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REGIONAL INPUT OUTPUT FORECASTING
A SYSTEMS APPROACH

by

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Abstract

REGIONAL INPUT OUTPUT FORECASTING
A SYSTEMS APPROACH

by

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Advisor: Professor Malcolm Galatin

This study describes an application of systems techniques to a real world problem; that of forecasting output by industry for 1980, 2000, and 2020 in the North Atlantic Region. These forecasts were developed in a regional input output forecasting system which incorporated both technological and spatial change in the input output coefficients. In addition to producing forecasts, the system allows for multiple scenario analysis. A number of general algorithms for input output coefficient sensitivity testing were formulated to simplify following efforts in the same area of analysis.

PREFACE

This study developed out of a path breaking project done by the North Atlantic Regional Water Resources Study Group of the North Atlantic Division, U.S. Army Corps of Engineers. The original project presented the first full application of sophisticated methodologies to a large Corps of Engineers study. Although similar work had been done by university research teams, this project was the first to be done by a regional office for actual implimentation to the construction stage.

My work to extend and improve the methods utilized in the initial study was financed for the first two years by the dissertation fellowship program set up by the Corps of Engineers to introduce " state of the art" theory into practical planning.

I am indebted to the principal framers of the initial study, Harry Schwarz, the chief engineer, and Dr. David Major, the chief economist, both of whom gave me encouragement and support in my tenure at the North Atlantic Divisional Office as a dissertation fellow and employee.

Dr. Major, in addition to his help as my supervisor, served as dissertation committee chairman for almost three years. In this period guidance was invaluable both at the technical and personal level. His interest and advice have helped me throughout the seven year span of my study.

The present chairman of my dissertation committee, Professor Malcolm Galatin, has for the past seven years provided continued advice and constructive criticism of the highest quality. Particularly in the area of trading pattern projection, his help and work have allowed me to clarify a subject that heretofore was never fully examined.

For the past four years Dr. Robert Ortner and the Bank of New York have kindly allowed me to use many of the Bank's resources. Without this help, much of the computer analysis would have taken substantially more time and effort.

Throughout this project, my wife, Lynn, has provided encouragement, advice, keypunching aid, typing services, editing assistance, company on long nights at the computer center, and much personal support when my resolve wained. Without her effort, this dissertation could have not been completed.

Lastly, I would like to express my thanks to the late Professor Alfred Conrad, my original advisor, and the individual who first introduced me to the subject of input-output analysis.

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CHAPTER I

INTRODUCTION

1. Introduction

This study will present a systems approach to the problem of forecasting regional economic activity as a step in the process of determining future resource demand. It will be shown that the systems approach makes it possible to construct and manipulate a consistent, relatively sophisticated forecasting model with limited time and staff. This approach is particularly useful to state or regional groups that have in the past decided to use simplified forecasting models in the belief that more sophisticated systems were beyond their resources. In addition to the greater realism possible with a systems approach, the ease in which forecasts may be altered or updated is significantly increased. This ease of manipulation is invaluable to ongoing planning studies that must interact with the public.

The term systems approach, which has many definitions, will be taken to mean the utilization of mathematical models in a computerized system.

Mathematical models are utilized to force the researcher to clearly present the full extent of his information about the subject being examined. Clear understanding of a phenomenon implies the ability to formalize its structure in a mathematical model. This does not mean that the model functions and relationships must be smooth, continuous, or linear in nature as they are in most simple systems. New methodologies, such as simulation, make it possible to describe any form of interaction.

Programming of the models for computer solution is an integral step in the systems approach since it allows for the use and manipulation of sophisticated mathematical models. This may seem merely a quantitative difference from the hand hewn results of previous years. But when it is realized that the computer makes it possible to handle tens of thousands of variables as opposed to perhaps five or ten, the difference seems to be one of almost a qualitative nature. In fact, the advent of the computer has made possible the solution of problems that were unsolvable by non-automated means. In studies that are limited in terms of time and staff the computerization of the solution algorithms is a crucial element of the systems approach. A small staff is able to work with very complex models if these models are part of a computerized system.

Mathematical models programmed for computer solution compel an explicit presentation of the underlying assumptions of a forecasting system. Methodologies not utilizing the systems approach may never face their basic assumptions since at no time is it necessary to formalize their structure. In addition, the systems approach provides for reproducibility which makes it amenable to varied applications and easy modification.

The benefits eminating from the systems approach to forecasting are particularly relevant to water resources planning efforts. Water resources planning studies are typically done by contract to private engineering firms or in civil service offices predominately staffed by engineers. Due to this predominence of engineering input, the studies tend to be imbalanced. That is, while sophisticated hydrolic simulation models and detailed construction costs are formed, the demand forecasts that determine the need for project construction are derived from highly simplified models. In many cases the projection models merely reflect freehand curve fitting combined with judgement factors. The rationale for such an approach is that the marginal cost of improving the forecasting methodology is less than the marginal benefit yielded by the improvement. Considering the inflexibility of the typical models used in many studies it is not clear if the above rationale is valid. Freehand curve fitting and judgement factors are impossible to update, reproduce, modify or even examine systematically, except possibly, by the individual who did the work. Despite this, if the rationale for limiting the input into forecasting is accepted as valid, the only approach to improving the methodology is the construction of more sophisticated models that are implementable with limited resources. The systems approach utilized in the present study will show that this is possible.

In order to illustrate the power of the systems approach an actual problem will be considered. Since the staff and time allotted to this task is significantly smaller than in even the most modest real world study, not all aspects of the problem will be examined in complete detail. In addition to the consideration of the general systems problem, a number of theoretical areas will be investigated.

The remainder of this chapter will describe the actual real world problem, some possible approaches to regional forecasting, a description of the prime methodological deficiencies of the projection approach originally used, an outline of the following study and a short summary of its major conclusions.

2. Real World Problem

The need for water demand forecasts by the North Atlantic Region (NAR) study group provides the real world problem that will be utilized to illustrate the power of the systems approach. By water demand we refer to the level of water used that will exist in the future assuming no changes in water price, unit consumption, or relative supply conditions. The projections are thus conditional forecasts and not demand in the strict sense of the term. Changes in utilization levels, due to variation in the factors assumed constant, can be dealt with by altering the parameters defining the system.

The North Atlantic Region stretches from Maine to Virginia and includes at least some portion of each of the original thirteen states. It lies eastward of the Ohio River Basin and north of the deep South. Its physical boundaries are defined on a hydrolic basis as encompassing a relatively independent water supply area. For the actual NAR study the area has been broken up into 21 river basins and 50 sub-basins.

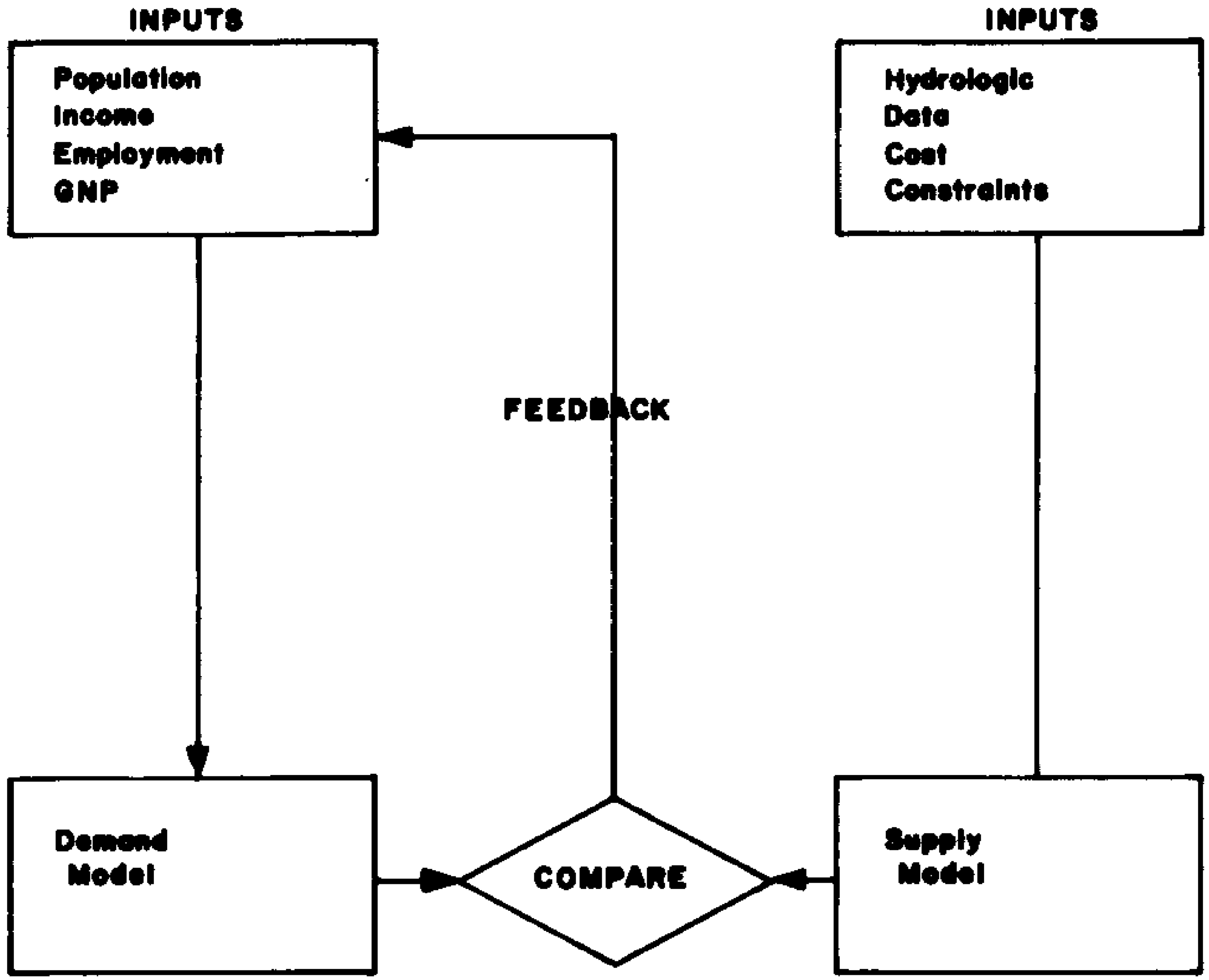
The primary purposes of the NAR study are to identify areas in the region where projected demands for water exceed projected supply, and suggest methods of eliminating these shortfalls. The first model, the Demand Model [1] - a regional input output (IO) forecasting model - yielded future water needs. The second model the Supply Model [2] - a linear programming optimization model - generated alternative ways and costs of meeting the projected demand.

The Demand Model was combined with the Supply Model to create a water resource forecasting and evaluation system. The system is simultaneous in nature, in that inputs into the Demand Model can be altered in response to output from the Supply Model. For instance, if the Supply Model indicates high costs (e.g., due to ecological constraints) of meeting a high level of demand, industrial growth rates could be altered to reduce the level of demand. Thus the complete system can act as a policy tool to assess the costs of particular policies. Schematically figure 1 describes the complete NAR system.

Since both the NAR and the present study utilize a regional input output model, the next section will consist of a discussion of the practicial and theoretical justification of the decision to use IO analysis.

Figure I

NAR Forecasting and Evaluation System



3.Approaches to Regional Forecasting

In the previous section the Demand Model was described as a regional input output model. Both the NAR project and the present project (RIOFS - Regional Input Output Forecasting System) utilize the input output approach to forecasting. The decision to employ input output is a function of the basic nature of resource utilization. The methodology is just as valid for oil, electricity, labor or any general input as it is for water. Although the remainder of the study will consider water as the relevant variable, the RIOFS is applicable to most general inputs.

Basic to forecasting of water demand is the assumption that industrial water use is related to economic activity in some systematic way. Water is used both as an input in the processing of certain goods (e.g., cooling for steel production) and as an integral part of other goods (e.g., soda bottling). In either case it is reasonable to expect, other things constant, that an increase in economic activity will increase the need for water. This assumption makes it possible to project economic activity as an intermediate step in the forecasting of water demand.

There are four types of models that are usually used to project regional economic activity; aggregate simultaneous equation models, dissaggregated independent equation models, disaggreated simultaneous equation models and regional input output models. Water requirements per unit of economic activity vary extensivly among industrial sectors, thus any aggregated model can mask the effects of differential growth rates in different industries. Independent equation systems suffer from the fact that they do not incorporate the interdependencies between sectors that exist in an economy. This can lead to inconsistencies between sectors (e.g., chemicals may be forecasted to reach a level that is not consistent with the growth of the industries utilizing chemicals in their production process). Disaggregated simultaneous equation systems, incorporating equations for each industry, do not suffer from the above theoretical drawbacks, but are very difficult to implement due to their extreme data requirements. Regional input output models provide an implementable, consistent, disaggregated structure that is ideally suited to projecting industrial economic activity.

Athough regional input output models are suitable for water demand forecasting, there are theoretical problems with their application. Following is a short description of regional input output forecasting and the problems existing in its implementation.

4. Regional Input Output Forecasting

The regional input output model utilized in the NAR study was static in nature. It was constructed using data from a single point in time and then assumed to remain structurally constant over the future. As shown later in this study, a static system such as the one developed for the NAR implies identical growth rates for all regional industries. This result provides little information as to the future resources requirements of a region and ignores existing data that can be used to better define the eventual structure of the region.

There are three forecasting tasks existing in the regional input output context: (1) final demand projection; (2) technology projection; and (3) trading pattern or industrial location projection.

Final demand projection has been subject to the most extensive consideration of the three areas. There are generally accepted approaches to final demand projection, and numerous projections are available [3]. Thus rather than construct a copy of previous final demand forecasts the RIOFS will adopt the NAR figures and concentrate on technology and trading pattern projection. These two areas, especially the latter, are areas in which little theoretical and practically no empirical work has been done.

The second forecasting task, that of technology projection, must be considered in relation to the length of the projection period. The target years for both the NAR and this study are 1980, 2000, and 2020. Forecasts of thirty and fifty years into the future are needed because of the long life of typical water supply projects. Rather than assume constant technology through 2020, as was done for the NAR, the RIOFS makes use of available information on technological change in a simple but reasonable model.

The last area of analysis, trading patterns, relates to the characteristic that distinguishes regional from national IO analysis. This refers to the fact that while the U.S. can be assumed to be a closed economy [4], a region such as the NAR will import and export significant amounts of commodities. The external trade results from the excess or absence of productive capacity in certain industries in relation to the input requirements of their customers. This trading makes it impossible to simply utilize the national input coeffcient matrix to represent the regional structure. Thus, unless it is possible to compile the massive amounts of data needed to produce a survey regional IO table, an adjustment procedure requiring limited amounts of primary data must be found. This procedure must yield regional input coefficients that give the amount of output produced in the region needed to manufacture a dollars' output from some industry. In essence, this means that the input coefficients must be given a spatial component in addition to the technological component which can be represented by the national coefficients.

In addition to providing regional input coefficients the adjustment procedure must lend itself to projection with a minimum of independent variables. This characteristic is essential to a long term study in the same way that technology projection is. The assumption of constant trading patterns is possibly less justifiable than the corresponding assumption about technology.

The RIOFS will thus concern itself with the forecasting of technology and trading patterns in the context of a regional input output model. More important than the specific methodology is the ease in which the projection system is formed and manipulated. This characteristic results directly from the utilization of the systems approach to solve this forecasting problem.

5. Study Outline

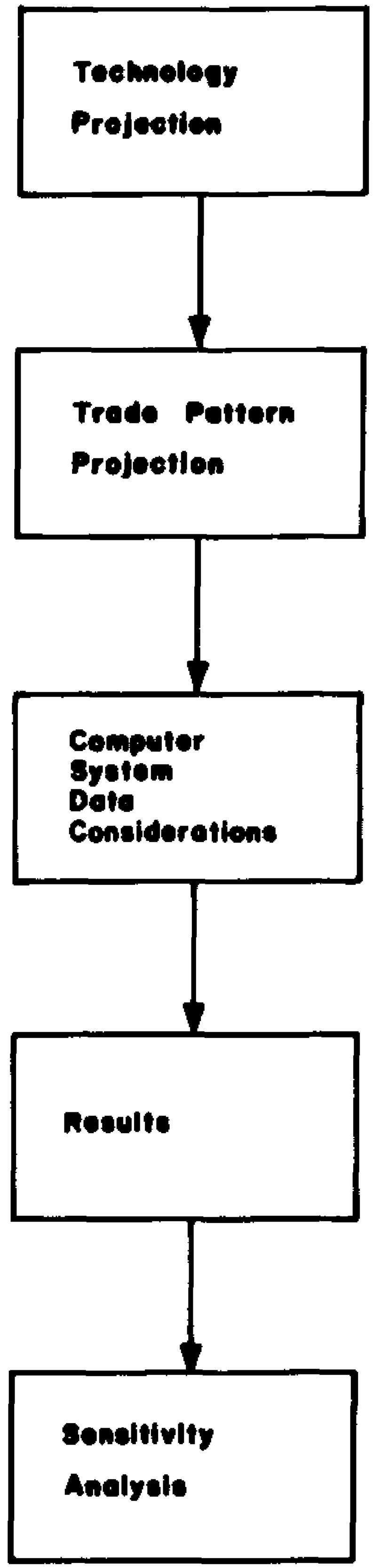
The structure of the study will follow the operational flow of the RIOFS. As shown in figure 2 the system consists of four steps.

Technology projection is the initial step in the forecasting procedure. In order to extrapolate technological change far into the future, a simple two point model is used. The observations consist of the 1947 and the 1963 IO tables. Although these tables are only 16 years apart they are the most widely separated studies done on a consistent basis. As more IO tables are available the structure of long run technological change will become more amenable to analysis. Since the observation period is shorter than the forecast periods, care was taken to assure that the projected tables were reasonable in an economic sense. For instance, column sums were constrained to be less than one and individual coefficients prevented from becoming negative. This was accomplished by transforming coefficient column sums into probability values and forecasting their change along a normal distribution function. This methodology, although not theoretically justifiable in most applications, provides the only reasonable procedure in an area of such limited data availability. Other approaches require extensive research and do not clearly produce superior results. Chapter II will present this methodology in detail and also examine the practical and theoretical advantages and disadvantages of the procedure vis-a'-vis other approaches.

The projected input coefficient matrices derived in step one must then be adjusted for changing trading patterns. Since direct information about regional trade is not available, the forecasting

Figure 2

Operational Flow of the RIOFS



scheme makes use of data on industrial location. The basic assumption underlying this approach is that the relative concentration of different industries in a region as compared to their concentration in the nation will determine the amount of external trade. Due to the lack of data on output by industry for individual regions over any time span, employment is utilized as a proxy variable. Lack of a computerized regional data base limited the number of observations considered to ten over a 21 year time span. Expanding the size of this data base would be relatively easy for a full study team.

The projected coefficient tables were adjusted row by row through the use of a set of location quotients. These location quotients were calculated as a function of projected regional and national employment and the future input coefficients. Under a number of rigid, but necessary assumptions, the projected location quotients show the relative ability of each regional industry to supply the needs of its customers. When the quotients indicated less than sufficient production capacity, the input coefficient row for that industry was reduced to reflect the fact that the necessary output to fill the gap between regional production and regional needs was being imported into the area. Industries showing excessive productive capacity were assumed to be exporters and, therefore, their input coefficients were not altered.

Despite the need for a restrictive set of assumptions to apply this methodology, the scheme makes use of data that in many cases show clear indications of future changes in industrial location. Assumption of constant trading patterns leads to wholely unrealistic forecasts that are merely blown up versions of the original situation.

Chapter three considers the theory of location quotients in detail since a literature search has shown the area to be one of great confusion. In this discussion the invalidity of most other approaches is clearly shown. Practical considerations of time and staff make the remaining alternatives not viable. The distinct limitations of the RIOFS approach are explicitly stated in this chapter.

The last step in the RIOFS consists of the integration and evaluation of steps one and two. It is in this step that the use of the systems approach clearly shows its advantages. Without the computerized system it would not be possible to examine more than one scenario. The RIOFS provides for the easy treatment of an almost unlimited number of alternatives. This study will only consider alternatives that highlight the effects of the forecasting procedures discussed in chapters two and three. Simple modifications of the system can be made to examine the consequences of changes in the exogenous variables, such as variations in GNP, population growth or final demand distribution.

The resulting output of four alternative configurations are examined in chapter four. These alternatives are:

1) No projection of either technology or industrial location

2) Technological change with no locational change

3) Location forecasts combined with constant technology

4) Both location and technology projections

Alternative one is the NAR Demand Model case and illustrates the consequences of assuming static conditions. The second and third alternatives show the independent effects of each projection methodology. Alternative four is the solution case that yields output forecast that can be used with resource coefficients to provide projections of resource demand.

Neither of the forecast procedures preclude the independent estimation of rows, columns or individual coefficients. In certain cases information may be available that can be used to provide more reliable forecasts of technological or spatial change. Usually such forecasts are complex and time consuming. Chapter V consists of a set of algorithms that test the sensitivity of industry output variations to changes in rows, columns or individual coefficients. The results obtained from these algorithms can minimize the effort put into independent forecasts by pinpointing critical elements of the coefficient matrix. They also indicate elements of the RIOFS that should be most closely monitored. Since the major portion of most resource utilization typically occurs in a subset of the total industry group, the number of critical elements is less than in the case where all industries are equally important. Due to time limitations the sensitivity routines were not applied in this study. The detailed presentation in chapter V is included to fill a gap in the literature on input output analysis. The only work done in this area, in connection with the Korean War [5], was discontinued in the 1950's due to lack of funds.

6. Conclusions

The system described in this study was developed and applied to an actual problem by one individual. This fact points to the potential of the systems approach since it proves that the application of sophisticated methodologies is not beyond the resources of even small study teams. Hence, in this sense the study must be considered a successful endeavor.

The actual results indicate that the assumption of no change in technological or spatial relationships leads to quite a different situation than would exist if changes are incorporated in the forecasting procedure. The RIOFS scenario (incorporating both types of change) resulted in significantly smaller gross output levels needed to support the given level of final demand, than the NAR alternative (incorporating the no change assumption). This result has important implications for water resource projects which in addition to costing millions of dollars have certain negative environmental effects. If the results of the RIOFS are accepted as valid, the cost benefit ratios developed under the NAR assumptions overstate the benefits, and so lead to non-optimal decisions.

Since the methodologies utilized to form the system were often only approximate, the results should be viewed as an indication of the need for further study. When the costs of a project are as high as they are in water resource development, large divergencies in results must be acknowledged as requiring additional examination, before decisions based on these results can be made.

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CHAPTER II

TECHNOLOGY PROJECTION

1. Introduction

The projection of regional technology is the initial step in the formation of a regional input output forecasting system. In this context technology refers to the productive structure of an economic system as represented by a matrix of input coefficients. The input coefficient matrix can be thought of as a set of column vectors, each column representing a linear homogenous production function with fixed factor proportions. Hence, input coefficient projections are in effect forecasts of future production functions.

Section two of this chapter considers empirical and theoretical evidence on the effects of input coefficient change upon the level of industrial output. Since input coefficients change in response to forces other than pure technological breakthrough, the theoretical discussion clearly differentiates the casual factors.

In as much as a directly compiled regional coefficient table is beyond the resources of most study groups, the national relationships are assumed to hold for the region. Section three presents a discussion of the basic assumptions that must be made to utilize the national table.

Section four develops the criteria that will be used to evaluate the projection methodologies. These criteria consist of resource limitations and a number of logical consistency constraints.

The next section is made up of a discussion of previous work on coefficient projection. Two approaches, the expert approach, and the RAS (row and column adjustment) approach, are presented. Since the latter methodology is closely related to the RIOFS method it is considered in some detail.

The form of the RIOFS methodology is examined in section six. It is essentially a two point forecasting scheme utilizing a transformation into probability space in order to satisfy the projection criteria.

The last part of the chapter (sections seven and eight) presents an analysis of the theoretical justification for the acceptance of a two point model. This discussion details the long term diffusion process that underlies most technological change (section seven). Also included in the section is an examination of the distinct limitations of the chosen methodology (section eight).

2. Empirical and Theoretical Evidence On the Importance of Coefficient Change

Before proceeding with any discussion of coefficient change it is important to consider the significance of possible changes in input coefficients on industrial output. Coefficient projection would only be necessary if empirical or theoretical evidence indicates that significant variations in output result from the coefficient changes that conceivably could occur over the study time frame.

Empirical evidence

Empirical studies have shown that, while certain of the coefficients are relatively stable, in general the changes that occur in the coefficients over time will effect the level and distribution of gross output given any level of final demand. For example, Beatrice N. Vaccara [1] compared the outputs required from each industry to produce a fixed set of final demands with the 1947 as compared to the 1958 inverse coefficient matrix. She found that, on the average (ignoring signs), there was a 16 percent difference in the output required from each industry to produce the 1958 bill of goods with the 1958 technology, as compared to the 1947 technology. Although the average impact on an individual industry's production requirements was 16 percent, for a few industries, the differences in output requirements exceeded 75 percent. Since a fixed level of final demand was utilized, industries selling primarily to final demand show smaller percentage changes in total output than industries selling primarily to intermediate demand. In consideration of this, Ms. Vacarra repeated the experiment concentrating on changes in intermediate rather than total output requirements. The average change (ignoring signs) under these conditions was almost 30 percent. Changes in intermediate output necessary to support a given level of total output under different technologies were also examined. This test was done using the 1958 level of total output and the 1947, 1958, and 1961 direct coefficient matrices [2]. Input requirements varied an average (ignoring signs) 28 percent over this time span.

Ann Carter [3] found similar results for the 1939, 1947 and 1958 input output tables.

These studies show that even in the relatively short period between 1947 and 1958, coefficient variation has been large enough to result in significant changes in the levels of industry output necessary to support a given set of final demands. Since the time frame for the RIOFS is much longer than that available for historical analysis, examination of the actual character of coefficient change helps clarify the long term outlook.

Theoretical evidence

There are two primary causes of coefficient variation: (1) changes in technology; and (2) changes in the composition of sectoral output.

Changes in technology refer to changes in the actual production functions. Observed vectors of output coefficients change in response to variations in factor prices and changes in productive technique.

Factor price relationships are the underlying cause of any change in the production function as represented by a column of input coefficients. Changes in productive technique are only adopted, and, therefore observable in the input coefficients when they result in lower costs. However, breakthroughs in production techniques are not necessary for variations in factor costs to result in new columns of input coefficients. These columns can change when new input prices cause the adoption of existing productive techniques that were not least costly at previous factor prices. Thus changes in the observed production function either occur when new techniques are less costly at existing prices or existing techniques are least costly at new prices.

Since factors include both intermediate and primary inputs it is reasonable to expect variations in their relative prices as factor supplies and demands vary over time. Physical exhaustion of many natural resources promises to cause large changes in factor prices. Also, the continuing drive towards more leisure and better living standards will alter the relationship between primary labor and intermediate input prices. These changes will induce technical innovation and adoption of new and existing techniques less intensive in the use of relatively expensive labor inputs. Hence, over time, alteration in input coefficients will occur as a result of the continuation of long term trends in resource supplies and the relative cost of labor.

In addition to technological change, whether due to changing factor prices or genuine breakthrough, input coefficients may vary in response to changes in sector composition. In theory an IO sector should consist of a number of plants producing an homogenous product using identical technology. In actuality, an input output sector is made up of a heterogeneous group of plants each producing similar but not identical products using different input structures. Thus the input coefficient column for any sector is a weighted average of the input coefficients for the original reporting units comprising that sector.

Any change in the distribution of output among the plants making up a sector will lead to variations in the coefficients for that sector. Such changes in distribution can result from alterations in the composition of final or intermediate demand for the products comprising a sector. Changes in sector composition over time must be expected since industries contract and expand their product lines in response to many factors such as demand changes and pollution control legislation.

From the preceding discussion it is clear on both empirical and theoretical grounds that an assumption of constant input structure would be unjustifiably naive. Hence, a model for the forecasting of coefficient change was formed for the RIOFS. The next section will present the basic assumptions required in order to forecast regional technology under the constraints put on most resource forecasting projects.

3. Assumption Required for Utilization of National Coefficients

There have been few directly compiled regional input coefficient matrices, and even those have been based on limited surveys. The inputs needed to compile the necessary data for such a matrix are beyond the resources of almost all study teams; hence, the RIOFS utilizes the national IO table. Three basic assumptions are required to adopt the national matrix for regional use. First the national economy must be assumed to approximate a closed system, and that system must be representative of regional technology.

Since foreign trade makes up a small portion of national economic activity, the published IO table closely approximates national technology. Although not utilized for this study, unpublished data is available to remove imported inputs from the flow matrix to form what is called a domestic base table. Succeeding studies utilizing the RIOFS could easily base their analysis on domestic base data. Limitations on the availability of a domestic base table for 1947 made it impossible to adopt such matrices for the present study.

The second assumption is that the production function for any product is the same in the region as in the nation. Although there are regional differences in production, it is reasonable to assume that for large areas the production functions will tend to be similar to those holding for the nation. In an area such as the NAR which produces almost one third of the total national output, the assumption of identical technology should be approximately valid since the national coefficients are highly influenced by the regional relationships.

As mentioned earlier, input output industry groupings are aggregrate categories encompassing many similar but not identical products. Thus, although the production function for a particular product in the region may be identical to that in the nation, the input structure for the IO sector containing that product may differ. This can occur when the distribution of products in any regional sector is not identical to the product composition for the corresponding national sector. The last assumption, therefore, is that the regional distribution of products in any industry category is identical to that existing in the nation.

The second and third assumptions assure that the use of national IO matrices will provide valid representations of regional technology. In as much as significant differences between regional and national input structures would make reasonably sized regional input output studies impossible, these assumptions are of prime importance. The validity of the assumptions increases with the size of the region studied since the national coefficients are in effect weighted averages of regional coefficients. As the weight of any region approaches one, its input structure becomes increasingly close to that for the nation. Hence, the assumption should be reasonably valid for a large and varied area such as the NAR.

The relatively closed nature of the U.S. economy combined with the size of the NAR makes it possible to utilize published IO tables to form a forecasting model for regional technology. In the next section the criteria for selection of this model will be discussed.

4. Projection Criteria

Any forecasting methodology must meet certain criteria. Some of these criteria will be specific to the study while others will be general statements of logical necessity. The input coefficient forecasting approach utilized in the RIOFS will be designed to satisfy two sets of criteria. The first set is derived from the specific nature of the NAR study. The second set are general mathematical constraints that would apply to any forecast of technology in an input output context.

The systems approach utilized for this study requires that the projection models be implementable by a small staff with limited time and resources. Since coefficient forecasting is only one step in the RIOFS, the chosen methodology must be limited in its requirements of study resources. Therefore, set up and solution ease is the first criteria used to evaluate alternative forecasting models.

Long run projection problems differ from those faced in short run forecasts, since over time many functions tend to either blow up or produce negative results. The target years of 1980, 2000, and 2020 make the RIOFS long run in nature. Thus any methodology used in the RIOFS must have stable long run characteristics. This requirement is stated in specific input output terms in the following set of mathematical criteria.

There are two logical contraints on a long run input coefficient forecasting procedure. These are:

1) the projected coefficient matrix should be nonnegative

2) column sums of the projected matrix should obey certain apriori constraints.

Negative coefficients would imply that output from a particular sector also results in output of goods belonging to some other sector. This situation, referred to as joint production, is impossible in the input output context used in this study. Hence, an acceptable projection methodology must insure the nonnegativity of future coefficients.

The last criteria is assymetrical in that it only requires the column sums to obey apriori constraints. This is justified from economic theory since the column sums represents the amount of intermediate output needed to produce one dollars' worth of output, while the row sums have no particular economic meaning. In a profit making industry the column sum must be less than one

The rest of this chapter considers alternative coefficient forecasting models and how well they meet the criteria presented in this section.

5. Previous Work

Previous work on input coefficient projection has been limited to two general approaches. The first approach was adopted by the Harvard Economic Research Project (HERP). Their methodology consisted primarily of detailed analysis and projection of individual rows, columns or single coefficients. The second forecasting model was developed by the Cambridge Growth Project. Their approach, the RAS or biproportional matrix model, is essentially a row and column extrapolation procedure. Since it is very similar to the RIOFS coefficient forecasting scheme, it is presented in some detail. Both of these methods have significant drawbacks and do not meet the criteria mentioned earlier. In addition to the discussion of the specific methodologies, the weaknesses of each approach is presented.

Expert approach

The HERP forecasts were based on studies of individual industries over time. Industry experts were questioned as to expected changes in input requirements. The expert opinions were used primarily to project individual coefficients. This approach, whatever its strengths may be, is not practical for a limited study. The time and staff needed to examine individual industry production patterns and single coefficients is beyond the resources of all but the largest study groups. An example of the magnitude of such an undertaking is the work done to determine the future of the steel sector [4]. This study, only a small portion of the total project, took one man year of effort. In addition to the excessive time and staff requirements, the HERP methodology does not lend itself to long term forecasting. The original work was done in the early 1960's with 1970 as the target year. Further extrapolations of the HERP approach, even if the resources were available, would probably lead to inconsistent and explosive situations. This type of problem is typical with procedures that use widely varied methodologies for different sectors.

Hence, the forecasting approach utilized by the HERP does not meet two of the RIOFS criteria. It cannot be used by relatively small study groups, and as an intermediate term model, does not lend itself to consistent long term forecasting.

RAS approach

The RAS (or biproportional) forecasting model was initially formulated to provide an estimated input coefficient matrix for a nonfuture year, for which no actual matrix existed. This was the problem faced by the Cambridge Growth Project in that a central purpose of the project was to estimate a detailed and comprehensive social accounting matrix for 1960. The most recent matrix was for 1954. In addition no earlier matrices existed which, together with that for 1954, could form a comparable time series that could be extrapolated to 1960.

The project was being completed after 1960, thus although no actual input coefficient matrix for 1960 existed, certain other relevant data items could be derived from available official statistics. These items were the levels of intermediate input, intermediate output, and final demand by sector for 1960. These figures combined with the existing matrix for 1954 provided the data set needed to estimate an input coefficient matrix for 1960. In addition to the formation of a coefficient matrix for a nonfuture year, the methodology suggested a procedure for coefficient projection utilizing the existing 1954 and the estimated 1960 matrices.

The summation of intermediate output and final demand yielded gross output by industry.

$$X = U + Y \qquad (2\text{-}1)$$

X - gross output vector
U - intermediate vector
Y - final demand vector

This was used to calculate vectors representing the row and column sums of the 1960 coefficient matrix.

$$\langle X \rangle^{-1} U = u \qquad (2\text{-}2)$$
$$\langle X \rangle^{-1} \quad = v \qquad (2\text{-}3)$$

U - intermediate output vector
u - coefficient row sum vector
V - intermediate input vector
v - coefficient column sum vector

Where (<X>) represents the diagonalized gross output vector which when inverted and multiplied by any conformable vector results in an element by element division of that vector.

Given the sum vectors, the problem was one of adjusting the 1954 input coefficient matrix to fit the known row and column sums. The study group decided that the adjustment process should exhibit certain desirable characteristics. These criteria were; (1) the estimated matrix should be nonnegative, (2) the estimated matrix should be a simple functional form of the given matrix, and (3) the estimated matrix should minimize a distance criterion, i.e., be nearest to the given matrix, in the class of matrices satisfying the row and column constraints.

In this form the adjustment process fits into the category of a biproportional constrained matrix problem [5]. The general biproportional constrained matrix model yields a nonnegative matrix that satisfies row and column constraints and is a simple transformation of a given matrix.

$$B \geq 0 \tag{2-4}$$

$$BJ = u \ , \ JB = V \tag{2-5}$$

J - the unit vector
u,v - row and column sum vectors

$$B = \langle R \rangle A \langle S \rangle \tag{2-6}$$

A - a given matrix
$\langle R \rangle, \langle S \rangle$ - diagonalized vectors

Vectors R and S can be estimated by an iterative approximation process [6], where the matrix A is alternately pre and post multiplied by $\langle R \rangle$ and $\langle S \rangle$, each time constraining the row and column sums to equal the given u and v vectors. The process can be shown [7] to approach a unique solution that results in a matrix, B, which is "near" the given matrix, A, in the sense that it minimizes a distance function [8]. This function is a positive linear transformation of the chi squared distribution function. That is,

$$B = \langle R \rangle A \langle S \rangle \tag{2-7}$$

minimizes

$$(b_{ij} - a_{ij})^2 / a_{ij} \quad \text{all } i,j \tag{2-8}$$

b_{ij} - i,jth element of matrix B

a_{ij} - i,jth element of matrix A

for the class of matrices satisfying the row and column constraints.

For the specific case of estimating the 1960 coefficient matrix, it was assumed that the 1960 matrix was biproportionally related to the 1954 matrix.

$$A1960 = \langle R \rangle A1954 \langle S \rangle \qquad (2\text{-}9)$$
$$A1960 \geq 0 \qquad (2\text{-}10)$$

A1960 - input coefficient matrix, 1960
A1954 - observed input coefficient matrix, 1954
$\langle R \rangle$ - diagonalized vector of row multipliers
$\langle S \rangle$ - diagonalized vector of column multipliers

Given the coefficient row and column sum vectors for 1960, u and v, the appropriate row and column multiplier vectors, R and S, were derived utilizing the iterative process mentioned earlier. This process constrained the 1960 matrix to have coefficients row and column sums equal to u and v, and be "near" the 1954 coefficient matrix. That is,

$$JA1960 = u \qquad (2\text{-}11)$$
$$A1960J = v \qquad (2\text{-}12)$$

and

$$(a_{ij}1960 - a_{ij}1954)^2 / a_{ij} \quad \text{all } i,j \qquad (2\text{-}13)$$

$a_{ij}1960$ - i,jth input coefficient for 1960

$a_{ij}1954$ - i,jth input coefficient for 1954

was minimized.

Since the Growth Project required a transactions matrix for 1960, the coefficient matrix was postmultiplied by the inverted diagonalized gross output vector for 1960.

$$T1960 = A1960 \langle X \rangle^{-1} \qquad (2\text{-}14)$$

T1960 - transaction matrix for 1960

The row and column multipliers, R and S can be interpreted to have meaning in an economic sense. Taking the biproportional relationship

$$B = \langle R \rangle A \langle S \rangle \tag{2-15}$$

the i,j,th element may be written,

$$b_{ij} = r_i a_{ij} s_j \tag{2-16}$$

r_i - ith element of vector R

s_j - jth element of vector S

The r_i can be considered a measure of the extent to which product i has substituted for other inputs or has been replaced by other products in the production process. This substitution effect is uniform across all industries in the biproportional formulation. The column multiplier s_j is interpreted as a measure of a fabrication effect, i.e., the extent to which industry j has increased or decreased its consumption of all intermediate inputs per unit of gross output. This effect is uniform over all industries selling to industry j.

The projection methodology suggested by the biproportional estimation procedure was based on the assumption that the trends observed during the interval between two coefficient matrices would continue. That is,

$$B2 = \langle R \rangle^t \, B1 \langle S \rangle^t \tag{2-17}$$

B2 - projected coefficient matrix, time, 2

B1 - observed coefficient matrix, time 1

where R and S are calculated from,

$$B1 = \langle R \rangle B0 \langle S \rangle \tag{2-18}$$

B0 - base coefficient matrix, time 0

This amounts to an exponential extrapolation with t representing the number of time periods of length (0,1). For instance, the 1966 coefficient matrix would be,

$$A1966 = \langle R \rangle A1960 \langle S \rangle \tag{2-19}$$

A1960 - projected coefficient matrix for 1966

where R and S satisfy

$$A1960 = \langle R \rangle A1954 \langle S \rangle \tag{2-20}$$

$$JA1960 = u \tag{2-21}$$

$$A1960J = v \tag{2-22}$$

The biproportional constrained matrix estimation procedure has become a widely accepted approach to coefficient estimation for nonfuture years [9]. The projection methodology suggested by the procedure was utilized by the Cambridge Group to forecast one period (six years) into the future. For short-term projections such as this, the scheme produces reasonable results. Under long- term conditions such as exist for the RIOFS study, exponential functions are inadequate since they are unbounded from above. This characteristic leads to "explosive" situations, in that the columns sums may exceed unity.

6. RIOFS Procedure

As mentioned earlier the chosen methodology had to be appropriate for long term forecasting, where other methods tend to "blow up". In the context of input output projection this problem refers to the situation where forecasted coefficient column sums take unreasonable values, that is, values less than zero or greater than one. In order to constrain future coefficient column sums to be within the reasonable range (0,1) a transformation of observed data points to a bounded function was applied. The transformed values were linearly extrapolated within this context and then transformed back into coefficient column sums. Since the function utilized is bounded from below by zero and above by one, the procedure insured against the projected sums taking unreasonable values. This approach is a modified version of an algorithm suggested by Michael Bachrach [10] for use by the Cambridge Growth Project. It belongs to the general class of half constrained matrix models in which only the row or column sums are held to some predetermined level. This is in contrast to the biproportional matrix models discussed in the previous section where both row and column sums are constrained.

The standard normal distribution function was utilized for the transformation. This function has a range between zero and one. It approaches these limits at a decreasing rate and hence exhibits the "S" shape sometimes assumed to be representative of long run economic change.

In more detail, let C1 and C2 represent observed coefficient column sum vectors one period apart, vectors T1 and T2 are defined by

$$\text{probability} \; (z \leq T1) = C1 \tag{2-23}$$

$$\text{probability} \; (z \leq T2) = C2 \tag{2-24}$$

with z being a vector whose elements are standardized normal variables. The forecasted column sum vector, CF, p periods in the future is equal to

$$\text{probability} \; (z \leq TP) \tag{2-25}$$

where

$$TP = T2 = p(T2 - T1) \tag{2-26}$$

That is, the rate of change during period (1,2) will continue into the future. Since the probability that the standard normal variable is less than or equal to any value lies between zero and one, the projected coefficient column sums are thus restricted to that range. It is possible to alter the upper limits for TP on an element by element basis by defining a vector k such that

$$\text{probability} \; (z \leq T1) = \langle k \rangle^{-1} C1 \tag{2-27}$$

$$\text{probability} \; (z \leq T2) = \langle k \rangle^{-1} C2 \tag{2-28}$$

and

$$CF = \langle k \rangle \text{probability} \; (x \leq TP) \tag{2-29}$$

with TP being derived in the same manner as before. In non-matrix terms, each columns sum is divided by the assumed upper bound and the resulting forecasted sum is multiplied by the same value. The value of this upper bound was assumed equal to one for the RIOFS. The possibility of using alternative values is discussed in the next section.

Since the matrix notation tends to obscure the operational procedure, the following example is stated in terms of a single industry. The data is taken from the work done by the Cambridge Growth Project. For industry one,

C1954 = .613 - sum of intermediate input coefficients, 1954

C1960 = .519 - sum of intermediate input coefficients, 1960

k is assumed equal to 1. Now T1 and T2 are calculated from

$$\text{probability} (z \leq T1) = .613 \quad (2\text{-}30)$$

$$\text{probability} (z \leq T2) = .519 \quad (2\text{-}31)$$

Using the standard normal distribution function

$$T1 = .29$$

$$T2 = .05$$

The target year, 1966, is 1 period into the future; thus, the projected column sum for industry one is

$$TP = .05 + 1 (.05 - .29) \quad (2\text{-}32)$$

$$= -.19$$

$$CF = \text{probability} (z \leq -.19) = .425 \quad (2\text{-}33)$$

Thus in order to derive column one for the 1966 matrix, column one from the 1960 matrix is reduced uniformly so that its elements sum to .425.

7. Theoretical Justification of RIOFS Approach

The basic assumption underlying the preceding methodology is that coefficient change is a slow moving process.

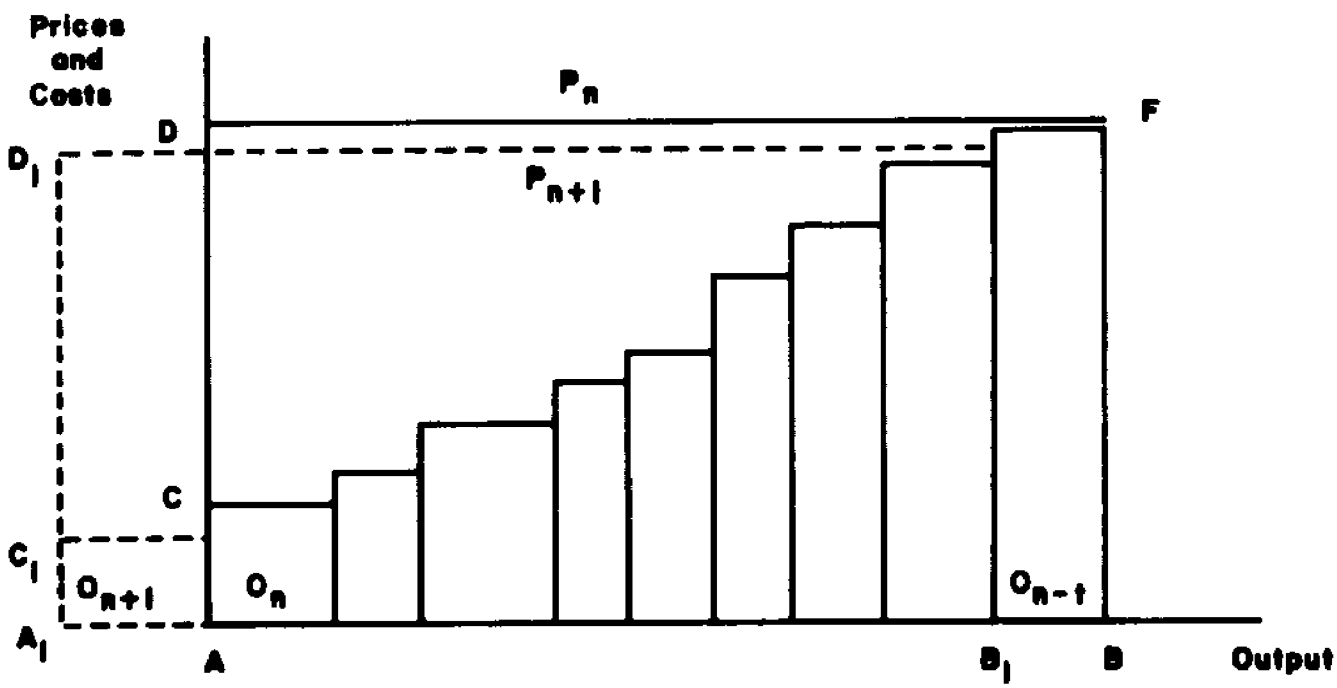
This section discusses the validity of this assumption. In addition the RIOFS coefficient forecasting model is evaluated with respect to the selection criteria presented earlier.

Technical input coefficients, as mentioned in the beginning of this chapter, represent a weighted average of the technologies embodied in the establishments contained in any particular IO sector. At any point in time there exists a temporal spectrum of plants, each plant embodying the best practice technique available at the time of its construction. Technological advance, either resulting from changing factor costs or new production methods, will lower the costs of fabricating the product. But adoption of the new process requires construction of plant and equipment, which only becomes feasible when the return from new process exceeds the return from the existing plants. This occurs when the reduction in operating costs due to the technological change is large enough to make up for the capital cost of producing plant and equipment embodying the advance. Since the existing plants vary in their costs, the least efficient ones will be replaced first. In time, reduced prices due to output expansion will lead to further scrappage of inefficient plants. This process is illustrated in figure 1.

C_n is the capacity output of plants built in the current period n. The current price P_n is composed of operating costs, AC, and capital

Figure 1

Capital Stock Adjustment Process



costs including normal profits, CD, of the best practice plants constructed in period n. That is, plants built in period n-t have operating costs, BF, which are only slightly below the current price. As technical knowledge or relative factor prices change, new best practice plants with operating cost, A_1C_1, and capital cost including normal profits, C_1D_1, are built thus expanding output until price declines to P_{n+1}. At this price, plants built in period n-t are no longer profitable and are scrapped. Equilibrium output at price P_{n+1} is then A_1B_1. The equilibrium is only momentary since technical advance and changes in factor prices provide a continuous stimulus producing a dynamic process that adjusts capital stock to the flow of technological change.

This process of continual adjustment in the mix of capital stock, in response to technical advance and relative factor price variation, is usually referred to as the diffusion of technological change. The extent of the time lag associated with the diffusion process is evidenced by the fact that even a highly profitable innovation, such as the basic oxygen steel furnace, introduced in 1954, only accounted for one third of the steel output in 1967 [11].

The input coefficients, which reflect the spectrum of plant and equipment existing at a particular moment, will change in response to the diffusion process. Since technological diffusion is by its very nature a slow moving long term process, changes in input coefficients will tend to occur slowly over long periods of time. This characteristic makes the utilization of a two point projection model to define the direction and approximate magnitude of coefficient variation a reasonable approach.

8. Critical Discussion

Earlier in this chapter a number of selection criteria were mentioned. These criteria required that the RIOFS technical projection methodology should:

1) be implementable by a small study group with limited resources,

2) have stable long run properties,

3) produce nonnegative forecasted matrices,

4) retain existing zero elements in the forecasted matrices,

5) have coefficient column sums that obey certain apriori constraints.

The two point forecasting methodology chosen for this study can be set up and solved within the resource limitation of a small study group since it requires only two observed coefficient matrices and a relatively simple computerized algorithm. The actual structure of the computer program used to calculate the projected matrices is described in the fourth chapter.

Long run properties of the RIOFS model are stable since the forecasted coefficient column sums are bounded from above by one. Thus the explosive results observable with exponential extrapolations [12] are impossible. The procedure also produces smooth changes in the coefficients since the forecasts are constrained to lie along a smooth continuous function.

The lower bounds of the transformation function insure against negative coefficient projections since the column sums must be greater than or equal to zero.

Zero elements in the base matrix will be maintained since the procedure utilizes column multipliers applied to the base year matrix uniformly across all industries.

The upper bounds of the probability distribution function insure that the projected coefficient column sums obey certain apriori constraints. Since the sum of a column of input coefficients represents the value of intermediate input per dollar of gross output, it must be less than one, (some part of the dollars' worth of output will be allocated for labor, profit, depreciation, and taxes). As discussed in the previous section, the upper constraint on the the forecasted sums is determined by the value of k. In this study k was assumed equal to one. Alternative values for k could be derived from observations on historical column sums. This would require detailed analysis by industry as to the minimum amounts of labor, profits, depreciation, and taxes that could exist under future conditions. Such analysis, although beyond the resources of this study, would be an area for further refinement in future studies.

In addition to fulfilling the above criteria, the half-constrained model also exhibits a minimum distance property similar to that discussed in connection to the biproportonal model.

Although the RIOFS forecasting model satisfies all the selection criteria, it still represents a rather gross approach to coefficient projection in that technological diffusion is assumed to uniformly affect coefficients along any column and this effect is constrained to conform to the shape of a normal distribution function.

This problem does not necessarily invalidate the projection procedure. Under the present extreme data limitations the two point methodology described in this chapter provides an acceptable alternative to the untenable assumption of constant technology. In time the existence of additional IO matrices will allow for more detailed analysis of the character of technological change.

The RIOFS methodology does not preclude the adoption of independent forecasts of columns, rows, or industrial coefficients. Chapter V will present a number of procedures designed to highlight areas in which independent estimation could result in significant effects on the overall outcome of the study.

Projection of trading patterns or industrial location, the second task in the RIOFS operational sequence, is discussed in the next chapter.

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CHAPTER III

TRADING PATTERN PROJECTION

1.Introduction

Although the national economy can be thought of as a closed system, the same assumption cannot be made for a regional economy. This difference results from a number of factors that tend to cause national economies to be more closed than regional economies. Obvious impediments to international as opposed to interregional trade are; monetary differences, language differences, cultural differences and the high level of transportation costs. Less obvious factors such as political differences and the need for self-sufficiency for security reasons also tend to cause national economies to be more closed than the regional economies that comprise the nation.

The relative openness of regional economies makes an analysis of the spatial structure of production an absolute necessity. The national production relationships, if utilized in a regional context, would overstate regional economic activity, since some part of the product consumed in the region is produced outside the region. This chapter presents a procedure designed to adjust the national relationships to more accurately reflect regional productive structure.

Section two of this chapter consists of an analysis of the consequences of interregional trade on IO input coefficients. Also discussed is the evidence that these spatial effects must be projected in order to accurately estimate future regional output. Since no time series studies on regional IO coefficients has been done, the evidence presented is taken from empirical and theoretical work on industrial location.

Sections three and four present critical discussions of existing approaches to the problem of adjusting national technological relationships to reflect regional trading patterns. All but one of the studies were not designed for forecasting purposes. In order to assess their potential for regional IO projection, modifications are suggested that would allow their use in forecasting. This discussion is segmented into two parts because of the divergence between the approaches to the problem. Part one, in section three, considers two procedures that do not utilize location quotients. The second part of the discussion on existing work, continued in section four, initially covers the general theory of location quotients in order to clarify an area in which a great deal of confusion seems to exist. This initial presentation simplifies the subsequent analysis of the adjustment procedures that utilize location quotients.

The projection methodology used in the RIOFS is presented in section five. It utilizes location quotients which are functions of projected industrial employment and the input coefficient discussed in Chapter II. The complexity of the problem of forecasting regional input output relationships over long time spans makes it necessary to adopt a number of relatively strong assumptions. To the extent that these assumptions do not exactly reflect reality, the projection methodology will only approximate the true situation. In spite of this, the projection procedure provides important information on the structure of the spatial changes that effect regional IO relationships. Section two shows that the alternative assumption of no change is completely untenable. In addition, the methodology makes use of the large body of existing industrial location data and so adds information to the final output forecasts.

The last section of this chapter consists of a critical discussion of the RIOFS spatial forecasting methodology. This procedure, just as the coefficient projection scheme, relies on the slow moving character of change. Hence the nature of spatial change will also be analyzed in this section. It is shown that locational variation as reflected in the regional coefficients conforms to a diffusion process in a similar manner as technological change.

2.Importance of Regional Trade

Whereas input coefficients observable in a closed economy represent material requirements in an engineering sense, those observed in an open economy also have a spatial element. That is, if a direct regional IO flow table was formed by surveying the input requirements of regional industries, the input coefficients derived from this table would not accurately reflect intraregional flows. Some part of the inputs reported by the respondents would be materials imported into the region and hence produced elsewhere. These imports would distort the IO matrices derived from a direct survey, in that gross output calculations would overstate actual regional production since they would include that output fabricated outside the region and imported. For some industries, excess output exists and exporting thus occurs. Exports do not distort IO relations since they are included in final demand.

Since estimates of future regional gross output are necessary in order to forecast resource demand, the regional input output matrices must reflect production within the region. Assuming that the national technological relationships reflect regional technology, the national input coefficients can be thought of as having two components; one, an intra-area flow element and the other a spatial element representing imports. That is,

$$a_{ij} = r_{ij} + m_{ij} \tag{3-1}$$

a_{ij} - amount of product i to produce one dollar's output of product j

r_{ij} - amount of product i produced in the region necessary to produce one dollar's output of product j in the region

m_{ij} - amount of product i imported into the region necessary to produce one dollar's output of product j in the region

The product produced in the region need not be consumed within the region since some portion of it may be exported. Given final demand (including exports) for products produced within the region, it is the r_{ij}, or regional input coefficients, that will yield gross regional output. Let YR be the regional final demand vestor, then

$$XR = (I - R)^{-1} YR \qquad (3\text{-}2)$$

XR - regional gross output vector
I - identity matrix
R - regional input coefficient matrix

R consists of the r_{ij} representing the within region production necessary to produce any output. The Leontief inverse $(I - R)^{-1}$ thus yields the output produced within the region necessary to support any level of final demand including export demand. It is the matrix, R, that must be estimated and projected. The procedure outlined in Chapter II yeilds the technological coefficient matrices which consist of the a_{ij}. These a_{ij} coefficient must be adjusted to reflect the imported portion of regional input requirements.

Since there is no direct information on the amount of regional trade by industry, some proxy must be utilized. Industrial location, as reflected by measures of economic activity such as employment or value added, is commonly used to provide information on external trade. The relative spatial concentration of certain industries in an area implies that the area will export the commodities produced by those industries. Conversely, if a region has a relative lack of capacity for production of a particular product, it would tend to be an importer of that commodity. Hence shifts in industrial location can be considered as indications that the structure of regional trade is altering and so also the regional input coefficients. The next part of this section will provide empirical and theoretical evidence that this process of changing industrial location has been occurring and can be expected to continue throughout the forecast period.

Empirical evidence

A great deal of descriptive work has concentrated on the character of changes in industrial location. Two particularly comprehensive studies were done by Fuchs [1] and Perloff [2]. Fuchs used shift analysis which measures the difference between the actual regional change and the change that would have occurred had the region grown at national rates. This difference is referred to as comparative gain or loss. Table 1 shows the comparative gain or loss in employment over two historical periods for the paper and allied products industry [3].

The results shown for this sector are indicative of the situation for most other industries. The observed shifts even over the short period from 1947 to 1954 are large enough to provide a strong indication that regional trading patterns have also changed. The Perloff study shows shifts of the same general magnitude for most sectors. The empirical evidence thus argues strongly against the assumption of no change in regional trading patterns and concomitantly, regional input coefficients.

Theoretical evidence

In addition to the empirical evidence of past spatial industrial change, there are theoretical reasons to expect this change to continue.

TABLE 1

Paper and Allied Products

Comparative gain or loss 1929-54

Division	Value Added ($ millions)	Value Added (percent)	Total Employment (thousands of employees)	Total Employment (percent)
New England	- 470	-47.6	-46	-39.5
Middle Atlantic	- 360	-26.3	-27	-17.6
East North Central	- 250	-17.9	-21	-13.4
West North Central	118	41.5	10	30.9
South Atlantic	385	60.4	30	47.2
East South Central	177	75.4	19	75.4
West South Central	189	66.2	20	67.7
Mountain	13	71.7	1	49.8
Pacific	190	40.5	15	35.3
Total net shift	1,072		95	

Comparative gain or loss 1947-54

Division	Value Added ($ million)	Value Added (percent)	Total Employment (thousands of employees)	Total Employment (percent)
New England	- 114	-18.1	-11	-13.9
Middle Atlantic	- 183	-15.7	-15	-10.6
East North Central	- 89	- 7.2	- 7	- 4.7
West North Central	86	30.3	6	19.6
South Atlantic	139	21.9	9	13.9
East South Central	48	20.6	7	26.6
West South Central	37	13.1	3	11.4
Mountain	10	56.8	1	40.3
Pacific	65	13.9	8	18.2
Total net shift	387		33	
U.S. total, 1954	4,580		530	

Standard regional economic analysis [4] shows that the primary variables affecting industrial location are; base price on inputs, transportation rates on inputs and final products, the geographical position of materials and markets, production functions, and demand functions. Changes in these variables occur in response to; technological changes in the input and transportation industries, the discovery, exploitation and exhaustion of natural resources, technological changes in the product industry, and variations in the tastes of consumers. All of these items that cause change in the variables relevant to industrial location have altered significantly. Thus industrial location and regional input coefficient variation can be expected to continue into the future. These changes must be factored into the regional output projections that are the primary results of this study.

The rest of the chapter will consider alternative approaches to the problem of incorporating projections of regional trading patterns into a general IO forecasting system. The next section presents a discussion of two methodologies that do not rely on the location quotient approach to the solution of the regional trading adjustment problem.

3.Previous Work Not Utilizing Location Quotients

Two approaches to the regional input output forecasting problem are presented in this section. In contrast to the procedures considered in the next section, neither approach is based on the utilization of location quotients. Since the second procedure discussed was not initially designed for forecasting purposes, a modified version more suited for projection is also presented.

Tiebout approach

The only attempt at projecting regional trading patterns was done by Charles Tiebout in his study of the state of Washington [5]. His approach depended on the validity of two assumptions; (1) that as a region's use of a product grows, relatively more will be produced locally, and (2) the future trading pattern of a region will be similar to the present trading pattern of a larger region. In this case the larger region was San Francisco Bay area for which an actual coefficient matrix existed. With the two assumptions, some actual Washington data, the San Francisco matrix and a "good deal of judgement" [6], a regional input coefficient matrix for Washington was formed.

Although the Tiebout approach is the only previously existing procedure specifically designed for spatial projection in an IO context, it is the least suitable for the RIOFS. The reliance on the availability of a regional input coefficient matrix for a larger area makes the methodology not generally applicable. For the NAR region the only available matrix for a larger area would be the national matrix. As discussed earlier, the national economy will tend to be more closed than any regional economy, hence the national matrix cannot be validly used to represent the future regional IO relationships for the NAR. In addition, the heavy reliance on judgement factors further limits the Tiebout approach to very specific circumstances.

Moore and Peterson approach

The second methodology discussed in this section, although not relying on location quotients, is an adjustment routine. The approaches presented in the next section also belong to the class of adjustment procedures. Adjustment procedures refer to the modification of national coefficients to reflect regional trading patterns. Since export demand is included in the total final demand that must be fulfilled, the coefficients need only be modified for imports. The adoption of the national technological structure constrains the regional input coefficients from above. That is, the regional coefficients can be at most equal to the national coefficients which reflect engineering relationships. In the case where some portion of the inputs are imported into the area, the regional coefficients will be smaller than the corresponding national coefficients. Hence the adjustment procedures are designed to lower certain national coefficients to reflect imports.

The adjustment procedure considered below was developed by Moore and Peterson [7] and utilized in the original NAR study. Its application relies on the validity of two assumptions; (1) that regions first supply internal intermediate and final demand, and (2) that imports are distributed among buyers in relation to their relative importance in terms of size of purchase. Data requirements for this model consist of a national coefficient matrix, estimated regional gross output and estimated final demand for regional product. The national coefficients are assumed to reflect regional technology. Utilizing the coefficient matrix and the estimated regional output, an IO transactions table is formed. This table represents the regional flows that would exist if the region were closed. The level of intermediate output necessary to support the regional gross output is then derived as the row sums of this flow matrix. Total demand, both intermediate and final, for commodities produced in the region is calculated by summing the derived intermediate output and the estimated regional gross output to determine industries in which regional production is greater than or less than total demand for products produced within the region. Negative balances represent imports while positive balances indicate exports. The imports are distributed across the rows in relation to the relative size of the flows in the transactions table derived earlier. These distributed imports are then subtracted from the original flows. The resulting adjusted regional transactions matrix reflects the flows between industries within the region. This matrix is easily transformed into an adjusted regional input coefficient matrix using the estimated regional gross output. The inverse coefficient table calculated from these adjusted regional input coefficients will yield the output produced within the region necessary to support any level of demand for regional final products. In the original NAR study this adjusted matrix was assumed to be invariant over the forecast period. Regional final demand forecasts were combined with the inverse matrix to form projections of regional gross output.

This methodology can be extended to provide projections of the regional coefficients. The national input coefficients (a_{ij}) when used in a regional context, can be thought of as being the sum of a regional coefficient (r_{ij}) and an import coefficient (m_{ij}). That is,

$$a_{ij} = r_{ij} + m_{ij} \qquad (3\text{-}3)$$

Applying the NAR procedure for two time periods yields two estimates for the m_{ij}. These import coefficients can then be projected using the two point methodology presented in Chapter II. The forecasted m_{ij} must be greater than or equal to zero and less than the projected technical coefficients. A simple subtraction produces a set of regional input coefficients.

The NAR methodology as modified for forecasting purposes suffers from the same drawbacks described in connection with the two point projection procedure discussed in Chapter II. Essentially the limitations of this approach are a consequence of having to postulate the shape of the curve on which the extrapolated values lie. In the case of technical input coefficient projection the need to adopt this assumed curve results from the lack of historical information about the structure of technological change. This is in contrast to locational change, where there is a good deal of employment data available. Changes in industrial regional employment over time provide information on the character of industrial locational change. Hence any procedure that ignores this data set would yield forecasts containing much less information than is available. The adjustment procedures discussed in the next two sections all utilize location quotients which are projected as functions of the extensive employment data set. As such they are optimal in terms of information content.

4. Previous Work Utilizing location Quotients

This section considers two procedures designed to adjust national input coefficients to reflect regional structure. Both utilize forms of location quotients. Although neither was used to form projections, simple extentions of the methodologies can be made for forecasting purposes.

In a regional input output context, the location quotient is utilized in two ways. First, it indicates which industries require imports to satisfy final demand. Secondly, it is used to adjust the input coefficient for those industries to reflect the effects of external trade. The theoretical implications of the location quotient approach to regional IO analysis have not been fully examined in the literature. The work that has been done seems confused and incomplete. To remedy this situation the initial part of this section will cover the character and significance of the location quotient as an indicator of external trade. Its use as an adjustment factor will be considered in the discussion of the approach taken in the present study.

Character and significance of location quotients

Location quotients have been used extensively in regional economic analysis to identify the orientation of regional industries engaging in external trade. Operationally they are utilized in basis-service models [8] to determine the export or basic industries whose activity levels are then forecasted. The service industries that serve these exporting groups can be projected as a function of the output in the basic sectors. The quotients have also been used in descriptive studies [9] to clarify industries as import, export, or local market oriented.

The general form of the location quotient is

$$LQ_i = \frac{e_i/b}{E_i/B} \qquad (3\text{-}4)$$

LQ_i - location quotient, industry i
e_i - regional activity measure, industry i
b - base measure for the region
E_i - national activity measure, industry i
B - base measure for the nation

Employment, or value added, is usually the industrial activity measure adopted for location quotient analysis due to its wide availability on a detailed regional basis. Typical base measures are; total employment [10], total population [11] and total personal income [12]. Since the present study deals with interindustry relationships which are most closely related to employment rather than income or population, the analysis will be made in terms of base levels of regional and national employment. Employment will also be utilized to measure regional and national activity by industry since more temporal data points are available than for value added.

Most of the studies utilizing location quotients to identify industrial orientation list a number of assumptions that must hold for the measures to be operationally valid. A typical list of assumptions [13] consists of; (1) the benchmark (national) economy must be closed, (2) consumption patterns in the subject (regional) economy must be similar to those in the benchmark economy, and (3) the community must first draw from its local production. Assumption two is expanded in some of the better discussions to include intermediate as well as final consumption patterns.

The identification of trade orientation is considered to be dependent on the value of the location quotient. Typically the critical value will be 1, although other values have been used [14]. Industries with quotients greater than one are identified as exporters while those with quotients less than one, importers.

In order to examine the validity of the location quotient as an indicator of trade orientation the following analysis is done within the context of a regional input output model. This will help clarify the limiting assumptions that are required to utilize location quotients to identify and adjust for importing.

Previous location quotient studies have implicitly assumed that it was sufficient to state that a value of less than one for the location quotient would imply import orientation. This is not logically complete since industries with locaton quotients greater than one may also import. That is, while a location quotient less than one implying importing is a sufficient condition, it is not a necessary condition. The necessary condition requires that importing imply that the location quotient is less than one. The assumptions required to make the necessity condition hold are much more stringent than the ones required for the sufficiency condition. It is coverage of this divergence between necessity and sufficiency that is completely lacking in the literature.

In input output terms the sufficient condition can be stated as,

$$LQ_i < 1 \text{ implies } \sum_j x_j a_{ij} + y_i > x_i \qquad (3\text{-}5)$$

LQ_i - location quotient, industry i
x_j - regional gross output, industry i
y_i - internal regional final demand, industry i

That is, a location quotient with a value less than one implies that the total regional internal demand for the output of the industry is greater than regional production for that industry. For the necessary condition it must hold that,

$$\sum_j x_j a_{ij} + y_i > x_i \text{ implies } LQ_i < 1 \qquad (3-6)$$

or if internal demand is greater than regional output the location quotient must be less than one.

The proceding discussion will describe the steps needed to derive the necessary and sufficient conditions, mentioned above, from a typical location quotient. In order to move from the location quotient to the above conditions, it is necessary to make a number of assumptions. The prime assumption required is later shown to be generally false; hence, demonstrating the invalidity of the typical location quotient as an indicator of industrial trade orientation. In addition to highlighting the limitations of typical location quotients, a more valid form of quotient is derived.

The location quotient most suited to IO utilizes employment as a proxy for economic activity for both the subject and benchmark economy. It has the form

$$LQ_i = \frac{e_i / e}{E_i / E} \qquad (3-7)$$

e_i - regional employment, industry i
e - total regional employment
E_i - national employment, industry i
E - total national employment

Employment is a proxy for economic activity - in this case gross output. In order to transform the above location quotient into an expression in gross output terms, an assumption must be made about the relationship between employment and gross output. For this analysis it is assumed that gross output is proportionally related to employment, with the industry proportions being identical in both region and nation. In addition, the relationship between total employment and total gross output in the region is assumed to be identical to that existing in the national economy. That is,

$$x_i = b_i e_i, \; X_i = b_i E_i \tag{3-8}$$

x_i - regional gross output, industry i
X_i - national gross output, industry i
b_i - constant, industry i

and

$$x = ce, \; X = cE \tag{3-9}$$

x - regional total gross output
X - national total gross output
c - constant

then the location quotient can be expressed as,

$$LQ_i = \frac{x_i / x}{X_i / X} \tag{3-10}$$

Industries for which this measure has a value less than 1 are considered to be import oriented. Letting LQ_i be less than 1 it follows that

$$\left(x_i / x\right) < \left(X_i / X\right) \tag{3-11}$$

Since total output is equal to the sum of the output of all industries,

$$x = \Sigma_j x_j, \; X = \Sigma_j X_j \tag{3-12}$$

it follows that

$$\frac{x_i}{\Sigma_j x_j} < \frac{X_i}{\Sigma_j X_j} \tag{3-13}$$

In order to continue the derivation it must be assumed that the inequality will continue to hold if each denominator element is multiplied by its corresponding input coefficient.

$$\frac{x_i}{\Sigma_j a_{ij} x_j} < \frac{X_i}{\Sigma_j a_{ij} X_j} \tag{3-14}$$

This assumption, which is not generally true, will be covered in greater detail later on in the chapter. For the present it will be assumed to hold generally.

Inequality (3-15) implies

$$x_i \Sigma_j a_{ij} X_j < X_i \Sigma_j a_{ij} x_j \tag{3-15}$$

Since $\Sigma_j a_{ij} X_j$ represents the intermediate output of industry's i's product, it can be substituted for in the inequality by, $X_i - Y_i$, gross output minus final demand.

$$x_i (X_i - Y_i) < X_i \; \Sigma_j a_{ij} x_j \tag{3-16}$$

$$x_i X_i < X_i \; \Sigma_j a_{ij} x_j + x_i Y_i \tag{3-17}$$

Let

$$g_i = \frac{x_i / y_i}{X_i / Y_i} \tag{3-18}$$

thus

$$x_i Y_i = g_i y_i X_i \tag{3-19}$$

substituting this into expression (3-17) yields

$$x_i X_i < X_i \sum_j a_{ij} x_j + g_i y_i X_i \qquad (3\text{-}20)$$

or

$$x_i < \sum_j a_{ij} x_j + g_i y_i \qquad (3\text{-}21)$$

This statement is identical to the sufficiency condition (3-5) except for the term g_i. Examination of the values of g_i that must hold to satisfy the sufficiency and necessity conditions, (3-5) and (3-6), results in a clarification of the limitations of location quotient analysis.

In order for (3-23) to imply the sufficiency condition (3-5), $g_i y_i$ must be less than or equal to y_i, or g_i must be less than or equal to one. Thus if $LQ_i < 1$ and,

$$\frac{x_i}{\sum_j a_{ij} x_j} < \frac{X_i}{\sum_j a_{ij} X_j} \qquad (3\text{-}22)$$

with

$$g_i < 1 \qquad (3\text{-}23)$$

it will be true that

$$x_i < \sum_j a_{ij} x_j + y_i \qquad (3\text{-}24)$$

For necessity, the conditions are more stringent since

$$x_i < \sum_j a_{ij} x_j + y_i \qquad (3\text{-}25)$$

must imply

$$LQ_i < 1 \text{ or } x_i < \sum_j a_{ij} x_j + g_i y_i \qquad (3\text{-}26)$$

which is only true when $g_i = 1$

The preceding derivation rests on the validity of the assumption that

$$\frac{x_i}{\sum_j x_j} < \frac{X_i}{\sum_j X_j} \tag{3-27}$$

implies

$$\frac{x_i}{\sum_j a_{ij} x_j} < \frac{X_i}{\sum_j a_{ij} X_j} \tag{3-28}$$

A simple counter example suffices to demonstrate the invalidity of this assumption in a general sense. Let

$x_1 = 2$	$X_1 = 20$	$a_{11} = .1$
$x_2 = 2$	$X_2 = 60$	$a_{12} = .5$
$x_3 = 6$	$X_3 = 10$	$a_{13} = .1$

then

$$\frac{x_1}{\sum_j x_j} = \frac{2}{10} < \frac{20}{50} = \frac{X_1}{\sum_j X_j} \tag{3-29}$$

but

$$\frac{x_1}{\sum_j a_{ij} x_j} = \frac{2}{1.8} > \frac{20}{38} = \frac{X_1}{\sum_j a_{ij} X_j} \tag{3-30}$$

Essentially the typical location quotient is inadequate because it relates the employment or output of a particular industry to total employment or total output in the area. It is intermediate demand for output of the relevant industry rather than total output that is important in determining trade requirements. Going from $\sum_j x_j$ to $\sum_j a_{ij} x_j$ represents a movement from total output to intermediate output. Thus the assumption that (3-27) implies (3-28), also implies that the relationship between intermediate and total output be the same in both the regional and national economies. This is not generally true since the distribution of output among regional industries differs from that in the nation.

In addition to illustrating the limitations of typical location quotients, the preceding discussion suggests a modified quotient that correctly identifies trade orientation. It was shown that if

$$\frac{x_i}{\sum_j a_{ij} x_j} < \frac{x_i}{\sum_j a_{ij} X_j} \tag{3-31}$$

and

$$g_i = \left[\frac{x_i}{y_i} \bigg/ \frac{X_i}{Y_i} \right] < 1 \tag{3-32}$$

then it will be true that

$$x_i < \sum_j a_{ij} x_j + y_i \tag{3-33}$$

This suggests a location quotient of the form

$$LQ_i = \frac{x_i}{\sum_j a_{ij} x_j} \bigg/ \frac{X_i}{\sum_j a_{ij} X_j} \tag{3-34}$$

Since this quotient is the basis for the methodology adopted for the RIOFS a critical discussion of its characteristics is contained in the last two sections of this chapter. The remainder of the present section considers two location quotient approaches to the regional adjustment problem.

CONSAD approach

The methodology used by the Consad Research Corporation [15] involved a location quotient of the form

$$LQ_i = \frac{x_i}{\sum_j x_j} \bigg/ \frac{X_i}{\sum_j X_j} \tag{3-35}$$

with the summations over all industries purchasing from industry i. This quotient was used to uniformly reduce coefficients along the rows of the national input coefficient matrix. The adjustment procedure is applied to sectors that are identified as importers. Sectors with quotients less than one, are assumed to be unable to supply their customers and hence must import. For example, if the location quotient has a value of .7, it is assumed that the selling industry in the region can only supply 70% of the purchasing industries requirements. The appropriate row of national coefficients is then multiplied, element by element, by .7 to account for the level of imports.

Since gross output is not available over time, further assumptions are necessary to form a projection methodology. One approach would be to use a similar quotient based on employment data. With the assumption that this quotient approximates the quotient based on gross output, a time series of quotients could be formed. This series could then be projected over time. An alternate approach would be to derive the gross output location quotients for two temporally distant points and apply the two point forecasting methodology suggested in Chapter II. These projected quotients would be constrained to take values between zero and one.

The CONSAD location quotient suffers from the same problems mentioned in connection with the typical location quotient used in many regional studies. That is, it relates the economic activity of a particular industry to the total activity of some group of industries rather than to the intermediate demand for the products of the particualr industry. As shown earlier it does not follow that the relationship between gross output of a sector and total gross output is identical to the relationship with intermediate demand. Since it is intermediate demand that determines (under certain assumptions) trade orientation, the

CONSAD approach is invalid and therefore not considered for the RIOFS.

Connecticut Planning Program approach

The last example of a prior approach to the use of location quotients to adjust for regional trading patterns is contained in the work done by the Connecticut Interregional Planning Program [16] for their Socio-Economic Growth Model. The location quotient used in the Connecticut Model was of the form

$$LQ_{ij} = \frac{e_i / e_j}{E_i / E_j} \qquad (3\text{-}36)$$

LQ_{ij} - location quotient, originating industry i, receiving industry j

e_i - regional employment, originating industry

e_j - regional employment, receiving industry

E_i - national employment, originating industry

E_j - national employment, receiving industry

The location quotient is used to adjust individual coefficients when it has a value of less than one. This application assumes that when the originating industry is relatively smaller as compared to the receiving industry in the region, than in the nation, imports are required. For instance, if $LQ_{ij} = .7$ it is assumed that regional industry i, can only supply 70% of industry j's requirements of it's product. The national input coefficient, a_{ij}, is then multiplied by the location quotient LQ_{ij} to account for the imported commodities.

This approach is easily adapted for forecasting purposes by deriving a time series of location quotients that can be projected using standard regression techniques.

The invalidity of the Connecticut location quotient is easily demonstrated by examining the logical implicatons of the adjustment procedure. For any given row certain receiving industries will be relatively more important (as compared to the originating industry) in the region than in the nation. The coefficients for these industries will be reduced, implying imports. But the location quotients for other receiving industries in the same row may be greater than one indicating export potential that could be used to supply the industries whose quotient values indicate import orientation. Thus the method may reduce national input coefficients in a regional industry that is a net exporter. This inability to correctly identify trade orientation by sector makes the Connecticut Model inappropriate for the RIOFS.

The next two sections consider the adjustment procedure used in the RIOFS.

5.RIOFS Approach

The regional adjustment methodology adopted for the RIOFS is presented in this section. A critical discussion of the approach follows in the next section. Since the assumptions required for the procedure to be completely valid are stringent, the critical discussion will be rather detailed.

The RIOFS approach utilizes a modified location quotient that is amenable to projection. It will be shown to be approximately valid under the typical situation existing in most resource demand projection systems.

In the preceding secton, a location quotient of the form

$$LQ_i = \frac{x_i}{\sum_j a_{ij} x_j} \bigg/ \frac{X_i}{\sum_j a_{ij} X_j} \qquad (3\text{-}37)$$

was shown to provide a valid determination of industrial trade orientation under certain assumptions. This quotient forms the basis for the study procedure. A number of further assumptions must be added to form a location quotient that can be used to adjust the national input coefficients to validly reflect regional relationships. These assumptions are covered in this and the next section.

The initial set of assumptions required to insure that the above quotient correctly identifies trade orientation are; (1) the region draws first from internal production to satisfy intermediate and final demand, and (2) the ratio of gross output to final demand for any industry is the same in the region as in the nation. The latter assumption is needed for the validity of both the necessary and sufficient conditions discussed in the previous section. Since it is a rather strong assumption, the consequences of its validity are covered in detail in the next section.

In its present form location quotient (3-37) is not useful for forecasting purposes since it is a function of the true values of regional gross output. For a future point in time the quotient would be

$$LQ_i^p = \frac{x_i^p}{\sum_j a_{ij}^p x_j^p} \Bigg/ \frac{X_i^p}{\sum_j a_{ij}^p X_j^p} \qquad (3\text{-}38)$$

where the superscript p indicates projected values. This quotient could be used to uniformly reduce those rows of the projected national input coefficient matrix that are identified as being importers. The form and validity of this adjustment procedure is examined in the following section.

Since, as mentioned above, this quotient requires knowledge of the true future values of gross output it must be transformed into a quotient that is a function of available or easily forecasted data. In order to make this transformation, an assumption about the equality of the ratio of projected gross output to projected employment in the region and in the nation is required. The exact validity of the assumption would provide an alternative means of forecasting regional gross output. However, this alternative procedure would lack the internal consistency of the input output approach. In addition, any invalidity of the assumption would cause much greater errors in the results using the alternative procedure, than would occur using the input output methodology. This is due to the bounded nature of the location quotients. The projected quotients are restricted to values between zero and one and are only utilized to adjust industries identified as importers. Hence, the regional IO approach is not as sensitive as would be any procedure based on a single assumption of the equality of gross output to employment ratios. The exact form of the required assumption is,

$$\frac{x_i^P}{e_i^P} = \frac{X_i^P}{E_i^P} \qquad (3\text{-}39)$$

i.e., the projected regional gross output to employment ratio is assumed to be equal to the projected national gross output to employment ratio for all industries. Under this condition, a location quotient of the form

$$LQ_i^P = \frac{e_i^P(x_i^P/e_i^P)}{\sum_j a_{ij}^P e_j^P (x_j^P/e_j^P)} \Bigg/ \frac{E_i^P(X_i^P/E_i^P)}{\sum_j a_{ij}^P E_j^P \; (X_j^P/E_j^P)} \qquad (3\text{-}40)$$

reduces to

$$LQ_i^P = \frac{x_i^P}{\sum_j a_{ij}^P x_j^P} \Bigg/ \frac{X_i^P}{\sum_j a_{ij}^P X_j^P} \qquad (3\text{-}41)$$

which has been shown to correctly identify regional trade organization under certain assumptions.

Hence, location quotient (3-40) is utilized to identify trade orientation and to adjust the projected national input coefficients for imported commodities. Industries whose location quotients have values less than one are considered importers. The rows of the national coefficient matrix corresponding to the sectors designated as requiring imports, are multiplied by the appropriate location quotient. That is, a diagonal matrix whose ith diagonal element is a diagonal matrix whose ith diagonal element is

$$LQ_i = \frac{e_i{}^P \dfrac{X_i{}^P}{E_i{}^P}}{\sum_j a_{ij}{}^P e_j{}^P \dfrac{X_j{}^P}{E_j{}^P}} \Bigg/ \frac{E_i{}^P \dfrac{X_i{}^P}{E_i{}^P}}{\sum_j a_{ij}{}^P E_j{}^P \dfrac{X_j{}^P}{E_j{}^P}} \qquad (3\text{-}42)$$

$$\text{if } LQ_i{}^P \leq 1$$

$$= 1$$

$$\text{if } LQ_i{}^P > 1$$

is postmultiplied by the projected input coefficient matrix to form the regional input coefficient matrix (RA),

$$RA^P = LQ^P A^P \qquad (3\text{-}43)$$

RA^P - projected regional input coefficient matrix
LQ^P - projected location quotient matrix
A^P - projected national input coefficient matrix

This adjustment procedure, which is examined more closely in the following section, implicitly assumes that all imports go to intermediate demand.

The projected regional input coefficient matrices are then postmultiplied by the corresponding projected regional final demand vectors to form the forecasted gross output vectors.

$$XR^P = RA^P YR^P \qquad (3\text{-}44)$$

XR^P - projected regional gross output vector
YR^P - projected regional final demand vector

Although this approach requires many stringent conditions in order to insure its theoretical validity, it is shown in the next section that for most applications the procedure provides a reasonable approximation to the true shape of locational change.

6. Critical Discussion

This section contains a detailed discussion of the strengths and weaknesses of the forecasting procedure introduced in the previous section. Initially, the characteristics required of the projection methodology will be covered. The discussion continues with an analysis of the validity of the coefficient adjustment scheme. The consequences and significance of each condition imposed on the procedure are presented next. The section concludes with a statement on the general philosophy of the RIOFS. In addition a numerical example, highlighting the magnitude of error resulting from the invalidity of the principal assumption adopted for the implimentation of the adjustment scheme, is presented as an appendix to this chapter.

Project criteria

Assuming that the projection model is theoretically reasonable, the principal criteria is the ease of implementation. Since the RIOFS was designed for use by small study teams, data requirements must be limited and solution procedures relatively simple. The long term nature of the study also requires that the projection procedure have stable long run properties. In addition, the methodology should be amenable to scenario analysis ;i.e., it should be easy to factor alternate assumptions, parameters, and exogeneous variables into the model. As mentioned in Chapter I these characteristics are to some extent more important than the exact theoretical validity of the methodologies, if strict adherance to theory causes the study team to adopt models that do not make optimum use of available information.

The approach outlined in this chapter fulfills the above criteria. Employment by industry, for both the region and nation, over some historical time period is the principal raw data set required for the procedure. The remaining data inputs are derived directly from other steps in the system. Since employment is the only available proxy for industrial location that exists on an industry and regional basis for more than one point in time, its use in the RIOFS represents a full utilization of available information. The location quotient estimation procedure, when programmed for solution by computer, is relatively simple. Employment forecasts are the output of a special program which is discussed in detail in Chapter IV. Essentially the forecasts are formed using simple multiple regression techniques. The exact form of the forecasting equation is tailored to prevent the type of problems that can occur in long term extrapolations.

In its computerized form the RIOFS regional adjustment model is ideally suited to scenario analysis since alternate values for any variable used to define the location quotient can easily be incorporated. This is particularly relevant in the case of the employment projections where alternative forecasts are often available.

Hence the adjustment procedure covered in this chapter fits the general requirements for use in limited resource forecasting studies. The remainder of this section considers the theoretical reasonableness of the approach. It is shown to be quite acceptable on this criteria.

Adjustment routine

The adjustment routine used to modify the national input coefficient for trading has, up to this point, been assumed to be valid. Its validity can be tested by applying a location quotient whose value is less than unity to a row of input coefficients.

$$r_{ij} = LQ_i a_{ij} \text{ , all } j \tag{3-45}$$

r_{ij} - adjusted (regional) coefficients

Multiplying this coefficient row by the appropriate levels of regional gross output and adding regional final demand yields an estimate of regional gross output. The equality of this estimate with the known output proves the validity of the procedure. In performing this test, regional gross outputs are assumed to be given, and the industry has been validly identified as an importer. Let x_i^* be the estimated regional gross output, which is equal to

$$\begin{aligned} x_i^* &= \Sigma_j r_{ij} x_j + y_i \\ &= LQ_i \Sigma_j a_{ij} x_j + y_i \end{aligned} \tag{3-46}$$

x_i^* - estimate of x_i

LQ_i is equal to

$$\frac{x_i}{\Sigma_j a_{ij} x_j} \Big/ \frac{X_i}{\Sigma_j a_{ij} X_j} \tag{3-47}$$

under the assumptions stated earlier. The estimate x_i^* then becomes

$$x_i^* = \left(\frac{x_i}{\Sigma_j a_{ij} x_j} \Big/ \frac{X_i}{\Sigma_j a_{ij} X_j} \right) \Sigma_j x_j a_{ij} x_j \tag{3-48}$$

or

$$x_i^* = \frac{x_i}{(X_i / \Sigma_j a_{ij} X_j)} + y_i \tag{3-49}$$

$\Sigma_j a_{ij} X_j$ is national intermediate output, or $X_i - Y_i$; hence,

$$x_i^* = \frac{x_i}{X_i / (X_i - Y_i)} + y_i \tag{3-50}$$

$$= \frac{x_i\left(X_i - Y_i\right)}{X_i} + y_i$$

$$= x_i - \frac{x_i Y_i}{X_i} + y_i$$

As discussed earlier in this chapter the valid determination of trade orientation using location quotient (3-36) requires that the ratio of internal final demand to gross output for each sector be the same in both the region and the nation. That is

$$g_i = \frac{x_i / y_i}{X_i / Y_i} = 1 \qquad (3\text{-}51)$$

This assumption was required to insure that insufficient or excessive internal final demand did not invalidate the determination of trade orientation yielded from examination of the value of the location quotient (which only compares relative levels of gross output to intermediate demand). Substituting this condition into the expression for estimated gross output,

$$x_i^* = x_i - \frac{x_i Y_i}{X_i} + y_i \qquad (3\text{-}52)$$

$$g_i = \frac{x_i / y_i}{X_i / Y_i} = 1 \text{ or } x_i Y_i = X_i y_i \qquad (3\text{-}53)$$

hence

$$x_i^* = x_i - \frac{X_i y_i}{X_i} + y_i \qquad (3\text{-}54)$$
$$= x_i$$

Thus, when $g_i = 1$, location quotient (3-37) will provide correct identification of trade orientation and a valid row multiplier to adjust the national input coefficients for this trade.

The following discussion considers the assumptions made to derive this adjustment procedure.

Before considering the assumptions adopted to insure the theoretical validity of the regional adjustment and forecasting model, it is important to reconsider its application within the total RIOFS. Regional adjustment and forecasting represents a single step in a four step resource forecasting system consisting of technology projection, trading pattern projection, final demand projection and resource coefficient projection. The first two areas, which are examined in this study, have had almost no coverage in previous studies. Data constraints have made it impossible to use other than approximate methodologies. Despite these limitations, the models adopted make use of available information and produce reasonable estimates of future conditions. The regional adjustment procedure, since it only is utilized to modify certain rows of the projected national input coefficient matrices, must not be evaluated as if it were the only step in a conventional modification of an existing approach. Hence the approximate nature of many of the assumptions discussed below does not invalidate the entire approach which certainly is more tenable than previous studies that assumed conditions of no change.

Explicit assumptions

The model described in the previous section rests on the validity of five assumptions, four explicitly stated earlier and one implied in the discussion. The four explicit assumptions are: (1) the region draws first from internal production to satisfy intermediate and final demand; (2) imports go exclusively to intermediate demand; (3) regional gross output to employment ratios by industry and total area are identical to corresponding national ratios; and (4) the regional ratios of gross output to final demand by sector are identical to the national ratios. The one assumption implied, but not explicitly stated, in the presentation of the RIOFS model, is the assumption that locational change and thus trading pattern change is a slow moving long term process.

The first assumption precludes the phenomenon known as cross hauling; that is, the simultaneous exporting and importing of a commodity by a particular region. In large well defined regions, such as the NAR, this assumption should be approximately valid. Even if this were not true, the data necessary to establish cross hauling patterns over a statistically significant time period is not available. Even one time point would be prohibitively expensive for all but the largest study teams.

Assumption two relates to the adjustment procedure, in that the location quotients are applied to the intermediate input coefficients. Alternate schemes would require explicit assumptions about the relationship of regional final demand to intermediate output. Use of national relationship of regional final demand to intermediate output would constitute an approach to modifying the adjustment procedure. Time constraints have made this additional work impractical for the present study. Applications of the RIOFS could easily incorporate modifications of the adjustment process to reflect the proportion of imports going to final demand.

The third assumption was discussed in detail earlier in the chapter in connection with the use of a location quotient of the form

$$LQ_i^P = \frac{e_i^P}{\sum_j a_{ij}^P e_j \frac{E_j^P}{X_j^P}} \Bigg/ \frac{E_i^P}{\sum_j a_{ij}^P E_j^P \frac{E_j^P}{X_j^P}} \qquad (3\text{-}55)$$

as an estimate of

$$LQ_i^P = \frac{x_i^P}{\sum_j a_{ij}^P x_j^P} \Bigg/ \frac{X_i^P}{\sum_j a_{ij} X_j^P} \qquad (3\text{-}56)$$

Although the exact equality of x_i^P / e_i^P to X_i^P / E_i^P provides knowledge of x_i^P, given the values of e_i^P, X_i^P, and E_i^P, the assumption is only used in an approximate sense. Since the regional adjustment process is utilized to provide an estimate of the shape of future trading patterns, errors in this assumption are not as important as they would be if the entire projection process depended upon the strict equality of regional and national quasi-productivities. In addition, the adoption of this assumption in the context of an input output system retains the consistency between sectors that would be lost in an alternative approach utilizing only the quasi-productivity ratios. Even if the regional and national ratios are not identical, on the average they should be approximately equal, since large divergences would produce forces tending to reduce the differences. Higher relative quasi-productivity would produce higher relative wages which would attract labor, thereby reducing quasi-productivity and equalizing the ratios. Also the size of a region such as the NAR reflects itself, in that national ratios of gross output to employment are significantly affected by the regional ratios.

The fourth explicit assumption requires that g_i, which is defined by,

$$g_i = \frac{x_i}{y_i} \bigg/ \frac{X_i}{Y_i} \tag{3-57}$$

be equal to unity in order for the study location quotient to correctly identify trade orientation and adjust for that trade. This condition insures that the location quotient, which compares the ratios of gross output to intermediate demand in the region and the nation, is not invalidated by relatively high or low levels of regional internal final demand. Although regional levels of internal final demand should be distributed in a similar manner as national final demand, in general, regional gross output will not be distributed in a similar manner as national gross output. Hence g_i will not be unity in many sectors, leading to estimation errors in those sectors. This problem, although unsolvable in the context of the present study limitations, will in most cases not cause significant errors in the output estimates of the high resource using basic industries. The basic industries such as chemicals, metals, textiles, and paper are primarily oriented towards intermediate demand, and as shown below the estimation error is a function of the national ratio of gross output to final demand. This can be seen by reconsidering the expression for estimated gross output (3-50).

$$x_i^* = x_i - \frac{x_i Y_i}{X_i} + y_i \tag{3-58}$$

The error term, d_i is equal to

$$d_i = x_i - x_i^* = x_i \frac{Y_i}{X_i} - y_i \tag{3-59}$$

Hence d_i will be smaller for industries that sell a relatively small portion of their output to final demand. For industries selling completely to intermediate demand d_i becomes zero and the estimation error disappears. Table 2 presents, for the high water using manufacturing sectors, data on the proportion of total output going to final purchases. Except for food products, all of sectors sell less than half their output to final demand. Since a significant portion of the water used by food products goes into soft drinks and beer, both local operations due to high transportation costs, the value of g_i for this sector should be close to one and the estimation error relatively small. Hence, although the identification and adjustment process is not an exact procedure, it will provide a close approximation for the basic processing sectors.

TABLE 2
HIGH WATER USING SECTORS

RIOFS Sector	Industry	National Final Demand as a percent of National Gross Output
10	Food Products	72%
12	Textile Products	14%
16	Paper and Allied Products	14%
18	Chemicals and Allied Products	32%
19	Petroleum Refining	46%
23	Primary Metals	3%

Source: Department of Commerce

Implicit assumptions

Throughout the preceding discussion it has been implicitly assumed that the locational change occurs relatively slowly over extended periods of time. This allows it to be projected as a function of basic demographic and economic variables. Many regional economists hold that this assumption is true. For instance Borts and Stein [17] refer to locational change as "glacier-like movements of regional economic development". Harry Richardson [18] contends that "locational movement, both of groups of individuals and of firms, tends to be long drawn out." Perloff, Dunn, Lampard, and Muth [19] base a major portion of their analysis on the assumption that "economic growth is an evolutionary process where the seeds of future development are to be found in past and present activities and decisions".

The assumption that locational change is a slow moving process can be examined in light of the earlier discussion as to the causes of locational movements. Locationally important variables are: input prices, transportation rates on inputs and final products, the geographical location of material sources and final markets, production functions, and demand functions. As mentioned at the beginning of this chapter, variations in these variables individually will occur slowly since they depend on: the diffusion of technology in the input and transportation industries, the discovery, exploitation and exhaustion of natural resources, the diffusion of technology in the product industry, and changes in consumer tastes. All of these factors tend to change slowly. Hence the total effect on relative costs and revenues of regional firms will be of a long-term nature.

Even if the net locational effect due to variations in the above variables was significant in the short run there are reasons to believe that the manifestation of this effect, in terms of actual locational change, would occur over a long period of time. Locational change for individual firms is actually spatial adjustment to changes in factors that alter the relative profitability of a particular location. In a sense this adjustment is similar to the variation in productive technique made by individual firms, to technological changes that alter the relative profitability of a particular production method. In both cases, the aggregate change will depend on the rate of diffusion of the individual changes. That is, in the same way that existing technology (as measured by the technical input coefficient) is actually a weighted average of a temporal spectrum of techniques, existing industrial location (as measured by the location quotient) is actually a weighted average of a temporal spectrum of location decisions. More specifically, as locationally important variables vary the relative profitability of different locations will also change. The change in the relative profitability of individual plants will depend on their present locations. Plants will only change their location when the increased profits resulting from the move more than offsets the initial cost of the new plant. The first plants to move will be those with the worse locations in terms of the present conditions. As the locationally important factors continue to change, plants that had been profitable to operate at their old locations are no longer profitable. Hence they are scrapped, and the new plants relocated. This process of locational change is similar to the diffusion process discussed in connection with technological change. Hence even if the locationally relevant variables change significantly in the short-run, the location quotients, which reflect the average location of an industry, will change slowly over a long timespan.

The above discussion neglects the possibility that technological change may alter the spatial adjustment process. That is, technological change may make it profitable to invest in a new plant before variations in spatial factors necessitate such investment. In this case the locational adjustment will be speeded up, since once the investment decision is made, the firm no longer has any ties to a particular location and will build its "state of the art" plant in the locationally optimal site.

Similarly, the earlier analysis of technological change neglected the effect of locationally important factors. If a firm is induced to change location, its new plant will not only be optimally located but will also utilize "state of the art" equipment.

The similarity in the character of technological and locational change allows both to be subjected to the same sort of sensitivity testing. Chapter V considers the theory of the application of a number of sensitivity routines to the problem of evaluating the importance of technological and locational change. Time constraints made actual empirical solution impossible.

The remainder of this chapter will consist of a statement on the general philosophy of the present study under the existing conditions of limited resources. In addition, a simple numerical example of the study adjustment procedure considered in this chapter is presented as an appendix to this chapter.

General philosophy

The models described in this and the previous chapter have many limitations. These limitations have been explicitly stated and discussed in detail. Despite the problems associated with the forecasting procedures, the total project represents an advancement of the state of the art in regional analysis. As mentioned previously, no existing study combines both technological and locational projection into a regional forecasting system. This systems approach makes scenario analysis possible. Examination of alternative long run situations is particualarly vital when one is making decisions on projects that cost hundreds of millions of dollars and last for decades. The RIOFS makes it possible to check the consequences of changes in any part of the system whether positive or normative.

Since the study team in this case was limited to one individual, the methodologies are necessarily only approximate in a theoretical sense. The project must be considered as a demonstration case, in that an attempt was made to use theoretically advanced theory in a computerized forecasting system. Rather than limit the methodologies to crude engineering estimates of economic quantities in a static structure, the RIOFS provides an example of the possibilities that exist when the systems approach is combined with advanced economic theory.

This approach is continued in the last chapter where a number of sensitivity procedures are presented. These routines allow one to identify elements within the RIOFS that are critical to the final results of the study. In this way a study team can apply its resources in an optimal way. Time limitations have made it impossible to actually implement the procedures on the RIOFS data sets. Future studies can easily utilize these routines in their own analysis.

The next chapter details the entire RIOFS as it was run to test the system. In addition, a discussion of the implications of both forecasting models is discussed.

APPENDIX

This appendix illustrates the application of the study methodology to a simple numerical problem. In addition the consequences of the relaxation of certain of model assumptions is examined.

Let A be the national input coefficient matrix,

$$A = \begin{bmatrix} 0.2 & 0.2 & 0.2 \\ 0.2 & 0.1 & 0.1 \\ 0.0 & 0.2 & 0.1 \end{bmatrix} \qquad (3A-1)$$

A - national input coefficient matrix

Also let X and x represent national and regional levels of gross output

$$X = \begin{bmatrix} 200 \\ 400 \\ 200 \end{bmatrix} \qquad x = \begin{bmatrix} 40 \\ 20 \\ 20 \end{bmatrix} \qquad (3A-2)$$

X - national gross output vector

x - regional gross output vector

The gross output levels were chosen to represent significantly different output distributions.

With this data and the assumption that the national economy is closed, national final and intermediate demand, and regional intermediate demand can be derived. This is done by forming dollar flow tables for both the nation and the region. Assuming that the national input coefficients represent regional technology, it is true that,

$$a_{ij} = X_{ij}/X_j = x_{ij}/x_j \qquad (3A-3)$$

a_{ij} - i, jth national input coefficient

X_{ij} - national dollar flow from sector i to j

X_j - national gross output, sector j

x_{ij} - regional dollar flow from sector i to sector j from all sources

x_j - regional dollar gross output, sector j

hence

$$x_{ij} = a_{ij}x_j \tag{3A-4}$$

$$X_{ij} = a_{ij}X_j \tag{3A-5}$$

and

$$FN = \begin{bmatrix} 40 & 80 & 0 \\ 40 & 40 & 20 \\ 0 & 80 & 20 \end{bmatrix} \tag{3A-6}$$

$$FR = \begin{bmatrix} 8 & 4 & 0 \\ 8 & 2 & 2 \\ 0 & 4 & 2 \end{bmatrix} \tag{3A-7}$$

FN - national dollar flow table
FR - regional dollar flow table

Intermediate demand is formed by summing across the rows of the flow tables,

$$Z = \begin{bmatrix} 120 \\ 100 \\ 100 \end{bmatrix} \qquad z = \begin{bmatrix} 12 \\ 12 \\ 6 \end{bmatrix} \tag{3A-8}$$

Z - national intermediate demand vector
z - regional intermediate demand vector

In a closed economy intermediate plus final demand will equal gross output; hence national final demand is simply,

$$Y = X - Z \quad \begin{bmatrix} 200 \\ 400 \\ 200 \end{bmatrix} - \begin{bmatrix} 120 \\ 100 \\ 100 \end{bmatrix} = \begin{bmatrix} 80 \\ 300 \\ 100 \end{bmatrix} \tag{3A-9}$$

Since the regional economy is not closed, a regional internal final demand vector must be assumed. The sectoral distribution of this vector is assumed to be similar to the national distribution.

$$yn = \begin{bmatrix} 18 \\ 15 \\ 14 \end{bmatrix} \tag{3A-10}$$

yn - regional internal final demand vector

Given this vector, regional gross output and intermediate demand, it is possible to derive a regional trading vector. This is done by adding intermediate demand to internal final demand, subtracting the result from gross output, with negative balances representing imports and positive balances representing exports.

$$x-(z+yn) = t$$

$$\begin{bmatrix} 40 \\ 20 \\ 20 \end{bmatrix} - \left(\begin{bmatrix} 12 \\ 12 \\ 6 \end{bmatrix} + \begin{bmatrix} 18 \\ 15 \\ 14 \end{bmatrix} \right) = \begin{bmatrix} 10 \\ -7 \\ 0 \end{bmatrix} \qquad (3A\text{-}11)$$

t - regional trading vector

Regional gross output is generated by total final demand from whatever source. Hence regional final demand is equal to the sum of internal final demand and export demand. Thus

$$yr = \begin{bmatrix} 28 \\ 15 \\ 14 \end{bmatrix} \qquad (3A\text{-}12)$$

yr - total regional final demand vector

With the information derived above, it is possible to examine a number of alternative situations. In order to evaluate the consequences of the invalidity of the assumption that g_2 is equal to one, three situations are considered. The gross output necessary to support total regional final demand is derived using; (1) the national input coefficient matrix, (2) the adjusted input coefficient matrix with $g_2=1$, and (3) the adjusted input coefficient matrix with $g_2=1$.

(1) To derive the gross output necessary to support total regional final demand given the national input coefficient matrix, the Leontief inverse must be formed.

$$(I-A)^{-1} = \begin{bmatrix} 1.3255 & .3020 & .0336 \\ .3020 & 1.2081 & .1432 \\ .0671 & .2685 & 1.1409 \end{bmatrix} \qquad (3A\text{-}13)$$

$(I-A)^{-1}$ - Leontief Inverse

Solving for gross output,

$$(I-A)^{-1}yr = x$$

$$\begin{bmatrix} 1.3255 & .3020 & .0336 \\ .3020 & 1.2081 & .1432 \\ .0671 & .2685 & 1.1409 \end{bmatrix} \begin{bmatrix} 28 \\ 15 \\ 14 \end{bmatrix} = \begin{bmatrix} 42.1141 \\ 28.4564 \\ 21.8792 \end{bmatrix} \qquad (3A\text{-}14)$$

The calculated gross outputs are too high especially for industry two, the sector known to be an importer. This results from the use of the national input matrix which assumes a closed economy. Since the regional economy in this example is not closed (industry two imports 7 units), the gross output estimates are biased upwards. In order to adjust for this bias, the input coefficients must be reduced to reflect the extent that demand for products of sector two are met by imports; that is, production in other regions.

(2) The section location quotients must be calculated in order to identify the trade orientation of each sector. The form of this quotient is

$$LQ_i = \frac{x_i}{\sum_j a_{ij} x_j} \Bigg/ \frac{X_i}{\sum_j a_{ij} X_j} \qquad (3A\text{-}15)$$

Although this quotient is not available directly, it was shown earlier that it can be estimated from employment and national gross output. To simplify this example the quotient will be used in its present form.

$$LQ_i = \frac{40}{12} \Bigg/ \frac{12}{120} = 2.000 \qquad (3A\text{-}16)$$

$$LQ_2 = \frac{20}{12} \Big/ \frac{12}{100} = .4167 \tag{3A-17}$$

$$LQ_3 = \frac{20}{6} \Big/ \frac{6}{100} = 1.6667 \tag{3A-18}$$

Industry two, with a quotient less than one, is identified as an importer. To adjust the input coefficient matrix so that it reflects these imports, row two is multiplied by LQ_i, element by element. In this sector the ratio of final demand to gross output for the reqion is equal to the corresponding ratio in the nation. Hence the estimated output levels should be identical to the actual levels. The regional adjusted matrix becomes

$$AR = \begin{bmatrix} 0.2000 & 0.2000 & 0.0000 \\ 0.0833 & 0.0417 & 0.0417 \\ 0.0000 & 0.2000 & 0.1000 \end{bmatrix} \tag{3A-19}$$

AR - regional input coefficient matrix

The regional internal flow table implied by this coefficient matrix is then,

$$FRI = \begin{bmatrix} 8.0000 & 4.0000 & 2.0000 \\ 3.3320 & 0.8334 & 0.8334 \\ 0.0000 & 4.0000 & 2.0000 \end{bmatrix} \tag{3A-20}$$

FRI - regional internal dollar flow table

Summing across the rows yields the intermediate output levels produced within the region necessary to support total final demand for products fabricated in the region.

$$zr = \begin{bmatrix} 12 \\ 5 \\ 6 \end{bmatrix} \tag{3A-21}$$

zr - intermediate output (demand)
within the region

Adding total regional final demand to this vector should yield regional gross output if the adjustment procedure is working properly.

$$x^* = yr + zr \quad \begin{bmatrix} 28 \\ 15 \\ 4 \end{bmatrix} + \begin{bmatrix} 12 \\ 5 \\ 6 \end{bmatrix} = \begin{bmatrix} 40 \\ 20 \\ 20 \end{bmatrix} \qquad (3A\text{-}22)$$

To test this procedure, regional gross output can be calculated directly from the regional input coefficient matrix and total regional final demand. Forming the Leontief inverse for the adjusted matrix and postmultiplying it by total regional final demand results in estimated regional gross output.

$$x^* = (I-A)^{-1}yr \quad \begin{bmatrix} 1.2780 & 0.2693 & 0.0125 \\ 0.1122 & 1.0773 & 0.0499 \\ 0.0249 & 0.2394 & 1.1222 \end{bmatrix} \begin{bmatrix} 28 \\ 15 \\ 14 \end{bmatrix} = \begin{bmatrix} 40 \\ 20 \\ 20 \end{bmatrix} \qquad (3A\text{-}23)$$

The regional adjusted input matrix AR yields gross output estimates that are identical to the actual assumed gross output levels. Hence, this matrix is the appropriate matrix to utilize in deriving regional gross output projections.

(3) The consequences of the inequality of regional and national ratios of gross output to final demand is examined under conditions of changing levels of final demand for products of the importing sector. This alters the ratio of regional gross output to final demand and so the value of g_2, without changing the location quotients. Each final demand vector is premultiplied by the regional Leontief inverse to form a gross output vector. Table A-1 presents the results of this operation for seven final demand vectors. Percentage errors are shown for each case. The gross output to final demand ratios vary from 50% to 100% with the value for which $g_2=1$ being 75%. In all cases the errors observed for nonadjusted sectors are insignificant. For the adjusted importing industry the errors are larger, but tend to be less than the change in

TABLE A-1

ALTERNATIVE ERROR SCENARIOS

Case #	Final Demand	Final Demand / Gross Output	% Change From 75%	Gross Output	Error %
	28			38.6533	- 3.4
1	10	.500	-33.3	14.6133	- 2.6
	14			18.8030	- 6.0
	28			39.1920	- 2.0
2	12	.600	-20.0	16.7680	-16.2
	14			19.2818	- 3.6
	28			39.7307	- 0.7
3	14	.700	- 6.7	18.9227	- 5.4
	14			19.7606	- 1.2
	28			40.0000	0.0
4	15	.750	0.0	20.0000	0.0
	14			20.0000	0.0
	28			40.2693	0.7
5	16	.800	6.7	21.0773	5.4
	14			20.2394	1.2
	28			40.8080	2.0
6	18	.900	20.0	23.2320	16.2
	14			20.7182	3.6
	28			41.3467	3.4
7	20	1.000	33.3	25.3867	26.9
	14			21.1970	6.0

gross output to final demand ratios. Since the intial ratio of 75% is very high, the magnitude of the observed errors represents an upper limit.

This example does not show the exact consequences of g_i not being equal to one. Such an exercise would be very complex and beyond the time constraints of this study. Despite this, it does indicate that the invalidity of the assumption that $g_i=1$ will not necessarily produce estimates of gross output that are significantly in error.

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CHAPTER IV

RESULTS

1. Introduction

This chapter presents a detailed discussion of the RIOFS in the context of an actual forecasting problem. In contrast to the theoretical analysis of chapters II and III, this chapter covers data considerations, computer programs and actual results of a practical application of the models discussed in the study.

As mentioned numerous times in the preceding chapters, this study is chiefly a test case of the efficiency of the systems approach to small group forecasting efforts. Any weakness in the projection models used in the present study does not invalidate the overall meaning of the work. The fact that a one man research effort could result in a complex interrelated computerized system utilizing large amounts of primary data in sophisticated forecasting models, is the prime result of this study. Hence the actual numerical results are not as relevant to an evaluation of the RIOFS, as is the form of the system itself. As discussed earlier, other projection models could have easily been substituted for the present models. This would only change the actual results and not the flexibility of the system to test alternate parameters and assumptions. Since the structure of the system is considered to be of paramount importance, the data considerations and numerical results are discussed principally in relation to their position in the overall structure of the RIOFS.

The remainder of this chapter contains a step by step presentation of the RIOFS. Section two presents a summary of the overall structure of the system. The next section discusses the implimentation of the technological projection model covered in chapter II. Section four details the system and data utilized to project employment. The fifth section presents a detailed discussion of the program that accepts the forecasted national coefficient matrices and employment vectors to form projected location quotients and gross output vectors. Section six demonstrates the flexibility of the RIOFS by presenting and comparing a set of output vectors based on alternative assumption patterns. Following these results is a summary section which discusses the overall meaning of the RIOFS in relation to the requirements of a study of its type. In addition, two appendices are included at the end of the next chapter. The first is a data appendix, and the second contains the principal computer programs conprising the RIOFS.

2. Overall Structure

The RIOFS was designed in a modular (sub-system) format to faciliate alterations in its structure. Essentially the system can be considered to consist of three modules. Module one contains the programs for forecasting technological change in the direct coefficients. The second module represents the trading pattern projection routines. Module number three consists of the solution and evaluation procedures that combine the output of the technological and trading pattern projection sub-systems. Figure 1 displays the modular structure of the RIOFS. Each module contains a number of computer programs that manipulate large quantities of data according to the theoretical models discussed in chapters two and three. Since the detailed structure of the RIOFS is somewhat complex, the remainder of this section consists of an outline of the operations and flows for each sub-system. Relevant data considerations are covered in latter sections.

(1) In order to project technological change two widely separated input coefficient matrices had to be formed. Since the sectoral aggregation used for the RIOFS is different than that published by the Office of Business Economics (OBE), the OBE matrices had to be aggregated. In order to do this the OBE coefficient matrices were transformed into dollar flow tables, reaggregated, and then retransformed into direct coefficient matrices. Since one of the OBE matrices was not available on a constant dollar basis, it had to be deflated. The resulting direct coefficient matrices were then used to form three projected coefficient matrices according to the forecasting methodology described in Chapter II. Figure 2 displays the operational flow of sub-system one.

Figure 1

Modular Structure of the RIOFS

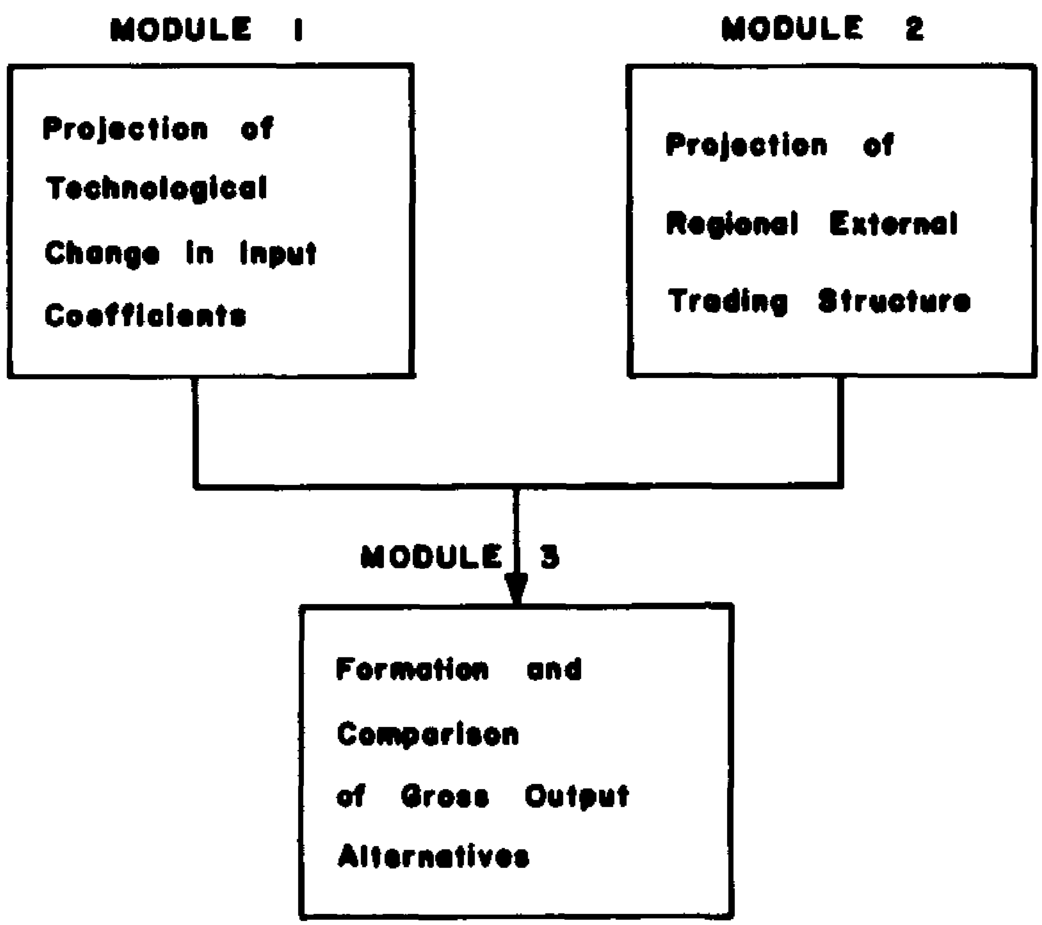
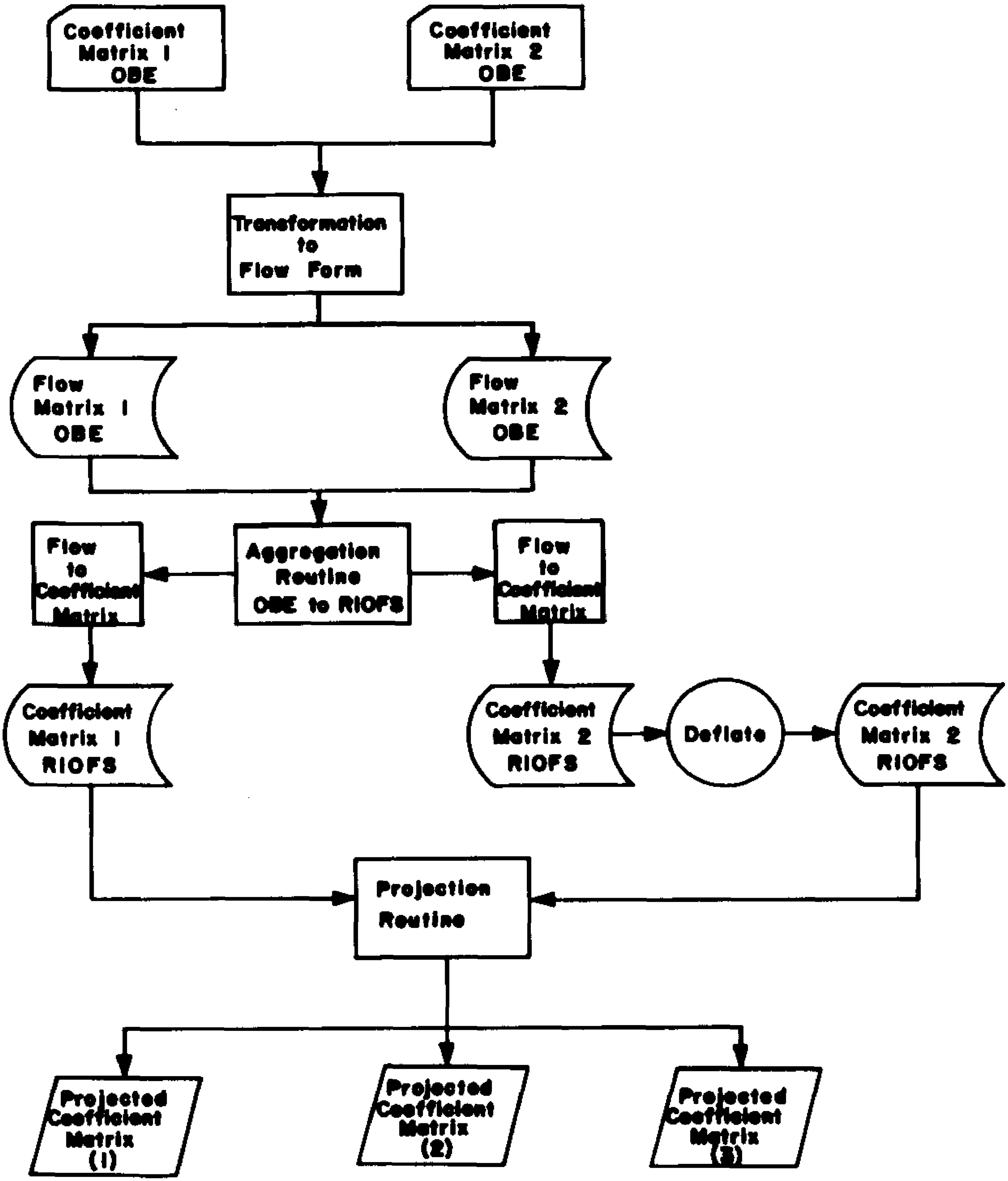


Figure 2

Coefficient Projection



(2) Sub-system two is concerned with trading pattern projection. As discussed in Chapter III, relative levels of employment are the primary determining variables in forecasting locational change and the structure of regional external trade. The actual location quotients, since they are a function of variables formed in module one are derived within the third module. The employment data which is available on a cross sectional basis by industry and state for a number of time periods, is first aggregated to the regional level and then transposed into time series form. Each time series represents regional employment in a particular industry for a number of time periods. Similar data is derived on the national level. Regional and national employment, by industry is then forecasted as a function of a number of basic economic and demographic variables. Figure 3 illustrates the operational flow of module two.

(3) The results of modules one and two are combined in the third module to form a number of alternative forecasts. The location quotients are calculated as an intermediate step in the production of these forecasts. The alternative forecasts which are detailed later in the chapter are based on a spectrum of assumptions that clarify the effect of each step in the RIOFS projection procedure. This clarification is facilitated by the cross comparisons made between the alternatives. Figure 4 illustrates the structure of this sub-system.

The next three sections present detailed discussions of the

Figure 3

Employment Projection

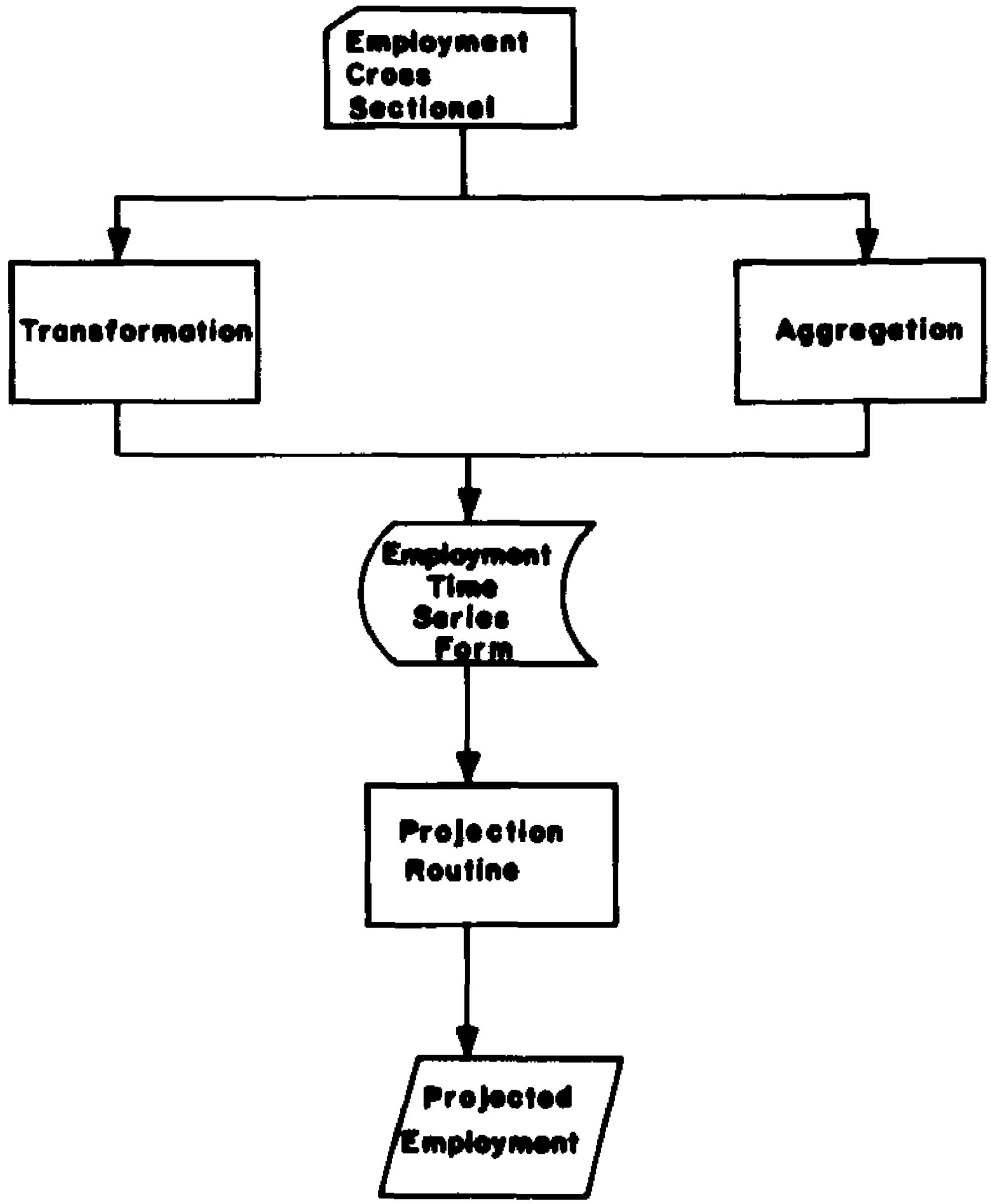
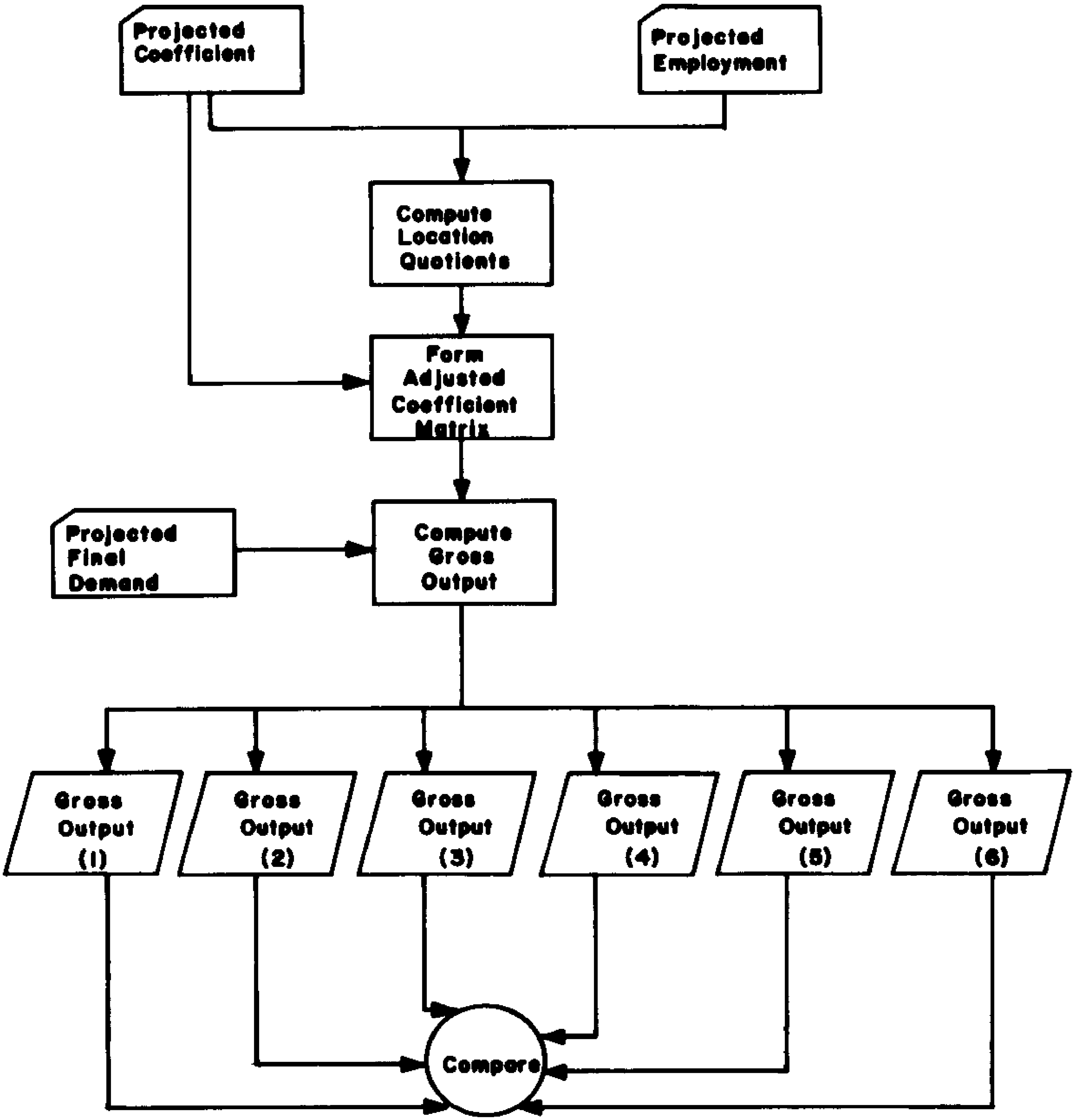


Figure 4

Output Formation



sub-systems outlined above. The principal concern of these sections is the practical aspects of applying the RIOFS to an actual problem. To illustrate the flexibility of the RIOFS, suggestions as to alterations in the system are included in each section.

3. Technologial Change In Direct Coefficients

Figure 2 describes the operational flow of the forecasting sub-system in Chapter II. The primary output of the module is a set of three direct coefficient matrices: one for each of the three target years, 1980, 2000, and 2020.

The principal input matrices to the projection model are the 1947 and 1963 national direct coefficient matrices. These two years represent the longest time span available for two consistent coefficient tables. The 1947 matrix was obtained from the offices of Ms. Beatrice Vicarra at the Census Bureau. It is a reworked version of the original 1947 study [1], made consistent with the 1958 study [2], in both sectoral aggregation and base prices. The 1963 coefficient matrix was taken directly from the published article. This matrix is aggregated in a manner consistent with the 1958 study and hence is also consistent with the 1947 table. The 1963 table was only available on a current price basis and thus required deflation for complete consistency with the 1947 matrix. A set of unpublished sectoral deflators was obtained from the OBE to accomplish the adjustment from 1963 prices to 1958 prices. Since these prices were only available on a two digit Standard Industrial Classification (SIC) [3] basis, the price adjustment routine was applied after the coefficient matrices were aggregated to the scheme used in the RIOFS. The RIOFS sector categories and the aggregation routine utilized to derive them is described below.

Since the final goal of the study was to forecast water intake, the sector aggregation utilized had to be consistent with the sources for data on water usage. The only source for industrial water intake figures by state for a number of time points [4] is organized on a two digit SIC basis. These two digit categories provide the primary sector categories for the RIOFS. Table 1 displays the RIOFS industry categories with cross references to SIC and OBE input output groupings.

Aggregation of the coefficient matrices requires a number of simple transformations. Initially each matrix was transformed into flow form. Since,

$$a_{ij} = \frac{x_{ij}}{x_j} \tag{4-1}$$

a_{ij} - input coefficient, sector i to sector j
x_{ij} - dollar flow, sector i to sector j
x_j - gross dollar output, sector j

the flow matrix elements are simply,

$$x_{ij} = a_{ij}x_j \tag{4-2}$$

Given flow tables for 1947 and 1963 on an 81 sector OBE basis, the aggregation routine was used to transform each into a 37 sector RIOFS basis table. To illustrate the aggregation process, assume that the original flow table consists of three sectors. A new table with two sectors, sector one being the sum of sectors one and two, is desired. The aggregation is a two step procedure.

	1	2	3	X_j
1	10	20	30	60
2	20	10	40	70
3	10	30	50	90

STEP I

	1	2	X_j
1	30	30	60
2	30	40	70
3	40	50	90

TABLE 1
SECTOR DEFINITIONS

RIOFS Sector	OBF Sector	SIC Sector	Description
1	1	1,2	Livestock and livestock products
2	2	1,2	Other agricultural products
3	3,4	7-9	Forestry and Fishery products and services
4	5,6	10	Iron, Ferroalloy and nonferrous metal ore mining
5	7	11,12	Coal
6	8	13	Crude petroleum and natural gas
7	9,10	14	Stone, clay, chemical and fertilizer mining
8	11,12	15-17	New Construction, maintenance and repair construction
9	13	19	Ordinance and accessories
10	14	20	Food and kindred products
11	15	21	Tobacco manufacturing
12	16,17	22	Textile products
13	18,19	23	Apparel
14	20,21	24	Lumber and wood products
15	22,23	25	Household furniture
16	24,25	26	Paper and allied products
17	26	27	Printing and publishing
18	27-30	28	Chemicals and allied products
19	31	29	Petroleum refining and related industries
20	32	30	Rubber and miscellaneous plastic products
21	33,34	31	Leather tanning and industrial related products
22	35,36	32	Glass and glass products, stone and clay products

TABLE 1 (Con't)

RIOFS Sector	OBE Sector	SIC Sector	Description
23	37,38	33	Primary iron and steel and nonferrous metal manufacturing
24	39-42	34	Fabricated metal products
25	43-52	35	Machinery, except electrical
26	53-58	36	Electrical machinery
27	59-61	37	Transportation equipment
28	62,63	38	Scientific and Controlling instruments
29	64	39	Miscellaneous manufacturing
30	68	49	Electric, gas, water, and sanitary services
31	65-67	40-49	Communications
32	69	50-59	Wholesale and retail trade
33	70,71	60-67	Finance, insurance and real estate
34	72-77	70-89	Services
35	78	NA	Federal Government enterprizes
36	79	NA	State and Local Government enterprizes
37	81	NA	Business travel, entertainment and gifts

STEP II

	1	2	X_j
1	60	70	130
2	40	50	90

After both matrices were aggregated to 37 sectors they were retransformed into coefficient form by dividing each flow element by the gross output of the corresponding column. That is,

$$a_{ij} = \frac{x_{ij}}{x_j} \tag{4-3}$$

As mentioned, the 1963 matrix was not available in 1958 prices. Since price deflators were available on a two digit level, it was possible to deflate the aggregated 1963 matrix. The deflated input coefficient is derived according to

$$a_{ij}{}^{d} = a_{ij}\frac{P_i}{P_j} \tag{4-4}$$

$a_{ij}{}^{d}$ - deflated input coefficient
a_{ij} - input coefficient in current period
P_i - price deflator, sector i
P_j - price deflator, sector j

All the above operations are simple matrix transformations which were done in a computer program called ADJUST. ADJUST is a modified form of a multi-purpose matrix manipulation program developed at the Lawrence Ration Laboratory computer center [5]. Since the program is similar to programs available on most systems it is not included in the appendix.

The two 37 sector constant dollar input coefficient matrices constituted the principal inputs to the coefficient projection program, PROJECT. This program produces coefficient matrices for 1980, 2000, and 2020 in 1958 dollars (included in the data appendix). It operates according to the projection procedure discussed in Chapter II. Given input matrices for 1947 and 1963. Let

C^{47} - column sum vector, 1947 input coefficients
C^{63} - column sum vector, 1963 input coefficients

$$\text{prob } (z \leq T^{47}) = C^{47} \tag{4-5}$$
$$\text{prob } (z \leq T^{63}) = C^{63} \tag{4-6}$$

z - vector of standardized normal variables
T^{47} - vector of values satisfying (4-5)
T^{63} - vector of values satisfying (4-6)

The projected value of T is

$$T^{P} = T^{47} + t(T^{63}-T^{47}) \tag{4-7}$$

$$t = (\text{future year} - 1963)/16 \tag{4-8}$$

Equation (4-7) represents a linear extrapolation of the changes occurring during the period of actual observation. The value of t is a linear function of the number of time periods making up the forecast span. Since the basic time period defined by the observations is 16 years, the values of t are,

$$T^{1980} = 17/16 = 1.0625 \tag{4-9}$$
$$T^{2000} = 37/16 = 2.3125 \tag{4-10}$$
$$T^{2020} = 57/16 = 3.5625 \tag{4-11}$$

Hence

$$T^{1980} = T^{47} + 1.0625\ (T^{63}-T^{47}) \tag{4-12}$$
$$T^{2000} = T^{47} + 2.3125\ (T^{63}-T^{47}) \tag{4-13}$$
$$T^{2020} = T^{47} + 3.5625\ (T^{63}-T^{47}) \tag{4-14}$$

Given the T values, corresponding values for c are derived from the normal distribution function as the probability of obtaining these values are; i.e.,

$$C^{80} = \text{prob } (z \leq T^{80}) \tag{4-15}$$
$$C^{00} = \text{prob } (z \leq T^{00}) \tag{4-16}$$
$$C^{20} = \text{prob } (z \leq T^{20}) \tag{4-17}$$

C^{80} - column sum vector, 1980 input coefficients
C^{00} - column sum vector, 2000 input coefficients
C^{20} - column sum vector, 2020 input coefficients

These projected column sum vectors are used to adjust the 1963 coefficient matrix so that the column sums of each forecasted matrix are equal to the sums derived in (4-14) to (4-16). Letting < > represent a diagonalized matrix, the projected matrices are then,

$$A^{80} = A^{63} \langle C^{80} \rangle \langle C^{63} \rangle^{-1} \quad (4\text{-}18)$$
$$A^{00} = A^{63} \langle C^{00} \rangle \langle C^{63} \rangle^{-1} \quad (4\text{-}19)$$
$$A^{20} = A^{63} \langle C^{20} \rangle \langle C^{63} \rangle^{-1} \quad (4\text{-}20)$$

A^{63} - input coefficient matrix, 1963
A^{80} - input coefficient matrix, 1980
A^{00} - input coefficient matrix, 2000
A^{20} - input coefficient matrix, 2020

If i represents the unit vector, it is true that

$$A^{80}i = C^{80} \quad (4\text{-}21)$$
$$A^{00}i = C^{00} \quad (4\text{-}22)$$
$$A^{20}i = C^{20} \quad (4\text{-}23)$$

The three matrices A^{80}, A^{00}, and A^{20} form the output from module one of the RIOFS. With the projected employment vectors from module two, which is discussed in the next section, they comprise the primary input data set for module three.

4. Regional Trading Projection

Sub-section two which is described in this section, does not itself produce projections of regional trading patterns. The location quotients that are utilized to adjust the coefficient matrices for trade, are functions of projected levels of national gross output and projected levels of national input coefficients as well as relative employment levels. Hence the actual projection of regional trading as represented by location quotients is accomplished in module 3 (see figure 4). Figure 3 displays the principal operations performed in module 2. The output of this sub-system consists of two sets of projected employment vectors; one for the region and one for the nation. As discussed in Chapter III the total output of this module or any subset of the output could be substituted for by independent forecasts.

In order to form employment projections that corresponded to the RIOFS sectors, a suitable data source had to be found. The only source consistent with the requirements of the RIOFS is County Business Patterns [6]. Data was available by state and two digit SIC sectors for ten data points between the years 1948 and 1969. This data base can easily be extended as more years become available.

The employment data was compiled into 43 two digit SIC groups by state (see table 2), which were then aggregated first across states and then into the 37 RIOFS categories. Although the following discussion will proceed as if data existed for all 37 sectors, it was possible to only obtain information for 32 sectors. Data for sectors one and two (livestock and agricultural products) and sectors 35, 36, and 37 (state and local, and federal enterprises, and business travel, entertainment

TABLE 2

STATES IN THE NAR

Maine

New Hampshire

Vermont

Massachusetts

Rhode Island

Connecticut

New York

New Jersey

Pennsylvania

Delaware

Maryland

West Virginia

District of Columbia

and gifts) was not available on a consistent basis with sectors 3 through 34. In addition, ranching and farming have special characteristics that make resource, especially water intake, estimation more appropriate for independent study. Sectors 35, 36, and 37 also are not typical in that the government enterprise sectors have no clear measures of output, and business travel, entertainment and gifts is a proxy industry for estimation errors in the other sectors.

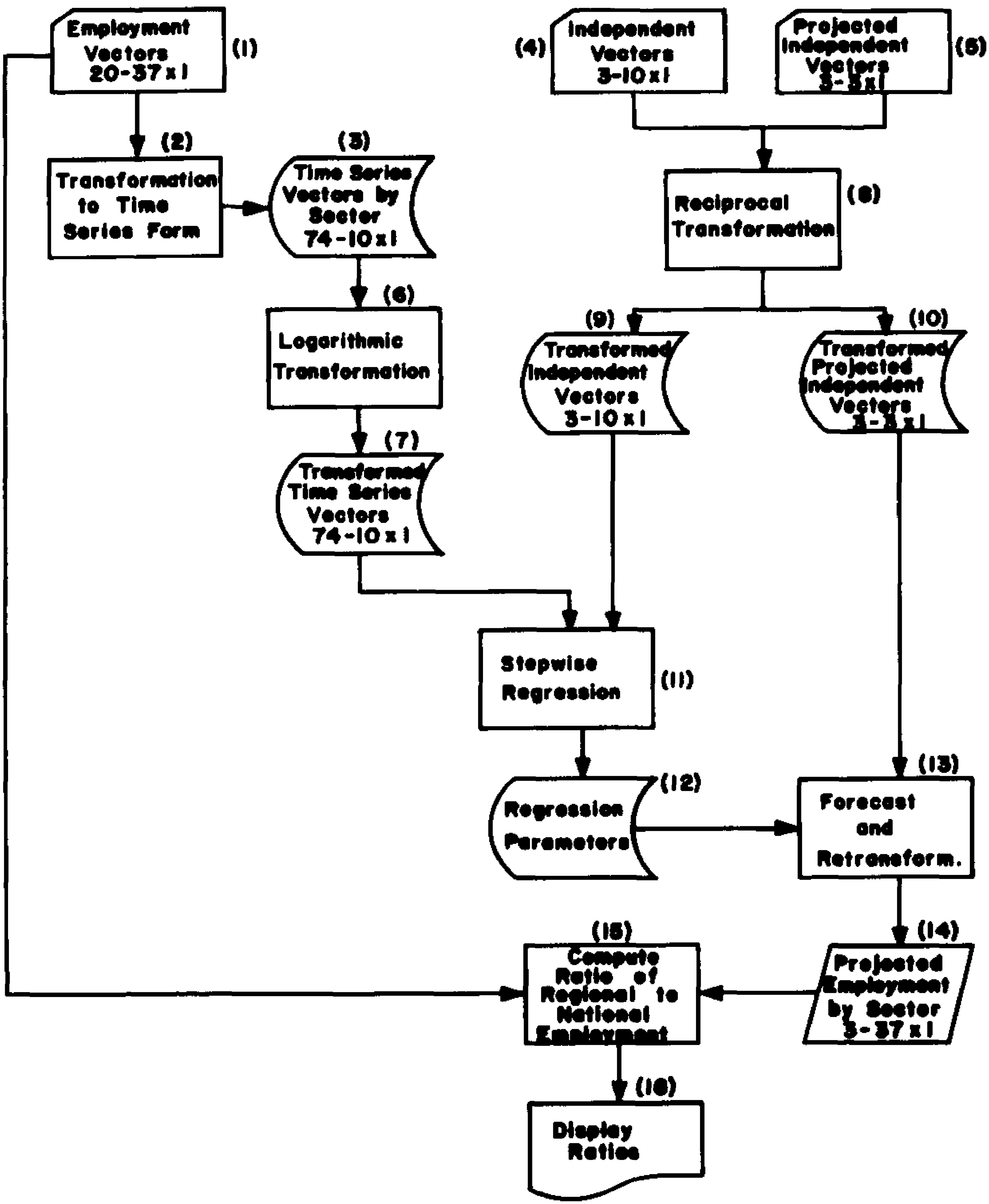
The aggregation from 43 to 37 sectors was done for each of the ten observation years in the program AGG, whose output consisted of 20, 37 element vectors, ten vectors for regional employment and ten vectors for national employment.

The rest of the projection procedure is accomplished in the program entitled EMPROJ. Essentially this program first transforms the 37 element vectors, one for each year, into 10 element time series vectors, one for each sector. These time series vectors; 37 for the region and 37 for the nation, make up the dependent variable set. This set of dependent variables and a set of independent variables is then transformed into a form that prevents the projection process from becoming explosive. Regressions are then run and the resulting parameter estimates are utilized to produce employment forecasts. These results are then tested for consistency. Since the actual program is more complex than the above description, a detailed flow chart is shown in Figure 5.

Box 1 refers to the employment vectors that are the output of program AGG. At this point in the program they are in cross sectional form; two

FIGURE 5

PROGRAM EMPROJ



37 element vectors by sector for each year, one for the region and one for the nation. Boxes 2 and 3 represent the transformation procedure that yields 74 time series vectors of dependent variables suitable for regression analysis. This data set is included as Table A-1 and A-2 in appendix A.

The independent variables utilized to estimate forecasting equations for employment are shown in boxes 4 and 5. This independent variable data set consists of past and future values of population, personal income and time for both the region and the nation. Although many other variables might influence future employment trends, it is important in forming a forecasting model to choose independent variables that can be projected with reasonable confidence. Particularly when projecting far into the future only the broad aggregate economic measures can be forecasted with any confidence. In addition to the above rationale, regional and national projections of population and personal income were made by the OBE for the original NAR study. Hence the choice of population, personal income, and time as independent variables are justifiable on both practical and theoretical grounds. The historical variables are included as table A-3 and the projected variables, table A-4 in appendix A.

Boxes 6 through 10 refer to the variable transformations that were necessary to prevent the employment projections from taking unreasonable values. Although the employment forecasts could have been done on an individual industry by industry basis, the limited nature of the study made a consistent forecasting methodology essential. Due to the excessive computation cost and general inavailability of nonlinear regression programs, ordinary least squares regressions were utilized. Within the context of the above restrictions, a number of nonlinear forms can be estimated by the use of appropriate transformations. Initial tests utilizing simple least squares without variable transformations yielded negative forecast results for a number of series that exhibited relatively sharp declines over the observation period. In addition the estimated relationships for industries that showed relatively sharp increases over the historical period produced extraordinarily high projected employment levels for 2000 and 2020. In essence, the untransformed simple least squares regressions were explosive, both upward and downward.

To remedy the situation a number of transformations were tested. Semilog. double log and reciprocal transformations ameliorated the problem, but a small number of sectors still had negative employment levels in 2020. The problem was solved by the adoption of a logarithmic, reciprocal transformation. In this transformation the independent variable was replaced by its log value while the dependent variables were substituted for by their reciprocals. In addition to alleviating the explosive character of the projections, the test ratios represented in boxes 15 and 16 and discussed below, took on more reasonable values than in previous cases.

Since it was impossible to estimate each of the 74 relationships on an industry by industry basis within the given time and staff limitations, a stepwise regression was adopted. The operation of this routine is shown in boxes 11 and 12. In addition to the maintenance of a high level of automation, the step-wise regression approach eliminates the difficult judgmental decisions required in ordinary regression programs. As mentioned numerous times, the preceding approach does not preclude the incorporation of outside estimates for certain sectors, although care must be taken to maintain consistency.

Since all of the independent variables contain positive long term trends they tend to be colinear. Test runs using low mininum significance requirements for variable inclusion tended to produce unstable results in certain cases where more than one variable was included. In order to eliminate this problem, the final runs only accepted variables that were significant at the 98% level. Hence in most cases only one independent was included in the estimated relationship. In cases where no variable is significant, the average value over the observation period is assumed to hold for the forecast period.

Boxes 13 and 14 refer to the projection process in which the projected independent variables are combined with estimated relationships to form employment forecasts. Since the dependent variable is in log form it must be transformed back into employment form. Tables 3 and 4 display the employment levels for regional and national industries in 1963, 1980, 2000, and 2020.

The last two boxes, 15 and 16, are concerned with the consistency of the employment projections. In order to test the results of the forecast procedure for overall reasonableness, the ratio of total regional employment to total national employment was compiled over both the historical and forecast period. The results of this calculation are shown in table 5. Over this historical period the ratios exhibit a

TABLE 3
PROJECTED EMPLOYMENT-REGION

Sector	Regional Employment 1963	Regional Employment 1963	Regional Employment 2000	Regional Employment 2020
1	0.	0.	0.	0.
2	0.	0.	0.	0.
3	33813.	58407.	95755.	156574.
4	4941.	6565.	6565.	6565.
5	86179.	32602.	13646.	5738.
6	8357.	4488.	2706.	1635.
7	19141.	20669.	20669.	20669.
8	839471.	921379.	992608.	1027609.
9	21486.	35487.	35487.	35487.
10	427311.	421628.	421628.	421628.
11	25454.	16760.	11716.	8206.
12	279984.	174133.	106632.	65467.
13	680685.	627920.	568889.	515678.
14	93981.	74579.	58555.	46034.
15	112917.	116514.	116514.	116514.
16	221103.	240585.	226445.	196124.
17	381076.	455902.	493049.	511353.
18	303534.	319488.	319488.	319488.
19	29026.	15253.	8351.	4587.
20	162171.	270080.	417006.	642378.
21	198967.	175456.	151568.	131034.
22	192785.	189439.	180405.	171845.
23	442976.	499125.	499125.	499125.
24	367085.	394221.	394221.	394221.
25	497780.	543260.	543260.	543260.
26	601373.	804244.	914964.	971567.
27	370230.	377263.	377263.	377263.
28	174380.	220219.	234590.	241594.
29	202126.	179777.	147877.	121763.
30	188746.	176140.	165696.	155922.
31	861810.	862711.	862711.	862711.
32	3792733.	4540000.	4906693.	5087300.
33	1105405.	1462745.	1681994.	1794950.
34	2631463.	5022242.	8867244.	15608817.
35	0.	0.	0.	0.
36	0.	0.	0.	0.
37	0.	0.	0.	0.

TABLE 4
PROJECTED EMPLOYMENT-NATION

Sector	National Employment 1963	National Employment 1980	National Employment 2000	National Employment 2020
1	0.	0.	0.	0.
2	0.	0.	0.	0.
3	141093.	265163.	368724.	427293.
4	71767.	67288.	55863.	48703.
5	140346.	63896.	55863.	14570.
6	223147.	251380.	251830.	251830.
7	106001.	108443.	108804.	108966.
8	2571423.	3181969.	2597965.	2000920.
9	242942.	414049.	515059.	583892.
10	1538558.	1700212.	1861596.	1990289.
11	75243.	61472.	55947.	53641.
12	857328.	704810.	366336.	200121.
13	1279624.	1382122.	1162122.	953205.
14	565328.	455424.	253470.	147745.
15	380044.	431497.	322520.	232295.
16	583678.	709134.	770393.	799469.
17	925385.	1149958.	1269986.	1327632.
18	748293.	854225.	676282.	518604.
19	150581.	110281.	78739.	61424.
20	417365.	749496.	1003605.	1143568.
21	325985.	298881.	261698.	237284.
22	563427.	619383.	646382.	658831.
23	1151851.	1244786.	1244786.	1244786.
24	1080182.	1125158.	1125158.	1125158.
25	1527567.	1641075.	1641075.	1641075.
26	1465767.	2296868.	2831442.	3109130.
27	1627597.	1681994.	1681994.	1681994.
28	310537.	424446.	474034.	498042.
29	369608.	424732.	424732.	424732.
30	586047.	635194.	657691.	668006.
31	2470762.	2480748.	2480748.	2480748.
32	11900935.	15339601.	13881840.	11680616.
33	2914936.	4269492.	5196765	5674139.
34	7288254.	15908863.	35016177.	62758833.
35	0.	0.	0.	0.
36	0.	0.	0.	0.
37	0.	0.	0.	0.

TABLE 5

REGIONAL TO NATIONAL EMPLOYMENT RATIOS

YEAR	REGIONAL EMPLOYMENT / NATIONAL EMPLOYMENT
1948	0.3849
1951	0.3761
1953	0.3679
1956	0.3551
1959	0.3542
1962	0.3494
1964	0.3443
1965	0.3416
1967	0.3338
1969	0.3242
1980	0.3155
2000	0.3006
2020	0.2966

downward movement. This corresponds to above average growth of the West and Far West over the period. The forecasted ratios show a continuation of this downward trend reflecting the expected relative decline of the NAR as a center of employment.

The employment vectors derived in the sub-system described above are combined with forecasted national input coefficients formed in module one and the projected quasi-productivities, to compute location quotients which are then utilized to adjust the coefficient matrices for changing patterns or regional trade. This process is described in the next section.

5. Module Three

Module three is described in this section. It details the operational flow of this subsystem. Since this study is primarily a test of the suitability of the systems approach to complex forecasting problems in the context of stringent time and staff constraints, the presentation of the results in the next section is somewhat limited. Initially the overall structure of module three will be discussed. A detailed discussion of the operational flow of the module follows.

Overall structure

Module three consists of one computer program. This program, FINAL, is a complex FORTRAN routine consisting of over one thousand statements, and requiring 140 thousand words of central core memory. It is structured so that modifications allowing it to be run on smaller computers can easily be made.

The general operational flow of program final is shown in Figure 4. Essentially the program utilizes the projected coefficient matrices and employment vectors formed in Modules 1 and 2 to form a number of regional gross output vectors. In order to derive these vectors, regional final demand vectors are computed from given levels of total regional and national gross output, employment, and input coefficients. The location quotients are combined with projected coefficient matrices to derive the adjusted or regional input coefficient matrices.

Five alternative scenarios are examined within the program. The alternatives are designed so that cross comparisons between the scenarios provide information on the consequences of the projected changes in technology and industrial location. Three of the alternatives are formed utilizing the regional input coefficient matrices while the remaining two are calculated from the unadjusted matrices. Detailed descriptions of the assumptions underlying each scenario are presented in the next section.

Detailed operational flow

Figure 6 shows a detailed flow chart of program FINAL. Boxes (1) through (6) refer to the procedure utilized to compute the national and regional final demand vectors. As mentioned in Chapter I, extensive research has been done in the area of final demand forecasting. Rather than reproduce these efforts, the RIOFS accepts the total regional and national final demand (gross national product) projections made by the OBE for the original NAR study (box 1). Total final demand figures must be distributed across the relevant industry sectors to form the regional and national final demand vectors. To conserve time it was assumed that the national final demand industry proportions observed in 1963 (box 2) would continue to hold throughout the forecast period for both the region and the nation. Hence for any year (box 3)

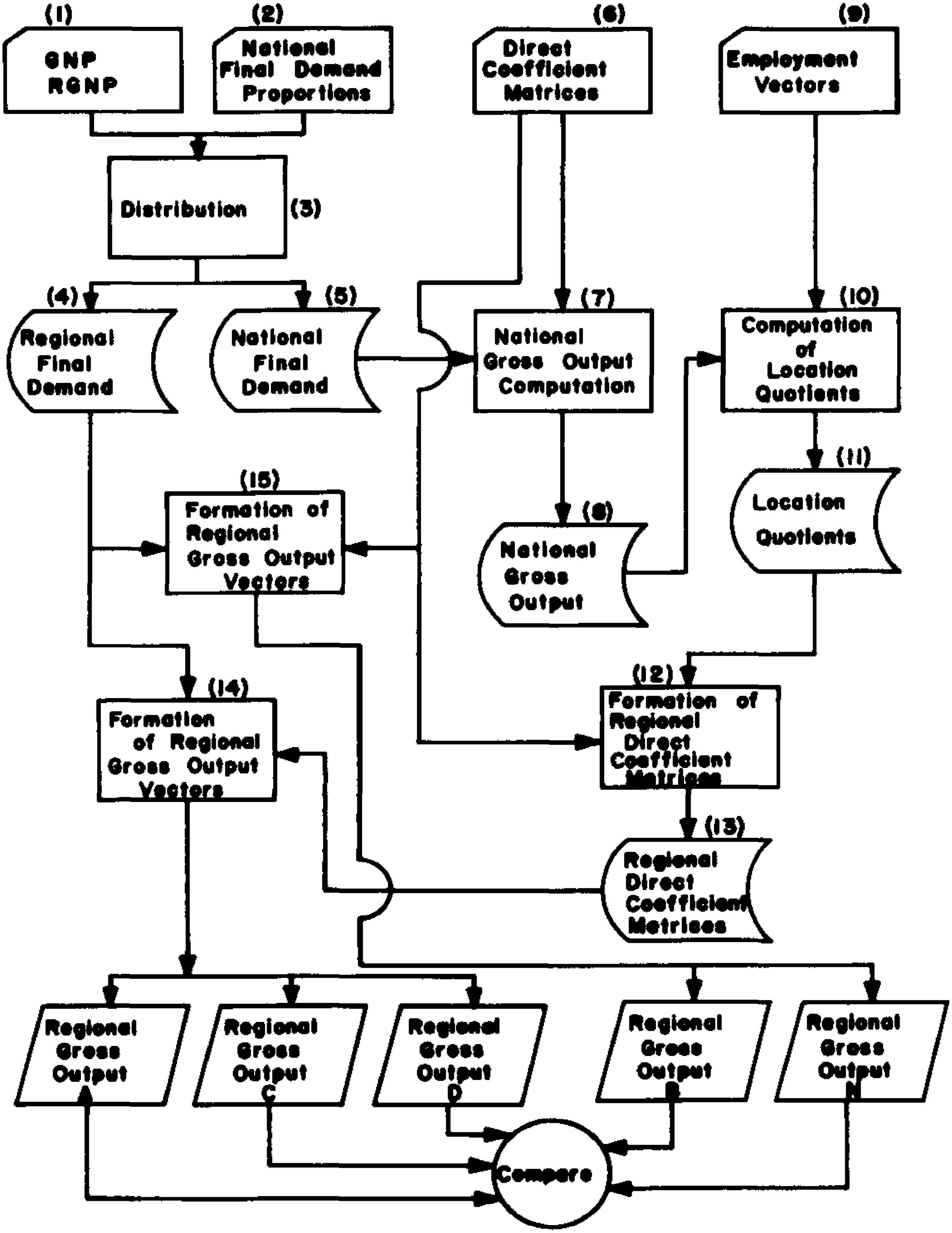
$$Y = GNP * YPCT \qquad (4\text{-}24)$$

$$YR = RGNP * YPCT \qquad (4\text{-}25)$$

* - multiplication symbol
Y - national final demand vector
GNP - total national final demand (gross national product)
YPCT - 1963 industry proportion vector
YR - regional final demand vector
RGNP - total regional final demand (gross national product)

FIGURE 6

PROGRAM FINAL



Boxes (4) and (5) refer to the vectors Y and YR. Alternate assumptions about the levels or distribution of final demand can easily be incorporated into this formulation. The actual data set is included as Table A-5 and A-6 in the data appendix.

The location quotients are formed in the routine represented by boxes (6) through (11). Each quotient is a function of projected national gross output by industry (box 8), projected regional and national employment by industry (box 9), and the forecasted input coefficients (box 6). The quotient for industry i in year p has the form,

$$LQ_i = \frac{e_i^p}{\sum_j a_{ij}^p e_j^p \dfrac{X_j^p}{E_j^p}} \Bigg/ \frac{E_i^p}{\sum_j a_{ij}^p E_j^p \dfrac{X_j^p}{E_j^p}} \qquad (4\text{-}26)$$

This formulation allows any set of employment forecasts to be tested by a simple input substitution.

Table 6 displays the location quotient vectors for 1963, 1980, 2000, and 2020 (box 11). Since no data was available, sectors 1 and 2, and 35 through 37 were assumed to have quotients equal to unity. Industries showing import orientation at some time during the study time frame are shown in table 7. Starred sectors have location quotients significantly different from one. Petroleum production in the NAR, both at the crude (sector 6) and the refined (sector 19) stage, is shown to be particularly inadequate as compared to the needs of the region. This gap continues to grow during the forecast period. Recent evidence of oil and natural gas reserves off the Atlantic Coast could influence the rate of decline of the location quotients for sectors 6 and 19.

Textiles (12) and paper (16) exhibit continued movement of productive facilities out of the region during the study time frame. Due to the

TABLE 6
LOCATION QUOTIENTS

Sector	Location Quotient 1963	Location Quotient 1980	Location Quotient 2000	Location Quotient 2020
1	1.0000	1.0000	1.0000	1.0000
2	1.0000	1.0000	1.0000	1.0000
3	0.9819	0.9298	0.9378	1.0000
4	0.2009	0.2707	0.3242	0.3680
5	1.0000	1.0000	1.0000	1.0000
6	0.1748	0.1033	0.0561	0.0362
7	0.5200	0.6009	0.5546	0.5467
8	0.9188	0.8868	1.0000	1.0000
9	0.3388	0.3634	0.3322	0.3263
10	0.9446	0.9361	0.9149	0.8933
11	0.9997	0.9979	0.9929	0.9842
12	0.7672	0.6790	0.7193	0.6866
13	1.0000	1.0000	1.0000	1.0000
14	0.5923	0.6133	0.7242	0.8964
15	0.8822	0.8840	1.0000	1.0000
16	1.0000	1.0000	0.9108	0.7761
17	1.0000	1.0000	1.0000	1.0000
18	1.0000	1.0000	1.0000	1.0000
19	0.5957	0.4545	0.3226	0.2237
20	1.0000	1.0000	1.0000	1.0000
21	1.0000	1.0000	1.0000	1.0000
22	1.0000	1.0000	0.8704	0.8054
23	1.0000	1.0000	1.0000	1.0000
24	1.0000	1.0000	1.0000	1.0000
25	1.0000	1.0000	1.0000	1.0000
26	1.0000	1.0000	1.0000	1.0000
27	0.9570	0.9789	1.0000	1.0000
28	1.0000	1.0000	1.0000	1.0000
29	1.0000	1.0000	1.0000	0.9870
30	0.9647	0.9119	0.8734	0.8213
31	1.0000	1.0000	1.0000	1.0000
32	0.9311	0.9444	1.0000	1.0000
33	1.0000	1.0000	1.0000	1.0000
34	1.0000	0.9999	0.8531	0.8584
35	1.0000	1.0000	1.0000	1.0000
36	1.0000	1.0000	1.0000	1.0000
37	1.0000	1.0000	1.0000	1.0000

TABLE 7

SECTORS SHOWING IMPORT ORIENTATION

RIOFS Sector	Description	LQ263	LQ280	LQ200	LQ220
3	Forestry and Fishery Products	0.9819	0.9298	0.9378	1.0000
4	Iron, Ferroallow and Non-Ferrous mining *	0.2009	0.2707	0.3242	0.3680
6	Crude petroleum and natural gas *	0.1748	0.1033	0.0561	0.0362
7	Stone, clay, glass and fertilizer mining *	0.5200	0.6009	0.5546	0.5467
8	New construction, maintenance and repair construction	0.9188	0.8868	1.0000	1.0000
9	Ordinance and accessories *	0.3388	0.3634	0.3322	0.3263
10	Food and kindred products	0.9446	0.9361	0.9149	0.8933
11	Tobacco manufacturing	0.9997	0.9979	0.9929	0.9842
12	Textile products *	0.7672	0.6790	0.7193	0.6866
14	Lumber and wood products *	0.5923	0.6133	0.7242	0.8964
15	Household furniture	0.8822	0.8840	1.0000	1.0000
16	Paper and allied products	1.0000	1.0000	0.8704	0.8054
19	Petroleum refining and related industries *	0.5957	0.4545	0.3226	0.2237
22	Glass products, stone and clay products *	1.0000	1.0000	0.8704	0.8054
27	Transportation equipment	0.9570	0.9789	1.0000	1.0000
29	Miscellaneous manufacturing	1.0000	1.0000	1.0000	0.9870
30	Electric, gas, water and sanitary services *	0.9467	0.9119	0.8734	0.8213
32	Wholesale and retail trade	0.9311	0.9444	1.0000	1.0000
34	Services	1.0000	0.9999	0.8531	0.8584

large amount of water used by these sectors the movement shown has important implications for future water supply project construction.

Although the mining sectors, metals (4) and the stone, clay and glass group (7), display strong import orientation, both become less dependent on the imports during the forecast period.

Other major sectors showing significant movements towards strong import orientation are the public utilities (30) and the stone, clay, and glass group (22).

Two related sectors, lumber and wood products (14) and household furniture (15), become less import oriented in the future.

The effects on regional gross output of the extent and changes in import orientation,as represented by the location quotients is described at the end of this section. But at this time,it is clear that these effects will be significant for both water supply needs and other basic resource requirements.

Boxes (12) and (13) represent the formulation of the projected adjusted or regional coefficient matrices. The regional coefficient matrix for any year is calculated as,

$$RA = LQ*A \qquad (4\text{-}27)$$

RA - regional (adjusted) input coefficient matrix

LQ - diagonal matrix with ith diagonal element being LQ_i

A - national input coefficient matrix

The last two number boxes in Figure 6, (14) and (15), refer to the calculation of the gross output vectors. Box (14) shows the computation process utilized to form the three alternate gross output sets that assume some type of adjustment for regional trade. These vector sets are designated as A, C, and D. The remaining two vector sets, B and N, which are formed in process box (15), utilize the unadjusted national coefficient matrices.

The five scenarios examined in program FINAL were chosen to test the consequences of applying the projection methodologies discussed in chapters II and III to a test case. In addition to this type of testing, the system is suited to the examination of a particular methodology under alternative forecasts of general economic activity. Since long run projections of even relatively stable variables such as population are subject to significant errors, it is vital to forecast under a number of assumed conditions. This provides a range of solutions that allows cost benefit evaluations to be made in a more realistic setting.

The five alternative gross output data sets and the comparison routine shown at the bottom of Figure 6 are discussed in detail in the next section.

6. Results

Introduction

Table 8 displays the computation formulas for each alternative gross output data set. The general formula for the scenarios utilizing input coefficient matrices adjusted for regional trade is,

$$XZ_{--} = RA_{--} * YR \quad (4\text{-}28)$$

$$RA_{--} = (I - RA_{--})^{-1} \quad (4\text{-}29)$$

with Z representing the scenario type, either A,C, or D. and the last two places of each neumonic indicating the time period (full dates are included in data tables), i.e.;

63-1963
80-1980
00-2000
20-2020

For scenarios using the unadjusted input matrices the general formulation is,

$$XZ_{--} = A_{--} * YR \quad (4\text{-}30)$$

$$A_{--} = (I - A_{--})^{-1} \quad (4\text{-}31)$$

with Z being either B or N, representing the national input coefficient matrices.

Since the quantity of output produced by program FINAL is particularly large, this section is divided into two parts to simplify the presentation. Initially the assumption set utilized to form each scenario is discussed. In as much as it is difficult to interpret the meaning of a set of vectors in level form, the actual output are not discussed and are only included in the data appendix.

TABLE 8
SCENARIO DESCRIPTIONS

ADJUSTED FOR REGIONAL TRADE	UNADJUSTED
SENARIO A	SENARIO B
XA63 = RA63 * YR63	XB63 = A63 * YR63
XA80 = RA80 * YR80	XB80 = A63 * YR80
XA00 = RA00 * YR00	XB00 = A63 * YR00
XA20 = RA20 * YR20	XB20 = A63 * YR20
SENARIO C	SENARIO N
XC63 = RA63 * YR63	XN63 = A63 * YR63
XC80 = RA80 * YR63	XN80 = A80 * YR80
XC00 = RA00 * YR63	XN00 = A00 * YR00
XC20 = RA20 * YR63	XN20 = A20 * YR20
SENARIO D	
XD63 = RA63 * YR63	
XC80 = RA63 * YR80	
XC00 = RA63 * YR00	
XC20 = RA63 * YR20	

In order to facilitate the evaluation, absolute and percentage differences were calculated for a number of logical paired alternative scenarios. In one case growth rates were computed over the forecast periods, 1963 to 1980, 1963 to 2000, and 1963 to 2020. A discussion of the above comparisons is included in the second part of this section.

Scenario description

Senario A incorporates the projections of technological and locational change described in chapters II and III. As such, it is the alternative that would be used to form a forecast of resource utilization, given a set of resource coefficients.

Senario C was designed to faciliate the analysis of the consequences of both projection procedures. By holding the final demand vector constant, the changes over the forecast periods due to the technological and locational projections are isolated.

The approach taken in the original NAR study is examined in Senario D. In this alternative, the national input coefficient matrix is adjusted for regional trading in a base period. This adjusted matrix is then assumed to remain stable with respect to both technological and trading change. The growth rates associated with this approach clearly illustrate the limitations of the assumption of no change.

Senario B is included principally as a control in that it assumes neither regional trade nor technological change; hence, it is useful in isolating the effects of the adjustment and projection procedures.

The last scenario, N, assumes technological change without adjustment for regional trading. It is useful for isolating the effects of both regional adjustment and technological change.

The projected vectors for each alternative are included in tables A-7 through A-11 in the data appendix.

Scenario comparisons

As shown in table 4 the scenarios are logically divided into: (1) alternatives using the adjusted regional matrices; and (2) alternatives solved using the unadjusted national matrix. Since the adjusted input coefficients are bounded from above by the national coefficients, gross output vectors derived from national matrices will (everything else held constant) always be larger, element by element, than gross output vectors derived from the adjusted matrices. Hence no comparisons between the adjusted and the unadjusted groups are presented.

Two scenario comparisons are discussed below. The first was made to examine the effect of projecting locational and technological change. This was accomplished by evaluating the differences between scenario A, the RIOFS forecast, with scenario D, the NAR-type forecast. The difference between these alternatives is the presence of technological and locational change in scenario A (see table 8). Technological change alone is covered in the second comparison, which was made between alternative B and N. The only divergence between output vectors XB and XN is the utilization of projected coefficient matrices in the fomation of the XN vectors. (see table 8).

Before presenting the above cross comparisons, two alternatives are examined individually. In order to completely isolate the effect of the RIOFS projection methodology from the final demand forecasts, scenario C is evaluated with respect to itself. That is, XC63 is compared with XC80, XC00 with XC20 (see table 8). Since Y63 is used in the calculation of each of the XC vectors any differences are the result of the changing input coefficient matrices. The second individual analysis is concerned with the logical consequences of not forecasting either technological or locational change. These consequences are made clear by presenting forecast period growth rates for the vectors formed in scenario D, which assumes no change from the base period adjusted matrix.

The vector comparisons are displayed in tabular form using a consistent format. The first and second column of each table contain the gross output vectors to be analyzed. The element by element difference between each vector is shown in column 3. Column 4 presents percentage differences using the column one vector as a base. In addition, summary statistics were computed to allow analysis of the overall divergence between any two vectors. These summary measures are then utilized to assess the magnitude and direction of overall differences over time. The statistics shown, consist of simple sum, simple average, standard deviation and a weighted average of the absolute percentage differences. Let X1 be the vector displayed in column one and X2 the vector in column two; then the summary statistics are defined as:

Sample Sum: $$SX1 = \sum_{i=1}^{n} X1_i \qquad (4\text{-}32)$$

$$SX2 = \sum_{i=1}^{n} X2_i \qquad (4\text{-}33)$$

$$SD = \sum_{i=1}^{n} D_i \ , \ D_i = X1_i - X2_i \qquad (4\text{-}34)$$

$$SP = \sum_{i=1}^{n} P_i \ , \ P_i = 100* \left[(X1_i - X2_i) / X1_i \right] \qquad (4\text{-}35)$$

Simple Average:

$$AX1 = SX1/n \qquad (4\text{-}36)$$

$$AX2 = SX2/n \qquad (4\text{-}37)$$

Standard Deviation:

$$SDD = \sum_{i=1}^{n} D_i^{\,2} \quad n \qquad (4\text{-}38)$$

$$SPD = \sum_{i=1}^{n} P_i^{\,2} \quad n \qquad (4\text{-}39)$$

Weighted Average:

$$WA = \left| D_i \right| * \left| P_i \right| \Big/ \sum_{i=1}^{n} \left| D_i \right| \qquad (4\text{-}40)$$

The notation used in the above expressions to designate each summary statistic was adopted as the standard notational system for the tables discussed below.

C to C

Tables 9A, B, and C display comparisons of gross output vector XC63,with XC80, XC00, and XC20. These vectors are defined, as in table 8, by:

$$XC63 = RA63 * Y63 \qquad (4\text{-}41)$$

$$XC80 = RA80 * Y63 \qquad (4\text{-}42)$$

$$XC00 = RA00 * Y63 \qquad (4\text{-}43)$$

$$XC20 = RA20 * Y63 \qquad (4\text{-}44)$$

Hence any changes in the output vectors are due to the forecasting procedures described in Chapters II and III.

An examination of the vector sums and averages indicates a clear decline in the total regional output needed to support a given level of final demand. With the 1980 matrix, the region required 1% less total gross output to produce the 1963 level of final demand than with the

TABLE 9A
C TO C
1980

Sector	Billions of 1958 Dollars			Percent
	XC1963	XC1980	D	D/XC1980
1	7172.13	7111.51	60.62	0.85
2	7548.89	7487.55	61.34	0.82
3	988.47	982.41	6.06	0.62
4	222.69	267.80	-45.12	-16.85
5	638.09	630.40	7.69	1.22
6	407.86	227.88	179.98	78.98
7	434.17	489.64	-55.48	-11.33
8	28221.61	27989.45	232.16	0.83
9	1942.83	1046.41	-3.58	-0.18
10	23749.74	23644.29	105.45	0.45
11	2434.21	2433.44	10.78	0.44
12	3511.05	3142.47	368.58	11.73
13	6940.84	6912.17	28.67	0.41
14	1504.98	1604.87	-99.89	-6.22
15	1894.19	1895.45	-1.27	-0.07
16	4610.25	4557.10	53.15	1.17
17	4427.81	4386.50	41.31	0.94
18	9665.51	9550.33	115.18	1.21
19	5486.36	4978.03	508.33	10.21
20	2750.22	2740.34	9.88	0.36
21	1433.29	1426.27	7.02	0.49
22	3209.58	3154.86	54.72	1.73
23	11445.31	11428.57	16.75	0.15
24	7067.57	7002.98	64.60	0.92
25	11522.05	11544.55	-22.50	-0.19
26	10032.99	10026.95	6.04	0.06
27	18722.90	18849.56	-126.66	-0.67
28	2094.95	2088.56	6.39	0.31
29	2109.85	2101.00	8.84	0.42
30	8253.83	7938.68	315.15	3.97
31	16623.35	16464.56	158.78	0.96
32	37600.25	37647.81	-47.56	-0.13
33	48853.82	48608.11	245.72	0.51
34	34030.25	33834.24	196.00	0.58
35	1985.88	1960.21	25.67	1.31
36	2262.98	2197.84	65.13	2.96
37	2355.31	2327.75	27.56	1.18
	SX1= 331801.	SX2= 32943.	SD= 2558.	SP= 88.97
	AX1= 9216.69	AX2= 9145.63	SDD= 147.93	SPD= 13.92
				WA= 8.47281

TABLE 9B
C TO C
2000

Sector	Billions of 1958 Dollars			Percent
	XC1963	XC2000	D	X/XC2000
1	7172.13	6078.40	1093.74	17.99
2	7548.89	6696.04	852.85	12.74
3	988.47	831.96	156.51	18.81
4	222.69	312.67	-89.98	-28.78
5	638.09	589.53	48.56	8.24
6	407.86	38.86	369.00	949.58
7	434.17	314.03	120.14	38.26
8	28221.61	26999.99	1221.62	4.52
9	1942.83	1942.52	0.31	0.02
10	23749.74	22232.66	1517.09	6.82
11	2434.21	2236.77	197.44	8.83
12	3511.05	2671.02	840.04	31.45
13	6940.84	6455.63	485.21	7.52
14	1504.98	1377.62	127.36	9.25
15	1894.19	1861.57	32.62	1.75
16	4610.25	3388.69	1221.55	36.05
17	4227.81	3566.05	861.76	24.16
18	9665.51	8441.40	1221.11	14.50
19	5486.36	4091.17	1395.19	34.10
20	2750.22	2566.69	183.52	7.15
21	1433.29	1298.39	134.90	10.39
22	3209.58	2015.23	1194.35	59.27
23	11445.31	11222.26	223.06	1.99
24	7067.57	6055.29	1012.28	16.72
25	11522.05	11915.15	-393.09	-3.30
26	10032.99	9973.30	59.69	0.60
27	18722.90	19371.24	-648.34	-3.35
28	2094.95	1965.72	129.23	6.57
29	2109.85	1947.65	162.19	8.33
30	8253.83	6600.15	1653.67	25.06
31	16623.35	14160.76	2462.59	17.39
32	37600.25	36207.97	1392.23	3.85
33	48853.82	45159.82	3694.00	8.18
34	34030.25	28576.57	5453.67	19.08
35	1985.88	1577.11	408.77	25.92
36	2262.98	1650.18	612.80	37.14
37	2355.31	1878.45	476.86	25.39
	SX1= 331801.	SX2= 302390.	SD= 29411.	SP= 1436.78
	AX1= 9216.69	AX2= 8399.72	SDD= 1398.64	SPD= 159.60
				WA= 29.18549

TABLE 9C
C TO C
2020

Sector	Billions of 1958 Dollars XC1963	XC2020	D	Percent D/XC2020
1	7172.13	5004.34	2167.79	43.32
2	7548.89	5753.24	1795.65	31.21
3	988.47	712.35	276.12	38.76
4	222.69	359.89	-137.21	-38.12
5	638.09	561.24	76.85	13.69
6	407.68	16.52	391.34	2368.85
7	343.17	213.79	220.38	103,08
8	28221.61	25936.93	2284.68	8.81
9	1942.83	1943.71	-0.88	-0.05
10	23749.74	21021.46	2728.29	12.98
11	2434.21	2098.25	335.96	16.01
12	3511.05	2109.06	1402.00	66.48
13	6940.84	6088.94	851.89	13.99
14	1504.98	1160.26	344.72	29.71
15	1894.19	1813.74	80.45	4.44
16	4610.25	2429.50	2180.75	89.76
17	4427.81	3059.30	1368.51	44.73
18	9665.51	7590.29	2075.22	27.34
19	5486.36	3697.79	1788.57	48.37
20	2750.22	2436.83	313.38	12.86
21	1433.29	1206.47	226.82	18.80
22	3209.58	1382.10	1827.48	132.22
23	11445.31	11302.31	143.00	1.27
24	7067.57	5444.84	1622.74	29.80
25	11522,05	12326.65	-804.59	-6.53
26	10032.99	10029.54	3.45	0.03
27	18722.90	19628.27	-905.37	-4.61
28	2094.95	1890.89	204.06	10.79
29	2109.85	1753.13	356.71	20.35
30	8253.83	5724.88	2528.94	44.17
31	16623.35	12773.70	3849.65	30.14
32	37600.25	34904.93	2695.32	7.72
33	48853.82	42302.35	6551.48	15.49
34	34030.25	26344.43	7685.82	29.17
35	1985.88	1302.88	679.99	52.07
36	2262.98	1272.58	990.40	77.83
37	2355.31	1579.12	776.19	49.15
	SX1= 331801.	SX2= 283600.	SD= 48200.	SP=3394.94
	AX1= 9216.69	AX2= 7877.79	SDD= 2221.42	SPD= 397.19
				WA= 51.98065

1963 matrix. With the 2000 matrix it was 9% less and with the 2020 matrix, 15% less. Although this is a hypothetical situation the magnitude of this reduction in internal gross output has significant implications for the level of future regional resource requirements and concomitantly, investment in resource production.

Although the total percentage change due to technological and locational change is 15% in 2020, many of the sectors exhibit much larger differences. This is evident in the weighted average figures which provide a more representative measure of the amount of individual sectoral divergence between vectors.

The most striking decline in gross output occurred in Sector 6, the crude petroleum and natural gas industry. As mentioned earlier in this chapter, the location quotients for this sector exhibit sharp secular declines over the forecast period. Other sectors showing significant declines in gross output levels required to produce a given level of final demand were; 12 (textile products), 16 (paper and allied products), 19 (petroleum refining) and 35 and 36 (government enterprises). The results for the service sectors 30 through 37 are less reliable than the figures for the other sectors due to the difficulty in measuring the output of industries providing services.

Three industries showed significant increases in their gross output levels over the projected periods. These were 4 (metal mining), 25 (nonelectrical machinery), and 27 (transportation equipment).

D to D

Alternative D presents a NAR type of scenario in that a base period adjustment is made for regional external trade. This adjusted coefficient matrix is then assumed to be constant over the study forecast period. In the notation of table 8, scenario D is described by,

$$XD63 = RA63 * Y63 \quad (4\text{-}45)$$

$$XD80 = RA63 * Y80 \quad (4\text{-}46)$$

$$XD00 = RA63 * Y00 \quad (4\text{-}47)$$

$$XD20 = RA63 * Y20 \quad (4\text{-}48)$$

Table 10 displays the compound annual rate of growth over the periods 1963 to 1980, 1963 to 2000, and 1963 to 2020 for the D alternative. This table is included to highlight the problems with the assumption of constant technology and constant regional trading patterns. Since under these assumptions the forecasting system is essentially a linear system, the growth rates shown in table 10 are the same for each industry during a given projection period. The growth rate slows down from 4.11% during the first period, to 4.00% over the second period, and to 3.94% for the whole 1963 to 2020 period.

The above results clearly illustrate the need for trading pattern and technology projection. Under no foreseeable conditions would each industry be expected to exhibit identical rates of output growth.

D to A

Alternative A incorporates both technology and regional trading pattern projection. In comparing it to the NAR type scenario, D, the effect of the projection methodologies is examined within the context of

TABLE 10
D TO D

(Annual Rate of Change,%)

Sector	DD1980	DD2000	DD2020
1	4.1087	4.003	3.9401
2	4.1087	4.003	3.9401
3	4.1087	4.003	3.9401
4	4.1087	4.003	3.9401
5	4.1087	4.003	3.9401
6	4.1087	4.003	3.9401
7	4.1087	4.003	3.9401
8	4.1087	4.003	3.9401
9	4.1087	4.003	3.9401
10	4.1087	4.003	3.9401
11	4.1087	4.003	3.9401
12	4.1087	4.003	3.9401
13	4.1087	4.003	3.9401
14	4.1087	4.003	3.9401
15	4.1087	4.003	3.9401
16	4.1087	4.003	3.9401
17	4.1087	4.003	3.9401
18	4.1087	4.003	3.9401
19	4.1087	4.003	3.9401
20	4.1087	4.003	3.9401
21	4.1087	4.003	3.9401
22	4.1087	4.003	3.9401
23	4.1087	4.003	3.9401
24	4.1087	4.003	3.9401
25	4.1087	4.003	3.9401
26	4.1087	4.003	3.9401
27	4.1087	4.003	3.9401
28	4.1087	4.003	3.9401
29	4.1087	4.003	3.9401
30	4.1087	4.003	3.9401
31	4.1087	4.003	3.9401
32	4.1087	4.003	3.9401
33	4.1087	4.003	3.9401
34	4.1087	4.003	3.9401
35	4.1087	4.003	3.9401
36	4.1087	4.003	3.9401
37	4.1087	4.003	3.9401

an actual forecast. The computational formulas for alternatives A and D are:

A	D	
XA63 = RA63 * Y63	XD63 = RA63 * Y63	(4-49)
XA80 = RA80 * Y80	XD80 = RA63 * Y80	(4-50)
XA00 = RA00 * Y00	XD00 = RA63 * Y00	(4-51)
XA20 = RA20 * Y20	XD20 = RA63 * Y20	(4-52)

Tables 11A through 11D display the vector comparisons between vectors XA and XD. On an overall basis the changing productive structure used in solving alternative A enables the same amount of final demand to be satisfied with less total gross output. The difference in total gross output necessary to produce a given level of final demand, is only 1% in 1980, 9% in 2000, and 15% in 2020.

These differences are identical to the ones described in the initial analysis involving scenario C. In addition, the percentage columns and percentage summary statistics are identical for the corresponding years. This peculiar situation is a result of the linear nature of the input output approach and the constant final demand distribution. For instance, the comparison between XC63 and XC00 shown in table 9B involves:

XC63 = RA63 * Y63, XC00 = RA00 * Y63 (4-53)

while that between XA00 and XD00 involves

XD00 = RA63 * Y00, XA00 = RA00 * Y00 (4-54)

Since Y00 is proportional to Y63, XD00 will be proportionally related

TABLE 11A
D TO A
1963

Sector	Billions of 1958 Dollars XD1963	XA1963	D	Percent D/XA1963
1	7172.13	7172.13	0.00	0.00
2	7548.89	7548.89	0.00	0.00
3	988.47	988.47	0.00	0.00
4	222.69	222.69	0.00	0.00
5	638.09	638.09	0.00	0.00
6	407.86	407.86	0.00	0.00
7	434.17	434.17	0.00	0.00
8	28221.61	28221.61	0.00	0.00
9	1942.83	1942.83	0.00	0.00
10	23749.74	23749.74	0.00	0.00
11	2434.21	2434.21	0.00	0.00
12	3511.05	3511.05	0.00	0.00
13	6940.83	6940.84	0.00	0.00
14	1504.98	1504.98	0.00	0.00
15	1894.19	1894.19	0.00	0.00
16	4610.25	4610.25	0.00	0.00
17	4427.81	4427.10	0.00	0.00
18	9665.51	9665.51	0.00	0.00
19	5486.36	5486.36	0.00	0.00
20	2750.22	2750.11	0.00	0.00
21	1433.29	1433.29	0.00	0.00
22	3209.58	3209.58	0.00	0.00
23	11445.31	11445.31	0.00	0.00
24	7067.57	7067.57	0.00	0.00
25	11522.05	11522.05	0.00	0.00
26	10032.99	10032.99	0.00	0.00
27	18722.90	18722.90	0.00	0.00
28	2094.95	2094.95	0.00	0.00
29	2109.85	2109.85	0.00	0.00
30	8253.83	8253.83	0.00	0.00
31	16623.35	16623.35	0.00	0.00
32	37600.25	37600.25	0.00	0.00
33	48853.82	48853.82	0.00	0.00
34	34030.25	34030.25	0.00	0.00
35	1985.88	1985.88	0.00	0.00
36	2262.98	2262.98	0.00	0.00
37	2355.31	2355.31	0.00	0.00
	SX1= 331801.	SX2= 331801.	SD= 0.	SP= 0.00
	AX1= 9216.69	AX2= 9216.69	SDD= 0.00	SPD= 0.00
			WA= 0.00000	

TABLE 11B
D TO A
1980

Sector	XD1980	XA1980	D	D/XA1980
	Billions of 1958 Dollars			Percent
1	14220.82	14100.62	120.20	0.85
2	14967.85	14846.22	121.63	0.82
3	1959.93	1947.91	12.02	0.62
4	441.54	531.00	-89.46	-16.85
5	1265.19	1249.95	15.24	1.22
6	808.70	451.83	356.87	78.98
7	860.87	970.86	-110.00	-11.33
8	55957.47	55947.15	460.33	0.83
9	3852.22	3859.33	-7.11	-0.18
10	47090.71	46881.63	209.08	0.45
11	4826.53	4805.16	21.37	0.44
12	6961.68	6230.86	730.82	11.73
13	13762.21	13705.36	56.85	0.41
14	2984.06	3182.11	-198.05	-6.22
15	3755.77	3758.28	-2.51	-0.07
16	9141.14	9035.76	105.38	1.17
17	8779.41	8697.51	81.90	0.94
18	19164.66	18936.28	228.38	1.21
19	10878.29	9870.37	1007.91	10.21
20	5453.10	5433.51	19.59	0.36
21	2841.91	2828.00	13.91	0.49
22	6363.92	6255.42	108.49	1.73
23	22693.64	22660.43	33.20	0.15
24	14013.50	13885.42	128.08	0.92
25	22845.79	22890.40	-44.61	-0.19
26	19893.29	19881.32	11.97	0.06
27	37123.54	37374.68	-251.14	-0.67
28	4153.85	4141.18	12.67	0.31
29	4183.38	4165.84	17.54	0.42
30	16365.59	15740.72	624.87	3.97
31	32960.58	32645.75	314.84	0.96
32	74553.33	74647.63	-94.30	-0.13
33	96866.79	96379.58	487.21	0.51
34	67474.77	67086.14	388.63	0.58
35	3937.57	3886.67	50.90	1.31
36	4487.00	4357.86	129.14	2.96
37	4670.08	4615.43	54.65	1.18
	SX1= 657891.	SX2= 652819.	SD= 5072.	SP= 88.97
	AX= 18274.74	AX2= 18133.85	SDD= 293.32	SPD= 13.92
				WA= 8.47281

TABLE 11C
D TO A
2000

Sector	Billions of 1958 Dollars XD2000	XA2000	D	Percent D/XA2000
1	30614.28	25945.65	4668.63	17.99
2	32222.44	28582.07	3640.37	12.74
3	4219.30	3551.23	668.06	18.81
4	950.53	1334.62	-384.09	-28.78
5	2723.67	2516.42	207.26	8.24
6	1740.96	165.87	1575.09	949.58
7	1853.25	1340.45	512.81	38.26
8	120464.01	115249.53	5214.48	4.52
9	8292.97	8291.65	1.33	0.02
10	101375.84	94900.15	6475.69	6.82
11	10390.45	9547.68	842.77	8.83
12	14986.94	11401.25	3585.70	31.45
13	29626.98	27555.87	2071.10	7.52
14	6424.01	5880.37	543.64	9.25
15	8085.34	7946.11	139.23	1.75
16	19678.84	14464.64	5214.21	36.05
17	18900.12	15221.71	3678.41	24.17
18	41257.25	36032.14	5225,10	14.50
19	23418.53	17463.15	5955.38	34.10
20	11739.30	10955.94	783.36	7.15
21	6117.99	5542.16	575.83	10.39
22	13700.10	8602.03	5098.07	59.27
23	48854.36	47902.24	952.12	1.99
24	30167.96	25847.03	4320.93	16.72
25	49181.91	50859.84	-1677.92	-3.30
26	42825.84	42571.06	254.78	0.60
27	79918.74	82686.20	-2767.46	-3.35
28	8942.31	8390.67	551.64	6.57
29	9005.88	8313.56	692.32	8.33
30	35231.48	28172.78	7058.71	25.06
31	70956.81	60445.23	10511.58	17.39
32	160496.76	154553.80	5942.95	3.85
33	208532.68	192764.81	15767.87	8.18
34	145258.19	121979.18	23279.01	19.08
35	8476.72	6731.88	1744.84	25.92
36	9659.52	7043.80	2615.72	37.14
37	10053.64	8018.18	2035.46	25.39
	SX1= 1416292.	SX2= 1290753.	SD= 125540.	SP= 1436.78
	AX1= 39341.45	AX2= 35834.24	SDD= 5970.09	SPD= 159.60
			WA= 29.18549	

TABLE 11D
D TO A
2020

Sector	Billions of 1958 Dollars			Percent
	XD2020	XA2020	D	D/XA2020
1	64907.20	45288.88	19618.32	43.32
2	68316.78	52066.34	16250.44	31.21
3	8945.59	6446.74	2498.85	38.76
4	2015.28	3257.02	-1241.73	-38.12
5	5774.63	5079.14	695.49	13.69
6	3691.11	149.51	3541.60	2368.85
7	3929.19	1934.81	1994.38	103.08
8	255403.12	234726.95	20676.76	8.81
9	17582.44	17590.42	-7.98	-0.05
10	214933.14	190242.38	24690.76	12.98
11	22029.42	18988.98	3040.45	16.01
12	31774.74	19086.78	12687.96	66.48
13	62813.97	55104.43	7709.54	13.99
14	13619.95	10500.28	3119.67	29.71
15	17142.23	16414.17	728.05	4.44
16	41722.32	21986.74	19735.58	89.76
17	40071.30	27686.37	12384.92	44.73
18	87472.02	68691.46	18780.56	27.34
19	49651.07	33464.71	16186.36	48.37
20	24889.22	22053.11	2836.10	12.86
21	12971.14	10918.43	2052.70	18.80
22	29046.42	12507.89	16538.53	132.22
23	103579.12	102284.96	1294.15	1.27
24	63960.94	49275.30	14685.64	29.80
25	104273.59	111555.08	-7281.50	-6.53
26	90797.69	90766.47	31.22	0.03
27	169440.62	177634.18	-8193.56	-4.61
28	18959.13	17112.44	1846.70	10.79
29	19093.92	15865.69	3228.23	20.35
30	74696.43	51809.70	22886.73	44.17
31	150439.88	115600.90	34838.98	30.14
32	340279.01	315886.62	24392.39	7.72
33	442122.91	382832.58	59290.32	15.49
34	307970.79	238414.81	69555.99	29.17
35	17972.01	11818.13	6153.88	52.07
36	20479.74	11516.71	8963.03	77.83
37	21315.34	14290.91	7024.43	49.15
	SX1= 3002768.	SX2= 2566559.	SD=436209.	SP= 3394.94
	AX1= 83410.22	AX2= 71293.31	SDD= 20103.65	SPD= 397.19
				WA= 51.98065

to XC63, and XA00 similarly related to XC00. Hence in computing percentage differences between XA00 and XD00 the proportionality constant cancels out, and the differences in percentages terms are identical to those between XC63 and XC00. The same holds for 1980 and 2020.

Hence the previous discussion involving scenario C applies to the comparisons between alternatives A and D, with the XD vectors in place of the XC63 vector and XA80, XA00, and XA20 in place of XD80, XD00, and XD20 respectively.

Since the detailed sector comparisons were covered earlier, the remainder of this discussion consists of some general observations on the effect of the projection procedures.

The tables (11A through 11D) illustrate the distortions that the assumption of no change cause. The manufacturing sectors (10 through 29) which use a high proportion of most natural resources (e.g., water, wood, petroleum) in almost all cases show significantly lower levels of gross output when technological and regional trading change is taken into consideration. Since most resource supply projects involve very large expenditures, it is vital to consider the effect of structural change on the needs for long term investment.

N to B

The last set of vector comparisons concerns scenarios N and B. Both alternatives are formed with unadjusted input coefficient matrices. B is computed under the assumption of constant technology while N allows for changing input coefficients. Hence the comparison between the two vector sets isolates the effect of technological change from trading change. The XN and XB vector sets are calculated from,

XN63 = A63 * Y63	XB63 = A63 * Y63	(4-55)
XN80 = A80 * Y80	XB80 = A63 * Y80	(4-56)
XN00 = A00 * Y00	XB00 = A63 * Y 00	(4-57)
XN20 = A20 * Y20	XB20 = A63 * Y20	(4-58)

Tables 12A through 12D displays the results of the above calculations. There is a clear decline in total output necessary to support the projected final demands. In 1980 the difference in total output requirements is less than 1%. By 2000 it is 9%, and in 2020 it becomes almost 15%.

Only two sectors show any significant increase in output requirements due to the projected coefficient change. Metal mining (sector 4) produces 12.8% more in 2020, under the 2020 technology, than under the 1963 technology and nonelectrical machinery (sector 25) 6.7% more.

While all the remaining sectors show negative or very small positive differences, a number of industries exhibit particularly large gross output declines using the projected technology. These sectors are crude petroleum (sector 6), stone, clay and glass mining (sector 7), textile products (sector 12), lumber and wood products (sector 14), stone, clay and glass products (sector 2), and state and local government enterprises.

These differences indicate that, even without adjustment for and projection of regional trade, the changes in productive structure due to alternations in technology will result in significant variations in

TABLE 12A
N TO B
1963

Sector	Billions of 1958 Dollars XN1963	XB1963	D	Percent D/XB1963
1	7351.32	7351.32	0.00	0.00
2	7805.39	7805.39	0.00	0.00
3	1157.29	1157.29	0.00	0.00
4	777.76	777.76	0.00	0.00
5	657.77	57.77	0.00	0.00
6	2917.76	2917.76	0.00	0.00
7	840.07	840.07	0.00	0.00
8	28930.40	28930.40	0.00	0.00
9	2089.38	2089.38	0.00	0.00
10	24285.56	24285.56	0.00	0.00
11	2436.55	2436.55	0.00	0.00
12	4832.54	4832.54	0.00	0.00
13	6977.29	6977.29	0.00	0.00
14	2896.75	2896.75	0.00	0.00
15	1945.42	1945.42	0.00	0.00
16	4730.93	4730.93	0.00	0.00
17	4473.04	4473.04	0.00	0.00
18	10060.49	10060.49	0.00	0.00
19	7199.53	7199.53	0.00	0.00
20	2814.98	2814.98	0.00	0.00
21	1437.10	1437.10	0.00	0.00
22	3320.57	3320.57	0.00	0.00
23	11743.70	11743.70	0.00	0.00
24	7259.06	7259.06	0.00	0.00
25	11687.67	11687.67	0.00	0.00
26	10133.91	10133.91	0.00	0.00
27	19201.27	19201.27	0.00	0.00
28	2116.74	2116.74	0.00	0.00
29	2126.30	2126.30	0.00	0.00
30	8684.68	8684.68	0.00	0.00
31	17104.14	17104.14	0.00	0.00
32	38585.64	38585.64	0.00	0.00
33	49882.57	49882.57	0.00	0.00
34	34480.64	34480.64	0.00	0.00
35	2029.27	2029.27	0.00	0.00
36	2345.24	2345.24	0.00	0.00
37	2440.07	2440.07	0.00	0.00
	SX1= 347319.	SX2= 347319.	SD= 0.	SP= 0.00
	AX1= 9647.74	AS2= 9647.74	SDD= 0.00	SPD= 0.00

WA= 0.00000

TABLE 12B
N TO B
1980

Sector	Billions of 1958 Dollars			Percent
	XN1980	XB1980	D	D/XB1980
1	14487.58	14576.10	-88.52	-0.61
2	15412.55	15476.44	-63.89	-0.41
3	2274.74	2294.66	-19.92	-0.87
4	1544.67	1542.14	2.53	0.16
5	1299.40	1304.22	-4.82	-0.37
6	5473.91	5785.30	-311.39	-5.38
7	1639.62	1665.68	-26.06	-1.56
8	57168.21	57362.86	-194.65	-0.34
9	4144.55	4142.79	1.76	0.04
10	48034.85	48153.13	-118.27	-0.25
11	4811.07	4831.16	-20.10	-0.42
12	9460.66	9581.90	-121.24	-1.27
13	13784.38	13834.48	-50.10	-0.36
14	5647.47	5743.65	-96.18	-1.67
15	3848.54	3857.36	-8.82	-0.23
16	9288.66	9380.43	-91.77	-0.98
17	8788.49	8869.09	-80.60	-0.91
18	19804.20	19947.83	-143.63	-0.72
19	14121.29	14275.14	-153.85	-1.08
20	5560.30	5581.50	-21.20	-0.38
21	2835.49	2849.47	-13.98	-0.49
22	6484.19	6583.98	-99.79	-1.52
23	23229.20	23285.27	-56.07	-0.24
24	14267.64	14393.17	-125.53	-0.87
25	23201.21	23174.28	27.03	0.12
26	20078.47	20093.41	-14.94	-0.07
27	38097.38	38072.06	25.32	0.07
28	4182.87	4197.04	-14.17	-0.34
29	4199.59	4216.00	-16.42	-0.39
30	17085.86	17219.89	-134.03	-0.78
31	33636.87	33913.88	-177.01	-0.82
32	76280.01	76507.16	-227.15	-0.30
33	98464.24	98906.57	-442.32	-0.45
34	68000.31	68367.80	-367.49	-0.54
35	13979.65	4023.61	-43.96	-1.09
36	4586.58	4650.12	-63.54	-1.37
37	4785.49	4838.14	-52.66	-1.09
	SX1= 685205.	SX2= 688659.	SD= -3455.	SP= -26.67
	AX1= 19033.46	AX2= 19129.43	SDD= 145.54	SPD= 1.18
				WA= 1.09901

TABLE 12C
N TO B
2000

Sector	Billions of 1958 Dollars XN2000	XB2000	D	Percent C/XB2000
1	26639.86	31379.11	-4739.25	-15.10
2	29401.88	33317.34	-3915.46	-11.75
3	4001.87	4939.89	-938.02	-18.99
4	3495.78	3319.89	175.90	5.30
5	2639.18	2807.69	-168.51	-6.00
6	2896.04	12454.46	-9558.42	-76.75
7	2582.52	3585.85	-1003.33	-27.98
8	115952.03	123489.50	-7537.47	-6.10
9	8998.39	8918.51	79.88	0.90
10	97664.25	103662.98	-5998.73	-5.79
11	9558.73	10400.42	-841.69	-8.09
12	15599.61	20627.69	-5028.08	-24.38
13	27665.01	29782.56	-2117.55	-7.11
14	8633.48	12364.80	-3731.32	-30.18
15	7963.29	8304.04	-340.75	-4.10
16	16557.90	20193.98	-3636.07	-18.01
17	15883.16	19093.18	-3210.03	-16.81
18	37272.63	42943.25	-5670.62	-13.20
19	25227.49	30731.21	-5503.72	-17.91
20	11184.98	12015.73	-830.74	-6.91
21	5563.99	6134.28	-570.29	-6.30
22	10306.64	14173.84	-3867.20	-27.28
23	48650.20	50128.01	-1477.81	-2.95
24	26260.30	30985.30	-4724.99	-15.25
25	51425.90	49888.87	1537.03	3.08
26	42885.91	43256.64	-370.73	-0.86
27	83207.09	81960.69	1246.40	1.52
28	8495.00	9035.30	-540.30	-5.98
29	8406.22	9076.12	-669.90	-7.38
30	31938.54	37070.59	-5132.05	-13.84
31	62002.77	73009.05	-11006.29	-15.08
32	155684.89	164702.92	-9018.03	-5.48
33	195278.90	212923.86	-17644.96	-8.29
34	132548.22	147180.69	-14632.46	-9.94
35	6934.42	8661.94	-1727.52	-19.94
36	7536.63	10010.67	-2474.05	-24.71
37	8359.62	10415.45	-2055.83	-19.74
	SX1= 1346944.	SX2= 1482531.	SD= 135587.	SP= -470.65
	AX1= 37415.10	AX2= 41181.41	SDD= 5683.86	SPD= 19.09

WA= 17.03193

TABLE 12D
N TO B
2020

Sector	XN2020	XB2020	D	D/XB2020
	Billions of 1958 Dollars			Percent
1	46509.24	66528.78	-20019.54	-30.09
2	53339.71	70638.13	-17298.42	-24.49
3	6810.16	10473.36	-3663.20	-34.98
4	7940.79	7038.69	902.10	12.82
5	5362.85	5952.76	-589.91	-9.91
6	2823.16	16405.46	-23582.30	-89.31
7	4207.83	7602.57	-3394.74	-44.65
8	235937.87	261817.65	-25879.77	-9.88
9	19231.78	18908.68	323.10	1.71
10	196398.58	210782.24	-23383.65	-10.64
11	19014.74	22050.57	-3035.82	-13.77
12	25253.97	43734.03	-18480.06	-42.26
13	55284.32	63143.83	-7859.51	-12.45
14	13368.89	26215.37	-12846.47	-49.00
15	16445.62	17605.89	-1160.28	-6.59
16	29836.27	42814.49	-12978.21	-30.31
17	28912.45	40480.63	-11568.18	-28.58
18	70996.98	91046.61	-20049.63	-22.02
19	47991.01	65155.11	-17164.10	-26.34
20	22582.61	25475.28	-2892.67	-11.35
21	10961.78	13005.66	-2043.88	-15.72
22	16918.70	30050.84	-13132.14	-43.70
23	104052.26	106279.47	-2227.22	-2.10
24	50111.64	65693.83	-15582.19	-23.72
25	112850.88	105772.44	7078.44	6.69
26	91445.40	91711.05	-265.65	-0.29
27	178948.05	173769.87	5178.17	2.98
28	17323.36	19156.29	-1832.93	-9.57
29	16842.94	19242.84	-2399.90	-12.47
30	60954.64	78595.63	-17640.99	-22.45
31	118566.52	154790.97	-36224.45	-23.40
32	318012.82	349196.76	-31183.94	-8.93
33	390852.40	451432.92	-60580.52	-13.42
34	261036.34	312046.80	-51010.46	-16.35
35	12203.06	18364.71	-6161.64	-33.55
36	12472.09	21224.25	-8752.15	-41.42
37	14939.65	22082.44	-71.42.79	-32.35
	SX1= 2681802.	SX2= 3143204.	SD= -461403.	SP= -739.32
	AX1= 74494.49	AX2= 87311.23	SDD= 19420.78	SPD= 28.00

WA= 24.50998

gross output and hence resource demand.

7. Summary

The RIOFS as described in this study illustrates the problem solving power of the systems approach. The system formed utilizes sophisticated forecasting models in a flexible modular computerized structure. This modular flexibility allows for the incorporation of alternate assumptions, independent forecasts, and forecasting models. This is in contrast to traditional approaches where one, basically hand hewn, alternative is formed. Since long term projections are particularly difficult to make, the ability to examine more than one scenario is vital.

Since the RIOFS was constructed by one individual, the study also proves that the systems approach to sophisticated economic forecasting is not beyond the smallest study teams. This is of particular importance since the rationale for the use of simple models in a traditional framework is often lack of manpower. In the last few years advances in data base management technology has made the construction of complex forecasting systems less resource intensive. While the RIOFS was formed in a batch process, fortran environment, similar studies done in the present time sharing, high level language environment, would require less resources.

In addition to illustrating the feasibility of the systems approach, the RIOFS provided useful insights about the future structure of the NAR economy. Despite the many limitations of the modeling procedures, mentioned throughout this study, important conclusions from the results can be formed. The output vectors discussed in this chapter clearly indicate that the assumption of no change in either technology or trading patterns produce substantially different results. All the results point to significant changes in the gross output requirements necessary to support projected levels of final demand. These differences are due to the structural variations in trading patterns and productive technology. Changes of the magnitude shown in this chapter have important implications for the size of planned resource supply projects. In the area of water supply, where investment is very expensive, and often disruptive, such variations in output forecasts must be considered in order to arrive at valid cost benefit calculations. Hopefully future studies will adopt the systems approach to their economic projections so that only truely necessary investment is made.

This chapter concludes the discussion of the operational aspects of the RIOFS. Chapter V, which follows, presents the sensitivity testing routines mentioned earlier in the study. Although it was not possible to incorporate these procedures in the RIOFS for this study, they are included to bridge a gap in the literature. Sensitivity testing holds potential for substantial improvement in input output forecasting projects since it provides an optimal allocation of limited time and manpower. These routines can easily be applied to future studies.

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CHAPTER V

SENSITIVITY TESTING

1. Introduction

Sensitivity testing is an integral part of the systems approach to problem solving. The system presented in this study relies on a number of mathematical models. These models must, necessarily, be simplifications of reality. The extent of these simplifications should depend on the importance of particular system elements to the output of the study. Critical elements of the system should be subject to more extensive research than elements that are not critical in terms of the study results. For instance, in projecting water demand the primary processing sector utilizes far more water per unit of output than the retail sector. Hence more time and effort should be utilized to estimate primary processing output than to estimate retail output In order to make decisions on the comparitive importance of system elements to the final system outputs, it is necessary to have information about the relative sensitivity of these outputs to variation in the system elements. This chapter presents a theoretical discussion of a number of sensitivity testing routines suited to input output type problems.

There are two general approaches to sensitivity testing: (1) system simulation, and (2) algebraic solution. Simulation in this context refers to numerical solutions with varied system elements. Algebraic solution results in mathematical relationships between system elements and system outputs. Ease of computation is the usual criteria utilized to decide between simulation and solution. Algebraic solutions usually provide more efficient sensitivity testing procedures. Simulations are for situations where it is either impossible, or prohibitively expensive to derive relationships between system elements and system outputs. Since they require a complete system solution for each change in a system element, they are generally inefficient.

These general statements about the relative efficiency of simulation and solution are detailed further on in the discussion of the actual testing problem. This discussion follows in the next section. In addition, that section describes the particular system elements to be tested and the general approach taken in the testing. The third section contains a derivation of a formula common to the individual testing routines presented in sections four through six.

As mentioned earlier, time limitations have made it impossible to proceed with the actual application of the sensitivitiy testing routines in the RIOFS. Despite this, their inclusion in the present study is justifiable, as the routines are unavailable from other sources. Other studies utilizing the systems approach will be able to make good use of the testing routines presented in this chapter.

2. System Elements and General Approach

There are two primary procedures or elements in the RIOFS. The first, described in Chapter II, was the technical input coefficient projection procedure. The second element consists of the trading pattern projection scheme covered in Chapter III. Essentially each procedure varies input coefficients. The former changes whole columns by a uniform percentage, while the latter varies complete rows. Hence the sensitivity testing routines must be able to evaluate the consequences of errors in the estimation of input columns and rows on the system outputs.

This type of approach is particularly appropriate in water resource demand studies where the system outputs consist of water demand levels, and less than ten IO sectors account for more than 90% of the industrial self supplied water intake. Thus sensitivity routines could be used to identify industry coefficient vectors or matrix elements that are critical to water usage. These vectors can then be subjected to more extensive analysis than is given to the less critical items. This can increase the reliability of the results by allowing more intensive research to be done on the most critical elements of the system.

The coefficient projection and adjustment procedures described in the last two chapters do not preclude the independent estimation of individual coefficient rows, columns, or matrix elements. In certain cases data is available for detailed estimation of technical or locational variation of certain coefficient vectors or matrix elements. A sensitivity testing routine can identify the vectors or matrix elements which are most critical to the study results and should be subjected to independent estimation or close monitoring.

The testing routines discussed in the remainder of this chapter provide partial tests in that they only evaluate the results of errors in one row, or one column, or one element of the relevant matrix. This is the only practical approach since the number of possible error combinations letting more than one row, column, or matrix element vary is huge.

Three routines form the testing package described in this chapter, The first two, covered in sections four and five, are designed to yield the variation in each industry's gross output due to erroneous estimation of a particular column or row. Section six presents a similar procedure that evaluates the change in each industry's output due to an error in the estimation of an individual coefficient. This last routine is easily applied to identify critical coefficients.

Utilization of the simulation approach to perform the above tests would require a matrix inversion for each system item examined. For the row and column tests 2n (n being the number of industry groups) inversions would have to be made. While this number of inversions might not be prohibitively large, the individual coefficient tests, which would require n^2 inversions, are not practical since inversions are among the most costly computer routines. Hence the algebraic solution approach presented in this chapter is the only reasonable alternative.

The next section presents a derivation of a general expression that is utilized in each testing routine.

3. Common Formula

This section presents the derivation of a formula that forms the basis of each of sensitivity routines described in sections four, five, and six. The derivation stems from work done by Frederick V. Waugh [1] on the effect of an equal absolute change in every direct coefficient.

Let A^* be a givan input coefficient matrix, and A another coefficient matrix that differs from A in either a single row, column, or element. Also let B^* stand for $(I-A^*)$ and B be equivalent to (I-A), with I being the identity matrix. Define E to be a matrix of differences between A^* and A; i.e., $E = A^*-A$, which is equivalent to $B-B^*$. In addition let D be the difference between B^{*-1} and B^{-1}, and F be equal to EB^{-1}. In equation form,

$$B^* = (I-A^*) \qquad (5\text{-}1)$$

$$B = (I-A) \qquad (5\text{-}2)$$

$$E = A^*-A = B-B^* \qquad (5\text{-}3)$$

$$D = B^{*-1}-B^{-1} \qquad (5\text{-}4)$$

$$F = EB^{-1} \qquad (5\text{-}5)$$

Now

$$\begin{aligned} D = B^{*-1}-B^{-1} &= (B-E)^{-1}-B^{-1} = \left[(I-EB^{-1})B\right]^{-1}-B^{-1} \\ &= \left[(I-F)B\right]^{-1}-B^{-1} = B^{-1}(I-F)^{-1}-B^{-1} \end{aligned} \qquad (5\text{-}6)$$

Since A^* and A only differ in a single row, column, or element, the matrix E will be sparce. If the differences between A and A^* are not too large, each column of the matrix F will have a sum of absolute value of its elements less than one [2]. The validity of this assumption can easily be checked numerically in any particular case. Then $(I-F)^{-1}$ can be expressed as an infinite series

$$(I-F)^{-1} = (I + F + F^2 + F^3 + \dots) \qquad (5\text{-}7)$$

Substituting (5-7) into (5-6) yields,

$$D = B^{-1}(I + F + F^2 + F^3 + \ldots) - B^{-1}$$
$$= B^{-1}(F + F^2 + F^3 + \ldots) \tag{5-8}$$

In the case of input coefficient sensitivity analysis it is possible to determine a scalar, s [3], for a matrix such as F, such that

$$F^2 = sF \tag{5-9}$$

Hence

$$F^3 = F^2F = sFF = sF^2 = s^2F \tag{5-10}$$

and

$$F^t = s^{t-1}F \tag{5-11}$$

This scalar multiplier, s, can be found in each of the three relevant cases. Its specific value depends on the assumed changes made in the coefficient matrix.

Factoring out F from expression (5-8) for D yields

$$D = B^{-1}F(I + F + F^2 + F^3 + \ldots) \tag{5-12}$$

Substituting (5-11) into (5-12) gives

$$D = B^{-1}F(1 + s + s^2 + s^3 + \ldots) \tag{5-13}$$

But

$$1 + s + s^2 + s^3 + \ldots = \frac{1}{1-s} \text{ if } |s|<1 \tag{5-14}$$

The absolute value of s is assumed to be less than 1 in each of the three cases. This assumption can be checked numerically to insure its validity; hence,

$$D = B^{-1}F\left(\frac{1}{1-s}\right) \tag{5-15}$$

Since $D=B^{*-1}-B^{-1}$, postmultiplying it by the final demand vector, Y, yields a vector, DX, that represents the change in gross output resulting from the difference in direct coefficient matrices defined by E in (5-3).

$$DX = \frac{1}{(1-s)} B^{-1}FY \qquad (5\text{-}16)$$

Expression (5-16) forms the basis for each of the three routines derived below. Since the procedures are relatively complex, they are described in full detail as well as in matrix notation. Column errors are evaluated in the next sector, row errors in section five, and individual coefficient errors in section six.

4. Column Errors

The methodology described to Chapter II, which is designed to forecast technological change in the input coefficients, operates uniformly on matrix columns, The sensitivity routine derived in this section evaluates the effect of errors in this projection procedure on the gross output of each industry.

Define E to be the difference between two matrices, one differing from the other only along a single column. That is,

$$E=A^{*}-A=\begin{bmatrix} 0 & \cdots & d_{1c} & \cdots & 0 \\ 0 & \cdots & d_{2c} & \cdots & 0 \\ \cdot & & \cdot & & \cdot \\ \cdot & & \cdot & & \cdot \\ \cdot & & \cdot & & \cdot \\ 0 & \cdots & d_{nc} & \cdots & 0 \end{bmatrix} \qquad (5\text{-}17)$$

Column c is assumed to contain the error, hence

$$d_{ic} = a_{ic}^{*} - a_{ic} \text{ , all } i \qquad (5\text{-}18)$$

Now following the general procedure outlined in the previous section,

$$F=EB^{-1}=\begin{bmatrix} 0 & \cdots & d_{1c} & \cdots & 0 \\ 0 & \cdots & d_{2c} & \cdots & 0 \\ \cdot & & \cdot & & \\ \cdot & & \cdot\cdot & & \\ \cdot & & \cdot & & \\ 0 & \cdots & d_{nc} & \cdots & 0 \end{bmatrix} \begin{bmatrix} b_{11} & b_{12} & \cdots & b_{1n} \\ b_{21} & b_{22} & \cdots & b_{2n} \\ & & & \\ b_{n1} & b_{n2} & \cdots & b_{nn} \end{bmatrix} \qquad (5\text{-}19)$$

b_{ij} = i, jth element of B^{-1}

hence

$$F=\begin{bmatrix} d_{1c}b_{c1} & \cdots & d_{1c}b_{cn} \\ \cdot & & \cdot \\ \cdot & & \cdot \\ \cdot & & \cdot \\ d_{nc}b_{c1} & \cdots & d_{nc}b_{cn} \end{bmatrix} \qquad (5\text{-}20)$$

$$F^{2}=\begin{bmatrix} d_{1c}b_{c1} & \cdots & d_{1c}b_{cn} \\ \cdot & & \cdot \\ \cdot & & \cdot \\ \cdot & & \cdot \\ d_{nc}b_{c1} & \cdots & d_{nc}b_{cn} \end{bmatrix} \begin{bmatrix} d_{1c}b_{c1} & \cdots & d_{1c}b_{cn} \\ \cdot & & \cdot \\ \cdot & & \cdot \\ \cdot & & \cdot \\ d_{nc}b_{c1} & \cdots & d_{nc}b_{cn} \end{bmatrix} \qquad (5\text{-}21)$$

or

$$F^2 = \begin{bmatrix} \sum_{i=1}^{n} d_{1c}b_{ci}d_{ic}b_{c1} & \cdots & \sum_{i=1}^{n} d_{1c}b_{ci}d_{ic}b_{cn} \\ \vdots & & \vdots \\ \sum_{i=1}^{n} d_{nc}b_{ci}d_{ic}b_{c1} & \cdots & \sum_{i=1}^{n} d_{nc}b_{ci}d_{ic}b_{cn} \end{bmatrix} \tag{5-22}$$

Scalar $\sum_{i=1}^{n} d_{ic}b_{ci}$ can be factored out from the expression for F^2. This is the scalar, s, mentioned in equation (5-9).

$$s = \sum_{i=1}^{n} d_{ic}b_{ci} \tag{5-23}$$

Hence

$$F^2 = \sum_{i=1}^{n} d_{ic}b_{ci}F \tag{5-24}$$

or

$$F^2 = sF \tag{5-25}$$

Using the procedure followed in equations (5-9) through (5-11) the expression for F^t is derived,

$$F^t = s^{t-1}F \tag{5-26}$$

In order to proceed further, the assumption that each column of F has a sum of absolute value of its elements less than 1 must be valid. Although this is not necessarily true in the general case, it probably holds in this case since each column element is a product of a difference between input coefficients and an inverse coefficient.

$$F_{ij} = d_{ic}b_{cj} \tag{5-27}$$

F_{ij} = element of F, row i, column j

As long as the difference between A^* and A is small, d_{ic} will be small enough to insure that the sum of the absolute values of the elements in each of the columns of F is less than one [2]. As mentioned in the previous section the assumption is easily checked using actual data.

It then follows from equations (5-6) through (5-13) that

$$D = B^{-1}F(1 + s + s^2 + s^3 + \ldots) \qquad (5\text{-}28)$$

Since $s=\sum_{i=1}^{n} d_{ic}b_{ci}$ in this case, and the d_{ic} are assumed to be relatively small, the absolute value of s should be less than 1. This assumption can also be checked using actual data. Then according to (5-14) and (5-15).

$$D = B^{-1}F\left(\frac{1}{1-s}\right) \qquad (5\text{-}29)$$

and

$$B^{-1}= \begin{bmatrix} b_{11} & \cdots & b_{1n} \\ \vdots & & \vdots \\ b_{n1} & \cdots & b_{nn} \end{bmatrix} \begin{bmatrix} d_{1c}b_{c1} & \cdots & d_{1c}b_{cn} \\ \vdots & & \vdots \\ d_{nc}b_{c1} & \cdots & d_{nc}b_{cn} \end{bmatrix} \qquad (5\text{-}30)$$

or

$$B^{-1}F= \begin{bmatrix} \sum_{i=1}^{n} b_{1i}d_{ic}b_{c1} & \cdots & \sum_{i=1}^{n} b_{1i}d_{ic}b_{cn} \\ \vdots & & \vdots \\ \sum_{i=1}^{n} b_{ni}d_{ic}b_{c1} & \cdots & \sum_{i=1}^{n} b_{ni}d_{ic}b_{cn} \end{bmatrix} \qquad (5\text{-}31)$$

Postmultiplying matrix D by the final demand vector yields the vector X which represents the errors in each industry's gross output due to the assumed errors in column c. This vector can be expressed as

$$DX = X^{*}-X = \frac{1}{1-s}B^{-1}FY \qquad (5\text{-}32)$$

X^{*} - gross output vector corresponding to A^{*}

X - gross output vector corresponding to A

or

$$B^{-1}FY = \begin{bmatrix} \sum_{i=1}^{n} b_{1c}d_{ic}b_{c1} & \cdots & \sum_{i=1}^{n} b_{1i}d_{ic}b_{cn} \\ \vdots & & \vdots \\ \sum_{i=1}^{n} b_{ni}d_{ic}b_{c1} & \cdots & \sum_{i=1}^{n} b_{ni}d_{ic}b_{cn} \end{bmatrix} \begin{bmatrix} y_1 \\ \vdots \\ y_n \end{bmatrix} \tag{5-33}$$

that is

$$B^{-1}FY = \begin{bmatrix} \sum_{i=1}^{n} b_{1c}d_{ic}b_{c1}Y_1 + & \cdots & + \sum_{i=1}^{n} b_{1i}d_{ic}b_{cn}Y_n \\ \\ \sum_{i=1}^{n} b_{ni}d_{ic}b_{c1}Y_n + & \cdots & + \sum_{i=1}^{n} b_{ni}d_{ic}b_{cn}Y_n \end{bmatrix} \tag{5-34}$$

The error in any sector's gross output is the product of the appropriate element of $D(B^{-1}FY)$ and the quantity $(1/1-s)$. Hence the output estimation error in sector q due to the errors in column c of the input coefficient matrix is

$$X_q^{*} - X_q = \left(\frac{1}{1-s}\right) \left[\sum_{i=1}^{n} b_{qi}d_{ic}b_{c1}Y_1 + \quad \cdots \quad + \sum_{i=1}^{n} b_{qi}d_{ic}b_{cn}Y_n\right] \tag{5-35}$$

This can be simplified by factoring out $\sum_{i=1}^{n} b_{qi}d_{ic}$,

$$X_q^{*} - X_q = \left(\frac{1}{1-s}\right) \sum_{i=1}^{n} b_{qi}d_{ic} \left[b_{c1}Y_1 + \quad \cdots \quad + b_{cn}Y_n\right] \tag{5-36}$$

The expression in the brackets is equal to Xc, hence (5-36) simplifies to,

$$X_q^{*} - X_q = \left(\frac{1}{1-s}\right) X_c \sum_{i=1}^{n} b_{qi}d_{ic} \tag{5-37}$$

or in percentage terms,

$$\frac{X_q^{*} - X_q}{X_q} = \frac{X_c}{X_q} \; \frac{\sum_{i=1}^{n} b_{qi}d_{ic}}{1-s} \tag{5-38}$$

Since s is equal to $b_{ic}d_{ic}$ in this case, (5-38) in final form becomes

$$\frac{X_q^{*} - X_c}{X_c} = \frac{X_c}{X_q} \; \frac{\sum_{i=1}^{n} b_{qi}d_{ic}}{1 - \sum_{i=1}^{n} b_{ic}d_{ic}} \tag{5-39}$$

Given any column estimation error vector d(=d_{ic}, all i), the resulting output error in any sector can be calculated from (5-39). This procedure can be applied to each column individually to identify columns whose accurate estimation is critical to the output level of important sectors.

The solution of (5-39) requires only one matrix inversion as compared to the simulation approach which would require n inversions. Hence expression (5-39) provides an efficient testing procedure for evaluating the sensitivity of gross output forecasts to errors in the projection of technological change. The next section describes a similar, but slightly more complex result, for the trading pattern problem.

5. Row Errors

The regional trading pattern forecasting model described in Chapter III varies individual rows of the input coefficient matrix according to the value of a projected location quotient. Hence, a routine to test the sensitivity of gross output to errors in the estimation of individual rows is needed to identify sectors that should be monitored closely. This routine, which is described below, is similar in its form to the preceding analysis done for column errors. Hence the exposition will be somewhat more sketchy.

The basic equations (4-1) through (4-5) still hold. Since the matrices A* and A differ along a single row, matrix E is a transposed version of (4-11). Letting r be the relevant row, matrix E has the form,

$$E = \begin{bmatrix} 0 & \cdots & 0 \\ \vdots & & \vdots \\ d_{r1} & \cdots & d_{rn} \\ \vdots & & \vdots \\ 0 & \cdots & 0 \end{bmatrix} \qquad (5\text{-}40)$$

The rest of the derivation, except for the last few steps follows directly and is analogous to the analysis of the previous section (equations 4-19 through 4-35). Therefore the relevant equations are presented without intervening discussion.

$$F = EB^{-1} = \begin{bmatrix} 0 & \cdots & 0 \\ \vdots & & \vdots \\ d_{r1} & & d_{r1} \\ \vdots & & \vdots \\ 0 & & 0 \end{bmatrix} \begin{bmatrix} b_{11} & \cdots & b_{1n} \\ \vdots & & \vdots \\ b_{n1} & & b_{nn} \end{bmatrix} \qquad (5\text{-}41)$$

$$F=EB^{-1}\begin{bmatrix} 0 & & 0 \\ \vdots & & \vdots \\ \sum_{i=1}^{n} d_{ri}b_{i1} & \cdots & \sum_{i=1}^{n} d_{ri}b_{in} \\ \vdots & & \vdots \\ 0 & \cdots & 0 \end{bmatrix} \tag{5-42}$$

$$F^2=\begin{bmatrix} 0 & \cdots & 0 \\ \vdots & & \vdots \\ \sum_{i=1}^{n} d_{ri}b_{i1} & \cdots & \sum_{i=1}^{n} d_{ri}b_{in} \\ \vdots & & \vdots \\ 0 & \cdots & 0 \end{bmatrix}\begin{bmatrix} 0 & \cdots & 0 \\ \vdots & & \vdots \\ \sum_{i=1}^{n} d_{ri}b_{i1} & \cdots & \sum_{i=1}^{n} d_{ri}b_{in} \\ \vdots & & \vdots \\ 0 & \cdots & 0 \end{bmatrix} \tag{5-43}$$

$$F^2=\begin{bmatrix} 0 & \cdots & 0 \\ \vdots & & \vdots \\ \left(\sum_{i=1}^{n} d_{ri}b_{ir} \quad \sum_{i=1}^{n} d_{ri}b_{i1}\right) & \cdots & \left(\sum_{i=1}^{n} d_{ri}b_{ir} \quad \sum_{i=1}^{n} d_{ri}b_{in}\right) \\ \vdots & & \vdots \\ 0 & \cdots & 0 \end{bmatrix} \tag{5-44}$$

$$F^2=\sum_{i=1}^{n} d_{ri}b_{ir}\begin{bmatrix} 0 & \cdots & 0 \\ \vdots & & \vdots \\ \sum_{i=1}^{n} d_{ri}b_{i1} & \cdots & \sum_{i=1}^{n} d_{ri}b_{in} \\ \vdots & & \vdots \\ 0 & \cdots & 0 \end{bmatrix} \tag{5-45}$$

$$s = \sum_{i=1}^{n} d_{ri}b_{ir} \tag{5-46}$$

$$F^2 = sF \tag{5-47}$$

$$F^t = s^{t-1}F \tag{5-48}$$

$$D = \left(\frac{1}{1-s}\right) B^{-1}F \tag{5-49}$$

$$B^{-1}F = \begin{bmatrix} b_{11} & \cdots & b_{1n} \\ \cdot & & \cdot \\ \cdot & & \cdot \\ b_{n1} & \cdots & b_{nn} \end{bmatrix} \begin{bmatrix} 0 & \cdots & 0 \\ \cdot & & \cdot \\ \cdot & & \cdot \\ \sum_{i=1}^{n} d_{ri}b_{i1} & \cdots & \sum_{i=1}^{n} d_{ri}b_{in} \\ \cdot & & \cdot \\ \cdot & & \cdot \\ 0 & & 0 \end{bmatrix} \tag{5-50}$$

$$B^{-1}F = \begin{bmatrix} b_{1r}\sum_{i=1}^{n} d_{ri}b_{i1} & \cdots & b_{1r}\sum_{i=1}^{n} d_{ri}b_{in} \\ \cdot & & \cdot \\ \cdot & & \cdot \\ b_{nr}\sum_{i=1}^{n} d_{ri}b_{i1} & \cdots & b_{nr}\sum_{i=1}^{n} d_{ri}b_{in} \end{bmatrix} \tag{5-51}$$

$$B^{-1}FY = \begin{bmatrix} b_{1r}\sum_{i=1}^{n} d_{ri}b_{i1} & \cdots & b_{1r}\sum_{i=1}^{n} d_{ri}b_{in} \\ \cdot & & \cdot \\ \cdot & & \cdot \\ b_{nr}\sum_{i=1}^{n} d_{ri}b_{i1} & \cdots & b_{nr}\sum_{i=1}^{n} d_{ri}b_{in} \end{bmatrix} \begin{bmatrix} Y_1 \\ \cdot \\ \cdot \\ \cdot \\ Y_n \end{bmatrix} \tag{5-52}$$

$$B^{-1}FY = \begin{bmatrix} b_{ir}Y_1\sum_{i=1}^{n} d_{ri}b_{i1} + \cdots + b_{1n}Y_n\sum_{i=1}^{n} d_{ri}b_{in} \\ \cdot \\ \cdot \\ \cdot \\ b_{nr}Y_n\sum_{i=1}^{n} d_{ri}b_{i1} + \cdots + b_{nr}Y_n\sum_{i=1}^{n} d_{ri}b_{in} \end{bmatrix} \tag{5-53}$$

$$X_q^* - X_q = \left(\frac{1}{1-s}\right)\left[b_{qr}Y_1\sum_{i=1}^{n} d_{ri}b_{i1} + \cdots + b_{qr}Y_n\sum_{i=1}^{n} d_{ri}b_{in}\right] \tag{5-54}$$

At this point the derivation diverges. In the previous derivation, it was possible to simplify the expression in the brackets (equations 5-36 and 5-37). The bracketed expression in (5-50) can only be simplified to the extent of factoring out b_{qr}. The finalized form is

$$\frac{X_q^* - X_q}{X_q} = \frac{b_{qr}}{X_r}\left[\frac{\sum_{t=1}^{n} Y_t \sum_{i=1}^{n} d_{ri}b_{it}}{1 - \sum_{i=1}^{n} d_{ri}b_{ir}}\right] \tag{5-55}$$

Despite the greater complexity of (5-55) than (5-39) only one matrix inversion is required to test error in the estimation on any row. Hence (5-55) is computionally efficient compared to the simulation approach which required n inversions.

6. Individual Coefficient Errors

The procedure used to evaluate the effect of errors in the estimation of individual coefficients is somewhat different from the previous two procedures. The initial derivation of a formula relating gross output variation to coefficient errors is almost identical to the preceding derivations. But given this formula an analogous testing routine to those used for column and row errors requires more computation. In the previous problems, n gross output variation calculations had to be made for each row or column change. Hence to examine all rows or columns of the coefficient matrix n^2 calculations must be done. In the case of individual coefficient analysis, n gross output variation calculations are necessary for each coefficient error. Since there are n^2 coefficients, n^3 calculations must be made. The problem is not as serious as it may seem since the calculations for each coefficient are much less complex than those for rows or columns. Because of its similarity to the analysis in sections 4 and 5, the derivation is presented directly without intervening discussion.

Let the estimation error be in row r and column c, then

$$E = \begin{bmatrix} 0 & \cdots & 0 & \cdots & 0 \\ \vdots & & \vdots & & \vdots \\ 0 & \cdots & d_{rc} & \cdots & 0 \\ \vdots & & \vdots & & \vdots \\ 0 & \cdots & 0 & \cdots & 0 \end{bmatrix} \tag{5-56}$$

$$F = EB^{-1} = \begin{bmatrix} 0 & \cdots & 0 & \cdots & 0 \\ \vdots & & \vdots & & \vdots \\ 0 & \cdots & d_{rc} & \cdots & 0 \\ \vdots & & \vdots & & \vdots \\ 0 & & 0 & & 0 \end{bmatrix} \begin{bmatrix} b_{11} & \cdots & b_{1n} \\ \vdots & & \vdots \\ b_{n1} & \cdots & b_{nn} \end{bmatrix} \tag{5-57}$$

$$F = \begin{bmatrix} 0 & \cdots & 0 \\ \vdots & & \vdots \\ d_{rc}b_{c1} & \cdots & d_{rc}b_{cn} \\ \vdots & & \vdots \\ 0 & & 0 \end{bmatrix} \quad (5\text{-}58)$$

$$F^2 = \begin{bmatrix} 0 & \cdots & 0 \\ \vdots & & \vdots \\ d_{rc}b_{c1} & \cdots & d_{rc}b_{cn} \\ \vdots & & \vdots \\ 0 & \cdots & 0 \end{bmatrix} \begin{bmatrix} 0 & \cdots & 0 \\ \vdots & & \vdots \\ d_{rc}b_{c1} & \cdots & d_{rc}b_{cn} \\ \vdots & & \vdots \\ 0 & \cdots & 0 \end{bmatrix} \quad (5\text{-}59)$$

$$F^2 = \begin{bmatrix} 0 & \cdots & 0 \\ \vdots & & \vdots \\ d^2_{rc}b_{cr}b_{c1} & \cdots & d^2_{rc}b_{cr}b_{cn} \\ \vdots & & \vdots \\ 0 & \cdots & 0 \end{bmatrix} \quad (5\text{-}60)$$

$$F^2 = d_{rc}b_{cr} \begin{bmatrix} 0 & \cdots & 0 \\ \vdots & & \vdots \\ d_{rc}b_{c1} & \cdots & d_{rc}b_{cn} \\ \vdots & & \vdots \\ 0 & & 0 \end{bmatrix} \quad (5\text{-}61)$$

$$s = d_{rc}b_{cr} \quad (5\text{-}62)$$

$$F^2 = sF \quad (5\text{-}63)$$

$$F^t = s^{t-1}F \tag{5-64}$$

$$D = \frac{1}{(1-s)}B^{-1}F \tag{5-65}$$

$$B^{-1}F = \begin{bmatrix} b_{11} & \cdots & b_{1n} \\ \vdots & & \vdots \\ b_{n1} & \cdots & b_{nn} \end{bmatrix} \begin{bmatrix} 0 & & 0 \\ \vdots & & \vdots \\ d_{rc}b_{c1} & \cdots & d_{rc}b_{cn} \\ \vdots & & \vdots \\ 0 & & 0 \end{bmatrix} \tag{5-66}$$

$$B^{-1}F = \begin{bmatrix} d_{rc}b_{c1}b_{1r} & \cdots & d_{rc}b_{cn}b_{1r} \\ \vdots & & \vdots \\ d_{rc}b_{c1}b_{nr} & \cdots & d_{rc}b_{cn}b_{nr} \end{bmatrix} \tag{5-67}$$

$$B^{-1}FY = \begin{bmatrix} d_{rc}b_{c1}b_{1r} & \cdots & d_{rc}b_{cn}b_{1r} \\ \vdots & & \vdots \\ d_{rc}b_{c1}b_{nr} & \cdots & d_{rc}b_{cn}b_{nr} \end{bmatrix} \begin{bmatrix} Y_1 \\ \\ Y_n \end{bmatrix} \tag{5-68}$$

$$B^{-1}Fy \quad \begin{bmatrix} d_{rc}b_{c1}b_{1r}Y_1 & + & \cdots & + & d_{rc}b_{cn}b_{1r}Y_n \\ \vdots & & & & \vdots \\ d_{rc}b_{c1}b_{nr}Y_1 & + & \cdots & + & d_{rc}b_{cn}b_{nr}Y_n \end{bmatrix} \tag{5-69}$$

$$X_q^* - X_q = \left(\frac{1}{1-s}\right)\left[d_{rc}b_{c1}b_{qr}Y_1 + \cdots + d_{rc}b_{cn}b_{qr}Y_n\right] \tag{5-70}$$

$$X_q^* - X_q = \left(\frac{1}{1-s}\right)d_{rc}b_{qr}\left[b_{c1}Y_1 + \cdots + b_{cn}Y_n\right] \tag{5-71}$$

$$X_q^* - X_q = \left(\frac{1}{1-s}\right) d_{rc} b_{qr} X_c \qquad (5\text{-}72)$$

$$\frac{X_q^* - X_q}{X_q} = \frac{X_c}{X_q} \left[\frac{d_{rc} b_{qr}}{1-d_{rc} b_{cr}}\right] \qquad (5\text{-}73)$$

For each coefficient tested, equation (5-73) must be calculated for each sector. Although this requires n^3 computations, each is relatively simple. Hence the increased number of calculations does not require n time as much computation time as the row or column analysis.

As mentioned at the beginning of this chapter time limitations have made it impossible to implement the above sensitivity testing procedures. Despite this, they are an important element of the systems approach as envisioned in the RIOFS since they provide for an optional allocation of limited resources.

REFERENCES

1 Waugh, Frederick, V. and Dwyer, Paul S., "On Errors in Matrix Inversion," Journal of the American Statistical Association (July, 1953).

2 Carl Christ, "A Review of Input-Output Analysis, "Input-Output: An Appraisal, A Report of the National Bureau of Economic Research, Princeton, N.J.: Princeton University Press, 1955, p. 152.

3 Ibid., p. 153.

APPENDIX A

DATA

TABLE A-1
REGIONAL EMPLOYMENT

	1	2	3	4	5
1948	0	0	14992	8216	341496
1951	0	0	17766	7672	297168
1953	0	0	19615	8399	239500
1956	0	0	22389	7322	170769
1959	0	0	25162	5654	123409
1962	0	0	29535	5134	92425
1964	0	0	33813	4941	86179
1965	0	0	37792	7607	83947
1967	0	0	36934	7752	61075
1969	0	0	40601	4536	74897

	6	7	8	9	10
1948	18174	20582	613323	20357	394259
1951	15633	21611	749584	32125	404914
1953	15383	20016	714687	38876	400269
1956	11105	23802	769153	49003	399825
1959	8866	22142	711772	59130	461103
1962	8629	20482	710528	68014	452472
1964	8357	19141	839471	21486	427311
1965	7075	19328	798777	21625	427848
1967	7236	18007	824008	32002	427214
1969	6982	22228	875001	42515	426442

	11	12	13	14	15
1948	45781	697641	785962	139906	108459
1951	38853	609558	825553	149497	120399
1953	37511	531835	810095	120409	118974
1956	34028	433277	765729	120507	117814
1959	30511	334303	710471	98475	109796
1962	24980	305172	722481	92533	109923
1964	25454	279984	680685	93981	112917
1965	21156	297666	714926	99985	124489
1967	22654	285422	708988	99985	124489
1969	22614	290539	651146	95358	125067

TABLE A-1 cont.

	16	17	18	19	20
1948	208112	315312	338304	76546	92145
1951	224564	333727	320366	65132	93205
1953	219434	335866	325409	62877	99559
1956	227019	349785	316955	55451	95743
1959	223029	419825	293627	35956	140900
1962	226276	364975	329859	31293	154987
1964	221103	381076	303534	29026	162171
1965	223579	391120	311242	27493	175144
1967	238818	413729	323203	26298	192060
1969	241629	420221	335163	26073	199741

	21	22	23	24	25
1948	255075	211429	521405	387007	547492
1951	252078	223880	510679	419445	562629
1953	243690	211781	552951	414668	612586
1956	239469	210508	543069	392356	554346
1959	222880	197673	495582	379882	473275
1962	220535	196473	475337	372467	501333
1964	198967	192785	442976	367085	497780
1965	203757	198793	487690	378130	522228
1967	201954	205448	499270	414108	602440
1969	191431	201970	472336	422047	576078

	26	27	28	29	30
1948	470241	287471	167509	289073	210034
1951	461269	319706	171286	310221	200057
1953	546517	447126	185793	297371	203405
1956	504928	399170	176156	283532	203159
1959	525452	364349	174700	206703	193981
1962	607582	358683	193732	210157	186121
1964	601373	370230	174380	202126	188746
1965	606175	386375	198410	252379	181897
1967	734928	445057	210245	225303	189354
1969	735109	427940	218807	214243	187983

TABLE A-1 cont.

	31	32	33	34	35
1948	978603	3289611	789117	1239952	0
1951	73563	3398376	822837	1248438	0
1953	769969	3470548	878756	1292215	0
1956	809360	3429050	942998	1372817	0
1959	835862	3524137	1002901	2119599	0
1962	704468	3654081	1068551	2424600	0
1964	861810	3792733	1105405	2631463	0
1965	940268	3927735	1129926	2754216	0
1967	1010517	4224338	1195230	3150751	0
1969	1004003	4442790	1306277	3478445	0

	36	37
1948	0	0
1951	0	0
1953	0	0
1956	0	0
1959	0	0
1962	0	0
1964	0	0
1965	0	0
1967	0	0
1969	0	0

TABLE A-2
NATIONAL EMPLOYMENT

	1	2	3	4	5
1948	0	0	50474	95665	538384
1951	0	0	65671	101335	451553
1953	0	0	75802	106351	355157
1956	0	0	90999	109867	260719
1959	0	0	106196	85191	194914
1962	0	0	126424	79482	151355
1964	0	0	141093	71767	140346
1965	0	0	148428	76507	135432
1967	0	0	169275	80054	123635
1969	0	0	182873	78784	124970

	6	7	8	9	10
1948	226950	85645	2007291	129482	1375453
1951	259799	89900	2345535	157542	1293938
1953	293568	96610	2367098	165251	1379627
1956	328783	116035	2501050	187599	1390355
1959	272531	113181	2483283	227215	1607370
1962	254760	104996	2410014	234980	1578305
1964	223147	106001	2571423	242942	1538558
1965	232192	107961	2799410	213159	1545154
1967	223898	108500	2933745	302089	1586182
1969	223796	109332	3162500	409204	1588882

	11	12	13	14	15
1948	101705	1336283	1182356	789262	352548
1951	89434	1273334	1269820	826956	377590
1953	87608	1184845	1267446	735750	372037
1956	81985	1057579	1234868	688550	374438
1959	80744	892769	1189431	578771	358438
1962	74557	874577	1252443	526622	364166
1964	75243	857328	1279624	565368	380044
1965	71929	883120	1333663	572562	407298
1967	68304	925159	1290846	570820	439118
1969	70547	965579	140785	586887	455456

TABLE A-2 cont.

	16	17	18	19	20
1948	458978	714762	700140	247826	256120
1951	508545	762327	720822	222606	256407
1953	519889	781217	775839	226429	271516
1956	558085	824974	794265	210648	258587
1959	560078	857556	732656	176605	359294
1962	587882	904208	772169	161367	387997
1964	583678	925385	748293	150581	417365
1965	595988	955345	775530	147001	448519
1967	641409	1029091	867201	138512	506611
1969	662625	1065503	880050	136804	560323

	21	22	23	24	25
1948	416666	498770	1223188	993305	1537849
1951	400950	533253	1243298	1083326	1608621
1953	391730	520025	1314901	1110781	1751921
1956	380377	530770	1325983	1088543	1693454
1959	350006	555967	1203353	1087672	1401761
1962	352919	548058	1168110	1062096	1445558
1964	325985	563247	1151851	1080182	1527567
1965	334502	577591	1246441	1132134	1635558
1967	337577	586422	1303067	1271085	1943140
1969	329853	602624	1280964	1291834	1962236

	26	27	28	29	30
1948	868629	1273672	249110	462123	523574
1951	916193	1471754	259748	495772	549685
1953	1113660	1942771	296934	491013	572313
1956	1060676	1786136	296397	488358	583162
1959	1183155	1646335	195424	368761	565213
1962	1405382	1541618	341796	369071	578767
1964	1465767	1627597	310537	369608	586047
1965	1540040	1708842	322894	389453	591586
1967	1905171	1953384	384954	413256	611232
1969	1930967	2020619	409989	429548	616808

TABLE A-2 cont.

	31	32	33	34	35
1948	2591564	9488147	1749395	3493848	0
1951	2106231	10002827	1894633	3490540	0
1953	2170606	10271512	2055096	3678763	0
1956	2228268	10730853	2284956	3960454	0
1959	2354850	10836151	2505432	5795905	0
1962	2431865	11284721	2723335	6615266	0
1964	2470762	11900935	2914936	7288254	0
1965	2627123	12398667	3814243	7709154	0
1967	2914235	13535368	3201271	8938459	0
1969	3086536	14627141	3509168	9974335	0

	36	37
1948	0	0
1951	0	0
1953	0	0
1956	0	0
1959	0	0
1962	0	0
1964	0	0
1965	0	0
1967	0	0
1969	0	0

TABLE A-3

HISTORICAL INDEPENDENT VARIABLES

REGIONAL

YEAR	POPULATION (MILLIONS)	PERSONAL INCOME (BILLIONS,$58)	TIME
1948	47,312	89,769	1
1951	48,660	99,920	4
1953	49,988	107,895	6
1956	51,888	120,210	9
1959	54,250	127,956	12
1962	56,412	141.020	15
1964	58.001	154,068	17
1965	58,711	162,913	18
1967	59,638	180,288	20
1969	60,206	196,995	22

NATIONAL

YEAR	POPULATION (MILLIONS)	PERSONAL INCOME (BILLIONS,$58)	TIME
1948	146,045	253,791	1
1951	154,060	285,812	4
1953	159,035	311,286	6
1956	168,043	348,607	9
1959	177,131	376,071	12
1962	185,890	419,628	15
1964	191,371	158,674	17
1965	193,815	492,591	18
1967	197,859	546,756	20
1969	201,306	602,809	22

TABLE A-4

PROJECTED INDEPENDENT VARIABLES

REGIONAL

YEAR	POPULATION (MILLIONS)	REGIONAL INCOME (BILLIONS,$58)	TIME
1980	68,150	312,540	33
2000	81,291	667,410	53
2020	100,583	1,415,220	73

NATIONAL

YEAR	POPULATION (MILLIONS)	REGIONAL INCOME (BILLIONS,$58)	TIME
1980	234,193	963,000	33
2000	306,757	2,169,648	53
2020	397,562	4,934,146	73

TABLE A-5
REGIONAL FINAL
DEMAND
(Billions of 1958 Dollars)

Sector	Y63	Y80	Y00	Y20
1	725	1437	3094	6559
2	2105	4174	8986	19052
3	115	228	491	1041
4	103	205	442	937
5	173	342	737	1562
6	10	19	41	87
7	38	76	164	347
8	23305	46208	99477	210906
9	1871	3710	7988	16935
10	17520	34739	74785	158557
11	1812	3593	7734	16397
12	723	1433	3085	6541
13	5190	10291	22154	46970
14	169	335	720	1527
15	1589	3152	6785	14384
16	684	1357	2922	6194
17	1323	2623	5647	11972
18	3298	6539	14076	29844
19	3342	6626	14265	30243
20	838	1661	3576	7583
21	1007	1996	4297	9109
22	297	589	1269	2689
23	426	844	1817	3852
24	901	1787	3846	8155
25	6565	13017	28022	59411
26	5842	11583	24937	52869
27	12370	24528	52803	111951
28	1110	2201	4739	10046
29	1380	2737	5892	12493
30	3687	7310	15738	33366
31	7570	15009	32310	68503
32	29294	58085	125043	265112
33	34857	69113	148785	315448
34	20548	40742	87708	185954
35	422	836	1800	3817
36	284	563	1211	2568
37	0	0	0	0

TABLE A-6
NATIONAL FINAL
DEMAND
(Billions of 1958 Dollars)

Sector	YN63	YN80	YN00	YN20
1	2196	4355	9374	19875
2	6379	12649	27230	57733
3	349	691	1488	3155
4	314	622	1339	2839
5	523	1037	2232	4732
6	29	58	124	263
7	116	230	496	1052
8	70621	140026	301444	639110
9	5671	11244	24209	51318
10	53092	105270	226622	480476
11	5490	10886	23436	49688
12	2190	4343	9350	19823
13	15728	31185	67134	142334
14	511	1014	2182	4627
15	4816	9550	20559	43589
16	2074	4113	8854	18771
17	4009	7949	17112	36280
18	9993	19814	42656	90438
19	10127	20079	43226	91647
20	2539	5034	10838	22977
21	3050	6048	13020	27605
22	901	1786	3844	8150
23	1290	2557	5506	11673
24	2731	5414	11656	24713
25	19893	39444	84915	180034
26	17703	35101	75566	160211
27	37486	74327	160010	339246
28	3364	6670	14359	30444
29	4183	8294	17856	37857
30	11173	22153	47690	101111
31	22938	45481	97910	207586
32	88771	176014	378919	803370
33	105626	209434	450864	955904
34	62266	123460	265782	563500
35	1278	2534	5456	11568
36	860	1705	3670	7782
37	0	0	0	0

TABLE A-7
OUTPUT VECTORS
SCENARIO C
(Billions of 1958 Dollars)

XC1963	XC1980	XC2000	XC2020	
7172.13	7111.51	6078.40	5004.34	1
7548.89	7487.55	6696.04	5753.24	2
988.47	982.41	831.96	712.35	3
222.69	267.80	312.67	359.89	4
638.09	630.40	589.53	561.24	5
407.86	227.88	38.86	16.52	6
434.17	489.64	314.03	213.79	7
28221.61	27989.45	26999.99	25936.93	8
1942.83	1946.41	1942.52	1943.71	9
23749.74	23644.29	22232.66	21021.46	10
2434.21	2423.44	2236.77	2098.25	11
3511.05	3142.47	2671.02	2109.06	12
6940.84	6912.17	6455.63	6088.94	13
1504.98	1604.87	1377.62	1160.26	14
1894.19	1895.45	1861.57	1813.74	15
4610.25	4557.10	3388.69	2429.50	16
4427.81	4386.50	3566.05	3059.30	17
9665.51	9550.33	8441.40	7590.29	18
5486.36	4978.03	4091.17	3697.79	19
2750.22	2740.34	2566.69	2436.83	20
1433.29	1426.27	1298.39	1206.47	21
3209.58	3154.86	2015.23	1382.10	22
11445.31	11428.57	11222.26	11302.31	23
7067.57	7002.98	6055.29	5444.84	24
11522.05	11544.55	11915.15	12326.65	25
10032.99	10026.95	9973.30	10029.54	26
18722.90	18849.56	19371.24	19628.27	27
2094.95	2088.56	1965.72	1890.89	28
2109.85	2101.00	1947.65	1753.13	29
8253.83	7938.68	6600.15	5724.88	30
16623.35	16464.56	14160.76	12773.70	31
37600.25	37647.81	36207.97	34904.93	32
48853.82	48608.11	45159.82	42302.35	33
34030.25	33834.24	28576.57	26344.43	34
1985.88	1960.21	1577.11	1305.88	35
2262.98	2197.84	1650.18	1272.58	36
2355.31	2327.75	1878.45	1579.12	37

TABLE A-8
OUTPUT VECTORS
SCENARIO D
(Billions of 1958 Dollars)

XD1963	XD1980	XD2000	XD2020	
7172.13	14220.82	30614.28	64907.20	1
7548.89	14967.85	32222.44	68316.78	2
988.47	1959.93	4219.30	8945.59	3
222.69	441.54	950.53	2015.28	4
638.09	1265.19	2723.67	5774.63	5
407.86	808.70	1740.96	3691.11	6
434.17	860.87	1853.25	3929.19	7
28221.61	55957.47	120464.01	255403.12	8
1942.83	3852.22	8292.97	17582.44	9
23749.74	47090.71	101375.84	214933.14	10
2434.21	4826.53	10390.45	22029.42	11
3511.05	6961.68	14986.94	31774.74	12
6940.84	13762.21	29626.98	62813.97	13
1504.98	2984.06	6424.01	13619.95	14
1894.19	3755.77	8085.34	17142.23	15
4610.25	9141.14	19678.84	41722.32	16
4427.81	8779.41	18900.12	40071.30	17
9665.51	19164.66	41257.25	87472.02	18
5486.36	10878.29	23418.53	49651.07	19
2750.22	5453.10	11739.30	24889.22	20
1433.29	2841.91	6117.99	12971.14	21
3209.58	6363.92	13700.10	29046.42	22
11445.31	22693.64	48854.36	103579.12	23
7067.57	14013.50	30167.96	63960.94	24
11522.05	22845.79	49181.91	104273.59	25
10032.99	19893.29	42825.84	90797.69	26
18722.90	37123.54	79918.74	169440.62	27
2094.95	4153.85	8942.31	18959.13	28
2109.85	4183.38	9005.88	19093.92	29
8253.83	16365.59	35231.48	74696.43	30
16623.35	32960.58	70956.81	150439.88	31
37600.25	74553.33	160496.76	340279.01	32
48853.82	96866.79	208532.68	442122.91	33
34030.25	67474.77	145258.19	307970.79	34
1985.88	3937.57	8476.72	17972.01	35
2262.98	4487.00	9659.52	20479.74	36
2355.31	4670.08	10053.64	21315.34	37

TABLE A-9
OUTPUT VECTORS
SCENARIO A
(Billions of 1958 Dollars)

XA1963	XA1980	XA2000	XA2020	
7172.13	14100.62	25945.65	45288.88	1
7548.89	14846.22	28582.07	52066.34	2
988.47	1947.91	3551.23	6446.74	3
222.69	531.00	1334.62	3257.02	4
638.09	1249.95	2516.42	5079.14	5
407.86	451.83	165.87	149.51	6
434.17	970.86	1340.45	1934.81	7
28221.61	55497.15	115249.53	234726.95	8
1942.83	3859.33	8291.65	17590.42	9
23749.74	46881.63	94900.15	190242.38	10
2434.21	4805.16	9547.68	18988.98	11
3511.05	6230.86	11401.25	19086.78	12
6940.84	13705.36	27555.87	55104.43	13
1504.98	3182.11	5880.37	10500.28	14
1894.19	3758.28	7946.11	16414.17	15
4610.25	9035.76	14464.64	21986.74	16
4427.81	8697.51	15221.71	27686.37	17
9665.51	18936.28	36032.14	68691.46	18
5486.36	9870.37	17463.15	33464.71	19
2750.22	5433.51	10955.94	22053.11	20
1433.29	2828.00	5542.16	10918.43	21
3209.58	6255.42	8602.03	12507.89	22
11445.31	22660.43	47902.24	102284.96	23
7067.57	13885.42	25847.03	49275.30	24
11522.05	22890.40	50859.84	111555.08	25
10032.99	19881.32	42571.06	90766.47	26
18722.90	37374.68	82686.20	177634.18	27
2094.95	4141.18	8390.67	17112.44	28
2109.85	4165.84	8313.56	15865.69	29
8253.83	15740.72	28172.78	51809.70	30
16623.35	32645.75	60445.23	115600.90	31
37600.25	74647.63	154553.80	315886.62	32
48853.82	96379.58	192764.81	382832.58	33
34030.25	67086.14	121979.18	238414.81	34
1985.88	3886.67	6731.88	11818.13	35
2262.98	4357.86	7043.80	11516.71	36
2355.31	4615.43	8018.18	14290.91	37

TABLE A-10
OUTPUT VECTORS
SCENARIO N
(Billions of 1958 Dollars)

XN1963	XN1980	XN2000	XN2020	
7351.32	14487.58	26639.86	46509.24	1
7805.39	15412.55	29401.88	53339.71	2
1157.29	2274.74	4001.87	6810.16	3
777.76	1544.67	3495.78	7940.79	4
657.77	1299.40	2639.18	5362.85	5
2917.76	5473.91	2896.04	2823.16	6
840.07	1639.62	2582.52	4207.83	7
28930.40	57168.21	115952.03	235937.87	8
2089.38	4144.55	8998.39	19231.78	9
24285.56	48034.85	97664.25	196398.58	10
2436.55	4811.07	9558.73	19014.74	11
4832.54	9460.66	15599.61	25253.97	12
6977.29	13784.38	27665.01	55284.32	13
2896.75	5647.47	8633.48	13368.89	14
1945.42	3848.54	7963.29	16445.62	15
4730.93	9288.66	16557.90	29836.27	16
4473.04	8788.49	15883.16	28912.45	17
10060.49	19804.20	37272.63	70996.98	18
7199.53	14121.29	25227.49	47991.01	19
2814.98	5560.30	11184.98	22582.61	20
1437.10	2835.49	5563.99	10961.78	21
3320.57	6484.19	10306.64	16918.70	22
11743.70	23229.20	48650.20	104052.26	23
7259.06	14267.64	26260.30	50111.64	24
11687.67	23201.21	51425.90	112850.88	25
10133.91	20078.47	42885.91	91445.40	26
19201.27	38097.38	83207.09	178948.05	27
2116.74	4182.87	8495.00	17323.36	28
2126.30	4199.59	8406.22	16842.94	29
8684.68	17085.86	31938.54	60954.64	30
17104.14	33636.87	62002.77	118566.52	31
38585.64	76280.01	155684.89	318012.82	32
49882.57	98464.24	195278.90	390852.40	33
34480.64	68000.31	132548.22	261036.34	34
2029.27	3979.65	6934.42	12203.06	35
2345.24	4586.58	7536.63	12472.09	36
2440.07	4785.49	8359.62	14939.65	37

TABLE A-11
OUTPUT VECTORS
SCENARIO B
(Billions of 1958 Dollars)

XB1963	XB1980	XB2000	XB2020	
7351.32	14576.10	31379.11	66528.78	1
7805.39	15476.44	33317.34	70638.13	2
1157.29	2294.66	4939.89	10473.36	3
777.76	1542.14	3319.89	7038.69	4
657.77	1304.22	2807.69	5952.76	5
2917.76	5785.30	12454.46	26405.46	6
840.07	1665.68	3585.85	7602.57	7
28930.40	57362.86	123489.50	261817.65	8
2089.38	4142.79	8918.51	18908.68	9
24285.56	48153.13	103662.98	219782.24	10
2436.55	4831.16	10400.42	22050.57	11
4832.54	9581.90	20627.69	43734.03	12
6977.29	13834.48	29782.56	63143.83	13
2896.75	5743.65	12364.80	26215.37	14
1945.42	3857.36	8304.04	17605.89	15
4730.93	9380.43	20193.98	42814.49	16
4473.04	8869.09	19093.18	40480.63	17
10060.49	19947.83	42943.25	91046.61	18
7199.53	14275.14	30731.21	65155.11	19
2814.98	5581.50	12015.73	25475.28	20
1437.10	2849.47	6134.28	13005.66	21
3320.57	6583.98	14173.84	30050.84	22
11743.70	23285.27	50128.01	106279.47	23
7259.06	14393.17	30985.30	65693.83	24
11687.67	23174.18	49888.87	105772.44	25
10133.91	20093.41	43256.64	91711.05	26
19201.27	38072.06	81960.69	173769.87	27
2116.74	4197.04	9035.30	19156.29	28
2126.30	4216.00	9076.12	19242.84	29
8684.68	17219.89	37070.59	78595.63	30
17104.14	33913.88	73009.05	154790.97	31
38585.64	76507.16	164702.92	349196.76	32
49882.57	98906.57	212923.86	451432.92	33
34480.64	68367.80	147180.69	312046.80	34
2029.27	4023.61	8661.94	18364.71	35
2345.24	4650.12	10010.67	21224.25	36
2440.07	4838.14	10415.45	22082.44	37

TABLE A-12
NATIONAL INPUT COEFFICIENT MATRIX
1963

A63

	1	2	3	4
1	0.1780000	0.0705390	0.0653000	0.0000000
2	0.3128540	0.0281900	0.1409560	0.0000000
3	0.0209930	0.0486480	0.0309890	0.0000000
4	0.0000000	0.0000000	0.0000000	0.1170450
5	0.0001920	0.0000180	0.0000000	0.0021390
6	0.0000000	0.0000000	0.0000000	0.0000000
7	0.0000770	0.0056590	0.0000000	0.0052970
8	0.0100900	0.0180820	0.0000000	0.0040130
9	0.0000000	0.0000000	0.0000000	0.0000000
10	0.1616290	0.0000970	0.0190610	0.0000000
11	0.0000000	0.0000000	0.0000000	0.0000000
12	0.0003950	0.0015780	0.0231360	0.0001840
13	0.0007940	0.0019300	0.0003080	0.0000070
14	0.0000900	0.0041010	0.0031950	0.0049850
15	0.0000000	0.0000000	0.0000000	0.0000000
16	0.0005490	0.0001650	0.0190620	0.0002400
17	0.0002270	0.0004040	0.0000930	0.0000470
18	0.0056280	0.0562140	0.0014870	0.0297750
19	0.0082080	0.0451590	0.0097460	0.0081240
20	0.0011350	0.0043640	0.0002300	0.0061620
21	0.0003430	0.0000000	0.0007020	0.0000070
22	0.0002840	0.0016220	0.0001110	0.0021510
23	0.0000350	0.0000350	0.0000000	0.0211300
24	0.0022840	0.0018480	0.0386010	0.0028400
25	0.0003650	0.0101660	0.0001530	0.0468830
26	0.0002580	0.0010110	0.0077730	0.0008410
27	0.0002850	0.0007630	0.0049100	0.0026640
28	0.0000000	0.0000000	0.0001690	0.0003720
29	0.0000690	0.0000810	0.0000480	0.0004270
30	0.0041590	0.0086540	0.0002210	0.0279760
31	0.0292050	0.0169690	0.0164900	0.0672830
32	0.0381400	0.0361510	0.0230480	0.0206510
33	0.0202420	0.1040930	0.0122010	0.0803190
34	0.0193130	0.0483010	0.0021860	0.0242140
35	0.0002020	0.0002020	0.0001660	0.0011550
36	0.0000250	0.0000380	0.0000890	0.0006550
37	0.0008270	0.0014660	0.0084080	0.0045740

TABLE A-12 cont.

A63

	5	6	7	8
1	0.0000000	0.0000000	0.0000000	0.0000000
2	0.0000000	0.0000000	0.0000000	0.0024720
3	0.0000000	0.0000000	0.0000000	0.0000300
4	0.0001920	0.0000210	0.0015780	0.0000000
5	0.1556100	0.0000000	0.0011800	0.0000000
6	0.0000000	0.0241800	0.0000000	0.0000000
7	0.0004640	0.0000000	0.0202770	0.0068570
8	0.0098780	0.0420560	0.0045880	0.0002910
9	0.0000000	0.0000000	0.0000000	0.0000450
10	0.0000000	0.0000000	0.0000210	0.0002540
11	0.0000000	0.0000000	0.0000400	0.0000000
12	0.0000000	0.0001940	0.0001180	0.0012840
13	0.0000000	0.0000000	0.0000000	0.0004360
14	0.0096090	0.0000110	0.0000950	0.0349660
15	0.0000000	0.0000000	0.0000000	0.0046790
16	0.0005580	0.0001550	0.0021310	0.0022480
17	0.0000340	0.0000380	0.0000360	0.0000270
18	0.0207680	0.0093420	0.0145910	0.0110930
19	0.0149830	0.0068350	0.0183790	0.0155000
20	0.0171740	0.0013470	0.0185370	0.0047170
21	0.0000000	0.0000000	0.0000090	0.0000120
22	0.0018050	0.0038170	0.0327090	0.0524970
23	0.0233880	0.0028840	0.0133150	0.0331000
24	0.0071520	0.0029810	0.0021090	0.0672800
25	0.0828240	0.0039720	0.0467930	0.0126760
26	0.0100420	0.0086200	0.0040400	0.0159100
27	0.0097160	0.0003110	0.0045460	0.0004560
28	0.0000000	0.0004390	0.0000000	0.0024660
29	0.0000000	0.0000000	0.0000180	0.0013890
30	0.0381530	0.0134060	0.0280320	0.0024630
31	0.0299760	0.0282610	0.0223190	0.0247620
32	0.0344080	0.0139520	0.0198600	0.0605780
33	0.0625200	0.2343770	0.0273710	0.0088330
34	0.0252890	0.0166450	0.0161740	0.0345080
35	0.0012260	0.0005860	0.0008740	0.0002560
36	0.0004080	0.0003340	0.0009830	0.0003500
37	0.0068520	0.0068870	0.0039490	0.0046430

TABLE A-12 cont.

A63

	9	10	11	12
1	0.0000000	0.1761020	0.0000000	0.0126040
2	0.0000000	0.0658130	0.1374970	0.0750660
3	0.0000000	0.0039080	0.0000000	0.0000000
4	0.0000000	0.0000000	0.0000000	0.0000000
5	0.0001420	0.0003610	0.0001690	0.0006760
6	0.0000000	0.0000000	0.0000000	0.0000000
7	0.0000000	0.0001660	0.0000000	0.0000270
8	0.0009530	0.0020420	0.0011360	0.0027210
9	0.0255100	0.0000000	0.0000000	0.0000000
10	0.0000000	0.1735400	0.0001670	0.0033980
11	0.0000000	0.0000000	0.2375300	0.0000000
12	0.0000000	0.0000430	0.0000000	0.3391460
13	0.0005710	0.0018070	0.0000000	0.0128760
14	0.0029650	0.0011140	0.0007590	0.0000200
15	0.0003220	0.0000000	0.0000000	0.0002220
16	0.0014100	0.0228810	0.0160490	0.0114100
17	0.0002060	0.0058010	0.0084050	0.0002470
18	0.0022710	0.0061890	0.0106560	0.0875970
19	0.0021110	0.0027710	0.0003000	0.0020470
20	0.0132030	0.0027030	0.0000090	0.0055580
21	0.0000400	0.0000390	0.0000110	0.0000930
22	0.0000000	0.0090110	0.0000000	0.0027840
23	0.0495330	0.0000360	0.0000000	0.0000270
24	0.0114030	0.0235550	0.0068150	0.0005260
25	0.0106470	0.0005980	0.0000300	0.0054790
26	0.0614350	0.0000780	0.0000000	0.0000760
27	0.2654210	0.0001730	0.0000100	0.0000300
28	0.0138530	0.0001190	0.0001350	0.0006510
29	0.0001170	0.0000960	0.0000300	0.0025220
30	0.0037610	0.0058480	0.0010750	0.0093060
31	0.0117030	0.0320650	0.0047420	0.0202640
32	0.0134930	0.0310440	0.0088630	0.0400120
33	0.0085110	0.0093270	0.0040030	0.0093380
34	0.0219550	0.0345430	0.0491480	0.0120550
35	0.0017320	0.0007060	0.0020210	0.0009990
36	0.0000690	0.0003280	0.0000220	0.0001910
37	0.0075110	0.0032700	0.0055380	0.0034290

TABLE A-12 cont.

A63

	13	14	15	16
1	0.0000000	0.0000000	0.0000000	0.0000000
2	0.0000000	0.0142110	0.0000000	0.0000000
3	0.0051340	0.0911860	0.0000000	0.0000000
4	0.0000000	0.0000000	0.0000000	0.0000000
5	0.0000610	0.0001560	0.0002500	0.0036510
6	0.0000000	0.0000000	0.0000000	0.0000000
7	0.0000000	0.0000330	0.0000000	0.0045270
8	0.0008880	0.0038620	0.0013580	0.0040410
9	0.0000000	0.0000000	0.0000000	0.0000180
10	0.0000070	0.0000580	0.0000150	0.0077800
11	0.0000000	0.0000000	0.0000000	0.0000000
12	0.2600420	0.0000190	0.0505590	0.0060030
13	0.1797940	0.0015210	0.0019290	0.0021610
14	0.0003340	0.3040100	0.1022490	0.0473190
15	0.0001150	0.0020650	0.0296740	0.0000410
16	0.0073480	0.0039860	0.0123970	0.2618250
17	0.0000690	0.0001430	0.0001290	0.0094440
18	0.0153110	0.0139240	0.0193300	0.0434590
19	0.0008560	0.0049070	0.0015570	0.0094060
20	0.0048040	0.0011270	0.0449970	0.0106080
21	0.0027080	0.0003390	0.0010570	0.0002120
22	0.0000950	0.0061140	0.0117920	0.0033110
23	0.0002290	0.0081040	0.0479890	0.0020190
24	0.0008840	0.0181050	0.0586120	0.0142250
25	0.0005350	0.0040810	0.0030080	0.0042110
26	0.0001130	0.0002740	0.0029230	0.0002130
27	0.0000570	0.0004470	0.0005630	0.0001080
28	0.0007530	0.0004400	0.0031990	0.0012790
29	0.0151150	0.0016670	0.0018790	0.0003500
30	0.0030980	0.0078900	0.0053420	0.0182070
31	0.0090850	0.0370590	0.0232450	0.0427450
32	0.0323420	0.0258620	0.0358720	0.0318200
33	0.0148780	0.0162400	0.0264980	0.0136260
34	0.0135290	0.0181600	0.0214300	0.0224740
35	0.0024410	0.0006730	0.0014140	0.0010350
36	0.0000590	0.0002200	0.0000950	0.0003770
37	0.0048640	0.0040340	0.0067760	0.0052300

TABLE A-12 cont.

A63

	17	18	19	20
1	0.0000000	0.0000720	0.0000000	0.0000000
2	0.0000000	0.0007450	0.0000000	0.0000000
3	0.0000000	0.0010310	0.0000000	0.0000000
4	0.0000000	0.0047840	0.0000000	0.0000000
5	0.0000260	0.0024460	0.0002640	0.0008760
6	0.0000000	0.0009420	0.3169790	0.0000000
7	0.0000000	0.0183570	0.0031580	0.0013380
8	0.0014920	0.0050670	0.0144500	0.0037290
9	0.0002100	0.0000060	0.0000000	0.0000490
10	0.0007330	0.0230520	0.0009360	0.0000380
11	0.0000000	0.0000000	0.0000080	0.0000000
12	0.0013460	0.0002100	0.0000000	0.0675710
13	0.0000000	0.0010170	0.0001310	0.0034730
14	0.0013400	0.0020800	0.0001590	0.0036550
15	0.0009560	0.0000000	0.0000000	0.0004270
16	0.1287440	0.0259930	0.0054840	0.0245440
17	0.1110200	0.0012240	0.0000340	0.0002340
18	0.0206030	0.2405200	0.0220530	0.2453220
19	0.0024010	0.0477540	0.0743000	0.0028320
20	0.0032170	0.0124730	0.0000560	0.0450200
21	0.0000450	0.0001560	0.0000850	0.0024590
22	0.0000000	0.0075750	0.0019170	0.0116030
23	0.0004040	0.0153810	0.0018510	0.0090870
24	0.0012970	0.0227110	0.0050890	0.0157490
25	0.0020650	0.0063100	0.0002530	0.0038790
26	0.0001010	0.0010000	0.0000280	0.0028370
27	0.0003840	0.0001020	0.0000310	0.0046220
28	0.0072530	0.0022350	0.0000490	0.0020270
29	0.0014740	0.0012280	0.0001240	0.0078180
30	0.0042510	0.0226910	0.0138510	0.0127490
31	0.0189430	0.0298740	0.0360400	0.0250540
32	0.0185560	0.0355760	0.0143080	0.0364020
33	0.0524780	0.0152820	0.0236110	0.0158200
34	0.0333720	0.0716050	0.0203390	0.0346360
35	0.0104550	0.0020860	0.0005790	0.0014300
36	0.0002770	0.0003780	0.0000940	0.0002490
37	0.0151800	0.0108850	0.0013110	0.0105440

TABLE A-12 cont.

A63

	21	22	23	24
1	0.0095100	0.0000000	0.0000000	0.0000000
2	0.0000000	0.0005130	0.0000000	0.0000000
3	0.0000000	0.0000000	0.0000000	0.0000000
4	0.0000000	0.0012290	0.0444570	0.0000220
5	0.0003580	0.0054210	0.0091970	0.0001370
6	0.0000000	0.0000000	0.0000000	0.0000000
7	0.0000670	0.0769390	0.0026790	0.0001310
8	0.0012030	0.0052670	0.0061010	0.0016760
9	0.0000000	0.0000000	0.0001240	0.0017110
10	0.0462090	0.0003500	0.0001680	0.0001900
11	0.0000000	0.0000000	0.0000000	0.0000000
12	0.0366670	0.0024200	0.0005930	0.0017720
13	0.0122070	0.0028940	0.0007050	0.0011540
14	0.0076310	0.0079270	0.0014420	0.0050420
15	0.0000460	0.0012680	0.0000070	0.0015660
16	0.0126960	0.0246990	0.0012940	0.0118470
17	0.0002370	0.0001530	0.0000720	0.0039740
18	0.0147480	0.0303820	0.0155250	0.0182660
19	0.0013550	0.0108120	0.0064440	0.0034580
20	0.0530430	0.0117760	0.0006730	0.0072780
21	0.2023990	0.0003240	0.0001890	0.0005020
22	0.0003540	0.1068510	0.0019290	0.0076500
23	0.0014550	0.0127630	0.2685420	0.2935770
24	0.0059710	0.0105470	0.0166110	0.0600240
25	0.0002360	0.0089670	0.0185450	0.0295620
26	0.0001510	0.0029270	0.0062730	0.0102600
27	0.0000560	0.0007080	0.0035390	0.0134500
28	0.0044560	0.0004570	0.0007360	0.0047430
29	0.0121900	0.0025260	0.0009770	0.0018930
30	0.0040320	0.0338410	0.0228620	0.0079520
31	0.0122350	0.0535400	0.0400430	0.0195900
32	0.0271060	0.0286660	0.0283200	0.0279970
33	0.0127300	0.0155280	0.0225090	0.0174500
34	0.0206760	0.0278870	0.0142820	0.0222200
35	0.0027390	0.0011450	0.0006550	0.0010810
36	0.0001190	0.0009750	0.0002870	0.0001310
37	0.0048050	0.0078570	0.0033960	0.0084920

TABLE A-12 cont.

A63

	25	26	27	28
1	0.0000000	0.0000000	0.0000000	0.0000000
2	0.0000000	0.0000000	0.0000000	0.0004690
3	0.0000000	0.0000000	0.0000000	0.0000000
4	0.0000000	0.0001460	0.0000000	0.0000150
5	0.0002290	0.0001920	0.0002480	0.0003100
6	0.0000000	0.0000000	0.0000000	0.0000000
7	0.0000250	0.0000360	0.0000370	0.0000250
8	0.0016780	0.0016650	0.0019620	0.0016930
9	0.0005570	0.0052150	0.0039020	0.0023870
10	0.0002270	0.0000090	0.0000000	0.0019240
11	0.0000000	0.0000000	0.0000000	0.0000000
12	0.0003430	0.0008740	0.0031010	0.0074690
13	0.0009160	0.0012170	0.0079810	0.0013130
14	0.0019130	0.0015510	0.0031570	0.0009590
15	0.0004870	0.0075260	0.0013550	0.0006140
16	0.0030660	0.0116800	0.0008660	0.0139550
17	0.0002290	0.0018110	0.0001150	0.0003960
18	0.0044710	0.0160250	0.0056670	0.0280950
19	0.0037800	0.0044330	0.0024560	0.0017870
20	0.0104290	0.0222360	0.0159410	0.0174260
21	0.0006610	0.0001900	0.0000390	0.0010990
22	0.0066250	0.0175250	0.0086140	0.0091770
23	0.1135570	0.0983330	0.0962760	0.0506980
24	0.0463990	0.0540640	0.0546770	0.0251480
25	0.1705030	0.0375020	0.0548910	0.0172740
26	0.0449060	0.1763310	0.0282490	0.0590500
27	0.0195540	0.0153070	0.2691320	0.0124530
28	0.0049650	0.0133730	0.0092340	0.0655800
29	0.0010070	0.0022960	0.0004160	0.0040870
30	0.0056990	0.0073330	0.0047330	0.0042780
31	0.0141770	0.0177840	0.0174320	0.0140070
32	0.0292490	0.0420910	0.0232250	0.0311530
33	0.0153460	0.0163190	0.0064850	0.0152990
34	0.0202770	0.0381000	0.0258720	0.0410370
35	0.0014230	0.0020430	0.0012660	0.0018980
36	0.0001190	0.0001820	0.0001230	0.0001580
37	0.0088350	0.0136930	0.0056940	0.0129160

TABLE A-12 cont.

A63

	29	30	31	32
1	0.0000000	0.0000000	0.0000300	0.0000000
2	0.0014210	0.0000000	0.0013530	0.0000000
3	0.0005710	0.0000000	0.0000000	0.0014500
4	0.0000000	0.0001030	0.0000000	0.0000080
5	0.0001540	0.0164110	0.0001200	0.0000150
6	0.0000000	0.0578070	0.0003230	0.0000520
7	0.0003030	0.0000000	0.0000150	0.0000310
8	0.0024420	0.0337500	0.0301510	0.0036140
9	0.0000300	0.0000000	0.0000000	0.0001560
10	0.0017840	0.0000610	0.0023190	0.0060130
11	0.0000000	0.0000100	0.0000000	0.0000280
12	0.0202550	0.0000570	0.0002820	0.0002670
13	0.0032740	0.0002250	0.0003340	0.0012160
14	0.0211310	0.0000660	0.0000450	0.0010610
15	0.0030810	0.0000000	0.0000000	0.0003580
16	0.0344480	0.0007070	0.0004640	0.0101230
17	0.0064560	0.0000740	0.0008960	0.0020190
18	0.0405450	0.0017470	0.0007780	0.0032240
19	0.0035250	0.0081350	0.0305300	0.0118640
20	0.0377250	0.0001660	0.0032150	0.0022700
21	0.0070720	0.0000000	0.0000000	0.0002180
22	0.0073960	0.0000280	0.0001960	0.0014720
23	0.0611740	0.0024770	0.0043030	0.0006810
24	0.0294590	0.0000100	0.0018150	0.0022300
25	0.0042910	0.0001080	0.0017250	0.0026900
26	0.0127830	0.0007770	0.0051060	0.0022230
27	0.0055590	0.0001050	0.0094530	0.0017200
28	0.0020380	0.0000000	0.0001120	0.0010440
29	0.0732300	0.0000100	0.0000200	0.0014690
30	0.0052910	0.1864100	0.0070200	0.0161960
31	0.0197380	0.0216820	0.0685320	0.0174180
32	0.0447010	0.0054740	0.0202840	0.0179000
33	0.0193280	0.0131480	0.0350040	0.0684140
34	0.0390530	0.0200620	0.0457330	0.0628740
35	0.0027520	0.0197740	0.0030970	0.0108260
36	0.0001280	0.1377390	0.0184410	0.0038580
37	0.0097990	0.0039840	0.0060900	0.0100830

TABLE A-12 cont.

A63

	33	34	35	36
1	0.0065220	0.0001600	0.0000000	0.0000000
2	0.0107580	0.0002460	0.0000000	0.0001610
3	0.0001360	0.0000000	0.0000000	0.0000780
4	0.0000890	0.0000020	0.0000000	0.0000000
5	0.0000590	0.0000710	0.0064170	0.0071120
6	0.0005440	0.0000070	0.0000000	0.0038800
7	0.0001530	0.0000190	0.0000000	0.0002140
8	0.0633780	0.0081840	0.0059270	0.1682740
9	0.0000130	0.0000060	0.0000070	0.0000000
10	0.0008310	0.0023610	0.0000000	0.0000000
11	0.0000490	0.0000000	0.0000000	0.0000000
12	0.0001580	0.0003870	0.0007630	0.0000000
13	0.0002900	0.0014160	0.0008190	0.0002870
14	0.0002190	0.0000000	0.0000000	0.0000000
15	0.0000660	0.0000000	0.0000070	0.0000000
16	0.0016250	0.0016170	0.0016880	0.0023550
17	0.0031230	0.0608810	0.0021580	0.0004800
18	0.0023110	0.0092410	0.0010860	0.0084700
19	0.0047590	0.0044840	0.0022790	0.0068530
20	0.0004990	0.0023520	0.0009870	0.0006560
21	0.0000820	0.0013460	0.0000620	0.0000000
22	0.0003060	0.0018890	0.0001400	0.0020880
23	0.0003040	0.0000540	0.0000360	0.0013330
24	0.0005880	0.0020080	0.0002330	0.0001110
25	0.0014560	0.0057880	0.0000650	0.0001710
26	0.0005440	0.0064270	0.0002280	0.0024270
27	0.0005180	0.0064320	0.0009810	0.0011980
28	0.0002680	0.0055050	0.0000820	0.0001170
29	0.0001000	0.0057100	0.0000990	0.0001680
30	0.0057330	0.0131460	0.0078060	0.0774720
31	0.0105260	0.0312790	0.0816370	0.0128860
32	0.0123750	0.0233080	0.0048810	0.0043570
33	0.1111420	0.0531940	0.0193670	0.0178250
34	0.0434840	0.0603520	0.0175320	0.0337800
35	0.0085010	0.0085720	0.0003500	0.0008580
36	0.0059510	0.0016500	0.0002970	0.0003800
37	0.0043460	0.0152010	0.0079230	0.0012310

TABLE A-12 cont.

A63

	37
1	0.0034780
2	0.0103740
3	0.0008610
4	0.0000000
5	0.0000000
6	0.0000000
7	0.0000000
8	0.0000000
9	0.0008620
10	0.2294470
11	0.0173130
12	0.0000000
13	0.0008700
14	0.0003220
15	0.0000000
16	0.0003750
17	0.0010960
18	0.0051880
19	0.0000000
20	0.0002970
21	0.0034710
22	0.0005250
23	0.0000000
24	0.0004990
25	0.0000000
26	0.0082920
27	0.0000000
28	0.0031110
29	0.0056340
30	0.0000000
31	0.3444800
32	0.0481320
33	0.0000000
34	0.1457450
35	0.0000000
36	0.0000000
37	0.0000000

TABLE A-13
NATIONAL INPUT COEFFICIENT MATRIX
1980

A80

	1	2	3	4
1	0.1802090	0.0703190	0.0637660	0.0000000
2	0.3167370	0.0281020	0.1376450	0.0000000
3	0.0212540	0.0484960	0.0302610	0.0000000
4	0.0000000	0.0000000	0.0000000	0.1198260
5	0.0001940	0.0000180	0.0000000	0.0021900
6	0.0000000	0.0000000	0.0000000	0.0000000
7	0.0000780	0.0056410	0.0000000	0.0054230
8	0.0102150	0.0180260	0.0000000	0.0041080
9	0.0000000	0.0000000	0.0000000	0.0000000
10	0.1636350	0.0000970	0.0186130	0.0000000
11	0.0000000	0.0000000	0.0000000	0.0000000
12	0.0004000	0.0015730	0.0225930	0.0001880
13	0.0008040	0.0019240	0.0003010	0.0000070
14	0.0000910	0.0040880	0.0031200	0.0051030
15	0.0000000	0.0000000	0.0000000	0.0000000
16	0.0005560	0.0001640	0.0186140	0.0002460
17	0.0002300	0.0004030	0.0000910	0.0000480
18	0.0056980	0.0560390	0.0014520	0.0304820
19	0.0083100	0.0450180	0.0095170	0.0083170
20	0.0011490	0.0043500	0.0002250	0.0063080
21	0.0003470	0.0000000	0.0006860	0.0000070
22	0.0002880	0.0016170	0.0001080	0.0022020
23	0.0000350	0.0000350	0.0000000	0.0216320
24	0.0023120	0.0018420	0.0376940	0.0029070
25	0.0003700	0.0101340	0.0001490	0.0479970
26	0.0002610	0.0010080	0.0075900	0.0008610
27	0.0002890	0.0007610	0.0047950	0.0027270
28	0.0000000	0.0000000	0.0001650	0.0003810
29	0.0000700	0.0000810	0.0000470	0.0004370
30	0.0042110	0.0086270	0.0002160	0.0286410
31	0.0295670	0.0169160	0.0161030	0.0688820
32	0.0386130	0.0360380	0.0225070	0.0211420
33	0.0204930	0.1037680	0.0119140	0.0822270
34	0.0195530	0.0481500	0.0021350	0.0247890
35	0.0002050	0.0002010	0.0001620	0.0011820
36	0.0000250	0.0000380	0.0000870	0.0006710
37	0.0008370	0.0014610	0.0082100	0.0046830

TABLE A-13 cont.

A80

	5	6	7	8
1	0.0000000	0.0000000	0.0000000	0.0000000
2	0.0000000	0.0000000	0.0000000	0.0024150
3	0.0000000	0.0000000	0.0000000	0.0000290
4	0.0001960	0.0000220	0.0015510	0.0000000
5	0.1590980	0.0000000	0.0011600	0.0000000
6	0.0000000	0.0248090	0.0000000	0.0000000
7	0.0004740	0.0000000	0.0199340	0.0067000
8	0.0100990	0.0431490	0.0045100	0.0002840
9	0.0000000	0.0000000	0.0000000	0.0000440
10	0.0000000	0.0000000	0.0000210	0.0002480
11	0.0000000	0.0000000	0.0000390	0.0000000
12	0.0000000	0.0001990	0.0001160	0.0012550
13	0.0000000	0.0000000	0.0000000	0.0004260
14	0.0098240	0.0000110	0.0000930	0.0341640
15	0.0000000	0.0000000	0.0000000	0.0045720
16	0.0005710	0.0001590	0.0020950	0.0021960
17	0.0000350	0.0000390	0.0000350	0.0000260
18	0.0212330	0.0095850	0.0143440	0.0108390
19	0.0153190	0.0070130	0.0180680	0.0151450
20	0.0175590	0.0013820	0.0182240	0.0046090
21	0.0000000	0.0000000	0.0000090	0.0000120
22	0.0018450	0.0039160	0.0321560	0.0512940
23	0.0239120	0.0029590	0.0130900	0.0323410
24	0.0073120	0.0030580	0.0020730	0.0657380
25	0.0846800	0.0040750	0.0460020	0.0123850
26	0.0102670	0.0088440	0.0039720	0.0155450
27	0.0099340	0.0003190	0.0044690	0.0004460
28	0.0000000	0.0004500	0.0000000	0.0024090
29	0.0000000	0.0000000	0.0000180	0.0013570
30	0.0390080	0.0137540	0.0275580	0.0024070
31	0.0306480	0.0289960	0.0219420	0.0241940
32	0.0351790	0.0143150	0.0195240	0.0591890
33	0.0639210	0.2404690	0.0269090	0.0086310
34	0.0258560	0.0170780	0.0159010	0.0337170
35	0.0012530	0.0006010	0.0008590	0.0002500
36	0.0004170	0.0003430	0.0009660	0.0003420
37	0.0070060	0.0070660	0.0038820	0.0045370

TABLE A-13 cont.

A80

	9	10	11	12
1	0.0000000	0.1749410	0.0000000	0.0125480
2	0.0000000	0.0653790	0.1364760	0.0747320
3	0.0000000	0.0038820	0.0000000	0.0000000
4	0.0000000	0.0000000	0.0000000	0.0000000
5	0.0001440	0.0003590	0.0001870	0.0006730
6	0.0000000	0.0000000	0.0000000	0.0000000
7	0.0000000	0.0001650	0.0000000	0.0000270
8	0.0009700	0.0020290	0.0011230	0.0027090
9	0.0259520	0.0000000	0.0000000	0.0000000
10	0.0000000	0.1723960	0.0001650	0.0033830
11	0.0000000	0.0000000	0.2349110	0.0000000
12	0.0000000	0.0000430	0.0000000	0.3376350
13	0.0005810	0.0017950	0.0000000	0.0128190
14	0.0030160	0.0011070	0.0007510	0.0000200
15	0.0003280	0.0000000	0.0000000	0.0002210
16	0.0014340	0.0227300	0.0158720	0.0113590
17	0.0002100	0.0057630	0.0083120	0.0002460
18	0.0023100	0.0061480	0.0105390	0.0872070
19	0.0021480	0.0027530	0.0002970	0.0020380
20	0.0134320	0.0026850	0.0000090	0.0055330
21	0.0000410	0.0000390	0.0000110	0.0000930
22	0.0000000	0.0089520	0.0000000	0.0027720
23	0.0503920	0.0000360	0.0000000	0.0000270
24	0.0116010	0.0234000	0.0067400	0.0005240
25	0.0108320	0.0005940	0.0000300	0.0054550
26	0.0625000	0.0000770	0.0000000	0.0000760
27	0.2700230	0.0001720	0.0000100	0.0000300
28	0.0140930	0.0001180	0.0001340	0.0006480
29	0.0001190	0.0000950	0.0000300	0.0025110
30	0.0038260	0.0058090	0.0010630	0.0092650
31	0.0119060	0.0318540	0.0046900	0.0201740
32	0.0137270	0.0308390	0.0087650	0.0398340
33	0.0086590	0.0092650	0.0039590	0.0092960
34	0.0223360	0.0343150	0.0486060	0.0120010
35	0.0017620	0.0007010	0.0019990	0.0009950
36	0.0000700	0.0003260	0.0000220	0.0001900
37	0.0076410	0.0032480	0.0054770	0.0034140

TABLE A-13 cont.

ABO

	13	14	15	16
1	0.0000000	0.0000000	0.0000000	0.0000000
2	0.0000000	0.0141990	0.0000000	0.0000000
3	0.0050650	0.0911110	0.0000000	0.0000000
4	0.0000000	0.0000000	0.0000000	0.0000000
5	0.0000600	0.0001560	0.0002480	0.0036470
6	0.0000000	0.0000000	0.0000000	0.0000000
7	0.0000000	0.0000330	0.0000000	0.0045220
8	0.0008760	0.0038590	0.0013470	0.0040360
9	0.0000000	0.0000000	0.0000000	0.0000180
10	0.0000070	0.0000580	0.0000150	0.0077710
11	0.0000000	0.0000000	0.0000000	0.0000000
12	0.2565690	0.0000190	0.0501590	0.0059960
13	0.1773930	0.0015200	0.0019140	0.0021590
14	0.0003300	0.3037600	0.1014390	0.0472660
15	0.0001130	0.0020630	0.0294390	0.0000410
16	0.0072500	0.0039830	0.0122990	0.2615330
17	0.0000880	0.0001430	0.0001280	0.0094330
18	0.0151070	0.0139130	0.0191770	0.0434110
19	0.0008450	0.0049030	0.0015450	0.0093960
20	0.0047400	0.0011260	0.0446410	0.0105960
21	0.0026720	0.0003390	0.0010490	0.0002120
22	0.0000940	0.0061090	0.0116990	0.0033070
23	0.0002260	0.0080970	0.0476090	0.0020170
24	0.0008720	0.0180900	0.0581480	0.0142090
25	0.0005280	0.0040780	0.0029840	0.0042060
26	0.0001110	0.0002740	0.0029000	0.0002130
27	0.0000560	0.0004470	0.0005590	0.0001080
28	0.0007430	0.0004400	0.0031740	0.0012780
29	0.0149130	0.0016660	0.0018640	0.0003500
30	0.0030570	0.0078840	0.0053000	0.0181870
31	0.0089640	0.0370290	0.0230610	0.0426970
32	0.0319100	0.0258410	0.0355880	0.0317840
33	0.0146790	0.0162270	0.0262880	0.0136110
34	0.0133480	0.0181450	0.0212600	0.0224490
35	0.0024080	0.0006720	0.0014030	0.0010340
36	0.0000580	0.0002200	0.0000940	0.0003770
37	0.0047990	0.0040310	0.0067220	0.0052240

TABLE A-13 cont.

A80

	17	18	19	20
1	0.0000000	0.0000720	0.0000000	0.0000000
2	0.0000000	0.0007400	0.0000000	0.0000000
3	0.0000000	0.0010240	0.0000000	0.0000000
4	0.0000000	0.0047510	0.0000000	0.0000000
5	0.0000260	0.0024290	0.0002500	0.0008930
6	0.0000000	0.0009350	0.2999400	0.0000000
7	0.0000000	0.0182300	0.0029880	0.0013640
8	0.0014850	0.0050320	0.0136730	0.0038020
9	0.0002090	0.0000060	0.0000000	0.0000500
10	0.0007290	0.0228930	0.0008860	0.0000390
11	0.0000000	0.0000000	0.0000080	0.0000000
12	0.0013400	0.0002090	0.0000000	0.0688910
13	0.0000000	0.0010100	0.0001240	0.0035410
14	0.0013340	0.0020660	0.0001500	0.0037260
15	0.0009530	0.0000000	0.0000000	0.0004350
16	0.1281240	0.0258140	0.0051890	0.0250230
17	0.1104860	0.0012160	0.0000320	0.0002390
18	0.0205040	0.2388590	0.0208680	0.2501140
19	0.0023890	0.0474240	0.0703060	0.0028870
20	0.0032020	0.0123870	0.0000530	0.0458990
21	0.0000450	0.0001550	0.0000800	0.0025070
22	0.0000000	0.0075230	0.0018140	0.0118300
23	0.0004020	0.0152750	0.0017520	0.0092640
24	0.0012910	0.0225540	0.0048150	0.0160570
25	0.0020550	0.0062660	0.0002390	0.0039550
26	0.0001010	0.0009930	0.0000260	0.0028920
27	0.0003820	0.0001010	0.0000290	0.0047120
28	0.0072180	0.0022200	0.0000460	0.0020670
29	0.0014670	0.0012200	0.0001170	0.0079710
30	0.0042310	0.0225340	0.0131060	0.0129980
31	0.0188520	0.0296680	0.0341030	0.0255430
32	0.0184670	0.0353300	0.0135390	0.0371130
33	0.0522250	0.0151760	0.0223420	0.0161290
34	0.0332110	0.0711110	0.0192460	0.0353120
35	0.0104050	0.0020720	0.0005480	0.0014580
36	0.0002760	0.0003750	0.0000690	0.0002540
37	0.0151070	0.0108100	0.0012410	0.0107500

TABLE A-13 cont.

A80

	21	22	23	24
1	0.0093720	0.0000000	0.0000000	0.0000000
2	0.0000000	0.0005150	0.0000000	0.0000000
3	0.0000000	0.0000000	0.0000000	0.0000000
4	0.0000000	0.0012350	0.0445480	0.0000220
5	0.0003530	0.0054460	0.0092160	0.0001370
6	0.0000000	0.0000000	0.0000000	0.0000000
7	0.0000660	0.0772870	0.0026840	0.0001310
8	0.0011860	0.0052910	0.0061140	0.0016740
9	0.0000000	0.0000000	0.0001240	0.0017090
10	0.0455370	0.0003520	0.0001680	0.0001900
11	0.0000000	0.0000000	0.0000000	0.0000000
12	0.0361340	0.0024310	0.0005940	0.0017700
13	0.0120290	0.0029070	0.0007060	0.0011530
14	0.0075200	0.0079630	0.0014450	0.0050370
15	0.0000450	0.0012740	0.0000070	0.0015650
16	0.0125110	0.0248110	0.0012970	0.0118360
17	0.0002340	0.0001540	0.0000720	0.0039700
18	0.0145340	0.0305200	0.0155570	0.0182490
19	0.0013350	0.0108610	0.0064570	0.0034550
20	0.0522720	0.0118290	0.0006740	0.0072710
21	0.1994550	0.0003250	0.0001890	0.0005020
22	0.0003490	0.1073350	0.0019330	0.0076430
23	0.0014340	0.0128210	0.2690930	0.2933070
24	0.0058840	0.0105950	0.0166450	0.0599690
25	0.0002330	0.0090080	0.0185630	0.0295350
26	0.0001490	0.0029400	0.0062860	0.0102510
27	0.0000550	0.0007110	0.0035460	0.0134380
28	0.0043910	0.0004590	0.0007380	0.0047390
29	0.0120130	0.0025370	0.0009790	0.0018910
30	0.0039730	0.0339940	0.0229090	0.0079450
31	0.0120570	0.0537820	0.0401250	0.0195720
32	0.0267120	0.0287960	0.0283780	0.0279710
33	0.0125450	0.0155980	0.0225550	0.0174340
34	0.0203750	0.0280130	0.0143110	0.0222000
35	0.0026990	0.0011500	0.0006560	0.0010800
36	0.0001170	0.0009790	0.0002880	0.0001310
37	0.0047350	0.0078930	0.0034030	0.0084840

TABLE A-13 cont.

A80

	25	26	27	28
1	0.0000000	0.0000000	0.0000000	0.0000000
2	0.0000000	0.0000000	0.0000000	0.0004630
3	0.0000000	0.0000000	0.0000000	0.0000000
4	0.0000000	0.0001460	0.0000000	0.0000150
5	0.0002320	0.0001930	0.0002480	0.0003060
6	0.0000000	0.0000000	0.0000000	0.0000000
7	0.0000250	0.0000360	0.0000370	0.0000250
8	0.0019050	0.0016700	0.0019630	0.0016710
9	0.0005650	0.0052300	0.0039050	0.0023560
10	0.0002300	0.0000090	0.0000000	0.0018990
11	0.0000000	0.0000000	0.0000000	0.0000000
12	0.0003480	0.0008770	0.0031030	0.0073730
13	0.0009290	0.0012200	0.0079870	0.0012960
14	0.0019410	0.0015550	0.0031590	0.0009470
15	0.0004940	0.0075480	0.0013560	0.0006060
16	0.0031110	0.0117130	0.0008870	0.0137760
17	0.0002320	0.0018160	0.0001150	0.0003910
18	0.0045360	0.0160710	0.0056710	0.0277340
19	0.0038350	0.0044460	0.0024580	0.0017640
20	0.0105820	0.0223000	0.0159530	0.0172020
21	0.0006710	0.0001910	0.0000390	0.0010850
22	0.0067220	0.0175750	0.0086200	0.0090590
23	0.1152190	0.0986150	0.0963470	0.0500460
24	0.0470780	0.0542190	0.0547170	0.0248250
25	0.1729990	0.0376100	0.0549320	0.0170520
26	0.0455630	0.1768370	0.0282700	0.0582910
27	0.0198400	0.0153510	0.2693310	0.0122930
28	0.0050380	0.0134110	0.0092410	0.0647370
29	0.0010220	0.0023030	0.0004160	0.0040340
30	0.0057820	0.0073540	0.0047370	0.0042230
31	0.0143850	0.0178350	0.0174450	0.0138270
32	0.0296770	0.0422120	0.0232420	0.0307520
33	0.0155710	0.0163660	0.0064900	0.0151020
34	0.0205740	0.0382090	0.0258910	0.0405090
35	0.0014440	0.0020490	0.0012670	0.0018740
36	0.0001210	0.0001830	0.0001230	0.0001560
37	0.0089640	0.0137320	0.0056980	0.0127500

TABLE A-13 cont.

A80

	29	30	31	32
1	0.0000000	0.0000000	0.0000300	0.0000000
2	0.0014180	0.0000000	0.0013470	0.0000000
3	0.0005700	0.0000000	0.0000000	0.0014380
4	0.0000000	0.0001020	0.0000000	0.0000080
5	0.0001540	0.0162720	0.0001190	0.0000150
6	0.0000000	0.0573180	0.0003220	0.0000520
7	0.0003020	0.0000000	0.0000150	0.0000310
8	0.0024370	0.0334640	0.0300110	0.0035830
9	0.0000300	0.0000000	0.0000000	0.0001550
10	0.0017800	0.0000600	0.0023080	0.0059620
11	0.0000000	0.0000100	0.0000000	0.0000280
12	0.0202100	0.0000570	0.0002810	0.0002650
13	0.0032670	0.0002230	0.0003320	0.0012060
14	0.0210840	0.0000650	0.0000450	0.0010520
15	0.0030740	0.0000000	0.0000000	0.0003550
16	0.0343710	0.0007010	0.0004620	0.0100360
17	0.0064420	0.0000730	0.0008940	0.0020020
18	0.0404540	0.0017320	0.0007740	0.0031960
19	0.0035170	0.0080660	0.0303890	0.0117620
20	0.0376400	0.0001650	0.0032000	0.0022510
21	0.0070560	0.0000000	0.0000000	0.0002160
22	0.0073790	0.0000280	0.0001950	0.0014590
23	0.0610370	0.0024560	0.0042830	0.0006750
24	0.0293930	0.0000100	0.0018070	0.0022110
25	0.0042810	0.0001070	0.0017170	0.0026670
26	0.0127540	0.0007700	0.0050820	0.0022040
27	0.0055470	0.0001040	0.0094090	0.0017050
28	0.0020330	0.0000000	0.0001110	0.0010350
29	0.0730660	0.0000100	0.0000200	0.0014560
30	0.0052790	0.1848320	0.0069880	0.0160570
31	0.0196940	0.0214980	0.0682150	0.0172690
32	0.0446010	0.0054280	0.0201900	0.0177470
33	0.0192850	0.0130370	0.0348420	0.0678290
34	0.0389650	0.0198920	0.0455210	0.0623360
35	0.0027460	0.0196070	0.0030830	0.0107330
36	0.0001280	0.1365730	0.0183560	0.0038250
37	0.0097770	0.0039500	0.0060620	0.0099970

TABLE A-13 cont.

A80

	33	34	35	36
1	0.0064270	0.0001590	0.0000000	0.0000000
2	0.0106020	0.0002440	0.0000000	0.0001600
3	0.0001340	0.0000000	0.0000000	0.0000770
4	0.0000880	0.0000020	0.0000000	0.0000000
5	0.0000580	0.0000700	0.0062030	0.0070580
6	0.0005360	0.0000070	0.0000000	0.0038510
7	0.0001510	0.0000190	0.0000000	0.0002120
8	0.0624560	0.0081230	0.0057290	0.1670000
9	0.0000130	0.0000060	0.0000070	0.0000000
10	0.0008190	0.0023430	0.0000000	0.0000000
11	0.0000460	0.0000000	0.0000000	0.0000000
12	0.0001560	0.0003840	0.0007380	0.0000000
13	0.0002860	0.0014050	0.0007920	0.0002850
14	0.0002160	0.0000000	0.0000000	0.0000000
15	0.0000650	0.0000000	0.0000070	0.0000000
16	0.0016010	0.0016050	0.0016320	0.0023370
17	0.0030780	0.0604280	0.0020860	0.0004760
18	0.0022770	0.0091720	0.0010500	0.0084060
19	0.0046900	0.0044510	0.0022030	0.0068010
20	0.0004920	0.0023350	0.0009540	0.0006510
21	0.0000810	0.0013360	0.0000600	0.0000000
22	0.0003020	0.0018750	0.0001350	0.0020720
23	0.0003000	0.0000540	0.0000350	0.0013230
24	0.0005790	0.0019930	0.0002250	0.0001100
25	0.0014350	0.0057450	0.0000630	0.0001700
26	0.0005360	0.0063790	0.0002200	0.0024090
27	0.0005100	0.0063840	0.0009480	0.0011890
28	0.0002640	0.0054640	0.0000790	0.0001160
29	0.0000990	0.0056680	0.0000960	0.0001670
30	0.0056500	0.0130480	0.0075450	0.0768850
31	0.0103730	0.0310460	0.0789110	0.0127880
32	0.0121950	0.0231350	0.0047180	0.0043240
33	0.1095260	0.0527980	0.0187200	0.0176900
34	0.0428520	0.0599030	0.0169470	0.0335240
35	0.0083770	0.0085080	0.0003380	0.0008520
36	0.0058640	0.0016380	0.0002870	0.0003770
37	0.0042830	0.0150880	0.0076580	0.0012220

TABLE A-13 cont.

A80

	37
1	0.0034080
2	0.0101640
3	0.0008440
4	0.0000000
5	0.0000000
6	0.0000000
7	0.0000000
8	0.0000000
9	0.0008450
10	0.2247960
11	0.0169620
12	0.0000000
13	0.0008520
14	0.0003150
15	0.0000000
16	0.0003670
17	0.0010740
18	0.0050830
19	0.0000000
20	0.0002910
21	0.0034010
22	0.0005140
23	0.0000000
24	0.0004890
25	0.0000000
26	0.0081240
27	0.0000000
28	0.0030480
29	0.0055200
30	0.0000000
31	0.3374980
32	0.0471560
33	0.0000000
34	0.1427910
35	0.0000000
36	0.0000000
37	0.0000000

TABLE A-14
NATIONAL INPUT COEFFICIENT MATRIX
2000

A00

	1	2	3	4
1	0.2085850	0.0658840	0.0360750	0.0000000
2	0.3666100	0.0263300	0.0778710	0.0000000
3	0.0246000	0.0454380	0.0171200	0.0000000
4	0.0000000	0.0000000	0.0000000	0.1728470
5	0.0002250	0.0000170	0.0000000	0.0031590
6	0.0000000	0.0000000	0.0000000	0.0000000
7	0.0000900	0.0052860	0.0000000	0.0078220
8	0.0118240	0.0168890	0.0000000	0.0059260
9	0.0000000	0.0000000	0.0000000	0.0000000
10	0.1894010	0.0000910	0.0105300	0.0000000
11	0.0000000	0.0000000	0.0000000	0.0000000
12	0.0004630	0.0014740	0.0127810	0.0002720
13	0.0009300	0.0018030	0.0001700	0.0000100
14	0.0001050	0.0038300	0.0017650	0.0073620
15	0.0000000	0.0000000	0.0000000	0.0000000
16	0.0006430	0.0001540	0.0105310	0.0003540
17	0.0002660	0.0003770	0.0000510	0.0000690
18	0.0065950	0.0525040	0.0008210	0.0439700
19	0.0096180	0.0421790	0.0053840	0.0119970
20	0.0013300	0.0040760	0.0001270	0.0091000
21	0.0004020	0.0000000	0.0003880	0.0000100
22	0.0003330	0.0015150	0.0000610	0.0031760
23	0.0000410	0.0000330	0.0000000	0.0312040
24	0.0026760	0.0017260	0.0213250	0.0041940
25	0.0004280	0.0094950	0.0000850	0.0692350
26	0.0003020	0.0009440	0.0042940	0.0012420
27	0.0003340	0.0007130	0.0027130	0.0039340
28	0.0000000	0.0000000	0.0000930	0.0005490
29	0.0000810	0.0000760	0.0000270	0.0006310
30	0.0048740	0.0080830	0.0001220	0.0413140
31	0.0342230	0.0158490	0.0091100	0.0993600
32	0.0446930	0.0337650	0.0127330	0.0304960
33	0.0237200	0.0972240	0.0067400	0.1186110
34	0.0226310	0.0451140	0.0012080	0.0357580
35	0.0002370	0.0001890	0.0000920	0.0017060
36	0.0000290	0.0000350	0.0000490	0.0009670
37	0.0009690	0.0013690	0.0046450	0.0067550

TABLE A-14 cont.

A00

	5	6	7	8
1	0.0000000	0.0000000	0.0000000	0.0000000
2	0.0000000	0.0000000	0.0000000	0.0013950
3	0.0000000	0.0000000	0.0000000	0.0000170
4	0.0002720	0.0000320	0.0010680	0.0000000
5	0.2207190	0.0000000	0.0007980	0.0000000
6	0.0000000	0.0373390	0.0000000	0.0000000
7	0.0006580	0.0000000	0.0137200	0.0038680
8	0.0140110	0.0649440	0.0031040	0.0001640
9	0.0000000	0.0000000	0.0000000	0.0000250
10	0.0000000	0.0000000	0.0000140	0.0001430
11	0.0000000	0.0000000	0.0000270	0.0000000
12	0.0000000	0.0003000	0.0000800	0.0007240
13	0.0000000	0.0000000	0.0000000	0.0002460
14	0.0136300	0.0000170	0.0000640	0.0197250
15	0.0000000	0.0000000	0.0000000	0.0026400
16	0.0007910	0.0002390	0.0014420	0.0012680
17	0.0000480	0.0000590	0.0000240	0.0000150
18	0.0294580	0.0144260	0.0098730	0.0062580
19	0.0212520	0.0105550	0.0124360	0.0087440
20	0.0243600	0.0020800	0.0125430	0.0026610
21	0.0000000	0.0000000	0.0000060	0.0000070
22	0.0025600	0.0058940	0.0221330	0.0296150
23	0.0331740	0.0044540	0.0090100	0.0186720
24	0.0101440	0.0046030	0.0014270	0.0379540
25	0.1174790	0.0061340	0.0316630	0.0071510
26	0.0142440	0.0133110	0.0027340	0.0089750
27	0.0137810	0.0004800	0.0030760	0.0002570
28	0.0000000	0.0006780	0.0000000	0.0013910
29	0.0000000	0.0000000	0.0000120	0.0007840
30	0.0541170	0.0207020	0.0189680	0.0013890
31	0.0425180	0.0436410	0.0151020	0.0139690
32	0.0488050	0.0215450	0.0134380	0.0341730
33	0.0886790	0.3619310	0.0185210	0.0049830
34	0.0358700	0.0257040	0.0109440	0.0194670
35	0.0017390	0.0009050	0.0005910	0.0001440
36	0.0005790	0.0005160	0.0006650	0.0001970
37	0.0097190	0.0106350	0.0026720	0.0026190

TABLE A-14 cont.

A00

	9	10	11	12
1	0.0000000	0.1509750	0.0000000	0.0114020
2	0.0000000	0.0564220	0.1066220	0.0679050
3	0.0000000	0.0033500	0.0000000	0.0000000
4	0.0000000	0.0000000	0.0000000	0.0000000
5	0.0001910	0.0003090	0.0001460	0.0006120
6	0.0000000	0.0000000	0.0000000	0.0000000
7	0.0000000	0.0001420	0.0000000	0.0000240
8	0.0012820	0.0017510	0.0008780	0.0024610
9	0.0343250	0.0000000	0.0000000	0.0000000
10	0.0000000	0.1487780	0.0001290	0.0030740
11	0.0000000	0.0000000	0.1835250	0.0000000
12	0.0000000	0.0000370	0.0000000	0.3067930
13	0.0007680	0.0015490	0.0000000	0.0116480
14	0.0039900	0.0009550	0.0005860	0.0000180
15	0.0004330	0.0000000	0.0000000	0.0002010
16	0.0018970	0.0196160	0.0124000	0.0103220
17	0.0002770	0.0049730	0.0064940	0.0002230
18	0.0030560	0.0053060	0.0082330	0.0792410
19	0.0028400	0.0023760	0.0002320	0.0018520
20	0.0177650	0.0023170	0.0000070	0.0050280
21	0.0000540	0.0000330	0.0000080	0.0000840
22	0.0000000	0.0077250	0.0000000	0.0025180
23	0.0666490	0.0000310	0.0000000	0.0000240
24	0.0153430	0.0201940	0.0052660	0.0004760
25	0.0143260	0.0005130	0.0000230	0.0049560
26	0.0826630	0.0000670	0.0000000	0.0000690
27	0.3571350	0.0001480	0.0000080	0.0000270
28	0.0186400	0.0001020	0.0001040	0.0005890
29	0.0001570	0.0000820	0.0000230	0.0022810
30	0.0050610	0.0050140	0.0008310	0.0084180
31	0.0157470	0.0274900	0.0036640	0.0183310
32	0.0181550	0.0266140	0.0068480	0.0361950
33	0.0114520	0.0079960	0.0030930	0.0084470
34	0.0295410	0.0296140	0.0379740	0.0109050
35	0.0023300	0.0006050	0.0015620	0.0009040
36	0.0000930	0.0002810	0.0000170	0.0001730
37	0.0101060	0.0028030	0.0042790	0.0031020

TABLE A-14 cont.

A00

	13	14	15	16
1	0.0000000	0.0000000	0.0000000	0.0000000
2	0.0000000	0.0139660	0.0000000	0.0000000
3	0.0036730	0.0896140	0.0000000	0.0000000
4	0.0000000	0.0000000	0.0000000	0.0000000
5	0.0000440	0.0001530	0.0002090	0.0035650
6	0.0000000	0.0000000	0.0000000	0.0000000
7	0.0000000	0.0000320	0.0000000	0.0044200
8	0.0006350	0.0037950	0.0011330	0.0039460
9	0.0000000	0.0000000	0.0000000	0.0000180
10	0.0000050	0.0000570	0.0000130	0.0075960
11	0.0000000	0.0000000	0.0000000	0.0000000
12	0.1860490	0.0000190	0.0421990	0.0058610
13	0.1286350	0.0014950	0.0016100	0.0021100
14	0.0002390	0.2987680	0.0853420	0.0462020
15	0.0000820	0.0020290	0.0247670	0.0000400
16	0.0052570	0.0039170	0.0103470	0.2556440
17	0.0000640	0.0001410	0.0001080	0.0092210
18	0.0109540	0.0136840	0.0161340	0.0424330
19	0.0006120	0.0048220	0.0013000	0.0091840
20	0.0034370	0.0011080	0.0375570	0.0103580
21	0.0019370	0.0003330	0.0008820	0.0002070
22	0.0000680	0.0060090	0.0098420	0.0032330
23	0.0001640	0.0079640	0.0400540	0.0019710
24	0.0006320	0.0177930	0.0489200	0.0138890
25	0.0003830	0.0040110	0.0025110	0.0041120
26	0.0000810	0.0002690	0.0024400	0.0002080
27	0.0000410	0.0004390	0.0004700	0.0001050
28	0.0005390	0.0004320	0.0026700	0.0012490
29	0.0108140	0.0016380	0.0015680	0.0003420
30	0.0022160	0.0077540	0.0044590	0.0177770
31	0.0065000	0.0364200	0.0194010	0.0417360
32	0.0231390	0.0254160	0.0299400	0.0310690
33	0.0106450	0.0159600	0.0221160	0.0133040
34	0.0096790	0.0178470	0.0178860	0.0219430
35	0.0017460	0.0006610	0.0011800	0.0010110
36	0.0000420	0.0002160	0.0000790	0.0003680
37	0.0034800	0.0039640	0.0056560	0.0051070

TABLE A-14 cont.

A00

	17	18	19	20
1	0.0000000	0.0000610	0.0000000	0.0000000
2	0.0000000	0.0006340	0.0000000	0.0000000
3	0.0000000	0.0008780	0.0000000	0.0000000
4	0.0000000	0.0040740	0.0000000	0.0000000
5	0.0000230	0.0020830	0.0000350	0.0011890
6	0.0000000	0.0008020	0.0424420	0.0000000
7	0.0000000	0.0156340	0.0004230	0.0018170
8	0.0013450	0.0043150	0.0019350	0.0050630
9	0.0001890	0.0000050	0.0000000	0.0000670
10	0.0006610	0.0196320	0.0001250	0.0000520
11	0.0000000	0.0000000	0.0000010	0.0000000
12	0.0012130	0.0001790	0.0000000	0.0917480
13	0.0000000	0.0008660	0.0000180	0.0047160
14	0.0012080	0.0017710	0.0000210	0.0049630
15	0.0008640	0.0000000	0.0000000	0.0005800
16	0.1160580	0.0221370	0.0007340	0.0333260
17	0.1000800	0.0010420	0.0000050	0.0003180
18	0.0185730	0.2048370	0.0029530	0.3331000
19	0.0021640	0.0406690	0.0099480	0.0038450
20	0.0029000	0.0106230	0.0000070	0.0611280
21	0.0000410	0.0001330	0.0000110	0.0033390
22	0.0000000	0.0064510	0.0002570	0.0157550
23	0.0003640	0.0130990	0.0002480	0.0123380
24	0.0011690	0.0193420	0.0006810	0.0213840
25	0.0018620	0.0053740	0.0000340	0.0052670
26	0.0000910	0.0008520	0.0000040	0.0038520
27	0.0003460	0.0000870	0.0000040	0.0062760
28	0.0065380	0.0019030	0.0000070	0.0027520
29	0.0013290	0.0010460	0.0000170	0.0106150
30	0.0038320	0.0193250	0.0018550	0.0173110
31	0.0170760	0.0254420	0.0048260	0.0340180
32	0.0167280	0.0302980	0.0019160	0.0494270
33	0.0473070	0.0130150	0.0031610	0.0214800
34	0.0300840	0.0609820	0.0027230	0.0470290
35	0.0094250	0.0017770	0.0000780	0.0019420
36	0.0002500	0.0003220	0.0000130	0.0003380
37	0.0136840	0.0092700	0.0001760	0.0143170

TABLE A-14 cont.

A00

	21	22	23	24
1	0.0066740	0.0000000	0.0000000	0.0000000
2	0.0000000	0.0005620	0.0000000	0.0000000
3	0.0000000	0.0000000	0.0000000	0.0000000
4	0.0000000	0.0013450	0.0463610	0.0000220
5	0.0002510	0.0059340	0.0095950	0.0001340
6	0.0000000	0.0000000	0.0000000	0.0000000
7	0.0000470	0.0842260	0.0027950	0.0001280
8	0.0008440	0.0057660	0.0063650	0.0016430
9	0.0000000	0.0000000	0.0001290	0.0016780
10	0.0324290	0.0003830	0.0001750	0.0001860
11	0.0000000	0.0000000	0.0000000	0.0000000
12	0.0257320	0.0026490	0.0006190	0.0017380
13	0.0085670	0.0031680	0.0007360	0.0011320
14	0.0053550	0.0086780	0.0015040	0.0049440
15	0.0000320	0.0013880	0.0000070	0.0015360
16	0.0089100	0.0270380	0.0013500	0.0116160
17	0.0001660	0.0001670	0.0000750	0.0038970
18	0.0103500	0.0332600	0.0161970	0.0179100
19	0.0009510	0.0118360	0.0067230	0.0033910
20	0.0372250	0.0128910	0.0007020	0.0071360
21	0.1420410	0.0003550	0.0001970	0.0004920
22	0.0002480	0.1169710	0.0020130	0.0075010
23	0.0010210	0.0139720	0.2801660	0.2878610
24	0.0041900	0.0115460	0.0173300	0.0588550
25	0.0001660	0.0098160	0.0193480	0.0289860
26	0.0001060	0.0032040	0.0065450	0.0100600
27	0.0000390	0.0007750	0.0036920	0.0131880
28	0.0031270	0.0005000	0.0007680	0.0046510
29	0.0085550	0.0027650	0.0010190	0.0018560
30	0.0028300	0.0370460	0.0238520	0.0077970
31	0.0085860	0.0586110	0.0417760	0.0192090
32	0.0190230	0.0313810	0.0295460	0.0274520
33	0.0089340	0.0169990	0.0234830	0.0171100
34	0.0145100	0.0305280	0.0149000	0.0217870
35	0.0019220	0.0012530	0.0006830	0.0010600
36	0.0000840	0.0010670	0.0002990	0.0001280
37	0.0033720	0.0086010	0.0035430	0.0083270

TABLE A-14 cont.

A00

	25	26	27	28
1	0.0000000	0.0000000	0.0000000	0.0000000
2	0.0000000	0.0000000	0.0000000	0.0003460
3	0.0000000	0.0000000	0.0000000	0.0000000
4	0.0000000	0.0001550	0.0000000	0.0000110
5	0.0002970	0.0002030	0.0002520	0.0002290
6	0.0000000	0.0000000	0.0000000	0.0000000
7	0.0000320	0.0000380	0.0000380	0.0000180
8	0.0024340	0.0017630	0.0019920	0.0012510
9	0.0007220	0.0055230	0.0039630	0.0017630
10	0.0002940	0.0000100	0.0000000	0.0014210
11	0.0000000	0.0000000	0.0000000	0.0000000
12	0.0004440	0.0009260	0.0031490	0.0055180
13	0.0011870	0.0012890	0.0081050	0.0009700
14	0.0024790	0.0016430	0.0032060	0.0007080
15	0.0006310	0.0079700	0.0013760	0.0004540
16	0.0039730	0.0123690	0.0009000	0.0103090
17	0.0002970	0.0019180	0.0001170	0.0002930
18	0.0057940	0.0169700	0.0057550	0.0207550
19	0.0048980	0.0046950	0.0024940	0.0013200
20	0.0135150	0.0235480	0.0161880	0.0128740
21	0.0008570	0.0002010	0.0000400	0.0008120
22	0.0085850	0.0185590	0.0087480	0.0067800
23	0.1471560	0.1041350	0.0977700	0.0374540
24	0.0601270	0.0572540	0.0555260	0.0185780
25	0.2209510	0.0397150	0.0557430	0.0127610
26	0.0581930	0.1867350	0.0286870	0.0436240
27	0.0253400	0.0162100	0.2733090	0.0092000
28	0.0064340	0.0141620	0.0093770	0.0484480
29	0.0013050	0.0024310	0.0004220	0.0030190
30	0.0073850	0.0077660	0.0048060	0.0031600
31	0.0183720	0.0188330	0.0177030	0.0103480
32	0.0379030	0.0445740	0.0235850	0.0230150
33	0.0196870	0.0172820	0.0065860	0.0113020
34	0.0262760	0.0403480	0.0262740	0.0303160
35	0.0018440	0.0021640	0.0012860	0.0014020
36	0.0001540	0.0001930	0.0001250	0.0001170
37	0.0114490	0.0145010	0.0057820	0.0095420

TABLE A-14 cont.

A00

	29	30	31	32
1	0.0000000	0.0000000	0.0000270	0.0000000
2	0.0013540	0.0000000	0.0012220	0.0000000
3	0.0005440	0.0000000	0.0000000	0.0011990
4	0.0000000	0.0000850	0.0000000	0.0000070
5	0.0001470	0.0135090	0.0001080	0.0000120
6	0.0000000	0.0475830	0.0002920	0.0000430
7	0.0002890	0.0000000	0.0000140	0.0000260
8	0.0023270	0.0277810	0.0272360	0.0029880
9	0.0000290	0.0000000	0.0000000	0.0001290
10	0.0017000	0.0000500	0.0020950	0.0049720
11	0.0000000	0.0000080	0.0000000	0.0000230
12	0.0193030	0.0000470	0.0002550	0.0002210
13	0.0031200	0.0001850	0.0003020	0.0010050
14	0.0201380	0.0000540	0.0000410	0.0008770
15	0.0029360	0.0000000	0.0000000	0.0002960
16	0.0328290	0.0005820	0.0004190	0.0083700
17	0.0061530	0.0000610	0.0008110	0.0016690
18	0.0386400	0.0014380	0.0007030	0.0026660
19	0.0033590	0.0066960	0.0275780	0.0098100
20	0.0359520	0.0001370	0.0029040	0.0018770
21	0.0067400	0.0000000	0.0000000	0.0001800
22	0.0070480	0.0000230	0.0001770	0.0012170
23	0.0582990	0.0020390	0.0038870	0.0005630
24	0.0280750	0.0000080	0.0016400	0.0018440
25	0.0040890	0.0000890	0.0015580	0.0022240
26	0.0121820	0.0006400	0.0046120	0.0018380
27	0.0052980	0.0000860	0.0085390	0.0014220
28	0.0019420	0.0000000	0.0001010	0.0008630
29	0.0697880	0.0000080	0.0000180	0.0012150
30	0.0050420	0.1534410	0.0063410	0.0133910
31	0.0188100	0.0178470	0.0619060	0.0144020
32	0.0426000	0.0045060	0.0183230	0.0148000
33	0.0184200	0.0108230	0.0316200	0.0565670
34	0.0372180	0.0165140	0.0413110	0.0519860
35	0.0026230	0.0162770	0.0027980	0.0089510
36	0.0001220	0.1133780	0.0166580	0.0031900
37	0.0093380	0.0032790	0.0055010	0.0083370

TABLE A-14 cont.

A00

	33	34	35	36
1	0.0046820	0.0001360	0.0000000	0.0000000
2	0.0077240	0.0002090	0.0000000	0.0001360
3	0.0000980	0.0000000	0.0000000	0.0000660
4	0.0000640	0.0000020	0.0000000	0.0000000
5	0.0000420	0.0000600	0.0028530	0.0060090
6	0.0003910	0.0000060	0.0000000	0.0032780
7	0.0001100	0.0000160	0.0000000	0.0001810
8	0.0455010	0.0069370	0.0026360	0.1421740
9	0.0000090	0.0000050	0.0000030	0.0000000
10	0.0005970	0.0020010	0.0000000	0.0000000
11	0.0000350	0.0000000	0.0000000	0.0000000
12	0.0001130	0.0003280	0.0003390	0.0000000
13	0.0002080	0.0012000	0.0003640	0.0002420
14	0.0001570	0.0000000	0.0000000	0.0000000
15	0.0000470	0.0000000	0.0000030	0.0000000
16	0.0011670	0.0013710	0.0007510	0.0019900
17	0.0022420	0.0516010	0.0009600	0.0004060
18	0.0016590	0.0078320	0.0004830	0.0071560
19	0.0034170	0.0038010	0.0010130	0.0057900
20	0.0003580	0.0019930	0.0004390	0.0005540
21	0.0000590	0.0011410	0.0000280	0.0000000
22	0.0002200	0.0016010	0.0000620	0.0017640
23	0.0002180	0.0000460	0.0000160	0.0011260
24	0.0004220	0.0017020	0.0001040	0.0000940
25	0.0010450	0.0049060	0.0000290	0.0001440
26	0.0003910	0.0054470	0.0001010	0.0020510
27	0.0003720	0.0054520	0.0004360	0.0010120
28	0.0001920	0.0046660	0.0000360	0.0000990
29	0.0000720	0.0048400	0.0000440	0.0001420
30	0.0041160	0.0111420	0.0034710	0.0654560
31	0.0075570	0.0265110	0.0363010	0.0108870
32	0.0088840	0.0197550	0.0021700	0.0036810
33	0.0797930	0.0450860	0.0086120	0.0150600
34	0.0312190	0.0511530	0.0077960	0.0285410
35	0.0061030	0.0072650	0.0001560	0.0007250
36	0.0042720	0.0013980	0.0001320	0.0003210
37	0.0031200	0.0128640	0.0035230	0.0010400

TABLE A-14 cont.

A00

	37
1	0.0014380
2	0.0042880
3	0.0003560
4	0.0000000
5	0.0000000
6	0.0000000
7	0.0000000
8	0.0000000
9	0.0003560
10	0.0948360
11	0.0071560
12	0.0000000
13	0.0003600
14	0.0001330
15	0.0000000
16	0.0001550
17	0.0004530
18	0.0021440
19	0.0000000
20	0.0001230
21	0.0014350
22	0.0002170
23	0.0000000
24	0.0002060
25	0.0000000
26	0.0034270
27	0.0000000
28	0.0012860
29	0.0023290
30	0.0000000
31	0.1423830
32	0.0198940
33	0.0000000
34	0.0602400
35	0.0000000
36	0.0000000
37	0.0000000

TABLE A-15
NATIONAL INPUT COEFFICIENT MATRIX
2020

A20

	1	2	3	4
1	0.2165320	0.0614730	0.0167050	0.0000000
2	0.3805780	0.0245670	0.0360590	0.0000000
3	0.0255370	0.0423960	0.0079280	0.0000000
4	0.0000000	0.0000000	0.0000000	0.2116130
5	0.0002340	0.0000160	0.0000000	0.0038670
6	0.0000000	0.0000000	0.0000000	0.0000000
7	0.0000940	0.0049320	0.0000000	0.0095770
8	0.0122740	0.0157580	0.0000000	0.0072550
9	0.0000000	0.0000000	0.0000000	0.0000000
10	0.1966170	0.0000850	0.0048760	0.0000000
11	0.0000000	0.0000000	0.0000000	0.0000000
12	0.0004810	0.0013750	0.0059190	0.0003330
13	0.0009660	0.0016820	0.0000790	0.0000130
14	0.0001090	0.0035740	0.0008170	0.0090130
15	0.0000000	0.0000000	0.0000000	0.0000000
16	0.0006680	0.0001440	0.0048760	0.0004340
17	0.0002760	0.0003520	0.0000240	0.0000850
18	0.0068460	0.0469890	0.0003800	0.0538320
19	0.0099850	0.0393550	0.0024930	0.0146880
20	0.0013810	0.0038030	0.0000590	0.0111410
21	0.0004170	0.0000000	0.0001800	0.0000130
22	0.0003450	0.0014140	0.0000280	0.0038890
23	0.0000430	0.0000310	0.0000000	0.0382020
24	0.0027780	0.0016100	0.0098750	0.0051350
25	0.0004440	0.0088590	0.0000390	0.0847630
26	0.0003140	0.0008810	0.0019880	0.0015210
27	0.0003470	0.0006650	0.0012560	0.0048160
28	0.0000000	0.0000000	0.0000430	0.0006730
29	0.0000840	0.0000710	0.0000120	0.0007720
30	0.0050590	0.0075420	0.0000570	0.0505800
31	0.0355270	0.0147880	0.0042180	0.1216450
32	0.0463960	0.0315050	0.0058960	0.0373360
33	0.0246240	0.0907140	0.0031210	0.1452140
34	0.0234940	0.0420930	0.0005590	0.0437780
35	0.0002460	0.0001760	0.0000420	0.0020880
36	0.0000300	0.0000330	0.0000230	0.0011840
37	0.0010060	0.0012780	0.0021510	0.0082700

TABLE A-15 cont.

A20

	5	6	7	8
1	0.0000000	0.0000000	0.0000000	0.0000000
2	0.0000000	0.0000000	0.0000000	0.0006740
3	0.0000000	0.0000000	0.0000000	0.0000080
4	0.0003170	0.0000410	0.0006890	0.0000000
5	0.2572990	0.0000000	0.0005150	0.0000000
6	0.0000000	0.0474850	0.0000000	0.0000000
7	0.0007670	0.0000000	0.0088560	0.0018690
8	0.0163330	0.0825910	0.0020040	0.0000790
9	0.0000000	0.0000000	0.0000000	0.0000120
10	0.0000000	0.0000000	0.0000090	0.0000690
11	0.0000000	0.0000000	0.0000170	0.0000000
12	0.0000000	0.0003810	0.0000520	0.0003500
13	0.0000000	0.0000000	0.0000000	0.0001190
14	0.0158880	0.0000220	0.0000410	0.0095310
15	0.0000000	0.0000000	0.0000000	0.0012750
16	0.0009230	0.0003040	0.0009310	0.0006130
17	0.0000560	0.0000750	0.0000160	0.0000070
18	0.0343400	0.0183460	0.0063730	0.0030240
19	0.0247740	0.0134230	0.0080270	0.0042250
20	0.0283970	0.0026450	0.0080960	0.0012860
21	0.0000000	0.0000000	0.0000040	0.0000030
22	0.0029850	0.0074960	0.0142860	0.0143090
23	0.0386720	0.0056640	0.0058150	0.0090220
24	0.0118260	0.0058540	0.0009210	0.0183390
25	0.1369480	0.0078000	0.0204370	0.0034550
26	0.0166040	0.0169280	0.0017640	0.0043370
27	0.0160650	0.0006110	0.0019850	0.0001240
28	0.0000000	0.0008620	0.0000000	0.0006720
29	0.0000000	0.0000000	0.0000080	0.0003790
30	0.0630850	0.0263270	0.0122430	0.0006710
31	0.0495650	0.0555000	0.0097480	0.0067490
32	0.0568930	0.0273990	0.0086740	0.0165120
33	0.1033760	0.4602760	0.0119540	0.0024080
34	0.0418150	0.0326880	0.0070640	0.0094060
35	0.0020270	0.0011510	0.0003820	0.0000700
36	0.0006750	0.0006560	0.0004290	0.0000950
37	0.0113300	0.0135250	0.0017250	0.0012660

TABLE A-15 cont.

A20

	9	10	11	12
1	0.0000000	0.1266490	0.0000000	0.0102100
2	0.0000000	0.0473310	0.0790670	0.0608100
3	0.0000000	0.0028110	0.0000000	0.0000000
4	0.0000000	0.0000000	0.0000000	0.0000000
5	0.0002270	0.0002600	0.0001080	0.0005480
6	0.0000000	0.0000000	0.0000000	0.0000000
7	0.0000000	0.0001190	0.0000000	0.0000220
8	0.0015240	0.0014690	0.0006510	0.0022040
9	0.0407860	0.0000000	0.0000000	0.0000000
10	0.0000000	0.1248060	0.0000960	0.0027530
11	0.0000000	0.0000000	0.1360950	0.0000000
12	0.0000000	0.0000310	0.0000000	0.2747370
13	0.0009130	0.0013000	0.0000000	0.0104310
14	0.0047410	0.0008010	0.0004350	0.0000160
15	0.0005150	0.0000000	0.0000000	0.0001800
16	0.0022540	0.0164560	0.0091950	0.0092430
17	0.0003290	0.0041720	0.0048160	0.0002000
18	0.0036310	0.0044510	0.0061050	0.0709610
19	0.0033750	0.0019930	0.0001720	0.0016580
20	0.0211090	0.0019440	0.0000050	0.0045020
21	0.0000640	0.0000280	0.0000060	0.0000750
22	0.0000000	0.0064810	0.0000000	0.0022550
23	0.0791940	0.0000260	0.0000000	0.0000220
24	0.0182310	0.0169400	0.0039050	0.0004260
25	0.0170230	0.0004300	0.0000170	0.0044380
26	0.0982240	0.0000560	0.0000000	0.0000620
27	0.4243600	0.0001240	0.0000060	0.0000240
28	0.0221480	0.0000860	0.0000770	0.0005270
29	0.0001870	0.0000690	0.0000170	0.0020430
30	0.0060130	0.0042060	0.0006160	0.0075390
31	0.0187110	0.0230600	0.0027170	0.0164160
32	0.0215730	0.0223260	0.0050780	0.0324130
33	0.0136080	0.0067080	0.0022940	0.0075650
34	0.0351020	0.0248430	0.0281600	0.0097660
35	0.0027690	0.0005080	0.0011580	0.0008090
36	0.0001100	0.0002360	0.0000130	0.0001550
37	0.0120090	0.0023520	0.0031730	0.0027780

TABLE A-15 cont.

A20

	13	14	15	16
1	0.0000000	0.0000000	0.0000000	0.0000000
2	0.0000000	0.0137300	0.0000000	0.0000000
3	0.0023970	0.0881010	0.0000000	0.0000000
4	0.0000000	0.0000000	0.0000000	0.0000000
5	0.0000280	0.0001510	0.0001710	0.0034820
6	0.0000000	0.0000000	0.0000000	0.0000000
7	0.0000000	0.0000320	0.0000000	0.0043180
8	0.0004150	0.0037310	0.0009270	0.0038540
9	0.0000000	0.0000000	0.0000000	0.0000170
10	0.0000030	0.0000560	0.0000100	0.0074210
11	0.0000000	0.0000000	0.0000000	0.0000000
12	0.1214010	0.0000180	0.0345210	0.0057260
13	0.0839370	0.0014700	0.0013170	0.0020610
14	0.0001560	0.2937240	0.0698140	0.0451330
15	0.0000540	0.0019950	0.0202610	0.0000390
16	0.0034300	0.0038510	0.0084650	0.2497270
17	0.0000420	0.0001380	0.0000880	0.0090080
18	0.0071480	0.0134530	0.0131980	0.0414510
19	0.0004000	0.0047410	0.0010630	0.0089710
20	0.0022430	0.0010890	0.0307230	0.0101180
21	0.0012640	0.0003280	0.0007220	0.0002020
22	0.0000440	0.0059070	0.0080510	0.0031580
23	0.0001070	0.0078300	0.0327660	0.0019260
24	0.0004130	0.0174920	0.0400200	0.0135680
25	0.0002500	0.0039430	0.0020540	0.0040160
26	0.0000530	0.0002650	0.0019960	0.0002030
27	0.0000270	0.0004320	0.0003840	0.0001030
28	0.0003520	0.0004250	0.0021840	0.0012200
29	0.0070560	0.0016110	0.0012830	0.0003340
30	0.0014460	0.0076230	0.0036470	0.0173660
31	0.0042410	0.0358050	0.0158710	0.0407700
32	0.0150990	0.0249870	0.0244930	0.0303500
33	0.0069460	0.0156910	0.0180930	0.0129960
34	0.0063160	0.0175460	0.0146320	0.0214360
35	0.0011400	0.0006500	0.0009650	0.0009870
36	0.0000280	0.0002130	0.0000650	0.0003600
37	0.0022710	0.0038980	0.0046270	0.0049880

TABLE A-15 cont.

A20

	17	18	19	20
1	0.0000000	0.0000510	0.0000000	0.0000000
2	0.0000000	0.0005270	0.0000000	0.0000000
3	0.0000000	0.0007290	0.0000000	0.0000000
4	0.0000000	0.0033820	0.0000000	0.0000000
5	0.0000210	0.0017290	0.0000010	0.0013670
6	0.0000000	0.0006660	0.0008940	0.0000000
7	0.0000000	0.0129790	0.0000090	0.0020880
8	0.0012090	0.0035830	0.0000410	0.0058200
9	0.0001700	0.0000040	0.0000000	0.0000760
10	0.0005940	0.0162980	0.0000030	0.0000590
11	0.0000000	0.0000000	0.0000000	0.0000000
12	0.0010910	0.0001480	0.0000000	0.1054570
13	0.0000000	0.0007190	0.0000000	0.0054200
14	0.0010860	0.0014710	0.0000000	0.0057040
15	0.0007760	0.0000000	0.0000000	0.0006660
16	0.1043180	0.0183780	0.0000150	0.0383050
17	0.0899570	0.0008650	0.0000000	0.0003650
18	0.0166940	0.1700550	0.0000620	0.3828690
19	0.0019450	0.0337630	0.0002100	0.0044200
20	0.0026070	0.0088190	0.0000000	0.0702620
21	0.0000360	0.0001100	0.0000000	0.0038380
22	0.0000000	0.0053560	0.0000050	0.0181090
23	0.0003270	0.0108750	0.0000050	0.0141820
24	0.0010510	0.0160570	0.0000140	0.0245790
25	0.0016730	0.0044610	0.0000010	0.0060540
26	0.0000820	0.0007070	0.0000000	0.0044280
27	0.0003110	0.0000720	0.0000000	0.0072130
28	0.0058770	0.0015800	0.0000000	0.0031630
29	0.0011940	0.0008680	0.0000000	0.0122010
30	0.0034440	0.0160430	0.0000390	0.0198970
31	0.0153490	0.0211220	0.0001020	0.0391010
32	0.0150360	0.0251530	0.0000400	0.0568120
33	0.0425220	0.0108050	0.0000670	0.0246900
34	0.0270410	0.0506270	0.0000570	0.0540560
35	0.0084710	0.0014750	0.0000020	0.0022320
36	0.0002240	0.0002670	0.0000000	0.0003890
37	0.0123000	0.0076960	0.0000040	0.0164560

TABLE A-15 cont.

A20

	21	22	23	24
1	0.0043160	0.0000000	0.0000000	0.0000000
2	0.0000000	0.0006070	0.0000000	0.0000000
3	0.0000000	0.0000000	0.0000000	0.0000000
4	0.0000000	0.0014550	0.0481950	0.0000210
5	0.0001620	0.0064170	0.0099700	0.0001520
6	0.0000000	0.0000000	0.0000000	0.0000000
7	0.0000300	0.0910800	0.0029040	0.0001260
8	0.0005460	0.0062350	0.0066140	0.0016120
9	0.0000000	0.0000000	0.0001340	0.0016460
10	0.0209730	0.0004140	0.0001820	0.0001830
11	0.0000000	0.0000000	0.0000000	0.0000000
12	0.0166420	0.0028650	0.0006430	0.0017050
13	0.0055400	0.0034260	0.0007640	0.0011100
14	0.0034640	0.0093840	0.0015630	0.0048510
15	0.0000210	0.0015010	0.0000080	0.0015070
16	0.0057620	0.0292390	0.0014030	0.0113980
17	0.0001080	0.0001810	0.0000780	0.0038230
18	0.0066940	0.0359660	0.0168310	0.0175740
19	0.0006150	0.0127990	0.0069860	0.0033270
20	0.0240750	0.0139400	0.0007300	0.0070020
21	0.0918640	0.0003840	0.0002050	0.0004830
22	0.0001610	0.1264890	0.0020910	0.0073600
23	0.0006600	0.0151090	0.2911240	0.2824540
24	0.0027100	0.0124850	0.0180080	0.0577500
25	0.0001070	0.0106150	0.0201040	0.0284420
26	0.0000690	0.0034650	0.0068000	0.0098710
27	0.0000250	0.0008380	0.0038370	0.0129400
28	0.0020220	0.0005410	0.0007980	0.0045630
29	0.0055330	0.0029900	0.0010590	0.0018210
30	0.0018300	0.0400610	0.0247840	0.0076510
31	0.0055530	0.0633800	0.0434100	0.0188480
32	0.0123030	0.0339350	0.0307010	0.0269360
33	0.0057780	0.0183820	0.0244020	0.0167890
34	0.0093840	0.0330120	0.0154830	0.0213780
35	0.0012430	0.0013550	0.0007100	0.0010400
36	0.0000540	0.0011540	0.0003110	0.0001260
37	0.0021810	0.0093010	0.0036820	0.0081700

TABLE A-15 cont.

A20

	25	26	27	28
1	0.0000000	0.0000000	0.0000000	0.0000000
2	0.0000000	0.0000000	0.0000000	0.0002440
3	0.0000000	0.0000000	0.0000000	0.0000000
4	0.0000000	0.0001620	0.0000000	0.0000080
5	0.0003500	0.0002140	0.0002550	0.0001610
6	0.0000000	0.0000000	0.0000000	0.0000000
7	0.0000380	0.0000400	0.0000380	0.0000130
8	0.0028700	0.0018530	0.0020210	0.0008810
9	0.0008510	0.0058040	0.0040190	0.0012420
10	0.0003470	0.0000100	0.0000000	0.0010010
11	0.0000000	0.0000000	0.0000000	0.0000000
12	0.0005240	0.0009730	0.0031940	0.0038850
13	0.0014000	0.0013540	0.0082200	0.0006830
14	0.0029230	0.0017260	0.0032510	0.0004990
15	0.0007440	0.0083760	0.0013960	0.0003190
16	0.0046850	0.0129990	0.0009130	0.0072590
17	0.0003500	0.0020150	0.0001180	0.0002060
18	0.0068320	0.0178340	0.0058370	0.0146140
19	0.0057760	0.0049330	0.0025290	0.0009300
20	0.0159360	0.0247460	0.0164180	0.0090650
21	0.0010100	0.0002110	0.0000400	0.0005720
22	0.0101230	0.0195030	0.0088720	0.0047740
23	0.1735180	0.1094340	0.0991570	0.0263720
24	0.0708990	0.0601670	0.0563130	0.0130810
25	0.2605330	0.0417360	0.0565340	0.0089860
26	0.0686180	0.1962370	0.0290940	0.0307170
27	0.0298790	0.0170350	0.2771860	0.0064780
28	0.0075870	0.0148830	0.0095100	0.0341130
29	0.0015390	0.0025550	0.0004280	0.0021260
30	0.0087080	0.0081610	0.0048750	0.0022250
31	0.0216630	0.0197920	0.0179540	0.0072860
32	0.0446930	0.0468430	0.0239200	0.0162050
33	0.0234490	0.0181610	0.0066790	0.0079580
34	0.0309840	0.0424010	0.0266460	0.0213470
35	0.0021740	0.0022740	0.0013040	0.0009870
36	0.0001820	0.0002030	0.0001270	0.0000820
37	0.0135000	0.0152390	0.0058640	0.0067190

TABLE A-15 cont.

#20

	29	30	31	32
1	0.0000000	0.0000000	0.0000240	0.0000000
2	0.0012910	0.0000000	0.0011040	0.0000000
3	0.0005190	0.0000000	0.0000000	0.0009850
4	0.0000000	0.0000680	0.0000000	0.0000050
5	0.0001400	0.0108430	0.0000980	0.0000100
6	0.0000000	0.0381960	0.0002640	0.0000350
7	0.0002750	0.0000000	0.0000120	0.0000210
8	0.0022180	0.0223000	0.0246010	0.0024550
9	0.0000270	0.0000000	0.0000000	0.0001060
10	0.0016200	0.0000400	0.0018920	0.0040840
11	0.0000000	0.0000070	0.0000000	0.0000190
12	0.0183970	0.0000380	0.0002300	0.0001810
13	0.0029740	0.0001490	0.0002730	0.0008260
14	0.0191920	0.0000440	0.0000370	0.0007210
15	0.0027980	0.0000000	0.0000000	0.0002430
16	0.0312680	0.0004670	0.0003790	0.0068760
17	0.0058640	0.0000490	0.0007330	0.0013710
18	0.0368250	0.0011540	0.0006350	0.0021900
19	0.0032020	0.0053750	0.0249100	0.0080590
20	0.0342640	0.0001100	0.0026230	0.0015420
21	0.0064230	0.0000000	0.0000000	0.0001480
22	0.0067170	0.0000190	0.0001600	0.0010000
23	0.0555620	0.0016370	0.0035110	0.0004630
24	0.0267560	0.0000070	0.0014810	0.0015150
25	0.0038970	0.0000710	0.0014070	0.0018270
26	0.0116100	0.0005130	0.0041660	0.0015100
27	0.0050490	0.0000690	0.0077130	0.0011680
28	0.0018510	0.0000000	0.0000910	0.0007090
29	0.0665120	0.0000070	0.0000160	0.0009980
30	0.0048060	0.1231690	0.0057280	0.0110010
31	0.0179270	0.0143260	0.0559180	0.0118310
32	0.0406000	0.0036170	0.0165500	0.0121590
33	0.0175550	0.0086870	0.0285610	0.0464700
34	0.0354700	0.0132560	0.0373150	0.0427070
35	0.0025000	0.0130660	0.0025270	0.0073540
36	0.0001160	0.0910100	0.0150470	0.0026210
37	0.0089000	0.0026320	0.0049690	0.0068490

TABLE A-15 cont.

A20

	33	34	35	36
1	0.0032470	0.0001140	0.0000000	0.0000000
2	0.0053570	0.0001760	0.0000000	0.0001140
3	0.0000680	0.0000000	0.0000000	0.0000550
4	0.0000440	0.0000010	0.0000000	0.0000000
5	0.0000290	0.0000510	0.0011040	0.0050380
6	0.0002710	0.0000050	0.0000000	0.0027480
7	0.0000760	0.0000140	0.0000000	0.0001520
8	0.0315570	0.0058420	0.0010200	0.1191990
9	0.0000060	0.0000040	0.0000010	0.0000000
10	0.0004140	0.0016850	0.0000000	0.0000000
11	0.0000240	0.0000000	0.0000000	0.0000000
12	0.0000790	0.0002760	0.0001310	0.0000000
13	0.0001440	0.0010110	0.0001410	0.0002030
14	0.0001090	0.0000000	0.0000000	0.0000000
15	0.0000330	0.0000000	0.0000010	0.0000000
16	0.0008090	0.0011540	0.0002900	0.0016680
17	0.0015550	0.0434560	0.0003710	0.0003400
18	0.0011510	0.0065960	0.0001870	0.0060000
19	0.0023700	0.0032010	0.0003920	0.0048540
20	0.0002480	0.0016790	0.0001700	0.0004650
21	0.0000410	0.0009610	0.0000110	0.0000000
22	0.0001520	0.0013480	0.0000240	0.0014790
23	0.0001510	0.0000390	0.0000060	0.0009440
24	0.0002930	0.0014330	0.0000400	0.0000790
25	0.0007250	0.0041310	0.0000110	0.0001210
26	0.0002710	0.0045870	0.0000390	0.0017190
27	0.0002580	0.0045910	0.0001690	0.0008490
28	0.0001330	0.0039290	0.0000140	0.0000830
29	0.0000500	0.0040760	0.0000170	0.0001190
30	0.0028550	0.0093830	0.0013430	0.0548780
31	0.0052410	0.0223260	0.0140420	0.0091280
32	0.0061620	0.0166370	0.0008400	0.0030860
33	0.0553390	0.0379690	0.0033310	0.0126270
34	0.0216510	0.0430780	0.0030160	0.0239280
35	0.0042330	0.0061190	0.0000600	0.0006080
36	0.0029630	0.0011780	0.0000510	0.0002690
37	0.0021640	0.0108500	0.0013630	0.0008720

TABLE A-15 cont.

A20

	37
1	0.0001870
2	0.0005590
3	0.0000460
4	0.0000000
5	0.0000000
6	0.0000000
7	0.0000000
8	0.0000000
9	0.0000460
10	0.0123630
11	0.0009330
12	0.0000000
13	0.0000470
14	0.0000170
15	0.0000000
16	0.0000200
17	0.0000590
18	0.0002800
19	0.0000000
20	0.0000160
21	0.0001870
22	0.0000280
23	0.0000000
24	0.0000270
25	0.0000000
26	0.0004470
27	0.0000000
28	0.0001680
29	0.0003040
30	0.0000000
31	0.0185610
32	0.0025930
33	0.0000000
34	0.0078530
35	0.0000000
36	0.0000000
37	0.0000000

APPENDIX B

COMPUTER PROGRAMS

PROGRAM EMPROJ

```
#FILE (BANNY)EMPROJ ON A01CUST
$SET LIMIT=100
$RESET FREE
FILE  5=EMPDAT,UNIT=DISK,RECORD=14,BLOCKING=30
FILE  6=EMPOUT,UNIT=PRINTER,RECORD=22
FILE  7=FINEMP,UNIT=DISK,RECORD=14,BLOCKING=30
      INTEGER YR
      REAL NEM,NEV,NEV63,NEV80,NEV00,NEV20
      DIMENSION SUMR(10),SUMN(10)
      DIMENSION REV(37),NEV(37)
      DIMENSION Y(10),YR(10)
      DIMENSION REV63(37),REV80(37),REV00(37),REV20(37)
      DIMENSION NEV63(37),NEV80(37),NEV00(37),NEV20(37)
      DIMENSION XVALR(3,3),XVALN(3,3)
      DIMENSION VALR(10,3),VALN(10,3)
      DIMENSION REM(10,37),NEM(10,37)
      DIMENSION SUMO(10)
      COMMON Y,YR,F,ISEC,ISEQ,KSEQ,IDW,ILN
      COMMON REV63,REV80,REV00,REV20,NEV63,NEV80,NEV00,NEV20,REV,NEV
      COMMON XVALR,XVALN
C-     RFV      REGIONAL EMPLOYMENT VECTOR
C-     NEV      NATIONAL EMPLOYMENT VECTOR
C-     REM      REGIONAL EMPLOYMENT MATRIX
C-     XVALR    REGIONAL PROJECTED INDEPENDENT VALUES MATRIX
C-     XVALN    NATIONAL PROJECTED INDEPENDENT VALUES MATRIX
       IRUN=1
      CALL ADRESS
      WRITE (6,1)
      READ(5,76) ITRAN,ILN,IDW
      WRITE(6,77) ITRAN,ILN,IDW
   76 FORMAT(6X,I1,5X,I1,5X,I1)
   77 FORMAT(1H0,6HITRAN=,I1,2X,4HILN=,I1,2X,4HIDW=,I1)
      READ (5,2) NSEC,NYR,NVAR
      WRITE(6,3)NSEC,NYR,NVAR
C- NSEC -NUMBER OF SECTORS
C- NYR  -NUMBER OF YEARS
C- NVAR -NUMBER OF INDEPENDENT VARIABLES
      READ(5,4) F
      WRITE(6,5)F
      WRITE(6,5)F
      DO 10 I=1,3
   10 READ (5,6) (XVALR(I,J),J=1,NVAR)
      DO 100 I=1,3
  100 READ (5,6) (XVALN(I,J),J=1,NVAR)
      WRITE(6,80)
      WRITE(6,7)
      DO 15 I=1,3
      L=1960+20*I
   15 WRITE (6,8) L,(XVALR(I,J),J=1,NVAR)
      WRITE (6,70)
      WRITE (6,7)
      DO 25 I=1,3
      L=1960+20*I
```

PROGRAM EMPROJ cont.

```
   25 WRITE (6,8) L,(XVALN(I,J),J=1,NVAR)
      DO 60 J=1,NVAR
   60 READ (5,108) (VALR(K,J),K=1,NYR)
      DO 79 J=1,NVAR
   79 READ (5,108) (VALN(K,J),K=1,NYR)
      WRITE(6,1492) IRUN,F,ITRAN,ILN
 1492 FORMAT(1H1//////////25X,5HIRUN=,I1,5X,2HF=,F8.4,5X,
     16HITRAN=,I1,5X,4HILN=,I1)
      WRITE(6,1)
      WRITE(6,1018)
 1018 FORMAT(1H0,9X,3HX1R,10X,3HX2R,10X,3HX3R,18X,3HX1N,10X,3HX2N,10X,
     13HX3N)
      DO 3300 I=1,NYR
      DO 3300 J=1,NVAR
      VALR(I,J)=1/VALR(I,J)
      VALN(I,J)=1/VALN(I,J)
 3300 CONTINUE
      DO 56 I=1,NYR
      WRITE(6,46) (VALR(I,J),J=1,NVAR),(VALN(I,J),J=1,NVAR)
   56 CONTINUE
      DO 3400 I=1,3
      DO 3400 J=1,NVAR
      XVALR(I,J)=1/XVALR(I,J)
      XVALN(I,J)=1/XVALN(I,J)
 3400 CONTINUE
      DO 57 I=1,3
      WRITE(6,46) (XVALR(I,J),J=1,NVAR),(XVALN(I,J),J=1,NVAR)
   57 CONTINUE
   46 FORMAT(1H0,1X,3(F10.4,3X),5X,3(F10.4,3X))
      IF (IRUN.GE.2) GO TO 20
      DO 20 K=1,NYR
      READ (5,9) YR(K)
      READ (5,11) (REV(I),I=1,NSEC)
      CALL XSUM(REV,SUM,N)
      SUMR(K)=SUM
      READ (5,11) (NEV(I),I=1,NSEC)
      IF (YR(K).EQ.1964) GO TO 300
      GO TO 310
  300 CONTINUE
      CALL BASE(REV,NEV,REV63,NEV63,NSEC)
  310 CONTINUE
      CALL XSUM(NEV,SUM,N)
      SUMN(K)=SUM
      WRITE(6,906) YR(K)
  906 FORMAT(1H1,54X,I4)
  907 FORMAT(1H1,54X,I4)
      CALL PRINTV(REV,NSEC,3HREV)
      WRITE(6,907) YR(K)
      CALL PRINTV(NEV,NSEC,3HNEV)
      DO 20 I=1,NSEC
      REM(K,I)=REV(I)
      NEM(K,I)=NEV(I)
   20 CONTINUE
  425 FORMAT(5F15.5)
```

PROGRAM EMPROJ cont.

```
      KSEQ=0
      ISEQ=0
      DO 120 J=3,34
      ISEC=J
      ISEQ=ISEQ+1
      WRITE (6,1)
      WRITE (6,207)
      WRITE(6,201) ISEQ,ISEC
      WRITE (6,202)
      DO 110 K=1,NYR
      Y(K)=ALOG(REM(K,J))
      WRITE (6,203)  YR(K),Y(K),(VALR(K,JJ),JJ=1,NVAR)
  110 CONTINUE
      CALL REGRES(NYR,NVAR,VALR,XVALR,XVALN)
  120 CONTINUE
      KSEQ=1
      ISEQ=0
      DO 130 J=3,34
      ISEC=J
      WRITE (6,1)
      ISEQ=ISEQ+1
      WRITE(6,208)
      WRITE(6,201) ISEQ,ISEC
      WRITE (6,202)
      DO 140 K=1,NYR
      Y(K)=ALOG(NEM(K,J))
      WRITE (6,203)  YR(K),Y(K),(VALN(K,JJ),JJ=1,NVAR)
  140 CONTINUE
      CALL REGRES(NYR,NVAR,VALN,XVALR,XVALN)
  130 CONTINUE
      DO 7311 I=1,2
      J=38-I
      REV80(I)=0.0
      REV80(J)=0.0
      NEV80(I)=0.0
      NEV80(J)=0.0
      REV00(J)=0.0
      REV00(I)=0.0
      NEV00(I)=0.0
      NEV00(J)=0.0
      REV20(I)=0.0
      REV20(J)=0.0
      NEV20(I)=0.0
      NEV20(J)=0.0
 7311 CONTINUE
      REV80(35)=0.0
      REV00(35)=0.0
      REV20(35)=0.0
      NEV80(35)=0.0
      NEV00(35)=0.0
      NEV20(35)=0.0
      WRITE(6,1)
       WRITE(6,66)
      DO 170 I=1,NSEC
```

PROGRAM EMPROJ cont.

```
      WRITE(6,68) I,REV63(I),REV80(I),REV00(I),REV20(I)
      WRITE(6,68) I,REV63(I),REV80(I),REV00(I),REV20(I)
  170 CONTINUE
      WRITE(6,1)
      WRITE(6,67)
      DO 180 I=1,NSEC
      WRITE(6,68) I,NEV63(I),NEV80(I),NEV00(I),NEV20(I)
      WRITE(6,68) I,NEV63(I),NEV80(I),NEV00(I),NEV20(I)
  180 CONTINUE
      CALL PUNCHV(REV63,NSEC)
      CALL PUNCHV(REV80,NSEC)
      CALL PUNCHV(REV00,NSEC)
      CALL PUNCHV(REV20,NSEC)
      CALL PUNCHV(NEV63,NSEC)
      CALL PUNCHV(NEV80,NSEC)
      CALL PUNCHV(NEV00,NSEC)
      CALL PUNCHV(NEV20,NSEC)
      WRITE(6,1)
      WRITE(6,2001)
 2001 FORMAT(1H0,10X,4HYEAR,5X,15HREG EMP/NAT EMP)
      DO 188 I=1,NYR
      IF (IRUN.GE.2) GO TO 8974
      SUMO(I)=SUMR(I)/SUMN(I)
      WRITE(6,646) YR(I),SUMO(I)
      WRITE(6,646) YR(I),SUMO(I)
 8974 CONTINUE
  188 CONTINUE
      CALL XSUM(REV80,SUM,N)
      SUMR(1)=SUM
      CALL XSUM(REV00,SUM,N)
      SUMR(2)=SUM
      CALL XSUM(REV20,SUM,N)
      SUMR(3)=SUM
      CALL XSUM(NEV80,SUM,N)
      SUMN(1)=SUM
      CALL XSUM(NEV00,SUM,N)
      SUMN(2)=SUM
      CALL XSUM(NEV20,SUM,N)
      SUMN(3)=SUM
      DO 189 I=1,3
      SUMN(I)=SUMR(I)/SUMN(I)
      L=1960+20*I
      WRITE(6,648) L,SUMN(I)
      WRITE(6,648) L,SUMN(I)
  648 FORMAT(1H0,10X,I4,10X,F6.4)
  189 CONTINUE
  646 FORMAT(1H0,10X,I4,10X,F6.4)
   66 FORMAT(15X,5HREV63,10X,5HREV80,10X,5HREV00,10X,5HREV20)
   67 FORMAT(15X,5HNEV63,10X,5HNEV80,10X,5HNEV00,10X,5HNEV20)
   68 FORMAT(8X,I2,1X,F10.0,5X,F10.0,5X,F10.0,5X,F10.0)
    1 FORMAT (1H1)
    2 FORMAT (5X,I2,5X,I2,6X,I2)
    3 FORMAT (1H0,4X,5HNSEC=,I2,2X,4HNYR=,I2,2X,5HNVAR=,I2)
    4 FORMAT (2X,F10.0)
```

PROGRAM EMPROJ cont.

```
    5 FORMAT (1H0,10X,2HF=,F7.4)
    6 FORMAT (3F10.0)
    7 FORMAT(1H0,10X,4HYEAR,8X,2HX1,8X,2HX2,7X,2HX3)
    8 FORMAT(1H0,10X,I4,3X,F10.3,F10.3,F8.0)
    9 FORMAT(I4)
   11 FORMAT (F14.2)
  108 FORMAT (F10.0)
   70 FORMAT (1H1,50X,30HNATIONAL INDEPENDENT VARIABLES)
   80 FORMAT (1H1,50X,30HREGIONAL INDEPENDENT VARIABLES)
  201 FORMAT(1H ,20X,16HEQUATION NUMBER ,I4,8X,4HROW ,I2/)
  202 FORMAT(1H ,19X,4HYEAR,5X,11H     Y     5X,11H     X1     5X,11H
     1  X2     5X,11H     X3     )
  203 FORMAT(1H ,20X,I4,1X,F11.7,4(5X,F11.7))
  207 FORMAT(50X,19HREGIONAL REGRESSION/)
  208 FORMAT(50X,19HNATIONAL REGRESSION/)
      CALL ADRESS
 1066 CONTINUE
      LOCK 7
      LOCK 6
      END
      SUBROUTINE PUNCHV(V,N)
      DIMENSION V(37)
      ISEQ=0
      DO 10 I=1,N
      ISEQ=ISEQ+1
   10 WRITE(7,1) V(I),ISEQ
    1  FORMAT(1F14.0,40X,I2)
      RETURN
      END
      SUBROUTINE REGRES(N,K,VAL,SVALR,SVALN)
      INTEGER YR
      REAL NEM,NEV,NEV63,NEV80,NEV00,NEV20
      DIMENSION SVALR(3,3),SVALN(3,3)
      DIMENSION TSTAT(5)
      DIMENSION DEV(10)
      DIMENSION  A(11,11),B(11),SB(11),SIGMA(11),SUMX(11),XBAR(11)
      DIMENSION  YPRED (50),X(10)
      DIMENSION Y(10),YR(10)
      DIMENSION VAL(10,3)
      DIMENSION REV63(37),REV80(37),REV00(37),REV20(37)
      DIMENSION NEV63(37),NEV80(37),NEV00(37),NEV20(37)
      DIMENSION XVALR(3,3),XVALN(3,3),BVAL(10,3)
      COMMON Y,YR,F,ISEC,ISEQ,KSEQ,IDW,ILN
      COMMON REV63,REV80,REV00,REV20,NEV63,NEV80,NEV00,NEV20,REV,NEV
C-   N IS THE NUMBER OF OBSERVATIONS
C-   K IS THE NUMBER OF INDEPENDENT VARIABLES
C-   F IS THE MARGINAL F VALUE USED TO TEST FOR INCLUSION OF A VARIABLE
      M=K+1
      XN=N
      JUNE=0
      DO 8 I=1,M
      SUMX(I)=0.0
      DO 8 J=1,M
    8 A(I,J)=0.0
```

PROGRAM EMPROJ cont.

```
      DO 30  L=1,N
      DO 60 LL=1,K
   60 X(LL)=VAL(L,LL)
      X(M)=Y(L)
      DO 618 I=1,M
      SUMX(I)=SUMX(I)+X(I)
      DO 618 J=I,M
      A(I,J)=A(I,J)+X(I)*X(J)
  618 A(J,I)=A(I,J)
   30 CONTINUE
      DO 20 I=1,M
   20 XBAR(I)=SUMX(I)/XN
      WRITE (6,1010) F
      DO 22 I=1,M
      DO 22 J=1,M
   22 A(I,J)=A(I,J)-SUMX(I)*SUMX(J)/XN
      SYY=A(M,M)
      DO 23 I=1,M
   23 SIGMA(I)=SQRT(A(I,I))
      DO 24 I=1,M
      DO 24 J=1,M
      IF(I-J) 25,26,25
   25 A(I,J)=A(I,J)/(SIGMA(I)*SIGMA(J))
      GO TO 24
   26 A(I,I)=1.0
   24 CONTINUE
      DF=XN-1.0
      WRITE (6,1024)
      DO 27 I=1,K
   27 WRITE (6,1011) (A(I,J),J=1,K)
      WRITE (6,1012) (A(M,J),J=1,K)
      ISTEP=0
  151 SE=SIGMA(M)*SQRT(A(M,M)/DF)
      VMIN=1.0E50
      VMAX=0.0
      NMIN=0
      NMAX=0
      DO 64 I=1,K
      IF (A(I,I)-0.0001)64,64,51
   51 VAR =A(I,M)*A(M,I)/A(I,I)
      IF(VAR) 57,64,53
   53 IF(VAR-VMAX) 75,75,54
   54 VMAX=VAR
      NMAX=I
   75 B(I)=0.0
      SB(I)=0.0
      GO TO 64
   57 B(I)=A(I,M)*SIGMA(M)/SIGMA(I)
      SB(I)=SE/SIGMA(I)*SQRT(A(I,I))
      ABVAR=ABS(VAR)
      ABVMIN=ABS(VMIN)
      IF(ABVAR-ABVMIN)81,64,64
   81 VMIN=VAR
      NMIN=I
```

PROGRAM EMPROJ cont.

```
 64 CONTINUE
    BZERO=0.0
    SUMBI=0.0
    DO 67 I=1,K
 67 SUMBI=SUMBI+B(I)*XBAR(I)
    BZERO=XBAR(M)-SUMBI
    IF(ISTEP) 301,301,99
301 STDEV=SQRT(SYY/DF)
    WRITE (6,1013) STDEV
    WRITE (6,1014) BZERO
 99 XF2=ABS(VMIN)*DF/A(M,M)
    IF (XF2-F) 92,93,93
 92 KR=NMIN
    DF=DF+1.0
    NVOUT=NMIN
    NVIN=0
    FLVLX=XF2
100 IF(ISTEP) 101,101,102
101 WRITE (6,1015)
    GO TO 201
102 WRITE (6,1016)
    WRITE (6,1017) ISTEP,FLVL,NXIN,NXOUT,SE
    WRITE (6,1018)
    DO 90 I=1,K
    IF(B(I)) 89,90,89
    TSTAT(I)=B(I)/SB(I)
    WRITE(6,1019) B(I),I,SB(I),TSTAT(I)
 89 WRITE (6,1019) B(I),I,SB(I),TSTAT(I)
 90 CONTINUE
    WRITE (6,1020) BZERO
    CC=SQRT(1.0-SE**2*DF/SYY)
    WRITE (6,1021) CC
    GO TO 97
 93 XF1=VMAX*(DF-1.0)/(A(M,M)-VMAX)
    IF(XF1-F) 200,200,94
 94 KR=NMAX
    DF=DF-1.0
    NVOUT=0
    NVIN=NMAX
    FLVLX=XF1
    IF (ISTEP) 97,97,100
 97 IF (JUNE)197,197,201
197 DO 109 I=1,M
    IF (I-KR) 98,109,98
 98 DO 109 J=1,M
    IF (J-KR) 108,109,108
108 A(I,J)= ((A(KR,KR)*A(I,J))-(A(I,KR)*A(KR,J)))/A(KR,KR)
109 CONTINUE
    DO 112 I=1,M
    IF (I-KR) 113,112,113
113 A(I,KR)=-A(I,KR)/A(KR,KR)
112 CONTINUE
    DO 114 J=1,M
    IF (J-KR) 115,114,115
```

PROGRAM EMPROJ cont.

```
 115 A(KR,J)=A(KR,J)/A(KR,KR)
 114 CONTINUE
     A(KR,KR)=1.0/A(KR,KR)
     ISTEP=ISTEP+1
     FLVL=FLVLX
     NXIN=NVIN
     NXOUT=NVOUT
     GO TO 151
 200 JUNE=1
     GO TO 100
 201 WRITE (6,1022)
     DW=0.0
     DTSQ=0.0
     DO 130 I=1,N
     YPRED(I)=0.0
     DO 131 II=1,K
 131 X(II)=VAL(I,II)
     X(M)=Y(I)
     DO 139 J=1,K
 139 YPRED(I)=YPRED(I)+(B(J)*X(J))
     YPRED(I)=YPRED(I)+BZERO
     DEV(I)=X(M)-YPRED(I)
     DTSQ=DTSQ+DEV(I)*DEV(I)
     IF(I-1)129,129,128
 128 DW=DW+(DEV(I)-DT)**2
 129 DT=DEV(I)
 130 WRITE(6,1023) I,X(M),YPRED(I),DEV(I)
     DW=DW/DTSQ
     WRITE (6,1025) DW
1010 FORMAT(15HOF VALUE INPUT F8.3)
1011 FORMAT(1X,10F9.5)
1012 FORMAT(83HOSIMPLE CORRELATION COEFFICIENT BETWEEN EACH\
         \ INDEPENDENT
    1 AND THE DEPENDENT VARIABLE / 1X,10F9.5)
1013 FORMAT(22HOSTANDARD DEVIATION Y F11.4)
1014 FORMAT(14H MEAN VALUE Y F14.4)
1015 FORMAT(31HO  NO VARIABLES MEET THE F TEST)
1016 FORMAT(55HOSTEP =     F VALUE     VAR IN     VAR OUT  STD ERROR Y\
         \)
1017 FORMAT(I4,6X,F9.3,6X,I2,10X,I2,4X,F10.3)
1018 FORMAT(1HO,3X,50HCOEFFICIENT  VARIABLE  STANDARD ERROR  T\
         \-STATISTI
    1C)
1019 FORMAT(1X,F14.2,2X,I3,4X,F14.2,1X,F12.2)
1020 FORMAT(10HOCONSTANT 2X,F22.4)
1021 FORMAT(25H CORRELATION COEFFICIENT F7.4)
1022 FORMAT(46HOOBSERVATION   ACTUAL    PREDICTED   DEVIATION /)
1023 FORMAT(3X,I4,4X,3(F10.2,2X))
1024 FORMAT(63HOSIMPLE CORRELATION COEFFICIENT AMONG THE INDEPENDENT\
         \ VA
    1RIABLES)
1025 FORMAT(1X/ 28H DURBIN - WATSON COEFFICIENT 4X,E13.6)
 567 CONTINUE
     CALL PREDIC (B,BZERO,N,K,SVALR,SVALN)
```

PROGRAM EMPROJ cont.

```
      RETURN
      END
      SUBROUTINE PREDIC(B,BZERO,N,K,SVALR,SVALN)
      INTEGER YR
      REAL NEM,NEV,NEV63,NEV80,NEV00,NEV20
      DIMENSION SVALR(3,3),SVALN(3,3)
      DIMENSION B(11)
      DIMENSION Y(10),YR(10)
      DIMENSION REV63(37),REV80(37),REV00(37),REV20(37)
      DIMENSION NEV63(37),NEV80(37),NEV00(37),NEV20(37)
      DIMENSION XVALR(3,3),XVALN(3,3)
      DIMENSION WVALR(3,3),WVALN(3,3)
      DIMENSION REV(37),NEV(37)
      COMMON Y,YR,F,ISEC,ISEQ,KSEQ,IDW,ILN
      COMMON REV63,REV80,REV00,REV20,NEV63,NEV80,NEV00,NEV20,REV,NEV
      DO 65 I=1,3
      DO 65 J=1,K
      XVALR(I,J)=SVALR(I,J)
      XVALN(I,J)=SVALN(I,J)
   65 CONTINUE
      IF (KSEQ.EQ.1) GO TO 100
      REV80(ISEC)=0.0
      REV00(ISEC)=0.0
      REV20(ISEC)=0.0
      DO 10 I = 1,K
      REV80(ISEC)=REV80(ISEC) + B(I)*XVALR(1,I)
      REV00(ISEC)=REV00(ISEC) + B(I)*XVALR(2,I)
      REV20(ISEC)=REV20(ISEC) + B(I)*XVALR(3,I)
   10 CONTINUE
      REV80(ISEC)=REV80(ISEC)+BZERO
      REV00(ISEC)=REV00(ISEC)+BZERO
      REV20(ISEC)=REV20(ISEC)+BZERO
      REV80(ISEC)=EXP(REV80(ISEC))
      REV00(ISEC)=EXP(REV00(ISEC))
      REV20(ISEC)=EXP(REV20(ISEC))
      GO TO 500
  100 CONTINUE
      NEV80(ISEC)=0.0
      NEV00(ISEC)=0.0
      NEV20(ISEC)=0.0
      DO 20 I=1,K
      NEV80(ISEC)=NEV80(ISEC) + B(I)*XVALN(1,I)
      NEV00(ISEC)=NEV00(ISEC) + B(I)*XVALN(2,I)
      NEV20(ISEC)=NEV20(ISEC) + B(I)*XVALN(3,I)
   20 CONTINUE
      NEV80(ISEC)=NEV80(ISEC)+BZERO
      NEV00(ISEC)=NEV00(ISEC)+BZERO
      NEV20(ISEC)=NEV20(ISEC)+BZERO
      NEV80(ISEC)=EXP(NEV80(ISEC))
      NEV00(ISEC)=EXP(NEV00(ISEC))
      NEV20(ISEC)=EXP(NEV20(ISEC))
  500 CONTINUE
      DO 66 I=1,3
      DO 66 J=1,K
```

PROGRAM EMPROJ cont.

```
      XVALR(I,J)=SVALR(I,J)
      XVALN(I,J)=SVALN(I,J)
   66 CONTINUE
      RETURN
      END
      SUBROUTINE PRINTV(X,N,TITLE)
      DIMENSION X(37)
      WRITE(6,2) TITLE
    2 FORMAT(56X,A6/)
      DO 10 I=1,N
   10 WRITE(6,3) X(I),I
    3 FORMAT(1H ,47X,F14.0,4X,I2)
      RETURN
      END
      SUBROUTINE BASE(REV,NEV,REV63,NEV63,NSEC)
      REAL NEV,NEV63
      DIMENSION REV(37),NEV(37),REV63(37),NEV63(37)
      DO 10 I=1,NSEC
      REV63(I)=REV(I)
      NEV63(I)=NEV(I)
   10 CONTINUE
      RETURN
      END
      SUBROUTINE TRANS(VAL,XVAL,N,K)
      DIMENSION VAL(10,3),XVAL(3,3)
      DO 10 I=1,N
      DO 10 J=1,K
      VAL(I,J)=ALOG(VAL(I,J))
   10 CONTINUE
      DO 20 I=1,3
      DO 20 J=1,K
      XVAL(I,J)=ALOG(XVAL(I,J))
   20 CONTINUE
      RETURN
      END
      SUBROUTINE XSUM(X,SUM,N)
      DIMENSION X(37)
      SUM=0.0
      DO 10 I=3,34
      SUM=SUM+X(I)
   10 CONTINUE
      RETURN
      END
      SUBROUTINE ADRESS
      DO 10 I=1,5
      WRITE(6,100)
  100    FORMAT(1H1,"PROPERTY OF IRA SILVER,BANK OF N.Y.")
   10  CONTINUE
       RETURN
       END
#
```

PROGRAM FINAL

```
#FILE (BANNY)FINAL ON A01CUST
$SET LIMIT=100
$RESET FREE
FILE  5=FINDAT,UNIT=DISK,RECORD=14,BLOCKING=30
FILE  9=FINOUT,UNIT=PRINTER,RECORD=22
FILE  7=FINEMP,UNIT=DISK,RECORD=14,BLOCKING=30
FILE  4=TTYIO,UNIT=REMOTE,IO,RECORD=14
FILE  8=FINTST,UNIT=DISK,RECORD=14,BLOCKING=30
FILE  6=FINDSK,UNIT=DISK,RECORD=14,BLOCKING=30
      REAL LQ163,LQ180,LQ100,LQ120,LQ263,LQ280,LQ200,LQ220
      DIMENSION REV63(37),NEV63(37),XUS63(37),RM(37)
      DIMENSION  LQ163(37),LQ180(37),LQ100(37),LQ120(37)
      DIMENSION  LQ263(37),LQ280(37),LQ200(37),LQ220(37)
      DIMENSION RS(37),RSA(37)
      DIMENSION PAB63(37),PAD63(37),PAN63(37),PND63(37)
      DIMENSION Y63(37),Y80(37),Y00(37),Y20(37)
      DIMENSION RA63(37,37),RA80(37,37),RA00(37,37),RA20(37,37)
      DIMENSION RB63(37,37),RB80(37,37),RB00(37,37),RB20(37,37)
      DIMENSION A63(37,37),A80(37,37),A00(37,37),A20(37,37)
      DIMENSION AA63(37,37)
      DIMENSION XA63(37),XA80(37),XA00(37),XA20(37)
       DIMENSION XB63(37),XB80(37),XB00(37),XB20(37)
      DIMENSION XC80(37),XC00(37),XC20(37),XC63(37)
      DIMENSION RDA80(37),RDA00(37),RDA20(37)
      DIMENSION RDB80(37),RDB00(37),RDB20(37)
      DIMENSION RDC80(37),RDC00(37),RDC20(37)
      DIMENSION RDN63(37),RDN80(37),RDN00(37),RDN20(37)
      DIMENSION RDD63(37),RDD80(37),RDD00(37),RDD20(37)
      DIMENSION PAB80(37),PAB00(37),PAB20(37)
      DIMENSION PAD80(37),PAD00(37),PAD20(37)
      DIMENSION PAN80(37),PAN00(37),PAN20(37)
      DIMENSION PND80(37),PND00(37),PND20(37)
      DIMENSION XD63(37),XD80(37),XD00(37),XD20(37)
      DIMENSION XN63(37),XN80(37),XN00(37),XN20(37)
      DIMENSION PCC80(37),PCC00(37),PCC20(37)
      DIMENSION YN63(37),YN80(37),YN00(37),YN20(37)
      COMMON A63,A80,A00,A20
      COMMON LQ163,LQ180,LQ100,LQ120,LQ263,LQ280,LQ200,LQ220
      COMMON GNP63,GNP80,GNP00,GNP20,YN63,YN80,YN00,YN20
      COMMON YP63(37),YP80(37),YP00(37),YP20(37)
      COMMON IRUN
      COMMON REV63,NEV63,XUS63
      IRUN=0
      WRITE(6,31)
      CALL ADRESS
      READ (5,172) N
  172 FORMAT(2X,I2)
      WRITE (6,11) N
   11 FORMAT(1H0,2HN=,I2)
      READ (5,2) IADJ,IN,IC,ID,IOUT,IPRO
      WRITE(6,6) IADJ,IN,IC,ID,IOUT,IPRO
      READ (5,3) GNP63,GNP80,GNP00,GNP20
      WRITE(6,4) GNP63,GNP80,GNP00,GNP20
```

PROGRAM FINAL cont.

```
      READ (5,3)PGNP63,PGNP80,PGNP00,PGNP20
      WRITE(6,5)PGNP63,PGNP80,PGNP00,PGNP20
      RGNP63=GNP63*PGNP63
      RGNP80=GNP80*PGNP80
      RGNP00=GNP00*PGNP00
      RGNP20=GNP20*PGNP20
      WRITE(6,7) RGNP63,RGNP80,RGNP00,RGNP20
      DO 51 I=1,N
   51 READ (5,1) Y63(I)
      CALL PRINTV(Y63,N,6HY63PCT)
      DO 52 I=1,N
   52 READ (5,1) Y80(I)
      CALL PRINTV(Y80,N,6HY80PCT)
      DO 53 I=1,N
   53 READ (5,1) Y00(I)
      CALL PRINTV(Y00,N,6HY00PCT)
      DO 54 I=1,N
   54 READ (5,1) Y20(I)
      CALL PRINTV(Y20,N,6HY20PCT)
      DO 10 J=1,N
   10 READ (5,9) (A63(I,J),I=1,N)
      DO 20 J=1,N
   20 READ (5,9) (A80(I,J),I=1,N)
      DO 30 J=1,N
   30 READ (5,9) (A00(I,J),I=1,N)
      DO 40 J=1,N
   40 READ (5,9) (A20(I,J),I=1,N)
      IF(IOUT.NE.1) GO TO 326
      CALL PRINTM(A63,N,3HA63)
      CALL PRINTM(A80,N,3HA80)
      CALL PRINTM(A00,N,3HA00)
      CALL PRINTM(A20,N,3HA20)
       IF(IOUT.EQ.1) GO TO 1066
  326 CONTINUE
      DO 7874 I=1,N
      YP63(I)=Y63(I)
      YP80(I)=Y80(I)
      YP00(I)=Y00(I)
      YP20(I)=Y20(I)
 7874 CONTINUE
      DO 100 I=1,N
      Y63(I)=Y63(I)*RGNP63
      Y80(I)=Y80(I)*RGNP80
      Y00(I)=Y00(I)*RGNP00
      Y20(I)=Y20(I)*RGNP20
  100 CONTINUE
      CALL PRINTV(Y63,N,3HY63)
      CALL PRINTV(Y80,N,3HY80)
      CALL PRINTV(Y00,N,3HY00)
      CALL PRINTV(Y20,N,3HY20)
 1776 CONTINUE
      WRITE(6,1492) IRUN,IADJ,IPRO
 1492 FORMAT(1H1//////////25X,5HIRUN=,I1,5X,5HIADJ=,I1,5X,
     1 5HIPRO=,I1)
```

PROGRAM FINAL cont.

```
      CALL LOCATQ(N)
      IF (IPRO.EQ.2) GO TO 1000
      DO 110 I=1,N
      DO 110 J=1,N
      RA63(I,J)=A63(I,J)*LQ163(I)
      RA80(I,J)=A80(I,J)*LQ180(I)
      RA00(I,J)=A00(I,J)*LQ100(I)
      RA20(I,J)=A20(I,J)*LQ120(I)
  110 CONTINUE
      GO TO 2000
 1000 CONTINUE
      DO 120 I=1,N
      DO 120 J=1,N
      RA63(I,J)=A63(I,J)*LQ263(I)
      RA80(I,J)=A80(I,J)*LQ280(I)
      RA00(I,J)=A00(I,J)*LQ200(I)
      RA20(I,J)=A20(I,J)*LQ220(I)
  120 CONTINUE
 2000 CONTINUE
      IF(IOUT.NE.1) GO TO 327
      CALL PRINTM(RA63,N,4HRA63)
      CALL PRINTM(RA80,N,4HRA80)
      CALL PRINTM(RA00,N,4HRA00)
      CALL PRINTM(RA20,N,4HRA20)
  327 CONTINUE
      DO 400 I=1,N
      DO 400 J=1,N
      AA63(I,J)=RA63(I,J)
  400 CONTINUE
      IF (IADJ.NE.1) GO TO 99
      IF(IPRO.EQ.2) GO TO 4027
      CALL ADJUST(RA63,A63,LQ163,RB63,Y63,RS,RSA,N)
      DO 500 I=1,N
      DO 500 J=1,N
      RA63(I,J)=A63(I,J)*LQ163(I)
  500 CONTINUE
      GO TO 5038
 4027 CONTINUE
      CALL ADJUST(RA63,A63,LQ263,RB63,Y63,RS,RSA,N)
      DO 523 I=1,N
      DO 523 J=1,N
      RA63(I,J)=A63(I,J)*LQ263(I)
  523 CONTINUE
 5038 CONTINUE
      CALL MULT(RB63,RA63,RS,RSA,N)
      CALL MULT(RB80,RA80,RS,RSA,N)
      CALL MULT(RB00,RA00,RS,RSA,N)
      CALL MULT(RB20,RA20,RS,RSA,N)
   99 CONTINUE
      IF (IADJ.NE.1) GO TO 999
      IF(IOUT.NE.1) GO TO 328
      CALL PRINTM(RB63,N,4HRB63)
      CALL PRINTM(RB80,N,4HRB80)
      CALL PRINTM(RB00,N,4HRB00)
```

PROGRAM FINAL cont.

```
      CALL PRINTM(RB20,N,4HRB20)
  328 CONTINUE
  999 CONTINUE
      CALL INVERT(RA63,N)
      CALL OUT(RA63,Y63,XA63,N)
      CALL INVERT(RA80,N)
      CALL OUT(RA80,Y80,XA80,N)
      CALL INVERT(RA00,N)
      CALL OUT(RA00,Y00,XA00,N)
      CALL INVERT(RA20,N)
      CALL OUT(RA20,Y20,XA20,N)
      WRITE(6,31)
      WRITE(6,32)
      DO 600 I=1,N
  600 WRITE(6,33) XA63(I),XA80(I),XA00(I),XA20(I),I
      CALL RGROWT(XA63,XA80,RDA80,17.0,N)
      CALL RGROWT(XA63,XA00,RDA00,37.0,N)
      CALL RGROWT(XA63,XA20,RDA20,57.0,N)
      IF (IADJ.NE.1) GO TO 98
   98 CONTINUE
      IF (IADJ.EQ.4) GO TO 2757
      DO 2747 I=1,N
       RM(I)=((REV63(I)/NEV63(I))*XUS63(I))/XA63(I)
 2747 CONTINUE
      DO 2757 I=1,N
      XB63(I)=RM(I)*XA63(I)
      XB80(I)=RM(I)*XA80(I)
      XB00(I)=RM(I)*XA00(I)
      XB20(I)=RM(I)*XA20(I)
 2757 CONTINUE
      IF (IADJ.EQ.4) GO TO 2787
      DO 2777 I=1,N
      RM(I)=XN63(I)/XA63(I)
 2777 CONTINUE
      DO 2787 I=1,N
      XB63(I)=RM(I)*XA63(I)
      XB80(I)=RM(I)*XA80(I)
      XB00(I)=RM(I)*XA00(I)
      XB20(I)=RM(I)*XA20(I)
 2787 CONTINUE
      IF (IC.NE.1) GO TO 998
      CALL OUT(RA63,Y63,XC63,N)
      CALL OUT(RA80,Y63,XC80,N)
      CALL OUT(RA00,Y63,XC00,N)
      CALL OUT(RA20,Y63,XC20,N)
      WRITE (6,31)
      CALL RGROWT(XC63,XC80,RDC80,17.0,N)
      CALL RGROWT(XC63,XC00,RDC00,37.0,N)
      CALL RGROWT(XC63,XC20,RDC20,57.0,N)
      CALL DIFF(XC63,XC80,PCC80,N)
      CALL DIFF(XC63,XC00,PCC00,N)
      CALL DIFF(XC63,XC20,PCC20,N)
      WRITE (6,35)
      DO 800 I=1,N
```

PROGRAM FINAL cont.

```
 800 WRITE (6,33) XC63(I),XC80(I),XC00(I),XC20(I),I
 998 CONTINUE
     IF (IN.NE.1) GO TO 97
     GO TO 961
     IF (IRUN.GE.1) GO TO 1727
     IF(IADJ.EQ.1) GO TO 961
 961 CONTINUE
  97 CONTINUE
     IF (ID.NE.1) GO TO 9999
     IF (IPRO.EQ.2) GO TO 4781
     DO 425 I=1,N
     DO 425 J=1,N
 425 RA63(I,J)=A63(I,J)*LQ163(I)
     GO TO 4787
4781 CONTINUE
     DO 4281 I=1,N
     DO 4281 J=1,N
4281 RA63(I,J)=A63(I,J)*LQ263(I)
4787 CONTINUE
     IF (IADJ.NE.1) GO TO 9998
     CALL MULT(RA63,RA63,RS,RSA,N)
9998 CONTINUE
     CALL INVERT(RA63,N)
     CALL OUT(RA63,Y63,XD63,N)
     CALL OUT(RA63,Y80,XD80,N)
     CALL OUT(RA63,Y00,XD00,N)
     WRITE(6,31)
     CALL OUT(RA63,Y20,XD20,N)
     WRITE(6,136)
     DO 185 I=1,N
 185 WRITE(6,33) XD63(I),XD80(I),XD00(I),XD20(I),I
     CALL RGROWT(XD63,XD80,RDD80,17.0,N)
     CALL RGROWT(XD63,XD00,RDD00,37.0,N)
     CALL RGROWT(XD63,XD20,RDD20,57.0,N)
     CALL DIFF(XD63,XA63,PAD63,N)
     CALL DIFF(XD80,XA80,PAD80,N)
     CALL DIFF(XD00,XA00,PAD00,N)
     CALL DIFF(XD20,XA20,PAD20,N)
9999 CONTINUE
     WRITE(6,31)
     WRITE (6,45)
     DO 750 I=1,N
 750 WRITE (6,46) RDA80(I),RDA00(I),RDA20(I),I
     IF (IADJ.NE.1) GO TO 266
     WRITE (6,31)
     WRITE (6,47)
     DO 751 I=1,N
 751 WRITE (6,46) RDB80(I),RDB00(I),RDB20(I),I
 266 CONTINUE
     IF (IC.NE.1) GO TO 267
     WRITE(6,31)
     WRITE(6,48)
     DO 752 I=1,N
 752 WRITE (6,46) RDC80(I),RDC00(I),RDC20(I),I
```

PROGRAM FINAL cont.

```
 267 CONTINUE
     IF (IN.NE.1) GO TO 268
     WRITE(6,31)
     WRITE(6,49)
     DO 753 I=1,N
 753 WRITE (6,46) RDN80(I),RDN00(I),RDN20(I),I
 268 CONTINUE
     IF (ID.NE.1) GO TO 269
     WRITE(6,31)
     WRITE(6,57)
     DO 754 I=1,N
 754 WRITE (6,46) RDD80(I),RDD00(I),RDD20(I),I
 269 CONTINUE
     CALL INVERT(A63,N)
     CALL OUT(A63,Y63,XB63,N)
     CALL INVERT(A80,N)
     CALL OUT(A63,Y80,XB80,N)
     CALL INVERT(A00,N)
     CALL OUT(A63,Y00,XB00,N)
     CALL INVERT(A20,N)
     CALL OUT(A63,Y20,XB20,N)
     WRITE(6,31)
     WRITE(6,34)
     DO 700 I=1,N
 700 WRITE(6,33) XB63(I),XB80(I),XB00(I),XB20(I),I
     CALL RGROWT(XB63,XB80,RDB80,17.0,N)
     CALL RGROWT(XB63,XB20,RDB20,57.0,N)
     CALL RGROWT(XB63,XB00,RDB00,37.0,N)
     CALL DIFF(XB63,XA63,PAB63,N)
     CALL DIFF(XB80,XA80,PAB80,N)
     CALL DIFF(XB00,XA00,PAB00,N)
     CALL DIFF(XB20,XA20,PAB20,N)
     CALL OUT(A80,Y80,XN80,N)
     CALL OUT(A63,Y63,XN63,N)
     CALL OUT(A00,Y00,XN00,N)
     CALL OUT(A20,Y20,XN20,N)
1727 CONTINUE
     IF ((ID.NE.1).OR.(IN.NE.1)) GO TO 9997
     CALL DIFF(XD63,XN63,PND63,N)
     CALL DIFF(XD80,XN80,PND80,N)
     CALL DIFF(XD00,XN00,PND00,N)
     CALL DIFF(XD20,XN20,PND20,N)
9997 CONTINUE
     CALL DIFF(XN63,XA63,PAN63,N)
     CALL DIFF(XN80,XA80,PAN80,N)
     CALL DIFF(XN00,XA00,PAN00,N)
     CALL RGROWT(XN63,XN80,RDN80,17.0,N)
     CALL DIFF(XN20,XA20,PAN20,N)
     CALL RGROWT(XN63,XN00,RDN00,37.0,N)
     CALL RGROWT(XN63,XN20,RDN20,57.0,N)
     WRITE(6,31)
     WRITE(6,134)
     DO 175 I=1,N
```

PROGRAM FINAL cont.

```
175 WRITE(6,33) XN63(I),XN80(I),XN00(I),XN20(I),I
    IF (IADJ.NE.4) GO TO 276
    CALL XPRINT(XA63,XB63,PAB63,6HXA1963,6HXB1963,N)
    CALL XPRINT(XA80,XB80,PAB80,6HXA1980,6HXB1980,N)
    CALL XPRINT(XA00,XB00,PAB00,6HXA2000,6HXB2000,N)
    CALL XPRINT(XA20,XB20,PAB20,6HXA2020,6HXB2020,N)
    CALL RPRINT(RDA80,RDB80,6HDA1980,6HDB1980,N)
    CALL RPRINT(RDA00,RDB00,6HDA2000,6HDB2000,N)
    CALL RPRINT(RDA20,RDB20,6HDA2020,6HDB2020,N)
276 CONTINUE
    IF (IN.NE.1) GO TO 278
    CALL XPRINT(XA63,XN63,PAN63,6HXA1963,6HXN1963,N)
    CALL XPRINT(XA80,XN80,PAN80,6HXA1980,6HXN1980,N)
    CALL XPRINT(XA00,XN00,PAN00,6HXA2000,6HXN2000,N)
    CALL XPRINT(XA20,XN20,PAN20,6HXA2020,6HXN2020,N)
    CALL RPRINT(RDA80,RDN80,6HDA1980,6HDN1980,N)
    CALL RPRINT(RDA00,RDN00,6HDA2000,6HDN2000,N)
    CALL RPRINT(RDA20,RDN20,6HDA2020,6HDN2020,N)
278 CONTINUE
    IF (ID.NE.1) GO TO 279
    CALL XPRINT(XA63,XD63,PAD63,6HXA1963,6HXD1963,N)
    CALL XPRINT(XA80,XD80,PAD80,6HXA1980,6HXD1980,N)
    CALL XPRINT(XA00,XD00,PAD00,6HXA2000,6HXD2000,N)
    CALL XPRINT(XA20,XD20,PAD20,6HXA2020,6HXD2020,N)
    CALL RPRINT(RDA80,RDD80,6HDA1980,6HDD1980,N)
    CALL RPRINT(RDA00,RDD00,6HDA2000,6HDD2000,N)
    CALL RPRINT(RDA20,RDD20,6HDA2020,6HDD2020,N)
279 CONTINUE
    IF (IC.NE.1) GO TO 286
    CALL XPRINT(XC80,XC63,PCC80,6HXC1980,6HXC1963,N)
    CALL XPRINT(XC00,XC63,PCC00,6HXC2000,6HXC1963,N)
    CALL XPRINT(XC20,XC63,PCC20,6HXC2020,6HXC1963,N)
286 CONTINUE
    IF ((ID.NE.1).OR.(IN.NE.1)) GO TO 287
    CALL XPRINT(XN63,XD63,PND63,6HXN1963,6HXD1963,N)
    CALL XPRINT(XN80,XD80,PND80,6HXN1980,6HXD1980,N)
    CALL XPRINT(XN00,XD00,PND00,6HXN2000,6HXD2000,N)
    CALL XPRINT(XN20,XD20,PND20,6HXN2020,6HXD2020,N)
    CALL RPRINT(RDN80,RDD80,6HDN1980,6HDD1980,N)
    CALL RPRINT(RDN00,RDD00,6HDN2000,6HDD2000,N)
    CALL RPRINT(RDN20,RDD20,6HDN2020,6HDD2020,N)
287 CONTINUE
    IF (IADJ.NE.4) GO TO 6227
    CALL DIFF(XN63,XB63,PAD63,N)
    CALL DIFF(XN80,XB80,PAD80,N)
    CALL DIFF(XN00,XB00,PAD00,N)
    CALL DIFF(XN20,XB20,PAD20,N)
    CALL XPRINT(XB63,XN63,PAD63,6HXB1963,6HXN1963,N)
    CALL XPRINT(XB80,XN80,PAD80,6HXB1980,6HXN1980,N)
    CALL XPRINT(XB00,XN00,PAD00,6HXB2000,6HXN2000,N)
    CALL XPRINT(XB20,XN20,PAD20,6HXB2020,6HXN2020,N)
    CALL RPRINT(RDB80,RDN80,6HDB1980,6HDN1980,N)
    CALL RPRINT(RDB00,RDN00,6HDB2000,6HDN2000,N)
    CALL RPRINT(RDB20,RDN20,6HDB2020,6HDN2020,N)
```

PROGRAM FINAL cont.

```
6227 CONTINUE
     IRUN=5
     IF(IRUN.EQ.2) IADJ=4
     IF(IRUN.EQ.3) IPRO=2
     IF(IRUN.EQ.4) IADJ=3
     IF (IRUN.EQ.5) GO TO 1066
     GO TO 1776
   1 FORMAT(1F14.0)
   2 FORMAT(5X,I1,4X,I1,4X,I1,4X,I1,6X,I1,6X,I1)
   3 FORMAT(4F10.0)
   4 FORMAT(1H0,5X,8HGNP1963=,F09.1,2X,8HGNP1980=,F09.1,2X,8HGNP2000=,
    1F09.1,2X,8HGNP2020=,F09.1)
   5 FORMAT(1H0,5X,9HPGNP1963=,F5.4 ,2X,9HPGNP1980=,F5.4 ,2X,
    19HPGNP2000=,F5.4 ,2X,9HPGNP2020=,F5.4 )
   6 FORMAT(1H0,5X,5HIADJ=,I1,2X,3HIN=,I1,2X,3HIC=,I1,2X,3HID=,I1,
    12X,5HIOUT=,I1,2X,5HIPRO=,I1)
   8 FORMAT(F14.0)
   7 FORMAT(1H0,5X,9HRGNP1963=,F09.1,2X,9HRGNP1980=,F09.1,2X,
    19HRGNP2000=,F09.1,2X,9HRGNP2020=,F09.1)
   9 FORMAT(7F10.4)
  31 FORMAT(1H1,16(/))
  32 FORMAT(8X,6HXA1963,10X,6HXA1980,10X,6HXA2000,10X,6HXA2020,/)
  33 FORMAT(4(4X,F12.2),4X,I2)
  34 FORMAT(8X,6HXB1963,10X,6HXB1980,10X,6HXB2000,10X,6HXB2020,/)
  35 FORMAT(8X,6HXC1963,10X,6HXC1980,10X,6HXC2000,10X,6HXC2020,/)
  45 FORMAT(3X,6HDA1980,3X,6HDA2000,3X,6HDA2020)
  46 FORMAT(3(2X,F8.4),2X,I2)
 134 FORMAT(8X,6HXN1963,10X,6HXN1980,10X,6HXN2000,10X,6HXN2020,/)
 136 FORMAT(8X,6HXD1963,10X,6HXD1980,10X,6HXD2000,10X,6HXD2020,/)
  47 FORMAT(3X,6HDB1980,3X,6HDB2000,3X,6HDB2020)
  48 FORMAT(3X,6HDC1980,3X,6HDC2000,3X,6HDC2020)
  49 FORMAT(3X,6HDN1980,3X,6HDN2000,3X,6HDN2020)
  57 FORMAT(3X,6HDD1980,3X,6HDD2000,3X,6HDD2020)
1066 CONTINUE
     CALL ADRESS
     LOCK 6
     LOCK 8
     END
     SUBROUTINE  LOCATQ(N)
     REAL LQ163,LQ180,LQ100,LQ120,LQ263,LQ280,LQ200,LQ220
     REAL NUMR63,NUMR80,NUMR00,NUMR20,NUMN63,NUMN80,NUMN00,NUMN20
     REAL NEV63,NEV80,NEV00,NEV20
     DIMENSION  LQ263(37),LQ280(37),LQ200(37),LQ220(37)
     DIMENSION  LQ163(37),LQ180(37),LQ100(37),LQ120(37)
     DIMENSION  A63(37,37),A80(37,37),A00(37,37),A20(37,37)
     DIMENSION  YN63(37),YN80(37),YN00(37),YN20(37)
     DIMENSION  REV63(37),REV80(37),REV00(37),REV20(37)
     DIMENSION  NEV63(37),NEV80(37),NEV00(37),NEV20(37)
     DIMENSION  Y63(37),Y80(37),Y00(37),Y20(37)
     DIMENSION  XUS63(37),XUS80(37),XUS00(37),XUS20(37)
     DIMENSION  B63(37,37),B80(37,37),B00(37,37),B20(37,37)
     DIMENSION  T(37,37)
     COMMON A63,A80,A00,A20
     COMMON LQ163,LQ180,LQ100,LQ120,LQ263,LQ280,LQ200,LQ220
```

```
      COMMON GNP63,GNP80,GNP00,GNP20,YN63,YN80,YN00,YN20
      COMMON YP63(37),YP80(37),YP00(37),YP20(37)
      COMMON IRUN
      COMMON REV63,NEV63,XUS63
      IF (IRUN.GE.1) GO TO 1066
      READ (7,1) (REV63(I),I=1,N)
      CALL PRINTV(REV63,N,5HREV63)
      READ (7,1) (REV80(I),I=1,N)
      CALL PRINTV(REV80,N,5HREV80)
      READ (7,1) (REV00(I),I=1,N)
      CALL PRINTV(REV00,N,5HREV00)
      READ (7,1) (REV20(I),I=1,N)
      CALL PRINTV(REV20,N,5HREV20)
      READ (7,1) (NEV63(I),I=1,N)
      CALL PRINTV(NEV63,N,5HNEV63)
      READ (7,1) (NEV80(I),I=1,N)
      CALL PRINTV(NEV80,N,5HNEV80)
      READ (7,1) (NEV00(I),I=1,N)
      CALL PRINTV(NEV00,N,5HNEV00)
      READ (7,1) (NEV20(I),I=1,N)
      CALL PRINTV(NEV20,N,5HNEV20)
 1066 CONTINUE
      DO 10 I=1,N
      YN63(I)=GNP63*YP63(I)
      YN80(I)=GNP80*YP80(I)
      YN00(I)=GNP00*YP80(I)
      YN20(I)=GNP20*YP20(I)
   10 CONTINUE
      CALL PRINTV(YN63,N,4HYN63)
      CALL PRINTV(YN80,N,4HYN80)
      CALL PRINTV(YN00,N,4HYN00)
      CALL PRINTV(YN20,N,4HYN20)
      DO 20 I=1,N
      DO 20 J=1,N
      T(I,J)=A63(I,J)
   20 CONTINUE
      CALL INVERT (A63,N)
      DO 30 I=1,N
      DO 30 J=1,N
      B63(I,J)=A63(I,J)
      A63(I,J)=T(I,J)
   30 CONTINUE
      DO 40 I=1,N
      DO 40 J=1,N
      T(I,J)=A80(I,J)
   40 CONTINUE
      CALL INVERT(A80,N)
      DO 50 I=1,N
      DO 50 J=1,N
      B80(I,J)=A80(I,J)
      A80(I,J)=T(I,J)
   50 CONTINUE
      DO 60 I=1,N
      DO 60 J=1,N
```

PROGRAM FINAL cont.

```
      T(I,J)=A00(I,J)
   60 CONTINUE
      CALL INVERT(A00,N)
      DO 70 I=1,N
      DO 70 J=1,N
      B00(I,J)=A00(I,J)
      A00(I,J)=T(I,J)
   70 CONTINUE
      DO 80 I=1,N
      DO 80 J=1,N
      T(I,J)=A20(I,J)
   80 CONTINUE
      CALL INVERT(A20,N)
      DO 90 I=1,N
      DO 90 J=1,N
      B20(I,J)=A20(I,J)
      A20(I,J)=T(I,J)
   90 CONTINUE
      CALL OUT(B63,YN63,XUS63,N)
      CALL PRINTV(XUS63,N,5HXUS63)
      CALL OUT(B80,YN80,XUS80,N)
      CALL PRINTV(XUS80,N,5HXUS80)
      CALL OUT(B00,YN00,XUS00,N)
      CALL PRINTV(XUS00,N,5HXUS00)
      CALL OUT(B20,YN20,XUS20,N)
      CALL PRINTV(XUS20,N,5HXUS20)
      LOCK 8
      DO 100 I=3,34
      NUMR63=REV63(I)*(XUS63(I)/NEV63(I))
      NUMR80=REV80(I)*(XUS63(I)/NEV63(I))
      NUMR00=REV00(I)*(XUS63(I)/NEV63(I))
      NUMR20=REV20(I)*(XUS63(I)/NEV63(I))
      NUMN63=NEV63(I)*(XUS63(I)/NEV63(I))
      NUMN80=NEV80(I)*(XUS63(I)/NEV63(I))
      NUMN00=NEV00(I)*(XUS63(I)/NEV63(I))
      NUMN20=NEV20(I)*(XUS63(I)/NEV63(I))
      DENR63=0.0
      DENR80=0.0
      DENR00=0.0
      DENR20=0.0
      DENN63=0.0
      DENN80=0.0
      DENN00=0.0
      DENN20=0.0
      DO 200 J=3,34
      DENR63=DENR63 + REV63(J)*A63(I,J)*(XUS63(J)/NEV63(J))
      DENR80=DENR80 + REV80(J)*A80(I,J)*(XUS63(J)/NEV63(J))
      DENR00=DENR00 + REV00(J)*A00(I,J)*(XUS63(J)/NEV63(J))
      DENR20=DENR20 + REV20(J)*A20(I,J)*(XUS63(J)/NEV63(J))
      DENN63=DENN63 + NEV63(J)*A63(I,J)*(XUS63(J)/NEV63(J))
      DENN80=DENN80 + NEV80(J)*A80(I,J)*(XUS63(J)/NEV63(J))
      DENN00=DENN00 + NEV00(J)*A00(I,J)*(XUS63(J)/NEV63(J))
      DENN20=DENN20 + NEV20(J)*A20(I,J)*(XUS63(J)/NEV63(J))
  200 CONTINUE
```

PROGRAM FINAL cont.

```
      LQ163(I)=(NUMR63/DENR63)/(NUMN63/DENN63)
      LQ180(I)=(NUMR80/DENR80)/(NUMN80/DENN80)
      LQ100(I)=(NUMR00/DENR00)/(NUMN00/DENN00)
      LQ120(I)=(NUMR20/DENR20)/(NUMN20/DENN20)
      IF (LQ163(I).LT.0.0.OR.LQ163(I).GT.1.0) LQ163(I)=1.0
      IF (LQ180(I).LT.0.0.OR.LQ180(I).GT.1.0) LQ180(I)=1.0
      IF (LQ100(I).LT.0.0.OR.LQ100(I).GT.1.0) LQ100(I)=1.0
      IF (LQ120(I).LT.0.0.OR.LQ120(I).GT.1.0) LQ120(I)=1.0
  100 CONTINUE
      DO 850 I=1,2
      LQ163(I)=1.0
      LQ180(I)=1.0
      LQ100(I)=1.0
      LQ120(I)=1.0
  850 CONTINUE
      DO 950 I=35,N
      LQ163(I)=1.0
      LQ100(I)=1.0
      LQ180(I)=1.0
      LQ120(I)=1.0
  950 CONTINUE
      CALL PRINTV(LQ163,N,5HLQ163)
      CALL PRINTV(LQ180,N,5HLQ180)
      CALL PRINTV(LQ100,N,5HLQ100)
      CALL PRINTV(LQ120,N,5HLQ120)
      DO 300 I=3,34
      NUMR80=REV80(I)*(XUS80(I)/NEV80(I))
      NUMR00=REV00(I)*(XUS00(I)/NEV00(I))
      NUMR20=REV20(I)*(XUS20(I)/NEV20(I))
      NUMN80=NEV80(I)*(XUS80(I)/NEV80(I))
      NUMN00=NEV00(I)*(XUS00(I)/NEV00(I))
      NUMN20=NEV20(I)*(XUS20(I)/NEV20(I))
      DENR80=0.0
      DENR00=0.0
      DENR20=0.0
      DENN80=0.0
      DENN00=0.0
      DENN20=0.0
      DO 400 J=3,34
      DENR80=DENR80 + REV80(J)*A80(I,J)*(XUS80(J)/NEV80(J))
      DENR00=DENR00 + REV00(J)*A00(I,J)*(XUS00(J)/NEV00(J))
      DENR20=DENR20 + REV20(J)*A20(I,J)*(XUS20(J)/NEV20(J))
      DENN80=DENN80 + NEV80(J)*A80(I,J)*(XUS80(J)/NEV80(J))
      DENN00=DENN00 + NEV00(J)*A00(I,J)*(XUS00(J)/NEV00(J))
      DENN20=DENN20 + NEV20(J)*A20(I,J)*(XUS20(J)/NEV20(J))
  400 CONTINUE
      LQ263(I)=LQ163(I)
      LQ280(I)=(NUMR80/DENR80)/(NUMN80/DENN80)
      LQ200(I)=(NUMR00/DENR00)/(NUMN00/DENN00)
      LQ220(I)=(NUMR20/DENR20)/(NUMN20/DENN20)
      IF (LQ263(I).LT.0.0.OR.LQ263(I).GT.1.0) LQ263(I)=1.0
      IF (LQ280(I).LT.0.0.OR.LQ280(I).GT.1.0) LQ280(I)=1.0
      IF (LQ200(I).LT.0.0.OR.LQ200(I).GT.1.0) LQ200(I)=1.0
      IF (LQ220(I).LT.0.0.OR.LQ220(I).GT.1.0) LQ220(I)=1.0
```

PROGRAM FINAL cont.

```
 300 CONTINUE
     DO 857 I=1,2
     LQ263(I)=1.0
     LQ280(I)=1.0
     LQ200(I)=1.0
     LQ220(I)=1.0
 857 CONTINUE
     DO 957 I=35,N
     LQ263(I)=1.0
     LQ280(I)=1.0
     LQ200(I)=1.0
     LQ220(I)=1.0
 957 CONTINUE
     CALL PRINTV(LQ263,N,5HLQ263)
     CALL PRINTV(LQ280,N,5HLQ280)
     CALL PRINTV(LQ200,N,5HLQ200)
     CALL PRINTV(LQ220,N,5HLQ220)
     WRITE(6,5)
     WRITE(6,2)
     DO 2003 I=1,N
     WRITE(6,3) LQ263(I),LQ280(I),LQ200(I),LQ220(I),I
2003 CONTINUE
     WRITE(6,5)
     WRITE(6,4)
     DO 2007 I=1,N
     WRITE(6,3) LQ163(I),LQ180(I),LQ100(I),LQ120(I),I
2007 CONTINUE
   2 FORMAT(23X,5HLQ163,7X,5HLQ180,7X,5HLQ100,7X,5HLQ120)
   3 FORMAT(20X,4(F8.4,4X),I2)
   4 FORMAT(23X,5HLQ263,7X,5HLQ280,7X,5HLQ200,7X,5HLQ220)
   5 FORMAT(1H1)
   1 FORMAT(1F14.0)
     RETURN
     END
     SUBROUTINE SUMR(A,C,N)
     DIMENSION A(37,37),C(37)
     DO 5 I=1,N
     C(I)=0.0
     DO 5 J=1,N
   5 C(I)=C(I)+A(I,J)
     RETURN
     END
     SUBROUTINE  OUT(A,Y,X,N)
     DIMENSION A(37,37),Y(37),X(37)
     DO 10 I=1,N
     X(I)=0.0
     DO 10 J=1,N
  10 X(I)=A(I,J)*Y(J)+X(I)
     RETURN
     END
     SUBROUTINE ADJUST(A,AA,Q,R,Y,RS,RSA,N)
     DIMENSION Q(37)
     DIMENSION X(37),Y(37),A(37,37),AA(37,37),RS(37),RSA(37)
     DIMENSION R(37,37),PCT(37)
```

PROGRAM FINAL cont.

```
      READ(5,1) (X(I),I=1,N)
      CALL PRINTV(X,N,3HX63)
      CALL PRINTV(Y,N,3HY63)
      DO 10 I=1,N
   10 RS(I)=X(I)-Y(I)
      CALL PRINTV(RS,N,2HRS)
      CALL INVERT(A,N)
      CALL OUT(A,Y,X,N)
      CALL PRINTV(X,N,1HX)
      DO 20 I=1,N
      DO 20 J=1,N
   20 A(I,J)=AA(I,J)*Q(I)*X(J)
      CALL PRINTM(A,N,4HFLOW)
      CALL SUMR(A,RSA,N)
      CALL MULT(R,A,RS,RSA,N)
      CALL DIFF(RSA,RS,PCT,N)
      CALL XPRINT(RS,RSA,PCT,2HRS,3HRSA,N)
      CALL PRINTV(RS,N,2HRS)
    1 FORMAT(F14.0)
      RETURN
      END
      SUBROUTINE INVERT(A,N)
      DIMENSION A(37,37),TEMP(200),IN(200)
      DO 87 J=1,N
      DO 88 I=1,N
   88 A(I,J) =-A(I,J)
   87 A(J,J)= A(J,J)+ 1.0
      IMAX=N
      ISING=0
      N=IMAX
      IMAXO=N-1
      I1=1
5     I3=I1
      IN(I1)=0
      SUM=ABS(A(I1,I1))
      DO 15 I=I1,N
      IF (SUM-ABS(A(I,I1))) 10,15,15
10    I3=I
      IN(I1)=I
      SUM=ABS(A(I,I1))
15    CONTINUE
      IF (I3-I1) 20,30,20
20    DO 25 J=1,N
      SUM=A(I1,J)
      A(I1,J)=A(I3,J)
25    A(I3,J)=SUM
30    I3=I1+1
      IF (A(I1,I1)) 35,155,35
35    DO 40 I=I3,N
40    A(I,I1)=A(I,I1)/A(I1,I1)
      J2=I1-1
      IF (J2) 45,55,45
45    DO 50 J=I3,N
      DO 50 I=1,J2
```

PROGRAM FINAL cont.

```
50   A(I1,J)=A(I1,J)-A(I1,I)*A(I,J)
55   J2=I1
     I1=I1+1
     DO 60 I=I1,N
     DO 60 J=1,J2
60   A(I,I1)=A(I,I1)-A(I,J)*A(J,I1)
     IF (I1-N) 5,65,5
65   IF (A(N,N)) 70,155,70
70   DO 85 JP=1,N
     J=N+1-JP
     A(J,J)=1.0/A(J,J)
     IF (J-1) 75,90,75
75   DO 85 IP=2,J
     I=J+1-IP
     IPO=I+1
     SUM=0.
     DO 80 L=IPO,J
80   SUM=SUM-A(I,L)*A(L,J)
85   A(I,J)=SUM/A(I,I)
90   DO 110 J=1,IMAXO
     JPO=J+1
     DO 110 I=JPO,N
     SUM=0.
     IMO=I-1
     DO 105 L=J,IMO
     IF (L-J) 95,100,95
95   SUM=SUM-A(I,L)*A(L,J)
     GO TO 105
100  SUM=SUM-A(I,L)
105  CONTINUE
110  A(I,J)=SUM
     DO 135 I=1,N
     DO 130 J=1,N
     TEMP(J)=0.0
     DO 125 K=I,N
     IF (K-J) 125,120,115
115  TEMP(J)=TEMP(J)+A(I,K)*A(K,J)
     GO TO 125
120  TEMP(J)=TEMP(J)+A(I,K)
125  CONTINUE
130  CONTINUE
     DO 135 J=1,N
135  A(I,J)=TEMP(J)
     DO 150 I=2,N
     M=N+1-I
     IF (IN(M)) 140,150,140
140  ISS=IN(M)
     DO 145 L=1,N
     SUM=A(L,ISS)
     A(L,ISS)=A(L,M)
145  A(L,M)=SUM
150  CONTINUE
     GO TO 160
155  ISING=1
```

PROGRAM FINAL cont.

```
160 IF (ISING.EQ.0) GO TO 285
    PRINT 330
    STOP
330 FORMAT (30H MATRIX IS SINGULAR-END OF RUN)
285 CONTINUE
    RETURN
    END
    SUBROUTINE PRINTM(A,N,TITLE)
    DIMENSION A(37,37)
    DO 25 J=1,N,4
    JB=J
    JE=JB+3
    IF(JE-N)15,20,20
 20 JE=N
 15 WRITE(9,40) TITLE,(K,K=JB,JE)
    DO 25 I=1,N
    WRITE(9,45) (I,(A(I,JS),JS=JB,JE))
 25 CONTINUE
 40 FORMAT(1H1,10(/),50X,A10,3(/),34X,I2,3(11X,I2))
 45 FORMAT (20X,I7,4F13.7)
    RETURN
    END
    SUBROUTINE PRINTV(X,N,TITLE)
    DIMENSION X(37)
    WRITE(6,1)
  1 FORMAT(1H1//////)
    WRITE (6,2) TITLE
  2 FORMAT(46X,A6/)
    DO 10 I=1,N
 10 WRITE (6,3) X(I),I
  3 FORMAT (1H ,40X,F14.4,4X,I2)
    RETURN
    END
    SUBROUTINE MULT(A,AA,CP,C,N)
    DIMENSION A(37,37),AA(37,37),CP(37),C(37)
    DIMENSION S(37)
    DIMENSION CPOC(37)
    DO 10 I=1,N
    DO 10 J=1,N
 10 A(I,J)=CP(I)/C(I)*AA(I,J)
    DO 30 I=1,N
    CPOC(I)=CP(I)/C(I)
 30 CONTINUE
    CALL PRINTV(CPOC,N,6HRS/RSA)
    DO 20 J=1,N
    S(J)=0.0
    DO 20 I=1,N
    S(J)=S(J)+A(I,J)
 20 CONTINUE
    CALL PRINTV(S,N,2HCS)
    RETURN
    END
    SUBROUTINE DIFF(G,H,P,N)
    DIMENSION G(37),H(37),P(37)
```

PROGRAM FINAL cont.

```
      DO 10 I=1,N
      IF(H(I).EQ.0.0) H(I)=0.0000001
   10 P(I)=((G(I)-H(I)) /H(I))*100.0
      RETURN
      END
      SUBROUTINE RGROWT(V1,V2,RD,YR,N)
      DIMENSION V1(37),V2(37),RD(37)
      DO 10 I=1,N
      X1=V1(I)
      X2=V2(I)
      IF(X1) 50,50,60
   50 X1=0.0000001
   60 IF(X2) 70,70,80
   70 X2=0.0000001
   80 CONTINUE
      RD(I)=((X2/X1)**(1.0/YR)-1.0)*100.0
   10 CONTINUE
      RETURN
      END
      SUBROUTINE RPRINT(R1,R2,TR1,TR2,N)
      DIMENSION R1(37),R2(37)
      DIMENSION D(37)
      WRITE(6,1)
      WRITE(6,2) TR1,TR2
      DO 10 I=1,N
      D(I)=R1(I)-R2(I)
   10 WRITE(6,3) R1(I),R2(I),D(I),I
      SR1=0.0
      SR2=0.0
      SD=0.0
      SSD=0.0
      NN=N-1
      DO 20 I=1,NN
      SR1=SR1+R1(I)
      SR2=SR2+R2(I)
      SD=SD+D(I)
      SSD=SSD+D(I)*D(I)
   20 CONTINUE
      XN=NN
      AR1=SR1/XN
      AR2=SR2/XN
      SDD=SQRT(SSD/XN)
      WRITE(6,4) SR1,SR2,SD
      WRITE(6,5) AR1,AR2,SDD
    1 FORMAT(1H1//////)
    2 FORMAT(12X,//,A6,8X,A6,10X,1HD)
    3 FORMAT(10X,//,F10.4,5X,F10.4,5X,F10.4,2X,I2)
    4 FORMAT(1H0,//,7X,4HSR1=,F8.4,3X,4HSR2=,F8.4,4X,3HSD=,F10.4)
    5 FORMAT(1H0,/,7X,4HAR1=,F8.4,3X,4HAR2=,F8.4,3X,4HSDD=,F8.4)
      RETURN
      END
      SUBROUTINE XPRINT(X2,X1,P,TX2,TX1,N)
      DIMENSION X1(37),X2(37),D(37),P(37)
      WRITE(6,1)
```

PROGRAM FINAL cont.

```
      WRITE(6,2) TX1,TX2,TX2
      DO 10 I=1,N
      D(I)=X1(I)-X2(I)
   10 WRITE(6,3)X1(I),X2(I),D(I),P(I),I
      SX1=0.0
      SX2=0.0
      SP=0.0
      SD=0.0
      SSD=0.0
      SSP=0.0
      WS=0.0
      SDA=0.0
      NN=N-1
      DO 20 I=1,NN
      SX1=SX1+X1(I)
      SX2=SX2+X2(I)
      SD=SD+D(I)
      SP=SP+P(I)
      SSD=SSD+D(I)*D(I)
      SSP=SSP+P(I)*P(I)
      WS=WS+ABS(D(I))*ABS(P(I))
      SDA=SDA+ABS(D(I))
   20 CONTINUE
      XN=NN
      WRITE(6,4) SX1,SX2,SD,SP
      AX1=SX1/XN
      AX2=SX2/XN
      SDD=SQRT(SSD/XN)
      SDP=SQRT(SSP/XN)
      WRITE(6,5) AX1,AX2,SDD,SDP
      IF(SDA.EQ.0) SDA=.000001
      WA=WS/SDA
      WRITE(6,6) WA
    4 FORMAT(1H0,7X,4HSX1=,F8.0,3X,4HSX2=,F8.0,4X,3HSD=,F8.0,4X,3HSP=,
     1 F8.2)
    5 FORMAT(1H0,7X,4HAX1=,F8.2,3X,4HAX2=,F8.2,3X,4HSDD=,F8.2,3X,4HSPD=\
         \,
     1 F8.2)
    6 FORMAT(1H0,44X,3HWA=,F10.5)
    1 FORMAT(1H1//////)
    2 FORMAT(12X,A6,9X,A6,12X,1HD,10X,2HD/,A6)
    3 FORMAT(10X,F10.2,5X,F10.2,5X,F10.2,5X,F10.2,2X,I2)
      RETURN
      END
      SUBROUTINE ADRESS
      WRITE(9,100)
  100 FORMAT(1H1,30(/),50X,"IRA SILVER",//,50X,"THE BANK OF N.Y.",
     1//,50X,"52 WILLIAM ST.",//,50X,"NEW YORK, N.Y., 10533")
       RETURN
       END
```

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