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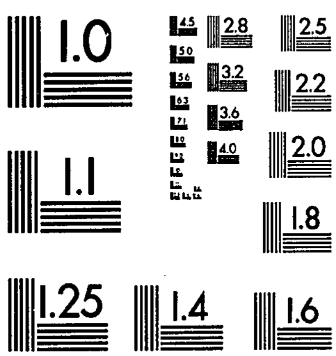

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INVESTIGATION OF SUPERSYMMETRIC LEFT-RIGHT THEORIES OF ELECTROWEAK INTERACTIONS

By

ASIM GANGOPADHYAYA

A dissertation submitted to the Graduate Faculty in Physics in partial fulfillment of the requirements for the degree of Doctor of Philosophy, The City University of New York.

1985

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ABSTRACT

SUPERSYMMETRIC LEFT-RIGHT THEORIES OF ELECTROWEAK INTERACTIONS

By

ASIM GANGOPADHYAYA

Adviser: Prof. Rabindra N. Mohapatra

We studied supergravity induced radiative breaking of $SU_R(2)XU(1)$ group in left-right symmetric theories of electroweak interactions. We find that in order to have a minimum that breaks $SU_R(2)XU(1)$ symmetry leaving $SU_L(2)$ intact, the parity has to be broken at a scale much above the Tev region. We show that if a parity odd singlet is introduced in the theory that picks up VEV at superheavy scale then $SU_R(2)XU(1)$ group breaks down

radiatively via a nonzero VEV acquired by right handed s-neutrino. We analyse the resulting mass spectrum of charged and neutral fermions and find that theory does allow two low mass fermionic states that are predominantly tau lepton and the corresponding neutrino.

Then calculating the K_L-K_S mass difference we find that supersymmetric contributions negate a well known constraint on right handed mass scale($M_{W_R} > 1.6$TeV). This implies that left-right theories could be a nontrivial alternative to standard model at low energies.

We prescribed a method of deriving renormalisation group equations(RGE) in explicitely broken supersymmetric theories using fully superspace methods. We claim that, to study the divergence structure of the theory efficiently, all supersymmetry breaking terms should be written in D-type form. Since calculating the divergent part is much easier in supergraphs than in component language, our prescriptions drastically reduce the work involved in derivation of RGE's. To corroborate our claims we derive the RGE's for a well known nontrivial model and reproduce all the equations that were obtained by component method.

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I also take this opportunity to thank the members of my thesis supervisory committee for the valuable time given and the energy they spent on this project.

Finally I would like to thank Alpana for the invaluable support extended to me during my frustrating graduate studies.

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INTRODUCTION:

Though the nonabelian gauge theories[1] were invented in early fifties they did not find much practical use until late sixties when Salam-Weinberg model[2] was proposed. This model, looking retrospectively, has enjoyed tremendous success. It was not just a renormalizable, phenomenologically consistent model of weak interactions but it was also the first time two different interactions were successfully blended into one. This sowed the seeds of a dream to be known later as Grand Unified Theories (GUTS).

Weak interaction, as we know, is a short range force and should be mediated by massive bosons(in gauge theories) as dictated by Yukawa theory. However, it is also well known that explicit mass term for the gauge bosons could not be introduced in the theory because that spoils renormalizablity. To avoid this problem in Salam-Weinberg model (Standard Model) Higgs-Kibble mechanism was used to give masses to bosons spontaneously. They conjectured that resulting theory is renormalizable which was later proved by t'Hooft in a series of excellent papers[3]. Thus standard model was on a solid theoretical rock by the end of '71. Major experimental success came in 73 when existence of neutral currents, an essential ingredient of standard model, was established experimentally.

However there are some complaints against the standard model. In standard model the parity violation is built into the theory by putting the left and right handed fermions in inequivalent representations and thus it does not provide any deeper understanding of the parity violation. Another major problem is the explanation of observed CP violation. To implement Kobayashi-Maskawa theory of CP violation in Standard model one needs atleast three generations of fermions to produce any physical phase. And also it does not give any reason for the smallness of the phase. We shall look at more of these in the next chapter.

These objections could be answered satisfactorily in the context of left-right symmetric(LRS) theories[4]. In these theories the parity is a good symmetry of the lagrangian. The right handed fields transform in a way exactly anagolous to the left handed fields under the gauge transformations. Many models based upon $SU_L(2) X SU_R(2) X U_{B-L}(1)$ group have been constructed. The parity violation in these models is generated by the asymmetry of the vacuum. The absence of the right handed neutrinos and V+A type charged currents at low energies are both explained by the same explanation. Also many models tie the CP violation phenomenon with the parity break down and thus several different phenomena can be described elegantly in these theories. We shall describe them in some what more detail in the next chapter.

Many authors[5] have studied the effect of supersymmetrisation of $SU_L(2)$ theories. However as one extends the $SU_L(2) X U_Y(1)$ in this direction some of the nice properties of the standard model are lost. Standard model conserves baryon and lepton numbers naturally. Symmetries of the theory prohibit introduction of any term with $\Delta B \neq 0$ or $\Delta L \neq 0$. Such conservations are not automatic in SUSY versions and one needs further symmetries to ban such terms from the lagrangian. In this regard we consider supersymmetric left-right models as natural extension of standard model. In left-right theories B-L quantum number is conserved as a result of the gauge symmetry. Lepton number then does break down spontaneously but such breaking is not arbitrary and is related to other phenomena. In this thesis I study supersymmetric left-right theories. The breaking patterns in these theories have been studied by several authors[6].

It is generally found that these theories the tree level potentials do not have asymmetric minima. The extrema are either symmetric or saddle points. It has been shown in literature[6] that if an odd parity singlet is introduced and given a super heavy VEV, an asymmetry is generated spontaneously between the two SU(2) groups at low energies. In such theories we find the $SU_R(2)XU(1)$ is broken down radiatively by nonzero vacuum expectation value of right handed s-neutrino. In this scenario R-parity is broken simultaneously with $SU_R(2)XU_{B-L}(1)$ and the matter fermions get mixed with gauginos and higgsinos through complicated mass matrices. Analysing the matrices we find that the theory allows for two fermionic particles that are predominantly tau lepton and corresponding neutrino.

Then we study the contributions of these new interactions to the k_L-K_S mass difference. We find that a stringent constraint obtained in non-SUSY case is rendered very weak by specifically supersymmetric contributions. Few authors have studied the breaking patterns in the supersymmetric left-right theories however still no extensive realistic calculation has been done.

I have mainly considered the supergravity models for their enhanced predictive power. Supergravity theories at low energy have very well defined supersymmetry breaking terms. At Planck scale, all the breakings are parametrized by just one parameter, the gravitino mass. This is the origin of the predictive power mentioned above. Supergravity theories have another very interesting feature. All the breaking terms that are induced belong to a certain class of interactions known as soft terms. These terms do not induce any quadratic divergence in the theory even though SUSY is broken. Thus one of the resons for which SUSY was introduced is not upset by these breaking terms.

In this thesis I review standard and left-right theories briefly. To make it self-contained I have added a chapter on supersymmetry in the appendix(A1).

We construct a supersymmetric left-right theory. This is described in chapter II. Under certain assumptions we show that the $SU_R(2) X U_{B-L}(1)$ group is broken radiatively to $U_Y(1)$ through nonzero vacuum expectation value (VEV) of right handed tau s-neutrino (scalar partner of right handed neutrino). We analyse, in detail, the resulting mass spectrum of charged and neutral fermions.

Then with a general case we show that a stringent constraint upon M_{W_R} ($M_{W_R} > 1.6$ TeV), coming from

K_L-K_S mass difference, is no longer valid when supersymmetric contributions are included. This is described in chapter III and relevant detailed calculations are done in appendix A2.

I also included a chapter on how to derive the renormalisation group equations using supergraph rules. The supersymmetry breaking terms in supergravity theories are, as mentioned above, all equal to $m_{3/2}$ at the Planck scale. The low energy parameters are then obtained by using these renormalisation group equations. Our prescriptions reduce this labor drastically. To demonstrate the practical applicability of our method we calculated the RGE's for a well known supersymmetric $SU_L(2)XU(1)$ model in the appendix(A3).

The original results in this thesis are in chapters II, III and IV. These have been done in colaboration with Prof. R. N. Mohapatra and Dr. Darwin Chang of University of Maryland.

CHAPTER ONE

STANDARD AND LEFT-RIGHT MODELS

STANDARD MODEL- A brief review:

This model of weak interaction is based on gauge group $SU(2)_L XU(1)_Y$. N_g generations of left handed leptons and quarks transform as doublets under the group $SU(2)_L$ where as their ringht handed counter parts are invariant. The particle content of the model is given by(The numbers in the parantheses represent the quantum numbers of the field under the gauge group $SU_L(2)XU_Y(1)$).

$$Q^0_{iL} = \begin{pmatrix} u_{iL} \\ d_{iL} \end{pmatrix} \qquad (\tfrac{1}{2}, \tfrac{1}{3}) \quad i = 1 \ldots N_g \qquad \begin{matrix} u_{iR} & (1, \tfrac{4}{3}) \\ d_{iR} & (1, -\tfrac{2}{3}) \end{matrix} \tag{1}$$

$$\Psi^0_{iL} = \begin{pmatrix} \nu_{iL} \\ e_{iL} \end{pmatrix} \qquad (\tfrac{1}{2}, -1) \quad i = 1 \ldots N_g \qquad e_{iR} \quad (1, -1) \tag{2}$$

The superscript '0' indicates that these fields are not mass eigenstates (They are called weak or interaction eigen states). The electric charge operator is given by

$$Q_{el} = T_{3L} + Y/2$$

where T_{iL} and Y are generaters of $SU(2)_L$ and $U(1)_Y$ respectively. The $SU(2)_L XU(1)_Y$ is broken down to $U_{EM}(1)$ by neutral component of a Higgs doublet Φ .

The transformation of the Higgs under $U(1)_Y$ is

$$\Phi = \begin{pmatrix} \phi_+ \\ \phi_o \end{pmatrix} \sim (\tfrac{1}{2}, 1) \quad (3)$$

When mass square is negative for Φ field the classical potential is minimized for following structure of VEV

$$\langle \Phi \rangle = \begin{pmatrix} 0 \\ v \end{pmatrix} \quad \text{(v is real)} \quad (4)$$

This break down of the gauge symmetry generates masses for gauge bosons and fermions spontaneously without affecting the renormalizability. The neutral gauge boson of $SU(2)_L$, W_3 , gets mixed with the boson B of $U_Y(1)$ and gauge boson mass eigen states are given by

$$W^{\pm} = \frac{W_1 \pm iW_2}{2} \qquad M_W^2 = \frac{1}{4}(gv)^2$$

$$Z = \frac{g\ W_3 - g'\ B}{(g^2+g'^2)} \qquad M_Z^2 = \frac{1}{4}(g^2+g'^2)v^2$$

$$A = \frac{g'W_3 + g\ B}{(g^2+g'^2)} \qquad M_A^2 = 0$$

Thus we end up with two massive charged and one massive neutral bosons. The remaining massless boson A corresponds to the unbroken group $U_{EM}(1)$ and is the photon field.

Defining $\frac{g'}{g} = \mathrm{Tan}\theta_W$ one can write the neutral bosons as

$$Z = W_3 \mathrm{Cos}\theta_W - B \mathrm{Sin}\theta_W$$

$$A = W_3 \mathrm{Sin}\theta_W + B \mathrm{Cos}\theta_W$$

Then one finds

$$m_Z = m_W / \mathrm{Cos}\theta_W$$

and hence a strict ratio is predicted between the two masses.

CURRENTS: The V-A character of the charged currents is built into the theory because the right handed fermions do not couple to $W^{\pm}$.

The coupling of the neutral bosons is given by

$$L_{neutral} = i\sqrt{(g^2+g'^2)}\,\bar{f}\; Z_\mu (T_3L - \mathrm{Sin}^2\theta_W Q_{el})\gamma_\mu f + \frac{gg'}{(g^2+g'^2)}(\bar{f}\; A_\mu Q_{el}\gamma_\mu f)$$

Or more elegantly

$$L_{neutral} = i\,\frac{g}{\mathrm{Cos}\theta_W}\,\bar{f}\, \not{Z}(T_3L - \mathrm{Sin}^2\theta_W Q_{el}) f + i\, e\, \bar{f} \not{A}\, Q_{el}\, f$$

where

$$L = \frac{1-\gamma_5}{2}$$

The neutral currents data predict $\sin\theta_W^2 \simeq .25$, a prediction which was verified by atomic experiments and ultimately by the explicit values of gauge boson masses at CERN.

LEFT RIGHT SYMMETRIC THEORIES:

With the simplicity and the compatibility with the low energy phenomenology, Standard model has acquired a unique position among theories of nature. With all that has been said so far one may wonder why any body would look for any other theory. However there are some short comings of standard model. The main ones are

a) The theory does not offer any deeper understanding of parity violation. Parity violation is generated by putting the left and right handed fermions in inequivalent representations.

b) U(1) generator lacks any physical meaning.

c) Kobayashi Maskawa model of CP violation requires at least 3 generations of fermions to generate any CP violating phase. And then there is no reason given for the smallness of the phase.

One way solve these problems is to extend the theory to left-right symmetric models. In left-right models the parity is broken spontaneously by non-invariance of the vacuum. In many models parity violation and CP violation are related. The U(1) generator here is nothing but the B-L quantum number. Finally if neutrino is found to have mass left-right theory provides an ideal setting to describe it.

In this section we would breifly review the left-right theories. For a detailed review I would refer the readers to original papers on the subject[1].

Let g_L, g_R and g' be the coupling of the gauge group $SU_L(2) X SU_R(2) X U_{B-L}(1)$. The fields of the theory are(The numbers in the parantheses represent the quantum numbers of the field under the gauge group $SU_L(2) X SU_R(2) X U_Y(1)$)

$$Q^0_{iL} = \begin{pmatrix} u_{iL} \\ d_{iL} \end{pmatrix} \qquad (\tfrac{1}{2}, 0, \tfrac{1}{3}) \quad i=1,..,N_g \tag{5}$$

$$\Psi^0_{iL} = \begin{pmatrix} \nu_{iL} \\ e_{iL} \end{pmatrix} \qquad (\tfrac{1}{2}, 0, -1) \quad i=1,..,N_g \tag{6}$$

$$Q^0_{iR} = \begin{pmatrix} u_{iR} \\ d_{iR} \end{pmatrix} \qquad (0, \tfrac{1}{2}, \tfrac{1}{3}) \quad i=1,..,N_g \tag{7}$$

$$\Psi^0_{iR} = \begin{pmatrix} \nu_{iR} \\ e_{iR} \end{pmatrix} \qquad (0, \tfrac{1}{2}, -1) \quad i=1,..,N_g \tag{8}$$

As before the superscript '0' indicates that these fields are not mass eigenstates. The elctric charge operator is now given by

$$Q_{el} = T_{3L} + T_{3R} + \frac{B-L}{2}$$

Where T_L, T_R and B-L are generators of $SU_L(2) X SU_R(2) X U_{B-L}(1)$. In order to generate fermion mass matrices one is lead to following Higgs multiplets

$$\Phi = \begin{pmatrix} \Phi^{o} & \Phi_1^{+} \\ \Phi_2^{-} & \Phi^{o'} \end{pmatrix} \quad \text{and} \quad \tilde{\Phi} = \begin{pmatrix} \Phi^{o'*} & -\Phi_2^{-*} \\ -\Phi_1^{*+} & \Phi^{*o} \end{pmatrix}$$

with transformation properties

$$=(\tfrac{1}{2}, \tfrac{1}{2}^{*}, 0) \quad \text{and} \quad =(\tfrac{1}{2}, \tfrac{1}{2}^{*}, 0)$$

From the representations it is clear that Φ connects the left and right handed fermionic multiplets and hence VEV of Φ genrates a mass tern for fermions. However this VEV would break the gauge group to U(1)XU(1) because

$$\langle\Phi\rangle = \begin{pmatrix} k & 0 \\ 0 & k' \end{pmatrix}$$

is invariant under the operator $(T_{3L} + T_{3R})$ i.e.

$$(T_{3L} + T_{3R})\langle\Phi\rangle = 0$$

Thus one introduces two more fields Δ_L and Δ_R

$$\Delta_{L,R} = \begin{pmatrix} \Delta_{L,R} & \Delta_{L,R} \\ \Delta_{L,R} & \Delta_{L,R} \end{pmatrix} \qquad \begin{matrix} \Delta_L \sim (1,0,2) \\ \Delta_R \sim (0,1,2) \end{matrix}$$

The theory is invariant under the parity operation P on the fields which is defined as

$$W_L \xleftrightarrow{P} W_R$$
$$f_L \xleftrightarrow{P} f_R$$
$$\Delta_L \xleftrightarrow{P} \Delta_R$$

Here $f_{L,R}$ stand for fermion doublets Q_i and Ψ_i. It is remarkable that clasical Higgs potential is invariant under the interchange $\Delta_L \xleftrightarrow{P} \Delta_R$ still an asymmetric solution could emerge as the minimum of the potential(2).

<u>Charged Currents</u>:

The charged gauge boson mass matrix is given by

$$\begin{array}{c} \\ W_L^- \\ \\ W_R^- \end{array} \begin{array}{c} \begin{array}{cc} W_L^+ & \qquad\qquad W_R^+ \end{array} \\ \begin{pmatrix} \frac{1}{4}g^2(k^2+k'^2) & -\frac{1}{4}g^2kk' \\ -\frac{1}{4}g^2kk' & \frac{1}{4}g^2(k^2+k'^2+v^2) \end{pmatrix} \end{array}$$

The physical eigen states are linear combination of W_L^+ and W_R^+. However, for a small $\frac{k'}{k}$, one finds such mixings can be safely ignored. That, in turn implies the usual Cabbibo like form of the Charged currents explicitely given by(with two generations)

$$J_L = \bar{\nu}_L \gamma_\mu e_L + \bar{\nu}_{\mu L} \gamma_\mu \mu_L + \overline{(uc)}_L \gamma_\mu O \binom{d}{s}_L$$
$$J_R = \bar{\nu}_R \gamma_\mu e_R + \bar{\nu}_{\mu R} \gamma_\mu \mu_R + \overline{(uc)}_R \gamma_\mu O \binom{d}{s}_R \qquad (9)$$

Where the mixing matrix 'O' is given by

$$O = \begin{pmatrix} \cos\theta_c & \sin\theta_c \\ -\sin\theta_c & \cos\theta_c \end{pmatrix} \qquad (10)$$

Where θ_c is the Cabibbo angle. The fact that the right and the left handed current's couplings are same implies that the right handed gauge boson must be very heavy.

<u>NEUTRAL CURRENTS</u>:

The mass matrix for the neutral gauge mesons is

$$\begin{array}{c} \\ W_L^3 \\ W_R^3 \\ B \end{array} \begin{array}{c} \begin{array}{ccc} W_L^3 & W_R^3 & B \end{array} \\ \begin{bmatrix} \frac{1}{4}g^2(k^2+k'^2) & -\frac{1}{4}g^2(k^2+k'^2) & 0 \\ -\frac{1}{4}g^2(k^2+k'^2) & \frac{1}{4}g^2(k^2+k'^2+v^2) & -\frac{1}{4}gg'v \\ 0 & -\frac{1}{4}gg'v^2 & \frac{1}{4}g'^2v^2 \end{bmatrix} \end{array}$$

The eigen values are(in the approx. $\frac{k^2+k'^2}{v^2} \ll 1$)

$$M_Z = \frac{M_{W_L}}{\cos\theta_W}$$

$$M_X = \frac{M_{W_R}\cos\theta_W}{\sqrt{\cos 2\theta_W}}$$

With the angle θ defined by

$$\sin^2\theta = \frac{g'^2}{(g^2+2g'^2)} \qquad (9)$$

The Neutral gauge bosons are given by

$$A = \sin\theta \; (W_L^3+W_R^3) + \sqrt{\cos 2\theta} \; B \qquad M_A^2 = 0$$

$$Z = \cos\theta \; W_L^3 - \sin\theta \; \tan\theta \; W_R^3 - \tan\theta \sqrt{\cos 2\theta} \; B$$

$$X = \frac{\sqrt{\cos 2\theta}}{\cos\theta} W_R^3 - \tan\theta \, B$$

From these one finds at low energies

$$J_\mu^Z = \frac{g}{\cos} \bar{f} \gamma_\mu (T_3 L - Q_{el} \sin^2\theta) f \qquad (10)$$

<u>Thus Left-Right symmetric theory reproduces all the predictions of standard model at low energies.</u>

However at short distances one should be able to see the deviation of this model from Weinberg-Salam. In fact it has been argued[3] that parity conservation should largely be restored at distances of the order of $(10^3 \text{ GeV})^{-1}$. The first constraint on the parity breaking scale was obtained from the analysis of the charged currents. They found that W_2 which couples predominantly to V+A terms could have a mass as low as three times the $SU_L(2)$ charged gauge boson with out affecting the known phenomenology. When contributions to K_L-K_S mass difference(ΔM_K) was calculated authors of Ref(4) found that the box digrams with both left and right bosons propagating contributed to ΔM_K with a sign opposite to that obtained in standard model and unless $M_{W_R} > 20 \, M_{W_L}$ the left-right theories predicted a wrong sign for the K_L-K_S mass difference. This would imply M_{W_R} is greater than 1.6 Tev. In the IIIrd chapter we shall study the effect of supersymmetrization on the above mass scale.

CHAPTER II

RADIATIVE BREAKING IN LEFT RIGHT SUPERGRAVITY THEORY

RADIATIVELY BROKEN LEFT-RIGHT SUGRA THEORIES

In this chapter we shall study the supersymmetric versions of left-right theories. The general way of arriving at a SUSY theory corresponding to a model is to replace each field by a superfield transforming identically under the internal symmetry group. All known fermions and scalar bosons are replaced by chiral and the vector bosons by vector superfields. Thus a supersymmetrized theory has many more fields than the one from which it is extended.

One major beauty of the SUSY is its constraints on the number of parameters. Many different interaction terms have equal coupling constants which receive identical renormalisation (including the finite parts) and hence remain equal under quantum corrections.

However, one pays a price for this attractive feature. In conventional theories one can introduce all the terms in the lagrangian that are consistent with internal symmetries. Many times this freedom is crucial in getting the desired symmetry breaking pattern. Lack of this freedom is one of the major hurdles in constructing realistic supersymmetric left-right[2-1] theories. Analysing this problem is the major goal of this chapter.

Supersymmetry is not an exact symmetry of the nature at least not at low energies. One may generate this violation either spontaneously or by introducing explicit soft[2] breaking terms. Both of these approaches have troubles. Spontaneous breaking leads to a serious constraint on the masses of the fields[3] ($\sum_J M_J^2 (-1)^{2J+1} = 0$) which is not seen at low energies. The explicit breakings on the other hand have too much arbitrariness and consequently very poor predictability. One beautiful solution is to make the supersymmetry local. It has been shown that if local SUSY is broken in Polonyi type hidden sector then the low energy effective theory behaves like a softly broken supersymmetric theories with all the breaking parameters being equal to the gravitino mass at the Planck scale[4].

We use a general supersymmetric model to carry out the analysis. We begin by describing the minimal model in detail. This is minimal in the sense that number of fields have been kept bare minimum. The particle content of the model is (the numbers in the parentheses describe the transformation under $SU_L(2) X SU_R(2) X U_{B-L}(1)$ internal group).

$$Q \sim (2,1,\tfrac{1}{3}) \quad ; \qquad Q^C \sim (1,2,-\tfrac{1}{3})$$

$$\Psi \sim (2,1,-1) \quad ; \qquad \Psi^c \sim (1,2,+1)$$

$$\phi_1 \sim (2,2,0) \quad ; \qquad \phi_2 \sim (2,2,0)$$

$$\Delta_L = \begin{pmatrix} \Delta_L^+ & \Delta_L^{++} \\ \Delta_L^0 & \Delta_L^+ \end{pmatrix} \sim (3,1,2)$$

$$\Delta_R = \begin{pmatrix} \Delta_R^- & \Delta_R^0 \\ \Delta_R^- & \Delta_R^- \end{pmatrix} \sim (1,3,-2)$$

$$\Delta_{\bar{L}} = \begin{pmatrix} \Delta_{\bar{L}}^- & \Delta_{\bar{L}}^0 \\ \Delta_{\bar{L}}^{--} & \Delta_{\bar{L}}^- \end{pmatrix} \sim (3,1,-2)$$

$$\Delta_{\bar{R}} = \begin{pmatrix} \Delta_{\bar{R}}^+ & \Delta_{\bar{R}}^{++} \\ \Delta_{\bar{R}}^0 & \Delta_{\bar{R}}^+ \end{pmatrix} \sim (1,3,+2)$$

One notices that in the SUSY case here the Higgs fields are introduced in pairs. One reason is anomaly cancellation because all the superfields have chiral fermions in them. Second reason is that one needs two different Φ's to give masses to up and down quarks. unlike the Non-SUSY case $\Phi^{\dagger}$ can not be used as it is not a chiral field. The minimal superpotential is chosen to be

$$W = \int \Big\{ m \left(\Delta_L \Delta_{\bar{L}} + \Delta_R \Delta_{\bar{R}}\right) + \frac{1}{2} m_\phi \phi^2 + h_M \left(\Psi^T \tau_2 \Delta_L \Psi + \Psi^{cT} \tau_2 \Delta_R \Psi^c\right) + h_\ell^{(i)} \Psi^T \tau_2 \phi_i \Psi^c + h_q^{(i)} Q^T \tau_2 \phi_i Q^c \Big\}$$

The part of the scalar potential relevant for symmetry breaking is given by

$$\begin{aligned} V = {} & \frac{1}{2}(m^2 + m_1^2)(\Delta_L^{\circ 2} + \Delta_R^2) \\ & + \frac{1}{2}(m^2 + m_2^2)(\Delta_{\bar{L}}^{\circ 2} + \Delta_{\bar{R}}^2) \\ & + m \tilde{m} \left(\Delta_L^\circ \Delta_{\bar{L}}^\circ + \Delta_R^\circ \Delta_{\bar{R}}^\circ\right) \\ & + \frac{1}{2}\left(m_\phi^{(i)2} + \tilde{\mu}^{(i)2}\right)\left(\phi_i^2 + \phi_i'^2\right) \\ & + \frac{1}{2} m^2 \left(\tilde{\nu}_\tau^2 + \tilde{\nu}_{\tau^c}^2\right) \\ & + \frac{1}{2} A_h m_{\frac{3}{2}} \left(\tilde{\nu}_\tau^2 \Delta_L + \tilde{\nu}_{\tau^c}^2 \Delta_R\right) \end{aligned}$$

Where

$$\begin{aligned} \frac{1}{2} D^2 = {} & \frac{1}{2} D^\alpha D_\alpha = \frac{1}{2} (g \phi^* T^\alpha \phi)(g \phi^* T_\alpha \phi) \\ = {} & \frac{1}{2} g_{BL}^2 \left| (\Delta_L^2 - \Delta_{\bar{L}}^2) - (\Delta_R^2 - \Delta_{\bar{R}}^2) - \frac{1}{2}(\tilde{\nu}_\tau^2 - \tilde{\nu}_{\tau^c}^2) \right|^2 \\ & + \frac{1}{2} g_{2L}^2 \left| (\Delta_L^2 - \Delta_{\bar{L}}^2) + \frac{1}{2} \tilde{\nu}_\tau^2 + \frac{1}{2} \sum_i (\phi_i^{\circ 2} - \phi_i'^{\circ 2}) \right|^2 \\ & + \frac{1}{2} g_{2R}^2 \left| (\Delta_R^2 - \Delta_{\bar{R}}^2) + \frac{1}{2} \tilde{\nu}_{\tau^c}^2 + \frac{1}{2} \sum_i (\phi_i^{\circ 2} - \phi_i'^{\circ 2}) \right|^2 \end{aligned}$$

Here the mass parameters with twidles at the top indicate their origin in supergravity and all are equal to $m_{3/2}$ at the planck scale

The low energy parameters are related to ones at Planck scale by the renomalisations group equations (RGE). In the fourth chapter we described a fully superspace method of deriving these RGE's in softly broken SUSY theories.

A detailed analysis reveals that this potential does not have stable asymmetric minimum. To arrive at asymmetric solutions one needs quartic terms that couple left handed fields with right handed ones eg $(\Delta_L^{0\dagger}\Delta_L^0)(\Delta_R^{0\dagger}\Delta_R^0)$, $\tilde{\nu}_\tau^2\,\tilde{\nu}_{\tau c}^2$ etc. Such terms arise only from D^2 terms and then they always have wrong sign that leads to a saddle point. For instance if extremal equations have solution $\langle\tilde{\nu}_{\tau c}\rangle \neq 0$ then the second derivative in the direction is always negative definite. This implies presence of a saddle point. Faced with this problem authors of reference(1) have argued that quantum corrections could turn the solution into a minimum. However we shall take a different approach and require that stability be achieved at the classical level.

Recently a very interesting idea, radiative breakdown of $SU_L(2)XU_Y(1)$ internal group has been successfully used in supergravity models of electro-weak interaction.

We investigate similar mechanism in left-right theories.

Since in the symmetric theories extremization equations always lead to unstable or symmetric vacua (asymmetric solutions always turn out to be saddle point configurations) we look for theories that are inherently asymmetric at low energies. Similar analysis was done by Hayaski and Murayama[5].

Recently a new way of parity breaking was discovered by Chang etal[6]. In the context of SO(10) models they showed that it was possible to dissociate the scale of parity break down from the breaking of $SU_R(2) X U_{B-L}(1)$ of the $SU_L(2) X SU_R(2) X U_{B-L}(1)$ subgroup. Consequently at intermediate energies the theory behaves asymmetrically between $SU_L(2)$ and $SU_R(2)$ groups.

To realise the above idea of bringing in assymetry we introduce a parity odd singlet η in the theory and give it a super heavy nonzero VEV. The only interaction term this field could have is of the type $g\,\eta\,(\Delta_L\Delta_{\bar{L}} - \Delta_R\Delta_{\bar{R}})$. One immediately finds that a nonzero VEV for η brings the asymmetry needed between $SU_L(2)$ and $SU_R(2)$ groups spontaneously. The effective masses of the fields $\Delta_{L,\bar{L}}$ and $\Delta_{R,\bar{R}}$ are $(m + g\langle\eta\rangle)$ and $(m - g\langle\eta\rangle)$ respectively. Now if $m \simeq g\langle\eta\rangle \gg m_{3/2}$ then left fields decouple from the low energy theory and the resulting theory has stable asymmetric minimum.

To find the low energy values of the parameters we use the following renormalisation group equations ($t=\ln(M_P/\mu)$)

$$\frac{dh_M}{dt} = -\frac{7}{4\pi^2}\cdot h_M^3 - \frac{h_M}{4\pi^2}\sum_i h_\ell^{(i)^2} + \frac{h_M}{16\pi^2}\left(3g_{BL}^2 M_{BL}^2 + 7g_R^2 M_R^2\right)$$

$$\frac{dA}{dt} = -\frac{7}{2\pi^2} h_M^2 A$$

$$\frac{dm^2_{\tilde{\nu}_\tau}}{dt} = -\frac{3}{2\pi^2} h_M^2 \left(2m^2_{\tilde{\nu}_\tau} + m^2_{\Delta_R} + m^2_{3/2}|A|^2\right) + \frac{1}{2\pi^2}\left(g_{BL}^2 \frac{M_{BL}^2}{4} + \frac{3}{4} g_R^2 M_R^2\right)$$

$$\frac{dM^2_{\Delta_R}}{dt} = -\frac{1}{2\pi^2} h_M^2 \left(2m^2_{\tilde{\nu}_\tau} + m^2_{\Delta_R} + m^2_{3/2}|A|^2\right) + \frac{1}{2\pi^2}\left(g_{BL}^2 M_{BL}^2 + 2g_{2R}^2 M_R^2\right)$$

Assuming $h_M \gg g$ we solve these equations numerically. The resulting solutions are depicted in Fig.(2-1) and Fig.(2-2). One finds from them that mass-squares for all the fields except $\tilde{\nu}_{\tau^c}$ remain positive at low energies thus signaling a breakdown of $SU_R(2) \times U_{B-L}(1)$ through nonzero VEV of s-neutrino. Now the second derivative in the $\tilde{\nu}_\tau$ direction is

$$\frac{\partial^2 V}{\partial \tilde{\nu}_\tau^2} = m^2_{\tilde{\nu}_\tau} - \frac{1}{2} g^2_{B-L} \tilde{\nu}^2_{\tau^c}$$

Since $\Delta_{L,\bar{L}}$ are absent in the low energies $m^2_{\tilde{\nu}_\tau}$ does not receive much renormalisation.

If $\quad m^2_{\tilde{\nu}_\tau} \simeq m^2_{3/2} > \frac{1}{2} g^2_{B-L} \tilde{\nu}_{\tau^c}$

then above second derivative is positive and the vacuum is stable. Another thing to be noted is that gaugino masses have to be bounded for radiative breakdown to occur. Gaugino terms tend to increase the mass parameters as the scale lowers. Assuming the mass-square of $\tilde{\nu}_{\tau^c}$ to be of the order of $m^2_{3/2}$, it's maximum positive value we get following inequalities

$$\frac{3}{2\pi^2} h^2_m (4m^2_{3/2}) \gtrsim \frac{1}{4}(g^2_R M^2_R + 3\, g^2_{B-L} M^2_{B-L})$$

Assuming $M_R \simeq M_{B-L}$ we get $\qquad M^2_R \leq \frac{12 h_M^2}{g^2_R} m^2_{3/2}$

The next level of breaking ($SU_L(2) X U_Y(1)$) is achieved by the nonzero VEV of $\Phi(2,2,0)$. Minimization equations relevant for discussing symmetry breaking are

$$\frac{\partial V}{\partial \phi^o} = \phi^o \{ m_\phi^2 + g_{2R}^2 \tilde{\nu}_{\tau c}^2 + (g_{2L}^2 + g_{2R}^2)(\phi^{o2} - \phi'^{o2}) \} + m_\phi \tilde{m}_\phi \phi'^o = 0$$

and

$$\frac{\partial V}{\partial \phi^{o\prime}} = \phi'^o \{ m_\phi^2 - g_{2R}^2 \tilde{\nu}_{\tau c}^2 + (g_{2L}^2 + g_{2R}^2)(\phi'^{o2} - \phi^{o2}) \} + m_\phi \tilde{m}_\phi \phi^o = 0$$

Which implies

$$\phi^{o2} = \left[\tilde{m}_\phi^2 / \{ 2 m_\phi^2 - \tilde{m}_\phi^2 + 2\sqrt{m_\phi^4 - m_\phi^2 \tilde{m}_\phi^2} \} \right] \phi'^{o2}$$

We see that ϕ^o and ϕ'^o have opposite signs for $m_\phi^2 <$ 0 and hence no real solutions exist for ϕ^o and ϕ'^o simultaneously. With $m_\phi^2 > \tilde{m}_\phi^2 > 0$ we do get positive solutions for both ϕ^{o2} and ϕ'^{o2}. Thus we would like consider the case where the mass m_ϕ^2 does not turn negative and always $m_\phi^2 > \tilde{m}_\phi^2$ is satisfied.

FERMION MASS MATRIX

The vacuum expectation value of $\tilde{\nu}_\tau^c$ breaks not only the $SU_R(2) X U_{B-L}(1)$ group but also R-parity. The conventional particles are assigned positive R-parity and their supersymmetric partners given negative R-parity. This is a symmetry of the lagrangian and breaksdown spontaneously as s-neutrino picks up VEV. R-parity is also called twidleness or s-property. The R-parity of any particle can be compactly written as $(-1)^{2S+L+3B}$. In a theory that is invariant under R-parity one always produces an even number of s-particles with the decay of every known particle. The phenomenology of the SUSY theories with R-parity is well known. Recently few authors have studied the effect of this parity breakdown in $SU_L(2) X U_Y(1)$ theories[6].

The break down of R-parity results in mixing of tau lepton with gauginos and higgsinos and the corresponding neutrino with neutral fermions(gauge and higgs). At this stage it is not at all clear if the neutrino and the tau lepton are going to be light and if their couplings with other fermions are in the allowable range. In this section we are going to analyse the resulting mass matrices for the fermions and identify the physical particles.

A) CHARGED FERMIONS:

The mass matrix of the charged fermions after the symmetry breaking is given by (we denote $\tilde{\nu}_{\tau}$ by v and m-g by M)

$$
\begin{array}{c} \\ \lambda_R^- \\ \lambda_L^- \\ \psi_{\Delta_R}^- \\ \psi_{\phi_1}^- \\ \psi_\tau^- \\ \psi_{\phi_2}^- \end{array}
\begin{array}{c}
\begin{array}{cccccc} \lambda_R^+ & \lambda_L^+ & \psi_{\Delta_R}^+ & \psi_{\phi_1}^+ & \psi_\tau^+ & \psi_{\phi_2}^+ \end{array} \\
\begin{pmatrix}
m_R & 0 & 0 & g_R k_1 & g_R v & g_R k_2 \\
0 & m_L & 0 & g_L k_1' & 0 & g_L k_2' \\
0 & 0 & M & 0 & -h_M v & 0 \\
g_R k_1' & g_L k_1 & 0 & -m_\phi^1 & 0 & \mu_{12} \\
0 & 0 & 0 & h_1^1 v & k_i h_1^{(i)} & h_1^{(2)} v \\
g_R k_2' & g_L k_1' & 0 & \mu_{12} & 0 & m_\phi^{(2)}
\end{pmatrix}
\end{array}
$$

Here k_i and k_i' are VEV's of ϕ_i^0 and $\phi_i'^0$ respectively. We assume the VEV's of ϕ_2 to be much smaller than that of ϕ_1, so that last row and coloumn practically decouple from the rest of the matrix. The resulting matrix is not Hermitian and can only be diagonalized by bi-unitary transformations. Let us denote the coloumn vector by Ψ^- and row by χ^+. Then the mass terms can be written as

$$L_M = \Psi^- M \chi^+ = \Psi^- U U M V V \chi^+$$

Where U and V are arbitrary Unitary matrices and one could choose them to be the ones that diagonalize the mass matrix M i.e. $U M V^\dagger = M_D$. The matrices U and V could be determined by solving the equations

$$U M M^\dagger U^\dagger = V^\dagger M^\dagger M V = M_D^2$$

We solve these equations by assuming

$$M_L, M_R, m_\phi, M \gg g_{2R} v \gg g_{2R} \kappa$$

The exact determinant of M_D^2 is

$$m_L^2 M^2 \Big[(m_\phi^2 + h_1^2 v^2) \{ (h_1 k)^2 m_R^2 + (g_R v . g_R k)^2 \} - \{ (g_R^2 + h_1^2) k v m_R - (g_R v)(m_R g_R k - m \; g_R k') \}^2 \Big] \tag{1}$$

In the limit $k, k' \rightarrow 0$ the determinant is zero and the product of the nonvanishing eigen values is

$$m_L^2 (m_\phi^2 + h_1^2 v^2) \quad (M^2 m_R^2 + v^2 (g_R^2 M^2 + h_M^2 m_R^2)) \tag{2}$$

Hence the small eigen value can be obtained by dividing the equation (1) by the equation (2). To find a rough estimate we set all big masses equal to M and we get

$$\begin{aligned} m_\tau^2 = \; & (h_1 k)^2 + (g_R v)^2 (g_R k')^2 / M^2 \\ & - (g_R^2 + h_1^2)(kv)^2 / M^2 \\ & - (g_R v)^2 \; g_R (k - k')^2 / M^2 \end{aligned}$$

Which is of the order $h_1 k$ in the limit of large M. To find the corresponding eigen vector we ignore k,k' dependent terms in $M^+ M$ and $M M^+$. We find

$$M\,M^+ = \begin{pmatrix} m_R^2+g_R^2v^2 & 0 & -h_M g_R v^2 & 0 & 0 \\ 0 & m_L^2 & 0 & 0 & 0 \\ -h_M g_R v^2 & 0 & h_M^2 v^2+M^2 & 0 & 0 \\ 0 & 0 & 0 & m_\phi^2 & -m_\phi h_1 v \\ 0 & 0 & 0 & -m_\phi h_1 v & h_1^2 v^2 \end{pmatrix}$$

and

$$M^+M = \begin{pmatrix} m_R^2 & 0 & 0 & 0 & m_R g_R v \\ 0 & m_L^2 & 0 & 0 & 0 \\ 0 & 0 & M^2 & 0 & -h_M M v \\ 0 & 0 & 0 & m_\phi^2+h_M^2 v^2 & 0 \\ m_R g_R v & 0 & -h_M M v & 0 & (g_R^2+h_M^2)v^2 \end{pmatrix}$$

The eigen vectors that corresponds to the zero mass are

a) for $M\,M^+$: $\quad \tau^- + \frac{h_\ell v}{m_\phi}\psi_\phi^-$

b) for M^+M: $\quad \tau^+ - \frac{g_R v}{m_R}\lambda_R^+ + \frac{h_M v}{M}\psi_{\Delta_{\bar{R}}}^+$

Hence the physical tau lepton is given by

$$\tau_{phys} = \begin{pmatrix} \tau^- + \frac{h_e v}{m_\phi} \psi_\phi^- \\ \left(\tau^+ - \frac{g_R v}{m_R} \lambda_R^+ + \frac{h_M v}{M} \psi_{\Delta_{\bar{R}}}^+ \right)^c \end{pmatrix}$$

Thus we see that physical tau lepton is predominantly constituted of τ^- and τ^+ which are third generation electron and positron weak eigenstates.

NEUTRAL FERMIONS:

Now let us turn to neutral fermion sector. Since the neutral fermions could have majorana masses the mass matrix is now much larger. We again solve it under the same assumptions i.e. Φ_2 practically decouples from the theory and gaugino and higgsino masses are much larger than other terms with dimension of mass. The mass matrix is given by

$$
\begin{array}{c|cccccccccc|}
 & \Psi_{N_R} & \nu_\tau & \lambda_{3R} & \lambda_{BL} & \Psi_{\Delta_R^0} & \Psi_{\phi_1^0} & \Psi_{\phi_1'^0} & \lambda_{3L} & \Psi_{\phi_2^0} & \Psi_{\phi_2'^0} \\
\Psi_{N_R} & 0 & h_l^{(i)}\frac{k_i}{2} & -g_R\frac{v}{2} & g_B\frac{v}{2} & h_M\frac{v}{2} & 0 & 0 & 0 & 0 & 0 \\
\nu_\tau & h_l^{(i)}\frac{k_i}{2} & 0 & 0 & 0 & 0 & 0 & h_l^{(1)}\frac{v}{2} & 0 & 0 & h_l^{(2)}\frac{v}{2} \\
\lambda_{3R} & -g_R\frac{v}{2} & 0 & m_R & 0 & 0 & g_R k_1 & -g_R k_1' & 0 & g_R k_2 & -g_R k_2' \\
\lambda_{BL} & g_B\frac{v}{2} & 0 & 0 & m_{BL} & 0 & 0 & 0 & 0 & 0 & 0 \\
\Psi_{\Delta_R^0} & h_M\frac{v}{2} & 0 & 0 & 0 & M & 0 & 0 & 0 & 0 & 0 \\
\Psi_{\phi_1^0} & 0 & 0 & g_R k_1 & 0 & 0 & 0 & m_\phi^{(1)} & g_L k_1 & 0 & \mu_{12} \\
\Psi_{\phi_1'^0} & 0 & h_l^{(1)}\frac{v}{2} & -g_R k_1' & 0 & 0 & m_\phi^{(1)} & 0 & -g_L k_1' & \mu_{12} & 0 \\
\lambda_{3L} & 0 & 0 & 0 & 0 & 0 & g_L k_1 & -g_L k_1' & m_L & g_L k_2 & g_L k_2' \\
\Psi_{\phi_2^0} & 0 & 0 & g\ k & 0 & 0 & 0 & \mu_{12} & g_L k_2 & 0 & m_\phi^{(2)} \\
\Psi_{\phi_2'^0} & 0 & h_l^{(2)}\frac{v}{2} & -g_R k_2' & 0 & 0 & \mu_{12} & 0 & g_L k_2' & m_\phi^{(2)} & 0
\end{array}
$$

Again to find the smallest eigen value we set k,k' equal to zero. We find them to be 0, $\pm\sqrt{m^2+h_1^2v^2}$,

m_L, m_R, m_{BL}, M and

$$\left(\frac{h_M^2}{M} + \frac{g_R^2}{m_R} - \frac{g_{BL}^2}{m_{BL}}\right) v^2$$

The eigen state belonging to the the '0' value is predominantly the weak eigenstate ν_τ. It is explicitely given by

$$\left(\nu_\tau - \frac{h_1 v}{m_\phi} \psi_\phi^\circ\right)$$

To find the corresponding eigen value we reintroduce k and k' in the matrix. We find the determinant of the matrix and divide it by product of the nonvanishing roots that we found earlier. The result is horrendous. To get a feeling for the magnitude of the eigen value we set m_L, m_R, m_{BL}, M and m_ϕ all equal to M. Then the tau neutrino mass comes out to be

$$m^2_{\nu_\tau} = \frac{(g_R v)^2 (h_1 k)^2}{M\ (h_M v)^2}$$

The corresponding eigen state, which is predominantly the physical neutrino is given by

$$\nu_\tau^{Phy.} \simeq \left(\nu_\tau - \frac{h_L v}{m_\phi} \psi_\phi^0\right)$$

The state that corresponds substantially to physical right handed neutrino is given by

$$N_R^{Phys.} = \Psi_{N_R} + \frac{g_R}{m_R}\lambda_{3R} - \frac{g_{BL}}{m_{BL}}\lambda_{BL} - \frac{h_M}{M}\psi_{\Delta_R}^\circ$$

and the corresponding eigen value is

$$m^2_{N_R} = (\frac{h}{M}M^2 + \frac{g_R}{m_R}^2 - \frac{g_{BL}}{m_{BL}}^2) v^2$$

The model has many unconventional implications. Because of the fact that R-parity is broken we can see odd number of s-particles being produced in the final state from the decay of an ordinary particle. As an example, we find the following new decay mode for τ^- (see fig. 2-4) if $m_{\tilde{\gamma}} \simeq m_\tau$

$$\tau^- \longrightarrow u\, d\, \tilde{\gamma}$$

with strength:

$$M(\tau^- \longrightarrow \tilde{\gamma}\, u\, d) = \frac{g_R v}{M_\lambda} \frac{g_R e}{m^2_{\tilde{d}_R}}$$

Again, for $m_{\tilde{d}_R} = M_W$, we expect this decay mode to exist with strength G_F. Note that there are no theoretical constaints.

Similarly, right handed squarks in this model could decay to quark and a charged lepton: (see fig. 2-5)

$$\tilde{u}_R \text{ —— } d_R\, \tau^+_L$$
$$u_R \text{ —— } d_R\, \mu^+_L$$
$$u_R \text{ —— } d_R\, e^+_L$$

with decay width:

$$\Gamma(\tilde{u}_R \rightarrow d_R l_L^+) \simeq \frac{g_R^4 \, v_{l,R}}{4\pi^2 \, M_R^2} \, m_{\tilde{\nu}_R}$$

Also the lightest s-particle is no longer a stable particle. For example we find

$$\lambda_R^+ \rightarrow \tau^+$$

Also we shall find disappearances of s-particles from the scene that is they may decay into ordinary particles through following type of reactions

$$\tilde{u}_R \longrightarrow d_R \tau_L^+$$

In the next chapter we shall study the impact of these theories on K_L-K_S mass difference and the bounds on $SU_R(2)$ breaking scale.

CHAPTER III

THE CONSTRAINTS ON BREAKING SCALE OF RIGHT HANDED GROUP FROM EXPERIMENTAL SIGNATURE OF K_L-K_S MASS DIFFERENCES

The tiny mass difference between long and short lived Kaons has played important roles in the development of field theories that describe the world at low energies. The prediction of the mass for the charmed quark is very well known.

The signature of the K_L-K_S mass difference (ΔM_K) had been used in constraining the breaking scale of $SU_R(2)$ group. Beall etal[1] showed that ΔM_K had wrong sign unless $M_{W_R} > 20\ M_{W_L}$.

Their calculation was based on the fact that the contribution of the box diagram, when both the left and right handed gauge bosons propagating was opposite to that obtained in standard model. Since standard model prediction[2] agrees with experimental value in signature (Assuming that the B factor is positive as is the case in vacuum insertion approximation), the authors of the ref(1) found the above constraint on M_{W_R}. However Chang etal[3] discovered that the calculation of Ref.3 was not complete. They made an interesting and very important observation that box diagrams in Left-Right gauge theories are not gauge invariant by themselves. They also predicted that the graphs involving flavor changing neutral higgs had to be introduced to maintain the gauge invariance. However detail calculations performed later[4] showed that the numerical constraint itself is not very much affected by these graphs though conceptually it was very important to include all of them.

We shall now study if the above constraint is valid when supersymmetric interactions are taken into account. For a definite prediction we consider a general left right symmetric model based on supergravity (SUGRA). With detailed calculations(detailed calculations are done in appendix A2) we find that the new arrivals,gluino box diagrams contribute to ΔM_K, with a sign opposite to that of the Left-Right box (L-R) diagram. And this is true for a wide range of values for these new parameters. The magnitude of this contribution depends upon the masses of gravitino, squark and gluino fields which we treat as independent input parameters. Thus they cut into the effectiveness of L-R to provide the above stringent constraint and consequently the constraint obtained[1] on M_{W_L}/M_{W_R} becomes much weaker. This raises the interesting possibility that the distinction of the model from $SU_L(2)XU(1)$ could be tested at low energies.

Major SUSY contributions to ΔM_K come from new flavor changing Squark-Gluino-quark interactions. To derive the explicit form of such interaction, here onwards we work with a minimal model based on Supergravity. Following the procedure developed in Ref(5) and using the fact that renormalization group equations are Left-Right symmetric we get following form of the down squark mass matrix.

$$m^2_{\tilde{d}} = \begin{pmatrix} \mu^2+m_d^2+Cm_u^2 & Am_g m_d \\ Am_g m_d & \mu^2+m_d^2+Cm_u^2 \end{pmatrix} \qquad (1)$$

where hermiticity of quark masses (dictated by L-R sym.) has been used. A is the soft[6] SUSY breaking parameter induced by SUGRA. C is a negative number and is related to one loop correction to squark mass. Since μ is of the order of several GeVs and other terms being proportional to the quark masses a near degenaracy of squark masses is predicted. In $SU_L(2)XU(1)$ theories similar diagonal blocks are not identical [7]. Thus left-right symmetric models have less number of parameters involved and we would be able to predict more decisively. The relevant interaction term of the lagrangian is

$$L_I(\lambda) = g_3 \, \tilde{d}^{0*}_a \, \bar{\lambda}^B \, T^B_{ab} \, d^0_b$$

Here d^0 and $\tilde{d}^0$ stand for quark and squark weak interaction eigenstates. λ is the gluino field. "B" and "a" are indices of SU(3) generator and color respetively. Let $U, \tilde{U}$ and $D, \tilde{D}$ be the unitary matrices that relate weak states to the mass eigenstates i.e.

$$d^0 = D\, d \;;\; u^0 = U\, u \;; \tilde{d}^0 = \tilde{D}\, \tilde{d};\; \tilde{u}^0 = \tilde{U}\, \tilde{u};$$

In terms of physical fields the interaction term becomes

$$L_I(\lambda) = g_3\, \tilde{d}^*_a\, \tilde{D}^+ D\, \bar{\lambda}^B\, T^B_{ab}\, d_b$$

Here $d = \binom{d_L}{d_R}$ is $2n_g$ dimensional vector with n_g being number of generations. D and $\tilde{D}$ are unitary matrices that diagonalize the mass matrices of quark and squark respectively. We can, without loss of generality, choose down quark mass matrix diagonal i.e. D=1. For $|c| \gtrsim 1$ it has been shown in literature[8], that matrix of equation (1) is diagonalised basically by the same matrix that diagonalizes up quark mass (with our choice of the quark basis the Kobayaski-Maskawa matrix $K = U^+$). In the case of two generations we find

$$D = \begin{pmatrix} K^+ & -K^+ \\ K^+ & K^+ \end{pmatrix} \quad \text{with} \quad K^+ = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \qquad (2)$$

from this gluino interaction term can be written explicitely as

$$L_I = \frac{g_3}{\sqrt{2}}\bar{\lambda}\left[\left\{(\tilde{d}^*_1, \tilde{d}^*_2) + (\tilde{d}^*_3, \tilde{d}^*_4)\right\} K\binom{d_R}{d_R} + \left\{(\tilde{d}^*_1, \tilde{d}^*_2) - (\tilde{d}^*_3, \tilde{d}^*_4)\right\} K\binom{d_L}{d_L}\right]$$

We shall define some integrals and functions for future need

$$g_{\alpha\beta} = \int \frac{d^4q}{(2\pi)^4} \frac{q^2}{(q^2+m_\lambda^2)^2(q^2+m_\alpha^2)(q^2+m_\beta^2)}$$

and

$$h_{\alpha\beta} = \int \frac{d^4q}{(2\pi)^4} \frac{1}{(q^2+m_\lambda^2)^2(q^2+m_\alpha^2)(q^2+m_\beta^2)}$$

Here m_α, m_β and m_λ are squarks and gluino masses respectively. Assuming near degenerateness of squark masses we get

$$g_{\alpha\beta} \simeq g_{\alpha\alpha} + i/(2X16\pi^2) \quad (m_\alpha^2-m_\beta^2)\ \tilde{g}_\alpha$$

and

$$h_{\alpha\beta} \simeq h_{\alpha\alpha} + i/(2X16\pi^2) \quad (m_\alpha^2-m_\beta^2)\ \tilde{h}_\alpha$$

where g and h are given by

$$\frac{(5m_\lambda^4-4m_\alpha^2 m_\lambda^2-m_\alpha^4)+2(m_\alpha^2 m_\lambda^2+2m_\alpha^4)\ln(m_\alpha^2/m_\lambda^2)}{(m_\alpha^2-m_\lambda^2)^4}$$

$$\frac{(m_\lambda^4+4m_\alpha^2 m_\lambda^2-5m_\alpha^4)+2(m_\alpha^2 m_\lambda^2+2m_\lambda^4)\ln(m_\alpha^2/m_\lambda^2)}{m_\alpha^2\ (m_\alpha^2-m_\lambda^2)^4}$$

We define two functions F_1 and F_2 of $g_{\alpha\beta}$ and $h_{\alpha\beta}$ by

$$F_1(g) = \sum_{\alpha,\beta} g_{\alpha\beta}(-1)^{\alpha+\beta} \qquad \text{where } \alpha,\beta = 1,\ldots,4$$

$$F_2(g) = (g_{11}-g_{12}-g_{13}+g_{14})$$

$$+ \text{ all cyclic replacement}$$

One finds that functions F_1 and F_2 vanish if all squark masses are equal and this property implies super GIM cancellation.

Now let us turn to calculation of ΔM_K . For detailed calculations we would refer the reader to the appendix(A2). The diagrams that contribute towards $H_{eff}^{\Delta S=2}(\lambda)$ are shown in Fig.(3-1) and (3-2). Fig.(3-1) arises from majorana type mass terms of gluino and Fig.(3-2) is due to Dirac type terms. From these one finds

$$
\begin{aligned}
H_{eff}^{\Delta S=2} = {} & \frac{\alpha_s}{8\pi^2} \sin^2\theta \cos^2\theta \\
& \frac{38}{9} F_1(g)(V_{LL}+V_{RR}) - m_\lambda^2/3 \; F_2(h)(T_{LL}+T_{RR}) \\
& + S_{LR}\left(\frac{136}{9} F_1(g) + \frac{80}{9} F_2(g) - \frac{112}{9} m_\lambda^2 F_1(h)\right) \\
& + V_{LR}\left(-8\, m_\lambda^2 \; F_1(h) + \frac{16}{3} m_\lambda^2 \; F_2(h) + \frac{28}{3} F_1(h)\right) \\
& - \frac{74}{9} m_\lambda^2 F_2(h) \; (S_{LL}+S_{RR})
\end{aligned}
$$

where

$$S_{AB} = (\bar{d} P_A s)(\bar{d} P_B s);$$
$$V_{AB} = (\bar{d}\, \gamma_\mu P_A s)(\bar{d}\, \gamma_\mu P_B s)$$
$$T_{AB} = (\bar{d}\, \sigma_{\mu\nu} P_A s)(\bar{d}\, \sigma_{\mu\nu} P_B s)$$

with P_A and P_B being the chirality projection operators.

The $K_L - K_S$ mass difference is linearly related to the matrix element of the operator $H_{eff}^{\Delta S=2}$ between K^0 and $\bar{K}^0$ states. To determine the ΔM_K we find the following matrix elements by vacuum insertion[2].

$S_{AA} = \frac{5}{24} R\, Q;$ $\quad S_{LR} = (\frac{5}{24} + \frac{R}{4})Q;$ $\quad V_{AA} = \frac{Q}{3};$

$V_{LR} = (-\frac{1}{4} + \frac{R}{6})Q;$ $\quad T_{AA} = -\frac{RQ}{4}$

Matrix elements of all other operators vanish. Here $Q=f_K^2 M_K$ and $R=\frac{6\rho+1}{4\rho+6}$ with f_K and ρ defined by $\langle 0|\bar{s}\gamma_\mu\gamma_5 d|\bar{K}^0\rangle = \frac{i f_K P_\mu}{2M}$ and $\langle k^0|S_{LR}|\bar{k}^0\rangle = \langle k^0|V_{LR}|\bar{k}^0\rangle \cdot \rho$

In the vacuum insertion approximation, one finds [3]

$$\rho = \frac{3}{4} M_K^2 \ (m_s+m_d)^{-2} + \frac{1}{8} \simeq 7.7$$

Now we shall evaluate ΔM_K. Assuming near degeneracy of squark masses one finds

$$F_1\binom{g}{h} = 4C(m_c^2-m_u^2)\binom{\tilde{g}}{\tilde{h}}$$

and

$$F_2\binom{g}{h} = 4A\ m_g(m_s-m_d)\ \text{Cos}2\theta_c\binom{\tilde{g}}{\tilde{h}}$$

Assuming $|C|$, $A \simeq o(1)$, as is the case in models with Polanyi type hidden sector, we calculate ΔM_K for wide range of values for the masses of squark, gluino and gravitino fields. We plot the result as functions of these masses in Fig(3-3). We used the following numerical values in the calculation. $\alpha_s = .1$, $m_u =$ 5MeV; m_c=1.5 GeV; m_d=25MeV; m_s=150MeV, f_K = .16 GeV, M_K = .5 GeV and $\sin\theta_c$=.23. The important point we see from the graphs is that SUSY contribution to ΔM_K has a sign opposite to that of the Left-Right box diagram. Consequently the experimental value of the K_L-K_S mass difference can no longer be used to constrain M_{W_L}/M_{W_R}. This negation of the constraint has strong implications. The above mentioned constraint implied $M_{W_L} < \frac{1}{20} M_{W_R}$.

Summary:SUSY sector of Left-Right model contributes to M_K with a sign such that, the famous constraint on M_R obtained from NONSUSY calculations is rendered much weaker. This ressurects the hope that Left-Right models could be a nontrivial alternative to $SU_L(2)XU(1)$ theory at low energies.

CHAPTER IV

RENORMALIZATION GROUP EQUATIONS IN SOFTLY BROKEN SUPERSYMMETRIC THEORIES USING SUPERSPACE METHODS

RGE'S IN BROKEN SUSY USING SUPERFIELDS

In global supersymmetric theories, the most amazing feature is the cancellation between bosonic and fermionic loop contributions first observed by Wess and Zumino. As a result all qudratic divergences vanish from the theory and only logarithmic infinities survive(1). A SUSY multiplet that includes bosonic and fermionic components can most easily be represented by a function, called superfield(2), in superspace which includes ordinary spacetime and 4 grassmannian cordinates. Superfield, besides providing an elegant formalism are also very useful tool in practical calculations. The superfield Feynman rules have been formulated and improved upon in the literature(For a review, see ref(3) and also listed in the appendix A1). However, relatively less attention has been paid to the application of superfield formalism to calculations in realistic models. The reason is quite obvious. There is no evidence of SUSY in our elementary particle physics phenomenology. If SUSY is relevant to nature it must be broken one way or another at the energy scale we have been observing so far. Apparently superfield formalism must be modified to take into account the effect of the SUSY breaking. A very simple artifact, called the spurion[4] technique, exists in the literature for this purpose. Since in the superfield language SUSY transformation is equivalent to translation in superspace, giving some superfield $\Phi(x, \theta, \bar{\theta})$ a fixed value results in SUSY breaking. The procedure is to introduce some external (spurion) superfields and let them couple to the quantum fields in SUSY fashion. The SUSY breaking is then achieved by giving the spurion a suitably chosen value. A SUSY breaking is called soft (SSB) breaking if it does not generate any quadratic divergence. The soft terms preserve the nice ultraviolet properties of SUSY theory while making it realistic. Girardello and Grisaru have used the spurion technique to analyse all possible soft breaking terms in N=1 SUSY theories[4]. Spurion technique is very useful in learning about the divergence structure of the realistic models. When one applies it to calculate a physical quantity, say the β-function of a coupling constant a question arises. Given a soft SUSY breaking term in the lagrangian, there is more than one way it can be written in the superfield form with spurion. All the superfield forms that corresponds to the same term in components, are equivalent. However some of them are more convenient and economical than others. It is the purpose of this chapter to shed light on this question and demonstrate it through toy models and realistic applications. In section II we illustrate the ambiguity in choosing a superfield soft breaking term in a toy example and demonstrate the way to fix it. In the section III, we show that all the soft SUSY breaking terms in a theory with supersymmetric interaction can be and should be written as D-type terms. This is the main result of the paper. The F-type terms that give rise to the same component forms can be discarded because they will automatically generate the corresponding D-terms with divergent coefficient at higher orders. Therefore the inclusion of F-terms is redundant once the right D-terms are chosen. In the appendix A3 we demonstrate an application of this approach in a realistic model--N=1 supergravity model with a SUSY breaking hidden sector. We derive the renormalisation group equations(RGE) in this model using our approach and spell out the specific supergraphs that contribute to each renormalisation constant. These RGE's have been derived earlier using conventional method[5]. We rederive it because we feel that the superfield version is simpler and it illustrates the points in the text very well.

We follow the convention of the textbook by WESS and BAGGER[6]. The only minor change is in the normalisation of the vector superfield. Their normalisation is more suitable for component reduction. We modified for the convenience of supergraph calculations.

2. Let us consider the following lagrangian as a toy example for the demonstration of the aforesaid ambiguities in selecting the breaking terms

$$L_1 = A^* \Box A - i\bar{\Psi}\sigma^m \partial_m \Psi - g^2 |AA^*|^2 + gA\,\Psi^2$$
$$ag\mu(A^3 + A^{*3}) + b\mu^2 AA^* \qquad (1)$$

Here "A" is a complex scalar field and is a Weyl spinor. The last two terms are SUSY breaking terms and in their absence reduces this lagrangian to that of the WESS-ZUMINO[7] (WZ) model.

Now we shall attempt to express this lagrangian by superfields. However, before we do that, we shall digress a bit and discuss about spurions. Spurions are classical superfields and function of θ only. Consequently they break the translation invariance in the superspace. Because of their classical nature they can appear only as the external legs and do not propagate inside the loops in supergraphs. Since the lagrangian in terms of spurions looks very similar to the SUSY one (SUSY breaking being completely hidden in the explicit value of the spurion field) all supergraph rules can still be applied the same way as in the supersymmetric case. This way spurions keep the powerful machinery of superfields rolling even when the SUSY is explicitely broken. To cast L_1 in the superfield form we employ a chiral superfield Φ($\Phi(x,\theta,\bar{\theta})$ $=A(x)+\Psi(x)\theta+\ldots$) and a spurion field $W=\mu\theta^2$ where μ characterizes the SUSY breaking scale and also keeps W dimensionless. The lagrangian can now be written as

$$L_2 = \int \phi^\dagger \phi \, d^4\theta + \frac{g}{3} \left\{ \int \phi^3 d^2\theta + H.c. \right\} + ag \int (\phi^3 W \, d^2\theta + H.C.) + b \int W W^\dagger \Phi \Phi^\dagger \quad (2)$$

It is obvious that introduction of explicit value for W yields the last two SUSY breaking terms of L_1. However this straight forward way of translating the SUSY breaking terms into superfield form turns out to be too naive. To start with, this lagrangian is not renormalizable because it generates a new logarithmically divergent term of the form $\int d^4\theta \, \Phi^\dagger \Phi (W+W^\dagger)$ through the graph depicted in Fig(4-1).

Hence we extend the lagrangian of equation (2) to include this term and arrive at

$$L_3 = \int d^4\theta \quad \Phi^\dagger \Phi + \frac{g}{3} \int (\Phi^3 d^2\theta + H.C.)$$
$$+ \alpha_1 g \int W \Phi^3 d^2\theta \qquad + H.C.$$
$$+ \alpha_2 \int (W^\dagger W) \Phi^\dagger \bar{\Phi} d^4\theta \tag{3}$$
$$+ \alpha_3 \int (W + W^\dagger) \bar{\Phi}^\dagger \Phi \, d^4\theta$$

A glimpse at this lagrangian immediately promts some questions. We started out with only two SUSY breaking terms in components and ended up with three terms in the superfield form. Apparently not all three parameters α_1, α_2 and α_3 could be independent physical parameters. There must be some redundancy involved. It is in fact not very hard to get rid of this redundancy. By nonrenormalization theorem we see that the F-term with the coupling constant α_1 will never be generated in the perturbation theory. We also note that the coefficients α_2 and α_3 together are sufficient to generate a and b terms of the component form in the equation (2). The obvious economical choice is to set $\alpha_1=0$. This illustrates the central theme of the paper. In practical calculations, there is a particular choice of terms involving spurions that are more economical than the others. Also note that, after setting $\alpha_1=0$, there are equal numbers of breaking terms in the component and the superfield versions. Even though verified in the last example and expected intuitively the proof of this one to one correspondence is not that obvious from our discussions so far. In the next section we shall generalize this result to include all soft breaking terms.

In the remainder of this section we shall simply push this toy example further and compare the RGE's obtained from the component and superfield approaches. Relations among a,b and α_2, α_3 can be obtained easily by substituting the value of W and integrating out θ to arrive at

$$a = -\alpha_3$$
$$b = \alpha_2 - \alpha_3^2 \qquad (4)$$

Graphs that contribute towards RGE's of a and b are depicted in Fig(4-2) and Fig(4-3) respectively. From them one gets

$$\beta_a = \mu \frac{\partial a}{\partial \mu} = \frac{12\ a\ g}{16\ \pi^2}$$

and (5)

$$\beta_b = \mu \frac{\partial b}{\partial \mu} = \frac{12\ a\ g(b-3a^2)}{16\ \pi^2}$$

Next we shall calculate RGE for α_2 and α_3 using supergraph technique~. Supergraphs that contribute are shown in Fig(4-4) and Fig(4-5). Before we get to their RGE's we shall like to share an interesting observation about supergraph calculations. We found that whenever the number of chiral super-propagator increases by one unit the sign of its contribution changes. For instance Fig(4-4a) has sign opposite to that of Fig(4-4b) and Fig(4-4c).

Fig(4-4) and Fig(4-5) contribute towards the vertex renormalisation of constants α_2 and α_3 respectively. One finds from them

$$\beta_{\alpha_2} = \frac{12\ g^2}{16\,\pi^2}\,(\ \alpha_2 -\ 2\,\alpha_3^2)$$

and (6)

$$\beta_{\alpha_3} = \frac{12\ g^2}{16\,\pi^2}\ \alpha_3$$

Using equation (6) and equation (4) one can calculate β_a and β_b and immediately finds that they are just the same as those in equation (5) derived from component methods. This verifies the equivalence of L_1 and L_3 when α_1 is set equal to zero.

3. In the last section we saw that soft SUSY breaking terms (SSB) when cast in D-type terms greatly simplifies the calculations of RGE's. In this section we shall

show that most general SSB when added to WZ model can always be written as D-terms. For that purpose consider the following lagrangian

$$L_C = A^* \Box A - i\bar{\psi}\sigma^m \partial_m \psi - m^2 AA^* - g^2 |AA^*|^2 + \{gA\,\psi^2 - mg\,A^2 A^* + \text{H.C.}\} + \{ag\mu A^3 + \tfrac{1}{2} b\mu^2 AA^* + c\mu^2 A^2 + d\mu^3 A + \text{H.C.}\} \tag{7}$$

The above lagrangian has been obtained by adding to WZ model, all possible SSB[4] terms (we are not considering gauge interactions at this point). The coefficients a,b,c and d are dimensionless SUSY breaking parameters. To write it in superfield form, a priori, we start with the following lagrangian

$$L_S = \tfrac{1}{2}\int d^4\theta\ \Phi^\dagger\Phi + \{\int d^2\theta\ (\tfrac{1}{2} m\Phi^2 + \tfrac{1}{3} g\Phi^3) + \text{H.c.}\} + \int d^2\theta\ \{\mu\beta_1 W\Phi^2 + \beta_2 W\Phi^3\} + \int d^4\theta\ (\alpha_1 W + \tfrac{1}{2}\alpha_2 W^\dagger W)\ \Phi^\dagger\Phi + \int d^4\theta\ \mu\,(\alpha_3 W^\dagger + \alpha_4 W^\dagger W)\ \Phi \tag{8}$$

Here as before, Φ's are chiral superfields and W is the spurion. α_i and β_i are SSB parameters. Substituting $\mu\theta^2$ for W and integrating out θ and the auxiliary fields one obtains following SUSY breaking terms

$$L_{SSB} = \mu^2(\alpha_4\mu - \alpha_3 m - \alpha_1\alpha_3\mu)\,A + \mu^2(\alpha_2 - \alpha_1^2)\,AA^* + \mu(\beta_1\mu - g\alpha_3\mu - \alpha_1 m)\,A^2 + \mu(\beta_2 - \alpha_1 g)\,A^3 \tag{9}$$

As is transparent from above, one could set $\beta_1 = \beta_2 = 0$ and still reproduce all the terms of lagrangian L_c. The nonrenormalization theorem then guarantees that these terms will never be generated at any order. Since all the divergences can be absorbed in α_i's one could derive correct RGE's ignoring all β_i's.

However one point of caution needs to be stated. All F-type terms can be dropped only in the presence of SUSY interactions. For instance one could not discard in equation(8) if g vanished. In this case one could show that D-terms $\int d^4\theta\, W \Phi^+\Phi$ and $\int d^4\theta\, W \Phi^+$ will not be generated with infinite coefficient and hence to derive RGE's choosing nonvanishing $\beta_1, \beta_2, \alpha_4$ and α_2 suffices. Note that in this case, when g=0 the SUSY breaking terms A^2 and A^3 do not get renormalized. Thus it is possible to break SUSY and still have SUSY breaking terms nonrenormalized. This observation may have important consequences on model building.

So far we confined ourselves to non-gauge type interactions. Gauge theories require much more careful attention. Since explicit breaking of gauge symmetry leads to nonrenormalizability, all SSB must be written in gauge invariant form. A realistic model that explains this concept beautifully has been worked out in detail in the appendix A3.

Thus we see that in the theories with softly broken supersymmetry the RGE's can be derived very easily using supergraph rules if all the breaking terms were written as D-terms with spurions. And we also observed that when supersymmetric interactions are present all soft SUSY breaking can indeed be cast in D-form. When SUSY interactions are absent, as would be the case if g=0 in the equation (9), we find that some F-type terms have to be introduced. These mandatory F-terms are protected from any renormalisation by nonrenormalization theorem. Which implies that, as we stated earlier, broken SUSY theories can be constructed such that some of the breaking parameters are not renormalized. Thus we only need to calculate the vertex renormalisation for only the D-terms. Hence our prescription should be that always write the SUSY breakings as D-terms except when they do not produce all the terms of the component description. The latter implies presence of some coupling constants that do not recieve any renormalization even though SUSY is broken.

The theories with gauge interactions require some simple but nontrivial modification, which involves writing the breakings as gauge invariant D-terms. To show the applicability of this approach in practical calculations, and also as an example with gauge interactions, we calculate the RGE's for a well-known supersymmetric SU(3)XSU(2)XU(1) model of electro-weak interaction in the appendix A3.

Conclusion: Superfields provide powerful, elegant and efficient tool for calculations even when supersymmetry is broken. In these cases supersymmetry breaking are represented by SPURION superfields. In general there are many different ways a breaking term can be written using spurions and they are not equivalent , at least when calculating renormalization group equations are concerned. We showed that the labor could be reduced drastically if SUSY breakings are all written as D-terms. Also we found that it is possible to construct broken SUSY theories where some of the breaking parameters do not receive any renormalization. We demonstrated the usefulness of the whole thing by deriving the RGE for a realistic model in the appendix.

APPENDIX A1

SUPERSYMMETRY

This beautiful symmetry was a prodigious discovery of mid seventies(1). Under most symmetries known in particle physics a boson (fermion) transforms in to a boson (fermion). Supersymmetry (SUSY) goes beyond this conventional wisdom and relates a bosonic object to a fermionic one.

The generators of SUSY Q_α and $\bar{Q}_{\dot\alpha}$ are spinorial objects (because they change the spin by half) and follow the following (anti) commutation rules.

$$(Q_\alpha, \bar{Q}_{\dot\alpha})_+ = 2\, \sigma^m_{\alpha\dot\alpha} P^m \quad (m=1,..,4 \text{ Lorentz indices})$$

$$(\alpha, \dot\alpha = 1,2 \text{ Spin indices}) \quad (1)$$

$$(Q_\alpha, Q_\beta)_+ = (\bar{Q}_{\dot\alpha}, \bar{Q}_{\dot\beta})_+ = 0 \quad (2)$$

$$(Q_\alpha, M^{mn})_- = i\,(\sigma^{mn} Q)_\alpha \quad (3)$$

Here $\sigma^m = (1, \vec{\sigma})$ where the $\vec{\sigma}$ are the Pauli matrices. This algebra is known as extended poincare or Supersymmetry algebra. This fusion of the internal symmetry group with the poincare group does not violate the Coleman-Mandula theorem because of the spinorial character of it's generators. Let us look at the physical implications of the algebra by acting the generator on a particle state of momentum p and spin s.

$$Q\ |p,s\rangle = a\ |p,\ s+\tfrac{1}{2}\rangle + b\ |p,s-\tfrac{1}{2}\rangle \tag{4}$$

Since Q_α commutes with p^m the new particle states have the same energy energy momentum as the original state i.e. they have the same mass. The spins of these states differ from that of the original by $\pm\frac{1}{2}$. These states of different spins and same mass form what is called a supermultiplet.

The equation(1) implies the intertwining of the SUSY with the ordinary space time translations. That is why, sometimes, the SUSY is called squareroot of translation. Multiplying the equation(1) by σ^0 and using the trace conditions

$Tr(\sigma^0\sigma^m) = -2\ g^{0m}$ one gets

$$\frac{1}{4}(Q_1Q_1 + Q_2Q_2 + Q_1Q_1 + Q_2Q_2) = H \tag{5}$$

or written more compactly

$$\frac{1}{2}(|Q_1|^2 + |Q_2|^2) = H$$

This equation has immediate consequences.

a) H is semi-positive definite i.e. eigenvalues of H are either zero or positive.

b) If a state is invariant under SUSY then Q_α annhilates such a state and hence by the equation(5) the energy of such a state is zero. If SUSY is spontaneously broken then the ground state energy of such a theory must be positive and vice versa.

Equation(3) simply implies the spinorial character of the generator.

Besides these there are some technical properties that make SUSY very interesting and relevant to the particle physics of today.

In absence of supersymmetry the masses of scalar particles receive quadratically divergent contribution in perturbation theory i.e. $\delta m_\phi^2 = g^2\ m_\phi^2\ (\Lambda^2/m^2)$.

This renormalisation property is very unwelcome in grand unification theories. In GUT's one needs to maintain two very different mass scales of scalar bosons. Renormalisation effects make such endevours very difficult and dirty. One has to redefine the parameters at each order to satisfy the constraints. This is the so called Hierarchy problem. The crux of the probem is unavailability of any symmetry that could keep bosonic masses unrenormalized.

In case of fermionic fields we know chiral symmetry prevents mass renormalization. And if chiral symmetry is broken the radiative correction to the mass is given by

$$\delta m_F = g^2\ m_F\ \ln(\Lambda/m_F)$$

where Λ is the scale at which chiral symmetry is broken. Since SUSY is a symmetry betewen fermions and bosons, it extends the blanket of protection, provided by the chiral symmetry, to bosons too. Thus in supersymmetric theories the bosonic masses receive exactly the same radiative correction as the fermions. Technically speaking the qudratically divergent cotribution to bosonic masses cancel between bosonic and fermionic loops and only logarithmic divergence persists. As we shall see later this is a general property of SUSY theory. Not only the mass terms rather all F-type (to be defined later) terms are protected from any intrinsic (vertex) renormalisation. They only receive wave function renormalisation which is always logarithmic.

Supersymetry is best realized in 8 dimensional superspace. First four dimensions of which are ordinary space time and remaining four are grassmannian. A position vector in this space is represented by $z = (x, t, \theta_1, \bar{\theta}_1, \theta_2, \bar{\theta}_2,)$. Supersymmetry is simply the translation symmetry in this superspace.

In the superspace the generators Q_α and $\bar{Q}_{\dot{\alpha}}$ are represented by

$$\left(\frac{\partial}{\partial\theta^\alpha} - i\,\sigma^m_{\alpha\dot{\alpha}}\bar{\theta}\,\partial_m\right)$$

and (6)

$$\left(\frac{\partial}{\partial\bar{\theta}^{\dot{\alpha}}} - i\theta^\alpha\,\sigma^m_{\alpha\dot{\alpha}}\,\partial_m\right)$$

These operators act upon superfields, general functions of the superspace coordinate z. These superfields are to be understood in terms of their expansion in θ and $\bar{\theta}$ i.e.

$$F(x,\theta,\bar{\theta}) = f(x) + \theta\,\Phi(x) + \bar{\theta}\,\bar{\chi}(x) + \theta^2 m(x) + \bar{\theta}^2 n(x) + \theta\sigma^m\bar{\theta}\, v_m(x) + \theta\theta\,\bar{\theta}\,\bar{\lambda}(x) + \bar{\theta}\bar{\theta}\,\theta\,\Psi(x) + \theta\theta\bar{\theta}\bar{\theta}\, d(x) \tag{7}$$

All higher terms vanish because of the grassmannian character of θ and $\bar{\theta}$. Under supersymmetry transformation(ξ is an infinitesimal grassmannian parameter)

$$\delta_\xi F = (\xi Q + \bar{\xi}\bar{Q})\, F \tag{8}$$

where Q and Q are differential operators given above. A superfield, in general, is reducible under supersymmetry transformation. The easiest way to see this is to recognize the following two differential operators D_α and $\bar{D}_{\dot{\alpha}}$ given by

$$D_\alpha = \frac{\partial}{\partial\theta_\alpha} + i\,\sigma^m\bar{\theta}^{\dot{\alpha}}\,\partial_m$$

and (9)

$$\bar{D}_{\dot{\alpha}} = \frac{\partial}{\partial\bar{\theta}^{\dot{\alpha}}} - i\,\theta^\alpha\sigma^m_{\alpha\dot{\alpha}}\,\partial_m$$

These operators obey following anti-commutation relations

$$(D_\alpha, Q_\beta)_+ = (D_\alpha, \bar{Q}_{\dot{\beta}})_+ = (\bar{D}_{\dot{\alpha}}, Q_\beta)_+ = (\bar{D}_{\dot{\alpha}}, \bar{Q}_{\dot{\beta}})_+ = 0 \tag{10}$$

Thus one can pick out irreducible representations of SUSY by operating polynomials of D_α and $\bar{D}_{\dot{\alpha}}$ on general superfields. Chiral and antichiral representations are defined by

$$\bar{D}_{\dot{\alpha}} S = 0 \text{ and } D_{\alpha} \bar{S} = 0 \tag{11}$$

The constraint on the chiral superfield can be solved to get

$$\begin{aligned} S(x,\theta,\bar{\theta}) &= A(x) + \sqrt{2}\,\theta\Psi(x) + \theta^2 F(x) \\ &+ i\theta\sigma^m\bar{\theta}\,\partial_m A - \tfrac{1}{2} i\,\theta^2 \partial_m \Psi \sigma^m \bar{\theta} \\ &+ \tfrac{1}{4}\theta^2\bar{\theta}^2 \partial^2 A(x) \end{aligned} \tag{12}$$

It can be written very simply using a convenient variable, called chiral coordinate, $y = x + i$

$$S(y,\theta,\bar{\theta}) = A(y) + \sqrt{2}\,\theta\,\Psi(y) + \theta^2 F(y) \tag{13}$$

Under SUSY the transformations of the component fields are given by

$$\begin{aligned} \delta_\xi A &= \sqrt{2}\,\xi\Psi \\ \delta_\xi \Psi &= \sqrt{2}\,F + i\sqrt{2}\,\sigma^n\bar{\xi}\,\partial_n A \\ \delta_\xi F &= -i\sqrt{2}\,(\partial_n\Psi)\,\sigma^n\bar{\xi} \end{aligned} \tag{14}$$

Another important irreducible representation is vector representation defined by

$$v = v^+ \tag{15}$$

Which gives the following expansion of the vector superfield in terms of the superspace coordinates

$$\begin{aligned} V(x,\theta,\bar{\theta}) &= C + i(\theta\chi - \bar{\theta}\bar{\chi}) + \tfrac{i}{2}\theta^2 (m+in) \\ &- \tfrac{i}{2}\bar{\theta}^2 (m-in) - \theta\sigma^m\bar{\theta}\,v_m \\ &+ i\,\theta^2\bar{\theta}\,(\bar{\lambda} + \tfrac{i}{2}\sigma^m\partial_m\chi) \end{aligned}$$

$$- i \bar{\theta}^2 \theta (\lambda + \frac{i}{2} \sigma^m \partial_m \bar{\chi})$$
$$+ \frac{1}{2} \theta^2 \bar{\theta}^2 (D + \frac{1}{2} \Box C) \qquad (16)$$

Sum of two or more chiral fields is chiral and that of vector fields is a vector field. However for a chiral field S, the sum $S + S^+$ is neither chiral nor anti-chiral rather it is a vector field.

$$S + S^+ = \ldots\ldots + i \; \theta \sigma^m \bar{\theta} \partial_m (A - A^*) + \ldots.$$

This suggests $V \longrightarrow V + S^+ + S$ is a possible extension of gauge transformations to the superfields. Under this transformation the components transform as

$$C \longrightarrow C + A + A^*$$
$$\chi \longrightarrow \chi - \sqrt{2}\, i \Psi$$
$$(m+in) \longrightarrow (m+in) - 2i\, F$$
$$v_m \longrightarrow v_m - i\, \partial_m (A - A^*)$$
$$\lambda \longrightarrow \lambda$$
$$D \longrightarrow D$$

First three components of the vector superfield can be gauged away by properly choosing the chiral gauge parameter S (This is known as Wess-Zumino or WZ gauge). The component v_m transforms exactly the same way as ordinary gauge field hence is the name vector superfield. The fields λ and D are gauge invariant. λ is called gauge fermion or gaugino and D is an auxiliary field. In WZ gauge

$$V = -\theta\sigma^m\bar{\theta}v_m + i(\theta^2\bar{\theta}\bar{\lambda} - \bar{\theta}^2\theta\lambda)$$
$$V^2 = -\frac{1}{2}\theta^2\bar{\theta}^2 v_m v^m$$
$$V^3 = 0$$

This gauge breakes SUSY because first three components are set zero against supersymmetric transformations.

SUPERFIELD LAGRANGIAN

Under supersymmetry transformation the last component of a superfield transforms into space derivatives of lower dimensional components eg. $\delta_\xi F = -i\sqrt{2}(\partial_n\psi)\,\sigma^n\xi$. Thus space-time integral of the higest component of a superfield is SUSY invariant.

Chiral Superfield:

a) Kinetic Energy:

The product S^+S is a vector superfield. It's highest term is the coefficient of $\theta^2\bar{\theta}^2$. This is called the D-term. This is denoted by $\int S^+S d^4\theta$ or $S^+S\big|_{\theta^2\bar{\theta}^2}$

$$L_{kin}^{Chiral} = -(\partial^n A)(\partial_n A) + i(\partial_n\psi)\,\sigma^n\psi + F^*F \tag{18}$$

thus one finds that L_{kin}^{Chiral} has kinetic energy terms for all the component fields contained in the superfield S.

b) Superpotential:

The coefficient of θ^2 term of chiral superfields, called F-term, transforms into a space derivative under supersymmetry transformations($\delta_\xi F \cong (\partial_n\psi)\sigma^n\xi$). Hence space time integration of the F-term of a product of chiral superfields is invariant under supersymmetry transformation. This term is denoted by $\int S^n d^2\theta$ or $S^n\big|_{\theta^2}$.

Thus one possible type of invariant interaction, called superpotential, is

$$W = \sum_n a_n \int (S)^n d^2\Theta \tag{19}$$

With dimensional arguments one finds that renormalizability requires $n \leq 3$. Also I would like to point out that in perturbation theory radiative corrections only generate D-type terms. F-Type terms are never generated(2) in perturbation theory. This is the so called nonrenormalization theorem. Let us express the following lagrangian in terms of the component fields.

$$L^{Chiral} = L^{Chiral}_{kin} + L^{Chiral}_{superpotential}.$$

Choosing the superpotential W(S) to be

$$W(S) = \frac{1}{2}\int d^2\theta\, m\, S^2 + \frac{1}{3}\, g \int d^2\theta\, S^3 + \text{H.C.} \tag{20}$$

We find

$$\begin{aligned} L^{Chiral} = & - (\partial^n A)^2 + i\, (\partial_n \overline{\Psi})\, \sigma^n \Psi + F F^* \\ & + m\, (A F + A^* F^* - \Psi\Psi - \overline{\Psi}\overline{\Psi}) \\ & + g\, (A^2 F + A \Psi^2 + A^{*2} F + A^* \overline{\Psi}^2) \end{aligned} \tag{21}$$

Since there is no kinetic energy term for the F field one eliminates the auxiliary field by using

$$\frac{\delta L}{\delta F} = F^* + m A + g A^2 \tag{22}$$

One arrives at

$$L = -(\partial^n A)^2 + i(\partial_n \Psi)\sigma^n \bar{\Psi} - m(\Psi\Psi + \bar{\Psi}\bar{\Psi})$$
$$- m^2 A A^* + g(A\Psi^2 + A^*\bar{\Psi}^2)$$
$$- g^2 (AA^*)^2 \tag{23}$$

Thus we see, once the auxiliary field is integrated out, $m S^2$ term generates same mass term for both fermion and boson. Also the potential term $V_F = F^* F \geq 0$, implies that SUSY is unbroken if and only if V_F at the minimum is zero.

In case of Gauge theory the Chiral superfield is coupled to a vector superfield. The gauge invariant kinetic energy of the chiral field is

$$L_{kin}^{Chiral} = \int d^4\theta \, S^+ e^{2gV} S \tag{24}$$

This lagrangian is invariant under following transformation

$$S \dashrightarrow e^{i\Lambda} S$$
$$S^+ \dashrightarrow S^+ e^{-i\Lambda^+}$$
$$V \dashrightarrow V + i/2g \, (\Lambda^+ - \Lambda)$$

where Λ is gauge parameter and itself is a chiral superfield. The kinetic energy of the vector superfield is written in terms of a chiral field strength W_α defined by

$$W_\alpha = -\frac{1}{4}\bar{D}^2 D_\alpha V$$

The kinetic energy is given by

$$L_{kin}^{vector} = \int d^2\theta \; W^\alpha W_\alpha \qquad (25)$$

In WZ gauge it goes into

$$L_{kin}^{vector} = -\frac{1}{4} v_{mn} \; v^{mn} - i\lambda\sigma^m \partial_m \bar{\lambda} + \frac{1}{2} D^2$$

Where $v_{mn} = (\partial_m v_n - \partial_n v_m)$ the usual field strength for the gauge boson. The vector superfield can not be given an explicit mass term because of the same reason as in non-SUSY case. However, one can give masses to gaugino without spoiling gauge invariance or renormalizability albeit supersymmetry is sacrificed.

FEYNMAN RULES FOR SUPERFIELDS

Before we get into the feynman rules we would like to give some rules of the D-algebra. These are very important in any calculation involving supergraphs. Our notations are the same as that of Wess & Bagger[3].
The superspace coordinates is represented by "z" and the covariant derivatives D_α and $\bar{D}_{\dot\alpha}$ are

$$D_\alpha = \frac{\partial}{\partial\theta^\alpha} + i\sigma^m_{\alpha\dot\alpha}\bar\theta^{\dot\alpha}\partial_m \text{ and } \bar{D}_{\dot\alpha} = \frac{\partial}{\partial\bar\theta^{\dot\alpha}} - i\theta^\alpha\sigma^m_{\alpha\dot\alpha}\partial_m$$

These operators are used to define irreducible representations. The Chiral(Antichiral) field satisfies $\bar{D}\Phi=0$ $(D\bar\Phi=0)$. Some useful identities obeyed by the D's are

$$D_{1\alpha}\delta^4(\theta_1-\theta_2)=D_\alpha(\theta_1,p)\ \delta^4(\theta_1-\theta_2)= -D_\alpha(\theta_2,-p)\ \delta^4(\theta_1-\theta_2)$$

$$D_1^2\bar{D}_1^2\ \delta^4(\theta_1-\theta_2)=\bar{D}_2^2D_2^2\ \delta^4(\theta_1-\theta_2)$$

$$D^2\bar{D}^2D^2 = 16\Box D^2$$

Functional derivatives with respect to chiral fields are obtained from

$$\frac{\delta}{\delta\Phi(z_1)}\ \Phi(z_2)=(-\frac{1}{4}\ D_1^2)\ \delta^8(z_1-z_2)$$

SUPERGRAPH RULES[4]:

(a) CHIRALPROPAGATOR:

$$D^2\bar{D}^2 \; \frac{\delta^4(\theta_1-\theta_2)\,\delta^4(x_1-x_2)}{16i\;(-\Box+m^2)}$$

(b) VECTOR SUPERFIELD PROPAGATOR (Feynman gauge =1)

$$\frac{i\;\delta^4(\theta_1-\theta_2)\,\delta^4(x_1-x_2)}{2\;(-\Box+m^2)}$$

(c) VERTEX RULES:

(i) To every vertex associate a $\int\delta^4\theta$ integral;

(ii) To every chiral leg(of vertex) attach a $(-\frac{1}{4}D_1^2)$ factor

(iii) When vertex is chiral(antichiral) omit the $-\frac{1}{4}D_1^2(-\frac{1}{4}\bar{D}_1^2)$ factor at one of the legs of the vertex.

BREAKING SUPERSYMMETRY EXPLICITELY

Supersymmetry can not be an exact symmetry of the nature, as there is no evidence for it at low energy. Thus if it is to play any role in physics it must be broken one way or another at the energy scale we have been observing so far. The spontaneous breaking of global SUSY leads to a very cumbersome mass relations $\sum_J (2J+1)(-1)^J m_J = 0$[4]. This condition is very difficult to be satisfied by particles at low energies. For instance it implies the existence of charged boson lighter than electron. Hence we shall mainly consider explicit breaking in this paper. Explicit breaking on the other hand is too arbitrary and in general very dangerous. It is quite possible that a supersymmetric theory may lose it's smooth divergence properties if one is not very careful in choosing the Explicit breaking terms. But the good news is that there exists a class of SUSY breaking terms (Called soft terms) that do not affect the nice ultraviolet properties of the theory.

Girardello and Grisaru[5] have classified all such terms in N=1 supersymmetric theory using superspace methods.

Supergravity theories, at low energies appear as softly broken theories. Attractive thing about it is that it removes the arbitrariness that is associated with soft explicit breakings. Once the superpotential is chosen, the soft explicit breakings are automatically fixed. We shall study the structure of such theories in the context of left right theories in chapter(IV).

APPENDIX: A2

DETAILED ANALYSIS OF K_L-K_S MASS DIFFERENCE

Here we furnish detailed calculations for the chapter III. Following are the conventions and notations used in this chapter.

Notations: $\beta = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ $\quad \vec{\alpha} = \begin{pmatrix} 0 & \vec{\sigma} \\ \vec{\sigma} & 0 \end{pmatrix}$

$$\vec{\gamma} = -i\beta\vec{\alpha} = \begin{pmatrix} 0 & -i\vec{\sigma} \\ i\vec{\sigma} & 0 \end{pmatrix}$$

$$\gamma_4 = \beta, \ \gamma_5 = \gamma_1\gamma_2\gamma_3\gamma_4 = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$$

$$\sigma_{\mu\nu} = -\frac{i}{2}(\gamma_\mu, \gamma_\nu)_-$$

With above definition we find

$$\gamma_\mu = \gamma_\mu^\dagger, \qquad (\gamma_\mu, \gamma_\nu)_+ = 2\,\delta_{\mu\nu}$$

The Charge conjugation matrix C has the propertes that

$$CC^+ = 1, \qquad C^T = -C,$$

$$C\gamma_\mu C^{-1} = -\gamma_\mu^T \qquad C\sigma_{\mu\nu}C^{-1} = -\sigma_{\mu\nu}^T$$

$$C\gamma_5 C^{-1} = \gamma_5^T \qquad C\gamma_5\gamma_\mu C^{-1} = +(\gamma_5\gamma_\mu)^T$$

The basis of Clifford matrices that we choose for the fierzing purposes are

$1, \gamma_\mu, \sigma_{\mu\nu}, i\gamma_\mu\gamma_5$ and γ_5

The general formula we use for fierzing is given by

$$(\bar{s}v)(\bar{u}d) = -\frac{1}{4}(\bar{s}M_a d)(\bar{u}M_a v)$$

Where M_a stands for the different elements of the Clifford basis we have chosen. This formula can be used for all the fierzing that we need to do in this chapter.

The free propagator for a fermionic field, in our notation is

$$\langle 0|T\psi(x)\bar{\psi}(y)|0\rangle = i\,S(x-y) = -i\int\frac{d^4p}{(2\pi)^4}\,\frac{(-i\not{p}+m)}{p^2+m^2}\,e^{ip.x}$$

If the field is majorana type i.e. $(x)^c = \psi(x)$ then

$$\langle 0|T\{\psi(x)\,\psi^T(y)\}|0\rangle = -i\,S(x-y)\,C^{-1}$$

and

$$\langle 0|T\{\bar{\psi}^T(x)\,\bar{\psi}(y)\}|0\rangle = i\,C\,S(x-y)$$

Now we shall calculate the effective interaction generated by diagrams of Fig(3-1) and Fig(3-2) of chapter III. Let $\lambda_{\alpha,\beta}$ denote the α,β components of a SU(3) generators. a=1, ,8 and $\alpha,\beta = 1,\cdots,3$ are generator and color indices respectively. The interaction term is

$$\tilde{d}^*_{i\alpha} O_{ij}\,\bar{g}^a\,\lambda^a_{\alpha\beta}\,d_j + \text{H.C.}$$

Here $\tilde{d}$ is a 4-dimensional squark vector and d is 4-

dimensional fermionic field given by $\binom{d_L}{d_R}$ with $d_L(d_R)$ representing a two generation coloumn vector of down and strange quarks.

The effective interaction generated from the Fig(3-1) is

$$(s^T_\beta \lambda^a_{\alpha\beta} \bar{g}^{a^T} \tilde{d}^*_\alpha)\ (\tilde{d}^*_\gamma g^b \lambda^b_{\gamma\delta} s_\delta)$$

$$(\bar{\tilde{d}}_\mu \lambda^{*c}_{\mu\nu} \bar{d}_\nu\ g^c)\ (\bar{\tilde{d}}_\rho \lambda^{*d}_{\rho\xi} \bar{d}_\xi g^d)^T$$

In the momentum space we get

$$\int \frac{d^4q}{(2\pi)^4} \frac{-iC(-i\not{q}+m\ )}{(q^2+m^2_\lambda)} \quad \frac{i\,\delta_{\alpha\mu}}{(q^2+m^2_\alpha)} \quad \frac{i\,\delta_{\gamma\rho}}{(q^2+m^2_\gamma)} \quad \frac{-i(-i\not{q}+m\ )C^{-1}}{(q^2+m^2_\lambda)} \ \delta_{ab}\ \delta_{cd}$$

It generates an interaction term (Combination factors and coupling constants have been suppressed)

$$I_M = \frac{-[s^T{}'C(-i\not{q}+m_\lambda)s]\,[\bar{d}\ (-i\not{q}+m_\lambda)\bar{d}^T_\xi]}{(q^2+m^2_\alpha)\,(q^2+m^2_\beta)\,(q^2+m^2_\lambda)^2} \lambda^a_{\alpha\beta}\, \lambda^a_{\gamma\delta}\, \lambda^c_{\nu\alpha} \lambda^c_{\xi\gamma}$$

Now we can simplify the effective interaction by using the identity

$$\lambda^a_{\alpha\beta}\, \lambda^a_{\gamma\delta}\quad \lambda^c_{\nu\alpha}\, \lambda^c_{\xi\gamma}$$

$$= 4\left\{ \delta_{\alpha\delta}\,\delta_{\beta\gamma} - \frac{\delta_{\alpha\beta}\,\delta_{\gamma\delta}}{3} \right\}\left\{ \delta_{\nu\gamma}\,\delta_{\alpha\xi} - \frac{\delta_{\nu\alpha}\,\delta_{\xi\gamma}}{3} \right\}$$

$$= 4\left\{ \frac{10}{9}\,\delta_{\xi\delta}\,\delta_{\beta\nu} - \frac{2}{3}\,\delta_{\beta\xi}\,\delta_{\delta\nu} \right\}$$

Substituting this in the expression for I_M we arrive at

$$I_M = -\int \frac{1}{(q^2+m_\alpha^2)(q^2+m_\gamma^2)(q^2+m_\lambda^2)^2} \times \frac{d^4q}{(2\pi)^4} \times$$

$$\Big[q^2 \Big\{\frac{10}{9}\Big((\bar{d}'s')(\bar{d}s)+\frac{1}{2}(\bar{d}'\gamma_\mu s')(\bar{d}\gamma_\mu s)$$
$$+\frac{1}{2}(\bar{d}'\gamma_\mu\gamma_5 s')(\bar{d}\gamma_\mu\gamma_5 s)-(\bar{d}'\gamma_5 s')(\bar{d}\gamma_5 s)\Big) -\frac{2}{3}\Big((\bar{d}'s)(\bar{d}s')$$
$$+\frac{1}{2}(\bar{d}'\gamma_\mu\gamma_5 s)(\bar{d}\gamma_\mu\gamma_5 s')+\frac{1}{2}(\bar{d}'\gamma_\mu s)(\bar{d}\gamma_\mu s')-(\bar{d}'\gamma_5 s)(\bar{d}\gamma_5 s')\Big)\Big\}$$
$$+ m^2 \Big\{\frac{10}{9}\Big((\bar{d}'s')(\bar{d}s)-(\bar{d}'\gamma_\mu s')(\bar{d}\gamma_\mu s)+(\bar{d}'\gamma_5 s')(\bar{d}\gamma_5 s)$$
$$+(\bar{d}'\gamma_\mu\gamma_5 s')(\bar{d}\gamma_\mu\gamma_5 s) -(\bar{d}'\sigma_{\mu\nu}s')(\bar{d}\sigma_{\mu\nu}s)\Big)$$
$$+ \frac{2}{3}\Big((\bar{d}'s)(\bar{d}s')-(\bar{d}'\gamma_\mu s)(\bar{d}\gamma_\mu s')-(\bar{d}'\sigma_{\mu\nu}s)(\bar{d}\sigma_{\mu\nu}s')$$
$$+(\bar{d}'\gamma_\mu\gamma_5 s)(\bar{d}\gamma_\mu\gamma_5 s')+(\bar{d}'\gamma_5 s)(\bar{d}\gamma_5 s')\Big)\Big\}\Big]$$

The effective interaction generated from the Fig(3-2) is

$$(\bar{d}_\alpha \lambda^a_{\alpha\beta} g^a \tilde{d}_\beta)\ (\tilde{d}^*_\gamma \lambda^b_{\gamma\delta}\bar{g}^b s_\delta)$$
$$(\bar{d}'_\theta \lambda^a_{\theta\eta} g^a \tilde{d}_\eta)\ (d^*_\mu \lambda^b_{\mu\nu}\bar{g}^b s_\nu)$$

With exactly similar analysis, as done for the Fig(3-1) for this graph we get

$$I_D = -\int \frac{1}{(q^2+m^2)(q^2+m^2)(q^2+m^2)^2} \times \frac{d^4q}{(2\pi)^4} \times$$

$$\frac{4}{9}\Big[q^2\Big\{-\frac{21}{4}(\bar{d}\gamma_\mu s')(\bar{d}'\gamma_\mu s)-\frac{1}{4}(\bar{d}s)(\bar{d}'s')$$
$$+\frac{1}{8}(\bar{d}\gamma_\mu s)(\bar{d}'\gamma_\mu s')-\frac{1}{8}(\bar{d}\gamma_\mu\gamma_5 s)(\bar{d}'\gamma_\mu\gamma_5 s')$$
$$+\frac{1}{4}(\bar{d}\gamma_5 s)(\bar{d}'\gamma_5 s')\Big\} +m^2\Big\{21(\bar{d}s')(\bar{d}'s)$$
$$+\frac{1}{4}\Big((\bar{d}s)(\bar{d}'s')+(\bar{d}\gamma_\mu s)(\bar{d}'\gamma_\mu s')+(\bar{d}\sigma_{\mu\nu}s)(\bar{d}'\sigma_{\mu\nu}s')$$
$$+(\bar{d}\gamma_5 s)(\bar{d}'\gamma_5 s')\Big)\Big\}\Big]$$

Using the expression for the I_M and I_D we shall one typical graph of each kind. Let us consider the Fig(3-4) as an example. We use the expression for I_M and substitute $s'=s_L, s=s_L$, $d'=d_L$ and $d=d_L$. We find that only the vector term contributes. Summing over α and β we get

$$L_I = 2\ {}^4C_2 \frac{(ig_3)^4}{4!} \sin^2\theta_c \cos^2\theta_c$$

$$\times \left\{ f_{11} + \ldots + f_{44} + 2\,(-f_{12} + f_{13} - f_{12} + \ldots) \right\}$$

Where $f_{\alpha\beta} = \frac{12}{27}\, g_{\alpha\beta}\, (\bar{d}_L \gamma_\mu s_L)^2$ and

$$g_{\alpha\beta} = -\int \frac{q^2}{(q^2+m_\alpha^2)(q^2+m_\beta^2)(q^2+m_\lambda^2)^2} \frac{d^4q}{(2\pi^4)}$$

For the Dirac type mass term a typical graph is shown in Fig(3-5). We use the expression for I_D and substitute $s'=s_L, s=s_L$, $d'=d_L$ and $d=d_L$. When summed over α and β we get the following contribution.

$$L_I = 2\ {}^4C_2 \left(\frac{ig}{4!}3\right)^4 \sin^2\theta_c \cos^2\theta_c \times$$

$$\left\{ f_{11} + \ldots + f_{44} + 2\,(-f_{12} + f_{13} - f_{12} + \ldots) \right\}$$

Where $f_{\alpha\beta} = -\frac{20}{9}\, g_{\alpha\beta}\, (\bar{d}_L \gamma_\mu s_L)^2$

To calculate the $K_L - K_S$ mass difference the fermionic s', s, d' and d are replaced by all possible combinations of s_L, s_R, d_L and s_R. There are 16 such graphs for each type of mass terms. When all these are taken into account we arrive at the effective Hamiltonian of chapter III of page(43).

APPENDIX:A3

APPLICATION OF CHAPTER IV IN REAL LIFE CALCULATION

As an application of the machinery developed in the text, we shall derive RGE's for a realistic supersymmetric SU(3)XSU(2)XU(1) model given by Alvarez-Gaumme etal[1]. The model is described by following lagrangian

$$L= \Sigma_{mn} (g_e^{mn} \hat{E}^m\hat{L}^n\hat{H} + g_d^{mn} \hat{D}^m\hat{Q}^n\hat{H} + g_u^{mn} \hat{U}^m\hat{Q}^n\hat{H}')$$
$$-m_{3/2}\Sigma_{mn} (A_e^{mn} g_e^{mn} E^m L^n H + A_d^{mn} g_d^{mn} D^m Q^n H$$
$$+ A_u^{mn} g_u^{mn} U^m Q^n H') - \Sigma_i m_i^2 |\Phi_i|^2 + \text{K.E.Terms} \tag{1}$$

Here fields with superhats indicate superfields and those without hats are corresponding scalarcomponents. Subscripts m,n are generation indices.

Since this is a gauge theory one must write all SSB in gauge invariant form. Using the procedure developed in the text we write down the breaking terms in the following form with Spurion W ($W= \mu\theta^2$).

$$L_{SSB} = \alpha^m_{1a}\int d^4\theta\ \Phi^{a+}_m (e^{-2g\ \vec{V}.\vec{T}})\ \Phi^a_m\ W^+W$$
$$+ \alpha^m_{2a}\int d^4\theta\ \Phi^{a+}_m (e^{-2g\ \vec{V}.\vec{T}})\ \Phi^a_m\ (W^+ + W) \tag{2}$$

where $\hat{\Phi}_\alpha$ stands for different superfields like $\hat{E}, \hat{Q}, \hat{U}, \hat{D}, \hat{H}'$ and $\hat{H}$. T_i are the gauge generators for the representation of $\hat{\Phi}_\alpha$ and g denotes the gauge coupling constants. Before we proceed further we shall, following Ref(1), assume that g_u^{33} (we shall denote it simply by g_u) is the only yukawa coupling that should be retained for RGE as others are much smaller. Integrating out θ and comparing with equation(1) we get

$$m_{H'}^2 = \mu^2(-\alpha_{1H'} + \alpha_{2H'}^2)$$
$$A_U^{mn} = (\alpha_{2H'} + \alpha_{2U}^m + \alpha_{2Q}^n) \tag{3}$$

Now we can easily calculate the wave function renormalization constant(WFC) using supergraph rules. The relavant graphs are shown in Fig (4-6). We find(Z_{Φ_a} denotes the WFC for the superfield $\hat{\Phi}_a$)

$$Z_Q = 1-\{g_u^2 - \frac{1}{2}(\frac{1}{9}g_1^2 + 3g_2^2 + \frac{16}{3}g_3^2)\}\frac{1}{16\pi^2\epsilon}$$

$$Z_U = 1-\{2g_u^2 - \frac{1}{2}(\frac{16}{9}g_1^2 + \frac{16}{3}g_3^2)\}\frac{1}{16\pi^2\epsilon}$$

$$Z_{H'} = 1-\{3g_u^2 - \frac{1}{2}(\frac{1}{9}g_1^2 + 3g_2^2)\}\frac{1}{16\pi^2\epsilon}$$

(4)

Since g_u has no vertex renormalization(Coeff. of F-type term) one easily derives the RGE for it by using

$$g_u^{Bare} = g_u^{Ren.} \; Z_{H'}^{-1/2} \; Z_Q^{-1/2} \; Z_U^{-1/2}$$

differentiating the equation above with respect to μ one gets

$$\beta_{g_u} = -\epsilon g_u + (6g_u^2 - \frac{13}{18}g_1^2 - \frac{3}{2}g_2^2 - \frac{8}{3}g_3^2)\frac{g_u}{8\pi^2}$$

Now we shall calculate the vertex renormalisation for $\alpha_{1H'}$ and $\alpha_{2H'}$. The graphs that contribute towards $Z_{\alpha_{1H'}}$ and $Z_{\alpha_{2H'}}$ (Vertex renorm. for the coeff. $\alpha_{1H'}$) are given in Fig(4-7) and Fig(4-8) respectively. From them we find

$$Z_{\alpha_{1H'}} = \left[1 + \frac{3g_U^2}{16\pi^2\epsilon}\left\{\frac{\alpha_{1Q} + \alpha_{1U} - 2(\alpha_{2Q}^2 + \alpha_{2U}^2 + \alpha_{2U}.\alpha_{2Q})}{\alpha_{1H'}}\right\} + \frac{g_1^2 + 3g_2^2}{32\pi^2\epsilon}\right]$$

$$Z_{\alpha_{2H'}} = \left[1 + \frac{3g_U^2}{16\pi^2\epsilon}\left\{\frac{\alpha_{2Q} + \alpha_{2U}}{\alpha_{2H'}}\right\} + \frac{1}{2}\cdot\frac{g_1^2 + g_2^2 . 3}{32\pi^2}\right] \qquad (5)$$

Using wave function renormalisation costants derived in equation(4), we can easily calculate the following RGE's

$$\mu\frac{\partial\alpha_{1H'}}{\partial\mu} = \frac{3g_U^2}{8\pi^2}\left[\alpha_{1Q} + \alpha_{1U} + \alpha_{1H'} - 2(\alpha_{2Q}^2 + \alpha_{2U}^2 + \alpha_{2U}.\alpha_{2Q})\right]$$

$$\mu\frac{\partial\alpha_{2H'}}{\partial\mu} = \frac{3g_U^2}{8\pi^2}\left[\alpha_{2Q} + \alpha_{2U} + \alpha_{2H'}\right] \qquad (6)$$

Now using the relationship derived in equation(3) one finds the following RGE's for $m_{H'}^2$ and A_U^{33}

$$\mu\frac{\partial m_{H'}^2}{\partial\mu} = \frac{3g_U^2}{8\pi^2}\left[m_{H'}^2 + m_Q^2 + m_U^2 + m_{3/2}^2|A_U|^2\right]$$

$$\mu\frac{\partial A_U^{33}}{\partial\mu} = \frac{6g_U^2}{8\pi^2}\cdot A_U^{33} \qquad (7)$$

In the similar way we could derive RGE's for m_Q^2 and m_U^2. Comparing with Ref(A3-1) one finds that our approach gives the same answer as derived by the conventional method.

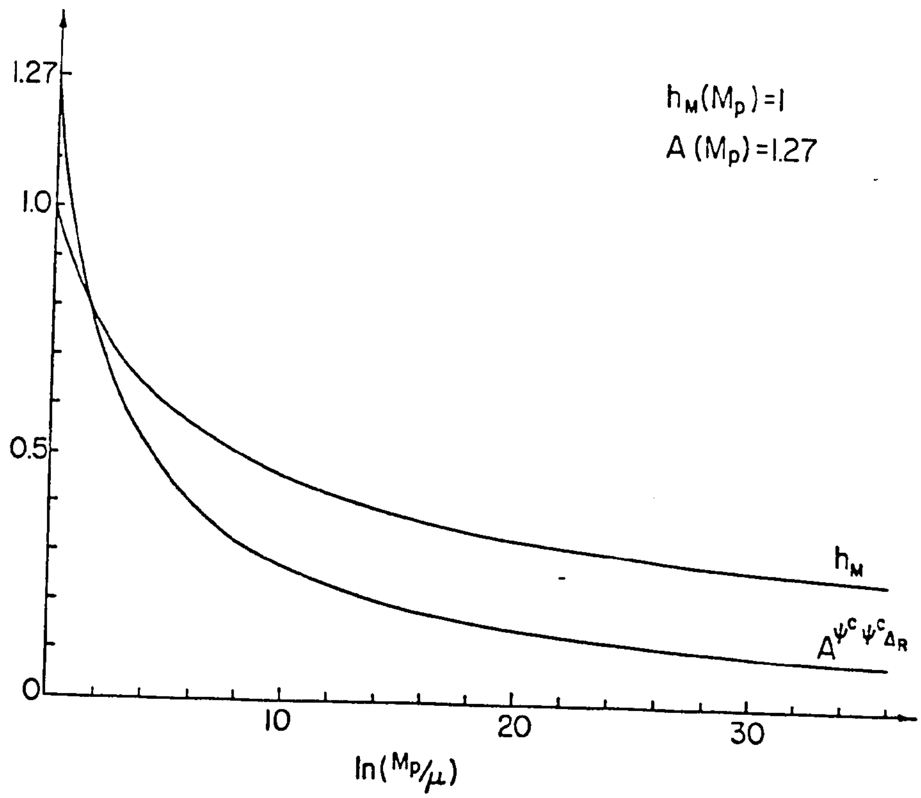


FIG (2-1)

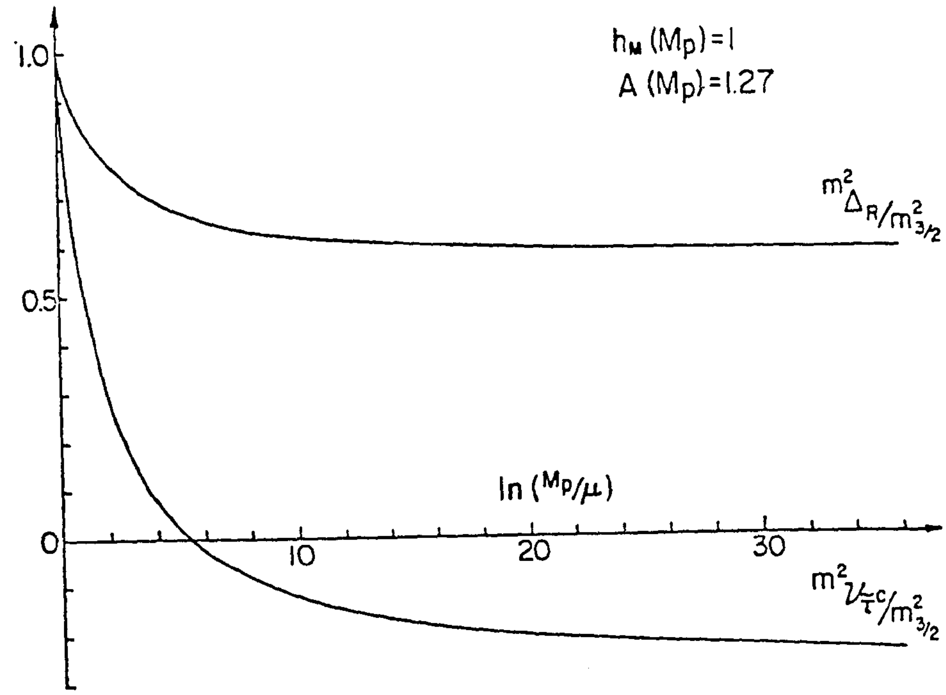

$h_M(M_p)=1$
$A(M_p)=1.27$
1.0
0.5
0
10
20
30
$\ln(M_p/\mu)$
$m^2_{\Delta_R}/m^2_{3/2}$
$m^2_{\nu_\tau^c}/m^2_{3/2}$

FIG (2-2)

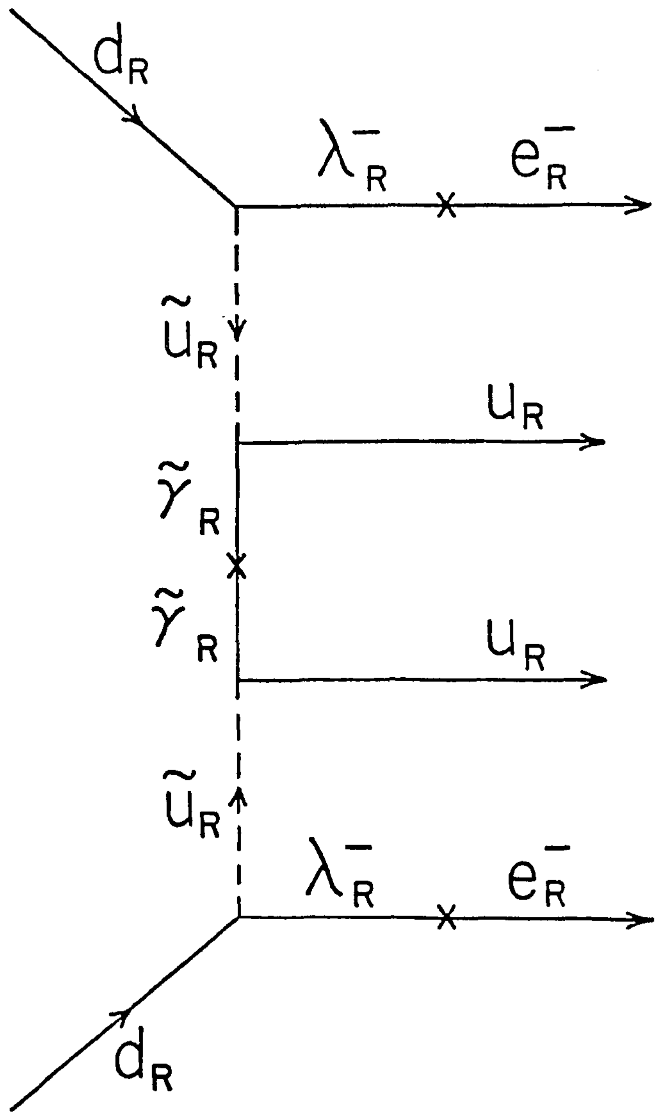
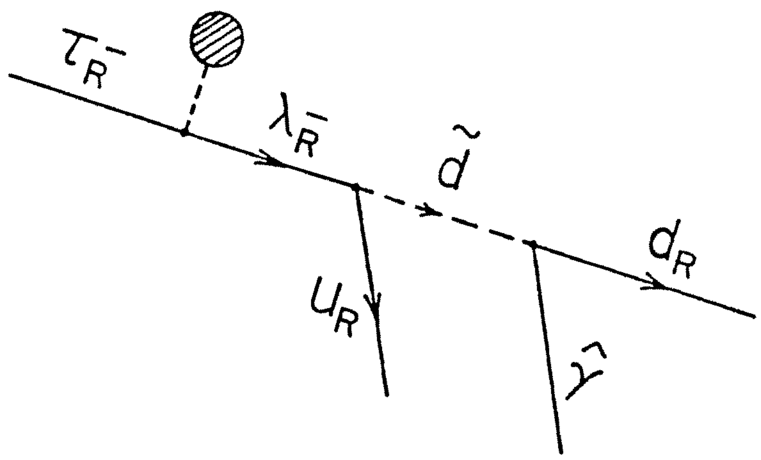
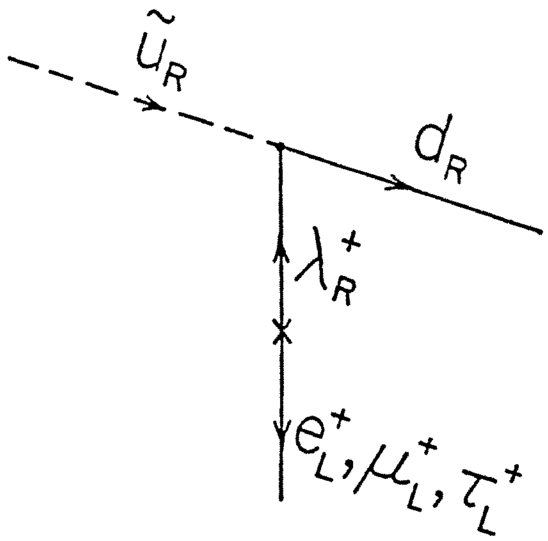

d_R
λ^-_R
e^-_R
$\tilde{u}_R$
u_R
$\tilde{\gamma}_R$
$\tilde{\gamma}_R$
u_R
$\tilde{u}_R$
λ^-_R
e^-_R
d_R

FIG (2-3)



τ_R^-
λ_R^-
$\tilde{d}$
d_R
u_R
$\hat{\gamma}$

FIG. (2-4)



$\tilde{u}_R$
d_R
λ_R^+
e_L^+, μ_L^+, τ_L^+

FIG (2-5)

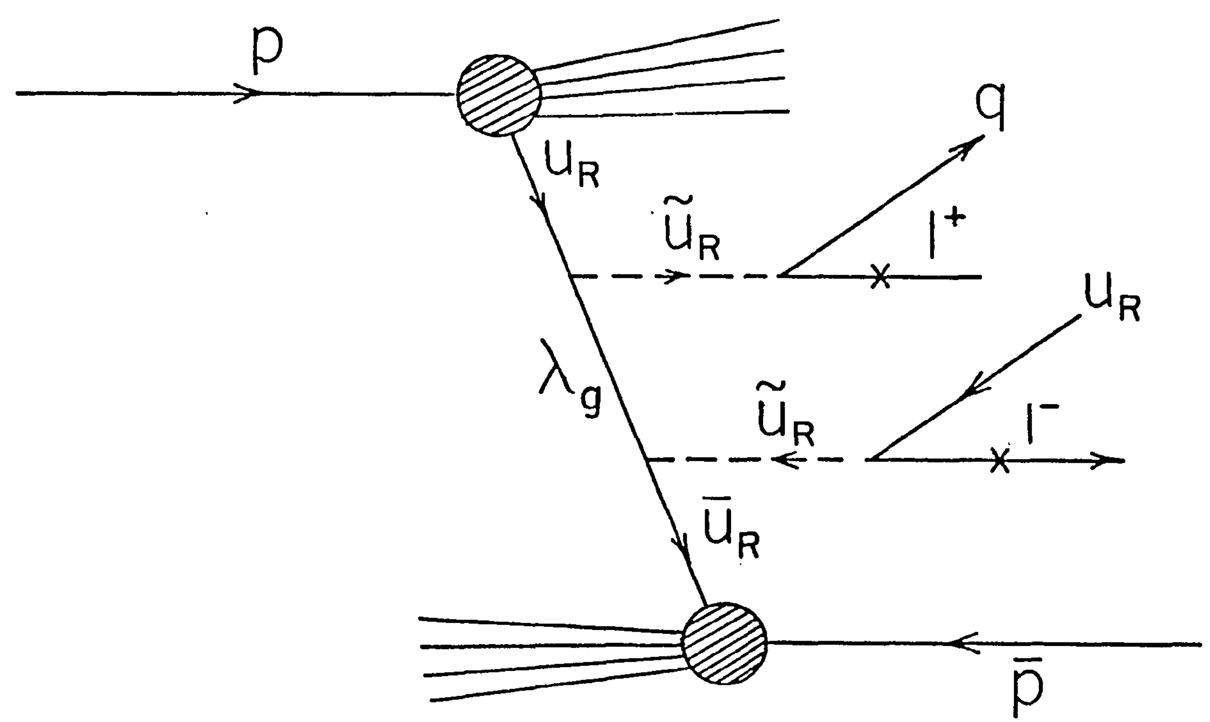
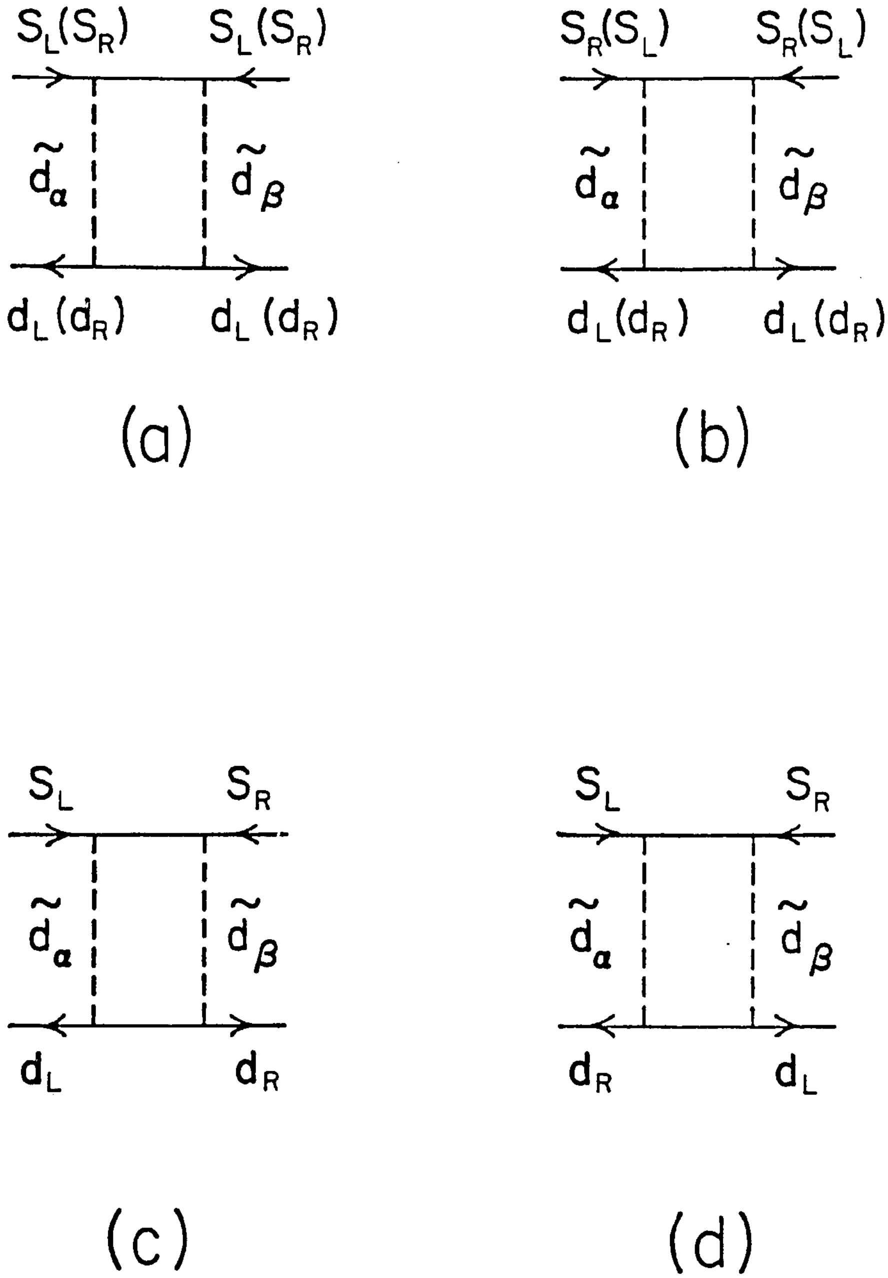
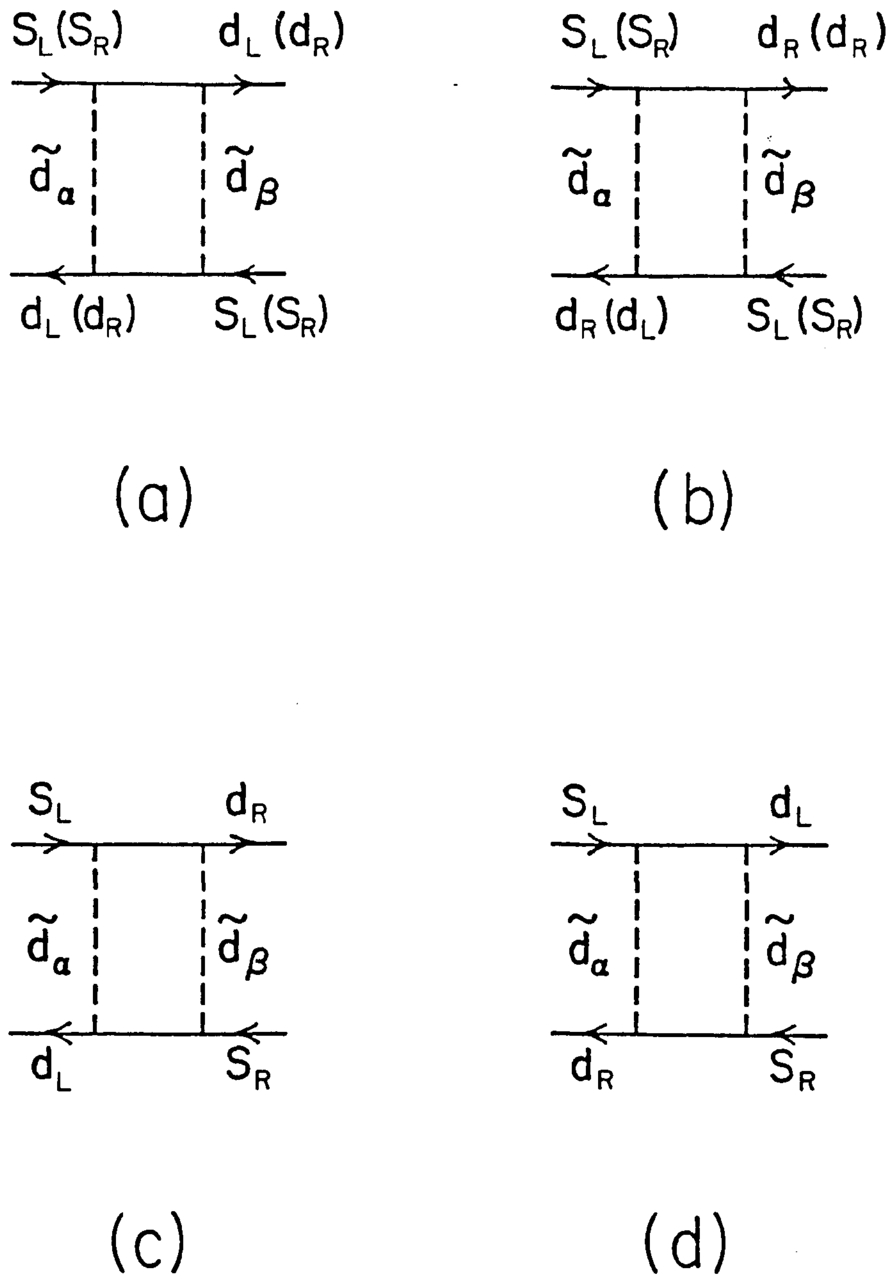


FIG (2-6)



S_L(S_R)
S_L(S_R)
d̃_α
d̃_β
d_L(d_R)
d_L(d_R)
(a)
S_R(S_L)
S_R(S_L)
d̃_α
d̃_β
d_L(d_R)
d_L(d_R)
(b)
S_L
S_R
d̃_α
d̃_β
d_L
d_R
(c)
S_L
S_R
d̃_α
d̃_β
d_R
d_L
(d)

FIG (3-1)



S_L(S_R)
d_L (d_R)
d̃_α
d̃_β
d_L (d_R)
S_L(S_R)
(a)
S_L (S_R)
d_R (d_R)
d̃_α
d̃_β
d_R(d_L)
S_L (S_R)
(b)
S_L
d_R
d̃_α
d̃_β
d_L
S_R
(c)
S_L
d_L
d̃_α
d̃_β
d_R
S_R
(d)

FIG (3-2)

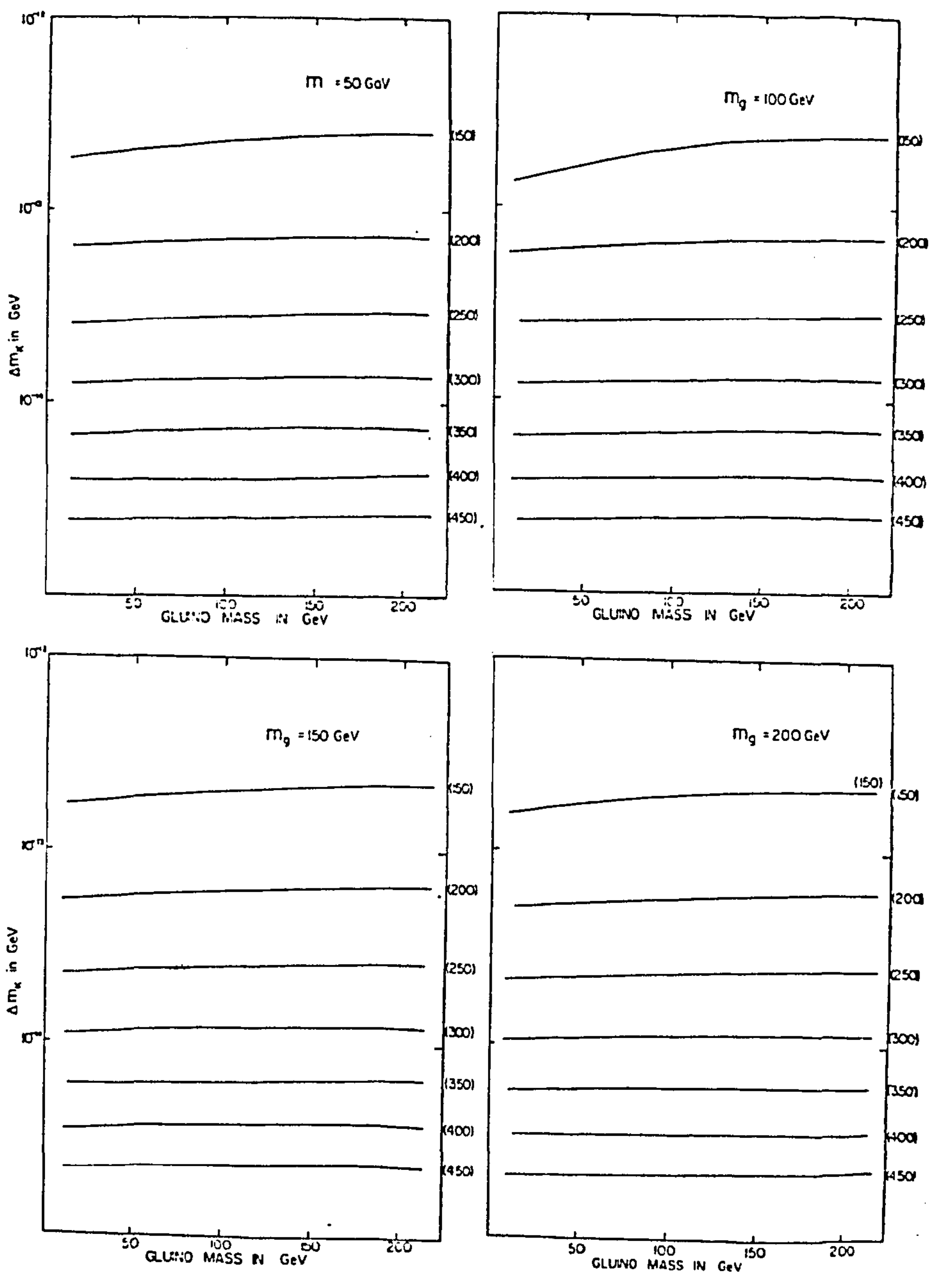

m = 50 GeV
m_g = 100 GeV
m_g = 150 GeV
m_g = 200 GeV
Δm_K in GeV
GLUINO MASS IN GeV
(150)
(200)
(250)
(300)
(350)
(400)
(450)
50
100
150
200

FIG (3-3)

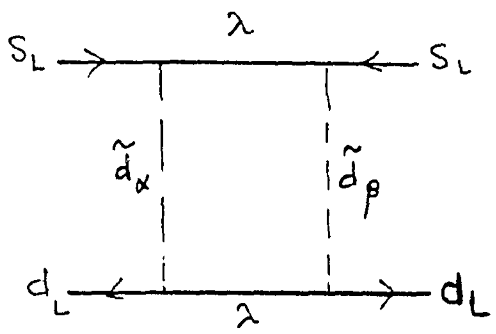

λ
S_L
S_L
$\tilde{d}_\alpha$
$\tilde{d}_\beta$
d_L
d_L
λ

FIG (3-4)

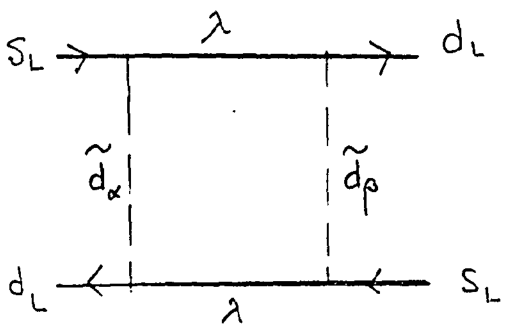
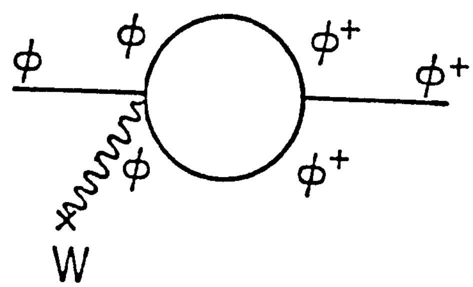

λ
S_L
d_L
$\tilde{d}_\alpha$
$\tilde{d}_\beta$
d_L
S_L
λ

FIG (3-5)



FIG(4-1)

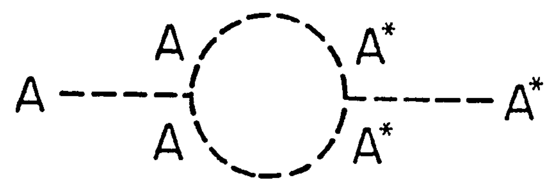


FIG (4-3a)

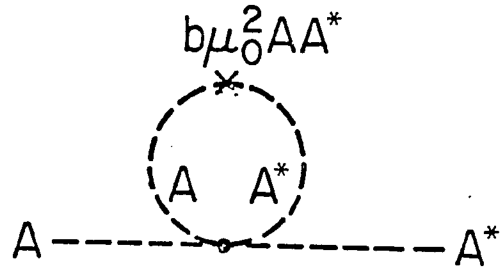


FIG (4-3b)

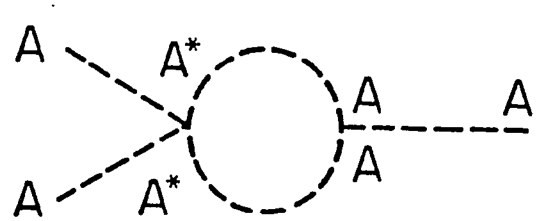


FIG (4-2)

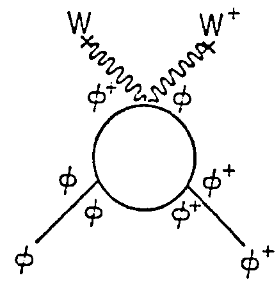


FIG (4-4 a)

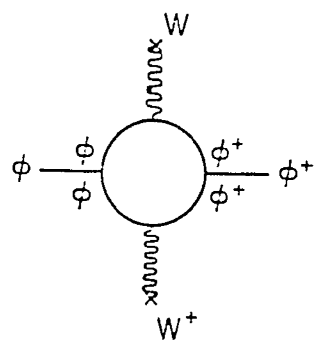


FIG (4-4 b)

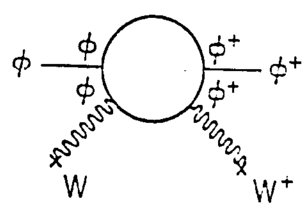


FIG (4-4 c)

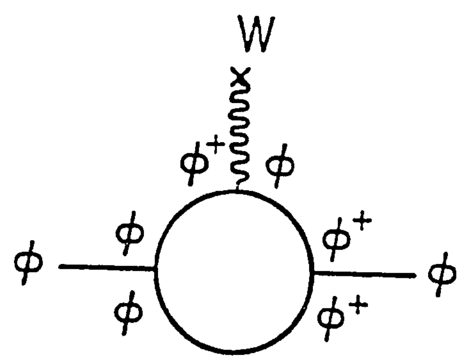
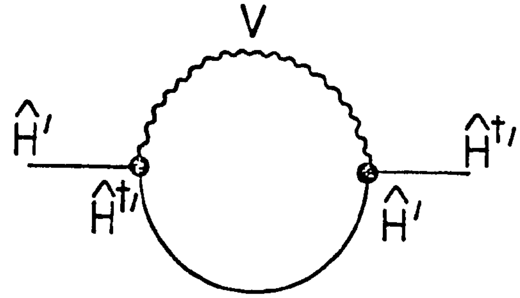
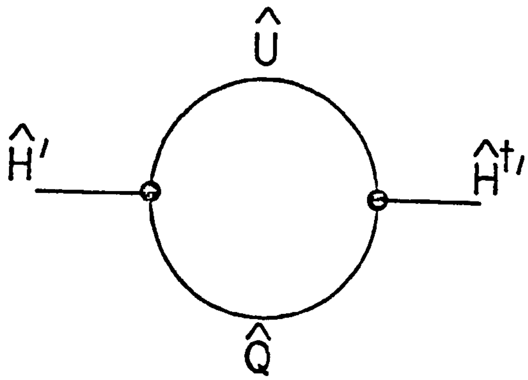


FIG (4-5)


V
$\hat{H}'$
$\hat{H}^{\dagger\prime}$
$\hat{H}^{\dagger\prime}$
$\hat{H}'$

(a)


$\hat{U}$
$\hat{H}'$
$\hat{H}^{\dagger\prime}$
$\hat{Q}$

(b)

FIG (4-6)

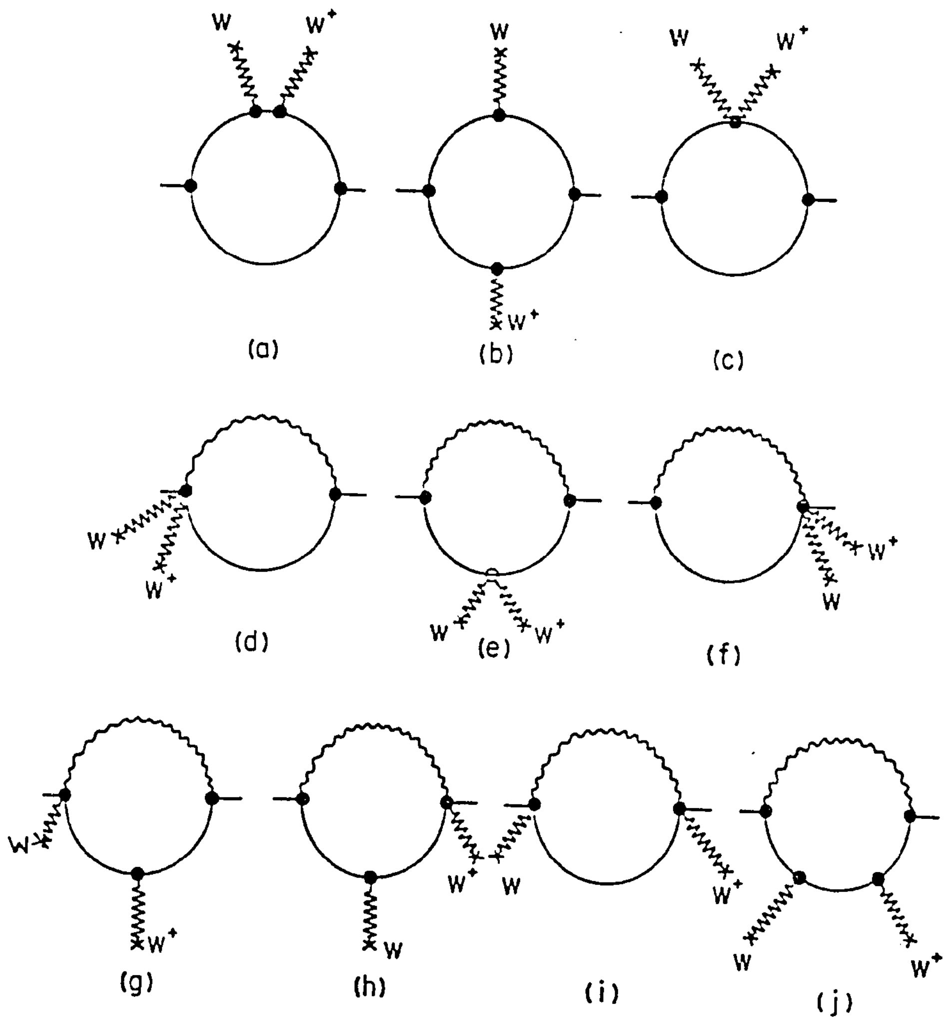


FIG (4-7)

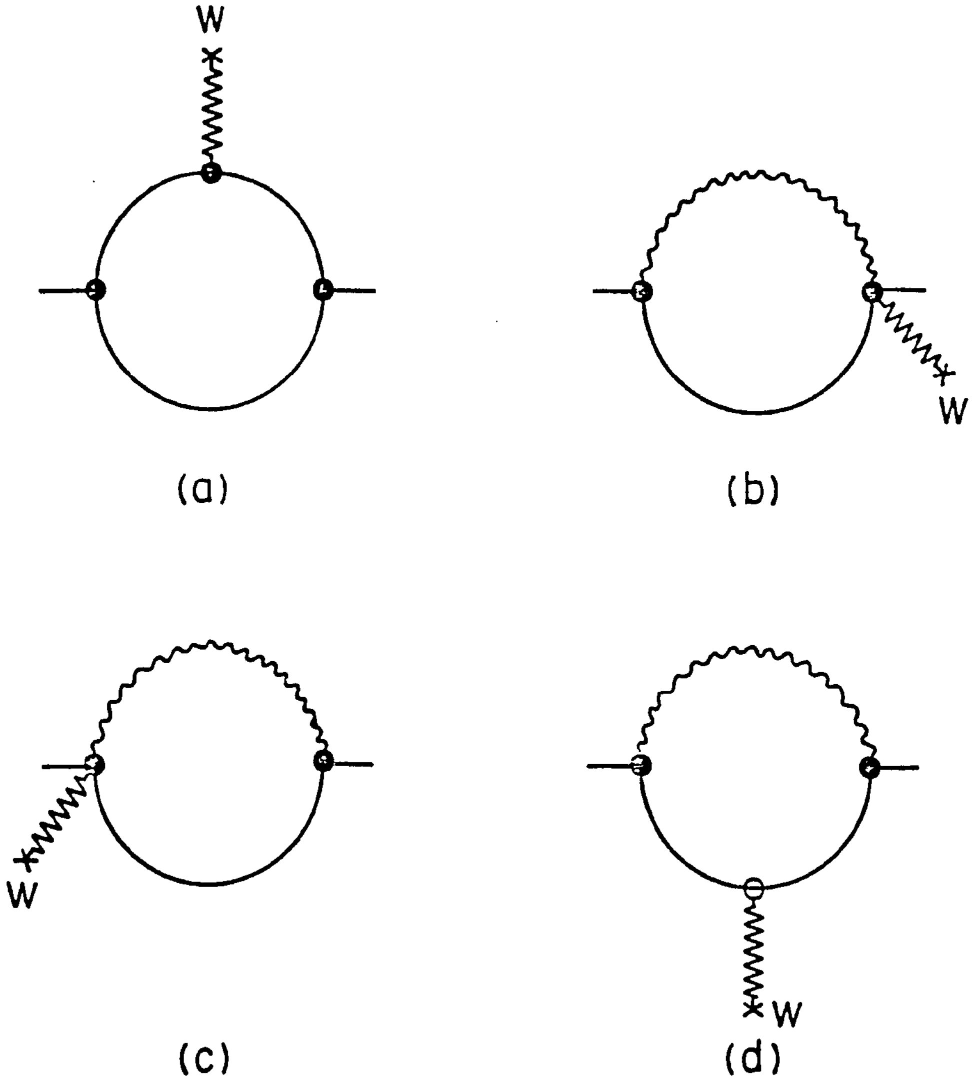


FIG (4-8)

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