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MONEY DEMAND ESTIMATION: AN APPLICATION OF THE BOOTSTRAP

by

KENNETH NARINE

A dissertation submitted to the Graduate Faculty in Economics in partial fulfillment of the requirements for the degree of Doctor of Philosophy, The City University of New York

1997

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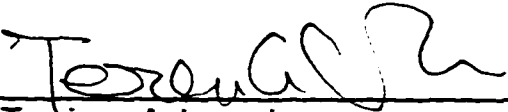
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ABSTRACT

MONEY DEMAND ESTIMATION: AN APPLICATION OF THE BOOTSTRAP

by

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Money demand studies attempt to quantify the linkages between money, income, prices, and some measure of the opportunity cost of holding money. The standard econometric models used in these studies are reduced form single equation models which produce estimates of the short run parameters. The long run parameters are deduced from the short run estimates.

This study was designed to:

- (1) Produce a reduced form single equation model of the long run demand for money which explicitly shows the long run coefficients, and, using parametric techniques, retrieve the coefficient estimates together with the relevant statistics for hypothesis testing.
- (2) Apply a non parametric procedure, the bootstrap, to estimate the same model, and to derive the relevant confidence intervals.

(3) Evaluate the results of the two procedures and compare the estimates with those produced by other researchers.

(4) Assess the non-parametric estimates with regards to stability, convergence, and sample size.

The literature is summarized. Conventional demand functions, time series models, and bootstrap techniques and models are analysed.

The long run model is developed and applied.

The results show:

(1) Parametric and bootstrap estimates of the model are of comparable magnitudes with those derived by other researchers in the field, and identify periods of instability.

(2) Confidence intervals produced by the bootstrap accept the regression values of the income elasticity parameters. These intervals are stable and converge rapidly.

(3) Strong conclusions cannot be made with regards the bootstrap and sample size.

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CHAPTER ONE

INTRODUCTION

1:1 Overview

A modern economy is characterized by extensive division of labour and specialization of function. These give rise to the production of a wide array of goods and services. Consequently, exchanges between individuals are restricted because of:

- (a) uncertainties introduced when participants in the exchange process lack crucial information about each other's credit worthiness, and,
- (b) given the large assortment of commodities - a function of the degree of specialization in the economy- there will necessarily exist the absence of a double coincidence of wants.

Such an economy requires some instrument that can serve as a medium of exchange, a specialized means of payment, which surmounts these problems and promotes efficient exchange. This instrument is money. This instrument is issued and controlled by the state. In pursuit of its goals the state conducts monetary policy: such policy invariably causes variations in the money supply.

1:2 The Demand for money.

This study focuses on the demand for money. This is of interest as it enables us to understand the implications and consequences of monetary policy. Changes in the money supply cause accommodating changes in the variables in the demand function - real income, interest rates, the general price level - which are in their own right objects of study as they have important implications for the economy.

Important questions pertaining to the long run behavior of the price level, the role of interest rates, the level of income, can be sensibly dealt with within the framework of analysis provided by the demand for money function.

If this function is stable we can predict the consequences of a change in monetary policy. If not, shifts in this function will represent independent sources of shocks to the macro-economy. Potential sources of instability arise from:

- (a) the trajectory of the economy over time, e.g., changes in regulations, innovations in technology and financial instruments, definitions of the monetary aggregates, and,
- (b) models used to capture agents' behaviour, together with the econometric methods used to estimate these models; eg. short run dynamics, function mis-specification, errors in ,and omission of variables, pobability distribution and structure of the error terms.

1:3 Objectives of this study.

The literature reveals that most studies of money demand have focused on the short run parameters of the demand function, deriving the long run parameters from the short run estimates. These derived parameter values are not accompanied by statistics, 't' values, confidence intervals, etc., hence various null hypotheses cannot be tested.

The methods used to estimate the short run function, regression methods, are parametric methods. These estimates rely on distributional assumptions of the error structure, generally normal, which is essential for calculating the statistics used for hypotheses testing.

If the actual error term does not have the assumed distribution then hypotheses tests on the parameters are invalid and we cannot make accurate probability statements about the null. The nature of variables in the demand function questions the assumption of normality.

Bounded aggregates such as aggregate income are generally skewed. The definition of monetary aggregate M1 has changed over the estimation period, 1959-1995. New components have been periodically introduced. The resulting aggregate would not necessarily be unimodal, hence the assumption of normality may not be appropriate.

Economic data derive from processes which are non-repeatable.

Observations, degrees of freedom, are precious. Estimation procedures should provide reliable estimates given this constraint.

Given these considerations, this study focuses on the long run demand for money along the following lines.

(a) A short run demand function is derived along traditional lines.

This function is transformed into a form in which the long run parameters are directly observed. This long run function is estimated using the instrumental variables technique. The relevant 't' statistics and confidence intervals are obtained.

(b) The bootstrap technique is applied to the regression model to derive an empirical distribution of the long run parameters and associated confidence intervals. This technique does not rely on any assumptions about the distribution of the error term.

U.S. data for the period 1959-1995 are analysed. Estimates are produced for the following sub-periods; 1959-1973, 1974-1983, 1984-1995, 1974-1995, and for the entire period. Recursive estimates for the entire period, beginning with 1961 are also produced. The analysis is carried by:

(1) Comparing estimates developed in this study with those of

Roley and Goldfeld, with respect to size and statistical significance

(2) Testing for instability of the demand function during the 1970-72 period.

(3) Developing bootstrap confidence intervals to test the parametric coefficient estimates.

(4) Investigating the stability and convergence of the bootstrap distribution of the income elasticity of money demand, developed for the period 1959-1995, as a function of the number of bootstrap iterations.

(5) Determining whether the regression coefficients are accepted by the bootstrap confidence intervals.

(6) Comparing the effects of sample size on the estimates.

1:4 Organization

This study is organized as follows.

Chapters Two and Three discuss the demand for money at the micro and macro level. The theoretical underpinnings of various models representing the demand for money are presented.

Chapter Four introduces and presents the rationales for short run dynamics and adjustment mechanisms.

Chapter Five focuses on empirical considerations in conventional models, concentrating on the functional specifications and discussing their relative merits.

Chapter Six continues the discussion of Chapter Four, focusing on time series models of co-integration and error correction.

Chapter Seven introduces the bootstrap methodology as a general procedure, and as applied to regression models. Select applications of this

technique to economic issues are reviewed. Bootstrap confidence intervals are introduced.

Chapter Eight defines and derives the parametric model to be estimated, together with a treatment of the instrumental variable estimation procedure.

Chapter Nine discusses the data and the variables used in the study. The estimation procedures, parametric and non-parametric, are described as they apply to the computer program developed for this study.

Chapter Ten presents and discusses the results of the regression and bootstrap methods.

Chapter Eleven presents the conclusions of this study.

The Appendix A contains the computer programme used in this study.

Chapter 2

Micro Demand for Money

2:1 Overview

In a modern economy money is essential for exchange to occur. Money balances held by individuals are a function of the transaction costs incurred in changing the size of these balances and the opportunity costs of holding them.

This chapter deals with a demand for money function derived from the perspective of transactions and opportunity costs. Models of the transactions demand for money are discussed. An example is cited - the demand for currency - in the context of a guide for the conduct of monetary policy. Another example is given where transaction costs do not fully explain the amount of money held.

The speculative demand for money, money held as an asset, is introduced.

The "buffer stock" notion, how individuals respond to an exogenous money supply, is examined. An empirical study is described where this notion is tested.

The functional form of the demand for money is derived from micro-economic considerations for an overlapping generations model, and from one that explicitly maximizes utility. These models generate money demand functions that depend on transaction and opportunity costs.

2:2 The transactions demand for money.

The demand for money at the micro level depends on:

- (a) The differential between the perceived yield on money and other assets.
- (b) The transaction costs associated with switching between money and other assets.
- (c) Asset price uncertainty.
- (d) The future level and variance of cash flows.

Traditional analyses along these lines¹ have given rise to

- (1) The "transactions" demand for money function, where for convenience, asset price uncertainty is suppressed and the function is derived by minimizing transaction costs, and,
- (2) the "speculative" demand function, where asset price uncertainty is explicit, and simplifying assumptions are made about transactions costs and the probability distributions of expenditure patterns.

Goodhart, C.A.E. (1989) Money, Information and Uncertainty (MIT Press) pp. 51-81.

Baumol's initial model of the inventory theory of the transactions demand for money ¹ gives rise to a demand function in the form

$$M = \sqrt{(2bT/i)}$$

where M is the average money balance held, T, the expenditure amounts withdrawn from assets, b, the fixed cost per withdrawal, and , i , the yield on a riskless asset.

An essential feature of this model is that it emphasizes a key role that money plays viz, as a specialized means of payment.² This model permits agents to substitute between goods and money balances as well as between money balances and other financial assets.

2:3 Demand for currency

Fama³ reintroduced the idea that monetary policy can be achieved by controlling currency, and that other forms of financial regulation need not be tools of policy.

¹ Baumol, W.J.(1952) " The Transactions Demand for Cash: An Inventory Theoretic Approach" *Quarterly Journal of Economics* 66 (November) pp. 545-556.

² Clower, R. (1967) " Reconsiderations on the Microfoundations of Monetary Theory " *Western Economic Journal (now Economic Enquiry)* 6, pp. 1-8.

³ Fama, E. (1980) " Banking and the Theory of Finance." *Journal of Monetary Economics* 6, pp. 39-57.

Fama, E.(1983)" Financial Intermediation and Price Level Control." , *Journal of Monetary Economics* 12 , pp. 7-28.

Dotsey analysed the behavior of real currency balances for the period 1921-80 using an extended version of the Baumol-Tobin inventory model of the transactions demand for money. The study shows that currency balances behave in a systematic and explainable manner over the sample period.¹

The period under consideration was characterized by numerous monetary regimes viz. gold standard with and without sterilization, interest rate pegging. The study further finds that the relevant scale variable should be the value of expenditures, not GNP.

Dotsey² notes that

“.. in view of the Lucas critique, one still needs to be cautious since direct control of currency was not attempted during the sample and explicit interest rates on demand deposits were regulated over most of the sample. ”

Extensions of the Baumol model, where optimal cash management procedures are employed under explicit conditions of expenditure pattern uncertainty, give rise to a functional form of the money demand equation which relates to the variance of the transactions and interest rate

¹ Dotsey, M. (1988) “ The Demand for Currency in the USA “ *The Journal of Money, Credit, and Banking*.(February) p. 22-40.

² Dotsey, M. (1988) p.23.

differentials. These "precautionary balances" are held to hedge cash flow uncertainty.

The basic model ¹ assumes the following:

- (a) There are two available assets, money and interest bearing bonds.
- (b) Transfer costs are fixed.
- (c) There are no overdraft facilities; cash balances cannot be negative.
- (d) Cash flows are generated by a stationary random walk.

The resulting demand equation is in the form:

$$M = \sqrt[3]{\frac{4}{3} \{3gm^2t / 4i\}}$$

where g is the transactions cost, m^2t , the transactions cost variance, i , the bond yield, m , the level of transactions, and t the number of periods in the interval in question.

As noted before, money is held as a specialized means of payment for future transaction purposes and the demand for these balances will be a function of the level and variance of future cash flows, transfer costs, and yields on alternative safe assets

¹ Goodhart, C.A.E. (1989) Chapter III p. 59

Orr, D. (1970) Cash Management and the Demand for Money (New York: Praeger) Chapter III

This model generates a wide range of income elasticities depending on the relationship between incomes and cash flows.

Furthermore, according to Sprenkle, this theory of money demand fails to account for data on money holdings¹.

Sprenkle finds that the percentage of the actual cash balances held by large firms that can be explained by the simple Baumol model is minuscule- in the order of 2.5 % of the total-, and notes that other research shows that State and Local Governments in the U.S.A. do appear to hold far larger balances than the Baumol model would suggest. He argues that the standard version of the inventory theory is inadequate as it overlooks crucial institutional features of the monetary system, viz. overdraft facilities, compensating balances, and cash management decentralization.

2:4 The speculative demand for money

The “speculative” demand for money derives from asset price uncertainty, which determines the yield differential between money, a capital certain asset, and risky assets. Borrowers are not committed to redeeming their liabilities upon demand at a fixed money value, and lenders require a premium in order to induce them to hold risky assets.

Portfolio allocation between risky and safe assets, and among safe

¹ Sprenkle, C.S. (1969) " The Uselessness of Transactions Demand Models" *Journal of Finance* 24(5) pp. 843-847

assets (of which money is a subset), is determined by this price uncertainty, transaction costs incurred in rearranging the portfolio structure, the length of the holding periods of the assets in the portfolio, assumptions about the probability distribution of the return on the risky asset¹, the functional form of the utility function of income and wealth, and the risk characteristics of the investor. Portfolio analysis of this type has given rise to a vast literature and application as The Capital Asset Pricing Theory².

Money is held in a portfolio of assets to satisfy risk and yield requirements. Long holding periods imply that money is dominated by quasi-safe assets, and for short holding periods, transaction costs become more important.

This separation between the transactions demand and that based on asset price uncertainty is a convenient artifice. It does not imply that the overall demand for money is an additive function, but merely represents the joint services provided by money.

Both transactions models and speculative demand models, in which money is dominated by interest bearing capital certain assets do

¹ Tobin, J.(1958) " Liquidity Preference and the Behavior Towards Risk" *Review of Economic Studies*, 25(2) No. 7, pp. 65-86.

² Gujarati, D (1988) Basic Econometrics. (Mc-Graw Hill Inc.) Chapter 5.

Sharpe, William E. (1978) Investments. Third Edition. (Prentice-Hall Inc.) pp.145-181.

not explain why large money balances are held. Goodhart ¹ notes that research into this matter has not proceeded apace as

“ One important reason for this has been a dearth of suitable data sets for studying such micro-level behavior. Individual money holdings and income and expenditure flows, both in currency form, where they can be used to hide the ‘black economy’ and illegal transactions, and at banks, which are subject to strict customer confidentiality, are not normally publicly revealed or available to a research worker.”

2:5 The buffer stock notion.

A more general method using the inventory theoretic approach has received attention as demand for money functions of the macro-economy failed. The aggregate money stock as defined by the authorities did not behave as predicted by these functions. An explanation for the failure in terms of “Disequilibrium Money” ², and the “Buffer Stock” notion³ is given in terms of “supply” shocks, whereby Banks and the Monetary Authorities behaved in such a way to cause an exogenous change in the

¹ Goodhart, C.A.E (1989) Money, Information and Uncertainty. p.75

² Goodhart, C.A.E. (1984a) Monetary Theory and Practice- The UK Experience (London: Macmillan) Chapter IV.

³ Laidler, D. (1984)“ The Buffer Stock Notion in Monetary Economics” *Conference Proceedings, Supplement to the Economic Journal* 94 (March). pp. 17-34

volume of money. In the United Kingdom a new policy regime was instituted in 1971- "Competition and Credit Control "- which removed direct credit controls and encouraged banks to engage in vigorous competition. This led to an explosion in bank lending. The accompanying growth in the money supply could not be predicted by the variables in these demand functions: current and lagged values of incomes and interest rates.

The "buffer stock" notion theorizes that the stock of money that individuals actually hold could be driven away from what would be normally held. Agents are seen to choose money balances as an inventory, or buffer stock, to cushion against unforeseen events. Given sluggish price adjustments, the immediate effect of a money supply shock is partly compensated by a change in financial conditions- the level of interest rates- and would result in an increase in money balances. This increase in agents' balances, above their underlying demand limits, would trigger an increase in expenditures which will eventually induce a rise in the price level, readjusting real money holdings to the original level.

The first implication of this "buffer" stock notion is that there is an adjustment mechanism at work. Secondly it allows for a reconciliation between the Monetarist view that expenditures can be influenced by factors other than interest rates; and the Keynesian view , that the world

is characterized by uncertainty with prices and wages being slow to adjust.

Lastrapes and Selgin¹ test the buffer stock notion by using a bi-variate Vector Auto Regression model. Their model shows real money as a function of lagged real money - the demand equation,- and an error term representing real permanent shocks. The money supply is modeled as nominal balances- a function of lagged values- and an error term representing transitory shocks. They found a significant short run effect on real M1 demand from nominal (supply) shocks, which they attribute to the buffer stock effect. Nominal shocks have no long run effect on the level of real holdings.

Milbourne and Otto², (applying the VAR methodology and the restrictions developed by Lastrapes and Selgin to currency) question this buffer-stock finding, in that they expect to see real M1 balances unaffected by nominal supply shocks if the restrictions imposed really identify the buffer stock phenomenon.

¹ Lastrapes, W. D., and Selgin, G. (1994) " Buffer-Stock Money: Interpreting Short-Run Dynamics using Long-Run Restrictions" *Journal of Money, Credit, and Banking* (February) pp. 34-54

² Milbourne, R, and Otto, G. (1996) " Buffer-Stock Money: Interpreting Short-Run Dynamics using Long-Run Restrictions" *Journal of Money, Credit, and Banking* (May) pp. 272-278.

2:6 Utility Maximization.

A more general formulation based on explicit utility maximization ¹ can be derived as follows.

Consider a household's multiperiod utility function given as

$$U(c_t, l_t) + \beta U(c_{t-1}, l_{t-1}) + \beta^2 U(c_{t-2}, l_{t-2}) + \dots$$

where c_t and l_t are household consumption of goods and leisure respectively at time t . We assume the utility function is well behaved with respect to continuity and curvature and that, $0 < \beta < 1$, i.e households have positive time preference² .)

The budget constraint is given as

$$P_t y + M_{t-1} + (1 + R_{t-1})B_{t-1} = P_t c_t + M_t + B_t$$

where M = money, B = bonds. The left hand side represents nominal resources available from current income, initial money balances, and loans made or borrowed; the right hand side being total expenditure on consumption and ending asset balances.

The constraints for each period can be expressed in terms of the prior period's ending bond balances, which when manipulated through a

¹ McCallum, Bennett. T. (1989) Monetary Economics: Theory and Policy. (Macmillan) pp. 35-41.

² Barro, Robert. (1984) Macroeconomics. (John Wiley and Sons Inc.) Chap. 16.

process of forward elimination produce a single equation of the household's entire intertemporal budget constraint.

The medium of exchange role of money is represented by the household's expenditure of time and energy in obtaining consumption goods. This representation is:

$$l_t = \phi(c_t, m_t)$$

where m_t is real money holdings, and $\phi_1 < 0$, and $\phi_2 < 0$.

The function U is maximized, with $\phi(\dots)$ substituted for l , subject to the intertemporal budget constraint. Optimality conditions lead to a demand equation in the form

$$m_t = L(c_t, R_t)$$

This form is that assumed by Aggregate demand functions.

2:7 Overlapping Generations Model

Consider an economy¹ with the following characteristics. Within the current period, t , N_t young persons earn an equal amount of the one good, y . When they become old, period $t + 1$, they rely on their 'savings', the economy's fiat money, m_t , obtained through trades made in period t . We let v_t be the value of consumption goods in terms of a unit of fiat money, i.e. the goods one must forego in order to obtain, say, \$1,

¹ Champ, Bruce. and Freeman, Scott. (1994) Modeling Monetary Economies, (J.Wiley & Sons) pp.16.

the unit of fiat money. For simplicity assume a constant population

$$N_{t-1} = N_t.$$

A young person born in period t maximizes a two period utility function where each period consumption is an argument.

Max $U(c_{1,t}, c_{2,t+1})$ subject to the budget constraints for the two periods.

$$1. \quad c_{1,t} + v_t m_t \leq y$$

$$2. \quad c_{2,t+1} \leq v_{t+1} m_t$$

Thus for the young, there is a real demand for fiat money represented by $v_t m_t = q_t$. Substituting for $v_t m_t$ we get

$$3. \quad q_t = y - c_{1,t}$$

$$4. \quad c_{2,t+1} = \left(\frac{v_{t+1}}{v_t}\right) v_t m_t = \left(\frac{v_{t+1}}{v_t}\right) q_t,$$

which when substituting into U we get the utility function in the form

$$U\left(y - q_t, \frac{v_{t+1}}{v_t} q_t\right),$$

where $\frac{v_{t+1}}{v_t}$

is the real gross return on fiat money. If we assume diminishing marginal rates of substitution of money for goods, this function is guaranteed a maximum at some \dot{y} , where $\delta v / \delta q_t = 0$.

The utility maximizing demand for money occurs where the marginal rate of substitution between first and second period consumption equals the rate of return on fiat money, i.e

$$\frac{U_1}{U_2} = \frac{v_{t+1}}{v_t}.$$

Solving for y_t , we arrive at the functional form of the demand for fiat money as

$$q_t = L(y, \frac{v_{t+1}}{v_t}) ,$$

again the form assumed in aggregate demand functions.

Chapter 3

Macro Demand

3:1 Overview.

In this chapter the macro demand function and the variables it contains are examined. Focus is on the variables and their potential to create instabilities in the function.

The independent variables derived and used in micro-demand functions are either unobservable, not available, or are difficult to measure at the macro level. Proxies are used for these variables.

The effects of these proxy variables on the demand for money are described. This is illustrated by the theoretical debate between Monetarists and Keynesians concerning the interest elasticity of demand and the transmission mechanism of monetary policy

Instabilities in the demand function are treated with respect to the left hand side variable, a monetary aggregate. This aggregate is defined by the monetary authority. The definition may periodically change. The aggregate is constructed by simple sum aggregation, which may present a problem when the function is estimated as the distribution is not normal.

A study of the demand for M2 is presented which shows that the demand for aggregate M2 remains unaffected by financial innovations and regulatory changes in the banking sector.

Instabilities with respect to the independent variables are revealed when the money supply is utilized to restrain inflationary pressures. The equation of exchange is analysed and shows that this control procedure crucially depends on the stability of velocity. Since velocity depends on the opportunity costs of holding money - interest rates- and if velocity is unstable then the money demand function is also unstable. The determinants of velocity are discussed

3:2 From micro-demand to macro-demand.

At the aggregate level the factors which determine the micro demand function - transfer costs, alternative safe assets, length of planning or holding periods, the level and variance of cash flows, expectations about relative yields - hardly appear. Some of these are difficult to measure, - transfer costs - some are not measurable, - relative yields expected to hold over an unknown planning or holding period - as portfolio choice depends on ex-ante expectations which cannot be measured by objective ex-post data.

Empirical measures to account for these unobservable expectational variables using the rational expectations hypothesis¹

¹ Lane, T.(1984) " Instability and Short-term Monetary Control." *Journal of Monetary Economics*, 14(2))

attempt to estimate future values of prices, income, and other relevant variables. These “constructed” variables can be entered in the demand function.

Fragility in the demand function based on other structural and related phenomena has diverted attention from this area of study.

At the macro level then, the demand for money is constructed with respect to:

- (1) Technological advances and their effect on the payments systems mechanism and cash management techniques¹.
- (2) The growing availability of alternative liquid assets².
- (3) Changes in the structural characteristics of the economy. urbanization, industrialization, globalization³.
- (4) Increases in Income and wealth⁴.

As a result, the macro level demand function is treated very generally as a simple asset demand function using proxies for the

¹ Garvy, G., and Blyn, M. (1969) The Velocity of Money (New York: Federal Reserve Bank of New York)

² Gurley, J. and Shaw, E.(1960) Money in A Theory of Finance (Washington: The Brookings Institution)

³ Ducca, John. V. (1995) “ Sources of Money Instability.” *The Federal Reserve Bank of Dallas: Economic Review*.(Fourth Quarter 1995) pp. 2-13.

⁴ Friedman, M., and Schwartz, Anna. J. (1963) “ A Monetary History of the United States, 1867-1960,. National Bureau of Economic Research (Princeton:Princeton University Press)

services that money provide: level of income as a proxy for transactions costs and cash flows; a market rate of interest as proxy for opportunity costs.

This generalized function takes the form of

$$M_d = L(Y, W, R_i)$$

where Y = Income, W = wealth, R_i = the monetary yield on the i th asset.

Since W is essentially the capitalization of future income, it is generally omitted from the estimating equation in order to avoid problems associated with multicollinearity. If money is narrowly defined as transactions balances, and the yield on money is zero, then the wealth elasticity should be zero if the transaction costs, volume of transactions, and the yield on alternative assets remain unchanged. (The amount of money balances held is determined in part by minimizing the opportunity costs of holding such balances.) If holding periods are long, or with a broader definition of money, this conclusion may not hold.

The economywide demand function is usually expressed as a function of Y , the nominal income, as scale variable. Movements in this measure are divided into two components, changes in the price level, P , and changes in real output, y , with the implication that the demand for money balances behaves differently depending whether P or y changes.

In the absence of money illusion, given nominal interest rates, a higher level of prices should correspond to an exactly proportional

increase in money balances i.e. homogeneity of degree one in prices. The function can be transformed into:

$$\frac{M}{P} = G(y, r)$$

While this hypothesis - price homogeneity-has empirical support,¹ demand functions generally include lagged right hand side variables. These lags represent adjustment costs which are distributed over time. The homogeneity postulate represents instantaneous responses in the endogenous variable.

Thus , as in the micro level, the demand for money at the macro level can be expressed as a function of transaction and opportunity costs. If this function is stable then the elasticities are of considerable interest to the monetary authorities and policy makers.

3:3 Elasticities

Per-capita real output rises because of technology and productivity gains. These advances are also reflected in the payments system mechanism which leads to lower transfer costs. Similarly higher incomes tend to induce better cash management techniques so that income elasticity should be less than one.

¹ Orr, D. (1970) Cash Management and The Demand for Money (New York: Praeger) pp. 197.

Since monetary policy can be effective whether the income elasticity is less than or greater than one, there are no major theoretical issues pertaining to this variable

The major schools of macro-theorists have had sharp disagreements over the value of the interest elasticity of demand. Monetary policy is effective as it causes changes in asset yields. These induce a spread between the calculated present value of real assets and their costs. If money balances were close substitutes for a subset of financial assets then the effect of a monetary shock would be to affect yields on the financial assets without affecting yields on the real assets.

The effect of money on the demand for real goods is the final stage in a "chain" reaction, the extent depending on the relative elasticities of the financial interest rates with respect to the monetary disturbance and the response of real assets to the changes in the rates.

Keynes' liquidity trap suggests an example where long rates do not respond to movements of short rates generated by operations in the monetary sector. The stability and predictability of the relationship between output and money would depend on the length and flexibility of this "chain" .

Monetarists argue that money is an asset that can be substituted for all assets, real or financial. Therefore the effects of monetary shocks

would occur by direct substitution, rather than being intermediated by financial markets.

Since Keynes' General Theory¹ the demand for money has been the center of the macro-policy debate. The necessary condition for money to exert a predictable influence on the aggregate economy is that there exists a stable demand function for money. This function is linear, and the arguments contain a small set of variables, each representing links to economic activity in the other sectors of the economy.²

3:4 Aggregation

This standard linear function has performed poorly³. Influences of money on the economy are studied using official monetary aggregates: M1, M2, M3, and L. These have been constructed by simple sum aggregation over arbitrary financial components. These definitions derive from the practice of commercial banks to construct aggregates from a list of those instruments considered to provide monetary services. As of 1992 the Federal Reserve Bank has 27 components of the entire collection of liquid financial assets. The component of each subset- $M1 \subset M2 \subset M3 \subset$

¹ J.M.Keynes, (1936) "The General Theory of Employment, Interest, and Money" (Harcourt Brace Jovanovich) pp.207.

² Judd, J., and Scadding. J. (1982) " The Search for a Stable Money Demand Function: a Survey of The Post 1973 Literature." *Journal of Economic Literature* 20(3) (September) pp. 993-1023.

³ Goldfeld, S. (1976)" The Case of the Missing Money" *Brookings Papers on Economic Activity* #3. pp. 683-730.

L- varies at different rates over time. There are fluctuations of these quantities, different for different assets, and new assets appear on the list from time to time complicating the calculation of the unique measure of "moneyness".

The simple sum aggregation:

$$M1 = \sum_i x_i$$

where x_i equals the i th. monetary component of the aggregate. The implication is that all the x_i are perfect substitutes. This procedure cannot distinguish income effects from substitution effects. This becomes a problem when there are significant changes in the relative prices of assets and the greater number of imperfect substitutes among the relevant short term assets.¹

The introduction of new accounts and the deregulation of deposit rates occurred in the late 1970's and early 1980's. In their study of the disaggregated demands of M1 and M2, Moore², Porter, and Small modeled M2 demands. Their models suggest that these innovations affected the components of M2 without materially affecting the M2

¹ Barnett, W.A., Fisher, D. and Serletis, A. (1992) "Consumer Theory and Demand for Money." *Journal of Economic Literature*. 30(4) pp. 2086-2119.

² Moore, G. R., Porter, R. D., and Small, D. H. (1990) " Modeling the Disaggregated Demands for M2 and M1: The U.S. Experience in the 1980's." in Financial Sectors in Open Economies: Empirical Analysis and Policy Issues. (ed.) Peter Hooper et al, (Washington: Board of Governors of The Federal Reserve System) pp.21-105.

aggregate. Their study showed that historical variations of M2 and the movement of funds among deposits within M2 can be explained by income and opportunity costs, and that demand adjusts promptly to changes in these determinants.

3:5 Inflation and Velocity

Central banks attempt to keep inflation in check so as to prevent major disruptions in economic activity. If there is a stable relationship between what can be seen to be controlled by the central bank - the money supply - and the level and rate of growth of economic activity, then the authorities can exploit this relationship. If this relationship changes then policy needs to be reconciled with these changes. Consider the equation of exchange.

$$MV = Py \quad \text{where}$$

y is real output, P is the price level and M a monetary aggregate.

Totally differentiate to obtain

$$V \frac{dM}{dt} + M \frac{dV}{dt} = P \frac{dy}{dt} + y \frac{dP}{dt}$$

Substituting $V = \frac{Py}{M}$, $M = \frac{Py}{V}$, and similarly for y and P we get

$$(a) \quad \frac{Py}{M} \frac{dM}{dt} + \frac{Py}{V} \frac{dV}{dt} = \frac{MV}{y} \frac{dy}{dt} + \frac{MV}{P} \frac{dP}{dt}$$

which implies the percentage change in the aggregate plus the percentage change in velocity is equal to the percentage change in real output plus the percentage change in prices.

Let $Y = Py$ be Nominal Income.

Totally differentiate Y , and divide the result by $Y=Py$ to get

$$\frac{1}{Y} \frac{dY}{dt} = \frac{1}{y} \frac{dY}{dt} + \frac{1}{P} \frac{dP}{dt}$$

which is the right hand side of (a), so that the percentage change in money plus the percentage change in velocity is equal to the percentage change in nominal income; GDP growth.

From (a) Inflation = Money supply growth + velocity growth - real Output Growth.

Total Output Growth = Nominal Output Growth - Inflation Growth.

Consequently if the US economy grows (typically 2.5% p.a.) then for growth in inflation to be zero then money supply growth = real output growth - growth in velocity.

Now money can be a predictor if and only if velocity growth is stable i.e. there is a predictable relationship between interest rates and velocity. Money holdings and velocity are inversely related as the opportunity costs of holding money rise as the yield differential between money, monetary assets, and other short term capital certain assets rise.

The M1 aggregate- checking deposits plus currency - by the mid 1970 had a velocity which was high by normal standards. The use of M1

in the post 1973 period underpredicted real GDP. During the early 1980's, as deregulation and financial innovation created new forms of narrow money, M1 became increasingly sensitive to interest rates and attention shifted to the broader M2 as that became less sensitive to short rates.

The relationship between the chosen aggregate and output will be stable if the underlying behavioral patterns remain unchanged. In a changing economic environment this stability will be fleeting as market forces and time induce changes in technology, demographics, and financial sophistication of individuals i.e. the structures of their risks and preferences.

Innovations in the arena of information technologies and computerized operations have allowed for the drastic reduction in transaction costs, monetary and convenience, associated with asset transfers. These new technologies and changing demands have produced new financial products (money market mutual funds, overnight money instruments, money market deposit accounts) and have made more products available to more people.

Thus with the appearance of no-load mutual funds, and in general lower transaction costs, and widespread availability of Treasury securities, electronic banking facilities have continuously degraded the relationship between M2 and output. Technological enhancements,

ATMs, and the increasing use of non-assets to meet shortfalls¹ (credit cards) have increased the choices available for transaction purposes, hence the components of M2 are being lowered while the composition also shifts.

Shifts in velocity, changes in the way people organize their consumption patterns and financial affairs , have produced instabilities in the demand for money.

¹ Ducca, J. and Whitesell, W. (1995)" Credit Cards and Money Demand: A Cross Sectional Study." *Journal of Money, Credit, and Banking* 27(May) pp.604-23.

Chapter 4

Theoretical considerations

4:1 Overview

The demand for money functions performed well in the period prior to the seventies. In subsequent periods their performance was questioned. This chapter focuses on how these functions were constructed.

Given a long run or equilibrium condition, short run dynamics are introduced. These dynamics produced three alternative specifications; a real adjustment, price level, and nominal adjustment. Combining the equilibrium condition with the short run dynamics gives a demand for money as an autoregressive distributed lag function.

Two representations of the function, the Koyck scheme, and the Pascal lag, are examined.

A behavioral rationale for the short run dynamics - adaptive expectations - is discussed. With certain specific assumptions a rational expectations model is derived from the adaptive expectations one.

4:2 Dynamic specifications

Prior to the 1970's, research on money demand indicated the existence of a stable relationship between the demand for money, M1, and a small set of macroeconomic variables.¹ These empirical relationships were inferred from a period that was relatively stable. In contrast, the subsequent period exhibited large swings in economic activity, high and volatile levels of prices, and supply shocks. Added to this the Federal Reserve began to formally use narrow money, M1, as an intermediate policy target in the early 1970's, adopting control procedures during 1979-1982, which for the first time sought to target the money stock.

Conventional demand functions, which had previously performed well, now over-predicted actual money balances² in the mid to late seventies. This led to research focusing on financial innovation and deregulation and reevaluation of money demand specifications. While these modified functions were successful in explaining the mid seventies they failed to account for the decline in velocity of M1 during 1982 and 1983.

¹ Goldfeld, S. (1973) "The Demand for Money Revisited." *Brookings Papers on Economic Activity* (3) pp. 573-638.

² Goldfeld, S. (1976) "The Case of the Missing Money" pp.690.

The conventional specification postulates a long run, equilibrium, demand for real transactions balances in the form

$$m_t^* = \beta_0 + \beta_1 r_t + \beta_2 y_t + XB$$

where m^* denotes desired real balances, r is a nominal rate on a riskless asset, y is real GDP, X is a row vector of possible explanatory variables which may include rates of return on long term assets, wealth, or expected inflation.

The short run dynamics are captured by using a partial adjustment mechanism,¹ developed by Chow, and is based on the notion that individuals adjust their actual real money stock to some desired level.

The functional form is:

$$m_t - m_{t-1} = \lambda(m^* - m_{t-1}) \quad 0 < \lambda \leq 1$$

where λ is the speed of adjustment, measuring the cost of "disequilibrium" relative to the cost of moving into "equilibrium" with respect to holding money balances. The Chow specification gives rise to an alternative price adjustment specification:²

¹ Chow, Gregory. C. (1966) "On the Long Run and Short Run Demand for Money" *Journal of Political Economy* (April) . pp. 111-131

² Laidler, D. E. W. (1983) " The 1981-82 Velocity Decline: A Structural Shift in Income or Money Demand?" *Monetary Targeting and Velocity, Conference Proceedings : Federal Reserve Bank of San Francisco* (December) pp. 100-03.

Gordon, R. J. (1984) " The Short Run Demand for Money: A Reconsideration" *Journal of Money Credit and Banking* (November) pp. 403-434.

$$P_t - P_{t-1} = \lambda(P_t^* - P_{t-1}).$$

This is rationalized by noting that in equilibrium the aggregate demand for money equals the aggregate supply. Since the nominal money stock is exogenous, equilibrium requires that $\frac{M}{P^*} = m^*$. Since M, nominal money balances, is given, the price level adjusts.

An alternative adjustment mechanism, suggested by Goldfeld¹, replaces real balances with nominal balances. His contention is that the real adjustment mechanism is defective on microeconomic grounds as it implies that individuals adjust real money fully to price level changes, but only partially to money demand changes i.e both the long run and short run demand for nominal money is unit elastic with respect to price.

Gordon² prefers the nominal specification as there are no adjustment costs to price level induced changes in money holdings as the only portfolio costs incurred result from adjusting nominal balances. If there is a supply shock that reduces real income then there is a proportional decrease in the demand for money: no portfolio disequilibrium occurs.

¹ Goldfeld, S. (1973) pp. 579.

² Gordon, R. J. (1984) "The Short Run Demand for Money: A Reconsideration" *Journal of Money Credit and Banking* (November) pp. 403-434.

When the nominal adjustment specification is used on aggregate data, Laidler¹ suggests a fallacy of composition in that the money supply is exogenous, determined by the Fed, and while individuals may adjust their individual balances, society as a whole cannot .

Hetzel² objects to the nominal specification as applied to aggregate data as this implies that money is endogenous and that the price level is exogenous. Monetary authorities supply money with a lag to the public hence λ , the speed of adjustment, represents the Federal reaction function rather than the speed of adjustment of money demand.

The conventional demand functions have as an equilibrium condition, real money balances as a function of income and interest rates. The short run dynamics have either real money or nominal money adjusting. One variant of the real adjustment implies a price level specification. The short run dynamics combined with the equilibrium condition results in a demand for money function that has distributed lags.

4:3 Distributed Lags

At the root of the partial adjustment specification is the distributed lag model:³

¹ Laidler, D. E. W. (1983) " The 1981-82 Velocity Decline: A Structural Shift in Income or Money Demand?" *Monetary Targeting and Velocity, Conference Proceedings : Federal Reserve Bank of San Francisco* (December) pp. 100-03.

² Hetzel, R. L. (1984) " Estimating Money Demand Functions" *Journal of Money Credit and Banking* (May) pp. 185-193.

³ Kmenta, J. "(1986) "Elements of Econometrics" (Macmillan) pp.527.

$$y_t = \alpha + \beta_0 x_t + \beta_1 x_{t-1} + \dots + \beta_k x_{t-k} + \epsilon_t ,$$

where x_t = a change in x that is maintained , k = number of lags, β_i , short run impact multipliers which give the change in average y given a change in x in the same period, $\beta_0 + \beta_1$ is the change in y in the next period.

$\sum_i \beta_i = B$, is the long run, or distributed lag multiplier.

In order to estimate a distributed lag model the following must be considered:

- 1: The length of the lags, k, is generally not derived from consistent theory, rather is determined in an ad hoc manner.
- 2: The number of degrees of freedom , given k, and small sample sizes result in poor estimates.
- 3: Multicollinearity and the associated large standard errors.

This model can be transformed and made tractable through the use of mathematical artifice.¹

4:3:1 The Koyck scheme

Assume an infinite distributed lag such that

Johnston, J. (1984)“ Econometric Methods” (McGraw Hill) pp.345.

¹ Koyck, L. M. (1954)“ Distributed Lags and Investment Analysis” (North Holland)

$$\beta_k = \beta_0 \lambda^k \quad 0 < \lambda < 1 \quad \text{where } \lambda \text{ is the rate of decline}$$

or decay, $1 - \lambda$ the speed of adjustment. Since $\beta_k < \beta_{k+1}$ the effect of the distant past becomes negligible. The coefficients depend on β_0 and λ , and the long run multiplier is given as:

$$\begin{aligned} \sum \beta_k &= \beta_0(1 + \lambda + \lambda^2 + \dots) \\ &= \beta_0 / (1 - \lambda) . \end{aligned}$$

The estimating equation:

$$Y_t = \alpha + \beta_0 x_t + \beta_0 \lambda + \beta_0 \lambda^2 + \dots + v_t ,$$

which when lagged once, multiplied by λ , and subtracted from the original equation gives:

$$Y_t = \alpha(1 - \lambda) + \beta_0 x_t + \lambda Y_{t-1} + \varepsilon_t .$$

This transformed equation is that of an autoregressive -moving average form, where the disturbance is serially correlated, i.e.:

$$\varepsilon_t = v_t - \lambda v_{t-1} .$$

The long run multiplier is estimated as $\beta_0 / (1 - \lambda)$. As noted before this model is a mechanical solution to a problem, one that is not based on theory.

This Partial adjustment -Koyck scheme can be justified by invoking technological and institutional rigidities, inertia, and high adjustment costs.

4:3:2 The Pascal Lag¹

It is sometimes reasonable to expect that the adjustment speeds, weights, would rise then decline, instead of consistently declining as in the partial adjustment model. This distribution of weights- an inverted V-lag distribution is called a Pascal Lag distribution. A model of this type is used to model the Swiss Demand for money function; discussed in the next Chapter; inter alia.

Let

$$Y_t = \alpha + \beta(w_0 X_t + w_1 X_{t-1} + \dots) + \varepsilon_t \quad ; \quad \varepsilon_t \sim N(0, \sigma^2),$$

where,

$$w_i = \binom{i+r-1}{i} (1-\lambda)^r \lambda^i \quad i = 0, 1, 2, \dots$$

and, r , is a positive integer, λ , is the adjustment parameter to be estimated. Upon substitution of the w_i the regression equation now becomes:

$$Y_t = \alpha + \beta(1-\lambda)^r \left[X_t + r\lambda X_{t-1} + \frac{r(r+1)}{2!} \lambda^2 X_{t-2} + \dots \right] + \varepsilon_t.$$

When $r = 1$ this reduces to a partial adjustment model of the type under discussion.

¹ Kmenta, J. "(1986) "Elements of Econometrics" (Macmillan) pp.537.

4:4 Adaptive Expectations and the partial adjustment specification.

The adaptive expectations, or error learning, model of Freedman¹ and Cagan² gives a behavioral rationale for the short run dynamics modeled as the partial adjustment specification.

Consider the following model:³

$$Y_t = \beta_0 + \beta_1 X_t^* + v_t$$

where Y is the demand for real money, X_t^* , the expected, equilibrium, unobservable, long run rate of interest. It is hypothesized that we form expectations adaptively according to

$$X_t^* = X_{t-1}^* + \lambda(X_{t-1} - X_{t-1}^*),$$

that is what is expected to occur during time t is equal what was the estimate for the prior period plus a fraction of the prior period surprise. We update our prior expectation by the surprise. If we substitute for the expected value in our original estimating equation, lag and multiply by λ and subtract we arrive at the following Autoregressive-Moving Average form:

¹ Friedman, Milton. (1957) "A Theory of the Consumption Function" National Bureau of Economic Research(Princeton University Press)

² Cagan, P. (1956) " The Monetary Dynamics of Hyper Inflation" "M.Friedman (ed), in *Studies in The Quantity Theory of Money*" (University of Chicago Press)

³ Gujarati, D. (1988) "Basic Econometrics" (McGraw Hill) pp 517.

$$Y_t = \beta_0 \lambda + \beta_1 \lambda X_{t-1} + (1 - \lambda) Y_{t-1} + v_t - (1 - \lambda) v_{t-1}$$

If we use the alternative form for the expectation,

$$X_t^* = \lambda X_t + (1 - \lambda) X_{t-1}^*,$$

we replace X_{t-1} with X_t

If this equation is estimated we would obtain the short run parameters. The long term parameters are deduced from the estimates, without the appropriate statistics necessary for hypothesis tests.

The Adaptive Expectations model can be justified as a Rational Expectations model¹ if X is stochastic and is generated by a first order moving average process.

Suppose:

$$X_t = X_{t-1} - X_{t-2} - \mu_t + \lambda \mu_{t-1}, \quad \mu \sim N(0, \sigma^2).$$

Substituting for $\mu_{t-1} = X_{t-1} - X_{t-2} - \lambda \mu_{t-2}$, and successive substitutions for μ_{t-i} give:

$$X_t = (1 - \lambda)(X_{t-1} + \lambda X_{t-2} + \lambda^2 X_{t-3} \dots) + \mu_t,$$

where:

$$E(X_t) = X_t^* = (1 - \lambda)(X_{t-1} + \lambda X_{t-2} + \lambda^2 X_{t-3} \dots).$$

Now substituting for X_t^* in the long run function we arrive at:

$$Y_t = \beta_0 + \beta_1 (1 - \lambda)(X_{t-1} + \lambda X_{t-2} \dots) + v_t.$$

¹ Kmenta, J. (1986). "Elements of Econometrics" P. 531.

Thus the Adaptive Expectations model is a Rational Expectations model when X is generated as postulated.

Uncertainty can be defined according to whether or not it is invariant to the participants- exogenous uncertainty versus behavioral uncertainty. We attempt to minimize behavioral uncertainty by creating institutions, establishing laws and customs: regularize interactions.

The Adaptive Expectations hypothesis is a simple way of modelling expectations - the way we attempt to deal with behavioral uncertainty in economic theory-. People learn from experience: there is inertia. They rely on past values in formulating expectations, and do not incorporate all information available.

CHAPTER 5

Empirical considerations.

5:1 Overview.

This chapter focuses on empirical studies that illustrate the variations in the forms of the conventional demand functions. Concerns that arise in estimating money demand functions are highlighted. In all the studies except one, the short run function is estimated, and the long run parameters are deduced from the short run estimates.

The discussion on short run dynamics and adjustment specifications previously referred to in Chapter 4 is enlarged to include empirical studies. Fair's test to determine whether nominal or real balances adjust is cited.

Roley's study of U.S. economy using both nominal and real specifications is presented. This study will later serve as a benchmark for comparing estimates developed by the models produced for this study.

The price level specification is developed and an application of this model - the Pascal lag variant - to the Swiss economy is analysed.

Wenninger's real adjustment specification applied to U.S. data is discussed. His results conclude that stability of the demand function is not the norm.

Thornton estimates the three specifications and concludes that none of the specifications produces parameter values that are consistent with theory for all the periods and sub-periods under consideration.

Stability and specification of the demand function are addressed. The problems associated with measurement errors, omitted variables and the transmission mechanism are illustrated.

The chapter concludes with a brief discussion of the Lucas critique and its implication for monetary policy effectiveness.

5:2 Real vs. Nominal Specification

A major question in the Literature has been whether the short run adjustments of actual to desired money holdings are in nominal or real terms. Fair¹ describes a procedure which tests the nominal vs. real hypothesis. The test is carried out for 27 countries and also investigates the structural stability and dynamic specification of conventional money demand functions. The results conclude in favour of the nominal adjustment hypothesis. He finds that although there is some evidence of

¹ Fair, Ray. C.(1987)" International Evidence on the Demand for Money" *Review of Economics and Statistics* pp. 473-480.

moderate structural instability, before and after 1973, this does not affect the conclusion .

The long run desired level of real money balances is a function of real income and a short term interest rate. The log specification is given as:

$$\text{Ln}\left(\frac{M_t^*}{P_t}\right) = \alpha + \beta \text{Ln}y_t + \gamma r_t$$

If the adjustment is in real terms it is given as:

$$m_t - m_{t-1} = \lambda(m_t^* - m_{t-1}) + \varepsilon_t \quad , \quad \text{where } m = \text{Ln}\left(\frac{M}{P}\right),$$

and if in nominal terms, as:

$$\text{Ln}(M_t) - \text{Ln}(M_{t-1}) = \lambda(\text{Ln}(M_t^*) - \text{Ln}(M_{t-1})) + \mu_t .$$

Combining the long run equilibrium condition with the short run dynamics leads to an estimating equation for the real adjustment as:

$$m_t = \lambda(\alpha + \beta \text{Ln}y_t + \gamma r_t) + (1 - \lambda)m_{t-1} + \varepsilon_t \quad , \quad (5:A)$$

and for the nominal adjustment as:

$$m_t = \lambda(\alpha + \beta \text{Ln}y_t + \gamma r_t) + (1 - \lambda)\text{Ln}\left(\frac{M_{t-1}}{P_t}\right) + \mu_t \quad . \quad (5:B)$$

The difference in these specifications is in the lagged money term. The test consists in putting the lagged money variables into an estimating equation which has real money balances as the dependent variable, and real GDP, nominal interest rates as independent variables. The object is to observe which one of the lagged money terms dominates.

The estimation was done by Two Stage Least Squares, corrected for serial correlation, and is based on the assumption that M_t is endogenous.

This test is in principle that of adding an irrelevant variable . If the variable is truly irrelevant then the variance-covariance matrix is unbiased. If the irrelevant variable is not orthogonal to the other independent variables then the variance-covariance matrix is larger and the estimates are not as efficient.¹

5:3 Estimates of Real vs. Nominal Specification

Using U.S. data, Roley² estimates the two basic versions of the money demand function- with real money adjusting, and with nominal money adjusting, for the periods 1959Q1-1973Q4, and 1974Q1-1983Q4.

The choice of samples was dictated by noting that the characteristics of the post 1974 economy were fundamentally changed as a result of deregulation, financial innovation, and especially in the change in monetary control procedures during the 1979-82 period.

The models estimated include as independent variables, a wealth variable, the quarterly average of the savings deposit rate, as well as the traditional short term T-bill yield and the quarterly average of GNP.

¹ Kennedy, Peter. (1989) "A Guide to Econometrics", 2nd. Ed.(MIT Press) pp.69.

²Roley, V.Vance. (1985) "Money Demand Predictability" *Journal of Money, Credit and Banking* Vol. 17 #4 Part 2 .pp. 611-641.

If we rearrange equation (5:B) by adding and subtracting $(1 - \lambda)LnP_{t-1}$

we get:

$$m_t = \lambda(\alpha + \beta Lny_t + \gamma r_t) + (1 - \lambda)m_{t-1} + (1 - \lambda)(p_{t-1} - p_t) + \mu_t, \quad (5:C)$$

this last term being the negative of actual inflation, and which, in the real adjustment model, the coefficient is zero. Roley notes that:¹

“ . if actual inflation serves as a proxy for expected inflation, .
a term such as $(p_{t-1} - p_t)$ may nevertheless appear a
statistically significant determinant in the real adjustment
model.”

This seems to imply that this term provides a test for the independent effect of inflation in the real adjustment model. Hafer² observes that it is unclear from the context whether we should reject the real adjustment in favour of the nominal adjustment, or accept that inflation has an independent effect on money demand. Hafer further notes that in estimating the nominal demand equation, (5:C), one cannot compare the relative merits of the two models as they are not nested.

The estimating equation used is:

$$m_t = b_0 + b_1(sdr)_t + b_2(tbill)_t + b_3y_t + b_4w_t + b_5m_{t-1} + b_6(p_{t-1} - p_t) + \varepsilon_t, \quad (5:D)$$

¹ Roley, V. Vance. (1985) pp.614.

²Hafer, R. W. (1985) “ Comment on Money demand Predictability” *Journal of Money, Credit and Banking* Vol. 17 #4 Part 2 pp. 642.

where sdr_t =short term deposit rate, $tbill_t$ = quarterly average of the T-bill rate, w_t =total real value of equities, m_t =real value of M1, p_t = price deflator. All variables are in natural logarithms. The coefficient b_6 is constrained to equal b_5 in the nominal specification. This equation is estimated using first differences. This is a type of specification test. If the model is properly specified then the coefficients should be robust over alternative orders of differences, and further, first differences may reduce spurious correlations due to trends.

The results for the log-level specification for 1959-74 show all coefficients except the savings rate are significant at the 5% level, with the correct signs for both nominal and real models. Serial correlation is seen to be a problem, implying problems with data or mis-specification. For the period 1974-83 income elasticity is significant at the 10% level, and lagged money at 5% for both models. Serial correlation seems to be absent. The coefficient of lagged money is 30% greater than for the prior period; which does not agree with the prediction of improved cash management techniques and economizing of transactions balances.

Results from the first difference form for 1959-73 are similar to those produced by the log-level model. In the post 1974 period for both nominal and real equations, the speed of adjustment coefficient,

$b_s = (1 - \lambda)$, declines from .9 to .3 . This implies that the speed of adjustment has risen¹ from 10 quarters to 1.4.

This study is concerned with short run parameters; long run parameters are derived, and their statistics are not determined.

Goldfeld (1973)² estimated the following equation using U.S. data for the period 1952-73:

$$\begin{aligned} \ln(M_t / P_t) = & a_0 + a_1 + \ln(GNP) + a_2 \ln(RMS)_t + a_3 \ln(RSAV)_t, \\ & + a_4 \ln(M_{t-1} - P_{t-1}) \end{aligned} \quad (5:E)$$

where M is currency and checkable deposits, P is the aggregate price level, GNP is real gross national product, RMS is a short term market rate of interest, and RSN is the rate of interest on savings deposits.

These results reported by Judd and Scadding³, together with Roley's results will serve as a basis for comparing estimates derived from this study and produced by the models, SHORTRUN and LONGRUN, discussed in Chapter 8.

5:4 Price Level Specification.

¹ λ is the speed of adjustment. The elasticities are short run elasticities., and $(1 - \lambda)$ is the coefficient of the lagged money term. Using quarterly data, the number of quarters to full adjustment $q_n = 1 / \lambda$.

² Goldfeld, S. (1973) "The Demand for Money Revisited." *Brookings Papers on Economic Activity* (3) pp. 573-638.

³ Judd, J., and Scadding, J. (1982) "The Search for a Stable Money Demand Function: A Survey of the Post 1973 Literature." *Journal of Economic Literature* 20(3) (September) pp. 996.

The conventional partial adjustment specification requires that in equilibrium, the money supply of real balances equals the money demand for real balances. If nominal money is exogenous then the equilibrium condition requires that:

$$\frac{M}{P^*} = m_d \quad ,$$

where P^* is the long run equilibrium Price Level. If M is exogenous and $M_t = M_{t-1}$, then if we substitute $(M / P^*) = m_d$ in the long run demand function

$$\text{Ln}(m_d) = f(Z) = \text{Ln}(M / P^*)$$

(where Z is a collection of independent variables- GDP, Interest Rates etc.) and in the adjustment equation

$$\text{Ln}(m_t) - \text{Ln}(m_{t-1}) = \lambda(\text{Ln}(m_d) - \text{Ln}(m_{t-1})) \quad ,$$

we get the price adjustment equation.

$$\text{Ln}(P_t) - \text{Ln}(P_{t-1}) = \lambda(\text{Ln}(P^*) - \text{Ln}(P_{t-1})) \quad .$$

Substituting for P^* we obtain the short run money demand function when nominal money is exogenous

$$\text{Ln}(M_t) - \text{Ln}(P_t) = \lambda f(Z) + (1 - \lambda)[\text{Ln}(M_t) - \text{Ln}(P_{t-1})]$$

5:5 Price level specification- Swiss Money Demand Estimates.

In 1973 the practice of pegging the exchange rate of the Swiss Franc to the U.S. dollar was discontinued. Under the Bretton Woods

agreement , the fixed exchange rate regime allowed U.S. inflation to be transmitted to the Swiss economy. In this regime inflation is determined to a large extent by the combined growth of the money supply of those countries participating in the exchange rate agreement. Prices are thus exogenous, the increases in international prices matched by increases in the domestic price level.

In order to maintain real balances the money stock is adjusted by the Swiss Central Bank by dollar purchases and the current account trade surplus.

After 1973, and with the new flexible exchange rate regime the monetary authorities could influence both money growth and inflation. Monetary targeting was central to the process , relying heavily on money demand estimates.

The policy goal was price stability, and to achieve this the demand for real balances had to be matched by growth in the money stock.

The underlying assumptions are that money stock is exogenous and that real balances adjust through price level adjustments.

Price level adjustments are not instantaneous as prices are "sticky". The partial adjustment specification is inadequate to capture in one speed of adjustment parameter, λ , the entire transmission

mechanism. Rotheli uses Pascal Lag model to estimate a Swiss money demand function in order to determine income elasticity.¹

Demand estimation becomes a part of the policymaking process since interest rates changes are unpredictable and income elasticity relatively stable. The product of this elasticity and growth in real income gives an estimate of money supply growth consistent with stable prices.

Because Prices are 'sticky' the adjustment process can be written as:

$$P_t = \sum_{i=0}^{\infty} w_i P_{t-i}^* ,$$

where $\sum_i w_i = 1$, and P_0^* is the equilibrium price at time zero.

Swiss estimates show price level responses to changes in money stock are characterized by slow initial speeds of adjustment, unlike the Koyck process where the adjustment speed is constant.

First the w_i is replaced with the Pascal Distribution giving:

$$P_t = (1-\lambda)^r \sum_{i=0}^{\infty} \binom{r+i-1}{i} \lambda^i P_{t-i}^* + \varepsilon_t .$$

When $r=1$, this gives the Koyck scheme. Empirically, using maximum likelihood procedures the order of r can be chosen - that which maximizes

R^2 . This specification implies an infinite adjustment process.

¹ Rotheli, Tobias. T. (1988) " Money Demand and Inflation in Switzerland: An Application of The Pascal Lag Techniques." *Federal Reserve Bank of St. Louis ' Review'* (May/June) pp.88.

Rewrite the equation to get

$$P_t = (1-\lambda) \sum_{i=0}^{t-1} \lambda^i P_{t-i}^* + (1-\lambda) \sum_{i=t}^{\infty} \lambda^i P_{t-i}^* + \varepsilon_t .$$

The second term reduces to $\lambda^t E(P_0)$. We can substitute another parameter for this expectation- the method of free parameters- or use the actual value of P at t=0- the method of determined parameters. Once the estimates of this equation are found, the equilibrium price level is replaced in the long run equation:

$$m_t - p_t^* = \alpha_0 + \alpha_1 y_t + \alpha_2 r_t , \text{ to get:}$$

$$\Delta p_t = (1-\lambda)^r \sum_{i=0}^{\infty} \binom{r+i-1}{i} \lambda^i (\Delta m_{t-1} - \alpha_1 y_{t-1} - \alpha_2 r_{t-1} + \varepsilon_0 .$$

Rotheli reports that $r=2$ is the best dynamic specification. The method of free parameters produces insignificant income elasticities. The method of determined parameters produces significant results. This model is correctly specified if and only if the initial values of P are approximately equal to the expected values. If not, the fit is poor, and there is significant correlation of residuals. Both methods produce consistent estimates of the long-run demand parameters and the adjustment parameter.

5:6 The Real Adjustment Specification.

Weninger¹ has an alternative explanation for the instability of the econometric money demand function estimates of M1 post 1970 and pre

¹ Weninger, John. (1988) " Money Demand; Some Long Run Properties" Federal Reserve Bank of New York, Review(Spring) pp. 23-40. "

1950. He suggests that the period 1950-1970 represented a unique period of parameter stability. For longer and different time periods the parameters are essentially unstable. The following equation was estimated for M1 and M2:

$$m_{t+} = b_0 + b_1(cpr)_t + b_2 y_t + b_3 m_{t-1} + \varepsilon_t ,$$

where cpr = commercial paper rate, y = real gross national product, and m = real money: all variables are in natural logarithms. This is essentially the real demand specification of Chow, the long run elasticities are β_1 and β_2 , obtained by derivation as $b_1 = \lambda\beta_1$; $b_2 = \lambda\beta_2$; and $b_3 = (1 - \lambda)$.

The equation was estimated for both real M1 and M2 for the period 1915Q1-1987Q3, and for the sub-periods 1915Q2-1949Q4, 1950Q1-1973Q4, and 1974Q1-1987Q3.

The study concludes that M1 was stable from 1950-73, confirming Goldfeld's results, (Goldfeld(1973)), but this stability was unique. Income and interest rate elasticities were low compared to the prior and later periods as well as the entire sample, implying that these were not representative of a general money demand function. In the longer term- for the other subperiods- M1 demonstrated persistent instability. Long run parameters are deduced; no statistics are presented.

5:7 Price Level, Real and Nominal Specification

Using U.S. data, Thornton¹ estimates the short run demand for the three adjustment specifications for the periods 1951Q2- 1961Q4; 1962Q1- 1973Q4; and 1974Q1-1984Q2. The first two periods correspond to the period when Goldfeld(1973) found a stable real money demand function.

In this period the Federal Reserve was relying on an interest rate target. After this period they focused on the monetary aggregates. In all periods the growth and variability of money and prices differ significantly. The estimating equations are in the conventional form. (Log specification, and lower case letters represent real magnitudes).

The long run function is given as:

$$m^* = f(x, \alpha, v) = f(z_t),$$

where x is a set of endogenous and exogenous variables, (real income, wealth and interest rates), α a vector of unknown parameters, and v an error term. The real adjustment specification is:

$$m_t = \lambda f(z_t) + (1 - \lambda)m_{t-1},$$

The Nominal specification is:

$$m_t = \lambda f(z_t) + (1 - \lambda)(M_{t-1} - P_t), \text{ reducing to}$$

$$M_t = \lambda f(z_t) + (1 - \lambda)M_{t-1} + \lambda P_t .$$

The price level specification is:

$$m_t = \lambda f(z_t) + (1 - \lambda)(M_t - P_{t-1}), \text{ reducing to}$$

¹ Thornton, Daniel. L. (1985) "Money Demand Dynamics; Some New Evidence" *Review-Federal Reserve Bank of St. Louis* Vol 67 # 3 pp. 14-23.

$$P_t = -\lambda f(z_t) + (1 - \lambda)P_{t-1} + \lambda M_t .$$

All equations contain the random error term v_t . As noted previously, these models cannot be compared statistically as the dependent variable is not the same in each case. The residual vector in each case is a different random variable, orthogonal to the vector space of the independent variables.

The results show that none of the specifications produce parameter values consistent with theory for all three periods. Stability tests depend critically on the inclusion of a passbook rate (interest rate variable), in the long run function, and the Nominal specification cannot be rejected at the 5% level in the last two periods. Long run parameters are deduced.

5:8 Measurement errors, omitted variables and the transmission mechanism.

The stability of the demand function, pioneered by the work of Friedman, Cagan, Brunner and Meltzer, and later confirmed by numerous other studies, produced a theory of the relationship between monetary policy and inflation.

Goldfeld's 1976 "The Case Of The Missing Money", questions this stability and subsequent researchers have identified reasons for this instability. As shown in the previous chapters these include: mis-specified functions and mis-specification, inadequate estimating techniques, changing economic institutions and environment, lags and short run

dynamics. These factors may not be adequate in explaining why conventional demand functions behave as erratically as they do.

As we have noted these functions begin with a long run relationship, which involves an expected value, and which is eliminated through a short run dynamic adjustment mechanism. The solution, an estimable function, is purportedly a structural form.

The estimates show a highly significant coefficient of the lagged dependent variable. The size of this coefficient is close to one, which implies extremely slow speeds of adjustment and high transaction costs. Goodfriend¹ suggested that the demand function contains measurement errors in the independent variables, which introduce serial correlation in the regression residuals and help to explain the significance in the lagged money coefficient. The conventional models use current income as a proxy for permanent income. He shows that models which include measurement errors and omitted variables produce serial correlation and significant lagged coefficient estimates which are biased toward one, if the correlations are positive.

Laidler² argues that the process whereby money holdings vary with a distributed lag in response to changes in the arguments of the demand

¹ Goodfriend, Marvin. S. (1985) "Reinterpreting Money Demand Regressions" *Carnegie-Rochester Conference Series on Public Policy* 22, pp. 207-242.

² Laidler, David. (1982) "Monetary Perspectives" (Harvard University Press) Chapter 2.

for money function cannot be appropriate in an economy where causation runs from changes in money supply to variations in those arguments.

These adjustments are better captured by describing a complete version of the transmission mechanism of monetary policy.

Laidler describes a simple model¹ to illustrate.

Consider an economy, all variables in log form, Y is the permanent component of Income, y is transitory, p is the general price level and m is money:

$$m_d = b_0 + b_1 Y + p \quad b_1 > 0$$

$$y = a(m_s - m_d) \quad a > 0$$

$$\Delta p = g y + \Delta p_{-1}^e \quad g > 0$$

$$\Delta p^e = d \Delta p + (1-d) \Delta p_{-1}^e \quad 0 < d < 1$$

This system reduces to:

$$m_s - p = \lambda(b_0 + b_1 Y) + (1-\lambda)(m_{s-1} - p_{-1}) + w, \text{ where:}$$

$$\lambda = (gad) / (1-d + ga), \text{ and:}$$

$$w = \left(\frac{1-d}{1-d+ga} \right) (\Delta m_s - \Delta p_{-1} - gab_1 \Delta Y).$$

This reduced form looks like the conventional form but with the omitted variable w .

¹Laidler, David. (1985) "Comment on Money Demand Predictability" *Journal of Money Credit and Banking*, Vol 17 #4 pp. 650.

It is noted that during times of relative economic stability, during the 1950/60, the w term would not be a source of major disturbances, and that the adjustment parameter, a complicated function of the monetary policy transmission mechanism, will also be stable under these conditions. Thus even though w is omitted the demand function can be identified and estimated.

If the demand function contains other variables apart from Y , as in this case, which vary with money growth, the omitted variable - which is now included in the disturbance term- is correlated with the explanatory variables, and estimates are biased and inconsistent. To quote Laidler¹

“.. if output and prices fluctuate with long and variable lags in response to variations in the money supply, then conventional approaches to estimating the short run demand function, so called, will involve attempts to compress the structure that underlies those lags into one parameter; the resulting relationship will not be a structural demand for money equation at all but a reduced form, and what appears to be a stable function under one policy regime will tend to shift under another,...”

¹ Laidler, David, W. (1985)“ Comments on Money Demand Predictability” *Journal of Money Credit and Banking*, Vol 17 #4 pp. 651.

However, when the money supply is subject to fluctuations akin to those experienced in the late 70's and early 80's, the effect of w can be felt. Furthermore as the focus is, in this period, on the money supply, the parameters of the transmission mechanism vary endogenously. This is a variation of the Lucas Critique ¹

5:9 The Lucas Critique

Econometric equations model expected change in inflation, π^e , as a distributed lag of past changes in cost of living² as $\pi_t^e = \sum_{i=1}^k \alpha_i \pi_{t-i}$, where $\sum_i \alpha_i = 1$, and the coefficients are estimated using historical - actual - past inflation results. It is assumed that reasonable forecasts of inflation are produced in the short run if monetary policy and other inflation influencing variables are stable.

Applied to the Phillips curve the equation assumes no knowledge of the effect of unemployment on the rate of inflation, and that inflation depends only on its past values and does not depend on the monetary policy objectives- whether or not anti-inflation policies are being pursued.

¹ Lucas, Robert. E. Jr. (1976) "Econometric Policy Evaluation" in "The Phillips Curve and Labour Markets" *Carnegie-Rochester Conference Series on Public Policy* Vol 1 : Karl Brunner and Allan H Meltzer Editors (Amsterdam : North Holland) pp. 19-46

² Fuhrer, Jeffrey. C. (1995) "The Phillips Curve is Alive and Well ", *New England Economic Review, Federal Reserve Bank of Boston*(March/April) pp. 41-57

Applied to demand for money equations, if changes in proposed monetary regimes rely on past estimates, prior period α 's, to produce forecasts, and if wage and price setters know about this proposed change, then they will change their expectations under the new regime, i.e. wage and price setters' expectations of future events may crucially depend on the behaviour of other agents in the economy, especially those who set monetary policy¹.

The conventional demand functions are reduced forms, derived from a structural model whose parameters are assumed to be invariant to policy. However, if agents know the reaction function of the authorities and if their expectations are formed rationally then these parameters will not be constant.

In order for monetary policy to be effective agents must be fooled. This is the monetary policy ineffectiveness proposition which suggests that the authorities define clear cut rules about the conduct of policy, rather than using discretionary policy to influence outcomes e.g. unemployment and output.

¹ Sargent, Thomas. J, and Wallace, Neil. (1975). " Rational Expectations and the Theory of Economic Policy" in *Studies in Monetary Economics 2 Federal Reserve Bank of Minneapolis*, (June).

CHAPTER 6

Time Series Econometrics and Money Demand

6:1 Overview.

This chapter deals with the problem associated with the empirical analysis of economic time series - non-stationarity-. The proposed solutions are discussed and applications to the field of money demand are cited.

The notion of stationarity is addressed, linking the concepts of polynomial lag functions and unit roots. The problems of nonsense correlations and spurious regressions lead to the notion of integrated series. Specification errors and the problems of inference are addressed.

The properties of integrated series are described, focusing on variables in the demand function. Unit root and co-integration tests are discussed.

The equation of exchange is used to illustrate the concept of co-integration and long term relationships.

Empirical demand for money studies using co-integration and error correcting models are discussed.

Miller's analysis of U.S. data- 1959-87 show that only M2 has a long run relation between real GDP, the price level, and the commercial paper rate. Transmission channels for monetary policy are different for the pre and post 1973 periods.

Hendry and Ericsson evaluate Friedman and Schwartz' model of the U.K. money demand function and conclude that money is endogenously determined by demand factors.

Dickey, Jansen, and Thornton, study U.S. data for the period 1953-88 and find a stable long run relationship between real M1, real GDP, and nominal interest rates. They conclude that price level stability can be achieved by controlling growth rates of either M1 or the monetary base.

Taylor's model tests and rejects Goodfriend's hypothesis that measurement errors give rise to the dynamic specification of demand for money functions . This study suggests that co-integration methods may be useful in testing the buffer-stock notion.

6:2 Stationarity

In standard econometric theory least squares estimation assumes stationarity of the explanatory variables. If this assumption is violated then whenever these procedures are used to estimate conventional money demand equations specification errors are committed. Re-evaluation of the models then becomes necessary.

Stationarity, "weakly stationarity", or "co-variance stationarity" requires that in a model such as:

$$Y_t = \alpha + \beta X_t + \varepsilon_t, \text{ where:}$$

$$E(\varepsilon_t) = 0, \dots V(\varepsilon_t) = \sigma_\varepsilon^2, \dots \text{Cov}(\varepsilon_t, \varepsilon_{t-1}) = 0, \dots t \neq s.$$

The error term is generated by a process whose mean and variance is unchanging. The co-variance depends on $|t-s|$, not on t , or s . Strong stationarity implies an unchanging distribution.

Ordinary Least Squares uses the assumption that :

$$Plim(1/n)(X'X) = Q,$$

where Q is a fixed matrix. Increasing the sample size result in sample moments converging to population values.

6:3 Autoregressive Processes

Economic time series are generally not stationary. The series:

$$y_t = \{y_0, y_1, y_2, \dots, y_t\},$$

is the realization of a non repeatable process. It is reasonable to assume that these variables are strongly auto-correlated, dependent on past values. This dependence is generally assumed to be Autoregressive of order 1, AR(1). This assumption dominates as higher order processes are mathematically difficult to analyse. As we cannot know a priori the correct order, the AR(1) formulation is a reasonable model for the underlying process.

The AR(1) process is described as:

$$\varepsilon_t = \rho\varepsilon_{t-1} + \mu_t,$$

where the μ represents a stationary process. Repeated substitutions give:

$$\varepsilon_t = \mu_t + \rho\mu_{t-1} + \rho^2\mu_{t-2} + \dots$$

If we allow the restriction $|\rho| < 1$ then ε_t becomes a stationary process. If s

becomes large then the variance of ε becomes $\frac{\sigma^2}{1-\rho^2}$, which is finite.

More generally we can represent an AR(1) process in terms of a polynomial lag function

$$(1 - \phi L)y_t = \varepsilon_t \quad \text{where } \varepsilon_t \text{ is a stationary process.}$$

Again:,
$$y_t = \frac{\varepsilon_t}{1 - \phi L} = \varepsilon_t + \phi\varepsilon_{t-1} + \phi^2\varepsilon_{t-2} + \dots$$

and:
$$E(y) = 0 \dots V(y) = \frac{\sigma_\varepsilon^2}{1 - \phi^2}.$$

Thus for the variance to be finite the condition $|\phi| < 1$ must hold.

If we define the characteristic function of the polynomial L to be

$$C(z) = 1 - \phi L = 0,$$

then the solution is:

$$z = (1/\phi).$$

Thus if y is a stationary series, then $|z| > 1$. This is the well known result that the root of the polynomial lies outside of the unit circle.

6:4 Spurious Regressions and Unit Roots.

Regression methods, appropriate in analysing stationary series, were assumed valid when also applied to non stationary series as the non

Regression methods, appropriate in analysing stationary series, were assumed valid when also applied to non stationary series as the non stationarity was of little consequence. However, recognizing potential problems in the offing, methods of transforming the data into stationary series; e.g. differencing, de-trending; were developed and applied. The problem with non stationary data was demonstrated by Yule's¹ "nonsense regressions", and the "spurious regressions" of Granger and Newbold² of 1974.

Consider a random walk without drift, defined as:

$$y_t = y_{t-1} + \varepsilon_t,$$

where ε_t is a stationary process. If the series is differenced then:

$$\Delta y_t = \varepsilon_t,$$

is a stationary process. The original series, an AR(1) is a non-stationary process - $\rho = 1$ -, the variance is infinite. The Characteristic polynomial

$$C(z) = 1 - \rho z = 0 \dots \Rightarrow z = 1/\rho, \text{ and } z=1.$$

The roots of this polynomial lie on the unit circle.

The preceding illustrates that if a series has an exact unit root, and if it is differenced then the resulting series is stationary. We define such a

¹ Yule, G. U. (1926) " Why Do We Sometimes Get Nonsense-correlations Between time Series? A Study in Sampling and the Nature of Time Series" *Journal of The Royal Statistical Society*, 89, pp. 1-64.

² Granger, C. W. J., and Newbold, P. (1974) " Spurious Regressions in Econometrics" *Journal of Econometrics* 2 pp.111-120.

series as integrated of order 1, $I(1)$. Generally if a series is $I(d)$, then a stationary series is produced if it is differenced d times.

Let y_t be a random walk without drift, and x_t another similar uncorrelated random walk, and assume further that there is no economic relationship between them. If the regression model:

$$y_t = \alpha + \beta x_t + \varepsilon_t ,$$

is estimated, spurious correlations may persist even in large samples.

That is $\beta \neq 0$ and the regression R^2 would not tend to zero.

If these series were stationary and the model estimated then the parameter estimate and the 't' statistics would tend to zero as the sample size increases.

Regressing two unrelated non-stationary series leads to results that cannot be interpreted within the conventional testing procedure. Phillips¹ provides analytical results. The conventionally calculated 't' statistics of these OLS parameter estimates do not have limiting distributions; in fact they diverge in distributions as the sample size increases. The null hypothesis- that there is no relationship between variables - will be rejected as the sample size increases. Similar divergence in distribution of the calculated 'F' statistic also leads to rejection of the null.

¹ Phillips, P. C. B. (1986) " Understanding Spurious Regressions in Econometrics" *Journal of Econometrics* #33 pp. 311-340.

While these tests reject the null, evidence of auto-correlation in the residuals, low Durbin-Watson statistics, suggests a mis-specification error.

Data with 'unit roots', $I(1)$ variables, pose problems which lead to serious errors of inference. Transformation to induce stationarity lead also to spurious results. Phillips (1986), demonstrated in Monte Carlo studies that standard 't' statistics do not have the standard 't' distribution, and that spurious correlations will tend to emerge even with deterministically trended random walks.

Therefore techniques to test for stationarity - orders of Integration - of a series and methods that apply to non-stationary data were developed.

Economic time series are generally non-stationary. They are usually integrated. Such series accumulate past effects; the future path depends on all past influences and their moments are time varying. The following is a list of some integrated economic series.

$I(1)$ series that grow at a constant rate. Stocks and flows that relate to population size etc. the monetary base, M1, M2.

$I(2)$ series that grow at an increasing rate, eg. price level data.

I(3) series that are unusual. Examples are found in data e.g. money supply, the price level, observed in hyperinflationary episodes.

Many economic variables contain an exact unit root. Roots slightly greater than one¹ imply non-stationarity, and are not integrated for any d . Roots substantially greater than one are explosive and are easily identified. Roots of series that are slightly less than one - near integrated series - are difficult to distinguish, in moderately sized samples, from those which have exact unit roots .

6:5 Tests for orders of Integration- Unit Root Tests

Unit roots in macroeconomic data have serious implications. If a variable such as Output is I(1) then shocks to this variable will have permanent effects. Schmidt and Waud² showed that there is no long run response to growth in real output to changes in M1. If output is I(1) then this conclusion should be reconsidered³

In the AR(1) process:

¹ Banerjee, A., Dolado, J., Galbraith, J. W., and Hendry, D. F. (1993) " Co-Integration, Error-Correction, and the Econometric Analysis of Non-Stationary Data" (Oxford University Press) Chap. 4.

² Schmidt, P., and Waud, R. (1973) " The Almon Lag Technique and Monetary vs. Fiscal Policy Debate" *Journal of the American Statistical Association* #68 pp. 11-19.

³ Greene, William. H. (1993) "Econometric Analysis" 2nd. Edition, (Macmillan Publishing. Co.) pp. 563.

$$y_t = \rho y_{t-1} + \varepsilon_t ,$$

the error term being stationary gives the OLS estimator , for $|\rho| < 1$,

$$Plim.. \hat{\rho} \rightarrow \rho , \text{ and that } \sqrt{T}(\hat{\rho} - \rho) \sim N(0, 1 - \rho^2) .$$

Evans & Savin¹., and Dickey & Fuller² showed that when $\rho=1$ then

$$T(\hat{\rho} - \rho) \sim v ,$$

where v is a random variable with a finite variance, and in finite samples the expectation of the estimate is less than one. The OLS estimator is biased downward and converges rapidly to its probability limit. Dickey-Fuller statistics adjust the OLS statistic to test the hypothesis that the correlation coefficient is equal to one.

The purpose of this and similar tests for unit roots is to test economic relationships and to avoid spurious relationships. Unit root tests are tests applied to univariate time series. Tests of relationships between Integrated series are Co-integration Tests.

6:6 Co-Integration

In the correctly specified model:

¹ Evans, G., and Savin, N. (1981) "Testing For Unit Roots I " *Econometrica* #49 pp.753-779.

Evans, G., and Savin, N. (1984) "Testing For Unit Roots II " *Econometrica* # 52 . pp.1241-1269.

² Dickey, D., and Fuller, W. (1979), " Distribution of Estimators for Autoregressive Time Series with a Unit Root" *Journal of the American Statistical Association* #74 pp.427-431.

$$y_t = \beta x_t + \varepsilon_t \quad , \quad \text{and } \varepsilon_t \text{ is stationary,}$$

then $(y_t - \beta x_t)$ must also be stationary. If x and y are integrated of different orders then there is an inconsistency, i.e. if they are of different orders they must be drifting apart. Linear combinations of series with different orders of integration give rise to an integrated series of the higher of the two orders.

If two series are integrated of order 1, as in the above example, then the difference between them may be stable around a fixed mean. If these series satisfy these requirements they are said to be Co-Integrated, and the vector $[1, -\beta]$ is the co-integrating vector.

If there is a theoretical relationship between economic variables, and if this is to hold, especially in the long run, then these variables should not drift apart.

In the short run there is room for divergence. Seasonal effects, information asymmetries, rigidities etc.- may contribute to this divergence. In the long run these conditions do not hold. If there is a divergence, it should be bounded, and furthermore this bounded interval should diminish over time.

Co-Integration expresses in statistical terms the nature of the long term relationship between two or more time series, each of which is individually non-stationary.

Consider the equation of exchange¹

$$\ln M + \ln V - \ln P - \ln Y \equiv 0$$

A theory of the demand for money converts this identity into an equation when v - the velocity - is a function of a number of economic variables.

If $v = g(r)$, then $\ln M + \ln(g(r)) - \ln P - \ln Y = \varepsilon_t$. If the residual is stationary, and the variables are integrated of the same order, then the vector $[1, 1, -1, -1]$, is the co-integrating vector, and we have a long run relationship given by economic theory. The unknown co-integrating vector must be estimated. Methods² developed involve locating the most stationary of the linear combinations out of all possible. This selection process can encounter distributional problems which result in loss of power.

A test for Co-integration can be performed by applying a conventional unit root test on the residual.

¹ Dickey, D. A., Jansen, D. W., and Thornton, D. L. (1991) "A Primer on Co-Integration with an Application to Money and Income" *The Federal Reserve Bank of St. Louis Review*, 73(2) pp.58-78.

² Engle, Robert. F., and Granger, C. W. J. (1987) "Co-Integration and Error Correction: Representation, Estimation and Testing" *Econometrica*, 55: pp.251-276.

Stock, James. H., and Watson, Mark. (1988) "Testing for Common Trends" *Journal of the American Statistical Association*, 83: pp.1097-1107.

Johansen, Soren. (1988) "Statistical Analysis of Cointegration Vectors" *Journal of Economic Dynamics and Control*, pp.231-254.

In general interpretation of these vectors must proceed with care. They are obtained from the reduced form of a system where all variables are assumed to be jointly endogenous.¹ These cannot be interpreted as structural equations because there is no way to go from reduced form to structural form. However they can be interpreted as the constraints that the economic structure imposes on the long run relationship among the jointly endogenous variables.

6:7 Co-Integration and error correction: Empirical studies

Consider a bi-variate case where there is a co-integrating relationship. The Co-integrating vector $[1, -\beta]$ captures the long run, low frequency, relationship between the variables, so that individual effects cancel producing the stationary vector of residuals. The residuals, $(y_t - \beta x_t)$, measure the high frequency, short run deviations from the long run relationship.

This suggests a procedure that produces estimates of the long run relationship, while at the same time retrieving information about the short run dynamics.

¹ Dickey, D., Jansen, D., and Thornton, D. (1991) "A Primer on Cointegration with an application to Money and Income" Review, Federal Bank of St. Louis, 73(2) pp. 76.

Miller¹, in his study of monetary dynamics summarizes the procedure in the following steps.

(1) Determine the orders of Integration for each of the variables under consideration- difference each series until a stationary series emerges.

(2) Estimate co-integration equations by OLS, using variables with the same orders of integration.

(3) Test the residuals of the co-integration regression for stationarity.

(4) Construct error-correction models. These are models where the first differences of each of the variables in the co-integration equation, endogenous and exogenous, are assumed to be functions of the distributed lags of the first differences of the variables, as well as the first differences of the residuals from the co-integration equation- this is the error correction term. The system is a Vector Auto-Regression, which is constrained by the error correction term.

All the variables in the system are stationary. Furthermore since all the variables are endogenous, questions about endogeneity of variables - the short run dynamic adjustment categories - whether in the short run

¹ Miller, Stephen. M, (1991) " Monetary Dynamics: An Application of Co-Integration and Error Correction Modelling." *Journal of Money, Credit, and Banking*. Vol. # 23, (May) pp. 139-154.

price, real money, or nominal money adjusts, become moot. All of these specifications can be incorporated into one model, which encompasses each of the specifications.

Hendry¹ notes that these error correction models only work within regimes and do not correct towards an equilibrium position that has changed.

Co-integration of more than two variables may lead to non-uniqueness of the integration vectors. Furthermore omission of variables may produce non co-integrating results.

Miller's analysis shows for US data for the period 1959i-87iv:

(1) Long run money demand, only M2 has a long run trend relationship with real GNP, the implicit Price deflator, and the four to six month commercial paper rate. The implication is that M2 should be used as a guide to policy.

(2) Different channels for the pre 1973 and post 1973 monetary policy effects. Pre 1973 results suggest that the bulk of the money market disequilibria was absorbed by the goods and service markets. This was a consequence of policy, which was to stabilize interest rates. Post 1973 results suggest that financial markets were allowed to respond to monetary disequilibria.

¹ Hendry, David. F. (1995) "Dynamic Econometrics" (Oxford University Press) pp. 213

Hendry and Ericsson¹ evaluate an empirical model of the UK money demand developed by Friedman and Schwartz. Their study finds mis-specification and suggests a better fitting dynamic error-correction co-Integration model. Their model is consistent with a structure in which nominal money is endogenously determined by demand factors, conditional on prices, incomes and interest rates.

Within this study, criteria for econometric modelling were enumerated, including the following.

(1) Theory Consistency. Economic Theory suggests the Co-integrating or Long run relationship

(2) Dynamic specification: The influence of model design- dynamic mis-specification- invalidates inference, hence dynamics cannot be ignored. Co-Integration implies and is implied by error-correction representations. These are discrepancies arising from agents' past decisions, and their attempts to correct them; alternatively, they arise from an optimal response to a dynamic environment.

(3) Exogeneity. This refers to the notion of being determined outside of the model. The omission of variables assuming exogeneity is illegitimate.

¹ Hendry, David. F, and Ericsson, Neil. R. (1991) " An Econometric Analysis of United Kingdom Demand in ' Monetary Trends in The United States and The United Kingdom, by Milton Friedman and Anna Schwartz' " *The American Economic Review*,(March)pp. 8-38.

(4) Parameter constancy. This implies structural stability. Economic systems are in a state of flux. Reduced form equations change when the underlying structure changes. While there is a need to identify models that have constant parameters, recursive estimation is recommended to investigate parameter constancy.

As previously noted, Dickey, Jansen, and Thornton, investigate the co-integration relationships between money income, and interest rates. Their study covers the U.S economy for the period 1953-1988. They argue that if M2 and nominal income are co-integrated, while M1 and nominal income are not, then a stationary long run relationship exists between M1 and the non M1 components of M2. They further argue that if M1, real income and nominal interest rates are co-integrated, then the same could be true for real income, nominal interest rates and the monetary base. Their tests for co-integration among real M1, real income and one of two interest rates using three procedures show results which are sensitive to the method used. The Johansen technique shows a single cointegrating relationship among these variables. While this vector cannot be interpreted as the long run demand for money, the estimated long run income and interest elasticities are consistent with hypothesized values and with previous empirical studies.

They find a stable long run relationship between real money, real income and nominal interest rates and conclude that there is the potential

for achieving price level stability by controlling the growth rates of either M1 or the monetary base.

Taylor¹ applies co-integration methods to estimate the short run demand for money function, and to test the hypothesis, Goodfriend's, that measurement errors in the demand function produces serial correlation in the residuals and are responsible for the significant results and magnitude of the coefficient of the lagged dependent variable. The period analysed, 1952-1972, was chosen because the demand function was stable. The tests strongly reject the hypothesis, suggesting that measurement errors are not solely responsible for the dynamic nature of empirical demand relationships.

Taylor² notes that theoretical underpinnings of the dynamics can be fruitfully investigated with the buffer stock notion in mind.

6:8 Conclusion.

Given the nature of economic data the question of non-stationarity must be taken seriously. The methods discussed attempt to resolve the difficulty this poses. The results produced can be contradictory: Miller's results versus those of Dickey, Jansen, and Thorton. The limitation noted

¹ Taylor, Mark, P. (1994) " On the Reinterpretation of Money Demand Regressions" *Journal of Money, Credit, and Banking* (November) pp. 851-866.

² Taylor, Mark, P. (1994) " On the Reinterpretation of Money Demand Regressions" *Journal of Money, Credit, and Banking* (November) pp. 861

by Hendry with respect to error correction models and changes in the equilibrium state must be borne in mind.

CHAPTER 7

THE BOOTSTRAP.

7:1 Overview.

This chapter focuses on the bootstrap, a non parametric estimation technique, developing the estimating procedure and the tools of inference, confidence intervals. Empirical studies are reviewed

First, fundamental weaknesses of parametric regression methods are highlighted, namely, assumptions have to be made about the distribution of the error terms, and that analytic formulae must exist in order to calculate the statistics of the coefficient estimates. The bootstrap, a non-parametric method, avoids making distributional assumptions and does not require analytic formulae. The general bootstrapping technique is introduced and is applied to regression models, where the random component in these models is the error term, not the data.

The bootstrap is diverse in application. The following summarizes the empirical studies reviewed.

(1)Application to determine measures of inequality.

(2) Generalized Least Squares application to study the demand for energy.

(3) A simulation study for prediction when the model is a stationary autoregressive process, and the error term has different distributions.

(4) Estimation of the Bartlett adjustment used in simultaneous equation modelling.

The bootstrap technique produces a sampling distribution of the estimate. Confidence intervals derived from this empirical distribution are described. These are The Normal approximation method, The Percentile method, The Percentile-t method, and the Bias corrected method.

7:2 Introduction.

Traditional estimation of linear econometric models requires strong assumptions about the probability distributions of the error term, and the existence of analytic formulae in order to calculate the statistics of the parameters. This becomes problematic when the formulae do not exist or are unavailable, and when the distributional assumptions are difficult to justify. The OLS estimator is fitted to the model:

$$Y = X\beta + \varepsilon ,$$

A stochastic structure is imposed in the form of assumptions underlying the variance-covariance matrix of the error term, e.g., $E(\varepsilon) = 0$, and the variance, $V(\varepsilon) = \sigma^2$, and additionally that, $\varepsilon \sim N(0, \sigma^2)$.

The Generalized Least Squares estimator calculates the parameter value β to be:

$$\beta_{GLS} = (X' \Sigma^{-1} X)^{-1} X' \Sigma^{-1} Y,$$

where the variance-covariance matrix Σ is unknown and is usually replaced by some estimate $\hat{\Sigma}$ through the use of iterative procedures. In finite samples, poor estimates of Σ are obtained giving rise to optimistic standard errors.

The bootstrap methodology differs from the traditional parametric approach in that it is a non-parametric technique whereby the sampling distribution of the coefficient estimate is directly estimated, allowing for inferences about the true parameter's properties.¹

The sampling distribution is influenced by the sample size and its measures of central tendencies. The object of the exercise is to use the limited information contained in the sample to make inferences about the underlying population. The Bootstrap technique uses the sample as if it were the population in order to construct "pseudo data" from which a sampling distribution of the parameter in question can be determined.

The scope of models in which the bootstrap can be applied include those with non-normal errors, autoregressive lag structures, and those that

¹ Freedman, David. A., and Peters, Stephen. C. (1984) "Bootstrapping a Regression Equation: Some Empirical Results" *Journal of The American Statistical Association*, Vol. 79, #.385. pp.97-105.

use Generalized Least Squares with estimated variance-covariance matrices.

Consider a parameter, θ , If the true probability distribution of θ is F_0 , and an estimator of the parameter is $\hat{\theta}$, whose distribution function is F_1 . The mean squared error is given as:

$$\sigma^2 = E(\hat{\theta} - \theta)^2,$$

or in terms of distributions:

$$= E\{\theta(F_1) - \theta(F_0)\}^2 / F_0.$$

The estimate $\hat{\theta}$ is a function of the distribution F_1 and of θ . Treating the sample as the population, the bootstrap estimate of θ , designated as θ_B . The mean squared error becomes:

$$\begin{aligned}\sigma_B^2 &= E(\theta_B - \theta)^2, \\ &= E\{\theta(F_2) - \theta(F_1)\}^2 / F_1,\end{aligned}$$

where F_2 is the bootstrap distribution.¹ We have replaced $\{F_0, F_1\}$ with $\{F_1, F_2\}$.

The probability statement describing the confidence interval:

$$P(\{\hat{\theta} - SE(\hat{\theta}) < \theta_0 < \hat{\theta} + SE(\hat{\theta})\}) = (1 - \alpha),$$

conditional on F_0 . becomes:

¹ Hall, Peter. (1992) " The Bootstrap and Edgeworth Expansion " (Springer Verlag.) pp.8

$$P\{\theta_B - SE(\theta_B) < \theta < \theta_B + SE(\theta_B)\} = (1 - \alpha),$$

conditional on F_1 .

More generally the intervals cover θ_0 , the true parameter, with a probability $(1 - \alpha)$ - referred to as nominal coverage. The coverage error is defined as the true coverage minus the nominal coverage, which converges to zero as the sample size increases.

The EFRON Bootstrap, named after B. Efron, who focused attention to the method's usefulness,¹ uses the technique of "resampling" the original sample, (of size n) by taking random samples of size n with replacement. At each resample the bootstrap statistic is calculated. The frequency distribution of the Bootstrap statistic, θ_{iB} , $i = 1, \dots, K$, is calculated from these K resamples and is an estimate of the distribution of the sample statistic $\hat{\theta}$. The basic procedure in the general case is given in the following section

7:3 The General Model

The basic procedure, given the sample $\{x_1, \dots, x_i, \dots, x_n\}$, is to:

- (i) Estimate $\hat{\theta}$.
- (ii) Select a random sample of size n with replacement from the sample and calculate θ_{iB} .

¹ Efron, B. (1992) "Bootstrap Methods: Another look at The Jackknife" *Annals of Statistics*, # 7, pp.1-26.

(iii) Repeat the random selection process K times, where K is a large number.¹

(iv) Construct the probability distribution of $\theta_{iB}, \dots, i = 1, \dots, K$ assuming the probability of each outcome is $1/K$.

This general bootstrapping procedure is used to recover the distribution of a statistic defined on the sample allowing for the recovery of standard errors, confidence intervals and permitting hypothesis tests.

7:4 An Application of the General Principle

An example of the general principle given by tests for measures of inequality.² The GINI coefficient defined as the arithmetic average of the absolute value of the difference between all pairs of income y , from a sample of size n , is given as:

$$G = 5n^2 \mu_y \sum_{i=1}^n \sum_{j=1}^n |y_i - y_j|.$$

This value is equal to one when inequality is at its maximum, and zero with an equal distribution. Their's entropy measures of inequality is defined as

¹ Efron, B, and Tibshirani, R.J. (1986) " Bootstrap Methods for Standard Errors, Confidence Intervals, and Other Measures of Statistical Accuracy." *Statistics Science*, #1 .pp. 54-77.

For standard errors; 50-100, and for confidence intervals; 1000.

² Mills, Jeffrey. A., and Zandvekili, S. (1996)" Statistical inference via Bootstrapping for Measures of Inequality" *Jerome Levy Economics Institute, Working Paper # 136*, pp. 3.

$$T = \sum_{i=1}^n s_i \text{Log}(ns_i), \text{ where } s_i = \frac{y_i}{\sum_{j=1}^n y_j}.$$

and is the relative share of the i th individual's income.

The bootstrap method described above was applied to the sample values to investigate the changes in equality from one year to the next and probability intervals calculated to test the significance of these changes. The results suggest that the bootstrapping technique appears to perform well.

7:5 Regression Models

For a regression model where $Y = X\beta + \varepsilon$, it is assumed that the X are nonstochastic. The bootstrap attempts to replicate the random component in the model, viz. the error term. The procedure is as follows.

- (1) Run the regression model and obtain the estimates of the regression coefficient, β . The original residual vector is calculated as:

$$\varepsilon_t = Y_t - \hat{Y}_t,$$

where Y_t is the observed value of the dependent variable, and \hat{Y}_t the predicted value.

- (2) Resample the residuals with replacement to obtain a new sample of the residuals denoted by:

$$\varepsilon_{tB} \dots t = 1, \dots, n; B = 1, \dots, K,$$

(3) A new vector of the dependent variable is calculated as:

$$\dot{Y}_{Bt} = \dot{Y}_t + \varepsilon_{Bt} \dots \dots \dots t = 1 \dots \dots n,$$

(4) This equation is estimated using the original data, the X matrix, and the new dependent variable calculated from the resamples in step (3). The parameter vector β_{1B}^{BS} is kept.

(5) This process - resampling the original residual vector, and calculating a new dependent variable vector, -is repeated K times to obtain the empirical distribution of β_e ., the original sample estimate.

7:6 Regression applications of the Bootstrap.

The demand for energy by industry was the object of this study by Freedman and Peters.¹ The procedure, as described above, the first application of the bootstrap to Generalized Least Squares estimation, revealed that conventional asymptotic formulae for estimating standard errors are optimistic by a factor of 3. The bootstrap estimates performed better than the conventional estimates.

In an application to the problem of prediction in a stationary Autoregressive process, Thombs and Schuchany² calculated a 100β%

¹ Freedman, David. A., and Peters, Stephen, P. (1994) " Bootstrapping a Regression Equation: Some Empirical Results." *Journal of The American Statistical Association*, March 1984, 79, #385 pp.100.

² Thombs, L.A., and Schuchany, W. R. (1990) " Bootstrap Prediction intervals for Autoregression." *Journal of the American Statistical Association*, 85, #410 pp.486-492

prediction interval for an AR(p) process. Conventional techniques assume an error sequence that is normal and rely on the fact that the conditional distribution of Y_{t+1} / Y_t is also normal. This study gives a non-parametric alternative to the conditional distribution. The method is similar to that used in linear models - as described above - as the residuals are resampled. This is a simulation process which studies bootstrap results given different error distributions. A normal distribution, an exponential distribution, and a mixed normal distribution were studied. They represent the ideal, skewed, and bi-variate skewed processes. Sample sizes vary from $n=24, 50, 75,$ and 100. For the normal error structure, the bootstrap and conventional techniques are not significantly different. For the exponential, the parametric estimates perform better. For estimates of the median, bootstrap estimates were consistently accurate. The bootstrap estimates performed far better than parametric estimates when the distribution was mixed normal.

In estimating simultaneous equation models the Seemingly Unrelated Regression method is very popular. Hypothesis testing in SUR uses the log-likelihood ratio - LLR - test. The distribution of this statistic is known. Given the small sample sizes in Economic studies, significance levels are too low, resulting in the rejection of too many null hypotheses.

If a constraint is placed on the parameter vector, the LLR statistic has a χ^2 distribution. This is not accurate in small samples. The

approximation is improved if it is assumed that the LLR statistic has a distribution which is a multiple of the χ^2 distribution - multiplied by a factor- the Bartlett adjustment factor- which eliminates the bias asymptotically. In a simulation study¹ this adjustment was determined by the bootstrap method. The LLR statistic's distribution was obtained under the null hypothesis constraint.

The study showed the bootstrap estimate of the Bartlett adjustment was considerably more accurate. Problems arise when lagged dependent variables are employed, however these problems are small and the performance of these estimates better than those of asymptotic estimates.

7:7 Confidence Intervals

Once the 'empirical' distribution of the parameter estimates is obtained, confidence intervals are constructed². While numerous intervals exist, the following are the most commonly used.

7:7:1 The Normal Approximation

¹ Rocke, David. M. (1989) "Bootstrap Bartlett Adjustment in Seemingly Unrelated Regressions" *Journal of The American Statistical Association*, 84, #406 pp.598-601.

² Mooney, C. Z., and Duvall, R. D. (1993) "Bootstrapping A Nonparametric Approach to Statistical Inference" (Sage University Paper 95) pp.36.

Hall, Peter. (1992) pp.8.

Efron, B. (1990) "More Efficient Bootstrap Computation" *Journal of The American Statistical Association*, 85, #409, pp.79-89.

The parametric approach assumes a distribution of θ , generally normally distributed. The sample estimate and its standard error determine the confidence interval: $\theta = \hat{\theta} \pm C_{\alpha/2} SE(\hat{\theta})$, where $C_{\alpha/2}$ is the critical point corresponding to the assumed distribution and the number of degrees of freedom.

The Normal approximation assumes the statistic is normally distributed but its standard error is unknown. Bootstrapping provides an estimate of the standard error. In this case full advantage of the technique is not utilized as only the standard error of the "empirical" distribution is used.

7:7:2 The Percentile Method.

Full advantage of the bootstrap is taken as the "empirical" distribution is used to develop the appropriate confidence intervals for the parameter. After the iterations are run, the vector of bootstrapped parameter estimates is ordered and the select percentile values of the distribution are read off. If, for example, there are 5000 iterations, then the lower confidence limit corresponds to the value of the $(\alpha / 2) \cdot 5000^{\text{th}}$ observation at a $(1 - \alpha)\%$ level, assuming a two tailed test. There is no assumption made about the unknown parameter's distribution.

This method is highly accurate and handles skewed distributions well. This method may perform poorly when the sample size is small, as the

tails of the distribution are of great significance.¹ It assumes that the “empirical” distribution is an unbiased estimate of the sampling distribution and may perform inadequately as it depends on an unknown variance. This dependence should be minimized².

7:7:3 The Percentile-t method.

Each Bootstrap estimate is transformed into a standardized variable:

$$t_{iK} = (\hat{\theta}_{iK} - \hat{\theta}) / SE(\hat{\theta}_{iK}), \text{ analogous to the student's -t}$$

distribution. The transformed variable determines the lower and upper confidence interval's percentile values and is used to calculate the coverage of the true parameter using the original estimate of the sample and the standard error of each bootstrap iteration as an estimate of the sample standard error. The interval is calculated according to:

$$P(\hat{\theta} - t_{(iK)\alpha/2} SE(\hat{\theta}_{iK}) < \theta < \hat{\theta} + t_{iK(1-\alpha/2)} SE(\hat{\theta}_{iK})) = 1 - \alpha .$$

This method is highly accurate and handles skewed distributions well.

7:7:4 The Bias-Corrected method.

We have θ_0 , the true value, $\hat{\theta}$, the sample estimate, and $\hat{\theta}_{BS}$, the bootstrap estimate. We require that the bootstrap is an unbiased estimate

¹ DiCiccio, T. J., and Romano J. P. (1988) “ A Review of Bootstrap Confidence Intervals” , *Journal of The Royal Statistical Society, Series B*, 50: pp.338-370

² Hall, Peter. (1992) pp.101-153.

of the sample estimate, and that the sample estimate is an unbiased estimate of the population value; that is both the distribution of $(\hat{\theta}_{BS} - \hat{\theta})$, and $(\hat{\theta} - \theta_0)$, are centered on zero.¹ Assume these distributions are centered around $z_0\sigma$, where σ is the standard error of $(\hat{\theta}_{BS} - \hat{\theta})$, and z_0 is a biasing constant. It is assumed that there is a monotonic transformation of the bootstrap sample estimate and true value, $\Phi_{BS}, \hat{\Phi}, \Phi_0$, such that:

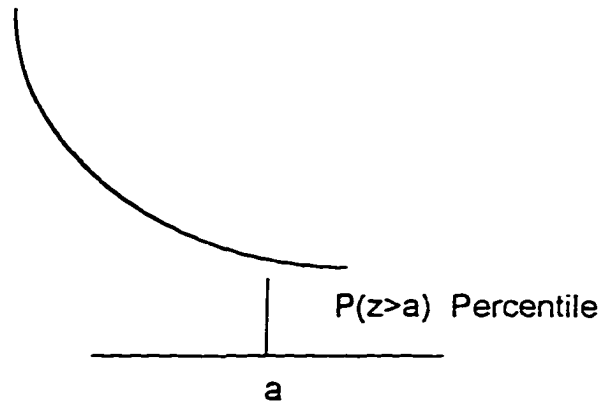
$$(\Phi_{BS} - \hat{\Phi}) \sim N(z_0\sigma, \sigma^2), \text{ and, } (\hat{\Phi} - \Phi_0) \sim N(z_0\sigma, \sigma^2).$$

The quantity $(\Phi_{BS} - \hat{\Phi})$ is a normal pivotal quantity with the same distribution, Normal, under the bootstrap distribution as well as the sample distribution.

The value of the biasing constant can be calculated from the Normal cumulative distribution function and the bootstrapped estimate transformed.

The constant z_0 is the value of the normal variate that corresponds to the proportion of the bootstrapped estimates that is less than or equal to the original sample estimate.

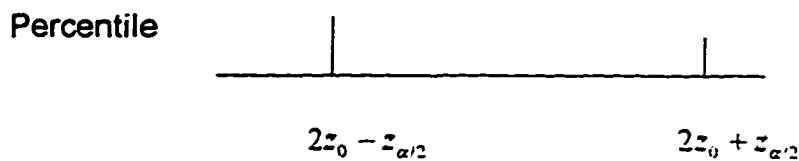
¹ Efron, B. (1982) "The Jackknife, The Bootstrap, and Other Resampling Plans" Monograph # 38, Society for Industrial and Applied Mathematics, (Philadelphia)



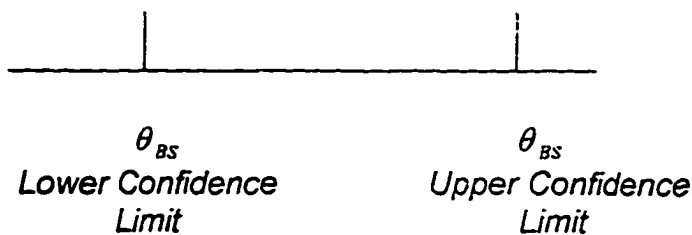
Adjust the upper confidence value at θ_{BS} at

$$\Phi(2z_0 + z_{\alpha/2}) \times 100\text{th. percentile} - \Phi(a) \times 100\text{th. percentile}$$

if $E(\theta_{BS}) = \hat{\theta} = z_0 = 0$, i.e the proportion of the bootstrap estimate that is less than or equal to the sample estimate is equal to .5, then $z_0 = 0$



Value of bootstrap estimate



The problems associated with this method are again a resort to parametric assumptions: the existence of a monotonic transformation, and the assumption that the bootstrap estimate is an unbiased estimate of the sample estimate, $\hat{\theta}$.

7:8 Conclusion.

The bootstrap technique can be applied in situations where parametric regression methods are inappropriate. It produces an empirical distribution of the sample estimate through resampling. The confidence intervals can be calculated to test various hypotheses. The studies cited show a very general application.

CHAPTER 8

Model Definition

8:1 Overview.

A demand for money function is derived from an equilibrium condition and a short run dynamic adjustment mechanism.

The function is transformed so that the long run coefficient estimates and their standard errors can be estimated directly.

The estimating equation contains contemporaneous independent variables on the right hand side which require the use of instrumental variables. The technique and the choice of instruments are discussed.

8:2 Model Definition

This model is developed to estimate the Long term parameters of the demand for money function using the "conventional " specification.

The long run , equilibrium, real money, $m_{0t} = \frac{M_{0t}}{P_{0t}}$, is represented as a desired or expected quantity. It is unobservable and is represented functionally as:

$$E(m_{0t}) = e^{\alpha} y_{0t}^{\beta} r_{0t}^{\gamma} . \quad (8:1)$$

where P_{0t} is the price level, GDP deflator, M_{0t} , nominal money balances-M1, and r_{0t} , a short term interest rate - 90day T-bill, and y_{0t} is a scale variable measured by real GDP.

In natural logarithms this becomes:

$$E(\ln(m_{0t})) = \alpha + \beta \ln(y_{0t}) + \gamma \ln(r_{0t}).$$

Let $\ln(m_{0t}) = m_t$, $\ln(y_{0t}) = y_t$, and $\ln(r_{0t}) = r_t$, $\ln(M_{0t}) = M_t$, and $\ln(P_{0t}) = P_t$, allowing us to simplify the equilibrium notation as

$$E(m_t) = \alpha + \beta y_t + \gamma r_t,$$

or as:

$$E(M_t) = \alpha + \beta y_t + \gamma r_t + P_t. \quad (8:A)$$

Short run conditions: adjustment costs, inertia, institutional rigidities, price inflexibility, give rise to departures of actual to desired money holdings. The short run dynamics are captured by having nominal money balances adjusting according to :

$$\frac{M_{0t}}{M_{0t-1}} = \left(\frac{E(M_{0t})}{M_{0t-1}} \right)^\lambda e^{\mu_t}.$$

The nominal adjustment specification is based on Fair's results, discussed in Chapter 5, pp. 47, which give it empirical support.

The adjustment mechanism is based on the Chow partial adjustment mechanism, also discussed in Chapter 5.

Taking logarithms and simplifying the notation as was done when the long run equilibrium was converted into log form, we get

$$M_t - M_{t-1} = \lambda(E(M_t) - M_{t-1}) + \mu_t$$

so that:

$$E(M_t) = \lambda^{-1}(M_t - (1 - \lambda)M_{t-1} - \mu_t), \quad (8:B)$$

where: $0 < \lambda \leq 1$.

Combining (8:A) and (8:B) we get

$$M_t = \lambda(\alpha + \beta y_t + \gamma r_t) + \lambda P_t + (1 - \lambda)M_{t-1} + \mu_t,$$

and if we subtract P_t from both sides this reduces to

$$m_t = \lambda(\alpha + \beta y_t + \gamma r_t) + (1 - \lambda)(M_{t-1} - P_t) + \mu_t, \quad (8:C)$$

This equation is an Autoregressive Distributed Lag- ADL(1,0,2)- whose general form is ADL(m,n,p), where m= number of lags in the dependent variable, n= number of lags in the independent variable, and p is equal to the number of independent variables. It is the conventional money demand specification with nominal money adjusting in the short run.

As noted before if the residuals are serially correlated then the lagged money is correlated with the residuals.

The short run parameters of this demand function are functions of the long run parameters and the speed of adjustment.

The long run parameters are deduced from the coefficient estimates, however their analytic standard errors are unknown.

8:3 The Transformed model.

If we multiply equation (8:C) by k , and subtract the result from (8:C) we get:

$$m_t = \lambda(1-k)(\alpha + \beta y_t + \gamma r_t) + (1-k)(1-\lambda)(M_{t-1} - P_t) + km_t + (1-k)\mu_t \quad (8:D)$$

Now λ is the speed of adjustment. If it is equal to one then the adjustment is instantaneous, and we would be observing the long run function (8:A); or an equivalent form, (8:D), which implies that k is equal to zero.

Choose the value of k so that:

$$1 = \lambda(1-k), \text{ and equivalently, } k = \frac{\lambda-1}{\lambda}.$$

Substituting for K in (8:D) we get:

$$m_t = \alpha + \beta y_t + \gamma r_t + \left(\frac{1-\lambda}{\lambda}\right)(M_{t-1} - P_t) + \left(\frac{\lambda-1}{\lambda}\right)m_t + \mu\lambda^{-1}.$$

Rearranging terms and simplifying we arrive at the following:

$$m_t = \alpha + \beta y_t + \gamma r_t - \frac{(1-\lambda)}{\lambda}(M_t - M_{t-1}) + \lambda^{-1}\mu_t, \text{ or:}$$

$$m_t = \alpha + \beta y_t + \gamma r_t - \left(\frac{1-\lambda}{\lambda}\right)(\Delta M_t) + \lambda^{-1}\mu_t \quad (8:E)$$

This is a special case of a general form of a transformed ADL(m,n,p) model.¹

Equations (8:C) and (8:E) are the estimating equations. The first, (8:C), is the conventional short run money demand function with nominal money adjusting. The short run parameters are retrieved from this equation.

¹ Bewley, R. (1979) "The Direct Estimation of the Equilibrium Response in a Linear Model" *Economics Letters* 3, pp.357-361.

The second (8:E), is the transformed model where the long run parameters are the coefficient estimates.¹

Since there is a contemporaneous value of the independent variable on the right hand side OLS estimation is not consistent.

Estimation to obtain consistent results requires an Instrumental Variables/ Two Stage Least Squares procedure.

8:4 Instrumental variables estimation

If the true model is given in Matrix terms as:

$$Y = X\beta + \varepsilon$$

and if X is stochastic and $E(X' \varepsilon) = 0$ then the OLS estimator given as

$$B = (X' X)^{-1} X' Y, \text{ is biased and inconsistent.}$$

Taking expectations we get:

$$E(B) = \beta + E(X' X)^{-1} X' \varepsilon.$$

The last term is the bias which does not approach zero as the sample size increases indefinitely.

This problem was resolved² by the introduction of a vector of

"Instruments" $[Z]_{m,1}$, such that:

$$(1) E(Z' \varepsilon) = 0 \quad \text{or} \quad (z' \varepsilon / T) \rightarrow 0, \dots \text{as} \dots T \rightarrow \infty.$$

¹ Banerjee, A., Doldado, J. J., Galbraith, J. W., and Hendry, D. (1994) "Co-integration, Error Correction and the Econometric Analysis of Non-Stationary Data" (Oxford University Press) pp.53-54.

² Sargan, J. D. (1958) "The Estimation of Economic Relationships using instrumental Variables" *Econometrica*, #26 P 397-415.

$$(2) V(Z_t) = 0$$

$$(3) \text{Cov}(Z_t, X_t) > 0, \text{ or: } (Z_t, X_t / T) \rightarrow \Sigma > 0 \dots \text{as } T \rightarrow \infty$$

Consider the case where there are k instruments $\{z_{t,1}, \dots, z_{t,k}\} = [Z]_{n,k}$ to be used in estimating the following equation:

$$[Y]_{n,1} = X_{n,k} \beta_{k,1} + \varepsilon_{n,1}, \text{ and the conditions (1) to (3) are met.}$$

Premultiplying by Z' gives $Z'Y = Z'X\beta + Z'\varepsilon$. If the rank of $T^{-1}Z'X = k$, then this is sufficient to identify β .

Then the estimator:

$$\begin{aligned} \beta_{iv} &= (Z'X)^{-1}Z'Y \\ &= \beta + [T^{-1}Z'X]^{-1}[T^{-1}Z'\varepsilon], \end{aligned}$$

exists, and is the instrumental variable estimator. This estimator is asymptotically unbiased as:

$$P\lim_{T \rightarrow \infty} \beta_{iv} = \beta + P\lim_{T \rightarrow \infty} (T^{-1}Z'X)^{-1} P\lim_{T \rightarrow \infty} (T^{-1}Z'\varepsilon) = \beta.$$

By Mann and Wald's Theorem and Cramer's Theorem¹

$$\sqrt{T}(\beta_{iv} - \beta) = (T^{-1}Z'X)^{-1}(\sqrt{1/T}Z'\varepsilon) \rightarrow N(0, \sigma^2 R),$$

where R is defined as:

$$P\lim_{T \rightarrow \infty} [T^{-1}X'Z(Z'Z)^{-1}Z'X]^{-1} = [A].$$

¹ Spanos, A. (1990) "Statistical Foundations of Econometric Modeling" (Cambridge University Press) pp. 197, pp 637-644.

Johnston, J. (1984) "Econometric Methods" (McGraw Hill) pp.363-366.

Standard OLS results show that the variance of the estimator depends on error and observation variances. $\hat{\beta}_n$ is a consistent estimator of β and can be deduced from sample evidence.

If :

$$\bar{\varepsilon} = Y - X\hat{\beta}_n ,$$

and:
$$V(\bar{\varepsilon}) = \overline{\sigma_\varepsilon^2} ,$$

then:
$$\text{in Plim } \hat{\beta}_n = \beta ,$$

and $\overline{\sigma_\varepsilon^2}$ is a consistent estimator of σ_ε^2 , the true residual variance.

Hence: $\sigma_\varepsilon^2 R = V(\hat{\beta}_n)$, can be estimated and is consistent.

The main problem encountered in practical implementation of the instrumental variable estimator is the choice of instruments. The success of this estimator requires that the instruments are correlated with the dependent variable and the explanatory variable, but not with the error term.

8:5 Choice of instruments

In Equation (8:C), the estimate \hat{m}_i is not correlated with the residual μ_i , but is highly correlated with m_i and with the explanatory variables. It seems appropriate to run the regression on (8:C), correcting for any serial correlation, and then use the predicted value as an instrument in the regression (8:E).

The model so far presented, assuming it is correctly specified, makes no assumption of the distribution of errors, has explicit lags, and may have correlated and heteroskedastic errors. In addition it may be used in circumstances where the data set is limited. These have been the major sources of technical difficulties associated with money demand system estimation. In order to avoid these problems the bootstrap methodology has been applied to recover the parameters and to perform the relevant hypothesis tests.

8:6 Conclusion

A money demand equation was derived from the conventional models whereby direct estimation of long run coefficients are possible. The instrumental variable technique is required to estimate this model. Hypotheses tests can be performed on these estimates.

CHAPTER 9

ESTIMATION9:1 Overview

The data are described, and the variables used in the estimating equation defined.

The estimating procedures are outlined. The derivation of the instruments, instrumental variable estimation, and the bootstrap application, are described.

9:2 Data

The data were obtained from the Federal Reserve Bank of St. Louis and is as follows

<u>Variable</u>	<u>Description</u>
NARM1	M1, narrow money as defined by the FED. These are quarterly averages, seasonally adjusted in \$ billions. M_{0t} in the notation of Chapter 8.
TBILL	Three month treasury bills ,annual percentage rates. Monthly averages; r_{0t} .

NOMGDP	Nominal GDP, Quarterly averages, seasonally adjusted in \$ billions.
PLVL	GDP price deflator, seasonally adjusted 1992 dollars.
POP	U.S. population annual data adjusted to derive quarterly data. First quarter in year

$$n = P_{n-1} + ((P_n - P_{n-1}) / 4)$$

The data covered the period 1959 first quarter to 1995 fourth quarter.

Regressions were run using RATS version 4.

9:3 Data transformation

NOMM1 = Per capita, nominal M1: M_{it}

$$ECM1 = \ln(M_{0t} - M_{0t-1}) = M_t - M_{t-1}$$

$$RLM1 = (M_{0t} / P_{0t}) = m_{0t}$$

$$LOGRLM1 = \ln\left(\frac{M_{0t}}{P_{0t}}\right) = m_t$$

PCGDP = Per capita GDP = Y_{0t}

$$RLGDP = Y_{0t} / P_{0t} = y_{0t}$$

$$LGRLGDP = \ln(y_{0t}) = y_t$$

$$TBILL = r_t$$

In estimating the demand function, the interest rate parameter is not in the log. form. As Fair¹ notes, a one percent change in interest rates leads to

¹ Fair, Ray. C. (1987) pp.473

four times the effect in logarithms when rates are low; the following table illustrates:

	<u>Change</u>	<u>Log of Int. rates</u>	<u>Change</u>
<u>Interest</u>			
<u>Rates</u>			
.02		-3.91	
.03	.01	-3.51	.41
.....
.10		-2.3	
.11	.01	-2.21	.1

9:4 Estimation procedure

STAGE A. The following equation was estimated recovering the short term parameters. The computer programme was written for Rats version 4.0. The complete programme is in Appendix A. The variable names are what were used in the programme.

$$\ln\left(\frac{M_t}{P_t}\right) = \lambda(\alpha + \beta y_t + \gamma r_t) + (1 - \lambda)\ln\left(\frac{M_{t-1}}{P_t}\right) + \mu_t \quad (9:A)$$

This is the nominal adjustment specification, equivalent to equation (8:C) in chapter 8.

This model is named SHORTRUN

The residuals are kept as the series RESIDS. The coefficients are stored in the coefficient vector OLSCOEF. Income elasticity, $\lambda\beta = \text{OLSCOEF}(2,2)$.

$$\begin{aligned} \text{M1HAT} &= \text{estimate of Nominal Money} \\ &= \ln(M_{0t} / P_{0t}) - \text{Residual} + \ln(\text{PLVL}) \\ &= (M_t - P_t - e_t + P_t) \\ &= \hat{M}_t. \end{aligned}$$

$$\text{ECM1HAT} = \text{M1HAT} - \text{M1HAT}(1) = (M_t - \hat{M}_{t-1})$$

The regression is run using the AR1 feature, correcting for serial correlation, using Maximum Likelihood estimates.

STAGE B : INSTRUMENTAL VARIABLES - Two Stage Least Squares is used to estimate the transformed equation , Equation (8:E) ,Chapter 8, and retrieve the long term parameters..

$$m_t = \alpha + \beta y_t + \gamma r_t - \frac{(1-\lambda)}{\lambda} (M_t - M_{t-1}) + \lambda^{-1} \mu_t \quad (9:B)$$

This model is named LONGRUN.

The instruments derived from the short term parameters estimation are the original GDP and interest rate, real money, and ECM1HAT.

The BOOTSTRAP Loop.

From Stage A the residuals, from the series RESIDS are used to obtain the predicted value of real money:

$$(M_t - P_{t0} - e_t) = \text{RLM1HAT}.$$

Bootstrap the residuals with replacement to obtain a new sample of the dependent variable:

$$\text{RLM1NEW} = \text{RLM1HAT} + b(e_t),$$
 the last term

representing the bootstrapped residual.

One observation was lost as a result of using a lagged variable in Stage A. The original observation of real money was used plus a random sample of the residual series RESIDS to obtain the missing observation. A complete, new, sample of data on real money is thus obtained.

Stage A and Stage B were repeated with the new data set, and the long term parameters and standard deviation are retrieved and stored for each iteration. The iterations go from 1 to nB, and the empirical distribution of the parameters can be calculated.

The bootstrap confidence intervals are computed given the empirical distribution.

CHAPTER 10

RESULTS

10:1 Overview.

A short run and a long run demand for money model, SHORTRUN, and LONGRUN, together with their bootstrap equivalents were developed to study the characteristics of the demand function and the estimators.

U.S. data for the period 1959-1995 are analysed. Estimates are produced for the following sub-periods; 1959-1973, 1974-1983, 1984-1995, 1974-1995, and for the entire period. Recursive estimates for the entire period, beginning with 1961 are also produced. The analysis is carried out by:

(i) Comparing income elasticity estimates developed by this study and those of Roley and Goldfeld, with respect to size and statistical significance. The results are found in sections 10:2, and 10:4. Sections 10:4-6 deal with estimates produced by LONGRUN and SHORTRUN., and summarized in Tables 1-2, 6-13.

(iiu) Testing for instability of the income elasticity, and interest rate coefficient, and the demand function, during the 1970-72 period. The

results are found in section 10:3, summarized in Tables 3, and 5, and Charts 1 and 2. Recursive estimates developed to test coefficient stability are found in Tables 16-19, and Charts 3 and 4.

(iii) Developing bootstrap confidence intervals to test the parametric coefficient estimates. These are summarized in Tables 19 and 20.

(iv) Investigating the stability and convergence of the bootstrap distribution of the income elasticity of money demand developed for the period 1959-1995, as a function of the number of iterations. Results are summarized in Tables 14 and 15.

(v) Comparing the effects of sample size on the estimates. The results are found in Table 22.

(vi) Evaluating the coefficient of the lagged dependent variable, the speed of adjustment. Results are summarized in Table 21, and Chart 5.

10:2 Period: 1959-1973

Tables 1 and 2 show the results of the nominal adjustment short run specification- SHORTRUN -for income elasticity, and the long run specification, LONGRUN, compared with the estimates developed by Roley ¹and Goldfeld². Goldfeld's results span the period 1952ii-1973iv, while the others are from 1959i-1973-iv.

¹Roley, Vance. V. (1985) "Money Demand Predictability." *Journal of Money Credit and Banking* MCB (November) pp. 616.

² Judd, J., and Scadding, J. (1982) "The Search for a Stable Demand Function: A Survey of the Post 1973 Literature." *Journal of Economic Literature*. (September) pp. 996.

(i). Roley and Goldfeld's estimates of the short run parameter for income elasticity $\lambda\beta$, differ from the estimates produced by the model

SHORTRUN by 69 and 44% respectively. These researchers ' models include more than one interest rate.

(ii) All models show significant 't' statistics.

(iii) The speed of adjustment parameters for all models show significant differences. Roley's value is approximately double that of Goldfeld, while, SHORTRUN is approximately two and one thirds times that of Goldfeld.

(iv) The long term parameters implied by the speed of adjustment and the short term parameters again are significantly different. SHORTRUN estimates being 134% greater than Roley's, and 43% higher than Goldfeld's.

(v) The direct estimates of the long term parameter produced by the model LONGRUN are statistically significant at the 95% level, and are comparable with Roley's. They are approximately 50% of what is implied by those produced by the model SHORTRUN.

(vi) The bootstrap estimates for 3000 iterations accept both the short term parameter estimates and the long term estimates at the 95% level for all the bootstrap intervals. The parametric interval- corresponding to that of the regression - for both models has the largest interval, while the percentile interval is the smallest for the LONGRUN - model

(vii) The speed of adjustment deduced from the direct estimates obtained from the model LONGRUN implies 4.4 quarters, while Roley's model gives 2.9 quarters and Goldfeld's 5.6. The model SHORTRUN gives 13 quarters.

TABLE 1
Estimation results: Income elasticity 1959-73

Regression Model	Period	Long term β	Short term $\lambda\beta$	Speed of Adjustment
Roley (i)	59ii.-73iv.	0.386	0.131	0.340
(t-value)			3.780	
Goldfeld (ii)	52ii.-73iv.	0.629	0.112	0.178
(t-value)			5.400	
SHORTRUN (iii)	59ii-73iv	0.905	0.070	0.077
(t-value)			4.798	
LONGRUN (iv)	59iv.73iv.	0.410	0.170	0.414
(t-value)	2.330			

Notes: (i) $\lambda\beta$ is equivalent to b_3 , and $(1-\lambda)$ is equivalent to b_5 in equation 5-D.
(ii) $\lambda\beta$ is equivalent to b_3 , and $(1-\lambda)$ is equivalent to b_5 in equation 5-E.
(iii) $\lambda\beta$, and $(1-\lambda)$ are coefficients in equation 8-C.
(iv) β , and $-(1-\lambda)/\lambda$ are coefficients in equation 8-E.

TABLE 2

Estimation results:1959-73. Bootstrap 95% confidence intervals;3000 iterations.

Interval	Lower	Upper	Length
Model: LONGRUN			
Parametric	0.065	0.755	0.689
Normal	0.158	0.661	0.503
Percentile	0.206	0.693	0.487
Percentile-t	0.172	0.679	0.507
Bias-Corrected	0.255	0.954	0.699
Model = SHORTRUN			
Parametric	0.041	0.099	0.057
Normal	0.060	0.080	0.020
Percentile	0.056	0.076	0.020
Percentile-t	0.058	0.075	0.017
Bias-Corrected	0.064	0.084	0.020

10:3 Pre - 1973 results:

Ronald L. Coopers 1972 survey of Econometric models concludes that evidence of structural change showed up in most financial variables . The evidence of 1971 indicated that the conventional demand function had shifted. Judd and Scadding ¹state that:

“ in retrospect Michael J. Hamburger (1973) showed that this was not true, but the episode left doubts whether this function was stable enough.”

Hamburger² examined the data for the period Q2 1971, which showed an acceleration of the growth of M1 in the face of rising interest rates and deceleration of GDP. While this is counter to the standard money demand theory, these results are consistent only if there are lags in the demand function: i.e they are sensitive to prior quarter's results.

Based on a comparison of actual and predicted M1 growth rates using data from 1955i-1969iv., it was shown that there was no significant underprediction in the second quarter of 1971. The results of the second half of 1971 yielded similar results, although there was some overestimation in the third quarter. Two other forecast equations were compared; one which showed overestimation in the third quarter. The

¹ Judd, J., and Scadding.,J. (1982) pp..P1006

² Hamburger, Michael, J. (1973)“ The Demand For Money in 1971: Was There a Shift?” By Michael .J. Hamburger, *Journal of Money Credit and Banking*, (May) pp.720-725.

general conclusion was that there was little evidence of substantial instability in the demand function.

The models - SHORTRUN, and LONGRUN - were used to investigate whether there was a shift in the demand function during the period in question. The results are shown on Tables 3 and 4, and are as follows:

The results for the long run parameter show:

- (i) A significant shift during the fourth quarter of 1971. This shift represents a 27% decline over the prior quarter's value.
- (ii) A 20 % decline during the first quarter 1971, bringing it to the level of Q4 1969.
- (iii) A decline of 22% during 1970.
- (iv).All the bootstrapped confidence intervals, with the exception of the normal approximation contain the regression parameter value estimate.

The results for the short run parameter show:

- (i) A significant shift during the fourth quarter of 1971. This shift represents a 17% decline over the prior quarter's value.
- (ii) A 22 % decline during the first quarter 1970.
- (iii) All the bootstrapped estimated intervals contain the parameter value.

CHART 1
Money demand: Recursive estimates- 1969-73

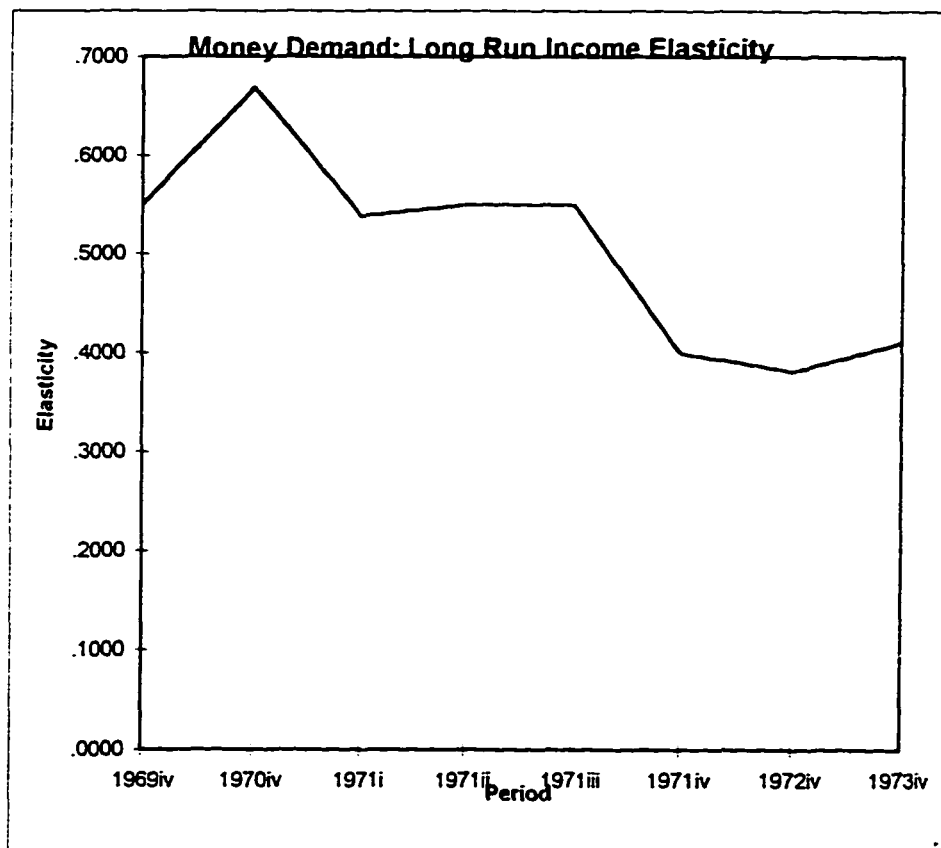


CHART 2

Money demand: Recursive estimates- 1969-73

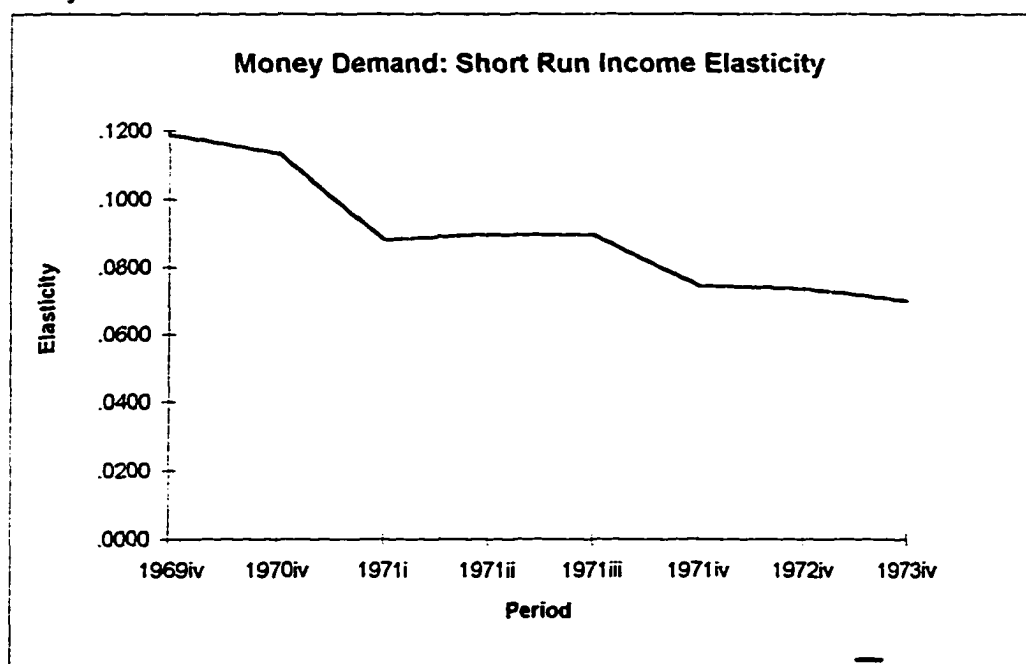


TABLE 3
Estimation results: Recursive estimates of long run income elasticity beginning 1959iii-

Ending date	Estimate	t-value	Percent change
1969iv	.550	3.31	
1970iv	.669	2.54	21.6
1971i	.537	2.06	-19.7
1971ii	.550	2.05	2.3
1971iii	.549	2.05	0
1971iv	.399	2.17	-27.3
1972iv	.381	-4.5	-4.6
1973iv	.410	2.33	7.6

Note: The long run elasticity is the coefficient β in equation 8E.

TABLE 4
Estimation results: Recursive estimates of short run income elasticity beginning 1959iii-

Ending date	Estimate	t-value	Percent change
1969iv	.1189	5.06	
1970iv	.1132	4.93	-4.8
1971i	.0881	4.38	-22.2
1971ii	.0897	4.66	1.8
1971iii	.0895	4.71	-.2
1971iv	.0745	3.86	-16.8
1972iv	.0736	4.65	-1.2
1973iv	.0700	4.7	-4.9

Note: The short run elasticity is thr coefficient $\lambda\beta$ in equation 8C.

TABLE 5
Estimation results: Recursive bootstrap estimates of long run income elasticity beginning 1959iii-

Ending date	Bootstrap mean	Percent change
1969iv	.608	
1970iv	.733	21.0
1971i	.746	2.0
1971ii	.680	-9.0
1971iii	.667	-2.0
1971iv	.608	-9.0
1972iv	.320	-47.0
1973iv	.399	25.0

Note: The long run elasticity is the coefficient β in equation 8E.

Both the long run estimates and short run estimates suggest there was a demand shift during the fourth quarter of 1971. Charts 1 and 2 show the shifts for the long and short term parameters.

The bootstrap estimates of the long run parameters show a marked decline in value for the period 1969-70, but does not show the instability shown by regression results produced by the models LONGRUN and SHORTRUN in Q4 1971. This instability, however, is revealed in 1972.

10:4 1974-1983.

Tables 6 and 7 show the results for the income elasticity estimates for 1974-1983.

For the long run parameters:

- (i) The LONGRUN direct estimates of the long run elasticity parameter are comparable with the derived estimates from Roley's study, and also with the Nominal adjustment, SHORTRUN, model.
- (ii) The 't'- statistics reject this parameter at the 95% levels.
- (iii) Bootstrap estimates, at 3000 iterations, with the obvious exception of the parametric estimates, accept the values at the 95% level. The Bias - corrected interval is the narrowest.
- (iv) The parameter values are almost double those of the period 1959-73.

For the short run parameters:

- (i) This estimate is directly comparable with Roley's estimate, and with that derived from the long run estimate.

- (ii) The 't' statistics for both studies reject the estimates at the 95% level.
 - (iii) Bootstrap estimates accept the parameter value, with the exception of the parametric interval.
 - (iv) The parameter values declined about 19% from those obtained in the period 1959-73, (from .070 to .059), compared to an almost 50% increase, (from .41 to .62), shown by the long run parameters.
 - (v) The speeds of adjustment are comparable for all models.
- The bootstrap estimates accept the values of the regression parameter estimates, indicating shifts in the function. Regression statistics reject the estimates at the 95% level.

TABLE 6
Estimation results: Income elasticity 1974-83

Regression Model	Long term β	Short term $\lambda\beta$	Speed of Adjustment
Roley (i)	.685	.062	.090
(t-value)		1.710	
SHORTRUN (ii)	.666	.059	.081
(t-value)		1.893	
LONGRUN (iii)	.615	.065	.107
(t-value)	1.421		

Notes: (i) $\lambda\beta$ is equivalent to b_3 , and $(1-\lambda)$ is equivalent to b_5 in equation 5-D.
(ii) $\lambda\beta$, and $(1-\lambda)$ are coefficients in equation 8-C.
(iii) β , and $-(1-\lambda)/\lambda$ are coefficients in equation 8-E.

TABLE 7

Estimation results:1974-83. Bootstrap 95% confidence intervals;3000 iterations.

Interval	Lower	Upper	Length
Model: LONGRUN			
Parametric	.233	1.462	1.229
Normal	.224	1.005	.782
Percentile	.342	1.239	.897
Percentile-t	.240	1.339	1.099
Bias- Corrected	.109	.784	.675
Model SHORTRUN			
Parametric	-.002	.120	.122
Normal	.033	.065	.032
Percentile	.032	.085	.053
Percentile-t	.036	.082	.046
Bias- Corrected	.029	.082	.053

10:5 1983-1995

Tables 8 and 9 show results for income elasticity for the period 1983- 1995.

For this period both the LONGRUN and the SHORTRUN estimates are rejected at all bootstrap confidence intervals, as well as by the regression statistics.

The speeds of adjustment implied by both models are comparable. The long run model gives 5.6 quarters to full adjustment, while the short term is 5.8.

These results are consistent with prior results that the conventional money demand functions perform poorly.

TABLE 8
Estimation results: Income elasticity 1983-95

Regression Model	Long term β	Short term $\lambda\beta$	Speed of Adjustment
SHORTRUN (i)	.079	.013	.171
(t-value)		.244	
LONGRUN (ii)	.011	.002	.179
(t-value)	.027		

Notes: (i) $\lambda\beta$, and $(1-\lambda)$ are coefficients in equation 8-C.
(ii) β , and $-(1-\lambda)/\lambda$ are coefficients in equation 8-E.

TABLE 9
Estimation results:1983-95. Bootstrap 95% confidence
intervals;3000 iterations.

Interval	Lower	Upper	Length
Model: LONGRUN			
Parametric	-.771	.793	1.564
Normal	-.147	.168	.315
Percentile	-.067	.224	.291
Percentile-t	-.082	.308	.390
Bias- Corrected	-.145	.088	.234
Model SHORTRUN			
Parametric	-.095	.121	.216
Normal	-.011	.038	.049
Percentile	-.004	.044	.048
Percentile-t	-.007	.047	.054
Bias- Corrected	-.014	.033	.047

10:6 1974-1995

Table 10 and 11 show the results for income elasticity for the period the 1974-1995.

For this period the following hold:

- (i) Direct estimates of both long and short run parameters are significant at all bootstrap intervals.
- (ii) Regression statistics accept both long run and short run estimates.
- (iii) The speed of adjustment parameter shows the long run estimates to be 46% higher than those of the short run. The number of quarters to full adjustment is, for the Long run, 5.3, compared with 7.6 for the short run.
- (iv) The short run parameter deduced from the direct estimates is 67% higher.

TABLE 10
Estimation results: Income elasticity 1974-95

Regression Model	Long term β	Short term $\lambda\beta$	Speed of Adjustment
SHORTRUN (i)	.572	.074	.131
(t-value)		2.960	
LONGRUN (ii)	.660	.124	.188
(t-value)	6.755		

Notes: (i) $\lambda\beta$, and $(1-\lambda)$ are coefficients in equation 8-C.
(ii) β , and $-(1-\lambda)/\lambda$ are coefficients in equation 8-E.

TABLE 11
Estimation results:1974-95. Bootstrap 95% confidence
intervals;3000 iterations.

Interval	Lower	Upper	Length
Model: LONGRUN			
Parametric	.470	.855	.385
Normal	.672	.708	.036
Percentile	.608	.690	.082
Percentile-t	.612	.688	.075
Bias- Corrected	.636	.718	.082
Model SHORTRUN			
Parametric	.025	.124	.099
Normal	.064	.086	.022
Percentile	.070	.093	.023
Percentile-t	.070	.094	.024
Bias- Corrected	.061	.079	.018

10:7 1959-1995

Tables 12 and 13 show results for the entire sample period, 1959-1995.

The overall estimates are in summary:

- (i) The direct estimate of the long run parameter has high t- values and is comparable with that deduced from the short run estimate.
- (ii) The bootstrap intervals - percentile, percentile-t, and marginally the bias corrected form, - reject this estimate at the 95% level.
- (iii) The speed of adjustment deduced from the long run parameters is 14% higher. The long run model suggests 8 quarters for full adjustment, compared with 9.1 for the short run.
- (iv) The short run estimates are accepted by the bootstrap intervals.

10:8 Bootstrap intervals: convergence and stability.

Tables 14 and 15 show convergence of the bootstrap intervals and select statistics of the empirical bootstrap distribution for the income elasticity coefficient. The intervals were compared using 10, 100, 500, and 3000 iterations. These intervals converge rapidly and are stable, This is in accordance with findings of other researchers.

The mean of the empirical distribution is shown to be stable as the number of iterations increases. The estimates are different from regression estimates from 2 to 12% depending upon the period, with the 1974-1995 period showing the least difference.

TABLE 12
Estimation results: Income elasticity 1959-95

Regression Model	Long term β	Short term $\lambda\beta$	Speed of Adjustment
SHORTRUN (i)	.400	.040	.110
(t-value)		5.45	
LONGRUN (ii)	.400	.050	.120
(t-value)	6.54		

Notes: (i) $\lambda\beta$, and $(1-\lambda)$ are coefficients in equation 8-C.

(ii) β , and $-(1-\lambda)/\lambda$ are coefficients in equation 8-E.

TABLE 13

Estimation results:1959-95. Bootstrap 95% confidence intervals;3000 iterations.

Interval	Lower	Upper	Length
Model: LONGRUN			
Parametric	.28	.52	.24
Normal	.38	.41	.03
Percentile	.43	.47	.04
Percentile-t	.42	.46	.03
Bias- Corrected	.41	.45	.04
Model SHORTRUN			
Parametric	.03	.06	.03
Normal	.04	.04	.00
Percentile	.04	.04	.00
Percentile-t	.04	.04	.00
Bias- Corrected	.04	.04	.00

TABLE 14
Estimation results: Bootstrap confidence intervals- long term income elasticity.

	n = 10			n = 100			n = 500			n = 3000		
Interval	Lower	Upper	Length	Lower	Upper	length	Lower	Upper	Length	Lower	Upper	Length
Parametric	.28	.52	.24	.28	.52	.24	.28	.52	.24	.28	.52	.24
Normal	.38	.42	.04	.38	.41	.03	.40	.42	.02	.38	.42	.04
Percentile	N/A	N/A	N/A	.43	.46	.03	.42	.46	.04	.43	.47	.04
Percentile-t	N/A	N/A	N/A	.42	.46	.04	.42	.46	.04	.43	.46	.03
Bias Corrected	.43	.44	.01	.42	.44	.02	.42	.45	.03	.41	.45	.04

TABLE 15**Estimates of long run income elasticity****The bootstrap distribution mean compared with the regression estimate.****Model: LONGRUN**

Period	# of iterations	Regression estimate	Bootstrap estimate	Bootstrap std. error
	100	.398	.445	.008
1959-1995	500	.398	.446	.009
	3000	.398	.446	.010
	100	.410	.391	.127
1959-1973	500	.410	.400	.126
	3000	.410	.391	.128
	100	.662	.651	.018
1974-1995	500	.662	.650	.021
	3000	.662	.650	.008

10:9 Recursive estimates: 1959-19XX

Table 16 shows the long run parameter estimates for the periods 1959:1 to 19XX:iv for each year beginning with 1970. This is the counterpart to Tables 3 and 4.

The income elasticity was stable for the periods 1971-73, 75-76. From 1977 to 1987 the 't' statistics were insignificant. Parameter estimates were erratic from 1977 and negative, through 1981. Stability and significant 't' values began to be re-established in 87-88, with the parameter values for the period 1989-95 returning to levels comparable to 1971-73.

The coefficient of the nominal interest rate shows low 't' statistics for the period 1970-1982, with exception in the years 1979-80. This coefficient in the post 1986 period has stable values and high 't' statistics, mirroring the income elasticity.

Chart 3, showing the long run elasticity and Chart 4, showing the long run interest rate coefficient illustrate these periods.

TABLE 16

Recursive estimates of long term parameters.

Beginning year 1959

Model : LONGRUN; equation 8E

Ending Year	Income elasticity β	t statisti c	Interest rate coefficient γ	t statistic
70	0.669	2.540	-0.03	2.00
71	0.399	2.170	-0.02	1.40
72	0.381	2.330	-0.02	1.42
73	0.410	2.330	-0.02	1.65
74	0.803	1.780	-0.04	1.64
75	1.188	4.390	0.00	0.38
76	1.183	4.380	0.00	0.38
77	-2.220	1.010	0.11	1.00
78	0.980	3.240	0.00	0.46
79	0.254	1.250	-0.02	2.74
80	0.064	0.400	-0.01	2.61
81	2.280	0.940	-0.08	1.09
82	1.460	1.490	-0.05	1.86
83	1.289	1.660	-0.05	2.11
84	1.103	1.680	-0.05	2.18
85	1.248	1.730	-0.05	2.21
86	2.171	1.320	-0.08	1.58
87	0.846	2.798	-0.04	3.67
88	0.614	3.630	-0.04	4.59
89	0.464	3.930	-0.03	4.98
90	0.416	4.070	-0.03	5.13
91	0.471	4.520	-0.03	5.55
92	0.470	4.990	-0.03	6.10
93	0.504	5.430	-0.03	6.57
94	0.453	6.370	-0.03	7.30
95	0.398	6.540	-0.03	7.45

Note: γ and β are coefficients defined in equation 8E

CHART 3

Money demand: recursive estimates : 1970-1995

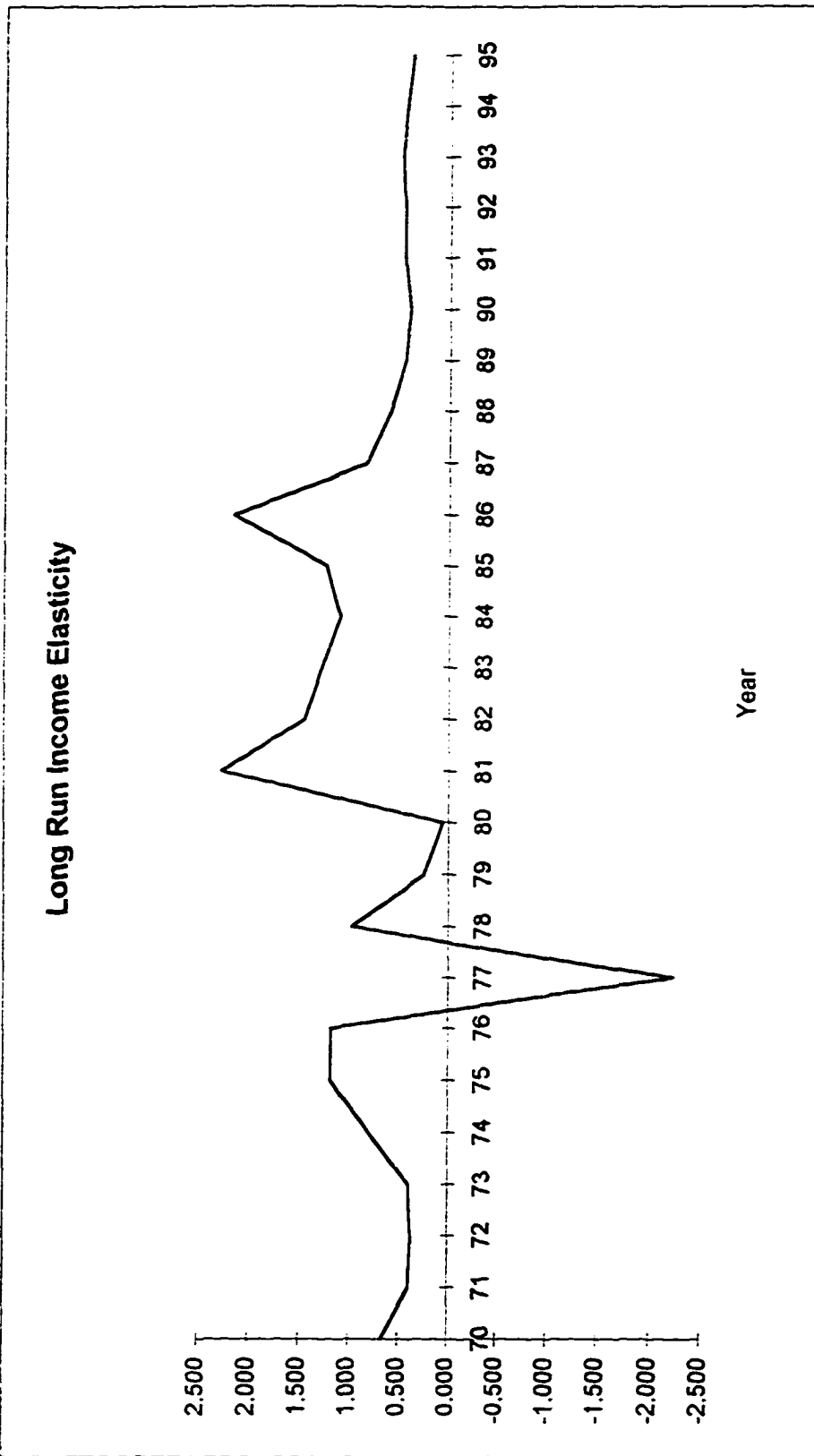
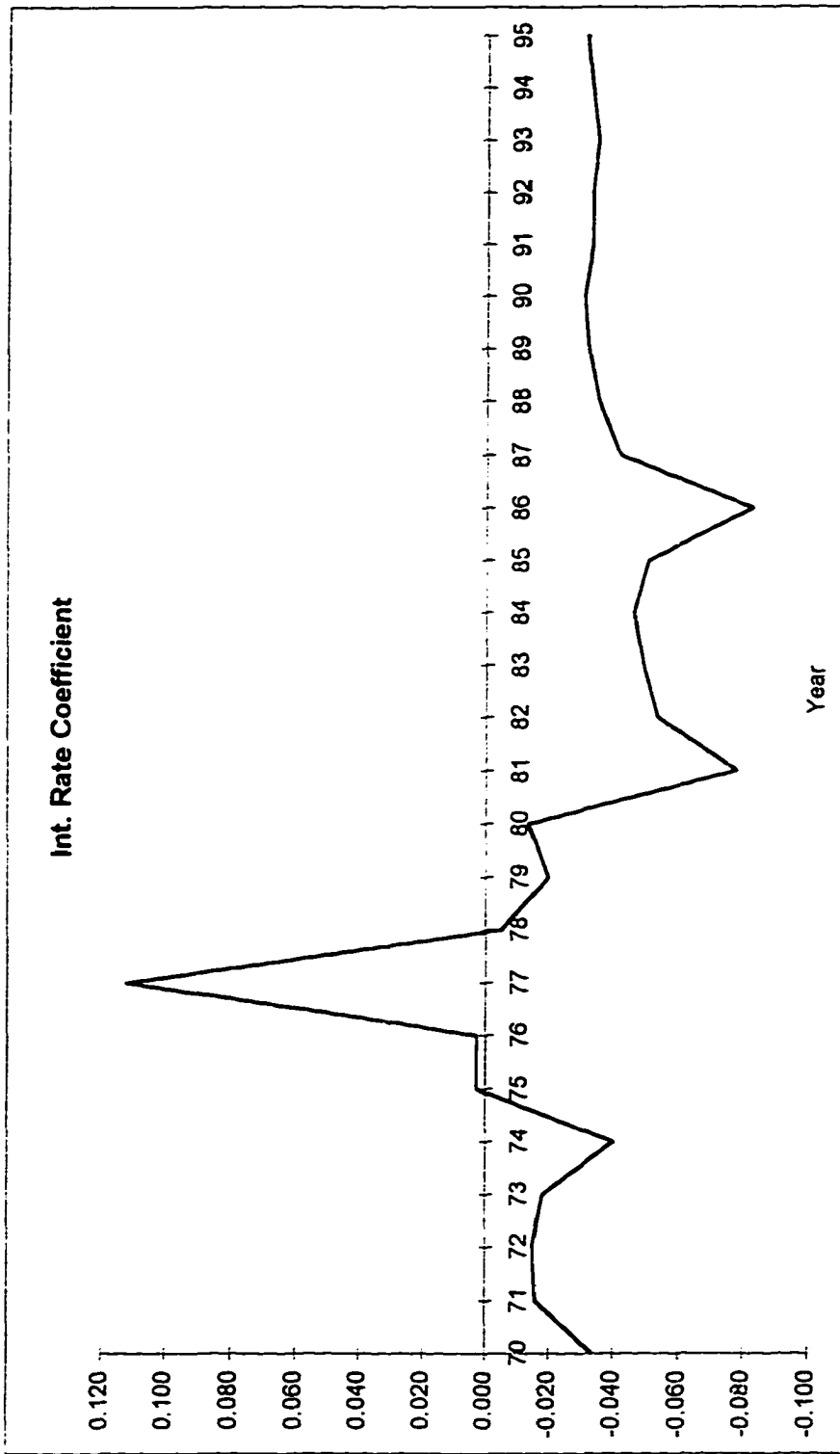


CHART 4

Money demand: recursive estimates: 1970-1995



10:10 Bootstrap estimates: Income elasticity

Table 17 compares the regression mean values and standard errors of the income elasticity with those of the bootstrap empirical distributions.

(i). Periods of stability and significant 't' statistics were 1971-73, 1975-1976, and 1987-1995.

(ii) The bootstrap empirical mean is greater on average than the regression mean. For the period 1989-95, the average difference was 8%.

(iii) The bootstrap standard error is smaller than the regression standard error. During 1987-95 it was on the average 80% smaller.

TABLE 17**Recursive estimates beginning 1959**

Regression estimates of long run income elasticity compared with bootstrap estimates.

Year	Regression model LONGRUN Income			Bootstrap		Change in	Change in Std.
	Elasticity a	t	Std. Error b	Mean c	Std. Error d	Mean c/a	Error d/b
70	0.669	2.540	0.263	0.733	0.285	1.096	1.084
71	0.399	2.170	0.183	0.605	0.309	1.516	1.684
72	0.381	2.330	0.163	0.320	0.104	0.839	0.639
73	0.410	2.330	0.176	0.399	0.139	0.973	0.790
74	0.803	1.780	0.451	0.793	0.222	0.988	0.492
75	1.188	4.390	0.271	2.022	1.281	1.702	4.733
76	1.183	4.380	0.270	-0.313	3.970	-0.265	14.708
77	-2.220	1.010	2.189	-0.694	2.614	0.313	1.194
78	0.980	3.240	0.302	-0.517	3.518	-0.528	11.662
79	0.254	1.250	0.202	1.778	2.765	7.000	13.688
80	0.064	0.400	0.162	0.011	0.556	0.172	3.439
81	2.280	0.940	2.416	0.189	0.876	0.083	0.362
82	1.460	1.490	0.977	0.646	0.387	0.442	0.397
83	1.289	1.660	0.774	0.702	0.602	0.545	0.777
84	1.103	1.680	0.654	0.654	0.472	0.593	0.722
85	1.248	1.730	0.720	0.941	0.371	0.754	0.516
86	2.171	1.320	1.637	1.711	0.370	0.788	0.226
87	0.846	2.798	0.302	1.467	0.249	1.734	0.824
88	0.614	3.630	0.169	0.853	0.124	1.389	0.734
89	0.464	3.930	0.118	0.488	0.022	1.052	0.186
90	0.416	4.070	0.102	0.446	0.017	1.072	0.165
91	0.471	4.520	0.104	0.475	0.015	1.008	0.144
92	0.470	4.990	0.094	0.491	0.018	1.045	0.188
93	0.504	5.430	0.093	0.553	0.021	1.097	0.222
94	0.453	6.370	0.071	0.518	0.017	1.143	0.244
95	0.398	6.540	0.061	0.446	0.010	1.121	0.158

Note: Income elasticity coefficient defined in equation 8E

10:11 Bootstrap estimates: Interest rate coefficient

Table 18 compares the regression mean values and standard errors for the interest rate coefficient with those of the bootstrap empirical distributions.

(i) The interest rate coefficient's 't' statistics are significant for the periods 1979-80, 1983-85, and 1987-95.

(ii) With the exception of 1979-80, and 1987, the bootstrap means are lower than the regression means. During 1988-95 the average difference was 15%.

(iii) The standard errors for the bootstrap distribution are lower for the comparable periods. During 1987-1995 the average difference was 84%.

TABLE 18**Recursive estimates beginning 1959**

Regression estimates of long run interest rate coefficient compared with bootstrap estimates.

Year	Regression model LONGRUN Interest			Bootstrap		Change in	Change in
	oefficien a	t	Std. Error b	Mean c	Std. Err d	Mean c/a	Std. Erro d/b
70	-0.034	2.000	0.017	-0.035	0.017	1.042	1.034
71	-0.016	1.400	0.012	-0.026	0.018	1.625	1.504
72	-0.015	1.420	0.011	-0.008	0.006	0.533	0.523
73	-0.018	1.650	0.011	-0.013	0.008	0.722	0.717
74	-0.040	1.640	0.025	-0.036	0.013	0.898	0.511
75	0.003	0.375	0.007	-0.080	0.081	-29.630	11.110
76	0.003	0.380	0.008	0.050	0.174	17.241	22.947
77	0.112	1.000	0.111	0.068	0.099	0.607	0.893
78	-0.005	0.460	0.010	0.055	0.153	-11.458	14.825
79	-0.020	2.740	0.007	-0.069	0.098	3.503	13.732
80	-0.014	2.610	0.005	-0.012	0.015	0.859	2.863
81	-0.078	1.090	0.071	-0.013	0.017	0.170	0.233
82	-0.053	1.860	0.029	-0.003	0.011	0.060	0.382
83	-0.049	2.110	0.024	-0.015	0.017	0.312	0.705
84	-0.046	2.180	0.021	-0.014	0.015	0.306	0.722
85	-0.050	2.210	0.023	-0.014	0.018	0.286	0.811
86	-0.082	1.580	0.052	-0.006	0.027	0.073	0.518
87	-0.041	3.670	0.011	-0.023	0.025	0.556	2.163
88	-0.035	4.590	0.008	-0.031	0.006	0.886	0.802
89	-0.032	4.980	0.006	-0.026	0.002	0.814	0.241
90	-0.030	5.130	0.059	-0.024	0.001	0.789	0.025
91	-0.033	5.550	0.060	-0.027	0.001	0.809	0.022
92	-0.033	6.100	0.005	-0.028	0.001	0.839	0.226
93	-0.035	6.570	0.005	-0.031	0.001	0.888	0.185
94	-0.033	7.300	0.004	-0.030	0.001	0.909	0.206
95	-0.031	7.450	0.004	-0.028	0.001	0.905	0.205

Note: Interest rate coefficient defined in equation 8E

10:12 Bootstrap confidence intervals

Tables 19 and 20 show the recursive estimates and the bootstrap confidence intervals. Table 19 shows the income elasticity parameter. For the period when the regression 't' values were significant, the bootstrap confidence intervals accept the regression estimates for 1970-73, and 1989-93, while rejecting them in 1994-95

Table 13 shows the bootstrap confidence intervals of the interest rate coefficient. These intervals reject all the regression parameter values.

10:13 Speed of adjustment: Lagged M1 coefficient

Table 21 compares the speed of adjustment derived from the long run model - LONGRUN - and from the short run one - SHORTRUN -. The speed of adjustment is derived from the lagged money coefficient in the models. For the LONGRUN model this coefficient is equal to $-(1 / (1 - \lambda))$, and for the SHORTRUN model it equals $1/\lambda$.

(i) The speed of adjustment, λ , derived from LONGRUN is in all cases greater than that for SHORTRUN.

(ii) During the 1987-95 period the average number of quarters to full adjustment for LONGRUN was 10.7, compared with 17.8 for SHORTRUN. The averages decreased for the period 1990-1995 to 9.5 and 11.4 respectively.

Chart 5 shows the long term coefficient of lagged money. This is comparable to Charts 3 and 4 , and shows that this coefficient was stable during 1970-1973, and 1990-1995.

TABLE 19
Recursive estimates beginning 1959
 Regression estimates of long run income elasticity compared with
 bootstrap estimates and confidence intervals.

Yea	Regression estimate	t	Bootstrap estimate	Bootstrap confidence intervals				Parametric
				Normal Approx.	Percentile	Percentil t	Bias Corrected	
70	0.669	2.540	0.733	a	a	a	a	a
71	0.399	2.170	0.605	r	a	a	a	a
72	0.381	2.330	0.320	a	a	a	a	a
73	0.410	2.330	0.399	a	a	a	a	a
74	0.803	1.780	0.793	a	a	a	a	r
75	1.188	4.390	2.022	r	a	a	a	a
76	1.183	4.380	-0.313	r	r	r	a	a
77	-2.220	1.010	-0.694	r	r	r	r	r
78	0.980	3.240	-0.517	r	r	r	a	a
79	0.254	1.250	1.778	r	a	a	a	r
80	0.064	0.400	0.011	r	r	r	a	r
81	2.280	0.940	0.189	a	r	r	a	r
82	1.460	1.490	0.646	a	a	r	a	r
83	1.289	1.660	0.702	a	r	r	a	r
84	1.103	1.680	0.654	a	r	r	a	r
85	1.248	1.730	0.941	a	a	r	a	r
86	2.171	1.320	1.711	a	a	a	a	r
87	0.846	2.798	1.467	a	a	a	a	a
88	0.614	3.630	0.853	a	r	r	r	a
89	0.464	3.930	0.488	a	a	a	a	a
90	0.416	4.070	0.446	a	a	a	a	a
91	0.471	4.520	0.475	a	a	a	a	a
92	0.470	4.990	0.491	a	a	a	a	a
93	0.504	5.430	0.553	r	a	a	a	a
94	0.453	6.370	0.518	a	r	a	a	a
95	0.398	6.540	0.446	a	r	r	r	a

Bootstrap 95% Confidence Intervals

a= accept Regression Parameter

r= reject Regression Parameter

TABLE 20**Recursive estimates beginning 1959**

Regression estimates of long run interest rate coefficient compared with bootstrap estimates and confidence intervals.

Year	Regression estimate	t	Bootstra estimate	Bootstrap confidence intervals				Parametric
				Normal Approx.	Percentile	Bias Corrected	t	
70	-0.034	2.000	-0.035	a	r	a	r	a
71	-0.016	1.400	-0.026	r	r	a	r	r
72	-0.015	1.420	-0.008	a	r	r	r	r
73	-0.018	1.650	-0.013	a	r	a	r	r
74	-0.040	1.640	-0.036	a	r	a	r	r
75	0.003	0.375	-0.080	r	r	a	r	r
76	0.003	0.380	0.050	r	r	r	r	r
77	0.112	1.000	0.068	r	r	r	r	r
78	-0.005	0.460	0.055	r	r	r	r	r
79	-0.020	2.740	-0.069	r	r	a	r	a
80	-0.014	2.610	-0.012	r	r	a	r	a
81	-0.078	1.090	-0.013	a	r	r	r	r
82	-0.053	1.860	-0.003	r	r	r	r	r
83	-0.049	2.110	-0.015	a	r	r	r	a
84	-0.046	2.180	-0.014	a	r	r	r	a
85	-0.050	2.210	-0.014	a	r	r	r	a
86	-0.082	1.580	-0.006	a	r	r	r	r
87	-0.041	3.670	-0.023	r	r	r	r	a
88	-0.035	4.590	-0.031	a	r	a	r	a
89	-0.032	4.980	-0.026	a	r	r	r	a
90	-0.030	5.130	-0.024	a	r	r	r	a
91	-0.033	5.550	-0.027	a	r	r	r	a
92	-0.033	6.100	-0.028	r	r	a	r	a
93	-0.035	6.570	-0.031	a	r	r	r	a
94	-0.033	7.300	-0.030	r	r	r	r	a
95	-0.031	7.450	-0.028	a	r	r	r	a

Bootstrap 95% Confidence Intervals

a= accept Regression Parameter

r= reject Regression Parameter

TABLE 21
Recursive estimates of lagged money coefficient beginning 1959

Year	Model :LONGRUN (Equation 8:E)				Model: SHORTRUN (Equation 8:C)		CHANGE	
	Lagged Money Coefficient	t	Speed Of Adjustment a	# Quarters to Adjust b	Speed Of Adjustment c	# Quarters to Adjust d	Speed Of Adjustment a/c	# Quarters to Adjust b/d
70	-2.99	1.80	0.25	3.99	0.05	18.81	4.72	0.21
71	-2.85	1.68	0.26	3.85	0.07	14.51	3.77	0.27
72	-2.71	1.73	0.27	3.71	0.08	13.20	3.56	0.28
73	-3.42	1.83	0.23	4.42	0.08	12.94	2.93	0.34
74	8.43	1.60	-0.13	-7.43	0.05	20.55	-2.77	-0.36
75	-0.05	0.09	0.96	1.05	-0.01	-100.00	-95.56	-0.01
76	-0.13	0.26	0.88	1.13	-0.02	-41.15	-36.32	-0.03
77	33.44	0.95	-0.03	-32.44	-0.02	-45.05	1.39	0.72
78	0.44	0.81	1.78	0.56	-0.01	-70.92	-126.20	-0.01
79	-1.68	2.03	0.37	2.68	0.00	2500.00	932.83	0.00
80	-1.45	2.37	0.41	2.45	0.02	52.49	21.44	0.05
81	-46.43	0.99	0.02	47.44	0.02	59.70	1.26	0.79
82	-29.37	1.65	0.03	30.37	0.02	45.58	1.50	0.67
83	-25.58	1.89	0.04	26.58	0.03	29.84	1.12	0.89
84	-23.53	1.97	0.04	24.53	0.04	27.67	1.13	0.89
85	-24.58	1.93	0.04	25.58	0.03	29.24	1.14	0.87
86	-35.98	1.35	0.03	36.98	0.02	48.05	1.30	0.77
87	-15.79	2.87	0.06	16.79	0.05	19.44	1.16	0.86
88	-12.02	3.72	0.08	13.02	0.07	15.14	1.16	0.86
89	-9.15	4.23	0.10	10.15	0.08	12.05	1.19	0.84
90	-8.56	4.44	0.10	9.56	0.09	11.45	1.20	0.83
91	-8.74	4.34	0.10	9.74	0.08	11.80	1.21	0.83
92	-8.84	4.45	0.10	9.84	0.08	11.88	1.21	0.83
93	-9.29	4.46	0.10	10.29	0.08	12.57	1.22	0.82
94	-8.26	5.18	0.11	9.26	0.09	11.07	1.20	0.84
95	-7.01	5.53	0.12	8.01	0.10	9.81	1.22	0.82

CHART 5

Money demand: recursive estimates: 1970-1995

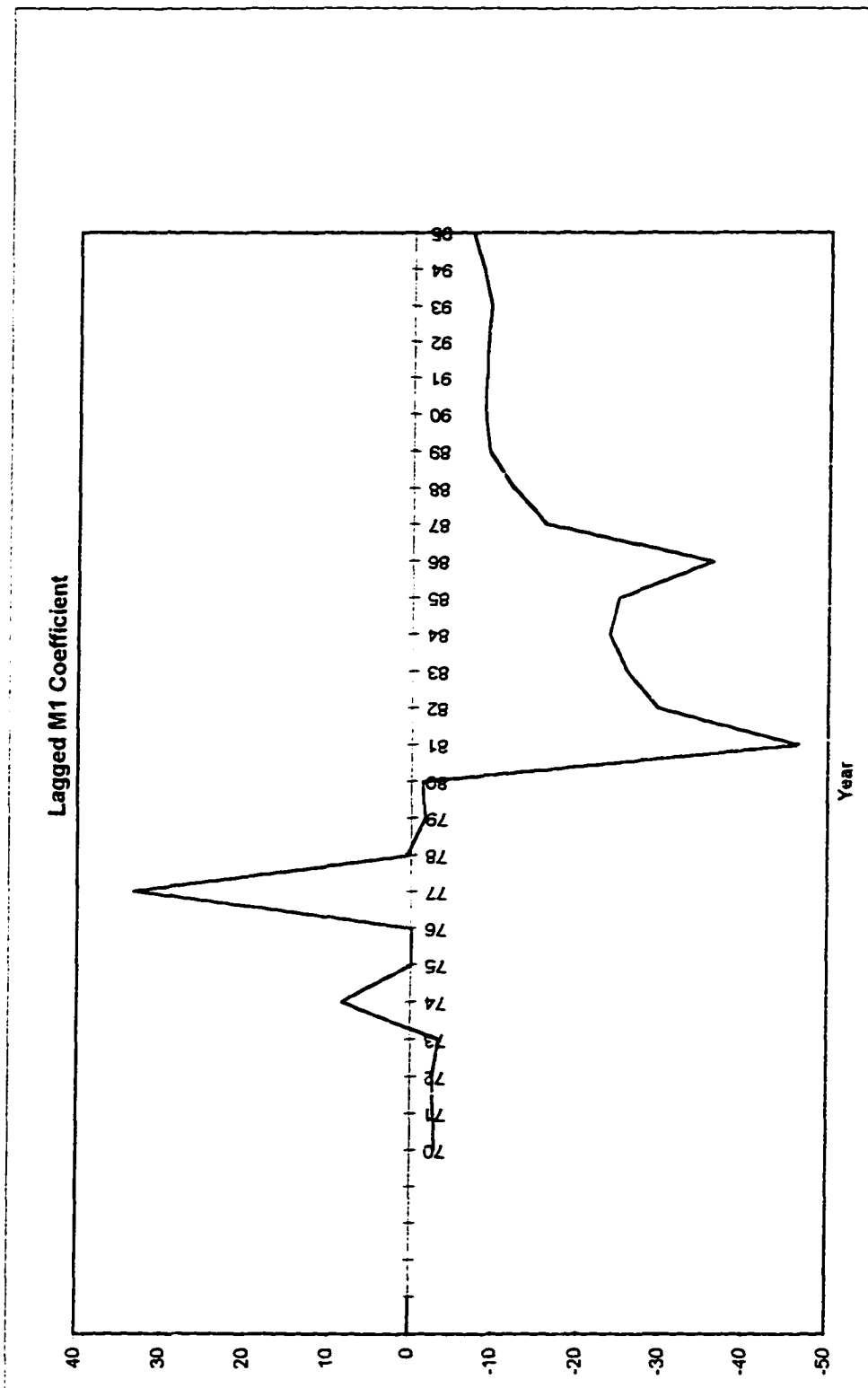


TABLE 22**Bootstrap recursive estimates of long run income elasticity: beginning year 1959**

Ending Year	Degrees of freedom	Bootstrap estimate	Bootstrap standard error	Regression estimate β	Standard error of β
69	36	.680	.630	.550	.170
68	32	.310	.170	-9.890	102.200
67	28	.120	.050	.720	.880
66	24	.160	.110	.980	1.750
65	20	.140	.100	1.00	2.030
64	16	.160	.140	.350	.140
63	12	.000	.160	.420	1.850
62	8	.160	.770	.130	.160
61	4	.100	.800	.220	.140

Note: β is the income elasticity coefficient defined in equation 8E.

10:14 Bootstrap estimates and sample size.

Table 22 compares the recursive estimates of income elasticity parameter derived from the bootstrap empirical distribution, with those from the long run model.

The bootstrap estimates have standard errors which are smaller than the parametric estimates. These appear to be stable for 32-16 degrees of freedom. However these results cannot reject the null hypothesis.

10:15 Summary

(i) The LONGRUN model transforms the conventional money demand equation, SHORTRUN, with nominal money adjustment, into a form where the long run-equilibrium- parameters, income elasticity, and the interest rate coefficient, are directly observed. The model produces results that lend support to a theoretical relationship between real M1, real GDP, and the 90 day T-Bill.

(ii) The statistics pertaining to these parameters are observed directly.

(iii) The parameters compare with estimates derived by other researchers.

(iv) The model detected instabilities in the demand function, and confirms prior results.

(v) The empirical distribution of the parameters is found by bootstrapping the LONGRUN model. Confidence intervals determined by

this method accept the regression values of the income elasticity parameters.

(vi) The regression interest rate coefficient is rejected by the bootstrap intervals .

(vii) The bootstrap confidence intervals are stable, and converge rapidly.

CHAPTER 11
CONCLUSIONS.

11:1 Overview.

This study was designed to:

- (1) Produce a reduced form single equation model of the long run demand for money which explicitly shows the long run coefficients, and, using parametric techniques, retrieve the coefficient estimates together with the relevant statistics for hypothesis testing.
- (2) Apply a non parametric procedure to estimate the coefficients of the same model.
- (3) Evaluate the results of the two procedures and compare these estimates with those produced by other researchers, .viz. Roley, and Goldfeld.
- (4) Assess the non-parametric estimates with regards to stability, convergence, and sample size.

The model was derived from a standard equilibrium formulation and a short run dynamic specification where nominal money adjusts.

These conditions were manipulated to produce the conventional short run function, named - SHORTRUN- which was then transformed to produce the long run model, named, -LONGRUN-. This model is

estimated using regression techniques, specifically, the instrumental variable procedure. The instruments are the estimates derived from the short run equation. The non-parametric technique is the bootstrap. It is applied to the models, SHORTRUN and LONGRUN, in order to produce empirical distributions of the parameter estimates produced by the regression .

A computer programme was developed, and using RATS 4.0 , applied to U.S. data for the period 1959-1995. The sample was split into the following sub-periods: 1959-1973, 1974-1983, 1983-1995, 1974-1995, and the entire sample.

The first period was chosen as it coincides with the period where the demand for money was assumed to be stable, and for which there are coefficient estimates to be used as benchmarks. The second period was when the stability of money demand functions was questioned. Roley's estimates cover both periods, while Goldfeld's cover the first. The subsequent periods were used to compare the regression estimates of the models SHORTRUN, and LONGRUN , with those derived by the bootstrapping method.

11:2 Estimation results

The long run parametric estimates of income elasticity produced by the LONGRUN model gave results that were of the same magnitude as Roley's, for both the 59-73, and 74-83 periods. The 't' statistics produced

were significant for the first period, and insignificant for the second period. This seems to be in accordance with the demand function deteriorating in the second period.

The short run model also produced results that show the function deteriorating in the second period.

The bootstrap 95% confidence intervals accept the long term and short term estimates produced by the regression for both periods, contradicting the regression statistics.

During the 1984-95 period, the parametric and bootstrap estimates are insignificant.

During the period 1974-1995, the bootstrap and parametric estimates are accepted at the 95% confidence level.

For the entire sample period, 1959-1995, the parametric long run and short run income elasticity coefficient has significant 't' values. The bootstrap confidence interval for the empirical distribution rejects these estimates.

Recursive estimation shows that periods of stability, significant 't' statistics generated by the parametric model correspond to those produced by the bootstrap. For income elasticity the means of the empirical distribution derived by bootstrapping are on average larger than those produced by the regression. The standard errors are smaller for the bootstrap.

For the interest rate coefficient, the comparable bootstrap means and standard errors are both lower than the regression values. The bootstrap intervals reject all the regression parameters values. Both bootstrap and regression models show stable coefficients in the post 1986 period.

The speed of adjustment parameter derived from the long run model was in all cases greater than those obtained from the short run model. This implies that the number of quarters to full adjustment to a monetary shock is lower. For the 87-95 period the number was 10.7 quarters, compared with the short run estimate of 18 quarters.

11:3 Bootstrap convergence and stability.

Focusing on the income elasticity coefficient, and for the entire sample period, estimates derived from different bootstrap iterations - 10, 100, 500, and 3000 - are generated in order to examine whether they converge and if they are stable.

The results show that the bootstrap intervals converge rapidly, and the means - coefficient estimates - are stable.

11:4 Bootstrap confidence intervals.

The bootstrap confidence intervals developed recursively for the entire period accept the income elasticity coefficient as significant for the periods 1970-1973, 1988-1993. These are periods where the regression 't' statistics are significant at the 95% level.

The interest rate coefficients are rejected for all intervals for the entire period. Already noted is the fact that the bootstrap mean and standard error is smaller than the regression value. This would imply a smaller interval.

11:5 The bootstrap and sample size.

Recursive estimates for income elasticity developed for the periods from 59-69, beginning in 61, show the bootstrap means and standard errors to be lower than the regression values, and that they are relatively stable for 32-16 degrees of freedom. Strong conclusions cannot be drawn as the confidence intervals do not reject the null.

Appendix A

The Bootstrap Programme.¹

CALENDAR 1959 1 4

ALL 1971:4

OPEN DATA FRBSTL.RAT

DATA(FORMAT=RATS,ORG=OBS) / NARM1 BRDM2 MBASE \$

NOMGDP PLVL TBILL LTBND POP

*

COMPUTE nB = 100

DECLARE VECTOR bb(nB)

DECLARE VECROR bib(nB)

DECLARE VECTOR sd(nB)

DECLARE VECTOR sid(nB)

*

BB AND SD ARE THE INCOME COEFFICIENT AND STD. ERROR.

SET NOMM1 = NARM1*1000/POP

SET LOGNOM1 =LOG(NOMM1)

SET ECM1 = LOGNOM1 - LOGNOM1{1}

¹ This is a significantly amended, corrected and modified version of that found in "Bootstrapping: a Non Parametric approach to Statistical Inference" by Mooney and Duvall. op. cit.

*

$$\text{SET NOMM2} = \text{BRDM2} * 1000 / \text{POP}$$
$$\text{SET LOGNOM2} = \text{LOG}(\text{NOMM2})$$
$$\text{SET ECM2} = \text{LOGNOM2} - \text{LOGNOM2}\{1\}$$

*

$$\text{SET PCMBASE} = \text{MBASE} * 1000 / \text{POP}$$
$$\text{SET LOGBASE} = \text{LOG}(\text{PCMBASE})$$

*

$$\text{SET RLM1} = \text{NOMM1} * 100 / \text{PLVL}$$
$$\text{SET LOGRLM1} = \text{LOG}(\text{RLM1})$$

*

$$\text{SET RLM2} = \text{NOMM2} * 100 / \text{PLVL}$$
$$\text{SET LOGRLM2} = \text{LOG}(\text{RLM2})$$

*

$$\text{SET NOMADM1} = \text{NOMM1}\{1\} * 100 / \text{PLVL}$$
$$\text{SET LOGNAD1} = \text{LOG}(\text{NOMADM1})$$

*

$$\text{SET NOMADM2} = \text{NOMM2}\{1\} * 100 / \text{PLVL}$$
$$\text{SET LOGNAD2} = \text{LOG}(\text{NOMADM2})$$

*

$$\text{SET PCGDP} = \text{NOMGDP} * 1000 / \text{POP}$$
$$\text{SET RLGDP} = \text{PCGDP} * 100 / \text{PLVL}$$

```
SET LGRLGDP = LOG(RLGDP)
```

```
SET PRICE = LOG(PLVL)
```

- * Obtaining the Short term parameters.

```
AR1(METHOD=MAXL) LOGRLM1 / RESIDS OLSCOE
```

```
#CONSTANT LGRLGDP TBILL LOGNAD1
```

```
STATISTICS RESIDS / SMPL
```

```
SET EMHAT = LOGRLM1 - RESIDS
```

```
SET MIHAT = LOGRLM1-RESIDS + LOG(PLVL)
```

```
SET ECMIHAT = MIHAT - MIHAT{1}
```

- * The Instrumental variable stage of the sample, obtaining the Long term Parameters

```
INSTRUMENTS CONSTANT EMHAT LGRLGDP TBILL ECMIHAT
```

```
AR1(METHOD=MAXL,INST) LOGRLM1 / RESINS OLSCOE
```

```
#CONSTANT LGRLGDP TBILL ECMI
```

- * The coefficients and standard errors .

```
COMPUTE OB = OLSCOE(2)
```

```
COMPUTE OBI = OLSCOE(3)
```

```
COMPUTE SEB = SQRT(%SEESQ*%XX(2,2))
```

```
COMPUTE SEIB = SQRT(%SEESQ*%XX(3,3))
```

```
COMPUTE PMLCI = OB - 1.96*SEB
```

```
COMPUTE PMUCI = OB + 1.96*SEB
```

```
COMPUTE PMILCI = OBI - 1.96*SEIB
```

```
COMPUTE PMIUCI = OBI + 1.96*SEIB
```

DISPLAY OB SEB OBI SEIB

* STATISTICS RESINS / SMPL

SET RLM1HAT = LOGRLM1 - RESIDS

* PRINT 1959:01 1960:02 RLM1HAT RESINS

* The Bootstrap Loop

DO I=1,NB

BOOT B / 1959:02 1971:04

SMPL 1959:01 1959:01

SET RLM1NEW = LOGRLM1 + RESIDS(B(T))

SMPL 1959:02 1971:04

BOOT B / 1959:02 1971:04

SET RLM1NEW = RLM1HAT + RESIDS(B(T))

SMPL 1959:01 1971:04

SET NEWM1 = RLM1NEW + LOG(PLVL)

SET ECMNEW = NEWM1 - NEWM1{1}

SET NAD1NEW = NEWM1{1} - LOG(PLVL)

* Bootstrap estimation . Estimates of Short term parameters of Resamples.

AR1(METHOD=MAXL,NOPRINT) RLM1NEW 1959:02 1971:04 RESIBS

OLSCOEF

#CONSTANT LGRLGDP TBILL NAD1NEW

* STATISTICS RESIBS / SMPL

SET EM2HAT = RLM1NEW - RESIBS

```

SET M1BSHAT = RLMINEW - RESIBS + LOG(PLVL)
SET ECM1BS = M1BSHAT - M1BSHAT{1}
*   Estimates of long term parameters of the resamples
INSTRUMENTS CONSTANT EM2HAT LGRLGDP TBILL ECM1BS
ARI(METHOD=MAXL,INST,NOPRINT) RLMINEW 1959:03 1971:04 IDERR
COEF
# CONSTANT LGRLGDP TBILL ECMNEW
*   Coefficients and standard errors of the resamples
COMPUTE BB(I) = COEF(2)
COMPUTE SD(I) = SQRT(%SEESQ*%XX(2,2))
COMPUTE BIB(I) = COEF(3)
COMPUTE SID(I) = SQRT(%SEESQ*%XX(3,3))
*   DISPLAY(UNIT = SCREEN) 'TRIAL ' I
    END DO I
*
SET C 1 NB = BB(T)
SET CSD 1 NB = SD(T)
STATISTICS CSD 1 NB
STATISTICS C 1 NB
*   PRINT 1 NB C
*   Normal Approximation Confidence interval
    COMPUTE NAUCI = OB + 1.96*SQRT(%VARIANCE)

```

```

COMPUTE NALCI = OB - 1.96*SQRT(%VARIANCE)

ORDER C 1 NB CSD

* PRINT 1 NB CSD

* Percentile and Percentile-t confidence intervals

COMPUTE IL = FIX((NB*.05)/2)

COMPUTE IU = NB - IL + 1

SET TSCORE 1 NB = (BB(T) - OB)/CSD(T)

ORDER TSCORE 1 NB CSD

COMPUTE PTLCI = OB + TSCORE(IL)*SEB

COMPUTE PTUCI = OB + TSCORE(IU)*SEB

SET PRT 1 NB = %IF(C(T) < OB,1,0)

* PRINT 1 NB PRT

STATISTICS(PRINT) PRT 1 NB

* Bias corrected confidence intervals

COMPUTE PROP = %MEAN

DECLARE REAL ZP

COMPUTE ZIT = -1.0

UNTIL ZP > PROP {

COMPUTE ZP = %CDF(ZIT)

COMPUTE ZIT = ZIT + .001

}

END UNTIL

```

```

COMPUTE ZIT = ZIT - .001

DISPLAY ZP ZIT PROP

COMPUTE UBC = FIX(%CDF(2*ZIT+1.96)*NB)
COMPUTE LBC = FIX(%CDF(2*ZIT-1.96)*NB)

IF LBC < 1 {
  COMPUTE LBC = 1
}

END IF

DISPLAY '          ' PTLCI PTUCI

DISPLAY 'PARAMETRIC ' PMLCI PMUCI PMUCI-PMLCI
DISPLAY 'NORMAL APPROX ' NALCI NAUCI NAUCI-NALCI
DISPLAY 'PERCENTILE ' C(IL) C(IU) C(IU)-C(IL)
DISPLAY 'PERCENTILE T ' PTLCI PTUCI PTUCI-PTLCI
DISPLAY 'BIAS-CORRECTED' C(LBC) C(UBC) C(UBC)-C(LBC)

```

* Repeat computations for interest variable

```
SET IC 1 NB = BIB(T)
```

```
SET CISD 1 NB = SID(T)
```

```
STATISTICS CISD 1 NB
```

```
STATISTICS IC 1 NB
```

* PRINT 1 NB C

* Normal Approximation confidence interval

```
COMPUTE NAIUCI = OBI + 1.96*SQRT(%VARIANCE)
```

```
COMPUTE NAILCI = OBI - 1.96*SQRT(%VARIANCE)
```

```
ORDER IC 1 NB CISD
```

```
* PRINT 1 NB CISD
```

```
* Percentile and Percentile-t method
```

```
COMPUTE ILI = FIX((NB*.05)/2)
```

```
COMPUTE IUI = NB - ILI + 1
```

```
SET ITSCORE 1 NB = (BIB(T) - OBI)/CISD(T)
```

```
ORDER ITSCORE 1 NB CISD
```

```
COMPUTE IPTLCI = OBI + ITSCORE(ILI)*SEIB
```

```
COMPUTE IPTUCI = OBI + ITSCORE(IUI)*SEIB
```

```
SET IPRT 1 NB = %IF(IC(T) < OBI,1,0)
```

```
* PRINT 1 NB IPRT
```

```
STATISTICS(PRINT) IPRT 1 NB
```

```
* Bias corrected confidence interval
```

```
COMPUTE IPROP = %MEAN
```

```
DECLARE REAL IZP
```

```
COMPUTE IZIT = -1.0
```

```
UNTIL IZP > IPROP {
```

```
COMPUTE IZP = %CDF(IZIT)
```

```
COMPUTE IZIT = IZIT + .001
```

```

}
END UNTIL
COMPUTE IZIT = IZIT - .001
DISPLAY IZP IZIT IPROP
COMPUTE IUBC = FIX(%CDF(2*IZIT+1.96)*NB)
COMPUTE ILBC = FIX(%CDF(2*IZIT-1.96)*NB)
IF ILBC < 1 {
COMPUTE ILBC = 1
}
END IF
DISPLAY '          ' IPTLCI IPTUCI
DISPLAY 'PARAMETRIC ' PMILCI PMIUCI PMIUCI-PMILCI
DISPLAY 'NORMAL APPROX ' NAILCI NAIUCI NAIUCI-NAILCI
DISPLAY 'PERCENTILE ' C(ILCI) C(IUCI) C(IUCI)-C(ILCI)
DISPLAY 'PERCENTILE T ' IPTLCI IPTUCI IPTUCI-IPTLCI
DISPLAY 'BIAS-CORRECTED' C(ILBC) C(IUBC) C(IUBC)-C(ILBC)
END

```

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