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Components of a Reading Comprehension Model of Mathematical Problem Solving and Their Relation to Problem Solving Success

by

Stephen J. Pape

A dissertation submitted to the Graduate Faculty in
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Abstract

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by

Stephen J. Pape

Advisor: Professor Carol Kehr Tittle

The NCTM Standards document (1989) set forth a call for the reform of school mathematics instruction. One component of this reform is the use of problem solving to teach mathematical concepts. The model developed for this study incorporates Mayer's (1992) analysis of mathematical problem solving within a model of reading comprehension (Ehri, 1995a, 1995b). This study examines the role of mathematical conceptual and procedural knowledge as well as reading processes in the representation and solution phases of mathematical problem solving.

Forty sixth-grade and 40 seventh-grade students from an urban public school participated. During the first session, students were individually videotaped "thinking-aloud" as they solved consistent (CL) and inconsistent language (IL) compare problems. In a second session, the students completed a computation test and a background questionnaire.

Total computation test score, number of problems solved using a meaningful approach, mean number of rereadings, and mean recall score accounted for 48% of the variance in problem solving success. On IL multiplication and CL division problems, high fraction knowledge students solved a greater number of problems, used a meaningful approach more frequently, recalled a greater number of elements and structure of the problems, and made fewer fraction of a number errors than the low fraction knowledge group. Problem solving success differed as a function of three problem variables: number of computational steps, language consistency, and arithmetic operation. Participants altered their behaviors on two-step versus one-step problems, only. Otherwise, the students did not alter their behaviors (i.e., use a meaningful approach or increase the number of rereadings) as a function of problem type.

Finally, in support of Lewis and Mayer's (1987) consistency hypothesis, students committed a significantly greater number of reversal errors on IL word problems than on CL word problems. However, contrary to Lewis and Mayer's findings, there was no main effect of language consistency for initial reading and total response time. Thus, this study furthers previous research by examining pattern of problem solving behavior, success rates, and problem representations of intermediate school students as a function of their level of mathematical conceptual and procedural knowledge and problem type.

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CHAPTER I

Introduction

The present reform movement in Mathematics education calls for the teaching of mathematics as problem solving (National Council of Teachers of Mathematics [NCTM], 1989). That is, one of the outcomes of mathematics instruction should include the ability to "use problem-solving approaches to investigate and understand mathematical content... Problem solving ... is a process that should permeate the entire program and provide the context in which concepts and skills can be learned" (NCTM, 1989, p. 23). The present study proposes to assess middle-school students' problem solving approaches in order to examine the role mathematical understanding plays in problem comprehension and solution. Specifically, the role of students' conceptual and procedural knowledge related to fractions will be assessed, and the behavior of individuals who do and do not have this knowledge will be examined as they attempt to understand and solve compare word problems.

Although it is clear that problem solving should play a prominent role in the mathematics classroom, individual teacher's conceptions of problem solving varies greatly and the goals or objectives of related instructional practices are not yet fully understood. In a review of the use of problem solving in the elementary grades, Schroeder and Lester (1989) highlight three views related to the teaching of problem solving: "(1) teaching *about* problem solving, (2) teaching *for* problem solving, and (3) teaching *via* problem solving" (p. 32). Middle school children are often asked to solve two distinct types of problems in mathematics classrooms: routine word problems and non-routine problems. Non-routine problems are typically problems for which several methods of solution may be used and no routine method is known. These problems are often used to increase the students' problem-solving or thinking skills. That is, the aim of instruction which uses non-routine problems is either to teach *about* problem solving or to teach *for* problem solving.

In the former case, routine problems may be employed to facilitate the development of children's conceptual knowledge related to arithmetic operations and other mathematical concepts (Carpenter, Hiebert, & Moser, 1981; Carpenter, 1986; Schroeder & Lester, 1989). Schroeder and Lester (1989) state that the learning of mathematics via problem solving

> ... can be viewed as a movement from the concrete (a real-world problem that serves as an instance of the mathematical concept or technique) to the abstract (a symbolic representation of a class of problems and techniques for operating with these symbols) (p. 33).

Well-formed conceptual knowledge, relational knowledge connecting various ideas and meaningfully learned concepts, supports the development of procedural knowledge (i.e., knowledge of the steps of an algorithm) (Baroody & Ginsburg, 1986). That is, when children understand the principles underlying an arithmetic operation and this knowledge is linked to the performance of individual steps of the algorithm, they are better able to remember the steps and perform them with fewer computational errors. Further, the use of routine word problems may allow for the investigation of children's understanding of the mathematical concepts underlying a given word problem.

Problem solving is frequently taught as an application of the arithmetic operations students are assumed to have mastered (Carpenter, et al., 1981; Schroeder & Lester, 1989), rather than using problem solving as a vehicle to support or develop conceptual understanding of these operations. For example, first-graders are typically instructed in the basic operations of arithmetic followed by the application of these algorithms to the solution of story problems. Later instruction seldom departs from this pattern. Word problems are frequently presented to students following instruction in an arithmetic algorithm rather than preceding this instruction. However, these routine problems can serve as effective means of accomplishing the objective, the development of a conceptual understanding, rather than serve only as means of assessing student understanding of the algorithm. Routine word problems are more appropriate for the development of conceptual knowledge than non-routine problems which serve the first objective of problem solving instruction, to impact student's general problem solving skills.

A related implication of these different objectives of problem solving instruction involves pedagogical practices. Problem solving may be presented as prescribed steps or as the application of heuristics such as the key word approach, searching for known key words in the text as indications of the necessary arithmetic operation. In such pedagogical approaches, problem solving is conceptualized as a simple, straight-forward process. In contrast, students need to develop an understanding of or belief about problem solving as a process of searching for meaning and patterns (Hegarty, Mayer, & Monk, 1995; NCTM, 1989). In the present study, children's behavior and processing as they read and solve routine mathematics word problems is examined for the purpose of testing a model of mathematics problem solving.

Researchers who examine mathematical problem solving postulate models of problem solving with two main phases: problem representation and problem solution (e.g., Mayer, 1992; see Figure 1). During the initial problem solving phase, the problem solver reads the elements of the problem and integrates them into a mental representation which serves as the framework to guide problem solution. Since problem-solving activities in schools are frequently presented as story problems based on written text, problem solving success is necessarily linked to reading comprehension ability. However, the relationship between reading comprehension and problem solving has not been established definitively (see Hembree, 1992; Goldin, 1992). With the increased use of traditional word problems as well as more non-routine problem-solving experiences as instructional practices, the investigation of patterns of reading comprehension and problem solving behaviors is increasing in importance.

The present study investigates specific components of a reading comprehension model of problem solving which are essential to the successful solution of mathematics word problems. Specifically, a comprehensive model of mathematical problem solving is proposed (Figure 3) which integrates the knowledge structures described in Mayer's (1992) analysis of mathematical problem solving (Figure 1) and text processing theory (Kintsch, 1994a, 1994b; Kintsch & Greeno, 1985; van Dijk & Kintsch, 1983) within the schematic diagram of reading comprehension (Figure 2) postulated by Ehri (1995a, 1995b). Thus, it is hypothesized that this model may be used to better explain children's behavior while reading and solving routine arithmetic word problems, in particular. This is especially significant with respect to classroom situations where word problems are often used to develop or to assess children's conceptual understanding of arithmetic operations.

The purpose of this research study is to investigate several key factors related to problem comprehension and solution. Briefly, the present study will investigate linguistic elements of particular problem types and their impact on student behavior, the role of mathematical conceptual and procedural knowledge in problem representation and subsequent solution, and the role of strategic behavior in problem solving success. The model of problem solving proposed in this study will be discussed in chapter II. First, an introductory overview of the model will be presented. Next, models of problem solving, text processing, and reading comprehension will be examined. Finally, the comprehensive model will be discussed in greater detail. In chapter III, selected research related to mathematical problem solving will be examined. This will include a discussion of previous research which has examined problems similar to those used in this study.

CHAPTER II

Interactive Model of Mathematical Problem Solving

Overview

Researchers in the field of mathematical problem solving have delineated two distinct yet interdependent phases of the problem solving process, problem representation and problem solution (e.g., Mayer, 1992). In order to solve an arithmetic word problem, the problem solver reads the problem and comes to an understanding of the individual sentences as well as the relationships between these sentences. That is, the problem solver must form a mental representation of the problem (Kintsch & Greeno, 1985; Mayer, 1992). The ability of the problem solver to do this depends upon factors related to reading comprehension. Based on the accuracy of this representation, the problem solver embarks upon a path toward problem solution. Success or failure, then, depends largely on the coherence of the mental representation that the problem solver forms. Thus, the focus of this study is on the representation phase of the problem solving process and, in turn, specific components of a reading comprehension model of problem solving which are hypothesized to influence the accuracy of the representation formed by the problem solver.

In this study, a comprehensive model of problem solving (Figure 3) has been developed to explain the problem solver's behavior as he or she reads the problem, comes to an understanding of the problem, and attempts to solve the problem based on this understanding. This model integrates two bodies of research: (1) models of mathematical problem solving; and (2) models of reading comprehension.

Mayer's (1992) analysis of mathematical problem solving will be discussed first, serving as a general model of problem solving. Next, text processing models (Kintsch, 1994a, 1994b; Kintsch & Greeno, 1985; van Dijk & Kintsch, 1983) will be examined. Researchers in this area have investigated and proposed models which explain how individuals process mathematical text and represent mathematical word problems. This research is incorporated to explain the factors which influence the problem solver's understanding of a mathematical word problem (i.e., the problem representation phase). Finally, reading researchers' efforts to understand reading comprehension provide a framework (e.g., Ehri, 1995a, 1995b) and a methodological perspective (e.g., Pressley & Afflerbach, 1995) for the present investigation of the problem solving process.

In summary, the model of mathematical problem solving (Figure 3) developed for this study integrates knowledge structures posited by Mayer (1992) in his analysis of mathematical problem solving (Figure 1) within the schematic diagram of reading comprehension (Figure 2) postulated by Ehri (1995a, 1995b). In addition, the proposed model expands upon the definitions of the knowledge structures hypothesized by Mayer (1992) to include elements of the text processing model of mathematical problem solving (Kintsch, 1994a, 1994b; Kintsch & Greeno, 1985; van Dijk & Kintsch, 1983) and Ehri's model. The proposed model will be presented in detail in the final section of this chapter.

Mayer's (1992) Analysis of Mathematical Problem Solving

Mayer's (1992) general model (see Figure 1) separates problem solving into two phases. The first phase, problem representation, is further divided into two subphases, translation and integration. According to this model, in order to mentally represent a word problem, the problem solver actively translates its elements into individual propositions and integrates them into a mental representation of the problem. For example, the propositions of a simple arithmetic problem might involve the number of objects in each set included in the problem. These propositions are integrated into a mental representation which reflects the relationships between these elements.

Three types of knowledge structures are postulated to account for and support these two processes. Translation is dependent upon linguistic and semantic knowledge, "knowledge of the English language", and "knowledge of facts about the world" (Mayer, 1992, p.458), respectively. Integration relies upon schematic knowledge; that is, "knowledge of problem types, such as knowing that area problems are based on the formula area = length x width" (Mayer, 1992, p.458).

A well-formed representation of the problem situation facilitates problem solution (Kintsch & Greeno, 1985), the second stage of Mayer's (1992) model. That is, the solution path pursued by an individual depends upon the representation that he or she forms while reading and understanding the problem. If the propositions and their relationships in the word problem are not represented correctly, then the problem solver will attempt to solve a miscomprehended problem (Cummins et al., 1988).

According to Mayer (1992), the problem solution phase may also be subdivided and is facilitated by the activation of two types of knowledge. During the planning and monitoring subphase, the problem solver relies on strategic knowledge or "techniques for how to use the various types of available knowledge in planning and monitoring the solution of problems" (Mayer, 1992, p. 459). Execution, the second subphase of problem solution, involves knowledge of how to perform an algorithm or procedural knowledge.

A Complementary Model of Text Processing

Kintsch and colleagues' model of text processing (Kintsch, 1994a, 1994b; Kintsch & Greeno, 1985; van Dijk & Kintsch, 1983) incorporates both theories of language comprehension and theories of mathematical problem solving. Therefore, this model can be used to further understand Mayer's (1992) problem representation phase. Kintsch hypothesizes that text comprehension consists of two components. First, as the reader processes the individual sentences of the text, he or she derives the propositional structure of the text, the text base. "The verbal input is transformed into a conceptual representation of its meaning, a list of propositions" (Kintsch & Greeno, 1985, p. 111) that represent the essential elements of the text. For example, according to this perspective, the young child who is solving a simple arithmetic word problem must represent the number of objects in each set mentioned in the problem (Kintsch & Greeno, 1985).

Next, these propositions or microstructures are organized into macrostructures which reflect the relationships between the problem elements as presented in the text (Kintsch & Greeno, 1985). The problem solver activates domain specific knowledge in order to organize these propositions into a coherent situational model of the problem. This process involves making inferences about the problem and depends upon the problem solver's prior knowledge. That is, the model formed "is a representation of the content of a text, independent of how the text was formulated and integrated with other relevant experiences" (Kintsch & Greeno, 1985, p. 110). For example, in several of the problems in the present study, if the student does not understand the concept of a fraction of a number, he or she will not be able to translate and then integrate the propositions of the word problem. Thus, the problem solver will fail to construct an accurate mental representation of the problem.

Kintsch and Greeno (1985) propose three types of knowledge structures necessary to form a coherent model of an arithmetic word problem: (1) a set of propositional frames, (2) general schemata for the properties and relationships of sets, and (3) specific schemata for arithmetic operations. Propositional frames facilitate the translation of text propositions. General schemata provide frameworks for particular problem types and aid the construction of macrostructures. Finally, schemata representing mathematical algorithms (e.g., arithmetic operations) are necessary to plan and carry out the solution plan for the problem. Each of these mental structures are specific to the mathematical content and domain of the particular problem and are constructed through repeated exposure to particular problem types (Kintsch & Greeno, 1985). In the present study, the knowledge hypothesized to aid the problem solver form a coherent representation of the word problems is proposed to include conceptual and procedural knowledge of the mathematical operations necessary to solve the different types of problems.

Kintsch and Greeno (1985) use the term schema to represent both knowledge of problem types and what other researchers (Resnick & Ford, 1981) have called procedural knowledge related to mathematical operations. The former concept is also discussed within schema theories of problem solving as an essential component of the problem understanding and solution processes (e.g., Marshall, 1995). In both cases, these schemata are defined as organizational structures or prototypes for specific problem types and are thought to be necessary to fully comprehend mathematical word problems. This concept is also similar to Mayer's (1992) schematic knowledge, but this definition is limited. This notion of schemata may be expanded or better defined if mathematical conceptual and procedural knowledge are separated from it. For example, in the problem types used in this study, children need to understand the concept of a fraction of a number. Without this knowledge, students will have difficulty forming an accurate representation of the problems.

Finally, Kintsch and Greeno (1985) propose a second important type of knowledge. Strategic knowledge is described as the student's understanding related to the employment of schema for the different problem types. It is proposed in the present study that this strategic knowledge is intricately connected to the individual's conceptual knowledge of arithmetic operations. That is, a well-formed conceptual understanding of a mathematical operation is inherently connected to the knowledge of when and how the operation is carried out.

According to Kintsch and Greeno's (1985) model, a well-formed mental representation of the statements of the problem and the relationships between these statements must be formulated in order to successfully solve the problem. These researchers state:

> ... the representation is a dual one: On one side we have the text base representing the textual input, and on the other side an abstract-problem representation, the problem model, which contains the problem-relevant information from the text base in a form suitable for calculational strategies that yield the problem solution (Kintsch & Greeno, 1985, p. 111).

The analysis of mathematical problem solving from a text processing perspective helps to explain the complexities of the representation process. In addition, this perspective highlights the importance of the representation constructed by the student thereby strengthening the claim that reading comprehension, particularly that which depends upon conceptual knowledge, strongly impacts the problem solving process.

Models of Reading Comprehension

Models of reading comprehension have evolved from a traditional view depicting the reader as an inactive participant whose task was to figure out the meaning imposed by the author, to an active participant who interacts with the text to construct meaning (e.g., Rosenblatt, 1994). These more elaborate models posit many factors which influence this constructive process. For example, Rosenblatt (1994) used the term "transaction" to highlight the mediational role of the reader in constructing meaning from text. Schema theory (Anderson, 1984; Marshall, 1995) posits that an individual's knowledge structures or schemata activated by the text function as a lens through which the reader interprets and constructs meaning from text. Finally, the interaction of both top-down and bottom-up processes have been theorized to influence reading comprehension (e.g., Ehri, 1995a, 1995b; Pressley & Afflerbach, 1995).

Each of these complementary models of reading will be examined briefly below. Subsequently, these models will be used in conjunction with the models of problem solving discussed above in the development of a comprehensive model of problem solving. This model is hypothesized to describe more fully the processes related to problem representation within the domain of mathematical problem solving.

Transactional model. Rosenblatt (1994) uses the term transaction, rather than interaction, to describe the relationship that exists between the reader and the text. This term emphasizes the reader's role as the mediator between the author's written word and the meaning of the text constructed by the individual. "The reader and the text are two aspects of a total dynamic situation. The meaning ... happens or comes into being during the transaction between reader and text" (Rosenblatt, 1994, p. 1063). This constructive process is dependent upon all that the reader brings to the reading event including prior experiences as well as the present situation or circumstance.

According to this perspective (Rosenblatt, 1994), the reader selects specific elements of the text to which he or she pays particular attention. Through this selection process, guided by the reader's expectations and previous experience, a framework or mental representation of the text is formed. The understanding of the text as constructed by the reader depends upon those elements of the text selected for attention, as well as the organization imposed on these elements by the reader. Both essential aspects of this constructive process, attention and organization, depend upon the reader's prior experience and knowledge. Similarly, a problem solver chooses to attend to particular aspects of the problem and organizes these elements according to the knowledge and experiences brought to the problem solving situation. This knowledge and experience includes one's understanding of arithmetic operations as well as one's prior experience solving specific problem types.

Constructively responsive reading. Predicated upon a careful review of studies which examined verbal protocols of strategic reading behavior, Pressley and Afflerbach (1995) propose that reading is a "constructively responsive" activity; "that is, good readers are always changing their processing in response to the text they are reading" (p. 2). The ability to appropriately change one's processing is necessarily dependent upon the degree to which the individual monitors understanding while reading.

Pressley and Afflerbach (1995) propose four indicators of "reader construction of meaning in the think-aloud protocols" (p. 98): (1) an active search for meaning; (2) a gradual coming to understanding with early misrepresentations due to interpretations based on prior knowledge but evaluated in light of growing information from the text; (3) a high level of engagement in the text; and (4) a store of prior knowledge that greatly influences processing of the text. This model of reading is based largely on a constructivist belief that individuals actively seek meaning. "New information is not simply received, but rather humans construct hypotheses about the meaning of new information and test those hypotheses against subsequent input" (Pressley & Afflerbach, 1995, p. 103).

The belief that individuals filter information provided by the text through their existing knowledge base has significant implications for the discussion of problem representation in problem solving. The extent to which students' mathematical knowledge represents well-formed conceptual knowledge will greatly influence the representation formed by the individual. The problem solver monitors this construction and, according to Pressley and Afflerbach's (1995) depiction, will alter his or her processing depending upon the degree to which the text is understood. This change in behavior depends upon whether the information in the text reflects existing knowledge structures or is in opposition to these ideas. In the present study, the problem solver who does not have an understanding of the concept of a fraction of a number and, therefore, has difficulty forming a meaningful representation of the problem, may alter his or her behavior by rereading (or other behavior) the text sentences to come to a meaningful understanding of the text.

Schema theory model. The organizational component of text comprehension described by Rosenblatt (1994) and the role of prior knowledge in the construction of meaning for text (Pressley & Afflerbach, 1995) is echoed in Anderson's (1984) schema theory of reading comprehension. From Anderson's perspective, text comprehension begins when the reader activates a schema or "organized knowledge of the world" (p. 469). The particular schema activated facilitates and guides the organization of the elements of the text. The representation of the text is fundamentally dependent upon the individual's knowledge within the domain (i.e. schemata).

From this perspective, a complete and coherent understanding of the text is not possible simply through an additive process at the word level. In other words, according to Anderson's model, comprehension "... is a matter of activating or constructing a schema that provides a coherent explanation of objects and events mentioned in a discourse" (Anderson, 1984, p. 473). Therefore, bottom-up processes (i.e., text-based processes) are not sufficient for text comprehension. These ideas are reiterated within the domain of mathematical problem solving (e.g., Marshall, 1995).

Interactive model. In her discussion of the development of reading processes, Ehri (1995a, 1995b) proposes an interactive model of reading comprehension (Figure 2). This model hypothesizes the simultaneous interaction of both bottom-up and top-down processes while reading. That is, these two levels of processing are essential for the construction of meaning from text and act in concert with one another as the reader processes text. Bottom-up processing is contingent upon two types of knowledge: knowledge of the grapho-phonic system and lexical knowledge. The former represents an understanding of the relationship between letters and their sounds and of the spelling system. This knowledge is necessary for the reader to sound out words that are not in his or her lexicon, or sightword vocabulary (Ehri, 1995a, 1995b). In Mayer's (1992) model, linguistic knowledge also includes domain specific vocabulary.

According to Ehri's (1995a, 1995b) description of the reading process, print acts as a stimulus which is interpreted by the reader as he or she interacts with the written text to construct meaning. In order to understand the text, the reader must also activate top-down processes. Three types of knowledge structures are proposed to facilitate these processes: knowledge of language, knowledge of the world, and metacognitive knowledge. "Knowledge of language enables readers to process sentences and their meanings" (Ehri, 1995, p. 11) and is also incorporated within Mayer's (1992) linguistic knowledge construct. The reader's knowledge of the world supports his or her understanding of the text and is developed through everyday experiences. Within the domain of mathematics word problems examined in this study, this knowledge construct is similar to the schematic knowledge as discussed earlier, but may be thought to be separate and different from conceptual and procedural knowledge of mathematics. For example, when reading math word problems which involve a particular mathematical concept (e.g., calculating a fraction of a number), the individual's conceptual understanding of that operation facilitates problem representation or understanding.

However, these constructs are distinct from knowledge of the world as defined by Ehri (1995a, 1995b) due to the domain specificity of conceptual knowledge of arithmetic operations. Thus, in the present study, conceptual and procedural knowledge are hypothesized to influence the problem solver's ability to construct a meaningful representation of a word problem. In the model presented in Figure 3 these constructs appear separate from Ehri's knowledge of the world construct. Mayer (1992) postulates three knowledge structures which coincide with Ehri's (1995a, 1995b) category of knowledge of the world: semantic knowledge, schematic knowledge, and procedural knowledge. This final type of knowledge is proposed only to affect the problem solver's execution of a problem solution and is not implicated in the representation processes (Mayer, 1992).

Finally, Ehri (1995b) proposes that "readers use their metacognitive knowledge to monitor the quality of their comprehension and to verify that the information makes sense and that it meets specific purposes" (p. 11). Similarly, Mayer's (1992) analysis postulates the necessity of strategic knowledge which aids the planning and monitoring of problem solution.

Interactive Model of Problem Solving

The model of problem solving proposed in this study (Figure 3) represents a more comprehensive analysis of mathematics problem solving processes than has been developed previously (e.g., Mayer, 1992). This is due to the inclusion of factors from theories of text processing and reading comprehension models in addition to theories of mathematics problem solving. In so doing, it expands upon and fully defines the knowledge structures Mayer originally proposed in his analysis.

The model proposed in this study (Figure 3) integrates knowledge structures from three sources: (1) Ehri's (1995a, 1995b) interactive model of reading, (2) Mayer's (1992) analysis of mathematical problem solving, and (3) domain specific mathematical knowledge. These groups of overlapping knowledge structures are depicted within the framework provided by Ehri's original model (refer to Figure 3) and described more fully below. The knowledge structures posited by Ehri are listed first in each box. These include knowledge of the grapho-phonic system, lexical knowledge, knowledge of language, knowledge of the world, and metacognitive knowledge. Mayer's (1992) constructs are depicted in Figure 3 as italic print. These knowledge structures include linguistic, semantic, schematic, procedural, and strategic knowledge. Finally, those components uniquely distinguished in this model are in bolded uppercase letters.

Mathematical conceptual and procedural knowledge are depicted separate from Ehri's knowledge of the world and from Mayer's schematic knowledge. This has been done to emphasize each as essential to the representation as well as the solution phase of problem solving. Also, by depicting mathematical knowledge as separate from Mayer's (and others') schematic knowledge, this model holds that both problem type schemata and more general mathematical knowledge are important to the problem solving process. Finally, the processing space in Ehri's model has been expanded to include Mayer's phases of the problem solving process.

Two additional expansions of Ehri's (1995a, 1995b) bottom-up constructs are depicted in the model: (1) Knowledge of the grapho-phonic system has been expanded to include knowledge of mathematical symbols, and (2) knowledge of words associated with mathematical concepts and operations has been included within the original construct lexical knowledge. The latter two additions are thought to expand rather than redefine the original constructs to take into account the specific domain of text processing of interest in this study (i.e., arithmetic word problems).

Following Mayer's analysis (1992), the present model depicts problem solving as consisting of two types of processes, problem representation and problem solution. However, these processes are posited to occur simultaneously and each is thought to feed into the other in an iterative fashion. That is, as the problem solver reads each sentence and forms a proposition for the information presented (Kintsch & Greeno, 1985) the text is interpreted through a filter of the prior knowledge of the student. As

Rosenblatt (1994) explained, the reader selectively attends to specific aspect of the text and organizes these elements into a mental representation. This selection and organization process depends greatly upon the individual's conceptual and procedural knowledge and prior experience.

According to Pressley and Afflerbach (1995), in order to form an accurate mental representation of the problem, the problem solver must be active. That is, the reader must monitor his or her comprehension of the problem and alter his or her problem solving behavior accordingly. In Figure 3, the metacognitive or strategic knowledge structures proposed (Ehri, 1995a, 1995b; Mayer, 1992) have been expanded to include the monitoring of problem representation as well as problem solution. Again, these processes feed back into one another in an iterative process. That is, based on an initial attempt to form a representation of the problem, the problem solver either rereads the problem or begins to solve it. The representation process is proposed to be gradual; initial understandings are altered based on the problem solver's conceptual and procedural knowledge as well as continued processing of the text.

While information from the problem's text enters the processing space the reader simultaneously activates two levels of knowledge structures (i.e., knowledge required for bottom-up and top-down processing). As each sentence is read, those knowledge structures proposed by Ehri (1995a, 1995b) and by Mayer (1992) as well as domain specific mathematical conceptual and procedural knowledge are hypothesized to influence the mental representation of the problem text base. Therefore, the model proposed here represents a more comprehensive analysis of mathematical problem solving in two ways. First, the model distinguishes between conceptual and procedural knowledge and Mayer's (1992) schematic knowledge. Second, by using a reading comprehension model as a framework for problem solving and by investigating problem solving behavior from reading researchers perspectives the proposed model provides for a better understanding of the components which influence the problem solving endeavor.

CHAPTER III

Research Related to Mathematical Problem Solving

Overview

Aspects of mathematics word problems which affect children's performance and have been investigated include: (1) the semantic structure of word problems (e.g., Carpenter, Ansell, Franke, Fennema, & Weisbeck, 1993; De Corte & Verschaffel, 1987; Riley & Greeno, 1988); (2) the effect of context (i.e., elaborate versus decontextualized problem context) (e.g., Davis-Dorsey, Ross, & Morrison, 1991; De Corte, Verschaffel, & De Win, 1985); and (3) different problem types and their associated levels of complexity (e.g., Briars & Larkin, 1984; Cummins et al., 1988). The present review is focused on research that has used one particular type of problem, compare problems, and has investigated the influence of language consistency within these problems. This research has contrasted the use of consistent language (CL) versus inconsistent language (IL) structures in the relational sentences of these arithmetic problems (e.g., Hegarty, Mayer, & Green, 1992; Hegarty et al., 1995; Lewis & Mayer, 1987; Lewis, 1989; Verschaffel, De Corte, & Pauwels, 1992; Verschaffel, 1994).

Mathematical word problems which incorporate relational statements have been classified as compare problems and have been shown to cause considerable difficulty for students of various levels (Carpenter et al., 1993;

Carpenter et al., 1981; De Corte & Verschaffel, 1987; Hegarty et al., 1995; Riley & Greeno, 1988). Two examples of compare problem at different levels of complexity are provided below:

1. Joe has 3 balloons. His sister Connie has 8 balloons. How many more balloons does Connie have than Joe? (Hiebert, Carpenter, & Moser, 1982, p. 87).
2. Joe runs 6 miles a week. Ken runs 3 times as many miles a week as Joe does. How many miles does Ken run in 4 weeks? (Lewis & Mayer, 1987, p. 366).

This second problem is an example of those which are investigated in the present study.

In an early study of the effect of relational statements, Huttenlocher and Strauss (1968) investigated "how one's understanding of a statement may depend upon the relation between that statement and the extra-linguistic situation it describes" (p. 300). Nursery-school, first-, and second-grade students were required to place blocks on a ladder-like apparatus following the relationship presented in the statement (i.e., "on top of" or "under"). These young students were able to form a meaningful representation of the elements of the relational statement when the movable object (i.e., the block that was to be placed above or below another object) was the subject rather than the object of the sentence.

The children in this study (Huttenlocher & Strauss, 1968) committed fewer errors when the relationship between the elements was expressed in a format that was consistent with the situation it represented. That is, problem-solving success is dependent upon the student's ability to map the relationship implied in the problem onto an organizational network or schema for the problem. This contention is supported by error pattern analysis studies in which error patterns have been shown to be a function of ill-formed mental constructions of the problem statements (e.g. Cummins et al., 1988; De Corte & Verschaffel, 1981, 1987). These contentions are also supported by theoretical investigations of problems solving (e.g., Kintsch & Greeno, 1985).

Research Related to Consistent and Inconsistent Language Compare Problems

Based on the premise that problem solving errors depend largely on the miscomprehension of story problems, Lewis and Mayer (1987) investigated problem comprehension, the translation and integration phases of Mayer's (1985, 1992; Mayer, Larkin, & Kadane, 1984) analysis of problem solving. These researchers used compare story problems involving consistent (CL) and inconsistent language (IL). These problems combine the complexity of relational statements and inconsistent language both of which have been found to cause difficulty for problem solvers.

CL and IL compare problems are three sentences long and involve either one or two computational steps. The three statements include an assignment statement, a relational statement, and a question (Lewis & Mayer, 1987; Mayer, 1981, 1982). The first sentence introduces the known quantity and assigns a value to that item. The second statement is the relational or compare sentence which presents the relationship between the unknown and the known quantities (i.e., states the quantity of the unknown in relation to that of the known quantity). The value of the unknown must be calculated using the information presented in the first two sentences of the problem. Finally, in the one-step compare problems, the third sentence states the question which requires the problem solver to simply state the value calculated for the unknown item. In two-step compare problems, the question posed in the final sentence involves the calculation of a multiple of the previously unknown quantity (i.e., a direct variation problem).

The relational sentence in these problems may be presented in either a consistent or an inconsistent form. In CL problems, the unknown is the subject of the comparison sentence, and the relational term is consistent with the arithmetic operation required to solve the problem (e.g., "more than" and "n times as many" signify the use of addition and multiplication, respectively). In IL problems, the unknown is the object of the comparison sentence, and the relational term is inconsistent with the arithmetic operation required (e.g., "more than" and “n times as many” signify subtraction and division, respectively). Below, two examples are provided to illustrate the differences between the language in consistent and inconsistent multiplication compare problems, respectively.

1. Joe runs 6 miles a week. Ken runs 3 times as many miles a week as Joe does. How many miles does Ken run in 4 weeks?
2. Joe runs 6 miles a week. He runs 1/3 as many miles a week as Ken does. How many miles does Ken run in 4 weeks? (Lewis & Mayer, 1987, p. 366).

Inconsistent language relational statements are hypothesized to require greater cognitive processing than CL problems. Lewis and Mayer (1987) proposed the consistency hypothesis to explain this difference. According to this hypothesis, the increased processing necessary results from the wording of the problems and a preference for the language of the CL problem. That is, an individual comes to the problem solving situation with a schema for the CL relational statement. When the language of the second sentence does not match this schema the problem solver reformulates the sentence by reversing its grammatical subject and object. Consequently, the relational term must also be changed to its opposite (e.g., in the IL problem above, “1/3 as many” is switched to “3 times as many”). In this way, the relational sentence is reformulated in the consistent format, but this reformulation results in a greater proportion of reversal errors. This type of error is made when the opposite arithmetic to that which is actually required to solve a problem is used to solve a given problem.

Several testable hypotheses can be derived from Lewis and Mayer's (1987) consistency hypothesis. First, a greater number of errors will be committed on IL than on CL problems. Further, a greater proportion of reversal errors will occur on IL than on CL problems. In addition, although not tested in their study, due to the extra processing necessary to reverse the relational sentence, problem solver response time will be greater for IL than for CL problems.

To test the consistency hypothesis, Lewis and Mayer (1987) conducted two experiments to investigate problem comprehension among undergraduate students. In Experiment 1, participants (N = 96) were required to solve two-step addition, subtraction, multiplication, and division problems and were instructed to show all work. Incorrect solutions were examined and coded for three types of errors. An error resulting from the use of the opposite arithmetic operation than required to solve the problem was coded as a reversal error. These errors were hypothesized to occur on IL problems but not on CL problems. An arithmetic error was coded when an arithmetic computation was carried out incorrectly. Finally, a goal monitoring error was coded when the second step of the problem (i.e., the direct variation) was ignored. Out of 768 solutions from the 96 participants only 53 reversal errors, 22 arithmetic errors, and 6 goal monitoring errors were made. This represents a very small proportion of the problems, probably a result of the sample of undergraduate students studied and the basic arithmetic operations in the problems used. That is, in order to isolate the effect of language consistency, Lewis and Mayer controlled for the difficulty of the mathematical content of the word problems.

ANOVA procedures revealed a significant main effect for language consistency; IL problems resulted in a greater number of reversal errors than CL problems (Lewis & Mayer, 1987, p. 366). In addition, the results supported a main effect of direction of operation (i.e., an increase versus a decrease) and a significant interaction between language consistency and direction of increase. That is, participants committed a greater number of reversal errors on addition and multiplication problems than subtraction and division problems. The greatest proportion of reversal errors were committed on IL addition and multiplication problems versus IL subtraction and division problems. Arithmetic errors were committed at the same rate for all problem types.

Lewis and Mayer (1987) discuss the salience of the terms "less", "shorter", or "younger" in inconsistent addition problems and "1/n as many" in inconsistent multiplication problems as the cause for the greater number of errors on these problems. That is, problem solvers are said to be influenced more by these marked terms (e.g., less, 1/n as many) than by unmarked terms (e.g., more, n times as many) (Clark, 1969; Lewis & Mayer, 1987). Lexically unmarked terms represent positive adjectives while marked terms are negative adjectives. According to Clark's (1969) theory, the cognitive representations of these terms differ in complexity, and, thus, problem solvers have greater difficulty surmounting the salience of marked terms resulting in a greater proportion of reversal errors on IL addition and multiplication problems.

An alternative explanation for these findings will be tested in the present study. Although the salience of lexically marked terms may explain the behavior of problem solvers on addition and subtraction problems, the complexity of the mathematical concepts underlying the multiplication and division problems may result in the greater number of errors on these problems. In other words, the present study will test the alternative hypothesis that the increased error rates depend more on the individual's conceptual understanding of the relationships embedded in compare problems (e.g., "1/n as many") rather than the lexical markedness of the relational terms. That is, among middle school children, the concept of "1/n as many" is not well formed. Thus, within this population, a greater number of errors is likely on multiplication and division problems which include the wording “1/n as many” in their relational sentence (i.e., IL multiplication and CL division problems) than on problems which do not include this wording (i.e., addition, subtraction, CL multiplication and IL division problems).

Finally, to explain the results of the study, Lewis and Mayer (1987) propose a model for the translation of compare problems based on the activation of schemata in the comprehension process (p. 368), and hypothesize probability estimates for reversal errors. According to this model, the individual who possesses a preference for consistent language reverses the order of the relational sentence when he or she reads a sentence involving inconsistent language.

Based on the data collected, Lewis and Mayer (1987) propose probability estimates of committing reversal errors on CL and IL problems. For CL problems, errors were estimated to be close to zero. The probability of making a reversal error on an IL problem was estimated to be 0.0833. Further, when the relational term in the IL problem is marked (i.e., “less than” or “1/n as many”) the probability of making an error was again estimated to be 0.0833. The probability of making a reversal error on IL subtraction and division problems (i.e., those which contain unmarked relational terms) were estimated at 8% and on IL addition and multiplication problems (i.e., those which contain marked relational terms) probability estimates were 17%. The actual data accurately fit these probability estimates with reversal errors occurring at a rate of 0%, 1%, 0% and 1% for CL addition, subtraction, multiplication and division problems, respectively. Further, participants committed reversal errors at a rate of 8% on both IL subtraction and division problems, while IL addition and multiplication problems were solved incorrectly at rates of 17% and 20%, respectively. The actual and predicted probabilities were highly correlated (r = .99) (Lewis & Mayer, 1987). Thus, the results substantiate the proposed model, but the estimated probabilities were derived from the data in the study. Thus, they need to be replicated within a similar population.

In addition, due to the undergraduate sample investigated the proportion of errors in general was quite low. As stated above, an alternative hypothesis for the larger proportion of errors on IL multiplication problems may lie in the complexity of the underlying mathematical concept (i.e., a fraction of a number) rather than in the lexical markedness of the term. The authors do not account for the differences in difficulty of the underlying mathematical concepts of the various problem types. Finally, the greater number of errors on IL problems may not result from a reversal of the relational sentence but may be the result of the application of a key-word problem solving strategy which results in accurate solutions on CL problems but not on IL problems.

In Experiment 2, 26 college students were presented with two assignment statements and were required to state the relationship between the two quantities (Lewis & Mayer, 1987). Two sets of these pairs of assignment statements were generated from the addition/subtraction and multiplication/division compare problems presented in the first experiment. The initial sentence of each was the first assignment statement of the original problem, and the second sentence consisted of an assignment statement for the unknown in the original problem that resulted from carrying out the arithmetic operation presented in the original relational statement. For example, the first problem illustrated above might be rewritten as the following (Lewis & Mayer, 1987, p. 369):

> Joe runs 6 miles a week.
>
> Ken runs 18 miles a week.

The participants' task was to write a statement reflecting the relationship between the two given quantities. Student responses were examined for a preference for unmarked versus marked relational terms. On all problems, the proportion of unmarked terms used in the relational statements was significantly greater than marked terms. The authors conclude:

> These results support the contention that students come to an arithmetic word-problem-solving task with an established set of schemata ... for comprehension and subsequent representation of

problem sentences. Problem representation forms that fail to adhere to the preferred information order can lead to miscomprehension and faulty representation (Lewis & Mayer, 1987, p. 370).

Further examination of the two sets of problems included in this investigation may clarify the findings of this experiment and suggest a potential alternative explanation for the results found. On problem sets 1-4 (Lewis & Mayer, 1987, p. 369), those derived from the addition and subtraction problems of the first experiment, no multiplication or division relational statements were written by the participants. On sets 5-8, those derived from the original multiplication and division problems, both addition/subtraction and multiplication/division problems were written. This finding may support the premise of the present study that the problem solver's conceptual understanding of the mathematical concept of a fraction of a number exerts a greater effect on the formation of the relational statement and, therefore, is potentially the cause of the larger proportion of reversal errors on IL multiplication problems than other problems.

Among the sample of college students in Experiment 1, arithmetic and goal setting errors were not common, and arithmetic errors appeared at an equal rate on all problem types. Therefore, Lewis and Mayer (1987) conclude that the linguistic elements of the target problems may be isolated as the cause of the misrepresentation and subsequent error patterns.

Based on the model of students' comprehension of relational statements proposed in Lewis and Mayer (1987) and the finding that problem comprehension is an important phase in problem solving, Lewis (1989) investigated the impact of diagramming instruction for compare problems on problem representation and problem solving success. In addition, the researcher hypothesized "that students who are instructed in identifying problem statements but not trained in how to integrate problem information will not improve in compare-problem comprehension" (Lewis, 1989, p. 524). Thus, the study set out, first, to replicate the findings of Lewis and Mayer (1987), and, second, to test the importance of both sub-phases (i.e., translation and integration) within the problem representation phase of Mayer's (1985, 1992; Mayer et al., 1984) model of problem solving.

Two hundred ninety-nine undergraduate students were pretested on CL and IL addition, subtraction, multiplication and division compare problems. The students were required to solve 22 math word problems including 8 target problems and 4 transfer problems. Participants' written solutions were analyzed. Ninety-six out of the 299 students committed at least one reversal error and were asked to continue with the training sessions (Lewis, 1989).

The greater proportion of reversal errors on inconsistent addition and multiplication problems found in the study (Lewis, 1989) supports Lewis and Mayer's (1987) consistency hypothesis. Those participants who continued with the treatment phase of the study (N = 96) were randomly assigned to one of three training groups (N=32 per group): (1) the diagram group; (2) the statement group; and (3) the control group. Participants in the first two groups were exposed to transparencies which defined the three types of sentences in compare problems (i.e., assignment, relational statement, and question). This treatment was hypothesized to influence the students' ability to translate the sentences in the problem. The diagramming group was also taught to use a number line to depict the relative magnitude of the quantities in the problem. This procedure was hypothesized to impact the integration phase of problem solving. Thus, the diagramming group received instruction in both translation and integration, while the statement group received training in translation only. The control group was exposed to the training problems, only.

The diagramming procedure as described by Lewis (1989) is somewhat mechanistic. In addition, the author states that it will not necessarily transfer to problem types other than compare problems but is potentially useful for examining the importance of problem representation (i.e., translation and integration). This is because the procedure facilitates the representation of the problem and the monitoring of solution progress. Further, transfer problems in this study were compare problems involving two relational statements and, therefore, two compare steps. These problems only differ from the target problems in their degree of complexity and are therefore examples of near transfer rather than far transfer. The use of the diagramming procedure is an example of a domain specific strategy useful only with these problem types.

The results supported the consistency hypothesis with a main effect for language consistency and a significant language consistency by direction of change interaction (Lewis, 1989). Also, the diagram group committed fewer reversal errors on both the posttest target and transfer problems and on transfer problem representation and solution. An interesting finding relative to the statement group involved their committing more errors on the filler posttest items than either the training or the control group. Lewis attributed these results to a focus on the surface structure fostered by paying exclusive attention to statement types. Thus, training in translating problem statements only (i.e., identifying problem statements in compare problems) does not lead to significant gains when compared to an intervention which impacts both sub-phases of problem comprehension, translation and integration.

Test of Lewis and Mayer's (1987) Results

To test Lewis and Mayer's (1987) consistency hypothesis, Verschaffel et al. (1992) conducted a series of three experiments contrasting adults versus children and one-step versus two-step compare problems among adults. In Experiment 1, 19 undergraduate students were administered 30 one-step addition and subtraction problems. The 16 target problems consisted of 8 CL and 8 IL addition and subtraction problems which were presented on a video monitor. Eye-fixation data was collected as the participants read the problem and stated the operation they would use to solve the problem. Thus, this experiment investigated the first three stages (i.e., translation, integration, and planning) of Mayer's (1985, 1992; Mayer et al., 1984) model. Further, eye-fixation protocols and time segments were parsed into two phases: (1) the initial reading of the first two sentences, and (2) the time from the first fixation on the third sentence until the participant stated his or her solution plan. Total response time was also calculated.

The performance of the undergraduate students did not support Lewis and Mayer's (1987) consistency hypothesis (Verschaffel et al., 1992). Due to the simplicity of addition and subtraction one-step problems, few errors, reversal or otherwise, were committed. Similarly, no effect of language consistency and a non-significant trend in the direction opposite to that hypothesized for total time and duration of the initial reading of the first two sentences were found (Verschaffel et al., 1992).

In Experiment 2, 15 third-grade students were administered 26 word problems including 16 target problems. The procedures were the same as in Experiment 1. The consistency hypothesis was strongly supported within this population. More reversal errors were committed on IL problems than on CL problems (31.6% vs. 3.3%, respectively). No difference was found for addition and subtraction CL problems; students committed reversal errors on approximately 3.3% of these problems. "... on IL problems pupils made more reversal errors on problems involving addition (41.6%) than on those involving subtraction (21.6%)" (Verschaffel et al., 1992, p. 90). These error probabilities are greater than those proposed by Lewis and Mayer (1987) which supports the alternative hypotheses discussed above. The students also took a significantly longer time to solve IL problems than CL problems and in reading the first two sentences of IL problems than CL problems. The eye-fixation data also support an effect of language consistency; participants fixated for longer periods of time on sentences one and two while reading IL problems than CL problems.

The results of the first two experiments contradict one another within two very different populations. The one-step addition and subtraction problems were not difficult for the undergraduate students and may not have placed significant demands on the cognitive abilities of these students. Thus, in the third experiment of this study (Verschaffel et al., 1992), 20 undergraduate students read and solved 24 two-step compare problems. These problems included 6 each of IL and CL addition, subtraction, multiplication, and division problems and no filler problems. All testing conditions were similar to the previous two studies except the participants were required to solve the problems. A serious limitation of this experiment involves the use of eye-fixation data equipment. Since participants were required to actually solve the problems while constantly looking at the video equipment so that eye-fixation data could be collected, the required calculations in the problems were greatly simplified. This simplification was necessary so that the participants could solve the problems without requiring pen and paper computations.

Although overall errors were quite low (7.5% vs. 1.7% for IL vs. CL problems, respectively), the number of reversal errors was greater on IL problems than on CL problems. Further, participants committed more reversal errors on IL addition and multiplication (13.3% errors) than on IL subtraction and division problems (1.7% errors) (Verschaffel et al., 1992). These results support the consistency hypothesis and the effect of markedness within compare problems, but the error rates were again lower than that originally predicted by Lewis and Mayer (1987).

Time variables also supported the effect of language consistency. Participants took longer to solve IL problems than CL problems, but there was no significant difference between IL and CL problems in initial reading times for the first two sentences. These results contradict Lewis and Mayer's (1987) model for the translation of the sentences in compare problems which postulates that the rearrangement of the relational sentence occurs during the initial reading of the problem. In addition, the authors note that only those protocols which included successive fixations on the first two sentences before fixating on the third sentence were analyzed. There were 80 out of 480 protocols that could not be included for the purposes of this analysis. "For some subjects, we even obtained empty cells for one or more of the problem types ..., which made it impossible to estimate the contrasts for the different sentences separately" (Verschaffel et al., 1992, p. 92). The authors do not indicate which problem types resulted in empty cells for some of the students. Finally, eye-fixation data indicated significantly longer fixations on sentence 2 for IL problems than for CL problems. This difference was not found for the first and the last sentences (Verschaffel et al., 1992).

Verschaffel et al. (1992) discuss potential alternative explanations for the contradictory findings in their three investigations. Experiment 1 findings seem to support van Dijk & Kintsch's (1983) idea of text coherence.

"According to this notion, interpretation and integration of new textual information about an agent is facilitated when this new information starts with the same agent" (Verschaffel, 1992, p. 93). According to this interpretation, IL relational sentences are easier to interpret than CL problems. This result is supported by the findings of Experiment 1. Second, the presence of pronouns in general in IL problems and not in CL problems may cause increased difficulty for the problem solver. Third, reversal errors may not be due to greater linguistic difficulty of IL problems but actually to the application of a simple strategy such as the key word strategy that is often taught in schools. Finally, the authors call for the use of alternative data collection procedures. "In the present study, the analysis of eye movement data was restricted to aggregated fixation times for individual sentences during the entire solution process and during one particular phase" (p. 94). The number of backward eye movements and/or the number of rereadings of each sentence may be better indicators of the difficulties students encounter with these problems.

In a follow-up study (Verschaffel, 1994), 10 and 11 year old students read, solved and recalled one-step addition and subtraction word problems. The students' recall followed solution and was prompted by showing them the numbers of the quantities in the problem. In support of the consistency hypothesis within a younger population, the number of reversal errors was greater on IL than CL problems. In addition, the participants took longer to solve IL than CL problems, and CL problems were retold correctly more often than IL problems. An interesting finding resulted from the inversions that students made when retelling the problems. As expected, IL problems were retold more often as CL problems than CL problems were retold as IL problems (61% vs. 1%, respectively). Verschaffel called for the use of think alouds in the future investigation of the problem representation and solution of compare word problems.

In a related study, Hegarty et al. (1992) attempted "to identify the locus of these comprehension difficulties more precisely by monitoring students' eye fixations as they read and prepared to solve arithmetic word problems" (p. 76). That is, the researchers were attempting to identify the specific phase of problem solving (i.e., translation, integration, planning or execution; Mayer, 1985, 1992; Mayer et al., 1984) during which comprehension difficulties were encountered by examining response time data.

> We predict that if students attempt to construct a model of the problem, then they will frequently reread lines of the problem, focusing on the specific numbers, variable names, and relational terms. If subjects have more difficulty in building a (nonquantitative) model of inconsistent problems than on consistent problems, we

> expect that they will reread the relevant variable names and relational terms (but not the numbers) more often for inconsistent problems than for consistent problems (Hegarty et al., 1992, p. 78).

Thirty-eight undergraduate students were asked to read and state a solution plan without carrying out the necessary computations. Only inconsistent and consistent two-step addition and subtraction problems were analyzed in this study (i.e., four problems involving the combination of language consistency and lexical markedness). Each of the target problems was presented on a video monitor equipped with eye-fixation data collection equipment. By interrupting the problem solving process just before the execution phase (i.e., by requiring only a solution plan), the researchers sought to isolate the sub-phase during which students perceive the inconsistency and attempt to rectify the discrepancy within either the translation, integration, or planning sub-phases of problem solving. According to Lewis and Mayer's (1987) hypothesized model for the translation of compare problems, if the problem solver comes to the problem solving situation with a schema for the language of the CL relational statement and the sentence is written in the inconsistent format, thus violating this framework, then the individual will attempt to rectify the discrepancy to fit his or her expectations. In this study, accurate and inaccurate problem solvers were investigated separately because poor problem solvers who are less sensitive to the wording of the problems may not take longer to read or plan IL problems than CL problems.

First, solution time analyses provided the researchers the opportunity to investigate whether greater processing is necessary for inconsistent problems. Second, eye-fixation data were parsed into two segments: (1) the initial reading of the problem; and (2) the time for integration and solution planning beginning from the end of the initial reading until the statement of a solution plan. This allowed the researchers to identify and segment the protocols into phases and to examine whether the translation phase or the integration and planning stages is the locus of student difficulty.

> If the solution plan is constructed immediately, then inconsistency between a relational statement and its operation should affect the time taken to initially read the problem. If the solution plan is constructed after the initial reading, then initial reading time should be equivalent for consistent and inconsistent problems (Hegarty et al., 1992, p. 78).

By eliminating the need for carrying out the solution, this study attempted to isolate the point of comprehension difficulty within the first three sub-phases of Mayer's (1985, 1992; Mayer et al., 1984) analysis of mathematical problem solving.

Eye-fixation protocols of two groups, high-accuracy (N = 15; one or no errors out of 4 target problems) and low-accuracy (N = 11; two or more errors out of 4 target problems), were analyzed separately. These groups were formed to test whether the consistency hypothesis was supported equally within each. Low-accuracy students are thought to be less metacognitively sensitive to problem features and, therefore, may not take longer to solve the inconsistent problems. Since they do not attend to the inconsistency of the problem's language, their response time on the IL problems may not be longer than for CL problems (Hegarty et al., 1992).

The proportions of reversal errors on CL and IL problems were compared. A significant difference was found for language consistency but not for lexical markedness (Hegarty et al., 1992). These findings in the absence of actually executing the solution plan support the hypothesis that comprehension difficulties are present prior to this last sub-stage of problem solving (i.e., the execution stage), but the lack of a significant main effect for direction of change contradicts earlier findings. Time analyses involving language consistency and lexical markedness support the researchers' hypotheses. IL problems and those involving lexically marked terms took longer to solve than CL problems and those containing unmarked terms, respectively.

Contrary to the original interpretation of the consistency hypothesis and the Lewis and Mayer (1987) model for translating compare problems, initial reading time was not different for CL and IL problems nor for problems with marked versus unmarked terms. Thus, the rearrangement that is hypothesized for IL problems may not occur during the initial reading of the problem. A difference in the time for the integration and planning phases among accurate students further clarifies the model proposed. Among these students, greater processing time was required for IL problems during the integration and planning phases. "The picture emerging from these results is that consistency affects the integration and planning phases rather than the initial reading of the problem or the final execution of the plan" (Hegarty et al., 1992, p. 81). Parallel analyses were conducted for the low-accuracy group.

Among low-accuracy students, there was no significant difference in total response time, initial reading time, nor integration and planning phase time for CL and IL problems (Hegarty et al., 1992). The first and third results contrast with those found for high-accuracy students while the lack of a difference for initial reading coincides with that found for the complementary group. This failure to attend to the differences in these problems and adjust processing time for the more difficult problems is attributed to low-accuracy students' lack of metacognitive awareness.

Hegarty et al. (1992) examined the eye-fixation protocols of a high-accuracy student in greater detail to reveal patterns of behavior on each of the four different types of problems, but the analyses presented are not comprehensive. On average, the participants read the numbers at the same rate for CL and IL problems, but on inconsistent problems they focused more on the relevant background information (i.e., variable names and relational terms). This indicates that the processing requirements necessary to form a mental representation of IL problems is greater than that for CL problems, but language consistency does not affect processing with respect to the specific quantities mentioned in the problem.

To further investigate the individual's pattern of behavior as they read and planned their solutions, extensive protocols of 15 high-accuracy students were developed. These transcripts consisted of a listing of each line the student looked at and the words in that line on which the individual fixated. All fixations which followed the initial reading of the problem were considered to be rereadings. From these protocols, three patterns were developed: the funnel effect, the selection effect, and the consistency effect. According to the funnel effect, individuals concentrate on progressively smaller proportions of the words of each sentence each time that they reread individual sentences. According to the selection effect, the problem solver refers to the numbers more than the other words in the sentences on successive rereadings. Finally, the consistency effect refers to the observation that individuals fixate on words other than the numbers more in IL than in CL problems.

Protocols of one high-accuracy student on a CL and an IL problem are presented. On the CL problem which did not cause the participant difficulty, the pattern of behavior follows what might be termed a direct translation approach (see later discussion of Hegarty et al., 1995). That is, the protocol exhibits a pattern of behavior involving 11 rereadings which mainly focused on the numerals. This pattern exemplifies the funnel and selection effects. In contrast, the high-accuracy student's protocol on an IL problem exhibited 13 rereadings and a greater focus on the words of the problems including the variable names and relational terms than on the numbers. Although this protocol also exhibits the funnel and selection effects, this pattern of behavior illustrates what may be called a meaningful approach to problem solving because the problem solver attempts to formulate a meaningful mental model of the problem (Kintsch & Greeno, 1985; Hegarty et al., 1995). The analyses carried out by Hegarty et al. (1992) seem to support the hypothesis that low-accuracy students tend to use a direct-translation approach rather than a meaningful approach as their predominant pattern of behavior. For example, although protocols of inaccurate students are not included, time data revealed that they did not spend additional time processing IL problems than CL problems. In contrast, students using a meaningful approach would spend more time on IL problems than on CL problems.

Contrasts Between the Behavior of Good and Poor Problem Solvers

Differences in the processing of successful and unsuccessful problem solvers were examined in an investigation of patterns of problem solving behaviors of good and poor problem solvers. Hegarty et al. (1995) propose that the comprehension process involves three stages: construction of the text base, construction of a mathematics-specific representation, and construction of a solution plan. This description parallels the depiction of the problem solving process presented in this study.

The first stage occurs in incremental phases and is based on knowledge of the three statement types for word problems (i.e., assignments, relational sentences, and questions). The following description is similar to that proposed by text processing theorists (e.g., Kintsch & Greeno, 1985).

> In constructing a text base, the problem solver must represent the propositional content of this [each] statement and integrate it with the other information in his or her current representation of the problem.... This integration involves making referential connections between the different statements in the problem.... Thus, a primary

> task of the problem solver is to translate each statement into an internal representation and to integrate this internal representation with the representation of other statements in the problem to construct a semantic network representation (Hegarty et al., 1995, p. 20).

During the second stage, the problem solver must create a mathematics-specific representation of the problem elements. An individual using a direct translation approach selects the numbers and relational terms from the text and directly translates these elements into their associated arithmetic operations without constructing a mental model for these propositions. Thus, an individual who uses the direct translation approach focuses on the numbers and relational terms in the word problem as opposed to the variable names. Conversely, an individual using a meaningful approach formulates an object-based representation of the situation or mental model of the relationships depicted in the problem. The mental model formed by the problem solver serves as the basis for a solution plan (Hegarty et al., 1995).

The main difference between these two patterns of behavior is hypothesized to result in differences in the accuracy of the representation formed and subsequent success or failure. This difference lies in the elements of the text the individual relies upon to represent the problem and plan a solution strategy. That is, using a direct translation approach, the individual bases the arithmetic operations performed on the key words in the problem. Using a meaningful approach, the problem solver bases his or her solution plan on the mental model formed for the specific problem.

In Experiment 1, eye movement data of 38 undergraduate students was analyzed to detect text processing differences between the two groups. In addition, participants were exposed to several sets of similar problems to examine whether successful and unsuccessful problem solvers would, in the absence of feedback, change their pattern of behavior over repeated exposures. Participants read four sets of 48 arithmetic word problems including 16 target and 32 filler problems from a video screen and were required to state their solution plan for each problem. The target problems included two-step consistent and inconsistent language addition and subtraction problems, only. The decision to have participants state their solution plan rather than solve the problems stemmed from the magnitude of the numbers in the problems and the data collection procedure used in the study. Eye-fixation data collection equipment placed similar limitations on this study (Hegarty et al., 1995) as in Verschaffel et al. (1992). Further, the use of addition and subtraction problems with an undergraduate population minimizes the influence of mathematical knowledge.

Participants were divided into two groups: successful and unsuccessful problem solvers. The former group consisted of those individuals who made zero or one error, and unsuccessful problem solvers were those who committed four or more errors. The analyses presented in this study were limited to the eight most successful and the eight least successful individuals.

Among unsuccessful problem solvers, the language consistency effect was supported. These participants made more errors on IL problems than CL problems. In addition to the type of errors committed, three objective measures were determined from the protocols for each target problem: the number of times the student looked back at a number, at a relational term, and at a variable name following the initial reading of the problem (Hegarty et al., 1995).

The percentage of rereadings that were focused on the numbers and relational terms was greater for unsuccessful students (66.3%) than for successful problem solvers (59.4%). This difference, although significant, is not large. For all subjects, the number of fixations on numbers and relational terms was greater than the number of fixations on variable names. "Unsuccessful problem solvers reexamined numbers an average of 16.3 times per problem as compared with 11.2 times for successful problem solvers" (Hegarty et al., 1995, p. 24). Again, although this difference is significant, it does not necessarily imply a difference in the predominant pattern of behavior of these two groups, and differences between the number of times successful and unsuccessful individuals fixated on the variable names in the problems were not found.

Since the problems were limited to addition and subtraction compare problems, differential patterns of behavior which may be more obvious on more difficult problem types (i.e., CL and IL multiplication and division problems) could not be investigated. Despite these limitations, the researchers suggest the following:

> The picture that emerged from this analysis was that unsuccessful problem solvers struggle more than do successful problem solvers to construct a representation of the problem but spent their additional effort mainly in reexamining numbers and relational terms rather than in reexamining other informative words (Hegarty et al., 1995, p. 24).

Successful problem solvers needed to reread the problem statements less frequently than unsuccessful problem solvers, and when they did reread they focused on the numbers and relational terms less often than unsuccessful problem solvers. However, examination of the data presented warrants further investigation of the conclusions put forth by the researchers.

An important aspect of this study (Hegarty et al., 1995) is that the participants were exposed to the problem sets four times. Problem solving behaviors of successful and unsuccessful students were compared to investigate changes in each group's behaviors over time. Unsuccessful problem solvers were found to remain at the same level of performance over the four trials. Although feedback was not provided which may be considered a serious limitation of the study, the authors state that these individuals did not learn from their mistakes. In contrast, successful problem solvers made few errors on the first two trials and even fewer (close to zero) errors on the third and fourth trials.

Over the four trials the unsuccessful problem solvers maintained their ineffective pattern of behavior. They continued to focus on the numbers and relational terms more than the variable names. Further, over the trials, successful participants needed to review the information in the problems less and did not focus on the numbers and relational terms more than the variable names in the problems. That is, they may have begun to take on a more meaningful approach over the four trials, but they still looked at the numbers and relational terms more than the variable names. In addition, the change in behavior of successful problem solvers may be an indication that they formed a schema for the problem types over repeated exposure.

In Experiment 2, Hegarty et al. (1995) further investigated the behaviors of these two groups by requiring participants to recall and to recognize the problems they had solved. Thirty-seven undergraduate students solved 12 arithmetic problems and were required to show all work needed to solve the problems. Four of these problems were target problems from the first experiment. Following completion of the problems, the participants were asked to recall the four target problems. The recall test consisted of a four prompts related to the context of each target problem (i.e., "gas", "butter", "package delivery", and "workers in fast-food restaurant) (Hegarty et al., 1995, p.27). Next, on the recognition test, participants were required to circle the problem they had solved given two options.

Two types of errors were calculated on both the recall and the recognition tests. A semantic error was coded when the recalled or recognized problem involved the opposite arithmetic operation from the original problem. "A literal error occurred when the student produced a problem on the recall test or circled a problem on the recognition test in which the relation between the two terms was consistent with the presented problem, but the wording of the relational term was changed" (Hegarty et al., 1995, p. 27). For example, a literal error consisted of accurately rearranging the elements of the IL relational statement to match the associated CL problem.

The results of this investigation showed that successful problem solvers made significantly fewer semantic errors than unsuccessful individuals, and unsuccessful problem solvers made significantly fewer literal errors. Hegarty et al. (1995) suggest that this pattern may be interpreted to support the contention that "the successful problem solvers were more likely than unsuccessful problem solvers to construct a problem model while comprehending the word problems, whereas unsuccessful problem solvers were more likely to use a direct-translation strategy for encoding word problems ..." (p. 27).

Summary of Previous Research Findings

Based on the premise that solution of mathematical word problems depends on the mental representation formed by the problem solver, Lewis and Mayer (1987) investigated problem solving success on CL and IL compare problems among undergraduate students. Compare problems involve two factors, relational sentences and inconsistent language, which cause problem solvers difficulty. These authors propose the consistency hypothesis and a model for the translation of the relational sentence in IL problems to explain greater error rates on these problems. Accordingly, the problem solver is said to come to the problem solving situation with an expectation for the language of the CL problem and, when this language is altered, he or she rearranges the structure of the relational sentence to fit this expectation. In addition, Lewis and Mayer (1987) found higher error rates for IL addition and multiplication than IL subtraction and division problems. The former problems contain relational terms (i.e., "less than" and "1/n as many") that are called lexically marked (Clark, 1969).

Subsequent research has supported these claims (e.g., Lewis, 1989), but a few studies (e.g., Verschaffel, 1994; Verschaffel et al., 1992) have resulted in some question as to the explanation provided by Lewis and Mayer (1987). In a review of the findings of previous studies, Verschaffel (1994) suggests that the results may be due not to a reversal of the relational sentence but to the application of the key word strategy. In addition, each of the studies reviewed here minimized the effect of mathematical conceptual and procedural knowledge on the problem representation process by investigating either young children as they solved addition and subtraction problems or undergraduate students. As a result, error rates reported in earlier studies are quite low. In addition, many of these studies have used eye-fixation data which is limiting due to the constraint that the problem solver must focus on a computer screen and, therefore, the calculations required to solve the word problems have been simplified.

Verschaffel (1994) investigated fifth-grade students as they solved one-step addition and subtraction compare problems. The results of this study lends additional support to the Lewis and Mayer (1987) explanation, but Verschaffel offers potential alternative explanations for the results and calls for the use of "new (combinations of) techniques for data gathering and data analysis" (p. 161). The present study addresses some of these concerns by employing video-taped verbal protocols of sixth- and seventh-grade students as they solve one- and two-step addition, subtraction, multiplication, and division problems. Moreover, requiring students to recall the word problems following solution will provide additional evidence related to the mental representation of the problem formed. Finally, it is hypothesized that the understanding of the concept of a fraction of a number will vary. Therefore, students' behavior and success will depend on the presence or absence of the phrase "1/n as many" within the relational sentence of the problem which appears in IL multiplication and CL division problems. Thus, it is hypothesized that error patterns within this group of students and for the problems examined in this study will differ from those which have been found in the past.

CHAPTER IV

Purposes and Hypotheses

Purposes of the Study

The primary purpose of this study was to investigate components of the Interactive Model of Problem Solving (Figure 3) proposed in this study. This model has been developed to account for reading comprehension factors related to mathematical word problem comprehension and representation. Mayer's (1985, 1992; Mayer et al., 1984) general analysis of mathematical problem solving posits a critical role of three types of knowledge, linguistic, semantic, and schematic knowledge, in the translation of word problem statements and in the formation of a mental representation based on these propositions. Ehri's (1995a, 1995b) interactive model proposes similar knowledge types necessary for comprehension of text. These include knowledge of language, knowledge of the world, metacognitive knowledge, knowledge of the grapho-phonic system, and lexical knowledge. Some of the terms used by these two theorists overlap. For example, both use the term semantic knowledge but define this term very differently.

The model of problem solving proposed in the present study incorporates two additional sources of knowledge. Mathematical conceptual and procedural knowledge are hypothesized to be highly interrelated and to be key components in the comprehension and representation processes which, in turn, have been shown to be important to problem solving success (e.g., Cummins, et al., 1988; De Corte et al., 1985). It was proposed that by incorporating these additional knowledge constructs this model explains the behavior and success rates of middle-school children as they solve consistent and inconsistent language compare problems better than the previous models investigated (e.g., Lewis & Mayer, 1987; Mayer, 1985, 1992).

Mayer's (1985, 1992; Mayer et al., 1984) general analysis of mathematical problem solving evolved, in part, from earlier studies of problem solving behavior of individuals solving consistent and inconsistent language compare problems (e.g., Lewis, 1989; Lewis & Mayer, 1987). Lewis and Mayer (1987) proposed the consistency hypothesis and a model for the translation of relational sentences in compare problems subsequent to the investigation of undergraduate students' problem solving behaviors. The results and implications of this investigation (Lewis & Mayer, 1987) have also been examined within samples of young children (e.g., third- and fifth-graders; Verschaffel, 1994; Verschaffel et al., 1992) as well as within additional samples of undergraduate students (e.g., Hegarty et al., 1992, 1995; Lewis, 1989; Verschaffel et al., 1992).

Each of these studies minimized the effect of mathematical conceptual and procedural knowledge on the problem representation process by examining either young children and undergraduates as they solved addition and subtraction compare problems or undergraduate students as they solved addition, subtraction, multiplication and division problems. As a result, error rates in each of these studies were found to be quite low. It is hypothesized that when the mathematical conceptual and procedural knowledge demands in a problem solving situation are not minimal (i.e., within a sample of middle school children solving multiplication and division compare problems), both Mayer's (1985, 1992; Mayer et al., 1984) knowledge constructs and the mathematical conceptual and procedural knowledge proposed here will be found to be critical factors related to accurate problem representation and successful problem solution.

Further, Hegarty et al. (1995) examined eye-movement protocol data to investigate successful and unsuccessful problem solvers' behavior. Poor problem solvers were found to use a predominant pattern of behavior called the direct translation approach. Accordingly, poor problem solvers focus more on the numbers and relational terms in the problem than the variable names and directly translate these elements into a solution plan. Conversely, successful problem solvers were hypothesized to use a meaningful approach (i.e., a focus on the variable names and background information as well as numbers and relational terms) in building a mental model of the problem elements. However, the data did not provide conclusive evidence of this pattern for all problem types (i.e., CL and IL addition, subtraction, multiplication and division compare problems). Similar to the behavior of the poor problem solvers, successful problem solvers focused more on the numbers and relational terms than the background information presented in the problem. Further, in this study Hegarty et al. (1995) assumed that the participants, undergraduate students, possessed the necessary mathematical knowledge to solve these word problems.

Therefore, the present study extends previous research in two ways. First, it is hypothesized that when the conceptual and procedural knowledge demands of a word problem are great both the successful and the unsuccessful problem solver may attempt to use a meaningful approach (i.e., may adjust behavior to model the behavior of successful problem solvers). However, in the absence of a well-formed conceptual understanding of the underlying mathematical content (e.g., the concept of "a fraction of a number" in IL multiplication and CL division problems), the problem solver will be unsuccessful in forming an accurate representation of the problem.

Within the sample chosen for this study, sixth- and seventh-grade students, participant's understanding of the concept of a fraction of a number will vary. If this concept is not well formed, a student may attempt to use a meaningful approach to represent the propositions of the problem. For example, he or she may reread whole sentences of the problem or restate the elements of the problem with appropriate context. However, the problem solver will be unable to form an accurate representation of the problem. Without a mental model of the problem, the child will fail to solve the problem. Therefore, overall problem solving success will vary depending upon mathematical knowledge, predominant pattern of problem solving behavior including the number of rereadings, and the quality of the representation formed. Further, on those problems which contain the fraction of a number wording (i.e., IL multiplication and CL division problems), students' problem solving success, pattern of behavior, number of rereadings, quality of problem recall, and types of errors committed will differ as a function of the students' knowledge of a fraction of a number.

Second, the effect of problem type on problem solving success, on the predominant pattern of observed problem solving behavior including the number of rereadings, on problem representation (i.e., problem recall), and on the types of errors committed will be examined. The cognitive demands associated with each type of problem varies as a function of a combination of three factors: (1) the number of computational steps (i.e., one- vs. two-step problems), (2) language consistency (i.e., CL vs. IL problems), and (3) the underlying mathematical concept in the wording of the relational statement of the problem. The cognitive demands of a particular type of problem are hypothesized to influence the problem solvers' predominant pattern of behavior, problem representation, and problem solution. Several of these analyses will test Lewis and Mayer's (1987) consistency hypothesis and their model of problem translation within a middle-school population.

In summary, four sets of analyses were performed. First, the relative influence of mathematical knowledge (i.e., total score on computation test), pattern of behavior including the number of rereadings, and problem representation (i.e., quality of recall) on problem solving success was investigated. Second, the influence of mathematical knowledge (i.e., knowledge of the concept of a fraction of a number) on middle-school children's problem solving success, pattern of behavior, number of rereadings, quality of recall, problem solving success, and types of errors committed was tested. Verbal protocols for IL multiplication and CL division problems were compared for participants who do and do not demonstrate an understanding of the concept of a fraction of a number. Third, the effect of problem type on problem solving success, predominant pattern of behavior including the number of rereadings, and problem representation was examined. Finally, this study directly tested hypotheses related to

Lewis and Mayer's (1987) consistency hypothesis. The number of reversal errors, initial reading time, and total response times were compared for CL versus IL problems.

Hypotheses

Tests of components of the Interactive Model of Problem Solving. The first hypothesis examines the relative influence of aspects of the proposed model of problem solving (Figure 3) on overall problem solving success.

H1: Problem solving success will vary as a function of mathematical knowledge (i.e., total computation test score), predominant pattern of behavior, number of rereadings, and problem representation (i.e., quality of recall).

The second set of hypotheses examine the influence of students' understanding of requisite mathematical knowledge on problem solving success, on the problem solver's pattern of behavior including the number of rereadings, on the resulting problem representation formed, and on the types of errors committed. On IL multiplication and CL division problems, those compare word problems which involve the wording "1/n as many" in their relational sentence, students who demonstrate a lack of understanding of the concept of a fraction of a number will attempt to use a meaningful approach, but will be unsuccessful in so doing.

H2: On IL multiplication and CL division problems, there will be a significant difference between students who demonstrate a lack of understanding of the concept of a fraction of a number and those who do not lack such understanding in the following directions -- students for whom this mathematical knowledge is not well formed will:

(1) solve fewer problems than students who have such knowledge;

(2) use a meaningful approach more often than students who have such knowledge;

(3) reread problem sentences more than students who have such understanding;

(4) recall fewer elements and structure of the original word problem than students who have such knowledge; and

(5) commit a greater number of fraction of a number mathematical errors than students who have such knowledge while solving these word problems.

Influence of problem type. The following hypotheses examined differences due to the cognitive complexity of the different problem types analyzed in this study. The four comparisons based upon the problem type include: (1) one-step versus two-step multiplication and division compare problems (four problems each); (2) CL versus IL compare problems (six problems each); (3) IL multiplication and CL division versus CL multiplication and IL division problems; and (4) addition and subtraction versus multiplication two-step problems.

H3: Problem solving success, predominant pattern of behavior, number of rereadings, and quality of recall will vary as a function of problem type.

Replication and extension of previous research findings. The final set of hypotheses examined Lewis and Mayer's (1987) consistency hypothesis and their model for the translation of compare problems within a sample of middle-school children.

H4: Problem solvers will commit a greater number of reversal errors on IL problems than on CL problems.

H5: Student initial reading time on IL problems will be greater than for CL problems.

H6: Student total response time on IL problems will be greater than for CL problems.

CHAPTER V

Methods

Participants

All sixth-grade (N = 288) and seventh-grade (N = 335) students who attend a public intermediate school in New York City were considered for participation in the study. The school's population is predominantly White (73%) and Hispanic (20%) with 5% Asian and 3% African American students. The researcher was introduced to the students during their weekly auditorium period at which time the researcher invited the students to participate by distributing and reviewing the Parent Consent Form (Appendix A) and the Student Consent Form (Appendix B).

Fifty sixth-grade and 56 seventh-grade students volunteered to participate in the study by returning both consent forms. Students who were Limited English Proficient, who were receiving English as a Second Language or Bilingual services, who were receiving special education services, or who could not decode the words of the verbal protocol stimulus were excluded from participation in this study. Seven sixth-grade students were excluded from the potential subject pool for various reasons (e.g., missing achievement test scores [1 student], LEP status [2 students], incomplete parental consent [1 student], discharged prior to the beginning of the study [1 student], receiving special education services [1 student], and recent admittance to the school and, therefore, no classroom grades or achievement test scores available [1 student]). Three seventh-grade students were excluded due to incomplete parental consent, and one seventh-grade student withdrew from participation on the day she was called for session one. Students who returned incomplete consent forms were asked to have these forms completed several times over approximately two months. Each of the student's final decision not to continue with the study was ascertained prior to excluding them from the sample.

All remaining 95 students, 43 sixth graders and 52 seventh graders (37.9% male, N = 36; 62.1% female, N = 59), participated in the study. From this final group, a random sample of 40 sixth- (42.5% male, N = 17; 57.5% female, N = 23) and 40 seventh-grade (30% male, N = 12; 70% female, N = 28) students was selected for data coding and statistical analyses. The mean age of the sixth graders was 11.37 years old (SD = 0.36), and that of the seventh graders was 12.42 years old (SD = 0.41).

Academic achievement data and demographic information are summarized in the first three tables. The participants were above average on both the mathematics and the reading achievement test scores (Table 1). The quartile scores indicate that the students scored higher in relation to the national norming sample on the standardized mathematics test than they did on the reading test. The ethnic background of the participants as identified by the student was 50% Caucasian, 20% Italian-American, 14% Hispanic, 8% Eastern-European, 5% Asian, 1% African American, and 2% other ethnicity (Table 2). Eighty-one percent spoke English as their first language at home (Table 3). The remaining students identified an Eastern-European language (6%), Italian (6%), Spanish (4%), Chinese (1%) and other (1%) as their primary language at home. All students were fluent in the English language, and all of the students could decode the sentences of the word problems.

Table 1

Academic Achievement Indicators: Mean (SD) and Quartile Scores

	Mean (SD)	Q1	Q2	Q3
Math Achievement (CAT) -- NPT	73.64 (25.14)	60.2	82.0	94.5
Reading Achievement (CTB) -- NPT	67.41 (23.82)	48.7	71.0	87.0
Math -- Feb 98 Report Card	82.86 (10.95)	75.0	85.0	90.0
English -- Feb 98 Report Card	84.74 (10.08)	80.0	88.0	91.0
Average -- Feb 98 Report Card	82.84 (9.14)	78.1	85.1	89.0
Computation Test Score (Max. = 24)	15.90 (4.27)	13.0	16.0	19.0

Note: NPT refers to national percentile score.

Table 2

Summary of Demographic Information

Ethnicity	Frequency	Percent (%)
Caucasian	40	50.0
Hispanic	11	13.8
Italian-American	16	20.0
Asian	4	5.0
Eastern European	6	7.5
African-American	1	1.3
Other	2	2.5

Table 3

Primary and Other Languages Spoken

	Primary Language		Other Language	
	Frequency	Percent (%)	Frequency	Percent (%)
None			48	60.0
English	65	81.3	15	18.8
Spanish	3	3.8	7	8.8
Italian	5	6.3	7	8.8
Chinese/Korean	1	1.3	2	2.5
Eastern European	5	6.3	0	0.0
Other	1	1.3	1	1.3

Materials

The materials for this study included: Parent Consent Form (Appendix A), Student Consent Form (Appendix B), Student Background Survey (Appendix C), Student Agreement Form (Appendix D), Verbal Protocol Stimuli (Appendices E & F), and Computation Test (Appendix G). The Background Survey (Appendix C) was used to collect information from the students' school records. Two measures served as the main instruments in this study: (1) Verbal Protocol Stimuli (Appendices E & F); and (2) Computation Test (Appendix G).

All materials were identified by a participant code number. A list of code numbers and names has been kept separate from student materials. This list was used to identify student school records, only.

Student background questionnaire. For each student, the following background information was recorded from their school records: date of birth, gender, grade and class in school, report card grades in Mathematics and Language Arts from the most recent report card distributed, and the most recent CTB Reading and CAT5 Mathematics standardized achievement test scores (Appendix C). Students' primary language spoken at home, other languages spoken, and ethnicity (optional) were identified by the student him or herself.

Verbal protocol stimulus. The Verbal Protocol Stimulus consisted of 16 mathematics word problems, and has two forms, Form A and Form B (Appendices E & F). Each form consisted of 12 target and 4 filler problems. The target problems were compare word problems, and the different problem types resulted from the combination of three problem factors: (1) number of computational steps (i.e., one- or two-step), (2) language consistency (i.e., CL or IL), and (3) mathematical operation (i.e., addition, subtraction, multiplication, or division). Table 4 presents the combination of problem type factors within the 12 target problems. The four filler problems were complex multi-step problems that do not include a relational statement and were included to avoid patterns of response.

Table 4

Table of Problem Types

Language Consistency	One-step		Two-step			
	Mult	Div	Add	Subtr	Mult	Div
CL	1	1	1	1	1	1
IL	1	1	1	1	1	1

Form A was created by randomly ordering the problem types resulting in a counterbalanced order of appearance. Form B was created from Form A by switching the order of the two-step problems such that the order of appearance with respect to language consistency (i.e., CL and IL problems) remained unchanged for each form while the required operation in each problem was changed systematically. That is, addition and multiplication problems were switched for one another, and subtraction and division problems were switched for one another. The use of two forms permitted an analysis of potential order of presentation effects. Forms A (N = 39) and B (N = 41) were distributed randomly such that approximately equal numbers of students received each form.

The 12 target compare word problems were three sentences long and required either one or two computational steps. Eight were two-step problems, four CL and four IL (i.e., addition, subtraction, multiplication and division). The remaining four target problems were one-step CL and IL multiplication and division compare problems. Only multiplication and division one-step problems were included to reduce the number of problems the students were asked to solve and because previous research has shown that one-step addition and subtraction problems do not pose considerable difficulty among populations other than young children (Verschaffel et al., 1992; Verschaffel, 1994).

Computation test. The computation test (Appendix G) was developed as an indicator of the participants' computational skills and knowledge related to the mathematical concepts and operations necessary to solve the word problems on the Verbal Protocol Stimulus. The mathematical content of these problems included: (1) converting one standard unit of measurement into another unit given the necessary conversion factor, (2) simplifying or reducing fractions to their lowest terms, (3) computing a fraction of a number, (4) adding two decimal numbers, (5) multiplying a decimal number by a whole number, and (6) visually depicting the multiplication of a fraction by a whole number and a fractional part of a set of objects. Two items per topic resulted in a total of 12 items.

Procedures

The researcher was introduced to the students during their weekly grade-level auditorium period. During this class period the researcher invited all students present to participate by distributing two copies each of the Parent Consent Form (Appendix A) and the Student Consent Form (Appendix B). Students and researcher reviewed the procedures for participation in the study. For two weeks following the initial distribution of the consent forms, the researcher was available to the students during their lunch period to answer questions, to explain the study to those who were absent on the day the consent forms were distributed, and to collect completed parent and student consent forms.

Participants completed all measures on two separate days at the students' school. On the first day, the researcher met with each student individually to complete the Verbal Protocol Stimulus (either Appendix E or

Appendix F). The first meeting took approximately one class period (45 minutes). In the event that a student did not complete the word problems within a class period, he or she was allowed to complete the task and was escorted to his or her next class by the researcher. The entire verbal protocol session, including reading the directions and practicing the "think-aloud," was videotaped. Video tapes were identified by participant number and, following data coding, were secured in a university professor's office (e.g., Dr. Carol Tittle, Chair of Dissertation Committee; CUNY Graduate School).

First, the student and researcher read the first page of the Verbal Protocol Stimulus together. Then, the student practiced "thinking-aloud" using two sample word problems. During this practice session, the researcher frequently prompted the student to say as much as possible, even if he or she was reading or rereading all or part of the word problem. This took the form of an initial question. If a student was silent during the practice session or the verbal protocol, the researcher asked the student what he or she was doing at that moment. The student was then reminded to do the identified behavior out-loud. For example, if a student responded that he or she was "thinking" or "reading," the student was reminded to think, read, or "do whatever you are doing" out-loud. The frequency of prompts during the practice session was sufficient to familiarize the student with the think-aloud procedure. The students were asked whether they understood the think-aloud and orally indicated their understanding prior to continuing to the first target problem.

The students were told that the researcher was constrained to saying only two things during the verbal protocol: (1) If the student was silent (for approximately 10 seconds), whispering, or inaudibly mumbling, he or she was reminded to say as much as possible out loud; and (2) Following a period of time during which the observed behavior indicated an inability to solve the problem or a lack of understanding of the problem, the student was reminded that he or she may explain the difficulty encountered and proceed to the next problem. In addition, the student was informed that during the protocol session the researcher could not answer questions which were related to the student's understanding of the problem, the mathematical concepts involved, or the student's solution for the problems. Once the student indicated that a problem was solved by turning the page over or stating that he or she could not solve the problem, the student recalled the problem sentences out loud without additional review of the problem. Following completion of the verbal protocol, each student was asked to indicate the primary language spoken at home, other languages spoken, and his or her ethnicity (optional) for the background questionnaire.

Each student then read and signed the Student Agreement Form (Appendix D).

During the second session, the students completed the Computation Test (Appendix G) in groups of approximately five. When the students completed the measures, they were thanked for their participation. Finally, the researcher examined school records to collect standardized achievement and report card grades.

Data Coding Procedures

The following procedures were used in coding the various sources of data.

Verbal protocol stimulus. The video-taped recordings of student behaviors while reading and solving the mathematics word problems were coded by the researcher. First, discrete sequences of behaviors (e.g., read sentence, reread sentence, refer to problem, calculate, etc.) were recorded producing a running log of problem solving behaviors. Eight indicators were coded from these lists of discrete behaviors. For each target problem, the following were recorded: (1) problem solving success, (2) type of error committed (when applicable), (3) number of rereadings, (4) pattern of problem solving behavior, (5) time at reading the first word of the problem, (6) time at reading the last word of the problem, (7) time at statement of solution or turning page over indicating problem completion, and (8) quality of problem recall.

Ten percent of the sample plus three practice video tapes (11 total video tapes) were selected at random and independently coded by a second rater. First, the rater was trained to parse the protocols into sequences of discrete behaviors and to calculate the seven variables by coding one protocol with the researcher's assistance. The next two students' videos were coded independently by the second rater, and the variables were calculated. The sequences of behaviors recorded by each rater were compared, and discrepancies discussed. Following training on these three protocols, the second rater independently coded eight additional protocols (10% of total number). Interrater reliability for type of error, pattern of behavior, number of rereadings, and problem recall were calculated.

The following sections describe the coding system used for the seven variables examined.

(1&2) Rate of success and type of error. First, each solution was coded as either correct or incorrect. Next, the solution paths (i.e., running logs of behaviors) for incorrect solutions were examined in detail. Four types of errors were coded: (1) a *reversal error* was coded when the student performed an arithmetic operation opposite to that required to solve the problem; (2) a *linguistic-other error* was coded when one or more of the text propositions was omitted from the calculations performed, when the student indicated that the problem could not be solved due to his or her inability to understand the problem, or when the student performed an erroneous mathematical computation resulting from a lack of attention to the units in the problem (e.g., multiplied by seven when the quantities in the problem were all given in terms of a week); (3) a *fraction of a number error* was coded when the error committed resulted from the student's lack of ability to compute a fraction of a number, or when the student indicated that he or she could not solve the problem because of the fraction in the problem; and (4) a *math-other* error was coded when the student made an arithmetic mistake.

(3) Number of rereadings. From the running logs of student behaviors, the total number of rereadings of each sentence of the problem was calculated. Any reading of a sentence or part of a sentence following the initial reading of the problem was coded as one rereading for that sentence.

(4) Pattern of problem solving behavior. The student's pattern of behavior for each target problem was coded into one of two categories: (1) direct translation approach or (2) meaningful approach (Hegarty et al., 1995). A participant's solution strategy was classified as exhibiting a direct

translation approach when one or more of the following was observed as the pattern of behavior within a given word problem:

(1) the student referred to and/or recorded elements of the problem without the appropriate context (i.e., units, associations, and relationship to other given information) provided in the text;

(2) the student repeatedly stated the given information only without relevant context (e.g., stated the numbers provided in the problem only);

(3) the student repeatedly reread individual sentences without recording or transforming the given information; and/or

(4) the student carried out mathematical computations without referring to nor rereading the text of the problem.

A participant's solution strategy was classified as a meaningful approach when one or more of the following was observed as the predominant pattern of behavior within a given word problem:

(1) the student read each sentence separately and recorded the given information with the appropriate context before reading the next sentence;

(2) the student first read the whole problem and then reread each individual sentence while recording the given information (i.e., wrote elements of problem in work space provided);

(3) following either rereading or referring to the text of the problem, the student stated and/or recorded given information within an appropriate context, or the student provided an explanation for the computational steps performed; and

(4) the student stated or wrote intermediate and/or final answers as a complete sentence or made an audible statement which incorporated the appropriate context for the elements of the problem and/or the answer stated (i.e., the student stated the answer in a manner which revealed an understanding of its meaning or relevance to the problem).

A continuous variable representing the number of problems solved using a meaningful approach was calculated.

(5&6) Time variables. Two time variables were calculated for each target problem. Initial reading time, duration from the reading of the first word of the problem until the utterance of the last word of the third sentence, was calculated. For those students whose initial reading of the problem was not continuous (i.e., all three sentences were not read prior to writing given information) the end reading time was judged by the coder as that time when the individual was no longer reading the problem and was beginning to solve the problem. Those who read each sentence separately and recorded information for individual sentences before reading the next sentence were not excluded from analyses related to initial reading time, but their initial reading time was considered to be an outlier. The total time to solve each problem was also calculated starting when the student read the first word of the problem and ending when the student indicated completion by stating his or her answer or by turning the question sheet over.

(7) Quality of recall. The quality of problem recall was coded to reflect the degree to which the retold problem incorporated and preserved the contents, relationships, and structure of the original problem. One of three codes was assigned for each target problem: (1) poor recall, (2) gist only recall, and (3) good recall. Good recall was assigned when the retold problem reflected the contents, relationships, and structure of the original problem and contained no omissions. Gist recall constituted recall which contained all of the essential elements of the propositions of the problem but either did not retain the correct order of the elements or omitted non-essential elements (e.g., the name or quantity stated in a problem). Finally, a student's recall was classified as poor when one or more essential elements were omitted, the order of the elements of the recalled problem did not reflect that of the presented problem, and/or the student could not recall all or a majority of the elements of the problem. This variable reflects the quality of the problem representation formed by the problem solver.

Composite verbal protocol variables. Each variable recorded for the individual target problems was collapsed across different problem types. The problems were grouped in the following ways (see Table 1): (1) one- versus two-step multiplication and division problems (four problems of each); (2) CL versus IL problems -- including addition, subtraction, multiplication and division one- and two-step problems (six problems of each); (3) IL multiplication and CL division versus CL multiplication and IL division problems -- including both one- and two-step problems (four problems of each); and (4) two-step addition and subtraction versus two-step multiplication and division problems (four problems each).

Computation test. Each of the twelve items was assigned zero, one, or two points. One point was recorded for setting up an appropriate solution strategy or procedure, and one point was recorded for correctly carrying out the solution and arriving at the correct answer. A total score for all problems (range = 0 to 24 points) and for problems related to the mathematical concept of a fraction and of a fraction of a number in particular (question #1b, 2, 3, and 6 -- range 0 to 14 points) were computed.

The latter variable was used to group the participants into two categories with respect to their fraction knowledge. Students' fraction of a number scores were approximately bimodal in distribution (Figure 4). Students whose scores on the fraction of a number subset of computation questions ranged from 0 to 7 were classified as low fraction knowledge group. Participants who scored between 8 and 14 points on this subtest were classified as high fraction knowledge group.

In summary, nine categories of variables were collected for analysis in this study: (1) demographic information, (2) academic achievement indicators, (3) computation skill, (4) problem solving success rates, (5) types of errors committed, (6) pattern of problem solving behavior, (7) number of rereadings, (8) duration of initial reading and total response time, and (9) quality of problem recall.

CHAPTER VI

Results

Data Analyses

Overview. Indicators of academic achievement and demographic information have been summarized above and in Tables 1, 2, and 3. Correlations and intercorrelations were examined next. To test the first hypothesis, regression analyses were performed with problem solving success as the dependent variable. The second hypothesis was examined using ANOVA procedures with fraction knowledge as the independent variable. The final hypotheses were examined using paired t-tests.

Interrater reliability analyses. Ten percent of the video tapes (N = 8) were selected at random and independently coded by a second rater. Interrater reliability for type of error and pattern of behavior was determined using Cramer's V coefficient. Spearman and Pearson correlation coefficients were calculated for the problem recall and the number of rereadings, respectively. Appendix H contains the coefficients calculated for each question. Cramer's V for type of error ranged from .82 to 1.00 (M = .94), and for pattern of behavior this measure of association ranged from .49 to 1.00 (M = .80). The relationship between the number of rereadings recorded by each rater ranged between 0.94 and 0.99 (M = .97). Finally, the correlation coefficient for problem recall ranged from .67 to 1.00 (M = .83).

Differences due to form, grade, and gender. ANOVA procedures were used to test differences due to protocol form (Appendix I), grade (Appendix J), and gender (Appendix K). Form A ($N = 39$) and Form B ($N = 41$) were randomly distributed to approximately equal numbers of participants. First, age, achievement test scores, most recent report card grades, and computations scores were tested to confirm the random distribution of each form. Analysis of variance outcomes revealed no significant differences between forms for age and for each of the academic indicators. Next, there were no significant differences due to form for each of the protocol variables. Since no differences were found, students' scores on the two forms were analyzed together.

ANOVA analyses revealed two significant differences due to grade (Appendix J) and one difference due to gender (Appendix K). First, seventh-grade computation test scores ($M = 17.00$) were significantly higher than the sixth-grade scores ($M = 14.80$), $F(1,79) = 5.63$, $p = 0.02$. In addition, seventh-grade students solved a greater number of problems using a meaningful approach ($M = 4.47$) than the sixth graders ($M = 2.98$), $F(1, 79) = 4.16$, $p = .045$. Also, female participants' grades in English ($M = 86.53$) were higher than male participants ($M = 81.59$), $F(1, 79) = 4.66$, $p = .034$. Since these were the only significant differences found due to gender and grade, all final analyses were carried out on the total sample.

Correlation Analyses

Intercorrelations between indicators of academic achievement, Computation Test scores, and verbal protocol variables are presented in Table 5. Academic achievement test scores and present grades were moderately to highly correlated, ranging from .51 to .77. The relationships between math achievement test scores and present grades (r = .60 to r = .77) were somewhat stronger than those between reading achievement test scores and present grades (r = .51 to r = .66). Scores on the computation test were moderately correlated with each of these indicators of academic achievement, ranging from .32 to .61.

Of the three verbal protocol scores investigated, the number of problems solved using a meaningful approach and the mean problem recall were positively associated with achievement test scores and computation test scores, ranging from .31 to .46. In addition, the mean recall scores were correlated with present report card grades, ranging from .32 to .43; however, the relationship between the number of problems solved using a meaningful approach and recent grades was weak. The mean number of rereadings was not significantly related with any of the indicators of academic achievement and computation test scores examined (r = -.10 to r = .04). Finally, number of rereadings was moderately associated with the number of problems solved using a meaningful approach (r = .45).

Table 5

Intercorrelations Between Variables Collected for Analysis in this Study

	1	2	3	4	5	6	7	8	9	10
1. Math achv test: Raw score	--	.80***	.71***	.60***	.77***	.55***	.52***	.31**	-.10	.44***
2. Reading achv test: Raw score		--	.56***	.51***	.66***	.47***	.41***	.32**	.04	.46***
3. Math report card grade			--	.62***	.87***	.58***	.61***	.27*	-.04	.41***
4. English report card grade				--	.85***	.32**	.35**	.22	.01	.32**
5. Average report card grade					--	.53***	.53***	.31**	.01	.43***
6. Computation test: Total score						--	.95***	.31**	-.02	.37**
7. Computation test: Fraction knowledge							--	.35**	.02	.36**
8. Number of problems solved using a MA								--	.45***	.26*
9. Mean number of rereadings									--	-.02
10. Mean recall										--

$^*p < 0.05$; $^{**}p < 0.01$; $^{***}p < 0.001$

Table 6 presents the correlations between each of these groups of variables and the total number of target problems on the verbal protocol solved correctly. Problem solving success was positively correlated with each of the indicators of academic success (r = .40 to r = .70), computation test scores (r = .57) and two out of three of the verbal protocol variables. While problem solving success was positively related to the number of problems solved using a meaningful approach (r = .30, *p* = .008) and mean recall scores (r = .48, *p* < .001), the relation between problem solving success and the mean number of rereadings was non-significant and negative in direction (r = -.20, *p* = .069).

Table 6

Correlations Between Variables Collected and Total Problem Solving Success

	Total Problem Solving
1. Math Achievement (CAT) -- RS	0.70***
2. Reading Achievement (CTB) -- RS	0.56***
3. Math -- Feb 98 Report Card	0.56***
4. English -- Feb 98 Report Card	0.40***
5. Average -- Feb 98 Report Card	0.53***
6. Computation Test Score	0.57***
7. Computation Test - Fraction of a Number	0.57***
8. Number of Problems Solved using a Meaningful Approach	0.30**
9. Mean number of Rereadings	-0.20
10. Mean Problem Recall Score	0.48***

*p < 0.05; **p < 0.01; ***p < 0.001.

Table 7

Summary of Regression Analysis for Variables Predicting Problem Solving Success (Stepwise Procedure)

Variable	B	SE B	ß	R^2
Step 1				
Computation test	0.29	0.07	0.39***	0.32
Step 2				
Mean recall	1.79	0.60	0.27**	0.41
Step 3				
Mean number of rereadings	-0.82	0.27	-0.29**	0.45
Step 4				
Number of problems solved using a meaningful approach	0.22	0.09	0.24*	0.48

Note. R^2 = 0.32 for Block 1; Change R^2 = 0.09 for Block 2; Change R^2 = 0.04 for Block 3; Change R^2 = 0.03 for Block 4 ($ps < 0.001$ for each step).
*$p < 0.05$; **$p < 0.01$; ***$p < 0.001$.

Regression Analyses With Problem Solving Success as the Dependent Variable

The first hypothesis predicted that problem solving success would vary as a function of mathematical knowledge, predominant pattern of behavior, number of rereadings, and quality of problem recall. To test this hypothesis, total problem solving scores were regressed on computation test scores, number of problems solved using a meaningful approach, mean number of rereadings, and mean recall score using a stepwise procedure.

The results of the regression analyses reported in Table 7 supported the first hypothesis. Each of the four variables examined significantly contributed to the prediction of problem solving total score. Together, total computation test score, number of problems solved using a meaningful approach, mean number of rereadings, and mean recall score accounted for 48% of the variance in problem solving success ($\underline{R} = 0.69$, $\underline{p} < .001$).

Differences Due to Fraction Knowledge

The second set of hypotheses postulated specific differences in performance on IL multiplication and CL division problems depending on the student's knowledge of the concept of a fraction of a number. Analysis of variance was performed to examine differences between low and high fraction of a number knowledge groups. Means and standard deviations for IL multiplication and CL division problems by knowledge group are given in Table 8. The following directional hypotheses were tested. Students for whom fraction knowledge is not well formed will:

(1) solve fewer problems than students who have such knowledge;

(2) use a meaningful approach more often than students who have such knowledge;

(3) reread problem sentences more than students who have such understanding;

(4) recall fewer elements and structure of the original word problem

than students who have such knowledge; and

(5) commit a greater number of fraction of a number mathematical errors than students who have such knowledge.

The first sub-hypothesis was supported. On average, the high knowledge group solved a greater number of problems ($M = 1.83$) than the low knowledge group ($M = 0.75$), $F(1,78) = 13.81$, $p < .001$. Counter to hypothesized, low knowledge students used a meaningful approach on significantly fewer problems ($M = 0.61$) than high knowledge students ($M = 1.25$), $F(1,78) = 8.02$, $p = .013$. In addition, the low and high knowledge groups did not statistically differ for mean number of rereadings.

Sub-hypotheses three and four were both supported. High knowledge individuals recalled a greater number of elements and structure of the problems ($M = 2.27$) than low knowledge students ($M = 1.99$), $F(1,78) = 5.08$, $p = .027$, and low knowledge students committed a greater number of fraction of a number errors ($M = 2.30$) than the high knowledge group ($M = 1.14$), $F(1,78) = 10.36$, $p = .002$.

Table 8

Mean (SD) for Problem Solving Variables on IL Multiplication and CL Division Problems by Fraction of Number Knowledge

	Fraction of a Number	
IL Multiplication & CL Division	Low N = 44	High N = 36
Problem Solving Success (max. = 4)	0.75 (1.16)	1.83*** (1.44)
Number of Problems Solved Using a Meaningful Approach	0.61 (0.97)	1.25* (1.27)
Mean Number of Rereadings	1.52 (1.41)	2.02 (1.43)
Mean Recall Score	1.99 (0.58)	2.27* (0.50)
Number of Fraction of a Number Errors	2.30 (1.68)	1.14** (1.50)

*$p < 0.05$; **$p < 0.01$; ***$p < 0.001$

Differences Due to Problem Type

The third set of hypotheses involved differences due to problem type. The cognitive demands associated with each type of problem varies as a function of a combination of three factors, the number of computational steps, language consistency, and the mathematical concept in the wording of the relational statement of the problem. The cognitive demands of a particular type of problem were hypothesized to influence the problem solvers' predominant pattern of behavior, problem representation, and problem solution.

Four comparisons were made: (1) one- versus two-step multiplication and division problems; (2) CL versus IL problems; (3) IL multiplication and CL division (i.e., those problems which involve the phrase "1/n as many" in their relational sentence) versus CL multiplication and IL division problems; and (4) addition and subtraction versus multiplication and division two-step problems. For each of these comparisons, problem solving success, number of problems solved using a meaningful approach, mean number of rereadings, mean quality of recall score, and mean initial reading and total response times were examined using paired $\underline{t}$ tests. Due to the large number of $\underline{t}$ tests, differences at the .05 level will be considered tentative. Differences at the .01 Alpha level will be held as significant. Mean values and standard deviations for each variable by problem type are presented in Table 9.

Table 9

Problem Solving Variables: Mean (SD) by Problem Type

	All Probls (12 total)	# Comp Steps (4 total)		Language Consistency (6 total)		Fr of # Lang in Problem (4 Total)		Arithmetic Operation (4 total)	
		1-Step	2-Step	CL	IL	CLM/ ILD	ILM/ CLD	ADD/ SUBTR	MULT/ DIV
Problem Solving Success	6.25 (3.13)	2.08 (1.31)	1.82* (1.25)	3.67 (1.73)	2.57*** (1.71)	2.66 (1.28)	1.24*** (1.40)	2.35 (1.28)	1.83** (1.25)
# of Probl Solved using MA	3.72 (3.35)	1.24 (1.22)	1.29 (1.21)	2.00 (1.76)	1.73* (1.81)	1.63 (1.41)	0.90*** (1.15)	1.20 (1.38)	1.29 (1.21)
Mean Recall Scores	2.15 (0.47)	2.28 (0.53)	2.07*** (0.54)	2.19 (0.48)	2.11 (0.55)	2.23 (0.52)	2.12* (0.56)	2.10 (0.50)	2.07 (0.54)
Mean Initial Reading Time (sec.)	15.91 (9.00)	16.39 (20.19)	15.26 (8.83)	15.41 (4.68)	16.40 (16.01)	16.56 (19.90)	15.06 (9.29)	16.07 (11.66)	15.26 (8.83)
Mean # of Rereadings	1.67 (1.12)	1.67 (1.15)	2.02** (1.55)	1.67 (1.13)	1.67 (1.29)	1.94 (1.38)	1.75 (1.43)	1.33 (1.18)	2.02*** (1.55)
Mean Total Response Time (sec.)	70.50 (21.29)	61.05 (27.54)	75.72*** (26.65)	71.04 (23.17)	69.96 (23.17)	66.73 (28.54)	70.04 (30.03)	74.73 (22.15)	75.72 (26.65)

* p < .05; ** p < .01; *** p < .001

One- versus two-step problems. There were three significant differences due to the number of computational steps in the problems. Participants reread two-step problems (*M* = 2.02) more frequently than one-step problems (*M* = 1.67), *t*(79) = 2.78, *p* = .007. Problem recall for one-step problems (*M* = 2.28) contained more elements and structure of the original problems than two-step problems (*M* = 2.07), *t*(79) = 4.64, *p* < .001. Finally, students took significantly longer to solve two-step (*M* = 75.72 sec.) than one-step problems (*M* = 61.05 sec.), *t*(79) = 4.81, *p* < .001.

CL versus IL problems. The only significant difference due to language consistency involved problem solving success. Participants solved a greater number of CL problems (*M* = 3.67) than IL problems (*M* = 2.57), *t* = 6.85, *p* < .001.

CL multiplication and IL division versus IL multiplication and CL division problems. There were two significant differences between those problems which do and do not involve a fraction of a number (i.e., "1/n as many") in the relational sentence of the problem. First, participants successfully solved CL multiplication and IL division problems (*M* = 2.66) more often than IL multiplication and CL division problems (*M* = 1.24), *t*(79) = 9.46, *p* < .001. Second, students used a meaningful approach on CL multiplication and IL division problems (*M* = 1.63) more often than on IL multiplication and CL division problems ($M = 0.90$), $t(79) = 5.31$, $p < .001$.

Addition and subtraction versus multiplication and division problems. Similar to the findings for the comparisons discussed above, there was a significant difference in the number of problems solved correctly due to arithmetic operation. Two-step addition and subtraction problems ($M = 2.35$) were solved successfully more often than multiplication and division problems ($M = 1.83$), $t(79) = 3.53$, $p = .001$. In addition, participants reread multiplication and division problems ($M = 2.02$) more often than addition and subtraction problems ($M = 1.33$), $t(79) = 4.73$, $p < .001$.

Tests of the Consistency Hypothesis

Finally, three analyses which test Lewis and Mayer's (1987) consistency hypothesis were performed. Differences due to language consistency were examined. Hypothesis four, problem solvers will commit a greater number of reversal errors on IL problems than on CL problems, was supported. On average, participants committed 0.21 reversal errors on CL problems versus 1.98 reversal errors on IL problems, $t(79) = 10.15$, $p < .001$. Hypotheses five and six proposed greater initial reading and total response times for IL than CL problems. There were no significant differences in reading or response times due to language consistency.

CHAPTER VII

Discussion

Overview

The primary purpose of this investigation was to examine components of the proposed Interactive Model of Problem Solving (Figure 3). Specifically, the study assessed middle-school students' problem solving approaches to investigate the role mathematical understanding plays in problem comprehension and subsequent solution. A premise of this analysis is that when the conceptual and procedural knowledge demands of a mathematics word problem are great the problem solver will alter his or her behavior to accurately represent and successfully solve the problem. As the student interacts with the text of the word problem a transaction occurs which is mediated through the individual's present knowledge structures (e.g., Ehri, 1995a, 1995b; Rosenblatt, 1994). If the problem solver detects difficulty in understanding the problem, Pressley and Afflerbach's (1995) analysis suggests that the individual will alter his or her behavior and processing in order to correct the miscomprehension or misrepresentation. That is, the student may attempt to use a meaningful approach or increase the number of times he or she reads the sentences of the problem in order to represent and solve the problem.

Sixth- and seventh-grade students' understanding of the concept of a fraction of a number was expected to vary. It was also expected that students whose understanding of this concept was not well formed might attempt to use a meaningful approach to represent the propositions of a problem. For example, the student may reread whole sentences of the problem or restate the elements of the problem with appropriate context. However, the problem solver may be unable to form an accurate representation of the problem. This was illustrated by many students in the low fraction knowledge group. Thus, the representation of the problem formed should depend on the individual's mathematical understanding as well as the problem solving or reading behaviors the individual chooses to carry out. Without a well-formed mental model of the problem, it was predicted that the problem solver will fail to solve the problem. Therefore, the first hypothesis predicted that problem solving success would vary as a function of mathematical knowledge, predominant pattern of problem solving behavior, number of rereadings, and the quality of the representation formed. The second hypothesis predicted that there would be differences resulting specifically from students' fraction knowledge.

A related premise involved differences due to problem type. The cognitive demands associated with each type of problem investigated in this study varied as a function of a combination of four factors: (1) the number of computational steps (i.e., one- vs. two-step problems), (2) language consistency (i.e., CL vs. IL problems), (3) the underlying mathematical concept in the wording of the relational statement of the problem (i.e., CL multiplication and IL division vs. IL multiplication and CL division); and (4) arithmetic operation (i.e., addition and subtraction vs. multiplication and division). The differential cognitive demands of each type of problem were hypothesized to influence the problem solvers' predominant pattern of behavior, problem representation, and problem solution. Several of these comparisons tested Lewis and Mayer's (1987) consistency hypothesis and their model of problem representation within a middle-school population.

Relationships Between Indices of Academic Achievement, Problem Solving Behavior, and Problem Solving Success

The relationship between indicators of academic achievement and computational skill as measured in this study were moderate to high (Table 5). The correlation between mathematics and reading achievement test scores was very high ($r = .80$), and the relationships between math achievement test scores and present grades or computation test scores were somewhat stronger than those between reading achievement test scores and these variables. However, both the computation total test score and the fraction knowledge score differentiated between Mathematics ($r = .58$ and $r = .61$) and English ($r = .32$ and $r = .35$) report card grades better than they did the standardized measures of mathematics (r = .55 and r = .52) and reading (r = .47 and r = .41) achievement. That is, the computation scores were equally correlated with mathematics grades and mathematics achievement test scores; however, the correlation between computation test scores and standardized reading achievement test scores was higher than that for the English report card grades.

Of the three verbal protocol scores investigated, problem recall was moderately associated with each of the indicators of academic achievement and computation skill. Although there were similar patterns of associations, the relationships between the number of problems solved using a meaningful approach and the five indicators of academic achievement and computation test scores were weaker overall. Therefore, the ability to recall the problems was more highly associated with the measures of academic achievement than was the number of problems solved using a meaningful approach. This may be accounted for by the number of students in the sample who were able to solve the problems without using such an approach. These students were able to directly translate the components of the problem into mathematical operations with little effort which may have weakened the predictive power of this variable. Yet, there was a significant positive relationship between the number of problems solved using a meaningful approach and several of the indicators of academic achievement and computational skill collected in this study.

The mean number of rereadings was not significantly related with any of the indicators of academic achievement and computation test scores examined. This may have been due to the different types of rereading associated with each of the two patterns of behavior. According to the operational definition of a direct translation approach, the student may reread the problem but does not transform the information included in the statement. This may be considered an ineffective rereading strategy. Alternatively, the student using a meaningful approach transforms or records the information contained in the problem sentences as he or she rereads them. This type of behavior may be indicative of a more active approach toward cognitive activities in general. Thus, since the type of rereading carried out was not distinguished when recorded, the relationships between the number of rereadings and each of the indicators of academic achievement ($r = -.10$ to $r = .04$) were nonsignificant and varied in their direction. Finally, the number of rereadings was significantly related to the number of problems solved using a meaningful approach ($r = .45$). This is very likely due to the operational definition of a meaningful approach which includes rereading the sentences of the word problem and transforming the indicated information as key components.

Problem solving success was highly correlated with mathematics achievement test scores (Table 6) and positively correlated with each of the other indicators of academic success, computation test scores, and two out of three of the verbal protocol variables. The use of meaningful approach and the ability to recall the problems successfully were directly related to problem solving success. However, although not significant, larger numbers of rereadings were associated with lower problem solving scores. This negative correlation is perhaps indicative of the unsuccessful problem solvers' attempt to make sense out of the word problems by increasing the number of times he or she rereads the problems, but rereading alone did not necessarily lead to problem solving success.

Regression Analyses With Problem Solving Success as the Dependent Variable

The first hypothesis, an overall test of the model proposed in this study, predicted that problem solving success would vary as a function of mathematical knowledge, predominant pattern of behavior, number of rereadings, and quality of problem recall. Regression analyses supported the importance of the students' mathematical understanding, pattern of behavior, and problem representation in the prediction of problem solving success ($R = .69$, $p < .001$). Students who scored higher on the computation test, who could recall more elements and structure of the problems, and who used a meaningful approach solved more problems correctly. However, contrary to expectations, a higher number of rereadings was indicative of lower problem solving scores. The negative relationship between the number of rereadings and problems solved provides evidence that to some extent the students altered their behaviors when they were experiencing difficulty. As indicated above, some students who experienced difficulty solving the problem reread the problems more but may not necessarily have used a meaningful approach to solve the problems. For example, they may have reread the problem sentences but did not transform the information as they reread the problem. Thus, simply rereading the problems did not facilitate problem solution.

Differences due to Fraction Knowledge

IL multiplication and CL division problems contain the expression "1/n as many" in the wording of their relational sentence. The existence of this phrase in the relational sentence is indicative of either multiplication or division depending upon whether the language of the relational sentence is in the inconsistent or consistent form. The second set of hypotheses postulated specific differences in performance on IL multiplication and CL division problems depending on the student's knowledge of the concept of a fraction of a number. These differences were hypothesized to be the result of the existence of this phrase rather than due to the actual mathematical operation necessary to solve the problem. Means and standard deviations for IL multiplication and CL division problems by knowledge group are given in Table 8.

As expected, the high knowledge group solved a greater number of these problems than the low knowledge group. These results may be interpreted in light of overall failure rates presented in Table 10. As shown, only two students failed to solve any of the problems correctly and both of these students were in the low fraction knowledge group. Almost half of the students (37 out of 80 students) failed to solve all four IL multiplication and CL division problems correctly, and 27 out of 37 of these students (73%) were in the low fraction knowledge group. In addition, only five students all of whom were in the low fraction group were unable to solve any of the CL multiplication and IL division problems. Therefore, overall failure rates were quite low while failure rates on those problems which included a fraction of a number in the wording of their relational sentence (i.e., IL multiplication and CL division problems) were much higher. Moreover, failure rates among the low fraction knowledge group were higher on these problems than for the high fraction knowledge group.

Table 10

Frequency (Percent) of Individuals Who Solved None of the Indicated Problems Correctly

		Fraction of a Number	
	Total Sample N = 80	Low N = 44	High N = 36
Problem Solving Total Score (max. = 12)	2 (2.5)	2 (4.5)	0 (0.0)
CL Multiplication and IL Division (max. = 4)	5 (6.3)	5 (11.4)	0 (0.0)
IL Multiplication and CL Division (max. = 4)	37 (46.3)	27 (61.4)	10 (27.8)

Counter to hypothesized, low knowledge students used a meaningful approach on significantly fewer problems than high knowledge students, and there were no differences between the low and high knowledge groups in the number of rereadings of the problem sentences. Low knowledge students solved approximately one in eight (0.61 out of 4) IL multiplication and CL division problem using a meaningful approach whereas high fraction knowledge students solved approximately 1.25 out of four of these problems using this approach. Thus, students in the low fraction knowledge group did not change their behaviors when trying to understand or solve these problems. Reading research (e.g., Pressley & Afflerbach, 1995) has indicated that individuals who recognize difficulties in comprehension will alter their behaviors in response to miscomprehension. However, in order to do so the individual must possess the knowledge to recognize the miscomprehension and the strategies to alter his or her behavior. This may not have been possible for the low fraction knowledge group individuals who may or may not recognize their miscomprehension and also may not have the knowledge of how to change their behavior. Further, high knowledge individuals recalled a greater number of elements and structure of the problems than low knowledge students, and low knowledge students committed a greater number of fraction of a number errors than the high knowledge group.

Differences Due to Problem Type

The third set of hypotheses examined differences due to the cognitive complexity of the different problem types which varied as a function of a combination of four factors: (1) the number of computational steps (i.e., one- vs. two-steps); (2) language consistency (i.e., CL vs. IL); (3) the wording of the relational statement of the problem (i.e., those which include or do not include "1/n as many" in their relational sentences), and (4) arithmetic operation (i.e., addition and subtraction vs. multiplication and division). The cognitive demands of a particular type of problem were hypothesized to influence problem solving success, problem solvers' pattern of behavior, number of rereadings, initial reading and total response times, and problem representation.

This hypothesis was only partially supported. Students successfully solved less cognitively demanding problems (i.e., CL, CL multiplication and IL division, and addition and subtraction) more frequently than their more cognitively demanding opposites (i.e., IL, IL multiplication and CL division, and multiplication and division). Therefore, each of these problem type differences (except for the number of computational steps which was significant at the $\underline{p} < .05$ level of significance) can be said to have caused the students to make a greater number of errors. This supports the hypothesis that these problem variables are related to the cognitive demands of the problems. However, except in one case (i.e., number of rereadings which differed due to number of computational steps and arithmetic operation), the students did not increase their activity levels in response to the increased cognitive demands of these different problem types.

In direct opposition to the premises of this study, students used a meaningful approach to solve CL multiplication and IL division problems significantly more often than the more cognitively demanding IL multiplication and CL division problems. It was hypothesized that on these latter problems the existence of the wording "1/n as many" would cause the students to change their behavior to model that of the meaningful approach. In fact, the data support the contention that when the students read this phrase they withdrew from the problem solving process. Additional support for this alternative hypothesis is the lack of a difference in total response times for each problem type comparison except for the number of computational steps. Anecdotally, while reading the problems several of the students indicated discomfort with the relational sentences which included a fraction of a number in their wording either through an inflection in the tone of their voice and/or facial expressions. Additionally, many students simply indicated that they could not solve these problems because they had not yet studied these topics during the present year.

Although counter to hypothesized, these findings support earlier research (e.g., Afflerbach, 1990; Pressley & Afflerbach, 1995) which has compared the reading behaviors of experts versus novices in a field. These studies have shown that individuals behave differently as a function of their knowledge in a given field which acts as a schema to support one's understanding of a text (Anderson, 1984). Without such knowledge an individual may either not realize that a miscomprehension has occurred or may not have the tools necessary to change his or her behavior accordingly.

Differences due to problem type were consistently found for one- and two-step multiplication and division problems, only. Although the difference in problem-solving success on these two types of problems is not considered significant for this study (t[79] = 2.06, p = .04), three differences due to the number of computational steps in the problems were significant. Participants reread two-step problems more frequently than one-step problems, recalled one-step problems with greater accuracy than two-step problems, and took significantly longer to solve two-step than one-step problems. Thus, only the number of computational steps resulted in the expected differences due to problem type.

Tests of the Consistency Hypothesis

Finally, differences due to language consistency were examined. These analyses served as direct tests of Lewis and Mayer's (1987) consistency hypothesis. In support of this hypothesis, participants committed a greater number of reversal errors on IL problems than on CL problems. On average, participants committed a reversal error on approximately 2 out of 6 IL problems which is ten times greater than on CL problems.

The last two analyses tested for differences in initial reading and total response times due to language consistency. According to Lewis and Mayer's (1987) consistency hypothesis, the problem solver reverses the order of the relational sentence in the IL problems as he or she reads the problem. In addition, due to the greater cognitive demands of these problems, IL problems should take longer to solve than CL problems. There have been mixed results in the literature regarding this difference. For example, Hegarty et al. (1992) found a significant effect due to language consistency for accurate and not for inaccurate problem solvers. In the present study, there were no significant differences in reading or response times due to language consistency. These results were counter to Lewis and Mayer's hypothesis but may also be explained in a similar way as were the lack of differences due to problem type.

Conclusions

This study sought to investigate components of the proposed Interactive Model of Mathematical Problem Solving (Figure 3). To do so, middle school children's problem solving approaches were assessed in order to examine the role mathematical conceptual and procedural knowledge and problem solving approach play in problem representation and subsequent problem solution. In this study, three main groups of analyses were performed to examine the variables collected.

First, almost half of the variance (48%) in problem solving success was accounted for by four main variables collected in this study, computation test score, number of problems solved using a meaningful approach, mean number of rereadings, and mean problem recall. Each of these variables contributed approximately equally to the prediction of problem solving success, except for computation test score which contributed slightly more than the other three variables in the regression equation. This analysis showed that those who used a meaningful approach solved a greater number of problems than those who did not use such an approach, and participants who were having difficulty did reread the problems more than those who were not experiencing difficulties. These results support Hegarty et al. (1995) who investigated the approaches of accurate versus inaccurate problem solvers. These authors conclude that

> the picture that emerges from this [their] analysis was that unsuccessful problem solvers struggle more than do successful problem solvers to construct a representation of the problem but spend their additional effort mainly in reexamining numbers and relational terms than in reexamining other informative words (p. 24).

Students in the present study may have reread the problem sentences when they were having difficulty, but this alone was not enough to help them form a mental representation of and solve the problems successfully.

The second set of analyses support the importance of conceptual and procedural knowledge in the representation and solution of IL multiplication and CL division problems. Students who lacked this knowledge were less successful in solving the problems, but these problem solvers also did not or were unable to alter their behavior in order to form an accurate mental representation of the problem and successfully solve the problem. Low fraction knowledge individuals' mean recall scores on these problems was significantly lower than the high knowledge group.

Finally, the analyses related to differences due to problem type yielded interesting results. These different sets of problems resulted in expected differences in problem solving success rates. However, except for differences due to the number of computational steps, there were few significant differences due to problem type with respect to students' behaviors. Thus, although CL problems, IL multiplication and CL division problems, and multiplication and division problems caused the students considerable difficulty, the differences in these problems did not translate into greater efforts or more strategic behavior on the part of the students.

Limitations and Future Research

One limitation of the present study is in the way the students' pattern of problem solving behavior was coded. This variable was operationally defined as two distinct groups (i.e., meaningful vs. direct translation). Hegarty et al. (1995) examined these patterns of behavior as possible differences between accurate and inaccurate problem solvers. Many of the students in this study were able to successfully solve the word problems without rereading or using a meaningful approach. In addition, many students reread the word problems but did not indicate the transformation of the provided information. Thus, these variables were interrelated ($\underline{r}$ = 0.45). This weakened the predictive value of both variables, yet the results of this study showed that both variables significantly accounted for unique variance in problem solving success. In future analyses, three categories of pattern of behavior should be coded or individuals for whom these problems do not pose a difficulty (i.e., those who can automatically represent the problems without rereading or transforming the information) should be excluded prior to analysis. Alternatively, the behaviors of accurate versus inaccurate problem solvers might be examined separately.

A second limitation of the study resulted from a methodological issue related to problem recall. In the present study, the students were required to recall the problems following solution. Many of these recalled problems did not match the problem as worded but directly matched the computational steps performed. In future analyses, the students should recall the problems before solution and/or following solution on a random basis. This methodological change would allow for more precise examination of the recalled problem as an indicator of the mental representation formed by the problem solver.

Also, in this study differences in behavior due to increased problem difficulty were not found. However, within the four "filler" problems there are problems which require more steps than the compare word problems investigated. Comparisons of the students' behaviors on these problems and the compare word problems may allow for a more in-depth examination of middle school children's behaviors as a function of problem difficulty. Individual case studies of student behaviors may also allow for greater understanding of the behaviors of those individuals who withdrew from the problem solving process. Finally, alternative ways of examining these behaviors (e.g., Artzt & Armour-Thomas, 1992; Schoenfeld, 1985) may help to elucidate the specific points during the problem solving process at which the components of the proposed model impact the problem solving endeavor.

This study provides several sources of data that have not yet been analyzed in this study. For example, an additional test of Lewis and Mayer's (1987) consistency hypothesis may be performed involving the types of errors committed on each of the different problem types. This more fine-grained analysis would allow for the examination of patterns of errors on these problems. In addition, the qualitative data contained on the videotapes collected for this study have not yet been examined. These protocols provide ample evidence of students' behaviors as well as their conceptions of fractions of a number.

Theoretical and Educational Implications

The implications of this study are both theoretical and educational. The proposed model represents a more comprehensive theoretical model of problem solving than previously proposed because it integrates an existing model of problem solving (Mayer, 1985, 1992; Mayer et al., 1984) within a model of reading comprehension (Ehri, 1995a, 1995b). Through the integration of these perspectives, the use of linguistically complex problems, and the use of verbal protocols, the present study extends previous research (Hegarty et al., 1992, 1995; Lewis, 1989; Lewis & Mayer, 1987; Verschaffel, 1994; Verschaffel et al., 1992) in two ways.

First, this study examined the influence of sixth- and seventh-grade students' prior knowledge (i.e., mathematical conceptual and procedural knowledge) on their solution processes. Despite students' problem solving efforts (e.g., the use of a meaningful approach versus a direct translation approach), in the absence of well-formed conceptual knowledge of the underlying mathematical concepts of the word problems, it was hypothesized that the problem solver would have great difficulty forming a meaningful representation of the problem and would fail to solve the problem. Second, the effect of problem type on problem representation, problem success, and pattern of problem solving behavior was investigated. Understanding the cognitive demands of different problem types and typical student efforts to solve these different problems has been shown to positively affect teachers' behaviors in the classroom (Fennema, Franke, Carpenter, & Carey, 1993).

From an educational perspective, the study of middle school children's problem solving behaviors from a reading comprehension perspective holds promise for enhancing our understanding of these behaviors. A better understanding of what students actually do to solve word problems may lead to the consistent use of more sound pedagogical practices. Therefore, there are several potential educational implications of this study. Specifically, the predominant use of a direct translation approach among students may reflect many years of key-word strategy instruction. This strategy leads almost without fail to a reversal error on IL compare problems. Understanding this consequence, teachers may alter their instruction so that it fosters a more active or meaningful approach to problem solving on both routine and non-routine problems.

Meaningful approaches to understanding word problems need to be explicitly modeled for students such that they may come to understand the nature of the problem solving process. If students are encouraged to understand and meaningfully represent mathematical word problems rather than directly translate the elements of the problems into seemingly corresponding mathematical operations, they may better comprehend the mathematical concepts embedded in these word problems. That is, a meaningful approach to problem solving may facilitate students' understanding of mathematical concepts. Word problems provide context for mathematical operations which may facilitate the students' development of meaning for these algorithms (Baroody & Ginsburg, 1986; Carpenter, 1986). Teaching children to think about the context of the problem (i.e., to take on a meaningful approach) and the reasonableness of the solution found, for example, will help them understand the essence of problem solving. In turn, through these experiences students will construct a well-formed base of conceptual knowledge that supports the mathematical operations being performed. This study, therefore, provides a specific example of where teachers may focus their instruction in order to fulfill the NCTM Standards document's (1989) vision of mathematics education and mathematics competence.

Figure 1

Mayer's (1992) Analysis of Mathematical Problem Solving

Stage/ Substages	Type of Knowledge	Mayer's description
Problem Representation		
Translation	Linguistic	Knowledge of the English Language; including recognizing words.
	Semantic	Knowledge of facts about the world; e.g., relationships between measurements (12in = 1ft).
Integration	Schematic	Knowledge of problem types, such as knowing that area problems are based on the formula A = LxW.
Problem Solution		
Planning and Monitoring	Strategic	Knowledge of techniques for how to use the various types of available knowledge in planning and monitoring the solution of problems; e.g., setting subgoals
Execution	Procedural	Knowledge of how to perform a sequence of operations, such as how to determine a fraction of a number.

Note. Adapted from Thinking, Problem Solving, and Cognition (p. 458-459), by R. E. Mayer, 1992, New York: W. H. Freeman and Company.

Figure 2

Interactive Model of Reading - Sources of Knowledge Used to Read Text (Ehri, 1995 - as adapted from Rumelhart, 1977).

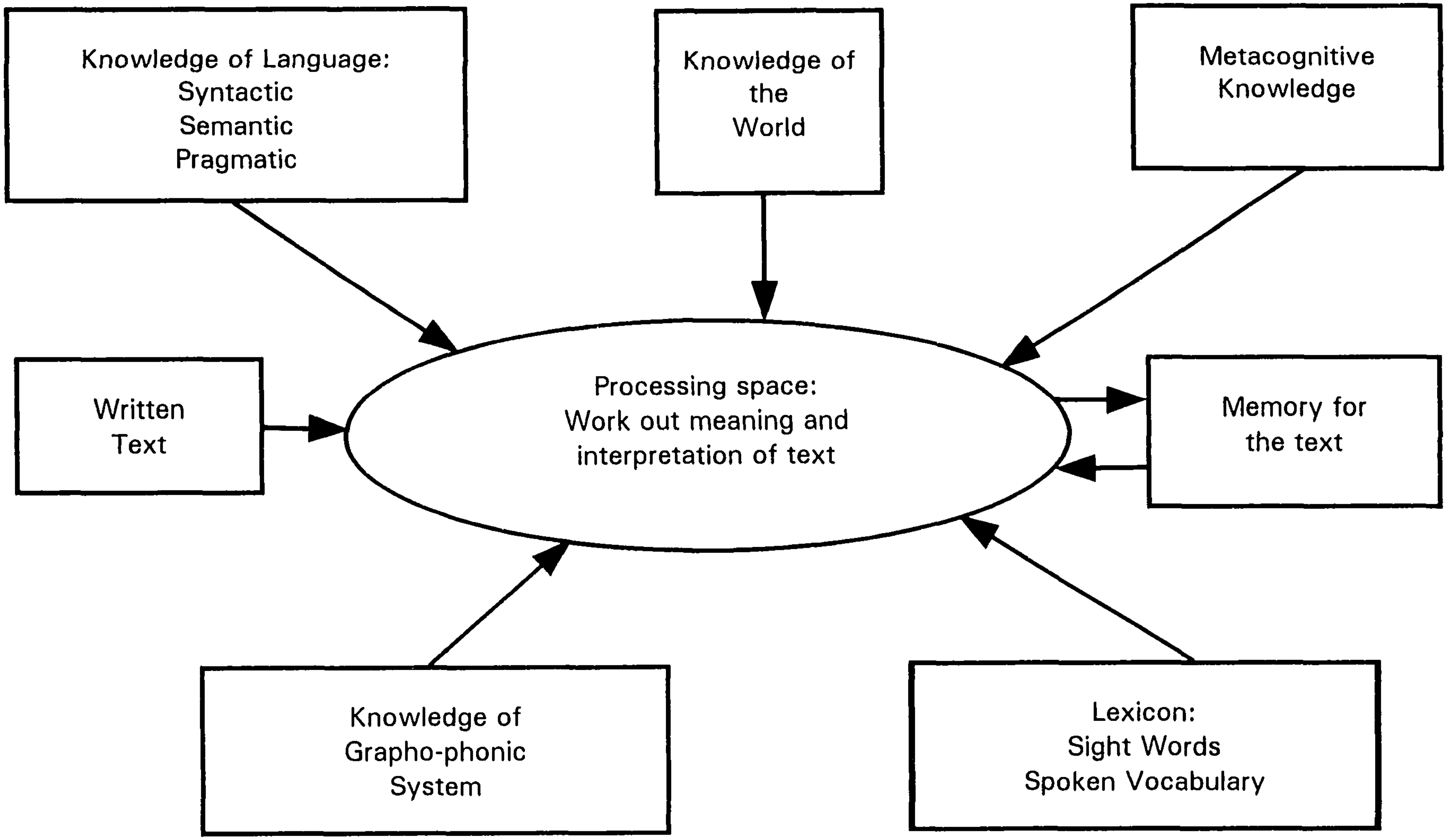


Figure 3

<u>Interactive Model of Mathematical Problem Solving</u>

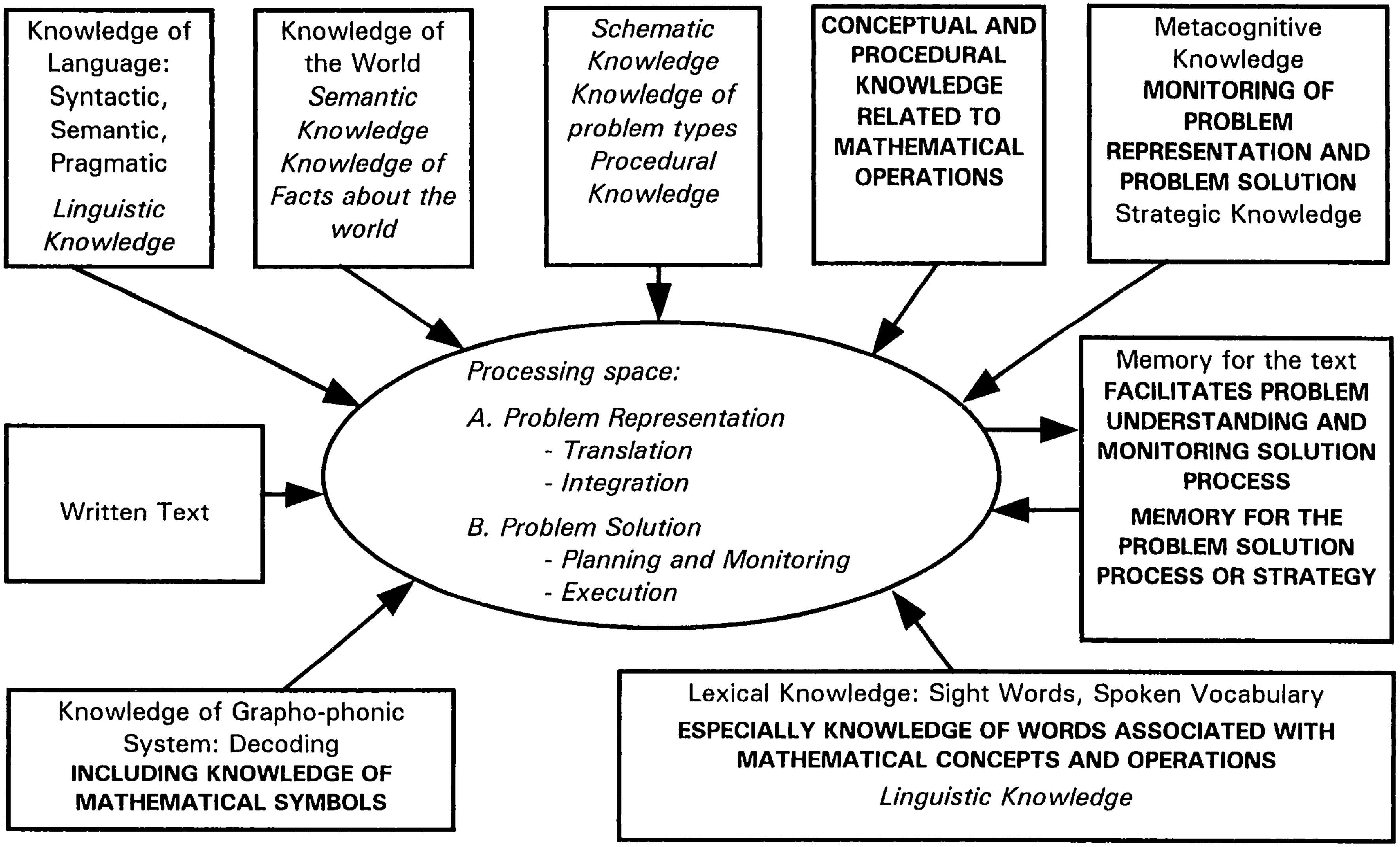
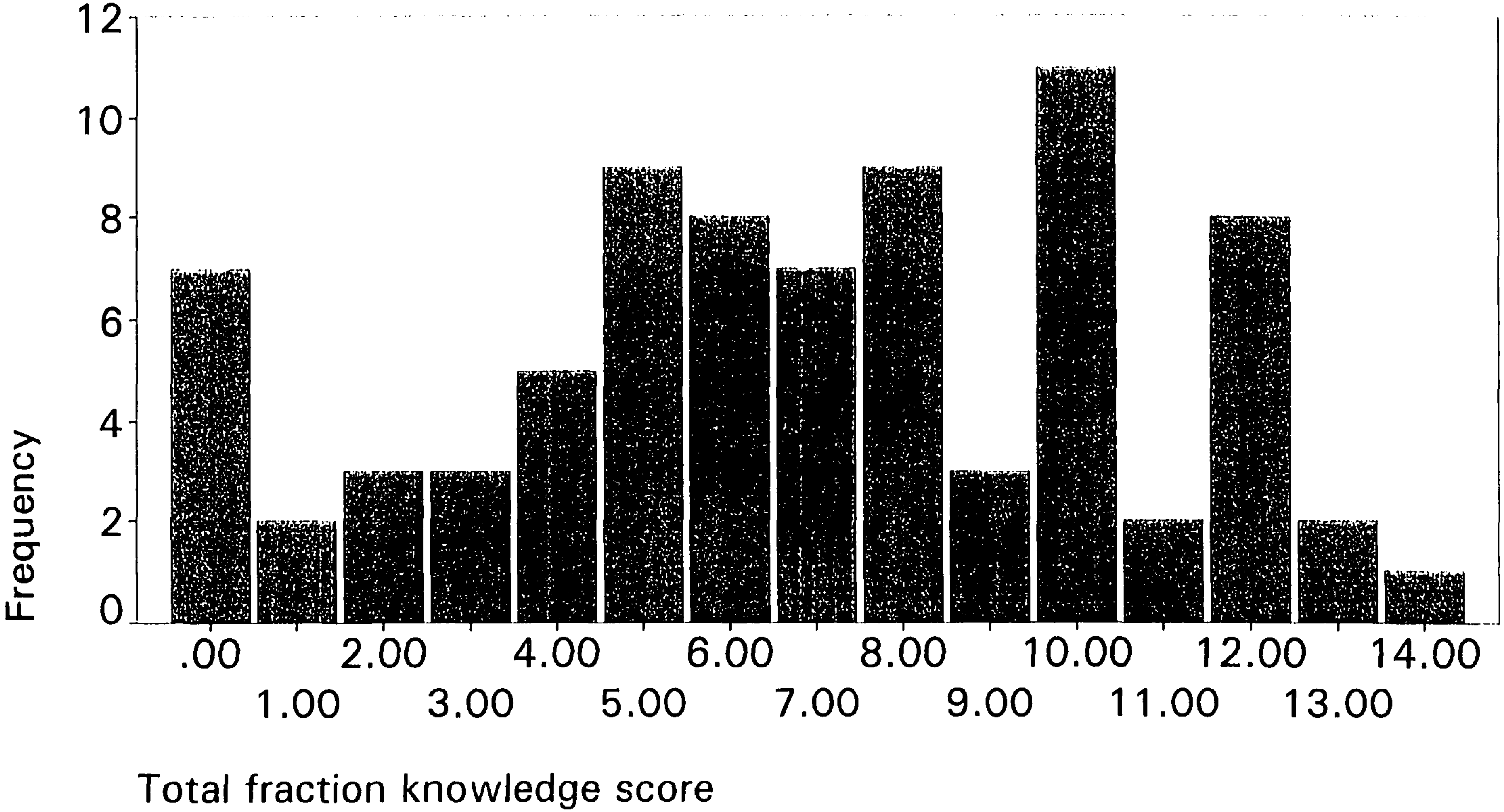


Figure 4

Total fraction knowledge score



Appendix A

Parent/Guardian Consent Form

Research Study - Mathematical Problem Solving

Dear Parent/Guardian:

My name is Stephen J. Pape, and I am a PhD student in Educational Psychology at the City University of New York, Graduate School. I would like to invite your son or daughter to participate in the research study for my dissertation.

The purpose of the study is to describe how sixth- and seventh-grade students solve mathematics word problems. Students who volunteer for the study will participate in two sessions. During the first session, the student will be videotaped as he or she solves typical mathematics word problems. The student will be given the opportunity to review the video, if he or she wishes. At the second session, the student will be asked to do computation (arithmetic) problems and to complete a questionnaire related to the ways in which he or she completes typical assignments for school. In addition, the student will be asked to state the language spoken at home, any other languages spoken, and ethnicity (optional). These procedures will take approximately two class periods. I will also collect standardized test scores and prior grades in Mathematics and Language Arts from the student's school records.

Your son or daughter's participation in this study is voluntary, and each student is free to withdraw from participation at any time during the study. Participation is **NOT** related to your son or daughter's school and will **NOT** affect his or her academic standing in any way. Non-participation in the study will also **NOT** affect his or her academic standing. All information will be kept strictly confidential by assigning a participant code number which will substitute for the student's name on all materials. No student names will be used in any report of the research. The videotapes will be used for research purposes, only. They may be shown to professional colleagues for the purposes of explaining the research; however, the administration and faculty from your son or daughter's school will **NOT** view them at any time. The videotapes will be secured in an office at the university for safe keeping.

At the conclusion of the study, a brief report of the findings will be delivered to the school administration and faculty. Any parent/guardian who would like a copy of this report should contact me directly. Summary data only will be reported. **No individual student information will be reported in this document.**

If you have any questions related to this study, please feel free to contact me in my office at (718) 997-5308 or my faculty advisor, Prof. Carol Tittle at (212) 642-2254. If you have any questions concerning your son or daughter's rights as a participant in this study, you may contact Sponsored Research, CUNY, Graduate School and University Center at (212) 642-2059.

Instructions for consent to participate: If you give permission for your child to participate, and he or she volunteers to participate by signing the Student Consent Form, you should do the following:

(1) **Initial** the bottom of the **first page** of this form;
(2) **Sign** this page on the line(s) provided **below**;
(3) Review and complete the Student Consent Form with your son or daughter;
(4) Keep the second copy of each of these forms for your records; and
(5) Return one copy of this form and the Student Consent Form to your child's school.

Only those students who return the signed forms will be considered for inclusion in the study, but returning the forms does not guarantee participation in the study. Thank you for your consideration to allow your son or daughter to participate in this project.

Sincerely yours,

Stephen Pape

Date: ____________ Student's Official Class : __________

I agree to let my child ________________________________ (please print student's name) participate in the study described above.

Parent/Guardian's signature: ______________________________________

Parent/Guardian's signature: ______________________________________

Appendix B

Student Consent Form

Research Study - Mathematical Problem Solving

Dear Student:

My name is Stephen J. Pape, and I am a student in Educational Psychology at the City University of New York Graduate School. I would like to invite you to participate in my research study for my dissertation.

The purpose of my study is to describe how sixth- and seventh-grade students solve mathematics word problems. If you volunteer for the study, you will participate in two sessions, one class period each. During the first session, I will videotape you as you solve typical mathematics problems. You will have an opportunity to review the videotape, if you wish. At the second session, you will be asked to do computation problems, to answer a questionnaire asking how you do some school assignments, the language you speak, and ethnicity (optional). I will also collect standardized test scores and prior grades in Mathematics and Language Arts from your school records.

Your participation in this study is voluntary, and you are free to withdraw from participating at any time. Participation is **NOT** related to your school and will **NOT** affect your academic standing in any way. Also, non-participation will **NOT** affect your academic standing. All information will be kept strictly confidential, and your name will **NOT** be used anywhere. The videotapes will be used for research purposes, only. They may be shown to professional colleagues for the purposes of explaining the study, but **NO ONE** from your school will see them. The videotapes will be kept in an office at the university for safe keeping.

A brief report of the findings of the study will be delivered to the school administration and faculty. **No student names or individual student information will be reported in this document.** Your parent or guardian may request a copy of this report directly from me.

If you have any questions related to this study, you and your parent or guardian can contact me in my office at (718) 997-5308 or my faculty advisor, Prof. Carol Tittle at (212) 642-2254. If you have any questions concerning your rights as a participant in this study, you and your parent or guardian can contact Sponsored Research, CUNY, Graduate School and University Center at (212) 642-2059.

Only those students who return the signed forms will be considered for inclusion in the study, but returning the forms does not guarantee your participation in the study. Thank you for considering volunteering for this study; I think you will find it interesting to do.

Sincerely yours,

Stephen Pape

Date:______________ Student Official Class: ____________

I ________________________________ (please print your name) fully understand what I will have to do to participate in this study and volunteer to participate in the study described above.

Student's Signature: ____

Appendix C

Student Background Survey:

Student's initials: ________

Participant Code: ________

Sign Date: ________

Video Date: ________

StratQ Date: ________

Tape#: ________

Form: ________

Tape Time: ____/____

Date of Birth: ________

Sex: ☐ M ☐ F

Class/Grade: ________

Primary Language Spoken at Home: ________

Other Languages Spoken: ________

Ethnicity (optional): ________

Report card grades:

	Previous Year Final Grade	Most Recent Grade
Mathematics		
Language Arts		

Achievement Test Score Data:

Date of Administration: ________

	Nat'l %	NCE score
CAT5 Mathematics		
CTB - Reading		

Appendix D

Student Agreement Form

I, ______________________________ , agree not to tell my friends, or any of the students in my class, about the study that I participated in until everyone has had time to participate. I understand that by talking to my peers about what transpired that I am giving them an advantage that I myself did not have, and that by sharing any information I would be jeopardizing the results of the study.

Date: ______________________________

Signed: ______________________________

Witnessed By : ______________________________

Appendix E

Verbal Protocol Stimulus (Form A)

The purpose of this exercise is to help me understand what students think about and do while reading and solving math word problems.

Directions for Mathematics Word Problems: Please read the following directions silently while I read them out loud.

I want to know what "comes to your mind" as you read, think about, and solve the word problems. To do this, I need you to say out loud everything that you think about or do including reading the problem, performing computations, and anything else that you think about. Try to **"think-out-loud"** -- in other words, say whatever you are thinking as you read and solve the word problems. If you cannot solve the problem, you may simply explain why you feel you cannot solve it and then go on to the next problem.

While you are working on the problems I can say only two things:

(1) If you are not talking out loud, I will remind you to say everything that comes to your mind; and

(2) If you are having difficulty solving the problem after a period of time, I will remind you that you may state the difficulty you are having with the problem and then you can go on to the next problem.

Also, while you are working on the problems, I **cannot** answer any questions which are related to understanding or solving the problem.

When you have completed the problem, you should turn the page over to indicate that you have solved it. After you have solved each problem, you will be asked to **recall** the problem out loud as best you can.

FORM A

Practice "Talk-Aloud" Problems

Here are two "practice problems." As you solve these practice problems, I will prompt you to say as much as possible. Remember that you should try to say out loud everything that you are thinking about or doing.

1. Jeremy bought a pair of hiking boots for $49.75 and a pair of wool socks for $3.19. If he gave the clerk $60, how much change should he expect to get back?

2. Erica bought a pair of sneakers for $52.89 and a pair of running shorts for $7.39. If she gave the clerk $65, how much change should she expect to get back?

FORM A

Note: In the protocol presented to the students, each of the following problems was presented on a separate page.

1. At Arco, gas sells for $1.13 per gallon. Gas at Chevron is 5 cents more per gallon than gas at Arco. How much does 5 gallons of gas cost at Chevron?

2. Apples come in large bags which contain about 36 pounds of apples each. Two large bags were delivered on Monday and almost 1/9 of the apples had to be discarded because they were damaged. Approximately how many pounds of apples were left for sale?

3. The local farm stand sells about 15 watermelons each day during the summer. The Supermarket sells 3 times as many as the farm stand a day. How many watermelons does the Supermarket sell in 5 days?

4. Last year the sixth grade sold 125 raffle tickets each day. That is 5 times as many tickets as the fifth grade sold per day. How many tickets did the fifth grade students sell in a day?

5. A group of 15 people are going camping for 3 days and need to carry their own water. They read in a guide book that 12.5 liters are needed for a party of 5 people for 1 day. How much water should they carry?

6. Pathmark sells 120 bottles of water a day. That is 2 times as many bottles as Waldbaum's sells in a day. How many bottles of water does Waldbaum's sell in 5 days?

7. At Pathmark a pound of sugar costs 89 cents. That is 20 cents more per pound than at Pace. How much do 5 pounds of sugar cost at Pace?

8. Mary runs about 6 miles per week. Sandy runs 3 times as many miles per week as Mary. How far does Sandy run in a week?

FORM A

9. Pace sells 50 pounds of potatoes a day. Pathmark sells 1/5 as many potatoes as Pace does in a day. How many pounds of potatoes does Pathmark sell in 4 days?

10. Sam needs to buy 4 dozen extra large eggs and 6 dozen small eggs each morning for his bakery. Extra large eggs are on sale for $1.99 a dozen. Small eggs are on sale for $1.39 a dozen. If Sam gives the cashier at the store a twenty dollar bill, how much change should he expect to get back?

11. At Pathmark a pound of pears cost $1.16. That is 15 cents less per pound than at Pace. How much does 5 pounds of pears cost at Pace?

12. Joe runs 6 miles a week. He runs 1/3 as many miles a week as Ken does. How many miles does Ken run in a week?

13. At Arco, gas sells for $1.13 per gallon. Gas at Chevron is 5 cents less per gallon than gas at Arco. How much does 5 gallons of gas cost at Chevron?

14. A large box of cereal sells for $4.29. How much change should you get back from a $10 bill if you buy 2 boxes?

15. Danny's Pizzeria sells 120 regular pizza pies a day. Angela's Pizzeria sells 1/3 as many regular pies as Danny's in a day. How many regular pizza pies does Angela's Pizzeria sell in a day?

16. Sam's Grocery sells 180 eggs a day. That is 1/3 as many eggs as Waldbaum's Supermarket sells a day. How many eggs does Waldbaum's sell in 3 days?

FORM A

Appendix F

Verbal Protocol Stimulus (Form B)

The purpose of this exercise is to help me understand what students think about and do while reading and solving math word problems.

Directions for Mathematics Word Problems: Please read the following directions silently while I read them out loud.

I want to know what "comes to your mind" as you read, think about, and solve the word problems. To do this, I need you to say out loud everything that you think about or do including reading the problem, performing computations, and anything else that you think about. Try to **"think-out-loud"** -- in other words, say whatever you are thinking as you read and solve the word problems. If you cannot solve the problem, you may simply explain why you feel you cannot solve it and then go on to the next problem.

While you are working on the problems I can say only two things:

(1) If you are not talking out loud, I will remind you to say everything that comes to your mind; and

(2) If you are having difficulty solving the problem after a period of time, I will remind you that you may state the difficulty you are having with the problem and then you can go on to the next problem.

Also, while you are working on the problems, I **cannot** answer any questions which are related to understanding or solving the problem.

When you have completed the problem, you should turn the page over to indicate that you have solved it. After you have solved each problem, you will be asked to **recall** the problem out loud as best you can.

FORM B

Practice "Talk-Aloud" Problems

Here are two "practice problems." As you solve these practice problems, I will prompt you to say as much as possible. Remember that you should try to say out loud everything that you are thinking about or doing.

1. Jeremy bought a pair of hiking boots for $49.75 and a pair of wool socks for $3.19. If he gave the clerk $60, how much change should he expect to get back?

2. Erica bought a pair of sneakers for $52.89 and a pair of running shorts for $7.39. If she gave the clerk $65, how much change should she expect to get back?

FORM B

Note: In the protocol presented to the students, each of the following problems was presented on a separate page.

1. The local farm stand sells about 15 watermelons each day during the summer. The Supermarket sells 3 times as many as the farm stand a day. How many watermelons does the Supermarket sell in 5 days?

2. Apples come in large bags which contain about 36 pounds of apples each. Two large bags were delivered on Monday and almost 1/9 of the apples had to be discarded because they were damaged. Approximately how many pounds of apples were left for sale?

3. At Arco, gas sells for $1.13 per gallon. Gas at Chevron is 5 cents more per gallon than gas at Arco. How much does 5 gallons of gas cost at Chevron?

4. Last year the sixth grade sold 125 raffle tickets each day. That is 5 times as many tickets as the fifth grade sold per day. How many tickets did the fifth grade students sell in a day?

5. A group of 15 people are going camping for 3 days and need to carry their own water. They read in a guide book that 12.5 liters are needed for a party of 5 people for 1 day. How much water should they carry?

6. At Pathmark a pound of sugar costs 89 cents. That is 20 cents more per pound than at Pace. How much do 5 pounds of sugar cost at Pace?

7. Pathmark sells 120 bottles of water a day. That is 2 times as many bottles as Waldbaum's sells in a day. How many bottles of water does Waldbaum's sell in 5 days?

8. Mary runs about 6 miles per week. Sandy runs 3 times as many miles per week as Mary. How far does Sandy run in a week?

FORM B

9. At Arco, gas sells for $1.13 per gallon. Gas at Chevron is 5 cents less per gallon than gas at Arco. How much does 5 gallons of gas cost at Chevron?

10. Sam needs to buy 4 dozen extra large eggs and 6 dozen small eggs each morning for his bakery. Extra large eggs are on sale for $1.99 a dozen. Small eggs are on sale for $1.39 a dozen. If Sam gives the cashier at the store a twenty dollar bill, how much change should he expect to get back?

11. Sam's Grocery sells 180 eggs a day. That is 1/3 as many eggs as Waldbaum's Supermarket sells a day. How many eggs does Waldbaum's sell in 3 days?

12. Joe runs 6 miles a week. He runs 1/3 as many miles a week as Ken does. How many miles does Ken run in a week?

13. Pace sells 50 pounds of potatoes a day. Pathmark sells 1/5 as many potatoes as Pace does in a day. How many pounds of potatoes does Pathmark sell in 4 days?

14. A large box of cereal sells for $4.29. How much change should you get back from a $10 bill if you buy 2 boxes?

15. Danny's Pizzeria sells 120 regular pizza pies a day. Angela's Pizzeria sells 1/3 as many regular pies as Danny's in a day. How many regular pizza pies does Angela's Pizzeria sell in a day?

16. At Pathmark a pound of pears cost $1.16. That is 15 cents less per pound than at Pace. How much does 5 pounds of pears cost at Pace?

FORM B

Appendix G

Computation Test   Participant Code:__________

Mathematics Computations: Answer each of the following problems in the space provided on this sheet. Show all work.

1a. How many feet are in 3 miles? (1 mi. = 5,280 ft.)

1b. How many feet are in one-third (1/3) of a mile? (1 mi. = 5,280 ft.)

2. Write each fraction in lowest terms or simplest form:

a) $\frac{16}{24}$

b) $\frac{25}{125}$

3. Compute:

a) $\frac{1}{5} \times 60 =$

b) $\frac{1}{4} \times 32 =$

4. Compute:

a) $\begin{array}{r} 1.76 \\ -\ \ .18 \\ \hline \end{array}$

b) $\begin{array}{r} 2.97 \\ +\ .08 \\ \hline \end{array}$

5. Compute:

a) $\begin{array}{r} 3.47 \\ \times\ \ \ 5 \\ \hline \end{array}$

b) $\begin{array}{r} 4.57 \\ \times\ \ \ 6 \\ \hline \end{array}$

6. Draw a picture to illustrate the following examples:

(a) $3 \times \frac{2}{5}$ (three times two-fifths)

(b) $\frac{1}{4} \times 16$ (one-fourth of sixteen)

Appendix H

Interrater reliability coefficients (N = 8)

Question Number	Type of Error	Pattern of Behavior	Number of Rereadings	Problem Recall
	Cramer's V	Cramer's V	Pearson	Spearman
01	1.00	1.00	0.99	1.00
03	1.00	0.77	0.98	0.84
04	1.00	0.49	0.99	0.84
06	0.87	0.75	0.95	0.73
07	0.84	0.77	0.99	0.73
08	1.00	0.77	0.99	0.67
09	0.83	1.00	0.98	0.68
11	1.00	0.77	0.95	0.84
12	1.00	0.65	0.99	1.00
13	0.82	1.00	0.94	0.82
15	1.00	0.65	0.97	0.80
16	0.88	1.00	0.97	1.00
M	0.94	0.80	0.97	0.83

Appendix I

Means (SD) and ANOVA Outcomes for Differences due to Form

	Form A	Form B		
Variable Name	Mean (SD)	Mean (SD)	F(1, 79)	Sig of F
Age	11.97 (0.70)	11.82 (0.60)	1.15	0.29
Math Achievement Test (CAT5): Raw Score	36.49 (9.85)	37.76 (9.06)	0.36	0.55
Reading Achievement Test (CTB): Raw Score	40.18 (7.35)	40.98 (6.43)	0.27	0.61
Math Report Card Grade	82.21 (12.15)	83.49 (9.78)	0.27	0.60
English Report Card Grade	84.54 (11.15)	84.93 (9.07)	0.03	0.86
Average Report Card Grade	82.43 (10.14)	83.22 (8.19)	0.15	0.70
Total Computation Test Score	15.74 (4.35)	16.05 (4.23)	0.10	0.75
Problem Solving Success	6.00 (2.91)	6.49 (3.35)	0.48	0.49
Number of problems solved using MA	3.72 (3.54)	3.73 (3.21)	0.00	0.99
Mean number of rereadings	1.72 (1.30)	1.62 (0.92)	0.16	0.69
Mean recall scores	2.19 (0.39)	2.11 (0.55)	0.49	0.49

Appendix J

Means (SD) and ANOVA Outcomes for Differences due to Grade

Variable Name	Grade 6 Mean (SD)	Grade 7 Mean (SD)	F(1, 79)	Sig of F
Math Report Card Grade	81.77 (12.03)	83.95 (9.77)	0.79	0.38
English Report Card Grade	85.63 (10.04)	83.85 (10.16)	0.62	0.43
Average Report Card Grade	83.86 (9.68)	81.81 (8.56)	1.00	0.32
Total Computation Test Score	14.80 (3.78)	17.00 (4.46)	5.63	0.02
Problem Solving Success	5.98 (3.14)	6.52 (3.14)	0.61	0.44
Number of problems solved using MA	2.98 (3.18)	4.48 (3.39)	4.16	0.05
Mean number of rereadings	1.60 (1.27)	1.75 (0.96)	0.36	0.55
Mean recall scores	2.13 (0.54)	2.17 (0.40)	0.12	0.73

Appendix K

Means (SD) and ANOVA Outcomes for Differences due to Gender

	Male	Female		
Variable Name	Mean (SD)	Mean (SD)	F(1, 79)	Sig of F
Age	11.77 (0.71)	11.97 (0.62)	1.70	0.20
Math Achievement Test (CAT5): Raw Score	39.62 (9.99)	35.73 (8.86)	3.25	0.08
Reading Achievement Test (CTB): Raw Score	40.62 (7.98)	40.57 (6.22)	0.00	0.97
Math Report Card Grade	80.45 (10.63)	84.24 (10.99)	2.25	0.14
English Report Card Grade	81.59 (12.58)	86.53 (7.92)	4.66	0.03
Average Report Card Grade	80.38 (9.77)	84.23 (8.55)	3.38	0.07
Total Computation Test Score	15.14 (3.79)	16.33 (4.49)	1.46	0.23
Problem Solving Success	6.90 (3.00)	5.88 (3.17)	1.96	0.17
Number of problems solved using MA	3.93 (3.41)	3.61 (3.35)	0.17	0.86
Mean number of rereadings	1.50 (1.09)	1.77 (1.13)	1.06	0.31
Mean recall scores	2.22 (0.44)	2.11 (0.49)	1.03	0.31

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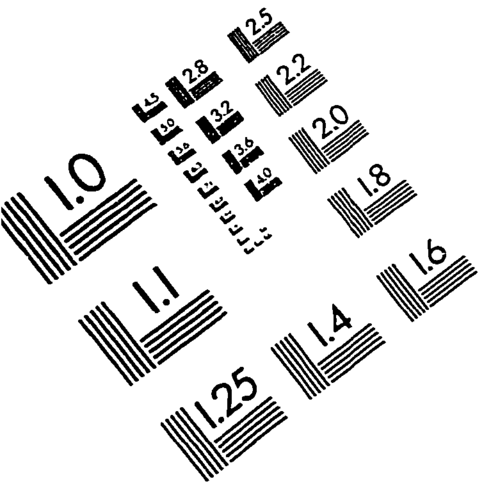
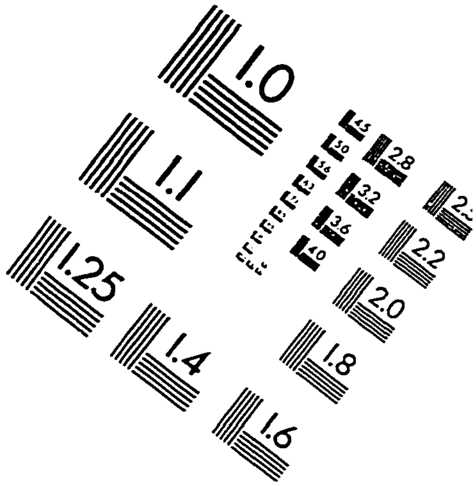
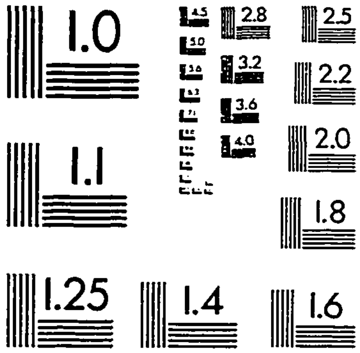
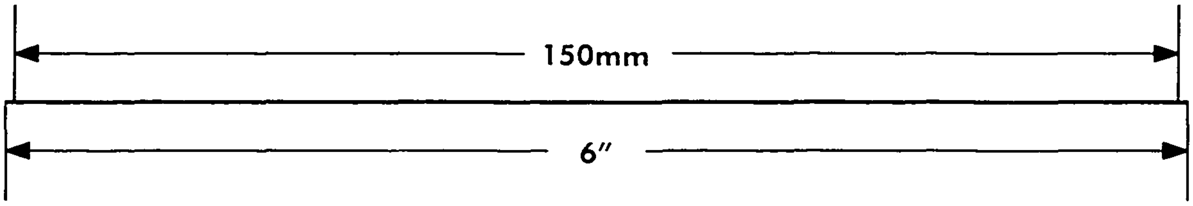
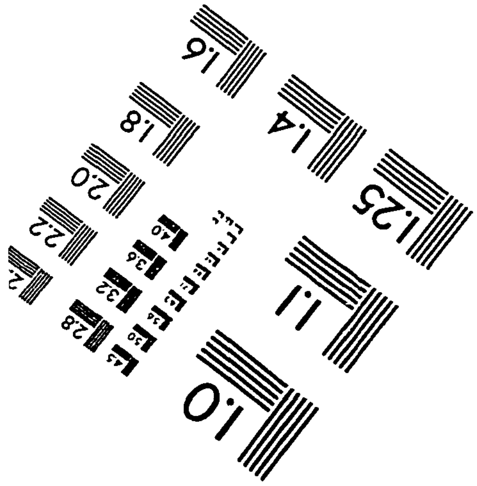
IMAGE EVALUATION
TEST TARGET (QA-3)











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