COINTEGRATION AND INDUSTRIAL CYCLES

by

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A dissertation submitted to the Graduate Faculty in Economics in partial fulfillment of the requirements for the degree of Doctor of Philosophy, The City University of New York.

1995

UMI Number: 9605667

UMI
300 North Zeeb Road
Ann Arbor, MI 48103

This manuscript has been read and accepted for the Graduate Faculty in Economics in satisfaction of the dissertation requirement for the degree of Doctor in Philosophy.

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Abstract

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Adviser : Prof. Dr. Michael Grossman

We investigate in this paper the degree of short-run and long-run comovements for ten industries. Based on the model developed by Johansen (1988, 1992) and Engle and Issler (1995), cointegration and common-cycle estimations are performed using per-capita output level of selected industries. The results show very similar cyclical behavior accross industries and indicate that they basically exhibit pro-cyclical behavior during the business cycles.

Acknowledgments

I would like to thank Prof. Dr. Jes Benhabib for his suggestions about the topic and support. My discussions with Faik Bilgili and his encouragement and support have been very helpful during the course of my study.

Financial support is provided by the Turkish Government, Ministry of Education, Youth and Sports.

The words are not enough to describe the importance of two person's presence in my life with their being on my side, and limitless understanding and support all the time during the course of my Ph. D. studies. My wife, my comrade, Tül Akbal Sualp, whose mind and heart I trust and respect the most. And my lovely son, Ömer, whose understanding was limitless at the age of ten, during our separation for the last two years. I am so grateful to them for their patience and encouragement.

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1.1 Introduction

Output and employment fluctuations have long been the concern of economic analysis. While different approaches have led to different analyses on business cycles, the common methodology used in most research programs have been in their quite aggregate structures.

The models developed, both at the theoretical and empirical level, use aggregate economic variables to explain the fluctuations in these variables and draw some conclusions about the multisector behavior of the economy.

At the very general level, the research programs can be classified into two categories, namely market clearing approach and market imperfections. The main goal of these research programs is to explain the underlying factors that lead to fluctuations in output and employment. However, as investigated in Bils (1985), Kydland and Prescott (1982), Murphy, Schleifer and Vishny (1989) and others, the interest in macroeconomic behavior extends beyond just the dynamics of output and employment to the analysis of the comovements of output and its components observed in macroeconomic variables.

Although the theoretical literature on comovements of output and its components is quite rich and has been advanced dramatically by the studies of Kydland and Prescott (1988), Long and Plosser (1983) and others, the empirical studies are not yet in line with these fast developments. Until recently the contemporary business cycle research has been carried out through the comparisons of rather simple statistical measures and characteristics to explain output fluctuations and that of its components. For this purpose, calibration models have been developed that mimic the observed movements in the economies with the persistence of aggregate shocks to the economy being the main focus.

The other line of research program puts the main stress on the market imperfections and increasing return technologies to understand the propagation mechanism and cyclical characteristics of output and employment, e.g. Murphy, Shleifer, and Vishny (1989), Stiglitz (1984), Bils (1986) among others. Still, as mentioned above, most of these studies conduct their analysis on the theoretical level and make use of the main features of the economies basically by looking at aggregate economic variables. The lack of empirical research using sectorally disaggregated data is one of the basic shortcomings of all the research programs outlined briefly.

Very recently, some studies such as Long and Plosser (1987), Pesaran (1993) and Engle and Issler (1995) focused on the analysis of disaggregated data. Especially Engle and Issler (1995), following Vahid and Engle (1993) and Engle and Kozicki (1993) analyzed the degree of short- and long-run movements found in the disaggregated data. By using the newly developed method by Granger (1986), Engle and Granger (1987 and Johansen (1988, 1991), called cointegration, they search for common trends as long term comovements of sectoral outputs and common cycles modeled as synchronized, persistent and transitory comovements of the sectoral outputs present in the data.

This study aims at the investigation of movements in output at the industrial level by looking at per-capita output series of a number of industries and using the method developed by Vahid and Engle (1993) and Engle and Issler (1995). We investigate the production series at a disaggregated level to examine whether the industries exhibit pro-cyclical or counter-cyclical behavior during the business cycles. In this context, the short-run dynamics of the system of the production series will be analyzed in terms of vector autoregressions. Beyond the developments that may cause permanent changes in the individual industries, the long-run relations among industries, which would be represented by linear combinations of the individual series, will be examined by the investigation of cointegration among the series. We hope that some more light may be shed on the understanding of long-run comovements and of business cycles.

Chapter 1 investigates the nonstationarity of the underlying series since the model works under the assumption that the data contain unit-roots. Chapter 2 reviews VAR modeling and the theoretical issues concerning cointegrating relations and presents the cointegration results applied to the industries. Chapter 3 briefly summarizes the so-called common features developed by Vahid and Engle (1993) and Engle and Issler (1995) and applies the method to the industries investigated to derive the cycles shared by the industries.

1.2. Notes on Data

The data for this study is the yearly industrial output series for the period 1947 to 1987 taken from Citibase Data Bank and in constant 1982 prices. Since almost all dynamic macroeconomic models are based on per-capita macroeconomic variables to analyze the dynamics of output, consumption and capital accumulation, the log levels of production of individual industries are converted into per-capita terms. Since all variables are transformed into logarithms the differences in levels indicates the growth rates of the industries' per-capita productions.

The industries selected are as follows (with the abbreviations used throughout the study in parentheses): Fabricated Metal Production (Fabmet), Electrical Machinery Production (Electric), Nonelectrical Machinery Production (Mach), Motor Vehicles and Parts Production (Motvehic), Stone Glass Production (Stonegla), Primary Metal Production (Primmeta), Lumber Production (Lumber), Chemicals (Chemicals), Printing (Print), Petroleum and Coal Products (Petrocoa), Furniture and Fixtures (Furniture), Rubber and Plastic Production (Rubblast), Leather and Products (Leather), Textile Mill Production (Textmill), Apparel Production (Apparel), and Miscellaneous Durable Manufacturing (Misc), Foods Production (Foods) and Instruments (Instrum). An informal check of the correlograms of the levels indicated that, among the log series chosen, the per-capita production level of Mach, Motvehic, Instrum, Textmill, Rubblast and Lumber exhibit either stationarity or near stationarity, therefore we excluded them from the analysis since the model is based on the assumption that the variables are integrated of order one. For the remaining 12 industries, Figure 1.1.1 through Figure 1.1.12 show the differenced series (growth rates). As it appears in the graphs, most series' growth rates fall during the recession periods, even though there are some considerable number of industries that exhibit some fluctuations in their growth rates in different recession periods.

Figure 1.1.1 Per-capita output growth rate of FABMET

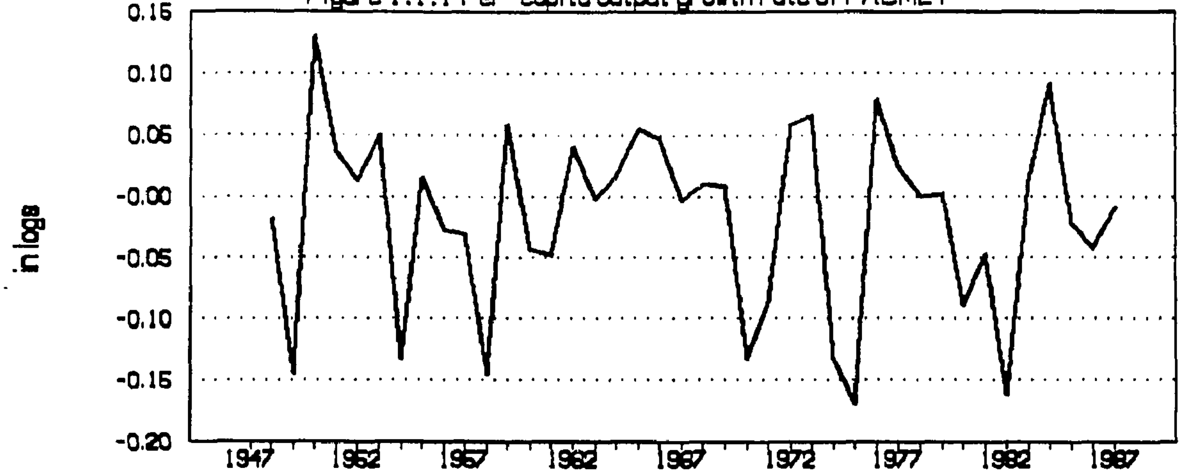


Figure 1.1.2 Per-capita output growth rate of ELECTRIC

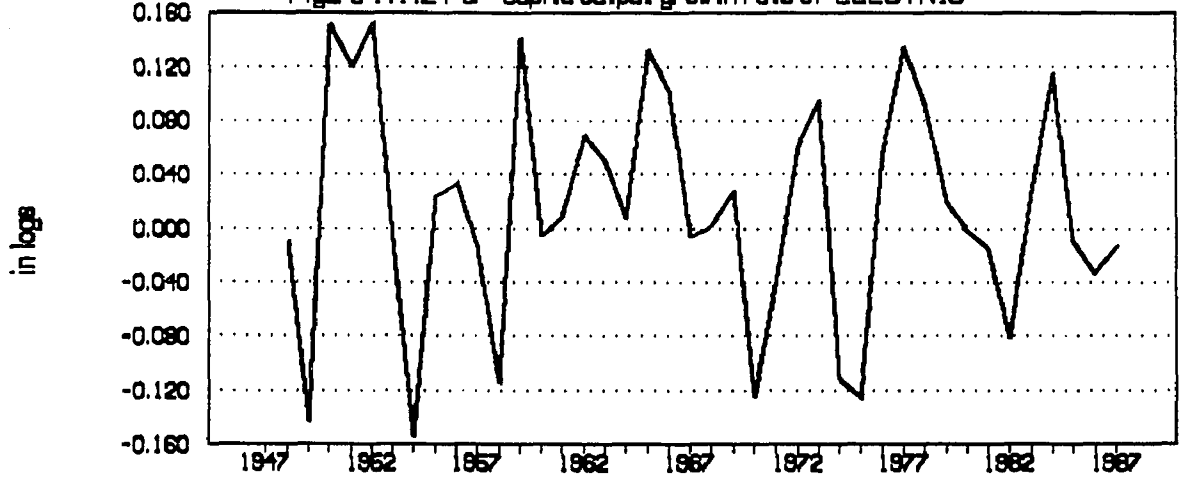
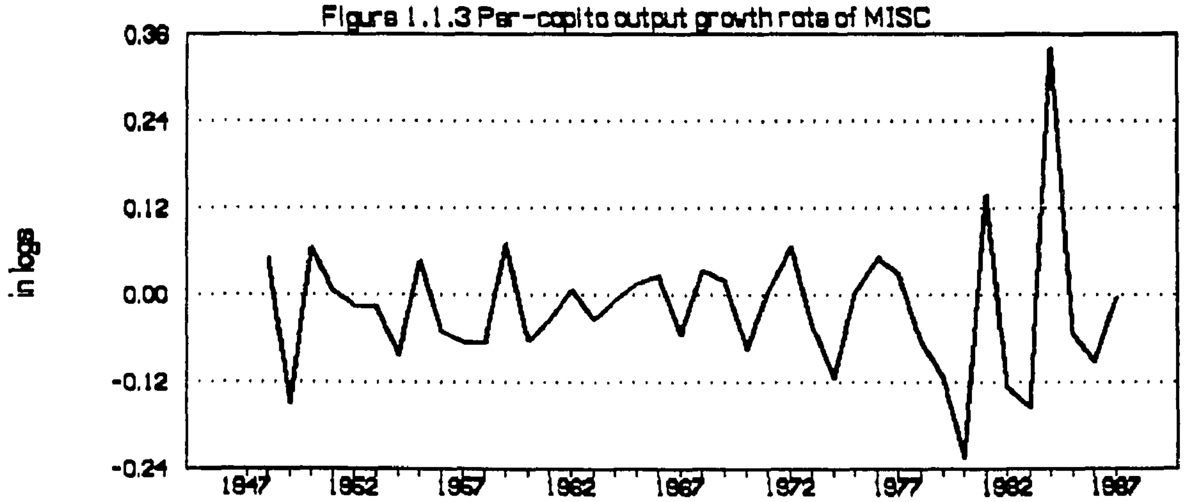


Figure 1.1.3 Per-capita output growth rate of MISC



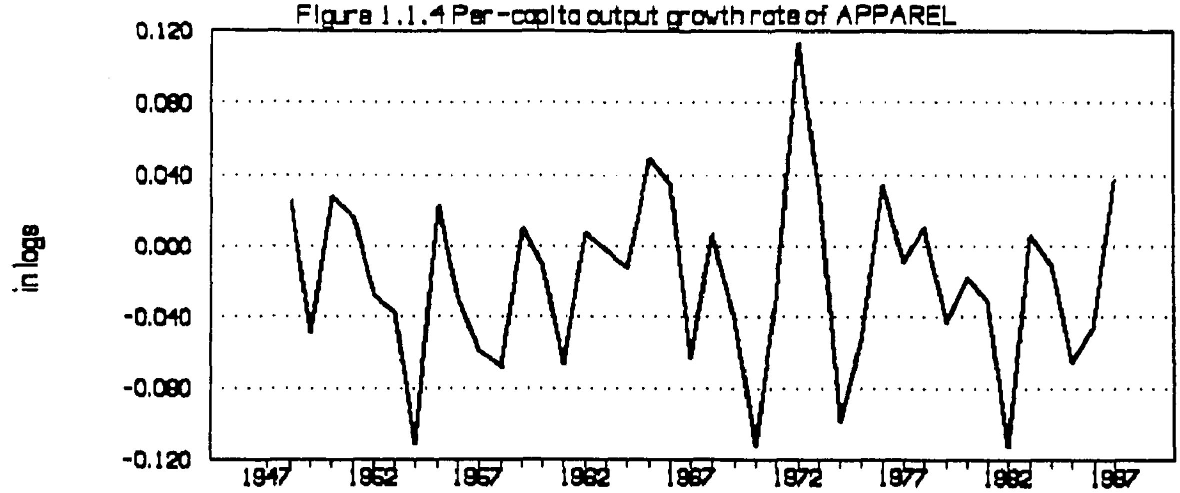


Figure 1.1.4 Per-capita output growth rate of APPAREL

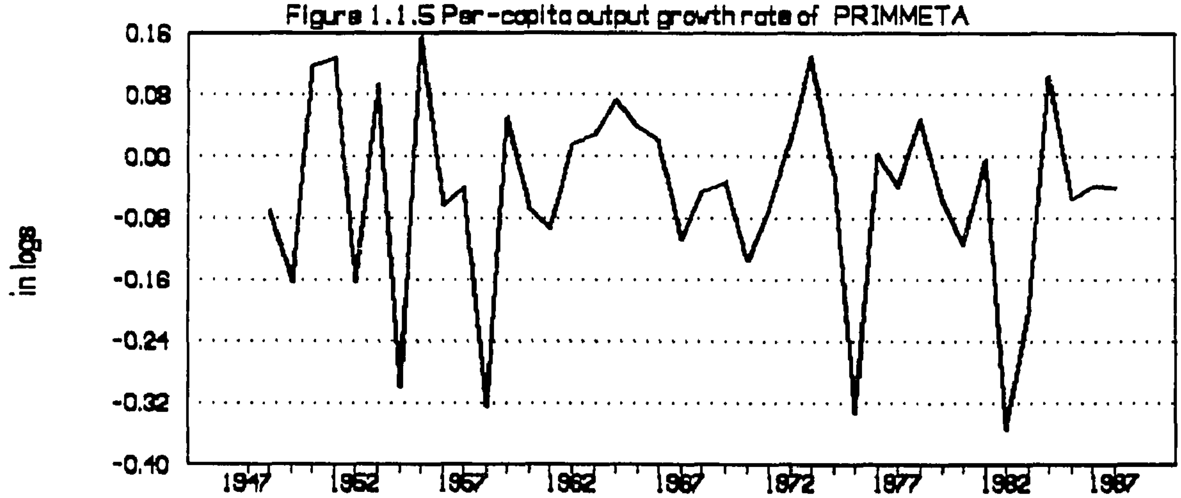


Figure 1.1.5 Per-capita output growth rate of PRIMMETA

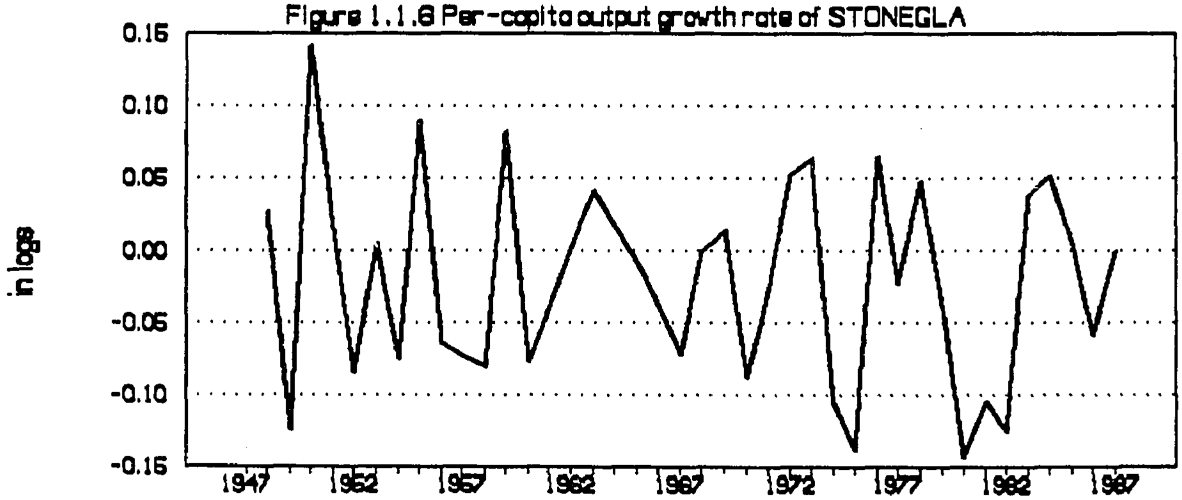


Figure 1.1.6 Per-capita output growth rate of STONEGLA

Figure 1.1.7 Per-capita output growth rate of PAPER

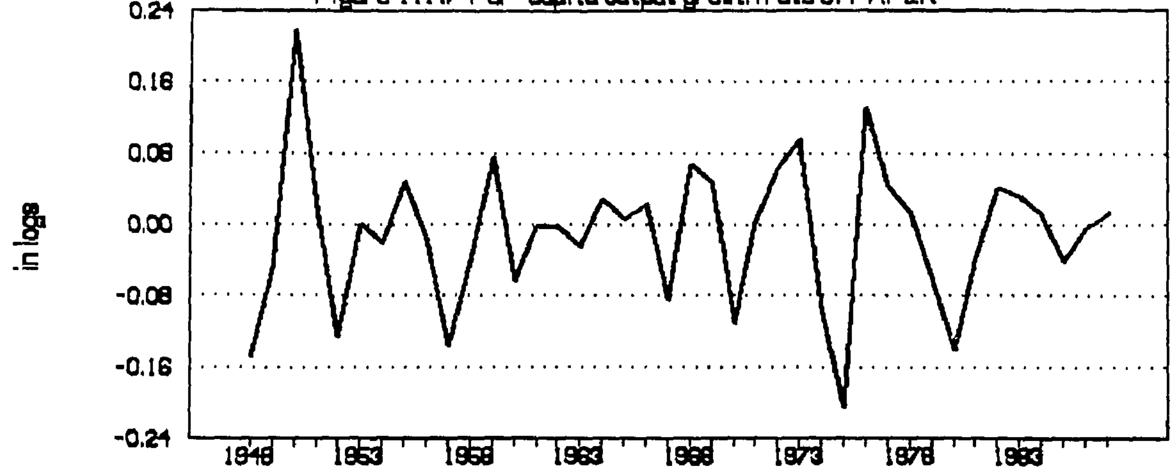


Figure 1.1.8 Per-capita output growth rate of CHEMICAL

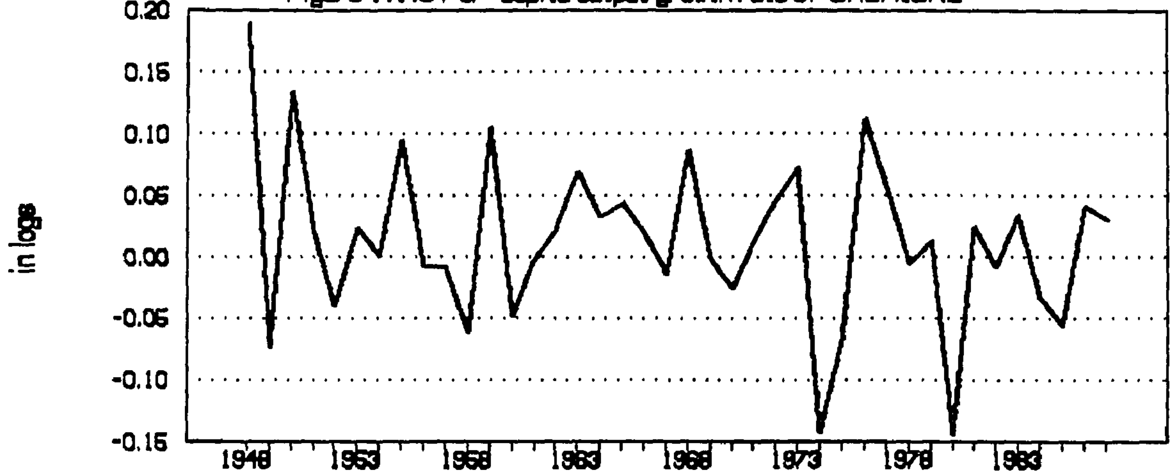


Figure 1.1.9 Per-capita output growth rate of PETROCOA

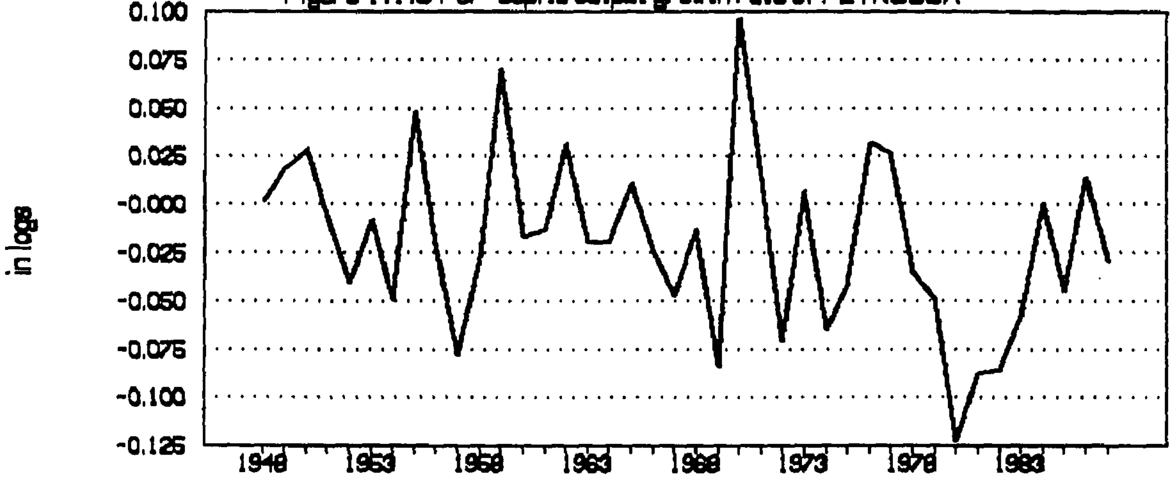


Figure 1.1.10 Per-capita output growth rate of LEATHER

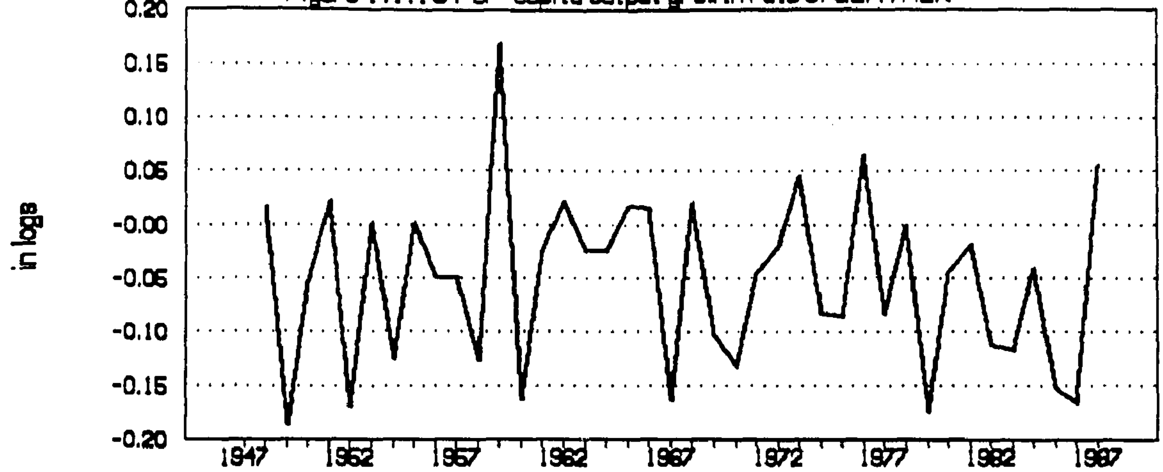


Figure 1.1.11 Per-capita output growth rate of FURNIT

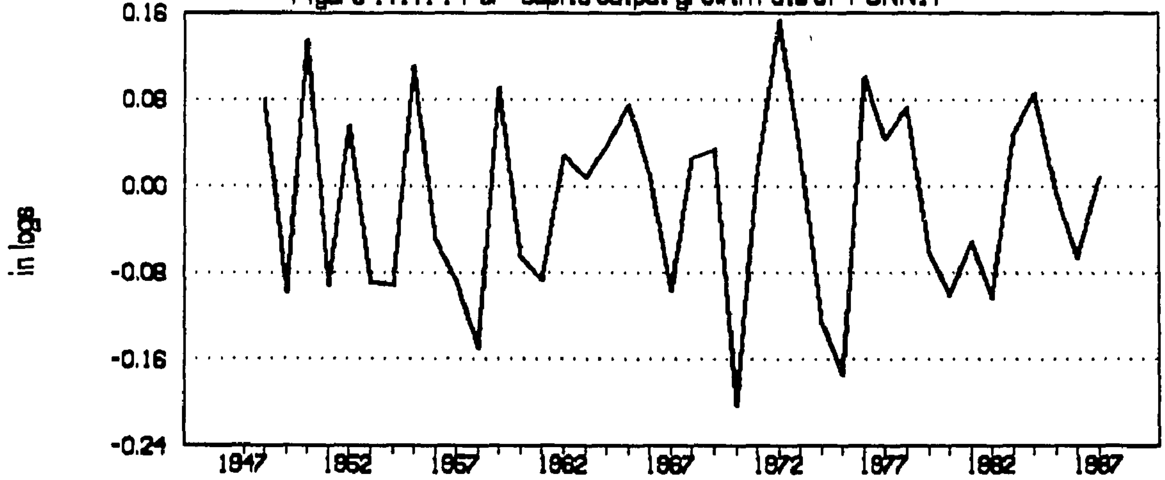
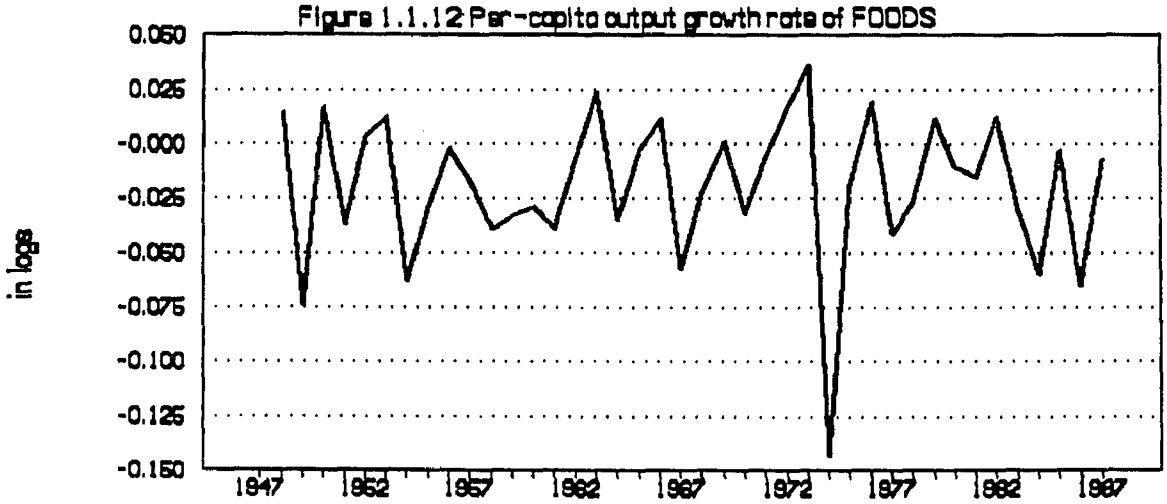


Figure 1.1.12 Per-capita output growth rate of FOODS



1.3. Investigation of Unit-Root Nonstationarity of the Series

Knowing whether the series under investigation are stationary and, if not, the nature of nonstationarity is important in the analysis of time series. Before testing the series for cointegration, it is important to ensure that each series demonstrates the same order of integration. In order to understand the nature of the nonstationarity in the series, a standard unit-root test is performed for each series. Because almost all the series show some trending features and, given the growing population and technological improvements, most of the series would be expected to exhibit a persistent trend, a linear trend component as well as a drift term are included in the unit-root tests. To account for the serial correlation in some of the series higher-order differences are taken into account in the testing process.

As can be easily seen from the correlograms in Figure 1.2.1 through Figure 1.2.24 for most of the series, the differences show no serial correlation; their autocorrelation lies within the confidence interval. For the series that have either autoregressive of moving average parts or both, to ensure that the error term in the test procedure is a white noise, we have included higher order lags. Various suggestions have been proposed for how to determine the order of the autoregressive processes when the process is regarded as ARIMA (p, 1, 0) with p unknown but finite. In order to determine the order of the processes used in unit-root test, we use the simplest approach regarding the fact that any ARIMA(p,1, q) process can be approximated by an ARIMA(l, 0, 0), therefore the unit-root test based upon a high order of AR approximation will produce the same results as Dickey-Fuller. The following regression is first run with a prespecified upper bound p= 4 for each series by OLS:

$$X_{i,t} = \xi_1 \Delta X_{i,t-1} + \xi_2 \Delta X_{i,t-2} + \ldots + \xi_{p-1} \Delta X_{i,t-p+1} + \alpha + \rho X_{i,t-1} + \varepsilon_{i,t} \qquad (1.2.1)$$

where X_i refers to series (in natural logs, so the differences of the series are the growth rates), Δ is the difference operator and $\varepsilon_{i,t}$ is the error term. The standard OLS t test of $\xi_{pl}=0$ is compared with the usual critical value for the t statistic from the usual t distribution table. If the null hypothesis that this coefficient is zero is accepted, the OLS test of the joint hypothesis that both ξ_{p-1} and ξ_{p-2} are zero is compared with the usual $F(2, T\text{-}k)$ distribution from the usual F table. The procedure has been carried out sequentially until the joint null hypothesis that $\xi_{p-1}, \xi_{p-2}, \ldots, \xi_{p-l}$ is rejected for some l. Once the optimum number of lags is determined, then the Dickey-Fuller unit-root test is performed by the following regression[1]:

$$\Delta X_{i,t} = \xi_1 \Delta X_{i,t-i} + \xi_2 \Delta X_{i,t-2} + \ldots + \xi_{p-l} \Delta X_{i,t-p+l} + \alpha + \beta X_{i,t-1} + \delta t + \upsilon_{i,t} \quad (1.2.2)$$

where $\upsilon_{i,t}$ is a white noise process and $\beta = \rho - 1$, while other variables are the same as in (1.2.1).

If autoregressive representation of $X_{i,t}$ contains a unit root (i.e., is integrated of order one), the t-ratio for β should be consistent with the hypothesis $\beta = 0$. At the same time if the true underlying process of the series contains a unit root with no

[1] The Akaike (1974) and Schwarz (1978) information criteria are also applied to check for the true order of AR in determining the model selection.

trend component, then the joint null hypothesis $\beta = \delta = 0$ should be consistent with F-ratio. Conventional t and F-tables are inappropriate for these hypothesis tests, therefore, the results of Dickey-Fuller (1981, p. 1063) and the tabulated distribution in Fuller (1976, p. 371 and p. 373) are applied to interpret the t- and F-ratios.

The test statistics, $\mathbf{\Phi_1}$, $\mathbf{\Phi_2}$, are constructed for the null hypothesis:

1) $H_0 : \beta = 0$

2) $H_0 : \beta = \delta = 0$

with the alternative hypothesis in each case being the stationarity of the series (i.e., $\beta < 0$).

If the tabulated test statistics are greater than the critical values, as tabulated in these tables, we reject the null hypothesis of unit-root. The results of the tests together with the optimum number of lags for each series are given in Table 1.1 and the test statistics are presented in Table 1.2 for the number of observations, T = 41. From the test results in Table 1.1, we conclude that, with the exception of furniture and food industries, we cannot reject the null hypothesis that the series exhibit unit-root processes with no linear trend at the 5 percent significance level. Based upon these results, the cointegration and common cycles analysis will be carried out for the 10 industries that are integrated of order one. Phillips (1987) indicates that the Dickey-Fuller tests are affected by the autocorrelations in the errors in Equation 1.2.2. The inspection of the correlograms of the errors (not included to save the space) and the Q-statistics do not indicate any autocorrelations, therefore no need is seen to refer to the modified Dickey-Fuller tests.

Table 1.1. Results of the unit root tests

Variables	# of Lags	Test Φ_1	Statistics Φ_2
FURNIT	0	- 3.922	7.694
FABMET	0	- 2.682	3.707
ELECTRIC	2	- 3.087	5.533
MISC	2	- 2.132	2.323
STONEGLA	0	- 3.119	5.093
PRIMMETA	0	- 2.515	3.373
FOODS	1	- 3.783	7.158
APPAREL	2	- 1.940	1.944
PAPER	0	-3.103	5.019
CHEMICAL	2	- 3.083	5.973
PETROCOA	0	- 1.240	2.699
LEATHER	1	- 0.937	0.721

Table 1.2. The critical values for the test statistics* (T ≈ 50)

Test Statistics	0.01	0.025	0.05	0.10
Φ_1	-4.15	-3.80	-3.50	-3.18
Φ_2	9.31	7.81	6.73	5.61

* The critical values for the statistics are from Fuller (1976), p. 373 and Dickey and Fuller (1981, p.1063).
All variables are estimated in natural logs.

Figure 1.2.1 AC Function of Log Diff of FABMET

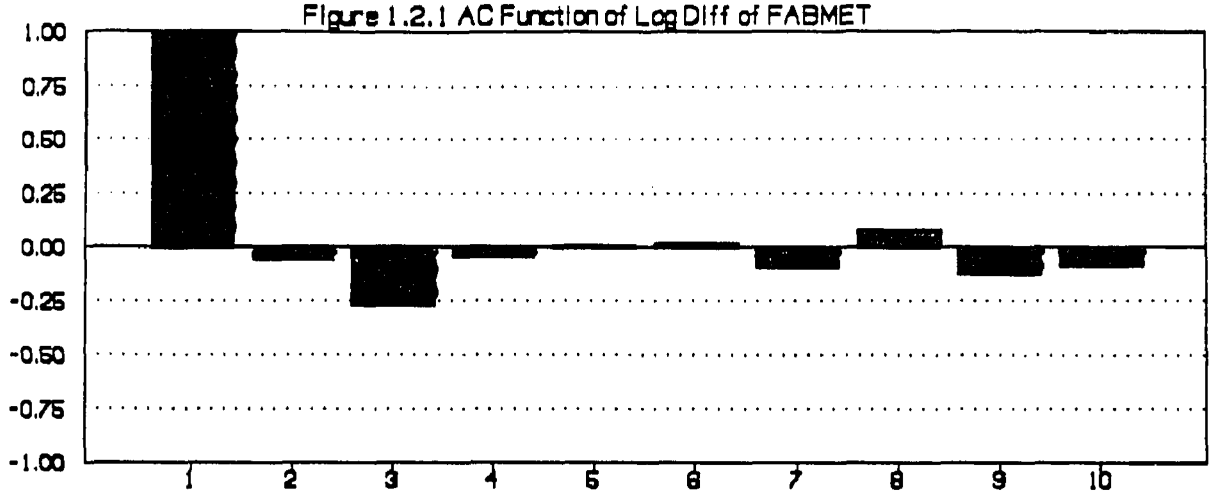


figure 1.2.2 AC Function of Log Diff of ELECTRIC

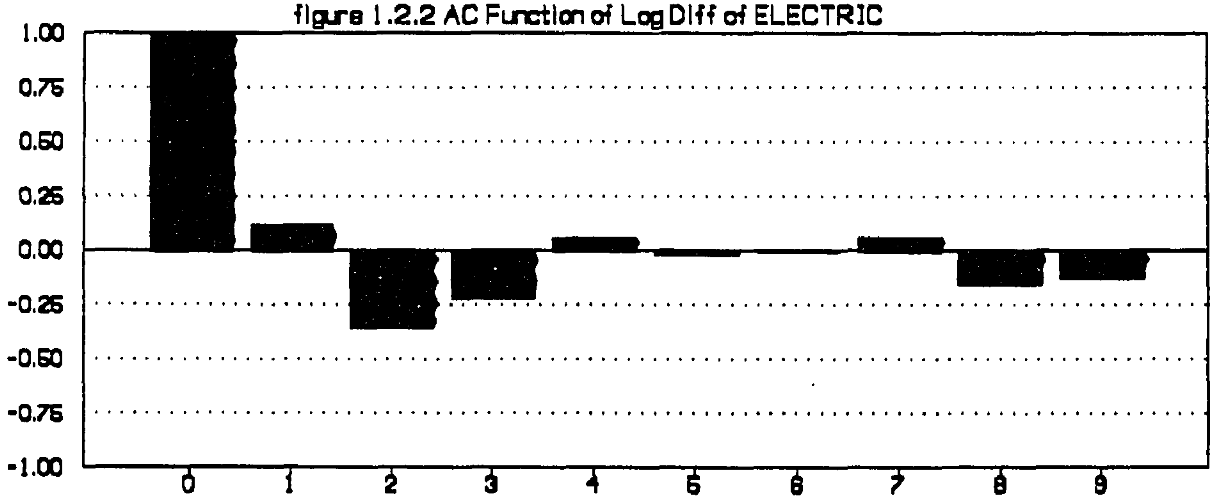


Figure 1.2.3 AC Function of Log Diff of MISC

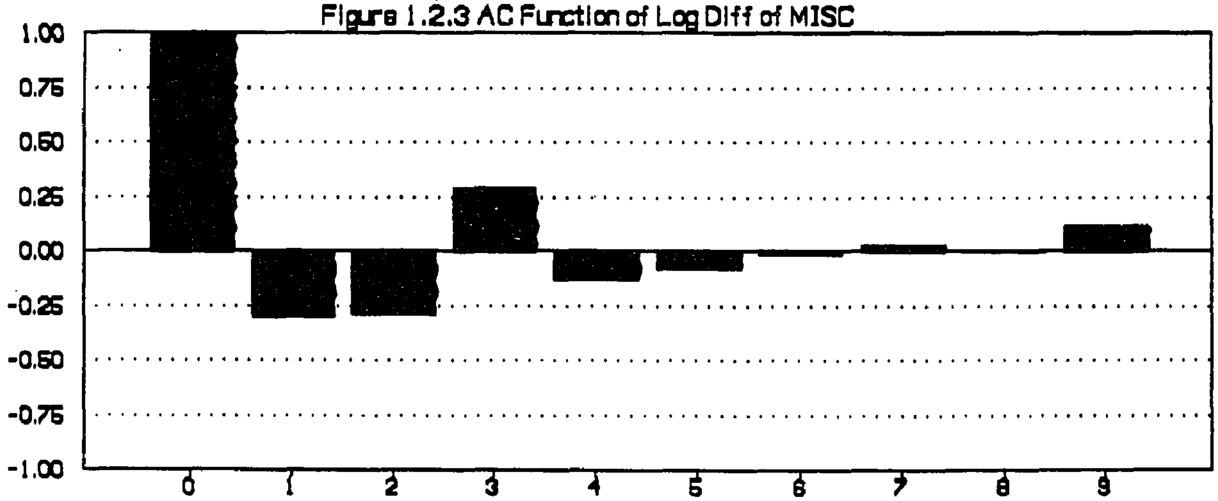


Figure 1.2.4 AC Function of Log Diff of APPAREL

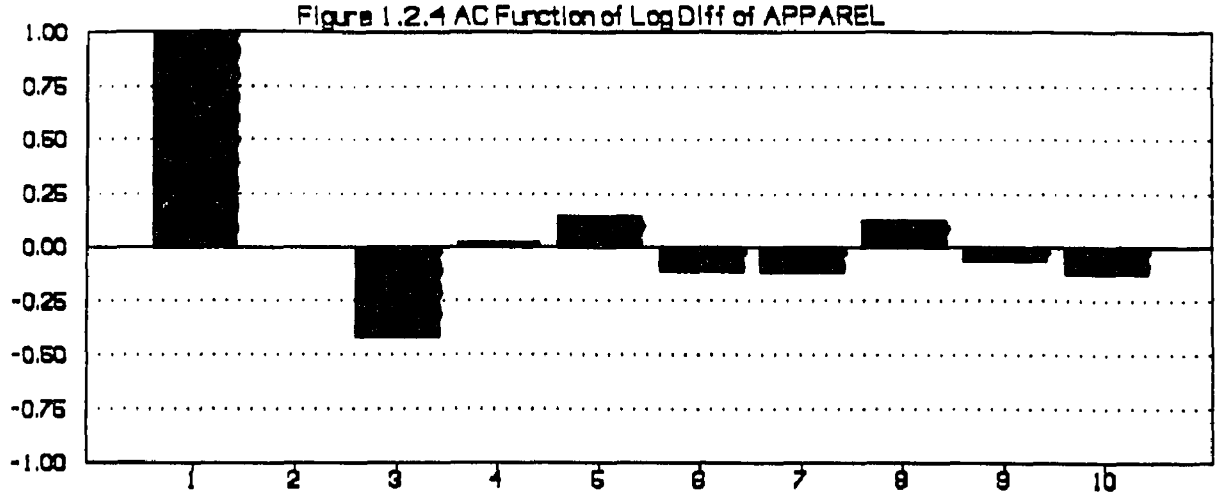


figure 1.2.5 AC Function of Log Diff of PRIMMETA

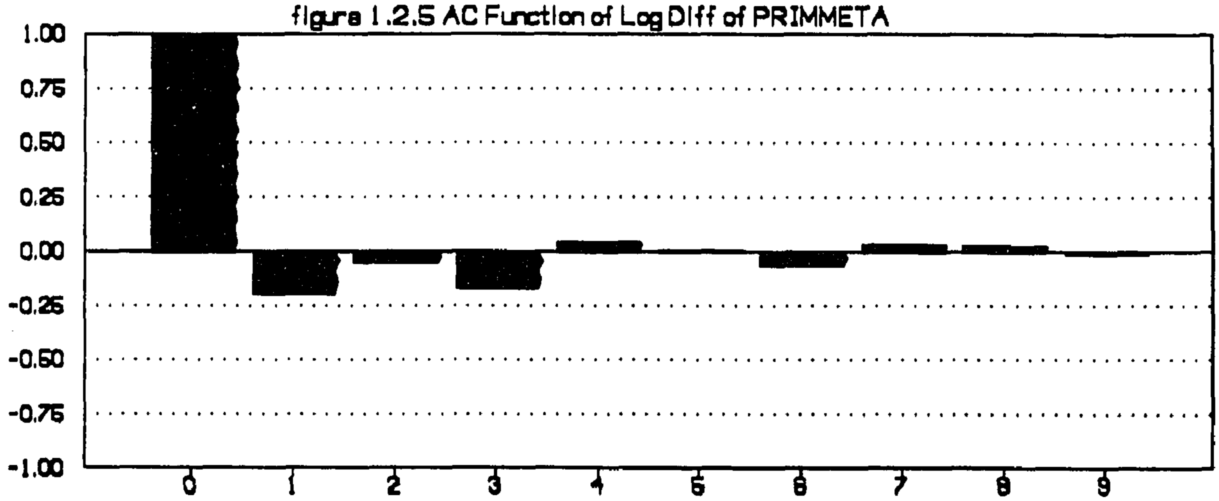


Figure 1.2.6 AC Function of Log Diff of STONEGLA

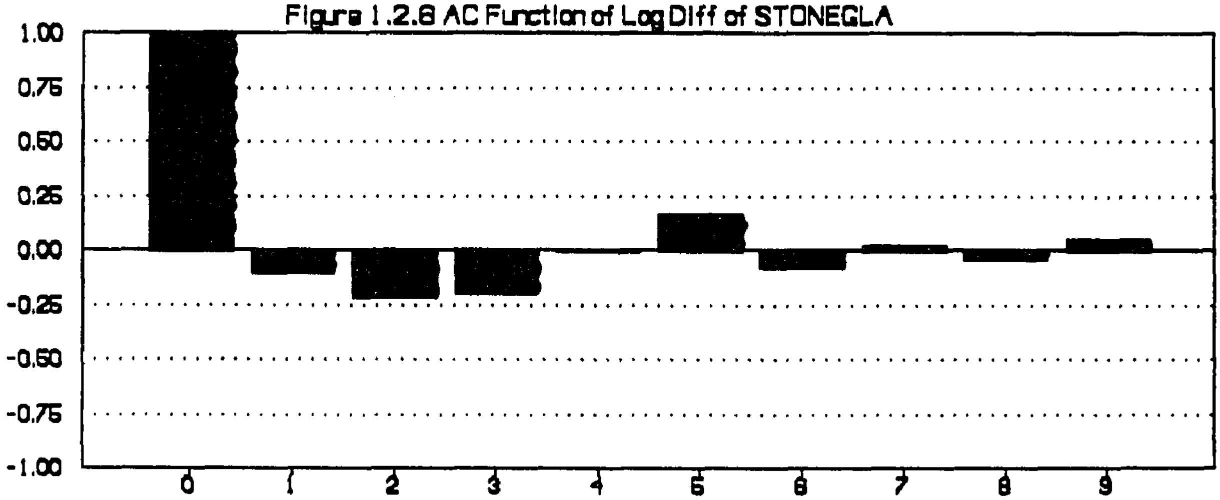


Figure 1.2.7 AC Function of Log Diff of PAPER

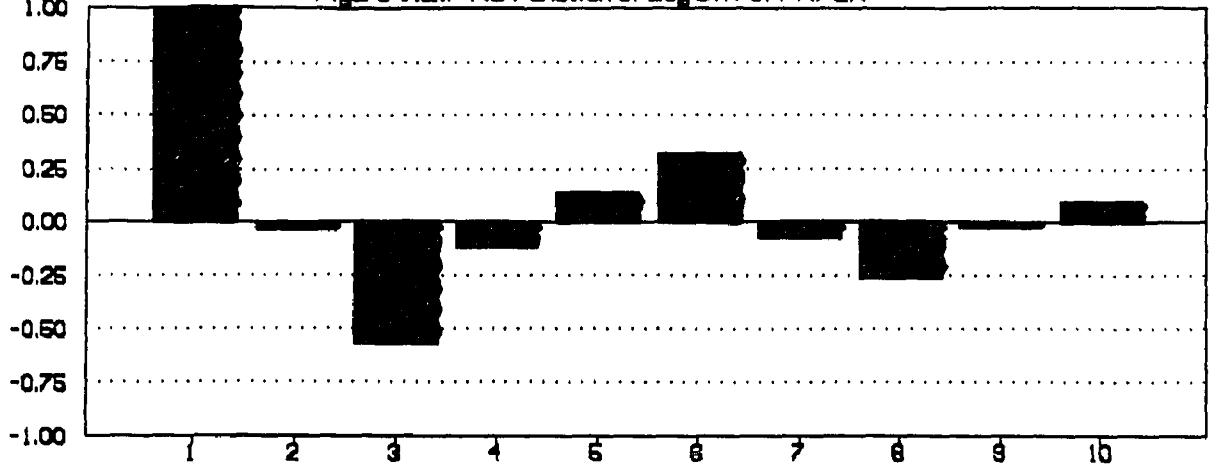


figure 1.2.8 AC Function of Log Diff of CHEMICAL

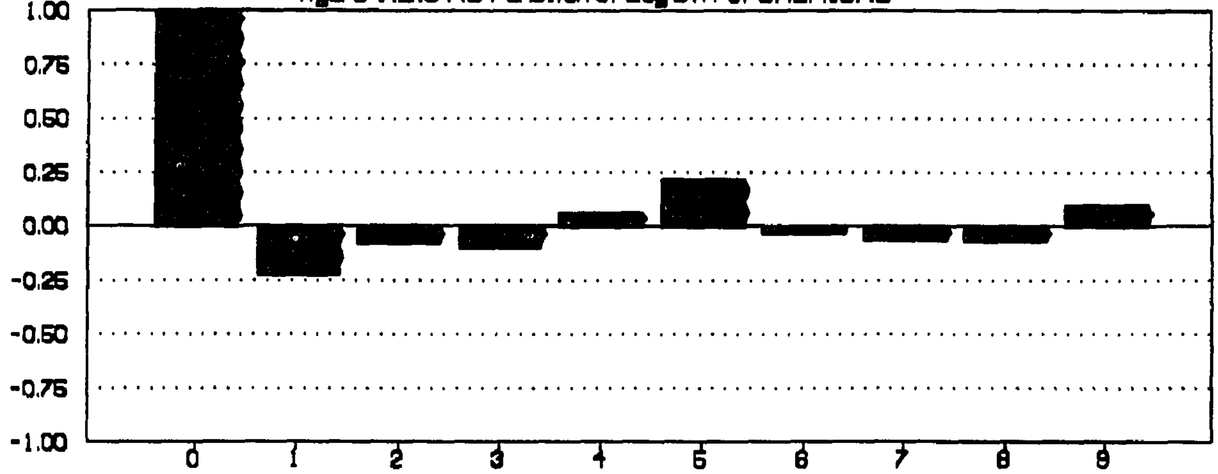


Figure 1.2.9 AC Function of Log Diff of PETROCOA

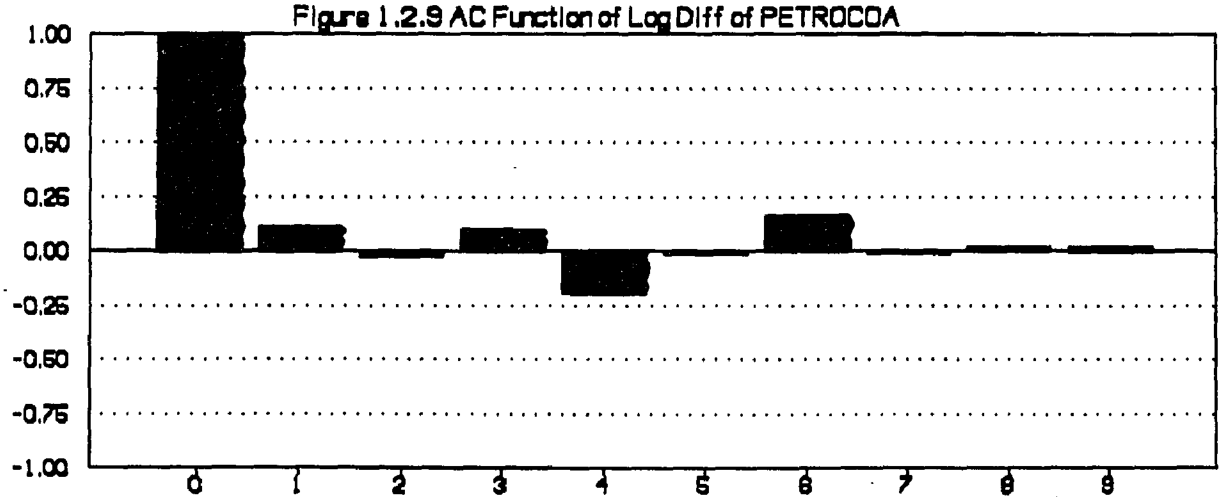


Figure 1.2.10 AC Function of Log Diff of LEATHER

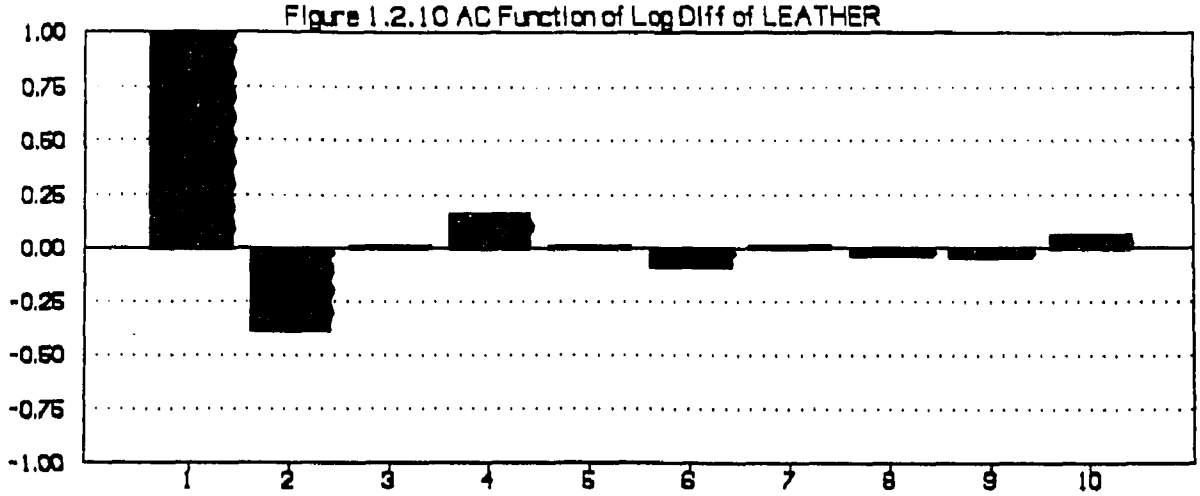


figure 1.2.11 AC Function of Log Diff of FURNIT

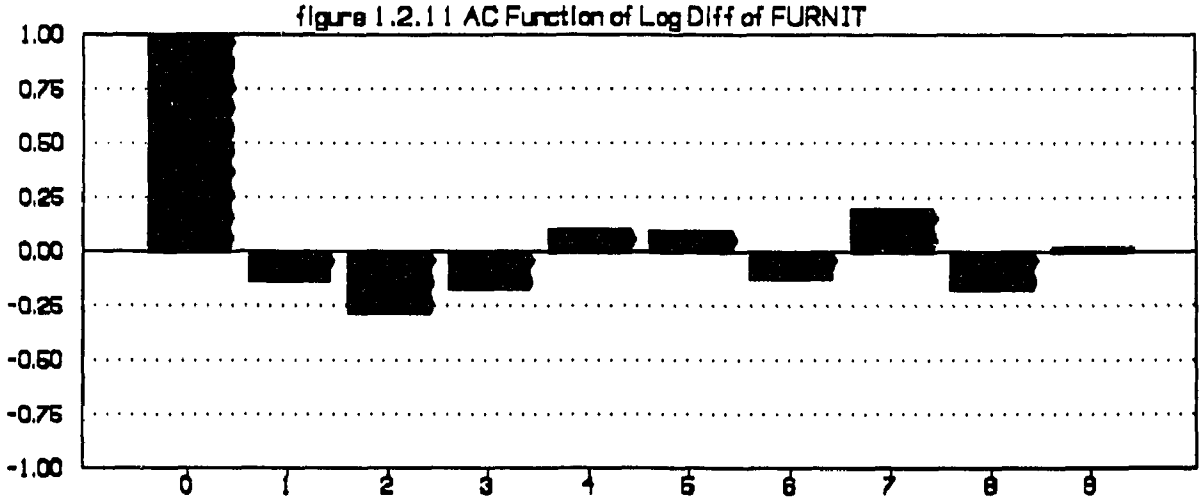


Figure 1.2.12 AC Function of Log Diff of FOODS

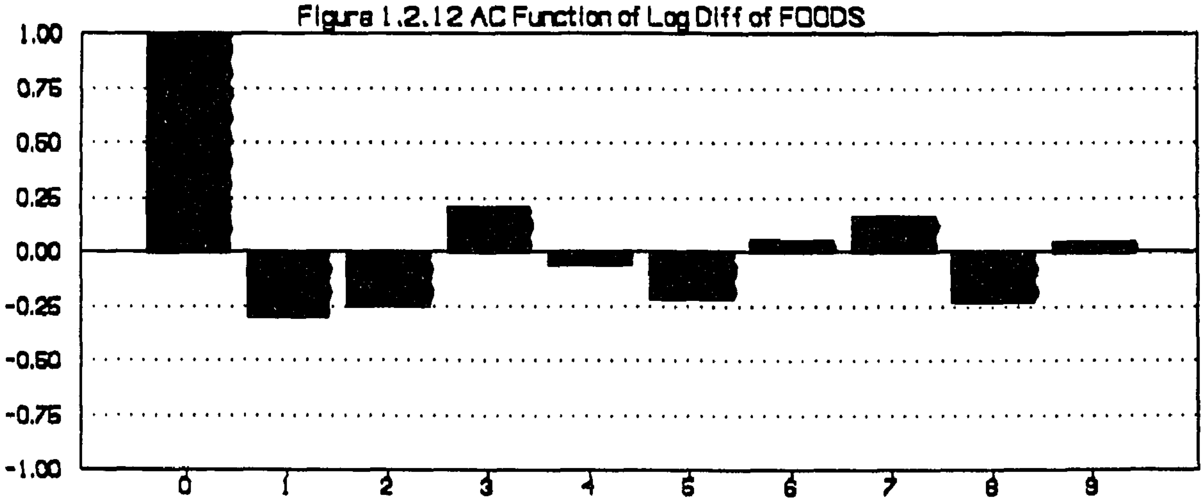


Figure 1.2.13 PAC Function of Log Diff of FABMET

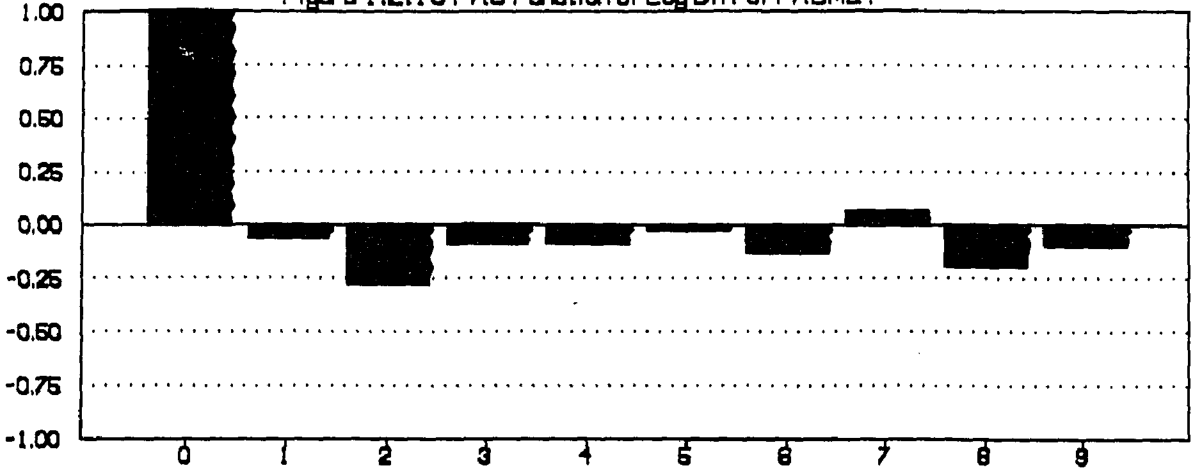


Figure 1.2.14 PAC Function of Log Diff of ELECTRIC

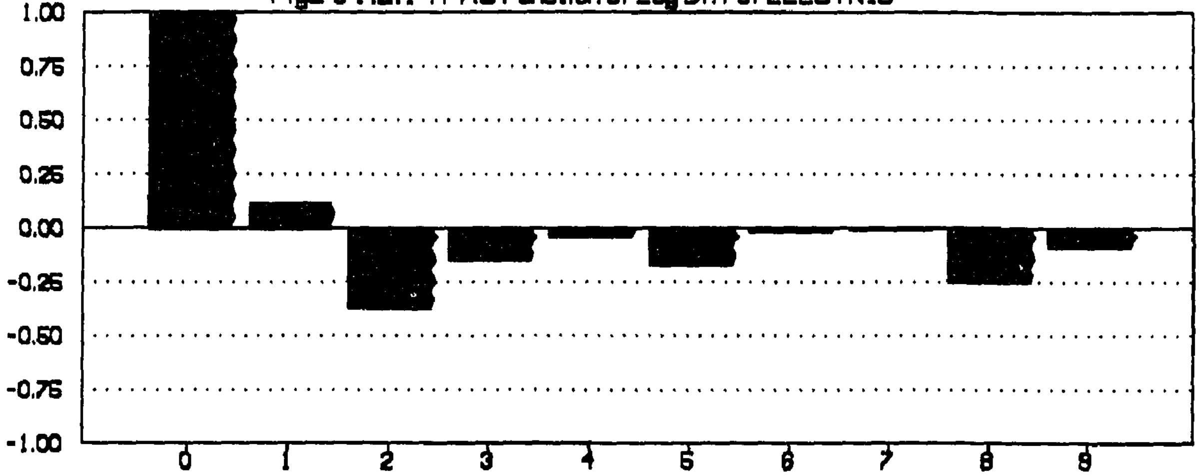


Figure 1.2.15 PAC Function of Log Diff of MISC

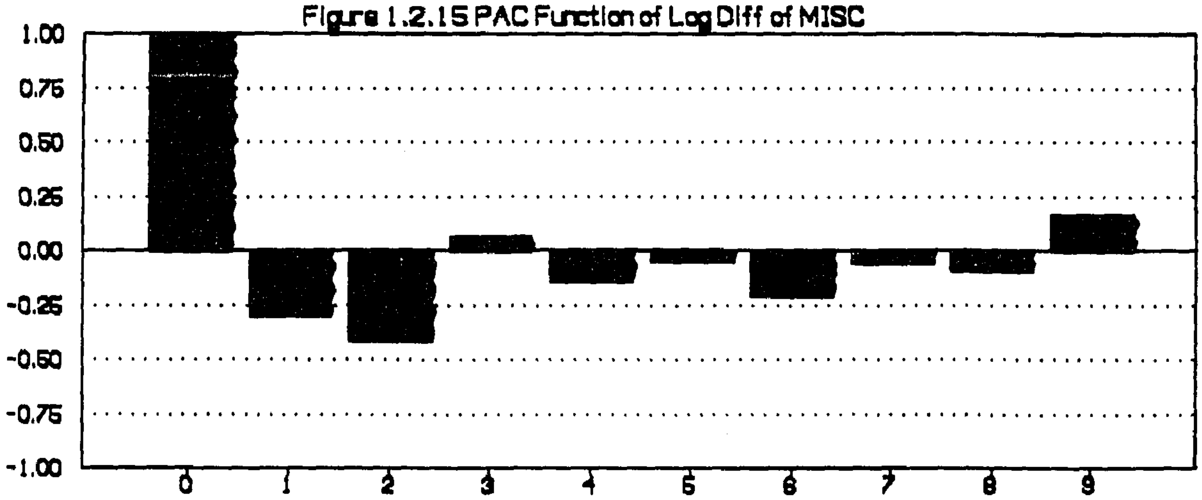


Figure 1.2.16 PAC Function of Log Diff of APPAREL

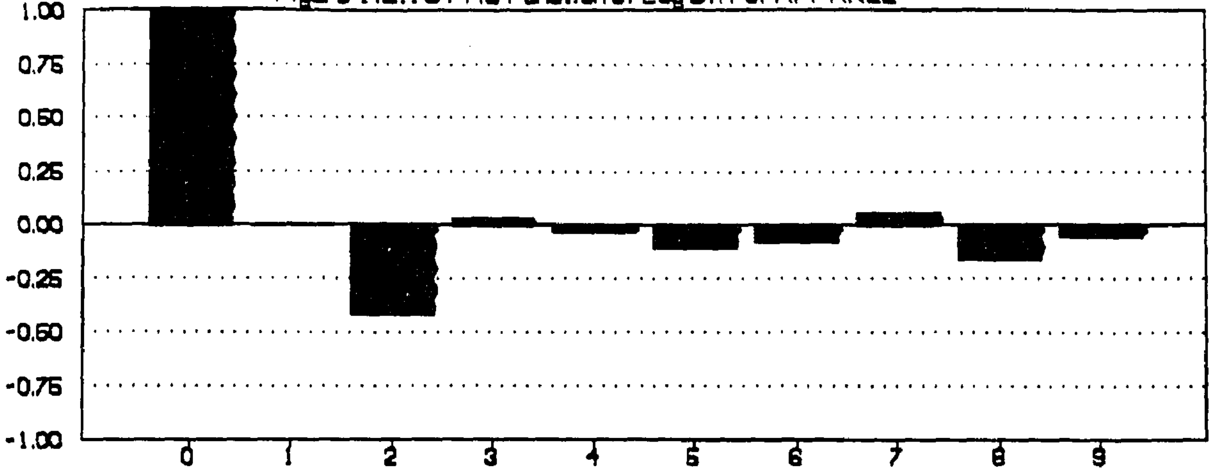


Figure 1.2.17PAC Function of Log Diff of PRIMMETA

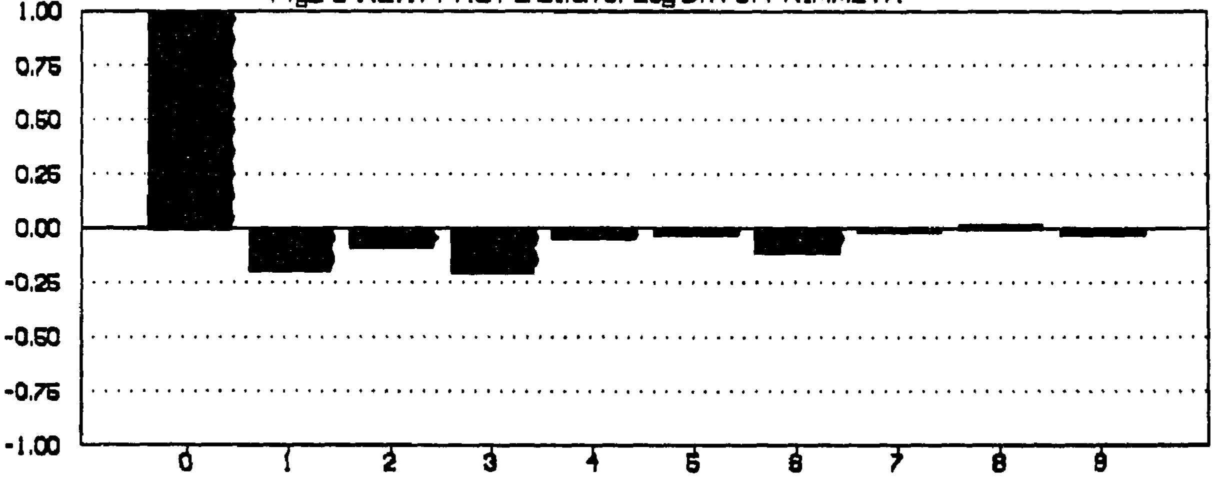


Figure 1.2.18 PAC Function of Log Diff of STONEGLA

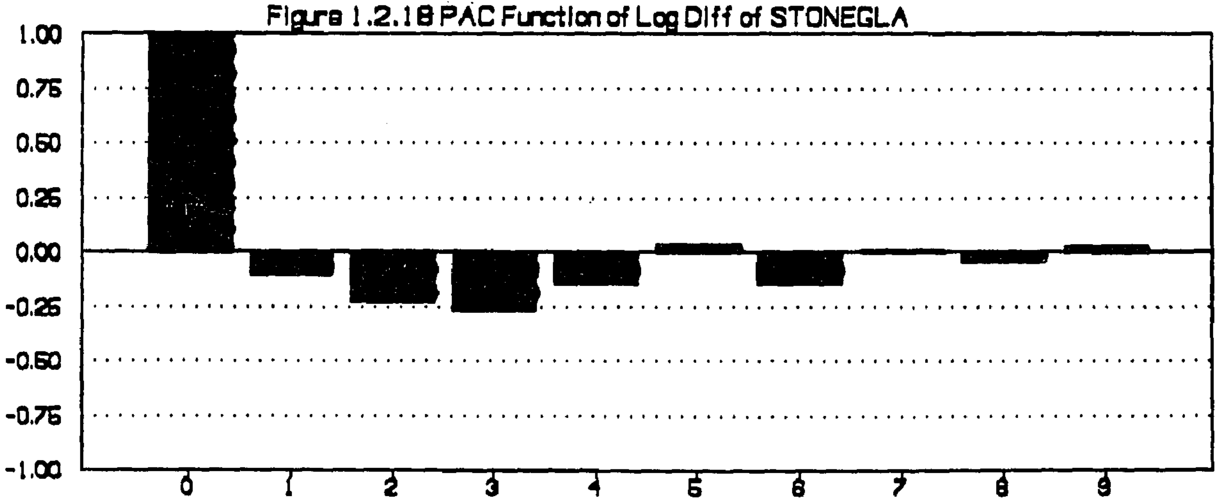


Figure 1.2.19 PAC Function of Log Diff of PAPER

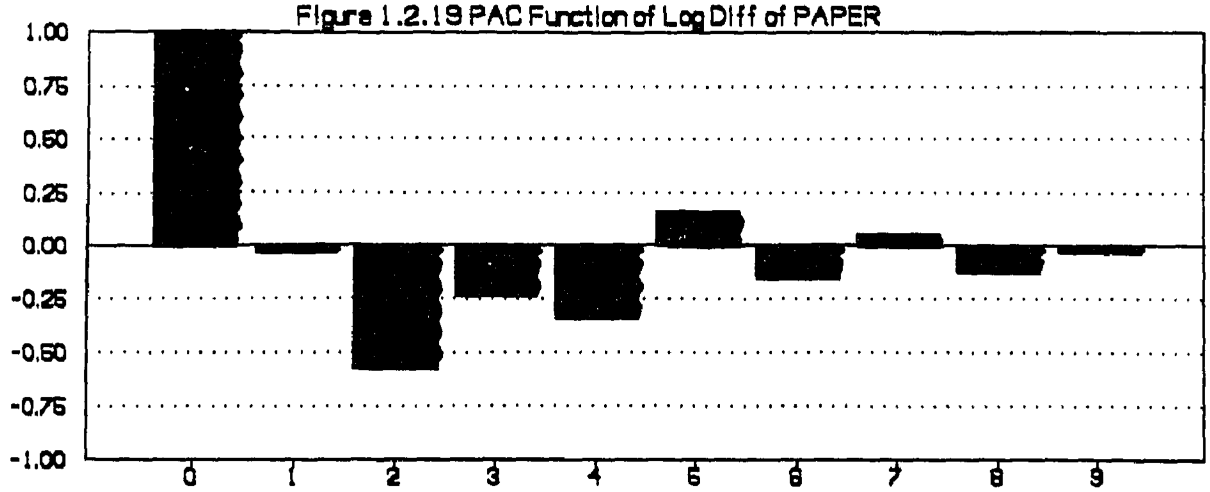


Figure 1.2.20 PAC Function of Log Diff of CHEMICAL

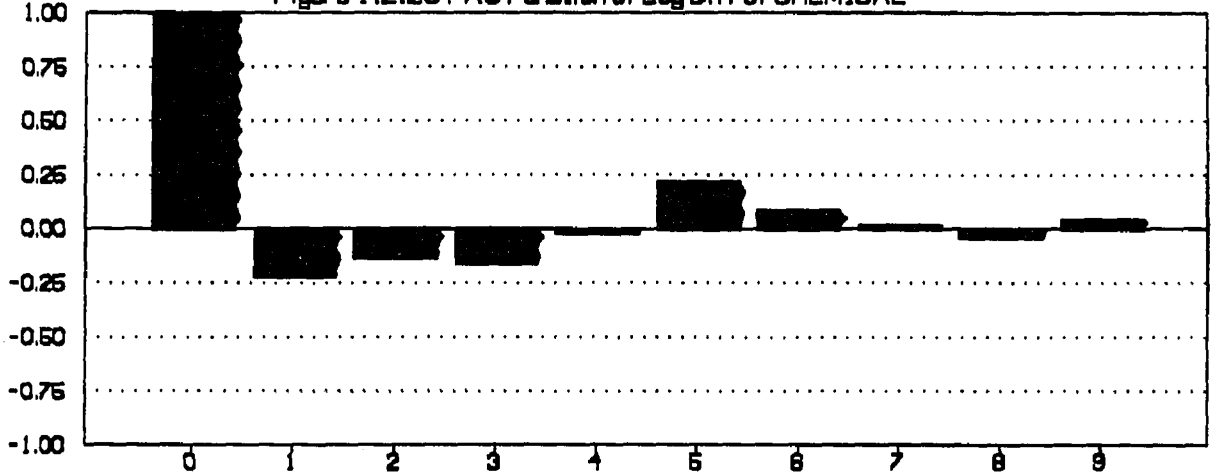


Figure 1.2.21 PAC Function of Log Diff of PETROCOA

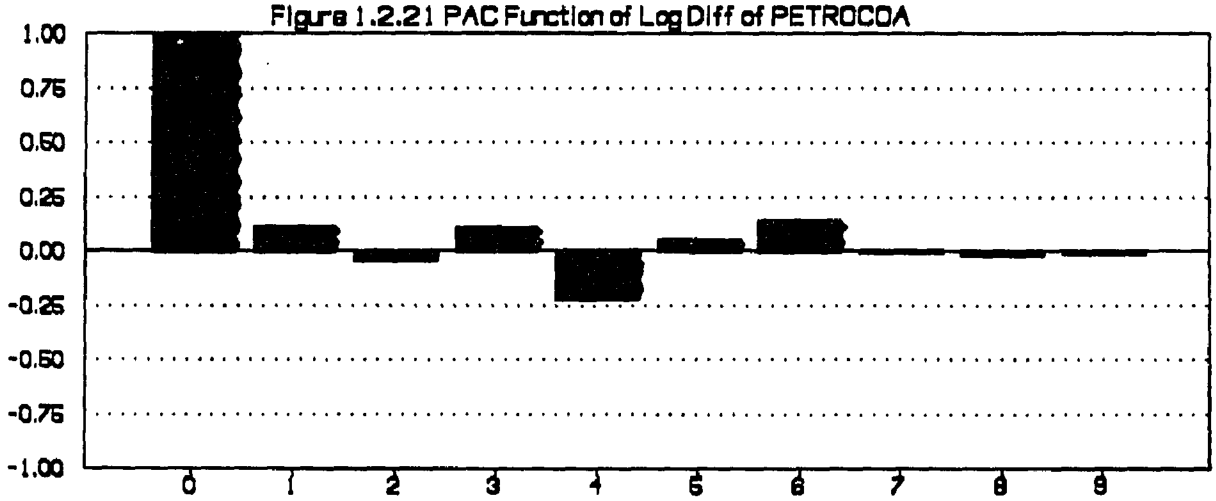
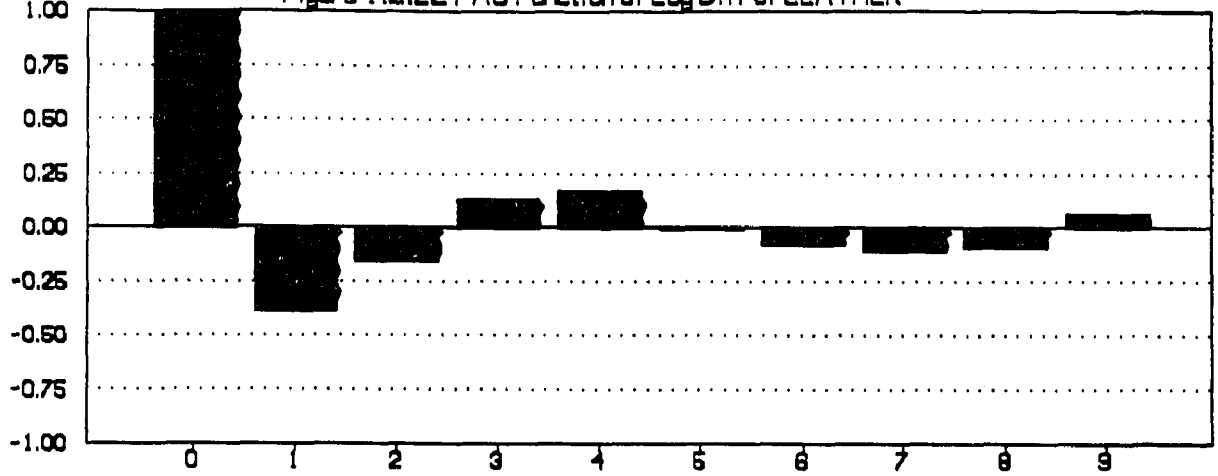


Figure 1.2.22 PAC Function of Log Diff of LEATHER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0.75
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0.25
0.00
-0.25
-0.50
-0.75
-1.00
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2
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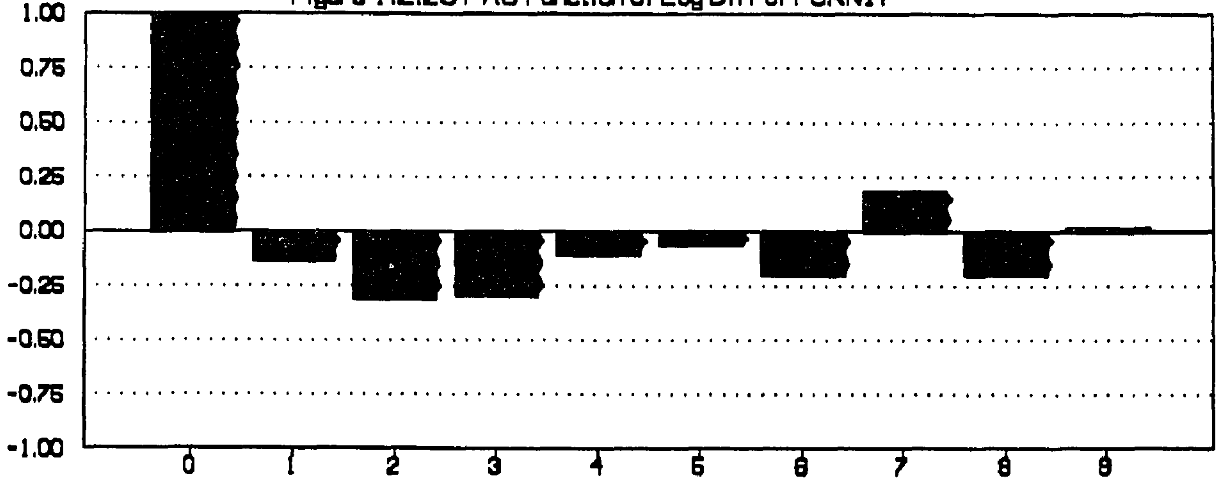
8
9

Figure 1.2.23 PAC Function of Log Diff of FURNIT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0.75
0.50
0.25
0.00
-0.25
-0.50
-0.75
-1.00
0
1
2
3
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8
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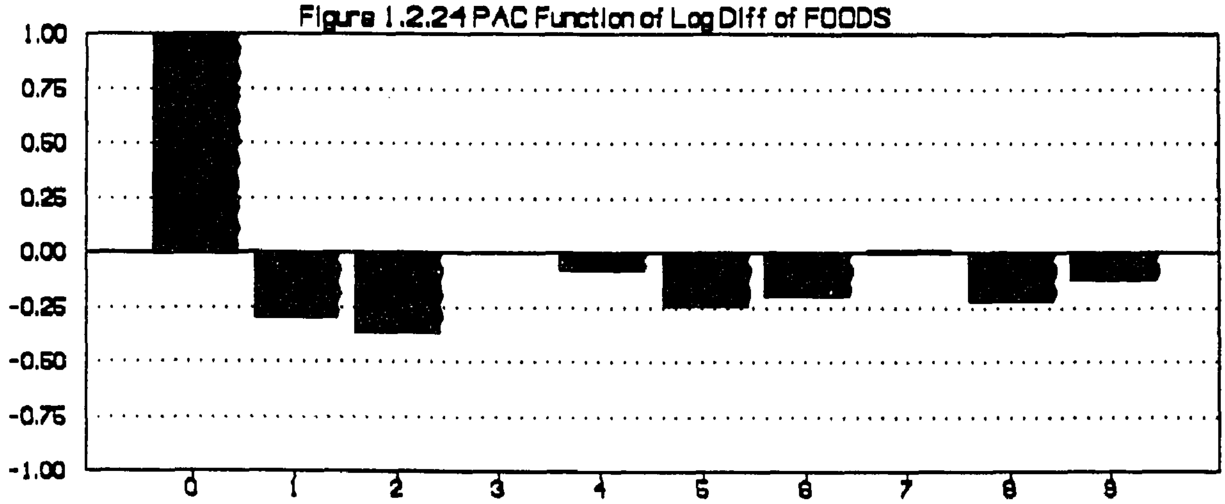
Figure 1.2.24 PAC Function of Log Diff of FOODS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0.50
0.25
0.00
-0.25
-0.50
-0.75
-1.00
0
1
2
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CHAPTER 2

In this chapter, we are going to examine a vector of variables $\mathbf{X}_t$, whose dynamics we would like to describe and whose elements are nonstationary. In other words, our goal is to characterize these dynamics in terms of a vector autoregression. Since, even though the individual series follow unitroot processes, some linear combinations of them may be stationary, we will investigate the cointegrating relationships between the industries. Section 2.1 makes a general introduction to the methods used in VAR modeling. In Section 2.2, we will give the technical details of cointegration, and discuss the motivations for developing and estimating the canonical relations to determine the cointegrating relations.

2.1. An Overview of Unit-Roots

There are basically three approaches in dealing with the nonstationary series. The first approach is to overlook the nonstationarity completely and estimate the VAR in levels and to evaluate the results based upon the standard t and F distributions for testing any hypothesis. The regression parameters obtained by this methodology would be asymptotically consistent in explaining the dynamics of the system at hand. Even if the true underlying process is a VAR in differences, certain functions of the parameters and hypothesis tests based on VAR in levels will have the same asymptotic distributions as will be the parameters of the differenced series (for technical details, see Hamilton (1994)).

The second approach would be to difference the nonstationary series before estimating VAR. The advantage of this approach is that if the true process of the series is a VAR in differences, differencing would improve especially the small sample performance of the estimates. Yet, an important point is that some of the series in VAR modeling may be stationary or it is possible that some linear combinations of the nonstationary series may prove to be stationary as in the cointegrated VAR. In this case it would be more appropriate to recognize the nature of the nonstationarity observed in the data.

As stated in Hamilton (1994, p.652):

"Yet a third approach is to investigate carefully the nature of the nonstationarity, testing each series individually for unit roots and then testing for possible cointegration among the series. Once the nature of the nonstationarity is understood, a stationary representation for the system can be estimated."

2.2. The VAR Modeling and the TEST for Cointegration : The Full Information Maximum Likelihood Estimation

In this section, based on the unit-root test results in the previous section, we are going to analyze whether the industrial outputs exhibit any cointegration. An $(n\times1)$ vector $\mathbf{X}_t$ is said to exhibit r cointegrating relations if there exist r linearly independent vectors $\beta_1, \beta_2, \ldots\ldots, \beta_r$ such that each $\beta'_i\mathbf{X}_t$ is stationary. In general, if it is known that there is only one such cointegrating vector, then it can be obtained by standard OLS estimation, put in the error correction form, which will be discussed below, and tested for whether it is stationary or not. However, the results obtained from this estimation would depend on the arbitrary assumptions as to which variable is going to be placed on the left side of the equation and what normalization conditions will be imposed on the elements of the cointegrating vectors. On the other hand, since any linear combination of the vectors $\beta_1, \beta_2, \ldots\ldots, \beta_r$ will be stationary, these cointegrating vectors are defined only up to a scale transformation.

As it is known, the Full Information Maximum Likelihood (FIML), developed by Johansen (1988, 1991), avoids this problem by estimating the linear space spanned by the cointegrating vectors $\beta_1, \beta_2, \ldots\ldots, \beta_r$ as well as it allows one to test for the number of cointegrating relations.

To describe Johansen's approach to FIML estimation of a system characterized by **r** cointegrating relations, let us suppose that $\mathbf{X}_t$ is an $(n \times 1)$ vector. The hypothesis here is that $\mathbf{X}_t$ follows a VAR(p) in levels as the following;

$$\mathbf{X}_t = \mu + \Phi_1\mathbf{X}_{t-1} + \Phi_2\mathbf{X}_{t-2} + \ldots + \Phi_p\mathbf{X}_{t-p} + \varepsilon_t \qquad (2.2.1)$$

or equivalently;

$$(\mathbf{I}_n - \Phi_1\mathbf{L} - \Phi_2\mathbf{L}^2 - \ldots - \Phi_p\mathbf{L}^p)\, \mathbf{X}_t = \mu + \varepsilon_t \qquad (2.2.2)$$

where Φ_s denotes an $(n \times n)$ matrix of coefficients for $s = 1, 2, \ldots, p$, μ is an $(n \times 1)$ vector of constants and ε_t are $(n \times 1)$ vector of white noise disturbances. Note that for any values of $\Phi_1, \Phi_2, \ldots, \Phi_p$, the following polynomials are equivalent:

$$(\mathbf{I}_n - \Phi_1\mathbf{L} - \Phi_2\mathbf{L}^2 - \ldots - \Phi_p\mathbf{L}^p) \qquad (2.2.3)$$

$$= (\mathbf{I}_n - \rho\mathbf{L}) - (\varsigma_1\mathbf{L} + \varsigma_2\mathbf{L}^2 + \ldots + \varsigma_{p-1}\mathbf{L}^{p-1})(\mathbf{I} - \mathbf{L})$$

where

$$\rho \equiv \Phi_1 + \Phi_2 + \ldots + \Phi_p \qquad (2.2.4)$$

$$\varsigma_s \equiv -(\Phi_{s+1} + \Phi_{s+2} + \ldots + \Phi_p) \tag{2.2.5}$$

for s = 1, 2, ... , p-1 and **I** is an (n × n) identity matrix.

Then it follows that any VAR(p) process can always be written as:

$$(\mathbf{I}_n - \rho \mathbf{L})\,\mathbf{X}_t - (\varsigma_1 \mathbf{L} + \varsigma_2 \mathbf{L}^2 + \ldots + \varsigma_{p-1} \mathbf{L}^{p-1})(1 - \mathbf{L})\,\mathbf{X}_t = \mu + \varepsilon_t \tag{2.2.6}$$

or

$$\mathbf{X}_t = \varsigma_1\, \Delta\, \mathbf{X}_{t-1} + \varsigma_2\, \Delta\, \mathbf{X}_{t-2} + \ldots + \varsigma_{p-1}\, \Delta\, \mathbf{X}_{t-p+1} + \mu + \rho\, \mathbf{X}_{t-1} + \varepsilon_t \tag{2.2.7}$$

Subtracting $\mathbf{X}_{t-1}$ from both sides of (2.2.7) and defining

$$\varsigma_0 \equiv \rho - \mathbf{I}_n = -(\mathbf{I}_n - \Phi_1 - \Phi_2 - \ldots - \Phi_p) = -\Phi(1) \tag{2.2.8}$$

equation (2.2.7) can be written in the form of;

$$\Delta\mathbf{X}_t = \varsigma_1\, \Delta\, \mathbf{X}_{t-1} + \varsigma_2\, \Delta\, \mathbf{X}_{t-2} + \ldots + \varsigma_{p-1}\, \Delta\, \mathbf{X}_{t-p+1} + \mu + \varsigma_0\, \mathbf{X}_{t-1} + \varepsilon_t \tag{2.2.9}$$

where ε_t is a (n × 1) vector of disturbances with;

$$E(\varepsilon_t) = 0$$

$$E(\varepsilon_t \varepsilon_\tau) = \begin{cases} \mathbf{\Omega} & \text{for } t = \tau \\ \mathbf{0} & \text{otherwise.} \end{cases}$$

The focus of cointegration analysis, in an analogy with the univariate case, is on the properties of the matrix ς_0. Based upon (2.2.9) there are three possible cases regarding ς_0 and they are related to the rank of this matrix:

I - Rank(ς_0) = n, i.e. the matrix ς_0 is of full rank. In this case the vector process $\mathbf{X}_t$ is stationary since all the elements of the vector process are the linear combinations of n linearly independent rows of ς_0 and standard estimation procedure applied to the level of the series is appropriate.

II - Rank(ς_0) = 0, i.e. ς_0 is a matrix of zeros such that no linear stationary combinations of the variables is possible. This corresponds to the traditional differenced vector time series models.

III - 0 < Rank (ς_0) = r < n. In this case, as will be discussed below, there exist ($n \times r$) matrices α and β such that

$$\varsigma_0 = \alpha\beta' \tag{2.2.10}$$

in other words, when the matrix ς_0 is of reduced rank, it is possible to decompose it so that $\beta'\mathbf{X}_t$ is stationary. The columns of β are the linearly independent cointegrating vectors and, therefore, the rank of the matrix ς_0 corresponds to the cointegrating rank of the system of variables $\mathbf{X}_t$. In this case, (2.2.9), written in the form of

$$\Delta\mathbf{X}_t = \varsigma_1 \Delta \mathbf{X}_{t-1} + \varsigma_2 \Delta \mathbf{X}_{t-2} + \ldots + \varsigma_{p-1} \Delta \mathbf{X}_{t-p+1} + \mu + \alpha\beta'\mathbf{X}_{t-1} + \varepsilon_t \tag{2.2.11}$$

is called the error correction model (see Engle and Granger (1987)).

Intuitively, the linear combinations of the variables in the system given by the columns of β can be considered as some long-run relationship between the integrated series of the vector $\mathbf{X}_t$ in the sense that, even though, at a certain time, the individual series may exhibit nonstationary movements due to some reasons such as shocks to the economy, in the long-run these series do not drift away too much from each other. The linear combinations determined by the cointegrating vectors play a role as an attractor every series in the system converges to. In this sense the elements of $\beta'\mathbf{X}_t$ are called equilibrium errors and the matrix α the adjustment factor.

An important point about estimation when there are both unit roots and some cointegration relationships among variables, as mentioned before, is that an estimation with an unrestricted VAR in the first differences of the variables will be misspecified since it does not take into account the true nature of the nonstationarity. An unrestricted VAR in levels , on the other hand, will cause some efficiency loss because it ignores some restriction put on the system by the previous equilibrium errors.

The determination of cointegrating rank of matrix ς_0 developed by Johansen is based on the maximum likelihood. In order to find the stationary linear combinations of the vector $\mathbf{X}_t$, one should find those linear combinations of the levels of the variables that correlate most highly with the stationary differences. As known from canonical correlation analysis, this turns out to be an eigenvalue problem as the following:

Suppose in the unrestricted VAR in (2.2.9), whose n elements are all integrated of order one, there are **r** linear combinations of the elements of $\mathbf{X}_t$ that are stationary. In this case it can be written in the error correction form of (2.2.11). If the errors are Gaussian then, for a sample of T observations, the log likelihood function of $\mathbf{X}_1, \mathbf{X}_2, \ldots\ldots, \mathbf{X}_T$ conditional on all previous periods' observations is written as

$$L(\Omega, \varsigma_1, \varsigma_2, \ldots\ldots, \varsigma_{p-1}, \alpha, \varsigma_0) = -(Tn/2)\log(2\pi) - (T/2)\log|\Omega|$$

$$-\left(\frac{1}{2}\right)\sum_{t=1}^{T}\left[\left(\Delta X_t - \varsigma_1 \Delta X_{t-1} - \varsigma_2 \Delta X_{t-2} \ldots\ldots - \varsigma_{p-1}\Delta X_{t-p+1} - \alpha - \varsigma_0 X_{t-1}\right)'\right.$$

$$\left.\times \Omega^{-1} \times \left(\Delta X_t - \varsigma_1 \Delta X_{t-1} - \varsigma_2 \Delta X_{t-2} \ldots\ldots - \varsigma_{p-1}\Delta X_{t-p+1} - \alpha - \varsigma_0 X_{t-1}\right)\right] \quad (2.2.12)$$

The purpose is to maximize (2.2.12) with respect to the parameters. The assumption that there are **r** cointegrating vector requires us to restrict $\varsigma_0 = \alpha\beta'$ but leaving the other parameters unrestricted in (2.2.11). As Johansen points out these parameters can be eliminated by regressing $\Delta\mathbf{X}_t$ and $\mathbf{X}_{t-1}$ on the lagged levels of each variable, a constant and other variables in the system by OLS, giving us the residual matrices $\mathbf{u}_t$ and $\mathbf{v}_t$ respectively. By concentrating the likelihood function this way, the concentrated likelihood function takes the form of reduced rank regression

$$\mathbf{u}_t = \alpha\beta' \mathbf{v}_t + \mathbf{e}_t \quad (2.2.13)$$

where $\mathbf{e}_t$ is the error matrix. Denoting the residual cross moment matrices by Ω_{uu}, Ω_{vv}, Ω_{uv} defined as

$$\Omega_{uu} = \sum_{t=1}^{T} u_t u_t \qquad (2.2.14)$$

$$\Omega_{vv} = \sum_{t=1}^{T} u_v u_v \qquad (2.2.15)$$

$$\Omega_{uv} = \sum_{t=1}^{T} u_t v_t \qquad (2.2.16)$$

For fixed β, (2.2.13) can be solved for α by regression

$$\alpha(\beta) = \Omega_{uv}\beta\,(\beta'\Omega_{vv}\,\beta)^{-1} \qquad (2.2.17)$$

and β is determined by solving the eigenvalue problem

$$|\,\lambda\,\Omega_{vv} - \Omega_{vu}\Omega_{uu}^{-1}\Omega_{uv}\,| = 0 \qquad (2.2.18)$$

which has n solutions ordered as $1 > \lambda_1 > \lambda_2 > \lambda_3 > \ldots\ldots > \lambda_n > 0$, with the corresponding eigenvectors, $\hat{A} = (\hat{a}_1, \hat{a}_2, \hat{a}_3, \ldots\ldots, \hat{a}_n)$, normalized by $\hat{A}\;\Omega_{vv}\,\hat{A} = \mathbf{I}_n$.

The maximum likelihood attained by the log likelihood function subject to the constraint that there are **r** cointegrating relations is given by

$$\mathbf{L}^{*} = -(Tn\ /\ 2)\log(2\pi - (T\ /\ 2) - \log|\hat{\Omega}_{uu}| - (T/2)\sum_{i=1}^{r}\log(1 - \hat{\lambda}_i) \qquad (2.2.19)$$

Given the form of the likelihood function in (2.2.19), one can form the hypothesis that there are **r** cointegrating relations against no cointegrating relation by likelihood ratio test of the form

$$2(\mathbf{L}_r^* - \mathbf{L}_0^*) = -T \sum_{i=r+1}^{n} \log(1 - \hat{\lambda}_i) \tag{2.2.20}$$

(2.2.20) is called the trace statistic. An alternative test, called λ_{max} statistic, is also constructed to test the hypothesis that there are **r** cointegrating relations against the alternative of **r**+1 cointegrating relations. Twice the log likelihood ratio is given by

$$2(\mathbf{L}_r^* - \mathbf{L}_0^*) = -T \log(1 - \hat{\lambda}_{r+1}) \tag{2.2.20}$$

As explained in Johansen (1990, 1992) in detailed this test statistics do not have the standard χ^2 distribution since the hypothesis involves the coefficient on $\mathbf{X}_{t-1}$, which depends upon the number of random walks, n- **r**. The tabulated tables are given Johansen (1990, 1992), Osterwald-Lenum (1992) and others for the different number of random walks present in the system. It should be noted that the asymptotic distribution of the test statistic will also depend upon the other variables included in the VAR such as deterministic trend, seasonal dummies (for a detailed discussion, see Hamilton (1994), Johansen (1992b).

2.3. Estimation of the Cointegrated Systems

In Engle and Issler (1995), they discuss the conditions for sectoral outputs to have cointegrating relations based on the model developed by Long and Plosser (1983). They prove that, given the model, for sectoral output series to have unit roots, the technology shocks should follow a unit-root process. Under this condition and if the labor is used in all production processes, then there is cointegration among sectoral outputs only if there is cointegration among the productivity shocks. On the other hand they assert that a necessary condition for sectoral outputs to have common cycles is that the matrix of input elasticities should be of reduced rank meaning basically that the outputs of some of the sectors are not used as inputs by other sectors. Based upon this propositions, we will now investigate the cointegration relations among 10 industries that exhibit unit-root processes in their per-capita productions.

Before conducting the cointegration analysis developed by Johansen (1988, 1991), we have examined the possibility of the presence of deterministic components in the system of 10 industries. Since the asymptotic distribution of the test statistic depends on the possible restrictions on deterministic components, this should be done before the analysis starts. Model (2.2.9) with two lags (one lag in differences) including a linear trend term is fitted to the industrial productions. The Likelihood Ratio test statistic obtained from the concentrated likelihood function of the form;

$$T\{\ln|\Omega_{uu}| - \ln|\Omega^{*}_{uu}| + \Sigma_{i=1}^{r^*} \ln(1-\lambda_i) - \Sigma_{i=1}^{r^*} \ln(1-\lambda_i^{*})\} \qquad (2.3.1)$$

is 36.489 , where (*) denotes the restricted regression, the model with no deterministic linear trend. In determining the deterministic structure in the VAR, the estimated cointegration rank for the unrestricted regression is used as suggested by Johansen (1990). The LR has a χ^2 distribution with (n - r) degrees of freedom, which in our case is 10-6=4. The obtained statistic rejects the null hypothesis that the VAR does not contain a linear trend at the 5 percent significance level. As a second step, we test whether the linear trend is restricted only inside the cointegrating relations, e.g. inside the error correction term or also outside this term. Again, using the likelihood ratio test in the form (3.1.1), the calculated test statistic is 12.36 which marginally accept the null hypothesis that the linear trend present in the data is only inside the error correction term at the 5 percent significance level, with 6 degrees of freedom. Having linear trend only in the cointegrating space means that the model probably have some trend stationary variables. Since the test result only marginally accepts the null hypothesis and from the results of the Dickey-Fuller test based upon which we excluded the trend stationary variables from the model, we concluded that the data is generated by a VAR process with unrestricted constant and a linear trend and model (2.2.9) with the addition of a linear term is fitted to the data. Table 2.3.1 show some test statistics for the normally, independently and identically distributed errors assumption for the model with linear trend and two lags. According to the table results all residuals pass the normality test at the 5 percent level, whereas the equation for ΔPETROCOA is almost significant. The multivariate normality

Table 2.3.1 Some Test Statistics for NIID Assumption for the Residuals in the Model

Variables	Test τ_1	Statistics τ_2
ΔFABMET	6.32	0.61
ΔELECTRIC	16.26	1.61
ΔMISC	11.77	1.32
ΔSTONEGLA	8. 72	1.19
ΔPRIMMETA	10.5 2	3.60
ΔAPPAREL	5.95	0.11
ΔPAPER	16.85	0.14
ΔCHEMICAL	14.71	0.50
ΔPETROCOA	18.53	0.28
ΔLEATHER	7.44	0.39

where $\tau_1=T\Sigma r_i^2$ $(I=1,\ldots\ldots,10) \sim \chi^2(10)$

$\tau_1=((T-m)/6)(SK^2+(EK^2/4)) \sim \chi^2(2)$

m is the number of regressors, SK is skewness and EK is excess kurtosis.

test statistic, as proposed in Hansen and Juselius (1995),[2] is found 26.26 which is approximately χ^2 -distributed with 2n degrees of freedom which is 20 in our case. Checking the roots of the companion matrix of the VAR in levels without a linear trend in (2.2.1) indicated that the models show explosive paths, on the other hand a model with 3 lags and linear trend cannot pass the autocorrelation tests for a number of residuals . As a consequence, we concluded that, in comparison to other model specifications, the

[2] The formula to calculate the multivariate normality test is rather long, therefore it is not given here. See Hansen and Juselius (1995) for the derivation of the test statistic which has χ^2 distribution with 2n degrees of freedom.

unrestricted VAR with 2 lags, unrestricted constant and a linear trend fits the data reasonably well.

On the basis of the model specification explained above, we have used Table 2 in Osterwald-Lenum (1992), presented in the next page, for the critical values of the asymptotic distributions for the trace and λ_{max} statistics for 5 percent significance level. The results are shown in Table 2.3.2 below.

Table 2.3.2 Eigenvalues and Estimated λ_{max} and Trace Statistics for 10 Industries

Null Hypotheses	Eigenvalues	λ_{max} Statistic	Trace Statistics
There are at most 0 cointegrating vectors	0.9159	96.54	392.56
There are at most 1 cointegrating vectors	0.8633	77.61	296.02
There are at most 2 cointegrating vectors	0.7855	60.04	218.41
There are at most 3 cointegrating vectors	0.7494	53.97	158.37
There are at most 4 cointegrating vectors	0.6185	37.58	104.40
There are at most 5 cointegrating vectors	0.4759	25.20	66.82
There are at most 6 cointegrating vectors	0.4445	22.92	41.62
There are at most 7 cointegrating vectors	0.2814	12.89	18.69
There are at most 8 cointegrating vectors	0.1356	5.69	5.81
There are at most 9 cointegrating vectors	0.0031	0.12	0.12

Table 2.3.3 Quantiles of the Asymptotic Distribution of the Cointegration

Rank Test Statistics *

Data Generating Process and Statistical Model

$$\Delta \mathbf{X}_t = \zeta_1 \Delta \mathbf{X}_{t-1} + \zeta_2 \Delta \mathbf{X}_{t-2} + \dots\dots\dots\dots + \zeta_{p-1} \Delta \mathbf{X}_{t-p+1} + \alpha\beta' \mathbf{X}_{t-1} + \mu + \delta\, \mathbf{t} + \psi\, \mathbf{D}_t + \varepsilon_t$$

$$\varepsilon_t \sim (\mathbf{0}, \Omega)$$

n - r	50%	80%	90%	95%	97.5%	99%	Mean	Var
				λ - max				
1	0.45	1.61	2.57	3.74	4.85	6.40	0.98	1.96
2	8.84	12.55	14.84	16.87	18.57	21.47	9.46	16.02
3	14.70	18.94	21.53	23.78	26.07	28.83	15.30	21.57
4	19.99	24.81	27.76	30.33	32.56	35.68	20.72	27.29
5	25.78	30.75	33.74	36.41	38.68	41.58	26.27	31.67
6	30.96	36.51	39.50	42.48	45.12	48.17	31.57	36.39
7	36.44	42.07	45.49	48.45	51.46	54.48	37.07	40.22
8	41.68	47.51	51.14	54.25	56.87	60.81	42.33	44.31
9	46.92	53.12	57.01	60.29	62.98	66.91	47.56	49.45
10	52.33	59.01	62.69	66.10	69.41	72.96	53.02	54.08
11	57.76	64.40	68.22	71.68	74.90	78.51	58.43	55.68
				Trace				
1	0.45	1.61	2.57	3.74	4.85	6.40	0.98	1.96
2	9.68	13.56	16.06	18.17	20.13	23.46	10.31	18.34
3	22.66	28.13	31.42	34.55	36.94	40.49	23.31	37.90
4	39.43	46.66	50.74	54.64	57.79	61.24	40.19	64.17
5	60.33	68.66	73.40	77.74	80.94	85.78	60.93	92.36
6	84.53	94.45	100.14	104.94	109.62	114.36	85.13	129.92
7	112.75	124.18	130.84	136.61	141.55	146.99	113.48	173.03
8	144.39	157.11	164.34	170.80	176.43	182.51	145.11	217.53
9	179.72	194.04	201.95	208.97	215.41	222.46	180.57	270.83
10	219.42	235.26	244012	250.84	256.60	263.94	220.05	332.87
11	262.30	279.31	288.08	295.99	303.98	312.58	262.92	398.78

* The Quantiles of the Asymptotic Distribution of the Cointegration Rank Test Statistics are taken from Osterwald-Lenum (1992, p. 470, Table 2, Case 2)

The formal procedure to test the hypothesis about cointegration relations are as follows:

After determining the significance level, which is 5 percent in most applications, one should start from the top of the table for either λ_{max} or trace statistics and compare them with the corresponding estimated values. If the calculated test statistic is bigger than the value in the distribution table, then we proceed with the second row. This procedure continues until we can no longer reject the hypotheses; in other words, the procedure stops when we come to the row where the calculated test statistic is smaller than the corresponding quantile. In general the test procedure is conducted sequentially beginning from nocointegration as against there is at the most one cointegrating relation. In this case the two statistics can be evaluated the same way as described above. Alternative test procedures related to the number of cointegrating relations can be performed by using the Trace Statistics. For example such that if we want to test the null hypothesis of cointegration rank of at most 3 against a cointegrating rank of at most 7, then we should look at the trace statistic which is given in line 4 of Table (2.3.3).

Based upon the sequential procedure we conclude that in the model we are analyzing there are 4 cointegrating relations, that is the cointegration rank is 4. Therefore the first four normalized eigenvectors corresponding to the first four eigenvalues give the stationary linear combinations of the level of productions of the ten industries under investigation. The normalized eigenvectors are presented in Table 2.3.4 below, normalized by $\hat{A}' \Omega_{vv} \hat{A} = \mathbf{I}_n$. Note that after this normalization, it is trivial to renormalize them so as to make the coefficient of one of the variables equal to 1 with the variable chosen arbitrarily since we can multiply each corresponding element of α by the same factor.

Table 2.3.4 Normalized Eigenvectors for the Industrial Production Data

Corresponding the First Largest Eigenvalues

	Eigenvectors			
Variables				
FABMET	0.786	-0.997	8.824	7.470
ELECTRIC	-0.491	0.486	-2.771	-3.513
MISC	-0.641	0.634	1.164	-1.550
STONEGLA	0.368	0.007	-7.403	-6.177
PRIMMETA	-0.424	-0.247	4.207	0.427
APPAREL	-0.240	0.052	-16.085	-0.629
PAPER	1.000	0.112	9.699	0.605
CHEMICAL	-0.199	1.000	-14.214	-2.202
PETROCOA	0.009	-0.583	1.000	0.784
LEATHER	0.316	0.213	5.759	1.000

We have selected the last four industries coefficients to be set equal to one in each eigenvectors, which corresponds to rows 7, 8, 9 and 10 respectively. In fact this renormalization should be done to construct and test the relations proposed by the theoretical models about economic variables. For example a study on money demand function would set money demand coefficient in the cointegrating relation to see whether the income elasticity of money demand is equal to 1. Since, in this study, we do not have any apriori knowledge on the relationships among industries, our selection may be done arbitrarily.

Once the cointegrating vectors are determined, they can be tested whether they contain any unit roots. Note that these vectors do not have to be white noise processes, all it is required is that they should be trend stationary. The standard Dickey-Fuller test can be applied in this case. Figure 2.3.1 through 2.3.4 show the detrended cointegrating vectors, $\beta_j' \mathbf{X}_t$ ($j = 1,2,3,4$). In order to extract their deterministic components, they were run on a constant and a linear trend. All of them seem to be well-behaved long-run relationships. An informal check of the correlograms of the detrended cointegrating vectors indicate that they are stationary.

Figure 2.3.1 DET_BETA1

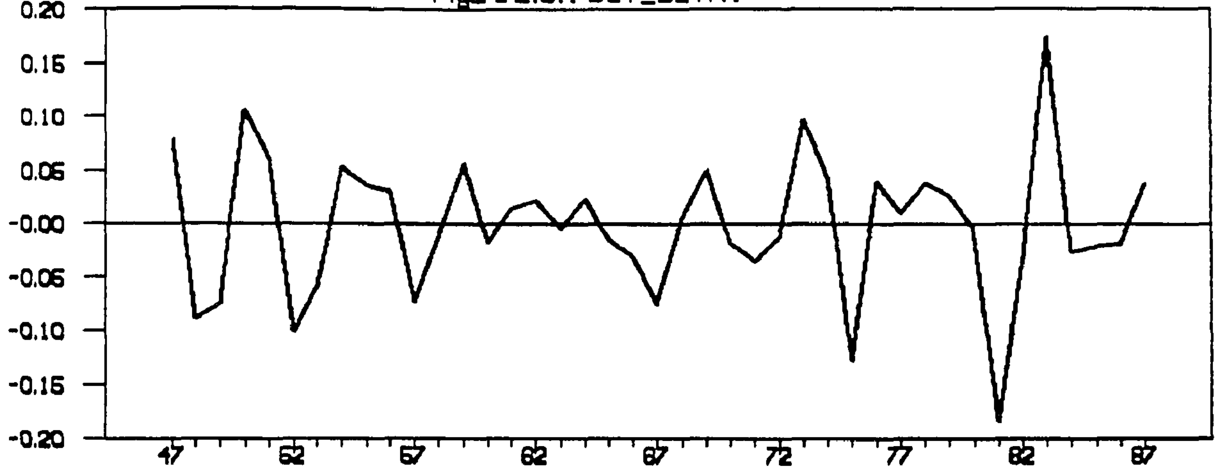


Figure 2.3.2 DET_BETA2

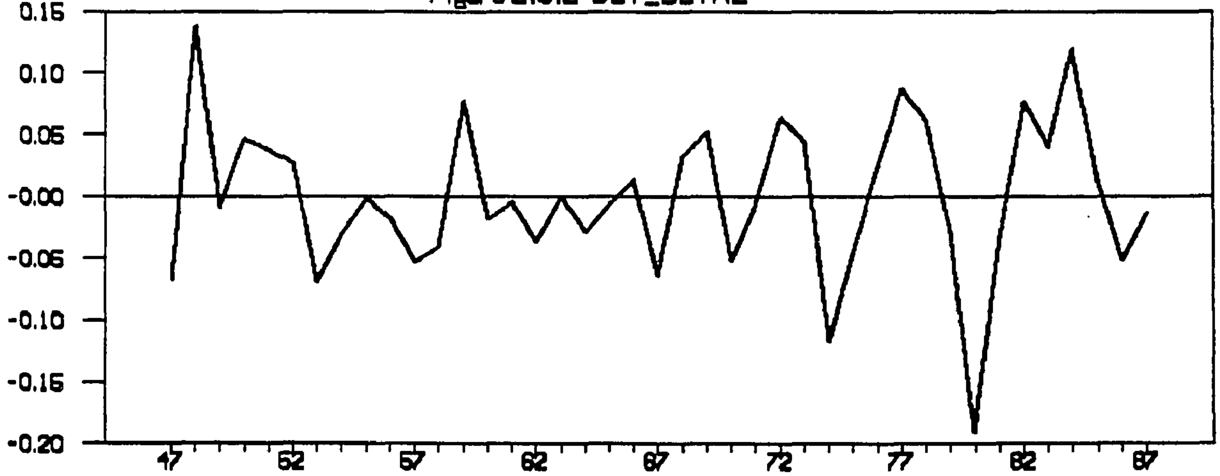
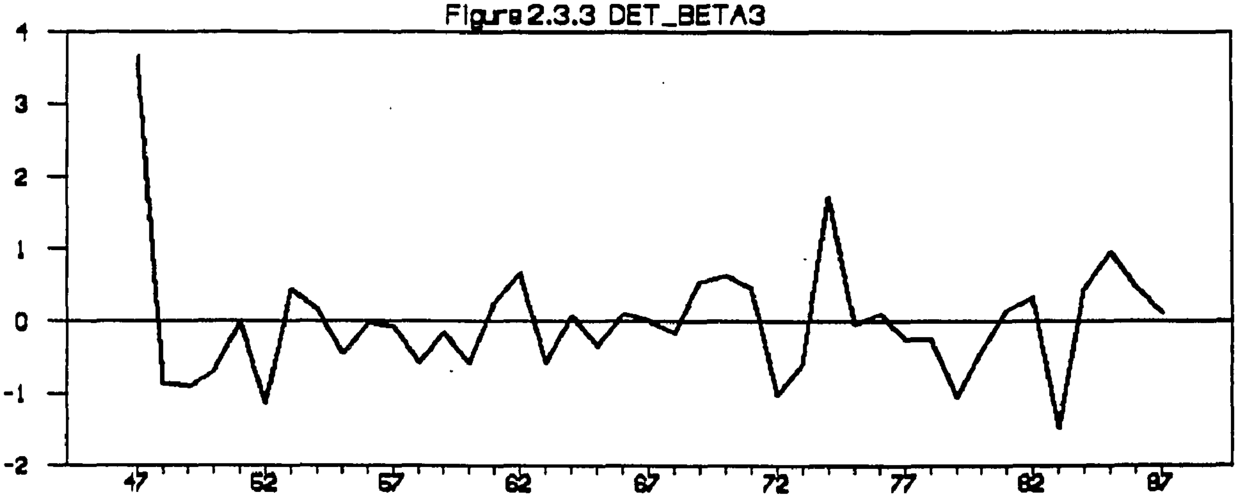


Figure 2.3.3 DET_BETA3



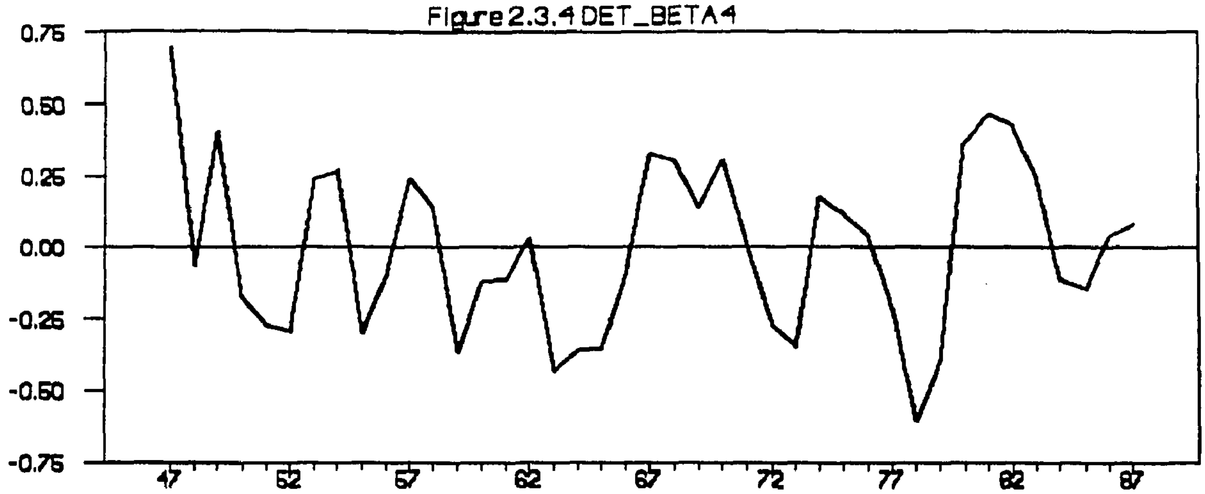


Figure 2.3.4 DET_BETA4
0.75
0.50
0.25
0.00
-0.25
-0.50
-0.75
47
52
57
62
67
72
77
82
87

By making use of (2.2.17) and using the normalization the adjustment matrix, α is estimated in the form of

$$\alpha = \Omega_{uv}\beta^{*} \tag{2.3.2}$$

and the other parameters of the VAR can be calculated as the linear combinations of the parameters estimated from the auxiliary regressions, regressing $\Delta\mathbf{X}_t$ and $\mathbf{X}_{t-1}$ on the lagged value of the differences and other deterministic components of the model. The maximum likelihood estimator of the variance is calculated as:

$$\hat{\Omega} = \left(\frac{1}{T}\right)\sum_{t=1}^{T}\left[\left(\hat{\mathbf{u}}_t - \hat{\varsigma}_t\hat{\mathbf{v}}_t\right)\left(\hat{\mathbf{u}}_t - \hat{\varsigma}_t\hat{\mathbf{v}}_t\right)'\right] \tag{2.2.3}$$

The estimated coefficients are given in Figure 2.3.5 through Figure 2.3.6 for the estimated cointegrating rank.

Table 2.3.5 Estimate of α

FABMET	-0.344	-0.315	-0.003	-0.062
t values	-1.540	-1.430	-0.145	-1.314
ELECTRIC	0.041	-0.489	-0.021	-0.070
t values	0.155	-1.853	-0.822	-1.236
MISC	0.643	-0.919	-0.011	0.225
t values	2.467	-3.573	-0.449	4.069
STONEGLA	-0.351	-0.120	0.003	0.006
t values	-1.793	-0.620	0.156	0.134
PRIMMETA	0.117	-0.899	-0.015	-0.114
t values	0.339	-2.651	-0.465	-1.561
APPAREL	-0.108	-0.450	0.020	-0.066
t values	-0.640	-2.712	1.273	-1.841
PAPER	-1.146	-0.334	-0.035	0.054
t values	-6.553	-1.937	-2.113	1.460
CHEMICAL	-0.792	-0.530	0.044	0.002
t values	-5.441	-3.685	3.202	0.059
PETROCOA	-0.063	0.017	0.036	0.001
t values	-0.421	0.116	2.584	0.034
LEATHER	0.124	-1.523	-0.010	-0.124
t values	0.696	-8.645	-0.571	-3.269

Table 2.3.6 Estimate of ζ_0 Matrix

	FAB	ELE	MISC	STO	PRI	APP	PAPE	CHE	PETR	LEA
FAB	-0.447	0.242	0.113	0.277	0.184	0.154	-0.446	-0.067	0.129	-0.255
ELE	-0.185	0.045	-0.252	0.596	-0.014	0.341	-0.256	-0.049	0.210	-0.280
MISC	3.001	-1.521	-1.355	-1.075	0.004	-0.166	0.569	-1.385	0.706	0.169
STO	-0.089	0.087	0.144	-0.186	0.193	0.028	-0.333	-0.103	0.074	-0.114
PRIM	0.006	-0.053	-0.486	0.850	0.061	0.238	-0.199	-0.458	0.421	-0.355
APP	0.052	0.009	-0.091	0.213	0.213	-0.280	-0.002	-0.570	0.230	-0.080
PAPE	-0.470	0.307	0.399	-0.501	0.446	0.782	-1.488	0.269	0.192	-0.580
CHE	0.306	0.004	0.221	-0.631	0.652	-0.544	-0.425	-0.999	0.347	-0.109
PETR	0.262	-0.065	0.092	-0.299	0.176	-0.569	0.292	-0.489	0.027	0.194
LEAT	0.608	-0.341	-0.864	0.869	0.230	0.122	-0.214	-1.140	0.782	-0.464

CHAPTER 3

Having determined the cointegration relations among the industries, we know that 10 industries we are investigating will share six common cycles. Engle and Issler (1995) argues that given the restrictions imposed by the cointegrating relations, a special decomposition can be performed to have both common trends and common cycles. They apply common cycles common trends decomposition to the sectoral GNPs to investigate their cyclical behavior.

In this chapter, we will attempt to use their methodology to understand the cyclical movements of the industrial sectors in relation to each other. It should be noted that the analysis is limited in the sense that it covers only very few industrial group, therefore it may not reflect the overall characterization of these industries. It should be evaluated within these limitations. In the first section, an overview of their model will be summarized. Section 2 gives the results of cofeature analysis applied to the industries under investigation.

3.1. Common Cycles

In this section we will briefly introduce the theoretical background for the common cycle analysis developed by Vahid and Engle (1993) and Engle and Issler (1995). They define a series to have a cycle if it has persistence in its first differences. If there is a linear combination of the first difference series that has no cycle, then it is called 'common cycle' or 'common feature'. If a series can be forecast ' based upon the past of all the series in the analysis' (Vahid and Issler (1995), p. 86), it is defined to be persistent.

In the error correction model in (2.2.11), all the serial correlation present in the data vector $\Delta \mathbf{X}_t$ is eliminated by the cointegrating vector(s) and the lag differences of the variables on the right hand side. This comes from the fact that the error term in (2.2.11) is white noise. If there is a vector, σ_j' that eliminates the serial correlation of $\Delta\mathbf{X}_t$, it is clear that such a vector should be orthogonal to $\alpha\beta'$ and ς_k for all k = 1,...,p-1 such that

$$\sigma_j' \alpha\beta' = \mathbf{0} \text{ and } \sigma_j' \varsigma_k = \mathbf{0} \qquad (3.1.1)$$

Vector σ_j' is called cofeature vector. Note that for this vector to eliminate the serial correlation both conditions should hold. In that sense the presence of cointegration does not imply common serial correlation neither prevents it. An important implication of the existence of a cofeature vector is that it eliminates the cyclical components of the level series, $\mathbf{X}_t$, and reveals only the deterministic and stochastic trends present in the data.

This can be seen from the observation that, putting aside the deterministic components for the notational simplicity, if the data have common serial correlation, then from (2.2.9) we have

$$\sigma_j^{'} \Delta \mathbf{X}_t = \sigma_j^{'} \varepsilon_t \tag{3.1.2}$$

and when integrated it will yield a random walk, with serially uncorrelated innovations. They also note that since random walk is an integrated of order one process, a cofeature vector should also be linearly independent from the cointegrating vectors which create series of integrated of order zero as discussed in the previous section. This means that if there are **r** linearly independent cointegrating vectors, then there can be at the most n - **r** cofeature vectors, where n denotes the dimension of the system, although this upper bound does not have to be reached.

When there are n - **r** cofeature vectors, however, the special trend-cycle decomposition is possible as they discussed. We know from Wold representation that the first differences of the integrated of order one series can be written in the following form

$$\Delta \mathbf{X}_t = \mathbf{C}\, \varepsilon_t \tag{3.1.3}$$

where $\mathbf{C}(L) = \mathbf{I} + \mathbf{C}_1 L^1 + \mathbf{C}_2 L^2 + \ldots\ldots\ldots\ldots$

(3.1.3) can be decomposed as

$$\mathbf{C}(L) = \mathbf{C}(1) + (1 - L)\,\mathbf{C}^*(L) \tag{3.1.4}$$

where $\mathbf{C}^*(L)$ is a matrix polynomial with $\mathbf{C}^*_0 = \mathbf{I} - \mathbf{C}(1)$ and $\mathbf{C}^*_j = \sum_{j>k} - \mathbf{C}_j$ for all k, k = 1, ..., p-1. Therefore (3.1.3) can be rewritten as

$$\Delta \mathbf{X}_t = \mathbf{C}(1)\varepsilon_t + (1-L)\mathbf{C}^*(L)\varepsilon_t \qquad (3.1.5)$$

Since $\mathbf{C}(1)\varepsilon_t$ is a multivariate white noise process all serial correlation of $\Delta \mathbf{X}_t$ should be captured by $(1-L)\mathbf{C}^*(L)\varepsilon_t$. Integration of (3.1.5) yields Beveridge-Nelson representation of the form

$$\mathbf{X}_t = \mathbf{C}(1)\sum_{i=0}^{\infty}\varepsilon_{t-i} + \mathbf{C}^*(L)\varepsilon_t \qquad (3.1.6)$$

Notice that if all the serial correlation is captured by $\mathbf{C}^*(L)$, then the cofeature vector(s) should satisfy

$$\sigma_j' \mathbf{C}^*(L) = 0 \text{ for all } (j) \qquad (3.1.7)$$

and since the cointegration vectors give all the stationary combinations we have

$$\beta_i' \mathbf{C}(1) = 0 \text{ for all } (i) \qquad (3.1.8)$$

If we denote σ as the matrix whose columns are all the linearly independent cofeature vectors then multiplying (3.1.6) by β' and σ respectively yields;

$$\beta' \mathbf{X}_t = \beta' \mathbf{C}^*(L)\varepsilon_t \qquad (3.1.9)$$

and

$$\sigma' \mathbf{X}_t = \sigma' \mathbf{C}(1)\sum_{i=0}^{\infty}\varepsilon_{t-i} \qquad (3.1.10)$$

Note that since cointegration vectors eliminates the stochastic trends from the data and cofeature vectors eliminates the cycles, (3.1.9) has no stochastic trends and (3.1.10) has no cycles.

In the special in which there are n - **r** cofeature vectors, the cointegration vectors and cofeature vectors together form a matrix of full rank with dimension (n × n) since they are linearly independent. We can write this matrix as the following:

$$\begin{bmatrix} \sigma' \\ \beta' \end{bmatrix} \mathbf{X}_t = \begin{bmatrix} \sigma' \mathbf{C}(1) \sum_{i=0}^{\infty} \varepsilon_{t-i} \\ \beta' \mathbf{C}^{*}(L) \varepsilon_t \end{bmatrix} \tag{3.1.11}$$

with σ' being ((n - **r**) × n) and β' being an (**r** × n) matrix. Denoting

$$\Lambda = \begin{bmatrix} \sigma' \\ \beta' \end{bmatrix}$$

we can partition Λ^{-1} conformable to the matrices in Λ as follows:

$$\Lambda^{-1} = \begin{bmatrix} \underset{n \times (n-r)}{\sigma'^{-}} & \underset{n \times r}{\beta'^{-}} \end{bmatrix}$$

Premultiplying (3.1.11) by Λ^{-1} creates the trend-cycle composition developed by Vahid and Engle (1993) in the form

$$\mathbf{X}_t = \sigma'^{-} \sigma' \mathbf{C}(1) \sum_{i=0}^{\infty} \varepsilon_{t-i} + \beta'^{-} \beta' \mathbf{C}^{*}(L) \varepsilon_t \tag{3.1.12}$$

where the first term in the right hand side gives the stochastic trend and the second term zero mean stationary cycles.

3.2. Estimation of Common Cycles

Once the restrictions imposed by the cointegrating vectors estimated, we can form a Vector Error Correction Model of the form (2.2.11). To determine the number of common cycles, we should look for the linear combinations of differenced series in the left-hand side of the equations that are uncorrelated with all the variables in the right hand side. These linear combinations can be obtained by the canonical correlation analysis between the differenced series and all the right hand side variables in the error correction model.

Each statistically zero canonical correlation would give a linear combination of $\Delta \mathbf{X}_t$ uncorrelated with all the linear combinations of the cointegrating vectors and the lag level of the differenced series. If it can be found that the number of statistically zero canonical correlations are equal to **n** - **r** , then the special decomposition of common trends and common cycles can be obtained.

These canonical correlations are given by the cross moment matrices of the variables in the model. As in the cointegration analysis they are given by the eigenvalues of combination of the variance-covariance matrices in the same manner discussed in the previous chapter. The trace and λ_{max} statistics would have χ^2 distributions with degrees of freedom T - (1/2)(n + (r+p-1)) + 3) as suggested by Barlett (1947).

The canonical correlation coefficients are presented below, in Table 3.2.1, with their probabilities.

Table 3.2.1 Canonical Correlations and Common Cycles Test

Squared Canonical Cor. (σ^2_i)	Pr > χ^2	Null Hypothesis
0.9801328	0.00000000	$\sigma_i = 0; \quad 1 \le i \le 10$
0.9650479	0.00000000	$\sigma_i = 0; \quad 2 \le i \le 10$
0.9061118	0.00006494	$\sigma_i = 0; \quad 3 \le i \le 10$
0.8525836	0.05355112	$\sigma_i = 0; \quad 4 \le i \le 10$
0.4981025	0.83959002	$\sigma_i = 0; \quad 5 \le i \le 10$
0.4087591	0.93468781	$\sigma_i = 0; \quad 6 \le i \le 10$
0.3320594	0.97599191	$\sigma_i = 0; \quad 7 \le i \le 10$
0.2212182	0.99553050	$\sigma_i = 0; \quad 8 \le i \le 10$
0.0467421	0.99985472	$\sigma_i = 0; \quad 9 \le i \le 10$
0.0120649	0.99746010	$\sigma_i = 0; \quad i=10$

As can be seen from the test results, the fourth canonical correlation is almost significant. Based upon these observation we conclude that the number of the cofeature vectors is 6. After normalizing them, the trend and cycles obtained for the industrial productions are given in Figure 3.2.1 to 3.2.20.

Upon the results obtained from our analysis, it is clear that industries cycles show the same phase for almost all the industries. The industries will share six common trends and four common cycles. Along with this observation, it can also be seen that industries basically show pro-cyclical behavior as the decreases in their production during the recession periods indicated by the vertical lines in the figures.

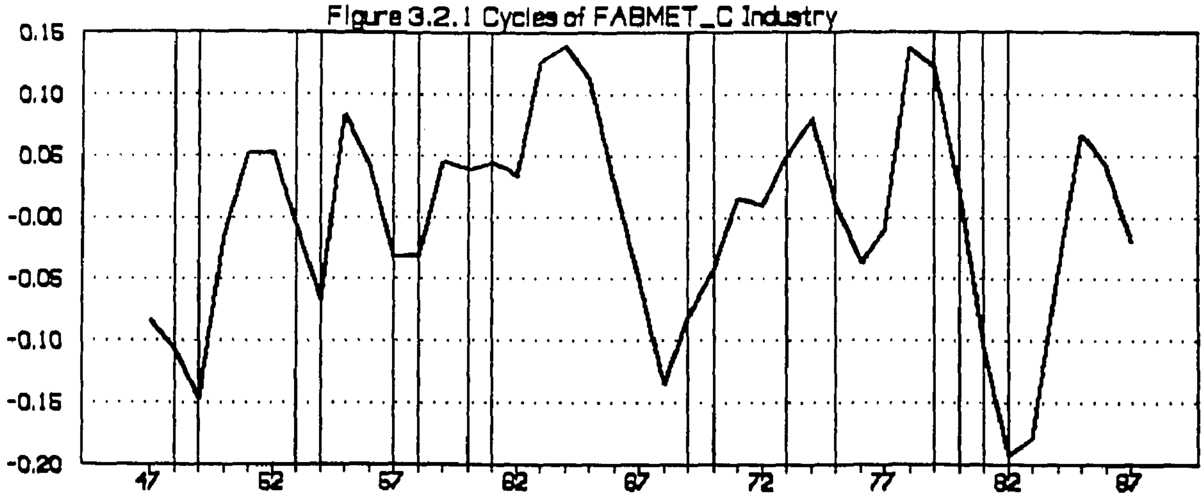


Figure 3.2.1 Cycles of FABMET_C Industry
0.15
0.10
0.05
-0.00
-0.05
-0.10
-0.15
-0.20
47
52
57
62
67
72
77
82
87

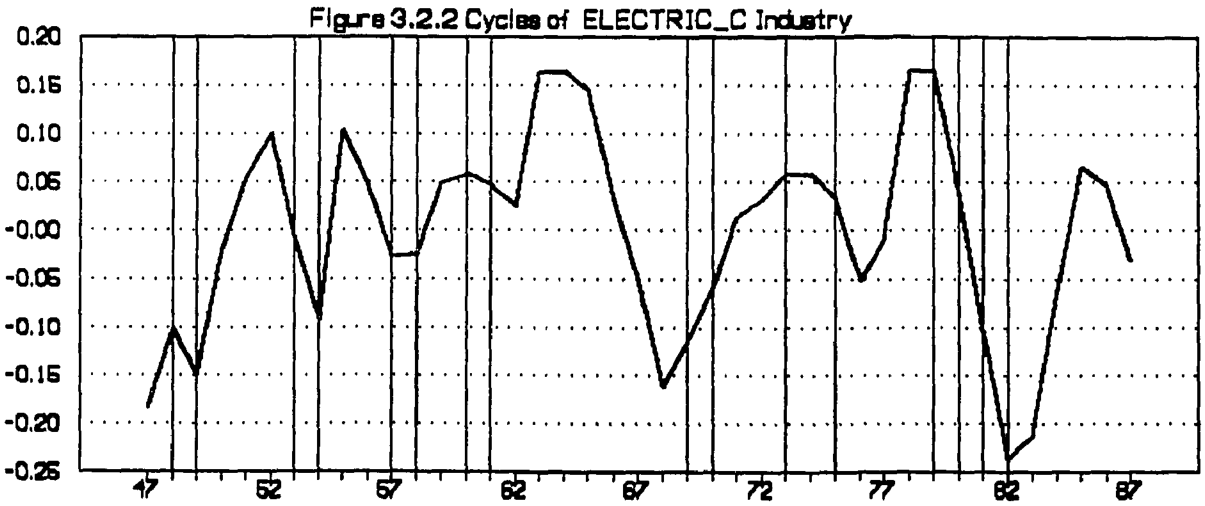


Figure 3.2.2 Cycles of ELECTRIC_C Industry
0.20
0.15
0.10
0.05
-0.00
-0.05
-0.10
-0.15
-0.20
-0.25
47
52
57
62
67
72
77
82
87

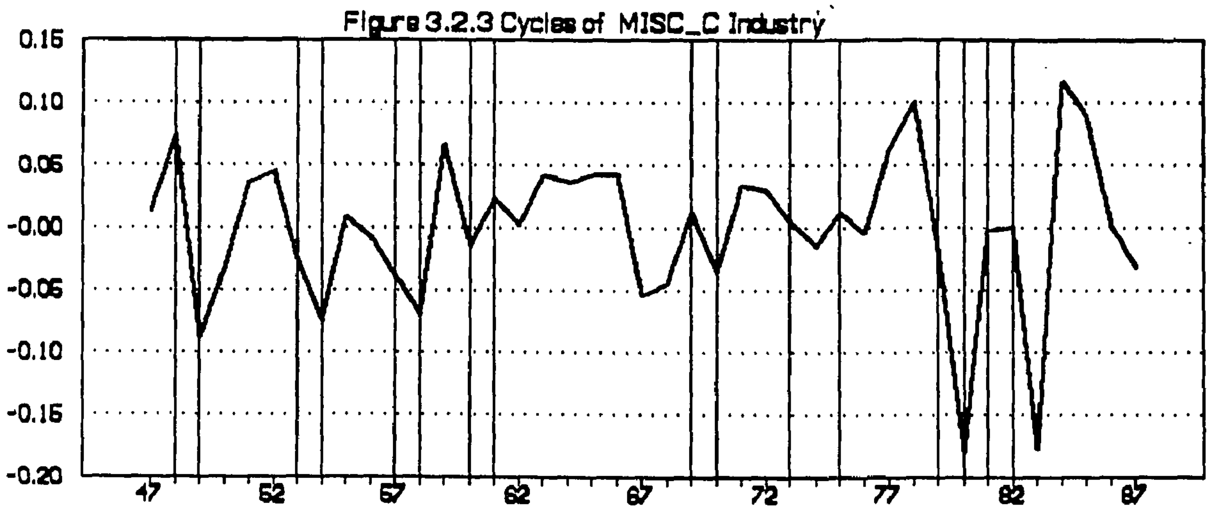


Figure 3.2.3 Cycles of MISC_C Industry
0.15
0.10
0.05
-0.00
-0.05
-0.10
-0.15
-0.20
47
52
57
62
67
72
77
82
87

Figure 3.2.4 Cycles of STONEGLA_C Industry

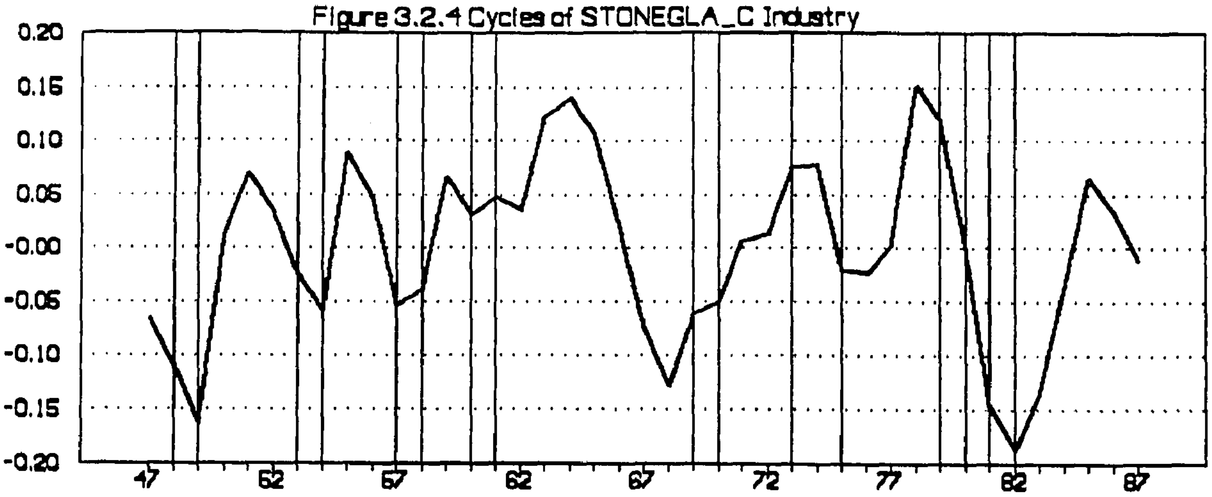


Figure 3.2.5 Cycles of PRIMMETA_C Industry

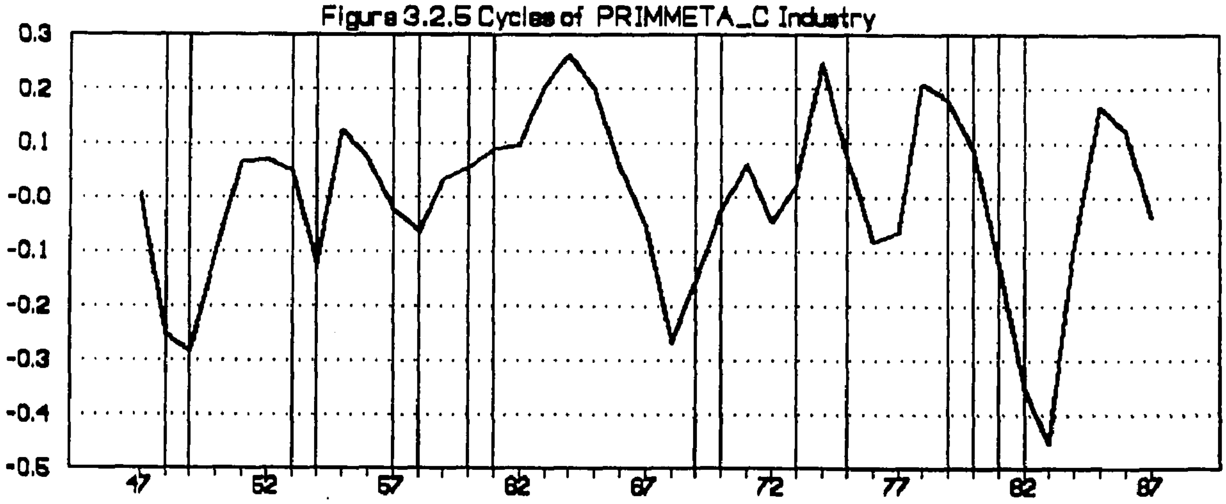


Figure 3.2.6 Cycles of APPAREL_C Industry

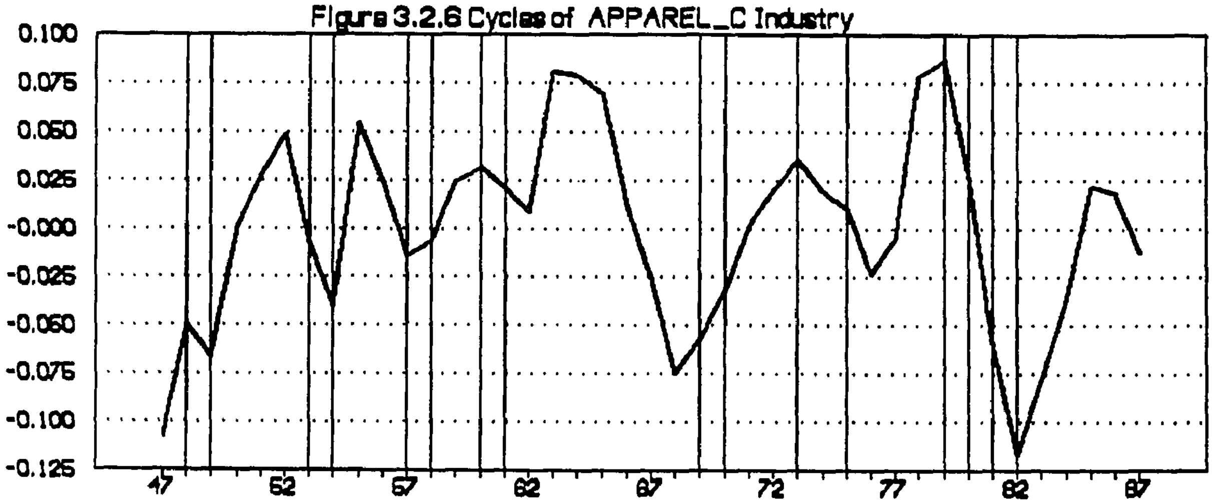


Figure 3.2.7 Cycles of PAPER_C Industry

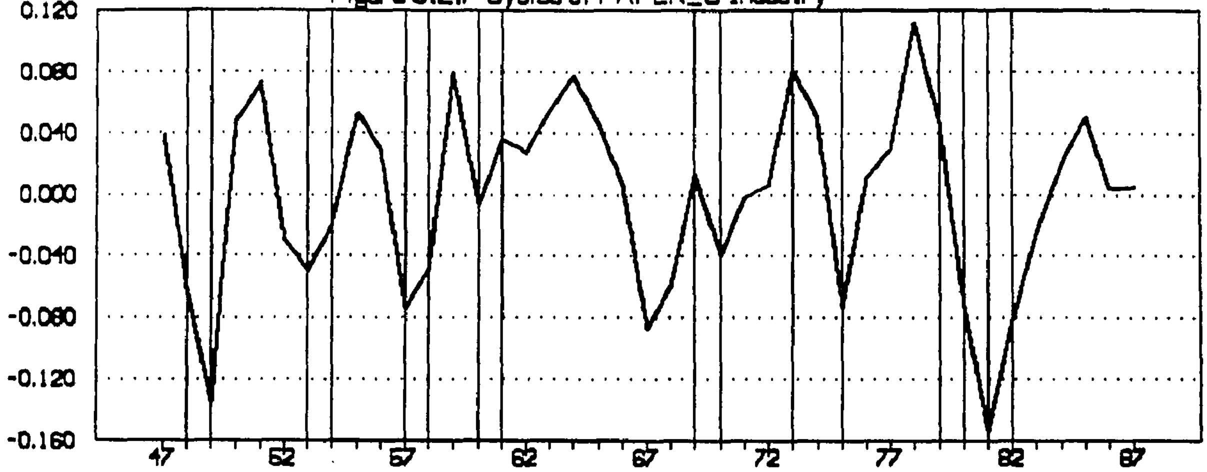


Figure 3.2.8 Cycles of CHEMICAL_C Industry

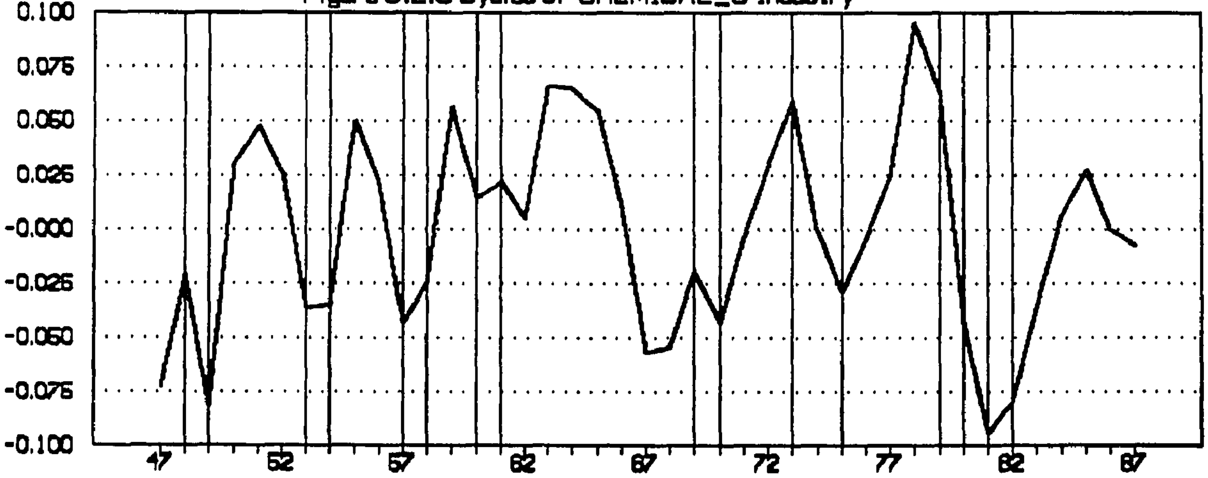
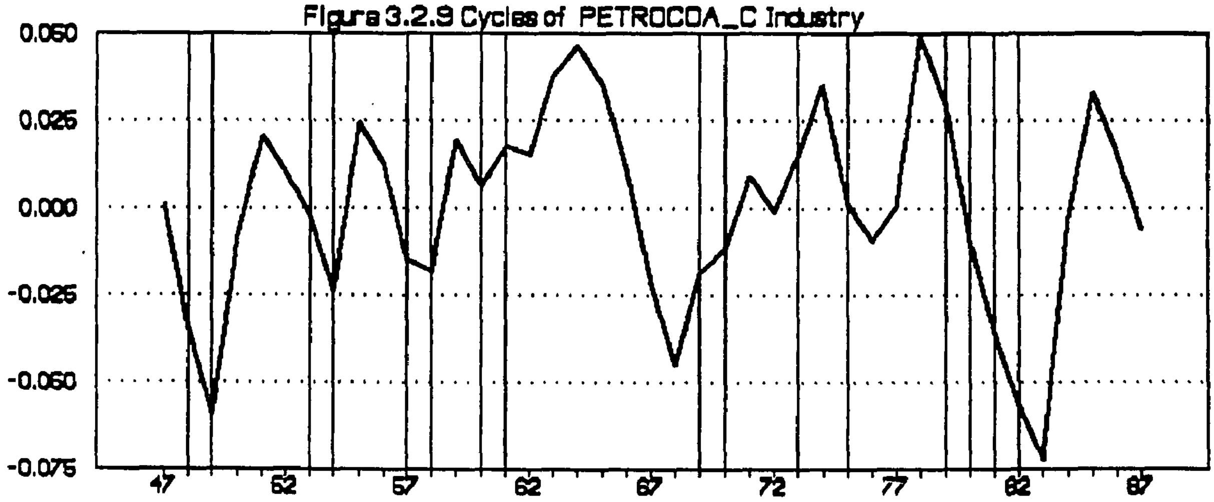


Figure 3.2.9 Cycles of PETROCOA_C Industry



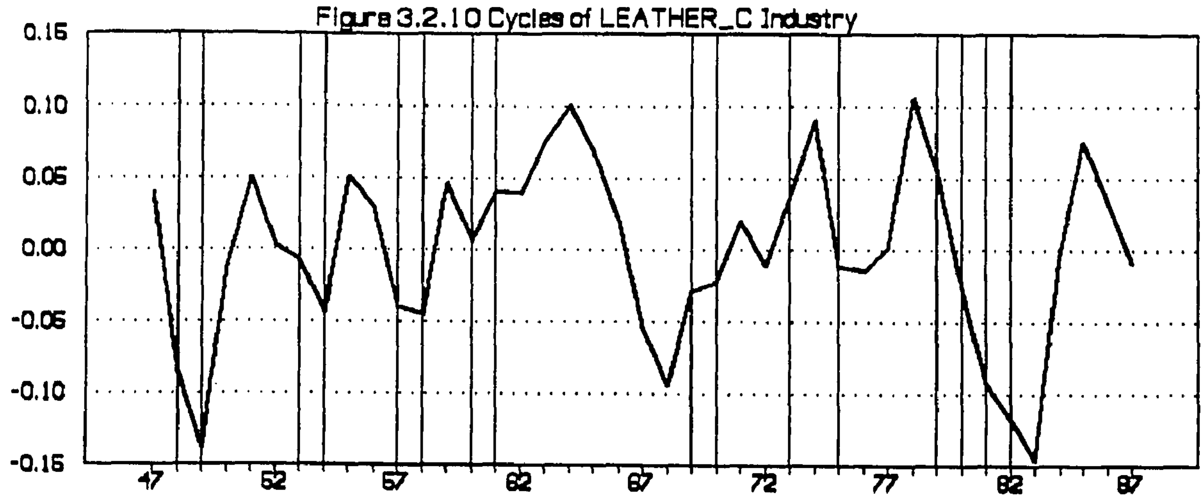


Figure 3.2.10 Cycles of LEATHER_C Industry
0.15
0.10
0.05
0.00
-0.05
-0.10
-0.15
47
52
57
62
67
72
77
82
87

Figure 3.2.11 Trends in FABMET Industry

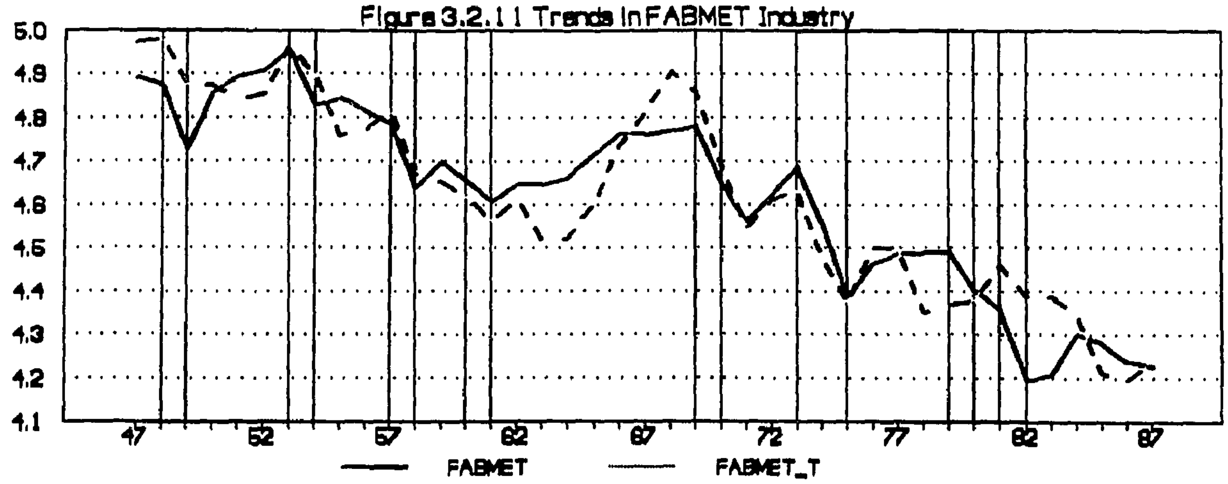


Figure 3.2.12 Trends in ELECTRIC Industry

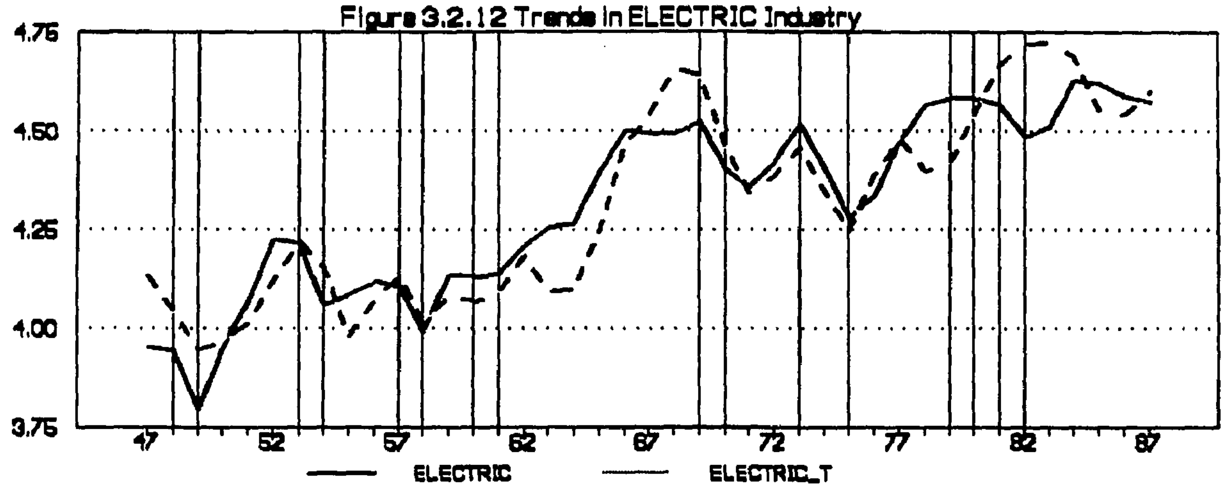


Figure 3.2.13 Trends in MISC Industry

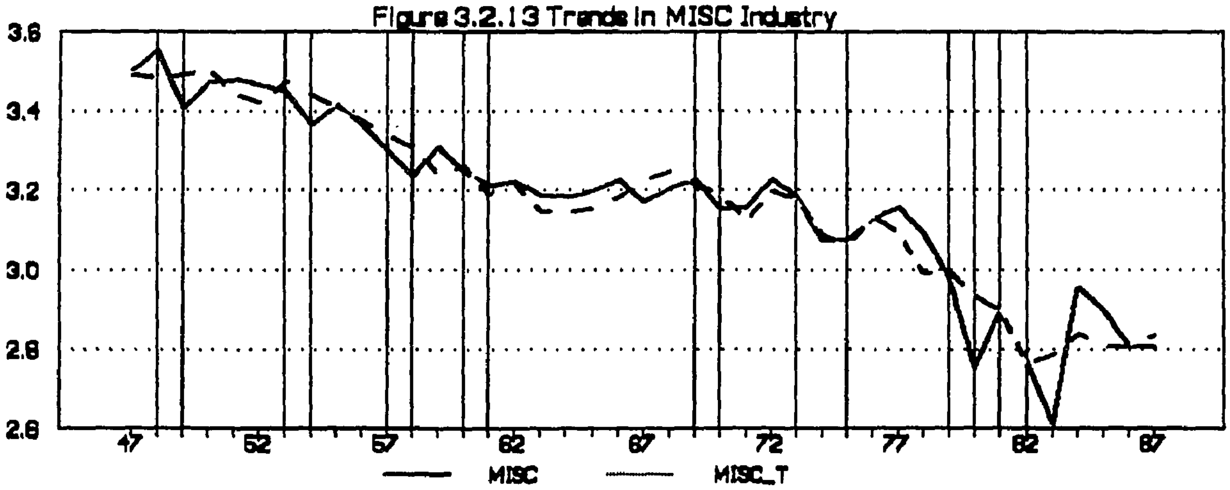


Figure 3.2.14 Trends in STONEGLA Industry

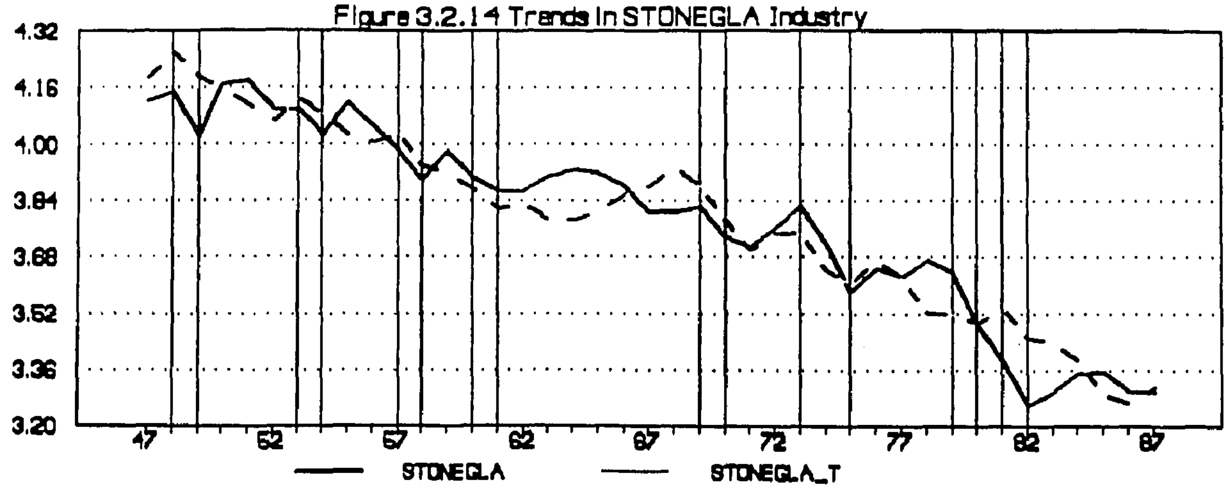


Figure 3.2.15 Trends in PRIMMETA Industry

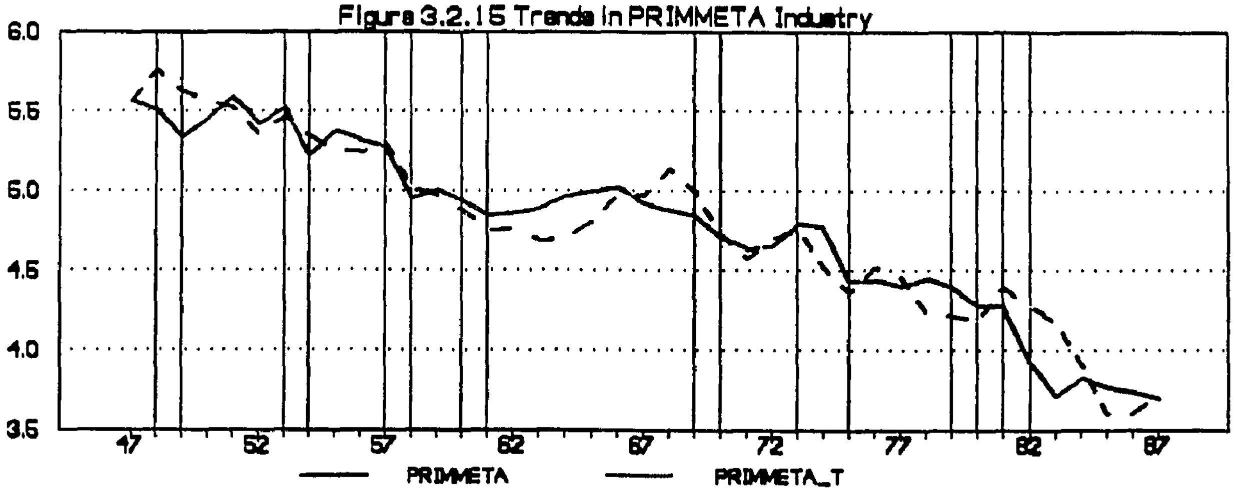
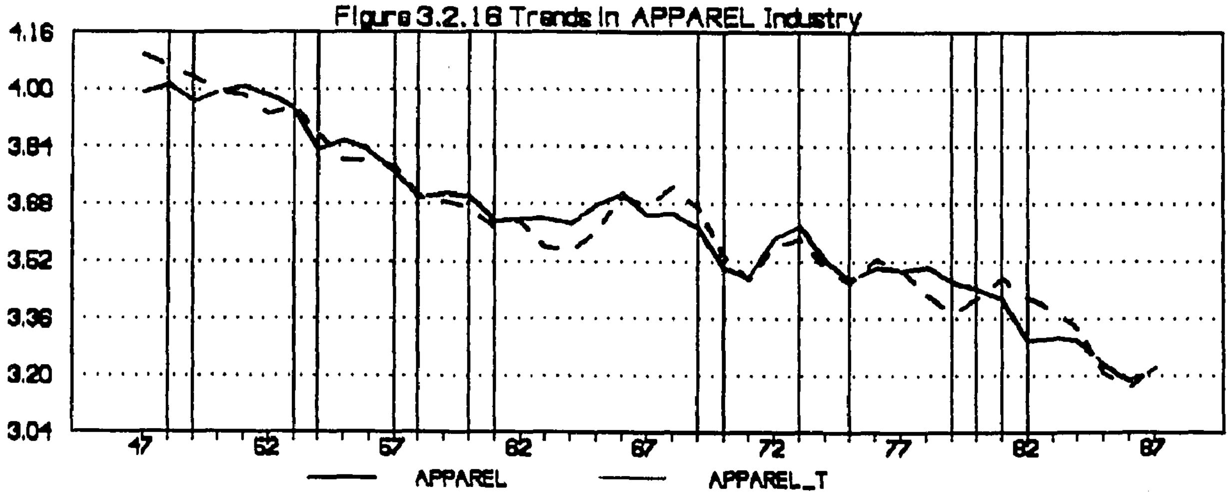


Figure 3.2.16 Trends in APPAREL Industry



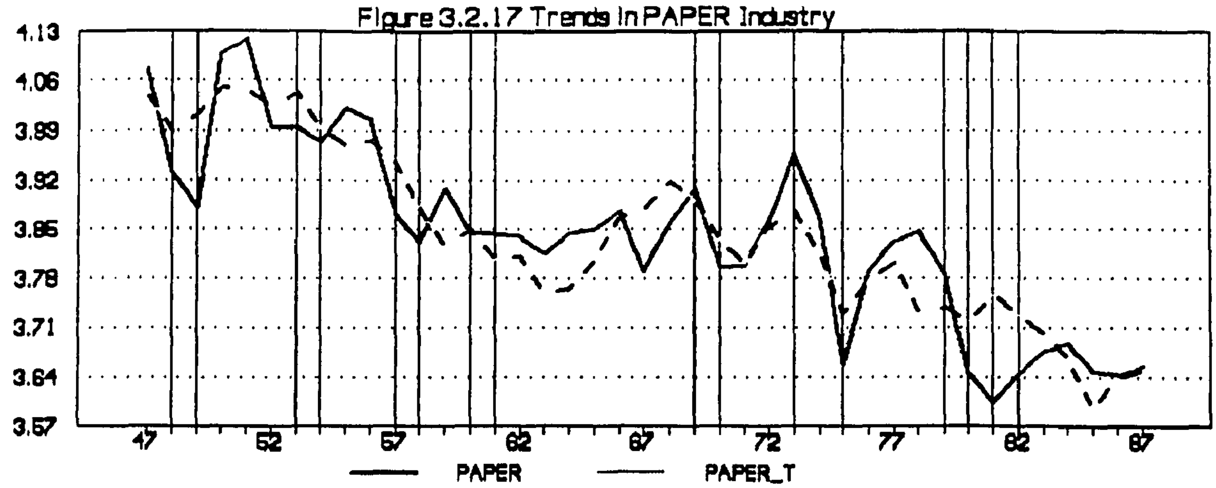


Figure 3.2.17 Trends in PAPER Industry
4.13
4.06
3.99
3.92
3.85
3.78
3.71
3.64
3.57
47
52
57
62
67
72
77
82
87
PAPER
PAPER_T

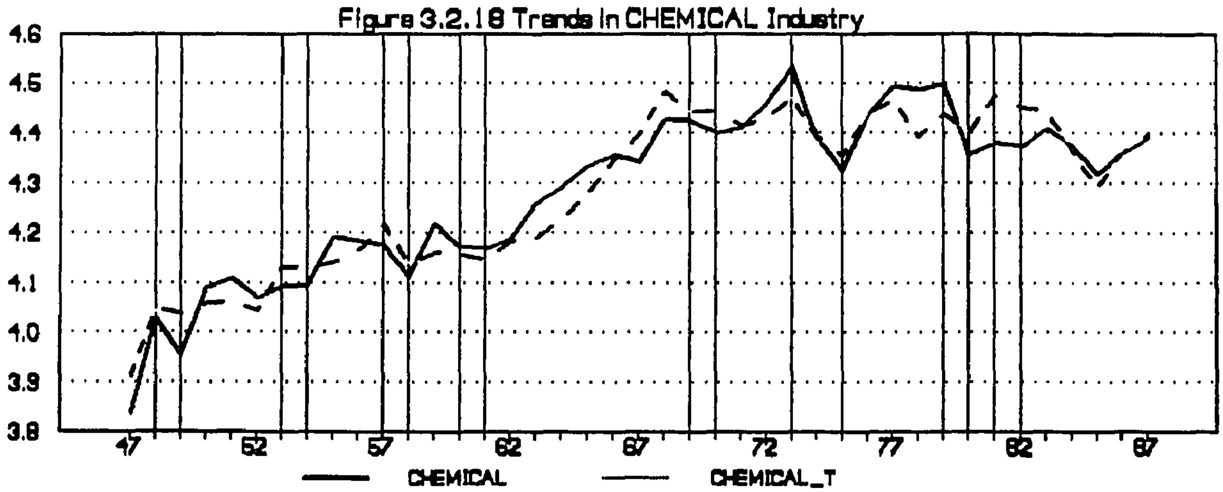


Figure 3.2.18 Trends in CHEMICAL Industry
4.6
4.5
4.4
4.3
4.2
4.1
4.0
3.9
3.8
47
52
57
62
67
72
77
82
87
CHEMICAL
CHEMICAL_T

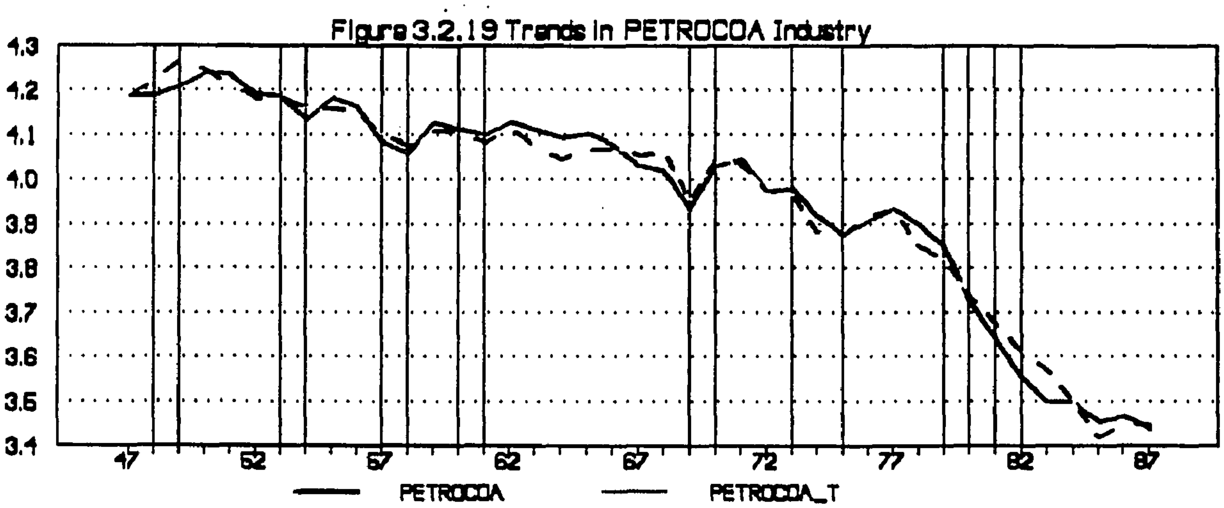
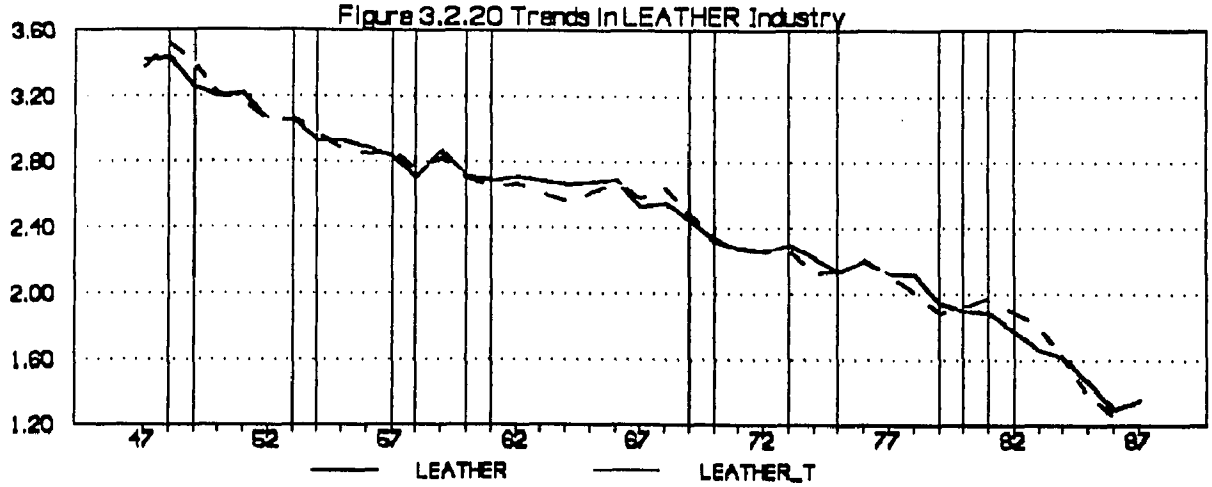


Figure 3.2.19 Trends in PETROCOA Industry
4.3
4.2
4.1
4.0
3.9
3.8
3.7
3.6
3.5
3.4
47
52
57
62
67
72
77
82
87
PETROCOA
PETROCOA_T

Figure 3.2.20 Trends in LEATHER Industry



Conclusions

In this work, we attempted a disaggregation model by looking at the industrial production data and decomposed them into their cyclical and trend components in a special way developed by Vahid and Engle (1993). Our decomposition shows that the industries investigated exhibit relatively high number of common cycles. They are basically procyclical, very similar in their phase and relatively different in amplitutes.

As we mentioned in the beginning, considering the number of industries in comparison to the whole, the conclusions drawn should be evaluated as a preliminiary attempt and a large base study should be realized. Such a study will be more conclusive in evaluating the industries performance.

Such a study is our goal to pursue in the future studies.

BIBLIOGRAPHY

Ahn, S.K. and G.C. Reinsel. (1988) ."Nested Reduced-rank autoregression model for multiple time series." *Journal of the American Statistical Association*, 83, 849-856.

Anderson, T. W. (1984). *An Introduction to Multivariate Statistical Analysis*. Second Edition. John Wiley & Sons.

Barlett, M. S. (1947) ."Multivariate Analysis." *J. Roy Statist. Soc.* B, 9, 176-197.

Beveridge, S. and C.R. Nelson. (1981) ."A new approach to decomposition of economic time series into permanent and transitory components with particular attention to measurement of the 'business cycle'." *Journal of Monetary Economics*, 7, 151-174.

Bils, Mark J. (1985). "Real Wages Over the Business Cycle: Evidence from Panel Data." *Journal of Political Economy*, 93, 666-689.

Bils, Mark J. (1986) "Cyclical Pricing of Durable Goods." *Mimeo.*

Blanchard, O.J. and D. Quah. (1989). "The dynamic effects of aggregate supply and demand disturbances. " *American Economic Review*, 79, 655-673.

Campbell, J.Y. and N. G. Mankiw. (1987). "Permanent and transitory components in macroeconomic fluctuations." *American Economic Review*, 77, 111-117.

Campbell, J. Y. and N. G. Mankiw. (1987). "Are output fluctuations transitory?" *Quarterly Journal of Economics*, 102, 857-880.

Campbell, J. Y. and R. J, Shiller. (1987). "Cointegration and tests of present value models." *Journal of Political Economy*, 95, 1062-1088.

Chrisiano, L. J., and L. Ljungqvist. (1988). "Money does Granger-cause output in the bivariate money- output relation."*Journal of Monetary Economics*, 22, 217-235.

Cochrane, J. H. (1988). "How big is the random walk in GNP?" *Journal of Political Economy*, 96, 893-920.

__________ (1991). "A critique of the application of unit root tests." *Journal of Economic Dynamics and Control*, 15, 275-284.

Davidson, J. H., and D.F. Hendry. (1981). "Interpreting econometric evidence: The behavior of consumer's expenditure in the U.K." *European Economic Review*, 16, 177-192.

Dickey, D. A. & Fuller, W. A. (1979). "Distribution of the Estimates for Autoregressive Time Series with a Unit Root." *Journal of the American Statistical Association*, 74, 427-431.

Dickey, D. A. & Fuller, W. A. (1981). "Likelihood Ratio Statistics for Autoregressive Time Series with a Unit Root." *Econometrica*, 49, 1057-1072.

Dickey, D.A., W. R. Bell, and R.B. Miller. (1986). "Unit roots in time series models: Tests and implications."*American Statistician*, 40, 12-26.

Durlauf, S.N. (1989). "Output persistence, economic structure, and the choice of stabilization policy." *Brookings Papers in Economic Activity*, 2, 69-136.

Durlauf, S. N. (1993). "Time series properties of aggregate output fluctuations." *Journal of Econometrics*, 56, 39-56.

Eichenbaum, M., and K.J. Singleton. (1986). "Do equilibrium real business cycle theories explain postwar U.S. business cycles?" *NBER Macroeconomics Annual* 1986, 91-135.

Engle, R. F. & Granger, C. W. J. (1987). "Co-integration and Error Correction: Representation , Estimation and Testing." *Econometrica*, 55, 251-276.

Engle, R. F. & Granger, C. W. J. (1991). *Long Run Econimic Relationships. Readings in Cointegration, Advanced Texts in Econometrics*. Oxford. Oxford University Press.

Engle, R. F. (1982). "Autoregressive Conditional Heteroscedasticity with Estimates of the Variance of United Kingdom Inflation." *Econometrica*, 50, 987-1007.

Engle, Robert F. and Joao Victor Issler. (1995). "Estimating common sectoral cycles." *Journal of Monetary Economics*, 35, 83-113.

Engle, R.F. and S. Kozicki. (1993) "Testing for common features. " *Journal of Business and Economic Statistics*, 11, 369-395, with discussions.

Engle, R. F., and B. S. Yoo. (1987). "Forcasting and testing in cointegrated systems." *Journal of Econometrics* 35, 143-159.

Evans, G. B. A., and N. E. Savin. (1981). "Testing for unit roots:1." *Econometrica* 49,753-779.

Fuller, W. A. (1976). *Introduction to Statistical Time Series*. New York: John Wiley

Granger, C. W. J. (1981). "Some properties of time series data and their use in econometric model specification." *Journal of Econometrics*, 16, 121-130.

Granger, C. W. J. (1986). "Developments in the Study of Cointegrated Economic Variables." *Oxford Bulletin of Economics and Statistics*, 48, 213-228.

Granger C. W. J., and P. Newbold. (1974). "Spurious regressions in econometrics." *Journal of Econometrics* 2: 111-120.

Granger, C. W. J. and A. A. Weiss. (1983). "Time series analysis of errorcorrecting models." In *Studies in Econometrics, Time Series, and Multivariate Statistics*, pp. 225-278. New York: Academic Press.

Hall, R. E. (1978). "Stochastic implications of the life cycle-permanent income hypothesis : Theory and evidence." *Journal of Political Economy*, 86, 971-987.

Hamilton, J. D. (1989). *Time Series Analysis*, Princeton University Press. Princeton. New Jersey.

Hamilton, J. D. (1989). "A new approach to the economic analysis of nonstationary time series and the business cycle." *Econometrica*, 57, 357-384.

Hansen, Henrik & Katarina Juselius. (1995). *Cats and Rats: Cointegration analysis of time series*. Evanston. Worlwide.

Harvey, A. C. (1989). *Forecasting, Structural time series models and the Kalman Filter*. Cambridge. Cambridge University Press.

Harvey, A. C. (1985). "Trends and cycles in macroeconomic time series." *Journal of Business and Economic Statistics*, 3, 216-227.

Hendry, D. F. (1986). "Econometric Modelling with Cointegrated Variables: An Overview." *Oxford Bulletin of Economics and Statistics*, 48, 201-213. Reprinted in Engle & Granger, 1991.

Hosking, J. R. M. (1980). "The Multivariate Portmanteau Statistic." *Journal of the American Statistical Association*, 75, 602-608.

Johansen, S. (1988). "Statistical Analysis of Cointegration Vectors." *Journal of Economic Dynamics and Control*, 12, 231-254.

Johansen, S. (1991). "Estimation and Hypothesis Testing of Cointegration Vectors in Gaussian Vector Autoregressive Models." *Econometrica*, 59, 1551-1580.

Johansen, S. (1992a). "Cointegration in Partial Systems and the Efficiency of Single-Equation Analysis." *Journal of Econometrics*, 52, 389-402.

Johansen, S. (1992b). "Determination of the Cointegration Rank in the Presence of a Linear Trend." *Oxford Bulletin of Economics and Statistics*, 54, 383-397.

Johansen, S. & Juselius, K. (1990). "The Full Information Maximum Likehood Procedure for Inference on Cointegration- with Applications to the Demand for Money." *Oxford Bulletin of Economics and Statistics*, 52, 169-210.

Johansen, S. & Juselius, K. (1994). "Testing Structural Hypotheses in a Multivariate Cointegration Analysis of the PPP and the UIP for UK." *Journal of Econometrics*, 53, 211-244.

King, R.G., C. I. Plosser, and S. Rebelo. (1988). "Production, growth and business cycles, II. New directions." *Journal of Monetary Economics*, 21, 309-341

King, R.G., C.I. Plosser, J.H. Stock, and M.W. Watson. "1991". "Stochastic trends and economic fluctuations." *American Economic Review*, 81, 819-840.

Kydland, F. and E. Prescott. (1982). "Time to build and aggregate fluctuations." *Econometrica* , 50, 1345-1370.

Kydland, F. and E. Prescott. (1988. "The Workweeks of Capital and its Cyclical Implications." *Journal of Monetary* Economics, 21, 343-360.

Long, J. B. and C.I. Plosser. (1983). "Real business cycles." *Journal of Political Economy*, 91, 39-69.

Long, J. B. and C. I. Plosser. (1987). "Sectoral vs. aggregate shocks in the business cycles." *American Economic Review*, 77, 333-336.

Mankiw.G. (1989). "Real business cycles: A new Keynesian perspective." *Journal of Economic Perspective*, 3, 79-90.

Mankiw. G., and M. D. Shapiro. (1985). "Trends, random walks, and tests of the permanent income hypothesis." *Journal of Monetary Economics*, 10, 139-162.

Murphy, M. K., A. Shleifer and R.W. Vishny. (1989). "Building Blocks of Market Clearing Business Cycle Models." *NBER Annual Conference on Macroeconomics*, 247-301, with discussions. Cambridge, Massachusetts.

Nelson, C. R. and C. I. Plosser. (1982). "Trends and random walks in macroeconomic time series." *Journal of Monetary Economics*, 10, 139-162.

Ogaki, M., and J. Y. Park. (1990). "A cointegration approach to estimating preference parameters." *Mimeo*, University of Rochester.

Osterwald-Lemum, M. (1992). "A note with quantiles of the asymptotic distribution of the maximum likehood cointegration rank test statistics." *Oxford Bulletin of Economics and Statistics*, 54, 461-472.

Ouliaris, S., J., Park, and P. C. B. Phillips. (1989). "Testing for a unit root in the presence of a maintained trend." In *Advances in Econometrics and Modelling*. B. Raj (ed.), pp. 7-28. Dordrecht: Kluwer Academic Publishers.

Pantula, S. G. (1989). "Testing for unit roots in time series data." *Econometric Theory*, 5, 256-271.

Park, J. Y., and P. C.B. Phillips. (1988). "Statistical inference in regressions with integrated processes: Part I." *Econometric Theory*, 4, 468-497.

Peasaran, M.H., R. G. Pierse, and K. C. Lee. (1993). "Persistence, Cointegration and Aggregation: A Disaggregated Analysis of Output Fluctuations in the U.S. Economy." *Journal of Econometrics*, 56, 57-88.

Perron, P. (1988). "Trends and random walks in macroeconomic time series: Further evidence from a new approach." *Journal of Economic Dynamics and Control*, 12, 297-332.

_______ (1989). "The great crash, the oil price shock and the unit root hypthesis." *Econometrica*, 57, 1361-1401.

_______ (1990). "Testing for a unit root in a time series with a changing mean." *Journal of Business and Economic Statistics* 8: 153-162.

Perron, P., and P. C. B. Phillips. (1987). "Does GNP have a unit root? A reevaluation." *Economics Letters*, 23, 139-145.

Phillips, A. W. (1954). "Stabilization policy in a closed economy." *Economic Journal*, 64, 290-323.

__________ (1987). "Time series regression with unit roots." *Econometrica*, 55, 277-302.

Phillips, P. C. B., and S. Ouliaris. (1988). "Testing for cointegration using principal components methods." *Journal of Economic Dynamics and Control*, 12, 205-230.

Phillips, P. C. B., and P. Perron. (1988). " Testing for a unit root in time series regression." *Biometrica*, 75, 335-346.

Rao, C. R. (1973). *Linear Statistical Inference and Its Applications*. (Second Edition). John Wiley & Sons.

Romer, C.D. (1991). "The cyclical behavior of individual production series." *Quarterly Journal of Economics*, 106, 1-31.

Said, S. E., and D. A. Dickey. (1984). "Testing for unit roots in autoregressive- moving average models of unkown order." *Biometrika*, 71, 599-608.

Sims, C. A., J. H., Stock, and M. W. Watson. (1990). "Inference in linear time series models with some unit roots. " *Econometrica*, 58, 113-144.

Stiglitz, Joseph. (1984). "Price Rigidities and Market Structure." *American Economic Review*, 7, 350-355.

Stock, J. H. (1987). "Asymptotic properties of least-squares estimators of cointegrating vectors." *Econometrica*, 55, 1035-1056.

Stock, J.H. and M.W. Watson. (1988). "Testing for common trends" *Journal of the American Statistical Association*, 83, 1097-1107.

_______ (1989). "Interpreting the evidence on money-income causality." *Journal of Econometrics*, 40, 161-182.

Stock, J. H., and K. D. West. (1988). "Integrated regressors and tests of the permanent income hypothesis." *Journal of Monetary Economics*, 21, 85-96.

Vahid, F. and R.F. Engle. (1993). "Common trends and common cycles." *Journal of Applied Econometrics*, 8, 341-360.

Watson, M. W. (1986). "Univariate detrending methods with stochastic trends." *Journal of Monetary Economics*, 18, 1-27

West, K. D. (1988). "Asymptotic normality when regressors have a unit root." *Econometrica*, 56, 1397-1418.