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A THEORY OF THE COLLECTIVE SOCIAL ORGANISM

by

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A dissertation submitted to the Graduate Faculty in Economics in partial fulfillment of the requirements for the degree of Doctor of Philosophy, The City University of New York.

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Abstract

A THEORY OF THE COLLECTIVE SOCIAL ORGANISM

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This paper seeks to argue that a collection of interacting organic agents behaves as if it were a separate entity in its own right. This result, although not new, will be cast in terms of a novel model that expands the usual formulations of the production constraint. This allows us to model a more general notion of an agent and thus the general equilibrium of a system. The paper will take advantage of an algebraic structure that is implicit in systems of exchange and transactions -- of information in general, and goods and services in particular. This will allow us to apply standard tools of analysis to the problem of choice for both producing and consuming agents, within the context of the algebra. The result is a partitioning of economic activity into easily derived closed form expressions. Of importance is the general equilibrium of the unit social organism. This organism, by behaving optimally and interacting with its environment, gives rise to a general equilibrium setting, which apply both to the micro unit and the macro system as a whole. The theoretical implications lead to iso-curve fields for both the micro and macro system analyzed. The model also solves for a relatively large number of relevant economic variables within a single framework. These hypotheses are tested against time series of aggregate and sectoral annual economic data for the U.S. Of interest is the finding of a negative and significant relationship between labor and a variable akin to the marginal

propensity to consume. This further implies that there is a tradeoff between labor and the amount of work that physical money performs in production. The output of the expanded production function is the turnover of goods and services as opposed to just goods and services within the time horizon. The model also finds that if the system reaches a steady state, then it must be a golden rule steady state where the rate of saving adjusts to the golden rule level. The paper concludes with a simulation and discussion of the comparative statics of a simple system.

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TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

PART ONE

THE GENERAL EQUILIBRIUM OF THE COLLECTIVE SOCIAL ORGANISM

INTRODUCTION

The body of standard economic theory can be characterized in general by two overriding principles. On the one hand, there is always some duality operating in any given economic problem. It is presumed that there exist opposite or contending forces acting upon economic reality where the balancing of forces is of central importance. On the other hand, some optimization principle is at work in the composition of the forces at play (Samuelson 1947; Hicks 1954). In the literature the maximization of utility subject to constraint, minimization of costs given some output to be produced or, cost-benefit considerations in general, are at the heart of all issues governing over rational economic behavior. Along with dualities and optimizing behavior we also find in economics the pervasiveness of homomorphisms. That is, structures with similar or analogous forms. For example, demand and supply analysis, arguably the most venerable of economic paradigms, mimics the cohesive and analytical soundness of classical mechanics. That this is the case is not surprising. Similar systems of analysis can be applied to explain very different phenomena. Physical constraints, binding behavioral imperatives in the form of some optimization principle and dualities are the hallmark of the existential context faced by a decision-making agent. We can thus identify an organic agent, which exists within the context of its environment. This environmental setting constitutes the most general constraint faced, by the agent. If, further, the organic agent is of a social nature, then we must add the effects of the network of social agents to the environmental setting. The impact of social interactions by transaction of information and/or goods. e.g., agglomeration economies resulting from trade (McDonald 1997), constitutes yet another constraint that must be explicitly taken into consideration in any economic analysis of the social organism(s).

The social organism within the context of its environment including the network of unit social organisms, is the focus of this paper. In order to analyze the behavior of this theoretical agent I make use of standard tools of economic analysis coupled with a more general notion of the production function. This approach to the production process. allows us to generalize our notion of an agent. where a set of agents can behave as if they were a separate entity altogether. A central concern will be the general equilibrium of the unit social organism within its environment. i.e., production function. The model developed for the micro (individual) unit can be directly aggregated for the macro (collective) behavior of the system. Here we find the most relevant homomorphism revealed. The relevance of evoking dualities, optimalities and homomorphisms is to underscore properties of the environmental setting faced by organic agents. If a system displays such properties and characteristics, then such may be identified as markers of the existence of an organic agent.

The property that the model can very easily be made operational by a Cobb-Douglas technology adds further. I believe. to its appeal. This will make econometric analysis feasible with direct interpretations of results, one of which is that there is empirical evidence of a negative and significant relationship in production between labor and a variable akin to the marginal propensity to consume. Because it can be shown that this MPC-relative is further related to the turnover speed of physical money in transactions, we can identify a measure of work being performed by money. This in turn implies a relationship in production between the work performed by physical money and labor. We can speak of a trade-off between the work performed by physical money and labor effort in the production of an environmental array of which goods and services are elements. The augmented production function brings these two variables together in a feasible production possibility frontier. Thus the production process is not only of goods and services but rapidity of turnover of those goods and services. From this, the general equilibrium of the agent and the economy can be derived. In essence the paper is about the general equilibrium of the organic entity relative its entire environment: natural-physical, social, economic.

I have tried to present as broad a scope as possible in the application of the model to several areas of interest such as labor, consumption, price level, etc. I also touch upon the dynamics for the sake of completeness. Finally, the full generality of the model proposed may have eluded the reader so far. The nature of the social organism is as broad as we may wish to make it. The main thesis thus is that, a collection of social organisms gives rise to another organism, which is as bound by physical and economic constraints as the organisms that make-up this broader unit. This allows us to aggregate blocks of sectors as a single agent in a general, simple, consistent, and cohesive system of analysis.

I begin with a brief survey of the general equilibrium theory literature and how it relates to the social organism. A discussion of the environment and the unit social organism follows. The network implications are next, where the novel algebraic structure used is derived. This is used to justify the general equilibrium equations. A couple of simulations are presented in this section. The important issues of the Jacobian and the stability of the system are covered in turn. Then the Constant Returns to Scale Cobb-Douglas technology is utilized for the derivation of a special case of the equilibrium conditions. This will allow us to derive and empirically test the notion of a labor effort-MPC-relative trade-off in equilibrium.

GENERAL EQUILIBRIUM AND ORGANIC NATURE

The ancient philosophical concept of animus finds easy expression in economic behavior and literature. A collection of forces with regular, stable, and balanced outcomes may reveal idiosyncratic patterns of behavior that give the collection of actions and reactions a particular animus and /or form. The Physiocrats and in particular Quesnay (Lewis 1934; Heimann 1945) realized that economic systems are but networks of transactions. The simple circular flow of goods and services that closes the system of transactions in an economy has indeed a long and enduring history (Knight 1933; Patinkin 1981). The network of the exchange of goods and services is in analogy with the human body's circulatory system. This is the fundamental insight provided by Quesnays' *Tableau Economique* (1758-59). Indeed biological underpinnings of economic behavior are well documented in other respects, as in the nature of preferences (Robson 2001) and socio-biological foundations (Becker 1976; Shotter 1999). However, organic behavior is a general concept that goes beyond biological nature. The behavior of an organic entity in its broadest sense is the result of the physical effects of the aggregate composition of smaller or separate, and possibly also organic, units.

Leon Walras provided the first comprehensive and rigorous analysis of the general equilibrium of an economic system (1874, 1954). His equations link all industries in the economy (labor included) where the quantities and relative prices of goods and services reveal themselves to be the relevant variables solved. In his model, (n) industries and (m) workers have dualities in the form of supply and demand relations (Marshall 1920) where the general equilibrium of the system is achieved when all excess demands are equal to zero. The description of the forces at play (supply and demand) and the conditions for equilibrium betray the markings of the behavior of the collection of all agents and industries within the general equilibrium setting. The collection of forces need not behave in the same manner, either in magnitude or quality, from the forces affecting an individual agent. This is what gives the organic entity of analysis its distinctive marks. If the individual forces and counter forces in equilibrium are identified as organic functional components, then the impact of the collection of all forces reveals an animus. The animus is provided by the collection of actions as result of the volition of unit agents and the constraints they face. Both the volitional and productive imperatives faced by agents in the agglomerative sense (Marshall), give the Walrasian system its agential existence. But agential existence is also a broad concept. General equilibrium can be characterized by paradigms other than the Walrasian. The Von Neumann (1945-46) distinction of production processes as opposed to the Walrasian and Leontieff (1941) decomposition (organic) into industries, still reveals the system's agential animus. The system as a whole, with all its network links and related agglomeration economies, gives rise to the distinct behavior of the aggregate. This, I believe, is behind Keynes' (1936) partial equilibrium system as characterized by Hicks (1937). There is an environment that the agent faces (physical constraints) and the desire to make the best of the constraints (Hicks 1952). The introduction of money is but an alteration of the environment faced by the agent. Thus, the issues of dichotomy, neutrality, and the inconsistency of the Walrasian system (Lange 1942) and its possible remedy (Patinkin 1949, 1966) do not negate the animus. Further, competitive economic systems with forward prices (Keynes 1923) display equilibrium tendencies (Arrow, Debreu 1954). Once again, this can be interpreted as markings of animus, as opposed to mere physical reaction. A hurled projectile or a reaction of compounds does not of itself reveal animus or volitional behavior under constraint -- the key element identifying animus. This is precisely what a general equilibrium system displays. Even models of fluctuations and propagation (Frisch 1933) may be seen as reaction of animus. The observed recurrences of behavioral patterns in the business cycle (Burns and Mitchell 1944; Fischer 1996) suggest its existence.

Clearly, the distinction between individual and aggregate behavior, micro and macro, may differ in quality and magnitude. The sheer force that a macro system can generate, if large enough, can easily overwhelm that of an individual's in the final relative outcomes. The harmonic or balanced states of an organic agent must be considered within the context of the general environment faced by the agent. This relationship is considered next.

THE ENVIRONMENT AND THE SOCIAL ORGANISM

1. The Environment Production Function.

Decision-making agents formulate choices within some environmental context. We assume that an organic agent exists and interacts within some environment, the states of which determine the set of options available to the agent. Interactions between agent and environment can bring forth a set of chain reactions in the environmental setting, such that a dynamic of available options is set into motion. This can affect the agent directly or indirectly by virtue of the effects of choices made by the agent. Natural processes, whether random or systematic, controllable or not by the agent, also have an effect on the set of options available to the agent. In either case the environment can be affected and therefore modify the constraints faced by the organic agent. Thus, the choices made available to the agent in the future are constrained in perhaps different ways. An analogy is useful. Let the agent be, for example, a farmer and let the environment be the general environmental setting faced by the agent, i.e., the natural state. The agent to ensure his survival, growth, and progeny makes use of his environment. From the environment he extracts the raw material needed to make tools, which in turn are used to make even better more effective and efficient tools. This act changes the natural state by the direct action(s) of the agent(s). The effect on the environment is the change in the primitive composition of the environment due to the actions taken. From a physical standpoint nothing has changed. Of course no mass or energy has been created., however, states of nature *have* changed. These states of nature may possess dynamics that imply certain sequences of natural states that in turn could become hostile or beneficial to the agent(s) per unit time. In the case of the primitive farmer the gathering of stones and wood to create tools affects the environment in, what can be said to be, negligible terms. This contrasts sharply with the effects of blocks of industrialized nations on a planet like Earth[1].

This leads to the possibility that the environment can be seen as a broadly defined production function. Of course the totality of the environment is composed of agents, input, as well as the physical/technological constraints on them. The environmental setting can thus be represented by a production function. A sub-production function can be defined that relates input and environmental conditions in the production of goods-commodities produced by the organic agent's efforts. Let this function be given by

[1] Landes (1969) in his breathtaking account of the history and development of the industrial revolution recounts the damage of the epoch. The scars that industrialization can leave on the environment are well documented in the historical record as well as in literature and folklore. However, the enviro- economic experience from those noted by Malthus to the Club of Rome (Khan 1976) and recently due to global warming (Duching and Lang 1994) may well prove immaterial due to the ability of economic systems to substitute resources. After all, a forward-looking capitalist is not bound to kill the goose (Earth) that lays golden eggs. However, shortsightedness may be a dooming factor.

$$f(\lambda, h, \kappa, \Omega^{*}) = q.$$

This production function is assumed to depend on labor-time per agent (λ), human capital per agent (h), the stock of physical capital per agent (κ) and the stock of environmental goods (Ω^{*}), that we may label the complement environment. Environmental goods will be defined to be a set or array of goods-commodities (of positive or negative properties relative to the agent) that may or may not be vital, or of interest, to the agent.

2. The Volition of the Social Organism: The Utility Function.

The level of interest assumed is embodied here by the preference structure of the agent. The preference structure will be characterized by a utility function with a consumption array (x), time array (t), a rate of saving out of available resources (s), and the stock of environmental good.

$$U(x, t, s, \Omega^{*}).$$

Both the production function and the utility function have the environmental array (Ω^{*}) as argument. The asterisk represents a one-period lag. Now, choices made by the agent are determined in part by the restrictions on output inherent in the production process. The output of the production function may alter the environment, changing the next period's environmental array. The current period thus may affect the change in the environmental array, i.e., ($\nabla\Omega$). Lagged effects will be ignored, but a more comprehensive treatment would allow for them.

Barring the possibility of cataclysmic changes that may affect the environment in very sharp terms, primitive conditions that any reasonable number of agents acting on the environment can have on the environment can be seen to be negligible. This can be expected to hold, of course, as long as Malthusian conditions are kept in check. That is, under primitive conditions $\nabla\Omega \approx 0$ per unit time. However, this may not hold true under all circumstances. This is clearly evident from the historical record throughout the ages yet not as prominently manifested as by the effects of the industrial revolution. Twentieth-century acceleration of industrial capacity and output has made its own mark on the environment, one that may yet prove deep and perhaps irreversible.

The model developed can be used to analyze a general equilibrium of environmental conditions or natural states under the economic imperatives of agent's choices and production technology. An input-output framework is easily applied to a model of the above. However, I will not look into those areas at present as it involves introducing unnecessary complexities. Instead I will exploit a simple model reduced to two variables in labor-time and a variable akin to the marginal propensity to consume out of available resources. This will allow for the demonstration of the modeling method proposed and to examine the conclusions derivable about the behavioral response of both the agent(s) and the environment under changing conditions. Thus, we assume for the rest of the paper $\nabla\Omega=0$. This can be relaxed with an explicit application to the problem of equilibrium between economic systems and environmental settings. Hence, q represents real output per capita where the environmental array has been assumed away from dynamically affecting (q), and vice versa.

SYNTHETIC INCOME

1. Overview of Process of Network Flows.

In order to study the structure of choices made by agents within an environmental setting, we need to focus on some sort of general equilibrium framework that brings together agent and environment within an harmonic or balanced interaction. To accomplish this I propose the application of an algebraic structure that is implicit in closed systems of exchange. The transfer of information given seepage or memory loss (or some other process of erosion of the information transferred, e.g., discounting) gives rise to an explicit algebraic structure. The initial reference signal transferred into a stream of transactions and/or exchanges gives rise to two multipliers that embody the cumulative effects of the transaction stream. In order to illustrate the algebraic structure of a network of transactions I will use a case in the context of the economic behavior of the organic agent. To justify the existence of the network of transactions we need to justify the behavior of the agent. I begin with a general overview of the problem of choice faced by the agent in three arguments. This will justify the sequence of transactions that lead to the algebraic structure that is implicit in the sequence of transactions, which are of closed form. After the algebraic structure is in place I use the result to explain the general equilibrium of the agent in relation to the production process, the environment, and the interactions with other agents. This process is to be manifested as the totality of transactions within a period of time to be considered, e.g., a year. That is, we will have a model that analyzes the social organic agent in the context of the network of relations and interactions; the production constraints and general environment faced, which affect the choices available to the agent and the network of agents in general.

2. The Problem of Choice.

Let the utility function have three arguments: current consumption (x), leisure (t), and the rate of saving out of available liquid resources (s). The function becomes

$$U(x, t, s).$$

For now let us call (s) the liquid-resource saving rate. Also assume that the utility function is strictly quasi-concave in the arguments. The agent is assumed to maximize utility given budget and time constraints, where household production may be included in the analysis (Becker 1965) but for simplicity will be ignored in this paper. Also I will ignore real money balances (Patinkin 1949, 1966) in the utility function. It will be shown that the general equilibrium system derived here solves for both absolute and relative prices.

The set of relevant variables of the model are defined as follows:

P = price level;

ω = nominal wage rate;

T = time available in period of analysis;

t = leisure;

λ = hours of market labor;

i = effective nominal interest rate on accumulated assets other than physical money ;

a = accumulated assets other than physical money;

μ = stock of physical money carried over by the agent (from end of last period to the beginning of the current period);

v = exogenous current resource (which may include returns to human capital); and

s = rate of saving out of available liquid resource.

An asterisk over a variable indicates a one-period lag. Now, let the constraints be, respectively, for budget and time[2],

$$Px = (1-s)\left(\omega\lambda + ia^{*} + \mu^{*} + v\right)$$

$$\text{and} \qquad T = t + \lambda .$$

Where the right-most term in bracket in the budget constraint is the available liquid resource, and (1 - s) is the liquid-resource rate of transfer. Further, I denote the rate of transfer of available liquid-resource by (c), c ≡ 1 - s.

As a particular case, let (s) be equal to the Marginal Propensity to Save (MPS). Although this is a misnomer I will nevertheless use it as its label throughout because it will help us contrast and distinguish the difference between the two, i.e., the Marginal

[2] This budget constraint is not entirely complete. It implicitly assumes that the agent does not tap into accumulated assets. It also assumes that the current period's end-of-period stock of money (μ) will be zero. In the appendix a more in-depth treatment of the budget constraint is discussed. For simplicity and without loss of generality we use this specification in the paper throughout. This is reasonable because one focus of the current paper is on pure positive growth behavior.

Propensity to Consume (MPC) and (c). This immediately implies that (c) is the MPC. The full distinction will be touched upon in the comparative statics. This will allow for the testing of the theory in terms of macroeconomic terminology and data of the national income and product accounts. Later on I will clarify the relationship between c and the true MPC as defined by Keynes (1936). The difference between the two variables is due to the definition of income used.

3. Transaction Sequencing

Suppose we have a representative agent faced with the above preference structure. prices. interest rate. composition of the available liquid resources. and assets. Also suppose that. within the social context. all agents transact in sequence such that the choices made for (x). (t). and (s) are the same across the board. This assumption is not necessary but is done for the simplicity of exposition that it affords. However. the analysis can be generalized to an arbitrary number of agents and types of agents. Finally. agents make choice decisions for a period at its beginning. Then we can identify a sequence of transactions relative to the stock of physical money per capita (μ). In view that both the utility function and the budget constraint are non-standard in form. I dedicate a section in the appendix to justify them. It can be shown that the above budget constraint allows us to analyze inter-temporal choice given the utility function as defined.

Thus. at the beginning of the period only (μ^*) of the available liquid-resource is realized. The other variables are determined in full by the end of the current period. We can then use (μ^*) as a piece of tangible information, i.e., a physical datum, and infer the effect that the inclusion of (μ) in available liquid-resource has in the aggregate. We drop the asterisk for ease of notation.

The agent hence transfers (μ) from the last period to the current period. This serves as a marker of a particular sequence of events. Because all agents are the same and react similarly we get that the introduction of (μ) to the current stream of expenditures, relative (μ), leads to the following stream:

$$\mu \rightarrow c\mu \rightarrow c^2\mu \rightarrow c^3\mu \rightarrow \ldots$$

Notice that as this sequence approaches infinity, the sum of all terms approaches a limit as long as $0 \leq c < 1$. Let this limit be expressed as,

$$II = (1 - c)^{-1}\mu$$

$$= U\,\mu.$$

We can call the above the Incident Income (or Incident Transaction Volume) that is generated by (μ). Let $U = (1 - c)^{-1}$ be called the physical-money Transfer Multiplier. This is the Keynesian multiplier under the current definition of (s) used. If a large amount of transactions occur, then the sum of the sequence approaches Incident Income. The above represents the immediate impact that (μ) generates. Actually (μ) does not generate income but serves instead as a reference measure of income (transfer) generated by the sequence of transactions.

4. Incident Saving.

The sequence of incident income, or more generally transactions, also generates saving relative to (μ). This sequence is

$$(1\text{-}c)\mu \rightarrow (1\text{-}c)c\mu \rightarrow (1\text{-}c)c^2\mu \rightarrow (1\text{-}c)c^3\mu \rightarrow \ldots$$

Thus, the sum of the sequence of savings in the incident transaction process is equal to (μ). That is in the limit, $(1-c)\sum_{k=0}^{\infty} c^k \mu = \mu$.

This suggests a principle of conservation. The introduction of (μ) in the transaction stream generates saving in the magnitude of (μ). Heuristically we can represent this process as $\mu \rightarrow$ {Exchange System}$\rightarrow \mu$.

Note that (Uμ) is generated in income (transactions). In general, we have, that for any physical quantity (X) introduced as liquid-resource into the income (transaction) stream, (UX) is generated as income and (X) in saving.

X → {Exchange System}→ X .

UX.

5. Re-Entry and Closure.

The generation of saving implies that (μ) has left the income (transaction) stream. Now, suppose that the amount saved is re-introduced back into the income (transaction) stream, in the form of investment at the rate of (r). That is (rμ) re-enters the income (transaction) stream. Therefore, ($Ur\mu$) will be generated as income, and $r\mu$ is generated as saving by the principle of conservation above. So,

$$r\mu \rightarrow \{\text{Exchange System}\} \rightarrow r\mu \ .$$

$$U(r\mu)$$

But the process of re-entry repeats itself, this time with $r(r\mu)$ as re-introduced amount. We can generalize this process as the sequence of marginal income (transactions) generated by the below schema.

$$\mu \rightarrow \{\text{Exchange System}\} \rightarrow \mu$$

$$U\mu$$

$$r\mu \rightarrow \{\text{Exchange System}\} \rightarrow r\mu$$

$$U(r\mu)$$

$$r^2\mu \rightarrow \{\text{Exchange System}\} \rightarrow r^2\mu$$

$$U(r^2\mu)$$

....

$$r^k\mu \rightarrow \{\text{Exchange System}\} \rightarrow r^k\mu$$

$$U(r^k\mu)$$

6. Cumulative Income (Transactions) Generated.

If $0 \leq r < 1$ then the sum of all income generated approaches in the limit

$$U(1 - r)^{-1} \mu.$$

Let us call this quantity Synthetic Income per capita and denote it by (z). Also let $W = (1 - r)^{-1}$ and call it the Time-Value multiplier. Therefore,

$$z \equiv UW\mu.$$

Now, because (μ) was transferred from the last period to the current period (μ) cannot be considered current income. So, by subtracting (μ) from (z) we get the actual gross nominal income generated in the current period of analysis. Let us denote this by (y) so.

$$y \equiv z - \mu$$

$$\equiv (UW - 1)\mu .$$

If we constrain (y) to be the actual gross nominal income per capita generated in the period, and (N) the number of agents, then aggregate nominal income is,

$$Y \equiv (UW - 1)\, m.$$

Where $\mu = m/N$, and m is the aggregate stock of physical money.

7. Investment-Saving.

Similarly as income, the cumulative amount of investment generated is not $W\mu$ because the initial introduced quantity (μ) is not investment. Instead ($W\mu - \mu$) is actually generated cumulatively as investment. Recall that the sequence of re-entry as investment is $r\mu \rightarrow r^2\mu \rightarrow r^3\mu \rightarrow r^4\mu \rightarrow \ldots$ That is, in equilibrium and in the aggregate

$$S \equiv (W - 1)\, m = PI.$$

Where (S) is gross nominal saving, (I) is gross real investment, and (P) is the price level as already noted.

8. The Theory of Synthetic Income.

The above framework provides an income space that I call "synthetic"[3]. In the aggregate we have thus,

Synthetic Income	$Z \equiv UWm$
Gross Nominal Income	$Y \equiv (UW - 1)m$
Gross Nominal Saving	$S \equiv (W - 1)\, m.$

These are the fundamental equations of the algebra. The algebraic structure that derives from the above relationships reveals, in addition to the above "shadow" values, a partitioning of an income space into relevant quantities and related sub-spaces. In particular, consumption is easily derivable by the income-consumption-saving identity. Let (C) represent aggregate gross nominal consumption, then we have the tautology $Y \equiv C + S$. Hence,

$$\begin{aligned} C &\equiv Y - S \\ &\equiv (UW - 1)\, m - (W - 1)\, m \\ &\equiv (UW - W)\, m. \end{aligned}$$

Therefore, real consumption per capita (x) must equal in equilibrium to

$$x \equiv (UW - W)\frac{\mu}{P}$$

and thus aggregate real consumption is:

$$X \equiv (UW - W)\frac{m}{P} \equiv \frac{C}{P}.$$

---

[3] This label of "synthetic" has no relation to Moor's Synthetic Economics.

9. Money.

The algebraic structure derived above gives rise, in particular, to two notions of money. The first one is a measure of the transactions volume of money activity. The magnitude of the value of the transactions is for goods and services. This quantity is defined to be $M_T \equiv Um.$

This measure is akin to (M1) plus reserves, if our measure of physical money is the monetary base. The income velocity of this money concept can be cast in terms of the multipliers. This velocity takes the form of

$$V_T \equiv \frac{UW-1}{U} \quad .$$

The second measure of money relates to time-value money. That is, money that is set aside and lent-out, hence the justifying label of time-value. This quantity and its velocity are, respectively, $M_S \equiv Wm$

$$V_T \equiv \frac{UW-1}{W} \quad .$$

If we were to add both measures of money we would have an additional quantity, which we may label as Mass-Money and denote it by (M). So,

$$M \equiv M_T + M_S \quad .$$

This last measure of money is akin to: (M2) plus reserves plus high-powered money. If, further, the ratio of this broad measure of money to the stock of physical money is some constant (ϕ) we get then, $\phi = U + W.$

This ratio, however, need not be constant. We can make this ratio a function of a variety of variables, like the nominal rate of interest and real income. For simplicity, I assume that the ratio is constant throughout.

CONSUMPTION EXPENDITURE SEQUENCING

The decision to purchase a volume of goods for a period of time can be accomplished by purchasing portions of the total volume throughout the period under consideration. The agent chooses to consume x for the current. However, he need not purchase the full amount at the very beginning of the period, or alternatively, consume the totality at the very end, if that is when consumption expenditure is realized. The agent can purchase real goods and services in an infinite sequence within the period and still end with the desired amount of consumption for the decision period. To see this, recall that consumption in our current model is the product of the MPC and real income.

$$x = cq .$$

This is equivalent to,

$$q = \frac{x}{c} .$$

$$= \frac{x}{(1+c)} + \frac{x}{(1+c)^2} + \ldots \quad .$$

The first term of the sequence can be viewed as the amount of consumption expenditure left over to be purchased,

$$\frac{x}{1+c} .$$

This would imply that the amount of goods and services actually purchased on the first round could be,

$$x - \frac{x}{1+c} .$$

This is equivalent to, $\frac{cx}{1+c}$.

The second round of possible consumption expenditure within the period is thus,

$$\frac{x}{1+c}-\frac{x}{(1+c)^2}\ .$$

This reduces then to, $\frac{cx}{(1+c)^2}$.

In general, $\frac{cx}{1+c}+\frac{cx}{(1+c)^2}+\ldots=$

$$\sum_{k=1}^{\infty}\left(\frac{1}{1+c}\right)^k(cx)=x\ .$$

Therefore it is possible for the agent to realize the planned total consumption expenditure within the envisioned period of time by making a series of expenditures within the period of time. This implies that the agent need not purchase the entire planned amount at once. The limiting case of infinite transactions is a convenient assumption for simple expressions of sums.

THE GENERAL EQUILIBRIUM

We now turn to the interaction between agent(s) and the environment characterized by the combination of the production function, the network of flows as described in the last section and the agent(s) preference structure. This encompasses the totality of the environment. Indeed, the agent(s) is/are but one element-organ characteristic of the environment as a whole. In order to analyze the interaction between the agent(s) and the rest of the system, we need to address the tensions and stresses that are inherent in the balancing of relevant forces. Although this framework will focus on economic forces, the reader is reminded that we can in general easily incorporate ecological and/or physical forces as well.

The overriding imperative invoked will be that of optimizing behavior under constraint. That is, both the production process and the choice of goods by agents are obtained by an optimization principle. As usual profit maximization and Constant Returns to Scale will be assumed for the production process of goods and services. Utility maximization is retained and elaborated on below.

For the production process we have the usual profit function of:

$$\pi = P\ f(\lambda, \kappa^*) - \omega\lambda - r^*\kappa^*.$$

Given P, κ^*, ω, and r^*, the production process maximizes profit when the marginal product of labor is equal to the real wage rate as the first order condition.

$$MPL = \omega / P.$$

The asterisk on r may indicate a known or accepted piece of information, e.g., an expectation of the rate of return.

Turning now to the agent, we saw that the agent has three arguments of choice. The first order conditions for consumption, leisure (or discretionary time), and the MPS are

$$U_x = \varphi P$$

$$U_t = \varphi (1 - s)\, \omega$$

$$U_s = \varphi \{\omega\lambda + i\, a^{*} + \mu^{*} + v\}.$$

Given the above first order conditions, we thus only need two Marginal Rates of Substitution (MRS) because the budget constraint completes the system in consumption, leisure, and the marginal propensity to save.

I) $$MRS_{tx} = \left(\frac{U-1}{U}\right)\frac{\partial f}{\partial \lambda}$$

II) $$MRS_{sx} = z / P \ .$$

Note that z (synthetic income per capita) can be shown to be equal to the right hand term of the equilibrium condition for the marginal utility of the MPS, i.e.,

$z = \omega\lambda + ia^{*} + \mu^{*} + v$. Also note that,

$$\frac{U-1}{U} \equiv (1 - s) \ .$$

The above first order conditions in conjunction with the equations derived in the previous section give us a general equilibrium setting between agent and environment; here the total environment given by the system of equations below.

Equation (1) equates the production function in equilibrium with per period income expressed in terms of the algebra of networks of the last section. Equation (2) is the equilibrium equation for the labor market. This combines the optimal condition for the agent, for consumption relative leisure, as the optimal condition for labor-hours.

Equation (3) equates the marginal rate of substitution between consumption and the MPS with real synthetic income per capita. This last term is expressed in terms of the algebra. Equations (4) through (7) have already been touched upon in the last section. They contain transformations of variables, constraints on variables and behavioral relations pertaining to consumption, the MPS, money balances, and time allocation. The result of two simulations are reported below to illustrate the model. In both simulations a Constant Returns to Scale Cobb-Douglas is assumed, where α is the capital share of income. In the first case, only labor and capital enter as argument. In the second simulation human capital per capita also enters as an argument. In this particular case the coefficients for the factors of production are those suggested by Mankiw (1995). In both cases the system was calibrated to resemble late 20th century behavior for the United States. In both cases constant MRS are also being assumed. This is a very strong assumption with interesting properties but with little empirical support. In these simulations stability by a first order Taylor approximation can occur if the time-value multiplier (W) is greater than the transaction multiplier (U) and the income share of labor is smaller than 0.03. Clearly this is highly unrealistic. The problem resides in the assumption of constant marginal rates of substitution, the economic implication that consumption, labor, and the MPS are all perfect substitutes effectively untenable.

GENERAL EQUILIBRIUM OF THE UNIT SOCIAL ORGANISM

Product

1) $$f(\lambda,\kappa)=(UW-1)\frac{\mu}{P}$$

Labor

2) $$MRS_{\alpha}(x,s,t)=\left(\frac{U-1}{U}\right)MPL(\lambda,\kappa)$$

Expenditure

3) $$MRS_{\alpha}(x,s,t)=UW\frac{\mu}{P}$$

Consumption

4) $$x=(UW-W)\frac{\mu}{P}$$

Transaction

5) $$U=\frac{1}{s}$$

Money

6) $$\phi=U+W$$

Time Allocation

7) $$T=t+\lambda$$

A more accurate description of the system would have convexities in operation. Indeed, the curvature of the system plays a major role in the stability of the model in equilibrium. However, as the simulations demonstrate, equilibrium under these restrictive conditions exists and is in magnitude consistent with historical performance.

Table 1

Simulations Results

	Factor Shares First Simulation		Factor Shares Second Simulation
	Labor Hrs 2/3 Human Cap. 0 Physical Cap 1/3		Labor Hrs 1/3 Human Cap. 1/3 Physical Cap 1/3
Leisure	0.436	Ratio	0.676
Labor Hours	0.564	Ratio	0.324
Consumption	26.598	Level	30.342
Saving Rate	0.300	Rate	0.202
U	3.333	Level	4.962
W	6.667	Level	5.038
Price Level	1.228	Level	1.381

THE JACOBIAN OF THE SYSTEM

The total differentials of the system have (dκ), (dμ), (dφ), and (dT) as exogenous variables, and (dλ), (dU), (dW), (dP), (dt), (ds), and (dx) as endogenous. They appear in the appendix as a system of equations. This system of differential equations can be compactly expressed as, $A\nabla Y = \nabla X$.

The Jacobian of the system, |A |, is the determinant of the square matrix A. For most relevant conditions the determinant of A is non-singular. This solves the system for changes in the endogenous variables, relative changes in the exogenous variables. The solution being simply, $\nabla Y = A^{-1}\nabla X$.

The above equation, in conjunction with the general equilibrium of the system, gives us means for the comparative statics. It can be easily verified that nominal physical money is neutral relative all real variables. The effects of changes in the money ratio (φ) on the variables of the system depend on the sign of the Jacobian and, the interaction between the marginal rates of substitution, the production function, and the multiplier structure of the network algebra. The stability issue (correspondence principle) is dealt with for a particular and very important case later on. Of interest are the conditions for a steady state scenario.

Let us define a global steady state to be that state wherein the change in all of the exogenous and endogenous variables is equal to zero. That is, both gradients are zero.

Thus, we can identify a state of inter-temporal equilibrium. If only the considered exogenous variables affect the endogenous variables then, within a small neighborhood of the original equilibrium, temporal change in the exogenous variables implies temporal change in the endogenous. In particular if $\nabla \mathbf{X} = \mathbf{0}$ then $\nabla \mathbf{Y} = \mathbf{0}$, and we have a global steady state given general equilibrium conditions when $|A| \neq 0$. The above is a necessary but not sufficient condition for a global steady state. In a later section, dedicated to growth, I discuss a particular set of conditions that yield a global steady state. The Jacobian will be revisited then, as it plays a major role in a first order Taylor approximation of the system in differentials.

REAL INCOME AND CONSUMPTION

1. Real Income.

In equilibrium the system solves for real income and consumption as well as savings and other real variables not explicitly present in the model. Recall that real income per capita in shadow values (or multiplier) form is.

$$q = (UW - 1)\frac{\mu}{P} \ .$$

Changes in the exogenous variables are transmitted to real income through (U), (W), (P), and (μ). Given a position of general equilibrium the change in real income can be expressed as.

$$dq = \left(\frac{W\mu}{P}\right)dU + \left(\frac{U\mu}{P}\right)dW - \left(\frac{UW-1}{P}\right)\left(\frac{\mu}{P}\right)dP + \left(\frac{UW-1}{P}\right)d\mu.$$

As noted before changes in μ are neutral in the real variables. This applies of course to q as well. For changes in μ we have that dU=dW=0 and dP is equal to $\frac{P}{\mu}d\mu$ which implies that $\frac{dq}{d\mu}=0$, the neo-classical result. Aggregate real income is simply qN. Thus aggregate real income is $Q=(UW-1)\frac{m}{P}$.

2. Consumption.

The shadow consumption function is expressed in terms of the multipliers and the incident signal μ. This shadow expression of the total consumption behavior of the system (agent or aggregate environment) can take on a variety of equivalent forms. That is, in equilibrium the consumption function is not unique, it can be expressed in a variety of ways. This may account for the different functional forms suggested by both a broad range of theoretical and empirical works (Keynes 1936; Friedman 1957; Modigliani-Brumberg 1954) on the consumption function. Recall that the multiplier form of consumption per capita is

4) $$x=(UW-W)\frac{\mu}{P}.$$

So, $$d\chi=\left(\frac{W\mu}{P}\right)dU+\left(\frac{U\mu-\mu}{P}\right)dW-\left(\frac{UW-W}{P}\right)\left(\frac{\mu}{P}\right)dP+\left(\frac{UW-W}{P}\right)d\mu.$$

By virtue of the algebra it can be shown that the shadow consumption function above is identically equal to a form akin to the Keynesian-Modigliani types, i.e.,

$$x = cq + c\frac{\mu}{P}.$$

Where, recall that c is the marginal propensity to consume and $\frac{\mu}{P}$ is wealth in the form of real money balances (in physical money.) Yet another form is more akin to Friedman-Modigliani types and can be derived from the multiplier form of the consumption function. This can be shown to have the proportional type form of.

$$x = \gamma q$$

where in this case γ is equal to the average propensity to consume (APC). In multiplier form the APC is equal to. $\dfrac{UW - W}{UW - 1}.$

Notice that this expression cannot be reduced further in terms of the multipliers. It is as if we had a ratio of prime numbers. The multipliers in different configurations have interpretations that are meaningful.

The above shows that the functional form of the consumption function is not unique. In fact it can be shown that consumption can be expressed in an infinite number of equivalent functional forms. Nevertheless, in equilibrium, real consumption is neutral in nominal physical money m.

THE MPC COMPLEX.

It is clear from the literature that the concept of the MPC is not unique. Depending on the definition of income used we can define a relevant MPC, e.g., the permanent income hypothesis identifies MPC's for both permanent and transitory income. However, as soon as the measure of income is properly defined we can identify a relevant MPC. So far I have not shown the relationship between what I have defined to be the MPC and what it actually means in the literature. I will show that (c) is only equal to the true MPC out of income when dc = dP =0. Keynes defined the marginal propensity to consume to be the change in consumption relative the change in disposable income. Now, recall that the agent faces the budget constraint of

$$x = c\left(q + \frac{\mu}{P}\right) = c\left(\frac{z}{P}\right) .$$

Differentiating the above gives the true MPC in equilibrium,

$$\frac{dx}{dq} = \frac{dc}{dq}\left(\frac{z}{P}\right) + c\left(1 - \frac{\mu}{P^2}\frac{dP}{dq}\right) .$$

We have then two notions of the MPC, the traditional one as above and that as defined earlier. The difference is that c is the marginal propensity to consume but out of synthetic income as opposed to income proper. as long as c does not change with changes in synthetic income. This should make clear the distinction between the two. In reality to label c as the MPC is somewhat of a misnomer because the true MPC is a special case of c. The Transaction Consumption Rate (TCR) would be a more appropriate term. However, we retain the notion that c is a liquid resource marginal propensity to consume and call it the MPC throughout unless otherwise noted.

The equilibrium Keynesian MPC can be obtained by the Jacobian of the system and the vector of exogenous changes. Cramer's rule shows that the Keynesian MPC is given by a ratio of determinants. Of interest is the result that small changes in any individual exogenous variable does not affect the Aggregate Keynesian MPC in equilibrium, as long as population growth is zero. However, if two or more of the exogenous variables are changing, then the Keynesian MPC is bound by those changes regardless of the rate of growth of population.

The equations for the Keynesian MPC are given in the last section of the paper. Of importance is the finding that there are two Keynesian MPC's that are obtained by the comparative statics. One relates to the micro unit and the other to the macro system, and further, they need not be equal. This underscores that although sharing the same fundamentals we find differences in response between the unit entity and the aggregate behavioral response. The difference in behavior between the macro system and the micro is that in essence we have one entity composed of the totality of unit organic agents. The multiplicity of a number of possible organic agents gives rise to the coalescing of agential animus. This is a manifestation of the collective social organism.

SOLVING FOR THE EQUILIBRIUM IN LABOR AND THE MPS

We can now solve the general equilibrium of the system by reducing the seven equations to two equations in labor as well as the MPS. These two equations will allow us to analyze the system in terms of labor-time and the marginal propensity to consume. We proceed as follows. Combine equations 1 and 3 to get

(1b) $$f(\lambda,\kappa) = MRSsx(x,s,t) - \frac{\dot{\mu}}{P} \quad .$$

Now use equations 4,5,6 and 7 to solve for U, W, x and t in equations 1b, 2 and 3. This yields

(1c) $$f(\lambda,\kappa) = MRS^{*}_{sx}(s,\lambda) - \frac{\mu^{*}}{P}$$

(2b) $$MRS^{*}_{tx}(s,\lambda) = (1-s)\frac{\partial f}{\partial \lambda}$$

(3b) $$MRS^{*}_{sx}(s,\lambda) = \left(\frac{\phi s - 1}{s^{2}}\right)\frac{\mu^{*}}{P} \quad .$$

Finally, use (3b) to solve away $\frac{\mu^{*}}{P}$ in (1c). The system is reduced to two equations in two unknowns. They are,

(I) $$f(\lambda,\kappa) = G(s,\phi)\mathrm{H}(s,\lambda,\kappa,\phi)$$

(II) $$\Lambda(s,\lambda,\kappa,\phi) = (1-s)\frac{\partial f(\lambda,\kappa)}{\partial \lambda}$$

where, $G = 1 - \frac{s^{2}}{\phi s - 1}$, $\mathrm{H} = MRS^{*}_{sx}$ and $\Lambda = MRS^{*}_{tx}$.

One of the appeals of equations I and II is that we have a direct association between the productive process and the preference structure of the agent. Note that if the MPS is close to zero, then MRSsx $\approx f(\lambda,\kappa)$ and MRStx $\approx$ MPL. In order to gain a unit of the marginal propensity to save, the agent would have to give up approximately the totality of output in consumption. Also in order to gain a unit of leisure the agent would have to give up approximately the marginal product of labor in consumption. Further, the marginal rates of substitution can be approximated by a first (or second) order Taylor expansion about equilibrium avoiding the need to specify a particular form of the utility function in equilibrium.

STABILITY

The stability conditions are given by the below differential equations:

$$\frac{ds}{d\tau} = \pi[(1-s)f_{\lambda} - \Lambda]$$

$$\frac{d\lambda}{d\tau} = \gamma[G(s,\phi)\mathrm{H} - f(\lambda,\kappa)] \ .$$

The functions $\pi(*)$ and $\gamma(*)$ are assumed to be positively sloped and equal to zero when evaluated at zero. This gives us the dynamic adjustment process, which can be further expressed as a first order Taylor approximation.

$$\frac{ds}{d\tau} = \pi'[(1-s)f_{\lambda\lambda} - \Lambda_{\lambda}](\lambda - \lambda^{e}) - \pi'[f_{\lambda} + \Lambda_{s}](s - s^{e})$$

$$\frac{d\lambda}{d\tau} = \gamma'[G(s,\phi)\mathrm{H}_{\lambda} - f_{\lambda}](\lambda - \lambda^{e}) + \gamma'[G_{s}\mathrm{H} + G(s,\phi)\mathrm{H}_{s}](s - s^{e}).$$

From the coefficients of the characteristic equation, we obtain the general necessary and sufficient conditions for stability. These are,

$$-\pi'[(1-s)f_{\lambda\lambda} - \Lambda_{\lambda}] > \gamma'[G_{s}\mathrm{H} + G\mathrm{H}_{s}]$$

$$[(1-s)f_{\lambda\lambda} - \Lambda_{\lambda}]\,[G_{s}\mathrm{H} + G\mathrm{H}_{s}] > -[G\mathrm{H}_{\lambda} - f_{\lambda}]\,[f_{\lambda} + \Lambda_{s}].$$

Particular sets of conditions that ensure that the above necessary and sufficient conditions are met are

a) $-\varepsilon G_s > \varepsilon H_s$

b) $\varepsilon H_\lambda > \varepsilon f_\lambda$

c) $\varepsilon \Lambda_s > -\frac{s}{1-s}$.

The first condition is that the negative of the MPS elasticity of G must be greater than the MPS elasticity of the MRS of consumption for the MPS. The magnitude of sensitivity to the MPS must be larger for G than for the marginal rate of substitution of consumption for MPS. If the MRS is zero then the condition is met. The second condition states that the labor elasticity of the MRS of consumption for the MPS must be greater than the labor elasticity of production.

The third condition states that the MPS elasticity of the MRS of consumption for labor must be greater than the negative of the ratio of the MPS to the MPC. The first and third conditions are easily met. The second condition, however, requires a strong MRS response relative to the marginal product of labor.

COBB-DOUGLAS CRS TECHNOLOGY AND GENERAL EQUILIBRIM

1. Cobb-Douglas Constant Returns to Scale.

If the production technology is Cobb-Douglas with Constant Returns to Scale we find an interesting result relating the marginal rates of substitution of the reduced Labor-MPS system. Notice that equations (I) and (II) of the Labor-MPS system may imply.

under certain conditions, multiple equilibria. By assuming a Constant Returns to Scale Cobb-Douglas production function we simplify somewhat the solution of the general equilibrium problem. Let the output per capita per unit time (q) be given by

$$\begin{aligned} q &= f(\lambda,\kappa) \\ &= A\lambda^{1-\alpha}\kappa^{\alpha} \end{aligned} .$$

Because the production function is Constant Returns to Scale (q), (λ), and (κ) can be measured in any ratio form in (Q), (L), and (K) (the aggregate counterparts) relative to some chosen reference. This reference can be a measure of time available in a period, or number of workers employed, or the labor-force proper. Here the reference used will be actual number of employed workers (N). As usual the marginal products for labor and capital respectively are

$$\begin{aligned} MPL &= (1-\alpha)\frac{q}{\lambda} \\ MPK &= \alpha\frac{q}{\kappa} \end{aligned} .$$

Second order conditions hold for profit maximization behavior and the factor coefficients are the factor output-income shares. But by Euler's Theorem the labor share of income is equal to its coefficient in the function. This production function implies that the labor elasticity of output is $1-\alpha$ as the labor share of income. That is,

$$(1-\alpha) = \frac{\frac{\partial f}{\partial \lambda}}{f(\lambda,\kappa)}\lambda \quad .$$

Now, by taking the ratio of equations I and II from the Labor-MPS system with the above Cobb-Douglas technology we get, after some re-arranging and substitutions of terms, the below equilibrium ratio. This ratio shows the equilibrium MRS between the MPS and leisure.

$$\text{(III)} \qquad \frac{MRS_{\alpha}}{MRS_{\alpha}} = \frac{(1-\alpha)(1-s)}{\lambda} G(s,\phi) \ .$$

The left hand term being equal to

$$\text{MRSts} = -\left(\frac{ds}{dt}\right)_0 = \left(\frac{ds}{d\lambda}\right)_0 .$$

This left-hand term represents the agent's manifested preference structure; in this case in terms of the marginal rate of substitution between the MPS and labor. That is the rate of change of the indifference curve at a given level of utility u_0. The right hand tem of equation (III) is the slope of the environmental production function.

$$\frac{ds}{d\lambda} = \frac{(1-\alpha)(1-s)G(s,\phi)}{\lambda} \ .$$

This equation is the result of the production function and the network taken together as a unified constraint. This may constitute the environment other than the agent. Both sides (forces) complete the general environment of the social organism proper. The above differential equation embodies the environmental constraint faced by the agent in terms of labor, production technology, money, and the marginal propensity to consume. Note that labor and the MPC are two goods that enter as "bads" in the utility function of the agent. The solution of the differential equation gives us a Labor-MPS possibility frontier. Given the assumed production function, the solution to the differential equation would give us the feasible set of combinations of labor and the MPS that the agent has at his disposal. The solution of the above differential equation is not straightforward however. Nevertheless, we can use an approximation to it by assuming (G) equal to 1.

2. The Labor-MPS Differential Equation Approximation.

For small values of (s), or very large values of (ϕ), we get that G(s,ϕ) is approximately equal to one. This is easy to see when we again use the expression of (G) in terms of the algebra. This can also be shown to be equal to the ratio of gross nominal income (Y) to nominal synthetic income, i.e., (Y/Z). Under all values of the multipliers (U) and (W) and in equilibrium, the ratio is always smaller than 1. In any event this suggests a first order differential equation in the MPS and labor in equilibrium. Taking the approximation that $G \approx 1$ we get,

$$\frac{ds}{d\lambda} \approx \frac{(1-\alpha)(1-s)}{\lambda} \quad .$$

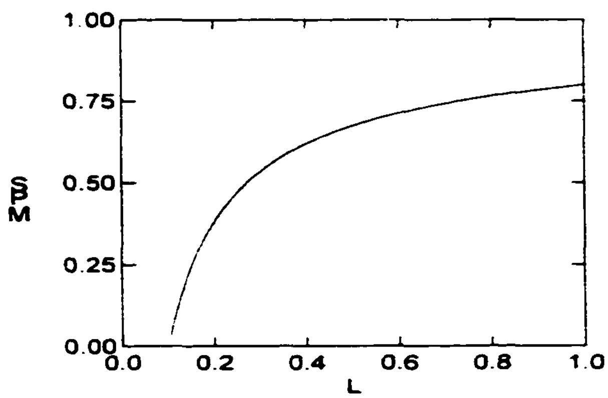
This last expression suggests a first order differential equation in s and λ which given initial conditions has as solution

$$s(\lambda) = 1-(1-\sigma)\lambda^{-(1-\alpha)} \quad .$$

The second derivative throughout implies concavity.

Now, because labor has an upper limit, i.e., the maximum level of time units available in a given period of time (T), the function looks as the one depicted below.

Figure 1



Note that in equilibrium the maximum marginal propensity to save is equal to σ,

$$Max\_s = \sigma .$$

Normalizing T to equal 1 obtains this result. So, labor is measured as a fraction of total time in a period ($0 \leq \lambda \leq 1$). In this paper the decision time period will be a year, thus λ is proportion of market-labor time in a year.

THE APPROXIMATE LABOR-MPC ENVELOPE

The Labor-MPS equilibrium relation is in fact an envelope equation of the Isoquant curve of the system or more precisely of the environment. By re-arranging terms we can reformulate the equation in terms of the marginal propensity to consume relative to labor.

Clearly,

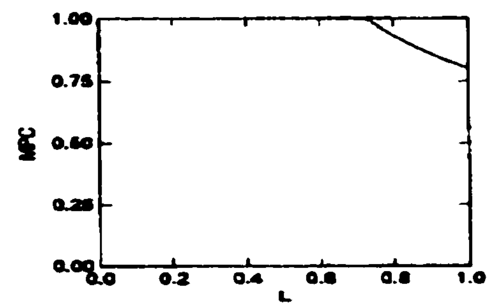
(IV) $$MPC = 1 - s = (1-\sigma)\lambda^{-(1-\alpha)}.$$

The minimum MPC is thus (1 - σ) per period of analysis. All of the above implies that in equilibrium ceteris paribus an increase in labor is associated with a fall in the MPC in production. The constant return to scale Cobb-Douglas technology yields a technological-environmental relationship between labor and the MPC. The simplicity of the function further allows for straightforward estimation of the coefficients by OLS.

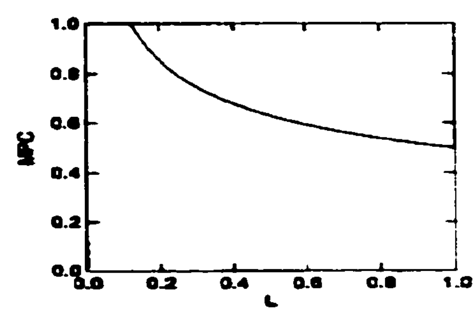
Of interest is the labor elasticity of the MPC. This is equal to the negative of the labor-share of income. Empirical support of the existence of this field would constitute evidence that the macro level is a reflection of the micro and vice versa. The micro unit is assumed to have a utility function in (x), (t), and (s) where an implicit utility function in equilibrium in (x), (λ), and 1-s would yield an indifference mapping between labor and the MPC ceteris paribus. The below graphs were calibrated at min MPC = 0.80 and labor share of income of 0.70 for the case of employment, i.e., number of workers. The man-hour case was calibrated at min MPC of 0.50 and labor share of 1/3.

Figures 2 and 3

EMPLOYMENT RATE



MAN-HOUR PROPORTION OF TIME HORIZON



THE FEASIBLE LABOR-MPC SET

The area above and to the right of the Labor-MPC envelope is the feasible set of points that the agent can achieve in labor and the MPC. The area is measured in labor-MPC units. This area is given by

$$FA = (1-\lambda_{min}) - \int_{\lambda_{min}}^{1} (1-\sigma)\lambda^{-(1-\alpha)} d\lambda$$

which reduces to $$(1-\lambda_{min}) - (1-\sigma)\left[\frac{1-\lambda_{min}^{\alpha}}{\alpha}\right].$$

This can be used as an index of potential general welfare. The area represents all possible combinations of labor-MPC that can be accomplished by the technological-environmental constraint. We may label this quantity the Feasible Set Index.
An index value of 1 unit labor-MPC can be considered to be in a sense Bliss. ceteris paribus. This is only achieved when both the minimum labor and minimum MPC are zero.

UNIQUENESS OF EQUILIBRIUM AND THE TRUE POSSIBILITY FRONTIER

The Labor-MPC envelope equation just seen is an approximation of the true possibility frontier equation. The slope of the true curve is given by

8) $$\frac{dMPC}{d\lambda} = -\frac{(1-\alpha)(1-s)G(s,\phi)}{\lambda}.$$

The equilibrium of the whole system depends on the agent reaching the lowest possible indifference curve in labor and the MPC, within the feasible set. This gives us the condition that the $MRS_{\lambda c}$ must equal the negative of the rate of change of the possibility frontier, - $\frac{dMPC}{d\lambda}$. Because the MPC and labor enter as bads in the utility function, the indifference curve would be concave to the origin. This implies that a solution to the general equilibrium of the system given CRS Cobb-Douglas technology is unique in all the endogenous variables, as long as the Labor-MPC envelope is convex to the origin. This condition is met if the second derivative of the true envelope function is positive. The second derivative of the envelope function can be shown to be

9) $$\frac{d^2MPC}{d\lambda^2} = (1-\alpha)\left(\frac{G}{\lambda^2}\right)\left[s(\varepsilon S_\lambda) + (1-s)(1-\varepsilon G_\lambda)\right].$$

The term in brackets is a weighted-sum of two elasticities and it is this that determines the sign of the derivative. The first one, the labor elasticity of the MPS, εS_λ, is easily verified to be strictly positive. If the labor elasticity of (G) is inelastic, i.e., $|\varepsilon G_\lambda| < 1$, then, the derivative is positive. In general if the term in brackets is positive then the possibility frontier is convex to the origin. This solves for the MPC and labor, and thus the MPS and leisure uniquely. Having solved for labor we thus solve for the real output and the marginal product of labor. The MPS in turn solves for the (U) multiplier and thus (W) is determined by equation (7). Given the multipliers and real output-income we solve for the general price level via equation (1). Finally having solved uniquely for the multipliers and the price level we can immediately solve for real consumption. Therefore, the endogenous variables of the system are determined uniquely, i.e.,

x, s, t, U, W, P, ω / P.

Notice that the model also uniquely determines all money balances, real or nominal, other than the nominal stock of physical money. Recall that the model takes the stock of nominal physical money per capita as exogenous at the beginning of the period. However, the model also determines the real value of this stock of money in the current period.

THE WORK OF PHYSICAL MONEY

The transaction multiplier here gives the work performed by physical money, in the transaction of goods and services. The number of hands that physical money changes hands by, thus defines work. Therefore, one dollar performs work of magnitude (U). The relationship between the MPC relative and labor implies that the transaction multiplier is also affected by labor. This relationship is that expressed in equation (8). We can express the same in terms of the multipliers. The (G) function in equation (8) expressed in the multipliers gives the labor elasticity of the MPC as,

10a) $$\frac{dMPC}{d\lambda}\frac{\lambda}{MPC} = -(1-\alpha)\frac{UW-1}{UW}.$$

This further implies that in production there is a relationship between labor and the transfer work of money given by (U). Recall that (U) is a function of the MPC. The labor elasticity of the transaction multiplier can be expressed as,

10b) $$\frac{dU}{d\lambda}\frac{\lambda}{U} = -(1-\alpha)\frac{(U-1)(UW-1)}{UW} \prec 0.$$

As labor increases in production, the transaction multiplier falls. Given the stock of capital in the production process, the amount and magnitudes of labor determine physical output for the time period considered. An economic interpretation of the relationship between labor and the multiplier illustrates the forces at work. An increase in labor in production, increases real output for a given level of capital. More output implies that money, for a fixed price level, must change hands more frequently. On the other hand, it may be useful to fix the real value of transactions to peg ideas. Recall that the volume of transaction money is $M_T = Um$. If the real value of transaction money is fixed, then we find a relationship between labor and the price level. As labor increases, output increases, thus increasing the supply of goods and services. More goods and services to be transacted require the turnover of money at higher frequencies for a fixed quantity of money (m). However, in production, the collective social organism is faced with a trade-off between transaction turnover and labor. Thus, an increase in labor in production is associated with a drop in the transaction multiplier. This, in turn, forces the price level to drop in order to keep real transaction money balances constant. Now, if labor increases, then the supply of goods and services increase as noted. Therefore, the real value of the stock of physical money must be higher in the sense that now a dollar commands more goods and services. By Walras' excess demand we can further argue that the price level would tend to decrease, increasing the real value of physical money $\frac{m}{P}$. Hence, physical money has to work less for the same level of real transactions. On the other hand, if labor decreases then output decreases and so the amount of embodied goods and services in physical money also decreases. This pushes the price level higher. Given a fixed stock of physical money, the real value of the stock falls. It has decreased in volume (or density). Therefore, in order to be able to transfer a fixed amount of real valued transactions the transaction multiplier must increase. Because physical money is in a sense a virtual vessel of goods and services, then, if the vessel carries less it would need to change hands more frequently to leave the level of real transactions of those goods and services fixed.

The tradeoff between labor and the transaction multiplier gives us a better picture of the shape of the environmental production function network inclusive. What is being produced goes beyond goods and services; it is more like rapidity of transfer of goods and services. In light of this we can clearly see that the transaction multiplier is a measure of the work performed by one unit of physical money.

THE RESOURCE AND SUBSTITUTION EFFECTS

We have seen that the general equilibrium of the system can be derived from the equilibrium between the MPC and labor-hours. When the marginal rate of substitution between the MPS for leisure is just tangent to the production-possibility frontier, we have the optimal combination between labor-hours and the marginal propensity to consume. However, changes in the production-possibility frontier have both a substitution effect and a resource effect. The change in the production-possibility frontier can have simultaneous changes in the net resource and in the relative prices of labor and the MPC.

The below figure identifies the total resource effect, the net resource effect, and the substitution effect. The movement from A to B is the substitution effect. From B to C we have the net resource effect akin to an income or wealth effect. The movement from A to C is thus the total resource change effect.

Figure 4

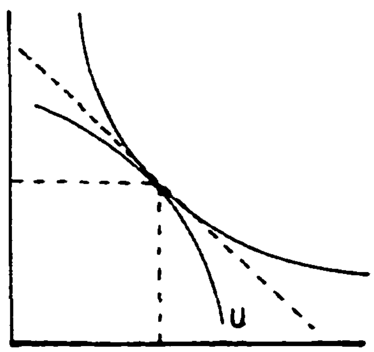
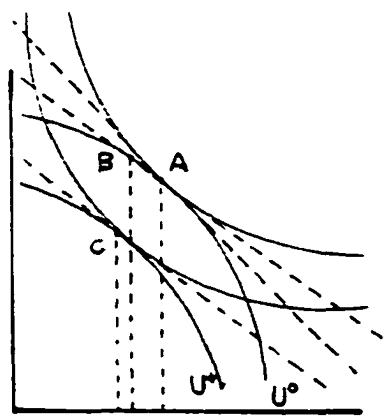


Figure 5



Clearly changes in the production-possibility frontier may result in either an increase or a decrease in utility depending on the shift and the magnitude of the shift. Nevertheless, ceteris paribus and especially holding consumption fixed. we can say that movements to the southwest corner are associated with a higher level of utility.

ESTIMATION OF THE MPC AND LABOR

Before the above models can be estimated and tested we need some adequate estimates of the variables to be used. The algebra of networks with its multipliers in combination with the national income accounts can generate estimates of the multipliers themselves. Data for the U.S. will be used where the sectors of government and the exterior will be incorporated to our general model. The inclusion of these sectors can be incorporated into the network algebra yielding its own very interesting algebraic and structural properties. This, however, at present would be an unnecessary complication to add to our model. By assuming that the social agent is composed of all consumers of the system including government then we can re-interpret our earlier result relative this broader entity.

1. The Multipliers and the MPC.

If we define aggregate consumption to be equal to the consumption expenditure of households and government, then

11) $$C = PX + PG$$

where (C) is nominal aggregate consumption, (X) is aggregate real consumption of households, (G) is real government expenditures, and as usual (P) is the price level.

Gross saving in an open economy is equal to investment plus net exports in equilibrium. Therefore nominal gross saving is equal in equilibrium to

12) $$S = PI + PNX \quad .$$

Where here (I) is gross real domestic investment and NX is real net exports. Given that we have defined consumption and saving we can calculate the values of (U) and (W) that are consistent with such definitions. Recall that the algebra of networks gives us equations for consumption and saving. We can replace these for the left-hand terms of the above equations. The national income accounts provide the values for the right hand side. We have thus a system of two equations in (U) and (W). For consumption and saving respectively we have

$$(UW - W)m = PX + PG$$
$$(W - 1)m = PI + PNX$$

The multipliers are thus,

$$13) \qquad Uy \equiv \frac{GDP + \frac{m}{P}}{I + NX + \frac{m}{P}} \qquad Wy \equiv \frac{P(I + NX) + m}{m}$$

Recall that the marginal propensity to consume is equal to (U-1)/U so we have a measure of the MPC for each observation. The GDP deflator will be used as the price level. The stock of physical money (m) will be made equal to the stock of the monetary base. currency in circulation, or any other suitable narrowly defined instrument.

There are other methods of estimating the multipliers. For example, if we take into account the money complex inferred by the algebra of networks, we find that there are estimates of U and W derivable from properly defined money. If we use as proxies of our money measures those mentioned earlier we get,

$$M_T \equiv Um \approx M1 + \text{Reserves}$$

$$M_S \equiv Wm \approx \text{Time Deposits} + \text{Monetary Base}.$$

Notice that the estimates of U and W are readily obtained by dividing through by m. Similarly as in the national income accounts variables, these money estimates will be denoted by subscript m. The money accounts are likely to reflect volume of transactions as opposed to final-goods value as the income measure used in this paper. Yet another measure would take the geometric mean of the above income and money estimates for each individual multiplier. The reader may verify that this last estimation method delivers an identity in income. However, only the national accounts (first one) estimates will be used in this paper.

The measure of labor that I will use will take on both a temporal measure and a proxy of such. The proxy of labor used will be the rate of employment. This is not a temporal unit but it is a measure of proportion of labor-force employed in a given time period, e.g., a year. The actual temporal unit measure will be average weekly (yearly) labor-hours. The data will be of sectoral behavior. The sectoral data of average labor-hours was obtained from Jorgenson, Gollop, and Fraumeni (1987).

The below two plots are for the MPC and the employment rate. This is given as a reference relationship for both the period from 1929 to 1975, and 1948 to 1975. For the period of 1948-1975 the correlation coefficient between the MPC and the employment rate is higher in absolute value than that for the broader period of 1929-1975. The correlation coefficient for the shorter period is –0.341 where the correlation was –0.249 for the broader period.

MPC RELATIVE AND THE RATE OF EMPLOYMENT 1929-1975 AND 1948-1975

Figure 6

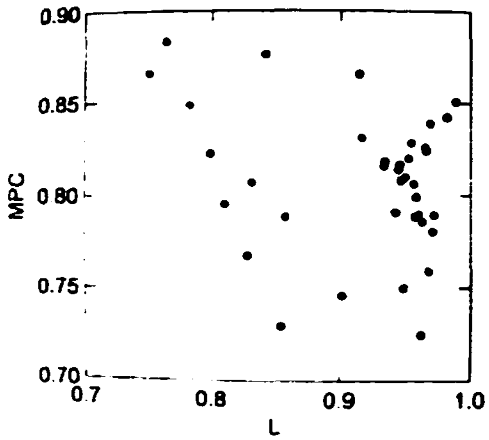
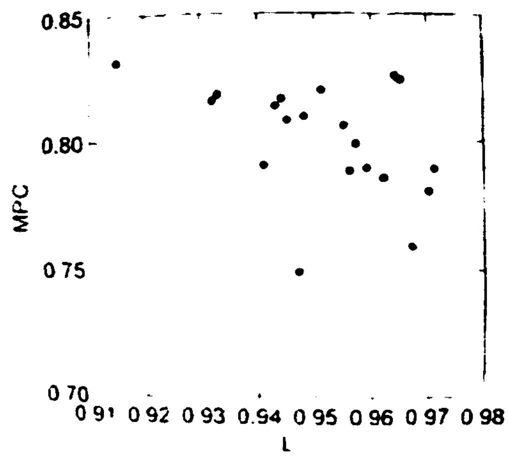


Figure 7



THE MPC LABOR CORRELATIONS

Estimated correlations between the estimates of the MPC and the measures of labor show that the signs expected hold for most measures of labor used. The correlation between the MPC and the employment rate were obtained for the period from 1929 to 1975. The correlation was –0.249. As the employment rate increased, the MPC fell on average. According to the theoretical results obtained earlier we would expect a negative relationship between the MPC and labor in particular labor-hours. The fact that the employment rate moved in the opposite direction of the MPC as calculated could be offered as a proxy result in support of the theory. More appropriate correlations would be between the MPC and labor-hours for a broad range of sectors. Using average weekly labor-hours from Jorgenson et.al. I obtain the Pearson correlation coefficient between the MPC as measured and labor-hours for 51 sectors from 1948 to 1975. The results are given in Table 2 below.

Table 2

PRIVATE SECTORS	SYMBOLS	CORRELATION	PARTIALS
Agricultural Production	AP	-0.869	-0.864
Agricultural Services	AS	-0.607	-0.534
Metal Mining	MM	-0.419	-0.260
Coal Mining	CM	0.573*	0.614
Crude Petroleum and Natural Gas	CPNG	0.394*	0.356
Non Metallic Mining and Quarrying	NMQ	0.048	0.377
Contract Construction	CC	-0.223	0.078
Food and Kindred Products	FKP	-0.870	-0.854
Tobacco Manufactures	TM	-0.135	-0.084
Textile Mill Products	TMP	0.419	0.566
Apparel & other Fabricated Textile Products	AOFTP	-0.153	0.042
Paper and Allied Products	PAP	-0.530	-0.463
Printing and Publishing	PP	-0.390	-0.248
Chemicals and Allied Products	CAP	-0.593	-0.518
Petroleum and Coal Products	PCP	0.087*	0.005
Rubber and Miscellaneous Plastic Products	RMPP	0.178*	0.441
Leather and Leather Products	LLP	0.036	0.312
Lumber and Wood Products, except furniture	LWPEF	-0.056	0.135
Furniture and Fixtures	FAF	-0.569	-0.485
Stone, Clay and Glass Products	SCGP	-0.061	0.197
Primary Metal Industries	PMI	-0.057	0.207
Fabricated Metal Industries	FMI	-0.297	-0.083
Machinery, except electrical	MEM	-0.446	-0.308
Electrical Machinery, equipment and supplies	EMES	-0.625	-0.561
Transportation Equipment and Ordinance, except motor vehicles	TEOEMH	-0.334	-0.202
Motor Vehicles and Equipment	MVE	-0.123	0.002
Professi'nl Photogrphic Equipmnt & Watches	PPEW	-0.446	-0.327
Miscellaneous Manufacturing Industries	MMI	-0.664	-0.607
Railroads and Rail Express Services	RRES	0.067	0.103
Street Railways, bus lines and taxicabs	SRBLT	0.067	0.103
Trucking Services and Warehousing	TSW	0.007	0.285
Water Transportation	WT	-0.397	-0.310
Air Transportation	AT	-0.611	-0.627
Pipelines, except natural gas	PENG	-0.516	-0.418
Transportation Services	TS	-0.436	-0.334
Telephone, Telegraph, and misc. Communication Services	TTMCS	0.251	0.319
Radio and Television Broadcasting	RTB	-0.474	-0.450
Electric Utilities	EU	-0.227	-0.075
Gas Utilities	GU	-0.438	-0.318
Water Supply and Sanitary Services	WSSS	-0.876	-0.863
Wholesale Trade	WHT	-0.587	-0.509
Retail Trade	RT	-0.759	-0.735
Finance, Insurance and Real Estate	FIRE	-0.709	-0.665
Services, except private households & instits.	SEPHI	-0.677	-0.638
Private Households	PH	-0.934	-0.926
Institution	INST	-0.375	-0.306

PUBLIC SECTORS			
Federal Public Administration	**FEDPA**	-0.716	-0.670
Federal Government Enterprise	**FEDGE**	-0.689	-0.727
State and Local Educational Services	**SLES**	-0.740	-0.726
State and Local Public Administration	**SLPA**	0.119	0.160
State and Local Government Enterprises	**SLGE**	-0.128	-0.092

* Fossil Fuels and Derivatives

Out of the 51 sectors considered only 9 had correlation between the MPC and labor-hours that were positive. The vast majority of sectors display the negative pattern suggested by the theory. Further, of the nine sectors with positive correlation four were in the fossil fuels and derivatives industries. The others involve public and quasi-public sectors.

Because the employment rate may have an effect on the average weekly labor-hours, I control for its effects. The last column of the table gives us the partial correlations between the MPC and labor-hours for each sector while controlling for the employment rate. Once again the vast majority of sectors have the right sign. However, the number of sectors with positive partial correlations doubles from 9 to 18. In the appendix we find the Labor-MPC plots with graphic representation of the above correlations for all sectors.

Since most sectors display the expected theoretical sign, I conclude that the historical data used lends some preliminary support to the theory. Those sectors that do not have the right sign seem to be those affected strongly by fossil fuels and derivatives industries. Given that the period of analysis saw dramatic changes in the energy producing industries we can speculate that these shocks de-stabilized the production function. This de-stabilization may have masked the true relationship between labor-hours and the MPC.

THE COEFFICIENTS OF THE APPROXIMATE ENVELOPE EQUATION

By taking logs of equation (IV) and by adding an error term we get the stochastic equation to be estimated,

$$\ln MPC = \ln(1-\sigma) - (1-\alpha)\ln\lambda + u$$

$$= \beta_0 + \beta_1 \ln\lambda + u .$$

We look at four different model specifications. Because of the presence of autocorrelation in the data, first order autoregressive processes are invoked. Also, the measures of labor-hours will be the proportion of average weekly labor-hours to total number of hours in a week. The models follow the below format.

Table 3
MODELS

	A	B	C	D
Dep.Variable	$LnMPC_t$	$lnMPC_t$-ρ$lnMPC_{t-}$	$LnMPC_t$ - $lnMPC_{t-}$	$LnMPC_t$ - $lnMPC_{t-}$
Constant	β_0	$(1-\rho)\beta_0$	θ_0	----
Slope for	$\ln\lambda_t$	$\ln\lambda_t - \rho\ln\lambda_{t-1}$	$\ln\lambda_t - \ln\lambda_{t-1}$	$\ln\lambda_t - \ln\lambda_{t-1}$

Model D is the unit root formulation with zero intercept $\Delta MPC = \beta\Delta\ln\lambda$.

Two data sets will be used. The first set (Figure 5) will use aggregate average weekly hours measured as the average of all monthly averages in a year (Bureau of Labor

Statistics 2002). We divide that average weekly hours by 168, which is the number of hours in a week. The ratio of these gives us our labor variable. This data set will span the years 1967 to 1996. This variable will be related to the constructed aggregate MPC relative. The second data set (see plots in appendix) will relate the MPC relative to the average weekly proportion of time dedicated to labor for the 51 sectors listed above. This will cover the period 1948 to 1975. The following graphs look at the relationship between the MPC relative and the average aggregate proportion of weekly hours dedicated to labor (first set). The first graph looks at the relationship directly. Notice that there seems to be no significant relationship between the two variables. The second scatter plot adds the dimension of year into the picture. We see a decrease in the labor variable as time progresses. This is just visual evidence of the well-established empirical finding that the average workweek has decreased with time (Killingsworth 1983). The relationship between time and the MPC relative is less significant. Indeed, there is no meaningful statistical relationship between the two variables. This would be counter to the evidence presented earlier over sectors of a negative relationship between the MPC relative and labor. If we plot the variables in the natural logs we still find no discernible pattern between them. However, notice that the third graph presented shows the scatter plot of the change in the natural logarithm. Here we clearly see that there is a linear and negative relationship between the two. This suggests that the labor elasticity of the MPC relative is constant and equal to the slope of the relationship. This is precisely what our model predicts.

We turn then to the regression estimate of the models A through D for the aggregate labor variable. The estimates are reported below, followed by the plot of residuals to estimate. Notice that model A delivers insignificant estimates. This, however, is not necessarily indicative of no relationship between the variables. As time passes, the relationship between the MPC relative and labor changes, that is, the production function shifts throughout time. This is evident by the second scatter plot. Estimates of model type B are no better. These estimates are unbiased but not efficient. Models C and D on the other hand deliver significant and meaningful estimates. Model C estimates the slope coefficient by a magnitude that is inconsistent with the prediction of the model, if Constant Returns to Scale is being assumed. Nevertheless we have an unbiased estimate of the coefficient with no evidence of autocorrelation present. However, this model includes a constant term when there is no particular theoretical reason for its inclusion.

Model D restricts the estimate to no constant term. The finding of a significant estimate with a meaningful slope coefficient supports the prediction of the model, where evidence of no autocorrelation is inconclusive at a significance level of 5%, but not at the 1% level. Further, we reject the null hypothesis that the slope coefficient is greater or equal to zero. We also cannot reject the hypothesis that the coefficient is greater or equal to negative one ,or whether it is equal to –2/3, a common estimate of the (negative) labor share of income. Further, we reject the hypothesis that the slope is –1/3 at the 5% level, but we cannot reject it at the 1% level.

MPC RELATIVE AND AGGREGATE AVERAGE HOURS LABOR PROPORTION

Figure 8

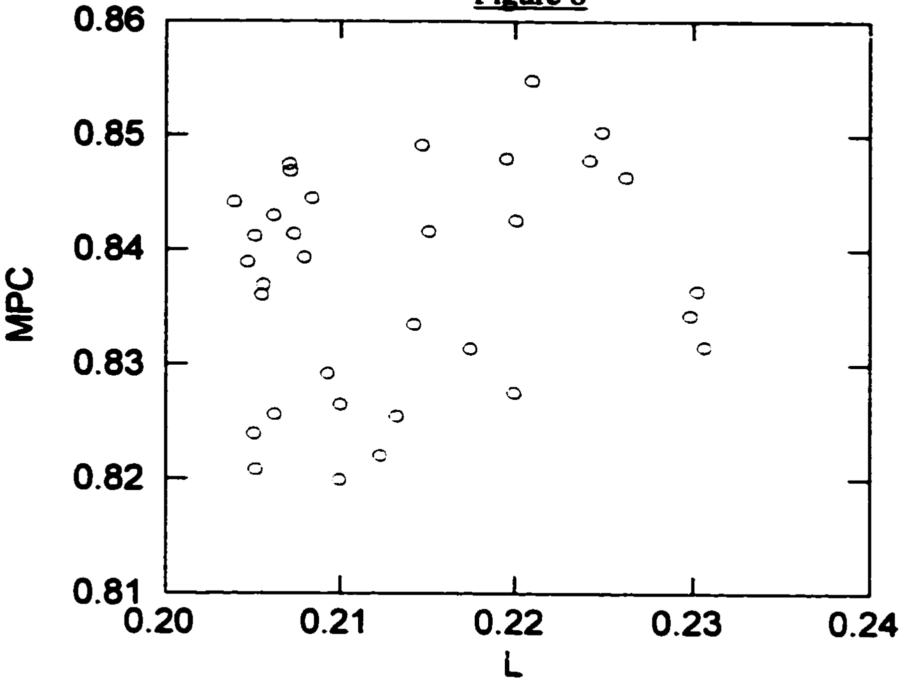
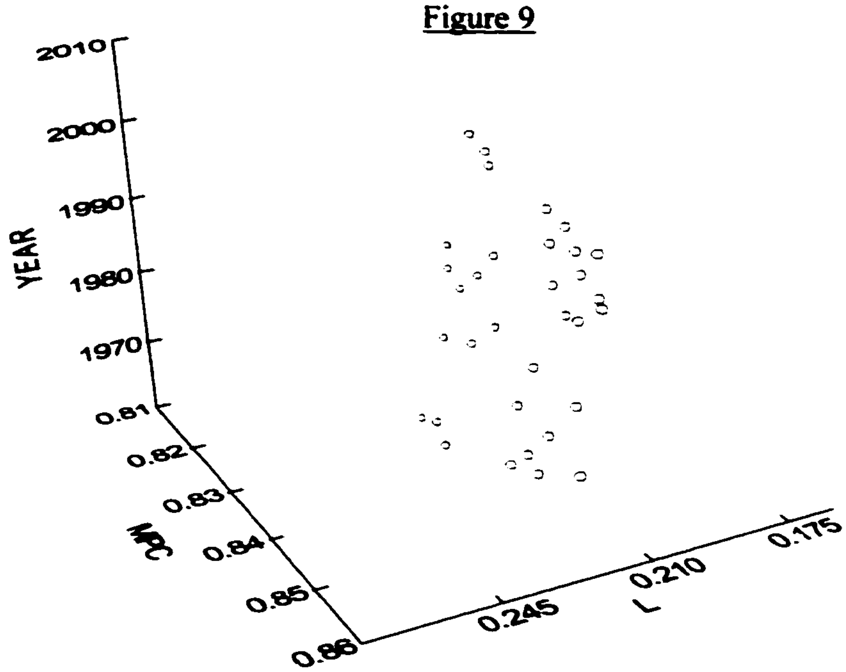


Figure 9



Dep Var: LN_MPC N: 33 Multiple R: 0.202 Squared multiple R: 0.041

Adjusted squared multiple R: 0.010 Standard error of estimate: 0.011

Effect	Coefficient	Std Error	t	P(2 Tail)
CONSTANT	-0.086	0.080	-1.076	0.290
LN_L	0.059	0.052	1.148	0.260

analysis of Variance:

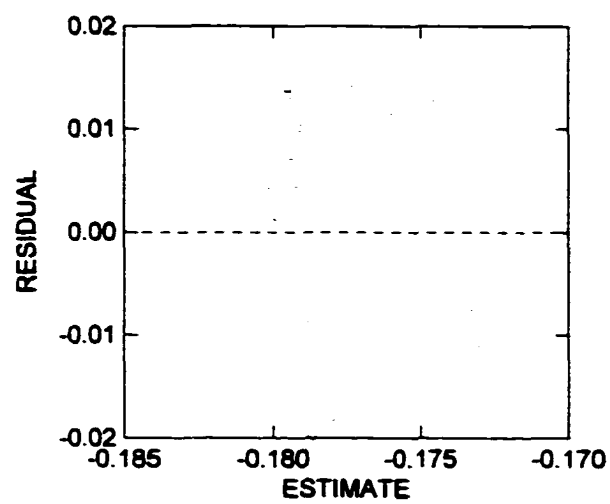
Source	Sum-of-Squares	df	Mean-Square	F-ratio
Regression	0.000	1	0.000	1.318
Residual	0.004	31	0.000	

Durbin-Watson D Statistic 0.827

First Order Autocorrelation 0.546

Figure 10

Plot of Residuals against Predicted Values



Dep Var: D_LN_MPC546 N: 32 Multiple R: 0.027 Squared_R: 0.001

Adjusted squared multiple R: 0.000 Standard error of estimate: 0.010

Effect	Coefficient	Std Error	t	P(2 Tail)
CONSTANT	-0.070	0.073	-0.954	0.348
D_LN_L546	0.016	0.104	0.151	0.881

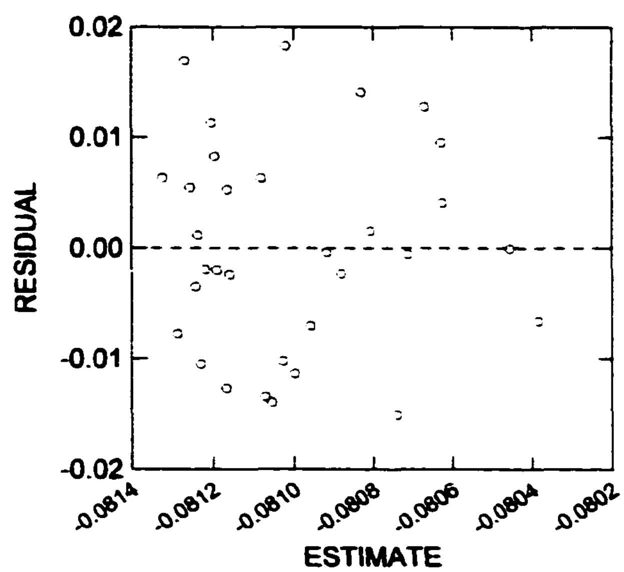
Analysis of Variance:

Source	Sum-of-Squares	df	Mean-Square	F-ratio
Regression	0.000	1	0.000	0.023
Residual	0.003	30	0.000	

Durbin-Watson D Statistic	1.564
First Order Autocorrelation	0.190

Figure 11

Plot of Residuals against Predicted Values



Dep Var: D_LN_MPC N: 32 Multiple R: 0.703 Squared _R: 0.494

Adjusted squared multiple R: 0.478 Standard error of estimate: 0.007

Effect	Coefficient	Std Error	t	P(2 Tail)
CONSTANT	-0.005	0.002	-3.282	0.003
D_LN_L	-1.234	0.228	-5.416	0.000

Analysis of Variance:

Source	Sum-of-Squares	df	Mean-Square	F-ratio
Regression	0.002	1	0.002	29.336
Residual	0.002	30	0.000	

Durbin-Watson D Statistic 1.605

First Order Autocorrelation 0.196

Figure 12

Plot of Residuals against Predicted Values

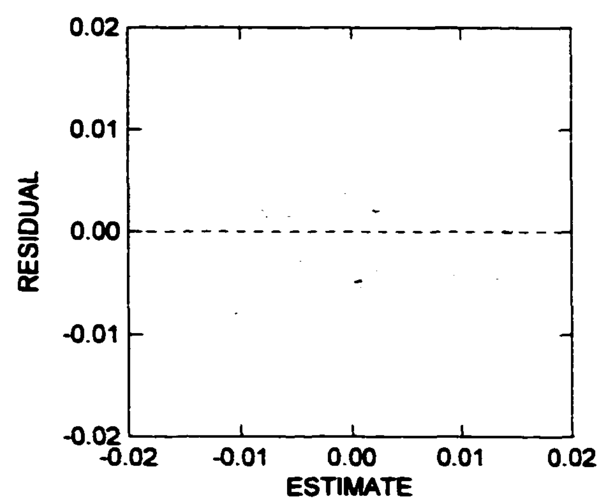
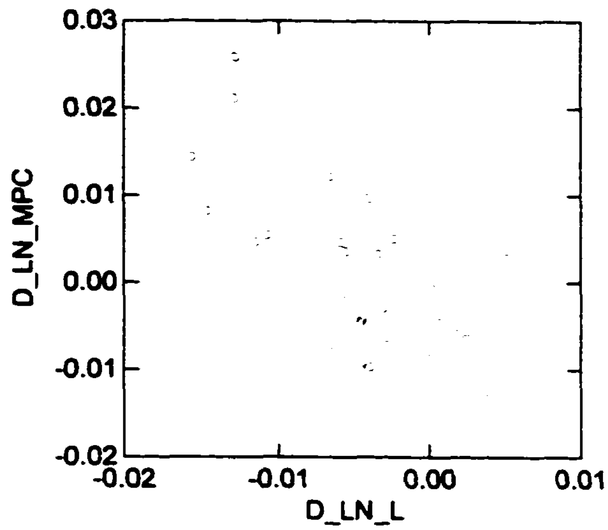


Figure 13

CHANGE IN LOG MPC RELATIVE CHANGE IN LOG LABOR



Model contains no constant

Dep Var: D_LN_MPC N: 32 Multiple R: 0.561 Squared__R: 0.315

Adjusted squared multiple R: 0.315 Standard error of estimate: 0.008

Effect	Coefficient	Std Error	t	P(2 Tail)
D_LN_L	-0.834	0.221	-3.777	0.001

Analysis of Variance:

Source	Sum-of-Squares	df	Mean-Square	F-ratio
Regression	0.001	1	0.001	14.269
Residual	0.002	31	0.000	

Durbin-Watson D Statistic 1.456

First Order Autocorrelation 0.265

Figure 14

Plot of Residuals against Predicted Values

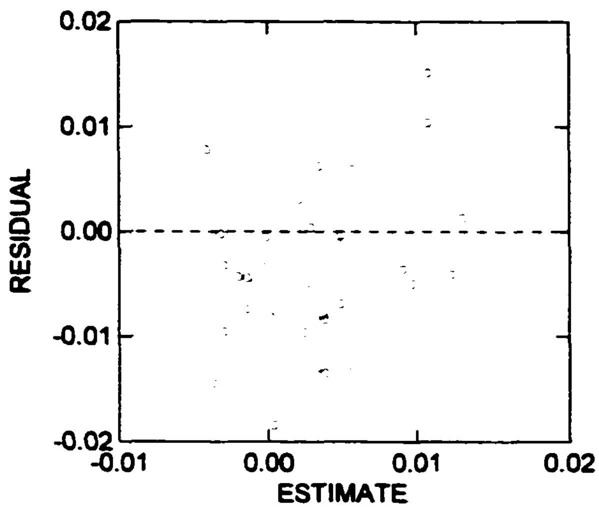
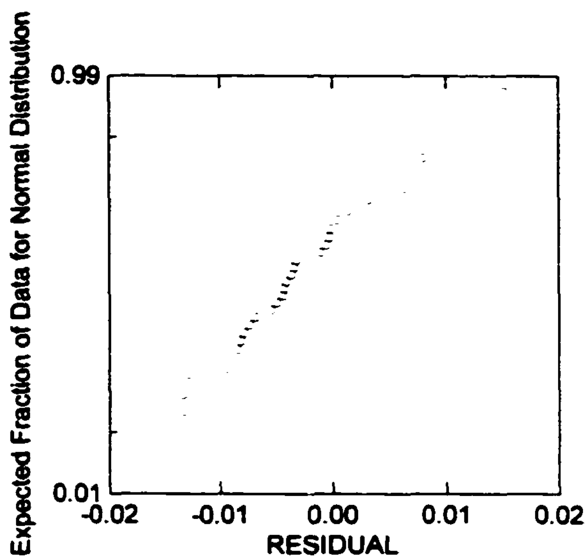


Figure 15



The second data set estimates are reported for model types A, C and D for the 51 sectors considered above. Notice that once again model type D has significant and meaningful estimate of the slope coefficient. The labor share of income varies from sector to sector but it is clear that it is of expected magnitudes for the vast majority of sectors. Of note a handful of enterprises where a positive coefficient is indicative of decreasing returns for the labor factor. This is clearly the case for State and Local governments.

Table 4

Log of MPC to Log of Labor Ratio

Sector	Constant	Slope	St.Error	t	F	D.W.	R-Sqrd	Auto Corr
L	-0.264	-0.836	0.458	-1.824	3.329	0.113	0.144	0.785
AP	-1.451	-0.960	0.109	-8.812	77.655	0.749	0.733	0.554
AS	-1.595	-0.896	0.239	-3.746	14.029	0.350	0.324	0.655
MM	-0.973	-0.512	0.220	-2.328	5.418	0.172	0.187	0.758
CM	0.182	0.256	0.074	3.464	11.999	0.316	0.667	0.420
CPNG	0.621	0.575	0.263	2.186	4.779	0.155	0.288	0.711
NMQ	-0.079	0.101	0.422	0.239	0.057	0.002	0.176	0.730
CC	-1.101	-0.579	0.494	-1.172	1.373	0.050	0.166	0.762
FKP	-3.001	-1.883	0.215	-8.776	77.011	0.748	0.450	0.671
TM	-0.569	-0.226	0.327	-0.690	0.476	0.018	0.149	0.758
TMP	0.599	0.554	0.236	2.350	5.521	0.175	0.523	0.518
AOFTP	-0.721	-0.318	0.408	-0.779	0.607	0.023	0.143	0.757
PAP	-1.817	-1.097	0.331	-3.311	10.964	0.297	0.255	0.710
PP	-1.669	-0.946	0.453	-2.086	4.353	0.143	0.375	0.700
CAP	-2.953	-1.824	0.495	-3.684	13.574	0.343	0.151	0.767
PCP	0.134	0.233	0.544	0.428	0.183	0.007	0.174	0.740
RMPP	0.274	0.336	0.348	0.964	0.930	0.035	0.274	0.689
LLP	-0.103	0.077	0.402	0.191	0.037	0.001	0.164	0.737
LWPEF	-0.385	-0.111	0.367	-0.302	0.091	0.003	0.143	0.760
FAF	-1.249	-0.698	0.204	-3.429	11.759	0.311	0.180	0.743
SCGP	-0.452	-0.156	0.525	-0.298	0.089	0.003	0.132	0.761
PMI	-0.341	-0.079	0.298	-0.266	0.071	0.003	0.129	0.758
FMI	-1.052	-0.562	0.365	-1.537	2.362	0.083	0.115	0.764
MEM	-1.165	-0.641	0.259	-2.477	6.133	0.191	0.152	0.742
EMES	-1.733	-0.997	0.251	-3.964	15.711	0.377	0.230	0.677
TEOEMH	-0.819	-0.396	0.226	-1.752	3.070	0.106	0.199	0.711
MVE	-0.399	-0.119	0.199	-0.597	0.357	0.014	0.153	0.738
PPEW	-1.307	-0.723	0.292	-2.477	6.135	0.191	0.176	0.707
MMI	-1.690	-0.980	0.223	-4.397	19.335	0.426	0.219	0.747
RRES	-0.107	0.078	0.220	0.352	0.124	0.005	0.172	0.713
SRBLT	-0.107	0.078	0.220	0.352	0.124	0.005	0.172	0.713

Table 4 cont'd.

Sector	Constant	Slope	St.Error	t	F	R-Sqrd	D.W.	AutoCorr
TSW	-0.204	0.012	0.557	0.022	0.000	0.000	0.157	0.742
WI	-1.247	-0.637	0.294	-2.163	4.677	0.152	0.213	0.730
AT	-1.500	-0.823	0.214	-3.845	14.782	0.362	0.250	0.704
PENG	-2.670	-1.595	0.531	-3.005	9.031	0.258	0.418	0.644
Is	-1.242	-0.666	0.275	-2.419	5.851	0.184	0.181	0.760
TTMCS	0.500	0.464	0.631	1.286	1.654	0.060	0.292	0.656
RTB	-3.111	-1.837	0.671	-2.738	7.499	0.224	0.303	0.700
EU	-1.630	-0.934	0.760	-1.228	1.509	0.055	0.124	0.802
GU	-2.938	-1.798	0.712	-2.525	6.374	0.197	0.155	0.823
WSSS	-4.091	-2.456	0.270	-9.082	82.490	0.760	1.306	0.211
WHT	-2.023	-1.219	0.339	-3.594	12.918	0.332	0.220	0.733
RT	-0.686	-0.316	0.507	-5.567	30.988	0.544	0.318	0.669
FIRE	-3.207	-1.913	0.382	-5.008	25.082	0.491	0.278	0.712
SEPHI	-1.059	-0.544	0.121	-4.505	20.294	0.438	0.254	0.704
PH	-0.548	-0.172	0.016	-11.109	123.400	0.826	0.663	0.542
INST	-1.121	-0.531	0.263	-2.023	4.094	0.136	0.209	0.732
FEDPA	-2.938	-1.702	0.333	-5.113	26.142	0.501	0.593	0.564
FEDGE	-2.314	-1.318	0.274	-4.820	23.230	0.472	0.685	0.524
SLES	-1.032	-0.444	0.083	-5.374	28.877	0.526	0.315	0.674
SLPA	0.255	0.300	0.403	0.743	0.552	0.021	0.122	0.802
SLGE	-0.466	-0.163	0.261	-0.624	0.890	0.015	0.191	0.723

Table 5

Change in Log of MPC to Change in Log of Labor Ratio

CAP	0.003	-1.322	0.365	-3.622	13.119	0.344	0.942	0.377
PCP	0.005	-0.350	0.238	-1.471	2.165	0.080	1.712	-0.015
RMPP	0.005	-0.281	0.109	-2.577	6.643	0.210	1.317	0.157
LIP	0.005	-0.134	0.139	-0.970	0.940	0.036	1.510	0.056
LWPEF	0.004	-0.330	0.156	-2.110	4.454	0.151	1.588	0.054
FAF	0.003	-0.396	0.129	-3.059	9.356	0.272	1.273	0.203
SCGP	0.004	-0.601	0.154	-3.906	15.253	0.379	1.129	0.315
PMU	0.005	-0.294	0.068	-4.329	18.736	0.428	1.412	0.160
FMI	0.004	-0.460	0.126	-3.660	13.398	0.349	1.192	0.249
MEM	0.004	-0.371	0.095	-3.918	15.349	0.380	1.210	0.258
EMES	0.004	-0.503	0.182	-2.769	7.667	0.235	1.282	0.166
TEOEMH	0.005	-0.019	0.167	-0.114	0.013	0.001	1.524	0.043
MVE	0.005	-0.077	0.073	-1.053	1.108	0.042	1.460	0.055
PPEW	0.004	-0.386	0.162	-2.388	5.701	0.186	1.270	0.172
MMI	0.003	-0.564	0.221	-2.557	6.536	0.207	1.200	0.253
RRES	0.004	-0.208	0.119	-1.744	3.043	0.109	1.926	-0.085
SRBIT	0.004	-0.208	0.119	-1.744	3.043	0.109	1.926	-0.085
TSW	0.004	-0.605	0.247	-2.451	6.008	0.194	1.464	0.079
WT	0.005	0.237	0.325	0.729	0.531	0.021	1.627	-0.005
AT	0.005	-0.186	0.371	-0.500	0.250	0.100	1.460	0.071
PENG	0.004	-0.322	0.206	-1.551	2.407	0.088	1.609	0.017
TS	0.004	-0.242	0.341	-0.709	0.503	0.020	1.488	0.082
TTMCS	0.005	-0.161	0.125	-1.029	1.651	0.062	1.651	-0.005
RTB	0.005	-0.403	0.321	-1.253	1.570	0.059	1.724	-0.060
EU	0.004	-0.939	0.335	-2.802	7.849	0.239	1.695	0.041
GU	0.004	-1.041	0.322	-3.230	10.433	0.294	1.674	0.055
WSSS	0.004	-0.573	0.291	-1.969	3.877	0.134	1.716	-0.061
WHT	0.004	-0.465	0.438	-1.062	1.127	0.043	1.502	0.052
RT	0.006	0.116	0.357	0.324	0.105	0.004	1.549	0.042
FIRE	0.004	-0.775	0.793	-0.978	0.956	0.037	1.417	0.100
SEPHI	0.005	-0.084	0.559	-0.151	0.023	0.001	1.523	0.042
PH	0.000	-0.218	0.104	-2.098	4.400	0.150	1.881	-0.125
INST	0.005	0.041	0.224	0.183	0.003	0.001	1.573	0.020
FEOPA	0.005	-0.211	0.269	-0.786	0.618	0.024	1.472	0.066
FEDGE	0.005	0.222	0.233	0.954	0.910	0.035	1.447	0.065
SLES	0.005	-0.010	0.315	-0.033	0.001	0.000	1.534	0.038
SLPA	0.004	0.565	0.170	3.322	11.037	0.306	1.568	0.137
SLGE	0.005	0.016	0.072	0.226	0.051	0.002	1.551	0.030

Table 6

Change in Log of MPC to Change in Log of Labor Ratio and No Intercept

Sector	Slope	St. Error	t	F	D.W.	R-Sqrd	AutoCorr
L	-0.529	0.168	-3.145	9.891	0.276	1.190	0.279
AP	-0.395	0.169	-2.342	5.484	0.174	1.444	0.083
AS	-0.324	0.143	-2.266	5.133	0.165	1.371	0.133
MM	-0.258	0.103	-2.513	6.313	0.195	1.126	0.273
CM	-0.151	0.044	-3.409	11.622	0.309	1.207	0.237
CPNG	-0.214	0.195	-1.098	1.206	0.044	1.291	0.141
NMQ	-0.483	0.154	-3.134	9.820	0.274	1.091	0.325
CC	-0.273	0.270	-1.011	1.023	0.038	1.331	0.127
FKP	-1.285	0.346	-3.711	13.768	0.346	1.409	0.113
TM	-0.200	0.152	-1.309	1.714	0.062	1.322	0.146
TMP	-0.279	0.102	-2.736	7.487	0.224	1.170	0.231
AOFTP	-0.276	0.155	-1.774	3.146	0.108	1.211	0.163
PAP	-0.472	0.183	-2.573	6.618	0.203	1.294	0.184
PP	-0.079	0.163	-0.482	0.233	0.009	1.398	0.081
CAP	-1.463	0.359	-4.069	16.558	0.389	0.863	0.404
PCP	-0.302	0.255	-1.185	1.404	0.051	1.400	0.105
RMPP	-0.284	0.117	-2.430	5.907	0.185	1.098	0.233
LLP	-0.153	0.146	-1.047	1.097	0.040	1.307	0.125
LWPEF	-0.375	0.161	-2.321	5.387	0.172	1.411	0.123
FAF	-0.443	0.130	-3.411	11.635	0.309	1.163	0.242
SCGP	-0.635	0.164	-3.881	15.061	0.367	0.956	0.379
PMI	-0.300	0.075	-4.032	16.254	0.385	1.137	0.272
FMI	-0.484	0.134	-3.614	13.064	0.334	1.005	0.317
MEM	-0.390	0.101	-3.844	14.778	0.362	1.008	0.334
EMES	-0.549	0.190	-2.893	8.371	0.244	1.111	0.223
TEOEMH	-0.054	0.175	-0.306	0.094	0.004	1.300	0.119
MVE	-0.083	0.077	-1.068	1.140	0.042	1.252	0.124
PPEW	-0.415	0.171	-2.436	5.933	0.186	1.079	0.238
MMI	-0.654	0.217	-3.008	9.047	0.258	1.109	0.281
RRES	-0.246	0.122	-2.017	4.066	0.135	1.767	-0.022
SRBLT	-0.246	0.122	-2.017	4.066	0.135	1.767	-0.022

Table 6 cont'd.

Sector	Slope	St.Error	t	F	R-Sqrd	D.W.	AutoCorr
TSW	-0.670	0.256	-2.612	6.825	0.208	1.281	0.149
WT	0.043	0.334	0.127	0.016	0.001	1.321	0.111
AT	-0.355	0.376	-0.944	0.891	0.033	1.242	0.146
PENG	-0.379	0.215	-1.761	3.100	0.107	1.442	0.078
IS	-0.452	0.325	-1.391	1.935	0.069	1.337	0.150
TTMCS	-0.168	0.133	-1.267	1.604	0.058	1.420	0.078
RTB	-0.482	0.334	-1.440	2.075	0.074	1.537	0.006
EU	-1.029	0.349	-2.951	8.709	0.251	1.506	0.117
CU	-1.134	0.334	-3.400	11.562	0.308	1.501	0.123
WSSS	-0.659	0.300	-2.196	4.821	0.156	1.562	-0.013
WHT	-0.725	0.408	-1.778	3.161	0.108	1.382	0.090
RT	-0.307	0.222	-1.385	1.919	0.069	1.433	0.042
FIRE	-1.291	0.690	-1.871	3.500	0.119	1.296	0.145
SEPHI	-0.604	0.384	-1.572	2.472	0.087	1.392	0.079
PH	-0.214	0.069	-3.096	9.585	0.269	1.872	-0.120
iNST	-0.053	0.232	-0.230	0.053	0.002	1.288	0.126
FEDPA	-0.301	0.277	-1.089	1.185	0.044	1.294	0.123
FEDGE	0.138	0.246	0.562	0.316	0.012	1.240	0.139
SLES	-0.311	0.262	-1.187	1.408	0.051	1.455	0.049
SLPA	0.610	0.177	3.446	11.876	0.314	1.362	0.219
SLGE	0.014	0.076	0.191	0.037	0.001	1.330	0.106

PART TWO

GROWTH

THE RATE OF INTEREST

The rate of interest both nominal and real will be analyzed given that a couple of assumptions hold. Firstly, let us assume that adjustments of deviations from equilibrium occur instantly. This implies that the adjustment processes have rates of change that approach infinity. This will allow us to view the system as one occurring in continuous time (Sargent 1987). The other assumption is that the quantities that we seek to measure are those that reflect in equilibrium the effect, in the limit, of actual physical forces.

1. The Real Rate.

The gross real rate of interest is equal to the marginal product of capital. This is what we can physically expect capital, given a production technology, to yield per period time. The net real rate of interest is thus the gross rate net of depreciation.

14) $$MPK = \delta + r \quad .$$

The rate of depreciation of capital is given by (δ) and (r) is the net real rate of interest. Therefore, given a position of equilibrium, where labor is determined, then the net real rate of interest is also determined, given a constant (or exogenous) rate of depreciation.

2. The Nominal Rate.

The nominal rate of interest is the sum of the net real rate of interest and the rate of inflation. Mathematically,

15a) $$i = r + \frac{dP}{d\tau}\frac{1}{P} .$$

From the Jacobian of the system and the comparative statics we can see that the actual rate of inflation is determined, given changes per unit time in the exogenous variables.

Explicitly, the model ignores the expected rate of inflation and in its stead uses the actual rate of growth of prices. This is a reasonable definition to use because this paper is concerned with the system in equilibrium. In the current model, actual values must coincide with those expected, otherwise there would not be equilibrium. If this were not so, agents would attempt to continually try to improve their position. A corner solution is not equilibrium proper but a sub-optimal solution.

If changes are taking place in discreet time then the nominal rate of interest is approximated by the Fisher equation,

15b) $$i \approx r + \frac{P - P^{*}}{P^{*}}.$$

Recall that the asterisk represents a period lag. In both cases and from a position of equilibrium the nominal rate of interest is endogenous, given changes in the exogenous variables.

DYNAMICS AND GROWTH

1. Physical Capital.

The process of accumulation of physical capital per capita will be characterized by the familiar differential equation with the modification that gross real investment will be expressed in multiplier form.

$$16) \quad (W-1)\frac{\mu}{P} = (\nu+\delta)\kappa + \frac{d\kappa}{d\tau}$$

Here ν is the rate of growth of population.

2. Human Capital.

The process of accumulation of human capital per capita will be characterized by the below equation.

$$17) \quad h = \int_{-\infty}^{T} \lambda e^{-\psi(\tau)} d\tau \quad .$$

Human capital per capita is equal to h. The rate of growth of aggregate human capital will be denoted by η. Aggregate human capital is.

$$\mathcal{H} = h\mathrm{N}.$$

Note that here human capital per capita is the sum of all past labor discounted, or eroded in memory or impact. The implication is that human capital is the result of the entirety of past labor efforts. The memory and impact of a particular labor effort falls, the earlier the labor effort occurred. Thus, more recent labor effort has a greater impact on the current stock of human capital per capita than earlier (or more dramatically: ancestral) efforts.

A more complete treatment of the issue of human capital would incorporate the decision to invest in human capital as an additional important variable to solve in the system. I consider this elsewhere (Maduro 2001).

3. The Complement Environment.

In this paper the effect of the complement environment has been assumed away. i.e., $\nabla\Omega = 0$. However, for the sake of completeness, the dynamic path of the complement environment is to be given by,

18) $$\Omega = \int_{-\infty}^{T} \Delta\Omega(q)\frac{dq}{d\tau}d\tau .$$

The assumption made makes Ω a constant. This simplifies the derivation of the model at the micro level. Recall that (Ω) appeared in both the production and utility functions. If, however, the effects of production affect the complement environment this could in turn affect, whether beneficially or adversely, the production process and preference structure of the agent(s).

4. Population.

The total number of agents is assumed to be the effect of forces that can be represented by a function in three key variables: aggregate human capital, complement environment, and aggregate physical output (production-transformation process).

19) $N=N(\mathcal{H},Q,\Omega)$.

As mentioned above, the population rate of growth is to be denoted by ν. A more detailed study of this function in relation to the current model is warranted, but will be left to a future work. But note that if we add the further assumption that the production function is CRS, then very interesting results are obtained. The structure relation, output per capita, as a function of human capital per capita, allows for rates of change of differing signs. So, human capital may affect output per capita in an ambiguous way. But the ambiguity is easily resolved by the actual production function being "well behaved." For a positive relationship all that is needed is that $N_H < 0$ and both N_Q and N_Ω positive.

5. Aggregate Physical Money and Physical Money Per Capita.

We assume the time path of physical money to be known. This takes the general form of,

20) $m=m(\tau)$.

Let the rate of growth of m be represented by upsilon (υ). An immediate result is obtained. If (N) and m grow at the same rate of growth then dμ=0. This is an important condition for steady state as discussed below.

GROWTH AND STEADY STATE

The derivation of time paths for all the variables in the system can be daunting. and in some cases impossible to determine in closed form, from any arbitrary set of behavioral functions. Fortunately closed form time paths for all of the endogenous variables are obtained for an important case. This case will be discussed in the appendix where we will again use the CRS Cobb-Douglas and constant MRS formulations. In this section I will focus on the steady state and in particular the golden rule steady state conditions and convergence. An additional observation is in place. Note that regardless of the form of the behavioral functions, the rate of growth of capital per capita is endogenous in a dynamic setting by virtue of equation (10).

The evolution of a system in time may lead to a steady state. or more specifically. a global steady state as defined earlier. Recall the condition for a global steady state that none of the exogenous variables change. This implies that given a position of equilibrium. where the Jacobian does not vanish. the following conditions provide us with a global steady state,

i) $$(W-1)\frac{\mu}{P}=(\nu+\delta)\kappa^{c}$$

ii) $\nu = \upsilon$

iii) $d\phi = 0$.

Now, whether the steady state is golden rule or not will depend on consumption per capita being optimal. At any steady state it is true that consumption per capita takes, as usual, the below form,

21) $$x^{*} = f(\lambda^{*},\kappa^{*}) - (\delta + \nu)\kappa^{*}.$$

Consumption per capita is optimized when $dx^{*} = 0$. This condition may or may not hold for

22) $$\frac{dx^{*}}{d\kappa^{*}} = f_{\lambda}\frac{d\lambda^{*}}{d\kappa^{*}} + f_{\kappa} - (\delta + \nu).$$

By definition, when in a steady state in labor and consumption per capita, the rates of change for these variables, relative steady state capital per capita, must both be equal to zero. This readily implies that, $f_{\kappa} = \delta + \nu$.

This is the condition for the golden rule steady state as in the Solow (1956) model (Swan 1956) with no technological growth. Note that a sufficient condition for a maximum is obtained when $\frac{d^{2}\lambda^{*}}{d\kappa^{*2}} \prec 0$.

The above, leads to the conclusion, that any global steady state is also a golden rule steady state. This is, as long as a change in steady state capital induces a negative change in the rate of change of steady state labor. As capital increases labor decelerates. More specifically, it implies that given a steady state in consumption and labor, then we have a golden rule steady state and the marginal product of capital is equal to the rate of depreciation of capital and the rate of growth of population. This result is similar to the Von Neumann steady state (Von Neumann 1945-46) but is in contrast to the Solow model. The difference with the simple Solow model in this respect is that it is labor and the endogenous rate of savings that are the determinants of the golden rule steady state. The rate of saving adjusts to the proper level consistent with a golden rule steady state.

The rate of convergence of the system also differs from the Solow model in that, as noted above, the rate of saving $(1-\gamma)$ is not assumed constant. From equation (16) we have the equivalent expression of

23) $$\frac{d\kappa}{d\tau} = (1-\gamma)f(\lambda,\kappa) - (\nu+\delta)\kappa$$

$$= S(\kappa) - (\nu+\delta)\kappa .$$

By a first order Taylor approximation we get,

$$\frac{d\kappa}{d\tau} \approx -(1-\varepsilon s\kappa)(\nu+\delta)(\kappa-\kappa^{e})$$
$$\approx -\varsigma(\kappa-\kappa^{e}) .$$

The rate of convergence (ζ) is equal to the product of one minus the capital elasticity of saving ($\varepsilon s\kappa$), times the sum of the rate of growth of population and the rate of depreciation. In the Solow model, the capital share of income replaces the above elasticity. For convergence, the capital elasticity of saving must be strictly smaller than one. By similar technique it can be shown that income converges at approximately the same rate as capital, i.e., (ζ). This last result is also a consequence of the Solow model.

A SIMULATION

1. The System .

We finally look at a simulation of the system in equilibrium. I will use the human capital specification of the production function, with the normalization of human capital per capita to be set at one. Following Mankiw (1995) the production function coefficients will be set at 1/3 for each of the three factors. We look at the behavior of the U.S. economy as registered in 1987. I also use currency in circulation as opposed to high-powered money as the reference stock of physical money. I do this to illustrate the flexibility of the model by integrating another definition of money, as measured by the central bank, into our current framework of analysis. The production function is Cobb-Douglas in λ, h, and κ.

Recall the system of seven equations of the unit social organism. The production function is CRS Cobb-Douglas in labor, human capital, and physical capital.

$$A\lambda^{1-\alpha-\beta}h^{\beta}\kappa^{\alpha} = (UW-1)\frac{\mu}{P}$$

$$\Lambda = \left(\frac{U-1}{U}\right)\left[(1-\alpha-\beta)A\left(\frac{h}{\lambda}\right)^{\beta}\left(\frac{\kappa}{\lambda}\right)^{\alpha}\right]$$

$$\mathrm{H} = (UW)\frac{\mu}{P}$$

$$x = (UW-W)\frac{\mu}{P}$$

$$s = \frac{1}{U}$$

$$1 = t + \lambda$$

$$\phi = U + W \; .$$

In order to solve the system, a set of values for the parameters and the exogenous variables must be given. Below are listed the exogenous quantities to be used. These correspond to magnitudes that are consistent with those of the economic experience of the U.S. in 1987. The assumption of constant MRS makes the solution process easier to perform, however, it is not implied that constant MRS is reflective of the U.S. for the historical period considered. It is nevertheless a convenient assumption. The MRS values chosen were used in order to calibrate the solutions to be consistent with historical experience.

Solve with values of parameters and endogenous variables:

H = 52.342 Λ = 59.303

κ = 106,725.40 capital per employed N = 112,466,200 employed

ϕ = 10.328 money ratio μ = 1,751.62 currency per worker

δ = 0.029303 rate of depreciation

Production function parameters

A = 1692.9 $\alpha \; \frac{1}{3}$ $\beta \; \frac{1}{3}$

Simulation Estimates

$vt = 0.761$ $\quad v\lambda = 0.239$ $\quad vx = 44{,}4317$ $\quad vs = 0.153$ $\quad vU = 6.522$

$vW = 3.806$ $\quad vP = 0.831$

Income-Output, the APC and the Synthetic Income MPC

$APC \quad \frac{U\cdot W \quad W}{U\cdot W \quad 1}$ $\qquad MPC \quad \frac{U \quad 1}{U}$ $\qquad q \quad (U\cdot W \quad 1)\cdot\frac{\mu}{P}$ $\qquad Q \quad q\cdot N$

APC = 0.882 MPC = 0.847 q = 50.233 Q = 5.65x1012

Money and Velocities

$MT \quad U\cdot(\mu\cdot N)$ $\qquad MS \quad W\cdot(\mu\cdot N)$ $\qquad M \quad MT \quad MS$ $\qquad M2 \quad M \quad (\mu\cdot N)$

MT = 1.285x1012 MS = 7.497x1011 M = 2.035x1012 M2 = .838x1012

VT = 3.652 VS = 6.26 VV = 2.307 V2 = 2.554

The below quantity is the simulated value for the physical money velocity of synthetic income.

$$V \quad U\cdot W$$

$$v = 24.822\ .$$

The gross nominal income velocity of physical money is thus 23.822.

The Real Rate of Interest

$$MPK \quad \alpha\cdot\frac{q}{\kappa} \qquad r = MPK - \delta$$

$$MPK = 0.157 \qquad r = 0.128$$

The Real Wage Rate

$$\omega \quad \frac{\Lambda}{MPC}$$

$$\omega = 70040\ .$$

Now, let labor receive the totality of its marginal product value. This is the sum of the wage bill and the returns to human capital. The labor-bill becomes,

$$LB = \omega\lambda + MPH \ .$$

MPH is the marginal product of human capital. With the proportion of labor-time in equilibrium of 0.239 the wage bill becomes 16,741.95. The returns to human capital would be 16,743.33. The total labor bill per worker is then 33,485.28.

Net Real Investment $$\Delta\kappa = \frac{(W-1)\mu}{P} - (\nu + \delta)\kappa$$

$$\Delta\kappa = 378.042 \ .$$

These predictions of the variables very closely resemble the aggregate performance of the U.S. economy in 1987. The below table summarizes observed values of selected variables for that year. In the appendix appears the complete simulation.

Table 7

US Aggregate Data 1987

Average Product of Labor (GDP per Worker)	Personal Consumption and Gvnmnt Expenditure Per worker	Average Propensity to Consume	Gross Real Investment and Net Exports Per Worker	Price Level in 1992 base	Transaction Multiplier	Time-Value Multiplier
50,233	44,352.88	0.883	5,915.55	83.1	6.523	3.805

Baumol & Blinder 1997

Where capital per worker was 106,725.40 and currency (the physical money definition used here) per worker was 1,751.62. The calibrated model used for the simulation approaches in most variables the magnitudes of the historical record of 1987. The issue of money becomes of extreme contrasting importance. The effect of money in the simulation represents total transaction volume impact, instead of merely final-goods volume, as assumed in our income approach to the multipliers.

The below table further lists observed values for key variables of the U.S. economy in 1986, 1987, and 1988.

Table 8

Macroeconomic Data / U.S. 1986-1988

	YEAR				
VARIABLES	1986		1987		1988
Real GDP 1992	5487.7b		5649.5b		5865.2b
Price Level 1992	80.58		83.06		86.1
Rate of Inflation		3.10%		3.70%	
Labor Force	117.8m		119.9m		121.7m
Unemployment Rate	7%		6.2		5.5
Employment (workers)	109.554m		112.466m		115.007m
Employment Rate of Growth		2.70%		2.3	
Labor Force Rate of Growth		1.02%		1.015	
Real GDP per Worker	50091.28		50232.96		50998.64
Personal Consumption	3708.7b		3822.3b		3972.7b
Government Expenditure	1135b		1165.9b		1180.9b
TOTAL CONSUMPTION	4843.7b		4988.2b		5153.6b
Consumption per Worker	44212.9		44352.96		44811.19
Gross Domestic Investment	811.8b		821.5b		828.2b
Net Foreign Exchange	-163.9b		-156.2b		-114.4b
TOTAL INVESTMENT	647.9b		665.3b		713.8b
Physical Capital per capita	106653.3		106725.4		107103.5
Nominal Interest Rate Aaa	9.02%		9.38%		9.71%
Net Real Rate		6.1%		5.805%	
CR	181b		197b		212b
MU_CR	1652.15		1751.64		1847.37
M1	724.7b		750.4b		787.5b
M2	2814.2b		2913.2b		3072.4b

2. The Jacobian.

The Jacobian of the CRS Cobb-Douglas constant MRS system takes on a straightforward expression.

$$\text{24)} \qquad \Delta = -\frac{(1-\alpha-\beta)(U^2W-U)\mu^3}{P^4\lambda^2}\left[(1-\alpha-\beta)(UW^2-W)+(\alpha+\beta)(U-1)(U-W)\right].$$

Note that the sign of the determinant is given by a weighted sum of terms within the bracketed expression. Under most relevant conditions this term is positive, making the determinant strictly negative. For the simulated case we get a value for the Jacobian of $-4.114\text{x}10^{14}$.

3. The Equilibrium Keynesian MPC.

As noted earlier the MPC is not unique because different definitions of income have their respective MPC. This is again evident when contrasting the MPC for income per capita as opposed to aggregate income. In the case of income per capita (the micro unit) the MPC is equal to

$$\text{25)} \qquad \frac{dx}{dq} = \frac{(W\frac{\mu}{P})\frac{\Delta_U}{\Delta} + \left[(U-1)\frac{\mu}{P}\right]\frac{\Delta_W}{\Delta} - (UW-W)\frac{\mu}{P^2}\frac{\Delta_P}{\Delta} + (UW-W)\frac{\mu}{P}\upsilon}{(W\frac{\mu}{P})\frac{\Delta_U}{\Delta} + \left[U\frac{\mu}{P}\right]\frac{\Delta_W}{\Delta} - (UW-1)\frac{\mu}{P^2}\frac{\Delta_P}{\Delta} + (UW-1)\frac{\mu}{P}\upsilon}.$$

We may label the above the micro MPC. By Cramer's rule we obtain the differential for all the endogenous changes. Note that $dU = \frac{\Delta_U}{\Delta}$, where the numerator is the determinant of the Cramer matrix. This matrix consists of the (A) matrix with the column associated with dU replaced by the column of exogenous changes. Similarly we obtain for (W) and (P).

The above MPC, however, may differ from the aggregate or macro MPC, which is given as

$$26) \qquad \frac{dX}{dQ} = \frac{dx + \nu x}{dq + \nu q} \ .$$

Recall that ν is the rate of growth of population. Note that if only physical money per capita is growing, then the aggregate MPC is identically equal to the APC for the macro system. Note that the APC is the same for both the micro or macro unit. The MPC for the micro unit, however, is not defined under the above conditions. This is so, due to neutrality. The micro MPC can be defined under multiplicity of changes in the exogenous variables.

For the simulated 1987 case we find a micro MPC of approximately 40. and a macro MPC of 0.813. These results are due to the assumption that aggregate capital is growing at a rate of 2.6%, physical money per capita is growing at a rate of 5.041%, and the employment rate of growth is 2.3%. The money ratio rate of growth is taken to be a negative 0.54%. These assumptions are consistent with the change in the exogenous variables from 1987 to 1988. It is important to note that the MPC's obtained by the above expressions can be positive or negative, smaller or greater than one, or even zero. However, the synthetic MPC as defined in this paper (, i.e., the transaction consumption rate), is always between zero and one. More importantly, we have that the behavior of the micro unit can be very different from the macro unit although they both share the same fundamentals.

4. The Rate of Inflation and the Nominal Interest Rate.

Given the changes in the exogenous variables assumed, the predicted rate of inflation becomes 4.8% from the equilibrium simulation. This is higher than the 3.7% observed for 1987-1988, the percentage change in price level (gdp deflator). We saw that the model generates a net real rate of interest of 12.8%, so by Fisher's equation we get a nominal rate of interest of 17.6 % for the current simulation values. This, perhaps, is too large. In 1987 the nominal rate of interest of high-grade corporate bonds was 9.38%. This may be due to the fact that the model has not taken explicitly the interest rate complex or the term structure of interest. Yet, the value generated is consistent with a speculative rate of interest at the peak of a speculative run. The rate simulated may have some bearing to the stock market crash of 1987.

CONCLUSION

I have set out to present in this paper a model that seeks to unify economic behavior in a simple and cohesive framework by use of the notion of an organic agent. The malleability of the nature of an agent, with its own imperatives and constraints, formed the fundamental building blocks of the structure derived. This agent is an organic entity, which exists due to the existence of component parts within an environmental setting. The theoretical consequences touch upon a great deal of variables that are of interest for both micro and macro economic theory. The variables include, but were not limited to, consumption, income, saving, the price level, the real wage rate, labor-hours, interest rates, money classes, and MPCs. The model presented is a new model of the general equilibrium of economic systems.

Of importance is the predicted relationship between the MPC-relative and labor. The theory deduces that it would be negative. The empirical evidence for a set of time periods for the U.S. lends some preliminary support to the theoretical prediction. This implies that there may be a tradeoff in production between labor and the amount of transactions (or work) that money would have to perform in the transaction of goods and services. The social organic agent has a production function of turnover of goods and services as opposed to just goods and services.

Further, simulations of a simple system with a Cobb-Douglas production technology and fixed MRSs can easily mimic historical behavior, given historical values for the exogenous components. A model was calibrated to capture late 20th century behavior for the United States. Further, the Mankiw factor shares, of one third for labor, human, and physical capital respectively, performs well in capturing the time period simulated.

The theory of growth was examined in light of the global steady state of the system. The current model does not assume that the economic system has a constant rate of saving, having endogenized this quantity. This contrasts with the simple Solow model. However, like the Solow model it shows that a golden rule steady state requires that the marginal product of physical capital must equal the rate of depreciation plus the rate of growth of population. However, the model also suggests that if parting from a position of equilibrium, and the system tends to a global steady state, then it approaches a golden rule steady state if labor decelerates with changes in capital per capita.

Underscoring the theoretical developments in the pape,r was the network of interactions between organic agents. From this we were able to deduce an algebraic structure that contained the physical consequences of the information/good-commodities exchanged. This algebraic structure was instrumental in making feasible closed-form expressions of many of the computations required. This algebraic structure partitions economic activity into expressions in a pair of multipliers and physical money. These expressions give us useful shadow values for many of the variables analyzed. This algebraic structure also reveals the dualities and homomorphism inherent in economic activity.

APPENDIX

A) THE INTER-TEMPORAL BUDGET CONSTRAINT

B) AN EQUILIBRIUM TIME PATH

C) A SIMULATION:

The Case of Labor in Efficiency units.

D) MPC RELATIVE LABOR PROPORTION PLOTS

THE INTER-TEMPORAL BUDGET CONSTRAINT

1. The Inter-Temporal Budget Constraint Transformation

Let the nominal wage rate be given by (ϖ), labor-hours (l), the nominal rate of return (R), and (v), which is the sum of all other components of current income. The amount of held physical units of money (, e.g.,, currency) is given by mu (μ). Asterisks represent a unit time lag. The price level of goods and services is (P) and accumulated nominal assets are given by (a). Changes in variables are denoted by delta (. e.g., Δa).

1a) $$\varpi l + (1 + R)a^{*} + v + \mu^{*} \equiv P\chi + a + \mu$$

So, $$\varpi l + Ra^{*} + v + \mu^{*} \equiv P\chi + \Delta a + \mu$$

1b) $$z \equiv P\chi + \Delta a + \mu$$

Hence,

1c) $$z - \Delta a \equiv P\chi + \mu\,.$$

2. States of Nominal Asset Accumulation

There are three states of accumulation of assets., i.e., states of positive, zero or negative growth. Given that z is the current liquid resource then there exists a fraction (s) such that the change in the stock of assets is proportional to the current resource.

$$\Delta a = sz$$

If no accumulation or depletion of resources is taking place then the change in accumulated assets is zero.

$$\Delta a = 0$$

On the other hand the depletion of accumulated assets is some proportion of of the stock. If this proportion is given by (1-ρ) then,

$$\Delta a = -(1-\rho)a^{*}$$

The three states of savings are mutually exclusive. The agent adds to, deplete or neither, the stock of accumulated assets held.

Thus,

2) $\Delta a = sz - (1-\rho)a^{*}$

Substituting into 1c) we get,

$$z - sz + (1-\rho)a^{*} \equiv P\chi + \mu .$$

Therefore the inter-temporal budget constraint becomes,

3) $P\chi + \mu \equiv (1-s)z + (1-\rho)a^{*}$.

Saving Behavior and the Saving Coefficient (σ)

Accumulation State

4) $s(\sigma) = \begin{cases} \sigma & \text{for } 0 < \sigma \le 1 \\ 0 & \text{Otherwise.} \end{cases}$

Notice that $s'(\sigma) = 1$ when $0 < \sigma \leq 1$ and also when $\sigma = 0$ and approached from the right. Otherwise the derivative is zero.

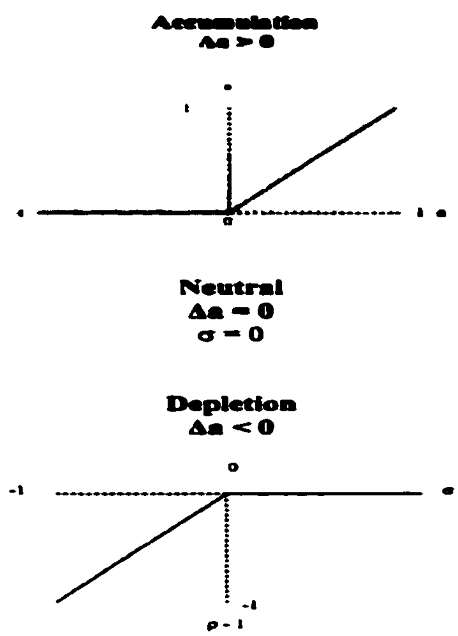
Depletion State

$$5) \qquad \rho(\sigma) - 1 = \begin{cases} \sigma & \text{for} \quad -1 < \sigma < 0 \\ 0 & \text{Otherwise.} \end{cases}$$

Also notice that $\rho'(\sigma) = 1$ when, $-1 \leq \sigma < 0$ and when $\rho = 0$ if approached from the left. Otherwise the derivative is zero.

Figures one and two graphically show the relationship between (s) and (ρ) in relation to the saving coefficient (σ).

<u>Figure 1A and 2A</u>



AN EQUILIBRIUM DYNAMIC PATH

This appendix explores the dynamic path of a simple system. This system is based on the assumption that from a position of equilibrium the time rate of change of the endogenous variables are proportional to the time rate of change of the exogenous variables. This is expressed as

$$\nabla Y = \varphi \nabla X$$

Now, we saw that

$$A\nabla Y = \nabla X$$

where |A| is the Jacobian of the system of equations. So,

$$\nabla Y = A^{-1}\nabla X$$

But by the assumption of proportionality above we have that $\varphi \nabla X = A^{-1}\nabla X$. Thus,

$$(A^{-1} - \varphi I)\nabla X = 0.$$

For any ∇X other than the trivial case we need

$$\det(A^{-1} - \varphi I) = 0.$$

Let C be a matrix of eigenvectors hence, if A^{-1} is diagonalizable then,

$$D = C^{-1}A^{-1}C$$

But D and A^{-1} are similar matrices (Hadley, 1961) therefore,

$$\nabla Y = D\nabla X.$$

The above and the constraints of the system (Cobb-Douglas) jointly imply the following set of time derivatives for the endogenous variables.

$$\frac{d\lambda}{d\tau} = \varphi_\lambda(\alpha\xi - \upsilon)q$$

$$\frac{dU}{d\tau} = \varphi_U\left[\left(\frac{U-1}{U}\right)\alpha(1-\alpha-\beta)\frac{q}{\lambda}\right]\xi$$

$$\frac{dW}{d\tau} = \varphi_W\frac{z}{P}\upsilon$$

$$\frac{dP}{d\tau} = -\varphi_P x\upsilon$$

$$\frac{dt}{d\tau} = 0$$

$$\frac{ds}{d\tau} = \varphi_s\phi\psi$$

$$\frac{dx}{d\tau} = 0$$

Where the (φ)s are the eigenvalues for each endogenous variable. Whether the time paths display oscillations or not will depend on whether the eigenvalues are real or complex. The salient property of the above is that consumption per capita and leisure are smoothed through out. By the constraint of the time horizon we have then that labor hours are also constant. This implies further that for the system to be consistent all but one of the exogenous changes must be parameterized to that variable. The below set of time derivatives are consequences of the above seven. They take into account the constraints imposed by the original general equilibrium set of equations.

$$\alpha\frac{d\kappa}{d\tau} = \upsilon\kappa$$

$$\frac{d\phi}{d\tau} = \frac{\varphi_U}{\varphi_s}\left[\left(\frac{U-1}{U^3}\right)(1-\alpha-\beta)\frac{q}{\lambda}\right]\upsilon$$

$$\frac{dW}{d\tau} = \varphi_W\frac{z}{P}\upsilon$$

$$\frac{dP}{d\tau} = -\varphi_P x\upsilon$$

This set of equations shows that if changes in the endogenous variables are proportional to the exogenous changes then steady state for the endogenous variables must imply that capital and money per capita must have rates of growth equal to zero.

A SIMULATION:

THE CASE OF LABOR IN EFFICIENCY UNITS

Figure 3A

SIMULATION WITH COBB - DOUGLAS TECHNOLOGY
GENERAL PRODUCTION
1987 US

Endogenous Variables $TOL := 0.00000001$

Guesses

$x := 44352.88 \quad s := 0.2 \quad g := 0.024 \quad t := \frac{1}{2} \quad \lambda := 0.201 \quad U := 6.523 \quad W := 3.805 \quad P := 0.831$

Exogenous $A := 1692.9\cdot(1.000000005)$

$h := 1 \quad \kappa := 106725.40 \quad \mu := 1751.64 \quad N := 112466000 \quad \alpha := \frac{1}{3} \quad \beta := \frac{1}{3}$

$\phi := 10.328$

Behavioral Equations

$H := 52342\cdot(1.000000005) \quad \Lambda := 59383\cdot(1.000000005) \quad F(\lambda,h,\kappa) := A\cdot[\lambda\cdot(1+g)]^{1-\alpha-\beta}\cdot h^{\beta}\cdot\kappa^{\alpha}$

Given

$$F(\lambda,h,\kappa) = (U\cdot W - 1)\cdot\frac{\mu}{P}$$

$$\Lambda = \left(\frac{U-1}{U}\right)\frac{d}{d\lambda}F(\lambda,h,\kappa) \qquad H = (U\cdot W)\cdot\frac{\mu}{P}$$

$$x = (U\cdot W - W)\cdot\frac{\mu}{P}$$

$$s = \frac{1}{U}$$

$$1 = t + \lambda$$

$$\phi = U + W$$

$$U > 1 \quad W > 1 \quad t > 0 \quad \lambda > 0 \quad P > 0$$

solve

$$G(H,\Lambda,\kappa,\mu,\phi,A,\alpha,\beta) := \text{find}(\lambda,U,W,P,t,s,x)$$

$\phi := 10.328 \quad \mu := 1751.62 \quad \kappa := 106725.40 \quad N := 112466200$

$\psi := -0.0061 \quad \upsilon := 0.05041 \quad \delta := 0.029302 \quad \eta := 0.02259$

Result

$$XX := G(H,\Lambda,\kappa,\mu,\phi,A,\alpha,\beta) \quad \begin{pmatrix} v\lambda \\ vU \\ vW \\ vP \\ vt \\ vs \\ vx \end{pmatrix} := XX$$

Figure 4A

ESTIMATES

$vt = 0.761$
$t := vt$

$v\lambda = 0.239$
$\lambda := v\lambda$

$vU = 6.522$
$U := vU$

$vW = 3.806$
$W := vW$

$vx = 4.4317 \times 10^{4}$
$x := vx$

$vs = 0.153$
$s := vs$

$vP = 0.831$
$P := vP$

$APC := \frac{U \cdot W - W}{U \cdot W - 1}$ $\quad$ $MPC := \frac{U - 1}{U}$ $\quad$ $q := (U \cdot W - 1) \cdot \frac{\mu}{P}$ $\quad$ $Q := q \cdot N$ $\quad$ $S := (q - x) \cdot N$

$APC = 0.882$ $\quad$ $MPC = 0.847$ $\quad$ $q = 5.0233 \times 10^{4}$ $\quad$ $Q = 5.65 \times 10^{12}$ $\quad$ $S = 6.654 \times 10^{11}$

MONEY AND VELOCITIES

$MT := U \cdot (\mu \cdot N)$ $\quad$ $MS := W \cdot (\mu \cdot N)$ $\quad$ $M := MT + MS$ $\quad$ $M2 := M - (\mu \cdot N)$ $\quad$ $m := \mu \cdot N$

$MT = 1.285 \times 10^{12}$ $\quad$ $MS = 7.497 \times 10^{11}$ $\quad$ $M = 2.035 \times 10^{12}$ $\quad$ $M2 = 1.838 \times 10^{12}$ $\quad$ $m = 1.97 \times 10^{11}$

$VT := \frac{Q \cdot P}{MT}$ $\quad$ $VS := \frac{Q \cdot P}{MS}$ $\quad$ $VV := \frac{U \cdot W - 1}{U + W}$ $\quad$ $V2 := \frac{Q \cdot P}{M2}$ $\quad$ $V := U \cdot W$

$VT = 3.652$ $\quad$ $VS = 6.26$ $\quad$ $VV = 2.307$ $\quad$ $V2 = 2.554$ $\quad$ $V = 24.822$

CAPITAL

$MPK := \frac{d}{d\kappa} F(\lambda, h, \kappa)$ $\quad$ $r := MPK - \delta$

$MPK = 0.157$ $\quad$ $r = 0.128$

$\Delta\kappa := (W - 1) \cdot \frac{\mu}{P} - (\eta + \delta) \cdot \kappa$

$\Delta\kappa = 378.148$ $\quad$ 378.1132

$\xi := \frac{\Delta\kappa}{\kappa}$

$\xi = 3.543 \times 10^{-3}$

REAL WAGE RATE

$\omega := \frac{\Lambda}{MPC}$ $\quad$ $\omega = 7.004 \times 10^{4}$

REAL LABOR BILL

$RLB := \omega \cdot \lambda + \beta \cdot q$ $\quad$ $RLB = 3.349 \times 10^{4}$

$z := U \cdot W \cdot \mu$

$z = 4.348 \times 10^{4}$

Figure 5A

THE PRODUCTION CHARACTERISTICS

$F\lambda := \frac{d}{d\lambda}F(\lambda,h,\kappa)$ $Fh := \frac{d}{dh}F(\lambda,h,\kappa)$ $F\kappa := \frac{d}{d\kappa}F(\lambda,h,\kappa)$

$F\lambda\lambda := \frac{d}{d\lambda}\frac{d}{d\lambda}F(\lambda,h,\kappa)$ $Fhh := \frac{d}{dh}\frac{d}{dh}F(\lambda,h,\kappa)$ $F\kappa\kappa := \frac{d}{d\kappa}\frac{d}{d\kappa}F(\lambda,h,\kappa)$

$F\lambda h := \frac{d}{d\lambda}\frac{d}{dh}F(\lambda,h,\kappa)$ $F\lambda\kappa := \frac{d}{d\lambda}\frac{d}{d\kappa}F(\lambda,h,\kappa)$ $F\kappa h := \frac{d}{d\kappa}\frac{d}{dh}F(\lambda,h,\kappa)$

THE JACOBIAN

$$A := \begin{bmatrix} -F\lambda & W\cdot\frac{\mu}{P} & U\cdot\frac{\mu}{P} & -\frac{q}{P} & 0 & 0 & 0 \\ -\left(\frac{U-1}{U}\right)\cdot F\lambda\lambda & -\frac{F\lambda}{U^2} & 0 & 0 & 0 & 0 & 0 \\ 0 & -W\cdot\frac{\mu}{P} & -U\cdot\frac{\mu}{P} & \frac{z}{P^2} & 0 & 0 & 0 \\ 0 & W\cdot\frac{\mu}{P} & (U-1)\cdot\frac{\mu}{P} & -\frac{x}{P} & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 & 0 & -\frac{1}{s^2} & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} \qquad X\Delta := \begin{bmatrix} F\kappa\cdot(\xi\cdot\kappa) - q\cdot\upsilon \\ \left(\frac{U-1}{U}\right)\cdot F\lambda\kappa\cdot(\xi\cdot\kappa) \\ \frac{z}{P}\cdot\upsilon \\ -x\cdot\upsilon \\ 0 \\ \phi\cdot\psi \\ 0 \end{bmatrix}$$

$$\Delta := |A|$$

$$\Delta = -4.114\times 10^{14}$$

Figure 6A

COMPARATIVE STATICS

$$Y\Delta := A^{-1} \cdot X\Delta \qquad \begin{pmatrix} d\lambda \\ dU \\ dW \\ dP \\ dt \\ ds \\ dx \end{pmatrix} := Y\Delta \qquad Y\Delta = \begin{pmatrix} -9.056 \times 10^{-4} \\ -0.133 \\ 0.07 \\ 0.04 \\ 9.056 \times 10^{-4} \\ 3.138 \times 10^{-3} \\ -164.255 \end{pmatrix}$$

$d\lambda = -9.056 \times 10^{-4}$

$dU = -0.133$

$dW = 0.07$

$dP = 0.04$

$dt = 9.056 \times 10^{-4}$

$ds = 3.138 \times 10^{-3}$

$dx = -164.255$

Figure 7A

INFLATION

$$\pi := dP\cdot\frac{1}{P} \qquad \pi = 0.048$$

$$i := r + \pi \qquad i = 0.176$$

MICRO MPC

$$Dxq := \frac{\left(W\cdot\frac{\mu}{P}\right)\cdot dU + \left[(U-1)\cdot\frac{\mu}{P}\right]\cdot dW - (U\cdot W - W)\cdot\frac{\mu}{P^2}\cdot dP + (U\cdot W - W)\cdot\frac{\mu}{P}\cdot\upsilon}{\left(W\cdot\frac{\mu}{P}\right)\cdot dU + \left(U\cdot\frac{\mu}{P}\right)\cdot dW - (U\cdot W - 1)\cdot\left(\frac{\mu}{P^2}\right)\cdot dP + (U\cdot W - 1)\cdot\frac{\mu}{P}\cdot\upsilon}$$

$$Dxq = 40.076$$

$$Exq := Dxq\cdot\frac{q}{x} \qquad Exq = 45.426$$

MACRO MPC

$$KMPC := \frac{\left[\left(W\cdot\frac{\mu}{P}\right)\cdot dU + \left[(U-1)\cdot\frac{\mu}{P}\right]\cdot dW - (U\cdot W - W)\cdot\frac{\mu}{P^2}\cdot dP + (U\cdot W - W)\cdot\frac{\mu}{P}\cdot\upsilon\right] + (\eta + g)\cdot x}{\left[\left(W\cdot\frac{\mu}{P}\right)\cdot dU + \left(U\cdot\frac{\mu}{P}\right)\cdot dW - (U\cdot W - 1)\cdot\left(\frac{\mu}{P^2}\right)\cdot dP + (U\cdot W - 1)\cdot\frac{\mu}{P}\cdot\upsilon\right] + (\eta + g)\cdot q}$$

$$KMPC = 0.813$$

$$EXQ := KMPC\cdot\frac{q}{x} \qquad EXQ = 0.922$$

<u>Figure 8A</u>

RATE OF CONVERGENCE

$$\Delta sS := -(W-1)\cdot\frac{\mu}{P^2}\cdot dP + (W-1)\cdot\frac{\mu}{P}\cdot\upsilon + \frac{\mu}{P}\cdot dW$$

$$Es\kappa := \frac{\Delta sS}{\Delta\kappa}\cdot\frac{\kappa}{(q-x)} \qquad \zeta := (1 - Es\kappa)\cdot(\eta+\delta)$$

$$Es\kappa = 7.64 \qquad \zeta = -0.345$$

$$ESK := \frac{\Delta sS + (q-x)\cdot(\eta+g)}{\Delta\kappa + \kappa\cdot(\eta+g)}\cdot\frac{\kappa}{(q-x)} \qquad \zeta\zeta := (1 - ESK)\cdot(\eta+\delta) \qquad EK := \frac{\Delta\kappa + \kappa\cdot(\eta+g)}{\kappa}$$

$$ESK = 1.469 \qquad \zeta\zeta = -0.024 \qquad EK = 0.05$$

DYNAMICS

$$E := \text{eigenvals}\left(A^{-1}\right)$$

$$E = \begin{pmatrix} -1.298 + 2.238i \\ -1.298 - 2.238i \\ 2.579 \\ 2.616\times10^{-3} \\ -1.485\times10^{-4} \\ -3.86\times10^{-5} \\ -9.387\times10^{-6} \end{pmatrix}$$

Figure 9A

DYNAMICS

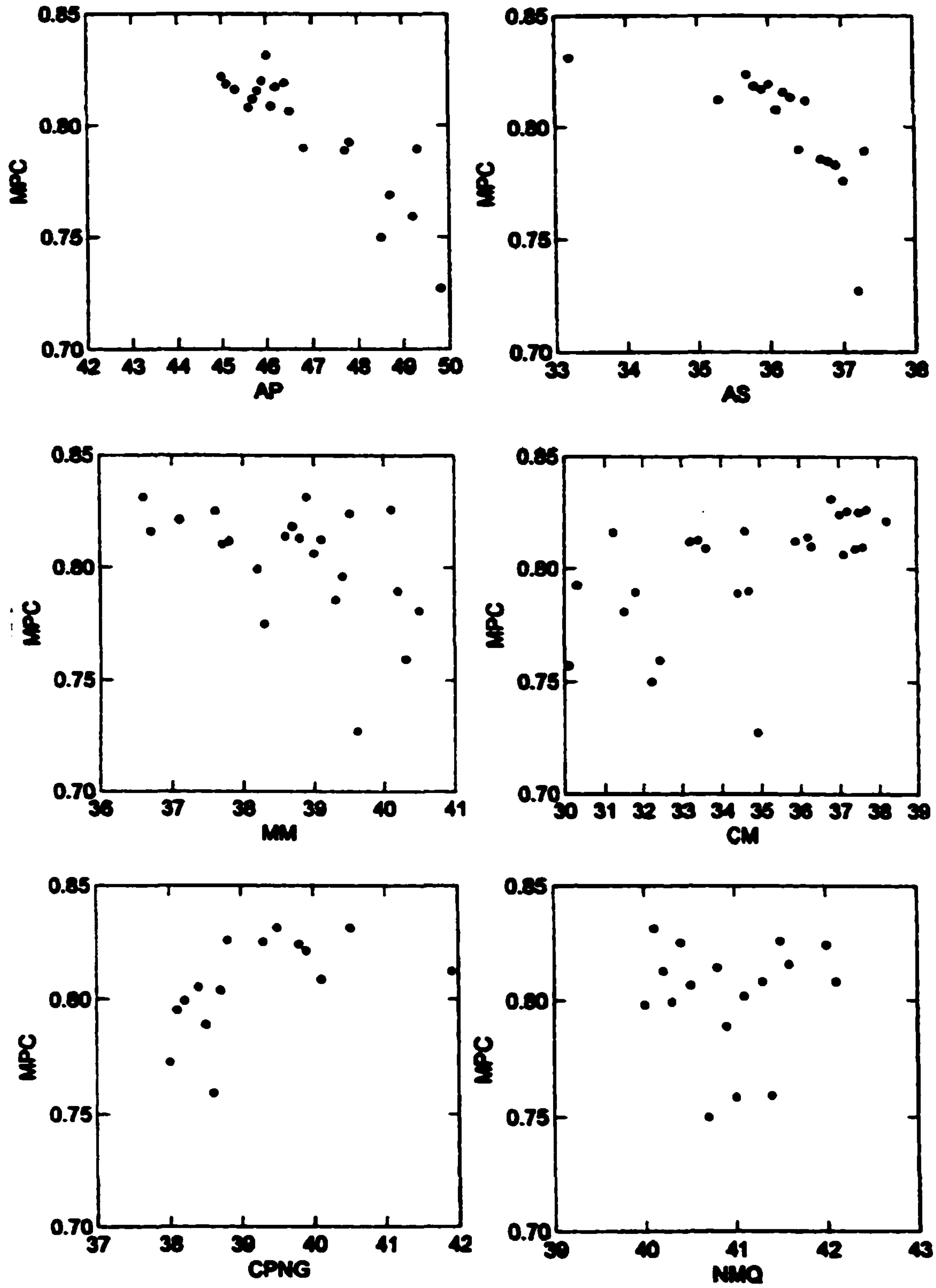
$$D := C^{-1} \cdot A^{-1} \cdot C$$

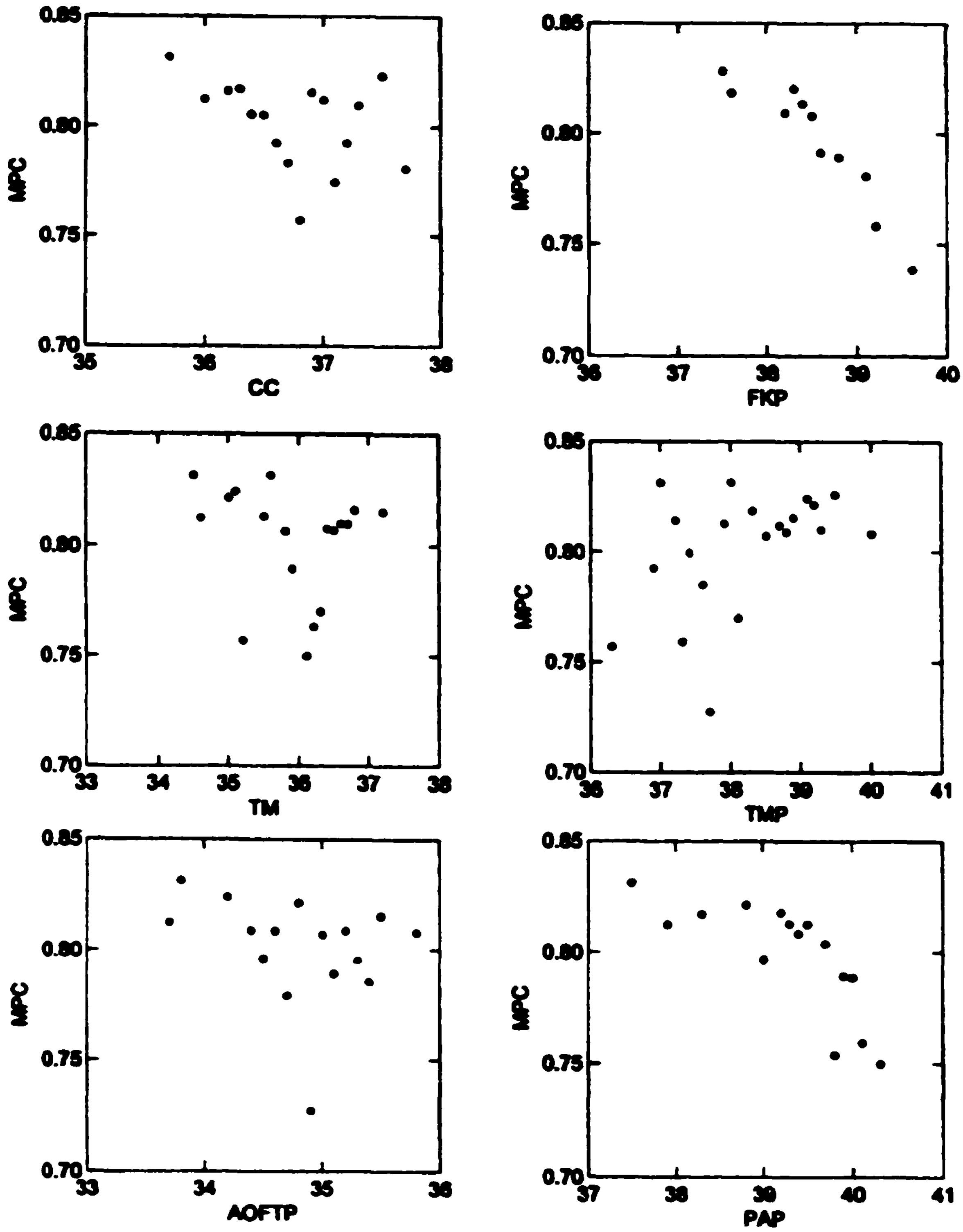
$$\begin{pmatrix} -1.298+2.238i & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1.298-2.238i & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2.579 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2.616\times 10^{-3} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1.485\times 10^{-4} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -3.86\times 10^{-5} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -9.387\times 10^{-6} \end{pmatrix}$$

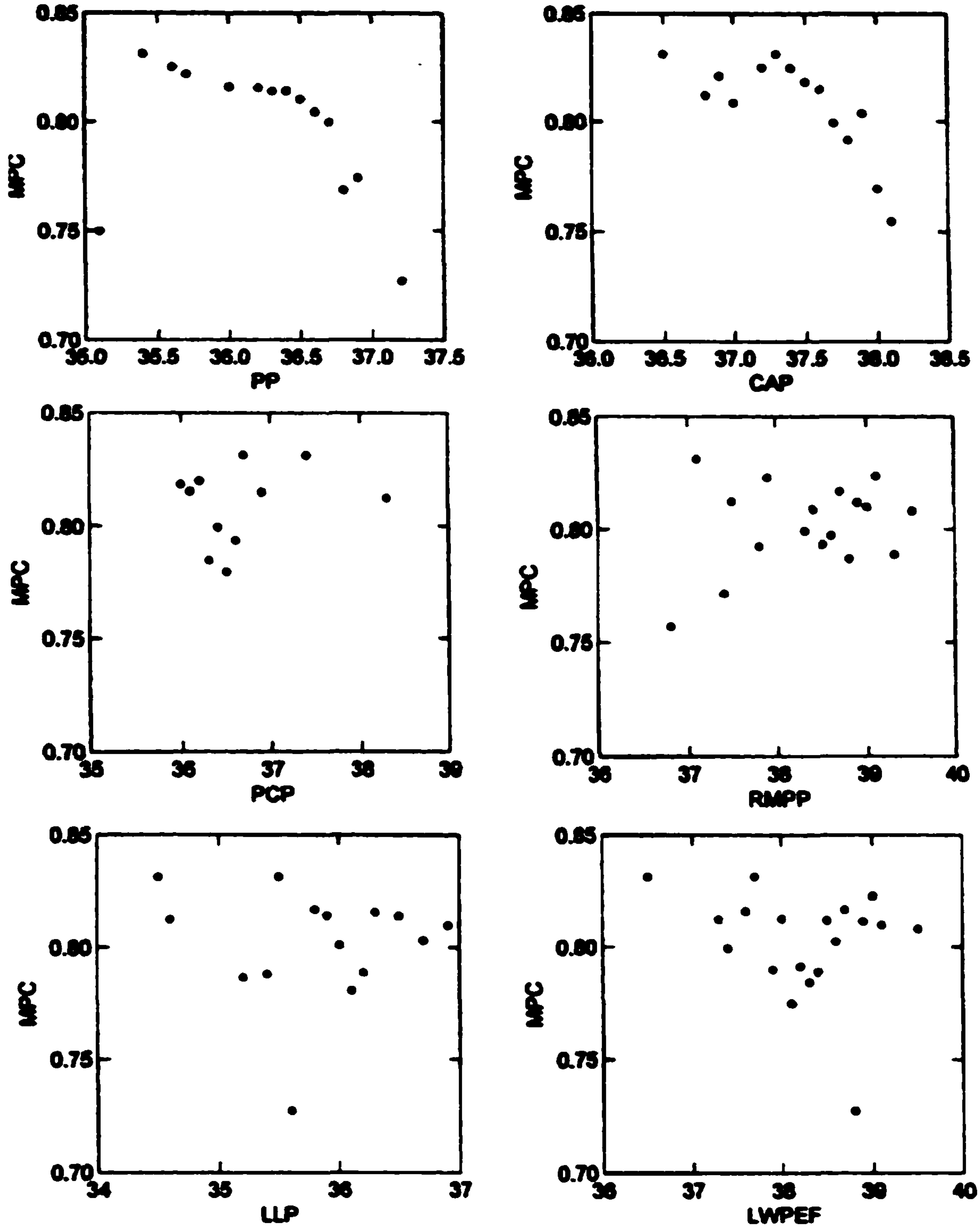
$|A| = -4.114\times 10^{14}$ $\quad |A^{-1}| = -2.43\times 10^{-15}$ $\quad |D| = -2.43\times 10^{-15}$

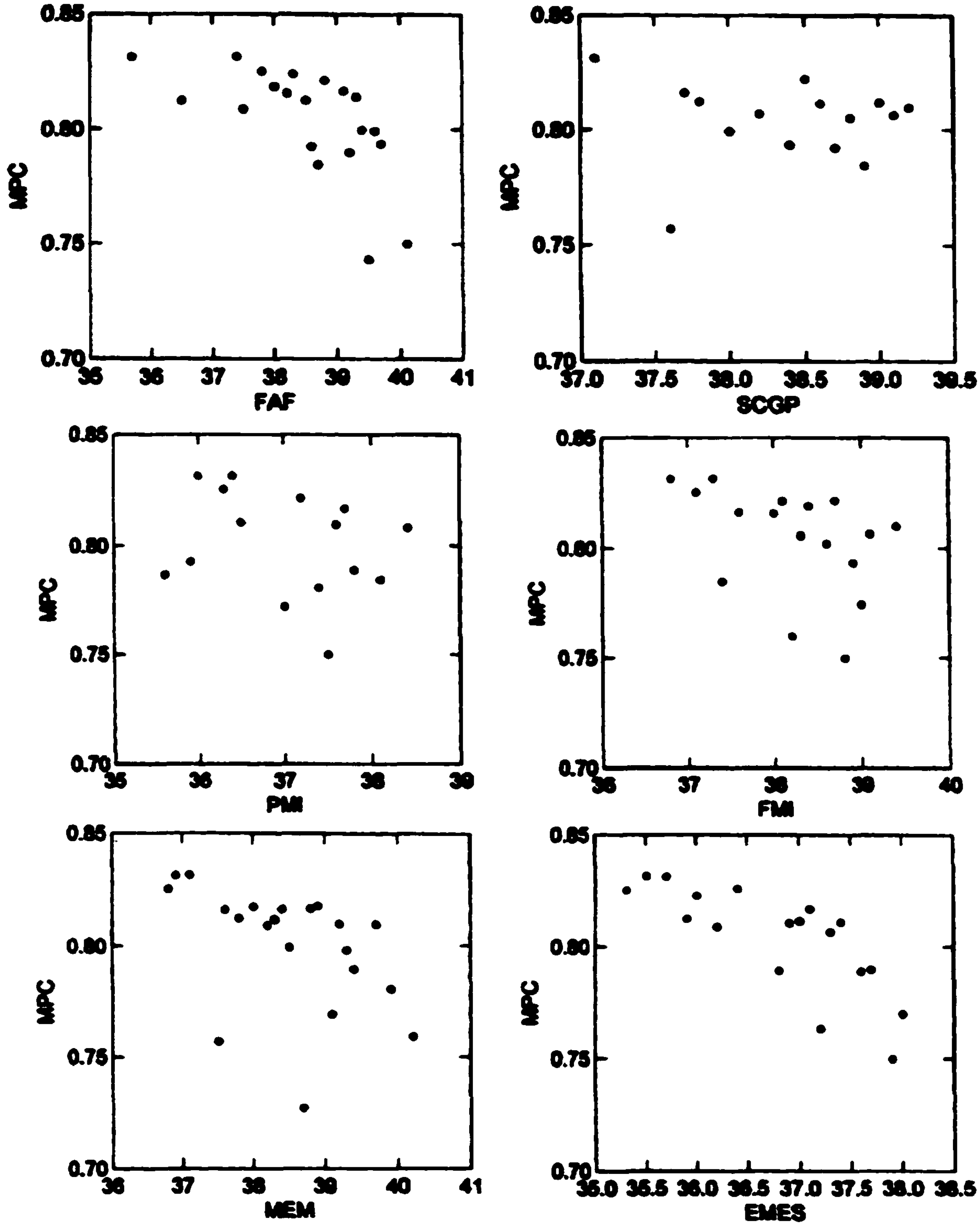
Figures 10A-60A -- PLOTS

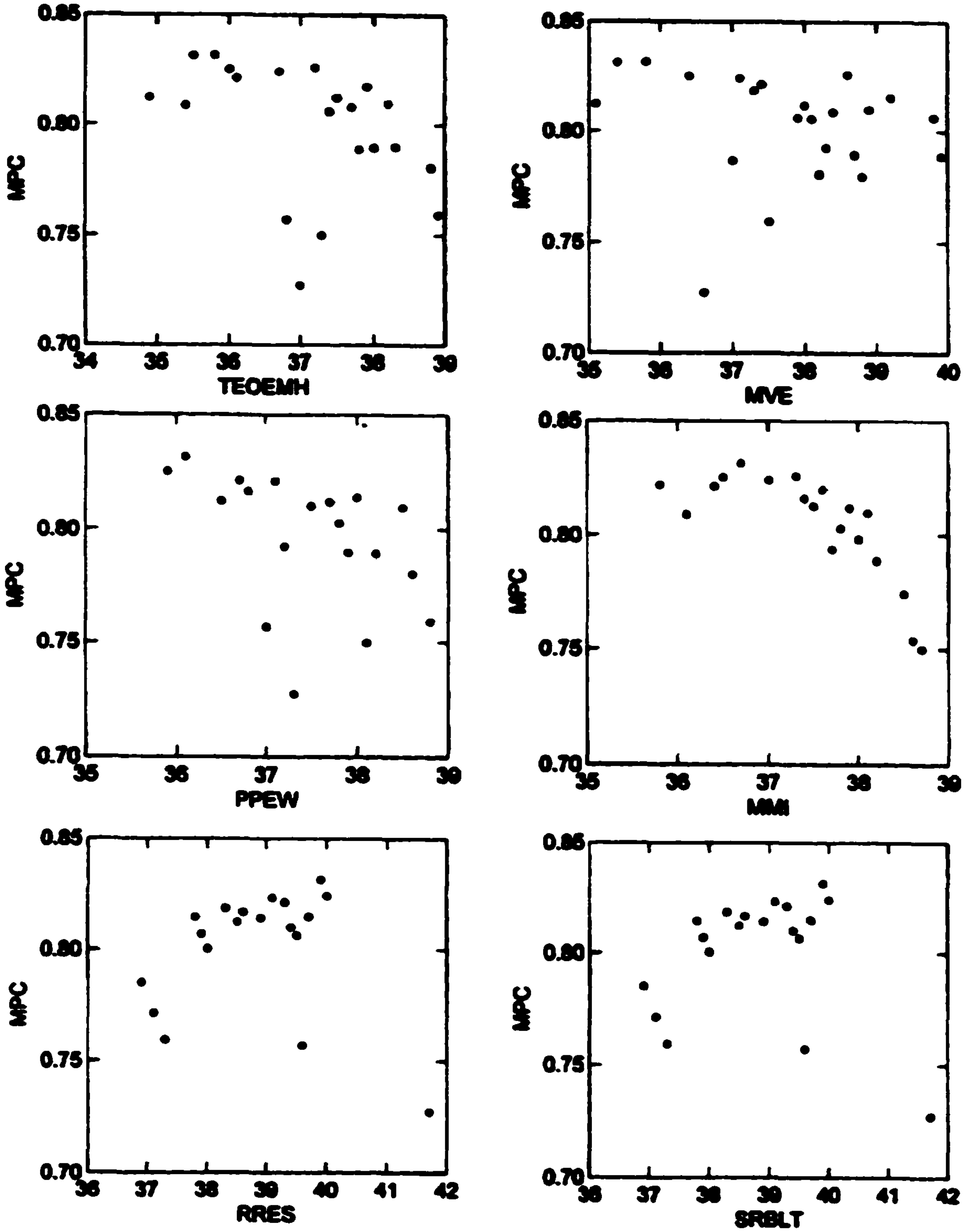
MPC RELATIVE TO LABOR IN HOURS

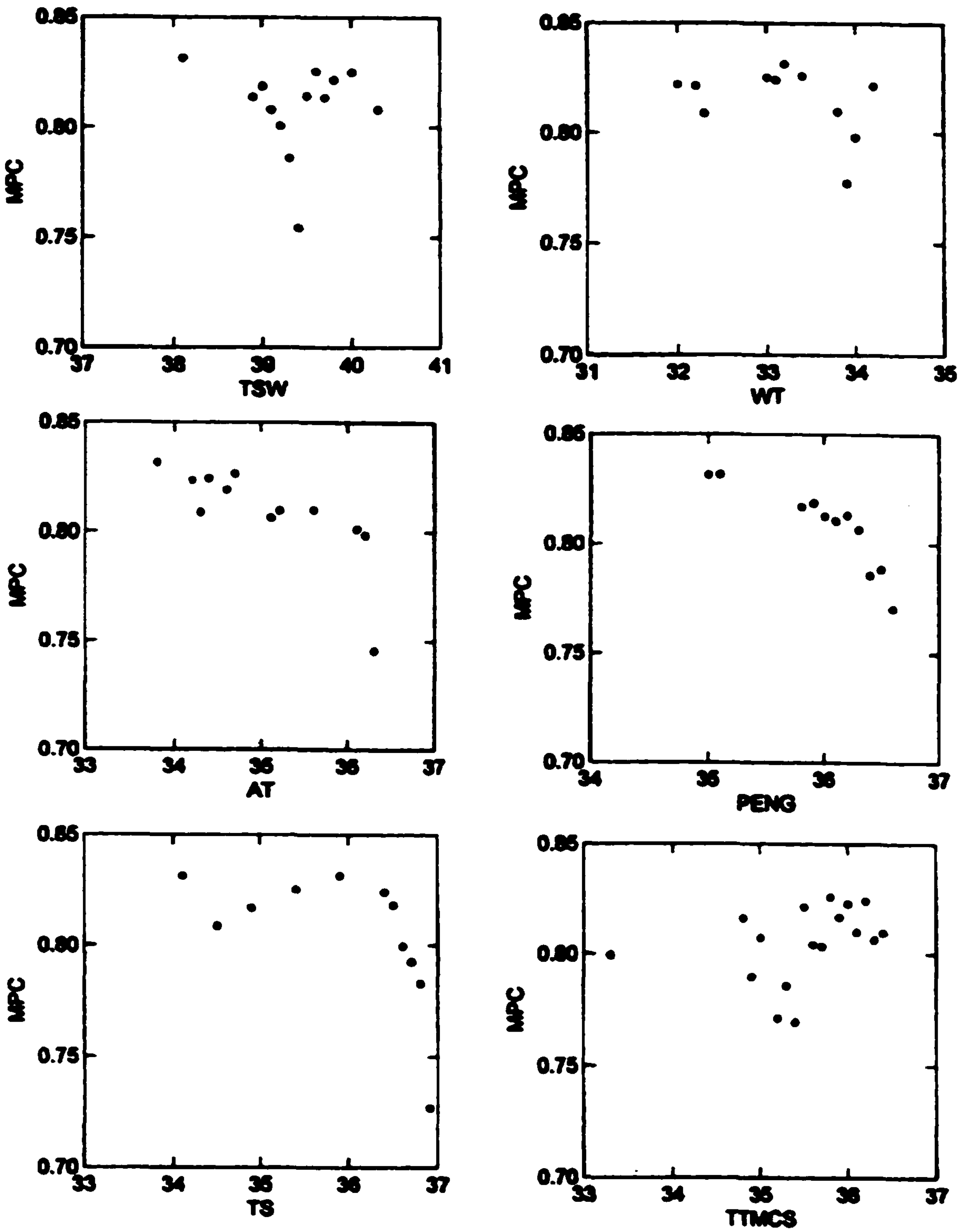



0.85
0.80
0.75
0.70
MPC
35
36
37
38
CC
39
40
FKP
33
34
TM
41
TMP
AOFTP
PAP

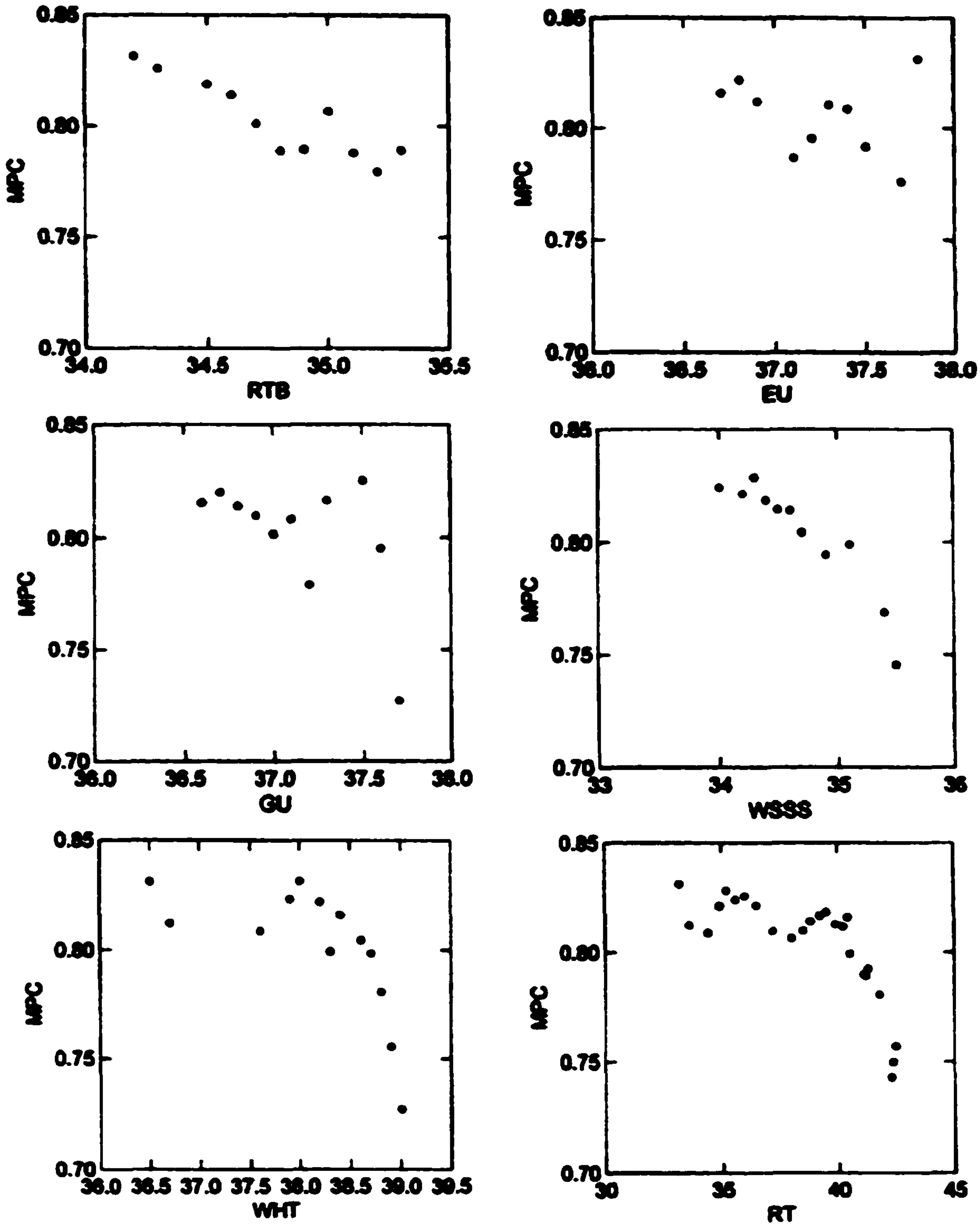

MPC
0.85
0.80
0.75
0.70
PP
CAP
PCP
RMPP
LLP
LWPEF


MPC
FAF
SCGP
PMI
FMI
MEM
EMES
0.85
0.80
0.75
0.70


MPC
TEOEMH
MVE
PPEW
RRES
SRBLT



MPC
0.85
0.80
0.75
0.70
37 38 39 40 41
TSW
31 32 33 34 35
WT
33 34 35 36 37
AT
34 35 36 37
PENG
33 34 35 36 37
TS
33 34 35 36 37
TTMCS


MPC
RTB
EU
GU
WSSS
WHT
RT
0.85
0.80
0.75
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34.0 34.5 35.0 35.5
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33 34 35 36
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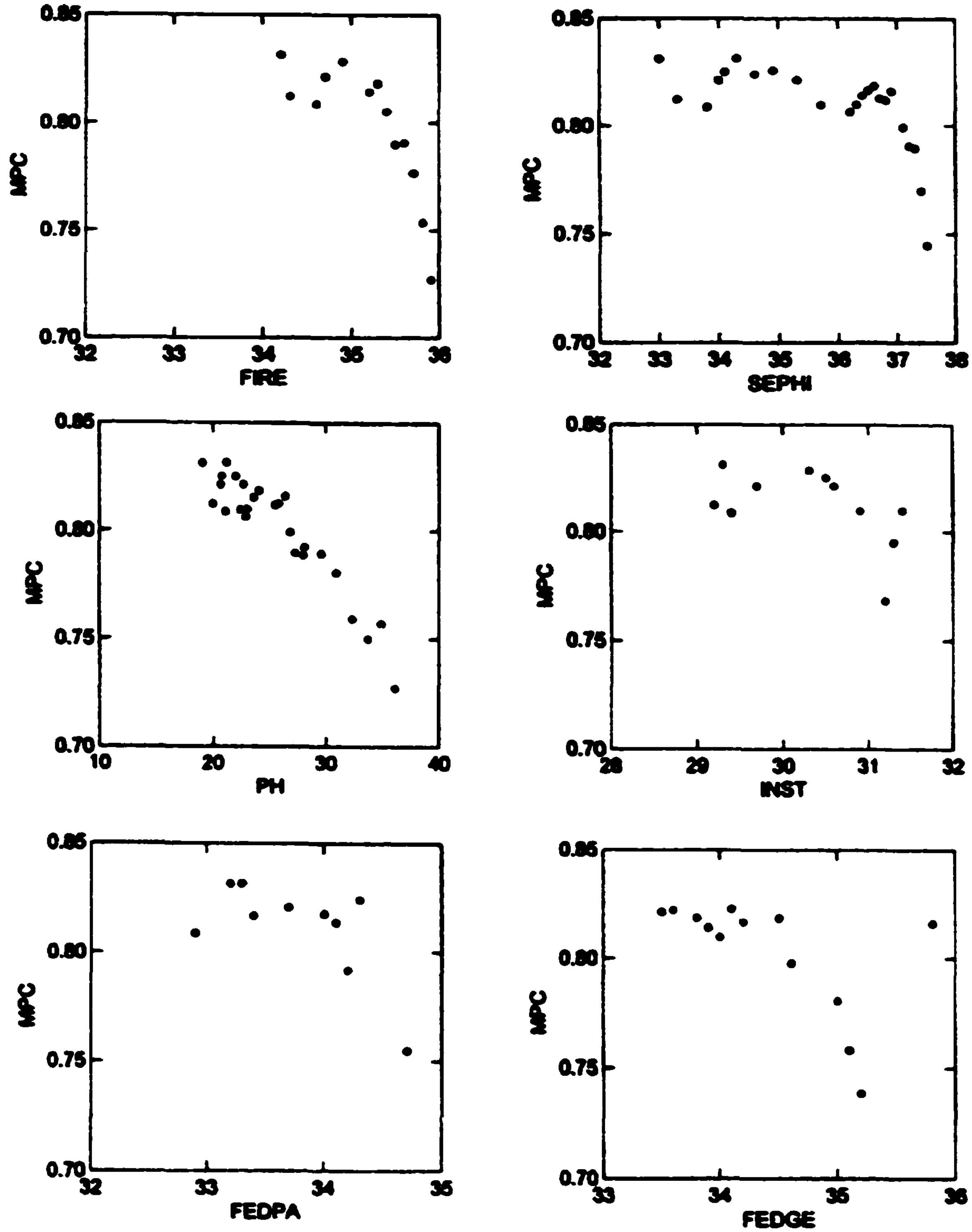
30 35 40 45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PH
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28 29 30 31 32
INST
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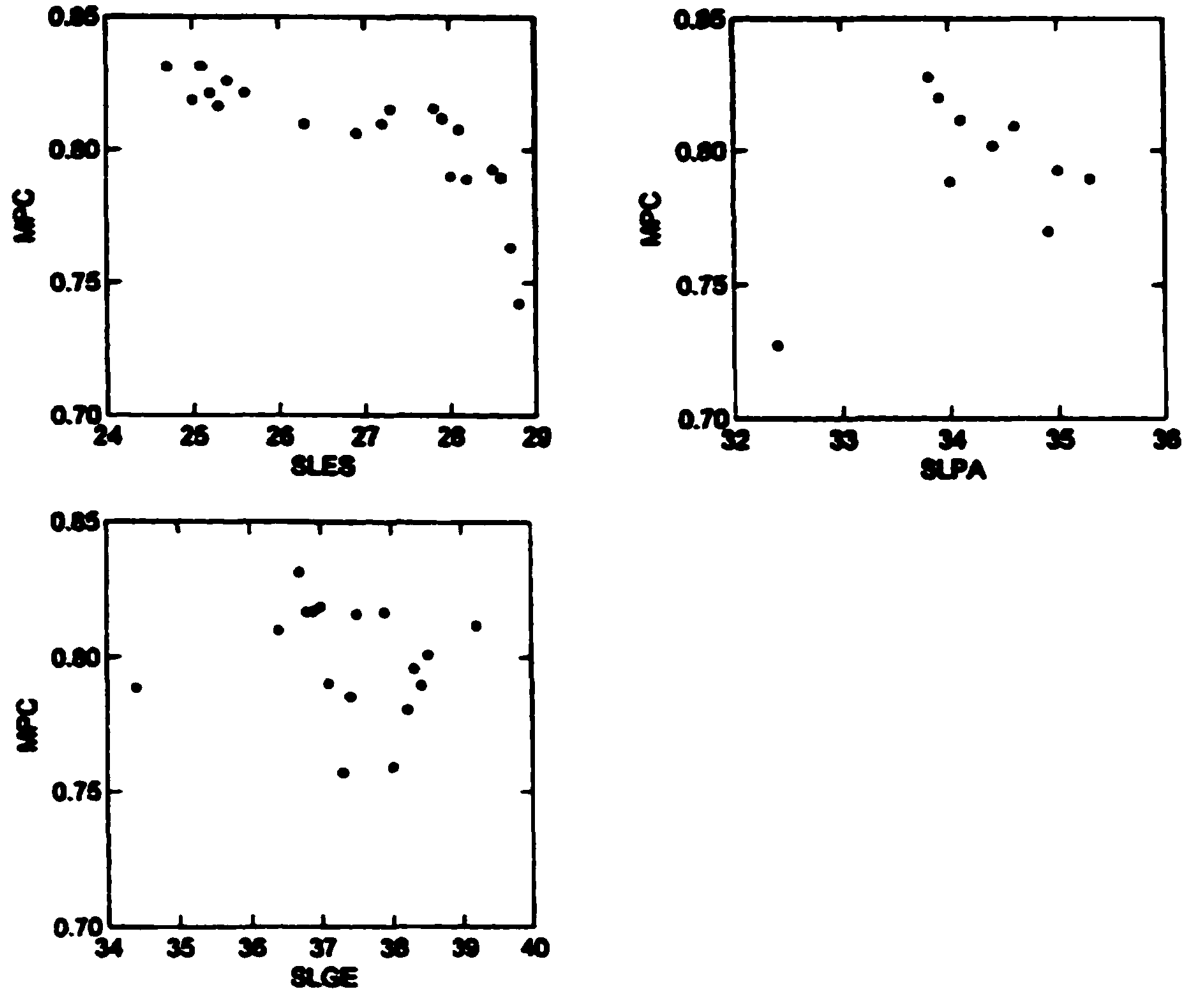
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REFERENCES

Allen, R.G.D.; Mathematical Economics, Macmillan, 1956.

Arrow, K.J. and G. Debreu; "Existence of an Equilibrium for a Competitive Economy." Econometrica, Vol. 22 no.3, 1954.

Becker, G.S.; "Altruism, Egoism and Genetic Fitness: Economics and Sociobiology" Journal of Economic Literature, 14, 1976.

--------------" A theory of the allocation of time," Economic Journal 75, no. 299 (September 1965).

Blanchard, O., Fischer, S.; Lectures on Macroeconomics, MIT Press, 1989.

Burn, A.F., Mitchell, W.C.; Measuring Business Cycles. New York: NBER, 1944.

Duchin, F., Lange, G-M.; The Future of The Environment, Oxford University Press, 1994.

Fischer, D.H.; The Great Wave, Oxford, 1996.

Fisher, A.C. and F.M. Peterson; "The Environment in Economics: A Survey," Journal of Economic Literature, vol. XIV, March 1976.

Fisher, I.; The Purchasing Power of Money, Macmillan, New York, 1918.

Friedman, M., Schwartz, A.; Monetary Trends in the United States and the United Kingdom, NBER, 1982.

Friedman, M.; A Theory of the Consumption Function, NBER, 1957.

Frisch, R.; "Propagation Problems and Impulse Problems in Dynamic Economics," in Economic Essays in Honor of Gustav Cassel, George Allen and Unwin, London, 1933. Reprinted in R.A. Gordon and L. R. Klein (eds.), Readings in Business Cycles, A.E.A. Series, George Allen and Unwin, London, 1966.

Greene, W.H.; Econometric Analysis 4th Ed., Prentice Hall, 2000.

Hadley, G.; Linear Algebra, Addison-Wesley Publishing, 1961.

Hansen, B.; A Survey of General Equilibrium Systems, McGraw Hill, 1970.

Heimann, E.; History of Economic Doctrines: An Introduction to Economic Theory, Oxford University Press, 1945.

Henderson, J.M., and R.E.Quandt; Microeconomic Theory: A Mathematical Approach, McGraw-Hill, 1980.

Hicks, J.R.; Value and Capital, Clarendon Press, Oxford 2nd Ed., 1954.

-------------; "'Mr. Keynes and the Classics': A Suggested Interpretation," Econometrica, 1937. Reprinted in J.R. Hicks, Critical Essays in Monetary Theory. Oxford University Press, 1967.

Howey, R.S.; The Rise of the Marginal Utility School 1870-1889, Morningside edition, 1989. Originally published by University of Kansas Press, 1960.

Jorgenson, D., Gollop, F., Fraumeni, B.; Productivity and U.S. Economic Growth, Harvard University Press, 1987.

Keynes, J.M.; The General Theory of Employment Interest and Money, Macmillan, 1936.

-----------A Tract on Monetary Reform, Macmillan, London, 1923.

Khan, H.; The Next 200 Years, Morrow, 1976.

Knight, F.H.; The Economic Organization, Chicago, 1933.

Landes, D.; The Unbound Prometheus, Cambridge, 1969.

Lange, O.; "Say's Law: A Restatement and Criticism," in O. Lange, et. al. (eds.) Studies in Mathematical Econometrics, University of Chicago Press, 1942.

Leontieff, W.W.; The Structure of the American Economy, Oxford University Press, Oxford,1941. Second enlarged edition, 1951.

Lewis, H.H.; History of Economic Thought, 4th ed., The Macmillan Co.. 1949. London. 1934. First Edition, 1907.

Maduro, G.A.; "The Partitioning of Nominal Income," manuscripts. 1997.

------------ "The Intertemporal Budget Constraint and Saving Behavior" manuscript. 2002.

------------ "An Economic Theory of Agential Existence," manuscript. 2001.

Mankiew. N.G.; "The Growth of Nations," Brookings Papers on economic activity, I:1995.

Marshall. A.; Principles of Economics, 8th ed., London. 1920.

McDonald, J.F.; Foundations of Urban Economics, Prentice-Hall. 1997.

Modigliani. F., R. Brumberg; "Utility Analysis and the Consumption Function: An Interpretation of Cross-Section Data," in K.K. Kurihara (ed.), Post-Keynesian Economics New Brunswick. NJ: Rutgers University Press. 1954.

Patinkin, D.; "The Indeterminacy of Absolute Prices in Classical Economic Theory," Econometrica, Jan. 1949. Reprinted in Essays on and in the Chicago Tradition, D. Patinkin. Duke University Press, 1981.

----------- Money, Interest and Prices: an Integration of Monetary and Value Theory, Row, Peterson, Evanston, Ill., 1956. Second edition, Harper and Row, New York, 1966.

Robson. A.J.; "The Biological Basis of Economic Behavior," Journal of Economic Literature, vol. XXXIX (March 2001).

Samuelson, P.A.; "The Stability of Equilibrium: Comparative Statics and Dynamics," Econometrica, Vol. 9 no. 1, 1947.

---------- Foundations of Economic Analysis, Harvard University Press, Cambridge Mass, 1947.

Sargent, T.; MacroeconomicTheory, 2nd ed, Academic Press, 1987.

Shotter, A.; Micro-Economics: A Modern Approach, 3rd ed., Addison-Weseley, 1996.

----------- "Productivity Under Group Incentives: An Experimental Study." (with H.R. Nalbantian), American Economic Review, Vol. 87. No.3. pp. 314-340, June 1997.

Solow, R.; "A Contribution to the Theory of Economic Growth," The Quarterly Journal of Economics, vol. 70, no. 1, 1956.

Swan, T.; "Economic Growth and Capital Accumulation," The Economic Record, vol. 32, no.63, 1956.

Varian, H.R.; Microeconomic Analysis, 3rd ed., Norton, 1992.

Von Neumann, J.; "A Model of General Economic Equilibrium." The Review of Economic Studies, vol. 13(1), no. 33, 1945-46.

Walras, L.; Elements of Pure Economics, George Allen and Unwin, London, 1954. English edition of Elements d' economie politique pure.

Wicksell, K.; Lectures on Political Economy, vol.1 and 2, Routledge and Kegan Paul.