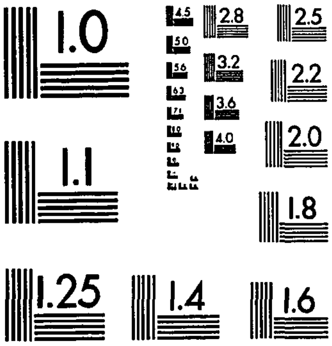
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STUDIES IN RELATIVISTIC NUCLEAR PHYSICS

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by

Avaroth Harindranath

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Abstract

STUDIES IN RELATIVISTIC NUCLEAR PHYSICS

by

Avaroth Harindranath

Advisor: Professor Carl M.Shakin

In the light of recent experimental advances made in the areas of nucleon-nucleus interactions and electron-nucleus interactions we investigate certain problems which have arisen in these fields.

In recent years various works have convincingly shown that nucleus is best treated as a relativistic system.Work in relativistic nuclear physics can be broadly divided into two categories.One is Dirac phenomenology where one uses several adjustable parameters to fit experimental data.The Relativistic Brueckner-Hartree-Fock (RBHF) approximation belongs to the second category where there are no adjustable parameters. When comparison is made between the strength of the relativistic potentials obtained in microscopic calculation and the (relativistic) phenomenological potentials it is found that the real parts of the potentials differ by about twenty-five percent. Usually, the phenomenological potentials are of only two kinds, (Lorentz) scalar and vector.Since for a finite system the self-energy operator

is non-local, it is clear that there are several other terms with different (Lorentz) structures that could be present in the general form of this operator. (It was shown recently that there are eight scalar invariants that determine the scattering of an off-shell nucleon from an on-shell spin-zero nucleus.) In part I of this thesis we discuss the calculation of the complete potential . Also we compare the microscopic calculation with the phenomenology and show how to resolve the apparent disagreement between the theoretical and phenomenological potentials.

{ For a historical survey and detailed description of relativistic nuclear dynamics we refer to the book :

Relativistic Nuclear Physics : Theories of Structure and Scattering

by L.S.Celenza and C.M.Shakin, to be published by World Scientific, Singapore.}

Recently we have seen some theoretical work which suggest that the properties of the nucleon are modified in the nuclear medium. Specifically we are interested in the recently developed model of hadron structure (Covariant Soliton Dynamics) which is able to provide parameter-free predictions of the modification of nucleon properties in nuclei.In part II of this work we investigate the predictions of this model in the analysis of (e,e') inclusive experiments near the quasi-elastic peak.

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PART I : MICROSCOPIC FOUNDATIONS OF DIRAC PHENOMENOLOGY

In recent years we have witnessed an ongoing effort to describe the nucleus as a relativistic system [I.1]. Starting from the nucleon - nucleon interaction in free space it is now possible to calculate the binding energy and saturation density of nuclear matter, the effective force in nuclei and the optical potential for continuum nucleons in a parameter-free approach. Much of this work was motivated by the highly successful fits to the nucleon - nucleus scattering data [I.2]. These fits were based on the use of Dirac equation for the description of nucleon motion.

The potentials which are used in Dirac phenomenology contain large (Lorentz) scalar and vector fields. Typically the strength of the scalar field is about - 400 MeV and the strength of the vector field is about + 300 MeV.

The physical origin of these large fields can be understood from a parameter-free approach to the nuclear many-body problem : The Relativistic Brueckner-Hartree-Fock (RBHF) approximation [I.1.1]. The potentials calculated in the RBHF approximation were quite strong but were smaller in magnitude when compared to the potentials used in Dirac phenomenology.

A general theory of the relativistic nuclear optical potential model (formulated in momentum space) appropriate for the finite system was developed in [I.3]. Here it was shown that the general optical potential in addition to scalar and vector terms contain six other terms. In the first part of this work we discuss the calculation of the complete potential. Also we compare the microscopic calculation with the phenomenology and show how to resolve the apparent disagreements.

In section 1 we outline the optical potential formalism. In section 2 we review the version of the RBHF approach which is appropriate for the study of an infinite system. In section 3 we review the relativistic optical model appropriate for a finite system. In section 4 we describe our model used for the calculation of the complete optical potential. Section 5 deals with the comparison with phenomenology and contains our conclusions.

Note :

In this work (both parts I and II) we use the conventions specified in J.D.Bjorken and S.D.Drell, Relativistic Quantum Mechanics (McGraw-Hill, New York, 1964).

Thus we have,

$$g^{\mu\nu} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

For any two four vectors A^μ and B^μ we denote the scalar product by

$$A.B = A^\mu B_\mu,$$

$$A^2 = A^\mu A_\mu.$$

For the Dirac matrices we used the explicit representation

$$\gamma^\mu = (\gamma^o , \vec{\gamma})$$

where

$$\gamma^o = \begin{bmatrix} I & 0 \\ 0 & -I \end{bmatrix},$$

$$\vec{\gamma} = \begin{bmatrix} 0 & \vec{\sigma} \\ -\vec{\sigma} & 0 \end{bmatrix},$$

and for γ^5 we have,

$$\gamma^5 = \begin{bmatrix} 0 & I \\ I & 0 \end{bmatrix} .$$

Here I is 2×2 unit matrix and σ is the 2×2 Pauli spin matrices. Thus

We denote the positive and negative-energy spinor for a Dirac particle with mass m and momentum $\vec{k}$ by $u(\vec{k},s)$ and $v(\vec{k},s)$ respectively. Thus we have,

$$u(\vec{k},s) = [\varepsilon/(2m)]^{\frac{1}{2}} \begin{bmatrix} \chi_s \\ \{\vec{\sigma}.\vec{k}/\varepsilon\}\chi_s \end{bmatrix}$$

$$v(\vec{k},s) = [\varepsilon/(2m)]^{\frac{1}{2}} \begin{bmatrix} \{\vec{\sigma}.\vec{k}/\varepsilon\}\chi_{-s} \\ \chi_{-s} \end{bmatrix} .$$

With † denoting the hermitian conjugate we also have,

$$\bar{u}(\vec{k},s) = [u(\vec{k},s)]^{\dagger}\gamma^{o},$$

$$\bar{v}(\vec{k},s) = [v(\vec{k},s)]^{\dagger}\gamma^{o}.$$

Here χ_s is the two-component Pauli spinor with spin projection s and

$$\varepsilon = (m^2 + \vec{k}^2)^{\frac{1}{2}} + m .$$

SECTION 1

Introduction to the Optical Potential Formalism :

We are interested in the problem of elastic scattering of a particle by a composite target containing many scatterers. Due to the complicated interactions between the projectile and the target particles the problem is indeed a difficult one. By introducing an optical potential this many- body problem can be converted into a one body problem , i.e., the scattering of a particle by a potential [I.4]. We illustrate this formalism in this section.

1.1 Optical potential method [non-relativistic formalism] :

Consider the Hamiltonian H of a system consisting of a projectile and a target containing many scatterers,

$$H = H_o + V, \tag{1}$$

$$H_o = K_p + \Sigma_j K_j + U. \tag{2}$$

Here K_p is the kinetic energy operator for the projectile, K_j is the kinetic energy operator for the j^{th} particle of the target and U is the interaction between N particles of the target. Further, V is the sum of all the interactions between and the projectile and the particles of the target.

Let us denote the initial asymptotic state of the target plus the incident particle by Φ_i and the corresponding energy by E_i i.e.,

$$H_o \Phi_i = E_i \Phi_i. \tag{3}$$

We start from the Lippmann- Schwinger equation whose solution is the many- body scattering wave function $\Psi^{(+)}$:

$$\Psi^{(+)} = \Phi_i + \frac{1}{E_i - H_o + i\varepsilon} V \Psi^{(+)}. \tag{4}$$

Since our interest is in the description of elastic scattering, we construct $\Psi^{(+)}_{el}$ from $\Psi^{(+)}$:

$$\Psi^{(+)}_{el} = \Pi_o \Psi^{(+)} , \tag{5}$$

where

$$\Pi_o = |0\rangle \langle 0| . \tag{6}$$

Here $|0\rangle$ describes the internal state of the target in its ground state. Operating from the left by Π_o , we have ,

$$\Psi^{(+)}_{el} = \Phi_i + \frac{1}{E_i - H_o + i\varepsilon} \Pi_o V \Psi^{(+)} . \tag{7}$$

We now introduce an operator F which reconstructs the full many-body wave function from its elastic-scattering part :

$$\Psi^{(+)} = F \, \Psi^{(+)}_{el} \, . \tag{8}$$

Then we have

$$\Psi^{(+)}_{el} = \Phi_i + \frac{1}{E_i - H_o + i\varepsilon} \, V_{opt} \, \Psi^{(+)}_{el} \, , \tag{9}$$

where we have introduced the optical potential V_{opt} :

$$V_{opt} = \langle O | VF | O \rangle \quad . \tag{10}$$

To construct V_{opt} we must calculate F.

F satisfies the following equation :

$$F = 1 + \frac{1}{E_i - H_o + i\varepsilon} \, (1 - \Pi_o) \, V \, F \tag{11}$$

Now define the exact two body scattering matrix t_j by

$$t_j = v_j + v_j \, \frac{1}{E_i - H_o + i\varepsilon} \, (1 - \Pi_o) \, t_j . \tag{12}$$

We can write

$$F = 1 + \frac{1}{E_i - H_o + i\varepsilon} \, (1 - \Pi_o) \, \Sigma_j \, t_j \, F_j \, , \tag{13}$$

with

$$F_j = 1 + \frac{1}{E_i - H_o + i\varepsilon} \, \Sigma_{k(\neq j)} \, (1 - \Pi_o) \, t_k \, F_k \, , \tag{14}$$

so that

$$V \, F = \Sigma_j \, t_j \, F_j ,$$

(15)

and

$$V_{opt} = \langle O|\ \Sigma_j\ t_j\ F_j\ |O\rangle . \tag{16}$$

As it stands the above equation represents an exact result . Even if we approximate F_j by 1 we still have a many-body problem to solve because of the presence of $\frac{1}{E_i - H_o + i\varepsilon}$ which contains the target Hamiltonian.

Remark :

Let us start from

$$V_{opt} \simeq \langle O|\ \Sigma_j\ t_j\ |O\rangle . \tag{17}$$

Written in more detail ,eq.(17) is

$$\langle \vec{k}'|\ V_{opt} |\vec{k}\rangle \simeq \int d\vec{Q}\ \mathrm{Tr}\ \Sigma_j\ \{\Psi_j(-\vec{k}-\vec{Q})\ \Psi_j^{\dagger}(-\vec{k}'-\vec{Q})\quad \langle \vec{k},\vec{k}-Q\ |t|\ \vec{k}',\vec{k}'-\vec{Q}\rangle\} . \tag{18}$$

Neglecting the $\vec{Q}$ dependence of t , we have

$$\langle \vec{k}'|\ V_{opt}\ |\vec{k}\rangle \quad \sim \quad \langle \vec{k}'\ |\ t\ |\vec{k}\rangle\ \ \rho(q) . \tag{19}$$

Here $\rho(q)$ is the Fourier transform of the coordinate-space matter density $\rho(r)$.

Further

$$\vec{q} = \vec{k} - \vec{k}' .$$

Assuming that V_{opt} is local in coordinate space , we find ,

$$V_{opt}(r) = \int dr' \, \rho(r') \, t\,(\vec{r} - \vec{r}'). \tag{20}$$

If we further assume that
we can approximate t ($\vec{r} - \vec{r}'$) by a contact interaction t_o, i.e.,

$$t\,(\vec{r} - \vec{r}') = t_o \, \delta(\vec{r} - \vec{r}'),$$

we have

$$V_{opt}(r) \sim \rho(r) \, t_o. \tag{21}$$

The above relation shows that the shape of optical potential is similar to the shape of the matter density of the target nucleus in the non-relativistic theory. Keeping in mind the approximations we have made , this now brings us to the domain of phenomenology.

1.2. Schrödinger phenomenology :

As long as only elastic scattering can occur ,the optical potential must be real. If inelastic scattering can occur the optical potential must be complex. This follows because the colliding system can leave the elastic channel ; no conservation of probability is required in the elastic channel. The complex central potential used in conventional phenomenology is written as

$$U_c(r) = V_o\, f_R(r) + i\, W_o\, f_I(r), \tag{22}$$

with the Woods- Saxon form factor,

$$f(r) = \frac{1}{1 + \exp\{\,(r - R)/a\,\}} \;.$$

The spin-orbit potential is also complex and is usually written as

$$U_{so}(r) = [\, V_{so}\, g_R + i\, g_I(r)\,]\, \vec{\sigma}.\vec{L} \tag{23}$$

with

$$g(r) = \lambda^2_\pi\, (1/r)\, [\, d/dr]\, f(r) \;. \tag{24}$$

(Here $\vec{\sigma}$ is the Pauli spin operator, $\vec{L}$ is the orbital angular momentum operator and λ_π is the pion Compton wavelength.) These potentials together with the Coulomb potential are inserted into Schrödinger equation which is then solved for phase shifts. Here we have twelve parameters and, in addition to the dependence on r , these potentials also depend on the energy variable [I.5].

1.3. Dirac phenomenology :

In Dirac phenomenology phase shifts are calculated by solving the

Dirac equation :

$$[\ \alpha.p + \beta \{ m_N + U_s(r) \} + U_o(r) + V_c(r)\ ]\ \Psi(r) = E\ \Psi(r)\ , \tag{25}$$

where

$$U_s(r) = V_s\ f_s^R(r) + i\ W_s\ f_s^I(r) \tag{26}$$

and

$$U_o(r) = V_o\ f_o^R(r) + i\ W_o\ f_o^I(r) \tag{27}$$

with

$$f = \frac{1}{1 + \exp\{\ (\ r - R\ )/a\ \}} \tag{28}$$

Thus, if we consider this to be a strictly phenomenological model, we have twelve adjustable parameters, the same number as for Schrodinger phenomenology. Also these potentials are energy dependent in addition to being dependent on r.

Note that a simple Dirac equation reduced to its Schrödinger equivalent form may contain many complicated terms. [Such a reduction is presented in appendix 1] . The remarkable success of Dirac phenomenology in providing excellent fits to the experimental obserables with simple potential forms indicates that the equivalent Schrödinger potentials are rather complicated.

1.4. Non-relativistic microscopic optical potential model

In this section we review microscopic approaches to the calculation of the optical potential. Before proceeding to the Relativistic Brueckner-Hartree-Fock (RBHF) method let us review the non-relativistic method.

Recall the expression for the ground-state energy of a uniform system of fermions in Hartree-Fock approximation :

$$E = \Sigma_{\vec{k}} \langle \vec{k},s \mid T \mid \vec{k},s \rangle$$

$$+ \tfrac{1}{2} \Sigma_{k,k'} \{ \langle \vec{k},s:\vec{k}',s' \mid v \mid \vec{k},s:\vec{k}',s' \rangle - \langle \vec{k},s:\vec{k}',s' \mid v \mid \vec{k}',s': \vec{k},s \rangle \} \qquad (29)$$

Here T is the kinetic energy operator and v represents the two body interaction. Further the self-consistent single-particle wave functions are plane waves. For a system of strongly interacting particles such as nucleons we have to replace v by the effective interaction or the reaction matrix G.

G obeys the equation :

$$G = v + v\,Q \frac{1}{e} G. \qquad (30)$$

Here Q is the Pauli Blocking operator and e is the 'energy denominator'. Thus for the ground-state energy we have the expression

$$E = \Sigma_{\vec{k}} \langle \vec{k} \mid T \mid \vec{k} \rangle +$$

$$(1/2)\ \Sigma_{\vec{k},\vec{k}'}\ \{\ <\vec{k},\vec{k}'|\ G\ |\vec{k},\vec{k}'> - <\vec{k},\vec{k}'|\ G\ |\vec{k}',\vec{k}>\}. \tag{31}$$

The optical potential in the Brueckner - Hartree - Fock approximation is given by

$$U^{BHF} = \Sigma_{\vec{k}'} < \vec{k}\ ,\ \vec{k}'\ |\ G\ |\ \vec{k}\ ,\ \vec{k}' >_A \tag{32}$$

Here the symbol A stands for the sum of direct and exchange contributions.

For details of Brueckner theory of nuclear matter see [I.6.1]. Non-relativistic microscopic optical potential calculations are discussed in [I.6.2] and [I.6.3] .

SECTION 2

Relativistic Microscopic Optical Potential Model

2.1. Self-energy operator for an infinite system :

The first step in formulating a relativistic microscopic optical potential model is to replace the single- particle wave function which is the solution of free Schrödinger equation by its relativistic counterpart , i.e., the solution of free Dirac equation . Thus one can introduce the 'semi- relativistic ' optical potential as

$$U'_{\text{rel.optical}} = [\, m_N/E_N(\vec{k})\,] \; \Sigma_{s'} \int d\vec{q}/((2\pi)^3) \; [m_N/E_N(\vec{q})\,]$$

$$<\bar{u}(\vec{k},s),\bar{u}(\vec{q},s') \mid G \mid u(\vec{k},s),u(\vec{q},s')>_A \; \theta(\, k_f - |\vec{q}|). \tag{33}$$

One can formally introduce the 'semi- relativistic ' self-energy operator $\Sigma^{(1)}$ at this level :

$$U'_{\text{rel.optical}} = [\, m_N/E_N(\vec{k})\,] \; \Sigma^{(1)++}. \tag{34}$$

Also

$$\Sigma^{(1)++}(\vec{k}) = \bar{u}(\vec{k},s) \; \Sigma^{(1)} \; u(\vec{k},s) \tag{35}$$

so that

$$\Sigma_{s'}\int d\vec{q}/((2\pi)^3)\ [\ m_N/E_N(\vec{q})\ ]\ \ <\vec{k},\bar{u}(\vec{q},s')|G|\vec{k},u(\vec{q},s')>_A\ \theta(\ k_f\ -|\vec{q}|).$$

(36)

However , the study of the relativistic nuclear many-body problem has convincingly shown that [I.1.1] the self-energy operator appropriate for the study of infinite nuclear matter is the one given by

$$\Sigma^{RBHF} =$$

$$\Sigma_s\int d\vec{q}/((2\pi)^3)\ [m_N/E_N(\vec{q})]\ <\vec{k},\bar{f}(\vec{q},s)|G^{RBHF}|\vec{k},f(\vec{q},s)>_A\ \theta(k_f\ -|\vec{q}|).$$

(37)

Here $f(\vec{q},s)$ is the appropriate single- particle wave function which satisfies

$$[\ \gamma.p\ -\ m_N\ -\ \Sigma(\{f(\vec{p},s)\},\vec{p})\ ]\ f(\vec{p},s)\ =\ 0.$$

(38)

This equation requires a self-consistent solution.

In these calculations G^{RBHF} is a solution of the following equation :

$$G^{RBHF} = U + U\ \ g_r^{++}\ \ G^{RBHF}.$$

(39)

U is chosen to be one of the many relativistic one-boson-exchange potentials developed by the Bonn group [I.7]. Further , g_r^{++} is the relativistic propagator for intermediate nucleons and includes the Pauli-blocking operator in its definition.

Remark :

In nuclear matter the most general form of self-energy is

$$\Sigma(\vec{p}) = A(\vec{p}) + \gamma^{0} B(\vec{p}) + \vec{\gamma}.\vec{p}\ C(p) \frac{1}{m_N} \tag{40}$$

The self-energy is local in momentum space because of translational invariance in a uniform medium.

2.2. Dispersion relation and effective potential :

To solve the Dirac equation containing the self-energy operator we write,

$$f(\vec{p},s) = a(\vec{p})\ u(\vec{p},s) + b(\vec{p})\ \Sigma_{s'} \langle s'|\vec{\sigma}.\hat{p}|s\rangle\ w(\vec{p},s') \tag{41}$$

where

$$[\ a(\vec{p})]^2 + [\ b(\vec{p})]^2 = 1 \tag{42}$$

Here $w(\vec{p},s) = v(-\vec{p},-s)$ of Bjorken and Drell.

Further we introduce the matrix elements of the self-energy operator in full Dirac basis:

in full Dirac basis:

$$\Sigma^{+\,+}_{s's}(\vec{p}) = \overline{u}(\vec{p},s')\ \Sigma\ u(\vec{p},s)\ , \tag{43}$$

$$\Sigma^{+\,-}_{s's}(\vec{p}) = \overline{u}(\vec{p},s')\ \Sigma\ w(\vec{p},s)\ , \tag{44}$$

$$\Sigma^{-\,+}_{s's}(\vec{p}) = \overline{w}(\vec{p},s')\ \Sigma\ u(\vec{p},s) \quad \text{and} \tag{45}$$

$$\Sigma^{-\,-}_{s's}(\vec{p}) = \overline{w}(\vec{p},s')\ \Sigma\ w(\vec{p},s)\ . \tag{46}$$

Inserting eq.(41) into the Dirac equation , we have

$$\begin{gathered}[\ \{\ E - E_N(\vec{p})\ \} - \gamma^0\ \Sigma\ ]\ u(\vec{p},s)\ a(\vec{p})\ + \\ [\ \gamma^0\ \{\ E + E_N\ \} - \Sigma\ ]\ \Sigma_{s'} <s'|\vec{\sigma}.\hat{p}\ |s>\ w(\vec{p},s')\ b(\vec{p}) = 0.\end{gathered} \tag{47}$$

We now use

$$\Sigma^{+\,-}_{s's} = \delta_{ss'} \qquad (-1)^{\{\frac{1}{2}\ -\ s\}} \qquad \Sigma^{+-} \tag{48}$$

and take $\vec{p}$ along the z axis.

We find

$$[\ E - E_N\ ]\ a = \{\ m_N/E_N\ \}\ [\ \Sigma^{++}\ a + \Sigma^{+-}\ b\ ] \tag{49}$$

and

$$[\ E + E_N\ ]\ b = \{\ m_N/E_N\ \}\ [\ \Sigma^{--}\ b + \Sigma^{-+}\ a\ ]\ . \tag{50}$$

Further, we define

$$\alpha = b/a$$

$$= [\ m_N/E_N\ ]\ \Sigma^{-+}\ \frac{1}{F1}$$

where

$$F1 = E + E_N - \{\ m_N/E_N\ \}\ \Sigma^{--}.$$

Using the above value of α ,

we have

$$E = E_N + \Sigma^{++}\ \{\ m_N/E_N\ \} + \{m_N/E_N\}^2\ \Sigma^{+-}\ \Sigma^{-+}\ \frac{1}{F1}\ . \tag{51}$$

This is an exact result.

Putting $E \simeq E_N + \{\ m_N/E_N\ \}\ \Sigma^{++}$ in the denominator,

we have

$$E = E_N + \{m_N/E_N\}\ \Sigma^{++} + (m_N/E_N)^2\ \Sigma^{+-}\ \Sigma^{-+}\ \frac{1}{F2}\ ,$$

where

$$F2 = 2\ E_N + \{m_N/E_N\}\ [\ \Sigma^{++} - \Sigma^{--}\ ]. \tag{52}$$

Also defining

$$E = E_N + U_{eff}(\vec{p})\ ,$$

We have ,

$$U_{eff}(\vec{p}) =$$

$$m_N/E_N\}\ \Sigma^{++}(p) + \{m_N/E_N\}^2\ \Sigma^{+-}\ \Sigma^{-+}\ \frac{1}{F2}. \tag{53}$$

Remark :

For future purpose it is useful to define an energy variable

$$\hat{\varepsilon}(\vec{p}) = E - m_N = \{\vec{p}^2/2\, m_N\} + U_{eff}(\vec{p}). \tag{54}$$

2.3. Comparison of RBHF with Dirac phenomenology:

We consider the comparison of the self- energy in the RBHF model with the optical potential of Dirac phenomenology.

We recall that in Dirac phenomenology the potential form is often limited to two terms: a (Lorentz) scalar, $U_o(r,E)$ and time-like component of a four-vector, $U_o(r,E)$.

To perform the comparison we have to identify the proper energy variable. The definition $\hat{\varepsilon} = \vec{p}^2/(2\, m_N) + U_{eff}(\vec{p})$ enables this identification to be made. Thus $\hat{\varepsilon}$ is the energy of the projectile nucleon and $\vec{p}$ is the momentum of the nucleon in nuclear matter, or in the nucleus , when the latter is approximated by a uniform system.

Values of A,B,C and U_{eff} are calculated [I.1.1] for a wide range of p { that is from p = .25 fm^{-1} to 3.25 fm^{-1} }. One may choose an incident energy $\tilde{E}$, and put $\hat{\varepsilon} = \tilde{E} = \vec{p}^2/(2m_N) + U_{eff}$. One finds U_{eff} and the corresponding p and then calculates the corresponding A,B and C. Now one can compare A with U_s and B with U_o. Such a comparison is made in [I.2].

The theoretical and phenomenological values of Im A and Im B agree well over the energy domain for which calculations are available, but it is found that the theoretical values of Re A and Re B are only about seventy-five percent of the phenomenological values. Now the question arises as to whether this poses a fundamental problem.

The RBHF self-energy for nuclear matter is local in momentum space, however a finite nucleus is not a uniform system and hence the self-energy operator must be nonlocal in its most general form. That means it is possible that the self-energy operator for a finite system has additional nonlocal terms that vanish in the nuclear-matter limit. On the other hand, phenomenological optical potentials usually contain only local (Lorentz) scalar and vector terms. Thus it seems that to make a fruitful comparison we must calculate the self-energy in its most general form.

SECTION 3

Construction of the Self-Energy Operator for Nucleon-Nucleus Scattering

3.1. A 'field-theoretic model' :

Let us consider the Bethe - Salpeter equation for the scattering of a nucleon from a nucleus. The nucleus is kept in its ground state so that the potential term in the Bethe - Salpeter equation is the optical potential. Let us write

$$M = K + K\,G_F\,M \tag{1}$$

where

M is the generalized relativistic scattering amplitude ,

K is the two-particle kernal ,

and

G_F is the Feynmann propagator for the nucleon and the nucleus (which is in the ground state).

Next we achieve a three dimensional reduction by writing ,

$$M = \Sigma + \Sigma\, g^{(+)}_o\, M \,, \tag{2}$$

$$\Sigma = K + K\,(\,G_F - g^{(+)}_o\,)\,\Sigma \,. \tag{3}$$

Here we have introduced the nucleon self-energy operator Σ and a modified propagator $g_o^{(+)}$. The corresponding wave equation is given in appendix 2.

3.2. General form of the self-energy operator:

Consider the form of self-energy operator appropriate for nucleon-nucleus scattering in a general frame. In such a frame let us denote the initial and final four momenta of the nucleon by p and p' respectively. The initial and final four momenta of the nucleus are denoted by P and P' respectively. Introduce a set of four vectors :

$$W_\mu = (p+P)_\mu = (p'+P')_\mu \tag{4}$$

$$W'_\mu = W_\mu / [\ s^{\frac{1}{2}}\ ] \tag{5}$$

$$\pi'_\mu = (P + P')_\mu / (2\ M_A) \tag{6}$$

$$q_\mu = (p - p')_\mu = (P' - P)_\mu \tag{7}$$

and

$$q'_\mu = q_\mu / m_N . \tag{8}$$

Here

$$s = (p+P)^2 = (p'+P')^2 \text{ is the square of the total energy.} \tag{9}$$

Further m_N is the nucleon mass and M_A is the nuclear mass. In terms of these four vectors we can write

$$
\begin{aligned}
(p',P'|\Sigma|p,P) = \; & a \\
& + b\,\gamma.W' \\
& + c\,\gamma.\pi' \\
& + d\, i\,\gamma.q' \\
& + e\, i\,\sigma^{\mu\nu}\, q'_{\mu}\, \pi'_{\nu} \\
& + f\, i\,\sigma^{\mu\nu}\, q'_{\mu}\, W'_{\nu} \\
& + g\,\sigma^{\mu\nu}\, \pi'_{\mu}\, W'_{\nu} \\
& + h\, i\,\varepsilon^{\mu\nu\rho\sigma}\,\gamma^{5}\, W'_{\mu}\,\gamma_{\nu}\, q'_{\rho}\, \pi'_{\sigma}\; .
\end{aligned}
\qquad (10)
$$

Further, define $\nu = W'.q'$.

We have introduced eight scalar functions a,b,c,d,e,f,g and h in our expression for Σ. These eight scalar functions are functions of s , q^2, $W'.\pi'$ and ν.

Let us discuss the invariance of Σ under the parity operation.

3.3. Parity invariance :

Parity invariance of Σ implies

$$
\begin{aligned}
& \gamma^{0}\,(-\vec{p}\,',-\vec{P}'|\Sigma|-\vec{p},-\vec{P})\,\gamma^{0} \\
& \qquad = (\vec{p}\,',\vec{P}'|\Sigma|\vec{p},\vec{P}).
\end{aligned}
\qquad (11)
$$

Under this parity transformation, $\nu \rightarrow \nu$ and each of the eight terms transforms into itself. Thus we see that the expression for Σ has been chosen so as to be invariant under the parity operation.

Next consider the implication of the hermiticity of V, where V is the optical potential.

3.4. Implication of the hermiticity of V :

We have

$$V = \gamma^{o}\ \Sigma. \tag{12}$$

where V is the "optical potential".

Now $V^{\dagger} = V$ implies

$$\gamma^{o}\ (\ p',P'|\Sigma|p,P\ )^{\dagger}\ \gamma^{o} = (p',P'|\Sigma|p,P). \tag{13}$$

i.e.,

$$\gamma^{o}\ (p,P|\Sigma^{\dagger}|p',P')\ \gamma^{o} = (p',P'|\Sigma|p,P). \tag{14}$$

Under the interchange of initial and final momentum variables, we also have,

$$\nu \rightarrow -\nu\ .$$

Thus hermiticity of V implies that

$$a^{*}(-\nu) = a(\nu),$$

$$b^{*}(-\nu) = b(\nu), \text{ and so on.}$$

Next let us discuss time reversal invariance.

3.5. <u>Time reversal invariance</u> :

Time reversal invariance requires that

$$\begin{aligned}&\bar{u}_{\alpha}(\vec{p}\,',s')\ \Sigma_{\alpha\beta}(\vec{p}\,',\vec{P}\,'|\vec{p},\vec{P})\ u_{\beta}(\vec{p},s)\\&= \eta_{s}^{*}\ \bar{u}_{\alpha}(-\vec{p},-s)\ \Sigma_{\alpha\beta}(-\vec{p},-\vec{P}|-\vec{p}\,',-\vec{P}\,')\\&u_{\beta}(-\vec{p}\,',-s')\ \eta_{s'}\end{aligned} \tag{15}$$

where

η_s and $\eta_{s'}$ are phase factors.

Introducing an operator $C = i\,\gamma^1\,\gamma^3$, we have;

$$\begin{aligned}&\eta_{s'}\ u_{\beta}(-\vec{p}\,',-s') =\\&\bar{u}_{\delta}(\vec{p}\,',s')\ C^{T}{}_{\delta\rho}\ \gamma^{o}{}_{\rho\beta}\end{aligned} \tag{16}$$

and

$$\begin{aligned}&\eta_{s}^{*}\ \bar{u}_{\alpha}(-\vec{p},-s) =\\&\gamma^{o}{}_{\alpha\delta}\ C^{*}{}_{\delta\chi}\ u_{\chi}(\vec{p},s)\ .\end{aligned} \tag{17}$$

Substituting eq. (16) and eq. (17) in eq. (15) , we have ,

$$\bar{u}_\delta(\vec{p}',s')\ \Sigma_{\delta\chi}(\vec{p}',\vec{P}'|\vec{p},\vec{P})\ u_\chi(\vec{p},s)$$

$$= \bar{u}_\delta(\vec{p}',s')\ C^T{}_{\delta\rho}\ \gamma^o{}_{\rho\beta}$$

$$\Sigma_{\alpha\beta}(-\vec{p},-\vec{P}|-\vec{p}',-\vec{P}')$$

$$\gamma^o{}_{\alpha\delta}\ C^*{}_{\delta\chi}\ u_\chi(\vec{p},s)\ , \tag{18}$$

from which it follows that

$$\Sigma_{\delta\chi}(\vec{p}',\vec{P}'|\vec{p},\vec{P}) =$$

$$C_{\delta\rho}\ \Sigma^T{}_{\rho\mu}(\vec{p},\vec{P}|\vec{p}',\vec{P}')\ C_{\mu\chi}\ . \tag{19}$$

Under the interchange of momentum variables

$$\nu \to -\nu.$$

Thus we arrive at

$$a(-\nu) = a(\nu),$$

$$b(-\nu) = b(\nu),$$

$$c(-\nu) = c(\nu),$$

$$d(-\nu) = -d(\nu),$$

$$e(-\nu) = e(\nu),$$

$$f(-\nu) = f(\nu),$$

$g(-\nu) = -g(\nu)$ and

$h(-\nu) = h(\nu)$.

Remark : Time reversal invariance together with hermiticity of V implies that

$a^*(\nu) = a(-\nu) \Rightarrow$ a is real.

$b^*(\nu) = b(\nu) \Rightarrow$ b is real.

$c^*(\nu) = c(\nu) \Rightarrow$ c is real.

$d^*(\nu) = -d(\nu) \Rightarrow$ d is imaginary.

$e^*(\nu) = e(\nu) \Rightarrow$ e is real.

$f^*(\nu) = f(\nu) \Rightarrow$ f is real.

$g^*(\nu) = -g(\nu) \Rightarrow$ g is imaginary.

$h^*(\nu) = h(\nu) \Rightarrow$ h is real.

3.6. The self-energy operator in the nucleon-nucleus center-of-momentum frame :

Now we specialize to the nucleon - nucleus center-of-momentum frame.

In this frame

$$W'^{\mu} = (1,0). \tag{20}$$

$$\pi'^{\mu} = [E_A(\vec{k}') + E_A(\vec{k}) , - (\vec{k}+\vec{k}')]/ 2 M_A. \tag{21}$$

$$q'^{\mu} = [E_A(\vec{k}') - E_A(\vec{k}) , \vec{k}-\vec{k}']/ m_N. \tag{22}$$

Then

$$\gamma.W' \to \gamma^o, \tag{23}$$

$$\gamma.\pi' \to [\gamma^o\{E_A(\vec{k}') + E_A(\vec{k})\} + \vec{\gamma}.(\vec{k}+\vec{k}')]/ 2 M_A, \tag{24}$$

$$i \gamma.q' \to i[\gamma^o\{E_A(\vec{k}') - E_A(\vec{k})\} - \vec{\gamma}.(\vec{k}-\vec{k}')]/m_N , \tag{25}$$

$$i \sigma^{\mu\nu} q'_{\mu} \pi'_{\nu} \to$$

$$- [\vec{\gamma}.\vec{k}' \gamma^o E_A(\vec{k}) + \gamma^o\vec{\gamma}.\vec{k}E_A(\vec{k}') + \vec{k}.\vec{k}' + \vec{\gamma}.\vec{k}'\vec{\gamma}.\vec{k}]/M_A m_N , \tag{26}$$

$$i\sigma^{\mu\nu}\, q'_{\mu} W'_{\nu} \rightarrow$$

$$- [\, \vec{\gamma}.\vec{k}'\, \gamma^{o} + \gamma^{o}\, \vec{\gamma}.\vec{k}\,]/m_{N} \;, \qquad (27)$$

$$i\sigma^{\mu\nu}\, \pi'_{\mu} W'_{\nu} \rightarrow$$

$$- [\, \vec{\gamma}.\vec{k}'\, \gamma^{o} - \gamma^{o}\, \vec{\gamma}.\vec{k}\,]/2\, M_{A} \;, \qquad (28)$$

$$i\, \gamma^{5}\, \varepsilon^{\mu\nu\rho\sigma}\, W'_{\mu}\, \gamma_{\nu}\, q'_{\rho}\, \pi'_{\sigma} \rightarrow$$

$$- i\, \gamma^{o} \vec{\Sigma}.(\vec{k}' \times \vec{k})/(M_{A} m_{N}). \qquad (29)$$

Now introduce a set of eight scalar functions A,B,C,D,E,F,G and H:

$$A = a, \qquad (30)$$

$$B = b + c\, [E_{A}(\vec{k}') + E_{A}(\vec{k})]/2\, M_{A} + i\, d\, [\, E_{A}(\vec{k}') - E_{A}(\vec{k})\,]/m_{N}, \qquad (31)$$

$$C = [m_{N}/M_{A}]\; c, \qquad (32)$$

$$D = d, \tag{33}$$

$$E = -2f - e\ [\ E_A(\vec{k}') + E_A(\vec{k})\ ]/M_A, \tag{34}$$

$$F = [\ m_N/M_A]\ g - i\ e\ [E_A(\vec{k}') - E_A(\vec{k})]/M_A, \tag{35}$$

$$G = [\ m_N/M_A\ ]\ e\ , \tag{36}$$

and

$$H = [\ m_N/M_A\ ]\ h\ . \tag{37}$$

Here $E_A(k) = (\vec{k}^2 + M_A^2)^{\frac{1}{2}}$.

Now we can write

$$< p', P' | \Sigma(W) | p, P> \rightarrow \quad <\vec{k}' | \Sigma(W) | \vec{k}> \ , \tag{38}$$

where

$$<\vec{k}' | \Sigma(W) | \vec{k}> = A$$

$$+ \gamma^o B$$

$$+ \vec{\gamma} . (\ \vec{k} + \vec{k}')\ C/(2\ m_N)$$

$$+ i \vec{\gamma}.(\vec{k}'-\vec{k})\ D/m_N$$

$$- \gamma^o \vec{\gamma}.(\vec{k}' - \vec{k})\ E/(2\ m_N)$$

$$- i \gamma^o \vec{\gamma}.(\vec{k}' + \vec{k})\ F/(2\ m_N)$$

$$+ i \vec{\Sigma}.(\vec{k}' \times \vec{k})\ G/m_N^2$$

$$- i \gamma^o \vec{\Sigma}.(\vec{k}' \times \vec{k})\ H/m_N^2. \quad (39)$$

Now hermiticity of V and time reversal invariance imply that A,B,C,E,G and H are real whereas D and F are imaginary.

Remark :

We consider the nuclear matter limit and put $\vec{k} = \vec{k}'$ so that $\nu = 0$.

Then

$$\Sigma = A + \gamma^o B + \vec{\gamma}.\vec{k}\ C/m_N + i \gamma^o \vec{\gamma}.\vec{k}\ F/m_N \quad (40)$$

or

$$V = \gamma^o A + B + \gamma^o \vec{\gamma}.\vec{k}\ C/m_N + i \vec{\gamma}.\vec{k}\ F/m_N. \quad (41)$$

Now

$$V^{\dagger} = \gamma^{0} A + B + \gamma^{0} \vec{\gamma}.\vec{k}\ C/m_N + i\ \vec{\gamma}.\vec{k}\ F/m_N . \tag{42}$$

$$= V\ , \quad \text{if } F^{*}(0) = F(0).$$

$F = m_N/m_A \ \ g$, when $\vec{k}=\vec{k}'$. But when $\vec{k}=\vec{k}'$, time reversal invariance implies that $g(0) = -\ g(-0)$ which means that $g=0$.

Thus we should put $F = 0$ in nuclear matter.

In nuclear matter self-energy has the form

$$\Sigma(\vec{k}) = A(\vec{k}) + \gamma^{0} B(\vec{k}) + \vec{\gamma}.\vec{k}\ C(\vec{k})/m_N . \tag{43}$$

This form of self- energy has been considered in the study of nuclear matter [I.1.1] .

SECTION 4

Microscopic Model for the Relativistic Optical Potential

In this section we discuss the model used for the calculation of the eight scalar invariants.

4.1. Direct and exchange terms in the Born approximation :

For the scattering of a spin - half nucleon from a spin - zero nucleus the calculation of microscopic optical potential in Born Approximation is represented by diagrams 1 and 2.

Here diagram 1 corresponds to the direct term and diagram 2 corresponds to the exchange term.

In these diagrams thin solid lines represent a nucleon, wavy lines represent an exchanged meson and crosses indicate particles put on mass shell.

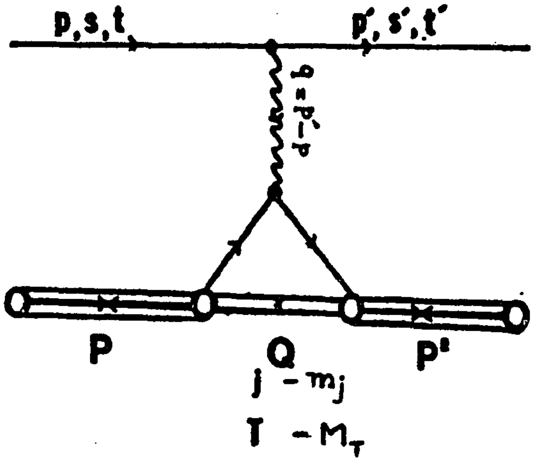


diagram 1

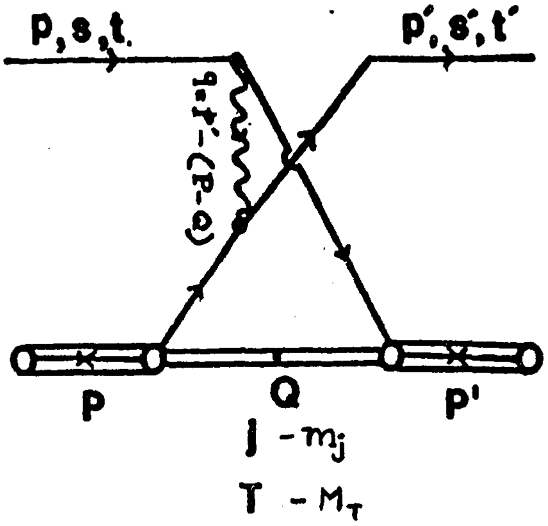


diagram 2

Here

P : Four momentum of the target before scattering.

P': Four momentum of the target after scattering.

Q : Four momentum of the spectator nucleus.

p, s, t : Four momentum , spin projection and isospin projection of the incident nucleon.

p', s', t' : Four momentum , spin projection and isospin projection of the scattered nucleon.

j, m_j , T, M_T : Angular momentum , angular momentum projection, isospin and isospin projection quantum numbers of the struck nucleon.

Let us approximate the nucleon-nucleus scattering amplitude by the exchange of four meson fields :

1) σ meson {scalar , isoscalar (i.e., spin zero , isospin zero) },

2) ω meson {vector , isoscalar (i.e., spin one , isospin zero) },

3) π meson {pseudoscalar , isovector (i.e., spin zero , isospin one)},

and

4) ρ meson {vector , isovector (i.e., spin one , isospin one)} .

Let m_A denote the mass of the exchanged meson.

Let P_A denote the appropriate propagator.

Let Γ^A denote the isospin-independent part of the nucleon - meson vertex.

Introduce I^A such that

$I^A = 1$ for isoscalar exchanges and

$I^A = \tau$ for isovector exchanges.

Introduce the two-component isospinor χ ,

the four-component Dirac spinor u,

and the four-component bound state wave function Ψ_{jm_j} .

Then the expression corresponding to diagram 1 is given by

$$
\begin{array}{l}
\Sigma^{+\,+}_{s's}(\vec{p}\,',\vec{p})_{\text{Direct}} = \\[6pt]
\bar{u}^{\frac{1}{2}}_{s'}(\vec{p}\,')_{\alpha'} \quad \Gamma^{A}(\vec{p}\,'-\vec{p})_{\alpha'\alpha} \quad u^{\frac{1}{2}}_{s}(\vec{p})_{\alpha} \\[6pt]
\chi^{\dagger}{}_{t'\rho'}\ I^{A}{}_{\rho'\rho}\ \chi_{t\ \rho} \\[6pt]
\Sigma_{jm_j} \quad \Sigma_{M_T} \int d\vec{Q}/\{(2\pi)^3\} \\[6pt]
[\ \bar{\Psi}_{jm_j}(\vec{P}\,'-\vec{Q},\vec{Q})_{\alpha'''} \quad P_A \quad \Gamma^{A}(\vec{P}\,'-\vec{P})_{\alpha'''\alpha''} \quad \Psi_{jm_j}(\vec{P}-\vec{Q},\vec{Q})_{\alpha''}\ ] \\[6pt]
\chi^{\dagger}_{M_T\ \rho'''}\ I^{A}{}_{\rho'''\ \rho''}\ \chi_{M_T\ \rho''}.
\end{array}
\tag{1}
$$

The expression corresponding to diagram 2 is given by

$$
\begin{array}{l}
\Sigma^{+\,+}_{s's}(\vec{p}\,',\vec{p})_{\text{Exchange}} = \\[6pt]
-\ \bar{u}^{\frac{1}{2}}_{s'}(\vec{p}\,')_{\alpha'} \quad \Sigma_{jm_j}\Sigma_{M_T}[\int d\vec{Q}/\{(2\pi)^3\} \quad \Gamma^{A}(-\vec{P}+\vec{Q}+\vec{p}\,')_{\alpha'\alpha''} \\[12pt]
\{\Psi_{jm_j}(\vec{P}-\vec{Q},\vec{Q})_{\alpha''}\}\{\chi^{\dagger}_{t'\ \rho'}\}\{I^{A}{}_{\rho'\rho''}\}\{\chi_{M_T\rho''}\}\{\bar{\Psi}_{jm_j}(\vec{P}\,'-\vec{Q},\vec{Q})_{\alpha'''}\} \\[12pt]
\Gamma^{A}(-\vec{p}+\vec{P}\,'-\vec{Q})_{\alpha'''\alpha}P_A\ ] \quad u^{\frac{1}{2}}_{s}(\vec{p})_{\alpha} \\[12pt]
\chi^{\dagger}_{M_T\ \rho'''} \quad I^{A}{}_{\rho'''\rho}\ \chi_{t\ \rho}\ .
\end{array}
\tag{2}
$$

Note: In nucleon-nucleus center-of-momentum frame the argument of Ψ_{jm_j} is $\vec{Q}$ - {(A-1)/A } $\vec{k}$ and the argument of $\overline{\Psi}_{jm_j}$ is $\vec{Q}$ - { (A-1)/A } $\vec{k}'$.

For the exchange of isoscalar particles we have

$$I^{A}{}_{\alpha\beta} = \delta_{\alpha\beta}. \tag{3}$$

We have

$$\chi^{\dagger}_{t'\ \rho'}\ I^{A}{}_{\rho'\rho}\ \chi_{t\ \rho}$$

$$= \delta_{t't}. \tag{4}$$

Further

$$\Sigma_{M_T}\ \chi^{\dagger}_{M_T\rho'''}\ I^{A}{}_{\rho'''\rho''}\ \chi_{M_T\ \rho''} = 2. \tag{5}$$

Then

$$\Sigma^{+\ +}_{s's}(\vec{p}',\vec{p})_{\text{Direct}} =$$

$$\overline{u}^{\frac{1}{2}}_{s'}(\vec{p}')\ \Gamma^{A}(\vec{p}'-\vec{p})\ u^{\frac{1}{2}}_{s}(\vec{p})$$

$$2\ \Sigma_{jm_j}[\int d\vec{Q}/\{(2\pi)^3\}\ \ \overline{\Psi}_{jm_j}(\vec{P}'-\vec{Q},\vec{Q})\quad P_A\ \ \Gamma^{A}(\vec{P}'-\vec{P})\quad \Psi_{jm_j}(\vec{P}-\vec{Q},\vec{Q})\ ]. \tag{6}$$

Introduce the density matrix of the target.(See appendix 3.)

$$\rho_j(\vec{Q}',\vec{Q}'') = \Sigma_{m_j}\quad \Psi_{jm_j}(\vec{P}-\vec{Q},\vec{Q})\quad \overline{\Psi}_{jm_j}(\vec{P}'-\vec{Q},\vec{Q}).$$

(7)

Also define $\rho = \Sigma_j \rho_j$.

Further recall the definition of $\Sigma^{+\ +}_{s's}(\vec{p}',\vec{p})$:

$$\Sigma^{+\ +}_{s's}(\vec{p}',\vec{p}) = \bar{u}^{\frac{1}{2}}_{s'}(\vec{p}')\ \Sigma(\vec{p}',\vec{p})\ u^{\frac{1}{2}}_{s}(\vec{p}).$$

(8)

Thus for the exchange of an isoscalar meson , for the direct diagram

$$\Sigma =$$

$$\Gamma^A(\vec{p}'-\vec{p})\ 2\ \Sigma_J[\int d\vec{Q}/\{(2\pi)^3\}\ \frac{\mathrm{Tr}}{4}\ \{\rho_J(\vec{Q}',\vec{Q}'')\ \Gamma^A(\vec{P}'-\vec{P})\ \}\ P_A\ ] .$$

(9)

For the exchange diagram we have

$$\Sigma_{M_T}\ \chi^{\dagger}_{t'\ \rho'}\ \delta_{\rho'\rho''}\ \chi_{M_T\ \rho'''}$$

$$\chi^{\dagger}_{M_T\rho'''}\ \delta_{\rho'''\rho}\ \chi_{t\ \rho}$$

$$= \delta_{t't}.$$

(10)

Then

$$\Sigma^{+\ +}_{s's}(\vec{p}',\vec{p})_{\mathrm{Exchange}} =$$

$$- \bar{u}^{\frac{1}{2}}_{s'}(p')_{\alpha'} \quad \Sigma_{jm_j} \int [\; d\vec{Q}/\{(2\pi)^3\}$$

$$\Gamma^A(-\vec{P}+\vec{Q}+\vec{p}\,')_{\alpha'\alpha''} \quad \Psi_{jm_j}(\vec{P}-\vec{Q},\vec{Q})_{\alpha''} \quad P_A \quad \bar{\Psi}_{jm_j}(\vec{P}'-\vec{Q},\vec{Q})_{\alpha''}$$

$$\Gamma^A(-\vec{p}+\vec{P}'-\vec{Q})_{\alpha''\alpha} \; u^{\frac{1}{2}}_{s}(p)_{\alpha} \;] \; . \tag{11}$$

Thus for the exchange of an isoscalar meson for the exchange diagram

$$\Sigma =$$

$$- \Sigma_J \; [\; \int d\vec{Q}/\{(2\pi)^3\} \quad \Gamma^A(-\vec{P}+\vec{Q}+\vec{p}\,')$$

$$\rho_J(\vec{Q}',\vec{Q}'') \; \Gamma^A(-\vec{p}+\vec{P}'-\vec{Q}) \; P_A \;] . \tag{12}$$

For isovector particle exchange

$$\chi^{\dagger}_{t'\;\rho'} \; I^A{}_{\rho'\rho} \; \chi_{t\;\rho}$$

$$\Sigma_{M_T} \; \chi^{\dagger}_{M_T\;\rho'''} \; I^A{}_{\rho'''\;\rho''} \; \chi_{M_T\;\rho''}$$

$$= \chi^{\dagger}_{t'} \; \tau^i \; \chi_t \quad \Sigma_{M_T} \; \chi^{+}_{M_T} \; \tau^i \; \chi_{M_T} . \tag{13}$$

$$= 0 \ . \tag{14}$$

Thus for the exchange of an isovector meson for the direct diagram

$$\Sigma = 0 \ . \tag{15}$$

For the exchange diagram

$$\Sigma_{M_T} \chi^{\dagger}_{t'} \tau^{i} \chi_{M_T} \chi^{\dagger}_{M_T} \tau^{i} \chi_{t}$$

$$= \chi^{\dagger}_{t'} \tau^{i} \tau^{i} \chi_{t} \tag{16}$$

$$= 3 \quad \delta_{t't} . \tag{17}$$

Thus

$$\Sigma^{+\ +}_{s's}(\vec{p}\,',\vec{p})_{\text{Exchange}} =$$

$$-3 \ \bar{u}^{\frac{1}{2}}_{s'}(\vec{p}\,') \qquad \Sigma_j \qquad \int d\vec{Q}/\{(2\pi)^3\}$$

$$[\ \Gamma^{A}(-\vec{P}+\vec{Q}+\vec{p}\,') \quad \rho_j(\vec{Q}\,',\vec{Q}\,'') \quad \Gamma^{A}(-\vec{p}+\vec{P}\,'-\vec{Q}) \quad P_A \quad u^{\frac{1}{2}}_{s}(\vec{p}) \ ]$$

$$(18)$$

i.e.,

for the exchange of an isovector meson , for the exchange diagram

$$\Sigma =$$

$$-3\ \Sigma_j \int d\vec{Q}/\{(2\pi)^3\}$$

$$[\ \Gamma^A(-\vec{P}+\vec{Q}+\vec{p}') \qquad \rho_j(\vec{Q}',\vec{Q}'') \qquad \Gamma^A(-\vec{p}+\vec{P}'-\vec{Q}) \qquad P_A\ ]\ . \tag{19}$$

4.2. Relativistic OBEP { HEA potential}:

The masses , coupling constants etc. of the potential [I.7.3] are presented in the following table.

TABLE I.1

Meson	Mass (MeV)	Isospin T	Spin Parity J^{π}	Coupling constant $g^2/4\pi$	f/g	Propagator P_A
σ	500.0	0	0^+	4.63	0	$(\Delta^2-m^2_{\sigma})^{-1}$
ω	782.8	0	1^-	14.0	0	$-g_{\mu\nu} \times (\Delta^2-m^2_{\omega})^{-1}$
π	138.5	1	0^-	13.0	0	$(\Delta^2-m^2_{\pi})^{-1}$
ρ	763.0	1	1^-	1.50	3.5	$-g_{\mu\nu} \times (\Delta^2-m^2_{\rho})^{-1}$

In OBEP models form factors are generally introduced to avoid divergences. For simplicity the following form factors are used in the potential HEA.

For scalar and pseudoscalar mesons use the cut off factor:

$$[\lambda^2/\{\lambda^2-\Delta^2\}]^2.$$

For the ω meson use the cut off factor :

$$[\lambda^2/\{\lambda^2-\Delta^2\}]^2 \qquad [m^2_\omega+\lambda^2_v]/[-\Delta^2+\lambda^2_v].$$

For the ρ meson use the cut off factor:

$$[\lambda^2/\{\lambda^2-\Delta^2\}]^2 \qquad [m^2_\rho+\lambda^2_v]/[-\Delta^2+\lambda^2_v].$$

Here Δ^2 is the square of the four - momentum transfer.

Further

$$\lambda = 1950 \text{ MeV}$$

and

$$\lambda_v = 1250 \text{ MeV}.$$

4.3. Scalar meson exchange :

For the exchange of a scalar meson we have the following expression for the self-energy operator :

$$<\vec{k}'|\Sigma|\vec{k}> =$$

$$\Gamma^A \{g_\sigma{}^2/[\ q^2 - m^2{}_\sigma\}\ ]\ F^2{}_\sigma\ 2\ \{Tr/4\}\{\int d\vec{Q}/[(2\pi)^3]\ \rho\ \Gamma^A\} \tag{20}$$

Here q is the 4-momentum carried by the exchanged meson.

Thus

$$<\vec{k}'|\Sigma|\vec{k}>^\sigma_{Direct} =$$

$$1\ \{g^2{}_\sigma/[\ q^2 - m^2{}_\sigma]\}\ F^2{}_\sigma(q^2)\ 2\ \{Tr/4\}\int d\vec{Q}/[(2\pi)^3]\rho 1 \tag{21}$$

$$= \{\ g^2{}_\sigma/[q^2 - m^2{}_\sigma]\}\ F^2{}_\sigma(q^2)\ 2\int d\vec{Q}/[(2\pi)^3]\ \rho_s \tag{22}$$

ρ_s is defined in the appendix 3.

Taking traces

$$A^\sigma_{Direct} =$$

$$\{\ g^2{}_\sigma\ /[q^2 - m^2{}_\sigma]\}\ F^2{}_\sigma\ 2\int d\vec{Q}/[(2\pi)^3]\ \ \rho_s\ , \tag{23}$$

$$B^\sigma_{Direct} = 0\ , \tag{24}$$

$$C^{\sigma}_{\mathrm{Direct}} = 0 , \tag{25}$$

$$D^{\sigma}_{\mathrm{Direct}} = 0 , \tag{26}$$

$$E^{\sigma}_{\mathrm{Direct}} = 0 , \tag{27}$$

$$F^{\sigma}_{\mathrm{Direct}} = 0 , \tag{28}$$

$$G^{\sigma}_{\mathrm{Direct}} = 0 , \tag{29}$$

and

$$H^{\sigma}_{\mathrm{Direct}} = 0. \tag{30}$$

Next consider the σ contribution to exchange diagram.

We have

$$\langle \vec{k}' | \Sigma | \vec{k} \rangle^{\sigma}_{\mathrm{Exchange}} =$$

$$- \; g^{2}{}_{\sigma} \; [\int \{ d\vec{Q}/(2\pi)^{3} \} \;\; F^{2}{}_{\sigma}(q^{2}) \;\; \{ 1/(q^{2} - m^{2}{}_{\sigma}) \} \; \rho \tag{31}$$

Here q represents the momentum carried by the meson in the exchange diagram.

Taking traces and refering to appendix 3 ,

$$A^{\sigma}_{\text{Exchange}} =$$

$$- g^{2}_{\sigma} \int [\ \{d\vec{Q}/(2\pi)^{3}\}\ \{1/(q^{2} - m^{2}_{\sigma})\}\ F^{2}_{\sigma}(q^{2})\ \rho_{s}\ ]. \tag{32}$$

$$B^{\sigma}_{\text{Exchange}} =$$

$$- g^{2}_{\sigma}\ [\int \{d\vec{Q}/(2\pi)^{3}\}\ \{1/(q^{2} - m^{2}_{\sigma})\}\ F^{2}_{\sigma}(q^{2})\ \rho^{o}_{V}\ ]\ . \tag{33}$$

With the definitions ,

$$I_{1} =$$

$$g^{2}_{\sigma}\ [\int \{d\vec{Q}/(2\pi)^{3}\}\ \{1/(q^{2} - m^{2}_{\sigma})\}\ F^{2}_{\sigma}(q^{2})\ \hat{k}.\vec{\rho}_{V}\ ] \tag{34}$$

and

$$I_{2} =$$

$$g^{2}_{\sigma}\ [\int \{d\vec{Q}/(2\pi)^{3}\}\ \{1/(q^{2} - m^{2}_{\sigma})\}\ F^{2}_{\sigma}(q^{2})\ \hat{k}'.\vec{\rho}_{V}\ ]\ ,$$

we have ,

$$C^{\sigma}_{\text{Exchange}} =$$

$$m_{N}\ [1/(kk'\sin^{2}\vartheta)]\ [\ (k'-\hat{k}.\hat{k}'\ k)\ I_{1} + (k-\hat{k}.\hat{k}'\ k')\ I_{2}\ ] \tag{35}$$

and

$$i\ D^{\sigma}_{\text{Exchange}} =$$

$$(1/2)m_N\ [1/(kk'\sin^2\vartheta)]\ [\ (k'+\hat{k}.\hat{k}'\ k)\ I_1\ -\ (k+\hat{k}.\hat{k}'\ k')\ I_2\ ].$$

(36)

With

$$I_3 =$$

$$(\tfrac{1}{2})\ \ g^2{}_\sigma\ [\int\{d\vec{Q}/(2\pi)^3\}\ \{1/(q^2\ -\ m^2{}_\sigma)\}\ F^2{}_\sigma(q^2)\ \hat{k}'.\vec{f}^{To}\ ]\ ,$$

(37)

and

$$I_4 =$$

$$(\tfrac{1}{2})\ \ g^2{}_\sigma\ [\int\{d\vec{Q}/(2\pi)^3\}\ \{1/(q^2\ -\ m^2{}_\sigma)\}\ F^2{}_\sigma(q^2)\ \hat{k}.\vec{f}^{To}\ ]\ ,$$

(38)

we have

$$E^\sigma_{\text{Exchange}} =$$

$$m_N\ [1/(kk'\sin^2\vartheta)]\ [\ -(k'+\hat{k}.\hat{k}'\ k)\ I_4\ +\ (k+\hat{k}.\hat{k}'\ k')\ I_3\ ]\ ,$$

(39)

$$i\ F^\sigma_{\text{Exchange}} =$$

$$m_N\ [1/(kk'\sin^2\vartheta)]\ [\ (k'-\hat{k}.\hat{k}'\ k)\ I_4\ +\ (k-\hat{k}.\hat{k}'\ k')\ I_3\ ]\ ,$$

(40)

$$G^\sigma_{\text{Exchange}} =$$

$$-m_N^2\ [1/(kk'\sin^2\vartheta)]\ g^2{}_\sigma\int[\{d\vec{Q}/(2\pi)^3\}\ \{1/(q^2\ -\ m^2{}_\sigma)\}\ F^2{}_\sigma(q^2)$$

$$(1/2)\ f^T(\ \hat{k}'.\hat{Q}'\ \hat{k}.\hat{Q}''\ -\ \hat{k}'.\hat{Q}''\hat{k}.\hat{Q}')\ ],$$

(41)

and

$$H^{\sigma}_{\mathrm{Exchange}} =$$

$$m_N^2\ [1/(kk'\sin^2\vartheta)]\ g^2{}_{\sigma}\int [\{d\vec{Q}/(2\pi)^3\}\ \{1/(q^2 - m^2{}_{\sigma})\}\ F^2{}_{\sigma}(q^2)$$

$$(1/2)\ f^{Av}(\ \hat{k}'.\hat{Q}'\ \hat{k}.\hat{Q}'' - \hat{k}'.\hat{Q}''\hat{k}.\hat{Q}')\ ].$$

(42)

4.4. Vector meson exchange:

First consider the direct diagram.

$$B^{\omega}_{\mathrm{Direct}} =$$

$$- g^2{}_{\omega}\{1/(q^2 - m^2{}_{\omega})\}\ F^2{}_{\omega}(q^2)[\int d\vec{Q}/\{(2\pi)^3\}\ 2\ \rho^{o}_{V}]$$

(43)

Since

$$\int d\vec{Q}\ \vec{\rho}_V = 0,$$

(44)

$$C^{\omega}_{\mathrm{Direct}} = 0,$$

(45)

and

$$D^{\omega}_{\mathrm{Direct}} = 0\ .$$

(46)

Further

$$A^{\omega}_{\mathrm{Direct}} = 0,$$

(47)

$$E^{\omega}_{\mathrm{Direct}} = 0,$$

(48)

$$F^{\omega}_{\mathrm{Direct}} = 0 \;,$$

(49)

$$G^{\omega}_{\mathrm{Direct}} = 0 \;,$$

(50)

and

$$H^{\omega}_{\mathrm{Direct}} = 0 \;.$$

(51)

Exchange contribution:

$$\Sigma^{\omega}_{\mathrm{Exchange}} =$$

$$(-1) (-) \; g^{2}_{\omega} \int d\vec{Q}/\{(2\pi)^{3}\} \; F^{2}_{\omega}(q^{2}) \; \{1/(q^{2} - m^{2}_{\omega})\}$$

$$\times \; \gamma_{\delta}[\; \rho_{s} + \gamma^{\mu}\rho_{v_{\mu}} + \sigma^{\mu\nu}\rho_{T_{\mu\nu}} + \gamma^{5}\gamma^{\mu}\rho_{av_{\mu}}] \; \gamma^{\delta}$$

(52)

Again, by taking traces, we calculate

$A^{\omega}_{\text{Exchange}}$,

$B^{\omega}_{\text{Exchange}}$, etc..

4.5. Pion exchange :

Since the pion carries isospin , evaluation of the direct diagram in our model yields zero. For the exchange diagram we have

$$\langle k' | \Sigma | k \rangle^{\pi}_{\text{exchange}} =$$

$$3/4 \; [\, g^2{}_{\pi} / \, m^2_N] \; [\{dQ/(2\pi)^3\} \; F^2{}_{\pi}(q^2) \; \{1/(q^2 - m^2{}_{\pi})\}$$

$$\times \, \gamma^5 \not{q}_1 \, \rho \quad \gamma^5(-\not{q}_1)] \tag{53}$$

Here $q_1 = P' - q - p$. (54)

Again taking traces we calculate

$A^{\pi}_{\text{exchange}}$,

$B^{\pi}_{\text{exchange}}$, etc..

Note : Here we have used pseudovector coupling for pion.

4.6. Rho exchange:

Again in this case the direct diagram yields zero.

For the exchange diagram we have

$$<\vec{k}'|\Sigma|\vec{k}>^{\rho}_{\text{exchange}} =$$

$$(-1)\ 3\ (-1)\ g^2_{\rho}\ [\int\{d\vec{Q}/(2\pi)^3\}\ F^2_{\rho}(q^2)\ \{1/(q^2 - m^2_{\rho})\}$$

$$\times\ J_{\mu}(p-P'+Q)\quad \rho \quad J^{\mu}(P'-Q-p)\ ]\ , \tag{55}$$

where

$$J_{\mu}(q) = \gamma^{\mu} + \{\ i\ f_{\rho}/2\ m_N\ g_{\rho}\}\ \sigma^{\mu\nu}\ q_{\nu}, \tag{56}$$

$$= \gamma^{\mu} - \alpha\ [\ \gamma^{\mu}\ \not{q}\ -\ \not{q}\ \gamma^{\mu}\ ], \tag{57}$$

and

$$\alpha = \{\ f_{\rho}/\ 4\ m_N\ g_{\rho}\ \}. \tag{58}$$

Again taking traces we calculate

$A^{\rho}_{\text{exchange}}$,

$B^{\rho}_{\text{exchange}}$ etc..

SECTION 5

Comparison with Dirac Phenomenology:

We perform calculations in the nucleon-nucleus center-of-mass frame. Since a large amount of data is available for proton- ^{40}Ca scattering at low energies we choose ^{40}Ca as the target nucleus.

By solving the Dirac equation with local (Lorentz) scalar and vector potentials we get a set of relativistic wave functions for the target nucleus. To be consistent, to get the wave functions to be used in this calculation one must solve the Dirac equation containing the nonlocal self- energy operator which has eight scalar invariants. Instead we have adjusted the parameters of our local scalar and vector potentials so as to yield the experimental charge radius and single nucleon separation energies.

5.1 Matrix elements of the self-energy operator in Dirac basis :

Since a direct comparison with phenomenology is not quite meaningful we propose the following method for the detailed comparison. First introduce the matrix elements of the self-energy operator Σ in the Dirac basis.

$$<\vec{k}',s'|\Sigma^{++}|\vec{k},s> = \bar{u}(\vec{k}',s') <\vec{k}'|\Sigma|\vec{k}> u(\vec{k},s) \quad , \tag{1}$$

$$<\vec{k}',s'|\Sigma^{+-}|\vec{k},s> = \bar{u}(\vec{k}',s')\ <\vec{k}'|\Sigma|\vec{k}>\ w(\vec{k},s) \quad , \tag{2}$$

$$<\vec{k}',s'|\Sigma^{-+}|\vec{k},s> = \bar{w}(\vec{k}',s')\ <\vec{k}'|\Sigma|\vec{k}>\ u(\vec{k},s) \quad , \tag{3}$$

$$<\vec{k}',s'|\Sigma^{--}|\vec{k},s> = \bar{w}(\vec{k}',s')\ <\vec{k}'|\Sigma|\vec{k}>\ w(\vec{k},s) \quad . \tag{4}$$

These are natural extensions of matrix elements defined in the study of nuclear matter (RBHF).

Next define 8 amplitudes S_1^{++}, S_2^{++}, etc. by

$$<\vec{k}',s'|\Sigma^{++}|\vec{k},s> = <s'|\{\ S_1^{++} + i\ \vec{\sigma}.(\vec{k}'\times\vec{k})/(\varepsilon'\varepsilon)S_2^{++}\ \}\ |s>, \tag{5}$$

$$<\vec{k}',s'|\Sigma^{--}|\vec{k},s> = <s'|\{\ S_1^{--} + i\ \vec{\sigma}.(\vec{k}'\times\vec{k})/(\varepsilon'\varepsilon)S_2^{--}\ \}\ |s>, \tag{6}$$

$$<\vec{k}',s'|\Sigma^{+-}|\vec{k},s> = <s'|\{\ \vec{\sigma}.\ (\vec{k}/\varepsilon+\vec{k}'/\varepsilon')\ S_1^{+-} + \vec{\sigma}.\ (\vec{k}'/\varepsilon'-\vec{k}/\varepsilon)\ S_2^{+-}\}|s>, \tag{7}$$

$$<\vec{k}',s'|\Sigma^{-+}|\vec{k},s> = <s'|\{\ \vec{\sigma}.\ (\vec{k}/\varepsilon+\vec{k}'/\varepsilon')\ S_1^{-+} + \vec{\sigma}.\ (\vec{k}'/\varepsilon'-\vec{k}/\varepsilon)\ S_2^{-+}\}|s>\ , \tag{8}$$

Where ,

$$\varepsilon = E_N + m_N = [\ m_N^2 + \vec{k}^2\ ]^{\frac{1}{2}} + m_N .$$

Here S_1^{++} is the leading term in the expression for the effective central potential and S_2^{++} is the leading term in the expression for the effective spin-orbit potential.

Define $N = [\, \varepsilon/(2\, m_N)]^{\frac{1}{2}}$ and $N' = [\, \varepsilon'/(2\, m_N)]^{\frac{1}{2}}$.

Now in terms of A , B , C , etc. we have ,

$$\begin{aligned}
S_1^{++}(\vec{k}',\vec{k}) = \; & N\, N' \{\, A\, [\, 1-\vec{k}.\vec{k}'\tfrac{1}{\varepsilon\varepsilon'}\,] + B\, [\, 1+\vec{k}.\vec{k}'\tfrac{1}{\varepsilon\varepsilon'}\,] \\
& + C\, /(2\, m_N)\, [\, \vec{k}'^2/\varepsilon' + \vec{k}^2/\varepsilon + \vec{k}.\vec{k}'\, (\, \varepsilon + \varepsilon')/(\varepsilon\varepsilon')\,] \\
& + i\, D\, /\, (2\, m_N)\, [\, 1 + \vec{k}.\vec{k}'/(\varepsilon\varepsilon')\,]\, [\, E_N(\vec{k}') - E_N(\vec{k})\,] \\
& + E\, /\, (2\, m_N)\, [\, \vec{k}'^2/\varepsilon' + \vec{k}^2/\varepsilon - \vec{k}.\vec{k}'\, (\varepsilon + \varepsilon')/(\varepsilon\varepsilon')\,] \\
& + i\, F\, /(2\, m_N)\, [\, 1 - \vec{k}.\vec{k}'/(\varepsilon\varepsilon')\,]\, [\, E_N(\vec{k}') - E_N(\vec{k})\,] \\
& - G\, /\, (\, m_N^2)\, [\, \vec{k}'^2\, \vec{k}^2 - (\vec{k}.\vec{k}')^2\,]\, /\, (\, \varepsilon\varepsilon') \\
& - H\, /\, (\, m_N^2)\, [\, \vec{k}'^2\, \vec{k}^2 - (\vec{k}.\vec{k}')^2\,]\, /\, (\, \varepsilon\varepsilon')\, \}.
\end{aligned} \tag{9}$$

By changing the signs of A,C,D, and G we obtain S_1^{--} from S_1^{++}.

$$\begin{aligned}
S_2^{++}(k',k\,) = \; & N\, N' \{\; - A + B + C\, [\, (\, \varepsilon + \varepsilon'\,)/\, (\, 2\, m_N)\,] \\
& + i\, D\, /\, (\, 2\, m_N)\, [\, E_N(\vec{k}') - E_N(\vec{k})\,] \; - E\, /(\, 2\, m_N)\, [\, \varepsilon + \varepsilon'] \\
& - i\, F\, /\, (\, 2\, m_N)\, [\, E_N(\vec{k}') - E_N(\vec{k})\,]
\end{aligned}$$

$$\begin{aligned}
&+ G / (m_N^2) \, [\, \varepsilon \, \varepsilon' + \vec{k} . \vec{k}'] \\
&- H / (m_N^2) \, [\, \varepsilon \varepsilon' - \vec{k} . \vec{k}' \,] \, \} \, .
\end{aligned} \tag{10}$$

By changing signs of A, C, D and G we obtain S_2^{--} from S_2^{++}.

$$\begin{aligned}
S_1^{+-}(\vec{k}',\vec{k}) = N \, N' \, \{ &- A + C - E / (2 \, m_N) \, [\, E_N(\vec{k}') - E_N(\vec{k}) \,] \\
&- i \, F / (2 \, m_N) \, [\, E_N(\vec{k}') + E_N(\vec{k}) \,] \\
&- G / (m^2_N) \, [\, \varepsilon \varepsilon'/2 \,] \, [\, (\vec{k}'/\varepsilon' - \vec{k}/\varepsilon)^2 \,] \\
&- H / m_N \, [\, \varepsilon' - \varepsilon \,] \, \} \, .
\end{aligned} \tag{11}$$

By changing the signs of E, F and H we obtain S_1^{-+} from S_1^{+-}.

$$\begin{aligned}
S_2^{+-}(\vec{k}',\vec{k}) = N \, N' \, \{ \, &B + C / (2 \, m_N) \, [\, E_N(\vec{k}') - E_N(\vec{k}) \,] \\
&+ i \, D / (2 \, m_N) \, [\, E_N(\vec{k}') + E_N(\vec{k}) \,] - E \\
&+ G / (m_N^2) \, [\, \varepsilon \varepsilon'/2 \,] \, [\, (\vec{k}'/\varepsilon')^2 - (\vec{k}/\varepsilon)^2 \,] \\
&+ H / (m_N^2) \, [\, E_N(\vec{k}') \, E_N(\vec{k}) - m_N^2 + \vec{k} . \vec{k}' \,] \, \} \, .
\end{aligned} \tag{12}$$

By changing the signs of B, E and H we obtain S_2^{-+} from S_2^{+-}.

5.2. $S_1^{++}(\vec{k},\vec{k})$ etc. from phenomenology :

We consider p - ^{40}Ca elastic scattering at T_p = 80 MeV.

[T_p is the proton laboratory energy.]

From a phenomenological fit to the data [I.8] we have

$$\mathrm{Re}\ U_s = -\ 422.04\ \mathrm{MeV}$$

and

$$\text{volume integral of Re } U_s = -\ 10.13 \times 10^4\ \mathrm{MeV\text{-}fm}^3.$$

At T_p = 80 MeV , $|\vec{k}| = 1.954\ \mathrm{fm}^{-1}$.

Further ,

$m_N = 4.75\ \mathrm{fm}^{-1}$, $\varepsilon = 9.89\ \mathrm{fm}^{-1}$,

and

$N^2 = (\varepsilon / 2\, m_N) = 1.04$ and $(\vec{k}^2/\varepsilon^2) = 0.04$.

We have

$$\langle\vec{k}|S_1^{++}|\vec{k}\rangle =$$

$$N^2 \{ [1 - \vec{k}^2/\varepsilon^2]\ U_s + [1 + \vec{k}^2/\varepsilon^2]\ U_v \}\ \langle\vec{k}|\ f\ |\vec{k}\rangle. \qquad (13)$$

Here we have defined $\langle\vec{k}|U_s|\vec{k}\rangle = U_s \langle\vec{k}|f|\vec{k}\rangle$ etc..

Further define

$$\langle\vec{k}|S_1^{++}|\vec{k}\rangle = S_1^{++}\ \langle\vec{k}|f|\vec{k}\rangle \text{ etc.,}$$

where

$$\langle\vec{k}|S_1^{++}|\vec{k}\rangle \text{ is in units of MeV-fm}^3,$$

and

$$S_1^{++} \text{ is in MeV.}$$

Now

$$S_1^{++} = N^2 \{\ [\ 1 - \vec{k}^2/\varepsilon^2]\ U_s + [\ 1 + \vec{k}^2/\varepsilon^2]\ U_v\ \} = -\ 68.64 \text{ MeV.} \tag{14}$$

$$S_2^{++} = N^2 \{\ U_s + U_v\ \} = 777 \text{ MeV.} \tag{15}$$

$$S_1^{+-} = -\ N^2\ U_s = -\ 1.04 \times (-1)\ 422.04 = 439 \text{ MeV.} \tag{16}$$

$$S_1^{-+} = S_1^{+-} = 439 \text{ MeV.} \tag{17}$$

$$S_2^{+-} = N^2\, U_v = 1.04 \times 325.5 \ .$$

$$= 339 \text{ MeV}. \tag{18}$$

$$S_2^{-+} = -\, N^2\, U_v$$

$$= -\ 339 \text{ MeV}. \tag{19}$$

$$S_2^{--} = N^2 \{ [\, 1 - \vec{k}^2/\varepsilon^2]\, (-U_s) + [\, 1 + \vec{k}^2/\varepsilon^2]\, U_s \}$$

$$= 774 \text{ MeV}. \tag{20}$$

$$S_1^{--} = N^2 [\, U_s + U_v \,]$$

$$= 1.04\ (\ -\ 422.04 + 325.5\ )$$

$$= -\ 100 \text{ MeV}. \tag{21}$$

5.3. $S_1^{++}(\vec{k},\vec{k})$ etc. from microscopic calculation :

In the following we present the table { taken from I.1.1 } that gives the values of $U_{eff}(p)$, $A(p)$, $B(p)$ and $C(p)$ at nuclear matter density for the potential HEA .

TABLE I.2

P (fm^{-1})	ReU_{eff} (MeV)	ImU_{eff} (MeV)	ReA (MeV)	ImA (MeV)	ReB (MeV)	ImB (MeV)	ReC (MeV)	ImC (MeV)
0.25	-79.2	0.0	-338.7	0.0	258.4	0.0	76.0	0.0
0.50	-75.9	0.0	-335.1	0.0	255.1	0.0	74.55	0.0
0.75	-70.9	0.0	-329.4	0.0	249.6	0.0	72.5	0.0
1.00	-64.9	0.0	-321.5	0.0	241.3	0.0	71.6	0.0
1.25	-58.3	0.0	-312.8	0.0	232.1	0.0	69.1	0.0
1.50	-52.3	-0.968	-303.3	0.50	221.0	-1.63	67.5	2.04
1.75	-45.5	-6.07	-292.78	4.1	209.3	-11.6	63.9	13.6
2.25	-27.5	-16.7	-268.5	22.05	191.6	-42.8	42.8	42.5
2.75	-13.1	-17.5	-248.8	44.9	177.0	-73.1	26.3	60.2
3.25	-2.68	-20.6	-231.6	66.6	160.8	-103.6	19.6	75.3

The following table gives values of the real part of the optical potential for nuclear matter as a function of the quasiparticle energy ε .

(All values in MeV units.) The potential is HEA.

{ taken from I.1.1 }

TABLE I.3

ε	$U_{el}(\rho_0)$	$U_{el}(\rho_0/2)$	ε	$U_{el}(\rho_0)$	$U_{el}(\rho_0/2)$
0.0	-50.55	-38.30	106.00	-20.75	-16.47
5.00	-48.92	-37.05	110.00	-19.60	-15.69
10.00	-47.31	-35.82	115.00	-18.48	-14.94
15.00	-45.72	-34.61	120.00	-17.38	-14.21
20.00	-44.14	-33.41	125.00	-16.31	-13.52
25.00	-42.59	-32.24	130.00	-15.28	-12.85
30.00	-41.05	-31.09	135.00	-14.28	-12.21
35.00	-39.54	-29.96	140.00	-13.30	-11.61
40.00	-38.05	-28.85	145.00	-12.36	-11.05
45.00	-36.58	-27.76	150.00	-11.45	-10.48
50.00	-35.13	-26.69	155.00	-10.58	-9.97
55.00	-33.70	-25.64	160.00	-9.74	-9.49
60.00	-32.30	-24.62	165.00	-8.94	-9.04
65.00	-30.92	-23.61	170.00	-8.17	-8.63
70.00	-29.56	-22.64	175.00	-7.45	-8.25
75.00	-28.23	-21.68	180.00	-6.76	-7.91
80.00	-26.92	-20.75	185.00	-6.11	-7.60
85.00	-25.63	-19.84	190.00	-5.50	-7.33
90.00	-24.37	-18.96	195.00	-4.93	-7.10
95.00	-23.14	-18.10	200.00	-4.41	-6.91
100.00	-21.93	-17.27			

We will take A,B and C from nuclear matter calculations which include correlations.

At T_p = 80 MeV we have to find corresponding p from

$$\hat{\varepsilon}_p = (\vec{p}^2 / 2\, m_N) + U_{eff}(\vec{p}) \; .$$

Using table I.3 , the corresponding Re $U_{eff}(\vec{p})$ = - 27 MeV.

Using table I.2 , the corresponding p = 2.25 fm^{-1} .

Using table I.2 for $|\vec{p}|$ = 2.25 fm^{-1} ,

Re A = - 269 MeV ,

Re B = + 192 MeV and

Re C = + 43 MeV.

To obtain estimates of D,E,F,G and H we rely on the calculation for the finite system. Remember that D,E,F,G and H are in units of MeV-fm^{-3} We have to convert them into quantities expressed in MeV units.

From finite- nucleus calculation

$i\,D \sim 0$,

E = - 34 MeV fm^{-3} ,

$i\,F \sim 0$,

$G \sim 0$

and H = -11 MeV fm^{-3}.

From phenomenology we have

Re U_s = - 422 MeV and

volume integral of Re U_s = - 10.13 x10^4 MeV-fm^{-3}.

We write

$$\langle\vec{k}|U_s|\vec{k}\rangle = U_s\,\langle\vec{k}|f_s|\vec{k}\rangle$$

$$\langle\vec{k}|f_s|\vec{k}\rangle = \langle\vec{k}|U_s|\vec{k}\rangle \frac{1}{U_s} \; . \qquad (22)$$

(23)

But

$$\langle\vec{k}|U_s|\vec{k}\rangle = [\ 1/(2\pi)^3\ ] \int d\vec{r}\ U_s(r)\ .$$

$$= -\ 408.47\ \text{MeV-fm}^3\ .$$

(24)

Thus

$$\langle\vec{k}|f_s|\vec{k}\rangle = 1/1.03 \quad \text{fm}^3\ .$$

(25)

Also

$$\text{Re}\ U_v = 325.5\ \text{MeV}$$

and

$$\text{volume integral of Re}\ U_v = 317.3\ \text{MeV-fm}^3.$$

We write

$$\langle\vec{k}|U_v|\vec{k}\rangle = U_v\ \langle\vec{k}|f_v|\vec{k}\rangle\ ,$$

(26)

with

$$\langle\vec{k}|f_v|\vec{k}\rangle = 1/1.03 \quad \text{fm}^3\ ,$$

(27)

so we can approximate

$$\langle\vec{k}|f_v|\vec{k}\rangle = \langle\vec{k}|f_s|\vec{k}\rangle$$

$$= \langle\vec{k}|f|\vec{k}\rangle.$$

(28)

Now write

$$\langle\vec{k}|D|\vec{k}\rangle = D' \langle\vec{k}|f|\vec{k}\rangle$$

and similar relations for E, F, G and H.

Thus $D' = \langle\vec{k}|D|\vec{k}\rangle \dfrac{1}{\langle\vec{k}|f|\vec{k}\rangle}$ MeV and so on.

Then

$$i\,D' \sim 0,$$

$$E' = -35 \text{ MeV},$$

$$i\,F' \sim 0,$$

$$G' \sim 0$$

and

$$H' = -11 \text{ MeV}.$$

Next we calculate S_1^{++} etc. (in MeV) .

$S_1^{++} = -52$ MeV [contributions from A, B and C] .

(29)

$S_2^{++} = 700$ MeV [contributions from A, B, C, E' and H'].

(30)

$S_1^{+-} = 323$ MeV [contributions from A and C]

(31)

$S_1^{-+} = 323$ MeV [contributions from A and C] .

(32)

$S_2^{+-} = 232$ MeV [contributions from B, E' and H'] .

(33)

$S_2^{-+} = -232$ MeV [contributions from B, E' and H'] .

(34)

$$S_1^{--} = 468 \text{ MeV } [\text{ contributions from A,B and C }] \ .$$

(35)

$$S_2^{--} = -\ 49 \text{ MeV } [\text{ contributions from A,B,C,E' and H' }] \ .$$

(36)

Next we compare the values of the effective central potential.

5.4. Effective central potential :

We have previously seen that

$$U_{eff} = [\ m_N \ / \ E_N(\vec{p})\ ]\ \Sigma^{++}(\vec{p})$$

$$+ \ [\ (m_N / \ E_N(\vec{p}))^2]\ \Sigma^{+-}(\vec{p})\ \Sigma^{-+}(\vec{p})\ \frac{1}{F(\vec{p})}$$

(37)

where

$$F(\vec{p}) = 2\ E_N(\vec{p}) + [\ m_N / \ E_N(\vec{p})\ ]\ [\ \Sigma^{++}(\vec{p}) - \Sigma^{--}(\vec{p})\ ].$$

(38)

For scattering in the forward direction

$$\Sigma^{++}(\vec{p}) = S_1^{++} \ ,$$

(39)

$$\Sigma^{+-}(\vec{p}) = 2\ \vec{\sigma}.\vec{p}\ \frac{1}{\varepsilon(\vec{p})}\ S_1^{+-} \ ,$$

(40)

$$\Sigma^{-+}(\vec{p}) = 2\ \vec{\sigma}.\vec{p}\ \frac{1}{\varepsilon(\vec{p})}\ S_1^{-+}$$

(41)

and

$$\Sigma^{--}(\vec{p}) = S_1^{--} \ . \tag{42}$$

Using our values

$$U_{eff}(\text{phenomenology}) = -\ 43 \text{ MeV} \ ,$$

and we find ,

$$U_{eff}(\text{theory}) \qquad = -\ 40 \text{ MeV} \ .$$

Thus the microscopic calculation agrees quite well with the phenomenological value when the relativistic correction is added.

5.5. Spin - orbit potential :

We have $A = -\ 268$ MeV ,

$B = 192$ MeV,

$U_s = -\ 422$ MeV and

$U_v = 325.5$ MeV.

Thus comparing A with U_s and B with U_v is not appropriate. Further with only A and B,

$$S_2^{++}(\text{theory}) = 460 \text{ MeV}$$

whereas

$$S_2^{++}(\text{phenomenology}) = 777 \text{ MeV}.$$

But with A,B,C,E' and H',

$$S_2^{++}(\text{theory}) = 700 \text{ MeV}.$$

Thus the coefficient of the leading contribution to the effective spin - orbit potential is nicely generated in this model. A more meaningful comparison should be made at the level of the effective spin - orbit potential appearing in the equivalent Schrodinger equation. But this requires a more detailed calculation than the one presented here.

5.6 Conclusions :

We have seen that the phenomenological potentials are best thought of as effective potentials in the sense that their values need to be adjusted to compensate for the use of a highly simplified form.

Our results which include the eight terms indicate that the values of the phenomenological potentials can be reproduced in a microscopic calculation . Thus , for example , if we want to get the correct spin - orbit potential in a microscopic calculation one must consider the optical potential in its most general form.

Further in our calculations the terms C,D,E,F,G and H arise when we calculate the exchange terms. In general since the relativistic density matrix contains scalar , vector (γ^0) and tensor terms, A,B and E could have non-zero direct terms. However in the model considered here the contribution of direct terms to E is zero. As the projectile energy increases, the exchange terms become progressively less important. The success of the parameter-free impulse approximation calculations [I.9] for energies greater than about 400 MeV is then, in part, due to the relative unimportance of exchange effects at higher energies.

The calculation presented here needs further improvements. Here the terms D,E,F,G and H are calculated in Born approximation. One must study the effect of correlations in modifying the values of D,E,F,G and H. Specifically the tensor correlations could affect the pion exchange terms significantly.

We have seen that there is excellent agreement for the effective central potential when a comparison is made between the phenomenological and theoretical values for this quantity. A similar comparison should be made for the effective spin - orbit potential . In order to include the contributions of the relativistic corrections to this potential we need a more detailed calculation. The general trend for the spin - orbit interaction is seen to be given correctly since the theoretical values and the phenomenological values of S_2^{++} are in rather good agreement.

PART II

EFFECTS OF MEDIUM-MODIFIED NUCLEON ELECTROMAGNETIC FORM FACTORS IN INELASTIC ELECTRON SCATTERING FROM NUCLEI

Inelastic electron scattering from nuclei has been used for studying various processes [II.1]. At low energy and momentum transfer one can study low-lying collective states and giant multipole resonances. As one goes to higher momentum transfer the main feature in the inelastic spectrum is the quasi-elastic peak. The term 'quasi-elastic' means that one can describe the inelastic electron-nucleus scattering in terms of elastic electron-nucleon scattering whereby, by sufficient transfer of energy and momentum, individual nucleons are knocked out from the nucleus. Recent experiments [II.2] which yield separated longitudinal and tranverse contributions in the cross section have created much excitement in this field.

In section 1 we discuss the general structure of electron interactions with the nuclei. In section 2 we present a relativistic finite nucleus model devised for the calculation of longitudinal and transverse response functions. In section 3 we present details of the modifications that are needed if we assume that the nucleon electromagnetic form factors are modified in the nuclear medium.Details of a relativistic Fermi gas model are presented in section 4. In section 5 we discuss our results and conclusions.

SECTION 1

Electron-Nucleus Interaction : General Considerations

Consider the electron nucleus interaction in the lowest order in photon exchange [II.3].

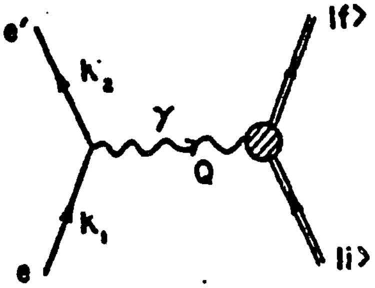


Figure 1.1

Refering to figure 1.1 we see that an electron with 4-momentum k_1 scatters from the nucleus by the exchange of a photon; k_2 is the final 4-momentum of the electron.The 4-momentum transfer is Q; $Q = (\omega , \vec{q})$, where ω is the energy transfer and $\vec{q}$ is the 3-momentum transfer. (q is the magnitude of the 3-momentum transfer). The square of 4-momentum transfer is space-like, i.e., $Q^2 = \omega^2 - q^2$ is less than or equal to zero.

We have three electron kinematic variables : Incident energy E_1, final electron energy E_2 and the scattering angle θ . Alternatively we can choose q , ω and θ as the three independent kinematic variables.

The electromagnetic cross section in lowest order can be factorized in to a electron tensor $\eta^{\mu\nu}$, a photon (virtual) propagator $(\frac{1}{Q^4})$ and a hadron tensor $W^{\mu\nu}$.

Thus the differential cross section for electron- nucleus scattering can be written as

$$d\sigma \sim \frac{1}{Q^4} \quad \eta_{\mu\nu} \quad W^{\mu\nu}. \tag{1}$$

(The $\frac{1}{Q^4}$ factor leads to $\frac{1}{\sin^4(\theta/2)}$ behaviour of the Mott cross section.)

Let us consider inclusive electron scattering where only the electron is detected.

Consider the electron vertex :

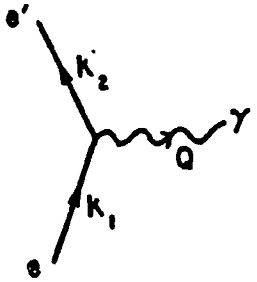


Figure 1.2

Using the fact that the electron electromagnetic current operator is a polar vector we have

$$< \vec{k}',s' \mid j^{\mu} \mid \vec{k},s > = \bar{u}(\vec{k}',s')\ \gamma^{\mu}\ u(\vec{k},s)$$

(2)

Hence

$$\eta^{\mu\nu} = k_1^{\mu} k_2^{\nu} + k_2^{\mu} k_1^{\nu} + g^{\mu\nu} Q^2/2$$

(3)

Next consider the hadron vertex :

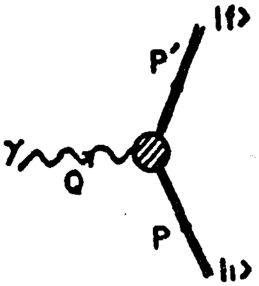


Figure 1.3

We have three 4-momenta P,P' and Q related by $P' = P + Q$. Hence we may choose two independent 4-vectors, P and Q.

Further $P'^2 = P^2 \; (= M_A^2) + Q^2 + 2\, P.Q$

Hence we have two independent scalars. We can choose the two scalars to be Q^2 and P.Q which in the lab frame is $M_A\,\omega$. Alternatively we can choose q and ω as the two independent scalars.

We have the most general form of the hadron tensor :

$$W^{\mu\nu} = W_1\, g^{\mu\nu} + W_2\, P^{\mu} P^{\nu} / (M_A^2)$$

$$+ A\, Q^{\mu} Q^{\nu} / (M_A^2)$$

$$+ B\, [\, P^{\mu} Q^{\nu} + Q^{\mu} P^{\nu} \,] / (M_A^2)$$

$$+ C [P^{\mu} Q^{\nu} - Q^{\mu} P^{\nu}] / (M_A^2)$$

$$. \qquad (4)$$

Using current conservation, i.e.,

$$Q_{\mu} W^{\mu\nu} = 0 ,$$

$$W^{\mu\nu} Q_{\nu} = 0 ,$$

we have

$$C = 0 ,$$

$$B = - [P. Q / Q^2] W_2 ,$$

$$A = - [M_A^2 / Q^2] W_1 + [(P.Q)^2 / Q^4] W_2 .$$

Thus

$$W^{\mu\nu} =$$

$$W_1 [g^{\mu\nu} - Q^{\mu} Q^{\nu} / Q^2]$$

$$+ W_2 \{ 1/ M_A^2 \} [P^{\mu} - Q.P Q^{\mu} / Q^2] [P^{\nu} - Q.P Q^{\nu} / Q^2] \qquad (5)$$

Response functions :

In terms of W_1 and W_2 the differential cross section can be written as

$$[d^2\sigma / d\Omega_2 dE_2] =$$

$$[Z^2 / M_A] \sigma_M [W_2 (q , \omega) - 2 W_1 (q , \omega) \tan^2\{ \theta/2 \}] ,$$

(6)

where σ_M is the Mott differential cross section.

The longitudinal response function is given by

$$S_L (q , \omega) = [Z^2 / (4 \pi)] [q^2 / Q^2]$$

$$\{ - W_1 (q , \omega) + [q^2 / Q^2] W_2 (q , \omega) \}$$

$$= [M_A / 4 \pi] R_L (q , \omega).$$

(7)

$R_L(q , \omega)$ is in units of MeV^{-1} .

The transverse response function is given by

$$S_T(q , \omega) =$$

$$[Z^2 / 4 \pi] [- 2 W_1 (q , \omega)]$$

$$= [M_A / 4 \pi] R_T (q , \omega).$$

(8)

$R_T (q , \omega)$ is in units of MeV^{-1} .

In terms of the hadron electromagnetic current matrix elements we have

$$W^{\mu\nu} = (2 \pi)^6 \{ M_A / Z^2 \}$$

$$\Sigma_{P_x} \; < P_x \mid J^N{}_\nu \mid P >^* < P_x \mid J^N{}_\mu \mid P >$$

$$\delta^4 \; [\; P + Q - P_x \;] \quad . \quad . \tag{9}$$

We have

$$W^{\mu\nu} = W_1 \; (\; g^{\mu\nu} - Q^\mu \, Q^\nu / Q^2 \;) + \hat{P}^\mu \, \hat{P}^\nu \, W_2 \quad . \tag{10}$$

Here

$$\hat{P}^\mu = [\; 1/M_A \;] \; [\; P^\mu - (P.Q) \; Q^\mu \, / \, Q^2 \;] \; . \tag{11}$$

Note that $P.P = 1 - (P.Q)^2 / \; [\; M_A^2 \; Q^2 \;]$ so that in the target rest frame

$$P.P = - \, q^2 \, / \, Q^2 \; .$$

Evaluating in the target rest frame ,

$$W_1 = \tfrac{1}{2} \; \{ \; W^{oo} - \Sigma_i \, W^{ii} + Q^2 \, / \, q^2 \quad W^{oo} \; \} \tag{12}$$

and

$$W_2 = \tfrac{1}{2} \; \{ \; 3 \; [\; Q^4 \, / \, q^4 \;] \; W^{oo} + [\; Q^2 \, / \, q^2 \;] \; [\; W^{oo} - \Sigma_i \, W^{ii} \;] \; \} \tag{13}$$

Then

$$S_L = [\; Z^2 \, / \, (\; 4 \; \pi) \;] \; W^{oo} \tag{14}$$

and

$$S_T = [\; Z^2 \, / \, (\; 4 \; \pi) \;] \; [\; \Sigma_i \, W^{ii} - (\; \omega^2 \, / \, q^2 \;) \; W^{oo} \;]$$

(15)

Since $Q_\mu J^\mu = 0$,

$\omega J^o = - \vec{q}.\vec{J}$ so that

$$J^2 - (\omega^2 / q^2) J^o = (\hat{q} \times \vec{J})^2.$$

Thus

S_L is proportional to the squared matrix element of ρ

and

S_T is proportional to the squared matrix element of $(\hat{q} \times \vec{J})$.

SECTION 2

Finite- Nucleus Model for Inclusive (e,e') Reactions

In this section we discuss a finite- nucleus model developed for the calculation of the response functions [II.4].

The basic diagram we evaluate is given in figure 2.1

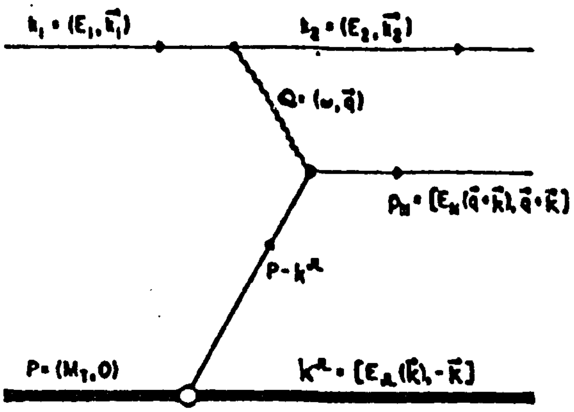


Figure 2.1

Here we have

$$< \vec{p}_N \ , \ s \ : \ - \vec{k} \ \ell jm \ | \ {J^N}_{\mu}(0) \ | \ \vec{P} > =$$

$$< \vec{p}_N \ , \ s \ |\bar{\psi}(0)| \ 0 \ > \Gamma_{\mu}(Q) \ < \ - \ \vec{k} \ \ell jm \ | \ \psi(0) \ | \ \vec{P} >$$

(1)

Here

$$\Gamma_\mu(Q) =$$

$$\gamma_\mu\ F_1\ (\ Q^2\ ) + i\ \sigma_{\mu\nu}\ Q^\nu\ [\ 1\ /\ (\ 2\ m_N\ )\ ]\ F_2\ (\ Q^2\ )\ .$$

(2)

Also ,

$$<\ \vec{p}_N\ ,\ s\ |\ \bar{\psi}(0)\ |\ O\ >\ =\ [\ m_N\ /\ E_N(\vec{p}_N)]^{\frac{1}{2}}\ (\ 2\ \pi\ )^{(-3/2)}\ \bar{u}_s(\ \vec{p}_N\ )$$

(3)

and

$$<\ -\vec{k}\ \ell jm\ |\ \psi(0)\ |\ \vec{P}\ >\ =\ (\ 2\ \pi\ )^{(-3/2)}\ \Psi_{\ell jm}\ (\ \vec{k}\ )\ .$$

(4)

The bound state Dirac wave function $\Psi_{\ell jm_j}(r)$ is given by

$$\Psi_{\ell jm_j}(\vec{r}) = (1/r) \begin{bmatrix} F_{\ell j}(r) & y_{\ell jm_j}(\hat{r}) \\ i\ G_{\ell' j}(r) & y_{\ell jm_j}(\hat{r}) \end{bmatrix}$$

(5)

Here $y_{\ell jm_j}$ is the generalized spherical harmonic defined in part I.

The momentum-space wave function $\Psi_{\ell jm_j}(\vec{k})$ is given by

$$\Psi_{\ell jm_j}(\vec{k}) = i^\ell \int d\vec{r}\ [\ (\ 2\ \pi\ )^{3/2}\ ]\ \exp\ \{\ -\ i\ \vec{k}.\vec{r}\ \}\ \Psi_{\ell jm_j}(\vec{r})$$

i.e.,

$$\Psi_{\ell j m_j}(\vec{k}) = \begin{bmatrix} R^u{}_{\ell j}(k) \quad y_{\ell j m_j}(\hat{k}) \\ \\ R^d{}_{\ell j}(k)\, \vec{\sigma}.\hat{k} \quad y_{\ell j m_j}(\hat{k}) \end{bmatrix} \tag{6}$$

Here

$$R^u{}_{\ell j}(k) = \int dr \ (2 / \pi)^{\frac{1}{2}} \ r \ F_{\ell j}(r) \ j_\ell(kr), \tag{7}$$

and

$$R^d{}_{\ell j}(k) = (- 1)^{(\ell' - \ell + 1)/2} \int dr \ (2 / \pi)^{\frac{1}{2}} \ r \ G_{\ell' j}(r) \ j_{\ell'}(kr). \tag{8}$$

Coming back to the response tensor $W^{\mu\nu}$, we can write

$$W^{\mu\nu} = \Sigma_{\ell j} \ W^{\mu}{}_{\ell j}^{\ \nu} \ ,$$

where

$$W^{\mu}{}_{\ell j}^{\ \nu} =$$

$$[\ M_A \ / \ Z^2 \ ] \int d\vec{k} \ [\ m_N / \ E_N(p_N) \ ] \ \delta \ \{ \ \omega + M_A - E_{\ell j}(\vec{k}) - E_N(\ \vec{p}_N) \ \}$$

$$T^{\mu}{}_{\ell j}^{\ \nu} \quad (\ 2 \ m_N \ )^{-1} \ . \tag{9}$$

Here

$$T^{\mu}{}_{\ell j}^{\ \nu} = \ \mathrm{Tr} \ \{ [\ \gamma^\mu \ F_1 - i \ \sigma^{\nu\rho} \ Q_\rho \ (\ 1/ \ (\ 2 \ m_N \ )) \ F_2]$$

$$[\ \not{p}_N + m_N \ ] \ [\ \gamma^\mu \ F_1 + i \ \sigma^{\mu\lambda} \ Q_\lambda \ (1/ \ (\ 2 \ m_N \ )) \ F_2 \ ] \ P_{\ell j}(k) \ \}. \tag{10}$$

The matrix

$$\begin{aligned} P_{\ell j}(k) &= \Sigma_{m_j} \; \Psi_{\ell j m_j}(\vec{k}) \;\; \overline{\Psi}_{\ell j m_j}(\vec{k}) \\ &= A_{\ell j}(k) + \not{B}_{\ell j}(k) \;, \end{aligned} \tag{11}$$

where

$$A_{\ell j}(k) = [\ (2j+1)/8\pi)\ ]\ [\ (\ R^{u}{}_{\ell j}(k))^{2} - (\ R^{d}{}_{\ell j}(k))^{2}\ ] / 2 \tag{12}$$

$$B^{o}{}_{\ell j}(k) = [\ (2j+1)/8\pi)]\ [\ (\ R^{u}{}_{\ell j}(k))^{2} + (\ R^{d}{}_{\ell j}(k))^{2}\ ] /2 \tag{13}$$

$$\vec{B}_{\ell j}(k) = [\ (2j+1)/8\pi)\ ]\ \hat{k}\ R^{u}{}_{\ell j}(k) \;\; R^{d}{}_{\ell j}(k) \;. \tag{14}$$

The trace is given by

$$\begin{aligned} & T^{\mu\ \nu}_{\ \ell j} = \\ & 4\ m_N\ \{\ [\ F_1{}^2 + F_2{}^2\ (\ 1/(2m_N))^2\ Q^2]\ A_{\ell j} \\ & + 2\ (\ 1/(2m_N))\ F_1\ F_2\ Q.\ [\ A_{\ell j}\ p_N/m_N\ - B_{\ell j}\ ] \\ & -\ [\ F_1{}^2 - F_2{}^2\ (\ 1/(2m_N))^2\ Q^2\ ]\ \ p_N.B_{\ell j}\ /\ m_N \\ & -\ 2\ F_2{}^2\ (\ 1/(2m_N))^2\ p_N.Q\ B_{\ell j}\ .\ Q\ /\ m_N\ \}\ \ g^{\mu\nu} \\ & + 4\ m_N\ \{\ [\ F_1{}^2 - F_2{}^2\ (\ 1/\ (2m_N))^2\ Q^2\ ] \\ & [\ p_N^{\mu}/m_N\ \ B^{\nu}{}_{\ell j}\ \ + B^{\mu}{}_{\ell j}\ \ p_N^{\nu}\ /\ m_N\ ]\ \} \end{aligned} \tag{15}$$

In this expression we have omitted terms containing Q^{μ}

and Q^{ν} anticipating further manipulations.

Further define

$$T^{\mu\ \nu}_{\ \ell j} =$$

$$4 \{ g^{\mu\nu} T_1 + (p_N^{\mu} B^{\nu}_{\ \ell j} + p_N^{\nu} B^{\mu}_{\ \ell j}) T_2 \} \tag{16}$$

To make $W^{\mu\nu}$ gauge invariant , define

$$\hat{T}^{\mu\ \nu}_{\ \ell j} =$$

$$[g^{\mu\lambda} - Q^{\mu} Q^{\lambda} / Q^2]$$

$$[T^{\lambda\ \gamma}_{\ \ell j}]$$

$$[g^{\gamma\nu} - Q^{\gamma} Q^{\nu} / Q^2] \tag{17}$$

$$= 4 [g^{\mu\nu} - Q^{\mu} Q^{\nu} / Q^2] T_1$$

$$+ 4 [\hat{p}_N^{\mu} \hat{B}^{\nu}_{\ \ell j} + \hat{p}_N^{\nu} \hat{B}^{\mu}_{\ \ell j}] T_2 , \tag{18}$$

where

$$\hat{p}_N^{\mu} = p_N^{\mu} - (p_N \cdot Q) Q^{\mu} / Q^2 . \tag{19}$$

Now define

$$F1 = 1 - (P.Q)^2 / (Q M_A)^2,$$

$$w_1^{\ell j} =$$

$$4\{ T_1 + T_2 \{ \hat{p}_N \cdot \hat{B}_{\ell j} - [(\hat{p}_N . P)(\hat{B}_{\ell j} . P)/(M_A^2 F1) \},$$

(20)

and

$$w^{\ell j}_{2} =$$
$$4\{\ T_2\ \{\ 3\ (\hat{p}_N . P)(\hat{B}_{\ell j} . P)/M_A^2\ -\ \hat{p}_N . \hat{B}_{\ell j}\ F1\ \}/(F1)^2\ \}\ .$$

(21)

Now

$$W^{\ell j}_{1,2} = [\ M_A / Z^2\ ]$$
$$\times \int d\vec{k}/(2\ E_N(\vec{P}_N))\ w^{\ell j}_{1,2}\ \delta[\ \omega + M_A - E_{\ell j}(\vec{k}) - E_N(\vec{p}_N)] .$$

(22)

Making use of the energy-conserving delta function ,

$$W^{\ell j}_{i} = [\ \pi/q\ ]\ [\ M_A\ /\ Z^2\ ]$$
$$\times \int_{k_d}^{k_u} k\ dk\ w^{\ell j}_{i}\ [\ q\ ,\ \omega\ ,\ k\ ,\ \cos\theta^{*}\ ]\ .$$

(23)

Here

$$\cos\theta^{*} = \{\ [\ \omega + M_A - E_{\ell j}(\vec{k})]^2 - [\ m_N^2 + \vec{k}^2 + q^2]\}/(2kq),$$

(24)

$$E_{\ell j}(\vec{k}) = [\ \vec{k}^2 + M^2_{\ell j}\ ]^{\frac{1}{2}},$$

(25)

$$M_{\ell j} = M_A - m_N + \varepsilon_{\ell j}\ .$$

(26)

Further, $\varepsilon_{\ell j}$ is the binding energy of the orbital ℓj ,

$$k_u = a + b,$$

(27)

$$k_d = |a - b| ,$$

with

$$a = [\ \omega + M_A\ ]/2$$
$$\times\ [\{\ 1 - (\ M_{\ell j} + m_N)^2 \Lambda^{-2}\}\ \{\ 1 - (M_{\ell j} - m_N)^2 \Lambda^{-2}\}]^{\frac{1}{2}} , \tag{28}$$

$$b = [\ q\ /\ 2\ ]\ [\ 1 + \{\ M^2_{\ell j} - m_N^2\ \}\ \Lambda^{-2}\ ] , \tag{29}$$

and

$$\Lambda^2 = (\ \omega + M_A\ )^2 - q^2\ .$$

The response functions calculated in this model have been compared with the experimental (separated) data of Altemus et al. [II.2.1]. While the transverse response function fairly agrees with the data , the longitudinal response function is overestimated by a factor of 2. This conclusion is in agreement with the result of a relativistic Fermi- gas calculation [II.5].

Various suggestions have been put forth to explain the quenching of the longitudinal response function. Noble [II.6] suggested that longitudinal response could be reduced by modification of the nucleon properties in the medium . Using a relativistic Fermi-gas model he fit the longitudinal response by changing the nucleon electromagnetic form factors . The required modifications indicated a larger nucleon size in the medium. In [II.4] it was suggested that an alternative mechanism, namely the depletion of shell-model orbitals through various short-range correlation effects, could account for the quenching. Susequently it was shown that the later argument is not viable [II.7]. Further, at higher momentum transfers, i.e., ($\sim \frac{1}{2}$GeV/c), the effective nucleon- nucleon interaction is very weak. Recently there have been calculations of collective modes of nuclear matter using realistic forces and the Tamm-Dancoff approximation [II.8]. These calculations show that at such large values of momentum transfer there is virtually no collective response. Thus the modification of nucleon properties in the medium appears to be the only possible physical mechanism responsible for the quenching of the londitudinal response. To avoid ambiguities we would like to investigate this phenomena in a parameter-free approach.

SECTION 3

Finite-Nucleus Model with Medium-Modified Nucleon Electromagnetic Form Factors

It is now a well established fact that the nucleons and mesons are made up of quarks. In traditional nuclear physics the relevant degrees of freedom are taken to be nucleons and mesons. An interesting question that can be asked now is how does one modify this picture to take into account the substructure of nucleons and mesons.

As we have observed in part I , the dominant fields required in the description of the nucleus as a relativistic system of interacting nucleons and mesons are of (Lorentz) scalar and vector character. These fields are quite strong in that the strength of the scalar field is about - 400 MeV and the strength of the vector field is about + 300 MeV. It seems quite possible that these fields may modify the properties of the nucleon in the nuclear medium. To discuss such modifications in a parameter-free manner one requires a covariant model of nucleon structure. Such a model was developed recently [II.9].

In the covariant soliton model quarks are coupled to a scalar field χ which serves to bind the system. Quarks are further coupled to the fields of the OBE model of the nucleon-nucleon interaction. Resulting nonlinear equations are solved for a self- consistent solution by iteration. This model reproduces rather well the single nucleon properties such as the proton charge radius , magnetic moments , axial vector coupling constant etc.

The next step is to consider the nucleon in nuclear matter. (An assumption was made that the scalar field χ, which serves to bind the system of quarks to form the nucleon, does not take part in nucleon-nucleon interaction). The introduction of strong scalar and vector fields amounts to the insertion of a quark self- energy term into the field equations. In a uniform system (nuclear matter) the scalar part of the self energy introduces a shift in the quark mass parameter and the vector part merely shifts the energy scale. Making use of a local- density approximation the quark mass parameter becomes a linear function of the local density of the nuclear matter.

When the resulting equations were solved it was found that the nucleon grew in size as a function of the local density of nuclear matter. There was an associated change in the nucleon electromagnetic form factors. The predicted increase in size is precisely what is needed for the explanation of the 'EMC effect' [II.10].

It is quite instructive to look for experimental evidence for the (predicted) modified nucleon electromagnetic form factors by studying those aspects of traditional nuclear physics that involve the nucleon form factors in their analysis. Thus naturally we come back to the analysis of inclusive (e , e') reactions near the quasi- elastic peak.

We stress the fact that at a fundamental level quarks are the elementary carriers of charge in nuclei. Thus the appropriate current operator is

$$J^{\mu}(x) = e\, \bar{q}(x) \quad [\ 1/6 + \tau_3^q/2\ ] \quad \gamma^{\mu} \quad q(x) \tag{1}$$

We are considering only up and down quarks; e is the magnitude of the electron charge and τ_3^q is 1 for the up quark and -1 for down quark.

Returning to consideration of the (e,e') reaction cross section we now turn our attention to the response tensor. The response tensor is proportional to the imaginary part of the forward (virtual) Compton scattering amplitude, which is the Fourier transform of the current-current correlation function [II.11]. In figure 3.1 the forward Compton scattering amplitude for virtual photon from a nucleus is shown.

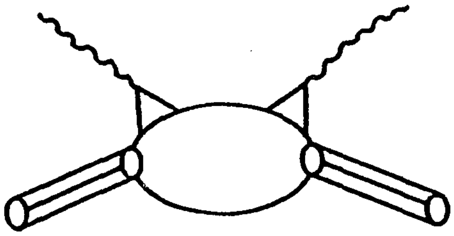


Figure 3.1

Here light lines represents quarks.

Now we make use of shell model concepts and assume that quarks are found in nucleons. This approximation is represented in figure 3.2.

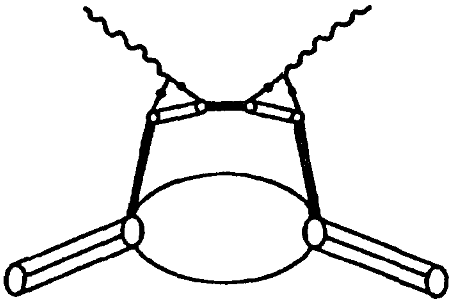


Figure 3.2

The modification of quark wave functions in nuclei leads to medium-modified nucleon electromagnetic form factors. Tables of the ratios

$$G_E^p(Q^2, \rho(r))/G_E^p(Q^2),$$
$$G_M^p(Q^2, \rho(r))/G_M^p(Q^2)$$
and $G_M^n(Q^2, \rho(r))/G_M^n(Q^2)$

are given in the following pages for three different values of matter densities :

1) nuclear matter density (0.172 fm^{-3})
2) (2/3) of the nuclear matter density (0.115 fm^{-3})

and

3) (1/3) of the nuclear matter density (0.057 fm^{-3}).

From these tables one can calculate

$$F_1^p(Q^2, \rho(r)),$$
$F_2^p(Q^2, \rho(r))$ etc.

TABLE II.1

MEDIUM-MODIFIED NUCLEON ELECTROMAGNETIC FORM FACTORS

Density ρ = 0.172 fm^{-3}

$-q^2$ (fm^{-2})	$\frac{G_E^p(q^2,\rho)}{G_E^p(q^2,0)}$	$\frac{G_M^p(q^2,\rho)}{G_M^p(q^2,0)}$	$\frac{G_M^n(q^2,\rho)}{G_M^n(q^2,0)}$
1.000	0.923	1.105	1.113
2.000	0.851	1.024	1.031
3.000	0.783	0.949	0.956
4.000	0.722	0.882	0.888
5.000	0.668	0.824	0.830
6.000	0.622	0.776	0.781
7.000	0.583	0.736	0.741
8.000	0.551	0.705	0.710
9.000	0.515	0.670	0.674
10.000	0.482	0.638	0.642
11.000	0.452	0.611	0.615
12.000	0.425	0.587	0.591
13.000	0.397	0.563	0.567
14.000	0.370	0.540	0.544
15.000	0.344	0.520	0.524
16.000	0.318	0.502	0.506
17.000	0.292	0.484	0.487
18.000	0.265	0.467	0.470
19.000	0.238	0.452	0.454
20.000	0.209	0.437	0.440
21.000	0.178	0.423	0.426
22.000	0.145	0.410	0.412
23.000	0.108	0.397	0.400
24.000	0.066	0.385	0.390

TABLE II.2

MEDIUM-MODIFIED NUCLEON ELECTROMAGNETIC FORM FACTORS

Density ρ = 0.115 fm^{-3}

$-q^2$ (fm^{-2})	$\frac{G_E^p(q^2,\rho)}{G_E^p(q^2,0)}$	$\frac{G_M^p(q^2,\rho)}{G_M^p(q^2,0)}$	$\frac{G_M^n(q^2,\rho)}{G_M^n(q^2,0)}$
1.000	0.958	1.069	1.074
2.000	0.918	1.028	1.032
3.000	0.879	0.989	0.993
4.000	0.842	0.952	0.956
5.000	0.809	0.920	0.939
6.000	0.778	0.890	0.892
7.000	0.751	0.865	0.869
8.000	0.726	0.844	0.848
9.000	0.700	0.821	0.825
10.000	0.675	0.800	0.803
11.000	0.651	0.781	0.784
12.000	0.629	0.764	0.767
13.000	0.605	0.746	0.749
14.000	0.582	0.729	0.732
15.000	0.558	0.714	0.717
16.000	0.535	0.700	0.703
17.000	0.511	0.685	0.688
18.000	0.485	0.672	0.675
19.000	0.458	0.659	0.662
20.000	0.430	0.647	0.650
21.000	0.398	0.635	0.638
22.000	0.363	0.623	0.626
23.000	0.324	0.612	0.615
24.000	0.278	0.602	0.610

TABLE II.3

MEDIUM-MODIFIED NUCLEON ELECTROMAGNETIC FORM FACTORS

Density ρ = 0.057 fm^{-3}

$-q^2$ (fm^{-2})	$\frac{G_E^p(q^2,\rho)}{G_E^p(q^2,0)}$	$\frac{G_M^p(q^2,\rho)}{G_M^p(q^2,0)}$	$\frac{G_M^n(q^2,\rho)}{G_M^n(q^2,0)}$
1.000	0.982	1.033	1.035
2.000	0.965	1.017	1.019
3.000	0.948	1.001	1.003
4.000	0.932	0.986	0.988
5.000	0.916	0.972	0.974
6.000	0.901	0.959	0.961
7.000	0.888	0.948	0.950
8.000	0.875	0.937	0.939
9.000	0.861	0.927	0.928
10.000	0.848	0.916	0.918
11.000	0.835	0.907	0.909
12.000	0.822	0.898	0.900
13.000	0.809	0.889	0.891
14.000	0.795	0.881	0.882
15.000	0.782	0.873	0.874
16.000	0.768	0.865	0.867
17.000	0.752	0.857	0.859
18.000	0.737	0.850	0.852
19.000	0.720	0.843	0.844
20.000	0.701	0.836	0.838
21.000	0.681	0.829	0.831
22.000	0.658	0.822	0.824
23.000	0.632	0.816	0.817
24.000	0.561	0.810	0.811

Calculation procedure :

Here we discuss the modifications needed in our calculation to incorporate the medium-modified form factors.

Since now $F_1(Q^2) \rightarrow F_1(Q^2, \rho_m(r))$ etc. ,

we define

$$\Gamma^1{}_{\ell jm_j}(\vec{r},Q^2) = F_1(Q^2, \rho_m(r))\, \Psi(\vec{r}), \tag{2}$$

$$\Gamma^2{}_{\ell jm_j}(\vec{r},Q^2) = F_2(Q^2, \rho_m(r))\, \Psi(\vec{r}) , \tag{3}$$

$$\Gamma^1{}_{\ell jm_j}(\vec{k},Q^2) = i^{\ell}\int d\vec{r}\, (2\pi)^{(-3/2)} \exp[\, i\, \vec{k}.\vec{r}\,]\, \Gamma^1{}_{\ell jm_j}(\vec{r},Q^2), \tag{4}$$

and

$$\Gamma^2{}_{\ell jm_j}(\vec{k},Q^2) = \int i^{\ell}\, d\vec{r}\, (2\pi)^{(-3/2)} \exp[\, i\, \vec{k}.\vec{r}\,]\, \Gamma^2{}_{\ell jm_j}(\vec{r},Q^2). \tag{5}$$

Now

$$T^{\mu\nu}{}_{\ell j} = \{\Sigma_{m_j} [\, \bar{\Gamma}^1{}_{\ell jm_j}(\vec{k},Q^2)\, \gamma^{\nu} - i\, \sigma^{\nu\rho}\, Q_{\rho}/(2m_N)\, \bar{\Gamma}^2{}_{\ell jm_j}(\vec{k},Q^2)\,]\, (\not{p}_N + m_N)\, [\, \Gamma^1{}_{\ell jm_j}(\vec{k},Q^2)\, \gamma^{\mu} + i\, \sigma^{\mu\lambda}\, Q_{\lambda}/(2m_N)\, \Gamma^2{}_{\ell jm_j}(\vec{k},Q^2)\,]\, \} \tag{6}$$

Define

$$\rho^{i\,j}_{\ell j} = \Sigma_{m_j}\ \Gamma^{i}_{\ell j m_j}(\ \vec{k},Q^2\ )\ \bar{\Gamma}^{j}_{\ell j m_j}(\ \vec{k},Q^2\ )\ . \tag{7}$$

We write

$$\Gamma^{i}_{\ell j m_j}(\ \vec{k},Q^2\ ) =. \begin{bmatrix} F^{i\,u}_{\ell j}(k) & y_{\ell j m_j}(\hat{k}) \\ & \\ F^{i\,d}_{\ell j}(k) & \vec{\sigma}.\hat{k}\ y_{\ell j m_j}(\hat{k}) \end{bmatrix} . \tag{8}$$

Here we have suppressed the Q^2 dependence of the F's.

Then

$$\rho^{i\,j}_{\ell j} = [(2j+1)/(8\pi)] \begin{bmatrix} F^{i\,u}_{\ell j}(k)\ F^{j\,u}_{\ell j}(k) & -\ F^{i\,u}_{\ell j}(k)\ F^{j\,d}_{\ell j}(k) \\ & \\ F^{i\,d}_{\ell j}(k)\ F^{j\,u}_{\ell j}(k) & -\ F^{i\,d}_{\ell j}(k)\ F^{j\,d}_{\ell j}(k) \end{bmatrix} \tag{9}$$

i.e.,

$$\rho^{i\,j}_{\ell\,j} = [(2j+1)/(8\pi)]\ [\ A^{i\,j}_{\ell j}(\ k,Q^2) + \gamma^{o}\ B^{i\,j}_{\ell j}(\ k,Q^2) - \vec{\gamma}.\vec{B}^{i\,j}_{\ell j}(\ k,Q^2) + \gamma^{o}\ \vec{\gamma}.\vec{C}^{i\,j}_{\ell j}(\ k,Q^2)\ ] \tag{10}$$

Taking traces ,we find ,

$$A^{i\,j}_{\ell j}(\ k,Q^2) = \tfrac{1}{2}\ \{\ F^{i\,u}_{\ell j}(k)\ F^{j\,u}_{\ell j}(k)\ -\ F^{i\,d}_{\ell j}(k)\ F^{j\,d}_{\ell j}(k)\ \} \tag{11}$$

$$B^{i\,\overset{\circ}{j}}_{\ell j}(\ k,Q^2) = \tfrac{1}{2}\ \{\ F^{i\,u}_{\ell j}(k)\ F^{j\,u}_{\ell j}(k)\ +\ F^{i\,d}_{\ell j}(k)\ F^{j\,d}_{\ell j}(k)\ \} \tag{12}$$

$$\vec{B}^{i\,j}_{\ell j}(\,k,Q^2) = \tfrac{1}{2}\,\hat{k}\ \{\ F^{i\,u}_{\ell j}(k)\ F^{j\,d}_{\ell j}(k)\ +\ F^{i\,d}_{\ell j}(k)\ F^{j\,u}_{\ell j}(k)\ \} \tag{13}$$

$$\vec{C}^{i\,j}_{\ell j}(\,k,Q^2) = \tfrac{1}{2}\,\hat{k}\ \{\ F^{i\,u}_{\ell j}(k)\ F^{j\,d}_{\ell j}(k)\ -\ F^{i\,d}_{\ell j}(k)\ F^{j\,u}_{\ell j}(k)\ \} \tag{14}$$

$$T^{\mu\,\nu}_{\ell j} = [(2j+1)/(8\pi)]$$

$$4\{g^{\mu\nu}\ [\ (\ A^{1\,1}_{\ell j}(\,k,Q^2) + Q^2/(4m_N^2)\ A^{2\,2}_{\ell j}(\,k,Q^2))\ m_N$$

$$-\ B^{1\,1}_{\ell j}(\,k,Q^2).p_N + (1/(4m_N^2))$$

$$(\ Q^2\ B^{2\,2}_{\ell j}(\,k,Q^2).p_N - 2\ B^{2\,2}_{\ell j}(\,k,Q^2).Q\ p_N.Q)$$

$$+\ (1/m_N)\ (\ A^{2\,1}_{\ell j}(\,k,Q^2)\ Q.p_N - Q.B^{2\,1}_{\ell j}(\,k,Q^2)\ +$$

$$(P/M_A).p_N\ C^{2\,1}_{\ell j}(\,k,Q^2).Q - (P/M_A).Q\ C^{2\,1}_{\ell j}(\,k,Q^2).p_N)\ ]$$

$$+\ [\ B^{1\,1\,\mu}_{\ell j}(\,k,Q^2) - (Q^2/(4m_N^2))\ B^{2\,2\,\mu}_{\ell j}(\,k,Q^2)\ +$$

$$(1/m_N)\ (\ P.Q/M_A\ C^{2\,1\,\mu}_{\ell j}(\,k,Q^2) - C^{2\,1}_{\ell j}(\,k,Q^2).Q\ P^{\mu}/(M_A))]p_N^{\nu}$$

$$+ [\ B^{1}{}_{\ell j}^{1\nu}(\ k,Q^2) - (Q^2/(4m))\ B^{2}{}_{\ell j}^{2\nu}(\ k,Q^2)$$

$$+ (1/m_N)\ (\ P.Q/M_A\ C^{2}{}_{\ell j}^{1\nu}(\ k,Q^2) - C^{2}{}_{\ell j}^{1}(\ k,Q^2).Q\ P^{\nu}/(M_A))]p_N^{\mu}\}$$

(15)

where again all terms in Q^μ and Q^ν are dropped.

Define

$$T^{\mu\nu}_{\ell j} = [\ (2j+1)/(8\pi)]\ \ 4$$

$$\{\ g^{\mu\nu}\ T_1 + (\ p_N^\mu\ B^\nu{}_{\ell j} + B^\mu{}_{\ell j}\ p_N^\nu)\ T_2\ \}$$

(16)

and again introduce $\hat{T}^{\mu\nu}_{\ell j}$.
The rest of the formalism is the same as before.

In this calculation we have represented the final nucleon by a (Dirac) plane wave whereas the initial nucleon is represented by the solution of Dirac equation containing a scalar and a vector potential. (We should in fact orthogonalize the outgoing wave to the bound orbitals. Also we have imposed gauge invariance in a particular way.)

One should check the model dependence of the results. A relativistic Fermi-gas model has the advantage that one can easily incorporate the Pauli principle. Also it is easy to maintain gauge invariance. Hence we discuss the relativistic Fermi-gas model in the next section.

SECTION 4

Relativistic Fermi- gas model

The best feature of a Fermi- gas model [II.12] is that we can arrive at analytic expressions for the response functions. In this model the electron scatters elastically from a nucleon in the Fermi sea. To satisfy the Pauli exclusion principle, the recoiling nucleon must occupy a state outside the Fermi sea. Both initial and final nucleon are taken to be free. In other words we neglect the interaction effects in the initial and final states. Now the hadron tensor is given by

$$W^{\mu\nu} = [\, M_A / Z^2 \,]\, [\, Z\, w^{\mu\nu}(p) + N\, w^{\mu\nu}(n) \,] \; ; \tag{1}$$

$$\begin{aligned} w^{\mu\nu}(p,n) &= V^{-1}\int d\vec{k}\, [\, m_N / E_N(\vec{k}') \,]\, [\, m_N / E_N(\vec{k}) \,] \\ &\times [\, 2 m_N \,]^{-1}\, T^{\mu\nu}\, [\, 2 m_N \,]^{-1} \\ &\quad \delta[\, \omega + E_N(\vec{k}) - E_N(\vec{k}') \,]\;\; \theta(\, k_F - |\vec{k}| \,)\;\; \theta(\, |\vec{k}'| - k_F \,) \, . \end{aligned} \tag{2}$$

The tensor $T^{\mu\nu}$ is given by

$$\begin{aligned} T^{\mu\nu} &= \mathrm{Tr}\, \{\, [\, \gamma^\mu F_1 - i\, \sigma^{\mu\rho} Q_\rho\, (\, 2 m_N \,)^{-1} F_2 \,] \\ &\times [\, \not{K}' + m_N \,]\, [\, \gamma^\nu F_1 + i\, \sigma^{\nu\lambda} Q_\lambda\, (\, 2 m_N \,)^{-1} F_2 \,]\, [\, \not{K} + m_N \,] \end{aligned} \tag{3}$$

with

$$k' = k + Q\ .$$

Evaluating the traces and doing the integrals we have the following analytic expression for the response functions.

$$R_L(q\ ,\ \omega) =$$

$$3\ [(4\ k_F\ q)^{(-1)}\ ]\{\ T_2$$

$$[(E_F^3 - E_{min}^3)/3 + \omega\ (E_F^2 - E_{min}^2)/2 + \omega^2(E_F - E_{min})/4]$$

$$-\ T_1\ [\ q^2\ /\ Q^2\ ]\ (\ E_F - E_{min})\ \}\ . \tag{4}$$

$$R_T\ (\ q\ ,\ \omega\ ) =$$

$$3\ [\ (4\ k_F\ q\ )^{-1}\ ]\ \{\ T_2$$

$$[\ (-Q^2/q^2)\ (E_F^3 - E_{min}^3)/3 + \omega(E_F^2 - E_{min}^2)/2$$

$$-\ (\ (Q^4)/(4q^2) + m_N^2\ )\ (\ E_F - E_{min}\ )\ ]$$

$$-\ 2\ T_1\ (\ E_F - E_{min}\ )\ \}\ . \tag{5}$$

Here

$$T_1 =$$
$$Q^2/2\ \{\ Z\ [\ F_1^p + F_2^p\ ]^2 + N\ [\ F_1^n + F_2^n\ ]^2\ \} \tag{6}$$

and

$$T_2 =$$

$$2 \{ Z [F_1^p - Q^2/(4m_N^2) F_2^p]^2 + N [F_1^n - Q^2/(4m_N^2) F_2^n]^2 \} \quad (7)$$

Further

$$E_F = [k_F^2 + m_N^2]^{\frac{1}{2}},$$

$$E_{min} = [k_{min}^2 + m_N^2]^{\frac{1}{2}},$$

and

$$k_{min} = - q/2 + \omega/q \; m_N \{ q^2/(-Q^2) + q^2/(4 m_N^2) \}^{\frac{1}{2}}.$$

Usually the Fermi gas model has two parameters :the Fermi momentum k_F and an energy shift ε.In the case where we incorporate the medium-modified form factors we let $F_1^p(Q^2) \rightarrow F_1^p(Q^2, \rho_{matter})$ etc.,where ρ_{matter} is the mean density of the appropriate nucleus.

The parameters used in our Fermi gas calculations are given in the following table :

TABLE II.4

Nucleus	k_F (MeV/c)	ρ_{matter} (fm^{-3})	ε (MeV)
^{12}C	220	0.089	20
^{40}Ca	251	0.102	28
^{56}Fe	260	0.117	35

Section 5

Results and conclusions :

Once again we stress the fact that the longitudinal response function in inclusive (e, e') reactions on nuclear targets is determined by the quark density - density correlation function. However if the momentum transfer is not too high ($\lesssim$ 1 GeV/c) we can make contact with standard nuclear physics models by assuming that quarks are located in nucleons. In this approximation we have further included the possiibility that the quark wave functions are modified in the nuclear medium. This in turn leads to medium-modified nucleon electromagnetic form factors. Since the longitudinal response function is known to be free (to a large extent) from processes like delta excitation, meson-exchange-current contributions etc., we now compare the data with our calculation of this function.

Longitudinal response function :

The longitudinal response function is plotted together with the data for the nuclei ^{40}Ca and ^{56}Fe [II.2.4] and the nucleus ^{12}C [II.2.3] at q = 550 MeV/c in figures 1-6. In these figures the dashed line represents the calculation with unmodified nucleon form factors. The solid line represents the calculation with medium-modified form factors. In the case of the finite- nucleus model the theoretical curves have been shifted downward in energy by 20 MeV.This shift is not fully understood at this time but such a shift may arise from mean- field interaction effects. It can be seen that the solid line provides a rather good fit to the data.

Next we consider the lower momentum transfer data.In figures 7 - 10 we show the comparison with the data at q = 410 MeV/c for the nuclei ^{40}Ca and ^{56}Fe. Here the agreement is less satifactory especially for the finite-nucleus model. The discrepancy may have its origin in the presence of residual nucleon - nucleon interaction ,Pauli blocking, ambiguity in the imposition of gauge invariance, etc. Obviously these effects are less and less important as we go to higher momentum transfers. Clearly the agreement is better with the Fermi gas model in the region of the peak. It appears that the energy-integrated longitudinal response is given quite well down to about 300 MeV/c [II.16] in the Fermi-gas model with medium-modified form factors.

From the comparison of the data with our calculation it appears that the quenching of the longitudinal response is a consequence of medium- modified nucleon electromagnetic form factors. Larger nucleon size also leads to an explanation of the 'EMC effect' [II.10].

Further the medium-modified form factors are found to be effective in resolving some famous problems encountered in relating experimental charge distributions to theoretical ones [II.13]. Thus a coherent picture seems to emerge regarding medium-modified nucleon properties.

Since we are discussing modification of nucleon properties in nuclei and since these effects appear to be rather important , we have to answer certain theoretical questions.

The first question deals with the shell-model description of nuclear structure. One can rely on a shell-model description if even after the nucleons grow in size the system is still dilute. This in fact turns out to be the case. Still there is the possibility that the outer regions of nucleons may overlap as they come closer. The question of distortion of quark wave functions seems to be an interesting problem about which only very little, if anything, is known at this time.

The second question deals with the reliability of the impulse approximation in this calculation. To answer this question it is instructive to look at the kinematics. Let us introduce the nucleon self energy $\Sigma = A + \gamma^0 B$. Typically $A \sim -400$ MeV and $B \sim +300$ MeV. Introducing the Dirac mass $\tilde{m} = m_N + A$, the energy of the nucleon is given by $\varepsilon = B + (\vec{p}^2 + \tilde{m}^2)^{\frac{1}{2}}$. This shows that even though the relativistic mass parameter is significantly modified, the nucleon inside a nucleus is not very far off from being on mass shell.

Transverse response function

The transverse response function is plotted together with the data in figures 11-16. In these figures we have also shown the difference between our calculation (which is based on the single-nucleon knock-out mechanism) and the experimental data. Here the agreement is less satisfactory.Notice that as we go to higher momentum transfers the modified form factors make this disagreement more pronounced.

Thus if the modified magnetic form factors are correct there is a large amplitude in the transverse response that is not described by the single-nucleon knock-out mechanism. The electroproduction of delta isobar is important in transverse response [II.14]. One can as well expect that the properties of delta will be modified in the nuclear medium. Thus the width of delta may be enhanced leading to effects at smaller values of energy transfer. In addition meson-exchange-current contributions are very important in transverse response especially at larger momentum transfer [II.15]. The understanding of the transverse response appears to be a problem worth detailed investigation in the future.

Response functions at higher momentum transfers :

At present the experimental data do not exist for momentum transfers greater than 600 MeV/c. In figures 17-24 , we present the longitudinal and transverse response functions for momentum transfers ranging from 400 MeV/c to 1 GeV/c with free form factors and with medium modified form factors for the nuclei ^{56}Fe and ^{12}C . Even though the experiments are very difficult to perform it is of extreme interest to see the pronounced quenching of the longitudinal response at higher momentum transfers as shown in our figures.

(e,e'p) experiments :

In (e,e'p) experiments , a proton is detected in coincidence with the outgoing electron. Since the photon is coupled to the quark, the interpretation of this experiment is affected by the medium modification of the quark wave functions of the nucleon. Since in our model the medium modification of form factors depends on the local density of nuclear matter, we can expect shell effects. For example, the quenching in s and p shells will have different magnitudes. Detailed analysis of (e,e') experiments are underway and these will include a study of the effects of the medium-modified form factors used in this work.

APPENDIX 1

Complicated Nature of the Effective Schrödinger Potentials :

Let us begin by considering the Dirac equation containing a local (Lorentz) scalar potential, $U_s(r)$, and a local (Lorentz) vector potential, $U_v(r)$:

$$\{\vec{\alpha}.\vec{p} + \beta[\, m_N + U_s(r) + \gamma^o\, U^o{}_v(r) - \vec{\gamma}.\hat{r}\, U^r{}_v(r) \,]\}\, \Psi(r) = E\, \Psi(r) . \tag{A1.1}$$

The wave function $\Psi(r)$, in terms of upper and lower components, is given by

$$\Psi(r) = \begin{bmatrix} \Psi_u(r) \\ \Psi_d(r) \end{bmatrix} . \tag{A1.2}$$

Substitution in to the Dirac equation yields,

$$\begin{bmatrix} 0 & \vec{\sigma}.\vec{p} \\ \vec{\sigma}.\vec{p} & 0 \end{bmatrix} \begin{bmatrix} \Psi_u(r) \\ \Psi_d(r) \end{bmatrix} + [m_N + U_s(r)] \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} \Psi_u(r) \\ \Psi_d(r) \end{bmatrix} - [E - U^o{}_v(r)] \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} \Psi_u(r) \\ \Psi_d(r) \end{bmatrix} - \begin{bmatrix} 0 & \vec{\sigma}.\hat{r} \\ \vec{\sigma}.\hat{r} & 0 \end{bmatrix} U^r{}_v(r) \begin{bmatrix} \Psi_u(r) \\ \Psi_d(r) \end{bmatrix} = 0. \tag{A1.3}$$

i.e., the equations satisfied by upper and lower components are given by

$$\vec{\sigma}.\vec{p}\ \Psi_d(r) + [\ m_N + U_s(r)\ ]\ \Psi_u(r)$$

$$- [\ E - U^o{}_v(r)\ ]\ \Psi_u(r) \quad - \vec{\sigma}.\hat{r}\ U^r{}_v(r)\ \Psi_d(r) = 0 \tag{A1.4}$$

and

$$\vec{\sigma}.\vec{p}\ \Psi_u(r) - [\ m_N + U_s(r)\ ]\ \Psi_d(r)$$

$$- [\ E - U^o{}_v(r)\ ]\ \Psi_d(r) \quad - \vec{\sigma}.\hat{r}\ U^r{}_v(r)\ \Psi_u(r) = 0\ . \tag{A1.5}$$

Eliminating the lower component in terms of the upper component , we have,

$$\Psi_d(r) = \{1/[\ (E + m_N)A(r)\ ]\}[\vec{\sigma}.\vec{p} - \vec{\sigma}.\hat{r}\ U^r{}_v(r)\ ]\ \Psi_u(r), \tag{A1.6}$$

where $\quad A(r) = [m_N + U_s + E - U^o{}_v(r)\ ]/[\ E + m_N\ ]\quad .$ (A1.7)

The equation obeyed by $\Psi_u(r)$ is

$$[\ (E - U^o{}_v(r))^2 - (m_N + U_s(r))^2 - F(r)\ ]\ \Psi_u(r) = 0\ , \tag{A1.8}$$

where

$$F(r) = A(r)\ [\ \vec{\sigma}.\vec{p} - \vec{\sigma}.\hat{r}\ U^r{}_v(r)\ ]\ [1/A(r)]\ [\vec{\sigma}.\vec{p} - \vec{\sigma}.\hat{r}\ U^r{}_v(r)\ ] \tag{A1.9}$$

i.e., $F(r) = \{- \vec{\nabla}^2 + (i/Ar)\ (\partial A/\partial r)\ (\hat{r}.\vec{p})$

$$- (1/Ar)\ (\partial A/\partial r)\ \vec{\sigma}.\vec{L} + U^r{}_v - (1/A)\ (\partial A/\partial r)\ i\ U^r{}_v +$$

$$i\ (2/r)\ U^r{}_v - (1/r)\ \hat{r}.\vec{p}\ U^r{}_v + (2/r)\ U^r{}_v\ \hat{r}.\vec{p}\ \} \ .$$

(A1.10)

Now write $\Psi_u(r) = K(r)\ \chi(r)$, (A1.11)

where $K(r) = A^{\frac{1}{2}} \exp\Big(\int [\ i\ d\vec{r}\ U^r{}_v\ ]$. (A1.12)

Defining

$$U_{Darwin} = (1/2E)\ [\ -\ \{1/(\ r^2\ A)\}(\partial/\partial r)\{\ r^2\ (\partial A/\partial r)\}$$

$$+\ \{\ 3/(4\ A^2\}\ (\partial A/\partial r)^2\}\ ]$$

(A1.13)

and

$$U_{so} = (\ 1/2E\ )\ \{\ -\ [\ 1/(Ar)\ ]\ [\ (\partial A/\partial r)]\ \}\ ,$$

(A1.14)

we have the equivalent Schrödinger equation :

$$[\ (\ E^2 - m^2{}_N\ ]\ \chi(r) =$$

$$[\ -\ \vec{\nabla}^2 + 2E\ (\ U_{eff} + U_{so}\ \vec{\sigma}.\vec{L}\ )\ ]\ \chi(r)\ .$$

(A1.15)

Here $U_{eff} = U_{Darwin}$ +

$$[\ 1/(2E)\ ]\ [\ 2E\ U^o_v + 2\ m_N\ U_s - (U^o_v)^2 + (U_s)^2\ ]\ .$$

(A1.16)

APPENDIX 2

Relativistic Scattering Equations:

We start from the equation

$$M = \Sigma + \Sigma\, g^{(+)}_{o}\, M\,. \tag{A2.1}$$

Here M is the generalized scattering amplitude,

Σ is the self-energy operator

and $g^{(+)}_{o}$ is the modified propagator.

Thus, $\langle p_3 p_4 | M | p_1 p_2 \rangle = \langle p_3 p_4 | \Sigma | p_1 p_2 \rangle +$

$$\iint \Big[\; d^4p_5\, d^4p_6\;\; \delta^4(P-p_5-p_6)\; \langle p_3 p_4 | \Sigma | p_5 p_6 \rangle\;\; g^{(+)}_{o}(p_5,p_6)\;\; \langle p_5 p_6 | M | p_1 p_2 \rangle \;\Big]. \tag{A2.2}$$

Here $P = p_1 + p_2 = p_3 + p_4 = p_5 + p_6$.

(See figure A.2.1).

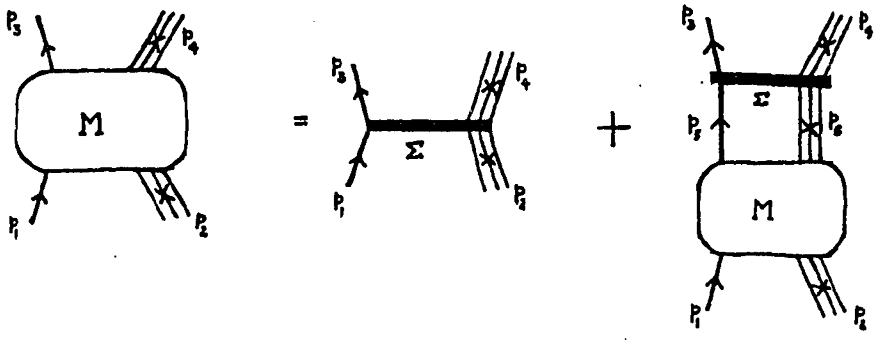


Figure A.2.1

Now introduce $p = (p_1-p_2)/2$,

$$p' = (p_3 - p_4)/2 , \qquad \text{and}$$

$$k'' = (p_5 - p_6)/2 .$$

Thus,

$$\langle p',P|M|p,P\rangle = \langle p',P|\Sigma|k'',P\rangle + \int [\, d^4k'' \, \langle p',P|\Sigma|k'',P\rangle \, g^{(+)}_o(k'',P) \, \langle k'',P|M|p,P\rangle \,] . \tag{A2.3}$$

Here $g^{(+)}_o(k'',P) =$

$$[i/(2\pi)^4] \quad \frac{1}{\gamma.(\tfrac{1}{2}P+k'')-m_N+i\varepsilon} \quad (\, - \, 2\pi i \,) \; \delta \{ \, (\, \tfrac{1}{2}P \, - \, k'')^2 \, - \, M_A^2 \, \} \, . \tag{A2.4}$$

In the center- of- momentum system $P = (W,0)$, where W is the total cm energy.

The equation

$$\langle p'|M(W)|p\rangle = \langle p'|\Sigma(W)|p\rangle + i/[(2\pi)^4] \int [\, dk'' \, \langle p'|\Sigma(W)|k''\rangle \, \frac{1}{\gamma.(\tfrac{1}{2}P+k'')-m_N+i\varepsilon} \, (\, - \, 2\pi i \,) \; \delta \{ \, (\, \tfrac{1}{2}P \, - \, k'')^2 \, - \, M_A^2 \, \} \quad \langle k''|M(W)|p\rangle \,] , \tag{A2.5}$$

upon evaluating the delta-function becomes

$$\langle p'|M(W)|p\rangle = \langle p'|\Sigma(W)|p\rangle + [1/(2\pi)^3] \int [\, d\vec{k}'' \quad (\gamma^o \, (\tfrac{1}{2}W+k''^o) - \vec{\gamma}.\vec{k}'' - m_N + i\varepsilon)^{-1} \quad \{1/(2E_A(\vec{k}''))\} \quad \langle \vec{k}''|M(W)|p\rangle \; \delta(k''^o - (W/2 - E_A(\vec{k}''))) \,] , \tag{A2.6}$$

with $k''^{o} = \frac{1}{2}W - E_A(\vec{k}'')$.

Also put $p^{o} = \frac{1}{2}W - E_A(\vec{p})$,

$$p'^{o} = \frac{1}{2}W - E_A(\vec{p}') \ .$$

Introducing the notation $k'' = [\ W - E_A(\vec{k}''), \vec{k}''\ ]$,

we have

$$\langle\vec{p}'|M(W)|\vec{p}\rangle = \langle\vec{p}'|\Sigma(W)|\vec{p}\rangle + [1/(2\pi)^3]\ \Big[\int d\vec{k}''\ \{1/(2E_A(\vec{k}''))\} \langle\vec{p}'|\Sigma(W)|\vec{k}''\rangle \ \frac{1}{\gamma . k'' - m_N + i\varepsilon}\ \langle\vec{k}''|M(W)|\vec{p}\rangle\ \Big] . \tag{A2.7}$$

Next introduce

$$\langle\vec{p}'|M'(W)|\vec{p}\rangle = [1/(2E_A(\vec{p}'))]^{\frac{1}{2}}\ \langle\vec{p}'|M(W)|\vec{p}\rangle\ [1/(2E_A(\vec{p}))]^{\frac{1}{2}}\ \frac{1}{(2\pi)^3} , \tag{A2.8}$$

$$\langle\vec{p}'|\Sigma'(W)|\vec{p}\rangle = [1/(2E_A(\vec{p}'))]^{\frac{1}{2}}\ \langle\vec{p}'|\Sigma(W)|\vec{p}\rangle\ [1/(2E_A(\vec{p}))]^{\frac{1}{2}}\ \frac{1}{(2\pi)^3} \tag{A2.9}$$

and

$$M'(W)|\vec{p}\rangle\ u(\vec{p},s) = \Sigma'(W)|\Psi^{(+)}_{\vec{p},s}\rangle \ . \tag{A2.10}$$

Multiplying (A2.7) by $u(\vec{p},s)$ and using the fact that

$$\int d\vec{p}''\ |\vec{p}''\rangle\langle\vec{p}''| = 1 \tag{A2.11}$$

and using (A2.8) - (A2.10) we get,

$$\int [\, d\vec{p}'' \;\; \langle \vec{p}'|\Sigma'(W)|\vec{p}''\rangle \; \langle \vec{p}''|\Psi^{(+)}_{\vec{p},s}\rangle \,] =$$

$$\int [\, d\vec{p}'' \;\; \langle \vec{p}'|\Sigma'(W)|\vec{p}''\rangle \; \langle \vec{p}''|\vec{p}\rangle \; u(\vec{p},s) \,] \;\; +$$

$$\iint [\;\; d\vec{p}'' \; d\vec{k}'' \;\; \langle \vec{p}'|\Sigma'(W)|\vec{k}''\rangle$$

$$\frac{1}{\gamma.k''-m_N+i\varepsilon} \;\; \langle \vec{k}''|\Sigma'(W)|\vec{p}''\rangle \;\; \langle \vec{p}''|\Psi^{(+)}_{\vec{p},s}\rangle \,] \;. \tag{A2.12}$$

ie., $\langle \vec{p}'|\Psi^{(+)}_{\vec{p},s}\rangle = \delta(\vec{p}'-\vec{p}) \; u(\vec{p},s)$

$$+\int [\;\; d\vec{k}'' \;\;\; \frac{1}{\gamma.k''-m_N+i\varepsilon}$$

$$\langle \vec{p}'|\Sigma'(W)|\vec{k}''\rangle \; \langle \vec{k}''|\Psi^{(+)}_{\vec{p},s}\rangle \,] \;. \tag{A2.13}$$

Using $(\gamma.p' - m_N) \; \delta(\vec{p}-\vec{p}') \; u(\vec{p},s) = 0$, (A2.14)

we arrive at the Dirac equation :

$$[\gamma^{o}\{W - E_A(\vec{p}')\} - \vec{\gamma}.\vec{p}' - m_N] \; \langle \vec{p}'|\Psi^{(+)}_{\vec{p},s}\rangle =$$

$$\int [\;\; d\vec{k}'' \;\; \langle \vec{p}'|\Sigma'(W)|\vec{k}''\rangle \; \langle \vec{k}''|\Psi^{(+)}_{\vec{p},s}\rangle \,] \;. \tag{A2.15}$$

APPENDIX 3

Decomposition of the Relativistic Density Matrix:

The relativistic density matrix (in momentum space) has the form:

$$\rho(\vec{Q}':\vec{Q}'') = \Sigma_{\ell j}\ \Sigma_{m_j}\ \Psi_{\ell j m_j}(\vec{Q}')\ \bar{\Psi}_{\ell j m_j}(\vec{Q}'') \tag{A3.1}$$

We start with the expression for momentum-space wave function $\Psi_{\ell j m_j}$:

$$\Psi_{\ell j m_j}(\vec{Q}') = \begin{bmatrix} R^u{}_{\ell j}(|\vec{Q}'|)\ y_{\ell j m_j}(\hat{Q}') \\ \\ R^d{}_{\ell j}(|\vec{Q}'|)\ \vec{\sigma}.\hat{Q}'\ y_{\ell j m_j}(\hat{Q}') \end{bmatrix} \tag{A3.2}$$

Hence

$$\bar{\Psi}_{\ell j m_j}(\vec{Q}'') = [\ R^u{}_{\ell j}(|\vec{Q}''|)\ y_{\ell j m_j}(\hat{Q}'') \quad - R^d{}_{\ell j}(|\vec{Q}''|)\ y^{\dagger}_{\ell j m_j}(\hat{Q}'')\ \vec{\sigma}.\hat{Q}''] \tag{A3.3}$$

and ρ =

$$\Sigma_{\ell j}\ \Sigma_{m_j} \begin{bmatrix} R^u{}_{\ell j}(|\vec{Q}'|)\ R^u{}_{\ell j}(|\vec{Q}''|) & - R^u{}_{\ell j}(|\vec{Q}'|)\ R^d{}_{\ell j}(|\vec{Q}''|) \\ y_{\ell j m_j}(\hat{Q}')\ y^{\dagger}_{\ell j m_j}(\hat{Q}'') & y_{\ell j m_j}(\hat{Q}')\ y^{\dagger}_{\ell j m_j}(\hat{Q}'')\ \vec{\sigma}.\hat{Q}'' \\ & \\ R^d{}_{\ell j}(|\vec{Q}'|)\ R^u{}_{\ell j}(|\vec{Q}''|) & - R^d{}_{\ell j}(|\vec{Q}'|)\ R^d{}_{\ell j}(|\vec{Q}''|) \\ \vec{\sigma}.\hat{Q}'\ y_{\ell j m_j}(\hat{Q}')\ y^{\dagger}_{\ell j m_j}(\hat{Q}'') & \vec{\sigma}.\hat{Q}'\ y_{\ell j m_j}(\hat{Q}')\ y^{\dagger}_{\ell j m_j}(\hat{Q}'')\ \vec{\sigma}.\hat{Q}'' \end{bmatrix} \tag{A3.4}$$

Now write

$$\rho = \rho_s + \gamma^\mu \rho_{V_\mu} + \sigma^{\mu\nu} \rho_{T_{\mu\nu}} + \gamma^5 \rho_{ps} + \gamma^5 \gamma^\mu \rho_{av_\mu} \tag{A3.5}$$

By taking traces we get

$$\begin{aligned} \rho_s = (1/4)\, \Sigma_{\ell j} \{ & \, 2\, R^u{}_{\ell j}(|\vec{Q}'|)\, R^u{}_{\ell j}(|\vec{Q}''|)\, A_{\ell j}\, P_\ell(\cos \vartheta) \\ & - 2\, R^d{}_{\ell j}(|\vec{Q}'|)\, R^d{}_{\ell j}(|\vec{Q}''|) \\ & [A_{\ell j} \cos \vartheta\, P_\ell(\cos \vartheta) + (B_{\ell j}/\sin \vartheta)\, P_\ell^{(1)}(\cos \vartheta) \sin^2 \vartheta] \} \;. \end{aligned} \tag{A3.6}$$

$$\begin{aligned} \rho^o{}_V = (1/4) \Sigma_{\ell j} \{ & \, 2\, R^u{}_{\ell j}(|\vec{Q}'|)\, R^u{}_{\ell j}(|\vec{Q}''|)\, A_{\ell j}\, P_\ell(\cos \vartheta) \\ & + 2\, R^d{}_{\ell j}(|\vec{Q}'|)\, R^d{}_{\ell j}(|\vec{Q}''|) \\ & [A_{\ell j} \cos \vartheta\, P_\ell(\cos \vartheta) + (B_{\ell j}/\sin \vartheta)\, P_\ell^{(1)}(\cos \vartheta) \sin^2 \vartheta] \} \;. \end{aligned} \tag{A3.7}$$

$$\begin{aligned} \vec{\rho}_V = (1/2)\, \Sigma_{\ell j} [[& \, \hat{Q}' \{ R^u{}_{\ell j}(|\vec{Q}'|)\, R^d{}_{\ell j}(|\vec{Q}''|) \\ & (B_{\ell j}/\sin \vartheta)\, P_\ell^{(1)}(\cos \vartheta) \\ & + R^d{}_{\ell j}(|\vec{Q}'|)\, R^u{}_{\ell j}(|\vec{Q}''|) \\ & [A_{\ell j}\, P_\ell(\cos \vartheta) - (B_{\ell j}/\sin \vartheta) \cos \vartheta\, P_\ell^{(1)}(\cos \vartheta)]\} \\ & + \hat{Q}'' \{ R^u{}_{\ell j}(|\vec{Q}'|)\, R^d{}_{\ell j}(|\vec{Q}''|) \\ & [A_{\ell j}\, P_\ell(\cos \vartheta) - (B_{\ell j}/\sin \vartheta) \cos \vartheta\, P_\ell^{(1)}(\cos \vartheta)] \\ & + R^d{}_{\ell j}(|\vec{Q}'|)\, R^u{}_{\ell j}(|\vec{Q}''|) \\ & (B_{\ell j}/\sin \vartheta)\, P_\ell^{(1)}(\cos \vartheta)\}]] \quad . \end{aligned} \tag{A3.8}$$

$$\begin{aligned} \rho^{oi}{}_T = (i/4)\, \Sigma_{\ell j}[[& \, \hat{Q}'^i \{ - R^u{}_{\ell j}(|\vec{Q}'|)\, R^d{}_{\ell j}(|\vec{Q}''|) \\ & (B_{\ell j}/\sin \vartheta)\, P_\ell^{(1)}(\cos \vartheta) \\ & + R^d{}_{\ell j}(|\vec{Q}'|)\, R^u{}_{\ell j}(|\vec{Q}''|) \\ & [A_{\ell j}\, P_\ell(\cos \vartheta) - (B_{\ell j}/\sin \vartheta) \cos \vartheta\, P_\ell^{(1)}(\cos \vartheta)]\} \end{aligned}$$

$$
\begin{aligned}
&+\hat{Q}''^{i}\{-R^{u}{}_{\ell j}(|\vec{Q}'|)\ R^{d}{}_{\ell j}(|\vec{Q}''|)\\
&[A_{\ell j}\ P_{\ell}(\cos\vartheta)\ -\ (B_{\ell j}/\sin\vartheta)\ \cos\vartheta\ P_{\ell}^{(1)}(\cos\vartheta)]\\
&+R^{d}{}_{\ell j}(|\vec{Q}'|)\ R^{u}{}_{\ell j}(|\vec{Q}''|)\\
&(B_{\ell j}/\sin\vartheta)\ P_{\ell}^{(1)}(\cos\vartheta)\}]]\ .
\end{aligned}
\tag{A3.9}
$$

$$
= (i/4)\ f^{To\ i}. \tag{A3.10}
$$

$$
\begin{aligned}
\rho^{ij}{}_{T} = (i/4)\ \varepsilon^{ijk}(\hat{Q}'\times\hat{Q}'')^{k}\\
&\Sigma_{\ell j}\ \{R^{u}{}_{\ell j}(|\vec{Q}'|)\ R^{u}{}_{\ell j}(|\vec{Q}''|)\\
&(B_{\ell j}/\sin\vartheta)\ P_{\ell}^{(1)}(\cos\vartheta)\\
&-R^{d}{}_{\ell j}(|\vec{Q}'|)\ R^{d}{}_{\ell j}(|\vec{Q}''|)\\
&[A_{\ell j}\ P_{\ell}(\cos\vartheta)\ -\ (B_{\ell j}/\sin\vartheta)\ \cos\vartheta\ P_{\ell}^{(1)}(\cos\vartheta)]\}\ .
\end{aligned}
\tag{A3.11}
$$

$$
= (i/4)\ \varepsilon^{ijl}(\hat{Q}'\times\hat{Q}'')^{l}\ f^{T}. \tag{A3.12}
$$

$$
\rho_{ps} = 0. \tag{A3.13}
$$

$$
\rho^{o}{}_{av} = 0\ , \tag{A3.14}
$$

$$
\begin{aligned}
\vec{\rho}_{av} = (i/2)\ (\hat{Q}'\times\hat{Q}'')\\
&\Sigma_{\ell j}\{R^{u}{}_{\ell j}(|\vec{Q}'|)\ R^{u}{}_{\ell j}(|\vec{Q}''|)\\
&(B_{\ell j}/\sin\vartheta)\ P_{\ell}^{(1)}(\cos\vartheta)\\
&+\ R^{d}{}_{\ell j}(|\vec{Q}'|)\ R^{d}{}_{\ell j}(|\vec{Q}''|)\\
&[A_{\ell j}\ P_{\ell}(\cos\vartheta)\ -\ (B_{\ell j}/\sin\vartheta)\ \cos\vartheta\ P_{\ell}^{(1)}(\cos\vartheta)]\}\ .
\end{aligned}
\tag{A3.15}
$$

$$
= (i/2)\ (\hat{Q}'\times\hat{Q}'')\ f^{av}\ . \tag{A3.16}
$$

In these expressions we have defined

$$y_{\ell j m_j} = \Sigma_{m_\ell m_s} C^{\ell \ \frac{1}{2} \ j}_{m_\ell m_s m_j} \quad Y_{\ell m_\ell} \chi_{\frac{1}{2} m_s} \tag{A3.17}$$

Here $Y_{\ell m_\ell}$ is the spherical harmonic function,

$\chi_{\frac{1}{2} m_s}$ is the Pauli spinor,

$$A_{\ell j} = (2j+1)/(8\pi) \qquad \text{for } j=\ell+\tfrac{1}{2}, \tag{A3.18}$$

$$B_{\ell j} = \begin{cases} (-1/4\pi) & \text{for } j=\ell+\frac{1}{2}, \\ (+1/4\pi) & \text{for } j=\ell-\frac{1}{2}, \end{cases} \tag{A3.19}$$

$$\cos \vartheta = \hat{Q}'.\hat{Q}'' , \tag{A3.20}$$

and

$$P_\ell^{(1)}(\cos \vartheta) = [(l+1)/(\sin \vartheta)] \; [\cos \vartheta \, P_\ell(\cos \vartheta) - P_{\ell+1}(\cos \vartheta)]. \tag{A3.21}$$

Here $P_\ell(\cos \vartheta)$ is the Legendre polynomial.

Further we have used the relations for

$j = \ell + \frac{1}{2}$:

$$\Sigma_{m_j}\ y_{\ell j m_j}(\hat{Q}')\ y^{\dagger}_{\ell j m_j}(\hat{Q}'')$$

$$= [(\ell+1)/(4\pi)]\ P_\ell(\cos\vartheta) - i\,\vec{\sigma}.(\ \hat{Q}' \times \hat{Q}''\ )\ \frac{1}{\sin\vartheta}\ \frac{1}{4\pi}\ P^{(1)}_\ell(\cos\vartheta) \tag{A3.22}$$

and for $j = \ell - \frac{1}{2}$:

$$\Sigma_{m_j}\ y_{\ell j m_j}(\hat{Q}')\ y^{\dagger}_{\ell j m_j}(\hat{Q}'')$$

$$= [1/(4\pi)]\ P_\ell(\cos\vartheta) + i\,\vec{\sigma}.(\ \hat{Q}' \times \hat{Q}''\ )\ \frac{1}{\sin\vartheta}\ \frac{1}{4\pi}\ P^{(1)}_\ell(\cos\vartheta). \tag{A3.23}$$

APPENDIX 4

Free Nucleon Electromagnetic Form Factors

Defining $k^2 = -Q^2 = q^2 - \omega^2$, we have

$$G_E^p(k^2) = \frac{1}{(1 + k^2/18.234)^2}, \tag{A4.1}$$

where

k is in fm^{-1}. Further,

$$G_E^n(k^2) = 1.91\,[k^2/(4 m_N^2)]\,\frac{1}{(1 + k^2/18.234)^2}, \tag{A4.2}$$

$$G_M^p(k^2) = 2.793\,\frac{1}{(1 + k^2/18.234)^2}, \tag{A4.3}$$

$$G_M^n(k^2) = -1.91\,\frac{1}{(1 + k^2/18.234)^2}, \tag{A4.4}$$

and

$$G_M(k^2) = F_1(k^2) + F_2(k^2), \tag{A4.5}$$

$$G_E(k^2) = F_1(k^2) - [k^2/4 m_N^2]\,F_2(k^2). \tag{A4.6}$$

FIGURES

LONGITUDINAL RESPONSE FUNCTION

FINITE NUCLEUS MODEL

Three momentum transfer q = 550 MeV/c

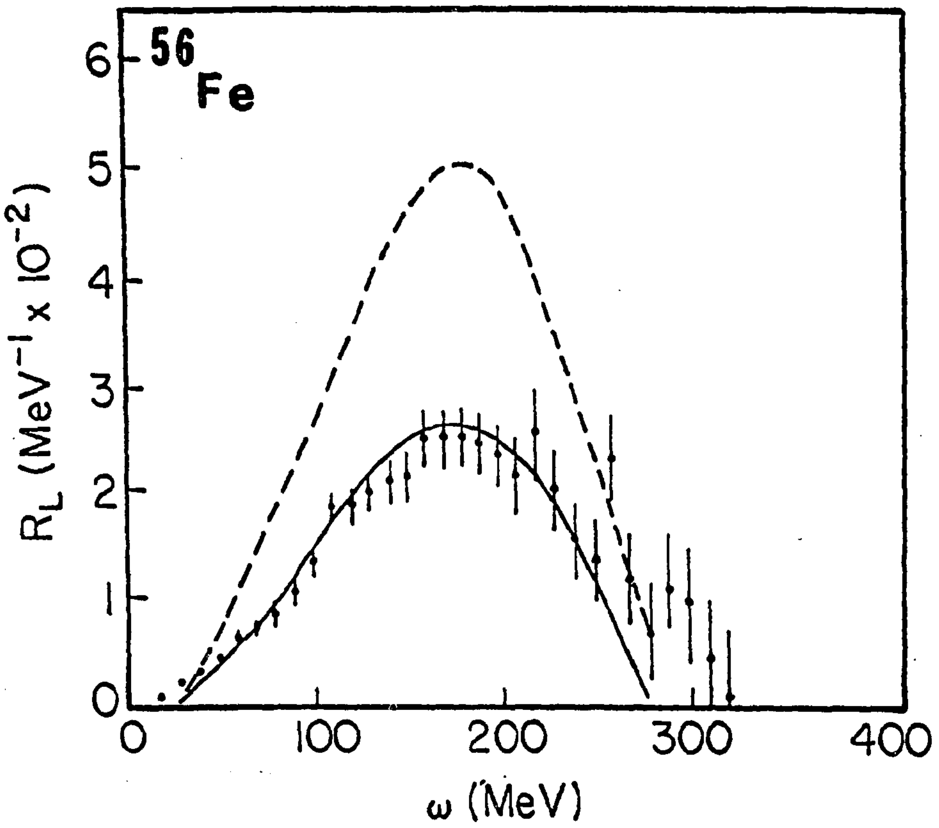


Fig. 1

LONGITUDINAL RESPONSE FUNCTION

FERMI GAS MODEL

Three momentum transfer q = 550 MeV/c

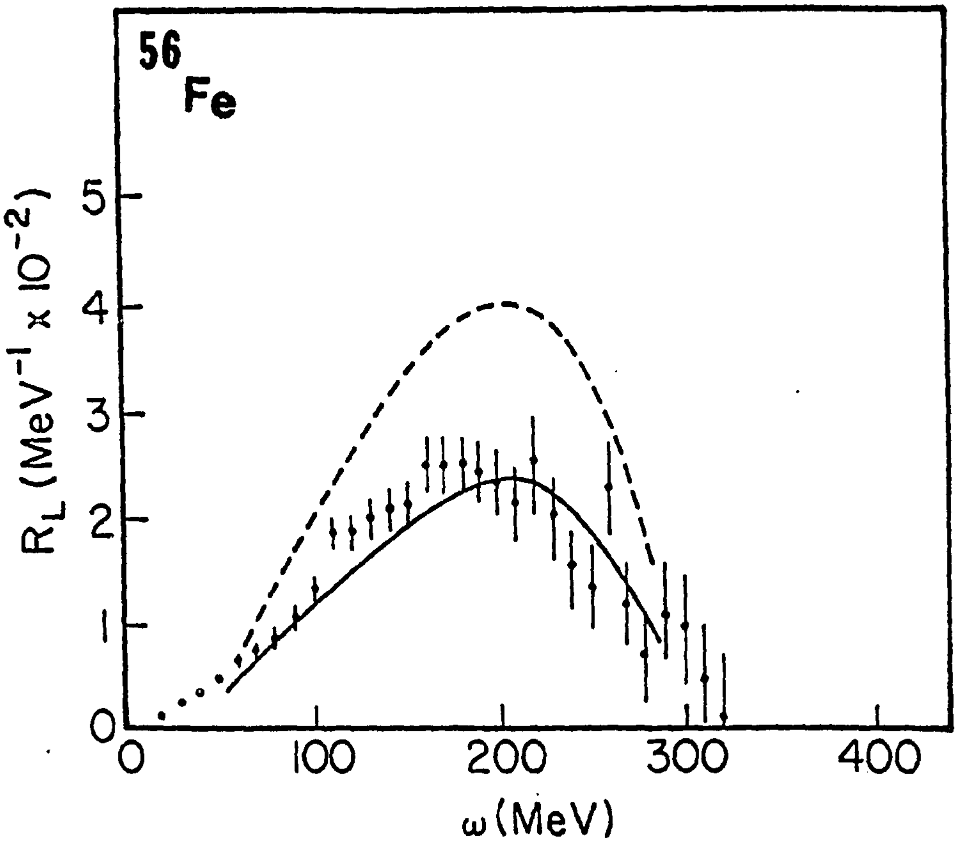


Fig. 2

LONGITUDINAL RESPONSE FUNCTION

FINITE NUCLEUS MODEL

Three momentum transfer q = 550 MeV/c

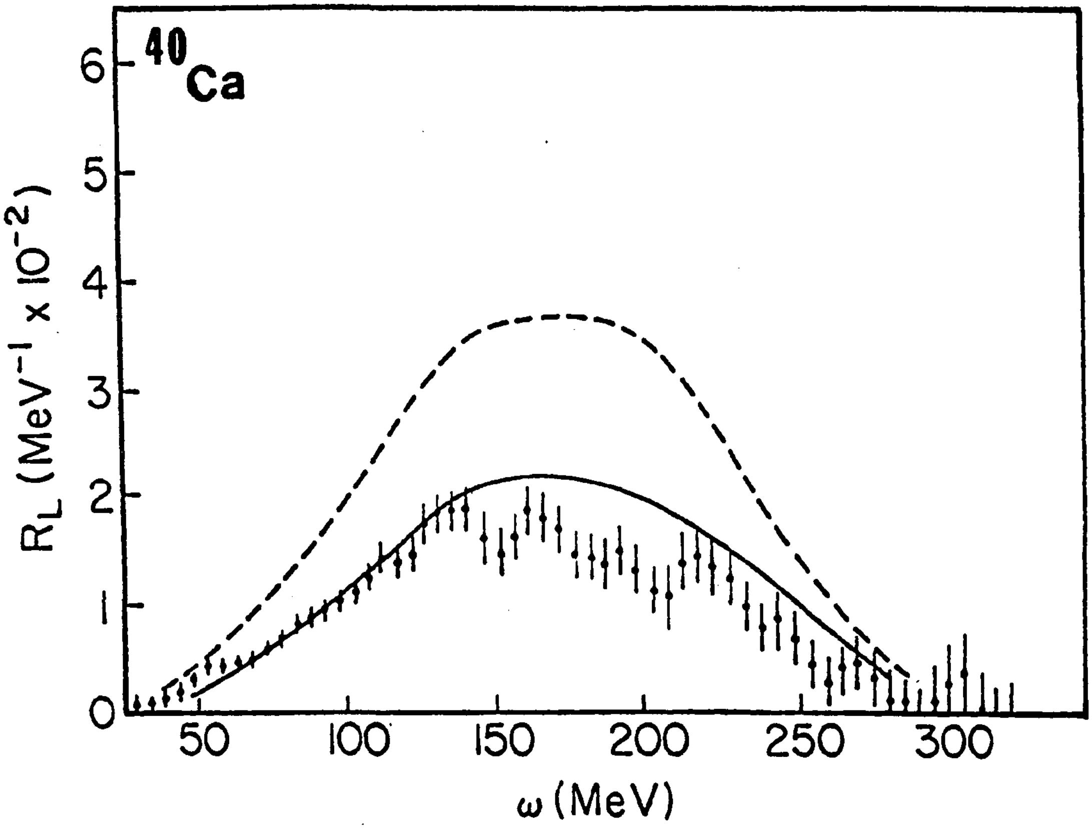


Fig. 3

LONGITUDINAL RESPONSE FUNCTION

FERMI GAS MODEL

Three momentum transfer q = 550 MeV/c

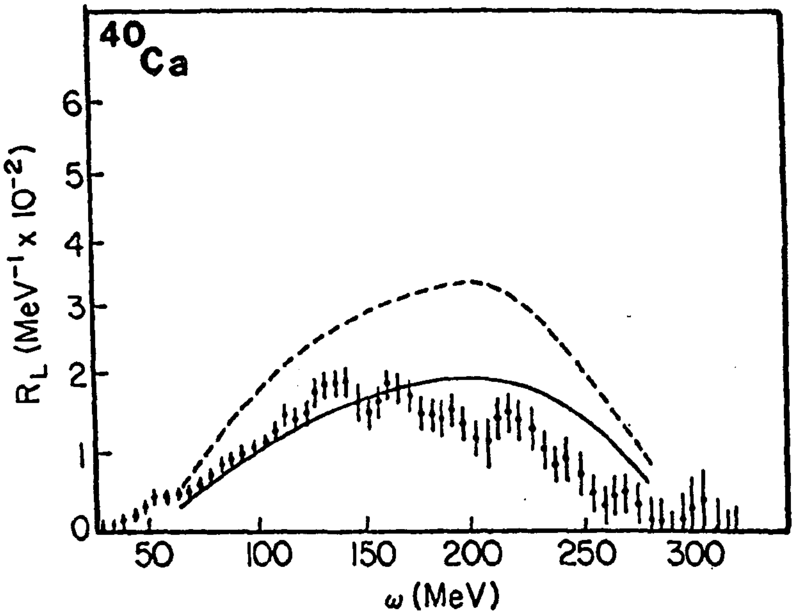


Fig. 4

LONGITUDINAL RESPONSE FUNCTION

FINITE NUCLEUS MODEL

Three momentum transfer q = 550 MeV/c

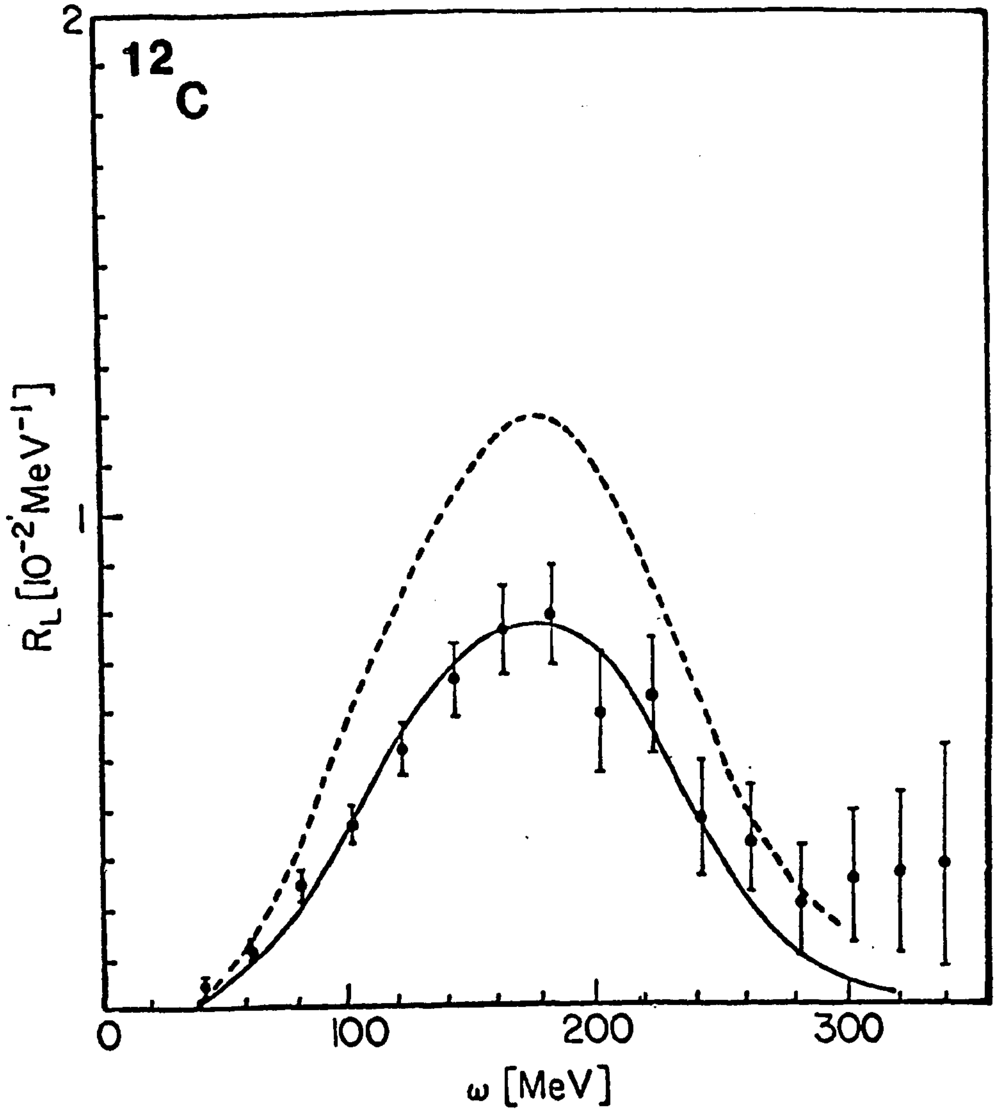


Fig. 5

LONGITUDINAL RESPONSE FUNCTION

FERMI GAS MODEL

Three momentum transfer q = 550 MeV/c

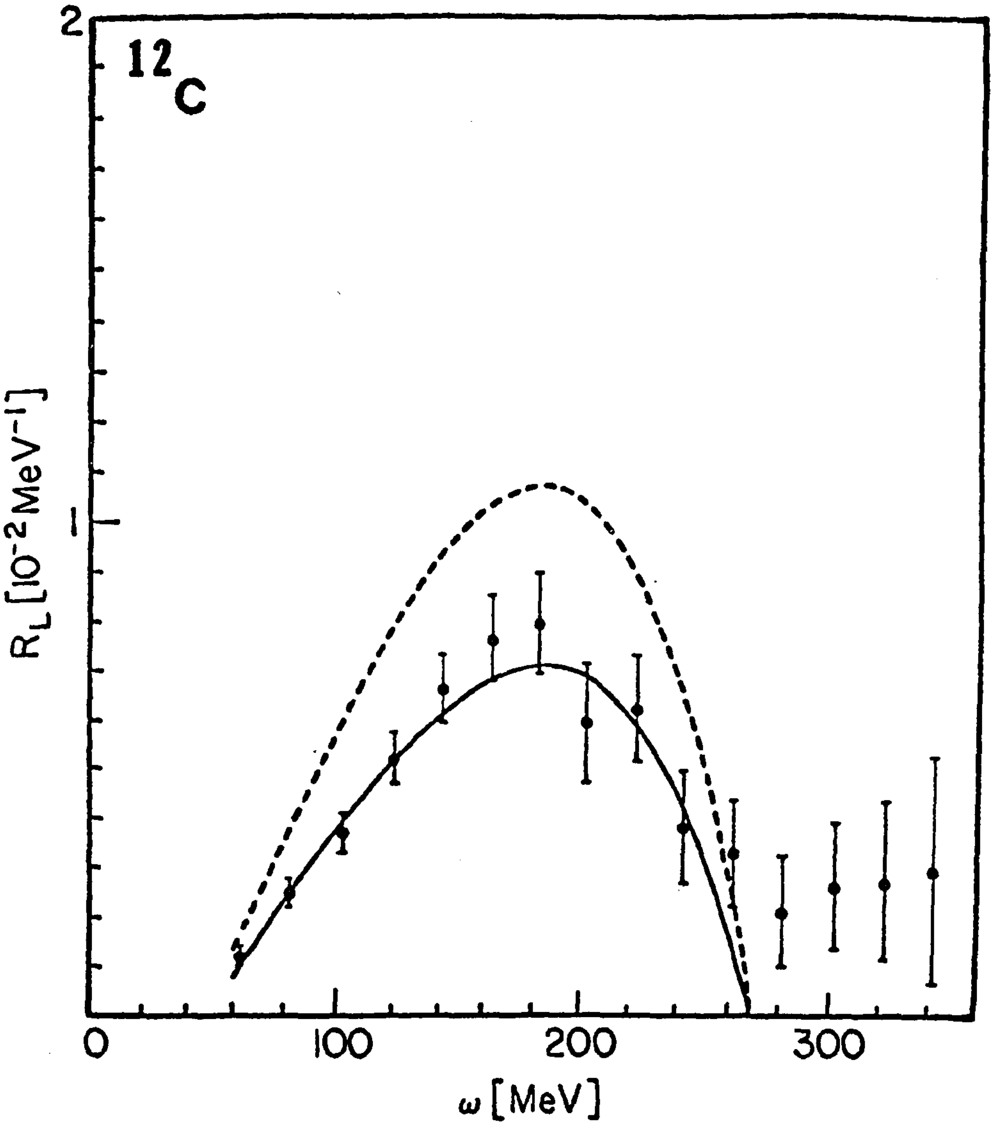


Fig. 6

LONGITUDINAL RESPONSE FUNCTION

FINITE NUCLEUS MODEL

Three momentum transfer q = 410 MeV/c

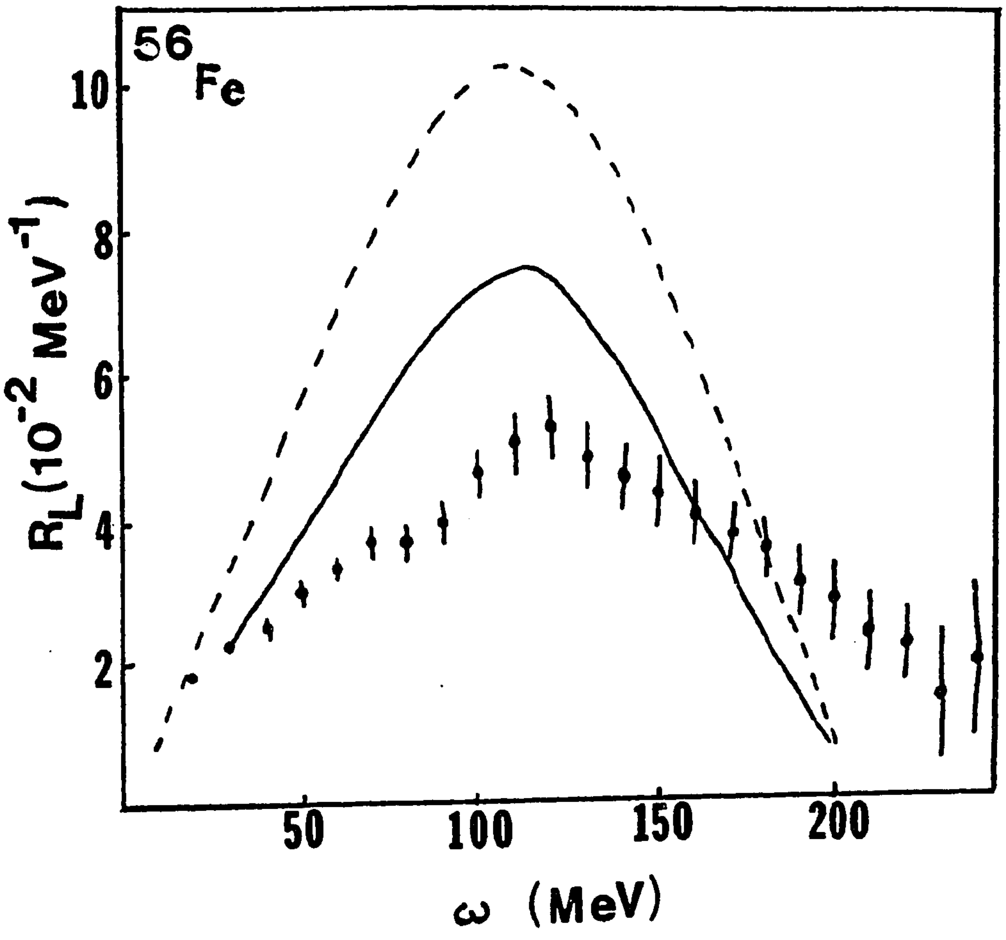


Fig. 7

LONGITUDINAL RESPONSE FUNCTION

FERMI GAS MODEL

Three momentum transfer q = 410 MeV/c

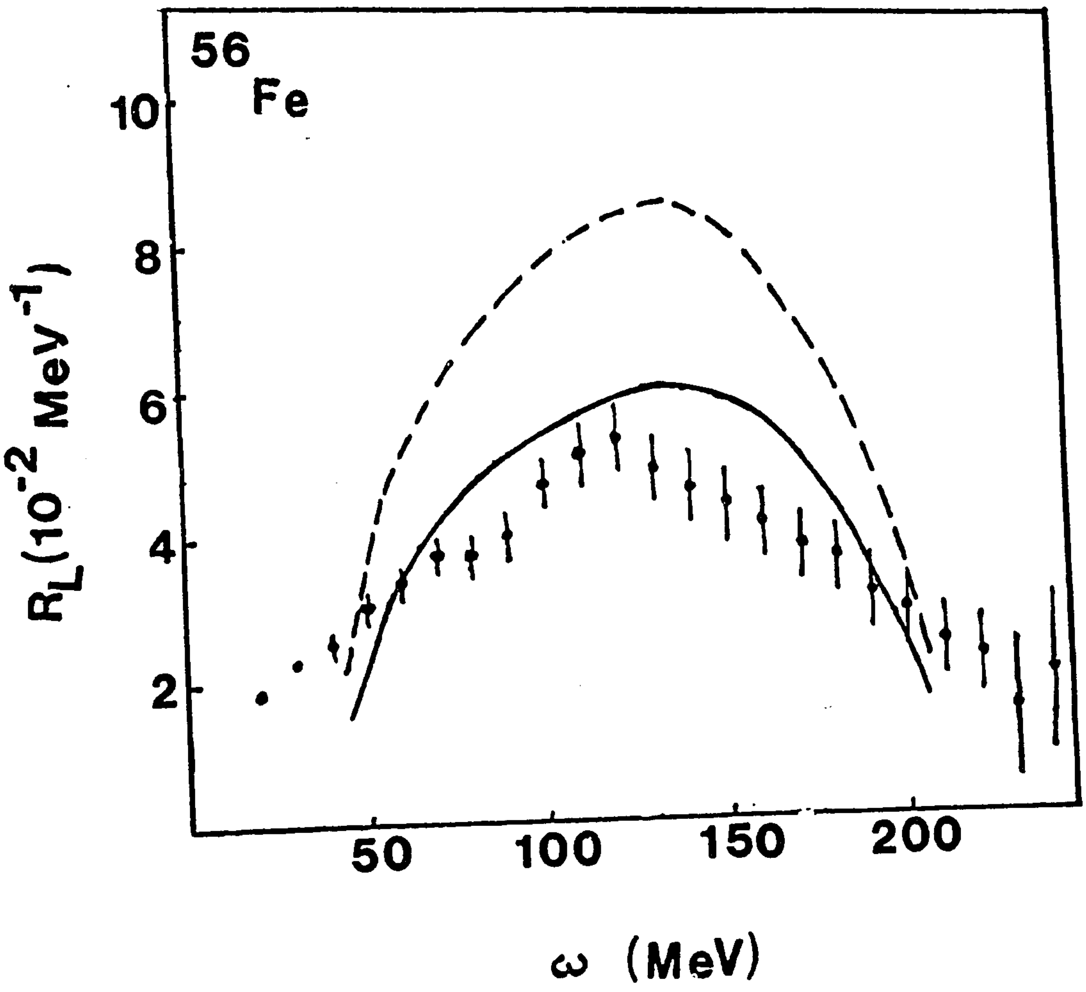
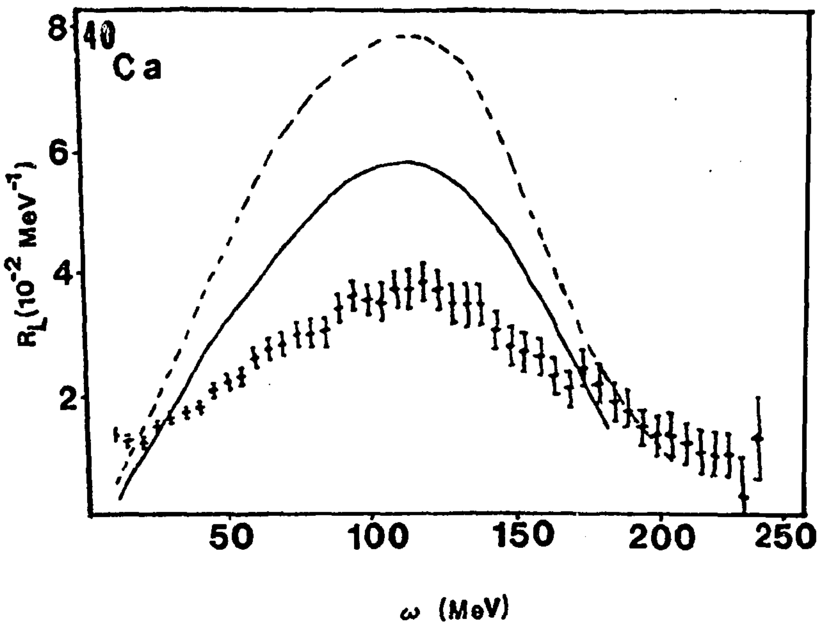


Fig. 8



LONGITUDINAL RESPONSE FUNCTION

FINITE NUCLEUS MODEL

Three momentum transfer q = 410 MeV/c

Fig. 9

LONGITUDINAL RESPONSE FUNCTION

FERMI GAS MODEL

Three momentum transfer q = 410 MeV/c

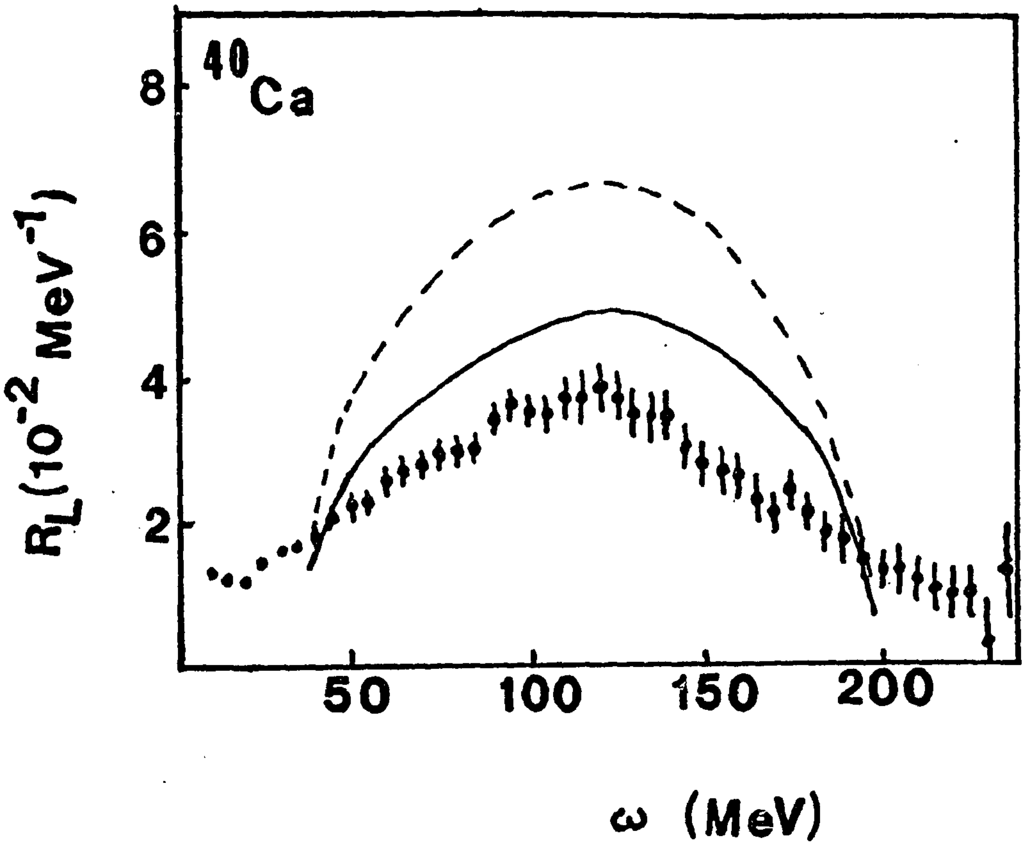


Fig. 10 .

TRANSVERSE RESPONSE FUNCTION

FINITE NUCLEUS MODEL

Three momentum transfer q = 550 MeV/c

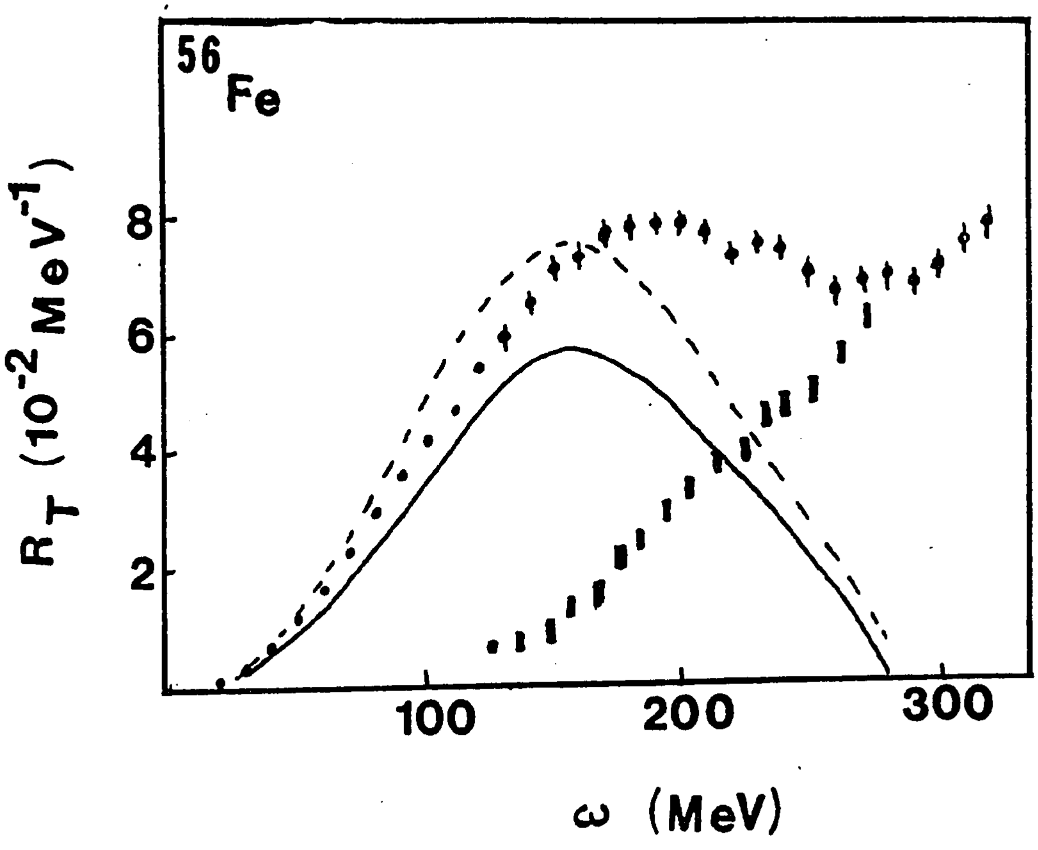


Fig. 11

TRANSVERSE RESPONSE FUNCTION

FINITE NUCLEUS MODEL

Three momentum transfer q = 410 MeV/c

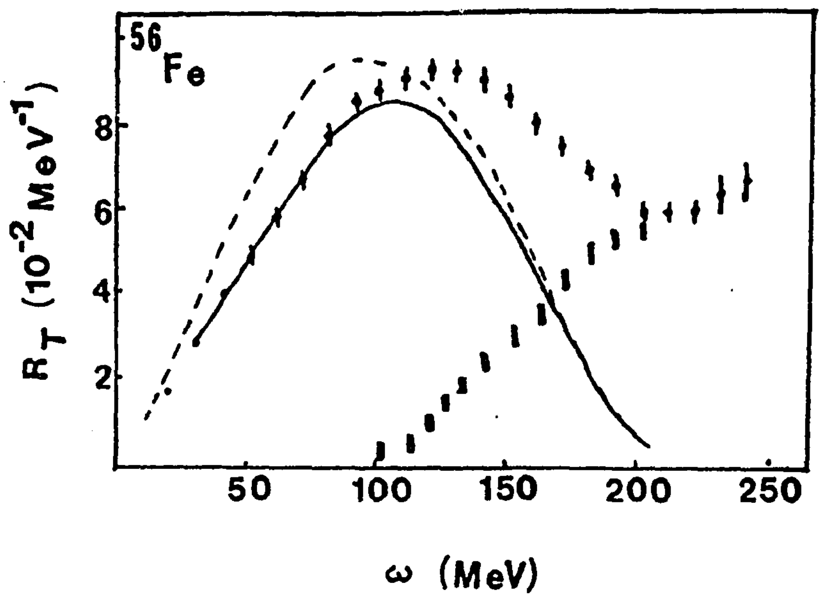


Fig. 12

TRANSVERSE RESPONSE FUNCTION

FINITE NUCLEUS MODEL

Three momentum transfer q = 550 MeV/c

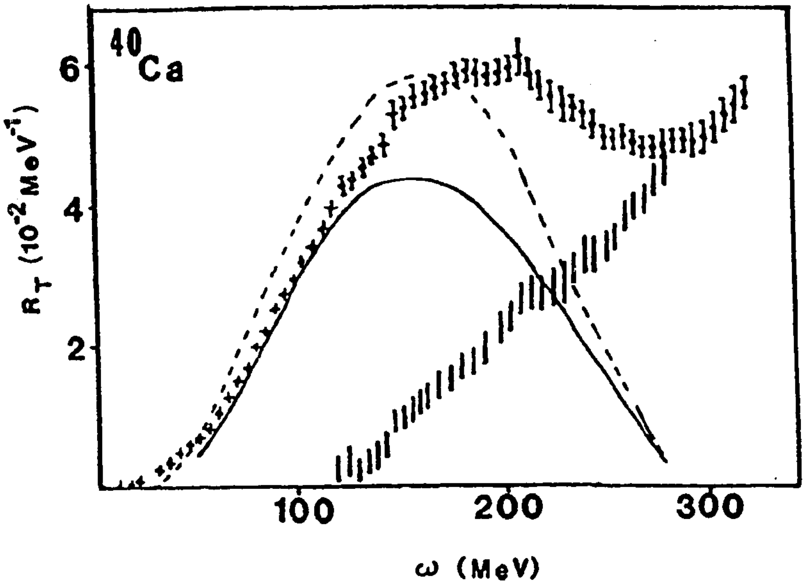


Fig. 13

TRANSVERSE RESPONSE FUNCTION

FINITE NUCLEUS MODEL

Three momentum transfer q = 410 MeV/c

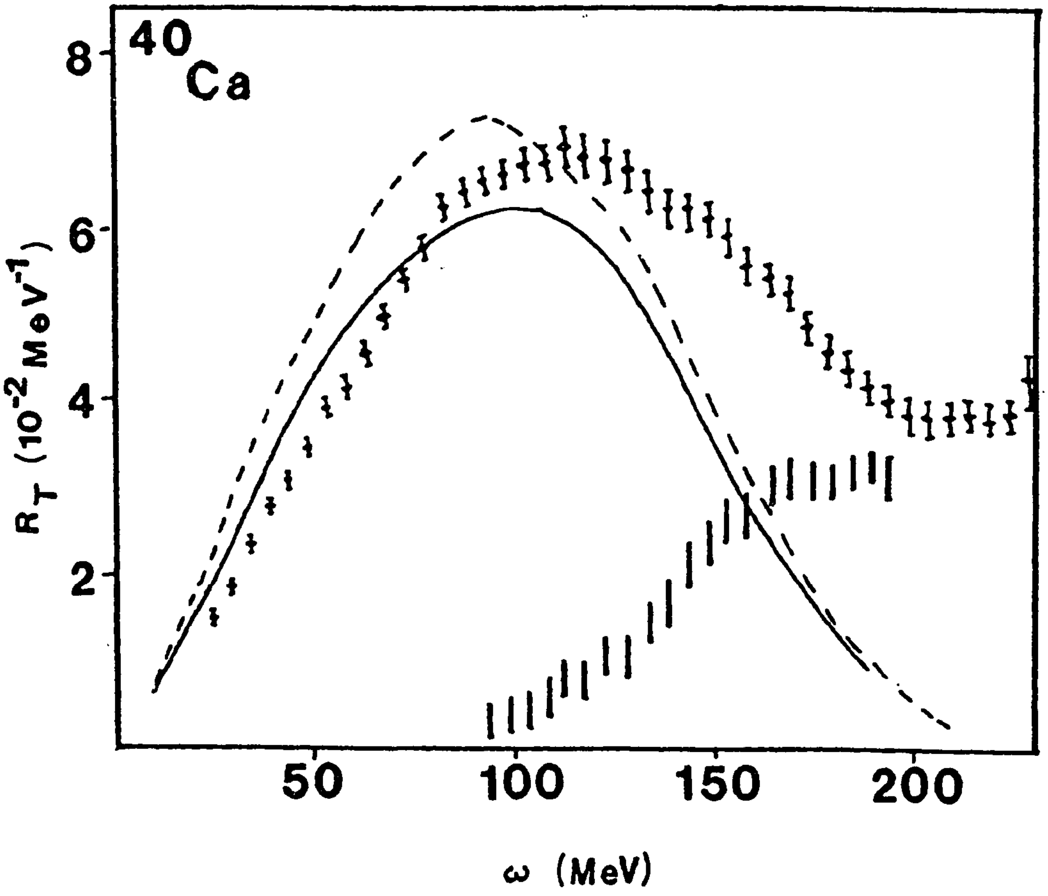


Fig. 14

TRANSVERSE RESPONSE FUNCTION

FERMI GAS MODEL

Three momentum transfer q = 550 MeV/c

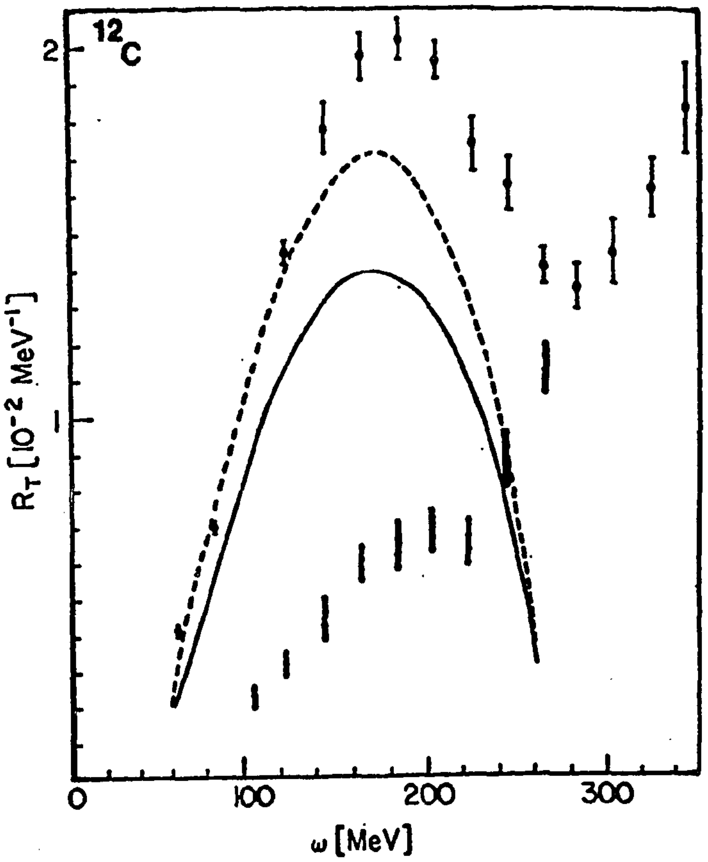


Fig. 15

TRANSVERSE RESPONSE FUNCTION

FERMI GAS MODEL

Three momentum transfer q = 400 MeV/c

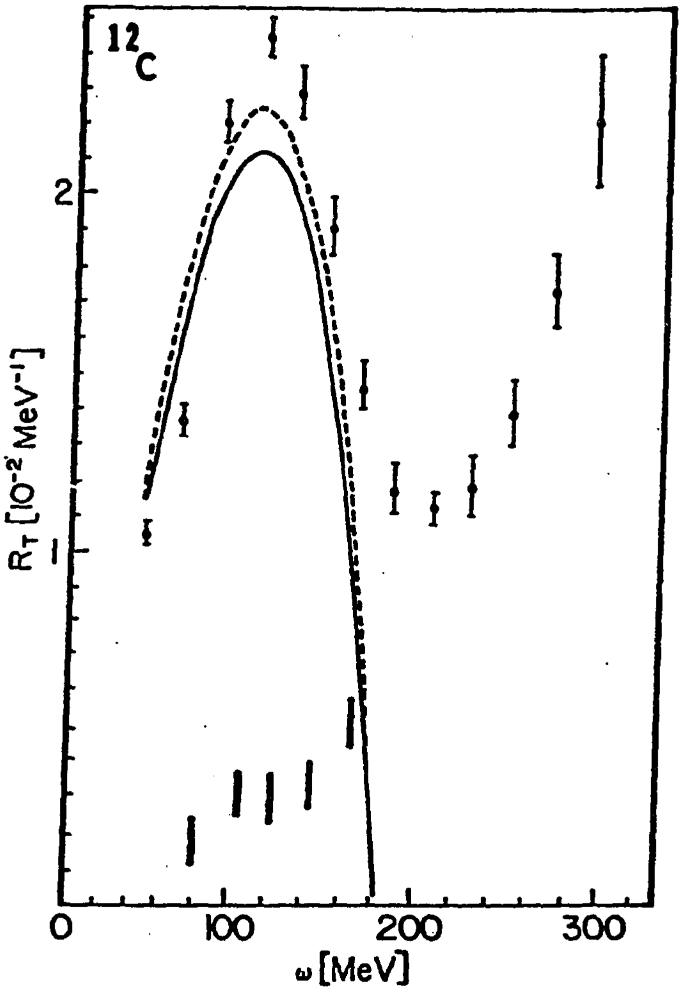


Fig. 16

Fig. 17 LONGITUDINAL RESPONSE FUNCTION

FERMI GAS MODEL

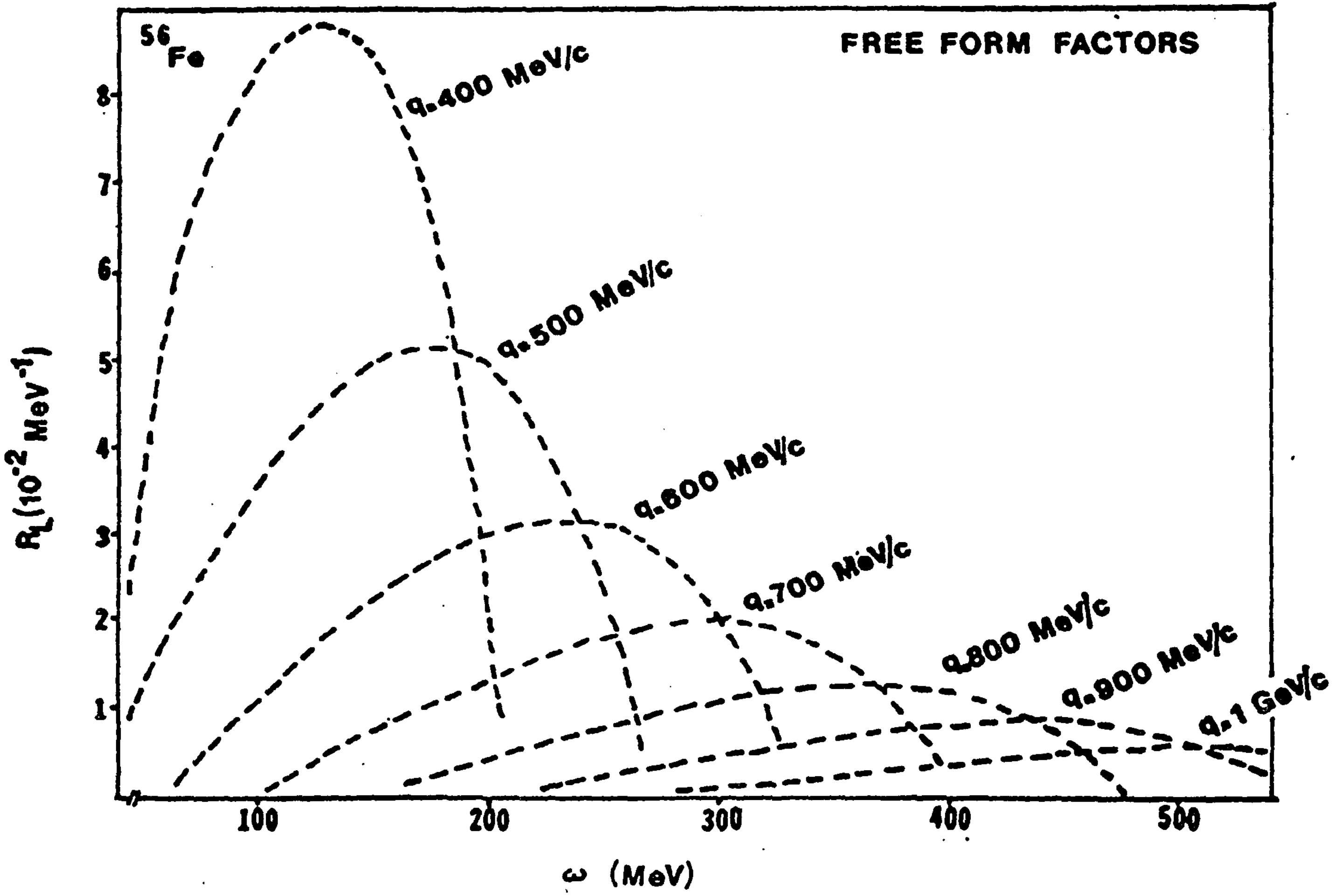


Fig. 18 LONGITUDINAL RESPONSE FUNCTION

FERMI GAS MODEL

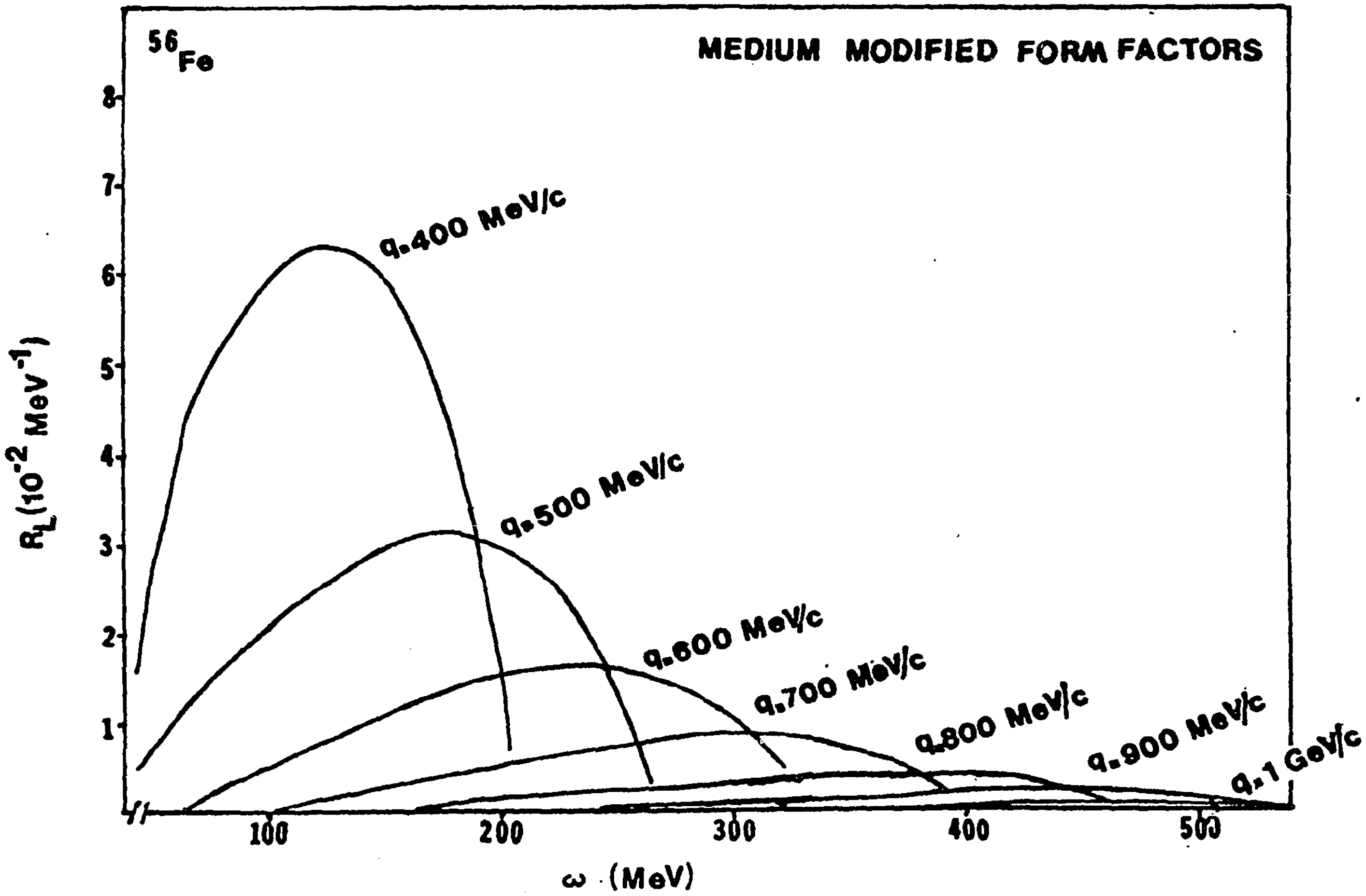


Fig. 19 TRANSVERSE RESPONSE FUNCTION

FERMI GAS MODEL

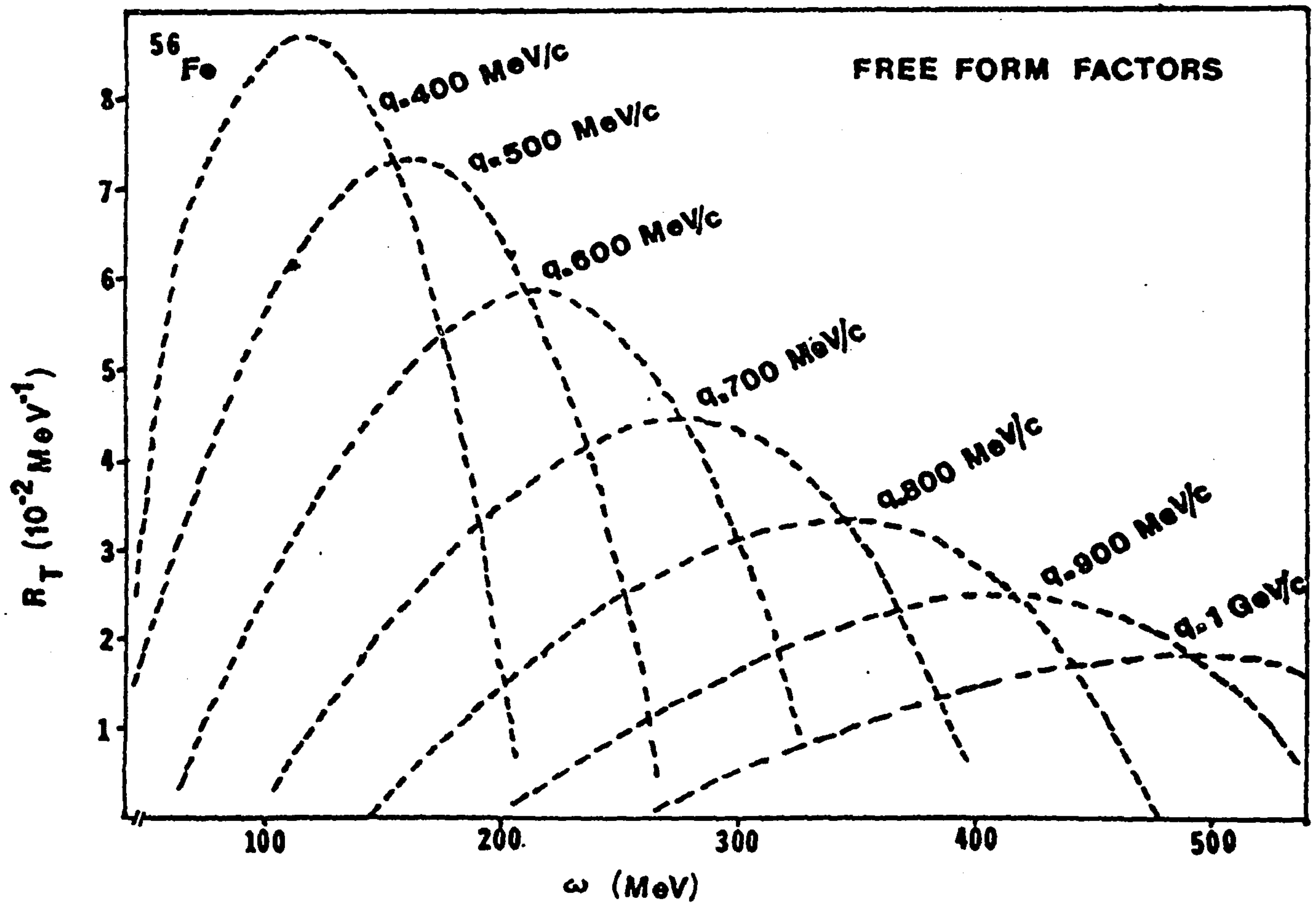


Fig. 20 TRANSVERSE RESPONSE FUNCTION

FERMI GAS MODEL

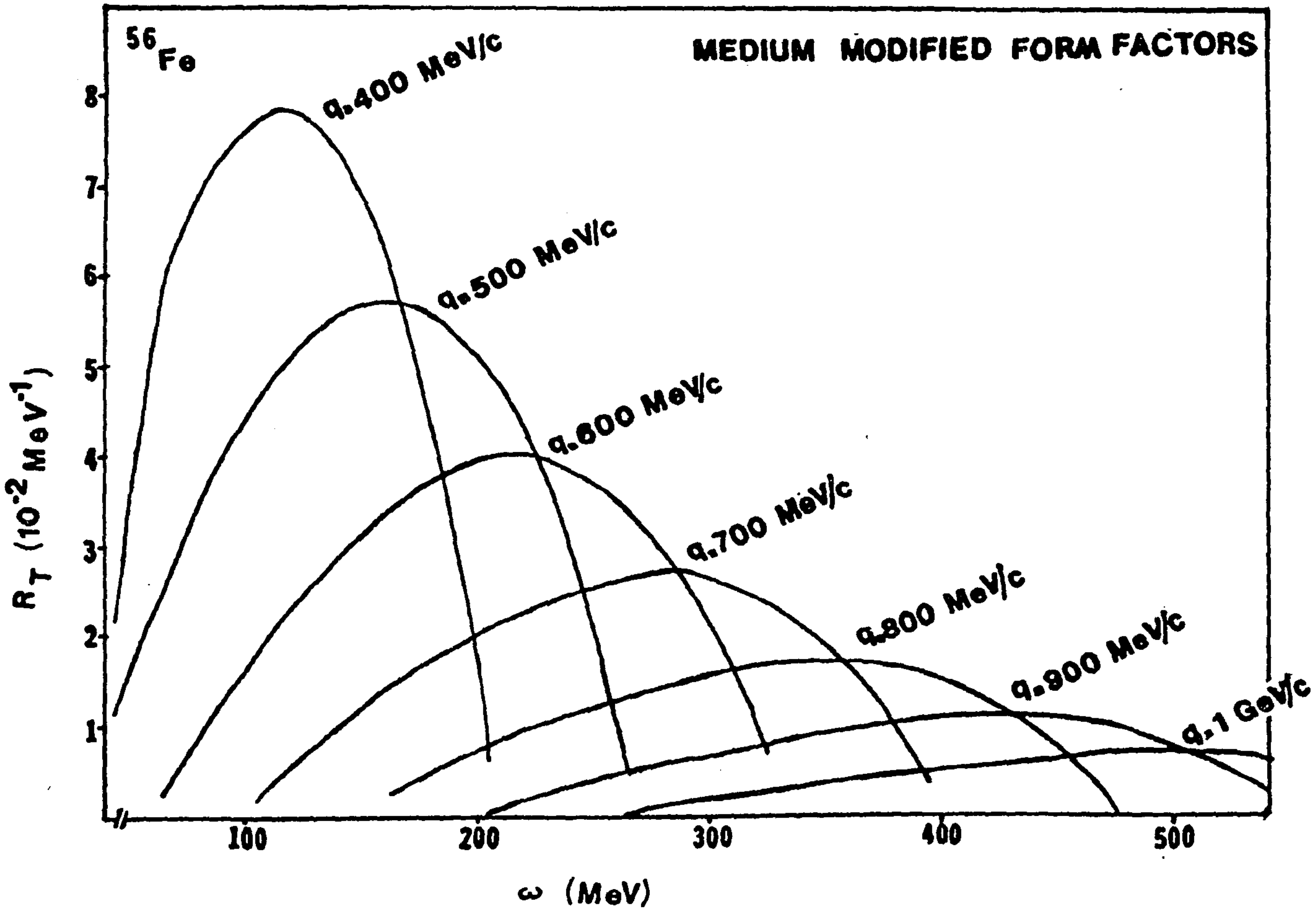


Fig. 21 LONGITUDINAL RESPONSE FUNCTION

FERMI GAS MODEL

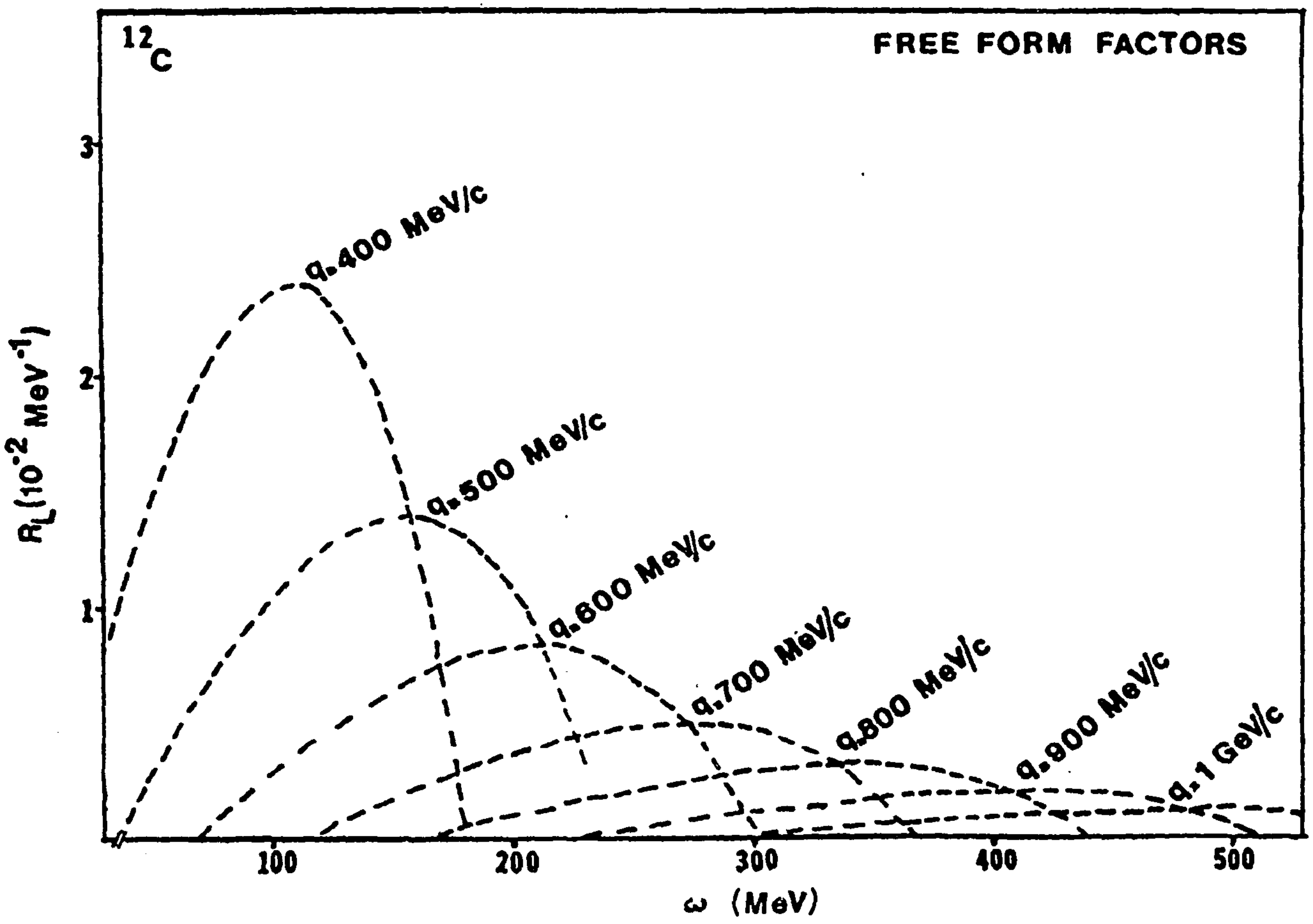
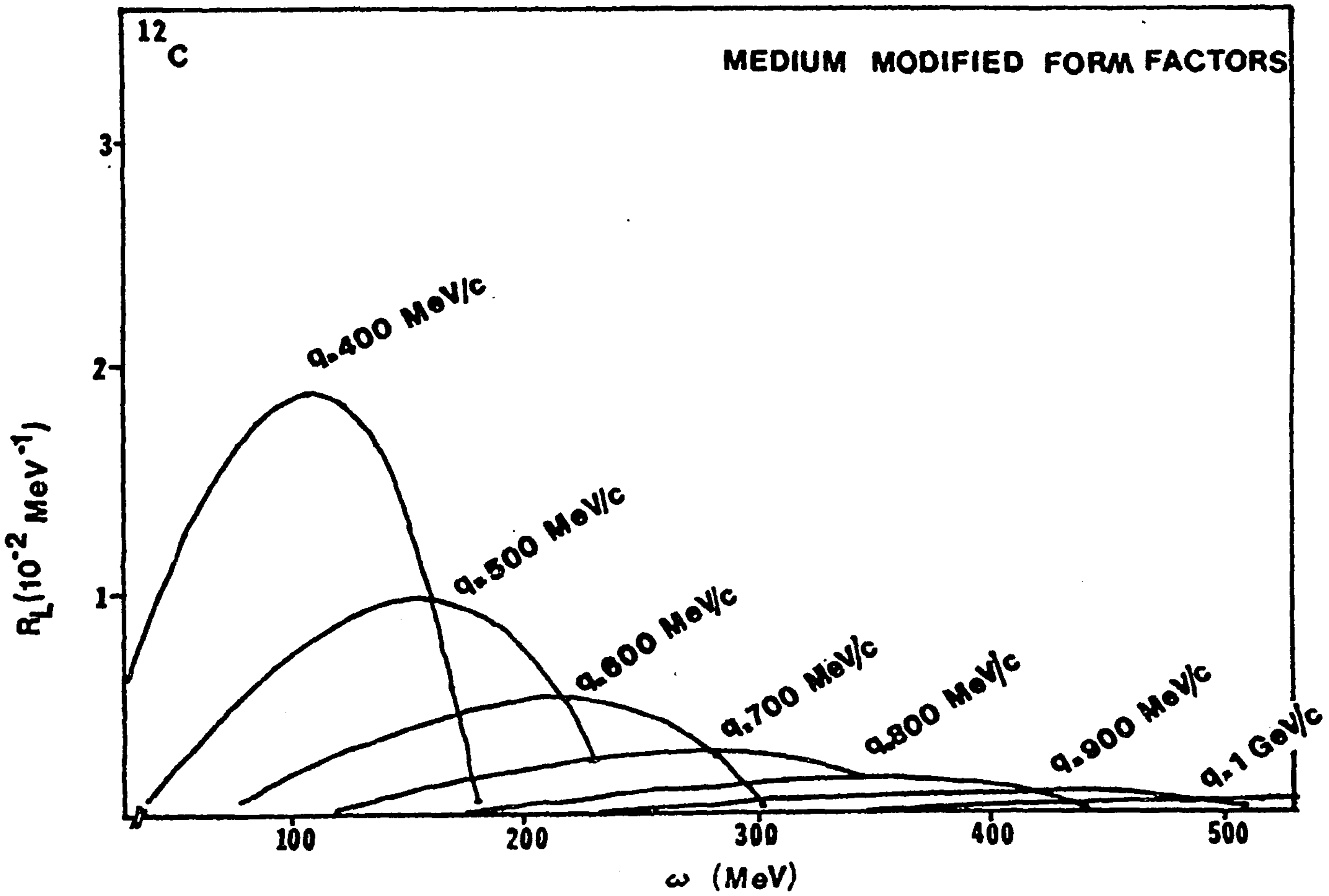


Fig. 22 **LONGITUDINAL RESPONSE FUNCTION**

FERMI GAS MODEL



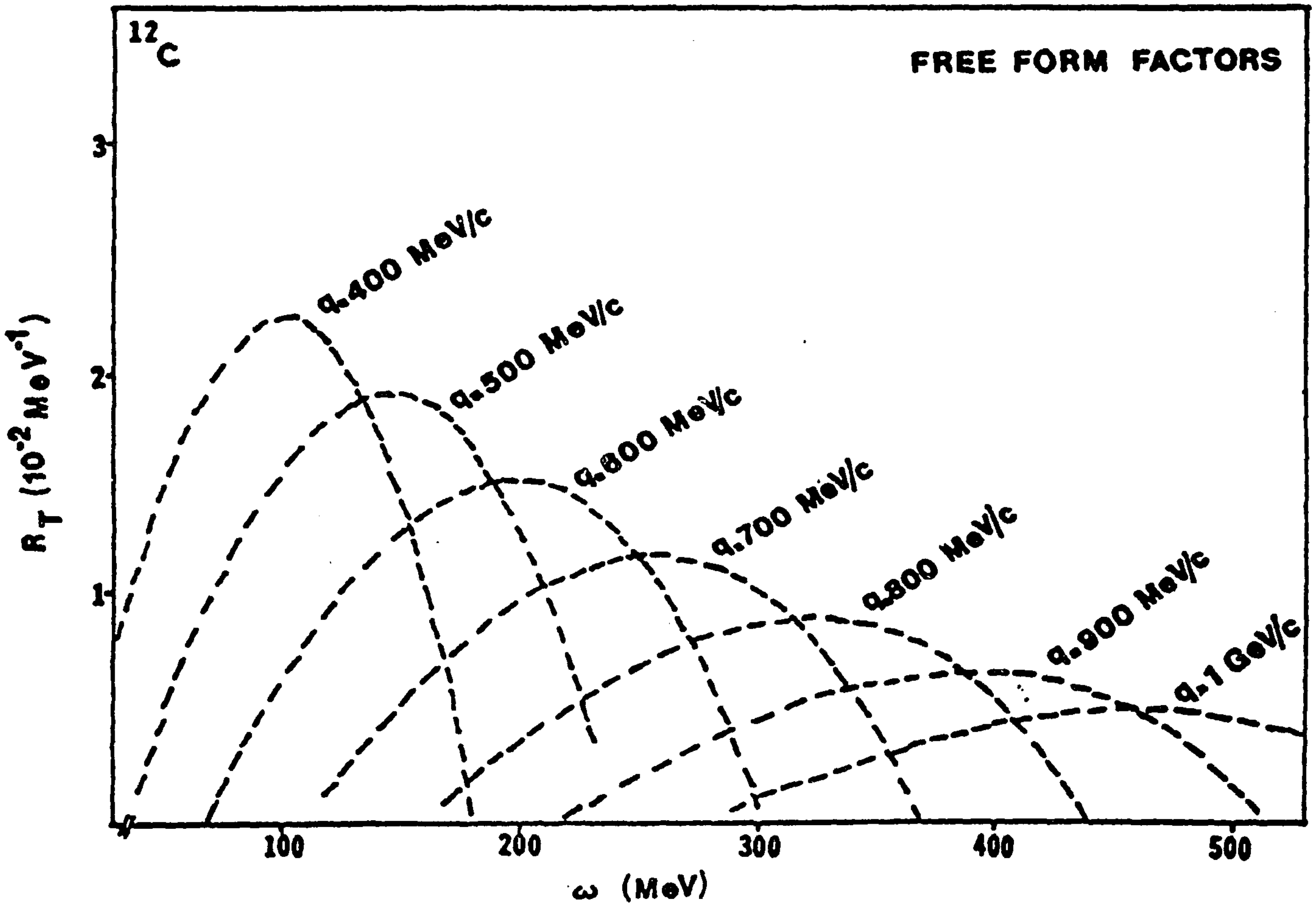
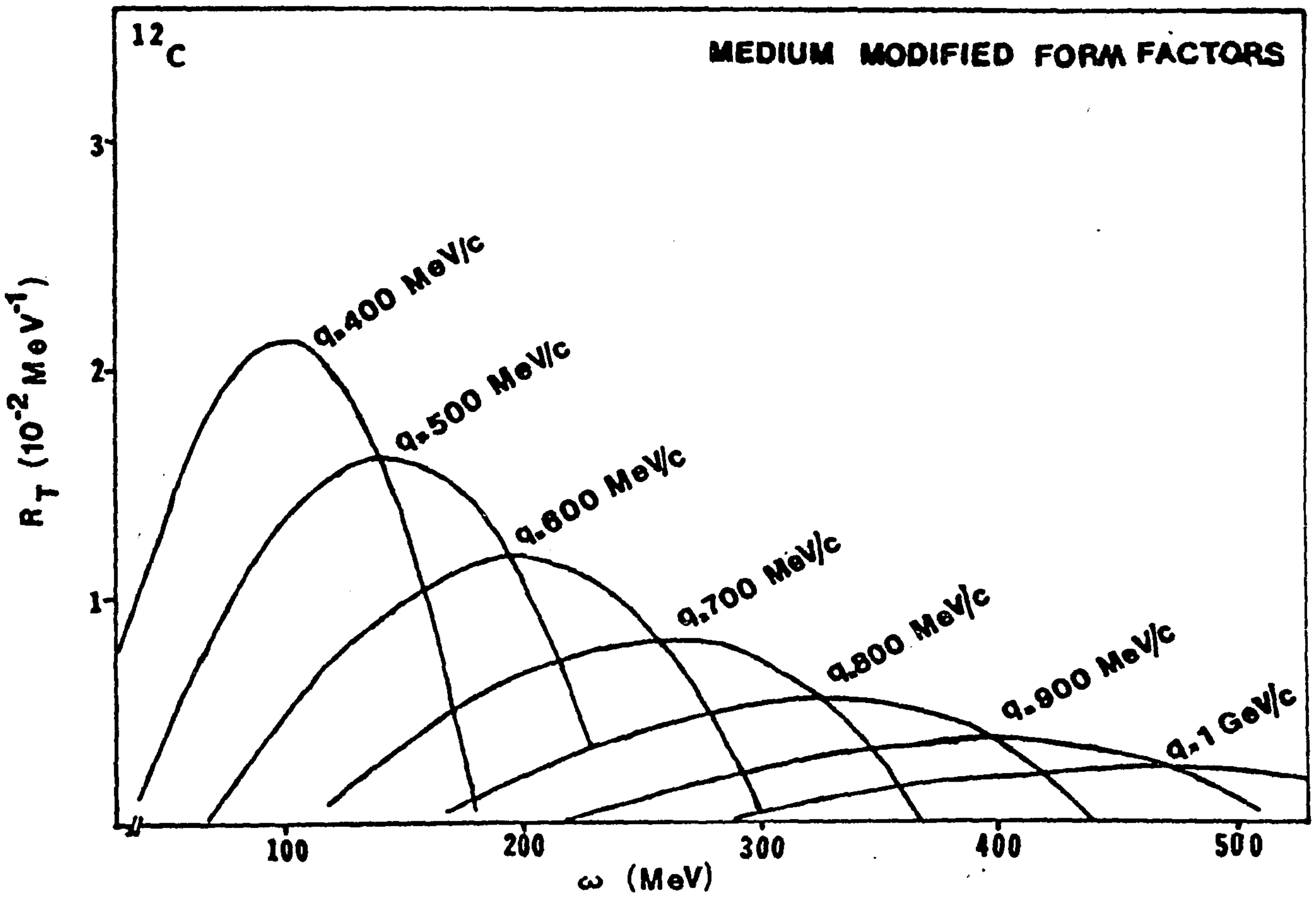


Fig. 23 TRANSVERSE RESPONSE FUNCTION

FERMI GAS MODEL

Fig. 24 **TRANSVERSE RESPONSE FUNCTION**

FERMI GAS MODEL



RFFERENCES TO PART I

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