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TRADING MODEL AND ASSET PRICE FORECAST:

A DISCRETE EVENT APPROACH

By

Li Li

A dissertation submitted to the Graduate Faculty in Economics in partial fulfillment of the requirements for the degree of Doctor of Philosophy, the City University of New York.

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THE CITY UNIVERSITY OF NEW YORK

Abstract**TRADING MODEL AND ASSET PRICE FORECAST: A DISCRETE EVENT
APPROACH**

By
Li Li

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This paper is an attempt to understand the trading market dynamics and the investment forecast processes.

The market trading activity is modeled as a Markovian process, where traders change their opinions which cause the buy/sell actions that drive the price movements. Based on this model, we investigate a series of complex phenomenon in asset markets, such as the turning points, the volume behaviors, and the collective behavior (the opinion polarization) through the information externalities.

This paper also develops certain types of empirical testing. The difficulties in financial forecasting can be partially overcome by making discrete, directional estimations. This leads to the discrete event approach. Computer programs have been designed to study the portfolio performance distributions, which is useful in investigating the probability behaviors of trading rules. Another tool, the polynomial approximation, is used to understand the empirical characteristics of the “break-up” patterns.

The approach presented here depends on some very loose assumptions. It is also very easy to program with computer, therefore it is highly applicable.

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I INTRODUCTION

This paper consists of a series of interrelated topics on market dynamics and market forecasts.

These studies serve the purpose of the so-called “active strategies” in the financial world. An “active strategy” means buy and sell securities by securities analysis. It usually includes market timing, security selection and hedging. As a money manager stated: “As the mutual fund industry becomes more saturated with funds, the importance of short-term performance as a competitive measurement intensifies. Mutual funds are measured monthly, quarterly and annually, as well as on a three-, five- and ten-year basis. To compete effectively, active management is required more today than in the past when a “buy and hold” strategy was efficient.” (Lisa A. Costa, “Using Technical Analysis to Identify Option Hedging Opportunities.” *Market Technicians Association Journal*, Fall-Winter 1995). Market forecast is critical to the active portfolio strategies. Many people find that even if they are daring, they still under perform the market in a limited time horizon. Bad forecasts cause money loss.

To try to forecast the markets actively does not conflict to the modern portfolio theory if we can tolerate higher risks. We do not claim that we can find out strategies that dominates the “efficient” strategy.

Many people use the “data mining” technique to find out the prediction rules. We here realize that the pure statistics “data mining” technique without knowing the market dynamics is easy to lose. Therefore we first suggest a dynamic model in chapter I as a framework for later studies. We use a partial equilibrium analysis and throw off the concept of Walrasian equilibrium. It is very different from the “standard” financial

modeling setups. However, our model is more convenient to handle certain types of problems, such as volume, volatility and the collective behavior in the financial markets that we meet everyday.

There have been mountains of evidence to indicate the nonlinear structure in the asset price processes. For example, Ball and Kothari (1989) showed that “negative serial correlation in relative returns is due almost entirely to variation in relative risks, and therefore expected relative returns, through time.” Poterba and Summers (1988) found that “our point estimates imply positive autocorrelation in returns over short horizons and negative autocorrelation over long horizons, although random-walk price behavior cannot be rejected at conventional statistical levels.” Romer (1993) further developed some models to show that “an important part of movement in asset prices may be caused by neither news nor irrationality, but by the revelation of information by the trading process itself.”

We define “market efficiency” in our model as market participants taking actions rapidly in responding to information. People’s options follow a Markov process. Options turning over generate supply and demand for the financial assets. This causes the asset prices to follow a time varying random walk.

Chapter II studies how to make a direction forecast with the “double momentum method” if we only know that the expected return is a curve (time-varying).

The word “momentum” is widely used by practitioners in lots of fields like prices, sales and earnings analysis. Momentum is often defined as the speed of the trend. The basic idea in applying momentum analysis is to compare it to car driving: you speed before you stop. You speed up your car after you start it. Therefore the pattern of

momentum can predict the trend turnover. However, such an analysis often fails because of the heavy noises in price series. To make use of momentum effectively, we first reveal the relationship between the momentum measure and the time-varying random walk model. Then we suggest our forecast rules based on the assumption that prices follow the time-varying random walk. The basic idea of our forecast rules is from the production theory in microeconomics: when average cost is rising, marginal cost must be above the average cost. When average product is declining, marginal product must be below the average product. These facts are true for any smooth, deterministic curve. If this is violated, it must be caused by temporary noises. This simple fact gives us a useful filter method to identify the price direction.

The empirical testing in chapter II shows how to identify the turning point patterns with our forecast rules. The testing tries to study the distribution of the turning patterns. We make our predictions based on the historical price series only and found that our forecast procedure can identify some turning point patterns although they have a wide performance divergence. In doing so, we have moved away from the orthodox “optional” forecast doctrine that requires predicting quantitative signals everywhere. Our predictions only give out rare and qualitative signals.

To improve the forecast quality, we need to include other variables besides historical prices. Trading volume is such a useful variable. It is widely accepted that trading volume can be used as filter to help price forecast. In chapter III we show how to understand the volume behavior in our model. Volume is the number of contracts that change hand during a specific time interval. In our trading model, a contract changes hand when traders with counter opinions change their options. Therefore the volume

behavior is straightforward and much easier to handle in our model framework than in the “heterogeneous trader model.”

Market participants are heterogeneous in opinion in our trading model. Trading exists because opinions differ. Asset prices are affected by the interactions among people. They not only watch the market information such as prices and volume, but also try to search all relevant information from all other people. This affected the price movements. Such an information search game is best described by Keynes: “...professional investment may be likened to those newspaper competitions in which the competitors have to pick out the six prettiest faces from a hundred photographs, the prize being awarded to the competitor whose choice most nearly corresponds to the average preference of the competitors as a whole; so that each competitor has to pick, not those which he himself finds prettiest, but those which he thinks likeliest to catch the fancy of the other competitor, all of whom are looking at the problem from the same point of view. It is not a case of choosing those which, to the best of one’s judgement, are really the third degree where we devote our intelligence to anticipating what average opinion expects the average opinion to be. And there are some, I believe, who practise the fourth, fifth and higher degrees.” In chapter V we investigate the possibility that the price stability may be broken when people are connected not only by the “economic” variables such as price but also by “behavioral” variables such as other people’s options.

The stability broken in price series is a typical nonlinear phenomenon. This kind of pattern can’t be used to predict even the price direction, but it is very important in market risk management. In chapter VI we develop an operational procedure to describe such a

pattern. We use a polynomial to approximate the expected return and use a stationary probability elasticity function to summarize the important properties of the pattern.

Finally we summarize the major points in this study and suggest further research projects in chapter VII.

II. A MARKOV TRADING MODEL

In this chapter we present a partial, non-Walrasian equilibrium trading model. We are going to start from modeling the public opinions with a Markov process. Information efficiency is defined as the rapid adjustment of the opinions to new information. Trading occurs when people change their opinions.

This trading model will be served as a basic framework for later applications. Generally, we found that the asset prices have a nonlinear expected return term.

II.1. REVIEW OF "STANDARD" FINANCIAL MODELING

The asset prices are affected by the forces of supply and demand. In order to model prices behavior, we must first model the demand and supply.

In most of the financial decision models (for example Grossman and Stiglitz, 1980), assets are classified into two groups: a safe asset yielding a return R , and a risky asset with a time varying random return rate u . Consider a two periods setup, let "0" as the initial of the period and let "1" as the end of the period. Each individual has an exponential utility function

$$V(W_{1i}) = -e^{-aW_{1i}},$$

(2.1) Utility Function

where $a > 0$ is the coefficient of absolute risk aversion. Each trader is endowed with two types of securities: M_i , the riskless asset, and X_i , the risky asset. The i th trader's budget constraint at the beginning(0) is

$$P \cdot X_i + M_i = W_{0i},$$

(2.2a) Budget Constraint

where P is the current price of risky assets and the price of risk free assets is set to be unity. If at the end of the period(1), the i th trader holds a portfolio (M_i, X_i) , his wealth will be

$$R \cdot M_i + u \cdot X_i = W_{i,t}.$$

(2.2b)

Maximizing (2.1) subject to (2.2)s yields a demand function. By equate the total demand to the supply of the asset, an Walrasian equilibrium price is obtained.

This type of financial model starts from the individual forecast on the risky asset return u . Different forecast of u results in different projected period-end wealth, hence affects one's utility function. To study the comparative dynamics, we suppose u receives a perturbation, the resulted equilibrium price change can then be investigated. In such a model, the perturbation is added on demand function only. The demand and supply are therefore asymmetric in this kind of model. Such a property seems OK in stock markets since there are fixed supply, the total shares outstanding. However, in markets like commodity futures, it is hard to say the supply is fixed. Both supply and demand sides in such markets are highly flexible.

II.2. A BINARY OPINION MODEL

We can model the supply and demand in a different way. We consider the statistical properties of the bullish and bearish opinions first. The supply/demand forces are created when opinions switching.

Endowments Suppose each participation in the market holds a unit (for example, 100 shares or 1 contract) of endowment. The eliminating of the wealth effect greatly simplifies the analysis of the dynamics and the collective behaviors.

Trading Opportunities Trade among individuals (markets) occurs either because of the heterogeneous tastes (risk aversions), the heterogeneous endowments, or the heterogeneous beliefs (opinions). For simplicity, we focus on the opinions behavior only.

Suppose people take actions simultaneously (rather than sequentially). Each person has two choices in *opinion* about the expected asset price: bullish or bearish. Denote the bullish opinion as "+" and the bearish opinion as "-". Trading *occurs* when a pair of players with counter opinions changes their opinions. The sequence of these opinion changes by the players constructs a *game*.

We further let n_+ and n_- represent the numbers of individuals who are in bullish opinion and in bearish opinion respectively. Suppose there exists a probability per unit time of the change of an investor from bullish opinion (+) to bearish opinion (-) or from bearish opinion (-) to bullish opinion (+). We denote these *transition probabilities* by $p_{+-}(n_+, n_-)$ and $p_{-+}(n_+, n_-)$, that's

$p_{+-}(n_+, n_-) \equiv$ transition probability that an investor turn his opinion from bullish to bearish,

$p_{-+}(n_+, n_-) \equiv$ transition probability that an investor turn his opinion from bearish to bullish.

The meanings for these transition probabilities are, for example, in a unit of time interval, 60% of bearish opinion traders turn their opinion to bullish, and 45% of bullish opinion traders turn their opinion to bearish. Note the sum of these transition probabilities does not necessarily be unity because of their definitions.

Also note that, in our model, the total number of participants N is not necessarily a constant over time. For example in a stocks market, a natural rule for the opinion choices would be like this: a "+" means a long position, and a "-" means a short position or a cash position (the "waiting people").

Tastes An individual i makes his bullish/bearish opinion based on his judgment variable h_i .

The real function h_i can be considered as the trader i 's objective function of decision, it can go from minus infinity to positive infinity. Here we can use the probability version of the rational choice theory. Suppose that each individual i 's decision process follows a stochastic rule:

$$O_i = +1 \text{ with probability } \text{Prob}(h_i); \quad O_i = -1 \text{ with probability } 1 - \text{Prob}(h_i).$$

In this theory, given the choice set $O_i = +1$ or -1 , "the probability that the choice $O_i = +1$ is selected exceeds 0.5" simply means "bullish opinion is preferred to bearish opinion".

Supply and Demand The supply/demand in a short time interval are simply defined as the sell/buy order flows in the short time interval. The direct formulas are:

$$\text{Demand} = W(\uparrow) = n_- \cdot p_{-,+}(n_-, n_+),$$

$$\text{Supply} = W(\downarrow) = n_+ \cdot p_{+,-}(n_+, n_-).$$

(2.3) Supply and Demand in the Binary Opinion Model

Thus the jump rates $W(\uparrow)$ and $W(\downarrow)$ represent the numbers of traders who turn their opinions from bearish to bullish and from bullish to bearish respectively in a short time interval. These are also called the opinions transition probabilities per unit time from bearish to bullish and from

bullish to bearish. These jump rates are further functions of other economic variables. We also require the comparative dynamics for the supply/demand functions as

$$\frac{\partial W(\uparrow)}{\partial Price} < 0, \quad \text{and} \quad \frac{\partial W(\downarrow)}{\partial Price} > 0.$$

(2.4) Stability Conditions

This says when other factors are fixed, prices away from the equilibrium point will be pulled back.

In our model, the supply and demand are symmetric. When a trader changes his opinion from bullish to bearish, a unit of demand is generated. A unit of supply occurs in the contra case.

The Market Dynamics Before time t traders decide their opinions based on the information set (Price, Private Message). At time t , the market price adjusts so that the buy/sell orders are balanced. For the adjustment dynamics we assume that the shortage side is fixed. For example, if there are 4 buy orders and 10 sell orders (each with the same amount of quantities) at time t , then price jumps down so that 4 orders are traded. Finally the trading volume at time t is determined by the short side.

This kind of market mechanism is not Walrasian. In the Walrasian mechanism, price is adjusted so that both sides of demand and supply would adjust to the equilibrium level. However, we found that this mechanism is not so appropriate in a dynamic setup.

This market dynamics is also different from many physics-chemistry processes. Although we use similar mathematics, one should be aware that the meaning in our model is very unique.

A Dynamics Example: Game Simulation Table 2.1 shows a simulated game and the statistics for the model described in this section.

Table 2.1

Traders	t=0	t=1	t=2	t=3
1	-	-	+	-
2	-	-	+	+
3	+	-	+	-
4	+	-	+	+
5	+	+	-	-
6	+	+	-	-
7	+	+	+	+
8	-	+	+	+
9	-	-	-	+
10	-	-	+	-
11	-	-	-	-

12	-	-	-	-
Time	0	1	2	3
Sell Orders(#s turn to bearish from bullish)		2	2	3
Buy Orders(#s turn to bullish from bearish)		1	5	1
Net Buy Orders		-1	3	-2
Trading Volume		1	2	1
$f(n_+)$	5	4	7	5

II.3 THE EVOLUTION OF OPINIONS: CONTINUOUS CASE

State Function $f(n_+)$ The model described in the last section is a typical game model. In each stage of the game we use a probability distribution function of n_+ , $f(n_+, t)$, to represent the state of the game at t .

Markov Game and the Information Efficiency Anything that can possibly affect the participants' actions is actually reflected in the state $f(n_+, t)$. In an information efficient market, the evolution of $f(n_+, t)$ must be Markovian.

The Evolution Equation for the Two-Level System Consider the evolution of the state variable $f(n+,t)$. By Markov property, the state variable $f(n+,t)$ must follow the master equation (a varied form of the Chapman-Kolmogorov equation):

$$\frac{df(n_+)}{dt} = \text{RatesGain} - \text{RatesLoss}.$$

Where

$$\text{Gain} = (n_+ + 1) \cdot p_{+,-}(n_+ + 1, n_- - 1) \cdot f(n_+ + 1) + (n_+ + 1) \cdot p_{-,+}(n_+ - 1, n_- + 1) \cdot f(n_+ - 1),$$

$$\text{Loss} = [n_+ \cdot p_{+,-}(n_+, n_-) + n_+ \cdot p_{-,+}(n_+, n_-)] \cdot f(n_+),$$

(2.5) Master Equation

and we have ignored the parameter t in the state function f . The transition probabilities $P_{+,-}$ and $P_{-,+}$ have been defined in section “Trading Opportunities”. The master equation (2.5) is a gain-loss equation for the probabilities of the state n_+ . The gain of state n_+ is resulted from the transitions from other states $(n_+)+1$ and $(n_+)-1$. This is a one-step process. So there are only two “other” states. The transition probability per unit time from state $(n_+)+1$ to state (n_+) , $(n_+ + 1) \cdot p_{+,-}(n_+ + 1, n_- - 1)$, is the jump rate that people turn their opinions from bullish to bearish. The transition probability per unit time from state $(n_+)-1$ to (n_+) , $(n_+ + 1) \cdot p_{-,+}(n_+ - 1, n_- + 1)$, is the jump rate that people turn their opinions from bearish to bullish. By the same reason, the loss of state n_+ is due to transitions from n_+ into other states $(n_+)+1$ and $(n_+)-1$. The transition probability per unit of time from state (n_+) to state $(n_+)-1$, $n_+ \cdot p_{+,-}(n_+, n_-)$, is the jump rates that people turn their opinions from bullish to bearish. . The

transition probability per unit of time from state (n_+) to state $(n_+)+1$, $n_+ \cdot p_{-,+}(n_+, n_+)$, is the jump rates that people turn their opinions from bearish to bullish.

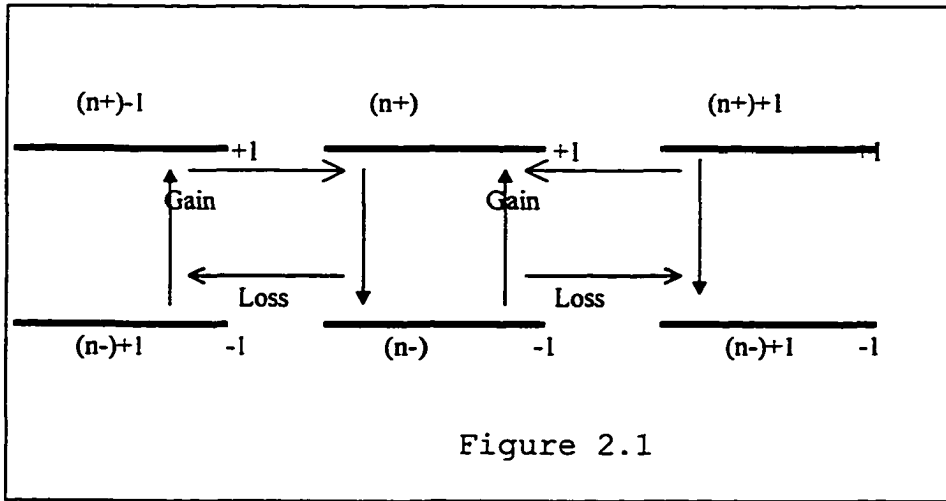


Figure 2.1 visualized the meaning of equation (2.5) by representing (2.5) in a two-level opinion space in which only

two states are possible: "+"(bullish opinion) or "-"(bearish opinion).

Assume that this space is homogeneous and note

$$W(\uparrow) \equiv (n_+ + 1) \cdot p_{-,+}(n_+ - 1, n_+ + 1) = n_+ \cdot p_{-,+}(n_+, n_+),$$

(2.6)

$$W(\downarrow) \equiv (n_+ + 1) \cdot p_{+,-}(n_+ + 1, n_+ - 1) = n_+ \cdot p_{+,-}(n_+, n_+).$$

(2.7)

Where $W(\uparrow)$ and $W(\downarrow)$ are the demand and supply as defined in section "Supply and Demand". Equation (2.6) and (2.7) simply say the transition probability per unit time (both of up and down) are independent of the original state(the numbers of people in the bullish opinion) in a small range. The master equation (2.5) is then simplified as

$$\frac{df(n_+)}{dt} = W(\downarrow) \cdot f(n_+ + 1) + W(\uparrow) \cdot f(n_+ - 1) - [W(\downarrow) + W(\uparrow)] \cdot f(n_+).$$

(2.8)

Note: in the former equations, we ignored all of the subindex "t" for simplicity. In reading, all of the variables should be understood as time varying.

The Diffusion of Opinions To make sense on the master equation (2.8), one must find out the solution of that equation. Unfortunately, only in rare cases is it possible to solve the master equation explicitly. One way to understand it is to approximate it in a *diffusion form*. We are going to express equation (2.8) in a diffusion form in this section.

Since n_+ is very big, we treat it as a continuous variable and introduce an variable b so that

$$n_+ = \lim_{b \rightarrow 0} \frac{x_+}{b}.$$

(2.9)

Define a new function $g(x_+)$ by

$$f(n_+) = \lim_{b \rightarrow 0} f\left(\frac{x_+}{b}\right) = \lim_{b \rightarrow 0} b g(x_+).$$

(2.10)

$g(x_+)$ is in fact the probability density function at x_+ . Introducing a new scale variable "a" so that,

$$f(n_{-} \pm I) = \lim_{b \rightarrow 0} f\left(\frac{x_{-} \pm a}{b}\right) = \lim_{b \rightarrow 0} b g(x_{-} \pm a).$$

(2.11)

Then (2.8) becomes

$$\frac{dg}{dt} = W(\downarrow) \cdot g(x_{+} + a) + W(\uparrow) \cdot g(x_{-} - a) - [W(\downarrow) + W(\uparrow)] \cdot g(x_{+}).$$

(2.12)

Expanding (2.12) up to second order in "a" yields

$$\frac{dg}{dt} = -[W(\uparrow) - W(\downarrow)] \cdot a \cdot \frac{dg}{dx} + [W(\uparrow) + W(\downarrow)] \cdot \frac{a^2}{2} \cdot \frac{d^2g}{dx^2}.$$

(2.13)

The corresponding stochastic differential equation representation is:

$$dx_{+} = [W(\uparrow) - W(\downarrow)] \cdot a \cdot dt + [W(\uparrow) + W(\downarrow)] \cdot \frac{a^2}{2} dB.$$

(2.14)

Where the dB is the standard Brownian process.

In (2.14), the drift term

$$\mu_{opinion} = [W(\uparrow) - W(\downarrow)] \cdot a,$$

(2.15)

measures *the average belief change* and the diffusion coefficient

$$\sigma^2_{opinion} = [W(\uparrow) + W(\downarrow)] \frac{a^2}{2}.$$

(2.16)

measures the dispersion of opinion change.

By the same principle, we can work out the diffusion equation for x_- as

$$dx_- = [W(\downarrow) - W(\uparrow)] \cdot a \cdot dt + [W(\uparrow) + W(\downarrow)] \cdot \frac{a^2}{2} dB.$$

(2.17)

II.4 THE EVOLUTION OF OPINIONS: DISCRETE CASE

Representative (Average) Opinion Each trader's opinion can only take two values at any time: +1 or -1. Note $n(\cdot)$ as the number of traders whose opinion is in state (\cdot), the mean opinion for a market at any time can be defined as an mean over all traders

$$\langle O \rangle = (+1) \frac{n_+}{n_+ + n_-} + (-1) \frac{n_-}{n_+ + n_-} = \frac{n_+ - n_-}{n_+ + n_-}.$$

(2.18) Representative Opinion

This number summarized the average opinion in the market and hence is called the "representative opinion".

A Two-State Markov Chain for Opinion Change Consider a *Markov chain* on two states "+" (bullish) and "-" (bearish). Suppose the transition probabilities are given as

$$Prob[s_t = + | s_{t-1} = +] = \pi_{++},$$

$$Prob[s_t = - | s_{t-1} = +] = \pi_{+-} = 1 - \pi_{++},$$

$$Prob[s_t = - | s_{t-1} = -] = \pi_{--},$$

$$Prob[s_t = + | s_{t-1} = -] = \pi_{-+} = 1 - \pi_{--}.$$

(2.19)

Using matrix, we note

$$\Pi = \begin{bmatrix} \pi_{++} & \pi_{-+} \\ \pi_{+-} & \pi_{--} \end{bmatrix} = \begin{bmatrix} \pi_{++} & 1 - \pi_{--} \\ 1 - \pi_{++} & \pi_{--} \end{bmatrix}$$

(2.20)

This two-state Markov chain is ergodic provided that $(\pi_{++}) < 1$, $(\pi_{--}) < 1$ and $(\pi_{++}) + (\pi_{--}) > 0$.

Consider a 2x1 state vector $\xi(t)$:

$$\xi_t = \begin{bmatrix} \xi_t^+ \\ \xi_t^- \end{bmatrix}$$

(2.21)

where $\xi^+ = \frac{n_+}{n_+ + n_-}$ is the fraction of traders in bullish opinions among the total population, and

$\xi^+ + \xi^- = 1$. By this notation, the representative opinion $\langle O \rangle$ can be expressed as

$$\langle O \rangle = \xi^+ - \xi^-.$$

(2.22)

Now suppose we start from a pure state that 100% traders are in bullish opinion or in bearish opinion. Note

$\xi_t = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ if $s(t) = "+"$, and $\xi_t = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ if $s(t) = "-"$. Thus, the conditional expectation of $\xi(t+1)$ given

$s(t) = "+"$ or $"-"$ is given by

$$E[\xi_{t+1} | s_t = +1] = \begin{bmatrix} \pi_{++} \\ \pi_{+-} \end{bmatrix},$$

(2.23a)

$$E[\xi_{t+1} | s_t = -1] = \begin{bmatrix} \pi_{-+} \\ \pi_{--} \end{bmatrix}.$$

(2.23b)

From the Markov property, it implies that

$$E[\xi_{t+1} | \xi_t, \xi_{t-1}, \dots] = E[\xi_{t+1} | \xi_t] = \Pi \cdot \xi_t,$$

(2.24) Markov Property

where Π is the transition matrix defined by (2.20). So it is possible to express a Markov chain in the form of a first-order vector autoregression for $\xi(t)$:

$$\xi_{t+1} = \Pi \cdot \xi_t + \varepsilon_{t+1}, \text{ with } E[\varepsilon_{t+1} | t] = 0.$$

(2.25)

In scalar form, we can write (2.25) as

$$\xi_{t+1}^+ = (1 - \pi_{-+}) + (\pi_{++} + \pi_{-+} - 1) \cdot \xi_t^+ + \varepsilon_{t+1}^+,$$

(2.26a)

$$\xi_{i+1} = (1 - \pi_{++i}) + (\pi_{++i} + \pi_{--i} - 1) \cdot \xi_i + \varepsilon_{i+1}$$

(2.26b)

Subtracting (2.26b) from (2.26a) and using (2.18), (2.22) yields

$$\langle O \rangle_{i+1} = (\pi_{++i} - \pi_{--i}) + (\pi_{++i} + \pi_{--i} - 1) \cdot \langle O \rangle_i + (\varepsilon_{i+1}^+ - \varepsilon_{i+1}^-)$$

(2.27)

This is the AR(1) representation for the representative opinion $\langle O \rangle$. Note when $(\pi_{++}) + (\pi_{--}) > 1$, the process is likely to persist in its current state and the variable $\langle O \rangle$ would be positively correlated, whereas when $(\pi_{++}) + (\pi_{--}) < 1$, the process is more likely to switch out of a state than to stay in it, produce negative serial correlation.

II.5 PRICE BEHAVIOR

Given the supply and demand functions, this section relates the price movements and the economic model.

The Price Adjustment Rule Suppose the price adjustment follows this dynamic rule:

$$\frac{dP}{dt} = D(\text{ExcessDemand}(P)),$$

(2.28) Price Adjustment

where D is some smooth sign-preserving function of excess demand. Because of our model setup, there is no wealth effect. For further simplicity, we make the following *linearity assumption*

Assumption 2.1 *Linear Price Adjustment Rule.* Let

$$D(z) = 1 \cdot z,$$

(2.29) Linear Rule

so that

$$\frac{dP}{dt} = \text{ExcessDemand}(P).$$

(2.30) Linear Price Adjustment Rule

Our next step is to connect the excess demand function with variables described in the former sections.

General Price Equation: Continuous Case A general relation between asset price and opinion can be obtained by observing the "stock" variables X_+ and X_- . Consider the excess demand during a short time interval dt . Since the demand is resulted from the numbers of traders that change their opinions from bearish to bullish during time dt , we have

$$\text{ExcessDemand} \cdot dt \propto dx_+ - dx_-.$$

(2.31)

Applying (2.31) on (2.30) we get

$$dP \propto dx_+ - dx_-.$$

(2.32)

Substituting (2.14) and (2.17) into the right hand side of (2.32) yields

$$dP = \theta \cdot [W(\uparrow) - W(\downarrow)]dt + \eta \cdot [W(\uparrow) + W(\downarrow)]dw,$$

(2.33) Stochastic Differential Equation for Price

where both θ and η are some positive constants, and dw is the standard Wiener process with properties:

$$E[dw] = 0, \text{ and } E[(dw)^2] = dt.$$

Formula (2.33) relates the price to the economic model variables $W(\uparrow)$ and $W(\downarrow)$. It shows that the expected change in P at time t is proportional to the difference of jump rates between up and down. From the stability conditions (2.4), the "mean reversion" (see, for example, Poterba and Summers(1988)) property is obvious. This is consistent with the results of some other models, like the noisy trader models. However, our model does not depend on the specific assumptions of "noisy traders".

The diffusion coefficient is a coefficient to "enlarge" the standard Brownian noise. Formula (2.33) shows that the diffusion is proportional to the total jump rates, or the total frequency of opinion changes. This means in our model volatility is caused by trading itself. (See Hull's (1993, page 230) brief discussion on this topic. Also see French and Roll(1986).)

General Price Time Series: Discrete Case In discrete case, the price change

$P_{t+1} - P_t$ is proportional to the excess demand $(n_+^{t+1} - n_-^{t+1}) - (n_+^t - n_-^t)$, which is proportional to the average opinion change $\langle O \rangle_{t+1} - \langle O \rangle_t$ ¹. We therefore can assume a linear relation between the average opinion and the price level:

$$P_t = \alpha + l \cdot \langle O \rangle_t + \varepsilon_t.$$

(2.34)

This relates the expected representative opinion at next period (t+1) to the expected price in the future (t+1). The expectation of representative opinion can be found from the evolution equation in section II.3. Combining (2.34) and (2.27) we have

$$E[P_{t+1}] = \alpha + l \cdot (\pi_{++t} - \pi_{-t}) - \alpha \cdot (\pi_{++t} + \pi_{-t} - 1) + (\pi_{++t} + \pi_{-t} - 1) \cdot P_t.$$

(2.35)

Note At time t, P_t is known, while P_{t+1} is a random variable. If we further note

$$r_t(\pi_{++t}, \pi_{-t}) \equiv \alpha + l \cdot (\pi_{++t} - \pi_{-t}) - \alpha \cdot (\pi_{++t} + \pi_{-t} - 1)$$

(2.36)

and add back a white noise term, then the former equation about P_{t+1} becomes

¹ From equation (2.18), the average opinion change $\langle O \rangle_{t+1} - \langle O \rangle_t$ is $\frac{n_+^{t+1} - n_-^{t+1}}{n_+^{t+1} + n_-^{t+1}} - \frac{n_+^t - n_-^t}{n_+^t + n_-^t}$.

The total number of participants in the market is basically a very big number and is constant (we count on the “waiting people”) over a short time. For example, suppose at time 0 there are 50 people in bullish and 50 people in bearish. At the next time period 10 people jumps up from bearish and 5 people jumps down from bullish. Now at time 1 there are $50 + 10 - 5 = 55$ people in bullish and $50 + 5 - 10 = 45$ people in bearish. The average opinion goes up 10%. In this example, we have 10 buy orders and 5 sell orders, the excess demand is then 5.

$$P_{t-1} = r_t(\pi_{+-t}, \pi_{-t}) + (\pi_{++t} + \pi_{-t} - 1) \cdot P_t + \varepsilon_{t-1}.$$

(2.37)

The exogenous parameters (π_{++}) and (π_{--}) in (2.37) may change drastically even in a very short time frame. The assumption of constant required rates thus smoothed out some very important information.

II.6 TIME-VARYING RANDOM WALK

We make the following important assumption:

Assumption 2.2 Assume that the transition probabilities

$$\pi_{++} \rightarrow 1,$$

$$\pi_{--} \rightarrow 1.$$

(2.38)

The meaning of assumption 2.2 is that the expected time that traders stay in their opinion status τ is very long relative to the unit time interval we are using. Since the probability that the trader stays in bullish opinion (given that he is already in bullish opinion) for one more period is π_{++} , and the probability for two more periods is $\pi_{++} \times \pi_{++}$, and so on. Suppose the time interval of each period is 1, then

$$\tau_+ = \sum_{i=1}^{\infty} \pi_{++}^i \cdot i = \frac{\pi_{++}}{(1 - \pi_{++})^2} \rightarrow \infty$$

(2.39)

when π_{++} approaches to 1². Similarly, the average time traders stay in bearish opinion under the assumption (2.38)

$$\tau_- = \sum_{t=1}^{\infty} \pi'_{--} \cdot t = \frac{\pi_{--}}{(1 - \pi_{--})^2} \rightarrow \infty.$$

(2.40)

The rationality of assumption 2.2 depends on two factors. The first factor is the unit of time intervals. For example, if we count on the “tick by tick” data, the unit time interval is the time each trade occur. (2.38) is easily satisfied. If we study yearly data, (2.38) is basically not correct. The second factor is “churning rate”. If a market has more long-term investors, (2.38) is easier to satisfy.

Use the above simplification, (2.37) now becomes

$$P_t \approx r_{t-1}(\pi_{++t-1}, \pi_{--t-1}) + P_{t-1} + \varepsilon_t.$$

(2.41)

Where

$$r_{t-1}(t-1) \approx \alpha + l \cdot (\pi_{++t-1} - \pi_{--t-1}).$$

(2.42)

²To see this, let $S = \sum \pi'_{++} \cdot t$. Divided both sides by π_{++} , we have $\frac{S}{\pi_{++}} = \sum \pi'^{t-1}_{++} \cdot t$. The right hand side happens to be $\frac{d}{d\pi_{++}} \sum \pi'_{++}$, which equals $\frac{d}{d\pi_{++}} \frac{1}{1 - \pi_{++}} = \frac{1}{(1 - \pi_{++})^2}$. Finally we get (2.39).

This is a random walk model with time-varying drift.³ The trend is basically determined by the difference between the up and down jump rates, or the transition probabilities in the market.

Conclusion In conclusion, we found that the asset prices in an efficient market follow a random walk with changing expected returns. This is supported by the empirical research like Ball and Kothari(1989) and Poterba and Summers(1988). The changing expected returns imply some important nonlinear phenomena, such as turning points, unstable patterns. We are going to investigate these patterns in the following chapters.

Notations:

f: probability distribution function.

n+: numbers of traders who are in bullish opinions.

n-: numbers of traders who are in bearish opinions.

O_i: trader i's opinion.

P-+: probability per unit time of the change of an investor from bearish opinion to bullish opinion(transition probability).

P+-: probability per unit time of the change of an investor from bullish opinion to bearish opinion(transition probability).

W(↑): the numbers of traders who turn their opinions from bearish to bullish. It is the buy order flow per unit time(jump rates).

W(↓): the numbers of traders who turn their opinions from bullish to bearish. It is the sell order flow per unit time(jump rates).

³The $r(t-1)$ term defined in (6.17) is an "absolute" return rates. One can substitute $\log P$ into P to get a "relative" return rates $R(t-1) = \log P(t) - \log P(t-1)$ as used by most of the people.

<O>: Average opinion over the market participants at a time point.

X+: Number of traders whose opinions are in bullish.

X-: Number of traders whose opinions are in bearish.

III. FORECAST TURNING POINTS WITH THE DOUBLE MOMENTUM METHOD

This chapter presents a method to identify the turning points in asset prices without presupposing the detail parameters of the expected returns.

III.1. DIRECTION FORECAST

Optimal Forecast and the Direction Forecast The first step in portfolio selection is to make probabilistic estimates of the future (expected) performances of the assets. The general understanding of “forecast” requires that at any time points, base on the historical information up to this point, we can predict the full probability distribution of the future prices. The performance of the forecast is evaluated by the “square error”. A forecast that minimizes the square error is called “optimal” forecast.

In practice, it’s very hard to satisfy the requirement of the optimal forecast. Many practitioners like to use simpler but still serve certain objectives methods. We here suggest a concept of direction forecast. In the direction forecast, we want to forecast the *direction* of the future prices only. We do not require making forecast at any time point.

One important argument in the direction forecast is that we care profit only. Suppose a trader is assumed to be a profit maximizer and he has a stochastic variable, opinion $O(t)$, at any time t . His “action” of buy or sell at any time t $A(t)$ would be the change of his opinion:

$$A_t = O_t - O_{t-1}.$$

(3. 1) define action

$A(t)$ indicates the trader's action and takes three discrete values: -1(sell), +1(buy) and 0(hold). As time t moving forward, if the trader does not change his opinion then $A(t) = 0$ and he is in a holding position. If he changes his opinion then he takes a buy($A(t)=1$) or sell($A(t)=-1$) action.

When $A(t) \neq 0$, the trader must have a forecast about his expected profits:

$$E[A_t \cdot (P_s - P_t)] > 0, s > t.$$

(3. 2) requirement for direction forecast

The forecast requirement for active strategy (3.2) is different from the standard optimal forecast requirement. For example, suppose there are two analysts A and B. A forecasted that the future return rate for a specific asset was 5%, B's forecast was -1%, and the realized return rate was 2%. By (0.1) both of them got the same forecast loss. A's performance is much better than B's, because A would long the asset and B would short the asset and hence got a loss.

Direction Forecast for Time Series To see the meaning of the forecast requirement (3.2) on the price equation (2.41), we sum up both sides of (2.41) from t to $t+s$. This yields:

$$\sum_{i=0}^s P_{t+i} = \sum_{i=0}^s r_{t+i-1} + \sum_{i=0}^s P_{t+i-1} + \sum_{i=0}^s \varepsilon_{t+i}.$$

(3. 3)

Rearranging the price terms, we get

$$P_{t+s} - P_{t-1} = \sum_{i=0}^s r_{t+i-1} + \sum_{i=0}^s \varepsilon_{t+i}.$$

(3.4)

Therefore

$$E[A_t \cdot (P_{t+s} - P_t) | t] = A_t \cdot \sum_{i=0}^s r_{t+i-1}.$$

(3.5)

So the condition (3.2) simply means

$$\sum_{i=0}^s r_{t+i-1} > 0, \forall A_t = 1; \text{ and } \sum_{i=0}^s r_{t+i-1} < 0, \forall A_t = -1.$$

(3.6)

The left-hand-side of (3.6) is geometrically the area under the drift curve in the interval $[t-1, t+s-1]$. The $r(t)$ s in the above equations are "absolute" return rates. If we use the more familiar definition $R(t) = \log P(t+1) - \log P(t)$, then (3.6) should be

$$E[A_t \cdot \log\left(\frac{P_{t+s}}{P_t}\right) | t] = A_t \cdot \sum_{i=0}^s R_{t+i-1}.$$

(3.6b)

Up to this point, we do not know the detail function form of the expected returns yet. To predict the future direction, we borrow the idea from the classical methods to the short-run production problem. The basic rules are: When the average product is in a declining stage, the marginal product must also be declining. When the average cost is in a rising stage, the marginal cost must also be declining. Generally, when the "longer

term” average of a curve is in rising (declining), the “shorter term” average should also be rising(declining). A striking property in the production theory is that, you need not necessarily to know everything about the production function. You need only some of the characteristics of the average and the first-order (marginal) derivative of the production function. In this method, we only require that the expected returns term is a curve. Obviously, this is a very loose requirement. This also make our procedure cost very low.

III.2. The Double Momentum Method

This section gives the forecasting procedure from the nonstationary time series property.

Using the Long-term “Momentum” as a Filter to Identify the Drift Term

In system (2.41), the price $P(t)$ is observable. The trader's first is to identify the unobservable deterministic function $r(t-1)$, given the time series $P(t)$ up to time t . This is a typical *filter problem*.

Given the observed time series $P(t)$ with a certain finite length and its generating mechanism (2.41)(suppose t is the current time), one can deduce the properties of the drift term by eliminating the noise.

Basically there are two ways to eliminate the white noise in a random process. One way is to use the zero mean property

$$\langle \varepsilon_i \rangle = \lim_{n \rightarrow \infty} \frac{\sum_{i=1}^{i=n} \varepsilon_i}{n+1} = 0.$$

(3.7)

Another way is to use the zero correlation property

$$\langle \varepsilon_i \varepsilon_j \rangle = \text{cov}[\varepsilon_i \varepsilon_j] = 0, i \neq j.$$

(3.8)

To make use of (3.7), we sum up both sides of (2.41) from the current time t to $t-n$ (suppose n is less than the available data length). This yields

$$\sum_{i=t}^{t-n} P_i = \sum_{i=t}^{t-n} r_{i-1} + \sum_{i=t}^{t-n} P_{i-1} + \sum_{i=t}^{t-n} \varepsilon_i.$$

If n is long enough, then (3.7) is held approximately. So the noise term in (2.41) is eliminated. We further move all of the price terms to the left-hand-side to get

$$P_t - P_{t-n-1} \approx \sum_{i=t}^{t-n} r_{i-1}.$$

(3.9)

In a log representation, it is

$$\log P_t - \log P_{t-n-1} \approx \sum_{i=t}^{t-n} R_{i-1}.$$

(3.9b)

Momentum The left-hand-side of (3.9) is exactly what the practitioners called "momentum" (with length of $n+1$). We note the momentum as M_n . Formula (3.9) thus relates the observed price series to the unobservable variable, which is the function of the time-varying drift term. To see its meaning, suppose first that one observes a decreasing momentum M_n , this means

$$M'_n < M^{t-1}_n \Leftrightarrow P_t - P_{t-n-1} < P_{t-1} - P_{t-n-2} \Leftrightarrow \sum_{i=t}^{t-n} r_{i-1} < \sum_{i=t}^{t-n} r_{i-2}.$$

(3.10)

The last inequality is equivalent to

$$r_{t-1} < r_{t-2-n}.$$

(3.11)

Geometrically, the right-hand-side in (3.9) is the area (with sign "+" or "-") under the drift curve in the interval $[t, t-n-1]$. Compare the two areas under the two momentums in (3.10).

As we are moving backward from the time interval $[t-1, t-n-1]$ to the interval $[t-2, t-2-n]$, we lose an area $r(t-1)$ and add an area $r(t-2-n)$. Therefore the last inequality in (3.10) implies (3.11).

By the same reason, if one observes an increasing momentum M_n , he can conclude that

$$M'_n > M^{t-1}_n \Leftrightarrow \sum_{i=t}^{t-n} r_{i-1} > \sum_{i=t}^{t-n} r_{i-2} \Leftrightarrow r_{t-1} > r_{t-2-n}.$$

Forecasting Rule Again, the output of our forecasting is qualitative than quantitative. This means partial information about the drift term is enough. For the purpose of direction forecast, we introduce a new momentum variable from the price data but with a short period k (for example $k=2$ to 5):

$$M_k = P_t - P_{t-k} = \sum_{i=t}^{t-k} r_{i-1} + \eta_t.$$

(3.12) short-term momentum

Where $\eta_t = \sum_{i=t}^{t-k} \varepsilon_i$ is a noise term that can not be eliminated, since k is short.

We have the following theorem:

Theorem 3.1. If $\sum_{i=t}^{t-n} r_{i-1}$ is monotonically decreasing (increasing) for $i \in [t, t-n]$. Then

$\sum_{i=t}^{t-k} r_{i-1}$ where $k \ll n$, must be less than $\sum_{i=t}^{t-n} r_{i-1}$. Here t is the current time. The return rate R

has the same result.

This theorem simply states that if the long-term momentum is monotonically decreasing (increasing), the short-term momentum, if there were no noises, should be below (above) the long-term momentum curve. Since there are always noises in the asset prices, the short-term momentum may be temporarily above (below) the long-term momentum curve. These situations create direction-forecast opportunities. For example, if one observe the long-term momentum is positive and rising but the short-term momentum is minus, then this must be caused by the short-term noise and the price is going to go up.

III.3 Empirical Testing

Testing Rules The testing rules are based on theorem 3.1.

In principle, (3.7) is correct only when n approaches to infinity. In reality n is often taken as 45 or even shorter, 21. The “long-term momentum” still has too much “whipsaws”. To identify the “trend” of the momentums, we use the concept of local extremes (maxima or minima). A local maximum/minimum means the maximum/minimum in a predefined length of period.

If we note the length of the period as m units, then a “rising long-term momentum” can be defined as when the long-term momentum reaches the m time unit local highest. A “declining long-term momentum” is defined as when the long-term momentum reaches the m time unit local lowest. Also, if the current long-term momentum is not in the local highest, it is defined as declining. If the current long-term momentum is not in the local lowest, it is defined as increasing.

We are not going to calculate the short-term momentum directly. Instead, we use the price series directly. It’s easy to see that if the current price is in the m -period local highest, then the momentums whose length from 1 to m must be no less than 0. And if the current price is in the m -period local lowest, then the shorter-term momentums must be no greater than 0.

Therefore we can suggest the following rules:

Sell If the current long-term momentum is not in m -period local new high and is negative, but the current price reaches in the m -period local new high.

Buy If the current long-term momentum is not in m -period local new low and is positive, but the current price reaches in the m -period local new low.

Since in reality it's very hard to trigger buy/sell signals by the above rules, we further release the negative/positive long-term momentum requirement. So we test the following rules:

Sell If the current long-term momentum is not in m-period local new high, but the current price reaches in the m-period local new high.

Buy If the current long-term momentum is not in m-period local new low, but the current price reaches in the m-period local new low.

This means, if there were no temporary random shocks, the short-term momentum is required to fall fast enough to be negative when the long-term momentum is declining (may or may not be negative). When the long-term momentum is rising (may or may not be positive), the short-term momentum is required to jump up enough to be positive. In the real world with random shocks, any violation is caused by the temporary noise and the price will follow the direction of the long-term momentum.

Data and Testing Method We applied the above rules on a portfolio with 545 stocks. The data are from DIAL/DATA (a commercial data vendor). We first randomly picked up a day and selected those stocks with buy/sell signals. Then we compared the portfolio performance before and after the test day.

In our Visual Basic program, the length of the long-term momentum was taken as 21 days. To show the portfolio performance, we took the test day as the base point and constructed two kinds of index. The simpler one, #Value index is constructed by equal weight:

$$\#Value_i = \frac{\sum P_i}{\sum P_0} - 1.$$

(3.13)#Value Index

In terms of percentage change, (3.13) gives more weight to the stock with higher price. If one believe that the penny stocks are easier to be manipulated, then the #Value index is a good choice to avoid this effect.

To give the same weight of percentage change to each stock, we need to use unequal portfolio weight. The #Portfolio% index is defined as:

$$\#Portfolio_i = \frac{\sum \left(\frac{\sum P_0}{P_0} \right) \cdot P_i}{\sum P_0} - 1 .$$

(3.14)#Portfolio% Index

Note that in both of (3.13) and (3.14), all the sums are on each stock.

Testing Results The testing dates cover from 1996 to 1997. During this period the market showed an up trend but in June-July of 1996 and March-April of 1997. Table 3.1 shows the selected portfolios.

Table 3.1

Testing Date	"Buy" Portfolio	"Sell" Portfolio
960425	ADMT,AMBC,BMPI,FFB C, MSIE,UNBJ,WMSI,WSCI	AB,AECO,ATG,BSC,CAN NY, CPY,DUF,EMLX,MACD, MAX,SVGI,WASH

960515	AMBC,BICO,CBCF,CHFC , CMED,ISYS,LEAF,SXI,U K, VHI,WSCI	ADP,CFBX,CHE,CNG,MG XI, VO,ZMX
960606	AMD,ASBC,BMET,CBSH, IMEX,KLIC,MXM,SGI,ST O, TYL,WYMN,X	AMFI,JUNO,MCC
960722	AKLM,BICO,BMPI,DYNT , GLAS,MEDI,MHCO,NOE L, PERS,PMK,PRLX,RHH,SP IR, TII,WSCI	(none)
960830	ADGE,ADMT,BITI,BLSC, CANNY,HPC,JUNO,KM, MSIE,MYR,NATS,RGC,S SW,WSCI	AIR,APGI,COT,IGT,KINN , MMM,OCC,UNBJ,ZMX
961010	AECO,BCP,GGNS,IMNX, MHCO,NETC,OCGT,PER	APD,ASBC,BGAS,CHFC, FSS,

	S, PIOG,VALU,WSCI,ZMX	MDD,PLCC,RXN,WMSI
961128	ADMT,AMBC,BMPI,CCO N, DTM,FFBC,LUTH,MHP,M SIE, SRGN,UNBJ	AMFI,BBOX,C,CADE,CL C, CPAK,CSL,GY,KAMNA, KMAG,SXI,UH,WAX
961210	AECO,AMBC,AXR,CLXX , DYNT,MHCO,MSIE,NAT S, NERX,PDC,PLCC,PRLX,S RGN,WASH	AMFI,CBH,CFBX,DUF,PJ C,SF
970108	ASYS,BFX,CGCO,FONR, HLM,KENT,MHCO,SPIR	ASPX,HCH,ISP,JUNO,PC OL, PTIS,RGC
970218	BITI,COKE,CPAK,ISYS,K T, LIFE,MHCO,MSIE,NSH,R OL, SMCC,SPIR	ABR,CBH,CBSH,CCE,CP Y, FFBC,GPS,GRA,H,KO,PG, RBNC,SGP,TII
970306	CANNY,CERB,GGNS,HD	FFBC,NATS,NSH,PIOG,T

	CO, IFMX,IRF,KENT,KIN,LU TH, MHCO,MSIE,NL,NOEL,P HX, QDEK,RHR,ROSS,RYL,S QNT	OL
970414	ABX,ATLI,AXR,AZA,BLS C, BMPI,BRBK,EQNX,FIGI, HCH, IS,JEC,JUNO,LUTH,MGN, MHCO,MXM,NATS,NOE L, PDG,WSCI	CHLN

Figure 3.1 (see the appendix) shows the testing portfolio performances. In most charts point “66” is the testing date and the index at that point is 0. The only exception is the “970306” portfolio, where the testing date is at point 45.

Conclusion Our forecast rules use the past twenty days closing prices only. The first property in our testing portfolio performances is their divergence. For example, the “9600606 Up” and the “961210 Down” show the worst performances. The “970306 Up”

and the “970306 Down” show very good results. If we assume that the return rates relative to the testing date across various testing portfolios are normal distribution, then we couldn’t prove that, for example, the “buy” portfolios’ return rates after the testing date are significantly greater than zero. This shows the high risk in our forecast rules. But this does not mean we can’t make money. If we can make good money in good situations and lose less money in bad situations, totally we still can make money. Figure 3.2 shows the summary results.

In figure 3.2 we give both the average (mean) performance and the median performance because we are not sure the normal distribution assumption. The results show the evidence of a biased return rates distribution.

Another interesting property is the significant change of the correlation between the testing performance and the SPX market index. For example, table 3.2 shows the correlation changes for the Value portfolio. The Percent Portfolio has the similar results. This shows the significance of the “turning points”.

Table 3.2

Value Portfolio Up

Correlation	Before the Testing Date	After the Testing Date
Median	-0.72	0.85
Average (mean)	-0.65	0.89

Value Portfolio Down

Correlation	Before the Testing Date	After the Testing Date
-------------	-------------------------	------------------------

Median	0.85	-0.78
Average (mean)	0.77	-0.53

IV. VOLUME BEHAVIOR

This chapter discusses the trading volume behavior in our binary opinion model.

IV.1. FACTS

Trading volume plays a minor role in the traditional representative agent paradigm. It became one of the major topics in capital markets only in recent years, under the field of heterogeneous traders model. However, practitioners have watched volume behaviors for many years.

We start by summarizing some basic facts about trading volume.

For practitioners, the "volume analysis" is often a topic of the so called "technical analysis". Readers can obtain a detail list of facts about volume behaviors observed by practitioners in, for example, "Technical Analysis Explained" by Martin J. Pring(1985), chapter 18.

Major relevant academic findings are listed below.

(1) Opinions and volume. In Larry H.P. Lang, Robert H.Litzenberger and Vicente Madrigal(1992), they did an interesting empirical test to try to distinguish between competitive (Walrasian), fully revealing rational expectations and noisy rational expectations equilibria by volume behaviors.

They distinguished these four types of models based on their predictions concerning trading volume around public information signals. Looking at a specific public-information event, period 0 is the period before this event, and period 1 is the period after this event. The four alternative hypotheses are nested in the equation

$$V^2 = \beta_1 \frac{1}{I} \sum_i (\Delta \frac{f_i - \bar{f}}{s^2})^2 + \beta_2 \cdot (\frac{\Delta p}{p})^2 + \beta_3 \cdot (\frac{\Delta \bar{f}}{\bar{f}})^2 + \beta_0$$

(4.1)

The first term is the change in dispersion. S is the end-of-the-year earnings per share. f_i is the i th analyst's forecast of the EPS. The second term is the price change. The third term is the change in average belief. And V is the total daily trading volume.

The value of the coefficient vector $(\beta_1, \beta_2, \beta_3, \beta_0)$ depends on the choice of hypothesis:

(I) under competitive (Walrasian) equilibrium with uncertain supply, $\beta_1 > 0$, $\beta_2 > 0$, $\beta_3 < 0$ and $\beta_0 = 0$;

(II) under competitive (Walrasian) equilibrium with constant supply, $\beta_1 > 0$ and $\beta_2 = \beta_3 = \beta_0 = 0$;

(III) under efficient markets (fully revealing rational expectations), $\beta_1 = \beta_2 = \beta_3 = \beta_0 = 0$; and

(IV) under noisy rational expectations, $\beta_1 > 0$, $\beta_2 > 0$, $\beta_3 < 0$, and $\beta_0 = 0$.

They showed that "(i) empirical results strongly support the noisy rational expectations hypothesis and reject the other hypotheses. This indicates that a significant amount of noise exists (so that private information has value), but not enough to obfuscate entirely the information content of price. This coincides with greater volatility of traders' equilibrium beliefs; (ii) the primary determinants of volume are the change in dispersion term and the price change term, which are orthogonal. This indicates that the dispersion of private information across traders has an impact on trading volume, but not on price. Thus prices can be determined separately using only aggregate and fundamental variables; (iii) the results uniformly consistent with the predictions of the noisy rational expectations model which depends on particular specifications. This indicates that the distribution and preference assumptions often used in

models of securities markets may be good approximate building blocks, at least as far as generating general hypotheses from these models is concerned."

(2) Volume and Returns. In Campbell, Grossman and Wang(1993), they found that "at the aggregate level returns accompany by high trading volume are more likely to be reversed than those accompany by low volume." They used the following regression:

$$r_{t+1} = \alpha + \left(\sum_{i=1}^5 \beta_i D_i + \gamma_1 V_t \right) r_t.$$

(4.2)

Where V_t is a "detrended relative volume",

$$V_t = \log\left(\frac{\text{Num. traded}}{\text{Num. outstanding}}\right) - \text{Mov}\left[\log\left(\frac{\text{Num. traded}}{\text{Num. outstanding}}\right), 1\text{year}\right].$$

The results showed that the coefficient on the product of volume and the stock return is significantly negative (both to one day ahead and two day ahead returns).

(3) Volume and Volatility. In Gallant Rossi and Tauchen(1992), they found a "positive correlation between conditional volatility and volume" by using a seminonparametric method. Similar findings were also obtained by other methods. For example, Lamoureux and Lastrapes(1991) also found a positive conditional volume-volatility relationship in models with Gaussian errors and GARCH-type volatility specifications. Larry H.P. Lang, Robert H.Litzenberger and Vicente Madrigal(1992) also showed the similar relations.

IV.2. Volume in the Binary Opinion Model

In our basic trading model, a trade occurs when a pair of traders with counter opinions change their opinions. Since the volume is defined as the number of contracts that change hands during a specific interval of time, it is simply one of the jump rates of the bullish/bearish groups.

From the market dynamics setup we have:

$$V_t = W(\downarrow), \forall P_t > P_{t-1};$$

$$V_t = W(\uparrow), \forall P_t < P_{t-1}.$$

(4.3) Definition of Volume in the Binary Opinion Model

In a bullish market, $P(t) > P(t-1)$, the demand for the asset is greater than the supply of the asset, hence trading volume is determined by the supply $W(\downarrow)$. In a bearish market, $P(t) < P(t-1)$, the supply of the asset is greater than the demand for the asset, therefore the trading volume is determined by the demand side $W(\uparrow)$.

IV.3. OPINIONS AND VOLUME BEHAVIOR

As mentioned in IV.2, the volume is determined by the jump rates at time t . To understand the relationship between opinions and volume behavior, we need to investigate the comparative dynamics of volume from (2.13) or (2.14).

The Comparative Dynamics of Volume Sum up equation (2.15) and (2.16), we get

$$\frac{\mu_{opinion}}{a} + \frac{2\sigma_{opinion}^2}{a^2} = 2W(\uparrow).$$

(4.4a)

Subtract (2.15) from (2.16) we have

$$\frac{2\sigma_{opinion}^2}{a^2} - \frac{\mu_{opinion}}{a} = 2W(\downarrow).$$

(4.4b)

Equation (4.4a) and (4.4b) and the calculation of volume (4.3)s indicate that, when market is *bullish*, $\mu_{opinion} > 0$, (4.4b) becomes

$$\frac{2\sigma_{opinion}^2}{a^2} - \frac{\mu_{opinion}}{a} = 2W(\downarrow) = 2V_t.$$

(4.5a)

When market is *bearish*, $\mu_{opinion} < 0$, (4.16a) becomes

$$\frac{\mu_{opinion}}{a} + \frac{2\sigma_{opinion}^2}{a^2} = 2W(\uparrow) = 2V_t.$$

(4.5b)

We therefore have the following important comparative dynamics:

Theorem IV.1. *Comparative Dynamics among the Trading Volume V , the Average Belief*

Change μ and the Dispersion of Opinion Change σ .

$$\frac{\partial V}{\partial \mu_{opinion}} < 0,$$

(4.6a)

and

$$\frac{\partial V}{\partial \sigma_{opinion}^2} > 0.$$

(4.6b)

Theorem IV.1 thus explains the fact of opinions and volume described in section IV.1. It says volume expands due to disagreements and volume shrinks due to agreements.

IV.4. Volume and Volatility

From (2.33) we can define the volatility as

$$Volatility = \eta \cdot [W(\uparrow) + W(\downarrow)].$$

(4.7) Definition of Volatility.

This means that the more orders flow in (both of buy and sell), the higher the volatility may be. However, the numbers of the executed order, the volume, are determined by the short side of the order flows. (4.7) thus implies the positive correlation between the trading volume and the volatility.

IV.5. Empirical Implications

Our results are consistent with the empirical evidence described in IV.1, although we use a very different model. An advantage of our model on volume behaviors is its simplicity. The conclusion is straightforward. It does not depend on any detail assumptions of trading incentives or assumptions of information flows.

Our result in section IV.3 shows that the effect of the average opinion change can be distinguished from the effect of the dispersion opinion change by observing the volume behavior.

The volume response to these two effects has a counter direction. Therefore, volume can be used as a filter to increase the quality of forecasts.

In constructing the real forecasting rules, one should note that a “disagreement” in opinion does not necessarily mean a price reversal. A high volume may indicate a price reversal or a price continuation. We need to combine other patterns like volatility to give a correct direction forecast.

Practitioners often use price "patterns" such as head & shoulders, triangle etc. together with volume patterns to forecast price turning points. In the Chartered Market Technician program, the "Methods of Pattern Recognition" includes a key topic—"Importance of Volume Confirmation with the Various Patterns". A common pattern in all of these "price reversal patterns" is a significant high trading volume in the beginning of these patterns. For example in a "head & shoulder" pattern, the left shoulder should be on heaviest volume, and the right shoulder should be on noticeably light volume. In a "double tops and bottoms" pattern, volume should tend to be heavier during the first and lighter on the second. In a "triangle" pattern, volume should diminish as the price swings narrow within the triangle. For all of these patterns, the volume condition must be met first. By theorem IV.1, a significant high volume in the beginning of the patterns means a significant disagreement on the original price trend, therefore a following price reversal is possible. These patterns also show volatility shrinkage before price reversals. Therefore these patterns do not use knowledge of past price series only (as often attacked by the “random walk” scholars).

V. INFORMATION EXTERNALITIES AND OPINIONS

PATTERN

One observes the price level distribution during a time period will find that it often shows a multi bell-shaped curve. The bulge at the peak of the curve indicates the price level where most trading takes place. The valley of the curve shows the area where trading is thin. In this chapter we try to investigate such a phenomenon by modeling the collective opinions pattern.

V.1. Searching for Private Information

Since information disperses among a large number of traders, the searching for private information must affect the price behavior. To investigate this problem, we first study the effect of private information searching on the formation of opinions.

Private Information In the standard model setup(see chapter II.1), the private information can be modeled by a deterministic component θ so that the return rate of the risky asset $u=\theta+\varepsilon$ where ε is the white noise term(Grossman and Stiglitz, 1980). It is generally accepted that private information holds important role because of the imperfect aggregation function of the prices. However, it is less studied how people search for private information. The most obvious fact we know is that there exist active markets for information in additional to the price information, such as advisory services and newsletters, and various "sentiment" indicators investigations.

Searching for Other People's Opinions We classify the ways people access private information into two types by its form: the first type is "by value". In this searching method, people try to access the real value of the economic variables about the asset (for example the θ in the above section). Another way to search private information is "by reference". That means we try to get the opinions from those people who may be able to access the private information set by value by observing their actions. Since we do not know the exact value of the economic variables, this method is less exact but is cheaper.

V.2 Information Externalities

When every traders try to know other people's opinions in order to decide their operations, there are strong interactions among traders. In this section we try to model this kind of information externality.

Decision Model As mentioned in chapter II.2 "Taste", we suppose that each individual i 's decision process follows a stochastic rule:

$$O_i = +1 \text{ with probability } \text{Prob}(h_i); \quad O_i = -1 \text{ with probability } 1 - \text{Prob}(h_i).$$

We can further specify $\text{Prob}(h_i)$ as *the sigmoid-shaped function*:

$$\text{Prob}(h_i) = \text{Prob}(h_i; \beta) \equiv \frac{1}{1 + \exp(-2\beta h_i)},$$

(5.1) Sigmoid-Shaped Function

where $\beta \in (0, \infty), h_i \in (-\infty, \infty)$.

A new parameter " β " is introduced in formula (5.1). β controls the steepness of the sigmoid near $h=0$ (that is, the degree of "easy to turn over"). At small β , $\text{Prob}(h;\beta)$ goes very smoothly from 0 to 1 as h goes from $-\infty$ to $+\infty$. In the limit $\beta=0$, the probability becomes 0.5, every decision about the next value of an O_i is completely random. A large β makes a sudden rise from near 0 to near 1 over a narrow range of h of width proportional to $1/\beta$. In the limit β approaches to infinity, $\text{Prob}(h;\beta)$ just reduces to a deterministic steep sign function.

β is in fact a measure of the market liquidity. The value of β is purely a collective phenomenon. Each person is assumed to face the same β . A low value of β reflects the "hot" market liquidities, and the traders are more crazy for buy/sell turnovers. A high β , on the other hand, simply means the "cool" market liquidities, traders are more "rational", more fundamental oriented in taking actions.

Externality Assume that each individual i form his bullish/bearish opinion O_i based on his connections with all other people j . The strength of the connection between i and j is denoted as W_{ij} . Bear in mind that j means "other". The trader i needs to determine the sign of the following inputs:

$$h_i = \sum_j W_{ij} \cdot O_j - T_i.$$

(5.2) Inputs for Trader i 's Decision

Equation (5.2) holds for every traders in the market, therefore it in fact described a group of *simultaneous equations*. The simultaneous equations reflect the externalities in decision. There are very strong interactions among traders.

There are two kinds of *linear rule* in (5.2). The first is the linear sum of the weighted summation term and the threshold term. The decision "threshold" T_i is determined by external economic environments. It is "fixed", independent of the dynamics of the system. Formula (5.2) shows that an individual i makes his bullish/bearish opinion based on the linear summation of all other people's opinions and his own judgment(which is a function of the economic variables). The second linear rule is the first term in (5.2) of the weighted sum of other people's opinions. The first term in the right hand side is induced by the internal strategic behavior of the market participants. It introduces a new phenomenon to the standard model.

Learning Rule How the decision weight is form is from the *learning rule*.

However, a general rule can be supposed to be

Assumption 5.1. Linear Rule of Weight.

$$W_{ij} \propto O_i \cdot O_j.$$

(5. 3) Linear Learning Rule

By its nature, W_{ij} is not symmetric. That's $W_{ij} \neq W_{ji}$. For example, suppose trader i forms his opinion by observing the advice of two of other advisors "a" and "b". By experience, i may find that A's advice often hit the markets with high likelihood, while B's opinion often goes contrarily to the markets. Trader i would assign a "+1" to the weight W_{ia} while assign a "-1" to the weight W_{ib} . The words " i is connected with all

other traders" do not mean i must find all of the weights with other people, his connections with other people who do not affect hi decision are simply set to zero.

V.3 Formation of Opinions

Equation (5.2) are in fact N (N is the total numbers of the participants in the market) nonlinear equations in N unknown stochastic variables O_j . To solve the problem, only an approximation is practicable.

Mean Field Approximation A way of approximation is the *mean field approximation*. This method approximates all other people's opinions into one "mean field" opinion.

To do this, we first replace the true fluctuating h_i by its average value

$$\langle h_i \rangle = \left(\sum_j W_{ij} \cdot \langle O_j \rangle \right).$$

(5.4)

Here and after we assume $T_j=0$ (the generalization is obvious). The eliminating of the threshold is to allow a more concise treatment of the most important features of our model.

On the other hand, we can calculate the average opinion for a typical person $\langle O_i \rangle$ as

$$\langle O_i \rangle \equiv \text{Prob}(+1) \cdot (+1) + \text{Prob}(-1) \cdot (-1).$$

(5.5)

From (5.1), we have $Prob(+1) = \frac{1}{1 + \exp(-2\beta h_i)}$, and

$Prob(-1) = \frac{1}{1 + \exp(2\beta h_i)}$. Substitute these functions into (5.5) we see that the

representative opinion $\langle O_i \rangle$ is a *hyperbolic tangent function*

$$\langle O_i \rangle = \frac{e^{\beta h_i} - e^{-\beta h_i}}{e^{\beta h_i} + e^{-\beta h_i}} \equiv \tanh(\beta h_i).$$

(5.6) Hyperbolic Tangent Function

To summarize, we have the following "mean field" theorem:

Theorem 5.1. Mean Field Theorem. The representative opinion $\langle O_i \rangle$ satisfies

$$\langle O_i \rangle = \tanh(\beta \cdot \langle h_i \rangle) = \tanh(\beta \cdot (\sum_j W_{ij} \cdot \langle O_j \rangle)).$$

(5.7) Mean Field Equation for $\langle O_i \rangle$

Opinions Congestion and Stability Collapse (5.7) are still N nonlinear equations in N unknowns, but now they no longer involve stochastic variables.

The next approximation is to eliminate the i, j index by taking further average. Let the normalization factor be 1/N, then linear rule (5.3) becomes

$$W_{ij} = \frac{O_i \cdot O_j}{N}.$$

(5. 8) Normalized Learning Rule

Substituting (5.8) into the mean field equation (5.7) yields

$$\langle O_i \rangle = \tanh\left(\frac{\beta}{N} \cdot \sum_j O_i \cdot O_j \cdot \langle O_j \rangle\right).$$

(5.9)

Summation is over j. To eliminate the summation, We make the following assumption:

Assumption 5.2. $\langle O_j \rangle$ is proportional to O_j :

$$\langle O_j \rangle = m \cdot O_j.$$

(5.10)

Then (5.9) becomes

$$m \cdot O_i = \tanh\left(\frac{\beta}{N} \cdot \sum_j O_i \cdot O_j \cdot m \cdot O_j\right) = \tanh(\beta \cdot m \cdot O_i),$$

(5.11)

where we make use the fact that O_j takes either value of +1 or -1. But because $\langle O_i \rangle$,

$\langle O_j \rangle$ have been replaced by one mean field opinion $\langle O \rangle$, we can eliminate the subindex.

Therefore we have

$$\langle O \rangle = \tanh(\beta \cdot \langle O \rangle).$$

(5.12) Equation for $\langle O \rangle$

Equation (5.12) can be solved by drawing a figure. Solutions are intersections between the straight line $\langle O \rangle = x/\beta$ and the sigmoid curve $\langle O \rangle = \tanh(x)$. When $\beta \leq 1$, there is only

one solution at $\langle O \rangle = 0$, whereas for $\beta > 1$ there are three solutions. One at zero, the other two are symmetric around the 0 point.

Near the 0 point, one can calculate the solutions algebraically. Note $\tanh(x)$ is an odd function around 0, that is, $\tanh(-x) = -\tanh(x)$, and

$$\tanh(x) \approx x - \frac{x^3}{3} + o[x^5],$$

(5.13)

when x is small. So, near 0, (5.12) can be expressed as

$$\langle O \rangle = \beta \cdot \langle O \rangle - \frac{\beta^3 \cdot \langle O \rangle^3}{3}.$$

(5.14)

The two solutions are

$$\langle O \rangle_1 = \sqrt{\frac{3(\beta - 1)}{\beta^3}}, \text{ and } \langle O \rangle_2 = -\sqrt{\frac{3(\beta - 1)}{\beta^3}}.$$

(5.15)

These two solutions are in stable states. They are areas that the representative opinion is biased to positive or negative. Or we can say that in these two states, the majority of opinions “congested” in bullish or bearish. As β goes through the 0 point, the system is now *broken* by the actually realized solution. The change of $\langle O \rangle$ around the 0 point is sharp and the system is experiencing a stability collapse process around the 0 point.

Robustness of the Collective Phenomenon The collective phenomenon is resulted from the connections among traders' decision process. It does not require the detail knowledge of interactions among traders. The only requirement is the existence of such connections. Therefore the result is robustness.

When the connections become zero, we go back to the standard representative agent model with endowments N . This "economic person" reacts to economic variables changes. However, the double stable states and the stability broken is a pure collective behavior.

VI APPROXIMATE THE PRICE MOVEMENTS BY POLYNOMIAL

The results in chapter V showed that, because of the externalities in the market, the mass opinions can show the collective behavior like polarization and stability broken. Chapter IV showed that the asset price movements are internally related to the opinion patterns.

VI.1 Price Equation with Polynomial Drift Term

Price Equation We suggest the following price movement equation:

$$dP = L^m(P)dt + \sigma(P)dB,$$

(6.1)

where dB is the standard Brownian motion. The corresponding backward Kolmogorov equation is

$$\frac{\partial H(P)}{\partial t} = \frac{\partial}{\partial P}[L^m(P) \cdot H(P)] + \frac{1}{2} \frac{\partial^2}{\partial P^2}[\sigma^2(P) \cdot H(P)],$$

(6.2)

where $H(P)$ is the probability density function and $L(P)$ is a m -degree polynomial,

$$L^m(P) = a_0 + a_1P + a_2P^2 + \dots + a_mP^m = \sum_{i=0}^m a_i P^i.$$

(6.3) m -Degree Polynomial

Stationary Price Distribution Suppose that the price adjustment speed towards the stationary distribution is faster than the time between shocks, the transitional densities would converge to the stationary density.

The absolute price distribution, although without information of time, still contains important information. The most important messages we are interested here are the positions of the modes and the antimodes. In the language of dynamics, the modes are stable region and the antimodes are unstable positions. To express in the probability words, the modes represent the normal state that the prices should return in higher probabilities, and the antimodes correspond the abnormal state that the prices would usually jump off in higher probability. Knowing these regions is of course invaluable to trading.

Theorem 6.1. Stationary Price Distribution. The stationary distribution of H^* is given by

$$H^*(P) = N \exp\left[-\int_0^P \frac{2L^m(\pi) + \frac{d\sigma^2(\pi)}{d\pi}}{\sigma^2(\pi)} d\pi\right].$$

(6.4) Stationary Price Distribution

Where $N = \frac{1}{\int_0^\infty (\exp[-\int_0^P \frac{2L^m(\pi) + d\sigma^2/d\pi}{\sigma^2(\pi)} d\pi]) dP}$, is the normalization constant. This is a

general exponential distribution (McDonald, 1984).

VI.2 The Stationary Probability Elasticity Function

The Elasticity Function Take a log to both sides of formula (6.4) and then take derivative, we have

$$\eta \equiv \frac{H''(P)}{H'(P)} = -\frac{2L^m(P) + \frac{d\sigma^2(P)}{dP}}{\sigma^2(P)}.$$

(6.5) Stationary Probability Elasticity Function

This function is called the stationary probability elasticity function with respect to price. It relates the stationary price distribution, the drift term and the diffusion term together. Since most of the important properties are related to the elasticity function only, it is enough to study this function η (rather than study the more complex function (6.4)).

For simplicity, we further assume that the volatility is a constant that is independent of the price level. Then

$$\frac{d\sigma^2(P)}{dP} = 0.$$

(6.6)

The stationary probability elasticity function η (6.5) is now simplified as

$$\eta \equiv \frac{H''}{H'} = -\frac{2L^m(P)}{\sigma^2} = c_0 + c_1 P + \dots + c_m P^m,$$

(6.7)

$$\text{where } c_i = -\frac{2a_i}{\sigma^2}.$$

The Number of Modes and Antimodes of the Stationary Density H^* Under assumption (6.6), the number of maxima and minima of H^* is given by the number of the roots of

$$L''(P) = 0.$$

(6.8)

The maxima or minima are determined by the second-degree conditions of $H^*(P)$.

Generalized Exponential Distribution and the Normal Distribution There is a simple relationship between the generalized exponential distribution (6.4) and the normal distribution. To see this, suppose P_0 is a mode of (6.4), then at this root, $[\log(H^*(P_0))]'' = 0$. Expand the exponential term of (6.4) in Taylor series up to the second approximation, we have

$$\log(H^*(P)) \approx \log H^*(P_0) + \frac{1}{2} \cdot (\log H^*(P_0))'' \cdot (P - P_0)^2.$$

(6.9)

Therefore,

$$H^*(P) = H^*(P_0) e^{\frac{(P-P_0)^2}{2 \left| \frac{1}{(\log H^*(P_0))''} \right|}}.$$

(6.10)

This says that if the variance is small enough, each mode of (6.4) closely resembles the normal distribution with

P_0 as mean and $\frac{1}{|[\log H^*(P_0)]''|}$ as variance.

Note that for the case of mode (maximum), $[\log H^*(P_0)]'' < 0$. And for an antimode (minimum), $[\log H^*(P_0)]'' > 0$.

This conclusion is very helpful in the determination of the range of modes. We know that approximately 68% of the area under the normal curve $N(0, \sigma^2)$ lies between the value of $-\sigma$ and the value of $+\sigma$. So for a stable mode at P_0 , given that the σ is small, there is about 68% probability that the price would fluctuate between $P_0 - \sigma(P_0)$ and $P_0 + \sigma(P_0)$. For an unstable antimode at P_0 , there is about 32% probability that the price would stay in the region between $P_0 - \sigma$ and $P_0 + \sigma$ variance. Where the σ is determined by

$$\sqrt{\frac{1}{|[\text{Log}(H^*(P_0))]''|}}$$

(6.11)

From our definition of the stationary probability elasticity η in (6.5), (6.11) can be written as

$$\sqrt{\frac{1}{\eta}}$$

(6.12)

This relates the η function to the forecast variable.

VI.3 Empirical Estimation of the Stationary Probability Elasticity Function η

We need to estimate the $m+1$ parameters in formula (6.7).

Moment Estimation(Loren Cobb, 1978) We are looking for a particular $m+1$ element parameter vector $c = [c_0, c_1, \dots, c_m]$ which minimizes the residual $S(c)$, a quadratic criterion

$$S(c) = E[(\eta(P) - \sum_{i=1}^{m+1} c_{i-1} \cdot P^{i-1})^2].$$

(6.13) quadratic criterion

$\eta(P)$ in (6.13) is the “true” elasticity.

The c^* which minimizes $S(c)$ can be solved by letting $\frac{\partial S}{\partial c} = 0$. Note that $S(c)$ is a scalar and

$$\frac{\partial S}{\partial c_i} = E[2 \cdot (\eta(P) - \sum_{j=1}^{m+1} c_{j-1} \cdot P^{j-1}) \cdot (-P^{i-1})]. \text{ Therefore we have}$$

$$E[P^{i-1} \cdot \eta(P)] = \sum c_{j-1}^* \cdot E[P^{i+j-2}].$$

(6.14) The First Order Condition

The true elasticity term $\eta(P)$ in (6.14) is unobservable and must be substituted by the other empirical terms. We can prove the following formula:

$$E[P^{i-1} \cdot \eta(P)] = (i-1) \cdot E[P^{i-2}], \text{ for } i \text{ from } 1 \text{ to } m+1.$$

(6.15)

In a matrix form, (6.14) can now be written as

$$\alpha^* = \Gamma^* \cdot c^*,$$

(6.16)

where α^* is a $((m+1) \times 1)$ vector as defined in (6.15) and Γ^* is a $((m+1) \times (m+1))$ matrix as defined in (6.14).

We can further show (Cobb, Koppstein and Chen, 1983) that the c^* estimated from (6.16) is consistent and asymptotically normal.

Theorem 6.2. The estimator $c^* = \Gamma^{*-1} \cdot \alpha^*$ is consistent, and $\sqrt{T} \cdot (c^* - c)$ is asymptotically multivariate normal with covariance matrix $[\Omega] = \Gamma^{-1} \cdot \Sigma \cdot \Gamma^{-1}$, where $\Sigma = \text{cov}[\alpha^* - \Gamma^* \cdot c]$.

An Empirical Example As an example to show the above calculations, we take the 30 year T-bond future price as our dataset. The data come from DIAL/DATA with 589 records.

These closing prices are from a “continuous” contract, which combines two most active future contract together when one contract is going to expire. Figure 6.1 shows the frequency chart.

Suppose we want to use an polynomial with $m=3$ to approximate the price distribution. We first compute the moments up to 6 and then construct the 4×4 matrix Γ

$$\Gamma = \begin{bmatrix} E(P^0) & E(P^1) & E(P^2) & E(P^3) \\ E(P^1) & E(P^2) & E(P^3) & E(P^4) \\ E(P^2) & E(P^3) & E(P^4) & E(P^5) \\ E(P^3) & E(P^4) & E(P^5) & E(P^6) \end{bmatrix}$$

and the 4×1 vector α

$$\alpha = \begin{bmatrix} 0 \\ 1 \\ 2 \cdot E(P^1) \\ 3 \cdot E(P^2) \end{bmatrix}.$$

By equation (6.16) we solve the vector c as

$$c = \begin{bmatrix} -1012.35635 \\ 27.95240 \\ -0.25682 \\ 0.00079 \end{bmatrix}.$$

Figure 6.2 shows the stationary probability elasticity function. From the calculation, we got the properties of the three important points as shown in table 6.1.

Table 6.1

Price	Extreme	η'	σ

101.25	Max.	0.0939	3.26
109.25	Min.	-0.0486	4.54
116.75	Max.	0.0916	3.30

Appendix 1: Prove of Theorem 6.1

From (6.2), the stationary density $H^*(P)$ can be found by setting

$$\frac{\partial H^*(P)}{\partial t} = 0.$$

(A.1)

Then (6.2) becomes an ordinary differential equation about H^* :

$$0 = \frac{d(L^m \cdot H^*(P))}{dP} + \frac{1}{2} \frac{d^2(\sigma^2 \cdot H^*)}{dP^2}.$$

(A.2)

Expanding the second term using the differential product rule yields

$$0 = \frac{d}{dP}(L^m(P) \cdot H^*) + \frac{1}{2} \frac{d}{dP}(H^* \cdot d\sigma^2 / dP + \sigma^2 \cdot dH^* / dP).$$

(A.3)

Integrating both sides with respect to P gives a first-order differential equation

$$0 = L^m(P) \cdot H^* - \frac{1}{2} [H^* \cdot d\sigma^2 / dP + \sigma^2 \cdot dH^* / dP].$$

(A. 4)

Rewrite (A.4) for dH^*/dP gives

$$\frac{1}{H^*} \frac{dH^*}{dP} = -\frac{2 L^m(P) + d\sigma^2 / dP}{\sigma^2}.$$

(A. 5)

Finally, the stationary density is given as the *solution* of (A.5)

$$H^*(P) = N \exp\left[-\int_0^P \frac{2 L^m(\pi) + \frac{d\sigma^2(\pi)}{d\pi}}{\sigma^2(\pi)} d\pi\right].$$

(A. 6)

This is formula (6.4).

Appendix 2: Prove of formula (6.15)

By definition

$$E[P^{i-1} \cdot \eta(P)] = \int_{-\infty}^{+\infty} P^{i-1} \cdot \eta(P) \cdot H^*(P) dP.$$

(A. 7)

Using (6.5), the right hand side of (A.7) becomes

$$\int_{-\infty}^{+\infty} P^{i-1} \cdot [-H^{**}(P)] dP$$

(A. 8)

Integrate expression (A.8) by parts:

$$-\int_{-\infty}^{+\infty} P^{i-1} \cdot H''(P) dP = [-P^{i-1} \cdot H'(P)]_{-\infty}^{+\infty} + \int_{-\infty}^{+\infty} [(P^{i-1})'] \cdot H'(P) dP.$$

(A. 9)

The first term in the right hand side of (A.9) is zero since the possibility of P vanishes when the price is in the positive and negative extreme levels. The second term in the right hand side of (A.9) is in fact

$$\int_{-\infty}^{+\infty} (P^{i-1})' \cdot H'(P) dP = E[(P^{i-1})'] = (i-1) \cdot E[P^{i-2}].$$

(A. 10)

This is exactly the formula (6.15).

VII SUMMARY AND FURTHER RESEARCH

The major points in this study are: the trading model, the discrete event approach in forecasting, the general procedure in identifying the turning points of a nonlinear process, the volume and volatility behaviors, the distributions of certain trading rules, the collective behaviors in the markets. The following text further discusses some points in depth.

The Trading Model In this study we start from the very beginning, the underlying supply and demand situation that drive the asset prices.

In doing so, we found it is necessary to use a different equilibrium concept other than the widely used Walrasian equilibrium concept. The Walrasian equilibrium originated from the thermodynamics in Physics is good for the slow change system but not appropriate for the fast evolution system. The asset trading market is exactly the latter case, where fierce competition causes rapid changes. We are handling the disequilibrium dynamics.

Our trading model based on the assumption that traders actions follow the Markov process. We then obtain the general stochastic equations for the opinions evolutions. Under some more assumptions, we get the general nonlinear equations for the asset prices movements.

The trading model presented here supplies a framework for further analysis. It can be extended to the more complicated situations like multi-periods and multi-assets.

Discrete Event Approach We adjust our concept of forecast. The investment process can be viewed as a kind of signal detection problem. However, the engineering

tools, such as the Kalman filters, ARIMA models have little power in trading analysis because of the extremely high noise level. Our solution is by observing that you don't need to make a quantitative estimation at each time point. Therefore we change our learning rule in modeling and forecasting the dynamic asset prices process.

The basic idea of our discrete event model can be described as follow.

Generally the value of an asset P can be expressed as a function of n "factors"

$$P = F(f_1, f_2, \dots, f_n).$$

(7.1) factors model

In order to forecast the future return rates, we use the total differential formula to expand

(7.1) as

$$\Delta P = \frac{\partial F}{\partial f_1} \Delta f_1 + \frac{\partial F}{\partial f_2} \Delta f_2 + \dots + \frac{\partial F}{\partial f_n} \Delta f_n,$$

(7.2) return factor model

where the coefficient before the factor return is the asset price sensitivity to the i th factor β_i . (7.2) is very similar to the APT formula. It decomposes the asset price return into the linear combination of the factor returns. By forecasting the factor returns, we obtain the predicted future asset return. This is what we need for an active strategy.

Here is our "direction forecast"(see III.1) concept. In order to make money, requirement (3.2) together with (7.2) gives

$$E[A_t \cdot (\frac{\partial F}{\partial f_1} \Delta f_1 + \frac{\partial F}{\partial f_2} \Delta f_2 + \dots + \frac{\partial F}{\partial f_n} \Delta f_n)] > 0.$$

(7.3) requirement for return direction forecast

This equation says that for an active strategy to be successful in the future interval, we only need to forecast the probabilities that the future price moves up or down.

We can further simplify our task by noting that we only need to know the sign of the factors return in order to satisfy (7.3). Define a three integers discrete variable which may take value of -1, 0 or +1. Let's call this an "event" when the variable is -1 or +1. When the value of the event is not zero, we say the event is *triggered*.

For example, a money manager may observe the ratio of the estimated earning growth to the P/E ratio and find that he can buy a specific stock when the ratio of this stock moves into the top 10 ranking. To study this strategy, he can run the following regression

$$\text{equation } R_{t+1} = \alpha + \beta \cdot \frac{\text{Estimated_Earning_Growth}_t}{(P/E)_t}.$$

If the money manager just takes care of the direction of the value, then this is a standard binary choice model with the dependent variable as a dichotomy one. To model this expert rule better, we can use a logic variable to indicate the event that "the ratio of the stock moves to the top 10 ranking". By doing so, we have a purely qualitative decision problem.

Up to this moment, we determined the final decision of A_t by the sensitivities (the *betas*) of the asset return to the factor return and the "event" if the factor is triggered. However, this idea is not robust and is difficult to implement in practice. The basic reason is that the qualitative factor generates +1 or -1 *sporadically*. Suppose factors are independent, then in a regression equation, the variance of the estimated beta would be

$$\text{Var}(\hat{\beta}_i) = \frac{\text{Var}(\text{residuals})}{\text{Var}(f_i)}.$$

Since in most of the time f_i is zero, $\text{Var}(f_i)$ is approximately

proportional to the numbers that f_i gives +1 or -1. But since f_i is sporadic, the estimated variance is naturally large, hence the t-test is often insignificant. This means the estimated $\hat{\beta}_i$ contains a lot of noise.

To overcome this problem, we transfer the betas into qualitative variables by the following rule:

$$\frac{\partial F}{\partial f_i} \rightarrow \text{sign}(\Delta f_i) \cdot \text{sign}(\Delta P).$$

(7.4)

(7.4) means that, if we find that the price went up(down) in responding to a factor which gives a “+1” signal, then we assign a +1(-1) to the corresponding beta coefficient value.

As an example, if you follow the “contra opinion” strategy, then you may take the “percentage bullish poll” as the factor for your forecast. Suppose you decide that the event “+1” in your model is triggered when more than 70% people are in bullish opinion, then you need to assign a -1 to the coefficient before your factor variable.

Our event modeling method also applies to hedging directly. “Hedging” is a technique to offset particular sources of risk. Since it requires to identify the sources of risks, it is often considered as an active strategy.

There is a qualitative problem in hedging. Consider the factor model (7.1), the linear expansion as (7.2) is not enough, one must also consider the risk sources from the nonlinear terms. Use Taylor’s expansion we have

$$\Delta P = \sum_{i=1}^n \left[\frac{\partial F}{\partial f_i} \Delta f_i + \frac{1}{2} \frac{\partial^2 F}{\partial f_i^2} (\Delta f_i)^2 + \text{crossterm} \right].$$

(7.5) return factors model for hedging

Often the n factors can be chosen so that they are independent of each other. Under such conditions the cross term in (7.5) is eliminated. To see the meaning of (7.5), we can take some examples from complex asset like derivatives. For stock options, if f_i is the underlying price, then the coefficient of the linear term in (7.5) is called “delta”, and the

coefficient of the second term is called “gamma”. For bond, f_i is the market interest rates and the coefficient of the linear term in (7.5) is called “modified duration”, and the coefficient of the second term is called “modified convexity”.

There is a cost-benefit problem in hedging: to hedge higher degree sources of risk requires higher cost. This depends on the significance of the future factor change Δf_i . A sufficiently small predicted future factor returns Δf_i results in a neglected higher degree terms of risks. Therefore we can use a vector H_1 to show the event that the higher degree of risks should be hedged :

$$H_1 = [1 \quad 1]$$

(7.6a)

And the event H_2 that the higher degree of risks are trivial:

$$H_2 = [1 \quad 0]$$

(7.6b)

Whether we should use H_1 or H_2 is a typical qualitative decision problem. Under the above notations, (7.5) is rewritten as

$$\Delta P = \sum_{i=1}^n H \cdot \left[\begin{array}{c} \frac{\partial F}{\partial f_i} \Delta f_i \\ \frac{1}{2} \frac{\partial^2 F}{\partial f_i^2} (\Delta f_i)^2 \end{array} \right],$$

(7.7)

where H equals H_1 or H_2 depends on how the events occur and we have eliminated the cross term. In practice the first degree risks are often hedged. The problem is the higher degree risks. An event of zero value means don't care about the higher degree risks. An event with value of +1 means we do care about the higher degree risks.

Such a discrete event method is extremely easy to program in modern computer. This is undoubtedly a big advantage.

Probability Distributions of Certain Trading Rules To simply use standard statistics like chi-squared, t-statistics, R-squared on the trading rule performance are dangerous. Each statistic carries with it a host of assumptions regarding underlying distribution that need to be satisfied for those statistics to be properly interpreted. In particular, they depend on the assumptions about the normal distribution.

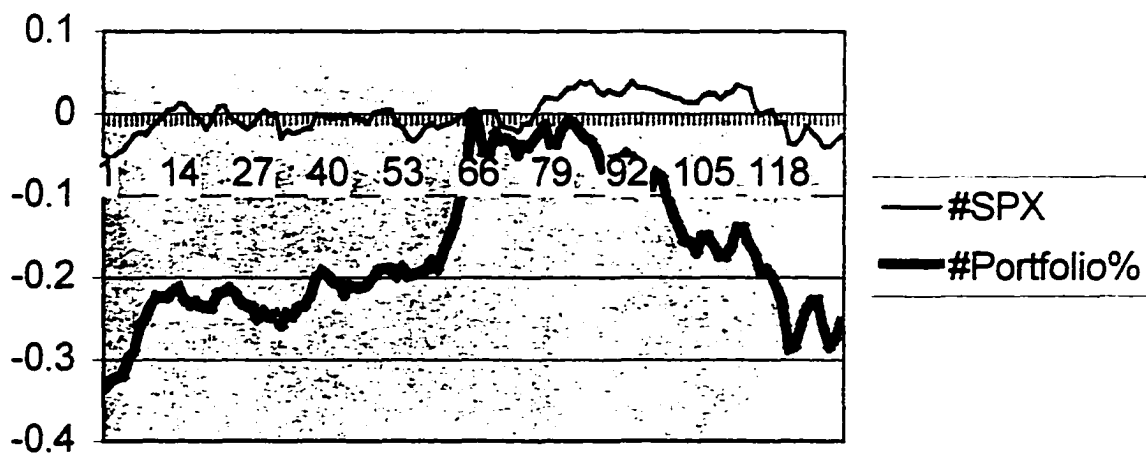
There are not many researches on the distributions of the portfolio returns under certain trading rules. Chapter III in this study did such an experiment. We examined the rationality of the so-called “non-confirmation” rule that is used by practitioners and gave preliminary results. The empirical research in this field will be beneficial.

The Collective Behavior and the Break-Out Rule The trading markets sometimes show sudden break patterns. They may be a break-up or a collapse. We investigate the dynamics by applying the “phase transformation” method and suggest an empirical way to measure it. Further research in this direction is required.

Figure 3.1 Testing Portfolio Performances

1(a)

960425 Down Signal



1(b)

960425 Down Signal

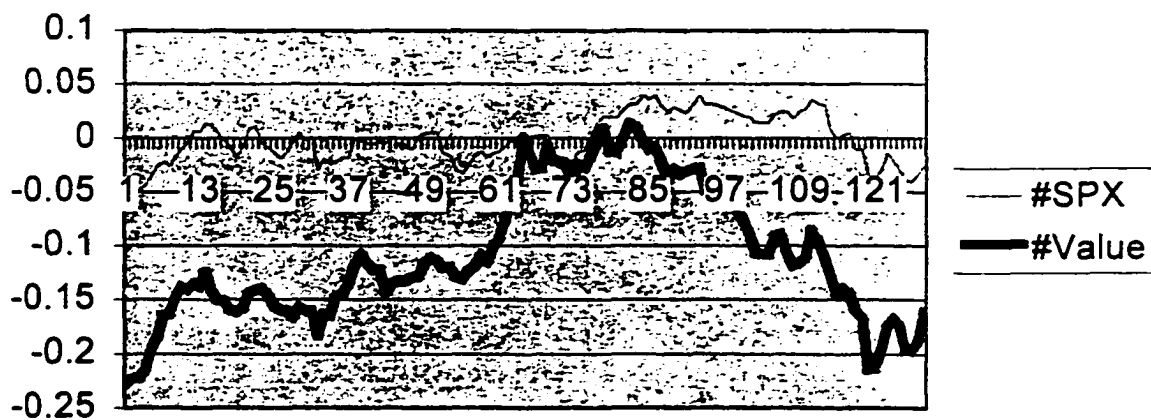
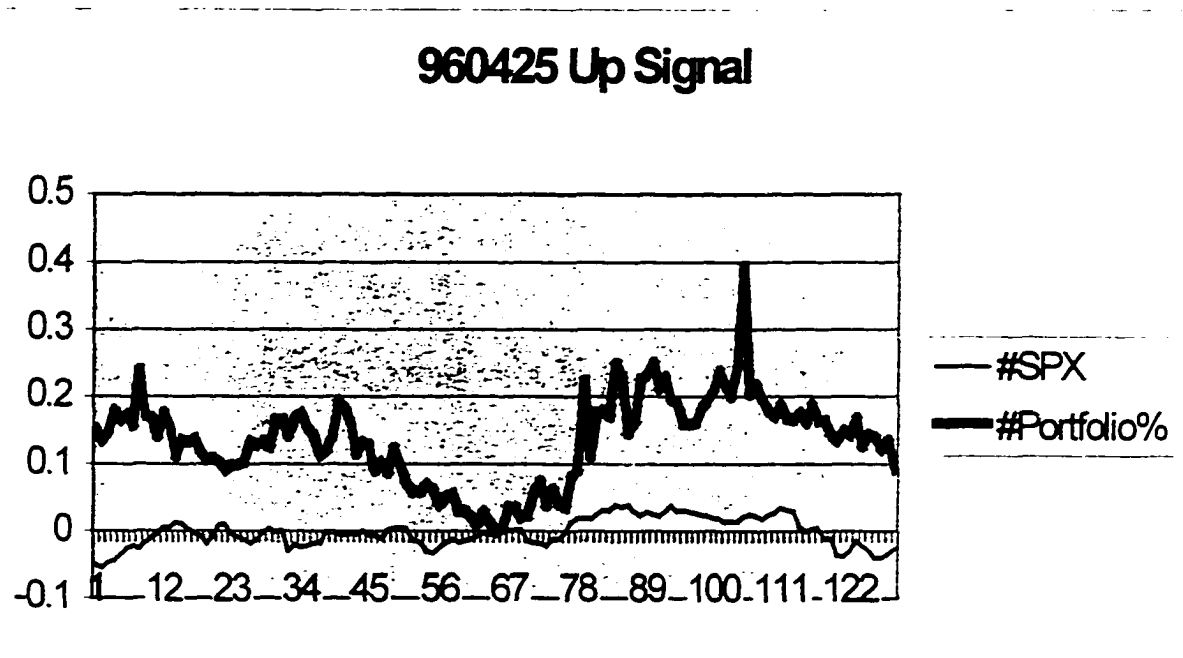
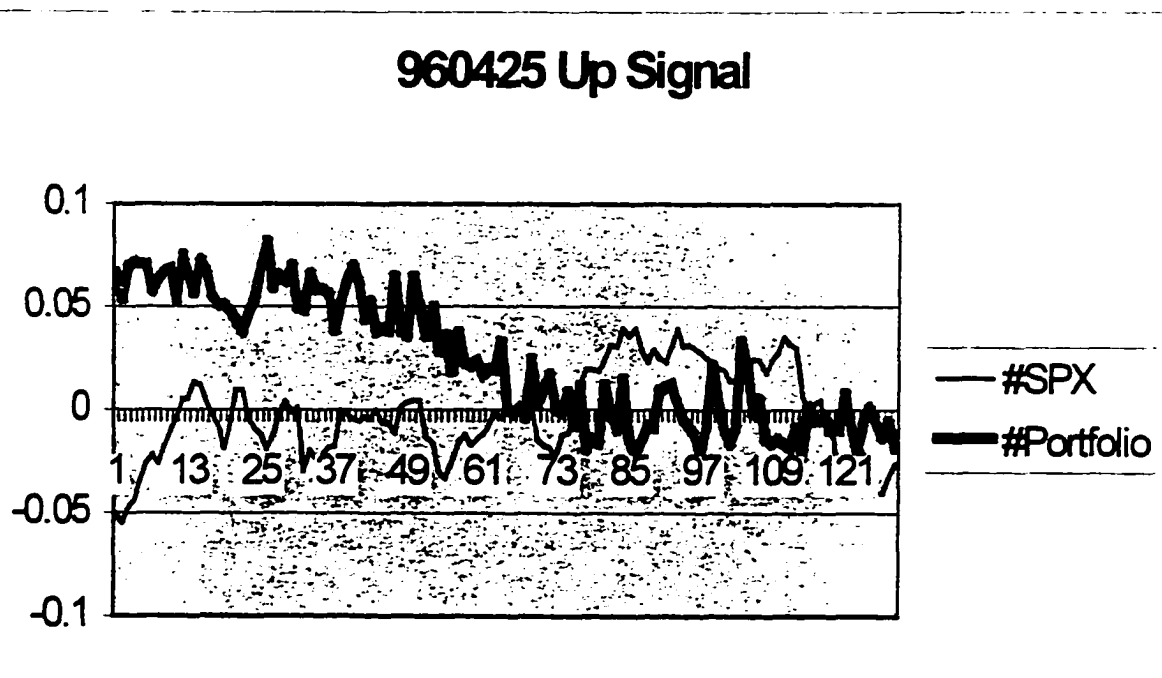


Figure 3.1 Testing Portfolio Performances

2(a)

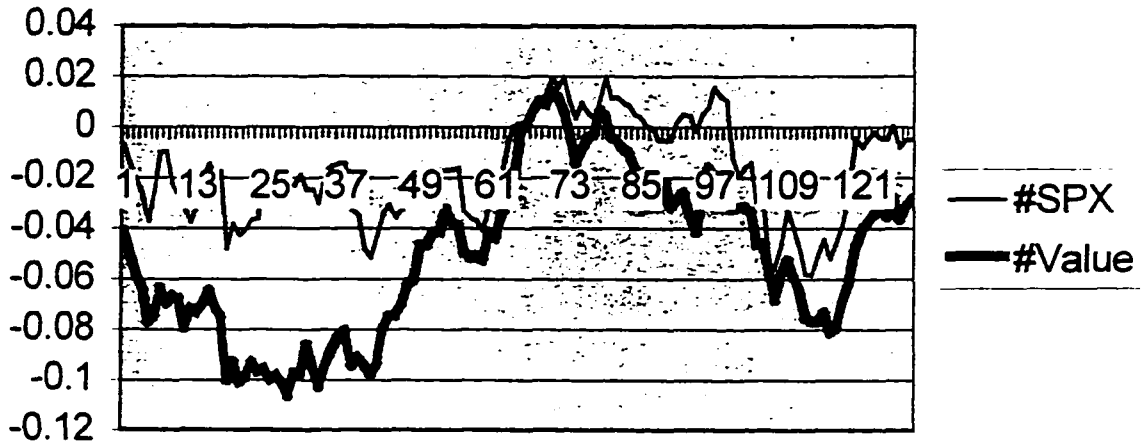


2(b)



3(a)

960515 Down Signal



3(b)

960515 Up Signal

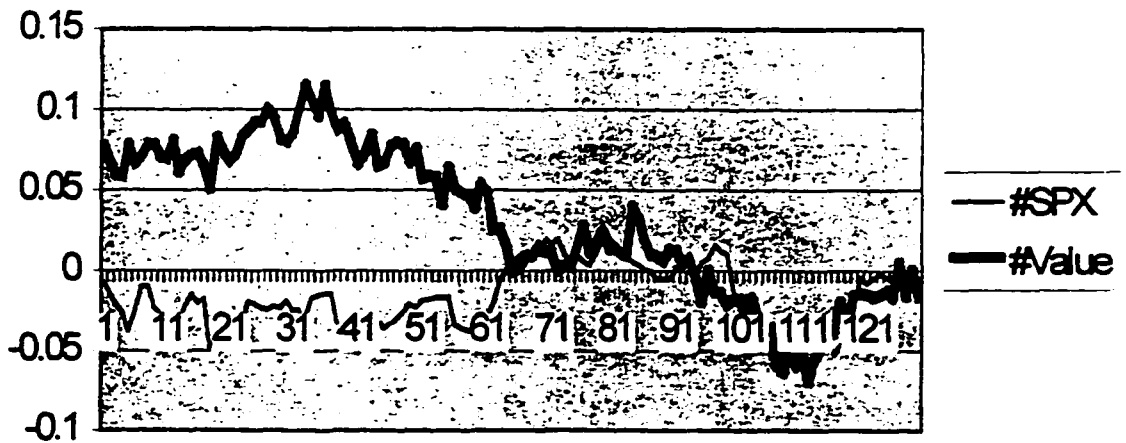
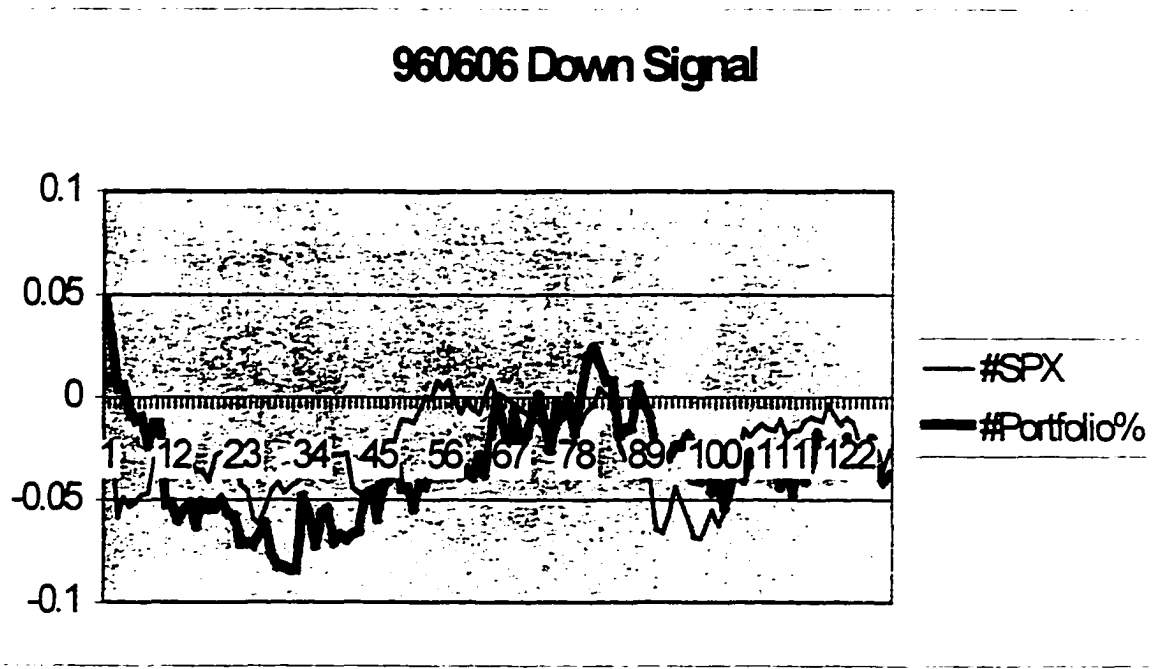


Figure 3.1 Testing Portfolio Performances

4(a)



4(b)

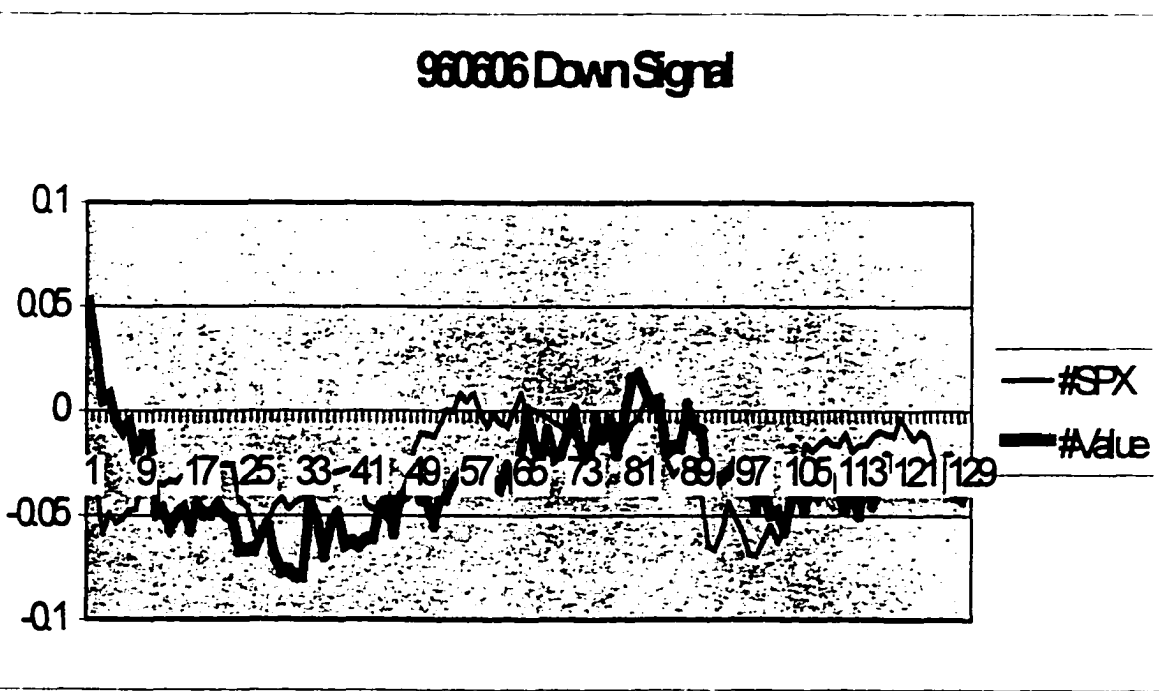
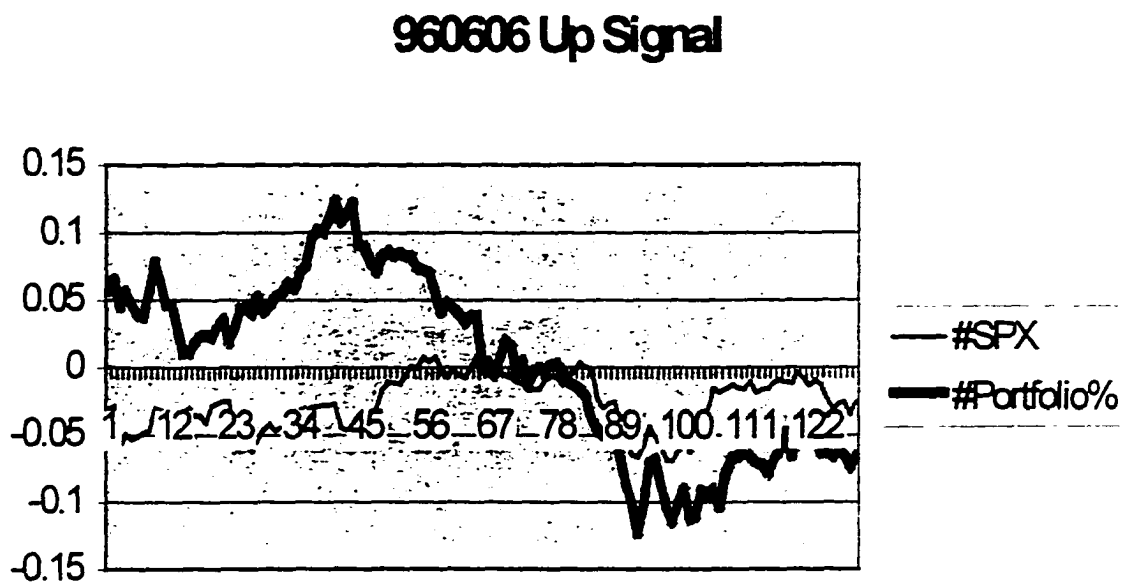


Figure 3.1 Testing Portfolio Performances

5(a)



5(b)

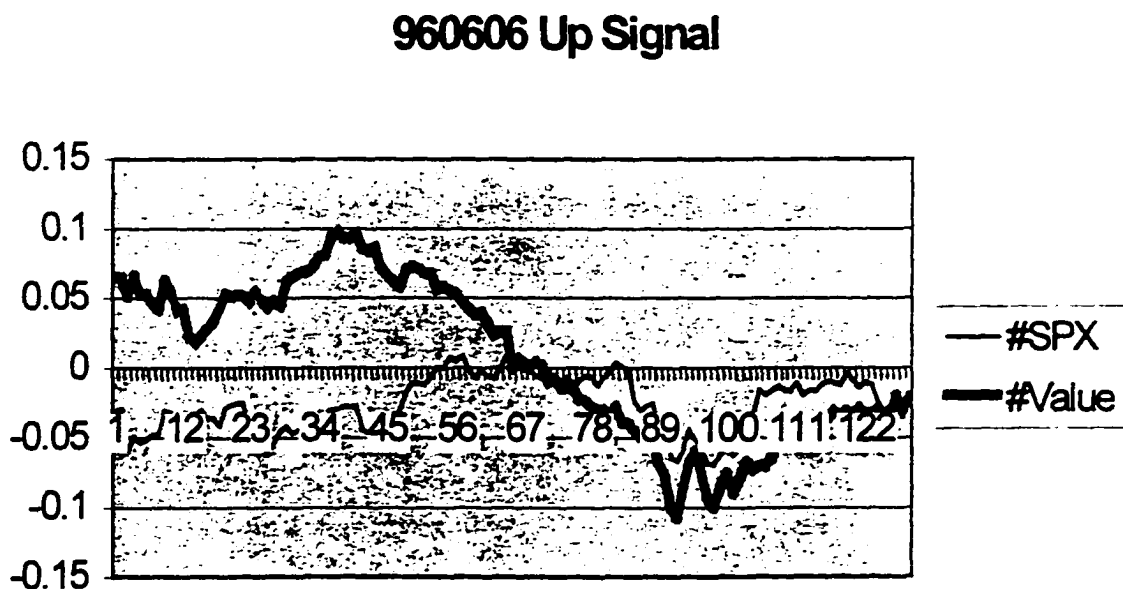
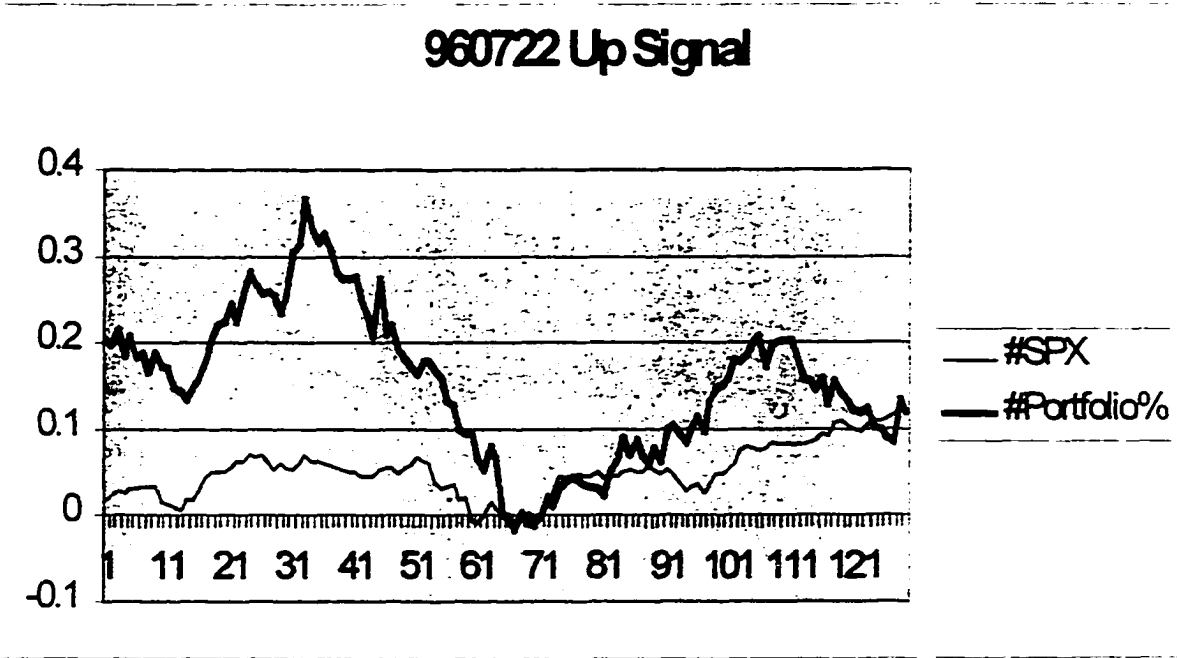


Figure 3.1 Testing Portfolio Performances

6(a)



6(b)

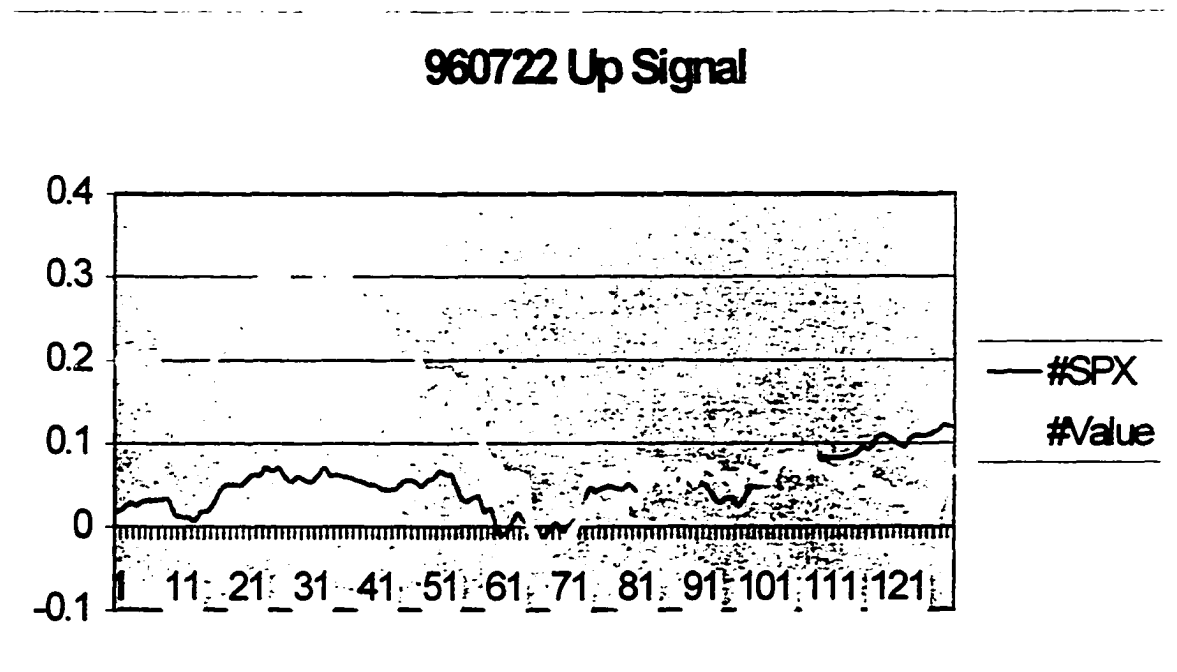
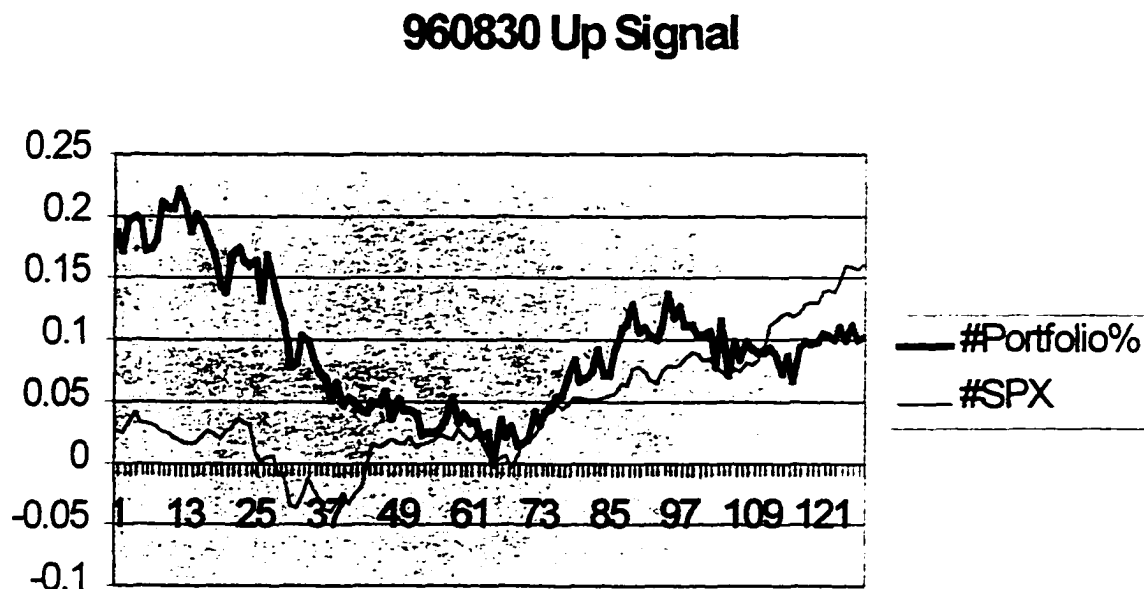


Figure 3.1 Testing Portfolio Performances

7(a)



7(b)

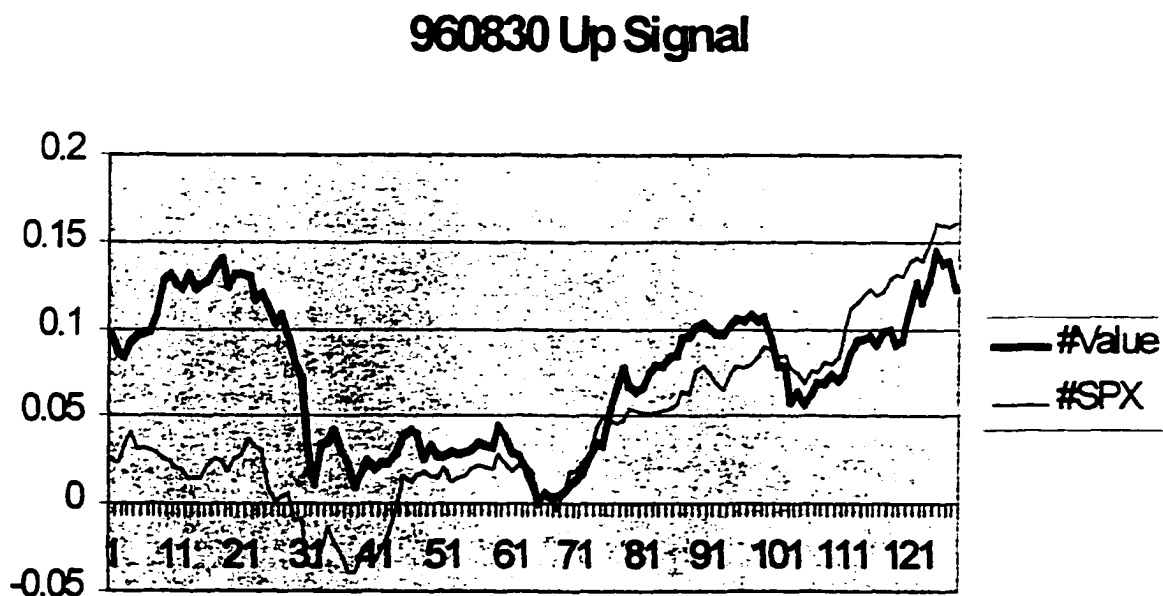
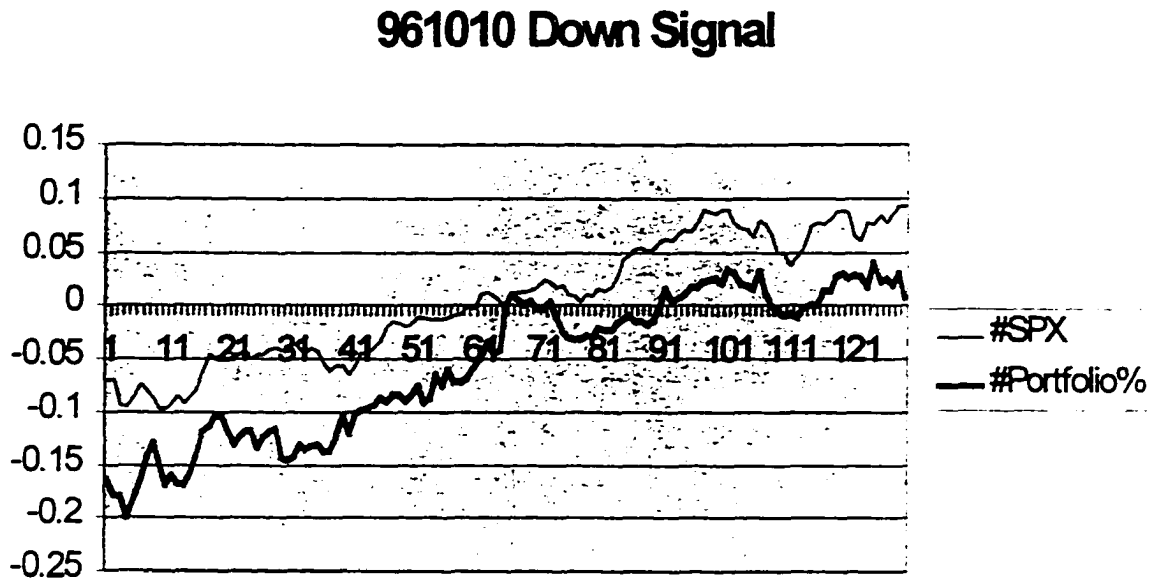


Figure 3.1 Testing Portfolio Performances

8(a)



8(b)

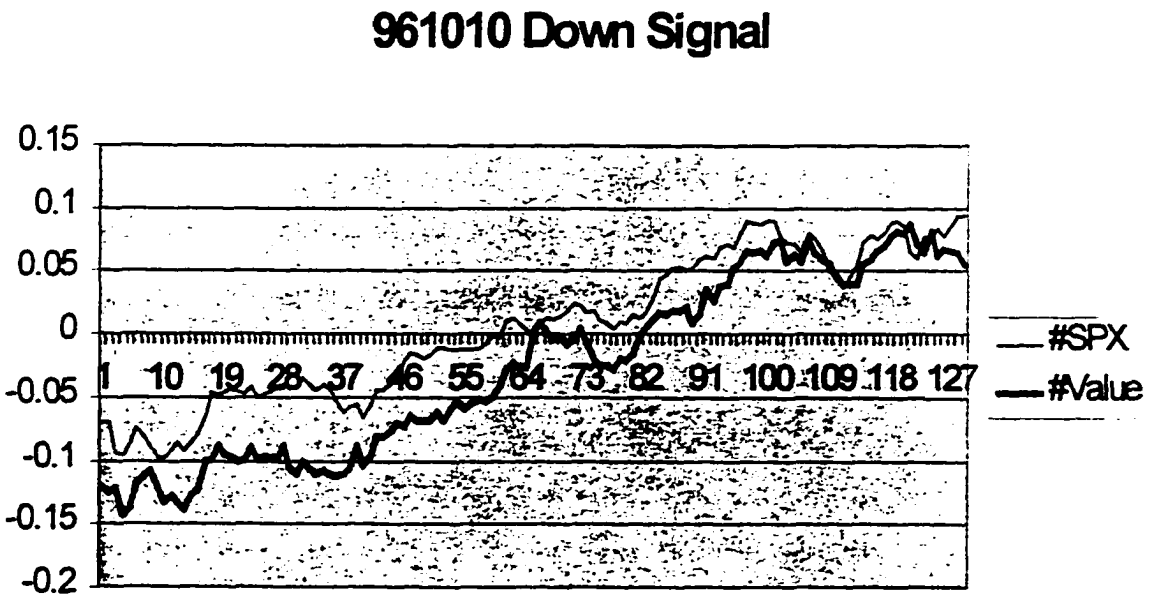
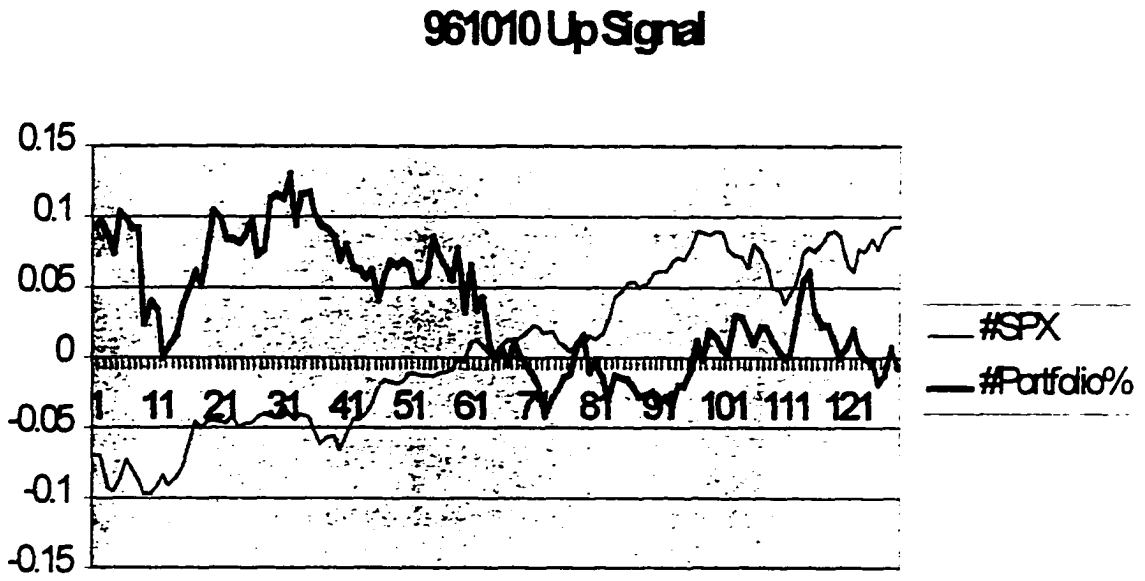
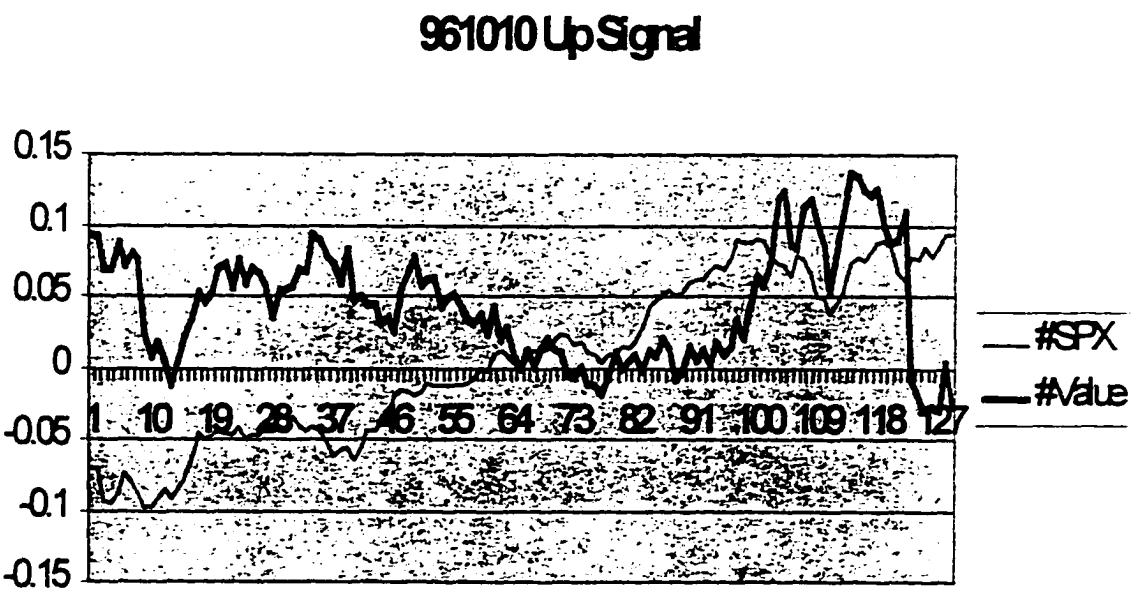


Figure 3.1 Testing Portfolio Performances

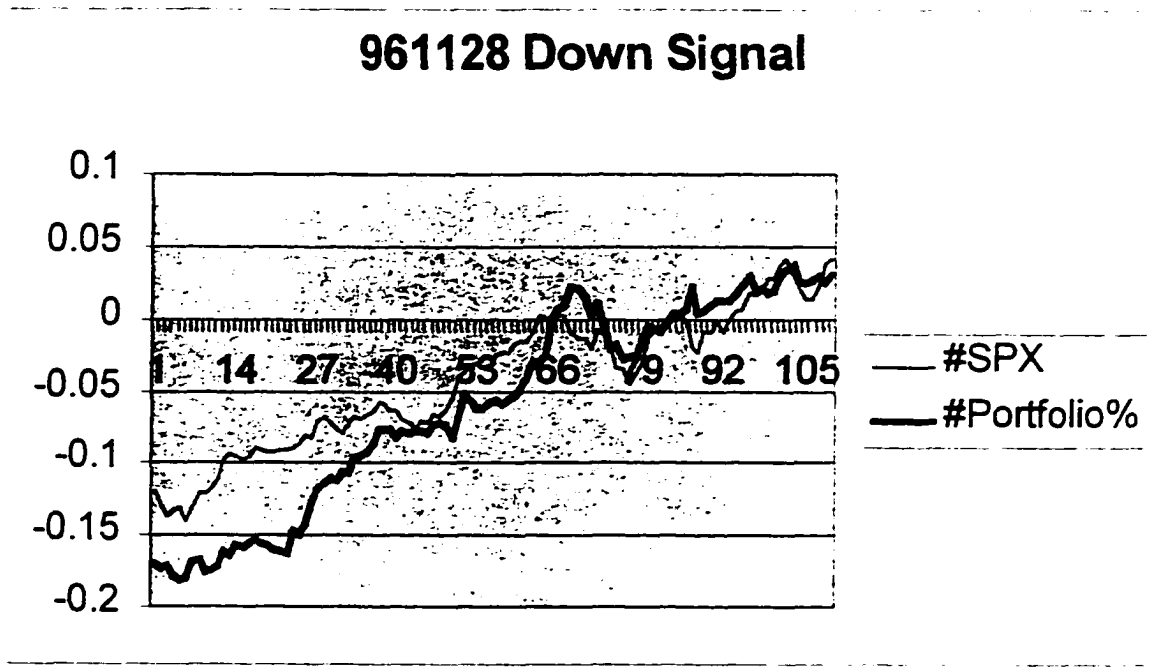
9(a)



9(b)



10(a)



10(b)

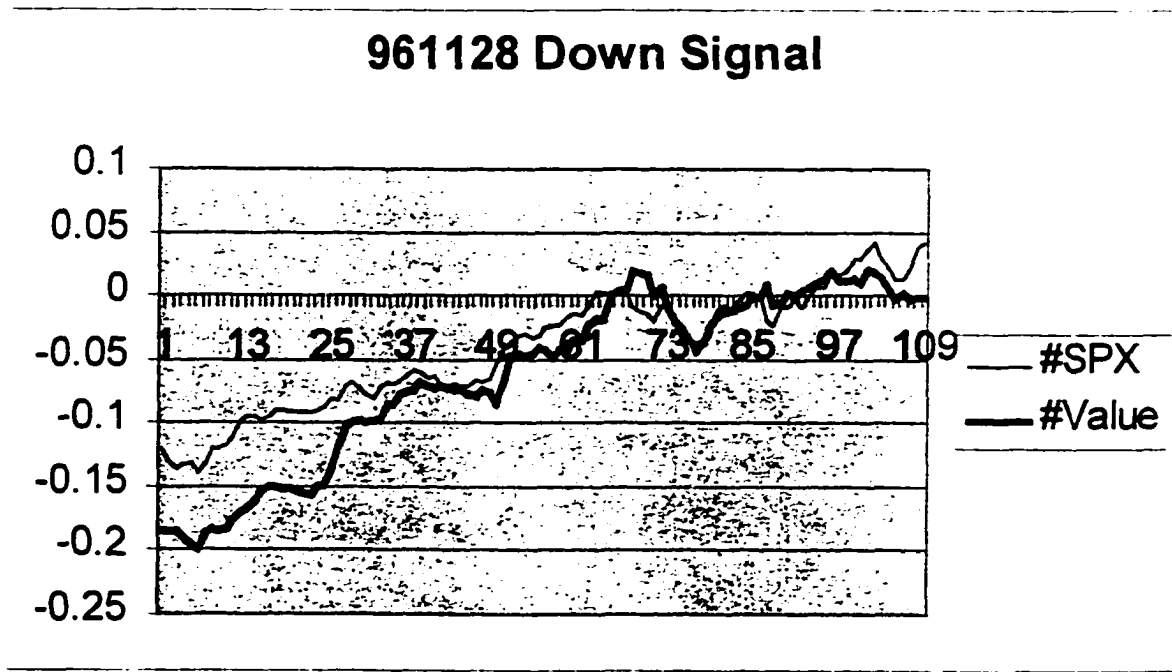
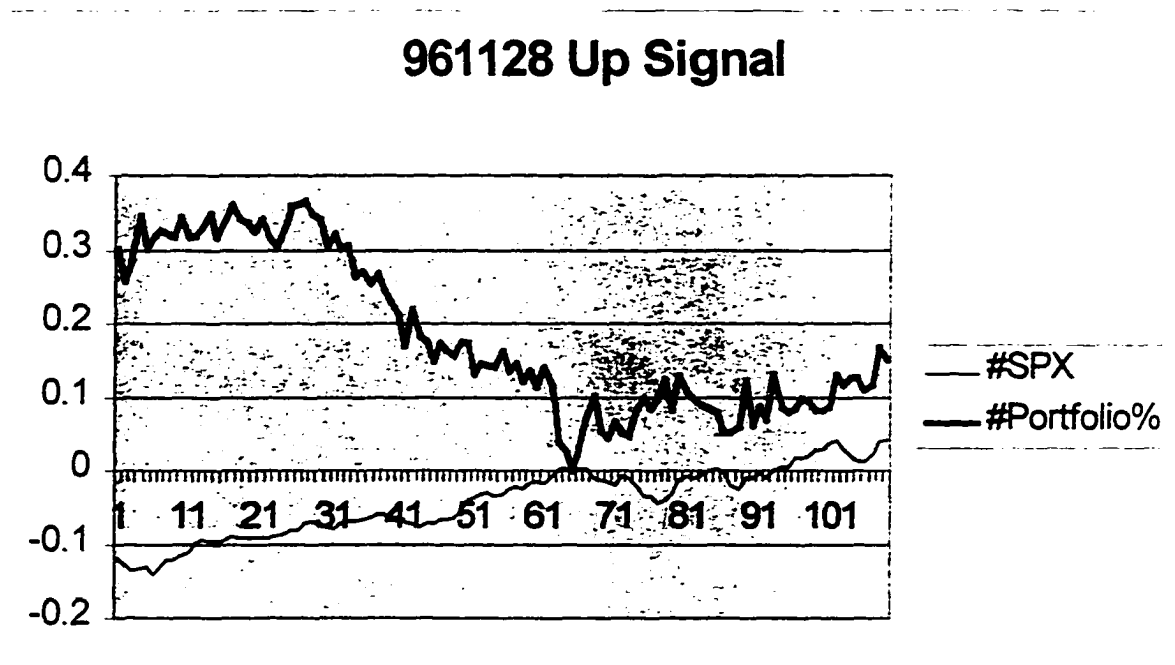


Figure 3.1 Testing Portfolio Performances

11(a)



11(b)

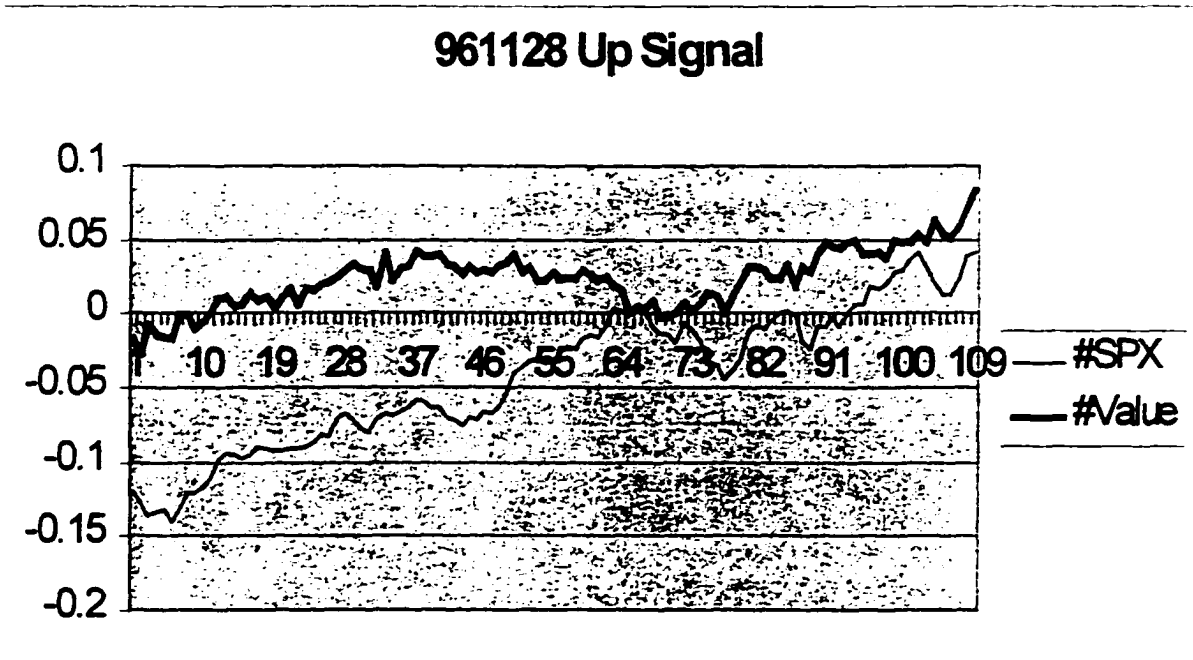
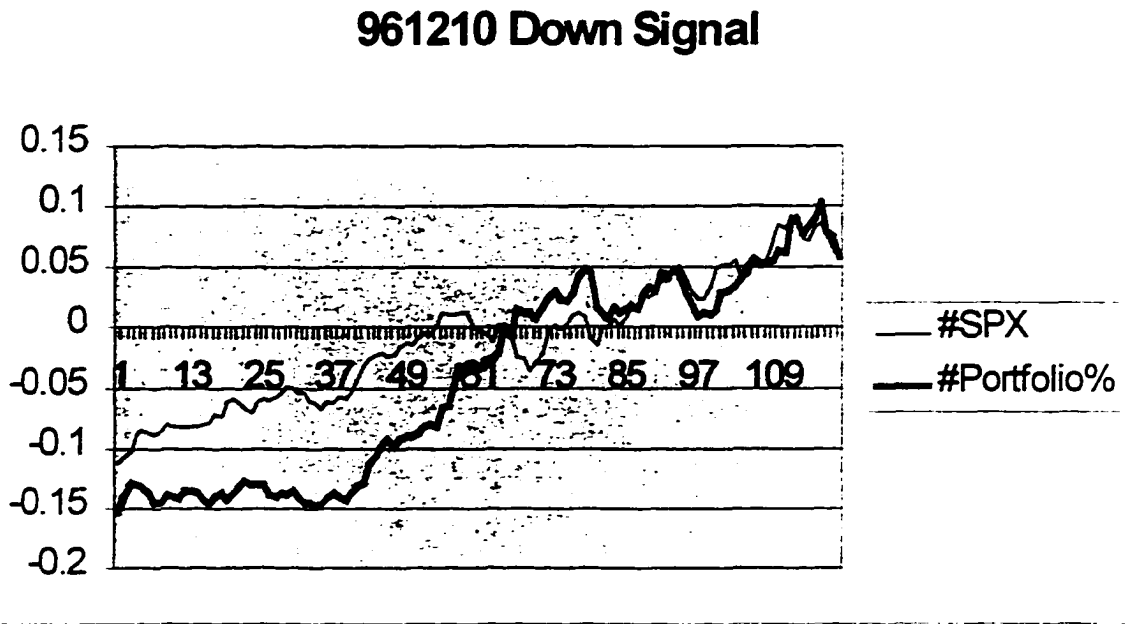


Figure 3.1 Testing Portfolio Performances

12(a)



12(b)

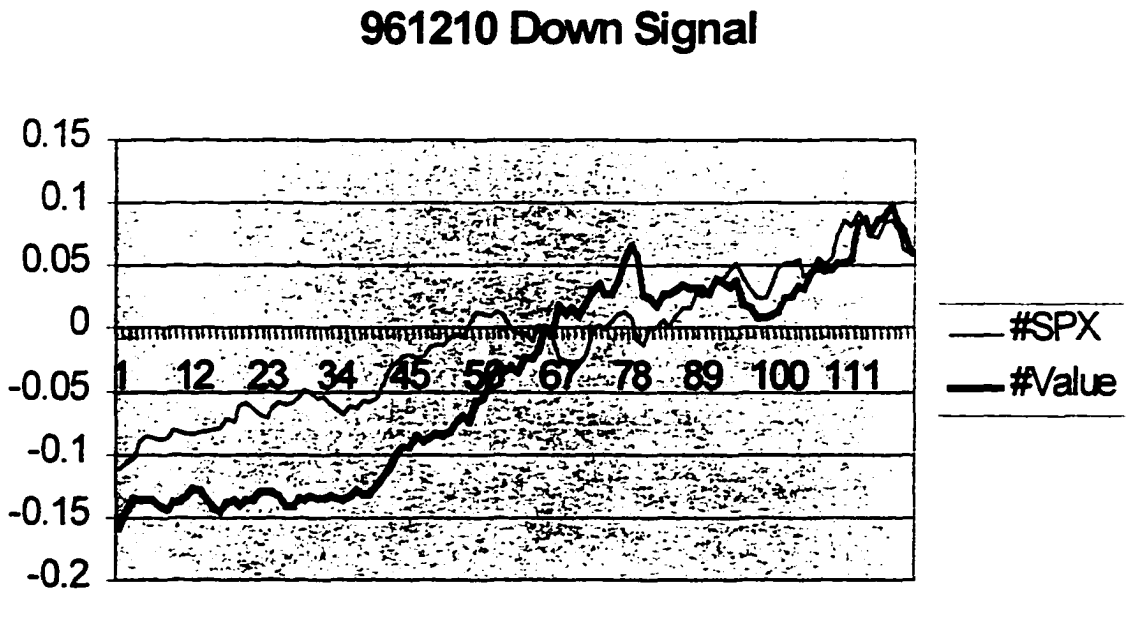
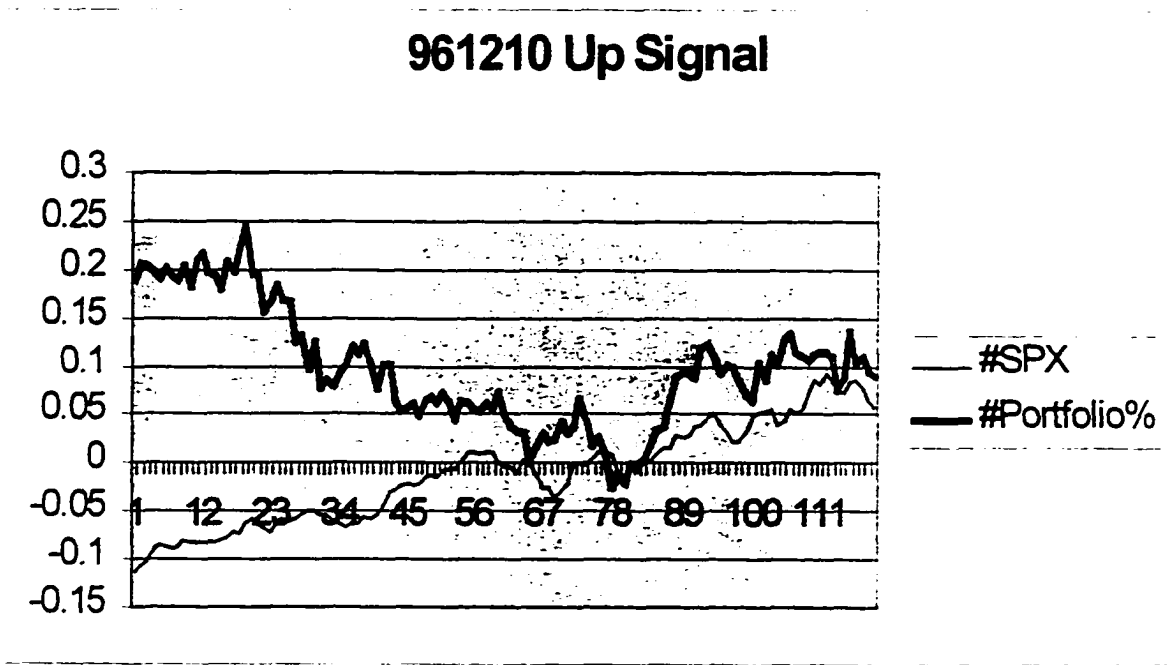


Figure 3.1 Testing Portfolio Performances

13(a)



13(b)

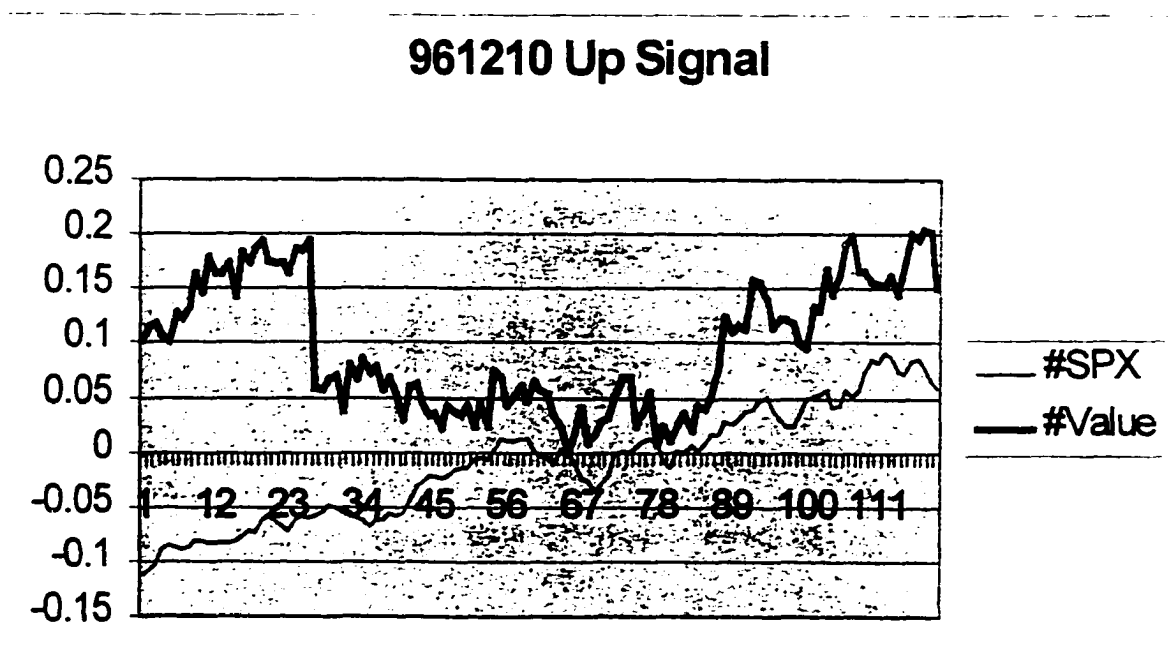
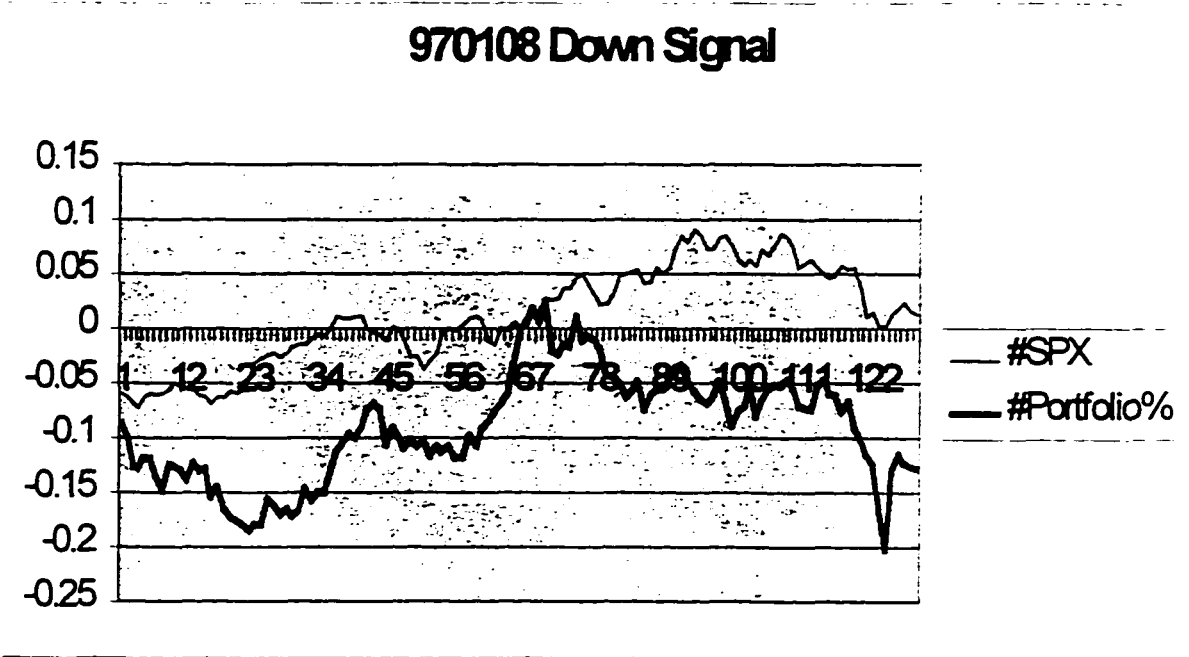


Figure 3.1 Testing Portfolio Performances

14(a)



14(b)

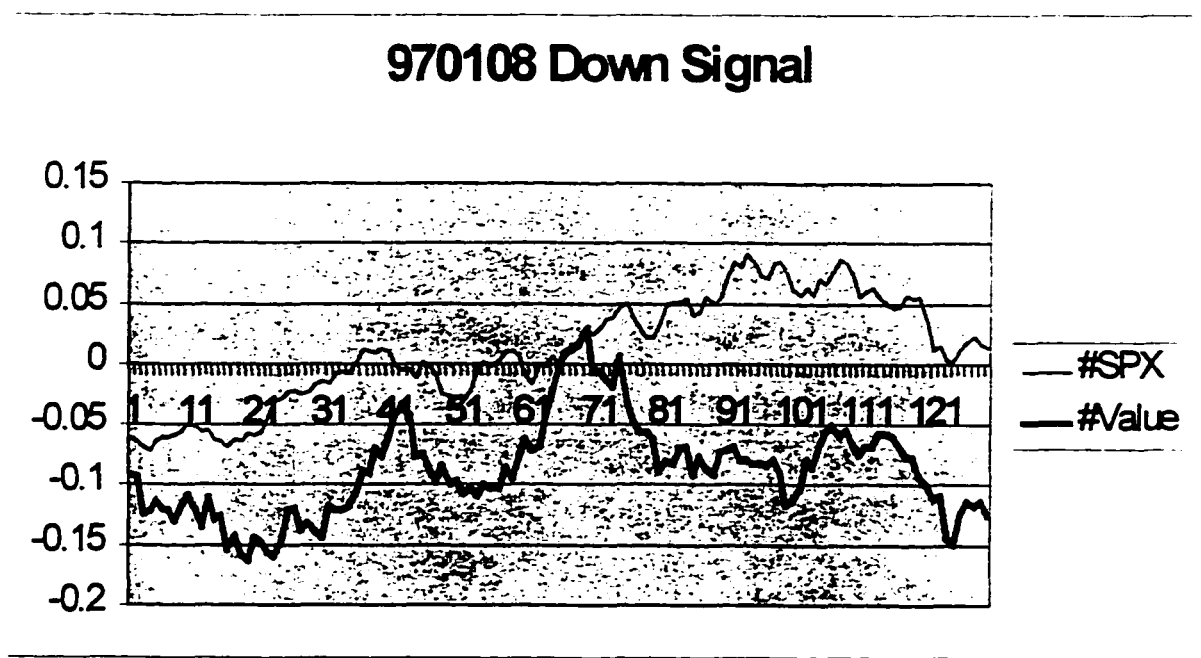
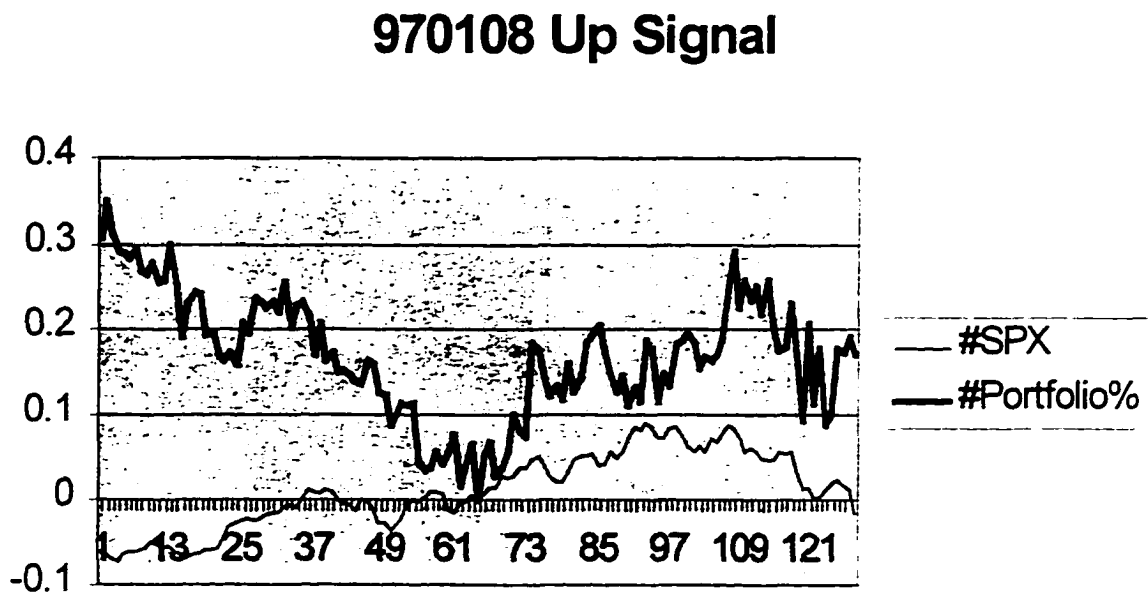


Figure 3.1 Testing Portfolio Performances

15(a)



15(b)

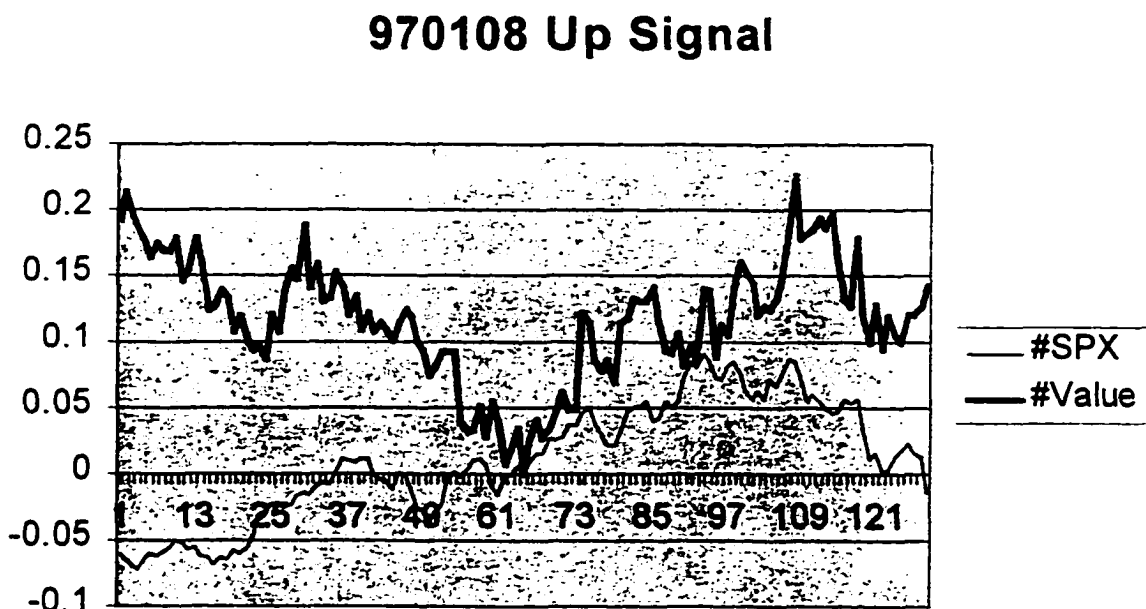
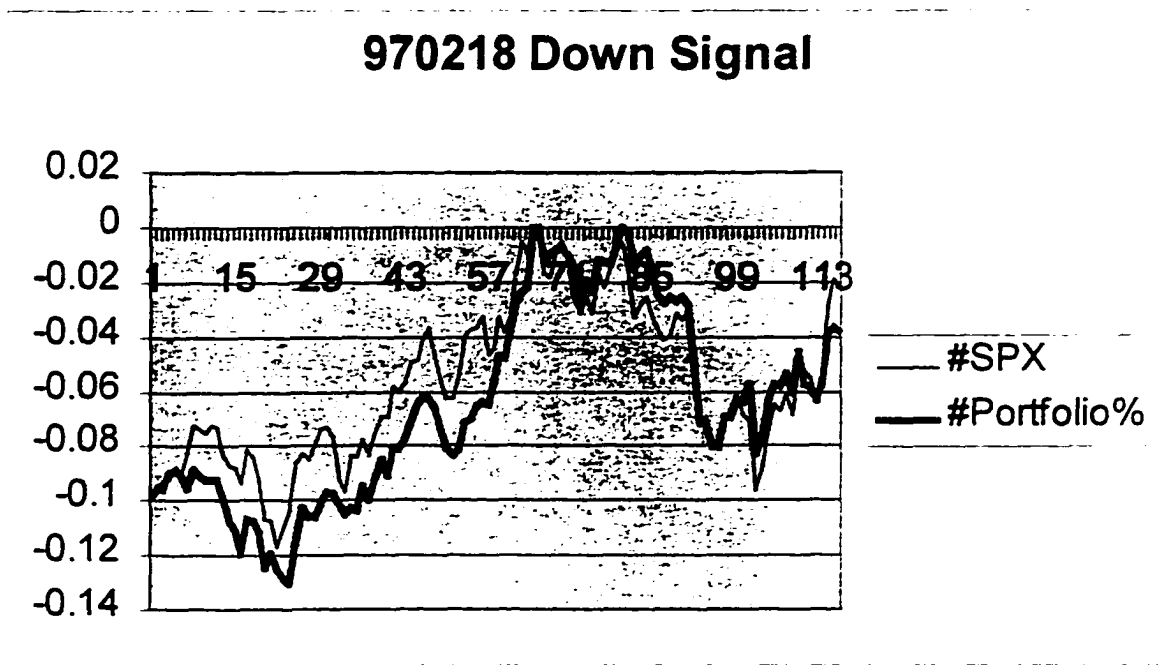


Figure 3.1 Testing Portfolio Performances

16(a)



16(b)

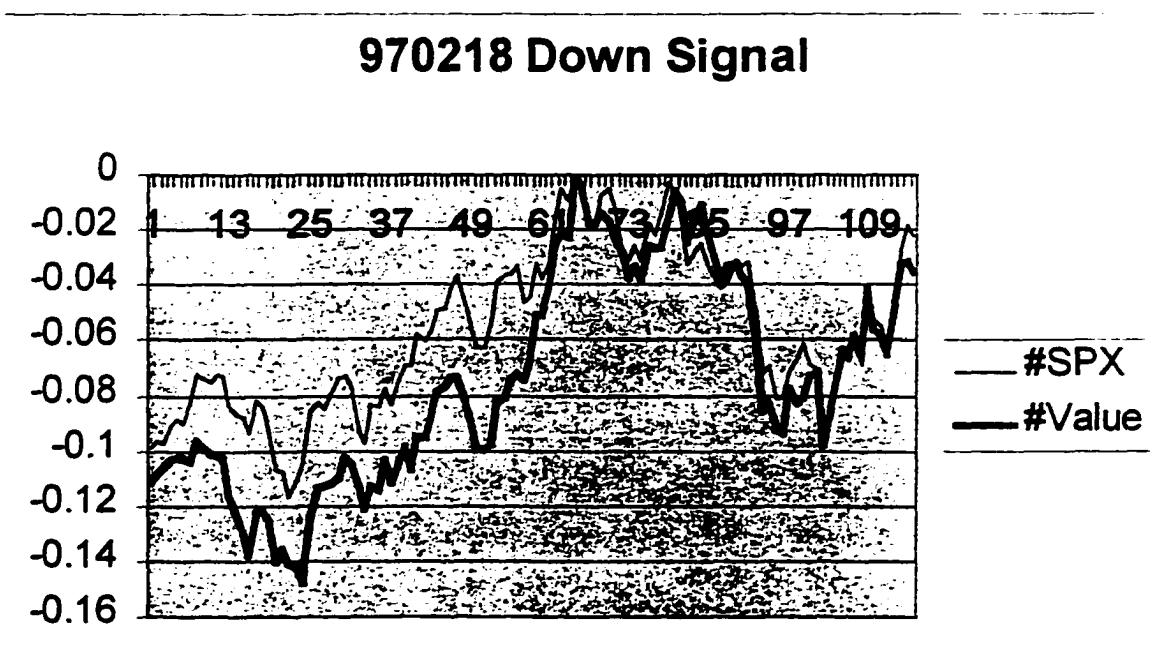
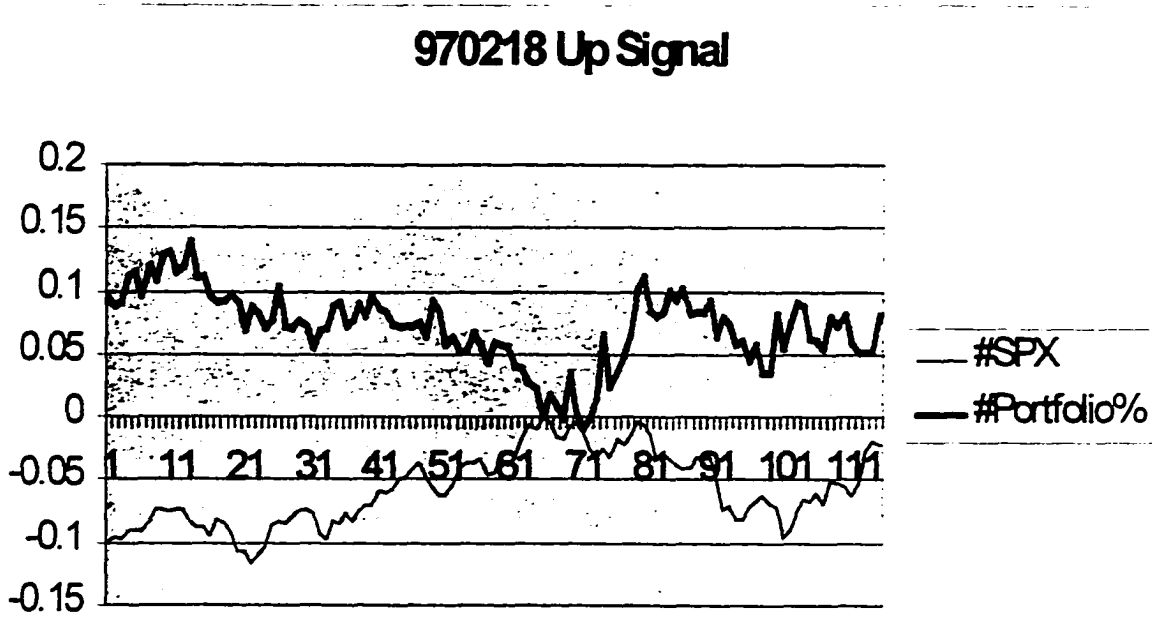


Figure 3.1 Testing Portfolio Performances

17(a)



17(b)

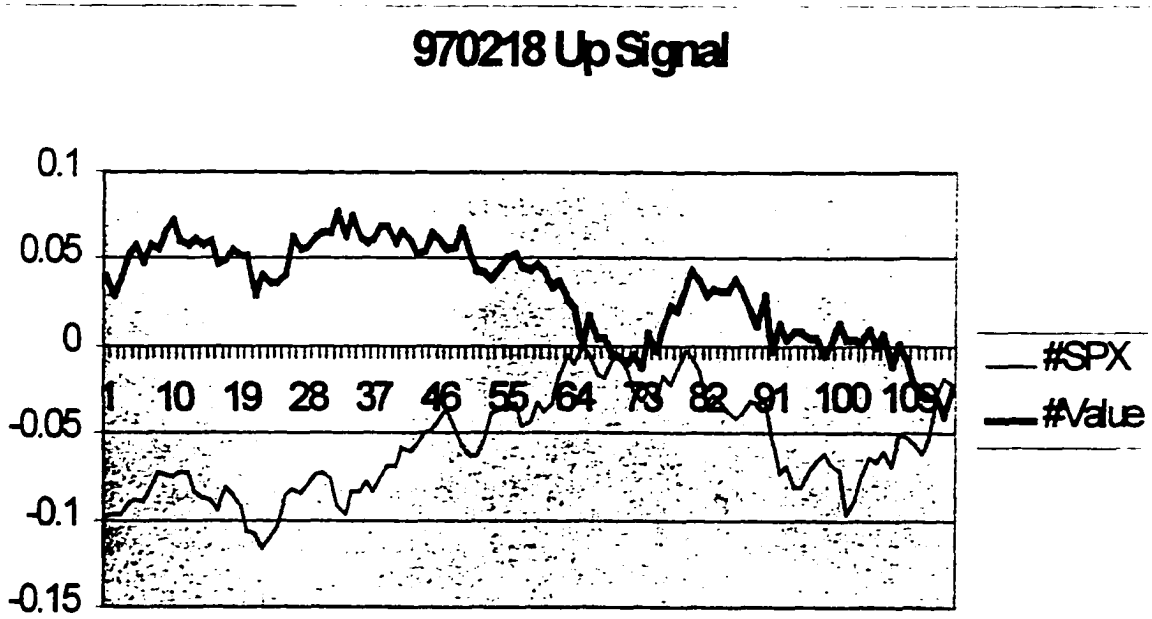
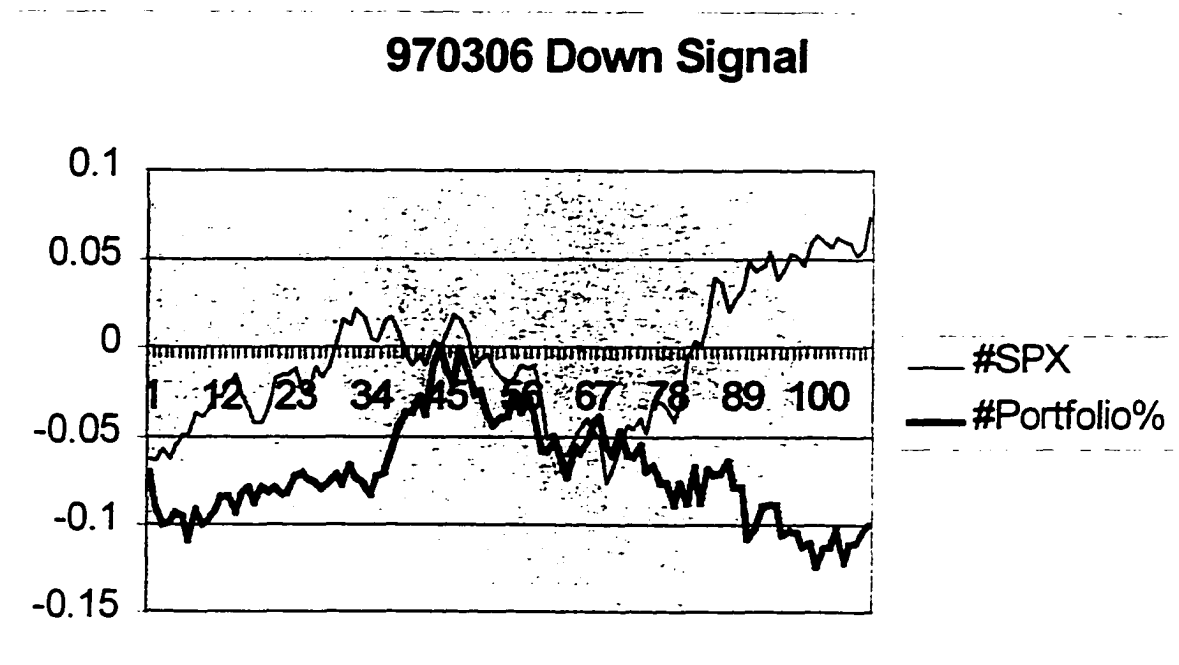


Figure 3.1 Testing Portfolio Performances

18(a)



18(b)

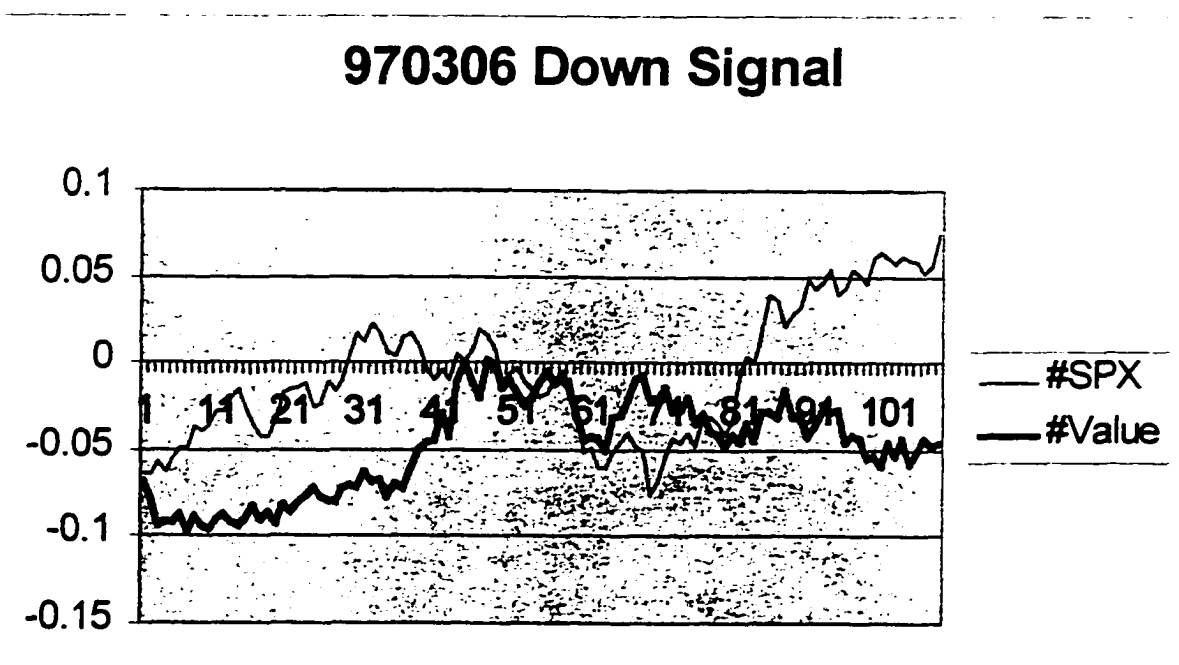
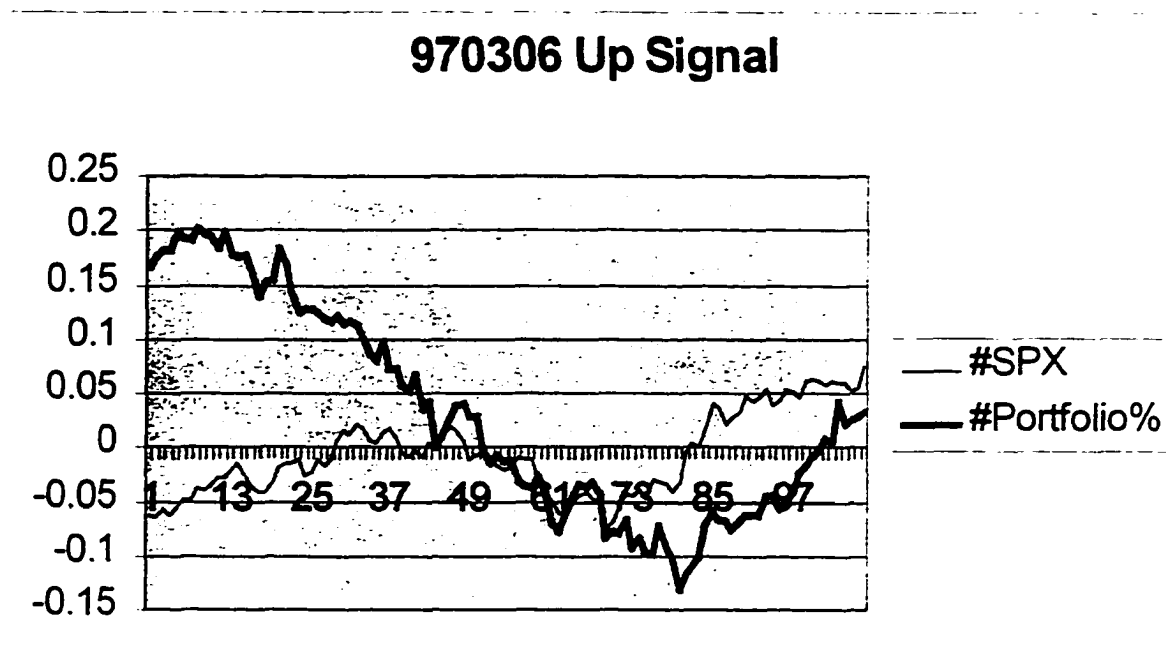


Figure 3.1 Testing Portfolio Performances

19(a)



19(b)

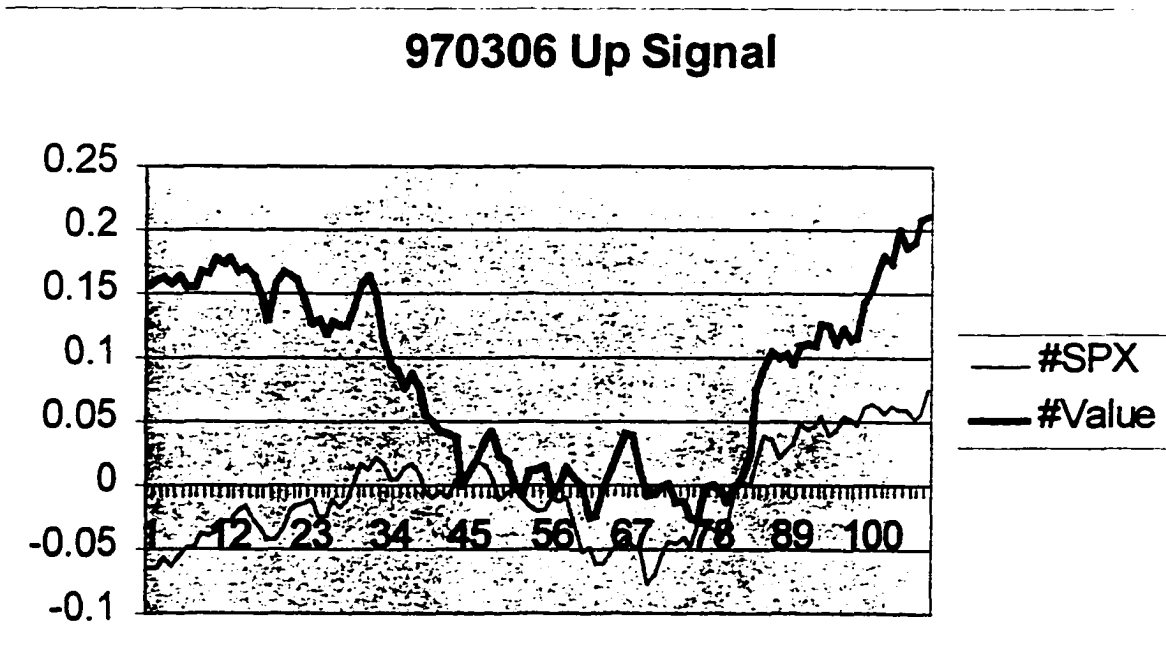
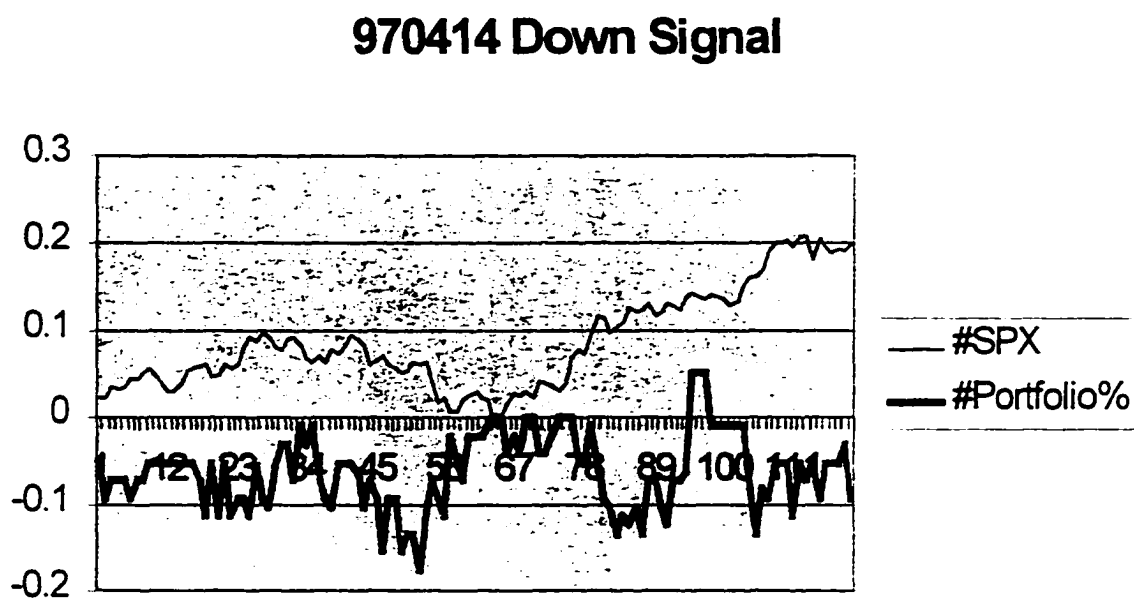
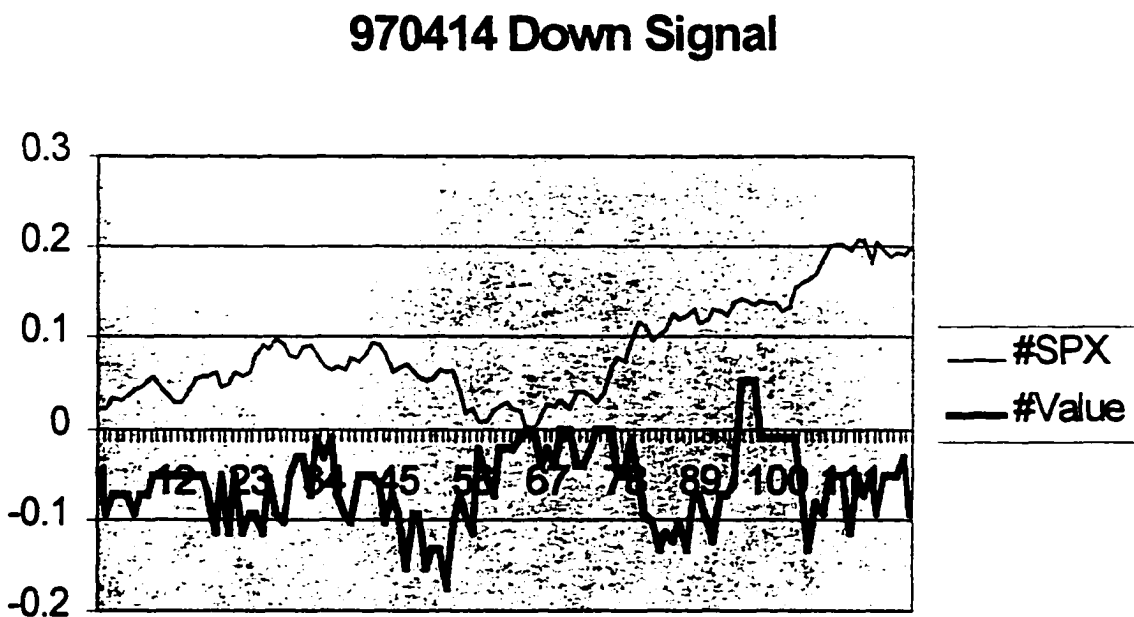


Figure 3.1 Testing Portfolio Performances

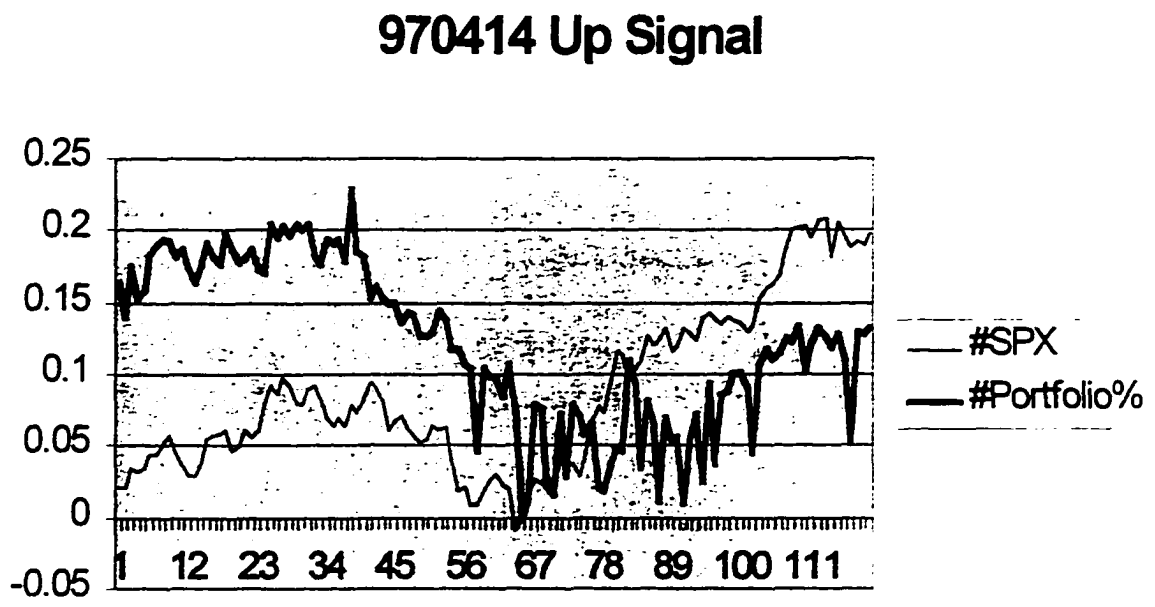
20(a)



20(b)



21(a)



21(b)

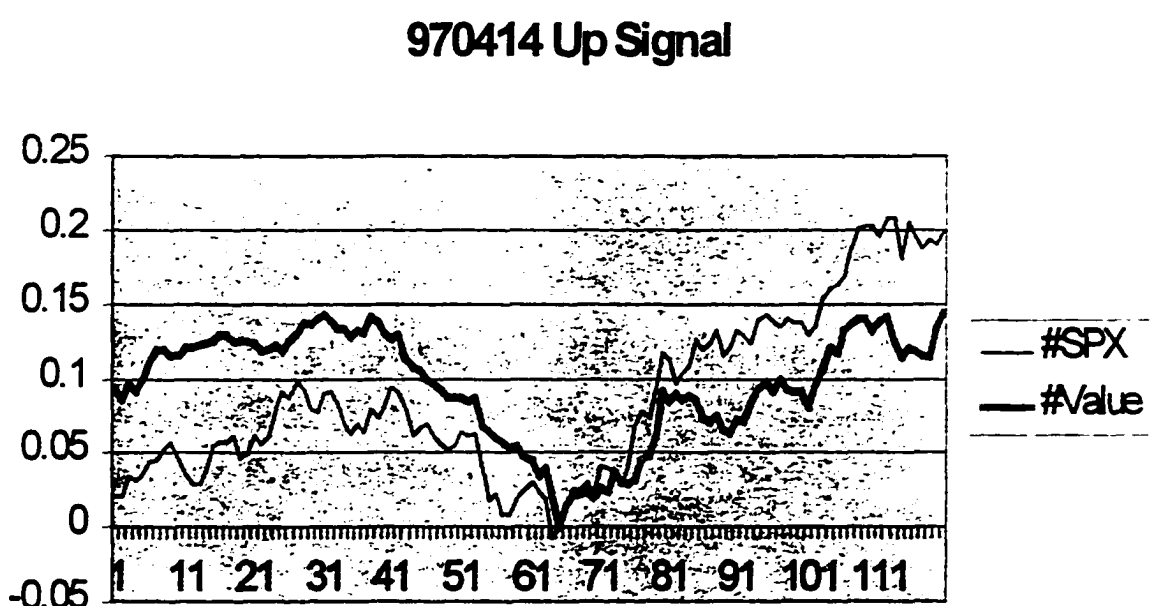
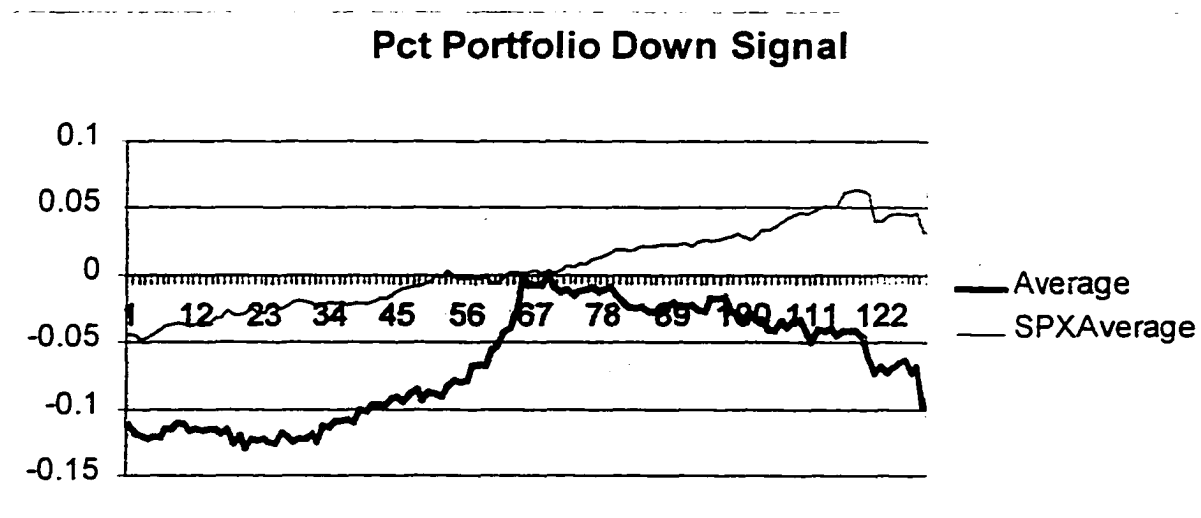


Figure 3.2 Total Performances

1(a)



1(b)

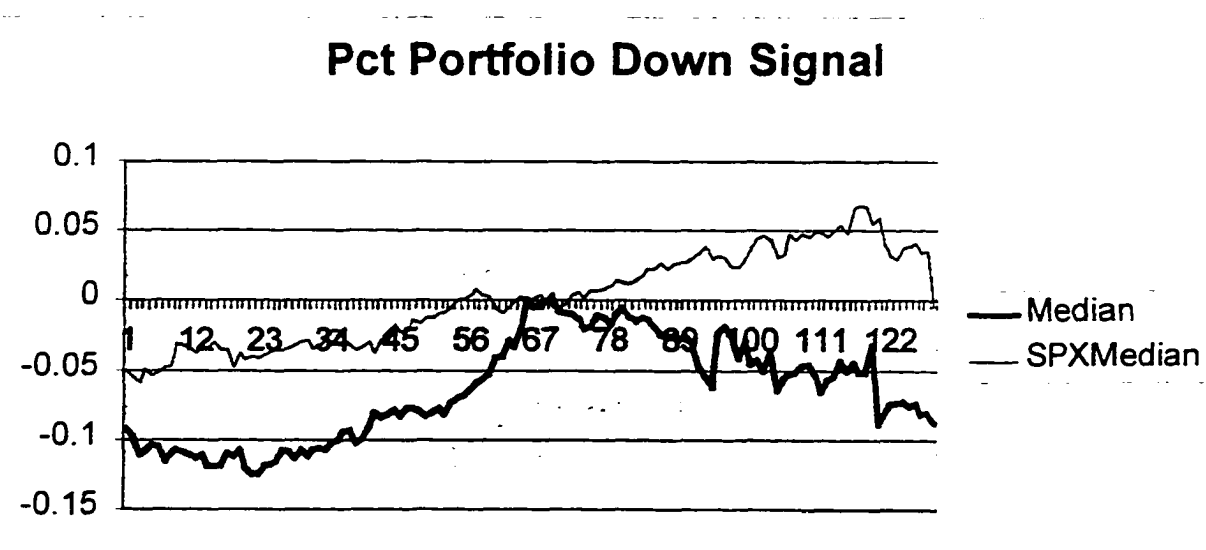
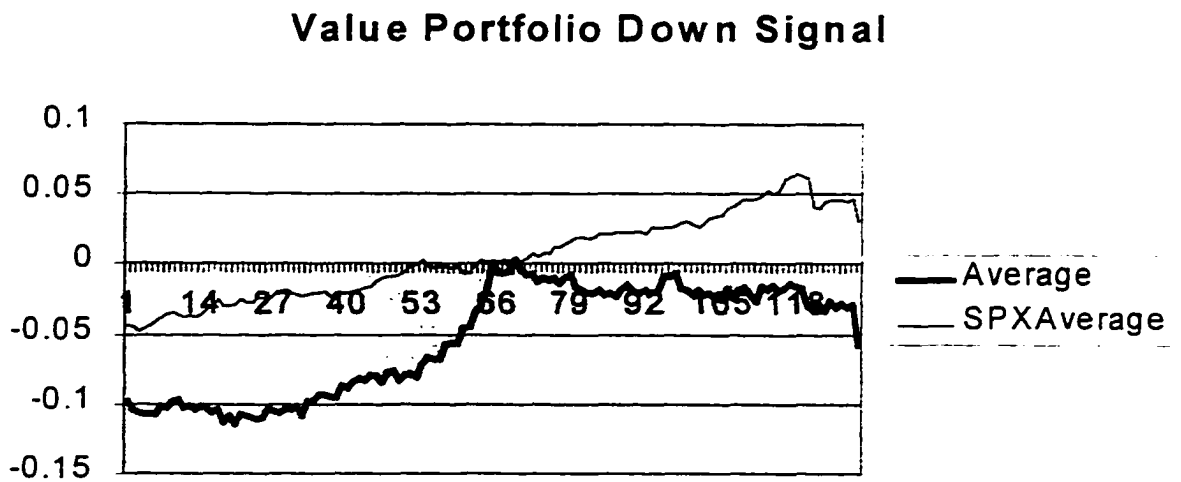


Figure 3.2 Total Performances

2(a)



2(b)

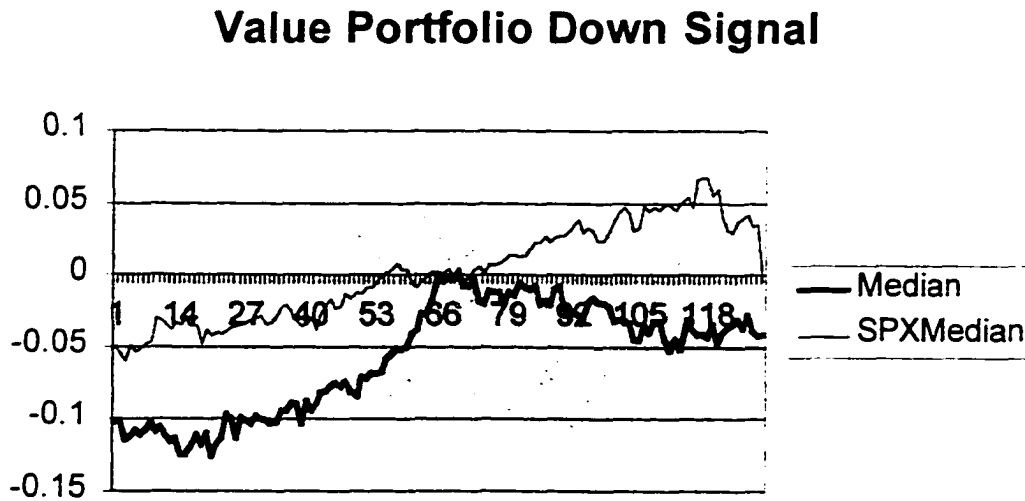
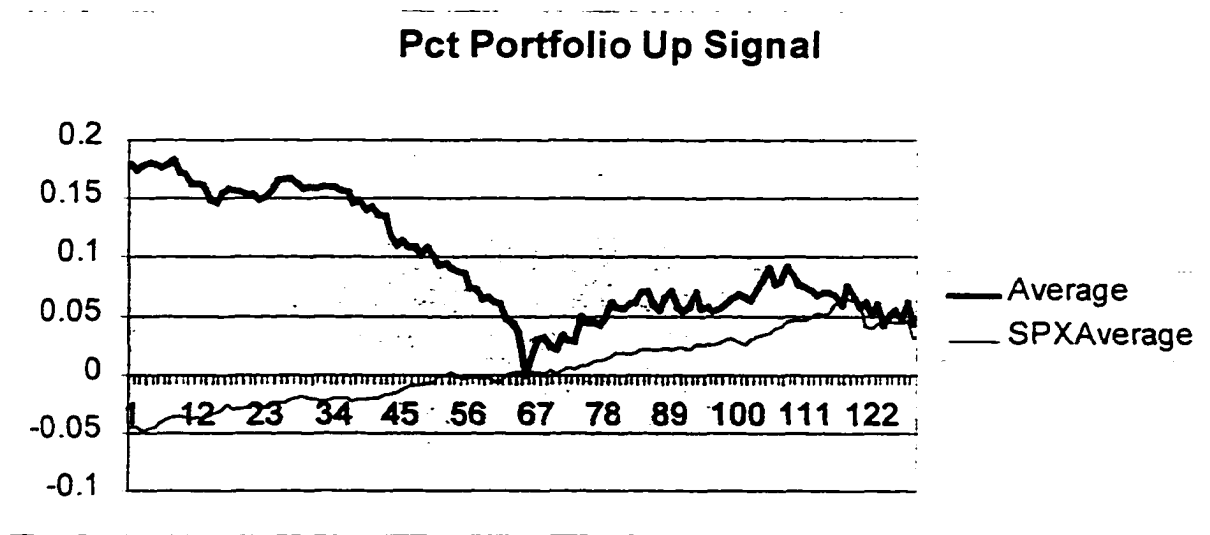


Figure 3.2 Total Performances

3(a)



3(b)

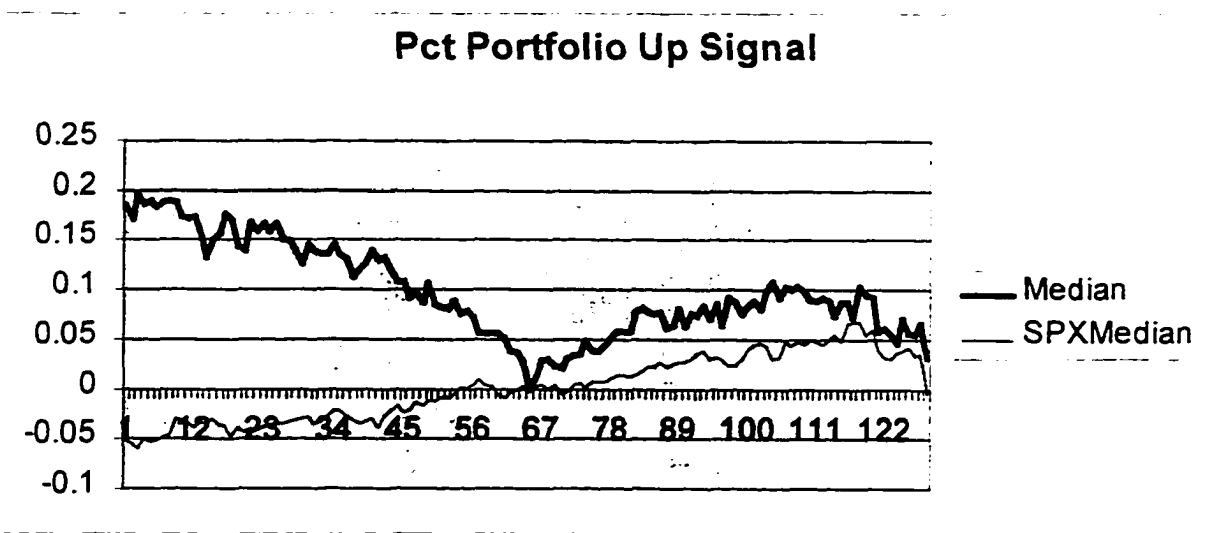
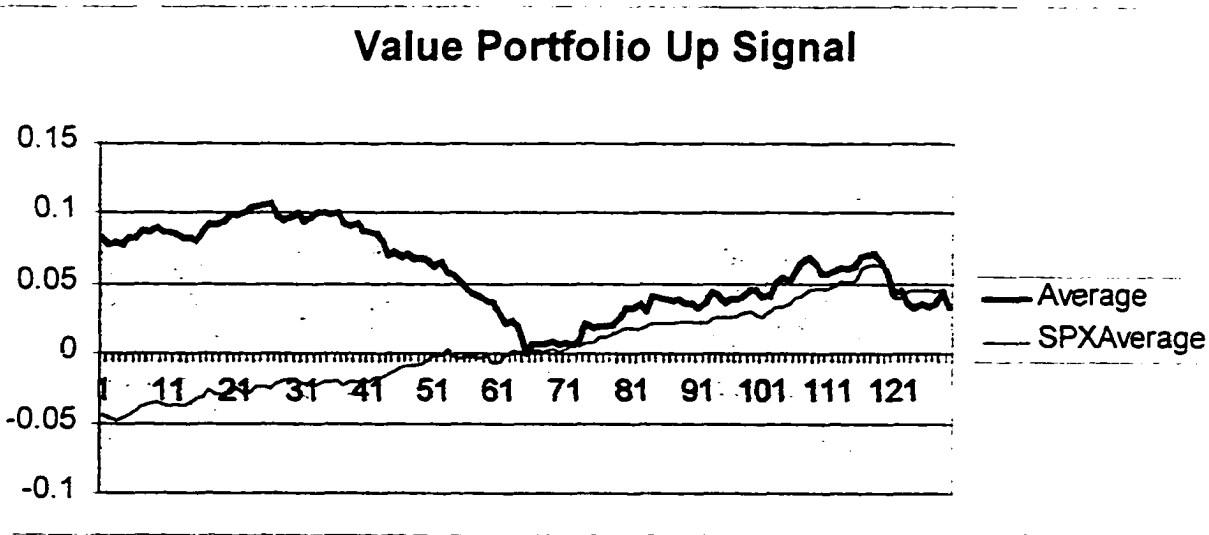


Figure 3.2 Total Performances

4(a)



4(b)

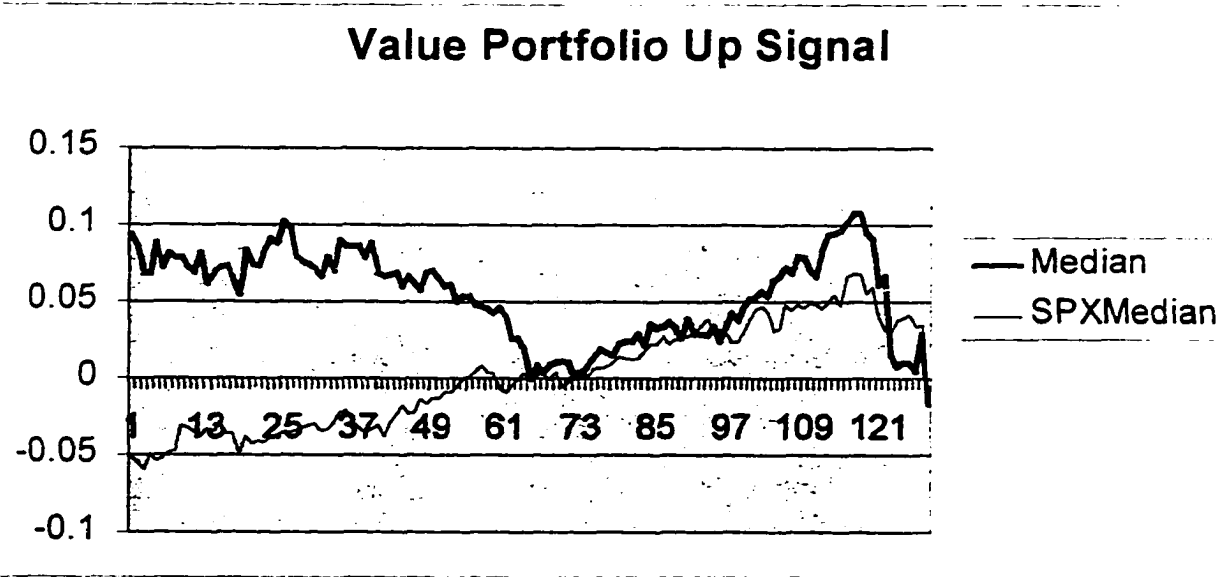


Figure 6.1 Frequency Chart

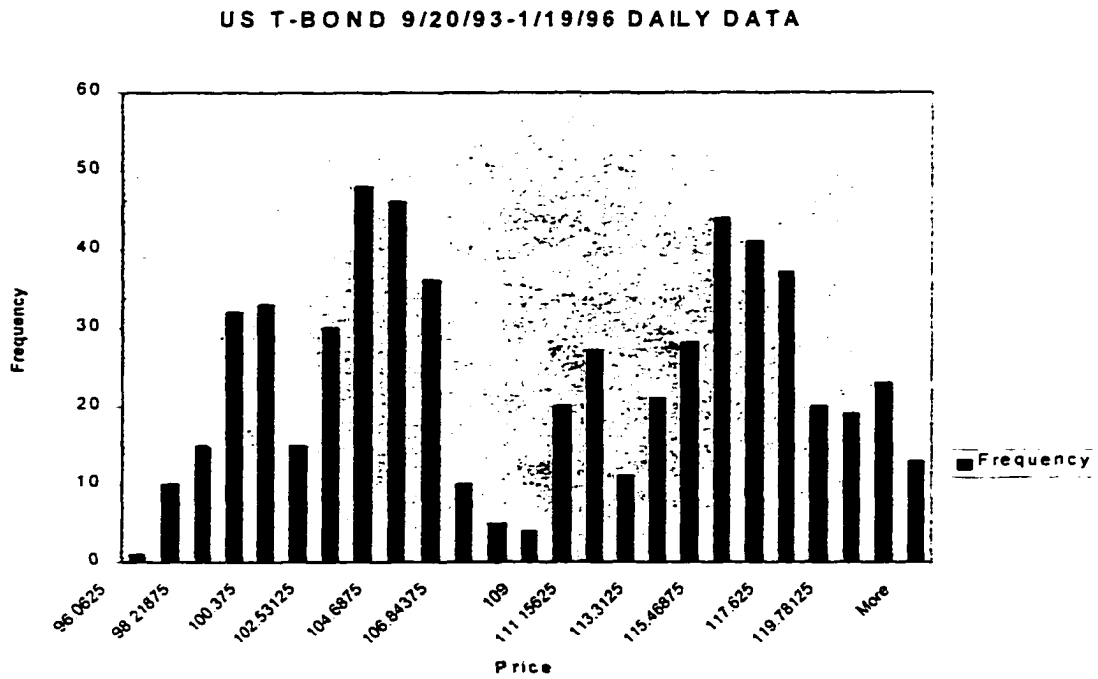
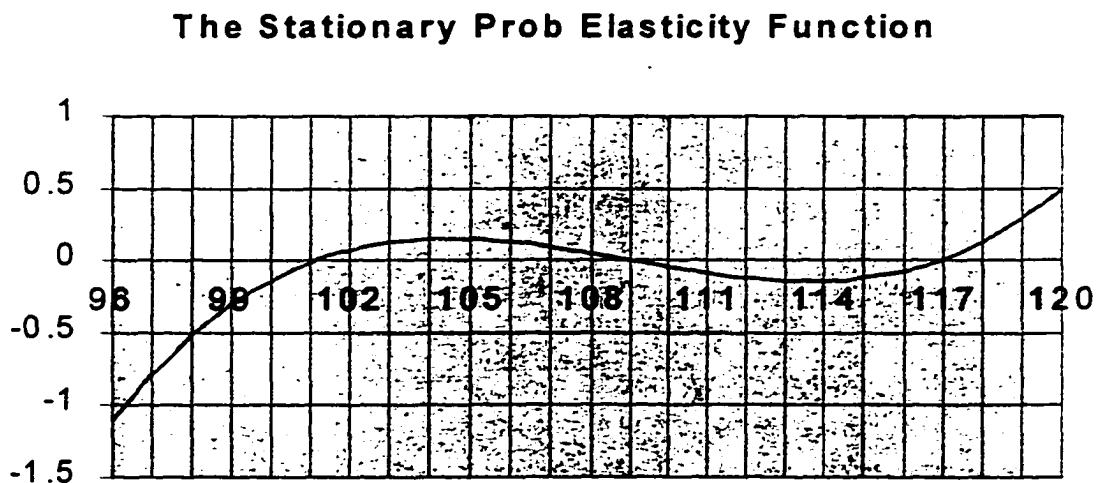


Figure 6.2 Stationary Prob Elasticity Function



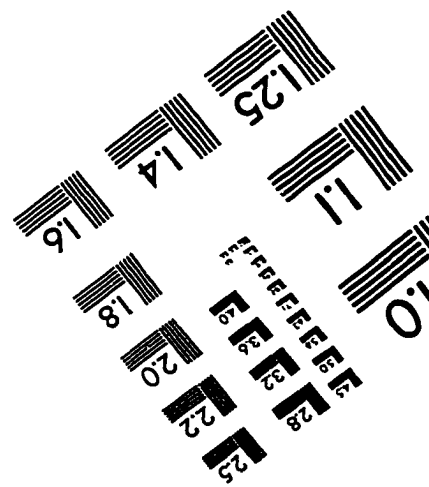
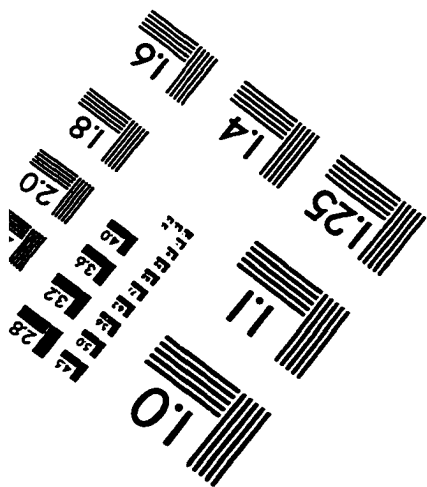
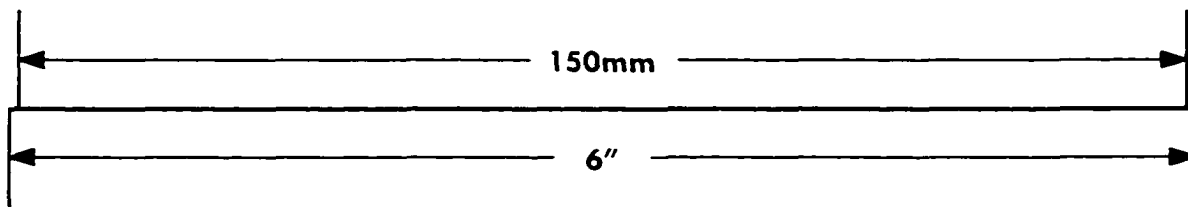
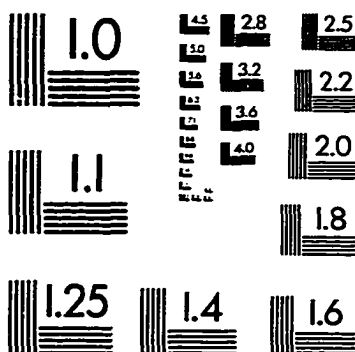
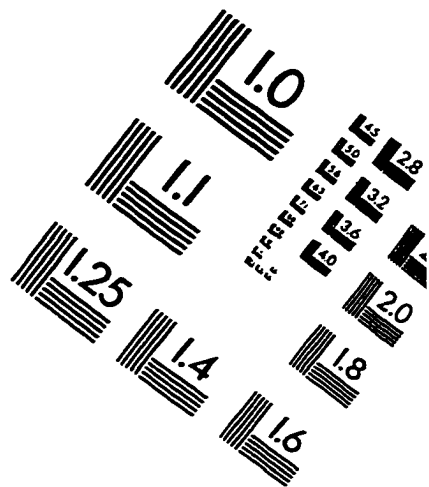
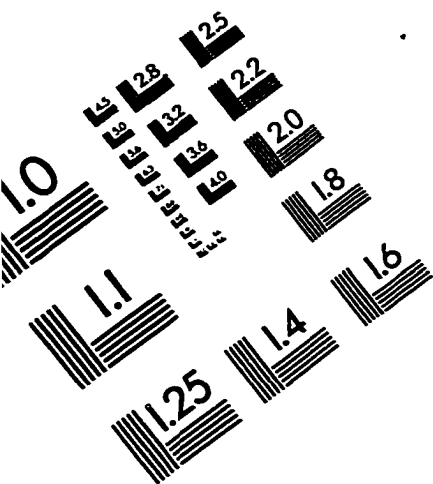
REFERENCES

- Ball, Ray. and Kothari, S.P. "Nonstationary Expected Returns. Implications for Tests of Market Efficiency and Serial Correlation in Returns." *Journal of Financial Economics*. 25.1989. Pp.51-74.
- Bodie, Zvi. and Kane, Alex. and Marcus, Alan. J. "Investments." 2nd Edition. *Irwin*. 1993.
- Brock, William. A. and Hsieh, David. H. and LeBaron, Blake. "Nonlinear Dynamics. Chaos and Instability. Statistical Theory and Economic Evidence." *The MIT press*. 1993.
- Brock, William. and Lakonishok, Josef. and LeBaron, Blake. "Simple Technical Trading Rules and the Stochastic Properties of Stock Returns." *The Journal of Finance*. Vol XLVII. No 5. Dec. 1992. Pp.1731-1764.
- Campbell, John. Y. and Grossman, Sanford. J. and Wang, Jiang. "Trading Volume and Serial Correlation in Stock Returns." *Quarterly Journal of Economics*. 1993. Pp.905-939.
- Cobb, Loren. and Koppstein, Peter. and Chen, N.H. "Estimation and Moment Recursion Relations for Multimodal Distributions of the Exponential Family." *Journal of the American Statistical Association*. Mar 1983. Vol 78. Theory and Methods section. Pp.124-130.
- Cobb, Loren. "Stochastic Catastrophe Models and Multimodal Distributions." *Behavioral Science*. Vol 23. 1978. Pp.360-374.
- Diebold, Francis. X. and Nason, James. A. "Nonparametric Exchange Rate Prediction?" *Journal of International Economics*. 28. (1990). Pp.315-332.

- Fama, Eugene. F. "Random Walks in Stock Market Prices." *Financial Analysts Journal*. Vol. 21. No.5. 1965. Pp.55-59.
- French, K. and Roll, R. "Stock Return Variances: The Arrival of Information and the Reaction of Traders." *Journal of Financial Economics*. 17. Sept. 1986.
- Gallant, Ronald. A. and Rossi, Peter. E. and Tauchen. George. "Stock Prices and Volume." *The Review of Financial Studies*. Vol 5. #2. 1992. Pp.199-242.
- Grinold, Richard. C. and Kahn, Ronald. N. "Active Portfolio Management. Quantitative Theory and Applications." *Probus*. 1995.
- Grossman, J. Sanford. and Stiglitz, Joseph.E. "On the Impossibility of Informationally Efficient Markets." *The American Economic Review*. June 1980. Pp.393-408.
- Kempen, N.G. Van. "Stochastic Processes in Physics and Chemistry." *North-Holland*. 1992.
- Lang, Larry H.P., Litzenberger, Robert H., Madrigal, Vicente. "Testing Financial Market Equilibrium under Asymmetric Information". *Journal of Political Economy*. 1992. Vol.100. No. 2. Pp.317-348.
- Lamoureux, C. and Lastrapes, W. "Heteroskedasticity in Stock Return Data: Volume versus GARCH Effects." *Journal of Finance*. 45. 1991.
- McDonald, James. "Some Generalized Functions for the Size Distribution of Income." *Econometrica*. Vol 52. #3. May. 1984. Pp.647-663.
- Neftci, Salih. N. "Naive Trading Rules in Financial Markets and Wiener-Kolmogorov Prediction Theory: A Study of 'Technical Analysis'." *Journal of Business*. Vol 64. No 4. 1991. Pp.549-571.

- Poterba, James. M. and Summers, Lawrence. H. "Mean Reversion in Stock Prices. Evidence and Implications." *Journal of Financial Economics*. 22(1988). Pp.27-59.
- Romer, David. "Rational Asset-Price Movements Without News." *The American Economic Review*. Dec. 1993. Pp.1112-1130.
- Shiller, Robert. J. "Fashions, Fads, and Bubbles in Financial Markets." *Knights, Raiders and Targets*. *Oxford Univ. Press*. 1988.
- Wang, Jiang. "A Model of Competitive Stock Trading Volume". *Journal of Political Economy*. 1994. Vol.102. No.11. pp.127-168.

IMAGE EVALUATION TEST TARGET (QA-3)



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