RAYLEIGH-SCHROEDINGER PERTURBATION THEORY APPLIED TO A NEW GROUND STATE WAVEFUNCTION FOR THE HYDROGEN MOLECULAR ION

by

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Abstract

RAYLEIGH-SCHROEDINGER PERTURBATION THEORY APPLIED TO A NEW GROUND STATE WAVEFUNCTION FOR THE HYDROGEN MOLECULAR ION

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The Geometric Mean (GM) variation function

$$\Phi = N\exp(-ZR\lambda/2)[2\alpha\cosh(ZR\mu/2) + \beta]$$

is used as a zeroth-order wavefunction for perturbation energy calculations on the ground state of the hydrogen molecular ion. The first order wavefunction is obtained and used to calculate the ground state energies of H_2^+ to third order. It is found that for a large range of values of the internuclear distance the third order energies lie within $1.2 \times 10^{-6}\ e^2/a_0$ of the exact values. The results are checked by several independent methods during the various stages of the calculations to insure as great an accuracy as possible.

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CHAPTER 1

THE HYDROGEN MOLECULAR ION

The hydrogen molecular ion, H_2^+, has been the subject of much theoretical interest since the introduction of quantum mechanics to the study of physical phenomena in 1925. The reason for this great interest is that the hydrogen molecular ion occupies the central position in the theory of molecular structure, similar to that enjoyed by the hydrogen atom in the theory of atomic structure.

If the energy be measured in units of e^2/a_0, and the distance measured in units of a_0, then the Schroedinger equation for the electron in the field of two stationary nuclei of unit charge becomes

$$\left(-\frac{1}{2}\nabla^2 - \frac{1}{r_a} - \frac{1}{r_b}\right)\Psi = E\Psi \tag{1-1}$$

where we have neglected the internuclear repulsion term, $1/R$. ∇^2 is the Laplacian operator and r_a and r_b are the respective distances of the electron from nuclei A and B.

It was first shown by Burrau[1] that the wave equation 1-1 is separable upon transforming to confocal elliptical coordinates (λ, μ, φ) where $\lambda R = r_a + r_b$, $\mu R = r_a - r_b$, and φ is the angle of rotation about the axis joining the nuclei. The introduction of these new coordinates transforms equation 1-1 to the following:

$$\frac{\partial}{\partial\lambda}\left[\left(\lambda^2 - 1\right)\frac{\partial\Psi}{\partial\lambda}\right] + \frac{\partial}{\partial\mu}\left[\left(1 - \mu^2\right)\frac{\partial\Psi}{\partial\mu}\right] + \left[\frac{1}{\lambda^2 - 1} + \frac{1}{1 - \mu^2}\right]\frac{\partial^2\Psi}{\partial\varphi^2}$$
$$+ \left[\frac{R^2E(\lambda^2 - \mu^2)}{2} + 2R\lambda\right]\Psi = 0. \tag{1-2}$$

The usual method for solving equations of this form is to write $\Psi(\lambda, \mu, \varphi) = L(\lambda)M(\mu)\Phi(\varphi)$. Then, letting $p^2 = -\frac{1}{2}R^2E$, we find that the equation is separable into three differential equations

$$\frac{d^2\Phi}{d\varphi^2} = -m^2\Phi \tag{1-3}$$

$$\frac{d}{d\lambda}\left[(\lambda^2 - 1)\frac{dL}{d\lambda}\right] + \left[A + 2R\lambda - p^2\lambda^2 - \frac{m^2}{\lambda^2 - 1}\right]L = 0 \tag{1-4}$$

$$\frac{d}{d\mu}\left[(1-\mu^2)\frac{dM}{d\mu}\right]+\left[-A+p^2\mu^2-\frac{m^2}{1-\mu^2}\right]M=0. \qquad (1\text{-}5)$$

The solution of the Φ equation leads to m values of 0, $\pm$ 1, $\pm$ 2.... The next step in the procedure for finding the energy consists of finding the relation between the separation constant A and the parameter p such that $M(\mu)$ possesses the properties of a satisfactory wave function. Once this relation is found, it is used to obtain the characteristic values of p from which the energy may be obtained.

Burrau[1] solved these equations for the case m = 0, the ground state of the ion, by a process of numerical integration. However, because the state of the art of numerical calculation in 1927 was quite below that of today, Burrau's results did not yield energies of as high an accuracy as would have been desired. Some years later, Hylleras[2] and Jaffe[3] managed to get explicit solutions to equations 1-4 and thus obtained energies accurate to four decimals.

With the advent of the new computers, energies accurate to seven decimals have been obtained by Wind[4] and to thirteen decimals by Peek[5]. Both of these calculations have shown that the equilibrium value of R, R_e, is $2.0a_0$. However, for all their accuracy, these authors still seem to have missed the true value of R_e. In 1941, Johnson*[6] calculated R_e as $1.997466a_0$, and a very recent calculation[7] has yielded an R_e equal to approximately $1.9972a_0$.

Inasmuch as the exact solutions of equations 1-4 and 1-5 presented so many difficulties, many theoreticians tried to surmount the obstacles by applying variation or perturbation techniques to simple wavefunctions in hope that they would give rise to acceptable values of the energy. For example, Pauling[8] used a linear combination of atomic hydrogen wave functions ($\Psi_{1sa} + \Psi_{1sb}$; LCAO) and obtained an equilibrium separation of $2.5a_0$, in error by 25 per cent, and a dissociation energy that was in error by approximately 36 per cent. Finkelstein

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*Johnson's calculation, however, included terms to correct for nuclear motion.

and Horowitz[9] used the same function as Pauling but introduced an effective nuclear charge Z. Their unnormalized function may be written as follows:

$$\varphi = \exp(-Zr_a) + \exp(-Zr_b). \tag{1-6}$$

Minimization of the energy with respect to the parameter Z led to an energy at $R_e = 2.0a_0$ in error by about 19 per cent, a big improvement over the Pauling function. Dickinson[10] decided to polarize the 1s orbitals of hydrogen by adding some 2p character to the wavefunction. The result was a rather complicated looking molecular orbital which led to energies in error by about five per cent.

Perhaps the most accurate wavefunction to be introduced at the early stages of the development of molecular structure theory was that of Guillemin and Zener[11]. They used a function of the form:

$$\varphi = \exp(-Z'r_a)\exp(-Z''r_b) + \exp(-Z''r_a)\exp(-Z'r_b). \tag{1-7}$$

The energies, as calculated by Kim et al.[12], agreed with the exact to within $0.0003e^2/a_0$ for the full range of internuclear separations.

The prime objectives of all the workers who used the variation functions described above was to design a wavefunction which, while simple in appearance and easy to handle, gave energies that are nearly exact. While almost all of the functions satisfy the first two objectives none of them satisfy the last as much as we would like. It was with this thought in mind that Lyon et al.[13], using the Guillemin-Zener function as their zeroth-order function, applied Rayleigh-Schroedinger perturbation theory to obtain energies for the hydrogen molecular ion accurate to six decimals.

Some time later, Radel et al.[14] published energies for the H_2^+ ion obtained from the following "geometric mean" variation function:

$$\varphi = \alpha(\Psi_{1sa} + \Psi_{1sb}) + \beta(\Psi_{1sa}\Psi_{1sb})^{1/2}. \tag{1-8a}$$

Upon transformation to confocal coordinates, this function becomes

$$\varphi = (Z^3/\pi)^{1/2}\exp(-ZR\lambda/2)[2\alpha\cosh(ZR\mu/2) + \beta]. \tag{1-8b}$$

In this function Z is an effective charge, while α and β are related through normalization. It was found that the Geometric Mean (GM) function gave even

better energies than the Guillemin-Zener function.

Possessing, as we did, a function as accurate as the GM function, it was decided to apply Rayleigh-Schroedinger perturbation theory to determine the energies for the ground state of the hydrogen molecular ion. As will be seen subsequently, the energy, corrected to third order, agrees with the results of Wind[4] to within $1.2 \times 10^{-6} e^2/a_0$ for virtually all internuclear separations. It is the calculation of the third order energy that will occupy us throughout the remainder of this thesis.

CHAPTER 2

THE RAYLEIGH-SCHROEDINGER PERTURBATION THEORY

In the next few paragraphs the salient features of perturbation theory, as they apply to the present calculations, will be described. The aim here is to fill the gaps found in the treatments of some of the well-known treatises on quantum mechanics[15,16,23,25]. An excellent survey of the recent developments in perturbation theory, replete with references, may be found in references (17) and (18).

The main assumption of perturbation theory is that a wavefunction for a given state Φ_k may be expanded as a power series[19] in some parameter λ as

$$\Phi_k = \sum_{n=0}^{\infty} \lambda^n \varphi_k^{(n)} = \varphi_k^{(0)} + \lambda\varphi_k^{(1)} + \lambda^2\varphi_k^{(2)} + \ldots . \tag{2-1}$$

Furthermore, we assume that the Hamiltonian and the energy can also be expanded in the same way, viz.:

$$H \cong H^{(0)} + \lambda H^{(1)} + \lambda^2 H^{(2)} + \ldots \quad = H^{(0)} + \lambda V \tag{2-2}$$

$$E_k = e_k^{(0)} + \lambda e_k^{(1)} + \lambda^2 e_k^{(2)} + \ldots . \tag{2-3}$$

The reason for performing these expansions is that we are studying a problem for which the Hamiltonian H is only slightly different from the Hamiltonian ($H^{(0)}$) of some problem whose solution is known exactly. This being the case, we assume that the Hamiltonian for the unsolved problem at hand will be the sum of the operator $H^{(0)}$ and some correction terms which we label $\lambda H^{(1)}$, $\lambda^2 H^{(2)}$, etc. This same type of reasoning is used to explain the expansion of Φ_k and E_k. That is, $\varphi_k^{(0)}$ and $e_k^{(0)}$ are approximate solutions to the problem at hand, and it is expected that the correction terms $\lambda\varphi_k^{(1)} + \lambda^2\varphi_k^{(2)} + \ldots$ and $\lambda e_k^{(1)} + \lambda^2 e_k^{(2)} + \ldots$ when added to the approximate solutions will give us eigenfunctions and eigenvalues, respectively, as exact as we wish for our actual problem. Thus, it is obvious that as λ approaches 0, our actual problem reduces to the problem for which the solution is known (i.e., the unperturbed problem). For these reasons, it is customary to refer to V as a "perturbation potential."

The Schroedinger equation for the perturbed problem is $H\Phi_k = E_k\Phi_k$, or

$$(H - E_k)\Phi_k = 0. \tag{2-4}$$

Suppose now that we have in our possession a wavefunction $\varphi_k^{(0)}$ which gives a good approximation to the energy for the problem at hand. Suppose further that we can find a Hamiltonian, $H^{(0)}$, such that

$$H^{(0)}\varphi_k^{(0)} = e_k^{(0)}\varphi_k^{(0)}. \tag{2-5}$$

If this be the case then $H^{(0)}$ is the unperturbed Hamiltonian and $\varphi_k^{(0)}$ is the unperturbed wavefunction. The determination of $H^{(0)}$ is, of course, a mathematical construction* which is necessary if we are to apply the formalism of perturbation theory as expressed in equations 2-1, 2-2 and 2-3.

Substituting equations 2-1, 2-2 and 2-3 into 2-4 gives

$$(H - E_k)\Phi_k = [H^{(0)} + \lambda V - \sum_{n=0}^{\infty} \lambda^n e_k^{(n)}] \sum_{n=0}^{\infty} \lambda^n \varphi_k^{(n)} = 0 \tag{2-6a}$$

or

$$[H^{(0)} + \lambda V - e_k^{(0)} - \lambda e_k^{(1)} - \lambda^2 e_k^{(2)} - \ldots][\varphi_k^{(0)} + \lambda\varphi_k^{(1)} + \lambda^2\varphi_k^{(2)} + \ldots] = 0. \tag{2-6b}$$

Collecting powers of λ we obtain

$$[H^{(0)}\varphi_k^{(0)} - e_k^{(0)}\varphi_k^{(0)}] + \sum_{n=1}^{\infty} \lambda^n [(H^{(0)} - e_k^{(0)})\varphi_k^{(n)} + V\varphi_k^{(n-1)} - \sum_{m=1}^{n} e_k^{(m)}\varphi_k^{(n-m)}] = 0. \tag{2-6c}$$

Making use of equation 2-5 gives

$$\sum_{n=1}^{\infty} \lambda^n \{[H^{(0)} - e_k^{(0)}]\,\varphi_k^{(n)} + V\varphi_k^{(n-1)} - \sum_{m=1}^{n} e_k^{(m)}\varphi_k^{(n-m)}\} = 0. \tag{2-6d}$$

If this last equation is to remain valid for all values of λ, then the coefficient of each power of λ must separately be identical to zero. Thus, dropping the subscript k in order to prevent the notation from becoming too clumsy, we find

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*That a Hamiltonian, $H^{(0)}$, can be found such that equation 2-5 is true will be shown in Chapter 3.

$$[H^{(0)} - e^{(0)}]\varphi^{(n)} + V\varphi^{(n-1)} = \sum_{m=1}^{n} e^{(m)}\varphi^{(n-m)}. \tag{2-7}$$

Placing n equal to 1 in equation 2-7 yields:

$$[H^{(0)} - e^{(0)}]\varphi^{(1)} + V\varphi^{(0)} = e^{(1)}\varphi^{(0)}. \tag{2-8a}$$

which becomes

$$[H^{(0)} - e^{(0)}]\varphi^{(1)} + [V - e^{(1)}]\varphi = 0 \tag{2-8b}$$

where the zeroth-order wavefunction is represented by φ; i.e., $\varphi = \varphi^{(0)}$. The solution of equation 2-8 gives us $\varphi^{(1)}$, and from $\varphi^{(1)}$ we can find the first, second and third order corrections to the unperturbed energy $e^{(0)}$.

For example, taking the scalar product of equation 2-8 with φ, and requiring that $\langle \varphi, \varphi^{(1)} \rangle = 0$, we find:

$$\langle \varphi, H^{(0)}\varphi^{(1)} \rangle - e^{(0)}\langle \varphi, \varphi^{(1)} \rangle + \langle \varphi, V\varphi \rangle = e^{(1)}\langle \varphi, \varphi \rangle$$

or

$$e^{(1)} = \langle \varphi, H^{(0)}\varphi^{(1)} \rangle + \langle \varphi, V\varphi \rangle \tag{2-9a}$$

since φ is normalized. As $H^{(0)}$ is an Hermitian operator, the following is true: $\langle \varphi, H^{(0)}\varphi^{(1)} \rangle = \langle \varphi^{(1)}, H^{(0)}\varphi \rangle = \langle \varphi^{(1)}, e^{(0)}\varphi \rangle = 0$. Hence

$$e^{(1)} = \langle \varphi, V\varphi \rangle. \tag{2-9b}$$

Letting n = 2 in equation 2-7 we find:

$$[H^{(0)} - e^{(0)}]\varphi^{(2)} + V\varphi^{(1)} = e^{(1)}\varphi^{(1)} + e^{(2)}\varphi. \tag{2-10}$$

We now take the scalar product of 2-10 with φ and integrate over all space. Again, using the Hermitian property of $H^{(0)}$, we find that $\langle \varphi, H^{(0)}\varphi^{(2)} \rangle$ $= \langle \varphi^{(2)}, H^{(0)}\varphi \rangle = \langle \varphi^{(2)}, e^{(0)}\varphi \rangle$. Cancelling and re-arranging terms gives the formula for the second order correction to the energy:

$$e^{(2)} = \langle \varphi, V\varphi^{(1)} \rangle. \tag{2-11}$$

In a similar manner, placing n = 3 in equation 2-7 leads to the following formula for $e^{(3)}$:

$$e^{(3)} = \langle \varphi, (V - e^{(1)})\varphi^{(2)} \rangle \tag{2-12a}$$

$$= \langle (V - e^{(1)})\varphi, \varphi^{(2)} \rangle \tag{2-12b}$$

From equation 2-8 we find that $(V - e^{(1)})\varphi = -(H^{(0)} - e^{(0)})\varphi^{(1)}$. Substitution of this into 2-12b, and making use of the Hermitian property of $H^{(0)}$, allows us to perform the following chain of operations:

$$e^{(3)} = -\langle (H^{(0)} - e^{(0)})\varphi^{(1)}, \varphi^{(2)} \rangle \tag{2-12c}$$

$$= -\langle \varphi^{(1)}, (H^{(0)} - e^{(0)})\varphi^{(2)} \rangle. \tag{2-12d}$$

However, from equation 2-10 we notice that $-(H^{(0)} - e^{(0)})\varphi^{(2)} = (V - e^{(1)})\varphi^{(1)} - e^{(2)}\varphi$. Using this relation in 2-12d we find that:

$$e^{(3)} = \langle \varphi^{(1)}, (V - e^{(1)})\varphi^{(1)} \rangle - \langle \varphi^{(1)}, e^{(2)}\varphi \rangle; \tag{2-12e}$$

and taking into account the requirement that $\langle \varphi, \varphi^{(1)} \rangle = 0$, we finally obtain:

$$e^{(3)} = \langle \varphi^{(1)}, (V - e^{(1)})\varphi^{(1)} \rangle. \tag{2-12f}$$

Equations 2-9b, 2-11 and 2-12f give the first, second and third order corrections, respectively, to the energy, and as can be seen, involve only the zeroth or first order wavefunctions. This is an example of the theorem[20] which states that if we know the n th order wavefunction we can calculate the energy accurate to the (2n + 1) st order. However, before any calculation of energies (other than the first order energy) can be effected, we must first solve equation 2-8 for $\varphi^{(1)}$.

In most treatises on quantum mechanics, the usual procedure of solution is to assume that $\varphi^{(1)}$ can be further expanded in terms of a complete set of known orthonormal functions φ_j; i.e.,

$$\varphi^{(1)} = \sum_j a_j \varphi_j \quad .$$

Substituting this expansion into equation 2-7 leads to the familiar formulas for the first, second and third order corrections to the energy (see references (15), (16), (25)). The method gives an exact solution for the energy of the hydrogen atom in an electric field. (Stark Effect: see reference (25), pages 191—198.) A recent application of the method[21] to the hydrogen molecular ion,

however, does not give results as accurate as one would like, even through fifth order. This does not mean that the method is not good. On the contrary, the lack of accuracy was attributed to the inadequacy of the zeroth-order function at large internuclear separations. Our calculation utilizes the geometric mean function (1-8) as the zeroth-order approximation and this function gives very good results for both large and small internuclear separations[14].

Hirschfelder et al. discuss another method[18] which permits us to solve the perturbation equation without explicitly stating the form of $\varphi^{(1)}$. The reasoning behind the method is to let the perturbation potential, V, shape the form of the wavefunction, and thus eliminate the need to guess the form before starting the calculations.* Thus, using their method, we let $\varphi^{(1)} = F\varphi$, where F is a scalar function of the coordinates to be determined. Substituting into equation 2-8, and simplifying notation such that $H^{(0)} = H_0$, we find:

$$(H_0 - e^{(0)})F\varphi + (V - e^{(1)})\varphi = 0. \tag{2-13}$$

Then, requiring that $\langle \varphi, F\varphi \rangle = 0$, substitution of $\varphi^{(1)} = F\varphi$ into equations 2-11 and 2-12f gives

$$e^{(2)} = \langle \varphi, VF\varphi \rangle \tag{2-14}$$

$$e^{(3)} = \langle F\varphi, (V - e^{(1)})F\varphi \rangle. \tag{2-15}$$

It is the solution of equations 2-13, 2-14 and 2-15 that constitutes the major portion of this thesis.

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*This same method is used in Appendix A to solve the Stark Effect problem for the hydrogen atom. It will be seen there that the exact solution is obtained without the labor involved in assuming an expansion interms of a spectral distribution of a complete function set.

CHAPTER 3

PERTURBATION THEORY AND THE GEOMETRIC MEAN VARIATION FUNCTION

In order to apply perturbation theory we need to find an H_0 and an $e^{(0)}$ such that equation 2-5 is satisfied; i.e., such that $H_0\varphi = e^{(0)}\varphi$ where φ, the zeroth-order wavefunction, is now taken as the Geometric Mean (GM) function 1-8. $e^{(0)}$, of course, will be the zeroth-order approximation to the actual ground state energy of the hydrogen molecular ion.

Let us begin by constructing a Sternheimer Hamiltonian[17]

$$H_0 = -\tfrac{1}{2}\nabla^2 + U \tag{3-1}$$

such that

$$H_0\varphi = -(Z^2/2)\varphi = e^{(0)}\varphi \tag{3-2}$$

In effect, we wish to construct H_0 such that the zeroth-order energy is that of a hydrogen-like system.

Combining equations 3-1 and 3-2 we find that

$$-\tfrac{1}{2}\nabla^2\varphi + U\varphi = -(Z^2/2)\varphi$$

or

$$U = -\frac{Z^2}{2} + \frac{\tfrac{1}{2}\nabla^2\varphi}{\varphi} \tag{3-3}$$

Operation on the GM function by ∇^2, followed by the appropriate substitutions into equation 3-3, ultimately yields the Sternheimer potential

$$U = -\frac{Z[4\lambda + ZR(1-\mu^2)]}{2R(\lambda^2-\mu^2)} + \frac{\alpha Z[ZR(1-\mu^2)\cosh(ZR\mu/2) - 4\mu\sinh(ZR\mu/2)]}{R(\lambda^2-\mu^2)[2\alpha\cosh(ZR\mu/2)+\beta]} \tag{3-4}$$

The true Hamiltonian for H_2^+, ignoring internuclear repulsion, is given by

$$H = -\frac{1}{2}\nabla^2 - \frac{1}{r_a} - \frac{1}{r_b} \tag{3-5a}$$

$$= -\frac{1}{2}\nabla^2 - \frac{4\lambda}{R(\lambda^2 - \mu^2)} \tag{3-5b}$$

If we use the Sternheimer Hamiltonian as our zeroth-order Hamiltonian (see equation 2-2), the perturbation potential is simply the difference between the true Hamiltonian and the Sternheimer Hamiltonian; i.e., assuming full perturbation,

$$V = H - H_0 \tag{3-6a}$$

$$= -\frac{4\lambda}{R(\lambda^2 - \mu^2)} - U \tag{3-6b}$$

after substitution of equations 3-1 and 3-5b. This is the potential which must be used in equation 2-8 as well as in the expression for the first, second and third order corrections to the energy.

The first order correction to the energy is given by equation 2-9b; i.e.,

$$e^{(1)} = \langle \varphi, V\varphi \rangle \tag{3-7a}$$

$$= \langle \varphi, (H - H_0)\varphi \rangle \tag{3-7b}$$

$$= \langle \varphi, H\varphi \rangle - e^{(0)} \tag{3-7c}$$

$$= \langle \varphi, H\varphi \rangle + Z^2/2. \tag{3-7d}$$

The integral $\langle \varphi, H\varphi \rangle$ is the energy corresponding to the GM function. This integral has been evaluated by Radel et al.[14] for various internuclear separations. In effect, then, $e^{(1)}$ has been determined.

The second and third order corrections to the energy (equations 2-11 and 2-12f) require a knowledge of $\varphi^{(1)}$. Hence, the evaluation of these corrections reduces to the problem of obtaining $\varphi^{(1)}$ by solving equation 2-8

$$[H_0 - e^{(0)}]\varphi^{(1)} + [V - e^{(1)}]\varphi = 0. \tag{3-8}$$

It is possible to separate this equation by making the substitution (see discussion on page 9)

$$\varphi^{(1)} = \varphi \cdot F(\lambda, \mu) \tag{3-9a}$$

$$= \varphi \cdot [L(\lambda) + M(\mu)] \tag{3-9b}$$

where $F(\lambda,\mu) = L(\lambda) + M(\mu)$. Substitution of 3-9 into 3-8 leads to the differential equation

$$[H_0 - e^{(0)}][L(\lambda) + M(\mu)]\varphi + [V - e^{(1)}]\varphi = 0 \tag{3-10}$$

Using H_0 as defined by equations 3-1 and 3-4, we find that equation 3-10 is separable in the coordinates λ and μ; i.e., if we let D be the separation constant, we can obtain the two differential equations:

$$\frac{d}{d\lambda}[(\lambda^2 - 1)\exp(-ZR\lambda)\frac{dL}{d\lambda}] = \exp(-ZR\lambda)\ [D - R(2 - Z)\lambda - R^2e^{(1)}\lambda^2/2], \tag{3-11}$$

and

$$\frac{d}{d\mu}[(1 - \mu^2)(2\alpha\cosh(ZR\mu/2) + \beta)^2\frac{dM}{d\mu}] = [2\alpha\cosh(ZR\mu/2) + \beta]^2 \tag{3-12}$$

$$\times\left\{-D + \frac{R^2\mu^2e^{(1)}}{2} + \frac{8\alpha ZR\mu\sinh(ZR\mu/2) + Z^2R^2\beta(1 - \mu^2)}{4[2\alpha\cosh(ZR\mu/2) + \beta]}\right\}.$$

The separation constant can easily be obtained by integrating equation 3-11 over all λ. Keeping in mind that $dL/d\lambda$ must always be finite, we obtain

$$D = \frac{e^{(1)}}{Z^2}[1 + ZR + Z^2R^2/2] + \frac{(2 - Z)(1 + ZR)}{Z}. \tag{3-13}$$

Equation 3-11 is now solved by integrating twice with respect to λ to yield

$$L(\lambda) = \left\{\frac{2 - Z}{Z} + \frac{e^{(1)}}{Z^2}\right\}\ln(1 + \lambda) + \frac{e^{(1)}R\lambda}{2Z} + F_1, \tag{3-14}$$

where F_1 is a constant of integration.*

In a similar fashion, we may integrate equation 3-12 once with respect to μ to obtain

$$(1 - \mu^2)[2\alpha\cosh a\mu + \beta]^2\frac{dM}{d\mu} = T_1\sinh 2a\mu + T_2\mu^2\sinh 2a\mu + T_3\sinh a\mu$$
$$+ T_4\mu^2\sinh a\mu + T_5\mu + T_6\mu^3 + T_7\mu\cosh 2a\mu$$

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*Substitution of $\lambda = 1$ into the result of the first equation shows that the first integration constant vanishes. F_1 is the second integration constant.

$$+ T_8 \mu \cosh a\mu + K = f(\mu) + K \tag{3-15}$$

where $a = ZR/2$ and K is an integration constant. The coefficients T_1 through T_8 depend on R, Z, α and β and are listed below:

$$T_1 = \frac{2\alpha^2}{Z^3R}[- Z^2D - Z^2 + e^{(1)}] \tag{3-16a}$$

$$T_2 = \frac{\alpha^2 R e^{(1)}}{Z} \tag{3-16b}$$

$$T_3 = \frac{4\alpha\beta}{Z^3R}[- 2DZ^2 + Z^4R^2/4 - 4Z^2 + 8e^{(1)}] \tag{3-16c}$$

$$T_4 = \frac{4\alpha\beta R}{Z}[e^{(1)} - Z^2/4] \tag{3-16d}$$

$$T_5 = - D[2\alpha^2 + \beta^2] + \frac{Z^2R^2\beta^2}{4} \tag{3-16e}$$

$$T_6 = \frac{R^2}{6}[- Z^2\beta^2/2 + 2\alpha^2 e^{(1)} + \beta^2 e^{(1)}] \tag{3-16f}$$

$$T_7 = \frac{2\alpha^2}{Z^2}[Z^2 - e^{(1)}] \tag{3-16g}$$

$$T_8 = \frac{4\alpha\beta}{Z^2}[2Z^2 - 4e^{(1)}] \tag{3-16h}$$

Inspection of $f(\mu)$ shows that $f(-\mu) = -f(\mu)$; i.e., $f(\mu)$ is an odd function. Hence, by requiring that $dM/d\mu$ remain finite at the limits $\mu = \pm 1$ we can easily show that when $\mu = -1$

$$K = -f(-1) = f(1) \tag{3-17a}$$

and when $\mu = +1$

$$K = -f(1) \tag{3-17b}$$

where

$$f(1) = (T_1 + T_2)\sinh 2a + (T_3 + T_4)\sinh a + T_5 + T_6 + T_7 \cosh 2a + T_8 \cosh a \tag{3-18}$$

Clearly, equation 3-17 can be valid only if

$$K = f(1) = 0 \tag{3-19}$$

All that remains in order to complete the specification of $M(\mu)$ is to integrate 3-15. This is an extremely difficult and tedious task, and the procedure will be described in detail in Chapter 4.

CHAPTER 4

SOLUTION OF THE μ EQUATION

Since $K = 0$, equation 3-15 becomes

$$(1 - \mu^2)[2\alpha \cosh a\mu + \beta]^2 \frac{dM}{d\mu} = T_1 \sinh 2a\mu + T_2\mu^2 \sinh 2a\mu + T_3 \sinh a\mu + T_4\mu^2 \sinh a\mu + T_5\mu + T_6\mu^3 + T_7\mu \cosh 2a\mu + T_8\mu \cosh a\mu \quad (4\text{-}1)$$

Hence,

$$M(\mu) = T_1\int X^{-1} \sinh 2a\mu\, d\mu + T_2\int X^{-1}\mu^2 \sinh 2a\mu\, d\mu + T_3\int X^{-1} \sinh a\mu\, d\mu + T_4\int X^{-1}\mu^2 \sinh a\mu\, d\mu + T_5\int X^{-1}\mu\, d\mu + T_6\int X^{-1}\mu^3\, d\mu + T_7\int X^{-1}\mu \cosh 2a\mu\, d\mu + T_8\int X^{-1}\mu \cosh a\mu\, d\mu + F_2 \quad (4\text{-}2)$$

where F_2 is an integration constant and $X = (1 - \mu^2)[2\alpha \cosh a\mu + \beta]^2$. Inspection of equation 4-2 shows that there are eight integrals that need to be evaluated in order to define $M(\mu)$. Let these integrals be denoted by T_1', T_2', ... T_8'. The procedure to be followed to effect the integrations is straightforward, albeit long and tedious. To conserve space, we decided to give the complete solution of only one of the integrals in detail. The other integrals may all be evaluated using the principles developed from this one solution.

We proceed now with the evaluation of the integral having the constant T_4 as its coefficient. This integral, T_4', is

$$T_4' = \int \frac{\mu^2 \sinh a\mu\, d\mu}{(1 - \mu^2)[2\alpha \cosh a\mu + \beta]^2} \quad (4\text{-}3a)$$

$$= \frac{1}{4\alpha^2}\int \frac{\mu^2 \sinh a\mu\, d\mu}{(1 - \mu^2)[\cosh a\mu + \beta/2\alpha]^2} \quad (4\text{-}3b)$$

$$= -\frac{\partial}{\partial a}\frac{1}{4\alpha^2}\int \frac{\mu\, d\mu}{(1 - \mu^2)[\cosh a\mu + \beta/2\alpha]} \quad (4\text{-}3c)$$

I.e., determination of T_4' reduces to the problem of integrating

$$I_5 = \int \frac{\mu\, d\mu}{(1 - \mu^2)[\cosh a\mu + \beta/2\alpha]} \quad (4\text{-}4)$$

We now distinguish three cases:

CASE 1.: When $\beta/2\alpha < 1$, let $\beta/2\alpha = \cos g$; i.e., $g = \arccos(\beta/2\alpha)$. Using the following identity[22]

$$\frac{1}{\cosh a\mu + \cos g} = \frac{\operatorname{cosec} g}{2i}\left\{\tanh\left(\frac{a\mu + ig}{2}\right) - \tanh\left(\frac{a\mu - ig}{2}\right)\right\} \tag{4-5}$$

in equation 4-4 gives

$$I_5 = \frac{\operatorname{cosec} g}{2i}\left\{\int \frac{\mu \tanh\left\{\frac{a\mu + ig}{2}\right\}}{(1-\mu^2)}\,d\mu - \int \frac{\mu \tanh\left\{\frac{a\mu - ig}{2}\right\}}{(1-\mu^2)}\,d\mu\right\} \tag{4-6a}$$

$$= \frac{\operatorname{cosec} g}{2i}\left\{\int \frac{\mu \tanh\,(+)}{(1-\mu^2)}\,d\mu - \int \frac{\mu \tanh\,(-)}{(1-\mu^2)}\right\}d\mu \tag{4-6b}$$

$$= \frac{\operatorname{cosec} g}{4i}\left[Q_1 - Q_2 - Q_3 + Q_4\right] \tag{4-6c}$$

where (+) and (–) are $(a\mu + ig)/2$ and $(a\mu - ig)/2$, respectively. The Q's are integrals given by

$$Q_1 = \int \frac{\tanh\,(+)}{1-\mu}\,d\mu \tag{4-7a}$$

$$Q_2 = \int \frac{\tanh\,(+)}{1+\mu}\,d\mu \tag{4-7b}$$

$$Q_3 = \int \frac{\tanh\,(-)}{1-\mu}\,d\mu \tag{4-7c}$$

$$Q_4 = \int \frac{\tanh\,(-)}{1+\mu}\,d\mu \tag{4-7d}$$

In order to determine the Q's, we follow Lyon et al.[13] and expand the hyperbolic tangent in the series

$$\tanh\,(\pm) = \tanh\,\frac{a\mu \pm ig}{2} = 4(a\mu \pm ig)\sum_{n=0}^{\infty}\left[(2n+1)^2\pi^2 + (a\mu \pm ig)^2\right]^{-1}. \tag{4-8}$$

Substitution of equation 4-8 into equations 4-7, interchanging the processes of summation and integration, expanding into partial fractions, and integrating term by term ultimately gives

$$Q_1 = -4(a+ig)\ln(1-\mu)\sum_{n=0}^{\infty} D_1^{-1} + 2(a+ig)\sum_{n=0}^{\infty} D_1^{-1}\ln X_+$$
$$-4\sum_{n=0}^{\infty}(2n+1)\pi D_1^{-1}\arctan\frac{a\mu + ig}{(2n+1)\pi} \tag{4-9a}$$

$$Q_2 = -4(a - ig)\ln(1 + \mu)\sum_{n=0}^{\infty} D_2^{-1} + 2(a - ig)\sum_{n=0}^{\infty} D_2^{-1}\ln X_+$$

$$+ 4\sum_{n=0}^{\infty} (2n + 1)\pi D_2^{-1} \arctan \frac{a\mu + ig}{(2n + 1)\pi} \tag{4-9b}$$

$$Q_3 = -4(a - ig)\ln(1 - \mu)\sum_{n=0}^{\infty} D_2^{-1} + 2(a - ig)\sum_{n=0}^{\infty} D_2^{-1}\ln X_-$$

$$- 4\sum_{n=0}^{\infty} (2n + 1)\pi D_2^{-1} \arctan \frac{a\mu - ig}{(2n + 1)\pi} \tag{4-9c}$$

$$Q_4 = -4(a + ig)\ln(1 + \mu)\sum_{n=0}^{\infty} D_1^{-1} + 2(a + ig)\sum_{n=0}^{\infty} D_1^{-1}\ln X_-$$

$$+ 4\sum_{n=0}^{\infty} (2n + 1)\pi D_1^{-1} \arctan \frac{a\mu - ig}{(2n + 1)\pi} \tag{4-9d}$$

where

$$D_1 = (2n + 1)^2\pi^2 + (a + ig)^2 \tag{4-10a}$$

$$D_2 = (2n + 1)^2\pi^2 + (a - ig)^2 \tag{4-10b}$$

$$X_+ = (2n + 1)^2\pi^2 + (a\mu + ig)^2 \tag{4-10c}$$

$$X_- = (2n + 1)^2\pi^2 + (a\mu - ig)^2 \tag{4-10d}$$

Substitution of the Q's (equations 4-9a through 4-9d) into equation 4-6c gives I_5. A final substitution of I_5 into equation 4-3c followed by the appropriate operations yields

$$T_4' = -\frac{1}{4\alpha^2}\frac{\operatorname{cosec} g}{4i}\frac{\partial}{\partial a}[Q_1 - Q_2 - Q_3 + Q_4] \tag{4-11a}$$

Expansion, followed by collection of terms (utilizing series 4-8, identity 4-5 and the additional identity $\ln(1 - \mu^2) = \ln(1 - \mu) + \ln(1 + \mu)$), gives

$$4\alpha^2 T_4' = \frac{1}{2}\frac{\partial}{\partial a}\frac{1}{\cosh a + \cos g}\ln(1 - \mu^2)$$

$$- \frac{\operatorname{cosec} g}{2i}\frac{\partial}{\partial a}\sum_{n=0}^{\infty}\left\{\frac{a + ig}{D_1} - \frac{a - ig}{D_2}\right\}\{\ln X_+ + \ln X_-\} \tag{4-11b}$$

$$+ \frac{\operatorname{cosec} g}{i}\frac{\partial}{\partial a}\sum_{n=0}^{\infty}(2n + 1)\pi\left\{\frac{1}{D_1} + \frac{1}{D_2}\right\}\left\{\arctan \frac{a\mu + ig}{(2n + 1)\pi}\right.$$

$$- \arctan \frac{a\mu - ig}{(2n+1)\pi} \Bigg\}$$

$$= - \frac{(\sinh a)\,(\ln[1-\mu^2])}{2(\cosh a + \cos g)^2} - \frac{\operatorname{cosec} g}{2i} \sum_{n=0}^{\infty} \left\{ \frac{1}{D_1} - \frac{1}{D_2} - \frac{2(a+ig)^2}{(D_1)^2} + \frac{2(a-ig)^2}{(D_2)^2} \right\}$$

$$\times \left\{ \ln X_+ + \ln X_- \right\} - \frac{2\operatorname{cosec} g}{i} \sum_{n=0}^{\infty} (2n+1)\pi \left\{ \frac{a+ig}{(D_1)^2} + \frac{a-ig}{(D_2)^2} \right\} \left\{ \arctan \frac{a\mu + ig}{(2n+1)\pi} \right.$$

$$\left. - \arctan \frac{a\mu - ig}{(2n+1)\pi} \right\} - \frac{\mu \operatorname{cosec} g}{i} \sum_{n=0}^{\infty} \left\{ \frac{a+ig}{D_1} - \frac{a-ig}{D_2} \right\} \left\{ \frac{a\mu + ig}{X_+} + \frac{a\mu - ig}{X_-} \right\}$$

$$+ \frac{\mu \operatorname{cosec} g}{i} \sum_{n=0}^{\infty} (2n+1)^2 \pi^2 \left\{ \frac{1}{D_1} + \frac{1}{D_2} \right\} \left\{ \frac{1}{X_+} - \frac{1}{X_-} \right\} \qquad (4\text{-}11c)$$

CASE 2.: When $\beta/2\alpha > 1$, let $\beta/2\alpha = \cosh g = \cos ig$. Equation 4-4 then becomes

$$I_5 = \int \frac{\mu\, d\mu}{(1-\mu^2)[\cosh a\mu + \cos ig]}$$

The previous results, obtained for Case 1, now apply here provided that g in equations 4-5 through 4-11 is replaced by ig.

CASE 3.: When $\beta/2\alpha = 1$, $g = 0$ and the preceding formulas cannot be used since the cosec g becomes infinite. Under these circumstances, equation 4-4 becomes

$$I_5 = \int \frac{\mu\, d\mu}{(1-\mu^2)[\cosh a\mu + 1]}$$

Using the identity

$$2\cosh^2 (a\mu/2) = \cosh a\mu + 1$$

gives

$$I_5 = \frac{1}{2} \int \frac{\mu \operatorname{sech}^2 (a\mu/2)\, d\mu}{(1-\mu)^2}$$

This integral was considered by Lyon et al. and the details may be found in reference (13).

On page 20 is a summary of the integrals T_1' through T_8' (defined on page 15) for the case $\beta/2\alpha < 1$. Replacement of g by ig gives the integrals when $\beta/2\alpha > 1$. Substitution of these integrals into equation 4-2 will, of course,

give $M(\mu)$. In this tabulation, $A_1 = (\cosh a\mu + \cos g)^{-1}$ and $X = (1 - \mu^2)(2\alpha\cosh a\mu + \beta)^2$. Symbols not defined below have been defined previously in the text. Case 3 ($\beta/2\alpha = 1$) did not occur in our calculations and we will not consider it further. We will see, however, that when $\beta/2\alpha \simeq 1$ the series solutions for Cases 1 and 2 are sufficiently perturbed so as to give erroneous energy results. In Chapter 5 we will handle this situation numerically using an integral representation for the series expansion for $M(\mu)$.

$$T'_1 = \int X^{-1} \sinh 2a\mu \, d\mu = \frac{1}{2\alpha^2}\left[I_2 + \frac{1}{a}A_1 \cos g - 2T'_4 \cos g\right]$$

$$T'_2 = \int X^{-1} \mu^2 \sinh 2a\mu \, d\mu = \frac{1}{2\alpha^2} I_3 - 2T'_4 \cos g$$

$$T'_3 = \int X^{-1} \sinh a\mu \, d\mu = -\frac{1}{4\alpha^2 a} A_1 + T'_4$$

$T'_4 = \int X^{-1}\mu^2 \sinh a\mu \, d\mu =$ (expression for T'_4 is given on page 17)

$$T'_5 = \int X^{-1}\mu \, d\mu = \frac{\operatorname{cosec}^2 g}{4\alpha^2}\left[P_3 - I_5 \cos g\right]$$

$$T'_6 = \int X^{-1}\mu^3 \, d\mu = \frac{1}{4\alpha^2}\left[(P_3 - I_5 \cos g) \operatorname{cosec}^2 g - I_4\right]$$

$$T'_7 = \int X^{-1}\mu \cosh 2a\mu \, d\mu = \frac{1}{4\alpha^2}\left[-\ln(1-\mu^2) + \cos 2g \operatorname{cosec}^2 g (P_3 - I_5 \cos g) - 4I_5 \cos g\right]$$

$$T'_8 = \int X^{-1}\mu \cosh a\mu \, d\mu = \frac{1}{4\alpha^2}\left[(1 + \operatorname{cosec}^2 g \cos^2 g) I_5 - P_3 \operatorname{cosec}^2 g \cos g\right]$$

where

$$I_2 = \int A_1 \frac{\sinh a\mu \, d\mu}{(1-\mu^2)} = \frac{1}{4}\left[Q_1 + Q_2 + Q_3 + Q_4\right]$$

$$I_3 = \int A_1 \frac{\mu^2 \sinh a\mu \, d\mu}{(1-\mu^2)} = -\frac{1}{a}\ln(\cosh a\mu + \cos g) + I_2$$

$$I_4 = \int (A_1)^2 \mu \, d\mu = -\frac{\operatorname{cosec}^2 g}{a^2}\ln(\cosh a\mu + \cos g) + \frac{\mu \operatorname{cosec}^2 g \sinh a\mu}{a(\cosh a\mu + \cos g)}$$

$$- \frac{1}{a} 2\mu \operatorname{cosec}^3 g \cos g \arctan\left[\tanh\frac{a\mu}{2} \tan\frac{g}{2}\right]$$

$$+ \frac{1}{a} 2\operatorname{cosec}^3 g \cos g \int_0^{\mu} \arctan\left[\tanh\frac{a\mu}{2} \tan\frac{g}{2}\right] d\mu$$

$$I_5 = \int A_1 \frac{\mu \, d\mu}{(1-\mu^2)} = \frac{\operatorname{cosec} g}{4i}\left[Q_1 - Q_2 - Q_3 + Q_4\right]$$

and

$$P_3 = \frac{\partial}{\partial a} I_2 = -\frac{1}{2}(A_1)^2 (1 + \cosh a \cos g) \ln(1 - \mu^2)$$

$$+ \frac{1}{2} \sum_{n=0}^{\infty} \left\{ \frac{1}{D_1} + \frac{1}{D_2} - \frac{2(a + ig)^2}{(D_1)^2} - \frac{2(a - ig)^2}{(D_2)^2} \right\} \left\{ \ln X_+ + \ln X_- \right\}$$

$$+ 2 \sum_{n=0}^{\infty} (2n + 1)\pi \left\{ \frac{a + ig}{(D_1)^2} - \frac{a - ig}{(D_2)^2} \right\} \left\{ \arctan \frac{a\mu + ig}{(2n + 1)\pi} \right.$$

$$\left. - \arctan \frac{a\mu - ig}{(2n + 1)\pi} \right\} + \mu \sum_{n=0}^{\infty} \left\{ \frac{a + ig}{D_1} + \frac{a - ig}{D_2} \right\} \left\{ \frac{a\mu + ig}{X_+} + \frac{a\mu - ig}{X_-} \right\}$$

$$- \mu \sum_{n=0}^{\infty} (2n + 1)^2 \pi^2 \left\{ \frac{1}{D_1} - \frac{1}{D_2} \right\} \left\{ \frac{1}{X_+} - \frac{1}{X_-} \right\}$$

CHAPTER 5

NUMERICAL CALCULATIONS

With the solutions for $L(\lambda)$ and $M(\mu)$ at our disposal, we are in a position to calculate the second and third order corrections to the energy, $e^{(2)}$ and $e^{(3)}$. However, as the solution for $M(\mu)$ is in the form of an infinite series, the integrals involved in the calculations become quite cumbersome. It is necessary, therefore, to obtain some idea as to the accuracy of the calculations. The following discussion details some of the problems encountered in the numerical solution of the M equation along with an analysis of the final results.

At the outset we notice that the various integrals (T_1' through T_8') in equation 4-2 possess terms involving $\ln(1-\mu^2)$; i.e., there is apparently a logarithmic singularity at the points $\mu = \pm 1$. One can show, however, that insertion of the expanded integrals into equation 4-2 followed by collection of terms involving $\ln(1-\mu^2)$ leads to the following expression for $M(\mu)$:

$$M(\mu) = m(\mu) + F_2 + \frac{\ln(1-\mu^2)}{2[2\alpha\cosh(ZR/2)+\beta]^2}\{(T_1+T_2)\sinh 2a + (T_3+T_4)\sinh a$$

$$+ T_5 + T_6 + T_7\cosh 2a + T_8\cosh a\} \qquad (5\text{-}1)$$

where $m(\mu)$ contains no singularities within the range of μ.

As shown by equations 3-18 and 3-19, the value of the bracketed sum is exactly equal to $f(1) = K = 0$. Therefore, the sum of the $\ln(1-\mu^2)$ terms that arise from the solution of 4-2 is exactly 0. This being the case, we can discuss the numerical solution of $M(\mu)$ and/or its component integrals by computing their values <u>without</u> including the terms involving $\ln(1-\mu^2)$.

Each of the integrals T_1' through T_8' (less the terms containing $\ln(1-\mu^2)$) can be evaluated by direct series solution. For example, the solution of T_3' was evaluated using 500 terms of the series and a computer printout was obtained to show its behavior upon the addition of each succeeding term. Some of the results are given in Table I (page 23) for $\mu = 1$. This table shows that the value of T_3' changes by only 1×10^{-10} as early as the 200th term for $R = 2$ and the 250th term

TABLE Ia. Values of the integral T_3' ($\mu = 1$) vs the number of terms (N) used in its series solution.[a,b]

N	$T_3'(R = 2)$	$T_3'(R = 2,$ single precision)	$T_3'(R = 4)$
1	.2540953857	.254199	.0126766055
50	.2515921760	.251697	.0083430549
51	.2515921530	.251697	.0083430139
99	.2515918067	.251697	.0083423985
100	.2515918049	.251697	.0083423953
149	.2515917623	.251697	.0083423196
150	.2515917619	.251697	.0083423189
199	.2515917508	.251697	.0083422991
200	.2515917507	.251697	.0083422989
249	.2515917465	.251697	.0083422916
250	.2515917465	.251697	.0083422915
299	.2515917446	.251697	.0083422881
300	.2515917446	.251697	.0083422880
349	.2515917436	.251697	.0083422863
350	.2515917435	.251697	.0083422863
399	.2515917430	.251697	.0083422852
400	.2515917430	.251697	.0083422852
449	.2515917426	.251697	.0083422846
450	.2515917426	.251697	.0083422846
499	.2515917424	.251697	.0083422842
500	.2515917424	.251697	.0083422842

TABLE Ib. d_{100} for the values in Table Ia.

N_{last}	d_{100} (R = 2)	d_{100} (R = 4)
200	542×10^{-10}	964×10^{-10}
300	61×10^{-10}	109×10^{-10}
400	16×10^{-10}	28×10^{-10}
500	6×10^{-10}	10×10^{-10}

[a] Terms containing $\ln(1 - \mu^2)$ were first subtracted from T_3' (see page 22).
[b] All distances are in Bohr radii.

for R = 4. The factor d_{100} gives us the difference between each successive 100 terms. Thus, at R = 4, we see that the value of the series for T_3' changes by 109×10^{-10} between the 200th and the 300th terms, but changes by only 10×10^{-10} between the 400th and the 500th terms. Furthermore, the increments are seen to be monotonically decreasing as would be expected for a well behaved function. On the basis of these results, we felt that it was reasonable to terminate our series with 400 term accuracy.

In order to check the above procedure, it was decided to compare the series results with those obtained by a direct numerical integration. Thus, for the integral T_3' we subtracted out the singularity and integrated the following expression by a fifteen point Gaussian scheme:

$$\int_0^1 \frac{1}{(1-\mu^2)} \left\{ \frac{\sinh a\mu}{(2\alpha \cosh a\mu + \beta)^2} - \frac{\mu \sinh a}{(2\alpha \cosh a + \beta)^2} \right\} d\mu \qquad (5\text{-}2)$$

The results of the Gaussian integration were compared with those of the series solution by evaluating the series at $\mu = 1$ and at $\mu = 0$ and subtracting these two values. As mentioned above, 400 terms were used to evaluate the series. Table II (page 25) compares the results for each of the integrals T_1' through T_8'. The values obtained in each case agree to within 1.2×10^{-10}.

In order to evaluate the energy corrections $e^{(2)}$ and $e^{(3)}$ from equations 2-11 and 2-12 we need the first order correction to the wavefunction; i.e., we must determine

$$\varphi^{(1)} = \varphi[L(\lambda) + M(\mu)] \qquad (5\text{-}3a)$$

$$= \varphi[1(\lambda) + m(\mu) + F^0] \qquad (5\text{-}3b)$$

where $F^0 = F_1 + F_2$, $m(\mu) = M(\mu) - F_2$, $1(\lambda) = L(\lambda) - F_1$, and φ is the Geometric Mean function (1-8). The constant F^0 may be determined from the condition that $\int \varphi \varphi^{(1)} d\tau = 0$ (see page 7). Thus,

$$F^0 = -\frac{\int \varphi^2 1(\lambda)\, d\tau + \int \varphi^2 m(\mu)\, d\tau}{\int \varphi^2 d\tau} \qquad (5\text{-}4a)$$

$$= -\int \varphi^2 1(\lambda) d\tau - \int \varphi^2 m(\mu) d\tau \qquad (5\text{-}4b)$$

TABLE II. Evaluation of the integrals T'_1 through T'_8 from $\mu = 0$ to $\mu = 1$.[a,b]

R = 2		
Integral No.	Value from series solution	Value from Gaussian quadrature
1	0.0082668155	0.0082668155
2	0.0084955554	0.0084955554
3	−0.0700192513	−0.0700192512
4	0.0528981380	0.0528981380
5	0.0128094570	0.0128094570
6	0.0121945550	0.0121945550
7	−0.0424470829	−0.0424470829
8	0.0174221196	0.0174221196

[a]Terms containing $\ln(1 - \mu^2)$ were first subtracted from integrals (see page 22).
[b]All distances are in Bohr radii.

since φ is normalized. Values of φ and $\Phi = \varphi + \varphi^{(1)}$ are given in Table III (page 27) and Appendix B for various internuclear separations. All values in these tables were computed numerically by a 15 point Gaussian scheme using the explicit form for $m(\mu)$; i.e., $m(\mu)$ was expressed as a series.

An alternate procedure for obtaining $\Phi = \varphi + \varphi^{(1)}$ involves using an integral representation for the series $m(\mu)$. From equations 5-3 and 5-4 we see that

$$\varphi^{(1)} = \varphi\{1(\lambda) + m(\mu) - \int\varphi^2 1(\lambda)\,d\tau - \int\varphi^2 m(\mu)\,d\tau\} \tag{5-5a}$$

$$= \varphi\{1(\lambda) + \int_0^\mu dm + m(0) - \int\varphi^2 1(\lambda)\,d\tau - \int\varphi^2[\int_0^\mu dm]\,d\tau$$

$$- \int\varphi^2 m(0)\,d\tau\} \tag{5-5b}$$

$$= \varphi\{1(\lambda) + \int_0^\mu dm - \int\varphi^2 1(\lambda)\,d\tau - \int\varphi^2[\int_0^\mu dm]\,d\tau\} \tag{5-5c}$$

since $\int\varphi^2 m(0)\,d\tau = m(0)\int\varphi^2\,d\tau = m(0)$. But, from equation 5-1,

$$\int_0^\mu dm = \int_0^\mu d(M - F_2) = \int_0^\mu dM = \int_0^\mu \frac{dM}{d\mu}d\mu \tag{5-6}$$

where $dM/d\mu$ is given by equation 3-15. Hence

$$\varphi^{(1)} = \varphi\{1(\lambda) + \int_0^\mu \frac{dM}{d\mu}\,d\mu - \int\varphi^2 1(\lambda)d\tau - \int\varphi^2[\int_0^\mu \frac{dM}{d\mu}]\,d\tau\} \tag{5-7}$$

Substitution of $dM/d\mu$ into equation 5-7, followed by numerical integration using a 15 point Gaussian scheme gives an alternate evaluation for $\varphi^{(1)}$. Table IV (page 29) compares selected values of $\Phi = \varphi + \varphi^{(1)}$ obtained in this manner with values obtained when $m(\mu)$ is expressed as a series. The results agree to within 1×10^{-6}. A more comprehensive tabulation of integral representation Φ values is given in Appendix C.

In Table V (page 30) we present values for the first order energy (Geometric Mean energy; E(1)), the second and third order corrections to the energy ($e^{(2)}$ and $e^{(3)}$), and the total energy corrected to second and third order (E(2) and E(3)). The values were obtained from a 15 point Gaussian scheme using the explicit expressions for $m(\mu)$ and compared to those of Wind[4] and Lyon et al.[13] where possible. The calculations were checked by replacing the

TABLE III. Calculated values of the normalized wavefunctions (series solution).[a,b]

		R = 0.2		R = 1.9972[c]		R = 2.0		R = 4.0		R = 9.0	
λ	μ	φ_{GM}	Φ	φ_{GM}	Φ	φ_{GM}	Φ	φ_{GM}	Φ	φ_{GM}	Φ
1.	0.0	1.26701	1.27513	0.31858	0.31519	0.31811	0.31473	0.12857	0.12682	0.01264	0.01167
	0.2	1.26733	1.25746	0.32360	0.32032	0.32314	0.31986	0.13543	0.13403	0.01631	0.01567
	0.4	1.26831	1.25843	0.33904	0.33599	0.33859	0.33554	0.15746	0.15689	0.03060	0.03085
	0.6	1.26994	1.26005	0.36604	0.36306	0.36562	0.36265	0.19927	0.19938	0.06826	0.06962
	0.8	1.27223	1.26232	0.40657	0.40305	0.40620	0.40269	0.26962	0.26901	0.16286	0.16449
	1.0	1.27518	1.26525	0.46363	0.45820	0.46334	0.45791	0.38325	0.37827	0.39873	0.39686
2.	0.0	1.04376	1.04076	0.08238	0.08274	0.08214	0.08250	0.01329	0.01301	0.00013	0.00011
	0.2	1.04403	1.04103	0.08368	0.08409	0.08344	0.08385	0.01399	0.01375	0.00017	0.00014
	0.4	1.04484	1.04184	0.08767	0.08820	0.08743	0.08796	0.01627	0.01610	0.00031	0.00029
	0.6	1.04618	1.04318	0.09465	0.09531	0.09441	0.09506	0.02059	0.02046	0.00071	0.00066
	0.8	1.04807	1.04506	0.10513	0.10581	0.10489	0.10556	0.02786	0.02761	0.00170	0.00157
	1.0	1.05050	1.04748	0.11989	0.12029	0.11965	0.12004	0.03961	0.03882	0.00416	0.00380
3.	0.0	0.85985	0.85974	0.02130	0.02077	0.02121	0.02068	0.00137	0.00123	0.00000	0.00000
	0.2	0.86007	0.85996	0.02164	0.02111	0.02154	0.02102	0.00144	0.00130	0.00000	0.00000
	0.4	0.86074	0.86063	0.02267	0.02214	0.02257	0.02205	0.00168	0.00152	0.00000	0.00000
	0.6	0.86185	0.86173	0.02447	0.02393	0.02438	0.02383	0.00212	0.00193	0.00000	0.00000
	0.8	0.86340	0.86329	0.02718	0.02657	0.02708	0.02646	0.00288	0.00261	0.00001	0.00001
	1.0	0.86540	0.86429	0.03100	0.03020	0.03089	0.03009	0.00409	0.00367	0.00004	0.00003
4.	0.0	0.70835	0.70935	0.70550	0.00507	0.00547	0.00504	0.00014	0.00010	0.00000	0.00000
	0.2	0.70853	0.70953	0.00559	0.00516	0.00556	0.00513	0.00014	0.00011	0.00000	0.00000
	0.4	0.70908	0.71008	0.00586	0.00541	0.00583	0.00538	0.00017	0.00013	0.00000	0.00000
	0.6	0.70999	0.71099	0.00632	0.00585	0.00629	0.00581	0.00022	0.00017	0.00000	0.00000
	0.8	0.71127	0.71228	0.00703	0.00649	0.00699	0.00646	0.00029	0.00023	0.00000	0.00000
	1.0	0.71292	0.71393	0.00801	0.00738	0.00797	0.00734	0.00042	0.00032	0.00000	0.00000

TABLE III. (continued)

		R = 0.2		R = 1.9972[c]		R = 2.0		R = 4.0		R = 9.0	
λ	μ	φ_{GM}	Φ	φ_{GM}	Φ	φ_{GM}	Φ	φ_{GM}	Φ	φ_{GM}	Φ
5.	0.0	0.58354	0.58482	0.00142	0.00121	0.00141	0.00120	0.00001	0.00000	0.00000	0.00000
	0.2	0.58369	0.58497	0.00144	0.00123	0.00143	0.00122	0.00001	0.00000	0.00000	0.00000
	0.4	0.58414	0.58543	0.00151	0.00129	0.00150	0.00128	0.00001	0.00001	0.00000	0.00000
	0.6	0.58489	0.58618	0.00163	0.00140	0.00162	0.00139	0.00002	0.00001	0.00000	0.00000
	0.8	0.58594	0.58724	0.00181	0.00155	0.00180	0.00154	0.00003	0.00001	0.00000	0.00000
	1.0	0.58730	0.58860	0.00207	0.00176	0.00206	0.00175	0.00004	0.00002	0.00000	0.00000
6.	0.0	0.48072	0.48191	0.00036	0.00028	0.00036	0.00028	0.00000	0.00000	0.00000	0.00000
	0.2	0.48084	0.48203	0.00037	0.00028	0.00037	0.00028	0.00000	0.00000	0.00000	0.00000
	0.4	0.48121	0.49240	0.00039	0.00030	0.00038	0.00030	0.00000	0.00000	0.00000	0.00000
	0.6	0.48183	0.48302	0.00042	0.00032	0.00041	0.00032	0.00000	0.00000	0.00000	0.00000
	0.8	0.48270	0.48389	0.00047	0.00036	0.00046	0.00036	0.00000	0.00000	0.00000	0.00000
	1.0	0.48382	0.48501	0.00053	0.00041	0.00053	0.00041	0.00000	0.00000	0.00000	0.00000
7.	0.0	0.39601	0.39695	0.00009	0.00006	0.00009	0.00006	0.00000	0.00000	0.00000	0.00000
	0.2	0.39612	0.39705	0.00009	0.00006	0.00009	0.00006	0.00000	0.00000	0.00000	0.00000
	0.4	0.39642	0.39736	0.00010	0.00006	0.00010	0.00006	0.00000	0.00000	0.00000	0.00000
	0.6	0.39693	0.39787	0.00010	0.00007	0.00010	0.00007	0.00000	0.00000	0.00000	0.00000
	0.8	0.39765	0.39859	0.00012	0.00008	0.00012	0.00008	0.00000	0.00000	0.00000	0.00000
	1.0	0.39857	0.39951	0.00013	0.00009	0.00013	0.00009	0.00000	0.00000	0.00000	0.00000
8.	0.0	0.32624	0.32687	0.00002	0.00001	0.00002	0.00001	0.00000	0.00000	0.00000	0.00000
	0.2	0.32632	0.32696	0.00002	0.00001	0.00002	0.00001	0.00000	0.00000	0.00000	0.00000
	0.4	0.32657	0.32721	0.00002	0.00001	0.00002	0.00001	0.00000	0.00000	0.00000	0.00000
	0.6	0.32699	0.32763	0.00002	0.00001	0.00002	0.00001	0.00000	0.00000	0.00000	0.00000
	0.8	0.32758	0.32822	0.00003	0.00001	0.00003	0.00001	0.00000	0.00000	0.00000	0.00000
	1.0	0.32834	0.32898	0.00003	0.00002	0.00003	0.00002	0.00000	0.00000	0.00000	0.00000

[a]All distances are in Bohr radii.
[b]$\Phi = \varphi_{GM} + \varphi^{(1)}$.
[c]Equilibrium separation.

TABLE IV. Comparison of selected values of $\Phi = \varphi + \varphi^{(1)}$ by different methods.[a]

Values obtained using series solution for $m(\mu)$.			Values obtained using integral representation for (μ). (15 point Gauss)		
μ	R = 2; λ = 1	R = 4; λ = 1	μ	R = 2; λ = 1	R = 4; λ = 1
0.0	0.3147314268	0.1268287867	0.0	0.3147314266	0.1268292817
0.2	0.3198650639	0.1340346899	0.2	0.3198650637	0.1340352129
0.4	0.3355496137	0.1568944633	0.4	0.3355496135	0.1568950752
0.6	0.3626548287	0.1993861110	0.6	0.3626548285	0.1993868883
0.8	0.4026933997	0.2690141881	0.8	0.4026933994	0.2690152371
1.0	0.4579172310	0.3782763156	1.0	0.4579172308	0.3782777917

[a]All distances are in Bohr radii.

TABLE V. Results of perturbation energy calculations for the ground state of the hydrogen molecular ion using the Geometric Mean variation function for the zeroth-order wavefunction.[a,b]

R	$-E(1) = -E(G.M.)$	$-e^{(2)}$	$e^{(3)}$	$-E(2)$	$-E(3)$	$-E(\text{Exact})$[c]	$-E(\text{Lyon})$[d]
0.2	1.9285375	0.0000729	0.0000002	1.9286105	1.9286103	1.9286202	1.9286203
0.6	1.6711852	0.0003027	0.0000044	1.6714879	1.6714834	1.6714846	1.6714846
1.0	1.4514847	0.0003075	0.0000068	1.4517923	1.4517854	1.4517863	1.4517860
1.2	1.3620291	0.0002846	0.0000068	1.3623138	1.3623068	1.3623078	
1.4	1.2840159	0.0002596	0.0000067	1.2842756	1.2842689	1.2842692	1.2842688
1.6	1.2157081	0.0002351	0.0000063	1.2159432	1.2159369	1.2159372	1.2159368
1.8	1.1556023	0.0002234	0.0000061	1.1558148	1.1558088	1.1558092	1.1558088
1.9	1.1282248	0.0002022	0.0000057	1.1284270	1.1284213	1.1284216	1.1284212
1.9972[e]	1.1031478	0.0001929	0.0000055	1.1033407	1.1033352		
2.0	1.1024466	0.0001926	0.0000054	1.1026393	1.1026338	1.1026342	1.1026338
2.1	1.0781463	0.0001828	0.0000052	1.0783301	1.0783249	1.0783254	1.0783250
2.2	1.0552139	0.0001757	0.0000050	1.0553897	1.0553847	1.0553851	1.0553847
2.4	1.0130626	0.0001617	0.0000045	1.0132243	1.0132198	1.0132203	
2.6	0.9753020	0.0001503	0.0000042	0.9754523	0.9754481	0.9754486	
2.8	0.9413612	0.0001411	0.0000038	0.9415024	0.9414985	0.9414989	
3.0	0.9107654	0.0001340	0.0000035	0.9108995	0.9108959	0.9108962	0.9108960
3.2	0.8831168	0.0001287	0.0000032	0.8832455	0.8832422	0.8832426	
3.4	0.8580795	0.0001248	0.0000030	0.8582044	0.8582014	0.8582017	
3.6	0.8353673	0.0001223	0.0000028	0.8354896	0.8354868	0.8354871	
3.8	0.8147341	0.0001197	0.0002922	0.8148539	0.8145616	0.8148526	
3.8*	0.8147341	0.0001208	0.0000026	0.8148550	0.8148524	0.8148526	
4.0	0.7959669	0.0001202	0.0000036	0.7960871	0.7960835	0.7960849	0.7960848
4.0*	0.7959669	0.0001203	0.0000025	0.7960872	0.7960847	0.7960849	0.7960848
4.2	0.7788792	0.0001726	0.0000227	0.7790539	0.7790311	0.7789974	
4.2*	0.7788792	0.0001747	0.0000225	0.7790539	0.7790313	0.7789974	
4.4	0.7633069	0.0001210	0.0000023	0.7634279	0.7634256	0.7634259	
4.6	0.7491041	0.0001219	0.0000021	0.7492261	0.7492239	0.7492241	

TABLE V. (continued)

R	$-E(1) = -E(G.M.)$	$-e^{(2)}$	$e^{(3)}$	$-E(2)$	$-E(3)$	$-E(\text{Exact})^c$	$-E(\text{Lyon})^d$
4.8	0.7361402	0.0001231	0.0000021	0.7362633	0.7362612	0.7362615	
5.0	0.7242978	0.0001243	0.0000020	0.7244221	0.7244201	0.7244203	0.7244202
5.2	0.7134705	0.0001255	0.0000020	0.7135961	0.7135940	0.7135943	
5.4	0.7035619	0.0001266	0.0000019	0.7036885	0.7036866	0.7036868	
5.6	0.6944841	0.0001275	0.0000019	0.6946117	0.6946097	0.6946100	
5.8	0.6861574	0.0001281	0.0000018	0.6862856	0.6862837	0.6862837	
6.0	0.6785089	0.0001285	0.0000018	0.6786374	0.6786355	0.6786357	0.6786355
6.2	0.6714724	0.0001285	0.0000018	0.6716009	0.6715991	0.6715993	
6.4	0.6649878	0.0001282	0.0000018	0.6651161	0.6651142	0.6651144	
6.6	0.6590007	0.0001277	0.0000018	0.6591284	0.6591266	0.6591268	
6.8	0.6534619	0.0001268	0.0000018	0.6535887	0.6535869	0.6535871	
7.0	0.6483272	0.0001256	0.0000018	0.6484528	0.6484510	0.6484511	
7.2	0.6435566	0.0001241	0.0000018	0.6436807	0.6436789	0.6436791	
7.4	0.6391145	0.0001224	0.0000017	0.6392369	0.6392351	0.6392353	
7.6	0.6349689	0.0001204	0.0000017	0.6350893	0.6350875	0.6350877	
7.8	0.6310912	0.0001182	0.0000017	0.6312095	0.6312077	0.6312079	
8.0	0.6274561	0.0001159	0.0000017	0.6275203	0.6275702	0.6275704	0.6275698
8.2	0.6240408	0.0001153	0.0000017	0.6241542	0.6241524	0.6241526	
8.4	0.6208251	0.0001107	0.0000017	0.6209359	0.6209341	0.6209343	
8.6	0.6177911	0.0001079	0.0000017	0.6178991	0.6178973	0.6178975	
8.8	0.6149228	0.0001051	0.0000017	0.6150279	0.6150262	0.6150164	
9.0	0.6122059	0.0001022	0.0000017	0.6123081	0.6123064	0.6123066	
9.2	0.6096277	0.0000992	0.0000017	0.6097270	0.6097252	0.6097254	
9.4	0.6071769	0.0000963	0.0000017	0.6072732	0.6072715	0.6072717	
9.6	0.6048434	0.0000933	0.0000017	0.6049367	0.6049350	0.6049352	
9.8	0.6026181	0.0000903	0.0000017	0.6027084	0.6027067	0.6027069	
10.0	0.6004929	0.0000873	0.0000017	0.6005803	0.6005785	0.6005787	0.6005785
10.2	0.5984605	0.0000843	0.0000017	0.5985449	0.5985432	0.5985434	
10.4	0.5965143	0.0000814	0.0000017	0.5964958	0.5965941	0.5965943	

TABLE V. (continued)

R	$-E(1) = -E(G.M.)$	$-e^{(2)}$	$e^{(3)}$	$-E(2)$	$-E(3)$	$-E(\text{Exact})^c$	$-E(\text{Lyon})^d$
10.6	0.5946485	0.0000786	0.0000017	0.5947271	0.5947253	0.5947256	
10.8	0.5928576	0.0000757	0.0000017	0.5929334	0.5929316	0.5929319	
11.0	0.5911368	0.0000730	0.0000017	0.5912098	0.5912081	0.5912083	
11.2	0.5894817	0.0000703	0.0000017	0.5895520	0.5895503	0.5895505	
11.4	0.5878882	0.0000676	0.0000017	0.5879559	0.5879542	0.5879545	
11.6	0.5863528	0.0000651	0.0000017	0.5864179	0.5864162	0.5864165	
11.8	0.5848720	0.0000626	0.0000017	0.5849347	0.5849330	0.5849332	
12.0	0.5834428	0.0000602	0.0000017	0.5835031	0.5835013	0.5835016	

[a]All distances are in Bohr radii and all energies are in units of e^2/a_0. The total energy may be obtained by adding the nuclear-nuclear repulsive energy 1/R.

[b]Starred results were obtained using an integral representation for $m(\mu)$. All other results were obtained using the series expression for $m(\mu)$.

[c]Exact from Ref. 4.

[d]Ref. 13.

[e]Equilibrium separation.

series expression for $m(\mu)$ with an integral representation. The energies obtained in this way compared very favorably to those obtained using the series form for $m(\mu)$. Some typical results are given in Table VI (page 34). More detailed tabulations for each of these methods are given in appendices B and C.

An examination of the preceding tabulations shows that when R lies between 3.8 and 4.2 Bohr radii, the series results differ from the "exact" results of Wind[4] and the results obtained by using an integral representation for $m(\mu)$. We believe this discrepancy arises from the fact that $\beta/2\alpha \simeq 1$ within this range ($\beta/2\alpha = 1.0000002$ at $R = 3.779420$). As mentioned on page 19, this causes the series solutions for Cases 1 and 2 to be perturbed and erroneous energy results are obtained. For this reason we have included integral representation values in Table V for R = 3.8, 4.0 and 4.2. These results compare favorably with those of Wind at R = 3.8 and 4.0, but we have no explanation for the discrepancy that occurs at R = 4.2.

All of the above calculations were done in both single and double precision on an XDS/Sigma 7 computer at Queens College of the City University of New York. Table VI (page 34) shows that the single precision values agree with the double precision values of the series solution to within 8×10^{-4} over the internuclear separations R = 1 to R = 10. A similar comparison for the integral representation solution shows even better agreement. The fact that the single precision results closely match those obtained by the use of double precision is indeed satisfying.

TABLE VI. Selected values of third order energies obtained by two different methods.[a]

	Values obtained using series solution for m(μ).		Values obtained using integral representation for m(μ) (15 point Gauss).	
R	Single precision	Double precision	Single precision	Double precision
1.0	−1.451802	−1.4517854294	−1.451762	−1.4517854294
2.0	−1.102574	−1.1026338401	−1.102619	−1.1026338401
4.0	−0.796855	−0.7960835542	−0.796083	−0.7960847168
10.0	−0.600578	−0.6005785783	−0.600578	−0.6005785783

[a]All distances are in Bohr radii and all energies are in units of e^2/a_0.

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APPENDIX A

THE STARK EFFECT FOR THE HYDROGEN ATOM: A NEW APPROACH

The Schroedinger equation for a hydrogen atom in an electric field of strength E acting along the z axis can be written as

$$-\frac{1}{2}\nabla^2\Psi - \frac{1}{r}\Psi + Ez\Psi = W\Psi \tag{A-1}$$

where atomic units are used throughout. Equation 2-9b tells us that the first order correction to the energy is $\langle \varphi, Ez\varphi \rangle$ if we treat the energy of interaction of the electron with the electric field as a perturbation. For the ground state ($\varphi = \varphi_{1s}$) it is easy to show that the above matrix element is zero[23], so that we must go to second order to calculate the effect of the field on the energy of the atom.

The second order correction to the energy is given by

$$e^{(2)} = \langle \varphi, Ez\varphi^{(1)} \rangle \tag{A-2}$$

where $\varphi^{(1)}$ is found by solving equation 2-13:

$$-\frac{1}{2}\nabla^2 - \frac{1}{r} + \frac{1}{2}\ \varphi^{(1)} + Ez\varphi = 0. \tag{A-3}$$

Following the procedure of Chapter 3, we let $\varphi^{(1)} = [L(\lambda) + M(\mu)]\varphi_{1s}$ where the parabolic coordinates λ, μ and α are related to the Cartesian coordinates by the following formulas:

$$x = \sqrt{\lambda\mu}\cos\alpha;\ y = \sqrt{\lambda\mu}\sin\alpha;\ z = \frac{\lambda - \mu}{2};\ r = \frac{\lambda + \mu}{2};\ d\tau = \frac{\lambda + \mu}{4}d\lambda d\mu d\alpha.$$

The limits for both λ and μ are 0 to ∞, while those for α are 0 to 2π. In this coordinate system $V = Ez = \frac{1}{2}E(\lambda - \mu)$. Substituting into equation A-3 we find that the differential equation is separated in the coordinates λ and μ as follows:

$$\lambda L'' + (1 - \lambda)L' - E\lambda^2/4 = K \tag{A-4}$$

$$\mu M'' + (1 - \mu)M' + E\mu^2/4 = -K \tag{A-5}$$

Solutions for these equations which also satisfy the relation $\langle \varphi_{1s}^{1}, \varphi^{(1)} \rangle = 0$ (see page 7) are found to be

$$L(\lambda) = -\frac{E}{4}[\lambda^2/2 + 2\lambda] \tag{A-6}$$

and

$$M(\mu) = \frac{E}{4}[\mu^2/2 + 2\mu] \tag{A-7}$$

with the separation constant, K, equal to $-E/2$.

These results lead to a corrected wavefunction of the form

$$\varphi = \varphi_{1s} + \varphi^{(1)} \tag{A-8a}$$

$$= \varphi_{1s}[1 + L(\lambda) + M(\mu)] \tag{A-8b}$$

$$= \varphi_{1s}[1 - Ez - \frac{E}{2}zr] \tag{A-8c}$$

and a corrected energy equal to

$$W = -\frac{1}{2} + e^{(1)} + e^{(2)} \tag{A-9a}$$

$$= -\frac{1}{2} + 0 - \frac{9E^2}{4}. \tag{A-9b}$$

The magnitude of the average dipole moment (μ) induced by the field is given by

$$\left|\frac{\partial W}{\partial E}\right| = \frac{9}{2}E \tag{A-10}$$

Since the magnitude of the induced dipole is also given by $\mu = \alpha E$, where α is the polarizability, we see that $\alpha = 9/2$ in atomic units. This, of course, is consistent with the accepted value of $9a_0^3/2$ in cgs units.

It is interesting to point out that Hassé[24] obtained the same value for the polarizability using a variation function of the form $\varphi = \varphi_{1s}(1 + Az + Bzr)$. Hassé's function is virtually identical to our derived function A-8c.

```
EXT. FORTRAN IV, VERSION DOC
      1.            COMMON /A/R,Z,ALPHA,BETA/AAA/ES
      2.              COMMON/ZOP/ERR
      3.     2        J = 1
      4.     6        READ(105,15) R,ERR
      5.     15       FORMAT(2F10.5)
      6.              IF(R.GT.12.) GO TO 200
      7.     4        Z = 1.
      8.      1       W = Z*R
      9.               A = EXP(-W)
     10.               B = EXP(-W/2.)
     11.               C = EXP(W/2.)
     12.               D = EXP(-2.*W)
     13.               AK = A*(1. + W)
     14.               AJ = -1./R + D/R*(1.+W)
     15.              S22 = A*(1. + W + W*W/3. )
     16.              S11 = 2. + 2.*S22
     17.               S12 = 2.*A/W*(C*(3.*W-3.) +B*(W+3.))
     18.               EZERO = Z*Z/2. -Z
     19.               SH = .5*(C-B)
     20.                CH = .5*(C+B)
     21.                AKA = A*(W*W -W -1.)
     22.               H11 = 2.*EZERO*(1.+2.*AK) + 2.*AJ - Z*Z*S22
     23.                H12 = 2.*AK*SH/R*(Z-4.)
     24.               H22 = AK*(EZERO-Z)
     25.               PS11Z = -2.*W*W*AK/(3.*Z)
     26.               PS12Z=-3.*AK*B*(C*C*(W-2.) + W + 2.)/(Z*W)
     27.               PS22Z = -W*W*AK/(3.*Z)
     28.              PEOZ = Z-1.
     29.               PAKZ = -W*R*A
     30.              PAJZ = -D*(1. +2.*W)
     31.               SPAKZ = R*R*A*(W-1.)
     32.               SPAJZ = -2.*R*(PAJZ+D)
     33.              PH11Z=2.*PEOZ*(1.+2.*AK)+4.*EZERO*PAKZ+2.*PAJZ-2.*Z*S22-Z*Z*PS22Z
     34.              PH12Z=2.*SH*(PAKZ*(Z-4.)+AK)/R+AK*CH*(Z-4.)
     35.               PH22Z=PAKZ*(EZERO-Z)+AK*(PEOZ-1.)
     36.                SPS11Z = 2.*W*W*AKA/(3.*Z*Z)
     37.              SPS12Z=3.*A/(2.*Z*Z*W)*(C*(W*W*W-W*W-4.*W-8.)+B*(3.*W*W*W + 9.*W*W
     38.             1+12.*W+8.))
     39.               SPS22Z =  W*W*AKA/(3.*Z*Z)
     40.              SPH11Z=2.*(1.+2.*AK)+8.*PEOZ*PAKZ+4.*EZERO*SPAKZ+2.*SPAJZ-2.*S22
     41.             1-4.*Z*PS22Z-Z*Z*SPS22Z
     42.               SPH12Z=2.*SH*(SPAKZ*(Z-4.)+2.*PAKZ)/R+2.*CH*(PAKZ*(  Z-4.)+AK)
     43.             1 +.5*R*SH*AK*(Z-4.)
     44.              SPH22Z=SPAKZ*(EZERO-Z) + 2.*PAKZ*(Z-2.) +AK
     45.                AA = S11*S22 - S12*S12
     46.               BB = 2.*S12*H12 - S11*H22 - H11*S22
     47.                CC = H11*H22 - H12*H12
     48.               DISC = BB*BB-4.*AA*CC
```

```
      SQRTDI = SQRT(DISC)
      PAAZ = S11*PS22Z + S22*PS11Z - 2.*S12*PS12Z
     PBBZ=2.*S12*PH12Z+2.*H12*PS12Z-S11*PH22Z-H22*PS11Z-H11*PS22Z-S22*
    1P-11Z
      PCCZ = H11*PH22Z +H22*PH11Z -2.*H12*PH12Z
     SPAAZ= S11*SPS22Z+S22*SPS11Z-2.*S12*SPS12Z+2.*PS11Z*PS22Z-2.*PS12
    1Z*PS12Z
     SPBBZ = 2.*S12*SPH12Z+4.*PS12Z*PH12Z+2.*H12*SPS12Z-S11*SPH22Z-2.*
    1 PH22Z*PS11Z-H22*SPS11Z-H11*SPS22Z-2.*PS22Z*PH11Z-S22*SPH11Z
     SPCCZ=H11*SPH22Z+H22*SPH11Z-2.*H12*SPH12Z+2.*PH11Z*PH22Z-2.*PH12Z
    1*PH12Z
      PDISCZ = 2.*BB*PBBZ - 4.*AA*PCCZ - 4.*CC*PAAZ
      ES = .5/AA*(-BB-SQRTDI)
     SPDISZ = 2.*(PBBZ*PBBZ + BB*SPBBZ) - 4.*(2.*PAAZ*PCCZ+AA*SPCCZ+
    1CC*SPAAZ)
       PSQTDI = .5*PDISCZ/SQRTDI
      SPQTDI = -.25*PDISCZ*PDISCZ/(DISC*SQRTDI) +.5*SPDISZ/SQRTDI
     PESZ = -.5*(PBBZ+PSQTDI)/AA -PAAZ*ES/AA
      SPESZ = -.5*(SPBBZ +SPQTDI)/AA-1./AA*(2.*PAAZ*PESZ+SPAAZ*ES)
      Z = Z-PESZ/SPESZ
     IF(ABS(PESZ)-.00000001) 100,100,5
5    J = J + 1
      IF(J-1000) 1,100,100
100   TOTAL= ES + 1./R
      RON=(H11-S11*ES)/(S12*ES-H12)
     ALPHA = 1./SQRT(S11+2.*S12*RON + S22*RON*RON)
     BETA = RON*ALPHA
     CALL ENERGY
      GO TO 2
200    CALL EXIT
       END
```

```
      SUBROUTINE ENERGY
      REAL*4 N
      REAL*4 INT1A,INT2A,INT3A,INT4A,INT5A,INT6A,INT7A,INT8A,INT9A
      REAL*4 INT10A,INT11A,INT12A
      REAL*4 INT1,INT2,INT3,INT4,INT5,INT6,INT7,INT8,INT9
      REAL*4 INTA,INTB,INTC,INTD,INTE,INTF
      COMMON /AAA/EZERO
      COMMON /ABC/G/ABB/COSG/ABD/COSEC/BBC/COSEC2/DEF/COSEC3/BASH/W1
      COMMON /A/R,Z,ALPHA,BETA/B/EP/C/EX/D/AL2/E/G1/F/G2
      COMMON/ZOR/RRR
      DARCS(X) = LOG( X + SQRT(X*X-1.) )
      EQ = EZERO + .5*Z*Z
      EP = EQ
      Z92 = -.5*Z*Z
      AL2 = LOG(2.)
      COSG = .5*BETA/ALPHA
      IF(COSG.LE.1.) G = ARCOS(COSG)
      IF(COSG.LE.1.) E = 1. - COSG*COSG
      IF(COSG.GT.1.0) G = DARCS(COSG)
      IF(COSG.GT.1.) E = COSG*COSG - 1.
      SI = SQRT(E)
      COSEC = 1./SI
        COSEC2 = COSEC*COSEC
        COSEC3 = COSEC2*COSEC
      EX = EXPI(R,Z)
      C1 = (2. - Z )/Z + EP/(Z*Z)
      C2 = EP*R*.5/Z
      W1 = Z*R
      G1 = EXP(-W1)
      G2 = EXP(2.*W1)
      G3 = COSH(W1)
      F1 = FINT1A(R,Z)
      F2 = FINT1B(R,Z)
      F3 = PSIS2U(1.,R,Z,ALPHA,BETA)
      F4 = SINT2A(R,Z)
      F5 = SINT2B(R,Z)
      FT1B = SINT2D(1.,R,Z,ALPHA,BETA)
      CALL DINB(1.,R,Z,ALPHA,BETA,ANS)
      FT2 = ANS
      F8 = F4
      F9 = F5
      F10 = -2.*FT1B/(Z*Z)
      IF(COSG.LE.1.) SUM6 = SERIE1(1.,R,Z,ALPHA,BETA)
      IF(COSG.GT.1.) SUM6 = SERIE2(1.,R,Z,ALPHA,BETA)
      SERIES = SUM6
      FUNCT2 = F3
      FUNCT4 =    FT2
      F11 = G1/W1
      FUNCT6 = F10
      API = 3.1415926535897932384
      A1 = .25*Z*Z*Z*R*R*R
```

```
      FUNCT9 = FUNCT4
      VLK =  (2.*Z - 4.)/R
      AREA3 = GAUSS(0.,1.,15,3)
      AREA6 = GAUSS(0.,1.,15,4)
      AREA9 = GAUSS(0.,1.,15,5)
      AREAA = GAUSS4(0.,1.,15,8)
      AREAB = GAUSS4(0.,1.,15,9)
      INT1 = 2.*VLK*A1*F3*(C1*F1 + C2*F2)
      INT2 = 2.*A1*VLK*F9*AREA3
      INT3 = 2.*VLK*A1*F9*F3
      INT4 = 2.*A1*FT2*(C1*F4 + C2*F5)
      INT5 = 2.*A1*F11*AREA6
      INT6 = 2.*A1*F11*FUNCT9
      INT7 = 2.*A1*(F3*AREAA - F10*AREAB)
      INT8 = 2.*A1*(F2*AREA3 - F11*AREA9)
      INT9 = 1.
      FZER0 = -INT7 - INT8
      ETW0 = INT1 + INT2 + INT4 + INT5 + FZER0*(INT3 + INT6 )
       AREA4 = GAUSS(0.,1.,15,1)
      SUM4 = AREA4
      AREA5 = GAUSS(0.,1.,15,2)
      SUM5 = AREA5
      AREA7 = GAUSS4(0.,1.,15,5)
      AREA8 = GAUSS4(0.,1.,15,6)
       INTA = A1*(AREA7*FUNCT2 - AREA8*FUNCT6)*2.
      INTB = A1*(F2*SUM4 - F11*SUM5)*2.
      INTC = FZER0*FZER0*INT9
       INTD = 4.*A1*( AREAA*AREA3 - AREAB*AREA9)
      INTE = 2.*FZER0*INT7
      INTF = 2.*FZER0*INT8
      AN0RM1= INTA+INTB+INTC+INTD+INTE+INTF  + INT9
       AN0RM = SQRT(AN0RM1)
      E2 = ETW0/AN0RM1
       AREA11 = GAUSS4(0.,1.,15,7)
       AREA12 = GAUSS(0.,1.,15,6)
      INT1A = 2.*A1*(VLK*F3*AREA11 + AREA8*FT2)
      INT2A = 2.*A1*(VLK*F9*AREA4 + F11*AREA12)
      INT3A = FZER0*FZER0*(INT3 + INT6)
      INT4A=4.*A1*( VLK*AREA3*(C1*F1+C2*F2)+ AREA6*(C1*F4+C2*F5)  )
      INT5A = 2.*FZER0*(INT2+INT5)
      INT6A = 2.*FZER0*(INT1 + INT4)
      ETHREE=(INT1A+INT2A+INT3A+INT4A+INT5A+INT6A-EG*(AN0RM1-INT9))
       E3 = ETHREE/AN0RM1
      RES1 = EZER0 + E2
       RES2 = RES1 + E3
       WRITE(108,906) R
  906  FORMAT(1X//'R = ',F8.4)
       WRITE(108,902) Z02,EZER0
  902  FORMAT(1X,'E(0)=-Z.Z/2=',F16.12,5X,'E1=EGM=',F16.12)
```

```
      WRITE(108,903)   EP
  9C3  FORMAT(1X,'E(1) = EGM + Z*Z/2 = ',F16.12)
      WRITE(108,904) E2,RES1
  9C4  FORMAT(1X,'E2 = ',F16.12,5X,'E(2) = ',F16.12)
      WRITE(108,905) E3,RES2
  9C5  FORMAT(1X,'E3 = ',F16.12,5X,'E(3) = ',F16.12/)
       IF(R.EQ.RRR) GO TO 3
        GO TO 31
    3  AN = SQRT(Z*Z*Z/API)
       WRITE(108,109)
  109  FORMAT(1X,'MU',5X,'LAMBDA',14X,'PHI GM',14X,'PHI(1)')
       DO 12 K = 0,10
       U = .1*K
       IF(COSG.LE.1.0) SERI = SERIE1(U,R,Z,ALPHA,BETA)
       IF(COSG.GT.1.0) SERI = SERIE2(U,R,Z,ALPHA,BETA)
       DO 12 NA = 1,10
       AL = 1.*NA
       A = .5*Z*R*AL
       F = EXP(-A)
       AA = 1. + AL
       ALA = C1*LOG(AA) + C2*AL
       F1 = F*AN*(2.*ALPHA*COSH(.5*Z*R*U) + BETA)
       F2 = SERI + FZERO + ALA
       TOTAL = F1*(1.0 + F2)/ANORM
   12  WRITE(108,108) U,AL,F1,TOTAL
  108  FORMAT(1X,F4.2,5X,F5.2,7X,F16.10,5X,F16.10)
   31   RETURN
        END
```

```
      FUNCTION SERIE1(U,R,Z,A,B)
       IMPLICIT COMPLEX*8(A-H,O-Z)
      REAL*4 COSEC3
      REAL*4 FI4A,FI4B,FI4C,FI4D,RESX,VAL, GAUSS4,FI4
      REAL*4 AP
     REAL*4 AA,AB,BB,U,R,Z,A,B,Z2,I,W2,API, FUNC7,FUNC7A,FUNC8,PSI
     REAL*4 COSG,A1A,G,E,SI,C0SEC,C0SEC2,C0S2G,Q11M,T1,T2,T3,T4,T5,T6
     REAL*4 T7,T8,ALAM,AL,EP
      REAL*4 W
      REAL*4 FAC, SERIE1,SERI
       COMMON /B/EP
       DIMENSION   ANS(8)
      FUNCT1 = 0.
      FUNCT2 = 0.
      FUNCT3 = 0.
      FUNCT4 = 0.
      FUNCT5 = 0.
      FUNCT6 = 0.
      FUNCT7 = 0.
      FUNCT8 = 0.
      FUNCT9 = 0.
      GUNCT1 = 0.
      GUNCT2 = 0.
      GUNCT3 = 0.
      GUNCT4 = 0.
     AA = A*A
     AB = A*B
     BB = B*B
      W = .5*Z*R
     Z2 = Z*Z
      I = EP/Z2*(1.+2.*W+2.*W*W) + (2.-Z)*(1.+Z*R)/Z
      W2 = 1./(W*W)
      AI = CMPLX(0.,1.)
     API = 3.14159265358979323846
     AP = API*API
      FUNC7 = COSH(U*W)
      FUNC7A = SINH(U*W)
     FUNC8 = 2.*A*FUNC7 + B
     PSI = FUNC8*FUNC8
      COSG = .5*B/A
      A1A = 1./(FUNC7+COSG)/W
      G = ARCOS(COSG)
      C = CMPLX(W,G)
      D = CONJG(C)
      CM = CMPLX(W*U,G)
      DM = CONJG(CM)
      E = 1.-COSG*COSG
      SI = SQRT(E)
      COSEC = 1./SI
      COSEC2 = COSEC*COSEC
        COSEC3 = COSEC2*COSEC
```

```
      C8S2G = C8S(2**G)
      G11M = LOG(FUNC7 + C8SG)
      CC = C*C
      DD = D*D
      CCM = CM*CM
      DDM = DM*DM
      T1 = 2.**AA*(-Z2*(I+1.)+EP)/((Z**3)*R)
      T2 = AA*EP*R/Z
      T3 = 4.**AB*(Z2*(-2.*I+.25*Z2<R*R-4.)+8.**EP)/(R*(Z**3))
      T4 = 4.**AB*R/Z*(EP-.25*Z2)
      T5 = -I*(2.*AA+BB) + Z2*R*R*BB*.25
      T6 = R*R*(-.5*Z2*BB+EP*(2.**AA+BB))**.16666666666666666
      T7 = 2.**AA/Z2*(Z2-EP)
      T8 = 8.**AB/Z2*(Z2-2.*EP)
      FI4A = VAL(U,R,Z,A,B)
      FI4B = -C8SEC2*C8SG*RESX(L,R,Z,A,B)
      IF(U.EQ.0.) FI4C = 0.
      IF(U.NE.0.) FI4C = GAUSS4(C,U,15,1)
      FI4D = 2.*C8SEC3*C8SG/W*FI4C
      FI4 = FI4A + FI4B + FI4D
      DO 1 N = 0,400
      L = 2*N + 1
      ALAM = API*L
      LL = L*L
      AL = AP*LL
      D1 = AL + CC
      D2 = AL + DD
      XPLUS = AL + CCM
      XMINUS = AL + DDM
      D3 = D1*D1
      D4 = D2*D2
      G1 = 1./D1
      G2 = 1./D2
      G3 = CC/D3
      G4 = DD/D4
      G5 = C/D3
      G6 = D/D4
      G7M = CLOG(XPLUS)
      G8M = CLOG(XMINUS)
      FUNC1 = G7M + G8M
      FUNC2 = (G1 + 2.*G4) - (G2 + 2.*G3 )
      G7 = C/D1
      G8 = D/D2
      FUNC3 = G7 - G8
      FUNC3A = G7 + G8
      G9 = AI*CM/ALAM
      G10 = 1. + G9
      G11 = 1. - G9
      G12 = G10/G11
```

```
      ARTN1 = .5/AI*CLOG(Q12)
     Q13 = AI*DM/ALAM
      C14 = 1. + Q13
     G15 = 1. - G13
     Q16 = Q14/Q15
      ARTN2 = .5/AI*CLOG(Q16)
      FUNC4 = ARTN1 - ARTN2
     FUNC5 = Q5 + Q6
      FUNC5A = Q5 - Q6
      FUNC6 = Q1 + Q2
     FUNC6A = Q1 - Q2
      Q9M = CM/XPLUS
      Q10M = DM/XMINUS
      FUNC9 = Q9M + Q10M
      FUNC10 = 1./XPLUS - 1./XMINUS
     FUNC2A = FUNC6 -2.*(Q3 + Q4)
      FUNCT1 = .5*FUNC2*FUNC1 + FUNCT1
      FUNCT2 = FUNC3*FUNC9 + FUNCT2
      FUNCT3 = ALAM*FUNC4*FUNC5 + FUNCT3
      FUNCT4 = AL*FUNC6*FUNC10 + FUNCT4
      BRAK1 = FUNC6 - 2.*(Q3+Q4)
      FUNCT5 = BRAK1*FUNC1 + FUNCT5
      FUNCT6 = ALAM*FUNC5A*FUNC4 + FUNCT6
      FUNCT7 = FUNC3A*FUNC9 + FUNCT7
      FUNCT8 = AL*FUNC6A*FUNC10 + FUNCT8
      FUNCT9 = FUNC3A*FUNC1 + FUNCT9
      GUNCT1 = ALAM*FUNC6A*FUNC4 + GUNCT1
       GUNCT2 = 2.*ALAM*FUNC4 + GUNCT2
      GUNCT3 = FUNC3*FUNC1 + GUNCT3
      GUNCT4 = ALAM*FUNC6*FUNC4 + GUNCT4
     P2 = FUNCT1 +2.*FUNCT3 + U*FUNCT2 - U*FUNCT4
      FP2 = -AI*C0SEC*P2
      FP3 = .5*FUNCT5 +2.*FUNCT6 +U*FUNCT7 -U*FUNCT8
      FI2 = .5*FUNCT9 -GUNCT1
      FI3 = -Q11M/W + FI2
      FI5 = -AI*C0SEC*( .5*GUNCT3 - GUNCT4 )
      FAC = .25/AA
   9  ANS(1)=T1*(2.*FI2+2.*C0SG*A1A+2.*C0SG*FP2)
   2  ANS(2)=T2*(2.*FI3+2.*C0SG*FP2)
   3  ANS(3)=-T3*(A1A+FP2)
   4  ANS(4)=-T4*FP2
   5  ANS(5)=T5*C0SEC2*(FP3-C0SG*FI5)
   6   ANS(6) = T6*( C0SEC2*(FP3-C0SG*FI5) - FI4)
   7  ANS(7)=T7*(C0SEC2*C0S2G*(FP3-C0SG*FI5)-4.*C0SG*FI5)
   8   ANS(8) =T8*(FI5*(1.+C0SEC2*C0SG*C0SG)-C0SG*C0SEC2*FP3)
       SER = 0.
      DO 13 J = 1,8
  13  SER = SER + ANS(J)
        SERIE1 = FAC*REAL(SER)
```

```
150.     1       CONTINUE
151.             RETURN
152.             END
```

```
      1.            FUNCTION SERIE2(U,R,Z,A,B)
      2.            REAL*4 I
      3.             COMMON /B/EP
      4.               DIMENSION ANS(6)
      5.            DARCS(X) = LOG( X + SQRT(X*X-1.) )
      6.            FUNCT1 = 0.
      7.            FUNCT2 = 0.
      8.            FUNCT3 = 0.
      9.            FUNCT4 = 0.
     10.            FUNCT5 = 0.
     11.            FUNCT6 = 0.
     12.            FUNCT7 = 0.
     13.            FUNCT8 = 0.
     14.            FUNCT9 = 0.
     15.            GUNCT1 = 0.
     16.            GUNCT2 = 0.
     17.            GUNCT3 = 0.
     18.            GUNCT4 = 0.
     19.           AA = A*A
     20.           AB = A*B
     21.           BB = B*B
     22.             W = .5*Z*R
     23.           Z2 = Z*Z
     24.            I = EP/Z2*(1.+2.*A+2.*W*W) + (2.-Z)*(1.+Z*R)/Z
     25.            W2 = 1./(W*W)
     26.              AI = 1.
     27.           API = 3.141592653589793284
     28.           AP = API*API
     29.            FUNC7 = COSH(U*W)
     30.            FUNC7A = SINH(U*W)
     31.           FUNC8 = 2.*A*FUNC7 + B
     32.           PSI = FUNC8*FUNC8
     33.            COSG = .5*B/A
     34.            A1A = 1./(FUNC7+COSG)/W
     35.           G = DARCS(COSG)
     36.           C = W + G
     37.           D = W - G
     38.           CC = C*C
     39.             DD = D*D
     40.           CM = U*W + G
     41.           DM = U*W - G
     42.            E = COSG*COSG - 1.
     43.            SI = SQRT(E)
     44.            COSEC = 1./SI
     45.            COSEC2 = COSEC*COSEC
     46.             COSEC3 = COSEC2*COSEC
     47.            COS2G = COSH(2.*G)
     48.            G11M = LOG(FUNC7 + COSG)
     49.             CCM = CM*CM
     50.           DDM = DM*DM
     51.            T1 = 2.**AA*(-Z2*(I+1.)+EP)/((Z**3)*R)
```

```
      T2 = AA*EP*R/Z
       T3 = 4.*AB*(Z2*(-2.*I+.25*Z2*R*R-4.)+8.*EP)/(R*(Z**3))
       T4 = 4.*AB*R/Z*(EP-.25*Z2)
       T5 = -I.*(2.*AA+BB) + Z2.*2.*R*R*BB**.25
      T6 = R*R*(-.5*Z2*BB+EP*(2.*AA+BB))**.16666666666666666
      T7 = 2.*AA/Z2*(Z2-EP)
       T8 = 8.*AB/Z2*(Z2-2.*EP)
       FI4A = VAL(U,R,Z,A,B)
       FI4B = -COSEC2*COSG*RESY(U,R,Z,A,B)
       IF(U.EQ.0.) FI4C = 0.
       IF(U.NE.0.) FI4C = GAUSS4(C.,U,15,4)
       FI4D = 2.*COSEC3*COSG/4.*FI4C
       FI4 = FI4A + FI4B + FI4D
       CO I N = 0.4CO
      L = 2*N+1
       ALAM = API*L
      LL = L*L
      AL = AP*LL
       D1 = AL + CC
       D2 = AL + DD
       XPLUS = AL + CCM
      XMINUS = AL + DDM
       D3 = D1*D1
       D4 = D2*D2
       G1 = 1./D1
       G2 = 1./D2
      G3 = CC/D3
        G4 = DD/D4
       G5 = C/D3
      G6 = D/D4
       G7M = LOG(XPLUS)
       G8M = LOG(XMINUS)
       FUNC1 = G7M + G8M
      FUNC2 = (G1 + 2.*G4) - (G2 + 2.*G3 )
      G7 = C/D1
      G8 = D/D2
       FUNC3 = G7 - G8
      FUNC3A = G7 + G8
      G9 = CM/ALAM
       ARTN1 = ATAN(G9)
      G13 = DM/ALAM
       ARTN2 = ATAN(G13)
       FUNC4 = ARTN1 - ARTN2
      FUNC5 = G5 + G6
       FUNC5A = G5 - G6
       FUNC6 = G1 + G2
      FUNC6A = G1 - G2
       G9M = CM/XPLUS
       G10M = DM/XMINUS
```

```
      101.            FUNC9 = Q9M + Q10M
      102.            FUNC10 = 1./XPLUS - 1./XMINUS
      103.           FUNC2A = FUNC6 -2.*(Q3 + Q4)
      104.            FUNCT1 = .5*FUNC2*FUNC1 + FUNCT1
      105.            FUNCT2 = FUNC3*FUNC9 + FUNCT2
      106.            FUNCT3 = ALAM*FUNC4*FUNC5 + FUNCT3
      107.            FUNCT4 = AL*FUNC6*FUNC10 + FUNCT4
      108.            BRAK1 = FUNC6 - 2.*(Q3+Q4)
      109.            FUNCT5 = BRAK1*FUNC1 + FUNCT5
      110.            FUNCT6 = ALAM*FUNC5A*FUNC4 + FUNCT6
      111.            FUNCT7 = FUNC3A*FUNC9 + FUNCT7
      112.            FUNCT8 = AL*FUNC6A*FUNC10 + FUNCT8
      113.            FUNCT9 = FUNC3A*FUNC1 + FUNCT9
      114.            GUNCT1 = ALAM*FUNC6A*FUNC4 + GUNCT1
      115.             GUNCT2 = 2.*ALAM*FUNC4 + GUNCT2
      116.            GUNCT3 = FUNC3*FUNC1 + GUNCT3
      117.            GUNCT4 = ALAM*FUNC6*FUNC4 + GUNCT4
      118.           P2 = FUNCT1 +2.*FUNCT3 + U*FUNCT2 - U*FUNCT4
      119.            FP2 = -AI*C0SEC*P2
      120.            FP3 = .5*FUNCT5 +2.*FUNCT6 +U*FUNCT7 -U*FUNCT8
      121.            FI2 = .5*FUNCT9 -GUNCT1
      122.            FI3 = -Q11M/4 + FI2
      123.            FI5 = -AI*C0SEC*( .5*GUNCT3 - GUNCT4 )
      124.            FAC = .25/AA
      125.     9     ANS(1)=T1*(2.*FI2+2.*C0SG+AI\-2.*C0SG*FP2)
      126.     2     ANS(2)=T2*(2.*FI3-2.*C0SG*FP2)
      127.     3     ANS(3) = -T3*(AIA - FP2)
      128.     4     ANS(4)= T4*FP2
      129.     5      ANS(5) = -T5*C0SEC2*(FP3+C0SG*FI5)
      130.     6     ANS(6) = -T6*(C0SEC2*(FP3+C0SG*FI5) - FI4 )
      131.     7      ANS(7) = T7*(C0SEC2*C0S2G*(FP3+C0SG*FI5)-4.*C0SG*FI5)*(-1.)
      132.     8      ANS(8)=-T8*(FI5*(1.-C0SEC2*C0SG*C0SG)-C0SG*C0SEC2*FP3)
      133.            SER = C.
      134.           DO 13 J = 1,8
      135.     13     SER = SER + ANS(J)
      136.            SERIL2 = FAC*SER
      137.     1       CONTINUE
      138.            RETURN
      139.             END
```

```
      1.          FUNCTION FUN4(U,R,Z)
      2.           COMMON /B/ EP
      3.          C1 = (2.-Z)/Z + EP/(Z*Z)
      4.          C2 = .5*EP*R/Z
      5.           U4 = U*U*U*U
      6.          F = 1. + 1./U
      7.          F1 = LOG(F)
      8.         F2 = C1*F1 + C2/U
      9.          W = Z*R/U
     10.          G = EXP(-W)
     11.          FUN4 = G*F2/U4
     12.          RETURN
     13.            END
```

```
1.            FUNCTION FUN5(U,R,Z)
2.            A = FUN4(U,R,Z)
3.          FUN5 = A*U*U
4.          RETURN
5.            END
```

```
      1.            FUNCTION GAUSS4(A,B,M,K)
      2.            COMMON /A/ R,Z,ALPHA,BETA
      3.           DIMENSION NPOINT(7),KEY(8),T(24),WEIGHT(24)
      4.           DATA NPOINT / 2,3,4,5,6,10,15 /
      5.           DATA KEY / 1,2,4,6,9,12,17,25 /
      6.           DATA T / 0.577350269,0.0,0.774596669,0.339981044,0.861136312,
      7.          10.0,0.538469310,0.906179846,0.238619186,0.661209387,0.932469514,
      8.          20.148874339,0.433395394,0.679409568,0.865063367,0.973906529,0.0,
      9.          3.201194093997435,.394151347077563,.570972172608539,.724417731360l7
     10.          40,.848206583410427,.937273392400706,.987992518020485 /
     11.            DATA WEIGHT / 1.0,0.888888889,0.555555556,0.652145155,.347854845,
     12.          10.568888889,0.478628671,0.236926885,0.467913935,0.360761573,
     13.          20.171324493,0.295524225,0.269266719,0.219086363,0.149451349,
     14.          30.066713441,.202578241925561,.198431485327111,.186161000115562,
     15.          4.166269205816994,.139570677926154,.107159220467172,.070366047488l0
     16.          58,.030753241996117 /
     17.            SUM = 0.0
     18.             I = 1
     19.    3      IF(M.EQ.NPOINT(I)) GO TO 2
     20.           I = I+1
     21.           IF(I.GT.7) GO TO 4
     22.            GO TO 3
     23.    4       GAUSS4 = 0.0
     24.           RETURN
     25.    2      JFIRST = KEY(I)
     26.           JLAST = KEY(I+1) - 1
     27.            C = .5*(B-A)
     28.           D = .5*(B+A)
     29.           GO TO (1,8,12,21,23,25,27,29,31),K
     30.    1      DO 5 J = JFIRST,JLAST
     31.           IF(T(J).EQ.0.0) SUM = SUM + WEIGHT(J)*H0PE(D,R,Z,ALPHA,BETA)
     32.    5      IF(T(J).NE.0.0) SUM = SUM+WEIGHT(J)*(H0PE(T(J)*C+D,R,Z,ALPHA,BETA)
     33.          1 + H0PE( -T(J)*C+D,R,Z,ALPHA,BETA))
     34.              GO TO 50
     35.    8      DO 9 J = JFIRST,JLAST
     36.           IF(T(J).EQ.0.0) SUM = SUM + WEIGHT(J)*F0FX(D,R,Z,ALPHA,BETA)
     37.    9      IF(T(J).NE.0.0) SUM = SUM+WEIGHT(J)*(F0FX(T(J)*C+D,R,Z,ALPHA,BETA)
     38.          1 + F0FX( -T(J)*C+D,R,Z,ALPHA,BETA))
     39.              GO TO 50
     40.    12     DO 13 J = JFIRST,JLAST
     41.           IF(T(J).EQ.0.0) SUM = SUM + WEIGHT(J)*F0FY(D,R,Z,ALPHA,BETA)
     42.    13     IF(T(J).NE.0.0) SUM = SUM+WEIGHT(J)*(F0FY(T(J)*C+D,R,Z,ALPHA,BETA)
     43.          1 + F0FY( -T(J)*C+D,R,Z,ALPHA,BETA))
     44.            GO TO 50
     45.    21     DO 22 J = JFIRST,JLAST
     46.           IF(T(J).EQ.0.0) SUM = SUM + WEIGHT(J)*H0PY(D,R,Z,ALPHA,BETA)
     47.    22     IF(T(J).NE.0.0) SUM = SUM+WEIGHT(J)*(H0PY(T(J)*C+D,R,Z,ALPHA,BETA)
     48.          1 + H0PY( -T(J)*C+D,R,Z,ALPHA,BETA))
     49.            GO TO 50
     50.    23     DO 24 J = JFIRST,JLAST
     51.            IF(T(J) .EQ.0.) SUM = SUM + WEIGHT (J)*FUN1(D,R,Z)
```

```
      24    IF(T(J).NE.0.) SUM =SUM+ WEIGHT(J)*(FUN1(T(J)*C+D,R,Z)+FUN1(
     1-T(J)*C+D,R,Z))
           GO TO 50
      25   DO 26 J = JFIRST,JLAST
           IF(T(J).EQ.0.) SUM = SUM + WEIGHT(J)*FUN2(D,R,Z)
      26   IF(T(J).NE.0.) SUM = SUM+WEIGHT(J)*(FUN2(T(J)*C+D,R,Z)+FUN2(
     1-T(J)*C+D,R,Z))
           GO TO 50
      27   DO 28 J = JFIRST,JLAST
           IF(T(J).EQ.0.) SUM = SUM + WEIGHT(J)*FUN3(D,R,Z)
      28   IF(T(J).NE.0.) SUM = SUM+WEIGHT(J)*(FUN3(T(J)*C+D,R,Z)+FUN3(
     1-T(J)*C+D,R,Z) )
           GO TO 50
      29   DO 30 J = JFIRST,JLAST
           IF(T(J).EQ.0.) SUM = SUM + WEIGHT(J)*FUN4(D,R,Z)
      30   IF(T(J).NE.0.) SUM = SUM+WEIGHT(J)*(FUN4(T(J)*C+D,R,Z)+FUN4(
     1-T(J)*C+D,R,Z))
           GO TO 50
      31   DO 32 J = JFIRST,JLAST
           IF(T(J).EQ.0.) SUM = SUM + WEIGHT(J)*FUN5(D,R,Z)
      32   IF(T(J).NE.0.) SUM= SUM+WEIGHT(J)*(FUN5(T(J)*C+D,R,Z)+FUN5(
     1-T(J)*C+D,R,Z))
      50    GAUSS4 = C*SUM
            RETURN
            END
```

```
      1.          FUNCTION FUNCT1(U,R,Z,A,B)
      2.           COSG = .5*B/A
      3.           IF(COSG.LE.1.) ANS = SERIE1(U,R,Z,A,B)
      4.           IF(COSG.GT.1.) ANS = SERIE2(U,R,Z,A,B)
      5.           W1 = .5*Z*R*U
      6.           Q1 = COSH(W1)
      7.          PSI = 2.*A*Q1 + B
      8.          PSI2 = PSI*PSI
      9.           F = ANS*ANS
     10.           FUNCT1 = PSI2*F
     11.          RETURN
     12.          END
```

```
1.          FUNCTION FUNCT2(U,R,Z,A,B)
2.          FUNCT2 = FUNCT1(U,R,Z,A,B)*U*U
3.           RETURN
4.           END
```

```
1.          FUNCTION FUNCT3(U,R,Z,A,B)
2.          COSG = .5*B/A
3.          IF(COSG.LE.1.) ANS = SERIE1(U,R,Z,A,B)
4.          IF(COSG.GT.1.) ANS = SERIE2(U,R,Z,A,B)
5.          PSI = 2.*A*COSH(.5*R*Z*U) + B
6.            FUNCT3 = PSI*PSI*ANS
7.          RETURN
8.          END
```

```
1.          FUNCTION FUNCT4(U,R,Z,A,B)
2.          COSG = .5*B/A
3.          PSI =  2.*A*COSH(.5*R*Z*U) + B
4.          IF(COSG.LE.1.) ANS = SERIE1(U,R,Z,A,B)
5.          IF(COSG.GT.1.) ANS = SERIE2(U,R,Z,A,B)
6.         F1 = FUNV*U(U,R,Z,A,B)
7.          FUNCT4 = F1*PSI*PSI*ANS
8.          RETURN
9.          END
```

```
1.          FUNCTION FUNCT5(U,R,Z,A,B)
2.           F1 = FUNCT3(U,R,Z,A,B)
3.          FUNCT5 = F1*U*U
4.          RETURN
5.          END
```

```
      FUNCTION FUNVMU(U,R,Z,A,B)
       W = .5*Z*R
      Q1 = COSH(U*W)
      Q2 = SINH(U*W)
      F1 = Z*Z+B*(1. - U*U)**.5
       F2 = 4.*A*Z*U*Q2/R
     PSI = 2.**A*Q1 + B
       FUNVMU = (F1 + F2 )/PSI
      RETURN
      END
```

```
      1.          FUNCTION GAUSS(A,B,M,K)
      2.           COMMON /A/ R,Z,ALPHA,BETA
      3.          DIMENSION NPOINT(7),KEY(8),T(24),WEIGHT(24)
      4.          DATA NPOINT / 2,3,4,5,6,10,15 /
      5.          DATA KEY / 1,2,4,6,9,12,17,25 /
      6.          DATA T / 0.577350269,0.0,0.774596669,0.339981044,0.861136312,
      7.         10.0,0.538469310,0.906179846,0.238619186,0.661209387,0.932469514,
      8.         20.148874339,0.433395394,0.679409568,0.865063367,0.973906529,0.0,
      9.         3.201194093997435,.394151347077563,.570972172608539,.72441773136017
     10.         40,.848206583410427,.937273392400706,.987992518020485 /
     11.           DATA WEIGHT / 1.0,0.888888889,0.555555556,0.652145155,.347854845,
     12.         10.568888889,0.478628671,0.236926885,0.467913935,0.360761573,
     13.         20.171324493,0.295524225,0.269266719,0.219086363,0.149451349,
     14.         30.066713443,.202578241925561,.198431485327111,.186161000115562,
     15.         4.166269205816994,.139570677926154,.107159220467172,.070366047488810
     16.         58,.030753241996117 /
     17.           SUM = 0.
     18.            I = 1
     19.     3     IF(M.EQ.NPOINT(I)) GO TO 2
     20.           I = I+1
     21.           IF(I.GT.7) GO TO 4
     22.            GO TO 3
     23.     4      GAUSS = 0.
     24.           RETURN
     25.     2     JFIRST = KEY(I)
     26.           JLAST = KEY(I+1) - 1
     27.            C = .5*(B-A)
     28.           D = .5*(B+A)
     29.           GO TO (1,8,6,10,12,14),K
     30.     1     DO 5 J = JFIRST,JLAST
     31.            IF(T(J).EQ.0.) SUM = SUM + WEIGHT(J)*FUNCT1(D,R,Z,ALPHA,BETA)
     32.     5      IF(T(J).NE.0.) SUM = SUM+WEIGHT(J)*(FUNCT1(T(J)*C+D,R,Z,ALPHA,
     33.         1BETA)   + FUNCT1(-T(J)*C+D,R,Z,ALPHA,BETA))
     34.            GO TO 50
     35.     8     DO 9 J = JFIRST,JLAST
     36.            IF(T(J).EQ.0.) SUM = SUM + WEIGHT(J)*FUNCT2(D,R,Z,ALPHA,BETA)
     37.     9      IF(T(J).NE.0.) SUM = SUM+WEIGHT(J)*(FUNCT2(T(J)*C+D,R,Z,ALPHA,
     38.         1BETA)   + FUNCT2(-T(J)*C+D,R,Z,ALPHA,BETA))
     39.            GO TO 50
     40.     6     DO 7 J = JFIRST,JLAST
     41.            IF(T(J).EQ.0.) SUM = SUM + WEIGHT(J)*FUNCT3(D,R,Z,ALPHA,BETA)
     42.     7      IF(T(J).NE.0.) SUM = SUM+WEIGHT(J)*(FUNCT3(T(J)*C+D,R,Z,ALPHA,
     43.         1BETA)  + FUNCT3(-T(J)*C+D,R,Z,ALPHA,BETA))
     44.            GO TO 50
     45.     10    DO 11 J = JFIRST,JLAST
     46.            IF(T(J).EQ.0.)  SUM = SUM + WEIGHT(J)*FUNCT4(D,R,Z,ALPHA,BETA)
     47.     11     IF(T(J).NE.0.) SUM = SUM+WEIGHT(J)*(FUNCT4(T(J)*C+D,R,Z,ALPHA,
     48.         1BETA)   + FUNCT4(-T(J)*C+D,R,Z,ALPHA,BETA))
     49.            GO TO 50
     50.     12    DO 13 J = JFIRST,JLAST
     51.            IF(T(J).EQ.0.)  SUM = SUM + WEIGHT(J)*FUNCT5(D,R,Z,ALPHA,BETA)
```

```
      13    IF(T(J).NE.0.) SUM = SUM+WEIGHT(J)*(FUNCT5(T(J)*C+D,R,Z,ALPHA,
     1BETA)   + FUNCT5(-T(J)*C+D,R,Z,ALPHA,BETA))
            GO TO 50
      14    DO 15 J = JFIRST,JLAST
            IF(T(J).EQ.0.) SUM = SUM + WEIGHT(J)*FUNCI2(D,R,Z,ALPHA,BETA)
      15    IF(T(J).NE.0.) SUM = SUM + WEIGHT(J)*(FUNCI2(T(J)*C+D,R,Z,ALPHA,
     1BETA)   + FUNCI2(-T(J)*C+D,R,Z,ALPHA,BETA))
            GO TO 50
      50    GAUSS = C*SUM
            RETURN
            END
```

```
      1.          FUNCTION FUN1(U,R,Z)
      2.          COMMON /B/ EP
      3.           C1 = (2.-Z)/Z + EP/(Z*Z)
      4.           C2 = .5*EP*R/Z
      5.            U4 = U*U*U*U
      6.           F= 1. + 1./U
      7.           F1 = LOG(F)
      8.          F2 = C1*F1 + C2/U
      9.           W = Z*R/U
     10.           G = EXP(-W)
     11.           FF = F2*F2
     12.           FUN1 = G*FF/U4
     13.           RETURN
     14.            END
```

```
1•          FUNCTION FUN2(U,R,Z)
2•          A = FUN1(U,R,Z)
3•           FUN2 = A*U*U
4•          RETURN
5•          END
```

```
1.            FUNCTION FUN3(U,R,Z)
2.             FUN3 = FUN1(U,R,Z)*U
3.           RETURN
4.            END
```

```
1.          FUNCTION FUNCI2(U,R,Z,A,B)
2.          F1 = FUNCT1(U,R,Z,A,B)
3.          F2 = FUNVMU(U,R,Z,A,B)
4.          FUNCI2 = F1*F2
5.           RETURN
6.           END
```

```
      1.          FUNCTION RESY(U,R,Z,A,B)
      2.         COMMON /ARC/G/ADD/COSEC
      3.          DATANH(X) = .5*LOG(  (1.+X)/(1.-X)  )
      4.          COSG = .5*B/A
      5.          G2 = .5*G
      6.          F1 = TANH(G2)
      7.          X = .25*Z*R*U
      8.          F2 = TANH(X)
      9.         F3 = F1*F2
     10.           F4 = DATANH(F3)
     11.           W = .5*R*Z
     12.          RESY = 2.*U*COSEC/W*F4
     13.          RETURN
     14.          END
```

```
      1.            FUNCTION HGPY(U,R,Z,A,B)
      2.            COMMON /ABC/G/ABB/COSG/ABC/COSEC
      3.           DATANH(X) = .5*LOG(  (1.+X)/(1.-X)  )
      4.           AA = A*A
      5.           AB = A*B
      6.           BB = B*B
      7.            Z2 = Z*Z
      8.            = .5*R*Z
      9.            W1 = .25*Z*R
     10.           F1 = TANH(U*W1)
     11.           G2 = .5*B
     12.           F2 = TANH(G2)
     13.            F3 = F1*F2
     14.            F4 = DATANH(F3)
     15.           HGPY = F4
     16.            RETURN
     17.             END
```

```
      FUNCTION RESX(U,R,Z,A,B)
      COMMON /ABC/G/ABD/COSEC
       COSG = .5*B/A
       G2 = .5*G
      F1 = TAN(G2)
      X = .25*Z*R*U
      F2 = TANH(X)
       F3 = F1*F2
      F4 = ATAN(F3)
      A = .5*R*Z
      RESX= 2.*U*COSEC/A*F4
       RETURN
        END
```

```
      1.            FUNCTION FOFY(U,R,Z,A,B)
      2.           COSG = .5*B/A
      3.          X1 = .5*R*Z*U
      4.          G1 = COSH(X1)
      5.           F1 = G1 + COSG
      6.             FOFY = U/F1
      7.           RETURN
      8.           END
```

```
      1•            FUNCTION HOPE(U,R,Z,A,B)
      2•            COMMON */ABC/G/AEB/COSG/ABD/COSEC
      3•           AA = A*A
      4•           AB = A*B
      5•           BB = B*B
      6•           Z2 = Z*Z
      7•           W = .5*R*Z
      8•            W1 = .25*Z*R
      9•           F1 = TAN(U+W1)
     10•            G2 = .5*G
     11•           F2 = TAN(G2)
     12•           F3 = F1*F2
     13•           F4 = ATAN(F3)
     14•             HOPE = F4
     15•           RETURN
     16•           END
```

```
      1.          FUNCTION VAL(U,R,Z,A,B)
      2.           COMMON /ABC/G/ABS/COSG/ARD/C0SEC
      3.          COMMON /BBC/C0SEC2/DEF/C0SEC3
      4.          G2 = .5*G
      5.             Z2 = Z*Z
      6.          AA = A*A
      7.          AB = A*B
      8.          BB = B*B
      9.          W = .5*Z*R
     10.          X1 = U*A
     11.          X2 = .5*U*W
     12.          G1 = COSH(X1)
     13.          G2 = SINH(X1)
     14.         F1 = G1 + COSG
     15.          F2 = LOG(F1)
     16.          W2 = 1./(W*W)
     17.          FUN1 = -C0SEC2*F2*W2
     18.         FUN2 = U*C0SEC2*G2/W/F1
     19.             VAL = FUN1 + FUN2
     20.          RETURN
     21.          END
```

```
1•              FUNCTION F9FX(U,R,Z,A,B)
2•             C0SG = .5*R/A
3•            X1 = .5*R*Z*U
4•            G1 = C0SH(X1)
5•             F1 = G1 + C0SG
6•              F2 = F1*F1
7•            F9FX = U/F2
8•             RETURN
9•             END
```

```
      1.            FUNCTION FINTIA(R,Z)
      2.            COMMON /C/EX/D/AL2/E/Q1/F/Q2
      3.           W = Z*R
      4.            W1 = 1./W
      5.            FINTIA = Q1*W1*(W1+(1.+W1)*AL2-Q2*EX*(1.-(2.*W-1.)*W1))
      6.           RETURN
      7.           END
```

```
1.            FUNCTION FINT1B(R,Z)
2.            COMMON /E/G1
3.           W = Z*R
4.            W1 = 1./
5.            W2 = W*W
6.           W3 = W2*W
7.            FINT1B = 2.*G1/W3*(1.+W+.5*W2)
8.           RETURN
9.           END
```

```
      FUNCTION PSISOU(U,R,Z,A,B)
      W = .5*Z**2
      W1 = Z*R
      AA = A*A
      BB = B*B
      AB = A*B
      C = SINH(2.*U*W)
      C1 = SINH(U*Z)
      PSISOU = 2.**AA/W1*C+8.**AB/W1*C1+U*(2.**AA+BB)
        RETURN
      END
```

```
      1.          SUBROUTINE DINB(U,R,Z,A,B,ANS)
      2.          A = .5*Z**2
      3.          W1 = R*Z
      4.          Y1 = U*W
      5.          Y2 = U*W1
      6.           C1 = COSH(Y1)
      7.           C2 = COSH(Y2)
      8.           C3 = SINH(Y1)
      9.           C4 = SINH(Y2)
     10.          A1 = W1**2
     11.          A2 = A1*R
     12.          AA = A*A
     13.          AB = A*B
     14.          BB = B*B
     15.          A3 = A2*Z
     16.          U2 = U*U
     17.          U3 = U*U*U
     18.          C1 = 2.*AA+BB
     19.          A4 = Z*R*R*R
     20.          A5 = Z*R*R
     21.          R2 = R*R
     22.             Z2 = Z**2
     23.           T1 = -4.*AA/(Z*R2*R)
     24.           T2 = 2.*AB*Z/R*(1. - 16./A1)
     25.           T3 = -2.*Z*AB/R
     26.           T4 = .5*Z2*BB
     27.           T5 = -Z2*BB**.16666666666666666
     28.           T6 = 4.*AA/R2
     29.           T7 = 16.*AB/R2
     30.     1     ANS1   = C4*T1
     31.     2     ANS2   = C3*T2
     32.     3     ANS3   = U2*C3*T3
     33.     4     ANS4   = U*T4
     34.     5     ANS5   = U3*T5
     35.     6     ANS6   = U*C2*T6
     36.     7     ANS7   = U*C1*T7
     37.           ANS = ANS1 + ANS2 + ANS3 + ANS4 + ANS5 + ANS6 + ANS7
     38.           RETURN
     39.           END
```

```
      FUNCTION EXPI(R,Z)
      DIMENSION EC0(36), XOC(36)
      DIMENSION EC1(61),EC2(70),XC1(61),XO2(70)
       DIMENSION EO3(50), XC3(50)
      DATA EOC /
     1 1.49334874693224C,1.34C8854448313?3,1.2225356C5080586,1.127045349
     2862316,1.047828008456CC6,.98C71472335839C5,.9229106324837305,.8724
     3573131594157,.8279334352735C85,.7852775692635576,.752678C20C295871
     4,.72C501506587045l,.691245397BC28315,.6645C47874381C68,.6399492266
     5392997,.6173C59205526C7C,.5963473623231941,.5768820704C5251C,.5587
     647556170236 4,.5418C49C14535823,.525934531894784 6,.511032883674047 6
     7  ,.497C07482616443,.4837861375242837,.4712925524860614,.45946756
     883131685,.44825666929153 0,.4376112831118401,.4274879752994281,
     9    .41784773267157 7,.4C8655596CCC5607,.39987977037797 6,.39149161
     1897C3562,.3834651C71497194,.375776539668 848,.3684042987138854 /
       DATA EC1 /
     1    .3613286168882226,.3545313797517170,.3479959534 7C7185,.34170703
     2389C5328,.3356505139529960,.3298133668814699,.3241835429648885,.31
     38749878109 6143,.3135020126C67469,.3C843C3187941878,.3C352583648598
     441,.2987802152032416,.2941856623768447,.2897348968C68591,.28542110
     5676C5018,.2812379121729163,.2771793304851186,.2732397457133851,.26
     6941386039566 8,.26569677C1046732,.2620837402553185,.25857C3849668 4
     728,.2551525477685169,.2518263C39623042,.2488579444773646,.24543396
     81069944C,.2423610327385172,.2393660132383432,.2364459195921340,.23
     9359792150455 8,.23C8193315950103,.2281075963956555,.22546028798531
     118,.2228756963044691,.2203498219926746,.2178823697628117,.21547074
     222475C7C,.2131130342810929,.21C8C74275813037,.2085521857982297,
     1 .2C63456499C10558,.2041862338758CC1,.2020724207096828,.2000C27586
     1399285,.19797585764676CC,.19599C3861721022,.1940450680471100,.1921
     2386796136470,.19C27C047C215C48,.1884380437CC5717,.1866415879757465
     3,.18487964C8339623,.1831512C382109C0,.1814553170635290,.1797910574
     4C59859,.1781575366533868,.1765538999222755,.1749793240740C63,.1734
     5330162445464,.1719142124525677,.17C422176284732 /
       DATA EC2 /
     1  .1675155916170776,.1647078767C47833,.1619940141687699,.159369323
     2612C981,.1568294335801275,.15437C2562925014,.1519879648307450,.149
     367897250602 1,.1474399141665211,
     1  .1452676292338860,.1431591462540558,.1411116690100267,.139122563
     24235193,.1371893461691771,.1353C96738395544,.1334813331918336,
     3131702232178 C76,.12997C3917123769,.1282839381003267,
     1  .1266410960766328,.1250401823371386,.1234795998702534,.121957831
     29914612,.12C4734377909765,.1190250472084111,.1176113567519010,.116
     3231125482322 2,.1148831712860667,.1135663674123762,
     1          .1122796392534993,.111C219613479563,.1097923545890153,
     1.1085898836221073,.1074136544163729,.1062628119968480,.10513653832
     249826,.1043C34050316254 6,.1C2954597984607C,.1C1897462704310 5,
     3.10C8619558806409,.C9984741592148153,.0986532C98026C409,.C97878728
     471997763,.C96923388329865 7,.C958662722929 24,.0950679C587161234,
     5.0941667055052613 9,.0932825271493381 0,.C924148906801926C,.09156333
     6393978 8C6,.C899066958733947 9,.C883C924437781962,.C8676784941570828
     7,.0852795986669841 4,.C838417788603459 5,.0824518589988171 1,.C811074
```

```
     52.          87525831423,.C798C641736726444,.C785466163C012471,.C773261331389123
     53.          9 /
     54.           DATA EC3 / .C761431489751727 3,.C749959557C34253,.C7388294786642593
     55.          1,.C728C2614,69C16544,.C717535335244624C,.C707343633254652 ,.0697438
     56.          2389235 1444,.C6878C76568C863 C,.C678440146689838 ,.C669325181834396
     57.          33,.C66C45265642834 8,.C651812998 C35035,.C643397133118 784,.063519
     58.          4645424C1636,.C6272C2791C740924,.C6194C83826443568,.C6118C585201044
     59.          537,.C6C434818275556 2,.C597178697184159 ,.C59CC81C36C855644,.C5831
     60.          679139933875 ,.C5764372314C63416,.C5698497991134497,.C5634115824436
     61.          7879,.C557117557434767 ,.C55C962923 9C9709,.C544943C915734226,
     62.          8 .C539C536716964754,.C53329C4631692088,.C527649444026250 ,.C522126
     63.          1741696867 ,.C516718724165552 ,.C511421791681213 ,.C5C623256897560 1
     64.          25,.C5C1147797954799 ,.C496164350830 196,.C491279223C66 5991,.C4864
     65.          38353C27C49C5,.C4817924965297609,.C477185454959 C84,.C466C449650238
     66.          4127,.C455413616545C517,.C445257236468 0038,.C435544646926 788,
     67.          5 .C42C2473437 C5852,.C417339215560 062,.C4087962975740386,.C400596
     68.          6555236403 ,.C3927196935293355,.C3251469884490402 /
     69.           DATA XCC / .2C,.25,.3C,.35,.4C,.45,.5C,.55,.6C,.65,.7C,.75,.8C,
     70.          1  .85,.9C,.95,1.00,1.C5,1.1C,1.15,1.20,1.25,1.30,1.35,1.40,1.45,
     71.          2  1.5C,1.55,1.60,1.65,1.7C,1.75,1.8C,1.85,1.9C,1.95 /
     72.            DATA XC1 /
     73.          1   2.C0,2.05,2.10,2.15,2.2C,2.25,2.30,2.35,2.40,2.45,2.5C,2.55,
     74.          2  2.6C,2.65,2.7C,2.75,2.8C,2.85,2.90,2.95,3.0C,3.05,3.10,3.15,
     75.          3  3.2C,3.25,3.3C,3.35,3.4C,3.45,3.5C,3.55,3.6C,3.65,3.7C,3.75,3.80
     76.          4,3.85,3.9C,3.95,
     77.          1   4.C0,4.05,4.10,4.15,4.2C,4.25,4.30,4.35,4.40,4.45,4.5C,4.55,
     78.          1  4.6C,4.65,4.7C,4.75,4.8C,4.85,4.90,4.95,5.0C /
     79.            DATA XC2 /
     80.          1  5.1,5.2,5.3,5.4,5.5,5.6,5.7,5.8,5.9,
     81.          1   6.CC,
     82.          1  6.1C,6.2C,6.3C,6.4C,6.5C,6.6C,6.7C,6.8C,6.9C,7.0C,7.1C,7.2C,7.3C
     83.          2,7.4C,7.5C,7.6C,7.7C,7.8C,7.9C,
     84.          2          8.0C,8.1C,8.2C,8.3C,8.4C,8.5C,8.6C,8.7C,8.8C,8.9C,9.0C,
     85.          19.1C,9.2C,9.3C,9.4C,9.5C,9.6C,9.7C,9.8C,9.9C,10.0C,10.2C,10.4C,
     86.          21C.6C,1C.8C,11.0C,11.2C,11.4C,11.6C,11.8C,12.CC /
     87.           DATA XC3 /
     88.          1 12.2,12.4,12.6,12.8,13.C,13.2,13.4,13.6,13.8,14.C,14.2,14.4,14.6,
     89.          2 14.8,15.C,15.2, 15.4,15.6,15.8,16.C,16.2,16.4,16.6,16.8,17.C,17.2,
     90.          3 17.4,17.6,17.8,18.C,18.2,18.4,18.6,18.8,19.C,19.2,19.4,19.6,19.8,
     91.          4 20.C,2C.5,21.C,21.5,22.C,22.5,23.C,23.5,24.C,24.5,25.C /
     92.           X = 2.**2.R
     93.           DC 9 K = 1,36
     94.           DIFFC = X - XCC(K)
     95.            X1C = XCC(K)
     96.            E1C = ECC(K)
     97.            IF(DIFFC.LT.C.)  GC TC 19
     98.     9      CONTINUE
     99.     6      DC 1 J = 1,61
    100.            DIFF1 = X - XC1(J)
```

```
      X11 = XC1(J)
       E11 = EC1(J)
       IF(DIFF1.LT.0.) GO TO 3
1     CONTINUE
      DO 2 I = 1,70
       DIFF2 = X-XO2(I)
       X12 = XO2(I)
      E12 = EC2(I)
       IF(DIFF2.LT.0.) GO TO 5
2     CONTINUE
      DO 30 M = 1,50
      DIFF3 = X - XC3(M)
       X13 = XC3(M)
       E13 = EC3(M)
       IF(DIFF3.LT.0.) GO TO 40
30    CONTINUE
40    EXPI = DEX(DIFF3,E13,X13,X)
      RETURN
19    EXPI = DEX( DIFFC,E1C,X1C,X)
      RETURN
3     EXPI = DEX(DIFF1,E11,X11,X)
      RETURN
5     EXPI = DEX(DIFF2,E12,X12,X)
      RETURN
      END
```

```
      1.          FUNCTION DEX(DIFF,E1,X1,>)
      2.          DIMENSION T(50)
      3.          SUM1 = 0.
      4.         DO 4 N = 1,50
      5.         K = N-1
      6.         IF(K.EQ.0) T(N) = DIFF/N*(E1+DIFF**K/((-X1)**N))
      7.         IF(K.NE.0) T(N) = DIFF/N*(T(K)+DIFF**K/((-X1)**N))
      8.         SUM1 = SUM1 + T(N)
      9.          G1 = T(N) - T(K)
     10.          D = ABS(G1)
     11.          IF(D.LE.1.E-16)GO TO 3
     12.    4     CONTINUE
     13.    3     EX1 = E1 + SUM1
     14.         G = DEXP(X)
     15.          DEX = -EX1/G
     16.          RETURN
     17.         END
```

```
      1.            FUNCTION SINT2A(R,Z)
      2.           COMMON /C/EX/D/AL2/E/G1/F/G2
      3.           W = Z*R
      4.           W1 = 1./W
      5.          SINT2A = G1*W1*(AL2-G2*EX)
      6.          RETURN
      7.          END
```

```
      1.          FUNCTION SINT2B(R,Z)
      2.          COMMON /E/G1
      3.          A = R*Z
      4.          W1 = 1./A
      5.          W2 = W1*W1
      6.        SINT2B = G1*W2*(1.+W)
      7.          RETURN
      8.          END
```

```
      1.          FUNCTION SINT2D(U,R,Z,A,B)
      2.          W = .5*Z*R
      3.           W1 = Z*R
      4.           W2 = W1*W1
      5.            W3 = W2*W1
      6.          AA = A*A
      7.          AB = A*B
      8.          BB = B*B
      9.           CS = COSH(U*W)
     10.           CSS = COSH(U*W1)
     11.           SIH = SINH(U*W)
     12.           SIHH = SINH(U*W1)
     13.          FUN2 = U*U*U/3.*(2.*AA+BB)+8.*AB*U*U*SIH/W1
     14.           FUN3 = 2.*AA*U*U*SIHH/W1-4.*AA*U*CSS/W2
     15.           FUN4 = 64.*AB*SIH/W3-32.*AB*U*CS/W2
     16.          FUN5 = 4.*AA*SIHH/W3
     17.          FUNCT1 = FUN2+FUN3+FUN4+FUN5
     18.           SINT2D = .5*Z*Z*FUNCT1
     19.          RETURN
     20.          END
```

R = .0500
E(0)=-Z*Z/2= -1.9885105365614 E1=EGM= -1.993974577396
E(1) = EGM + Z*Z/2 = -.0054640417800
E2 = -.0000005869716 E(2) = -1.993975167112
E3 = -.0000000006474 E(3) = -1.993975173586

R = .1000
E(0)=-Z*Z/2= -1.9602173360014 E1=EGM= -1.978225899025
E(1) = EGM + Z*Z/2 = -.0180085638811
E2 = -.0002307310682 E(2) = -1.97824663C9C7
E3 = .0000000880702 E(3) = -1.97824655C2C6

R = .2000
E(0)=-Z*Z/2= -1.8734125865784 E1=EGM= -1.928537622873
E(1) = EGM + Z*Z/2 = -.0551250370900
E2 = -.0009287587120 E(2) = -1.928610609993
E3 = .0000002869916 E(3) = -1.928610340077

MU	LAMBDA	PHI GM	PHI(1)
.00	1.00	1.2670115378	1.2571391669
.00	2.00	1.0437663851	1.0407697749
.00	3.00	.8598566265	.8597441970
.00	4.00	.7083514363	.7093534375
.00	5.00	.5835411861	.5848273497
.00	6.00	.4807228C9	.4819106280
.00	7.00	.3960198802	.3965537695
.00	8.00	.32624189CC	.3268789752
.00	9.00	.2687586561	.2691127230
.00	10.00	.2214038646	.2215136708
.10	1.00	1.2670930455	1.2572201225
.10	2.00	1.0438335313	1.0408367967
.10	3.00	.8599119416	.8597995612
.10	4.00	.7083970051	.7093991171
.10	5.00	.5835787277	.5848650103
.10	6.00	.4807532061	.4819416611
.10	7.00	.3960453564	.3969793318
.10	8.00	.3262628774	.3269000249
.10	9.00	.2687759455	.2691300528
.10	10.00	.2214181076	.2215279354
.20	1.00	1.2673375993	1.2574630079
.20	2.00	1.0440349951	1.0410378775

.20	3.00	.8600779079	.8599656667
.20	4.00	.7085337284	.7095361664
.20	5.00	.5836913606	.5849780007
.20	6.00	.4808459933	.4820347677
.20	7.00	.3961217947	.3970560244
.20	8.00	.3263758473	.3269431789
.20	9.00	.2688278203	.2691820462
.20	10.00	.2214008422	.2215707326
.30	1.00	1.2677452909	1.2578678791
.30	2.00	1.0443708522	1.0413730639
.30	3.00	.8603545876	.8602425520
.30	4.00	.7087616576	.7097646172
.30	5.00	.5838791291	.5851663471
.30	6.00	.4810006773	.4821899693
.30	7.00	.3962492236	.3971838653
.30	8.00	.3264308236	.3270684519
.30	9.00	.2689142999	.2692687153
.30	10.00	.2215320842	.2216420722
.40	1.00	1.2683162737	1.2584348295
.40	2.00	1.0448412288	1.0418424333
.40	3.00	.8607420847	.8606302812
.40	4.00	.7090808784	.7100845225
.40	5.00	.5841421038	.5854300933
.40	6.00	.4812173164	.4824073019
.40	7.00	.3964276913	.3973628839
.40	8.00	.3265778455	.3272158682
.40	9.00	.2690354169	.2693900802
.40	10.00	.2216318606	.2217419709
.50	1.00	1.2690507621	1.2591639896
.50	2.00	1.0454463017	1.0424460942
.50	3.00	.8612405448	.8611289440
.50	4.00	.7094915108	.7104959564
.50	5.00	.5844803834	.5857693003
.50	6.00	.4814959917	.4826868159
.50	7.00	.3966572645	.3975931220
.50	8.00	.3267669684	.3274054621
.50	9.00	.2691912166	.2695461689
.50	10.00	.2217002087	.2218704517
.60	1.00	1.2699490320	1.2600555276
.60	2.00	1.0461862980	1.0431841863
.60	3.00	.8618501551	.8617386561
.60	4.00	.7099937089	.7109990143
.60	5.00	.5848940951	.5861840470
.60	6.00	.4818368082	.4830285763
.60	7.00	.3969380297	.3978746328
.60	8.00	.3269982633	.3276372774
.60	9.00	.2693817578	.2697370178
.60	10.00	.2219171768	.2220275443

•70	1•CC	1•2710114209	1•2611096490
•70	2•CC	1•0470614960	1•0440568801
•70	3•CC	•8625711447	•8624595585
•70	4•CC	•7105876614	•7115938129
•70	5•CC	•5853833943	•5866744294
•70	6•CC	•4822398937	•4834326622
•70	7•CC	•3972700923	•3982074818
•70	8•CC	•3272718170	•3275113681
•70	9•CC	•2696071118	•2699626710
•70	10•CC	•2221028239	•2222132853
•80	1•CC	1•2722383280	1•2623265968
•80	2•CC	1•0480722242	1•0450643776
•80	3•CC	•8634037845	•8632918183
•80	4•CC	•7112735915	•7122804900
•80	5•CC	•5859484648	•5872405614
•80	6•CC	•4827053995	•4838991675
•80	7•CC	•3976535771	•3985917460
•80	8•CC	•3275877324	•3282277976
•80	9•CC	•2698673635	•2702231805
•80	10•CC	•2223172198	•2224277177
•90	1•CC	1•2736302142	1•2637066516
•90	2•CC	1•0492186626	1•0462069117
•90	3•CC	•8643483872	•8642356284
•90	4•CC	•7120517570	•7130592050
•90	5•CC	•5865895188	•5878825743
•90	6•CC	•4832335013	•4844282004
•90	7•CC	•3980886280	•3990275147
•90	8•CC	•3279461282	•3285866394
•90	9•CC	•2701626106	•2705186077
•90	10•CC	•2225604448	•2226708911
1•00	1•CC	1•2751876024	1•2652501317
1•00	2•CC	1•0505018418	1•0474847468
1•00	3•CC	•8654053078	•8652912074
1•00	4•CC	•7129224500	•7139301383
1•00	5•CC	•5873067974	•5886006171
1•00	6•CC	•4838243966	•4850198836
1•00	7•CC	•3985754087	•3995148891
1•00	8•CC	•3283471390	•3289879766
1•00	9•CC	•2704929640	•2708490202
1•00	10•CC	•2228325906	•2229428619

R = •3000

E(0)=-Z•Z/2= -1•7852566476665 E1=EGM= -1•8665411539920

E(1) = EGM + Z••2/2 = -•0812845063270

E2 = -•0001723034561 E(2) = -1•8667134574430

E3 = •0000161293900 E(3) = -1•8667119280530

R = .4000
E(0)=-Z*Z/2= -1.692087111364 E1=EGM= -1.800525423037
E(1) = EGM + Z*Z/2 = -.107538311672
E2 = -.000231837536 E(2) = -1.800757260573
E3 = .000002774823 E(3) = -1.800754485751

R = .5000
E(0)=-Z*Z/2= -1.606182250088 E1=EGM= -1.734714716425
E(1) = EGM + Z*Z/2 = -.128532466337
E2 = -.000271737853 E(2) = -1.734986454308
E3 = .000003527476 E(3) = -1.734982926832

MU	LAMBDA	PHI GM	PHI(1)
.00	1.00	.9096812573	.8996524989
.00	2.00	.5811550061	.5822061324
.00	3.00	.3712741561	.3733927103
.00	4.00	.2371905904	.2383071436
.00	5.00	.1515305475	.1516232334
.00	6.00	.0968061456	.0962619713
.00	7.00	.0618451526	.0610152825
.00	8.00	.0395101248	.0386246828
.00	9.00	.0252412663	.0244248389
.00	10.00	.0161255254	.0154314529
.10	1.00	.9100115996	.8999808409
.10	2.00	.5813660472	.5824186039
.10	3.00	.3714089809	.3735289744
.10	4.00	.2372767240	.2383941107
.10	5.00	.1515855745	.1516785675
.10	6.00	.0968412999	.0962971027
.10	7.00	.0618676111	.0610375512
.10	8.00	.0395244726	.0386387803
.10	9.00	.0252504324	.0244337541
.10	10.00	.0161313813	.0154370858
.20	1.00	.9110032900	.9009662934
.20	2.00	.5819995942	.5830562962
.20	3.00	.3718137260	.3739379452
.20	4.00	.2375352976	.2386551258
.20	5.00	.1517507657	.1518446418
.20	6.00	.0969468333	.0964025424
.20	7.00	.0619350317	.0611043863
.20	8.00	.0395675446	.0386810910
.20	9.00	.0252779492	.0244605112
.20	10.00	.0161489605	.0154539917
.30	1.00	.9126583198	.9026101364

.30	2.00	.5830569193	.58412C0434
.30	3.00	.3724892041	.3746201588
.30	4.00	.2379668306	.2390905308
.30	5.00	.152026453C	.1521216736
.30	6.00	.0971229577	.0965784280
.30	7.00	.0620475497	.0612158744
.30	8.00	.0396394274	.0387516694
.30	9.00	.0253238719	.C245C51444
.30	10.00	.0161782985	.0154821920
.40	1.00	.9149800124	.9049145056
.40	2.00	.5845401457	.5856112372
.40	3.00	.3734367717	.3755765095
.40	4.00	.2385721895	.2397C08962
.40	5.00	.15241319CC	.1525100250
.40	6.00	.0973700267	.C968249885
.40	7.00	.0622053912	.C613721603
.40	8.00	.0397402653	.C388506068
.40	9.00	.0253882928	.C245677111
.40	10.00	.0162194542	.0155217229
.50	1.00	.9179730299	.9078823955
.50	2.00	.5864522518	.5875318287
.50	3.00	.3746583315	.3768C82512
.50	4.00	.2393525899	.2404870219
.50	5.00	.1529117531	.153C1C2C39
.50	6.00	.0976885366	.0971425454
.50	7.00	.0624088731	.0615734467
.50	8.00	.03987026C8	.0389780308
.50	9.00	.0254713412	.C246482916
.50	10.00	.0162725101	.0155726347
.60	1.00	.9216433825	.9115176642
.60	2.00	.5887970773	.5898843321
.60	3.00	.3761563364	.3783169998
.60	4.00	.2403095968	.2414499386
.60	5.00	.1535231437	.1536228646
.60	6.00	.0980791269	.0975315126
.60	7.00	.0626584039	.0618199949
.60	8.00	.040029675C	.0391341061
.60	9.00	.0255731838	.C247469894
.60	10.00	.0163375728	.0156349924
.70	1.00	.9259984405	.9158250391
.70	2.00	.5915793307	.5926718281
.70	3.00	.3779337947	.3801047349
.70	4.00	.2414451381	.2425909092
.70	5.00	.1542485894	.1543488C88
.70	6.00	.0985425820	.C975923974
.70	7.00	.0629544848	.C621121252
.70	8.00	.0402188279	.0393190343
.70	9.00	.0256940253	.C248639314

•70	10•CC	•0164147731	•0157088756
•80	1•CC	•9310469491	•9208101246
•80	2•CC	•5948045989	•5958979685
•80	3•CC	•3799942755	•3821738036
•80	4•CC	•2427614879	•2439114307
•80	5•CC	•1550695469	•1551869871
•80	6•CC	•0990798324	•0985258014
•80	7•CC	•0632977102	•0624502172
•80	8•CC	•0404380995	•0395330546
•80	9•CC	•0258341081	•0249992678
•80	10•CC	•0165042658	•0157943787
•90	1•CC	•9367990460	•9264794103
•90	2•CC	•5984793585	•5995669824
•90	3•CC	•3823419164	•3845269238
•90	4•CC	•2442612915	•2454132368
•90	5•CC	•1560477049	•1561445001
•90	6•CC	•0996919571	•0991324209
•90	7•CC	•0636887695	•0628347103
•90	8•CC	•0406879299	•0397764437
•90	9•CC	•0259937137	•0251531727
•90	10•CC	•0166062307	•0158916110
1•00	1•CC	•9432662819	•9328402608
1•00	2•CC	•6026109887	•6036836823
1•00	3•CC	•3849814317	•3871671882
1•00	4•CC	•2459475607	•2470983005
1•00	5•CC	•1571249875	•1572166002
1•00	6•CC	•1003801852	•0998130485
1•00	7•CC	•0641284479	•0632661047
1•00	8•CC	•0409688209	•0400495167
1•00	9•CC	•0261731625	•0253258442
1•00	10•CC	•0167208726	•0160006969

R = .6000
E(0)=-Z*Z/2= -1•526260460244 E1=EGM= -1•671185327835
E(1) = EGM + Z*Z/2 = -•144924867591
E2 = -•00036271413 5 E(2) = -1•671488041970
E3 = •00000449264 9 E(3) = -1•671483549321

R = .7000
E(0)=-Z*Z/2= -1•453335065668 E1=EGM= -1•610884736452
E(1) = EGM + Z*Z/2 = -•157549670785
E2 = -•00031884684 5 E(2) = -1•611203583298
E3 = •00000553423 0 E(3) = -1•611198049067

R = .8000
E(0)=-Z*Z/2= -1.3869997C0768 E1=EGM= -1.554166175507
E(1) = EGM + Z*Z/2 = -.167166474739
E2 = -.00032175674? E(2) = -1.554487932254
E3 = .0000C63C1642 E(3) = -1.554481630613

R = .9000
E(0)=-Z*Z/2= -1.3266668674851 E1=EGM= -1.5C1C71782041
E(1) = EGM + Z*Z/2 = -.1744C31C715C
E2 = -.000316666826 E(2) = -1.501388448867
E3 = .0000C6717117 E(3) = -1.501381731750

MU	LAMBDA	PHI GM	PHI(1)
.00	1.00	.6368166444	.6295985958
.00	2.CC	.3C59659171	.3C78643648
.00	3.C0	.1470048612	.1476890452
.00	4.CC	.07C6301846	.C7011C5809
.00	5.CC	.0339350885	.033C533247
.00	6.CC	.0163045055	.0155027012
.00	7.00	.0078336881	.CC724C5855
.00	8.CC	.0037637860	.0033693443
.00	9.CC	.0018083545	.0015625991
.00	10.CC	.0008688448	.0007223318
.10	1.00	.6374719951	.6302543717
.10	2.CC	.3062807879	.3081849626
.10	3.00	.1471561445	.1478428455
.10	4.CC	.07070287C4	.C701836030
.10	5.0C	.0339700113	.033C877585
.10	6.CC	.0163212845	.0155188561
.10	7.0C	.0078417498	.0072481335
.10	8.CC	.0037676593	.0033728581
.10	9.C0	.0018102155	.0015642295
.10	10.CC	.0008697389	.0007230858
.20	1.00	.6394415699	.6322241073
.20	2.CC	.3072270929	.3091479424
.20	3.00	.1476108077	.1483048153
.20	4.CC	.07C9213186	.C7C4029380
.20	5.0C	.0340749672	.0331911855
.20	6.CC	.0163717118	.0155673793
.20	7.00	.0078659782	.0072708041
.20	8.CC	.0037793CC1	.0033834119
.20	9.00	.0018156084	.0015691263
.20	10.CC	.0008724261	.0007253506
.30	1.00	.6427359562	.6355150375
.30	2.CC	.3088099188	.3107568696

.30	3.00	.1483712947	.1490766637
.30	4.00	.0712867034	.0707693929
.30	5.00	.0342505206	.0333639829
.30	6.00	.0164560585	.0156484455
.30	7.00	.0079065035	.0073086780
.30	8.00	.0037987710	.0034010425
.30	9.00	.0018251634	.0015773061
.30	10.00	.0008769209	.0007291336
.40	1.00	.6473728625	.6401392561
.40	2.00	.3110377741	.3130177040
.40	3.00	.1494416935	.1501612480
.40	4.00	.0718009889	.0712843170
.40	5.00	.0344976150	.0336067815
.40	6.00	.0165747778	.0157623474
.40	7.00	.0079635436	.0073618299
.40	8.00	.0038261766	.0034258116
.40	9.00	.0018383307	.0015887972
.40	10.00	.0008832473	.0007344475
.50	1.00	.6533772143	.6461137689
.50	2.00	.3139226343	.3159388263
.50	3.00	.1508277579	.1515625857
.50	4.00	.0724665395	.0719496073
.50	5.00	.0348175787	.0339204678
.50	6.00	.0167285081	.0159094962
.50	7.00	.0080374051	.0074306293
.50	8.00	.0038616642	.0034578060
.50	9.00	.0018553812	.0016036390
.50	10.00	.0008914393	.0007413103
.60	1.00	.6607812874	.6534605684
.60	2.00	.3174800067	.3195310750
.60	3.00	.1525369386	.1532858728
.60	4.00	.0732881352	.0727677172
.60	5.00	.0352121316	.0343061888
.60	6.00	.0169180756	.0160904237
.60	7.00	.0081284850	.0075151412
.60	8.00	.0039054245	.0034971378
.60	9.00	.0018764064	.0016218826
.60	10.00	.0009015411	.0007497450
.70	1.00	.6696248816	.6622067290
.70	2.00	.3217290138	.3238077937
.70	3.00	.1545784232	.1553375062
.70	4.00	.0742689900	.0737416673
.70	5.00	.0356833947	.0347653561
.70	6.00	.0171444994	.0163057845
.70	7.00	.0082372729	.0076157271
.70	8.00	.0039576930	.0035439452
.70	9.00	.0019015193	.0016435906
.70	10.00	.0009136069	.0007597800

.80	1.00	.6799555352	.6723845235
.80	2.00	.3266924957	.3287848890
.80	3.00	.1569631854	.1577251113
.80	4.00	.0754147766	.0748750579
.80	5.00	.0362339011	.0352996524
.80	6.00	.0174089966	.0165563588
.80	7.00	.0083643536	.0077327463
.80	8.00	.0040187504	.0035983924
.80	9.00	.0019308551	.0016688381
.80	10.00	.0009277016	.0007714491
.90	1.00	.6918287795	.6840315601
.90	2.00	.3323971332	.3344808980
.90	3.00	.1597040444	.1604575747
.90	4.00	.0767316540	.0761720847
.90	5.00	.0368666100	.0359110383
.90	6.00	.0177129869	.0168430557
.90	7.00	.0085104102	.0078666176
.90	8.00	.0040889250	.0036606712
.90	9.00	.0019645713	.0016977121
.90	10.00	.0009439010	.0007847918
1.00	1.00	.7053084381	.6971905397
1.00	2.00	.3388735910	.3409170668
1.00	3.00	.1628157335	.1635450814
1.00	4.00	.0782267009	.0776375562
1.00	5.00	.0375849226	.0366017609
1.00	6.00	.0180581105	.0171669170
1.00	7.00	.0086762279	.0080178205
1.00	8.00	.0041685940	.0037310010
1.00	9.00	.0020028492	.0017303124
1.00	10.00	.0009622920	.0007998530

R = 1.0000
E(0)=-Z*Z/2= -1.27172499181^ E1=EGM= -1.451484730634
E(1) = E_{GM} + Z*Z/2 = -.179759738824
E2 = -.000307581645 E(2) = -1.451792312299
E3 = .000068826866 E(3) = -1.45178542941 3

MU	LAMBDA	PHI GM	PHI(1)
.00	1.00	.5895329505	.5828881491
.00	2.00	.2655812306	.2673675886
.00	3.00	.1196428291	.1200167794
.00	4.00	.0538984119	.0532349382
.00	5.00	.0242809270	.0234254953
.00	6.00	.0109384191	.0102458282
.00	7.00	.0049276954	.0044586529
.00	8.00	.0022196967	.0019314440
.00	9.00	.0010000517	.0008330692

.00	10.CC	.0004505176	.0003577784
.10	1.CC	.590259748C	.5836168259
.10	2.CC	.2659086487	.2677017461
.10	3.CC	.1197903291	.1201667844
.10	4.CC	.0539648598	.0533014890
.10	5.CC	.0243108614	.0234547899
.10	6.CC	.0109519044	.0102586465
.10	7.CC	.0049337704	.0044642339
.10	8.CC	.0022226354	.0019338631
.10	9.CC	.0010012846	.0008341133
.10	10.CC	.0004510730	.0003582272
.20	1.CC	.5924447643	.5858060585
.20	2.CC	.2668929861	.2687056985
.20	3.CC	.1202337675	.1206174625
.20	4.CC	.0541646262	.0535014334
.20	5.CC	.0244008551	.0235428010
.20	6.CC	.0109924460	.0102971564
.20	7.CC	.0049520342	.0044810004
.20	8.CC	.0022308632	.0019411303
.20	9.CC	.0010049911	.0008372499
.20	10.CC	.0004527428	.0003595753
.30	1.CC	.5961019005	.5894654706
.30	2.CC	.2685405051	.2703838940
.30	3.CC	.1209759654	.1213708076
.30	4.CC	.0544989822	.0538356502
.30	5.CC	.0245514805	.0236899113
.30	6.CC	.0110603019	.0103615229
.30	7.CC	.0049826029	.0045090229
.30	8.CC	.0022446342	.0019532757
.30	9.CC	.0010111949	.0008424915
.30	10.CC	.0004555375	.0003618280
.40	1.CC	.6012544233	.5946111584
.40	2.CC	.2708616872	.2727437725
.40	3.CC	.1220216447	.1224301543
.40	4.CC	.0549700547	.0543056087
.40	5.CC	.0247636960	.0238967607
.40	6.CC	.0111559038	.0104520222
.40	7.CC	.0050256710	.0045484199
.40	8.CC	.0022640361	.0019703495
.40	9.CC	.0010199353	.0008498595
.40	10.CC	.0004594751	.0003649941
.50	1.CC	.6079351130	.6012657754
.50	2.CC	.2738712998	.2757958051
.50	3.CC	.1233774580	.1238001957
.50	4.CC	.0555806409	.0549133769
.50	5.CC	.0250388517	.0241642500
.50	6.CC	.0112796598	.0105690431
.50	7.CC	.0050815125	.0045993576

.50	8.00	.0022891924	.0019924223
.50	9.00	.0010312681	.0008593635
.50	10.00	.0004645804	.0003690861
.60	1.00	.4161864722	.6094586514
.60	2.00	.2775884900	.2795535492
.60	3.00	.1250520310	.1254870084
.60	4.00	.0563352265	.0556616323
.60	5.00	.0253786981	.0244935452
.60	6.00	.0114329587	.0107130887
.60	7.00	.0051504827	.0046620514
.60	8.00	.0023202631	.0020195857
.60	9.00	.0010452653	.0008711020
.60	10.00	.0004708860	.0003741201
.70	1.00	.6260609959	.6192259443
.70	2.00	.2820369066	.2840337194
.70	3.00	.1270560173	.1274980838
.70	4.00	.0572380108	.0565536756
.70	5.00	.0257853973	.0248860837
.70	6.00	.0116161744	.0108847791
.70	7.00	.0052330203	.0047367667
.70	8.00	.0023574458	.0020519522
.70	9.00	.0010620159	.0008850626
.70	10.00	.0004784321	.0003861159
.80	1.00	.6376215058	.6306108258
.80	2.00	.2872446504	.2892562736
.80	3.00	.1294021662	.1298423670
.80	4.00	.0582549376	.0575934476
.80	5.00	.0262615368	.0253435816
.80	6.00	.0118306725	.0110848548
.80	7.00	.0053296505	.0048238202
.80	8.00	.0024009771	.0020896563
.80	9.00	.0010816265	.0009013219
.80	10.00	.0004872666	.0003870972
.90	1.00	.6509415498	.6436636998
.90	2.00	.2932454542	.2952445149
.90	3.00	.1321054040	.1325303021
.90	4.00	.0595127308	.0587855503
.90	5.00	.0268101457	.0258680424
.90	6.00	.0120776177	.0113141799
.90	7.00	.0054409879	.0049235818
.90	8.00	.0024511340	.0021328551
.90	9.00	.0011042219	.0009199459
.90	10.00	.0004974457	.0003950915
1.00	1.00	.6661058699	.6584424543
1.00	2.00	.3000766939	.3020252084
1.00	3.00	.1351829286	.1355738847
1.00	4.00	.0608591380	.0601352673
1.00	5.00	.0274347142	.0264617665

APPENDIX B, PART 2: SERIES SOLUTION RESULTS 96

1.00	6.00	.0123591822	.0115737466
1.00	7.00	.0055677411	.0050364758
1.00	8.00	.0025082356	.0021817287
1.00	9.00	.0011299456	.0009410105
1.00	10.00	.0005090341	.0004041305

R = 1.1000
E(0)=-Z*Z/2= -1.221582597974 E1=EGM= -1.405211911403
E(1) = EGM + Z*Z/2 = -.183629313427
E2 = -.000296598515 E(2) = -1.405508509918
E3 = .000006915443 E(3) = -1.405501594476

R = 1.2000
E(0)=-Z*Z/2= -1.175709681000 E1=EGM= -1.362029063770
E(1) = EGM + Z*Z/2 = -.186319382770
E2 = -.000284667134 E(2) = -1.362313730903
E3 = .000006682934 E(3) = -1.362306847970

R = 1.3000
E(0)=-Z*Z/2= -1.133634298935 E1=EGM= -1.321705255542
E(1) = EGM + Z*Z/2 = -.188070956607
E2 = -.000272860504 E(2) = -1.321977516046
E3 = .000006210804 E(3) = -1.321971705242

R = 1.4000
E(0)=-Z*Z/2= -1.094942246392 E1=EGM= -1.284015842513
E(1) = EGM + Z*Z/2 = -.189073596121
E2 = -.000259696876 E(2) = -1.284275539389
E3 = .000006703810 E(3) = -1.284268835578

R = 1.5000
E(0)=-Z*Z/2= -1.059271826031 E1=EGM= -1.248748949974
E(1) = EGM + Z*Z/2 = -.189477123943
E2 = -.000247240654 E(2) = -1.248996190628
E3 = .000006561935 E(3) = -1.248989628644

R = 1.6000

E(0)=-Z*Z/2= -1.02630771567C E1=EGM= -1.2157083C6979
E(1) = EGM + Z*Z/2 = -.1894CC5913C9
E2 = -.000235114907 E(2) = -1.215943421886
E3 = .000C6387551 E(3) = -1.215937034335

R = 1.7000
E(0)=-Z*Z/2= -.995774927965 E1=EGM= -1.184714045029
E(1) = EGM + Z*Z/2 = -.1889391170C4
E2 = -.000283494203 E(2) = -1.184937539232
E3 = .000C6186C39 E(3) = -1.184931353193

R = 1.5000
E(0)=-Z*Z/2= -.967433286568 E1=EGM= -1.155C2387750
E(1) = EGM + Z*Z/2 = -.1883169107181
E2 = -.000212501463 E(2) = -1.155814889213
E3 = .000CC5965C19 E(3) = -1.1558C8924CC0

R = 1.9000
E(0)=-Z*Z/2= -.941072515479 E1=EGM= -1.128224766755
E(1) = EGM + Z*Z/2 = -.187152251285
E2 = -.000202212165 E(2) = -1.128426978940
E3 = .000C05732422 E(3) = -1.128421246519

R = 1.9972
E(0)=-Z*Z/2= -.917172934938 E1=EGM= -1.10314780894S
E(1) = EGM + Z*Z/2 = -.1853748740C7
E2 = -.000192920340 E(2) = -1.103340729286
E3 = .000CC5502043 E(3) = -1.103335227243

MU	LAMBDA	PHI GM	PHI(1)
.00	1.CC	.3185804075	.3151918942
.00	2.CC	.0823839618	.0827476515
.00	3.0C	.0213042516	.0207772620
.00	4.CC	.0055092172	.CC5C789066
.00	5.C0	.0014246674	.0012155289
.00	6.CC	.0003684148	.0002851306
.00	7.C0	.0000952710	.0000654453
.00	8.CC	.0000246368	.0000146349
.00	9.C0	.0000063710	.0000031629
.00	10.CC	.0000016475	.0000006510

.10	1.00	.3198306215	.3164697295
.10	2.00	.0827072635	.0830829619
.10	3.00	.0213878565	.0208615352
.10	4.00	.0055308371	.0050995455
.10	5.00	.0014302583	.0012204820
.10	6.00	.0003696606	.0002862969
.10	7.00	.0000956449	.0000657144
.10	8.00	.0000247335	.0000146955
.10	9.00	.0000063960	.0000031761
.10	10.00	.0000016540	.0000006537
.20	1.00	.3236041674	.3203207600
.20	2.00	.0836830914	.0824035142
.20	3.00	.0216402027	.0211155049
.20	4.00	.0055960931	.0051617386
.20	5.00	.0014471333	.0012354058
.20	6.00	.0003742244	.0002898103
.20	7.00	.0000967733	.0000665247
.20	8.00	.0000250253	.0000148780
.20	9.00	.0000064715	.0000032160
.20	10.00	.0000016735	.0000006620
.30	1.00	.3299701766	.3267978521
.30	2.00	.0853293228	.0857932485
.30	3.00	.0220659133	.0215426414
.30	4.00	.0057061807	.0052663193
.30	5.00	.0014756016	.0012604946
.30	6.00	.0003815862	.0002957146
.30	7.00	.0000986771	.0000678859
.30	8.00	.0000255176	.0000151842
.30	9.00	.0000065988	.0000032827
.30	10.00	.0000017064	.0000006760
.40	1.00	.3390452748	.3359900908
.40	2.00	.0876761166	.0882056558
.40	3.00	.0226727874	.0221487919
.40	4.00	.0058631165	.0054146922
.40	5.00	.0015161848	.0012960757
.40	6.00	.0003920810	.0003040840
.40	7.00	.0001013910	.0000698141
.40	8.00	.0000262194	.0000156177
.40	9.00	.0000067803	.0000033771
.40	10.00	.0000017534	.0000006956
.50	1.00	.3509957179	.3480242443
.50	2.00	.0907664662	.0913641651
.50	3.00	.0234719427	.0229422771
.50	4.00	.0060697758	.0056088556
.50	5.00	.0015696263	.0013426148
.50	6.00	.0004059068	.0003150234
.50	7.00	.0001049647	.0000723321
.50	8.00	.0000271436	.0000161829

.50	9.00	.0000070192	.0000035000
.50	10.00	.0000018152	.0000007211
.60	1.00	.3660404385	.3630668219
.60	2.00	.0946569870	.0953126674
.60	3.00	.0244780200	.0239340253
.60	4.00	.0063699444	.0058514333
.60	5.00	.0016369051	.0014007233
.60	6.00	.0004232989	.0003286711
.60	7.00	.0001094639	.0000754699
.60	8.00	.0000283070	.0000168863
.60	9.00	.0000073201	.0000036525
.60	10.00	.0000018930	.0000007527
.70	1.00	.3844550564	.3813267378
.70	2.00	.0994189534	.1001063199
.70	3.00	.0257094506	.0251377473
.70	4.00	.0066483887	.0061457166
.70	5.00	.0017192539	.0014711675
.70	6.00	.0004445940	.0003451998
.70	7.00	.0001149707	.0000792651
.70	8.00	.0000297311	.0000177354
.70	9.00	.0000076884	.0000038361
.70	10.00	.0000019882	.0000007905
.80	1.00	.4065769281	.4030586032
.80	2.00	.1051396047	.1058122175
.80	3.00	.0271887943	.0265701515
.80	4.00	.0070309427	.0064957148
.80	5.00	.0018181812	.0015548806
.80	6.00	.0004701763	.0003648201
.80	7.00	.0001215862	.0000837633
.80	8.00	.0000314418	.0000187397
.80	9.00	.0000081308	.0000040527
.80	10.00	.0000021026	.0000008349
.90	1.00	.4328113268	.4285666789
.90	2.00	.1119237435	.1125106384
.90	3.00	.0289431528	.0282512024
.90	4.00	.0074846147	.0069062167
.90	5.00	.0019354995	.0016529765
.90	6.00	.0005005145	.0003877827
.90	7.00	.0001294316	.0000890188
.90	8.00	.0000334706	.0000199102
.90	9.00	.0000086554	.0000043042
.90	10.00	.0000022383	.0000008862
1.00	1.00	.4636388678	.4582095407
1.00	2.00	.1198956554	.1202761799
1.00	3.00	.0310046659	.0302044256
1.00	4.00	.0080177159	.0073828624
1.00	5.00	.0020733579	.0017667661
1.00	6.00	.0005361643	.0004143827

APPENDIX B, PART 2: SERIES SOLUTION RESULTS 1CO

1.00	7.CC	.0001386505	.C00C950954
1.00	8.CC	.0000356546	.C00C212601
1.00	9.CC	.0000092719	.C00CC45931
1.00	10.CC	.0000023977	.C000CC9448

R = 2.0000

E(0)=-Z*Z/2= -.9165C8375355 E1=EGM= -1.1C2446672381

E(1) = EGM + Z*Z/2 = -.185938597026

E2 = -.C0019g(63154 E(2) = -1.102639335547

E3 = .000CC5495401 E(3) = -1.10263384C146

MU	LAMBDA	PHI GM	PHI(1)
.00	1.CC	.3181149226	.3147314268
.00	2.CC	.082148079C	.0825091081
.00	3.CC	.0212134241	.02C6866372
.00	4.CC	.0054780266	.0C5C490342
.00	5.CC	.0014146125	.0012064811
.00	6.CC	.0003653010	.CC02825481
.00	7.CC	.0000943331	.0000647422
.00	8.CC	.00002436CC	.CCCC144515
.00	9.CC	.0000C629C6	.CCCCC31170
.00	10.CC	.0000016244	.CCCCCC6400
.10	1.CC	.3193662326	.316C1C4410
.10	2.CC	.0824712082	.0828442500
.10	3.CC	.0212966670	.C2077C7434
.10	4.CC	.0054995744	.0056696009
.10	5.CC	.0014201769	.0012114C92
.10	6.CC	.0003667379	.0002837066
.10	7.CC	.0000947042	.0000650091
.10	8.CC	.0000244558	.0000145115
.10	9.CC	.0000C63153	.0000C31301
.10	10.00	.0000016308	.0000006427
.20	1.CC	.3231431162	.3198650639
.20	2.CC	.0834465278	.0838543045
.20	3.CC	.0215487276	.021C242125
.20	4.CC	.0055646134	.0051315771
.20	5.CC	.0014369722	.0012262577
.20	6.CC	.0003710750	.0002871966
.20	7.CC	.0000958242	.0000658127
.20	8.CC	.0000247451	.0000146921
.20	9.CC	.0000C639CC	.0000C31694
.20	10.CC	.0000016501	.0000006509
.30	1.CC	.3295149160	.3263483309
.30	2.CC	.0850919429	.0855532364
.30	3.CC	.0219736297	.0214505156
.30	4.CC	.0056743375	.0052357951
.30	5.CC	.0014653067	.0012512204

.30	6.00	.0003783919	.0002930619
.30	7.00	.0000977137	.0000671626
.30	8.00	.0000252330	.0000149953
.30	9.00	.0000065160	.0000032354
.30	10.00	.0000016827	.0000006647
.40	1.00	.3385986069	.3355496137
.40	2.00	.0874376604	.0879645785
.40	3.00	.0225793736	.0220555016
.40	4.00	.0058307612	.0053836577
.40	5.00	.0015057006	.0012866237
.40	6.00	.0003886230	.0003013759
.40	7.00	.0001004073	.0000690748
.40	8.00	.0000259266	.0000154243
.40	9.00	.0000066956	.0000033287
.40	10.00	.0000017290	.0000006841
.50	1.00	.3505609486	.3475960932
.50	2.00	.0905267433	.0911218206
.50	3.00	.0233770798	.0228474937
.50	4.00	.0060367560	.0055771606
.50	5.00	.0015588954	.0013329320
.50	6.00	.0004025597	.0003122436
.50	7.00	.0001039546	.0000715721
.50	8.00	.0000268446	.0000159239
.50	9.00	.0000069382	.0000034501
.50	10.00	.0000017901	.0000007092
.60	1.00	.3656215481	.3626548287
.60	2.00	.0944159017	.0950689553
.60	3.00	.0243813985	.0238374243
.60	4.00	.0062961036	.0058189247
.60	5.00	.0016258678	.0013907548
.60	6.00	.0004196543	.0003258021
.60	7.00	.0001084206	.0000746841
.60	8.00	.0000279979	.0000166802
.60	9.00	.0000072300	.0000036008
.60	10.00	.0000018670	.0000007403
.70	1.00	.3840568906	.3809354365
.70	2.00	.0991765332	.0998611859
.70	3.00	.0256107493	.0250390100
.70	4.00	.0066135653	.0061122375
.70	5.00	.0017078472	.0014608564
.70	6.00	.0004410241	.0003422238
.70	7.00	.0001138874	.0000784483
.70	8.00	.0000294096	.0000175209
.70	9.00	.0000075946	.0000037822
.70	10.00	.0000019612	.0000007776
.80	1.00	.4062054153	.4026933997
.80	2.00	.1048960345	.1055658006
.80	3.00	.0270877188	.0264689670

.80	4.00	.0069949690	.0064611039
.80	5.00	.0018063386	.0015441676
.80	6.00	.0004664580	.0003617180
.80	7.00	.0001204553	.0000829100
.80	8.00	.0000311056	.0000185152
.80	9.00	.0000080325	.0000039962
.80	10.00	.0000020743	.0000008214
.90	1.00	.4324737287	.4282340443
.90	2.00	.1116794052	.1122632231
.90	3.00	.0288394155	.0281472711
.90	4.00	.0074473166	.0068703081
.90	5.00	.0019231501	.0016417992
.90	6.00	.0004966227	.0003845346
.90	7.00	.0001282448	.0000881231
.90	8.00	.0000331172	.0000196742
.90	9.00	.0000085520	.0000042447
.90	10.00	.0000022084	.0000008719
1.00	1.00	.4633440694	.4579172310
1.00	2.00	.1196511757	.1200482534
1.00	3.00	.0308979974	.0300974630
1.00	4.00	.0079789123	.0073454348
1.00	5.00	.0020604262	.0017550589
1.00	6.00	.0005320720	.0004109673
1.00	7.00	.0001373991	.0000941512
1.00	8.00	.0000354811	.0000210108
1.00	9.00	.0000091624	.0000045302
1.00	10.00	.0000002366	.0000009297

R = 2.1000
E(0)=-Z*Z/2= -.893577487656 E1=EGM= -1.078146415478
E(1) = EGM + Z*Z/2 = -.184568927822
E2 = -.000183862439 E(2) = -1.078330277916
E3 = .000005259671 E(3) = -1.078325018245

R = 2.2000
E(0)=-Z*Z/2= -.872137206189 E1=EGM= -1.055213898514
E(1) = EGM + Z*Z/2 = -.183076612325
E2 = -.000175796317 E(2) = -1.055389696831
E3 = .000005029404 E(3) = -1.055384667427

R = 2.3000
E(0)=-Z*Z/2= -.852636396902 E1=EGM= -1.033549448204
E(1) = EGM + Z*Z/2 = -.181489051302

```
 E2 =   -.000168446655      E(2) =  -1.033717894859
 E3 =    .000004807376      E(3) =  -1.033713087483


R =   2.4000
 E(0)=-Z*Z/2=  -.833233918848      E1=EGM=  -1.013062734895
 E(1) = EGM + Z*Z/2 =   -.179828816047
 E2 =   -.000161776150      E(2) =  -1.013224511045
 E3 =    .000004595243      E(3) =  -1.013219915802


R =   2.5000
 E(0)=-Z*Z/2=  -.845557239131      E1=EGM=  -.993671788189
 E(1) = EGM + Z*Z/2 =   -.178114549058
 E2 =   -.000155751958      E(2) =   -.993827540146
 E3 =    .000004392876      E(3) =   -.993823146631


R =   2.6000
 E(0)=-Z*Z/2=  -.798940428928      E1=EGM=  -.975302109362
 E(1) = EGM + Z*Z/2 =   -.176361680434
 E2 =   -.000150337968      E(2) =   -.975452447330
 E3 =    .000004203430      E(3) =   -.975448243920


R =   2.7000
 E(0)=-Z*Z/2=  -.783302875675      E1=EGM=  -.957885876306
 E(1) = EGM + Z*Z/2 =   -.174583000631
 E2 =   -.000145498251      E(2) =   -.958031374556
 E3 =    .000004023946      E(3) =   -.958027350611


R =   2.8000
 E(0)=-Z*Z/2=  -.768577111667      E1=EGM=  -.941361234270
 E(1) = EGM + Z*Z/2 =   -.172789120604
 E2 =   -.000141197566      E(2) =   -.941502431836
 E3 =    .000003855106      E(3) =   -.941498576730


R =   2.9000
 E(0)=-Z*Z/2=  -.754682821481      E1=EGM=  -.925671664709
 E(1) = EGM + Z*Z/2 =   -.171028843228
```

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E2 = -.00013740188I E(2) = -.92580906659C
E3 = .000C03696534 E(3) = -.9258C537C056

R = 3.0000

E(0)=-Z*Z/2= -.741575959922 E1=EGM= -.910765424385

E(1) = E_{GM} + Z*Z/2 = -.16°189464463

E2 = -.0001340C78349 E(2) = -.91C899502733
E3 = .000CC3E47E14 E(3) = -.91C895954920

MU	LAMBDA	PHI GM	PHI(1)
.00	1.CC	.1978572778	.1956180287
.00	2.CC	.0318416468	.0316648705
.00	3.00	.0051243527	.0048066847
.00	4.CC	.0008246744	.0006988973
.00	5.CC	.0001327168	.C000975996
.00	6.CC	.0000213564	.0000129952
.00	7.CC	.0000034373	.0000016175
.00	8.CC	.0000005532	.0000001795
.00	9.CC	.0000000890	.0000000153
.00	10.CC	.0000000143	.0000000002
.10	1.CC	.1994059885	.1972270888
.10	2.CC	.0320908845	.0319252575
.10	3.CC	.0051644631	.0048463256
.10	4.CC	.0008311294	.C007046925
.10	5.CC	.0001337557	.0000984158
.10	6.CC	.0000215256	.0000131053
.10	7.CC	.0000034642	.0000016315
.10	8.CC	.0000005575	.0000001811
.10	9.CC	.0000000897	.0000000154
.10	10.CC	.0000000144	.0000000002
.20	1.CC	.2041C39466	.2020967672
.20	2.CC	.0328469382	.0327133057
.20	3.00	.0052861366	.0049662808
.20	4.CC	.0008507107	.0007222246
.20	5.00	.0001369069	.0001008841
.20	6.CC	.0000220328	.0000134382
.20	7.00	.0000035458	.0000016738
.20	8.CC	.0000005706	.0000001860
.20	9.00	.0000000918	.0000000159
.20	10.CC	.0000000148	.0000000002
.30	1.00	.0121083636	.2103559164
.30	2.CC	.0341351082	.0340498977
.30	3.00	.0054934449	.0051696813
.30	4.CC	.0008840733	.0007519381
.30	5.00	.0001422760	.0001050642
.30	6.CC	.0000228968	.0000140012

.30	7.CC	.0000036848	.0000017453
.30	8.CC	.0000005930	.0000001942
.30	9.CC	.0000000954	.0000000167
.30	10.CC	.0000000154	.0000000003
.40	1.CC	.2236870979	.2222238340
.40	2.CC	.0359985017	.0359705741
.40	3.CC	.0057933253	.0054618551
.40	4.CC	.0009323332	.0007945895
.40	5.CC	.0001500427	.0001110576
.40	6.CC	.0000241467	.0000148070
.40	7.CC	.0000038860	.0000018472
.40	8.CC	.0000006254	.0000002059
.40	9.CC	.0000001006	.0000000178
.40	10.CC	.0000000162	.0000000003
.50	1.CC	.2392276183	.2380171406
.50	2.CC	.0384994750	.0385266489
.50	3.CC	.0061958129	.0058504939
.50	4.CC	.0009971071	.0008512705
.50	5.CC	.0001604668	.0001190111
.50	6.CC	.0000258243	.0000158740
.50	7.CC	.0000041560	.0000019817
.50	8.CC	.0000006688	.0000002212
.50	9.CC	.0000001076	.0000000192
.50	10.CC	.0000000173	.0000000003
.60	1.CC	.2592499700	.2581595627
.60	2.CC	.0417217201	.0417867959
.60	3.CC	.0067143766	.0063458894
.60	4.CC	.0010805607	.0009234410
.60	5.CC	.0001738972	.0001291203
.60	6.CC	.0000279857	.0000172265
.60	7.CC	.0000045038	.0000021514
.60	8.CC	.0000007248	.0000002607
.60	9.CC	.0000001166	.0000000209
.60	10.CC	.0000000188	.0000000004
.70	1.CC	.2844241773	.2831948331
.70	2.CC	.0457730657	.0458391412
.70	3.CC	.0073663694	.0069612445
.70	4.CC	.0011854875	.0010129730
.70	5.CC	.0001907834	.0001416362
.70	6.CC	.0000307032	.0000172957
.70	7.CC	.0000049411	.0000023598
.70	8.CC	.0000007952	.0000002635
.70	9.CC	.0000001280	.0000000289
.70	10.CC	.0000000206	.0000000004
.80	1.CC	.3155926656	.3134030432
.80	2.CC	.0507890854	.0507939152
.80	3.CC	.0081736095	.0077130475
.80	4.CC	.0013153986	.0011222055

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.80	5.00	.0002116903	.0001568719
.80	6.00	.0000340678	.0000292?4
.80	7.00	.0000054826	.0000026110
.80	8.00	.0000008823	.0000002912
.80	9.00	.0000001420	.0000000252
.80	10.00	.0000000229	.0000000004
.90	1.00	.3537984516	.3508209339
.90	2.00	.0569376343	.0567867443
.90	3.00	.0091631102	.0090[illegible]04
.90	4.00	.0014746413	.0014540?0
.90	5.00	.0002373176	.0001752114
.90	6.00	.0000381921	.0000233482
.90	7.00	.0000061463	.0000029102
.90	8.00	.0000009891	.0000003238
.90	9.00	.0000001592	.0000000278
.90	10.00	.0000000256	.0000000004
1.00	1.00	.4003200470	.3952667837
1.00	2.00	.0644244663	.0639826889
1.00	3.00	.0103679840	.0097117178
1.00	4.00	.0016685446	.0014118844
1.00	5.00	.0002685209	.0001971205
1.00	6.00	.0000432140	.0000262365
1.00	7.00	.0000069548	.0000032636
1.00	8.00	.0000011192	.0000003616
1.00	9.00	.0000001801	.0000000307
1.00	10.00	.0000000290	.0000000004

R = 3.1000
E(0)=-Z*Z/2= -.729[illegible]8C26533 E1=EGM= -.896595047172
E(1) = EGM + Z*Z/2 = -.167397018639
E2 = -.[illegible]13[illegible]195289 E(2) = -.896726242461
E3 = .000003408526 E(3) = -.896722833935

R = 3.2000
E(0)=-Z*Z/2= -.71750C-22440 E1=EGM= -.883116^01560
E(1) = EGM + Z*Z/2 = -.165616479121
E2 = -.000128722093 E(2) = -.8830456^3654
E3 = .000003278264 E(3) = -.88324234538?

R = 3.3000
E(0)=-Z*Z/2= -.7C6438874269 E1=EGM= -.870290797481
E(1) = EGM + Z*Z/2 = -.163851923212
E2 = -.000126629121 E(2) = -.87C417426602

E3 = .000003156640 E(3) = -.87C41426?962

R = 3.4000
E(0)=-Z*Z/2= -.695972968457 E1=EGM= -.858C79636772
E(1) = EGM + Z*Z/2 = -.1-2106668315
E2 = -.000124887602 E(2) = -.8582C4524375
E3 = .000003C4?286 E(3) = -.858201481089

R = 3.5000
E(0)=-Z*Z/2= -.6860657172?7 E1=EGM= -.846449102219
E(1) = EGM + Z*Z/2 = -.160383384922
E2 = -.000123469572 E(2) = -.846572571790
E3 = .000002937888 E(3) = -.846569633903

R = 3.6000
E(0)=-Z*Z/2= -.676683189832 ?1=EGM= - ?35367380733
E(1) = EGM + Z*Z/2 = -.158684190901
E2 = -.000122347818 E(2) = -.835489728550
E3 = .?000?840318 E(3) = -.835486888232

R = 3.7000
E(0)=-Z*Z/2= -.667794186120 E1=EGM= -.824804916770
E(1) = EGM + Z*Z/? = -.157010730650
E2 = -.000121495912 E(2) = -.824926412682
E3 = .000002752800 E(3) = -.824923?59881

R = 3.8000
E(0)=-Z*Z/2= -.659369950588 E1=EGM= -.?14734192574
E(1) = EGM + Z*Z/2 = -.155364241986
E2 = -.000119741094 E(2) = -.814?53933668
E3 = .000292277206 E(3) = -.814561?64042

R = 3.9000
E(0)=-Z*Z/2= -.651383919185 E1=EGM= -.8C5129532272
E(1) = EGM + Z*Z/2 = -.153745613087
E2 = -.000120388452 E(2) = -.80524992?723

E3 = .000008247443 E(3) = -.8052416732?0

R = 4.0000

E(0)=-Z*Z/2= -.643811495879 E1=EGM= -.795966927219

E(1) = EGM + Z*Z/2 = -.152155431?4?

E2 = -.000120?650.? E(2) = -.79608719230?

E3 = .000003638054 E(3) = -.796083554254

MU	LAMBDA	P-I GM	PHI(1)
.00	1.00	.1285769590	.1266287867
.00	2.00	.0132?06143	.0130180383
.00	3.00	.0013738109	.0012318801
.00	4.00	.0001420067	.0001096720
.00	5.00	.0000146788	.0000091245
.00	6.00	.0000015173	.0000006870
.00	7.00	.0000001568	.0000000424
.00	8.00	.0000000162	.0000000013
.00	9.00	.0000000017	-.0000000002
.00	10.00	.0000000002	-.0000000001
.10	1.00	.1302707320	.1286113386
.10	2.00	.0134656945	.0132010851
.10	3.00	.0013919084	.0012493026
.10	4.00	.0001438774	.0001112403
.10	5.00	.0000148722	.0000092575
.10	6.00	.0000015373	.0000006574
.10	7.00	.0000001589	.0000000431
.10	8.00	.0000000164	.0000000013
.10	9.00	.0000000017	-.0000000002
.10	10.00	.0000000002	-.0000000001
.20	1.00	.1354396637	.1340346899
.20	2.00	.0139999915	.0137579873
.20	3.00	.0014471371	.0013022948
.20	4.00	.0001495?62	.0001160078
.20	5.00	.0000154623	.0000096614
.20	6.00	.0000015983	.0000007288
.20	7.00	.0000001652	.0000000452
.20	8.00	.0000000171	.0000000014
.20	9.00	.0000000018	-.0000000002
.20	10.00	.0000000002	-.0000000001
.30	1.00	.1443511242	.1433297824
.30	2.00	.0149211424	.0147124255
.30	3.00	.0015?63537	.0013930680
.30	4.00	.0001594285	.0001241665
.30	5.00	.0000164796	.0000103514
.30	6.00	.0000017035	.0000007824
.30	7.00	.0000001761	.0000000488

.30	8.00	.0000000182	.0000000015
.30	9.00	.0000000019	-.0000000002
.30	10.00	.0000000002	-.0000000001
.40	1.00	.1574660709	.1568944633
.40	2.00	.0162767951	.0161051948
.40	3.00	.0016824834	.0015254312
.40	4.00	.0001739133	.0001360468
.40	5.00	.0000179769	.0000113538
.40	6.00	.0000018582	.0000008599
.40	7.00	.0000001921	.0000000539
.40	8.00	.0000000199	.0000000017
.40	9.00	.0000000021	-.0000000002
.40	10.00	.0000000002	-.0000000001
.50	1.00	.1754628922	.1753132795
.50	2.00	.0181370775	.0179962332
.50	3.00	.0018747747	.0017049799
.50	4.00	.0001937898	.0001521338
.50	5.00	.0000200315	.0000137071
.50	6.00	.0000020706	.0000009639
.50	7.00	.0000002140	.0000000607
.50	8.00	.0000000221	.0000000040
.50	9.00	.0000000023	-.0000000002
.50	10.00	.0000000002	-.0000000001
.60	1.00	.1992724983	.1993861110
.60	2.00	.0205982000	.0204675567
.60	3.00	.0021291741	.0019393682
.60	4.00	.0002200863	.0001730909
.60	5.00	.0000227497	.0000144638
.60	6.00	.0000023516	.0000010980
.60	7.00	.0000002431	.0000000692
.60	8.00	.0000000251	.0000000025
.60	9.00	.0000000026	-.0000000003
.60	10.00	.0000000000	-.0000000001
.70	1.00	.2301264735	.2301668742
.70	2.00	.0237874828	.0236272256
.70	3.00	.0024588407	.0022386764
.70	4.00	.0002541630	.0001997900
.70	5.00	.0000262721	.0000166929
.70	6.00	.0000027157	.0000012670
.70	7.00	.0000002807	.0000000798
.70	8.00	.0000000290	.0000000027
.70	9.00	.0000000030	-.0000000003
.70	10.00	.0000000003	-.0000000001
.80	1.00	.2696207817	.2690141881
.80	2.00	.0278698909	.0276145358
.80	3.00	.0028808270	.0026158910
.80	4.00	.0002977824	.0002333567
.80	5.00	.0000307809	.0000194830

.80	6.CC	.0000031817	.0000014767
.80	7.CC	.0000003289	.0000000927
.80	8.CC	.0000000340	.0000000031
.80	9.CC	.0000000035	-.0000000004
.80	10.CC	.0000000004	-.0000000001
.90	1.CC	.3197983199	.3176566593
.90	2.CC	.0334565924	.0326067085
.90	3.CC	.0034169607	.0030875235
.90	4.CC	.0003532010	.0002752146
.90	5.CC	.0000365093	.0000229466
.90	6.CC	.0000037739	.0000017347
.90	7.CC	.0000003901	.0000001083
.90	8.CC	.0000000403	.0000000034
.90	9.CC	.0000000042	-.0000000005
.90	10.CC	.0000000004	-.0000000001
1.00	1.CC	.3832545894	.3782763156
1.00	2.CC	.0396158765	.0388274387
1.00	3.CC	.0040949742	.0036744003
1.00	4.CC	.0004232852	.0003271608
1.00	5.CC	.0000437537	.0000272244
1.00	6.CC	.0000045227	.0000020505
1.00	7.CC	.0000004675	.0000001268
1.00	8.CC	.0000000483	.0000000038
1.00	9.CC	.0000000050	-.0000000006
1.00	10.CC	.0000000005	-.0000000002

R = .1000
E(0)=-Z*Z/2= -.636629854719 E1=EGM= -.787223880310
E(1) = EGM + Z*Z/2 = -.150594025591
E2 = -.000120263669 E(2) = -.787344143979
E3 = .000027989905 E(3) = -.787341345074

R = 4.2000
E(0)=-Z*Z/2= -.629817764281 E1=EGM= -.778879267265
E(1) = EGM + Z*Z/2 = -.149061502985
E2 = -.000174699238 E(2) = -.779039666503
E3 = .000022723540 E(3) = -.779031242963

R = 4.3000
E(0)=-Z*Z/2= -.623355431778 E1=EGM= -.770913213122
E(1) = EGM + Z*Z/2 = -.147557781345
E2 = -.000120654802 E(2) = -.771033867925
E3 = .000002388677 E(3) = -.771031479248

R = 4.4000
E(0)=-Z*Z/2= -.6172243e4539 E1=EGM= -.763306982379
E(1) = EGM + Z*Z/2 = -.144082617840
E2 = -.000121C15523 E(2) = -.76[illegible]27997902
E3 = .000CC2306933 E(3) = -.763425690969

R = 4.5000
E(0)=-Z*Z/2= -.611407246875 E1=EGM= -.756042881394
E(1) = EGM + Z*Z/2 = -.144635634519
E2 = -.000121461640 E(2) = -.756164343034
E3 = .000CC2246879 E(3) = -.756162096156

R = 4.6000
E(0)=-Z*Z/2= -.6C5587830636 E1=EGM= -.749104171798
E(1) = EGM + Z*Z/2 = -.143216341162
E2 = -.000121974950 E(2) = -.749226146748
E3 = .000CC2198049 E(3) = -.749223948699

R = 4.7000
E(0)=-Z*Z/2= -.6C0650838007 E1=EGM= -.742474993795
E(1) = EGM + Z*Z/2 = -.141824155788
E2 = -.000122536C95 E(2) = -.742597531890
E3 = .000CC2166123 E(3) = -.742595375767

R = 4.8000
E(0)=-Z*Z/2= -.595681875235 E1=EGM= -.736140298324
E(1) = EGM + Z*Z/2 = -.140458423090
E2 = -.000123134735 E(2) = -.736263433059
E3 = .000CC2119097 E(3) = -.736261313962

R = 4.9000
E(0)=-Z*Z/2= -.59C367356174 E1=EGM= -.730C85787152
E(1) = EGM + Z*Z/2 = -.139118430974
E2 = -.000123749639 E(2) = -.730209536785
E3 = .000CC2085927 E(3) = -.730207450858

R = 5.0000
E(0)=-Z*Z/2= -.58649443466? E1=EGM= -.72429786C035
$E(_1)$ = EGM + Z*Z/2 = -.1378C3425368
E2 = -.0001243687?6 E(2) = -.724422228741
E3 = .0000C2C55539 E(3) = -.724420172743

MU	LAMBDA	PHI GM	PHI(1)
.00	1.00	.2834092542	.0819185044
.00	2.00	.2055630360	.0053387706
.00	3.00	.2003710304	.0003158834
.00	4.00	.2000247461	.0000172231
.00	5.00	.2000016505	.0000008431
.00	6.00	.2000001101	.0000000336
.00	7.00	.2000000073	.0000000006
.00	8.00	.2000000005	-.0000000001
.00	9.00	.2000000000	-.0000000000
.00	10.00	.2000000000	-.0000000000
.10	1.00	.2851064519	.0837226189
.10	2.00	.2056762317	.0054565571
.10	3.00	.2003785801	.0003229215
.10	4.00	.2000252497	.0000176143
.10	5.00	.2000001684	.0000008630
.10	6.00	.2000001123	.0000000345
.10	7.00	.2000000075	.0000000011
.10	8.00	.2000000005	-.0000000001
.10	9.00	.2000000000	-.0000000000
.10	10.00	.2000000000	-.0000000000
.20	1.00	.2903232315	.0892475213
.20	2.00	.2060241683	.0058172361
.20	3.00	.2004017859	.0003444636
.20	4.00	.2000267974	.0000188106
.20	5.00	.2000017873	.0000009237
.20	6.00	.2000001192	.0000000371
.20	7.00	.2000000080	.0000000007
.20	8.00	.2000000005	-.0000000001
.20	9.00	.2000000000	-.0000000000
.20	10.00	.2000000000	-.0000000000
.30	1.00	.2994443869	.0988392301
.30	2.00	.2066325099	.0064433099
.30	3.00	.2004423597	.0003818251
.30	4.00	.2000295035	.0000208820
.30	5.00	.2000019678	.0000010284
.30	6.00	.2000001312	.0000000416
.30	7.00	.2000000088	.0000000008
.30	8.00	.2000000006	-.0000000001

.30	9.CC	.0000000CC	-.0000000000
.30	10.CC	.CCCCCCCCC	-.CCCCCCCCC
.40	1.CC	.1131427015	.1131025812
.40	2.CC	.0075461781	.0073741114
.40	3.CC	.0005032941	.0004373052
.40	4.CC	.0000335675	.0000238508
.40	5.CC	.0000022388	.0000011829
.40	6.CC	.0000001493	.0000000482
.40	7.CC	.0000000100	.0000000010
.40	8.CC	.0000000007	-.0000000001
.40	9.CC	.0000000000	-.0000000000
.40	10.CC	.0000000000	-.0000000000
.50	1.CC	.1324285740	.1329448047
.50	2.CC	.0088224123	.0086686365
.50	3.CC	.0005890836	.0005143518
.50	4.CC	.0000392893	.0000282005
.50	5.CC	.0000026204	.0000013957
.50	6.CC	.0000001748	.0000000571
.50	7.CC	.0000000117	.0000000012
.50	8.CC	.0000000008	-.0000000001
.50	9.CC	.0000000001	-.0000000000
.50	10.CC	.0000000000	-.0000000000
.60	1.CC	.1587245472	.1596399019
.60	2.CC	.0105862397	.0104097267
.60	3.CC	.0007060563	.0006178047
.60	4.CC	.0000470909	.0000338882
.60	5.CC	.0000031408	.0000016787
.60	6.CC	.0000002095	.0000000689
.60	7.CC	.0000000140	.0000000015
.60	8.CC	.0000000009	-.0000000001
.60	9.CC	.0000000001	-.0000000000
.60	10.CC	.0000000000	-.0000000000
.70	1.CC	.1939702269	.1949182870
.70	2.CC	.0129369740	.0127098899
.70	3.CC	.0008626401	.0007542330
.70	4.CC	.0000575477	.0000413626
.70	5.CC	.0000038362	.0000020461
.70	6.CC	.0000002560	.0000000839
.70	7.CC	.0000000171	.0000000018
.70	8.CC	.0000000011	-.0000000001
.70	9.CC	.0000000001	-.0000000000
.70	10.CC	.0000000000	-.0000000000
.80	1.CC	.2407653680	.2410884340
.80	2.CC	.0160580072	.0157192003
.80	3.CC	.0010709995	.0009323935
.80	4.CC	.0000714310	.0000510880
.80	5.CC	.0000047641	.0000025253
.80	6.CC	.0000003177	.0000001031

.80	7.00	.0000000212	.0000000022
.80	8.00	.0000000014	-.0000000002
.80	9.00	.0000000001	-.0000000000
.80	10.00	.0000000000	-.0000000000
.90	1.00	.3025616150	.3011999459
.90	2.00	.0201795492	.0196358872
.90	3.00	.0013458885	.0011638440
.90	4.00	.0000897649	.0000636762
.90	5.00	.0000059869	.0000031385
.90	6.00	.0000003993	.0000001272
.90	7.00	.0000000266	.0000000026
.90	8.00	.0000000018	-.0000000002
.90	9.00	.0000000001	-.0000000000
.90	10.00	.0000000000	-.0000000000
1.00	1.00	.3839171075	.3792607035
1.00	2.00	.0256056082	.0247204367
1.00	3.00	.0017077832	.0014637615
1.00	4.00	.0001139017	.0000799291
1.00	5.00	.0000075968	.0000039245
1.00	6.00	.0000005067	.0000001575
1.00	7.00	.0000000338	.0000000030
1.00	8.00	.0000000023	-.0000000003
1.00	9.00	.0000000002	-.0000000001
1.00	10.00	.0000000000	-.0000000000

R = 5.1000
E(0)=-Z*Z/2= -.582250944757 E1=EGM= -.718763568150
E(1) = EGM + Z*Z/2 = -.136512623393
E2 = -.000124979048 E(2) = -.718888547198
E3 = .000002028900 E(3) = -.718886518298

R = 5.2000
E(0)=-Z*Z/2= -.578228346064 E1=EGM= -.713470573081
E(1) = EGM + Z*Z/2 = -.135245225015
E2 = -.000125548934 E(2) = -.713596142015
E3 = .000002004325 E(3) = -.713594137690

R = 5.3000
E(0)=-Z*Z/2= -.574406627418 E1=EGM= -.708407110655
E(1) = EGM + Z*Z/2 = -.134000423237
E2 = -.000126127820 E(2) = -.708533238475
E3 = .000001982026 E(3) = -.708531256449

R = 5.4000
E(0)=-Z*Z/2= -.57074546114 E1=EGM= -.703561959031
E(1) = EGM + Z*Z/2 = -.132777412915
E2 = -.000126446321 E(2) = -.703688605352
E3 = .000001961794 E(3) = -.703686643557

R = 5.5000
E(0)=-Z*Z/2= -.56734901216? E1=EGM= -.698924410440
E(1) = EGM + Z*Z/2 = -.131575396272
E2 = -.000127115134 E(2) = -.699051526625
E3 = .000001943443 E(3) = -.699049583182

R = 5.6000
E(0)=-Z*Z/2= -.56409C646873 E1=EGM= -.694484246077
E(1) = EGM + Z*Z/2 = -.130393599199
E2 = -.000127530254 E(2) = -.694611776332
E3 = .000001926806 E(3) = -.694609849526

R = 5.7000
E(0)=-Z*Z/2= -.561000457238 E1=EGM= -.690231713640
E(1) = EGM + Z*Z/2 = -.129231256402
E2 = -.000127882492 E(2) = -.690359596068
E3 = .000001911733 E(3) = -.690357684335

R = 5.8000
E(0)=-Z*Z/2= -.558049871592 E1=EGM= -.686157507110
E(1) = EGM + Z*Z/2 = -.128087635518
E2 = -.000128167629 E(2) = -.686285674731
E3 = .000001898093 E(3) = -.686283776638

R = 5.9000
E(0)=-Z*Z/2= -.555290718111 E1=EGM= -.682252748372
E(1) = EGM + Z*Z/2 = -.126962030261
E2 = -.000128381629 E(2) = -.682381129992
E3 = .000001885738 E(3) = -.682379244254

R = 6.0000
E(0)=-Z.Z/2= -.552455205633 E1=EGM= -.678508970357
E(1) = EGM + Z*Z/2 = -.125853764724
E2 = -.000128521211 E(2) = -.678637491569
E3 = .000001874574 E(3) = -.678635616995

MU	LAMBDA	PHI GM	PHI(1)
.00	1.00	.0530513737	.0517217543
.00	2.00	.0022642610	.0021182497
.00	3.00	.0000966309	.0000776241
.00	4.00	.0000041246	.0000025575
.00	5.00	.0000001760	.0000000712
.00	6.00	.0000000075	.0000000013
.00	7.00	.0000000003	-.0000000000
.00	8.00	.0000000000	-.0000000000
.00	9.00	.0000000000	-.0000000000
.00	10.00	.0000000000	-.0000000000
.10	1.00	.0546229170	.0534058412
.10	2.00	.0023313353	.0021874832
.10	3.00	.0000995026	.0000802003
.10	4.00	.0000042468	.0000026451
.10	5.00	.0000001813	.0000000739
.10	6.00	.0000000077	.0000000013
.10	7.00	.0000000003	-.0000000000
.10	8.00	.0000000000	-.0000000000
.10	9.00	.0000000000	-.0000000000
.10	10.00	.0000000000	-.0000000000
.20	1.00	.0594951811	.0586040704
.20	2.00	.0025392862	.0024011485
.20	3.00	.0001083781	.0000881456
.20	4.00	.0000046256	.0000029148
.20	5.00	.0000001974	.0000000819
.20	6.00	.0000000084	.0000000015
.20	7.00	.0000000004	-.0000000000
.20	8.00	.0000000000	-.0000000000
.20	9.00	.0000000000	-.0000000000
.20	10.00	.0000000000	-.0000000000
.30	1.00	.0681568801	.0677692751
.30	2.00	.0029089721	.0027777505
.30	3.00	.0001241565	.0001021319
.30	4.00	.0000052991	.0000033884
.30	5.00	.0000002262	.0000000959
.30	6.00	.0000000097	.0000000018
.30	7.00	.0000000004	-.0000000000
.30	8.00	.0000000000	-.0000000000
.30	9.00	.0000000000	-.0000000000

.30	10.CC	.0000000000	-.0000000000
.40	1.CC	.0814768285	.0817067873
.40	2.CC	.0034774746	.0033501993
.40	3.CC	.0001484205	.0001233546
.40	4.CC	.0000063347	.0000041045
.40	5.CC	.0000027C4	.0000001170
.40	6.00	.0000000115	.0000000023
.40	7.CC	.0000000005	-.0000000000
.40	8.CC	.0000000000	-.0000000000
.40	9.CC	.0000000000	-.0000000000
.40	10.00	.0000000000	-.0000000000
.50	1.CC	.1007910882	.1016540057
.50	2.CC	.0043018176	.0041690597
.50	3.CC	.0001836039	.0001536498
.50	4.00	.0000078363	.0000051225
.50	5.CC	.0000003345	.0000001466
.50	6.00	.0000000143	.0000000029
.50	7.CC	.0000000006	-.0000000000
.50	8.00	.0000000000	-.0000000000
.50	9.CC	.0000000000	-.0000000000
.50	10.00	.0000000000	-.0000000000
.60	1.CC	.1280369824	.1294009188
.60	2.00	.0054646869	.0053074698
.60	3.CC	.0002332358	.0001956722
.60	4.00	.0000099546	.0000065280
.60	5.CC	.0000004249	.0000001871
.60	6.00	.0000000181	.0000000037
.60	7.CC	.0000000008	-.0000000000
.60	8.00	.0000000000	-.0000000000
.60	9.CC	.0000000000	-.0000000000
.60	10.00	.0000000000	-.0000000000
.70	1.CC	.1659474195	.1674636572
.70	2.00	.0070827246	.0068682210
.70	3.CC	.0003022945	.0002531505
.70	4.00	.0000129021	.0000084414
.70	5.00	.0000005507	.0000002417
.70	6.00	.0000000235	.0000000047
.70	7.CC	.0000000010	-.0000000001
.70	8.00	.0000000000	-.0000000000
.70	9.CC	.0000000000	-.0000000000
.70	10.00	.0000000000	-.0000000000
.80	1.CC	.2183250186	.2193292610
.80	2.00	.0093182286	.0089937440
.80	3.CC	.0003977070	.0003312473
.80	4.00	.0000169743	.0000110287
.80	5.CC	.0000007245	.0000003147
.80	6.00	.0000000309	.0000000061
.80	7.CC	.0000000013	-.0000000001

APPENDIX B, PART 2: SERIES SOLUTION RESULTS 118

.80	8.00	.0000000001	-.0000000000
.80	9.00	.0000000000	-.0000000000
.80	10.00	.0000000000	-.0000000000
.90	1.00	.2904235319	.2897974646
.90	2.00	.0123954317	.0118800541
.90	3.00	.0005290437	.0004370628
.90	4.00	.0000225799	.0000145182
.90	5.00	.0000009637	.0000004121
.90	6.00	.0000000411	.0000000078
.90	7.00	.0000000018	-.0000000001
.90	8.00	.0000000001	-.0000000000
.90	9.00	.0000000000	-.0000000000
.90	10.00	.0000000000	-.0000000000
1.00	1.00	.3894748249	.3854553739
1.00	2.00	.0166229939	.0157961246
1.00	3.00	.0007094783	.0005803341
1.00	4.00	.0000302809	.0000192225
1.00	5.00	.0000012924	.0000005421
1.00	6.00	.0000000552	.0000000100
1.00	7.00	.0000000024	-.0000000002
1.00	8.00	.0000000001	-.0000000000
1.00	9.00	.0000000000	-.0000000000
1.00	10.00	.0000000000	-.0000000000

R = 6.1000
E(0)=-Z*Z/2= -.550155906480 E1=EGM= -.674918101390
E(1) = EGM + Z*Z/2 = -.124762194910
E2 = -.000128683930 E(2) = -.675046685319
E3 = .000001864486 E(3) = -.675044820833

R = 6.2000
E(0)=-Z*Z/2= -.547785740912 E1=EGM= -.671472450517
E(1) = EGM + Z*Z/2 = -.123686709605
E2 = -.000128568068 E(2) = -.671601018605
E3 = .000001855378 E(3) = -.671599163227

R = 6.3000
E(0)=-Z*Z/2= -.545537962902 E1=EGM= -.668164693595
E(1) = EGM + Z*Z/2 = -.122626730693
E2 = -.000128472694 E(2) = -.668293166290
E3 = .000001847157 E(3) = -.668291319133

R = 6.4000
E(0)=-Z*Z/2= -.543406146982 E1=EGM= -.664987859963
E(1) = EGM + Z*Z/2 = -.121581712981
E2 = -.000128973459 E(2) = -.665116157351
E3 = .000CC1899740 E(3) = -.665114317611

R = 6.5000
E(0)=-Z*Z/2= -.541384175922 E1=EGM= -.661935319568
E(1) = EGM + Z*Z/2 = -.120551143645
E2 = -.000128042379 E(2) = -.662063361946
E3 = .000CC1833C51 E(3) = -.662061528895

R = 6.6000
E(0)=-Z*Z/2= -.539466229079 E1=EGM= -.659000770439
E(1) = EGM + Z*Z/2 = -.119534541360
E2 = -.000127068381 E(2) = -.659128478820
E3 = .000CC1827018 E(3) = -.659126651802

R = 6.7000
E(0)=-Z*Z/2= -.537646771239 E1=EGM= -.656178226425
E(1) = EGM + Z*Z/2 = -.118531455185
E2 = -.000127096560 E(2) = -.656305522985
E3 = .000CC1821575 E(3) = -.656303701410

R = 6.8000
E(0)=-Z*Z/2= -.535920541859 E1=EGM= -.653462005137
E(1) = EGM + Z*Z/2 = -.117541463278
E2 = -.000126608476 E(2) = -.653588813613
E3 = .000CC1816662 E(3) = -.653586996952

R = 6.9000
E(0)=-Z*Z/2= -.534282544601 E1=EGM= -.650846716076
E(1) = EGM + Z*Z/2 = -.116564171474
E2 = -.000126246030 E(2) = -.650972962105
E3 = .000CC1812223 E(3) = -.650971149882

APPENDIX B, PART 2: SERIES SOLUTION RESULTS 120

R = 7.0000
E(0)=-Z*Z/2= -.5327280371C9 E1=EGM= -.648327248902
E(1) = EGM + Z*Z/2 = -.115599211793
E2 = -.000125611414 E(2) = -.648452560316
E3 = .000C01808297 E(3) = -.64P451C521C8

MU	LAMBDA	PHI G-	PHI(1)
.00	1.CC	.0331159998	.0319120896
.00	2.CC	.000893+043	.0008110744
.00	3.CC	.0000241023	.0000181828
.00	4.CC	.000001e5C2	.0000003560
.00	5.CO	.0000000175	.0000000054
.00	6.CC	.000000C005	.0000000000
.00	7.CC	.0000000000	-.0000000000
.00	8.CC	.0000000000	-.0000000000
.00	9.CC	.0000000000	-.0000000000
.00	10.CC	.0000000000	-.0000000000
.10	1.00	.0344741828	.0333779716
.10	2.CC	.000935C454	.0008485765
.10	3.CC	.0000250908	.0000190429
.10	4.CC	.0000006769	.0000003737
.10	5.CC	.0000000183	.0000000057
.10	6.CC	.000000C005	.0000000000
.10	7.CC	.0000000000	-.0000000000
.10	8.CC	.0000000000	-.0000000000
.10	9.CC	.0000000000	-.0000000000
.10	10.CC	.0000000000	-.0000000000
.20	1.CC	.0387279362	.0379450494
.20	2.CC	.0010448033	.0009653839
.20	3.CO	.0000241867	.0000217191
.20	4.CC	.0000007604	.0000004286
.20	5.CO	.0000000205	.0000000066
.20	6.CC	.000000C006	.0000000000
.20	7.CC	.0000000000	-.0000000000
.20	8.CC	.0000000000	-.0000000000
.20	9.CO	.0000000000	-.0000000000
.20	10.CC	.0000000000	-.0000000000
.30	1.CC	.0464385178	.0461446911
.30	2.CC	.0012523195	.0011749870
.30	3.CO	.0000337966	.0000265127
.30	4.CC	.0000009118	.0000005266
.30	5.CO	.0000000246	.0000000083
.30	6.CC	.000000C0C7	.0000000000
.30	7.CO	.0000000000	-.0000000000
.30	8.CC	.0000000000	-.0000000000
.30	9.CO	.0000000000	-.0000000000
.30	10.CC	.0000000000	-.0000000000

.40	1.00	.0586232937	.0589411622
.40	2.00	.0015815407	.0015018684
.40	3.00	.0000426668	.0000339705
.40	4.00	.0000011511	.0000006782
.40	5.00	.0000000311	.0000000108
.40	6.00	.0000000008	.0000000001
.40	7.00	.0000000000	-.0000000000
.40	8.00	.0000000000	-.0000000000
.40	9.00	.0000000000	-.0000000000
.40	10.00	.0000000000	-.0000000000
.50	1.00	.0768899737	.0778574327
.50	2.00	.0020743397	.0019846945
.50	3.00	.0000559616	.0000449562
.50	4.00	.0000015097	.0000009004
.50	5.00	.0000000407	.0000000145
.50	6.00	.0000000011	.0000000001
.50	7.00	.0000000000	-.0000000000
.50	8.00	.0000000000	-.0000000000
.50	9.00	.0000000000	-.0000000000
.50	10.00	.0000000000	-.0000000000
.60	1.00	.1036487395	.1051712648
.60	2.00	.0027962324	.0026812868
.60	3.00	.0000754370	.0000607605
.60	4.00	.0000020351	.0000012180
.60	5.00	.0000000549	.0000000196
.60	6.00	.0000000015	.0000000001
.60	7.00	.0000000000	-.0000000000
.60	8.00	.0000000000	-.0000000000
.60	9.00	.0000000000	-.0000000000
.60	10.00	.0000000000	-.0000000000
.70	1.00	.1424302535	.1442081343
.70	2.00	.0043842870	.0036760452
.70	3.00	.0001036627	.0000832661
.70	4.00	.0000027966	.0000016676
.70	5.00	.0000000754	.0000000268
.70	6.00	.0000000020	.0000000002
.70	7.00	.0000000001	-.0000000000
.70	8.00	.0000000000	-.0000000000
.70	9.00	.0000000000	-.0000000000
.70	10.00	.0000000000	-.0000000000
.80	1.00	.1983515095	.1997714719
.80	2.00	.0053511321	.0050908541
.80	3.00	.0001443630	.0001151896
.80	4.00	.0000038946	.0000023016
.80	5.00	.0000001051	.0000000368
.80	6.00	.0000000028	.0000000002
.80	7.00	.0000000001	-.0000000000
.80	8.00	.0000000000	-.0000000000

.80	9.00	.0000000000	-.0000000000
.80	10.00	.0000000000	-.0000000000
.90	1.00	.2787909898	.2787694876
.90	2.00	.0075212305	.0071009760
.90	3.00	.0002029079	.0001604354
.90	4.00	.0000054741	.0000031954
.90	5.00	.0000001477	.0000000506
.90	6.00	.0000000040	.0000000003
.90	7.00	.0000000001	-.0000000000
.90	8.00	.0000000000	-.0000000000
.90	9.00	.0000000000	-.0000000000
.90	10.00	.0000000000	-.0000000000
1.00	1.00	.3943622131	.3911246518
1.00	2.00	.0106391139	.0099581229
1.00	3.00	.0002870223	.0002246088
1.00	4.00	.0000077433	.0000044570
1.00	5.00	.0000002089	.0000000699
1.00	6.00	.0000000056	.0000000004
1.00	7.00	.0000000002	-.0000000000
1.00	8.00	.0000000000	-.0000000000
1.00	9.00	.0000000000	-.0000000000
1.00	10.00	.0000000000	-.0000000000

R = 7.1000
E(0)=-Z*Z/2= -.5312525200961 E1=EGM= -.645698761864
E(1) = EGM + Z*Z/2 = -.114646240903
E2 = -.000124907070 E(2) = -.646023668934
E3 = .000001804568 E(3) = -.646021864366

R = 7.2000
E(0)=-Z*Z/2= -.5299517318000 E1=EGM= -.643556670376
E(1) = EGM + Z*Z/2 = -.113704938576
E2 = -.000124135644 E(2) = -.643680806020
E3 = .000001801262 E(3) = -.643679004759

R = 7.3000
E(0)=-Z*Z/2= -.5288216295944 E1=EGM= -.641296635758
E(1) = EGM + Z*Z/2 = -.112775006162
E2 = -.000123299950 E(2) = -.641419357C8
E3 = .000001798249 E(3) = -.641418137459

R = 7.4000
E(0)=-Z*Z/2= -.5[illegible] E1=EGM= -.639114554160
E(1) = EGM + Z*Z/2 = -.111856165091
E2 = -.000122402935 E(2) = -.639236957095
E3 = .000001795496 E(3) = -.639235161599

R = 7.5000
E(0)=-Z*Z/2= -.526058330257 E1=EGM= -.637006545694
E(1) = EGM + Z*Z/2 = -.110948155436
E2 = -.000121445690 E(2) = -.637127991384
E3 = .000001786842 E(3) = -.637126192842

R = 7.6000
E(0)=-Z*Z/2= -.52491820924[illegible] E1=EGM= -.634968943778
E(1) = EGM + Z*Z/2 = -.110050734533
E2 = -.000120435815 E(2) = -.635089379593
E3 = .000001794551 E(3) = -.635087585042

R = 7.7000
E(0)=-Z*Z/2= -.523934609081 E1=EGM= -.632998284740
E(1) = EGM + Z*Z/2 = -.109163675660
E2 = -.000117265508 E(2) = -.633115550268
E3 = .000001045507 E(3) = -.633114504761

R = 7.8000
E(0)=-Z*Z/2= -.522[illegible] E1=EGM= -.631091297682
E(1) = EGM + Z*Z/2 = -.108286766804
E2 = -.000118262831 E(2) = -.631209560512
E3 = .000001788396 E(3) = -.631207772117

R = 7.9000
E(0)=-Z*Z/2= -.52122[illegible] E1=EGM= -.629244894630
E(1) = EGM + Z*Z/2 = -.107419809492
E2 = -.000117106199 E(2) = -.629362000819
E3 = .000001785935 E(3) = -.629360214885

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R = 8.0000

E(0)=-Z*Z/2= -.520893543303 E1=EGM= -.627456161001

E(1) = EGM + Z*Z/2 = -.104562617694

E2 = -.000115907C61 E(2) = -.627572068063

E3 = .0000C1783752 E(3) = -.627570284311

MU	LAMBDA	PHI GM	PHI(1)
.00	1.CC	.0204715862	.0193868392
.00	2.CC	.0003451829	.0003027132
.00	3.CC	.0000582C3	.0000041089
.00	4.CC	.0000000981	.0000000471
.00	5.CC	.0000000017	.0000000004
.00	6.CC	.0000000000	-.0000000000
.00	7.CC	.0000000000	-.0000000000
.00	8.CC	.0000000000	-.0000000000
.00	9.CC	.0000000000	-.0000000000
.00	10.CC	.0000000000	-.0000000000
.10	1.CC	.0215826579	.0205944452
.10	2.CC	.0003639173	.0003217630
.10	3.CC	.0000061362	.0000043761
.10	4.CC	.0000001035	.0000000504
.10	5.CC	.0000000017	.0000000004
.10	6.CC	.0000000000	-.0000000000
.10	7.CC	.0000000000	-.0000000000
.10	8.CC	.0000000000	-.0000000000
.10	9.CC	.0000000000	-.0000000000
.10	10.CC	.0000000000	-.0000000000
.20	1.CC	.0251036599	.0243973321
.20	2.CC	.0004232868	.0003817269
.20	3.CC	.0000071373	.0000052160
.20	4.CC	.0000012C3	.0000000607
.20	5.CC	.0000000020	.0000000005
.20	6.CC	.0000000000	-.0000000000
.20	7.CC	.0000000000	-.0000000000
.20	8.CC	.0000000000	-.0000000000
.20	9.CC	.0000000000	-.0000000000
.20	10.CC	.0000000000	-.0000000000
.30	1.CC	.0316296915	.0313675136
.30	2.CC	.0005333259	.0004915471
.30	3.CC	.0000089927	.0000067505
.30	4.CC	.0000001516	.0000000795
.30	5.CC	.0000000026	.0000000007
.30	6.CC	.0000000000	-.0000000000
.30	7.CC	.0000000000	-.0000000000
.30	8.CC	.0000000000	-.0000000000
.30	9.CC	.0000000000	-.0000000000
.30	10.CC	.0000000000	-.0000000000
.40	1.CC	.0422637445	.0425673693

.40	2.00	.0007126326	.0006678342
.40	3.00	.0000120161	.0000092060
.40	4.00	.0000002026	.0000001094
.40	5.00	.0000000034	.0000000010
.40	6.00	.0000000001	-.0000000000
.40	7.00	.0000000000	-.0000000000
.40	8.00	.0000000000	-.0000000000
.40	9.00	.0000000000	-.0000000000
.40	10.00	.0000000000	-.0000000000
.50	1.00	.0588031249	.0597274096
.50	2.00	.0009915124	.0009376444
.50	3.00	.0000167184	.0000129513
.50	4.00	.0000002819	.0000001546
.50	5.00	.0000000048	.0000000014
.50	6.00	.0000000001	.0000000000
.50	7.00	.0000000000	-.0000000000
.50	8.00	.0000000000	-.0000000000
.50	9.00	.0000000000	-.0000000000
.50	10.00	.0000000000	-.0000000000
.60	1.00	.0840432226	.0855322840
.60	2.00	.0014170997	.0013429435
.60	3.00	.0000238945	.0000185581
.60	4.00	.0000004029	.0000002217
.60	5.00	.0000000068	.0000000020
.60	6.00	.0000000001	.0000000000
.60	7.00	.0000000000	-.0000000000
.60	8.00	.0000000000	-.0000000000
.60	9.00	.0000000000	-.0000000000
.60	10.00	.0000000000	-.0000000000
.70	1.00	.1222499720	.1240651586
.70	2.00	.0020613251	.0019475416
.70	3.00	.0000347572	.0000268951
.70	4.00	.0000005861	.0000003209
.70	5.00	.0000000099	.0000000028
.70	6.00	.0000000002	.0000000000
.70	7.00	.0000000000	-.0000000000
.70	8.00	.0000000000	-.0000000000
.70	9.00	.0000000000	-.0000000000
.70	10.00	.0000000000	-.0000000000
.80	1.00	.1798860560	.1814887355
.80	2.00	.0030330716	.0028477243
.80	3.00	.0000511423	.0000392717
.80	4.00	.0000008623	.0000004670
.80	5.00	.0000000145	.0000000041
.80	6.00	.0000000002	-.0000000000
.80	7.00	.0000000000	-.0000000000
.80	8.00	.0000000000	-.0000000000
.80	9.00	.0000000000	-.0000000000

.80	10.00	.0000000000	-.0000000000
.90	1.00	.2666763116	.2670822447
.90	2.00	.0044965783	.0041884495
.90	3.00	.0000756193	.0000576586
.90	4.00	.0000012784	.0000006829
.90	5.00	.0000000216	.0000000059
.90	6.00	.0000000004	-.0000000000
.90	7.00	.0000000000	-.0000000000
.90	8.00	.0000000000	-.0000000000
.90	9.00	.0000000000	-.0000000000
.90	10.00	.0000000000	-.0000000000
1.00	1.00	.3973060020	.3948160303
1.00	2.00	.0066991985	.0061879524
1.00	3.00	.0001129589	.0000850224
1.00	4.00	.0000019047	.0000010026
1.00	5.00	.0000000321	.0000000085
1.00	6.00	.0000000005	-.0000000000
1.00	7.00	.0000000000	-.0000000000
1.00	8.00	.0000000000	-.0000000000
1.00	9.00	.0000000000	-.0000000000
1.00	10.00	.0000000000	-.0000000000

R = 8.1000
E(0)=-Z*Z/2= -.520007329569 E1=EGM= -.625722346375
E(1) = EGM + Z*Z/2 = -.105715018806
E2 = -.000114668616 E(2) = -.625837014991
E3 = .000001781780 E(3) = -.625835233211

R = 8.2000
E(0)=-Z*Z/2= -.519164012905 E1=EGM= -.624040855607
E(1) = EGM + Z*Z/2 = -.104876842703
E2 = -.000113393971 E(2) = -.624154249579
E3 = .000001779968 E(3) = -.624152469610

R = 8.3000
E(0)=-Z*Z/2= -.518361299432 E1=EGM= -.622409240284
E(1) = EGM + Z*Z/2 = -.104047940852
E2 = -.000112086189 E(2) = -.622521326473
E3 = .000001778280 E(3) = -.622519548193

R = 8.4000

```
 E(0)=-Z*Z/2=   -.517597025020      E1=EGM=   -.620825190519
 E(1) = EGM + Z*Z/2 =    -.102228165499
 E2 =    -.000110748264      E(2) =    -.620935938783
 E3 =     .0000C1776686      E(3) =    -.620934162096


R =   8.5000
 E(0)=-Z*Z/2=   -.516869148193      E1=EGM=   -.619286527105
 E(1) = EGM + Z*Z/2 =    -.102417378912
 E2 =    -.000109383114      E(2) =    -.619395910219
 E3 =     .0000C1775166      E(3) =    -.619394135053


R =   8.6000
 E(0)=-Z*Z/2=   -.516175743336      E1=EGM=   -.617791194018
 E(1) = EGM + Z*Z/2 =    -.101615450682
 E2 =    -.000107993577      E(2) =    -.617899187596
 E3 =     .0000C1773702      E(3) =    -.617897413894


R =   8.7000
 E(0)=-Z*Z/2=   -.515514994192      E1=EGM=   -.616337251271
 E(1) = EGM + Z*Z/2 =    -.100822257079
 E2 =    -.000106582403      E(2) =    -.616443833674
 E3 =     .0000C1772282      E(3) =    -.616442061392


R =   8.8000
 E(0)=-Z*Z/2=   -.514885187656      E1=EGM=   -.614922868112
 E(1) = EGM + Z*Z/2 =    -.100037680456
 E2 =    -.000105152249      E(2) =    -.615028020361
 E3 =     .0000C1770897      E(3) =    -.615026249464


R =   8.9000
 E(0)=-Z*Z/2=   -.514284707865      E1=EGM=   -.613546316565
 E(1) = EGM + Z*Z/2 =    -.099261608701
 E2 =    -.000103705680      E(2) =    -.613650022245
 E3 =     .0000C1769538      E(3) =    -.613648252707


R =   9.0000
```

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E(0)=-Z*Z/2= -.513712030582 E1=EGM= -.612205965312

E(1) = EGM + Z*Z/2 = -.099493934731

E2 = -.0001C2245159 E(2) = -.61230821C471

E3 = .0000C1768200 E(3) = -.612306442271

MU	LAMBDA	PHI GM	PHI(1)
.00	1.00	.0126417137	.0116774864
.00	2.CC	.0001320883	.0001112402
.00	3.CC	.0000013801	.0000009071
.00	4.CC	.0000000144	.0000000060
.00	5.CC	.0000000002	.0000000000
.00	6.CC	.0000000000	-.0000000000
.00	7.CC	.0000000000	-.0000000000
.00	8.CC	.0000000000	-.0000000000
.00	9.CC	.0000000000	-.0000000000
.00	10.CC	.0000000000	-.0000000000
.10	1.0C	.0135138101	.0126324527
.10	2.CC	.0001412005	.0001204750
.10	3.00	.0000014753	.0000009860
.10	4.CC	.0000000154	.0000000066
.10	5.CC	.0000000002	.0000000000
.10	6.CC	.0000000000	-.0000000000
.10	7.0C	.0000000000	-.0000000000
.10	8.CC	.0000000000	-.0000000000
.10	9.00	.0000000000	-.0000000000
.10	10.CC	.0000000000	-.0000000000
.20	1.00	.0163147096	.0156759583
.20	2.CC	.0001704660	.0001498885
.20	3.CC	.0000017811	.0000012368
.20	4.CC	.0000000186	.0000000084
.20	5.00	.0000000002	.0000000000
.20	6.CC	.0000000000	-.0000000000
.20	7.0C	.0000000000	-.0000000000
.20	8.CC	.0000000000	-.0000000000
.20	9.CC	.0000000000	-.0000000000
.20	10.C0	.0000000000	-.0000000000
.30	1.00	.0216373224	.0213839417
.30	2.CC	.0002260798	.0002049930
.30	3.00	.0000023622	.0000017052
.30	4.CC	.0000000247	.0000000119
.30	5.C0	.0000000003	.0000000001
.30	6.CC	.0000000000	-.0000000000
.30	7.00	.0000000000	-.0000000000
.30	8.CC	.0000000000	-.0000000000
.30	9.CC	.0000000000	-.0000000000
.30	10.CC	.0000000000	-.0000000000
.40	1.0C	.0306083695	.0308545120
.40	2.CC	.0003198148	.0002963022

.40	3.00	.0000033416	.0000024781
.40	4.00	.0000000349	.0000000175
.40	5.00	.0000000004	.0000000001
.40	6.00	.0000000000	-.0000000000
.40	7.00	.0000000000	-.0000000000
.40	8.00	.0000000000	-.0000000000
.40	9.00	.0000000000	-.0000000000
.40	10.00	.0000000000	-.0000000000
.50	1.00	.0451268929	.0459380392
.50	2.00	.0004715131	.0004415313
.50	3.00	.0000049267	.0000037026
.50	4.00	.0000000515	.0000000263
.50	5.00	.0000000005	.0000000001
.50	6.00	.0000000000	-.0000000000
.50	7.00	.0000000000	-.0000000000
.50	8.00	.0000000000	-.0000000000
.50	9.00	.0000000000	-.0000000000
.50	10.00	.0000000000	-.0000000000
.60	1.00	.0682662558	.0696203678
.60	2.00	.0007132871	.0006492593
.60	3.00	.0000074529	.0000056150
.60	4.00	.0000000779	.0000000399
.60	5.00	.0000000008	.0000000002
.60	6.00	.0000000000	-.0000000000
.60	7.00	.0000000000	-.0000000000
.60	8.00	.0000000000	-.0000000000
.60	9.00	.0000000000	-.0000000000
.60	10.00	.0000000000	-.0000000000
.70	1.00	.1059247297	.1066434276
.70	2.00	.0010963171	.0010248582
.70	3.00	.0000011455	.0000085906
.70	4.00	.0000001197	.0000000609
.70	5.00	.0000000013	.0000000003
.70	6.00	.0000000000	-.0000000000
.70	7.00	.0000000000	-.0000000000
.70	8.00	.0000000000	-.0000000000
.70	9.00	.0000000000	-.0000000000
.70	10.00	.0000000000	-.0000000000
.80	1.00	.1628623882	.1644992206
.80	2.00	.0017016848	.0015799961
.80	3.00	.0000177802	.0000132216
.80	4.00	.0000001858	.0000000933
.80	5.00	.0000000019	.0000000004
.80	6.00	.0000000000	-.0000000000
.80	7.00	.0000000000	-.0000000000
.80	8.00	.0000000000	-.0000000000
.80	9.00	.0000000000	-.0000000000
.80	10.00	.0000000000	-.0000000000

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.90	1.00	.2543438028	.2550140428
.90	2.00	.0026575379	.0024477593
.90	3.00	.0000277676	.0000204423
.90	4.00	.0000002901	.0000001435
.90	5.00	.0000000030	.0000000007
.90	6.00	.0000000000	-.0000000000
.90	7.00	.0000000000	-.0000000000
.90	8.00	.0000000000	-.0000000000
.90	9.00	.0000000000	-.0000000000
.90	10.00	.0000000000	-.0000000000
1.00	1.00	.3987342779	.3988691955
1.00	2.00	.0041662170	.0038069283
1.00	3.00	.0000435312	.0000317291
1.00	4.00	.0000004548	.0000002217
1.00	5.00	.0000000048	.0000000010
1.00	6.00	.0000000000	-.0000000000
1.00	7.00	.0000000000	-.0000000000
1.00	8.00	.0000000000	-.0000000000
1.00	9.00	.0000000000	-.0000000000
1.00	10.00	.0000000000	-.0000000000

R = 9.1000
E(0)=-Z*Z/2= -.513165717861 E1=EGM= -.610900273888
E(1) = EGM + Z*Z/2 = -.097734556027
E2 = -.000100773063 E(2) = -.611001046942
E3 = .000001766880 E(3) = -.610999280062

R = 9.2000
E(0)=-Z*Z/2= -.512644412994 E1=EGM= -.609627787205
E(1) = EGM + Z*Z/2 = -.096983374210
E2 = -.000099291625 E(2) = -.609727078830
E3 = .000001765573 E(3) = -.609725313257

R = 9.3000
E(0)=-Z*Z/2= -.512146835728 E1=EGM= -.608387130370
E(1) = EGM + Z*Z/2 = -.094240294642
E2 = -.000097803038 E(2) = -.608484933408
E3 = .000001764278 E(3) = -.608483169130

R = 9.4000
E(0)=-Z*Z/2= -.511671777747 E1=EGM= -.607177003813

```
 F(1) = EGM + Z*Z/2 =   -.0955C5226066
 E2 =   -.0000963C9352      E(2) =   -.607273313164
 E3 =    .000CC1762903      E(3) =   -.607271550171


R =   9.5000
 E(0)=-7*Z/2=  -.511218C98401      E1=EGM=  -.605996178679
 E(1) = EGM + Z*Z/2 =   -.C947780C0277
 E2 =   -.0000948125C4      E(2) =   -.606090991242
 E3 =    .000CC1761730      E(3) =   -.6060C89229512


R =   9.6000
 E(0)=-7*Z/2=  -.51C784720690      E1=EGM=  -.604843492508
 F(1) = EGM + Z*Z/2 =   -.C94C58771818
 E2 =   -.0000933144l6      E(2) =   -.604936806924
 E3 =    .000CC1760469      E(3) =   -.604935046474


R =   9.7000
 E(0)=-7*Z/2=  -.51C37C627463      E1=EGM=  -.603717845165
 F(1) = EGM + Z*Z/2 =   -.C93347217696
 E2 =   -.0000918167B3      E(2) =   -.6038C9661948
 E3 =    .000CC1759192      E(3) =   -.6038C7902756


R =   9.8000
 E(0)=-7*Z/2=  -.5C9974857881      E1=EGM=  -.602618195015
 F(1) = EGM + Z*Z/2 =   -.C92643337135
 E2 =   -.0000903212B7      E(2) =   -.6027C8516303
 E3 =    .000CC1757942      E(3) =   -.602706758361


R =   9.9000
 E(0)=-7*Z/2=  -.5095965C4C05      E1=EGM=  -.601543555341
 E(1) = EGM + Z*Z/2 =   -.C91947C51336
 E2 =   -.0000888294B4      E(2) =   -.601632384834
 E3 =    .000CC1756732      E(3) =   -.601630628133


R =  10.0000
 E(0)=-7*Z/2=  -.5C923470770?      E1=EGM=  -.600492990971
```

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F(1) = F6M + Z*7/2 = -.091258283269

E2 = -.000087342373 E(2) = -.600580333844

E3 = .000001755471 E(3) = -.600578578373

MU	LAMBDA	PHI GM	PHI(1)
.00	1.00	.0078442968	.0069990452
.00	2.00	.0005048C1	.0004404783
.00	3.00	.0000003249	.0000001970
.00	4.00	.0000000021	.0000000007
.00	5.00	.0000000000	.0000000000
.00	6.00	.0000000000	-.0000000000
.00	7.00	.0000000000	-.0000000000
.00	8.00	.0000000000	-.0000000000
.00	9.00	.0000000000	-.0000000000
.00	10.00	.0000000000	-.0000000000
.10	1.00	.0085079509	.0077319545
.10	2.00	.0005475C9	.0004480B8
.10	3.00	.0000003523	.0000002195
.10	4.00	.0000000023	.0000000008
.10	5.00	.0000000000	.0000000000
.10	6.00	.0000000000	-.0000000000
.10	7.00	.0000000000	-.0000000000
.10	8.00	.0000000000	-.0000000000
.10	9.00	.0000000000	-.0000000000
.10	10.00	.0000000000	-.0000000000
.20	1.00	.0106715068	.0100984145
.20	2.00	.0006687A0	.0005B7792
.20	3.00	.0000004419	.0000002919
.20	4.00	.0000000028	.0000000012
.20	5.00	.0000000000	.0000000000
.20	6.00	.0000000000	-.0000000000
.20	7.00	.0000000000	-.0000000000
.20	8.00	.0000000000	-.0000000000
.20	9.00	.0000000000	-.0000000000
.20	10.00	.0000000000	-.0000000000
.30	1.00	.0148976318	.0146488341
.30	2.00	.0009567C2	.0008856043
.30	3.00	.0000006170	.0000004303
.30	4.00	.0000000040	.0000000018
.30	5.00	.0000000000	.0000000000
.30	6.00	.0000000000	-.0000000000
.30	7.00	.0000000000	-.0000000000
.30	8.00	.0000000000	-.0000000000
.30	9.00	.0000000000	-.0000000000
.30	10.00	.0000000000	-.0000000000
.40	1.00	.0222853970	.0224638929
.40	2.00	.0001434124	.0001315994
.40	3.00	.0000009229	.0000006665

.40	4.00	.0000000059	.0000000028
.40	5.00	.0000000000	.0000000000
.40	6.00	.0000000000	-.0000000000
.40	7.00	.0000000000	-.0000000000
.40	8.00	.0000000000	-.0000000000
.40	9.00	.0000000000	-.0000000000
.40	10.00	.0000000000	-.0000000000
.50	1.00	.0347561081	.0354325097
.50	2.00	.0002236648	.0002078027
.50	3.00	.0000014393	.0000010559
.50	4.00	.0000000093	.0000000045
.50	5.00	.0000000001	.0000000000
.50	6.00	.0000000000	-.0000000000
.50	7.00	.0000000000	-.0000000000
.50	8.00	.0000000000	-.0000000000
.50	9.00	.0000000000	-.0000000000
.50	10.00	.0000000000	-.0000000000
.60	1.00	.0555529721	.0567322601
.60	2.00	.0003574991	.0003327763
.60	3.00	.0000023006	.0000016918
.60	4.00	.0000000148	.0000000071
.60	5.00	.0000000001	.0000000000
.60	6.00	.0000000000	-.0000000000
.60	7.00	.0000000000	-.0000000000
.60	8.00	.0000000000	-.0000000000
.60	9.00	.0000000000	-.0000000000
.60	10.00	.0000000000	-.0000000000
.70	1.00	.0900845445	.0916417870
.70	2.00	.0005797179	.0005373440
.70	3.00	.0000037306	.0000027288
.70	4.00	.0000000240	.0000000115
.70	5.00	.0000000002	.0000000000
.70	6.00	.0000000000	-.0000000000
.70	7.00	.0000000000	-.0000000000
.70	8.00	.0000000000	-.0000000000
.70	9.00	.0000000000	-.0000000000
.70	10.00	.0000000000	-.0000000000
.80	1.00	.1473313103	.1489024533
.80	2.00	.0009481160	.0008725355
.80	3.00	.0000061014	.0000044224
.80	4.00	.0000000393	.0000000185
.80	5.00	.0000000003	.0000000000
.80	6.00	.0000000000	-.0000000000
.80	7.00	.0000000000	-.0000000000
.80	8.00	.0000000000	-.0000000000
.80	9.00	.0000000000	-.0000000000
.80	10.00	.0000000000	-.0000000000
.90	1.00	.2421812020	.2429924358

.90	2.00	.0015850002	.0014228627
.90	3.00	.0000100294	.0000071962
.90	4.00	.0000000645	.0000000300
.90	5.00	.0000000004	.0000000001
.90	6.00	.0000000000	-.0000000000
.90	7.00	.0000000000	-.0000000000
.90	8.00	.0000000000	-.0000000000
.90	9.00	.0000000000	-.0000000000
.90	10.00	.0000000000	-.0000000000
1.00	1.00	.3993014435	.3979218231
1.00	2.00	.0025696105	.0023284897
1.00	3.00	.0000165361	.0000117524
1.00	4.00	.0000001064	.0000000487
1.00	5.00	.0000000007	.0000000001
1.00	6.00	.0000000000	-.0000000000
1.00	7.00	.0000000000	-.0000000000
1.00	8.00	.0000000000	-.0000000000
1.00	9.00	.0000000000	-.0000000000
1.00	10.00	.0000000000	-.0000000000

R = 10.1000
E(0)=-7*7/2= -.5038838657651 E1=EGM= -.599465615132
E(1) = EGM + Z*Z/2 = -.0905769574 81
E2 = -.0000854842836 E(2) = -.599551477936
E3 = .0000011754252 E(3) = -.599549723685

R = 10.2000
E(0)=-7*7/2= -.5038557566570 E1=EGM= -.598460586486
E(1) = EGM + Z*Z/2 = -.0899029999 16
E2 = -.0000845905728 E(2) = -.598544977064
E3 = .0000011753044 E(3) = -.598543224020

R = 10.3000
E(0)=-7*7/2= -.5038407626029 E1=EGM= -.597471106366
E(1) = EGM + Z*Z/2 = -.0892936337758
E2 = -.0000837927396 E(2) = -.597560033762
E3 = .0000011751849 E(3) = -.597558281913

R = 10.4000
E(0)=-7*7/2= -.5037997516429 E1=EGM= -.596514416181
E(1) = EGM + Z*Z/2 = -.0886576899289

E2 = -.0000814743[illegible] E(2) = -.596595890561
E3 = .000001750668 E(3) = -.596594139893

R = 10.5000
E(2)=-7*7/2= -.5C74471[illegible]1235 E1=EGM= -.595571794993
E(1) = EGM + Z*7/2 = -.087924613757
E2 = -.0000800325[illegible] E(2) = -.595651827558
E3 = .000001749502 E(3) = -.595650078056

R = 10.6000
E(2)=-7*7/2= -.5C7349145981 E1=EGM= -.594648557244
E(1) = EGM + Z*7/2 = -.087279411263
E2 = -.0000784[illegible]2915 E(2) = -.594728716C159
E3 = .000001748353 E(3) = -.594725411806

R = 10.7000
E(2)=-7*7/2= -.5C7102827984 E1=EGM= -.5937440[illegible]638
E(1) = EGM + Z*7/2 = -.086641222652
E2 = -.0000771863[illegible] E(2) = -.593821236949
E3 = .000001747222 E(3) = -.593819489727

R = 10.8000
E(2)=-7*7/2= -.5C6847674787 E1=EGM= -.592857654150
E(1) = EGM + Z*7/2 = -.086009979423
E2 = -.0000757835[illegible] E(2) = -.592933437718
E3 = .000001746110 E(3) = -.592931691608

R = 10.9000
E(2)=-7*7/2= -.5C6600162527 E1=EGM= -.591988776175
E(1) = EGM + Z*7/2 = -.085385613649
E2 = -.0000743[illegible] E(2) = -.592063171604
E3 = .000001745019 E(3) = -.592061426585

R = 11.0000
E(2)=-7*7/2= -.5C6368794890 E1=EGM= -.591136852791
E(1) = EGM + Z*7/2 = -.084768057901

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FP = -.0000730225e7 F(2) = -.5912C9475358

E3 = .000CC1743949 F(3) = -.591208131408

MU	LAMBDA	PHI GM	PHI(1)
.00	1.CC	.0049092201	.0041759584
.00	2.CC	.0000193744	.0000146039
.00	3.CC	.0000000765	.0000000421
.00	4.CC	.0000000003	.0000000001
.00	5.CC	.0000000000	-.0000000000
.00	6.CC	.0000000000	-.0000000000
.00	7.CC	.0000000000	-.0000000000
.00	8.CC	.0000000000	-.0000000000
.00	9.CC	.0000000000	-.0000000000
.00	10.CC	.0000000000	-.0000000000
.10	1.CC	.0054024296	.0047262348
.10	2.CC	.0000213209	.0000165870
.10	3.CC	.0000000841	.0000000484
.10	4.CC	.0000000003	.0000000001
.10	5.CC	.0000000000	.0000000000
.10	6.CC	.0000000000	-.0000000000
.10	7.CC	.0000000000	-.0000000000
.10	8.CC	.0000000000	-.0000000000
.10	9.CC	.0000000000	-.0000000000
.10	10.CC	.0000000000	-.0000000000
.20	1.CC	.0070370515	.0065277101
.20	2.CC	.0000277720	.0000230717
.20	3.CC	.0000001096	.0000000688
.20	4.CC	.0000000004	.0000000002
.20	5.CC	.0000000000	.0000000000
.20	6.CC	.0000000000	-.0000000000
.20	7.CC	.0000000000	-.0000000000
.20	8.CC	.0000000000	-.0000000000
.20	9.CC	.0000000000	-.0000000000
.20	10.CC	.0000000000	-.0000000000
.30	1.CC	.0103267729	.0100849314
.30	2.CC	.0000407550	.0000358528
.30	3.CC	.0000001608	.0000001088
.30	4.CC	.0000000006	.0000000003
.30	5.CC	.0000000000	.0000000000
.30	6.CC	.0000000000	-.0000000000
.30	7.CC	.0000000000	-.0000000000
.30	8.CC	.0000000000	-.0000000000
.30	9.CC	.0000000000	-.0000000000
.30	10.CC	.0000000000	-.0000000000
.40	1.CC	.0163054034	.0164213131
.40	2.CC	.0000643498	.0000585740
.40	3.CC	.0000002540	.0000001795
.40	4.CC	.0000000010	.0000000004

.40	5.00	.0000000000	.0000000000
.40	6.00	.0000000000	-.0000000000
.40	7.00	.0000000000	-.0000000000
.40	8.00	.0000000000	-.0000000000
.40	9.00	.0000000000	-.0000000000
.40	10.00	.0000000000	-.0000000000
.50	1.00	.2268517542	.2273969323
.50	2.00	.0001059714	.0000978580
.50	3.00	.0000004182	.0000003012
.50	4.00	.0000000017	.0000000008
.50	5.00	.0000000000	.0000000000
.50	6.00	.0000000000	-.0000000000
.50	7.00	.0000000000	-.0000000000
.50	8.00	.0000000000	-.0000000000
.50	9.00	.0000000000	-.0000000000
.50	10.00	.0000000000	-.0000000000
.60	1.00	.2452800630	.2462797087
.60	2.00	.0001786993	.0001653347
.60	3.00	.0000007052	.0000005091
.60	4.00	.0000000028	.0000000013
.60	5.00	.0000000000	.0000000000
.60	6.00	.0000000000	-.0000000000
.60	7.00	.0000000000	-.0000000000
.60	8.00	.0000000000	-.0000000000
.60	9.00	.0000000000	-.0000000000
.60	10.00	.0000000000	-.0000000000
.70	1.00	.2773815077	.2787554642
.70	2.00	.0003053888	.0002812296
.70	3.00	.0000012052	.0000008649
.70	4.00	.0000000048	.0000000022
.70	5.00	.0000000000	.0000000000
.70	6.00	.0000000000	-.0000000000
.70	7.00	.0000000000	-.0000000000
.70	8.00	.0000000000	-.0000000000
.70	9.00	.0000000000	-.0000000000
.70	10.00	.0000000000	-.0000000000
.80	1.00	.3324411100	.3347013676
.80	2.00	.0005256524	.0004806667
.80	3.00	.0000020753	.0000014751
.80	4.00	.0000000082	.0000000037
.80	5.00	.0000000000	.0000000000
.80	6.00	.0000000000	-.0000000000
.80	7.00	.0000000000	-.0000000000
.80	8.00	.0000000000	-.0000000000
.80	9.00	.0000000000	-.0000000000
.80	10.00	.0000000000	-.0000000000
.90	1.00	.3842294009	.2312920427
.90	2.00	.0009093720	.0008247154

.90	3.00	.0000035889	.0000025252
.90	4.00	.0000000142	.0000000063
.90	5.00	.0000000001	.0000000000
.90	6.00	.0000000000	-.0000000000
.90	7.00	.0000000000	-.0000000000
.90	8.00	.0000000000	-.0000000000
.90	9.00	.0000000000	-.0000000000
.90	10.00	.0000000000	-.0000000000
1.00	1.00	.3994568802	.3984395265
1.00	2.00	.0015764702	.0014197501
1.00	3.00	.0000062216	.0000043383
1.00	4.00	.0000000246	.0000000107
1.00	5.00	.0000000001	.0000000000
1.00	6.00	.0000000000	-.0000000000
1.00	7.00	.0000000000	-.0000000000
1.00	8.00	.0000000000	-.0000000000
1.00	9.00	.0000000000	-.0000000000
1.00	10.00	.0000000000	-.0000000000

R = 11.1000
E(0)=-7*Z/2= -.5061441C0944 E1=EGM= -.590301346131
E(1) = EGM + Z*Z/2 = -.0841572451B6
E2 = -.000071665599 E(2) = -.590373011730
E3 = .0000C1742934 E(3) = -.590371268827

R = 11.2000
E(0)=-7*Z/2= -.5059286339B4 E1=EGM= -.589481742877
E(1) = EGM + Z*Z/2 = -.0835531088?1
E2 = -.000070325075 E(2) = -.589552067952
E3 = .0000C1741[illegible] E(3) = -.589550326069

R = 11.3000
E(0)=-7*Z/2= -.5057219700C91 E1=EGM= -.588677552831
E(1) = EGM + Z*Z/2 = -.0829855827[illegible]C
E2 = -.000069001490 E(2) = -.58874655432C
E3 = .0000C1740[illegible] E(3) = -.588744613431

R = 11.4000
E(0)=-7*Z/2= -.5055237068[illegible] E1=EGM= -.587883075?4
E(1) = EGM + Z*Z/2 = -.0824646000745
E2 = -.000067695283 E(2) = -.587956002877

E3 = .000CC1739923 E(3) = -.587954262953

R = 11.5000
E(0)=-7*Z/2= -.505333462144 E1=EGM= -.587113559325
E(1) = EGM + Z*Z/2 = -.C81780097181
E2 = -.000C66406842 E(2) = -.587179966168
E3 = .000CC1738958 E(3) = -.587178277180

R = 11.6000
E(0)=-7*Z/2= -.506150873038 E1=EGM= -.586352879583
E(1) = EGM + Z*Z/2 = -.C81202006551
E2 = -.000C65136505 E(2) = -.586418016088
E3 = .000CC1738024 E(3) = -.586416278005

R = 11.7000
E(0)=-7*Z/2= -.504978594671 E1=EGM= -.585605858239
E(1) = EGM + Z*Z/2 = -.C80630263569
E2 = -.000C63684562 E(2) = -.585669742801
E3 = .000CC1737213 E(3) = -.585668055568

R = 11.8000
E(0)=-7*Z/2= -.504307259327 E1=EGM= -.584872102462
E(1) = EGM + Z*Z/2 = -.C80064803135
E2 = -.000C62651261 E(2) = -.584934753723
E3 = .000CC1736379 E(3) = -.584933017345

R = 11.9000
E(0)=-7*Z/2= -.503645675439 E1=EGM= -.584151235770
E(1) = EGM + Z*Z/2 = -.C79505560331
E2 = -.000C61436806 E(2) = -.584212672576
E3 = .000CC1735580 E(3) = -.584210936996

R = 12.0000
E(0)=-7*Z/2= -.502990426737 E1=EGM= -.583442897140
E(1) = EGM + Z*Z/2 = -.C78952470403
E2 = -.000C60241364 E(2) = -.583503138504

APPENDIX B, PART 2: SERIES SOLUTION RESULTS 140
F3 = .0000C1734821 E(3) = -.5835C14C36P3

MU	LAMBDA	PHI GM	PHI(1)
.00	1.CC	.0031069624	.CC24745073
.00	2.CC	.0000074971	.CCCCC52102
.00	3.CC	.0000000181	.CCCCCC0088
.00	4.CC	.0000000000	.CCCCCCCCCC
.00	5.CC	.0000000000	-.CCCCCCCCCC
.00	6.CC	.0000000000	-.CCCCCCCCCC
.00	7.CC	.0000000000	-.CCCCCCCCCC
.00	8.CC	.0000000000	-.CCCCCCCCCC
.00	9.CC	.0000000000	-.CCCCCCCCCC
.00	10.CC	.0000000000	-.CCCCCCCCCC
.10	1.CC	.0034667211	.0028811467
.10	2.CC	.0000083652	.CCCCC61034
.10	3.CC	.0000000282	.CCCCCCC105
.10	4.CC	.0000000000	.CCCCCCCCCC
.10	5.CC	.0000000000	-.CCCCCCCCCC
.10	6.CC	.0000000000	-.CCCCCCCCCC
.10	7.CC	.0000000000	-.CCCCCCCCCC
.10	8.CC	.0000000000	-.CCCCCCCCCC
.10	9.CC	.0000000000	-.CCCCCCCCCC
.10	10.CC	.0000000000	-.CCCCCCCCCC
.20	1.CC	.0046806774	.0042313273
.20	2.CC	.0001129+5	.CCCCC9C641
.20	3.CC	.0000000273	.CCCCCCC162
.20	4.CC	.0000000001	.CCCCCCCCCC
.20	5.CC	.0000000000	.CCCCCCCCCC
.20	6.CC	.0000000000	-.CCCCCCCCCC
.20	7.CC	.0000000000	-.CCCCCCCCCC
.20	8.CC	.0000000000	-.CCCCCCCCCC
.20	9.CC	.0000000000	-.CCCCCCCCCC
.20	10.CC	.0000000000	-.CCCCCCCCCC
.30	1.CC	.0072C329C3	.0069719736
.30	2.CC	.0000173816	.CCCC15C596
.30	3.CC	.0000000419	.CCCCCCC276
.30	4.CC	.0000000001	.CCCCCCCCCC
.30	5.CC	.0000000000	.CCCCCCCCCC
.30	6.CC	.0000000000	-.CCCCCCCCCC
.30	7.CC	.0000000000	-.CCCCCCCCCC
.30	8.CC	.0000000000	-.CCCCCCCCCC
.30	9.CC	.0000000000	-.CCCCCCCCCC
.30	10.CC	.0000000000	-.CCCCCCCCCC
.40	1.CC	.0119789CC	.0120428499
.40	2.CC	.0000289C52	.CCCC261263
.40	3.CC	.0000000697	.CCCCCCC485
.40	4.CC	.0000000002	.CCCCCCCCC1
.40	5.CC	.0000000000	.CCCCCCCCCC

.40	6.0E	.0000000000	-.0000000000
.40	7.0E	.0000000000	-.0000000000
.40	8.0E	.0000000000	-.0000000000
.40	9.0E	.0000000000	-.0000000000
.40	10.0E	.0000000000	-.0000000000
.50	1.0E	.2207954145	.0212239264
.50	2.0E	.0000501794	.0000461215
.50	3.0E	.0000001211	.0000000860
.50	4.0E	.0000000003	.0000000001
.50	5.0E	.0000000000	.0000000000
.50	6.0E	.0000000000	-.0000000000
.50	7.0E	.0000000000	-.0000000000
.50	8.0E	.0000000000	-.0000000000
.50	9.0E	.0000000000	-.0000000000
.50	10.0E	.0000000000	-.0000000000
.60	1.0E	.3695330C0	.0377858308
.60	2.0E	.0000891665	.0000821290
.60	3.0E	.0000002152	.0000001532
.60	4.0E	.0000000005	.0000000002
.60	5.0E	.0000000000	.0000000000
.60	6.0E	.0000000000	-.0000000000
.60	7.0E	.0000000000	-.0000000000
.60	8.0E	.0000000000	-.0000000000
.60	9.0E	.0000000000	-.0000000000
.60	10.0E	.0000000000	-.0000000000
.70	1.0E	.2665014841	.0676941239
.70	2.0E	.0001604684	.0001470627
.70	3.0E	.0000003872	.0000002739
.70	4.0E	.0000000009	.0000000004
.70	5.0E	.0000000000	.0000000000
.70	6.0E	.0000000000	-.0000000000
.70	7.0E	.0000000000	-.0000000000
.70	8.0E	.0000000000	-.0000000000
.70	9.0E	.0000000000	-.0000000000
.70	10.0E	.0000000000	-.0000000000
.80	1.0E	.1205016830	.1218260794
.80	2.0E	.0002907712	.0002644609
.80	3.0E	.0000007016	.0000004915
.80	4.0E	.0000000017	.0000000007
.80	5.0E	.0000000000	.0000000000
.80	6.0E	.0000000000	-.0000000000
.80	7.0E	.0000000000	-.0000000000
.80	8.0E	.0000000000	-.0000000000
.80	9.0E	.0000000000	-.0000000000
.80	10.0E	.0000000000	-.0000000000
.90	1.0E	.2191695164	.2200447108
.90	2.0E	.0005288572	.0004773031
.90	3.0E	.0000012761	.0000008850

.90	4.00	.0000000031	.0000000013
.90	5.00	.0000000000	.0000000000
.90	6.00	.0000000000	-.0000000000
.90	7.00	.0000000000	-.0000000000
.90	8.00	.0000000000	-.0000000000
.90	9.00	.0000000000	-.0000000000
.90	10.00	.0000000000	-.0000000000
1.00	1.00	.3994424675	.3986897001
1.00	2.00	.0009638567	.0008642328
1.00	3.00	.0000023258	.0000015993
1.00	4.00	.0000000056	.0000000024
1.00	5.00	.0000000000	.0000000000
1.00	6.00	.0000000000	-.0000000000
1.00	7.00	.0000000000	-.0000000000
1.00	8.00	.0000000000	-.0000000000
1.00	9.00	.0000000000	-.0000000000
1.00	10.00	.0000000000	-.0000000000

```
 EXT. FORTRAN IV, VERSION DOC
      1.          COMMON /A/R,Z,ALPHA,BETA/AAA/ES
      2.            COMMON/BR/RRR
      3.     2    J = 1
      4.     6      READ(105,15) R,RRR
      5.    15      FORMAT(2F10.5)
      6.            IF(R.GT.12.) GO TO 200
      7.     4      Z = 1.
      8.     1    W = Z*R
      9.            A = EXP(-W)
     10.            B = EXP(-W/2.)
     11.            C = EXP(W/2.)
     12.            D = EXP(-2.*W)
     13.            AK = A*(1. + W)
     14.            AJ = -1./R + D/R*(1.+W)
     15.           S22 = A*(1. + W + W*W/3. )
     16.           S11 = 2. + 2.*S22
     17.            S12 = 2.*A/W*(C*(3.*W-3.) +B*(W+3.))
     18.           EZERO = 2*Z/2. -Z
     19.            SH = .5*(C-B)
     20.             CH = .5*(C+B)
     21.             AKA = A*(W*W -W -1.)
     22.            H11 = 2.*EZERO*(1.+2.*AK) + 2.*AJ - Z*Z*S22
     23.             H12 = 2.*AK*SH/R-(Z-4.)
     24.            H22 = AK*(EZERO-Z)
     25.            PS11Z = -2.*W*W*AK/(3.*Z)
     26.            PS12Z=-3.*AK*B*(C*C*(W-2.) + W + 2.)/(Z*Z)
     27.            PS22Z = -W*W*AK/(3.*Z)
     28.           PEQZ = Z-1.
     29.            PAKZ = -W*R*A
     30.           PAJZ = -D*(1. +2.*W)
     31.            SPAKZ = R*R*A*(W-1.)
     32.            SPAJZ = -2.*R*(PAJZ+D)
     33.           PH11Z=2.*PEQZ*(1.+2.*AK)+4.*EZERO*PAKZ+2.*PAJZ-2.*Z*S22-Z*Z*PS22Z
     34.           PH12Z=2.*SH*(PAKZ*(Z-4.)+AK)/R+AK*CH*(Z-4.)
     35.            PH22Z=PAKZ*(EZERO-Z)+AK*(PEQZ-1.)
     36.             SPS11Z = 2.*W*W*AKA/(3.*Z*Z)
     37.           SPS12Z=3.*A/(2.*Z*Z*W)*(C*(W*W*W-W*W-4.*W-8.)+B*(3.*W*W*W + 9.*W*W
     38.          1+12.*W+8.))
     39.             SPS22Z =  W*W*AKA/(3.*Z*Z)
     40.           SPH11Z=2.*(1.+2.*AK)+8.*PEQZ*PAKZ+4.*EZERO*SPAKZ+2.*SPAJZ-2.*S22
     41.          1-4.*Z*PS22Z-Z*Z*SPS22Z
     42.            SPH12Z=2.*SH*(SPAKZ*(Z-4.)+2.*PAKZ)/R+2.*CH*(PAKZ*(  Z-4.)+AK)
     43.          1 +.5*R*SH*AK*(Z-4.)
     44.           SPH22Z=SPAKZ*(EZERO-Z) + 2.*PAKZ*(Z-2.) +AK
     45.             AA = S11*S22 - S12*S12
     46.            BB = 2.*S12*H12 - S11*H22 - H11*S22
     47.             CC = H11*H22 - H12*H12
     48.             DISC = BB*BB-4.*AA*CC
     49.             SQRTDI = SQRT(DISC)
```

```
      PAAZ = S11*PS22Z + S22*PS11Z - 2.*S12*PS12Z
     PBBZ=2.*S12*PH12Z+2.*H12*PS12Z-S11*PH22Z-H22*PS11Z-H11*PS22Z-S22*
    1PH11Z
      PCCZ = H11*PH22Z +H22*PH11Z -2.*H12*PH12Z
     SPAAZ= S11*SPS22Z+S22*SPS11Z-2.*S12*SPS12Z+2.*PS11Z*PS22Z-2.*PS12
    1Z*PS12Z
     SPBBZ = 2.*S12*SPH12Z+4.*PS12Z*PH12Z+2.*H12*SPS12Z-S11*SPH22Z-2.*
    1 PH22Z*PS11Z-H22*SPS11Z-H11*SPS22Z-2.*PS22Z*PH11Z-S22*SPH11Z
     SPCCZ=H11*SPH22Z+H22*SPH11Z-2.*H12*SPH12Z+2.*PH11Z*PH22Z-2.*PH12Z
    1*PH12Z
      PDISCZ = 2.*BB*PBBZ - 4.*AA*PCCZ - 4.*CC*PAAZ
      ES = .5/AA*(-BB-SQRTDI)
     SPDISZ = 2.*(PBBZ*PBBZ + BB*SPBBZ) - 4.*(2.*PAAZ*PCCZ+AA*SPCCZ+
    1CC*SPAAZ)
      PSQTDI = .5*PDISCZ/SQRTDI
     SPQTDI = -.25*PDISCZ*PDISCZ/(DISC*SQRTDI) +.5*SPDISZ/SQRTDI
     PESZ = -.5*(PBBZ+PSQTDI)/AA -PAAZ*ES/AA
      SPESZ = -.5*(SPBBZ +SPQTDI)/AA-1./AA*(2.*PAAZ*PESZ+SPAAZ*ES)
      Z = Z-PESZ/SPESZ
      IF(ABS(PESZ)-.00CCCOC1) 100,100,5
5     J = J + 1
      IF(J-1000) 1,100,100
100   TOTAL= ES + 1./R
      RON=(H11-S11*ES)/(S12*ES-H12)
      ALPHA = 1./SQRT(S11+2.*S12*RON + S22*RON*RON)
     BETA = RON*ALPHA
     CALL ENERGY
      GO TO 2
200    CALL EXIT
       END
```

```
      SUBROUTINE ENERGY
      REAL*4 N
      REAL*4 INT1A,INT2A,INT3A,INT4A,INT5A,INT6A,INT7A,INT8A,INT9A
      REAL*4 INT10A,INT11A,INT12A
      REAL*4 INT1,INT2,INT3,INT4,INT5,INT6,INT7,INT8,INT9
      REAL*4 INTA,INTB,INTC,INTD,INTE,INTF
      COMMON /AAA/EZERO
      COMMON /ABC/G/ABB/COSG/ABD/COSEC/BBC/COSEC2/DEF/COSEC3/BASH/W1
      COMMON /A/R,Z,ALPHA,BETA/B/EP/C/EX/D/AL2/E/G1/F/G2
      COMMON/ZOR/ERR
      DARCS(X) = LOG( X + SQRT(X*X-1.) )
      EQ = EZERO + .5*Z*Z
      EP = EQ
       Z82 = -.5*Z*Z
      AL2 = LOG(2.)
      COSG = .5*BETA/ALPHA
      IF(COSG.LE.1.) G = ARCOS(COSG)
      IF(COSG.LE.1.) E = 1. - COSG*COSG
      IF(COSG.GT.1.0) G = DARCS(COSG)
      IF(COSG.GT.1.0) E = COSG*COSG - 1.
      SI = SQRT(E)
      COSEC = 1./SI
        COSEC2 = COSEC*COSEC
        COSEC3 = COSEC2*COSEC
      EX = EXPI(R,Z)
      C1 = (2. - Z )/Z + EP/(Z*Z)
      C2 = EP*R**.5/Z
      W1 = Z*R
      G1 = EXP(-W1)
       G2 = EXP(2.*W1)
      G3 = COSH(W1)
      F1 = FINT1A(R,Z)
      F2 = FINT1B(R,Z)
      F3 = PSISQU(1.,R,Z,ALPHA,BETA)
      F4 = SINT2A(R,Z)
      F5 = SINT2B(R,Z)
      FT1B = SINT2D(1.,R,Z,ALPHA,BETA)
      CALL DINR(1.,R,Z,ALPHA,BETA,ANS)
      FT2 = ANS
      F8 = F4
      F9 = F5
      F10 = -2.*FT1B/(Z*Z)
      IF(COSG.LE.1.) SUM6 = SERIE1(1.,R,Z,ALPHA,BETA)
      IF(COSG.GT.1.) SUM6 = SERIE2(1.,R,Z,ALPHA,BETA)
      SERIES = SUM6
      FUNCT2 = F3
      FUNCT4 =    FT2
       F11 = G1/W1
       FUNCT6 = F10
      API = 3.1415926535897932384
      A1 = .25*Z*Z*Z*R*R*R
```

```
      FUNCT9 = FUNCT4
      VLK =  (2.*Z - 4.)/R
      AREA3 = GAUSS(0.,1.,15,3)
      AREA6 = GAUSS(0.,1.,15,4)
      AREA9 = GAUSS(0.,1.,15,5)
      AREAA = GAUSS4(0.,1.,15,8)
      AREAB = GAUSS4(0.,1.,15,9)
      INT1 = 2.*VLK*A1*F3*(C1*F1 + C2*F2)
      INT2 = 2.*A1*VLK*F9*AREA3
      INT3 = 2.*VLK*A1*F9*F3
      INT4 = 2.*A1*FT2*(C1*F4 + C2*F5)
      INT5 = 2.*A1*F11*AREA6
      INT6 = 2.*A1*F11*FUNCT9
      INT7 = 2.*A1*(F3*AREAA - F10*AREAB)
      INT8 = 2.*A1*(F2*AREA3 - F11*AREA9)
      INT9 = 1.
      FZERO = -INT7 - INT8
      ETH0 = INT1 + INT2 + INT4 + INT5 + FZERO*(INT3 + INT6 )
       AREA4 = GAUSS(0.,1.,15,1)
      SUM4 = AREA4
       AREA5 = GAUSS(0.,1.,15,2)
      SUM5 = AREA5
       AREA7 = GAUSS4(0.,1.,15,5)
       AREA8 = GAUSS4(0.,1.,15,6)
        INTA = A1*(AREA7*FUNCT2 - AREA8*FUNCT6)*2.
      INTB = A1*(F2*SUM4 - F11*SUM5)*2.
      INTC = FZERO*FZERO*INT9
       INTD = 4.*A1*( AREAA*AREA3 - AREAB*AREA9)
      INTE = 2.*FZERO*INT7
      INTF = 2.*FZERO*INT8
      ANORM1= INTA+INTB+INTC+INTD+INTE+INTF  + INT9
       ANORM = SQRT(ANORM1)
      E2 = ETH0/ANORM1
       AREA11 = GAUSS4(0.,1.,15,7)
       AREA12 = GAUSS(0.,1.,15,9)
      INT1A = 2.*A1*(VLK*F3*AREA11 + AREA8*FT2)
      INT2A = 2.*A1*(VLK*F9*AREA4 + F11*AREA12)
      INT3A = FZERO*FZERO*(INT3 + INT6)
      INT4A=4.*A1*( VLK*AREA3*(C1*F1+C2*F2)+ AREA6*(C1*F4+C2*F5)  )
      INT5A = 2.*FZERO*(INT2+INT5)
      INT6A = 2.*FZERO*(INT1 + INT4)
      ETHREE=(INT1A+INT2A+INT3A+INT4A+INT5A+INT6A-EG*(ANORM1-INT9))
       E3 = ETHREE/ANORM1
      RES1 = EZERO + E2
       RES2 = RES1 + E3
       WRITE(10,906) R
  906  FORMAT(1X//'R = ',E8.4)
       WRITE(10,902) Z02,EZERO
  902  FORMAT(1X,'E(0)=-Z*Z/2=',F16.12,5X,'E1=EGM=',F16.12)
```

```
      WRITE(10?,903)   EP
  903  FORMAT(1X,'E(1) = EGM + Z*Z/2 = ',F16.12)
      WRITE(10?,904) E2,RFS1
  904  FORMAT(1X,'E2 = ',F16.12,5X,'E(2) = ',F16.12)
      WRITE(10?,905) E3,RFS2
  905  FORMAT(1X,'E3 = ',F16.12,5X,'E(3) = ',F16.12,/)
       IF(R.EQ.RRR) GO TO 3
        GO TO 31
    3  AN = SQRT(Z*Z*Z/API)
       WRITE(108,109)
  109  FORMAT(1X,'MU',5X,'LAMBDA',14X,'PHI GM',14X,'PHI(1)')
       DO 12 K = 0,10
       U = .1*K
       IF(COSG.LE.1.0) SERI = SERIE1(U,R,Z,ALPHA,BETA)
       IF(COSG.GT.1.0) SERI = SERIE2(U,R,Z,ALPHA,BETA)
       DO 12 NA = 1,10
       AL = 1.*NA
       A = .5*Z*R*AL
       E = EXP(-A)
       AA = 1. + AL
       ALA = C1*LOG(AA) + C2*AL
       F1 = E*AN*(2.*ALPHA*COSH(.5*Z*R*U) + BETA)
       F2 = SERI + FZERO + ALA
       TOTAL = F1*(1.0 + F2)/ANORM
   12  WRITE(10?,108) U,AL,F1,TOTAL
  108  FORMAT(1X,F4.2,5X,F5.2,7X,F16.10,5X,F16.10)
   31   RETURN
       END
```

APPENDIX C, PART 1: INTEGRAL REPPESENTATION LISTING 148

```
      1.            FUNCTION FUN*(U,R,Z)
      2.             COMMON /B/ EP
      3.            C1 = (2.-Z)/Z + EP/(Z*Z)
      4.            C2 = .5*EP*R/Z
      5.             U4 = U*U*U*U
      6.            F = 1. + 1./U
      7.            F1 = LOG(F)
      8.           F2 = C1*F1 + C2/U
      9.            X = Z*R/U
     10.            Q = EXP(-X)
     11.            FUN4 = Q*F2/U4
     12.            RETURN
     13.              END
```

```
      1.            FUNCTION FUN5(U,R,Z)
      2.              A = FUN4(U,R,Z)
      3.           FUN5 = A*U*U
      4.           RETURN
      5.             END
```

```
APPENDIX C, PART 1: INTEGRAL REPRESENTATION LISTING  150
      1.          FUNCTION GAUSS4(A,B,*,K)
      2.          COMMON /A/ R,Z,ALPHA,BETA
      3.         DIMENSION NPOINT(7),KEY(8),T(24),WEIGHT(24)
      4.         DATA NPOINT / 2,3,4,5,6,10,15 /
      5.         DATA KEY / 1,2,4,6,9,12,17,25 /
      6.         DATA T / .57735026918962576,0.0,0.774596669,0.339981043,0.861136312,
      7.        1 0.0,0.538469310,0.906179846,0.238619186,0.661209386,0.932469514,
      8.        2 0.148874338,0.433395394,0.679409568,0.865063367,0.973906529,0.0,
      9.        3 0.201194093,0.394151347,0.770637,0.570721726,0.8539,0.72441773136017
     10.        4 0,0.842048833410427,0.937273392400706,0.987992518020485 /
     11.          DATA WEIGHT / 1.0,0.888888889,0.555555556,0.652145155,0.347854845,
     12.        1 0.568888889,0.478628671,0.236926885,0.467913935,0.360761573,
     13.        2 0.171324492,0.295524925,0.269266719,0.219086363,0.149451349,
     14.        3 0.066671344,0.202578241925561,0.198431485327111,0.186161000115562,
     15.        4 0.160228928168994,0.138570677926154,0.107159220467172,0.703660474881O
     16.        58,0.030753241996117 /
     17.          SUM = 0.0
     18.           I = 1
     19.    3     IF(M.EQ.NPOINT(I)) GO TO 2
     20.          I = I+1
     21.          IF(I.GT.7) GO TO 4
     22.           GO TO 3
     23.    4      GAUSS4 = 0.0
     24.          RETURN
     25.    2     JFIRST = KEY(I)
     26.          JLAST = KEY(I+1) - 1
     27.           C = .5*(B-A)
     28.          D = .5*(B+A)
     29.          GO TO (1,8,12,21,23,25,27,29,31),K
     30.    1     DO 5 J = JFIRST,JLAST
     31.          IF(T(J).EQ.0.0) SUM = SUM + WEIGHT(J)*H0PE(D,R,Z,ALPHA,BETA)
     32.    5     IF(T(J).NE.0.0) SUM = SUM+WEIGHT(J)*(H0PE(T(J)*C+D,R,Z,ALPHA,BETA)
     33.        1 + H0PE( -T(J)*C+D,R,Z,ALPHA,BETA))
     34.           GO TO 50
     35.    8     DO 9 J = JFIRST,JLAST
     36.          IF(T(J).EQ.0.0) SUM = SUM + WEIGHT(J)*F0FX(D,R,Z,ALPHA,BETA)
     37.    9     IF(T(J).NE.0.0) SUM = SUM+WEIGHT(J)*(F0FX(T(J)*C+D,R,Z,ALPHA,BETA)
     38.        1 + F0FX( -T(J)*C+D,R,Z,ALPHA,BETA))
     39.           GO TO 50
     40.   12     DO 13 J = JFIRST,JLAST
     41.          IF(T(J).EQ.0.0) SUM = SUM + WEIGHT(J)*F0FY(D,R,Z,ALPHA,BETA)
     42.   13     IF(T(J).NE.0.0) SUM = SUM+WEIGHT(J)*(F0FY(T(J)*C+D,R,Z,ALPHA,BETA)
     43.        1 + F0FY( -T(J)*C+D,R,Z,ALPHA,BETA))
     44.           GO TO 50
     45.   21     DO 22 J = JFIRST,JLAST
     46.          IF(T(J).EQ.0.0) SUM = SUM + WEIGHT(J)*H0PY(D,R,Z,ALPHA,BETA)
     47.   22     IF(T(J).NE.0.0) SUM = SUM+WEIGHT(J)*(H0PY(T(J)*C+D,R,Z,ALPHA,BETA)
     48.        1 + H0PY( -T(J)*C+D,R,Z,ALPHA,BETA))
     49.           GO TO 50
     50.   23     DO 24 J = JFIRST,JLAST
     51.           IF(T(J) .EQ.0.) SUM = SUM + WEIGHT (J)*FUN1(D,R,Z)
```

```
      24      IF(T(J).NE.C.) SUM =SUM+ WEIGHT(J)*(FUN1(T(J)*C+D,R,Z)+FUN1(
     .1-T(J)*C+D,R,Z))
              GO TO 50
      25     DO 26 J = JFIRST,JLAST
             IF(T(J).EQ.C.) SUM = SUM + WEIGHT(J)*FUN2(D,R,Z)
      26     IF(T(J).NE.C.) SUM = SUM+WEIGHT(J)*(FUN2(T(J)*C+D,R,Z)+FUN2(
     1-T(J)*C+D,R,Z))
             GO TO 50
      27     DO 28 J = JFIRST,JLAST
             IF(T(J).EQ.C.) SUM = SUM + WEIGHT(J)*FUN3(D,R,Z)
      28     IF(T(J).NE.C.) SUM = SUM+WEIGHT(J)*(FUN3(T(J)*C+D,R,Z)+FUN3(
     1-T(J)*C+D,R,Z) )
              GO TO 50
      29     DO 30 J = JFIRST,JLAST
             IF(T(J).EQ.C.) SUM = SUM + WEIGHT(J)*FUN4(D,R,Z)
      30     IF(T(J).NE.C.) SUM = SUM+WEIGHT(J)*(FUN4(T(J)*C+D,R,Z)+FUN4(
     1-T(J)*C+D,R,Z))
              GO TO 50
      31     DO 32 J = JFIRST,JLAST
             IF(T(J).EQ.C.) SUM = SUM + WEIGHT(J)*FUN5(D,R,Z)
      32     IF(T(J).NE.C.) SUM= SUM+WEIGHT(J)*(FUN5(T(J)*C+D,R,Z)+FUN5(
     1-T(J)*C+D,R,Z))
      50      GAUSS4 = C*SUM
              RETURN
              END
```

APPENDIX C, PART 1: INTEGRAL REPRESENTATION LISTING 152

```
      1.          FUNCTION FUNCT1(U,R,Z,A,B)
      2.           COSG = .5*B/A
      3.           IF(COSG.LE.1.) ANS = SERIE1(U,R,Z,A,B)
      4.           IF(COSG.GT.1.) ANS = SERIE2(U,R,Z,A,B)
      5.           X1 = .5*Z*R*U
      6.           Q1 = COSH(X1)
      7.          PSI = 2.**A*Q1 + B
      8.          PSI2 = PSI*PSI
      9.           F = ANS*ANS
     10.           FUNCT1 = PSI2*F
     11.          RETURN
     12.          END
```

```
      1.          FUNCTION FUNCT2(U,R,Z,A,B)
      2.          FUNCT2 = FUNCT1(U,R,Z,A,B)*U*U
      3.           RETURN
      4.           END
```

APPENDIX C, PART 1: INTEGRAL REPRESENTATION LISTING 154

```
      1.          FUNCTION FUNCT3(U,R,Z,A,B)
      2.          COSG = .5*B/A
      3.          IF(COSG.LE.1.) ANS = SERIE1(U,R,Z,A,B)
      4.          IF(COSG.GT.1.) ANS = SERIE2(U,R,Z,A,B)
      5.          PSI = 2.*A*COSH(.5*R*Z*U) + B
      6.            FUNCT3 = PSI*PSI*ANS
      7.          RETURN
      8.          END
```

```
      1.          FUNCTION FUNCT4(U,R,Z,A,B)
      2.          COSG = .5*B/A
      3.          PSI =  2.*A*COSH(.5*R*Z*U) + B
      4.          IF(COSG.LE.1.) ANS = SERIE1(U,R,Z,A,B)
      5.          IF(COSG.GT.1.) ANS = SERIE2(U,R,Z,A,B)
      6.         F1 = FUNVMU(U,R,Z,A,B)
      7.          FUNCT4 = F1*PSI*PSI*ANS
      8.          RETURN
      9.          END
```

APPENDIX C, PART 1: INTEGRAL REPRESENTATION LISTING 156

```
      1.          FUNCTION SERIE1(U,R,Z,A,B)
      2.        SERIE1 = GAUSS1(C,,U,15,7)
      3.          RETURN
      4.        END
```

```
      1•            FUNCTION SERIE2(C,R,Z,A,B)
      2•            SERIE2 = GAUSS1(C,,U,15,7)
      3•              RETURN
      4•            END
```

APPENDIX C, PART II: INTEGRAL REPRESENTATION LISTING 15R

```
      1.            FUNCTION DRZ(U,R,Z,A,B)
      2.              REAL*4 I
      3.              COMMON /B/ EP
      4.             IF(U.EQ.1.0)  GO TO 31
      5.            AA = A*A
      6.            AB = A*B
      7.            BB = B*B
      8.            Z2 = Z*Z
      9.             I = EP/Z2*(1.+2.**W+2.**W*W) + (2.-Z)*(1.+Z*R)/Z
     10.             W = .5*Z*R
     11.             P = 1./(W*W)
     12.             A1 = Z*R
     13.             T1 = 2.**AA*(-Z2*(I+1.)+EP)/((Z**3)*R)
     14.            T2 = AA*EP*R/Z
     15.             T3 = 4.**AB*(Z2*(-2.*I+.25*Z2*R*R-4.)+8.*EP)/(R*(Z**3))
     16.            T4 = 4.*AB*R/Z*(EP-.25*Z2)
     17.            T5 = -I*(2.*AA+BB)+Z2*R*R*BB**.25
     18.            T6 = R*R*(-.5*Z2*BB+EP*(2.*AA+BB))*.166666666666666666
     19.             T7 = 2.**AA/Z2*(Z2-EP)
     20.            T8 =   8.*AB/Z2*(Z2-2.*EP)
     21.             G1 =  COSH(U*W)
     22.             G1A =  COSH(W)
     23.             G2 =  COSH(U*W1)
     24.             G2A =  COSH(W1)
     25.             G3 =  SINH(U*W)
     26.             G3A =  SINH(W)
     27.             G4 =  SINH(U*W1)
     28.             G4A =  SINH(W1)
     29.             G5 = (2.*A*G1 + B)**2
     30.             G5A = (2.*A*G1A + B)**2
     31.             G6 = 1.    - U*U
     32.            DMDU = G4*(T1+U*U*T2)+ G3*(T3+U*U*T4) + U*(T5+U*U*T6) + U*T7*G2
     33.           1  + U*T8*G1
     34.             A4K = G4A*(T1+T2) + G3A*(T3+T4) + T5+T6 +T7*G2A + T8*G1A
     35.             DRZ = DMDU/(G6*G5) - U*A4K/(G6*G5A)
     36.              RETURN
     37.     31      DRZ = 0.
     38.              RETURN
     39.             END
```

```
      FUNCTION GAUSS1(A,B,M,K)
      COMMON /A/ R,Z,ALPHA,BETA
      DIMENSION NPOINT(7),KEY(8),T(24),WEIGHT(24)
      DATA NPOINT / 2,3,4,5,6,10,15 /
      DATA KEY / 1,2,4,6,9,12,17,25 /
      DATA T / 0.57735026918962576,0.0,0.77459666924148337,0.33998104358485626,0.86113631159405257,
     1 0.0,0.53846931010568309,0.90617984593866399,0.23861918608319690,0.66120938646626451,0.93246951420315205,
     2 0.14887433898163121,0.43395394849921755,0.67940956829902441,0.86506336668898451,0.97390652851717172,0.0,
     3 0.20119409399743452,0.39415134707756337,0.57097217260853884,0.72441773136017004,
     4 0.84820658341042721,0.93727339240070590,0.98799251802048543 /
      DATA WEIGHT / 1.0,0.88888889,0.55555556,0.65214515,0.34785485,
     1 0.56888889,0.47862867,0.23692689,0.46791393,0.36076157,
     2 0.17132449,0.29552422,0.26926672,0.21908636,0.14945135,
     3 0.06671344,0.20257824192556127,0.19843148532711157,0.18616100011556294,
     4 0.16627620526816942,0.13957067792615431,0.10715922046717194,0.70366047488108,
     5 0.03075324199611727 /
        SUM = 0.
        I = 1
    3   IF(M.EQ.NPOINT(I)) GO TO 2
        I = I+1
        IF(I.GT.7) GO TO 4
        GO TO 3
    4     GAUSS1 = 0.
          RETURN
    2   JFIRST = KEY(I)
        JLAST = KEY(I+1) - 1
        C = .5*(B-A)
        D = .5*(B+A)
        IF(K.GT.1) GO TO 1
        GO TO 50
    1   DO 5 J = JFIRST,JLAST
        IF(T(J).EQ.0.)    SUM = SUM + WEIGHT(J)*QRZ(D,R,Z,ALPHA,BETA)
    5   IF(T(J).NE.0.)    SUM = SUM+WEIGHT(J)*(QRZ(T(J)*C+D,R,Z,ALPHA,BETA)
     1 + QRZ(-T(J)*C+D,R,Z,ALPHA,BETA))
          GO TO 50
   50     GAUSS1 = C*SUM
          RETURN
        END
```

APPENDIX C, PART 1: INTEGRAL REPRESENTATION LISTING 160

```
      1.          FUNCTION FUNCT5(U,R,Z,A,B)
      2.      .    F1 = FUNCT3(U,R,Z,A,B)
      3.          FUNCT5 = F1*U*U
      4.          RETURN
      5.          END
```

```
      1.            FUNCTION FUNVMU(U,R,Z,A,B)
      2.             W = .5*Z*R
      3.            G1 = COSH(U*W)
      4.            G2 = SINH(U*W)
      5.            F1 = Z*Z*R*(1. - U*U)**.5
      6.             F2 = 4.*A*Z*U*G2/R
      7.           PSI = 2.*A*G1 + B
      8.             FUNVMU = (F1 + F2 )/PSI
      9.            RETURN
     10.            END
```

APPENDIX C, PART 1: INTEGRAL REPRESENTATION LISTING 162

```
      1.          FUNCTION GAUSS(A,B,N,K)
      2.           COMMON /A/ R,Z,ALPHA,BETA
      3.          DIMENSION NPOINT(7),KEY(8),T(24),WEIGHT(24)
      4.          DATA NPOINT / 2,3,4,5,6,10,15 /
      5.          DATA KEY / 1,2,4,6,9,12,17,25 /
      6.          DATA T / .577350269,0.0,.774596669,.339981044,.861136312,
      7.         1.0,.538469310,.906179846,.238619186,.661209387,.932469514,
      8.         2.148874339,.433395394,.679409568,.865063367,.973906529,0.0,
      9.         3.201194093997435,.394151347,.770563,.570721726,.8539,.724417731360170
     10.         4.,.848206583410427,.937273392400706,.987992518020485 /
     11.          DATA WEIGHT / 1.0,0.,.888888889,0.,.555555556,0.,.652145155,.347854845,
     12.         1.0,.568888889,0.,.478628671,0.,.236926885,0.,.467913935,0.,.360761573,
     13.         2.0,.171324492,0.,.295524225,0.,.269266719,0.,.219086363,0.,.149451349,
     14.         3.0,.066713443,.202578241925561,.198431485327111,.186161000115562,
     15.         4.,.166269205816994,.139570677926154,.107159220467172,.070366047488108
     16.         5.,.030753241996117 /
     17.           SUM = 0.
     18.            I = 1
     19.     3     IF(N.EQ.NPOINT(I)) GO TO 2
     20.           I = I+1
     21.           IF(I.GT.7) GO TO 4
     22.            GO TO 3
     23.     4     GAUSS = 0.
     24.           RETURN
     25.     2     JFIRST = KEY(I)
     26.           JLAST = KEY(I+1) - 1
     27.            C = .5*(B-A)
     28.           D = .5*(B+A)
     29.           GO TO (1,8,6,10,12,14),K
     30.     1     DO 5 J = JFIRST,JLAST
     31.            IF(T(J).EQ.0.) SUM = SUM + WEIGHT(J)*FUNCT1(D,R,Z,ALPHA,BETA)
     32.     5      IF(T(J).NE.0.) SUM = SUM+WEIGHT(J)*(FUNCT1(T(J)*C+D,R,Z,ALPHA,
     33.         1BETA)   + FUNCT1(-T(J)*C+D,R,Z,ALPHA,BETA))
     34.            GO TO 50
     35.     8     DO 9 J = JFIRST,JLAST
     36.            IF(T(J).EQ.0.) SUM = SUM + WEIGHT(J)*FUNCT2(D,R,Z,ALPHA,BETA)
     37.     9      IF(T(J).NE.0.) SUM = SUM+WEIGHT(J)*(FUNCT2(T(J)*C+D,R,Z,ALPHA,
     38.         1BETA)   + FUNCT2(-T(J)*C+D,R,Z,ALPHA,BETA))
     39.            GO TO 50
     40.     6     DO 7 J = JFIRST,JLAST
     41.            IF(T(J).EQ.0.) SUM = SUM + WEIGHT(J)*FUNCT3(D,R,Z,ALPHA,BETA)
     42.     7      IF(T(J).NE.0.) SUM = SUM+WEIGHT(J)*(FUNCT3(T(J)*C+D,R,Z,ALPHA,
     43.         1BETA)   + FUNCT3(-T(J)*C+D,R,Z,ALPHA,BETA))
     44.            GO TO 50
     45.     10    DO 11 J = JFIRST,JLAST
     46.            IF(T(J).EQ.0.)  SUM = SUM + WEIGHT(J)*FUNCT4(D,R,Z,ALPHA,BETA)
     47.     11     IF(T(J).NE.0.) SUM = SUM+WEIGHT(J)*(FUNCT4(T(J)*C+D,R,Z,ALPHA,
     48.         1BETA)   + FUNCT4(-T(J)*C+D,R,Z,ALPHA,BETA))
     49.            GO TO 50
     50.     12    DO 13 J = JFIRST,JLAST
     51.            IF(T(J).EQ.0.)  SUM = SUM + WEIGHT(J)*FUNCT5(D,R,Z,ALPHA,BETA)
```

```
      52.     13     IF(T(J).NE.C.) SUM = SUM+WEIGHT(J)*(FUNCT5(T(J)*C+D,R,Z,ALPHA,
     53.          1BETA)    + FUNCT5(-T(J)*C+D,R,Z,ALPHA,BETA))
      54.            GO TO 50
      55.     14     DO 15 J = JFIRST,JLAST
      56.            IF(T(J).EQ.C.) SUM = SUM + WEIGHT(J)*FUNCI2(D,R,Z,ALPHA,BETA)
      57.     15     IF(T(J).NE.C.) SUM = SUM + WEIGHT(J)*(FUNCI2(T(J)*C+D,R,Z,ALPHA,
      58.          1BETA)    + FUNCI2(-T(J)*C+D,R,Z,ALPHA,BETA))
      59.            GO TO 50
      60.     50     GAUSS = C*SUM
      61.            RETURN
      62.            END
```

APPENDIX C, PART 1: INTEGRAL REPRESENTATION LISTING 164

```
      1.            FUNCTION FUN1(U,R,Z)
      2.           COMMON /R/ EP
      3.            C1 = (2.-Z)/Z + EP/(Z*Z)
      4.            C2 = .5*EP*R/Z
      5.             U4 = U*U*U*U
      6.            F= 1. + 1./U
      7.            F1 = LOG(F)
      8.           F2 = C1*F1 + C2/U
      9.            A = Z*R/2
     10.            Q = EXP(-A)
     11.            FF = F2*F2
     12.            FUN1 = Q*FF/U4
     13.            RETURN
     14.             END
```

```
      1.            FUNCTION FUN2(U,R,Z)
      2.            A = FUN1(U,R,Z)
      3.             FUN2 = A*U*U
      4.            RETURN
      5.            END
```

```
      1.          FUNCTION FUN3(U,R,Z)
      2.            FUN3 = FUN1(U,R,Z)*U
      3.          RETURN
      4.          END
```

```
      1.          FUNCTION FUNCI2(U,R,Z,A,B)
      2.          F1 = FUNCT1(U,R,Z,A,B)
      3.          F2 = FUNCT2(U,R,Z,A,B)
      4.          FUNCI2 = F1*F2
      5.           RETURN
      6.           END
```

APPENDIX C, PART 1: INTEGRAL REPRESENTATION LISTING 168

```
      1.          FUNCTION RESY(U,R,Z,A,B)
      2.         COMMON /ABC/G/ABD/COSEC
      3.          DATANH(X) = .5*LOG(  (1.+X)/(1.-X)  )
      4.          COSG = .5*B/A
      5.          G2 = .5*G
      6.          F1 = TANH(G2)
      7.          X = .25*Z*R*U
      8.          F2 = TANH(X)
      9.         F3 = F1*F2
     10.           F4 = DATANH(F3)
     11.           W = .5*R*Z
     12.          RESY = 2.*U*COSEC/W*F4
     13.          RETURN
     14.          END
```

```
      1.            FUNCTION HAPY(U,R,Z,A,B)
      2.            COMMON /ABC/G/ABB/COSG/ABD/COSEC
      3.           DATANH(X) = .5*LOG(  (1.+X)/(1.-X)  )
      4.           AA = A*A
      5.           AB = A*B
      6.           BB = B*B
      7.            ZP = Z*Z
      8.           W = .5*R*Z
      9.            W1 = .25*Z*R
     10.           F1 = TANH(U*W1)
     11.           G2 = .5*G
     12.           F2 = TANH(G2)
     13.            F3 = F1*F2
     14.            F4 = DATANH(F3)
     15.           HAPY = F4
     16.            RETURN
     17.             END
```

APPENDIX C, PART 1: INTEGRAL REPRESENTATION LISTING 170

```
      1.          FUNCTION RESX(U,L,Z,A,B)
      2.          COMMON /ABC/G/ABD/COSEC
      3.           COSG = .5*B/A
      4.           G2 = .5*G
      5.          F1 = TAN(G2)
      6.          X = .25*7*R*U
      7.          F2 = TANH(X)
      8.           F3 = F1*F2
      9.          F4 = ATAN(F3)
     10.          R = .5*R*7
     11.          RESX= 2.*L*COSEC/A*F4
     12.           RETURN
     13.            END
```

```
      1.            FUNCTION FOFY(U,R,Z,A,B)
      2.           COSG = .5*B/A
      3.           X1 = .5*R*Z*U
      4.           G1 = COSH(X1)
      5.           F1 = G1 + COSG
      6.             FOFY = U/F1
      7.           RETURN
      8.           END
```

```
      1.            FUNCTION HBPE(U,R,Z,A,B)
      2.            COMMON /ABC/G/ABB/COSG/ABD/COSEC
      3.           AA = A*A
      4.           AB = A*B
      5.           BB = B*B
      6.           ZZ = Z*Z
      7.           R = .5*R*Z
      8.            W1 = .25*Z*R
      9.           F1 = TAN(U*1)
     10.            G2 = .5*G
     11.           F2 = TAN(G2)
     12.           F3 = F1*F2
     13.           F4 = ATAN(F3)
     14.             HBPE = F4
     15.           RETURN
     16.           END
```

```
      1.          FUNCTION VAL(U,R,Z,A,B)
      2.           COMMON /ABC/G/ABB/COSG/ARD/COSEC
      3.          COMMON /BBC/COSEC2/DEF/CHSEC3
      4.          GP = .5*R
      5.             ZZ = 2.*Z
      6.          AA = A*A
      7.          AB = A*B
      8.          BB = B*B
      9.          X = .5*Z*R
     10.          X1 = U*W
     11.          X2 = .5*U*W
     12.          Q1 = COSH(X1)
     13.          Q2 = SINH(X1)
     14.         F1 = Q1 + COSG
     15.          F2 = LOG(F1)
     16.          W2 = 1./(W*W)
     17.          FUN1 = -COSEC2*F2*W2
     18.         FUN2 = U*COSEC2*Q2/W/F1
     19.             VAL = FUN1 + FUN2
     20.          RETURN
     21.          END
```

APPENDIX C, PART 1: INTEGRAL REPRESENTATION LISTING 174

```
      1.            FUNCTION FEFX(U,R,Z,A,B)
      2.           COSG = .5*B/A
      3.           X1 = .5*R*Z*L
      4.           G1 = COSH(X1)
      5.            F1 = G1 + COSG
      6.              F2 = F1*F1
      7.           FEFX = U/F2
      8.            RETURN
      9.            END
```

```
      1.            FUNCTION FINT1A(R,Z)
      2.            COMMON /C/EX/D/AL2/E/G1/F/Q2
      3.            W = Z**2
      4.            W1 = 1./W
      5.            FINT1A = C1*W1*(W1+(1.+W1)*AL2-Q2*EX*(1.-(2.*W-1.)*W1))
      6.            RETURN
      7.            END
```

APPENDIX C, PART 1: INTEGRAL REPRESENTATION LISTING 176

```
      1.          FUNCTION FINT16(R,Z)
      2.          COMMON /F/G1
      3.          A = Z+R
      4.          W1 = 1./A
      5.          W2 = W*W
      6.          W3 = W2*W
      7.          FINT16 = 2.*G1/W3*(1.+W+.5*W2)
      8.          RETURN
      9.          END
```

```
      1.          FUNCTION PSISQU(U,R,Z,A,B)
      2.         W = .5*Z*R
      3.         W1 = Z*R
      4.         AA = A*A
      5.         BB = B*B
      6.          AB = A*B
      7.          C = SINH(2.*U*W)
      8.          C1 = SINH(U*W)
      9.         PSISQU = 2.*AA/W1*C+8.*AB/W1*C1+U*(2.*AA+BB)
     10.            RETURN
     11.          END
```

APPENDIX C, PART 1: INTEGRAL REPRESENTATION LISTING 178

```
      1.          SUBROUTINE DIRB(U,R,Z,A,B,ANS)
      2.          W = .5*Z**2
      3.          W1 = R*Z
      4.          Y1 = U*W
      5.          Y2 = U*W1
      6.           Q1 = COSH(Y1)
      7.           Q2 = COSH(Y2)
      8.           Q3 = SINH(Y1)
      9.           Q4 = SINH(Y2)
     10.          A1 = W1*W1
     11.          A2 = A1*R
     12.          AA = A*A
     13.          AB = A*B
     14.          BB = B*B
     15.          A3 = A2*Z
     16.          U2 = U*U
     17.          U3 = U*U*U
     18.          C1 = 2.*AA+BB
     19.          A4 = Z*R**2*R
     20.          A5 = Z*R**2
     21.          R2 = R*R
     22.              Z2 = Z**2
     23.           T1 = -4.*AA/(Z*R2*R)
     24.           T2 = 2.*AB*Z/R*(1. - 16./A1)
     25.           T3 = -2.*Z*AB/R
     26.           T4 = .5*Z2*BB
     27.           T5 = -Z2*BB*.166666666666666666
     28.           T6 = 4.*AA/R2
     29.           T7 = 16.*AB/R2
     30.    1     ANS1    = 3.*T1
     31.    2     ANS2    = Q3*Y2
     32.    3     ANS3    = U2*Q3*T3
     33.    4     ANS4    = Q*T4
     34.    5     ANS5    = U3*T5
     35.    6     ANS6    = U*Q2*T6
     36.    7     ANS7    = U*Q1*T7
     37.          ANS = ANS1 + ANS2 + ANS3 + ANS4 + ANS5 + ANS6 + ANS7
     38.          RETURN
     39.          END
```

```
      FUNCTION EXPI(R,Z)
      DIMENSION EC0(36), XC0(36)
      DIMENSION EC1(61),EC2(70),XC1(61),XC2(70)
      DIMENSION EC3(50), XC3(50)
      DATA EC0 /
     1 1.49334874693224C,1.34C8854448313931,1.2225356050805861,1.127045349
     2 2842316,1.047828CC8456CC6,.98C71472335839C5,.92291C6324837305,.8724
     3 3573131594157,.8279334352735C88,.7882775692635576,.7526780200295871
     4 4,.72C5015C65870451,.691245397802831S,.6645C4787431C68,.6399492266
     5 5392997,.61730592C9526C7C,.5963473623231941,.5768820704052510,.5587
     6 6.755617C23644,.5418C49C14535823,.525934531947846,.5110328836740476
     7 7  ,.497C097482616443,.4837861375242837,.4712925524860814,.45946756
     8 883131685,.44825666929153BC,.43761128311184C1,.42748797529942B1,
     9 9   .41784777326715771,.4C8655596C0C5607,.39987977037797261,.39149161
     1 1897C3562,.3834651C71497194,.375776539668B248,.3684042987138854 /
       DATA EC1 /
     1   .3613296166882226,.3545313797517170,.3479959534707185,.3417070 3
     2 2389C5328,.3356505139529960,.3298133668814699,.3241835429648885,.31
     3 3874987810936143,.3135082C1266C7469,.3C843C3187941878,.3035258364859B
     4 441,.2987882152032416,.2941856623768447,.2897348968068591,.28542110
     5 5676C5C18,.2812379121729163,.2771793304851186,.2732397457133851,.26
     6 6941388C3956638,.2658967701046732,.2620837402553185,.25857C38496684
     7 728,.2551595477685169,.2518263C39623C42,.2485879444773646,.24543396
     8 81C69944C,.2423610327385172,.2393660132383432,.2364459195921340,.23
     9 9389792150455781,.23C8193315980103,.2281C75963956555,.22546C2879853 1
     A 118,.2228750963044691,.22C3498219926746,.2178823697628117,.2154707 4
     B 2224750705,.2131130342810929,.2105074275813037,.2085521857982297,
     C 1 .2063456499010558,.2041862338758C01,.2020C7242070968261,.2000027586
     D 1398285,.1979758576467ECC,.1959903661721022,.1940450680471100,.1921
     E 2386796130670,.1902700470215C48,.1884380437005717,.1866415879757465
     F 3,.1849079648339623,.1831512C38210900,.1814553170635290,.1797910574
     G 4C53859,.1781575366533868,.1765538999222755,.1749793240740C63,.1734
     H 5330162445464,.1719142124525677,.17042217628473Z /
       DATA EC2 /
     1 .1675155916170776,.1647C78767047833,.1619940141687699,.159369323
     2 26120981,.1568294335801275,.1543702562925014,.1519879648307450,.149
     3 3678987250600241,.1474399141665211,
     4 1 .1452674929338860,.1431591462840558,.1411116690100267,.139122563
     5 2483519B,.1371893461691771,.1353C96738395544,.1334813331918336,.
     6 313170223217830760,.12997C3917123769,.1282839381003267,
     7 1 .1266410960766328,.1250401823871386,.1234795998702534,.121957831
     8 29914612,.1204734377909765,.1190250472084111,.1176113567519010,.116
     9 3231125482322222,.1148831712866667,.1135663674123762,
     A 1 .1122796392534993,.1110219613479563,.1097923545890153,
     B 1.1086569883622107B,.1074136544163729,.1062628119968480,.1051365383 2
     C 249826,.1043034050316254G,.1029545979846070,.1018974627043105,
     D 3.1008619555504409,.0998474159214815B,.0988532C98026C409,.097878728
     E 471997763,.0969233883229865 7,.0958662722929240,.0950679058716123 4,
     F 5.0941667C55C526139,.0932825271493381C,.0924148906801926C,.09156333
     G 6393978806,.0899066958733947B,.0883C92443778196 2,.0867678494157082 8
     H 7,.0852795926666984 4,.0838417788 6C34595,.0824518589988171 1,.0811074
```

```
      8759581423,.0798064173672644 4,.0785466163001247 1,.07732613313891 23
      9 /
       DATA E03 / .07614314897517273,.07499595570342 53,.07386294786642593
      1,.07202614690165 44,.0717535335244624 0,.07073436332946521,.0697438
      2392381444,.068780765680863 20,.06784401466898389,.06693251818 34396
      33,.06045265642834 48,.06518129981035035,.06433971331189784,.063519
      4645421626,.06272027910740924,.06194083826443568,.06118058520104 4
      537,.06043981827555632,.05971786971841596,.05900810360855644,.05831
      6791398338751,.05764372314063416,.05698497991134497,.0563411582443 6
      7879,.0557117557434767 3,.05509629235909709,.05449430915734226,
      8 .05385347169647 54,.0533290463169208 5,.05276494440262507,.0522125
      1761698677,.05167187241655524,.05114217916812135,.05062325689756 01
      25,.05014779795479 92,.04961643508305196,.04912792230640599 1,.04864
      3895307C4905,.04817924965297609,.04771854549596084,.0466044965023 8
      4127,.04554136165450517,.04452572364680038,.04355446469261788,
      5 .04212473437505852,.04173392155600062,.04087962975740886,.0400596
      6555230403 2,.03927196935293355,.0365146988449 0402 /
       DATA X00 / .20,.25,.30,.35,.40,.45,.50,.55,.60,.65,.70,.75,.80,
      1  .85,.90,.95,1.00,1.05,1.10,1.15,1.20,1.25,1.30,1.35,1.40,1.45,
      2  1.50,1.55,1.60,1.65,1.70,1.75,1.80,1.85,1.90,1.95 /
        DATA XC1 /
      1   2.00,2.05,2.10,2.15,2.20,2.25,2.30,2.35,2.40,2.45,2.50,2.55,
      2  2.60,2.65,2.70,2.75,2.80,2.85,2.90,2.95,3.00,3.05,3.10,3.15,
      3  3.20,3.25,3.30,3.35,3.40,3.45,3.50,3.55,3.60,3.65,3.70,3.75,3.80
      4,3.85,3.90,3.95,
      1   4.00,4.05,4.10,4.15,4.20,4.25,4.30,4.35,4.40,4.45,4.50,4.55,
      1  4.60,4.65,4.70,4.75,4.80,4.85,4.90,4.95,5.00 /
        DATA XC2 /
      1  5.1,5.2,5.3,5.4,5.5,5.6,5.7,5.8,5.9,
      1   6.00,
      1  6.10,6.20,6.30,6.40,6.50,6.60,6.70,6.80,6.90,7.00,7.10,7.20,7.30
      2,7.40,7.50,7.60,7.70,7.80,7.90,
      2             8.00,8.10,8.20,8.30,8.40,8.50,8.60,8.70,8.80,8.90,9.00,
      19.10,9.20,9.30,9.40,9.50,9.60,9.70,9.80,9.90,10.00,10.20,10.40,
      210.60,10.80,11.00,11.20,11.40,11.60,11.80,12.00 /
       DATA X03 /
      1 12.2,12.4,12.6,12.8,13.0,13.2,13.4,13.6,13.8,14.0,14.2,14.4,14.6,
      2 14.8,15.0,15.2, 15.4,15.6,15.8,16.0,16.2,16.4,16.6,16.8,17.0,17.2,
      3 17.4,17.6,17.8,18.0,18.2,18.4,18.6,18.8,19.0,19.2,19.4,19.6,19.8,
      4 20.0,20.5,21.0,21.5,22.0,22.5,23.0,23.5,24.0,24.5,25.0 /
       X = 2.*Z*P
       DO 9 K = 1,36
       DIFF0 = X - X00(K)
        X10 = X00(K)
        E10 = E00(K)
        IF(DIFF0.LT.0.)  GO TO 19
    9   CONTINUE
    6  DO 1 J = 1,61
        DIFF1 = X - X01(J)
```

```
      X11 = XC1(J)
       E11 = E01(J)
       IF(DIFF1.LT.0.) G9 TO 3
1      CONTINUE
      DO 2 I = 1,70
       DIFF2 = X-X02(I)
       X12 = X02(I)
      E12 = E02(I)
       IF(DIFF2.LT.0.) G9 TO 5
2     CONTINUE
      DO 30 M = 1,50
      DIFF3 = X - X03(M)
       X13 = X03(M)
       E13 = E03(M)
       IF(DIFF3.LT.0.) G0 TO 4C
30    CONTINUE
4C    EXPI = DFX(DIFF3,E13,X13,X)
      RETURN
19    EXPI = DFX( DIFFC,E1C,X1C,X)
      RETURN
3     EXPI = DFX(DIFF1,E11,X11,X)
      RETURN
5     EXPI = DFX(DIFF2,E12,X12,X)
      RETURN
      END
```

```
APPENDIX C. PART 1: INTEGRAL REPRESENTATION LISTING  182
      1.            FUNCTION DEX(DIFF,E1,X1,X)
      2.            DIMENSION T(50)
      3.            SUM1 = 0.
      4.           DO 4 N = 1,50
      5.           K = N-1
      6.           IF(K.EQ.0) T(N) = DIFF/N*(E1+DIFF**K/((-X1)**N))
      7.           IF(K.NE.0) T(N) = DIFF/N*(T(K)+DIFF**K/((-X1)**N))
      8.           SUM1 = SUM1 + T(N)
      9.            D1 = T(N) - T(K)
     10.            D = ABS(D1)
     11.            IF(D.LE.1.E-16)GO TO 3
     12.     4      CONTINUE
     13.     3      EX1 = E1 + SUM1
     14.           Q = DEXP(X)
     15.            DEX = -EX1/Q
     16.           RETURN
     17.           END
```

```
      1.            FUNCTION SINT2A(R,Z)
      2.            COMMON /C/EX/D/AL2/E/Q1/F/Q2
      3.            X = Z*R
      4.            W1 = 1./X
      5.            SINT2A = Q1*W1*(AL2-Q2*EX)
      6.            RETURN
      7.            END
```

APPENDIX C, PART 1: INTEGRAL REPRESENTATION LISTING 184

```
      1.            FUNCTION SINT2B(R,Z)
      2.            COMMON /E/Q1
      3.            A = R*Z
      4.            W1 = 1./A
      5.            W2 = W1*A1
      6.          SINT2B = Q1*W2*(1.+W)
      7.            RETURN
      8.            END
```

```
      FUNCTION SINT2D(U,R,Z,A,B)
      W = .5*Z*R
       W1 = Z*R
       W2 = W1*W1
       W3 = W2*W1
      AA = A*A
      AB = A*B
      BB = B*B
       CS = COSH(U*W)
       CSS = COSH(U*W1)
       SIH = SINH(U*W)
       SIHH = SINH(U*W1)
      FUN2 = U*U*U/3.*(2.*AA+3B)+R.*AB*U*U*SIH/W1
       FUN3 = 2.*AA*U*U*SIHH/W1-4.*AA*U*CSS/W2
       FUN4 = 64.*AB*SIH/W3-32.*AB*U*CS/W2
      FUN5 = 4.*AA*SIHH/W3
      FUNCT1 = FUN2+FUN3+FUN4+FUN5
       SINT2D = -.5*Z*Z*FUNCT1
      RETURN
      END
```

R = .0500
F(0)=-7*Z/2 =-1.9885[illegible]3.6616 E1=EGM= -1.993974577396
E(1) = EGM + Z*Z/2 = -.C0F464C41780
E2 = -.000CC0F89645 E(2) = -1.993975167C61
E3 = -.000C0C0C5554 E(3) = -1.993975172614

R = .1000
F(0)=-7*Z/2 -1.96C217336014 E1=EGM= -1.978225899825
E(1) = EGM + Z*Z/2 = -.C1R0C8E63811
E2 = -.000CF0730341 E(2) = -1.9782466C787
E3 = .000CC0C8C631 E(3) = -1.978246550156

R = .2000
F(0)=-7*Z/2 -1.F78412585784 E1=EGM= -1.928537622873
E(1) = EGM + Z*Z/2 = -.C5012503709C
E2 = -.000C72987C84 E(2) = -1.928610609960
E3 = .000CC0F697C0 E(3) = -1.928610340260

MU	LAMBDA	PHI GM	PHI(1)
.00	1.CC	1.2670115378	1.2571389170
.00	2.CC	1.0437663851	1.0407695680
.00	3.CC	.8598566265	.8597440261
.00	4.CC	.7083514363	.7093532965
.00	5.CC	.5835711881	.5848272334
.00	6.CC	.4807222809	.4819105322
.00	7.CC	.3961988C2	.3969536906
.00	8.CC	.3262+189C0	.3268789102
.00	9.CC	.2687586561	.2691126695
.00	10.CC	.2214038646	.2215136268
.10	1.CC	1.2670930455	1.2572198726
.10	2.CC	1.0438335313	1.0408365898
.10	3.CC	.8599119416	.8597993903
.10	4.CC	.7083970051	.7093989761
.10	5.CC	.5835787277	.5848648940
.10	6.CC	.4807832061	.4819415653
.10	7.CC	.3960-53564	.3969792529
.10	8.CC	.3262628774	.3268999599
.10	9.CC	.2687759455	.2691299993
.10	10.CC	.2214181076	.2215278914
.20	1.CC	1.2673375993	1.2574627580
.20	2.CC	1.0440349951	1.0410376706

•20	3•CC	•8600779079	•8599654958
•20	4•CC	•7085337284	•7095360254
•20	5•CC	•5836913606	•5849778844
•20	6•CC	•4808459933	•4820346719
•20	7•CC	•3961217947	•3970559455
•20	8•CC	•3263258473	•3269631139
•20	9•CC	•2688278203	•2691819927
•20	10•CC	•2214608422	•2215706886
•30	1•CC	1•2677452909	1•2578676291
•30	2•CC	1•0443708522	1•0413728569
•30	3•CC	•8603545876	•8602423810
•30	4•CC	•7087616576	•7097644761
•30	5•CC	•5838791291	•5851662308
•30	6•CC	•4810006773	•4821898734
•30	7•CC	•3962492236	•3971837863
•30	8•CC	•3264308236	•3270683869
•30	9•CC	•2689142959	•2692686618
•30	10•CC	•2215320842	•2216420282
•40	1•CC	1•2683162737	1•2584345794
•40	2•CC	1•0448412288	1•0418422262
•40	3•CC	•8607420847	•8606301101
•40	4•CC	•7090808784	•7100843814
•40	5•CC	•5841421038	•5854299769
•40	6•CC	•4812173164	•4824072060
•40	7•CC	•3964276913	•3973628050
•40	8•CC	•3265778455	•3272158032
•40	9•CC	•2690354169	•2693900266
•40	10•CC	•2216318606	•2217419268
•50	1•CC	1•2690507621	1•2591637394
•50	2•CC	1•0454463017	1•0424458870
•50	3•CC	•8612405448	•8611287729
•50	4•CC	•7094915108	•7104958152
•50	5•CC	•5844803834	•5857691639
•50	6•CC	•4814959917	•4826867200
•50	7•CC	•3966572645	•3975930430
•50	8•CC	•3267669684	•3274053970
•50	9•CC	•2691912166	•2695461154
•50	10•CC	•2217602087	•2218704076
•60	1•CC	1•2699490320	1•2600552773
•60	2•CC	1•0461862980	1•0431839790
•60	3•CC	•8618501551	•8617384848
•60	4•CC	•7099937089	•7109988730
•60	5•CC	•5848940951	•5861839305
•60	6•CC	•4818368082	•4830284803
•60	7•CC	•3969380297	•3978745538
•60	8•CC	•3269982633	•3276372123
•60	9•CC	•2693817578	•2697369642
•60	10•CC	•2219171768	•2220275002

APPENDIX C, PART 2: INTEGRAL REPRESENTATION RESULTS 188

.70	1.00	1.2710114209	1.2611093985
.70	2.00	1.0470614960	1.0440566727
.70	3.00	.8625711447	.8624593871
.70	4.00	.7105876614	.7115936715
.70	5.00	.5853833943	.5866743128
.70	6.00	.4822398937	.4834325662
.70	7.00	.3972700923	.3982074026
.70	8.00	.3272718170	.3279113029
.70	9.00	.2696071118	.2699626173
.70	10.00	.2221028239	.2222132412
.80	1.00	1.2722383280	1.2623263461
.80	2.00	1.0480722242	1.0450641700
.80	3.00	.8634037845	.8632916468
.80	4.00	.7112735915	.7122803485
.80	5.00	.5859484648	.5872404447
.80	6.00	.4827053998	.4838990714
.80	7.00	.3976535771	.3985916668
.80	8.00	.3275877324	.3282277324
.80	9.00	.2698673635	.2702231271
.80	10.00	.2223172198	.2224276735
.90	1.00	1.2736302142	1.2637064007
.90	2.00	1.0492188626	1.0462067040
.90	3.00	.8643483872	.8642354568
.90	4.00	.7120517570	.7130590634
.90	5.00	.5868595188	.5878824575
.90	6.00	.4832335013	.4844281042
.90	7.00	.3980886280	.3990274355
.90	8.00	.3279461282	.3285865741
.90	9.00	.2701626106	.2705185540
.90	10.00	.2225604448	.2226708469
1.00	1.00	1.2751876024	1.2652498505
1.00	2.00	1.0505018418	1.0474845388
1.00	3.00	.8654053078	.8652910356
1.00	4.00	.7129224500	.7139299966
1.00	5.00	.5873067974	.5886005002
1.00	6.00	.4838243966	.4850197873
1.00	7.00	.3985754087	.3995148097
1.00	8.00	.3283471390	.3289879113
1.00	9.00	.2704929640	.2708489664
1.00	10.00	.2228325906	.2229428177

R = .3000
F(0)=-Z*Z/2 =-1.78525664766E E1=EGM= -1.866541153992
E(1) = EGM + Z*Z/2 = -.0812845063?7
E2 = -.0001723030?? E(2) = -1.866713457084
E3 = .0000015179?? E(3) = -1.866711939142

R = .4000
E(0)=-Z**2/2 -1.698987111366 E1=EGM= -1.800525423037
E(1) = EGM + Z**2/2 = -.107538311672
E2 = -.000231837527 E(2) = -1.800757260564
E3 = .000002774817 E(3) = -1.800754485747

R = .5000
E(0)=-Z**2/2 -1.706182250088 E1=EGM= -1.734714716425
E(1) = EGM + Z**2/2 = -.128532466337
E2 = -.000271737878 E(2) = -1.734986454303
E3 = .000003527471 E(3) = -1.734982926831

MU	LAMBDA	PHI GM	PHI(1)
.00	1.00	.9096812573	.8996524916
.00	2.00	.5811550061	.5822061277
.00	3.00	.3712741561	.3733927073
.00	4.00	.2371905904	.2383071417
.00	5.00	.1515305475	.1516232322
.00	6.00	.0968061456	.0962619705
.00	7.00	.0618451526	.0610152820
.00	8.00	.0395101248	.0386246825
.00	9.00	.0252412663	.0244248387
.00	10.00	.0161255254	.0154314528
.10	1.00	.9100115996	.8999808336
.10	2.00	.5813660472	.5824185991
.10	3.00	.3714089809	.3735289713
.10	4.00	.2372767240	.2383941088
.10	5.00	.1515855745	.1516765662
.10	6.00	.0968412999	.0962971019
.10	7.00	.0618676111	.0610375507
.10	8.00	.0395244726	.0386387800
.10	9.00	.0252504324	.0244337539
.10	10.00	.0161313813	.0154370857
.20	1.00	.9110032900	.9009662861
.20	2.00	.5819995942	.5830562914
.20	3.00	.3718137260	.3739379422
.20	4.00	.2375352976	.2386551238
.20	5.00	.1517507657	.1518446406
.20	6.00	.0969468333	.0964025416
.20	7.00	.0619350317	.0611043858
.20	8.00	.0395675446	.0386810907
.20	9.00	.0252779492	.0244605110
.20	10.00	.0161489605	.0154539915
.30	1.00	.9126583198	.9026101291

.30	2.00	.5830569193	.5841200387
.30	3.00	.3724892041	.3746201557
.30	4.00	.2379668306	.2390905288
.30	5.00	.1520264530	.1521216724
.30	6.00	.0971229577	.0965784272
.30	7.00	.0620475497	.0612158739
.30	8.00	.0396394274	.0387516691
.30	9.00	.0253238719	.0245051442
.30	10.00	.0161782985	.0154821919
.40	1.00	.9149800124	.9049144983
.40	2.00	.5845401457	.5856112324
.40	3.00	.3734367717	.3755765064
.40	4.00	.2385721895	.2397008942
.40	5.00	.1524131900	.1525100238
.40	6.00	.0973700267	.0968249877
.40	7.00	.0622053912	.0613721598
.40	8.00	.0397402653	.0388506065
.40	9.00	.0253882928	.0245677109
.40	10.00	.0162194542	.0155217228
.50	1.00	.9179730299	.9078923882
.50	2.00	.5864522518	.5875316240
.50	3.00	.3746583315	.3768082482
.50	4.00	.2393525899	.2404870200
.50	5.00	.1529117531	.1530102027
.50	6.00	.0976885366	.0971425446
.50	7.00	.0624088731	.0615734462
.50	8.00	.0398702608	.0389780305
.50	9.00	.0254713412	.0246462914
.50	10.00	.0162725101	.0155726345
.60	1.00	.9216433825	.9115176568
.60	2.00	.5887970773	.5898843274
.60	3.00	.3761563364	.3783169967
.60	4.00	.2403095988	.2414499367
.60	5.00	.1535231437	.1536228634
.60	6.00	.0980791269	.0975315118
.60	7.00	.0626584039	.0618199944
.60	8.00	.0400296750	.0391341058
.60	9.00	.0255731838	.0247469893
.60	10.00	.0163375728	.0156349922
.70	1.00	.9259844C5	.9158250318
.70	2.00	.5915793307	.5926718233
.70	3.00	.3779337947	.3801047318
.70	4.00	.2414451381	.2425909072
.70	5.00	.1542485894	.1543428076
.70	6.00	.0985425820	.0979923966
.70	7.00	.0629544848	.0621121247
.70	8.00	.0402188279	.0393190340
.70	9.00	.0256940253	.0248639312

.70	10.00	.0164147731	.0157088755
.80	1.00	.9310469491	.9208101173
.80	2.00	.5948045989	.5958979637
.80	3.00	.3799942755	.3821738006
.80	4.00	.2427614879	.2439114287
.80	5.00	.1550895469	.1551889859
.80	6.00	.0990798324	.0985258006
.80	7.00	.0632977102	.0624502167
.80	8.00	.0404380995	.0395330543
.80	9.00	.0258341081	.0249992676
.80	10.00	.0165042658	.0157943786
.90	1.00	.9367990460	.9264794030
.90	2.00	.5984793585	.5995669776
.90	3.00	.3823419164	.3845269207
.90	4.00	.2442612315	.2454132349
.90	5.00	.1560477049	.1561444988
.90	6.00	.0996919571	.0991324201
.90	7.00	.0636887695	.0628347098
.90	8.00	.0406879299	.0397764434
.90	9.00	.0259937137	.0251531725
.90	10.00	.0166062307	.0158916109
1.00	1.00	.9432662819	.9328402734
1.00	2.00	.6026109887	.6036836775
1.00	3.00	.3849814317	.3871671851
1.00	4.00	.2459475607	.2470982985
1.00	5.00	.1571249875	.1572165990
1.00	6.00	.1003801852	.0998130477
1.00	7.00	.0641284479	.0632661042
1.00	8.00	.0409688209	.0400495164
1.00	9.00	.0261731625	.0253258440
1.00	10.00	.0167208726	.0160006968

R = .6000
E(0)=-Z*Z/2 -1.526260460244 E1=EGM= -1.671185327835
E(1) = EGM + Z*Z/2 = -.144924867591
E2 = -.000302714131 E(2) = -1.671488041966
E3 = .000004492644 E(3) = -1.671483549321

R = .7000
E(0)=-Z*Z/2 -1.453335065668 E1=EGM= -1.610884736452
E(1) = EGM + Z*Z/2 = -.157549670785
E2 = -.000318846842 E(2) = -1.611203583295
E3 = .000005634225 E(3) = -1.611198049070

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R = .8000
E(0)=-Z*Z/2 =-1.386959700768 E1=EGM= -1.554166175507
E(1) = EGM + Z*Z/2 = -.167166474739
E2 = -.0003217567+4 E(2) = -1.554487932251
E3 = .0000C6301634 E(3) = -1.554481630617

R = .9000
E(0)=-Z*Z/2 =-1.326563674851 E1=EGM= -1.5C1071782C41
E(1) = EGM + Z*Z/2 = -.174403107190
E2 = -.0003166766823 E(2) = -1.501386448864
E3 = .0000C6717105 E(3) = -1.501381731759

MU	LAMBDA	PHI GM	PHI(1)
.00	1.CC	.4368166444	.6295985948
.00	2.00	.3059659171	.3078643643
.00	3.CC	.1470048612	.1476890450
.00	4.00	.0706301846	.0701105808
.00	5.CC	.0339350885	.0330533246
.00	6.00	.0163045055	.0155027011
.00	7.CC	.0078336881	.CC724C5855
.00	8.00	.0037637860	.0033693443
.00	9.CC	.0018083545	.0015625991
.00	10.0C	.0008688448	.0007223318
.10	1.CC	.4374719951	.6302543707
.10	2.0C	.3062807879	.3081849621
.10	3.CC	.1471561445	.1478428453
.10	4.0C	.0707028704	.0701836029
.10	5.CC	.0339700113	.0330877584
.10	6.CO	.0163212845	.0155188561
.10	7.CC	.0078417498	.CC72481334
.10	8.00	.0037676593	.0033728581
.10	9.CC	.0018102155	.0015642295
.10	10.00	.0008697389	.0007230858
.20	1.CC	.4394415699	.6322241063
.20	2.00	.3072270929	.3091479419
.20	3.CC	.1476108077	.1483048150
.20	4.00	.0709213186	.0704029379
.20	5.CC	.0340749672	.0331911854
.20	6.CC	.0163717118	.0155673792
.20	7.CC	.0078659782	.CC72708040
.20	8.00	.0037793001	.0033834119
.20	9.CC	.0018158084	.0015691262
.20	10.00	.0008724261	.0007253506
.30	1.CC	.4427359562	.6355150365
.30	2.00	.3088099188	.3107568691

.30	3.00	.1483712947	.1490766635
.30	4.00	.0712867034	.0707693928
.30	5.00	.0342505206	.0333639829
.30	6.00	.0164560585	.0156484455
.30	7.00	.0079065035	.0073086779
.30	8.00	.0037987710	.0034010425
.30	9.00	.0018251634	.0015773061
.30	10.00	.0008769209	.0007291336
.40	1.00	.4473728625	.6401392551
.40	2.00	.3110377741	.3130177035
.40	3.00	.1494416035	.1501612477
.40	4.00	.0718009889	.0712843169
.40	5.00	.0344976150	.0336067814
.40	6.00	.0165747778	.0157623474
.40	7.00	.0079635436	.0073618899
.40	8.00	.0038261766	.0034258116
.40	9.00	.0018383307	.0015887972
.40	10.00	.0008832473	.0007344475
.50	1.00	.4533772143	.6461137679
.50	2.00	.3139226343	.3159388258
.50	3.00	.1508277579	.1515625855
.50	4.00	.0724669395	.0719496072
.50	5.00	.0348175787	.0339204678
.50	6.00	.0167285081	.0159094962
.50	7.00	.0080374051	.0074306293
.50	8.00	.0038616642	.0034578060
.50	9.00	.0018553812	.0016033690
.50	10.00	.0008914393	.0007413103
.60	1.00	.4607812874	.6534605674
.60	2.00	.3174800067	.3195310745
.60	3.00	.1525369386	.1532858726
.60	4.00	.0732881352	.0727677171
.60	5.00	.0352121316	.0343061888
.60	6.00	.0169180756	.0160904237
.60	7.00	.0081284850	.0075151412
.60	8.00	.0039054245	.0034971378
.60	9.00	.0018764064	.0016218826
.60	10.00	.0009015411	.0007497450
.70	1.00	.4696248816	.6622067280
.70	2.00	.3217290138	.3238077932
.70	3.00	.1545784232	.1553375060
.70	4.00	.0742689900	.0737416672
.70	5.00	.0356833947	.0347653561
.70	6.00	.0171444994	.0163057844
.70	7.00	.0082372729	.0076157271
.70	8.00	.0039576930	.0035439451
.70	9.00	.0019015193	.0016435906
.70	10.00	.0009136069	.0007597800

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.80	1.CC	.4799555352	.6723845225
.80	2.CC	.3266924957	.3287848885
.80	3.CC	.1569631854	.1577251111
.80	4.CC	.0754147766	.0748750578
.80	5.CC	.0362339011	.0352996523
.80	6.CC	.0174089966	.0165563588
.80	7.CC	.0083643536	.0077327463
.80	8.CC	.0040187504	.0035983924
.80	9.CC	.0019308551	.0016688381
.80	10.CC	.0009277016	.0007714491
.90	1.CC	.4918287795	.6840315591
.90	2.CC	.3323971332	.3344808975
.90	3.CC	.1597040444	.1604575745
.90	4.CC	.0767316540	.0761720846
.90	5.CC	.0368666100	.0359110383
.90	6.CC	.0177129889	.0168430557
.90	7.CC	.0085104102	.0078666176
.90	8.CC	.0040888250	.0036606712
.90	9.CC	.0019645713	.0016977121
.90	10.CC	.0009439010	.0007847918
1.00	1.CC	.7053084381	.6971909387
1.00	2.CC	.3388735910	.3409170663
1.00	3.CC	.1628157335	.1635450812
1.00	4.CC	.0782267009	.0776375561
1.00	5.CC	.0375849226	.0366017609
1.00	6.CC	.0180581105	.0171669170
1.00	7.CC	.0086762279	.0080178205
1.00	8.CC	.0041685940	.0037310010
1.00	9.CC	.0020028492	.0017303124
1.00	10.CC	.0009629920	.0007998530

R = 1.0000
E(0)=-Z*Z/2 =-1.5772439181C E1=EGM= -1.45148473C634
E(1) = EGM + Z*Z/2 = -.1775973882A
E2 = -.0003C7581661 E(2) = -1.4517923122 95
E3 = .0000C6283455 E(3) = -1.451785429427

MU	LAMBDA	PHI GM	PHI(1)
.00	1.0C	.5895329505	.5828881484
.00	2.CC	.2655123C6	.2673675883
.00	3.CC	.1196428291	.12CC167793
.00	4.CC	.0538984119	.0532349382
.00	5.00	.0242809270	.0234254952
.00	6.CC	.0109384191	.0102458282
.00	7.CC	.0049276954	.CC44586529
.00	8.CC	.0022198987	.0019314440
.00	9.CC	.0010000517	.CCC8330692

•00	10•CC	•0C04505176	•C003577784
•10	1•C0	•5902597480	•5836168252
•10	2•CC	•2459086487	•2677017457
•10	3•CC	•1197903291	•1201667842
•10	4•CC	•0539648598	•0533014889
•10	5•CC	•0243108614	•0234547898
•10	6•CC	•0109519044	•0102586465
•10	7•CC	•0049337704	•0044642339
•10	8•CC	•0022226354	•0019938631
•10	9•CC	•0010012846	•0008341133
•10	10•CC	•0004510730	•0003582272
•20	1•CC	•5924447643	•5858060578
•20	2•CC	•2668929861	•2687056982
•20	3•CC	•1202337675	•1206174424
•20	4•CC	•0541546262	•0535014334
•20	5•CC	•0244008551	•0235428010
•20	6•CC	•0109924460	•0102971563
•20	7•CC	•0049520342	•0044810003
•20	8•CC	•0022308632	•0019411303
•20	9•CC	•0010049911	•0008372499
•20	10•CC	•0004527428	•0003595753
•30	1•00	•5961019005	•5894654699
•30	2•CC	•2685405051	•2703838937
•30	3•CC	•1209759654	•1213708075
•30	4•CC	•0544089822	•0538356502
•30	5•CC	•0245514805	•0236899112
•30	6•CC	•0110603019	•0103615228
•30	7•CC	•0049826029	•0044909229
•30	8•CC	•0022446342	•0019532757
•30	9•CC	•0010111949	•0008424915
•30	10•CC	•0004555375	•0003618280
•40	1•CC	•6012544233	•5946111576
•40	2•CC	•2708616872	•2727437722
•40	3•CC	•1220216447	•1224301541
•40	4•CC	•0549700547	•0543056087
•40	5•CC	•0247636960	•0238967607
•40	6•CC	•0111559038	•0104520222
•40	7•CC	•0050256710	•0045484199
•40	8•CC	•0022640361	•0019703495
•40	9•CC	•0010199353	•0008498595
•40	10•CC	•0004594751	•0003649941
•50	1•CC	•6079351130	•6012657747
•50	2•CC	•2738712398	•2757958048
•50	3•CC	•1233774580	•1238001956
•50	4•CC	•0555808409	•0549133769
•50	5•CC	•0250388517	•0241642500
•50	6•CC	•0112798598	•0105690431
•50	7•CC	•0050815125	•0045993576

.50	8.00	.0022591924	.0019924223
.50	9.00	.0010312681	.0008593835
.50	10.00	.0004645804	.0003690561
.60	1.00	.6161864722	.6094586507
.60	2.00	.2775884900	.2795535489
.60	3.00	.1250520310	.1254870083
.60	4.00	.0563352265	.0556616322
.60	5.00	.0253786981	.0244935452
.60	6.00	.0114329587	.0107130287
.60	7.00	.0051504827	.0046620514
.60	8.00	.0023202631	.0020195857
.60	9.00	.0010452653	.0008711020
.60	10.00	.0004708860	.0003741201
.70	1.00	.6260609959	.6192259436
.70	2.00	.2820369066	.2840337190
.70	3.00	.1270560173	.1274980837
.70	4.00	.0572380108	.0565536755
.70	5.00	.0257853973	.0248860837
.70	6.00	.0116161744	.0108847791
.70	7.00	.0052330203	.0047367667
.70	8.00	.0023574458	.0020519522
.70	9.00	.0010620159	.0008850626
.70	10.00	.0004784321	.0003801159
.80	1.00	.6376215058	.6306108251
.80	2.00	.2872448504	.2892562733
.80	3.00	.1294021662	.1298423669
.80	4.00	.0582949376	.0575934478
.80	5.00	.0262615368	.0253435816
.80	6.00	.0118306725	.0110848548
.80	7.00	.0053296505	.0048238202
.80	8.00	.0024009771	.0020896563
.80	9.00	.0010816265	.0009013219
.80	10.00	.0004872666	.0003870972
.90	1.00	.6509415498	.6436636992
.90	2.00	.2932454542	.2952445145
.90	3.00	.1321054040	.1325303019
.90	4.00	.0595127308	.0587855502
.90	5.00	.0268101457	.0258680423
.90	6.00	.0120778177	.0113141799
.90	7.00	.0054409879	.0049235818
.90	8.00	.0024511340	.0021328551
.90	9.00	.0011042219	.0009199459
.90	10.00	.0004974457	.0003950915
1.00	1.00	.6661058699	.6584424536
1.00	2.00	.3000768939	.3020252080
1.00	3.00	.1351829286	.1355738846
1.00	4.00	.0608991380	.0601352673
1.00	5.00	.0274347142	.0264617665

1.00	6.00	.0123591882	.0115737466
1.00	7.00	.0055677411	.0050364758
1.00	8.00	.0025082356	.0021817286
1.00	9.00	.0011299458	.0009410105
1.00	10.00	.0005090341	.0004041305

R = 1.1000
E(0)=-7*Z/2 -1.[illegible] E1=EGM= -1.4052119114O3
E(1) = EGM + Z*Z/2 = -.183429313427
E2 = -.0002965985[illegible] E(2) = -1.4055C8E09913
E3 = .0000C6915417 E(3) = -1.40550159449G

R = 1.2000
E(0)=-7*Z/2 -1.175[illegible] E1=EGM= -1.362C29C63770
E(1) = EGM + Z*Z/2 = -.184319382770
E2 = -.0002[illegible]667127 E(2) = -1.36231373C897
E3 = .0000C6882901 E(3) = -1.3623C6847996

R = 1.3000
E(0)=-7*Z/2 -1.133634298935 E1=EGM= -1.32170525554[illegible]
E(1) = EGM + Z*Z/2 = -.188[illegible]0956607
E2 = -.0002[illegible]C496 E(2) = -1.3219775160[illegible]
E3 = .0000C6810767 E(3) = -1.32197C705272

R = 1.4000
E(0)=-7*Z/2 -1.0[illegible]246392 E1=EGM= -1.284C15842513
E(1) = EGM + Z*Z/2 = -.1890[illegible]596121
E2 = -.0002[illegible]96867 E(2) = -1.28427553938C
E3 = .0000C6703770 E(3) = -1.28426883561[illegible]

R = 1.5000
E(0)=-7*Z/2 -1.0[illegible]A2[illegible]031 E1=EGM= -1.24874894997[illegible]
E(1) = EGM + Z*Z/2 = -.189477123943
E2 = -.0002[illegible]C6[illegible] E(2) = -1.24899619C619
E3 = .0000C6561944 E(3) = -1.248989628676

R = 1.6000

E(0)=-Z*Z/2 -1.02637715670 E1=EGM= -1.21570830979
E(1) = EGM + Z*Z/2 = -.18940059139
E2 = -.00023511483 E(2) = -1.21594342187
E3 = .00000638751 E(3) = -1.21593703436

R = 1.7000
E(0)=-Z*Z/2 -.99577492796 E1=EGM= -1.18471404502
E(1) = EGM + Z*Z/2 = -.18493911706
E2 = -.00022349419 E(2) = -1.18493753922
E3 = .00000618600 E(3) = -1.18493135322

R = 1.8000
E(0)=-Z*Z/2 -.96743328568 E1=EGM= -1.15560238775
E(1) = EGM + Z*Z/2 = -.18816910716
E2 = -.00021250145 E(2) = -1.15581428926
E3 = .00000596450 E(3) = -1.15580832426

R = 1.9000
E(0)=-Z*Z/2 -.94107951547 E1=EGM= -1.12822476675
E(1) = EGM + Z*Z/2 = -.18715225128
E2 = -.00020221217 E(2) = -1.12842697893
E3 = .00000573239 E(3) = -1.12842124654

R = 1.9972
E(0)=-Z*Z/2 -.91717293493 E1=EGM= -1.10314780894
E(1) = EGM + Z*Z/2 = -.18597487400
E2 = -.00019292033 E(2) = -1.10334072928
E3 = .00000550218 E(3) = -1.10333522726

MU	LAMBDA	PHI GM	PHI(1)
.00	1.00	.3185804075	.3151918940
.00	2.00	.0823839618	.0827476514
.00	3.00	.0213042516	.0207772620
.00	4.00	.0055092172	.0050789066
.00	5.00	.0014246674	.0012155289
.00	6.00	.0003684148	.0002851306
.00	7.00	.0000952710	.0000654453
.00	8.00	.0000246368	.0000146349
.00	9.00	.0000063710	.0000031629
.00	10.00	.0000016475	.0000006510

.10	1.00	.3198306215	.3164697292
.10	2.00	.0827072635	.0830829619
.10	3.00	.0213878565	.0208615352
.10	4.00	.0055308371	.0050995455
.10	5.00	.0014302563	.0012204820
.10	6.00	.0003698606	.0002862969
.10	7.00	.0000956449	.0000657144
.10	8.00	.0000247335	.0000146955
.10	9.00	.0000063960	.0000031761
.10	10.00	.0000016540	.0000006537
.20	1.00	.3236041674	.3203207598
.20	2.00	.0836830914	.0840935141
.20	3.00	.0216402087	.0211155049
.20	4.00	.0055960931	.0051617385
.20	5.00	.0014471333	.0012354058
.20	6.00	.0003742244	.0002898103
.20	7.00	.0000967733	.0000665247
.20	8.00	.0000250253	.0000148780
.20	9.00	.0000064715	.0000032160
.20	10.00	.0000016735	.0000006620
.30	1.00	.3299701766	.3267978519
.30	2.00	.0853293228	.0857932484
.30	3.00	.0220659133	.0215426414
.30	4.00	.0057061807	.0052663193
.30	5.00	.0014756016	.0012604946
.30	6.00	.0003815863	.0002957146
.30	7.00	.0000986771	.0000678859
.30	8.00	.0000255176	.0000151842
.30	9.00	.0000065988	.0000032827
.30	10.00	.0000017064	.0000006760
.40	1.00	.3390452748	.3359900906
.40	2.00	.0876761166	.0882056557
.40	3.00	.0226727874	.0221487919
.40	4.00	.0058631165	.0054146922
.40	5.00	.0015161848	.0012960757
.40	6.00	.0003920810	.0003040840
.40	7.00	.0001013910	.0000698141
.40	8.00	.0000262194	.0000156177
.40	9.00	.0000067803	.0000033771
.40	10.00	.0000017534	.0000006956
.50	1.00	.3509957179	.3480242441
.50	2.00	.0907664662	.0913641650
.50	3.00	.0234719427	.0229422770
.50	4.00	.0060697758	.0056088556
.50	5.00	.0015696263	.0013426148
.50	6.00	.0004059008	.0003150234
.50	7.00	.0001049647	.0000723321
.50	8.00	.0000271436	.0000161829

APPENDIX C, PART 2: INTEGRAL REPRESENTATION RESULTS 200

.50	9.00	.0000070192	.0000350C0
.50	10.00	.0000018152	.0000007211
.60	1.00	.3660404385	.3630668216
.60	2.00	.0946569870	.0953126873
.60	3.00	.0244780200	.0239340253
.60	4.00	.0063299444	.0058514333
.60	5.00	.0016369051	.0014007233
.60	6.00	.0004232989	.0003286711
.60	7.00	.0001094639	.0000754699
.60	8.00	.0000283070	.0000168863
.60	9.00	.0000073201	.0000036525
.60	10.00	.0000018930	.0000007527
.70	1.00	.3844550564	.3813267376
.70	2.00	.0994189534	.1001063198
.70	3.00	.0257094506	.0251377473
.70	4.00	.0066483887	.0061457166
.70	5.00	.0017192539	.0014711675
.70	6.00	.0004445940	.0003451998
.70	7.00	.0001149707	.0000792651
.70	8.00	.0000297311	.0000177354
.70	9.00	.0000076884	.0000038361
.70	10.00	.0000019882	.0000007905
.80	1.00	.4065769281	.4030586030
.80	2.00	.1051396047	.1058122174
.80	3.00	.0271887943	.0265701515
.80	4.00	.0070309427	.0064957148
.80	5.00	.0018181812	.0015548806
.80	6.00	.0004701763	.0003648201
.80	7.00	.0001215862	.0000837633
.80	8.00	.0000314418	.0000187397
.80	9.00	.0000081308	.0000040527
.80	10.00	.0000021026	.0000008349
.90	1.00	.4328113268	.4285666787
.90	2.00	.1119237435	.1125106383
.90	3.00	.0289431528	.0282512024
.90	4.00	.0074846147	.0069062167
.90	5.00	.0019354995	.0016529765
.90	6.00	.0005005145	.0003877827
.90	7.00	.0001294316	.0000890188
.90	8.00	.0000334706	.0000199102
.90	9.00	.0000086554	.0000043042
.90	10.00	.0000022383	.0000008862
1.00	1.00	.4436388672	.4582C95405
1.00	2.00	.1198956554	.1202961798
1.00	3.00	.0310046659	.0302044256
1.00	4.00	.0080177159	.0073828624
1.00	5.00	.0020733579	.0017667661
1.00	6.00	.0005361643	.0004143827

1.00	7.00	.0001386505	.0000950954
1.00	8.00	.0000358546	.0000212601
1.00	9.00	.0000092719	.0000045931
1.00	10.00	.0000023977	.0000009448

R = 2.0000

E(0)=-Z*Z/2 = -.9165080075355 E1=EGM= -1.1012446672381

E(1) = EGM + Z*Z/2 = -.1859385970026

E2 = -.0001928663161 E(2) = -1.1026393335541

E3 = .0000054953746 E(3) = -1.1026338640165

MU	LAMBDA	PHI GM	PHI(1)
.00	1.00	.3181149286	.3147314266
.00	2.00	.0821480790	.0825091081
.00	3.00	.0212134241	.0206866372
.00	4.00	.0054780266	.0050490342
.00	5.00	.0014146125	.0012064811
.00	6.00	.0003653010	.0002825481
.00	7.00	.0000943331	.0000647422
.00	8.00	.0000243600	.0000144515
.00	9.00	.0000062906	.0000031170
.00	10.00	.0000016244	.0000006400
.10	1.00	.3193662326	.3160104407
.10	2.00	.0824712082	.0828442499
.10	3.00	.0212968670	.0207707434
.10	4.00	.0054995744	.0050696009
.10	5.00	.0014201769	.0012114092
.10	6.00	.0003667379	.0002837066
.10	7.00	.0000947042	.0000650091
.10	8.00	.0000244558	.0000145115
.10	9.00	.0000063153	.0000031301
.10	10.00	.0000016308	.0000006427
.20	1.00	.3231431162	.3198650637
.20	2.00	.0834465278	.0838543045
.20	3.00	.0215487276	.0210242125
.20	4.00	.0055646134	.0051315771
.20	5.00	.0014369722	.0012262577
.20	6.00	.0003710750	.0002871966
.20	7.00	.0000958242	.0000658127
.20	8.00	.0000247451	.0000146921
.20	9.00	.0000063900	.0000031694
.20	10.00	.0000016501	.0000006509
.30	1.00	.3295149160	.3263483306
.30	2.00	.0850919429	.0855532364
.30	3.00	.0219736297	.0214505156
.30	4.00	.0056743375	.0052357951
.30	5.00	.0014653067	.0012512204

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.30	6.00	.0003783919	.0002930619
.30	7.00	.0000977137	.0000671626
.30	8.00	.0000252330	.0000149953
.30	9.00	.0000065160	.0000032354
.30	10.00	.0000016827	.0000006647
.40	1.00	.3385986069	.3355496135
.40	2.00	.0874376604	.0879645785
.40	3.00	.0225793736	.0220555016
.40	4.00	.0058307612	.0053836577
.40	5.00	.0015057006	.0012866237
.40	6.00	.0003888230	.0003013759
.40	7.00	.0001004073	.0000690748
.40	8.00	.0000259286	.0000154243
.40	9.00	.0000066956	.0000033287
.40	10.00	.0000017290	.0000006841
.50	1.00	.3505609486	.3475960930
.50	2.00	.0905267433	.0911218205
.50	3.00	.0233770798	.0228474937
.50	4.00	.0060367560	.0055771606
.50	5.00	.0015588954	.0013329320
.50	6.00	.0004025597	.0003122436
.50	7.00	.0001039546	.0000715721
.50	8.00	.0000268446	.0000159839
.50	9.00	.0000069322	.0000034501
.50	10.00	.0000017901	.0000007092
.60	1.00	.3656215481	.3626548285
.60	2.00	.0944159017	.0950689552
.60	3.00	.0243813925	.0238374243
.60	4.00	.0062961036	.0058189247
.60	5.00	.0016258678	.0013907548
.60	6.00	.0004196543	.0003258021
.60	7.00	.0001084206	.0000746841
.60	8.00	.0000279979	.0000166802
.60	9.00	.0000072300	.0000036008
.60	10.00	.0000018670	.0000007403
.70	1.00	.3840568906	.3809354362
.70	2.00	.0991765332	.0998611859
.70	3.00	.0256107493	.0250390099
.70	4.00	.0066135653	.0061122375
.70	5.00	.0017078472	.0014608564
.70	6.00	.0004410241	.0003422238
.70	7.00	.0001138874	.0000784483
.70	8.00	.0000294096	.0000175209
.70	9.00	.0000075946	.0000037822
.70	10.00	.0000019612	.0000007776
.80	1.00	.4062054153	.4026933994
.80	2.00	.1048960345	.1055658005
.80	3.00	.0270877188	.0264689670

•80	4•CC	•CC6994960	•0064611039
•80	5•CC	•CC18063386	•0015441676
•80	6•CC	•CC4664580	•0003617180
•80	7•CC	•CC01204553	•0000829100
•80	8•CC	•CC0311056	•CCCC185152
•80	9•CC	•CC00080325	•CCCCC39962
•80	10•CC	•CCCC20743	•CCCCCC8214
•90	1•CC	•4324737287	•4282340441
•90	2•CC	•1116794052	•1122632230
•90	3•CC	•CC88394155	•0281472711
•90	4•CC	•CC74473166	•CC687C3081
•90	5•CC	•CC192315C1	•CC16417992
•90	6•CC	•CC4966227	•CCC3845346
•90	7•CC	•CC01282448	•CCCC881231
•90	8•CC	•CCC0331172	•CCCC196742
•90	9•CC	•CCCC8552C	•CCCCC42447
•90	10•CC	•CCCCC22084	•CCCCCC8719
1•00	1•CC	•4633440694	•4579172308
1•00	2•CC	•1196511757	•1200482533
1•00	3•CC	•CCC8979974	•C300974630
1•00	4•CC	•CC79789123	•0073454848
1•00	5•CC	•CC20604262	•CC17550589
1•00	6•CC	•CC0532072C	•CCC4109673
1•00	7•CC	•CC01373991	•CCCC941512
1•00	8•CC	•CC0354811	•CCCC210108
1•00	9•CC	•CC00091624	•CCCCC45302
1•00	10•CC	•CCCCC23660	•CCCCCC9297

R = 2•1000
E(0)=-7•7/2 =-•P3557748765[illegible] E1=EGM= -1•C78146415476
E(1) = EGM + Z•7/2 = -•184568927822
E2 = -•CCC1838[illegible] E(2) = -1•C7833C277911
E3 = •CCCCC[illegible] E(3) = -1•C7832[illegible]

R = 2•2000
E(0)=-7•7/2 =-•[illegible] E1=EGM= -1•C[illegible]
E(1) = EGM + Z•7/2 = -•1830766123[illegible]
E2 = -•CCC1757[illegible] E(2) = -1•C55389696826
E3 = •CCCCC[illegible] E(3) = -1•C55384667443

R = 2•3000
E(0)=-7•7/2 =-•[illegible] E1=EGM= -1•C33549448204
E(1) = EGM + Z•7/2 = -•18148905130[illegible]

F2 = -.00016844665 E(2) = -1.033717894855
F3 = .00000480735? E(3) = -1.033713087498

R = 2.4000
E(0)=-7*Z/2 =-.8932[illegible] E1=EGM= -1.013062734895
E(1) = EGM + Z*Z/2 = -.17982881604?
F2 = -.00016177614[illegible] E(2) = -1.013224511041
E3 = .00000469522[illegible] E(3) = -1.013219916817

R = 2.5000
E(0)=-7*Z/2 =-.81555729[illegible] E1=EGM= -.9936717884189
E(1) = EGM + Z*Z/2 = -.17811454905[illegible]
F2 = -.00015575129[illegible] E(2) = -.9938275400143
E3 = .00000439361[illegible] E(3) = -.9938231463255

R = 2.6000
E(0)=-7*Z/2 =-.79895[illegible] E1=EGM= -.9753021093[illegible]
E(1) = EGM + Z*Z/2 = -.17636168043[illegible]
F2 = -.00015033794[illegible] E(2) = -.9754824473[illegible]
E3 = .00000420340[illegible] E(3) = -.9754482439[illegible]

R = 2.7000
E(0)=-7*Z/2 =-.78339[illegible] E1=EGM= -.9578858763[illegible]
E(1) = EGM + Z*Z/2 = -.17458300063[illegible]
F2 = -.00014549824[illegible] E(2) = -.9580313745[illegible]
E3 = .00000402392[illegible] E(3) = -.9580273506[illegible]

R = 2.8000
E(0)=-7*Z/2 =-.76857211[illegible] E1=EGM= -.9413612342[illegible]
E(1) = EGM + Z*Z/2 = -.17278912060[illegible]
F2 = -.00014119756[illegible] E(2) = -.9415024318[illegible]
E3 = .00000385507[illegible] E(3) = -.9414985767[illegible]

R = 2.9000
E(0)=-7*Z/2 =-.75463[illegible] E1=EGM= -.9256716647[illegible]
E(1) = EGM + Z*Z/2 = -.17098884322[illegible]

F2 = -.0001374C1874 E(2) = -.9258C9266585
F3 = .0000C3696501 E(3) = -.9258C537CCA4

R = 3.0000
F(0)=-Z*Z/2 = -.7415759599?2 E1=EGM= -.910765424385
F(1) = EGM + Z*Z/2 = -.1691589464463
F2 = -.0001340783?3 E(2) = -.910899502728
F3 = .0000C3547772 E(3) = -.910895954956

MU	LAMBDA	PHI GM	PHI(1)
.00	1.CC	.1878572778	.1956180281
.00	2.CC	.0318416468	.03166487C4
.00	3.CC	.0051243527	.0048C66847
.00	4.CC	.0008246744	.0006988973
.00	5.CC	.0001327168	.CCOC975996
.00	6.CC	.0000213564	.0000129952
.00	7.CC	.0000034373	.CCOCC16175
.00	8.CC	.0000005532	.0000001795
.00	9.CC	.0000000890	.CCOOOOO153
.00	10.CC	.0000000143	.CCOCOOOOO2
.10	1.CC	.1894C59885	.1972270882
.10	2.CC	.0320908845	.0319252574
.10	3.CC	.0051644631	.CC48463256
.10	4.CC	.0008311294	.0007046925
.10	5.CC	.0001337557	.CCOC984158
.10	6.CC	.0000215256	.0000131053
.10	7.CC	.0000034642	.CCOCC16315
.10	8.CC	.0000005575	.0000001811
.10	9.CC	.0000000897	.0000000154
.10	10.CC	.0000000144	.CCOOOOOOC2
.20	1.CC	.1841039466	.2020967665
.20	2.CC	.0328469382	.0327133056
.20	3.CC	.0052861366	.CC49662808
.20	4.CC	.0008507107	.0007222246
.20	5.CC	.0001369069	.CCO1OC8841
.20	6.CC	.0000220328	.CCOC134382
.20	7.CC	.0000035458	.CCOC016738
.20	8.CC	.0000005706	.0000001860
.20	9.CC	.0000000918	.0000000159
.20	10.CC	.0000000148	.CCOCCOOOO2
.30	1.CC	.1761083636	.2103559157
.30	2.CC	.0331351082	.0340498976
.30	3.CC	.0054934449	.CC51696813
.30	4.CC	.0008840733	.CCO7519381
.30	5.CC	.0001422760	.CCO1C5C642
.30	6.CC	.0000228968	.CCOC140012

.30	7.CC	.00C0036848	.0CCCC17453
.30	8.CC	.0000C0593C	.0000001942
.30	9.CC	.0000C00954	.C00CCC0167
.30	10.CC	.0000000154	.0000000003
.40	1.CC	.2236870979	.2222238334
.40	2.CC	.0359985017	.0359705740
.40	3.CC	.0057933253	.CC5461855C
.40	4.CC	.0009323338	.0007945895
.40	5.CC	.0001500427	.0001110576
.40	6.CC	.0000241467	.0000148C7C
.40	7.CC	.0000C03886	.0000018472
.40	8.CC	.0000006254	.0000002059
.40	9.CC	.0000001006	.0000000176
.40	10.CC	.0000000162	.0000000003
.50	1.CC	.2392276183	.238C171399
.50	2.CC	.0384994756	.0385266488
.50	3.CC	.0061958129	.C058504939
.50	4.CC	.0009971071	.0008512705
.50	5.CC	.0001604668	.0001190111
.50	6.CC	.0000258243	.0000158740
.50	7.CC	.0000041566	.0000019817
.50	8.CC	.0000006688	.0000002212
.50	9.CC	.0000001076	.0000000192
.50	10.CC	.0000000173	.0000000003
.60	1.CC	.2592499700	.2581595619
.60	2.CC	.0417217201	.0417867958
.60	3.CC	.0067143766	.0063458894
.60	4.CC	.0010805607	.0009234410
.60	5.CC	.0001738972	.0001291203
.60	6.CC	.0000279857	.0000172265
.60	7.CC	.0000045038	.0000021514
.60	8.CC	.0000007248	.0000002403
.60	9.CC	.0000001166	.0000000209
.60	10.CC	.0000000188	.0000000004
.70	1.CC	.2844241773	.2831948323
.70	2.CC	.0457730657	.0458391411
.70	3.CC	.0073663694	.0069612445
.70	4.CC	.0011854875	.0010129730
.70	5.CC	.0001907834	.0001416362
.70	6.CC	.0000307032	.0000188957
.70	7.CC	.0000049411	.0000023598
.70	8.CC	.0000007952	.0000002635
.70	9.CC	.0000001280	.0000000229
.70	10.CC	.0000000206	.0000000004
.80	1.CC	.3155926656	.3138030423
.80	2.CC	.0507890854	.0507939150
.80	3.CC	.0081736095	.0077130675
.80	4.CC	.0013153986	.0011222055

.80	5.00	.0002116903	.0001568719
.80	6.00	.0000340678	.0000209204
.80	7.00	.0000054826	.0000026110
.80	8.00	.0000008823	.0000002912
.80	9.00	.0000001420	.0000000252
.80	10.00	.0000000229	.0000000004
.90	1.00	.3537984516	.3502209330
.90	2.00	.0569376343	.0567867441
.90	3.00	.0091631102	.0086216603
.90	4.00	.0014746413	.0012540120
.90	5.00	.0002373176	.0001752114
.90	6.00	.0000381921	.0000233482
.90	7.00	.0000061463	.0000029102
.90	8.00	.0000009891	.0000003238
.90	9.00	.0000001592	.0000000278
.90	10.00	.0000000256	.0000000004
1.00	1.00	.4003200470	.3952467827
1.00	2.00	.0644244663	.0639826888
1.00	3.00	.0103679340	.0097117178
1.00	4.00	.0016685446	.0014118844
1.00	5.00	.0002685229	.0001971205
1.00	6.00	.0000432140	.0000262365
1.00	7.00	.0000069545	.0000032636
1.00	8.00	.0000011192	.0000003616
1.00	9.00	.0000001801	.0000000307
1.00	10.00	.0000000290	.0000000004

R = 3.1000
E(0)=-Z*Z/2 = -.7291982028533 E1=EGM= -.8965950471?2
E(1) = EGM + Z*Z/2 = -.1673970?18639
E2 = -.0001319528? E(2) = -.8967262424?4
E3 = .0000034084?? E(3) = -.8967228339?3

R = 3.2000
E(0)=-Z*Z/2 = -.7175000422440 E1=EGM= -.8831169?1560
E(1) = EGM + Z*Z/2 = -.1654164791?1
E2 = -.0001287220?5 E(2) = -.8832456236?5
E3 = .0000037?81?9 E(3) = -.8832423454?6

R = 3.3000
E(0)=-Z*Z/2 = -.7064388?4269 E1=EGM= -.8702907974?1
E(1) = EGM + Z*Z/2 = -.1638519232?2
E2 = -.0001266291?0 E(2) = -.8704174265?1

E3 = •000CC3156529 E(3) = -•87C41427C062

R = 3.4000
E(0)=-Z•Z/2 -•695972968457 E1=EGM= -•858C79636772
E(1) = EGM + Z•Z/2 = -•1691C6668315
E2 = -•0C0124887588 E(2) = -•858204524360
E3 = •000CC3C43108 E(3) = -•858201481252

R = 3.5000
E(0)=-Z•Z/2 -•686C65717297 E1=EGM= -•846449102219
E(1) = EGM + Z•Z/2 = -•16?383384922
E2 = -•000123469551 E(2) = -•846572571769
E3 = •000CC2937555 E(3) = -•8465696?4214

R = 3.6000
E(0)=-Z•Z/2 -•6766?3189832 E1=EGM= -•835367380733
E(1) = EGM + Z•Z/2 = -•15?684190901
E2 = -•000122347779 E(2) = -•835489728512
E3 = •000CC2?38509 E(3) = -•835486689009

R = 3.7000
E(0)=-Z•Z/2 -•667794186120 E1=EGM= -•82480491677C
E(1) = EGM + Z•Z/2 = -•157?1C73065C
E2 = -•000121495782 E(2) = -•824926412551
E3 = •000CC2748548 E(3) = -•824923663963

R = 3.8000
E(0)=-Z•Z/2 -•659369950588 E1=EGM= -•814734192574
E(1) = EGM + Z•Z/2 = -•15?364241986
E2 = -•000120?88340 E(2) = -•814855C8C914
E3 = •000CC2664550 E(3) = -•814852416364

R = 3.9000
E(0)=-Z•Z/2 -•651383919185 E1=EGM= -•8C5129532272
E(1) = EGM + Z•Z/2 = -•157?45613087
E2 = -•000120499140 E(2) = -•80525C031412

E3 = •0000C2586737 E(3) = -•805247444625

R = 4.0000

E(0)=-Z•Z/2 -•643811495879 E1=EGM= -•795966927219

E(1) = EGM + Z•Z/2 = -•152155431340

E2 = -•0001203C4695 E(2) = -•796087231914

E3 = •0000C2515C29 E(3) = -•796C84716832

MU	LAMBDA	PHI GM	PHI(1)
•00	1•CC	•1285769590	•1268292817
•00	2•CC	•0132906143	•C130180891
•00	3•CC	•0013738109	•0012318849
•00	4•CC	•0001420067	•CCC1C96725
•00	5•CC	•0000146788	•CCCCC91246
•00	6•CC	•0000015173	•CCCCC06870
•00	7•CC	•0000001568	•0000000424
•00	8•CC	•0000000162	•0000000013
•00	9•CC	•0000000017	-•0000000002
•00	10•CC	•0000000002	-•0000000001
•10	1•CC	•1302707320	•1286118405
•10	2•CC	•0134656945	•C132011366
•10	3•CC	•0013919084	•0012493075
•10	4•CC	•0001438774	•CCC1112407
•10	5•CC	•0000148722	•0000092575
•10	6•CC	•0000015373	•CCCCC06974
•10	7•CC	•0000001589	•0000000431
•10	8•CC	•0000000164	•0000000013
•10	9•CC	•0000000017	-•0000000002
•10	10•CC	•0000000002	-•0000000001
•20	1•CC	•1354396637	•1340352129
•20	2•CC	•0139999915	•0137580410
•20	3•CC	•0014471371	•0013022999
•20	4•CC	•0001495862	•CCC1160083
•20	5•CC	•0000154623	•CCCCC96614
•20	6•CC	•0000015983	•0000007288
•20	7•CC	•0000001652	•0000000452
•20	8•CC	•0000000171	•0000000014
•20	9•CC	•0000000018	-•0000000002
•20	10•CC	•0000000002	-•0000000001
•30	1•CC	•1443511242	•1433C3415
•30	2•CC	•0149211424	•0147124829
•30	3•CC	•0015423537	•0013930735
•30	4•CC	•0001594285	•CCC124167C
•30	5•CC	•0000164796	•0000103515
•30	6•CC	•0000017035	•0000007824
•30	7•CC	•0000001761	•0000000488

APPENDIX C, PART 2: INTEGRAL REPRESENTATION RESULTS 210

.30	8.00	.0000000122	.0000000015
.30	9.00	.0000000019	-.0000000002
.30	10.00	.0000000002	-.0000000001
.40	1.00	.1574660709	.1568950752
.40	2.00	.0162767951	.0161052576
.40	3.00	.0016824834	.0015254372
.40	4.00	.0001739133	.0001360473
.40	5.00	.0000179769	.0000113539
.40	6.00	.0000018582	.0000008599
.40	7.00	.0000001921	.0000000539
.40	8.00	.0000000199	.0000000018
.40	9.00	.0000000021	-.0000000002
.40	10.00	.0000000002	-.0000000001
.50	1.00	.1754628922	.1753139631
.50	2.00	.0181370725	.0179963034
.50	3.00	.0018747747	.0017049866
.50	4.00	.0001937898	.0001521344
.50	5.00	.0000200315	.0000127071
.50	6.00	.0000020706	.0000009639
.50	7.00	.0000002140	.0000000607
.50	8.00	.0000000221	.0000000020
.50	9.00	.0000000023	-.0000000002
.50	10.00	.0000000002	-.0000000001
.60	1.00	.1992724983	.1993868883
.60	2.00	.0205982000	.0204676366
.60	3.00	.0021291741	.0019393758
.60	4.00	.0002200863	.0001730916
.60	5.00	.0000227497	.0000144638
.60	6.00	.0000023516	.0000010980
.60	7.00	.0000002431	.0000000692
.60	8.00	.0000000251	.0000000023
.60	9.00	.0000000026	-.0000000003
.60	10.00	.0000000003	-.0000000001
.70	1.00	.2301264735	.2301677716
.70	2.00	.0237874828	.0236273178
.70	3.00	.0024588407	.0022386852
.70	4.00	.0002541630	.0001997916
.70	5.00	.0000262721	.0000166929
.70	6.00	.0000027157	.0000012670
.70	7.00	.0000002807	.0000000798
.70	8.00	.0000000290	.0000000027
.70	9.00	.0000000030	-.0000000003
.70	10.00	.0000000003	-.0000000001
.80	1.00	.2696207817	.2690152371
.80	2.00	.0278698909	.0276146435
.80	3.00	.0028808270	.0026159013
.80	4.00	.0002977824	.0002333576
.80	5.00	.0000307809	.0000194831

.80	6.00	.0000031817	.0000014767
.80	7.00	.0000003289	.0000000927
.80	8.00	.0000000340	.0000000031
.80	9.00	.0000000035	-.0000000004
.80	10.00	.0000000004	-.0000000001
.90	1.00	.3197983199	.3176578984
.90	2.00	.0330565924	.0326068357
.90	3.00	.0034169607	.0030875356
.90	4.00	.0003532010	.0002752157
.90	5.00	.0000365093	.0000229466
.90	6.00	.0000037739	.0000017347
.90	7.00	.0000003901	.0000001083
.90	8.00	.0000000403	.0000000034
.90	9.00	.0000000042	-.0000000005
.90	10.00	.0000000004	-.0000000001
1.00	1.00	.3832545894	.3782777917
1.00	2.00	.0396158765	.0388275903
1.00	3.00	.0040949742	.0036744147
1.00	4.00	.0004232852	.0003271621
1.00	5.00	.0000437537	.0000272245
1.00	6.00	.0000045227	.0000020505
1.00	7.00	.0000004675	.0000001268
1.00	8.00	.0000000483	.0000000038
1.00	9.00	.0000000050	-.0000000006
1.00	10.00	.0000000005	-.0000000002

R = 4.1000
E(0)=-Z*Z/2 = -.4366[illegible]54719 E1=E_{GM}= -.787223880310
E(1) = E_{GM} + Z*Z/2 = -.150594025591
F2 = -.0001208[illegible]1525 F(2) = -.787344161835
E3 = .0000024490[illegible]72 E(3) = -.787341712763

R = 4.2000
E(0)=-Z*Z/2 = -.4298177642[illegible] E1=E_{GM}= -.778879267265
E(1) = E_{GM} + Z*Z/2 = -.149061502985
F2 = -.0001747082[illegible] F(2) = -.779053975407
E3 = .0000228690976 E(3) = -.779031384431

R = 4.3000
E(0)=-Z*Z/2 = -.4233554317[illegible] E1=E_{GM}= -.770913213122
E(1) = E_{GM} + Z*Z/2 = -.147557781345
F2 = -.0001206[illegible]9419 F(2) = -.771033872541
E3 = .0000022332719 E(3) = -.771031539822

R = 4.4000
E(0)=-Z*Z/2 =-.1722496453 9 E1=EGM= -.763306982379
E(1) = EGM + Z*Z/2 = -.144082617840
E2 = -.000121217999 E(2) = -.763428000378
E3 = .000002281684 E(3) = -.763425718695

R = 4.5000
E(0)=-Z*Z/2 =-.1140729468 75 E1=EGM= -.756042881394
E(1) = EGM + Z*Z/2 = -.144635634519
E2 = -.000121462991 E(2) = -.756164344385
E3 = .000002234966 E(3) = -.756162109419

R = 4.6000
E(0)=-Z*Z/2 =-.0758578306 36 E1=EGM= -.749104171798
E(1) = EGM + Z*Z/2 = -.149216341162
E2 = -.000121975695 E(2) = -.749226147493
E3 = .000002192250 E(3) = -.749223955243

R = 4.7000
E(0)=-Z*Z/2 =-.0506504360 7 E1=EGM= -.742474993795
E(1) = EGM + Z*Z/2 = -.141824155788
E2 = -.000122538509 E(2) = -.742597533304
E3 = .000002163233 E(3) = -.742595379071

R = 4.8000
E(0)=-Z*Z/2 =-.3956318758 35 E1=EGM= -.736140298324
E(1) = EGM + Z*Z/2 = -.140458423090
E2 = -.000123134966 E(2) = -.736263433290
E3 = .000002117629 E(3) = -.736261315661

R = 4.9000
E(0)=-Z*Z/2 =-.3909673561 78 E1=EGM= -.730085787152
E(1) = EGM + Z*Z/2 = -.139118430974
E2 = -.000123749769 E(2) = -.730209536915
E3 = .000002085169 E(3) = -.730207451746

R = 5.0000
E(0)=-Z*Z/2 =-.5864344346E7 E1=EGM= -.72429786CC35
E(1) = EGM + Z*Z/2 = -.1378C3425368
E2 = -.0001243687E0 E(2) = -.724422228814
E3 = .0000020556CC E(3) = -.724420173214

MU	LAMBDA	PHI GM	PHI(1)
.00	1.CC	.0834C92542	.0819185C44
.00	2.CC	.0055630360	.0053387706
.00	3.CC	.0037103C4	.0003158834
.00	4.CC	.0000247461	.0000172231
.00	5.CC	.0000165C5	.0000C08431
.00	6.CC	.0000011101	.0000C00336
.00	7.CC	.0000000073	.0000C00006
.00	8.CC	.0000000005	-.0000000001
.00	9.CC	.0000000000	-.0000000000
.00	10.CC	.0000000000	-.0000000000
.10	1.CC	.0851064519	.0837226189
.10	2.CC	.0056762317	.0054565571
.10	3.CC	.0037858C1	.0003229215
.10	4.CC	.0000252497	.0000176143
.10	5.CC	.0000016840	.0000008630
.10	6.CC	.0000001123	.0000000345
.10	7.CC	.0000000075	.0000000006
.10	8.CC	.0000000005	-.0000000001
.10	9.CC	.0000000000	-.0000000000
.10	10.CC	.0000000000	-.0000000000
.20	1.CC	.0803232315	.0892475213
.20	2.CC	.0060241683	.0058172361
.20	3.CC	.0004017859	.0003444636
.20	4.CC	.0000267974	.0000188106
.20	5.CC	.0000017873	.0000009237
.20	6.CC	.0000001192	.0000000371
.20	7.CC	.0000000080	.0000000007
.20	8.CC	.0000000005	-.0000000001
.20	9.CC	.0000000000	-.0000000000
.20	10.CC	.0000000000	-.0000000000
.30	1.CC	.0894443869	.0988392301
.30	2.CC	.0066325099	.0064433099
.30	3.CC	.0004423597	.0003818251
.30	4.CC	.0000295035	.0000208820
.30	5.CC	.0000019678	.0000010284
.30	6.CC	.0000001312	.0000000416
.30	7.CC	.0000000088	.0000000008
.30	8.CC	.0000000006	-.0000000001

.30	9.CC	.0000000000	-.0000000000
.30	10.CC	.0000000000	-.0000000000
.40	1.CC	.1131427015	.1131025812
.40	2.CC	.0075461281	.0073741114
.40	3.CC	.0005032941	.0004373052
.40	4.CC	.0000335675	.0000239508
.40	5.CC	.0000022388	.0000011429
.40	6.CC	.0000001493	.0000000482
.40	7.CC	.0000000100	.0000000010
.40	8.CC	.0000000007	-.0000000001
.40	9.CC	.0000000000	-.0000000000
.40	10.CC	.0000000000	-.0000000000
.50	1.CC	.1324285740	.1329448046
.50	2.CC	.0088324123	.0086686365
.50	3.CC	.0005890836	.0005143518
.50	4.CC	.0000392893	.0000282005
.50	5.CC	.0000026204	.0000013957
.50	6.CC	.0000001748	.0000000571
.50	7.CC	.0000000117	.0000000012
.50	8.CC	.0000000008	-.0000000001
.50	9.CC	.0000000001	-.0000000000
.50	10.CC	.0000000000	-.0000000000
.60	1.CC	.1587245452	.1596399019
.60	2.CC	.0105862397	.0104097267
.60	3.CC	.0007060563	.0006178047
.60	4.CC	.0000470909	.0000338882
.60	5.CC	.0000031408	.0000016787
.60	6.CC	.0000002095	.0000000689
.60	7.CC	.0000000140	.0000000015
.60	8.CC	.0000000009	-.0000000001
.60	9.CC	.0000000001	-.0000000000
.60	10.CC	.0000000000	-.0000000000
.70	1.CC	.1939702269	.1949182869
.70	2.CC	.0129369740	.0127098899
.70	3.CC	.0008628401	.0007542330
.70	4.CC	.0000575477	.0000413626
.70	5.CC	.0000038382	.0000020481
.70	6.CC	.0000002560	.0000000839
.70	7.CC	.0000000171	.0000000018
.70	8.CC	.0000000011	-.0000000001
.70	9.CC	.0000000001	-.0000000000
.70	10.CC	.0000000000	-.0000000000
.80	1.CC	.2407653680	.2410384339
.80	2.CC	.0160580072	.0157192003
.80	3.CC	.0010709995	.0009323935
.80	4.CC	.0000714310	.0000510880
.80	5.CC	.0000047641	.0000025253
.80	6.CC	.0000003177	.0000001031

.80	7.CC	.00CCCCC212	.C000000022
.80	8.00	.0000000014	-.C000000002
.80	9.CC	.00C0000001	-.C000000000
.80	10.C0	.0000000000	-.C000000000
.90	1.CC	.3025616150	.3011999459
.90	2.CC	.0201795492	.C196358872
.90	3.CC	.0013456885	.C011638440
.90	4.CC	.0000897649	.0000636762
.90	5.CC	.00CCC59869	.C000031385
.90	6.CC	.0000003993	.C000001272
.90	7.CC	.0000000266	.C000000026
.90	8.CC	.0000000018	-.0000000002
.90	9.CC	.00C0000001	-.C000000000
.90	10.CC	.0000000000	-.C000000000
1.00	1.CC	.3839171075	.3792607035
1.00	2.CC	.0256056082	.0247204367
1.00	3.CC	.0017C77832	.0014637615
1.00	4.CC	.0001139017	.C000799291
1.00	5.CC	.0000075968	.C000039245
1.00	6.CC	.0000005067	.0000001575
1.00	7.CC	.0000000338	.0000000030
1.00	8.CC	.0000000023	-.0000000003
1.00	9.CC	.0000000002	-.0000000001
1.00	10.CC	.0000000000	-.0000000000

R = 5.1000
E(0)=-Z*Z/2 = -.582250944757 E1=EGM= -.718763568150
E(1) = EGM + Z*Z/2 = -.134512623393
E2 = -.000124979090 E(2) = -.718888547240
E3 = .000CC2C28636 E(3) = -.718886512554

R = 5.2000
E(0)=-Z*Z/2 = -.578225348066 E1=EGM= -.713470573081
E(1) = EGM + Z*Z/2 = -.135245225015
E2 = -.000125568959 E(2) = -.713596142039
E3 = .000CC2CC4206 E(3) = -.713594137833

R = 5.3000
E(0)=-Z*Z/2 = -.574406687418 E1=EGM= -.7C8407110655
E(1) = EGM + Z*Z/2 = -.134000423237
E2 = -.000126127835 E(2) = -.708533236490
E3 = .000CC198195R E(3) = -.708531256532

R = 5.4000
E(0)=-Z*Z/2 =-.57C764546116 E1=EGM= =-.7C3561959031
E(1) = EGM + Z*Z/2 = =-.132777412915
E2 = =-.000126646330 E(2) = =-.703688605361
E3 = .000001961752 E(3) = =-.703686643609

R = 5.5000
E(0)=-Z*Z/2 =-.567349012168 E1=EGM= =-.698924410440
E(1) = EGM + Z*Z/2 = =-.131575398272
E2 = =-.000127116190 E(2) = =-.699051526631
E3 = .000001943415 E(3) = =-.699049583216

R = 5.6000
E(0)=-Z*Z/2 =-.564090646878 E1=EGM= =-.694484246077
E(1) = EGM + Z*Z/2 = =-.130393599199
E2 = =-.000127530259 E(2) = =-.694611776336
E3 = .000001925706 E(3) = =-.694609849551

R = 5.7000
E(0)=-Z*Z/2 =-.561000457238 E1=EGM= =-.690231713640
E(1) = EGM + Z*Z/2 = =-.129231256402
E2 = =-.000127882431 E(2) = =-.690359596071
E3 = .000001911717 E(3) = =-.690357684355

R = 5.8000
E(0)=-Z*Z/2 =-.558069871592 E1=EGM= =-.686157507110
E(1) = EGM + Z*Z/2 = =-.128087635518
E2 = =-.000128167625 E(2) = =-.686285674734
E3 = .000001898940 E(3) = =-.686283776654

R = 5.9000
E(0)=-Z*Z/2 =-.555290718111 E1=EGM= =-.682252748372
E(1) = EGM + Z*Z/2 = =-.124962030261
E2 = =-.000128381622 E(2) = =-.682381129995
E3 = .000001885726 E(3) = =-.682379244269

R = 6.0000
E(0)=-7+7/2 =-.552655EC5633 E1=EGM= -.678508970357
E(1) = EGM + Z+7/2 = -.1254253764724
E2 = -.000128521214 E(2) = -.678637491571
E3 = .000C01874549 E(3) = -.678635617008

MU	LAMBDA	PHI GM	PHI(1)
.00	1.00	.0530513737	.0517217542
.00	2.00	.0022642610	.0021182497
.00	3.00	.0000966399	.0000776241
.00	4.00	.0000041246	.0000025575
.00	5.00	.0000001760	.0000000712
.00	6.00	.0000000075	.0000000013
.00	7.00	.0000000003	-.0000000000
.00	8.00	.0000000000	-.0000000000
.00	9.00	.0000000000	-.0000000000
.00	10.00	.0000000000	-.0000000000
.10	1.00	.0546229170	.0534058411
.10	2.00	.0023313353	.0021874832
.10	3.00	.0000995026	.0000802003
.10	4.00	.0000042468	.0000026451
.10	5.00	.0000001813	.0000000739
.10	6.00	.0000000077	.0000000013
.10	7.00	.0000000003	-.0000000000
.10	8.00	.0000000000	-.0000000000
.10	9.00	.0000000000	-.0000000000
.10	10.00	.0000000000	-.0000000000
.20	1.00	.0594951811	.0586040703
.20	2.00	.0025392862	.0024011485
.20	3.00	.0001083781	.0000881456
.20	4.00	.0000046256	.0000029148
.20	5.00	.0000001974	.0000000819
.20	6.00	.0000000084	.0000000015
.20	7.00	.0000000004	-.0000000000
.20	8.00	.0000000000	-.0000000000
.20	9.00	.0000000000	-.0000000000
.20	10.00	.0000000000	-.0000000000
.30	1.00	.0648156881	.0677692750
.30	2.00	.0029089721	.0027777505
.30	3.00	.0001241565	.0001021319
.30	4.00	.0000052991	.0000033884
.30	5.00	.0000002262	.0000000959
.30	6.00	.0000000097	.0000000018
.30	7.00	.0000000004	-.0000000000
.30	8.00	.0000000000	-.0000000000
.30	9.00	.0000000000	-.0000000000

.30	10.00	.0000000000	-.0000000000
.40	1.00	.0814768285	.0817067872
.40	2.00	.0034774746	.0033501993
.40	3.00	.0001484205	.0001233546
.40	4.00	.0000063347	.0000041045
.40	5.00	.0000002704	.0000001170
.40	6.00	.0000000115	.0000000023
.40	7.00	.0000000005	-.0000000000
.40	8.00	.0000000000	-.0000000000
.40	9.00	.0000000000	-.0000000000
.40	10.00	.0000000000	-.0000000000
.50	1.00	.1007910882	.1016540056
.50	2.00	.0043018176	.0041690597
.50	3.00	.0001836039	.0001536498
.50	4.00	.0000078363	.0000051225
.50	5.00	.0000003345	.0000001466
.50	6.00	.0000000143	.0000000029
.50	7.00	.0000000006	-.0000000000
.50	8.00	.0000000000	-.0000000000
.50	9.00	.0000000000	-.0000000000
.50	10.00	.0000000000	-.0000000000
.60	1.00	.1280369824	.1294009187
.60	2.00	.0054646869	.0053074698
.60	3.00	.0002332358	.0001956722
.60	4.00	.0000099546	.0000065280
.60	5.00	.0000004249	.0000001871
.60	6.00	.0000000181	.0000000037
.60	7.00	.0000000008	-.0000000000
.60	8.00	.0000000000	-.0000000000
.60	9.00	.0000000000	-.0000000000
.60	10.00	.0000000000	-.0000000000
.70	1.00	.1659474195	.1674636571
.70	2.00	.0070827246	.0068682210
.70	3.00	.0003022945	.0002531505
.70	4.00	.0000129021	.0000084414
.70	5.00	.0000005507	.0000002417
.70	6.00	.0000000235	.0000000047
.70	7.00	.0000000010	-.0000000001
.70	8.00	.0000000000	-.0000000000
.70	9.00	.0000000000	-.0000000000
.70	10.00	.0000000000	-.0000000000
.80	1.00	.2183250186	.2193292609
.80	2.00	.0093182286	.0085937440
.80	3.00	.0003977070	.0003312473
.80	4.00	.0000169745	.0000110287
.80	5.00	.0000007245	.0000003147
.80	6.00	.0000003069	.0000000061
.80	7.00	.0000000013	-.0000000001

.80	8.CC	.0CCCCCCCC1	-.CCCCCCCCCC
.80	9.CC	.00CCCCCCCC	-.0000000000
.80	10.CC	.CCCCCCCCCC	-.CCCCCCCCCC
.90	1.00	.2904235319	.2897974644
.90	2.CC	.0123954317	.0118800541
.90	3.CC	.0005290437	.0004370628
.90	4.CC	.0000225799	.0000145182
.90	5.CC	.0000009637	.0000004121
.90	6.CC	.0000000411	.0000000078
.90	7.00	.0000000018	-.0000000001
.90	8.CC	.0000000001	-.0000000000
.90	9.00	.0000000000	-.0000000000
.90	10.CC	.0000000000	-.0000000000
1.00	1.CC	.3894748249	.3854553736
1.00	2.CC	.0166229939	.0157961246
1.00	3.CC	.0007094783	.0005803341
1.00	4.CC	.0000302809	.0000192225
1.00	5.CC	.0000012924	.0000005421
1.00	6.CC	.0000000552	.0000000100
1.00	7.CC	.0000000024	-.0000000002
1.00	8.CC	.0000000001	-.0000000000
1.00	9.CC	.0000000000	-.0000000000
1.00	10.CC	.0000000000	-.0000000000

R = 6.1000
E(0)=-Z*Z/2 = -.55C1559C648C E1=EGM= -.6749181C1390
E(1) = EGM + Z*Z/2 = -.124762194910
E2 = -.0001286839?? E(2) = -.675046685322
E3 = .000001R64477 E(3) = -.675044820845

R = 6.2000
E(0)=-Z*Z/2 = -.5477?5740912 E1=EGM= -.67147245C517
E(1) = EGM + Z*Z/2 = -.123686709605
E2 = -.0001285620?0 E(2) = -.6716C1?166C7
E3 = .000001R553?R E(3) = -.671599163239

R = 6.3000
E(0)=-Z*Z/2 = -.5455?79629C2 E1=EGM= -.668164693595
E(1) = EGM + Z*Z/2 = -.122626730693
E2 = -.0001284?8607 E(2) = -.668293166292
E3 = .000001R47148 E(3) = -.668291319144

APPENDIX C, PART 2: INTEGRAL REPRESENTATION RESULTS 220

R = 6.4000
F(0)=-7.7/2 =-.5434[illegible]6146982 E1=EGM= -.664987859963
E(1) = EGM + Z.7/2 = -.121581712981
E2 = -.000128[illegible]97391 E(2) = -.665116157363
E3 = .000001839739 E(3) = -.665114317621

R = 6.5000
F(0)=-7.7/2 =-.5413[illegible]175922 E1=EGM= -.661935319562
E(1) = EGM + Z.7/2 = -.120651143645
E2 = -.000128[illegible]423[illegible] E(2) = -.662063361948
E3 = .000001[illegible]33043 E(3) = -.662061528905

R = 6.6000
F(0)=-7.7/2 =-.5394[illegible]220079 E1=EGM= -.659000770439
E(1) = EGM + Z.7/2 = -.119534541360
E2 = -.000127[illegible]83[illegible] E(2) = -.659128478822
E3 = .000001827010 E(3) = -.659126651812

R = 6.7000
F(0)=-7.7/2 =-.5376[illegible]771239 E1=EGM= -.656178226425
E(1) = EGM + Z.7/2 = -.118531455185
E2 = -.000127[illegible]965[illegible] E(2) = -.656305522987
E3 = .000001821568 E(3) = -.656303701419

R = 6.8000
F(0)=-7.7/2 =-.5359[illegible]541859 E1=EGM= -.653462005137
E(1) = EGM + Z.7/2 = -.117541463278
E2 = -.000126[illegible]8478 E(2) = -.653588813615
E3 = .000001816655 E(3) = -.653586996960

R = 6.9000
F(0)=-7.7/2 =-.5342[illegible]644601 E1=EGM= -.650846716076
E(1) = EGM + Z.7/2 = -.116564171474
E2 = -.000126[illegible]6039 E(2) = -.650972962107
E3 = .000001812216 E(3) = -.650971149891

R = 7.0000
E(0)=-Z/2 =-.5327[illegible]37109 E1=EGM= -.648327248902
E(1) = EGM + Z**2/2 = -.115599211793
E2 = -.000125611416 E(2) = -.648452866318
E3 = .000018082C1 E(3) = -.648451052117

MU	LAMBDA	PHI GM	PHI(1)
.00	1.CC	-.331159938	.0315120896
.00	2.C0	-.0008934043	.0008110744
.00	3.CC	-.0000241083	.0000181828
.00	4.CC	-.0000006502	.0000003560
.00	5.CC	-.0000000175	.0000000054
.00	6.CC	-.0000000005	.0000000000
.00	7.CC	-.0000000000	-.0000000000
.00	8.CC	-.0000000000	-.0000000000
.00	9.CC	-.0000000000	-.0000000000
.00	10.C0	-.0000000000	-.0000000000
.10	1.CC	-.0344741828	.0333779715
.10	2.CC	-.0009300454	.0008485765
.10	3.CC	-.0000250908	.0000190429
.10	4.CC	-.0000006769	.0000003737
.10	5.CC	-.0000000183	.0000000057
.10	6.C0	-.0000000005	.0000000000
.10	7.CC	-.0000000000	-.0000000000
.10	8.CC	-.0000000000	-.0000000000
.10	9.CC	-.0000000000	-.0000000000
.10	10.C0	-.0000000000	-.0000000000
.20	1.CC	-.0357279362	.0379450494
.20	2.CC	-.0010448033	.0009653839
.20	3.CC	-.0000281867	.0000217191
.20	4.CC	-.0000007604	.0000004286
.20	5.CC	-.0000002C5	.0000000066
.20	6.CC	-.0000000006	.0000000000
.20	7.CC	-.0000000000	-.0000000000
.20	8.C0	-.0000000000	-.0000000000
.20	9.CC	-.0000000000	-.0000000000
.20	10.C0	-.0000000000	-.0000000000
.30	1.CC	-.0464385178	.0461446911
.30	2.C0	-.0012528195	.0011749870
.30	3.CC	-.0000337986	.0000265127
.30	4.0C	-.0000009118	.0000005266
.30	5.CC	-.0000000246	.0000000083
.30	6.00	-.0000000007	.0000000000
.30	7.CC	-.0000000000	-.0000000000
.30	8.00	-.0000000000	-.0000000000
.30	9.CC	-.0000000000	-.0000000000
.30	10.C0	-.0000000000	-.0000000000

•40	1•CC	•0586232937	•C589411622
•40	2•CC	•0015815407	•0015C18684
•40	3•CC	•00C0426668	•CCOC339705
•40	4•00	•00C0011511	•C000C06782
•40	5•CC	•00CCCC0311	•C000C00108
•40	6•0C	•00C0000008	•C000000001
•40	7•CC	•00CCCCCCCC	-•CCCCCCCCC
•40	8•CC	•00C0000000	-•CCCCCCCCC
•40	9•CC	•00CCCCCCCC	-•CCCCCCCCC
•40	10•CC	•00C0000000	-•CCCCCCCCC
•50	1•CC	•0768899737	•C778574327
•50	2•0C	•0020743397	•0019846945
•50	3•CC	•00CC559616	•CCCC449562
•50	4•CC	•00C0015097	•000C0C9004
•50	5•CC	•00CCCC04C7	•C000C00145
•50	6•CC	•00C0000011	•C000000001
•50	7•CC	•00CCCCCCCC	-•CCCCCCCCC
•50	8•00	•00C0000000	-•CCCCCCCCC
•50	9•CC	•00CCCCCCCC	-•CCCCCCCCC
•50	10•00	•00C0000000	-•CCCCCCCCC
•60	1•CC	•1036487395	•1051712647
•60	2•0C	•0027962384	•0C26812868
•60	3•CC	•00CC75437C	•CCCC6C7605
•60	4•CC	•00CCC20351	•C000012180
•60	5•CC	•00CCCC0549	•C000C00196
•60	6•CC	•00C0000015	•C000000001
•60	7•CC	•00CCCCCCCC	-•CCCCCCCCC
•60	8•CC	•00C0000000	-•CCCCCCCCC
•60	9•CC	•00CCCCCCCC	-•CCCCCCCCC
•60	10•0C	•00C0000000	-•CCCCCCCCC
•70	1•CC	•1424302535	•1442081342
•70	2•CC	•0038424870	•0C3676C452
•70	3•CC	•00C1C36627	•CCCC832661
•70	4•CC	•00C0027966	•C000016676
•70	5•CC	•00CCCC0754	•C000C00268
•70	6•CC	•00C0000020	•C000000002
•70	7•CC	•00CCCC0001	-•CCCCCCCCC
•70	8•00	•00C0000000	-•CCCCCCCCC
•70	9•CC	•00CCCCCCCC	-•CCCCCCCCC
•70	10•00	•00C0000000	-•CCCCCCCCC
•80	1•CC	•1983515095	•1997714718
•80	2•00	•0053511321	•0C5C9C8541
•80	3•CC	•00C144363C	•CCC1151896
•80	4•00	•00CC038946	•C000C23016
•80	5•CC	•00CCCC1051	•C000000368
•80	6•CC	•00C0000028	•C000000002
•80	7•CC	•00CCCCCCC1	-•CCCCCCCCC
•80	8•CC	•00CC000000	-•CCCCCCCCC

.80	9.CC	.00CCCCCCCC	-.CCCCCCCCCC
.80	10.CC	.00CCCCCCCC	-.CCCCCCCCCC
.90	1.CC	.2787909898	.2787694875
.90	2.CC	.0075212305	.0071009760
.90	3.CC	.0002029079	.0001604354
.90	4.CC	.0000054741	.0000031954
.90	5.CC	.0000001477	.0000000506
.90	6.CC	.0000000040	.0000000003
.90	7.CC	.0000000001	-.CCCCCCCCCC
.90	8.CC	.0000000000	-.CCCCCCCCCC
.90	9.CC	.0000000000	-.CCCCCCCCCC
.90	10.CC	.0000000000	-.CCCCCCCCCC
1.00	1.CC	.3943622131	.3911246516
1.00	2.CC	.0106391139	.0099581229
1.00	3.CC	.0002870223	.0002246088
1.00	4.CC	.0000077433	.0000044570
1.00	5.CC	.0000020889	.0000000699
1.00	6.CC	.0000000056	.0000000004
1.00	7.CC	.0000000002	-.CCCCCCCCCC
1.00	8.CC	.0000000000	-.CCCCCCCCCC
1.00	9.CC	.0000000000	-.CCCCCCCCCC
1.00	10.CC	.0000000000	-.CCCCCCCCCC

R = 7.1000
E(0)=-Z*Z/2 -.53125252C961 E1=E_{GM}= -.645898761864
E(1) = E_{GM} + Z*Z/2 = -.114462409C3
E2 = -.00012907072 E(2) = -.646023668936
E3 = .0000C1804562 E(3) = -.6460218E4374

R = 7.2000
E(0)=-Z*Z/2 -.529851731800 E1=E_{GM}= -.64355667C376
E(1) = E_{GM} + Z*Z/2 = -.113704936576
E2 = -.00012413564E Z(2) = -.643680806022
E3 = .0000C1EC1254 E(3) = -.643679C04767

R = 7.3000
E(0)=-Z*Z/2 -.528521629596 E1=E_{GM}= -.641296635755
E(1) = E_{GM} + Z*Z/2 = -.113775C06162
E2 = -.00012329995? E(2) = -.641419935710
E3 = .0000C1798243 E(3) = -.641418137466

R = 7.4000
E(0)=-7*Z/2 -.527258389070 E1=EGM= -.639114554160
E(1) = EGM + Z*Z/2 = -.111256165091
E2 = -.000122402934 E(2) = -.639236957097
E3 = .000001795490 E(3) = -.639235161607

R = 7.5000
E(0)=-7*Z/2 -.526058389257 E1=EGM= -.637006545694
E(1) = EGM + Z*Z/2 = -.110948155436
E2 = -.000121447794 E(2) = -.637127993488
E3 = .000001792993 E(3) = -.637126200495

R = 7.6000
E(0)=-7*Z/2 -.524918209245 E1=EGM= -.634968943778
E(1) = EGM + Z*Z/2 = -.110650734533
E2 = -.000120437305 E(2) = -.635089381083
E3 = .000001790654 E(3) = -.635087590430

R = 7.7000
E(0)=-7*Z/2 -.523834409081 E1=EGM= -.632998284740
E(1) = EGM + Z*Z/2 = -.110363675660
E2 = -.000117266584 E(2) = -.633115551324
E3 = .000001042769 E(3) = -.633114508555

R = 7.8000
E(0)=-7*Z/2 -.522804530877 E1=EGM= -.631091297682
E(1) = EGM + Z*Z/2 = -.110086766804
E2 = -.000118263579 E(2) = -.631209561260
E3 = .000001786469 E(3) = -.631207774791

R = 7.9000
E(0)=-7*Z/2 -.521825085138 E1=EGM= -.629244894630
E(1) = EGM + Z*Z/2 = -.109819809492
E2 = -.000117106719 E(2) = -.629362001350
E3 = .000001784578 E(3) = -.629360216771

R = 8.0000
E(0)=-7+7/2 =-.62C8935433C8 E1=EGM= -.627456161001
E(1) = EGM + 2+7/2 = -.104562617694
E2 = -.0001159C743B E(2) = -.627572068439
E3 = .000CC1782734 E(3) = -.627570285643

MU	LAMBDA	PHI GM	PHI(1)
.00	1.CC	.0204715862	.0193868394
.00	2.CC	.00C3451829	.C003027132
.00	3.CC	.0000058203	.0000041089
.00	4.CC	.00C0000981	.0000000471
.00	5.CC	.00C0000017	.0000000004
.00	6.CC	.00C0000000	-.0000000000
.00	7.CC	.0000000000	-.0000000000
.00	8.CC	.00C0000000	-.0000000000
.00	9.CC	.0000000000	-.0000000000
.00	10.CC	.00C0000000	-.0000000000
.10	1.CC	.0215826579	.0205944454
.10	2.CC	.00C3639173	.C003217630
.10	3.CC	.00C0061362	.0000043761
.10	4.CC	.00C0001035	.0000000504
.10	5.CC	.0000000017	.0000000004
.10	6.CC	.00C0000000	-.0000000000
.10	7.CC	.0000000000	-.0000000000
.10	8.CC	.00C0000000	-.0000000000
.10	9.CC	.0000000000	-.0000000000
.10	10.CC	.00C0000000	-.0000000000
.20	1.00	.0251036599	.0243973323
.20	2.CC	.00C4232868	.C003817269
.20	3.CC	.0000071373	.0000052160
.20	4.CC	.00C0001203	.0000000607
.20	5.CC	.0000000020	.0000000005
.20	6.CC	.00C0000000	-.0000000000
.20	7.CC	.0000000000	-.0000000000
.20	8.CC	.00C0000000	-.0000000000
.20	9.CC	.0000000000	-.0000000000
.20	10.CC	.00C0000000	-.0000000000
.30	1.CC	.0316296915	.0313675138
.30	2.CC	.00C5333259	.C004915471
.30	3.CC	.0000089927	.0000067505
.30	4.CC	.00C0001516	.0000000795
.30	5.CC	.0000000026	.0000000007
.30	6.CC	.00C0000000	-.0000000000
.30	7.CC	.0000000000	-.0000000000
.30	8.CC	.00C0000000	-.0000000000
.30	9.CC	.0000000000	-.0000000000
.30	10.CC	.00C0000000	-.0000000000
.40	1.CC	.0422637445	.0425673697

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.40	2.CC	.00C7126326	.CC06678342
.40	3.CC	.00C0120161	.CCCC092060
.40	4.CC	.00C0002026	.CCCC001094
.40	5.CC	.00C0000034	.0000C00010
.40	6.CC	.00C0C0000C1	-.0CCCCC0000
.40	7.CC	.00000000C0	-.CC00000000
.40	8.CC	.00C0000CCC	-.CCCCCCCC00
.40	9.CC	.00C000000C	-.CCCCC00000
.40	10.CC	.00C0000000	-.CCCCCCCCC0
.50	1.CC	.0588031249	.0597274101
.50	2.CC	.00C9915124	.CCC9376444
.50	3.CC	.0000167184	.CCCC129513
.50	4.CC	.00C0CC2819	.CCCCC01546
.50	5.CC	.00C0000048	.0000C00014
.50	6.CC	.00C0C0C0C1	.CCCCCCCCCC
.50	7.CC	.00000000C0	-.0CCCCCC0000
.50	8.CC	.00C000000C	-.CCCCCCC0000
.50	9.CC	.00C0000000	-.0CCCCCC0000
.50	10.CC	.00C0C0C0CC	-.CCCCCCC0000
.60	1.CC	.0840432226	.0855322847
.60	2.CC	.0014170997	.CC13429435
.60	3.CC	.0000238945	.CCCC185581
.60	4.CC	.00C0004029	.CCCCCC2217
.60	5.CC	.00C0000068	.CCCC000020
.60	6.CC	.00C0C0C0C1	.CCCCCCCCCC
.60	7.CC	.00C0000CCC	-.CCCCCCC0000
.60	8.CC	.00C0C0C0CC	-.CCCCCCCC00
.60	9.CC	.00C0000000	-.0CCCCCC0000
.60	10.CC	.00C0C0000C	-.CCCCCCCCC0
.70	1.CC	.1222499720	.1240651595
.70	2.CC	.0020613251	.CC19475416
.70	3.CC	.0000347572	.CCCC268951
.70	4.CC	.00C0C05861	.CCCCCC3209
.70	5.CC	.00C0000099	.CCCCC00028
.70	6.CC	.00C0C0C0C2	.CCCCC00000
.70	7.CC	.00C000000C	-.CCCCCC0000
.70	8.CC	.00C0C0C0CC	-.CCCCCC0000
.70	9.CC	.00C000000C	-.CCCCCC0000
.70	10.CC	.00C0C0C0CC	-.CCCCCCCCC0
.80	1.CC	.1798808560	.1814867369
.80	2.CC	.0030330716	.CC28477243
.80	3.CC	.0000511423	.CCCC392717
.80	4.CC	.00C0008623	.CCCCC0467C
.80	5.CC	.00C0000145	.CCCCC00041
.80	6.CC	.00C000C0C2	-.CCCCC00000
.80	7.CC	.00C000000C	-.CCCCCC0000
.80	8.CC	.00C0C0C0CC	-.CCCCCC0000
.80	9.CC	.00C0000000	-.CCCCCCC000

.80	10.00	.0000000000	-.0000000000
.90	1.00	.2466763116	.2670822467
.90	2.00	.0044965783	.0041884495
.90	3.00	.0000758193	.0000576586
.90	4.00	.0000012784	.0000006829
.90	5.00	.0000000216	.0000000059
.90	6.00	.0000000004	-.0000000000
.90	7.00	.0000000000	-.0000000000
.90	8.00	.0000000000	-.0000000000
.90	9.00	.0000000000	-.0000000000
.90	10.00	.0000000000	-.0000000000
1.00	1.00	.3973060020	.3948160333
1.00	2.00	.0066991985	.0061879525
1.00	3.00	.0001129589	.0000850224
1.00	4.00	.0000019047	.0000010026
1.00	5.00	.0000000321	.0000000085
1.00	6.00	.0000000005	-.0000000000
1.00	7.00	.0000000000	-.0000000000
1.00	8.00	.0000000000	-.0000000000
1.00	9.00	.0000000000	-.0000000000
1.00	10.00	.0000000000	-.0000000000

R = 8.1000
F(0)=-7*7/2 -.520007329569 E1=EGM= -.625722346375
E(1) = EGM + Z*Z/2 = -.105715016806
E2 = -.000114668883 E(2) = -.625837015258
E3 = .000001781105 E(3) = -.625835234152

R = 8.2000
F(0)=-7*7/2 -.519164012905 E1=EGM= -.624040855607
E(1) = EGM + Z*Z/2 = -.104876842703
E2 = -.000113994161 E(2) = -.624154249769
E3 = .000001779492 E(3) = -.624152470277

R = 8.3000
F(0)=-7*7/2 -.518361299432 E1=EGM= -.622409240284
E(1) = EGM + Z*Z/2 = -.104047940852
E2 = -.000112886325 E(2) = -.622521326609
E3 = .000001777942 E(3) = -.622519548666

R = 8.4000

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F(0)=-Z*Z/2 =-.5175970250C0 E1=EGM= =-.62C8251905l9
F(1) = EGM + Z*Z/2 = =-.103228165499
F2 = =-.0001107483G1 E(2) = =-.62C935938879
E3 = .0000C1776447 E(3) = =-.62C9341624G2

R = R.5000
F(0)=-Z*Z/2 =-.5168591481193 E1=EGM= =-.6192865271C5
F(1) = EGM + Z*Z/2 = =-.102417378912
F2 = =-.0001C9883183 E(2) = =-.619395910288
E3 = .0000C1774995 E(3) = =-.619394135293

R = R.6000
F(0)=-Z*Z/2 =-.5161756743836 E1=EGM= =-.617791194018
F(1) = EGM + Z*Z/2 = =-.101615450682
F2 = =-.0001C7993627 E(2) = =-.617899187645
E3 = .0000C1773580 E(3) = =-.617897414065

R = R.7000
F(0)=-Z*Z/2 =-.5155145694192 E1=EGM= =-.6163372G1271
F(1) = EGM + Z*Z/2 = =-.100822257079
F2 = =-.0001C6582439 E(2) = =-.616443833710
E3 = .0000C1772195 E(3) = =-.616442061515

R = R.8000
F(0)=-Z*Z/2 =-.5148251876G6 E1=EGM= =-.614922868112
F(1) = EGM + Z*Z/2 = =-.100037680456
F2 = =-.0001C5152275 E(2) = =-.615028020387
E3 = .0000C1770834 E(3) = =-.615026249553

R = R.9000
F(0)=-Z*Z/2 =-.5142547078G5 E1=EGM= =-.613546316565
F(1) = EGM + Z*Z/2 = =-.09926160870l
F2 = =-.0001C3705698 E(2) = =-.613650022264
E3 = .0000C1769492 E(3) = =-.613648252772

R = 9.0000

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E(0)=-7*7/2 =-.5137?0C3C582 E1=EGM= -.6122C5965312

E(1) = EGM + Z*Z/2 = -.C9?493934731

FP = -.0001C?245173 E(2) = -.612306210485

E3 = .0000C1768167 E(3) = -.612306442318

MU	LAMBDA	PHI GM	PHI(1)
.00	1.CC	.?126417137	.C116774864
.00	2.00	.0001320883	.C001112402
.00	3.CC	.?CC013801	.C000C09071
.00	4.00	.0000000144	.0000000060
.00	5.CC	.?CC000CCC2	.CCCCCC0000
.00	6.CC	.0000000000	-.0000000000
.00	7.CC	.?CC0C00CCC	-.0CCCC00000
.00	8.00	.0000000CCC	-.C000C00000
.00	9.CC	.?CCCCCCCCC	-.0CCCCCCCCC
.00	10.00	.0000000CCC	-.C000C00000
.10	1.CC	.?135138101	.C126324527
.10	2.CC	.0001412CC5	.0001204750
.10	3.CC	.?CC0014753	.0C00C9860
.10	4.CC	.0000C00154	.C000000066
.10	5.CC	.?CC0C0CCC2	.0C00C00000
.10	6.CC	.0000000CCC	-.0000C00000
.10	7.CC	.?CC0000CCC	-.CCCCC00C00
.10	8.CC	.?000000CCC	-.C000C00000
.10	9.CC	.?CC0C0CCCC	-.C00C0C0C0C
.10	10.CC	.0000000CCC	-.C000C00000
.20	1.CC	.?163147096	.C156759583
.20	2.00	.0001704660	.C001498885
.20	3.CC	.?CC0017811	.0000012368
.20	4.00	.0000000186	.0000000084
.20	5.CC	.?CC0C00CC2	.0000C00000
.20	6.00	.0000000CCC	-.0000C00000
.20	7.CC	.?CC00C0CC	-.0C0C000000
.20	8.00	.0000000000	-.C000C00000
.20	9.CC	.?CCCC00CCC	-.C000C00000
.20	10.00	.0000000000	-.C000C00000
.30	1.CC	.?216373224	.0213839417
.30	2.00	.0002260798	.C002049930
.30	3.CC	.?CC0C23622	.0000017052
.30	4.00	.0000000247	.0000000119
.30	5.CC	.?CCCC00003	.0000000001
.30	6.00	.0000000000	-.0CCC000000
.30	7.CC	.?CC0C0000C	-.0000000000
.30	8.00	.0000C00000	-.0000C00000
.30	9.CC	.?CCCCCCCCC	-.CCCCCCCCCC
.30	10.00	.0000000000	-.0000C00000
.40	1.CC	.?3C683695	.C3C8545120
.40	2.CC	.0003198148	.0002963022

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.40	3.00	.0000033416	.0000024781
.40	4.00	.0000000349	.0000000175
.40	5.00	.0000000004	.0000000001
.40	6.00	.0000000000	-.0000000000
.40	7.00	.0000000000	-.0000000000
.40	8.00	.0000000000	-.0000000000
.40	9.00	.0000000000	-.0000000000
.40	10.00	.0000000000	-.0000000000
.50	1.00	.0451268929	.0459380392
.50	2.00	.0004715131	.0004415313
.50	3.00	.0000049267	.0000037026
.50	4.00	.0000000515	.0000000263
.50	5.00	.0000000005	.0000000001
.50	6.00	.0000000000	-.0000000000
.50	7.00	.0000000000	-.0000000000
.50	8.00	.0000000000	-.0000000000
.50	9.00	.0000000000	-.0000000000
.50	10.00	.0000000000	-.0000000000
.60	1.00	.0482662558	.0696203678
.60	2.00	.0007132871	.0006692593
.60	3.00	.0000074529	.0000056150
.60	4.00	.0000000779	.0000000399
.60	5.00	.0000000008	.0000000002
.60	6.00	.0000000000	-.0000000000
.60	7.00	.0000000000	-.0000000000
.60	8.00	.0000000000	-.0000000000
.60	9.00	.0000000000	-.0000000000
.60	10.00	.0000000000	-.0000000000
.70	1.00	.0049247297	.0066434276
.70	2.00	.0001096317	.0001248582
.70	3.00	.0000011455	.0000085906
.70	4.00	.0000001197	.0000000609
.70	5.00	.0000000013	.0000000003
.70	6.00	.0000000000	-.0000000000
.70	7.00	.0000000000	-.0000000000
.70	8.00	.0000000000	-.0000000000
.70	9.00	.0000000000	-.0000000000
.70	10.00	.0000000000	-.0000000000
.80	1.00	.0142862388	.0164992206
.80	2.00	.0017016848	.0015799961
.80	3.00	.0001778C2	.0000132216
.80	4.00	.0000001858	.0000000933
.80	5.00	.0000000019	.0000000004
.80	6.00	.0000000000	-.0000000000
.80	7.00	.0000000000	-.0000000000
.80	8.00	.0000000000	-.0000000000
.80	9.00	.0000000000	-.0000000000
.80	10.00	.0000000000	-.0000000000

.90	1.CC	.2543438C28	.2550140-28
.90	2.CC	.0026575379	.0024477E93
.90	3.CC	.0000277676	.0000C4423
.90	4.CC	.0000002901	.0000001435
.90	5.CC	.0000000C3C	.0000000007
.90	6.CC	.0000000000	-.0000000000
.90	7.CC	.0000000000	-.0000000000
.90	8.CC	.0000000000	-.0000000000
.90	9.CC	.0000000000	-.0000000000
.90	10.CC	.0000000000	-.0000000000
1.00	1.CC	.3387342779	.3968691055
1.00	2.CC	.0041662170	.0038069283
1.00	3.CC	.0000435312	.0000317E91
1.00	4.CC	.0000004548	.0000002217
1.00	5.CC	.0000000048	.0000000010
1.00	6.CC	.0000000000	-.0000000000
1.00	7.CC	.0000000000	-.0000000000
1.00	8.CC	.0000000000	-.0000000000
1.00	9.CC	.0000000000	-.0000000000
1.00	10.CC	.0000000000	-.0000000000

R = 9.1000
E(0)=-Z*Z/2 -.513165717861 E1=EGM= -.610900273888
E(1) = EGM + Z*Z/2 = -.0977345560C7
E2 = -.0001C0773063 E(2) = -.611001046952
E3 = .0000C1766855 E(3) = -.61099928C097

R = 9.2000
E(0)=-Z*Z/2 -.512644412994 E1=EGM= -.6C9627787205
E(1) = EGM + Z*Z/2 = -.0948337421C
E2 = -.0000[illegible]99291633 E(2) = -.609727076838
E3 = .0000C1765[illegible] E(3) = -.609725313223

R = 9.3000
E(0)=-Z*Z/2 -.5121[illegible]46A3[illegible]57[illegible]28 E1=EGM= -.6C8387130370
E(1) = EGM + Z*Z/2 = -.0942[illegible]0294642
E2 = -.0000[illegible]978C3C44 E(2) = -.608484933414
E3 = .0000C1764264 E(3) = -.608483169150

R = 9.4000
E(0)=-Z*Z/2 -.511671777747 E1=EGM= -.6C7177003813

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E(1) = EGM + Z*Z/2 = -.C955C5226066
E2 = -.0000C9630C9356 E(2) = -.607273313169
E3 = .000CC17629B2 E(3) = -.60727155C187

R = 9.5000
E(0)=-Z*Z/2 -.5112180984C1 E1=EGM= -.6C5996178679
E(1) = EGM + Z*Z/2 = -.C94778C80277
E2 = -.0000C94812567 E(2) = -.606090991246
E3 = .000CC1761782 E(3) = -.6060B9229525

R = 9.6000
E(0)=-Z*Z/2 -.51078472C690 E1=EGM= -.6C4843492508
E(1) = EGM + Z*Z/2 = -.C94C58771818
E2 = -.0000C93814419 E(2) = -.6049368C6927
E3 = .000CC176C443 E(3) = -.604935C46484

R = 9.7000
E(0)=-Z*Z/2 -.51037062746 9 E1=EGM= -.603717845165
E(1) = EGM + Z*Z/2 = -.C93347217696
E2 = -.0000C91816785 E(2) = -.60380966195C
E3 = .000CC175913 6 E(3) = -.603807902765

R = 9.8000
E(0)=-Z*Z/2 -.5099748578 81 E1=EGM= -.6C2618195015
E(1) = EGM + Z*Z/2 = -.C92643337135
E2 = -.0000C90321289 E(2) = -.6027C85163C5
E3 = .000CC175793 7 E(3) = -.6027C6758368

R = 9.9000
E(0)=-Z*Z/2 -.5095965040C5 E1=EGM= -.6C1543555341
E(1) = EGM + Z*Z/2 = -.C91947C51336
E2 = -.0000C88629496 E(2) = -.601632384836
E3 = .000CC1756697 E(3) = -.6016C3628139

R = 10.0000
E(0)=-Z*Z/2 -.5092347077C2 E1=EGM= -.6CC492990971

E(1) = EGM + Z**2 = -.C9125828326 9

E2 = -.000087342875 E(2) = -.600580333846

E3 = .000C1755447 E(3) = -.600578578379

MU	LAMBDA	PHI GM	PHI(1)
.00	1.CC	.0078442968	.0069990452
.00	2.CC	.0005048C1	.0004C4783
.00	3.CC	.0000003249	.0000001970
.00	4.CC	.0000000021	.0000000007
.00	5.CC	.0000000000	.0000000000
.00	6.CC	.0000000000	-.0000000000
.00	7.CC	.0000000000	-.0000000000
.00	8.CC	.0000000000	-.0000000000
.00	9.CC	.0000000000	-.0000000000
.00	10.CC	.0000000000	-.0000000000
.10	1.CC	.0085079509	.0077319545
.10	2.CC	.0005475C9	.0004448088
.10	3.CC	.0000003523	.0000002195
.10	4.CC	.0000000023	.0000000008
.10	5.CC	.0000000000	.0000000000
.10	6.CC	.0000000000	-.0000000000
.10	7.CC	.0000000000	-.0000000000
.10	8.CC	.0000000000	-.0000000000
.10	9.CC	.0000000000	-.0000000000
.10	10.CC	.0000000000	-.0000000000
.20	1.CC	.0106715068	.0100984145
.20	2.CC	.0006848674	.0000587792
.20	3.CC	.0000004419	.0000002919
.20	4.CC	.0000000028	.0000000012
.20	5.CC	.0000000000	.0000000000
.20	6.CC	.0000000000	-.0000000000
.20	7.CC	.0000000000	-.0000000000
.20	8.CC	.0000000000	-.0000000000
.20	9.CC	.0000000000	-.0000000000
.20	10.CC	.0000000000	-.0000000000
.30	1.CC	.0148976318	.0146488341
.30	2.CC	.0009587C2	.0000856043
.30	3.CC	.0000006170	.0000004303
.30	4.CC	.0000000040	.0000000018
.30	5.CC	.0000000000	.0000000000
.30	6.CC	.0000000000	-.0000000000
.30	7.CC	.0000000000	-.0000000000
.30	8.CC	.0000000000	-.0000000000
.30	9.CC	.0000000000	-.0000000000
.30	10.CC	.0000000000	-.0000000000
.40	1.CC	.0222853970	.0224638929
.40	2.CC	.0001434124	.0001315994
.40	3.CC	.0000009229	.0000006665

.40	4.00	.00000059	.0000000028
.40	5.00	.0000000000	.0000000000
.40	6.00	.0000000000	-.0000000000
.40	7.00	.0000000000	-.0000000000
.40	8.00	.0000000000	-.0000000000
.40	9.00	.0000000000	-.0000000000
.40	10.00	.0000000000	-.0000000000
.50	1.00	.0347561081	.0354325097
.50	2.00	.0002236648	.0002078027
.50	3.00	.0000014393	.0000010559
.50	4.00	.0000000093	.0000000045
.50	5.00	.0000000001	.0000000000
.50	6.00	.0000000000	-.0000000000
.50	7.00	.0000000000	-.0000000000
.50	8.00	.0000000000	-.0000000000
.50	9.00	.0000000000	-.0000000000
.50	10.00	.0000000000	-.0000000000
.60	1.00	.0555529721	.0567322601
.60	2.00	.0003574981	.0003327763
.60	3.00	.0000023006	.0000016918
.60	4.00	.0000000148	.0000000071
.60	5.00	.0000000001	.0000000000
.60	6.00	.0000000000	-.0000000000
.60	7.00	.0000000000	-.0000000000
.60	8.00	.0000000000	-.0000000000
.60	9.00	.0000000000	-.0000000000
.60	10.00	.0000000000	-.0000000000
.70	1.00	.0900845445	.0916417870
.70	2.00	.0005797179	.0005373440
.70	3.00	.0000037306	.0000027288
.70	4.00	.0000000240	.0000000115
.70	5.00	.0000000002	.0000000000
.70	6.00	.0000000000	-.0000000000
.70	7.00	.0000000000	-.0000000000
.70	8.00	.0000000000	-.0000000000
.70	9.00	.0000000000	-.0000000000
.70	10.00	.0000000000	-.0000000000
.80	1.00	.1473131103	.1489024532
.80	2.00	.0009481160	.0008725355
.80	3.00	.0000061014	.0000044224
.80	4.00	.0000000393	.0000000185
.80	5.00	.0000000003	.0000000000
.80	6.00	.0000000000	-.0000000000
.80	7.00	.0000000000	-.0000000000
.80	8.00	.0000000000	-.0000000000
.80	9.00	.0000000000	-.0000000000
.80	10.00	.0000000000	-.0000000000
.90	1.00	.2421812020	.2429924357

.90	2.00	.0015585002	.0014228627
.90	3.00	.0000100294	.0000071962
.90	4.00	.0000000645	.0000000300
.90	5.00	.0000000004	.0000000001
.90	6.00	.0000000000	-.0000000000
.90	7.00	.0000000000	-.0000000000
.90	8.00	.0000000000	-.0000000000
.90	9.00	.0000000000	-.0000000000
.90	10.00	.0000000000	-.0000000000
1.00	1.00	.3993014435	.3979218230
1.00	2.00	.0025961005	.0023284897
1.00	3.00	.0000165361	.0000117524
1.00	4.00	.0000001064	.0000000487
1.00	5.00	.0000000007	.0000000001
1.00	6.00	.0000000000	-.0000000000
1.00	7.00	.0000000000	-.0000000000
1.00	8.00	.0000000000	-.0000000000
1.00	9.00	.0000000000	-.0000000000
1.00	10.00	.0000000000	-.0000000000

R = 10.1000
E(0)=-Z*Z/2 -.5088888657651 E1=E_{GM}= -.599465615132
E(1) = E_{GM} + Z*Z/2 = -.0900576957481
E2 = -.0000085868806 E(2) = -.599551477938
E3 = .000001754248 E(3) = -.599549723690

R = 10.2000
E(0)=-Z*Z/2 -.5085557586570 E1=E_{GM}= -.598460586486
E(1) = E_{GM} + Z*Z/2 = -.0899002999916
E2 = -.0000084390570 E(2) = -.598544977065
E3 = .000001753040 E(3) = -.598543224025

R = 10.3000
E(0)=-Z*Z/2 -.5082407666080 E1=E_{GM}= -.597477106366
E(1) = E_{GM} + Z*Z/2 = -.0899236337758
E2 = -.0000082927398 E(2) = -.597560033764
E3 = .000001751846 E(3) = -.597558281918

R = 10.4000
E(0)=-Z*Z/2 -.5079376516892 E1=E_{GM}= -.596514416181
E(1) = E_{GM} + Z*Z/2 = -.0898576899289

E2 = -.0000814743:1 E(2) = -.59659589C562
E3 = .000001750665 E(3) = -.596594139897

R = 10.5000
E(0)=-7*Z/2 = -.5C76-7181235 E1=EGM= -.595571794993
E(1) = EGM + Z*Z/2 = -.087924613757
E2 = -.0000800362E47 E(2) = -.59565182756C
E3 = .000001749500 E(3) = -.595650078060

R = 10.6000
E(0)=-7*Z/2 = -.5073:2145981 E1=EGM= -.594648557244
E(1) = EGM + Z*Z/2 = -.087279411263
E2 = -.0000786C2814 E(2) = -.59472716C16C
E3 = .000001748351 E(3) = -.594725411809

R = 10.7000
E(0)=-7*Z/2 = -.5071:2827986 E1=EGM= -.5937440C0638
E(1) = EGM + Z*Z/2 = -.086641222652
E2 = -.0000771863:3 E(2) = -.593821236950
E3 = .000001747220 E(3) = -.593819489731

R = 10.8000
E(0)=-7*Z/2 = -.5064-7674727 E1=EGM= -.5928576541 5C
E(1) = EGM + Z*Z/2 = -.0860C9979423
E2 = -.0000757835:0 E(2) = -.592933437719
E3 = .000001746108 E(3) = -.592931691612

R = 10.9000
E(0)=-7*Z/2 = -.5056:3162527 E1=EGM= -.591988776175
E(1) = EGM + Z*Z/2 = -.085385613649
E2 = -.0000743954:9 E(2) = -.592C63171605
E3 = .000001745016 E(3) = -.592061426588

R = 11.0000
E(0)=-7*Z/2 = -.5C48:8794890 E1=EGM= -.591136852791
E(1) = EGM + Z*Z/2 = -.084768057901

E2 = -.0000730228548 E(2) = -.5912C9875359

E3 = .0000C1743947 E(3) = -.591208131412

MU	LAMBDA	PHI GM	PHI(1)
.00	1.CC	.0049C92201	.0041759584
.00	2.CC	.0000193744	.0000146039
.00	3.CC	.0000C00765	.0000C00421
.00	4.CC	.0000000003	.0000000001
.00	5.CC	.0000000000	-.0000000000
.00	6.CC	.0000000000	-.0000000000
.00	7.CC	.0000000000	-.0000000000
.00	8.CC	.0000000000	-.0000000000
.00	9.CC	.0000000000	-.0000000000
.00	10.CC	.0000000000	-.0000000000
.10	1.CC	.0054024296	.0047262348
.10	2.CC	.0000213209	.0000165870
.10	3.CC	.0000000841	.0000000484
.10	4.CC	.0000000003	.0000000001
.10	5.CC	.0000000000	.0000000000
.10	6.CC	.0000000000	-.0000000000
.10	7.CC	.0000000000	-.0000000000
.10	8.CC	.0000000000	-.0000000000
.10	9.CC	.0000000000	-.0000000000
.10	10.CC	.0000000000	-.0000000000
.20	1.CC	.0070370515	.0065277101
.20	2.CC	.0000277720	.0000230717
.20	3.CC	.0000001096	.0000000688
.20	4.CC	.0000000004	.0000000002
.20	5.CC	.0000000000	.0000000000
.20	6.CC	.0000000000	-.0000000000
.20	7.CC	.0000000000	-.0000000000
.20	8.CC	.0000000000	-.0000000000
.20	9.CC	.0000000000	-.0000000000
.20	10.CC	.0000000000	-.0000000000
.30	1.CC	.0103267729	.0100849314
.30	2.CC	.0000407550	.0000358528
.30	3.CC	.0000001608	.0000001088
.30	4.CC	.0000000006	.0000000003
.30	5.CC	.0000000000	.0000000000
.30	6.CC	.0000000000	-.0000000000
.30	7.CC	.0000000000	-.0000000000
.30	8.CC	.0000000000	-.0000000000
.30	9.CC	.0000000000	-.0000000000
.30	10.CC	.0000000000	-.0000000000
.40	1.CC	.0163C54034	.0164213131
.40	2.CC	.0000643498	.0000585740
.40	3.CC	.0000002540	.0000001795
.40	4.CC	.0000000010	.0000000004

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.40	5.00	.0000000000	.0000000000
.40	6.00	.0000000000	-.0000000000
.40	7.00	.0000000000	-.0000000000
.40	8.00	.0000000000	-.0000000000
.40	9.00	.0000000000	-.0000000000
.40	10.00	.0000000000	-.0000000000
.50	1.00	.2268517542	.0273969323
.50	2.00	.0001059714	.0000978580
.50	3.00	.0000004182	.0000003012
.50	4.00	.0000000017	.0000000008
.50	5.00	.0000000000	.0000000000
.50	6.00	.0000000000	-.0000000000
.50	7.00	.0000000000	-.0000000000
.50	8.00	.0000000000	-.0000000000
.50	9.00	.0000000000	-.0000000000
.50	10.00	.0000000000	-.0000000000
.60	1.00	.2452800630	.0462797087
.60	2.00	.0001786993	.0001653347
.60	3.00	.0000007052	.0000005091
.60	4.00	.0000000028	.0000000013
.60	5.00	.0000000000	.0000000000
.60	6.00	.0000000000	-.0000000000
.60	7.00	.0000000000	-.0000000000
.60	8.00	.0000000000	-.0000000000
.60	9.00	.0000000000	-.0000000000
.60	10.00	.0000000000	-.0000000000
.70	1.00	.2773615077	.0787554642
.70	2.00	.0003053888	.0002812296
.70	3.00	.0000012052	.0000008649
.70	4.00	.0000000048	.0000000022
.70	5.00	.0000000000	.0000000000
.70	6.00	.0000000000	-.0000000000
.70	7.00	.0000000000	-.0000000000
.70	8.00	.0000000000	-.0000000000
.70	9.00	.0000000000	-.0000000000
.70	10.00	.0000000000	-.0000000000
.80	1.00	.3132441100	.1347013676
.80	2.00	.0005258524	.0004806667
.80	3.00	.0000020753	.0000014751
.80	4.00	.0000000082	.0000000037
.80	5.00	.0000000000	.0000000000
.80	6.00	.0000000000	-.0000000000
.80	7.00	.0000000000	-.0000000000
.80	8.00	.0000000000	-.0000000000
.80	9.00	.0000000000	-.0000000000
.80	10.00	.0000000000	-.0000000000
.90	1.00	.3304224409	.2312920426
.90	2.00	.0009093720	.0008247154

.90	3.00	.0000035829	.0000025252
.90	4.00	.0000000142	.0000000063
.90	5.00	.0000000001	.0000000000
.90	6.00	.0000000000	-.0000000000
.90	7.00	.0000000000	-.0000000000
.90	8.00	.0000000000	-.0000000000
.90	9.00	.0000000000	-.0000000000
.90	10.00	.0000000000	-.0000000000
1.00	1.00	.3994568802	.3984395264
1.00	2.00	.0015764702	.0014197501
1.00	3.00	.0000062216	.0000043383
1.00	4.00	.0000000246	.0000000107
1.00	5.00	.0000000001	.0000000000
1.00	6.00	.0000000000	-.0000000000
1.00	7.00	.0000000000	-.0000000000
1.00	8.00	.0000000000	-.0000000000
1.00	9.00	.0000000000	-.0000000000
1.00	10.00	.0000000000	-.0000000000

R = 11.1000
E(0)=-Z*Z/2 = -.506144100946 E1=EGM= -.590301346131
E(1) = EGM + Z*Z/2 = -.084157245186
E2 = -.000071665600 E(2) = -.590373011731
E3 = .000001742902 E(3) = -.590371268830

R = 11.2000
E(0)=-Z*Z/2 = -.505928633986 E1=EGM= -.589481742877
E(1) = EGM + Z*Z/2 = -.083553108891
E2 = -.000070325076 E(2) = -.589552067953
E3 = .000001741881 E(3) = -.589550326072

R = 11.3000
E(0)=-Z*Z/2 = -.505721970091 E1=EGM= -.588677552831
E(1) = EGM + Z*Z/2 = -.082955582740
E2 = -.000069001490 E(2) = -.588746554321
E3 = .000001740857 E(3) = -.588744813434

R = 11.4000
E(0)=-Z*Z/2 = -.505523706848 E1=EGM= -.587888307594
E(1) = EGM + Z*Z/2 = -.082364600745
E2 = -.000067695284 E(2) = -.587956002878

APPENDIX C, PART 2: INTEGRAL REPRESENTATION RESULTS 240

E3 = .000001739982 E(3) = -.587954262956

R = 11.5000
E(0)=-7*Z/2 -.505333462144 E1=EGM= -.587113559325
E(1) = EGM + Z*Z/2 = -.C81780C97181
E2 = -.000066406842 E(2) = -.587179966168
E3 = .000001738976 E(3) = -.587178227182

R = 11.6000
E(0)=-7*Z/2 -.505150873032 E1=EGM= -.586352879583
E(1) = EGM + Z*Z/2 = -.C812C2006551
E2 = -.000065136506 E(2) = -.586418016089
E3 = .000001738C62 E(3) = -.586416278007

R = 11.7000
E(0)=-7*Z/2 -.504975594671 E1=EGM= -.585605858239
E(1) = EGM + Z*Z/2 = -.C80630263569
E2 = -.000063884563 E(2) = -.585669742802
E3 = .000001737212 E(3) = -.585668005591

R = 11.8000
E(0)=-7*Z/2 -.504807299327 E1=EGM= -.584872102462
E(1) = EGM + Z*Z/2 = -.C800648031 35
E2 = -.000062651262 E(2) = -.584934753724
E3 = .000001736376 E(3) = -.584933017347

R = 11.9000
E(0)=-7*Z/2 -.504645675439 E1=EGM= -.584151235770
E(1) = EGM + Z*Z/2 = -.C795C5560331
E2 = -.000061436807 E(2) = -.584212672577
E3 = .000001735578 E(3) = -.584210936998

R = 12.0000
E(0)=-7*Z/2 -.504490426737 E1=EGM= -.583442897140
E(1) = EGM + Z*Z/2 = -.C7895247043
E2 = -.000060241364 E(2) = -.583503138504

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F3 = .0000C1734820 E(3) = -.5835C1403685

MU	LAMBDA	PHI GM	PHI(1)
.00	1.CC	.0031069624	.0024745073
.00	2.CC	.0000074971	.0000052102
.00	3.CC	.0000000181	.0000000088
.00	4.CC	.0000000000	.0000000000
.00	5.CC	.0000000000	-.0000000000
.00	6.CC	.0000000000	-.0000000000
.00	7.CC	.0000000000	-.0000000000
.00	8.CC	.0000000000	-.0000000000
.00	9.CC	.0000000000	-.0000000000
.00	10.CC	.0000000000	-.0000000000
.10	1.CC	.0034667211	.0028811467
.10	2.CC	.0000083652	.0000061034
.10	3.CC	.0000000202	.0000000105
.10	4.CC	.0000000000	.0000000000
.10	5.CC	.0000000000	-.0000000000
.10	6.CC	.0000000000	-.0000000000
.10	7.CC	.0000000000	-.0000000000
.10	8.CC	.0000000000	-.0000000000
.10	9.CC	.0000000000	-.0000000000
.10	10.CC	.0000000000	-.0000000000
.20	1.CC	.0046806774	.0042313273
.20	2.CC	.0000112945	.0000090641
.20	3.CC	.0000000273	.0000000162
.20	4.CC	.0000000001	.0000000000
.20	5.CC	.0000000000	.0000000000
.20	6.CC	.0000000000	-.0000000000
.20	7.CC	.0000000000	-.0000000000
.20	8.CC	.0000000000	-.0000000000
.20	9.CC	.0000000000	-.0000000000
.20	10.CC	.0000000000	-.0000000000
.30	1.CC	.0072032903	.0069719736
.30	2.CC	.0000173816	.0000150596
.30	3.CC	.0000000419	.0000000276
.30	4.CC	.0000000001	.0000000000
.30	5.CC	.0000000000	.0000000000
.30	6.CC	.0000000000	-.0000000000
.30	7.CC	.0000000000	-.0000000000
.30	8.CC	.0000000000	-.0000000000
.30	9.CC	.0000000000	-.0000000000
.30	10.CC	.0000000000	-.0000000000
.40	1.CC	.0119789CC	.0120428499
.40	2.CC	.0000289052	.0000261263
.40	3.CC	.0000000697	.0000000485
.40	4.CC	.0000000002	.0000000001
.40	5.CC	.0000000000	.0000000000

.40	6.CC	.nnCCCCCCCC	-.CCCCCCCCCC
.40	7.CC	.nnCCCCCCCC	-.CCCCCCCCCC
.40	8.CC	.nnCCCCCCCC	-.CCCCCCCCCC
.40	9.CC	.nnCCCCCCCC	-.CCCCCCCCCC
.40	10.CC	.nnCCCCCCCC	-.CCCCCCCCCC
.50	1.CC	.nnC7954145	.0212239264
.50	2.CC	.nnCC501794	.CCCC461215
.50	3.CC	.nnCCCC1211	.CCCCCCC860
.50	4.CC	.nnCCCCCCC3	.CCCCCCCCC1
.50	5.CC	.nnCCCCCCCC	.CCCCCCCCCC
.50	6.CC	.nnCCCCCCCC	-.CCCCCCCCCC
.50	7.CC	.nnCCCCCCCC	-.CCCCCCCCCC
.50	8.CC	.nnCCCCCCCC	-.CCCCCCCCCC
.50	9.CC	.nnCCCCCCCC	-.CCCCCCCCCC
.50	10.CC	.nnCCCCCCCC	-.CCCCCCCCCC
.60	1.CC	.na69533CCC	.0377858308
.60	2.CC	.nnCC891685	.CCCC821290
.60	3.CC	.nnCCCC2152	.CCCCC01532
.60	4.CC	.nnCCCCCCC5	.CCCCCCCCC2
.60	5.CC	.nnCCCCCCCC	.CCCCCCCCCC
.60	6.CC	.nnCCCCCCCC	-.CCCCCCCCCC
.60	7.CC	.nnCCCCCCCC	-.CCCCCCCCCC
.60	8.CC	.nnCCCCCCCC	-.CCCCCCCCCC
.60	9.CC	.nnCCCCCCCC	-.CCCCCCCCCC
.60	10.CC	.nnCCCCCCCC	-.CCCCCCCCCC
.70	1.CC	.na65014841	.0676941239
.70	2.CC	.nnC1604684	.CCC1470627
.70	3.CC	.nnCCCC3872	.CCCCC02739
.70	4.CC	.nnCCCCCCC9	.CCCCCCCCC4
.70	5.CC	.nnCCCCCCCC	.CCCCCCCCCC
.70	6.CC	.nnCCCCCCCC	-.CCCCCCCCCC
.70	7.CC	.nnCCCCCCCC	-.CCCCCCCCCC
.70	8.CC	.nnCCCCCCCC	-.CCCCCCCCCC
.70	9.CC	.nnCCCCCCCC	-.CCCCCCCCCC
.70	10.CC	.nnCCCCCCCC	-.CCCCCCCCCC
.80	1.CC	.n2C501683C	.1218260793
.80	2.CC	.nnC2907712	.CCC26446C9
.80	3.CC	.nnCCCC7016	.CCCCCC4915
.80	4.CC	.nnCCCCCC17	.CCCCCCCCC7
.80	5.CC	.nnCCCCCCCC	.CCCCCCCCCC
.80	6.CC	.nnCCCCCCCC	-.CCCCCCCCCC
.80	7.CC	.nnCCCCCCCC	-.CCCCCCCCCC
.80	8.CC	.nnCCCCCCCC	-.CCCCCCCCCC
.80	9.CC	.nnCCCCCCCC	-.CCCCCCCCCC
.80	10.CC	.nnCCCCCCCC	-.CCCCCCCCCC
.90	1.CC	.n191695164	.2220447108
.90	2.CC	.nnC5288572	.CCC4773031
.90	3.CC	.nnCCCC12761	.CCCCCC8850

.90	4.CC	.0CC0C00031	.C000CC0013
.90	5.CC	.0C0000000C	.0000CC0000
.90	6.CC	.0C0000000C	-.C00C000000
.90	7.CC	.0C0000000C	-.0000C00000
.90	8.CC	.0C0000000C	-.C000C00000
.90	9.CC	.0C0000000C	-.C00C000000
.90	10.CC	.0C0000000C	-.C000C00000
1.00	1.CC	.3994424675	.3986897CC0
1.00	2.CC	.0C09638567	.C008642328
1.00	3.CC	.0C00023258	.C00C015993
1.00	4.CC	.0C000000S6	.C000C00024
1.00	5.CC	.0C0000000C	.0000C00000
1.00	6.CC	.0C0000000C	-.C000C00000
1.00	7.CC	.0C0000000C	-.0000C00000
1.00	8.CC	.0C0000000C	-.C000C00000
1.00	9.CC	.0C0000000C	-.C000C00000
1.00	10.CC	.0C0000000C	-.C000C00000

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EXIT