On the Role of Information and Regimes in Asset Pricing

by
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A dissertation submitted to the Graduate Faculty in Business in partial fulfillment of the requirements for the degree of Doctor of Philosophy, The City University of New York

2010

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This manuscript has been read and accepted for the Graduate Faculty in Business in satisfaction of the dissertation requirement for the degree of Doctor of Philosophy.

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Abstract

On the Role of Information and Regimes in Asset Pricing

by

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Adviser: **Prof. Christos Giannikos**

The aim of this work is to shed further light on the role of asset pricing in macroeconomics. Starting from the long run risk hypothesis and agents with a recursive utility, I study the role of information processing on the state of the economy and its relation with asset pricing figures in an pure exchange setting. A richer setup is also studied, introducing the production side in the economy and modelling the business cycles with a regime switching process.

In particular, the first part of the thesis is concerned with the role of information in the long run risk model. Here I document an unpleasant feature of the stylized model economy of long-run risk type now popular in asset pricing. In the second part of the thesis I study the asset pricing implications of the model introduced above, focusing on the relation between information on the state of the economy and the equity risk premium. The last part of the thesis provides an analysis of the asset pricing implications of the studied model in a real business cycle setting.

A mio Papa'

Acknowledgements

All the work presented in this dissertation is based on two independent research projects both started in 2006. The first one is in collaboration with my adviser, Prof. Christos Giannikos, and is concerned with Real Business Cycle models and asset pricing. The second one, with Frode Brevik, is related with the study of information processing when agents have recursive preferences.

I am indebted to both of them for our academic collaboration.

The first two chapters were the starting point for a series of three papers in collaboration with Frode Brevik. One of these papers titled "Information Quality and Stock Returns Revisited", was accepted by the *Journal of Financial and Quantitative Analysis* for publication.

Contents

List of Figures

List of Tables

1

Introduction

In the last twenty years, the financial literature produced a consistent amount of empirical evidence challenging the basic assumption underlying consumption based asset pricing models. In fact, the role of asset prices in macroeconomics has gained huge popularity since the seminal contribution of Mehra and Prescott (1985). Their well celebrated "equity premium puzzle" implies an implausibly high level of risk aversion, for an agent with power utility preferences, in order to match the US postwar equity premium. The most successful strand of literature that tries to reconcile economic theory and financial figures is based on the "long run risk" models. These models feature a slow moving predictable component in the consumption path, coupled with a recursive structure of the preferences first introduced by Kreps and Porteus (1978). The separation of the preference parameters governing the elasticity of intertemporal substitution and relative risk aversion permitted by this preferences specification, referred to as the Epstein-Zin utility function after the contribution by Epstein and Zin (1989), has proven very fruitful in Asset Pricing. In fact, recent successful asset pricing models rely on calibrations of the utility function where the representative agent has both a moderate level of risk aversion and a high elasticity of intertemporal substitution. In particular, in a seminal paper Bansal and Yaron (2004) forcefully demonstrate that when the trend growth rate of the endowment good follows a trend stationary process with low conditional volatility but high unconditional volatility (the long run risk component), such parameterizations allow to match both a low risk free rate and a plausible risk premium for equity.

The aim of this thesis is to shed further light on the role of Asset Pricing in Macroeconomics, when agents have a recursive utility specifications, starting from the long run risk hypothesis. In particular I study the role of information processing on the state of the economy, and its relation with Asset Pricing figures.

The first part of the thesis is concerned with the role of information in the long run risk model. In the first chapter I document an unpleasant feature of the stylized model economy of long-run risk type now widespread in Asset Pricing: Investors with preference parameters commonly described as indicating a "preference for early resolution of uncertainty" (see Kocherlakota (1990)) achieve higher utility levels if they can commit to ignore information on the state of the business cycle. In particular, by calibrating the model with parameter choices similar to those used to explain asset prices, I show how a consumer can achieve utility gains equivalent to a more than 40% increase in life-time consumption by committing to ignore information on the trend growth rate of the endowment good. Pushing this analysis further, I investigate the lack of firm knowledge about the prevailing state of the economy in a (slightly) different setup: I study a variation of Lucas' (1978) exchange economy where the economy switches between booms and recessions at random intervals and agents are embedded with Epstein-Zin preferences. The two main results obtained by calibrating the model to match the properties of the postwar US data, are apparently contradictory: (i) consumers experience a (modest) utility gain when they are provided with information beyond what is already incorporated in their endowment process; (ii) there are large utility gains from either committing to a slow learning rule or assuming a simplified model for the economy. These results can be reconciled by noting the different mechanism behind each of the two: in the second case, the lack of knowledge about the state of the economy removes some of the positive covariance between realized consumption and expected future consumption. For preferences such as those used in the Asset Pricing literature this leads to a utility gain. In the first case, the information is orthogonal to the current consumption and adding more of it does not change the covariance between realized consumption and expected utility. Here the expected result obtains: consumers with a preference for early resolution of uncertainty will also benefit from more information.

The second part of the thesis moves towards the Asset Pricing implications of the latter model. Starting from an important contribution by Veronesi (2000), I investigate the relation between information on the state of the economy and equity risk premium. Veronesi (2000) studies the link between information quality and risk premia in an exchange economy where the trend growth rate follows a hidden Markov process and agents have a power utility function. He documents two surprising relationships between the information on the state of the economy and stock returns: (i) if the representative investor has a relative risk aversion larger than unity, then the risk premium increases in the amount of information contained in the signals; (ii) unless the signals contain complete information on the state of the economy, the equity premium is bounded above, independently of investor relative risk aversion. By revisiting Veronesi's model, I show numerically, that both results are reversed for a range of plausible parameters in an economy calibrated to U.S. data. In particular, I use the setup where investors have Epstein-Zin preferences and the economy randomly switches between booms and recessions. I am able to establish two key results: first, investors with high elasticity of intertemporal substitution require lower excess returns for holding stocks if they have better information on the state of the economy. Second, this also holds for investors with moderate elasticity of intertemporal substitution if they are sufficiently risk averse. Also, in such a model there is no global maximum for the required equity premium as a function of investor risk aversion.

The last part of the thesis provides an analysis of the Asset Pricing implications of the model above in a Real Business Cycle setting.

The starting point is that besides the equity premium puzzle, the main implication of Mehra and Prescott (1985) contribution is that, in a simple endowment economy where agents have a power utility function, by arbitrarily increasing the risk aversion parameter, one can make an agent sufficiently risk averse to offset the observed low volatility of consumption and thus obtain a high risk premium. On the contrary, in a standard RBC model, an agent can smooth her consumption path by substituting between labor and leisure in response to productivity shocks. So, by just increasing her coefficient of risk aversion, we are not able solve the equity premium puzzle.

This is one flaw of standard RBC models that are highly capable of explaining the main features of business cycles, but need the introduction of restrictive assumptions to explain observed financial figures. In fact, the existing RBC literature circumvents this problem by introducing rigidities on the production side of the model such as limited intersectorial mobility of factors or capital adjustment costs, coupled with habit formation preferences.

I try to dispense of all such rigidities. Instead, I provide an extension to the prior RBC literature similar to the model studied in the previous chapters. In particular, I model an economy that switches between booms and busts where technological shocks follow a hidden two state Markov chain, in conjunction with Epstein-Zin preferences for consumers.

I show that a reasonable parametrization of this “rigidity-free” model conveys plausible financial figures that are in line with empirical observations over the U.S. postwar economy. The last part of this chapter also provides a detailed theoretical and numerical analysis on the model's predictions. This allows me to clarify the role played in such a model by the risk aversion and the elasticity of intertemporal substitution in determining asset prices and thus the equity premium.

2

The role of information in long-run risk models

Abstract

I document an unpleasant feature of Epstein-Zin preferences in a stylized model economy of the long-run risk type now widespread in Asset Pricing: Agents with preference parameters commonly described as indicating a "preference for early resolution of uncertainty" achieve higher utility levels if they can commit to ignoring information on the state of the business cycle. For parameter choices similar to those used to explain asset prices, an agent can achieve utility gains equivalent to a more than 40 % increase in life-time consumption by committing to ignore information on the trend growth rate of the endowment good. I show that opting for such a coarser information set can be implemented and supported as an equilibrium strategy.

The separation of the preference parameters governing the elasticity of intertemporal substitution and relative risk aversion permitted by the Epstein-Zin utility function (Epstein and Zin, 1989; Weil, 1989) has proven very fruitful in the asset pricing literature. Recent successful asset pricing models rely on calibrations of the utility function where the representative agent has both a high level of risk aversion and a high elasticity of intertemporal substitution. To name a few, Campbell and Vuolteenaho (2004) use such a calibration to explain stock market anomalies, Piazzesi and Schneider (2007) use such a calibration to explain the average shape of the yield curve, and Lettau et al. (2008) use such a calibration to explain the run up in stock prices during the late nineties. In a seminal paper Bansal and Yaron (2004) forcefully demonstrate that in an exchange economy with a long run risk component in consumption, that is when the growth rate of the endowment good follows a trend stationary process with low conditional volatility but high unconditional volatility, such utility specifications produce both a low risk free rate and a plausible risk premium for equity. Their paper has spawned a large body of research which I will refer to as the long-run risk literature.[1]

In this chapter I go through the following thought experiment: I place an agent with Epstein-Zin preferences in a stylized endowment economy of the type analyzed in the long-run risk literature and give her the option not to incorporate any type of news when forming posterior beliefs about the current state of the trend consumption growth rate. If she chooses to do so, her information set includes all the hyper-parameters of the economy and her current consumption level, but does not include any information that would help her determine the current level of the stochastic trend growth rate of

[1] With no pretension of a complete list: Kaltenbrunner and Lochstoer (2008) and Croce (2008) show how a predictable component to consumption growth rates can arise in production economies. Colacito and Croce (2008) and Bansal and Shaliastovich (2009) study international linkages in open economies when each country has a small predictable component in its consumption growth rate trend. Hasseltoft (2008), Doh (2008), and Wu (2008) look at interest rate implications of long-run risk models. Constantinides and Ghosh (2008) and Rangvid et al. (2009) provides explicit estimation of long run risk models. Bansal et al. (2007) and Cederburg and Hore (2008) analyzes the extent to which a long-run risk model can explain the predictability in the cross-secional data on security returns.

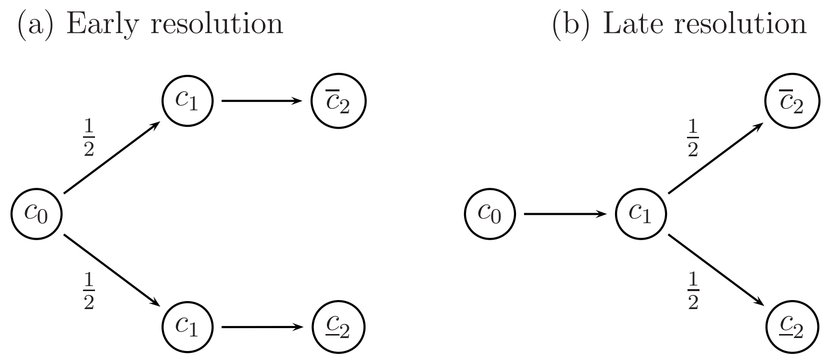
the endowment good. I assume that her preference parameters are the same as those of the representative agent of the economy. Knowing that her information is coarser than that of other agents she does not trade actively in a way that can be exploited by more informed agents. She keeps all her assets in the market portfolio and consumes the same as the representative agent. That is, she holds only claims to the Lucas tree and consumes its fruit every period. The consumption profile of this agent will mirror that of the representative agent in the economy. The only way the coarser information set she uses influences her utility level is through the timing of information about future consumption.

I find that, for model parameters similar to those used in the asset pricing literature, the continuation value for the coarser information set is much higher than for an agent whose information set also includes the current trend growth level of consumption. For a calibration that draws on Bansal and Yaron (2004), I find utility gains from committing to using a coarser information set equivalent to a 40 % increase in lifetime consumption.

To us, the numbers I find are not only surprising in their magnitude but also in their direction. The parameterization I look at are such that the agent would be classified as having *preference for early resolution of uncertainty* according to a common taxonomy. (See e.g. Kocherlakota, 1990 or Skiadas, 1998.) The presumed plausibility of utility functions generating a preference for early resolution of uncertainty seems to lend credence to Epstein-Zin preferences.[1] The concept is illustrated in Figure 2.1 which is taken from Kocherlakota (1990). An agent with a preference for early resolution of

[1] It is important to keep in mind that we are discussing early resolution of uncertainty about consumption itself. The seminal article by Kreps and Porteus (1978) motivates the preference for early resolution over lotteries by stating that it is natural to prefer to know your income earlier so that you can better budget it for consumption purposes. In the endowment economy I am considering, equilibrium consumption of the representative agent is always going to be equal to the endowment stream, so early resolution of uncertainty cannot provide any means for better budgeting since consumption is unaffected by it. Any preference for resolution of uncertainty in such economies must come directly from the way the distribution of possible consumption paths is aggregated to a certainty equivalent.

Figure 2.1
Preferences for resolution of uncertainty



$$(a) \succ (b) \iff EIS > 1/CRRA$$

$$(a) \prec (b) \iff EIS < 1/CRRA$$

uncertainty would prefer tree (a) to tree (b): the two trees offer the same distribution of outcomes at each point in time, but in tree (a) time 2 consumption is revealed one period earlier. The label "preference for early resolution of uncertainty" seems to suggest that an agent would like to process any information on the current state of the economy because it reduces uncertainty about her future consumption. For the parameters used in the long run risk asset pricing literature, I show that always processing information is optimal in the sense of being a Nash strategy. However, if there is a persistent component in the consumption growth rate trend, consumers can achieve an even higher utility level by committing to not processing information at any point in the future.

One way to understand this result is by noting that agents with relatively high risk aversion also dislike a positive correlation between current consumption growth and expected consumption growth (Piazzesi and Schneider, 2007). One the one hand, the consumer faces more consumption uncertainty when she relies on the coarser information set. On the other hand, relying on the coarser information set also shut down any correlation between current and expected future consumption growth rates. In the simple economy I study, the second effect dominates, so ignorant agents achieve, on average, a higher utility level.

The chapter is structured as follows: section I. introduces the stylized long run risk model I use in this analysis. Section II. analyzes the process of information acquisition by an agent facing the possibility not to incorporate any type of news about the current state of the trend consumption growth rate. Section III. shows that learning the growth rate of the economy is a Nash strategy, but that ignorance can be supported as an equilibrium strategy when it yields a higher utility level. Section IV. quantifies the utility gains from ignorance using standard calibrations from the long run risk asset pricing literature. Section V. concludes.

I. A stylized economy with long-run risk

The laboratory is a simple endowment economy where the growth rate of the log of the representative agent's consumption is the sum of an AR(1) component and a white noise shock. The setup is based on Hansen et al. (2008) and the exposition closely follow theirs.

A. Endowment process

Let ϵ_t and w_t be two series of i.i.d. standard normal innovation terms. Log consumption follows a random walk plus a time varying drift. The first difference of the drift is given by

$$c_{t+1} - c_t = \mu_c + x_{t+1} + \sigma_c \epsilon_{t+1}. \tag{2.1}$$

That is, the log consumption growth rate trend at time t is a combination of a constant (μ_c) and a time varying component x_t. x follows an AR(1) process given by:

$$x_{t+1} = \kappa x_t + \sigma_x w_{t+1}. \tag{2.2}$$

B. Preferences

All agents in the economy are ex-ante equal with preferences over consumption paths given by the recursion:

$$V_t = \left[(1-\beta)\left(C_t\right)^{1-\rho} + \beta \mathcal{R}_t\left(V_{t+1}\right)^{1-\rho}\right]^{\frac{1}{1-\rho}} \tag{2.3}$$

where ρ is equal to the reciprocal of the Elasticity of Intertemporal Substitution (EIS). The risk adjustment $\mathcal{R}_t$ is also of the constant elasticity of substitution type:

$$\mathcal{R}_t\left(V_{t+1}\right) = \mathrm{E}_t\left[\left(V_{t+1}\right)^{1-\theta}\right]^{\frac{1}{1-\theta}} \tag{2.4}$$

where θ is the Coefficient of Relative Risk Aversion (CRRA). Given the process assumptions above, V_t is homogeneous of degree 1 in the level of consumption. Let v_t denote the logarithm of the continuation value normalized by the consumption level. I can rewrite the recursion above as

$$v_t = \frac{1}{1-\rho}\log\left[(1-\beta) + \beta\exp[(1-\rho)\mathcal{Q}_t(v_{t+1} + c_{t+1} - c_t)]\right], \tag{2.5}$$

where the operator $\mathcal{Q}_t$ is given by

$$\mathcal{Q}_t(v_{t+1}) = \frac{1}{1-\theta}\log E_t\left[\exp\left((1-\theta)v_{t+1}\right)\right]$$

I distinguish two main information sets for the consumer: Under the coarser information set $\mathcal{F}_t^I$, the consumer is endowed with information about all the model hyper parameters and the current consumption level. The alternative information set $\mathcal{F}_t^N$ is a refinement of $\mathcal{F}_t^I$ where the consumer also knows the current level of the consumption growth trend x_t.[1] For analytical tractability, I focus on the case $\rho = 1$ as in Tallarini (2000). This assumption, in conjunction with the Gaussian shock processes I assume, allows for simple closed form solutions for the value function under the two information sets. The $\rho = 1$ limit of recursion (2.5) is

$$v_t = \frac{\beta}{1-\theta} \log E\left(\exp[(1-\theta)(v_{t+1} + c_{t+1} - c_t)]\right). \tag{2.6}$$

II. Optimal information acquisition

A. Alternative value functions

1. Updating every period (Nash)

I denote the log continuation value when the consumer observes x_t and expects to always learn x_t by v_t^N. As we will see below, always choosing to acquire information is a Nash equilibrium in a game that the agent plays against her future selves. In this case the continuation value from equation (2.6) is given by

$$v_t^N = \mu_v + U_v x_t, \tag{2.7}$$

[1]The trend growth rate follows a Markov process, so the most recent trend level is a sufficient statistic for the predictive information of the sigma algebra formed by past levels of the consumption growth rate trend.

where

$$\mu_v = \frac{\beta}{1-\beta}\left(\mu_c + \frac{1-\theta}{2}\left(\sigma_c^2 + \frac{1}{(1-\kappa\beta)^2}\sigma_x^2\right)\right) \tag{2.8}$$
$$U_v = \frac{\kappa\beta}{1-\kappa\beta}.$$

The term μ_v is the unconditional expectation of the scaled log continuation value. It is given by the discounted present value of the long run consumption growth μ_c and a correction for the variance of the consumption growth rate that depends on the coefficient of relative risk aversion parameter θ.[1] The coefficient U_v gives the discounted present value of the temporary increase in log-consumption growth induced by a unit change in the mean reverting trend component x_t.

1. Never updating

I now turn to the agent's value function if she has no information on the current level of x and she can commit to never learning anything about x in the future. In the next section, I show how this can be supported as an equilibrium strategy. When no information is revealed about x, the only variable in the agent's information set which changes over time is the current consumption level C_t. It follows that v_t is constant. I denote its value by v^I, where the superscript I reflects the relative ignorance of the consumer under this information set. From equation (2.6), it follows that v^I satisfies

$$v^I = \frac{\beta}{(1-\beta)(1-\theta)} \log E\left[\exp[(1-\theta)(c_{t+1} - c_t]\right] \tag{2.9}$$

Unconditionally, $\Delta c_{t+1} \sim N\left(\mu_c, \sigma_c^2 + \sigma_x^2/(1-\kappa^2)\right)$. Solving for the expectation on

[1] As we will see in Section IV., the long run risk asset pricing literature assumes that $\theta > 1$, so that the correction is negative.

the right hand side of equation (2.9) gives

$$v^I = \frac{\beta}{1-\beta}\left[\mu_c + \frac{1-\theta}{2}\left(\sigma_c^2 + \frac{1}{1-\kappa^2}\sigma_x^2\right)\right]. \tag{2.10}$$

The first term in equation (2.10) is the discounted present sum of future mean growth rates. The second term in the squared parenthesis of equation (2.10) is a risk adjustment which is proportional to unconditional variance of consumption growth rates. With log utility ($\theta = 1$) this term is zero. When the coefficient of risk aversion is greater than 1 the risk correction lowers the continuation value.

1. Interpretation

Under the coarser information set, the unconditional and conditional variance of consumption growth rates are equal. For $\kappa \in (0, 1)$, this means that the consumer faces a higher conditional consumption volatility under the coarser information set. By itself this will increase the perceived riskiness of the consumption path and gives the consumer an incentive to choose the finer information set.

Under the finer information set the conditional variance of trend innovations enters the consumer's continuation value with the scaling factor $1/(1-\beta\kappa)^2$. This reflects that any shock w_{t+1} to the trend growth rate is sticky. A shock w_{t+1} will increase consumption growth at $t+1+n$ by $\kappa^n w_{t+1}$. Relative to an increase in time $t+1$ consumption growth, consumption growth at $t+1+n$ is valued at β^n. The factor $\sum_{n=0}^{\infty}\beta^n\kappa^n = 1/(1-\beta\kappa)$ scales the effect of shocks to trend consumption shocks to take account for the stickiness of the trend.

Under the coarser information set, the agent effectively finds herself living in an economy where consumption growth is a random walk with a drift. In this economy consumption growth rates are more volatile, which is reflected in the scaling factor $1/(1-\kappa^2)$ on the conditional variance of the trend growth rate of consumption.

For $\theta > 1$, the consumer profits from the lower conditional variance, but suffer from the larger impact of innovations to x_{t+1} on her continuation value.

1. When does ignorance pay off?

It is only interesting for the consumer to opt for the coarser information set when her ex-ante continuation value is higher without information on the trend growth rate. This is the case when

$$v^I = \mathcal{Q}(v^I) \geq \mathcal{Q}(v^N) = \mu_v + \frac{1-\theta}{2} (U_v)^2 \frac{\sigma_x^2}{1-\kappa^2}$$

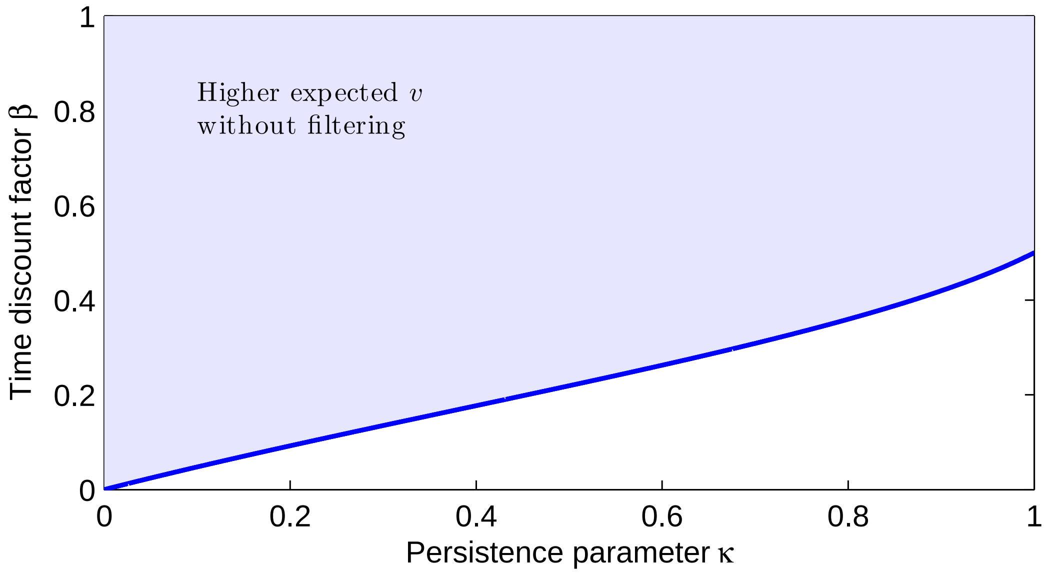
Since v^I is constant, it is equal to its certainty equivalent (i.e. $v^I = \mathrm{Q}(v^I)$.) v^N depends on the normally distributed trend growth rate, so its certainty equivalent $\mathrm{Q}(v^N)$ corrects for the influence of the trend growth rate through the last term on the right hand side of the above equation. Figure 2.2 provides an graphical analysis of the agent's options: the shaded area in the figure gives combinations of β and κ where v^I is higher than $\mathrm{Q}(v^N)$. That is, it gives parameter combinations for which an agent would prefer to commit to not learning the trend growth rate. For the high time discount factors used in the long-run risk literature (see Section IV.), the figure indicates that the agent would prefer to commit to ignorance regardless of the value of the persistence parameter κ.

III. Implementability

For ignorance to be an equilibrium strategy, I need to show that it is feasible and individually rational.

Figure 2.2
Parameter regions where information lowers utility when $\theta > 1$

The shaded area give parameter values for which there is an expected utility loss from always learning the trend growth rate compared to the case of ignorance. The CRRA is fixed to a value of 10.



A. Feasibility

Since we are in a complete markets endowment economy, one feasible investment strategy for the consumer is to invest all her wealth in a consumption claim (i.e. invest in a claim which pays dividends proportional to aggregate consumption) and every period consume its dividends. Because I am assuming that the preferences of the consumer are identical to those of the representative agent in the economy, this is the same consumption and portfolio choice she will achieve in equilibrium if she chooses to learn the trend growth rate every period.

B. Individual rationality

Always filtering is a Nash equilibrium

Assume that the agent knows x_{t-1} and that she expects she will always include the current value of x in her information set in future periods. Her certainty equivalent if she chooses to learn x_t also this period is

$$\begin{aligned} \mathcal{Q}(v_t \mid x_{t-1}) &= \mathcal{Q}[\mu_v + U_v x_t \mid x_{t-1}] \\ &= \mu_v + U_v \kappa x_{t-1} + \frac{1-\theta}{2}\frac{\beta^2\kappa^2}{(1-\beta\kappa)^2}\sigma_x^2. \end{aligned} \tag{2.11}$$

her continuation value if she does not learn x_t is given by

$$\begin{aligned} v_t &= \frac{\beta}{1-\theta}\log E\left(\exp[(1-\theta)(v_{t+1}+c_{t+1}-c_t] \mid x_{t-1}\right) \\ &= \mu_v + U_v \kappa x_{t-1} + \frac{1-\theta}{2}\frac{\beta\kappa^2}{(1-\kappa\beta)^2}\sigma_x^2. \end{aligned} \tag{2.12}$$

The two expressions differ only in a factor β in the last term. For $\theta > 1$, the right hand side of equation (2.12) is strictly lower than the certainty equivalent when the agent learns the state of the economy given by equation 2.11. This implies that the agent suffers a utility loss if she deviates by not updating her information on the trend growth rate in a single period.

Supporting ignorance strategy by threat of Nash

Consider the following strategy for the agent who has no information on the trend growth rate. As long as she has never learned the state of the economy in the past, she will never choose to learn it. She promises herself that, should she ever deviate from this strategy by learning the growth rate trend, she will always keep learning it in future periods. The threat is credible, since it amounts to playing a Nash strategy.

Her continuation value conditional on never updating in the future is given by v^I. Her expected scaled continuation value if she deviates is given by

$$\mathcal{Q}[v_t^N] = \mu_v + \frac{1-\theta}{2} U_v^2 \frac{\sigma_x^2}{1-\kappa^2}.$$

As long as the preference and process parameters belong to the shaded area in Figure 2.2, the agent will never choose to deviate, so never learning x_t is an equilibrium strategy.

IV. Numerical results

In this section I quantify the utility gains that an agent could achieve by committing to ignorance using two parameterizations taken from successful asset pricing models: One taken employed by Bansal and Yaron (2004) and one employed by Hansen (2007).

I measure the utility gains from ignorance by solving for the percentage change in consumption level an agent, who is forced to play the Nash strategy of always learning the trend growth rate of consumption, would require to make him equally happy ex-ante as an agent who is allowed to commit to ignorance.

For $\rho = 1$ the utility gain is computed using the closed form solutions provided in equations 2.7 and 2.10. In particular I subtract from v^I the certainty equivalent $\mathcal{Q}(v_t^N)$. For $\rho \neq 1$ I use a Gaussian quadrature with 300 nodes to approximate the law of motion for the trend growth rate and solve for v_t on the nodes of the quadrature. Here the certainty equivalent is computed by applying the operator $\mathcal{Q}$ to the values of v_t on the grid using the ergodic state probabilities implied by the discretized law of motion. (See Tauchen and Hussey (1991) for a discussion of this method.)

To match asset prices, all the proposed parametrizations share a high level of persistence (κ) for the consumption process. Such high levels of κ generates large utility gains from committing to ignore the state of the trend growth rate, because it magnifies

Table 2.1
Utility gains

Reported are the estimated utility gains for agent that is not processing the available information in the analyzed experimental economy. The first column is based on the calibration introduced by Bansal and Yaron (2004), where the parameters for the utility function are as following: the CRRA is set to 10, the EIS is set to 1.5, and the discount factor β is set to 0.998. For this case the gains are also computed with the closed solution case of $EIS = 1$. The last column is calculated with the set of parameters specified in Hansen (2007) which has a CRRA of 2 and an EIS set to 1. All reported gains are in percentage points.

	Bansal and Yaron (2004)	Hansen (2007)
Process parameters:		
μ_c	0.0015	0.0056
σ_c	0.0078	0.0054
σ_x	0.0003	0.0005
κ	0.9720	0.9800
Utility parameters:		
β	0.998	0.998
$1/\rho$	1.5	1.0
γ	10	2
Utility Gains:		
EIS$\neq$ 1	42.986	
EIS= 1	35.021	12.573

both the effect of the information of the trend growth rate on the conditional variance of consumption growth and the larger impact of innovations to x_{t+1} on her continuation value.

The quantitative results are reported in Table 2.1. The first column gives the gain from commiting to ignorance for the calibration used by Bansal and Yaron (2004) which has an elasticity of intertemporal substitution ($1/\rho$) of 1.5. Results are striking, with a 43% increase in lifetime consumption obtained by ignoring the trend growth rate. The last line of the table gives the same figure in the case the elasticity of intertemporal substitution is 1.. The utility gains from committing to ignore the information on the trend growth rate are still sizable at 35 %. The second column of Table 2.1 gives results for the parametrization used by Hansen (2007). He sets the risk aversion parameter to 2 and the elasticity of substitution to 1. By itself, reducing θ from 10 to 2, reduces the utility gains from committing to ignorance by 88 %, but the higher standard deviation of innovations to the trend growth rate still produces a utility gain equivalent to a 12 % increase in lifetime consumption from committing to ignorance.

V. Conclusion

In this chapter I have documented an unknown feature of the family of recursive preferences known as Epstein-Zin preferences that arise in long-run risk models used heavily in asset pricing. I have shown that an agent can achieve large utility gains from committing to ignoring information on the state of the trend growth rate.

The feature I document is surprising as far as the preference parameters used are known to produce a preference for early resolution of uncertainty. The model of the agent's decision problem as a repeated game against her future selves shows that such a commitment to ignorance can be implemented and supported as an equilibrium strategy.

3

The role of information in an endowment economy with recursive preferences

Abstract

This chapter investigates the relation between information on the state of the economy and equity risk premium. I use a setup where investors have Epstein-Zin preferences and the economy randomly switches between booms and recessions. I am able to establish two key results: First, investors with high elasticity of intertemporal substitution will require lower excess returns for holding stocks if they are provided with better information on the state of the economy. Second, I find this also holds for investors with moderate elasticity of intertemporal substitution if they are sufficiently risk averse.

I. Introduction

Publicly available signals might contain more or less information on the underlying state of the economy. High-quality signals will enable investors to make high-quality forecasts on the state of the economy, so it is natural to expect risk premia to vary with the amount of information signals contain.

In an important contribution, Veronesi (2000) studies the link between information quality and risk premia in an exchange economy where the trend growth rate follows a hidden Markov process. Assuming that the representative investor is a power utility maximizer, he establishes several surprising relations between the information on the state of the economy and stock returns, including the following: (i) If the representative investor has a relative risk aversion (RA) larger than unity, then the risk premium is *increasing* in the amount of information contained in the signals and (ii) unless the signals contain complete information on the state of the economy, the equity premium is *bounded above* independently of investor RA. The second result implies that, even with extremely risk-averse investors, the model would not be capable of replicating the empirically observed risk premium.

The first result has been reviewed in different setups by Li (2005) and Ai (2005). Li (2005) introduces a separate process for dividends and looks at two special cases. When the conditional mean growth rate of both consumption and dividends is the same, he obtains the same results as Veronesi: When only the dividend growth rate is time varying, better information lowers the equity premium. Ai (2005) looks at a production economy where equity is a claim to one unit of capital and there are no adjustment costs. He finds a negative relation between information quality and the required return to capital. Since the price of capital in terms of the consumption good is constant in his model, it is not clear how his results would translate to the exchange economy setting investigated by Veronesi.

In this article, I maintain both the pure exchange economy setting and the assumption that equity is a claim to aggregate consumption. I revisit the relation between information quality and stock returns by introducing Epstein-Zin preferences to Veronesi's model. Because Epstein-Zin preferences nest the power utility function as a special case, I am able to build on Veronesi's work and provide a direct comparison with his results. As I show numerically, both results are reversed for a range of plausible parameters in an economy calibrated to U.S. data. Although there is a large literature exploring the asset-pricing implications of alternative preference specifications, I am not aware of anyone who specifically addresses the topic of information quality.[1]

The main finding of this chapter is that, for a wide range of plausible parameterizations of the utility function in an economy calibrated to U.S. data, the conditional equity premium is decreasing in the quality of information available to investors. This range covers both a domain where this reversal has been predicted in the literature, which is when the EIS is greater than 1, as well as an important domain where the EIS is smaller than 1, provided that investors are sufficiently risk averse. Both results are important, since there is considerable controversy with respect to the appropriate parameter value for the EIS.[2]

[1]One particularly prominent line of research looks at the asset-pricing implications of including habits in the utility function (e.g., Constantinides (1990); Abel (1990); Gali (1994); Jermann (1998), Campbell and Cochrane (1999); and Boldrin et al. (2001)). Another line of research, started by Epstein and Zin (1989, 1991) and Weil (1989), looks at generalizations of the power utility function that relax the link between RA and the elasticity of intertemporal substitution (EIS). This class of utility functions, referred to as Epstein-Zin preferences, has been widely used in recent research in asset pricing (see Campbell and Viceira (2001); Campbell et al. (2003); Guvenen (2005); and many others), including some featuring the same kind of Bayesian learning I am assuming (Brandt et al. (2004), and Lettau et al. (2008)).

[2]Empirical estimates vary strongly with the assumptions made on the structure of the economy. One line of empirical research uses a representative agent setup and estimates the EIS parameter using aggregate consumption data. This approach typically leads to estimated EIS coefficients in the range of 0 to 1 (see, e.g., Hall (1988); Campbell and Mankiw (1989, 1991); Hahm (1998); Yogo (2004); and Zhang (2006)). Another line of research seeks to avoid potential biases, introduced by using aggregate data, by relying on microeconomic survey data. For stockholders, these studies find EIS parameters around or above 1 (see Beaudry and van Wincoop (1996); Vissing-Jørgensen (2002); Vissing-Jørgensen and Attanasio (2003); and Guvenen (2005)). Recent literature on asset pricing relies on the higher EIS estimates of the previously cited literature (e.g., Bansal and Yaron (2004), and Lettau et al. (2008),

An alternative to introducing hidden growth rate regimes is to model consumption growth rates as having a slow-moving predictable component. This is the approach taken by Bansal and Yaron (2004). Although they also use Epstein-Zin preferences, their results are different from ours. They report that if investors have a preference for early resolution of uncertainty, an EIS larger than 1 is required for the equity premium to be increasing with uncertainty. The reason for the discrepancy with my results lies in a different understanding of uncertainty. In Bansal and Yaron's model, the state of the economy is observable and uncertainty is understood as conditional consumption volatility. In my model uncertainty is lack of knowledge about the prevailing state of the economy.

Another finding is that in my setup there is no global maximum for the required equity premium as a function of investor RA. This is different from what obtains in a regime-switching economy, where investors are power utility maximizers (see proposition 3b in Veronesi (2000)). Unless the EIS parameter is very low, increasing investor RA in my model leads monotonically to a higher required equity premium, at least for the economies I considered numerically.

While I allow for a more general utility function than Veronesi (2000), I remain close to his model in terms of the dynamics of the model economy. I assume that the trend growth rate follows a two-state Markov switching process. The current trend growth rate is a hidden variable, so investors have to rely on the information embedded in dividend growth rates and other signals for pricing equities and bonds. [1]

This chapter is organized as follows: Section II introduces the general model and

both calibrate their models with an EIS greater than 1).

[1] Because hidden Markov models of this class are able to capture regularities found in the data that are missed by linear models (see the discussion in Hamilton (2005)), they have been widely used in economics since Hamilton (1989). In particular, in the asset-pricing literature, the implications of a Markov switching process in the conditional mean of the endowment process are analyzed by Cecchetti et al. (1990, 1993); Kandel and Stambaugh (1991); Abel (1994); Abel (1999); Veronesi (1999); Whitelaw (2000); Lettau et al. (2008); and many others.

the properties of the external signal, Section III shows the estimation of the process parameters for the U.S. economy, and Section IV presents pricing formulas, some qualitative results, and some conjectures for some special cases. Quantitative results for a wide range of preference parameters are provided in Section V. Section VI presents my conclusions. Proofs, algebraic derivations, and additional results are provided in the appendices.

II. Model

I assume a pure exchange economy as in Lucas (1978). The economy is populated by a continuum of identical agents with Epstein-Zin preferences given by

$$V_t = \left[(1-\beta)C_t^{1-\frac{1}{\psi}} + \beta \mathcal{R}_t(V_{t+1})^{1-\frac{1}{\psi}}\right]^{\frac{1}{1-\frac{1}{\psi}}}. \tag{3.1}$$

The operator $\mathcal{R}_t$ makes a risk adjustment to the date $t+1$ continuation value. The risk adjustment is given by

$$\mathcal{R}_t(V_{t+1}) = \left(\mathrm{E}_t\left[V_{t+1}^{1-\gamma}\right]\right)^{\frac{1}{1-\gamma}}.$$

The parameter γ is the coefficient of RA, while the EIS is given by ψ. The function reduces to a monotone transformation of the standard power utility function for $\psi = \gamma^{-1}$. Dividends (the endowment good) grow according to the process

$$C_t = C_{t-1}\mathrm{e}^{\mu_i + \sigma_c \epsilon_t}, \tag{3.2}$$

where μ_i denotes the mean log consumption growth rate in state i, σ_c is its state-invariant standard deviation, and ϵ_t is an independent and identically distributed (i.i.d.) standard normal noise term. The underlying state of the economy s_t follows an ergodic two-state Markov chain with transition probability matrix between time t and $t+1$

given by

$$\Theta = \begin{pmatrix} \theta_1 & (1-\theta_2) \\ (1-\theta_1) & \theta_2 \end{pmatrix}. \tag{3.3}$$

The element (i, j) of the matrix denotes $\Pr(s_{t+1} = i \mid s_t = j)$. In general, I will assume that $\theta_i > 0.5$. For identification, I assume $\mu_1 > \mu_2$, so that the first state has the natural interpretation of a boom state, while the second state is a recession state.

The state of the economy is not directly observable but agents have various sources of information at hand for inferring it. The most obvious of these sources is the growth rate of the dividends themselves. Given the structure of the economy, which is assumed known to the agents, high growth rates indicate a high probability of being in the boom state, whereas the reverse is true for low growth rates.

All information in addition to that contained in dividend growth rates is aggregated as an independent signal. For convenience, I let the signal in state i take the form

$$y_t = \mu_i + \sigma_y \nu_t, \tag{3.4}$$

where ν_t is an i.i.d. standard normal noise term. The precision of the external signal is typically defined as $1/\sigma_y$. As σ_y goes to zero the precision of the external signal goes to infinity and as σ_y grows the precision goes to zero. Instead of working directly with the precision of the signal, I prefer working with the variable $h \in [0, 1]$, which I call the strength of the signal. The strength of the signal is defined as the percentage reduction of the probability of receiving a signal in state i that has a higher likelihood in state j. Working with the strength of the signal has the advantage of being easier to interpret: When $h = 0$, the signal contains no information, whereas when $h = 1$ the signal is strong enough to reveal the state of the economy with certainty.

In the case where the external signal is pure noise, the probability of assigning a lower posterior probability to the true state based on only one realization of the signal (i.e., the probability of making a type I error) is 50%. For intermediate levels of signal strength, I let h be the percentage reduction in this probability relative to the pure noise case.[1]

Figure 3.1 illustrates that, based on a single observation, a higher probability is assigned to the state where the density is highest for that observation. Thus, the probability of a type I error is given by the area of the shaded region. It converges to 0.5 as the means of the two distributions converge.

III. Data and Estimation

The sample period chosen for calibrating the model spans the first quarter of 1952 to the last quarter of 2006. Prices and dividends are on the S&P 500 composite, while the risk-free rate is the yield on 1-year treasury bills. These series were taken from Robert J. Shiller's website.[2] Consumption is quarterly real total personal consumption expenditures (NIPA Table 2.3.6, line 1), as stated on the Bureau of Economic Analysis (BEA) website.[3] Finally, I use the official recession dates as reported on the website of the National Bureau of Economic Research (NBER).[4]

The dataset is a standard one and the descriptive statistics are similar to those reported elsewhere in the literature. The average return on equity is 12.2% on an annual basis with a standard deviation of 11.7%. Compared with the mean risk-free

[1] There is a one-to-one relation between the strength of the signal (h) and the precision of the signal ($1/\sigma_y$). This is implicitly given by

$$\sigma_y = -\frac{\mu_1 - \mu_2}{2F^{-1}\left(\frac{1-h}{2}\right)},$$

where F denotes the cumulative distribution function for a standard normal.

[2] http://www.econ.yale.edu/~shiller/data.htm.

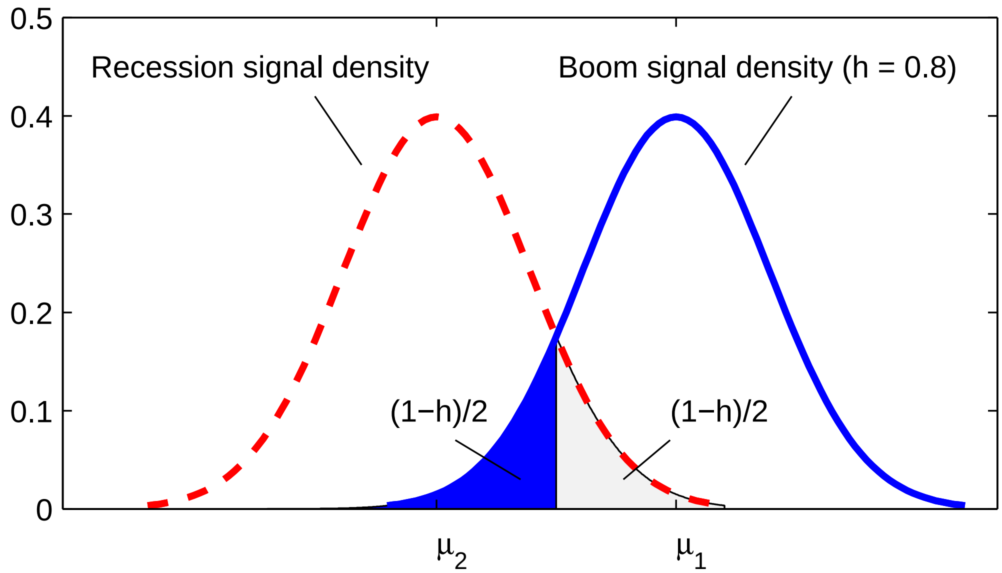
[3] http://www.bea.doc.gov/.

[4] http://www.nber.org/cycles.html/.

Figure 3.1

Signal Strength and Signal Densities

Plotted are the densities of the signal in the two states when the signal has strength $h = 0.8$. The area of the dark shaded region, which is $(1 - h)/2$, gives the probability of assigning a higher likelihood to the recession state, based on a single observation when the true state of the economy is a boom. Conversely, the area of the light shaded region gives the probability of assigning a higher likelihood of being in a boom when the true state of the economy is a recession.



rate of 2.8%, this yields an equity premium around 9%. These numbers are summarized in Table 3.1.

Financial market lore contends that prices move procyclically with the business cycle. To verify this conjecture, I calculate the correlation matrix among the cyclical components of the U.S. economic and financial series. For this, all series were expressed in real terms, logged, and then filtered with the HP filter (Hodrick and Prescott, 1997). As shown in Table 3.2, the cyclical components of all the series are strongly positively correlated, with a correlation coefficient ranging from 0.26 for the GDP and the price-dividend ratio to 0.94 for stock prices and the price-dividend ratio.

Table 3.1

Descriptive Statistics

This table summarizes annualized means and standard deviations in percentage points for key U.S. time series. Equity return is the real return from holding the S&P 500 composite, the risk-free rate is the real yield on 1-year treasury bills, and consumption is real per capita personal consumption expenditures (Q1:1952–Q4:2006; sources BEA and Robert Shiller's website).

	Equity Return	Risk-Free Rate	Consumption Growth
Mean	12.2	2.8	2.3
Std. deviation	11.7	1.1	1.4

Table 3.2

Cyclical Correlations

This table reports the correlation matrix for the cyclical component of some U.S. financial and economic series, as obtained by detrending them with the Hodrick-Prescott (HP) filter. The S&P 500 is the level of the S&P 500 composite, the PD ratio is the price-dividend ratio for the S&P 500 composite, the GDP is the real per capita gross domestic product, and consumption is real per capita personal consumption expenditures (Q1:1952–Q4:2006; sources BEA and Robert Shiller's webpage).

	S&P 500	PD Ratio	GDP	Consumption
S&P 500	1.000			
PD ratio	0.938	1.000		
GDP	0.370	0.257	1.000	
Consumption	0.439	0.345	0.878	1.000

Table 3.3

Estimated Model Parameters

The estimated model economy parameters reported here are based on an MCMC algorithm from Kim and Nelson (1999) using real quarterly per capita personal consumption expenditures (Q1:1952–Q4:2006; source BEA). Standard errors are reported in parentheses.

	μ_i	σ_c	$(1-\theta_i)$
Boom	0.0074		0.0630
	(0.0005)	0.0061	(0.0255)
Recession	-0.0009	(0.0003)	0.2308
	(0.0010)		(0.0780)

Parameter estimates for the regime-switching model for the consumption series were found by a Markov Chain Monte Carlo (MCMC) algorithm similar to the one described in Section 9.1 of Kim and Nelson (1999). The resulting estimates are given in Table 3.3.

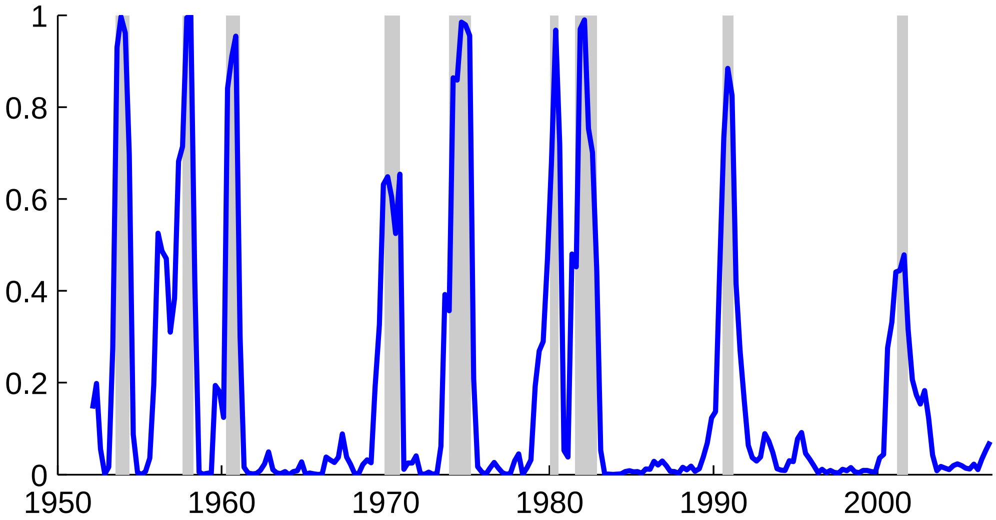
The probabilities of switching from the two states are 6.30% and 23.08%, respectively. These probabilities imply an average duration of 15.9 quarters for booms and 4.3 quarters for recessions.

Figure 3.2 shows that the Markov switching model is able to capture fairly well U.S. recessions as chronicled in the official NBER business cycle reference dates: The gray areas indicate the official recession periods while the solid line gives the smoothed probabilities of being in a recession state.

Figure 3.2

Model Implied Recession Probabilities

Plotted are the smoothed recession probabilities, computed by applying the Hamilton (1989) filter to the U.S. consumption series, coupled with the official NBER recession dates (shaded areas).



IV. Log-Linear Results and Conjectures

One of the key results of Epstein and Zin (1989) is that the stochastic discount factor for the recursive utility function in Equation (3.1) can be expressed as

$$M_{t+1} = \beta^{\kappa} \left(\frac{C_{t+1}}{C_t} \right)^{-\frac{\kappa}{\psi}} (R^e_{t+1})^{\kappa - 1}, \tag{3.5}$$

where $\kappa \equiv (1-\gamma)/(1-1/\psi)$ and R^e_{t+1} is the equilibrium gross return to aggregate wealth between t and $t+1$. Denoting the price-consumption ratio at time t by W_t, R^e_{t+1} can be expressed as $(C_{t+1} + C_{t+1}W_{t+1})/(C_t W_t)$.

Using Equation (3.5), I can find expressions for the equity premium, as well as the one-period real risk-free rate. As usual, the gross risk-free rate is given by the inverse of the expected value of the stochastic discount factor, or

$$R_t^f = \mathrm{E}_t\left[\beta^\kappa \left(\frac{C_{t+1}}{C_t}\right)^{-\gamma} \left(\frac{1+W_{t+1}}{W_t}\right)^{\kappa-1}\right]^{-1}. \tag{3.6}$$

Thus, in this setting, the real interest rate will fluctuate not only with the expected growth rate of consumption but also with the expected changes in the price-consumption ratio.

I use the Euler equation for the claim to aggregate consumption to derive its price. Substituting for M_{t+1} and R_{t+1}^e in

$$1 = \mathrm{E}_t\left[M_{t+1}R_{t+1}^e\right] \tag{3.7}$$

and multiplying both sides by W_t^κ gives

$$W_t^\kappa = \mathrm{E}_t\left[\beta^\kappa \left(\frac{C_{t+1}}{C_t}\right)^{1-\gamma} (1+W_{t+1})^\kappa\right]. \tag{3.8}$$

I follow Lettau et al. (2008) and solve this equation numerically, using the property that the boom state probability is a sufficient statistic for the investors' time t information set.[1]

Since we are in an endowment economy, equilibrium requires that consumption be always equal to the dividends of the claim to the aggregate endowment, so the price-dividend ratio is also given by Equation (3.8). I will use the two terms interchangeably.

For the discussion in the next section, one key concern is to determine whether the prices predicted by the model are moving procyclically or countercyclically with the (perceived) state of the economy. I restrict my attention to the relevant case where the probability of the economy remaining in the same state, on a period-by-period basis, is

[1] The details of the numerical procedure are provided in Appendix A.

higher than the probability of a regime switch. Appendix B establishes in the following proposition:

Proposition 1. *For* $\theta_1, \theta_2 > \frac{1}{2}$ *if* $\psi > 1$ $(\psi < 1)$ *the price-dividend ratio is increasing (decreasing) in investors' posterior boom probability. If* $\psi = 1$*, the price-dividend ratio is constant and equal to* $1/(1-\beta)$*.*

Proof. See Appendix B. □

A. Expected Returns

I will now provide an intuition for Proposition 1 by showing that the expected growth rate of dividends influence not only expected future cash flows but also the rate at which they are discounted. Log-linearizing the Euler equation for equity and solving it for expected returns gives us a simplified framework to analyze it. The conditional expected log returns to equity ($\mathrm{E}_t\left[r^e_{t+1}\right]$) and the risk-free rate (r^f_t) can be expressed as

$$\mathrm{E}_t\left[r^e_{t+1}\right] = -\log\beta + \frac{1}{\psi}\,\mathrm{E}_t\left[g_{t+1}\right] - \frac{\kappa}{2}\left[\frac{1}{\psi^2}\sigma^2_t(g_{t+1}) + \sigma^2_t(r^e_{t+1}) - 2\frac{1}{\psi}\mathrm{cov}_t\left(g_{t+1}, r^e_{t+1}\right)\right], \tag{3.9}$$

$$r^f_t = -\log\beta + \frac{1}{\psi}\,\mathrm{E}_t\left[g_{t+1}\right] - \frac{\kappa}{2}\left[\frac{1}{\psi^2}\sigma^2_t(g_{t+1}) + \left(\frac{1}{\kappa} - 1\right)\sigma^2_t(r^e_{t+1})\right], \tag{3.10}$$

where $\sigma^2_t(g_{t+1})$ and $\sigma^2_t(r^e_{t+1})$ denote the conditional variance of the log consumption growth rate and the log return to equity, respectively. Their conditional covariance is denoted by $\mathrm{cov}_t\left(g_{t+1}, r^e_{t+1}\right)$.

The main difference between the two states is that the expected conditional growth rate of dividends is higher in booms than in recessions, so that the second term of Equation (3.9) will be higher. On the one hand, an upward revision of the conditional expected growth rate of dividends increases the expected payoffs of equity, increasing its value to investors. On the other hand, investors prefer consumption profiles which are smooth over time. Given an upward revision in the conditional expected dividend growth rate, investors would like to smooth their intertemporal consumption profile by shifting consumption from the future to the present. Since the model does not allow for any aggregate saving or dissaving, equilibrium can only be obtained if the conditional expected returns on all assets increases sufficiently to check the investors' desire to sell them off in order to finance consumption increases. The amount by which conditional expected returns will have to increase to maintain equilibrium depends on how tolerant investors are to consumption variations over time (i.e., on their EIS). If $\psi < 1$, an upward adjustment of the conditional expected dividend growth rate causes an even larger upward adjustment of the required return to equity. This leads to a drop in prices. If $\psi > 1$, an upward adjustment of conditional expected consumption growth rates is matched by a less than one-to-one adjustment of the required return to equity; hence prices would be increasing in the boom probability. By Proposition 1, I know that this result from the log-linear approximation holds generally.

B. Equity Returns

Having clarified how the conditional expected log return is determined in my setup, I turn my focus to the relation I want to analyze: the influence of information quality on the conditional equity premium. The following proposition provides an approximate analytical expression for assessing it.[1]

Proposition 2. *If consumption growth and asset returns are conditionally jointly log*

[1] The quantitative results in the next section do not rely on this linear approximation but, rather, on precise numerical algorithms.

normal, the conditional equity premium is given by

$$\mathrm{E}_t\left[r^e_{t+1}\right] - r^f_t + \frac{1}{2}\sigma^2_t(r^e_{t+1}) = \gamma\sigma^2_t(g_{t+1}) + (1-\kappa)\,\sigma^2_t(\omega_{t+1}) + ((1-\kappa)+\gamma)\,\mathrm{cov}_t\left(g_{t+1}, \omega_{t+1}\right), \tag{3.11}$$

where $\sigma_t(g_{t+1})$ *is the conditional standard deviation of the log consumption growth,* $\sigma^2_t(\omega_{t+1})$ *is the conditional variance of* $\log\frac{1+W_{t+1}}{W_t}$, *and* $\mathrm{cov}_t\left(g_{t+1}, \omega_{t+1}\right)$ *is their conditional covariance.*

Proof. See Appendix B. □

Below I analyze the terms in Equation (3.11) individually.

C. Information and Equity Premium

The first part of Equation (3.11), $\gamma\sigma^2_t(g_{t+1})$, is the familiar textbook formula for the conditional equity premium. Increasing the quality of the signal decreases the conditional volatility of consumption independently of RA and the EIS. Information quality also affects returns through their conditional variance and their conditional covariance with consumption. I will look at each of the two in turn.

3. Contribution of Returns Variance

I now conjecture that if $\psi \neq 1$, the conditional variance of equity returns net of dividend growth ($\log[(1+W_{t+1})/W_t]$) is increasing in signal quality.[1] One way to think about this conjecture is along the lines of Shiller (1981), who shows that the perfect foresight price of a stock will be more variable than the price with a smaller information set because expectations are smoother than realizations. The analogy to Shiller's argument does not carry over perfectly to my model, because the price-dividend ratio is nonlinear and

[1] In Section V I show that the conjecture holds numerically for all the parameterizations with $\psi \neq 0$ I analyze. The returns-based Euler equation I use in the log-linearization is not defined at $\psi = 1$.

because I am analyzing returns instead of prices directly. Instead I can appeal to the following reasoning.

From Equation (3.8) I know that the current price-dividend ratio is uniquely determined by the posterior beliefs that investors assign to each of the two states. In the extreme case where the signal is strong enough to reveal the state of the economy with certainty (i.e., $h = 1$), the price-dividend ratio will be constant as long as the underlying state of the economy does not change. Whenever the state of the economy switches, this will be detected immediately and the price-dividend ratio will jump straight to its new value. This implies a relatively large deviation of $\log[(1 + W_{t+1})/W_t]$ from its conditional mean.

The noisier the external signal, the harder it will be to detect regime changes. Learning about the new state of the economy will be protracted, resulting in smaller deviations of $\log[(1 + W_{t+1})/W_t]$ from its conditional mean as W moves toward the value predicted by the new state of the economy. Because the variance of a random variable is the expectation of the *squared* deviations of the variable from its mean, this is likely to lead to a lower variance.

If the conditional variance of returns net of dividend growth increases with the quality of the external signal, the effect of information quality on the conditional equity premium through this channel depends on the coefficient $(1 - \kappa)$. The following conjecture immediately follows.

Conjecture 1. *I conjecture that the contribution of better information to the conditional equity premium through the conditional variance of returns is*

- *Negative if* $\kappa > 1$
- *Zero if* $\kappa = 1$
- *Positive if* $\kappa < 1$

Corollary 1. *(a) If investors have a preference for early resolution of uncertainty ($\gamma > 1/\psi$) and a coefficient of relative RA greater than unity, then the contribution of better information through the conditional variance of returns is*

- *Negative if $\psi < 1$*
- *Positive if $\psi > 1$*

(b) In the power utility case the contribution of better information through the conditional variance of returns is zero.

3. Contribution of the Covariance of Returns and Consumption

In addition to the effect of the conditional variance of returns net of dividend growth, Equation (3.11) shows that the conditional covariance with consumption growth rates will also have an effect on the equilibrium equity premium. It follows from Proposition 1 that the conditional covariance will be positive whenever ψ is greater than 1 and negative otherwise.

I conjecture that a better external signal reduces (in absolute terms) the conditional covariance of returns net of dividend growth and the consumption growth rate. By equation (3.8), the price-dividend ratio is fully determined by the probabilities that investors assign to the two growth states. The more informative the external signal is, the less weight investors give to consumption growth rates when updating their beliefs on the state of the economy. Thus, the conditional covariance of prices with consumption growth rates decreases (in absolute terms) with information quality.

The impact of information quality on the required risk premium will depend on the sign of the conditional covariance and on the sign of its coefficient. The conditional covariance is positive if returns net of dividend growth are procyclical (when $\psi > 1$) and negative if returns net of dividend growth are countercyclical (when $\psi < 1$). Solving for the sign of the coefficient $(1-\kappa+\gamma)$, this implies the following.

Conjecture 2. *(a) If investors have a preference for early resolution of uncertainty ($\gamma > 1/\psi$) and an RA parameter larger than unity ($\gamma > 1$), I conjecture that the contribution of better information quality on the conditional equity premium through the conditional covariance of returns net of growth and consumption growth rates is*

- *Positive if* $\psi \leq 1/2$
- *Negative if* $1/2 < \psi < 1$ *and* $\gamma > 1/(2\psi - 1)$
- *Negative if* $\psi > 1$

(b) In the power utility case, the contribution is negative if $\psi > 1$.

An implication of this intuition-based conjecture is that if returns are procyclical ($EIS > 1$), the condition that investors have an RA parameter above unity is sufficient to ensure that the effect of better information through the covariance channel lowers the conditional equity premium. To generate the same effect when returns are countercyclical ($EIS \in (0.5, 1)$), we need a higher RA. In particular, we need the investors to have a sufficiently strong preference for early resolution of uncertainty that $(1-\kappa)+\gamma < 0$.

3. Two Special Cases

In general, we cannot determine the direction of the effect of better information on the conditional equity premium through the variance and covariance channels conjectured previously. Two important exceptions are:

> *Power utility:* With power utility, κ equals 1. So the conditional equity premium does not depend on the conditional variance of returns net of growth. Moreover, the coefficient on the conditional covariance term simplifies to γ, which is always

positive. By Conjecture 2, the conditional equity premium is conjectured to decrease with the quality of the external signal if $\psi > 1$ (which implies $\gamma < 1$).

Moderate EIS and relatively high RA: When $1/2 < \psi < 1$ but the relative RA is sufficiently high ($\gamma > 1/(2\psi - 1)$), the conditional equity premium is conjectured to decrease in the quality of the external signal.

V. Numerical Results

The log-linearized approximation in the last section allows us to conjecture the direction of the effect of information quality on the equilibrium equity premium in some special cases. In this section I extend the analysis in several directions: First, I analyze the quantitative implications of information on stock returns for a particular parameter choice. Second, I show how the effect of information varies with the relative magnitude of the EIS and the RA parameters. The latter allows us to establish a parameter region where better information lowers the equity premium. Third, I provide a numerical analysis on the Sharpe ratio for a wide range of parameters.

A. Benchmark Calibration

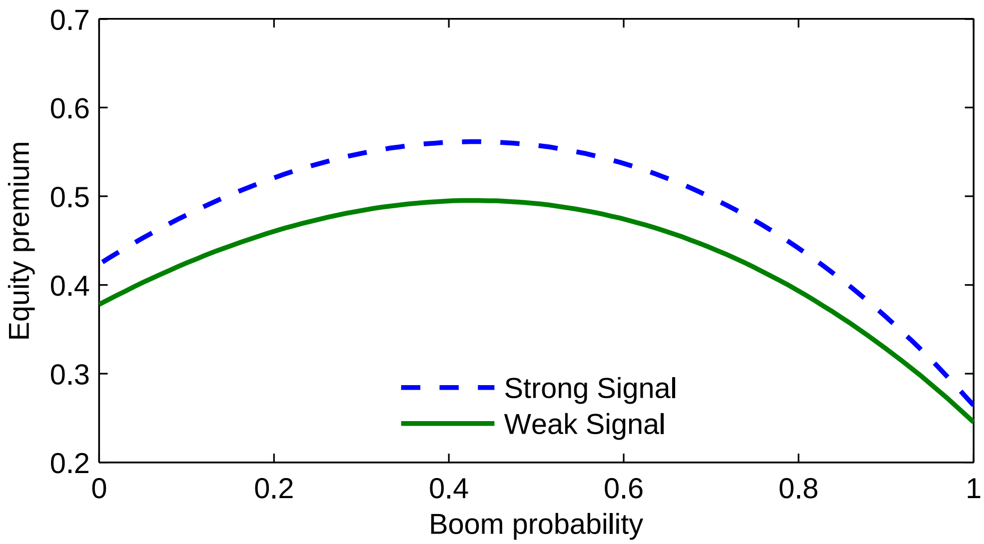
In the first part of this section, I look at how the conditional equity premium changes with two variables: the current state uncertainty and the quality of the external signal. This lays the foundation for understanding the impact of information quality on the average equity premium.

The benchmark parametrization follows Bansal and Yaron (2004): Investor RA is given by $\gamma = 10$, investor EIS is given by $\psi = 1.5$, and the time discount rate is given by $\beta = 0.9925$. The persistence of the estimated consumption process is lower than

Figure 3.3

Conditional Equity Premium and Signal Strength

This figure gives the conditional log return equity premium ($\mathrm{E}_t\left[r^e_{t+1}\right] - r^f_t + \frac{1}{2}\sigma^2_t(r^e_{t+1})$ in % p.a.) predicted by the model as a function of the probability investors assign to the boom state. The dashed line is obtained with a very strong signal ($h = 0.99$); the solid line is for the case where investors only have access to the consumption signal ($h = 0$). The utility parameters are set to the benchmark values ($\gamma = 10$, $\psi = 1.5$, and $\beta = 0.9925$).



that of the process they postulate, so these parameter values do not allow us to match the empirical equity premium. [1]

I start off by looking at the equity premium conditional on given beliefs about the state of the world for the benchmark calibration. The solid line in Figure 3.3 shows the conditional log return equity premium from equation (3.11) as a function of the

[1]To match both the empirical equity premium and the average risk-free rate for the benchmark EIS of 1.5, I would need to set γ and β to 375 and 0.9785, respectively (an interesting analysis on the magnitude of Epstein and Zin utility parameters can be found in Campanale et al. (2007)). A higher ψ increases the riskiness of equity by making returns more procyclical, but even for very high values of ψ extreme RA is needed for the model to match the postwar equity premium. If I use $\psi = 20$, which is an order of magnitude greater than what has been reported in the literature (see footnote 1), I still need an RA parameter of 295 and a time discount rate of 0.9815 to match the average risk-free rate and equity premium in the postwar data.

probability that investors assign to the boom state when they have access only to the consumption signal ($h = 0$). The dashed line in the figure shows the conditional log return equity premium they demand when they also have access to a strong external signal ($h = 0.99$). Three facets of the figure are particularly noteworthy: First, the conditional log return equity premium is hump shaped in the probability investors assign to the boom state. Second, the conditional log return equity premium is lower in booms than in recessions. Third, for given current beliefs about the state of the economy, the conditional log return equity premium is uniformly higher when the investors have access also to the strong external signal. All three facets are related to the conditional variance of equity returns. In the benchmark calibration, the variance of equity returns enters equation (3.11) with a positive coefficient, so the conditional log return equity premium will be higher the higher the variance of equity returns. From equation (3.8), we know that the price-dividend ratio is a function of the current beliefs about the state of the economy. The more beliefs are expected to be revised in the immediate future, the higher is the conditional variance of equity returns.

Even when investors only have access to the consumption signal, their beliefs typically gravitate quickly towards the true state of the economy. Periods when there is great uncertainty about the state of the economy are in expectation followed by learning about the true state of the economy and such learning will lead to price movements. This contributes to the hump-shaped pattern for the equity premium: When uncertainty is high, expected return volatility is also high and investors require a high premium to hold equity.

A related effect contributes to the fact that the equity premium is higher in recessions than in booms. The estimated average duration of a boom is about 4 years, whereas recessions on average last only about a year (see Table 3.3). In booms, investors do not expect to receive signals that will greatly change their beliefs about the state because the state itself is unlikely to change. In contrast, during recessions, the state itself is likely to switch and investors know that such a switch will be reflected in the signals they receive. This makes the conditional volatility of equity returns higher in recessions than in booms.

As discussed previously, the quality of the external signal also affects the volatility of prices. When the signal is very strong, investors know that it will accurately reflect the state of the economy. If they are currently uncertain about the state of the economy, they expect to either believe firmly that they are in a boom after they receive next period's signal or to believe firmly that they are in a recession. Both events entail revisions of their current beliefs and thus return volatility. Even when investors are very certain about the prevailing state of the economy, stronger signals increase the conditional volatility of returns. With a strong external signal, it is easier to detect switches in the underlying state of the economy and such switches would entail larger price movements. The higher conditional return variance with a strong signal contributes to the upward shift of the conditional equity premium from the solid line to the dashed line in Figure 3.3.

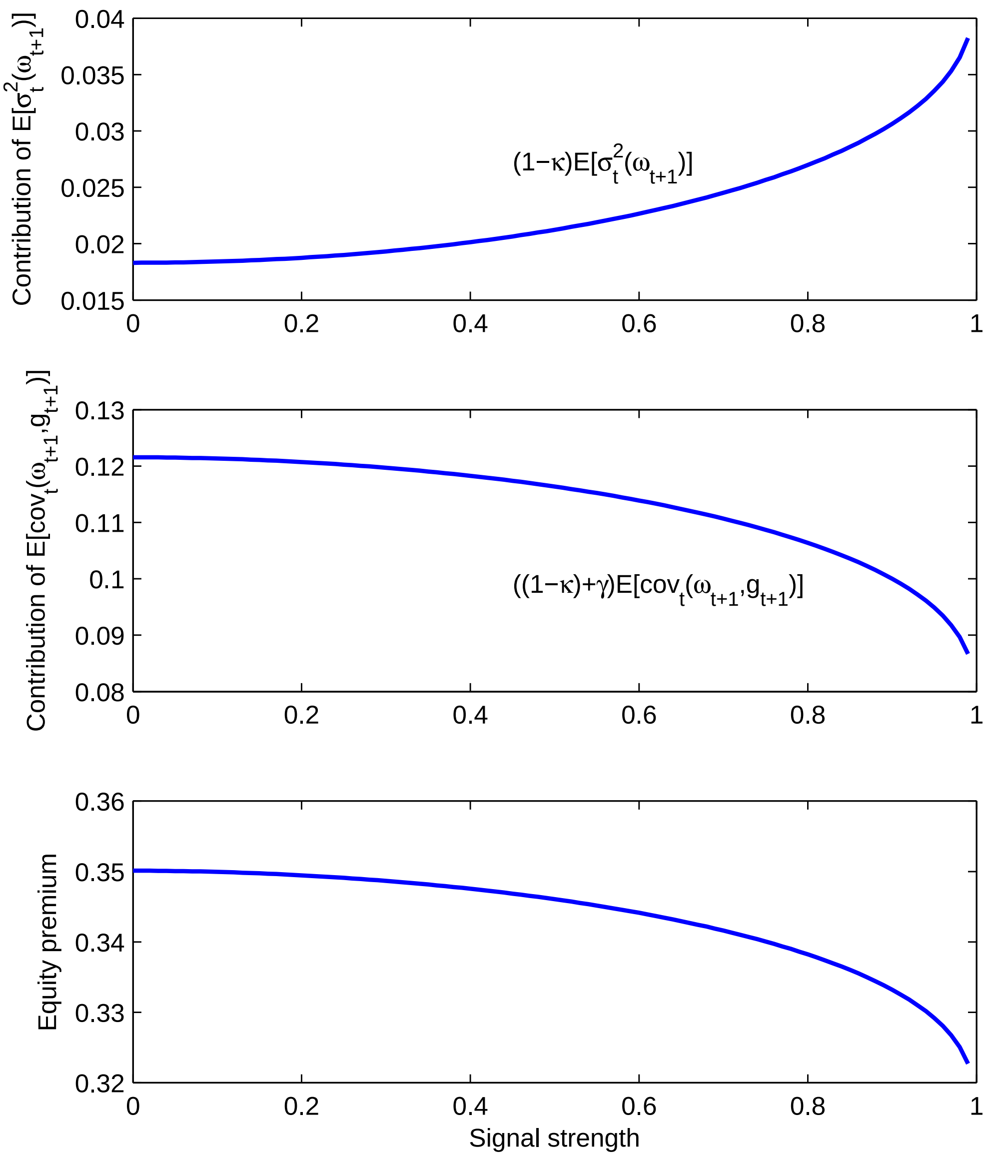
The higher *conditional* log return equity premium (equation 3.11) with high signal quality does not translate into a higher *average* log return equity premium, $\mathrm{E}\left[r^e_{t+1} - r^f_t + \frac{1}{2}\sigma^2_t(r^e_{t+1})\right]$. This is because with a strong signal investors will typically be certain about whether they are in a boom or a recession state. From Figure 3.3, we know that this implies a relatively low conditional equity premium. With a weaker signal, investors will be more uncertain about the state of the economy. Since the conditional equity premium is hump shaped in the boom-state probability, this will drive up the average equity premium.

This is confirmed in Figure 3.4, which decomposes the average log return equity premium as a function of signal strength. Graphs A and B show the contribution of re-

Figure 3.4

Effect of Information Quality on the Expected Equity Premium

This figure decomposes the average log return equity premium ($\mathrm{E}\left[r^e_{t+1} - r^f_t + \frac{1}{2}\sigma^2_t(r^e_{t+1})\right]$ in % p.a.) as a function of signal strength. The signal strength is increasing along the x axis, where 0 indicates a completely noisy signal and 1 a perfect signal. Graph A gives the contribution of the conditional variance of returns to the equity premium. Graph B gives the contribution from the conditional covariance of returns and consumption growth rates. The equity premium (Graph C) aggregates these two terms to the contribution from the conditional variance of consumption growth rates. The utility parameters are set to the benchmark values ($\gamma = 10$, $\psi = 1.5$, and $\beta = 0.9925$).



turn variance and the covariance of returns and consumption growth rates to the equity premium, respectively, while Graph C shows the average log return equity premium. As I conjecture in Section IV.B.3, the variance of returns increases with the quality of the external signal, while the covariance of returns and consumption decreases with it. The net effect is an average log return equity premium which increases with the quality of the external signal.

B. Other Parameterizations

2. EIS> 1

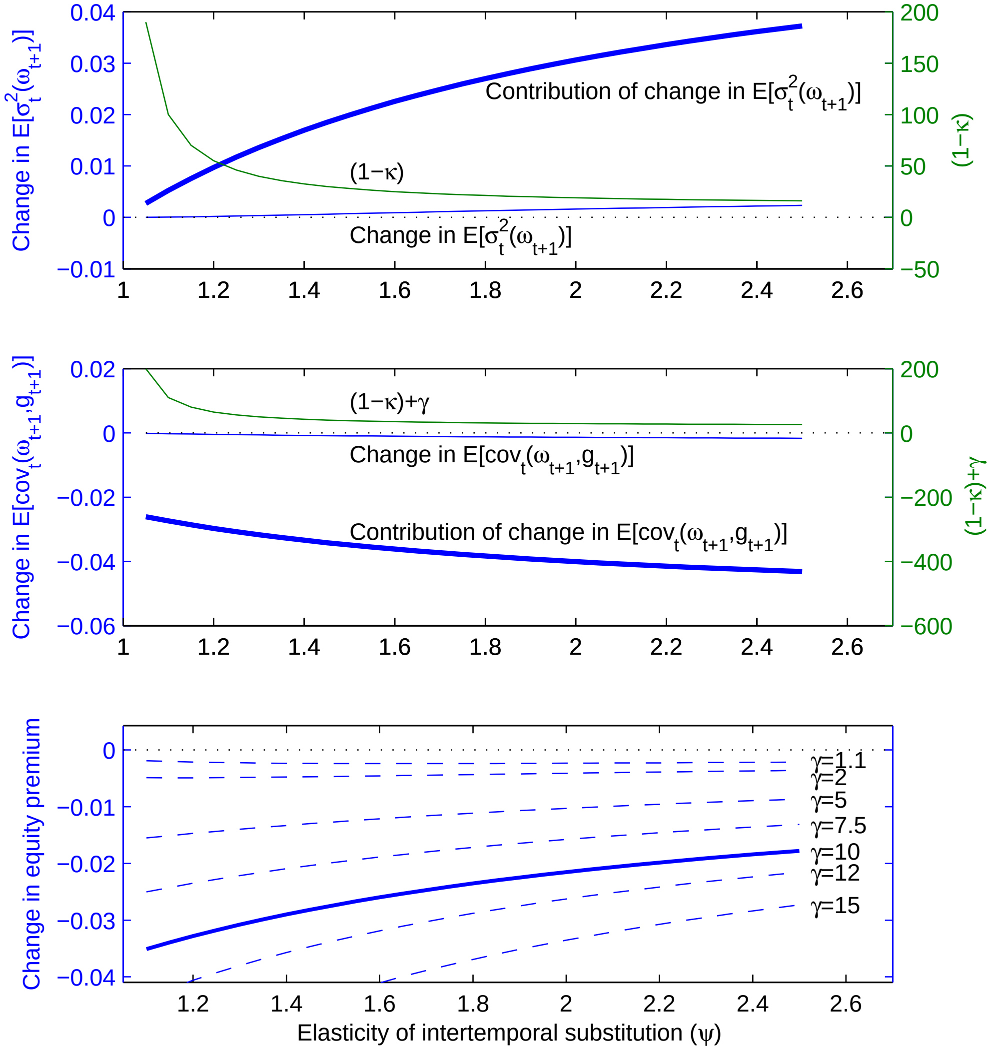
With $\psi > 1$ prices are procyclical and better information lowers the covariance of returns net of dividends growth and consumption growth rates, while, as always, a higher signal quality increases the variance of returns net of dividends. For this parametrization, the two forces pull in opposite directions, so I need to resort to numerical techniques to determine the total impact.

Graphs A and B of Figure 3.5 decompose the change in the average log return equity premium a representative investor would require when given access to a high-quality signal (h goes from 0.01 to 0.99). In these two graphs, the investor's RA parameter is set to $\gamma = 10$. As we move along the abscissa, the EIS parameter ψ of the representative investor varies from 1.05 to 2.5. The contribution of the reduction in the average conditional variance $\mathrm{E}\left[\sigma_t^2(\omega_{t+1})\right]$ is given in Graph A. As argued previously, information increases the variance of returns. For $\psi > 1$, this increases the required equity premium (Graph A). Better information quality also reduces the covariance between consumption growth and returns. This lowers the required equity premium (Graph B). The solid line in Graph C aggregates these terms with the contribution from the change in the consumption growth rate variance term, which is independent

Figure 3.5

Improved Information and the Equity Premium (EIS > 1)

This figure decomposes the change in the average log return equity premium $(\mathrm{E}\left[r^e_{t+1} - r^f_t + \frac{1}{2}\sigma^2_t(r^e_{t+1})\right]$ in % p.a.) that results from changing the strength of the external signal from $h = 0.01$ (very uninformative) to $h = 0.99$ (very informative). As we move along the abscissa, investors become increasingly tolerant to shifting consumption over time. In Graphs A and B, the other parameters are fixed to their benchmark values ($\gamma = 10$, $\beta = 0.9925$). Graph A shows the contribution to the equity premium that comes from the higher variance of returns, while Graph B shows the contribution from the change in the covariance between returns and consumption that results from a stronger signal. The solid line in the Graph C aggregates these terms with the contribution from the change in the consumption growth rate variance term (which is independent of ψ). The dashed lines in Graph C show the change in equity premium for alternative levels of RA.



of ψ. The dashed lines in Graph C show the change in equity premium for alternative levels of RA. For all parameter values plotted, the net effect is an average log return equity premium which decreases with the quality of the external signal.

2. EIS< 1

The result for $\psi > 1$ is important but not surprising, since $\psi > 1$ is the condition for procyclical prices. In the last section, I conjecture that the same effect obtains when $1/2 < \psi < 1$ and $\gamma > 1/(2\psi - 1)$. If these conditions are not satisfied, I cannot rely on the intuition-driven conjecture from last section because the impact of a change in the information quality through the variance and covariance channels pull the equilibrium equity premium in opposite directions.

Nevertheless, I can establish a parameter region where improved information quality lowers the equilibrium return to equity. One way to do so is to fix the level of RA and investigate how the effect of information on the equity premium changes as I vary the willingness of investors to substitute consumption over time.

Figure 3.6 illustrates this procedure and links the numerical solution to the discussion of the terms in the log-linear approximation. It decomposes the change in the average log return equity premium changes when agents are provided with a high-quality signal (h goes from 0.01 to 0.99). In Figure 3.6, γ is set to the benchmark value of 10, while I let ψ vary from 0.15 to 0.9.[1] The contribution of the reduction in variance on the required equity premium is given in Graph A. Better information increases the variance of returns, which, in most of the range plotted, lowers the required equity premium. The effect is reversed for EIS parameters below $1/\gamma$, or 0.1 in my benchmark parametrization.

Graph B provides the analogous decomposition for the effect of the change in co-

[1] I avoid going too close to 1: The returns-based Euler equation is not defined at $\psi = 1$ and there is no reason to expect the log-linearization to be a good approximation around this point.

Figure 3.6

Improved Information and the Equity Premium (EIS < 1)

This figure decomposes the change in the average log return equity premium (E $\left[r^e_{t+1} - r^f_t + \frac{1}{2}\sigma^2_t(r^e_{t+1})\right]$ in % p.a.) that results from changing the strength of the external signal from $h = 0.01$ (very uninformative) to $h = 0.99$ (very informative). As we move along the abscissa, investors become increasingly tolerant to shifting consumption over time, while the other utility parameters are fixed to their benchmark values ($\gamma = 10$, $\beta = 0.9925$). Graph A shows the contribution to the equity premium that comes from the higher return variance, while Graph B shows the contribution from the change in the covariance between returns and consumption that results from a stronger signal. The total effect (Graph C) aggregates these two terms with the contribution from the change in the variance of consumption growth rates.

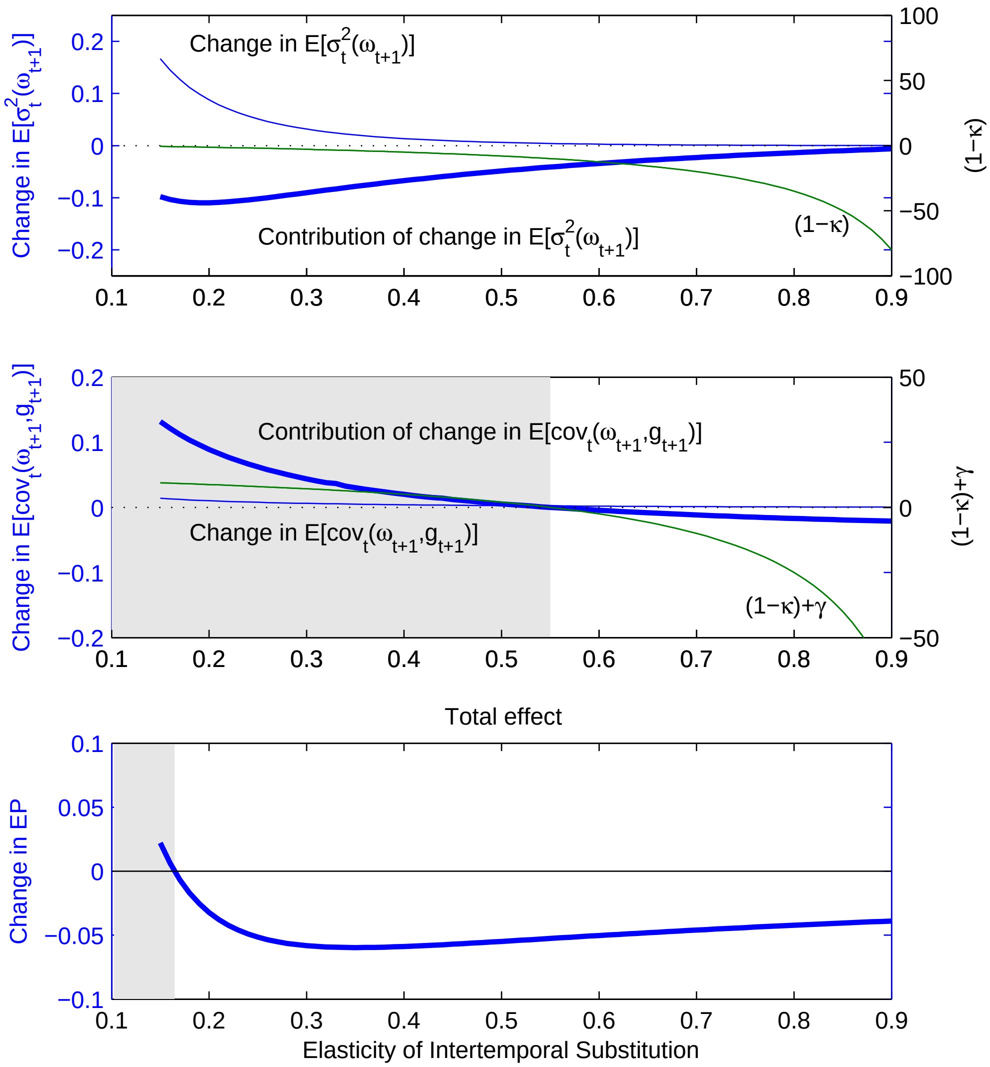
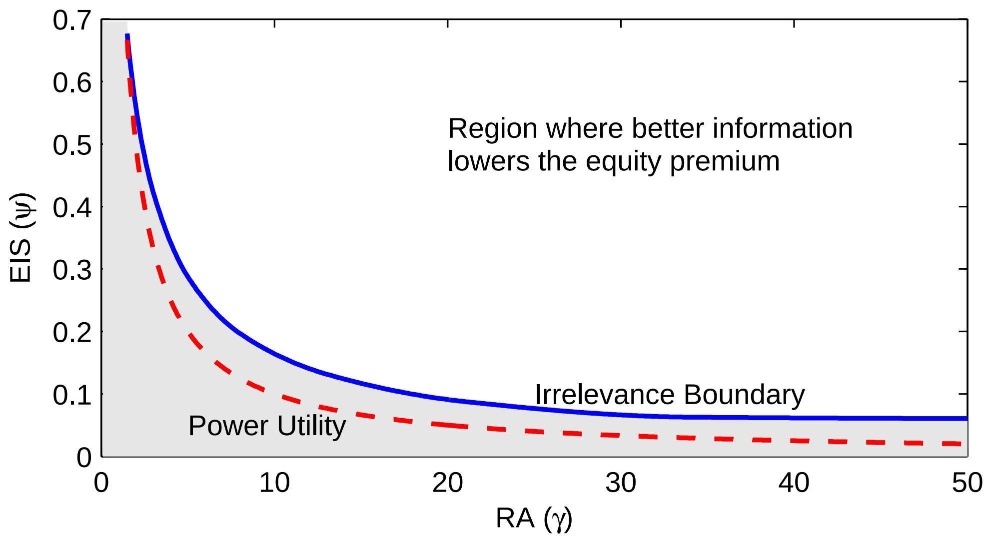


Figure 3.7

Information Quality and Equity Premium by Parameter Regions

This figure groups parameter constellations by the effect of better information on the average log return equity premium ($\text{E}\left[r^e_{t+1} - r^f_t + \frac{1}{2}\sigma^2_t(r^e_{t+1})\right]$ in % p.a.). The shaded region depicts parameter constellations where better information increases the equity premium. For the domains considered, power utility specifications fall in this region. The time discount rate is set to 0.9925.



variance between consumption and returns when the signal quality varies. As argued previously, returns covary less with consumption when the signal quality is high. For $\psi < 1$, the covariance is negative. So an increase in the signal quality leads to a less negative covariance. The lower ψ, the more pronounced the effect. The effect on the change in the covariance is shown by the bold line in Graph B. It is the product of the change in the covariance and its coefficient in Equation (3.11). The gray area delimits the range where the effect of lower signal precision through this component leads to a lower required equity premium.

The net effect of the variance and covariance terms is given in Graph C. This is the sum of the two effects described previously and the effect of the variance of the dividend growth rates term, which is constant.

Repeating the same exercise as in Figure 3.6 for various levels of investor RA yields a locus that marks the boundary at which the effect of information quality on the average log return equity premium reverses. In Figure 3.7, I mark such points of intersection for a set of RA parameters with a solid line. It separates the two regions of interest: Above the line, the relation is negative; below it is positive. The dashed line in sets out parameter combinations for the power utility case.

C. Information Quality and the Sharpe Ratio

In this section I extend the numerical analysis to the effect of information quality on the unconditional log return equity Sharpe ratio defined as the average log return equity premium divided by the unconditional variance of equity log returns

$$\mathrm{SR} = \frac{\mathrm{E}\left[r^e_{t+1} - r^f_t + \frac{1}{2}\sigma^2_t(r^e_{t+1})\right]}{\sigma(r^e_{t+1})}. \tag{3.12}$$

Obviously, the effect of information quality on the Sharpe ratio will depend on its effect on both the equity premium and the standard deviation of equity returns. It is helpful to decompose the unconditional variance of equity log returns as

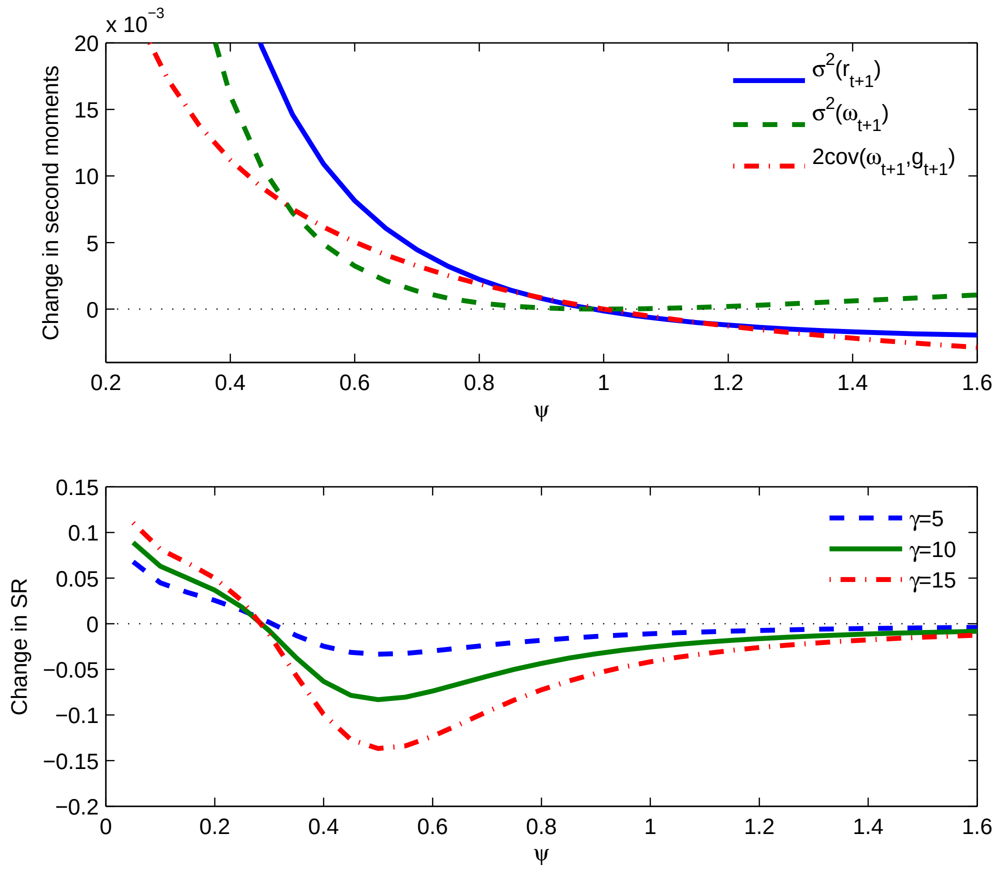
$$\sigma^2(r^e_{t+1}) = \sigma^2(g_{t+1}) + \sigma^2(\omega_{t+1}) + 2\mathrm{cov}\left(g_{t+1}, \omega_{t+1}\right). \tag{3.13}$$

Consumption growth is exogenous so information quality will only impact the denominator in Equation (3.12) through the last two terms of Equation (3.13). Graph A of Figure 3.8 plots the effect of increasing the precision of the external signal from $h = 0.01$ to $h = 0.99$ on the terms in Equation (3.13) for different values of ψ when the other model parameters are kept at their benchmark values ($\gamma = 10$, $\beta = 0.9925$). We know from Proposition 1 that the price-dividend ratio is constant at $\psi = 1$. At this

Figure 3.8

Improved Information and the Sharpe Ratio

This figure decomposes the change in the unconditional log return Sharpe ratio (as defined in Equation (3.12)) that results from changing the strength of the external signal from $h = 0.01$ (very uninformative) to $h = 0.99$ (very informative). As we move along the abscissa, investors become increasingly tolerant to shifting consumption over time. In Graph A, the other parameters are fixed to their benchmark values ($\gamma = 10$, $\beta = 0.9925$). Graph A shows the effect of better information on the variance of returns (solid line). The effect is decomposed into the variance of returns net of dividend growth, and their covariance with consumption growth. The Graph B shows the corresponding change in the Sharpe ratio for alternative levels of RA.



point the variance of equity returns is not affected by information quality. As we move the EIS away from 1, the two last terms on the right-hand side of Equation (3.13) come into play. The solid line gives the total effect of better information on the unconditional variance of equity log returns.

The solid line in Graph B gives the corresponding change in the unconditional log return equity Sharpe ratio defined in equation (3.12). For moderate and high levels of ψ better information lowers the Sharpe ratio, whereas for low levels of ψ the better information quality actually leads to a higher Sharpe ratio for equity. The seemingly erratic behavior of this relation as $\psi \longrightarrow 0$ is due to the fact that the equity premium becomes negative for low levels of ψ.[1] In this case, an increase in the variance of returns increases the average Sharpe ratio by bringing it closer to zero. Increasing the coefficient of relative RA from the benchmark calibration magnifies the pattern. This is illustrated by the two other lines in Graph B. The response of the Sharpe ratio for $\gamma = 5$ is more subdued (dashed line), while the response for $\gamma = 15$ is more pronounced (dash-dotted line).

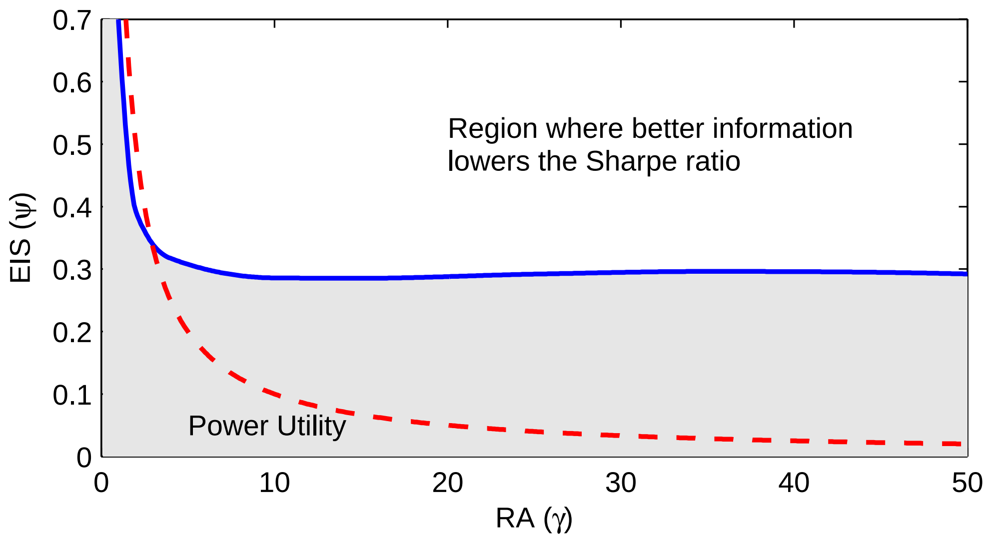
As I did for the equity premium in Figure 3.6, I repeat the exercise in Figure 3.8 for different levels of γ and collect the points where the impact of information quality on the unconditional log return equity Sharpe ratio defined in equation (3.12) changes sign. These points are plotted as the solid line in Figure 3.9. It separates the two regions of interest: For parameter combinations that are above the line, better information leads to a lower Sharpe ratio, while for the parameter combinations in the shaded region below the line, better information leads to a higher Sharpe ratio. The dashed line sets out parameter combinations for the power utility case. The link between the EIS and RA parameters places us in the shaded region when the RA is high. Here, the counterintuitive effect of better information quality on the log return equity premium

[1]At low levels of ψ the price-consumption ratio is sufficiently countercyclical to make equity a hedge against consumption risk. See the negative premia reported in Table 3.4 below.

Figure 3.9

Information Quality and Sharpe Ratio by Parameter Region

This figure groups parameter constellations by the effect of better information on the unconditional log return Sharpe ratio as defined in Equation (3.12). The shaded region gives parameter constellations where better information leads to a higher Sharpe ratio. The time discount rate is set to 0.9925. The dashed line gives parameter combinations where the utility function simplifies to a power utility function.



under power utility is inherited by the Sharpe ratio.

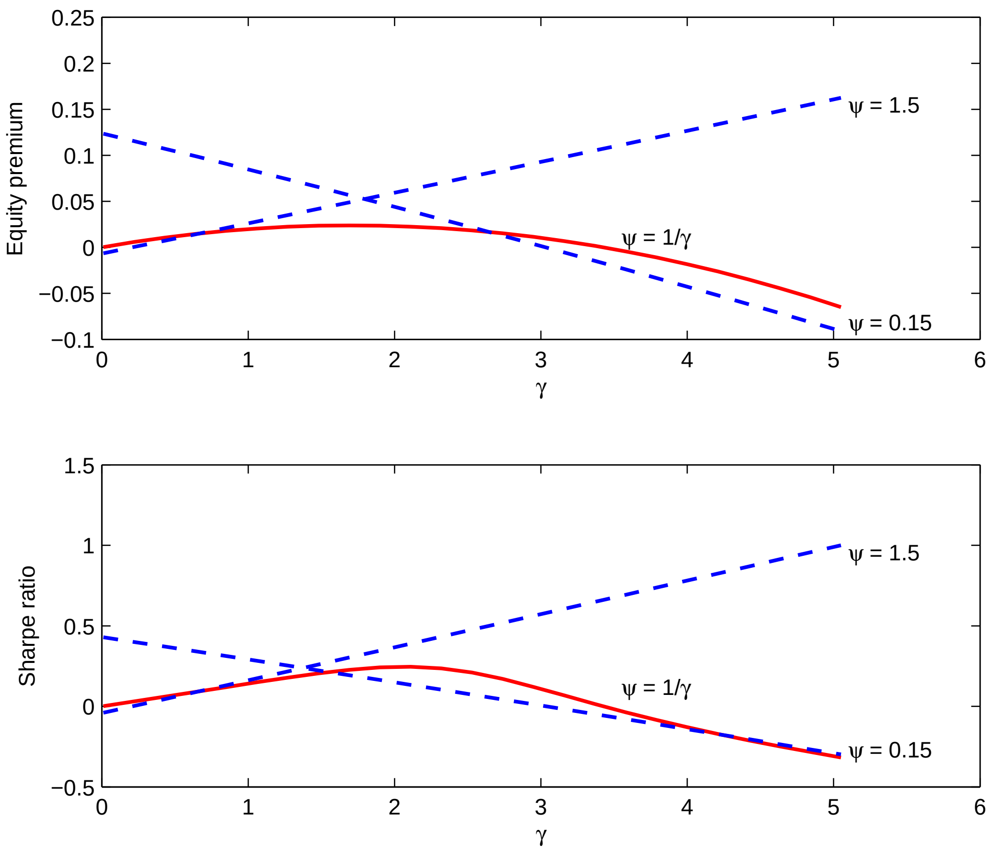
D. Unbounded Equity Premium and Sharpe Ratio

As noted in the introduction, another feature Veronesi (2000) identified for the power utility case in this setting, is that the equity premium is bounded above at a low value when signals are noisy. This seems to aggravate the equity premium puzzle, because the model predicts a negative equity premium for high levels of RA. The solid line in Graph A of Figure 3.10 replicates that in Panel B of Figure 2 from Veronesi (2000). The external signal is set to be moderately informative ($h = 0.5$). The line displays a well-defined global maximum. Such a maximum makes it even more difficult to

Figure 3.10

Equity Premium and Utility Parameters

This figure plots the average log return equity premium (Graph A) and the unconditional log return Sharpe ratio (Graph B) against the coefficient of relative RA with Epstein-Zin preferences (dashed lines) and power utility (solid line). The expected log return equity premium is calculated as $\mathrm{E}\left[r^e_{t+1} - r^f_t + \frac{1}{2}\sigma^2_t(r^e_{t+1})\right]$ and the Sharpe ratio is calculated using Equation (3.12). With Epstein-Zin preferences, the EIS parameter can be held fixed. With power utility, the EIS parameter is given by the inverse of RA. All lines are plotted for a time discount factor β of 0.9925 and assuming an exogenous signal which is moderately informative ($h = 0.5$).



solve the equity premium puzzle of Mehra and Prescott (1985), because increasing the RA parameter would lower the equity premium beyond this point. As a means of comparison, the graph also plots the average log return equity premium with Epstein-Zin preferences. In this setup, increasing the RA parameter does not affect the EIS and hence leaves the cyclicality of returns largely unaffected. The upshot is that for most values of ψ a practically linear and increasing relation between the average log return equity premium and RA obtains. Only for very small values of ψ is this relation reversed. For such small values, returns become negatively correlated with the pricing kernel. Thus, the more risk-averse investors are, the greater the returns they are willing to give up to hold such a claim.[1]

Graph B shows that the relation between the RA parameter and the average log return equity premium is reflected in the unconditional log return Sharpe ratio. With power utility (solid line), the Sharpe ratio is hump-shaped in the RA parameter, while with Epstein-Zin preferences (dashed lines) the Sharpe ratio changes in a seemingly linear way with the RA parameter.

The patterns from Figure 3.10 are confirmed for a larger set of parameter values in Table 4. Except for the lowest level of the EIS parameter (leftmost column), there is a positive relation between γ and both the average log return equity premium and the unconditional log return Sharpe ratio. This relation is stronger the further ψ is away from 1, reflecting that the EIS parameter governs the cyclicality of returns.

[1] In general, the average log return equity premium is not zero, even when investors have an RA parameter of zero. The exception is the curve graphing the power utility case where the average log return equity premium at $\gamma = 0$ is exactly zero. Mathematically we can see that this holds by noting that Equation (3.11) equals $-\frac{1}{\psi - 1}(\sigma_\omega^2 + \sigma_{\omega,g})$ at $\gamma = 0$. Unless we are in the power utility case, where $\psi \longrightarrow +\infty$ as $\gamma \longrightarrow 0$, the average log return equity premium will not be zero at $\gamma = 0$.

Table 3.4

Predicted Equity Premium and Sharpe Ratio

This table reports the average log return equity premium (E $\left[r^e_{t+1} - r^f_t + \frac{1}{2}\sigma^2_t(r^e_{t+1})\right]$) and the Sharpe ratio for a range of RA (γ) and EIS (ψ) parameters. The Sharpe ratio is computed according to Equation 3.12. All values are in % p.a., computed with a time discount factor of 0.9925 and assuming a signal which is moderately informative ($h = 0.5$).

	ψ					
γ	0.25	0.50	0.75	1.20	1.75	2.50
Panel A. Equity Premium						
2	0.01	0.02	0.04	0.05	0.06	0.07
4	-0.02	0.05	0.09	0.12	0.14	0.15
10	-0.13	0.13	0.23	0.32	0.36	0.39
25	-0.59	0.29	0.62	0.87	1.01	1.10
50	-1.64	0.53	1.29	1.87	2.18	2.38
75	-2.24	0.90	1.96	2.76	3.19	3.46
100	-2.28	1.38	2.60	3.52	4.00	4.31
Panel B. Sharpe Ratio						
2	0.00	0.02	0.03	0.04	0.04	0.04
4	-0.01	0.05	0.07	0.08	0.08	0.08
10	-0.06	0.12	0.18	0.21	0.21	0.21
25	-0.26	0.26	0.49	0.56	0.58	0.58
50	-0.69	0.48	1.03	1.20	1.23	1.25
75	-1.02	0.82	1.56	1.78	1.83	1.85
100	-1.17	1.27	2.04	2.28	2.34	2.36

VI. Conclusions

This chapter focuses on the implications of changes in the quality of information on asset prices in a pure exchange economy. Matching empirical figures with model predictions has been a challenging aim ever since the seminal contribution of Mehra and Prescott (1985). When variations in information quality are introduced, the model predictions become even more puzzling. Veronesi (2000) has shown that if investors maximize a power utility function, the required risk premium is increasing in the quality of information. He also shows that, in this case, there is a strict and small upper bound for the attainable equity premium.

I generalize Veronesi's model by using the Epstein-Zin utility specification. This allows us to revisit the relation between information quality and the equity premium for a broader set of parameter combinations. I provide both an analytical and a numerical analysis of the relation and find a large region of parameter values for which his result is overturned. I also find that the upper bound on the equity premium is an artifact of the restriction embedded in the power utility specification and that there is no apparent local maximum on the equity premium with Epstein-Zin preferences.

The parameter region where the equity premium is decreasing in signal quality contains both cases where investors have an EIS greater than 1 and an EIS smaller than 1, provided that they are sufficiently risk averse. When the EIS is less than 1, the interplay between the utility parameters switches the signs of the relevant second moments, allowing the model to predict an equity premium which is decreasing in signal quality. When the EIS is greater than 1, the result is mainly driven by the capability of the model to generate procyclical prices. The degree to which this procyclicality translates into a positive covariance between consumption and returns, and hence high risk premia, depends on the quality of the signals available to investors. The better the external information available, the less prices will be driven by the information embedded in consumption growth rates, and the smaller the covariance will be.

VII. Computational Details

A. Discretized Dynamics

For a given model economy, equation (3.8) relates the current price-consumption ratio to expectations of future consumption growth rates and future price-consumption ratios. The only state variable the time t consumption-price ratio depends on is the investors posterior boom probability given their t information set. Lacking a functional form for the price-consumption ratio, I solve for price-consumption ratios in a discretized version of the model.

Denote the posterior boom probability of the representative investor, given the time t information set, by ξ_t and his posterior boom probability, given the time t information set less the external signal, by ξ_t^c. Conceptually, I think of the investor as first observing the consumption growth rate and updating his belief based on this realization and then observing the external signal: After observing the consumption growth rate, his posterior boom probability is ξ_{t+1}^c; after observing the external signal, it is updated to ξ_{t+1}. I discretize the interval $[0, 1]$, the support of the posterior boom probability into n equally spaced grid points. ξ_i, the ith grid point, is at $(i - 1)/(n - 1)$.

Let $F(\xi_{t+1}^c|\xi_t)$ denote the one period ahead distribution function for the posterior boom probability given the consumption signal. I start by approximating this distribution function with the discrete distribution

$$\hat{F}(\xi_i^c|\xi_t = \xi_j) = \begin{cases} \frac{F(\xi_i^c|\xi_t=\xi_j)+F(\xi_{i+1}^c|\xi_t=\xi_j)}{2} & 1 \leq i \leq n \\ 1 & i = n. \end{cases}$$

Analogously, let $G(\xi_{t+1}|\xi^c_{t+1})$ denote the distribution of the posterior boom probabilities after the consumption signal is observed, but before the external signal is observed. I approximate this distribution with the distribution function

$$\hat{G}(\xi_i|\xi^c_{t+1}=\xi_j) = \begin{cases} \frac{G(\xi_i|\xi^c_{t+1}=\xi_j)+G(\xi_{i+1}|\xi^c_{t+1}=\xi_j)}{2} & 1 \leq i \leq n \\ 1 & i = n. \end{cases}$$

Let $\hat{\Theta}^c$ be the matrix of transition probabilities between the grid points given only the consumption signal. The element $\hat{\Theta}^c_{i,j}$ gives the probability of the representative investors posterior boom probability at time $t+1$ being ξ_i given that his posterior probability at time t was ξ_j and the model parameters. I compute the elements of $\hat{\Theta}^c$ by

$$\hat{\Theta}^c_{i,j} = \begin{cases} \hat{F}(\xi_i|\xi_t=\xi_j) & i=1 \\ \hat{F}(\xi_i|\xi_t=\xi_j) - \hat{F}(\xi_{i-1}|\xi_t=\xi_j), & 2 \leq i \leq n. \end{cases}$$

Analogously, let $\hat{\Theta}^y$ be the matrix of transition probabilities between posterior boom probabilities given only the consumption signal and the posterior boom probabilities given both the external signal and the consumption signal. The elements of Θ^y are computed in the same manner as those of Θ^c, using the approximate conditional distribution $\hat{G}$ instead of $\hat{F}$.

Finally, I collect the transition probabilities $\Pr(\xi_{t+1}=\xi_i \mid \xi_t=\xi_j)$ in the matrix $\hat{\Theta}$ computed as

$$\hat{\Theta} = \hat{\Theta}^y\hat{\Theta}^c.$$

The ergodic discretized distribution of ξ solves

$$\pi^\star = \hat{\Theta}\pi^\star,$$
$$1 = \sum_s \pi^\star(s).$$

I solve for the gross consumption growth rate that takes investor beliefs from $\xi_t = \xi_j$ to ξ^c_{t+1} and collect them in the matrix $\hat{C}$:

$$\hat{C}_{i,j} = \mathcal{F}(\xi^c_{t+1} = \xi_i, \xi_t = \xi_j).$$

In my discretized economy, the vector of state price-consumption ratios $\hat{W}$ now solves:

$$\hat{W}_i = \beta^\kappa \sum_{j=1}^{n} \sum_{k=1}^{n} \Theta^y_{j,k} \Theta^c_{k,i} (1 + \hat{W}_j)^\kappa \hat{C}^{1-\gamma}_{k,i}.$$

Equity returns depend on both the consumption signal and on the external signal. If the consumption signal takes the investor's beliefs from ξ_j to state ξ_k and the external signal takes the investor's beliefs to ξ_i, I take the realized return to equity to be:

$$R^e(\xi_{t+1} = \xi_i, \xi^c_{t+1} = \xi_k, \xi_t = \xi_j) = \frac{1 + \hat{W}_i}{\hat{W}_j} \hat{C}_{j,k}$$

B. Financial Statistics

The conditional gross risk free rate and the gross expected return to equity in the model can be calculated using:

$$R_t^f(\xi_t = \xi_j) = \left(\beta^\kappa \sum_{i=1}^n \sum_{k=1}^n \hat{\Theta}_{i,k}^y \hat{\Theta}_{k,j}^c \hat{C}_{k,j} \left(\frac{1 + \hat{W}_i}{\hat{W}_j} \right)^{\kappa - 1} \right)^{-1},$$

$$\mathrm{E}_t \left[R_{t+1}^e \mid \xi_t = \xi_j \right] = \sum_{i=1}^n \sum_{k=1}^n \hat{\Theta}_{i,k}^y \hat{\Theta}_{k,j}^c \frac{1 + \hat{W}_i}{\hat{W}_j} \hat{C}_{k,j}.$$

The conditional moments of the variables in the log-linear approximation are found by weighing their values in each of the possible paths with the probability of the paths:

$$\mathrm{E} \left[\omega_{t+1} \mid \xi_t = \xi_j \right] = \sum_{i=1}^n \hat{\Theta}_{i,j} \log(1 + \hat{W}_i) - \log \hat{W}_j,$$

$$\mathrm{E} \left[\omega_{t+1}^2 \mid \xi_t = \xi_j \right] = \sum_{i=1}^n \hat{\Theta}_{i,j} (\log(1 + \hat{W}_i) - \log \hat{W}_j)^2,$$

$$\mathrm{E} \left[g_{t+1} \mid \xi_t = \xi_j \right] = \sum_{i=1}^n \hat{\Theta}_{i,j}^c \log \hat{C}_{i,j},$$

$$\mathrm{E} \left[g_{t+1}^2 \mid \xi_t = \xi_j \right] = \sum_{i=1}^n \hat{\Theta}_{i,j}^c (\log \hat{C}_{i,j})^2,$$

$$\mathrm{E} \left[g_{t+1} \omega_{t+1} \mid \xi_t = \xi_j \right] = \sum_{i=1}^n \sum_{k=1}^n \hat{\Theta}_{i,k}^y \hat{\Theta}_{k,j}^c (\log(1 + \hat{W}_i) - \log \hat{W}_j + \log \hat{C}_{k,j}).$$

The conditional variances and covariances in Equation (3.11) are then found by the general formulas

$$\sigma_t^2(x_{t+1}) = \mathrm{E}_t[x_{t+1}^2] - \mathrm{E}_t[x_{t+1}]^2,$$

$$\mathrm{cov}_t \left(x_{t+1}, y_{t+1} \right) = \mathrm{E}_t[x_{t+1} \; y_{t+1}] - \mathrm{E}_t[x_{t+1}] \mathrm{E}_t[y_{t+1}].$$

The reported average log equity premium is found by weighing the conditional equity premium for each of the belief states ξ_i with its ergodic probability or

$$\mathrm{E}\left[r^e_{t+1} - r^f_t + \frac{1}{2}\sigma^2_t(r^e_{t+1})\right] = \sum_{i=1}^{n} \pi^\star_i \left(\mathrm{E}\left[r^e_{t+1} \mid \xi_t = \xi_i\right] - r^f_t + \frac{1}{2}\sigma^2_t(r^e_{t+1})\right).$$

The reported Sharpe ratio is found by dividing the average equity premium by the unconditional variance of log equity returns:

$$\sigma^2(r^e_{t+1}) = \sigma^2(\omega_{t+1}) + \sigma^2(g_{t+1}) + 2\mathrm{cov}\left(g_{t+1}, \omega_{t+1}\right).$$

VIII. Proofs and Derivations

Proof of Proposition 1. Denoting the wealth (assets) of the representative investor by A_t, I rewrite the value function of the representative investor as

$$V_t(A_t) = \left((1-\beta)C_t^{1-\frac{1}{\psi}} + \beta\mathcal{R}(V_{t+1}(A_{t+1}))^{1-\frac{1}{\psi}}\right)^{\frac{1}{1-\frac{1}{\psi}}},$$

where $A_{t+1} = (A_t - C_t)R^e_{t+1}$. Taking the first order condition for optimal consumption C_t, rearranging, and using the fact that the risk adjustment $\mathcal{R}$ is homogeneous of degree 1 gives

$$\frac{A_t}{C_t} = \frac{1}{1-\beta}\left(\frac{V_t}{C_t}\right)^{1-\frac{1}{\psi}}.$$

This equation links the wealth consumption ratio to the scaled continuation value of the representative investor. In equilibrium, the representative investor's portfolio consists of one unit of the claim to the aggregate consumption, so the left hand side of the equation above equals the quantity W_t in the text. I now need to establish that the continuation value V_t is monotonically increasing in the boom probability. Let ξ and ξ' be two possible boom probabilities with $\xi < \xi'$. Given the process assumption and $\theta_1, \theta_2 > 0.5$, the distribution of future consumption conditional on ξ is stochastically dominated by the distribution of future consumption conditional on ξ'. It follows that the continuation value V_t is higher under ξ' than under ξ. This establishes that V_t is increasing in the boom probability if the two states are sufficiently persistent. It follows immediately that W_t is increasing in the boom probability if $\psi > 1$ and that it is decreasing in the boom probability if $\psi < 1$. For $\psi = 1$, the exponent on the right hand side of the last equation is zero, making W_t constant and equal to $1/(1-\beta)$.

□

Proof of Proposition 2. Here and henceforth I define $g_{t+1} = \log \frac{C_{t+1}}{C_t}$. In my setting, the return to any asset will obey the Euler equation $\mathrm{E}_t\left[M_{t+1}R_{t+1}\right] = 1$. When g_{t+1} and r^e_{t+1} are conditionally jointly log normal, I can log-linearize the Euler equations for the return to equity (R_e) and the risk-free rate (R_b). I obtain the following system of equations:

$$\begin{aligned}
0 =& \kappa \log(\beta) - \frac{\kappa}{\psi}\mathrm{E}_t\left[g_{t+1}\right] + \kappa\,\mathrm{E}_t\left[r^e_{t+1}\right] \\
&+ \frac{1}{2}\left[\left(\frac{\kappa}{\psi}\right)^2 \sigma^2_t(g_{t+1}) + \kappa^2\sigma^2_t(r^e_{t+1}) - \frac{2\kappa^2}{\psi}\mathrm{cov}_t\left(g_{t+1}, r^e_{t+1}\right)\right],
\end{aligned}$$

$$\begin{aligned}
0 =& \kappa \log(\beta) - \frac{\kappa}{\psi}\mathrm{E}_t\left[g_{t+1}\right] + (\kappa-1)\,\mathrm{E}_t\left[r^e_{t+1}\right] + r^f_t \\
&+ \frac{1}{2}\left[\left(\frac{\kappa}{\psi}\right)^2 \sigma^2_t(g_{t+1}) + (\kappa-1)^2\sigma^2_t(r^e_{t+1}) - \frac{2\kappa(\kappa-1)}{\psi}\mathrm{cov}_t\left(g_{t+1}, r^e_{t+1}\right)\right],
\end{aligned} \tag{3.14}$$

where $\sigma^2_t(g_{t+1})$ and $\sigma^2_t(r^e_{t+1})$ denote the conditional variance of the consumption growth rate and the return to equity, respectively and $\mathrm{cov}_t\left(g_{t+1}, r^e_{t+1}\right)$ denotes their conditional covariance. Lowercase letters denote logged variables. Taking the difference between the two equations, I get[1]

$$\mathrm{E}_t\left[r^e_{t+1}\right] - r^f_t + \frac{1}{2}\sigma^2_t(r^e_{t+1}) = (1-\kappa)\,\sigma^2_t(r^e_{t+1}) + \frac{\kappa}{\psi}\mathrm{cov}_t\left(g_{t+1}, r^e_{t+1}\right), \tag{3.15}$$

that is, the equity premium plus a Jensen's inequality term.

Now I can use the definition of log returns, $r^e_{t+1} = \omega_{t+1} + g_{t+1}$, to calculate the second moments in Equation (3.15):

$$\begin{aligned} \sigma^2_t(r^e_{t+1}) &= \sigma^2_t(\omega_{t+1}) + \sigma^2_t(g_{t+1}) + 2\mathrm{cov}_t\left(\omega_{t+1}, g_{t+1}\right), \\ \mathrm{cov}_t\left(g_{t+1}, r^e_{t+1}\right) &= \sigma^2_t(g_{t+1}) + \mathrm{cov}_t\left(\omega_{t+1}, g_{t+1}\right). \end{aligned} \tag{3.16}$$

The expression for the equity premium follows by substituting Equation (3.16) in equation (3.15). □

[1] This corresponds to Equation (8.3.7) on page 320 of Campbell et al. (1997).

4

The role of regimes for asset pricing in RBC models with recursive preferences

Abstract

Unless extreme forms of rigidity are allowed, standard Real Business Cycle models are well known to generate counterfactual asset pricing implications. This chapter proposes an approach to circumvent all such rigidities by providing a simple extension to the prior literature. I set up an economy that follows a regimes process both in the mean and the volatility, in conjunction with Epstein-Zin preferences for the consumers. I provide a detailed theoretical and numerical analysis on the model's predictions. I also show that a reasonable parametrization of the "rigidity-free" model conveys plausible financial figures that are in line with empirical observations over the U.S. postwar economy. Furthermore, I provide evidence to support the necessity of modelling the decline of macroeconomic risk in this class of models.

I. Introduction

The role of asset prices in macroeconomics gained huge popularity after Mehra and Prescott (1985). Besides the well celebrated equity premium puzzle, the main implication of their seminal contribution is that, in a simple endowment economy where agents have a power utility function, by arbitrarily increasing the risk aversion parameter, one can make an agent sufficiently risk averse to offset the observed low volatility of consumption and thus obtain a high risk premium. On the contrary, in a standard RBC model, an agent can smooth her consumption path by substituting between labor and leisure in response to productivity shocks. So, by just increasing her coefficient of risk aversion, we are not able to solve the equity premium puzzle[1].

This is one serious flaw of standard RBC models that enjoy a high capability of explaining the main features of business cycles, but need the introduction of restrictive assumptions to explain observed financial figures. This chapter proposes a way to overcome this problem by providing a simple extension to the prior RBC literature. I introduce an economy that switches between booms and busts where technological shocks follow a hidden two state Markov chain, in conjunction with recursive preferences for the consumers.

Assuming a different utility specification is not novel in the literature. In fact, alternatives to the power utility specification and their implications in the asset pricing literature have been studied since the late eighties. One particularly prominent line of research examines the asset-pricing implications of including habits in the utility function (e.g. Constantinides, 1990; Abel, 1990; Gali, 1994; Jermann, 1998, Campbell and Cochrane, 1999; and Boldrin et al., 2001.) Another strand of this research, which emanated from Epstein and Zin (1989), Epstein and Zin (1991), and Weil (1989), examines generalizations of the power utility function that unlink risk aversion and the

[1] for an exhaustive analysis on the role of asset prices in RBC models see Lettau (2003).

elasticity of intertemporal substitution.

Here, I adopt this utility function, referred to as Epstein-Zin preferences. Two reasons drive this choice. First, this form of utility function is widely used in the latest asset-pricing research (see Bansal and Yaron, 2004; Campbell and Viceira, 2001; Campbell et al., 2003; Brandt et al., 2004; Lettau et al., 2008; Guvenen, 2005 among others). Second, since Epstein-Zin preferences nest the power utility function as a special case, these are particularly useful to provide a closer comparison with the standard models based on power utility specification.

In the RBC literature, the first analysis on asset prices, (Danthine et al., 1992, Rouwenhorst, 1995) while unsuccessful in explaining the behavior of returns over the business cycle, provided useful insights on what would be a necessary ingredient of a successful model. Along this line, to improve the capability in explaining financial figures, the main literature on asset pricing with a non-trivial production side (Jermann, 1998, Boldrin et al., 2001), introduced some form of rigidity in the model. While both Boldrin et al. (2001) and Jermann (1998) specify a habit utility for consumers, the former contribution relies on a limited mobility of production's factors and the latter introduces capital adjustment costs[1].

The assumption of a recursive but time non-separable felicity function is not novel in this literature. Tallarini (2000), or more recently, Gomes and Michaelides (2008), assume Epstein-Zin utility function, and both document the inability to generate reasonable return's figures without introducing some production frictions in the model.

I take a different approach on the production side of the model and instead of imposing any kind of restriction, I provide a simple extension of a standard RBC model where the economy switches between booms and busts. This is accomplished by letting the economy follow a hidden markov chain. Most of the literature studies the implica-

[1] A different approach can be found in Cochrane (1991) who evaluates asset pricing implications from the producer's first order conditions.

tion of a Markov switching process in the conditional mean and in the volatility of the endowment process[1]. Differently, here the regimes are introduced via the production side by allowing the technology shocks to follow a latent two states process both in the mean and the volatility[2].

This work is closely related with the main literature on asset pricing with a non-trivial production side cited above, and it contributes a novel theoretical framework where a sizeable equity premium can be obtained without imposing any kind of rigidity on the production side of the model and without the need of an implausibly high value for the risk aversion of the agents.

The remainder of the chapter is organized as follows: Section II. introduces the general model, derives equilibrium asset prices, and analyzes the determinants of the equity premium predicted by the model. Section III. discusses model calibration and provides numerical analysis of asset returns' properties over the business cycle, while section IV. provides a sensitivity analysis of the results' determinants. Section V. concludes. Proofs, algebraic derivations, and additional results are provided in Appendix VI..

[1]Regime switching is widely used in economics since the seminal contribution by Hamilton (1989). In particular, in the asset pricing literature, the implications of a Markov switching process in the conditional mean of the endowment process are analyzed by Cecchetti et al. (1990); Kandel and Stambaugh (1991); Cecchetti et al. (1993); Abel (1994); Abel (1999). Recently, the time series properties of the second moments gained popularity in this framework: by setting up an equilibrium economy where the endowment process follows a latent two state regime switching process, Veronesi (1999) shows a better explanatory power of volatility clustering than a model without regimes. In the same setting, Whitelaw (2000) introduces time-varying transition probabilities between regimes, finding a complex nonlinear relation between expected returns and stock market volatility. A recent contribution that studies the impact of regime switches in the volatility of the endowment process is in Lettau et al. (2008).

[2]In a different setting Cagetti et al. (2002) model the technology shocks as a Markov switching model in the first moment.

II. Model

A standard production economy with two actors is considered. Consumers are modeled via a representative, risk averse, agent which derives utility from consumption, while the production side is modeled through a standard representative firm that maximizes its shareholders' value. There are two securities in the economy: a riskless bond that agents can use for transferring their wealth to the future, and equity, which provides a claim on firm's profits.

A. Firms and technology

Each period the firm has to decide how much human capital to employ and how much capital to invest in physical assets.

In particular, there is only one traded good which is produced through a constant return to scale technology. Analytically, production can be described using a Cobb-Douglas production function with human and physical capital as factors:

$$Y_t = A_t K_t^{\theta} H_t^{1-\theta}, \tag{4.1}$$

where θ is the share of physical capital.

The human capital H evolves according to:

$$H_{t+1} = (1 - \delta_H)H_t + E_t, \tag{4.2}$$

where δ_H is its depreciation rate, and E_t is the investment in education.[1]

[1] As in Barro and Sala-I-Martin (2004) we can think of human capital as the number of workers multiplied by the human capital of a typical worker.

The capital stock's K evolution is governed by:

$$K_{t+1} = (1 - \delta_K)K_t + I_t, \tag{4.3}$$

where δ_K is its depreciation rate, and I_t indicates capital investment.

If we consider the productivity shock (A) in a regime switching model, we can express its law of motion as a process with stochastic parameters depending on the state of the economy:

$$\Delta \log A_t = \mu(s_t) + \sigma(s_t)\varepsilon_t, \tag{4.4}$$

where μ and σ define the mean and the volatility of the process, and s_t indicates the state of the economy. I assume that s_t follows a hidden Markov chain with transition probabilities matrix P (see Hamilton (1989)). The evolution of the state of the economy in terms of state beliefs (ξ_{t+1}) can be expressed as realizations of the equation:

$$\xi_{t+1} = P\xi_t + \epsilon_t. \tag{4.5}$$

The agents cannot directly observe the state of the economy, s_t, and they have to rely on interpreting external signals. The agents update their belief according to the posterior probabilities computed as

$$\hat{\xi}_{t+1|t} = P\frac{\hat{\xi}_{t|t-1} \odot \zeta_t}{\mathbf{1}'\left(\hat{\xi}_{t|t-1} \odot \zeta_t\right)}, \tag{4.6}$$

where $\odot$ denotes the Hadamard product, ζ_t is a vector that stacks the conditional densities of the technological shocks' growth rates:

$$\zeta_t = \begin{bmatrix} f\left(\Delta \log A_t \mid s_t = 1, \Omega_{t-1}\right) \\ \vdots \\ f\left(\Delta \log A_t \mid s_t = n, \Omega_{t-1}\right) \end{bmatrix} \tag{4.7}$$

with the density of $\Delta \log A_t$ conditional on state s_t is defined as:

$$f\left(\Delta \log A_t \mid s_t = i, , \Omega_{t-1}\right) = \frac{1}{\sqrt{2\pi}\sigma(s_t)} \exp\left\{-\frac{\left(\Delta \log A_t - \mu(s_t)\right)^2}{2\sigma(s_t)^2}\right\}, \tag{4.8}$$

where Ω denotes the information set.

B. Consumers

The representative agent has preferences defined over current consumption and future utility. Following Epstein and Zin (1989, 1991), the utility function is defined recursively by:

$$U(C_t, \mathbb{E}_t(U_{t+1})) = \left[(1-\beta)C_t^{\frac{1-\gamma}{\alpha}} + \beta(\mathbb{E}_t(U_{t+1}^{1-\gamma}))^{\frac{1}{\alpha}}\right]^{\frac{\alpha}{1-\gamma}}, \tag{4.9}$$

where C_t indicates aggregate consumption, β is the time preference parameter, and $\alpha \equiv (1-\gamma)/(1-1/\psi)$, with $\gamma > 0$.

The parameter γ is the coefficient of relative risk aversion (RA), while the elasticity of intertemporal substitution (EIS) is given by ψ [1].

[1] An interesting feature of this utility function specification is that it nests the power utility. In fact, when $\gamma = \frac{1}{\psi}$ equation 4.9 can be solved forward to get the standard power utility function.

C. Equilibrium

To begin, I set up a social planner problem where the resource constraint is defined as:[1]

$$C_t + I_t \leq Y_t. \tag{4.10}$$

Following the standard literature, I can characterize the equilibrium allocation by solving the social planner problem where the welfare function is defined as:

$$\mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t U_t \right]. \tag{4.11}$$

The solution can be found by maximizing 4.11 with respect to 4.1, 4.2, 4.3, 4.4 and 4.10.

3. Asset prices

Next, I derive the equilibrium asset prices implied by the model. We know from Hansen and Richard (1987) that optimality of the consumers' solution implies the existence of a unique pricing kernel (Q) for pricing all the available assets (i.e. the Euler equation holds true for any asset return (R) as $\mathbb{E}_t \left[Q_{t+1} R_{t+1} \right] = 1$).

Epstein and Zin (1989) show that the stochastic discount factor for the case of utility over consumption is given by

$$Q_{t+1} = \beta^{\alpha} \left(\frac{C_{t+1}}{C_t} \right)^{-\frac{\alpha}{\psi}} \left(R_{c,t+1} \right)^{\alpha - 1}, \tag{4.12}$$

where $R_{c,t+1}$ is the equilibrium gross return to consumption claim between t and $t+1$.

I examine two different types of assets: a one period asset that yields one unit of consumption (i.e. a "risk-free" asset), and a claim to the physical capital (i.e. an

[1]This implicitly implies that consumption includes education expenditures. That is once the optimal consumption is chosen, the education expenditure is determined by the budget constraint.

"equity asset"). If firms only use retained earnings to finance their capital the equity holder gets a dividend $D_t = \frac{\partial Y_t}{\partial K_t} K_t - I_t = \theta Y_t - I_t$. This implies that the unlevered stock market of this economy represents the value of the capital stock.

Proposition 3. *When the economy meets a "balanced growth condition"* [1]*, then the gross dividends' growth rate can be expressed as:*

$$\frac{D_{t+1}}{D_t} = \frac{A_{t+1}}{A_t}\left[\frac{K_{t+1}}{K_t}\right]^{\theta}\left[\frac{H_{t+1}}{H_t}\right]^{1-\theta} = \frac{A_{t+1}}{A_t}\lambda_t^{\theta}\eta_t^{1-\theta}, \tag{4.13}$$

where $\eta_t = \left(1 - \delta_H + \frac{E_t}{H_t}\right)$ *and* $\lambda_t = \left(1 - \delta_K + \frac{I_t}{K_t}\right)$.

Moreover, when the economy meets a "balanced growth condition", consumption and dividends share the same gross growth rate.

Given proposition 3 I can express the price to consumption ratio (PC) of the consumption claim $R_{c,t+1} = \left(\frac{P^c_{t+1}+C_{t+1}}{P^c_t}\right)$ as:

$$PC_t^{\alpha} = \mathbb{E}_t\left[\beta^{\alpha}\left(\frac{A_{t+1}}{A_t}\lambda_t^{\theta}\eta_t^{1-\theta}\right)^{1-\gamma}(PC_{t+1}+1)^{\alpha}\right]. \tag{4.14}$$

In the same fashion I can express the price to dividend ratio (PD) of the dividend claim $R_{e,t+1} = \left(\frac{P^e_{t+1}+D_{t+1}}{P^e_t}\right)$ as

$$PD_t = \mathbb{E}_t\left[\beta^{\alpha}\left(\frac{A_{t+1}}{A_t}\lambda_t^{\theta}\eta_t^{1-\theta}\right)^{1-\gamma}(PC_{t+1}+1)^{\alpha-1}(PC_t)^{1-\alpha}(PD_{t+1}+1)\right] \tag{4.15}$$

Proof. see Appendix VI. □

Following the approach of Lettau et al. (2008), I solve Equations 4.14 and 4.15

[1] The same result follows if the ratio of investment to output is assumed to be constant between two periods (i.e. $\frac{I_{t+1}}{Y_{t+1}} = \frac{I_t}{Y_t}$).

numerically. Thus I can get the equity return from:

$$R_{e,t+1} = \frac{1 + PD_{t+1}}{PD_t}\frac{D_{t+1}}{D_t}, \tag{4.16}$$

and calculate its second moment as

$$\mathbb{E}_t\left[R_{e,t+1}^2\right] - \mathbb{E}_t\left[R_{e,t+1}\right]^2. \tag{4.17}$$

Finally, the risk free rate can be expressed as: $R_{f,t+1} = \left(\mathbb{E}_t\left[Q_{t+1}\right]\right)^{-1}$, from which I can calculate both its first and second moment.

D. The equity premium

In this section I provide a first grasp on the determinants of the equity premium implied by the model. I do that by log-linearizing the Euler equations for the equity asset and the risk free asset respectively, and solving them for the expected excess return. Thus, I can study the role played by the interplay of the utility function parameters, namely the risk aversion and the elasticity of intertemporal substitution.

4. The role of utility parameters

Following the analysis in Campbell et al. (1997), if consumption growth rates and asset returns are homoskedastic and jointly lognormal, the equity premium can be expressed as[1]

$$EP = \gamma\sigma_g^2 + \left(1 - \frac{1-\gamma}{1-1/\psi}\right)\sigma_\omega^2 + \left(1 - \frac{1-\gamma}{1-1/\psi} + \gamma\right)\sigma_{\omega,g} \tag{4.18}$$

[1] Even if the log-linearization does not strictly hold for this non-linear economy, the implications that follow are valuable.

where σ_g^2 is the variance of the log consumption growth, σ_ω^2 is the variance of log $\frac{1+PDt+1}{PD_t}$, and $\sigma_{\omega,g}$ is their covariance[1].

The first component of equation 4.18, $\gamma\sigma_g^2$, is the determinant of the equity premium when an agent has a power utility function. Clearly, as has already been established in the literature, the only way to increase the equity premium through this term is to increase the coefficient of risk aversion, leading to the well known finance puzzles.

The second component links the variance of increases in the price-dividend ratio with the utility function parameters. The variance is always positive so, we can focus on the coefficient $\left(1 - \frac{1-\gamma}{1-1/\psi}\right)$ to analyze the contribution of this term to the equity premium. To have a positive contribution of the above coefficient we need the term $\left(\frac{1-\gamma}{1-1/\psi}\right)$ to be less than 1. When the elasticity of intertemporal substitution (EIS) is larger than 1, this implies a higher RA parameter relative to the inverse of the EIS, or a preference for early resolution of uncertainty, in the language of Kocherlakota, 1990.

It is important to note that an EIS larger than 1, implies procyclical prices (see Brevik and d'Addona (2006)). Thus, in order to have a positive contribution by the variance of prices to the equity premium we need an agent with a preference for the early resolution of uncertainty coupled with procyclical prices.

In the same fashion we can analyze the third component of equation 4.18. Focusing on the case when both RA and EIS are greater than 1, the prociclycality of prices leads to a positive covariance between consumption growth rates and prices themselves. So a positive contribution of this third term is assured if the numerator of its coefficient, rewritten as $\frac{\gamma(2-1/\psi)-1/\psi}{1-1/\psi}$ is positive. This turns out to be always true for the case I am focusing on.

The above analysis can be used to define the restrictions I can impose on the model to expect a better performance in explaining financial figures. That is: in order to have

[1]For a detailed loglinear analysis in a similar framework see Brevik and d'Addona (2006).

a positive contribution by all the terms in equation 4.18 to the equity premium, it is sufficient to assume preference for the early resolution of uncertainty for the representative agent and a EIS parameter strictly greater than 1.

Interestingly enough, both the preference for the early resolution of uncertainty and prociclycal prices are not necessary requirements for a positive contribution by the two second moments to the equity premium. In fact it is straightforward to see that, when ψ is less than 1, we still can have a positive value of the second component in equation 4.18, provided that agents have a preference for the late resolution of uncertainty (that is $\gamma < 1/\psi$). Moreover, the covariance of prices and consumption growth rates would be negative in this case. So a sufficient condition for having a positive contribution from the last term in equation 4.18 is to have a RA parameter greater than 1. Summarizing, if prices are countercyclical, then the equity premium is monotonically increasing in both the variance of prices and the (negative) covariance between consumption growth rates and prices themselves, if agents have preference for late resolution of uncertainty and have a RA parameter greater than 1.

III. Empirical analysis

A. Estimation

Having provided the theoretical implications given by my framework, I can now turn to the numerical analysis of the model.

The data used for the calibration span from the beginning of 1952 to the end of 2004. The dataset is expressed in real terms with a quarterly frequency. The financial series (prices and dividends) are on the S&P 500 composite, while the risk-free rate is the yield on 1 year treasury bills. These series are from Robert J. Shiller's webpage[1]. The

[1] http://www.econ.yale.edu/~shiller/data.htm.

main economic series are downloaded from the Bureau of Economic Analysis' website[1]. Consumption is quarterly real total personal consumption expenditures (NIPA table 2.3.6, line 1), GDP is quarterly real gross domestic income (NIPA table 1.1.6, line 1), investment in physical capital is quarterly non residential fixed investment (NIPA table 5.3.5, line 2), and education expenditures are personal education and research expenditures (NIPA table 2.5.5, line 104). Both the human capital and the physical capital series are constructed using the perpetual inventory method. Finally, I use the official recession dates as reported on the website of the National Bureau of Economic Research[2].

To estimate the technology shocks, a standard technique based on the growth accounting framework is applied. In particular, with constant return to scale, it is possible to decompose the output growth in two parts, and thus the change in technology shocks can be estimated as:

$$\Delta \log A = \Delta \log Y - \theta \Delta \log K - (1-\theta)\, \Delta \log H. \tag{4.19}$$

Figure 4.1 shows the time series of the estimated technological shocks. A procyclical behavior in the series is clearly evident.

The regime switching specification for the US economy with two possible states both for the mean and the volatility of the productivity shocks is also estimated. Parameter estimates for the model were computed using a Markov-Chain Monte-Carlo (MCMC) procedure following Kim and Nelson (1999).

The results from this analysis are given in Table 4.1. An important finding from my estimation is the high persistence of the states associated with the mean. In fact, the probabilities for the first moment of switching from the two states are 5.67% and

[1] http://www.bea.doc.gov/.

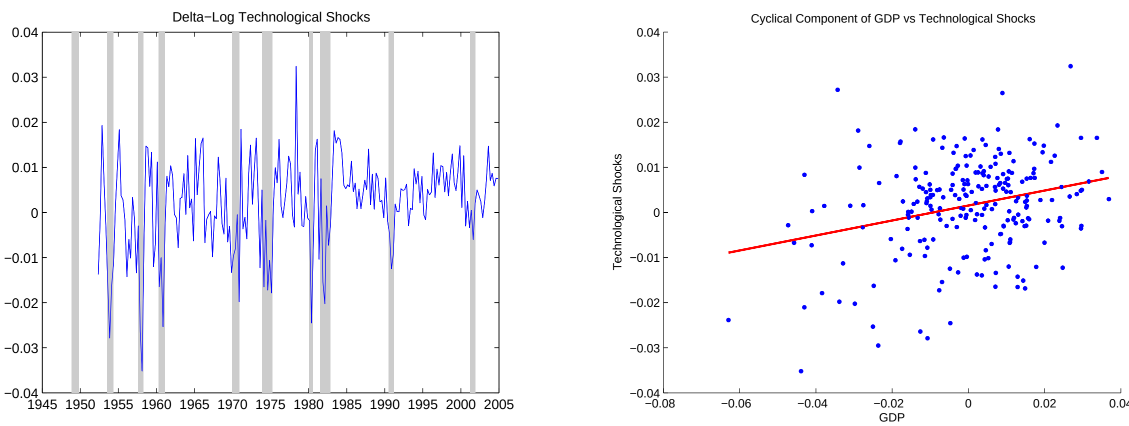
[2] http://www.nber.org/cycles.html/.

Figure 4.1

Estimated technological shocks

This figure plots the empirical estimate of technological shocks. Data, transformed with logarithms, are quarterly starting from I-1952 to IV-2004. Panel A plots the estimated series of technological shocks, coupled with the recession periods according to NBER (shadow areas in the graph). Panel B shows a scatter-plot of GDP cyclical component, estimated with a HP filter, versus the shocks.

Panel A | Panel B



19.72%, respectively. This implies an average duration of more than four years (17.7 quarters) for high mean states (associated with booms), and more than one year (5.3 quarters) for low mean states (associated with busts). Hence, if we find ourselves in either of the two states, we expect to stay there for several periods. The results are more striking for the second moment. Looking at its switching probabilities, it is clear how the volatility state is extremely persistent, so if we find ourselves in either one of the two states for the volatility, it is very well the case that we will face that state for the majority of a sample period.

To get a final grasp on the estimation of the regime switching economy, I investigate the capability of the model in picking up the historical business cycles of the US postwar economy. Figure 4.2 reports this analysis by plotting the estimated posterior probability, associated with the mean of the productivity shock process, of being in the recession state. It is evident how the Markov switching model is able to capture fairly

Table 4.1

Estimation of regime switching economy

This table reports the estimated parameters of a two state Markov switching model for the US postwar economy. The estimates are based on a MCMC algorithm from Kim and Nelson (1999) with both the mean and the volatility of technological shocks being different in the two possible states. The estimation is performed using real quarterly data (Q1:1952–Q4:2004; source: BEA). Standard errors are reported in parenthesis.

Technological shocks' process estimation				
State	$\mu(s)$	$\sigma(s)$	p^{μ}_{ij}	p^{σ}_{ij}
High (s=high)	0.0047	0.0101	0.0567	0.0191
	(0.0010)	(0.0009)	(0.0227)	(0.0164)
Low (s=low)	-0.0109	0.0046	0.1972	0.0209
	(0.0069)	(0.0005)	(0.0680)	(0.0207)

well the US recessions as chronicled by the official NBER business cycle dates (the gray areas in the graph).

Figure 4.3 reports a similar analysis by plotting the estimated posterior probabilities, associated with the volatility of the productivity shock process, of being in the low state. The obtained graph is consistent with the declining macroeconomic volatility starting in the mid eighties and documented widely in the literature (see Blanchard and Simon, 2001, and Lettau et al. (2008) among others), also named as "the great moderation" by Stock and Watson (2002).

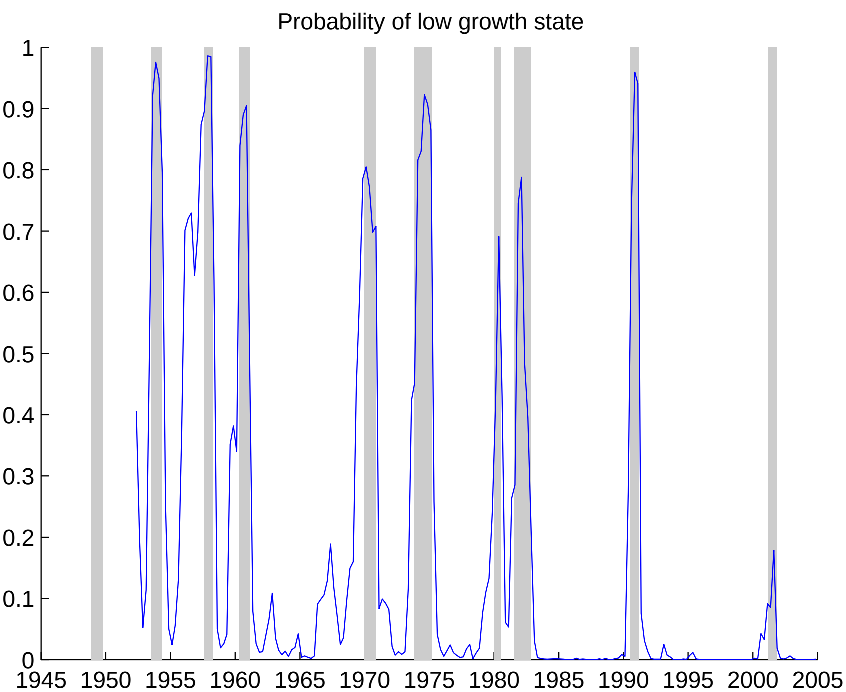
B. Calibration

The basic calibration sets the model's parameters as follows: the share of physical capital θ, is fixed to 0.36 which is the standard approach taken in the literature on business cycles. The depreciation rate for physical capital is fixed at 0.021 , which is also standard in the literature. For human capital, I follow Heckman (1976) by choosing a value of 0.009.

The ratios of investment to physical capital and the ratio of education expenses to human capital are set to replicate the difference in GDP growth rates during the two states of the economy. This gives investment to capital ratios of 0.0365 and 0.002

Figure 4.2
Posterior probabilities of a recession

This figure shows the estimated posterior probabilities of being in a recession coupled with the official NBER recession dates (shadow area). Data employed in the estimation are quarterly starting from I-1952 to IV-2004.



during booms and during busts respectively. Similarly, education expenses to human capital ratios are set to 0.0145 and 0.001 respectively.

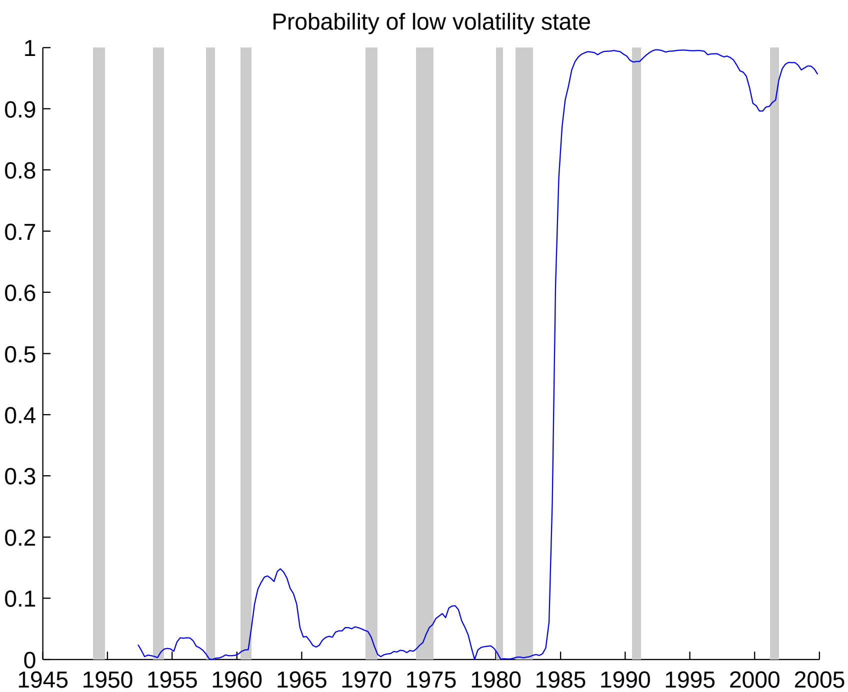
C. Results

In this subsection I present the results from solving the model numerically. As a first comparison, I set the utility function parameters as in Lettau et al. (2008): $\gamma = 30$, $\psi = 1.5$, and I fix $\beta = 0.9925$.

The basic results from this calibration are presented in table 4.2. It reports the set of estimates discarding the first five years of data, in order to address the well known

Figure 4.3
Posterior probabilities of the low volatility state

This figure shows the estimated posterior probabilities of being in a low volatility state coupled with the official NBER recession dates (shadow area). Data employed in the estimation are quarterly starting from I-1952 to IV-2004.



critique to the perpetual inventory methodology used in the capital estimations.

As shown in this first set of results, the model overshoots in estimating the mean equity premium. This is mainly due to the poor performance in matching the risk free rate. In fact, with the proposed parametrization, I obtain a real risk free rate that is negative and big (almost -7%).

Again, we can use a loglinear approximation as in Campbell et al. (1997), to interpret this quite odd result. Loglinearizing the Euler equation, and solving it for the risk free return, gives us a simplified framework to analyze it. The expected risk free rate can

Table 4.2

Financial series of the US economy using Lettau et al. (2008) utility parameters

This table shows the asset returns implied by the model calibrated on the US postwar economy. The estimation is based on real quarterly data (Q1:1952–Q4:2004; source: BEA). Reported are the estimates obtained by calculating capital and education investments on the whole sample and discarding the first five years of data to address the critiques to the perpetual inventory methodology. The market dataset is from Professor Robert J. Shiller webpage (http://www.econ.yale.edu/ shiller/data.htm). The coefficient of risk aversion is set to 30, the EIS is set to 1.5.

Series	Mean Data	Mean Model	Std. Data	Std. Model
Equities	0.072	0.084	0.074	0.030
Risk Free	0.013	-0.067	0.006	0.002
Equity premium	0.058	0.152	0.074	0.043
$\rho(r^f; r^e)$	0.047	-0.062	-	-
$\rho(r^e_{t+1}; r^e_t)$	0.105	0.594	-	-
$\rho(r^f_{t+1}; r^f_t)$	0.890	0.611	-	-

be expressed as:

$$r_{f,t+1} = -\log\beta + \frac{1}{\psi}\mathrm{E}_t\left[g_{t+1}\right] - \frac{\alpha}{2}\left[\frac{1}{\psi^2}\sigma_g^2 + \left(\frac{1}{\alpha} - 1\right)\sigma_{pc}^2\right] \tag{4.20}$$

where $\mathrm{E}_t\left[g_{t+1}\right]$ is the expected consumption growth rate, and σ_g and σ_{pc} denote the volatility of the log consumption growth rate and the volatility of the log return to the consumption claim, respectively.

By inspection, we can see that the coefficient on the volatility of the consumption claim, which is $(.5(\alpha - 1))$ is negative with the parametrization adopted above. So, this is the term that is pulling down the required risk free in the model, given the positivity of volatilities.

Given the unsatisfactory performance of the previous calibration, especially regarding the risk free rate, I perform a simple exercise: I fix the risk aversion parameter to 10, and I let the elasticity of intertemporal substitution vary to match the mean risk free rate of the postwar US economy[1]. The results for this calibration, obtained discarding

[1]It is well known that Mehra and Prescott (1985) indicate 10 as an upper bound for an acceptable RA parameter in their setting. But is is important to point out that even if a risk aversion parameter

Table 4.3

Empirical series of US financial markets matching the risk free rate

This table shows the asset returns implied by the model calibrated on the US postwar economy. The estimation is based on real quarterly data (Q1:1952–Q4:2004; source: BEA). Reported are the estimates obtained by calculating capital and education investments on the whole sample and discarding the first five years of data to address the critiques to the perpetual inventory methodology. The market dataset is from Professor Robert J. Shiller webpage (http://www.econ.yale.edu/ shiller/data.htm). The coefficient of risk aversion is set to 10. The obtained EIS by matching the empirical mean of the risk free rate is 1.371

Series	Mean Data	Mean Model	Std. Data	Std. Model
Equities	0.072	0.065	0.074	0.035
Risk Free	0.013	0.013	0.006	0.009
Equity premium	0.058	0.052	0.074	0.050
$\rho(r^f; r^e)$	0.047	0.061	-	-
$\rho(r^e_{t+1}; r^e_t)$	0.105	0.598	-	-
$\rho(r^f_{t+1}; r^f_t)$	0.890	0.610	-	-

the first five years of data, are reported in table 4.3.

As shown in this table, the model performs fairly well in matching the mean equity premium and the first moment of the equity asset. In fact the model predicts a yearly equity return of 6.5%, and I match the real risk free return to a level of 1.3%, with a EIS parameter of 1.371. This leads to a predicted yearly equity premium of 5.2%. The matching of second moments is also satisfactory. In particular, both the standard deviations of the risk free asset and the equity asset are of the same magnitude of their empirical counterparts. Regarding the correlation and the autocorrelation of the assets, the model performs well in matching the autocorrelation of the risk free asset as well as the correlation between equity and the risk free asset. Less satisfaction is derived from the performance in matching the equities' autocorrelation. This is probably due to the nature of the model in which prices have a strong persistence with respect to the states of the economy.

higher than 10 can be perceived as implausible in a standard power utility setting, the parameter's implications in a Epstein-Zin utility framework change dramatically with respect to the power utility case. For a detailed theoretical characterization of these implications see Campanale et al. (2007).

The drastic improvement in the model performance is due to the decrease of both the risk aversion and the elasticity of intertemporal substitution assumed for the representative agent. By lowering the agent risk aversion we directly influence the equity claim: a lower value of γ makes equities more appealing; this leads to higher prices and so decreases the equity return. Instead, the risk free claim is directly influenced by the elasticity of substitution: the lower is the EIS the lower is the willingness of an agent to transfer her wealth overtime; this leads to lower prices for a risk free asset and so to a higher risk free return.

Before moving ahead in the empirical analysis, it is worth analyzing the value obtained for the EIS parameter to match the risk free return, given that the empirical estimates in the literature vary considerably. One approach of the empirical research focuses on a representative agent setup and uses aggregate consumption data. This leads to estimates of the EIS coefficient below 1, and even close to 0 (see e.g. Hall, 1988; Campbell and Mankiw, 1989, 1991; Hahm, 1998; Yogo, 2004; and Zhang, 2006). Another strand of research relies on microeconomic survey data to avoid potential biases in the aggregate data. If a stockholder is considered, these studies find EIS parameters around or above 1. (See Beaudry and van Wincoop, 1996; Vissing-Jørgensen, 2002; Vissing-Jørgensen and Attanasio, 2003; and Guvenen, 2005). Even if there is this lack of consensus in the economic literature, the recent asset-pricing literature relies on the higher EIS estimates of the latter literature, in fact both Bansal and Yaron (2004) or Lettau et al. (2008) calibrate their models with an EIS greater than one. As discussed above (cf. section 4.) this choice is mainly linked to the capability of generating procyclical prices when agents have a recursive utility function. Consequently, we can consider the value of 1.371 for the EIS, obtained by matching the risk free rate, in line with the latter literature, and theoretically well grounded.

Table 4.4

Estimated parameters for the restricted regime switching model

Reported are the estimated parameters for the restricted version of the model. The estimates are based on a MCMC algorithm from Kim and Nelson (1999) with the mean of technology shocks being different in two possible states. The estimation is performed using real quarterly data (Q1:1952–Q4:2004; source: BEA). Standard errors are reported in parenthesis.

State	$\mu(s)$	σ	p_{ij}^{μ}
High (s=high)	0.0051		0.0630
	(0.0007)	0.0074	(0.0219)
Low (l=low)	-0.0112	(0.0004)	0.2244
	(0.0019)		(0.0692)

D. The role of macroeconomic risk

Loosely speaking, the insight we can gather from Lettau et al. (2008) is that the reduction in macroeconomic volatility in the last twenty years can account for a good portion of assets' valuations in the recent past. So, it is natural to ask ourselves what is the role of the volatility regimes in the economy I am studying. I then re-estimate the regime switching economy, imposing the restriction of a single state for the volatility of the productivity shocks.

The parameters for the restricted regime switching model are again obtained using the same Markov-Chain Monte-Carlo (MCMC) algorithm used for the unrestricted model. The resulting estimates are given in table 4.4.

As expected the estimated value for the volatility is in between the values I obtained for the two states version of the regime switching process. The main departure from the unrestricted estimation is that the difference between the means in the two states is sharper. Furthermore, the estimated probabilities of switching from the two mean states are 6.30% and 22.08%, respectively. These probabilities confirm the persistence of each state also in the restricted setting.

I can now move to analyze the performance of the model, when the decline in the macroeconomic risk is not considered. I re-estimate the main financial figures, implied by the model with a restricted regime switching economy, fixing the utility parameters at $\gamma = 10$ and $\psi = 1.371$. The results for this calibration, obtained by discarding the first five years of data, are presented in table 4.5. While the equity returns are not effected by disregarding the decline in the macroeconomic risk, the implied risk free rate moves in the expected direction. In fact I obtained an estimate of 2.2% on an annual basis, that is about 90 basis point higher, than the observed risk free rate on which I calibrated the EIS parameter in the unrestricted version of the model (cf. table 4.3).

This result can be better interpreted if we focus on the role played by the volatility of the underlying state of the economy in determining the prices and thus the returns. When we shut down the regime on the volatility, we are preventing the agent from entering a persistent "high risk" regime associated with a high volatility of the economy, but we are also depriving her of the possibility of entering a persistent "low risk" regime. This creates an economy with a smoother path for the consumption claim. Such a path is appealing for the representative agent of this economy, and this would push the price of the risk free asset down increasing its return. So, an economy with a volatility regime gives an higher incentive to the representative agent to use the risk free asset to transfer consumption overtime, pushing its prices up and lowering the risk free return.

A mathematical interpretation follows from equation 4.20: in the parametrization adopted above, the risk free rate is positively related with the volatility of the consumption growth rate (σ_g) and negatively related with the volatility of the consumption claim (σ_{pc}). When we move from an economy with no regimes for the volatility of the underlying state to an economy that models such possibility, we don't expect any change to the (unconditional) volatility of consumption growth rate. To the contrary, the volatility of the consumption claim is expected to increase in the latter economy, given the

Table 4.5

Empirical series of US financial markets: restricted regime switching model

This table shows the asset returns implied by the restricted model calibrated on the US postwar economy. The estimation is based on real quarterly data (Q1:1952–Q4:2004; source: BEA). Reported are the estimates obtained by calculating capital and education investments on the whole sample and discarding the first five years of data to address the critiques to the perpetual inventory methodology. The market dataset is from Professor Robert J. Shiller webpage (http://www.econ.yale.edu/ shiller/data.htm). The coefficient of risk aversion is set to 10, the EIS is set to 1.371.

Series	Mean Data	Mean Model	Std. Data	Std. Model
Equities	0.072	0.061	0.074	0.035
Risk Free	0.013	0.022	0.006	0.010
Equity premium	0.058	0.039	0.074	0.049
$\rho(r^f; r^e)$	0.047	0.064	-	-
$\rho(r^e_{t+1}; r^e_t)$	0.105	0.540	-	-
$\rho(r^f_{t+1}; r^f_t)$	0.890	0.540	-	-

higher volatility of prices. This, given the above mentioned negative relation of σ_{pc} and the risk free, pulls down the risk free rate.

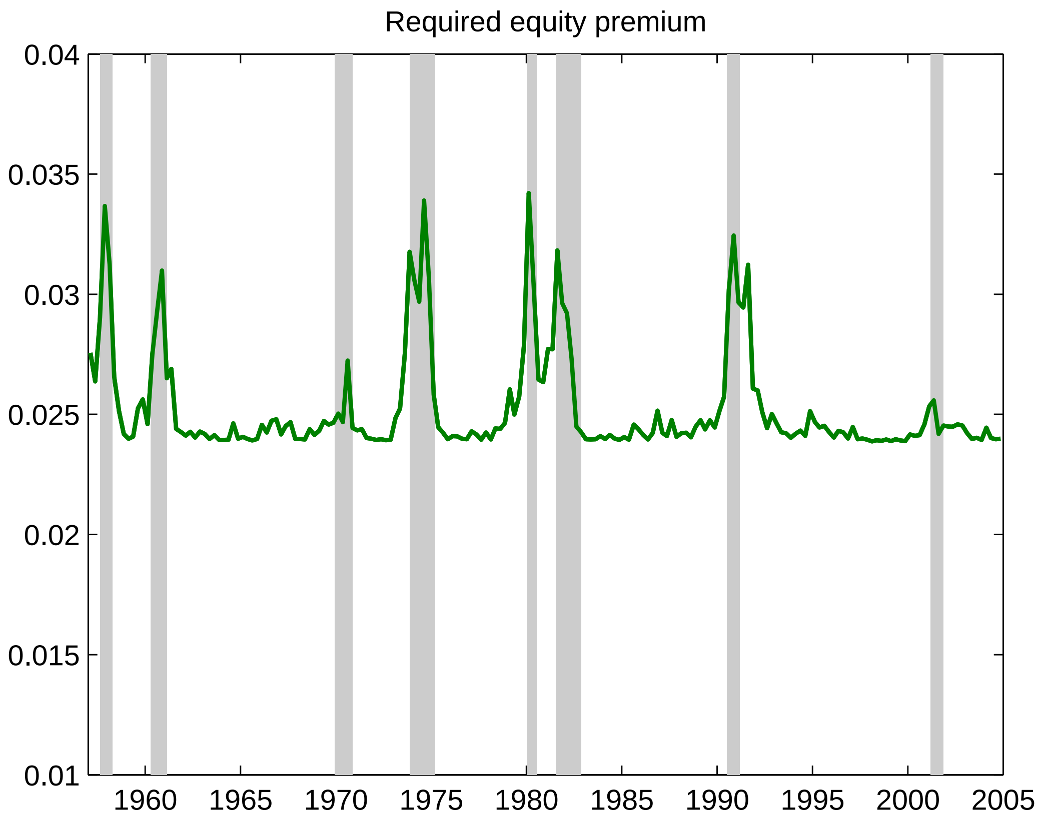
E. Time varying properties of the equity premium

Another notable feature of the model is the ability of replicating the behavior of the equity premium over the business cycle. The model delivers a higher risk premium in recessions than in booms: the implied annualized equity premium during recession varies from 3.46% under the high volatility state to 3.32% under the low volatility state. During booms the model predicts a much lower level of equity premium on an annual basis: 2.39% in the high volatility state and 2.25% in the low volatility state.

As a thoughtful exercise, I can calculate the expected equity premium by an investor that only observes consumption realizations. In practice, at each point in time, I let investors only have access to consumption growth rates. They know the structure of the economy, but they can use only consumption data in order to infer the current state of the economy. The probabilities the investors attach to each of the two states

Figure 4.4
The time varying properties of the equity premium

This figure shows the time-series of the expected equity premium by an agent that only observes consumption realizations. The state probabilities are those inferred using the Hamilton's 1989 filter. The equity premium is calculated fixing the utility parameters to the benchmark values (i.e. $\gamma = 10$, $\psi = 1.371$, and $\beta = 0.9925$).



correspond to the Hamilton's filtered state probabilities.

Figure 4.4 shows the expected equity premium for the US postwar period. The plotted line clearly picks up in recessions, confirming the expected coutercyclical behavior of the required premium an investor would ask in order to hold a risky asset.

IV. Sensitivity analysis

In order to get a better grasp on the forces driving my results, I provide a sensitivity analysis of the main financial figures implied by the model to the three most relevant parameters: risk aversion (γ) and elasticity of intertemporal substitution (ψ).

Figure 4.5 provides a general overview of how the equity premium is affected by different values of the parameters. The figure shows the mesh of the equity premium by changing RAs and EISs. As expected the equity premium is monotonically increasing in the risk aversion. The same increasing relation is displayed for the intertemporal substitution and the equity premium itself, but somewhat less pronounced.

After having discussed the general behavior of the implied equity premium with respect to risk aversion and intertemporal elasticity of substitution, it could be interesting to focus on the relevance of the utility function's parameters. Figure 4.6 delivers some insights on this. Panel A plots the obtained equity return by letting the risk aversion vary, keeping the remaining parameters fixed to the benchmark case. As expected the equity return is increasing in γ with a higher rate for lower values of γ. The same exercise, but on the risk free return, is reported in Panel B by letting the elasticity of intertemporal substitution vary. Again the relation is as expected: The higher the EIS the higher the willingness of an agent to transfer her wealth overtime; this leads to higher prices for a risk free asset and so to a lower risk free return.

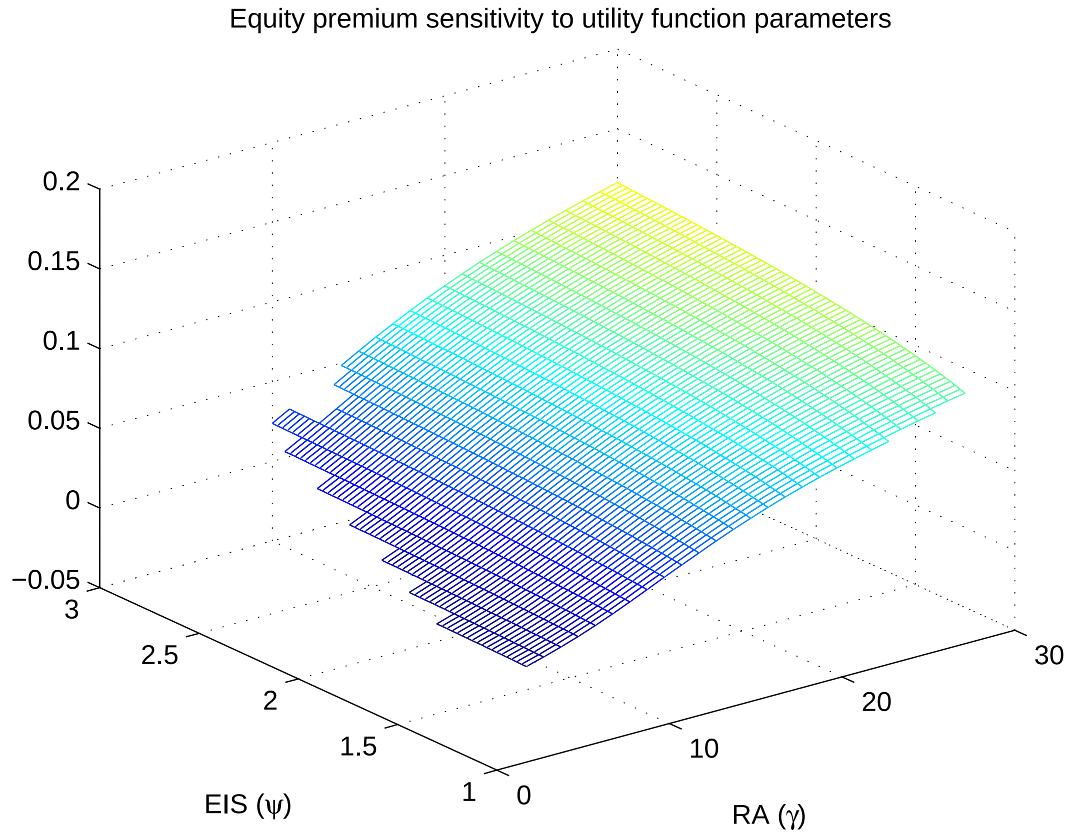
V. Conclusion

This chapter deals with asset pricing implications of production economies. In particular, I propose a simple extension to a standard Real Business Cycle (RBC) model where the economy switches between booms and busts and consumers have a recursive utility.

Figure 4.5

Equity premium's sensitivity

This figure shows the sensitivity of the implied equity premium to the relevant parameters used in the model. It plots the role of both Risk Aversion and Elasticity of Intertemporal Substitution in determining the equity premium.

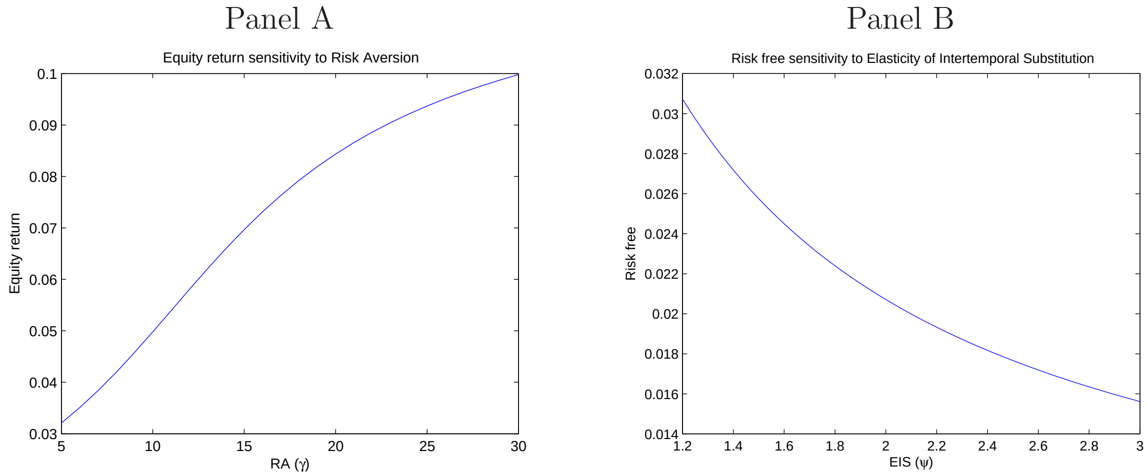


My first contribution is a detailed theoretical analysis of the equity premium's determinants in the proposed framework. Secondly, I show that a plausible parametrization of the model conveys financial figures that are in line with the empirical observations on the postwar U.S. data. A detailed analysis on the relative contribution of prominent parameters of the model is also provided. This allows us to clarify the role of different choices on the utility function. In particular, I investigate the role of risk aversion and elasticity of intertemporal substitution in determining asset prices and thus, in deter-

Figure 4.6

Returns' sensitivity to risk aversion and intertemporal substitution

This figure shows the sensitivity of assets' returns to utility function's parameters. Panel A plots the role of Risk Aversion in determining the required return on a risky asset. Panel B analyzes the influence of Elasticity of Intertemporal Substitution in determining the required risk free return.



mining the equity premium. Furthermore, I study the role of macroeconomic risk in the proposed economy, providing evidence in favor of modelling the "great moderation" observed in the last two decades.

The model, presented herein, is also shown to be able to replicate the empirical evidence of higher, and sizeable, required premia during a downturn of the economy, by simply letting the agent infer the state of the economy from consumption realizations.

VI. Deriving Asset Prices

Starting from Equation 4.12, I can rewrite it as

$$\begin{aligned} Q_{t+1} = \beta^{\alpha} \left(\frac{C_{t+1}}{C_t}\right)^{-\frac{\alpha}{\psi}} \left(\frac{P^c_{t+1} + C_{t+1}}{P_t}\right)^{\alpha-1} = \\ = \beta^{\alpha} \left(\frac{C_{t+1}}{C_t}\right)^{-\frac{\alpha}{\psi}} \left(\frac{P^c_{t+1}}{C_{t+1}} + 1\right)^{\alpha-1} \left(\frac{P^c_t}{C_t}\right)^{1-\alpha} \left(\frac{C_{t+1}}{C_t}\right)^{\alpha-1} = \\ = \beta^{\alpha} \left(\frac{C_{t+1}}{C_t}\right)^{-\frac{\alpha}{\psi}+\alpha-1} \left(\frac{P^c_{t+1}}{C_{t+1}} + 1\right)^{\alpha-1} \left(\frac{P^c_t}{C_t}\right)^{1-\alpha} \end{aligned} \tag{4.21}$$

giving us an expression for the stochastic discount factor as a function of consumption and price of its claim.[1]

First I price the consumption claim $R_{c,t+1} = \left(\frac{P^c_{t+1}+C_{t+1}}{P^c_t}\right)$

$$\mathbb{E}_t \left[\beta^{\alpha} \left(\frac{C_{t+1}}{C_t}\right)^{-\frac{\alpha}{\psi}+\alpha-1} \left(\frac{P^c_{t+1}}{C_{t+1}} + 1\right)^{\alpha-1} \left(\frac{P^c_t}{C_t}\right)^{1-\alpha} \left(\frac{P^c_{t+1} + C_{t+1}}{P^c_t}\right)\right] = 1 \tag{4.22}$$

Define $PC_t = \frac{P^c_t}{C_t}$

$$PC^{\alpha}_t = \mathbb{E}_t \left[\beta^{\alpha} \left(\frac{C_{t+1}}{C_t}\right)^{-\frac{\alpha}{\psi}+\alpha} (PC_{t+1} + 1)^{\alpha}\right]. \tag{4.23}$$

[1] It is worth noting that Q_{t+1} can be further simplified:

$$\begin{aligned} Q_{t+1} = \beta^{\alpha} \left(\frac{C_{t+1}}{C_t}\right)^{\alpha(1-\frac{1}{\psi})-1} (PC_{t+1} + 1)^{\alpha-1} (PC_t)^{1-\alpha} \\ = \beta^{\alpha} \left(\frac{C_{t+1}}{C_t}\right)^{\frac{1-\gamma}{1-1/\psi}(1-\frac{1}{\psi})-1} (PC_{t+1} + 1)^{\alpha-1} (PC_t)^{1-\alpha} = \\ \beta^{\alpha} \left(\frac{C_{t+1}}{C_t}\right)^{-\gamma} (PC_{t+1} + 1)^{\alpha-1} (PC_t)^{1-\alpha} \end{aligned}$$

where PC indicates the Price Consumption ratio on the consumption claim

Then I solve for the dividend claim $R_{e,t+1} = \left(\frac{P^e_{t+1}+D_{t+1}}{P^e_t}\right)$

$$\mathbb{E}_t \left[\beta^\alpha \left(\frac{C_{t+1}}{C_t}\right)^{-\frac{\alpha}{\psi}+\alpha-1} \left(\frac{P^c_{t+1}}{C_{t+1}}+1\right)^{\alpha-1} \left(\frac{P^c_t}{C_t}\right)^{1-\alpha} \left(\frac{P^e_{t+1}+D_{t+1}}{P^e_t}\right)\right] = 1 \tag{4.24}$$

Define $PD_t = \frac{P^e_t}{D_t}$

$$PD_t = \mathbb{E}_t \left[\beta^\alpha \left(\frac{C_{t+1}}{C_t}\right)^{-\frac{\alpha}{\psi}+\alpha-1} (PC_{t+1}+1)^{\alpha-1} (PC_t)^{1-\alpha} \left(PD_{t+1}+1\right) \left(\frac{D_{t+1}}{D_t}\right)\right]. \tag{4.25}$$

Finally, using proposition 3, I can plug $\frac{A_{t+1}}{A_t}\lambda_t^\theta \eta_t^{1-\theta}$ in place of the growth rates of consumption and dividends in Equations 4.23 and 4.25 for getting Equations 4.14 and 4.15 respectively.

I can solve this set of equations using the same technique as Lettau et al. (2008) and the estimation on the $\Delta \log A_{t+1}$ process (i.e. solve for the fix point and then compute expected PC and PD using posterior probabilities from the Hamilton (1989) filter.

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