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AN EXTENSION OF THE CHOU-YANG MODEL TO INCLUDE SPIN

by

EDWARD KONIG

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1/25/71 ______ N. Chang ______

date — Chairman of Examining Committee

1/22/71 ______ Marvin H. Mittleman ______

date — Executive Officer

Professor Alberto Sirlin

Professor R. Lea

Professor Huan Lee

Supervisory Committee

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Section I Introduction

There has been considerable interest in the theory of high energy scattering, especially in view of the recent construction of the National Accelerator and of storage rings for colliding beam experiments. The question that has been posed is the limiting behavior of scattering cross sections as the energy of the incident particle becomes infinite. There are two schools of thought on this matter.

One is the Regge school of thought, in which the behavior of infinite energy scattering cross-sections is governed by a Regge pole and/or a Regge cut in the crossed channel. In this way of thinking, there is generally speaking a shrinking diffraction peak in the forward direction, unless the Regge pole has an intercept exactly equal to one and constant, in which case the elastic diffraction peak will not shrink. Predictions for infinite energy then depend on the parameters of the Regge pole and cuts in the t-channel.

The other school of thought is the diffractive approach to scattering at infinite energy. The physics underlying this approach reflects the belief that at infinite energy, all the scattering processes that survive are the result of fragmentation. In this picture, the target as well as the projectile are looked upon as fragmenting upon collision, with a definite limit to the fragmentation probabilities as energy goes to infinity. Elastic scattering then takes place as a shadow of the limiting fragmentation.

Yang and co-workers[1,2,3] have successfully applied this picture to proton-proton (p-p) scattering and have predicted spectacular features of p-p scattering based upon this philosophy. Their applications however assume a zero spin for the nucleon, which they believe a good approximation to the data at high energies. We have attempted in this work to improve upon the work of Chou and Yang(CY), by removing this approximation.

The basis of our extension of the CY model is the optical model, which is an extremely simple picture of infinite energy scattering.

In this description, the incident beam, e^{ikz}, is attenuated by the hadronic matter that it goes through, so that the emerging beam will be given by

$$S(\vec{b})\, e^{ikz} \tag{1.1}$$

where $\vec{b}$ is the impact parameter. In particular, the scattered wave would be

$$(S(\vec{b})-1)\, e^{ikz} \tag{1.2}$$

By an application of Huyghens' principle, the resulting scattering amplitude is given by

$$\frac{ik}{2\pi}\int d^2b\; e^{-i\vec{k}_\perp\cdot\vec{b}}(S(\vec{b})-1) \tag{1.3}$$

In CY model, their explicit assumption is that the "survival" amplitude $S(b)e^{ikz}$ is to be represented by

$S(\vec{b})=e^{-\chi(b)}$, with χ (b) defined as being proportional to the hadronic matter of the interacting particles. They assume, on the grounds of symmetry between projectile and target, that

(1.4) $$\chi(b) = \eta \int d^2b' D(\vec{b}') D(\vec{b}'-\vec{b})$$

where η is a real parameter, and

(1.5) $$D(b) = \int_{-\infty}^{+\infty} dz\, \rho(\vec{b},z)$$, $\rho(\vec{b},z)$ being the hadron density.

In terms of Feynman language, they assume an effective contact interaction between the two hadrons at infinite energy, which when iterated gives their transition matrix element. The density ρ is chosen to be the electromagnetic charge density. This gives the expression:

(1.6) $$S_{pp}(\vec{b}) = e^{-\eta\int d^2b' D(\vec{b}')D(\vec{b}'-\vec{b})}$$

Ignoring spin effects, this form of the effective S-matrix element leads to the well-known relation between p-p elastic scattering cross-sections and the electromagnetic form factors of the proton.

It is at this point that we make the improvement of including spin effects realistically and we are able to show that the effects of spin do not spoil the remarkable fit of the CY model to p-p elastic scattering.

Section II Diffractive Picture of P-P Scattering

Consider the optical model of very high energy scattering of a proton by a proton. Let us view this scattering in the lab frame, where the target is at rest. Let the incident beam be described by the wave function

$$\chi^{(1)} e^{ikz}$$

where $\chi^{(1)}$ is the Pauli spinor describing the spin state of the incident proton. Now in going through the target, the projectile as well as the target are likely to break up, so that the amplitude for finding the projectile and the target in the same state as before is diminished by the fragmentation process. From the point of view ofthe p-p channel, there is absorption into other channels. Let us represent the state that emerges from scattering as:

(2.1) $$S_{\sigma\tau,\sigma'\tau'}(\vec{b})\, \chi^{(1)}_{\sigma'}\, \chi^{(2)}_{\tau'}\, e^{ikz}$$

where the matrix $S(\vec{b})$ is the "survival" probability amplitude, and we should in general allow for a spin transfer between the projectile and the target in the process of scattering. Notice that we are already assuming a limit for the transition amplitude at infinite energy, and in general the amplitude will be a function of b, the impact parameter. Parity invariance implies a restriction on the form of the amplitude as follows:

(2.2) $$(i\sigma_2)_{\sigma\sigma'}\,(i\sigma_2)_{\tau\tau'}\, S_{\sigma'\tau';\xi'\eta'}(\vec{b})\,(-i\sigma_2)_{\xi'\xi}\,(-i\sigma_2)_{\eta\eta'} = S_{\sigma\tau;\xi\eta}(\vec{b}^{R})$$

where if $\vec{b}=(b_1,b_2)$, $\vec{b}^R=(b_1,-b_2)$. This follows from combining the usual parity with a rotation of 180° around the y axis. Time reversal invariance implies the relation

$$\left(S_{\sigma\tau,\xi\eta}(-\vec{b}^R)\right)^* = S_{\sigma\tau,\xi\eta}(\vec{b}) \tag{2.3}$$

where again we have combined the usual time reversal operation with a rotation of 180° around the y axis. The reason for having to do an extra rotation is due to the following: under parity, $\vec{k}\rightarrow-\vec{k}$, so that whereas particle 1 was coming in from the left, it would now come in from the right. Our transition amplitudes however have been defined with particle 1 coming in from the left, so we use an extra rotation of 180° around the y axis to bring the scattering configuration back to essentially the original configuration, with the exception of having been reflected in the xz plane. Scattering amplitudes are invariant under rotation, so the restrictions (2.2) and (2.3) are equivalent to the usual requirements of parity and time reversal invariance.

The most general form of $S(\vec{b})$ that satisfies P and T invariance can be represented in terms of 5 amplitudes[4]. We choose to represent it as

$$S(\vec{b})=e^{-\hat{\chi}(\vec{b})} \tag{2.4}$$

where

$$\hat{\chi}(\vec{b}) = d(b^2)\,\hat{1} + e(b^2)\,\vec{\sigma}_1\cdot\hat{p}\,\vec{\sigma}_2\cdot\hat{p} + i f(b^2)(\vec{\sigma}_1+\vec{\sigma}_2)\cdot\hat{p} + g(b^2)\times$$
$$\times\,\vec{\sigma}_1\cdot\hat{q}\,\vec{\sigma}_2\cdot\hat{q} + h(b^2)\,\vec{\sigma}_1\cdot\hat{r}\,\vec{\sigma}_2\cdot\hat{r} \tag{2.5}$$

is the representation of the scattering amplitude in the canonical Pauli spinor representation. Here the basis vectors consist of the direct product of the two single particle state vector spaces. $\sigma^{(1)}$ ($\sigma^{(2)}$) operate on the state vectors of particle 1 (2) only, while 1 is the unit operator. $\hat{p}$, $\hat{q}$, $\hat{r}$ are an orthonormal set of basis vectors in b space, while $d(b^2),\ldots,h(b^2)$ are scalar functions of b^2. $\hat{\chi}(\vec{b})$ is denoted the opacity, and is a positive definite Hermitian matrix in spin space to account for absorption.

The observation by CY however is that the opacity function $\hat{\chi}(\vec{b})$ should be proportional to the hadron "stuff" in the incident projectile as well as to the stuff in the target. Their suggestion is that

$$\chi(\vec{b}) = \eta \int d^2b' \, D_I(\vec{b}') \, D_{II}(\vec{b}'-\vec{b}) \tag{2.6}$$

on the grounds that this form of $\chi(\vec{b})$ satisfies the criteria:

(i) it is proportional to the "stuff" the projectile sees in going through the target.

(ii) it is symmetrical under interchange of projectile and target co-ordinates, so that $\chi(b)$ is the same if the projectile is at rest and the target is in motion.

In their original application of these ideas, CY assumed zero spin for the nucleons, so that the density distribution D , is a scalar. Our idea was to ask what extension should be made in the realistic case where the particles have spin.

Clearly, the most reasonable assumption to make is the natural one, viz., that (2.6) should be replaced by

$$\hat{\chi}(b) = \eta \int d^2b' \, \hat{D}(\vec{b}') \, \hat{D}(\vec{b}'-\vec{b}) \tag{2.7}$$

where $D(\vec{b})$ is the two dimensional particle density, which at infinite energies has only two components, namely the z and t components.

B.W.Lee (5) and others have also conjectured generaligations of the CY model, except that they have put in a propagator between the two currents, rather than the contact interaction assumed by CY and by us here.

Cheng and Wu (6) and others have also studied the infinite energy limit of a field theory of neutral vector mesons and find an effective S-matrix operator at infinite energy. It is

$$S(y) = e^{-\frac{ig^2}{2}\int d^2x \, \sigma^R(x) \Delta(x-y) \sigma^L(y)} \tag{2.8}$$

$$\sigma^R(x_\perp) = \frac{1}{2}\int_{-\infty}^{+\infty} dz \left(j_3(x_\perp, z, z) + j_0(x_\perp, z, z)\right)$$

$$\sigma^L(x_\perp) = \frac{1}{2}\int_{-\infty}^{+\infty} dz \left(j_3(x_\perp, z, -z) - j_0(x_\perp, z, -z)\right)$$

where j_μ^R is the current for the right-moving hadron in the center of momentum frame, and j_μ^L is the current for the left moving hadron. Their result is of course model dependent, although as B.W.Lee (5) has shown, their result is equivalent to the CY model for elastic scattering, except for the absence of a propagator in the CY model.

At this point, it will be instructive to rewrite some of the preceeding results in the optical potential formalism. We introduce our notation in Fig.1.

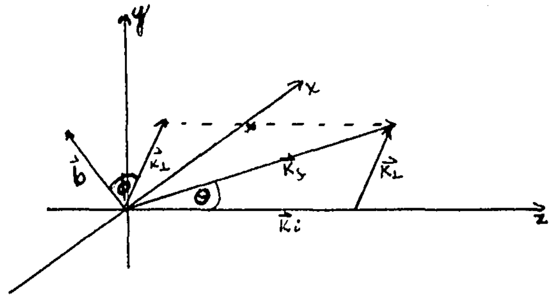


Fig.1

where: $\vec{K}_i \equiv$ initial momentum of particle 1 in the center of momentum (COM) system.

$\vec{K}_f \equiv$ final momentum of particle 1 in COM.

$k \equiv |\vec{K}_\perp| \equiv$ transverse momentum transfer $\equiv |\vec{K}_f - \vec{K}_i|$.

$b \equiv |\vec{b}| \equiv \frac{\ell + 1/2}{|\vec{K}_i|} \equiv$ magnitude of impact parameter in COM

As above, particle 1 is the incident particle in the lab frame, The x-y plane is denoted the transverse plane.

For the spinless case, the expression for the scattering amplitude in the eikonal approximation (high energy forward scattering) is given by:(7)

$$(2.9) \quad a(k) = \frac{1}{2\pi} \int \left(1 - e^{-i\int_{-\infty}^{+\infty} V(\vec{b} + \hat{K}_i z)\, dz}\right) e^{i\vec{k}\cdot\vec{b}}$$

where $\hat{K}_i \equiv \vec{K}_i / |\vec{K}_i|$, and V is the pptical potential, which is related to the Schrodinger potential through the scaling $V_{SCHR} = kV$, k =COM momentum. For the case $V = 0$, $e^{-i \cdot 0} = 1$, and $a(k) = 0$.

The elastic scattering in the optical picture again appears as the "shadow" of the inelastic processes. In this picture, to account for fragmentation into other channels, the effective potential is complex, and at infinite energies, totally negative imaginary. We may define the scattering operator S(b) in configuration space as:

$$S(\vec{b}) \equiv e^{-i\int_{-\infty}^{+\infty} V(\vec{b}+\hat{k}_i z)\,dz} \tag{2.10}$$

and define:

$$\chi(\vec{b}) \equiv i\int_{-\infty}^{+\infty} V(\vec{b}+\hat{k}_i z)\,dz \tag{2.11}$$

For V negative imaginary, $\chi(\vec{b})$ is positive and real, so

$$a(k) = \frac{1}{2\pi}\int \left(1 - e^{-\chi(b)}\right) e^{i\vec{k}\cdot\vec{b}}\, d^2b \tag{2.12}$$

Now V negative and imaginary corresponds to the presence of absorptive processes only, so the amplitude of (2.9) displays the elastic scattering as the difference between no scattering and the "survival amplitude, which is the diffraction picture described above.

The absorption constant η of (2.6) is fixed by requiring that the calculated cross section in the forward direction agree with the value predicted from the optical theorem from a total cross section of 40 mb, the value for p-p scattering at current energies. η is the only free parameter in the theory, and since this is fixed by experiment, it may be called a no parameter theory.

A dramatic feature of the CY theory is the alternation of the signs of the contributions of the terms in the expansion of the exponential to the scattering amplitude. Physically, the alternating sign comes from the geometrical shadow corrections since the contribution from the pieces of hadron stuff in the shadow will have been overestimated in adding up theamplitudes due to each part of the hadron.

The alternating signs result in the prediction of two breaks in the differential cross section at present energies, which should become more pronounced as the energy gets higher.

Our intent here is not to find the most general form of the scattering amplitude for p-p scattering, but rather to find the simplest generalisation of the simple and elegant idea of CY to the case of spin. Off hand, it is not clear if this will improve or even worse, destroy the predictions of the CY model. What is gratifying therefore, is that the generalisation that we have proposed has turned out not to destroy the fit, but to improve it, as we shall show.

Section III Extension of the Chou Yang Model

In accordance with our discussion in Section II, the extension of the Chou-Yang model to include spin amounts to the replacement of a scalar $\chi(\vec{b})$ by a matrix for $\chi(b)$.

We introduce the notation:

$$(3.1) \quad \langle f \rangle \equiv \frac{1}{2\pi}\int f(b)\, e^{i\vec{k}\cdot\vec{b}}\, d^2b = \frac{1}{2\pi}\int_0^\infty\int_0^{2\pi} f(b)\, e^{i\vec{k}\cdot\vec{b}}\, d\phi\, b\, db$$

where ϕ is the angle between k and b , as shown in Fig.1. We also define:the 2 dimensional convolution:

$$(3.2) \quad \int D_1(\vec{b}-\vec{b}')\, D_2(\vec{b}')\, d^2b' \equiv (D_1 \otimes D_2)$$

of two functions $D_1(\vec{b})$ and $D_2(\vec{b})$.

Then we have as a theorem:

$$(3.3) \quad \langle D_1 \otimes D_2 \rangle = \langle D_1 \rangle \langle D_2 \rangle$$

We define the scattering amplitude $a(k)$ so that:

$$(3.4) \quad \frac{d\sigma}{dt} = \pi\, |\hat{a}(k)|^2$$

Then the scattering amplitude of (2.12) can be written:

$$(3.5) \quad \hat{a}(k) = \langle 1 - e^{-\hat{\chi}(\vec{b})} \rangle = \langle \hat{\chi}(\vec{b}) - \frac{1}{2!}\hat{\chi}(\vec{b})\hat{\chi}(\vec{b}) + \frac{1}{3!}\hat{\chi}(\vec{b})\hat{\chi}(\vec{b})\hat{\chi}(\vec{b}) - \cdots \rangle$$

From a practical point of view, the $\hat{\chi}(\vec{b})$ we shall use may be regarded as the two-dimensional Fourier transform of the Born term in the perturbation expansion as energy goes to infinity, except that the overall χ is real, rather than as in an ordinary field theory imaginary. From the point of view of field theory, of course, this $\hat{\chi}(\vec{b})$ is not the Born term, but is the result of all the inelastic channels; however its effect is, crudely speaking, to change the Born term into an imaginary eikonal (i.e. a real $\hat{\chi}(\vec{b})$).

It will now be necessary to introduce the most general amplitude for nucleon-nucleon scattering compatible with P and T reversal invariance. We will take as our basis of representation, the direct product space of Pauli spinors for the two particles. In this representation, the scattering amplitude can be written as:[4]:

(3.6) $$\hat{a}(k)=\alpha(k)\,\hat{1}+\beta(k)\,\vec{\sigma}^1\cdot\hat{n}\,\vec{\sigma}^2\cdot\hat{n}+i\gamma(k)\,(\vec{\sigma}^1+\vec{\sigma}^2)\cdot\hat{n}+\delta(k)\,\vec{\sigma}^1\cdot\hat{m}\,\vec{\sigma}^2\cdot\hat{m}+\epsilon\,\vec{\sigma}^1\cdot\hat{\ell}\,\vec{\sigma}^2\cdot\hat{\ell}$$

where:

(3.7) $$\hat{\ell}=\frac{\hat{k}_i+\hat{k}_f}{2\cos\theta/2}\,;\quad \hat{m}=\frac{\hat{k}_i-\hat{k}_f}{2\sin\theta/2}\,;\quad \hat{n}=\frac{\hat{k}_i\times\hat{k}_f}{\sin\theta}$$

and θ is the angle between $\vec{k}_i$ and $\vec{k}_f$ (Fig.1). $\vec{\sigma}^1$ ($\vec{\sigma}^2$) operates only on the spinor of particle 1(2), respectively. $\alpha(k),\beta(k),\ldots,\epsilon(k)$ are scalar functions of k.

As energy goes to infinity, the vectors in (3.7) assume limiting forms. We illustrate the situation in Fig. 2.

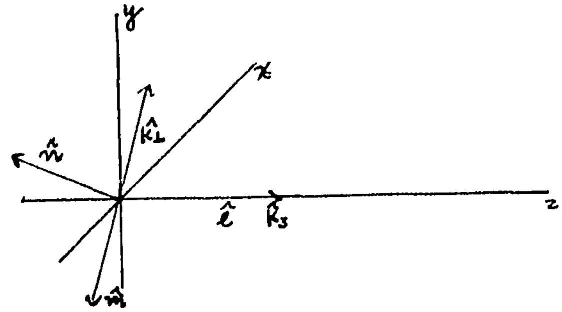


Fig. 2

The $\hat{\ell}$, $\hat{m}$, $\hat{n}$ shown in the diagram are the infinite energy limits of those defined in (3.7). Explicitly, as the energy becomes infinite, we have:

$$
\begin{aligned}
&\hat{\ell} = \hat{k}_3 \\
&\hat{m} = -\hat{k}_\perp \\
(3.8\ ) \quad &\hat{n} = \hat{k}_3 \times \hat{k}_\perp
\end{aligned}
$$

with $\quad \hat{m} \times \vec{\ell} = \hat{n}$

We henceforth use $\hat{\ell}$, $\hat{m}$, $\hat{n}$ to denote the infinite energy limit of $\hat{\ell}$, $\hat{m}$, $\hat{n}$; i.e. those vectors shown in Fig. 2, and equations (3.8), and not those in (3.7).

Now $\alpha(k), \beta(k), \ldots, \epsilon(k)$ may be expressed in terms of the spin dependent T matrix elements;

$$
\begin{aligned}
&\alpha = \tfrac{1}{4}\left(2\, T_{++,++} + T_{+-,+-} + T_{-+,-+}\right) \\
&\beta = \tfrac{1}{4}\left(-2\, T_{++,--} + T_{+-,-+} + T_{-+,+-}\right) \\
(3.9\ ) \quad &\gamma = \tfrac{1}{4}\left(T_{++,+-} + T_{++,-+} + T_{+-,++} + T_{-+,++}\right) \\
&\left.\begin{matrix}\delta \\ \epsilon\end{matrix}\right\} = \tfrac{1}{4}\left(T_{++,++} + T_{++,--} - \tfrac{1}{2}\left(T_{+-,+-} + T_{-+,-+} - T_{+-,-+} - T_{-+,+-}\right)\right)
\end{aligned}
$$

Here the notation is: $T_{f,i}$,where f=final spin configuration. + stands for spin along the direction of quantization and correspondingly for -.

For completeness, the expansion of $\alpha(k), \beta(k), \ldots, e(k)$ in terms of Singlet-Triplet amplitudes is given in Appendix F .

It remains to give a method for the evaluation of $\hat{\chi}(\vec{b})$, so as to be able to calculate the scattering amplitude $\hat{a}(k)$. We will take:

(3.10) $$\hat{\chi}(\vec{b}) = \hat{\chi}(\vec{b})\big|_{B.A.} \equiv \langle \hat{a}(k)|_{BA} \rangle .$$

(3.11) $$\hat{a}(k)|_{BA} = \alpha|_{BA} \cdot \hat{1} + \beta|_{BA}\, \vec{\sigma}_1\cdot\hat{n}\,\vec{\sigma}_2\cdot\hat{n} + i\gamma|_{BA}\,(\vec{\sigma}_1+\vec{\sigma}_2)\cdot\hat{n} + \delta|_{BA}\,\vec{\sigma}_1\cdot\hat{m}\,\vec{\sigma}_2\cdot\hat{m} + \epsilon_{BA}\,\vec{\sigma}_1\cdot\hat{l}\,\vec{\sigma}_2\cdot\hat{l}$$

is given by (3.9) with the infinite energy Born approximation used for the T matrix elements on the right hand side. We denote these as $T_{S,J}|_{BA}$.

To begin a systematic discussion of $\hat{a}(\vec{k})|_{BA}$, we introduce the following notation:

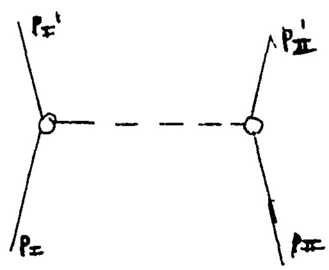


Fig. 3

where $p_I(p_I')$ stand for the initial and final four momenta of the incident initial (scattered) particle I, and similarly, $p_{II}(p_{II}')$ are the corresponding quantities for the target particle II. The circles denote the electromagnetic vertices for the particles involved, while the dotted line joining them denotes the form of the interaction to be given below. We define initial and final states for the two particle system as:

$$(3.12)\quad |i\rangle = |u^{s}(p_{I})\rangle |u^{r}(p_{II})\rangle$$

$$|f\rangle = |u^{s'}(p_{I}')\rangle |u^{r'}(p_{II}')\rangle$$

We take the quantity $T_{f,i}|_{BA}$ to be defined by:

$$(3.13)\quad T_{f,i}|_{B.A.} \equiv \lim_{momentum \to \infty} \eta \langle f | J_{I}^{\mu}(0)_{E.M.}\, J_{\mu\, E.M.}(0)_{II} | i \rangle$$

$$\equiv \lim_{momentum \to \infty} \eta \langle p_{II}' r' | J_{\mu}(0)_{E.M.} | p_{II}, r\rangle \langle p_{I}' s' | J^{\mu}_{E.M.}(0) | p_{I}, s\rangle$$

$$= \lim_{momentum \to \infty} \eta \cdot \bar{u}^{r'}(p_{II}')(F_1 \gamma_\mu + i\sigma_{\mu\nu}(p_{II}'-p_{II})^{\nu} F_2) u^{r}(p_{II}) \cdot \bar{u}^{s'}(p_{I}')(F_1 \gamma^{\mu} + i\sigma^{\mu\nu}(p_{I}'-p_{I})_{\nu} F_2) u^{s}(p_{I})$$

where u is the usual Dirac spinor, $\bar{u} \equiv u^{*}\gamma^{0}$, $\sigma_{\mu\nu} = \frac{i}{2}[\gamma_\mu, \gamma_\nu]$, and F_1 and F_2 are form factors, later to be identified with the electromagnetic form factors in the 1 photon exchange approximation.

We take the infinite momentum limit of (3.13) in Appendix B, and find:

$$(3.14)\quad T_{NO\ SPIN\ FLIP} = F_1^2$$

$$T_{++,-+ \atop +-,++} = (\pm) K F_1 F_2$$

$$T_{++,-- \atop --,++} = K^2 F_2^2$$

$$T_{+-,-+ \atop -+,+-} = -K^2 F_2^2$$

Here the subscript "no spin flip" indicates that all non spin flip amplitudes are equal in the infinite momentum limit.

As a special case, we may now investigate the limit in which spin is neglected. We may take this limit by setting F_2=0. Then we find:

$$(3.15)\quad \hat{\chi}(b) = \eta \langle F_1 \cdot F_1 \rangle \hat{1}$$

The function D is, in our treatment, the "hadron density" effective in absorbtion. We may assume that this quantity is proportional to the electric charge density of the hadron; in other words, that mass density is proportional to charge density, as in the QED of a point electron. Then, speaking loosely, the mass-energy associated with the mass distribution of the hadron is due mainly to the prosence of strongly interacting hadron "stuff". Hence we are led to associate the "hadronic density" with the charge density.

Now in the non-relativistic limit, the Fourier transform of the form factor F_1 may be associated with the electric charge distribution density in configuration space [8]. Hence, we may define the charge distribution density of (1.5) by:

(3.16) $$\rho(x,y,z) = \frac{1}{(2\pi)^{3/2}} \iiint F_1(q^2)\, e^{i\vec{q}\cdot(\vec{b}+\hat{k}_i z)}\, dq_x\, dq_y\, dq_z$$

The two-dimensional density distribution may be defined as before (1.5), to give:

(3.17) $$D(x,y) = \int_{-\infty}^{+\infty} \rho(x,y,z)\, dz = \frac{1}{(2\pi)^{1/2}} \iint F_1(k^2)\, e^{i\vec{q}_\perp\cdot\vec{b}}\, dq_x\, dq_y$$

The above discussion gives us:

(3.18) $$\hat{\chi}(\vec{b}) = \eta \langle F_1 \cdot F_1 \rangle = \langle \langle D_1 \rangle \langle D_1 \rangle \rangle = \langle\langle D_1 \otimes D_1 \rangle\rangle = D_1 \otimes D_1$$

which is what CY [2] assumed. We see that our Eq.(3.15) thus reproduces the CY result in the limit that spin is neglected, in operator form.

In terms of the infinite momentum T matrix elements of (3.14), the infinite momentum BA scattering amplitude becomes:

(3.19).

$$a(k) = F_1^2 \cdot \hat{1} - k^2 F_2^2 \vec{\sigma}^1 \cdot \hat{n} \, \vec{\sigma}^2 \cdot \hat{n} - ik F_1 F_2 (\vec{\sigma}^1 + \vec{\sigma}^2) \cdot \hat{n}$$

We next discuss how to evaluate the products of $\chi(\vec{b})$ which appear in the expansion of the exponential, and how to take the Fourier transform in (3.5) to find the scattering amplitude.

Section IV Fourier Transforms and Spin Algebra

We begin by re-writing (3.19) as:

(4.1) $a(k) = F_1^2 \cdot 1 - k^2 F_2^2 \vec{\sigma}^1 \cdot \hat{n}\, \vec{\sigma}^2 \cdot \hat{n} - ik F_1 F_2 (\vec{\sigma}^1 + \vec{\sigma}^2) \cdot \hat{n}$

We must now evaluate the two dimensional Fourier transform. Because (4.1) contains the vector n, this expression is dependent on angle. With the exception of factors of k_i or $k_i k_j$, the integrand of the Fourier transform is invariant under rotations, so we may use the requirement of covariance to evaluate the angular integration. If the left hand side transforms as a vector or tensor, then so must the right hand side, and the result must thus be a linear combination of the vectors or tensors available when the d^2k integration has been performed.

We therefore consider the integration for the following forms: let f,g,h be scalar functions of k^2 where k will stand for k from now on. Then we may write:

$$\frac{1}{2\pi}\int f\, e^{-ik\cdot b}\, d^2k = \lambda(b^2)$$

(4.2)
$$\frac{1}{2\pi}\int g\, \hat{k}_i\, e^{-i\vec{k}\cdot\vec{b}}\, d^2k = \mu(b^2)\, \hat{b}_i$$

$$\frac{1}{2\pi}\int h\, \hat{k}_i \hat{k}_j\, e^{-i\vec{k}\cdot\vec{b}}\, d^2k = \nu(b^2)\, \delta_{ij} + \pi(b^2)\, \hat{b}_i \hat{b}_j$$

where λ, μ, ν, π are scalar functions of b^2.

We may evaluate these otherwise unknown functions by taking the inner product with b and by taking the trace as well in the case of the tensor term. This gives:

$$\lambda(b) = \frac{1}{2\pi}\int f\, e^{-i\vec{k}\cdot\vec{b}}\, d^2k$$

(4.3)
$$\mu(b) = \frac{1}{2\pi}\int g\cdot(\hat{k}\cdot\hat{b})\, e^{-i\vec{k}\cdot\vec{b}}\, d^2k$$

$$2\nu + \pi = \frac{1}{2\pi}\int h\, e^{-i\vec{k}\cdot\vec{b}}\, d^2k$$

$$\nu + \pi = \frac{1}{2\pi}\int h\, (\hat{k}\cdot\hat{b})^2 e^{-i\vec{k}\cdot\vec{b}}\, d^2k$$

We may represent the last three integrals as:

(4.4)
$$\mu = -i\frac{\partial}{\partial b}\int_0^\infty g\, J_0(kb)\, k\,dk$$

$$2\nu + \pi = \int_0^\infty h\, J_0(kb)\, k\,dk$$

$$\nu + \pi = -\frac{\partial^2}{\partial b^2}\int \frac{h}{k^2}\, J_0(kb)\, k\,dk$$

where we have used the representation of the zeroth order Bessel function.

(4.5)
$$J_0(kb) = \frac{1}{2\pi}\int_0^{2\pi} e^{-ikb\cos\phi}\, d\phi$$

We may simplify the second and last equation in (4.4) by using:

(4.6)
$$\frac{\partial J_0(kb)}{\partial b} = -k J_1(kb)$$

$$\frac{\partial^2 J_0(kb)}{\partial b^2} = -\frac{k^2}{2}\left(J_0(kb) - J_2(kb)\right)$$

to give:

(4.7)
$$\mu = -i\int g\, J_0(kb)\, k\,dk$$

$$\nu + \pi = \frac{1}{2}\int h\, \left(J_0(kb) - J_2(kb)\right) k\,dk$$

We may now solve for ν and π. Introducing the notation for an arbitrary integrable function l:

(4.8)
$$\int_0^\infty \ell(k^2)\, J_{\begin{pmatrix}0\\1\\2\end{pmatrix}}(kb)\, k\,dk \equiv \langle \ell \rangle_{\begin{pmatrix}0\\1\\2\end{pmatrix}}$$

Then our equations become:

(4.9)
$$\lambda = \langle f \rangle_0$$
$$\mu = -i\langle g \rangle_1$$
$$2\nu + \pi = \langle h \rangle_0$$
$$\nu + \pi = \tfrac{1}{2}\left(\langle h \rangle_0 - \langle h \rangle_2\right)$$

We may now solve for ν and π to get

(4.10)
$$\nu = \tfrac{1}{2}\left(\langle h \rangle_0 + \langle h \rangle_2\right)$$
$$\pi = -\langle h \rangle_2$$

Comparing (4.3) with (4.1), we see that:

(4.11)
$$f = F_1^2$$
$$g = -i F_1 F_2$$
$$h = -F_2^2$$

Thus, in order to find λ, μ, ν, π, we must evaluate the Bessel transformations of F_1^2, $-kF_1F_2$ and $k^2F_2^2$.

As originally done by CY[3], we parametrise the functions F_1^2, F_1F_2, F_2^2 as a linear combination of Gaussians.

(4.12)
$$F_1^2 = .8113\, e^{-5.26|t|} + .18\, e^{-1.36|t|} + .0085\, e^{-.51|t|} + .00015\, e^{-.163|t|}$$

$$F_1F_2 = .8300\, e^{-5.36|t|} + .118\, e^{-1.62|t|} + .0057\, e^{-.669|t|} + .000058\, e^{-.264|t|}$$

$$F_2^2 = .7\, e^{-6.54|t|} + .213\, e^{-2.65|t|} + .008\, e^{-.948|t|} + .000065\, e^{-.429|t|}$$

We have taken F_1^2 above as given by CY[3], and have plotted F_1F_2 and F_2^2 and constructed Gaussian fits to the data ourselves. The details will be discussed in Appendix E. We then perform the integration using the formula [11]:

(4.13)

$$\int_0^\infty e^{-ak^2} k^{\mu-1} J_\nu(bk)\,dk = \frac{\Gamma(\frac{1}{2}(\mu+\nu))\left(\frac{b}{2\sqrt{a}}\right)^\nu}{2(\sqrt{a})^\mu\,\Gamma(\nu+1)} M\left(\tfrac{1}{2}(\mu+\nu), \nu+1, -b^2/4a\right)$$

where M is the Confluent Hypergeometric Function.

For the special case of $\frac{1}{2}(\mu+\nu) = \nu+1 = m$, we have:

(4.14) $M(m, m, -b^2/4a) = e^{-b^2/4a}$

In this way, λ , μ , ν ,and π are explicitly evaluated. Under the Fourier transformation, the vectors $\hat{l},\hat{m},\hat{n}$ transform as follows. Since $\hat{l}$ is in the z direction, it is unaffected by the d^2k integration. $\vec{\sigma}^1\cdot\hat{n}$ is rewritten $\vec{\sigma}^1\cdot\hat{n} = \vec{\sigma}^1\cdot(\hat{k}_3\times\hat{k}_\perp) = \hat{k}_3\cdot(\hat{k}_\perp\times\vec{\sigma}^1) = \hat{l}\cdot(\hat{k}_\perp\times\vec{\sigma}^1)$.

Under the Fourier transformation, $\vec{k}\rightarrow\vec{b}$, as is obvious from (4.2). The net result is that $\hat{n}$ transforms to a new vector $\hat{k}_3\times\hat{b} = \hat{p}$. Thus we define a new orthonormal set of vectors in b space, $\hat{p},\hat{q},\hat{r}$, with:

(4.15)
$$\hat{r}\equiv\hat{k}_3$$
$$\hat{q}\equiv-\hat{b}$$
$$\hat{p}\equiv\hat{r}\times\hat{q}$$

In terms of these, the results of our Fourier transformation may be written in terms of $\chi|_{BA}$ of (3.10) as:

(4.16) $\hat{\chi}(b)|_{BA} = \lambda(b^2)\,\hat{1} + \mu(b^2)(\vec{\sigma}^1+\vec{\sigma}^2)\cdot\hat{p} + \nu(b^2)\,\vec{\sigma}^1\cdot\vec{\sigma}^2 + \pi(b^2)\,\vec{\sigma}^1\cdot\hat{p}\,\vec{\sigma}^2\cdot\hat{p}$

We will find it especially convenient to rewrite the $\vec{\sigma}^1\cdot\vec{\sigma}^2$ term above as:

(4.17) $\sigma^1\cdot\sigma^2 = \vec{\sigma}^1\cdot\hat{p}\,\vec{\sigma}^2\cdot\hat{p} + \vec{\sigma}^1\cdot\hat{q}\,\vec{\sigma}^2\cdot\hat{q}$

to give:
(4.18) $\hat{\chi}(b^2)|_{BA} = \lambda(b^2)\,\hat{1} + (\pi(b^2)+\nu(b^2))\,\vec{\sigma}^1\cdot\hat{p}\,\vec{\sigma}^2\cdot\hat{p} + \nu(b^2)\,\vec{\sigma}^1\cdot\hat{q}\,\vec{\sigma}^2\cdot\hat{q} + \mu(\vec{\sigma}^1+\vec{\sigma}^2)\cdot\hat{p}$

to give:
(4.18) $\hat{\chi}(b^2)|_{BA} = \lambda(b^2)\,\hat{1} + (\pi(b^2)+\nu(b^2))\,\vec{\sigma}^1\cdot\hat{p}\,\vec{\sigma}^2\cdot\hat{p} + \nu(b^2)\,\vec{\sigma}^1\cdot\hat{q}\,\vec{\sigma}^2\cdot\hat{q} + \mu(\vec{\sigma}^1+\vec{\sigma}^2)\cdot\hat{p}$

We may now evaluate the function λ, μ, ν and π by using the formulas given above. We find:

(4.19)

$$A_1 = .07\pi\, e^{-.0475 b^2} + .0662\, e^{-.1838 b^2} + .0085\, e^{-.5006 b^2} + .00046\, e^{-1.534 b^2},$$

$$A_2 = .7 \times b^2/(8\times(6.54)^3)\, e^{-b^2/4\times 6.54} + \frac{.213\, b^2}{8\times(2.65)^3}\, e^{-\frac{b^2}{4\times 2.65}} + \frac{.008\, b^2}{8\times(.948)^3}\, e^{-\frac{b^2}{4\times .948}} + \frac{.000065 b^2}{8\times(.429)^3}\, e^{-\frac{b^2}{4\times .429}}$$

$$A_3 = 0.$$

$$A_4 = 0.$$

$$A_5 = -\frac{.7}{4\times(6.54)^2}\, e^{-\frac{b^2}{4\times 6.54}} - \frac{.213}{4\times(2.65)^2}\, e^{-\frac{b^2}{4\times 2.65}} - \frac{.008}{4\times(.948)^2}\, e^{-\frac{b^2}{4\times .948}} - \frac{.000065}{4\times(.429)^2}\, e^{-\frac{b^2}{4\times .429}}$$

$$A_6 = -b\times\left(\frac{.83}{4\times(5.36)^2}\, e^{-\frac{b^2}{4\times 5.36}} + \frac{.118}{4\times(1.62)^2}\, e^{-\frac{b^2}{4\times 1.62}} + \frac{.0057}{4\times(.669)^2}\, e^{-\frac{b^2}{4\times .669}} + \frac{.000058}{4\times(.264)^2}\, e^{-\frac{b^2}{4\times .264}}\right)$$

For details, see Appendix C.

We henceforth drop the subscript BA on $\chi|_{BA}$, regarding it as understood in the following.

We next turn to the problem of calculating the products of χ's, which involves the consideration of the full complexity of the algebra of the spin matrices.

We see that we can demonstrate the correctness of this approach with the aid of the two Lemmas proved below. We introduce the definitions:

(4.20)

$$\Sigma_{11} = \vec{\sigma}^1\cdot\hat{p}\,\vec{\sigma}^2\cdot\hat{p}$$

$$\Sigma_{22} = \vec{\sigma}^1\cdot\hat{q}\,\vec{\sigma}^2\cdot\hat{q}$$

$$\Sigma_{33} = \vec{\sigma}^1\cdot\hat{r}\,\vec{\sigma}^2\cdot\hat{r}$$

$$\Sigma_{12} = \vec{\sigma}^1\cdot\hat{p}\,\vec{\sigma}^2\cdot\hat{q}$$

$$\Sigma_{23} = \vec{\sigma}^1\cdot\hat{q}\,\vec{\sigma}^2\cdot\hat{r}$$

$$\Sigma_{13} = \vec{\sigma}^1\cdot\hat{p}\,\vec{\sigma}^2\cdot\hat{r}$$

where the first subscript refers to the component of the sigma matrix operating in the subspace of particle 1, and the second subscript refers to the sigma matrix operating in the subspace of particle 2.

We now introduce the quantities $A_i, i=1,\ldots,6$ and the expressions (4.20), and rewrite (4.19) as:

(4.21)

$$\chi(b) = A_1 \cdot \hat{1} + A_2 \Sigma_{11} + A_3 \Sigma_{22} + A_6 (\vec{\sigma}^1 + \vec{\sigma}^2) \cdot \hat{p}$$

In order to evaluate the scattering amplitude, we must first be able to calculate the product of an arbitrary number of factors of the form (4.21). First, we tabulate the products of sigmas in the multiplication table below.

<u>Table 1</u>

$$\Sigma_{11} \Sigma_{22} = \Sigma_{22} \Sigma_{11} = -\Sigma_{33}$$

$$\Sigma_{22} \Sigma_{33} = \Sigma_{33} \Sigma_{22} = -\Sigma_{11}$$

$$\Sigma_{11} \Sigma_{33} = \Sigma_{33} \Sigma_{11} = -\Sigma_{22}$$

$$(\sigma^1 + \sigma^2) \cdot \hat{p} \, (\sigma^1 + \sigma^2) \cdot \hat{p} = 2(1 + \Sigma_{11})$$

$$(\sigma^1 + \sigma^2) \cdot \hat{p} \, \Sigma_{11} = \Sigma_{11} (\sigma^1 + \sigma^2) \cdot \hat{p} = (\vec{\sigma}^1 + \vec{\sigma}^2) \cdot \hat{p}$$

$$(\sigma^1 + \sigma^2) \cdot \hat{p} \, \Sigma_{22} = i (\Sigma_{32} + \Sigma_{23})$$

$$\Sigma_{22} (\sigma^1 + \sigma^2) \cdot \hat{p} = -i (\Sigma_{32} + \Sigma_{23})$$

$$(\sigma^1 + \sigma^2) \cdot \hat{p} \, \Sigma_{33} = -i (\Sigma_{23} + \Sigma_{32})$$

$$\Sigma_{33} (\sigma^1 + \sigma^2) \cdot \hat{p} = +i (\Sigma_{23} + \Sigma_{32})$$

Next, we consider the product of two factors of the form of (4.21), i.e. of two factors of χ . However, we will, for the sake of generality, allow the Σ_{ij}'s to have arbitrary coefficients. Introducing $B_i, i=1,\ldots,6$ for this purpose, we have; using Table 1:

$$
\begin{aligned}
(4.22)\quad & \left(A_1 1 + A_2\Sigma_{11} + A_3\Sigma_{22} + A_4\Sigma_{33} + A_6(\vec{\sigma}^1+\vec{\sigma}^2)\cdot\hat{p}\right)\times \\
& \left(B_1 1 + B_2\Sigma_{11} + B_3\Sigma_{22} + B_4\Sigma_{33} + B_6(\vec{\sigma}^1+\vec{\sigma}^2)\cdot\hat{p}\right) \\
&= A_1B_1\hat{1} + A_1B_2\Sigma_{11} + A_1B_3\Sigma_{22} + A_1B_4\Sigma_{33} + A_1B_6(\vec{\sigma}^1+\vec{\sigma}^2)\cdot\hat{p} \\
&+ A_2B_1\Sigma_{11} + A_2B_2\hat{1} - A_2B_3\Sigma_{33} - A_2B_4\Sigma_{22} + A_2B_6(\vec{\sigma}^1+\vec{\sigma}^2)\cdot\hat{p} \\
&+ A_3B_1\Sigma_{22} - A_3B_2\Sigma_{33} + A_3B_3\hat{1} - A_3B_4\Sigma_{11} + A_3B_6\left(-i(\Sigma_{23}+\Sigma_{32})\right) \\
&+ A_4B_1\Sigma_{33} - A_4B_2\Sigma_{22} - A_4B_3\Sigma_{11} + A_4B_4\hat{1} + A_4B_6\left(i(\Sigma_{23}+\Sigma_{32})\right) \\
&+ A_6B_1(\vec{\sigma}^1+\vec{\sigma}^2)\cdot\hat{p} + A_6B_2(\vec{\sigma}^1+\vec{\sigma}^2)\cdot\hat{p} + A_6B_3\left(i(\Sigma_{23}+\Sigma_{32})\right) \\
&+ A_6B_4\left(-i(\Sigma_{23}+\Sigma_{32})\right) + A_6B_6(2+2\Sigma_{11}).
\end{aligned}
$$

We see from the above that imaginary terms would arise in general for arbitrary choices of A_i, B_i. However, we will see that they will not arise when A_i and B_i are generated as will be described below. Collecting these terms for future reference, we have:

$$(4.23) \quad \left(A_6(B_3-B_4)+(A_4-A_3)B_6\right)\cdot i\left(\Sigma_{23}+\Sigma_{32}\right)$$

The terms which do contribute are:

$$(4.24) \quad \begin{aligned} &\left(A_1 1 + A_2\Sigma_{11} + A_3\Sigma_{22} + A_4\Sigma_{33} + A_6(\vec{\sigma}^1+\vec{\sigma}^2)\cdot\hat{p}\right) \times \\ &\left(B_1 1 + B_2\Sigma_{11} + B_3\Sigma_{22} + B_4\Sigma_{33} + B_6(\vec{\sigma}^1+\vec{\sigma}^2)\cdot\hat{p}\right) \\ &= \left(A_1B_1 + A_2B_2 + A_3B_3 + A_4B_4 + 2A_6B_6\right) 1 \\ &+ \left(A_1B_2 + A_2B_1 - A_3B_4 - A_4B_3 + 2A_6B_6\right)\Sigma_{11} \\ &+ \left(A_1B_3 + A_3B_1 - A_2B_4 - A_4B_2\right)\Sigma_{22} \\ &+ \left(A_1B_4 + A_4B_1 - A_2B_3 - A_3B_2\right)\Sigma_{33} \\ &+ \left((A_1+A_2)B_6 + A_6(B_1+B_2)\right)(\vec{\sigma}^1+\vec{\sigma}^2)\cdot\hat{p} \end{aligned}$$

Our next observation is that the algebra (4.24) is closed. Hence, we may proceed as follows. We begin by setting A_1, A_2, A_3, A_6 to their appropriate values, those given in (4.19), and $B_1=A_1, B_2=A_2, B_3=A_3, B_6=A_6$ and taking the product. The result is given by (4.24). As can be seen, the coefficients of the sigma matrices in the product are merely linear combinations of bilinear products of A's and B's. This gives χ^2. To get χ^3, we set B_1= coefficient of 1, B_2= coefficient of Σ_{11}, etc., and re-apply (4.24). The result is now $\chi^3 = (\chi)\times(\chi^2)$. This process may be repeated any number of times to generate any arbitrary power of χ.

The electronic computer is programmed to perform the process described above, and thus is able to "do" the sigma algebra.

We now prove two lemmas which will be important in establishing the validity of the above procedure.

Lemma I

We show that the terms of (4.23) always add to zero.

Proof: Let us introduce the notation B_i^n as the value resulting for B_i in (4.22) when n multiplications have been done and the result stored in the B_i's. This means that there are n+1 factors of χ in the product.

1) Consider the case of n=1, i.e. χ^2. To form the coefficients of the sigmas for the result of the product of χ times χ, we take $A_i=B_i$, i=1,6, and find that the coefficient of $i(\Sigma_{23}+\Sigma_{32})$ is:

(4.25) $$A_6(A_3-A_4)+(A_4-A_3)A_6 \equiv 0$$

This cancellation occurs, of course, because $A_i=B_i$. This condition will no longer be true for higher n. For higher n, although $A_1,\ldots,A_6$ retain their initial values, we have nevertheless, e.g.

(4.26) $$B_3^n=(A_1B_3^{n-1}+A_3B_1^{n-1}-A_2B_4^{n-1}-A_4B_2^{n-1})+A_3$$

2) Hence, we assume that the coefficient of vanishes for order n, and prove that this implies the vanishing of the coefficient for order n+1. Thus our assumption is:

(4.27) $$A_6(B_3^n-B_4^n)+(A_4-A_3)B_6^n=0. \qquad \text{for n.}$$

Then for n+1, we have:

$$A_6\left(B_3^{n+1} - B_4^{n+1}\right) + (A_4 - A_3)\, B_6^{n+1} \tag{4.28}$$

For the B's in (4.28), using our multiplication table, we find:

$$\begin{aligned} B_3^{n+1} &= \left(A_1 B_3^n + A_3 B_1^n - A_2 B_4^n - A_4 B_2^n\right) \\ B_4^{n+1} &= \left(A_1 B_4^n + A_4 B_1^n - A_2 B_3^n - A_3 B_2^n\right) \\ B_6^{n+1} &= \left((A_1 + A_2) B_6^n + A_6 (B_1^n + B_2^n)\right) \end{aligned} \tag{4.29}$$

Substituting these in (4.28), we find:

$$\begin{aligned} & A_6\left(A_1 B_3^n + A_3 B_1^n - A_2 B_4^n - A_4 B_2^n - A_1 B_4^n - A_4 B_1^n + A_2 B_3^n + A_3 B_2^n\right) \\ & + (A_4 - A_3)\left((A_1 + A_2) B_6^n + A_6 (B_1^n + B_2^n)\right) \\ & = (A_1 + A_2)\left[A_6 \left(B_3^n - B_4^n\right) + (A_4 - A_3)\, B_6^n\right] \end{aligned} \tag{4.30}$$

and the final term in square brackets vanishes by hypothesis. Since the remaining factor is not infinite, the entire expression, which is the coefficient of $i(\Sigma_{13} + \Sigma_{32})$, vanishes for all n. Hence, this is a proof by mathematical induction.

Lemma II

The purpose of this Lemma is tb demonstrate that the results of our algebraic manipulations above are independent of the choice of sign in the exponential of (2.12). This is not obvious, under $i \to -i$ in the exponential, the function μ of (4.4) would change sign. This would change the sign of μ in (4.18), which is A_6 of (4.21). This change might affect other terms in (4.24) to return a different final result. Of course, this arbitrariness is allowed, since it just corresponds to our choice of phase in writing the Fourier transformation. Hence, we investigate the property of invariance of our results under reversal of sign of A_6, $A_6 \to -A_6$. We proceed as in Lemma I, and give the proof by mathematical induction. We use our previous notation.

1) For n=1:

$$A_6 = B_6 = B_6^1 \tag{4.31}$$

Therefore, if $A_6 \to -A_6, B_6 \to -B_6$.

2) Assume that in order n, the contributions to 1 and Σ_{11} are invariant under $A_6 \to -A_6$. We need consider these terms only, since these are the only places where contributions from a factor of A_6 appear.(4.24). The condition 2) requires that $A_6 B_6^n$ be invariant under $A_6 \to -A_6$, which implies:

(4.32) $$B_6^n \rightarrow -B_6^n$$

(4.32) therefore is our assumption about n. To order n+1, the contribution to consider is $A_6 B_6^{n+1}$. As above:

(4.33) $$B_6^{n+1} = \left((A_1 + A_2) B_6^n + A_6 (B_1^n + B_2^n) \right)$$

Under $A_6 \rightarrow -A_6, B_6 \rightarrow -B_6$ by hypothesis, therefore:

(4.34) $$B_6^{n+1} \rightarrow B_6^{n+1}$$

Hence in order n+1,

(4.35) $$A_6 B_6^{n+1} \rightarrow A_6 B_6^{n+1}$$ under $A_6 \rightarrow -A_6$.

Thus, the contributions to terms other than $(\vec{\sigma}^1 + \vec{\sigma}^2) \cdot \hat{p}$ are independent of the sign of A_6 , which proves the Lemma.

In this way, we can instruct the computer to perform the sigma algebra. However, we may only consider a finite number of terms in the expansion of the exponential $e^{-\chi(b)}$. In order to decide how many terms are needed to insure convergence of the expansion, we consider the series of Fourier transforms of powers of χ . Thus, we take the Fourier transform of each power of χ separately, and check that the series does indeed converge. We may write any term, of arbitrary power in χ , as

(4.36) $$C_1 \mathbf{1} + C_2\Sigma_{11} + C_3\Sigma_{22} + C_4\Sigma_{33} + C_6(\vec{\sigma}_1 + \vec{\sigma}_2)\cdot\hat{p}$$

where $C_1, \ldots, C_6$ are the computed coefficients.

The angular integration now proceeds similarly as in the case of the original Fourier transformation from k space to b space at the beginning of this section, the difference being that the roles of b and k are reversed. However, unlike the situation which obtained then, the integrand is no longer available as a sum of gaussians times powers of k. Hence, we must do the Bessel transformation numerically. Having done this, we add up all the coefficients of each sigma operator separately, with alternating signs as indicated in the expansion of the exponential, to get a convergent answer for the amplitudes as a function of k. We then compute the cross section.

Section V Numerical Results and Discussion

We first summarize our procedures as given above. Starting with expressions for $\alpha|_{BA}$, $\beta|_{BA}$, ..., $\epsilon|_{BA}$ in the most general form of the amplitude for scattering of two spin $\frac{1}{2}$ particles in the infinite momentum Born approximation, we Fourier transform to obtain the quantity $\chi|_{BA}$. as a linear combination of operators in the direct product spin space of the two particles. Because the algebra of these operators is closed, we are able to write an algorithm for the calculation, numerically, of all the powers of χ that appear in the expansion of the exponential $e^{-\chi}$. We then Fourier transform this expression back into k space. This last integration has two parts: integration over angles is done in the same way as done in going from k space to b space in the first part of the calculation. This is done merely by taking appropriate linear combinations of Bessel transforms. However, the Bessel transforms must be done numerically.

We may check the correctness of our manipulations, since the first term in our final answer is merely the double Fourier transform of $\hat{f}(x)|_{BA}$, for which we have our Gaussian fits. Hence, we compare this item of our output, and test that they are in fact equal. It was found that to obtain convergence of the series of Fourier transforms, fifteen terms or less were needed.[1)]

We fit η by finding what value of η is needed to make the calculated value of $d\sigma/dt$ at t=0 equal to some definite value. The value needed to achieve agreement with a forward amplitude calculated with the optical theorem from a total cross section of 40 mb is $\eta \cong 11.5$. While this is in good agreement with the forward cross section at current energies, it gives a cross section which is too large for higher values of t. We find that $\eta \cong 10.5$ gives the best fit over all t, is not very different from the 11.5 fit in the near forward direction, and corresponds to a total cross section of 35.7 mb. We note that a value of 35.7 mb has been predicted for p-p scattering by Barger (13) using a Regge pole model. $\eta = 10.5$ gives the result plotted in Fig. 6.

We also do the Chou-Yang calculation as a special case by inserting a multiplicative parameter in our calculation:

$$(5.1) \quad \hat{f}(k)\Big|_{BA} = \alpha\Big|_{BA}\,\hat{1} + (\text{Parameter})\left[\beta\,\vec{\sigma}_1\cdot\hat{n}\,\vec{\sigma}_2\cdot\hat{n} + \cdots + \epsilon\,\vec{\sigma}_1\cdot\hat{\ell}\,\vec{\sigma}_2\cdot\hat{\ell}\right]$$

When this is set equal to 0, all the spin flip amplitudes are suppressed leaving only α. Although the results are not given here, we have also done the calculation for various other values of this parameter intermediate between 0 and 1, implying various degrees of suppression of the spin flip amplitudes by a momentum independent factor. Since, if a mechanism which suppresses

1) For this number of terms, we have convergence to six significant figures.

spin flip amplitudes exists, there is no reason to suppose a priori that it is momentum independent, we must regard this parametrization as a property of our method of calculation, rather than a property of physical reality, We do not present the graphs here. We only report that, in this way, we can achieve a continuous transition between the Chou-Yang calculation and ours. This will be discussed below in connection with "breaks" (abrupt changes) in the differential cross section.

As is well known, the original paper of Chou and Yang(3) predicted <u>zeroes</u> in the differential cross section for p-p scattering at about the same values of t for which "breaks" or dips are observed at current energies. We redo the CY calculation, as shown in Fig.7.

We find that, as expected, when spin effects are taken into account, as in our calculation, that breaks, which are observed, still appear, but that zeroes, which are not observed, do not.(Fig.6).

The zeroes in the region of the experimentally observed breaks are due, in theChou-Yang calculation, to the well known cancellation of the terms in the expansion of $e^{-\chi}$. In our calculation, the breaks are due to the numerically smaller amplitudes having under gone just such a cancellation some distance in t before the break, giving a result of zero there. The break itself is due to the amplitudes rising from that zero,

with a reversal of sign, just as in the Chou-Yang calculation. Their contribution to the cross section thus increases rapidly enough to decrease the magnitude of the negative slope, and forms "shoulders" in the differential cross section, at about the same points that zeroes were previously predicted in the Chou-Yang calculation.
By means of the parameter described above, we may rederive the Chou-Yang results for the parameter =0, and watch them turn into our results.

To summarise, by increasing our parameter to 1 by intervals, the irregularities are observed to stay in about the same place, but the zeroes in thecross section are "filled in" by the non- vanishing spin flip amplitudes, and the "shoulders" are due to the rapid rise of spin flip amplitudes after going through zeroes.(Fig.6).

We next discuss the case of $p\bar{p}$ scattering, and assume that, although the electromagnetic current for a $\bar{p}$ is opposite in sign to that for p, in the diffraction picture of scattering, only the absolute magnitudes of the "hadronic density" are effective in the absorptive process. Hence, we consider that the amplitudes for $p\bar{p}$ have the same momentum dependence as those for pp. The only difference will be that η is larger in order to agree with the experimentally observed larger forward cross section for $p\bar{p}$. This larger value of η results in a steeper slope for the pp cross section, so much so that it sinks below that for pp at $t \cong .5$ $(Gev/c)^2$, as may be seen in Fig. 7 . This phenomenon is actually observed at current energies. However, $p\bar{p}$ cross sections have not been measured at energies much above 5 Gev/c incident Lab momentum, so this experimental data is not presented here. The curves in Fig.7 are presented only to illustrate the cross over at $t \cong .5$ $(Gev/c)^2$, predicted under the assumptions above.

As noted in Appendix G, we predict a zero value for the polarization parameter P_w defined there. This is a consequence of our model, which predicts real values for α, β, ..., ϵ . The latest experiment of which we are aware is that of Ref.(12). We note in Appendix G, that the quantity P_0 measured there may not be the P_w which we discuss in our theory. The deviation would be of the same order of magnitude as the difference between $|P_T(up)|$ and $|P_T(down)|$, which these Authors quote as being about 10%, would allow their measured P_0 to differ from ours by about 10%. Further, many of their values are affected by large errors, so that we may conclude that our prediction is not definitely ruled out by this experiment.

We next consider the connection with the Regge treatment of high energy scattering. Since the helicity amplitudes are the ones which are Reggeized, we have derived them in Appendix D, and find that they are equal in magnitude to our spin flip amplitudes. To proceed further, we relate our assumptions to the Regge formalism by referring to the work of Huang and Pinsky (15), who present a Reggeization of the helicity flip amplitudes for p-p scattering. We note that our helicity flip amplitudes are energy independent, while theirs have a multiplicative factor of $(s-u)^{\alpha-1}$. Thus, our assumptions correspond to a choice of $\alpha = 1$, or a single zero slope Pomeranchuk trajectory.

We have made several attempts to extrapolate this calculation to finite energies. We first attempted to abandon the assumption $p_{\perp} = \sqrt{-t}$, and instead calculate the transverse momentum from t and the energies at which the Allaby (9) experiment was performed, $p_{incident}$=19.20 and 21.12 Gev/c, and plotting the result as a function of t. The effect of this modification was to increase $d\sigma/dt$ to an unacceptable level for large t. We tried in addition, calculating $T|_{SA}$ without taking the infinite momentum limit, but this also led to the same trouble as above. However, we would expect that the assumption that the eikonal can be expressed as $\hat{\chi} = \langle \hat{\delta} | BA \rangle$ would have to be essentially modified at finite energies. Possibly we can no longer assume that a current-current form of the interaction holds, at finite energies.

Appendix A Evaluation of Transition Amplitudes in the Born Approximation in the Infinite Momentum Limit

We evaluate the matrix element (3.13) for the different possible spin transition amplitudes in the infinite momentum limit. Our representation of γ 's and spinors is:

(A.1)
$$\gamma^0 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad \vec{\gamma} = \begin{pmatrix} 0 & \vec{\sigma} \\ -\vec{\sigma} & 0 \end{pmatrix}, \quad u^s(p) = \begin{pmatrix} \chi_s \\ \frac{\vec{\sigma}\cdot\vec{p}}{p_0+m}\chi_s \end{pmatrix}$$

$$\bar{u}^s(p) = u^{s+}(p)\gamma^0 = \left(\chi_s^+, -\chi_s^+ \frac{\vec{\sigma}\cdot\vec{p}}{p_0+m}\right); \quad \chi_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad \chi_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$\sigma_{\mu\nu} = \frac{i}{2}[\gamma_\mu, \gamma_\nu]_-$$

We have, using the Dirac equation:

(A.2)
$$\langle p's' | J^\mu_{EM\,I}(0) | ps \rangle = \bar{u}^{s'}(p')\left(F_1\gamma^\mu + i\sigma^{\mu\nu} q_\nu F_2\right) u^s(p)$$

$$= \bar{u}^{s'}(p')\left(G_m\gamma^\mu - (p'+p)^\mu F_2\right) u^s(p)$$

$$q^\mu = p'^\mu - p^\mu$$

where $G_m = F_1 + 2MF_2$, M is the proton mass, and I denotes the particle incident in the Lab frame. Using an obvious notation for spin up and spin down (parallel or anti-parallel to the direction of quantization), we have:

(A.3) $\langle p',\uparrow | J_I^0 | p,\uparrow\rangle = \langle p',\downarrow | J_I | p,\downarrow\rangle =$

$$= \left(G_m \left(1 + \frac{p_3 p_3'}{(p_0'+m)(p_0+m)} \right) - (p'+p)_0 F_2 \left(1 - \frac{p_3 p_3'}{(p_0'+m)(p_0+m)} \right) \right)$$

for particle I, same for particle II.

(A.4) $\langle p' \updownarrow | J_I^0 | p \downarrow\!\uparrow\rangle =$

$$= \left[\frac{p_3 G_m}{(p_0'+m)(p_0+m)} + \frac{p_3 (p'+p)_0}{(p_0'+m)(p_0+m)} \right] \left[\mp p_\perp e^{\mp i\phi} \right]$$

for particle I, same for particle II.

(A.5) $\langle p' \updownarrow | J_I^3 | p \updownarrow\rangle =$

$$= \left[(p'+p)_3 F_2 \left(\frac{p_3' p_3}{(p_0'+m)(p_0+m)} - 1 \right) + G_m \left(\frac{p_3'}{p_0'+m} + \frac{p_3}{p_0+m} \right) \right]$$

for particle I, overall change of sign for particle II.

(A.6)

$\langle p' \updownarrow | J_I^3 | p, \downarrow\!\uparrow\rangle =$

$$= \left[\frac{G_m}{p_0+m} + F_2 \frac{p_3 (p'+p)_3}{(p_0'+m)(p_0+m)} \right] \left(\mp p e^{\mp i\phi} \right)$$

for particle I, overall change of sign for particle II.

The following matrix elements will not survive in the infinite momentum limit, but are included here for completeness.

(A.7)
$$\langle p', \updownarrow | J^2 | p, \updownarrow \rangle = \left[\frac{G_m}{p_0' + m} \left(\pm i \, p_\perp e^{\mp i\phi} \right) - p_2' F_2 \left(1 - \frac{p_3 p_3'}{(p_0' + m)(p_0 + m)} \right) \right]$$

for particle I, overall change of sign for particle II.

(A.8)
$$\langle p' \updownarrow | J^2 | p, \downarrow\uparrow \rangle = \left[i G_m \left(\frac{p_3}{p_0 + m} - \frac{p_3'}{p_0' + m} \right) + \frac{p_2' p_3 F_2}{(p_0' + m)(p_0 + m)} (\mp p_\perp) e^{\mp i\phi} \right]$$

for particle I, overall change of sign for particle II .

(A.9)
$$\langle p' \updownarrow | J^1 | p \updownarrow \rangle = \left[\frac{G_m}{p_0' + m} \left(p_\perp e^{\mp i\phi} \right) - p_1' F_2 \left(1 - \frac{p_3' p_3}{(p_0' + m)(p_0 + m)} \right) \right]$$

for particle I, overall change of sign for particle II.

(A.10)
$$\langle p' \updownarrow | J^1 | p, \downarrow\uparrow \rangle = \left[(\mp) G_m \left(\frac{p_3}{p_0 + m} - \frac{p_3'}{p_0' + m} \right) + \frac{p_1' F_2 p_3 (\mp p_\perp) e^{\mp i\phi}}{(p_0 + m)(p_0' + m)} \right]$$

for particle I, overall change of sign for particle II.

The matrix elements given above are for the transitions of particle I. The matrix elements for the transitions of particle II, the target in the Lab frame, are obtained from these by making the changes noted in the statements to the right. The net result is to change the overall sign of the matrix elements in (A.5) to (A.10) inclusive. Since we use the metric:

(A.11)
$$g^{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

to give the product:

(A.12)
$$j^{\mu} j_{\mu} = j^{0} j_{0} - j^{3} j_{3} - j^{2} j_{2} - j^{1} j_{1}$$

Thus, this overall change of sign for the matrix elements of $J^3{}_{EM,II}$ result in the minus signs in (A.12) being changed to plus signs (matrix elements of J^2 and J^1 vanish in the infinite momentum limit).

In taking the infinite momentum limit, we will need the expansion:

(A.13)
$$\frac{p_3}{p_0+m} \cdot \frac{p_3'}{p_0'+m} = \frac{p_3\, p_3'}{(p_0+m)(p_0'+m)} \left(1 - \frac{2m}{p_0} + \frac{3m^2}{p_0^2} + \mathcal{O}\left(\frac{1}{p_0^3}\right)\right)$$

The infinite momentum limit is characterized by $p_0 \longrightarrow \infty$, $p_3 \longrightarrow \infty$. Hence,

(A.14)
$$p_0^2 = p_3^2 + p_\perp^2 + m^2 \cong p_3^2$$

in this limit.

Hence, the transition amplitude (3.13) is given by:

(A.15)
$$T_{s_I', s_{II}'; s_I, s_{II}}\Big|_{BA} = \lim_{\text{momentum} \to \infty} \Big[\langle p_I', s_I' | J_I^{0}(0) | p_I, s_I \rangle \langle p_{II}', s_{II}' | J_{II}^{0}(0) | p_{II}, s_{II} \rangle -$$
$$- \langle p_I', s_I' | J_I^{3}(0) | p_I, s_I \rangle \langle p_{II}', s_{II}' | J_{II}^{3}(0) | p_{II}\, s_{II} \rangle \Big]$$

minus similar terms for J^2 and J^1.

The extra minus sign due to the matrix elements for particle II changes the minus signs from the space components of $g_{\mu\nu}$ to plus signs.

Inspecting (A.7) to (A.10), we see that these amplitudes vanish as $p_0, p_3 \longrightarrow \infty$, so we concentrate our attention on the J^0 and J^3 terms.

(A.16)

$$T_{\uparrow\uparrow\leftarrow\uparrow\uparrow} = T_{\downarrow\downarrow\leftarrow\downarrow\downarrow} = T_{\uparrow\downarrow\leftarrow\uparrow\downarrow} = T_{\downarrow\uparrow\leftarrow\downarrow\uparrow} =$$

$$= \left(G_m \left(1 + \frac{p_3 p_3'}{(p_0+m)(p_0'+m)} \right) - (p'+p)_0 F_2 \left(1 - \frac{p_3 p_3'}{(p_0+m)(p_0'+m)} \right) \right)^2 + \left((p'+p)_3 F_2 \left(\frac{p_3 p_3'}{(p_0+m)(p_0'+m)} - 1 \right) + G_m \left(\frac{p_3'}{p_0'+m} + \frac{p_3}{p_0+m} \right) \right)^2$$

$$\longrightarrow \left(2 G_m - \frac{2m}{p_0} F_2 (2p_0) \right)^2 + \left((p'+p)_3 F_2 \left(-\frac{2m}{p_0} \right) + 2 G_m \right)^2$$

$$\longrightarrow 8 G_m^2 - 32 m G_m F_2 + 32 F_2^2 = 8 F_1^2$$

where (A.13) and the definition of G_M have been used.

We summarise our results below.

(A.17) $T_{\text{no spin flip}} = F_1^2$

(A.18) $T \begin{pmatrix} \uparrow\leftarrow\downarrow \\ \downarrow\leftarrow\uparrow \end{pmatrix} = (\mp) e^{\mp i\phi} F_1 F_2$, the other particle unaffected.

(A.19) $T_{\substack{\uparrow\uparrow\leftarrow\downarrow\downarrow \\ \downarrow\downarrow\leftarrow\uparrow\uparrow}} = p_\perp^2 F_2^2 e^{\pm 2i\phi}$

(A.20) $T_{\substack{\uparrow\downarrow\leftarrow\downarrow\uparrow \\ \downarrow\uparrow\leftarrow\uparrow\downarrow}} = -p_\perp^2 F_2^2$

Here we have dropped the factor of 8, since the coefficient η will include this if it were kept, and η will be determined later by comparison with experiment.

Appendix B Infinite Momentum Limit of Vectors $\hat{l}, \hat{m}, \hat{n}$

In order to derive (3.8), we need the infinite momentum limit of the following basis vectors:

$$\hat{n} = \frac{\hat{k} \times \hat{k}_f}{\sin\theta}, \quad \hat{m} = \frac{\hat{k} - \hat{k}_f}{2 \sin\theta/2}, \quad \hat{l} = \frac{\hat{k} + \hat{k}_f}{2 \cos\theta/2}$$

1) Case of n in the infinite momentum limitl

Define k_z = z component of COM momentum of incident par- cle, in the initial state.

k'_z = z component of COM momentum of the same particle, in the final state.

k' = transverse momentum of the same particle.

Also by definition:

(B.1) $$\hat{k}_i \equiv \frac{\vec{k}_z}{|k_z|} \; ; \; \hat{k}_f \equiv \frac{\vec{k}'_\perp + \vec{k}'_z}{|\vec{k}'_\perp + \vec{k}'_z|}$$

where: $\sin\theta \equiv \frac{|\vec{k}'_\perp|}{|k'_z|}$

Then:

(B.2) $$\hat{n} = \frac{k_z \times (k'_\perp + k'_z)}{|k_z|\,|k'_\perp + k'_z| \cdot \frac{|\vec{k}'_\perp|}{|\vec{k}'_z|}} = \frac{\vec{k}_z}{|\vec{k}_z|} \times \frac{\vec{k}'_\perp}{(k'^2_\perp + k'^2_z)^{1/2}} \cdot \frac{1}{\frac{|k'_\perp|}{|k'_z|}}$$

Expanding the denominator:

(B.3) $$\frac{1}{(k'^2_\perp + k'^2_z)^{1/2}} = \frac{1}{|k_z|}\left(1 - \frac{1}{2}\frac{k'^2_\perp}{k_z^2} + \cdots\right)$$

This gives:

(B.4)
$$\hat{n} = \frac{k_z}{|\vec{k}_z|} \times \frac{k_\perp'}{|k_z'|}\left(1 - \tfrac{1}{2}\frac{k_\perp'^2}{k_z^2} + \cdots\right)\frac{1}{\frac{|k_\perp'|}{|\vec{k}_z'|}} \cong \frac{\vec{k}_z}{|\vec{k}_z|} \times \frac{\vec{k}_\perp'}{|\vec{k}_\perp'|} \equiv$$
$$\equiv \vec{k}_z \times \vec{k}_\perp'$$

Hence:

(B.5) $\hat{n} = \hat{k}_z \times \hat{k}_\perp'$

2) Case of $\hat{m} = \dfrac{\hat{k} - \hat{k}_s}{2 \sin \theta/2}$

We have:

(B.6)
$$\hat{m} = \left(\frac{\vec{k}_3}{|\vec{k}_3|} - \frac{\vec{k}_\perp' + \vec{k}_3'}{(k_\perp'^2 + k_3'^2)^{1/2}}\right)\frac{1}{2 \sin \theta/2}$$

Since:
$$2 \sin \theta/2 \cong \theta = \sin \theta \doteq \frac{|\vec{k}_\perp'|}{|\vec{k}_3'|}$$

(B.7)
$$\hat{m} = \left(\frac{\vec{k}_3}{|\vec{k}_3|} - \frac{(\vec{k}_\perp' + \vec{k}_3')}{|\vec{k}_3'|\left(1 + \frac{k_\perp'^2}{k_3'^2}\right)^{1/2}}\right)\frac{1}{\frac{|\vec{k}_\perp'|}{|\vec{k}_3'|}} = \left(\frac{\vec{k}_3}{|\vec{k}_3|} - \left(\frac{k_\perp'}{|\vec{k}_3'|} + \frac{k_3'}{|\vec{k}_3'|}\right)\left(1 - \tfrac{1}{2} \times\right.\right.$$
$$\left.\left.\times \frac{k_\perp'^2}{k_z^2} + \cdots\right)\right)\frac{1}{\frac{|k_\perp'|}{|k_3'|}} \cong \frac{\vec{k}_3}{|\vec{k}_\perp'|} - \left(\frac{k_\perp'}{|\vec{k}_\perp'|} + \frac{k_3'}{|\vec{k}_\perp'|}\right)\left(1 - \tfrac{1}{2}\frac{k_\perp'^2}{k_z^2}\right)$$

Since $\dfrac{\vec{k}_3}{|k_\perp'|} \cong \dfrac{|\vec{k}_3'|}{|k_\perp'|}$ for high energies, (B.7) becomes:

$$\hat{m} = -\hat{k}_\perp' + \tfrac{1}{2}\,\hat{k}_z' \cdot \frac{|\vec{k}_\perp|}{|\vec{k}_z|}$$

The second term is negligible as $k_z \to \infty$. Hence:

(B.8) $\hat{m} = -\hat{k}_\perp'$

3) Case of $\hat{l} \equiv \dfrac{\hat{k}_4 + \hat{k}_5}{2\cos\theta/2}$.

Again using the definitions (B.1), we get:

(B.9)

$$\hat{l} = \left(\frac{\vec{k}_3}{|\vec{k}_3|} + \frac{\vec{k}_\perp' + \vec{k}_3'}{|k_3'|\left(1 + \frac{k_\perp^2}{k_3^2}\right)^{1/2}}\right) \cdot \frac{1}{2}\left(1 + \tfrac{1}{2}\frac{|\vec{k}_\perp'|}{|\vec{k}_3'|}\right).$$

Since: $\dfrac{1}{2\cos\theta/2} \simeq \dfrac{1}{2}\left(1 + \dfrac{|\vec{k}_\perp'|}{2|\vec{k}_3|}\right)$

Then:

(B.10)

$$\hat{l} = \frac{1}{2}\left(\hat{k}_3 + \hat{k}_3'\right)\left(1 + \mathcal{O}\left(\frac{1}{|k_3|}\right)\right) + \frac{\vec{k}_\perp'}{|\vec{k}_3|}\left(1 + \mathcal{O}\left(\frac{1}{k_3'}\right)\right).$$

Hence, as $|k_3| \to \infty$,

(B.11)

$$\hat{l} = \frac{2\hat{k}_3}{2} = \hat{k}_3$$

Summarising,

(B.12)

$$\hat{l} = \hat{k}_3$$
$$\hat{m} = -\hat{k}_\perp'$$
$$\hat{n} = \hat{k}_3 \times \hat{k}_\perp'$$
$$\hat{n} \times \hat{m} = \hat{l}.$$

as illustrated in Fig. 2.

Appendix C Integration of Gaussian Fits to the Form Factors F_1 and F_2

One of the advantages of parametrizing the product of the products of form factors as a sum of Gaussians is that we con perform the k integration exactly. In this Appendix, we use (4.13) and (4.14) to explicitly derive the general form of (4.19). We have fitted the products of form factors F_1^2, F_1F_2, and F_2^2 by sums of Gaussians in Appendix E , and take the general term to be:

(C.1) $$A e^{-ak^2}$$

1) Case of λ , which corresponds to the integration of $\alpha \cdot \hat{1}$. Here the integrand is F_1^2. Using (4.13) and (4.14), we get:

(C.2)
$$\int_0^\infty A e^{-ak^2} J_0(kb)\, k\, dk = \frac{A \cdot \Gamma(1)}{2 \cdot a \cdot \Gamma(1)} M(1, 1, -b^2/4a) = \frac{A}{2a} e^{-b^2/4a}$$

This is called A_1 in our notation. The first part of (4.19) is derived from (C.2) by substituting A and a from the fits of (4.12).

2) Case of μ term, which corresponds to the integrand $i\, \gamma(\vec{\sigma}_1 + \vec{\sigma}_2) \cdot \hat{n}$. The integrand here is

$-kF_1F_2$, with F_1F_2 parametrized as a sum of Gaussians in (4.12), the general termbeing defined as Ae^{-ak^2} .

Then (4.13) and (4.14) imply:

(C .3)

$$A\int_0^\infty e^{-ak^2} J_1(kb)\, k^2 dk = \frac{A\,\Gamma(2)\,\frac{b}{2\sqrt{a}}}{2(\sqrt{a})^3\,\Gamma(2)} M(2,2,-b^2/4a)$$

$$= \frac{A.\,b}{4a^2} e^{-b^2/4a}$$

This gives the A_6 term of (4.19).

3) Case of ν and π terms from the integrand $\beta\,\vec{\sigma}^1\cdot\hat{n}\,\vec{\sigma}^2\cdot\hat{n}$. We recall the definition:

(C .4) $$\langle f\rangle_{\substack{0\\2}} \equiv \int f(k)\, J_{\substack{0\\2}}(kb)\, k\, dk$$

and Eqs.

(C.5) $$\nu = \tfrac{1}{2}\left(\langle f\rangle_0 + \langle f\rangle_2\right)$$

$$\pi = -\ \langle f\rangle_2$$

for sòme integrable function f.

Now our integrand is $\beta\ =-k^2F_2^2$ here, so we need:

(C.6) $$\langle\beta\rangle_0 = \int(-k^2F_2^2)J_0(kb)\,k\,dk = -\int F_2^2 J_0(kb)\, k^3 dk$$

Parametrizing F_2^2 by a Gaussian, we find:

(C.7) $$-\int A e^{-ak^2} J_0(kb)\, k^3 dk = -\frac{A\,\Gamma(2)}{2a^2\Gamma(1)} M(2,1,-b^2/4a)$$

Using the recurrence relations for the Confluent Hypergeometric Function, M, we have:

(C.8)

$$M(2,1,-b^2/4a) = (1-b^2/4a)\,M(1,1,-b^2/4a) = (1-b^2/4a)\,e^{-b^2/4a}$$

So:

(C.9)

$$\langle\beta\rangle_0 = -\frac{A}{2a^2}\left(1-b^2/4a\right)e^{-b^2/4a}$$

We also need:

(C.10)

$$\langle\beta\rangle_2 = \int \beta\, J_2(kb)\, dk = -\int F_2^{\,2}\, J_2(kb)\, k^3 dk = -A\int e^{-ak^2} J_2(kb)\, k^3 dk$$

$$= -\frac{A\,\Gamma(3)}{2a^2\,\Gamma(3)}\cdot\frac{b^2}{4a}\cdot M(3,3,-b^2/4a) = -\frac{Ab^2}{8a^3}\,e^{-b^2/4a}$$

Thus:

(C.11)

$$\nu = \tfrac{1}{2}\left(\langle\beta\rangle_0+\langle\beta\rangle_2\right) = -\frac{A}{2}\left(\frac{1}{2a^2}\left(1-b^2/4a^2\right)+\frac{b^2}{8a^3}\right)e^{-b^2/4a}$$

$$= -A/2\cdot\frac{1}{2a^2}\,e^{-b^2/4a}$$

$$\nu = -\frac{A}{4a^2}\,e^{-b^2/4a}$$

and

(C.12)

$$\pi = -\langle\beta\rangle_2 = +\frac{Ab^2}{8a^3}\,e^{-b^2/4a}$$

ν appears in (4.16) as the coefficient of $\vec{\sigma}_1\cdot\vec{\sigma}_2$, denoted A_5, while π is the coefficient of Σ_{11}, denoted A_2. Eq. (4.19) is written out in terms of A_5, even though A_5 is immediately absorbed into A_2 and

A_3, because this corresponds to our procedure when actually doing the calculation by computer. We then absorb A_5 explicitly into A_2 and A_3 , i.e. go from (4.16) to (4.18), during the calculation.

Summarizing, our expression for A_2 in 4.19 is just π in(C.12) above, while A_5 appears in (4.19) as expression (C .11) above.

Appendix D Helicity Formalism

In this appendix, we derive amplitudes for helicity flip transitions in the infinite momentum limit.

The direction of incident particle I is chosen as the axis of quantization. We choose the canonical representation of γ matrices,

(D.1)

$$\gamma^0 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} , \quad \vec{\gamma} = \begin{pmatrix} 0 & \vec{\sigma} \\ -\vec{\sigma} & 0 \end{pmatrix}$$

which imply the canonical form for the Dirac spinors:

We rotate these about the y axis to the final direction, and so get the helicity spinors:

(D.2)

$$u_{\lambda_I} = N \begin{pmatrix} 1 \\ \frac{2p\lambda_I}{E_p+m} \end{pmatrix} \chi_{\lambda_I} \; ; \quad u_{\lambda'_I} = N \begin{pmatrix} 1 \\ \frac{2p'\lambda'_I}{E_{p'}+m} \end{pmatrix} e^{-i\sigma_y\theta/2} \chi_{\lambda'_I}$$

$$u_{\lambda_{II}} = N \begin{pmatrix} 1 \\ \frac{2p\lambda_{II}}{E_p+m} \end{pmatrix} \chi_{-\lambda_{II}} \; ; \quad u_{\lambda'_{II}} = N \begin{pmatrix} 1 \\ \frac{2p'\lambda'_{II}}{E_{p'}+m} \end{pmatrix} e^{-i\sigma_y\theta/2} \chi_{-\lambda'_{II}}$$

$$2p\lambda = \vec{\sigma}\cdot\vec{p}$$

where χ is an eigenstate of $\frac{1}{2}\sigma_z$ with $\lambda = \pm \frac{1}{2}$ as eigenvalues (for either particle), and where N is a normalization factor which will be dropped for the same reasons as noted in footnote 1) of Appendix A. Using the representation of J^{EM} in terms of matrices as in (A.2), we may write out the representation explicitly.

We first treat the case of $J^{EM}_{0,I}$. We have:

(D.3) $\langle p' | J^{0\,EM}_{I(0)} | p \rangle =$

$$= \chi^{+}_{\lambda'_I}\, e^{i\sigma_y^{(I)}\theta/2} \Big[\big(G_m - (p'_I + p_I)_0 F_2 \big) + \big(G_m + (p'_I + p_I)_0 \cdot \frac{2 p_I \lambda_I \cdot 2 p'_I \lambda'_I}{(E_p + m)(E_{p'} + m)} \Big] \chi_{\lambda_I}$$

$$= \chi^{+}_{\lambda'_I} \Big[\cos\theta/2 \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \Big(\big(G_m - (p'_I + p_I)_0 F_2 \big) + \big(G_m + (p'_I + p_I)_0 F_2 \cdot \frac{4 p_I p'_I \lambda_I \lambda'_I}{(E_p + m)(E_{p'} + m)} \Big] \chi_{\lambda_I}$$

$$+ \chi^{+}_{\lambda'_I} \Big[\sin\theta/2 \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \Big(\big(G_m - (p'_I + p_I)_0 F_2 \big) + \big(G_m + (p'_I + p_I)_0 F_2 \cdot \frac{4 p_I p'_I \lambda_I \lambda'_I}{(E_p + m)(E_{p'} + m)} \Big] \chi_{\lambda_I}$$

Diagonal elements are: $(\lambda'_I, \lambda_I) = (+\tfrac{1}{2}, +\tfrac{1}{2})$ or $(-\tfrac{1}{2}, -\tfrac{1}{2})$.

(D.4)

$$\cos\theta/2 \left(2\, G_m + (p'_I + p_I)_0\, F_2 \left(\frac{4 p_I p'_I \lambda_I \lambda'_I}{(p_0 + m)(p'_0 + m)} - 1 \right) \right)$$

for particle I

Off diagonal elements are:

(D.5)

$$\{\pm\} \sin\theta/2 \left(2\, G_m - (p'_I + p_I)_0\, F_2 \left(1 - \frac{4 p_I p'_I \lambda_I \lambda'_I}{(p_0 + m)(p'_0 + m)} \right) \right) \quad \text{FOR} \begin{Bmatrix} \langle + | J | - \rangle \\ \langle - | J | + \rangle \end{Bmatrix}$$

for particle I

For particle II, we have:

(D.6)

$$\cos\theta/2\left(-2\,G_m + (p'_{II}+p_{II})_0\,F_2\left(\frac{4pp'\lambda_{II}\lambda'_{II}}{(p_0+m)(p_0'+m)} - 1\right)\right)$$

for diagonal elements.

(D.7)

$$(\mp)\sin\theta/2\left(2\,G_m + (p'_{II}+p_{II})_0\,F_2\left(\frac{4p'p\lambda'_{II}\lambda_{II}}{(p_0+m)(p_0'+m)} - 1\right)\right) = \begin{Bmatrix} \langle +|J^0|-\rangle \\ \langle -|J^0|+\rangle \end{Bmatrix}.$$

for off diagonal elements.

We next consider $J^{EM}_{3,I}$. We have:

(D.8) $\langle p'_I|\,J^{EM}_{3(0)}\,|p_I\rangle =$

$$\left(\chi^+_{\lambda'_I}\,e^{i\sigma_y\theta/2},\ -\chi^+_{\lambda'_I}\,e^{+i\sigma_y\theta/2}\,\frac{2p'\lambda'_I}{p_0'+m}\right)\begin{pmatrix} -(p'_I+p_I)_3F_2 & G_m\sigma_3 \\ -G_m\sigma_3 & -(p'_I+p_I)_3 \end{pmatrix}\begin{pmatrix} \chi_{\lambda_I} \\ \frac{2p\lambda_I}{p_0+m}\chi_{\lambda_I} \end{pmatrix}$$

$$= \chi^+_{\lambda'_I}\,e^{i\sigma_y\theta/2}\left(-(p'_I+p_I)_3F_2 + G_m\sigma_3\frac{2p\lambda_I}{p_0+m} + \frac{2p'\lambda'_I}{p_0'+m}G_m\sigma_3 + (p'_I+p_I)_3F_2\left(\frac{4p'\lambda'_I p\lambda_I}{(p_0+m)(p_0'+m)}\right)\right)\chi_{\lambda_I}$$

$$= \chi^+_{\lambda'_I}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\cos\theta/2\left[(p'+p)_3F_2\left(\frac{4p\lambda_I p'\lambda'_I}{(p_0'+m)(p_0+m)} - 1\right) + G_m\sigma_3\left(\frac{2p\lambda_I}{p_0+m} + \frac{2p'\lambda'_I}{p_0'+m}\right)\right]\chi_{\lambda_I}$$

$$+ \chi^+_{\lambda'_I}\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\sin\theta/2\left[(p'+p)_3F_2\left(\frac{4pp'\lambda_I\lambda'_I}{(p_0+m)(p_0'+m)} - 1\right) + G_m\sigma_3\left(\frac{2p\lambda_I}{p_0+m} + \frac{2p'\lambda_I}{p_0'+m}\right)\right]\chi_{\lambda_I}.$$

(D.9) $\langle p'_{\text{II}} \,|\, J^{em}_{3,\text{II}} \,|\, p \rangle =$

$$= \chi^{+}_{-\lambda'_{\text{II}}} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \cos\theta/2 \Big[(p'_{\text{II}}+p_{\text{II}})_3 F_2 \Big(\frac{4\, p'_{\text{II}} p_{\text{II}} \lambda'_{\text{II}} \lambda_{\text{II}}}{(p_0'+m)(p_0+m)} - 1 \Big) + \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} G_m \Big(\frac{2p\lambda_{\text{II}}}{p_0+m} + \frac{2p'\lambda'_{\text{II}}}{p_0'+m} \Big) \Big] \chi_{-\lambda_{\text{II}}}$$

$$+ \chi^{+}_{-\lambda'_{\text{II}}} \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \sin\theta/2 \Big[(p'_{\text{II}}+p_{\text{II}})_3 F_2 \Big(\frac{4\, p'_{\text{II}} p \lambda'_{\text{II}} \lambda_{\text{II}}}{(p_0'+m)(p_0+m)} - 1 \Big) + \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} G_m \Big(\frac{2p\lambda_{\text{II}}}{p_0+m} + \frac{2p'\lambda'_{\text{II}}}{p_0'+m} \Big) \Big] \chi_{-\lambda_{\text{II}}}$$

$$+ \chi^{+}_{-\lambda'_{\text{II}}} \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \sin\theta/2 \Big[(p'_{\text{II}}+p_{\text{II}})_3 F_2 \Big(\frac{4\, p'_{\text{II}} p \lambda'_{\text{II}} \lambda_{\text{II}}}{(p_0'+m)(p_0+m)} - 1 \Big) + \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} G_m \Big(\frac{2p\lambda_{\text{II}}}{p_0+m} + \frac{2p'\lambda'_{\text{II}}}{p_0'+m} \Big) \Big] \chi_{-\lambda_{\text{II}}}$$

We now enumerate the special cases.

1) No helicity flip:

The $J^{em}_{0,\text{I}} J^{em}_{0,\text{II}}$ term and the $J^{em}_{3,\text{I}} J^{em}_{3,\text{II}}$ term become:

(D.10)

$$\cos^2\theta/2 \Big[4G_m^2 + 4G_m F_2 (p'+p)_0 \Big(\frac{p'p}{(p_0+m)(p_0'+m)} - 1 \Big) + (p'+p)_0^2 F_2^2 \Big(\frac{p'p}{(p_0+m)(p_0'+m)} - 1 \Big)^2$$

$$+ (p'+p)_3^2 F_2^2 \Big(\frac{p'p}{(p_0+m)(p_0'+m)} - 1 \Big)^2 + G_m^2 \Big(\frac{p}{p_0+m} + \frac{p'}{p_0+m} \Big)^2 + 2(p'+p)_3 F_2 \,\times$$

$$\times \Big(\frac{p'p}{(p_0+m)(p_0'+m)} - 1 \Big) G_m \Big(\frac{p}{p_0+m} + \frac{p'}{p_0'+m} \Big) \Big]$$

Using the approximate expansions for infinite momentum:

(D.11) $p_3 = p_0\left(1 - \frac{m^2}{2p_0^2} + \cdots\right)$; $\frac{p'}{p_0'+m} = 1 - \frac{m}{p_0} + \frac{m^2}{2p_0^2}$

$p_3' = p_0'\sqrt{1 - \frac{p_\perp^2 + m^2}{p_0^2}}$

$$\frac{p'\,p}{(p_0'+m)(p_0+m)} \cong 1 - \frac{2m}{p_0}$$

(D.10) becomes

(D.12) $8F_1^2 \cos^2\theta/2$

We briefly summarise this by introducing the symbol for the helicity flip amplitude:

(D.13) $\langle \lambda_{II}', \lambda_I' | \phi | \lambda_{II}, \lambda_I \rangle$

Since, as momentum becomes infinite, $\cos^2\theta/2 \to 1$ we have in this limit:

(D.14) $\langle ++|\phi|++\rangle = \langle --|\phi|--\rangle = \langle +-|\phi|+-\rangle = \langle -+|\phi|-+\rangle = 8F_1^2$

2) Helicity 1 flip. The amplitude becomes:

(D.15) $\langle \pm, \pm|\phi|\pm,\mp\rangle = \{\langle \pm|J_I|\mp\rangle(\langle +|J_{II}|+\rangle \text{ or } \langle -|J|-\rangle)$

$$= \pm \sin\theta/2 \cos\theta/2 \Big[\Big(4G_m^2 + 2G_mF_2\,(p'+p)_0 \times (-2) + (p'+p)_0^2 F_2^2 \times \left(1 - \frac{p'^2p^2}{(p_0'+m)^2(p_0+m)^2}\right) + (p'+p)_3^2 F_2^2\left(1 - \frac{p'^2p^2}{(p_0+m)^2(p_0'+m)^2}\right) + G_m^2\left(\frac{p^2}{(p_0+m)^2} - \frac{p'^2}{(p_0'+m)^2}\right) + (p'+p)_3 F_2 G_m\left(\frac{-2p'}{p_0'+m}\right)\left(2 - \frac{2m'}{p_0}\right)\Big]$$

where we have used:

(D.16)
$$1-\frac{p'^2 p^2}{(p_0'+m)^2(p_0+m)^2}=\frac{4m}{p_0}\left(1-\frac{m}{p_0}\right)$$

to expand the cross terms. In the infinite momentum limit,

(D.17)
$$\sin\theta/2 \simeq \frac{p_+'}{2p_3}\ ,\ \cos\theta/2 \simeq 1\ .$$

giving:

(D.18)
$$\phi_{1\,\text{spin flip}}=\langle\pm,\pm|\phi|\pm,\mp\rangle=\mp p_+' F_1 F_2 .$$

3) Double helicity flip.

The calculation proceeds as in the above two cases, and we quote only the result here:

(D.19)
$$\langle--|\phi|++\rangle=\langle++|\phi|--\rangle=-8|p_\perp'|^2 F_2^{\,2}$$
$$\langle+-|\phi|-+\rangle=\langle-+|\phi|+-\rangle=+8|p_\perp|^2 F_2^{\,2}.$$

Appendix E Form Factor Fits

In this appendix, we discuss our fits to the form factor data.

The most attractive possibility for deriving Gaussian fits to F_1^2, F_1F_2, and F_2^2 would have been to make use of the work of Chou and Yang.(3). In their article, four Gaussian fits to $F_{1\,p}^2$ and $(G_{Mp}/\mu)^2$ are given to four significant f gures. However, this possibility proved to be unsatisfactory for the following reason.

We can derive F_2 from theCY fits as follows. Using the definition of G_E and G_M :

(E.1) $$G_E \equiv F_1 - |t|/2m\, F_2$$

$$G_M = F_1 + 2m\, F_2$$

we can take linear combinations of these equations to find:

(E.2) $$G_E + G_M = 2F_1 + (-|t|/2m + 2m)F_2$$

$$G_E - G_M = (-|t|/2m - 2m)F_2$$

Subtracting, we get:

(E.3) $$2\,G_M = 2F_1 + (-|t|/2m + 2m + |t|/2m + 2m)\,F_2 = 2F_1 + 4mF_2$$

or

(E.4) $$F_2 = \frac{G_M - F_1}{2m}$$

We now can tabulate the F_2 implied by the two CY fits for F_1^2 and $(G_M/\mu)^2$ taken simultaneously and fit the result with Gaussians. When we tabulate the results, however, we find that the two CY fits noted above give an F_2^2 with a distinct "shoulder" at about 10 $(\mathrm{Gev/c})^2$, and a less obvious shoulder in F_1F_2.

Since, except for the effects of the multiplication, our answer is similar to the form factor squared, the presence of such a "shoulder" where none appears in the experimental data would make the appearance of similar "shoulders" in our answer, which we actually derive, highly suspicious. We must thus conclude that the CY fits to F_1^2 and $(G_M/\mu)^2$ have implications which are not consistent with experiment, and we must use another procedure.

We turn to the recent experiment of Coward et al. (13), who derive G_M/μ for the proton from electron-proton scattering under the assumption:

(E.5) $$G_E = 1/\left(1 + |t|/(.71)^2\right)^2$$

which holds for low momentum transfers. The reason for referring to this experiment is that the authors tabulate G_M/μ for momentum transfers up to 25 $(\mathrm{Gev/c})^2$.

We may use (E.1) to find:

(E.6) $$G_E - G_m = -\frac{1}{2M}\left(|t| + 4M^2\right)F_2$$

or

(E.7)
$$F_2 = \frac{2m\,(G_m - G_E)}{|t| + 4m^2}$$

with

(E.8)
$$F_1 = G_m - 2mF_2$$

Thus, we may extract F_1 and F_2 from the Coward data.

In view of the above, we have decided on a compromise procedure. We accept the fit of CY for F_1^2 as given in (3), but extract F_2 from the Coward data. We do not use the F_1 implied by this data, using it only for the purpose of obtaining F_2.

Alternatively, we could ignore the CY fits entirely, and use only the Coward data. Since we are reluctant to abandon the CY fits, and since our results are not too sensitive to the fine details of these fits, we adopt the compromise above.

The fit achieved with this compromise is illustrated in Figs. 4 and 5. The errors shown there are computed from the given statistical errors in Ref.13 and include an estimated 6% systematic error given there. The fit could be improved by using an arbitrary number of Gaussians, but we consider the accuracy enough for the values of t at which we expect good agreement.

Appendix F Singlet-Triplet and Spin Transition Amplitudes

The Singlet-Triplet states for the scattering of two spin one-half particles are denoted $|S,S_3\rangle$, where S total spin, and S_3 is the projection of total spin along the axis of quantization. $|+\rangle$, $|-\rangle$ denote single particle spin states with spin along (opposite) to the direction of quantization. We give the relations between these bases for completeness:

(F.1)

$$|0,0\rangle = \frac{1}{\sqrt{2}}\left(|+\rangle|-\rangle - |-\rangle|+\rangle\right) \equiv |S\rangle$$

$$|1,-1\rangle = |-\rangle|-\rangle$$

$$|1,+1\rangle = |+\rangle|+\rangle$$

$$|1,0\rangle = \frac{1}{\sqrt{2}}\left(|+\rangle|-\rangle + |-\rangle|+\rangle\right)$$

Using these, we find for the Singlet-Triplet amplitudes defined below an expansion in terms of spin flip amplitudes $T_{f,i}$.

(F.2)

$$T_{SS} = \frac{1}{2}\left(T_{+-,+-} + T_{-+,-+} - T_{+-,-+} - T_{-+,+-}\right) = F_1^2 + k^2 F_2^2$$

$$T_{1,1} = \langle 1,1|T|1,1\rangle = T_{++,++} = F_1^2$$

$$T_{-1,-1} = \langle 1,-1|T|1,-1\rangle = T_{--,--} = F_1^2$$

$$T_{1,0} = \langle 1,1|T|1,0\rangle = \frac{1}{\sqrt{2}}\left(T_{++,+-} + T_{++,-+}\right) = -\sqrt{2}F_1F_2$$

$$T_{0,1} = \sqrt{2}\,kF_1F_2$$

$$T_{0,0} = \frac{1}{2}\left(T_{+-,+-} + T_{-+,-+} + T_{+-,-+} + T_{-+,+-}\right) = F_1^2 - k^2F_2^2$$

In the above, the +(-) signs in the spin flip amplitude are in the usual notation, and where we have substituted the values of the spin flip amplitudes from Appendix A .

The Singlet-Triplet amplitudes in (F.2) have the same phase as the corresponding amplitudes calculated in the infinite energy Born approximation, and given in Appendix A . These factors cancel, and we obtain the Singlet-Triplet amplitudes of Ref. (4). We may easily derive alternatively the information contained in (3.9), by using the formulas of Ref. (4) and (F.2) above to give:

(F.3)

$$\alpha = \frac{1}{4}\left(2T_{1,1} + T_{0,0} + T_{S,S}\right) = F_1^2$$

$$\beta = \frac{1}{4}\left(-2T_{1,-1} + T_{0,0} - T_{S,S}\right) = -k^2 F_2^2$$

$$\gamma = \frac{\sqrt{2}}{4}\left(T_{1,0} - T_{0,1}\right) = -k F_1 F_2$$

$$\binom{\delta}{\epsilon} = \frac{1}{4}\left(T_{1,1} + T_{1,-1} - T_{S,S}\right) \mp \sec\theta\left(T_{1,1} - T_{1,-1} - T_{0,0}\right) = 0$$

Finally, by substituting the expressions for the Singlet-Triplet amplitudes of (F.2) into (F.3), we derive the expressions given as (3.9).

Appendix G Treatment of Polarization

In this appendix, we relate the usual treatment of polarization (4) to the "polarization parameter" of Borghini et al. (12), and show:

1) That the usual polarization parameter P_0 will vanish in any model with real amplitudes, as in our case.

2) That in the case of an unpolarized beam incident on a polarized target, a quantity related to the polarization of the target particle is non-vanishing in our case, but is not obtainable from the polarization experiments on p-p scattering to date.

For the problem of the scattering of two spin ½ particles, we have: (4)

(G.1) $$\left.\frac{d\bar{\sigma}}{d\Omega}\right|_{P_i} = \left.\frac{d\bar{\sigma}}{d\Omega}\right|_{u}\left(1 + \vec{P}_u \cdot \vec{P}_i\right)$$

Here, $\left.\frac{d\sigma}{d\Omega}\right|_u$ is the cross section summed over final spins when the target and beam are unpolarized.

(G.2) $\vec{P}_i$ is the polarization of the target in the initial state.

$\vec{P}_u$ is the polarization of either particle in the final state when both the initial state incident and target beams are unpolarized.

(G.3) $\vec{P}_u = Tr\left(\hat{f}^{+}\vec{\sigma}\hat{f}\right) / Tr\left(\hat{f}^{+}\hat{f}\right)$

where $\hat{f}$ is the operator

(G.4) $\hat{f} = \alpha\hat{1} + \beta\,\vec{\sigma}^1\cdot\hat{n}\;\vec{\sigma}^2\cdot\hat{n} + i\gamma\,(\vec{\sigma}^1 + \vec{\sigma}^2)\cdot\hat{n} + \delta\,\vec{\sigma}^1\cdot\hat{m}\;\vec{\sigma}^2\cdot\hat{m} + \epsilon\,\vec{\sigma}^1\cdot\hat{l}\;\vec{\sigma}^2\cdot\hat{m}.$

We will show below that the quantity P_0 measured by the authors of Ref. (12) is equal to the magnitude of the quantity P_u defined above under certain conditions, which, however, are not met in the experiment of Ref.(12).

We define the polarization of the original target, now in the final state, as:

(G.5) $$\vec{P}(\vec{P}_i) = \frac{\vec{P}_u + \vec{\vec{P}}\cdot\vec{P}_i}{1 + \vec{P}_u\cdot\vec{P}_i}$$

$\vec{\vec{P}}$ is a dyadic, so that $\vec{\vec{P}}\cdot\vec{P}_i$ is a vector.

(G.6) $$\vec{\vec{P}} = \frac{Tr\left(\vec{\sigma}\hat{f}\vec{\sigma}\hat{f}^{+}\right)}{Tr\left(\hat{f}\hat{f}^{+}\right)}$$

The experiment of Ref. (12) is performed using an unpolarized beam and a partially polarized target. Defining the polarization vector of the target as P_T, only particles scattered in the plane perpendicular to P_T are counted. Then the direction of polarization is reversed (but with a different magnitude, see below), and particles are again counted in the plane perpendicular to P_T. At each scattering angle in this plane, the following quantity is measured:

(G.7)

$$P_o = \frac{2\epsilon}{\left(|\vec{P}_{T(up)}| - |\vec{P}_{T(down)}|\right) - \left(|\vec{P}_{T(up)}| - |P_{T(down)}|\right)\epsilon}$$

where $N_{(up)}$ is the number of events when the target polarization is up, and correspondingly for $N_{(down)}$, in:

(G.8)

$$\epsilon = \frac{N_{(up)} - N_{(down)}}{N_{(up)} + N_{(down)}}$$

$P_{T(up)}$ is the magnitude of the target polarization, when it is polarized in the up direction and correspondingly for down.

By using (G.1), we see that:

(G.9)

$$N_{(up)} = \frac{d\sigma}{d\Omega}\left(1 + \vec{P}_u \cdot \vec{P}_i\right)$$

for a certain angle. Similarly for $N_{(down)}$. We may then substitute for ϵ, and find for P_0 :

(G.10)

$$P_o = \frac{2\vec{P}_o \cdot \left(\vec{P}_{T\,up} - \vec{P}_{T\,down}\right)}{2\left(|\vec{P}_{T\,up}| + |\vec{P}_{T\,down}|\right) + 2\vec{P}_o \cdot \left(\vec{P}_{T\,up}|\vec{P}_{T\,down}| + \vec{P}_{T\,down}|\vec{P}_{T\,up}|\right)}$$

If the polarizations of the target in the initial state were equal in magnitude and opposite in direction before and after reversal, we would have:

(G.11) $$\vec{P}_{T\,up} = -\vec{P}_{T\,down}$$
$$|\vec{P}_{T\,up}| = |\vec{P}_{T\,down}|$$

In this case, the second term in the denominator of (G.10) would equal zero, and we would have:

(G.12) $$\vec{P}_{T\,up} - \vec{P}_{T\,down} = 2\,\vec{P}_{T\,up}$$

giving:

(G.13) $$P_o = \frac{2\,\vec{P}_o \cdot (2\,\vec{P}_{T\,up})}{2 \cdot 2\,|\vec{P}_{T\,up}|} = \vec{P}_o \cdot \frac{\vec{P}_{T\,up}}{|\vec{P}_{T\,up}|}$$

Since in this experiment, the scattering plane is always perpendicular to P_T, and since any non-zero polarization is perpendicular to the scattering plane, it follows that P_0 is parallel or antiparallel to $P_{T(up)}$. Then we would have:

(G.14) $$P_o = |\vec{P}_o|$$

However, $|\vec{P}_{T\,up}| \neq |\vec{P}_{T\,down}|$ in this experiment. Although the authors take note of this fact, they do not give the numerical differences in the magnitudes of $P_{T(up)}$ and $P_{T(down)}$.

The importance of the above discussion for our work is that the experimentally measured parameter P_0

as defined above in (G.7), and which differs from zero by about 10 %, may not be the quantity predicted to be zero in our model, P_u in the usual terminology. Furthermore, the experiment above was done at 10.0 GeV/c max., while our prediction is for infinite energy.

We now explicitly evaluate $\vec{P}_u$ in our model. Omitting the details, we find:

(G.15)
$$\vec{P}_u = -8\left(\mathrm{Im}\,(\alpha^* + \beta^*)\,\gamma\right) = 0.$$

Since α, β, and γ are real in our calculation, (Appendix A), this quantity vanishes identically.[1]

We next evaluate the quantity $\vec{P}(\vec{P}_i)$, the polarization, in the final state, of the target initially polarized with polarization $\vec{P}_i$ in the initial state. We again quote only the result:

(G.16)
$$\vec{P}(\vec{P}_i) = \frac{|\alpha|^2 + |\beta|^2 + 2|\gamma|^2 - |\delta|^2 - |e|^2}{|\alpha|^2 + |\beta|^2 + 2|\gamma|^2 + |\delta|^2 + |e|^2}$$

which we get by explicitly taking the traces implied in (G.6) and substituting in (G.5). We are not aware of any experimental measurement of this quantity.

1) It would, of course, vanish if our amplitudes were pure imaginary, which is the usual phase convention.

Appendix H Angular Integration and Basis Vectors in b Space.

In this appendix, we give an illustration of the transformation of the Σ_{ij}'s in the expansion of $\mathcal{X}$, under the Fourier transform from k space into b space.

We choose the direction of the velocity vector of the incident particle in the Lab frame as the positive z direction. We first consider the term:

(H.1) $$k_i k_j \sigma^1_l \sigma^2_m$$

where the indices i,j,l,m,=1,2, or 3.

Using Eq.(4.2), this becomes:

(H.2) $$\nu(b^2)\,\sigma^1_l \sigma^2_m \delta_{ij} + \pi(b^2)\,\sigma^1_l \sigma^2_m b_i b_j$$

Next, we consider:

(H.3) $$\hat{n} = \frac{\hat{k}\times\hat{k}_f}{\sin\theta} \longrightarrow \hat{k}_z \times \hat{k}_\perp$$

in the infinite momentum limit(see Appendix B) It is understood that the infinite momentum form of $\hat{n}$ is used below. Since k is parallel to the z component of k_f (denoted $k_\perp$), the cross product of k with k_{f_z} vanishes, and we have:

(H.4) $$\vec{\sigma}^1\cdot\hat{n}\;\vec{\sigma}^2\cdot\hat{n} = \vec{\sigma}^1\cdot(\hat{k}\times\hat{k}_\perp)\;\vec{\sigma}^2\cdot(\hat{k}\times\hat{k}_\perp)$$

We may use the properties of the mixed product to rewrite (H.4) as:

(H.5) $$\vec{\sigma}^1\cdot\hat{n}\;\vec{\sigma}^2\cdot\hat{n} = (\hat{k}_\perp\times\vec{\sigma}^{(1)})_z\;(\hat{k}_\perp\times\vec{\sigma}^{(2)})_z = \left(\hat{k}_1\sigma^{(1)}_2 - \hat{k}_2\sigma^{(1)}_1\right)\left(k_1\sigma^{(2)}_2 - k_2\sigma^{(2)}_1\right)$$

or:

(H.6)

$$\sigma^1 \cdot n \, \sigma^2 \cdot n = \hat{k}_1 \hat{k}_1 \vec{\sigma}^{(1)}_2 \vec{\sigma}^2_2 + \hat{k}_2 \hat{k}_2 \sigma^{(1)}_1 \sigma^{(2)}_1 - \hat{k}_1 \hat{k}_2 \sigma^{(2)}_2 \sigma^{(1)}_1 - \hat{k}_1 \hat{k}_2 \sigma^{(1)}_2 \sigma^{(2)}_1$$

where $\hat{k}_{1(2)}$ denote the 1(2) components of the unit vector $\hat{k}_\perp$. Now each of the terms in (H.6) transforms according to the last line of (4.2). We have explicitly:

(H.7) $$\frac{1}{2\pi}\int \beta \; k_i k_j e^{-i\vec{k}\cdot\vec{b}} d^2k = \nu \delta_{ij} + \pi \hat{b}_i \hat{b}_j$$

where $\hat{b}_{1(2)}$ denote the 1(2) components of $\hat{b}$ in the basis in b space to be derived below.

Hence, (H.7) explicitly becomes:

(H.8)
$$\nu \delta_{11} \vec{\sigma}^1_2 \vec{\sigma}^2_2 + \pi \hat{b}_1 \hat{b}_1 \vec{\sigma}^1_2 \vec{\sigma}^2_2$$
$$+ \nu \delta_{22} \sigma^1_1 \sigma^2_1 + \pi \hat{b}_2 \hat{b}_2 \vec{\sigma}^1_1 \vec{\sigma}^2_1$$
$$- \pi \hat{b}_1 \hat{b}_2 \vec{\sigma}^{(2)}_2 \vec{\sigma}^{(1)}_1$$
$$- \pi \hat{b}_1 \hat{b}_2 \vec{\sigma}^{(1)}_1 \vec{\sigma}^{(2)}_2$$

When these terms are combined, we have:

(H.9) $$\nu (\vec{\sigma}^1 \cdot \vec{\sigma}^2) + \pi (\hat{b}_1 \vec{\sigma}^{(1)}_2 - \hat{b}_2 \vec{\sigma}^{(1)}_1)(\hat{b}_1 \sigma^{(2)}_2 - b_2 \sigma^{(2)}_1)$$

Re-arranging the mixed product $\vec{k}_3\cdot(\hat{b}\times\vec{\sigma})$ as before, we have:

(H.10) $$\nu(\vec{\sigma}^1\cdot\vec{\sigma}^2)+\pi\,\vec{\sigma}^1\cdot(\hat{b}_3\times\hat{b})\,\vec{\sigma}^{(2)}\cdot(\hat{b}_3\times\hat{b}).$$

Hence, we are led to define:

(H.11) $$\hat{p}\equiv\hat{b}_3\times\hat{b}$$

which gives us:

(H.12) $$\frac{1}{2\pi}\int\beta(k^2)\,\vec{\sigma}^1\cdot\hat{n}\,\vec{\sigma}^2\cdot\hat{n}=\nu(\vec{\sigma}^1\cdot\vec{\sigma}^2)+\pi\,\vec{\sigma}^1\cdot\hat{p}\,\vec{\sigma}^2\cdot\hat{p}$$

This term is of the same form as $\hat{f}$ in our original amplitude in k space. $\hat{p}$ is a vector in the transverse plane. To have a complete orthonormal basis in b space, we must choose a third vector perpendicular to both p and k_z(k_z is left unaffected by our Fourier transformation in the transverse plane). The natural choice is $\hat{q}\equiv-\hat{b}$, This is because the vector $\hat{m}$ which appears in our representation of $\hat{f}$, equals $-\hat{k}_\perp$, which thus goes to $-\hat{b}$, when, as seen above, $\hat{k}\rightarrow\hat{b}$.

Then, defining $\hat{k}_z\equiv\hat{r}$, we have:

(H.13)

We now set up our convention: 1=$\hat{p}$ direction, 2 = $\hat{q}$ direction, and 3 = $\hat{r}$ direction. This is the convention used in the definitions of the Σ_{ij}'s, (4.20).

When, after doing the sigma algebra in b space, we integrate over angles in b space, the situation is as illustrated above, with the only difference being that the roles of b and k are interchanged. Thus; in this latter case we find:

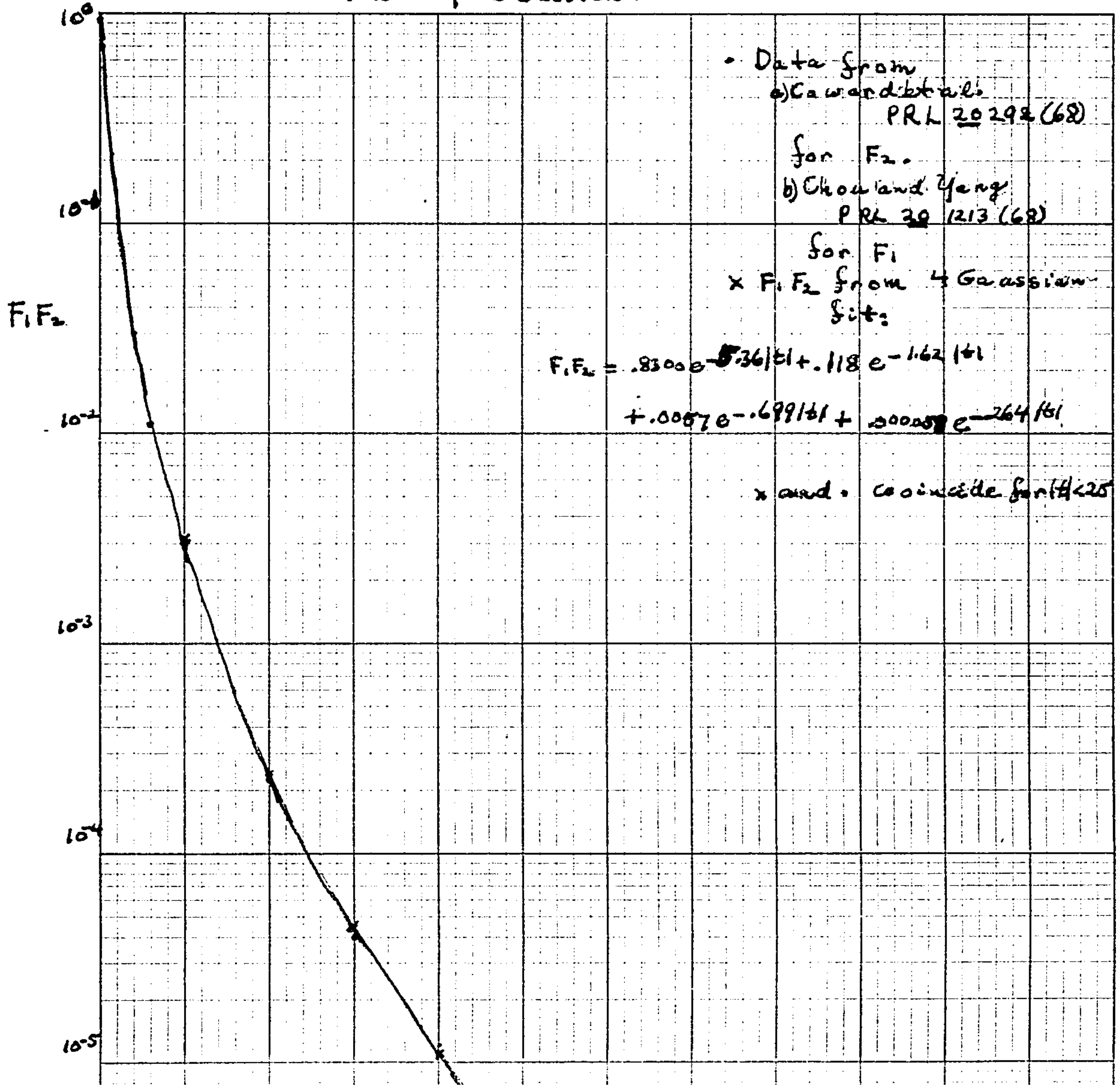
(B 14)

$$\vec{\sigma}^{1}\cdot\hat{p}\ \vec{\sigma}^{2}\cdot\hat{p} \rightarrow \vec{\sigma}^{1}\cdot\hat{n}\ \vec{\sigma}^{2}\cdot\hat{n}$$

$$\vec{\sigma}^{1}\cdot\hat{q}\ \vec{\sigma}^{2}\cdot\hat{q} \rightarrow \vec{\sigma}^{1}\cdot\hat{m}\ \vec{\sigma}^{2}\cdot\hat{m}$$

$$\vec{\sigma}^{1}\cdot\hat{r}\ \vec{\sigma}^{2}\cdot\hat{r} \rightarrow \vec{\sigma}^{1}\cdot\hat{l}\ \vec{\sigma}^{2}\cdot\hat{l}$$

Fig. 4
F_1F_2 4 Gaussian Fit.



$F_1F_2 = .8300 e^{-5.36|t|} + .118 e^{-1.62|t|} + .0087 e^{-.699|t|} + .00005 e^{-.264|t|}$

72

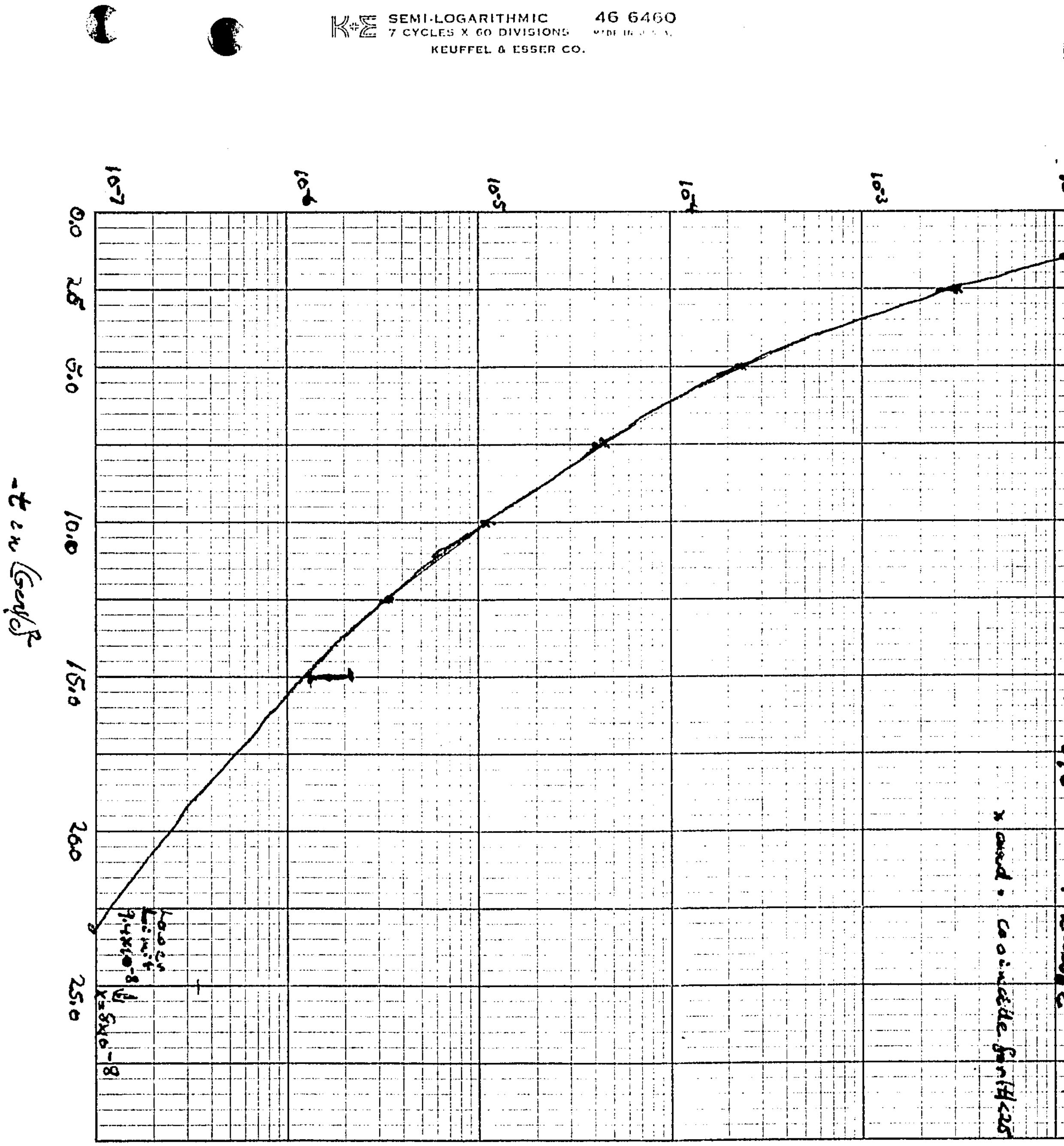
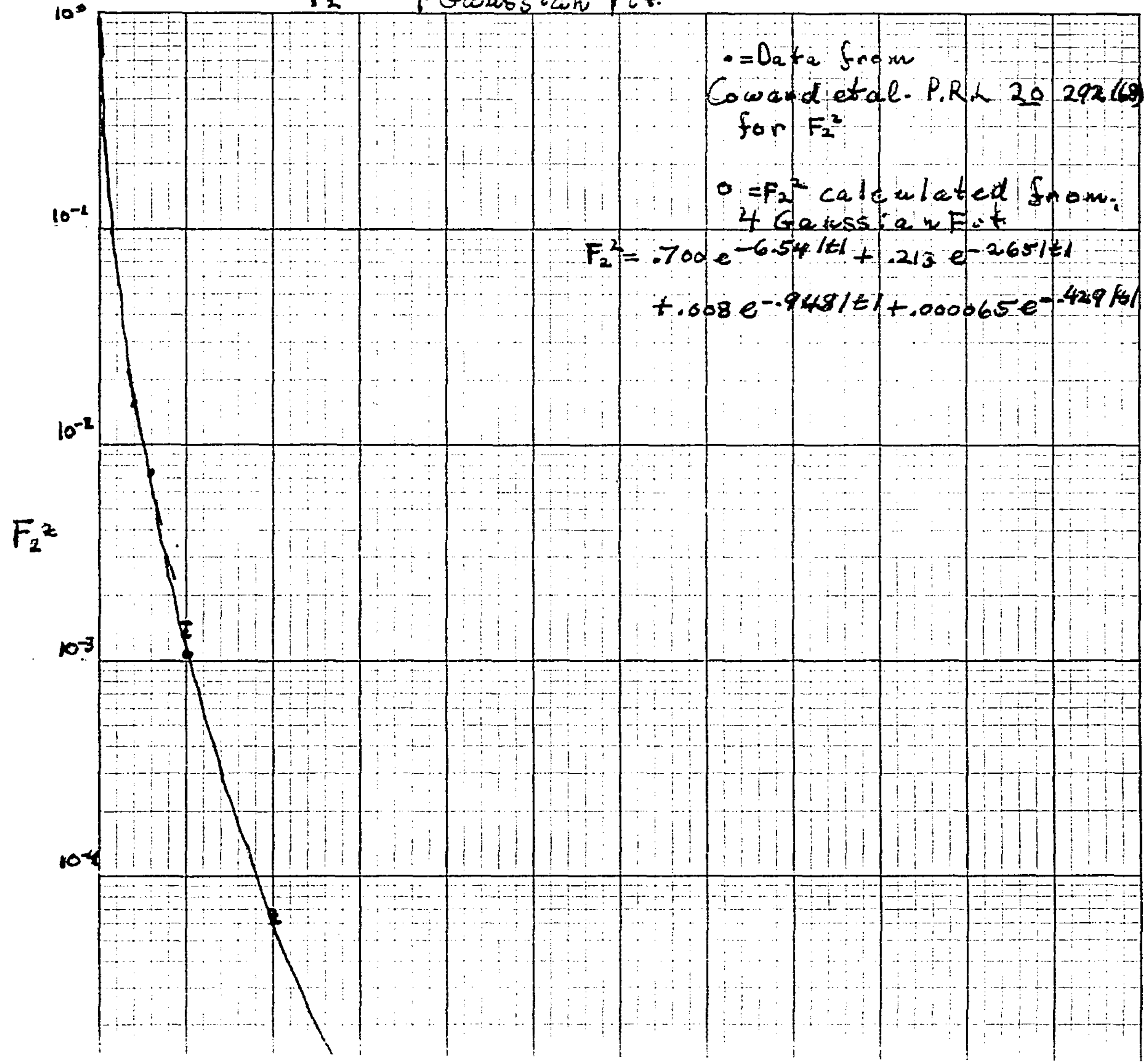

K+E SEMI-LOGARITHMIC 46 6460
7 CYCLES X 60 DIVISIONS
KEUFFEL & ESSER CO.
10-7
10-6
10-5
10-4
10-3
0.0
2.5
5.0
10.0
15.0
20.0
25.0
-t in (GeV/c)²
x and • coincide for t<25

Fig. 5


F_2^2 4 Gaussian Fit.
• = Data from Coward et al. P.R.L. 20 292 (68) for F_2^2
o = F_2^2 calculated from 4 Gaussian Fit
$F_2^2 = .700\,e^{-6.54|t|} + .213\,e^{-2.65|t|} + .608\,e^{-.948|t|} + .000065\,e^{-.429|t|}$
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
F_2^2

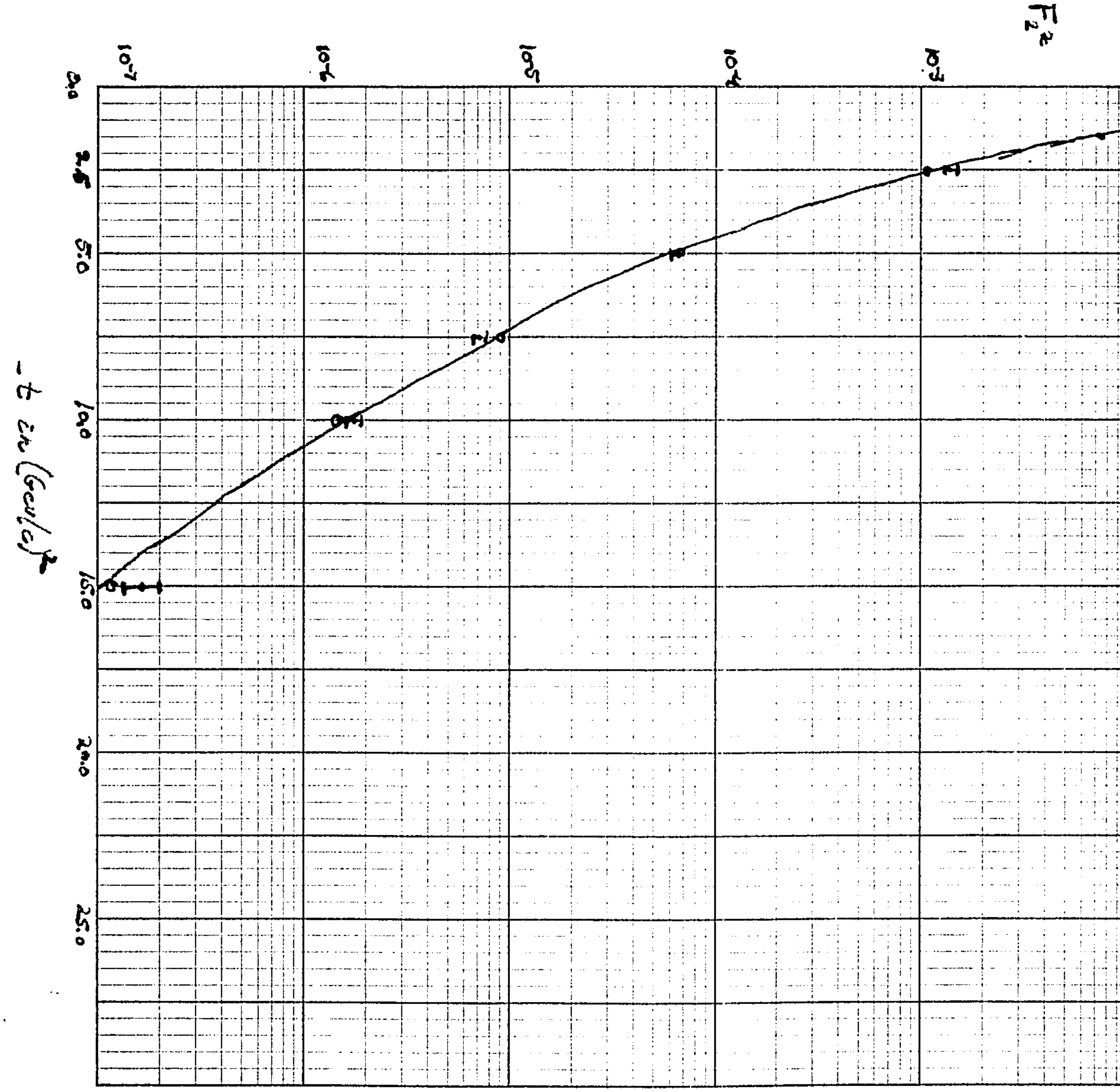
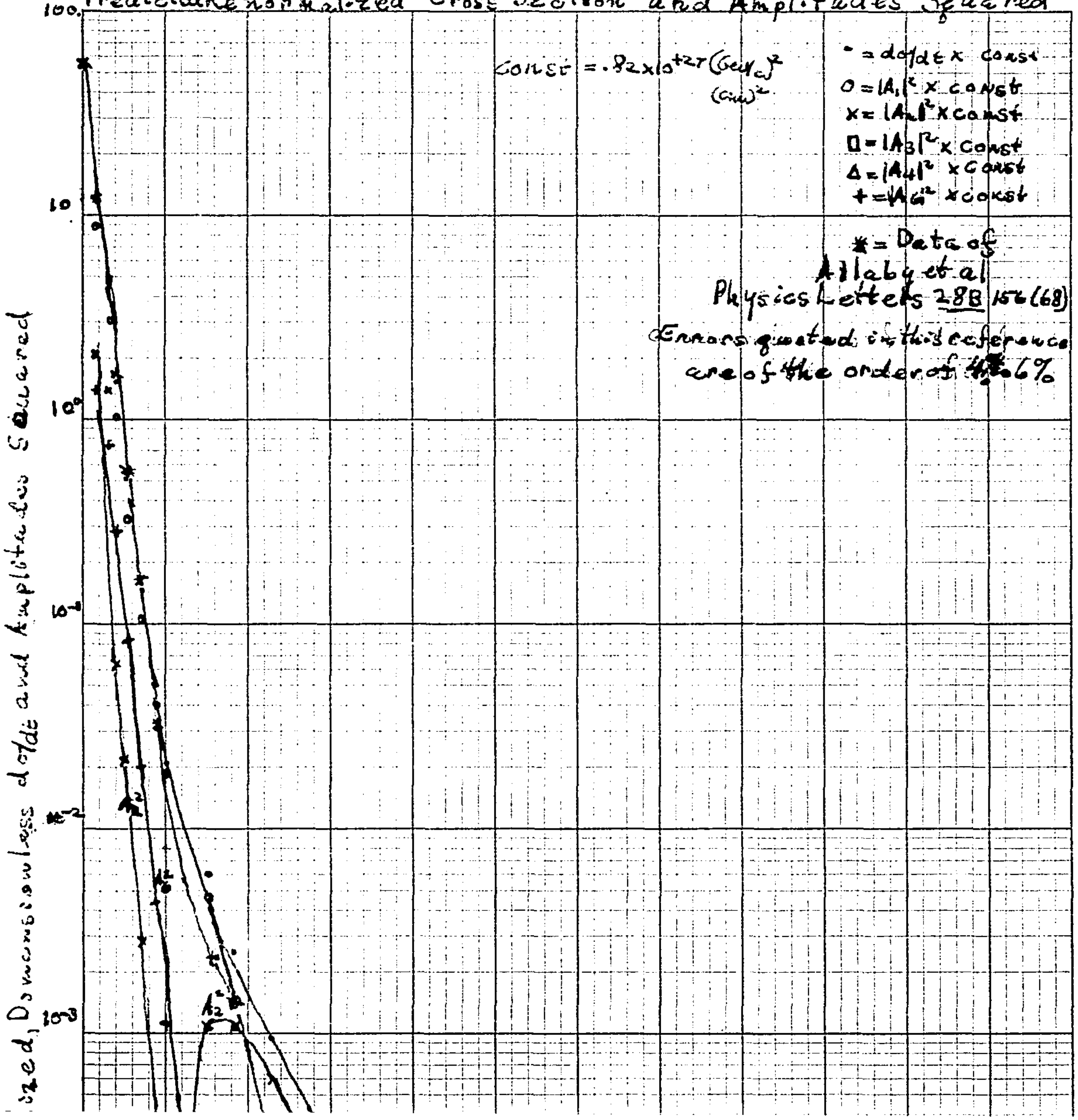
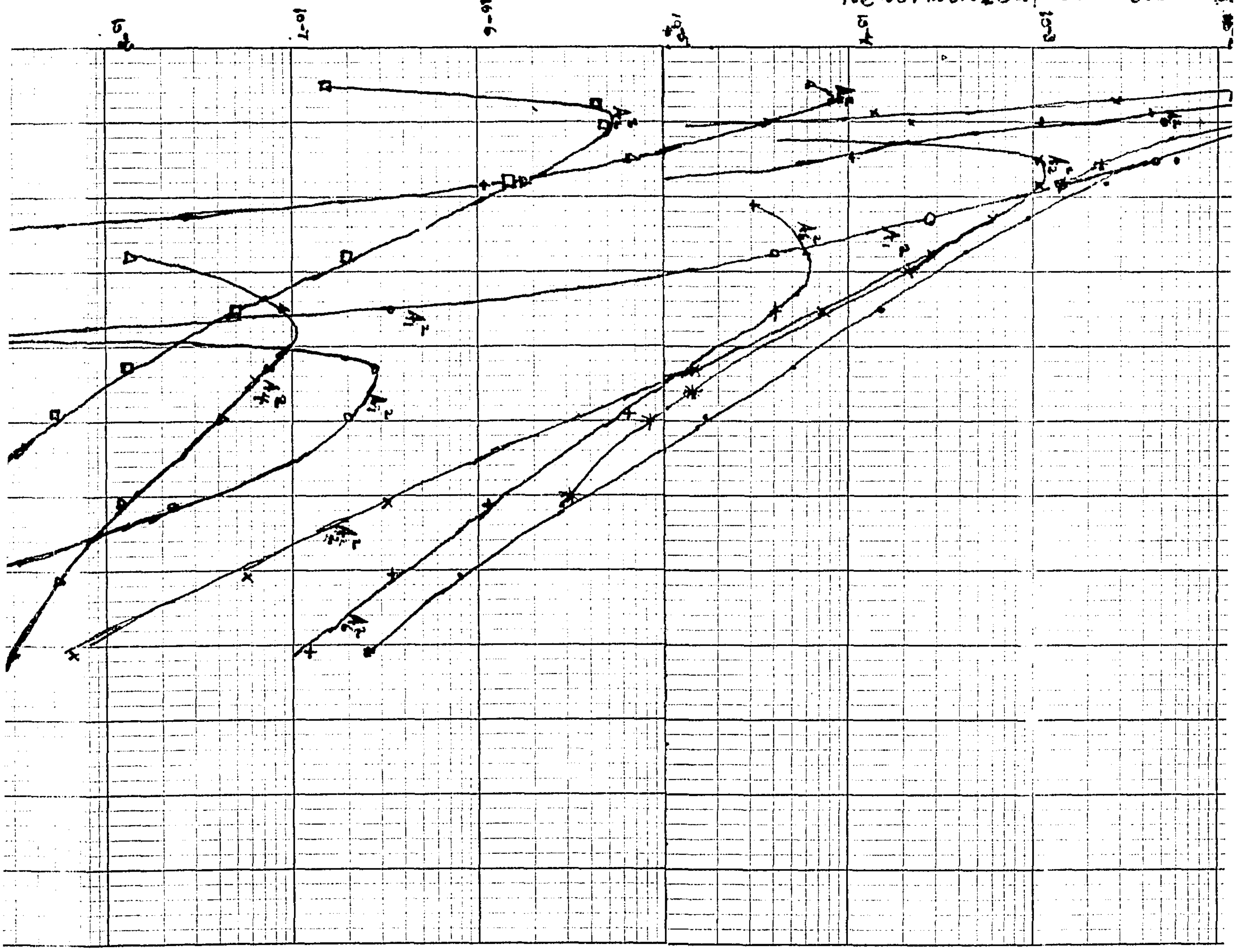

F_2^2
$-t$ in $(GeV/c)^2$
10^{-7}
10^{-6}
10^{-5}
10^{-4}
10^{-3}
0.0
2.5
5.0
10.0
15.0
20.0
25.0

Fig 6

Predicted Renormalized Cross Section and Amplitudes Squared



Renormalized, Dimensionless



K+E SEMI-LOGARITHMIC 46 6460
7 CYCLES X 60 DIVISIONS MADE IN U.S.A.
KEUFFEL & ESSER CO.

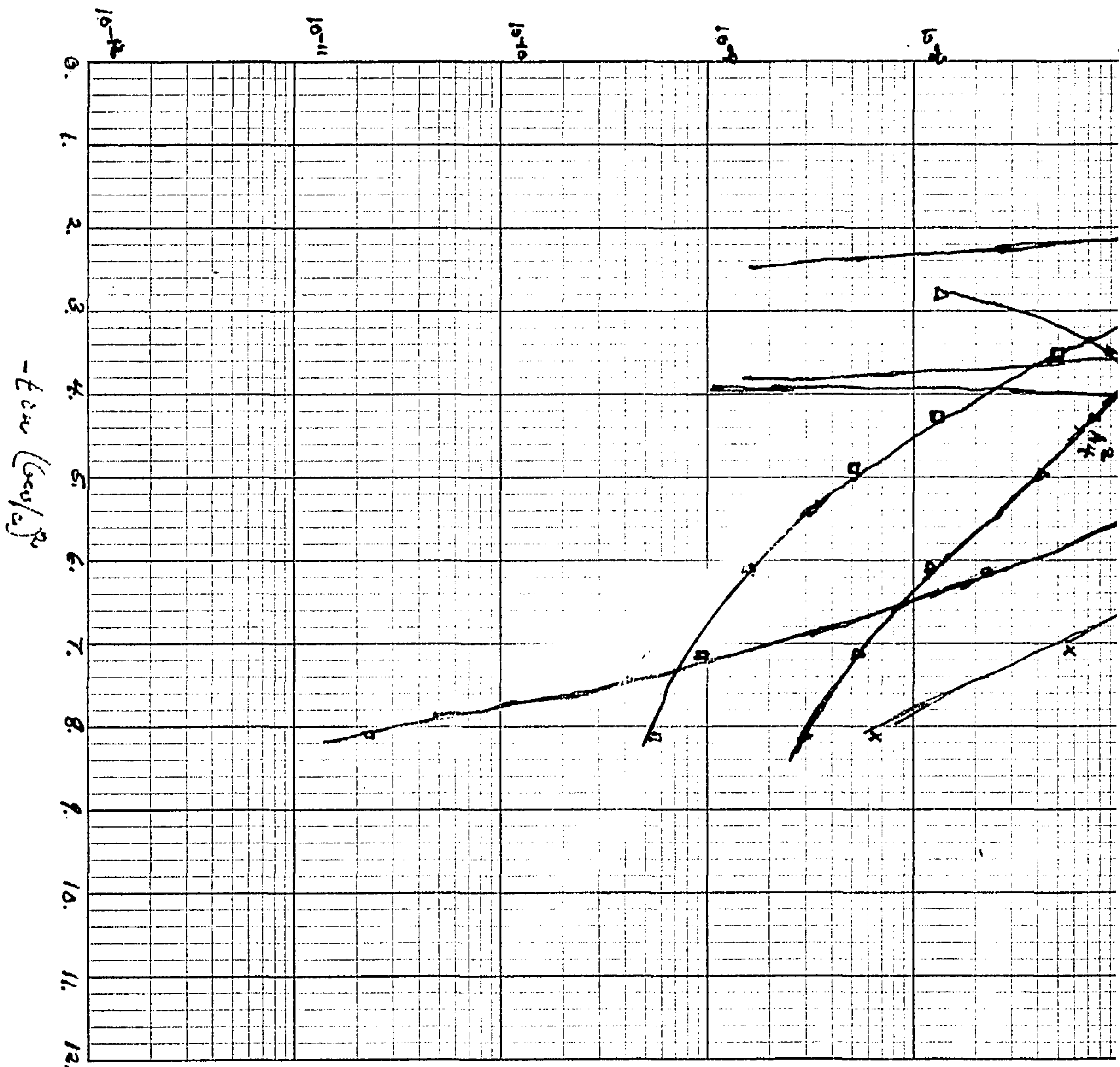
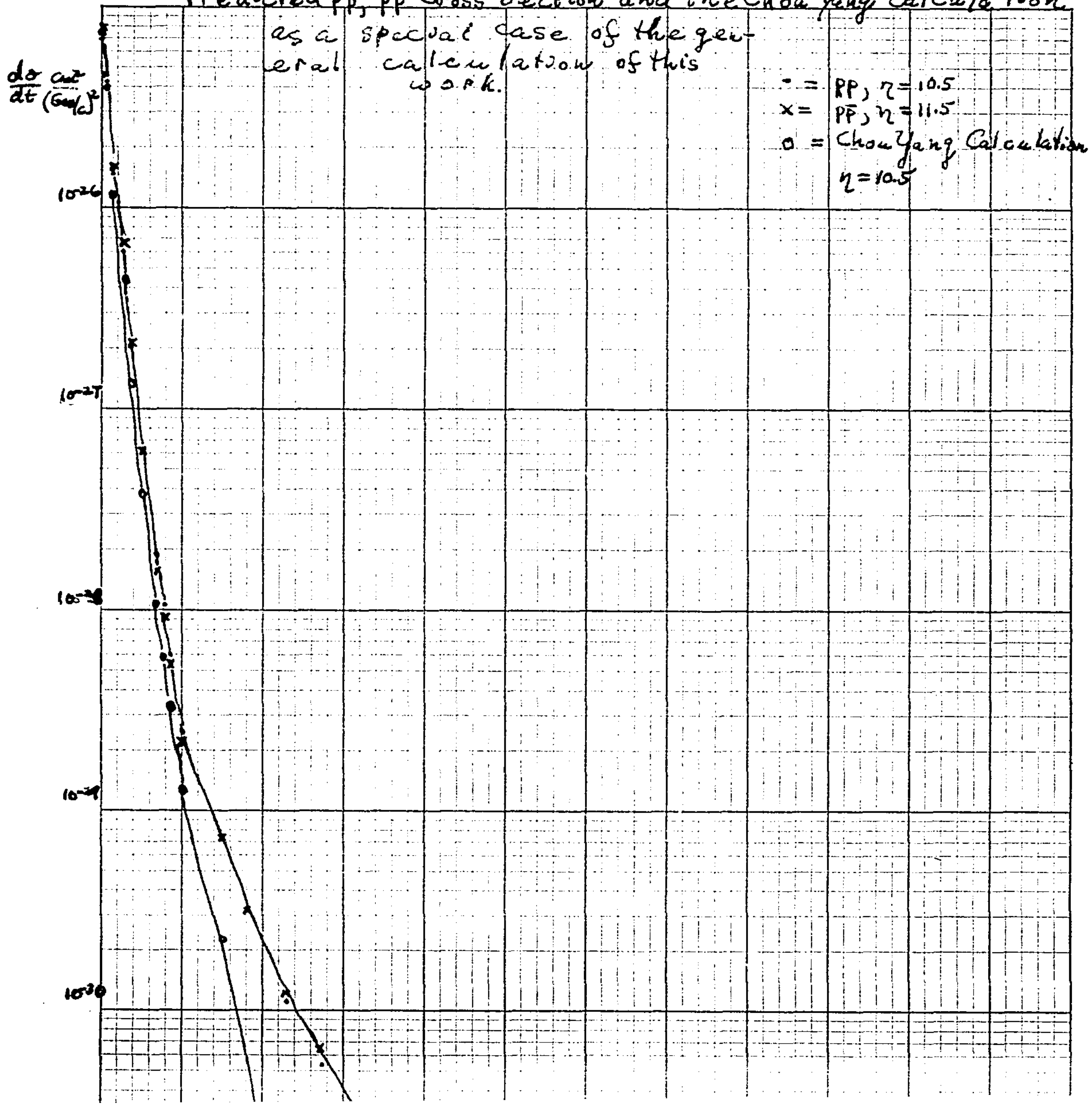
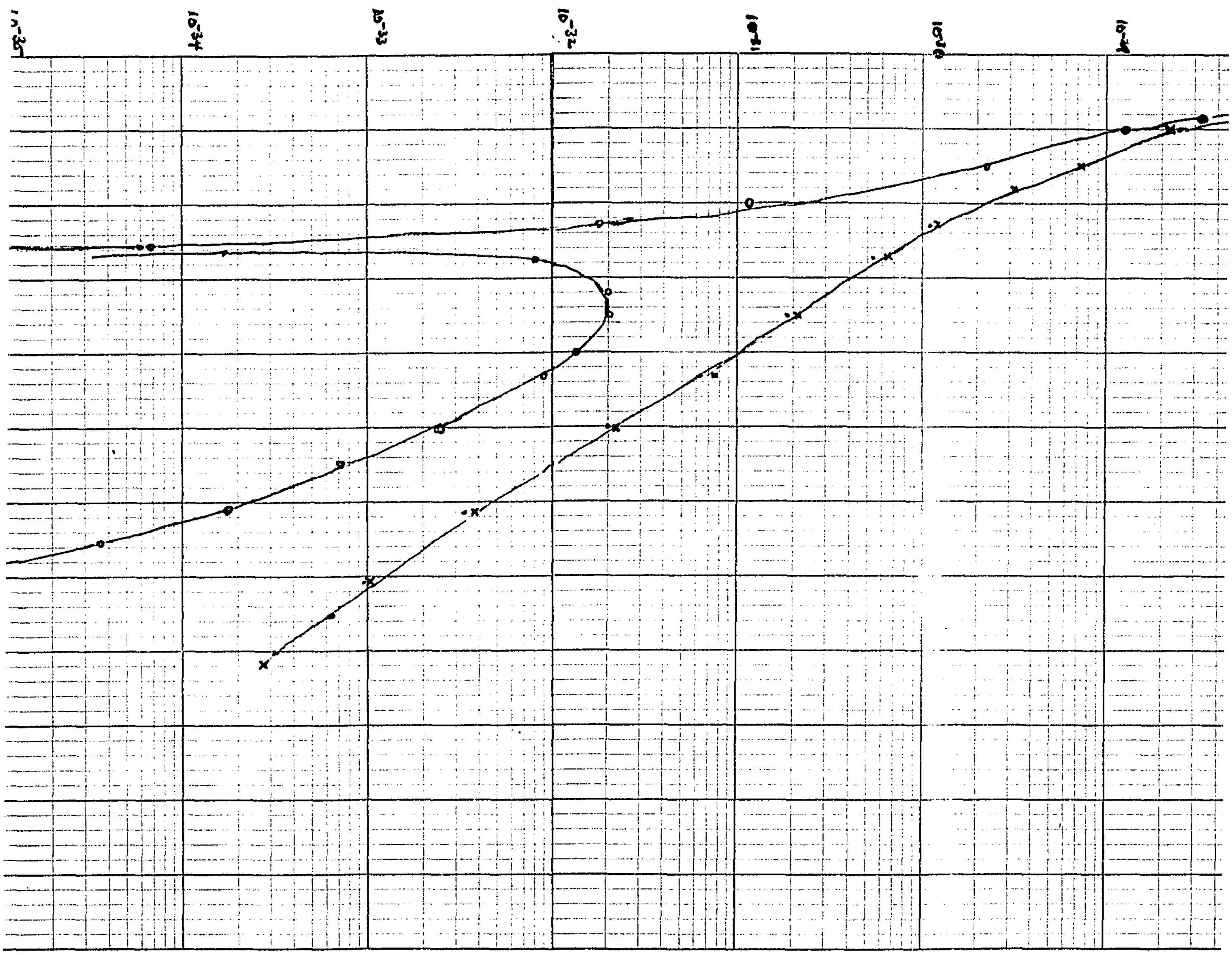


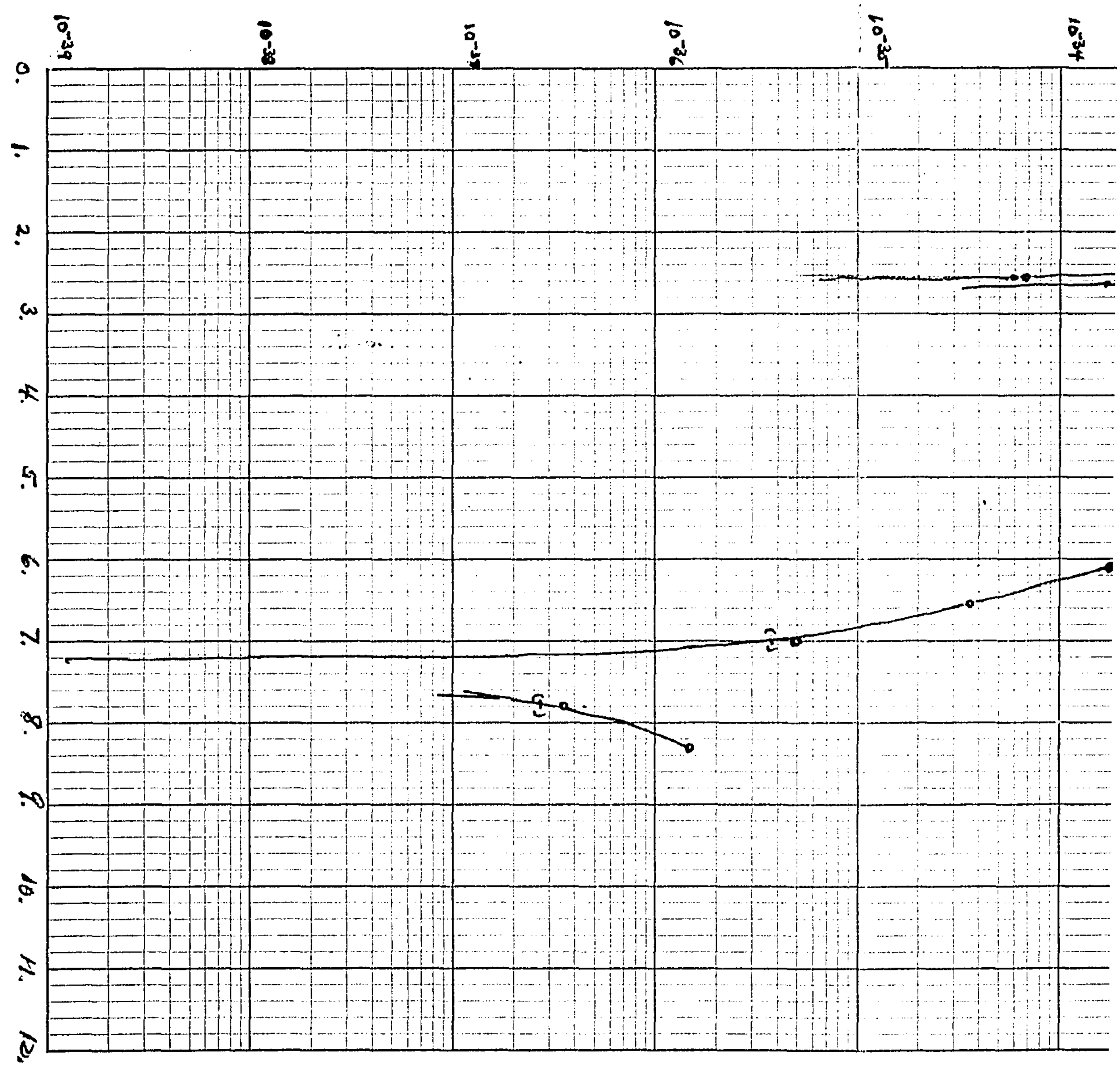
Fig 7

Predicted pp, p$\bar{p}$ Cross Section and the Chou-Yang Calculatoon as a special case of the general calculation of this work.



K+E SEMI-LOGARITHMIC 46 6460
7 CYCLES X 60 DIVISIONS MADE IN U.S.A.
KEUFFEL & ESSER CO.




10-39
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10-36
10-35
10-34
0.
1.
2.
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12.

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