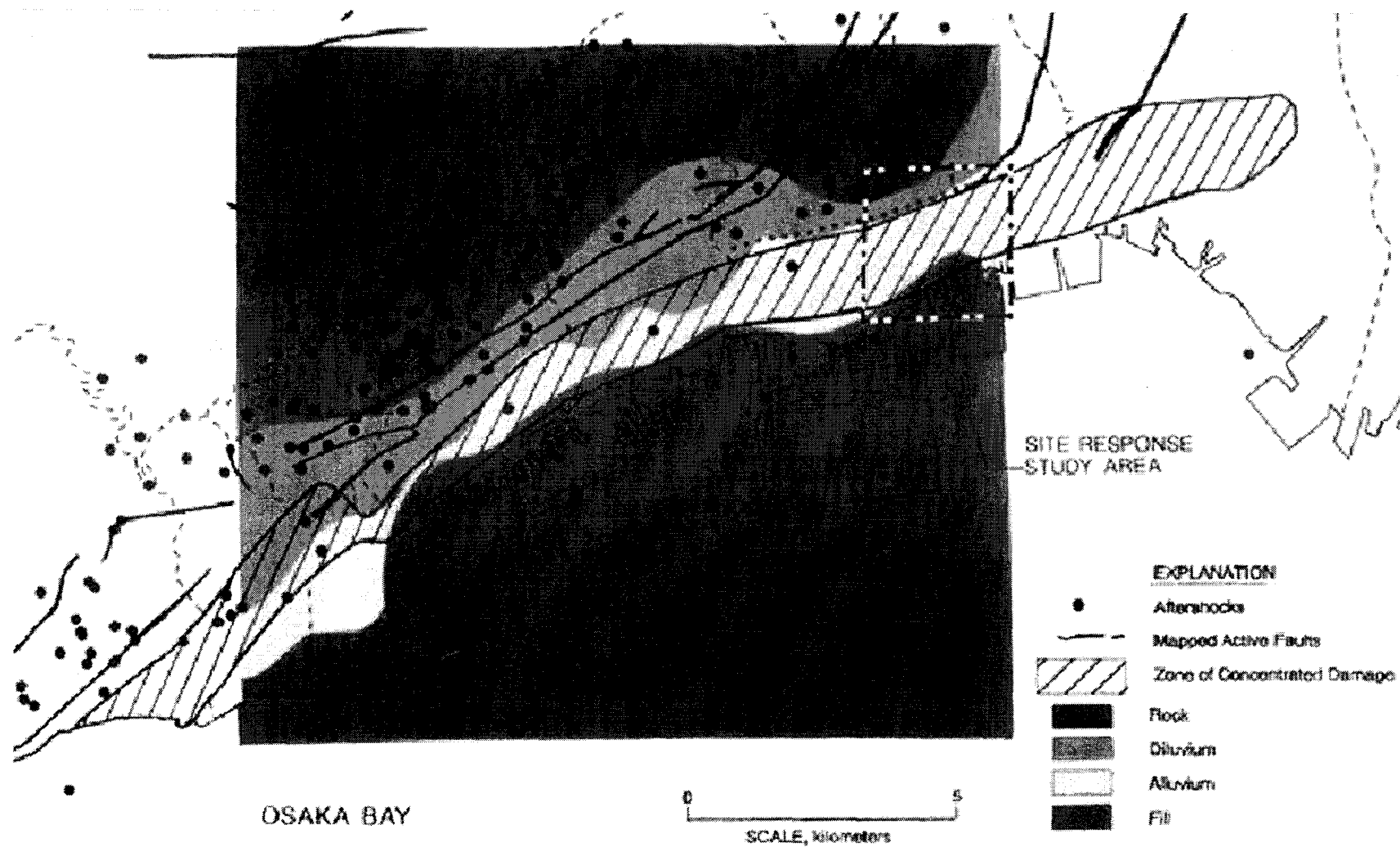


FIGURE 6.3: Location of zone of intense destruction in the Kobe area, and its relationship to the location of active faults, the aftershock distribution, and geological site conditions (modified from Kohketsu, 1995)

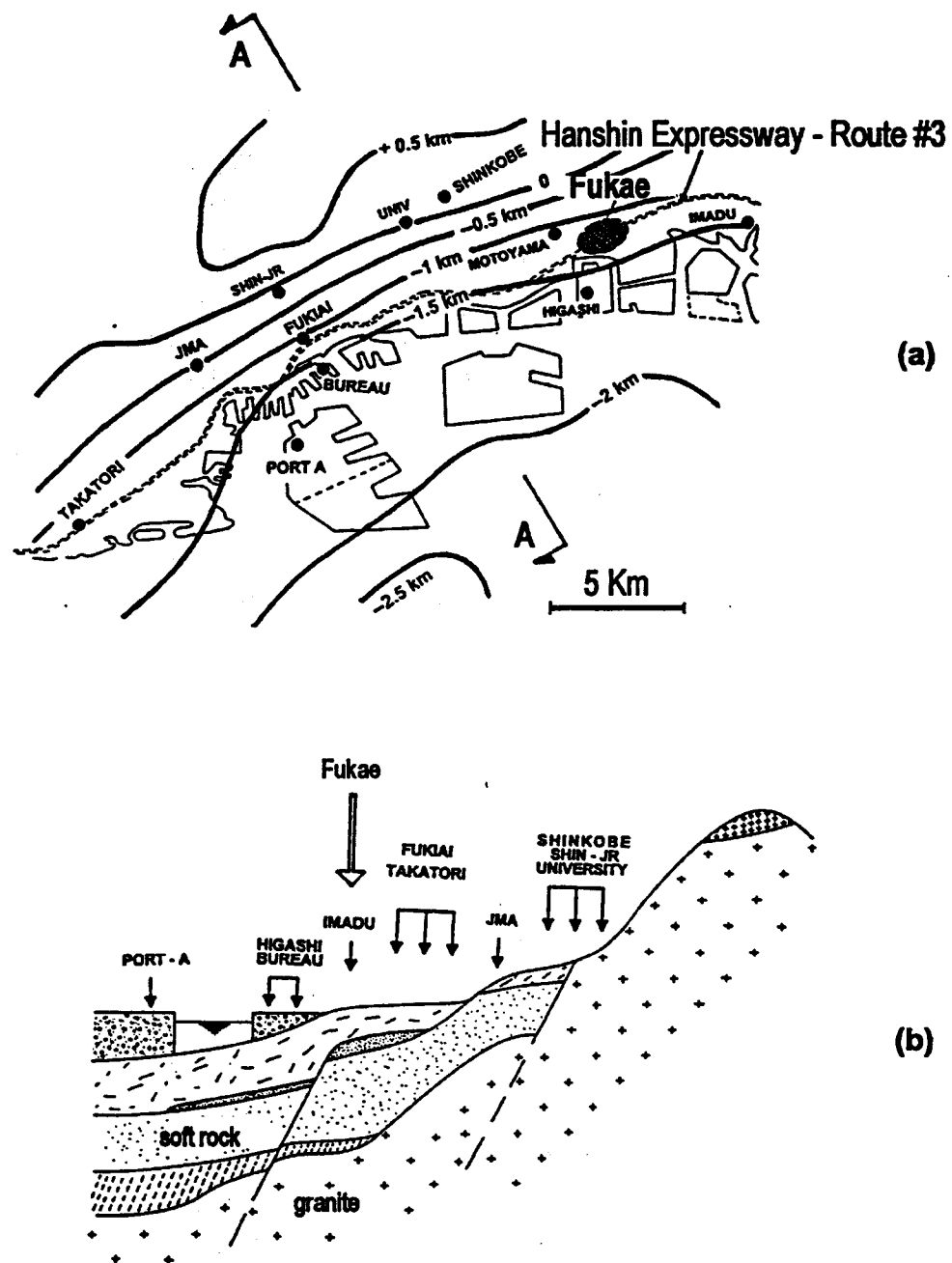


FIGURE 6.4: (a) Contours of bedrock elevation and location of accelerometers; (b) approximate geologic section A-A

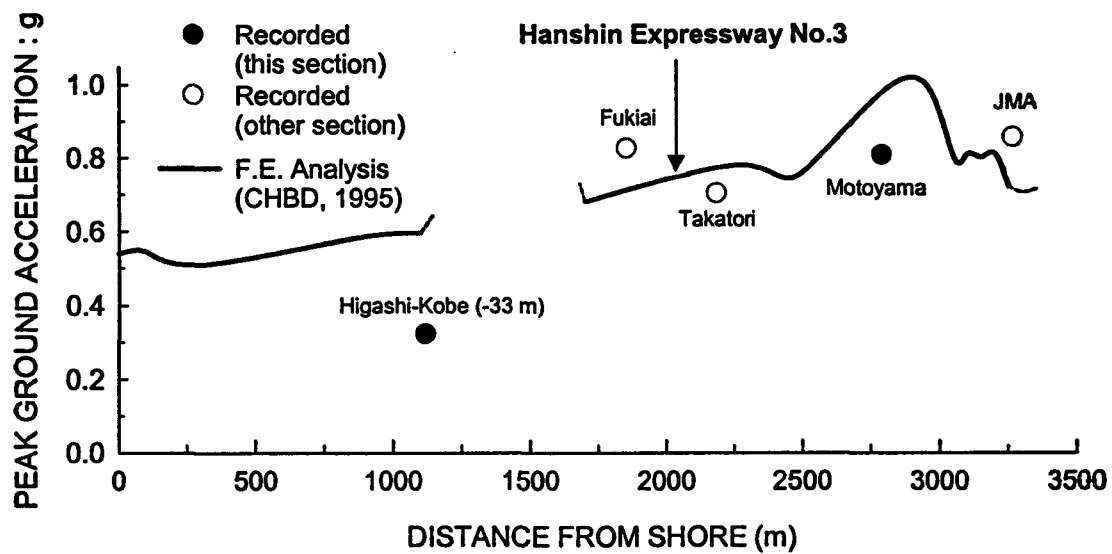


FIGURE 6.5: Distribution of peak ground acceleration in a cross section perpendicular to shore and fault at Fukae. (Modified from Iwasaki et al 1995)

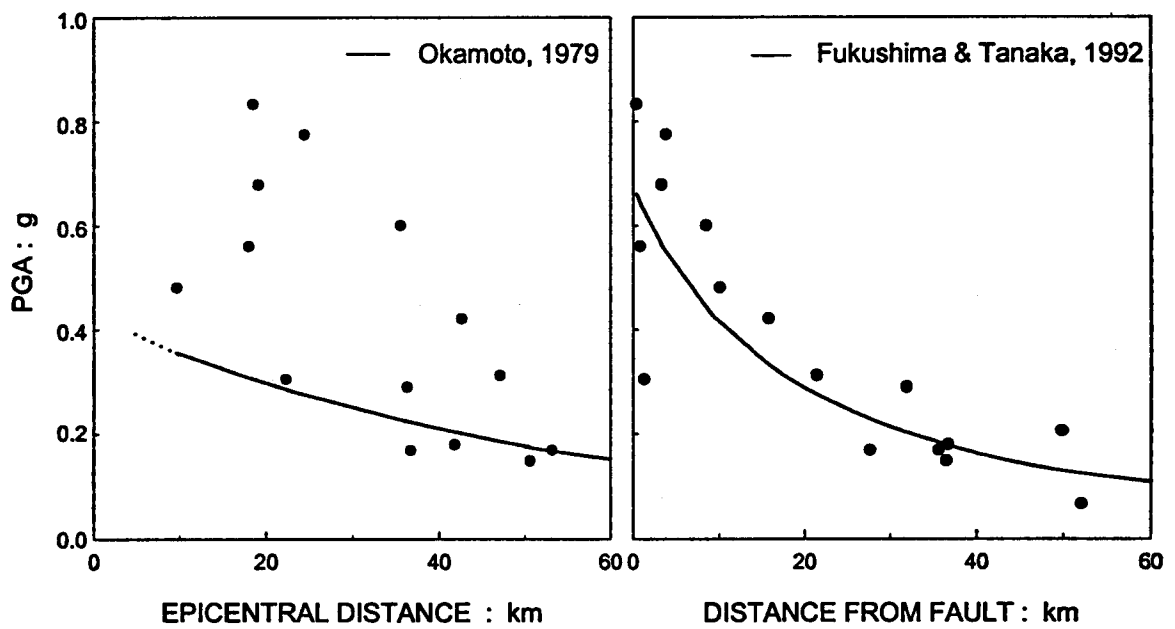


FIGURE 6.6: Attenuation of peak horizontal ground acceleration with distance from source. Note the poor correlation of epicentral distance with PGA (after Gazetas et al 1996).

It is evident that peak ground acceleration is statistically independent to epicentral distance. For instance, in a radius of about 40 km from the epicenter, PGA varies from about 0.6g in Kobe and Nishinomiya, to 0.15g in the southern part of Osaka. In contrast, distance from fault generates much less scattering (Figure 6.6b), in agreement with existing attenuation relations (Ejiri et al. 2000, Fukushima et al 2000).

Variability in local soil conditions among the recording stations might be partly responsible for the significant differences in intensity and frequency content of the recorded motions, as clearly shown in Figures 6.7 and 6.8. Three additional effects, however, seem to have affected the surface motions in Kobe: *forward rupture directivity*, *basin effects*, and *soil liquefaction*.

The first is of a seismological nature, affecting ground shaking at near-fault sites located in the direction of fault rupture propagation (Somerville 1998, Mavroeidis and Papageorgiou 2003). The effect of forward fault-rupture directivity is primarily to increase the intensity of the horizontal component of the motion *normal* to the fault strike, at periods longer than about 0.5 sec. The resulting differences between Fault–Normal (FN) and Fault–Parallel (FP) components in the recorded motions (Figure 6.9) and the corresponding response spectra (Figure 6.10) are indeed striking.

The 2D basin (valley) effect has been shown to increase or decrease the intensity, duration, and frequency characteristics of ground motion depending on the proximity to the edge of the valley, the dipping angle, the frequency content of the excitation, and the incidence wave angles (Bielak et al. 1999). Finally, soil liquefaction results in significant reduction of high-frequency acceleration peaks, increase of dominant period of vibration, and in large permanent deformations if static (permanent) shear stresses exist in the

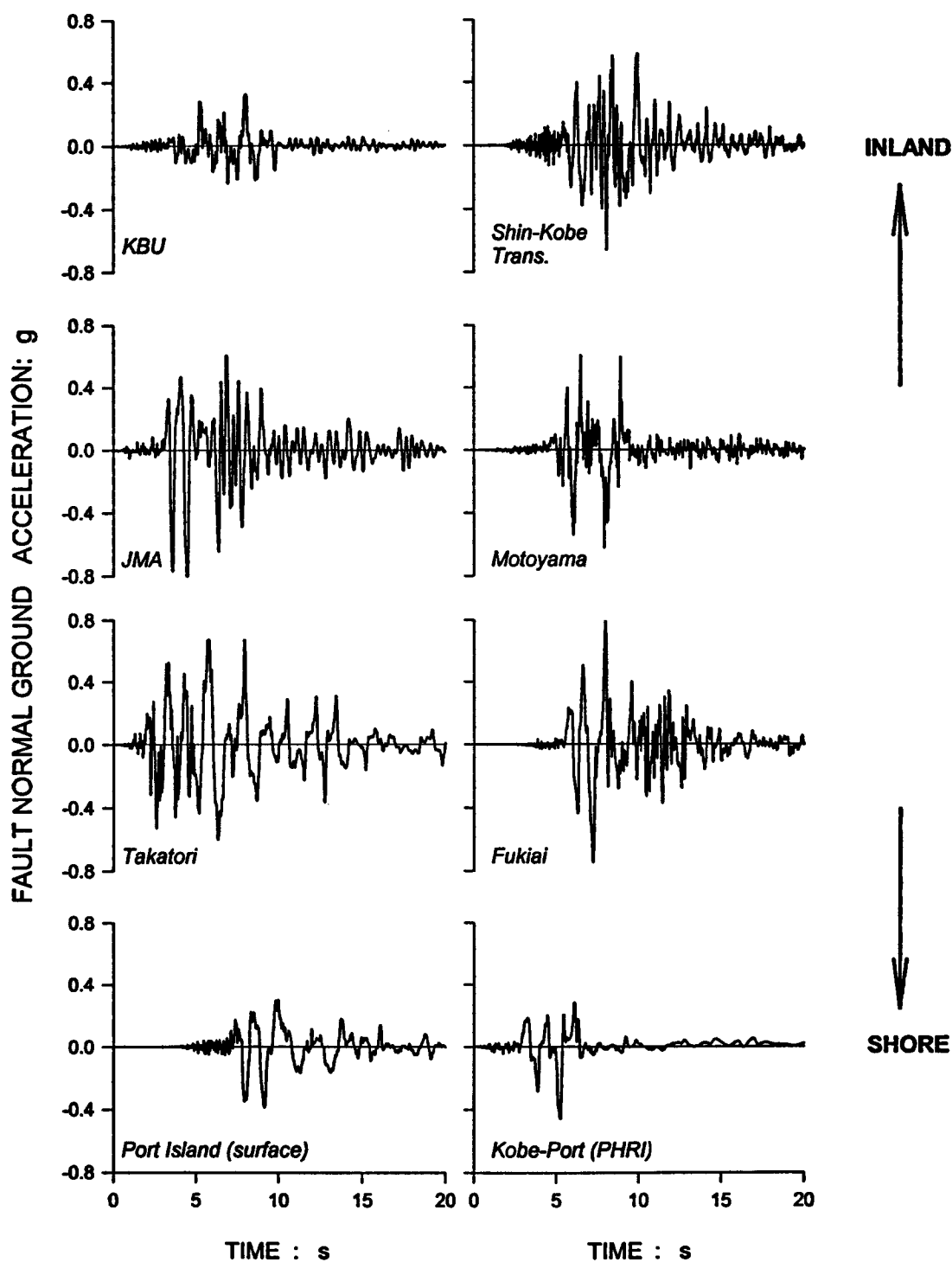


FIGURE 6.7: Selected accelerograms from Kobe earthquake. All time histories represent fault normal components (E127N)

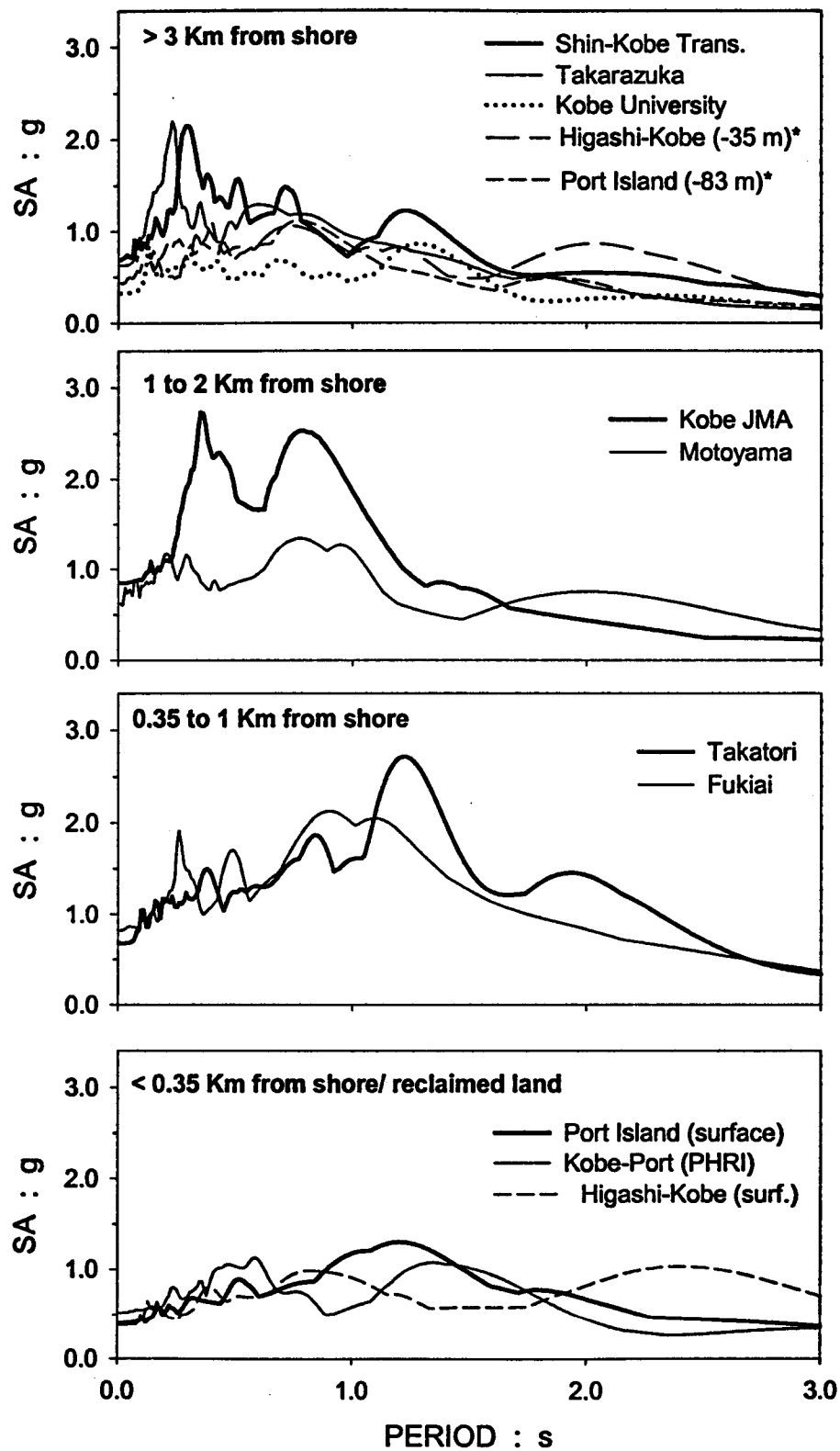


FIGURE 6.8: Fault-normal acceleration spectra with respect to distance from shore. Note the difference in predominant periods among the various groups. [* denotes motions at depth]; $\zeta = 5\%$

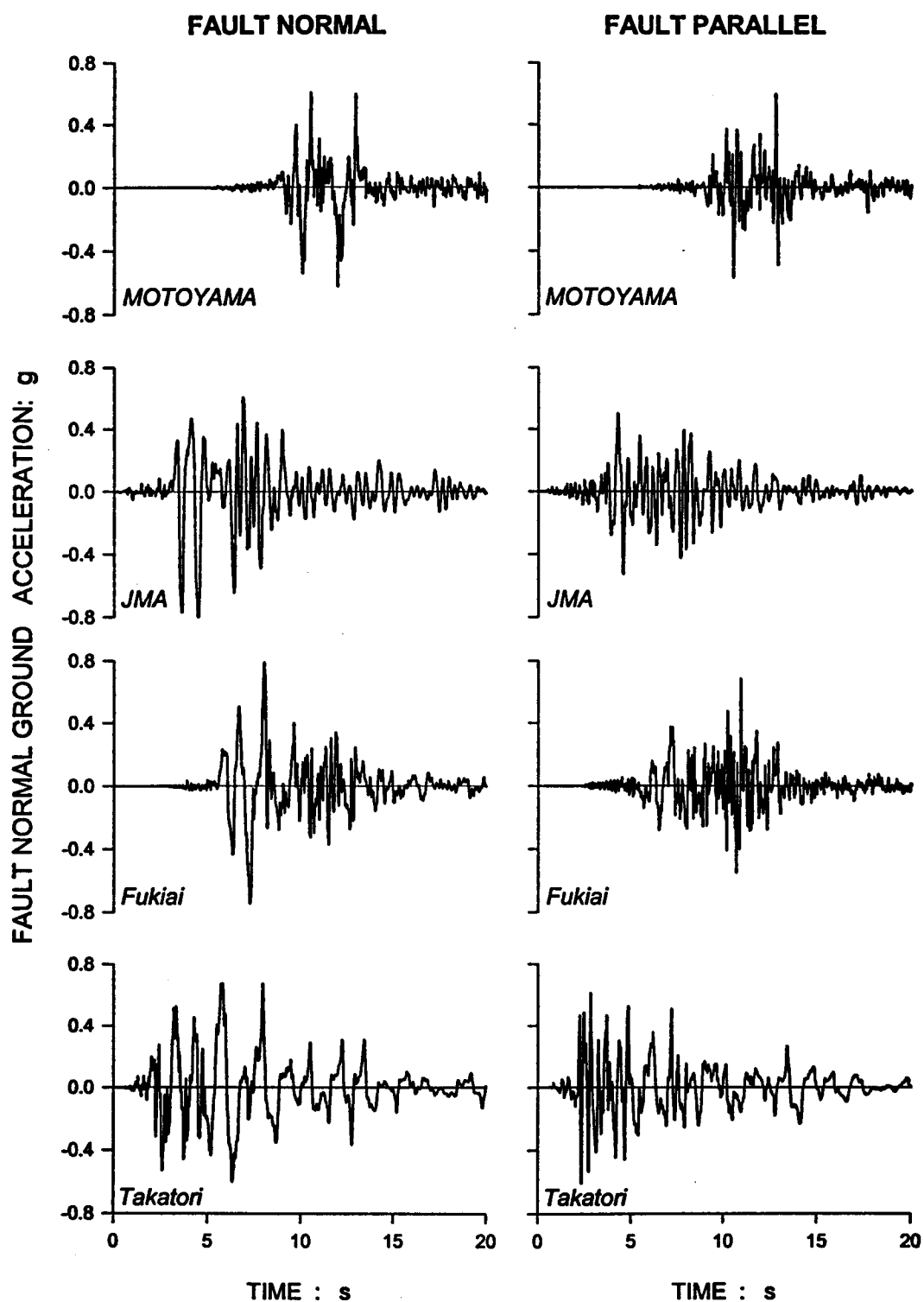


FIGURE 6.9: Selected near-fault accelerograms from Kobe earthquake. Note the differences between fault-normal (E127N) and fault-parallel components (E37N)

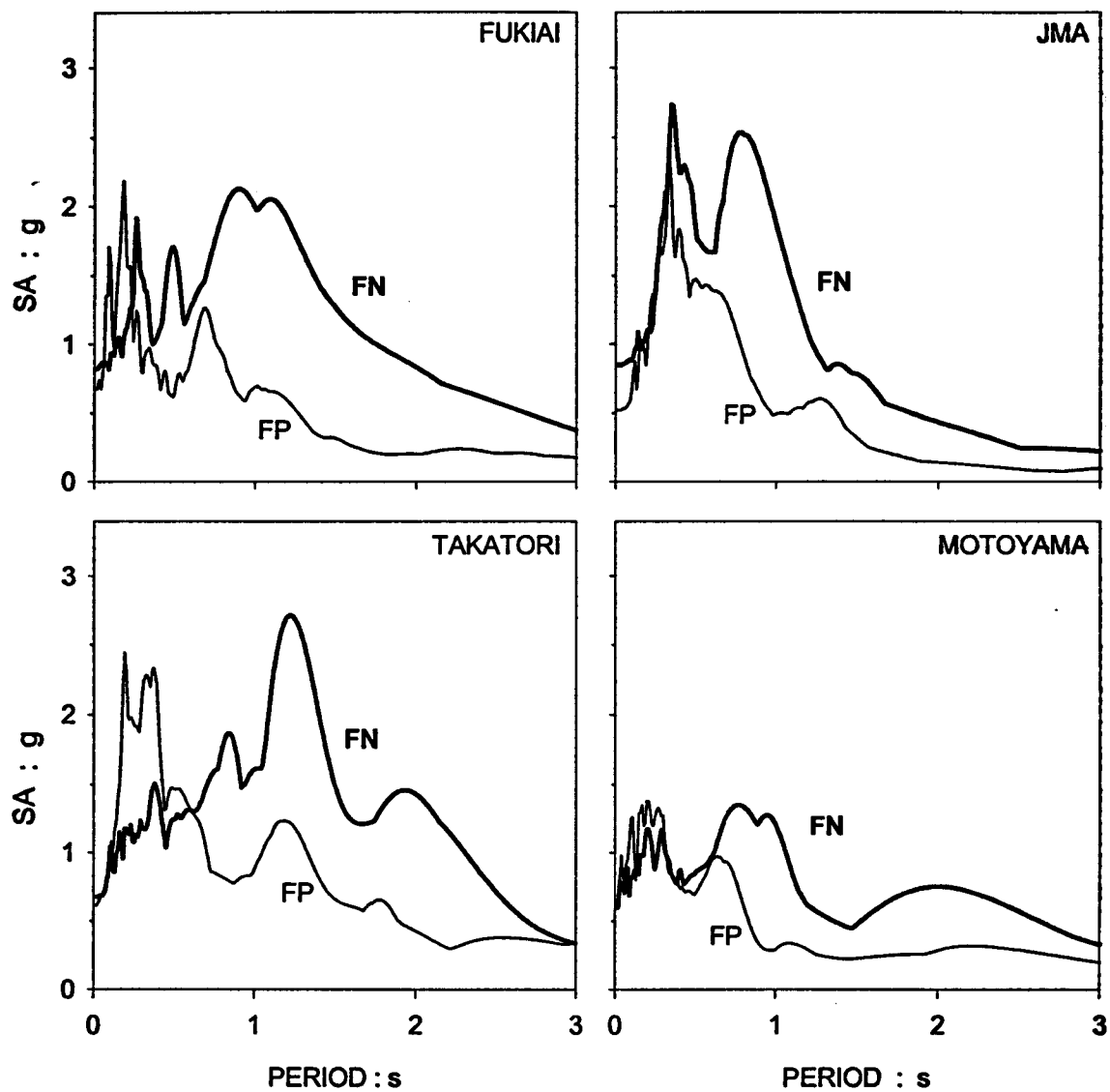


FIGURE 6.10: Elastic response spectra of selected records at fault-normal and fault parallel orientations; $\zeta = 5\%$

ground. Such effects became particularly pronounced in the near shore region (JGS 1998).

Additional evidence on source directivity is given in Figure 6.11: *Polar plots* of spectral accelerations in horizontal plane are presented for six acceleration records and three selected periods. The fault normal (FN) and fault parallel (FP) directions are indicated in the graphs. It is evident that long-period acceleration components ($T > 0.6s$) attain their maxima in the fault-normal direction and their minima in the fault-parallel direction. The opposite seems to be true with the short-period component (with the exception of JMA record). In contrast, PGA is essentially independent of azimuth. It is worth mentioning that these attributes seem to be independent of local soil conditions, as evident from the Port Island record (Ejiri et al. 2000).

An alternative representation is provided in Figure 6.12, which depicts acceleration response spectra in the fault-normal and fault-parallel directions. Envelopes of spectral values at all azimuths are also shown. Evidently, the fault normal curve practically coincides with the envelope for periods longer than 0.6s, while the same is true for the fault-parallel curve for periods less than 0.3s.

All these effects have contributed (more or less) to the differences in ground motions seen in Figure 6.8. Evidently, the closer the site to the shore, the deeper and softer the soil deposit, thereby leading to a longer predominant period and a flatter spectrum. Interestingly, the site groups in Figure 6.8 differ not only with respect to distance from shore, and flexibility of soil, but also with respect to distance from fault (recall that the fault trace is essentially parallel to the shoreline). It is important to mention here that the site categories in Figure 6.8 would have been different if the

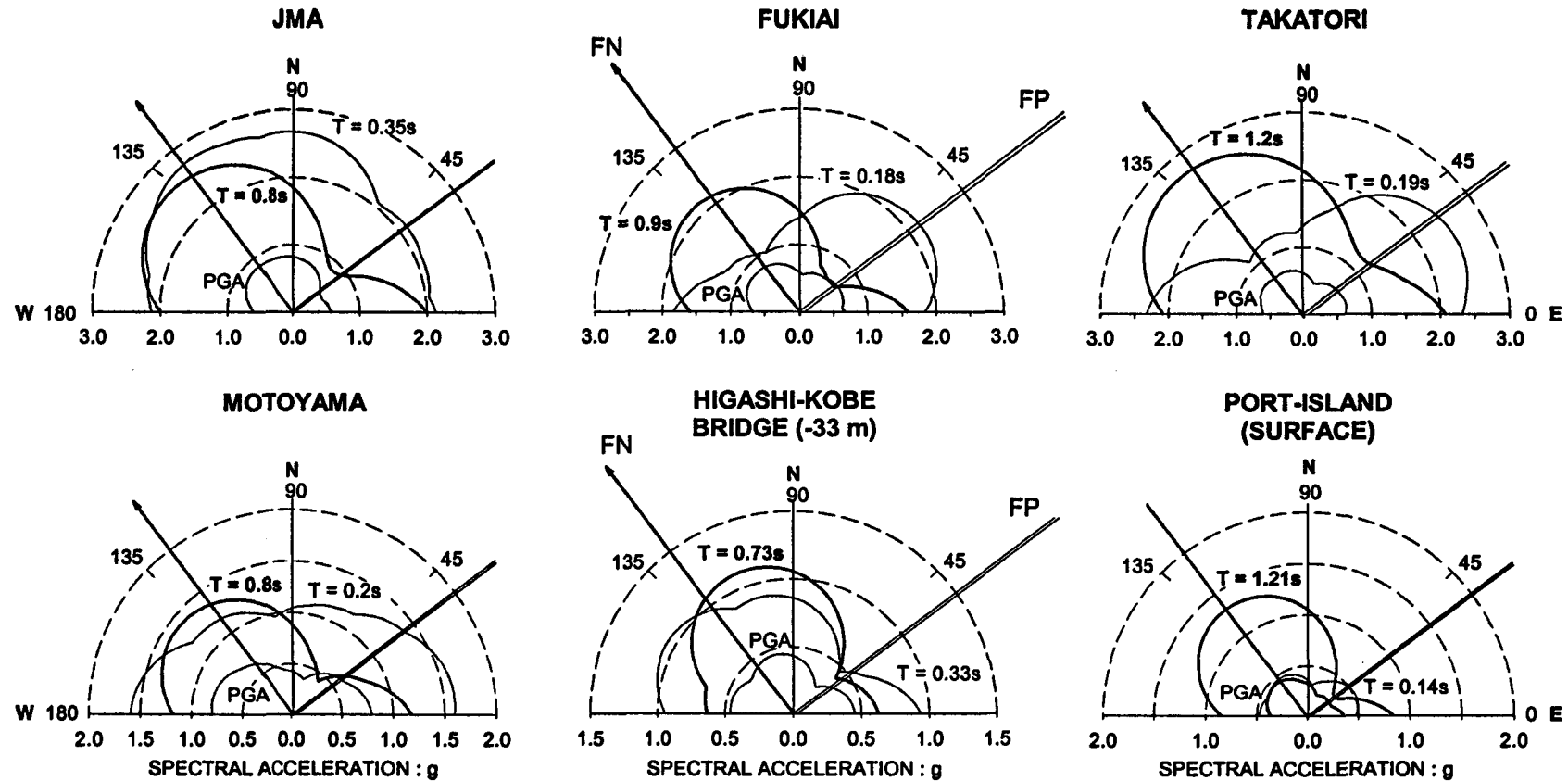


FIGURE 6.11: Polar plots of spectral accelerations for six records. Selected periods correspond to peak spectral values in the short and long period range. Note the pronounced values in the fault normal (FN) direction at long periods; $\zeta = 5\%$

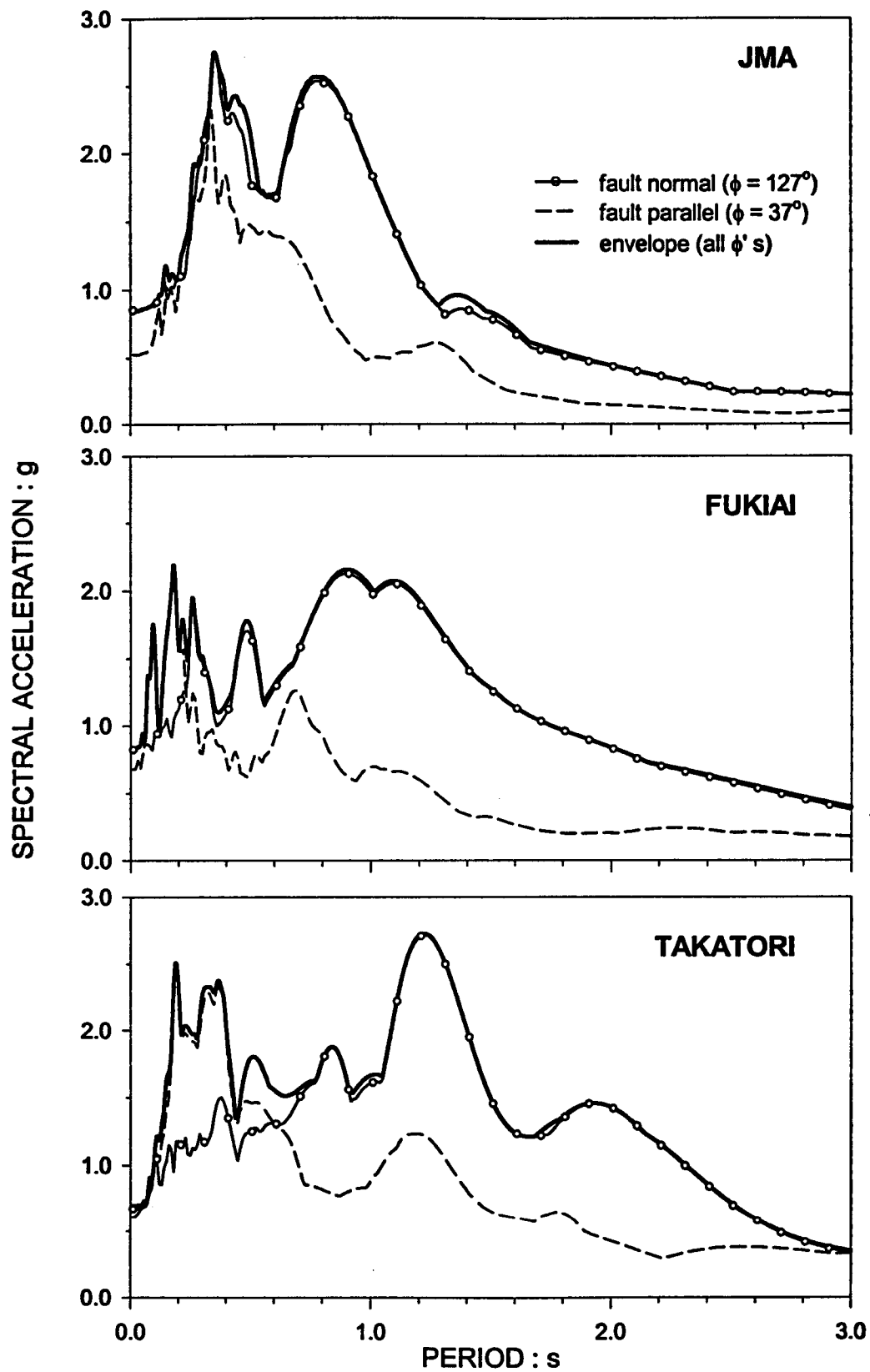


FIGURE 6.12: Elastic response spectra of main records. Angles are measured counter-clockwise with respect to East; $\zeta = 5\%$

stiffness of the top 30 meters of soil had been used as the classification criterion (NEHRP 2001). For instance the Takarazuka site (a shallow site located approximately 6 km inland), would classify in the same group as the much deeper and softer Takatori site, due to the presence of soft soil close to the surface. Similar “misclassifications” would occur with the Kobe Port (PHRI), Fukiai and Takatori records, which would classify in the same group with records on shallower-stiffer deposits, thereby having much different frequency and amplitude content. Evidently, site thickness has an important influence of surface ground motions, which is not adequately recognized by existing codes.

Tabulated results for peak ground motion parameters are given in Table 6.1. These indices include:

- Periods of peak spectral acceleration and velocity (T_a and T_g)
- Arias intensity (I_A)
- Bracketed duration (D_b)
- Destructiveness potential (D_p)
- Housner Intensity (I_H)

The above parameters are defined as (Uang and Bertero 1988, Kramer 1995):

$$I_A = \frac{\pi}{2g} \int_0^4 \ddot{u}_g^2 dt \quad (6.1)$$

$$I_H = \int_{0.2}^{2.5} SV dT \quad (6.2)$$

$$D_b = \text{time lag between first and last exceedance of a threshold acceleration} \quad (6.3)$$

TABLE 6.1. Tabulated results for peak recorded ground motions during Kobe earthquake: PGA, PGV, PGD, Arias intensity (I_A), Housner intensity (I_H), Bracketed duration (D_b), Destructiveness potential factor (D_p).

Station	Orientation	PGA (g)	PGV (m/s)	PGD (m)	SA _{max} (g)	SV _{max} (m/s)	T _a (s)	T _s (s)	I _A (m/s)	I _H (m)	D _b (s)	D _p (m/s ²)
AMAGASAKI (CEORKA)	FN	0.36	0.491	0.431	1.20	1.72	0.83	1.68	2.38	2.42	19.98	0.022
	FP	0.37	0.495	0.955	1.43	1.44	0.47	0.97	1.61	2.02	23.74	0.018
	Envelope	0.42	0.500	0.985	1.45	1.73						
FUKIAI	FN	0.82	1.147	1.542*	2.14	3.24	0.90	1.17	7.02	5.56	6.77	0.118
	FP	0.68	0.637	3.183*	2.20	1.34	0.18	2.60	3.06	1.88	7.11	0.019
	Envelope	0.83	1.242	3.488*	2.20	3.28			7.13	5.60		0.364
HIGASHI (UG1)	FN	0.44	0.813	0.319	1.07	2.95	0.73	2.19	2.29	3.60	7.22	0.115
	FP	0.33	0.621	0.275	0.73	1.72	0.29	2.63	1.07	1.90	12.37	0.056
	Envelope	0.46	0.851	0.340	1.12	2.97			2.34	3.61		0.141
KBU	FN	0.32	0.440	0.132	0.86	1.70	1.29	1.40	1.17	2.05	6.68	0.018
	FP	0.24	0.376	0.072	0.72	1.16	0.41	1.20	0.87	1.43	6.02	0.016
	Envelope	0.32	0.551	0.139	0.96	1.84			1.24	2.17		0.028
JMA	FN	0.85	0.958	0.243	2.73	3.38	0.35	0.85	9.52	4.04	9.54	0.299
	FP	0.53	0.544	0.110	2.34	1.43	0.34	1.27	4.31	1.83	8.14	0.107
	Envelope	0.86	0.958	0.249	2.74	3.39			9.63	4.06		0.321
MOTOYAMA	FN	0.62	0.747	0.679	1.36	2.40	0.78	2.23	2.95	3.61	7.73	0.011
	FP	0.60	0.632	0.639	1.38	1.20	0.21	2.57	1.97	1.69	8.93	0.007
	Envelope	0.80	0.850	0.820	1.66	2.60			3.29	3.74		0.013
NISHI-AKASI	FN	0.47	0.335	0.109	2.54	1.77	0.43	0.46	3.79	1.46	9.54	0.060
	FP	0.33	0.373	0.121	1.25	1.24	0.22	0.69	1.83	1.46	11.18	0.033
	Envelope	0.54	0.385	0.124	2.57	1.82			3.83	1.50		0.069
PORT ISL. (GL 0)	FN	0.39	0.849	0.453	1.31	2.32	1.21	1.86	2.45	3.82	7.42	0.034
	FP	0.14	0.216	0.108	0.49	0.89	0.14	1.96	0.39	1.20	17.11	0.008
	Envelope	0.39	0.855	0.454	1.31	2.39			2.45	3.84		0.036
PORT ISL. (GL -83)	FN	0.70	0.498	0.223	1.13	1.48	0.77	1.85	2.34	2.40	9.17	0.018
	FP	0.33	0.273	0.117	1.58	0.87	0.36	0.36	1.64	1.15	10.24	0.012
	Envelope	0.74	0.526	0.242*	1.60	1.57			2.52	2.53		0.019
SHIN-KOBE TRANS	FN	0.66	1.418	8.074*	2.16	2.40	0.29	1.25	4.96	3.58	7.57	0.002
	FP	0.52	0.591	2.159*	1.93	1.86	0.35	0.72	4.51	2.30	10.53	0.003
	Envelope	0.70	1.418	8.355*	2.69	2.40			6.22	3.68		0.004
TAKARAZUKA	FN	0.63	0.714	0.211	2.20	1.78	0.23	1.18	2.90	2.84	5.08	0.043
	FP	0.68	0.846	0.261	2.88	2.09	0.47	0.48	4.10	3.28	3.38	0.100
	Envelope	0.76	0.896	0.285	2.89	2.10			4.17	3.30		0.118
TAKATORI	FN	0.68	1.688	0.450	2.73	4.99	1.22	1.27	10.49	6.98	10.80	0.911
	FP	0.61	0.613	0.226	2.44	2.61	0.19	1.22	6.33	3.31	11.87	0.592
	Envelope	0.70	1.699	0.451	2.74	5.03			10.51	6.98		0.967

* uncorrected displacements

$$D_p = \frac{I_A}{v_o^2} \quad (6.4)$$

where $\ddot{u}_g$ = ground acceleration; t_d = total duration of ground motion; SV = spectral velocity; v_o = intensity of zero crossings (characteristic frequency) in the acceleration record.

Results for twelve records are provided in Table 6.1. Note the very high spectral accelerations (above 2.5g) and spectral velocities (above 3 m/s) for some of the motions. As a final remark, the intensity of vertical ground motion is examined in Figure 6.13, in terms of the ratio of vertical and horizontal spectral accelerations. It is seen that with the exception of very short periods (less than 0.1s), vertical spectral acceleration is significantly smaller than the corresponding horizontal acceleration in all records examined. This suggests a rather minor participation of vertical ground motion in the damage induced by the earthquake. This observation is important given that the recordings were obtained in the *near field*, where vertical ground motion is expected to be particularly pronounced. Newmark's "seventy percent" rule seems to be conservative for periods above 0.3s but highly unconservative at smaller periods.

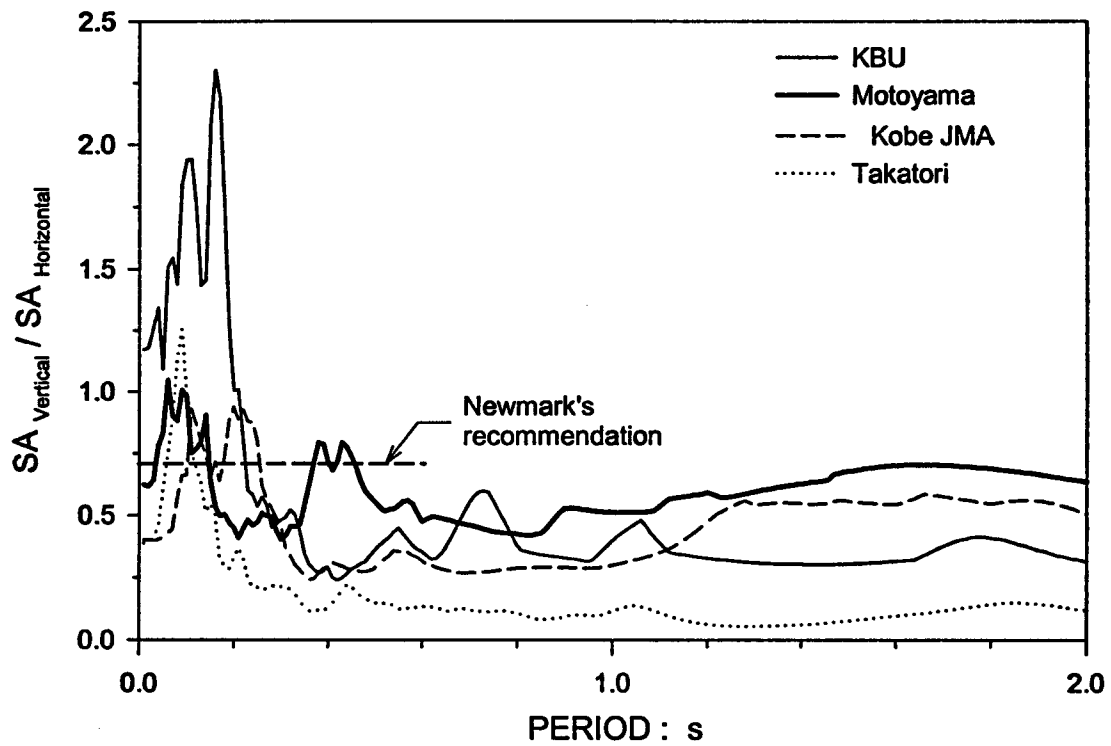


FIGURE 6.13: Vertical to horizontal spectral acceleration ratios of the four main records ; $\zeta = 5\%$

6.3.1 Ground Shaking at the Site

Unfortunately, no records were obtained at Fukae during the main shock. The closest stations were Motoyama and Higashi Kobe, both located at distances in excess of 1 km from the site. The first instrument (Motoyama), is a velocity meter stationed at a school building about 1500m to the northwest of the collapsed segment. The instrument went off scale during the earthquake. The clipped record was subsequently restored by Kagawa et al (1996). The second instrument is part of a vertical array installed at the east abutment of Higashi Kobe Bridge, located about 1.3km south of the collapsed bridge.

The uncertainty in the characteristics of the ground motion and the soil conditions at the location of the bridge dictated the use of plausible scenarios. From available borehole data at the Hanshin site, soil is deemed as a relatively deep deposit of moderately stiff to soft layers of sand and gravel with low-strain shear-wave velocity ($V_{s, \max}$) of the order of 200 to 300 m/s for the upper 20 meters (Figures 6.14-6.16). No significant permanent deformations or evidence of liquefaction were observed after the earthquake. The similarities in shear-wave velocities with those at Fukiai and Takatori stations (Figure 6.16) are noteworthy.

Six acceleration records with different peak ground accelerations and frequency characteristics are examined (Figures 6.8, 6.9, Table 6.1):

- The accelerogram FUKIAI, with PGA of about 0.83 g and PGV of 115 cm/s in the fault normal direction, recorded on a medium-soft and relatively deep deposit [60 m of soil with average $V_{s, \max}$ less than 400 m/s (Figure 6.16)]

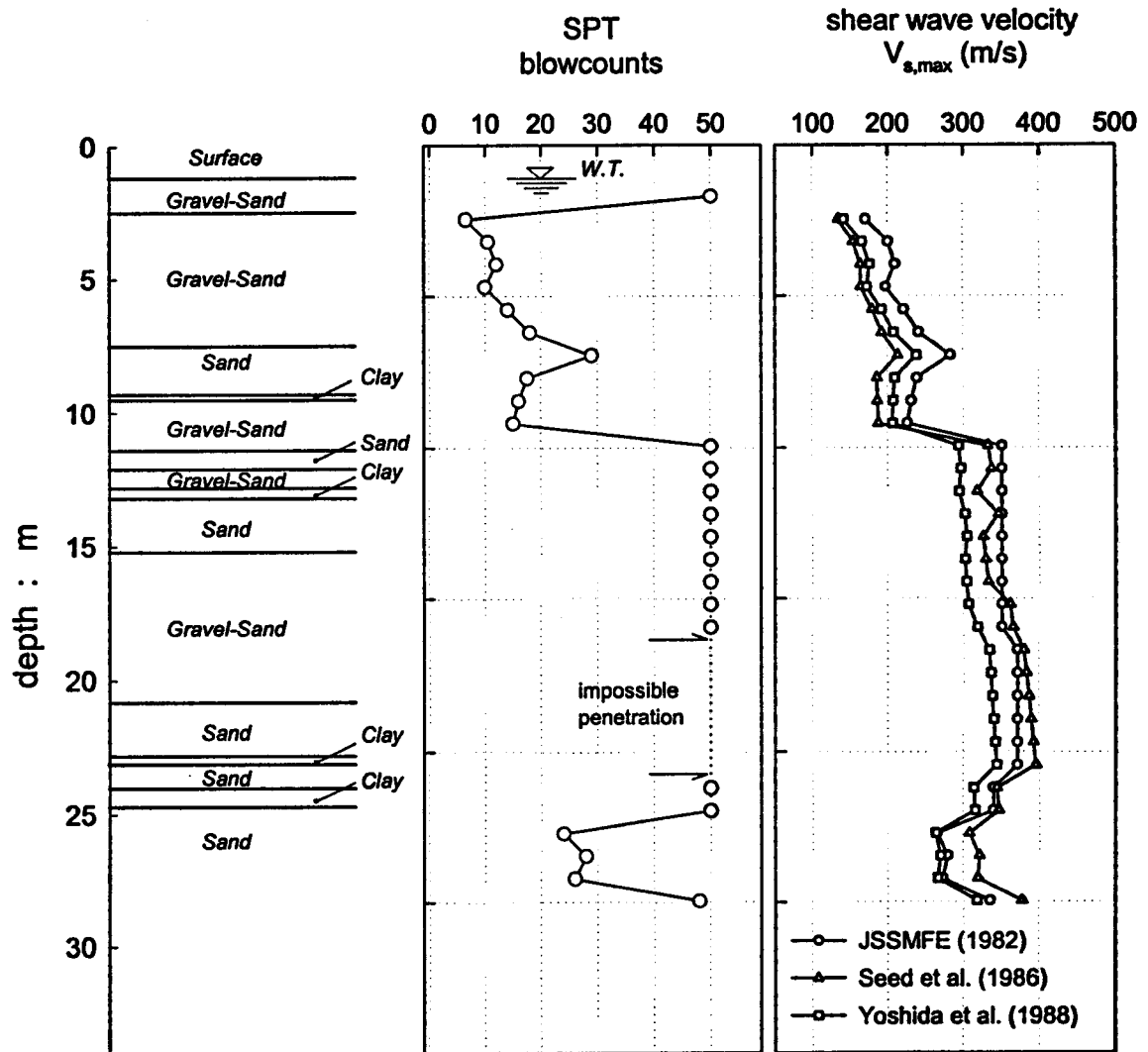


FIGURE 6.14: Soil profile at pier 126 (Adachi, 2001)

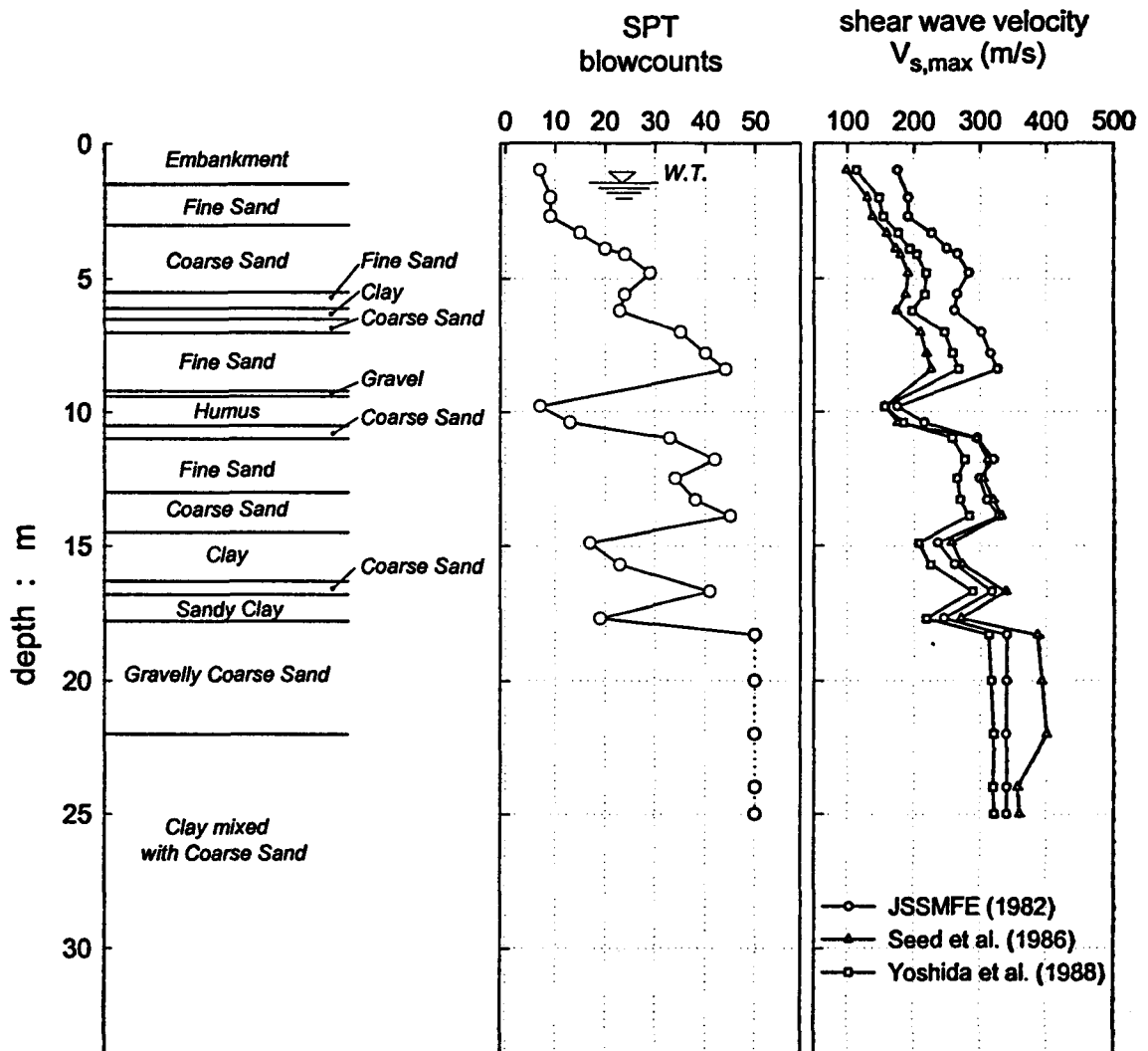


FIGURE 6.15: Soil profile at pier 138 (Iwasaki et al 1995)

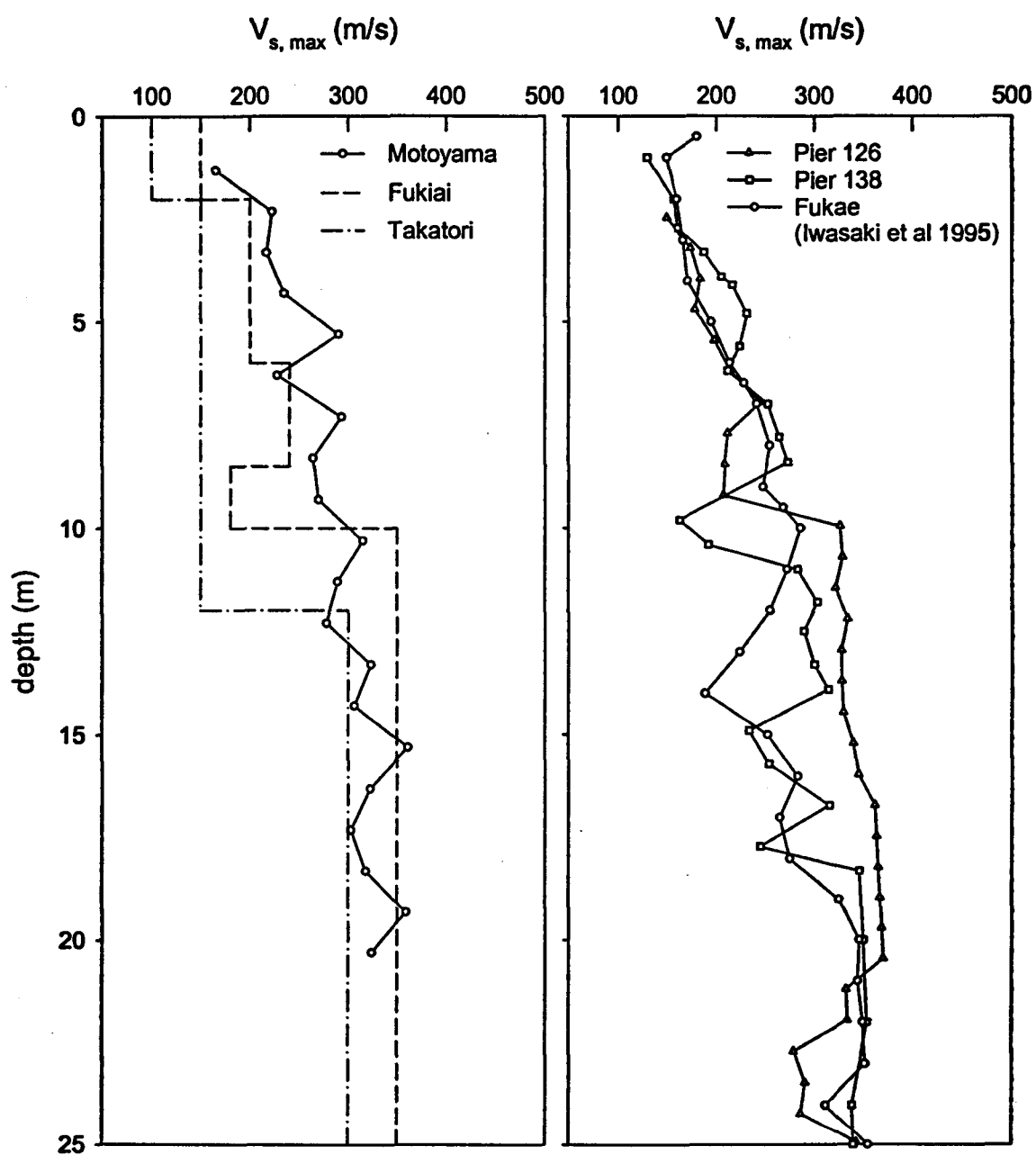


FIGURE 6.16: Low-strain shear wave velocities considered

- The accelerogram TAKATORI, with PGA of 0.68 g and PGV of 169 cm/s in the fault-normal direction, recorded on a soft and deep deposit [80 m of soil with $V_{s, \max}$ less than 400 m/s (Figure 6.16)]
- The accelerogram JMA, with PGA of 0.83 g and PGV of 96 cm/s in the fault-normal direction, recorded on a stiffer soil formation [10-15 m of stiff soil (Figure 6.16)]
- The accelerogram MOTOYAMA, with PGA of 0.62 g and PGV of 75 cm/s recorded on a shallow soil site (soil thickness of about 20m), about 1 km to the northwest of the bridge (Figure 6.16)
- The accelerogram HIGASHI Kobe, with PGA of 0.44 g and PGV of 81 cm/s recorded, at a depth of 35 meters, on a stiff layer below a liquefied layer.
- A synthetic accelerogram (SYNTHETIC) derived by Matsushima and Kawase (1999) based on a multiple asperity model and a 3-D basin structure. The time history was obtained at the location of the collapsed bridge, on the surface of a “reference” rock stratum with $V_{s, \max} = 400\text{m/s}$.

Of the above records, the first two, although recorded far from the bridge, are believed to be the most representative of the motion at the site, because: (a) of their similar distance from fault and shoreline, (b) of the similar orientation with respect to rupture as the collapsed segment. Indeed, judging from the geology of Figures 6.4a and 6.16 soil conditions at the bridge site seem to be closer to those of FUKIAI and TAKATORI than to any other station. The third accelerogram, JMA, was selected because it has been invariably used (often as the only record) in previous studies of the collapse (e.g., Park 1996, Kawashima and Unjoh 1997). It has been obtained much closer

to fault and on much stiffer soil to be representative of the Fukae motion. The last three records (MOTOYAMA, HIGASHI Kobe, SYNTHETIC) were selected because of their proximity to the structure. Of these, Motoyama record was first de-convoluted using one-dimensional wave propagation theory (Schnabel et al. 1972) to a depth of 20m to obtain the pertinent rock motion. The three “rock” motions and the corresponding acceleration spectra are depicted in Figure 6.17. Note the similarities in the waveforms and spectra.

As a next step, the rock motions were convoluted using one-dimensional wave propagation theory (Schnabel et al. 1972) to derive free-field surface motions. Three rock depths (25m, 50m, 75m) were utilized to this end. The first depth is consistent with the thickness of holocene deposits in the area (Iwasaki et al. 1995). The last is a probable upper bound of the thickness of alluvial and dilluvial deposits. Finally, the depth of 50m was selected as an “intermediate” scenario. Shear wave velocities were derived from SPT data based on different correlations (Figures 6.14, 6.15). The correlation by Yoshida et al. 1988 was adopted in the analyses.

In Figures 6.18 and 6.19, acceleration spectra in the fault normal direction and corresponding amplification spectra (RRS) between surface and rock are shown for the three amplified records. For each record, three different curves are presented corresponding to different site thickness. As expected, the deeper profiles exhibit peak amplifications at longer periods. It is evident that the actual motions (Higashi and Motoyama) exhibit similar amplification patterns. The amplified motions derived from these records for an average site thickness of 50m are used in the subsequent analyses.

An average site-specific spectrum based on the amplified Motoyama and Higashi records is depicted in Figure 6.20 along with several actual spectra. Both acceleration

(SA) and composite (PSA-SD) spectra are provided. It can be seen that the computed spectrum has distinct similarities with the actual spectra. Specifically: (1) it has similar peaks as the JMA record around 1s; yet its predominant periods are longer (an anticipated difference given the different soil thickness in the two sites). (2) Its overall shape strongly resembles that of Fukiai record. (3) it exhibits a secondary peak at around 2s, analogous to that of Takatori record. The latter feature is important for the inelastic response, which is examined next.

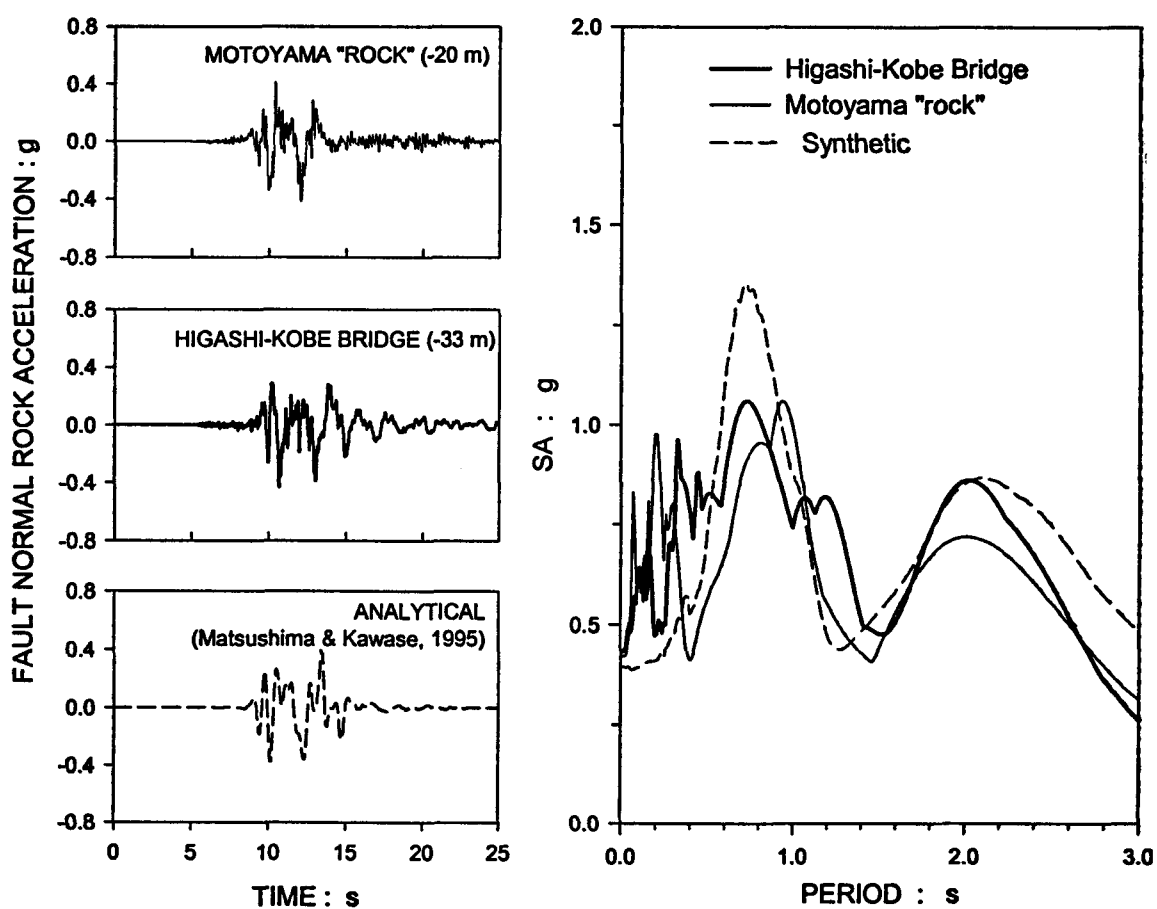


FIGURE 6.17: Fault-normal bedrock motions close to the bridge site, and corresponding 5%-damped acceleration spectra

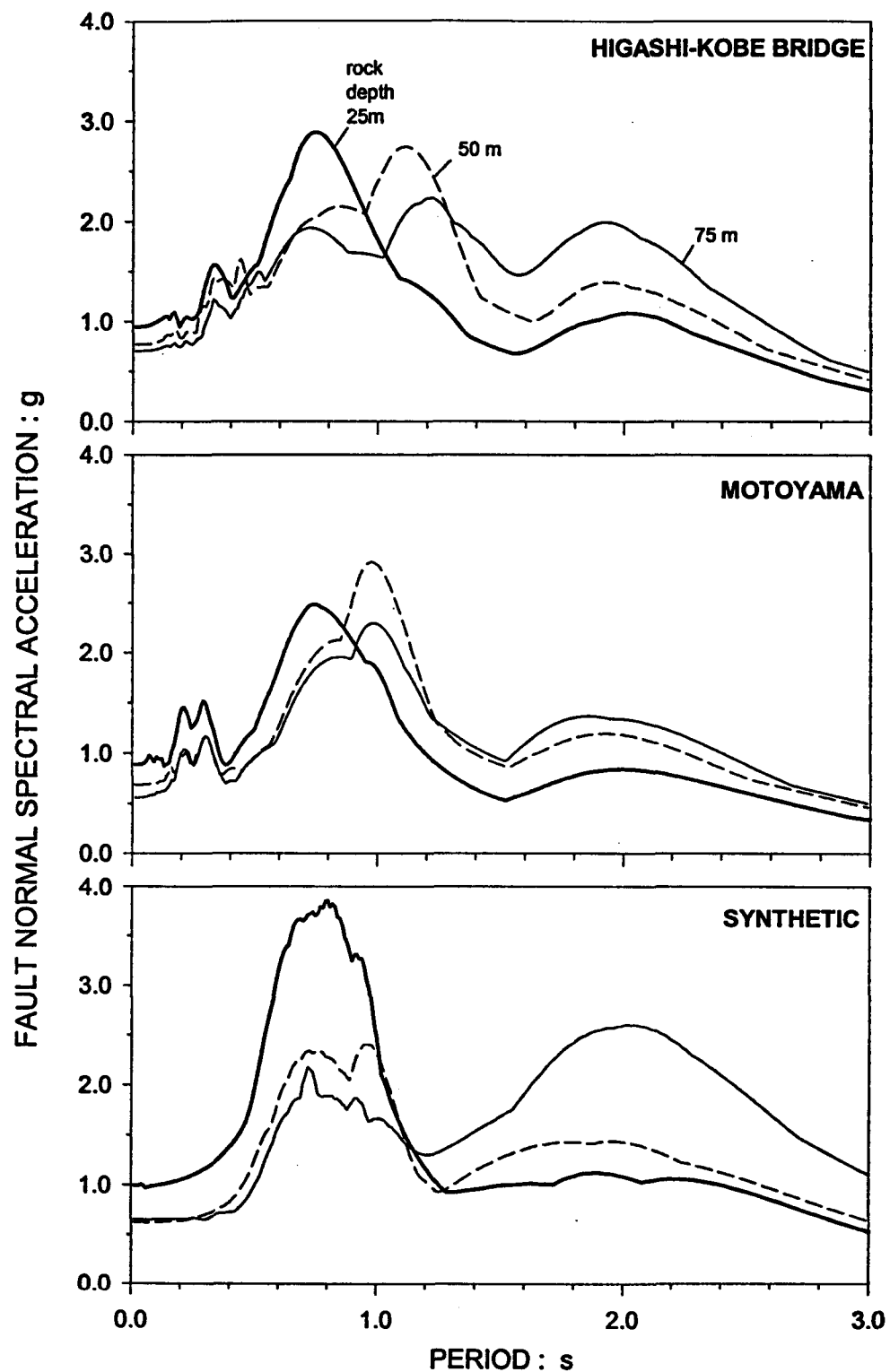


FIGURE 6.18: Fault normal acceleration spectra of surface motions convoluted based on the bedrock motions of Figure 6.17, assuming different rock depths; $\zeta = 5\%$

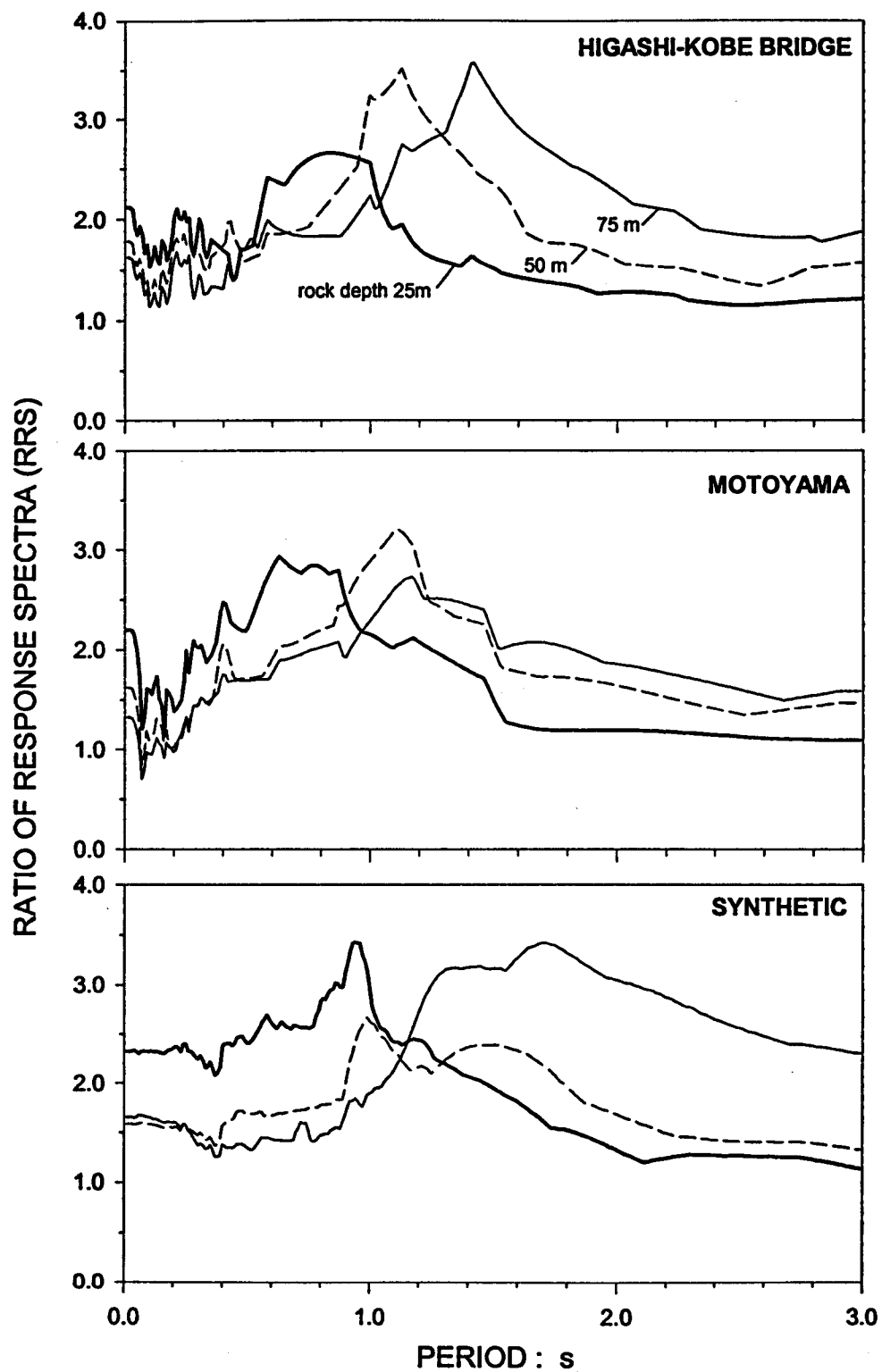


FIGURE 6.19: Ratio of response spectra between surface and rock motions along fault-normal direction; $\zeta = 5\%$

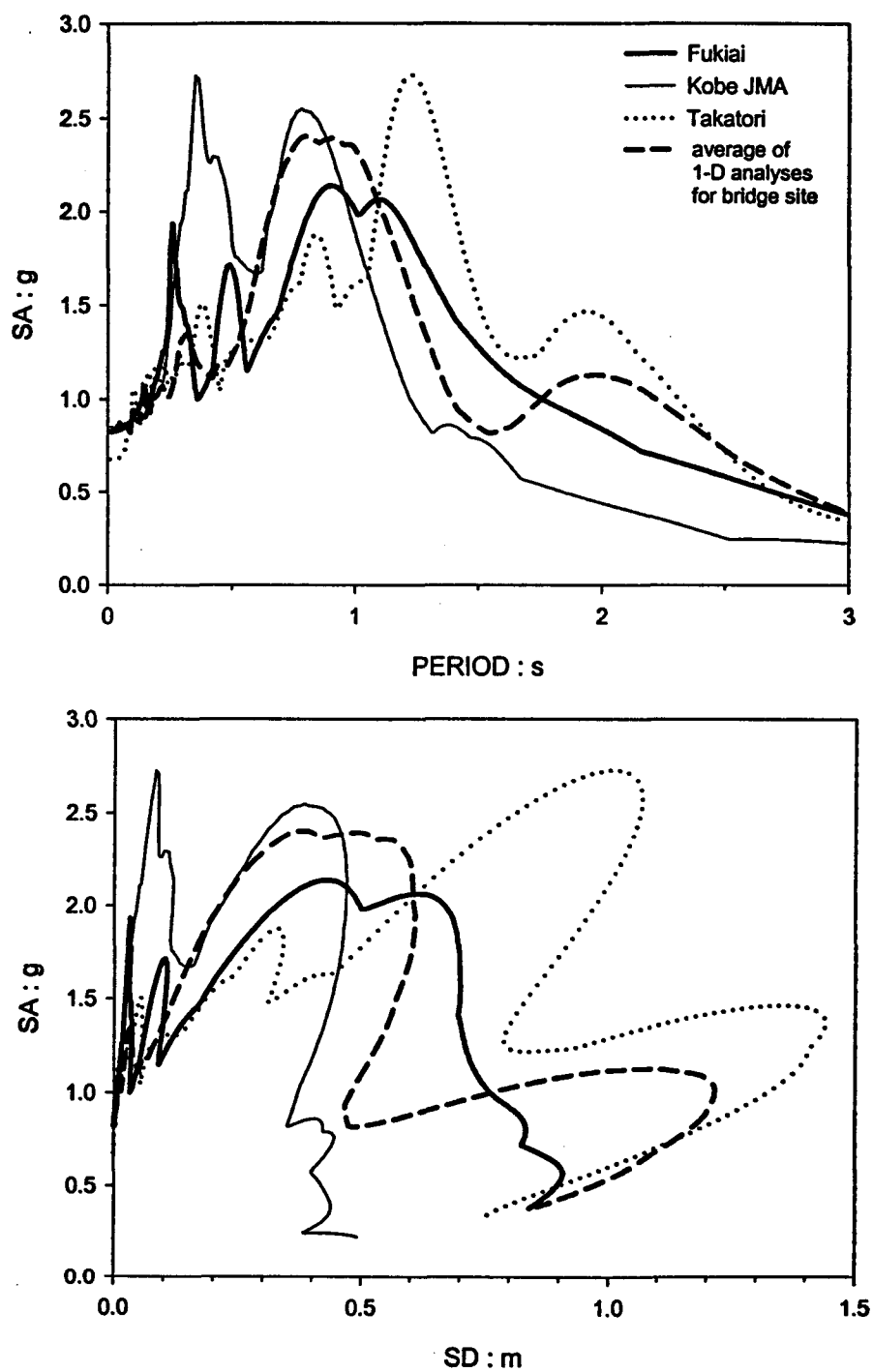


FIGURE 6.20: Comparison of derived site specific spectrum for Fukae with actual spectra; fault normal components; $\zeta = 5\%$

6.4 RESPONSE OF FOUNDATION-SUPERSTRUCTURE SYSTEM

The bridge consisted of 18 single circular piers (P126 to P143), measuring about 12m in height and 3.1m in diameter¹, founded on groups of 17 cast-in-place concrete piles (Figures 6.2a,b). The piers were monolithically connected to a 2-hinge prestressed concrete deck spanning a length of 35m. The piles were connected to a large pile cap of planar dimensions 9.6mx10.6m and thickness of 2.1m. Post-failure laboratory investigations provided an average concrete strength of 35Kpa and a Young's modulus of 28Gpa. For steel reinforcement, the corresponding figures were 5.3Mpa (yield at 3.5Mpa) and 200Gpa, respectively.

Longitudinal reinforcement in the pier consisted of 3 concentric rings composed of 60 D35 mm (#11) bars, a total of 180 bars at the bottom of the columns. The inner ring terminated at 2.5m above the pile cap, leaving 120 bars, which continued to the top of the pier. The concrete cover was 100mm thick and the spacing between the rings 100mm.

Transverse reinforcement consisted of three hoops of D 16 mm (#5). The inner hoop was spaced at 400mm and stopped at 2.5m above the cap. The two outside hoops were spaced at 200mm for the first 2.5m, then spaced at 300 mm for the next 4.2m, and then again spaced at 200mm for the remaining 4.3m (Iwasaki et al., 1995).

Structural parameters for the foundation-superstructure system used in previous studies are summarized in Table 6.2. Despite the differences in the cross-sectional moment of inertia and (especially) the stiffness of the bridge pier among the various studies, the variation in fixed-base natural period (T_{fixed}) is rather small, ranging between

¹ These dimensions correspond to Pier 138, which is analyzed here.

0.55 to 0.75 seconds. Considering SSI, natural period (T_{SSI}), is longer varying between 0.75 and 0.93 seconds. Differences in pier strength are considerable with the normalized yielding strength $C_y (= F_y / M / g)$ ranging between 0.5 and 0.7 depending primarily on the assumed concrete strength and structure mass. This strength is deemed high given the year of the design (1964) and the adopted seismic coefficient of 0.2 (Kawashima and Unjoh 1997). Estimated displacement ductility capacity of the pier ranges between 1.6 to 3.2 depending on the assumptions. Additional parameters listed in Table 6.2 are discussed below.

Following several earlier studies, a single pier model was adopted, as shown in Figure 6.21. This idealization is justified because of: (i) the almost identical geometry of all piers in the segment; (ii) the presence of moment releases in each span; (iii) the uniform ground conditions along the bridge; (iv) the almost identical failure pattern observed in all piers. Two different boundary conditions were examined: fixed-base and flexible base, as clearly indicated in Figure 6.21. The first model incorporates two dynamic degrees of freedom (translation and rotation of the deck), and the second four degrees of freedom. A more sophisticated model based on program DRAIN-2DX will be described later on. For the purposes of the investigation, the pier is considered to be either elastic, or elastic-fully plastic. The structure is excited in the out-of-plane direction (fault normal), which coincides with the direction of maximum ground motion and structural failure. Response to vertical ground motion was found to be small, as it induces dynamic vertical normal stresses in the pier of approximately 2% the design strength of unconfined concrete (Kitrilakis et al 1997).

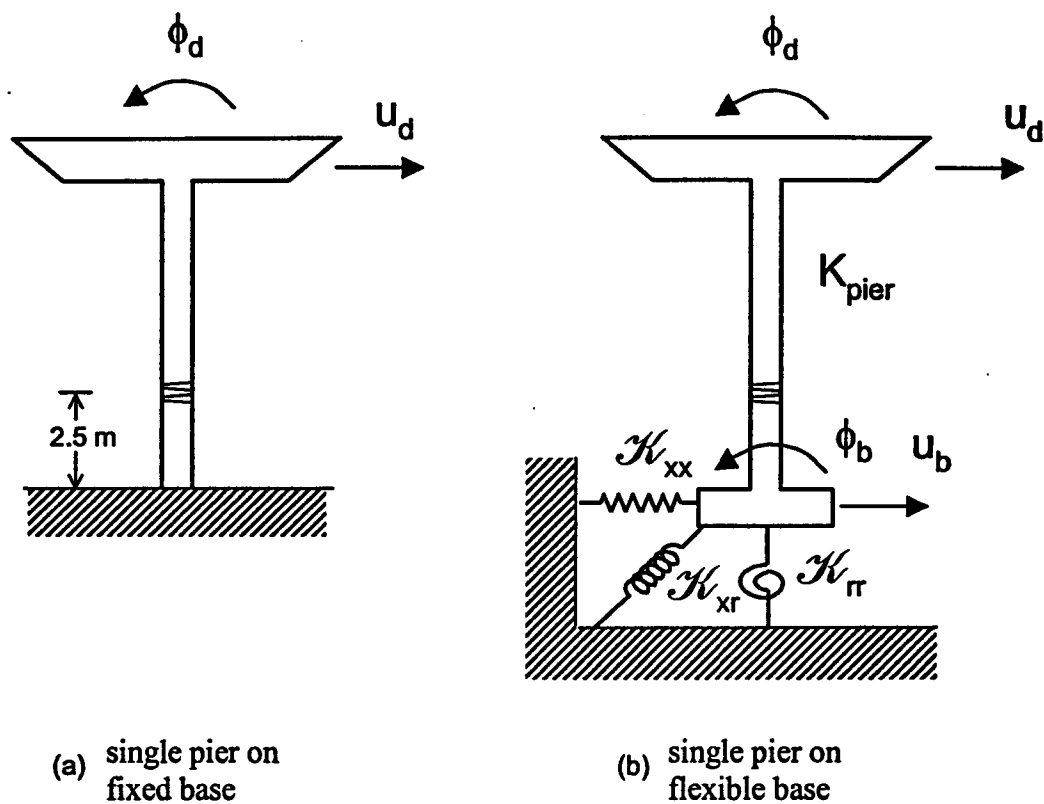


FIGURE 6.21: Pier models used in this study: (a) fixed-base pier with two degrees of freedom; (b) pier on flexible base with four degrees of freedom.

P- Δ effects were found to affect the pre-failure response of the massive pier by an insignificant amount (about 1%). In the subsequent analyses, both vertical excitation and P- Δ effects are neglected.

6.4.1 Vibrational Characteristics of Fixed-Base Pier

Detailed calculations performed by the Author suggest a participating deck mass of about 1000 Mg, a rotational deck moment of inertia of approximately 32,300 Mg m², and a pier mass of about 226 Mg (Table 6.3). Following Seible et al (1995), the cross-sectional moment of inertia of the cracked pier was taken at about 40% of its gross value.

Using this information, the fixed-base natural period of the bridge, modeled as a generalized simple oscillator, can be estimated from the Rayleigh quotient (Appendix G):

$$T_{\text{fixed}} = 2\pi \sqrt{\frac{M_{\text{deck}} + \phi_M M_{\text{pier}} + \phi_D^2 I_{\text{deck}}}{K_{\text{pier}}}} \quad (6.5)$$

in which $K_{\text{pier}} = 3 E I / L^3$ denotes the lateral stiffness of the cantilever pier. It should be noted that the “mushroom” shape of the piers allows for rotation of the deck. The dimensionless coefficients ϕ_M and ϕ_D account for the participation in the overall mass of the pier mass and the rotational inertia of the deck. They are computed from the following energy expressions:

$$\phi_M = \frac{1}{L} \int_0^L \psi^2 dx \quad (6.6)$$

$$\phi_D = \left. \frac{d\psi}{dx} \right|_{x=L} \quad (6.7)$$

where $\psi = \psi(x)$ stands for the deflected shape of the pier.

For fixed-base, free-head conditions, it is sufficient to assume²

$$\psi(x) = \frac{1}{2} \left(\frac{x}{L} \right)^2 \left[3 - \left(\frac{x}{L} \right) \right] \quad (6.8)$$

which corresponds to the deflected shape of a uniform elastic column subjected to a static lateral head force. In the above equation, x denotes the vertical distance from the base.

² $\psi(x)$ in Equation (6.8) is consistent with the stiffness of $3EI/L^3$ adopted for the pier

TABLE 6.2: Structural parameters used in previous studies

Reference	Seible et al. (1995)	Park (1996)	Kawashima & Unjoh (1997)	Michaelides & Gazetas (1998)	Anastasopoulos (1999)	Sun et al. (2003)
Model	Single pier on rigid foundation	Single pier on rigid foundation	Multiple piers on flexible foundation	Single pier on flexible foundation	Single pier on flexible foundation	Single pier on rigid foundation
L (m)	12.3	12	—	11	12	13.5
d (m)	3.1	3.1	3.1-3.3	3.1	3.1	3.3
E (Gpa)	—	30.1	27.8	20	25	—
I / I _{gross}	0.4	0.45	0.59	0.75	1	—
K _{pier} (MN/m)	80	107	128	155	157	126
M _{deck} * (Mg)	1100	1121	—	1200	1100	1262
I _{deck} (Mg m ²)	0	0	—	40000	40000	—
M _{cap} (Mg)	0	0	—	0	0	—
I _{cap} (Mg m ²)	0	0	—	0	0	—
T _{fixed} (s)	0.75	0.64	0.55**	0.68	0.68	0.63
T _{SSI} (s)	—	—	0.75	0.93	0.93	—
F _y (kN)	5407	6640	4673	8240 (bottom)	7280 (bottom)	—
C _y	0.5	0.6	0.43**	0.7	0.66	—
force-displ. relation	—	elastic-perfectly plastic	Takeda	elastic-perfectly plastic	BIAX (Park)	Extended Distinct Element Method
α	—	0	0.07-0.15	0	0	—
ζ _{pier} (%)	—	5	—	5	5	3
ζ _{SSI} (%)	—	—	—	7.5	7.5	—
μ _{capacity}	2.4	2.2	3.2	1.6	3.6	3.9
excitation	—	JMA	JMA	JMA, Fukiai	JMA, Fukiai, Takatori	JMA
μ _{demand}	—	> 2.2	> 3.2	1.3 to 1.7	> 3.6	>3.9

— not reported , * includes portion of pier mass

** estimated by the Author considering M_{deck} = 1100 Mg

TABLE 6.3: Structural parameters considered

L (m)	12				
d (m)	3.1				
E (Gpa)	27.8				
I / I _{gross}	0.4	0.5	1	0.5	0.5
K _{col} (MN/m)	88	109	219	109	109
M _{deck} (Mg)	1000	1000	1000	1000	1000
I _{deck} (Mg m ²)	32300	32300	32300	0	0
M _{cap} (Mg)	750	750	750	750	0
I _{cap} (Mg m ²)	9000	9000	9000	0	0
M _{pier} (Mg)	226 [†]	226 [†]	226 [†]	226 [†]	226 [†]
K _{xx} (MN/m)	310 (950)*				
K _{xr} (MN)	1090 (3750)*				
K _{rr} (MN m)	48300 (70000)*				
ζ _{xx}	0.31 (0.11)*				
ζ _{xr}	0.20 (0.08)*				
ζ _{rr}	0.03 (0.03)*				
T _{fixed} (s)	0.84	0.75	0.53	0.62	0.62
T _{SSI} (s)	1.04	0.98	0.84	0.89	0.87
ζ _{SSI} (%)	9.7	10.3	12.2	10.3	10.3
χ _{fixed}	0.72	0.72	0.73	1.07	1.07
χ _{SSI}	0.94	0.97	1.04	1.16	1.04

† participating mass approximately: $\left(\frac{33}{140}\right) \times 226Mg = 53Mg$ see Eqn. 6.9

* estimated by Michaelides & Gazetas (1998)

Carrying out the integration in equation (6.6) and using equation (6.7), one obtains

$$T_{\text{fixed}} = 2 \pi \sqrt{\frac{M_{\text{deck}} + \left(\frac{33}{140}\right) M_{\text{pier}} + \left(\frac{3}{2L}\right)^2 I_{\text{deck}}}{K_{\text{pier}}}} \quad (6.9)$$

Equation (6.9) yields,

$$T_{\text{fixed}} \cong 0.84 \text{ s} \quad (6.10)$$

in agreement with Table 6.3. The difference from the periods estimated by Seible et al. (1995) and Park (1996) (Table 6.2) is attributed to the inclusion of the rotational inertia of the deck. Note that the overall effective mass of the bridge according to the present analysis is

$$M = 1000 \text{ Mg} + \left(\frac{33}{140}\right) \times 226 \text{ Mg} + \left(\frac{3}{2 \times 12\text{m}}\right)^2 \times 32300 \text{ Mg m}^2 = 1557 \text{ Mg} \quad (6.11)$$

nearly 40% higher than in the aforementioned studies.

Owing to the presence of deck rotational inertia and pier mass in the analysis, the following modal participation factor is defined

$$\chi = \frac{M_{\text{deck}} + \left(\frac{1}{L} \int_0^L \psi \, dx\right) M_{\text{pier}}}{M} \quad (6.12)$$

pertaining to the horizontal seismic excitation assumed.

Equation (6.12) yields

$$\chi \cong 0.72 \quad (6.13)$$

which elucidates the role of deck rotational inertia in reducing the input in the system. A rigorous eigenvalue analysis of the fixed-base pier performed by the Author suggests that the error in the above estimates is only about 2 percent. The same analysis showed that the second mode of the system has a small contribution to the response and, thereby, can be neglected without significant loss of accuracy.

6.4.2 Vibrational Characteristics of Pier on Flexible Base

The compliance of the foundation further increases both the natural period, T_{SSI} , and the damping, ζ_{SSI} , of the system. Modeling the bridge as a generalized simple oscillator accounting for the horizontal translation and rotation both of the superstructure and the pile cap, good estimates of natural period and damping can be obtained from the energy expressions (Appendix G):

$$T_{SSI} = 2\pi \sqrt{\frac{M_{deck} + \phi_M M_{pier} + \phi_{21}^2 M_{cap} + \phi_{31}^2 I_{deck} + \phi_{41}^2 I_{cap}}{\phi_K K_{pier} + \phi_{21}^2 K_{xx} + \phi_{41}^2 K_{\pi\pi} + 2\phi_{21}\phi_{41} K_{x\pi}}} \quad (6.14)$$

$$\zeta_{SSI} = \phi_K \zeta_{pier} + \phi_{21}^2 \zeta_{xx} + \phi_{41}^2 \zeta_{\pi\pi} + 2\phi_{21}\phi_{41} \zeta_{x\pi} \quad (6.15)$$

in which ϕ_{21} , ϕ_{31} , ϕ_{41} , ϕ_K , and ϕ_M are dimensionless factors given by

$$\phi_{21} = K_{pier} (K_{\pi\pi} + K_{xx} L) \lambda^{-1} \quad (6.16a)$$

$$\phi_{31} = -\left[(1 - \phi_{21})\phi_D + \frac{1}{2}\phi_{41}\right] \quad (6.16b)$$

$$\phi_{41} = K_{\text{pier}} (K_{xr} + K_{xx} L) \lambda^{-1} \quad (6.16c)$$

$$\lambda = (-K_{xr}^2 + K_{rr} K_{xx}) + K_{\text{pier}} [K_{rr} + (2 K_{xr} L + K_{xx} L^2)] \quad (6.16d)$$

$$\phi_K = \frac{L^3}{3} \int_0^L \left(\frac{d^2 \psi_{\text{SSI}}}{dx^2} \right)^2 dx \quad (6.16e)$$

In the above equation ψ_{SSI} is a modified shape function expressing the deflected shape of the flexibly supported structure under a static lateral seismic load; it relates to ψ through

$$\psi_{\text{SSI}}(x) = (1 - \phi_{21} + L \phi_{41}) \psi(x) + \phi_{21} - x \phi_{41} \quad (6.17)$$

Equations (6.14) and (6.15) differ from similar formulations developed for structures on surface foundations (Veletsos 1977, Wolf 1985), due to the presence of cross terms K_{xr} and ζ_{xr} in the foundation, and rotational inertia in the deck, I_{deck} and pile cap, I_{cap} . These parameters are important for the structure at hand, given the large rotational inertia of the mushroom-like ("piltz") deck, the massive pile cap and the presence of piles in the foundation. Note that with increasing K_{xx} and K_{rr} , equation (6.14) duly reduces to equation (6.5). The participation factor of the flexible-base generalized oscillator is given by:

$$\chi_{\text{SSI}} = \frac{M_{\text{deck}} + \left(\frac{1}{L} \int_0^L \psi_{\text{SSI}} dx \right) M_{\text{pier}} + \phi_{21} M_{\text{cap}}}{M} \quad (6.18)$$

where M is the total effective mass of the system (equation (6.14)).

Using the finite-element code K-PAX (Chapters 2, 3) in conjunction with the pile-to-pile interaction model of Mylonakis and Gazetas (1999), estimates of foundation stiffness have been obtained as shown in Figure 6.22 and Table 6.3. These values pertain to low-excitation frequencies (less than about 1Hz) and peak soil shear strains in the free-field soil of 5×10^{-3} or higher. Corresponding values for low strains obtained by Michaelides and Gazetas (1998) are also provided. The differences between the predictions, particularly in the swaying mode, are as expected.

Based on the parameters listed in Table 6.3, the natural period, damping, and modal participation factor of the flexibly-supported system are estimated from equations (6.14) to (6.18) as

$$T_{SSI} \approx 1.05s \quad (6.19)$$

$$\zeta_{SSI} \approx 0.10 \quad (6.20)$$

$$\chi_{SSI} \approx 0.93 \quad (6.21)$$

which are indicative of the role of SSI in increasing natural period by an appreciable 20% and damping by 100%. The participation factor of the generalized mode is higher than in the fixed-base system. Note that the damping estimate in equation 6.15 does not account for inelastic action in the pier.

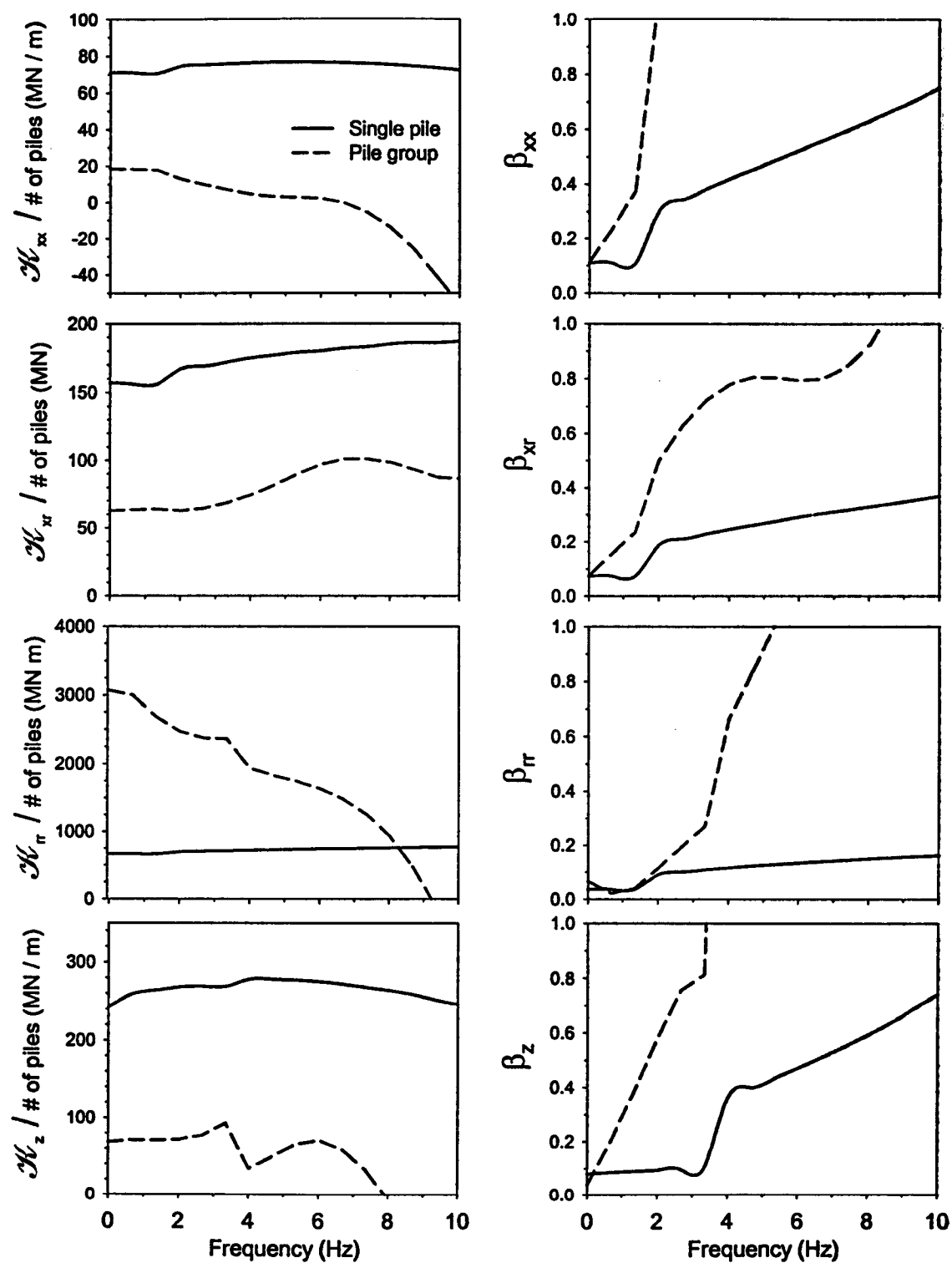


FIGURE 6.22: Real part of stiffness and damping ratio for a single pile and the 17-pile group of pier 138.

Figure 6.23 depicts the sensitivity of the system's natural period depending on pier and foundation stiffness. The most probable range for natural pier period is between 0.9s and 1.1s, accounting for base flexibility and concrete cracking.

Detailed eigenvalue analyses performed by the Author suggest that the participation of higher modes in the response is small and can be ignored without significant loss in accuracy. This becomes apparent in Figure 6.24, referring to steady-state translational oscillations of the bridge deck due to harmonic ground excitation.

These analyses were carried out using the computer code SPIAB (Mylonakis 1995), which takes into account the frequency-dependence of foundation stiffness and damping. Evidently, peak transfer functions between deck and free-field response hardly exceed 2.5 at second resonance and 0.75 at third resonance, justifying a single mode analysis.

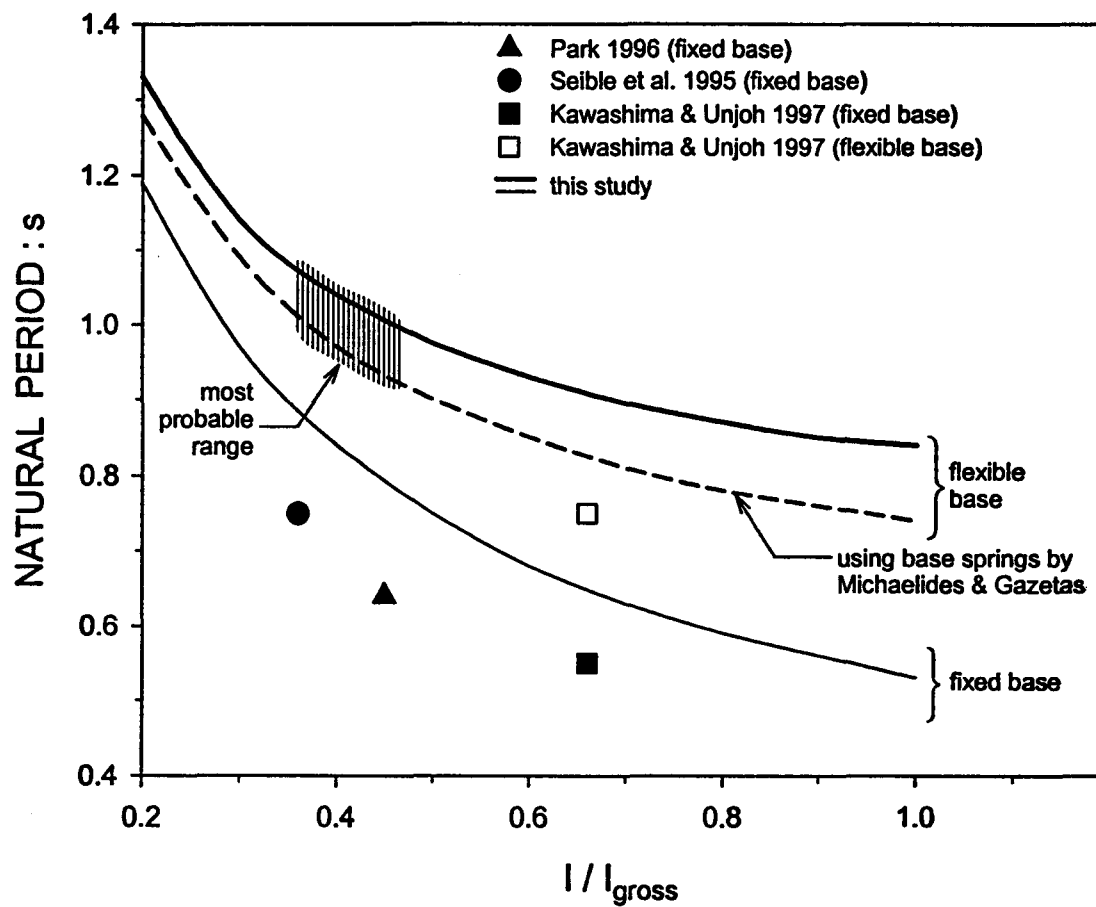


FIGURE 6.23: Fundamental natural period of collapsed pier as function of column cross-sectional moment of inertia. Comparison of analytical results from various studies.

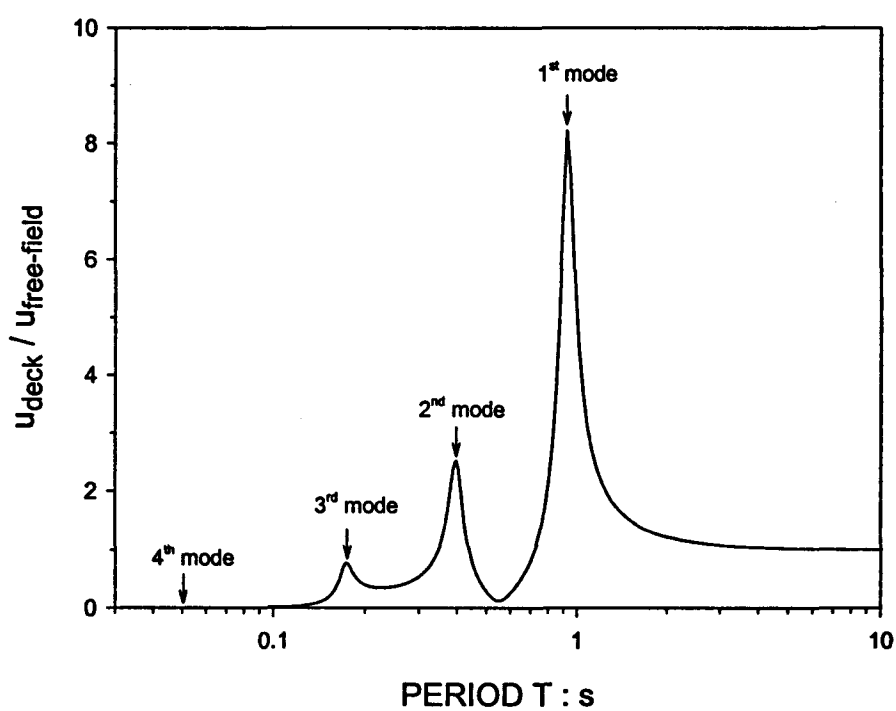


FIGURE 6.24: Harmonic steady-state transfer function between deck translation and free-field surface displacement. Structural properties refer to set (1) in Table 6.3.

6.4.3 Simplified Elastic Analysis

From the elastic spectra of Figure 6.25, the influence of SSI on the response starts becoming apparent. For instance, if the actual excitation was similar to the JMA record, the increase in period due to SSI and the progressive cracking of the pier would tend to slightly reduce the response, as indicated by the decreasing trend of the spectrum beyond about 0.8 sec. In contrast, with either Fukiai or Takatori motions (undoubtedly more likely surrogate motions to the unknown real ones), SSI would lead to equal or higher response. The trend becomes more apparent with the Higashi-Kobe and Motoyama amplified records (site thickness of 50m), for which elastic response at T_{SSI} may exceed 2.5g.

As a first approximation, for the somewhat conservative estimate of $SA \approx 0.93 \times (2.1g) \times (5/9.5)^{0.4} = 1.47$ g, which is derived from the Fukiai spectrum and accounts for both the modal participation factor of the generalized mode and the increased damping due to SSI (Table 6.3), the force reduction factor based on a calculated strength ratio, C_y , of the column of about 0.5 would be equal to approximately $1.47 / 0.5 \approx 3$. Taking the *equal displacement* rule as approximately valid, the ductility demand on the system, $\mu^{(system)}_{demand}$, would be:

$$\mu^{(system)}_{demand} \approx R \approx 3 \quad (6.22)$$

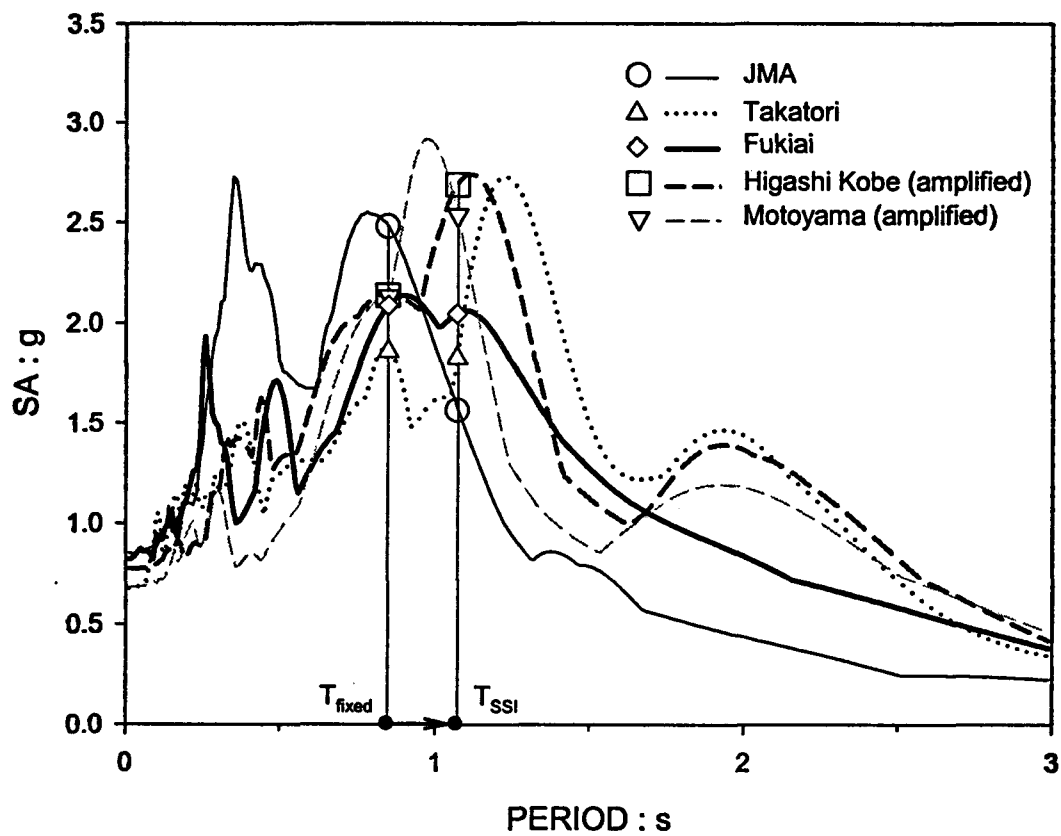


FIGURE 6.25: Acceleration response spectra of selected ground motions in the fault normal direction; $\zeta = 5\%$ (modified from Gazetas & Mylonakis 1998)

The ductility demand on the *pier*, $\mu^{(\text{pier})}_{\text{demand}}$, is obtained by considering only pier deformations³. For an elastic-perfectly plastic system, this can be done using the expression (Mylonakis and Gazetas 2000)

$$\mu^{(\text{pier})} = (1+c) \mu^{(\text{system})} - c \quad (6.23)$$

where c is a dimensionless factor expressing the relative flexibility of foundation and superstructure

$$c = K_{\text{pier}} \frac{L^2 K_{xx} + 2 L K_{xr} + K_{rr}}{K_{xx} K_{rr} - K_{xr}^2} \quad (6.24)$$

For the problem at hand, $c = 0.76$; thus,

$$\mu^{(\text{pier})}_{\text{demand}} = (1 + 0.7) \times 3.1 - 0.7 = 4.6 \quad (6.25)$$

which is 50% higher than system ductility and far exceeds the ductility capacity of the pier (Table 6.2).

On the other hand, ignoring SSI, and for the conservative value of

$$SA \approx 0.72 \times 2.1 \text{ g} = 1.51 \text{ g} \quad (6.26)$$

which accounts for the participation factor of the generalized system in Table 6.3, the spectra of Figure 6.25 would yield a ductility demand of

³ Recall that the response of the flexibly-supported pier involves rigid body movements, which do not inflict structural damage. Design implications of this effect are discussed in Ciampoli & Pinto (1995) and in Mylonakis & Gazetas (2000).

$$\mu_{\text{demand}}^{(\text{pier})} \approx R = \frac{1.51}{0.5} = 3 \quad (6.27)$$

which, although conservatively estimated, does not exceed the upper bound ductility capacity of 2 to 3.2 (Table 6.2) and, thereby, could hardly explain the spectacular failure of the bridge.

Although approximate, the above results indicate that the role of soil in the collapse could have been multiple: *First*, soil modified (in 1D or 2D fashion) the seismic waves so that the frequency content of the surface motion at the site became disadvantageous for the particular structure (i.e., similar to Fukiai or Takatori, rather than JMA). *Second*, the compliance of soil at the foundation increased the period of the system and moved it to a region of stronger response and, hence, higher inelastic demand. *Third*, the compliance of the foundation increased the participation of the fundamental mode inducing stronger response. *Fourth*, the ductility demand in the pier was substantially higher than the ductility demand of the system, as evident in Equations (6.26) and (6.27).

6.4.4 Simplified Inelastic Analysis

A more rigorous analysis can be employed by means of the composite spectrum technique, as shown in Figure 6.26. The method is well known (Reinhorn 1997; Chopra and Goel 1999; Fajfar 1999); only results are presented here. Employing the iterative scheme based on the Takeda damping model (Lin and Chang 2003), ductility demands for the inelastic pier were computed (Figure 6.26b). Note the jumps in the curves for Takatori, Higashi Kobe and Motoyama records at around 0.85s to 1s, which practically coincides with the period of the flexibly supported structure. These demands are so high

that could not be possibly sustained by the inadequately designed pier. On the other hand, the Fukiai and JMA records exhibit a monotonic decrease in ductility demand beyond 0.35s. These findings are in qualitative agreement with those of the previous analyses, providing additional evidence as to a potentially detrimental role of SSI for the particular structure. It is worth mentioning that these jumps are not associated with the spectral peaks close to the period of 1s, but with those at around 2s (Figure 6.26a). In the realm of the composite spectrum analysis, this suggests a period lengthening of the order of 2 due to inelastic action in the pier.

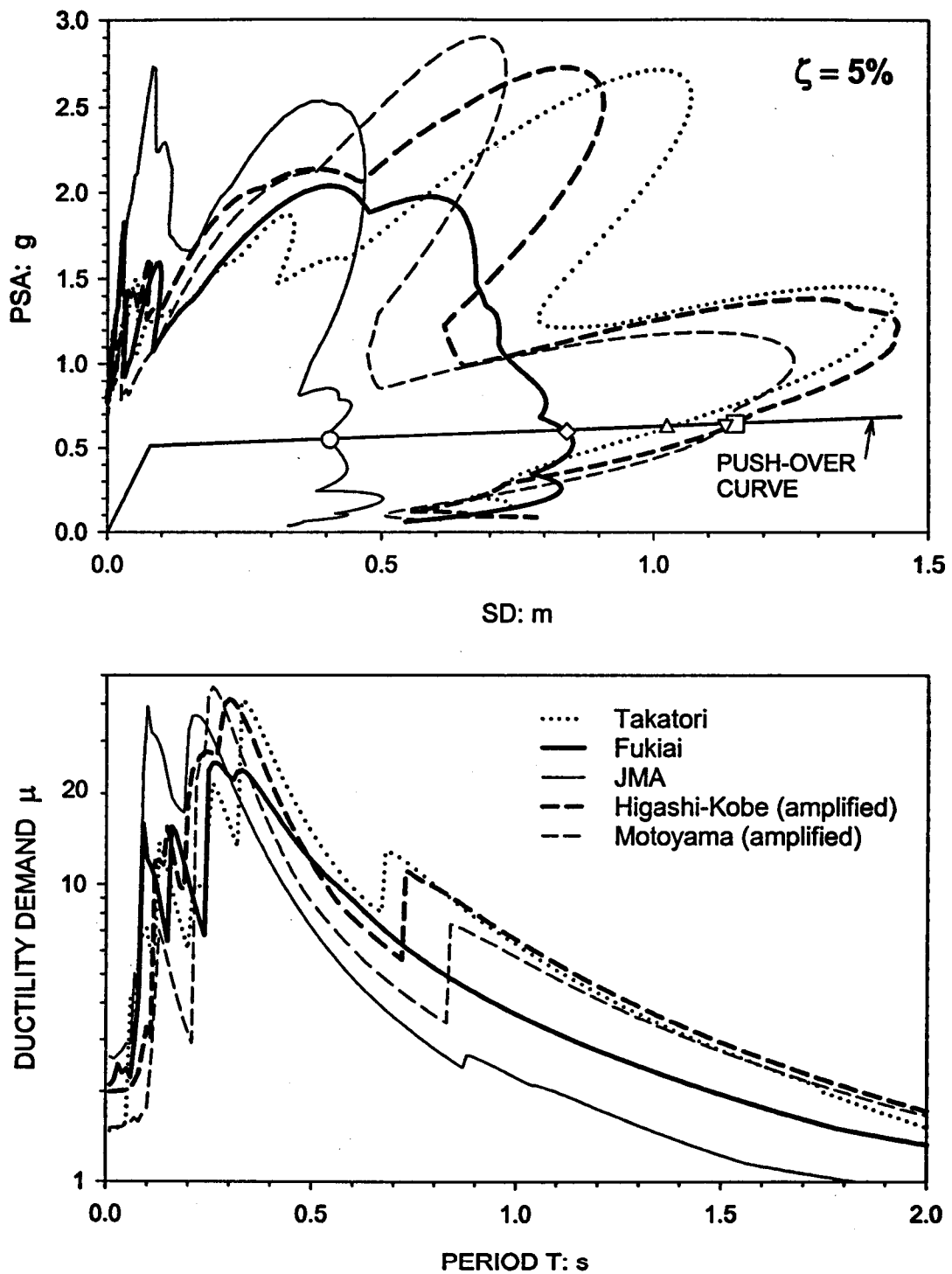


FIGURE 6.26: (a) Composite spectra of main acceleration records and structural push-over curve; (b) corresponding pier ductility demands using Takeda's damping model; $C_y = 0.5$

6.4.5 Nonlinear Inelastic Analyses

To gain further insight on the importance of SSI in inelastic response, a series of non-linear time-domain inelastic analyses were conducted. As a first step, a fixed-base elastoplastic SDOF oscillator was considered. The response was determined by means of the Bouc-Wen model (Appendix E), in conjunction with a state-space numerical formulation based on the Adams Moulton algorithm. Results for three excitation motions and three pier strengths are provided in Figure 6.27 plotted in terms of ductility demand versus elastic structural period.

To account in an approximate manner for the effect of rotational inertia of the deck (which is not incorporated in the SDOF system), the input acceleration should be multiplied by the participation factor of the generalized simple oscillator (i.e., $\chi \cong 0.72$, see eq. 6.13). An equivalent approach is to consider a modified strength for the oscillator,

$$C_y^* = C_y / \chi = 0.5 / 0.72 = 0.7 \quad (6.28)$$

which is about 40% higher than the actual yielding strength. This increase in strength suggests a beneficial influence of the rotational inertia of the deck for structures of this type.

For flexible base conditions, the corresponding strength is (Equation 6.21):

$$C_y^* = C_y / \chi_{SSI} = 0.5 / 0.94 = 0.53 \quad (6.29)$$

In the light of this information, the ductility demand in the fixed-base pier for JMA, Fukiai, and Takatori records is about 2, 3 and 2.2, respectively. For the flexible-base system, using, as an approximation, a SDOF system with a modified (longer) natural

period and strength, the corresponding system ductilities are read from Figure 6.27 at approximately 2.2, 3.3 and 4.

The ductility demand in the pier can be extracted from the above values by means of eq. (6.23). This yields the estimates of 2.9, 5 and 6.3 for the three records, respectively.

Although approximate, the above analysis indicates a strong detrimental influence of SSI on inelastic response. This is in agreement with the previous discussions (except for the observed increase for JMA motion). The results for Fukiai motion ($\mu_{\text{fixed}} = 3$, $\mu_{\text{SSI}} = 5$) are in accord with those in Equations (6.27) and (6.25).

Proceeding to a more advanced model, a multi-degree of freedom inelastic model of the pier was developed using the computer code DRAIN-2DX (Prakash et al. 1993). In this model, the column was divided into four two-noded inelastic beam elements, each having one translational and one rotational degree of freedom at each end. Concentrated plasticity at the ends of the elements was adopted. The influence of shear forces in the development of plastic deformations was neglected. The compliance of the foundation was modelled using a series of springs and dashpots attached to the base of the pier (Fig. 6.21). Assuming initial yielding 2.5 m above the cap, a yielding force of 5,636 kN was established, corresponding to a deck acceleration of about 0.5 g. The inherent (non-SSI) damping of the structure was assumed of the Rayleigh form, taken equal to 5% of critical. The SSI dashpots at each degree of freedom were computed from the linear coefficients ζ_{ij} of the foundation impedance, at the characteristic period T_{SSI} (Fig. 6.22). Eigenvalue analyses provided the values $T_{\text{fixed}} = 0.88\text{s}$ and $T_{\text{SSI}} = 1.07\text{s}$ which are in good agreement with the results of the simplified model in Table 6.3. Results obtained with five earthquake records are depicted in Table 6.4.

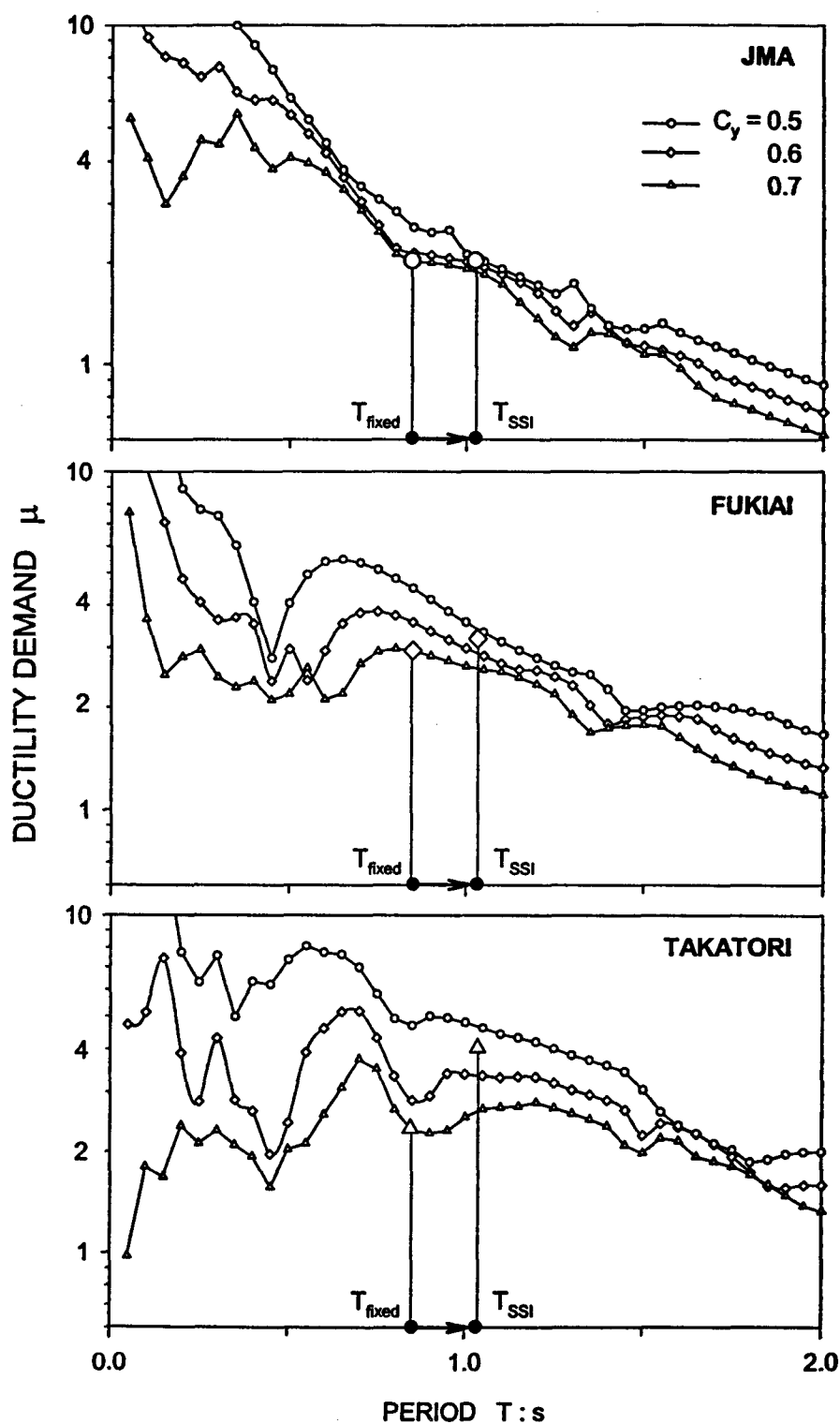


FIGURE 6.27: Ductility demand of a SDOF oscillator subjected to the fault-normal component of three main records; $\zeta = 5\%$

For the JMA record, SSI appears to play a beneficial role, as column ductility demand decreases from 2.5 for the fixed-base pier to 2.2 for the flexibly supported one. In contrast, with Fukiai and Takatori motions, SSI is clearly detrimental, increasing substantially the ductility demand in the pier. In the case of the Fukiai record, the agreement between the numerical results and those in equations (6.25) and (6.27) is encouraging for the simple analysis. The strongest SSI effect is observed with the Takatori record: μ increases from 3.2 for the fixed-base structure to the astonishing 7.3 for the flexibly supported—a somewhat fortuitous consequence of the strong peak at about $T \approx 1.2$ seconds.

Substantial increase in ductility demand with SSI is also observed with the amplified Higashi–Kobe motion (Figure 6.28), while with Motoyama its role is rather minor. The ranges in computed ductility values for the Motoyama and Higashi stem from the different scenarios of soil thickness used in the amplification analyses. Again, the trends obtained with the simple analysis are in qualitative agreement with the numerical study.

The excessive seismic demand computed with Fukiai, Takatori, and Higashi–Kobe records may explain the spectacular failure of the 18 piers of the bridge, especially if one considers the simultaneous deleterious action of the cyclic shear force in the cracked and plastically deforming column cross-section. This suggests that the actual excitation at the site may have indeed resembled the Fukiai, Takatori, or amplified Higashi–Kobe motions much more than the JMA or amplified Motoyama accelerograms.

TABLE 6.4. Tabulated results from DRAIN-2DX and simplified analyses of inelastic bridge response

Excitation	Pier Ductility Demand		Increase (%)		Role of SSI	Predicted Performance
	Fixed-Base (A)	Deformable Base (B)	DRAIN-2DX (columns A, B)	Simple model (Eqs. 6.23 to 6.27)		
Fukiai	3.1	4.1	+ 32	+ 54	detrimental	failure
Takatori	3.2	7.3	+ 128	+ 42	very detrimental	failure
Motoyama [†]	3.5 to 3.7	3.2 to 3.5	– 5 to – 9	+ 92	≈ minor	probably failure
Higashi [†]	3.9 to 4.6	4.8 to 6.4	+ 23 to + 39	+100	detrimental	failure
JMA	2.5	2.2	– 12	– 12	slightly beneficial	heavy damage

[†] Amplified to account for soil effects

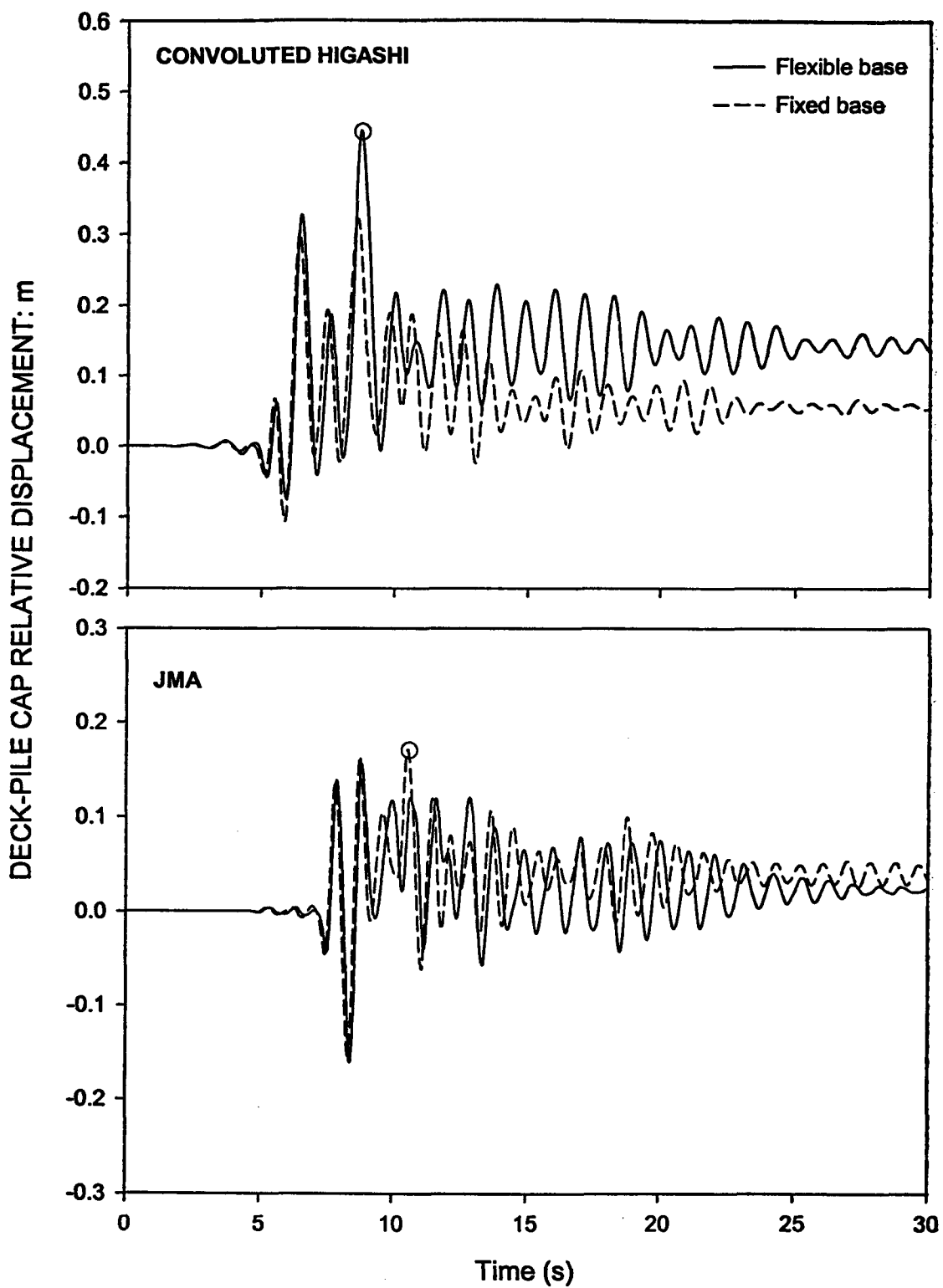


FIGURE 6.28: Pile-cap to deck relative displacement from DRAIN-2DX non-linear analyses of flexible and fixed base systems using fault-normal components of JMA and convoluted Higashi records.

6.4.6 A Final look: The Role of Reconstruction and Seismic Isolation⁴

The collapsed Fukae section was replaced by a completely new design. With the exception of the capped pile foundation, which was extremely difficult to replace and was found only slightly damaged, everything else has been rebuilt. The cylindrical piers were replaced by large orthogonal 6.0 m x 3.0 m “shear walls”. In addition, the monolithically mounted concrete deck was replaced by a steel structure seismically-isolated from the supporting pier.

The performance of the new design has been evaluated by Anastasopoulos and Gazetas (1999). The following analysis is due to them and Syngros et al. (2003). The pier is found to possess a yielding strength $F_y \geq 15\,000$ kN, leading to a normalized strength, C_y , of at least 1.50. Its huge stiffness would lead to a period $T_{\text{fixed}} \approx 0.20$ s, if the deck had not been seismically isolated. Taking SSI into account, and following the logic of section 6.4.3, the period of the system becomes $T_{\text{SSI}} \approx 0.55$ sec, for which Figure 6.24 shows that most response spectra are about 1.5 g. And since the ductility capacity of the new pier is found to exceed 5, the new pier would not collapse in a repeat of the Kobe earthquake, even if the deck had not been seismically isolated. In fact, accepting that $C_y = 1.5$, only a spectral acceleration of the order of $\approx 1.5 \times 5.0 = 7.5$ g could lead to collapse! However, seismic isolation was also utilized, presumably for two reasons: (a) to reduce the horizontal force and overturning moment onto the piled foundation, and thus avoid the risk of further pile damage, and (b) to minimize the likelihood that the pier will suffer

⁴ Based on analyses by Anastasopoulos and Gazetas (1999).

even minor damage. Two Lead Rubber Bearings (LRB) isolate the two beams of the deck from the pier. The stiffness of the LRBs can be estimated as:

$$K_{LRB} = \frac{2GA}{h} \quad (6.30)$$

where A is the LRB's cross sectional area, h its effective height, and G its effective shear modulus. With $G = 1$ Mpa, $A = 1.07$ m², and $h = 0.183$ m, $K_{LRB} = 12.5$ MN/m. The stiffness of the system is then obtained from:

$$K_{isolated} = \frac{K_{pier} K_{LRB}}{K_{pier} + K_{LRB}} \quad (6.31)$$

With $K_{pier}=1608$ Mn/m, and $K_{LRB} = 12.5$ MN/m, the stiffness of the system is $K_{isolated} = 12.4$ MN/m, and the period of the isolated structure becomes:

$$T_{isolated, fixed} = 2\pi \sqrt{m / K_{isolated}} \approx 2s \quad (6.32)$$

Taking account of SSI (according to section 6.4.3) the period of the system now increases only marginally to $T_{isolated,SSI} \approx 2.10$ sec. Going back to Figure 6.24, we observe that for all the records the spectral accelerations are between 0.90 g and 1.3 g. So, the isolated structure would behave linearly, without any damage.

The logical question that arises is what would have happened if the original deck were seismically isolated. Following the logic of equations (6.14) to (6.20), and for the same LRBs, we find that the period of the original bridge would be equal to $T_{isolated,SSI} \approx 2.15$ sec. As in section 6.4.3, a first approximation of the spectral acceleration would lead to $SA \approx 0.8$ g $(5/10)^{0.4} \approx 0.61$ g with the Fukiai excitation. Accepting $C_y = 0.5$, the

ductility demand would be equal to $0.61/0.50 = 1.22$. Following Equations (6.22) to (6.27), the ductility demand on the pier $\mu_{\text{demand}}^{(\text{pier})} = 1.35$, which is well within the ductility capacity of the pier (Table 6.2: worst estimate $\mu_{\text{capacity}} = 2.2$). To confirm the validity of this result, the isolated system was analysed numerically. The results of the non-linear inelastic analysis confirm that the bridge would not have failed if its deck were seismically isolated. The maximum acceleration was found to be 0.64 g, which is very close to the former approximation. The maximum curvature did not exceed 0.1%, a value indicating only small straining. The pier would simply not have failed if the deck was seismically isolated. However, the relative displacement between pier and deck reaches 55 cm. The analysis does not take into account the possibility of LRB failure. Given that the height of the LRB is 18 cm, a relative displacement of 55 cm would cause shear deformations higher than 250 %, which is at, or just below, its ultimate deformation limit. This means that the LRB would have sustained damage or perhaps barely failed in such a case. So, the bridge would have survived the earthquake with some half-a-meter permanent lateral displacement and its LRBs more-or-less seriously damaged — not an unacceptable damage, in midst of this unprecedented disaster.

The Fukae section was built in 1969, a time at which the LRB had not yet been established. However, slider-bearings were used widely much earlier than the 60s, for accommodating temperature induced strain. It is interesting that the section adjacent to the collapsed Fukae section survived the earthquake with only minor pier damage. The “temperature” bearings were seriously damaged and the deck exhibited permanent displacements of the order of 30 cm. Piers and foundation of both sections were practically identical. The only difference was the connection of the deck to the pier.

Contrary to the Fukae section, where pier and deck were monolithically connected, the deck of the unharmed section was simply supported. This compliance, accompanied by the failure of the pinned bearings allowed the superstructure to seismically isolate itself, protecting the piers from serious failure. When the deck is simply supported through slider bearings it tends to follow the pier only if the acceleration is a fraction of gravity less than the friction coefficient, μ . The slider prevents higher accelerations from developing, and hence limits the inertia forces transmitted on the pier.

The *seismic isolation using* low friction sliding bearings, with $\mu \leq 0.05$, is a relatively new technology. However, sliding bearings with $\mu = 0.30$ were surely available at the time and such an intermediate type of isolation would have most likely saved the bridge. This kind of isolation would not aim at isolating the bridge completely; for that purpose low friction sliding bearings would be needed. It aims at acting as a “fuse” protecting the structure from higher accelerations than the ones it is designed for. By comparing the two analysed cases of seismic isolation, it becomes evident that the “fuse”-type isolation performs at least as well as the LRB. Even the permanent displacement, which is usually the main problem of such solutions, is limited to merely 15 cm. Observe that the pier behaves elastically while with the LRB some inelasticity (leading to minor pier cracking) might have taken place. Thus, the “fuse” protected bridge would suffer no structural damage provided of course that the seating of the deck was large enough to accommodate the 0.5m of maximum displacement. Nevertheless, some “re-centering” of the deck would have to be accomplished after the earthquake.

6.5 CONCLUSIONS

Analytical and recorded evidence is presented on a quadruple detrimental role of soil in the collapse of Hanshin Expressway at Fukae. *First*, the soil modified the incoming seismic waves such that the resulting ground surface motion became very severe for the particular bridge. *Second*, the presence of compliant soil at the foundation resulted to an increase in natural period of the bridge, which moved to a region of stronger response; *Third*, the compliance of the foundation increased the participation of the fundamental mode, inducing stronger response. *Fourth*, ductility demand in the pier was higher than the ductility demand of the system, as suggested by Equation (6.25). All four phenomena might have simply worsen an already dramatic situation for the bridge due to: (i) its proximity to the fault and the strong forward rupture directivity effects which produced very high long-period acceleration normal to the fault, which is exactly in the transverse direction of the bridge; and (ii) the structural deficiencies of the pier which were almost unavoidable given the time of design of the bridge (1964). The bridge would not have collapsed if it had been seismically isolated. If LRBs were utilized to isolate the structure (as are today) the bearings would have been significantly distressed and the pier would have experienced some cracking. Had the bridge been seismically isolated with high friction ($\mu = 0.3$) fuse-type sliding bearings it would have survived the earthquake with no visible damage other than a permanent offset of the deck of no more than about 20cm. The above conclusions contradict a widespread view among practicing engineers, of an always-beneficial role of soil-structure-interaction in seismic response.

CHAPTER 7

CONCLUSIONS AND RECOMMENDATIONS FOR FUTURE RESEARCH

7.1 Summary and Conclusions

The aim of this research was to explore certain aspects of the dynamic soil-structure interaction problem with applications to the seismic analysis of bridge piers supported on pile foundations.

In the first chapter, an overview of the SSI problem is presented; the definition and the methods available for treating it are discussed in a concise yet unambiguous way. The substructure approach is followed, in which the complete SSI problem is split into two parts: kinematic interaction and inertial interaction. Emphasis is given on the inertial interaction problem, more specifically to its two sub-components, the foundation impedance problem and the foundation-superstructure-interaction problem.

The second chapter deals with certain aspects of the foundation impedance problem, namely the impedance of a single pile subjected to an axial harmonic head-load. The problem is examined using a rigorous elastodynamic finite element formulation equipped with wave transmitting boundaries and special integration schemes for axisymmetric geometries. Different soil profiles are analyzed, ranging from a homogeneous half-space to profiles that are inhomogeneous both with respect to depth (vertically) and with radial distance from the pile (horizontally-radially).

The effect of radial inhomogeneity on pile response was explored. For static conditions, it was shown that accounting for radial inhomogeneity may increase pile head

settlement by as much as 50%. For dynamic loading, the concept of a substitute (“statically equivalent”) soil profile was introduced, which yields the same static pile settlement as the actual inhomogeneous profile. For statically equivalent profiles, it was shown that radiation damping decreases as soil becomes more inhomogeneous. Given that actual deposits are rarely homogeneous, radiation damping estimated based on a homogeneous half-space solution is usually unrealistically high. It was also shown that assuming a smaller elastic modulus for the soil is conservative (due to increased settlements) for static analysis, but may render a dynamic analysis un-conservative due to increased radiation damping. Finally it was shown that soil profiles with different radial inhomogeneity but identical vertical structure exhibit similar soil reactions along the pile. On the other hand, soils with different vertical structure, but identical radial inhomogeneity yield similar values for Winkler springs along the pile.

In the third chapter, the case of horizontal pile loading was examined, for two limiting conditions: a pile whose head is fixed against rotation; and a pile whose head is free to rotate. For static loading it was observed that active pile length is about 25% higher for fixed-head conditions, as compared to the (usually assumed) free head conditions. The trend is anticipated since a fixed head pile is stiffer and, thereby, mobilizes soil reaction at higher depths. In addition, a new definition was provided for the active pile length, based on the soil reaction along a flexible pile. According to the new definition, the active pile length is defined by the point (+ 1 to 2 d) where soil reaction to pile movement becomes zero. In the case of dynamic loading, it was shown that radiation damping decreases with increasing soil inhomogeneity – in agreement with the results of the previous chapter. In addition, it is shown that the cut-off frequency of a

soil deposit increases with increased levels of inhomogeneity, thus radiation damping appears at higher frequencies. Finally, normalized Winkler springs and dashpots were determined, to be used in simplified Beam-on-Winkler-Foundation models.

In the fourth chapter, comparisons were made between the theoretical results of the first two chapters and experimental results from seven (7) field experiments of actual pile foundations. Results from the analysis of single piles were used in conjunction with wave-based superposition methods to predict the dynamic response of pile groups. In all cases examined, accounting for radial inhomogeneity in the soil resulted in improved predictions of foundation stiffness and damping. However, in some of the tests, predicted damping was higher than measured regardless of inhomogeneity level. No satisfactory explanation was found for this discrepancy.

In the fifth chapter, the recorded response of a pile-supported pier of an actual bridge (Ohba Ohashi) is examined. This massive pier is founded on extremely soft clay ($V_s < 100\text{m/s}$) thus making necessary to account for soil-structure interaction in dynamic analyses of the superstructure. It is shown that the response of the bridge can be well simulated by means of simple models accounting for foundation compliance and the effect of the nearby piers. However, free-field response and pile bending strains could be perfectly simulated by the methods employed in this study, due to complex wave scattering resulting from valley effects.

In the sixth chapter the response of a collapsed segment of a concrete bridge (Hanshin expressway) is examined accounting for non-linear soil structure interaction. It is shown that SSI played a detrimental role in the performance of the pier. First, the soil modified the incoming seismic waves such that the resulting ground surface motion

became very severe for the particular bridge. The motion at the site was less severe (in terms of ground accelerations) than motions recorded a few kilometres away, but was significantly different in frequency content, which made it very detrimental for the particular bridge. Second, the presence of compliant soil at the foundation resulted to an increase in natural period of the bridge which moved it to a region of stronger response. These effects, together with the inadequate reinforcement of the piers, seem to have aggravated the inelastic behavior of the pier leading to the spectacular collapse. Simple techniques were proposed to analyse this SSI effect. These techniques make use of generalized SDOF models that take into account the foundation stiffness and the mass moment of inertia of the deck to determine the SSI stiffness and damping. It is shown that using simple analytical formulas in conjunction with elastic acceleration spectra yields results that agree qualitatively with more rigorous non-linear analyses.

7.2 Recommendations for future research

The recommendations for future research focus on the substructure method used in the SSI analysis. These are organized in three categories: a) the kinematic response problem b) the foundation impedance problem and c) the foundation- superstructure problem.

- Regarding kinematic response, the effect of radial inhomogeneity in determining foundation input motion has not been studied. The finite element procedure followed in the present work can be used to investigate this effect. The additions that need to be made is to assign a harmonic displacement at the nodes at the bottom of the mesh, or equivalent body forces (inertial forces) within the elements. This formulation can also

be extended to account for transient motions by analyzing the excitation into Fourier harmonics. This formulation will help not only study the effect of radial inhomogeneity in the foundation input motion but also to study the development of bending moments and shears along the pile during earthquake excitation.

- In the case of the foundation impedance problem the current work focuses on elastic solutions which are valid for low strains using a linearization scheme. More work is needed to account for nonlinear effects in the soil. Such effects include gapping and sliding between pile and soil due to large pile-head displacements that cause the soil to separate from the pile. The gap can be simulated using contact elements. Such elements will force the contact nodes to separate once the maximum soil shear stress is exceeded. Owing to the linear nature of the frequency domain solution adopted in this work, nonlinearity can be incorporated only in an approximate way.
- To improve pile-group analysis, the research proposed will have to focus in two main directions: (1) obtain improved interaction factors between two piles, (2) analyze axisymmetric pile groups in the form of concentric rings of densely spaced piles. In the present work, an approximate method was employed to account for the radial inhomogeneity around the pile by estimating the interaction factors using several “equivalent homogeneous” profiles with reduced properties. Unfortunately, the axisymmetric formulation is inappropriate to use for solving the non-axisymmetric problem of two piles embedded in a soil profile that varies radially with respect to the centers of both piles. For this case, bipolar coordinates or a full a three-dimensional formulation has to be employed.

- In the case of the foundation-superstructure problem, the recommendations focus on simple techniques that will evaluate the non-linear response of the superstructure taking into account the non-linear response of the soil. Based on the 2-DOF representation described in appendix E, a parametric study could be performed in conjunction with regression analyses to obtain average $R-\mu$ (force reduction – ductility demand) relations for flexible base systems subjected to strong earthquake excitation. These relations could replace generic $R-\mu$ relations, for structures on soft soil.

APPENDIX A: F.E. FORMULATION FOR AXIAL RESPONSE

A.1 Isoparametric formulation of the F.E. model

In cylindrical coordinates, the stress-strain relationships of a linearly elastic isotropic material are:

$$\begin{aligned}
 \sigma_{rr} &= \lambda \frac{1-\nu}{\nu} \epsilon_{rr} + \lambda \epsilon_{\theta\theta} + \lambda \epsilon_{zz} \\
 \sigma_{\theta\theta} &= \lambda \frac{1-\nu}{\nu} \epsilon_{\theta\theta} + \lambda \epsilon_{rr} + \lambda \epsilon_{zz} \\
 \sigma_{zz} &= \lambda \frac{1-\nu}{\nu} \epsilon_{zz} + \lambda \epsilon_{rr} + \lambda \epsilon_{\theta\theta} \\
 \tau_{rz} &= \lambda \frac{1-2\nu}{2\nu} \gamma_{rz} \\
 \tau_{r\theta} &= 0 \\
 \tau_{z\theta} &= 0
 \end{aligned} \tag{A-1}$$

where λ denotes the first Lamé's constant and ν is the Poisson's ratio

$\lambda = E \frac{\nu}{(1+\nu)(1-2\nu)}$ and E is the complex valued element's modulus of elasticity analyzed

further down.

The corresponding field (strain – displacement) relationships are:

$$\begin{aligned}
 \epsilon_{rr} &= \frac{\partial u}{\partial r} \\
 \epsilon_{\theta\theta} &= \frac{u}{r} \\
 \epsilon_{zz} &= \frac{\partial w}{\partial z} \\
 \gamma_{rz} &= \frac{\partial u}{\partial z} + \frac{\partial w}{\partial r} \\
 \gamma_{r\theta} &= 0 \\
 \gamma_{z\theta} &= 0
 \end{aligned} \tag{A-2}$$

where r and z denote the radial and vertical coordinates respectively.

The constitutive relationship in matrix form is:

$$\boldsymbol{\sigma} = \mathbf{D} \boldsymbol{\varepsilon} \quad (\text{A-3})$$

From (A-1), writing in matrix form $\boldsymbol{\sigma}$ is the matrix holding the element's stresses

$$\boldsymbol{\sigma} = \begin{bmatrix} \sigma_{rr} \\ \sigma_{\theta\theta} \\ \sigma_{zz} \\ \tau_{rz} \end{bmatrix} \quad (\text{A-4})$$

$\mathbf{D}$ is the constitutive (stress-strain) matrix

$$\mathbf{D} = \lambda \begin{bmatrix} \frac{1-\nu}{\nu} & 1 & 1 & 0 \\ 1 & \frac{1-\nu}{\nu} & 1 & 0 \\ 1 & 1 & \frac{1-\nu}{\nu} & 0 \\ 0 & 0 & 0 & \frac{1-2\nu}{2\nu} \end{bmatrix} \quad (\text{A-5})$$

and $\boldsymbol{\varepsilon}$ is the matrix containing the element's strains

$$\boldsymbol{\varepsilon} = \begin{bmatrix} \varepsilon_{rr} \\ \varepsilon_{\theta\theta} \\ \varepsilon_{zz} \\ \gamma_{rz} \end{bmatrix}$$

From eq. (A-2), writing in matrix form, the strain-displacement relationship is:

$$\boldsymbol{\varepsilon} = \mathbf{S} \mathbf{u} \quad (\text{A-6})$$

where $\mathbf{S}$ is the strain-compatibility matrix relating strains to stresses

$$\mathbf{S} = \begin{bmatrix} \frac{\partial}{\partial r} & 0 \\ \frac{1}{r} & 0 \\ 0 & \frac{\partial}{\partial z} \\ \frac{\partial}{\partial z} & \frac{\partial}{\partial r} \end{bmatrix} = \frac{1}{r} \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} \frac{\partial}{\partial r} & 0 \\ 0 & 0 \\ 0 & \frac{\partial}{\partial z} \\ \frac{\partial}{\partial z} & \frac{\partial}{\partial r} \end{bmatrix} = \frac{1}{r} \mathbf{S}_0 + \mathbf{S}_1 \quad (\text{A-7})$$

and $\mathbf{u}$ contains the element displacements.

$$\mathbf{u} = \begin{bmatrix} u \\ w \end{bmatrix} \quad (\text{A-8})$$

The displacements within the element are related to the nodal displacements through an interpolation matrix $\mathbf{N}_{QT}$

$$\mathbf{u} = \mathbf{N}_{QT} \mathbf{U} \quad (\text{A-9})$$

where $\mathbf{N}_{QT}$ is the displacement interpolation matrix and $\mathbf{U}$ contains the element's nodal displacements. In this study quadratic shape functions are used to interpolate the element's displacements.

$$\mathbf{N}_{QT} = \begin{bmatrix} N_{QT1} & 0 & N_{QT2} & 0 & \dots & N_{QT6} & 0 \\ 0 & N_{QT1} & 0 & N_{QT2} & \dots & 0 & N_{QT6} \end{bmatrix} \quad (\text{A-10})$$

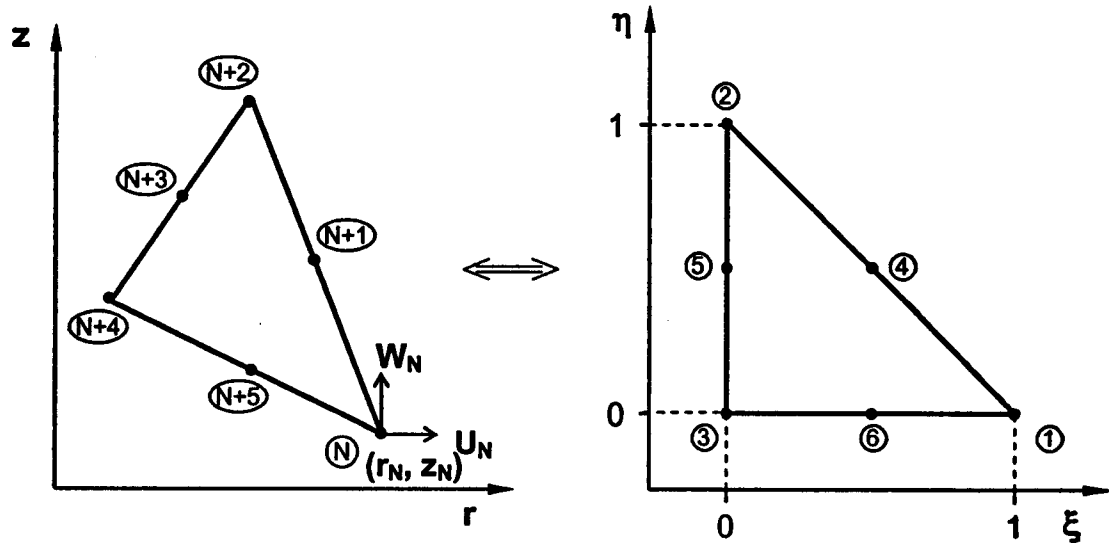


FIGURE A.1: Quadratic 6-noded triangle; model and computational space representation

The quadratic shape functions for the six-noded triangular element used (QT : Quadratic Triangle), are the following (figure A.1)

$$\begin{aligned}
 N_{QT1} &= \xi(2\xi-1) \\
 N_{QT2} &= \eta(2\eta-1) \\
 N_{QT3} &= (1-\xi-\eta)(1-2\xi-2\eta) \\
 N_{QT4} &= 4\xi\eta \\
 N_{QT5} &= 4\eta(1-\xi-\eta) \\
 N_{QT6} &= 4\xi(1-\xi-\eta)
 \end{aligned} \tag{A-11}$$

Also

$$\mathbf{U} = \begin{bmatrix} U_1 & W_1 \\ \vdots & \vdots \\ U_6 & W_6 \end{bmatrix} \tag{A-12}$$

where U_i is the radial displacement of the i -th node and W_i is the vertical displacement of the i -th node of the element

Now, substituting (A-9) into (A-6) yields

$$\boldsymbol{\varepsilon} = \mathbf{S} \mathbf{u} = \mathbf{S} (\mathbf{N}_{QT} \mathbf{U}) = (\mathbf{S} \mathbf{N}_{QT}) \mathbf{U} = \mathbf{B} \mathbf{U} \quad (\text{A-13})$$

where $\mathbf{B}$ is the strain-displacement matrix.

$$\mathbf{B} = \mathbf{S} \mathbf{N}_{QT} = \left(\frac{1}{r} \mathbf{S}_0 + \mathbf{S}_1 \right) \mathbf{N}_{QT} = \frac{1}{r} \mathbf{S}_0 \mathbf{N}_{QT} + \mathbf{S}_1 \mathbf{N}_{QT} = \frac{1}{r} \mathbf{B}_0 + \mathbf{B}_1 \quad (\text{A-14})$$

For the evaluation of matrix $\mathbf{B}_1$ a change of variables is performed to jump from the r, z space to the ξ, η .

$$\mathbf{B}_1 = [\mathbf{S}_{1,\xi} \quad \mathbf{S}_{1,\eta}] \begin{bmatrix} \mathbf{N}_{QT,\xi} \\ \mathbf{N}_{QT,\eta} \end{bmatrix} \quad (\text{A-15})$$

where the notation $\mathbf{A}_{,x}$ means the differentiation of the matrix $\mathbf{A}$ with respect to the variable x . The terms in the matrices $\mathbf{S}_{1,\xi}$ and $\mathbf{S}_{1,\eta}$ are calculated by inverting the Jacobian operator which relates the natural coordinates derivatives (ξ, η in computational space) to the local coordinate derivatives (r, z in model space).

$$\begin{bmatrix} \frac{\partial}{\partial \xi} \\ \frac{\partial}{\partial \eta} \end{bmatrix} = \mathbf{J} \begin{bmatrix} \frac{\partial}{\partial r} \\ \frac{\partial}{\partial z} \end{bmatrix} = \begin{bmatrix} \frac{\partial r}{\partial \xi} & \frac{\partial z}{\partial \xi} \\ \frac{\partial r}{\partial \eta} & \frac{\partial z}{\partial \eta} \end{bmatrix} \begin{bmatrix} \frac{\partial}{\partial r} \\ \frac{\partial}{\partial z} \end{bmatrix} \quad (\text{A-16})$$

It is straight forward to show that the Jacobian is

$$\mathbf{J} = \begin{bmatrix} N_{QT1,\xi} & \cdots & N_{QT6,\xi} \\ N_{QT1,\eta} & \cdots & N_{QT6,\eta} \end{bmatrix} \begin{bmatrix} r_1 & z_1 \\ \vdots & \vdots \\ r_6 & z_6 \end{bmatrix} \quad (\text{A-17})$$

the terms in the matrices $S_{1,\xi}$ and $S_{1,\eta}$ are derivatives with respect to the natural coordinates so the inverse of the Jacobian (called Γ) needs to be evaluated and obtain the appropriate terms from it

$$\mathbf{J}^{-1} = \Gamma = \begin{bmatrix} \frac{\partial \xi}{\partial r} & \frac{\partial \eta}{\partial r} \\ \frac{\partial \xi}{\partial z} & \frac{\partial \eta}{\partial z} \end{bmatrix} \quad (\text{A-18})$$

thus (A-15) becomes

$$\mathbf{B}_1 = [S_{1,\xi} \quad S_{1,\eta}] \begin{bmatrix} N_{QT,\xi} \\ N_{QT,\eta} \end{bmatrix} = \begin{bmatrix} \Gamma(1,1) & 0 & \Gamma(1,2) & 0 \\ 0 & 0 & 0 & 0 \\ 0 & \Gamma(2,1) & 0 & \Gamma(2,2) \\ \Gamma(2,1) & \Gamma(1,1) & \Gamma(2,2) & \Gamma(1,2) \end{bmatrix} \begin{bmatrix} N_{QT,\xi} \\ N_{QT,\eta} \end{bmatrix} \quad (\text{A-19})$$

The calculation of $\mathbf{B}_0$ is straight-forward. The radial coordinate r is interpolated with the help of the interpolation matrix $\mathbf{N}$ as

$$\mathbf{r} = \mathbf{N}_{QT} \begin{bmatrix} r_1 \\ \vdots \\ r_6 \end{bmatrix} \quad (\text{A-20})$$

and then $\mathbf{B}_0$ is computed from (A-14) and (A-7)

$$\mathbf{B}_0 = \mathbf{S}_0 \mathbf{N}_{QT} = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} N_{QT1} & 0 & N_{QT2} & 0 & \dots & N_{QT6} & 0 \\ 0 & N_{QT1} & 0 & N_{QT2} & \dots & 0 & N_{QT6} \end{bmatrix} \quad (\text{A-21})$$

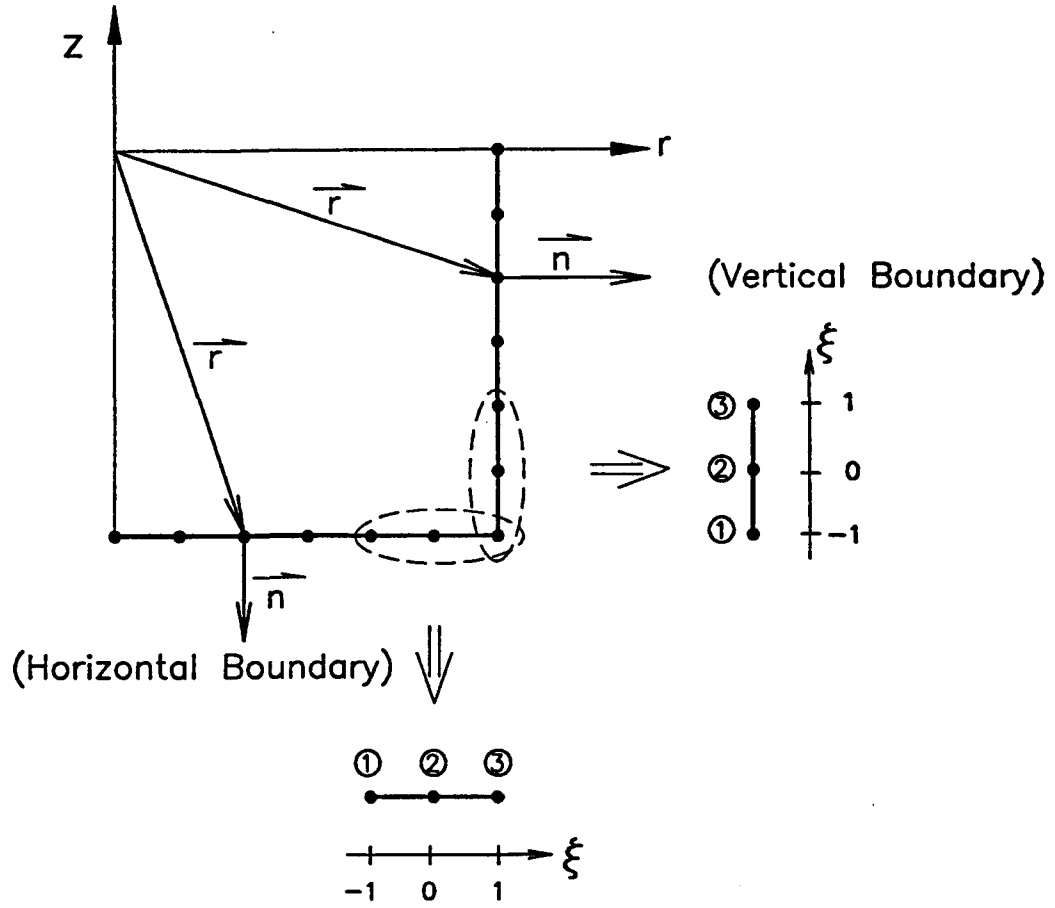


FIGURE A.2: Implementing the local transmitting boundaries at the F.E. mesh.

Transition from model space (r, z) to computational space (ξ)

A.2 Local Transmitting boundaries - Vertical boundary

The integral in eq. (2.19) is evaluated as follows: the outward unit vector, $\bar{n}$ (figure A.2) is given as

$$\bar{\mathbf{n}} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \quad (\text{A-22})$$

and the wave direction vector is

$$\bar{\mathbf{r}} = \begin{bmatrix} r \\ 0 \\ |z| \end{bmatrix} \frac{1}{R} \quad (\text{A-23})$$

where $|z|$ is used, since in Kellezi's formulation, the positive z direction is downwards, and R is

$$R = \sqrt{r^2 + z^2} \quad (\text{A-24})$$

Using, (A-22), (A-23) and (A-24) one obtains

$$\bar{\mathbf{n}} \cdot \bar{\mathbf{r}} = \frac{r}{R} \quad (\text{A-25})$$

Also,

$$\mathbf{N} = \bar{\mathbf{n}} \cdot (\bar{\mathbf{n}})^T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad (\text{A-26})$$

and

$$\{[\mathbf{I}] - [\mathbf{N}]\} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (\text{A-27})$$

Substituting (A-25), (A-26) and (A-27) in (2.18) and carrying out the algebraic computations yields

$$\mathbf{D}_K = \frac{\rho V_p^2}{R} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \beta^2 & 0 \\ 0 & 0 & \beta^2 \end{bmatrix} \xrightarrow{\text{AXISYMMETRIC ANALYSIS}} \mathbf{D}_K = \frac{\rho V_p^2}{R} \begin{bmatrix} 1 & 0 \\ 0 & \beta^2 \end{bmatrix} \quad (\text{A-28})$$

where

$$\beta^2 = \left(\frac{V_s}{V_p} \right)^2 = \frac{1-2\nu}{2-2\nu} \quad (\text{A-29})$$

From the known relationships

$$G = \frac{E}{2(1+\nu)}, \quad M = E \frac{(1-\nu)}{(1+\nu)(1-2\nu)} \quad \text{and} \quad M = \rho V_p^2$$

it can easily be shown that

$$\rho V_p^2 = \frac{G}{\beta^2} \quad (\text{A-30})$$

So, (A-28) becomes

$$\mathbf{D}_K = \frac{G}{R} \begin{bmatrix} \frac{1}{\beta^2} & 0 \\ 0 & 1 \end{bmatrix} \quad (\text{A-31})$$

now, the stiffness integral can be rewritten as

$$\mathbf{K}^e = \int_{\Gamma_e} \mathbf{N}_Q^T \mathbf{D}_K \mathbf{N}_Q \, d\Gamma = \iint \mathbf{N}_Q^T \mathbf{D}_K \mathbf{N}_Q \, r \, d\theta \, dz \Leftrightarrow$$

$$\Leftrightarrow \mathbf{K}^e = r \Delta\theta \int_{z_1}^{z_3} \frac{G(z)}{\sqrt{r^2 + z^2}} \mathbf{N}_Q^T \begin{bmatrix} \frac{1}{\beta^2} & 0 \\ 0 & 1 \end{bmatrix} \mathbf{N}_Q dz \quad (\text{A-32})$$

where $G = G(z)$ is in general a function of depth.

To move from the model space (z) to the computational space (ξ), a change of variables is performed based on the following quadratic shape functions:

$$z = \mathbf{N}_{Q-1D} \mathbf{Z} = \begin{bmatrix} N_{Q1} & N_{Q2} & N_{Q3} \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} \quad (\text{A-33})$$

where

$$\begin{aligned} N_{Q1} &= -\frac{1}{2}\xi(1-\xi) \\ N_{Q2} &= 1-\xi^2 \\ N_{Q3} &= \frac{1}{2}\xi(1+\xi) \end{aligned} \quad (\text{A-34})$$

Assuming $z_2 = \frac{z_3 - z_1}{2}$, results in

$$dz = \frac{dz}{d\xi} d\xi = \frac{z_3 - z_1}{2} d\xi \quad (\text{A-35})$$

Also, the variable interpolation matrix $\mathbf{N}_Q$ (Q : Quadratic), is

$$\mathbf{N}_Q = \begin{bmatrix} N_{Q1} & 0 & N_{Q2} & 0 & N_{Q3} & 0 \\ 0 & N_{Q1} & 0 & N_{Q3} & 0 & N_{Q3} \end{bmatrix} \quad (\text{A-36})$$

Before evaluating the integral the variable shear modulus $G(z)$ needs to be evaluated. The shear modulus is interpolated linearly as

$$G(z) = G(\xi) = \frac{G_1 + G_3}{2} + \frac{G_3 - G_1}{2} \xi \quad (\text{A-37})$$

Finally, using eq. (A-37), (A-36), (A-35) and (A-33) in (A-32) integral for the element stiffness at the vertical boundary (VB) becomes

$$\mathbf{K}_{\text{VB}}^e = r \Delta \theta \left(\frac{z_3 - z_1}{2} \right) \int_{-1}^1 \left(\frac{G_1 + G_3}{2} + \frac{G_3 - G_1}{2} \xi \right) \frac{1}{\sqrt{r^2 + z(\xi)^2}} \mathbf{N}_Q(\xi)^T \begin{bmatrix} \frac{1}{\beta^2} & 0 \\ 0 & 1 \end{bmatrix} \mathbf{N}_Q(\xi) d\xi \quad (\text{A-38})$$

The damping matrix is treated similarly.

Substituting in (2.22) and carrying out the computations yields

$$\mathbf{D}_c = \frac{r}{R} \sqrt{G \rho} \begin{bmatrix} \frac{1}{\beta} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \xrightarrow[\text{ANALYSIS}]{\text{AXISYMMETRIC}} \mathbf{D}_c = \frac{r}{R} \sqrt{G \rho} \begin{bmatrix} \frac{1}{\beta} & 0 \\ 0 & 1 \end{bmatrix} \quad (\text{A-39})$$

substituting (A-39) in (2.20) one obtains

$$\mathbf{C}^e = \int_{\Gamma_e} \mathbf{N}_Q^T \mathbf{D}_c \mathbf{N}_Q d\Gamma = \iint \mathbf{N}_Q^T \mathbf{D}_c \mathbf{N}_Q r d\theta dz = r^2 \Delta \theta \sqrt{\rho} \int_{z_1}^{z_3} \frac{\sqrt{G(z)}}{\sqrt{r^2 + z^2}} \mathbf{N}_Q^T \begin{bmatrix} \frac{1}{\beta} & 0 \\ 0 & 1 \end{bmatrix} \mathbf{N}_Q dz$$

and using the previous results

$$\mathbf{C}_{\text{VB}}^e = r^2 \Delta \theta \sqrt{\rho} \left(\frac{z_3 - z_1}{2} \right) \int_{-1}^1 \frac{\sqrt{\left(\frac{G_1 + G_3}{2} + \frac{G_3 - G_1}{2} \xi \right)}}{\sqrt{r^2 + z(\xi)^2}} \mathbf{N}_Q(\xi)^T \begin{bmatrix} \frac{1}{\beta} & 0 \\ 0 & 1 \end{bmatrix} \mathbf{N}_Q(\xi) d\xi \quad (\text{A-40})$$

A.3 Local Transmitting boundaries - Horizontal boundary

The outward unit vector $\bar{n}$ is in this case

$$\bar{n} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \quad (A-41)$$

while the wave direction vector remains the same (eq. (A-23))

$$\bar{n} \cdot \bar{r} = \frac{|z|}{R} \quad (A-42)$$

In addition,

$$N = \bar{n} \cdot (\bar{n})^T = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (A-43)$$

$$\{[I] - [N]\} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad (A-44)$$

Substituting (A-42), (A-43) and (A-44) in (2.18) and carrying out the algebraic computations yields

$$D_K = G \frac{|z|}{r} \frac{1}{R} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \frac{1}{\beta^2} \end{bmatrix} \xrightarrow{\text{AXISYMMETRIC ANALYSIS}} D_K = G \frac{|z|}{r} \frac{1}{R} \begin{bmatrix} 1 & 0 \\ 0 & \frac{1}{\beta^2} \end{bmatrix} \quad (A-45)$$

With reference to stiffness, the corresponding integral is rewritten as

$$\mathbf{K}^e = \int_{\Gamma} \mathbf{N}_Q^T \mathbf{D}_K \mathbf{N}_Q d\Gamma = \iint \mathbf{N}_Q^T \mathbf{D}_K \mathbf{N}_Q r d\theta dr = |z| \Delta\theta \int_{r_1}^{r_3} \frac{G(z)}{\sqrt{r^2+z^2}} \mathbf{N}_Q^T \begin{bmatrix} 1 & 0 \\ 0 & \frac{1}{\beta^2} \end{bmatrix} \mathbf{N}_Q dr \quad (\text{A-46})$$

Again a change of variable must be performed to go from model space (r) to computational space (ξ). To this end, quadratic shape functions are employed to interpolate r

$$r = \mathbf{N}_{Q-1D} \mathbf{R} = \begin{bmatrix} N_{Q1} & N_{Q2} & N_{Q3} \end{bmatrix} \begin{bmatrix} r_1 \\ r_2 \\ r_3 \end{bmatrix}$$

In the same spirit as in the previous section,

$$r_2 = \frac{r_3 - r_1}{2} \text{ and } dr = \frac{dr}{d\xi} d\xi = \frac{r_3 - r_1}{2} d\xi$$

The variable interpolation matrix $\mathbf{N}$ remains the same (eq. (A-36)).

Regarding the shear modulus (G), it can be kept constant for two reasons: (a) at the bottom of the domain all the nodes are in the same depth so there is no need in varying G with depth is out of the question. (b) the radial reduction of the shear modulus in the soil happens due to installation effects of the pile and at the bottom of the domain these effects don't take place. This way, the shear modulus is constant also with respect to the radial distance from the pile. Nevertheless, accounting for a radial variation of the shear modulus yields

$$G(r) = G(\xi) = \frac{G_1 + G_3}{2} + \frac{G_3 - G_1}{2} \xi$$

but it should be mentioned that in the calculations in chapters 2 and 3 a constant value was used.

Substituting in (A-46) yields the following element stiffness at the horizontal boundary (HB)

$$\mathbf{K}_{HB}^e = |z| \Delta\theta \frac{r_3 - r_1}{2} \int_{-1}^1 \frac{G}{\sqrt{r(\xi)^2 + z^2}} \mathbf{N}(\xi)^T \begin{bmatrix} 1 & 0 \\ 0 & \frac{1}{\beta^2} \end{bmatrix} \mathbf{N}(\xi) d\xi \quad (\text{A-47})$$

The damping matrix is treated similarly.

Substituting in (2.19) yields

$$\mathbf{D}_c = \frac{|z|}{R} \sqrt{G\rho} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \frac{1}{\beta} \end{bmatrix} \xrightarrow[\text{AXISYMMETRIC ANALYSIS}]{} \mathbf{D}_c = \frac{|z|}{R} \sqrt{G\rho} \begin{bmatrix} 1 & 0 \\ 0 & \frac{1}{\beta} \end{bmatrix} \quad (\text{A-48})$$

and substituting (A-48) in (2.20) results in

$$\mathbf{C}^e = \int_{r_o} \mathbf{N}_Q^T \mathbf{D}_c \mathbf{N}_Q d\Gamma = \iint \mathbf{N}_Q^T \mathbf{D}_c \mathbf{N}_Q r d\theta dr = |z| \Delta\theta \sqrt{\rho} \int_{r_1}^{r_3} \frac{r G(r)}{\sqrt{r^2 + z^2}} \mathbf{N}_Q^T \begin{bmatrix} 1 & 0 \\ 0 & \frac{1}{\beta} \end{bmatrix} \mathbf{N}_Q dr$$

and for a constant shear modulus,

$$\mathbf{C}_{HB}^e = |z| \Delta\theta \sqrt{G\rho} \frac{r_3 - r_1}{2} \int_{-1}^1 \frac{r(\xi)}{\sqrt{r(\xi)^2 + z^2}} \mathbf{N}_Q(\xi)^T \begin{bmatrix} 1 & 0 \\ 0 & \frac{1}{\beta} \end{bmatrix} \mathbf{N}_Q(\xi) d\xi \quad (\text{A-49})$$

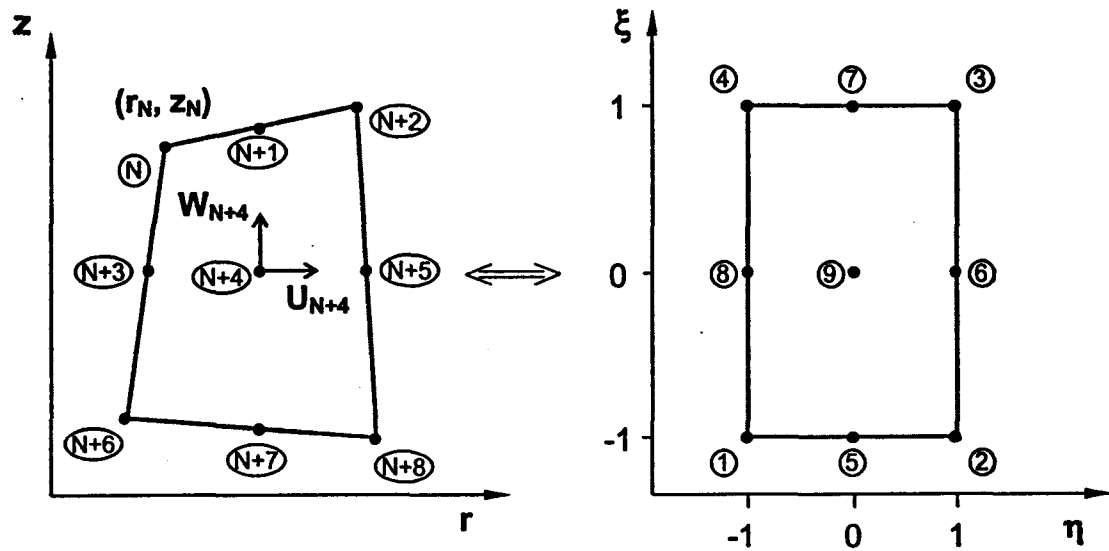


FIGURE A.3: Quadrilateral 9-noded element; model and computational space representation

A.4 Infinite elements

The quadratic shape functions used for the 9-noded quadrilateral element (QQ : Quadratic Quadrilateral) (figure A.3) are

$$N_{QQ1} = \frac{\xi \eta (1-\xi)(1-\eta)}{4}$$

$$N_{QQ2} = \frac{-\xi \eta (1+\xi)(1-\eta)}{4}$$

$$N_{QQ3} = \frac{\xi \eta (1+\xi)(1+\eta)}{4}$$

$$N_{QQ4} = \frac{-\xi \eta (1-\xi)(1+\eta)}{4}$$

$$N_{QQ5} = \frac{-\eta (1-\xi^2)(1-\eta)}{2}$$

$$N_{QQ6} = \frac{-\xi (1-\eta^2)(1+\xi)}{2}$$

$$N_{QQ7} = \frac{\eta (1-\xi^2)(1+\eta)}{2}$$

$$N_{QQ8} = \frac{-\xi (1-\eta^2)(1-\xi)}{2}$$

$$N_{QQ9} = (1-\xi^2) (1-\eta^2) \quad (A-50)$$

and the displacement interpolation matrix N_{QQ} is

$$N_{QQ} = \begin{bmatrix} N_{QQ1} & 0 & N_{QQ2} & 0 & \cdots & N_{QQ9} & 0 \\ 0 & N_{QQ1} & 0 & N_{QQ2} & \cdots & 0 & N_{QQ9} \end{bmatrix} \quad (A-51)$$

Note that this interpolation matrix is not used in the evaluation of the Jacobian. For that purpose, growth shape functions are used that are applied to the geometry.

$$M_1 = 2 \xi \eta \frac{1-\eta}{(1-\xi)^2}$$

$$M_2 = 0$$

$$M_3 = 0$$

$$M_4 = 2 \xi \eta \frac{1+\eta}{(1-\xi)^2}$$

$$M_5 = -(1+\xi) \eta \frac{1-\eta}{(1-\xi)^2}$$

$$M_6 = 0$$

$$M_7 = (1+\xi) \eta \frac{1+\eta}{(1-\xi)^2}$$

$$M_8 = 2\xi \frac{\eta^2 - 1}{(1-\xi)}$$

$$M_9 = -(1+\xi) \frac{\eta^2 - 1}{(1-\xi)}$$

(A-52)

Similarly as before

$$\mathbf{J} = \begin{bmatrix} M_{1,\xi} & \cdots & M_{9,\xi} \\ M_{1,\eta} & \cdots & M_{9,\eta} \end{bmatrix} \begin{bmatrix} r_1 & z_1 \\ \vdots & \vdots \\ r_9 & z_9 \end{bmatrix}$$

(A-53)

Using this result for the Jacobian, the stiffness matrix is evaluated. An interesting topic arises where 3 of the 9 nodes are “placed” at infinity. This is taken care by removing the rows and columns belonging to the nodes at infinity and reducing the 18 x 18 matrix to a 9 x 9. The reduced matrix is assembled to the total stiffness matrix as 6-noded element with nodes that exist in the boundary.

APPENDIX B: F.E. FORMULATION FOR LATERAL RESPONSE

B.1 Isoparametric formulation of the F.E. model

The constitutive relationship in matrix form is:

$$\boldsymbol{\sigma} = \mathbf{D} \boldsymbol{\varepsilon} \quad (\text{B-1})$$

where, writing in matrix form $\boldsymbol{\sigma}$ is the matrix holding the element's stresses

$$\boldsymbol{\sigma} = [\sigma_{rr} \quad \sigma_{\theta\theta} \quad \sigma_{zz} \quad \tau_{rz} \quad \tau_{r\theta} \quad \tau_{z\theta}]^T \quad (\text{B-2})$$

$$\mathbf{D} = \begin{bmatrix} \lambda+2\mu & \lambda & \lambda & & & \\ \lambda & \lambda+2\mu & \lambda & & & 0 \\ \lambda & \lambda & \lambda+2\mu & & & \\ & & & \mu & & \\ & 0 & & & \mu & \\ & & & & & \mu \end{bmatrix} \quad (\text{B-3})$$

and

$$\boldsymbol{\varepsilon} = [\varepsilon_{rr} \quad \varepsilon_{\theta\theta} \quad \varepsilon_{zz} \quad \gamma_{rz} \quad \gamma_{r\theta} \quad \gamma_{z\theta}]^T \quad (\text{B-4})$$

Expressed in matrix form, the strain-displacement relationship is

$$\boldsymbol{\varepsilon} = \mathbf{S} \mathbf{u} \quad (\text{B-5})$$

where $\mathbf{S}$ is the strain-compatibility matrix relating strains to stresses

$$S = \begin{bmatrix} \frac{\partial}{\partial r} & 0 & 0 \\ \frac{1}{r} & \frac{1}{r} \frac{\partial}{\partial \theta} & 0 \\ 0 & 0 & \frac{\partial}{\partial z} \\ \frac{\partial}{\partial z} & 0 & \frac{\partial}{\partial r} \\ \frac{1}{r} \frac{\partial}{\partial \theta} & \left(\frac{\partial}{\partial r} - \frac{1}{r} \right) & 0 \\ 0 & \frac{\partial}{\partial z} & \frac{1}{r} \frac{\partial}{\partial \theta} \end{bmatrix} = \frac{1}{r} S_0 + S_1 + \frac{1}{r} S_2 \frac{\partial}{\partial \theta} \quad (B-6)$$

where

$$S_0 = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, S_1 = \begin{bmatrix} \frac{\partial}{\partial r} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \frac{\partial}{\partial z} \\ \frac{\partial}{\partial z} & 0 & \frac{\partial}{\partial r} \\ 0 & \frac{\partial}{\partial r} & 0 \\ 0 & \frac{\partial}{\partial z} & 0 \end{bmatrix} \text{ and } S_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (B-7)$$

Now one can relate the displacements of any point within the element to the nodal displacements by using a pertinent interpolation matrix N . Accordingly

$$u = N U \quad (B-8)$$

where N is given by:

$$\mathbf{N} = \begin{bmatrix} N_1 & 0 & 0 & N_2 & 0 & \cdots & 0 \\ 0 & N_1 & 0 & 0 & N_2 & \cdots & 0 \\ 0 & 0 & N_1 & 0 & 0 & \cdots & N_6 \end{bmatrix} \quad (\text{B-9})$$

In this study quadratic shape functions are used to interpolate the element's displacements. In the spirit of eq. (3.3), (B-8) can be written as

$$\mathbf{u} = \begin{bmatrix} u \\ v \\ w \end{bmatrix} = \sum_{n=0}^{\infty} (\mathbf{N}_n \bar{\mathbf{U}}_n) \quad (\text{B-10})$$

where $\mathbf{N}_n$ is the new interpolation matrix depending solely on θ

$$\mathbf{N}_n = e^{in\theta} \mathbf{N} \quad (\text{B-11})$$

and $\bar{\mathbf{U}}_n$ is a matrix containing the (complex) nodal displacement amplitudes,

$$\bar{\mathbf{U}}_n = [\bar{\mathbf{U}}_n \quad \bar{\mathbf{V}}_n \quad \bar{\mathbf{W}}_n] \quad (\text{B-12})$$

Note that the nodal displacement amplitudes are independent of θ .

Substituting (B-10) into (B-5) yields

$$\boldsymbol{\varepsilon}_n = \mathbf{S} \mathbf{u}_n = \mathbf{S} (\mathbf{N}_n \bar{\mathbf{U}}_n) = (\mathbf{S} \mathbf{N}_n) \bar{\mathbf{U}}_n = \mathbf{B}_n \bar{\mathbf{U}}_n \quad (\text{B-13})$$

where $\mathbf{B}_n$ is the strain-displacement matrix for the n-th mode

$$\mathbf{B}_n = \mathbf{S} \mathbf{N}_n = \left(\frac{1}{r} \mathbf{S}_0 + \mathbf{S}_1 + \frac{1}{r} \mathbf{S}_2 \frac{\partial}{\partial \theta} \right) e^{in\theta} \mathbf{N} = e^{in\theta} \left(\frac{1}{r} \mathbf{S}_0 \mathbf{N} + \mathbf{S}_1 \mathbf{N} + i \frac{n}{r} \mathbf{S}_2 \mathbf{N} \right)$$

or

$$\mathbf{B}_n = e^{in\theta} \left(\frac{1}{r} \mathbf{B}_1 + n i \mathbf{B}_2 \right) \quad (\text{B-14})$$

For the evaluation of matrix $\mathbf{B}_1$ (not to be confused with matrix $\mathbf{B}_n$ for $n=1$) we need to perform a change of variables to go from the r, z space to the ξ, η (see appendix A)

$$\mathbf{B}_1 = \frac{1}{r} \mathbf{S}_0 \mathbf{N} + \begin{bmatrix} S_{1,\xi} & S_{1,\eta} \end{bmatrix} \begin{bmatrix} N_{,\xi} \\ N_{,\eta} \end{bmatrix} \quad (\text{B-15})$$

From eq. (2.8) the element stiffness matrix for the n -th node can be derived as

$$\mathbf{K}_n = \int_{-\pi}^{\pi} \iint \mathbf{B}_n^T \mathbf{D} \mathbf{B}_n r |J| d\xi d\eta d\theta \quad (\text{B-16})$$

where $\mathbf{B}_n$ is the strain-compatibility (strain-displacement) matrix for the n -th mode and $\mathbf{D}$ is the constitutive matrix.

Substituting eq. (B-14) in (B-16) and carrying out the calculations we end up with

$$\mathbf{K}_n = 2\pi \left[\mathbf{K}_1 + n^2 \mathbf{K}_2 + n i \mathbf{K}_3 \right] \quad (\text{B-17})$$

where

$$\begin{aligned} \mathbf{K}_1 &= \iint \mathbf{B}_1^T \mathbf{D} \mathbf{B}_1 r |J| d\xi d\eta \\ \mathbf{K}_2 &= \iint \mathbf{B}_2^T \mathbf{D} \mathbf{B}_2 r |J| d\xi d\eta \\ \mathbf{K}_3 &= \iint (\mathbf{B}_1^T \mathbf{D} \mathbf{B}_2 - \mathbf{B}_2^T \mathbf{D} \mathbf{B}_1) r |J| d\xi d\eta \end{aligned} \quad (\text{B-18})$$

($\mathbf{K}_1$ should not be confused with $\mathbf{K}_n$ for $n=1$)

The matrices K_1 , K_2 and K_3 are independent of n and θ and need to be evaluated only once.

Similarly, the element mass matrix is determined as (eq. 2.7)

$$\mathbf{M}_i = \int_{-\pi}^{\pi} \int \int \rho \mathbf{N}^T \mathbf{N} r |J| d\xi d\eta d\theta \quad (\text{B-19})$$

For the n -th node, using eq. (B-11)

$$\mathbf{M}_n = \int_{-\pi}^{\pi} \int \int \rho \mathbf{N}_n^T \mathbf{N}_n r |J| d\xi d\eta d\theta = 2\pi \int \int \rho \mathbf{N}^T \mathbf{N} r |J| d\xi d\eta = \mathbf{M}_{\text{AXISYMMETRIC}} \quad (\text{B-20})$$

which is identical to the one in the axisymmetric analysis.

B.2 Infinite Elements

In the formulation of the infinite elements matrices $\mathbf{S}$, $\mathbf{B}$ and $\mathbf{D}$ along with the shape functions remain the same. The only difference is that the displacement interpolation matrix $\mathbf{N}_{QQ}$ has now 3 rows and 27 columns

$$\mathbf{N}_{QQ} = \begin{bmatrix} N_{QQ1} & 0 & 0 & N_{QQ2} & 0 & \dots & 0 \\ 0 & N_{QQ1} & 0 & 0 & N_{QQ2} & \dots & 0 \\ 0 & 0 & N_{QQ1} & 0 & 0 & \dots & N_{QQ9} \end{bmatrix} \quad (\text{B-21})$$

B.3 Local transmitting boundaries

The boundary equations remain the same as in chapter 2. The only difference is that the constitutive matrices are 3x3 instead of 2x2. Thus for the vertical boundary the stiffness matrix will be

$$\mathbf{K}_{\text{VB}}^e = r \Delta\theta \left(\frac{z_3 - z_1}{2} \right) \int_{-1}^1 \left(\frac{G_1 + G_3}{2} + \frac{G_3 - G_1}{2} \xi \right) \frac{1}{\sqrt{r^2 + z(\xi)^2}} \mathbf{N}(\xi)^T \begin{bmatrix} \frac{1}{\beta^2} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \mathbf{N}(\xi) d\xi \quad (\text{B-22})$$

and the corresponding damping matrix

$$\mathbf{C}_{\text{VB}}^e = r^2 \Delta\theta \sqrt{\rho} \left(\frac{z_3 - z_1}{2} \right) \int_{-1}^1 \frac{\sqrt{\left(\frac{G_1 + G_3}{2} + \frac{G_3 - G_1}{2} \xi \right)}}{\sqrt{r^2 + z(\xi)^2}} \mathbf{N}(\xi)^T \begin{bmatrix} \frac{1}{\beta} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \mathbf{N}(\xi) d\xi \quad (\text{B-23})$$

For the horizontal boundary, the stiffness matrix is

$$\mathbf{K}_{\text{HB}}^e = |z| \Delta\theta \frac{I_3 - I_1}{2} \int_{-1}^1 \frac{G}{\sqrt{r(\xi)^2 + z^2}} \mathbf{N}(\xi)^T \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \frac{1}{\beta^2} \end{bmatrix} \mathbf{N}(\xi) d\xi \quad (\text{B-24})$$

and the damping matrix,

$$\mathbf{C}_{\text{HB}}^e = |z| \Delta\theta \sqrt{G \rho} \frac{I_3 - I_1}{2} \int_{-1}^1 \frac{r(\xi)}{\sqrt{r(\xi)^2 + z^2}} \mathbf{N}(\xi)^T \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \frac{1}{\beta} \end{bmatrix} \mathbf{N}(\xi) d\xi \quad (\text{B-25})$$

where for the above four equations, the interpolation matrix $N(\xi)$ is

$$N = \begin{bmatrix} N_1 & 0 & 0 & N_2 & 0 & 0 & N_3 & 0 & 0 \\ 0 & N_1 & 0 & 0 & N_2 & 0 & 0 & N_3 & 0 \\ 0 & 0 & N_1 & 0 & 0 & N_2 & 0 & 0 & N_3 \end{bmatrix} \quad (B-26)$$

instead of N_Q in appendix A, and the associated shape functions are

$$\begin{aligned} N_1 &= -\frac{1}{2}\xi(1-\xi) \\ N_2 &= 1-\xi^2 \\ N_3 &= \frac{1}{2}\xi(1+\xi) \end{aligned} \quad (B-27)$$

APPENDIX C: PILE ON DYNAMIC WINKLER FOUNDATION

C.1 Axial Response

Modeling a vertical elastic pile as an axially loaded rod supported on Winkler springs and dashpots leads to the differential equation:

$$E_p A_p \frac{d^2 W(z)}{dz^2} + m \omega^2 W(z) - (k_z + i\omega c_z) W(z) = 0 \quad (C-1)$$

where E_p is the modulus of elasticity of the pile, A_p is the pile's cross-sectional area, m is the mass per unit pile length, ω is the exciting frequency, k_z is the stiffness of the - frequency dependent, continuously distributed along the pile- springs, an c_z are the equivalent dashpots. $W(z)$ denotes the axial displacement of the pile.

The general solution of eq. (C-1) is

$$W(z) = A e^{\lambda z} + B e^{-\lambda z} \quad (C-2)$$

where

$$\lambda = \left[\frac{k_z + i\omega c_z - m\omega^2}{E_p A_p} \right]^{1/2} \quad (C-3)$$

and λ is a complex wave number describing the attenuation of pile displacement with depth.

In a homogeneous medium, integration constants A and B are determined from the boundary conditions

$$z = 0 \text{ (pile head): } W(0) = 1 \quad (C-4)$$

$$z = L \text{ (pile tip): } K_b W(L) = -E_p A_p \left(\frac{dW}{dz} \right)_{z=L} \quad (\text{C-5})$$

where K_b is a complex-valued impedance representing the vertical soil reaction at the pile-tip.

Based on the above, the complex dynamic impedance of a harmonically-oscillating pile in a homogeneous stratum underlain by a deformable base is derived as (Mylonakis & Gazetas, 1998)

$$K_z = E_p A_p \lambda \frac{\Omega + \tanh(h\lambda)}{1 + \Omega \tanh(h\lambda)} \quad (\text{C-6})$$

where h is the thickness of the soil layer and Ω is

$$\Omega = \frac{K_b}{E_p A_p \lambda} \quad (\text{C-7})$$

This solution holds for a homogeneous soil medium but it can be extended to layered soil as proposed by Mylonakis & Gazetas (1998).

C.2 Interaction factors for axial loading

The interaction factor between two piles can be determined as follows:

- i) Determine the complex-valued displacement profile along a single “source” pile, $W_{11}(z)$
- ii) Determine the complex-valued displacement profile in the “free-field” at a distance s from the “source” pile.

$$U(s, z) \approx \psi(s) W_{11}(z)$$

- iii) The axial stiffness of the “receiver” pile combined with the soil reaction at its base gives rise to an interaction between the pile and the surrounding soil, leading to diffraction of the arriving wave field.

For a receiver pile, the dynamic equilibrium of an infinitesimal pile segment yields the governing equation:

$$E_p A_p \frac{d^2 W_{21}(z)}{dz^2} + m \omega^2 W_{21}(z) - (k_z + i\omega c_z) [W_{21}(z) - U(s, z)] = 0 \quad (C-8)$$

where

$$U(s, z) = \psi(s) W_{11}(z) = \psi(s) (A_{11} e^{\lambda z} + B_{11} e^{-\lambda z}) \quad (C-9)$$

The solution to (C-8) is:

$$W_{21}(z) = \frac{k_z + i\omega c_z}{2\lambda E_p A_p} \psi(s) z (-A_{11} e^{\lambda z} + B_{11} e^{-\lambda z}) + A_{21} e^{\lambda z} + B_{21} e^{-\lambda z} \quad (C-10)$$

Using the boundary conditions (zero force at the pile head and continuity of force and displacement at the various interfaces one obtains

$$W_{21}(z) = \frac{k_z + i\omega c_z}{k_z - m\omega^2 + i\omega c_z} \frac{\psi(s)}{2} \left[\frac{K}{E_p A_p \lambda} (\lambda z \cosh(\lambda z) - \sinh(\lambda z)) - \lambda z \sinh(\lambda z) \right. \\ \left. + \frac{2h\lambda + \sinh(2\lambda z) + \Omega^2 [\sinh(2\lambda z) - 2h\lambda] + 2\Omega [\cosh(2\lambda z) - 1]}{\sinh(2\lambda z) + \Omega^2 \sinh(2\lambda z) + 2\Omega \cosh(2\lambda z)} \cosh(\lambda z) \right] W_{11}(0) \quad (C-11)$$

The ratio $W_{21}(z=0) / W_{11}(z=0)$ is the desired interaction factor and is written as a product of two complex-valued functions:

$$\alpha = \psi(s) \zeta(h\lambda, \Omega) \quad (C-12)$$

where the function $\zeta = \zeta(h\lambda, \Omega)$ is given by (Mylonakis & Gazetas, 1998)

$$\zeta = \frac{k_z + i\omega c_z}{k_z - m\omega^2 + i\omega c_z} \frac{2h\lambda + \sinh(2h\lambda) + \Omega^2 [\sinh(2h\lambda) - 2h\lambda] + 2\Omega [\cosh(2h\lambda) - 1]}{2\sinh(2h\lambda) + \Omega^2 \sinh(2h\lambda) + 4\Omega \cosh(2h\lambda)} \quad (C-13)$$

Accordingly, for the interaction between bases

$$\alpha_b = \psi_b(s) \zeta_b(h\lambda, \Omega) \quad (C-14)$$

where

$$\zeta_b = \frac{2\Omega}{\sinh(2h\lambda)(\Omega^2 + 1) + 2\Omega \cosh(2h\lambda)} \quad (C-15)$$

This solution holds for a homogeneous soil medium but it can be extended to layered soil as proposed in Mylonakis & Gazetas (1999).

C.3 Lateral Response

Modeling a single pile as a laterally loaded beam supported on Winkler springs leads to the following differential equation:

$$\frac{d^4 Y(z)}{dz^4} + 4\lambda(z, \omega)^4 Y(z) = \frac{q(z)}{EI} \quad (C-16)$$

where $Y(z, t) = Y(z)\exp[i\omega t]$ is the lateral harmonic deflection of the pile, $\lambda(\omega, z)$ is given by

$$\lambda(\omega, z) = \left[\frac{k(z, \omega) - m\omega^2}{4EI} \right]^{1/4} \quad (C-17)$$

where EI and m = flexural rigidity and mass per unit length of the pile, respectively; $q(z)$ = distributed forces along the pile; $\mathcal{H}(z, \omega) = k(z, \omega) + i \omega c(z, \omega)$ = complex soil impedance encompassing the stiffness, inertia, radiation and hysteretic action of and in the soil; λ = a complex wavenumber.

In the special case of a pile of length L embedded in a homogeneous soil of thickness $h=L$, the solution to eq. (C-16) yields, after some straightforward algebra and enforcement of boundary conditions, the following closed-form expressions for the complex impedances, $\mathcal{H}_{ij}(z, \omega) = K_{ij}(z, \omega) + i \omega C_{ij}(z, \omega)$ at the head of the pile (Mylonakis, 1995):

$$\mathcal{H}_{xx} = 4EI \lambda^3 \frac{\sin(2\lambda h) + \sinh(2\lambda h)}{\pm 2 + \cos(2\lambda h) + \cosh(2\lambda h)} \quad (C-18)$$

$$\mathcal{H}_{xr} = 2EI \lambda^2 \frac{-\cos(2\lambda h) + \cosh(2\lambda h)}{\pm 2 + \cos(2\lambda h) + \cosh(2\lambda h)} \quad (C-19)$$

$$\mathcal{H}_{rr} = 2EI \lambda \frac{-\sin(2\lambda h) + \sinh(2\lambda h)}{\pm 2 + \cos(2\lambda h) + \cosh(2\lambda h)} \quad (C-20)$$

where $\mathcal{H}_{xx}$, $\mathcal{H}_{rr}$ and $\mathcal{H}_{xr}$ are the swaying, rocking and cross-swaying-rocking impedances, respectively. The plus sign in the denominator corresponds to the special case of a pile completely free of stresses at the tip, whereas the minus sign corresponds to a perfectly fixed base pile. As in the case of axial loading, this formulation can be extended for the case of multilayered soils (Mylonakis, 1995).

C.4 Interaction factors for lateral and moment loading

The interaction factors can be obtained using the “wave interference model” which is based on the following three steps

- i) Determine the deflected shape of the source pile $Y_{11}(z)$ due to a unit lateral force or moment applied at its head
- ii) Determine the free-field soil displacement at a distance s of the “source” pile and angle θ from the direction of loading.

$$U(s, z, \theta) \approx \psi(s, \theta) Y_{11}(z)$$

- iii) The inertial and flexural stiffness of the “receiver” pile lead to a diffraction of the arriving wave field.

For a receiver pile, the dynamic equilibrium of an infinitesimal pile segment yields the governing equation:

$$\frac{d^4 Y_{21}(z)}{dz^4} + 4 \lambda^4 Y_{21}(z) = \frac{k(z, \omega) \psi(s, \theta) Y_{11}(z)}{EI} \quad (C-21)$$

The forcing term on the right side of eq. (C-21) is equal to the attenuated free-field soil displacement times the dynamic soil impedance divided by the flexural rigidity of the pile. Solving eq. (C-16) we get for Y_{11}

$$Y_{11}(z) = e^{\lambda z} (A_{11} \sin(\lambda z) + B_{11} \cos(\lambda z)) + e^{-\lambda z} (C_{11} \sin(\lambda z) + D_{11} \cos(\lambda z)) \quad (C-22)$$

where A_{11} , B_{11} , C_{11} and D_{11} are integration constants determined from the boundary conditions of the source pile. The solution to eq. (C-21) is

$$Y_{21}(z) = \frac{k(\omega)}{k(\omega) - m\omega^2} \psi(s, \theta) \frac{\lambda z}{4} Y_{11}(z) + e^{\lambda z} (A_{21} \sin(\lambda z) + B_{21} \cos(\lambda z)) + e^{-\lambda z} (C_{21} \sin(\lambda z) + D_{21} \cos(\lambda z)) \quad (C-23)$$

where A_{21} , B_{21} , C_{21} and D_{21} are integration constants to be determined from the boundary conditions of the receiver pile (zero shear force and bending moment at the pile head and continuity of force, moment, displacement and rotation at the various interfaces (case of multilayered soil). Differentiating eq. (C-23) with respect to z yields the flexural rotations, $\Theta_{21}(z)$ along the receiver pile. The interaction factors are calculated as:

$$\begin{aligned} \alpha_{uP} &= \frac{Y_{21}(0)}{Y_{11}(0)} \\ \alpha_{\varphi P} &= \frac{\Theta_{21}(0)}{\Theta_{11}(0)} \\ \alpha_{uM} &= \frac{Y_{21}(0)}{Y_{11}(0)} \\ \alpha_{\varphi M} &= \frac{\Theta_{21}(0)}{\Theta_{11}(0)} \end{aligned} \quad (C-24)$$

The interaction factors can be obtained in closed form and can be written as a product of two complex-valued functions (Mylonakis, 1995)

$$\begin{bmatrix} \alpha_{uP} & \alpha_{uM} \\ \alpha_{\varphi P} & \alpha_{\varphi M} \end{bmatrix} = \psi(s, \theta, \omega) \begin{bmatrix} \zeta_{uP} & \zeta_{uM} \\ \zeta_{\varphi P} & \zeta_{\varphi M} \end{bmatrix} \quad (C-25)$$

where ζ_{uP} , ζ_{uM} , $\zeta_{\varphi P}$ and $\zeta_{\varphi M}$ are given by

$$\zeta_{uP} = \frac{3}{4} \frac{k(\omega)}{k(\omega) - m\omega^2} \quad (C-26)$$

$$\zeta_{uM} = \zeta_{\varphi P} = \frac{1}{2} \frac{k(\omega)}{k(\omega) - m\omega^2} \quad (C-27)$$

$$\zeta_{\varphi M} = \frac{1}{4} \frac{k(\omega)}{k(\omega) - m\omega^2} \quad (C-28)$$

As in the case of axial loading, this formulation can be extended for the case of multilayered soils (Mylonakis, 1995).

APPENDIX D: PILE GROUP FORMULATION

D.1 Derivation

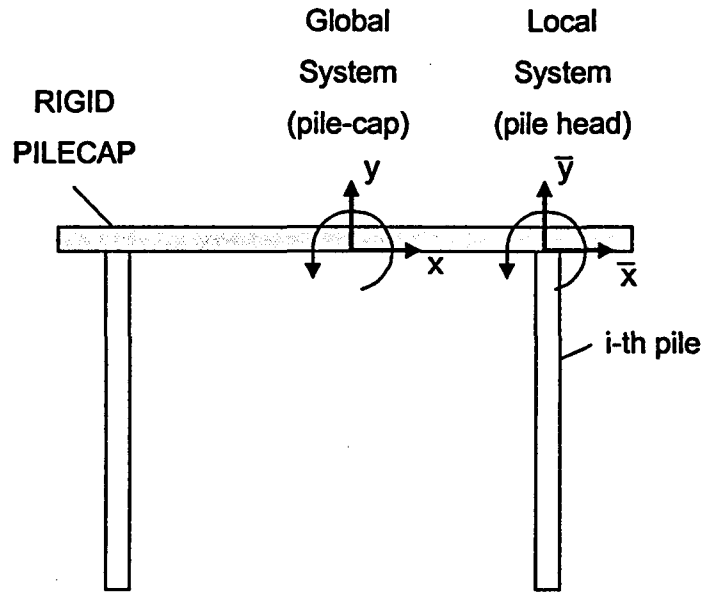


FIGURE D.1: Pile group cross-section and coordinate systems considered

Considering planar oscillations, the response of pile i in the group can be written as

$$\begin{Bmatrix} D_1 \\ D_2 \\ D_3 \end{Bmatrix}_i = \sum_{j=1}^m [\alpha]_{ij} [F]_j \begin{Bmatrix} P_1 \\ P_2 \\ P_3 \end{Bmatrix}_j \quad (D-1)$$

in which vector $\{D\}_i$ is the response at the head of pile i , $[\alpha]_{ij}$ is a matrix containing the interaction factors between piles i and j , and $[F]_j$ is the flexibility matrix of pile j and m is the total number of piles. For the whole group, eqn. (D-1) takes the form

$$\begin{Bmatrix} \{D\}_1 \\ \vdots \\ \{D\}_m \end{Bmatrix} = \begin{bmatrix} [F]_{11} & [AF]_{12} & \dots & [AF]_{1m} \\ [AF]_{21} & [F]_{22} & \dots & [AF]_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ [AF]_{m1} & [AF]_{m2} & \dots & [F]_{mm} \end{bmatrix} \begin{Bmatrix} \{P\}_1 \\ \vdots \\ \{P\}_m \end{Bmatrix} \quad (D-2)$$

where $[AF]_{ij}$ is the product of the interaction matrix $[\alpha]_{ij}$ between piles i and j , and the flexibility matrix $[F]_j$ of pile j .

Denoting the matrix in the right-hand side of eqn. (D-2) as $[F_{TOT}]$, the above equation can be inverted providing the stiffness of the group without the influence of the cap:

$$\{D\} = [K_{TOT}] \{P\}, \text{ with } [K_{TOT}] = [F_{TOT}]^{-1} \quad (D-3)$$

For a rigid cap, the three components of global response generate all the local responses through the following equation:

$$\begin{Bmatrix} \{D\}_1 \\ \vdots \\ \{D\}_m \end{Bmatrix} = \begin{bmatrix} T \\ T \\ T \end{bmatrix} \begin{Bmatrix} D_G(1) \\ D_G(2) \\ D_G(3) \end{Bmatrix} \quad (D-4)$$

in which $[T]$ is a $3m \times 3$ matrix given by

$$\begin{bmatrix} T \\ T \\ T \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & x_1 & 1 \\ \vdots & \vdots & \vdots \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & x_m & 1 \end{bmatrix} \quad (D-5)$$

where x_i denoted the x coordinate of pile i measured from the global system. The final 3x3 stiffness matrix $[K_G]$ is obtained after imposing the geometric condition (D-4) into (D-3).

$$\begin{Bmatrix} P_G(1) \\ P_G(2) \\ P_G(3) \end{Bmatrix} = \begin{bmatrix} & & \\ & & \\ T & & \end{bmatrix}^T [K_{TOT}] \begin{bmatrix} & & \\ & & \\ T & & \end{bmatrix} \begin{Bmatrix} D_G(1) \\ D_G(2) \\ D_G(3) \end{Bmatrix}$$

or

$$\{P_G\} = [K_G] \{D_G\} \quad (D-6)$$

where

$$[K_G] = \begin{bmatrix} & & \\ & & \\ T & & \end{bmatrix}^T [K_{TOT}] \begin{bmatrix} & & \\ & & \\ T & & \end{bmatrix}$$

Additional information on the derivation of eq. (D-6) can be found in El-Marsafawi et al. (1992) and in Mylonakis (1995).

APPENDIX E: BOUC-WEN MODEL

E.1 Description

The Bouc-Wen hysteretic model is expressed as (Bouc, 1967; Wen 1975, 1976)

$$F_s = \alpha k y + (1-\alpha) k y_0 J \quad (\text{E-1})$$

where $J = J(y, \dot{y})$ (dot denotes differentiation with respect to time) is given by the following differential equation:

$$y_0 \ddot{J} + \gamma |\dot{y}| |J|^{n-1} \dot{J} + \beta \dot{y} |J|^n - A \dot{y} = 0 \quad (\text{E-2})$$

As will be shown in the sequel, α , y_0 and k can be interpreted as post-yielding stiffness ratio, yielding deflection and initial stiffness respectively.

A , β , γ , n are dimensionless constants to be determined in the sequel. Note that, in the original model of Bouc (1967), n was taken equal to 1.

- *Initial stiffness k*

Eliminating the time variable from eq. (E-2) we obtain (see section A.2)

$$y_0 \frac{dJ}{dy} = A \pm (\beta \pm \gamma) J^n \quad (\text{E-3})$$

For $J \ll 1$ the behavior is elastic. At this stage

$$J^n \rightarrow 0 \quad (J \ll 1, n > 1) \quad (\text{E-4})$$

From eq. (E-3) and (E-4),

$$\frac{dJ}{dy} = \frac{A}{y_0} \quad (E-5)$$

Differentiating eqn. (E-1) with respect to y and substituting eq. (E-5) yields:

$$\frac{dF_s}{dy} = \alpha k + (1-\alpha) k y_0 \frac{dJ}{dy} = \alpha k + (1-\alpha) k A = [\alpha + (1-\alpha) A] k \quad (E-6)$$

The above expression implies that k will be the initial stiffness of the system if and only if

$$A = 1 \quad (E-7)$$

- *Yielding displacement y_0 and maximum force F_{smax}*

For a monotonic displacement input, eqns. (E-2) and (E-3) are written as

$$y_0 \frac{dJ}{dy} = A - (\beta + \gamma) J^n, \quad y > 0, \quad \dot{y} > 0 \quad (E-8)$$

The maximum value J_{max} is obtained when $\frac{dJ}{dy} = 0$

$$J_{max} = \left(\frac{A}{\beta + \gamma} \right)^{1/n}, \quad \beta + \gamma \neq 0 \quad (E-9)$$

From which it is seen that J_{max} is bounded provided that the sum $\beta + \gamma$ is different than zero. Substituting eqn. (E-9) into eqn. (E-1) we get

$$F_{smax} = \alpha k y_{max} + (1-\alpha) k y_0 \left(\frac{A}{\beta + \gamma} \right)^{1/n} \quad (E-10)$$

where $F_{s \max}$ denotes the ultimate resistance of the system. If the post-yielding stiffness ratio α is zero, eqn. (E-10) reduces to

$$F_{s \max} = k y_0 \left(\frac{A}{\beta + \gamma} \right)^{1/n}, \quad \alpha = 0 \quad (\text{E-11})$$

It is evident that y_0 can be interpreted as the “yielding” displacement of a non-hardening elastoplastic system only if the term $\left(\frac{A}{\beta + \gamma} \right)^{1/n}$ is equal to unity. Since A has been already shown to be 1, this is equivalent to

$$\beta + \gamma = 1 \quad (\text{E-12})$$

In such a case, $J_{\max}$ is bounded between -1 and 1 i.e.,

$$|J| \leq 1 \quad (\text{E-13})$$

and (Constantinou & Adnane, 1987)

$$F_{s \max} = k y_0, \quad F_s = F_{s \max} J \quad (\text{E-14})$$

● *Post yielding stiffness ratio α*

With increasing displacement y , the first term in eqn. (E-1) dominates which implies that α can be interpreted as the post-yielding stiffness of the system.

Constantinou & Adnane (1987) showed that the Bouc-Wen model is an extension of the Ozdemir model of viscoplasticity. A discussion on this property of the model can be found in that publication.

E.2 Time-independent representation

To elucidate the development of eqn. (E-3), the governing equation (E-1) can be written as

$$y_0 \dot{J} = \left[A - |J|^n \left(\beta + \gamma \frac{|\dot{y}|}{\dot{y}} \frac{J}{|J|} \right) \right] \dot{y} \quad , \quad \dot{y} \neq 0 \quad , \quad J \neq 0 \quad (E-15)$$

which, in turn, can be expressed in the more convenient form

$$y_0 \dot{J} = \left[A - |J|^n \left(\beta + \gamma \operatorname{sgn}(\dot{y} J) \right) \right] \dot{y} \quad , \quad \dot{y} \neq 0 \quad , \quad J \neq 0 \quad (E-16)$$

which is a time-dependent representation. Using the chain rule of differentiation

$$\dot{J} = \frac{dJ}{dt} = \frac{dJ}{dy} \frac{dy}{dt} = J' \dot{y} \quad (E-17)$$

eqn. (E-16) becomes

$$y_0 J' = A - |J|^n \left[\beta + \gamma \operatorname{sgn}(\dot{y} J) \right] \quad , \quad \dot{y} \neq 0 \quad , \quad J \neq 0 \quad (E-18)$$

which is the time-independent representation in eqn. (E-3).

Note that the function $\dot{y}$ in eq. (E-18) does not represent the first time derivative of deformation with respect to time. It can be interpreted as the direction (“arrow”) of the (positive or negative) deformation.

E.3 Elastic-Fully plastic model ($\alpha = 0$)

Considering the special case $\alpha = 0$, eqn. (E-1) becomes

$$F_s = k y_0 J$$

Upon determining the elastic stiffness of the system,

$$K \equiv \frac{dF_s}{dy} = k y_0 J' \quad (E-19)$$

two distinct cases can be identified:

a) For $|J| < 1$ the term J^n tends to zero; therefore

$$|J| < 1 \Rightarrow J' = \frac{A}{y_0} \quad \text{and} \quad K = k \quad (E-20)$$

b) For $|J| \rightarrow 1$ the term J^n tends to unity. If the signum operator is positive, as in the case of virgin loading, we get

$$y_0 J' = A - 1 \cdot [\beta + \gamma \cdot 1] = A - [\beta + \gamma] = 1 - 1 = 0 \quad (E-21)$$

which means that the system has yielded.

When the motion reverses (after the system has yielded), eqn. (E-21) becomes

$$y_0 J' = A - 1 \cdot [\beta + \gamma \cdot (-1)] = A - [\beta - \gamma] = 1 - [\beta - \gamma] = 2 \gamma$$

The following interesting comments can be made on the last equation:

- i) The derivative J' is always positive (which corresponds to positive stiffness K), provided that γ is positive.

- ii) If γ is larger than 0.5 (as recommended by Constantinou & Adnane, 1987; Badoni & Makris, 1996) the resulting unloading stiffness $K = 2 \gamma k$, is larger than the loading elastic stiffness $K = k$.
- iii) In the particular case where $\beta = \gamma = 0.5$, the loading and unloading stiffnesses become equal to k .

In this study the following parameters will be used:

$A = 1$, $\beta = \gamma = 0.5$, $n = 21$ and $\alpha = 0.03$. For more details on the selection of these parameters the reader is referred to Constantinou & Adnane, 1987.

APPENDIX F: SSI PERIOD OF A 2-DOF SYSTEM

F.1 SSI period of a 2-DOF system

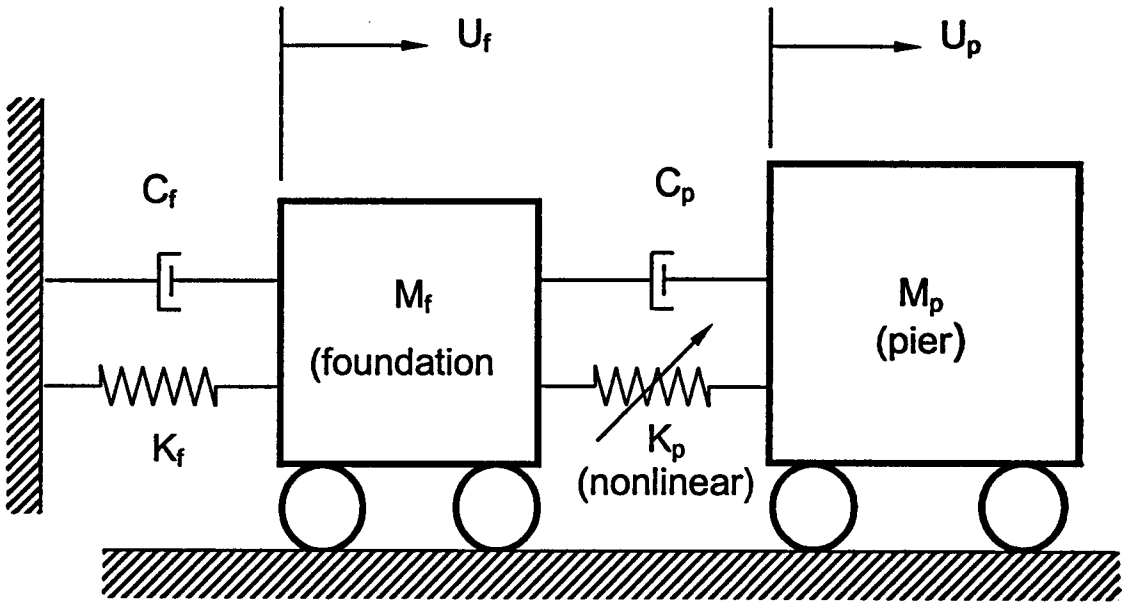


FIGURE F.1: The 2-DOF system used in this study

The equation of motion of the 2-DOF system in Figure F.1 in matrix form is:

$$\begin{bmatrix} M_p & 0 \\ 0 & M_f \end{bmatrix} \begin{Bmatrix} \ddot{U}_p \\ \ddot{U}_f \end{Bmatrix} + \begin{bmatrix} C_p & -C_p \\ -C_p & C_p + C_f \end{bmatrix} \begin{Bmatrix} \dot{U}_p \\ \dot{U}_f \end{Bmatrix} + \begin{bmatrix} K_p & -K_p \\ -K_p & K_p + K_f \end{bmatrix} \begin{Bmatrix} U_p \\ U_f \end{Bmatrix} = \begin{Bmatrix} P_p \\ P_f \end{Bmatrix} \quad (\text{F-1})$$

Considering undamped free vibrations, and using normalized expressions

$$M_f = b^* M_p \quad (\text{F-2})$$

and

$$K_f = c^{**} K_p \quad (\text{F-3})$$

($c^{**} = 1/c^*$ for simplicity in the calculations) one obtains

$$\begin{bmatrix} K_p - M_p \omega^2 & -K_p \\ -K_p & K_p (1+c^{**}) - M_p b^* \omega^2 \end{bmatrix} \begin{Bmatrix} U_p \\ U_f \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \quad (F-4)$$

setting the determinant of the above matrix equal to zero yields:

$$\begin{aligned} (K_p - M_p \omega^2) [K_p (1+c^{**}) - M_p b^* \omega^2] - K_p^2 &= 0 \Rightarrow \\ M_p^2 b^* (\omega^2)^2 - M_p K_p (1+b^*+c^{**}) \omega^2 + K_p^2 c^{**} &= 0 \end{aligned} \quad (F-5)$$

From which the natural frequencies of the system, $\omega_{1,2}$, are obtained

$$\begin{aligned} \omega_{1,2}^2 &= \frac{M_p K_p (1+b^*+c^{**}) \pm \sqrt{M_p^2 K_p^2 (1+b^*+c^{**})^2 - 4 M_p^2 b^* K_p^2 c^{**}}}{2 M_p^2 b^*} \Rightarrow \\ \omega_{1,2}^2 &= \frac{K_p}{M_p} \frac{1 + c^{**}/b^* + 1/b^* \pm \sqrt{(1+c^{**}/b^* + 1/b^*)^2 - 4 c^{**}/b^*}}{2} \end{aligned}$$

or

$$\omega_{1,2}^2 = \frac{K_p}{M_p} \frac{1 + b^* + c^{**} \pm \sqrt{(1+b^*+c^{**})^2 - 4c^{**} b^*}}{2 b^*} \quad (F-6)$$

Defining T_n , as the natural period of the pier with a fixed base,

$$T_n = 2 \pi \sqrt{\frac{M_p}{K_p}} \quad (F-7)$$

The fundamental period, T_{SSI} , of the 2-DOF system, can then be written as

$$T_{SSI} = T_n f_T^{-1} \quad (F-8)$$

where

$$f_T = \sqrt{\frac{1 + b^* + \frac{1}{c^*} + \sqrt{\left(1 + b^* + \frac{1}{c^*}\right)^2 - 4 \frac{b^*}{c^*}}}{2 b^*}} \quad (F-9)$$

F.2 Limiting cases

- $M_f \rightarrow 0$ corresponding to a massless foundation, coefficient b^* tends to zero and f_T in eq. (F-7) becomes undefined.

Applying L' Hospital's rule to eq. (F-6) we obtain

$$\omega_1^2 = \frac{K_p}{M_p} \frac{c^{**}}{1+c^{**}} = \frac{K_p}{M_p} \frac{1}{1+c^*} \quad (F-10)$$

and $\omega_2^2 = \infty$

- Letting $K_f \rightarrow \infty$ corresponding to a rigid foundation, coefficients c^{**} and c^* tend to infinity and zero, respectively, leading to

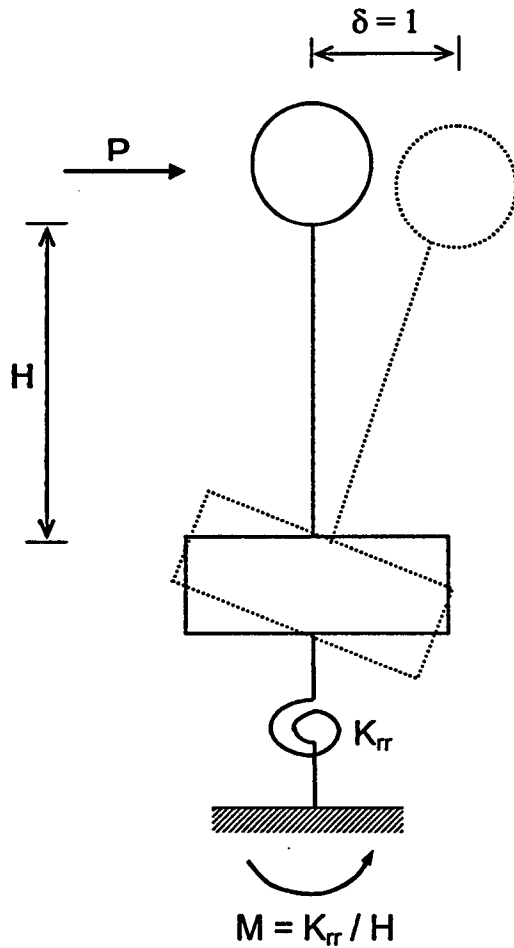
$$\omega_1^2 = \frac{K_p}{M_p} \frac{2 b^*}{2 b^*} = \omega_n^2 \quad (F-11)$$

and

$$\omega_2^2 = \infty$$

APPENDIX G: VIBRATIONAL CHARACTERISTICS OF FLEXIBLY-SUPPORTED BRIDGE MODELS

G.1 Stiffness of a rigid bridge pier due to foundation rocking



The equivalent stiffness of the flexibly-supported pier is defined as

$$\tilde{K} = \frac{P}{\delta} \quad (G-1)$$

Applying moment equilibrium yields

$$P = \frac{K_r}{H^2} \quad (G-2)$$

from eq. (G-2) and (G-1) one obtains

$$\tilde{K} = \frac{K_r}{H^2} \quad (G-3)$$

Note that, in order to include damping in eq. (G-3), the rocking stiffness K_r can be expressed as a complex quantity

$$\mathcal{K}_r = K_r (1 + 2i\beta_r).$$

FIGURE G.1

In general, eq. (G-3) can be written as

$$\tilde{K} = \alpha \frac{K_r}{H^2} \quad (G-4)$$

where

$$\alpha = \begin{cases} 1 & , \text{ deck free to rotate} \\ \frac{1}{2} & , \text{ deck fixed against rotation} \end{cases} \quad (\text{G-4b})$$

G.2 Stiffness of a rigid pier due to uncoupled foundation swaying and rocking

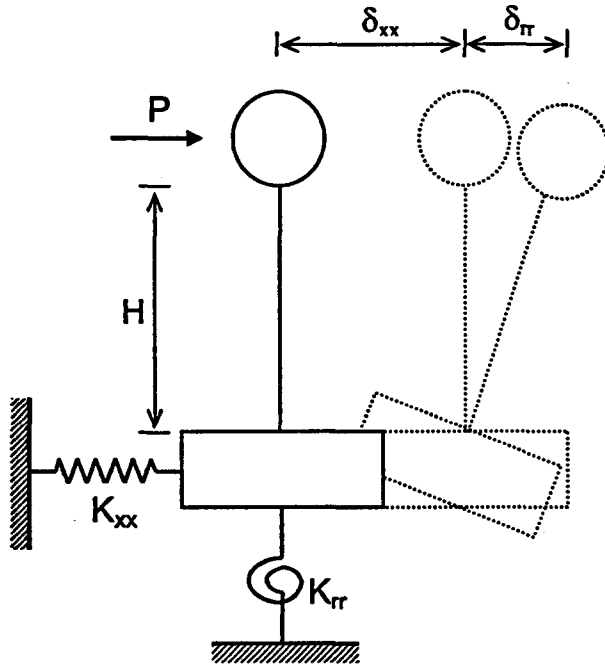


FIGURE G.2

The translational and rotational springs are connected in series so, the total displacement is the sum of the two individual displacements

$$\delta_{\text{tot}} = \delta_{xx} + \delta_{rr} \quad (\text{G-5})$$

where

$$\delta_{\text{tot}} = \frac{P}{\tilde{K}} \quad (\text{G-6})$$

$$\delta_{xx} = \frac{P}{K_{xx}} \quad (\text{G-7})$$

and

$$\delta_{rr} = \alpha \frac{P H^2}{K_{rr}} \quad (\text{G-8})$$

Substituting eq. (G-8), (G-7) and (G-6) in (G-5) and after some algebra, one obtains

$$\tilde{K} = \frac{K_{xx} \cdot K_{rr}}{\alpha K_{xx} H^2 + K_{rr}} \quad (\text{G-9})$$

where α is given by eq. (G-4b).

To account for damping, the swaying and rocking stiffnesses, K_{xx} and K_{rr} , in eq. (G-9) are replaced by the complex quantities

$$\mathcal{K}_r = K_r (1 + 2i\beta_r), \mathcal{K}_{xx} = K_{xx} (1 + 2i\beta_{xx}).$$

G.3 Stiffness of a rigid bridge pier due to coupled foundation swaying and rocking

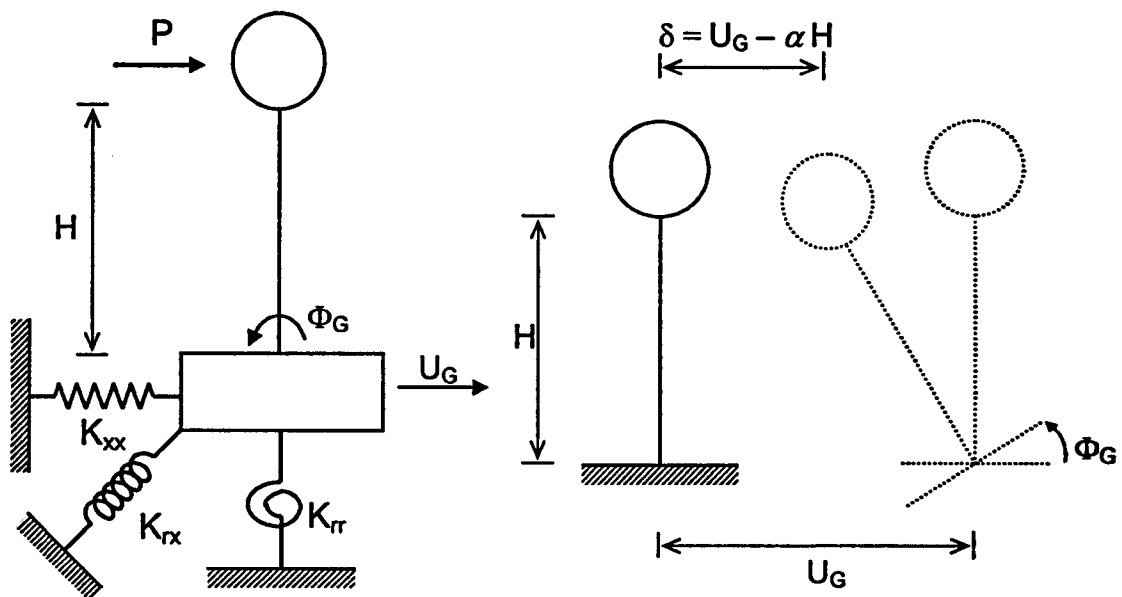


FIGURE G.3

The equivalent SSI stiffness is defined as

$$\tilde{K} = \frac{P}{\delta} \quad (G-10)$$

From geometry

$$\delta = U_G - \alpha H \Phi_G \quad (G-11)$$

Combining the equations of force and moment equilibrium of the base with the corresponding Hooke's equations, one obtains the matrix equation

$$\begin{Bmatrix} P \\ -P \cdot H \end{Bmatrix} = \begin{bmatrix} K_{xx} & K_{xr} \\ K_{rx} & K_{rr} \end{bmatrix} \begin{Bmatrix} U_G \\ \Phi_G \end{Bmatrix} \quad (G-12)$$

Upon solving eq. (G-12) for displacements and rotations

$$\begin{Bmatrix} U_G \\ \Phi_G \end{Bmatrix} = \frac{1}{K_{xx} K_{rr} - K_{rx}^2} \begin{Bmatrix} K_{rr} + H K_{rx} \\ -K_{rx} - K_{xx} H \end{Bmatrix} P \quad (G-12b)$$

and substituting eq. (G-12b) and (G-11) in (G-10) yields

$$\bar{K} = \frac{P}{U_G - \alpha H \Phi_G} = \frac{K_{xx} K_{rr} - K_{rx}^2}{\alpha K_{xx} H^2 + (\alpha + 1) K_{rx} H + K_{rr}} \quad (G-13)$$

where α is given by eq. (G-4b).

To include damping, the swaying, rocking and cross term stiffness K_{xx} , K_{rr} and K_{rx} should be replaced by the complex quantities

$$\mathcal{K}_{rr} = K_{rr} (1 + 2i\beta_{rr}), \mathcal{K}_{xx} = K_{xx} (1 + 2i\beta_{xx}), \mathcal{K}_{rx} = K_{rx} (1 + 2i\beta_{rx}) .$$

G.4 Stiffness of a flexible bridge pier due to coupled foundation swaying and rocking

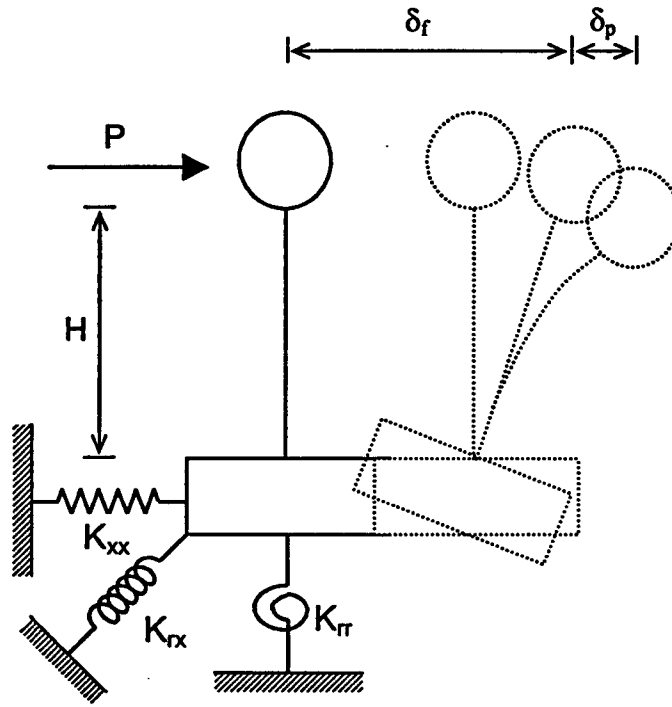


FIGURE G.4

In this case, the results of the previous section will be used.

Basis of the analysis is that the stiffness (spring) of the pier and the equivalent SSI stiffness (base) of the foundation due to coupled swaying and rocking are connected in series, so

$$\delta_{\text{tot}} = \delta_f + \delta_p \quad (\text{G-14})$$

where

$$\delta_{\text{tot}} = \frac{P}{\tilde{K}} \quad (\text{G-15})$$

$$\delta_p = \frac{P}{K_p} \quad (\text{G-16})$$

and

$$\delta_f = \frac{P}{K_f} \quad (\text{G-17})$$

Combining the above equations yields the well-known result

$$\tilde{K} = \frac{K_p K_f}{K_p + K_f} \quad (\text{G-18})$$

where, K_f is given by eq. (G-13)

$$K_f = \frac{K_{xx} K_{\pi} - K_{rx}^2}{\alpha K_{xx} H^2 + (\alpha+1) K_{rx} H + K_{\pi}} \quad (G-19)$$

in which

$$\alpha = \begin{cases} 1 & , \text{ deck free to rotate} \\ \frac{1}{2} & , \text{ deck fixed against rotation} \end{cases}$$

and

$$K_p = \begin{cases} \frac{3 EI}{H^3} & , \text{ deck free to rotate} \\ \frac{12 EI}{H^3} & , \text{ deck fixed against rotation} \end{cases}$$

And like before, the stiffness terms K_{xr} , K_{π} , K_{xx} and K_p can be expressed as complex numbers in order to include damping.

G.5 Bridge pier on flexible foundation as a generalized SDOF oscillator

G.5.1 SSI stiffness, period and damping

With reference to Figure G.5, this model consists of:

- 2 translational (U_1 , U_2) and 2 rotational (U_3 , U_4) degrees of freedom at the base and the top respectively
- 2 lumped masses (m_1 , m_2) and 2 moments of inertia (I_1 , I_2)
- distributed mass ($\bar{m}$) along the height of the pier (L)

- swaying (K_{xx} , C_{xx}), rocking ($K_{\pi\pi}$, $C_{\pi\pi}$) and cross-swaying-rocking ($K_{x\pi}$, $C_{x\pi}$) lumped base springs and dashpots
- a lumped dashpot at the top (C_1)

The external force is of the form of a ground acceleration $\ddot{U}_g(t)$ imposed at the base of the springs.

Figure G.5 depicts this generalized pier in position in time (t). The basis of the proposed method is that the deformation in Fig. G.5 follows a time independent shape function $\psi(x)$.

With reference to the infinitesimally small section dx , taking the equilibrium of moments about the center of the section yields:

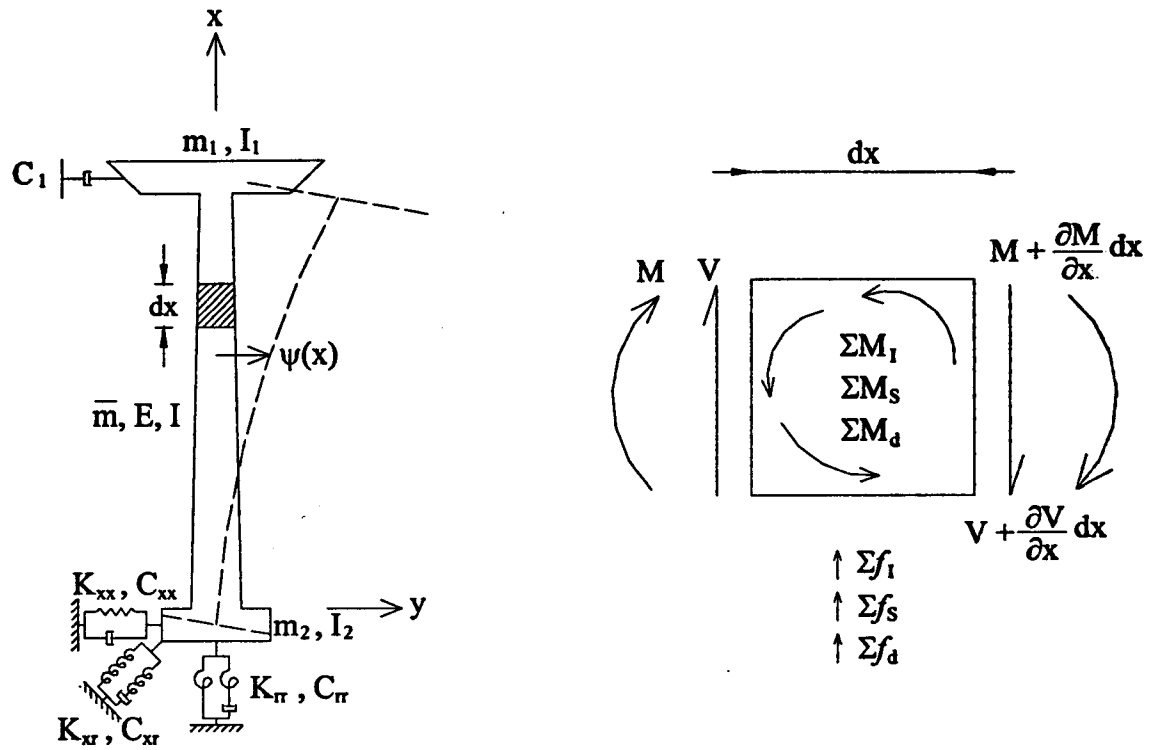
$$\begin{aligned} M + Vdx - \left(M + \frac{\partial M}{\partial x} dx \right) - I_2 \frac{\partial^3 y}{\partial t^2 \partial x} \delta(x) dx - I_1 \frac{\partial^3 y}{\partial t^2 \partial x} \delta(x-L) dx \\ - K_{x\pi} y \delta(x) dx - K_{\pi\pi} \frac{\partial y}{\partial t} \delta(x) dx - C_{x\pi} \frac{\partial y}{\partial t} \delta(x) dx - C_{\pi\pi} \frac{\partial^2 y}{\partial t \partial x} \delta(x) dx = 0 \end{aligned} \quad (G-20)$$

Taking the equilibrium of forces in the horizontal direction yields:

$$\begin{aligned} V - \left(V + \frac{\partial V}{\partial x} dx \right) + m_1 \delta(x-L) dx \left(\frac{\partial^2 y}{\partial t^2} + \ddot{U}_g \right) + m_2 \delta(x) dx \left(\frac{\partial^2 y}{\partial t^2} + \ddot{U}_g \right) \\ + \bar{m} dx \left(\frac{\partial^2 y}{\partial t^2} + \ddot{U}_g \right) + K_{xx} y \delta(x) dx + K_{\pi\pi} \frac{\partial y}{\partial t} \delta(x) dx + C_{xx} \frac{\partial y}{\partial t} \delta(x) dx \\ + C_{x\pi} \frac{\partial^2 y}{\partial t \partial x} \delta(x) dx + C_1 \frac{\partial y}{\partial t} \delta(x-L) dx = 0 \end{aligned} \quad (G-21)$$

According to beam theory, bending moment is related to curvature through

$$M = -EI y'' \quad (G-22)$$



$$\sum M_I = I_1 \frac{\partial^3 y}{\partial t^2 \partial x} \delta(x-L) dx + I_2 \frac{\partial^3 y}{\partial t^2 \partial x} \delta(x) dx$$

$$\sum M_S = K_{xx} y \delta(x) dx + K_{\pi} \frac{\partial y}{\partial x} \delta(x) dx$$

$$\sum M_d = C_{xr} \frac{\partial y}{\partial x} \delta(x) dx + C_{\pi} \frac{\partial^2 y}{\partial t \partial x} \delta(x) dx$$

$$\sum f_I = m_1 \delta(x-L) dx \left(\frac{\partial^2 y}{\partial t^2} + \ddot{U}_s \right) + m_2 \delta(x) dx \left(\frac{\partial^2 y}{\partial t^2} + \ddot{U}_s \right) + \bar{m} dx \left(\frac{\partial^2 y}{\partial t^2} + \ddot{U}_s \right)$$

$$\sum f_S = K_{xx} y \delta(x) dx + K_{xr} \frac{\partial y}{\partial t} \delta(x) dx$$

$$\sum f_d = C_{xx} \frac{\partial y}{\partial t} \delta(x) dx + C_{xr} \frac{\partial^2 y}{\partial t \partial x} \delta(x) dx + C_I \frac{\partial y}{\partial t} \delta(x-L) dx$$

FIGURE G.5: Pier model considered

Differentiating (G-20) with respect to x and plugging (G-21) and (G-22) into (G-20)

yields the governing partial differential equation:

$$\begin{aligned}
 & \frac{\partial^2}{\partial x^2} \left(E I \frac{\partial^2 y}{\partial x^2} \right) + m_1 \delta(x-L) \left(\frac{\partial^2 y}{\partial t^2} + \ddot{U}_g \right) + m_2 \delta(x) \left(\frac{\partial^2 y}{\partial t^2} + \ddot{U}_g \right) \\
 & + \bar{m} dx \left(\frac{\partial^2 y}{\partial t^2} + \ddot{U}_g \right) + K_{xx} y \delta(x) + K_{\pi} \frac{\partial y}{\partial t} \delta(x) + C_{xx} \frac{\partial y}{\partial t} \delta(x) \\
 & + C_{\pi} \frac{\partial^2 y}{\partial t \partial x} \delta(x) + C_1 \frac{\partial y}{\partial t} \delta(x-L) - I_1 \frac{\partial}{\partial x} \left(\frac{\partial^3 y}{\partial t^2 \partial x} \delta(x-L) \right) \\
 & - I_2 \frac{\partial}{\partial x} \left(\frac{\partial^3 y}{\partial t^2 \partial x} \delta(x) \right) - K_{\pi} \frac{\partial}{\partial x} (y \delta(x)) - K_{\pi} \frac{\partial}{\partial x} \left(\frac{\partial y}{\partial t} \delta(x) \right) \\
 & - C_{\pi} \frac{\partial}{\partial x} \left(\frac{\partial y}{\partial t} \delta(x) \right) - C_{\pi} \frac{\partial}{\partial x} \left(\frac{\partial^2 y}{\partial t \partial x} \delta(x) \right) = 0
 \end{aligned} \tag{G-23}$$

Introducing separation of variables, the response function $y(x,t)$ can be written as

$$y(x,t) = \psi(x) U(t) \tag{G-24}$$

where U is a function of time having units of length and y is a dimensionless function of x .

Substituting (G-24) into (G-23), multiplying both sides with $\psi(x)dx$ and integrating over the height of the pier yields:

$$\begin{aligned}
& U \int_0^L \frac{\partial^2}{\partial x^2} \left(EI \frac{\partial^2 \psi}{\partial x^2} \right) \psi dx + \ddot{U} \int_0^L m_1 \psi^2 \delta(x-L) dx + \ddot{U}_g \int_0^L m_1 \psi \delta(x-L) dx \\
& + \ddot{U} \int_0^L m_2 \psi^2 \delta(x) dx + \ddot{U}_g \int_0^L m_2 \psi \delta(x) dx + \ddot{U} \int_0^L \bar{m} \psi^2 dx \\
& + \ddot{U}_g \int_0^L \bar{m} \psi dx - \ddot{U} \int_0^L I_2 \frac{\partial}{\partial x} (\psi' \delta(x)) \psi dx - \ddot{U} \int_0^L I_1 \frac{\partial}{\partial x} (\psi' \delta(x-L)) \psi dx \\
& + U \int_0^L K_{xx} \psi^2 \delta(x) dx + U \int_0^L K_{\pi} \psi' \delta(x) \psi dx - U \int_0^L K_{\pi} \frac{\partial}{\partial x} (\psi \delta(x)) \psi dx \\
& - U \int_0^L K_{\pi} \frac{\partial}{\partial x} (\psi' \delta(x)) \psi dx + \dot{U} \int_0^L C_{xx} \psi^2 \delta(x) dx + \dot{U} \int_0^L C_{\pi} \psi' \delta(x) \psi dx \\
& + \dot{U} \int_0^L C_1 \psi^2 \delta(x-L) dx - \dot{U} \int_0^L C_{\pi} \frac{\partial}{\partial x} (\psi' \delta(x)) \psi dx \\
& - \dot{U} \int_0^L C_{\pi} \frac{\partial}{\partial x} (\psi'' \delta(x)) \psi dx = 0
\end{aligned} \tag{G-25}$$

Evaluating the integrals results in the simpler expression:

$$\begin{aligned}
& \ddot{U} \left[m_1 \psi^2(L) + m_2 \psi^2(0) + I_1 \psi'(L)^2 + I_2 \psi'(0)^2 + \int_0^L \bar{m} \psi^2(x) dx \right] \\
& + \dot{U} \left[C_{xx} \psi^2(0) + C_{\pi} \psi'(0)^2 + 2 C_{\pi} \psi(0) \psi'(0) + C_1 \psi(L) \right] \\
& + U \left[K_{xx} \psi^2(0) + K_{\pi} \psi'(0)^2 + 2 K_{\pi} \psi(0) \psi'(0) + \int_0^L EI (\psi'')^2 dx \right] = \\
& = -\ddot{U}_g \left[m_1 \psi(L) + m_2 \psi(0) + \int_0^L \bar{m} \psi(x) dx \right]
\end{aligned} \tag{G-26}$$

Equation (G-26) can be rewritten as

$$\bar{M} \ddot{U} + \bar{C} \dot{U} + \bar{K} U = -\bar{L} \ddot{U}_g \tag{G-27}$$

in which the generalized mass $\bar{M}$, dashpot $\bar{C}$, stiffness $\bar{K}$ and influence coefficient $\bar{L}$ are given by

$$\bar{M} = m_1 \psi^2(L) + m_2 \psi^2(0) + I_1 \psi'(L)^2 + I_2 \psi'(0)^2 + \int_0^L \bar{m} \psi^2(x) dx \tag{G-28}$$

$$\bar{C} = C_{xx} \psi^2(0) + C_{\pi} \psi'(0)^2 + 2 C_{xr} \psi(0) \psi'(0) + C_1 \psi(L) \quad (G-29)$$

$$\bar{K} = K_{xx} \psi^2(0) + K_{\pi} \psi'(0)^2 + 2 K_{xr} \psi(0) \psi'(0) + \int_0^L EI(\psi'')^2 dx \quad (G-30)$$

$$\bar{L} = m_1 \psi(L) + m_2 \psi(0) + \int_0^L \bar{m} \psi(x) dx \quad (G-31)$$

It follows that the SSI period of the generalized oscillator is equal to:

$$T_{SSI} = 2\pi \sqrt{\frac{m_1 \psi^2(L) + m_2 \psi^2(0) + I_1 \psi'(L)^2 + I_2 \psi'(0)^2 + \int_0^L \bar{m} \psi^2(x) dx}{K_{xx} \psi^2(0) + K_{\pi} \psi'(0)^2 + 2 K_{xr} \psi(0) \psi'(0) + \int_0^L EI(\psi'')^2 dx}} \quad (G-32)$$

The associated modal participation factor is equal to

$$\psi_{SSI} = \frac{\bar{L}}{\bar{M}} = \frac{m_1 \psi(L) + m_2 \psi(0) + \int_0^L \bar{m} \psi(x) dx}{m_1 \psi^2(L) + m_2 \psi^2(0) + I_1 \psi'(L)^2 + I_2 \psi'(0)^2 + \int_0^L \bar{m} \psi^2(x) dx} \quad (G-33)$$

Following eq. (G-30) the equivalent hysteretic SSI damping ratio is equal to

$$\zeta_{SSI} = \beta_{xx} \psi^2(0) + \beta_{\pi} \psi'(0)^2 + 2 \beta_{xr} \psi(0) \psi'(0) + \beta_{pier} \int_0^L (\psi'')^2 dx \quad (G-34)$$

where β_{xx} , β_{π} , β_{xr} denote the swaying, rocking and cross swaying-rocking damping ratios of the foundation and β_{pier} is the damping ratio of the pier.

G.5.2 Shape function

The model under investigation has 2 translational and 2 rotational degrees of freedom (DOF's) in the two ends of the pier. Let's assume that the translational DOF's are U_1 (top) and U_2 (bottom) and the rotational DOF's are U_3 (top) and U_4 (bottom). The

deflected shape that will be employed here is the one of a beam that is supported by springs at its base and is deformed under a static horizontal load P applied at the top like in figure G.4.

Assuming that the length of the beam is L , the beam rigidity is EI , and the deflected shape $\psi(x)$, it is a simple matter to show that the deflected shape of the beam is:

$$\psi(x) = \frac{1}{EI} \left(P \frac{x^3}{6} - P \frac{L x^2}{2} + C_1 x + C_2 \right) \quad (G.35)$$

Subject to the boundary conditions:

$$\psi(0) = U_2 \quad (G.36)$$

$$\psi'(0) = -U_4 \quad (G.37)$$

$$\psi(L) = 1 \quad (G.38)$$

Enforcing the conditions in equations (G.36), (G.37) and (G.38) yields the solution:

$$P = -(1 - U_2 + L U_4) \frac{3EI}{L^3} \quad (G.39)$$

and

$$\psi(x) = (1 - U_2 + L U_4) \left(\frac{3}{2} \frac{x^2}{L^2} - \frac{x^3}{2 L^3} \right) + U_2 - x U_4 \quad (G.40)$$

In the above equation, base translation (U_2) and rotation (U_4) of of forces and moments depend on the foundation stiffness K_{xx} , K_{rr} and K_{xr} .

Taking the equilibrium at the base leads to the following matrix equation:

$$P \begin{bmatrix} 1 \\ -L \end{bmatrix} = \begin{bmatrix} K_{xx} & K_{xr} \\ K_{rx} & K_{rr} \end{bmatrix} \begin{bmatrix} U_2 \\ U_4 \end{bmatrix} \quad (G.41)$$

Solving eq. (G.41) with respect to U_2 and U_4 , yields

$$U_2 = \frac{K_{\pi} + L K_{\kappa}}{K_{\kappa} K_{\pi} - K_{\kappa}^2} P \quad (G.42)$$

and

$$U_4 = -\frac{K_{\kappa} + L K_{\kappa\kappa}}{K_{\kappa} K_{\pi} - K_{\kappa}^2} P \quad (G.43)$$

Thus

$$P = -\frac{3EI (K_{\kappa}^2 - K_{\kappa\kappa} K_{\pi})}{(K_{\kappa} K_{\pi} - K_{\kappa}^2) L^3 + 3EI (K_{\pi} + 2L K_{\kappa} + K_{\kappa\kappa} L^2)} \quad (G.44)$$

$$U_2 = \frac{3EI (K_{\pi} + L K_{\kappa})}{(K_{\kappa} K_{\pi} - K_{\kappa}^2) L^3 + 3EI (K_{\pi} + 2L K_{\kappa} + K_{\kappa\kappa} L^2)} \quad (G.45)$$

$$U_4 = -\frac{3EI (K_{\kappa} + L K_{\kappa\kappa})}{(K_{\kappa} K_{\pi} - K_{\kappa}^2) L^3 + 3EI (K_{\pi} + 2L K_{\kappa} + K_{\kappa\kappa} L^2)} \quad (G.46)$$

which suffice for evaluating the vibrational characteristics of the system.

APPENDIX H: INELASTIC RESPONSE OF HANSHIN EXPRESSWAY PIER 138 USING A 2-DOF MODEL

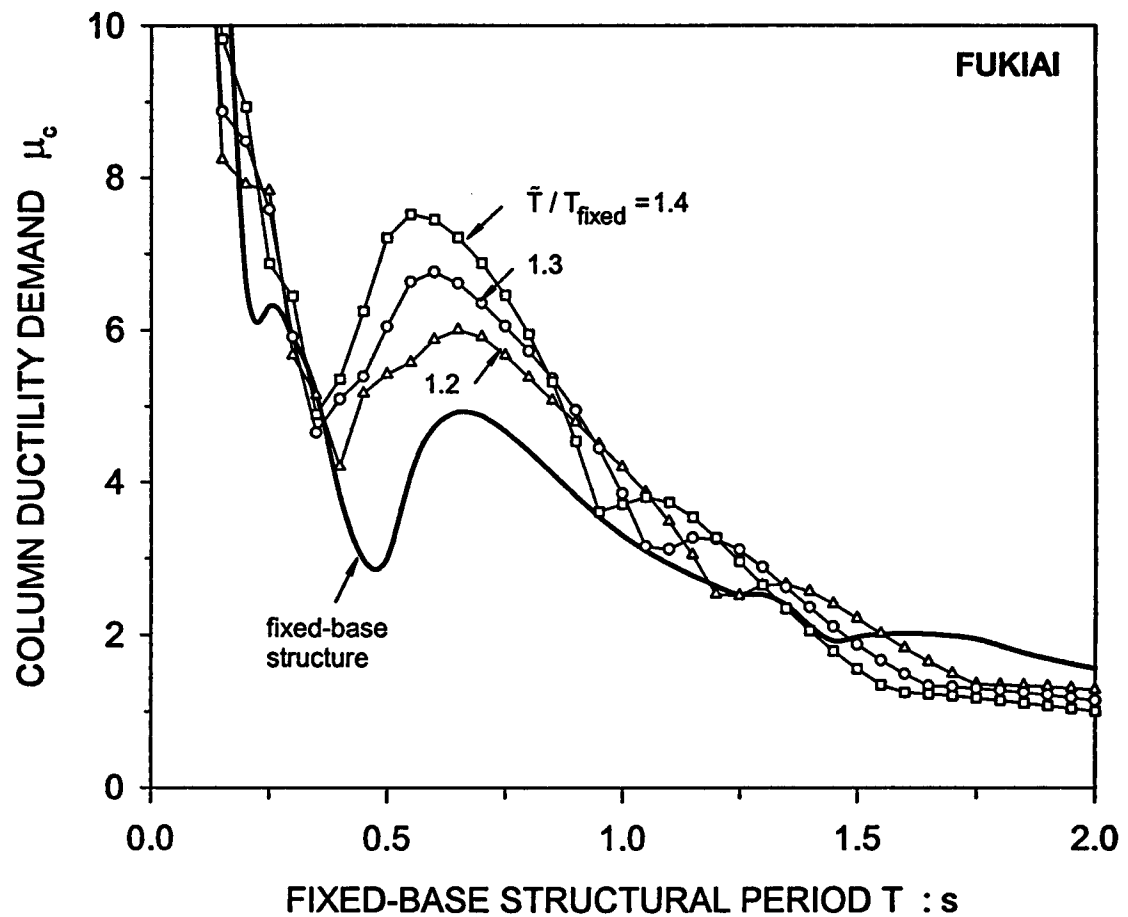


FIGURE H.1: Effect of SSI on ductility demand of a bridge pier subjected to the fault-normal component of Fukiai record; $C_y = 0.5$, $\alpha = 0.05$, $\zeta = 5\%$

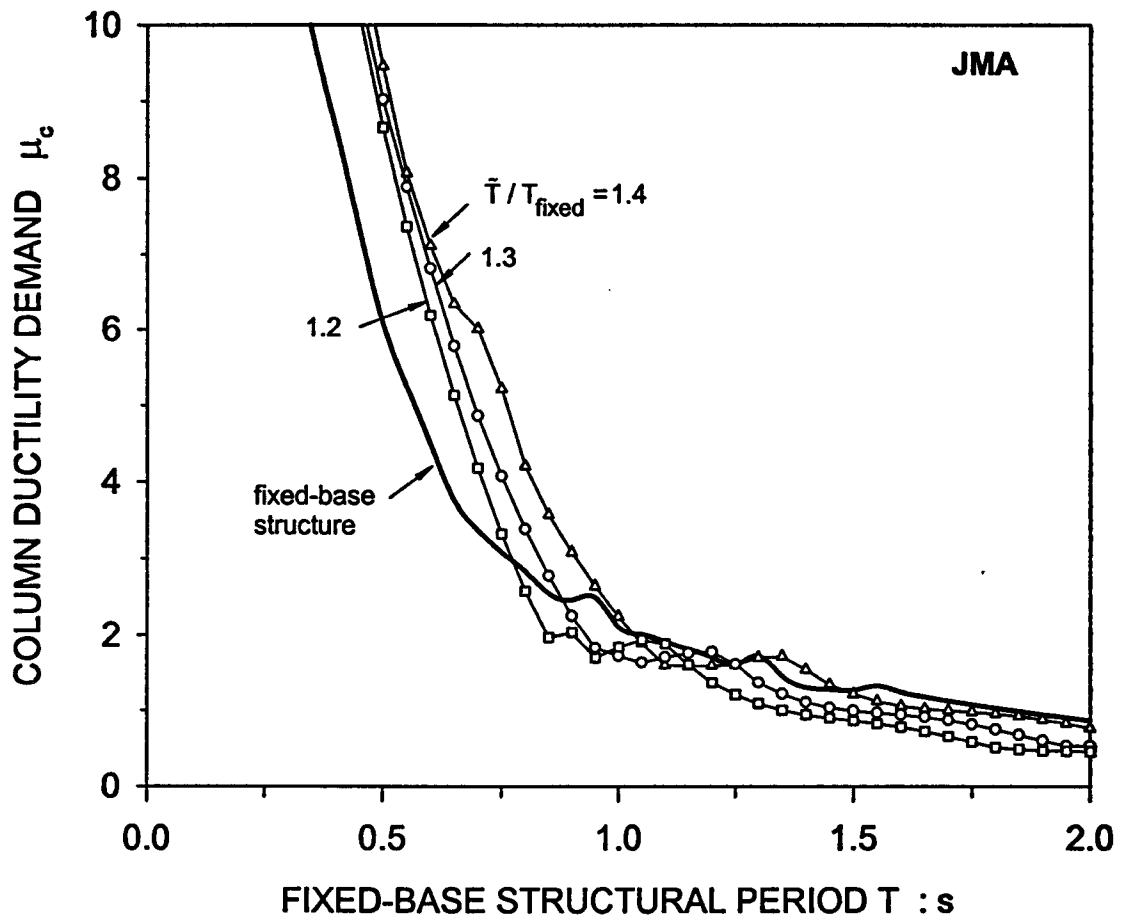


FIGURE H.2: Effect of SSI on ductility demand of a bridge pier subjected to the fault-normal component of JMA record; $C_y = 0.5$, $\alpha = 0.05$, $\zeta = 5\%$

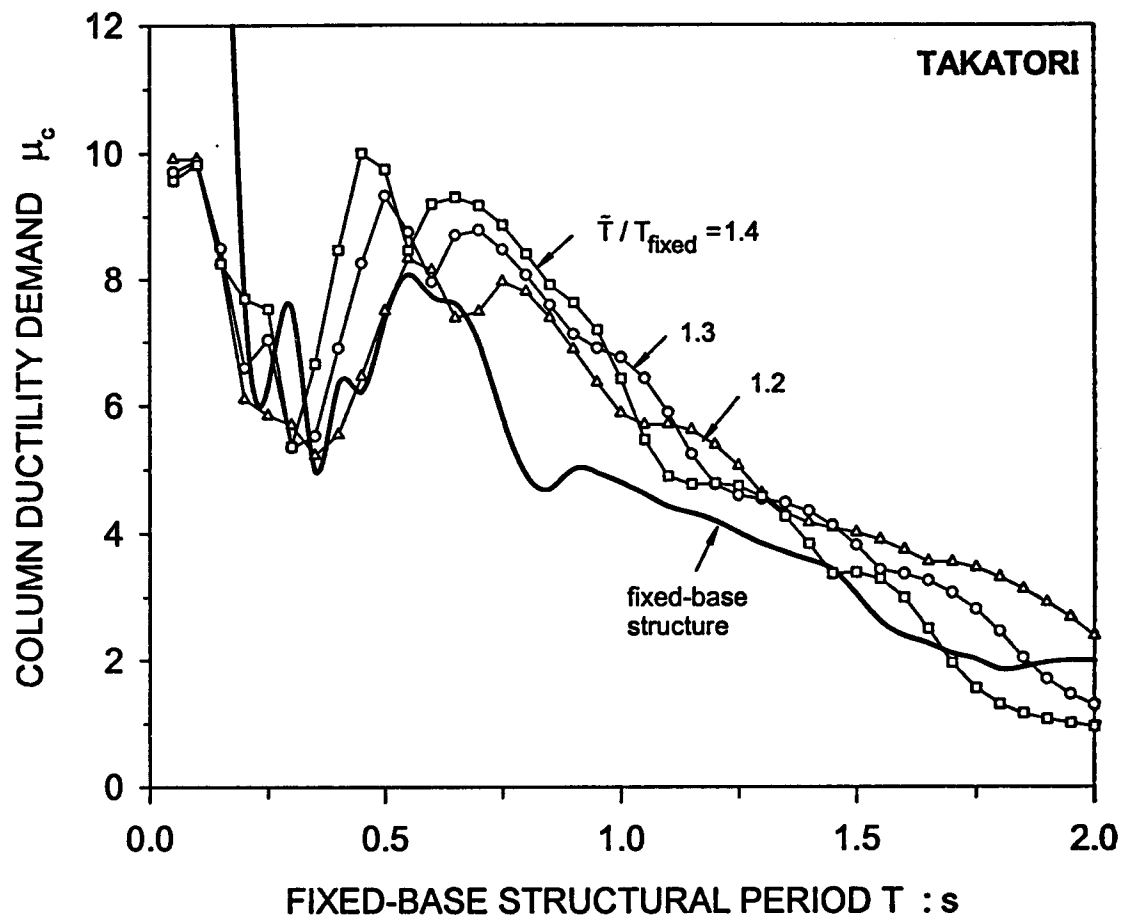


FIGURE H.3: Effect of SSI on ductility demand of a bridge pier subjected to the fault-normal component of Takatori record; $C_y = 0.5$, $\alpha = 0.05$, $\zeta = 5\%$

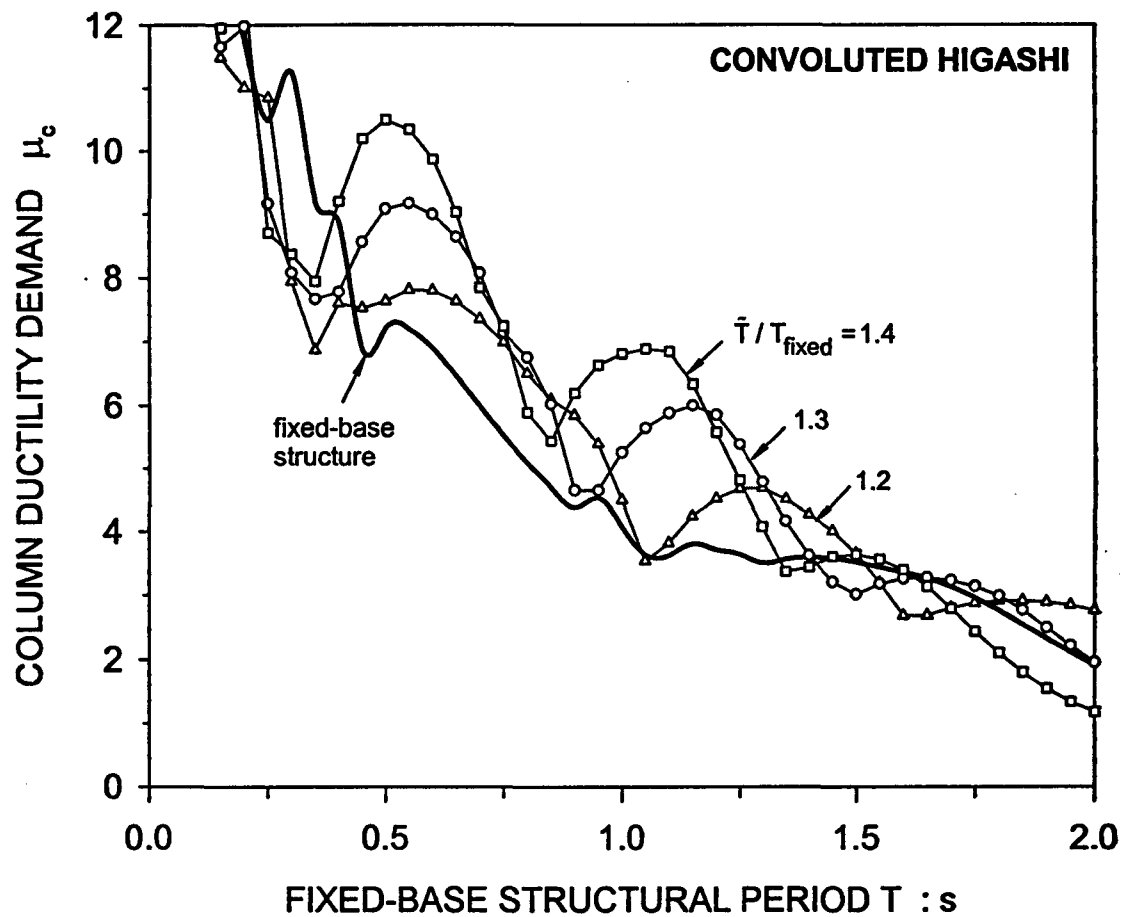


FIGURE H.4: Effect of SSI on ductility demand of a bridge pier subjected to the fault-normal component of the convoluted Higashi record; $C_y = 0.5$, $\alpha = 0.05$, $\zeta = 5\%$

APPENDIX I: NOTATION

SYMBOL	EXPLANATION
A	radial inhomogeneity factor
A_p	pile's cross-sectional area
$A_p, \gamma_p, \beta_p, \eta_p$	dimensionless parameters of the Bouc-Wen model
B	strain-displacement matrix
b^*	mass proportionality coefficient
B_0, B_s	components of the strain-displacement matrix B
C	total damping matrix
C_∞	damping matrix accounting for the damping of the infinite medium at the boundaries
C_{VB}^e	element damping matrix at vertical boundary
C_{HB}^e	element damping matrix at horizontal boundary
C_f	foundation damping
C_p	compression wave velocity of the soil, pier damping
C_s	shear wave velocity of the soil
C_y	normalized pier yielding strength
c	dimensionless factor expressing the relative flexibility of foundation and superstructure
c^*	foundation flexibility coefficient
c_z	distributed vertical dashpots along the pile
c_r	distributed dashpots along the pile accounting for radiation damping
c_m	distributed dashpots along the pile accounting for material damping
D	constitutive (stress-strain) matrix
D	pier diameter
D_b	bracketed duration
D_p	destructiveness potential
d	pile diameter, pier diameter

D_C	constitutive damping matrix
D_K	constitutive stiffness matrix
E	complex valued element's modulus of elasticity, pier modulus of elasticity
E_s	modulus of elasticity for homogeneous soil profile
E_{sL}	modulus of elasticity at a depth equal to the pile length L for a linear soil profile
E_{sd}	modulus of elasticity at a depth equal to the pile diameter d for a linear soil profile
$E_{s\infty}$	modulus of elasticity at far field ($r = \infty$) for a soil profile that is radially inhomogeneous
$E_{sL,\infty}$	modulus of elasticity at far field ($r = \infty$) and at a depth equal to the pile length (L) for a soil profile that is radially and vertically inhomogeneous
$E_{sd,\infty}$	modulus of elasticity at far field ($r = \infty$) and at a depth equal to the pile diameter (d) for a soil profile that is radially and vertically inhomogeneous
E_p	pile's modulus of elasticity
$\bar{T}$	equivalent (SSI) period
E_s	soil's modulus of elasticity
F_p	pier's resisting force
f_s	frictional capacity of the pile-soil interface
$F_{y,p}, \delta_{y,p}$	pier's yielding strength and displacement
FN	fault normal component of ground motion
FP	fault parallel component of ground motion
G	shear modulus of elasticity
g	acceleration of gravity
h	thickness of the soil layer
H	mesh height, pier height
I_A	Arias intensity
I_H	Housner Intensity

I, I_p	the cross-sectional second moment of inertia of the pile; of the pier
I_p, PI	soil's plasticity index
I_u, I_f	kinematic displacement factors
I_{gross}	gross value of second moment of inertia of pier
I_{deck}	mass moment of inertia of deck
I_{cap}	mass moment of inertia of pile-cap
$[I]$	identity matrix.
J	Jacobian matrix
K	total stiffness matrix
K_b	complex-valued impedance representing the vertical soil reaction at the pile-tip
K_{eq}	equivalent stiffness
K_f	foundation stiffness
$K_{xx}, K_{\pi\pi}, K_{x\pi}$	static foundation stiffness for swaying, rocking and cross-swaying rocking, respectively
$\mathcal{H}_{xx}, \mathcal{H}_{\pi\pi}, \mathcal{H}_{x\pi}$	swaying, rocking and cross-swaying-rocking impedances, respectively
K_p	pier elastic stiffness
K_{pier}, K_{col}	pier elastic stiffness
$K_{ss}, K_{sb}, K_{bs}, K_{bb}$	stiffness submatrices corresponding to the superstructure (s) and base (b)
k_z	vertical stiffness of distributed springs along the pile
K_∞	stiffness matrix accounting for the stiffness of the infinite medium at the boundaries
K_{HB}^e	element stiffness matrix at horizontal boundary
K_{VB}^e	element stiffness matrix at vertical boundary
$[\mathcal{H}_{ij}]$	pile-group dynamic impedances
$k(z, \omega)$	complex soil impedance encompassing the stiffness, inertia, radiation and hysteretic action of the soil

L	length of the pile, pier height
L_a	critical (active) pile length
M	effective bridge mass
$\mathbf{M}$	total mass matrix
M_0	moment
M_1	growth shape function corresponding to node #1 in computational space for the infinite Quadrilateral element
M_f	foundation mass
M_{JMA}	earthquake magnitude according to the Japanese Meteorological Agency
M_p	pier mass
M_s	earthquake magnitude
M_{deck}	deck mass
M_{cap}	pile-cap mass
M_{pier}	pier mass
$[m_{ss}], [m_{bb}]$	mass-submatrices of the superstructure and the foundation, respectively
N_{L1}	shape function corresponding to node #1 in computational space for the linear triangular element
N_{linear}	interpolation matrix containing linear shape functions for a linear triangular element
$\bar{n}$	normalized vector, normal to the boundary
$\bar{r}$	normalized wave direction vector
N_{Q-1D}	interpolation matrix containing quadratic shape functions for a 1-D element
N_{QQ}	interpolation matrix containing quadratic shape functions for a quadratic quadrilateral element
N_{QQ1}	shape function corresponding to node #1 in computational space for the quadratic quadrilateral element
N_{QT}	displacement interpolation matrix containing quadratic shape functions for the quadratic triangular element

N_{QT1}	shape function corresponding to node #1 in computational space for the quadratic triangular element
$[N]$	transformation matrix ($= \bar{n} \cdot (\bar{n})^T$)
P	total load vector
P_{r1}	radial load at node #1 (in computational space)
P_{z1}	vertical load at node #1 (in computational space)
PGA	Peak Ground Acceleration
PGV	Peak Ground Velocity
PSA	pseudo spectral acceleration
p	horizontal and/or vertical force per pile length along pile shaft
$q(z)$	distributed forces along pile
Q_x	horizontal load
R	length of the (r, z) vector, force reduction factor
R	force reduction factor
r	radial coordinate
s	axis to axis distance between the piles
S	mesh width
S	strain-compatibility matrix
S_1, S_2, S_3	components of the strain-compatibility matrix S
SA	spectral acceleration
SD	spectral displacement
SV	spectral velocity
S_u	undrained shear strength of the soil
t	time variable
T	load vector
T	return period of oscillator
T_a	period of peak spectral acceleration
T_g	period of peak spectral velocity
T_{fixed}	natural period of a fixed base bridge pier
T_{SSI}	natural period of a flexible base bridge pier
u	element's displacement matrix

$\mathbf{U}$	element's nodal displacement matrix
$\mathbf{U}$	global nodal displacement vector
u	radial node displacement
u_{deck}	displacement on the top of the pier (total)
$u_{\text{free-field}}$	displacement at the free-field
$\ddot{u}_g$	ground acceleration
U_f	displacement of the foundation (total)
U_g	ground displacement
U_i	radial displacement of node I
U_i	maximum absolute relative inelastic displacement of pier
U_k, ϕ_k	translation and rotation of the kinematic response of the foundation
U_p	displacement on the top of the pier (total)
$\bar{\mathbf{U}}_n$	displacement matrix containing the amplitudes of nodal displacements
$\bar{u}_n, \bar{v}_n, \bar{w}_n$	amplitudes for radial, tangential and vertical displacements
$\bar{\mathbf{U}}_n, \bar{\mathbf{V}}_n, \bar{\mathbf{W}}_n$	amplitudes for radial, tangential and vertical nodal displacements
v	angular node displacement
v_0	intensity of zero crossings in the acceleration record
V_s	shear wave velocity
$V_{s, \text{max}}$	low strain shear wave velocity
V_{sL}	soil shear wave velocity evaluated using modulus of elasticity E_{sL}
V_{sd}	soil shear wave velocity evaluated using modulus of elasticity E_{sd}
$V_{sL, \infty}$	soil shear wave velocity evaluated using modulus of elasticity $E_{sL, \infty}$
V_p	compression wave velocity
w	vertical node displacement
$W(z)$	axial displacement of the pile
W_{11}	axial displacement of the "source" pile
W_{21}	axial displacement of the "receiver" pile
W_i	vertical displacement of node i
Y	lateral deflection of pile

Y_{11}	lateral deflection of “source” pile
Y_{12}	lateral deflection of “receiver” pile
z	vertical coordinate
Z_p	Bouc-Wen parameter
α	reduction factor, interaction factor, pier’s post yielding stiffness coefficient
α_0	dimensionless frequency
$\alpha_{0,d}$	dimensionless frequency normalized with shear wave velocity $V_{s,d}$
$\alpha_{0,L}$	dimensionless frequency normalized with shear wave velocity $V_{s,L}$
$\alpha_{0,\infty}$	dimensionless frequency normalized with shear wave velocity $V_{s,\infty}$
$\alpha_{0,d\infty}$	dimensionless frequency normalized with shear wave velocity $V_{s,d\infty}$
$\alpha_{0,L\infty}$	dimensionless frequency normalized with shear wave velocity $V_{s,L\infty}$
α_p	pier’s post yielding stiffness coefficient
$\alpha_{uP}, \alpha_{\phi M}, \alpha_{uM}, \alpha_{\phi P}$	swaying; rocking interaction factor and cross-swaying-rocking interaction factors;
β, β_{ij}	material damping
$\beta_{xx}, \beta_{rr}, \beta_{xr}$	swaying; rocking and cross-swaying-rocking damping ratio of foundation
Γ	inverse of Jacobian matrix
Γ	notation to indicate boundary surface
γ_{ij}	shear strain component
δ	normalized Winkler modulus
δ_{max}	maximum normalized Winkler modulus
ε	element’s strain matrix
ε_{ij}	normal strain component
ζ	coefficient that accounts for the effect of diffraction, damping ratio
ζ_{SSI}	damping ratio of a bridge pier with flexible base

$\zeta_{uP}, \zeta_{\phi M}, \zeta_{uM}, \zeta_{\phi P}$	swaying; rocking and cross-swaying-rocking diffraction functions
$\zeta_{xx}, \zeta_{\pi\pi}, \zeta_{xr}$	swaying; rocking and cross-swaying-rocking damping ratio of foundation
$\zeta, \zeta_{\text{pier}}$	pier damping ratio
$\Theta_{11}(z)$	flexural rotation of “source” pile
$\Theta_{21}(z)$	flexural rotation of “receiver” pile
θ	angular coordinate, “aperture” angle between the direction of loading and the line connecting the pile centers on the horizontal plane
Λ	loading intensity factor
λ	1 st Lamé’s constant, function of the plasticity index, complex wave number describing the attenuation of pile displacement with depth.
μ	2 nd Lamé’s constant, displacement ductility ratio
μ_d	ductility demand
μ_{sys}	system (total) ductility
$\mu, \mu_{\text{demand}}, \mu^{(\text{pier})}_{\text{demand}}$	pier ductility demand
$\mu^{(\text{system})}_{\text{demand}}$	system ductility demand
μ_{capacity}	ductility capacity
ν	Poisson’s ratio
ξ	damping ratio
ξ, η	computational space coordinates
ρ	mass density
σ	element’s stress matrix
σ_{ij}	normal stress component
τ_{ij}	shear stress component
$\phi_{ij}, \phi_M, \phi_D$	dimensionless factors
χ_{fixed}	modal participation factor of 1 st mode for a fixed-base bridge-pier
χ_{SSI}	modal participation factor of 1 st mode for a flexible-base bridge-pier

ψ	free-field displacement attenuation function, shape function of fixed-base pier
ψ_{SSI}	shape function of flexible-base pier
Ω	pile's stiffness related parameter $\Omega = \frac{K_b}{E_p A_p \lambda}$
ω	frequency of excitation

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