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Covariant Soliton Dynamics

by

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Abstract

COVARIANT SOLITON DYNAMICS

by

Ramesh Babu Thayyullathil

Advisor: Professor Carl M. Shakin

We provide a fully covariant analysis of a nontopological soliton model of haron structure and make application to the structure of various mesons. We study the rho and omega mesons, charmonium, and upsilon system. The model describes quarks coupled to a scalar field which plays the role of an order parameter of the QCD vacuum. There are a few parameters in this model, a flavor-dependent constituent quark mass, a mass parameter for the scalar field, a coupling constant which determines the strength of the coupling of the quarks to the scalar field, and a cut-off parameter. The mass parameter of the scalar field and the scalar-quark coupling constant are taken from our study of nucleon structure. Therefore, once a value is chosen for the high-momentum cut-off, only single parameter is varied in this analysis, the flavor-dependent (constituent) quark mass. A resonably good fit is obtained to a series of mesonic states of quite different masses in this extremely simple model, indicating

that a unified aproach to hadron structure is possible. (At this point, we have not attempted to model the confinement mechanism. Further, our Hamiltonian has continuum solutions and given our method of calculation, these solutions prevent us from studying all but the low-lying states of charmonium and the upsilon system, for example.)

We have also modified our Lagrangian in order to study gluon-exchange effects, however, the study of such effect requires the introduction of additional parameters. By fitting these new parameters to the mass splitting of the lowest 0^- and 1^- states of the charmonium system, we are able to make prediction for corresponding splitting in the upsilon system.

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CHAPTER I

1.1 Introduction

The vast amount of experimental data collected in the last one or two decades supports the theoretical concept that hadrons, on a subnuclear scale, manifest substructures which are called quarks after Gell-Mann and Zweig [1,2]. There have been many attempts to derive strong interaction physics from the fundamental interaction between quarks.

A very promising candidate for a theory of strong interaction is quantum chromodynamics (QCD), which is the physics of quarks and gluons. This is a nonabelian gauge theory [3] with gluons as gauge bosons. Unlike QED, this theory is highly nonlinear due to the fact that gluons exhibit self interaction, and carry color charge. This is a general feature of any nonabelian gauge theory. Quantum chromodynamics has the remarkable property of being aymptotically free [4]. This feature leads to vanishing of the effective coupling constant at high momenta and gives the correct description of Bjorken scaling and many other high-energy process [5,6].

On the other hand the low-energy behavior of this theory is not well understood. There has been some progress in understanding

QCD obtained recently from Lattice Gauge Theory calculations. Such conclusions are still being developed and extended.

Before we go to the description of various models it is worth noting that the QED, the very well understood theory of charged particles interacting with the electromagnetic field, has its own difficulties when applied to a bound-state problem. It is possible to solve a bound-state problem by partially summing a class of diagrams in the theory [7], however a full fledged field theory of positronium is definitely beyond the scope of the present formalism.

Recent developments in nonlinear physics has led many to think of bound states as solitons [8]. Solitons are finite energy, spacially localized and nondissipative solutions to a certain class of covariant nonlinear field equations. Even though a soliton has an extended structure, we can provide a covariant model and can make Lorentz transformations. The velocity of a particle can be taken as the velocity of the soliton rest frame, for example.

Now in strong interaction physics the difficulties are twofold. First, there is the problem of finding the dynamics and second, the problem of describing the bound states. The choice of the right degrees of freedom for a dynamics is very important as can be seen in the case of the QED. There, starting from a simple Bohr model we are able to go to the Dirac description and finally to a calculation of the Lamb shift in a smooth way. We are able to do this because we have the right starting point. Also the standard model of Weinberg-Salam gives the low-energy limit which is the same as the phenomenological description of the Fermi theory of beta decay. In the case of strong interaction physics the developments are to some extent in a reverse order.

In QCD we start with a theory with right asymptotic limit. In that limit the relevant degrees of freedom are quarks and gluons and the theory is comparitively well understood through perturbation techniques. From here we are trying to interpolate it to a regime where the relavent degrees of freedom are unknown.

This has a consequence that there are "too many" phenomenological descriptions of hadrons. The prototypes are the bag model [9] and the various recent modifications of the model [10]. One unsatisfactory feature of such models is their noncovariance. Even if one can make corrections to the static aproximation such corrections are either involved or ambiguous.

The importance of translational invariance for a model of nucleon structure has been discussed earlier [11]. In this model a fully covariant analysis of a simplified version of Friedberg-Lee nontopological soliton model [12] was made and the fit to nucleon observables obtained was quite good. In a covariant analysis one is able to calculate the properties of the nucleon when the nucleon is in a nucleus [13]. With these modified nucleon properties one was able to explain the EMC effect and the quenching of the longitudinal response observed in (e , e') inclusive reactions near nucleon quasi-elastic peak [14].

Here we apply the same simple nontopological soliton model to study the structure of mesons. In this model the quarks are coupled to a scalar field which plays the role of an order parameter of the QCD vacuum. (This is a formalism somewhat analogous to the Ginzburg-Landau theory of superconductivity, where an order parameter is used in the description of the system [15].)

The analysis is carried forward through the definition of various covariant amplitudes and the specification of an integral equation which determines these amplitudes. Even though we truncate our Hilbert space by considering meson states, quark states and antiquark states only, we do not violate translational invariance.

We organize this thesis as follows: In section 1.2 we describe various Dirac matrices and spinors and the metric used. In Chapter 2 we present the Lagrangian and the field equations required for our analysis. In Chapter 3 we introduced various invariant amplitudes and in Chapter 4 we present form factors. In chaptor 5 we study the integral equation for the various covariant amplitudes. Finally, in Chapter 6 we present our method of solution, numerical results, and conclusions.

1.2 Dirac Matrices and Notation:

We use the Dirac matrices and metric as in the texts of Bjorken and Drell [16]. Thus we have,

$$g^{\mu\nu} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \tag{1.1}$$

In our notation all repeated indices are summed. For any two four vectors A^{μ} and B^{μ} we denote the scalar product by

$$A \bullet B = g_{\mu\nu} A^{\mu} B^{\nu} = A^{\mu} B_{\mu} = A^{o} B^{o} - \vec{A} \bullet \vec{B}, \tag{1.2}$$

$$A^{2} = A \bullet A = A^{\mu} A_{\mu} = (A^{o})^{2} - \vec{A}^{2}. \tag{1.3}$$

For the Dirac matrices we used the explicit representation

$$\gamma^{\mu} = (\gamma^{o} , \vec{\gamma}) \tag{1.4}$$

where

$$\gamma^{o} = \begin{bmatrix} I & 0 \\ 0 & -I \end{bmatrix}, \quad (1.5)$$

$$\vec{\gamma} = \begin{bmatrix} 0 & \vec{\sigma} \\ -\vec{\sigma} & 0 \end{bmatrix}, \quad (1.6)$$

and for γ^5 we have,

$$\gamma^{5} = \begin{bmatrix} 0 & I \\ I & 0 \end{bmatrix}. \quad (1.7)$$

Here I is 2×2 unit matrix and σ is the 2×2 Pauli spin matrices. Thus for any four vector A^{μ} we have,

$$\not{A} = A\cdot\gamma = A^{\mu}\gamma_{\mu} = \begin{bmatrix} A^{o} & -\vec{A}.\vec{\sigma} \\ \vec{A}.\vec{\sigma} & A^{o} \end{bmatrix}. \quad (1.8)$$

We denote the positive and negative-energy spinor for a Dirac particle with mass m_q and momentum $\vec{K}$ by $U(\vec{K},s)$ and $V(\vec{K},s)$ respectively. Thus we have,

$$U(\vec{K},s) = [\varepsilon_q(\vec{K})/(2m_q)]^{\frac{1}{2}} \begin{bmatrix} \chi_s \\ \{\vec{\sigma}\cdot\vec{K}/\varepsilon_q(\vec{K})\}\chi_s \end{bmatrix}, \quad (1.9)$$

$$V(\vec{k},s) = [\varepsilon_q(\vec{k})/(2m_q)]^{\frac{1}{2}} \begin{bmatrix} \{\vec{\sigma}.\vec{k}/\varepsilon_q(k)\}\chi_{-s} \\ \\ \chi_{-s} \end{bmatrix} \qquad (1.10)$$

With + denoting the hermitian conjugate we also have,

$$\overline{U}(\vec{k},s) = [U(\vec{k},s)]^{+}\gamma^{o}, \qquad (1.11)$$

$$\overline{V}(\vec{k},s) = [V(\vec{k},s)]^{+}\gamma^{o}. \qquad (1.12)$$

Here χ_s is the two-component Pauli spinor with spin projection s and

$$\varepsilon_q(\vec{k}) = (m_q^2 + \vec{k}^2)^{\frac{1}{2}} + m_q \; . \qquad (1.13)$$

We denote the Dirac indices by Greek letters and the isospin indices by i, j, k. We also denote the magnitude of a three vector, $\vec{V}$ by V itself when it can be done without ambiguity.

CHAPTER II

The Lagrangian and Hamiltonian

Here we discuss the model Lagrangian we used in our analysis. In order to make the theory fully covariant we start from a (Lorentz) scalar Lagrangian. First we consider a simple Lagrangian density where quarks are coupled to a scalar field. Later we will discuss the Lagrangian for the theory with gluonic degrees of freedom.

2.1 The Lagragian and the Operator Equations of Motion:

The Lagrangian density of our model is

$$\mathcal{L}(x) = \bar{q}(x)[\, i\gamma^{\mu}\partial_{\mu} - m_{q} - g_{\chi}\, \chi(x)\,]q(x) + (1/2)[\, \partial_{\mu}\chi(x)\partial^{\mu}\chi(x) - m_{\chi}^{2}\, \chi^{2}(x)\,]. \quad (2.1)$$

Here $q(x)$ is the Dirac-field operator and $\chi(x)$ the scalar field. The Lagrangian we are considering can be thought of as an effective Lagrangian describing low-energy hadron physics [15]. In other words this is a theory where the Hamiltonian is is written in terms of an order-parameter field, $\chi(x)$, which effectively describes the gross feature of low-energy hadron physics. (As we mentioned earlier in Chapter 1 this analysis analogous to the Ginzburg-Landau theory of superconductivity.)

The parameter m_q in the Lagrangian density of Eq.(2.1) has to be taken as effective mass rather than the current quark mass, which is about 5-10 Mev for up and down quarks. This effective quark mass is thought to have its origin through the formation of vacuum condensate. To demonstrate this we rewrite our Lagrangian using the transformation,

$$\phi(x) = \phi_{vac} + \chi(x) \tag{2.2}$$

where $\chi(x)$ represents the fluctuation of the ϕ field around the vacuum value ϕ_{vac}. Using Eq.(2.2) in Eq.(2.1) the Lagrangian density becomes,

$$\mathcal{L}(x) = \bar{q}(x)[\, i\gamma^{\mu}\partial_{\mu} - m_q^{cur} - g_{\chi}\, \phi(x)\,]q(x) + (1/2)[\, \partial_{\mu}\phi(x)\partial^{\mu}\phi(x) - m_{\chi}^{2}\, \{\phi(x) - \phi_{vac}\}^{2}\,], \tag{2.3}$$

where

$$m_q^{cur} = m_q - g_{\chi}\phi_{vac}. \tag{2.4}$$

This new quark mass parameter m_q^{cur} can be interpreted as the current quark mass.

It is important to note that even in the limit $m_q^{cur} = 0$ the Lagrangian given by Eq.(2.3) is not invariant under chiral transformation. Thus the chiral symmetry is broken at the outset.

Eventually we would like to describe the pion in our model as a composite particle. Here we do not consider the pion as the chiral partner of the field $\chi(x)$. Thus to restore the chiral symmetry to our model we would have to introduce some unphysical pseudoscalar particle in our formalism. This would introduce more parameters and we have not considered such an extension of our model.

It is worthwhile to note that the Lagrangian given by Eq.(2.1) is the simplest one can consider for an interacting fermion systems with minimum number of parameters. Our goal is to analyse this Lagrangian in a fully covariant way. We did not attempt to make any detailed fit of various observables present in different mesonic systems. We do wish to demonstrate how the implementation of the right Lorentz transformation properties simplifies the analysis. We also demonstrate how the various physical quantities can be calculated in a less ambigous way in this formalism.

The Lagrangian density in Eq.(2.1), and the associated Euler-Lagrange equations, yield the equations of motion:

$$[i\gamma_{\mu}\partial^{\mu} - m_{q}]q(x) = g_{\chi}\, q(x)\chi(x), \qquad (2.5)$$

$$[\partial_{\mu}\partial^{\mu} + m_{\chi}^{2}]\chi(x) = -g_{\chi}\, \bar{q}(x)q(x). \qquad (2.6)$$

As in the case of all field-theoretic operator equations, to get a meaningful expression we have to take the matrix elements of these equations. (This is done in Chapter 5.) In the limit $g_\chi = 0$ we get the free field equations. In this limit note that the parameter entering in the free Dirac equations given by Eq.(2.5) is the effective quark mass m_q. Thus the mass appearing in various spinors in our analysis will be m_q. Also note that we have no confinement mechanism in our model.

It is clear from Eqs.(2.5) and (2.6) that the quark scalar density is the source of χ field and the coupling of quark field with the scalar field renders the equations of motion highly nonlinear. Our non-perturbative analysis of these equations yield nonlinear equations for the soliton structure and will require that we do the calculations in a self-consistent manner. The details of such calculation are given in Chapters 5 and 6.

2.2 Lagrangian with Gluon Degrees of Freedom:

The inclusion of gluonic degrees of freedom to the Lagrangian described by Eq.(2.1) is ambiguous for various reasons. The main problem is overcounting. The expression given by Eq.(2.1) is considered to be an effective Lagrangian density derived from QCD after integrating out the relevant gluonic degrees of freedom, but at the same time we consider some "gluon-exchange" effects. One way to overcome this ambiguity is to assume that the expression in Eq.(2.1) effectively represent the multigluon [15] and nonlinear gauge coupling in QCD. Thus we add to the expression in Eq.(2.1) a gluon coupling term and neglet terms describing the self-interaction of gluons. Thus we have,

$$\mathcal{L}(x) = \bar{q}(x)[\, i\gamma^{\mu}\partial_{\mu} - m_q - g_{\chi}\chi(x)\,]q(x)$$
$$+(1/2)[\, \partial_{\mu}\chi(x)\partial^{\mu}\chi(x) - m_{\chi}^2\, \chi^2(x)\,]$$
$$-g\, \bar{q}(x)\gamma^{\mu}(\lambda^a/2)q(x)\, A_{\mu}^a(x) \;\; -(1/4)F^{\mu\nu a}(x)F_{\mu\nu}^{\;\;a}(x), \qquad (2.7)$$

where

$$F^{\mu\nu a}(x) = \partial^{\mu}A^{\nu a}(x) \;\; -\partial^{\nu}A^{\mu a}(x). \qquad (2.8)$$

Here a is the color index (a = 1,....8), g is the coupling of quark field with gluon field $A^{\mu a}(x)$, and $\lambda^a/2$ is the (matrix) generator of color SU(3). Again using the Euler-Lagrange equation we have,

$$[i\gamma_{\mu}\partial^{\mu} - m_q\,]q(x) = g_{\chi}\; q(x)\chi(x) \;\; -g\;\gamma^{\mu}(\lambda^a/2)q(x)A_{\mu}^a(x), \qquad (2.9)$$

$$[\partial_{\mu}\partial^{\mu} \;\; +m_{\chi}^2\,]\chi(x) = -g_{\chi}\; \bar{q}(x)q(x), \qquad (2.10)$$

$$\partial_{\nu}\partial^{\nu}A_{\mu}^a(x) = g\; \bar{q}(x)(\lambda^a/2)\gamma_{\mu}q(x). \qquad (2.11)$$

Here the quark color current is the source of gluon field, as can be seen from Eq.(2.10). These equations are analysed in Appendix E.

2.3 The Hamiltonian:

From the Lagrangian densites described by Eqs.(2.1) and (2.7) it is easy to construct the Hamiltonian densities using the usual canonical procedure. For a Lagrangian density $\mathcal{L}(\phi, \partial_\mu \phi)$ we define the canonical mometum as usual,

$$\pi(x) = \partial \mathcal{L}(x)/\partial \dot{\phi}(x), \tag{2.12}$$

where

$$\dot{\phi}(x) = \partial_t \phi(x) \ . \tag{2.13}$$

Then the Hamiltonian density is given by

$$\mathcal{H}(x) = \dot{\phi}(x)\pi(x) - \mathcal{L}(x). \tag{2.14}$$

Now using Eqs.(2.12) and (2.13) and the Lagrangian described by Eq.(2.1) we have,

$$\mathcal{H}(x) = \bar{q}(x)[-i\vec{\gamma}\cdot\vec{\nabla} + m_q + g_\chi \chi(x) \ ]q(x)$$

$$+(1/2) \ [\ \dot{\chi}^2(x) + |\nabla\chi(x)|^2 + m_\chi^2 \ \chi^2(x) \ ]. \tag{2.15}$$

The Hamiltonian H is given by

$$H = \int d\vec{x}\, \mathcal{H}(x)$$

$$= \int d\vec{x}\, \{\bar{q}(x)[-i\vec{\gamma}\cdot\vec{\nabla} + m_q + g_\chi \chi(x)\ ]q(x)$$

$$+(1/2)[\ \dot{\chi}^2(x) + |\vec{\nabla}\chi(x)|^2 + m_\chi^2\ \chi^2(x)\ ]\ \}, \qquad (2.16)$$

where

$$\dot{\chi}(x) = \partial_t \chi(x). \qquad (2.17)$$

Using the equations of motion given by Eq.(2.5) this can be written as,

$$H = \int d\vec{x}\, \{\bar{q}(x)\ (i\gamma^o)\ \dot{q}(x)$$

$$+(1/2)\ [\ \dot{\chi}^2(x) + |\vec{\nabla}\chi(x)|^2 + m_\chi^2\ \chi^2(x)\ ]\ \}, \qquad (2.18)$$

where

$$\dot{q}(x) = \partial_t q(x). \qquad (2.19)$$

The evaluation of this Hamiltonian is given in Appendix D.

An exactly similar calculation using the Lagrangian density in Eq.(2.7) with gluonic interaction gives a Hamiltonian H_G which is,

$$H_G = \int d\vec{x}\, \{\ \bar{q}(x)\ (i\gamma^o)\ \dot{q}(x)$$

$$+ (1/2)\ [\ \dot{\chi}^2(x) + |\vec{\nabla}\chi(x)|^2 + m_\chi^2\ \chi^2(x)\ ]$$

$$+ F^{\mu\nu a}(x)F_{\mu\nu}^{\ a}(x)/4 \quad -F_{o\nu}^{a}(x)A^{\nu a}(x)\ \} \qquad (2.20)$$

we write,

$$H_G = H + H'_G, \qquad (2.21)$$

where H is given by Eq.(2.18) and

$$H'_G = \int d\vec{x} \, [\, F^{\mu\nu a}(x) F_{\mu\nu}^{\ a}(x)/4 \quad - F_{o\nu}^{a}(x) A^{\nu a}(x) \,] \, . \qquad (2.22)$$

The evaluation of this Hamiltonian is described in Appendix E.

CHAPTER III

Invariant Amplitudes

In order to analyse the field equations obtained from the Lagrangian we have to define the matrix elements of various operators. Here we specify the matrix element of the quark field operator in a manifestly covariant manner. The matrix element we are considering is the decay amplitude for a meson to go into an off-shell quark and an on-shell antiquark. The amplitude for a meson going into an off-shell antiquark and on-shell quark can be obtained from the previous amplitude by charge conjugation. Before we go into the definition of these amplitudes we specify our notation and the normalization of state vectors.

3.1 Normalization of State Vectors:

We denote a quark state by $|\vec{k}\ s\ t\rangle$, where $\vec{k}$ is the quark momentum, s the spin projection and t the isospin projection. The corresponding antiquark state is denoted by $|\overline{\vec{k}\ s\ t}\rangle$. When we consider the quarks without isospin we will drop the isospin index. For these states we choose the normalization:

$$\langle \vec{k}'\ s'\ t' |\ \vec{k}\ s\ t\rangle = \delta_{s's}\delta_{t't}\delta(\ \vec{k}'-\vec{k}\ ), \tag{3.1}$$

$$\langle \overline{\vec{k}'\ s'\ t'} |\ \overline{\vec{k}\ s\ t}\rangle = \delta_{s's}\delta_{t't}\delta(\ \vec{k}'-\vec{k}\ ). \tag{3.2}$$

For pseudoscalar mesons we denote the state by $|\vec{p}\, M_T\rangle$ where $\vec{p}$ is the meson momentum and M_T is the isospin projection. The isospin index will be absent when we consider particles having no isospin. Thus we have,

$$\langle \vec{p}'\, M'_T |\, \vec{p}\, M_T\rangle = \delta_{M_T M'_T}\, \delta(\, \vec{p}'-\vec{p}\,)\,. \tag{3.3}$$

In the case of vector mesons with momentum $\vec{p}$, helicity projection λ, and isospin projection M_T, we denote the state by $|\vec{p}\, \lambda\, M_T\rangle$. We have,

$$\langle \vec{p}'\, \lambda'\, M'_T |\, \vec{p}\, \lambda\, M_T\rangle = \delta_{\lambda'\lambda}\, \delta_{M_T M'_T}\, \delta(\, \vec{p}'-\vec{p}\,)\,. \tag{3.4}$$

The choice of a helicity basis instead of the canonical basis is due to the simplicity of relativistic analysis in this basis. Now we define various invariant amplitudes for pseudoscalar and vector mesons.

3.2 Invariant Amplitude for Pseudoscalar Mesons:

Here we consider the decay of a pseudoscalar meson into an on-shell quark and an off-shell antiquark. The relevant matrix element is $\langle \vec{k}\; s\; t | \bar{q}_{\alpha i}(0) | \vec{p}\, M_T\rangle$ where $q_{\alpha i}(0)$ is the quark field operator with α and i as Dirac and isospin indices respectively. We want to represent this amplitude in a manifestly covariant way. We denote the meson mass by m and quark mass by m_q. Thus we have,

$$p^2 = p^\mu p_\mu = m^2, \tag{3.5}$$

$$k^2 = k^\mu k_\mu = m_q^2, \tag{3.6}$$

$$p^o = \omega(\vec{p}) = (m^2 + \vec{p}^2)^{\frac{1}{2}}, \tag{3.7}$$

$$k^o = E_q(\vec{k}) = (m_q^2 + \vec{k}^2)^{\frac{1}{2}}. \tag{3.8}$$

Now from the available Lorentz tensors, and using the fact that

$$\not{k}\ U(\vec{k},s) = m_q\ U(\vec{k},s), \tag{3.9}$$

where $U(\vec{k},s)$ is the positive-energy spinor solution of the free Dirac equation, we can write

$$\begin{aligned} \langle \vec{k}\ s\ t|\overline{q}_{\alpha i}(0)|\vec{p}\ M_T\rangle = {} & [1/(2\pi)^3][1/2\omega(\vec{p})]^{\frac{1}{2}}[m_q/E_q(\vec{k})]^{\frac{1}{2}} \\ & \times[\ \overline{U}(\vec{k},s)(A+B\ \not{p}/m)\gamma^5]_\alpha\ [\chi_t^+ \vec{\tau}\cdot\hat{e}_{M_T}]_i. \end{aligned} \tag{3.10}$$

Here $\vec{\tau}$ and χ_t are Pauli matrices and Pauli spinor with projection t respectively and $\hat{e}_{M_T}$ is a unit vector in the spherical basis. Also , the two independent amplitudes A and B are Lorentz scalar functions.

Since the meson and one quark are on mass shell, the only available Lorentz scalar variable is p•k. For later convenience we choose the amplitudes to be functions of $[(p\bullet k/m_q)^2-m^2]^{\frac{1}{2}}$. The factor $[1/(2\pi)^3][1/2\omega(\vec{p})]^{\frac{1}{2}}[m_q/E_q(\vec{k})]^{\frac{1}{2}}$ in Eq.(3.10) is due to the normalization given in Eqs.(3.1)-(3.3). Thus the invariant structure of Eq.(3.10) is very clear. The appearence two more independent amplitudes without a γ^5 factor is forbidden by parity considerations (See Appendix A). Also the amplitudes with a $\not{k}$ factor can be eliminated by using Eq.(3.9). Thus, Eq.(3.10) is the most general expression we can write.

Now using charge conjugation (See Appendix B) we can write

$$\overline{<\vec{k}\ s\ t|} q_{\alpha i}(0)|\vec{p}\ M_T> = [1/(2\pi)^3][1/2\omega(\vec{p})]^{\frac{1}{2}}[m_q/E_q(\vec{k})]^{\frac{1}{2}}$$

$$\times[\ \gamma^5(E+F\not{p}/m)V(\vec{k},s)]_\alpha\ [(\ \vec{\tau}\bullet\hat{e}^*_{M_T})^T\chi_{-t}\eta_{-t}]_i, \tag{3.11}$$

where E = A, F = -B, and η_t is a phase factor. Here $V(\vec{k},s)$ is the negative-energy spinor solution of the free Dirac equation. Again the scalar invariants E and F are functions of $[(p\bullet k/m_q)^2-m^2]^{\frac{1}{2}}$.

Our whole analysis depend on the structure of the amplitude Eqs.(3.10) and (3.11), so it is important to look at these amplitudes more closely. In the meson rest frame, $\vec{p}$ = 0, the amplitude given in Eq.(3.11) becomes:

$$\overline{\langle\vec{k}\ s\ t}|q(0)|\ \vec{p}=0\ M_T\rangle = [1/(2\pi)^3]\{\varepsilon_q(\vec{k})/[E_q(\vec{k})4m]\}^{\frac{1}{2}}$$

$$\times\begin{bmatrix}\{E(k')-F(k')\}\chi_{-s}\\ \\ \{E(k')+F(k')\}\{k/\varepsilon_q(\vec{k})\}\ \vec{\sigma}\cdot\hat{k}\ \chi_{-s}\end{bmatrix}\times[(\vec{\tau}\cdot\hat{e}^*_{M_T})^T\chi_{-t}\eta_{-t}], \tag{3.12}$$

where $k = |\vec{k}|$, $\varepsilon_q(\vec{k}) = E_q(\vec{k})+m_q$ and χ_{-s} is a two-component Pauli spinor with projection -s. The dependence of E and F on $k' = mk/m_q$ is due to the fact that they are functions of $[(p\cdot k/m_q)^2-m^2]^{\frac{1}{2}}$. If we compare the structure of Eq.(3.12) with the Dirac spinor of the hydrogen atom problem the physical interpretation of various terms is very clear. Thus we define wave functions as follows,

$$\hat{R}_u(k) = \{[4\pi/\{m(2\pi)^3\}][\varepsilon_q(\vec{k})/E_q(\vec{k})]\}^{\frac{1}{2}}\{E(k')-F(k')\}, \tag{E.13}$$

$$\hat{R}_l(k) = \{[4\pi/\{m(2\pi)^3\}][\varepsilon_q(\vec{k})/E_q(\vec{k})]\}^{\frac{1}{2}}\{E(k')+F(k')\}\ k/\varepsilon_q(\vec{k}) \tag{3.14}$$

and we choose the normalization:

$$\int dk\quad \vec{k}^2[\{\hat{R}_u(k)\}^2+\{\hat{R}_l(k)\}^2] = 1. \tag{3.15}$$

Here the integration is over the variable $k=|\vec{k}|$. The choice of this normalization gives the correct charge for the meson. (See Appendix C).

3.3 Invariant Amplitude for Vector Mesons:

Here we consider the decay of a vector meson into an on-shell quark and an off-shell antiquark. The description of this amplitude is more involved due to the spin of the meson. This is because of the presence of the the polarization vector of the meson, which makes an additional Lorentz vector available to construct various invariant amplitudes. We denote the polarization of a vector meson with momentum $\vec{p}$ and helicity projection λ by $\xi_\lambda^{\ \mu}$. Thus we have,

$$\xi_\lambda^{\ \mu} = \xi_\lambda^{\ \mu}(\vec{p}). \tag{3.16}$$

These vectors have the following properties [17]:

$$\xi_{\lambda'}^{*} \cdot \xi_\lambda = \xi_{\lambda'\mu}^{*} \ \xi_\lambda^{\ \mu} = -\delta_{\lambda'\lambda}, \tag{3.17}$$

$$\xi_\lambda \cdot p = \xi_\lambda^{\ \mu} \ p_\mu = 0, \tag{3.18}$$

$$\Sigma_\lambda \quad \xi_\lambda^{*\mu} \ \xi_\lambda^{\ \nu} = -g^{\mu\nu} \ + p^\mu p^\nu / m^2. \tag{3.19}$$

It is useful to note that in the meson rest frame, p = 0 we have,

$$\xi_\lambda^{\ \mu} = (\ 0, \ \vec{\xi}_\lambda \ ). \tag{3.20}$$

A convenient representation of these polarization vectors is given by

$$\xi_\pm^{\ \mu} = \{ \ 0, \ -[\ \pm\hat{e}_1(\hat{p}) \ + \ \hat{e}_2(\hat{p})]/\surd 2\}, \tag{3.21}$$

$$\xi_o^\mu = \{|\vec{p}|/m \ , \ \hat{p}\ \omega(\vec{p})/m\}. \tag{3.22}$$

Here $\hat{p}$ is unit vector in the direction of $\vec{p}$ and $\hat{p}$, $\hat{e}_1(\hat{p})$ and $\hat{e}_2(\vec{p})$ form three mutually-orthogonal unit vectors. Consistent with the parity transformation (See Appendix A) and the relation in Eq. (3.18), we can write

$$\langle \vec{k}\ s\ t|\bar{q}_{\alpha i}(0)|\vec{p}\ \lambda\ M_T\rangle = [1/(2\pi)^3][1/2\omega(\vec{p})]^{\frac{1}{2}}[m_q/E_q(\vec{k})]^{\frac{1}{2}}$$
$$\times\{\ \bar{U}(\vec{k},s)[\xi_\lambda\bullet k/m_q\ (C+D\ \not{p}/m)+\not{\xi}_\lambda(\tilde{A}+\tilde{B}\ \not{p}/m)]\}_\alpha[\chi_t^+\ \vec{\tau}\bullet\hat{e}_{M_T}]_i. \tag{3.23}$$

Again the invariant functions C, D, $\tilde{A}$, and $\tilde{B}$ are functions of $[(p\bullet k/m_q)^2-m^2]^{\frac{1}{2}}$. The expression in Eq. (3.23) is the most general relation one can write which is consistent with general invariance of the matrix element. Now using charge conjugation (See Appendix B) we can write

$$\overline{\langle \vec{k}\ s\ t|}q_{\alpha i}(0)|\vec{p}\ \lambda\ M_T\rangle = -\ [1/(2\pi)^3][1/2\omega(\vec{p})]^{\frac{1}{2}}[m_q/E_q(\vec{k})]^{\frac{1}{2}}$$
$$\times\{[(\xi_\lambda\bullet k/m_q)(C-D\ \not{p}/m)-\not{\xi}_\lambda(\tilde{A}+\tilde{B}\ \not{p}/m)]V(\vec{k},s)\}_\alpha[(\ \vec{\tau}\bullet\hat{e}^*_{M_T})^T\chi_{-t}\eta_{-t}]_i. \tag{3.24}$$

We now look at the amplitude in the meson rest frame, $\vec{p} = 0$. To get a multiplet structure in the meson rest frame we arrange our amplitude so that

$$A_1 = C = D = -(\tilde{A}+\tilde{B})/[1+p\cdot k/(mm_q)] \ . \tag{3.25}$$

With this aproximation the amplitude given in Eq.(3.24) in the meson rest frame, $\vec{p} = 0$ becomes

$$\langle\overline{\vec{k}\ s\ t}|q(0)|\ \vec{p} = 0\ \lambda\ M_T\ \rangle = [1/(2\pi)^3]\{\varepsilon_q(\vec{k})/[E_q(\vec{k})4m]\}^{\frac{1}{2}}$$

$$\times \begin{bmatrix} \{\tilde{A}(k') - \tilde{B}(k')\}\ \vec{\xi}_\lambda\cdot\vec{\sigma}\ \chi_{-s} \\ \\ \{\tilde{A}(k') + \tilde{B}(k')\}\{k/\varepsilon_q(\vec{k})\}\ \vec{\sigma}\cdot\hat{k}\vec{\xi}_\lambda\cdot\vec{\sigma}\ \chi_{-s} \end{bmatrix} \times [(\vec{\tau}\cdot\hat{e}^{*}_{M_T})^T \chi_{-t}\eta_{-t}] . \tag{3.26}$$

Note that the structure given by Eq.(3.26) is valid only if we make the aproximation given in Eq.(3.25). From Eq.(3.26) we can identify the various wave functions. Thus we have,

$$\hat{R}_u(k) = \{[4\pi/\{m(2\pi)^3\}][\varepsilon_q(\vec{k})/E_q(\vec{k})]\}^{\frac{1}{2}}\{\tilde{A}(k') - \tilde{B}(k')\}, \tag{3.27}$$

$$\hat{R}_l(k) = \{[4\pi/\{m(2\pi)^3\}][\varepsilon_q(\vec{k})/E_q(\vec{k})]\}^{\frac{1}{2}}\{\tilde{A}(k') + \tilde{B}(k')\}\ k/\varepsilon_q(\vec{k}) \ . \tag{3.28}$$

We normalize the amplitude (See Appendix C) so that

$$\int dk \quad \vec{k}^2[\{\hat{R}_u(k)\}^2+\{\hat{R}_l(k)\}^2] = 1 \ . \tag{3.29}$$

3.4 Related Invariant Amplitudes:

By taking the complex conjugate of Eqs.(3.10), (3.(11), (3.23), and (3.24) we can get various amplitudes which are needed for the calculation of various physical quantities. Thus for pseudoscalar mesons we have,

$$<\vec{p}\ M_T|q_{\alpha i}(0)|\vec{k}\ s\ t> = <\vec{k}\ s\ t|q^+_{\alpha i}(0)|\vec{p}\ M_T>^*$$

$$= <\vec{k}\ s\ t|\bar{q}_{\alpha i}(0)|\vec{p}\ M_T>^* \gamma^o_{\beta\alpha}$$

$$= -\ [1/(2\pi)^3][1/2\omega(\vec{p})]^{\frac{1}{2}}[m_q/E_q(\vec{k})]^{\frac{1}{2}}$$

$$\times[\gamma^5(E-F\ \not{p}/m)U(\vec{k},s)]_\alpha[\ \vec{\tau}\cdot\hat{e}^*_{M_T}\chi_t]_i, \tag{3.30}$$

$$<\vec{p}\ M_T|\bar{q}_{\alpha i}(0)|\overline{\vec{k}\ s\ t}> = <\overline{\vec{k}\ s\ t}|q_{\beta i}(0)|\vec{p}\ M_T>^* \gamma^o{}_{\beta\alpha}$$

$$= -\ [1/(2\pi)^3][1/2\omega(\vec{p})]^{\frac{1}{2}}[m_q/E_q(\vec{k})]^{\frac{1}{2}}$$

$$\times[\bar{V}(\vec{k},s)(E+F\ \not{p}/m)\gamma^5]_\alpha[\chi^+_{-t}(\ \vec{\tau}\cdot\hat{e}_{M_T})^T\eta^*_{-t}]_i. \tag{3.31}$$

Similarly for vector mesons we have,

$$<\vec{p}\ \lambda\ M_T|q_{\alpha i}(0)|\vec{k}\ s\ t> = [1/(2\pi)^3][1/2\omega(\vec{p})]^{\frac{1}{2}}[m_q/E_q(\vec{k})]^{\frac{1}{2}}$$

$$\times\{[(\xi^*_\lambda\cdot k/m_q)A_1(1+\not{p}/m)+(\tilde{A}+\tilde{B}\ \not{p}/m)\not{\xi}^*_\lambda]U(\vec{k},s)\}_\alpha[\ \vec{\tau}\cdot\hat{e}^*_{M_T}\chi_t]_i, \tag{3.32}$$

$$<\vec{p}\ \lambda\ M_T|\bar{q}_{\alpha i}(0)|\vec{k}\ s\ t> = -\ [1/(2\pi)^3][1/2\omega(\vec{p})]^{\frac{1}{2}}[m_q/E_q(\vec{k})]^{\frac{1}{2}}$$

$$\times\{\bar{V}(\vec{k},s)[(\xi_\lambda^*\cdot k/m_q)A_1(1-\not{p}/m)-(\tilde{A}+\tilde{B}\ \not{p}/m)\not{\xi}_\lambda^*]\}_\alpha[\chi_{-t}^+(\vec{\tau}\cdot\hat{e}_{M_T})^T\eta_{-t}^*]_i, \tag{3.33}$$

where

$$\not{\xi}_\lambda^* = \xi_\lambda^{*\mu}\ \gamma_\mu. \tag{3.34}$$

CHAPTER IV

Form Factors

The analysis of the field equations in Chapter 3 requires the definition of various form factors. Here we define them for pseudoscalar and vector mesons in detail. It is important to note that these definitions are exact. Only when we calculate the various form factors in terms of the invariant amplitudes E and F in Eq.(3.11), or A_1, A, and B in Eq.(3.24), do we have to make some approximation. The evaluation also gives us an idea of how various quantities are calculated in our formalism.

4.1 Form Factor of Pseudoscalar Mesons:

For a pseudoscalar meson of mass m we define the form factor $F_s(q^2)$, which is a Lorentz scalar,

$$<\vec{p}'\, M'_T|\bar{q}(0)q(0)|\vec{p}\, M_T> = [1/(2\pi)^3]\,\delta_{M_T M'_T}[2m/\{\omega(\vec{p})\omega(\vec{p}')\}^{\frac{1}{2}}]\;F_s(q^2), \tag{4.1}$$

where

$$\omega(\vec{p}) = [\; m^2 + \vec{p}^2 \;]^{\frac{1}{2}}, \tag{4.2}$$

$$\omega(\vec{p}') = [\; m^2 + \vec{p}'^2 \;]^{\frac{1}{2}}, \tag{3.3}$$

and

$$q^2 = (p'-p)^{\mu}(p'-p)_{\mu} = (p'-p)^2 = 2(m^2-p\bullet p). \tag{4.4}$$

It is clear from the structure of L.H.S. of Eq.(4.1) and the available Lorentz tensors that we can construct only one Lorentz scalar function, $F_s(q^2)$. The factor $[2m/\{\vec{\omega}(p)\omega(\vec{p}')\}^{\frac{1}{2}}]$ in the R.H.S. of Eq.(4.1) is due to the normalization condition given in Eq.(3.3). The dependence of F_s on q^2 can also be easily understood when one notes the fact that the mesons are on the mass shell. Thus we have $p^2 = m^2$, $p'^2 = m^2$, and the only avaliable Lorentz scalar is $p \bullet p'$.

We now calculate this form factor in terms of the invariant amplitudes E and F. We start from the definition given in Eq.(4.1) and insert a set of quark and antiquark states between the two quark field operators. Thus we have,

$$<\vec{p}'\ M'_T|\bar{q}(0)q(0)|\vec{p}\ M_T> = \Sigma_{s,t}\int d\vec{K}$$

$$\times[<\vec{p}'\ M'_T|q_{\alpha i}(0)|\overline{\vec{K}\ s\ t}><\overline{\vec{K}\ s\ t}|q_{\alpha i}(0)|\vec{p}\ M_T>$$

$$-<\vec{p}'\ M'_T|q_{\alpha i}(0)|\vec{K}\ s\ t><\vec{K}\ s\ t|\bar{q}_{\alpha i}(0)|\vec{p}\ M_T>]. \qquad (4.5)$$

Here we used a factorization which is an approximate completeness relation. (This is expected to be a good approximation provided the sea quark effects are negligible.)

Using Eqs.(3.10), (3.11), (3.30) and (3.31) and the relations:

$$\Sigma_t\ [\vec{\tau}\bullet\hat{e}^*_{M'_T}\chi_t]_i\ [\chi_t^+\ \vec{\tau}\bullet\hat{e}_{M_T}]_i = 2\ \delta_{M_T M'_T}, \qquad (4.6)$$

$$\Sigma_t \, [\chi_{-t}^{+}(\vec{\tau}\cdot\hat{e}_{M'_T})^T]_i \, [(\vec{\tau}\cdot\hat{e}_{M_T}^{*})^T \chi_{-t}]_i = 2\,\delta_{M_T M'_T}, \tag{4.7}$$

$$\Sigma_s \, U(\vec{k},s)\,\overline{U}(\vec{k},s) = (\not{k} + m_q)/(2m_q), \tag{4.8}$$

$$\Sigma_s \, V(\vec{k},s)\,\overline{V}(\vec{k},s) = (\not{k} - m_q)/(2m_q), \tag{4.9}$$

we write Eq.(4.5) as

$$\langle\vec{p}'\, M'_T|\overline{q}(0)q(0)|\vec{p}\, M_T\rangle = 2\,\delta_{M_T M'_T}[1/(2\pi)^3] \int[d\vec{k}/(2\pi)^3][m_q/E_q(\vec{k})]$$
$$\times[1/\{4\omega(\vec{p})\omega(\vec{p}')\}^{\frac{1}{2}}]\mathrm{Tr}[(E'+F'\not{p}'/m)(E+F\not{p}/m)(-\not{k}+m_q)/(2m_q)$$
$$+(E-F\not{p}/m)(E'-F'\not{p}'/m)(\not{k}+m_q)/(2m_q)], \tag{4.10}$$

where the trace is over the Dirac matrices. Here E and F are functions of $[(p\cdot k/m_q)^2-m^2]^{\frac{1}{2}}$ and E' and F' are functions of $[(p'\cdot k/m_q)^2-m^2]^{\frac{1}{2}}$. After taking the trace in Eq.(4.10) we get

$$\langle\vec{p}'\, M'_T|\overline{q}(0)q(0)|\vec{p}\, M_T\rangle = [1/(2\pi)^3]\,\delta_{M_T M'_T}[2m/\{\omega(\vec{p})\omega(\vec{p}')\}^{\frac{1}{2}}]F_s(q^2), \tag{4.11}$$

where $F_s(q^2)$ is given by

$$F_s(q^2) = (2/m) \int[d\vec{k}/(2\pi)^3][m_q/E_q(\vec{k})]$$
$$\times[EE'+FF'\; p'\cdot p/m^2 - E'F\; p\cdot k/(mm_q) - EF'p'\cdot k/(mm_q)]. \tag{4.12}$$

Note the invariant structure of the integral in the Eq.(4.12) is as we expected. The whole integral is an invariant scalar function since E and F are scalar invariant functions and the three-dimensional integral in Eq.(4.12) can be easily made to a four-dimensional integral by noting the fact that quarks on the mass shell. This can be done through the relation,

$$\int [d\vec{k}/E_q(\vec{k})] = 2 \int d^4k\ \delta(k^2 - m_q^2)\ \theta(k^o) \tag{4.13}$$

where $\theta(k^o)$ is a step function.

4.2 Form Factor of Vector mesons:

The form factors for vector mesons are more involved due to the spin of the meson. This is partly because of the availability of one more Lorentz vector, $\xi_\lambda{}^\mu$, the polarization vector of the meson, which can be used to make various invariant functions. To make the notation simple we again denote the polarization of a particle with momentum $\vec{p}$ as $\xi_\lambda{}^\mu$, and the polarization of a particle with momentum $\vec{p}'$ as $\xi'_\lambda{}^\mu$. Thus we have,

$$\xi_\lambda{}^\mu = \xi_\lambda{}^\mu(\vec{p}), \tag{4.14}$$

$$\xi'_\lambda{}^\mu = \xi_\lambda{}^\mu(\vec{p}'), \tag{4.15}$$

$$\xi_\lambda \bullet p = 0, \tag{4.16}$$

$$\xi'_{\lambda} \bullet p' = 0. \tag{4.17}$$

The relations (4.16) and (4.17) reduces the number of independent invariant functions we can construct from the avaliable Lorentz tensors. Thus for a vector meson we define the various form factors through the relation,

$$<\vec{p}'\ \lambda'\ M'_T|\bar{q}(0)q(0)|\vec{p}\ \lambda\ M_T> = [1/(2\pi)^3]\delta_{M_T M'_T}[2m/\{\omega(\vec{p})\omega(\vec{p}')\}^{\frac{1}{2}}]$$
$$\times[\ \{(\xi'^{*}_{\lambda'}\bullet p\xi_{\lambda}\bullet p')/m^2\}\ F_1(q^2) + (\xi'^{*}_{\lambda'}\bullet\xi_{\lambda})\ F_2(q^2)\ ]. \tag{4.18}$$

Equation (4.18) is the most general form consistent with the relations in Eqs.(4.16) and (4.17) and the condition that the matrix elements be bilinear in ξ_{λ} and $\xi'^{*}_{\lambda'}$. The evaluation of these form factors proceeds exactly as in the pseudoscalar meson calculation. Thus we have,

$$<\vec{p}'\ \lambda'\ M'_T|\bar{q}(0)q(0)|\vec{p}\ \lambda\ M_T> = \Sigma_{s,t}\int d\vec{k}$$
$$\times[<\vec{p}'\ \lambda'\ M'_T|\bar{q}_{\alpha i}(0)|\overline{\vec{k}\ s\ t}><\overline{\vec{k}\ s\ t}|q_{\alpha i}(0)|\vec{p}\ \lambda\ M_T>$$
$$-<\vec{p}'\ \lambda'\ M'_T|q_{\alpha i}(0)|\vec{k}\ s\ t><\vec{k}\ s\ t|\bar{q}_{\alpha i}(0)|\vec{p}\ \lambda\ M_T>]\ . \tag{4.19}$$

Again using the relations in Eqs.(3.23), (3.24), (3.32), (3.33), performing the spin sum over s and the isospin sum over t and using the relations in Eqs.(4.6)-(4.9) we get,

$$<\vec{p}'\ \lambda'\ M'_T|\bar{q}(0)q(0)|\vec{p}\ \lambda\ M_T>$$

$$= 2\ \delta_{M_T M'_T}[1/(2\pi)^3][1/\{4\omega(\vec{p})\omega(\vec{p}')\}^{\frac{1}{2}}] \int [d\vec{k}/(2\pi)^3][m_q/E_q(\vec{k})]$$

$$\times \mathrm{Tr}\{[(\not{k}-m_q)/(2m_q)][(\xi'^{*}_{\lambda'}\bullet k/m_q)\ A'_1(1-\not{p}'/m)-(\tilde{A}'+\tilde{B}'\ \not{p}/m)\not{\xi}'^{*}_{\lambda'}]$$

$$\times[(\xi_\lambda\bullet k/m_q)\ A_1(1-\not{p}/m)-\not{\xi}_\lambda(\tilde{A}+\tilde{B}\ \not{p}/m)]$$

$$+[-(\not{k}+m_q)/(2m_q)][(\xi_\lambda\bullet k/m_q)\ A_1(1+\not{p}/m)+\not{\xi}_\lambda(\tilde{A}+\tilde{B}\ \not{p}/m)]$$

$$\times[(\xi'^{*}_{\lambda'}\bullet k/m_q)\ A'_1(1+\not{p}'/m)+(\tilde{A}'+\tilde{B}'\ \not{p}'/m)\not{\xi}'^{*}_{\lambda'}]\}. \tag{4.20}$$

Here again A_1, A and B are functions of $[(p\bullet k/m_q)^2-m^2]^{\frac{1}{2}}$ and A'_1, A' and B' are functions of $[(p'\bullet k/m_q)^2-m^2]^{\frac{1}{2}}$. After taking the trace over the Dirac matrices in Eq.(4.20) we have,

$$<\vec{p}'\ \lambda'\ M'_T|\bar{q}(0)q(0)|\vec{p}\ \lambda\ M_T>$$

$$= 8\ \delta_{M_T M'_T}[1/(2\pi)^3][1/\{4\omega(\vec{p})\omega(\vec{p}')\}^{\frac{1}{2}}] \int [d\vec{k}/(2\pi)^3][m_q/E_q(\vec{k})]$$

$$\times[\{\xi'^{*}_{\lambda'}\bullet p\xi_\lambda\bullet k/(mm_q)\}\ f_3+(\xi'^{*}_{\lambda'}\bullet\xi_\lambda)\ f_4\ +\{(\xi'^{*}_{\lambda'}\bullet p\xi_\lambda\bullet p')/m^2\}\ f_5$$

$$+\{\xi'^{*}_{\lambda'}\bullet k\xi_\lambda\bullet k/m_q^2\}f_1+\{\xi_\lambda\bullet p'\xi'^{*}_{\lambda'}\bullet k/(mm_q)\}\ f_2 \tag{4.21}$$

where

$$f_1 = -A_1A'_1[1+\{(p+p')\bullet k/(mm_q)\}+(p\bullet p'/m^2)]$$

$$-A'_1\tilde{A} - A_1\tilde{A}' - (p\bullet p'/m^2 \; (A'_1\tilde{B}) + A_1\tilde{B}'), \qquad (4.22)$$

$$f_2 = A'_1\tilde{B} \; [p\bullet k/(mm_q)] \; -\tilde{B}'\tilde{A} \; -A'_1\tilde{A}, \qquad (4.23)$$

$$f_3 = A_1\tilde{B}' \; [p'\bullet k/(mm_q)] \; -\tilde{B}\tilde{A}' \; -A_1\tilde{A}', \qquad (4.24)$$

$$f_4 = -\tilde{A}\tilde{A}' \; -\tilde{B}\tilde{B}' \; (p'\bullet p/m^2) \; +\tilde{A}'\tilde{B} \; \{p\bullet k/(mm_q))\}$$

$$+\tilde{A}\tilde{B}' \; (p'\bullet k/(mm_q))\}, \qquad (4.25)$$

$$f_5 = \tilde{B}\tilde{B}' \; . \qquad (4.26)$$

Again the invariant nature of the integral in Eq.(4.21) is clear. This invariance can be used to perform the integrations in Eq.(4.21). For the evaluation of these integrals we define

$$\tilde{\pi}^\mu = (p'+p)^\mu/(2m), \qquad (4.27)$$

$$\tilde{q}^\mu = (p'-p)^\mu/(2m), \qquad (4.28)$$

where as usual

$$q^2 = (p'-p)^2. \qquad (4.29)$$

Now consider the integral

$$\int d\vec{k}[m_q/E_q(\vec{k})]\ (\xi'^{*}_{\lambda'}\cdot k\xi_\lambda\cdot k/m_q^2)\ f_1 \ = \xi'^{*\mu}_{\lambda'}\ \xi_\lambda^{\ \nu}\ \ I_{\mu\nu}. \qquad (4.30)$$

Since the function f_1 is an invariant function and the integral is invariant, due to the relation Eq.(4.13), the quantity $I_{\mu\nu}$ is a Lorentz symmetric tensor of second rank. Thus we can write,

$$I^{\mu\nu} = I_1(q^2)\tilde{\pi}^\mu\tilde{\pi}^\nu + I_2(q^2)\tilde{q}^\mu\tilde{q}^\nu$$

$$+I_3(q^2)(g^{\mu\nu} - \tilde{\pi}^\mu\tilde{\pi}^\nu/\tilde{\pi}^2 - \tilde{q}^\mu\tilde{q}^\nu/\tilde{q}^2). \qquad (4.31)$$

Here $I_1(q^2)$, $I_2(q^2)$ and $I_3(q^2)$ are Lorentz scalar functions. In writing Eq.(4.31) we used the fact that $I^{\mu\nu}$ is symmetric under the interchange of μ with ν and p^μ with p'^μ and that the mesons are on the mass shell. Thus Eq.(4.31) is the most general relation we can write. Equation (4.31) can be inverted to get $I_1(q^2)$, $I_2(q^2)$ and $I_3(q^2)$ in terms of $I^{\mu\nu}$. Thus we find,

$$I_1(q^2) = \int d\vec{k}[m_q/E_q(\vec{k})]\ \{(\tilde{\pi}\cdot k)^2/(m_q^2\tilde{\pi}^4)\}\ \ f_1, \qquad (4.32)$$

$$I_2(q^2) = \int d\vec{k}[m_q/E_q(\vec{k})]\ \{(\tilde{q}\cdot k)^2/(m_q^2\tilde{q}^4)\}\ \ f_1, \qquad (4.33)$$

$$I_3(q^2) = \int d\vec{k}[m_q/E_q(\vec{k})]$$

$$\times[1 - \{(\tilde{\pi}\cdot k)^2/(m_q^2\tilde{\pi}^2)\} - \{(\tilde{q}\cdot k)^2/(m_q^2\tilde{q}^2)\}\ ]\ \ f_1/2\ . \qquad (4.34)$$

After substituting Eq.(4.31) into Eq.(4.30) we have,

$$\int d\vec{k}[m_q/E_q(\vec{k})]\ (\xi'^*_{\lambda'}\bullet k\xi_\lambda\bullet k/m_q^2)\ f_1$$

$$= \{(\xi'^*_{\lambda'}\bullet p\xi_\lambda\bullet p')/m^2\}[I_1(q^2) - I_2(q^2) - I_3(q^2)/\tilde{\pi}^2 - I_3(q^2)/\tilde{q}^2]/4$$

$$+ (\xi'^*_{\lambda'}\bullet\xi_\lambda)\ I_3(q^2)\ , \qquad (4.35)$$

where we used the relations in Eq.(4.16) and (4.17). Similarly we consider the integral:

$$\int d\vec{k}[m_q/E_q(\vec{k})]\ \{\xi'^*_{\lambda'}\bullet k\xi_\lambda\bullet p'/(m_q m)\}\ f_2 = \{\xi_\lambda\bullet p'/m\}\ \xi'^{*\mu}_{\lambda'}\ I_\mu \qquad (4.36)$$

where

$$I_\mu = \int d\vec{k}[m_q/E_q(\vec{k})]\ k_\mu\ f_2. \qquad (4.37)$$

Again from general invariance properties we have,

$$I^\mu = \tilde{\pi}^\mu\ I_4(q^2) + \tilde{q}^\mu\ I_5(q^2)\ , \qquad (4.38)$$

where $I_4(q^2)$ and $I_5(q^2)$ are Lorentz scalar functions. If the function f_2 is symmetric under the interchange of p and p' the function $I_5(q^2)$ will be zero. Equation (4.38) can be solved for $I_4(q^2)$ and $I_5(q^2)$ to find,

$$I_4(q^2) = \int d\vec{k}[m_q/E_q(\vec{k})] \; \{(\tilde{\pi}\bullet k)/(m_q\tilde{\pi}^2)\} \; f_2, \qquad (4.39)$$

$$I_5(q^2) = \int d\vec{k}[m_q/E_q(\vec{k})] \; \{(\tilde{q}\bullet k)/(m_q\tilde{q}^2)\} \; f_2. \qquad (4.40)$$

Now sustituting Eq.(4.38) into Eq.(4.36) gives us,

$$\int d\vec{k}[m_q/E_q(\vec{k})] \; \{\xi'^{*}_{\lambda'}\bullet k\xi_{\lambda}\bullet p'/(m_q m)\} \; f_2$$

$$= \{(\xi'^{*}_{\lambda'}\bullet p\xi_{\lambda}\bullet p')/m^2\}[\; I_4(q^2) \; - \; I_5(q^2) \;]/2. \qquad (4.41)$$

Similar considerations leads us to

$$\int d\vec{k}[m_q/E_q(\vec{k})] \; \{\xi'^{*}_{\lambda'}\bullet p\xi_{\lambda}\bullet k/(m_q m)\} \; f_3$$

$$= \{(\xi'^{*}_{\lambda'}\bullet p\xi_{\lambda}\bullet p')/m^2\}[\; I_6(q^2) \; + \; I_7(q^2) \;]/2 \; , \qquad (4.42)$$

where

$$I_6(q^2) = \int d\vec{k}[m_q/E_q(\vec{k})] \; \{(\tilde{\pi}\bullet k)/(m_q\tilde{\pi}^2)\} \; f_3, \qquad (4.43)$$

$$I_7(q^2) = \int d\vec{k}[m_q/E_q(\vec{k})] \; \{(\tilde{q}\bullet k)/(m_q\tilde{q}^2)\} \; f_3. \qquad (4.44)$$

Now we use Eqs.(4.35), (4.41) and (4.42) in Eq.(4.21) and compare the result with the definition given in Eq.(4.18). This leads us to

$$F_1(q^2) = (2/m) \int [d\vec{k}/(2\pi)^3][m_q/E_q(\vec{k})]$$

$$\times \Big[f_1 \; [\{(\tilde{\pi}\cdot k)^2/(m_q^2\tilde{\pi}^4)\} - \{(\tilde{q}\cdot k)^2/(m_q^2\tilde{q}^4)\}]/4$$

$$+f_1 \; [1 - \{(\tilde{\pi}\cdot k)^2/(m_q^2\tilde{\pi}^2)\} - \{(\tilde{q}\cdot k)^2/(m_q^2\tilde{q}^2)\}][1/\tilde{q}^2 - 1/\tilde{\pi}^2]/8$$

$$+f_2 \; [\{(\tilde{\pi}\cdot k)/(m_q\tilde{\pi}^2)\} - \{(\tilde{q}\cdot k)/(m_q\tilde{q}^2)\}]/2$$

$$+f_3 \; [\{(\tilde{\pi}\cdot k)/(m_q\pi^2)\} + \{(\tilde{q}\cdot k)/(m_q\tilde{q}^2)\}]/2 + f_5 \Big], \qquad (4.45)$$

$$F_2(q^2) = (2/m) \int [d\vec{k}/(2\pi)^3][m_q/E_q(\vec{k})]$$

$$\times \Big[f_4 + f_1 \; [1 - \{(\tilde{\pi}\cdot k)^2/(m_q^2\tilde{\pi}^2)\} - \{(\tilde{q}\cdot k)^2/(m_q^2\tilde{q}^2)\}]/2 \Big]. \qquad (4.46)$$

Again the invariant structure of the functions $F_1(q^2)$ and $F_2(q^2)$ is clear from the Eqs.(4.45) and (4.46). (It is useful remember that the form factors are quadratic in the invariant amplitude. This is true for the pseudoscalar meson case also.)

CHAPTER V

Dynamical Equations

We now develop an integral equation which can be used to determine the invariant functions E, F, A and B given in Chapter 3. This is done through the analysis of Eqs.(2.5) and (2.6). We have,

$$[i\gamma_\mu \partial^\mu - m_q]q(x) = g_\chi\, q(x)\chi(x), \tag{5.1}$$

$$[\partial_\mu \partial^\mu + m_\chi^2]\chi(x) = -g_\chi\, \overline{q}(x)q(x). \tag{5.2}$$

Here, for simplicity, we are considering the case where the explicit gluonic interaction has been neglected. (A covariant description of gluon-exchange is presented in the Appendix E.) We first analyse the equation for pseudoscalar mesons which is relatively simple.

5.1 Integral Equation for Pseudoscalar Mesons:

We now form the matrix element of Eq.(5.1) with meson and antiquark states. Thus we have,

$$[i\gamma_\mu \partial^\mu - m_q]\langle\overline{\vec{k}\ s\ t}|q(x)|\vec{p}\, M_T\rangle = g_\chi\, \langle\overline{\vec{k}\ s\ t}|q(x)\chi(x)|\vec{p}\, M_T\rangle. \tag{5.3}$$

Similarly by forming the matrix element of Eq.(5.2) with two meson states we get,

$$[\partial_\mu \partial^\mu + m_\chi^2]\langle \vec{p}' M'_T|\chi(x)|\vec{p} M_T\rangle = -g_\chi \langle \vec{p}' M'_T|\bar{q}(x)q(x)|\vec{p} M_T\rangle. \tag{5.4}$$

By using the relations

$$\overline{\langle \vec{k}\ s\ t|}q(x)|\vec{p} M_T\rangle = e^{-ix\cdot(p-k)} \overline{\langle \vec{k}\ s\ t|}q(0)|\vec{p} M_T\rangle \tag{5.5}$$

and

$$\langle \vec{p}' M'_T|\chi(x)|\vec{p} M_T\rangle = e^{-ix\cdot(p-p')} \langle \vec{p}' M'_T|\chi(0)|\vec{p} M_T\rangle, \tag{5.6}$$

we can simlplify Eqs.(5.3) and (5.4) to find,

$$[\not{p} - \not{k} - m_q] \overline{\langle \vec{k}\ s\ t|}q(0)|\vec{p} M_T\rangle = g_\chi \overline{\langle \vec{k}\ s\ t|}q(0)\chi(0)|\vec{p} M_T\rangle, \tag{5.7}$$

$$[-q^2 + m_\chi^2]\langle \vec{p}' M'_T|\chi(0)|\vec{p} M_T\rangle = -g_\chi \langle \vec{p}' M'_T|\bar{q}(0)q(0)|\vec{p} M_T\rangle, \tag{5.8}$$

where $q^2 = (p'-p)^2$. Upon inserting a set of mesonic states between the operators $q(0)$ and $\chi(0)$ in Eq.(5.7) we obtain,

$$[\not{p} - \not{k} - m_q] \overline{\langle \vec{k}\ s\ t|}q(0)|\vec{p} M_T\rangle.$$

$$= g_\chi \Sigma_{M'_T} \int d\vec{p}' \overline{\langle \vec{k}\ s\ t|}q(0)|\vec{p}' M'_T\rangle\langle \vec{p}' M'_T|\chi(0)|\vec{p} M_T\rangle. \tag{5.9}$$

It is clear that this factorization is approximate, but we expect that the main contribution is from this particular set of states. Now using the definition of the form factor in Eq. (4.1), Eq. (5.8) becomes:

$$\langle \vec{p}\,' M'_T | \chi(0) | \vec{p}\, M_T \rangle$$

$$= -\delta_{M_T M'_T} [1/(2\pi)^3] [g_\chi/(m_\chi^2 - q^2)] [2m/\{\omega(\vec{p})\omega(\vec{p}\,')\}^{\frac{1}{2}}] \; F_s(q^2). \quad (5.10)$$

Now from the expression for $F_s(q^2)$ in terms of the invariant amplitude in Eq. (4.12) it is evident that the form factor is a functional of the amplitude we are trying to obtain. Thus Eqs. (5.9) and (5.10) form a set of nonlinear coupled integral equations. This set of equations can be combined pictorially as in Fig. 1, where the nonlinear nature of the problem is apparent.

As it stands, Eq. (5.9) is a covariant equation. Thus we can analyse it in any frame. This equation becomes very simple in the antiquark rest frame, $\vec{k} = 0$. We again stress the point that this choice is only a matter of convenience. Different frames are connected to each other by Lorentz boosts. Since the amplitudes are defined in an invariant way it is easy to go from one frame to another. It is this covariant formalism which makes our analysis very elegant. Now using the definition of invariant amplitudes in Eq.(3.11), we can rewrite Eq.(5.9) as,

$$[\not{p} - \not{k} - m_q][\gamma^5(E+F\ p/m)V(\vec{k},s)]/\sqrt{\omega(\vec{p})}$$

$$=g_\chi \int [d\vec{p}'/\sqrt{\omega(\vec{p}')}][\gamma^5(E'+F'\not{p}'/m)V(\vec{k},s)]\langle\vec{p}'\ M_T|\chi(0)|\vec{p}\ M_T\rangle, \tag{5.11}$$

where we performed the sum over the isospin index M'_T and cancelled the isospin factors on both sides of Eq.(5.9). Here E and F are functions of $[(p\bullet k/m_q)^2-m^2]^{\frac{1}{2}}$ and E' and F' are functions of $[(p'\bullet k/m_q)^2-m^2]^{\frac{1}{2}}$. In the antiquark rest frame, where $\vec{k}=0$, E and F become functions of $|\vec{p}|$ and E' and F' are functions of $|\vec{p}'|$. After putting $\vec{k}=0$ in Eq.(5.11), using the explicit representation of Dirac matrices in Eqs.(1.4)-(1.8), and using the fact that

$$V(\vec{k}=0,s) = \begin{bmatrix} 0 \\ \chi_{-s} \end{bmatrix}, \tag{5.12}$$

where χ_{-s} is a two component spinor, we get

$$[1/\sqrt{\omega(\vec{p})}] \begin{bmatrix} \omega(\vec{p})-2m_q & -\vec{p}\cdot\vec{\sigma} \\ \vec{p}\cdot\vec{\sigma} & -\omega(\vec{p}) \end{bmatrix} \begin{bmatrix} \{E(p) - F(p)\omega(\vec{p})/m\}\chi_{-s} \\ -F(p)\ (p/m)\ \hat{p}\cdot\vec{\sigma}\,\chi_{-s} \end{bmatrix}$$

$$= g_\chi \int d\vec{p}'\ 1/[\sqrt{\omega(\vec{p}')}] \begin{bmatrix} \{E(p')-F(p')\omega(\vec{p}')/m\}\chi_{-s} \\ \{-F(p')\ (p/m)\}\ \hat{p}'\cdot\vec{\sigma}\,\chi_{-s} \end{bmatrix} \langle\vec{p}'\ M_T|\chi(0)|\vec{p}\ M_T\rangle \tag{5.13}$$

Here we denote $|\vec{p}|$ and $|\vec{p}'|$ by p and p' respectively . Further, $\hat{p}$ and $\hat{p}'$ are unit vectors in the direction of $\vec{p}$ and $\vec{p}'$, respectively; $\vec{\sigma}$ is the Pauli spin matrix. For the analysis of equation (5.13) it is useful to define,

$$R_u(p) = [1/\sqrt{\omega(\vec{p})}][\ E(p) - F(p)\ \omega(\vec{p})/m\ ], \tag{5.14}$$

$$R_l(p) = -[1/\sqrt{\omega(\vec{p})}][p/m]\ F(\vec{p}), \tag{5.15}$$

$$R_u(p') = [1/\sqrt{\omega(\vec{p}')}][\ E(p') - F(p')\ \omega(\vec{p}')/m\ ], \tag{5.16}$$

$$R_l(p') = -[1/\sqrt{\omega(\vec{p}')}][p'/m]\ F(p'). \tag{5.17}$$

Using the definitions in Eqs.(5.14)-(5.17), Eq.(5.13) becomes,

$$\begin{bmatrix} \{[\omega(\vec{p})-2m_q]R_u(p) - pR_l(p)\}\ \chi_{-s} \\ \{pR_u(p) - \omega(\vec{p})R_l(p)\}\ \hat{p}\cdot\vec{\sigma}\ \chi_{-s} \end{bmatrix}$$

$$= g_\chi \int d\vec{p}\,' \begin{bmatrix} R_u(p')\quad \chi_{-s} \\ R_l(p')\ \hat{p}'\cdot\vec{\sigma}\ \chi_{-s} \end{bmatrix} \langle \vec{p}\,'\ M_T | \chi(0) | \vec{p}\ M_T \rangle \tag{5.18}$$

Equation (5.18) can be written as two coupled 2×2 matrix equations. We multiply the upper component from the right by χ_{-s}^{+} and the lower part from the right by $-\chi_{-s}^{+}\ \hat{p}\cdot\vec{\sigma}$. We then perform a sum over the spin index s. Finally we take the trace over the 2×2 matrix in the upper and the lower part separately. Thus we have,

$$\begin{bmatrix} \{\omega(\vec{p})-2m_q\}\ R_u(p) - p\ R_l(p) \\ - p\ R_u(p) + \omega(p)\ R_l(p) \end{bmatrix}$$

$$= g_\chi \int d\vec{p}\,' \begin{bmatrix} R_u(p') \\ - R_l(p')\ \hat{p}\cdot\hat{p}' \end{bmatrix} \langle \vec{p}\,'\ M_T | \chi(0) | \vec{p}\ M_T \rangle\ , \tag{5.19}$$

where we used the fact that

$$\mathrm{Tr}\ [\ \vec{\sigma}\cdot\vec{A}\ \vec{\sigma}\cdot\vec{B}\ ] = 2\vec{A}\cdot\vec{B}. \tag{5.20}$$

Finally using the definition in Eq.(5.10), Eq.(5.19) can be written as:

$$\omega(\vec{p}) \begin{bmatrix} R_u(p) \\ \\ R_l(p) \end{bmatrix} = \begin{bmatrix} 2m_q & p \\ \\ p & 0 \end{bmatrix} \begin{bmatrix} R_u(p) \\ \\ R_l(p) \end{bmatrix}$$

$$- g_\chi^2 \int [d\vec{p}'/(2\pi)^3]\ [2m/\{\omega(\vec{p})\omega(\vec{p}')\}^{\frac{1}{2}}]\,[F_s(q^2)/(m_\chi^2 - q^2)]$$

$$\times \begin{bmatrix} 1 & 0 \\ \\ 0 & -\hat{p}\cdot\hat{p}' \end{bmatrix} \begin{bmatrix} R_u(p') \\ \\ R_l(p') \end{bmatrix} \tag{5.21}$$

This is the central equation for all of our analysis. Note the hermicity of the kernal in Eq.(5.21), which is symmetric under the interchange of $\vec{p}$ and $\vec{p}'$. This has the effect of reducing this problem to an eigenvalue problem with real eigenvalues. Equation (5.21) can be solved for $R_u(p)$ and $R_l(p)$ and thus for E and F. The details of the calculation are presented in Chapter 6. Since we do not make a static approximation the "effective potential" term in the kernal is nonlocal. Also note that our equation is homogeneous, which is what we expect for a bound-state problem.

5.2 Integral Equation for Vector Mesons:

As in the case of pseudoscalar mesons, we form the matrix elements of equation (5.1) with vector meson and antiquark states and develop Eq.(5.2) with vector meson states. Thus we have,

$$[\not{p} - \not{k} - m_q] \overline{\langle \vec{k}\ s\ t|} q(0) | \vec{p}\ \lambda\ M_T\rangle$$

$$= g_\chi \Sigma_{\lambda' M'_T} \int d\vec{p}\,' \overline{\langle \vec{k}\ s\ t|} q(0) | \vec{p}\,'\ \lambda'\ M'_T\rangle \langle \vec{p}\,'\ \lambda'\ M'_T | \chi(0) | \vec{p}\ M_T\rangle, \tag{5.22}$$

and

$$[\ -q^2 + m_\chi^2\ ] \langle \vec{p}\,'\ \lambda'\ M'_T | \chi(0) | \vec{p}\ \lambda\ M_T\rangle$$

$$= -g_\chi \langle \vec{p}\,'\ \lambda'\ M'_T | \bar{q}(0) q(0) | \vec{p}\ \lambda\ M_T\rangle. \tag{5.23}$$

Using the definition of form factors in Eq.(4.18), Eq.(5.23) becomes,

$$\langle \vec{p}\,'\ \lambda'\ M'_T | \chi(0) | \vec{p}\ \lambda\ M_T\rangle = -\delta_{M_T M'_T} [1/(2\pi)^3] [g_\chi/(m_\chi^2 - q^2)]$$

$$\times [2m/\{\omega(\vec{p})\omega(\vec{p}\,')\}^{\frac{1}{2}}] [\{(\xi'^{*}_{\lambda'} \bullet p \xi_\lambda \bullet p')/m^2\}\ F_1(q^2) + (\xi'^{*}_{\lambda'} \bullet \xi_\lambda)\ F_2(q^2)]. \tag{5.24}$$

After using the definition of the invariant amplitude in Eq.(3.24), and cancelling the isospin factors on both sides of Eq.(5.22), that equation becomes,

$$[\not p - \not k - m_q]\{[(\xi_\lambda \cdot k/m_q)\ A_1(1-\not p/m) - \not\xi_\lambda(\tilde{A}+\tilde{B}\ \not p/m)]V(\vec{k},s)\}/\sqrt{\omega(\vec{p})}$$

$$= g_\chi\ \Sigma_{\lambda'} \int [d\vec{p}'/\sqrt{\omega(\vec{p}')}]$$

$$\times[(\xi'_{\lambda'} \cdot k/m_q)\ A'_1(1-\not p'/m) - \not\xi'_{\lambda'}\ (\tilde{A}'+B'\tilde{\not p}'/m)]V(\vec{k},s)$$

$$\times\langle \vec{p}'\ \lambda'\ M_T | \chi(0) | \vec{p}\ M_T\rangle\ . \tag{5.25}$$

Using Eq.(5.24) and performing the λ' sum making use of

$$\Sigma_{\lambda'}\ \xi'^{*\mu}_{\lambda'}\ \xi'^{\nu}_{\lambda'} = -g^{\mu\nu} + p'^{\mu}p'^{\nu}/m^2\ , \tag{5.26}$$

Eq.(5.25) yields,

$$[\not p - \not k - m_q][\{(\xi_\lambda \cdot k/m_q)\ A_1(1-\not p/m) - \not\xi_\lambda(\tilde{A}+\tilde{B}\ \not p/m)\}V(\vec{k},s)]/\sqrt{\omega(\vec{p})}$$

$$= -g_\chi^2 \int [d\vec{p}'/(2\pi)^3]\ [2m/\{\omega(\vec{p})\omega(\vec{p}')\}^{\frac{1}{2}}][1/(m_\chi^2 - q^2)][1/\sqrt{\omega(\vec{p}')}]$$

$$\times\Big[(\xi_\lambda \cdot p'/m)F_1(q^2)[\{-p\cdot k/(mm_q)+(p\cdot p'/m^2)p'\cdot k/(mm_q)\}A'_1(1-\not p'/m)$$

$$-\{(-\not p/m) + (p\cdot p'/m^2)(\not p'/m)\}(\tilde{A}'+\tilde{B}'\ \not p'/m)]V(\vec{k},s)$$

$$+F_2(q^2)[\{(-k\cdot\xi_\lambda/m_q) + (p'\cdot\xi_\lambda/m)(k\cdot p')/(mm_q)\}A'_1(1-\not p'/m)$$

$$-\{-\not\xi_\lambda + (p'\cdot\xi_\lambda/m)\ (\not p'/m)\}(\tilde{A}'+\tilde{B}'\ \not p'/m)]V(\vec{k},s)\Big]\ . \tag{5.27}$$

In Eqs.(5.25) and (5.27) A_1, $\widetilde{A}$ and $\widetilde{B}$ are functions of $[(p\cdot k/m_q)^2-m^2]^{\frac{1}{2}}$ and A'_1, $\widetilde{A}'$ and $\widetilde{B}'$ are functions of $[(p'\cdot k/m_q)^2-m^2]^{\frac{1}{2}}$. Again we analyse Eq.(5.27) in the antiquark rest frame, $\vec{k} = 0$. In this frame A_1, $\widetilde{A}$ and $\widetilde{B}$ are functions of $|\vec{p}|$ and A'_1, $\widetilde{A}'$ and $\widetilde{B}'$ are functions of $|\vec{p}'|$. For the analysis of Eq.(5.27) it is useful to define,

$$R_u(p) = [1/\sqrt{\omega(\vec{p})}][\ A(p) - B(p)\ \omega(\vec{p})/m\ ], \tag{5.28}$$

$$R_l(p) = -[1/\sqrt{\omega(\vec{p})}][p/m]\ B(p), \tag{5.29}$$

$$R_u(p') = [1/\sqrt{\omega(\vec{p}')}][\ A(p') - B(p')\ \omega(\vec{p}')/m\ ], \tag{5.30}$$

$$R_l(p') = -[1/\sqrt{\omega(\vec{p}')}][p'/m]\ B(p'). \tag{5.31}$$

Here, for simplicity, we denote $|\vec{p}|$ and $|\vec{p}'|$ by p and p', respectively.

The analysis of the Eq.(5.27) is very involved due to the presence of the polarization vector $\xi_\lambda{}^\mu$. It is simple to consider the transverse polarization $\lambda=\pm$ and the longitudinal polarization $\lambda=0$ separately. For the tranverse polarization $\lambda=\pm$ the time component $\xi^o_\pm=0$. Thus we have,

$$\xi_\pm{}^\mu = (0\ ,\ \vec{\xi}_\pm), \tag{5.32}$$

$$\xi_{\pm}\bullet p \;=\; 0, \tag{5.33}$$

$$\xi_{\pm}\bullet p' \;=\; -\vec{\xi}_{\pm}\bullet\vec{p}'. \tag{5.34}$$

Using Eqs.(5.28)-(5.34) and (5.12) we can write Eq.(5.27) for $\lambda=\pm$ as:

$$\begin{bmatrix} \{(\omega-2m_q)R_u(p) - pR_1(p)\}\ \vec{\xi}_{\pm}\bullet\vec{\sigma}\,\chi_{-s} \\ \{pR_u(p) - \omega R_1(p)\}\ \hat{p}\bullet\vec{\sigma}\ \vec{\xi}_{\pm}\bullet\vec{\sigma}\ \chi_{-s} \end{bmatrix}$$

$$= -g_\chi^2 \quad [d\vec{p}'/(2\pi)^3]\ [2m/\{\omega(\vec{p})\omega(\vec{p}')\}^{\frac{1}{2}}]\ [1/(m_\chi^2 - q^2)]$$

$$\times \begin{bmatrix} R_u(p')\{-F_1(q^2)(\vec{p}'\bullet\vec{\xi}_{\pm}/m)\ M_1 + F_2(q^2)\ M_2\}\ \chi_{-s} \\ \{B(p')/\surd\omega'\}\{-F_1(q^2)(\vec{p}'\bullet\vec{\xi}_{\pm}/m)\ M_3 + F_2(q^2)\ M_4\}\ \chi_{-s} \end{bmatrix}, \tag{5.35}$$

where

$$M_1 = -\{1/(1+\omega'/m)\}\{-\omega/m + (\omega'/m)(p\bullet p'/m^2)\}(\vec{p}'\bullet\vec{\sigma}/m)$$

$$-\{\vec{p}\bullet\vec{\sigma}/m - (p\bullet p'/m^2)(\vec{p}'\bullet\vec{\sigma}/m)\},$$

$$M_2 = \{1/(1+\omega'/m)\}(\omega'/m)(\vec{p}'\bullet\vec{\xi}_\pm/m)(\vec{p}'\bullet\vec{\sigma}/m)$$

$$-\{\vec{\xi}_\pm\bullet\vec{\sigma} + (\vec{p}'\bullet\vec{\xi}_\pm/m)(\vec{p}'\bullet\vec{\sigma}/m)\},$$

$$M_3 = \{-(\vec{p}\bullet\vec{\sigma}/m) + (p\bullet p'/m^2)(\vec{p}'\bullet\vec{\sigma}/m\}\ \vec{p}'\bullet\vec{\sigma}/m$$

$$-(1+\omega'/m)\{-\omega/m + (p\bullet p'/m^2)(\omega'/m)\},$$

$$M_4 = \{-\vec{\xi}_\pm\bullet\vec{\sigma} - (\vec{p}'\bullet\vec{\xi}_\pm/m)(\vec{p}'\bullet\vec{\sigma}/m)\}(\vec{p}'\bullet\vec{\sigma}/m)$$

$$+(1+\omega'/m)(\omega'/m)(\vec{p}'\bullet\vec{\xi}_\pm/m)\ .$$

We have used the relations,

$$A_1 = -(\tilde{A}+\tilde{B})/[1+p\bullet k/(mm_q)]\ , \tag{5.36}$$

$$A'_1 = -(\tilde{A}'+\tilde{B}')[1+p'\bullet k/(mm_q)]\ , \tag{5.37}$$

and the notation

$$\omega = \omega(\vec{p}), \tag{5.38}$$

$$\omega' = \omega(\vec{p}'). \tag{5.39}$$

Equation (5.35) can be decomposed into two 2×2 coupled matrix equations for $R_u(p)$ and $R_l(p)$. Now we multiply both the upper and the lower part of the Eq.(5.35) from the right by χ^+_{-s} and from the left by $\vec{\xi}_\pm\bullet\vec{\sigma}$ separately and perform the sum over the spin index s. Finally we take the trace of the upper and lower part of the resulting equations separately and perform a sum over the transverse polarization $\lambda=\pm$. For the trace evaluation of the 2×2 matrices we used the following relations:

$$\vec{\xi}_{\pm}^{\,*}\times\hat{p} = -\pm i\hat{p}\,\vec{\xi}_{\pm}^{\,*}, \tag{5.40}$$

$$\vec{\xi}_{\pm}^{\,*}\times\vec{\xi}_{\pm} = i\pm\,\hat{p}, \tag{5.41}$$

and

$$\mathrm{Tr}[\,\vec{\sigma}\cdot\vec{A}\;\vec{\sigma}\cdot\vec{B}\;\vec{\sigma}\cdot\vec{C}\,] = 2i(\vec{A}\times\vec{B})\cdot\vec{C}. \tag{5.42}$$

Using the relations in the Eqs.(5.40)-(5.42) we perform the required traces in Eq.(5.35). We can write that equation as,

$$2\begin{bmatrix}(\omega-2m_q)\;R_u(p)\;-\;p\;R_l(p)\\ \\ p\;R_u(p)\;\;-\;\;\omega\;R_l(p)\end{bmatrix}$$

$$= -g_\chi^2\int[d\vec{p}'/(2\pi)^3]\;[2m/\{\omega(\vec{p})\omega(\vec{p}')\}^{\frac{1}{2}}]\;[1/(m_\chi^2-q^2)]$$

$$\times\left\{F_1(q^2)\begin{bmatrix}-R_u(p')(\omega'/m-1)[\omega'/m+(\vec{p}\cdot\vec{p}'/m^2)][1-(\hat{p}\cdot\hat{p}')^2]\\ \\ -R_l(p')(pp'/m^2)[1-(\hat{p}\cdot\hat{p}')^2]\end{bmatrix}\right.$$

$$\left.+F_2(q^2)\begin{bmatrix}-R_u(p')\{(\omega'/m-1)[1-(\hat{p}\cdot\hat{p}')^2]+2\}\\ \\ -R_l(p')\;2\;\hat{p}\cdot\hat{p}'\end{bmatrix}\right\}. \tag{5.43}$$

In deriving the relation in Eq.(5.43) from Eq.(5.35) we made the replacement,

$$|\vec{p}'\cdot\vec{\xi}_{\pm}|^2 = |\vec{p}'|^2[1 - (\hat{p}\cdot\hat{p}')^2]/2 \ , \tag{5.44}$$

where $\hat{p}$ and $\hat{p}'$ are unit vectors in the direction of $\vec{p}$ and $\vec{p}'$, respectively.

We now consider Eq.(5.27) for the longitudinal polarization, λ=0. For longitudinal polarization we have [See Eq.(3.22)],

$$\xi_o^{\mu} = [\,|\vec{p}|/m \ , \ \hat{p}\ \omega(\vec{p})/m\ ] \ . \tag{5.45}$$

Using Eq.(5.45) and considering the equation in the antiquark rest frame ($\vec{K}$ = 0), Eq.(5.27) becomes,

$$\begin{bmatrix} \{(\omega-2m_q)R_u(p) - pR_l(p)\}\ \hat{p}\cdot\vec{\sigma}\ \chi_{-s} \\ \\ \{pR_u(p) - \omega R_l(p)\}\ \chi_{-s} \end{bmatrix}$$

$$= -g_\chi^2 \quad [d\vec{p}'/(2\pi)^3]\ [2m/\{\omega(\vec{p})\omega(\vec{p}')\}^{\frac{1}{2}}]\ [1/(m_\chi^2 - q^2)]$$

$$\times \begin{bmatrix} R_u(p')\{F_1(q^2)(p'\cdot\xi_o/m)\ M_5 + F_2(q^2)\ M_6\}\ \chi_{-s} \\ \\ \{B(p')/\sqrt{\omega'}\}\{F_1(q^2)(p'\cdot\xi_o/m)\ M_7 + F_2(q^2)\ M_8\}\ \chi_{-s} \end{bmatrix} \tag{5.46}$$

where

$$M_5 = -\{1/(1+\omega'/m)\}\{-\omega/m + (\omega'/m)(p\cdot p'/m^2)\}\ \vec{p}'\cdot\vec{\sigma}/m$$

$$-\{(\vec{p}\cdot\vec{\sigma}/m) - (p\cdot p'/m^2)(\vec{p}'\cdot\vec{\sigma}/m)\}\ ,$$

$$M_6 = \{-1/(1+\omega'/m)\}\{-\xi_o^o + (\omega'/m)(p'\cdot\xi_o/m)\}(\vec{p}'\cdot\vec{\sigma}/m)$$

$$-\{\vec{\xi}_o\cdot\vec{\sigma} - (p'\cdot\xi_o/m)(\vec{p}'\cdot\vec{\sigma}/m)\}\ ,$$

$$M_7 = \{(-\vec{p}\cdot\vec{\sigma})/m + (p\cdot p'/m^2)(\vec{p}'\cdot\vec{\sigma}/m)\}(\vec{p}'\cdot\vec{\sigma}/m)$$

$$-(1+\omega'/m)\{(-\omega/m) + (p\cdot p'/m^2)(\omega'/m)\}\ ,$$

$$M_8 = \{-\vec{\xi}_o\cdot\vec{\sigma} + (p'\cdot\xi_o/m)(\vec{p}'\cdot\vec{\sigma}/m)\}(\vec{p}'\cdot\vec{\sigma}/m)$$

$$-(1+\omega'/m)\{-\xi_o^o + (\omega'/m)(p'\cdot\xi_o/m)\}\ .$$

We now multiply the upper part of this equation from the left by $\hat{p}\cdot\vec{\sigma}$ and the right by χ_{-s}^{+}. Similarly we multiply the lower part from the right by χ_{-s}^{+}. After performing the sum over the index s and taking traces of the upper and lower part separately Eq.(5.46) becomes,

$$\left| \begin{array}{l} \{(\omega-2m_q)\ R_u(p)\ -\ p\ R_l(p)\} \\ \\ \{p\ R_u(p)\ \ -\ \ \omega\ R_l(p)\} \end{array} \right.$$

$$= -g_\chi^2 \int [d\vec{p}'/(2\pi)^3]\ [2m/\{\omega(\vec{p})\omega(\vec{p}')\}^{\frac{1}{2}}]\ [1/(m_\chi^2 - q^2)]$$

$$\times \left\{ F_1(q^2) \left[\begin{array}{l} R_u(p')[(\omega\omega'/m^2)(\vec{p}\bullet\vec{p}'/m^2) - (\hat{p}\bullet\hat{p}')^2(\omega'^2\vec{p}^2 + \omega^2\vec{p}'^2)/m^4 \\ \quad +(\hat{p}\bullet\hat{p}')^3(\omega\omega' pp'/m^4) + \{(\hat{p}\bullet\hat{p}')^2-1\}(p^2\omega'-\omega\vec{p}\bullet\vec{p}')/m^3] \\ \\ R_l(p')[\{\omega p'-\omega' p\ \hat{p}\bullet\hat{p}')(\omega' p-\omega p'\ \hat{p}\bullet\hat{p}')/m^4] \end{array} \right] \right.$$

$$\left. +F_2(q^2) \left[\begin{array}{l} R_u(p')[(\vec{p}\bullet\vec{p}'/m^2)-(\omega\omega'/m^2)(\hat{p}\bullet\hat{p}')^2-(\omega/m)\{1-(\hat{p}\bullet\hat{p}')^2\}] \\ \\ R_l(p')[(pp'/m^2) - (\omega\omega'/m^2)\hat{p}\bullet\hat{p}'] \end{array} \right] \right\} . \tag{5.47}$$

We can eliminate the polarization dependence of the equations by combining Eq.(5.43) for transverse polarization and Eq.(5.47) for longitudinal polarization. The resulting equation is,

$$\omega \begin{bmatrix} R_u(p) \\ R_l(p) \end{bmatrix} = \begin{bmatrix} 2m_q & p \\ p & 0 \end{bmatrix} \begin{bmatrix} R_u(p) \\ R_l(p) \end{bmatrix}$$

$$-(g_\chi^2/3) \quad [d\vec{p}'/(2\pi)^3] \;\; [2m/\{\omega(\vec{p})\omega(\vec{p}')\}^{\frac{1}{2}}] \;\; [1/(m_\chi^2 - q^2)]$$

$$\times \left\{ F_1(q^2) \begin{bmatrix} V_{11}^1 & V_{12}^1 \\ V_{21}^1 & V_{22}^1 \end{bmatrix} \begin{bmatrix} R_u(p') \\ R_l(p') \end{bmatrix} \right.$$

$$\left. + F_2(q^2) \begin{bmatrix} V_{11}^2 & V_{12}^2 \\ V_{21}^2 & V_{22}^2 \end{bmatrix} \begin{bmatrix} R_u(p') \\ R_l(p') \end{bmatrix} \right\} \qquad (5.48)$$

where

$$V_{12}^1 = V_{21}^1 = V_{12}^2 = V_{21}^2 = 0 \; , \qquad (5.49)$$

$$V_{11}^{1} = -(\omega\vec{p}'^2 + \omega'\vec{p}^2)/m^3 + (\vec{p}\cdot\vec{p}'/m^2)[(\omega/m) + (\omega'/m) + (\omega\omega'/m^2) - 1]$$

$$- (\hat{p}\cdot\hat{p}')^2(\vec{p}^2\vec{p}'^2/m^4)[(\omega/m) + (\omega'/m)]$$

$$+ (\hat{p}\cdot\hat{p}')^3(pp'/m^2)[1 - (\omega/m) - (\omega'/m) + (\omega\omega'/m^2)] \ , \tag{5.50}$$

$$V_{22}^{1} = (pp'/m^2)[1 - (\omega\omega/m^2)] + (\hat{p}\cdot\hat{p}')(\vec{p}^2\omega'^2 + \vec{p}'^2\omega^2)/m^4$$

$$- (\hat{p}\cdot\hat{p}')^2(pp'/m^2)[(\omega\omega'/m^2) + 1] \ , \tag{5.51}$$

$$V_{11}^{2} = -1 - (\omega + \omega')/m + \vec{p}\cdot\vec{p}'/m^2$$

$$+ (\hat{p}\cdot\hat{p}')^2[(\omega/m) + (\omega'/m) - (\omega\omega'/m^2) - 1] \ , \tag{5.52}$$

$$V_{22}^{2} = -(pp'/m^2) + \hat{p}\cdot\hat{p}'[(\omega\omega'/m^2) + 2] \ . \tag{5.53}$$

In deriving Eq.(5.48) from Eqs.(5.43) and (5.47) we changed the sign of the lower part of the resulting combined equations. This is done to exhibit a hermitian interaction for this problem. Here again we notice the symmetry of the kernal in Eq.(5.48) under the interchange of $\vec{p}$ and $\vec{p}'$. This is the dynamical equation we have to consider for the vector meson. The method of solution of this equation is exactly similar to the pseudoscalar case and is described in Chapter 6.

5.3 Equations of Motion for Particles without Isospin:

For the charmonium and upsilon systems the quarks does not carry isospin. In our whole analysis the isospin factors drop out from the equations. Thus we can extend our analysis to particles without isospin. This is because all our amplitudes are defined to within a constant multiplicative factor and it is the normalization which fixes this multiplicative factor. In this case we define the amplitude in a slightly different way so that the expressions for various physical quantities under consideration are same as in the case the quarks have isospin. Thus for pseudoscalar meson we define,

$$< \vec{K}\, s\ |\bar{q}_{\alpha}(0)|\vec{p}> = [\ \sqrt{2}]\times[1/(2\pi)^{3}]\,[1/2\omega(\vec{p})]^{\frac{1}{2}}[m_{q}/E_{q}(\vec{K})]^{\frac{1}{2}}$$

$$\times[\ \bar{U}(\vec{K},s)(A+B\ \not{p}/m)\gamma^{5}]_{\alpha}\ , \qquad (5.54)$$

where $|\vec{K}\ s>$ is the quark state and $|\vec{p}>$ the meson state. The charge conjugate amplitude can be derived from this in a manner exactly similar to the case when there is isospin [See Appendix B]. Thus we have,

$$< \overline{\vec{K}\ s}\ |q_{\alpha}(0)|\vec{p}> = \eta\ \ [\sqrt{2}]\times[1/(2\pi)^{3}]\,[1/2\omega(\vec{p})]^{\frac{1}{2}}[m_{q}/E_{q}(\vec{K})]^{\frac{1}{2}}$$

$$\times[\ \gamma^{5}(E+F\ \not{p}/m)V(\vec{K},s)]_{\alpha}\ , \qquad (5.55)$$

where E = A ,F = -B and η is a phase factor. As in Eqs.(3.13) and (3.14), if we define,

$$\hat{R}_u(k) = \{[4\pi/\{m(2\pi)^3\}][\varepsilon_q(\vec{k})/E_q(\vec{k})]\}^{\frac{1}{2}}\{E(k') - F(k')\} , \qquad (5.56)$$

$$\hat{R}_l(k) = \{[4\pi/\{m(2\pi)^3\}][\varepsilon_q(\vec{k})/E_q(\vec{k})]\}^{\frac{1}{2}}\{E(k') + F(k')\} k/\varepsilon_q(\vec{k}) , \qquad (5.57)$$

where $k' = mk/m_q$, we can choose a normalization:

$$\int dk \quad \vec{k}^2[\{\hat{R}_u(k)\}^2+\{\hat{R}_l(k)\}^2] = 1. \qquad (5.58)$$

With this normalization the new problem is the same as the old problem with isospin. Similarly, in the case of vector meson we define,

$$< \vec{k}\ s\ |\overline{q}_\alpha(0)|\vec{p}\ \lambda > = [\surd 2]\times[1/(2\pi)^3][1/2\omega(\vec{p})]^{\frac{1}{2}}[m_q/E_q(\vec{k})]^{\frac{1}{2}}$$

$$\times\{\overline{U}(\vec{k},s)[\xi_\lambda\bullet k/m_q\ A_1(1+\not{p}/m)+\not{\xi}_\lambda(\tilde{A}+\tilde{B}\ \not{p}/m)]\}_\alpha . \qquad (5.59)$$

We have,

$$\hat{R}_u(k) = \{[4\pi/\{m(2\pi)^3\}][\varepsilon_q(\vec{k})/E_q(\vec{k})]\}^{\frac{1}{2}}\{\tilde{A}(k') - \tilde{B}(k')\} , \qquad (5.60)$$

$$\hat{R}_l(k) = \{[4\pi/\{m(2\pi)^3\}][\varepsilon_q(\vec{k})/E_q(\vec{k})]\}^{\frac{1}{2}}\{\tilde{A}(K') + \tilde{B}(k')\} k/\varepsilon_q(\vec{k}) , \qquad (5.61)$$

$$\int dk \quad \vec{k}^2[\{\hat{R}_u(k)\}^2+\{\hat{R}_l(k)\}^2] = 1. \qquad (5.62)$$

CHAPTER VI

Numerical Results and Discussion

Here we discuss our calculational procedure and the results of our analysis. First we comment on the parameters of our model. We consider the case where we do not include the gluonic degrees of freedom explicitly. In this case we have three parameters in our model. These are the constituent quark mass, m_q, mass of the scalar field, m_χ, and the coupling constsant, g_χ, which determines the strength of the coupling of the quarks to the scalar field. In addition, we include a cut-off parameter in the form factors. This cut-off is used to regulate the behavior of the theory at high momentum transfer. The details of this cut off is given in Section 6.3. Of these three parameters m_χ, and g_χ are fixed from our previous calculation of nucleon structure [11,13]. Thus the only free parameter in our model is the flavour-dependent constituent quark mass, m_q. The inclusion of explicit gluonic degrees of freedom introduces another parameter in our model. (The details of this modification are given in Section 6.4.)

We now describe the procedure we used in the numerical calculations. The central equations in our formalism are given in

Chapter 5. These are Eq.(5.21) for pseudoscalar mesons and Eq.(5.48) for vector mesons. The method of solution for both these equations are the same. Thus we consider the equations for pseudoscalar mesons in detail.

6.1 Mass of the Meson as an Eigenvalue:

Mathematically Eq.(5.21) is a homogeneous coupled nonlinear integral equation. This is due the fact that the amplitude we are trying to obtain appears in the expression for the form factors [See Eq.(4.12)]. (This is clear from the Figure 1.) Another very important feature is the appearence of the mass of the meson 'm' (which is an unknown quantity) in a nonlinear fashion in Eq.(5.21). It is useful to note that this mass, m, also appears in the expression for the form factors given in Eq.(4.12).

Thus for a given g_χ, m_χ and m_q, Eq.(5.21) represents a nontrivial eigenvalue problem. It is a homogeneous equation for $R_u(p)$ and $R_l(p)$ and it does not yield a nontrivial solution for all values of m. Thus the value of m is determined so that one has a nontrivial solutions to Eq.(5.21). This is true for the vector meson case also where we consider Eq.(5.48) instead of Eq.(5.21). We denote this value of m which yields a nontrivial solution as m(theory). [See Tables 1-2]

In actual calculation we cast our equations in the form of an eigenvalue equation by replacing $\omega(p)$ in the L.H.S of Eq.(5.21) by $(\lambda^2 + \vec{p}^2)^{\frac{1}{2}}$. With an initial guess for m and for the scalar form factors we now iterate Eq.(5.21) until the whole procedure is self-consistent. It is important to note that the initial guessed form for $F_s(q^2)$ is only used in the first iteration. After full iteration, Eq.(5.21) will give a new self-consistent form factor. We achieve full self-consistency by making the eigenvalue λ the same as the input mass m, [denoted as m(theory) in Table 1-2].

We adjust our quark mass parameter, m_q, so that we get self-consistency for the ground states of the ρ, J/Ψ and Υ systems. Now when considering the excited states (for each flavour sector) we do not change any parameter. We just look for the next self-consistent eigenvalue of Eq.(5.21) with the same parameters as used to construct the ground state. Thus the properties of the excited states are predictions of our model. For vector mesons a similar analysis can be made using Eq.(5.48).

As a by-product of this self-consistent analysis of Eq(5.21), we get the eigenvectors $R_u(p)$ and $R_l(p)$ for each eigenvalue m. Using Eqs.(5.14)-(5.17) we can now find the invariant amplitudes E and F. Once we have the invariant amplitude we can calculate the meson rest frame wave functions $\hat{R}_u(k)$ and $\hat{R}_l(k)$ using Eqs.(3.13) and (3.14). The arbitrariness in the amplitude due to the homogeneity of Eq.(5.21) can be removed by using the normalization condition of Eq.(3.15). The same analysis may be used for vector mesons as well.

In our analysis we use g_χ = 7.0 and m_χ = 500Mev. The constituent quark mass, m_q, for various flavour sectors are presented in Table 1.

6.2 Coordinate Wave Functions and Radii:

Once we have the self-consistent invariant amplitudes and the momentum-space wave functions, $\hat{R}_u(k)$ and $\hat{R}_l(k)$, as described in Section 6.1 we can calculate the coordinate-space wave functions by Fourier transformation. Thus we have,

$$\hat{R}_u(r) = (2/\pi)^{\frac{1}{2}} \int \vec{k}^2 \, dk \; R_u(k) \; j_o(kr), \tag{6.1}$$

$$\hat{R}_l(r) = (2/\pi)^{\frac{1}{2}} \int \vec{k}^2 \, dk \; R_l(k) \; j_1(kr). \tag{6.2}$$

Here $j_o(kr)$ and $j_1(kr)$ are Bessel functions of order 0 and 1 respectively. Plots of coordinate wave functions for various systems are shown in Figures 2-5.

Once we have the coordinate-space wave functions we can find the size of the soliton. Thus we have the root-mean-square radii of baryon and scalar densities:

$$<r^2>_B = \int r^2\, dr\, r^2[\{\hat{R}_u(r)\}^2 + \{\hat{R}_l(r)\}^2], \tag{6.3}$$

$$<r^2>_S = \int r^2\, dr\, r^2[\{\hat{R}_u(r)\}^2 - \{\hat{R}_l(r)\}^2]. \tag{6.4}$$

Values for these-root mean-square radii are presented in Table 1.

For charged particles (for example ρ and π) we can also define charge radii, $<r^2>_{em}$, by taking the appropriate slope of the electromagnetic form factors. Thus for pseudoscalar mesons we have,

$$<r^2>_{em} = -(1/6)\ dF_S^{em}(q^2)/dq^2|_{q^2=0} \tag{6.5}$$

and for vector meson

$$<r^2>_{em} = -(1/6)\ dF_1^{em}(q^2)/dq^2|_{q^2=0}\ , \tag{6.6}$$

where $F_S{}^{em}(q^2)$ and $F_1{}^{em}(q^2)$ are given by Eqs.(C.18) and (C.72) respectively. The values for these radii for π and ρ mesons are given in Table 1.

6.3 Form Factors:

From the (self-consistent) analysis of Eq.(5.21) for pseudoscalar mesons and Eq.(5.48) for vector mesons, as described in Section 6.1, we get the self-consistent form factors $F_s(q^2)$, $F_1(q^2)$ and $F_2(q^2)$, which were defined in Chapter 4. As we mentioned earlier, there is a cut-off in our model to regulate the high momentum components in the theory. We chose a dipole form:

$$f_c(q^2) = [\Lambda^2/(\Lambda^2-q^2)]^3 \tag{6.7}$$

with $\Lambda = 10\text{fm}^{-1} \simeq 2\text{Gev}$ for this cut-off. During iteration we modulate all of our scalar form factors, $F_s(q^2)$, $F_1(q^2)$ and $F_2(q^2)$ with this factor. Note that this effects only the high q^2 region of the actual form factors. These modulated form factors for various systems are ploted in Figures 6-9. It is important to remember that the self-consistent solution we present are obtained with the modulated form facators. The use of form factors does not effect the self-consistency of our solution.

6.4 Mass of the Meson as an Expectation Value of the Hamiltonian:

We have calculated the expectation value of the Hamiltonian for various mesons in Appendix D. Using the self-consistent amplitudes we can explicitly evaluate this expectation value. Thus for pseudoscalar mesons, using Eqs.(D.15) and (D.16), we can calculate

$$\langle H\rangle = m_H = 2[m - \langle E_q\rangle] + \varepsilon_\chi^s . \tag{6.8}$$

The values for $\langle E_q\rangle$ and ε_χ^s are given in Table 2 for various mesons. For vector mesons we have,

$$\langle H\rangle = m_H = 2[m - \langle E_q\rangle] + \varepsilon_\chi^v , \tag{6.9}$$

where $\langle E_q\rangle$ and ε_χ^v are given in Eqs.(D.29) and (D.30) respectively. The values for $\langle E_q\rangle$ and ε_χ^v for various vector mesons are also presented in Table 2.

6.5 Numerical Results with Gluon-Exchange Correction:

We now investigate the effect of gluon-exchange in our model by considering the equations given in Appendix E. Here we consider Eqs.(E.32) and (E.40) for pseudoscalar and vector mesons, respectively. These equations are similar to the homogeneous integral equation we considered in Section 6.1 except for the fact that the kernal due to gluon exchange does not depend on the amplitude we are trying to obtain.

From Eqs.(E.32) and (E.40) it is clear that this correction introduces one more parameter, g, which is quark-gluon coupling strength. In addition we have a new cut-off function $f_G(q_k^2)$ which is needed to make the kernal due to gluon exchange finite. For this cut-off function we chose,

$$f_G(q_k^2) = [\Lambda_k^2/(\Lambda_k^2 - q_k^2)]^n \qquad (6.10)$$

where $\Lambda_k = 30\ fm^{-1}$ and $n = 6$. We have a larger cut off for Λ_k for this function compared to the previous one due to the fact that we want to keep the high-momentum components of the gluon exchange as this is expected to be a relatively short-range effect.

With the new parameter g and the cut-off function $f_G(q_k^2)$ we analysed Eqs.(E.32) and (E.40) as described in Section 6.1. We fix the parameter g such that we have the right mass splitting between J/Ψ(1S) and the χ_c(1S). With these parameter fixed we now have a prediction for the splitting between J/Ψ(2S) and χ_c(2S) and between various pseudoscalar and vector states in the upsilon systems. In carrying out this program we change our quark mass parameter, m_q, to give a new self-consistent solution using the techniques described in Section 6.1.

In our calculation we used $g^2 = 6.08$ or $\alpha_c = g^2/(4\pi) = 0.48$. The result of these new self-consistent calculations are given in Table 3. Here $\langle r^2\rangle_B$ and $\langle r^2\rangle_S$ are the same quantities as defined Section 6.2 except that we now calculate these quantities using the new self-consistent amplitudes.

In Table 4 we present the mass of the meson calculated by taking the expectation value of the Hamiltonian [See Eqs.(E.67) and (E.102)]. In the table the various contributions are tabulated seperately.

6.6 Discussion:

We have presented a simple field-theoretic model of nontopological solitons with a minimum number of parameters and demonstrated the simplicity and the power of the covariant analysis. We did not try to make a detailed fit to various levels by doing an extensive parameter search. From Tables 1 and 2 we have a resonable fit to various levels of the mesonic systems. The main discrepancy is the too large seperation between the various states of charmonium and upsilon systems. There is also a difference between m(theory) calculated from the dynamical equation and the m_H calculated from the Hamiltonian as can be seen from Tables 2 and 4. It is interesting to note that the largest discrepancies between m and m_H appear for the mesons with the smallest size. The discrepancy becomes smaller as the meson size increases in a systematic fashion indicating that for quite large objects one could achieve consistency for m and m_H. As we pointed out earlier we did not attempt an extensive parameter search and it may be possible to remedy the defects noted here partially by carrying out such a search.

From Table 1 it is also interesting to note the degeneracy in masse between the pseudoscalar and vector mesons, eventhough the equations which determine the invariant amplitudes for pseudoscalar and vector mesons are very different [See Eqs.(5.21) and (5.48)]. The inclusion of gluon-exchange removes this degeneracy, as can be seen from Table 3. Thus we have the predictions for mass splitting in the upsilon system: $m[\Upsilon(1S)]_{J=1}-m[\Upsilon(1S)]_{J=0}=35$ Mev. The splitting of $\Upsilon(2S)$ and $\Upsilon(3S)$ states are quite small and probably not significant, given the numerical accuracy of our calculation. It is worthwhile noting that inclusion of gluon-exchange gives an overall attraction in both pseudoscalar and vector channels, but the attractive force is stronger in the pseudoscalar channel.

In our formalism without gluon-exchange the pion is degenerate with the ρ and ω mesons. The inclusion of gluon exchange in the pion channel has some interesting properties. It leads to a quite strong attraction [18] in this channel and we were not able to find a stable solution. (The numerical results became very unstable in this case.) It is true that any satisfactory model of pion structure requires some understanding of the breaking of chiral symmetry in QCD and our effective Lagrangian in Eq.(2.3) does not exhibit chiral symmetry. But within the broken symmetry configuration it is interesting to see the large attractive contribution obtained from gluon-exchange effects. Ofcourse, the description of the pion in our model as quark antiquark pair has its own limitations. It may be possible to improve our model by expanding the Hilbert space to include the sea quark effects. It is also true that we did not consider the Goldstone boson natutre of pion in our model [19].

Another serious limitation of the model is that we do not have a confinement mechanism. A specific coavariant confinement model is described in [15]. We are now investigating the consequences of this model.

APPENDIX A

Parity.

In this Appendix we investigate the properties of the amplitudes given by Eqs.(3.10) and (3.23) under parity transformations. With $\underset{\sim}{P}$ as the parity operator we have [16,17],

$$\underset{\sim}{P}\; q(0)\; \underset{\sim}{P}^{-1} = \gamma^{o}\; q(0), \tag{A.1}$$

$$\underset{\sim}{P}\; |\vec{k}\; s\; t> = e^{-i\phi(k,s)}\; |-\vec{k}\; s\; t>, \tag{A.2}$$

where $\phi(k,s)$ is a phase factor. Now we consider the transformation of pseudoscalar and vector mesons seperately.

A.1 Pseudoscalar Mesons:

For a canonical state we have [17],

$$\underset{\sim}{P}\; |\vec{p}\; M_T> = -|-\vec{p}\; M_T>. \tag{A.3}$$

Consider now the amplitude given by Eq.(3.10)

$$<\vec{k}\; s\; t|\bar{q}_{\alpha i}(0)|\vec{p}\; M_T> = f_i(k,p,t,M_T)\times[\; \bar{U}(\vec{k},s)(A+B\; \not{p}/m)\gamma^5]_\alpha \tag{A.4}$$

where

$$f_i(k,p,t,M_T) = [1/(2\pi)^3][1/2\omega(\vec{p})]^{\frac{1}{2}}[m_q/E_q(\vec{K})]^{\frac{1}{2}}\ [\chi_t^{+}\ \vec{\tau}\cdot\hat{e}_{M_T}]_i. \tag{A.5}$$

Using Eqs.(A.1)-(A.3) we have,

$$<\vec{K}\ s\ t|\bar{q}_{\alpha i}(0)|\vec{p}\ M_T> = <\vec{K}\ s\ t|\underset{\sim}{P}^{-1}\underset{\sim}{P}\bar{q}_{\alpha i}(0)\underset{\sim}{P}^{-1}\underset{\sim}{P}|\vec{p}\ M_T>$$

$$= -e^{i\phi(k,s)}\ <-\vec{K}\ s\ t\ |\bar{q}_{\beta i}(0)|-\vec{p}\ M_T>\ (\gamma^o)_{\beta\alpha}. \tag{A.6}$$

Using the fact that A and B are functions of $[(p\cdot k/m_q)^2-m^2]^{\frac{1}{2}}$ we can write Eq.(A.6) as:

$$<\vec{K}\ s\ t|\bar{q}_{\alpha i}(0)|\vec{p}\ M_T> = -\ e^{i\phi(k,s)}\ f_i(k,p,t,M_T)$$

$$\times\ [\overline{U}(-\vec{K},s)(A + B\ \tilde{\not{p}}/m)\gamma^5\gamma^o]_\alpha \tag{A.7}$$

where

$$\tilde{p}^\mu = (p^o\ ,\ -\vec{p}). \tag{A.8}$$

Now using

$$\gamma^o\ \gamma^5 = -\gamma^5\ \gamma^o, \tag{A.9}$$

$$\gamma^o\ \tilde{\not{p}}\ \gamma^o = \not{p}\ , \tag{A.10}$$

Eq.(A.7) becomes:

$$<\vec{K}\ s\ t|\bar{q}_{\alpha i}(0)|\vec{p}\ M_T> = e^{i\phi(k,s)}\ f_i(k,p,t,M_T)$$

$$\times\ [\overline{U}(-\vec{K},s)\gamma^o(A + B\not{p}/m)\gamma^5]_\alpha. \tag{A.11}$$

Further with the relation

$$U(-\vec{k},s) = e^{i\phi(k,s)}\ \gamma^{o}\ U(\vec{k},s) \qquad (A.12)$$

we can recover Eq.(A.4) from (A.11). [Note the important role of γ^5 in Eq.(A.4).]

A.2 Vector mesons:

For a state with helicity λ we have [17],

$$\underset{\sim}{P}\ |\vec{p}\ \lambda\ M_T\rangle = \eta\ e^{-i\pi}\ |-\vec{p}\ -\lambda\ M_T\rangle \qquad (A.13)$$

where η is the intrinsic parity of the vector particle. We now consider the amplitude given by Eq.(3.23),

$$\langle\vec{k}\ s\ t|\bar{q}_{\alpha i}(0)|\vec{p}\ \lambda\ M_T\rangle = f_i(k,p,t,M_T)$$
$$\times[\bar{U}(\vec{k},s)\{(\xi_\lambda\bullet k/m_q)A_1(1 + p/m) + \xi_\lambda(\tilde{A} + \tilde{B}\ p/m)\}]_\alpha\ . \qquad (A.14)$$

Also we have,

$$\langle\vec{k}\ s\ t|\bar{q}_{\alpha i}(0)|\vec{p}\ \lambda\ M_T\rangle = \langle\vec{k}\ s\ t|\underset{\sim}{P}^{-1}\underset{\sim}{P}q_{\alpha i}(0)\underset{\sim}{P}^{-1}\underset{\sim}{P}|\vec{p}\ \lambda\ M_T\rangle$$
$$= e^{-i\pi}\ e^{i\phi(k,s)}\ \langle-\vec{k}\ s\ t|\bar{q}_{\beta i}(0)|-\vec{p}\ -\lambda\ M_T\rangle\ (\gamma^{o})_{\beta\alpha} \qquad (A.15)$$

where we have used Eqs.(A.1), (A.2) and (A.13). Now using Eq.(A.14) we can rewrite Eq.(A.15) as,

$$\langle\vec{k}\ s\ t|\bar{q}_{\alpha i}(0)|\vec{p}\ \lambda\ M_T\rangle = \eta\ e^{-i\pi}e^{i\phi(k,s)}\ f_i(k,p,t,M_T)$$

$$\times[\bar{U}(-\vec{k},s)\{(\tilde{\xi}_{-\lambda}\bullet\tilde{k}/m_q)A_1(1 + \tilde{\not{p}}/m) + \tilde{\not{\xi}}_{-\lambda}(\tilde{A} + \tilde{B}\ \tilde{\not{p}}/m)\}\gamma^o]_\alpha\ ,$$

(A.16)

where

$$\tilde{k}^\mu = (k^o\ ,\ -\vec{k}), \tag{A.17}$$

and

$$\tilde{\xi}_\lambda{}^\mu = \{\xi^o_\lambda(-p)\ ,\ \vec{\xi}_\lambda(-p)\}. \tag{A.18}$$

Making use of the relation

$$\xi_{-\lambda}{}^\mu = (\xi^o_\lambda\ ,\ -\vec{\xi}_\lambda) \tag{A.19}$$

we find

$$\tilde{\xi}_{-\lambda}\bullet\tilde{k} = \xi_\lambda\bullet k, \tag{A.20}$$

$$\gamma^o\,\tilde{\not{\xi}}_{-\lambda}\ \gamma^o = \not{\xi}_\lambda\ . \tag{A.21}$$

We can recover Eq.(A.14) from Eq.(A.16) by noting that the intrinsic parity of the vector particle $\eta = -1$.

APPENDIX B

Charge Conjugation

Here we derive Eq.(3.11) from Eq.(3.10), and Eq.(3.24) from Eq.(3.23). With $\underset{\sim}{C}$ as the charge conjugation operator we have [16,17,20,21],

$$\underset{\sim}{C}\ \bar{q}_{\alpha i}(0)\ \underset{\sim}{C}^{-1} = C_{\alpha\beta}\ q_{\beta i}(0), \tag{B.1}$$

$$\underset{\sim}{C}\ |\vec{k}\ s\ t\rangle = \eta_t\ \overline{|\vec{k}\ s\ -t\rangle} \tag{B.2}$$

where $C = i\,\gamma^2\,\gamma^o$ and $\overline{|\vec{k}\ s\ -t\rangle}$ is the antiparticle state corresponding to the particle state $|\vec{k}\ s\ t\rangle$. In addition, for the mesonic states we have [16,17,20,21],

$$\underset{\sim}{C}\ |\vec{p}\ M_T\rangle = (-1)^{M_T}\ |\vec{p}\ -M_T\rangle, \tag{B.3}$$

$$\underset{\sim}{C}\ |\vec{p}\ \lambda\ M_T\rangle = (-1)^{1+M_T}\ |\vec{p}\ \lambda\ -M_T\rangle. \tag{B.4}$$

From Eqs.(B.3) and (B.4) it is clear that the neutral pseudoscalar and vector mesons are eigenstates of the charge conjugation operator with eigenvalues +1 and -1, respectively. For the pseudoscalar meson we now consider the amplitude given in Eq(3.10),

$$\langle \vec{k}\ s\ t|\bar{q}_{\alpha i}(0)|\vec{p}\,M_T\rangle = f(k,p)\ [\bar{U}(\vec{k},s)(A + B\not{p}/m)\gamma^5]_\alpha\ [\chi_t^+\ \vec{\tau}\cdot\hat{e}_{M_T}]_i\ , \tag{B.5}$$

where

$$f(k,p) = [1/(2\pi)^3][1/2\omega(\vec{p})]^{\frac{1}{2}}[m_q/E_q(\vec{k})]^{\frac{1}{2}}. \tag{B.6}$$

We also have,

$$\langle \vec{k}\ s\ t|\bar{q}_{\alpha i}(0)|\vec{p}\,M_T\rangle = \langle \vec{k}\ s\ t|\mathcal{C}^{-1}\mathcal{C}q_{\alpha i}(0)\mathcal{C}^{-1}\mathcal{C}|\vec{p}\,M_T\rangle$$

$$= (-1)^{1+M_T}\ \eta_t^*\ \langle \vec{k}\ s\ -t|q_{\beta i}(0)|\vec{p}\ -M_T\rangle\ C_{\beta\alpha}^{-1}\ , \tag{B.7}$$

where we used the relations in Eqs.(B.1)-(B.3). We can now rewrite Eq.(B.7) by using Eq.(B.5):

$$\overline{\langle \vec{k}\ s\ t}|q_{\beta i}(0)|\vec{p}\,M_T\rangle = (-1)^{1+M_T}\ \eta_{-t}\ \langle \vec{k}\ s\ -t|\bar{q}_{\beta i}(0)|\vec{p}\ -M_T\rangle\ C_{\alpha\beta}$$

$$= (-1)^{1+M_T}\ \eta_{-t}f(k,p)\ [\bar{U}(\vec{k},s)(A + B\not{p}/m)\gamma^5]_\alpha C_{\alpha\beta}$$

$$\times[\chi_{-t}^+\vec{\tau}\cdot\hat{e}_{-M_T}]_i. \tag{B.8}$$

Now,

$$V(\vec{k},s) = C\ [\bar{U}(\vec{k},s)]^T, \tag{B.9}$$

$$C\ \gamma_\mu\ C^{-1} = -\ \gamma_\mu^{\ T}, \tag{B.10}$$

$$\hat{e}_{M_T}^* = (-1)^{M_T}\ \hat{e}_{-M_T}, \tag{B.11}$$

$$C^2 = -1. \tag{B.12}$$

We can also rewrite Eq.(B.8) as,

$$\overline{\langle \vec{k}\, s\, t|} q_{\alpha i}(0)|\vec{p}\, M_T\rangle = f(k,p)\ [\gamma^5(E + F\not{p}/m)V(\vec{k},s)]_\alpha$$

$$\times\ [(\vec{\tau}\cdot\hat{e}^*_{M_T})^T \chi_{-t}\eta_{-t}]_i\ , \qquad \text{(B.13)}$$

where

$$E = A\ , \qquad \text{(B.14)}$$

$$F = -B\ . \qquad \text{(B.15)}$$

This is the relation given in Eq.(3.11).

Using Eqs.(B.1), (B.2) and (B.4), a similar analysis for the vector meson yields,

$$\overline{\langle \vec{k}\, s\, t|} q_{\beta i}(0)|\vec{p}\ \lambda\ M_T\rangle = (-1)^{M_T}\eta_{-t}\ \langle\vec{k}\, s\ -t|\overline{q}_{\beta i}(0)|\vec{p}\ \lambda\ -M_T\rangle\ C_{\beta\alpha}. \qquad \text{(B.16)}$$

Now the right-hand side of Eq.(B.17) can be written in terms of the invariant amplitudes using the definition in Eq.(3.23). Thus we have,

$$\overline{\langle \vec{k}\, s\, t|} q_{\beta i}(0)|\vec{p}\ \lambda\ M_T\rangle = f(k,p)\ (-1)^{M_T}\eta_{-t}$$

$$\times\{\overline{U}(\vec{k},s)[(\xi_\lambda\cdot k/m_q)\ A_1(1+\not{p}/m) + \not{\xi}_\lambda(\tilde{A}+\tilde{B}\ \not{p}/m)]\}_\alpha\ C_{\alpha\beta}$$

$$\times[\chi^+_{-t}\ \vec{\tau}\cdot\hat{e}_{-M_T}]_i\ . \qquad \text{(B.17)}$$

Now using Eqs.(B.9)-(B.12), Eq.(B.17) can be rewritten as,

$$\overline{\langle \vec{K}\ s\ t|} q_{\alpha i}(0)|\vec{p}\ \lambda\ M_T\rangle = -f(k,p)$$

$$\times\{[(\xi_\lambda \cdot k/m_q)\ A_1(1-\not{p}/m)\ -\xi_\lambda(\tilde{A}+\tilde{B}\not{p}/m)]V(\vec{K},s)\}_\alpha[(\vec{\tau}\cdot\hat{e}^*_{M_T})^T\chi_{-t}\eta_{-t}]_i, \tag{B.18}$$

which is the relation previously given in Eq.(3.24).

APPENDIX C

Electromagnetic Form Factors

Here we calculate the electromagnetic form factors of various mesons and check the consistency of the normalization given by Eqs.(3.15) and (3.29).

C.1 The Charge Operator:

The charge operator, Q is given by

$$Q \quad = \quad \int d\vec{x} \;\; :\bar{q}(x)\; \gamma^{o}[(1/6) + (\tau_3/2)]\; q(x): \;, \tag{C.1}$$

where τ_3 is the third component of the isospin operator and :: represents a normal ordering prescription. For a pseudoscalar meson we have,

$$<\vec{p}'\; M'_T|Q|\vec{p}\; M_T> \;\; = M_T\; \delta_{M_T M'_T}\; \delta(\vec{p}'-\vec{p}). \tag{C.2}$$

Now denoting the momentum operator as $\underset{\sim}{P}^{\mu}$ we can write

$$q(x) = e^{i\underset{\sim}{P}\bullet x} q(0) e^{-i\underset{\sim}{P}\bullet x}. \tag{C.3}$$

Using Eq.(C.3) we can write Eq.(C.1) as,

$$\langle\vec{p}'\ M'_T|Q|\vec{p}\ M_T\rangle$$
$$= (2\pi)^3\ \delta(p'-p)\ \langle\vec{p}'\ M'_T|\ \bar{q}(0)\ \gamma^o[(1/6)\ +\ (\tau_3/2)]\ q(0):|\vec{p}\ M_T\rangle. \tag{C.4}$$

If we compare Eq.(C.4) and Eq.(C.2) it is clear that the relevant matrix element is given by $(2\pi)^3\langle\vec{p}'\ M'_T|\bar{q}(0)\gamma^{\mu}[(1/6)+(\tau_3/2)]q(0)|\vec{p}\ M_T\rangle$. The charge of the particle can be obtained from this by considering the time component, $\mu = 0$. It is also useful to remember that $\bar{q}(0)\gamma^{\mu}[(1/6)+(\tau_3/2)]q(0)$ is the electromagnetic current operator.

C.2 Electromagnetic Form Factor for Pseudoscalar Mesons:

The matrix element of the quark electromagnetic current between mesonic states is

$$(2\pi)^3\langle\vec{p}'\ M'_T|\bar{q}(0)\gamma^{\mu}[(1/6)+(\tau_3/2)]q(0)|\vec{p}\ M_T\rangle$$
$$= M_T\ \delta_{M_T M'_T}[(p'+p)^{\mu}/(2m)]\ \ [m/\{\omega(\vec{p})\omega(\vec{p}')\}^{\frac{1}{2}}]\ F_s^{em}(q^2). \tag{C.5}$$

Here $q^2 = (p'-p)^2$ and $F_s^{em}(q^2)$ is a (Lorentz) scalar function. It is evident that this is the only invariant we can construct from the available Lorentz tensors. When we take the time component, μ=0, of Eq.(C.5) and go to the meson rest frame, $\vec{p}$=0, $\vec{p}'$=0 we have q^2=0. In this way we can exhibit the charge of the particle. Therefore the normalization has to be such that

$$F_s^{em}(q^2=0) = 1. \tag{C.6}$$

We now check the consistency of the relation in Eq.(C.6) with the normalization condition given in Eq.(3.15). For this we consider

$$\begin{aligned}&<\vec{p}'\,M'_T|:\bar{q}(0)\,\gamma^\mu[(1/6)+(\tau_3/2)]\,q(0):|\vec{p}\,M_T> = \Sigma_{s,t}\int d\vec{k}\\ &\times\{<\vec{p}'\,M'_T|\bar{q}_{\alpha i}(0)|\overline{\vec{k}\,s\,t>}(\gamma^\mu)_{\alpha\beta}[(1/6)+(\tau_3/2)]_{ij}\overline{<\vec{k}\,s\,t|}q_{\beta j}(0)|\vec{p}\,M_T>\\ &-<\vec{p}'\,M'_T|q_{\beta j}(0)|\vec{k}\,s\,t>(\gamma^\mu)_{\alpha\beta}[(1/6)+(\tau_3/2)]_{ij}<\vec{k}\,s\,t|\bar{q}_{\alpha i}(0)|p\,M_T>\}\end{aligned} \tag{C.7}$$

where we have inserted a set of quark and antiquark states between the quark operators. Now using Eqs.(3.10), (3.11), (3.30) and (3.31) and the following relations:

$$\begin{aligned}&\Sigma_t[\vec{\tau}\cdot\hat{e}^*_{M'_T}\chi_t]_i[(1/6)+(\tau_3/2)]_{ij}[\chi_t^+\,\vec{\tau}\cdot\hat{e}_{M_T}]_j\\ &\qquad = 2\,\delta_{M_T M'_T}[(1/6)-(M_T/2)],\end{aligned} \tag{C.8}$$

$$\begin{aligned}&\Sigma_t[\chi_{-t}^+(\vec{\tau}\cdot\hat{e}_{M'_T})^T]_i[(1/6)+(\tau_3/2)]_{ij}[(\vec{\tau}\cdot\hat{e}^*_{M_T})^T\chi_{-t}]_j\\ &\qquad = 2\,\delta_{M_T M'_T}[(1/6)+(M_T/2)],\end{aligned} \tag{C.9}$$

$$\Sigma_s U(k,s)\,\bar{U}(k,s) = (\not{k} + m_q)/(2m_q), \tag{C.10}$$

$$\Sigma_s V(k,s)\,\bar{V}(k,s) = (\not{k} - m_q)/(2m_q) \tag{C.11}$$

we can write Eq.(C.7) as,

$$<\vec{p}'\, M'_T|:\bar{q}(0)\ \gamma^\mu[(1/6)+(\tau_3/2)]\ q(0):|\vec{p}\, M_T>$$

$$=\delta_{M_T M'_T}\int[d\vec{k}/(2\pi)^3][m_q/E_q(\vec{k})]\quad[1/\{\omega(\vec{p})\omega(\vec{p}')\}^{\frac{1}{2}}]$$

$$\times\mathrm{Tr}\Big\{\{(E'+F'\ \not{p}'/m)\gamma^5\gamma^\mu\gamma^5(E+F\ \not{p}/m)[(-\not{k}+m_q)/(2m_q)][(1/6)+(M_T/2)]\}$$

$$+\{(E-F\ \not{p}/m)\gamma^5\gamma^\mu\gamma^5(E'-F'\ \not{p}'/m)[(\not{k}+m_q)/(2m_q)][(1/6)-(M_T/2)]\}\Big\}. \tag{C.12}$$

Here the trace is over the Dirac matrices. Note that E and F are functions of $[(p\bullet k/m_q)^2-m^2]^{\frac{1}{2}}$ and E' and F' are functions of $[(p'\bullet k/m_q)^2-m^2]^{\frac{1}{2}}$. After taking the trace we have,

$$(2\pi)^3\ <\vec{p}'\, M'_T|\bar{q}(0)\gamma^\mu[(1/6)+(\tau_3/2)]q(0)|\vec{p}\, M_T>$$

$$= 2\ M_T\delta_{M_T M'_T}\ [1/\{\omega(\vec{p})\omega(\vec{p}')\}^{\frac{1}{2}}]\int[d\vec{k}/(2\pi)^3][m_q/E_q(\vec{k})]$$

$$\times[EE'(k^\mu/m_q)-E'F(p^\mu/m)-F'E(p'^\mu/m)$$

$$+FF'\{(p'^\mu p\bullet k)/(m^2m_q)\ -(p\bullet p'/m^2)(k^\mu/m_q)+(p'\bullet kp^\mu)/(m^2m_q)\}]$$

$$= M_T\ \delta_{M_T M'_T}\ [m/\{\omega(\vec{p})\omega(\vec{p}')\}^{\frac{1}{2}}]\ I^\mu\ , \tag{C.13}$$

where I^μ is given by,

$$I^{\mu} = (2/m) \int [d\vec{k}/(2\pi)^3]\,[m_q/E_q(\vec{k})]$$

$$\times [EE'(k^{\mu}/m_q) - E'F(p^{\mu}/m) - F'E(p'^{\mu}/m)$$

$$+FF'\{(p'^{\mu}p\cdot k)/(m^2 m_q) - (p\cdot p'/m^2)(k^{\mu}/m_q) + (p'\cdot k p^{\mu})/(m^2 m_q)\}]. \tag{C.14}$$

Due to the structure of the integral in Eq.(C.14) we can write

$$I^{\mu} = [(p'+p)^{\mu}/(2m)]\; I_o(q^2) \tag{C.15}$$

where $I_o(q^2)$ is (Lorentz) scalar function. Equation(C.15) can be solved for $I_o(q^2)$ in terms of I^{μ} and is given by

$$I_o(q^2) = [4m^2/(p'+p)^2]\; [(p'+p)^{\mu}/(2m)]\; I_{\mu}. \tag{C.16}$$

Now using Eqs.(C.13)-(C.16) we can rewrite Eq.(C.12) as,

$$(2\pi)^3 \langle \vec{p}'\, M'_T | q(0)\gamma^{\mu}[(1/6)+(\tau_3/2)]q(0) | \vec{p}\, M_T \rangle$$

$$= M_T\, \delta_{M_T M'_T}\; [m/\{\omega(\vec{p})\omega(\vec{p}')\}^{\frac{1}{2}}]\; [(p'+p)^{\mu}/(2m)]\; F_s^{em}(q^2), \tag{C.17}$$

where

$$F_s^{em}(q^2) = (2/m) \int [d\vec{k}/(2\pi)^3]\,[m_q/E_q(\vec{k})]$$

$$\times [(EE'+FF')\{k\cdot(p'+p)/(mm_q)\}\{m^2/(m^2+p\cdot p')\} - E'F - EF']. \tag{C.18}$$

If we take $\vec{p}=0$, $\vec{p}'=0$ (and thus $q^2=0$) in Eq.(C.18) we find

$$F_s^{em}(q^2=0) = (2/m) \int [d\vec{k}/(2\pi)^3]\,[m_q/E_q(\vec{k})] \times [(E^2+F^2)\{E_q(\vec{k})/m_q\} - 2EF]. \qquad (C.19)$$

In Eq.(C.19), E and F are functions (km_q/m). By using the definition of wave functions $\hat{R}_u(k)$ and $\hat{R}_l(k)$ given in Eqs.(3.13)-(3.14) and the relation in Eq.(C.6) we can get the following relation from Eq.(C.19)

$$\int dk \quad \vec{k}^2[\{\hat{R}_u(k)\}^2+\{\hat{R}_l(k)\}^2] = 1. \qquad (C.20)$$

This is the required normalization as given in Eq.(3.15).

C.3 Electromagnetic Form Factor for Vector Mesons:

We now turn to the definition of the various electromagnetic form factors of vector mesons. This calculation is more involved than that for the pseudoscalar mesons as we noted in Chapter 4. We start with the definition

$$J^{\mu} = (2\pi)^3 \langle \vec{p}'\ \lambda'\ M'_T | \bar{q}(0) \gamma^{\mu} [(1/6)+(\tau_3/2)] q(0) | \vec{p}\ \lambda\ M_T \rangle . \tag{C.21}$$

Now we can write

$$J^{\mu} = M_T\ \delta_{M_T M'_T}\ [m/\{\omega(\vec{p})\omega(\vec{p}')\}^{\frac{1}{2}}]$$
$$\times[(\xi'^{*}_{\lambda'}\bullet\xi_{\lambda})\ \tilde{\pi}^{\mu}\ F_1^{em}(q^2) + \{(\xi'^{*}_{\lambda'}\bullet p \xi_{\lambda}\bullet p')/m^2\}\ \tilde{\pi}^{\mu}\ F_2^{em}(q^2)$$
$$+\{\xi_{\lambda}{}^{\mu}(\xi'^{*}_{\lambda'}\bullet p/m) + \xi'^{*\mu}_{\lambda'}(\xi_{\lambda}\bullet p'/m)\}\ F_3^{em}(q^2)]. \tag{C.22}$$

where $F_1^{em}(q^2)$, $F_2^{em}(q^2)$ and $F_3^{em}(q^2)$ are (Lorentz) scalar functions and $\tilde{\pi}^{\mu} = (p'+p)^{\mu}/(2m)$. The $F_1^{em}(q^2)$, $F_2^{em}(q^2)$ and $F_3^{em}(q^2)$ are the only three independent scalar functions we can construct consistent with the relations,

$$\xi_{\lambda}\bullet p = 0, \tag{C.23}$$

$$\xi'^{*}_{\lambda'}\bullet p' = 0, \tag{C.24}$$

and

$$q_{\mu}J^{\mu} = 0 . \tag{C.25}$$

Here $q^{\mu} = (p'-p)^{\mu}$. The relations in Eqs.(C.23) and (C.24) are properties of the polarization vectors and the relation given by Eq.(C.25) is due to current conservation. Upon inserting quark and antiquark states between the quark operators in Eq.(C.21) we can write

$$J^{\mu} = (2\pi)^3 \ \Sigma_{s,t} \int d\vec{k}$$

$$\times\{<\vec{p}' \ \lambda' \ M'_T|\bar{q}_{\alpha i}(0)|\overline{\vec{k} \ s \ t}>(\gamma^{\mu})_{\alpha\beta}[(1/6) + (\tau_3/2)]_{ij}$$

$$\times<\overline{\vec{k} \ s \ t}|q_{\beta j}(0))|\vec{p} \ \lambda \ M_T>$$

$$-<\vec{p}' \ \lambda' \ M'_T|q_{\beta j}(0)|\vec{k} \ s \ t>(\gamma^{\mu})_{\alpha\beta}[(1/6) + (\tau_3/2)]_{ij}$$

$$\times<\vec{k} \ s \ t|\bar{q}_{\alpha i}(0))|\vec{p} \ \lambda \ M_T>\}. \qquad (C.26)$$

Now using the definition of invariant amplitudes given in Eqs.(3.23), (3.24), (3.32) and (3.33) and the relations in Eq.(C.8)-(C.11) we find

$$J^{\mu} = 2 \ \delta_{M_T M'_T} \int [d\vec{k}/(2\pi)^3][m_q/E_q(\vec{k})] \quad [1/\{\omega(\vec{p})\omega(\vec{p}')\}^{\frac{1}{2}}]$$

$$\times Tr\{[(\not{k}-m_q)/(2m_q)][(\xi'^{*}_{\lambda'}\bullet k/m_q)A'_1(1-\not{p}/m)-(\tilde{A}'+\tilde{B}' \ \not{p}'/m)\not{\xi}'^{*}_{\lambda'}]$$

$$\times\gamma^{\mu}[(\xi_{\lambda}\bullet k/m_q)A_1(1-p/m)-\not{\xi}_{\lambda}(\tilde{A}+\tilde{B} \ p/m)][(1/6)+(M_T/2)]$$

$$-[(\not{k}+m_q)/(2m_q)][(\xi_{\lambda}\bullet k/m_q)A_1(1+\not{p}/m)+\not{\xi}_{\lambda}(\tilde{A}+\tilde{B} \ \not{p}/m)]\gamma^{\mu}$$

$$\times[(\xi'^{*}_{\lambda'}\bullet k/m_q)A'_1(1+\not{p}'/m)+(\tilde{A}'+\tilde{B}' \ \not{p}'/m)\not{\xi}'^{*}_{\lambda'}][(1/6)-(M_T/2)]\} \qquad (C.27)$$

where

$$\not{\xi}'^{*}_{\lambda'} = \xi'^{*\mu}_{\lambda'} \ \gamma_{\mu}. \qquad (C.28)$$

Here A_1, $\tilde{A}$, and $\tilde{B}$ are functions of $[(p\cdot k/m_q)^2-m^2]^{\frac{1}{2}}$ and A'_1, $\tilde{A}'$, and $\tilde{B}'$, are functions of $[(p'\cdot k/m_q)^2-m^2]^{\frac{1}{2}}$. After taking the trace over the Dirac matrices we get

$$J^\mu = 2\ M_T\ \delta_{M_T M'_T}\ [1/\{\omega(\vec{p})\omega(\vec{p}')\}^{\frac{1}{2}}] \int [d\vec{k}/(2\pi)^3][m_q/E_q(\vec{k})]$$

$$\times\{\xi'^{*\sigma}_{\lambda'}\ \xi_\lambda{}^\nu[(\tilde{q}^2 k^\mu k_\sigma k_\nu/m_q^2)f_1 + \tilde{\pi}^\mu(k_\sigma k_\nu/m_q^2)f_2 + \tilde{q}^\mu(k_\sigma k_\nu/m_q^2)f_3]$$

$$+(\xi_\lambda{}^\mu \xi'^{*\nu}_{\lambda'})(k_\nu/m_q)f_4 + (\xi'^{*\mu}_{\lambda'}\xi_\lambda{}^\nu)(k_\nu/m_q)f'_4$$

$$+\xi_\lambda{}^\mu(\xi'^{*}_{\lambda'}\cdot p/m)\ f_5 + \xi'^{*\mu}_{\lambda'}(\xi_\lambda\cdot p'/m)\ f'_5$$

$$+(\xi_\lambda\cdot p'/m)\ \xi'^{*\nu}_{\lambda'}[(k^\mu k_\nu/m_q^2)f_6 + (\tilde{\pi}^\mu-\tilde{q}^\mu)(k_\nu/m_q)f_7]$$

$$+(\xi'^{*}_{\lambda'}\cdot p/m)\ \xi_\lambda{}^\nu[(k^\mu k_\nu/m_q^2)f'_6 + (\tilde{\pi}^\mu+\tilde{q}^\mu)(k_\nu/m_q)f'_7]$$

$$+(\xi'^{*}_{\lambda'}\cdot\xi_\lambda)[(k^\mu/m_q)f_8 + \tilde{\pi}^\mu\ f_9 + \tilde{q}^\mu\ f_{10}]$$

$$+[(\xi'^{*}_{\lambda'}\cdot p\xi_\lambda\cdot p')/m^2]\ (k^\mu/m_q)\ f_{11}\} \qquad (C.29)$$

where

$$\tilde{q}^\mu = (p'-p)/(2m), \qquad (C.30)$$

$$f_1 = 2A_1A'_1, \qquad (C.31)$$

$$f_2 = A_1A'_1[(p'+p)\cdot k/(mm_q)] + A'_1\tilde{B} + A_1\tilde{B}',$$

$$+ A_1\tilde{A}' + A'_1\tilde{A} + 2A_1A'_1, \qquad (C.32)$$

$$f_3 = -A_1A'_1[(p'-p)\cdot k/(mm_q)] - A'_1\tilde{B} + A_1\tilde{B}'$$

$$- A_1\tilde{A}' + A'_1\tilde{A}, \qquad (C.33)$$

$$f_4 = -A'_1\tilde{B}[k\cdot p/(mm_q) + (p\cdot p'/m^2)] + A'_1\tilde{A}[k\cdot p'/(mm_q)]$$

$$+A'_1\tilde{A} + \tilde{A}\tilde{A}' - \tilde{B}\tilde{B}'(p\cdot p'/m^2), \qquad (C.34)$$

$$f'_4 = -A_1\tilde{B}'[k\cdot p'/(mm_q) + (p\cdot p'/m^2)] + A_1\tilde{A}'[k\cdot p/(mm_q)]$$

$$+A_1\tilde{A}' + \tilde{A}\tilde{A}' - \tilde{B}\tilde{B}'(p\cdot p'/m^2), \qquad (C.35)$$

$$f_5 = \tilde{B}\tilde{B}'[k\cdot p'/(mm_q)] - \tilde{A}'\tilde{B}, \qquad (C.36)$$

$$f'_5 = \tilde{B}\tilde{B}'[k\cdot p/(mm_q)] - \tilde{A}\tilde{B}', \qquad (C.37)$$

$$f_6 = -A'_1\tilde{A}, \qquad (C.38)$$

$$f'_6 = -A_1\tilde{A}', \qquad (C.39)$$

$$f_7 = \tilde{B}\tilde{B}' + A'_1\tilde{B}, \qquad (C.40)$$

$$f'_7 = \tilde{B}\tilde{B}' + A_1\tilde{B}', \qquad (C.41)$$

$$f_8 = -\tilde{A}\tilde{A}' + \tilde{B}\tilde{B}'(p\cdot p'/m^2), \qquad (C.42)$$

$$f_9 = -\tilde{B}\tilde{B}'[(p'+p)\cdot k/(mm_q)] + \tilde{A}\tilde{B}' + \tilde{A}'\tilde{B}, \qquad (C.43)$$

$$f_{10} = \tilde{B}\tilde{B}'[(p'-p)\cdot k/(mm_q)] + \tilde{A}\tilde{B}' + \tilde{A}'\tilde{B}, \qquad (C.44)$$

$$f_{11} = -\tilde{B}\tilde{B}'. \qquad (C.45)$$

The functions in Eq.(C.31)-(C.45) are invariant functions of $\vec{p}$, $\vec{p}'$ and $\vec{k}$. They are of three different types. The first is symmetric, the second is antisymmetric and the third has no symmetry at all under the interchange of p^{μ} and p'^{μ}. We denote these quantities by F^S, F^A and F, respectively. Thus we have,

$$F^S = F^S(p,p',k) = F^S(p',p,k), \tag{C.46}$$

$$F^A = F^A(p,p',k) = -F^S(p',p,k), \tag{C.47}$$

$$F = F(p,p',k) \neq F(p',p,k) = F'. \tag{C.48}$$

To evaluate the various integrals in Eq.(C.29) we used the following relations, which follow from the Lorentz transformation properties of the integral we are considering. They are

$$\int d\vec{k}[m_q/E_q(\vec{k})]\ F^A = 0\ , \tag{C.49}$$

$$\int d\vec{k}[m_q/E_q(\vec{k})]\ (k^{\mu}/m_q)\ F^S = I_o^S(q^2)\ \tilde{\pi}^{\mu}\ , \tag{C.50}$$

where

$$I_o^S(q^2) = \int d\vec{k}[m_q/E_q(\vec{k})]\ \{(\tilde{\pi}\cdot k)/(m_q\tilde{\pi}^2)\}\ F^S. \tag{C.51}$$

Also,

$$\int dk[m_q/E_q(\vec{k})]\ (k^{\mu}/m_q)\ F^A = \tilde{q}^{\mu}\ I_o^A(q^2), \tag{C.52}$$

where

$$I_o^A(q^2) = \int d\vec{k}[m_q/E_q(\vec{k})]\ \{(\tilde{q}\cdot k)/(m_q\tilde{q}^2)\}\ F^A. \tag{C.53}$$

and

$$\int d\vec{k}[m_q/E_q(\vec{k})]\ (k^\mu/m_q)\ F = I_1(q^2)\ \tilde{\pi}^\mu + I_2(q^2)\ \tilde{q}^\mu \tag{C.54}$$

where

$$I_1(q^2) = (1/2) \int d\vec{k}[m_q/E_q(\vec{k})]\ \{(\tilde{\pi}\cdot k)/(m_q\tilde{\pi}^2)\}\ [F + F'] , \tag{C.55}$$

$$I_2(q^2) = (1/2) \int d\vec{k}[m_q/E_q(\vec{k})]\ \{(\tilde{q}\cdot k)/(m_q\tilde{q}^2)\}\ [F - F']. \tag{C.56}$$

Further,

$$\int d\vec{k}[m_q/E_q(k)]\ (k^\mu/m_q)\ F' = I_1(q^2)\ \tilde{\pi}^\mu - I_2(q^2)\ \tilde{q}^\mu. \tag{C.57}$$

Eqs.(C.54)-(C.57) follow from the fact that

$$F = [(F+F') + (F-F')]/2, \tag{C.58}$$

$$F' = [(F+F') - (F-F')]/2 . \tag{C.59}$$

Now,

$$\int d\vec{k}[m_q/E_q(\vec{k})]\ (k^\mu k^\nu/m_q^2)\ F^S = I_1^S(q^2)\ \tilde{\pi}^\mu\tilde{\pi}^\nu + I_2^S(q^2)\ \tilde{q}^\mu\ \tilde{q}^\nu + I_3^S(q^2)\ [g^{\mu\nu} - (\tilde{\pi}^\mu\tilde{\pi}^\nu/\tilde{\pi}^2) - (\tilde{q}^\mu\tilde{q}^\nu/\tilde{q}^2)], \tag{C.60}$$

where

$$I_1^S(q^2) = \int d\vec{k}[m_q/E_q(\vec{k})]\ \{(\tilde{\pi}\cdot k)^2/(m_q^2\tilde{\pi}^4)\}\ F^S, \qquad (C.61)$$

$$I_2^S(q^2) = \int d\vec{k}[m_q/E_q(\vec{k})]\ \{(\tilde{q}\cdot k)^2/(m_q^2\tilde{q}^4)\}\ F^S, \qquad (C.62)$$

$$I_3^S(q^2) = \int d\vec{k}[m_q/E_q(\vec{k})]\ F^S$$

$$\times[1 - \{(\tilde{\pi}\cdot k)^2/(m_q^2\tilde{\pi}^2)\} - \{(\tilde{q}\cdot k)^2/(m_q^2\tilde{q}^2)\}] \qquad (C.63)$$

and

$$\int dk[m_q/E_q(\vec{k})]\ (k^\mu k^\nu/m_q^2)\ F^A = I_1^A(q^2)[p'^\mu\ \ p'^\nu - p^\mu p^\nu]/m^2 \qquad (C.64)$$

where

$$I_1^A(q^2) = \int d\vec{k}[m_q/E_q(\vec{k})]\ [(\tilde{\pi}\cdot k\tilde{q}\cdot k)/(2m_q^2\tilde{\pi}^2\tilde{q}^2)]\ F^A. \qquad (C.65)$$

The integral $\int d\vec{k}[m_q/E_q(\vec{k})]\ F\ \ (k^\mu k^\nu/m_q^2)$ can be evaluated using the fact that

$$F = [(F+F') + (F-F')]/2. \qquad (C.66)$$

Finally we have,

$$\int d\vec{k}[m_q/E_q(\vec{k})]\ F^S\ (k^\mu k^\nu k^\sigma/m_q^3))$$

$$= I_1^T(q^2)\ \tilde{\pi}^\mu\tilde{\pi}^\nu\tilde{\pi}^\sigma + I_2^T(q^2)\ [\tilde{\pi}^\mu\tilde{q}^\nu\tilde{q}^\sigma + \tilde{\pi}^\nu\tilde{q}^\sigma\tilde{q}^\mu + \tilde{\pi}^\sigma\tilde{q}^\mu\tilde{q}^\nu]$$

$$+ I_3^T(q^2)\ [\tilde{\pi}^\mu g^{\nu\sigma} + \tilde{\pi}^\nu g^{\sigma\mu} + \tilde{\pi}^\sigma g^{\mu\nu} - (3/\tilde{\pi}^2)(\tilde{\pi}^\mu\tilde{\pi}^\nu\tilde{\pi}^\sigma)$$

$$-(1/\tilde{q}^2)(\tilde{\pi}^\mu\tilde{q}^\nu\tilde{q}^\sigma + \tilde{\pi}^\nu\tilde{q}^\sigma\tilde{q}^\mu + \tilde{\pi}^\sigma\tilde{q}^\mu\tilde{q}^\nu)] \qquad \text{(C.67)}$$

with

$$I_1^T(q^2) = \int d\vec{k}[m_q/E_q(\vec{k})]\ F^S\ \{(\tilde{\pi}\cdot k)^3/(\tilde{\pi}^6 m_q^3)\}, \qquad \text{(C.68)}$$

$$I_2^T(q^2) = \int d\vec{k}[m_q/E_q(\vec{k})]\ F^S\ \{(\tilde{q}\cdot k)^2/(m_q^2\tilde{q}^4)\}$$

$$\times\{(\tilde{\pi}\cdot k)/(m_q\tilde{\pi}^2)\}, \qquad \text{(C.69)}$$

$$I_3^T(q^2) = (1/2)\int d\vec{k}[m_q/E_q(\vec{k})]\ F^S\ [\{(\tilde{\pi}\cdot k)/(m_q\tilde{\pi}^2)\}$$

$$-\{(\tilde{\pi}\cdot k)^3/(\tilde{\pi}^4 m^3)\} - \{(\tilde{\pi}\cdot k)/(m_q\tilde{\pi}^2)\}\{(\tilde{q}\cdot k)^2/(m_q^2\tilde{q}^4)\}].$$

(C.70)

Using Eqs.(C.49)-(C.70) the integrals in Eq.(C.29) can be completed. Thus we find,

$$J^{\mu} = M_T \, \delta_{M_T M'_T} \, [m/\{\omega(\vec{p})\omega(\vec{p}')\}^{\frac{1}{2}}]$$

$$\times[(\xi'^{*}_{\lambda'}\bullet\xi_{\lambda})\ \tilde{\pi}^{\mu}\ F_1^{em}(q^2) + \{(\xi'^{*}_{\lambda'}\bullet p\xi_{\lambda}\bullet p')/m^2\}\ \tilde{\pi}^{\mu}\ F_2^{em}(q^2)$$

$$+\{\xi_{\lambda}^{\mu}(\xi'^{*}_{\lambda'}\bullet p/m) + \xi'^{*\mu}_{\lambda'}(\xi_{\lambda}\bullet p'/m)\}\ F_3^{em}(q^2)]. \qquad (C.71)$$

where

$$F_1^{em}(q^2) = (2/m)\int[d\vec{k}/(2\pi)^3][m_q/E_q(\vec{k})]\Big[\{(\tilde{\pi}\bullet k)/(m_q\tilde{\pi}^2)\}f_8 + f_9$$

$$+[q^2\{(\tilde{\pi}\bullet k)/(m_q\tilde{\pi}^2)\}f_1+f_2][1-\{(\tilde{\pi}\bullet k)^2/(m_q^2\tilde{\pi}^2)\}-\{(\tilde{q}\bullet k)^2/(m_q^2\tilde{q}^2)\}]/2\Big], \qquad (C.72)$$

$$F_2^{em}(q^2) = (2/m)\int[d\vec{k}/(2\pi)^3][m_q/E_q(\vec{k})]$$

$$\times\Big[\ [-\tilde{q}^2\{(\tilde{\pi}\bullet k)/(m_q\tilde{\pi}^2)\}\ f_1 - f_2]\{(\tilde{q}\bullet k)^2/(m_q^2\tilde{q}^4)\}/4$$

$$+[\tilde{q}^2\{(\tilde{\pi}\bullet k)/(m_q\tilde{\pi}^2)\}\ f_1 + f_2 + 2f_6 + 2f'_6][\{(\tilde{\pi}\bullet k)^2/(m_q^2\tilde{\pi}^4)\}]/4$$

$$+[\tilde{q}^2\{(1/\tilde{q}^2) - (3/\tilde{\pi}^2)\}\ \{(\tilde{\pi}\bullet k)/(m_q\tilde{\pi}^2)\}\ f_1$$

$$+\{(1/\tilde{q}^2) - (1/\tilde{\pi}^2)\}\ f_2\ -\{2(f_6+f'_6)/\tilde{\pi}^2\}]$$

$$\times[1 - \{(\tilde{\pi}\bullet k)^2/(m_q^2\tilde{\pi}^4)\} - \{(\tilde{q}\bullet k)/(m_q\tilde{q}^2)\}]/8$$

$$+(f_7+f'_7)\{(\tilde{\pi}\bullet k)/(m_q\tilde{\pi}^2)\}/2 \quad -(f_7-f'_7)\{(\tilde{q}\bullet k)/(m_q\tilde{q}^2)\}/2$$

$$-(f_6-f'_6)[\tilde{\pi}\bullet k\tilde{q}\bullet k/(2\tilde{\pi}^2\tilde{q}^2)]\Big], \qquad (C.73)$$

$$F_3^{em}(q^2) = (2/m) \int [d\vec{k}/(2\pi)^3]\,[m_q/E_q(\vec{k})] \Big[(f_5+f'_5)/2$$

$$+(f_4+f'_4)\{(\tilde{\pi}\bullet k)/(m_q\tilde{\pi}^2)\}/2 - (f_4-f'_4)\{(\tilde{q}\bullet k)/(m_q\tilde{q}^2)\}/2$$

$$+[f_1\; q^2\{(\tilde{\pi}\bullet k)/(m_q\tilde{\pi}^2)\} + f_6 + f'_6]$$

$$\times[1 - \{(\tilde{\pi}\bullet k)^2/(m_q^2\tilde{\pi}^2)\} - \{(\tilde{q}\bullet k)^2/(m_q^2\tilde{q}^2)\}]/4 \Big] . \qquad (C.74)$$

If we take the time component, $\mu=0$, and go to the meson rest frame, $\vec{p}=0$ and $\vec{p}'=0$ $(q^2=0)$ we get,

$$J^{\mu=0} = M_T\; \delta_{M_T M'_T}\; \xi^*_{\lambda}\bullet\xi_\lambda\; F_1^{em}(q^2=0). \qquad (C.75)$$

Using $\xi^*_{\lambda}\bullet\xi_\lambda = -\delta_{\lambda'\lambda}$, we see that for the correct normalization we need

$$F_1^{em}(q^2=0) = -1. \qquad (C.76)$$

From Eq.(C.72) we have

$$F_1^{em}(q^2=0) = (2/m) \int [d\vec{k}/(2\pi)^3]\,[m_q/E_q(\vec{k})]$$

$$\times[-(\tilde{A}^2+\tilde{B}^2)E_q(\vec{k})/m_q + 2\tilde{A}\tilde{B}] \qquad (C.77)$$

where A and B are functions of km/m_q. Now using the definitions of $\hat{R}_u(k)$ and $\hat{R}_l(k)$ given in Eqs.(3.27) and (3.28) we get the normalization condition

$$\int dk \quad \vec{k}^2 [\{\hat{R}_u(k)\}^2 + \{\hat{R}_l(k)\}^2] \quad = \quad 1. \tag{C.78}$$

APPENDIX D

Hamiltonian

Here we introduce a new mass m_H which is calculated by taking the expectation value the Hamiltonian H, given in Eq.(2.18), between two meson states.

D.1 Pseudoscalar Mesons:

From the definition of the Hamiltonian, H, we have,

$$\langle \vec{p}\,' M'_T | H | \vec{p}\, M_T \rangle = \delta_{M_T M'_T} \delta(\vec{p}\,' - \vec{p}\,)\ \omega_H(\vec{p}\,), \tag{D.1}$$

where

$$\omega_H(\vec{p}\,) = [m_H^2 + \vec{p}\,^2]^{\frac{1}{2}}. \tag{D.2}$$

Here m_H is the mass of the particle we calculate using the Hamiltonian given in Eq.(2.18). We denote it by m_H to distinguish it from the mass which appears in the rest of our analysis. For a truly consistent theory m_H has to equal to m. For any operator $Q(x)$, using the momentum operator $\underset{\sim}{P}$, we have the formal relation:

$$Q(x) = e^{i\underset{\sim}{P}\cdot x} Q(0) e^{-i\underset{\sim}{P}\cdot x}. \tag{D.3}$$

Using this relation we have,

$$<\vec{p}'\ M'_T|H|\vec{p}\ M_T> \ = \int d\vec{x}\ \{\ <\vec{p}'\ M'_T|\bar{q}(x)\ \ (i\gamma^0)\ \ \dot{q}(x)|\vec{p}\ M_T>$$

$$+(1/2)\ <\vec{p}'\ M'_T|\ [\ \dot{\chi}^2(x)\ +\ |\vec{\nabla}\chi(x)|^2\ +\ m_\chi^2\ \chi^2(x)\ \ ]\ |\vec{p}\ M_T>\}$$

$$=\ (2\pi)^3\ \ \delta(\ \vec{p}'-\vec{p}\ )\ <\vec{p}\ M'_T|\ \mathcal{H}(0)\ |\vec{p}\ M_T>\ , \qquad \text{(D.4)}$$

where

$$<\vec{p}\ M'_T|\ \mathcal{H}(0)\ |\vec{p}\ M_T> \ = <\vec{p}\ M'_T|\bar{q}(0)\ \ (i\gamma^0)\ \ \dot{q}(0)\ |\vec{p}\ M_T>$$

$$+(1/2)\ <\vec{p}\ M'_T|\ [\ \dot{\chi}^2(0)\ +\ |\vec{\nabla}\chi(0)|^2\ +\ m_\chi^2\ \chi^2(0)]\ |\vec{p}\ M_T>. \qquad \text{(D.5)}$$

As it stands Eq.(D.5) is a formal relation since it contain quadratic operators with derivatives. Its meaning will be clear when we insert various states between the operators. Comparing Eqs.(D.4) and (D.1) we have,

$$\delta_{M_T M'_T}\ \omega_H(\vec{p})\ =\ (2\pi)^3\ <\vec{p}\ M'_T|\ \mathcal{H}(0)\ |\vec{p}\ M_T>. \qquad \text{(D.6)}$$

Now we can define our new mass m_H by considering Eq.(D.6) in the meson rest frame, $\vec{p}$=0. Thus we have,

$$\delta_{M_T M'_T}\ m_H\ =\ (2\pi)^3\ <\vec{p}=0\ M_T\ \ |\ \mathcal{H}(0)\ |\vec{p}=0\ M_T>. \qquad \text{(D.7)}$$

After inserting the appropriate states between various operators in Eq. (D.5) we have,

$$<\vec{p}\ M'_T|\mathcal{H}(0)|\vec{p}\ M_T> = i\ \Sigma_{s,t}\int d\vec{k}$$

$$\times[<\vec{p}\ M'_T|\bar{q}_{\alpha i}(0)|\overline{\vec{k}\ s\ t}>(\gamma^o)_{\alpha\beta}<\overline{\vec{k}\ s\ t}|\dot{q}_{\beta i}(0)|\vec{p}\ M_T>$$

$$-<\vec{p}\ M'_T|\dot{q}_{\beta i}(0)|\vec{k}\ s\ t>(\gamma^o)_{\alpha\beta}<\vec{k}\ s\ t|\bar{q}_{\alpha i}(0)|\vec{p}\ M_T>]$$

$$+\ (1/2)\ \Sigma_{M''_T}\int d\vec{p}''$$

$$\times[<\vec{p}\ M'_T|\dot{\chi}(0)|\vec{p}''\ M''_T><\vec{p}''\ M''_T|\chi(0)|\vec{p}\ M_T>$$

$$+<\vec{p}\ M'_T|\vec{\nabla}\chi(0)|\vec{p}''\ M''_T>\bullet<\vec{p}''\ M''_T|\vec{\nabla}\chi(0)|\vec{p}\ M_T>$$

$$+m_\chi^2<\vec{p}\ M'_T|\chi(0)|\vec{p}''\ M''_T><\vec{p}''\ M''_T|\chi(0)|\vec{p}\ M_T>]. \qquad (D.8)$$

Using the relation in Eq. (D.3) and the fact that we are in the meson rest frame (p=0) we can write Eq. (D.8) as,

$$<\vec{p}\ M'_T|\mathcal{H}(0)|\vec{p}\ M_T> = i\ \Sigma_{s,t}\int d\vec{k}\ [m\ -\ E_q(\vec{k})]$$

$$\times[<\vec{p}\ M'_T|\bar{q}_{\alpha i}(0)|\overline{\vec{k}\ s\ t}>\gamma^o{}_{\alpha\beta}<\overline{\vec{k}\ s\ t}|q_{\beta i}(0)|\vec{p}\ M_T>$$

$$+<\vec{p}\ M'_T|q_{\beta i}(0)|\vec{k}\ s\ t>\gamma^o{}_{\alpha\beta}<\vec{k}\ s\ t|\bar{q}_{\alpha i}(0)|\vec{p}\ M_T>]$$

$$+(1/2)\ \Sigma_{M''_T}\int d\vec{p}''\ \ [(q^o)^2\ +\vec{q}^{\,2}\ +\ m_\chi^2]$$

$$\times[<\vec{p}\ M'_T|\chi(0)|\vec{p}''\ M''_T><\vec{p}''\ M''_T|\chi(0)|\vec{p}\ M_T>] \qquad (D.9)$$

where

$$q_o = q^o = m - \omega(\vec{p}\,'') \ , \tag{D.10}$$

and

$$\vec{q} = -\vec{p}\,'' \ . \tag{D.11}$$

Using the definitions of the amplitudes given in Eqs.(3.10), (3.11), (3.30) and (3.31) and Eq.(5.10), we have from Eq.(D.9)

$$\langle \vec{p}\, M'_T | \mathcal{H}(0) | \vec{p}\, M_T \rangle = \delta_{M_T M'_T} [1/(2\pi)^3] \ (1/m)$$
$$\times \int [d\vec{k}/(2\pi)^3] [m_q/E_q(\vec{k})] [m-E_q(\vec{k})]$$
$$\times \mathrm{Tr}[(E+F \not{p}/m)\gamma^o(E+F \not{p}/m)\{(\not{K} - m_q)/(2m_q)\}$$
$$+(E-F \not{p}/m)\gamma^o(E-F\ p/m)\{(\not{K} + m_q)/(2m_q)\}]$$

$$+2\ \delta_{M_T M'_T} [1/(2\pi)^6] \int d\vec{p}\,'' \ [m/\omega(\vec{p}\,'')]$$
$$\times[2(q^o)^2 + m_\chi^2 - q^2] \ [g_\chi^2/(m_\chi^2 - q^2)^2] \ [F_s(q^2)]^2 . \tag{D.12}$$

Where we have performed the sum over the spin index s and isospin index t in Eq.(D.9) using Eqs.(4.6)-(4.9). After taking the trace over the Dirac matrices in Eq.(D.12) we have,

$$<\vec{p}=0\ M'_T\ |\mathcal{H}(0)|\vec{p}=0\ M'_T> = 2\ [1/(2\pi)^3]\ \delta_{M_T M'_T}\ (2/m)$$

$$\times \int [d\vec{k}/(2\pi)^3][m_q/E_q(\vec{k})]\ [m-E_q(\vec{k})][(E^2+F^2)E_q(\vec{k})/m_q\ -2EF]$$

$$+2\ \delta_{M_T M'_T}\ [1/(2\pi)^6]\ \int d\vec{p}''\ [m/\omega(\vec{p}'')]$$

$$\times[2(q^o)^2 + m_\chi^2 - q^2]\ [g_\chi^2/(m_\chi^2 - q^2)^2]\ [F_s(q^2)]^2. \qquad (D.13)$$

Finally using the definition of the wave functions in Eqs.(3.13)-(3.14), and the normalization condition in Eq.(3.15), Eq.(D.7) becomes:

$$m_H = 2\ [\ m - <E_q>\ ] + \mathcal{E}_\chi^s\ , \qquad (D.14)$$

where

$$<E_q> = \int dk\ \vec{k}^2\ E_q(\vec{k})[\{\hat{R}_u(k)\}^2+\{\hat{R}_l(k)\}^2], \qquad (D.15)$$

$$\mathcal{E}_\chi^s = 2 \int [d\vec{p}''/(2\pi)^3][m/\omega(\vec{p}'')][g_\chi^2/(m_\chi^2 - q^2)^2]$$

$$\times[2(q^o)^2 + m_\chi^2 - q^2]\ [F_s(q^2)]^2. \qquad (D.16)$$

In principle $[m-<E_q>]$ can be negative. However $\mathcal{E}_\chi^s$ which represents one aspect of the scalar field energy is always positive.

D.2 Vector Mesons:

Proceeding as in the case of the pseudoscalar meson calculation, we have for vector mesons,

$$<\vec{p}'\ \lambda'\ M'_T|H|\vec{p}\ \lambda\ M_T> = \delta_{M_T M'_T}\delta_{\lambda'\lambda}\delta(\vec{p}'-\vec{p})\ \omega_H(\vec{p}), \qquad (D.17)$$

$$m_H = \omega_H(\vec{p}=0). \qquad (D.18)$$

As in Eq.(D.7) we now have,

$$\delta_{M_T M'_T}\ \delta_{\lambda'\lambda}\ m_H = (2\pi)^3\ <\vec{p}=0\ \lambda'\ M'_T\ |\mathcal{H}(0)|\vec{p}=0\ \lambda\ M_T> \qquad (D.19)$$

where

$$<\vec{p}\ \lambda'\ M'_T|\mathcal{H}(0)|\vec{p}\ \lambda\ M_T> = i\ \Sigma_{s,t}\int d\vec{k}\ [m - E_q(\vec{k})]$$

$$\times[<\vec{p}\ \lambda'\ M'_T|\bar{q}_{\alpha i}(0)|\overline{\vec{k}\ s\ t}>\gamma^o_{\alpha\beta}<\overline{\vec{k}\ s\ t}|q_{\beta i}(0)|\vec{p}\ \lambda\ M_T>$$

$$+<\vec{p}\ \lambda'\ M'_T|q_{\beta i}(0)|\vec{k}\ s\ t>\gamma^o_{\alpha\beta}<\vec{k}\ s\ t|\bar{q}_{\alpha i}(0)|\vec{p}\ \lambda\ M_T>]$$

$$+(1/2)\ \Sigma_{\lambda'' M''_T}\int d\vec{p}''\ \ [(q^o)^2 + \vec{q}^2 + m_\chi^2]$$

$$\times[<\vec{p}\ \lambda'\ M'_T|\chi(0)|\vec{p}''\ \lambda''\ M''_T><\vec{p}''\ \lambda''\ M''_T|\chi(0)|\vec{p}\ \lambda\ M_T>] \qquad (D.20)$$

with

$$q_o = q^o = m - \omega(\vec{p}'') \qquad (D.21)$$

and

$$\vec{q} = -\vec{p}''\ . \qquad (D.22)$$

In writing the Eqs.(D.20)-(D.22) we used the fact that we are in in the meson rest frame, $\vec{p}=0$. Using the definition of the invariant amplitudes in Eqs.(3.23), (3.24), (3.32) and (3.33) and Eq.(5.24), Eq.(D.20) becomes,

$$\langle \vec{p}\,\lambda'\, M'_T | \mathcal{H}(0) | \vec{p}\,\lambda\, M_T \rangle = [1/(2\pi)^3]\ \delta_{M_T M'_T}$$

$$\times \int [d\vec{k}/(2\pi)^3]\,[m_q/E_q(\vec{k})]\ [m-E_q(\vec{k})]$$

$$\times\ \mathrm{Tr}\{[(\not{k}-m_q)/(2m_q)]\,[(\xi_{\lambda'}^{*}\bullet k/m_q)\ A_1(1-\not{p}/m)-(\tilde{A}+\tilde{B}\ \not{p}/m)\not{\xi}_{\lambda'}^{*}]$$

$$\times\gamma^{o}[(\xi_{\lambda}\bullet k/m_q)\ A_1(1-\not{p}/m)-\not{\xi}_{\lambda}(\tilde{A}+\tilde{B}\ \not{p}/m)]$$

$$+[(\not{k}+m_q)/(2m_q)]\,[(\xi_{\lambda}\bullet k/m_q)\ A_1(1+\not{p}/m)+\not{\xi}_{\lambda}(\tilde{A}+\tilde{B}\ \not{p}/m)]$$

$$\times\gamma^{o}[(\xi_{\lambda'}^{*}\bullet k/m_q)\ A_1(1+\not{p}/m)+(\tilde{A}+\tilde{B}\ \not{p}/m)\not{\xi}_{\lambda'}^{*}]\}$$

$$+2\ \delta_{M_T M'_T}\int[d\vec{p}\,''/(2\pi)^6]\,[m/\omega(\vec{p}\,'')]\,[2(q^o)^2+m_\chi^2-q^2]\,[g_\chi^2/(m_\chi^2-q^2)^2]$$

$$\times\{[\xi_\lambda\bullet p''\xi_{\lambda'}^{*}\bullet p''/m^2]\,[\{(p''\bullet p/m^2)F_1(q^2)+F_2(q^2)\}^2-\{F_2(q^2)\}^2]$$

$$+\xi_\lambda\bullet\xi_{\lambda'}^{*}\ [F_2(q^2)]^2\ \}. \qquad (D.23)$$

Here we have performed the sum over s and t by using Eqs.(4.6)-(4.9) and performed the sum over λ'' using the relation in Eq.(3.19). We also used the fact that the meson momentum is $\vec{p}=0$. Thus we have,

$$\xi_\lambda^{\ \mu} = (0\ ,\ \vec{\xi}_\lambda), \qquad (D.24)$$

$$\vec{\xi}_{\lambda} \cdot \vec{\xi}^{\,*}_{\lambda'} = \delta_{\lambda'\lambda} \,, \tag{D.25}$$

$$\int d\hat{p}'' \; \vec{\xi}_{\lambda} \cdot \vec{p}'' \; \vec{\xi}^{\,*}_{\lambda'} \cdot \vec{p}'' = \delta_{\lambda'\lambda} \, (4\pi/3) \, \vec{p}''^{\,2} . \tag{D.26}$$

In Eq.(D.26) we performed angular integral over $\vec{p}''$. Using Eqs.(D.24)-(D.26) and performing the trace in Eq.(D.23) we get,

$$\begin{aligned} <\vec{p}=0 \; \lambda' \; M'_T \; |\mathcal{H}(0)| \vec{p}=0 \; \lambda \; M_T > &= 2 \; [1/(2\pi)^3] \delta_{\lambda'\lambda} \delta_{M_T M'_T} \\ &\times (2/m) \int [d\vec{k}/(2\pi)^3] [m_q/E_q(\vec{k})] \; [m - E_q(\vec{k})] \\ &\times [(\tilde{A}^2 + \tilde{B}^2) E_q(\vec{k})/m_q - 2\tilde{A}\tilde{B}] \\ &+ \delta_{\lambda'\lambda} \delta_{M_T M'_T} [1/(2\pi)^3] (1/\pi^2) \int \vec{p}''^{\,2} \, dp'' \\ &\times [2(q^o)^2 + m_\chi^2 - q^2] [g_\chi^2/(m_\chi^2 - q^2)^2] \\ &\times [\{\vec{p}''^{\,2}/(3m^2)\}\{[\omega(\vec{p}'')/m \; F_1(q^2) + F_2(q^2)]^2 - [F_1(q^2)]^2\} \\ &+ \{F_2(q^2)\}^2] . \end{aligned} \tag{D.27}$$

Finally using the definition of the wave functions $\hat{R}_u(k)$ and $\hat{R}_l(k)$ in Eqs.(3.27)-(3.29) and using Eq.(D.19) we get,

$$m_H = 2 \; [\; m - <E_q>] \; + \; \mathcal{E}_\chi^v \tag{D.28}$$

where

$$\langle E_q \rangle = \int dk\, \vec{k}^2\, E_q(\vec{k})[\{\hat{R}_u(k)\}^2 + \{\hat{R}_l(k)\}^2], \tag{D.29}$$

$$\mathcal{E}_\chi^V = (1/\pi^2) \int \vec{p}''^2\, dp''\, [2(q^o)^2 + m_\chi^2 - q^2][g_\chi^2/(m_\chi^2 - q^2)^2]$$
$$\times [\{\vec{p}''^2/(3m^2)\}\{[\omega(\vec{p}'')/m\; F_1(q^2) + F_2(q^2)]^2 - [F_1(q^2)]^2\}$$
$$+ \{F_2(q^2)\}^2]. \tag{D.30}$$

APPENDIX E

Gluon Exchange-Corrections

In this Appendix we consider the modification of the analysis to include the effect of "gluon" exchange. For this we consider the Lagrangian density of Eq.(2.7) and the equations of motion given as Eq.(2.9)-(2.11). They are

$$[i\gamma^{\mu}\partial_{\mu} - m_q]q(x) = g_{\chi}q(x)\chi(x) + g\,\gamma^{\mu}(\lambda^a/2)q(x)\,A^a_{\mu}(x), \tag{E.1}$$

$$[\partial^{\mu}\partial_{\mu} + m^2_{\chi}]\chi(x) = -g_{\chi}\,\bar{q}(x)q(x), \tag{E.2}$$

$$\partial^{\mu}\partial_{\mu}\,A^a_{\nu}(x) = g\,\bar{q}(x)\gamma_{\nu}(\lambda^a/2)q(x). \tag{E.3}$$

E.1 Pseudoscalar Mesons:

We introduce the invariant amplitude and make explicit reference to color. Thus we have,

$$\langle\vec{k}\ s\ t\ b|\bar{q}_{\alpha ic}(0)|\vec{p}\ M_T\rangle = (\delta_{cb}/\sqrt{3})\ [1/(2\pi)^3][1/2\omega(\vec{p})]^{\frac{1}{2}}[m_q/E_q(\vec{k})]^{\frac{1}{2}}$$
$$\times[\bar{U}(\vec{k},s)(A+B\,\not{p}/m)\gamma^5]_{\alpha}[\chi^{+}_{t}\,\vec{\tau}\cdot\hat{e}_{M_T}]_i\ . \tag{E.4}$$

Here α is the Dirac index and i the isospin index, as before. Further, b and c are the color labels. This representation is same as that used previously except for the color factor $\delta_{cb}/\sqrt{3}$. With this choice we have the previous expressions for form factors in Eq.(4.12) and the normalization condition given by Eq.(3.15) remains the same. It is easy to find the charge conjugate amplitude starting with Eq.(E.4) [See Appendix B]. It is given by,

$$\overline{<\vec{k}\ s\ t\ b}|q_{\alpha ic}(0)|\vec{p}\ M_T> = (\delta_{cb}/\sqrt{3})\ [1/(2\pi)^3][1/2\omega(\vec{p})]^{\frac{1}{2}}[m_q/E_q(\vec{k})]^{\frac{1}{2}}$$

$$\times[\gamma^5(E+F\ \not{p}/m)V(\vec{k},s)]_\alpha[(\ \vec{\tau}\cdot\hat{e}^*_{M_T})^T\chi_{-t}\eta_{-t}]_i \qquad (E.5)$$

where E=A and F=-B and all the invariant amplitudes are functions of $[(p\cdot k/m_q)^2-m^2]^{\frac{1}{2}}$ as before. If we define the upper and lower components of the wave functions as:

$$\hat{R}_u(k) = \{[4\pi/\{m(2\pi)^3\}][\varepsilon_q(\vec{k})/E_q(\vec{k})]\}^{\frac{1}{2}}[E(k')-F(k')], \qquad (E.6)$$

$$\hat{R}_l(k) = \{[4\pi/\{m(2\pi)^3\}][\varepsilon_q(\vec{k})/E_q(\vec{k})]\}^{\frac{1}{2}}[E(k')+F(k')][k/\varepsilon_q(\vec{k})] \qquad (E.7)$$

with $k'=km/m_q$ we can choose the normalization:

$$\int dk\quad \vec{k}^2[\{\hat{R}_u(k)\}^2+\{\hat{R}_l(k)\}^2] = 1. \qquad (E.8)$$

We now analyse Eqs.(E.1)-(E.3) in the same manner as in Chapter 5. Thus from Eq.(E.3) we have,

$$-q_k^2\langle\overline{\vec{k}'\ s'\ t'\ b'}|A_\mu^a(0)|\overline{\vec{K}\ s\ t\ b}\rangle$$

$$=\langle\overline{\vec{k}'\ s'\ t'\ b'}|\overline{q}(0)\gamma_\mu(\lambda^a/2)q(0)|\overline{\vec{K}\ s\ t\ b}\rangle\ , \qquad \text{(E.9)}$$

where

$$q_k^2 = (k'-k)\bullet(k'-k) = (k'-k)^2. \qquad \text{(E.10)}$$

From Lorentz invariance we can write,

$$\langle\overline{\vec{k}'\ s'\ t'\ b'}|\overline{q}(0)\gamma_\mu(\lambda^a/2)q(0)|\overline{\vec{K}\ s\ t\ b}\rangle = -\delta_{t't}(\lambda^a/2)_{b'b}[1/(2\pi)^3]$$

$$\times\ [m_q^2/\{E_q(\vec{k})E_q(\vec{k}')\}]^{\frac{1}{2}}\ f_G(q_k^2)\ \overline{V}(\vec{K},s)\gamma_\mu V(\vec{k}',s'), \qquad \text{(E.11)}$$

where $f_G(q_k^2)$ is an Lorentz scalar function. This is a new cut-off needed to regulate the high momentum behaviour of our model. (This is required to make the various integrals in the dynamical equation converge.) With in our formalism we do not have any prescription to evaluate this new function. Thus it introduces a new parameter in our model.

Upon inserting meson states between the quark operator and the χ field, and antiquark states between gluon and quark field operators Eq.(E.1) yields,

$$[\not{p} - \not{k} - m_q]\langle\overline{\vec{k}\ s\ t\ b}|q_{\alpha ic}(0)|\vec{p}\ M_T\rangle$$

$$= g_\chi\ \Sigma_{M'_T} \int d\vec{p}'\ \langle\overline{\vec{k}\ s\ t\ b}|q_{\alpha ic}(0)|\vec{p}'\ M'_T\rangle\langle\vec{p}'\ M'_T|\chi(0)|\vec{p}\ M_T\rangle$$

$$+g\ \Sigma_{s',t',b'} \int d\vec{k}'\ \langle\overline{\vec{k}\ s\ t\ b}|A^a_\mu(0)|\overline{\vec{k}'\ s'\ t'\ b'}\rangle$$

$$\times(\gamma^\mu)_{\alpha\beta}(1/2)(\lambda^a)_{cc'}\langle\overline{\vec{k}'\ s'\ t'\ b'}|q_{\beta ic'}(0)|\vec{p}\ M_T\rangle. \qquad \text{(E.12)}$$

Using Eqs.(E.5), (E.9) and (E.11) and the relation

$$\Sigma_{a=1,8}\ [\lambda^a/2]^2_{bb'} = (4/3)\ \delta_{bb'} \qquad \text{(E.13)}$$

we can factor out the color and the isospin factors from the Eq.(E.12). This gives us,

$$[\not{p} - \not{k} - m_q][\gamma^5(E+F\ \not{p}/m)V(\vec{k},s)]/\sqrt{\omega(\vec{p})}$$

$$= g_\chi \int d\vec{p}'\ [1/\sqrt{\omega(\vec{p}')}]$$

$$\times[\gamma^5(E'+F'\ \not{p}'/m)V(\vec{k},s)]\ \langle\vec{p}'\ M_T|\chi(0)|\vec{p}\ M_T\rangle$$

$$-(4/3)\ g^2\ \Sigma_{s'} \int [d\vec{k}'/(2\pi)^3][m_q/E_q(\vec{k}')]\ [f_G(q_k^2)/(-q_k^2)]$$

$$\times[\gamma^\mu\gamma^5(E''+F''\ \not{p}/m)V(\vec{k}',s')/\sqrt{\omega(p)}]\overline{V}(\vec{k}',s')\gamma_\mu V(\vec{k},s).$$

$$\text{(E.14)}$$

Here E and F are functions $[(p\bullet k/m_q)^2-m^2]^{\frac{1}{2}}$ and E' and F' are functions of $[(p'\bullet k/m_q)^2-m^2]^{\frac{1}{2}}$. But E" and F" are functions of $[(p\bullet k'/m_q)^2-m^2]^{\frac{1}{2}}$. This is clear from Eq.(E.12), where the matrix elements are defined. We again analyse this equation in the antiquark rest frame where k=0. In this frame the functions E and F are functions of $|\vec{p}|$ and E' and F' are functions of $|\vec{p}'|$. Thus we have,

$$[\not p - \not k - m_q][\gamma^5(E+F\,\not p/m)V(0,s)]/\sqrt{\omega(\vec{p})}$$

$$= g_\chi \int d\vec{p}' \;[1/\sqrt{\omega(\vec{p}')}]$$

$$\times[\gamma^5(E'+F'\,\not p'/m)V(0,s)]\;\langle\vec{p}'\,M_T|\chi(0)|\vec{p}\,M_T\rangle$$

$$-(4/3)\;g^2 \int [d\vec{k}'/(2\pi)^3][m_q/E_q(\vec{k}')]\;[f_G(q_k^2)/(-q_k^2)]$$

$$\times[2E'' - 2F''\{p\bullet k'/(mm_q)\} - E''\,(\not k'/m_q) + F''\,(\not p/m)]\gamma^5V(0,s) \tag{E.15}$$

where we have used the relations:

$$\gamma^\mu\gamma_\mu \quad = 4, \tag{E.16}$$

$$\gamma^\mu \not A \gamma_\mu \quad = -2\not A, \tag{E.17}$$

$$\gamma^\mu \not A \not B \gamma_\mu = 4A\bullet B, \tag{E.18}$$

$$\gamma^\mu \not A \not B \not C \gamma_\mu = -2\not C \not B \not A \tag{E.19}$$

and performed the sum over the spin index using Eq.(4.9). Using the represetation of the Dirac matrices given in Eq.(1.4)-(1.7) we have from Eq.(E.15),

$$\begin{bmatrix} \{[\omega(\vec{p})-2m_q]R_u(p) - pR_l(p)\}\ \chi_{-s} \\ \{pR_u(p) - \omega(\vec{p})R_l(p)\}\ \hat{p}\cdot\vec{\sigma}\ \chi_{-s} \end{bmatrix}$$

$$= g_\chi \int d\vec{p}' \begin{matrix} R_u(p') \quad \chi_{-s} \\ R_l(p')\ \hat{p}'\cdot\vec{\sigma}\ \chi_{-s} \end{matrix} \langle \vec{p}'\ M_T|\chi(0)|\vec{p}\ M_T\rangle$$

$$-(4/3)\ g^2 \int [d\vec{k}'/(2\pi)^3][m_q/E'_q][f_G(q_k^2)/(-q_k^2)]$$

$$\times(1/\sqrt{\omega}) \begin{bmatrix} \{2[E'' - F''\ p\cdot k'/(mm_q)] - E''\ E'_q/m_q + F''\ \omega/m\}\chi_{-s} \\ -\{E''\ \vec{k}'\cdot\vec{\sigma}/m_q - F''\ \vec{p}\cdot\vec{\sigma}/m\}\chi_{-s} \end{bmatrix} \tag{E.20}$$

Here again $R_u(p)$, $R_l(p)$, $R_u(p')$ and $R_l(p')$ are same as the functions given in Eqs.(5.14)-(5.17). We also have used the notation

$$\omega = \omega(\vec{p}), \tag{E.21}$$

$$\omega' = \omega(\vec{p}'), \tag{E.22}$$

$$E_q = E_q(\vec{k}), \tag{E.23}$$

$$E'_q = E_q(\vec{k}'). \tag{E.24}$$

Equation (E.20) differs somewhat from Eq.(5.18), where we neglected the gluonic degrees of freedom. This is due to the different integration variable appearing in the second term in the right-hand side of Eq.(E.20) as compared to the first term. Also the dependence of E" and F" on $[(p\cdot k'/m_q)^2-m^2]^{\frac{1}{2}}$ makes the analysis different. We can deal with this new aspect of the problem by making a change of variables in the second integral [11]. Thus from the integration variable $\vec{k}'$ we go to a new variable $\vec{p}'$ such that

$$\vec{p}' = \vec{p} + \vec{k}'[\vec{k}'\cdot\vec{p}/\{\varepsilon_q(\vec{k}')m_q\} - \omega/m_q]. \tag{E.25}$$

The time component p_o' of this vector can be written as:

$$\omega' = [\, \vec{p}'^2 + m^2 \,]^{\frac{1}{2}} = p_o' = p\cdot k'/m_q. \tag{E.26}$$

Thus the dependence of E" and F" on $[(p\cdot k'/m_q)^2-m^2]^{\frac{1}{2}}$ simplifies to only a dependence on $|\vec{p}'|$, upon using Eq.(E.26). From Eqs.(E.25) and (E.26) it follows that:

$$\vec{k}'/\varepsilon_q(k') \quad = \quad (\vec{p}'-\vec{p})/(\omega'+\omega), \tag{E.27}$$

$$\omega = E'_q\omega'/m_q + \vec{p}'\cdot\vec{k}'/m_q, \tag{E.28}$$

$$|\vec{p}| = \vec{p}'\cdot\hat{p} + (\hat{p}\cdot\vec{k}'\ \vec{k}'\cdot\hat{p}')/(\varepsilon' m_q) + \omega'\ \hat{p}\cdot\vec{k}'/m_q\ . \tag{E.29}$$

Here for simplicity we denoted $\varepsilon_q(\vec{k}')$ as ε'. The evaluation of the Jacobian for the transformation in Eq.(E.25) is straight-forward [11]. We have,

$$d\vec{k}' \; [m_q/E'_q] \;=\; d\vec{p}' \; [m/\omega'] \; (m_q^3/m)[(\omega'+\omega)^2/(\omega'\omega+m^2+\vec{p}\cdot\vec{p}')^2]. \tag{E.30}$$

After performing the traces over 2×2 matrices as we have done in the derivation of Eq.(5.21), Eq.(E.20) yields,

$$\begin{bmatrix} \{\omega(\vec{p})-2m_q\} \; R_u(p) \; - \; p \; R_l(p) \\ - \; p \; R_u(p) \; + \; \omega(\vec{p}) \; R_l(p) \end{bmatrix}$$

$$= g_\chi \int d\vec{p}' \begin{bmatrix} R_u(p') \\ - \; R_l(p') \; \hat{p}\cdot\hat{p}' \end{bmatrix} \times\langle\vec{p}' \; M_T|\chi(0)|\vec{p} \; M_T\rangle$$

$$-(4/3) \; g^2 \int [d\vec{k}'/(2\pi)^3][m_q/E'_q][f_G(q_k^2)/(-q_k^2)]$$

$$\times(1/\sqrt{\omega}) \begin{bmatrix} \{2[E'' \; -F'' \; p\cdot k'/(mm_q)] \; -E'' \; E'_q/m_q \; +F'' \; \omega/m\} \\ -\{E'' \; \vec{k}'\cdot\hat{p}/m_q \; - \; F'' \; |\vec{p}|/m\} \end{bmatrix} \tag{E.31}$$

Finally using the relations in Eq.(5.10) and Eqs.(E.25)-(E.30), Eq.(E.31) becomes,

$$\omega(\vec{p}) \begin{bmatrix} R_u(p) \\ R_l(p) \end{bmatrix} = \begin{bmatrix} 2m_q & p \\ p & 0 \end{bmatrix} \begin{bmatrix} R_u(p) \\ R_l(p) \end{bmatrix}$$

$$- g_\chi^2 \quad [d\vec{p}'/(2\pi)^3] \; [2m/\{\omega(\vec{p})\omega(\vec{p}')\}^{\frac{1}{2}}] [F_s(q^2)/(m_\chi^2 - q^2)]$$

$$\times \begin{bmatrix} 1 & 0 \\ 0 & -\hat{p}\cdot\hat{p}' \end{bmatrix} \begin{bmatrix} R_u(p') \\ R_l(p') \end{bmatrix}$$

$$-(4/3) \; g^2 \quad [d\vec{p}'/(2\pi)^3] [1/(\omega\omega')^{\frac{1}{2}}] [f_G(q_k^2)/(-q_k^2)]$$

$$\times [m_q^3(\omega'+\omega)^2/(\omega'\omega+m^2+\vec{p}'\cdot\vec{p})^2]$$

$$\times \begin{bmatrix} 2-E'_q/m_q & -\hat{p}'\cdot\vec{k}'/m_q \\ \vec{k}'\cdot\hat{p}/m_q & \hat{p}\cdot\hat{p}'+\hat{p}\cdot\vec{k}'\hat{p}'\cdot\vec{k}'/(\varepsilon' m_q) \end{bmatrix} \begin{bmatrix} R_u(p') \\ R_l(p') \end{bmatrix} \qquad \text{(E.32)}$$

In writing these equations we used the fact that the dependence of E" and F" on $[(p\cdot k'/m_q)^2-m^2]^{\frac{1}{2}}$ can be reduced to only a dependence on $|\vec{p}'|$ using the transformations in Eqs.(E.25) and (E.26). Note again the symmetry of the kernal in the second term in Eq.(E.32) under the interchange of $\vec{p}$ and $\vec{p}'$. From Eq.(E.27) it is clear that under the interchange of $\vec{p}$ and $\vec{p}'$, $\vec{k}'$ changes sign. Thus E'_q does not change since it is quadratic in $\vec{k}'$. To retain the full symmetry under the interchange of $\vec{p}$ and $\vec{p}'$ we have to interchange the off-diagonal matrix element in the kernal. Equation (E.32) is the central equation we used to study gluonic effect in pseudoscalar mesons. Calculational details are given in Chapter 6.

D.2 Vector Mesons:

Proceeding as in the case of the pseudoscalar mesons, for vector mesons we find,

$$\langle\vec{k}\ s\ t\ b|\bar{q}_{\alpha ic}(0)|\vec{p}\ \lambda\ M_T\rangle$$

$$= (\delta_{cb}/\sqrt{3})\ [1/(2\pi)^3][1/2\omega(\vec{p})]^{\frac{1}{2}}[m_q/E_q(\vec{k})]^{\frac{1}{2}}$$

$$\times\{\bar{U}(\vec{k},s)[(\xi_\lambda\cdot k/m_q)\ A_1(1+\not{p}/m)+\not{\xi}_\lambda(\tilde{A}+\tilde{B}\ \not{p}/m)]\}_\alpha[\chi_t^+\ \vec{\tau}\cdot\hat{e}_{M_T}]_i \tag{E.33}$$

and the charge conjugate amplitude

$$\langle \overline{\vec{k}\ s\ t\ b} | q_{\alpha ic}(0) | \vec{p}\ \lambda\ M_T \rangle$$

$$= -(\delta_{cb}/\sqrt{3})\ [1/(2\pi)^3][1/2\omega(\vec{p})]^{\frac{1}{2}}[m_q/E_q(\vec{k})]^{\frac{1}{2}}$$

$$\times\{[(\xi_\lambda \cdot k/m_q)\ A_1(1-\not{p}/m)\ -\not{\xi}_\lambda(\tilde{A}+\tilde{B}\ \not{p}/m)]V(\vec{k},s)\}_\alpha[(\vec{\tau}\cdot\hat{e}^*_{M_T})^T \chi_{-t}\eta_{-t}]_i. \tag{E.34}$$

Then we have the various wave functions,

$$\hat{R}_u(k) = \{[4\pi/\{m(2\pi)^3\}][\varepsilon_q(\vec{k})/E_q(\vec{k})]\}^{\frac{1}{2}}[\tilde{A}(k')-\breve{B}(k')], \tag{E.35}$$

$$R_l(k) = \{[4\pi/\{m(2\pi)^3\}][\varepsilon_q(\vec{k})/E_q(\vec{k})]\}^{\frac{1}{2}}[\tilde{A}(k')+\tilde{B}(k')][k/\varepsilon_q(\vec{k})] \tag{E.36}$$

with $k'=km/m_q$ and the normalization,

$$\int dk\quad \vec{k}^2[\{\hat{R}_u(k)\}^2+\{\hat{R}_l(k)\}^2] = 1. \tag{E.37}$$

We now analyse the equations of motion given by Eqs.(E.1)-(E.3) by taking the matrix element between various states. As in Eq.(E.12) we now have,

$$[\not{p} - \not{k} - m_q]\langle \overline{\vec{k}\ s\ t\ b} | q_{\alpha ic}(0) | \vec{p}\ \lambda\ M_T \rangle$$

$$= g_\chi \Sigma_{\lambda', M'_T} \int d\vec{p}' \langle \overline{\vec{k}\ s\ t\ b} | q_{\alpha ic}(0) | \vec{p}'\ \lambda'\ M'_T \rangle \times \langle \vec{p}'\ \lambda'\ M'_T | \chi(0) | \vec{p}\ \lambda\ M_T \rangle$$

$$+ g\ \Sigma_{s', t', b'} \int d\vec{k}' \langle \overline{\vec{k}\ s\ t\ b} | A^a_\mu(0) | \overline{\vec{k}'\ s'\ t'\ b'} \rangle \times (\gamma^\mu)_{\alpha\beta} (1/2) (\lambda^a)_{cc'} \langle \overline{\vec{k}'\ s'\ t'\ b'} | q_{\beta ic'}(0) | \vec{p}\ \lambda\ M_T \rangle. \qquad \text{(E.38)}$$

Using Eqs.(5.24), (E.9) and (E.11), performing the sum over M'_T, λ', s' and t', and using the relations in Eq.(3.19) and (4.9), we obtain from Eq.(E.38),

$$[\not{p} - \not{k} - m_q][\{(\xi_\lambda \cdot k/m_q)\ A_1(1-p/m)\ -\not{\xi}_\lambda(A+B\ \not{p}/m)\}V(\vec{k},s)]/\sqrt{\omega(\vec{p})}$$

$$= -g_\chi^2 \int [d\vec{p}'/(2\pi)^3]\ [2m/\{\omega(\vec{p})\omega(\vec{p}')\}^{\frac{1}{2}}][1/(m_\chi^2 - q^2)][1/\sqrt{\omega(\vec{p}')}]$$

$$\times \Big[(\xi_\lambda \cdot p'/m) F_1(q^2)[\{-p \cdot k/(mm_q) + (p \cdot p'/m^2) p' \cdot k/(mm_q)\} A'_1 (1-\not{p}'/m)$$

$$-\{-\not{p}/m + (p \cdot p'/m^2)\ \not{p}'/m\}(\tilde{A}' + \tilde{B}'\ \not{p}'/m)]V(\vec{k},s)$$

$$+F_2(q^2)[\{(-k \cdot \xi_\lambda/m_q)\ +(p' \cdot \xi_\lambda/m)(k \cdot p')/(mm_q)\}A'_1(1-\not{p}'/m)$$

$$-\{-\not{\xi}_\lambda + (p' \cdot \xi_\lambda/m)\ (\not{p}'/m)\}(\tilde{A}' + \tilde{B}'\ \not{p}'/m)]V(\vec{k},s) \Big]$$

$$-(4/3)\ g^2 \int d\vec{k}'\ [m_q/E'_q][f_G(q_k^2)/(-q_k^2)][1/\sqrt{\omega}]$$

$$\times[(\xi_\lambda \cdot k'/m_q) A''_1\{-\not{k}'/m_q\ -2\ -2p \cdot k'/(mm_q)\ -\not{p}/m\}$$

$$-\{\tilde{A}''(2\xi_\lambda \cdot k'/m_q + \not{\xi}_\lambda)\ -\tilde{B}''\ \not{k}'\ \not{p}\ \not{\xi}_\lambda/(mm_q)\ -2\tilde{B}'' p \cdot \xi_\lambda/m\}]V(\vec{k},s). \tag{E.39}$$

In obtaining Eq.(E.39) from Eq.(E.38) we have factored out the color and isospin factors and used the relations in Eqs.(E.16)-(E.19). Here again A_1, A and B are functions of $[(p\bullet k/m_q)^2-m^2]^{\frac{1}{2}}$ and A'_1, A' and B' are functions of $[(p'\bullet k/m_q)^2-m^2]^{\frac{1}{2}}$. Similarly, A''_1, A" and B" are functions of $[(p\bullet k'/m_q)^2-m^2]^{\frac{1}{2}}$. We again analyse this equation in the antiquark rest frame ($\vec{k}$=0). To simplify the analysis we consider the transverse and longitudinal polarization seperately, as in the derivation of the Eq.(5.48). Following through the same procedure described in Eqs.(5.27) to Eq.(5.47) and making a change of variable as described in Eqs.(E.25)-(E.30), Eq.(E.39) yields,

$$\omega \begin{bmatrix} R_u(p) \\ R_l(p) \end{bmatrix} = \begin{bmatrix} 2m_q & p \\ p & 0 \end{bmatrix} \begin{bmatrix} R_u(p) \\ R_l(p) \end{bmatrix}$$

$$-(g_\chi^2/3) \int [d\vec{p}'/(2\pi)^3] \; [2m/\{\omega(\vec{p})\omega(\vec{p}')\}^{\frac{1}{2}}] \; [1/(m_\chi^2 - q^2)]$$

$$\times \left\{ F_1(q^2) \begin{bmatrix} V_{11}^1 & V_{12}^1 \\ V_{21}^1 & V_{22}^1 \end{bmatrix} \begin{bmatrix} R_u(p') \\ R_l(p') \end{bmatrix} \right.$$

$$\left. + F_2(q^2) \begin{bmatrix} V_{11}^2 & V_{12}^2 \\ V_{21}^2 & V_{22}^2 \end{bmatrix} \begin{bmatrix} R_u(p') \\ R_l(p') \end{bmatrix} \right\}$$

$$-(4/9) \; g^2 \int [d\vec{p}'/(2\pi)^3] [1/(\omega\omega')^{\frac{1}{2}}] [f_G(q_k^2)/(-q_k^2)] m_q^3$$

$$\times [(\omega'+\omega)^2/(\omega'\omega + m^2 + \vec{p}'\cdot\vec{p})^2] \begin{bmatrix} V_{11}^G & V_{12}^G \\ V_{21}^G & V_{22}^G \end{bmatrix} \begin{bmatrix} R_u(p') \\ R_l(p') \end{bmatrix} \quad . \quad (E.40)$$

The functions $R_u(p)$, $R_l(p)$, $R_u(p')$, and $R_l(p')$ are given in Eqs.(5.28)-(5.31) and V^1_{11}, V^1_{12}, V^1_{21}, V^1_{22}, V^2_{11}, V^2_{21}, V^2_{12} and V^2_{22} are given in Eqs.(5.49)-(5.53). The new matrix elements, which gives the explicit gluonic interaction terms are given by,

$$V^G_{11} = 2 + \omega/m - [m/(m+\omega')][-\vec{k}'^2/m_q^2 + (\vec{k}'\cdot\hat{p})^2/m_q^2$$

$$+\{(p/m)(E'_q/m_q) - (\hat{p}\cdot\vec{k}'/m_q)(\omega/m)\}\{\hat{p}\cdot\vec{k}'/m_q + p/m\}], \quad (E.41)$$

$$V^G_{12} = [m/(m+\omega)][(p'/m)(E'_q/m_q) + (\hat{p}'\cdot\vec{k}'/m_q)(\omega'/m)][E'_q/m_q + \omega'/m]$$

$$-p'/m, \quad (E.42)$$

$$V^G_{21} = [m/(m+\omega')][(p/m)(E'_q/m_q) - (\hat{p}\cdot\vec{k}'/m_q)(\omega/m)][E'_q/m_q + \omega/m]$$

$$-p/m, \quad (E.43)$$

$$V^G_{22} = (m/p')[\{(p/m)(E'_q/m_q) - (\hat{p}\cdot\vec{k}'/m_q)(\omega/m)\}\{E'_q/m_q + \omega/m - 4\}$$

$$-(\omega'/m)(p/m) + \hat{p}\cdot\vec{k}'/m_q]. \quad (E.44)$$

Again the matrix elements are symmetric under the interchange of $\vec{p}$ and $\vec{p}'$. This symmetry also requires the interchange of V^G_{12} and V^G_{21} after the replacement of $\vec{p}$ and $\vec{p}'$. The expressions for V^G_{11} and V^G_{22} are symmetric although that fact is not obvious from the expression given above. Equation (E.40) is the equation we have to consider for the vector meson with gluonic interaction. Calculational details are given in Chaptor 6.

E.3 The Hamiltonian:

Here we calculate the contribution to the total Hamiltonian from the gluonic interaction terms. [The additional term due to gluonic exchange is given in Eq.(2.22).] Thus from Eqs.(2.20)-(2.22) we have,

$$H_G = H + H'_G \tag{E.45}$$

where

$$H = \int d\vec{x} \{ [\bar{q}(x)\ (i\gamma^o)\ \dot{q}(x)]$$

$$+(1/2)[\ \dot{\chi}^2(x) + |\vec{\nabla}\chi(x)|^2 + m_\chi^2\ \chi^2(x)\ ]\}, \tag{E.46}$$

$$H'_G = \int d\vec{x}\ [F^{\mu\nu a}(x)F_{\mu\nu}^{\ a}(x)/4 \quad -F^a_{o\nu}(x)A^{\nu a}(x)]. \tag{E.47}$$

We have already calculated the contribution from H to the total Hamiltonian in Appendix D. Thus, here we only consider the term H'_G. Using Eq.(2.8), Eq.(E.47) yields,

$$H'_G = \int d\vec{x}\ [\partial^\mu A^{\nu a}(x)\partial_\mu A^a_\nu(x)\ -(1/2)\partial^\mu A^{\nu a}(x)\partial_\nu A^a_\mu(x)$$

$$-\{\partial^o A^{\nu a}(x) - \partial^\nu A^{oa}(x)\}\partial^o A^a_\nu(x)]. \tag{E.48}$$

In order to facilitate formal manipulation of Eq.(E.48), we have to eliminate the gluon field $A^a_\mu(x)$ in favour of the color current source $j^{\mu a}(x)$. The current, $j^{\mu a}(x)$ is defined through the relation,

$$j_\mu^a(x) = \bar{q}(x)\gamma_\mu(\lambda^a/2)q(x). \tag{E.49}$$

Now Eq.(E.3) can be solved to give,

$$A_\mu^a(x) = g \int D(x-x')\ j_\mu^a(x')\ d^4x' \tag{E.50}$$

where $D(x-x')$ satisfies,

$$\partial^\mu\partial_\mu\ D(x-x') = \delta^4(x'-x). \tag{E.51}$$

We introduce the Fourier decomposition,

$$D(x-x') = [1/(2\pi)^4] \int d^4q\ e^{-iq\bullet(x-x')}\ [1/(-q^2)]. \tag{E.52}$$

Using Eqs.(E.49)-(E.52), Eq.(E.48) yields,

$$\begin{aligned} H'_G = -[g^2/(2\pi)^8] \int & d^4q\ d^4q'\ d^4x'\ d^4x''\ [1/(q^2q'^2)] \\ & \times[(1/2)\ q^\mu q'_\mu j^{\nu a}(x')j_\nu^a(x'')\ -(1/2)\ q^\mu q'_\nu j^{\nu a}(x')j_\mu^a(x'') \\ & -q^o q'^o j^{\nu a}(x')j_\nu^a(x'')\ +q^\nu q'^o j^{oa}(x')j_\nu^a(x'')] \\ & \times[e^{-iq\bullet(x-x')}e^{-iq'\bullet(x-x'')}] \end{aligned} \tag{E.53}$$

E.4 Hamiltonian for Pseudoscalar Mesons:

We now form the expectation value of the Eq.(E.53) between meson states. In taking this expectation value we encounter matrix elements of the form,

$$I_{\mu\nu}(x',x'') = <\vec{p}'\, M'_T|j^a_\mu(x')j^a_\nu(x'')|\vec{p}\, M_T> \tag{E.54}$$

We now rearrange the current operator in Eq.(E.54) using Eq.(E.49) so that we can form meanigful factorizations. Thus we have,

$$I^{\mu\nu}(x',x'') = \Sigma_{s,t,b,s't'b'} \int d\vec{k}\, d\vec{k}'$$
$$\times[<\vec{p}'\, M'_T|\bar{q}_{\alpha ic}(x')|\overline{\vec{k}\ s\ t\ b}>(\gamma^\mu)_{\alpha\beta}(\lambda^a/2)_{cc'}$$
$$\times<\overline{\vec{k}\ s\ t\ b}|j^{\nu a}(x'')|\overline{\vec{k}'\ s'\ t'\ b'}><\overline{\vec{k}'\ s'\ t'\ b'}|q_{\beta ic'}(x')|\vec{p}\, M_T>$$
$$-<\vec{p}'\, M'_T|q_{\beta ic'}(x')|\vec{k}\ s\ t\ b>(\gamma^\mu)_{\alpha\beta}(\lambda^a/2)_{cc'}$$
$$\times<\vec{k}\ s\ t\ b|j^{\nu a}(x'')|\vec{k}'\ s'\ t'\ b'><\vec{k}'\ s'\ t'\ b'|\bar{q}_{\alpha ic}(x')|\vec{p}\, M_T>] \tag{E.55}$$

Using charge conjugation (See Appendix B) and transalational invariance, Eq.(E.55) becomes,

$$I_{\mu\nu}(x',x'') = 2\Sigma_{s,t,b,s',t',b'} \int d\vec{k}\, d\vec{k}'\ [e^{ix'\cdot(p'-k+k'-p)}\ e^{ix''\cdot(k-k')}]$$
$$\times[<\vec{p}'\, M'_T|\bar{q}_{\alpha ic}(0)|\overline{\vec{k}\ s\ t\ b}>(\gamma^\mu)_{\alpha\beta}(\lambda^a/2)_{cc'}$$
$$\times<\overline{\vec{k}\ s\ t\ b}|j^{\nu a}(0)|\overline{\vec{k}'\ s'\ t'\ b'}><\overline{\vec{k}'\ s'\ t'\ b'}|q_{\beta ic'}(0)|\vec{p}\, M_T>]. \tag{E.56}$$

Substituting Eq.(E.56) into Eq.(E.53) and using the relations

$$\int d^4x\ e^{ip\cdot x} = (2\pi)^4\delta^4(p), \tag{E.57}$$

$$\int d\vec{x}\ e^{i\vec{p}\cdot\vec{x}} = (2\pi)^3\ \delta(\vec{p}). \tag{E.58}$$

Eq.(E.48) yields,

$$\begin{aligned}
&<\vec{p}'\ M'_T|H'_G|\vec{p}\ M_T> \\
&=2\ (2\pi)^3\delta(\vec{p}'-\vec{p})g^2\ \Sigma_{s,t,b,s't'b'}\int d\vec{k}\ d\vec{k}'\ [q_k^2 - 2(q_k^o)^2]/q_k^4 \\
&\times[<\vec{p}\ M'_T|\bar{q}_{\alpha ic}(0)|\overline{\vec{k}\ s\ t\ b}>(\gamma^\mu)_{\alpha\beta}(\lambda^a/2)_{cc'} \\
&\quad\times<\overline{\vec{k}\ s\ t\ b}|j^{\nu a}(0)|\overline{\vec{k}'\ s'\ t'\ b'}><\overline{\vec{k}'\ s'\ t'\ b'}|q_{\beta ic'}(0)|\vec{p}\ M_T>],
\end{aligned} \tag{E.59}$$

where

$$q_k^\mu = (k'-k)^\mu. \tag{E.60}$$

In deriving Eq.(E.58) we used current-conservation,

$$\partial^\mu j_\mu^a(x) = 0, \tag{E.61}$$

so that

$$q_k^\mu<\overline{\vec{k}'\ s'\ t'\ b'}|j_\mu^a(0)|\overline{\vec{k}\ s\ t\ b}> = 0. \tag{E.62}$$

Equation (E.62) follows from Eq.(E.61) upon use of transalational invariance.

Using Eq.(E.5), the complex conjugate equation, and Eqs.(4.6)-(4.9), and the following relations,

$$\overline{\langle\vec{k}'\ s'\ t'\ b'|}j_\mu^a(0)\overline{|\vec{k}\ s\ t\ b\rangle} = -\delta_{t't}(\lambda^a/2)_{b'b}[1/(2\pi)^3]$$

$$\times f_G(q_k^2)[m_q^2/\{E_q(\vec{k})E_q(\vec{k}')\}]^{\frac{1}{2}}\overline{V}(\vec{k},s)\gamma_\mu V(\vec{k}',s') \qquad (E.63)$$

$$\Sigma_{a=1,8}(\lambda^a/2)^2_{cc'} = (4/3)\delta_{cc'} \qquad (E.64)$$

Eq.(E.59) yields,

$$\langle\vec{p}'\ M'_T|H'_G|\vec{p}\ M_T\rangle = -(4/3)\ g^2\ \delta(\vec{p}'-\vec{p})\ \delta_{M_T M'_T}[1/\omega(\vec{p})]$$

$$\times \int[d\vec{k}\ d\vec{k}'/(2\pi)^6][m_q^2/(E_q E'_q)]\ f_G(q_k^2)[-q_k^2 + 2(q_k^o)^2]/(q_k^4)$$

$$\times \mathrm{Tr}[\gamma_\mu\gamma^5(E'+F'\ \not{p}/m)\{(\not{k}-m_q)/(2m_q)\}\gamma^\mu\{(\not{k}-m_q)/(2m_q)\}(E+F\ \not{p}/m)\gamma^5]. \qquad (E.65)$$

Here E' and F' are functions of $[(p\bullet k'/m_q)^2-m^2]^{\frac{1}{2}}$ and E and F are functions of $[(p\bullet k/m_q)^2-m^2]^{\frac{1}{2}}$. Using the relations (E.16)-(E.19) and performing the trace in Eq.(E.65) we have,

$$\langle\vec{p}'\ M'_T|H'_G|\vec{p}\ M_T\rangle = (8/3)\ g^2\ \delta(\vec{p}'-\vec{p})\ \delta_{M_T M'_T}[1/\omega(\vec{p})]$$

$$\times \int[d\vec{k}\ d\vec{k}'/(2\pi)^6][m_q^2/(E_q E'_q)]\ f_G(q_k^2)[-q_k^2 + 2(q_k^o)^2]/(q_k^4)$$

$$\times[EE'\{2 - k\bullet k'/m_q^2\} + E'F\{E'_q/m_q - 2E_q/m_q\}$$

$$+EF'\{E_q/m_q - 2E_q/m_q\} + FF'\{2E_q E'_q/m_q^2 - 1\}]. \qquad (E.66)$$

Comparing the Eq.(E.66) with Eqs.(D.1)-(D.7) and noting the fact that we are in the meson rest frame (p=0) we find,

$$m_H = 2[m - \langle E_q\rangle] + \mathcal{E}^s_\chi + \mathcal{E}^s_G , \qquad (E.67)$$

where $\langle E_q\rangle$ and $\mathcal{E}^s_\chi$ are given in Eqs.(D.15) and (D.16) and $\mathcal{E}^s_G$ is given by,

$$\mathcal{E}^s_G = [8/(3m)]g^2$$

$$\times \int [d\vec{k}\, d\vec{k}'/(2\pi)^6][m_q^2/(E_q E'_q)] f_G(q_k^2)[-q_k^2 + 2(q_k^o)^2]/(q_k^4)$$

$$\times [EE'\{2 - k\bullet k'/m_q^2\} + E'F\{E'_q/m_q - 2E_q/m_q\}$$

$$+EF'\{E_q/m_q - 2E_q/m_q\} + FF'\{2E_q E'_q/m_q^2 - 1\}]. \qquad (E.68)$$

In Eq.(E.68) E' and F' are functions of mk'/m_q and E and F are functions of mk/m_q, since we are in the meson rest frame where $\vec{p}=0$. The integral over the azimuthal angles in equation (E.68) can be done using the following procedure. We consider a general integral,

$$I(k,k') = \int d\hat{k}\, d\hat{k}'\; F(k,k',\hat{k}\bullet\hat{k}'). \qquad (E.69)$$

We can now expand $F(k,k',\hat{k}\bullet\hat{k})$ in Legendre polynomials, $P_l(\hat{k}\bullet\hat{k}')$. Thus we have,

$$F(k,k',k\bullet k') = \Sigma_l\, f_l(k,k')\; P_l(\hat{k}\bullet\hat{k}'), \qquad (E.70)$$

where the $f_l(k,k')$ are given by

$$f_l(k,k') = [(2l+1)/2] \int d(\hat{k}\cdot\hat{k}') \; F(k,k',\hat{k}\cdot\hat{k}') \; P_l(\hat{k}\cdot\hat{k}'). \tag{E.71}$$

Using the spherical-harmonic addition theorem we have,

$$P_l(\hat{k}\cdot\hat{k}') = [4\pi/(2l+1)] \; \Sigma_m Y^*_{lm}(\hat{k}) Y_{lm}(\hat{k}'). \tag{E.72}$$

Substituting Eqs.(E.70) and (E.72) into Eq.(E.69), and using the orthogonality properties of spherical harmonics, we obtain,

$$I(k,k') = (4\pi)^2 \; f_o(k,k'). \tag{E.73}$$

From Eq.(E.71) we have,

$$f_o(k,k') = (1/2) \int d(\hat{k}\cdot\hat{k}') \quad F(k,k',\hat{k}\cdot\hat{k}'). \tag{E.74}$$

Thus using Eq.(E.69)-(E.74), Eq.(E.68) yields,

$$\begin{aligned} \mathcal{E}^s_G = {} & [g^2/(3m\pi^4)] \int \vec{k}^2 dk \; \vec{k}'^2 \; dk' [m_q^2/(E_q E'_q)] \; d(\hat{k}\cdot\hat{k}') \\ & f_G(q_k^2)[-q_k^2 + 2(q_k^o)^2]/(q_k^4) \\ & \quad \times [EE'\{2 - k\cdot k'/m_q^2\} + E'F\{E'_q/m_q - 2E_q/m_q\} \\ & \qquad + EF'\{E_q/m_q - 2E_q/m_q\} + FF'\{2E_q E'_q/m_q^2 - 1\}]. \end{aligned} \tag{E.75}$$

E.5 Hamiltonian for Vector Mesons:

Following a similar procedure as in Eqs.(E.54)-(E.62) in the case of pseudoscalar mesons, we have for vector mesons,

$$<\vec{p}'\ \lambda'\ M'_T|H'_G|\vec{p}\ \lambda\ M_T> = (2\pi)^3\delta(\vec{p}'-\vec{p})\ 2\ g^2$$

$$\times\ \Sigma_{s,t,b,s't'b'} \int d\vec{k}\ d\vec{k}'\ [q_k^2 - 2(q_k^o)^2]/(q_k^4)$$

$$\times[<\vec{p}\ \lambda'\ M'_T|\bar{q}_{\alpha ic}(0)|\overline{\vec{k}\ s\ t\ b}>(\gamma^\mu)_{\alpha\beta}(\lambda^a/2)_{cc'}$$

$$\times<\overline{\vec{k}\ s\ t\ b}|j^a_\mu(0)|\overline{\vec{k}'\ s'\ t'\ b'}><\overline{\vec{k}'\ s'\ t'\ b'}|q_{\beta ic'}(0)|\vec{p}\ \lambda\ M_T>]. \tag{E.76}$$

Using the Eq.(E.34), its complex conjugate, and the relations in Eqs.(4.6)-(4.9), Eq.(E.76) yields,

$$<\vec{p}'\ \lambda'\ M'_T|H'_G|\vec{p}\ \lambda\ M_T> = \delta(\vec{p}'-\vec{p})\ \delta_{M_T M'_T}\ [4g^2/3\omega(\vec{p})]$$

$$\times \int[d\vec{k}\ d\vec{k}'/(2\pi)^6][m_q^2/(E_q E'_q)]\ f_G(q_k^2)[-q_k^2 + 2(q_k^o)^2]/(q_k^4)$$

$$\times \mathrm{Tr}\{\gamma^\mu[(\xi_\lambda \bullet k'/m_q)A'_1(1-\not{p}/m) - \not{\xi}_\lambda(\tilde{A}'+\tilde{B}'\not{p}/m)]$$

$$\times[(\not{k}'-m_q)/(2m_q)]\gamma_\mu[(\not{k}-m_q)/(2m_q)]$$

$$\times[(\xi^*_{\lambda'}\bullet k/m_q)A_1(1-\not{p}/m) - (\tilde{A}+\tilde{B}\not{p}/m)\not{\xi}^*_{\lambda'}]\}. \tag{E.77}$$

Using the relations in Eq.(E.16)-(E.19) and the fact that we are in the meson rest frame ($\vec{p}$=0) we may perform the trace over the Dirac matrices to obtain from Eq.(E.77),

$$<\vec{p}'\ \lambda'\ M'_T|H_G|\vec{p}\ \lambda\ M_T> = \delta(\vec{p}'-\vec{p})\ [8g^2/(3m)]$$

$$\times \int [d\vec{k}\ d\vec{k}'/(2\pi)^6][m_q^2/(E_q E'_q)]\ f_G(q_k^2)[-q_k^2 + 2(q_k^o)^2]/(q_k^4)$$

$$\times\ [(\xi_\lambda \cdot k' \xi_{\lambda'}^{*} \cdot k/m_q^2)\ f_1 + \xi_\lambda \cdot \xi_{\lambda'}^{*}\ f_2 + (\xi_{\lambda'}^{*} \cdot k' \xi_\lambda \cdot k/m_q^2)\ f_3$$

$$+ (\xi_{\lambda'}^{*} \cdot k \xi_\lambda \cdot k/m_q^2)\ f_4 + (\xi_{\lambda'}^{*} \cdot k' \xi_\lambda \cdot k'/m_q^2)\ f'_4], \qquad (E.78)$$

where

$$f_1 = -\tilde{A}\tilde{A}' + 2\tilde{B}\tilde{B}' + A'_1 A_1\ (\vec{k} \cdot \vec{k}'/m_q^2), \qquad (E.79)$$

$$f_2 = -[\tilde{A} - \tilde{B}\ E_q/m_q][\tilde{A}' - \tilde{B}'\ E'_q/m_q] - \tilde{B}\tilde{B}'(\vec{k} \cdot \vec{k}'/m_q^2), \qquad (E.80)$$

$$f_3 = -\tilde{B}\tilde{B}', \qquad (E.81)$$

$$f_4 = -\tilde{A}'A_1 + \tilde{B}'A_1\ E'_q/m_q, \qquad (E.82)$$

$$f'_4 = -\tilde{A}A'_1 + \tilde{B}A'_1\ E_q/m_q. \qquad (E.83)$$

The angular integrations in Eq.(E.78) can be performed using the properties of the polarization vector in the meson rest frame, $\vec{p}=0$. We have,

$$\xi_\lambda{}^\mu = \{0\ ,\ \vec{\xi}_\lambda\}, \qquad (E.84)$$

$$\xi_{\lambda'}{}^\mu = \{0\ ,\ \vec{\xi}_{\lambda'}\}. \qquad (E.85)$$

Now using the properties of the spherical harmonics we can write,

$$\xi_\lambda \cdot k = -\vec{\xi}_\lambda \cdot \vec{k}$$

$$= -|\vec{k}| \ [4\pi/3]^{\frac{1}{2}} \ Y_{1\lambda}(\hat{k}), \tag{E.86}$$

$$\xi_{\lambda'}^{*} \cdot k = -|\vec{k}| \ [4\pi/3]^{\frac{1}{2}} \ Y_{1\lambda'}^{*}(\hat{k}). \tag{E.87}$$

Consider the following general integrals,

$$I_1 = \int d\hat{k} \ d\hat{k}' \ F_1(k,k',\hat{k}\cdot\hat{k}') \ \xi_{\lambda'}^{*} \cdot k \ \xi_\lambda \cdot k. \tag{E.88}$$

Using Eqs.(E.86) and (E.88) we have,

$$I_1 = \int d\hat{k} \ d\hat{k}' \ F_1(k,k',\hat{k}\cdot\hat{k}') \ \vec{k}^2 (4\pi/3) \ Y_{1\lambda'}^{*}(\hat{k}) \ Y_{1\lambda}(\hat{k}). \tag{E.89}$$

As in Eqs.(E.69)-(E.74) we now expand $F_1(k,k',\hat{k}\cdot\hat{k}')$ as,

$$F_1(k,k',k\cdot k') = \Sigma_{l,m} \ f_l^1(k,k') \ [4\pi/(2l+1)] \ Y_{lm}(\hat{k}) Y_{lm}^{*}(\hat{k}'), \tag{E.90}$$

where

$$f_l^1(k,k') = [(2l+1)/2] \int d(\hat{k}\cdot\hat{k}') \ F_1(k,k',\hat{k}\cdot\hat{k}') \ P_l(\hat{k}\cdot\hat{k}'). \tag{E.91}$$

Substituting Eq.(E.89) into Eq.(E.88), and using the orthogonality properties of spherical harmonics,

$$\int d\hat{k} \quad Y_{lm}^{*}(\hat{k}) Y_{l'm'}(\hat{k}) = \delta_{ll'} \delta_{mm'} \ , \tag{E.92}$$

we obtain

$$I_1 = (4\pi)(4\pi/3) \ f_o^1(k,k') \ \delta_{\lambda'\lambda} \ \vec{k}^2. \tag{E.93}$$

Using Eq.(E.91), Eq.(E.93) can be written as,

$$I_1 = (4\pi)(4\pi/3)\ \delta_{\lambda'\lambda}(1/2) \int d(\hat{k}\cdot\hat{k}')\ F_1(k,k',\hat{k}\cdot\hat{k}')\ \vec{k}^2. \tag{E.94}$$

Again we consider,

$$I_2 = \int d\hat{k}\ d\hat{k}'\ F_2(k,k',\hat{k}\cdot\hat{k}')\ \xi_{\lambda'}^{*}\cdot k\ \xi_{\lambda}\cdot k'$$

$$= \int d\hat{k}\ d\hat{k}'\ \ F_2(k,k',\hat{k}\cdot\hat{k}')\ kk'(4\pi/3)\ Y_{1\lambda'}^{*}(\hat{k})\ Y_{1\lambda}(\hat{k}') \quad . \tag{E.95}$$

Using Eqs.(E.90) and (E.91) and orthogonality relations, we have,

$$I_2 = (4\pi/3)^2\delta_{\lambda'\lambda}\ (3/2) \int d(\hat{k}\cdot\hat{k}')\ F_2(k,k',\hat{k}\cdot\hat{k}')\ kk'. \tag{E.96}$$

Similarly we find

$$I_3 = \int d\hat{k}\ d\hat{k}'\ F_3(k,k',\hat{k}\cdot\hat{k}')\ \xi_{\lambda'}^{*}\cdot k'\ \xi_{\lambda}\cdot k'$$

$$= (4\pi)(4\pi/3)\delta_{\lambda'\lambda}\ (1/2) \int d(\hat{k}\cdot\hat{k}')\ F_3(k,k',\hat{k}\cdot\hat{k}')\ \vec{k}'^2, \tag{E.97}$$

$$I_4 = \int d\hat{k}\ d\hat{k}'\ F_4(k,k',\hat{k}\cdot\hat{k}')\ \xi_{\lambda'}^{*}\cdot k'\ \xi_{\lambda}\cdot k$$

$$= (4\pi/3)^2\delta_{\lambda'\lambda}\ (3/2) \int d(\hat{k}\cdot\hat{k}')\ F_4(k,k',\hat{k}\cdot\hat{k}')\ kk'. \tag{E.98}$$

Further, using the relations in Eqs.(E.88)-(E.98), Eq.(E.78) yields,

$$\langle \vec{p}'\ \lambda'\ M'_T|H_G|\vec{p}\ \lambda\ M_T\rangle = \delta_{\lambda'\lambda}\ \delta_{M_T M'_T}\ \delta(\vec{p}'-\vec{p})\quad \mathcal{E}_G^{v} \tag{E.99}$$

where

$$\mathcal{E}_G^v = (16g^2/9) \quad [1/(2\pi)^4] \int dk\ \vec{k}^2\ dk'\ \vec{k}'^2\ d(\hat{k}\cdot\hat{k}')\,[m_q^2/(E_q E'_q)]$$

$$\times f_G(q_k^2)\ [-q_k^2 + 2(q_k^o)^2]/(q_k^4)$$

$$\times[(\vec{k}\cdot\vec{k}'/m_q^2)(f_1+f_3) + (\vec{k}^2/m_q^2)\ f_4$$

$$+(\vec{k}'^2/m_q^2)\ f'_4\ -3f_2] \qquad \text{(E.100)}$$

Here we used the relation

$$\xi_{\lambda'}^{*}\cdot\xi_{\lambda} = -\delta_{\lambda'\lambda}\ . \qquad \text{(E.101)}$$

Finally, we have,

$$m_H = 2\{m - \langle E_q\rangle] + \mathcal{E}_\chi^v + \mathcal{E}_G^v\ , \qquad \text{(E.102)}$$

where $\langle E_q \rangle$ and $\mathcal{E}_\chi^v$ are given in Eqs.(D.29) and (D.30), respectively.

TABLES

Table I

Result of calculations based upon the Lagrangian of Eq.(2.1). The quark masses, m_q, are fixed so that the corresponding underlined mssses are equal. The baryon and scalar radii are defined in Section 6.2. The electromagnetic radius is also defined in Section 6.2 in terms of the slope of the appropriate form factors.

Meson	J^{π}	m(expt) (Mev)	m(theory) (Mev)	m_q (Mev)	Baryon Density Radius (fm)	Scalar Density Radius (fm)	Electro-magnetic Radius (fm)
ρ,ω	1^-	<u>775</u>	<u>775</u>	471	1.38	1.21	0.949
π	0^-	140	782	471	1.39	1.21	0.936
$J/\Psi(1S)$	1^-	<u>3100</u>	<u>3100</u>	2025	0.484	0.434	----
$J/\Psi(2S)$	1^-	3685	3795	2025	1.21	1.19	----
$\chi_c(1S)$	0^-	2980	3101	2025	0.483	0.431	----
$\chi_c(2S)$	0^-	3590	3794	2025	1.21	1.19	----
$\Upsilon(1S)$	1^-	<u>9460</u>	<u>9460</u>	5700	0.272	0.263	----
$\Upsilon(2S)$	1^-	10025	10355	5700	0.531	0.524	----
$\Upsilon(3S)$	1^-	10355	10900	5700	0.893	0.886	----
$\Upsilon(1S)$	0^-	----	9460	5700	0.271	0.262	----
$\Upsilon(2S)$	0^-	----	10356	5700	0.534	0.527	----
$\Upsilon(3S)$	0^-	----	10900	5700	0.889	0.883	----

Table II

Results of calculations based upon the Lagrangian of Eq.(2.1). The various quantities $\langle E_q \rangle$, ε_χ^s, ε_χ^v and m_H are defined in Appendix D.

Meson	J^π	m(expt) (Mev)	m(theory) (Mev)	$\langle E_q \rangle$ (Mev)	ε_χ^s (Mev)	ε_χ^v (Mev)	$\langle H \rangle = m_H$ (Mev)
ρ, ω	1^-	775	775	524	----	240	742
π	0^-	140	782	527	224	----	734
J/Ψ(1S)	1^-	3100	3100	2125	----	735	2685
J/Ψ(2S)	1^-	3685	3795	2103	----	339	3721
χ_c(1S)	0^-	2980	3101	2126	726	----	2676
χ_c(2S)	0^-	3590	3794	2105	341	----	3719
Υ(1S)	1^-	9460	9460	5806	----	1236	8545
Υ(2S)	1^-	10025	10355	5847	----	905	9922
Υ(3S)	1^-	10355	10900	5831	----	588	10726
Υ(1S)	0^-	----	9460	5806	1234	----	8542
Υ(2S)	0^-	----	10356	5847	906	----	9924
Υ(3S)	0^-	----	10900	5830	590	----	10710

Table III

Results of calculations including the gluon-exchange potential. See Section 6.5 and Appendix E. The quark masses are fixed so that the underlined theoretical and experimental masses are equal.

Meson	J^{π}	m(expt) (Mev)	m(theory) (Mev)	m_q (Mev)	Baryon Density Radius (fm)	Scalar Density Radius (fm)	Electro-magnetic Radius (fm)
ρ,ω	1^-	<u>775</u>	<u>775</u>	619	0.781	0.641	----
π	0^-	140	----	----	----	----	----
$J/\Psi(1S)$	1^-	<u>3100</u>	<u>3100</u>	2299	0.337	0.288	----
$J/\Psi(2S)$	1^-	3685	3961	2299	0.763	0.741	----
$\chi_c(1S)$	0^-	<u>2980</u>	<u>2980</u>	2299	0.310	0.264	----
$\chi_c(2S)$	0^-	3590	3910	2299	0.713	0.692	----
$\Upsilon(1S)$	1^-	<u>9460</u>	<u>9460</u>	6037	0.210	0.200	----
$\Upsilon(2S)$	1^-	10025	10505	6037	0.419	0.412	----
$\Upsilon(3S)$	1^-	10355	11152	6037	0.672	0.665	----
$\Upsilon(1S)$	0^-	----	9425	6037	0.208	0.199	----
$\Upsilon(2S)$	0^-	----	10503	6037	0.415	0.408	----
$\Upsilon(3S)$	0^-	----	11157	6037	0.674	0.668	----

Table IV

Results of calculations including the gluon-exchange potential. [See Appendix E for the definition of m_H and $\mathcal{E}_G$.] The quantities $\mathcal{E}_\chi$ and $<E_q>$ are defined Appendix D for pseudoscalar and vector mesons.

Meson	J^π	m(expt) (Mev)	m(theory) (Mev)	$<E_q>$ (Mev)	$\mathcal{E}_\chi$ (Mev)	$\mathcal{E}_G$ (Mev)	$<H>=m_H$ (Mev)
ρ, ω	1^-	775	775	755	362	306	708
π	0^-	140	----	----	----	----	----
J/Ψ(1S)	1^-	3100	3100	2485	758	570	2556
J/Ψ(2S)	1^-	3685	3961	2469	509	318	3811
χ_c(1S)	0^-	2980	2980	2544	770	768	2409
χ_c(2S)	0^-	3590	3910	2508	533	298	3736
Υ(1S)	1^-	9460	9460	6208	1264	664	8431
Υ(2S)	1^-	10025	10505	6263	1023	442	9948
Υ(3S)	1^-	10355	11152	6252	771	308	10980
Υ(1S)	0^-	----	9425	6216	1272	698	8388
Υ(2S)	0^-	----	10503	6266	1009	454	9937
Υ(3S)	0^-	----	11157	6251	750	314	10874

FIGURES

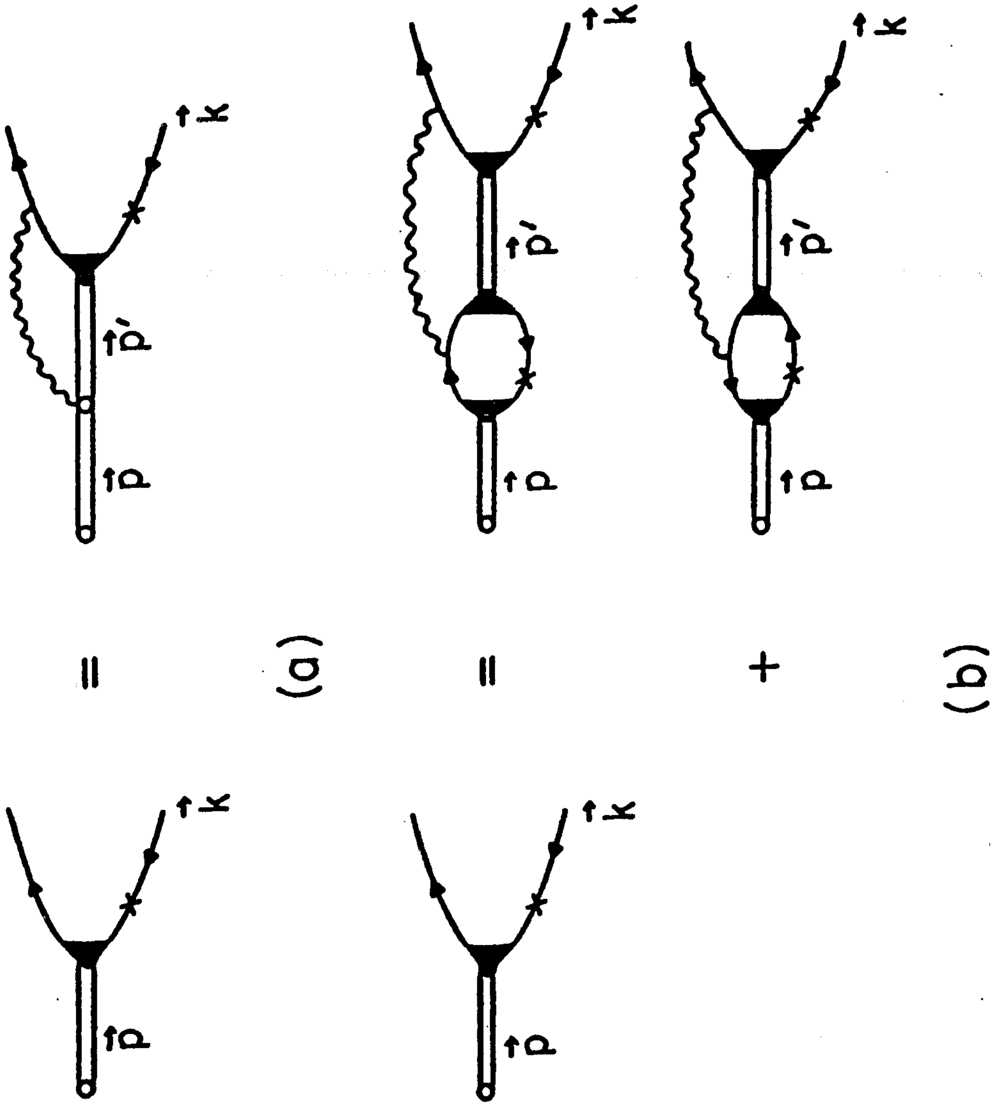


Fig.1

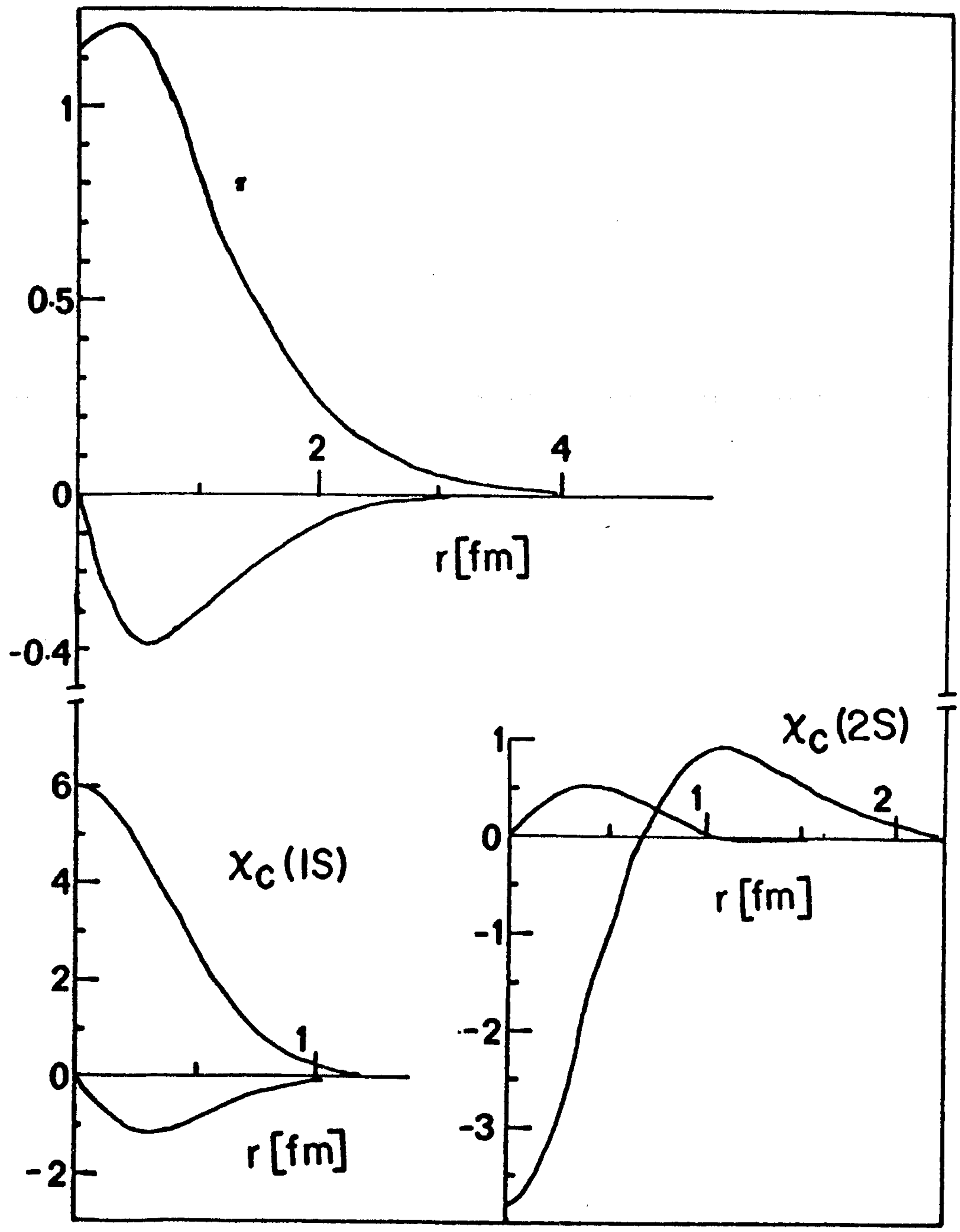


Fig.2

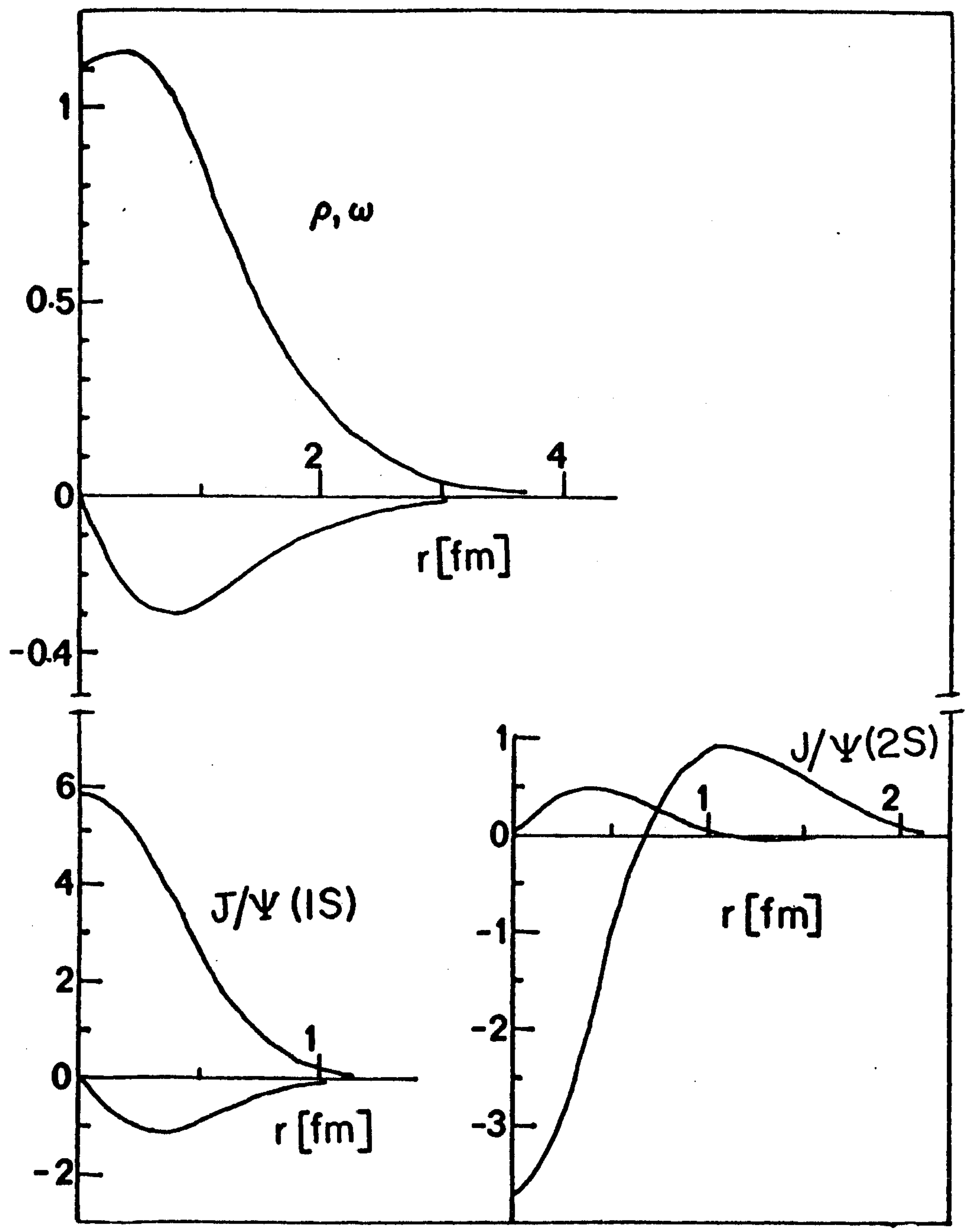
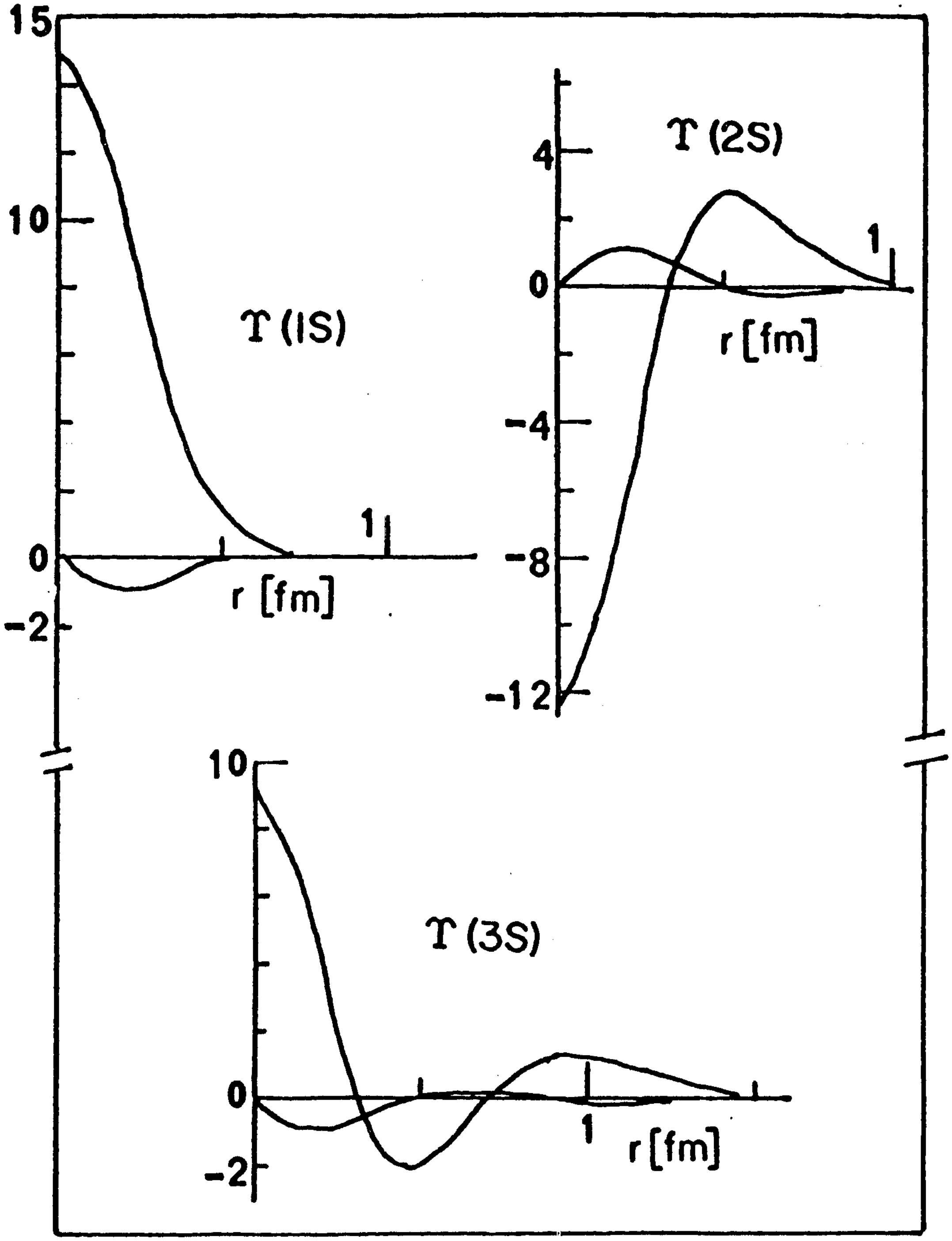


Fig. 3



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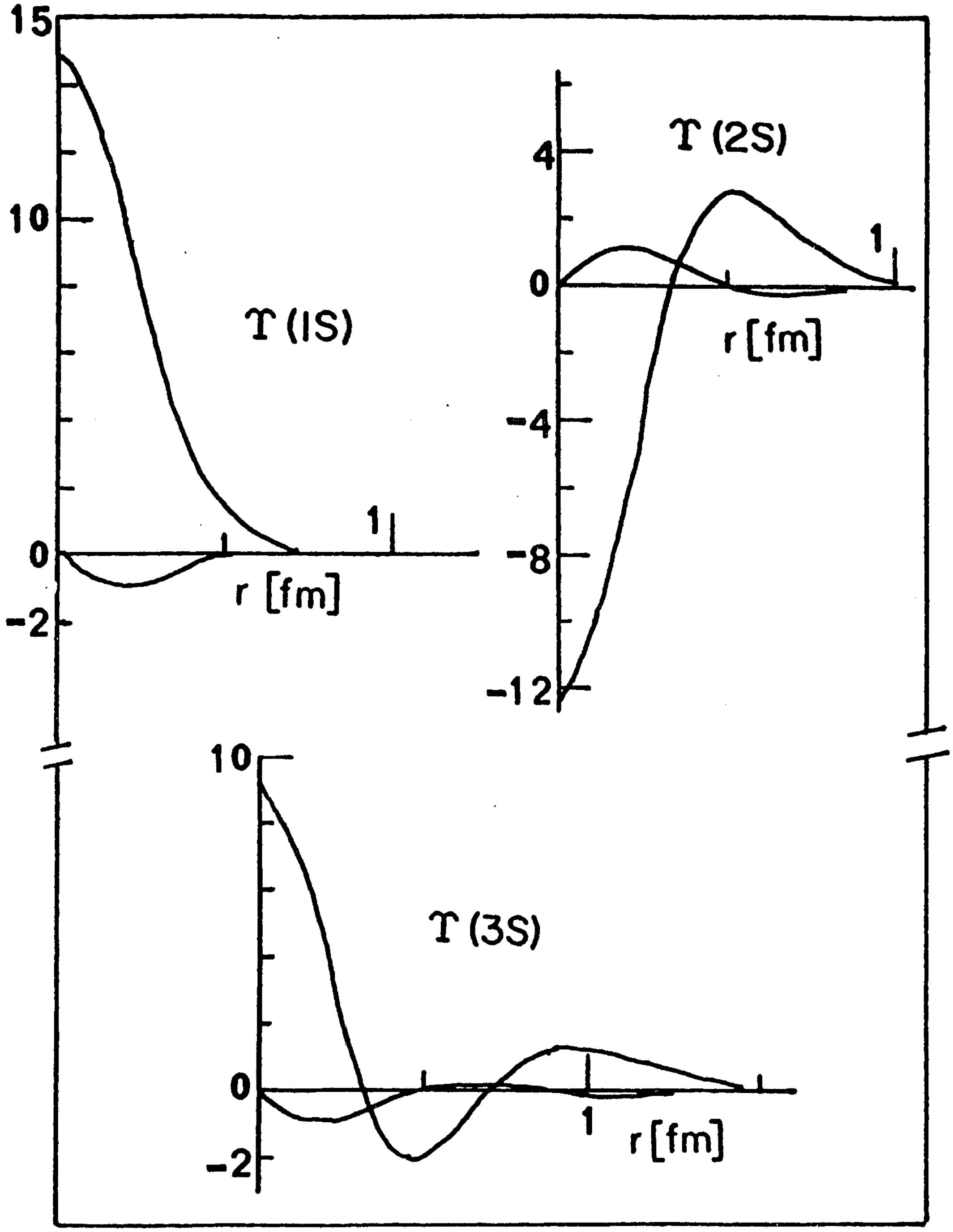

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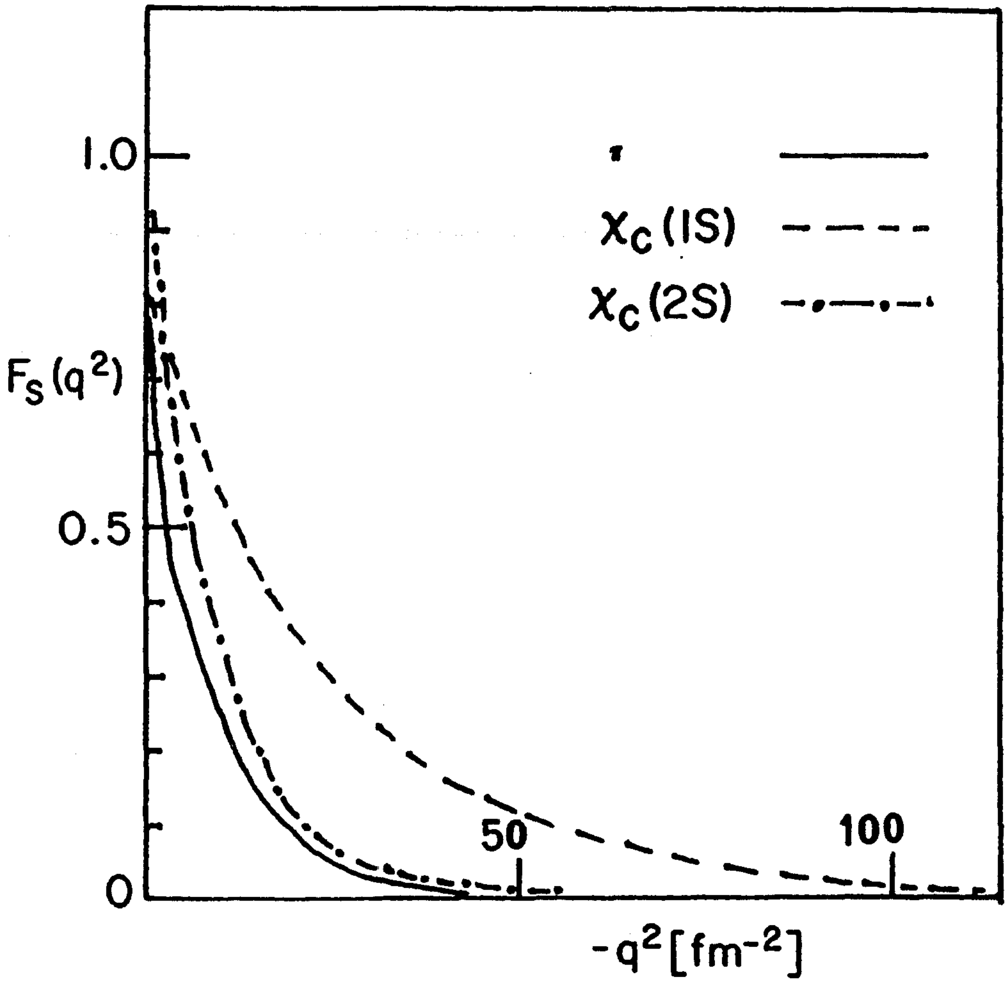

Fig.6

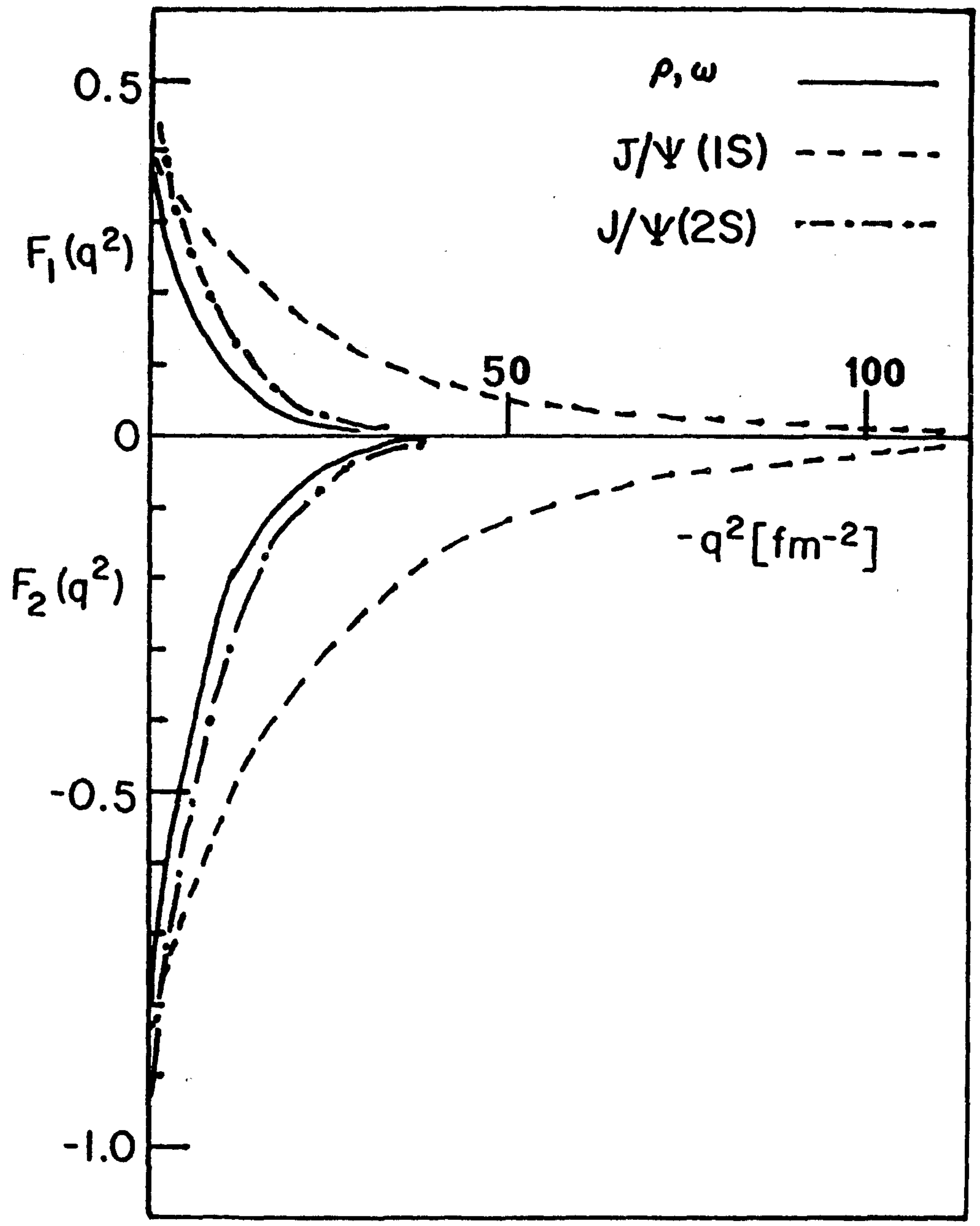

Fig. 7

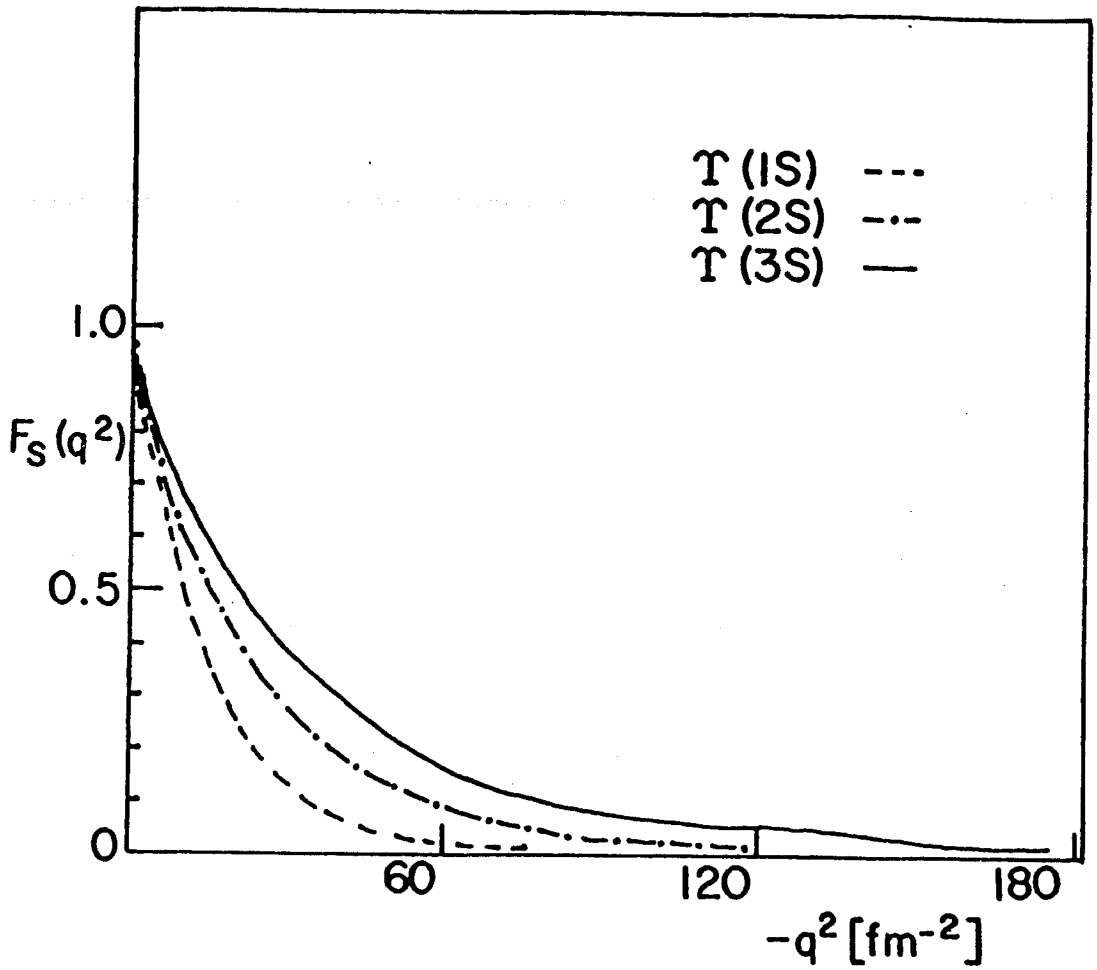

Fig. 8

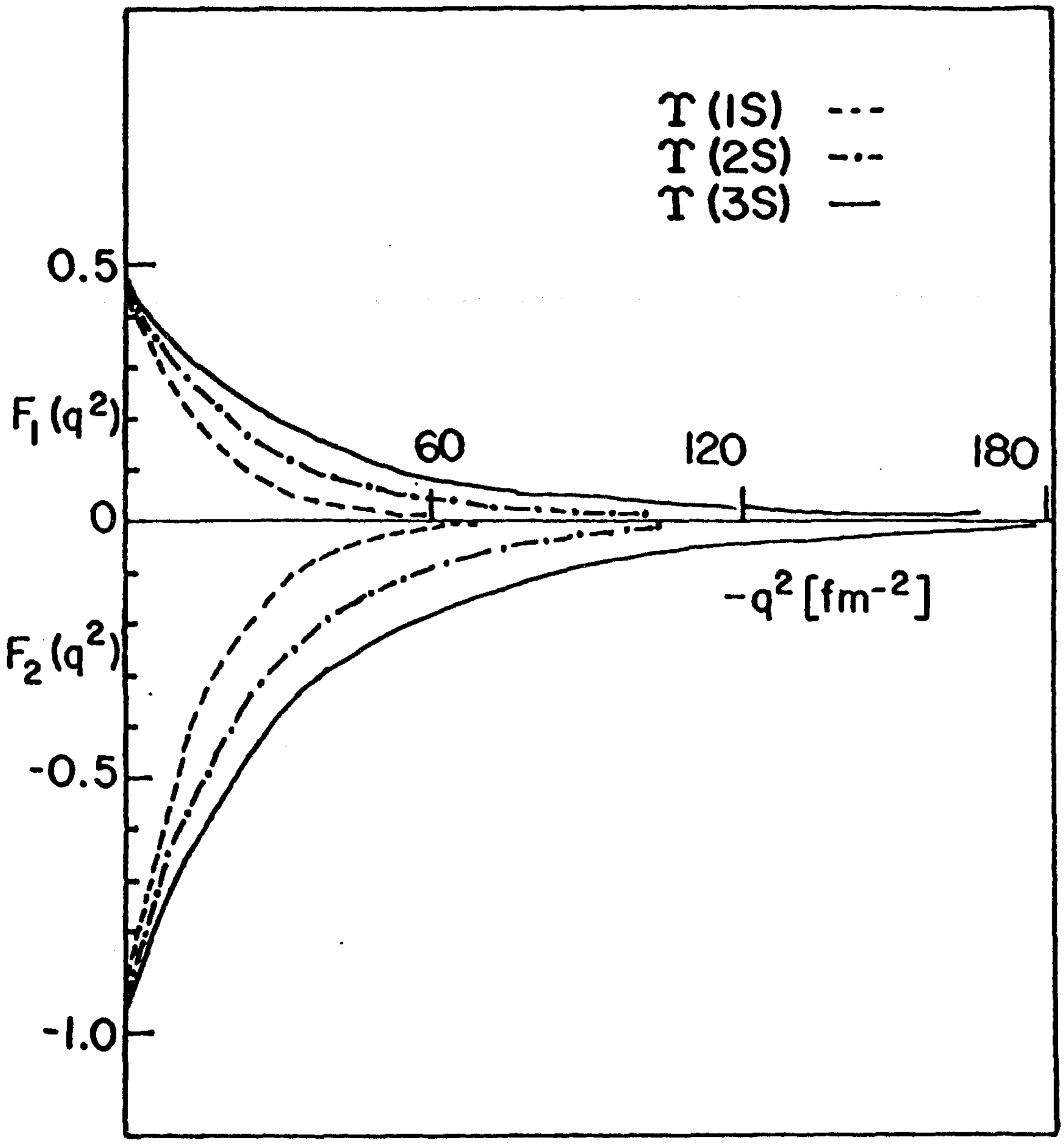

Fig.9

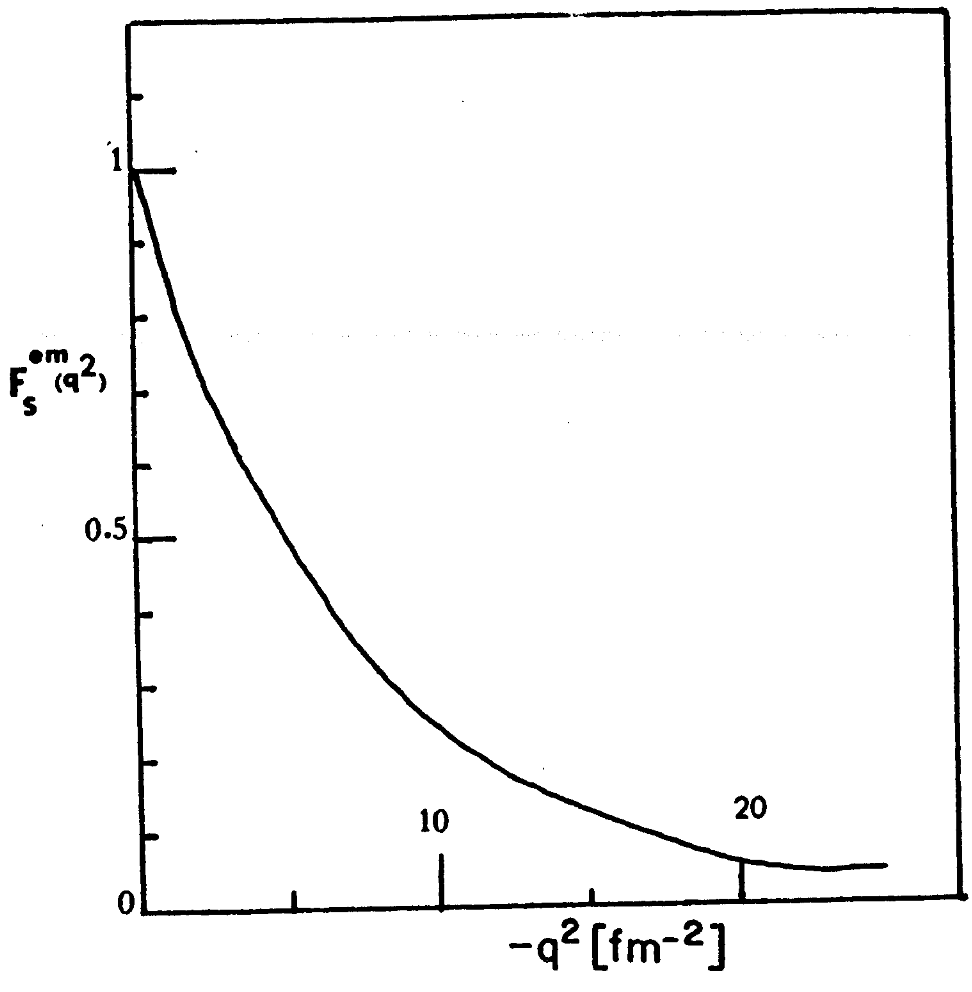

Fig. 10

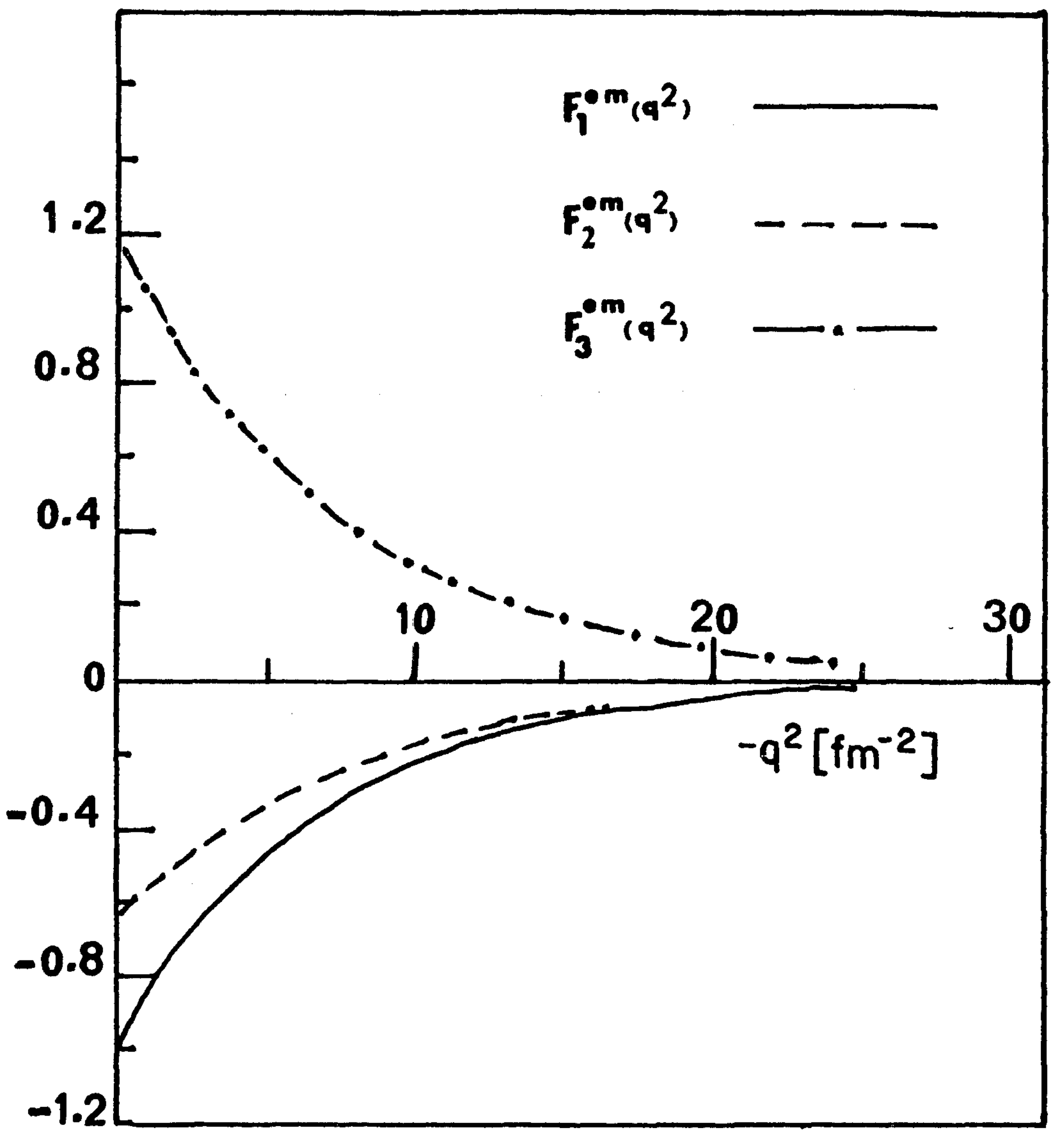

Fig. 11

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