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THE ROLE OF ILLUSTRATIVE ERRORS IN LEARNING TO SOLVE PROBLEMS

by

Stanley Fisher

A dissertation submitted to the Graduate Faculty in Education in partial fulfillment of the requirement for the degree of Doctor of Philosophy, The City University of New York.

1977

This manuscript has been read and accepted for the Graduate Faculty in Education in satisfaction of the dissertation requirement for the degree of Doctor of Philosophy.

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Abstract

THE ROLE OF ILLUSTRATIVE ERRORS IN LEARNING TO SOLVE PROBLEMS

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Although problem solving is obviously an important category of learning, education currently lacks effective methods for teaching the heuristics of problem solving. This research explored one approach which had been suggested by the results of experiments in human learning, as different as maze and rule learning. The present study was an extension of the research which has focused on the influence on learning of instructional contacts with exemplars and non-exemplars of the behavior to be acquired.

Tutorial presentations, either in textbook or lecture format, typically use only ideal (appropriate) solutions to illustrate heuristic approaches to problem solving. This strategy is pervasive and can be found at all levels of education. However, there is no empirical evidence to support sustained adherence to this practice, or that of excluding illustrative problems with plausible, but erroneous, solutions. The subjects were 120 high school students of above average ability, 48 males and 72 females, whose mean age was 16 years and three months. They were taught to solve problems by working their way through pairs of

illustrative solutions, appropriate only, erroneous only, appropriate/erroneous and erroneous/appropriate. Those who had tutorial contact with matched pairs of illustrative solutions (appropriate/erroneous and erroneous/appropriate) were expected to solve more problems, derive more perfect solutions, and produce more necessary solution steps than students in any of the other groups. It was further expected that the students who had vicariously experienced error via contact with erroneous solutions would produce fewer inappropriate behaviors, and cancel a greater proportion, than the students who had had instructional contact with only appropriate solutions.

Although contact with pairs of illustrative solutions differentially influenced problem solving performance on a transfer task, it did so in a manner which had been only partially predicted. Treatment did not make a difference in the students' production of or response to erroneous behaviors; it only influenced the problem solving aspects of behavior. Those students whose first instructional contact was with an appropriate solution significantly outperformed those whose first contact was with an erroneous solution. When a more parsimonious model of problem solving was derived, post-hoc, it was found that those whose instructional contacts were with first an appropriate and then an erroneous solution solved significantly more problems than the students in any of the other treatment groups. It was further found that less than 25% of the

variance in problem solving performance could be explained by both treatment condition and the level of prerequisite knowledge, concepts and principles.

The implications of these findings for education and research are discussed.

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CHAPTER I

INTRODUCTION

A primary objective of formal education is the optimization of the student's ability to utilize acquired knowledge to solve substantive problems. A high school student engaged in proving a theorem in Euclidian geometry, a computer science major programming a solution to the "Towers of Hanoi" puzzle, or a medical student attempting to diagnose a patient, are all participating in instructional activities expected to enhance their problem solving skills. Although problem solving is a critical aspect of learning in most academic disciplines, relatively little is known regarding the influence of tutorial presentations on the students' acquisition of the required skill. Landa (1976, p. 172) noted that education currently lacks effective methods for teaching the heuristics of problem solving. Over 30 years ago, Wertheimer (1945) urged psychologists and educators to develop more effective methods for teaching students to think productively.

In a discussion of this critical issue, Ausubel (1968, pp. 166-168) recommended that educators develop means to enhance their students' abilities to acquire prerequisite facts, concepts and principles, rather than focus directly on problem solving. He asserted that problem solving behaviors were less susceptible to educational manipulation than other

forms of learning: intra-personal factors such as drive, persistence, flexibility, anxiety, intellectual curiosity, cognitive style, and problem sensitivity, have been shown to be significant sources of variance in problem solving ability (1968, pp. 546-551). Furthermore, success in solving problems related to specific subject-matter is highly correlated with the students' level of prerequisite knowledge (Gangler, 1967; Mayer, 1975).

However, several investigators have observed that even when students have adequately demonstrated their acquisition of prerequisite material, some fail to solve problems while others succeed (Bloom & Broder, 1950, pp. 25-31; Scandura, Durnin & Wulfeck, 1974). Therefore, Scandura et al. have argued that researchers should focus directly on problem solving: (1) to identify the sources of student failures and (2) to find instructional means to remedy them.

A potential source of student failure may be the limited scope of the sample problems which are used as illustrations (Ausubel, 1968, p. 547; Merrill & Boutwell, 1973). Tutorial presentations, either in textbook or lecture form, typically use only ideal solutions to illustrate heuristic approaches to subject-matter problems. This strategy is pervasive and can be found at all levels of education. However, Merrill and Boutwell have observed that there is no empirical evidence to support sustained adherence to this practice, or that of

excluding illustrative problems with plausible, but erroneous, solutions. Furthermore, they noted that transfer to new instances of a concept was substantially enhanced when the tutorial presentation contained both exemplars and non-exemplars (Tennyson, 1973; Tennyson, Woeley & Merrill, 1972). Extrapolating from the results of studies in concept learning, Merrill and Boutwell posed the following question: Will transfer in problem solving be enhanced by the tutorial presentation of both correct and erroneous illustrative solutions to sample problems?

The present study is an initial step toward developing an answer to that question. It is an extension of the research in human learning which has focused on the influence of instructional contacts with exemplars and non-exemplars of the behavior to be acquired. This study examined the influence of earliest possible tutorial contacts with illustrative solutions, ideal and/or erroneous, on students' performance in a problem solving transfer task. Specifically, the experimental design contrasted four tutorial treatments; these were contacts with: (1) only appropriate (ideal) solutions, (2) only erroneous solutions, (3) first an appropriate solution and then an erroneous one, and (4) first an erroneous solution and then an appropriate one.

Clarification of Terms

Problem Solving

Because "problem solving" has been used as a classification for a wide variety of human behaviors (see Getzels, 1964 or Ray, 1955, for examples), its use in this research must be delineated. This research focused on one specific class of problems, those which require heuristic solutions. Furthermore, within that category, the focus was on the types of problems which are typically found in school subject-matter. That is, those in which successful problem solving has as a critical antecedent the acquisition of relevant facts, concepts and principles.

According to Newell and Simon (1972), a person is confronted with a problem when he is presented with a designated objective and the series of actions required to achieve that objective are not immediately known. Gagné (1974) would furthermore require that the situation be a novel one for the problem solver. Although these statements define what is meant by "a problem," they cannot be used to distinguish between problems which must be solved heuristically and those which may be solved in other ways. Psychologists with an information processing point of view (e.g., Newell, Shaw & Simon, 1963) have observed that humans use two distinct types of processes in solving problems: algorithmic and heuristic.

Algorithmic Problem Solving. The algorithmic approach utilizes a specified series of actions which has been precisely defined over the problem domain. If an algorithm is carried out as specified and a solution exists, the production of a correct solution is guaranteed. Algorithmic solution processes are strictly rule-governed behaviors of which the only prerequisite for successful performance is the knowledge of and correct application of the rules.

For example, Scandura (1974) taught students to solve "complex trading problems" by teaching them an algorithm for forming composite trading rules. First he taught all of the students simple trading rules, e.g., a group of red chips could be traded for a group of pencils. After they had all learned the simple rules, he taught half of the group an algorithm for constructing composite rules. Composite rules were formed by the concatenation of two simple rules, under the constraint that the output of the first must match the input of the second, i.e., if the two simple rules were "three red chips can be traded for two pencils," and "two pencils can be traded for ten paper clips," then a composite rule could be constructed which stated that "three red chips can be traded for ten paper clips." Scandura then presented the students with composite trading problems. Only those who had learned the algorithm for constructing composite rules were able to solve the problems.

The solution algorithm used by the group who had received training may have looked something like this:

- Step 1. Select all simple trading rules whose input matches the problem input. Consider these first-rule potentials.
- Step 2. Select all simple trading rules whose output matches the problem output. Consider these second-rule potentials.
- Step 3. Select and apply to the problem that pair of potentials in which the output of the first rule matches the input of the second.

Note that the solution made use of two basic principles:

(1) that the output of the first rule must match the input of the second; (2) that the input of the first must match the input of the problem and that the output of the second must match the output of the problem. The derivation of a correct solution would not necessarily depend upon the sequence in which these rules were applied. In this case, steps one and two of the algorithm could have been reversed. A correct solution would be obtained as long as the rules had been correctly applied.

However, there are many classes of problems which are not amenable to algorithmic approaches. One example is the diagnosing of illness. A simple algorithm for diagnosing any disease would be to try all known medical tests until either the disease has been identified, or every test has been tried. The shortcomings of such an algorithm are obvious. Although it may often be possible to develop an algorithm to solve a specific class of problems, i.e., diagnosing diseases, the situational variants to be analyzed and tested may be so numerous that humans are simply unable to construct efficient algorithms. For example, humans do not have efficient

algorithms which can be used in the following problem solving situations: playing chess, designing a building, writing a computer program, determining the chemical composition of an unknown organic specimen, teaching students, designing an experiment, interpreting research results.

The Heuristic Approach. Heuristic solution processes, unlike the algorithmic, provide no guarantee that a correct solution will be produced even if one exists. In the heuristic approach the solution must often be derived by a series of approximations (Duncker, 1945). The problem solver considers these approximations plausible, but provisional, solutions. Each solution approach is conceived, tried, and evaluated. The results of that evaluation serve as the generator for the next attack. The adequacy of any of the specific steps used in constructing a solution can be evaluated only in terms of its relationship to the hypothesized solution. However, neither the approach, nor the solution it provides, can be adequately evaluated until after the provisional solution has been actually derived and verified. Because a correct solution is not guaranteed, an integral element of the heuristic process is verification (see Klausmeier & Loughlin, 1961, for examples). If the problem solver is to succeed, he must have the facility to recognize and correct his own errors.

Like the algorithmic, the heuristic approach requires that the problem solver correctly apply a sequence of rules. Although the correct application of an applicable rule is a necessary corequisite of a correct solution, at any stage

in the solution process the determination of which rule to apply next is not strictly rule-governed. Problem solvers often find themselves far from a solution, even though they have correctly applied a sequence of applicable rules. Therefore, the heuristic approach must be applied whenever the problem solver does not have an algorithm available for determining the "best" next step. In formal education, as in life, there are many situations in which students are required to solve problems when the likelihood of obtaining a solution solely by randomly applying a sequence of acquired rules is virtually zero. For example, in trigonometry students are often required to prove that two statements are equivalent:

Prove that:

$$\frac{\tan \theta - \cot \theta}{\tan \theta + \cot \theta} = 2\sin^2 \theta - 1$$

In attempting to solve the problem the students are free to substitute trigonometric identities for any of the expressions or apply any algebraic transformation rule. There are probably more than a dozen transformations which could be correctly applied to the expression shown above: e.g., the students could add a 1 to both sides of the equation; they could multiply both sides of the equation by $\tan \theta + \cos \theta$; they could substitute $1/\cos \theta$ or $\sin \theta/\cos \theta$ for one or all of the $\tan \theta$ s; they could substitute $1/\tan \theta$ for $\cot \theta$. However, only a small percentage of permissible transformations would lead to a solution of the problem. Yet students have

been able to solve such problems without the aid of an algorithm, by utilizing subject-matter knowledge in a goal-directed manner (Carpenter, Moore, Synder & Lisansky, 1961; Moore & Anderson, 1954a).

For this research, problem solving is defined as that category of learning in which students learn to utilize acquired subject-matter knowledge in a goal-directed manner, to derive a previously unknown series of actions which can accomplish a substantive, pre-designated objective. Students are usually taught to solve these types of problems by reviewing illustrative examples of ideal solutions. The ability to solve problems heuristically is that learning outcome which Gagné (1974) has labelled "Cognitive Strategies" and Wertheimer (1959) has called "Productive Thinking."

The Constructs "Exemplar" and "Non-Exemplar"

The terms "exemplar" and "non-exemplar" were initially used in concept learning to designate members and non-members of a concept class. Occasionally, synonyms such as "positive example" and "negative example" have been used in the literature. Recently, Merrill and Boutwell (1973) expanded the meanings of these terms to include illustrative examples of correct and incorrect applications of a rule, and appropriate and erroneous solutions of a problem.

Exemplars and Non-Exemplars as Models of Behavior. In the context of this research, exemplars and non-exemplars

are viewed as models of the behaviors the learner is expected to acquire or avoid. The presentation of an instance which has been designated an example of the concept, e.g., "The boy RUNS; RUN is a verb.", provides the learner with some information about the characteristics a word should exhibit whenever it is identified as "a verb." Conversely, the presentation of a non-exemplar, a word which is not an example of the concept, e.g., "The boy runs WILLINGLY; WILLINGLY is not a verb.", is one way to demonstrate the behaviors to be avoided. In effect, the learner is told "You will be wrong if you identify 'willingly' as a verb." Tutorial presentations which have been designated as exemplars or non-exemplars provide information which is equivalent to that which the learner could obtain from observing another student's successes and failures at attempting to classify the set of instances.

The Scope of Illustrative Instances. Tutorial presentations can contain either similar or very different examples to illustrate the behavior to be acquired. In concept learning, the exemplars may be either convergent or divergent in the attributes of classification. For example, if the concept is VERB and sentences are used to illustrate the concept, the exemplars would be convergent if the same verb was used in a variety of sentences; they would diverge if different verbs and different tenses of the verbs were used. Exemplars and non-exemplars could be either matched or unmatched on

their irrelevant attributes (Tennyson, Wooley & Merrill, 1972). However, negative instances must bear some plausible resemblance to positive instances if they are to be didactically useful. If the concept to be acquired is ADVERB, the non-exemplars should be verbs or adjectives, but not clauses or paragraphs. Although the latter are erroneous instances, the differences are too obvious. If the concept is RED STAR, the non-exemplars should be such figures as a red diamond or blue star, rather than a striped ball.

In rule learning, the illustrative problems can be heterogeneous or homogeneous (Traub, 1966). Heterogeneous problems are those in which the attributes of the situations in which the rule is to be applied are different. Problems in graphic addition may use only negative numbers, only positive numbers, or a combination; solutions may have both negative and positive results. Homogeneous problems would use only the positive numbers. Here too, the non-exemplars should bear a plausible resemblance to the exemplars. Thatcher (Note 11) used prior students' errors to derive non-exemplars which would be plausible to the learner. Those instances closely resembled the type of mistakes the learners were likely to make.

There were an infinite variety of non-exemplars which could have been selected for the present experiment. Students produced many different sets of erroneous solution behaviors for the same problem (Fisher, Note 4). The exemplars and

non-exemplars used here were homogeneous; they were all based on a single problem. The erroneous exemplars closely resembled completed incorrect solutions created by new learners. The behaviors which were presented were drawn from the attempts of new learners who tried to solve similar problems. At first glance, these erroneous solutions might be mistakenly considered correct.

CHAPTER II

BACKGROUND OF THE RESEARCH

Research contrasting the role of exemplars and non-exemplars in learning has substantially contributed toward an understanding of school subject concept acquisition (Clark, 1971; Merrill & Boutwell, 1973). However, this strategy has not yet been applied to school subject problem solving (Merrill & Boutwell, 1973). Nonetheless, psychologists have proffered definitive positions regarding the use of erroneous solutions as exemplars in instructional presentations. For example, Skinner posited the view that instructional contact with erroneous solutions would inhibit learning. In an article summarizing the educational contributions of teaching machines, he wrote:

An appropriate teaching machine will have several important features. The student must compose his response rather than select it from a set of alternatives, as in multiple choice. . . . one reason for this is that . . . effective multiple choice materials must contain plausible wrong responses, which are out of place in the delicate process of 'shaping' behavior because they strengthen unwanted forms. (Skinner, 1958, p. 970) [Emphasis added.]

In his book on the technology of teaching, Skinner made it clear that he considered both the solution to a problem and the steps which produced that solution to be responses. He asserted that students should not be exposed to erroneous suggestions or presentations of erroneous behaviors (Skinner, 1968,

pp. 132-133). However, Skinner accepted the idea that occasionally there might be valid reasons for placing the students in contact with erroneous responses; this should occur only when the response set to be acquired directly relates to making choices between correct and erroneous items.

Miller (1957), in an article discussing the role of graphic communication in education, hypothesized that presentations which contrasted correct responses with illustrative errors would enhance learning:

In addition to showing the correct response, it should be useful to demonstrate errors, labelled as such. . . . It seems probable that this will help the student to identify errors and be specifically motivated to correct them. It may also help the student avoid certain wrong responses and help him to discriminate the correct cues and responses by contrasting them with the incorrect cues and responses. Finally, demonstrating errors may decrease the amount of stimulus generalization required, by making the film learning more like the real life situation in which errors must be made and corrected. (Miller, 1957, p. 108)

It is clear, both because of the nature and application of the materials under discussion, that Miller also used the term "response" for a wide range of behaviors, both verbal and non-verbal.

In the first section of this review the contribution of exemplars and non-exemplars to various aspects of learning will be examined, in situations as different as maze and rule learning. The sequence of presentation will follow Gagné's (1970) hierarchy of learning, when feasible. In the second section

the role of errors in learning will be reviewed. Both theoretical viewpoints and empirical results will be presented. The role of errors and the role of non-exemplars will be differentiated, and then re-associated using the observational learning paradigm. In the third section the relationship between problem solving and transfer of learning will be examined. Issues in transfer of learning will be described and parallels drawn between problem solving and transfer. Experiments will be described in which the influence of illustrative examples on transfer were examined.

In Chapter III, a methodological tool for exploring school-related heuristic problem solving, the Moore and Anderson (1954a) Symbolic Logic Task, will be described in detail. The instructional materials used in this research were derived from that task. The task description will be followed by a review of experimental applications which highlight the multi-faceted aspects of the problem solving process.

The Contributions of Exemplars and Non-Exemplars to Learning

Empirical investigations contrasting the contributions of exemplars and non-exemplars to learning have rarely appeared in the literature, except for studies of concept learning. Studies of the roles of exemplars and non-exemplars in problem solving were not to be found. There have been some studies in which the students' critical examination of the behavior of others was chosen as the method to enhance

learning. These studies tended to use complex didactic strategies and undesigned exemplars and non-exemplars. For example, Oswalt (1962) found that proof-reading other students' exercises by checking the words in a dictionary significantly improved spelling achievement scores for subjects with average and low I.Q.s. Selz (1935) enhanced the performance of retarded children in sentence completion, analogy and number series tasks by having the children criticize each others' errors.

The studies which are described below examined the influence of designated contact with exemplars and non-exemplars on various aspects and outcomes of learning. Achievement, learning, re-learning, and transfer were studied for educational outcomes which are presented in the following order: motor skills, verbal information, concepts, rules, and a teaching skill.

Motor Skills

Gagné (1974, pp. 66-67) noted that motor skills were essential elements of the school curriculum. Students must print and write letters, pronounce sounds in foreign languages, and acquire procedures for handling equipment in the laboratory. Evans (Note 2) used exemplars and non-exemplars to improve the writing of adequate numerals among young students. He was interested in errorless learning. Each of the students started training with an inadequate,

but established, level of numeral writing. Evans showed each child a series of well written and poorly written numerals, labelled to indicate which was which. He reported that the childrens' numeral writing was improved without overt practice. He suggested that the acquisition of a means of cognitively discriminating between good and poor numerals was all that was required to improve performance.

Waters conducted a series of experiments using a stylus maze (Waters, 1931; Waters & Ellis, 1931). These investigations contrasted the effects of differing aspects and degrees of guidance on the acquisition of a motor skill. College students, individually matched in respect to sex, college year and approximate age, were distributed among three experimental groups.

The "Free Learning" group, which served as the control, was required to learn the maze by trial-and-error. The experimenters kept a detailed record of the first trials for each subject in this group, noting the steps each subject took in traversing the maze and the traversal time.

The "Erroneous Path" group of subjects was told that on their first trial they would be guided through the maze, and would follow the path of a student who had learned the maze without any help. The instructions warned them that they would be led "into blind alleys, have to retrace steps, etc., just as he had done." They were then led along the

path which had been recorded for their individually matched control.

The "Correct Path" group of subjects were also guided, but they were guided along the most direct path to the goal. The guided traversal of that route took 20 seconds. Each subject in the group repeated the guided trials until the amount of time was equivalent to that which had been taken by their individually matched control on the first trial.

The criterion of mastery was four perfect, successive runs within an hour and a half. The subjects who met criterion were asked to return a week later for a retention test. At that time all subjects were required to relearn the maze without guidance.

In contrast to the "Free Learning" control group, the "Erroneous Path" group required significantly fewer trials to achieve mastery during learning. There were no significant differences found between the performances of the two groups which had been guided, or between the performances of the "Correct Path" group and the control group, during the first session. There were no significant differences between any of the groups' performance in relearning the maze. However, the smaller number of trials to mastery required by those in the "Erroneous Path" group when contrasted with the number of trials required by the control group, did approach statistical significance.

Waters reported that a prominent characteristic of the

"Erroneous Path" group was cautiousness. The attitude was observed during both learning and relearning. Although the "Erroneous Path" group had a higher mean error rate during relearning than either of the other two groups, an analysis of that group's learning curve indicated that most of the errors occurred in the early trials. Rapidly thereafter, the mean number of errors the group made fell below those of the other groups.

The results from these studies appear to support Miller's (1957, p. 108) view that when students are shown examples of errors, they are more motivated to avoid or to correct errors. That type of self-motivation was reflected in the attitude of the subjects who had been guided along erroneous paths while learning the maze. The young subjects in Evans' experiment must also have been motivated to avoid error if their numeral writing improved. Gagné (1974, p. 65) considered attitudes as a distinct class of educational outcomes, paralleling knowledge and intellectual skills as desirable educational outcomes. An appropriate attitude toward error is critical in problem solving. Duncker (1945) indicated that an attitude which combines expectancy and acceptance of error tends to foster success. It is important to note that exemplars and non-exemplars of the behaviors to be acquired appear to provide more than information; they may be successfully used in influencing the acquisition of the important affective components of learning -- motivation and attitudes.

Verbal Information

In most school subjects, students are required to acquire verbal information, e.g., the names of objects and symbols, facts, and events which have occurred, before they begin to study concepts, rules and problem solving. They are often required to demonstrate their knowledge via examinations testing recall and recognition. Skinner (1958) opposed the use of multiple choice materials in teaching verbal information because he felt that the presence of plausible wrong answers would strengthen "unwanted forms."

Kulhavy and Anderson (1972) explored the influence of instructional contacts with correct and plausible erroneous responses in a multiple choice test of verbal information on a subject's performance on the same test after a one-week interval. The subjects were high school students who were randomly assigned to eight experimental groups. Two consecutive days were assigned for the experimental treatment.

The treatment activities were:

1. TEST - take the test.
2. REVIEW - review a copy of the test in which the correct and incorrect answers were appropriately designated.
3. JUDGE - looking at a copy of the test, recall your responses to the questions.
4. DELAY - the activity which follows occurs on the following day.

The eight experimental groups were designated by the treatment activities in which they were engaged:

1. TEST-REVIEW
2. TEST-REVIEW-JUDGE
3. TEST-DELAY-REVIEW
4. TEST-DELAY-REVIEW- JUDGE
5. REVIEW-TEST
6. REVIEW-DELAY-TEST
7. REVIEW
8. DELAY-REVIEW

A week later, all of the students took the same test.

The treatment activity REVIEW appeared to perform two substantially different functions depending upon the sequence in which it occurred. When REVIEW occurred after the test, it served as feedback; when it occurred before the test, it served as a tutorial.

The REVIEW-TEST and the REVIEW-DELAY-TEST groups received the highest scores on the test which was given a week later. Both of these groups' mean scores were significantly higher than those of the TEST-REVIEW, TEST-REVIEW-JUDGE, and REVIEW groups. The REVIEW-DELAY-TEST group's mean score was also significantly higher than that of the TEST-DELAY-REVIEW-JUDGE group. The results indicated that exposure to correct and incorrect answers was most enhancing when they occurred tutorially, rather than as a result of feedback. That effect

seemed to be most pronounced when the tutorial presentation was followed by practice.

Concept Learning

School learning often deals with the acquisition of descriptive terms which represent classes of objects, ideas, and relationships: i.e., concepts. Gagné (1974, pp. 57-60) identified two classes of concepts: (1) "concrete concepts," partitionings of objects, object qualities, and events, which can be shown; and (2) "defined concepts," partitionings of object qualities, events, and relationships, which cannot be shown. School subject matter tends to deal with defined, rather than concrete concepts. Subject matter concepts are normally defined over infinite sets, and therefore there is no way to specify all of the attributes of members and non-members of a concept class.

Although the findings from the psychological laboratory have extended our understanding of the role of exemplars and non-exemplars in acquiring a concept, they have had only limited applicability in education (Clark, 1971). The typical experiment in concept learning tends to deal with concrete concepts which have been defined over some limited set of objects. The usual didactic strategy used in laboratory concept learning is unguided discovery (see Bourne, Ekstrand & Dominowski, 1971 for a review of the literature). Nonetheless, the psychological laboratory has uncovered

material which appears pertinent to this research. These findings will be presented first, and will be followed by a review of studies in school subject-matter concept learning.

In a comprehensive examination of the effect of positive and negative instances in promoting the acquisition of concepts, Bourne and Guy (1968) observed that in the typical laboratory experiment, learning was most enhanced when the instances which were presented were from the smallest or most homogeneous category. It made no difference whether or not the smallest group contained exemplars or non-exemplars. However, Wells (1967) reported that the presentation of contrasting exemplar pairs, one positive and one negative, induced performance which was superior to that induced by the presentation of only negative or only positive pairs. Performance was superior both in regard to the number of trials and number of instances.

Tennyson, Woolley and Merrill (1972) and Tennyson (1973), in a series of experiments, examined the role of negative and positive instances in the acquisition of concepts defined over infinite classes. Seventy-six undergraduates served as the subjects in the first experiment. The concept to be acquired, TROCHAIC METER, was communicated via a self-instructional booklet. The booklet opened with a definition, which specified the concept's critical attributes. This was followed by eight pairs of exemplars and non-exemplars,

two per page. In each pair the first instance was always an exemplar. The members of a pair were appropriately designated as an "Example" or "Not an Example." There was also an indication as to whether or not the pair was matched, i.e., as similar as possible on the irrelevant attributes. The subject was also told whether or not the positive exemplars appearing on the same page were divergent, i.e., differed as much as possible on the irrelevant attributes. Prior to their incorporation in the text the instances had been ranked according to their probability of being correctly identified by a novice. The text did not require student responses.

Four treatment groups were established in order to test the Tennyson et al. hypotheses. Each group was presented with a different set of instances. It was hypothesized that subjects presented with: (1) matched and divergent instances, at all levels of probability, would make the least number of classification errors; (2) unmatched and divergent instances, at all levels of probability, would over-generalize; (3) matched and divergent instances, with a high level of probability, would under-generalize; (4) unmatched and convergent instances would misconceive the concept. The post-test contained 30 new selections of poetry, exemplars and non-exemplars, which had been scrambled so that pairings of any type would not be obvious. All levels of probability of identification were represented. An Expected Error Matrix,

indicating by treatment group the mean number and type of errors the subjects were expected to make, was established prior to the experiment. Each subject's grade point average was used as a covariate in the analysis of the data.

As predicted, the subjects who had been provided with the matched and divergent instances, at all levels of probability, made significantly fewer errors. The group which made the next lowest error score had also been provided with matched and divergent instances, but those were only at the highest level of probability. Tennyson et al.'s predictions of the type of errors made by each of the treatment groups were statistically in accord with the results.

Although Tennyson et al. had demonstrated that exemplars and non-exemplars matched on irrelevant attributes enhanced learning and transfer, the question of whether contacts with non-exemplars were required for transfer remained to be explored. To study the issue as well as add to the external validity of the previous findings, Tennyson (1973) replicated the prior experiment using both a different population and task. The subjects were 260 seventh and eighth graders. The subject-matter was grammar and the concept was "ADVERB." For one group of subjects the instructional treatments were those of the previous experiment, except of course for the topic. Another group of subjects was given the materials with all negative instances deleted. An external control group received no training. Tennyson hypothesized that the group lacking negative instances would not differ

significantly from the control group in their performance on the transfer task.

The groups whose treatment conditions were those of the previous experiment replicated the results of that study. There were significant differences between their scores and those of the control group. The groups lacking negative instances had error scores which did not significantly differ from those of the control group. Tennyson concluded that:

Negative instances facilitate learning of concepts by requiring the subject to concentrate on the critical attributes when presented with a matched relationship of exemplars and non-exemplars. . . thereby focusing attention on the information to be communicated. (Tennyson, 1973, p. 248)

Whether or not these results accurately reflected the contributions made by the non-exemplars, or whether they reflected a lack of equal time on the task, is not clear. However, the results did indicate that instructional contacts with eight positive instances, in itself, did nothing to foster a level of learning which was significantly different than that attained by subjects in the control group who had not received instruction.

Klausmeier, Ghatala and Frayer (1974, pp. 195-202) reported two studies of the role of exemplars and non-exemplars in concept acquisition. In the first, the subjects were 95 sixth graders who were taught the concepts of POPULATION, HABITAT, and COMMUNITY via a self-instruction method. There were four versions of the presentation:

1. A matched set of positive and negative exemplars.
2. Positive instances only, equal in number to those given in treatment 1.
3. Two positive instances, and no negative.
4. Placebo lessons.

Aside from the subject matter, these presentations differed from those used by Tennyson in a critical area; definitions of the concepts were not provided. Each concept was presented in a separate lesson. At the end of each lesson the subject was required to correctly identify new instances and non-instances and to select, from a number of choices, the correct definition of the concept. Upon completion of all of the lessons, the subjects were given a test which attempted to measure their understanding of the inter-relationships among the concepts and which included the task of matching concept names to definitions. The dependent measures were the number of instances correctly identified, the number of correctly matched definitions and the score on the inter-relationship test.

Significant differences in the number of instances correctly identified were found. Those who had been presented with matched positive and negative instances correctly identified more new instances than those who had been presented with an equal number of only positive instances. These findings supported the Tennyson position regarding the contribution of matched negative instances. Unexpectedly, those subjects who had been given only two positive instances out-performed those who had been

presented with many more. The two groups did not differ on any other measures. Klausmeier et al. hypothesized that the group receiving the larger number of positive instances had an information over-load which hampered discrimination.

The second experiment in the series used the same paradigm, except all of the self-instruction booklets now contained concept definitions. The subjects were 102 sixth graders. While the performance of all three instructional treatment groups was significantly superior to that of the control group in the identification of new instances, there were no longer any significant differences between the treatment groups. Klausmeier et al. hypothesized that the presence of a definition compensated for the lack of negative instances.

The results of the experiments reported by Klausmeier et al. and the Tennyson experiments would appear to be contradictory. In the Klausmeier et al. experiment, the presentation of a definition eliminated the differences between the group that had been presented with contrasting sets of positive and negative exemplars and the group that had been presented with positive exemplars only. It would appear that for some concepts a definition and/or some positive instances are enough to enable the learner to compose an algorithm which can accurately identify new instances. However, for some concepts, such as TROCHAIC METER or ADVERB a definition accompanied by a few positive instances may

simply not be enough. The results of the series of experiments could be interpreted to indicate that learning can be maximized whenever an adequate definition of the concept has not been provided, by providing the students with instructional contact with matched exemplar and non-exemplar pairs.

Rule Learning

School subject matter often requires that students be able to transform classes of objects into classes of responses: e.g., solve the following equation for the value of x , $x^2 - 1 = 0$. A rule is a well defined series of operations to be applied to a class of objects which produces a specified result. A learner has acquired a rule when he can select the situations in which the rule is applicable and can correctly apply it (Gagné, 1974, p. 61). Complex rules are derived by the appropriate input/output linking of simpler rules. The response class of the first rule must be the object class of the rule to which it is linked.

There are very few studies contrasting the influence of exemplars and non-exemplars on the acquisition of a rule, despite a general feeling that a review of non-exemplars would enhance tutorial presentations. For example, Worthen (1968) in exploring the contrast between the expository and discovery approaches to mathematics, handled the issue of rule misapplications in the following manner:

The teacher warns the student of common errors made in applying the principle. He points out specifically the types of problems the students are likely to make errors on and then gives examples of each kind of error (Worthen, 1968, p. 5)[Emphasis added.]

Traub (1966) studied the acquisition of a rule which dealt with the graphic addition of integers. The subjects were sixth graders. Contrasting a practice program containing a heterogeneous set of sample problems with one containing a homogeneous set, Traub found that practice with the heterogeneous set significantly enhanced positive transfer. The heterogeneous set contained both negative and positive numbers as objects and answers, while the homogeneous set dealt only with the addition of positive numbers. Although the subjects in the heterogeneous group made significantly more errors on treatment problems, their transfer scores were significantly higher. The subjects provided their own feedback when they solved the same problems by regular, rather than graphic, addition. Traub's results suggested that the heterogeneous problems fostered a more extensive error experience, but also enhanced transfer.

Thatcher (Note 11) modified Traub's materials so that they could be used to study the influence on transfer of contact with correct and incorrect applications of a rule. Thatcher used the stereotypic errors of Traub's students to create realistic examples of rule misapplication. In the tutorial presentation he preceded each practice problem with presentation of either a matched pair of correct and incorrect

examples, or with a single example of a correct application. After each practice problem, the student was shown the correct solution. There were three experimental groups. Heterogeneous practice problems were given to two of the groups: one group was presented with matched pairs of correct and incorrect examples, and one group was presented with only single correct examples. The third group was given homogeneous practice problems and was presented with only single correct examples. Only the graphic addition of integers was studied. Thatcher's subjects, 76 fifth graders, were naive in regard to addition with positive and negative integers.

No significant differences in performance on the transfer task were found. Assuming that the extra time spent in the contrasting pairs treatment group was a strong indication that the non-exemplars had been processed by the subjects, Thatcher found it difficult to explain why there were no differences. He could offer no explanation as to why Traub's results had not been replicated. At first, Thatcher thought that perhaps the revised task brought all subjects to the same level of learning. During an attempt to replicate, Thatcher (Note 10) observed behaviors which could account for the lack of difference among the groups. Students could leave as soon as they finished. He noted that as soon as some of the students left, those in the contrasting pairs treatment group who needed more time than the rest to complete their programs hurried through the

remaining material so that they too could leave. Thatcher found no group differences in the replication study.

Verbal Warnings Regarding Rule Applicability. Exemplars show students what has to be done. What do non-exemplars do? The only knowledge available regarding what non-exemplars contribute to learning comes from Waters' research; erroneous exemplars instill a sense of cautiousness. If that is what non-exemplars do, and nothing else, the question arises: Why not include warnings in all instructional presentations? Perhaps these warnings, in and of themselves, could alert students to the possibility of error and instill a sense of cautiousness. Worthen had suggested that teachers should warn students of the common errors they are likely to make in applying a rule, and also show them examples of errors.

McQuarrie and Grotelueschen (1971) and Logan and Wodtke (Note 6) examined the influence of verbal warnings on preventing the misapplication of a rule. Logan and Wodtke observed that many school subjects contain rules which are used in solving problems. Although many of the rules tend to have limited applicability, they have a great deal of utility for students. However, teaching only the rules is not enough.

In spite of the great utility of rules in problem solving, most instructors would agree that teaching students to solve problems by a set of rules is not sufficient. Although the rules provide an immediate parsimony in achieving solutions to problems, rules applied in 'cook book' fashion do

not provide the basic understanding of the processes involved in the solution. For example, students taught to solve statistical problems by means of the rote application of formulas would not be expected to perform well in a transfer situation which required understanding of a higher order principle. Instruction which provided the basic understanding of the principles underlying the rules, and an understanding of why the rules worked in some problems and not others, should facilitate transfer to problems in which the rule did not apply. (Logan & Wodtke, Note 6, p. 53)

Logan and Wodtke randomly assigned 79 college students with minimal knowledge of significant figures, to three treatment groups: No-Rules, Rules-Early, and Rules-Late. Each was supplied with a programmed text which explained why a specified number of significant digits were retained when multiplying two numbers. This text contained no "rule-of-thumb." A problem solving practice session followed the instructional presentation. The Rules-Early group, was given instruction on a simple rule-of-thumb which was applicable only to the practice problems before doing them. The Rules-Late group was instructed in the same "rule-of-thumb," but only after they had completed the practice problems. In both "rule" conditions, the students were issued two didactic warnings regarding the limited applicability of the "rule-of-thumb." One of the warning statements stated specifically that the "rule-of-thumb" would not be applicable to the transfer task. For the No-Rules group, a transfer post-test, measuring the subjects' ability to determine the correct number of significant digits in problems not specifically taught in the program, was given immediately after the completion of the practice session.

The results indicate that both "rule" conditions served to significantly depress performance on the transfer task. In contrasting the behavior of the rule and no-rule groups, these decrements in performance could be directly attributed to the application of the rule in situations in which it did not apply. Contrary to the authors' hypothesis that Rules-Late would interfere less than Rules-Early because of the lack of practice, the misuse of the "rule-of-thumb" occurred more frequently in the Rules-Late group. However, this difference was not statistically significant. Logan and Wodtke felt that the poor performance of both treatment groups resulted from an overgeneralized view of the rule's applicability. They concluded that didactic warnings do not provide the required training in discrimination.

McQuarrie and Grotelueschen (1971) studied the effect of the intensity of the verbal warning to see if the level of intensity would proportionately influence performance. Eighty-six undergraduates were randomly assigned, within college classes, to eight experimental groups. There were four levels of verbal warnings, and a Test/No Test measure of original learning. Half of the subjects in each verbal warning group took the test, while the other half engaged in an unrelated activity. The instructional materials were those used by Logan and Wodtke for the No-Rule and Rules-Early groups; however, the latter was modified so that it now contained one of three intensities of

warning statements. In a scaling of 27 warning statements the three which were used placed at the two extremes and at the mid-point. The statement which fell in the mid-range was the warning used by Logan and Wodtke, thereby more closely relating the results of the two experiments.

The transfer task performance measures included a measure of rule misuse, a measure of rule recall, and the number of correct solutions. To assess original learning, half of each group was given a test containing items which were similar to those used in training. An analysis of the data indicated that the level of original learning did not account for the differences found between groups on the transfer task. The higher scores obtained by the No-Rule and Strong Warning groups could best be explained by the Strong Warning group's high level of recall as to when the "rule-of-thumb" was not to be applied, and both groups' low level of rule misapplications.

McQuarrie and Grotelueschen found a significant multivariate effect attributable to the treatment conditions. In contrasting the groups on the misuse of the rule they found that the Medium Warning (the Logan and Wodtke statement) was most effective. The Mild Warning group misused the rule the most, followed by the Strong Warning and Medium Warning treatment groups. The difference between the latter pair was small. In recalling when the rule could not be applied, in contrast to

rule misuse, the Strong Warning Group was the best, while the Mild Warning group was the worst; their levels of recall were equi-distant from the recall level of the Medium Warning group, and as would be expected, in opposite directions. The findings of Logan and Wodtke, regarding the effect of verbal warnings on applying a rule of limited applicability, were replicated.

McQuarrie and Grotelueschen hypothesized that to the extent that the general principle was not mastered, the subjects did not have a real choice between applying the general principle or the "rule-of-thumb." Under the circumstance, that explanation appears to be inadequate. First, because the instructions specifically directed the students, at least temporarily, not to rely on the "rule-of-thumb."

...[A]s with many rules, there are occasional exceptions where the rule gives an incorrect answer. Therefor you are strongly advised to check ANY rule-result by using the basic reasoning of significant figures until you get a feeling for when the rule works and when it doesn't -- say at least for a week or so. (Logan & Wodtke, Note 6, p. 57)

Furthermore, although the mean score on the test of original learning was low, 38%, all groups had achieved the same level of mastery. However, on the transfer task, only the No-Rule group was able to attain a mean score, 53.3%, which surpassed their original learning score. The scores of the other groups on the transfer task were all lower than 38%. Klausmeier, Ghatala and Frayer (1974) indirectly suggested

a more adequate explanation of these results. They indicated that too much information that does not foster discrimination early in learning overloads the learner's capacity for processing the information, and thereby inhibits learning.

Teaching Skill

One important aspect of teaching is the ability to pose questions which require the students to analyze information before they respond, and not just repeat what they have heard. Koran, Koran and McDonald (1972) contrasted the influence of positive and negative models on the acquisition of that skill. The treatments were presentations of filmed classroom interactions, with a teacher acting as either a positive or negative model. Separate cameras recorded either the teacher's actions, and the student's actions, or the combined teacher-student interactions. In the "positive treatment" groups the subjects were asked to identify the positive aspects of the behavior they viewed. In the "negative treatment" groups they were asked to identify the negative aspects. The pre- and post-tests were 30 minute micro teaching sessions.

The subjects were teaching interns, randomly assigned to six treatment groups. The experimental factors were:

- (1) Source of Information which had three categories, observation of teacher-student interactions, observation of teacher's actions only, and observation of student's actions

only; and (2) Type of Information, the negative or positive behaviors of the teaching model. The positive behaviors, which were also the criterion measures, were the frequency and variety of process oriented questions -- those which require the students to analyze information. Negative behaviors were defined as the posing of content oriented questions -- those for which the student need only recall information.

The groups with positive models produced a significant increase in the number of categories of process-eliciting questions from pre- to post-test, while those with negative models did not. However, subjects in the negative model groups did show a significant increase in the frequency of process questions. Koran et al. felt that the difference between the groups could be attributed to the limited amount of information communicated by the negative models and the amount of additional processing which was required to derive positive information from negative models. They argued that given the independent contribution to learning of both positive and negative models, both in combination would probably exceed the effect of either alone.

Summary

The contributions to learning of exemplars and non-exemplars have been empirically examined at various levels of

the hierarchy of learning. Although the evidence is not conclusive, the findings suggest that instructional contact with plausible models of erroneous behavior tended to enhance, rather than inhibit, learning and transfer. There were indications that each exemplar type, positive and negative, made its unique contribution to learning. There were indications that non-exemplars instilled a sense of cautiousness, an awareness of the potential for error, which was not induced by exemplars (Waters, 1931; Waters & Ellis, 1931). They appeared to help the students to identify critical aspects of what was being learned (Tennyson, 1973; Klausmeier et al., 1974), and motivated them to avoid error (Evans, Note 2). Positive exemplars appeared to increase the scope of acquired knowledge. The results from the various studies appeared to indicate that learning and transfer were most likely to be enhanced when instructional contact was provided with matched exemplar and non-exemplar pairs.

However, oversimplified explanations of the results tended to lead to contradictions. The complexity of what was to be learned appeared to be an important issue. Klausmeier et al. found that matched exemplar and non-exemplar pairs enhanced transfer, but not in every case. When an adequate definition was provided, matched exemplar and non-exemplar pairs made little difference. The interpretations of these results were seldom based on any transferable,

theoretical view of learning, making it almost impossible to generalize. A theoretical framework for generalizing these results will be explored in the next two sections of this report.

The Role of Errors in Learning

Skinner (1968, pp. 132-133) regarded errors and non-exemplars as inhibitors of learning and believed that they strengthen unwanted forms of behavior. He called for their total elimination through the use of behavior shaping techniques. Skinner's position that a low error rate enhanced learning has been supported by numerous experiments which focused on the acquisition of verbal information (Angell & Terry, 1962; Kaess & Zeaman, 1960; Keislar, 1960; Meyer, 1960). Nonetheless, a number of psychologists have questioned the desirability of always minimizing errors in learning (Amsel, 1960; Estes, 1960; Travers, 1972, p. 56).

Learning As a Dual Process

Travers described learning as a dual process in which new, appropriate behaviors are acquired and undesirable behaviors eliminated (1972, p. 56). These undesirable behaviors are caused by response tendencies which foster misperception and mislearning (Harlow, 1959). Amsel (1960) recommended that in those situations in which the tendency toward error is detrimental, erroneous behaviors be

elicited and systematically non-reinforced. One approach which has been tried is negative practice, the purposeful repetition of errors. Megn, Moore and Smith (Note 7) studied the use of negative practice in spelling. That is, words which were spelled incorrectly were practiced in the incorrect form. They found that while negative and positive practice were generally equivalent in their effect on spelling improvement, a small group of subjects did significantly better with words learned through negative practice. Guthrie (1935) explained that through negative practice involuntary error responses could be changed to voluntary responses.

Errors and The Acquisition of Verbal Information

Although there is a common belief that under the right conditions errors can enhance learning, little is known about the processes involved or how to use errors effectively (Glaser, 1971). A number of investigators have supported an approach to error which is intuitively appealing (Crowder, 1960; Lewis & Pask, 1965). In the programmed-instruction approach developed by Crowder, the learners' responses are used to select alternative sequences of tutorial frames. The method assumes that the inappropriate response generated by the learner provides the key to his misconceptions; "knowing" the reason for the erroneous response, the teaching programs branch the learner to some error-specific remedial

path. Some questions have arisen however regarding the validity of that approach.

In a comprehensive study of programmed instruction, Day, Kulhavy, Anderson and Malczynski (1971) did not find any differences in verbal information acquisition which could be attributed to "error specific" branching. The subjects were 1,120 undergraduate psychology students matched on ACT score and randomly distributed among the treatment groups. The subject-matter was "The Structure and Function of the Eye." The material was presented in 30 small step, programmed-instruction frames. Six illustrations were included in the presentations. The ten treatment groups represented the following self-instructional presentation methods:

1. Base Control
2. Covert Responding
3. Overt Responding
4. Auto-elucidative
5. Machine Presented Branching
6. Machine Presented Branching Without Remediation
7. Machine Presented Branching Without Feedback
8. Book Presented Branching
9. Machine Presented Instruction Minus Overt Responding
10. Auto-elucidative Minus Overt Responding

Remedial frames were restatements of the original information and were varied according to the nature of the misunderstanding implied by the erroneous response

selected. The dependent measures were the number of items answered correctly on a test administered after the completion of training, program time, and the number of program errors. The test scores, and the scores from a separate control group, were used to calculate a simulated measure of gain. An efficiency ratio -- Gain score/Program Time -- was calculated for each subject. Day et al. (pp. 38,55) found no significant differences on any of the dependent measures, either between treatment or scholastic ability groups, except for a significant increase in program time for the lowest scholastic range. Although branching programs tend to be most efficient, subjects acquired and retained equivalent amounts of information in programs with or without remediation or feedback. When the number of items in the multiple choice questions of the instructional program were reduced from three (one right and two wrong) to two (one right and one wrong), significant differences attributable to feedback were obtained. With only two answers to select from, there was a substantial increase in the efficiency ratio of the groups which had received feedback. These results replicated the findings of Kaess and Zeaman (1960).

McKeachie (1974) might well have predicted the latter result! In a commentary on "the decline and fall of the laws of learning," he suggested the reason why results of human learning experiments often appear contradictory, especially

in regard to the role of errors in learning. He argued that the apparently contradictory findings resulted from the application of theoretical conceptualizations derived from animal experiments to human learning. According to McKeachie, models of human learning will have to account for the differences between human and animal abilities to conceptualize, relate and remember. He observed that the results of learning experiments with human subjects could not be adequately predicted if they were conceptualized solely in terms of the learner's motivation and knowledge of results. McKeachie argued that feedback enhances learning only when the knowledge of results is informative and the learner knows what to do to correct his errors. In the Day et al. experiment, when there were only two possible answers the students knew what they needed to do to correct their errors. Under those conditions, contact with plausible, but erroneous, exemplars enhanced rather than inhibited learning.

Errors and Problem Solving

The ability to contend with error is critical in problem solving. Duncker (1945) noted that the very essence of successful problem solving requires that the student expect and accept errors as a consequence of the problem solving process. He must develop the ability to utilize knowledge gained through error if he is to derive a correct solution. Bloom and Broder (1950, pp. 25-31) reported that those students who failed to solve problems often failed because

they lacked persistence in the face of exploratory errors. Klausmeier and Loughlin (1961) found that successful problem solvers consistently used verification procedures to detect and correct errors.

Duncker envisioned problem solving as a continuous reformulation of the problem through knowledge gained in the commission of errors. The final form of the solution is typically obtained through iterative mediating phases, each of which in retrospect possesses the character of a solution, and in prospect that of a problem. At each phase an overall solution is attempted. The results are then evaluated and the erroneous sections of the solution attempt are identified. The problem the learner now faces is different from the original problem. He must now find ways of modifying only the erroneous sections of the solution. And so the solution process continues.

Duncan (1959) argued that for the purpose of facilitating transfer the subject should have contact with heterogeneous exemplars and have experienced error. Szekely (1950) found that transfer in problem solving was enhanced in the experimental versus the lecture mode when the subjects during the experiment found their predictions in error. Schulz (1960) asserted that the learner can only become a successful problem solver after his tendencies to make inappropriate responses have been eliminated.

Sources of Error

A variety of intra-subject characteristics has been identified as sources of error in human learning; set or direction (Maier, 1930); functional fixedness (Duncker, 1945); anxiety (Boutwell & Tennyson, Note 1) the limited capacity of short term memory (Pasqual-Leone, 1970; Posner & Warren, 1972); interference in the coding and decoding of information (Posner & Warren, 1972); alterations in the subject's knowledge structure generated by the vocalization of thought (Luria, 1969); the inability to simultaneously attend to a variety of physiological and psychological operations (Sperry, 1966).

Cognitive Aspects. Fisher (Note 3) conducted a study of concept learning from an information processing point of view and found that many of the errors were a natural consequence of the limitations of the human information processing system, rather than a consequence of the learner's state of knowledge. The experiment was an introspective investigation in which the learner was urged to verbalize his thought. The subjects were presented with a series of 18 objects for classification, on a computer driven Cathode Ray Tube screen. The presentation program contained a series of classification rules which were used to generate dichotomized classes of the object set. The subject had to guess whether or not an object was a member of a class, and the computer provided feedback as to the accuracy of his response. The subject was asked for a running commentary about what he was doing.

The subjects could repeat the presentation of a series controlled by the same rule as often as they wanted. The object displays were under their control. A new item was presented only after the subject responded to the previous display and had been given feedback.

The subjects appeared to be inappropriately influenced by their own verbal feedback when, as inferred from previous actions, their vocal descriptions did not accurately express their state of knowledge. For example, one subject indicated that he no longer considered position as a relevant classification dimension by commenting on the irrelevancy of one of the two values of that attribute, "bottom doesn't matter." However, shortly thereafter, he considered the alternative value, "maybe it's the top." The subject acted as if he had not previously and validly concluded that the "top/bottom" dimension was irrelevant. This type of error occurred repeatedly with a number of subjects whenever their vocal descriptions and actions were incompatible.

Here is another example. Subjects entered their classification of an object by pressing one of two adjacent keys. (A paradigm which resembles the discrimination learning experiments with animals.) In order to interpret the feedback which simply indicated right or wrong, the subject needed to know which key had been depressed. It was observed that whenever the subject failed to focus his attention on the key depressing action, there were often errors in recall. Perhaps if the two keys had been further apart, kinesthetic

feedback would have reduced the large number of errors of this type. All twelve subjects made a number of logical errors, which seemed to reflect a tendency toward error in the human information processing system.

Affective Aspects. Skinner (1968) and Papert (Note 8) have both reported that a pejorative attitude toward errors has been found in many formal educational institutions. Therefore, errors are likely to have affective outcomes. The mix of affective and cognitive influences on learning provided by errors is reflected in the results of a study by Marks (1951). Eighty-two undergraduates, matched on general achievement scores, were randomly assigned to eight experimental groups. Some of the groups, just prior to the experimental sessions, were given special training in general approaches to problem solving, in a regular psychology class. They were taught the "correct" way to analyze a problem, ways to determine the requirements of a solution, and how to develop hypotheses. Problems which were not related to the experiment were used to illustrate each of the techniques.

In the training phase of the experimental sessions, all subjects were taught to compute square roots using a calculator and an auxiliary table of factors. After training, every subject was required to execute four practice exercises to familiarize himself with the technique. However, Marks had changed the entries in the factor table for problems two

and four, so that the solutions were always incorrect. He then assigned the subjects to either a Personal (real-life) or Impersonal (text book) treatment. In the Personal situation the subjects' four test exercises, in which the second and fourth solutions were marked as incorrect, were returned. The subject was asked to ascertain the source of the errors. In the Impersonal situation, the subjects were given a typed narrative concerning one John Jones, who had attempted to find solutions to the four test exercises and had failed to get the correct solutions to the second and fourth exercises.

After working for 10 minutes, some of the Personal and Impersonal sub-groups were furnished with a list of the potential sources of error: (1) the operator, (2) the calculator, (3) the method, (4) the table of factors. They were told that the error was attributable to only one source. Subjects who were in the Personal group, but who had not received special training or the list, were given special aid. At regular intervals the experimenter told them, "I want you to ask yourself, what are the elements of my problem? What are the only ways in which a mistake can occur?" Whenever the subjects strayed from this analytic framework, the statements were repeated. The dependent measures were: the number of vocalizations -- vocalizing that one of the elements might be a source of error; solving a problem -- showing that the table was in error and producing a corrected

solution. A 25 minute limit was set for both problems.

Marks hypothesized that the impersonal situation would evoke more vocalizations and more solutions than the personal situation. He also posited that this difference would be eliminated when subjects were furnished a list showing the potential sources of error. There was no Impersonal-Aid group. Marks found that subjects in the impersonal situation did produce significantly more vocalizations and solutions than subjects in the personal situation. However, contrary to his hypothesis, he found the difference remained, even when personal groups were furnished a list of possible sources of error. He also found that those who received aid in the Personal situation did significantly better than those who did not. Neither the problem solving lecture, nor the list, fostered any significant differences.

Marks suggested that the differences in performance between subjects in the impersonal and personal situations were a function of the subjects' awareness of the possible sources of error. According to Marks, this awareness could not be developed by simply providing the subject with a list. It depended on the subject's ability to analyze the problem situation. Marks felt that the personal situation tended to be disruptive and interfered with the subject's analytical capacity. The successful performance fostered by the provision of aid was explained as a refocusing of the subject's

activity which put him back on an analytical track.

Marks' findings indicated that as long as the student has difficulty accepting personal errors, his error handling capacity can best be trained in an impersonal situation. However, the impersonal situation must communicate the following critical aspects to the learner: it must show the learner how to correct his own errors (McKeachie, 1974), it must foster a thoughtful, cautious approach (Waters, 1931), and it must generate an expectance and acceptance of error as a consequence of dealing with complex and difficult material (Duncker, 1945; Papert, Note 8).

Observational Learning

The psychological framework in which the relationship between errors in learning and contact with exemplars of erroneous behaviors may best be drawn is observational learning. Bandura (1971a) has noted that much of a student's learning occurs through the observation of symbolic models. For example, texts that describe in detail how and how not to behave in particular situations directly influence the student's acquisition of effective behavior in those situations, without needless trial-and-error. Bandura (1971b) has shown that the observation of a model's behavior, which has been designated as appropriate or erroneous, often effects learning and transfer in a manner equivalent to a direct experience. Furthermore, Bandura (1971a) reported

that these effects do not differ even when the modality of presentation was changed. As long as the critical aspects of the model's performance were effectively communicated, real-life or abstract symbolic models were equivalent in their influence on the observer's learning. This would mean that even when a learner's responses were correct, he could still experience error through contacts with designated exemplars of erroneous behaviors. For example, when a subject reviews a programmed instruction frame containing a designated erroneous exemplar, even if his response to the frame is correct he will vicariously experience the performance error represented in the contents of the frame. Observational learning permits greater instructional control over the learner's experience of error than that which is generally available in the typical problem solving situation.

Problem Solving and Transfer of Learning

An understanding of the relationship between problem solving and transfer of learning is critical in regard to the rationale for this study. In order to solve a problem, the students must be able to transfer skills to a new task. Researchers thus far have explored the influence of contrasting exemplars on transfer only in domains of learning other than problem solving. Nonetheless, the parallels between problem solving and transfer of learning, which will be drawn in this section, suggest that the factors which influence

transfer of learning in any area will also influence problem solving.

Problem Solving and Transfer Paradigms

The topics "problem solving" and "transfer of learning" had been traditionally treated as separate domains in the psychology of human learning (Duncan, 1959). Nonetheless, analysis has shown that many studies of problem solving were in fact studies of transfer (Ellis, 1965, pp. 66-69; Schulz, 1960). Ellis (p. 3) defines transfer of learning as the influence a prior experience has on the performance of a subsequent task. Many studies of problem solving, such as Birch and Rabinowitz (1951), Luchins and Luchins (1950), and Maier (1930) also involved two phases: (1) some type of preliminary activity which was expected to influence performance on the criterion task; (2) a criterion task in which the learner had to discover appropriate responses for solving the problem. These phases are similar to the elements in the most current transfer of learning paradigm.

Early studies of transfer usually involved a control and an experimental group whose performances were measured on a new, transfer task. Normally, the control group either rested or engaged in an irrelevant activity, while the experimental group engaged in an experience which was directly related to the transfer task. More recent research has focused on the influence of differential

prior experience, and a control group, as such, has been eliminated (Schulz, 1960; Travers, 1972, pp. 154-181).

Concepts and Issues in Transfer of Learning. Transfer takes three different forms, according to Ellis (1965): positive transfer -- a prior experience facilitates performance on a subsequent task; negative transfer -- a prior experience inhibits performance on a subsequent task; zero transfer -- a prior experience neither facilitates nor inhibits performance on a subsequent task (p. 3). Gagné (1970, pp. 237-241) presented a structural view of transfer. He viewed organized knowledge as a hierarchy in which prerequisite entities, both in the logical and psychological sense, must be acquired before the learner can be moved to the next superordinate level. Prerequisites influence later learning in two ways. The first is "lateral transfer," which is the generalization of acquired skills to new situations at the same level of complexity. The second is "vertical transfer," which influences the learner's acquisition of more advanced knowledge (pp. 335-338). For Gagné, the psychological prerequisites of vertical transfer to problem solving were the acquired verbal skills, concepts and principles which are used to formulate higher order rules.

In a recent review of the transfer of learning literature, Travers (1972, pp. 155-181) distinguished two basic types of transfer processes. The first relates to the effect of response, stimulus and relational similarities,

and accounts for transfer effects in the simpler learning processes. The second, which Travers identifies as "non-specific transfer," relates to the task-to-task carry-over of intellectual skills and problem solving strategies.

Osgood's "transfer surface" (Hulse, Deese, & Egeth, 1975, pp. 343-344; Travers, 1972, pp. 168-169) seems to provide a sound basis for predicting transfer effects on many of the simpler learning tasks.

Osgood used a three dimensional surface to show the relationship between stimulus and response similarity across tasks and transfer effects. Two dimensions of the Osgood transfer surface are used to represent the gradient of stimuli and response similarity for both tasks. The third dimension represents the direction and degree of transfer to be expected. If stimuli and responses in two tasks are similar, then a high degree of positive transfer is expected. If the stimuli are similar and the responses dissimilar a high degree of negative transfer between tasks is expected. The constructs of stimulus and response generalization have been extended to include semantic similarities, as well as morphic similarities. The essence of semantic generalization is that stimuli or responses conditioned to a particular word will transfer to words with similar meanings.

Another simple form of transfer which is included in the similarity paradigm, and yet in some ways is distinguished from it, is "transposition." Here the similarity is determined

along a relational dimension. For example, if on the first task a response is conditioned to the larger of three objects, on the transfer task the subject, rather than respond to the same object he responded to on the transfer task, will also respond to the largest one. No proposed theory of transposition has been able to account for all of the experimental results (Travers, 1972, p. 163). One theory suggested that subjects note the relationships between the stimuli and recognize when the same relationships exist in both tasks. Another explanation deals with the tendency in man to select dissimilar objects. When the same objects plus a new one appear in the transfer task, it is the unfamiliar object that subjects tends to choose. Although the reason remains undetermined, Travers (p. 164) reported that transposition is most likely to occur when the first task is mastered and when transposition tasks have been practiced.

In the simpler type of transfer, some quite identifiable characteristic of the situation or a response to a task have a subsequent influence on the performance of a later task. However, not all forms of transfer effects are a result of simple task similarities. Often they involve a carry-over of strategies or skills from task to task. For example, Travers (p. 170) reported that the subject's speed of learning poetry, facts, dates and even nonsense syllables improves with additional practice on a variety of similar tasks. Furthermore, he noted that the phenomena can still

be demonstrated after considerable amounts of practice. Travers indicated that the learning-to-learn phenomenon represents a permanent kind of learning which influences performance on subsequent tasks. He stated that for this type of non-specific transfer to occur, the subject must acquire some set of higher-order rules or strategies which deal with learning efficiencies and relationships, rather than objects (pp. 170-174). The subject must also encode this knowledge in a form which permits the derivation of abstract and symbolic similarities. Travers reported that the relationship between task difficulty and transfer tends to be complex. He indicated that transfer was most probable when the strategy to be acquired could be related to knowledge and situations which were familiar. For simple tasks, optimum positive transfer occurs from the easier to the more difficult problem; for complex tasks, optimum positive transfer occurs from the more difficult to the easier problem (pp. 179-180).

Links Between Problem Solving and Transfer. Until recently, Gagné (1970,1972) included only algorithmic problem solving in his structural view of learning and transfer. He now has described a more comprehensive structure with five domains of learning: Motor Skills, Verbal Information, Intellectual Skills, Cognitive Strategies, and Attitudes (Gagné, 1974, pp. 49-70). While

concept, rule, and high-order rule learning have been designated as intellectual skills, heuristic problem solving is classified, in the main, as a cognitive strategy (pp. 64-65). Gagné defines "Cognitive Strategies" as internally organized capabilities which the learner uses to guide his attending, learning, remembering, and thinking. While intellectual skills are oriented to the learner's environment, cognitive strategies govern the learner's behavior in dealing with the environment (p. 64). Gagné's description of the role of Cognitive Strategies in problem solving can be used in part to draw the required parallel between transfer of learning and heuristic problem solving tasks.

When a novel problem is posed for a learner, he must bring to bear . . . the retained effects of prior learning in the form of previously learned information and intellectual skills. Although these capabilities are necessary they are not sufficient. In addition, the learner must have a strategy or approach to the new problem -- or possibly he may choose among several alternative strategies. (Gagné, 1974, pp. 64-65).

The learner when faced with a new example in a transfer situation must also bring to bear prior knowledge of and similar experiences in the substantive domain. He must also select a strategy for approaching the new example. A learner who has acquired knowledge of a complex concept must usually have acquired a strategy for identifying new instances. Sometimes it is impossible to derive a definition for the concept from which one can construct an algorithmic procedure for identifying new instances. For example, the concept WORK has many meanings. When that condition

occurs, concept identification and heuristic problem solving will tend to have much in common, as do subjects who are learning or even relearning a maze. The factors which influence transfer in any domain of learning, e.g., positive and negative exemplars, may well influence the acquisition of heuristic problem solving skill in a similar manner.

Travers (1972, pp. 155) noted that transfer effects were not limited solely to intellectual behaviors; transfer effects have also been demonstrated for attitudinal behaviors. Attitudes are an important aspect of learning and transfer (Gagné, 1974, pp. 65-67). Citing the Logan and Wodtke experiment with verbal warning, Travers indicated that negative transfer can be induced by the problem solver's willingness to acquire the most simple procedure, in contrast to one which requires verification, even when there is an element of risk.

Illustrative Exemplars and Transfer. The task environment may not be the sole determinant of whether the learner uses the algorithmic or heuristic approach to solving a problem. For example, in the Scandura (1974) experiment on "complex tradin rules," the control group's lack of knowledge regarding the construction of higher-order rules would foster a heuristic approach in those who would succeed. However, in the typical instructional environment, the range of applicable knowledge would usually have been communicated prior to the

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problem solving task. Furthermore, unlike the laboratory situation, the student's attention would have already been focused on the relevant elements of the problem solving task. What then is left to be communicated? What instructional function is served by presenting the learner with illustrative examples? In what way does contact with these examples influence performance on a transfer task? The influences exerted by the exemplars used in training have been explored by a number of investigators.

Divesta and Walls (1967) explored the influence on a transfer task of rules acquired during training. They hypothesized that when the relationships necessary for effective solutions were congruent in the training and transfer tasks, performance in the transfer task would be facilitated. Conversely, if the relationships were not congruent, performance on the transfer task would be inhibited. The subjects were 75 undergraduates from an introductory psychology course who were randomly assigned to five training conditions. The training exemplars were eight-letter, nonsense word anagrams. Each anagram was preceded by a discriminative stimulus, and topped by a set of symbols. For example, the group trained for Associative Facilitation was shown the following example:

-+-+--+
* EINOZPUH

and told that the rule for solution was, "First the minus and then the plus letters." The discriminative stimulus for this

rule was the Asterisk (*). For the Associative Facilitation group the discriminative stimulus and the rule for solution remained the same, on both the training and transfer tasks, although the symbols above the anagrams were changed, for example, to numbers rather than signs. The rule would then only have to be changed to "First the odd and then the even letter."

The Warm-Up Control group, using numbers, was taught formulas to solve training problems. Rules, as such, were not provided. On the transfer tasks, the discriminative stimuli, the symbols, and the solution approaches were completely different. The Classical Interference Group in the training tasks learned the same formulas as the Warm-Up Control group. However, while the discriminative stimuli remained the same, the solution formulas were not related. The Associative Interference group received the same training as the Associative Facilitation group. However, on the transfer tasks the pairings of the discriminative stimulus and its rule were changed. The Re-Paired Control group received the same training as the Warm-Up Control group. On the transfer tasks, however, only the pairings of the discriminative stimuli and the formuli were switched.

The subjects practiced the training tasks until they were able to produce solutions without being exposed to either a rule or a formula. On the transfer task the subjects were required to solve 16 eight-letter, English

word anagrams. A maximum of 120 seconds was allowed for each anagram. If a subject did not produce a correct solution within that time, he was shown the solution rule. There were no significant differences in the mean time required to meet criterion during training. On the transfer tasks, however, there was a significant difference in the mean time to solution.

In support of their hypotheses, DiVesta and Walls found the mean time to solution for the Associative Facilitation group was significantly lower than that of all the other groups. In contrast to the Classical Interference and Warm-Up Control groups, the Re-Paired Control and Associative Interference groups also required less time.

DiVesta and Walls hypothesized that the differences were generated by the learning demands of the transfer task. At the poorest performance levels, new solution rules had to be acquired, linked with new discriminative stimuli, and integrated with the symbols in a solution approach. It should be noted in support of that explanation that after the first four trials the differences in the mean time to solution rapidly diminished. DiVesta and Walls posited that the results reflected the associative strength of the rules across tasks. From this investigator's point of view an alternative interpretation would also appear to be plausible. Travers had already noted the willingness of the problem solver to apply already acquired algorithmic processes to

new situations even when they were unlikely to produce correct results. Furthermore, the DiVesta and Wall results highlighted a general weakness in the algorithmic approach to teaching problem solving. Students often believe that the procedures they are taught have wide applicability. Despite warnings to use their general understanding until it was clear under what conditions the procedure would be effective, the learners showed preference for the simpler process (McQuarrie and Grotelueschen, 1971; Logan and Wodtke, Note 6). Wertheimer's (1945) plea for the educational enhancement of productive thinking suggests that, especially early in learning, experiences must be provided which foster an ability to evaluate the applicability of the procedures being taught. Once DiVesta and Walls' subjects made errors and were informed how they were to be corrected, performance improved.

Hilgard, Edgren and Irvine (1954) studied errors made on a transfer task, following "learning with understanding" of Katona's card-tricks. All the card tricks to be performed had a standard format. The deck, which was always face-down, had to be arranged in a sequence so that the cards which were turned face-up formed a particular pattern. The allowable actions were either turning a card from the top face-up, or burying a specified number of cards at the bottom. The experimenter specified the pattern to be produced, the number of cards to be turned face-up and the

number of cards to be buried, at each turn. For example, the experimenter might have said, "Produce a sequence in which the face-up cards are alternating red and black. Do this by turning the first top card face-up and then burying the next one."

The subjects, 140 high school students, were randomly assigned to one of five training groups. The subjects in each of the groups were taught a different method of determining the deck arrangement. Each used a unique method of recording while they made their way through the trick in order to understand how a trick was constructed. The subjects' task was to organize the deck so that the required display could be produced, using the actions specified by the experimenter. Two tricks were used for training, while four were used as the transfer task. There were significant differences between groups in performance of only the most difficult transfer task, the third trick. In that task the subject for the first time was required to bury two cards between displays instead of one; during training the subjects had only been required to bury one card.

Hilgard, et al., had anticipated that differential errors would be produced by the differences in the training experience. In accordance with their predictions, specific error types could be directly attributed to particular training methods. The most common error (63 out of 109 errors) was burying one card, instead of two, between displays. Thirteen subjects made the perseverative error of burying only one card

throughout. The remaining subjects made this type of error when there were only two cards left in the deck. While most of the subjects tended to make errors as if they had lost count at the end of the trick, those who were trained in the most easily acquired method tended always to bury one card. Half the subjects in this group who made errors (10 out of 20), made this type of error. They persistently skipped one card with great confidence, repeating the same error over and over again. The lack of difference between groups on the second transfer task in which two cards had to be buried, the fourth trick, was directly attributable to the substantial reduction in burying errors. These results support McKeachie's (1974) view that knowledge of results is most effective when the learner knows what to do to correct the errors.

Hilgard, et al., were unable to suggest any explanation as to why the most easily acquired method fostered a constant error condition, the perseverative burying of only one card. Perhaps the reason lay in the level of confidence held in the method, which appears to be a function of ease of acquisition. Estes (1972) reported that subjects tend to repeat those behaviors that appear to have a high probability of reward. Wason (1968) and Fisher (Note 3) reported that in concept learning, when the subject's level of confidence in his hypothesis is at a peak, recognition of errors tends to be inhibited. Therefore, feedback information may be

either misinterpreted or ignored.

The results of the DiVesta and Walls, and Hilgard, et. al., experiments, suggested that the problem solving strategies used by the new learner were strongly influenced by the exemplars of solution used in training. The learner appeared to use each new exemplar to verify the strategy he had developed. During the training experience, the learner continued to develop and modify his strategy until he was able to derive correct solutions for all of the training exemplars. When the range of training exemplars was limited, learning with understanding was easily transformed into an algorithmic or reproductive process. Solidification of thinking was most likely to occur when the learner's level of confidence in his method was high. Therefore, it would appear that the exemplars used in training must communicate the critical components of a successful course of action: how decisions are guided and the need for continued verification (Gagné, 1959).

Summary and Discussion

Research contrasting the roles of exemplars and non-exemplars in learning has seldom been based on any transferable, theoretical framework. The findings occasionally appeared contradictory. McKeachie (1974) suggested that the application of theoretical conceptualizations derived from animal experiments to human learning often leads

to conflicting interpretations, especially in regard to the role of errors in learning. He asserted that models of human learning will have to account for the differences between the abilities of animals and humans to conceptualize, relate and remember.

Bandura's (1971a, 1971b) conceptual framework, "observational learning," was used as the link between the research concerned with the role of errors in learning and the influence of non-exemplars. According to Bandura, any representation which communicates the essential characteristics and results of an action would be an effective model for observation learning. For exemplars and non-exemplars to serve as vicarious learning models, they only need be designated as such. That is, students are informed that exemplars represent the behaviors to be acquired, while non-exemplars represent the behaviors to be avoided. Bandura found that when the model clearly indicated whether or not the behavior was acceptable, the behavior of the learner was influenced in a manner which was equivalent to direct participation in a similar interaction. Furthermore, he found that vicarious experience was sometimes more effective than direct experience in its influence on learning.

Psychologists often disagree about the role of errors in learning. Although there is a common belief that under the right conditions students can learn from their errors, relatively little is known about the processes involved

or how to use errors effectively (Glaser, 1971). Human error has often been seen as an inhibitor of learning, and some psychologists have argued for the use of behavior shaping techniques to totally eliminate errors in learning (Angell & Terry, 1962; Keislar, 1960; Meyer, 1960; Skinner, 1968). Others question the desirability of totally eliminating errors in learning (Amsel, 1960; Estes, 1960). Many psychologists considered learning to be a dual process in which new, appropriate behaviors are to be acquired and involuntary error response tendencies eliminated or changed to voluntary responses; they asserted that inappropriate behaviors have to be elicited and non-reinforced (Amsel, 1960; Guthrie, 1935; Estes, 1960; Harlow, 1959; Travers, 1972, pp. 56-60). Estes has argued that learning cannot be maximized without the learner experiencing error.

Day et al. studied the role of error in the acquisition of verbal information within a programmed-instruction framework. They examined the influence of feedback and planned remediation. Day et al. (1971) obtained what would appear to be contradictory results—feedback made no difference in one situation and enhanced learning in another. McKeachie (1974) could probably have predicted what they found; that is, learning was enhanced when the learners knew what they had to do to correct their errors.

Duncker's (1945) research raised serious questions regarding either the desirability or possibility of

eliminating error experiences in the teaching of problem solving. He asserted that the very essence of successful problem solving requires that the problem solver expect and accept error as one of the consequences of a complex process. The variety of cognitive and affective characteristics of the human information processing system which were sources of error in human learning, raised questions about the probability of totally eliminating human errors through behavioral shaping or non-reinforcement techniques. Papert (Note 8) has asserted that humans need to acquire effective strategies for contending with their erroneous response tendencies. Given the pejorative attitude toward error often found in many educational institutions (Skinner, 1968; Papert, Note 8). Marks' (1951) study suggested that students were more likely to acquire effective strategies for dealing with errors vicariously, in an impersonal situation, than they would in a personal situation; that is, early in learning, students were better able to deal with someone else's errors, rather than their own.

In order to view the results of the various theoretical and empirical explorations as an entity, one more theoretical linkage needed to be studied -- the relationship between transfer of learning and heuristic problem solving. Ellis (1965) and Schulz (1960) had noted that many problem solving studies were actually transfer of learning studies. Certainly there were many issues in the study of transfer

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which were also issues in the study of problem solving. For example, how do the exemplars used in training students influence performance on a transfer task?

Gagné's (1974) recent reformulation of his structural view of learning provided the theoretical tools needed to bridge the gap between transfer of learning and problem solving. The algorithmic approach to problem solving, classified by Gagné as "Higher-Order Rule Learning," now was assigned to the learning domain "Intellectual Skills." There was a new domain of learning, "Cognitive Strategies," which encompassed both non-specific transfer and heuristic problem solving. Although the knowledge acquired at lower levels of the learning hierarchy was still seen as influencing the acquisition of higher level knowledge, the primary focus of this new domain was on the cognitive strategies the learner used when applying prior knowledge and experiences to new activities. How did previously acquired learning strategies influence behavior in subsequent tasks? Gagné had included "Attitudes" as one of the valid educational outcomes. The research which was reviewed suggested that contacts with exemplars and non-exemplars often influenced attitudes as well as strategies.

Travers (1972) observed that transfer effects influenced attitudes as well as intellectual behaviors. Divesta and Walls (1967) suggested the exemplars used in training

students developed expectations that the behavior which had proved successful during training would also be effective in dealing with new problems. Estes (1972) hypothesized that the learner's choice of a strategy was often determined by his evaluation of the expected probability of reinforcement associated with the selection. Travers (1972) noted that learners often applied algorithmic strategies even when they were inappropriate. Hilgard et al.'s (1954) results indicated that even learning with understanding can be transformed by the learner into an algorithmic approach. Verification procedures tended to be excluded from a newly acquired method when there was minimal contact with error during learning. The greater the learners' confidence that the method will produce correct results, the greater the likelihood that erroneous applications will persevere.

A critical factor in learning to solve problems is the ability to expect, accept and use error. Klausmeier and Loughlin (1961) observed that this ability was most meaningful when the learner had a planned rather than a random approach to a problem. Some of the reasons why illustrative correct and erroneous solutions, designated as such, do enhance learning and transfer may be that: (1) the learner can use these sample solutions as models to test and correct his own problem solving strategies; (2) they provide information regarding common errors the learner might make and what to do to correct these errors; (3) being presented with an

error someone else has made may help to remove some of the stigma associated with making mistakes while trying to solve a problem.

Implications and General Hypothesis

The available evidence suggested that tutorial presentations of problem solving should include illustrative presentations of erroneous behavior if learning is to be maximized. Therefore, as a general hypothesis, it is expected that students' abilities to solve problems would be enhanced through instructional contacts with plausible, but erroneous, solutions, especially in matched exemplar and non-exemplar pairs. A vicarious experience of error should reduce students' inappropriate behavior, or at least increase the students' abilities to recognize and correct their mistakes.

CHAPTER III

A METHODOLOGICAL TOOL FOR STUDYING SCHOOL RELATED PROBLEM SOLVING

Tasks which contain the many dimensions of the type of problem solving found in educational settings are extremely rare. Although human problem solving has been the subject of extensive study, in much of the research problem solving tasks have involved puzzles (see Ray, 1955, for examples). Subject matter knowledge is usually a prerequisite for educational problem solving, but not for solving puzzles. Gagné (1959) considered the Moore and Anderson (1954a) Symbolic Logic Task as one of the few which were acceptable. ". . . Used as specified, these materials are self-contained and appear to have many of the desirable characteristics for studying the course of actions in sequential tasks" (p. 157).

The Symbolic Logic Task

Moore and Anderson adapted twelve tautological inference schemes from propositional logic to serve as an artificial subject domain for exploring problem solving. (Propositional logic has often been used to represent the developmental structure of thought, see Ennis, 1975.) In order to keep problem solving efforts free of "logical" conclusions, and as a device to maintain interest, they treated the language as a cryptic code without any reference to the source. The

Moore and Anderson version of the sentential calculus was a self-contained mini-course. Prior knowledge of the subject matter was neither expected nor required. Prior to learning to solve problems the students first acquired a set of grammatical concepts and transformation rules. These were the prerequisite domains of knowledge for the problem solving task. Since the Moore and Anderson approach served as the model for a number of experiments it will be used to introduce the general nature of the task. However, only the four transformation rules used in the present research will be used to illustrate the discussion which follows.

The Language

The language has four elements. Letters, such as P, Q, and R; Connectors, the Horseshoe " \supset ", Wedge " \vee " and Dot " \cdot "; a Sign, the Tilde " \sim "; Punctuation, parenthesis "(" "). Well formed, valid expressions in the language are built according to a series of grammatical rules. See Appendix A for the rules of the grammar, expressed in Backus Normal Form (Newell & Simon, 1972, p. 45).

Another major component of the language is the set of transformation rules. The rules delimit the procedures to be used for changing existing expression into new expressions. The symbolic representations of the rules and their corresponding identification in the calculus, are shown below.

	<u>Rule</u>	<u>Identification</u>
1.	$A \supset B \leftrightarrow \sim A \vee B$	Conditional
2.	$A \vee (B \circ C) \leftrightarrow (A \vee B) \circ (A \vee C)$	Distribution
3.	$A \circ B \rightarrow A$	Conjunctive Simplification
	$A \circ B \rightarrow B$	
4	$A \supset B \rightarrow B$ A	Modus Ponens

The arrowheads indicate the permissible direction of the transformation. Double headed arrows indicate that a transformation can go either way (i.e., Rule 1, $A \supset B \rightarrow \sim A \vee B \rightarrow A \supset B$).

Figure 1 shows how the transformation rules are applied. For example, the expression on Line 4, $P \supset Q$ was generated by applying Rule 1 to the expression on Line 3, $\sim P \vee Q$. The letters which appear in the rules are variable names and represent expressions, rather than themselves. They are called the "A-part," "B-part" and "C-part." When Line 3 was transformed into Line 4, " $\sim P$ " was substituted for the A-part in the right-hand expression of Rule 1, and " Q " was substituted for the B-part. In a similar manner, Rule 1 was used to transform the expression on Line 5, $\sim R \vee P$, into the expression on Line 6, $R \supset P$.

Figure 1

A SAMPLE PROBLEM AND SOLUTION

<u>Sector I</u>		<u>Sector II</u>	
<u>Line</u>			
<u>1</u>	$(\sim P \vee R) \circ (\sim P \vee Q)$		
<u>2</u>	$(\sim R \vee P) \circ (\sim R \circ R)$		Q
<u>Sector III</u>		<u>Sector IV</u>	
		<u>Rule #</u>	<u>Line #</u>
<u>3</u>	$\sim P \vee Q$	<u>3</u>	<u>1</u>
<u>4</u>	$P \supset Q$	<u>3</u>	<u>1</u>
<u>5</u>	$\sim R \vee P$	<u>3</u>	<u>2</u>
<u>6</u>	$R \supset P$	<u>1</u>	<u>5</u>
<u>7</u>	$\sim R \circ R$	<u>3</u>	<u>2</u>
<u>8</u>	R	<u>3</u>	<u>7</u>
<u>9</u>	P	<u>4</u>	<u>6,8</u>
<u>10</u>	Q	<u>4</u>	<u>4,9</u>
<u>11</u>			

q.e.d.

Rules 1, 2, and 3 are single line transformation rules, while Rule 4 is a two line rule. Rule 4 can be applied when any two lines (not necessarily adjacent) meet the transformation criteria.

The complexity of the language makes it particularly useful for investigating instructional issues in school subject problem solving. As in most school subjects, before a student can solve problems he must first acquire knowledge of associated concepts and rules. With these materials, the student must first learn concepts such as "A-part," "B-part," "valid expressions," and how to apply the transformation rules.

An Example of Problem Solving in the Calculus. Figure 1 shows a problem and its solution. The example is divided into four sectors, labelled I, II, III and IV. Sectors I and II contain the problem. The subject is to "decode" the message which appears in Sector I. "Decoding" means that the subject is to use the transformation rules to show that the expression which appears in Sector II can be derived from the message (the expressions in Sector I). In Sector III and IV the subject records the decoding or solution steps, line by line. The transformed expressions are recorded in Sector III and each is justified on a matching line in Sector IV. The justification for an expression is the number of the transformation rule which was used to create the expression, and the number(s) of the line(s) to which the rule was applied.

In the sample problem, the subject applied Rule 3 to Line 1 to get the expression " $P \supset Q$ " appearing on Line 4.

In Line 10, the final line of the proof, the subject applied Rule 4, which is a two line rule, to lines 4 and 9, to get "Q", the expression in Sector II. Although there can be many valid mediating steps, not all of the possible rule applications lead to a solution.

The Moore and Anderson Training and Testing Procedure

Training in the use of the rules was presented via a tape recording; there were also other training aids. Each subject was issued a Rule sheet, showing all of the transformation rules; a work book; and a deck of plastic cards, each showing one rule and some examples of how it could be applied. At the end of the first training session, Moore and Anderson gave the subjects a Rule Application test. Each item of the test consisted of a well-formed expression and a rule number. The subjects indicated whether or not the rule was applicable to the given expression. The two alternate forms of the Rule Application test each yielded Kuder-Richardson estimates of reliability greater than .90. During the teaching and testing phases, the subject was permitted the free use of all training aids.

A recorded presentation explained the nature of the problem solving task, using a solution which was displayed. No attempt was made to explain the solution heuristics. The subjects were then presented with problems to be solved, one at a time. The maximum solution time permitted for each

problem was fixed, and records were kept on the actual time it took each subject to complete a problem.

Measures of Behavior. All of the decoding problems selected by Moore and Anderson (1954b) have at least one principal or perfect solution; some of the problems have more than one (see Newell & Simon, 1972). A perfect solution is one in which only the necessary rules were used and each was correctly applied. In addition to rule misapplication, there are a number of other inappropriate behaviors which may occur. However, none of these invalidates a solution. The inappropriate behaviors observed by both Gangler (1967) and Moore and Anderson (1954b), are illustrated in Figure 2.

Figure 2

*EXAMPLES OF INAPPROPRIATE BEHAVIORS
IN PROBLEM SOLUTION

Sector I		Sector II	
Line			
<u>1</u>	$(\neg P \vee R) \circ (\neg P \vee Q)$		
<u>2</u>	$(\neg R \vee P) \circ (\neg R \circ R)$		Q
Sector III		Sector IV	
		Rule #	Line #
<u>3</u>	$\neg P \vee Q$	<u>3</u>	<u>1</u>
<u>4</u>	$P \supset Q$	<u>1</u>	<u>3</u>
<u>5</u>	$\neg P \vee Q$	<u>1</u>	<u>4</u>
<u>6</u>	$\neg P \vee R$	<u>3</u>	<u>1</u>
<u>7</u>	$\neg R \vee (P \circ R)$	<u>2</u>	<u>2</u>
<u>8</u>	$P \supset Q$	<u>1</u>	<u>3</u>
<u>9</u>	⋮		
<u>10</u>	⋮		
<u>11</u>	⋮		

*Cycling, repeat of Line 1

*Introduction of
Extraneous materials

*Incorrect Application
of a rule

*Repetition

1. Cycling. The subject applies a rule to create an expression that is already there (see Lines 3 and 5 in Figure 2).

2. Repetition. The subject re-applies the same rule to the same line (see Lines 4 and 8).

3. Introduction of Extraneous Material. The subject creates an expression which is not part of the solution (see Line 6).

4. Line Fixation. The subject repeatedly uses only a small portion of the expressions in Sector I.

5. Rule Fixation. The subject attempts to use the same rule over and over again, whether or not it is applicable.

These measures parallel the cul-de-sac and retracing errors which Waters (1931) observed in the maze learning experiment. While not all are independent, Moore and Anderson noted that the following quantitative measures could be collected from each subject.

1. Number of Solutions
2. Mean Time to Solutions
3. Number of Correct Steps
4. Number of Rule Misapplications
5. Number of Cycles
6. Number of Repetitions
7. Measures 4-6/per minute

They noted that these measures would be useful in contrasting the behaviors of experimental groups.

Experiments in Problem Solving

Moore and Anderson (1954b) conducted the first study of problem solving behaviors using these materials. Their study explored the differences in group and individual problem solving. The subjects were 24 Navy enlisted men, of slightly above average intelligence, who were taught the use of the rules and subsequently tested on their ability to apply this knowledge. The Rule Application Test scores were used to match six individuals to six groups of three individuals. Individuals and groups were given one problem per day for ten days, with 30 minutes allowed to arrive at its solution.

Each of the groups chose a recorder. A solution was recorded only when all of the members of the group agreed. Five criterion measures were collected: the number of solutions; mean time to solution; the number of correct steps produced; the number of errors; the number of repetitions. Of these, only the difference in the number of repetitions was found to be statistically significant. Moore and Anderson concluded that the social interactions in the groups served to depress the number of repetitive steps, a measure of redundant, trial-and-error behavior.

Anderson (1957) used a similar group of enlisted men to explore the effect on problem solving of the number of goals included in a task. That is, Sector II of the problem could contain one or more messages to be decoded. The subjects either had to prove that the message could be derived from the information in Sector I, or reject that

possibility. Only one out of the two, three or four messages found in Sector II was provable. Anderson found that as the number of messages increased, the number of correct solutions decreased. The additional messages simply added to the complexity of the task.

Carpenter, Moore, Snyder and Lisansky (1961) used the calculus task to explore the effects of varying amounts of alcohol on problem solving. Seventy-first and second-year medical students volunteered to be subjects. After the completion of training, the subjects were run individually in four problem solving sessions a week apart. Four separate problems, with a 15-minute time limit per problem, were presented at each session. A linked, hyper-greco-latin square with subplots was used in the experimental design, so that the four alcohol consumption treatments and the problem sets were balanced, and each subject could serve as his own control. The subjects were run individually, and the experimenter served as the recorder. Whenever the subject requested the incorrect application of a rule, the experimenter notified him of his error and did not apply the transformation. The dependent measures were: total activity; the number of correct rule applications and violations; the number of different rules used; the number of repetitions; the use of irrelevant rules; the number of shifts (e.g., instead of using the last line, going back to use prior expressions).

The results indicated that there was a significant linear relationship between alcohol consumption and total activity. With total activity as a covariate, problem solving efficiency appeared as a curvilinear function of dose size, the smallest dose facilitating and the largest inhibiting. Repetitions also increased linearly with dose size. The relation of the size of the dosage to the use of irrelevant rules was complex. There was a sharp drop at the lowest dose level, which paralleled the increase in the number of solutions. With additional consumption, the level of use was equivalent to that found prior to imbibing. Although subjects as a factor showed significant differences on all problem solving measures, no attempt was made to relate individual characteristics to performance.

Gangler (1967) explored how individual factors, such as a subject's level of scholastic achievement and experience with mathematics, interacted with performance on the calculus task. Three hundred and eleven freshman mathematics students served as subjects, and were distributed among various experimental groups. Those in an "Overt Participation" group used the Moore and Anderson rule cards and recorded constructed-responses to questions in a notebook. The correct answers were found on the reverse side of the cards. Those in a "Covert Participation" group had only covert practice with the rules. Following a small delay after each question, during which the subjects were asked to think about the answers, the

teacher wrote the correct answer on a blackboard. Gangler devised a pre-test, which he used as an independent measure of subject variability in learning a set of symbolic rules. The pre-test was used to measure the subject's ability to learn rules similar to, but simpler than, the transformation rules of the calculus. The other experimental factors were SAT score, number of previous classes in mathematics, time of day for training, and whether a subject was told that the test would or would not influence his final grade. The dependent measures were pre-test score, score on the Rule Application test, the number of solutions, the mean number of correct steps per minute, and the mean numbers of errors per minute. Although Gangler reported infrequent occurrences of cycling, repetitions, and rule and line fixations, he made no attempt to analyze these behaviors.

In the data analysis, pre-test score was used as the covariate for rule learning, and Rule Application Test score was used as the covariate for problem solving. Although the correlation between the Rule Application and pre-test scores was significant, it was low, $r=.26$. Gangler found that the subject's SAT score, his mathematical background and absence of concern as to the test's influence on his final grade, significantly enhanced learning of the rules. These factors also significantly enhanced his production of correct steps. None of these factors, either singly or in combination, significantly influenced the number of problems

solved. Their influence on problem solving was mediated through the enhancement of rule acquisition. The scores on the Rule Application Test were significantly correlated with the number of correct solutions and the mean number of correct steps per minute. The mean number of errors and correct steps per minute were both significantly correlated with the number of correct solutions, although in opposite directions. However, there was almost no relationship with each other. The correlation between the mean number of correct steps and the mean number of errors was small and not significant.

Gangler hypothesized that the number of errors a subject made did not necessarily reflect his ability to apply the rules. He suggested that a possible explanation of this behavior lies in the variable of "direction" (Maier, 1930). Once the subject established a line of attack, he frequently forced an answer by incorrectly applying a rule. Rather than change direction, the subject would commit an error. This type of error was common and only overt practice with the rules was influential in significantly reducing the occurrence.

Successful problem solving appears to require a combination of knowledge and skill. The successful problem solver must know how to use the rules, be able to devise and execute a plan, evaluate the results of the implementation, and recognize and correct his errors. Aside from enhancing a

subject's knowledge of the rules, can training influence any of the other behaviors? The results of a self-instructional pilot experiment (Fisher, Note 4), suggested that contact with contrasting presentation of appropriate and plausible, but erroneous, solutions, may enhance a subject's strategic planning.

Fisher converted the Moore and Anderson materials into a text for a mini-course. At the end of each section of the text there were questions, with spaces for writing responses. This permitted the subjects to participate overtly, covertly, or not at all. The text described the language, explained how the rules were applied and how they could be used in solving problems. However, there were no examples of solved problems. After reviewing the text, the subjects were given the Rule Application Test. They were then randomly assigned to four experimental groups. Group 1 was given additional practice in applying the rules. Group 2 was asked to study two appropriate solution examples, which were so designated. Group 3 was asked to study an appropriate and an erroneous solution. Group 4 was asked to study two erroneous solution examples. The groups worked for 20 minutes. The subjects were then given a test containing four problems, and were permitted 12 minutes per problem to develop a solution.

The subjects were 54 high school students, approximately 16 years old. About half were foreign born, and about 40%

were from families associated with the United Nations. Their teachers had asked for volunteers to participate in an experiment. The reactions of the subjects during the long (2.5 hours) experimental session, as well as a review of the protocols, made it clear that the material was too difficult for most of the students. Only two were able to solve any problems. After studying the protocols, Fisher found it possible to assign each of the subjects to one of the following categories, based upon their solution to the second problem.

1. Made No Solution Attempt. The subjects assigned to this category were representative of two sub-categories: those who left their problem sheets blank or simply recopied the given expressions; those who imitated the form, if not the content, of solutions. This second sub-group wrote expressions, along with line and rule numbers in the appropriate places on the form. However, on examination it was easy to see that neither the rule numbers, nor the line numbers, nor the expressions, had anything to do with each other or the problem. Twenty subjects were assigned to this category.
2. Applied Rules in A Trial-and-Error Sequence. The subjects assigned to this group appeared to be searching for a correct solution by applying the applicable rules. Although the rules were usually correctly applied, no planning was evident. However, some subjects tried every expression with every rule, while others focused on a single rule or a single expression. Most simply seemed to try to find a rule and an expression that went together. Sequentially linked rule applications tended to be scarce. Twenty-two subjects were assigned to this group.
3. Demonstrated Planning. The subjects assigned to this group demonstrated some planning. Some, starting from the final line which is known, tried to derive the expression(s) which must be created prior to the final step. Some built chains of expressions leading to the correct final line.

However, somewhere in the sequence some rules were incorrectly applied, indicating that the subject was forcing a solution. Twelve subjects were assigned to this group, including two who had actually solved one problem.

Fisher found a significant difference between those who had received additional practice, and those who were given examples of solutions. In this complex situation, for which the subjects were not adequately prepared, additional practice with the rules fostered a strategic approach which was both unplanned and nondirected. For most of the subjects, solution attempts were simply trial-and-error rule applications.

Fisher, in post-hoc analysis, noted what appeared to be significant differences which might be attributable to treatment condition. Those exposed to only appropriate solutions were equally distributed among the three behavioral categories; contact with only erroneous solutions appeared to foster either a no-attempt or trial-and-error approach. The split in that group was almost equal. In the group which had been exposed to the paired appropriate and erroneous solutions, only one subject used a trial-and-error approach; the others were assigned to either the no attempt or planning group. Fisher suggested that the differential influences on transfer which appeared to be related to treatment condition indicated that additional investigations with a revised version of this material might indeed prove fruitful.

Summary and Discussion

The Symbolic Logic Task (Moore & Anderson, 1954a) has many of the characteristics associated with subject-matter problem solving. The acquisition of concepts and rules are critical prerequisites of problem solving and can be experimentally controlled. Furthermore, the task provides experimental access to the process as well as the products of problem solving. Early research with the task provided examples of common erroneous behavior, but little else which was relevant to the present research. However, the studies of Carpenter et al. (1961) and Gangler (1967) were pertinent. Carpenter et al.'s results showed that problem solving could be directly influenced in a manner which was independent of the level of prerequisite knowledge, even if it was only by the consumption of alcohol. They made no attempt to identify the concomitant psychological changes which may have taken place.

Gangler studied the influence of psychological and educational factors on problem solving. Of those factors he studied, he found that only prior achievement and subject aptitude influenced problem solving performance, and those did so in an indirect manner. They had a direct influence on the subjects' acquisition of prerequisite knowledge, i.e., the transformation rules. Gangler found that the subjects' level of knowledge was not the only factor which influenced performance. He found that the number of rule mis-applications

which occurred during problem solving was in no way indicative of the subjects' knowledge of the rules. Instead, he suggested that rule mis-applications reflected the subjects' efforts to force a planned solution. Had the subjects been able to contend effectively with their errors, successful problem solving might have ensued. After all, the ability to develop a plan is critical in problem solving.

Fisher (Note 4) observed that a planned approach to problem solving appeared to be most often fostered by instructional contact with paired examples of correct and erroneous solutions. That specific instructional approach almost totally eliminated a trial-and-error approach to problem solving. However, many students in the treatment group made no attempt to solve problems. The proportion of subjects making no attempt was highest for the group whose only contact was with non-exemplars. Additional practice with rule applications fostered the highest proportion of trial-and-error attempts to solve the problem. Those subjects who had had contact with only correct solutions were equally distributed among all of the behavioral categories -- the planned approach, the trial-and-error approach, and the no attempt. There was no way of determining if the planned approach would have made a significant difference in problem solving as only two students solved any problems -- the task was too difficult. Development of a viable set of instructional materials and tasks was needed to continue the exploration.

The revised set of materials used in this research are described in the Method section of the next chapter.

CHAPTER IV

THE EXPERIMENT

The experiment was conducted as a two session programmed instruction mini-course, in which the students were to learn problem solving solely from a text. In the first session the students all used the same text to learn the components of the problem solving language and how to apply the transformation rules which were used to solve problems. The language was called PROBE. It resembles algebra in its linguistic elements and rules, all of which are symbolic, and geometry in the format of problems and solution strategies. Problem solutions are developed in the same way one develops geometric proofs.

The text for the second session taught problem solving by taking the student through a sample pair of illustrative solutions step by step. There were four different texts, one for each instructional treatment. A treatment provided either (1) a pair of appropriate solutions, (2) a pair of erroneous solutions, (3) first an appropriate solution and then an erroneous one, or (4) first an erroneous solution and then an appropriate one.

Multivariate statistical analysis was used to study the influence of treatment condition on two aspects of performance on a problem solving test: problem solving and

the production of and response to erroneous behavior.

Method

Subjects

The subjects were 120 high school students of above-average ability, 48 males and 72 females, 13 to 18 years old. Their mean age was 16 years and 3 months. There were 22 ninth-graders, 39 tenth-graders, 38 eleventh-graders and 21 twelfth-graders. The subjects were selected from 300 students who had been asked by their schools to volunteer to participate in the experiment. The subjects were selected solely on the basis of their performance on a pre-experimental training examination, the Rule Application Test (see Figure 3). To be taken as an experimental subject, the student had to have attained a score of 75% or higher. Four students who had achieved criterion did not appear at the experimental session. None of the subjects had to be removed from the analysis because of prior contact with symbolic logic (see Procedure).

The Schools. The subjects were drawn from four schools, two in New York State and two in Peurto Rico. While more than 40 public and private schools had been asked to participate, only these four accepted. Seven of the subjects were selected from a New York surburban-community public school. Eleven were selected from a major New York metropolitan-area parochial school. Sixty-three were

selected from a major Puerto Rican metropolitan-area parochial school. Thirty-nine were selected from a small Puerto Rican industrial-community parochial school. The latter two schools are considered "American Schools," in Puerto Rico, because major subjects are taught in English.

Recruiting Volunteers. Each of the schools made its own arrangements for recruiting volunteers. One of the schools offered extra credit. The students were simply directed to report at a specific time and place to participate in an experiment in problem solving.

Task Materials

The task materials were derived from the Moore and Anderson (1954a) Symbolic Logic Task. Selected portions of the training materials were converted to programmed instruction presented in a linear format. A matching set of task materials was also developed. Despite these major revisions, the language, concepts, and problem solving heuristics to be acquired remained essentially the same as Moore and Anderson's.

Figure 3 shows the presentation sequence of the instructional and task materials, and should be referred to during the discussion which follows.

Figure 3

The Procedure for the PROBE Experiment

Day 1 First Session:
Language & Rules Text

VOLUNTEERS
WITHIN A
SCHOOL.

INTRODUC-
TION TO
THE EX-
PERIMENT

STUDY
PROGRAMMED
TEXT
..
PROBE
LANGUAGE &
RULES

RULE
APPLIC-
ATION
TEST

Day 2
Assignment of Subjects

RULE APPLICATION
TESTS SCORED

RANDOM ASSIGNMENT OF
SUBJECTS TO TREATMENT
GROUPS WITHIN TEST SCORE
STRATIFICATIONS

Day 3 Second Session:
Problem Solving Text

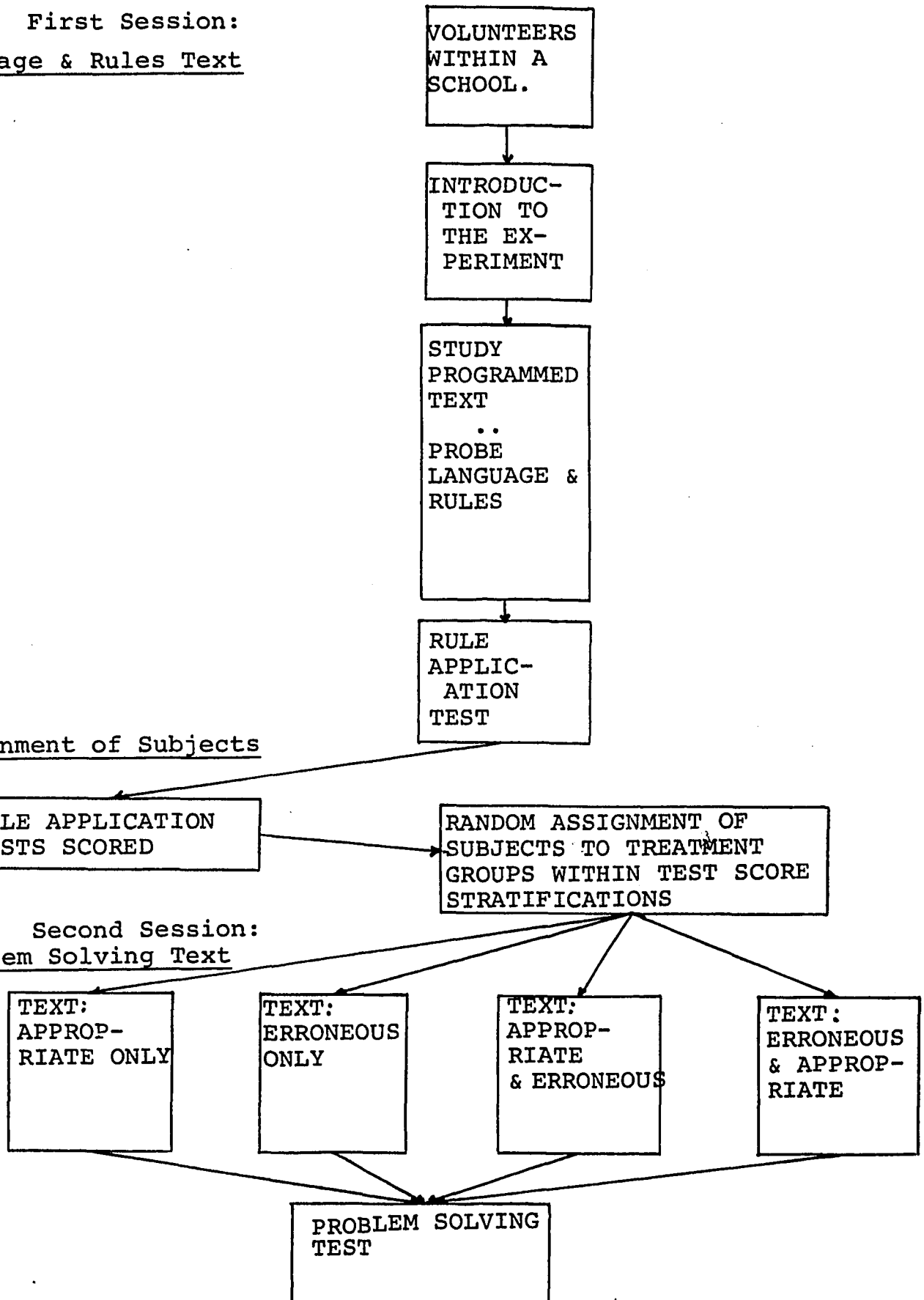
TEXT:
APPROP-
RIATE ONLY

TEXT:
ERRONEOUS
ONLY

TEXT:
APPROP-
RIATE
& ERRONEOUS

TEXT:
ERRONEOUS
& APPROP-
RIATE

PROBLEM SOLVING
TEST



The Language and Rules of PROBE. Only the language and rules portion of the PROBE text was issued at the first session (see Figure 3). This section contained 42 constructed response frames. The frames introduced the grammar and transformation rules of the language. Feedback, in the form of the correct answer, was provided after every frame. The answers, which were printed on the reverse side of the presentation frame, came into view when the page was turned.

Instructions on how to use the text cautioned the student to record his response before he turned the page to check his answer. The students were not given any pre-established time limits for completing the text. A copy of the language and rules portion of the PROBE text appears in Appendix B.

The PROBE Panelbook. The panelbook was an adjunct to the text and contained two illustrative panels. Panel I showed symbolic description of the four transformation rules; Panel II had an example of a PROBE problem. This panel was used to explain only the format of the problem solving task and, therefore, did not include a solution. Panel I, which described the transformation rules, was usually referenced in the response frame discussions of the rules. It was also used by the student as a primary reference source during the Rule Application and Problem Solving Tests. A copy of this booklet appears in Appendix B.

The Student Response Booklet. The student's constructed responses to questions in the text were recorded in a separate answer booklet, rather than in the text. The student used the same booklet to record his responses for both of the training sessions. The first page of the booklet contained student identification information, such as name, sex, school, grade, age in years and months, and the number of the PROBE text he was using. There was a brief questionnaire which the student completed at the end of the second (experimental) session. The questionnaire asked if the student had ever had prior experience with the type of material presented; his opinion of the training materials; and what his teachers think when he makes a mistake.

The Rule Application Test. The scores on the Rule Application Test were used to select the subjects for the experiment. This test was given during the first session, after the student had completed his study of the language and rules portion of the text. The test contained 16 rule application problems; there were four problems associated with each of the four rules. A problem contained a rule number and one or more PROBE expressions. A subject had to determine whether or not the given rule could be applied to the expression(s). If the subject thought that the rule was applicable, he applied the transformation rule and created a new expression. If he thought that the rule was

not applicable, he was required to provide a number of an applicable rule and create a new expression by applying the rule. The given rules were applicable 50% of the time. The problems were not in sequence by rule, but were randomly distributed. To be considered a subject in the experiment, the student had to solve at least 12 of the 16 problems. There was no time limit set for this test. A copy of the Rule Application Test appears in Appendix B.

The Experimental Treatment Texts. There were four versions of the experimental treatment portion of the text (see Figure 3). All versions discussed the format of PROBE problems, and contained a problem example with two illustrative solutions. The same problem was used in every treatment. The treatment portions of the text each contained 30 frames, 26 of which brought the subject into direct contact with either the whole, or some portion, of an illustrative solution. The treatment portions were appended to the language and rules text after the first training session.

The illustrative solutions were described as examples of homework assignments, prepared by students learning PROBE. The solutions were introduced a step at a time, and all previous steps were included in the current frame. All treatment booklets were constructed using only four illustrative solutions, two appropriate and two plausible, but erroneous, solutions. The frames of these solutions were so numbered that the solutions could be identified as

Appropriate 1, Appropriate 2, Erroneous 1, and Erroneous 2. The illustrative solutions were presented in the following orders in the four treatment conditions (see Figure 3):

<u>Treatment Condition</u>	<u>Exemplar Order</u>
1. Appropriate Only	Appropriate 1, Appropriate 2
2. Erroneous Only	Erroneous 1, Erroneous 2
3. Appropriate/Erroneous	Appropriate 1, Erroneous 2
4. Erroneous/Appropriate	Erroneous 1, Appropriate 2

Although the response frame questions asked the student to apply rules provided in the solution or to evaluate a solution step, the student was never asked to create any portion of the solution. The number of questions posed, and the number of solution lines in the illustrations, were matched for each frame in all treatment conditions. For example, Figure 4 shows a matched presentation frame, F 51, from Appropriate 1 and Erroneous 1. In both, the illustrative solution contained two lines, 3 and 4. The format of the questions is also essentially the same, i.e., "Can Rule X be applied to Line Y to form the expression Andy wrote on Line Z?" The expected response(s) were often different. Appendix C contains two of the treatment portions of the text, the Appropriate Only and Erroneous Only portions. Nonetheless, they contain all four illustrative solutions.

Figure 4

Frame F51 - Appropriate 1 (top); Erroneous 1 (bottom)

F51.0

Can Rule 1 be applied
to Line 3 to form the
expression that Andy
wrote on Line 4?

<u>Sector I</u>		<u>Sector II</u>	
Line			
<u>1</u>	$(\neg P \vee R) \circ (\neg P \vee Q)$		
<u>2</u>	$(\neg R \vee P) \circ (\neg R \circ R)$		Q
<u>Sector III</u>		<u>Sector IV</u>	
		Rule #	Line #
<u>3</u>	$\neg P \vee Q$	<u>3</u>	<u>1</u>
<u>4</u>	$P \supset Q$	<u>1</u>	<u>3</u>
<u>5</u>			
<u>6</u>			
<u>7</u>			
<u>8</u>			
<u>9</u>			
<u>10</u>			
<u>11</u>			

F51.0

Can Rule 2 be applied
to Line 2 to form the
expression that Andy
wrote on Line 4?

<u>Sector I</u>		<u>Sector II</u>	
Line			
<u>1</u>	$(\neg P \vee R) \circ (\neg P \vee Q)$		
<u>2</u>	$(\neg R \vee P) \circ (\neg R \circ R)$		Q
<u>Sector III</u>		<u>Sector IV</u>	
<u>3</u>	$\neg P \vee Q$	<u>3</u>	<u>1</u>
<u>4</u>	$\neg R \vee (P \circ R)$	<u>2</u>	<u>2</u>
<u>5</u>			
<u>6</u>			
<u>7</u>			
<u>8</u>			
<u>9</u>			
<u>10</u>			
<u>11</u>			

The Problem Solving Test. The Problem Solving Test, which was the transfer task, contained four PROBE problems. All treatment groups took the same test. The cover sheet notified the subject that the test had to be completed in 60 minutes, and that he was to spend no more than 15 minutes on any problem. The subject was also told that the problems had to be done sequentially. The student recorded the start and stop times for each problem as he went along. Steps in the solution were recorded in ink, and the subjects were directed not to cross out any of the lines. However, unwanted lines could be eliminated, if the subject chose to do so. To eliminate a line, the subject placed an "X" over the line number as an indication that the line was crossed out. A copy of the Problem Solving Test, and one of the correct solutions for each of the problems, appears in Appendix D.

Testing the Experimental Materials. The text and task materials, along with a scoring procedure (see Appendix E), were extensively tested in a pilot study. The subjects for that study were 55 high school students, of both sexes, who were participants in two National Science Foundation summer science programs. The students were drawn from suburban and inner-city schools in a major New York State metropolitan area. Table 1 shows the maximum possible and actual average scores achieved by the students on each of the components.

Table 1

Maximum Possible & Average Actual
Scores for
Instructional & Task Materials

	<u>Maximum Possible</u>	<u>Actual Mean</u>	<u>SD</u>	<u>SE</u>
<u>Text Responses Frames</u>				
First Session Language & Rules Text	63	60.8	2.25	.30
Second Session Problem Solving Text	30	28.6	1.54	.21
First & Second Combined	93	89.2	3.76	.51
<u>Tasks</u>				
Rule Application Test (Criterion Task)	16	12.4	3.02	.41
Problem Solving Test	4	2.67	1.23	.17

N=55

The response frame scores show that the texts fostered almost error-free learning. On the average the students constructed correct responses to over 95% of the questions posed in the texts' frames. Furthermore, no significant error differences were found between the appropriate or erroneous set of solution exemplars, $t < 1.00$. The scores attained by the students on the tests demonstrated that the materials were appropriate for the experimental target population of above average ability high school students. The students in the pilot study solved an average of 2.67 problems, out of a maximum of 4, and 69% met criterion on the Rule Application Test. The Kuder-Richardson Formula 20 Alpha measure of item reliability for the Rule Application Test was .789 (Ebel, 1972).

Three raters, two female and one male, were used to establish the inter-rater reliability coefficients for scoring the text response frames, and the Rule Application and Problem Solving tests, (see Table 2). The scoring procedures are detailed in Appendix E.

Table 2

Inter-Rater Reliability
Coefficients for Instructional
and Task Components.

<u>Component</u>	<u>Reliability Coefficient</u>
Text Response Frames	.792
Rule Application Test	.976
Problem Solving Test	.999

Procedure

The First Session - Learning the Language & Rules.

When the students were seated, the experimenter read the instructions which follow (in English in New York; in Spanish in Puerto Rico):

You are going to participate in an experiment in problem solving, which requires two sessions. Each takes approximately two hours. Like most experiments, it is a test of the experimenter's ideas. Each of you will be taught to solve problems, but in slightly different ways. We would like to learn which way is best. In a few minutes, we will pass out the textbooks. These will teach you all you need to know. Take your time and read the material carefully. Please do not rush -- no one will be permitted to leave until everyone is finished. The material is difficult, but you will not be misled or fooled. Do the best you can. What we are trying to find out is how people learn to solve problems.

Text and response booklets were passed out and the experimenter continued:

Please fill out all of the information asked for at the top of the response booklet, in pen. Open

the PROBE text to the first page and begin. Read the instructions carefully and do exactly what they say. Please do not write in the text, write your answers in the response booklet. Raise your hands if you have any questions.

If the students asked any specific questions about PROBE, they were asked to find the answers in the text. Whenever a student asked a general question, i.e., "Should I correct my answers when they're wrong?" -- an announcement was made to the class, e.g., "Please do not cross-out or change anything you have written; we would like to learn how you answered the question."

When a student completed the text, he or she received a Rule Application Test. The student's name and text number were recorded at the top of the test booklet. When they completed the test, the students' materials were collected and they were told to sit and work quietly until dismissed.

The Second Session -- Learning to Solve Problems. The Rule Application Tests were scored using the procedure in Appendix E. The tests were then sorted in order by scores, with the highest score on top, and the subjects were randomly assigned to treatment groups. When the subjects appeared for the second session they picked up the extended treatment text and response booklet. The treatment text contained both the Language & Rules and Problem Solving texts. When everyone was seated the experimenter made the following announcement:

When you complete this portion of the text, you will be given four PROBE problems to solve in an open book test. Start where you left off, on page F45, but feel free to review any part of the text. You can begin now.

Once the students started raising their hands, requesting the problem solving test, the group was stopped, and the following announcement made:

When you do the problem solving test be sure that you take no more than 15 minutes for each problem. If you haven't finished, just go on to the next problem -- do not go back. Please show all of your work! Do not erase or cross out anything. The instructions tell you how to get rid of a line you do not want, and still show how you solved the problem. That is what we need to know for the experiment. Be sure that you record your start and stop time for each problem.

Portions of that announcement were repeated as needed. Whenever a student finished the test, he was reminded to complete the questionnaire at the end of the response booklet. All of the students materials were collected and the students asked to work quietly, until dismissed. When the experiment was over, the students were thanked for their participation and handed an answer sheet.

Procedural Variations for the Second Session. Scheduling problems generated between-school variations which affected the time between the first and second sessions. In two of the schools, the second session took place two days after the first, as originally planned. One of these schools was in New York, and one in Puerto Rico. In the second

Puerto Rican school, the second session took place on the day after the first. In the last school, the first and second sessions were separated by eight days. Eleven subjects came from this school. To reacquaint these students with the first session's material, the following revisions were made to the second session procedure:

1. The session was opened with the following announcement:

It has been a long time since our last session. For the next 15 minutes please review pages F18 to F43 in the language and rules portion of the text. Please do not start the new portion of the text until signaled to do so.

2. At the end of 15 minutes, the experimenter directed the students to stop their review, and initiated the standard second-session procedure.

Scoring the Experiment. Appendix E contains the scoring procedures for all components of the experiment. Before scoring, a subject's response to the prior experience with symbolic logic questions was checked. None of the subjects reported prior contact with symbolic logic. The following raw scores were available for each subject:

1. First Session Response Score
2. First Session Training Time
3. Second Session Response Score
4. Second Session Training Time
5. Rule Application Test Score
6. Rule Application Test Time

For each problem:

7. Total Number of Lines
8. Number of Incomplete Lines
9. Number of Incorrect Rule Applications
10. Number of Extra Lines
11. Number of Incomplete Lines Cancelled
12. Number of Incorrect Rule Applications Cancelled
13. Number of Extra Lines Cancelled
14. Number of Necessary Lines Cancelled
15. Problem Solved? Yes = 1; No = \emptyset
16. Problem Solving Time

Hypothesis To Be Tested

This research has two principal areas of interest, problem solving and erroneous behaviors. Treatment condition are expected to differentially influence each of these domains. Therefore, separate sets of outcome measures were derived to explore the hypotheses relating to problem solving and those relating to erroneous behaviors.

As a general hypothesis, it is expected that students' abilities to solve problems would be enhanced through instructional contacts with plausible, but erroneous, solutions, especially in matched exemplar and non-exemplar pairs. A vicarious experience of error should reduce students' inappropriate behavior, or at least increase the students' abilities to recognize and correct their mistakes.

Outcome Measures. The followin measures were used in testing the hypotheses relating to problem solving:

- a. Total Solutions -- the total number of problems solved.
- b. Total Perfect Solutions -- the total number of solutions either lacking inappropriate steps or in which all inappropriate steps had been cancelled.
- c. Total Necessary Steps -- The total number of steps which led to a correct solution.

The following set was used to test hypotheses relating to the production of and response to erroneous behaviors:

- a. Total Rule Misapplications -- the total number of times that rules had been incorrectly applied.
- b. Proportion of Rule Misapplications Cancelled -- Total Rule Misapplications was divided into the total number of rule misapplications cancelled. This proportion was set to 1.0 for those students who had never misapplied a rule.
- c. Total Extra Steps -- the total number of steps which could not have led to a solution, but in which the rule was correctly applied.
- d. Proportion of Extra Steps Cancelled -- Total Extra Steps was divided into the total number of extra steps cancelled. This proportion was set to 1.0 for those students who had taken no extra steps.

Specific Hypotheses. The following hypotheses regarding the differential effects of treatment conditions on problem solving and erroneous behaviors had been stipulated:

1. Problem Solving

- a. The Appropriate/Erroneous and the Erroneous/ Appropriate treatment groups will solve more problems, have more perfect solutions, and produce more necessary steps, than the Appropriate Only treatment group. This expectation is based on Miller's (1957) proposition that instructional contact with designated appropriate and erroneous exemplars will help the learner acquire correct cues and responses, and avoid incorrect responses. Duncan (1959) has argued that for purposes of transfer students should have contact with heterogeneous exemplars and experience error. Waters (Waters, 1931) found that contact with the erroneous behavior of others instills cautiousness in the learner. Furthermore, Tennyson (1973) observed that instructional contact with exemplar and non-exemplar pairs improves performance in identifying new instances of a concept; lacking contact with negative instances, the students' performances were equivalent to that of students who had not received training.

b. The Appropriate/Erroneous and the Erroneous/
Appropriate treatment groups will solve more
problems, have more perfect solutions, and
produce more necessary steps than the
Erroneous Only treatment group. This
expectation is based on Koran et al.'s (1972)
findings that erroneous exemplars did not
provide students with the information they
needed to derive appropriate behaviors.

2. Erroneous Behaviors

a. The Appropriate Only group will produce
a larger number of rule misapplications and
extra steps, and cancel a smaller proportion
than the combined populations of the
Erroneous Only, Appropriate/Erroneous and
Erroneous/Appropriate treatment groups.

This expectation is based on the Waters and
Ellis' (Waters, 1931; Waters & Ellis, 1931)
findings that instructional contact with the
erroneous behavior of others leads to a
reduction in errors. Hilgard et al. (1954)
found that when a problem solving strategy
was acquired with a minimal experience of
error the learner was more confident regarding
his approach thereby increasing the number of
perseverative errors.

- b. The Appropriate Only treatment group will produce a larger number of rule misapplications and extra steps, and cancel a smaller proportion than the Appropriate/Erroneous and the Erroneous/Appropriate treatment groups. This expectation is based on Miller's (1957) view that instructional contact with designated appropriate and erroneous exemplars will help the learner avoid incorrect responses. Waters and Ellis (1931) noted that contact with the erroneous behavior of others tends to reduce errors.
- c. The Erroneous Only treatment group will produce a smaller number of rule misapplications and extra steps, and cancel a smaller proportion than the Appropriate/Erroneous and Erroneous/Appropriate treatment groups. Waters and Ellis (Waters, 1931; Waters & Ellise, 1931) found that instructional contact with the erroneous behavior of others instills cautiousness and leads to a reduction in errors. However, Day et al. (1971) and McKeachie (1974) have observed that an experience of error may not provide learners with the information they need to correct their errors.

A multivariate comparison of the Appropriate/Erroneous and Erroneous/Appropriate treatment groups was scheduled as the first step in the analysis, in order to determine whether those groups could be combined or should be treated separately.

CHAPTER V

RESULTS AND DISCUSSION

This research focused on the instructional influences of exemplars and non-exemplars in learning to solve problems. Since this was a new experimental domain, relatively few conventions regarding the outcome measures had been established. Outcome measures were derived which represented somewhat different but related aspects of the problem solving process, i.e., Total Solutions and Total Perfect Solutions, Total Rule Misapplications and Total Extra Steps. These were but a sample of the behaviors which might be affected by the experimental manipulation. The literature had provided only slight indications as to how these behaviors might be affected. At this juncture, we were interested in knowing how treatment condition influenced selected combinations of variables.

Harris (1975, p. 5) indicated that multivariate statistical techniques are most appropriate when the domain of exploration is relatively new and the researcher is interested in determining how the experimental manipulation influences a variety of measures. By considering a number of measures simultaneously, the research can shed light on the sources of difference, if any are found to exist, and on the relationships among the measures

(Tatsuoka, 1973). The results which follow were primarily derived using MLIN, a multivariate linear model statistical program based on Roy's Union-Intersection criteria

(Note 5).

Results

Treatment Groups for Analysis

In order to determine whether the populations of the Appropriate/Erroneous and Erroneous/Appropriate treatment groups could be combined or should be treated separately for the purpose of analysis, a multivariate comparison was performed on these groups' problem solving outcome measures. The results indicated that the two groups represented significantly different populations, and should be treated separately, $F\phi(3,1,114)^1 = 11.99, p < .025$. The Appropriate/Erroneous treatment group solved more problems, $F\phi(3,1,114) = 8.92, p < .05$, and produced more perfect solutions, $F\phi(3,1,114) = 11.51, p < .025$, than the Erroneous/Appropriate treatment group. Therefore, the four treatment groups: Appropriate Only, Erroneous Only, Appropriate/Erroneous, and Erroneous/Appropriate, were maintained.

¹ $F\phi$ is the symbol for the generalized F statistic which was used for testing significance. Bock (1975) derived critical values for this distribution from the non-central probability distribution of the greatest characteristic roots, Roy's Union-Intersection criteria for multivariate significance testing. Three degree-of-freedom parameters are associated with the generalized F statistic, $F\phi(S,R,T)$; $S = \min(Kr,P)$, $R = |Kr - P| + 2$, and $T = Ns - Kg - P + 1$, where $Kr = \text{Rank of The Hypothesis Matrix}$, $P = \text{Number of Dependent Measures}$, $Ns = \text{Number of Subject}$ and $Kg = \text{Number of Treatment Groups}$.

Problem Solving Outcomes

The outcome measures, Total Solutions, Total Perfect Solutions and Total Necessary Steps, were simultaneously examined using multivariate analytical techniques (Harris, 1975), in order to test the overall hypothesis that the sets of illustrative solutions used in the self-instructional presentations would differentially influence performance on a problem solving transfer task. The treatment groups' means for the problem solving outcome measures, and the univariate F statistic, are presented in Table 3.

Table 3

Univariate Statistics for Problem Solving
Outcome Measures by Treatment Group

<u>Treatment</u> <u>Group</u>	<u>Outcome Measures</u>		Total Perfect Solutions		Total Necessary Steps		<u>N</u>
	<u>M</u>	<u>SD</u>	<u>M</u>	<u>SD</u>	<u>M</u>	<u>SD</u>	
Appropriate Only	2.47	1.25	2.4	1.22	13.67	5.81	30
Erroneous Only	2.38	1.1	2.25	1.11	12.91	4.67	32
Appropriate/ Erroneous	3.14	1.16	2.97	1.18	15.93	4.86	29
Erroneous/ Appropriate	2.17	1.36	1.90	1.29	12.69	5.49	29
Univariate <u>F</u>	3.43*		4.00*		2.37		

* $p < .05$

Discriminant Analysis of Total Solutions, Total Perfect Solutions and Total Necessary Steps. The initial overall test for treatment effect was performed using Discriminant Analysis. Although the differences among the groups approached statistical significance, F_{ϕ} (3.1,114) = 12.35, $p > .07$, the null hypothesis of "no treatment effects" could not be rejected. This was unexpected, in light of the univariate results; the overall, univariate F_s for both Total Solutions and Total Perfect Solutions were statistically significant (see Table 3). However, the extremely high within-group correlations for the problem solving outcome measures (see Table 4), suggested that Profile Analysis would be a more appropriate statistical technique for examining treatment effects.

Table 4

Within-Group Correlation for
Problem Solving Outcome Measures

<u>Outcome Measure</u>	Total Solutions	Total Perfect Solutions	Total Necessary Steps
Total Solutions	1.0	.946	.901
Total Perfect Solutions		1.0	.849
Total Necessary Steps			1.0

Treatment Effects on Problem Solving Outcomes. The profiles of the treatment groups' mean response scores on the problem solving outcome measures are shown in Figure 5. The shapes exhibited by these curves forcefully suggested that each measure represented a repeated observation of the same behavior, rather than an observation of an independent behavior; each of the curves appeared to be parallel to the others.

Profiles Analysis provides a means of testing for multivariate group differences when the dependent measures are highly correlated. There are three statistically independent tests, which are analogous to the tests of significance in a two-way, univariate ANOVA in which the response measures are treated as a factor. The test of parallelism was computed first, in order to examine the Treatment X Measure interaction. The rejection of the null hypothesis of parallelism would terminate the analysis, as the results of the remaining tests would be uninterpretable. The null hypothesis was not rejected, $F_{\phi}(2,1,115) p > .30$; there were no significant Treatment X Measure interactions. The test for

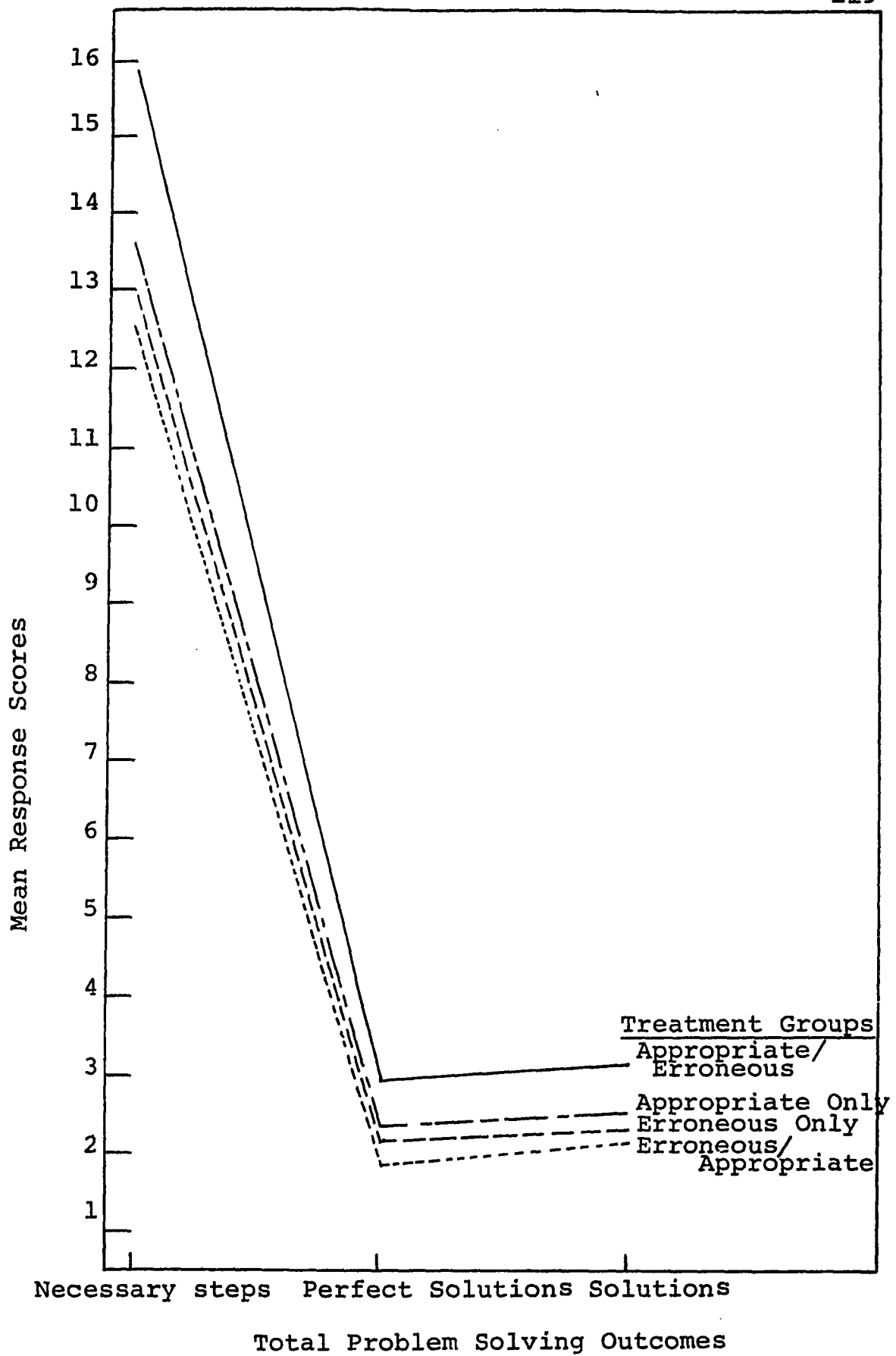


Figure 5. Profiles of Problem Solving Outcomes for Each Treatment Group

flatness, an independent evaluation of difference among the response measures, was not performed as it was obvious that the grand means of these measures were different (see Table 3).

The dependent measure for testing the main effect of treatment in Profile Analysis is the unit-weighted, linear combination of the subjects' scores on each of the measures in the analysis. When the unit-weighted, linear combination of Total Solutions, Total Perfect Solutions, and Total Necessary Steps was used as the dependent measure, there was a significant difference among the treatment groups on the problem solving transfer task, $F_{\phi}(1,3,116) = 2.89$, $p < .038$. The null hypothesis of equal treatment effects was rejected. However, there was some question about the way the scores were weighted in the combined measure. Given the mean values for each of the problem solving outcome measures (see Table 3), Total Necessary Steps had four times the weight of either Total Solutions or Total Perfect Solutions. In order to make all of the measures comparable (Harris, 1975, p. 81), population-standardized scores were substituted for raw scores in the combined dependent measure and the hypothesis of equal treatment effects was retested. The main effect of treatment differences using this transformation was even more pronounced, $F_{\phi}(1,3,116) = 3.42$ $p < .02$.

The Effect of a Combined Experience With Appropriate and Erroneous Solution Exemplars on Problem Solving Performance. The following a priori contrasts were evaluated to test the general hypothesis that instructional contact with a matched set of appropriate and erroneous exemplars would induce superior performance on a problem solving transfer task.

1. The Appropriate/Erroneous vs the Appropriate Only group.
2. The Appropriate/Erroneous vs the Erroneous Only group.
3. The Erroneous/Appropriate vs the Appropriate Only group.
4. The Erroneous/Appropriate vs the Erroneous Only group.

The dependent measure of performance was the unit-weighted, linear combination of population-standardized scores for Total Solutions, Total Perfect Solutions, and Total Necessary Steps.

Although the Appropriate/Erroneous treatment group attained a higher mean score on the dependent measure than both the Appropriate Only and Erroneous Only treatment groups, only the difference between its score and that of the Erroneous Only group was statistically significant, $F_{\phi}(1,1,116) = 5.90, p < .017$. However, the difference between the Appropriate/Erroneous and Appropriate Only

treatment groups' mean scores just missed being significant by a small margin, $F_{\phi}(1,1,116) = 3.74, p < .056$. In contrast to the Appropriate/Erroneous group, the Erroneous/Appropriate treatment group did not do well. While there were no significant differences between its performance and that of the Appropriate Only and Erroneous Only treatment groups, $F_{\phi}(1,1, 116) < 1.00$, the Erroneous/Appropriate group had the lowest scores.

Analyses of Erroneous Behaviors

An Overall Test of Treatment Effects. To test the hypothesis that treatment conditions would differentially influence the production of and response to erroneous behaviors on a problem solving transfer task, the outcome measures Total Rule Misapplications, Proportion of Rule Misapplications Cancelled, Total Extra Steps, and Proportion of Extra Steps Cancelled, were simultaneously examined via Discriminant Function Analysis. The hypothesis that the groups' mean response vectors were equal could not be rejected. When, in accord with Harris' (1975, p. 103) recommendations, a priori contrasts were evaluated, no significant differences could be found.

Table 5 shows the means and standard deviations for each of the treatment groups on the four outcome measures. There was no indication in the univariate statistics that the groups differed in their erroneous behaviors in any way. These results were in accord with those of the Discriminant Analysis.

Table 5

Univariate Statistics for Erroneous Behaviors
by Treatment Group

<u>Treatment Groups</u>	<u>Outcome Measures</u>								
	Total Rule Mis- applications		Proportion Cancelled		Total Extra Steps		Proportion Cancelled		<u>N</u>
	<u>M</u>	<u>SD</u>		<u>SD</u>	<u>M</u>	<u>SD</u>	<u>M</u>	<u>SD</u>	
Appropriate Only	2.47	2.58	.46	.43	3.23	2.85	.60	.43	30
Erroneous Only	2.97	3.21	.45	.42	3.69	2.48	.55	.37	32
Appropriate/ Erroneous	1.90	2.60	.63	.46	2.72	2.55	.69	.44	29
Erroneous/ Appropriate	3.28	3.56	.40	.43	2.93	2.70	.56	.45	29
Univariate F	1.17		1.54		.767		.69		

The Analysis of Other Issues

The results of the a priori comparisons suggested some other issues which should be explored. For example, the magnitude of the difference between groups suggested that the relationship between errors made during training and performance should be explored. It also suggested that factors such as school location and sex may have interacted

with performance on the problem solving task and that that interaction merits investigation. The fact that the problem solving measures were highly correlated, suggested that there was likely to be a more parsimonious model for exploring group differences rather than using the unit-weighted linear combination of total solutions, total perfect solutions and total necessary steps. The results of analysis of these issues follow.

The Relationship Between Errors During Training and Performance. Although treatment condition had not significantly influenced the subjects' erroneous behaviors during problem solving, we were interested in studying the relationship between the errors the students made during training and their performance in the Rule Application and Problem Solving Tests. There was also interest in the relationship between the students' scores on the Rule Application Test and their performance in the transfer task. First, it was necessary to determine if either the assignment to treatment groups or the difference in treatment texts had significantly influenced the treatment groups' means for the number of errors made during training; and to determine if either of these factors had influenced the groups' mean scores on the Rule Application Test. A series of one-way analyses showed that the treatment groups did not differ significantly on any of the training measures or on their mean scores on the Rule Application Test (see Table 6).

Table 6

Univariate Statistics for Training Outcome
Measures by Treatment Group

<u>Treatment Group</u>	<u>Correct Responses -- Language & Rules Text</u>		<u>Correct Responses -- Problem Solving Text</u>		<u>Rule Application Test</u>	
	<u>M</u>	<u>SD</u>	<u>M</u>	<u>SD</u>	<u>M</u>	<u>SD</u>
Appropriate Only	60.13	2.46	28.20	1.94	14.03	1.22
Erroneous Only	60.94	1.83	27.69	2.10	14.19	1.33
Appropriate/ Erroneous	60.55	2.47	28.00	1.67	14.28	1.39
Erroneous / Appropriate	60.73	1.44	27.79	2.04	14.03	1.45
Univariate F	1.64		0.42		0.23	

*Maximum Possible Scores
 Language & Rule Text 63
 Problem Solving Text 30
 Rule Application Test 16

During training, the subjects were almost error-free; in responding to questions in the texts the rate of error was slightly less than 4.5%, an average of four incorrect responses per student. The intercorrelations between training outcomes and performance on the Rule Application and Problem Solving Tests are presented in Table 7. The problem solving measure was the unit-weighted, linear combination of population-standardized scores for Total Solutions, Total Perfect Solutions, and Total Necessary Steps.

Table 7

Significant Intercorrelations Between
Training Outcomes and Performance

	Correct Responses-- Language and Rules Text	Rule Application Test	Correct Responses-- Problem Solving Text
Rule Application Test	.162	1.000	
Number of Correct Responses on Problem Solving Text	.311**	-.028	1.000
Problem Solving	.193*	.323**	.185*

* $p < .05$; ** $p < .01$

There appeared to be only a slight relationship between the number of errors subjects made in answering questions in the Language & Rules text and their Rule Application Test scores, $r = .162$, $p < .10$. However, the errors the subjects made in answering questions in the Problem Solving text were in no way related to their scores on the Rule Application Test, $r = -.028$. Students who made errors in answering questions in the Language & Rules text, also tended to make errors in answering questions in the Problem Solving text, $r = .311$, $p < .01$, despite the fact that the Problem Solving texts for each treatment condition were different.

There were significant and positive correlations between:

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the number of correct responses to questions in the Problem Solving text, $r = .185$, $p < .05$; and students' scores on the Rule Application Test, $r = .323$, $p < .01$; performance on the problem solving task and the number of correct responses to questions in the Language & Rules text, $r = .193$, $p < .05$. The subjects who made the least number of errors during training tended to do best on the problem solving task. After the variance in problem solving performance accounted for by treatment conditions (8.83%) had been partialled out (see Nie, Hull, Jenkins, Steinbrenner & Bent, 1975, for the multiple-regression procedure), the students' scores during training accounted for the following proportions of the variation in performance -- Rule Application Test Score: 9.45%; Correct Responses-Problem Solving Text: 1.54%. The proportion of the variance in problem solving performance accounted for by treatment condition was almost equivalent to that accounted for by the level of prerequisite knowledge, as reflected in the students' scores on the Rule Application Test.

The students' inappropriate behaviors appeared to be the major source of variance in problem solving performance. The number of rule misapplications during problem solving accounted for 44.93% of the variance in performance, even after the variance accounted for by the training and treatment measures (23.1%) had been partialled out. The students' abilities to recognize and correct inappropriate behavior was also important; success in problem solving was significantly

correlated with the proportion of rule misapplications cancelled, $r = .664$, $p < .001$, and the proportion of extra steps cancelled, $r = .473$, $p < .001$. The students' abilities to correct inappropriate behavior appeared to require more than just a knowledge of the rules. There was almost no relationship between the students' scores on the Rule Application Test and the proportion of rule misapplications cancelled, $r = .13$, $p > .10$.

Demographic X Treatment Condition Interactions. In order to evaluate the influence of school location and sex on problem solving performance, the unit-weighted, linear combination of population-standardized scores for Total Solutions, Total Perfect Solutions, and Total Necessary Steps was used as the dependent measure in two-way, unequal N, Analysis of Variance. (Nie et al., 1975, pp. 405-408). Table 8 shows the results from those univariate tests. There were no significant main effects, or interactions found for either school location or sex. However, the Treatment Condition X School Location interaction did approach statistical significance, $F(3,112) = 2.38$, $p .072$. The Means and SDs for the School Location X Treatment Condition interactions are presented in Table 9. The distribution of the means suggests that there may be some cultural differences which interacted with treatment. However, the variance is large, and the number of subjects in the New York group too small, to draw any conclusions.

Table 8
Demographic X Treatment Condition
Evaluations of the Combined, Standardized
Measure of Problem Solving

<u>Effect</u>	<u>F</u>	<u>Significance</u>	<u>DF</u>
Treatment	3.48	p<.018	3,112
School Location (New York or Puerto Rico)	.747	p>.999	1,112
Treatment X School Location	2.38	p<.072	3,112

Treatment	3.50	p<.018	3,112
Sex	1.65	p>.198	1,112
Treatment X Sex	1.75	p>.159	3,112

Toward a More Parsimonious Model of Group Differences.

Thus far, Total Solutions, Total Perfect Solutions, and Total Necessary Steps have all been given equal weight in the analyses of group differences. With the linear combination outcome measure, differences in performance of equal magnitude were treated as equivalent, no matter what the source. It is difficult to imagine any educational context in which all of these measures would be equally important. If a rank order of importance could be

Table 9

Demographic X Treatment Condition Interaction Means
and Standard Deviations for the Combined, Standardized
Measures of Problem Solving

<u>Demographic Factors</u>	Appropriate Only			Erroneous Only			<u>Treatment Conditions</u> Appropriate/ Erroneous			Erroneous/ Appropriate			Total		
	<u>M</u>	<u>SD</u>	<u>N</u>	<u>M</u>	<u>SD</u>	<u>N</u>	<u>M</u>	<u>SD</u>	<u>N</u>	<u>M</u>	<u>SD</u>	<u>N</u>	<u>M</u>	<u>SD</u>	<u>N</u>
School Location															
New York	-2.55	2.35	5	-.71	4.28	4	.33	2.46	4	.83	1.25	5	-.56	2.81	18
Puerto Rico	.45	2.84	25	-.35	2.32	28	1.52	2.74	25	-1.23	3.19	24	.09	2.91	102
<hr/>															
Sex															
Male	-1.36	3.09	13	-.70	2.46	14	.90	2.59	11	-.14	3.23	10	-.40	2.87	48
Female	.94	2.50	17	-.14	2.65	18	1.64	2.80	18	-1.25	2.94	19	.26	2.90	72
<hr/>															
Total	-.05	2.96	30	-.39	2.54	32	1.36	2.70	29	-.87	3.03	29	.00	2.89	120

established for each of the individual measures, it might be possible to construct a more parsimonious model than the linear combination.

For the purpose of this investigation, what rank order should be assigned to each of the problem solving outcome measures? A primary objective of this research was to find more effective methods of teaching students to solve problems, even if they made mistakes along the way. From both an educational and research perspective it was felt that Total Solutions should be ranked first. Justifying the selection of the second rank measure was more difficult. Total Perfect Solutions and Total Necessary Steps have different foci: the first is concerned with perfection, and the second with partial solutions. Either might have been given the second rank position. However, the procedure which was selected for model construction produces the most accurate results when the measure assigned to a next highest rank has at least some potential for producing an independent contribution to group differences. The univariate results (see Table 3) indicated that although Total Perfect Solutions had the potential for making an independent contribution to group differences, Total Necessary Steps did not. Therefore, the outcome measures were ranked as follows: (1) Total Solutions; (2) Total Perfect Solutions; (3) Total Necessary Steps.

These measures were entered in rank order sequence into a Stepdown Discriminant Function procedure in order to derive the most parsimonious model of group differences (Finn, 1974, pp. 322-324). In this post-hoc evaluation, Total Solutions was entered first and, as was expected, showed a significant difference among the groups, $F(3,116) = 3.43, p < .019$. That, of course, was the equivalent of a univariate test. Total Perfect Solutions was entered in Step 2, but did not make a significant independent contribution to group differences, $F(2,116) = 1.77, p < .15$; therefore, the stepdown analysis was terminated.

The most parsimonious model of treatment group differences was the single variate Total Solutions. A priori contrasts between groups were re-examined for Total Solutions. The Appropriate/Erroneous treatment group solved significantly more problems than both the Appropriate Only, $F\phi(1,1,116) = 4.47, p < .037$, and Erroneous Only treatment groups, $F\phi(1,1,116) = 5.95, p < .016$. Those findings, along with the prior results, indicated that the Appropriate/Erroneous group solved more problems than all of the others. The difference between the Appropriate/Erroneous and Appropriate Only treatment groups had only approached statistical significance using the linear combination model.

There were no significant differences in the number of problems solved between the Erroneous/Appropriate treatment group and either the Appropriate Only, $F\phi(1,1,116) < 1.0$,

or Erroneous Only treatment groups, $F_{(1,1,116)} < 1.0$.

Discussion

Students were taught to solve problems by working their way through pairs of illustrative solutions, appropriate only, erroneous only, appropriate/erroneous, and erroneous/appropriate. The measure of problem solving performance was a unit-weighted, linear combination of population-standardized scores for Total Solutions, Total Perfect Solutions, and Total Necessary Steps; this measure was used because the variables were highly correlated. Those who had tutorial contact with matched pairs of illustrative solutions, the Appropriate/Erroneous and Erroneous/Appropriate treatment groups, were expected to solve more problems, derive more perfect solutions, and produce more necessary steps toward solution than the students in either the Appropriate Only or Erroneous Only treatment groups. It was further expected that those who had vicariously experienced error via contact with erroneous examples of solutions, the Erroneous Only, Appropriate/Erroneous and Erroneous/Appropriate treatment groups, singly and in combination, would produce fewer rule misapplications and extra steps, and cancel a greater proportion than those students who did not have a vicarious experience of error, the Appropriate Only treatment group.

Although contact with pairs of illustrated solutions

differentially influenced problem solving performance on a transfer task, it did so in a manner which had been only partially predicted. This research found that the group which was exposed to the Appropriate/Erroneous solutions solved more problems than any of the other groups. The Appropriate/Erroneous treatment group attained the highest mean score for the linear combination of problem solving measures; a score which was significantly superior to those attained by the Erroneous Only and Erroneous/Appropriate treatment groups. Moreover, when a more parsimonious model of problem solving, derived post-hoc, was used -- that of Total Solutions alone -- the performance of the Appropriate/Erroneous treatment group was also significantly superior to that of the Appropriate Only treatment group. There were no significant differences in the problem solving performances of the Appropriate Only, Erroneous Only, and Erroneous/Appropriate treatment groups. Treatment condition made no difference in the students' production of or response to erroneous behavior during problem solving.

Although treatment condition significantly influenced transfer on the problem solving task, its overall effect was comparatively small; treatment condition accounted for only 8.83% of the variance in the students' scores on the unit-weighted, linear combination of problem solving outcome measures. The students' knowledge of the rules, as measured by their scores on the Rule Application Test, was able to account for roughly the equivalent amount of variance.

The major source of variance in the students' problem solving ability was their propensity for error, along with their ability to recognize and correct inappropriate and erroneous behaviors. These factors still accounted for 53.7% of the variance in problem solving ability, even after treatment and training effects, and knowledge of the rules, had been partialled out.

First Contact Order Effects

In contrast to the instructional influence of the appropriate models of solutions which are typically used to illustrate problem solving, tutorial contact with erroneous models enhanced the problem solving ability of some students, but inhibited it in others. Depending on specific aspects of their experience, those who had vicariously experienced error landed at both extremes and at the midpoint of the performance continuum. At the extremes, those whose first instructional contact was with an appropriate solution followed by an erroneous one tended to do best, while those whose first contact was with an erroneous solution followed by an appropriate one, tended to do most poorly. The sequence in which the erroneous exemplars were contacted appeared to be a critical determinant of success in problem solving.

An examination of all of the groups' mean scores on the problem solving outcome measures (see Table 3) suggested that students benefited the most when the first instructional

contact was with an appropriate exemplar. To test that supposition, the students in the Appropriate Only and Appropriate/Erroneous treatment groups were contrasted with the students in the Erroneous Only and Erroneous/Appropriate treatment groups. The evidence in support of a first contact order effect was unequivocal; the students whose first contact was with an appropriate solution solved more problems, $F_{\phi}(1,1,116) = 5.63, p < .019$, and had higher scores for the linear combination of outcome measures, $F_{\phi}(1,1,116) = 6.28, p < .014$. When first instructional contact was with an appropriate solution, problem solving performance was enhanced, no matter what type of exemplar was used as the second member of the pair.

Unique Contributions of Appropriate and Erroneous Exemplars

Although first exemplar contacts have been shown to be most critical, that finding in and of itself does not explain all of the results. For example, the performance of the Appropriate Only group was not significantly different from that of the Erroneous Only and Erroneous/Appropriate treatment groups. Furthermore, the Appropriate/Erroneous group solved more problems than any of the other groups, and just missed being significantly superior to the rest in overall performance.

A more comprehensive explanation is that contact with each type of solution produces some independent contribution

to learning, a contribution which is mediated by the sequence in which the particular type of solution was apprehended. This explanation is congruent with an information processing view of human learning in which both affective and information contents have to be considered (McKeachie, 1974). The information content of the exemplar sets presented to both the Appropriate/Erroneous and Erroneous/Appropriate treatment groups were equivalent from an external viewpoint. Furthermore, the average time spent on studying this information was relatively brief, 26-27 minutes. Nonetheless, as the results indicated, the two groups did not derive the same information.

Koran et al. (1972) have argued that each type of exemplar, negative and positive, makes unique contributions to learning, and both in combination exceed the effect of either one alone. That position has been supported in concept learning. When the influence on learning of positive/negative pairs was contrasted with the influence of positive pairs alone, the performance on a transfer task for those who had had contact with mixed pairs was superior (Klausmeier et al., 1974, pp. 195-202; Tennyson, 1973). However, negative instances had never been used as the first member of a pair (Tennyson, Note 9).

The learner uses the illustrative exemplars as models for constructing procedures or strategies with which

to approach new problems (DiVesta & Walls, 1967); Hilgard et al., 1954). The second instructional contact with an appropriate exemplar seemed to provide little additional information which the students could use to modify their procedures. Perhaps contact with the second appropriate solution simply made the students feel more confident, and therefore less likely to learn from error (Fisher, Note 3). Hilgard et al. reported that even in learning with understanding, the problem solving procedure which was acquired with relatively little contact with error during learning, rapidly degenerated into an algorithmic approach on a transfer task. Those procedures which were more difficult to acquire, and therefore provided a broader experience with error during training, left the students more heuristically oriented. They were more likely to modify their strategic approach whenever they made an identifiable error.

Those whose first contact was with an appropriate solution followed by an erroneous one, might have used the erroneous exemplar to modify some shortcomings in the strategy which had been derived from the first contact. Lack of knowledge due to misunderstanding especially in regard to common errors, might be clarified. This investigator would argue that verbal warnings are not enough (McQuarrie & Grotelueschen, 1971; Logan & Wodtke, Note 6); the student should be provided with an experience of error.

However, the timing of the error experience is critical. The pejorative attitude toward error found in many educational environments tends to make it hard for the student to attempt to use error as a source of knowledge. Koran et al. (1972) observed that it was difficult for the students to derive appropriate behaviors from erroneous exemplars, although some managed to do so. It is likely that exemplars in tutorial presentations contain affective, as well as informational, elements. Although it is not clear what message was being communicated when the first contact was with an erroneous exemplar, it is clear that the message inhibited the students' use of the available information. Although the student was free to move back and forth between the exemplars, the order of contact rather than information content made the difference.

The incorporation of illustrative correct and plausible erroneous solutions enhanced learning, perhaps for some of the following reasons: (1) the sample solutions were models against which the learner could test his own problem solving strategy; (2) they contained information regarding what would have to be done to correct common errors; (3) the presentation of errors made by another learner helped to remove some of the stigma associated with trying and failing. However, when the learner's first contact was with an erroneous solution, they might

have concluded that all attempts to solve complex problems tend to end in failure -- so, why bother to learn? (4) The combination of an exemplar and non-exemplar permit the learner to sort out problem solving strategies, maintaining those which are successful and eliminating those which lead to error (Trabasso & Bower, 1968).

Erroneous Behavior.

Treatment condition appeared to influence the students' general problem solving strategy, rather than production of or response to erroneous and inappropriate behaviors. The treatment groups differed neither in production of rule misapplication and extra steps, nor in the proportion of these behaviors which were cancelled. The major source of variation in the students' problem solving performance was the characteristics which a student brought to the problem solving situation which would relate to their making errors and their ability to recognize and correct them. There was no way to determine how traits such as functional fixedness, lack of persistence, or the acceptance of errors in a complex task, influenced the students' problem solving performance. Further study of these issues will be necessary before it will be possible to determine effective strategies for modifying the students' production of errors and response to errors during problem solving.

Implications for Education and Research

Papert (Note 8) has argued that education should provide

experiences by which learners can acquire techniques for recognizing and correcting errors. Students should be taught how to use error effectively as a source of knowledge and accept error as an expected component of learning. The inclusion of illustrative errors in tutorial presentations may help. An important implication of the results of the present experiment is that the students' initial experience of error must be carefully programmed. Although the subjects in this study were bright, they appeared to require some basic understanding of how problems were to be solved before they could effectively make use of the vicarious error experience.

This finding could be interpreted to suggest that you cannot teach students what not to do, before you teach them what to do. Once students have acquired some basic knowledge and understanding, vicarious experiences of error can help to maximize learning. They can help students become more effective problem solvers by alerting them to both the positive and negative consequences of error. Error experiences early in learning with understanding may prevent the degeneration of newly acquired heuristic strategies, and inhibit the students' tendency to transform these strategies into algorithmic procedures.

Scandura et al. (1974) asserted that researchers should focus directly on problem solving in order to develop effective means of teaching students to think productively.

Ausubel's (1968, pp. 166-168) position differed. He argued that problem solving behaviors were less susceptible to educational manipulations and recommended that psychologists and educators focus on enhancing the acquisition of prerequisite knowledge. The findings of this research conflict with the viewpoint that psychologists should not focus directly on problem solving. Although the influence of treatment condition on problem solving performance was relatively small, it was equivalent to that portion which could be accounted for by prior knowledge. There appears to be little reason for not attacking problem solving directly. Relatively little effort has been expended to either develop or explore various alternatives in teaching problem solving. The lack of effort in this domain has served to limit psychologists' understanding of the sources of student failures, and has also prevented the development of instructional means for remedying them. The results of this research indicate that instructional contact with matched combinations of appropriate and erroneous solutions could be used to enhance the problem solving skills of medical students, computer science majors, chemists, researchers, teachers, etc. Contrasting exemplars are an effective means for communicating the heuristics of problem solving.

Recommendations for Future Research. McKeachie (1974) has indicated that psychologists need to develop laws of human learning that accurately reflect people's ability to conceptualize, relate and remember. An aspect of learning

that must be considered in establishing a new perspective is the role of error. Education and psychology have too long been dominated by the view derived from studies of animals, that error experiences, either vicarious or direct, have no place in the delicate process of shaping behavior. That view is not congruent with our current understanding of how humans learn. Humans tend to make mistakes, especially while they are learning complex tasks; errors are a consequence of the learning process. Psychologists need to learn how errors can be used effectively to teach students to think productively.

The following studies on the role of errors in learning to solve problems are some recommended next steps.

1. Studying Order and Density Effects -- Klausmeier et al.'s (1974) study of concept learning which dealt with information overload, suggested a strategy for exploring some questions raised by the results of the current study. Is there an optimum number of positive exemplars which should be presented before the students are provided with a negative exemplar? If more than two illustrative examples are presented before students are presented with problem solving task, which combinations would be most effective? The first of what could be a series of experiments would start with four illustrative examples of solutions. Treatment conditions would contain the following sequences of exemplars and non-exemplars:

- a. Appropriate-Appropriate-Appropriate-Appropriate
- b. Appropriate-Erroneous-Appropriate-Erroneous
- c. Erroneous-Appropriate-Erroneous-Appropriate
- d. Appropriate-Appropriate-Erroneous-Erroneous
- e. Erroneous-Erroneous-Appropriate-Appropriate
- f. Erroneous-Erroneous-Erroneous-Erroneous

The Appropriate Only sequence would serve as a control for the "information load," and would be needed in validating the results of the current study. The proposed study would serve to extend our understanding in regard to the amount of positive experiences the students must have before they can make effective use of illustrative errors. There are a variety of variations in presentation sequence and volume which could be used in follow-up studies.

2. The Use of Contrasting Exemplars -- Another study could be used to examine the students' ability to relate and remember. It would contrast the influence on learning of simultaneous and sequential presentations. There would be four treatment conditions, two of which would be the current study's Appropriate Only and Appropriate/Erroneous treatments. In the other two treatments the students would be taken step-by-step through both exemplar pairs simultaneously. Having both exemplars in a side-by-side presentation, the

students would not have to turn the pages or rely on their memories in order to see how the solutions differed or how to correct errors.

3. Studying Cognitive and Affective Treatment Influences --

Waters' studies (Waters, 1931; Waters & Ellis, 1931) indicated that instructional contact with errors made the students cautious. It is not clear whether their situational anxiety (State-Anxiety) increased, or they simply became aware of the possibility of error and therefore tended to verify their actions. Tobias (1973) studied State-Anxiety in programmed instruction by introducing a brief five item questionnaire at various points in the instructional sequence. This brief A-State Scale could be administered at various points in the current experiment. It could first be introduced to the students immediately after the Rule application Test. The students could then be matched on both the Rule Application Test and A-State Scale scores, and randomly assigned to treatment groups. The scale could then be re-administered before and after each illustrative exemplar presentation. It would then be possible to determine if the various exemplar types and sequences differentially influenced State-Anxiety. The more extensive exploration of this aspect of treatment influences would require new instructional materials. The new materials would have to include a procedure for verifying

solutions. The students could be taught to verify solutions and a record would be kept to show how often and when the procedure was applied. By incorporating the A-State Scale into the instructional sequence we would be able to determine if contact with error heightened State-Anxiety or just made the students more aware of the possibility of error.

There is almost an infinite variety of issues which merit study in attempting to fully understand the role of errors in learning. We will briefly sketch out a few of the more prominent topics.

1. The role of error in rule learning. The Language and Rules Text could be revised so that the students would be presented with illustrative examples of correct and incorrect applications of a rule.
2. Order effects in rule learning. The proposed revision which was described above could also vary the presentation sequence of the positive and negative exemplars.
3. Cross-cultural attitudes toward error. Although the differences between the New York and Puerto Rican students' performances were not significant, there still is a strong possibility that different cultural attitudes toward error would make a difference in how treatment condition would influence performance. The New York and

Puerto Rican students in this experiment had many common educational experiences. The schools in Puerto Rico used text and teachers from the United States. If the teachers and texts had been of Puerto Rican origin, would the results have been the same?

This research has opened the door to a series of research issues relating to the role of error in learning. It has demonstrated that, under some circumstances, an experience with error can enhance learning. Further research is needed to elucidate the conditions under which exposure to error will facilitate or inhibit learning. Psychologists need to understand more fully how students can learn to use error effectively, what processes are involved, and what instructional experience can be used to transmit to students the appropriate attitude toward error.

APPENDIX

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A-1

Appendix AThe Probe Language Defined in Backus Normal Form

Backus Normal Form (BNF) is a meta-language used in formal definition of linguistic productions (see Newell & Simon, 1972, p. 45). The symbols of the language have specified meanings:

<u>Symbol</u>	<u>Interpretation</u>
[]	Brackets enclose "identifiers."
::=	Translates as "is defined as."
/	Translates as "or."

The definitions of an identifier can either be a string of identifiers and/or symbols concatenated together, or a single identifier or symbol. Each identifier definition is separated by the symbol "/". For example, the definition of the identifier LETTER is:

[LETTER] ::= A/B/C/D/.... /Z

The interpretation of this expression is that LETTER is defined as the symbol A or B or C or D or

The BNF definition of the Probe Language is as follows.

```
[LETTER] ::= A/B/C/D/.... /Z
[CONNECTOR] ::= %/√/∩
[SIGN] ::= ~
[PUNCTUATION] ::= (/)
[VARIABLE] ::= [LETTER]/[SIGN] [LETTER]
[COMPLEX VARIABLE] ::= [VARIABLE] [CONNECTOR] [VARIABLE]
[EXPRESSION] ::= [VARIABLE]/[SIGN] ([SIGN] [LETTER])/[SIGN]
                ([COMPLEX VARIABLE])
```

A-2

```
[VALID PROBE EXPRESSION] ::= [COMPLEX VARIABLE]/[EXPRESSION]/  
[EXPRESSION] [CONNECTOR] [EXPRESSION]/  
([COMPLEX VARIABLE]) [CONNECTOR] [EX-  
PRESSION]/  
[EXPRESSION] [CONNECTOR] ([COMPLEX  
VARIABLE])/  
([COMPLEX VARIABLE]) [CONNECTOR]  
([COMPLEX VARIABLE])
```

APPENDIX B.1

The Probe Programmed Text, Language Portion

Probe Text# 3713

PROBE

A

Programmed-Instruction Mini-Course

for studying

Problem Solving

**Derived from materials created
by O.K. Moore and S.B. Anderson*

Stanley Fisher
Columbia University

1975

B.1-0

B.1-1
PROBE

I1.0

A Programmed Text

For most of us, problem solving is a daily activity. We are faced with a problem each time we want to accomplish something and do not know what sequence of actions will get us to our goal. Solving a riddle, deciding who to invite to a party, or proving a geometric theorem, are all examples of situations in which we use our problem solving skills.

More and more, symbolic languages are being used to represent complicated problem situations. The symbols are a shorthand notation for representing complex ideas, objects, and operations. These symbolic languages have become popular because they are precise and easier to manipulate than the natural languages such as English. Moreover, any problem which can be formulated in a symbolic language can be manipulated both by humans and by computers.

In the lessons which follow, you will be taught the symbolic language called PROBE. When you complete your study of this text, you will be tested on your ability to solve symbolic problems. Before you begin, please check to see that you have completed all of the blanks at the top of the response booklet.

Instructions for Using This Text

The materials in this text are presented in a form which is known as programmed instruction. The material to be learned is presented as a

I2.0

series of mini-lessons or frames, each of which requires you to answer one or more questions. The correct answers to these questions are always found on the next page. Please write you answers in the response booklet. Do not write in the text.

In order to get the maximum benefit from the programmed text, write your answer first and then look at the correct answer. Do not change your answer at any time.

Please follow this sequence of actions while studying the programmed materials:

1. Turn to Frame 1 in this text. (There is a "F1.0" in the upper-right-hand corner of that page.)
2. Read Frame 1.
3. Write your answer to Frame 1 next to F1.0 in the response booklet.
4. After writing your response, turn the page to check your answer. Notice that the page containing the answer to Frame 1 has an "AF1.0" (Answer Frame 1.0) in the upper-right-hand corner.
5. Now read Frame 2.
6. Write your answer next to F2.0 in the response booklet.
7. Turn the page and check your answer to Frame 2 on page AF2.0
8. Continue the procedure until you are directed to stop.
9. If you have any questions about how to proceed, raise your hand and a proctor will try to answer them.

B.1-2

F1.0

Write the time, to the nearest minute, in hours and minutes:

:

Hours : Minutes

AF1.0

The correct answer to this frame would be the correct time, and the answer would normally be recorded here. Of course, there was no way to know in advance what that would be, so the correct answer could not be recorded. Occasionally, you will be asked to write a response and you will not receive any correct answer. However, in most cases, such as the next frame, the correct answer will be provided.

No attempt has been made to trick you on any of the questions. While some may be easy and some may be hard, a careful reading of the frame should lead you to the correct answer. Look at the next page and try the next frame.

B.1-3

F2.0

PROBE is a symbolic problem-solving language. Probe expressions look like this:

 $R \vee T$ $S \supset P$ T $\sim P$ $\sim R \circ Q$ $T \vee (S \circ P)$

The symbols shown above are called Valid Probe Expressions, which we will abbreviate as VPEs. (The space indicates that you are to fill in the information which is missing.)

AF2.0

VPEs

B.1-4

F3.0

One way to form a VPE is to place a single capital letter on each side of a Connector. Some examples of that type of VPE are shown below:

R \supset SP \circ TS \vee TM \vee NG \circ LC \supset S

Write the symbols for the 3 Probe connectors, which are:

- a) the Wedge _____
 b) the Dot _____
 c) the Horseshoe _____

AF3.0

- a) the Wedge \vee
 b) the Dot \circ
 c) the Horseshoe \supset

B.1-5

F4.0

Look at this expression: $R \bullet S$

Read it as " R dot S."

Write a VPE using the capital letters

R and T and:

- a) the Horseshoe connector _____
- b) the Wedge connector _____
- c) the Dot connector _____

AF4.0

a) $R \supset T$

b) $R \vee T$

c) $R \bullet T$

B.1-6

F5.0

We know that one way to form a VPE is by placing a capital letter on both sides of a connector. Capital letters standing alone are also VPEs.

Which of the following expressions are VPEs?

T
S ◦ T
S ◡
◡
R

AF5.0

T
S ◦ T
R

B.1-7

F6.0

Another way to form a VPE is by placing a VPE on both sides of a connector.

Form a VPE from the VPEs R and (S • T) using the Wedge connector.

AF6.0

R v (S • T)

B.1-8

F7.0

In Probe, parentheses, (), are needed whenever a VPE appearing on either side of a connector has more than one letter. For example, in the VPE:

$$R \supseteq (T \vee S)$$

the VPE $T \vee S$ has to be enclosed in parentheses.

Form a VPE from the VPEs T and $R \supseteq S$ using the Dot connector.

AF7.0

$$T \cdot (R \supseteq S)$$

B.1-9

F8.0

Form a VPE from the VPEs $Q \circ T$ and $R \circ T$
using the Wedge connector.

AF8.0

$(Q \circ T) \vee (R \circ T)$

B.1-10

F9.0

The Major Connector

Whenever a VPE contains more than one connector, one of the connectors is considered the major connector. The connector which ties together the whole expression is the major connector. In the expression $T \vee (R \supset P)$ the Wedge is the major connector.

Which are the major connectors in the VPEs shown below?

a) $(Q \vee T) \supset (R \cdot P)$ _____

b) $(Q \supset P) \cdot Q$ _____

AF9.0

a) $(Q \vee T) \supset (R \cdot P)$: \supset or the Horseshoe

b) $(Q \supset P) \cdot Q$: \cdot or the Dot

F10.0

B.1-11

When we use the term "the whole expression" we mean all of the symbols on a single line which form a VPE. Whole expressions can usually be separated into parts, where each part is also a VPE. For example, we can separate the whole expression $(S \bullet Q) \supset R$ into two parts:

1. $(S \bullet Q)$ and 2. R .

The expression $(S \vee T) \vee (P \supset Q)$ can also be separated into two parts:

1. _____ and 2. _____ .

AF10.0

1. $(S \vee T)$ and 2. $(P \supset Q)$.

B.1-12

F11.0

Negative and Positive Expressions

A VPE is considered negative whenever the Tilde symbol, \sim , appears to the left of the VPE, and is applicable to the whole expression. For example, the Tilde to the left of the parenthesis in the expression $\sim (R \vee T)$ means that the whole expression $(R \vee T)$ is considered _____.

AF 11.0

negative

F12.0

B.1-13

For an expression to be considered as negative, the Tilde must apply to the whole expression and not just to one of the parts. For example, in the expression $\sim R \vee T$ the Tilde is only applicable to one of the parts. Although the $\sim R$ is negative, the whole expression $\sim R \vee T$ is considered positive. Any expression which is not negative is positive. Next to each of the following expressions, indicate whether the whole expression is negative or positive.

- a) $\sim R$: _____
 b) R : _____
 c) $\sim(S \vee T)$: _____
 d) $\sim S \vee T$: _____

AF12.0

- a) $\sim R$: negative
 b) R : positive
 c) $\sim(S \vee T)$: negative
 d) $\sim S \vee T$: positive

B.1-14

F13.0

A VPE and its Tilde, \sim , are always treated as a single unit.
 That is why the VPEs R and $\sim R$ are not the same. If we
 were to separate the expression

$$S \supset \sim T$$

into two parts, each of which is a single unit VPE, one of them
 would be S and the other would be _____ .

AF13.0

$$\underline{\sim T}$$

B.1-15

F14.0

To change the sign of a VPE from positive to negative, we place a Tilde, \sim , to the left of the expression and enclose the VPE within parentheses. Parentheses should not be used when the VPE is a single capital letter.

Change the positive expressions shown below to negative expressions by changing the sign of the whole expression.

- a) R : _____
 b) $R \vee T$: _____
 c) $Q \supset \sim R$: _____
 d) $\sim Q \vee T$: _____

AF14.0

- a) $\sim R$
 b) $\sim(R \vee T)$
 c) $\sim(Q \supset \sim R)$
 d) $\sim(\sim Q \vee T)$

B.1-16

F15.0

To change the sign of the whole expression from negative to positive the whole process is reversed. The Tilde sign is removed, along with any parentheses that are no longer needed.

Change the following negative expressions to positive expressions:

a) $\sim (R \vee \sim T)$: _____

b) $\sim R$: _____

c) $\sim (S \vee (R \supset T))$: _____

AF15.0

a) $R \vee \sim T$

b) R

c) $S \vee (R \supset T)$

B.1-17

F16.0

When we talk about changing the sign of a VPE we mean that negative VPEs become positive (remove the Tilde) and positive VPEs become _____.

AF16.0

negative

B.1-18

F17.0

Congratulations! Now that you know how to form Valid Probe Expressions, you are ready to study the Probe Recoding Rules.

Please turn to the next
presentation frame.

AF17.0

Not here, on the next page.

B.1-19

F18.0

Look at PANEL I in The PROBE Panelbook. Those are the Probe Recoding Rules. The rules tell you how to form new VPEs from old VPEs. They tell you that when you have an expression which is like the expression to the left of the arrow, you may recode that expression so that it looks like the expression to the right of the arrow. However, some of the arrows are double-headed. That tells you that any recoding may take place in both directions.

Write the numbers of the rules which permit recoding, going only from left to right. _____ . What are the numbers of the rules which permit recoding in both directions? _____

AF18.0

Rules 3 and 4 permit recoding only from left to right.

Rules 1 and 2 permit recordings in both directions.

B.1-20

F19.0

The letters A, B and C, which appear in the rules, stand for the VPEs which are the parts of a whole expression. For example, if we look at the expression $\sim R \vee (Q \circ S)$ according to Rule 1:

$$\begin{aligned} \text{(Rule 1)} \quad & \sim A \vee B \\ & \sim R \vee (Q \circ S) \end{aligned}$$

then $\sim R$ would be the A part and $(Q \circ S)$ would be the B part.

If we look at the same expression according to Rule 2:

$$\begin{aligned} \text{(Rule 2)} \quad & A \vee (B \circ C) \\ & \sim R \vee (Q \circ S) \end{aligned}$$

then $\sim R$ would be the A part, and _____ would be the B part and _____ would be the C part.

AF19.0

Q would be the B part

S would be the C part

B.1-21

F20.0

In applying a rule to an expression, you have to decide which part of the expression will be the same part as the letter in the rule. Then the VPE which stands for that letter is always put in place of that letter in applying the rule. For example, if we were looking at the expression $(X \vee Y) \circ (R \supset S)$, according to Rule 3, Section 3.1: $A \circ B \rightarrow A$, and decided that $(X \vee Y)$ was the A part and $(R \supset S)$ was the B part, which VPE would we put in place of the letter A on the right hand side of the arrow?

AF20.0

 $(X \vee Y)$

B.1-22

F21.0

Look at Rule 1. Rule 1 states that if you have an expression that is like $\sim A \vee B$, then you can apply Rule 1 to form a new expression which is like $A \supset B$. Rule 1 also states that if you have an expression that is like $A \supset B$ you can recode that expression so that it looks like _____.

AF21.0

 $\sim A \vee B$

B.1-23

F22.0

In the expression $\sim (X \supset Y) \vee (Q \supset S)$, according to Rule 1,
which is the A part? _____. Which is the B part?
_____.

Remember what we said in Frame 13: a VPE and its sign are
always treated as a single unit.

AF22.0

The A part is $\sim (X \supset Y)$

The B part is $(Q \supset S)$

B.1-24

F23.0

Look at Rule 1 again. Rule 1 states that when we change the major connector, which connects the whole expression, from a Wedge, \vee , to a Horseshoe, \supset , we must change the sign of the A part of the expression. If the sign of the A part is negative, we must change the sign of the A part to _____ .

AF23.0

positive

B.1-25

F24.0

Form new VPEs by applying Rule 1 to the following expressions:

a) $\sim R \vee T$: _____

b) $S \vee R$: _____

AF24.0

a) $R \supset T$

b) $\sim S \supset R$

B.1-26

F25.0

Rule 1 also states that when we change the major connector from a Horseshoe to a _____ we must also change the sign of the A part of the expression.

AF25.0

Wedge or V

B.1-27

F26.0

Apply Rule 1 to these expressions to form new expressions:

a) $R \supset S$: _____

b) $\sim Q \supset R$: _____

AF26.0

a) $\sim R \vee S$

b) $Q \vee R$

B.1-28

F27.0

Rule 1 may be applied to VPEs such as those written on Lines 1 and 2 below. In problem-solving, VPEs are almost always placed on numbered lines. You will soon find out why this is done.

Line

1	$\sim T \vee (R \circ Q)$
2	$(R \circ S) \supset Q$

Apply Rule 1 to Line 1 and write the expression that is formed on Line 3.

3	_____
---	-------

Apply Rule 1 to Line 2 and write the expression that is formed on Line 4.

4	_____
---	-------

AF27.0

3	$T \supset (R \circ Q)$
4	$\sim (R \circ S) \vee Q$

B.1-29

F28.0

If the expressions we wrote in Lines 3 and 4 were part of a solution to a Probe problem, we would have to fill in 2 other items on each line. We would have to write the number of the rule which was applied to form the expression appearing on that line, and the number of the line which contained the expression to which the rule was applied. For example, because Rule 1 was applied to Line 1 to form Line 3, Line 3 would look like this:

Line		Rule #	Line #
<u>3</u>	<u>$T \supset (R \circ Q)$</u>	<u>1</u>	<u>1</u>

What would you fill in under Rule # and Line # for Line 4?

<u>4</u>	<u>$\sim (R \circ S) \vee Q$</u>	_____	_____
----------	---	-------	-------

AF28.0

<u>4</u>	<u>$\sim (R \circ S) \vee Q$</u>	<u>1</u>	<u>2</u>
----------	---	----------	----------

B.1-30

F29.0

Rule 2. Rule 2 states that if you have a whole expression in which the major connector is a Wedge, and the A part is connected to a VPE which has a B part connected to a C part by a Dot connector, then you can change the major connector to a Dot by forming the expression $(A \vee B) \cdot (A \vee C)$.

Rule 2 also states that if you have an expression that is like $(A \vee B) \cdot (A \vee C)$ you can apply Rule 2 and form a new expression which is like _____ .

AF29.0

$A \vee (B \cdot C)$

B.1-31

F30.0

Rule 2 may be applied to expressions such as:

$$\begin{array}{l} \underline{1} \quad \underline{T \vee (S \cdot Q)} \\ \underline{2} \quad \underline{(Q \vee \sim P) \cdot (Q \vee T)} \end{array}$$

In the expression on Line 1, according to Rule 2, which is the:

A part? _____
B part? _____
C part? _____

AF30.0

The A part is T .

The B part is S .

The C part is Q .

B.1-32

F31.0

In the expression on Line 2:

$$\underline{2} \quad \underline{(Q \vee \sim P) \cdot (Q \vee T)}$$

if the A part is Q, what is the B part? _____

What is the C part? _____

AF31.0

The B part is $\sim P$.

The C part is T .

B.1-35

F34.0

Rule 3. Rule 3 has two sections, 3.1 and 3.2. In order to apply either section of Rule 3, the total expression must be positive, and the Dot, \bullet , must be the major connector.

Section 3.1 states that when you have a positive expression which looks like $\underline{A} \bullet \underline{B}$ then, by applying Rule 3, you can form a new line which is just the A part of the expression.

In the expression $(R \supset S) \bullet (T \supset R)$, which is the A part of the expression?

AF34.0

$R \supset S$

B.1-36

F35.0

Remember, in Probe, when parentheses are no longer needed, they may be removed. If we created a new line by applying Rule 3, Section 3.1 to the expression $(R \supset S) \bullet (T \supset R)$, the expression we would write would be

AF35.0

R \supset S

B.1-37

F36.0

Section 3.2 of Rule 3 states that if you have an expression in the form $A \circ B$, then by applying Section 3.2 of Rule 3, you can form a new expression from the part.

AF36.0

B part

B.1-38

F37.0

In solving Probe problems, new lines may be formed by applying Rule 3, either Section 3.1 or Section 3.2, to any previous line or lines. The values found on Line 3, in the example below, tell us that Rule 3, Section 3.1, was applied to Line 2 to form Line 3.

Line		Rule #	Line #
<u>1</u>	<u>S • T</u>		
<u>2</u>	<u>(Q ⇒ S) • R</u>		
<u>3</u>	<u>Q ⇒ S</u>	<u>3</u>	<u>2</u>
<u>4</u>	<u> </u>	<u> </u>	<u> </u>

Form a new expression on Line 4 by applying Rule 3, Section 3.2 to Line 1. Fill in all three blank spaces.

AF37.0

<u>4</u>	<u> T </u>	<u>3</u>	<u>1</u>
----------	------------------------------	----------	----------

B.1-39

F38.0

Rule 4. Rule 4 is very different. All the previous rules could be applied to only one line at a time in order to form a new expression. Rule 4 may only be applied when the VPEs which look like $A \supset B$ and A are on _____.
(the same line/different lines)

AF38.0

Rule 4 may be applied only when the VPEs which look like $A \supset B$ and A are on different lines.

B.1-40

F39.0

In order to apply Rule 4, the expressions which look like $A \supset B$ and A must appear on two separate lines and the A part of both lines must be exactly the same. In the example below which VPE is like the A part of the expression in Rule 4?

2 R \supset Q

5 R

AF39.0

R

B.1-41

F40.0

Rule 4 may be applied as long as there are any two lines,
 in any sequence, which look like $A \supset B$ and A . For example,
 Rule 4 can be applied to the lines shown below.

1	$R \circ T$
6	$(R \circ T) \supset R$

Which line contains the expression which looks like the
first line of Rule 4?

AF40.0

Line 6

B.1-42

F41.0

The following lines are part of a problem proof.

<u>7</u>	<u>R \supset T</u>
<u>8</u>	<u>R \cdot T</u>
<u>9</u>	<u>\sim R</u>
<u>10</u>	<u>R</u>

Record the line numbers and the expressions to which Rule 4
may be applied.

AF41.0

<u>7</u>	<u>R \supset T</u>
<u>10</u>	<u>R</u>

B.1-43

F42.0

Apply Rule 4 to Lines 7 and 10 shown above, and write the expression which is formed on Line 11. Fill in all of the information which would be needed in a problem solution.

Line	Rule #	Line #
<u>11</u>	_____	_____

AF42.0

Line	Rule #	Line #
<u>11</u>	<u>T</u>	<u>4</u> <u>7, 10</u>

B.1-44

F43.0

Congratulations! You have just completed your study of the Probe rules. This is almost the end of the first session. However, before you leave, you will be tested on your knowledge of the rules. This will be an open-book test, as are all tests in this mini-course. You are free to use your Probe text and Panelbook. As soon as you are ready, write the time, to the nearest minute, in hours and minutes, next to F43 in the response booklet.

Hours : Minutes

When you have done this, raise your hand. The proctor will bring you the Rules Application Test. Take all the time you need to complete this test. Next session you will be taught to solve Probe problems.

AF43.0

Good Luck!

See you at the next session. Remember to check your appointment time for that session.

APPENDIX B.2

The Probe Panelbook

PROBE*

PANELBOOK

for

The PROBE Text

Stanley Fisher
Columbia University
1975

PANEL I

PROBE Recoding Rules

<u>RULE #</u>	<u>RULE</u>
1	$\sim A \vee B \longleftrightarrow A \supset B$
2	$A \vee (B \circ C) \longleftrightarrow (A \vee B) \circ (A \vee C)$
3	3.1 $A \circ B \longrightarrow A$ 3.2 $A \circ B \longrightarrow B$ Use only with <u>positive</u> <u>total expressions</u> and <u>major Dot connectors</u> . <u>No sign changes.</u>
4	$A \supset B \longrightarrow B$ A The expressions <u>A</u> and <u>A \supset B</u> represent two separate lines. The <u>total expression</u> <u>A \supset B</u> must be <u>positive</u> .

PANEL II

A Sample PROBE Problem

<u>Sector I</u>		<u>Sector II</u>	
<u>Line</u>			
1	P		
2	$\sim P \vee (R \circ S)$		R
<u>Sector III</u>		<u>Sector IV</u>	
		<u>Rule#</u>	<u>Line#</u>
3			
4			
5			
6			
7			
8			
9			
10			
11			
12			
13			
14			
15			
16			

B.2-1

APPENDIX B.3

The Rule Application Test

Name _____ B.3-1 Probe Text # _____

PROBE

Rule Application Test

This booklet contains 16 rule application problems. You can use any of the materials you have been given and work on each problem as long as you like. For each problem you will be given a rule number and one or more expressions. Here is a sample problem:

Sample Problem I

Rule# 1	<u> $A \vee (T \circ R)$ </u>
Rule# _____	_____

If you think that the rule which has been given, in this case Rule 1, can be applied to the expression(s), apply the rule and write the new expression which is formed in the space provided. If you think that the rule cannot be applied to the given expression(s), select a rule that can be applied. Record the number of that rule in the space provided and write the expression which is formed when that rule is applied. Try to solve Sample Problem I.

Rule 1 can be applied to the given expression; therefore the correct answer looks like this:

Rule# _____ $\sim A \supset (T \circ R)$.

The space for the rule number was not filled in because the given rule was applied.

Let's try another example:

Sample Problem II

Rule# 3.2	<u> $T \supset R$ </u>
	<u> T </u>
Rule# _____	_____

The only rule which can be applied to the given expressions is Rule 4. Therefore, the correct answer to this problem is:

Rule# 4 R

Remember, feel free to use any of the materials and work as long as you like. Before you start the problems, write the time in the Start Box, below. When you finish, write your finishing time in the Stop Box. As soon as you record the stop time, raise your hand. The proctor will come and collect your materials and you will be free to work quietly on anything you want to. However, please remain in your seat so that others who are still working will not be disturbed. Good Luck! You can start as soon as you record the time.

Start Time

	:	
Hours:		Min.

Stop Time

	:	
Hours:		Min.

B.3-2
RULE APPLICATION PROBLEMS

- | | |
|---|---|
| <p>1. Rule# 3.1 <u>$\sim(S \circ R) \vee T$</u>
Rule# _____
**</p> | <p>9. Rule# 2 <u>$(S \vee R) \supset C$</u>
<u>$S \vee R$</u>
Rule# _____
**</p> |
| <p>2. Rule# 4 <u>$\sim R$</u>
<u>$\sim R \supset (S \vee T)$</u>
Rule# _____
**</p> | <p>10. Rule# 3.2 <u>$(S \supset T) \circ \sim R$</u>
Rule# _____
**</p> |
| <p>3. Rule# 4 <u>$(P \vee R) \circ (P \vee S)$</u>
Rule# _____
**</p> | <p>11. Rule# 1 <u>$\sim R \circ (Q \supset R)$</u>
Rule# _____
**</p> |
| <p>4. Rule# 2 <u>$\sim(Q \circ R) \vee (Q \circ C)$</u>
Rule# _____
**</p> | <p>12. Rule# 1 <u>$(A \vee B) \circ (R \supset T)$</u>
Rule# _____
**</p> |
| <p>5. Rule# 3.1 <u>$(S \vee R) \circ (S \vee T)$</u>
Rule# _____
**</p> | <p>13. Rule# 2 <u>$(S \vee R) \circ (S \vee Q)$</u>
Rule# _____
**</p> |
| <p>6. Rule# 4 <u>$Q \vee (S \circ T)$</u>
Rule# _____
**</p> | <p>14. Rule# 3.2 <u>$\sim Q \supset (T \circ P)$</u>
<u>$\sim Q$</u>
Rule# _____
**</p> |
| <p>7. Rule# 2 <u>$R \vee (S \circ T)$</u>
Rule# _____
**</p> | <p>15. Rule# 4 <u>$P \vee T$</u>
<u>$(P \vee T) \supset \sim R$</u>
Rule# _____
**</p> |
| <p>8. Rule# 1 <u>$(P \circ R) \vee \sim R$</u>
Rule# _____
**</p> | <p>16. Rule# 2 <u>$T \supset (R \circ S)$</u>
Rule# _____
**</p> |

APPENDIX C.1

Introduction to Problem Solving

C.1-1

F44.0

Welcome back for the second and last session of PROBE. This session you will be learning how to solve Probe problems. Before turning the page to the next presentation frame, please record the time, to the nearest minute, in hours and minutes, in the space next to F44 on the response booklet.

_____ : _____
Hours : Minutes

AF44.0

Not here, the next page.

C.1-2

F45.0

PROBE Problems

Turn to Panel II in the PROBE PANELBOOK. This is an example of the types of problems you will be asked to solve. Notice that each sector of the problem outline is numbered with a Roman Numeral. How many sectors are there in the problem solving form?

AF45.0

Four (I, II, III and IV)

C.1-3

F46.0

Sectors I and II contain the statement of the problem. Sector I contains the Valid Probe Expressions you are given at the beginning of the problem. Sector II contains the answer. Your job is to apply rules to the VPEs and eventually end up with the answer that is shown in Sector II. You do this by performing a series of steps and writing each step down as you go along. Each step is an application of one of the rules to the prior expressions (usually just one, sometimes two). You start with the expressions in Sector I and end when the last line contains the same expression as that in Sector II. In solving the sample problem shown in Panel II, we would have to find the series of rule applications which eventually forms the VPE _____.

AF46.0

R

C.1-4

F47.0

For each problem, you will have to show how the VPE in Sector II can be formed from the VPEs in Sector I. The expression which is formed at each step of the solution will be recorded on the next available line in Sector _____. The number of the rule which was applied to create the new expression and the number of the line containing the expression to which the rule was applied, will be recorded in Sector _____, on the same line as the expression.

AF47.0

The new VPE is recorded in Sector III.

The Rule # and the Line # are recorded in Sector IV.

F48.0

C.1-5

The problem solver normally writes q.e.d., which in Latin means "it has been shown," on the final line of the solution. The q.e.d. is written to the right of the Line # in Sector IV and it indicates that the problem is solved. For example, the last line of a completed solution might look like this:

7 R 3 6 q.e.d.

The VPE which appears on the last line of a completed solution should be exactly like the VPE in which sector of the problem solving form? _____

AF48.0

Sector II

C.1-6

F49.0

Now you are ready to learn how to solve Probe problems. One of the best ways to learn is to see what other students have done. You will be shown examples of what some students like you did on one of their homework assignments.

Work through each example carefully. After you have finished studying these examples, you will be given Probe problems to solve.

AF49.0

It's now up to you.

APPENDIX C.2

The Probe Programmed Text,
Appropriate Only Treatment

C.2-1

F50.0

<u>Sector I</u>		<u>Sector II</u>	
Line			
1	$(\sim P \vee R) \circ (\sim P \vee Q)$		
2	$(\sim R \vee P) \circ (\sim R \circ R)$		Q
<u>Sector III</u>		<u>Sector IV</u>	
		Rule #	Line #
3		3	1
4			
5			
6			
7			
8			
9			
10			
11			

Andy started his solution to the problem by filling in Line 3 as shown.

Write the expression that Andy formed when he applied Section 3.2 of Rule 3 to Line 1.

AF50.0

 $\sim P \vee Q$

C.2-2

F51.0

Can Rule 1 be applied to
Line 3 to form the
expression that Andy
wrote on Line 4?

<u>Sector I</u>		<u>Sector II</u>	
Line			
1	$(\neg P \vee R) \bullet (\neg P \vee Q)$		
2	$(\neg R \vee P) \bullet (\neg R \bullet R)$		Q
<u>Sector III</u>		<u>Sector IV</u>	
		Rule #	Line #
3	$\sim P \vee Q$	3	1
4	$P = Q$	1	3
5			
6			
7			
8			
9			
10			
11			

AF51.0

Yes

C.2-3

F52.0

What Section of Rule 3, Section 3.1 or Section 3.2, would have to be applied to Line 2 to form the expression on Line 5?

<u>Sector I</u>		<u>Sector II</u>	
Line			
1	$(\sim P \vee R) \cdot (\sim P \vee Q)$		
2	$(\sim R \vee P) \cdot (\sim R \cdot R)$		Q
<u>Sector III</u>		<u>Sector IV</u>	
		Rule #	Line #
3	$\sim P \vee Q$	3	1
4	$P \supset Q$	1	3
5	$\sim R \vee P$	3	2
6			
7			
8			
9			
10			
11			

AF52.0

Section 3.1

C.2-4

F53.0

Look at Line 6. According to the Rule # and Line # shown, what connector should be filled in to complete the expression?

<u>Sector I</u>		<u>Sector II</u>	
Line			
1	$(\sim P \vee R) \bullet (\sim P \vee Q)$		
2	$(\sim R \vee P) \bullet (\sim R \bullet R)$		Q
<u>Sector III</u>		<u>Sector IV</u>	
Line		Rule #	Line #
3	$\sim P \vee Q$	3	1
4	$P \supset Q$	1	3
5	$\sim R \vee P$	3	2
6	$R \quad P$	1	5
7			
8			
9			
10			
11			

AF53.0

the Horseshoe : \supset

C.2-5

F54.0

An "extra step" is a line which is not needed to solve the problem. The expression on that line cannot be used to form a solution. Is the expression on Line 6 an extra or a necessary step?

Before you attempt to answer this question, try to figure out how the final expression, which is Q, can be formed.

<u>Sector I</u>		<u>Sector II</u>	
Line			
1	$(\sim P \vee R) \circ (\sim P \vee Q)$		
2	$(\sim R \vee P) \circ (\sim R \circ R)$		Q
<u>Sector III</u>		<u>Sector IV</u>	
		Rule #	Line #
3	$\sim P \vee Q$	3	1
4	$P \supset Q$	1	3
5	$\sim R \vee P$	3	2
6	$R \supset P$	1	5
7			
8			
9			
10			
11			

AF54.0

It is necessary.

C.2-6

F55.0

Is the expression on
Line 7 extra or
necessary?

<u>Sector I</u>		<u>Sector II</u>	
Line			
1	$(\sim P \vee R) \circ (\sim P \vee Q)$		
2	$(\sim R \vee P) \circ (\sim R \circ R)$		Q
<u>Sector III</u>		<u>Sector IV</u>	
		Rule #	Line #
3	$\sim P \vee Q$	3	1
4	$P \supset Q$	1	3
5	$\sim R \vee P$	3	2
6	$R \supset P$	1	5
7	$\sim R \circ R$	3	2
8			
9			
10			
11			

AF55.0

Necessary

C.2-7

F56.0

What is the major
connector in the
expression on Line 7?

Can Rule 3 be applied
to the expression on
Line 7 to form Line 8?

<u>Sector I</u>		<u>Sector II</u>	
Line			
<u>1</u>	$(\sim P \vee R) \circ (\sim P \vee Q)$		
<u>2</u>	$(\sim R \vee P) \circ (\sim R \circ R)$		Q
<u>Sector III</u>		<u>Sector IV</u>	
		Rule #	Line #
<u>3</u>	$\sim P \vee Q$	<u>3</u>	<u>1</u>
<u>4</u>	$P \supset Q$	<u>1</u>	<u>3</u>
<u>5</u>	$\sim R \vee P$	<u>3</u>	<u>2</u>
<u>6</u>	$R \supset P$	<u>1</u>	<u>5</u>
<u>7</u>	$\sim R \circ R$	<u>3</u>	<u>2</u>
<u>8</u>	R	<u>3</u>	<u>7</u>
<u>9</u>			
<u>10</u>			
<u>11</u>			

AF56.0

The Dot : \circ

Yes

C.2-8

F57.0

If you want to apply Rule 4, using Line 4 as the first line and Line 9 as the second line, what VPE will you have to form on Line 9?

<u>Sector I</u>		<u>Sector II</u>	
Line			
<u>1</u>	$(\sim P \vee R) \bullet (\sim P \vee Q)$		
<u>2</u>	$(\sim R \vee P) \bullet (\sim R \bullet R)$		Q
<u>Sector III</u>		<u>Sector IV</u>	
		Rule #	Line #
<u>3</u>	$\sim P \vee Q$	<u>3</u>	<u>1</u>
<u>4</u>	$P \supset Q$	<u>1</u>	<u>3</u>
<u>5</u>	$\sim R \vee P$	<u>3</u>	<u>2</u>
<u>6</u>	$R \supset P$	<u>1</u>	<u>5</u>
<u>7</u>	$\sim R \bullet R$	<u>3</u>	<u>2</u>
<u>8</u>	R	<u>3</u>	<u>7</u>
<u>9</u>		<u>4</u>	<u>6, 8</u>
<u>10</u>			
<u>11</u>			

AF57.0

P

C.2-9

F58.0

<u>Sector I</u>		<u>Sector II</u>	
Line			
1	$(\sim P \vee R) \circ (\sim P \vee Q)$		
2	$(\sim R \vee P) \circ (\sim R \circ R)$		Q
<u>Sector III</u>		<u>Sector IV</u>	
		Rule #	Line #
3	$\sim P \vee Q$	3	1
4	$P \supset Q$	1	3
5	$\sim R \vee P$	3	2
6	$R \supset P$	1	5
7	$\sim R \circ R$	3	2
8	R	3	7
9	P	4	6, 8
10	Q	4	4, 9 <i>q.e.d.</i>
11			

Is the VPE, Q, which
is on Line 10, the
correct final expression
for this problem?

AF58.0

Yes

C.2-10

F59.0

In this solution,
how many times did
Andy incorrectly
apply the rules?

<u>Sector I</u>		<u>Sector II</u>	
Line			
1	$(\sim P \vee R) \circ (\sim P \vee Q)$		
2	$(\sim R \vee P) \circ (\sim R \circ R)$		Q
<u>Sector III</u>		<u>Sector IV</u>	
		Rule #	Line #
3	$\sim P \vee Q$	3	1
4	$P = Q$	1	3
5	$\sim R \vee P$	3	2
6	$R = P$	1	5
7	$\sim R \circ R$	3	2
8	R	3	7
9	P	4	6, 8
10	Q	4	4, 9 <i>g.s.d.</i>
11			

AF59.0

Zero; None

C.2-11

F60.0

Are there any extra
steps in this solution,
that is, steps in which
the expressions were
correctly formed but
which were not needed?

(yes/no)

<u>Sector I</u>		<u>Sector II</u>	
Line			
1	$(\neg P \vee R) \cdot (\neg P \vee Q)$		
2	$(\neg R \vee P) \cdot (\neg R \cdot R)$		Q
<u>Sector III</u>		<u>Sector IV</u>	
		Rule #	Line #
3	$\sim P \vee Q$	3	1
4	$P \supset Q$	1	3
5	$\sim R \vee P$	3	2
6	$R \supset P$	1	5
7	$\sim R \cdot R$	3	2
8	R	3	7
9	P	4	6, 8
10	Q	4	4, 9 <i>g.e.d.</i>
11			

AF60.0

No

C.2-12

F61.0

After the teacher
checked the solution,
he told Andy it was

(correct/incorrect)

<u>Sector I</u>		<u>Sector II</u>	
Line			
1	$(\sim P \vee R) \circ (\sim P \vee Q)$		
2	$(\sim R \vee P) \circ (\sim R \circ R)$		Q
<u>Sector III</u>		<u>Sector IV</u>	
		Rule #	Line #
3	$\sim P \vee Q$	3	1
4	$P \supset Q$	1	3
5	$\sim R \vee P$	3	2
6	$R \supset P$	1	5
7	$\sim R \circ R$	3	2
8	R	3	7
9	P	4	6, 8
10	Q	4	4, 9 g.l.d.
11			

AF61.0

Correct

C.2-13

F62.0

John worked on the same problem, but came up with a different solution. Fill in the major connector that John will write when he completes the new expression on Line 3.

<u>Sector I</u>		<u>Sector II</u>	
Line			
<u>1</u>	$(\sim P \vee R) \bullet (\sim P \vee Q)$		
<u>2</u>	$(\sim R \vee P) \bullet (\sim R \bullet R)$		Q
<u>Sector III</u>		<u>Sector IV</u>	
		Rule #	Line #
<u>3</u>	$\sim P (R \bullet Q)$	<u>2</u>	<u>1</u>
<u>4</u>	_____	_____	_____
<u>5</u>	_____	_____	_____
<u>6</u>	_____	_____	_____
<u>7</u>	_____	_____	_____
<u>8</u>	_____	_____	_____
<u>9</u>	_____	_____	_____
<u>10</u>	_____	_____	_____
<u>11</u>	_____	_____	_____

AF62.0

The Wedge : \vee

C.2-14

F63.0

<u>Sector I</u>		<u>Sector II</u>	
Line			
1	$(\sim P \vee R) \circ (\sim P \vee Q)$		
2	$(\sim R \vee P) \circ (\sim R \circ R)$		0
<u>Sector III</u>		<u>Sector IV</u>	
		Rule #	Line #
3	$\sim P \vee (R \circ Q)$	2	1
4	$P \supset (R \circ Q)$	1	3
5			
6			
7			
8			
9			
10			
11			

Look at the connectors
in the expression on
Line 3. Can Rule 1 be
applied to Line 3 to
form the expression John
wrote on Line 4?

AF63.0

Yes

C.2-15

F64.0

<u>Sector I</u>		<u>Sector II</u>	
Line			
1	$(\sim P \vee R) \circ (\sim P \vee Q)$		
2	$(\sim R \vee P) \circ (\sim R \circ R)$		Q
<u>Sector III</u>		<u>Sector IV</u>	
		Rule #	Line #
3	$\sim P \vee (R \circ Q)$	2	1
4	$P \supset (R \circ Q)$	1	3
5	$\sim R \vee P$	3	2
6			
7			
8			
9			
10			
11			

Is Line 5 an extra or a necessary step?

Remember, an extra step is one which can not be used in forming a solution.

AF64.0

Necessary

C.2-16

F65.0

Apply Rule 1 to Line 5
to form the expression
that John wrote on
Line 6.

<u>Sector I</u>		<u>Sector II</u>	
Line			
1	$(\sim P \vee R) \cdot (\sim P \vee Q)$		
2	$(\sim R \vee P) \cdot (\sim R \cdot R)$		Q
<u>Sector III</u>		<u>Sector IV</u>	
		Rule #	Line #
3	$\sim P \vee (R \cdot Q)$	2	1
4	$P \supset (R \cdot Q)$	1	3
5	$\sim R \vee P$	3	2
6		1	5
7			
8			
9			
10			
11			

AF65.0

 $R \supset P$

C.2-17

F66.0

What Section of Rule 3,
Section 3.1 or
Section 3.2 was used
to form the expression
on Line 7?

<u>Sector I</u>		<u>Sector II</u>	
Line			
1	$(\sim P \vee R) \cdot (\sim P \vee Q)$		
2	$(\sim R \vee P) \cdot (\sim R \cdot R)$		Q
<u>Sector III</u>		<u>Sector IV</u>	
		Rule #	Line #
3	$\sim P \vee (R \cdot Q)$	2	1
4	$P \supset (R \cdot Q)$	1	3
5	$\sim R \vee P$	3	2
6	$R \supset P$	1	5
7	$\sim R \cdot R$	3	2
8			
9			
10			
11			

AF66.0

Section 3.2

C.2-18

F67.0

<u>Sector I</u>		<u>Sector II</u>	
Line			
1	$(\sim P \vee R) \circ (\sim P \vee Q)$		
2	$(\sim R \vee P) \circ (\sim R \circ R)$		Q
<u>Sector III</u>		<u>Sector IV</u>	
		Rule #	Line #
3	$\sim P \vee (R \circ Q)$	2	1
4	$P \supset (R \circ Q)$	1	3
5	$\sim R \vee P$	3	2
6	$R \supset P$	1	5
7	$\sim R \circ R$	3	2
8	R	3	7
9			
10			
11			

Is Line 8 an extra or
a necessary step?

AF67.0

Necessary

C.2-19

F68.0

<u>Sector I</u>		<u>Sector II</u>	
Line			
1	$(\sim P \vee R) \cdot (\sim P \vee Q)$		
2	$(\sim R \vee P) \cdot (\sim R \cdot R)$		Q
<u>Sector III</u>		<u>Sector IV</u>	
		Rule #	Line #
3	$\sim P \vee (R \cdot Q)$	2	1
4	$P \supset (R \cdot Q)$	1	3
5	$\sim R \vee P$	3	2
6	$R \supset P$	1	5
7	$\sim R \cdot R$	3	2
8	R	3	7
9		4	6, 8
10			
11			

Can Rule 4 be applied
to the expressions on
Lines 6 and 8?

AF68.0

Yes

C.2-20

F69.0

Sector I		Sector II	
Line			
1	$(\sim P \vee R) \bullet (\sim P \vee Q)$		
2	$(\sim R \vee P) \bullet (\sim R \bullet R)$		0
Sector III		Sector IV	
Line		Rule #	Line #
3	$\sim P \vee (R \bullet Q)$	2	1
4	$P \supset (R \bullet Q)$	1	3
5	$\sim R \vee P$	3	2
6	$R \supset P$	1	5
7	$\sim R \bullet R$	3	2
8	R	3	7
9	P	4	6, 8
10		4	4, 9
11			

Apply Rule 4 to Lines
4 and 9 to form the
expression John wrote
on Line 10.

AF69.0

R • Q

C.2-21

F70.0

Does the final expression
in the solution match
the VPE in Sector II?

<u>Sector I</u>		<u>Sector II</u>	
Line			
1	$(\sim P \vee R) \cdot (\sim P \vee Q)$		
2	$(\sim R \vee P) \cdot (\sim R \cdot R)$		Q
<u>Sector III</u>		<u>Sector IV</u>	
Line		Rule #	Line #
3	$\sim P \vee (R \cdot Q)$	2	1
4	$P \supset (R \cdot Q)$	1	3
5	$\sim R \vee P$	3	2
6	$R \supset P$	1	5
7	$\sim R \cdot R$	3	2
8	R	3	7
9	P	4	6, 8
10	$R \cdot Q$	4	4, 9
11	Q	3	10 <i>f.l.d.</i>

AF70.0

Yes

C.2-22

F71.0

<u>Sector I</u>		<u>Sector II</u>	
Line			
1	$(\sim P \vee R) \circ (\sim P \vee Q)$		
2	$(\sim R \vee P) \circ (\sim R \circ R)$		Q
<u>Sector III</u>		<u>Sector IV</u>	
		Rule #	Line #
3	$\sim P \vee (R \circ Q)$	2	1
4	$P \supset (R \circ Q)$	1	3
5	$\sim R \vee P$	3	2
6	$R \supset P$	1	5
7	$\sim R \circ R$	3	2
8	R	3	7
9	P	4	6, 8
10	$R \circ Q$	4	4, 9
11	Q	3	10 g.e.d.

Look at John's solution
to the problem. How many
times did John incorrectly
apply the rules?

AF71.0

Zero ; None

C.2-23

F72.0

Are there any steps
in this solution where
the rule was correctly
applied but the
expression was not
needed in the solution?
That is, are there any
extra steps?

_____ (yes/no)

<u>Sector I</u>		<u>Sector II</u>	
Line			
1	$(\neg P \vee R) \cdot (\neg P \vee Q)$		
2	$(\neg R \vee P) \cdot (\neg R \cdot R)$		Q
<u>Sector III</u>		<u>Sector IV</u>	
		Rule #	Line #
3	$\sim P \vee (R \cdot Q)$	2	1
4	$P \supset (R \cdot Q)$	1	3
5	$\sim R \vee P$	3	2
6	$R \supset P$	1	5
7	$\sim R \cdot R$	3	2
8	R	3	7
9	P	4	6, 8
10	$R \cdot Q$	4	4, 9
11	Q	3	10 <i>q.e.d.</i>

AF72.0

No

C.2-24

F73.0

After the teacher
checked John's
solution, she told
him it was

(correct/incorrect)

<u>Sector I</u>		<u>Sector II</u>	
Line			
1	$(\sim P \vee R) \circ (\sim P \vee Q)$		
2	$(\sim R \vee P) \circ (\sim R \circ R)$		Q
<u>Sector III</u>		<u>Sector IV</u>	
		Rule †	Line †
3	$\sim P \vee (R \circ Q)$	2	1
4	$P \supset (R \circ Q)$	1	3
5	$\sim R \vee P$	3	2
6	$R \supset P$	1	5
7	$\sim R \circ R$	3	2
8	R	3	7
9	P	4	6,8
10	$R \circ Q$	4	4,9
11	Q	3	10 <i>g.d.</i>

AF73.0

Correct

F74.0

C.2-25

You have just seen two examples of students' attempts to solve Probe problems. Their solutions are repeated on the next two pages. Look at them again.

After you have completed your study of these examples, write down the time in your answer booklet next to F74, then raise your hand.

The proctor will give you a set of Probe problems to solve. You are free to use any of the materials while trying to solve the problems.

Good luck on the problem solving!

AF74.1

Andy's
solution

<u>Sector I</u>		<u>Sector II</u>	
Line			
1	$(\sim P \vee R) \bullet (\sim P \vee Q)$		
2	$(\sim R \vee P) \bullet (\sim R \bullet R)$		Q
<u>Sector III</u>		<u>Sector IV</u>	
		Rule #	Line #
3	$\sim P \vee Q$	3	1
4	$P \supset Q$	1	3
5	$\sim R \vee P$	3	2
6	$R \supset P$	1	5
7	$\sim R \bullet R$	3	2
8	R	3	7
9	P	4	6, 8
10	Q	4	4, 9 q. e.d.

C.2-26

AF74.2

John's
solution

<u>Sector I</u>		<u>Sector II</u>	
Line			
1	$(\sim P \vee R) \cdot (\sim P \vee Q)$		
2	$(\sim R \vee P) \cdot (\sim R \cdot R)$		Q
<u>Sector III</u>		<u>Sector IV</u>	
		Rule #	Line #
3	$\sim P \vee (R \cdot Q)$	2	1
4	$P \supset (R \cdot Q)$	1	3
5	$\sim R \vee P$	3	2
6	$R \supset P$	1	5
7	$\sim R \cdot R$	3	2
8	R	3	7
9	P	4	6, 8
10	$R \cdot Q$	4	4, 9
11	Q	3	10 q.e.d.

APPENDIX C.3

The Probe Programmed Text,
Erroneous Only Treatment

C.3-1

F50.0

Andy started his solution to the problem by filling in Line 3 as shown. Write the expression that Andy formed when he applied Section 3.2 of Rule 3 to Line 1.

<u>Sector I</u>		<u>Sector II</u>	
Line			
<u>1</u>	$(\sim P \vee R) \cdot (\sim P \vee Q)$		
<u>2</u>	$(\sim R \vee P) \cdot (\sim R \cdot R)$		Q
<u>Sector III</u>		<u>Sector IV</u>	
		Rule #	Line #
<u>3</u>	_____	<u>3</u>	<u>1</u>
<u>4</u>	_____		
<u>5</u>	_____		
<u>6</u>	_____		
<u>7</u>	_____		
<u>8</u>	_____		
<u>9</u>	_____		
<u>10</u>	_____		
<u>11</u>	_____		

AF50.0

$\sim P \vee Q$

C.3-2

F51.0

Can Rule 2 be applied
to Line 2 to form the
expression that Andy
wrote on Line 4?

Sector I		Sector II	
Line			
1	$(\sim P \vee R) \bullet (\sim P \vee Q)$		
2	$(\sim R \vee P) \bullet (\sim R \bullet R)$		Q
Sector III		Sector IV	
		Rule #	Line #
3	$\sim P \vee Q$	3	1
4	$\sim R \vee (P \bullet R)$	2	2
5			
6			
7			
8			
9			
10			
11			

AF51.0

No

C.3-3

F52.0

What Section of Rule 3,
Section 3.1 or
Section 3.2 would have
to be applied to Line 1
to form the expression
on Line 5?

<u>Sector I</u>		<u>Sector II</u>	
Line			
<u>1</u>	$(\sim P \vee R) \cdot (\sim P \vee Q)$		
<u>2</u>	$(\sim R \vee P) \cdot (\sim R \cdot R)$		Q
<u>Sector III</u>		<u>Sector IV</u>	
		Rule #	Line #
<u>3</u>	$\sim P \vee Q$	<u>3</u>	<u>1</u>
<u>4</u>	$\sim R \vee (P \cdot R)$	<u>2</u>	<u>2</u>
<u>5</u>	$\sim P \vee R$	<u>3</u>	<u>1</u>
<u>6</u>			
<u>7</u>			
<u>8</u>			
<u>9</u>			
<u>10</u>			
<u>11</u>			

AF52.0

Section 3.1

C.3-4

F53.0

<u>Sector I</u>		<u>Sector II</u>	
Line			
1	$(\sim P \vee R) \circ (\sim P \vee Q)$		
2	$(\sim R \vee P) \circ (\sim R \circ R)$		Q
<u>Sector III</u>		<u>Sector IV</u>	
		Rule #	Line #
3	$\sim P \vee Q$	3	1
4	$\sim R \vee (P \circ R)$	2	2
5	$\sim P \vee R$	3	1
6	$P R$	1	5
7			
8			
9			
10			
11			

Look at Line 6. According to the Rule # and Line # shown, what connector should be filled in to complete the expression?

AF53.0

the Horseshoe : \supset

C.3-5

F54.0

An "extra step" is a line which is not needed to solve the problem. The expression on that line cannot be used to form a solution. Is the expression on Line 6 an extra or a necessary step?

Before you attempt to answer this question, try to figure out how the final expression, which is Q, can be formed.

<u>Sector I</u>		<u>Sector II</u>	
Line			
1	$(\sim P \vee R) \cdot (\sim P \vee Q)$		
2	$(\sim R \vee P) \cdot (\sim R \cdot R)$		Q
<u>Sector III</u>		<u>Sector IV</u>	
		Rule #	Line #
3	$\sim P \vee Q$	3	1
4	$\sim R \vee (P \cdot R)$	2	2
5	$\sim P \vee R$	3	1
6	$P \supset R$	1	5
7			
8			
9			
10			
11			

AF54.0

It is an "extra step."

C.3-6

F55.0

Is the expression on
Line 7 extra or
necessary?

<u>Sector I</u>		<u>Sector II</u>	
Line			
1	$(\sim P \vee R) \cdot (\sim P \vee Q)$		
2	$(\sim R \vee P) \cdot (\sim R \cdot R)$		Q
<u>Sector III</u>		<u>Sector IV</u>	
		Rule #	Line #
3	$\sim P \vee Q$	3	1
4	$\sim R \vee (P \cdot R)$	2	2
5	$\sim P \vee R$	3	1
6	$P \supset R$	1	5
7	$P \supset Q$	1	3
8			
9			
10			
11			

AF55.0

Necessary

C.3-7

F56.0

What is the major
connector in the
expression on Line 4?

Can Rule 3 be applied
to the expression on
Line 4 to form Line 8?

<u>Sector I</u>		<u>Sector II</u>	
Line			
1	$(\sim P \vee R) \cdot (\sim P \vee Q)$		
2	$(\sim R \vee P) \cdot (\sim R \cdot R)$		Q
<u>Sector III</u>		<u>Sector IV</u>	
		Rule #	Line #
3	$\sim P \vee Q$	3	1
4	$\sim R \vee (P \cdot R)$	2	2
5	$\sim P \vee R$	3	1
6	$P \supset R$	1	5
7	$P \supset Q$	1	3
8	$P \cdot R$	3	4
9			
10			
11			

AF56.0

the Wedge : \vee

no

C.3-8

F57.0

If you wanted to apply Rule 4, using Line 7 as the first line and Line 9 as the second line, what VPE would you have to form on Line 9?

<u>Sector I</u>		<u>Sector II</u>	
Line			
<u>1</u>	$(\sim P \vee R) \bullet (\sim P \vee Q)$		
<u>2</u>	$(\sim R \vee P) \bullet (\sim R \bullet R)$		Q
<u>Sector III</u>		<u>Sector IV</u>	
		Rule †	Line †
<u>3</u>	$\sim P \vee Q$	<u>3</u>	<u>1</u>
<u>4</u>	$\sim R \vee (P \bullet R)$	<u>2</u>	<u>2</u>
<u>5</u>	$\sim P \vee R$	<u>3</u>	<u>1</u>
<u>6</u>	$P = R$	<u>1</u>	<u>5</u>
<u>7</u>	$P = Q$	<u>1</u>	<u>3</u>
<u>8</u>	$P \bullet R$	<u>3</u>	<u>4</u>
<u>9</u>		<u>3</u>	<u>8</u>
<u>10</u>			
<u>11</u>			

AF57.0.

P

C.3-9

F58.0

Is the VPE Q, which is
on Line 10, the correct
final expression for
this problem?

<u>Sector I</u>		<u>Sector II</u>	
Line			
<u>1</u>	$(\sim P \vee R) \cdot (\sim P \vee Q)$		
<u>2</u>	$(\sim R \vee P) \cdot (\sim R \cdot R)$		Q
<u>Sector III</u>		<u>Sector IV</u>	
		Rule #	Line #
<u>3</u>	$\sim P \vee Q$	<u>3</u>	<u>1</u>
<u>4</u>	$\sim R \vee (P \cdot R)$	<u>2</u>	<u>2</u>
<u>5</u>	$\sim P \vee R$	<u>3</u>	<u>1</u>
<u>6</u>	$P \supset R$	<u>1</u>	<u>5</u>
<u>7</u>	$P \supset Q$	<u>1</u>	<u>3</u>
<u>8</u>	$P \cdot R$	<u>3</u>	<u>4</u>
<u>9</u>	P	<u>3</u>	<u>8</u>
<u>10</u>	Q	<u>4</u>	<u>7, 9 g.e.d.</u>
<u>11</u>			

AF58.0

Yes

C.3-10

F59.0

In this solution, how many times did Andy incorrectly apply the rules?

<u>Sector I</u>		<u>Sector II</u>	
Line			
1	$(\sim P \vee R) \bullet (\sim P \vee Q)$		
2	$(\sim R \vee P) \bullet (\sim R \bullet R)$		Q
<u>Sector III</u>		<u>Sector IV</u>	
		Rule #	Line #
3	$\sim P \vee Q$	3	1
4	$\sim R \vee (P \bullet R)$	2	2
5	$\sim P \vee R$	3	1
6	$P \supset R$	1	5
7	$P \supset Q$	1	3
8	$P \bullet R$	3	4
9	P	3	8
10	Q	4	7, 9 g.l.d.
11			

AF59.0

2 times

C.3-11

F60.0

Are there any extra steps in this solution, that is, steps in which expressions were correctly formed but which were not needed?

(yes/no)

<u>Sector I</u>		<u>Sector II</u>	
Line			
1	$(\sim P \vee R) \cdot (\sim P \vee Q)$		
2	$(\sim R \vee P) \cdot (\sim R \cdot R)$		Q
<u>Sector III</u>		<u>Sector IV</u>	
		Rule #	Line #
3	$\sim P \vee Q$	3	1
4	$\sim R \vee (P \cdot R)$	2	2
5	$\sim P \vee R$	3	1
6	$P \supset R$	1	5
7	$P \supset Q$	1	3
8	$P \cdot R$	3	4
9	P	3	8
10	Q	4	7, 9 g. l. d.
11			

AF60.0

Yes

C.3-12

F61.0

After the teacher
checked the solution,
he told Andy that it
was _____
(correct/incorrect)

<u>Sector I</u>		<u>Sector II</u>	
Line			
1	$(\sim P \vee R) \bullet (\sim P \vee Q)$		
2	$(\sim R \vee P) \bullet (\sim R \bullet R)$		Q
<u>Sector III</u>		<u>Sector IV</u>	
		Rule #	Line #
3	$\sim P \vee Q$	3	1
4	$\sim R \vee (P \bullet R)$	2	2
5	$\sim P \vee R$	3	1
6	$P \supset R$	1	5
7	$P \supset Q$	1	3
8	$P \bullet R$	3	4
9	P	3	8
10	Q	4	7, 9, l.l.
11			

AF61.0

Incorrect

C.3-13

F62.0

John worked on the same problem, but came out with a different solution. Fill in the major connector that John will write when he completes the new expression on Line 3.

<u>Sector I</u>		<u>Sector II</u>	
Line			
1	$(\sim P \vee R) \circ (\sim P \vee Q)$		
2	$(\sim R \vee P) \circ (\sim R \circ R)$		Q
<u>Sector III</u>		<u>Sector IV</u>	
		Rule †	Line †
3	$\sim P (R \circ Q)$	2	1
4			
5			
6			
7			
8			
9			
10			
11			

AF62.0

the Wedge : \vee

C.3-14

F63.0

Look at the connectors
in the expression on
Line 2. Can Rule 2 be
applied to Line 2 to
form the expression John
wrote on Line 4?

<u>Sector I</u>		<u>Sector II</u>	
Line			
<u>1</u>	$(\sim P \vee R) \cdot (\sim P \vee Q)$		
<u>2</u>	$(\sim R \vee P) \cdot (\sim R \cdot R)$		Q
<u>Sector III</u>		<u>Sector IV</u>	
		Rule #	Line #
<u>3</u>	$\sim P \vee (R \cdot Q)$	<u>2</u>	<u>1</u>
<u>4</u>	$\sim R \vee (P \cdot R)$	<u>2</u>	<u>2</u>
<u>5</u>			
<u>6</u>			
<u>7</u>			
<u>8</u>			
<u>9</u>			
<u>10</u>			
<u>11</u>			

AF63.0

No

C.3-15

F64.0

Is Line 5 an extra or a necessary step?

Remember, an extra step is one which can not be used in forming a solution.

<u>Sector I</u>		<u>Sector II</u>	
Line			
<u>1</u>	<u>$(\sim P \vee R) \cdot (\sim P \vee Q)$</u>		
<u>2</u>	<u>$(\sim R \vee P) \cdot (\sim R \cdot R)$</u>		Q
<u>Sector III</u>		<u>Sector IV</u>	
		Rule #	Line #
<u>3</u>	<u>$\sim P \vee (R \cdot Q)$</u>	<u>2</u>	<u>1</u>
<u>4</u>	<u>$\sim R \vee (P \cdot R)$</u>	<u>2</u>	<u>2</u>
<u>5</u>	<u>$R = (P \cdot R)$</u>	<u>1</u>	<u>4</u>
<u>6</u>			
<u>7</u>			
<u>8</u>			
<u>9</u>			
<u>10</u>			
<u>11</u>			

AF64.0

Extra

C.3-16

F65.0

Apply Rule 1 to Line 3
to form the expression
that John wrote on
Line 6.

<u>Sector I</u>		<u>Sector II</u>	
Line			
1	$(\sim P \vee R) \cdot (\sim P \vee Q)$	Q	
2	$(\sim R \vee P) \cdot (\sim R \cdot R)$		
<u>Sector III</u>		<u>Sector IV</u>	
		Rule #	Line #
3	$\sim P \vee (R \cdot Q)$	2	1
4	$\sim R \vee (P \cdot R)$	2	2
5	$R \supset (P \cdot R)$	1	4
6		1	3
7			
8			
9			
10			
11			

AF65.0

$$\underline{P \supset (R \cdot Q)}$$

C.3-17

F66.0

What Section of Rule 3,
Section 3.1 or
Section 3.2 was used
to form the expression
on Line 7?

<u>Sector I</u>		<u>Sector II</u>	
Line			
1	$(\sim P \vee R) \cdot (\sim P \vee Q)$		
2	$(\sim R \vee P) \cdot (\sim R \cdot R)$		Q
<u>Sector III</u>		<u>Sector IV</u>	
		Rule #	Line #
3	$\sim P \vee (R \cdot Q)$	2	1
4	$\sim R \vee (P \cdot R)$	2	2
5	$R \supset (P \cdot R)$	1	4
6	$P \supset (R \cdot Q)$	1	3
7	$\sim P \vee Q$	3	1
8			
9			
10			
11			

AF66.0

Section 3.2

C.3-18

F67.0

<u>Sector I</u>		<u>Sector II</u>	
Line			
1	$(\sim P \vee R) \cdot (\sim P \vee Q)$		
2	$(\sim R \vee P) \cdot (\sim R \cdot R)$		Q
<u>Sector III</u>		<u>Sector IV</u>	
Line		Rule #	Line #
3	$\sim P \vee (R \cdot Q)$	2	1
4	$\sim R \vee (P \cdot R)$	2	2
5	$R \supset (P \cdot R)$	1	4
6	$P \supset (R \cdot Q)$	1	3
7	$\sim P \vee Q$	3	1
8	$\sim R \cdot R$	3	2
9			
10			
11			

Is Line 8 an extra or
a necessary step?

AF67.0

Necessary

C.3-19

F68.0

Can Rule 4 be applied
to the expressions on
Lines 5 and 8?

<u>Sector I</u>		<u>Sector II</u>	
Line			
<u>1</u>	$(\neg P \vee R) \cdot (\neg P \vee Q)$		
<u>2</u>	$(\neg R \vee P) \cdot (\neg R \cdot R)$		Q
<u>Sector III</u>		<u>Sector IV</u>	
		Rule #	Line #
<u>3</u>	$\sim P \vee (R \cdot Q)$	<u>2</u>	<u>1</u>
<u>4</u>	$\sim R \vee (P \cdot R)$	<u>2</u>	<u>2</u>
<u>5</u>	$R \supset (P \cdot R)$	<u>1</u>	<u>4</u>
<u>6</u>	$P \supset (R \cdot Q)$	<u>1</u>	<u>3</u>
<u>7</u>	$\sim P \vee Q$	<u>3</u>	<u>1</u>
<u>8</u>	$\sim R \cdot R$	<u>3</u>	<u>2</u>
<u>9</u>		<u>4</u>	<u>5, 8</u>
<u>10</u>			
<u>11</u>			

AF68.0

No

C.3-20

F69.0

Apply Rule 1 to Line 7
to form the expression
that John wrote on
Line 10.

<u>Sector I</u>		<u>Sector II</u>	
Line			
<u>1</u>	$(\sim P \vee R) \cdot (\sim P \vee Q)$		
<u>2</u>	$(\sim R \vee P) \cdot (\sim R \cdot R)$		Q
<u>Sector III</u>		<u>Sector IV</u>	
Line		Rule †	Line ‡
<u>3</u>	$\sim P \vee (R \cdot Q)$	<u>2</u>	<u>1</u>
<u>4</u>	$\sim R \vee (P \cdot R)$	<u>2</u>	<u>2</u>
<u>5</u>	$R \supset (P \cdot R)$	<u>1</u>	<u>4</u>
<u>6</u>	$P \supset (R \cdot Q)$	<u>1</u>	<u>3</u>
<u>7</u>	$\sim P \vee Q$	<u>3</u>	<u>1</u>
<u>8</u>	$\sim R \cdot R$	<u>3</u>	<u>2</u>
<u>9</u>	$P \cdot R$	<u>4</u>	<u>5, 8</u>
<u>10</u>		<u>1</u>	<u>7</u>
<u>11</u>			

AF69.0

P \supset Q

Q.3-21

F70.0

Does the final expression
in the solution match
the VPE in Sector II?

<u>Sector I</u>		<u>Sector II</u>	
Line			
1	$(\sim P \vee R) \circ (\sim P \vee Q)$		
2	$(\sim R \vee P) \circ (\sim R \circ R)$		Q
<u>Sector III</u>		<u>Sector IV</u>	
		Rule #	Line #
3	$\sim P \vee (R \circ Q)$	2	1
4	$\sim R \vee (P \circ R)$	2	2
5	$R \supset (P \circ R)$	1	4
6	$P \supset (R \circ Q)$	1	3
7	$\sim P \vee Q$	3	1
8	$\sim R \circ R$	3	2
9	$P \circ R$	4	5, 8
10	$P \supset Q$	1	7
11	Q	4	9, 10 <i>l.l</i>

AF70.0

Yes

C.3-22

F71.0

Look at John's solution
to the problem. How many
times did John incorrectly
apply the rules?

<u>Sector I</u>		<u>Sector II</u>	
Line			
1	$(\sim P \vee R) \circ (\sim P \vee Q)$		
2	$(\sim R \vee P) \circ (\sim R \circ R)$		Q
<u>Sector III</u>		<u>Sector IV</u>	
		Rule #	Line #
3	$\sim P \vee (R \circ Q)$	2	1
4	$\sim R \vee (P \circ R)$	2	2
5	$R \supset (P \circ R)$	1	4
6	$P \supset (R \circ Q)$	1	3
7	$\sim P \vee Q$	3	1
8	$\sim R \circ R$	3	2
9	$P \circ R$	4	5, 8
10	$P \supset Q$	1	7
11	Q	4	9, 10 <i>g.e.d.</i>

AF71.0

3 times

C.3-23

F72.0

Are there any steps in
this solution where the
rule was correctly applied
but the expression was
not needed in the solution?
That is, are there any
extra steps?

(yes/no)

<u>Sector I</u>		<u>Sector II</u>	
Line			
1	$(\neg P \vee R) \cdot (\neg P \vee Q)$		
2	$(\neg R \vee P) \cdot (\neg R \cdot R)$		Q
<u>Sector III</u>		<u>Sector IV</u>	
		Rule #	Line #
3	$\sim P \vee (R \cdot Q)$	2	1
4	$\sim R \vee (P \cdot R)$	2	2
5	$R = (P \cdot R)$	1	4
6	$P = (R \cdot Q)$	1	3
7	$\sim P \vee Q$	3	1
8	$\sim R \cdot R$	3	2
9	$P \cdot R$	4	5, 8
10	$P = Q$	1	7
11	Q	4	9, 10 g. 2. 2

AF72.0

Yes

C.3-24

F73.0

After the teacher
checked John's
solution, she told
him it was

(correct/incorrect)

<u>Sector I</u>		<u>Sector II</u>	
Line			
1	$(\sim P \vee R) \circ (\sim P \vee Q)$		
2	$(\sim R \vee P) \circ (\sim R \circ R)$		Q
<u>Sector III</u>		<u>Sector IV</u>	
		Rule #	Line #
3	$\sim P \vee (R \circ Q)$	2	1
4	$\sim R \vee (P \circ R)$	2	2
5	$R \circ (P \circ R)$	1	4
6	$P \circ (R \circ Q)$	1	3
7	$\sim P \vee Q$	3	1
8	$\sim R \circ R$	3	2
9	$P \circ R$	4	5, 8
10	$P \circ Q$	1	7
11	Q	4	9, 10 g. e. d.

AF73.0

Incorrect

C.3-25

F74.0

You have just seen two examples of students' attempts to solve Probe problems. Their solutions are repeated on the next two pages. Look at them again.

After you have completed your study of these examples, write down the time in your answer booklet next to F74, then raise your hand.

The proctor will give you a set of Probe problems to solve. You are free to use any of the materials while trying to solve the problems.

Good luck on the problem solving!

VEAV'I

AF74.1

Andy's solution

<u>Sector I</u>		<u>Sector II</u>	
Line			
1	$(\sim P \vee R) \circ (\sim P \vee Q)$	Q	
2	$(\sim R \vee P) \circ (\sim R \circ R)$		
<u>Sector III</u>		<u>Sector IV</u>	
		Rule #	Line #
3	$\sim P \vee Q$	3	1
4	$\sim R \vee (P \circ R)$	2	2
5	$\sim P \vee R$	3	1
6	$P \supset R$	1	5
7	$P \supset Q$	1	3
8	$P \circ R$	3	4
9	P	3	8
10	Q	4	7, 9 q.o.d.
11			

C.3-26

AF74.2

John's
solution

<u>Sector I</u>		<u>Sector II</u>	
Line			
1	$(\sim P \vee R) \cdot (\sim P \vee Q)$		
2	$(\sim R \vee P) \cdot (\sim R \cdot R)$		Q
<u>Sector III</u>		<u>Sector IV</u>	
		Rule #	Line #
3	$\sim P \vee (R \cdot Q)$	2	1
4	$\sim R \vee (P \cdot R)$	2	2
5	$R \supset (P \cdot R)$	1	4
6	$P \supset (R \cdot Q)$	1	3
7	$\sim P \vee Q$	3	1
8	$\sim R \cdot R$	3	2
9	$P \cdot R$	4	5, 8
10	$P \supset Q$	1	7
11	Q	4	9, 10 q.e.d.

APPENDIX D

The Transfer Task -- A Problem Solving Test

Name _____ D-1 Probe Text # _____

PROBE

Problem Solving Test

This booklet contains four Probe problems. You are allowed 15 minutes for each one, and are free to use any of the materials you have been given. Each problem appears on a separate page which looks like the example in Panel II. Sectors I and II contain the expressions you have been given to solve the problem. Your solution is to be written in Sectors III and IV.

Before you start any problem, fill in the Start Time at the top of the page. After you have finished, or at the end of 15 minutes, record the Stop Time. Whether or not you have completed your solution, go on to the next problem after 15 minutes. When you have gone through all the problems, complete the questionnaire which is on the last page of the Response Booklet. When that is done, raise your hand and the proctor will collect all of your materials.

PLEASE DO NOT CROSS OUT ANY LINES YOU HAVE WRITTEN!!!

In this experiment, the complete record of what you have done is very important. However, you can indicate which lines you would like to remove, by placing an "X" over the line number. This will remind you not to use that line. For example, if you didn't want Line 3 to count:

~~X~~ 3 P = R 3 2

The "X" over the line number means that that line does not count.

So that we can know when you decided to cross out a line, please write the number(s) of the line(s) you just crossed out in the right margin, next to the last line you had written. If for example, you crossed out Line 3 after you wrote Line 6, then Line 6 would look like this:

6 R = P 4 1,5 | 3

Line 6 remains the same, but now a 3 appears in the right margin. You can remove, but not cross out, any lines you do not want.

If you need any scrap paper, feel free to use any of the margins or the back of the sheet. Try your best to solve each of the problems in the least number of steps. Be sure to spend no more than 15 minutes on any problem. This test must be done in ink. Please record your start and stop times to the nearest minute.

You can start whenever you are ready. Good Luck!

D-2

Start Time: _____

Stop Time: _____

PROBE Problem #1

<u>Line#</u>	<u>Sector I</u>	<u>Sector II</u>	
<u>1</u>	<u>P</u>	<u>R</u>	
<u>2</u>	<u>$\sim P \vee (R \circ S)$</u>		
<u>Sector III</u>		<u>Sector IV</u>	
		<u>Rule #</u>	<u>Line #</u>
<u>3</u>	_____	_____	_____
<u>4</u>	_____	_____	_____
<u>5</u>	_____	_____	_____
<u>6</u>	_____	_____	_____
<u>7</u>	_____	_____	_____
<u>8</u>	_____	_____	_____
<u>9</u>	_____	_____	_____
<u>10</u>	_____	_____	_____
<u>11</u>	_____	_____	_____
<u>12</u>	_____	_____	_____
<u>13</u>	_____	_____	_____
<u>14</u>	_____	_____	_____
<u>15</u>	_____	_____	_____
<u>16</u>	_____	_____	_____
<u>17</u>	_____	_____	_____
<u>18</u>	_____	_____	_____
<u>19</u>	_____	_____	_____
<u>20</u>	_____	_____	_____

If you need more room, please use the back of this sheet.

D-4

Start Time: _____

Stop Time: _____

PROBE Problem #3

<u>Line#</u>	<u>Sector I</u>	<u>Sector II</u>	
<u>1</u>	<u>$\sim T \circ R$</u>		
<u>2</u>	<u>$(\sim R \vee Q) \circ (T \vee S)$</u>	$\sim T \vee S$	
<u>3</u>	<u>$Q \supset (T \supset S)$</u>		
<u>Sector III</u>		<u>Sector IV</u>	
		<u>Rule #</u>	<u>Line #</u>
<u>4</u>	_____	_____	_____
<u>5</u>	_____	_____	_____
<u>6</u>	_____	_____	_____
<u>7</u>	_____	_____	_____
<u>8</u>	_____	_____	_____
<u>9</u>	_____	_____	_____
<u>10</u>	_____	_____	_____
<u>11</u>	_____	_____	_____
<u>12</u>	_____	_____	_____
<u>13</u>	_____	_____	_____
<u>14</u>	_____	_____	_____
<u>15</u>	_____	_____	_____
<u>16</u>	_____	_____	_____
<u>17</u>	_____	_____	_____
<u>18</u>	_____	_____	_____
<u>19</u>	_____	_____	_____
<u>20</u>	_____	_____	_____

If you need more room, please use the back of this sheet.

D-5

Start Time: _____

Stop Time: _____

PROBE Problem #4

<u>Line#</u>	<u>Sector I</u>	<u>Sector II</u>	
<u>1</u>	<u>P \circ T</u>	$T \supset Q$	
<u>2</u>	<u>$\sim(P \circ T) \vee (T \supset (R \circ Q))$</u>		
	<u>Sector III</u>	<u>Sector IV</u>	
		<u>Rule #</u>	<u>Line #</u>
<u>3</u>	_____	_____	_____
<u>4</u>	_____	_____	_____
<u>5</u>	_____	_____	_____
<u>6</u>	_____	_____	_____
<u>7</u>	_____	_____	_____
<u>8</u>	_____	_____	_____
<u>9</u>	_____	_____	_____
<u>10</u>	_____	_____	_____
<u>11</u>	_____	_____	_____
<u>12</u>	_____	_____	_____
<u>13</u>	_____	_____	_____
<u>14</u>	_____	_____	_____
<u>15</u>	_____	_____	_____
<u>16</u>	_____	_____	_____
<u>17</u>	_____	_____	_____
<u>18</u>	_____	_____	_____
<u>19</u>	_____	_____	_____
<u>20</u>	_____	_____	_____

If you need more room, please use the back of this sheet.

Perfect Problem Solutions

Problem 1

1. P R
 2. $\sim P \vee (R \circ S)$

 3. $P \supset (R \circ S)$ B L
 4. $R \circ S$ 1 2
 5. R 3 4 q.e.d.

Problem 2

1. $Q \supset (P \circ T)$ Q \supset P

 2. $\sim Q \vee (P \circ T)$ R L
 3. $(\sim Q \vee P) \circ (\sim Q \vee T)$ 1 1
 4. $\sim Q \vee P$ 2 2
 5. $Q \supset P$ 3 3

Problem 3

1. $\sim T \circ R$
 2. $(\sim R \vee Q) \circ (T \vee S)$ $\sim T \vee S$
 3. $Q \supset (T \supset S)$

 4. $\sim R \vee Q$ R L
 5. $R \supset Q$ 3 2
 6. R 1 4
 7. Q 3 1
 8. $T \supset S$ 4 5,6
 9. $\sim T \vee S$ 4 3,7
 1 8
 q.e.d.

Problem 4

1. $P \circ T$ T \supset Q
 2. $\sim(P \circ T) \vee (T \supset (R \circ Q))$

 3. $(P \circ T) \supset (T \supset (R \circ Q))$ R L
 4. $T \supset (R \circ Q)$ 1 2
 5. $\sim T \vee (R \circ Q)$ 4 1,3
 6. $(\sim T \vee R) \circ (\sim T \vee Q)$ 1 4
 7. $\sim T \vee Q$ 2 5
 8. $T \supset Q$ 3 6
 1 7
 q.e.d.

APPENDIX E

Scoring Procedures

SCORING PROCEDURES

PROBE Experiment

The form "Subject's Data Record" (Figure 1), is used for recording the student's demographic characteristics and his scores for the (a) Response Booklet, (b) Rule Application Test, (c) Problem Solving Test. The headings on the form indicate which scores are to be recorded. The procedures for obtaining these scores follows.

1. The Response Booklet.

A master scoring form has been prepared for each treatment conditions' Response Booklet, showing:

- a. the preferred responses
- b. acceptable alternatives
- c. errors

If a scorer has any questions about the correctness of a response, the learner's Text#, question number, response and the scoring of the response, is to be brought to the attention of the project director, as soon as possible.

Scoring Rules:

a. General Approach -- a response is counted as correct if the answer indicates that the learner knew the correct response to the question, and the answer did not violate any rules of the Probe language.

For example, in F4.0 the learner is required to associate

SUBJECT'S DATA RECORD

Subject's Data Record

PROBE Experiment

Subject's Name _____ Probe Text# _____ Treatment _____

Sex M F Age _____ : _____ Grade _____ Scorer _____ Date _____

(circle one) Years Months

Response Booklet Scores

Session	# Right	# Wrong	Items Answered Incorrectly	Time
First (63 Items)				
Second (30 Items)				

Has the subject had previous contact with Symbolic Logic? _____

Rule Application Test Score (16 Items)

# Right	# Wrong	Items Answered Incorrectly	Time

Problem Solving Test

Problem #	1	2	3	4	Total
Total Lines Coded					
Number of IC Lines					
IC Lines Cancelled					
Number of ER Lines					
ER Lines Cancelled					
Number of EX Lines					
EX Lines Cancelled					
Necessary Steps Cancelled					
Problem Solved?(Y or N)					
Time to Solution					

Figure 1.

E-3

the connector symbols with their names, and to show how these symbols can be used to construct a VPE in Probe. A response is marked as an error whenever the learner has associated the incorrect symbol and name.

	<u>Preferred Responses</u>	<u>Acceptable Responses</u>	<u>Errors</u>
a.	R \supset T	R \supset S	R \vee T
b.	R \vee T	R \vee S	R \circ T
c.	R \circ T	R \circ S	R \supset T

Another example. In F48.0, the learner is asked to indicate the number of the sector, from Panel II of the Probe Panelbook, which contains the expression appearing in the final line of a correct solution. While the preferred answer to F48.0 is "Sector II", sometimes the expression in Sector II, e.g., "R", rather than the sector number, is recorded. As the expression "R" is found only in Sector II, responding with that expression is scored as a correct response.

b. Specific Scoring Rules

1. Where the correct expression contains more than one connector, thereby requiring the presence of parentheses, all left parentheses must be present for the expressions to be correct. For example, in F6.0:

<u>Preferred Response</u>	<u>Acceptable Response</u>	<u>Error</u>
R \vee (S \circ T)	R \vee (S \circ T	R \vee S \circ T

2. Letter substitutions are acceptable only when there are no violations of the basic response to the question. For example, in F4.0, the student is required to construct VPE's using the letters "R" and "T", and the Horseshow, Wedge and Dot connectors. If the student responds: $R \supset S$, $R \vee S$, and $R \circ S$, consistence in the use of the substitute letter is maintained, without violating the basic response.

3. The correct connectors and all signs must be properly located, in regard to each other and parenthetical delimiters, if the answer is to be counted as correct. For example, in F14.0 c.:

<u>Preferred Response</u>	<u>Acceptable Response</u>	<u>Error</u>
$\sim (Q \supset \sim R)$	$\sim (Q \supset \sim R)$	$(\sim Q \supset \sim R)$
		$\sim Q \supset R$

2. The Rule Application Test.

The Rule Application Test master scoring sheet shows all of the possible correct answers to a problem, in preferred response form. However, certain responses which do not exactly match any of those shown on the master will still be considered as correct, if they are in accord with the following:

1. The rule number may either be filled in correctly, or left blank in those problems in which the given rule is

E-5

to be applied, e.g., 2, 5, 7, 8, 10, 13, and 15.

2. An item is to be marked correct if only right parentheses are missing.

3. An item is to be marked correct if extra parentheses are present, but are around two valid expressions which are separated by a connector.

4. An item is to be marked incorrect if there are letter substitutions.

3. The Problem Solving Test.

The Problem Solving Test master scoring sheet shows all of the acceptable solutions to each of the problems, but in preferred form. In general, the following discrepancies from preferred form should be ignored: writing errors (corrections to a single character or symbol); Rule #3, can be written with or without a section number; missing right parentheses are to be ignored. Score the problem solving test through the use of the following steps:

a. Count the Total Lines Coded -- Include in the count any line on which either an expression, part of an expression, a line number, or a rule number has been recorded. If a line has been removed, 'X-ed' out, and either used in a non-removed line or checked to show that it has been reentered, it should be counted twice.

b. Count the Number of IC Lines -- Mark any line on which the expression is incomplete, or the rule or line

E-6

number is blank with an "IC". An incomplete expression is one in which the expression to the right or left of a connector is missing. Examples of incomplete expressions follow.

$(\vee T) \circ (Q \vee S)$ $(S \supset T) \vee$ $Q \supset$

c. Count the IC Lines Cancelled -- Any IC line with an "X" through the line number is a cancelled IC line.

d. Count the Number of ER Lines -- Mark any line which was created by the misapplication of a rule with an "ER". The special scoring rules for determining if a Probe rule has been misapplied follows:

1. If only one line number appears alongside a Rule #4 application, the line should be marked ER.

2. If any letters, signs, connectors or left parentheses are missing, or are incorrect, or if there are an excess of parentheses, the line should be marked ER. For example, if the correct expression is $T \vee (R \supset Q)$, but the expression which appears is:

a) $(T \vee (R \supset Q)$ This is an ER line.

b) $T \vee (R \supset Q$ This is not an ER line.

3. Any line which attempts to apply a rule to a line which does not contain a valid Probe expression, except where the expression was in error because of excessive parentheses, should be marked ER.

E-7

Expressions To Which The

Rule Has Been AppliedAction

- a) $Q \circ T \vee S$ Missing Parentheses Mark this line ER.
- b) $(T \vee (R \supset Q))$ Excess Parentheses Do not mark this line ER.

4. When the major connector is a DOT, switching the expressions to either side is permitted, and the line should not be marked ER.

Should be: $(Q \vee T) \circ (Q \supset S)$

Is: $(Q \supset S) \circ (Q \vee T)$ This line is not an ER.

5. Common Errors -- To always be marked ER.

<u>Line</u>	<u>Expression</u>	<u>Rule #</u>	<u>Line #</u>	
3	$(\sim Q \vee P) \circ (\sim Q \vee T)$	x	x	
4	$(Q \supset P) \circ (Q \supset T)$	1	3	This line is ER.
5	$(Q \supset P) \circ (\sim Q \vee T)$	1	3	This line is a correct rule application.
6	$(\sim Q \vee P) \circ (Q \supset T)$	1	3	This line is a correct rule application.
2	$Q \vee (P \circ T)$	x	x	
3	$(\sim Q \vee P) \circ (\sim Q \circ T)$	2	2	This line is ER.
5	$R \circ S$	4	<u>3</u>	This line is ER.
3	$T \supset (R \circ Q)$	x	x	
4	$T \supset \underline{R}$	3	3	This line is ER.

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e. Count the ER Lines Cancelled -- Any ER line with an "X" through the line number is a cancelled ER line.

f. Count the Number of EX Lines -- These are lines in which the rule has been correctly applied, but are not a required portion of the solution.

1. A line which is never referenced by a later line should be marked EX.

2. A line which duplicates an already existing expression should be marked EX.

3. Any line which has to be counted twice in the count of Total Lines Coded, if it is not incomplete or in error, should be counted once as an EX line.

4. Any line which does not match a line in a selected ideal solution, should be marked EX. To select an ideal solution for comparison, first check for a problem solution using the procedure described in the next section. If a solution is matched, mark those lines which do not match lines in the solution with an EX. If a correct solution cannot be matched, use the following rules of thumb -- select the best match on the first few solution steps; select that solution for comparison which maximizes the learner's score on the number of necessary solution steps produced.

g. Count the EX Lines Cancelled -- Any EX line with an "X" through the line number is a cancelled EX line.

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h. The master solution set has been organized so that the most frequently produced solutions appear first. In most cases the form of the solution is determined in the first few steps.

Check the solution presented against the master set. A solution excludes all IC, ER and "X-ed" out lines. If the remaining lines match a master solution, the problem has been solved. For a solution to be matched neither the line sequences, nor the line reference numbers need match. EX lines can be included in a solved problem.

Master Scoring Form
For Response Booklets

Name _____ E-10 Age _____ : _____ Grade _____
years months

School _____ Sex M F Probe Text # _____
circle one

PROBE

Mini-Course
Response Booklet

First Session

F 1.0 *Start Time* _____ : _____ , to the nearest minute.
Hours Minutes

F 2.0 v PEs | Acceptable VPE

F 3.0	a) the Wedge <u>v</u>	Acceptable Letter <u>v</u> Letter
	b) the Dot <u>o</u>	" " " "
	c) the Horseshoe <u>⊃</u>	" " " "

F 4.0	a) the Horseshoe connector <u>R⊃T</u>	Acceptable <u>R⊃S</u>	Error Wrong connector symbol; connector name rather than symbol.
	b) the Wedge connector <u>RvT</u>	<u>RvS</u>	
	c) the Dot connector <u>RoT</u>	<u>RoS</u>	

F 5.0 T, S o T, R | Acceptable: Any sequence referring to the 1st, 2nd and 5th items.

F 6.0	<u>Rv(S o T)</u>	Acceptable <u>Rv(S o T)</u>	Error <u>RvS o T</u>
-------	------------------	--------------------------------	-------------------------

F 7.0	<u>T o (R⊃S)</u>	<u>T o (R⊃S)</u>	<u>T o R⊃S</u>
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F 8.0	<u>(Q o T)v (R o T)</u>	<u>(Q o T)v (R o T); (Q o T)v (R o T)</u>	<u>(Q o T)v R o T</u>
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F 9.0 a) (Q v T)⊃ (R o P) ⊃ , Horseshoe

b) (Q⊃P) o Q o , Dot

F 10.0	1. <u>(SvT)</u>	Acceptable <u>SvT, (SvT)</u>
--------	-----------------	---------------------------------

	2. <u>(PvQ)</u>	<u>P⊃Q, (P⊃Q) (P⊃Q)</u>
--	-----------------	-------------------------

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-2-

F11.0 Negative

F12.0 a) $\sim R$: Negative

b) R : Positive

c) $\sim(S \vee T)$: Negative

d) $\sim S \vee T$: Positive

F13.0 $\sim T$

	Acceptable	Error
F14.0 a) $R : \sim R$		
b) $R \vee T : \sim(R \vee T)$	$\sim(R \vee T)$	$\sim R \vee \sim T$ $\sim R \vee T$
c) $Q \supset \sim R : \sim(Q \supset \sim R)$	$\sim(Q \supset \sim R)$	$\sim Q \supset \sim R$ $\sim Q \supset R$
d) $\sim Q \vee T : \sim(\sim Q \vee T)$	$\sim(\sim Q \vee T)$	$\sim Q \vee \sim T$ $Q \vee \sim T$
F15.0 a) $\sim(R \vee \sim T) : R \vee \sim T$	$(R \vee \sim T), (R \vee \sim T)$	

b) $\sim R$: R

c) $\sim(S \vee (R \supset T)) : S \vee (R \supset T)$ Error
 $S \vee R \supset T$

F16.0 Negative

F18.0 The rules which permit recoding only from left to right are:

	Acceptable
3, 4	3.1, 3.2, 4

The rules that permit recoding in both directions are:

1, 2

F19.0 B part Q

C part S

F20.0 $(X \vee Y)$ | Acceptable
 $X \vee Y$

F21.0 $\sim A \vee B$

F22.0 A part	$\sim (X \supset Y)$	Acceptable $\sim (X \supset Y)$	Error $(X \supset Y); X \supset Y$ $\sim X \supset Y$
B part	$(Q \supset S)$	$Q \supset S; (Q \supset S)$	

F23.0 Positive

F24.0 a)	$\sim R \vee T : R \supset T$	Acceptable $R \supset B$
b)	$S \vee R : \sim S \supset R$	

F25.0 \vee , Wedge

F26.0 a) $R \supset S : \sim R \vee S$

b) $\sim Q \supset R : Q \vee R$

F27.0	3	$T \supset (R \circ Q)$	Acceptable $T \supset (R \circ Q)$	Error $T \supset R \circ Q$
	4	$\sim (R \circ S) \vee Q$	$\sim (R \circ S \vee Q)$	$(\sim R \circ S) \vee Q$ $\sim R \circ S \vee Q$

F28.0 Rule # Line #

1 2

F29.0 $A \vee (B \circ C)$ | Acceptable
 $A \vee (B \circ C)$

F30.0 A part? T

B part? S

C part? Q

F31.0 B part? $\sim P$

C part? T

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-4-

Line	Rule #	Line #
F32.0	3	3
	PRE 3	1
	ACC 3	1
F33.0	ERR 3	1
F34.0	A part (R \supset S)	Error
F35.0	R \supset S	(R \supset S); (R \supset S)
F36.0	B part	
F37.0	ACC 4	3.2
	PRE 4	3
F38.0	different lines	(the same line/different lines)
F39.0	R	
F40.0	6	
F41.0	7 R \supset T	
	10 R	
F42.0	11	
	T	
	4	
	-7,10	
	Acceptable	
	7&10; 10,7	
F43.0	*Stop Time*	
	Hours	
	Minutes	
	, to the nearest minute.	

End of The First Study Session

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-5-

Second Study Session

F44.0 *Start Time* _____:_____, to the nearest minute.

	Hours	Minutes
	Acceptable	
F45.0	Four; 4; IV	I, II, III, IV

F46.0 VPE R _____.

F47.0 Sector III, 3, Three_____

Sector IV, 4, Four_____

	Acceptable
F48.0	Sector <u>II, 2, Two</u> <u> R </u>

	Error
F50.0	<u> ~ P ∨ Q </u> <u>(~ P ∨ Q); ~ (P ∨ Q)</u>

F51.0 Yes _____

F52.0 3.1 _____

F53.0 ⊃, Horseshoe _____

F54.0 Necessary _____

F55.0 Necessary _____

F56.0 Major connector? o , Dot _____

Can Rule 3 be applied? Yes _____

F57.0 P _____

F58.0 Yes _____

F59.0 ∅, None, Zero _____

F60.0 No (yes/no)

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-6-

F61.0 Correct, Right (correct/incorrect)F62.0 ∇, WedgeF63.0 YesF64.0 NecessaryF65.0 R ⊃ P | Error
(R ⊃ P)F66.0 Rule 3, Section 3.2; 2F67.0 NecessaryF68.0 YesF69.0 R ⊃ Q | Error
(R ⊃ Q)F70.0 YesF71.0 ∅ ; None; ZeroF72.0 No (yes/no)F73.0 Correct; Right (correct/incorrect)F74.0 *Stop Time* : , to the nearest minute.
Hours MinutesPLEASE COMPLETE THE FOLLOWING QUESTIONNAIRE AFTER
YOU FINISH THE PROBLEM SOLVING TEST.

1. Have you ever worked on this type of problem before? When? Where?

Master Scoring Form
Erroneous Only

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-5-

Second Study Session

F44.0 *Start Time* _____:_____, to the nearest minute.

	Hours	Minutes
	Acceptable	
F45.0	Four; 4; IV	I, II, II, IV

F46.0 VPE R

F47.0 Sector III, 3, Three

Sector IV, 4, Four

	Acceptable
F48.0	Sector <u> II, 2, Two </u> <u> R </u>

	Error
F50.0	<u> ~ P V Q </u> <u> (~ P V Q); ~ (P V Q) </u>

F51.0 No

F52.0 3.1

F53.0 ☞, Horseshoe

F54.0 Extra, Unnecessary

F55.0 Necessary

F56.0 Major connector? ✓, Wedge

Can Rule 3 be applied? No

F57.0 P

F58.0 Yes

	Acceptable
F59.0	<u> 2 times </u> <u> ☞ Number ≤ 2 </u>

F60.0 Yes (yes/no)

F61.0 Incorrect; Wrong (correct/incorrect)

F62.0 v , Wedge

F63.0 No

F64.0 Extra, Unnecessary

F65.0	<u>$P \supset (R \circ Q)$</u>	<u>Acceptable</u>
		<u>$P \supset (R \circ Q)$</u>

F66.0 Rule 3, Section 3.2; 2

F67.0 Necessary

F68.0 No

F69.0	<u>$P \supset Q$</u>	<u>Error</u>
		<u>($P \supset Q$)</u>

F70.0 Yes

F71.0	<u>3 Times</u>	<u>Acceptable</u>
		<u>0 Number 3</u>

F72.0 Yes (yes/no)

F73.0 Incorrect; Wrong (correct/incorrect)

F74.0 *Stop Time* : , to the nearest minute.
Hours Minutes

PLEASE COMPLETE THE FOLLOWING QUESTIONNAIRE AFTER YOU FINISH THE PROBLEM SOLVING TEST.

1. Have you ever worked on this type of problem before?When?Where?

Master Scoring Form
Appropriate/Erroneous

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Second Study Session

F44.0 *Start Time* _____:_____, to the nearest minute.
Hours Minutes

F45.0 Four; 4; IV | Acceptable
I, II, III, IV

F46.0 VPE R _____.

F47.0 Sector III, 3, Three _____

Sector IV, 4, Four _____

F48.0 Sector II, 2, Two | Acceptable
R

F50.0 ~PVQ _____ | Error
(~PVQ); ~(PVQ)

F51.0 Yes _____

F52.0 3.1 _____

F53.0 ∩, Horseshoe _____

F54.0 Necessary _____

F55.0 Necessary _____

F56.0 Major connector? •, Dot _____

Can Rule 3 be applied? Yes _____

F57.0 P _____

F58.0 Yes _____

F59.0 ∅, None, Zero _____

F60.0 No _____ (yes/no)

E-19

-6-

F61.0 Correct, Right (correct/incorrect)F62.0 ∇, WedgeF63.0 NoF64.0 Extra, UnnecessaryF65.0 $P \supset (R \circ Q)$ | Acceptable
 $P \supset (R \circ Q)$ F66.0 Rule 3, Section 3.2: 2F67.0 NecessaryF68.0 NoF69.0 $P \supset Q$ | Error
 $(P \supset Q)$ F70.0 YesF71.0 3 times | Acceptable
~~6~~ Number ~~4~~ 3F72.0 Yes (yes/no)F73.0 Incorrect; Wrong (correct/incorrect)F74.0 *Stop Time* : , to the nearest minute.
Hours MinutesPLEASE COMPLETE THE FOLLOWING QUESTIONNAIRE AFTER
YOU FINISH THE PROBLEM SOLVING TEST.

1. Have you ever worked on this type of problem before? When? Where?

Master Scoring Form
Erroneous/Appropriate

E-20

-5-

Second Study Session

F44.0 *Start Time* _____: _____, to the nearest minute.

	Hours	Minutes
		Acceptable
F45.0	Four; 4; IV	I, II, III, IV

F46.0 VPE R

F47.0 Sector III, 3, Three

Sector IV, 4, Four

	Acceptable
F48.0	Sector <u>II, 2, Two</u> R

	Error
F50.0	<u>~ P V Q</u> (<u>~ P V Q</u>); <u>~(P V Q)</u>

F51.0 No

F52.0 3.1

F53.0 ∩, Horseshoe

F54.0 Extra, Unnecessary

F55.0 Necessary

F56.0 Major connector? ∨, Wedge

Can Rule 3 be applied? No

F57.0 P

F58.0 Yes

	Acceptable
F59.0	<u>2 times</u> <u>∠ Number ≠ 2</u>

F60.0 Yes (yes/no)

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- F61.0 Incorrect; Wrong (correct/incorrect)
- F62.0 ∨ , Wedge
- F63.0 Yes
- F64.0 Necessary
- F65.0 R ⊃ P Error
(R ⊃ P)
- F66.0 Rule 3, Section 3.2; 2
- F67.0 Necessary
- F68.0 Yes Error
- F69.0 R • Q (R • Q)
- F70.0 Yes
- F71.0 ∅ ; None; Zero
- F72.0 No (yes/no)
- F73.0 Correct; Right (correct/incorrect)
- F74.0 *Stop Time* : , to the nearest minute.
Hours Minutes

PLEASE COMPLETE THE FOLLOWING QUESTIONNAIRE AFTER
YOU FINISH THE PROBLEM SOLVING TEST.

1. Have you ever worked on this type of problem before?When?Where?

Master Scoring Form
Rule Application Test

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RULE APPLICATION PROBLEMS

- | | |
|--|---|
| <p>1. Rule# 2 $\frac{R \vee (S \cdot T)}{(R \vee S) \cdot (R \vee T)}$
Rule# <u> </u> $\frac{\quad}{**}$</p> | <p>9. Rule# 2 $\frac{(S \vee R) \supset C}{S \vee R}$
Rule# 4 $\frac{\quad}{C}$
$\frac{\quad}{**}$</p> |
| <p>2. Rule# 4 $\frac{\sim R}{\sim R \supset (S \vee T)}$
Rule# <u> </u> $\frac{S \vee T}{**}$</p> | <p>10. Rule# 3.2 $\frac{(S \supset T) \cdot \sim R}{\sim R}$
Rule# <u> </u> $\frac{\quad}{**}$</p> |
| <p>3. Rule# 4 $\frac{(P \vee R) \cdot (P \vee S)}{P \vee R}$
Rule# $\frac{3.1}{3.2}$ $\frac{P \vee S}{P \vee (R \cdot S)}$
$\frac{\quad}{2}$ $\frac{\quad}{**}$</p> | <p>11. Rule# 1 $\frac{\sim R \cdot (Q \supset R)}{\sim R}$
Rule# $\frac{3.1}{3.2}$ $\frac{Q \supset R}{\sim R \cdot (\sim Q \vee R)}$
$\frac{\quad}{I}$ $\frac{\quad}{**}$</p> |
| <p>4. Rule# 2 $\frac{\sim(Q \cdot R) \vee (Q \cdot C)}{(Q \cdot R) \supset (Q \cdot C)}$
Rule# $\frac{1}{2}$ $\frac{\quad}{(\sim(Q \cdot R) \vee Q) \cdot (\sim(Q \cdot R) \vee C)}$</p> | <p>12. Rule# 1 $\frac{(A \vee B) \cdot (R \supset T)}{A \vee B}$
Rule $\frac{3.1}{3.2}$ $\frac{\quad}{R \supset T}$
$\frac{\quad}{**}$</p> |
| <p>5. Rule# 3.1 $\frac{(S \vee R) \cdot (S \vee T)}{S \vee R}$
Rule# <u> </u> $\frac{\quad}{**}$</p> | <p>13. Rule# 2 $\frac{(S \vee R) \cdot (S \vee Q)}{S \vee (R \cdot Q)}$
Rule# <u> </u> $\frac{\quad}{**}$</p> |
| <p>6. Rule# 4 $\frac{Q \vee (S \cdot T)}{(Q \vee S) \cdot (Q \vee T)}$
Rule# 1 $\frac{\quad}{\sim Q \supset (S \cdot T)}$
$\frac{\quad}{**}$</p> | <p>14. Rule# 3.2 $\frac{\sim Q \supset (T \cdot P)}{\sim Q}$
Rule# 4 $\frac{\quad}{T \cdot P}$
$\frac{\quad}{**}$</p> |
| <p>7. Rule# 3.1 $\frac{\sim(S \cdot R) \vee T}{(S \cdot R) \supset T}$
Rule# <u> </u> $\frac{\quad}{**}$</p> | <p>15. Rule# 4 $\frac{P \vee T}{(P \vee T) \supset \sim R}$
Rule# <u> </u> $\frac{\quad}{\sim R}$
$\frac{\quad}{**}$</p> |
| <p>8. Rule# 1 $\frac{(P \cdot R) \vee \sim R}{\sim(P \cdot R) \supset \sim R}$
Rule# <u> </u> $\frac{\quad}{**}$</p> | <p>16. Rule# 2 $\frac{T \supset (R \cdot S)}{\sim T \vee (R \cdot S)}$
Rule# 1 $\frac{\quad}{**}$</p> |

Master Scoring Form
 Problem Solving Test

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SCORING MASTER

PROBLEM SOLVING TEST

Problem 1

Given: 1. P R

2. $\sim P \vee (R \circ S)$

3. $P \supset (R \circ S)$	$\frac{R}{1}$	$\frac{L}{2}$
4. $R \circ S$	4	1,3
5. R	3	4

3. $(\sim P \vee R) \circ (\sim P \vee S)$	$\frac{R}{2}$	$\frac{L}{2}$
4. $\sim P \vee R$	3	3
5. $P \supset R$	1	4
6. R	4	1,5

3. $(\sim P \vee R) \circ (\sim P \vee S)$	$\frac{R}{2}$	$\frac{L}{2}$
4. $(P \supset R) \circ (\sim P \vee S)$	1	3
5. $P \supset R$	3	4
6. R	4	1,5

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Problem 2Given: 1. $Q \supset (P \circ T)$ $Q \supset P$

	R	L
2. $\sim Q \vee (P \circ T)$	I	I
3. $(\sim Q \vee P) \circ (\sim Q \vee T)$	2	2
4. $\sim Q \vee P$	3	3
5. $Q \supset P$	1	4

* * * *

	R	L
2. $\sim Q \vee (P \circ T)$	I	I
3. $(\sim Q \vee P) \circ (\sim Q \vee T)$	2	2
4. $(Q \supset P) \circ (\sim Q \vee T)$	1	3
5. $Q \supset P$	3	4

* * * *

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Problem 3

Given: 1. $\sim T \circ R$

2. $(\sim R \vee Q) \circ (T \vee S)$

3. $Q \supset (T \supset S)$

$\sim T \vee S$

	R	L		R	L
4. $\sim R \vee Q$	3	2	4. $(R \supset Q) \circ (T \vee S)$	1	2
5. $R \supset Q$	1	4	5. $R \supset Q$	3	4
6. R	3	1	6. R	3	1
7. Q	4	5,6	7. Q	4	5,6
8. $T \supset S$	4	3,7	8. $Q \supset (\sim T \vee S)$	1	3
9. $\sim T \vee S$	1	8	9. $\sim T \vee S$	4, 7, 8	
****			****		
4. $(R \supset Q) \circ (T \vee S)$	R	L	4. $\sim R \vee Q$	R	L
	1	2		3	2
5. $R \supset Q$	3	4	5. $R \supset Q$	1	4
6. R	3	1	6. R	3	1
7. Q	4	5,6	7. Q	4	5,6
8. $T \supset S$	4	3,7	8. $Q \supset (\sim T \vee S)$	1	3
9. $\sim T \vee S$	1	8	9. $\sim T \vee S$	4	7,8
****			****		

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Problem 4Given: 1. $P \circ T$ 2. $\sim(P \circ T) \vee (T \supset (R \circ Q))$ $T \supset Q$

		R	L			R	L
		I	2			I	2
3.	$(P \circ T) \supset (T \supset (R \circ Q))$			3.	$(P \circ T) \supset (T \supset (R \circ Q))$		
4.	$T \supset (R \circ Q)$	4	1,3	4.	$(P \circ T) \supset (\sim T \vee (R \circ Q))$	1	3
5.	$\sim T \vee (R \circ Q)$	1	4	5.	$\sim T \vee (R \circ Q)$	4	1,4
6.	$(\sim T \vee R) \circ (\sim T \vee Q)$	2	5	6.	$(\sim T \vee R) \circ (\sim T \vee Q)$	2	5
7.	$\sim T \vee Q$	3	6	7.	$(\sim T \vee R) \circ (T \supset Q)$	1	6
8.	$T \supset Q$	1	7	8.	$T \supset Q$	3	7
*****				*****			
3.	$(P \circ T) \supset (T \supset (R \circ Q))$	R	L	3.	$(P \circ T) \supset (T \supset (R \circ Q))$	R	L
		I	2			I	2
4.	$(P \circ T) \supset (\sim T \vee (R \circ Q))$	1	3	4.	$(P \circ T) \supset (\sim T \vee (R \circ Q))$	1	3
5.	$(P \circ T) \supset ((\sim T \vee R) \circ (\sim T \vee Q))$	2	4	5.	$\sim T \vee (R \circ Q)$	4	1,4
6.	$(\sim T \vee R) \circ (\sim T \vee Q)$	4	1,5	6.	$(\sim T \vee R) \circ (\sim T \vee Q)$	2	5
7.	$\sim T \vee Q$	3	6	7.	$\sim T \vee Q$	3	6
8.	$T \supset Q$	1	7	8.	$T \supset Q$	1	7
*****				*****			
3.	$(P \circ T) \supset (T \supset (R \circ Q))$	R	L	3.	$(P \circ T) \supset (T \supset (R \circ Q))$	R	L
		I	2			I	2
4.	$(P \circ T) \supset (\sim T \vee (R \circ Q))$	1	3	4.	$(P \circ T) \supset (\sim T \vee (R \circ Q))$	1	3
5.	$(P \circ T) \supset ((\sim T \vee R) \circ (\sim T \vee Q))$	2	4	5.	$(P \circ T) \supset ((\sim T \vee R) \circ (\sim T \vee Q))$	2	4
6.	$(\sim T \vee R) \circ (\sim T \vee Q)$	4	1,5	6.	$(P \circ T) \supset ((\sim T \vee R) \circ (T \supset Q))$	1	5
7.	$(\sim T \vee R) \circ (T \supset Q)$	1	6	7.	$(\sim T \vee R) \circ (T \supset Q)$	4	1,6
8.	$T \supset Q$	3	7	8.	$T \supset Q$	3	7
*****				*****			

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Problem 4
(continued)

Given: 1. $P \circ T$ 2. $\sim(P \circ T) \vee (T \supset (R \circ Q))$ $T \supset Q$

	R	L		R	L
3. $(P \circ T) \supset (T \supset (R \circ Q))$	I	2	3. $\sim(P \circ T) \vee (\sim T \vee (R \circ Q))$	I	2
4. $(P \circ T) \supset (\sim T \vee (R \circ Q))$	1	3	4. $\sim(P \circ T) \vee ((\sim T \vee R) \circ (\sim T \vee Q))$	2	3
5. $\sim T \vee (R \circ Q)$	4	1,4	5. $\sim(P \circ T) \vee ((\sim T \vee R) \circ (T \supset Q))$	1	4
6. $(\sim T \vee R) \circ (\sim T \vee Q)$	2	5	6. $(P \circ T) \supset ((\sim T \vee R) \circ (T \supset Q))$	1	5
7. $(\sim T \vee R) \circ (T \supset Q)$	1	6	7. $(\sim T \vee R) \circ (T \supset Q)$	4	1,6
8. $T \supset Q$	3	7	8. $T \supset Q$	3	7
* * * *			* * * *		
3. $\sim(P \circ T) \vee (\sim T \vee (R \circ Q))$	R	L	3. $\sim(P \circ T) \vee (\sim T \vee (R \circ Q))$	1	2
4. $\sim(P \circ T) \vee ((\sim T \vee R) \circ (\sim T \vee Q))$	I	2	4. $\sim(P \circ T) \vee ((\sim T \vee R) \circ (\sim T \vee Q))$	2	3
5. $(P \circ T) \supset ((\sim T \vee R) \circ (\sim T \vee Q))$	2	3	5. $(P \circ T) \supset ((\sim T \vee R) \circ (\sim T \vee Q))$	1	4
6. $(\sim T \vee R) \circ (\sim T \vee Q)$	1	4	6. $(\sim T \vee R) \circ (\sim T \vee Q)$	4	1,5
7. $\sim T \vee Q$	4	1,5	7. $(\sim T \vee R) \circ (T \supset Q)$	1	6
8. $T \supset Q$	3	6	8. $T \supset Q$	3	7
* * * *			* * * *		
3. $\sim(P \circ T) \vee (\sim T \vee (R \circ Q))$	R	L			
4. $\sim(P \circ T) \vee ((\sim T \vee R) \circ (\sim T \vee Q))$	I	2			
5. $(P \circ T) \supset ((\sim T \vee R) \circ (\sim T \vee Q))$	1	4			
6. $(P \circ T) \supset ((\sim T \vee R) \circ (T \supset Q))$	2	3			
7. $(\sim T \vee R) \circ (T \supset Q)$	1	5			
8. $T \supset Q$	4	1,6			
	3	7			

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