

INFORMATION TO USERS

The most advanced technology has been used to photograph and reproduce this manuscript from the microfilm master. UMI films the text directly from the original or copy submitted. Thus, some thesis and dissertation copies are in typewriter face, while others may be from any type of computer printer.

The quality of this reproduction is dependent upon the quality of the copy submitted. Broken or indistinct print, colored or poor quality illustrations and photographs, print bleedthrough, substandard margins, and improper alignment can adversely affect reproduction.

In the unlikely event that the author did not send UMI a complete manuscript and there are missing pages, these will be noted. Also, if unauthorized copyright material had to be removed, a note will indicate the deletion.

Oversize materials (e.g., maps, drawings, charts) are reproduced by sectioning the original, beginning at the upper left-hand corner and continuing from left to right in equal sections with small overlaps. Each original is also photographed in one exposure and is included in reduced form at the back of the book.

Photographs included in the original manuscript have been reproduced xerographically in this copy. Higher quality 6" x 9" black and white photographic prints are available for any photographs or illustrations appearing in this copy for an additional charge. Contact UMI directly to order.

U·M·I

University Microfilms International
A Bell & Howell Information Company
300 North Zeeb Road, Ann Arbor, MI 48106-1346 USA
313 761-4700 800 521-0600



Order Number 9108085

The Monte Carlo approach to the valuation of mortgage-backed securities

Cai, Fuchun, Ph.D.

City University of New York, 1990

Copyright ©1990 by Cai, Fuchun. All rights reserved.

U·M·I
300 N. Zeeb Rd.
Ann Arbor, MI 48106

A

**The Monte Carlo Approach
To the Valuation of Mortgage-backed Securities**

by

Fuchun Cai

A dissertation submitted to the Graduate
Faculty in Economics in partial fulfillment
of the requirements for the degree of Doctor
of Philosophy, The City University of New York

1990

© 1990

Francis Fuchun Cai

All Rights Reserved

This manuscript has been read and accepted for the Graduate Faculty in Economics in satisfaction of the dissertation requirement for the degree of Doctor of Philosophy.

September 17, 1990

Date

Ronald W. Anderson

Chair of Examining Committee

September 17, 1990

Date

Michael Grossman

Executive Officer

Prof. Ronald W. Anderson

Prof. Salih N. Neftci

Prof. Michael Grossman

Supervisory Committee

The City University of New York

Abstract**The Monte Carlo Approach
To the Valuation of Mortgage-backed Securities**

by

Fuchun Cai

Advisor: Professor Ronald W. Anderson

The valuation model of mortgage-backed securities is one of the important topics in the finance theory and its application. The purpose of this paper is to provide a Monte Carlo approach to valuing the GNMA mortgage-backed securities. Two interest rate movement models, modified binomial interest rate model and full information interest rate model, have been introduced into the GNMA valuation framework. We found that the monte carlo simulation approach provides a very good alternative to valuing GNMA mortgage-backed securities in terms of accuracy of the simulation. Furthermore, the Monte Carlo approach is quite powerful in a sense that stochastic state variables other than diffusion process as well as various stochastic prepayment functions can be easily introduced into the Monte Carlo simulation.

CONTENTS

CHAPTER 1

PART I	INTRODUCTION	1
§ I-1	Background of GNMA Mortgage-backed Pass-through Securities	2
§ I-2	Literature on Prepayment Experience	4
§ I-3	Literature on Valuation Model of Mortgage-backed Securities Static Economy Approach; Average Life Method; Curley and Guttentag Method; Dynamic Economy Approach; Dunn and McConnell's Model;	5
PART II	METHODOLOGY	14
§ II-1	Monte Carlo Approach	14
§ II-2	The Binomial Model for Pricing the GNMA Mortgage-backed Pass-through Securities	17
PART III	AN APPLICATION OF MONTE CARLO APPROACH	20
§ III-1	Specifications	20
§ III-2	The Shape of Term Structure	23
§ III-3	Term to Maturity	25
PART IV	CONCLUSION FOR CHAPTER 1	28

CHAPTER 2

PART I	INTRODUCTION	38
PART II	ARBITRAGE-FREE RATE MOVEMENTS MODEL	39
§-II-1	Assumption	39
§-II-2	AR Model	40
§-II-3	Modification and Improvement of Ho and Lee's Model	44

PART III	APPLICATION OF AR MODEL IN VALUING FIXED-INCOME SECURITIES	47
§-III-1	The Risk-neutral Pricing Formula	47
§-III-2	Implementation of AR Model	49
§-III-3	Simulation and Analysis of the Results	51
CHAPTER 3		
PART I	INTRODUCTION	67
PART II	DATA DESCRIPTION	68
§-II-1	FAMA-BLISS Discount Bond Price Data and Treasury Bill Term Structure Files	68
§-II-2	GNMA DATA	69
PART III	CORRELATION ANALYSIS	69
§-III-1	Single Factor Interest Rate Model Results	70
§-III-2	Arbitrage Free Rate Model Results	70
§-III-3	Correlation Analysis	70
PART IV	CONCLUSION	72
BIBLIOGRAPHY		97

CHAPTER 1

PART I INTRODUCTION

The valuation model of mortgage-backed securities is one of the important topics in the finance theory and its application. The purpose of this paper is to provide a Monte Carlo approach to valuing the GNMA mortgage-backed securities by incorporating a modified stochastic binomial model. Dunn and McConnell (1981b) gave a valuation model for GNMA mortgage-backed securities. They derived a second order partial differential equation which the value of GNMA mortgage-backed securities must satisfy, and they solved this partial differential equation by finite difference method. To solve this second order partial differential equation by finite difference method, Dunn and McConnell imposed the optimal call policy which is not appropriate (the reason will be explained later in this paper) to the GNMA mortgage-backed security. By Monte Carlo simulation method, we don't have to impose this unnecessary optimal call policy to solve the partial differential equation. Furthermore, the Monte Carlo approach is quite powerful in a sense that stochastic state variables other than diffusion process as well as various stochastic prepayment functions can be easily introduced into the Monte Carlo simulation.

§ I-1 Background of GNMA Mortgage-backed Pass-through Securities¹

Government National Mortgage Association (GNMA) mortgage-backed securities are issued by the mortgage bankers, which are approved by the Federal Housing Administration (FHA). According to the regulations of GNMA, the mortgage bankers must generate a pool of mortgages with the same coupon interest rate and with the same original term to maturity (Current terms to maturity may differ for a pool within certain bounds) Additionally, the mortgages should be either insured by FHA or guaranteed by the Veterans Administration (VA).

Each month, the mortgage issuers collect the agreed upon interest payments and partial principal⁽²⁾ from the mortgage borrowers (known as mortgagors) and after taking 50 basis points as service fee (to the mortgage issuers) and insurance fee (to GNMA) of the securities, "pass through" the rest of the payments to the holders of the mortgage-backed securities. When the mortgagors default on the scheduled monthly payments, GNMA is responsible for the scheduled payments to the security holders. Generally, GNMA mortgage-backed pass-through securities are regarded as risk free

¹ Interested readers should refer to Dunn and McConnell (1981a,1981b), and Lucy and Anderson (1987) for the discussion of this part

² GNMA mortgage-backed securities are fully amortizing. So the monthly payment on the mortgage includes interest on the outstanding principal balance and a partial repayment of principal.

fixed-income securities.

However, the valuation of GNMA mortgage-backed pass-through securities is more complicated than that of ordinary risk-free fixed-income securities because of the prepayment characteristic of the GNMA securities. Mortgagors can prepay (call) the loans in part, or in full at any time prior to the maturity date without being charged for the prepayment decision. The GNMA mortgage-backed pass-through securities are best viewed by the investors as a portfolio consisting of a fully amortizing default risk free bond plus an implicit call options. As mentioned in Lucy and Anderson's paper (1987), many of the mortgagors don't exercise their call option even when they can refinance their loan with a similar loan that has a lower coupon interest rate. On the other hand, many of the mortgagors exercise their call option when the market interest is above the contract rate on the loans. Thus, the prepayment decision cannot be completely explained from the economic point of view, because not only economic factors but also noneconomic factors will influence the prepayment decision. Obviously, various prepayment decisions will greatly affect the valuation of GNMA mortgage-backed securities. This is why prepayment functions are always the primary interest in the valuation of mortgage-backed securities. Lucy and Anderson (1987) summed up three categories of factors that will determine the prepayment actions of mortgagors. These categories are: (1) financial incentives to refinance a

mortgage loan, (2) the history of the prepayment process, and (3) demographic or geographic changes.

§ I-2 Literature on Prepayment Experience

The early work on prepayment function of GNMA mortgage-backed securities was done by Curley and Guttentag (1974). They developed a regression model in which the prepayment rate (termination rate, defined as the proportion of mortgages in a mortgage pool prepaid within certain period) is explained by: (1) the existing pool rate relative to the rate on the new mortgages, (2) market discounts on new mortgages, and (3) the relationship between the original maturity to the age of the pool. Basically, they assumed that the prepayment decision is determined by economic considerations only.

Dunn and McConnell (1981) extended Curley and Guttentag's work by introducing two kinds of prepayment decisions, "optimal" and "suboptimal". Namely, Prepayment is "optimal" if the rate of refinancing is less than the coupon rate of the existing mortgage. Otherwise, the prepayment decision is "suboptimal". The "suboptimal" prepayment occurs according to a Poisson process whose mean is taken to be a function of the time to maturity.

Lucy and Anderson (1987) argued that demographic and geographic characteristics of the mortgage should be taken into consideration. They thoroughly investigated three categories of the prepayment factors and established a

functional dependency between a combination of economic and noneconomic factors and the parameters of the prepayment stochastic process.

§ I-3 Literature on Valuation Model of Mortgage-backed Securities

Since the valuation of GNMA mortgage-backed securities caught the attention in academics, several different approaches have been introduced to the research in this field. Basically, these approaches can be classified into the following two categories: one is static approach (including (1) nonstochastic prepayment function, and STATIC economy approach, (2) and stochastic prepayment function , and STATIC economy approach), the other is dynamic approach (or (3) stochastic prepayment function and DYNAMIC economy approach. Simple average life method as well as Curley and Guttentag (1974, 1977) method falls into the first category, and Dunn and McConnell (1981a, 1981b) one state of economy variable model, and Schwartz and Torous two state of economy variables model belong to the second category.

A. Static Economy Approach

The valuation of GNMA mortgage-backed pass-through securities depends on the market condition as well as the prepayment behavior of mortgagors. In most of the researches in this field, the market condition is represented by the

current level of risk-free interest rate. Static economy approach mainly emphasizes the prepayment function, and assumes the market condition is static, namely, ignores the stochastic change of the interest rates. That is, they treat the mortgage-backed securities as if the only relevant variable in valuing the securities is the prepayment function. The static approach is best represented by (1) simple average life approach (nonstochastic prepayment function and STATIC economy) and by (2) Curley and Guttentag method (stochastic prepayment function and STATIC economy).

A.1 Average Life Method

Average life method handles the prepayment problem by assuming that the mortgage loan will be prepaid when the period of the mortgage is equal to the "average life" of a portfolio of "comparable mortgages". The standard assumption about average life is that 30-year mortgages will be prepaid in twelve years on an average.³ Yields are calculated because it's convenient to compare yields across different pools which differ with respect to principal outstanding and to coupon rate. The formula to compute the yield is given by equation (1.1)

$$P_0 = \sum_1^T [CF_k / (1+R)^k], \quad (1.1)$$

where

³ The twelve year average life assumption is based upon the past FHA experience for mortgage terminations.

- P_0 -- the current market price of the newly-issued mortgage
- CF_k -- monthly payment of interest and partial payment of the principal (when $k=1,2,\dots,TA-1$)
- CF_{TA} -- full payment of the remaining principal
- TA -- average number of periods to prepay for a newly-issued security.
- R -- yield to maturity on a newly-issued mortgage-backed security

In (1.1), all the other variables except the yield R are known. So R can be determined by solving (1.1). By assuming that the yields and the average life of the old securities and new securities are the same, the price of any currently outstanding mortgages can be derived by (1.1)'

$$P_t = \sum_1^{TA-t} [CF_k / (1+R)^k], \quad (1.1)'$$

where

- P_t -- the price of the currently outstanding mortgage which was issued t periods ago.

This traditional average life method has some serious drawbacks. The twelve year average life assumption is far from the actual experience with most GNMA mortgages when interest rates change dramatically. For example, the securities issued in the early 1980's with very high interest rates were in large part prepaid when interest rates fell.

A.2 Curley and Guttentag Method

Curley and Guttentag extended average life method by allowing that the mortgage loan will be prepaid anytime (either before or after the average life) with some prepayment probabilities throughout the life of the securities. Then the yield is computed by the equation (1.2)

$$P_0 = \sum_{T=1}^N \alpha_T (\sum_{k=1}^T [CF_k / (1+R)^k]), \quad (1.2)$$

where

- P_0 -- the current market price of the newly-issued mortgage
- α_T -- the proportion of a mortgages in the pool prepaid after T periods ($T=1,2, \dots, N$)
- CF_k -- monthly payment of interest and partial payment of the principal (when $k=1,2, \dots, T-1$),
- CF_T -- full payment of the proportion of α_T mortgages of the remaining principal,
- N -- number of periods for a newly-issued security,
- R -- yield to maturity on a newly-issued mortgage-backed security.

By the same assumption about the yields as average life method, that is, the yields are the same for all the mortgage-backed securities, the price of any currently outstanding mortgages securities can be derived by the equation (1.2)'

$$P_t = \sum_{T=1}^{N-t} \alpha_T (\sum_{k=1}^T [CF_k / (1+R)^k]), \quad (1.2)'$$

where

- P_t -- the price of the currently outstanding mortgage

which was issued t periods ago.

B. Dynamic Economy Approach

Dynamic economy approaches introduce the stochastic dynamic economy state variables which the prepayment function is based upon. Dunn and McConnell developed one dynamic variable model for GNMA securities. Schwartz and Torous developed two dynamic variables model for the GNMA securities. These models assume that the relevant information for pricing the GNMA securities can be summarized either by one state variable, instantaneous risk-free interest rate, as in Dunn and McConnell, or by two state variables, one short-term rate and one long-term rate as in Schwartz and Torous. The dynamics of the state variable(s) is (are) described as following the stochastic diffusion process(s). Then, by standard arbitrage arguments developed by Black and Scholes (1973) and Merton (1973), a second-order nonstochastic partial differential equation is derived.

B.1 Dunn and McConnell's Model

Dunn and McConnell (1981) developed a contingent-claims model for pricing the mortgages. In their model, a GNMA security is treated as a single default-free mortgage loan. The model assumed that all the uncertainty about pricing the mortgage can be summarized by the instantaneous risk-free borrowing and lending rate, whose dynamics follows a mean-

reverting square root diffusion process (see Cox, Ingersoll, and Ross (1978)). The stochastic process is given by

$$dr = k(m-r)dt + \sigma(r)dz \quad (1.3)$$

where

- r -- the instantaneous risk-free rate,
- m -- the steady-state mean of the process,
- $\sigma(r)$ -- instantaneous standard deviation,
- k -- the speed of adjustment parameter,
- dz -- standard Wiener process.

Let the value of the default-free fixed income security, $V(r, \tau)$, be the function of only of the current instantaneous risk-free interest rate, $r(t)$, and the time to maturity, $\tau = T - t$, where T is the maturity date. Using Ito's Lemma (McKean (1969)), the instantaneous return on the default-free fixed income security is then given by

$$dV = V_r dr + V_\tau d\tau + V_{rr} (dr)^2, \quad (1.4)$$

where

- V_r -- the first partial derivative with respect to r ,
- V_{rr} -- the second partial with respect to r ,
- V_τ -- the first partial with respect to time to maturity.

From (1.3), we have $dr^2 = \sigma(r)^2 dt$, and $d\tau = -dt$. Then, (1.4) can be rewritten

$$dV/V = [(k(m-r)V_r + \frac{1}{2}\sigma(r)^2 V_{rr} - V_\tau)/V]dt + (\sigma(r)V_r/V)dz.$$

(1.4)' Standard arbitrage arguments developed by Black and Scholes (1973), and Merton (1973), will give the following partial differential equation which the value of default-free

fixed income security must satisfy:

$$\frac{1}{2}\sigma(r)^2V_{rr} + [k(m-r)-p(r)\sigma(r)]V_r -V\tau -rV +C(\tau) =0, \quad (1.5)$$

where $C(\tau)$ is the continuous payment of from the mortgage securities.

If the prepayment decision of GNMA securities is characterized by the Poisson-driven process, then the value of the GNMA securities is the function of instantaneous risk-free interest rate $r(t)$, the time to maturity τ , and the Poisson random variable y . And by Ito's Lemma, the instantaneous return on the GNMA securities is

$$\begin{aligned} \frac{dV}{V} = & [(\frac{1}{2}\sigma(r)^2V_{rr} + k(m-r)V_r -V\tau +C(\tau) +\delta(r, \tau)(B(\tau)-V))/V]dt \\ & + (\sigma(r)V_r/V)dz + (B(\tau)-V)dy. \end{aligned} \quad (1.6)$$

Then no arbitrage arguments will give the following partial second-order differential equation:

$$\begin{aligned} \frac{1}{2}\sigma(r)^2V_{rr} + [k(m-r)-p(r)\sigma(r)]V_r -V\tau -rV \\ +C(\tau) +\delta(r, \tau)[B(\tau) -V]=0, \end{aligned} \quad (1.7)$$

where

p -- market price of interest rate,

δ -- the probability of a suboptimal prepayment,

$B(\tau)$ -- Face value of GNMA securities or the remaining principal balance.

It's almost impossible to find an analytical solution to equation (1.7). The numerical solution has to be found. One widely used numerical approach is so called finite difference method. Writing finite differences in place of partial

derivatives, the differential equation (1.7) can be approximated by ⁴

$$a_i V_{i-1,j} + b_i V_{i,j} + c_i V_{i+1,j} = d_i V_{i,j-1} + C_j, \quad i=1, \dots, n-1$$

$$j=1, \dots, m$$

(1.7)' where $a_i, b_i, c_i, d_i,$ and C_j are known.

Solving equation (1.7) by finite difference method requires the following set of the boundary conditions:

$$V(r, 0) = 0, \quad (1.8)$$

$$\lim_{r \rightarrow \infty} V(r, \tau) = 0, \quad (1.9)$$

$$V(r, \tau) \leq B(\tau). \quad (1.20)$$

Because GNMA mortgage-backed securities is fully amortizing,

the outstanding balance of the mortgage loan at maturity is zero. So, (1.8) states that the value of the securities at maturity is zero. (1.9) states that as the interest rate becomes very high the value of the securities will be near zero. $B(\tau)$ in (1.20) is the remaining balance of the mortgage loan. (1.20) states that the market value of the securities will never exceed its remaining outstanding principal balance.⁵

⁴ See Eduardo S. Schwartz (1977)

⁵ This assumption is by no means appropriate to the actual experiences of mortgage-backed securities.

In Dunn and McConnell's paper, the inequality of (1.20) is used to solve the equation (1.7) by the finite difference method. However, it should be noted that the inequality of (1.20) is not appropriate to actual experiences of the mortgage-backed securities because it doesn't always hold in reality. In other words, it does not reflect the actual mortgage experience.

The purpose of this paper is to examine Monte Carlo method in valuing the mortgage-backed securities. That is, we solve the equation (1.7) by the Monte Carlo method instead by the finite difference method. In order to show that the Monte Carlo method provides another numerical approach to valuing the GNMA mortgage-backed securities, and in order to compare the approaches and to provide a reference for further extension, we follow Dunn and McConnell (1981b) and apply our approach to valuing a variety of fixed income securities including GNMA mortgage-backed securities by incorporating a modified stochastic binomial model into Monte Carlo simulation method. Then, we perform the same analysis as Dunn and McConnell did in their paper. When underlying assumptions are the same as Dunn and McConnell's, we arrive at very similar values. This advantage of Monte Carlo method is that stochastic state variables other than diffusion process can be easily introduced into the Monte Carlo simulation. Finally, if we carefully examine the boundary condition of (1.20), we see that it violates the actual GNMA mortgage-backed security

experience. As mentioned in the previous section, the mortgagor doesn't always exercise his call option even when he can refinance his loan with a similar loan that has a lower coupon interest rate. That is, the mortgagor often retains the mortgage loan even when it appears "optimal" to prepay it. Consequently, the mortgage-backed securities may be traded above par. With the Monte Carlo method, we don't have to impose this inequality in order to get the solution to (1.7). Hence, the Monte Carlo result more accurately reflects the reality.

The organization of this paper is as follows: section I gives an introduction. Section II describes the methodology. Section III shows an application of the Monte Carlo approach. Section IV concludes with some guide line for the use of Monte Carlo Approach.

II METHODOLOGY

§II-1 Monte Carlo Approach [See Hammersley and Handscomb (1964) and Jack P.C. Kleijnen (1975) for theoretical discussion and Phelim P. BOYLE (1977) for its application in option]

Monte Carlo method is defined in a wide sense as any technique for the solution of a model using random numbers or pseudorandom numbers [Jack P.C. Kleijnen (1974)]. Suppose we want to calculate the following one-dimensional integral

$$\phi = \int_0^1 f(x) dx , \quad (2.1)$$

and let D be the integral domain of $(0, 1)$, and $f(x)$ an integrable function.

To apply the Monte Carlo method, we think of the variable x as a random variable with the uniform distribution over the interval $(0, 1)$. Then, ϕ becomes the expectation of $f(x)$. The approximation of the expectation ϕ can be obtained by the following so-called Crude Monte Carlo method.

Picking N numbers $(x_i, i \text{ from } 1 \text{ to } N)$ independently from the uniform distribution over $(0, 1)$, we get N quantities $f_i = f(x_i)$. Then, the expectation of f_i is ϕ . An unbiased estimator of ϕ is

$$\bar{f} = (1/N) \sum_1^N f_i , \quad (2.2)$$

and the standard deviation of the estimator

$$\sigma_f = \sigma / \sqrt{N} \quad (6). \quad (2.3)$$

In practice, σ (the standard deviation of $f(x)$) is replaced by the estimator s , where

$$s^2 = (1/(N-1)) \sum_1^N (f_i - \bar{f})^2. \quad (2.4)$$

Then, (2.3) becomes

$$\sigma_f = s / \sqrt{N} . \quad (2.5)$$

From (2.5) we can see that if we want to reduce the standard error of the estimator, we have to increase the N ,

⁶. σ^2 is the variance of $f(x)$.

the number of the simulation. The error can be reduced half by increasing the number of observations N to $4*N$. In practice, neither the cost nor the computation time will allow us to increase dramatically the number of observations. So, variance reduction methods other than increasing the number of observations should be used. One of the variance reduction method is known as antithetic variate method and we will use this method in our simulation. (Interested readers can refer to chapter III of Kleijnenher (1974) and Chapter 5 of Hammersley and Handscomb (1964) to have more exposures of other sampling methods).

The antithetic variate technique tries to create negative correlation between observations, generating one observation from the random number x and the other observation from its "antithetic" partner $(1-x)$. One practical approach to generate antithetic variate is as follows: Generating one run from the random numbers over the interval of $(0,1)$ x_1, x_2, \dots , and the other run, or antithetic run from the random numbers $(1-x_1), (1-x_2), \dots$.⁷ Negative correlation between the two responses is desirable since it decreases the variance of the estimated response. We use $\frac{1}{2}(\bar{f}(x)+\bar{f}(1-x))$ to estimate . The variance of the estimator is given by

⁷ $(1-x_1)$ is also random number.

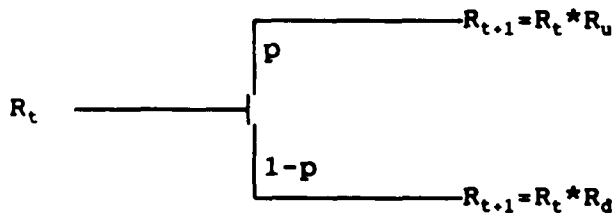
$$\text{Var}[\frac{1}{2}(\bar{f}(x)+\bar{f}(1-x))] = \frac{1}{4}\text{Var}(\bar{f}(x)) + \frac{1}{4}\text{Var}(\bar{f}(1-x)) + \frac{1}{2}\text{Cov}[\bar{f}(x),\bar{f}(1-x)].^{\circ} \quad (2.6)$$

The variance of $\frac{1}{2}[\bar{f}(x)+\bar{f}(1-x)]$ decreases if $\bar{f}(x)$ and $\bar{f}(1-x)$ are negatively correlated.

§ II-2 The Binomial Model for Pricing the GNMA Mortgage-backed Pass-through Securities

Assume that the interest rate movement obeys a binomial generating process as shown in Figure 2.1 where

Figure 2-1



- R_t -- the interest rate at time t ,
- R_{t+1} -- the interest rate at time $t+1$,
- p -- the probability that the interest rate will go up,
- $1-p$ -- the probability that the interest rate will go down,
- R_u -- the multiplicative upward movement in the interest rate,
- R_d -- the multiplicative downward movement in the interest rate.

⁶ When the following inequality is satisfied,

$\frac{1}{4}\text{Var}(f(\bar{x})) + \frac{1}{4}\text{Var}(f(1-\bar{x})) + \frac{1}{2}\text{Cov}[f(\bar{x}),f(1-\bar{x})] < \text{Var}(f(\bar{x}))$,
we achieve the desired goal.

rate.

At time t , the interest rate is R_t . At time $t+1$, the interest rate might go up to $R_t \cdot R_u$ with the probability of p and might go down to $R_t \cdot R_d$ with the probability of $(1-p)$. In our model, an exception to the standard binomial model is the assumption about p . The probability that the interest rate goes up or down is assumed to be non-stationary and to have the mean reverting characteristic, that is:

$$p_t = \frac{1}{2} [1 + \delta (\mu - R_{t-1})], \quad (2.7)$$

where

$$\delta = K / \sqrt{(\mu^2 + R_{t-1}^2)}, \quad (2.8)$$

K -- the speed of the adjustment parameter,

μ -- the long-term mean of the interest rate R .

The up and down movements of R_u and R_d are determined by the following relationships:⁹

$$R_u = \exp(\sigma \sqrt{T/N}) \quad (2.9)$$

and
$$R_d = \exp(-\sigma \sqrt{T/N}), \quad (2.10)$$

where σ is the annual standard deviation of the risk-free interest rate (or the return of short term government bonds), T is the time from the first period to the last period, and N is the number of time intervals. Equations (2.9) and (2.10) are used to translate the continuous time variables such as standard deviation σ into the discrete variables such as R_d

⁹ Please see Cox, Ross, and Rubinstein (1979) for the discussion of this part.

and R_u .¹⁰

Suppose the number of the periods is M . Under the assumption that the interest rates follow the binomial model, theoretically we can generate the whole interest rate structure. And we can get all the interest rate paths. Then, after we determine the series of cash flows associated with every path, we can compute the present value by the Discount Cash Flow method. However, If M is 360, 2^M sample paths will be too time consuming to calculate⁽¹¹⁾. So, we apply Monte Carlo method to calculate the value of the securities.

Suppose we generate a path (or a series of interest rates) R_1, R_2, \dots, R_M . In each period of this particular path, the cash flow will be either the scheduled monthly payment if the mortgage is not prepaid, or the remaining principal if the mortgage is prepaid according to the stochastic prepayment function. Consequently, we can determine a series of cash flows for this particular interest rate path. The present value of this series of cash flows associated with this particular path is computed in this way that the discount factors are just the reciprocal of the risk free interest rates.

¹⁰ Of course, some other approaches such as MLE method can be used to get the estimates of R_d and R_u .

¹¹ The number of paths will be 2^M if the number of periods is M .

The justification is stated as follows. Suppose the economy is frictionless. Given a particular path of interest rates (Note that there is no uncertainty about this particular path because it's already given.) , pick up any number i ($i=1,2,\dots,N$). The product of one period forward rates

$$(1+R_1)(1+R_2)\cdots(1+R_i) \quad (2.8)$$

should be equal to the yield to the maturity of the risk free bond with i period maturity.¹² There are two rationales about this justification: 1) One is rational expectation hypothesis and risk neutral investors hypothesis. Under these hypotheses, all fixed income securities will follow expectation process for given probability distribution; and 2) because markets are dynamically complete so there exists martingale pricing process for given probability distribution. By using the risk neutral investor assumption, we can solve all values by expectation hypothesis. So YTM, where

$$(1+YTM_i)^i = (1+R_1)(1+R_2)\cdots(1+R_i), \quad (2.9)$$

can be used to evaluate this cash flow in the i th period.

PART III AN APPLICATION OF MONTE CARLO APPROACH

§ III-1

In this part of the research, we treat the mortgage-backed securities as if it were a single default-free mortgage

¹² No arbitrage requirement will guarantee this conclusion, also known as unbiased expectation.

loan.¹³ Four types of different interest rate path dependent securities are investigated. These four types of securities are, 1) nonamortizing, noncallable bonds, 2) nonamortizing, callable bonds, 3) amortizing, noncallable bonds, and 4) amortizing, callable bonds. GNMA security is belonging to the last type of the security. We perform the sensitivity analysis as Dunn and McConnell did in their research. Namely, under various current risk-free interest rates, and under different current terms to maturity, we calculated the values of these four types of securities. For the GNMA securities, we computed the values under the optimal policy, and under the suboptimal policy, that is, under the assumption of the prepayment rates of 100% FHA level, and 200% FHA level.

In their research, Dunn and McConnell imposed optimal call policy for all the computations of GNMA securities under different prepayment assumptions. However, as mentioned before, this restriction is not appropriate to the actual experience of GNMA mortgage-backed securities. So in our research, we don't impose this restriction in the Monte Carlo simulation. One advantage of our approach as mentioned in the previous section, is that various prepayment functions can be easily introduced into the simulation. We calculated the value under the optimal call policy. We also calculated the

¹³ Later, we will investigate the valuation of the pool of the mortgages based on the result of the individual mortgage.

values of GNMA securities under the 100% of FHA experience and 200% of FHA experience without the imposition of the optimal call policy.

Table-I lists the values of four types of securities as a function of the current interest rate (interest rates from one-percent to twenty-percent) when the coupon rate is eight-percent and the term to maturity is thirty years. Table-II shows the values of the four types of securities as a function of the term to maturity when the current interest rate is twelve-percent and the coupon rate is eight-percent. These results are calculated by the parameters: $K=1.0$, $\sigma=0.09$, and $\mu=0.055$, $R_u=\exp(1.375\sigma)$, and $R_d=\exp(-.785\sigma)$. The coupon interest rates of all the four different securities are 8-percent. The reported results are simulated by crude Monte Carlo method. Only the standard deviations of the estimates of the value of nonamortizing, noncallable bonds are reported. ¹⁴

Figure-II uses the diagrams to show the results of the four types of the securities when the coupon rate is eight-percent and the term to maturity is thirty years. In Figure-III are the results of the four types of securities as a function of the current term to maturity when the current

¹⁴ Because the underlying interest rate processes are the same for all of the bonds discussed here, and the number of the simulation are the same, the deviations of all these estimates have quite similar magnitude. So, readers should think of the estimates of other bonds having the same accuracy as the reported nonamortizing, noncallable bonds.

interest rate is twelve-percent and the coupon rate is eight-percent.

§ III-2 The Shape of Term Structure

III-2-1 The Call Option

Obviously, noncallable bonds should be more valuable than the otherwise identical callable bonds. The results support this assertion.

At every level of the instantaneous interest rate the value of the call option can be calculated by subtracting the value of a callable security from the value of an otherwise identical noncallable one. The value of the call options is higher when the interest rates are low than when the interest rates are high. This is because when the current interest rate is very low, the call option is deeply in the money, so the value of the call option is very high. On the other hand, when the interest rate is very high, the call option is not deeply in the money, the value of the option of course becomes small. When the current interest rate becomes very high, the value of the call option is expected to be near zero.

III-2-2 The Amortizing Feature

The amortizing feature can have either positive impact or negative impact on value of the bonds. This can be seen by calculating the nonamortizing, noncallable bond with the

amortizing, noncallable bond.

When the interest rates are very low, let's say, below the contract coupon rate, the value of the nonamortizing, noncallable bonds are higher than that of amortizing, noncallable bonds. The concept of opportunity cost can be used to explain this phenomenon. The security holders get back higher monthly payments from the amortizing bonds than from nonamortizing bonds before the last payment. Reinvestment of this cash flow will get lower return due to very low current interest rate. The opportunity cost forgone by getting the higher payments now is very high for amortizing bonds. Thus, the value of amortizing bonds are lower than nonamortizing ones when current interest rates are lower than the contracted coupon rate.

The same argument can be applied to the amortizing impact on the value of the bond when the current interest rate is very high.

III-2-3 Suboptimal Prepayments

The effect of suboptimal prepayments is to increase the value of the GNMA securities. When the current interest rates are very low, (to say, when the interest rates are below the contract coupon rate, here eight percent) the GNMA with 200% of FHA experience will have smaller increase in value than that with 100% of FHA experience. When the current interest rate is above contract coupon rate, the GNMA security with 200% of FHA experience will have a greater increase in value

than that with 100% of FHA experience. We don't see the contrast between these two different experiences in Dunn and McConnell's paper when the current interest rates are very low. Because they impose the "Optimal Policy" when they tried to solve the differential equation. Thus, they gave the same value for these two different FHA experiences when the current interest rates are very low.

§ III-3 Term to Maturity

We performed the sensitivity analysis for the four types of securities when the term to maturity changes from one year to thirty years under the assumption of twelve-percent current risk-free interest rate and eight-percent coupon rate. The results are shown in the TABLE-II and the FIGURE-III.

The values of all four types of bonds decrease when the term to maturity increases with the assumed current market conditions. The results also show that when the term to maturity increase to a quite large number, the value decreases much slower. Eventually, the values of these four types of bonds will converge to some limits with the increase of the current term to maturity.

III-3-1 The Amortization Feature

As mentioned above, the values of the four types of bonds approach to the limits when the term to maturity increases. This phenomenon tells us that the sensitivity of

the values is very small when the term to maturity increases to a large number. So we look at the results when the current term to maturity is short. Look at the TABLE II, the amortizing, noncallable bonds is more sensitive to the change of the current term to maturity than the nonamortizing, noncallable bonds. Again, this phenomenon can be explained by the concept of opportunity cost. When the current interest rate is very high (compared with the contract coupon rate), the opportunity cost of putting money into a low rate of return project is also very high. Amortizing, noncallable bonds get back larger cash flows sooner than nonamortizing, noncallable bonds, and thus, the opportunity cost is small. So the corresponding values are high.

III-3-2 The Call Option

From the option pricing knowledge, we know that the value of a option can be divided into two parts: one is the time value of the option, and the other is the intrinsic value of the option. That is, with the increase of the term to maturity, the option will have a higher time value. Our results prove this assertion. When the current term to maturity is very short, the difference between the nonamortizing, callable bonds and nonamortizing, noncallable bonds is very small. But when the current time to maturity becomes larger, the differences are becoming larger. This means that with the increase of the current time to maturity,

the time value of the call option is large.

III-3-3 Suboptimal Prepayments

In the reported tables, we listed the standard deviation of the results for the nonamortizing, noncallable bonds. The standard deviation is very large when the current term to maturity is large.¹⁵ So, the results with a shorter current term to maturity is more believable than those with a long current term to maturity. Again, that the suboptimal prepayment function is to increase the value of the GNMA can be seen from the table.

¹⁵ For the purpose of saving space, we listed the standard deviation for the nonamortizing, noncallable bonds only.

VI CONCLUSION OF CHAPTER 1

First of all, the Monte Carlo method can easily treat different kinds of prepayment functions. Secondly, Monte Carlo method can easily treat the penalty charge for the prepayment decision if we want to introduce the service fee associated with the prepayment decision.¹⁶

Obviously, better variance reduction method should be found in Monte Carlo simulation. The standard error of the estimates are too high from the crude Monte Carlo simulation, especially, for the long-term securities. The average time for computing these values on WYLBUR system is 40 seconds when the number of trials N equals 2,000. If we increase the number of trials, the computing time will be longer. For the sake of increasing the accuracy of the estimates and at the same time not increasing computing time, we are doing the antithetic method and control variate method to reduce the standard error.

When current risk free interest rates are very high, the results from Monte Carlo simulation are higher than those in Dunn and McConnell's paper. One possible explanation is that the binomial interest process in our model does not perfectly match their (Dunn and McConnell) short rate assumption. The ranges of current interest rates from 1-percent to 12-percent of two short term interest rate processes seem to fit quite

¹⁶ Some mortgage agencies do charge a fee for early prepayment.

well. But the higher range of the interest rate might not match. Thus, we achieve very similar results with Dunn and McConnell's for one range of current interest rates, but arrive at different results from Dunn and McConnell's for the other range.

Better interest rate process should be found. We will try to estimate the interest rate process by introducing all the relevant bond yields within the term to maturity of the GNMA securities. Then the discount factors we use in the calculation of the present value of a specific interest rate path will be more accurate. Consequently, the valuation will be better.

REFERENCES FOR CHAPTER 1

- Black, F. and M.J. Scholes, 1973, The pricing of options and corporate liabilities, Journal of Political Economy 81, 637-659.
- Boyle, Phelim P., 1977, Options: a monte carlo approach, Journal of Financial Economics 4, 323-338
- Cox, J.C., J.E. Ingersoll Jr., and S.A. Ross, 1978, A theory of term structure of interest rates, Unpublished manuscript, Stanford University.
- Curley, Anthony J. and Jack M. Guttentag, 1974, The yield on insured residential mortgages, Exploration in Economic Research, (Summer), pp.114-161.
- and Jack M. Guttentag, "Value and Yield Risk on Outstanding Insured Residential Mortgages," Journal of Finance, Volume 32, Number 2 (May 1977), pp.403-412.
- Dunn, K.B. and J.J. McConnell, 1981a, A comparison of alternative models for pricing GNMA mortgage-backed securities, Journal of Finance 36, 471-484.
- and J.J. McConnell, 1981b, Valuation of GNMA mortgage-backed securities, Journal of Finance 36, 599-616.
- Hammersley, J.M. and D.C. Handscomb, 1964, Monte Carlo methods (John Wiley, New York).
- Jacob, David P. and Graham Lord and James A. Tilley, (1985), Price, duration and convexity of mortgage-backed securities, The Hand Book of Mortgage-backed Securities, Probus, Chicago.

- Kleijnen, Jack P.C., 1975, Statistical techniques in simulation, (Marcel Dekker, New York).
- McKean, H.P., Jr., 1969, Stochastic Integrals (Academic Press, New York).
- Merton, R.C., 1973, The theory of rational option pricing, Bell Journal of Economics and Management Science 4, 141-193.
- Robert Lucy and Ronald W. Anderson, "Pool-specific Studies in GNMA Pass-through Securities," (November, 1987), unpublished manuscript.
- Schwartz, Eduardo S., 1977, The valuation of warrents: implementing a new approach, Journal of financial economics 4(1977) 79-93 .

TABLE I

Percentage Current Interest rate	Nonamortizing Noncallable Bond	Standard Deviation (N=2000)	Nonamortizing Callable Bond	Amortizing Noncallable Bond	GNMA Security With optimal Call Policy	GNMA Security at 100% of FHA Experience	GNMA Security at 200% of FHA Experience
1	112.523	4.152	100	112.51	100	111.78	110.5726
2	108.561	4.111	100	108.619	100	108.426	107.669
3	106.091	4.32	100	106.184	100	106.209	105.737
4	104.227	4.198	100	104.33	100	104.585	104.232
5	102.647	4.035	100	102.755	100	103.141	102.953
6	101.13	4.025	99.9503	101.256	100	101.83	101.745
7	99.84	4.139	98.0705	99.9832	100	100.574	100.621
8	98.5854	3.986	97.4771	98.7374	98.3483	99.515	99.658
9	97.4	4.059	96.3921	97.5558	97.353	98.2995	98.523
10	96.2576	3.896	94.5153	96.4343	95.8842	97.2526	97.6106
11	95.1307	3.913	93.0421	95.3134	94.9648	96.1506	96.5916
12	94.6003	3.794	93.0896	94.8843	93.9183	95.2859	95.7898
13	93.1449	3.913	92.3164	93.3323	92.962	94.4169	94.9311
14	92.063	3.789	91.1377	92.241	92.0128	93.4796	94.0937
15	91.1509	3.875	89.9193	91.3604	90.8213	92.508	93.2508
16	90.0347	3.681	88.3591	90.2468	89.948	91.6079	92.3374
17	89.0583	3.682	87.4129	89.2809	89.0893	90.6736	91.5054
18	88.156	3.637	86.4269	88.3879	87.9125	89.8067	90.6844
19	87.1893	3.6073	85.968	87.4274	87.1813	88.8654	89.8555
20	86.3169	3.6002	85.2531	86.5563	86.1813	88.005	89.0173

TABLE-11

Term To Maturity	Nonamortizing Noncallable Bond	Standard Deviation (N=2,000)	Nonamortizing Callable Bond	Amortizing Noncallable Bond	GNMA Security With optimal Call Policy	GNMA Security at 100% of FNA Experience	GNMA Security at 200% of FNA Experience
1	97.1944	1.303	97.1172	98.2984	98.1813	98.2569	98.2572
2	96.0068	2.461	95.9856	97.3573	97.3327	97.3654	97.3947
3	95.5028	3.228	95.4298	96.7419	96.7194	96.7836	96.886
4	95.1848	3.776	95.0011	96.3544	96.3445	96.4764	96.6258
5	94.9838	4.167	94.349	95.9645	95.9666	96.3175	96.4559
6	95.0055	4.292	94.9316	95.7882	95.7952	96.03	96.2526
7	94.9065	4.38	94.8926	95.6172	95.6548	95.9423	96.2249
8	94.8537	4.48	94.9385	95.4569	95.5589	95.7844	96.0803
9	94.8676	4.55	94.9569	95.3483	95.4858	95.7486	96.0382
10	94.9165	4.493	94.9416	95.2497	95.4063	95.7065	96.0354
11	94.9592	4.436	94.9577	95.19	95.3591	95.5501	95.9694
12	95.1296	4.099	95.064	95.129	95.401	95.554	95.9874
13	94.9291	4.515	94.9436	95.0501	95.2533	95.6258	96.0559
14	94.9318	4.528	94.8723	94.9756	95.1569	95.6228	96.0252
15	94.837	4.63	94.8834	94.8371	95.1351	95.6496	96.0121
16	94.937	4.58	94.884	94.7689	95.111	95.6813	96.0108
17	94.9353	4.488	94.8462	94.7379	95.1468	95.6902	96.0883
18	94.8921	4.533	94.7057	94.666	95.084	95.5647	95.997
19	94.8444	4.414	94.6908	94.6433	95.0635	95.5571	95.9718
20	94.4881	4.293	94.03	94.5182	95.1794	95.6268	96.0362
21	94.9358	4.49	93.9121	94.5656	95.0459	95.5916	95.9121
22	95.0345	4.359	93.9879	94.6032	95.1072	95.5505	95.9355
23	94.7386	4.677	93.9101	94.4767	95.0217	95.4412	95.9214
24	95.0363	4.333	93.8785	94.3963	94.9809	95.3762	95.8891
25	94.8615	4.474	93.8957	94.4587	94.9889	95.328	95.8506
26	94.7809	4.592	93.8646	94.4032	94.9474	95.3264	95.8425
27	95.0124	4.555	93.893	94.36	94.9684	95.3264	95.8579
28	94.8709	4.698	93.9022	94.2918	94.9709	95.2138	95.8337
29	95.0326	4.466	93.8555	94.278	94.9191	95.2368	95.8609
30	94.7341	4.467	93.0676	94.3732	95.1213	95.2873	95.7192

FIGURE 11 VALUES OF THE FOUR TYPES OF BONDS

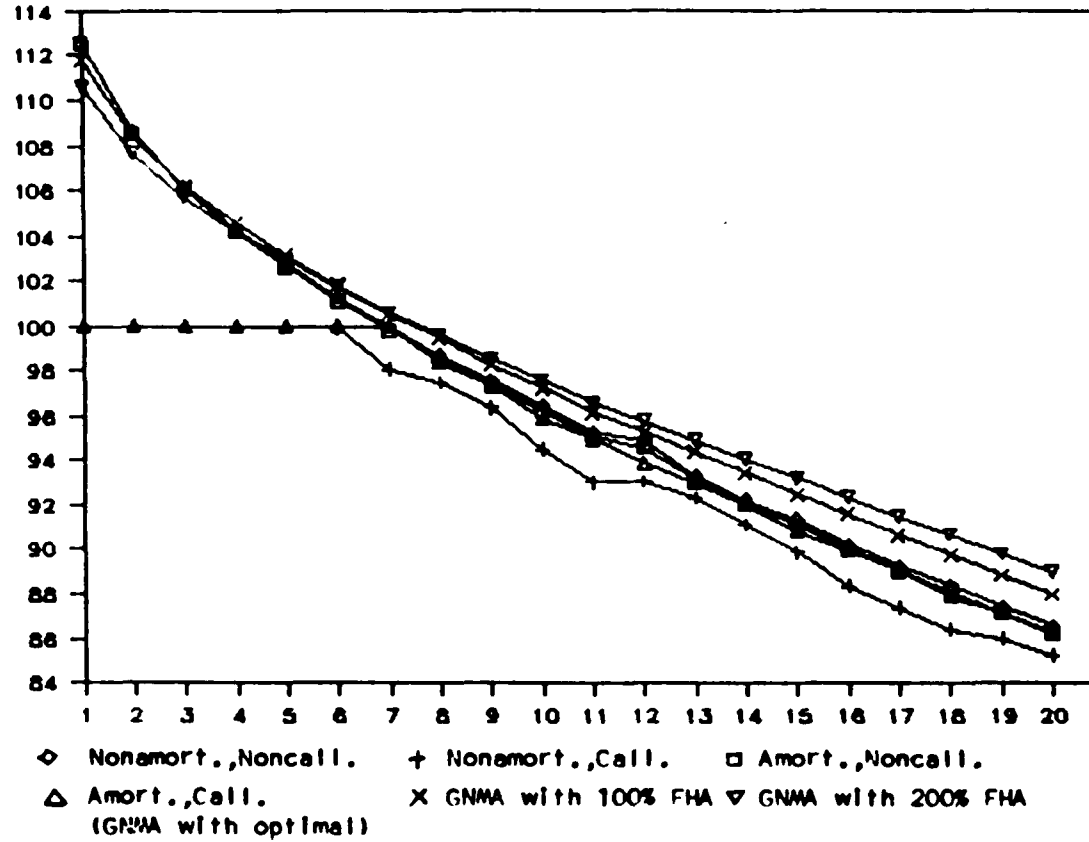
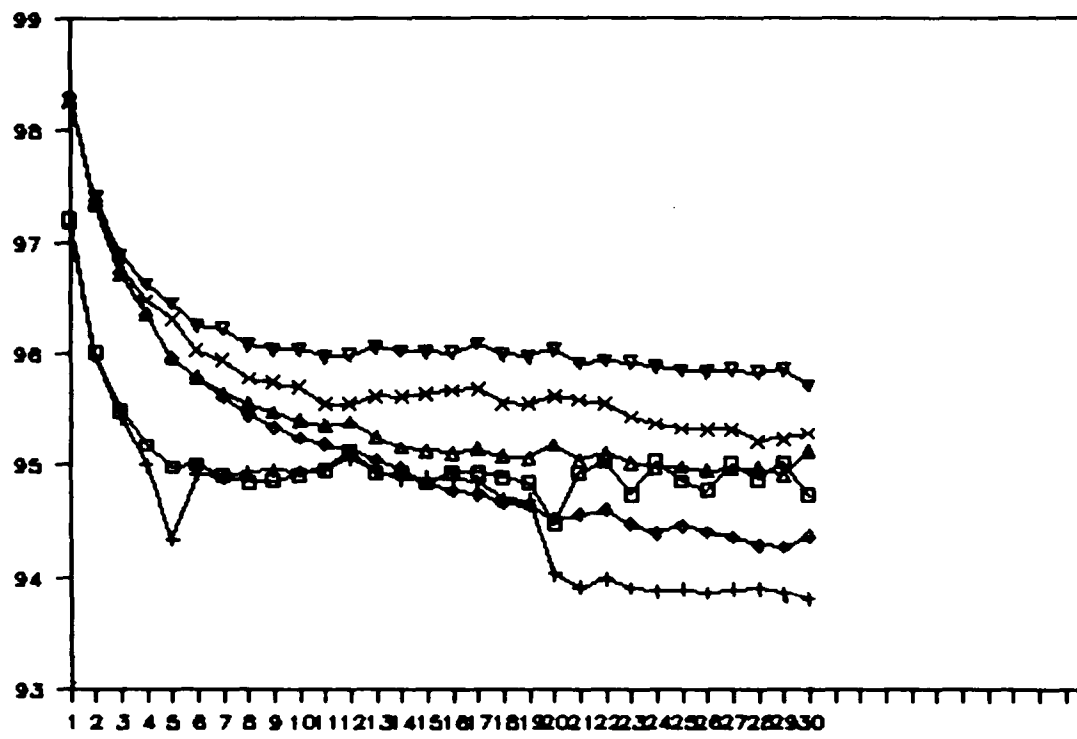


FIGURE 111 VALUES OF THE FOUR TYPES OF BONDS



◇ Nonmort., Noncall. + Nonmort., Call. □ Amort., Noncall.
 △ Amort., Call. x GNMA with 100% FHA ▽ GNMA with 200% FHA
 (GNMA with optimal)

Table 1 (from Dunn and McConnell (1981b))

Prices of Various Fixed-rate Securities as a Function of the Current Interest Rate when the Coupon Rate of the Bonds is 8 percent and the Term to Maturity is 30 years

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Percentage Current Interest Rate	Percentage 30 Year Interest Rate	Nonamortizing Noncallable Bond	Nonamortizing Callable Bond	Amortizing Noncallable Bond	GNMA Security with the Optimal Call Policy	GNMA Security with Suboptimal Prepayments at 100% of FHA Experience	GNMA Security with Suboptimal Prepayments at 200% of FHA Experience
0.0000	7.5268	113.29805	100.00000	113.11888	100.00000	100.00000	100.00000
1.0000	7.5863	111.52800	100.00000	111.37348	100.00000	100.00000	100.00000
2.0000	7.6459	109.78614	100.00000	109.65568	100.00000	100.00000	100.00000
3.0000	7.7054	108.07378	100.00000	107.96676	100.00000	100.00000	100.00000
4.0000	7.7649	106.38861	100.00000	106.30642	100.00000	100.00000	100.00000
5.0000	7.8244	104.73284	99.98574	104.67083	100.00000	100.00000	100.00000
6.0000	7.8840	103.10400	99.27023	103.06378	99.37159	99.62100	99.75824
7.0000	7.9435	101.50229	98.13678	101.48319	98.29127	98.89237	98.92891
8.0000	8.0030	99.92686	96.85764	99.92836	97.05001	97.55658	97.86710
9.0000	8.0625	98.37738	95.51956	98.39893	95.74209	96.33082	96.70242
10.0000	8.1221	96.85347	94.15900	96.89456	94.40717	95.06459	95.48851
11.0000	8.1816	95.35476	92.79359	95.41488	93.06444	93.78187	94.25497
12.0000	8.2411	93.88086	91.43269	93.95850	91.72411	92.49552	93.01308
13.0000	8.3006	92.43135	90.08169	92.52803	90.39207	91.21296	91.77215
14.0000	8.3602	91.00581	88.74380	91.12006	89.07187	89.93871	90.53723
15.0000	8.4197	89.60381	87.42101	89.73514	87.76572	88.67562	89.31155
16.0000	8.4792	88.22487	86.11456	88.37284	86.47499	87.42552	88.09717
17.0000	8.5387	86.86852	84.82519	87.03268	85.20054	86.18959	86.89548
18.0000	8.5982	85.53428	83.55332	85.71419	83.94289	84.96857	85.70734
19.0000	8.6578	84.22222	82.29961	84.41745	82.70275	83.76333	84.53372
20.0000	8.7173	82.93240	81.06464	83.14252	81.48074	82.57460	83.37541
Selected Interest Rate Elasticities of the Bond Prices							
6.0000	7.8840	-0.09398	-0.05968	-0.09277	-0.05557	-0.04376	-0.03650
8.0000	8.0030	-0.12508	-0.10862	-0.12346	-0.10565	-0.09755	-0.09180
12.0000	8.2411	-0.18683	-0.17803	-0.18434	-0.17488	-0.16673	-0.16026

Table 11 (from Dunn and McConnell (1981b))

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Years to Maturity	Percentage Yield-to-Maturity of a Discount Bond	Nonamortizing Noncallable Bond	Nonamortizing Callable Bond	Amortizing Noncallable Bond	GNMA Security with the Optimal Call Policy	GNMA Security with Suboptimal Prepayments at 100% of FHA Experience	GNMA Security with Suboptimal Prepayments at 200% of FHA Experience	Annual Prepayment Probabilities at 100% of FHA Experience
0	12.0000	100.00000	100.00000	100.00000	100.00000	100.00000	100.00000	.00000
1	11.0864	97.12317	97.08360	98.27117	98.25939	98.43507	98.57899	.32444
2	10.4334	95.60206	95.37638	97.19669	97.12534	97.50224	97.79363	.24495
3	9.9622	94.79564	94.28749	96.44427	96.26721	96.74149	97.10418	.15885
4	9.6170	94.36739	93.55308	95.90570	95.59501	96.20273	96.66045	.15037
5	9.3597	94.13973	93.03641	95.51170	95.05344	96.79050	96.33231	.14224
6	9.1644	94.01863	92.66152	95.21722	94.60796	96.45815	96.07090	.13459
7	9.0133	93.95418	92.38434	94.99257	94.23701	96.18049	95.85009	.12733
8	8.8942	93.91986	92.17448	94.81783	93.92274	94.94301	95.65635	.12049
9	8.7987	93.90162	92.01275	94.67944	93.65248	94.73589	95.48189	.11399
10	8.7208	93.89190	91.88702	94.56804	93.41841	94.55234	95.32201	.10785
11	8.6562	93.88673	91.78890	94.47703	93.21484	94.38657	95.17380	.10208
12	8.6021	93.88398	91.71218	94.40169	93.03722	94.21396	95.00179	.09174
13	8.5560	93.88251	91.65214	94.33860	92.88167	94.04752	94.83072	.08358
14	8.5165	93.88173	91.60513	94.28521	92.74407	93.89060	94.66681	.07665
15	8.4821	93.88131	91.56832	94.23962	92.62090	93.74542	94.51423	.07099
16	8.4520	93.88109	91.53949	94.20040	92.51009	93.61551	94.37781	.06687
17	8.4255	93.88098	91.51692	94.16640	92.41020	93.50229	94.26041	.06408
18	8.4018	93.88092	91.49923	94.13676	92.32010	93.40574	94.16281	.06235
19	8.3807	93.88089	91.48537	94.11079	92.23880	93.32517	94.08464	.06145
20	8.3617	93.88087	91.47437	94.08791	92.16540	93.25891	94.02380	.06112
21	8.3444	93.88086	91.46553	94.06768	92.09910	93.20172	93.97206	.06063
22	8.3288	93.88086	91.45835	94.04972	92.03918	93.16005	93.94139	.06159
23	8.3145	93.88086	91.45250	94.03373	91.98499	93.12710	93.92032	.06221
24	8.3014	93.88086	91.44772	94.01945	91.93594	93.09666	93.88900	.06202
25	8.2893	93.88086	91.44382	94.00665	91.89151	93.06893	93.87892	.06189
26	8.2782	93.88086	91.44062	93.99516	91.85125	93.02219	93.82418	.05774
27	8.2679	93.88086	91.43801	93.98483	91.81473	92.98124	93.72754	.05128
28	8.2583	93.88086	91.43587	93.97552	91.78159	92.86122	93.58875	.04431
29	8.2494	93.88086	91.43412	93.96711	91.75148	92.72586	93.39147	.03172
30	8.2411	93.88086	91.43269	93.95950	91.72411	92.49552	93.01308	.00840

Mortgage-Backed Securities

CHAPTER 2

PART I INTRODUCTION

In this chapter, we will value the mortgage-backed securities in the binomial interest rate movements model of Ho and Lee (1986), the model that uses full information of initial term structure. We will discuss how we can generate the reasonable Ho and Lee's AR model and how we can apply the AR model to valuing the GNMA mortgage-backed securities. The reason why we want to incorporate all available information is obvious. More independent information can determine more parameters of the term structure. Think of the interest rate movements as a very complicated function. Suppose we want to approximate this function by a polynomial with some degrees. (Because the assumption of complicated function, the polynomial should have more than one degrees in order to have a better fit). The n degree polynomial needs at least n independent initial conditions in order to be determined. From this point of view, the interest rate movements that use all available initial information, such as Ho and Lee's AR model, are better than the single factor interest rate movements model.

The organization of this chapter is as follows: first, we will briefly introduce Ho and Lee AR (Arbitrage-free Rate movements) model; second, we will discuss some problems in Ho and Lee's model and the modifications and improvements;

finally, we value the GNMA mortgage-backed securities in the modified AR model, and again, the application of Monte Carlo method is discussed.

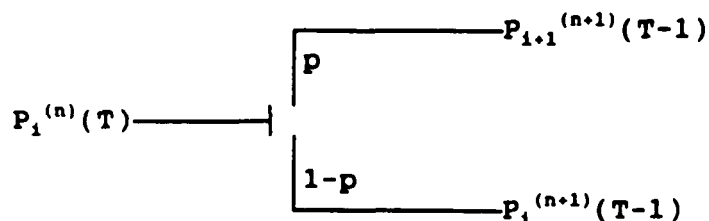
PART II ARBITRAGE-FREE RATE MOVEMENT MODEL (see Ho and Lee 1986))

§-II-1 Assumptions

Unlike the single factor interest rate models such as the binomial model we discussed in the first chapter, or the mean reverting stationary Markov process model, the Ho and Lee's interest rate movement model starts from a set of bond prices, not just one short term bond price, or current short term rate. First, we assume that bond market is complete. There exists a discount zero coupon bond for each maturity n ($n=1, 2, \dots, T$).¹ There are a finite number of states of world at each time n . Let $P_i^{(n)}(T)$, which is called discount function, denote the equilibrium price of the discount bond of maturity T for state i at time n ($i=1, 2, \dots, n; n=1, 2, \dots, T$). The set of discount functions forms a binomial lattice. That is, the discount function follows a binomial process as in figure-I.

¹ It's fine if there does not exist a zero coupon bond, but there is coupon bond for each maturity n . We can replicate zeros by coupon bonds.

Figure-I



Let the implied forward function, $F_1^{(n)}(T)$, be equal to $P_1^{(n+1)}(T)/P_1^{(n)}(1)$. In an uncertainty world in which the interest rates move according to a binomial lattice, $F_1^{(n)}(T)$ will not equal $P_1^{(n)}(T)$. Ho and Lee introduced two perturbation functions, $h(T)$ and $h^*(T)$, which specify the deviations of the discount functions from the implied forward function, where in the upstate,

$$P_{1.1}^{(n+1)}(T) = F_1^{(n)}(T) * h(T), \quad (1)$$

and in the downstate,

$$P_1^{(n+1)}(T) = F_1^{(n)}(T) * h^*(T). \quad (2)$$

If we know the value of $h(T)$ and $h^*(T)$, we can calculate $P_{1.1}^{(n+1)}(T)$ and $P_1^{(n+1)}(T)$. Thus, to determine the whole term structure process is equivalent to determine the set of the perturbation functions.

§ II-2 AR Model

In order to determine the set of perturbation functions, Ho and Lee impose arbitrage free condition and path independent condition on the discount zero coupon bond

function $P_i^{(n)}(\cdot)$ over the binomial lattice. From arbitrage free condition, we can get the following equation:

$$\pi * h(T) + (1-\pi) * h^*(T) = 1, \quad (3)$$

where, π is called implied binomial probability in Ho and Lee's paper and is constant throughout the time horizon. For the sake of completeness, we give the proof as follows:²

Consider a portfolio of one discount bond with maturity T and ϕ discount bonds with maturity t at any time n and state i . Because the result is irrelevant to n and i , we drop all the indices (n, i) : In the beginning, the portfolio has the value $V = P(T) + \phi P(t)$.

At the end of the period, the portfolio will have two possible outcomes. When an upstate prevails, the portfolio will have by equation (1)

$$V^u = [P(T)h(T-1) + \phi P(t)h(t-1)]/P(1), \quad (4)$$

and when a downstate prevails, the portfolio will be by equation (2):

$$V^d = [P(T)h^*(T-1) + \phi P(t)h^*(t-1)]/P(1). \quad (5)$$

The portfolio can be made into a risk-free hedge by choosing an appropriate ϕ , e.g., the portfolio will have the same value in upstate and downstate. This hedge ratio ϕ can be derived by equating equation (4) and (5):

$$[P(T)h(T-1) + \phi P(t)h(t-1)] = [P(T)h^*(T-1) + \phi P(t)h^*(t-1)].$$

Solving the above equation, we get

² This part can be found in Ho and Lee (1986) paper.

$$\phi = P(T)[h(T-1) - h^*(T-1)] / P(t)[h^*(t-1) - h(t-1)] \quad (6)$$

Because the portfolio has the same value at the end the period, the return to the hedge portfolio becomes riskless. Therefore the portfolio must earn the riskfree rate, i.e., the one-period discount bond return in our problem, $1/P(1)$. That is,

$$V^u * P(1) = P(T) + \phi P(t); \quad (7)$$

or equivalently,

$$[P(T)h^*(T-1) + \phi P(t)h^*(t-1)] = P(T) + \phi P(t). \quad (7)'$$

To solve, substitute equation (6) into equation (7)' and we get

$$[1 - h^*(T-1)] / [h(T-1) - h^*(T-1)] = [1 - h^*(t-1)] / [h(t-1) - h^*(t-1)]$$

$$\text{for any } T \text{ and } t > 0. \quad (8)$$

The left hand and the right hand of equation (8) have the same functions except the time indexes T and t . Let π be equal to $[1 - h^*(T-1)] / [h(T-1) - h^*(T-1)]$ and this π is independent of the time index. In another word, π is constant over the time period.

From path independent condition, we get the following equation:

$$h(T+1)h^*(T)h^*(1) = h^*(T+1)h(T)h(1).^3 \quad (9)$$

³ We give the proof as follows: The concept of the interest rate path independence is that at any state i and time n , a downward move followed by an upward move of the interest rate equals an upward move followed by a downward move of the interest rate.

In order to get equation (9), let's start at time n and state i . The discount function for a bond with maturity time $T+2$ is

Combining equation (3) and (9), we get a first-order difference equation,

$$1/h(T+1) = \delta/h(T) + b, \quad (10)$$

where a and b are constants such that

$$\delta = (1-\pi h(1))/(1-\pi)h(1), \quad (11)$$

$$b = \pi(h(1)-1)/(1-\pi)h(1). \quad (12)$$

The general solution to the first-order difference equation (10) is

$$h(T) = 1/(\pi + c\delta^T). \quad (13)$$

A discount zero-coupon bond is worth \$1 at the maturity date.

That is

$$P_i^{(n)}(0) = 1 \quad \text{for all } i \text{ and } n. \quad (14)$$

From (1) and (14), we get the initial condition $h(0)=1$ for

(8). Given (14), we can specify a unique solution for (13)

$$h(T) = 1/(\pi + (1-\pi)\delta^T). \quad (15)$$

From equation (3), we get

$$h^*(T) = \delta^T/(\pi + (1-\pi)\delta^T). \quad (16)$$

From (1), (2), (15), and (16), we can get the explicit expression for

$P_i^{(n)}(T+2)$. After two periods, an upward move followed by a downward move of the bond price will give us :

$$\begin{aligned} P_{i+1}^{(n+2)}(T) &= [P_{i+1}^{(n+1)}(T+1)h^*(T)]/P_{i+1}^{(n+1)}(1) \\ &= [P_i^{(n)}(T+2)h(T+1)h^*(T)]/[P_{i+1}^{(n+1)}(1)P_i^{(n)}(1)] \\ &= [P_i^{(n)}(T+2)h(T+1)h^*(T)]/[P_i^{(n)}(2)h(1)] \end{aligned} \quad (F1)$$

Similarly, a downward move followed by an upward move of the bond price will give us :

$$P_{i+1}^{(n+2)}(T) = [P_i^{(n)}(T+2)h^*(T+1)h(T)]/[P_i^{(n)}(2)h^*(1)] \quad (F2)$$

From the assumption of interest rate path independence, we get:

$$h(T+1)h^*(T)h^*(1) = h^*(T+1)h(T)h(1). \text{ This is equation (9).}$$

$P_i^{(n)}(T)$ for any state i and time n ,

$$P_i^{(n)}(T) = \frac{P(T+n)h(T+n-1)h(T+n-2)\cdots h(T)\delta^{T(n-1)}}{P(n)h(n-1)h(n-2)\cdots h(1)}. \quad (17)$$

Conclusion of this section: Given the initial term structure, and the parameter π and δ , we can determine the subsequent term structure by (17). When we calculate the GNMA mortgage backed securities in the following section, we need the one period discount function. Let $T=1$, equation (17) becomes

$$P_i^{(n)}(1) = [P(n+1)/P(n)]\delta^{n-i}/(\pi + (1-\pi)\delta^n) \quad (18)$$

§-II-3 Modification and Improvement

Ritchken and Boenawan (1990) gives a numerical example showing that Ho and Lee's model provides negative interest rates for some values of the two parameters given the initial term structure. In Ho and Lee's model, the rate (or the yield) is defined as to be

$$r(T) = -\ln P(T)/T. \quad (19)$$

Negative rate means that the discount function $P(T)$ is big than one, $P(T) > 1$. Here, we will show the general case of negative rates that might occur in Ho and Lee's model. First, let's see the one period short term interest rate $T=1$, that is $r(1) = -\ln P(1)$. The discount zero coupon bond price for one period is

$$P_i^{(n)}(1) = [P(n+1)/P(n)]\delta^{n-i}/(\pi + (1-\pi)\delta^n) \quad (18)$$

where, $A=P(n+1)/P(n)$. If $[(P(2)/P(1) - \pi)/(1-\pi)] > \delta$, then $P_1^{(1)} > 1$ for $n=1$ and $i=1$, or $r_1^{(1)}(1) < 0$. In the same way as

above example, we can see that $P_i^{(n)}(T)$ will be greater than 1 for the some subsequent n's and i's given the initial term structure. $P_2^{(2)}(1)$, for example, is equal to

$$P_2^{(2)}(1) = [P(3)/P(2)]/(\pi + (1-\pi)\delta^2). \quad (20)$$

If $[P(3)/P(2) - \pi]/(1-\pi) > \delta^2$, then $P_2^{(2)}(1) > 1$, or the short rate $r_2^{(2)}(1) < 0$, and so on.

Given the initial term structure, equation (15) tells us that the discount bond prices are determined by the two parameters π and δ . If we hold one parameter π (δ) fixed, then the other parameter δ (π) can be chosen to make sure that the discount function will not be greater than 1, namely, the interest rate will not be negative. Ritchken and Boenawan (1990) add one more constraint to the Ho and Lee's AR model to make sure that the interest rates will not be in excess of unit. They require that the parameter δ be within certain range in order to get positive interest rates for all n, i, and T. As a matter of fact, following the proposition II of Ritchken and Boenawan, we can either restrict δ or restrict π to ensure the positive interest rate.

PROPOSITION I ⁴: If all single-period zero coupon bonds are priced below one, and if no single period riskfree arbitrage opportunities exist, then at each node of the vertex the discount function must be monotone nonincreasing.

⁴ Please see Ritchken and Boenawan (1990)

The proposition says that the following expression is true

$$1 = P_i^{(n)}(0) \geq P_i^{(n)}(1) \geq P_i^{(n)}(2) \geq \dots \geq P_i^{(n)}(T) \quad \text{for any } i \text{ and } n. \quad (21)$$

In other words, if we can ensure $P_i^{(n)}(1) \leq 1$ for all n and i , then the proposition II will guarantee all the discount bonds will be priced below unit, and thus, all the interest rates will be positive. From the definition of the binomial lattice, we have $P_{i+1}^{(n)}(1) \geq P_i^{(n)}(1)$ for all $i=1,2,\dots,n-1$. That is, $P_i^{(n)}(1) \leq 1$ for all $i=1,2,\dots,n-1$ if $P_n^{(n)}(1) \leq 1$. So, in order to avoid risk free arbitrage opportunities, the restriction $P_n^{(n)}(1) \leq 1$ must be added for all $n=0,1,2,\dots,T-1$. The above restriction is equivalent to the following, by (19),

$$[P(n+1)/P(n)] / (\pi + (1-\pi)\delta^n) \leq 1 \quad \text{for } n=0,1,\dots,T-1. \quad (22)$$

If we have a predetermined π , then we require

$$\delta \geq \{ [P(n+1)/P(n) - \pi] / (1 - \pi) \}^{1/n} \quad \text{for } 0 \leq n \leq T-1. \quad (23)$$

If we have a predetermined δ , then we require

$$\pi \geq \{ [P(n+1)/P(n) - \delta^n] / (1 - \delta^n) \} \quad \text{for } 0 \leq n \leq T-1. \quad (24)$$

Let

$$\delta_1(k) = \{ [P(n+1)/P(n) - \pi] / (1 - \pi) \}^{1/n}$$

and

$$\pi_1(k) = \{ [P(n+1)/P(n) - \delta^n] / (1 - \delta^n) \} \quad \text{for } 0 \leq n \leq T-1.$$

Then, (18) and (19) are equivalent to $\delta \geq \delta^1$ and $\pi \geq \pi^1$, where

$$\delta_1 = \max_k [\delta_1(k)] \quad \text{and} \quad \pi_1 = \max_k [\pi_1(k)].$$

PART III APPLICATION OF AR MODEL IN VALUING FIXED INCOME
SECURITIES

§ III-1 The Risk Neutral Pricing Formula

III-1-1 Lemma I

Consider any interest rate contingent claim $C(n, i)$, where $C(n, i)$ is the contingent claim price at the vertex (n, i) of the binomial lattice, that can be bought and sold in a conventional perfect financial market.⁵ And assume the contingent claims are interest rate path independent. If no arbitrage profit is to be realized in holding any portfolio of the contingent claim and the discount bonds, the following equation must hold:

$$C(n, i) = (\pi[C(n+1, i+1) + X(n+1, i+1)] + (1-\pi)[C(n+1, i) + X(n+1, i)])P_i^{(n)}(1), \quad (2.1)$$

where $X(n, i)$ is the payoff from the contingent claim to its holder at state-time (n, i) .

Remark: On the left hand of (2.1) is the value at time n . On the right hand of the (2.1), all the components except for the discount factor have the time index of $n+1$. The value of the contingent claims at time n can be derived by the values and the payoffs at the next period $n+1$. In other words, the value of the interest rate path independent contingent claims can be calculated by the backward substitution procedure.

⁵ See Ho and Lee (1990)

III-1-2

From the discussion of chapter 1, we know that GNMA mortgage backed securities are interest rate path dependent contingent claims.⁶ We cannot just simply apply the backwards method in Lemma 1 to our valuation model. In order to calculate the interest rate path dependent contingent claims, we should modify Lemma 1 so that the interest rate path dependent contingent claims can be appropriately valued by the Ho and Lee AR model. One way to calculate the interest rate path dependent contingent claims is simply to calculate all possible interest rate paths, and then calculate the average. This is practical when the time period is not long. But as mentioned in the first chapter, when the time period is very long, the number of the interest rate paths will be tremendously large. It will be too time consuming to calculate all possible interest rate paths. A good alternative method will be the Monte Carlo approach. That is, we pick up some interest rate paths randomly. For each interest rate path, we calculate the cash flows associated with each node for this particular path. Of course, we have to consider the history of the refinancing behavior for the GNMA mortgage backed securities when we determine the cash flows. For each such picked interest rate path, we calculate the present value of

⁶ Because the prepayment behavior depends upon the historical interest rate path.

the interest rate path dependent contingent claims. The one period discount function derived from equation (18) will be used to compute the present value of the cash flows for each path. We repeat this present value calculation n times. The average of the n times calculations will be the value of the mortgage backed securities. Let's call this approach as Lemma I'.

Lemma I': Suppose we want to calculate the interest rate path dependent contingent claims, and suppose the underlying interest rate movements are constructed by Ho and Lee (1986) AR model, then the Monte Carlo approach can be used to calculate the values of the interest rate path dependent contingent claims.

§-III-2 Implementation of AR Model

We know that with the initial yield curve and the two constant parameters ρ and σ , we can get the subsequent yield curve. But still, we have one question unanswered. That is, how do we determine the two parameters? In this section, we will discuss the implementation of the AR model. Specifically, we will discuss how we can estimate the parameters σ and ρ .

III-2-1 Estimation of the probability p that The Interest rate will go up

Let's start from equation (18),

$$P_1^{(n)}(1) = [P(n+1)/P(n)]\delta^{n-1}/(\pi + (1-\pi)\delta^n). \quad (18)$$

Suppose that we have the historical data of one period interest rate $X_0, X_1, X_2, \dots, X_N$. The indexes 1 to N stand for time, i.e., X_1 is the short term interest rate observed at time 1, and so on.

Let q_1, q_2, \dots, q_N be the binary variables which take the values of either zero or one. The value of q is determined in the following way: for any $q_i, i=1, 2, N,$

$$\begin{aligned} q_i &= 1, & \text{if } X_i \text{ is greater than } X_{i-1}, \\ q_i &= 0, & \text{otherwise.} \end{aligned} \quad (2.2)$$

Let small letter p be the probability that the interest rate will go up, and $1-p$ be the probability that the interest rate will go down.

The maximum likelihood function is

$$L = p^{\sum q_i} \cdot (1-p)^{\sum (1-q_i)} \quad (2.3)$$

$$\ln(L) = \sum q_i \cdot \ln(p) + \sum (1-q_i) \cdot \ln(1-p) \quad (2.3)'$$

Solve (2.3), we get:

$$p = \sum q_i / N. \quad (2.4)$$

III-2-2 Trial and Error Method to Get π and δ

Suppose we have a series of actual government bond price data, $X_0, X_1, X_2, \dots, X_N$. Pick up some initial values of δ and π which satisfy the condition (23) or condition (24). By Lemma I or Lemma I', we can calculate the theoretical values of the corresponding bond price. Let the calculated bond prices be Y_1, Y_2, \dots, Y_N .

Define the root mean squared error as follows:

$$\text{RMSE} = \sqrt{(\sum (X_i - Y_i)^2) / N}. \quad (2.5)$$

Try different sets of the parameters π and δ , we get different series of Y . Compute RMSE for each series Y . Choose the set of the parameters δ and π which makes the smallest RMSE.

§-III-3 Simulation and Analysis of the Results

We give the results of two sets of simulations. One simulation is under the assumption of $\delta=0.9999$ and $\pi=0.4945$; the other simulation is under the assumption of $\delta=0.9999$ and $\pi=0.495$. Each estimate was obtained using 5000 trials where each trial corresponds to a randomly selected interest rate path.

The results of the first simulation are reported from Table II-1 to Table II-6. And the results of the other simulations are reported from Table II-7 to Table II-12.

Under the same value of δ , the values of various interest rate contingent claims increase when π decreases. This is because the current interest rate (12%) is quite high compared with the coupon rate (8%). To increase π is to make the term structure bent downward. In other words, the interest rates will go down more likely than the interest rates will go up. When the interest rate goes down, the bond price will go up. Therefore, the discount price goes up, and the present values of the cash flows computed by the discount prices goes up. That's why the values of Table 2-1 to Table 2-6 are smaller than the values from Table 2-7 to Table 2-12.

III-3-1

One question that might be raised here is that : there is a mean-reverting mechanism in Dunn and McConnell's research, but there isn't such explicit expression in AR model. Then, how could we get the similar results in such a degree that the biggest difference in values got from two approaches is less than one dollar? For example, let's look at the last lines (the maturity of thirty years) of Table 2-1 to Table 2-6. When the $\pi=0.4945$ and $\delta=0.9999$, the value of nonamortizing, noncallable bond is \$92.9945, and the value we see in Dunn and McConnell's table is \$93.8808. The difference is 0.8863. The value of nonamortizing, callable bond in Table 2-2 is \$91.7659 and the value of Dunn and McConnell (thereafter D&M) is 91.4326, resulting a difference of 0.3333. The differences of amortizing, noncallable bond, amortizing, callable bond (GNMA with optimal call policy), GNMA with 100 % FHA experience, GNMA with 200% FHA experience are 0.6799, 0.6293, 0.00992, and 0.1606 respectively. So none of the results have bigger than one dollar difference. The results with shorter time to maturity have smaller difference between two approaches.

There might be two possible explanations. The initial yield curve we used in our simulations is exactly the same as D&M (1981b). Because this entire initial term structure is got

from the mean reverting stochastic process, this initial term structure itself might have the characteristic of mean reverting mechanism. The other explanation is that we have carefully selected the two parameters κ and δ by trial and error method which made the value of one interest rate contingent claim quite close to the value by D&M. So we got the very similar results for other contingent claims.

III-3-2

When the term to maturity is less than or equal to two years, the values of nonamortizing, noncallable bonds and amortizing, noncallable bonds are equal to the values of corresponding callable ones. This can be explained as follows: Because the current interest rate is pretty high. It takes quite a long time for the interest rate to get to the rate below the coupon rate. Thus the incentive to call the bonds is really small. When the term to maturity is less than or equal to two years, there is no incentive to prepay the bond at all. Therefore, both noncallable bond and callable bond have the same results when the time to maturity is very short. From the technical point of view, when the time to maturity is very short, there are not many monthly time periods for the binomial process. For example, when the time to maturity is one year, there are only twelve monthly time intervals. With the parameter $\delta=0.9999$ and $\kappa=0.495$, the binomial process won't have any interest rates below the coupon rate at any binomial

lattice. If the interest rate will not drop to below the coupon rate (8%), there will be no optimal call incentives. Consequently, there will be no difference between callable bond and noncallable one under this circumstance.⁷ This is why we see the same values for the first two year's simulations.

The Table II-5 and Table II-6 as well as the Table II-11 and Table II-12 show that even when the time to maturity is very short, the values of 100% FHA experience and 200 % FHA experience have the different values from the amortizing, noncallable bonds. This is due to the fact of suboptimal call policy. We know that when the current interest rate is very high compared with the bond coupon rate, the opportunity cost of holding the bond is very high. With the suboptimal call policy, the GNMA securities might be called even when the refinancing rate is very high.⁸ Thus, the GNMA values increase. Two hundred percent FHA experience will have the higher values than those of one hundred percent FHA experience.

⁷ If the parameters and have different values, the results of two different bonds might be different even with very short time period.

⁸ Please see the discussion of chapter 1.

TABLE II-1

VALUE OF NONAMORTIZING, NONCALLABLE BOND
 ($\alpha = 0.9999$, $\beta = 0.4945$)

YEARS TO MATURITY	NONAMORTIZING NONCALLABLE BOND	STANDARD DEVIATION OF MONTE CARLO ESTIMATES
1	9.700163E-01	1.522152E-03
2	9.544003E-01	4.031897E-03
3	9.460768E-01	7.038549E-03
4	9.416052E-01	1.054611E-02
5	9.389851E-01	1.423074E-02
6	9.375047E-01	1.809964E-02
7	9.365298E-01	2.201220E-02
8	9.358501E-01	2.608775E-02
9	9.353200E-01	2.966524E-02
10	9.348219E-01	3.343830E-02
11	9.344649E-01	3.710353E-02
12	9.341070E-01	4.171039E-02
13	9.338406E-01	4.393235E-02
14	9.333894E-01	4.712922E-02
15	9.332352E-01	5.013711E-02
16	9.328000E-01	5.242098E-02
17	9.325069E-01	5.581900E-02
18	9.322095E-01	5.845408E-02
19	9.319072E-01	6.105025E-02
20	9.316666E-01	6.359565E-02
21	9.316030E-01	6.571931E-02
22	9.314313E-01	6.775790E-02
23	9.309686E-01	6.063199E-02
24	9.307413E-01	7.109834E-02
25	9.307649E-01	7.330004E-02
26	9.304294E-01	7.478935E-02
27	9.303963E-01	7.648987E-02
28	9.302226E-01	7.084003E-02
29	9.300360E-01	7.908744E-02
30	9.299455E-01	8.024389E-02

TABLE II-2

VALUE OF NONAMORTIZING, CALLABLE BOND
 ($\alpha = 0.9999$, $\beta = 0.4945$)

YEARS TO MATURITY	NONAMORTIZING CALLABLE BOND	STANDARD DEVIATION OF MONTE CARLO ESTIMATES
1	9.700163E-01	1.522152E-03
2	9.544003E-01	4.031897E-03
3	9.460706E-01	7.017270E-03
4	9.414763E-01	1.023697E-02
5	9.385229E-01	1.317090E-02
6	9.364725E-01	1.587411E-02
7	9.347493E-01	1.836784E-02
8	9.332298E-01	2.085083E-02
9	9.319489E-01	2.302344E-02
10	9.306154E-01	2.523617E-02
11	9.294838E-01	2.740315E-02
12	9.283960E-01	2.944960E-02
13	9.274759E-01	3.142624E-02
14	9.263432E-01	3.313642E-02
15	9.256537E-01	3.491769E-02
16	9.245888E-01	3.526558E-02
17	9.239487E-01	3.804845E-02
18	9.231349E-01	3.944793E-02
19	9.224951E-01	4.093775E-02
20	9.217816E-01	4.229031E-02
21	9.213448E-01	4.343168E-02
22	9.208292E-01	4.436671E-02
23	9.202473E-01	4.561710E-02
24	9.189839E-01	4.652619E-02
25	9.194794E-01	4.742289E-02
26	9.188869E-01	4.017852E-02
27	9.186511E-01	4.908984E-02
28	9.183263E-01	4.979970E-02
29	9.179561E-01	5.049509E-02
30	9.176591E-01	5.102993E-02

TABLE II-3

VALUE OF AMORTIZING, NONCALLABLE BOND
 ($r = 0.9999$, $\sigma = 0.4945$)

YEARS TO MATURITY	AMORTIZING NONCALLABLE BOND	STANDARD DEVIATION OF MONTE CARLO ESTIMATES
1	9.818776E-01	7.380559E-04
2	9.707883E-01	1.763018E-03
3	9.630399E-01	3.096922E-03
4	9.575137E-01	4.681889E-03
5	9.534004E-01	6.413877E-03
6	9.503320E-01	8.341838E-03
7	9.479128E-01	1.040561E-02
8	9.460150E-01	1.26034E-02
9	9.444473E-01	1.475245E-02
10	9.431116E-01	1.702635E-02
11	9.420186E-01	1.936310E-02
12	9.410785E-01	2.181244E-02
13	9.402661E-01	2.407514E-02
14	9.394482E-01	2.643520E-02
15	9.388794E-01	2.874187E-02
16	9.381813E-01	3.043859E-02
17	9.375941E-01	3.341456E-02
18	9.370488E-01	3.578437E-02
19	9.365396E-01	3.807959E-02
20	9.360905E-01	4.053931E-02
21	9.358331E-01	4.267490E-02
22	9.354080E-01	4.478018E-02
23	9.348470E-01	4.685549E-02
24	9.344957E-01	4.897651E-02
25	9.343053E-01	5.097832E-02
26	9.338425E-01	5.285206E-02
27	9.336357E-01	5.488124E-02
28	9.333482E-01	5.479257E-02
29	9.330177E-01	5.844807E-02
30	9.327951E-01	6.013919E-02

TALE II-4

VALUE OE GNMA WITH OPTIMAL CALL POLICY
 (= 0.9999, = 0.4945 .)

YEARS TO MATURITY	GNMA WITH OPTIMAL CALL POLICY	STANDARD DEVIATIN OF MONTE CARLO ESTIMATES
1	9.818776E-01	7.380559E-01
2	9.707883E-01	1.763018E-03
3	9.630341E-01	3.083225E-03
4	9.574655E-01	4.606646E-03
5	9.532893E-01	6.201897E-03
6	9.500778E-01	7.843815E-03
7	9.474347E-01	9.468019E-03
8	9.452440E-01	1.106767E-02
9	9.433498E-01	1.262540E-02
10	9.416281E-01	1.416084E-02
11	9.401102E-01	1.571387E-02
12	9.387190E-01	1.726532E-02
13	9.374709E-01	1.360031E-02
14	9.361821E-01	2.018771E-02
15	9.351927E-01	2.163675E-02
16	9.341081E-01	2.276520E-02
17	9.330177E-01	2.448941E-02
18	9.320164E-01	2.586481E-02
19	9.311156E-01	2.728114E-02
20	9.301756E-01	2.869655E-02
21	9.294803E-01	2.990399E-02
22	9.286879E-01	3.106276E-02
23	9.278595E-01	3.245724E-02
24	9.266891E-01	3.362003E-02
25	9.265933E-01	3.472563E-02
26	9.257863E-01	3.577898E-02
27	9.252725E-01	3.697231E-02
28	9.246631E-01	3.804714E-02
29	9.240831E-01	3.903395E-02
30	9.235341E-01	3.993959E-02

TABLE II-5

VALUE OF GNMA WITH 100 % FHA EXPERIENCE
 ($\rho = 0.9999$, $\sigma = 0.4945$)

YEARS TO MATURITY	GNMA WITH 100 % FHA EXPERIENCE	STANDARD DEVIATION OF MONTE CARLO ESTIMATES
2	9.752481E-01	1.048547E-02
3	9.688809E-01	1.286951E-02
4	9.627454E-01	1.295322E-02
5	9.585638E-01	1.467488E-02
6	9.558736E-01	1.643809E-02
7	9.539995E-01	1.797890E-02
8	9.523169E-01	2.023801E-02
9	9.512582E-01	2.063025E-02
10	9.502350E-01	2.054781E-02
11	9.493324E-01	2.261945E-01
12	9.485249E-01	2.442634E-02
13	9.475632E-01	2.490614E-02
14	9.454694E-01	2.396426E-02
15	9.408842E-01	2.032611E-02
16	9.393227E-01	2.706298E-02
17	9.364642E-01	2.120071E-02
18	9.350768E-01	2.251966E-02
19	9.338174E-01	2.390502E-02
20	9.325824E-01	2.539429E-02
21	9.317430E-01	2.663752E-02
22	9.308083E-01	2.790423E-02
23	9.300052E-01	2.891912E-02
24	9.293398E-01	2.984270E-02
25	9.284061E-01	3.123084E-02
26	9.276680E-01	3.226050E-02
27	9.269492E-01	3.322938E-02
28	9.257816E-01	3.054858E-02
29	9.255380E-01	3.576335E-02
30	9.248560E-01	3.729030E-02

TABLE II-6

VALUE OE GNMA WITH 200 % FHA EXPERIENCE
 (= 0.9999, = 0.4945)

YEARS TO MATURITY	GNMA WITH 200 % FHA EXPERIENCE	STANDARD DEVIATIN OF MONTE CARLO ESTIMATES
1	9.833109E-01	5.361613E-03
2	9.767497E-01	1.135772E-02
3	9.710390E-01	1.416799E-02
4	9.657431E-01	1.544302E-02
5	9.622839E-01	1.778828E-02
6	9.602324E-01	1.945636E-02
7	9.590182E-01	2.089305E-02
8	9.578954E-01	2.251535E-02
9	9.545871E-01	1.984527E-02
10	9.519753E-01	2.021813E-02
11	9.502016E-01	2.172427E-02
12	9.491350E-01	2.354080E-02
13	9.479036E-01	2.439428E-02
14	9.470955E-01	2.547814E-02
15	9.463908E-01	2.542118E-02
16	9.460271E-01	2.396911E-02
17	9.454237E-01	2.823317E-02
18	9.450200E-01	2.917804E-02
19	9.446890E-01	2.987973E-02
20	9.442078E-01	3.176527E-02
21	9.439124E-01	3.144016E-02
22	9.434038E-01	3.223011E-02
23	9.435262E-01	3.226949E-02
24	9.434595E-01	3.377455E-02
25	9.433157E-01	3.306910E-02
26	9.410696E-01	3.127347E-02
27	9.369833E-01	2.726011E-02
28	9.343800E-01	2.602407E-02
29	9.328706E-01	2.620866E-02
30	9.317374E-01	2.673486E-02

TABLE II-7

VALUE OF NONAMORTIZING, NONCALLABLE BOND
 ($\alpha = 0.9999$, $\beta = 0.495$)

YEARS TO MATURITY	NONAMORTIZING NONCALLABLE BOND	STANDARD DEVIATION OF MONTE CARLO ESTIMATES
1	9.700195E-01	1.522191E-03
2	9.544119E-01	4.031997E-03
3	9.461023E-01	7.038761E-03
4	9.416444E-01	1.054752E-02
5	9.390501E-01	1.423113E-02
6	9.375970E-01	1.810151E-02
7	9.366296E-01	2.201596E-02
8	9.359581E-01	2.509364E-02
9	9.354922E-01	2.967276E-02
10	9.350287E-01	3.344857E-02
11	9.346941E-01	3.711727E-02
12	9.343961E-01	4.672811E-02
13	9.341299E-01	4.395454E-02
14	9.337085E-01	4.715645E-02
15	9.335890E-01	5.016972E-02
16	9.331941E-01	5.245951E-02
17	9.329143E-01	5.586491E-02
18	9.326389E-01	5.850608E-02
19	9.323734E-01	6.111106E-02
20	9.321571E-01	6.366316E-02
21	9.322122E-01	6.579419E-02
22	9.319687E-01	6.784242E-02
23	9.315376E-01	6.972510E-02
24	9.313322E-01	7.130013E-02
25	9.313832E-01	7.341206E-02
26	9.310577E-01	7.490975E-02
27	9.310521E-01	7.661986E-02
28	9.308934E-01	7.798553E-02
29	9.307323E-01	7.923704E-02
30	9.306593E-01	8.010285E-02

TABLE II-8

VALUE OF NONAMORTIZING, CALLABLE BOND
 ($r = 0.9999$, $\delta = 0.495$)

YEARS TO MATURITY	NONAMORTIZING CALLABLE BOND	STANDARD DEVIATION OF MONTE CARLO ESTIMATES
1	9.700195E-01	1.522191E-03
2	9.544119E-01	4.031997E-03
3	9.460951E-01	7.016975E-03
4	9.415187E-01	1.023304E-02
5	9.385777E-01	1.315942E-02
6	9.365936E-01	1.587119E-02
7	9.349176E-01	1.836545E-02
8	9.335062E-01	2.079051E-02
9	9.322037E-01	2.300137E-02
10	9.309554E-01	2.519936E-02
11	9.298305E-01	2.735747E-02
12	9.288675E-01	2.934732E-02
13	9.279002E-01	3.134453E-02
14	9.268486E-01	3.303126E-02
15	9.261482E-01	3.481665E-02
16	9.255756E-01	3.609077E-02
17	9.244667E-01	3.791516E-02
18	9.237334E-01	3.931591E-02
19	9.230706E-01	4.077309E-02
20	9.225348E-01	4.205911E-02
21	9.219521E-01	4.324479E-02
22	9.215123E-01	4.418937E-02
23	9.208725E-01	4.540984E-02
24	9.200145E-01	4.625534E-02
25	9.201053E-01	4.719762E-02
26	9.196349E-01	4.796525E-02
27	9.193292E-01	4.884376E-02
28	9.192313E-01	4.947037E-02
29	9.186235E-01	5.023296E-02
30	9.184595E-01	5.077048E-02

TABLE II-9

VALUE OF AMORTIZING, NONCALLABLE BOND
 ($r = 0.9999$, $\sigma = 0.495$)

YEARS TO MATURITY	AMORTIZING NONCALLABLE BOND	STANDARD DEVIATION OF MONTE CARLO ESTIMATES
1	9.818791E-01	7.378366E-04
2	9.707948E-01	1.762402E-03
3	9.630501E-01	3.096914E-03
4	9.575294E-01	4.182065E-03
5	9.534244E-01	6.404328E-03
6	9.503685E-01	8.342410E-03
7	9.479559E-01	1.040678E-02
8	9.460826E-01	1.260426E-02
9	9.445574E-01	1.575440E-02
10	9.431997E-01	1.701974E-02
11	9.421259E-01	1.936751E-02
12	9.412061E-01	2.181308E-02
13	9.404126E-01	2.401209E-02
14	9.396071E-01	2.544471E-02
15	9.390648E-01	2.875300E-02
16	9.383731E-01	3.045206E-02
17	9.378125E-01	3.343169E-02
18	9.372942E-01	3.580439E-02
19	9.368001E-01	3.810306E-02
20	9.363731E-01	4.056604E-02
21	9.361401E-01	4.270601E-02
22	9.357509E-01	4.480614E-02
23	9.351949E-01	4.589613E-02
24	9.348640E-01	4.902219E-02
25	9.346870E-01	5.102932E-02
26	9.342445E-01	5.290848E-02
27	9.340671E-01	5.494309E-02
28	9.337839E-01	5.086015E-02
29	9.334970E-01	5.852276E-02
30	9.332970E-01	6.022016E-02

TABLE II-10

VALUE OF GNMA WITH OPTIMAL CALL POLICY
 ($\alpha = 0.9999$, $\beta = 0.495$)

YEARS TO MATURITY	GNMA WITH OPTIMAL CALL POLICY	STANDARD DEVIATION OF MONTE CARLO ESTIMATES
1	9.818791E-01	7.378366E-04
2	9.707948E-01	1.762442E-03
3	9.630444E-01	3.083182E-03
4	9.574812E-01	4.606534E-03
5	9.533067E-01	5.198324E-03
6	9.500106E-01	7.840570E-03
7	9.474890E-01	9.464640E-03
8	9.453348E-01	1.104743E-02
9	9.434540E-01	1.261830E-02
10	9.417589E-01	1.414800E-02
11	9.402640E-01	1.569925E-02
12	9.389246E-01	1.720014E-02
13	9.376877E-01	1.877153E-02
14	9.364413E-01	2.015536E-02
15	9.354596E-01	2.059745E-02
16	9.345978E-01	2.270219E-02
17	9.333293E-01	2.443835E-02
18	9.323680E-01	2.580676E-02
19	9.314771E-01	2.721008E-02
20	9.306442E-01	2.857976E-02
21	9.298813E-01	2.981818E-02
22	9.291536E-01	3.097352E-02
23	9.283018E-01	3.235529E-02
24	9.273888E-01	3.346536E-02
25	9.270401E-01	3.460535E-02
26	9.263200E-01	3.566480E-02
27	9.257767E-01	3.683207E-02
28	9.253276E-01	3.784713E-02
29	9.245963E-01	3.887643E-02
30	9.241487E-01	3.978631E-02

TABLE II-11

VALUE OF GNMA WITH 100 $\frac{1}{2}$ FHA EXPERIENCE
 (= 0.9999, = 0.495)

YEARS TO MATURITY	GNMA WITH 100 $\frac{1}{2}$ FHA EXPERIENCE	STANDARD DEVIATION OF MONTE CARLO ESTIMATES
1	9.829538E-01	4.718926E-03
2	9.752518E-01	1.048319E-02
3	9.688853E-01	1.086500E-02
4	9.627611E-01	1.294603E-02
5	9.585736E-01	1.466659E-02
6	9.558987E-01	1.642334E-02
7	9.540321E-01	1.795561E-02
8	9.523630E-01	2.120162E-02
9	9.513263E-01	2.058097E-02
10	9.503207E-01	2.147698E-02
11	9.494136E-01	2.254514E-02
12	9.486473E-01	2.430849E-02
13	9.476808E-01	2.480530E-02
14	9.456037E-01	2.383234E-02
15	9.410400E-01	2.023233E-02
16	9.396733E-01	2.681466E-02
17	9.366734E-01	2.110939E-02
18	9.353574E-01	2.240227E-02
19	9.340938E-01	2.383456E-02
20	9.330412E-01	2.527446E-02
21	9.320644E-01	2.656558E-02
22	9.312250E-01	2.779950E-02
23	9.304153E-01	2.854671E-02
24	9.300815E-01	2.970327E-02
25	9.288727E-01	3.113916E-02
26	9.282192E-01	3.210631E-02
27	9.274689E-01	3.312300E-02
28	9.265317E-01	3.438449E-02
29	9.260762E-01	3.564450E-02
30	9.254936E-01	3.710016E-02

TABLE II-12

VALUE OE GNMA WITH 200 § FHA EXPERIENCE
 (= 0.9999, = 0.495)

YEARS TO MATURITY	GNMA WITH 100 § FHA EXPERIENCE	STANDARD DEVIATIN OF MONTE CARLO ESTIMATES
1	9.833130E-01	5.361043E-03
2	9.767532E-01	1.135517E-02
3	9.710434E-01	1.416321E-02
4	9.657605E-01	1.543539E-02
5	9.622920E-01	1.777900E-02
6	9.602527E-01	1.944025E-02
7	9.590402E-01	2.087102E-02
8	9.579161E-01	2.240902E-02
9	9.540320E-01	2.981405E-02
10	9.520271E-01	2.016403E-02
11	9.502599E-01	2.167324E-02
12	9.492062E-01	2.348353E-02
13	9.480026E-01	2.431706E-02
14	9.472095E-01	2.535566E-02
15	9.465190E-01	2.631467E-02
16	9.463669E-01	2.360515E-02
17	9.455515E-01	2.811030E-02
18	9.451895E-01	2.900736E-02
19	9.448228E-01	2.974958E-02
20	9.444469E-01	3.154090E-02
21	9.440625E-01	3.129005E-02
22	9.436255E-01	3.199710E-02
23	9.437096E-01	3.207932E-02
24	9.439684E-01	3.329674E-02
25	9.435085E-01	3.286709E-02
26	9.413660E-01	3.098730E-02
27	9.371883E-01	2.709763E-02
28	9.347299E-01	2.580135E-02
29	9.331490E-01	2.609061E-02
30	9.321323E-01	2.655780E-02

CHAPTER III

PART I INTRODUCTION

So far, we have got one set of simulations generated by mean-reverting binomial model with Monte Carlo method in chapter 1, and one set of simulations with full information interest rate model by Monte Carlo method in chapter 2. We also have one set of simulation results of Dunn and McConnell, that is the results from the contingent claims model. In this chapter, we will make some comparisons between the results from different models and actual observed GNMA data. Specifically, we are going to question whether one of the models is superior to others, or which model or which approach gives more accurate results, or which result is closer to the actually observed GNMA data.

The data we used are the GNMA data which are from Wall Street Journal and IRS, and the Fama-Bliss discount bond data from CRSP Bond File. In order to make the comparisons, we use the following criterion: the smaller the root of sum of the square, the more accurate the results, and thus, the better the approach.

The organization of this chapter is as follows: Part II gives a description about the GNMA data and Fama-Bliss discount bond price data. Part III gives the results of the regression of simulated data on the actual data in order to compare the correlation between them. Part IV concludes this chapter.

PART II DATA DESCRIPTION

Two sets of observation data are used. One data set is CRSP Bond data, the other is GNMA data. The following is the description about these data sets.

§-II-1 FAMA-BLISS DISCOUNT BOND PRICE DATA AND TREASURY BILL TERM STRUCTURE FILES

The data of Fama-Bliss discount bond prices and Treasury bill term structure files are taken from the Fama files, which is a subset of the CRSP Bond files. Fama-Bliss discount bonds files have the artificial discount bonds with one year to five years to maturity, constructed after first extracting the term structure from a filtered subset of the available bonds. We have taken the data for the period of January 1978 to December 1983, because we have the GNMA data for only that period. These files use only fully taxable, non-callable, non-flower bonds. Four filters were used to select from amongst the remaining bonds a subset from which to construct a term structure. The detailed information about how to get the discount bonds is provided in the CRSP publication "BOND FILES GUIDE", December 1988.

We use monthly steps in calculating the GNMA values. In Famabliss discount bond files, there is no monthly discount price available. So the yearly discount bond prices in Fama-Bliss bond files have to be transferred to monthly bond

prices. We did this by linear interpolation.

Treasury bill term structure files have three sets of data, bid, asked, and average quotes. We picked the average data. The data in the treasury bill term structure files are monthly data, which is suitable for our calculation. Each file in treasury bill term structure files is built by selecting for each month the bill closest to either 6 months to maturity or twelve months to maturity. We took the price data for one to six months to maturity from the data set for 6 months to maturity, and took the price data for 7 months to 11 months to maturity from the data set for twelve months to maturity.

§-II-2 GNMA DATA

The GNMA mortgage-backed securities data are taken from Wall Street Journal and Internal Revenue Services. The data we have in the tapes are from 1978 to 1983. Basically these GNMA data are daily records. Because the discount bond prices from the CRSP bond files are monthly data.

§-III-1 SINGLE FACTOR INTEREST RATE MODEL RESULTS

Chapter one gives the results of modified binomial interest rate movement model and we also printed the results of contingent claims model of Dunn and McConnell. Both of these two models are considered single factor models, because they have only single (one) state of economy variable.

Table-1 and Table-2 on page 32 and page 33 in chapter one are the results of Binomial model. The Dunn and McConnell's results are printed in the tables on page 36 and page 37.

§-III-2 ARBITRAGE FREE RATE MODEL RESULTS

Table III-1 to Table III-10 are the simulation results of arbitrage free interest rate movement model. Table III-11 to Table III-20 are the actually observed GNMA data.

§-III-3 CORRELATION ANALYSIS

Table III-22, Table III-23, and Table III-24 are the regression results. In these tables, the variable ACTUAL is the actual observation data, BINOMI stands for the results from binomial model, SIM is the variable for full information model, and DM stands for Dunn and McConnell.

R-squared in table III-24 is 0.673228, which is the result of the regression of simulation of full information model on the actual data. Table III-22 and table III-23 give the results for the other two models. R-squared of the

binomial model is 0.283001, and R-squared of D&M is 0.144354.

If we assume that the smaller the R-squared, the better fit is the model, we will rank the model in the following order: 1) full information model; 2) binomial model; 3) D&M model. Further more, the full information model is definitely better than single interest factor model.

PART IV CONCLUSION

The full information model needs a lot more data than the single factor model. Does this mean that it needs lot more computaion time? The answer is not necessary. From a theoreticl point of view, we need three hundred sixty discount bond prices for each computaion. We cannot do that in the reality. The market is changing so quickly that when you finish inputting the three hundred sixty data, you might have already lost your money.

As a matter of fact, we only use six different time to maturity discount bond prices in our simulatins. But the results show that the prediction is quite good in a sense the simulation results are better than the single factor model which is widely used in the marketplace.

Further researches should be carried on in: 1) getting more GNMA data, in order to see whether the same conclusion hold for a large data bank; 2) incorporating more sophisticated prepayment function, which might give more accurate reults than the one calculated with 100% FHA prepayment experience.

TABLE III-1

VALUE OF GNMA WITH 100 % FHA EXPERIENCE
 ($d = 0.9999$, $p = 0.495$, $N = 2,000$)
 (Interest rate=6%)

DATE OF OBS.	GNMA WITH 100 % FHA EXPERIENCE	STANDARD DEVIATION OF MONTE CARLO ESTIMATES
19780131	1.001378E-02	6.108621E-04
19780228	1.001352E-02	4.955282E-04
19780428	1.001352E-02	4.921144E-04
19780531	1.001288E-02	1.849515E-04
19800630	9.977800E-01	4.014398E-03
19820831	6.921181E-01	4.863188E-02

TABLE III-2

VALUE OF GNMA WITH 100 % FHA EXPERIENCE

($d = 0.9999$, $p = 0.495$, $N = 2,000$)

(Interest rate=7%)

DATE OF OBS.	GNMA WITH 100 % FHA EXPERIENCE	STANDARD DEVIATION OF MONTE CARLO ESTIMATES
19780331	1.000753E-02	6.421781E-04
19800530	9.956621E-01	3.779250E-02
19820930	1.000689E-02	3.289934E-04

TABLE III-3

VALUE OF GNMA WITH 100 % FHA EXPERIENCE
 ($d = 0.9999$, $p = 0.495$, $N = 2,000$)
 (Interest rate=8%)

DATE OF OBS.	GNMA WITH 100 % FHA EXPERIENCE	STANDARD DEVIATION OF MONTE CARLO ESTIMATES
19780929	9.938803E-02	3.289934E-03
19821029	1.000065E-02	3.675257E-04
19821130	8.029309E-02	6.577569E-02
19821231	9.489584E-01	1.416754E-01
19830131	8.221011E-01	1.416754E-01
19830429	8.333994E-01	7.090473E-02
19830531	7.974130E-01	6.491226E-02
19830630	7.855818E-01	6.317854E-02
19830831	7.273546E-01	5.401424E-02

TABLE III-4

VALUE OF GNMA WITH 100 % FHA EXPERIENCE
 ($d = 0.9999$, $p = 0.495$, $N = 2,000$)
 (Interest rate=9%)

DATE OF OBS.	GNMA WITH 100 % FHA EXPERIENCE	STANDARD DEVIATION OF MONTE CARLO ESTIMATES
19781130	9.639093E-01	6.647491E-02
19781229	9.225662E-01	1.006006E-01
19790629	9.813202E-01	4.851551E-02
19790731	9.624371E-01	8.378965E-02
19791231	9.672618E-01	3.696745E-02
19800430	9.744744E-01	2.083524E-03
19800829	7.501814E-01	5.804539E-02
19811231	6.402533E-01	4.162440E-02
19811231	6.402533E-01	4.162440E-02
19820730	6.531493E-01	4.301121E-02
19830729	7.388025E-01	5.555689E-02
19830930	7.546494E-01	5.821638E-02
19831230	7.484740E-01	5.721309E-02

TABLE III-5

VALUE OF GNMA WITH 100 % FHA EXPERIENCE
 ($d = 0.9999$, $\rho = 0.495$, $N=2,000$)
 (Interest rate=10%)

DATE OF OBS.	GNMA WITH 100 % FHA EXPERIENCE	STANDARD DEVIATION OF MONTE CARLO ESTIMATES
19790330	9.829202E-01	2.108113E-03
19790531	9.682237E-01	6.123893E-02
19790831	9.367740E-01	9.632349E-02
19790928	9.113127E-01	1.066834E-01
19791130	9.674112E-01	2.077385E-03
19811131	9.697278E-01	2.081269E-03

TABLE III-6

VALUE OF GNMA WITH 100 % FHA EXPERIENCE

($d = 0.9999$, $p = 0.495$, $N = 2,000$)

(Interest rate=11%)

DATE OF OBS.	GNMA WITH 100 % FHA EXPERIENCE	STANDARD DEVIATION OF MONTE CARLO ESTIMATES
19800131	8.075562E-01	1.448853E-01
19800930	7.365971E-01	6.458888E-02
19801030	6.952368E-01	4.988605E-02

TABLE III-7

VALUE OF GNMA WITH 100 % FHA EXPERIENCE
($d = 0.9999$, $p = 0.495$, $N = 2,000$)
(Interest rate=12%)

DATE OF OBS.	GNMA WITH 100 % FHA EXPERIENCE	STANDARD DEVIATION OF MONTE CARLO ESTIMATES
19801231	9.538618E-01	3.305138E-03
19820129	6.304811E-01	1.759450E-01
19820226	6.390292E-01	1.892468E-01
19820430	6.531754E-01	1.856339E-01

TABLE III-8

VALUE OF GNMA WITH 100 % FHA EXPERIENCE
($d = 0.9999$, $\rho = 0.495$, $N = 2,000$)
(Interest rate=13%)

DATE OF OBS.	GNMA WITH 100 % FHA EXPERIENCE	STANDARD DEVIATION OF MONTE CARLO ESTIMATES
19810331	6.677028E-01	4.533958E-02
19810930	5.731403E-01	3.309280E-02
19811031	6.565257E-01	3.668734E-02

TABLE III-9

VALUE OE GNMA WITH 100 % FHA EXPERIENCE
($d = 0.9999$, $p = 0.495$, $N = 2,000$)
(Interest rate=14%)

DATE OF OBS.	GNMA WITH 100 % FHA EXPERIENCE	STANDARD DEVIATION OF MONTE CARLO ESTIMATES
19800229	6.923065E-01	4.993378E-02
19810130	6.941064E-01	4.973546E-02
19810227	6.569925E-01	4.412225E-02
19810430	6.249214E-01	3.982803E-02

TABLE III-10

VALUE OF GNMA WITH 100 % FHA EXPERIENCE
($d = 0.9999$, $p = 0.495$, $N = 2,000$)
(Interest rate=15%)

DATE OF OBS.	GNMA WITH 100 % FHA EXPERIENCE	STANDARD DEVIATION OF MONTE CARLO ESTIMATES
19800331	6.915694E-01	4.989523E-02

TABLE III-11

ACTUAL PRICE OF GNMA
(Interest rate=6%)

DATE OF OBS.	GNMA WITH 100 % FHA EXPERIENCE
19780131	N/A
19780228	N/A
19780428	N/A
19780531	N/A
19800627	84.938
19820831	70.969

TABLE III-12
ACTUAL PRICE OF GNMA
(Interest rate=7½)

DATE OF OBS.	GNMA WITH 100 ½ FHA EXPERIENCE
--------------------	--------------------------------------

19780331	N/A
19800530	83.750
19820930	73.875

TABLE III-13

ACTUAL PRICE OF GNMA
(Interest rate=8%)

DATE OF OBS.	GNMA WITH 100 % FHA EXPERIENCE
19780929	92.469
19821029	N/A
19821130	N/A
19821231	N/A
19830131	N/A
19830429	N/A
19830531	N/A
19830630	N/A
19830831	N/A

TABLE III-14

ACTUAL PRICE OF GNMA
(Interest rate=9%)

DATE OF OBS.	GNMA WITH 100 % FHA EXPERIENCE
19781130	91.875
19781229	90.000
19790629	91.625
19790731	90.500
19791231	82.063
19800430	81.313
19800829	75.625
19811231	62.750
19820730	66.781
19830729	N/A
19830930	N/A
19831230	N/A

TABLE III-15

ACTUAL PRICE OF GNMA
(Interest rate=10%)

DATE OF OBS.	GNMA WITH 100 % FHA EXPERIENCE
--------------------	--------------------------------------

19790330	N/A
19790531	N/A
19790831	N/A
19790928	992.469
19791130	91.875
19811130	68.75

TABLE III-16

ACTUAL PRICE OF GNMA

(Interest rate=11%)

DATE OF OBS.	GNMA WITH 100 % FHA EXPERIENCE
19800131	77.375
19800930	75.438
19801030	72.125

TABLE III-17

ACTUAL PRICE OF GNMA
(Interest rate=12%)

DATE OF OBS.	GNMA WITH 100 % FHA EXPERIENCE
19801231	71.656
19820129	62.122
19820226	63.375
19820430	65.063

TABLE III-18

ACTUAL PRICE OF GNMA
(Interest rate=13½)

DATE OF OBS.	GNMA WITH 100 ½ FHA EXPERIENCE
19810331	69.00
19810930	56.313
19811031	57.875

TABLE III-19

ACTUAL PRICE OF GNMA
(Interest rate=14%)

DATE OF OBS.	GNMA WITH 100 % FHA EXPERIENCE	STANDARD DEVIATION OF MONTE CARLO ESTIMATES
19800229	72.188	
19810130	71.938	
19810227	68.375	
19810430	63.625	

TABLE III-20

ACTUAL PRICE OF GNMA
(Interest rate=15%)

DATE OF OBS.	GNMA WITH 100 % FHA EXPERIENCE
19800331	71.031

Table III-21

obs	D1	D2	D3
1	14.84000	16.06200	14.6800
2	-1.758003	30.86100	28.65100
3	15.81600	16.82000	14.94230
4	26.12500	26.69500	24.81500
5	6.919098	7.045998	5.081001
6	4.524200	6.424004	4.455002
7	2.255997	8.299004	6.330002
8	6.504997	6.674004	4.705002
9	5.742996	7.799004	5.830002
10	14.66309	16.23600	14.26700
11	16.13440	16.98600	15.01700
12	-0.606903	22.67400	20.70500
13	1.275299	35.54900	33.58000
14	1.466095	31.51801	29.54900
15	-1.337730	4.782997	2.590996
16	4.866097	5.376999	3.184998
17	28.22270	28.50000	26.31000
18	3.380600	18.77500	16.40500
19	-1.778305	20.71200	18.34200
20	-2.601402	24.02500	21.65500
21	23.73010	23.62400	20.83400
22	0.926098	33.15800	30.36800
23	0.527901	31.90500	29.11500
24	0.254494	30.21700	27.42699
25	-2.229721	25.41000	22.21000
26	1.001301	38.09700	34.89700
27	7.777496	36.53500	33.33500
28	2.957405	21.28200	17.74200
29	-2.527405	21.53200	17.99200
30	-2.675797	25.09500	21.55500
31	-1.132900	29.84500	26.30500
32	1.874100	21.46900	0.000000

- D1 -- The difference between the results from full informatin model and the actual data
- D2 -- The difference between the results from binomial model and the actual data
- D3 -- The difference between the reults from D&M and the actual data

Table III-22

```

SMPL      1  32
32 Observations
LS // Dependent Variable is DM
=====
VARIABLE  COEFFICIENT  STD. ERROR  T-STAT.   2-TAIL SIG.
=====
C          80.655134   5.9089337   13.649694  0.000
GNMA       0.1747684   0.0776845   2.2497210  0.032
=====
R-squared                0.144354
Mean of dependent var    93.81260
Adjusted R-squared       0.115833
S.D. of dependent var    5.071568
S.E. of regression       4.768804
Sum of squared residual  682.2447
Durbin-Watson stat       0.795206   F-statistic  5.061245
Log likelihood           -94.36047
=====
                          Covariance Matrix
=====
C,C                      34.91550
C,GNMA                   - 0.454337
GNMA,GNMA                0.006035
=====
Residual Plot           obs RESIDUAL  ACTUAL
=====

```

Table III-23

```

SMPL      1          32
32 Observations
LS // Dependent Variable is BINOMI
=====
VARIABLE COEFFICIENT   STD. ERROR   T-STAT.   2-TAIL SIG.
=====
C              87.854371    2.6403755  33.273438  0.000
GNMA           0.1194500    0.0347129  3.4410859  0.002
=====
R-squared          0.283001 Mean of dependent var 96.84719
Adjusted R-squared 0.259101 S.D. of dependent var 2.475635
S.E. of regression 2.130915 Sum of squared resid 136.2239
Durbin Watson stat 0.572998 F-statistic      11.84107
Log likelihood     -68.5830
=====
                          Covariance Matrix
=====
C,C              6.971583 C,GNMA          0.090717
GNMA, GNMA      0.001205

```

Table III-24

```

SMPL      1                      32
32 Observations
LS // Dependent Variable is SIM
=====
VARIABLE  COEFFICIENT  STD.ERROR  T-STAT.   2-TAIL  SIG.
=====
GNMA      1.0691187     0.0204530  52.271983  0.000
=====
R-squared 80.36463
Adjusted R-squared 0.673228 S.D. of dependent var 15.39518
S.E. of regression 8.800491 Sum of squared resid 2400.908
Durbin-Watson stat 1.750478 Log likelihood -114.4919
=====
Covariance Matrix
=====
C,C      121.6315   C,GNMA    -1.582726
GNMA,GNMA 0.021023
=====

```

BIBLIOGRAPHY

- Black, F. and M.J. Scholes, 1973, The pricing of options and corporate liabilities, Journal of Political Economy 81, 637-659.
- Boyle, Phelim P., 1977, Options: a monte carlo approach, Journal of Financial Economics 4, 323-338
- Cox, J.C., J.E. Ingersoll Jr., and S.A. Ross, 1978, A theory of term structure of interest rates, Unpublished manuscript, Stanford University.
- Curley, Anthony J. and Jack M. Guttentag, 1974, The yield on insured residential mortgages, Exploration in Economic Research, (Summer), pp.114-161.
- and Jack M. Guttentag, "Value and Yield Risk on Outstanding Insured Residential Mortgages," Journal of Finance, Volume 32, Number 2 (May 1977), pp.403-412.
- Dunn, K.B. and J.J. McConnell, 1981a, A comparison of alternative models for pricing GNMA mortgage-backed securities, Journal of Finance 36, 471-484.
- and J.J. McConnell, 1981b, Valuation of GNMA mortgage-backed securities, Journal of Finance 36, 599-616.
- Hammersley, J.M. and D.C. Handscomb, 1964, Monte Carlo methods (John Wiley, New York)
- Ho, Thomas S. Y. and Sang-bin Lee, 1986, Term structure movements and pricing interest rate contingent claims, Journal of Finance, 41, 1011-1029

- Jacob, David P. and Graham Lord and James A. Tilley, (1985),
Price, duration and convexity of mortgage-backed
securities, The Hand Book of Mortgage-backed Securities,
Probus, Chicago.
- Kleijnen, Jack P.C., 1975, Statistical techniques in
simulation, (Marcel Dekker, New York)
- McKean, H.P., Jr., 1969, Stochastic Integrals (Academic Press,
New York)
- Merton, R.C., 1973, The theory of rational option pricing,
Bell Journal of Economics and Management Science 4, 141-
193
- Ritchken, Peter and Kiekie Boenawan, 1990, On arbitrage-free
pricing of interest rate contingent claims, Journal of
Finance 45, 259-264.
- Robert Lucy and Ronald W. Anderson, "Pool-specific Studies in
GNMA Pass-through Securities," (November, 1987),
unpublished manuscript.
- Schwartz, Eduardo S., 1977, The valuation of warrents:
implementing a new approach, Journal of Financial Economics
4, 79-93.