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by

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Abstract

AUDITORY INTENSITY DISCRIMINATION WITH BURSTS OF REPRODUCIBLE NOISE

by

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It is often assumed that sensitivity in detection and discrimination tasks is limited by variability of some sort. Two major sources of variability are fluctuations in the stimuli themselves and in neural events ("internal noise"). Three studies of auditory intensity discrimination were conducted in an attempt to specify the nature of auditory neural variability and its relationship to stimulus parameters.

In Experiment 1, bursts of pseudo-random (reproducible) noise were employed. (Although their waveforms can be generated in unvarying fashion from one presentation to the next, these stimuli sound like bursts of random noise.) Weber fractions were obtained at a number of energy levels with stimuli that differed in intensity, duration and bandwidth. Weber's law was approximated in all cases. In addition, discriminability was found to be somewhat dependent on burst duration and relatively independent of noise bandwidth; the Weber fractions improved when duration was increased but not when bandwidth was increased. Taken together, these results indicate that reproducible noise bursts which are equal in energy do not always yield equal discriminabilities.

The above results cannot be explained by stimulus-oriented theories, since these theories would predict infinitely good intensity discrimination in all instances where stimulus fluctuations are absent. The results also present problems for McGill and Goldberg's (1968a, 1968b) model which suggests that: (1) reproducible stimuli which are equal in energy yield equal discriminabilities, and (2) the Weber fraction improves as energy level is increased (i.e., a "near-miss" to Weber's law should obtain).

In Experiment 2, 1000-Hz tone bursts were employed and the "near-miss" to Weber's law was confirmed. However, both discriminability and the "nearness of the miss" were clearly dependent on burst duration. While the Weber fractions at low levels were better for 100-msec stimuli than for 10-msec stimuli, the rate of improvement with level was greater for the latter; the Weber functions approached each other at higher levels.

The results of the two experiments suggest that discriminability with reproducible stimuli can be described by the relationship $\Delta E/E^{m} = k$, where the parameters, m and k, depend upon stimulus conditions. This relationship, together with the assumption that evoked neural counts (impulses) fluctuate in Poisson fashion, is the core of a model which describes the mapping of tone bursts onto neural counts in terms of a power law, and which describes mapping of reproducible noise bursts onto neural counts in terms of a log-square law. The model describes our results when all parameters of the mapping functions are reclaimed directly from the data.

Discriminability data obtained with reproducible stimuli provide us with direct estimates of internal variability. The results of a third experiment, in which real noise bursts were employed, also provided us with estimates of internal noise. The two sets of estimates were very similar. Disparities between our characterization of internal noise and that of Green (1960a) -- whose results suggest that neural variability mimics stimulus variability -- were also considered.

The results of our experiments lead to specific propositions concerning the nature of neural variability: With noises, neural fluctuations mimic stimulus fluctuations with respect to the effects of intensity. As a result, the two sources of variability can be simply added to support a stimulus-oriented model of discrimination. However, this is not the case when the effects of duration and bandwidth are considered. Nor is it so for discrimination with tones. We conclude that the interactions between internal noise and stimulus parameters are complex.

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The author dedicates this thesis to his wife, Diane and daughter, Melissa.

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Chapter I

INTRODUCTION

It is well known that sensitivity in detection and discrimination tasks is limited and it is often assumed that some source of variability acts to impose this limitation. Three types of models have been developed which deal with this matter of variability in psychophysics. One group of models considers the constraints imposed by fluctuations in the stimuli themselves (e.g., quantal fluctuations in light). These models specify the behavior of optimal observers whose performance is limited solely by stimulus parameters. A second set of models is concerned primarily with neural effects. In these models, the nature of neural variability is hypothesized and the resulting limitations on optimal behavior are then derived. In a third set of models, both stimulus fluctuations and neural events are considered jointly in the description of optimal performance. In these last models, furthermore, transformations of stimulus parameters into neural effects are explicitly described.

Examples of these three types of models are reviewed in this chapter. The reader will note that the classification is somewhat arbitrary and the boundaries of the classes are not altogether clear. In the chapters that follow, we describe an empirical attempt to specify the nature of auditory neural variability by employing two types of noise stimuli: random and pseudo-random (reproducible) waveforms.

Stimulus-Oriented Theories

The "physical quantum theory" of Hecht, Schlaer and Pirenne (1942) is the earliest stimulus-oriented approach to signal detection. This theory, based on the statistical nature of photon absorption at the retina, accounts for absolute sensitivity to light in the following manner: Repeated flashes of a light of "fixed" energy contain some average number of quanta. Since the absorption of a single quantum at the retina is a discrete, independent, random event of low probability, the number of quanta absorbed, from flash to flash, is distributed in Poisson fashion. Two assumptions are made: (1) At low levels of illumination the fluctuation in absorption of quanta far outweighs any "biological" fluctuations. (2) Some critical number (n_c) of quanta must be absorbed on each trial for a visual effect to occur. The relationship between stimulus energy (mean number of quanta) and visual sensitivity (percent detections) can then be derived as one of a family of Poisson sums after taking into account attenuation of the stimulus by the ocular media. A plot of "frequency of seeing" against stimulus energy (the empirical psychometric function) can be used to ascertain the critical number of quanta absorbed (n_c).

Whereas the model of Hecht and his associates deals with absolute sensitivity, Mueller (1950) considers how quantum fluctuations can account for differential sensitivity -- i.e., discrimination between two luminances. Here, the parameters of two quantum absorption distributions (both Poisson) must be considered. For the more intense of two stimuli (I_2) to have a greater visual effect than the less intense stimulus (I_1), it is necessary that the difference in number of absorbed quanta equal or exceed some fixed criterion. It follows from Mueller's formulation that intensity discrimination is described by the "square root law" ($\Delta I = k\ I^{\frac{1}{2}}$) rather than by Weber's law ($\Delta I = k\ I$).

Variability in quantum absorptions has little significance in the analysis of auditory detection and discrimination. Hecht, Schlaer and Pirenne (1942) reported that under ideal viewing conditions the threshold energies for seven observers ranged from 54 to 148 quanta. Further calculations indicated that as few as 5 to 14 quanta were actually absorbed at absolute threshold. By way of contrast, threshold sounds can contain as many as 10^{12} quanta. This follows from the fact that a burst of tone (f = 10 kHz) at absolute threshold involves energies of roughly 10^{-10} ergs. The energy of one acoustic quantum at 10 kHz equals approximately 10^{-22} ergs. Since the variance of a Poisson distribution equals its mean, relative variability (sigma/mean) at threshold is only one part in 10^6. As a result, stimulus-oriented theories in hearing deal specifically with situations in which the stimulus fluctuations are more molar.

Modern stimulus-oriented theories are best represented by the Signal Detection Theory of Peterson, Birdsall and Fox (1963) who develop models of "optimum receivers" for a number of situations in which signals are to be detected against a background of noise. Human sensory systems as such are not considered in the paper by

Peterson, et.al.; signals and noises are represented as voltages that vary as a function of time. The receiver's task is to decide whether or not the signal occured during a prescribed observation interval. Peterson, Birdsall and Fox demonstrate that, for all of the cases considered, "a receiver which calculates the likelihood ratio for each receiver input is the optimum receiver for detecting signals in noise." (p. 182). The optimal receiver bases its decisions on likelihood ratio or on some quantity which is monotonic with likelihood ratio. For example, Peterson, Birdsall and Fox show that a cross-correlator is the optimal receiver in instances where the signal is known exactly; an envelope-detector is optimal when the signal is known exactly except for phase; and when the signal itself is a sample of noise, the optimal receiver is an energy-detector.

These three models have been applied to problems in auditory detection -- the translation from voltage waveforms to acoustic waveforms is straightforward. In the first experiments carried out by the Michigan group, the listener's task was to detect a pulsed sinusoid against a background of white Gaussian noise. Psychometric functions and ROC curves were compared with those derived from the cross-correlator model (cf., Green, 1960b). Data from similar experiments have also been compared to predictions based on envelope-detection (Jeffress, 1964, 1967) and to those derived from an energy-detector (Pfafflin & Mathews, 1962). It should be noted that performance is almost identical for envelope- and energy-detectors (Jeffress, 1967; Pfafflin & Mathews, 1962).

Green (1960a) has studied the case in which the signal is itself a sample of noise and has compared the performance of real and ideal (energy-detector) observers. Since this case is closely related to the central problem of this thesis, it will be reviewed below.

In comparing the behavior of human observers to that of stimulus-oriented models, discrepancies are invariably encountered. In many cases the discrepancies are systematic; e.g., the performance of real observers parallels that of the optimal observer but is consistently poorer. "Internal noise" is usually invoked to account for these differences (see, for example, Green, 1960a). Stimulus-oriented theories are limited in another respect. They cannot predict or account for behavior -- they say nothing -- in situations where the stimuli are reproducible and exhibit no energy fluctuations to speak of (e.g., tone bursts, clicks, high-intensity flashes, repeated noise waveforms). For these reasons, neural theories have been developed.

Neural Theories

Neural theories of detection and discrimination propose that sensitivity is limited because of fluctuations in neural coding. This view was formalized by Crozier who strongly disagreed with Hecht's stimulus-oriented approach. As Blackwell (1963) points out, these theories are "neural by inference only" in that they rarely specify the actual nature and locus of neural activity.

In a classic paper, Crozier (1940) suggests that <u>availability</u>

of neural elements plays an important role in discrimination between stimulus intensities. This neural-availability theory begins with the assumption that the population of "elements of neural effect" is normally distributed with respect to log threshold. Thus, a given neural effect (E) is related to the intensity of stimulation (I) by a log-normal ogive. When the entire population of elements is excited by a stimulus, "maximal" neural effect (E_{max}) occurs. Presentation of a submaximal stimulus (I_1) results in some excitation (E_1) that is less than maximal and a proportion of the population of elements ($E_{max} - E_1$) remains available for stimulation. The magnitude of the just noticable difference in intensity (ΔI) is assumed to be inversely proportional to availability since the ability to excite is directly related to the size of the pool of available elements. Thus, as I_1 increases, ΔI increases, and the relative increase in ΔI with respect to I_1 determines the form of the Weber function. It has been noted that the descriptive equations of this model generate an unrealistic situation: viz., that ΔE, the just noticeable difference in neural effect, is a decreasing function of E_1 (cf., Raab, 1968).

Crozier's neural-availability theory does not consider variability in the magnitudes of the neural effects evoked by I_1 and I_2. However, Crozier (1936) was one of the first to emphasize the importance of such fluctuations when he suggested that the observer distinguishes between two intensities on the basis of "a statistical comparison of their effects." That is, "two intensities are not compared; their effects are, and these effects are variable." (p. 414). In determination of the difference limen, the observer compares the effects evoked by I_1 with those evoked by the various values of I_2 that are presented from trial to trial. Since these effects are variable, ΔI is defined statistically; its value is determined by the parameters of the two distributions of neural effect.

The neural-variability approach is developed in greater detail by Blackwell (1963) in his theory of visual intensity discrimination. Blackwell is more explicit than Crozier in defining the nature of the distributions of neural effect and in describing the decision process.

Blackwell, after ruling out a number of alternate possibilities, concludes that the function relating neural effect (E) to stimulus intensity (I) is a hyperbola. Distributions of neural effect are assumed to be normal, with variability of neural effect decreasing as E increases.

The observer's task is to determine whether the neural effect on a given trial belongs to one or the other of two distributions: the E_0 distribution which is generated by the background stimulus (I_0), or the ($E_0 + \Delta E$) distribution associated with the incremented stimulus ($I_0 + \Delta I$). The observer employs a neural criterion (E_c) in making judgments about the presence or absence of a stimulus increment on each trial. This criterion is set so that the probability that I_0 alone will produce a value of E_0 greater than E_c is small. Thus, "sensory false alarms" will rarely occur and the proportion of correct detections is given by the tail area of the ($E_0 + \Delta E$) distribution above E_c.

In their "decision-making theory of visual detection," Tanner and Swets (1954) also consider distributions of neural effect evoked by noise alone and by noise plus a signal. The model generates ROC curves on the basis of the effects of such parameters as the location of the observer's criterion, the magnitudes of the variances of the two distributions, and the distance between the means of the two distributions. In this model, the observer's criterion can vary. Its location along the neural decision axis depends on such biasing variables as the *a priori* probability of a signal occuring and the relative rewards for correct and incorrect responses. In this threshold-less theory, all decision outcomes ("hits", "misses", correct rejections and "false alarms") result from sensory events.

Neurally-oriented Signal Detection Theory is severly limited for the present purposes. Behavior is described solely in terms of ROC curves, whereas other discrimination data (e.g., psychometric functions, difference limens, Weber functions, intensity-duration reciprocities) are not predicted. However, these predictions can be made when the neural decision axis of the model is replaced by one based on stimulus energy (or some other stimulus measure). Stimulus-oriented models have already been considered (see pp. 2 ff). One of these, Green's model for energy-detection of noise signals, is most pertinent to the problems of this thesis. We consider it in detail in the next section.

Detection of a Noise Signal

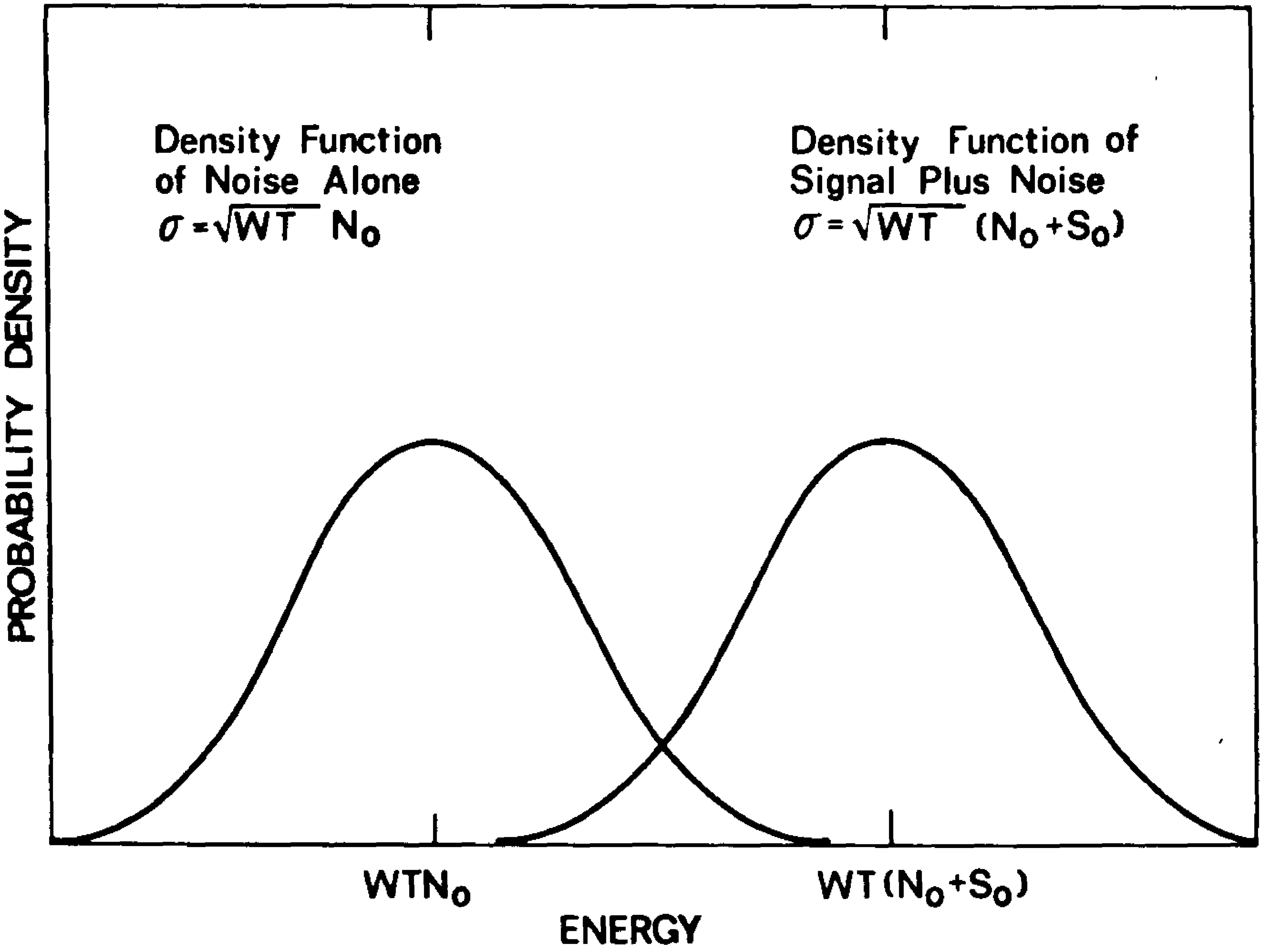
Green (1960a) describes an energy-detection scheme for the case in which the signal is a burst of noise. The signal is added to a continuous background of noise during either the first or the second of two time intervals. The observer's task is to report which of the two intervals contained the signal. The model requires the observer to have exact knowledge of starting times and signal duration and bandwidth.

Energy distributions for the noise and signal-plus-noise waveforms are given in Figure 1. The left-hand distribution is the probability density function for the energy of the noise alone. The right hand distribution is that for signal plus noise. The parameters of these two distributions are derived as follows: If the noise waveforms are white, Gaussian, and Fourier-series band-limited, then they can be approximated by a discrete representation consisting of 2WT terms, where W and T are, respectively, the bandwidth (in Hz) and duration (in seconds) of the signal. Green notes that the probability distributions in Figure 1 are approximately normal (if 2WT is greater than 30); means and variances are as given in the figure.

The decision rule for optimal forced-choice detection is to report that the signal occured in the interval containing the greater energy. Thus, discriminability for the ideal observer can be predicted from the probability that a random observation from the signal-plus-noise distribution is greater than one from the noise distribution. This probability is easily computed from the difference distribution. Since both energy distributions are approximately normal, the differ-

Figure 1

Theoretical energy distributions for signal plus noise and noise alone. S_o and N_o are, respectively, average power per Herz for signal and for noise. The signal and noise waveforms are statistically independent (average $r = 0$).


Density Function of Noise Alone $\sigma = \sqrt{WT}\ N_0$
Density Function of Signal Plus Noise $\sigma = \sqrt{WT}\ (N_0 + S_0)$
PROBABILITY DENSITY
WTN_0
$WT(N_0 + S_0)$
ENERGY

ence distribution is also approximately normal with mean equal to the difference between the means and variance equal to the sum of the two variances.

Green shows that the mean of the normalized difference distribution is

$$M = \frac{WTS_o}{\left[WTN_o{}^2 + WT(N_o + S_o)^2\right]^{\frac{1}{2}}}$$

$$= (WT/2)^{\frac{1}{2}} \frac{S_o}{N_o} \frac{1}{\left[1 + (S_o/N_o) + \frac{1}{2}(S_o/N_o)^2\right]^{\frac{1}{2}}} . \qquad (1)$$

This is approximately

$$M \approx (WT/2)^{\frac{1}{2}} \frac{S_o}{N_o} , \quad \text{when} \quad \frac{S_o}{N_o} \ll 1; \qquad (2)$$

S_o and N_o are the power densities for the signal and noise waveforms, respectively. Since the measure of sensitivity d' is equal to $\sqrt{2}$ times the normal deviate, it follows that

$$d' \approx (WT)^{\frac{1}{2}} (S_o/N_o). \qquad (3)$$

Thus: (a) The form of the psychometric function -- percent correct as a function of S_o/N_o -- is specified to be a normal ogive. (b) Weber fractions are determined by signal bandwidth and duration. At a level of about 75% correct responses (d' = 1), S_o/N_o is approximately equal to $1/(WT)^{\frac{1}{2}}$. (c) There is perfect reciprocity between W and T; neither d' nor the Weber fraction is affected by trading W and T for each other. Finally, (d) for a fixed N_o, the reciprocity between S_o and either W or T is such that a tenfold increase in either W or T yields a half log-unit improvement in the Weber fraction.

Green compares the behavior of the model to that of human listeners and reports the following: Human psychometric functions are steeper than those suggested by the model and human Weber fractions are poorer than optimal by 5 to 6 dB. When obtained Weber fractions (in dB) are plotted against log T, a straight line results whose slope is -0.5. For durations between 3 and 300 msec, the real listener is consistently 5 to 6 dB worse than his ideal counterpart. Analogous results obtain when W is manipulated.

Green invokes internal noise (a constant attenuation factor) to account for the consistent discrepancy between real and ideal behavior. By assuming that stimulus variability and neural noise are additive, it follows that internal fluctuations must mimic stimulus fluctuations with respect to the effects of duration and bandwidth. This is a strong statement; a 6 dB difference between the real and the ideal implies that <u>there is three times as much internal noise as acoustic noise</u>.

But there is even more internal noise than this. The filter that defines signal bandwidth for Green's model has a rectangular response curve. A more realizable filter, is shown by Mathews and Pfafflin (1965) to produce energy distributions whose sigma-to-mean ratios equal $1/(2WT)^{\frac{1}{2}}$. With ideal performance 1.5 dB better than that given by Green, internal noise is greater than stimulus noise by a factor of more than 4.5!

The psychoacoustic literature contains three other studies which provide data on the relationship between noise-signal intensity and duration. Campbell (1963) confirms Green's finding that $\Delta I/I$ increases by about 5 dB for a tenfold increase in duration. On the other hand, Raab, Osman and Rich (1963) obtained an improvement of approximately 7 dB in $\Delta I/I$ per log unit of duration and Rochester (1971) reports a change of 8 dB per decade of duration. If the discrepancy between real and ideal listeners is not constant at all durations, a fixed "attenuation factor" cannot be employed -- the situation with respect to internal noise becomes complicated.

Hybrid Theories

We turn now to consider another class of ideal detection schemes in which both stimulus fluctuations and neural fluctuations interact in specified ways. These models explicitly consider the relationship between neural effects and the stimuli that evoke them. Unlike stimulus-oriented models, these theories can account for discriminations involving stimuli whose energies are "fixed;" unlike such strictly neural theories as those of Crozier (1940) and Blackwell (1963), these models do not ignore stimulus fluctuations, where they occur. Hybrid models have been developed by Treisman (1964, 1966) for visual discrimination, and by Siebert (1965, 1968) and McGill (1967; McGill & Goldberg, 1968a, 1968b) for auditory discrimination.

Treisman begins by hypothesizing that each quantum of light that is absorbed at the retina evokes neural firings in a sensory "channel." Thus, the number of channels activated on any trial equals the number of quanta that are absorbed on that trial. It follows that the distribution of the number of activated channels is Poisson, with mean proportional to stimulus intensity.

A second assumption is that the distributions of firings in the activated channels are all Gaussian and have identical means and identical standard deviations. All of these parallel channels are also considered to be equally responsive on a given trial; all channels that are activated fire the same number of impulses.

The number of impulses evoked by each "constant" intensity flash, is the product of the number of channels activated (Poisson) by the number of impulses per channel (Gaussian). Treisman (1966) computed the mean and variance of this distribution of neural effect and has generated the distribution itself by computer simulation.

This description of Treisman's model is somewhat simplified. The complete model contains expressions for spontaneous firings ("dark light") and for adaption, to deal, respectively, with absolute thresholds and the effects of constant backgrounds.

By considering discrimination between two flashes to be based on the difference in the neural effect each evokes, the model describes the form of the Weber function. The well known (for vision) transition from the square-root law at low intensity levels to Weber's law at high levels is also obtained.

Siebert (1965, 1968) presents a model of auditory intensity discrimination which also generates psychophysical data by comparing the number of neural impulses evoked by standard and incremented stimuli. The model includes an idealization of the "peripheral auditory system" and an "ideal decision computer." For simplicity, we consider the model to be composed of three segments. The first contains a series of linear, time-invariant systems and represents the transformations performed on acoustic inputs beginning with the ear drum and ending with the displacement of the cochlear partition. The equations for this portion of the model are based on data reported by Bekesy (1960). The next segment of the model considers the conversion of acoustic stimulation into impulses in the auditory nerve. The structure and parameters of this portion of the model were chosen so that the output, at this point, is quantitatively and qualitatively similar to data obtained by Kiang and his associates (Kiang, Wattanabe, Thomas & Clark, 1965) on evoked activity in single units of the auditory nerve in cats. The final segment is an "ideal decision computer" which makes the "best possible discriminations" given the output of the "auditory nerve."

"Internal noise" is introduced in the second stage of the model. Kiang, et.al. (1965) demonstrate that the activity evoked in single fibers varies statistically from presentation to presentation of an identical acoustic waveform. Siebert (1968) shows that these fluctuations in coding, across the entire auditory nerve as well as within individual units, approximate a Poisson process. Thus, given an estimate of the mean frequency of discharge evoked in the nerve, the variance is established.

"Physiological Weber functions" derived from simulated distributions of neural effect are similar to human psychophysical data. Predicted Weber functions, for both noise and tonal stimuli, show increasing differential sensitivity with increasing intensity level up to some intermediate range of intensities. Beyond this level, Weber's law obtains. The model also yields the "pedestal effect" (cf., Pfafflin & Mathews, 1962) when discrimination between pulsed tonal stimuli occurs against a background of noise.

Siebert's model represents an approach based on what has been called Mountcastle's "linearity hypothesis" (Taub, 1969). This hypothesis (Mountcastle, 1967) states that the coding of stimulus intensity into neural impulses is nonlinear at the level of receptor and first-order cells; succeeding transformations performed by higher-order cells, are all linear. The decision axis employed by real observers, then, is a linear transform of an axis based on activity in first-order cells. Support for the linearity hypothesis comes from similarities between Weber functions determined for tactile sensation in humans and those computed for individual first-order touch fibers in cats and monkeys (Werner & Mountcastle, 1965). It should be noted that the hypothesis of linearity is a strong one. The similarities in the Weber functions can be accounted for by invoking any monotonic transformation followed by an appropriate decision criterion.

Siebert's formulation is noteworthy as an attempt at modelling human behavior on the basis of physiological data compiled from many single units rather than from the activity of individual units. Sampling errors are markedly reduced in Siebert's "auditory nerve." His reconstruction of the nerve, it should be noted, is based on data from approximately 5000 units from about 150 cats.

McGill (1967) also presents a hybrid model in which discrimination between two acoustic stimuli is based on the number of impulses evoked by each. The model includes a "neural counting mechanism" which observes the "mass flow" of impulses in a sensory pathway over short, fixed, periods of time. The mechanism counts the total number of impulses arriving from a number of information channels, but is incapable of differentiating between the inputs from the various channels. One result of this "smearing" process is that the number of counts evoked by a stimulus of "fixed" energy varies in Poisson fashion from observation to observation. More specifically, McGill relates the mean of this Poisson distribution of counts ($\overline{N}$) to the energy (E) of the stimulus, as follows:

$$\overline{N} = bE. \tag{4}$$

According to the model, stimulus fluctuations and neural fluctuations interact to limit discrimination performance. When the energy of the stimulus varies from presentation to presentation, the mean number of counts also fluctuates. That is, the number of counts is sampled from a Poisson distribution whose mean is "driven" by the energy fluctuations of the stimulus.

McGill has applied the model to intensity discrimination with pure tones and noise stimuli and to detection of a sinusoid against a noise background. It is interesting to note that in the latter two cases, the behavior of McGill's model is similar to that of the stimulus-oriented energy-detector model -- it predicts signal-to-noise ratios that are constant for fixed detectability.

McGill's model is of particular interest when it deals with pure-tone intensity discrimination. With these stimuli, only neural fluctuations (Poisson) limit performance. Equation (4) then describes the relationship between fixed stimulus energies and mean neural effects.

Two unrealistic consequences result from the linear-mapping model as presented thus far. First, since the standard deviation of a Poisson distribution is equal to the square root of its mean, the model yields a square-root law for intensity discrimination with tonal stimuli. Empirically, this is not the case. Second, the model implies that the dynamic range of the auditory nervous system must equal the intensity range of audible stimuli. That a sound 130 dB above absolute threshold evokes 10^{13} times the number of impulses as a threshold sound, is very unlikely.

To obviate these problems, the model was modified (McGill & Goldberg, 1968a, 1968b) so that the mapping of stimulus energy onto neural counts is via a power law. (See also Stevens, 1970). Specifically, if

$$\overline{N} = bE^{c}, \qquad 0 < c \leqslant 1, \tag{5}$$

the dynamic range of neural counts is compressed, for values of c less than 1. The authors then show that the Weber function generated by the Poisson counting mechanism moves from the "square root" law (when c = 1) towards Weber's law (as c approaches 0). If -- as is likely -- the dynamic range of neural firings is heavily "compressed" from 130 dB (see McGill & Goldberg, 1968b, p. 580), the Weber function for tones will be closer to Weber's law than to the square-root law.

In this connection, McGill and Goldberg demonstrate that the mapping exponent, c, can be reclaimed from the empirically determined slope, m, of the intensity—discrimination function.

$$\log \Delta I = m \log I + \log k. \tag{6}$$

Here, k represents the average Weber fraction for the range of values of I. Specifically, it is shown that

$$c = 2(1 - m) \tag{7}$$

(see Appendix I). Since c can assume values only between 0 and 1.0, m must lie between 1.0 (Weber's law) and 0.5 (square-root law"). A "near-miss" to Weber's law, is indicated by a value of m which is close to 1.0.

The behavior of the model has been compared to that of human listeners discriminating between tonal stimuli. McGill and Goldberg (1968a, 1968b) obtained slopes of approximately 0.9 ($c \approx 0.2$) with 1000-Hz tone bursts. A similar finding was observed by Dimmick and Olson (1941) and may also be seen in data reported by Campbell and Lasky (1967). These departures from Weber's law are not dismissed as errors of some sort. They are, in fact, confirmations of the theory (see equations 5 and 7).

Discrimination with Reproducible Stimuli

The possibility that McGill's model can be applied to intensity discrimination with reproducible stimuli other than tones is attractive. Click-intensity discrimination has been studied by Raab and Taub (1969) who found that discriminability is poorer in the region of 35 dB SL than at higher or lower levels -- there is a hump in the Weber function. However, it is clear from the Weber functions that discriminability consistently improves as the level of the standard click increases above 40 dB. It should be noted that McGill and Goldberg (1968a, 1968b) report that the "near-miss" to Weber's law with tonal stimuli does not hold below 20 dB SL.

Pseudo-random noise waveforms constitute another class of precisely reproducible stimuli. One advantage of working with these noises is that they are richer in information than clicks or tones. Clicks are virtually unidimensional; they vary chiefly in intensity. Tonal stimuli of given frequency and phase have two parameters: intensity and duration. With reproducible noise stimuli, the effects of three variables -- intensity, duration and bandwidth -- can be examined. Quantification of these effects will, hopefully, help to clarify the nature of auditory neural coding.

Reproducible noise waveforms have been employed in a number of studies of auditory discrimination. Some of these studies are summarized next.

In a group of experiments, Green (1964) measured the consistency of individual listeners' responses to repeated presentations of tape recorded noise stimuli. Pairs of intervals with a signal in one of the two intervals were interspersed with pairs of intervals without a signal. On some tapes, the signals were noise increments; on others they were sinusoids. Percent correct responses on "signal" trials provided a measure of detection performance. Responses on "no-signal" trials provided a measure of consistency. Average agreement was approximately 66% for all conditions studied. Green shows that if external and internal variabilities are additive, this result implies that the standard deviations of external and internal noise are approximately equal.

There are two problems with Green's method. First, stimulus variability is not completely removed when the same tape is repeatedly played back. Second, repeated presentations of the stimuli in the same order allows response bias to interfere with the estimate of internal noise. Bell and Nixon (1970), who also employed tape recorded stimuli in a study of consistency of judgments, avoided the problem of response bias by randomizing the orders of presentation.

Pfafflin and Mathews (1965) and Pfafflin (1968) employed computer-generated waveforms as stimuli, in order to present precisely reproducible waveforms. Twelve samples of noise were used; the signal to be detected was a sinusoid. In one set of experiments (Pfafflin & Mathews, 1965), forced-choice trials were presented with either the same noise waveform or two different noise waveforms, as backgrounds on the two halves of each trial. A general finding was that the probability of choosing one interval (as containing the signal) in preference to the other was a function of the relative energies of the waveforms in the two intervals. In a second set of experiments, Pfafflin (1968) demonstrated that detection of the signal was better when the noise waveform was fixed for a series of 288 trials than when the noise sample varied randomly from trial to trial.

Raab and Leshowitz (1968) studied intensity discrimination with random and with reproducible noises. A computer of average transients (CAT-400B) was employed to store and play back noise waveforms. Preliminary data (in the form of Weber fractions) were obtained for each of two experimental conditions. In one condition, the sample of noise varied from trial to trial but was identical in waveform (except for amplitude) on the two halves of each trial. In the second condition, a completely new waveform was presented on each half of each trial; here, within-trials variability was the same as between-trials variability. The effects of varying intensity, noise bandwidth and burst duration were examined.

Raab and Leshowitz report that discriminability was improved when energy variations within trials were removed. Improvement was greater when more stimulus variability was removed.

With noises varying both within and between trials, discriminability was closest to that predicted for an ideal energy-detector in those instances where stimulus fluctuations were greatest (narrow-band, short-duration waveforms). A similar observation was made by Ronken (1969) in a study of intensity discrimination with tone bursts whose amplitudes were varied in Rayleigh fashion.

Raab and Leshowitz report that increasing either the bandwidth or the duration of the noise stimuli resulted in improved Weber fractions. With waveforms that did not vary within trials, this result can only be ascribed to internal matters -- stimulus fluctuations were absent.

Unfortunately, the noise stimuli, as stored by the CAT, were only approximately Gaussian; they had a crest factor of only two. Use of the CAT imposed another limitation. Waveforms could be stored as no more than 400 amplitudes. Thus, only waveforms with 2WT less than 400 could be reasonably represented in this discrete fashion.

Purpose of the Present Experiments

In planning the experiments of this thesis, we sought to specify -- hopefully in a quantitative fashion -- the nature of the neural fluctuations that limit auditory discriminability. In its approach, the present investigation is an extension of the very preliminary work of Raab and Leshowitz (1968).

Analysis of Raab and Leshowitz's findings is limited by the nature of the stimuli employed. The present research employs a pseudo-random noise generator whose output has a crest factor exceeding three, and has a bandwidth-duration product greater than 6500 ($2WT > 13{,}000$).

The present study is a parametric one. Discriminability for reproducible noise bursts is examined at a number of intensity levels, at two stimulus durations (in one instance, three durations), and at two bandwidths. In addition, stimuli were selected and parameters arranged so that all three reciprocities among intensity, duration and bandwidth could be examined.

Discriminability for random noise bursts was also tested and the results are compared with those obtained for reproducible stimuli. Special attention is paid to the effects of duration and bandwidth in these comparisons.

A two-interval, forced-choice discrimination procedure is employed in the present experiments. In all instances, the background noises are not continuous but are gated for the durations of the observation intervals. Similarly, the bandwidths of the background waveforms always equalled those of the signals. These procedures allow us to compare our results with those predicted by energy-detector models. These models assume that the observer summates energy only within the bandwidth and duration of the increment. If real observers cannot do this (cf. Green & Sewall, 1962; Leshowitz, Taub & Raab, 1968), then matters are simplified if the noise is gated and filtered to match the signal.

Chapter II

METHOD

A forced-choice procedure was used to determine the magnitude of the difference limen (DL) and the form of the Weber function in three studies of auditory intensity discrimination.

(1) In Experiment 1, the stimuli were bursts of reproducible noise. (These stimuli sound like bursts of random noise, although their waveforms can be generated in unvarying fashion from one presentation to the next.) Difference thresholds were obtained at a number of energy-levels with stimuli that differed in intensity, duration and bandwidth. The effects of these three variables on intensity discrimination were examined.

(2) The results of Experiment 1 differed somewhat from previous findings concerning intensity discrimination with tonal stimuli (McGill & Goldberg, 1968a, 1968b). In Experiment 2, therefore, intensity discrimination for tones (f = 1000 Hz) was studied.

(3) In Experiment 3, DLs obtained with "real" noise-bursts were compared with those obtained with bursts of reproducible noise. This experiment constitutes an attempt to specify the nature of "internal noise" by analyzing discrimination data obtained with the two kinds of stimuli.

Subjects

The two subjects in all three experiments were the author (I.G.)

and a female undergraduate student at Brooklyn College (P.J.). Both subjects received considerable training before obtaining the DLs to be reported here.

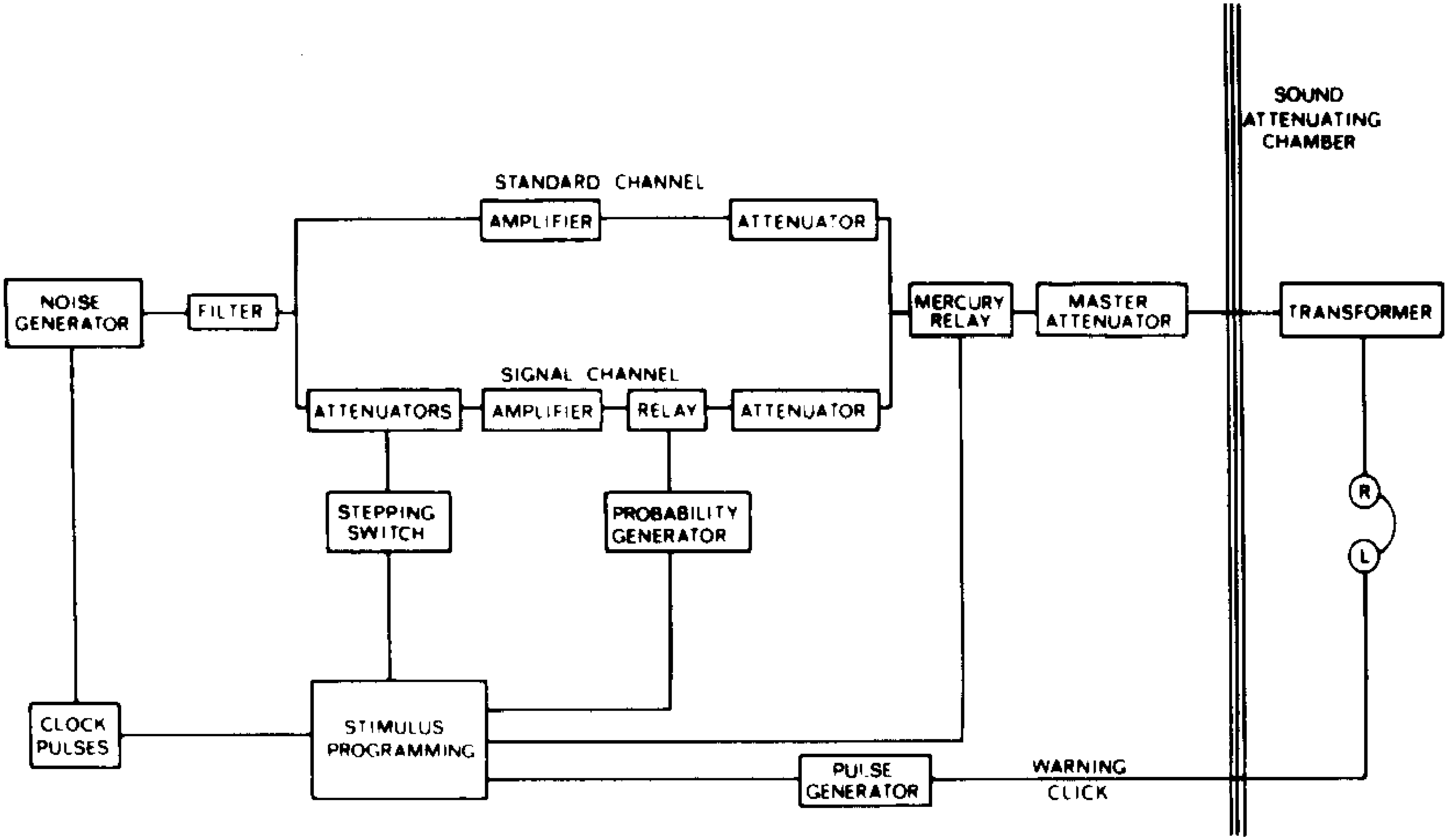
Apparatus

A block diagram of the apparatus used in Experiment 1 is presented in Figure 2. Minor modifications of the system were made for Experiments 2 and 3. These are described later.

The stimulus programming system included a digital counter (Iconix 6255) and a series of preset counters (Iconix 6010). Clock pulses (f = 163841 pulses per second) were supplied to the counters by an Eico 377 oscillator. The same clock pulses were also delivered to a Hewlett Packard 3722A digital noise generator, which generated the pseudo-random noise stimuli. With the sequence-length of the noise generator output equal to 131071 pulse-counts, a noise waveform lasting exactly 800 milliseconds (131071/163841 = 0.79999 sec) was generated repeatedly. By appropriate timing of close ("on") and open ("off") pulses from the programming system to a bistable mercury-wetted relay (Potter & Brumfield, JML 5430-82), two noise bursts of equal duration could be presented on each trial. The durations of both bursts were set at 10, 31.6 or 100 milliseconds. Irrespective of burst duration, the interval between the start of the first burst and the start of the second burst was always exactly 800 milliseconds, i.e. the Iconix presets were always set such that 131071 counts occurred

Figure 2

Block diagram of the apparatus employed in Experiment 1.


SOUND ATTENUATING CHAMBER
STANDARD CHANNEL
AMPLIFIER
ATTENUATOR
NOISE GENERATOR
FILTER
MERCURY RELAY
MASTER ATTENUATOR
TRANSFORMER
SIGNAL CHANNEL
ATTENUATORS
AMPLIFIER
RELAY
ATTENUATOR
STEPPING SWITCH
PROBABILITY GENERATOR
R
L
CLOCK PULSES
STIMULUS PROGRAMMING
PULSE GENERATOR
WARNING CLICK

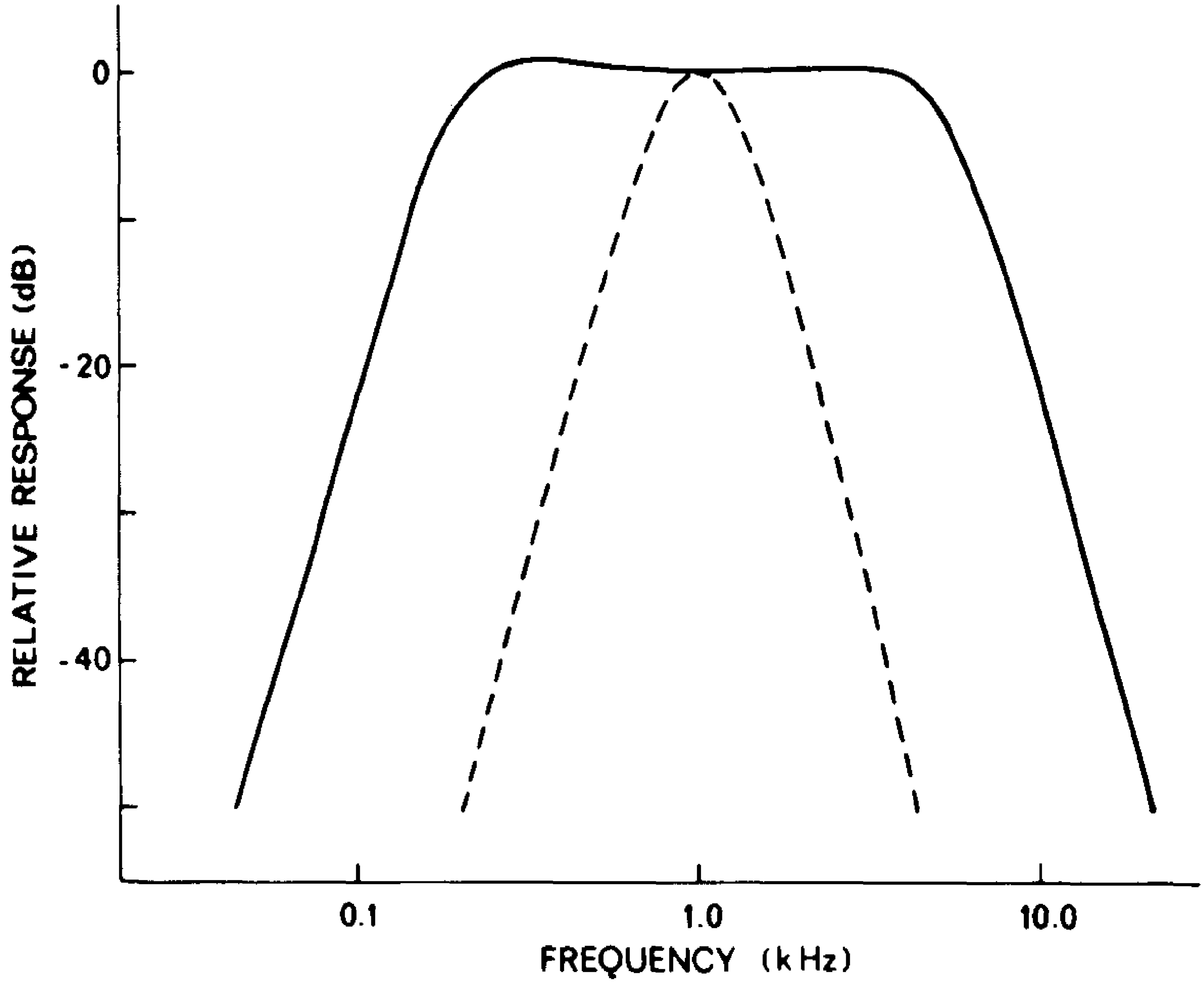
between the two "on" pulses to the relay. Thus, two identical noise bursts were generated on each forced-choice trial. Each trial was started by the subject's response for the preceding trial. Since the subject's responses were aperiodic, the noise waveforms varied from trial to trial with the constraint that the noise bursts were samples of the 800-millisecond, repeating waveform.

The output of the noise generator was band-limited by means of a Krohn-Hite 310-CR filter. Two such filters were used; the pass-band for each was centered at 1000 Hz. One filter was set for a bandwidth of 500 Hz (f = 781 to f = 1281 Hz) between the half-power points. The other filter was set for a 5000-Hz bandwidth (f = 194 to f = 5194 Hz). The two bandwidths, therefore, differed by one decade. (See Figure 3.)

The output of the filter was split into two channels -- one for the standard (V), the other for the increment signal (ΔV). Each channel contained an amplifier (Scott 140B) and level-setting attenuators. The signal channel also contained a series of 12 attenuators connected in cascade. A Lehigh Valley (1427) two-way stepping switch and associated mercury relays made it possible to change the magnitude of ΔV between blocks of trials. A probability generator (BRS-PP1) was used to close the contacts of a "signal" relay throughout one or the other of the two stimulus intervals on each trial. As a result, ΔV was randomly added to the first or the second noise burst on each trial. The outputs of the two channels were added in phase by means of a resistive mixing network. The sum was passed through a master attenuator, a transformer and, finally, to the subject's right earphone (Permoflux PDR-10 mounted in an MX/41-AR cushion). The mercury-wetted

Figure 3

Response curves of the bandpass filters employed in Experiments 1 and 3. The solid curve is for the broad-band filter (W = 5000 Hz). The dashed curve is for the narrow-band filter (W = 500 Hz). For both filters the center frequency equalled 1000 Hz.


RELATIVE RESPONSE (dB)
0
-20
-40
0.1
1.0
10.0
FREQUENCY (kHz)

relay used for gating the noise stimuli was connected after the mixture point.

Procedure

Procedures were basically the same in all three experiments. Difference thresholds were determined using a blocked, "up-down," two-interval, forced-choice procedure (cf. Campbell & Lasky, 1968). A faint warning click to the subject's left earphone marked the beginning of each trial. The two stimulus bursts, spaced 800 milliseconds apart, were presented in succession 1.5 seconds after the warning click. The standard burst (V) was presented on both halves of each trial. The increment signal (ΔV) was added in phase to one of the two standard bursts. By pressing one or the other of two microswitch buttons, the subject reported which of the two bursts was "louder." Indicator lights on the subject's response panel signalled the stimulus intervals and provided immediate knowledge of results on each trial. As indicated earlier, each response initiated the sequence of events comprising the next trial. This self-pacing procedure yielded one trial approximately every five seconds.

At the beginning of each threshold determination, ($V + \Delta V$) and V were set so as to be easily discriminated. If the subject voted correctly on five out of six trials at a given level of ΔV, the signal was attenuated by 3dB for the next block of six trials. On the other hand, if the subject voted incorrectly on two of the six trials in a block, ΔV was increased by 3 dB. This "up-down" procedure allowed the subject to cross and recross his threshold (the value of ΔV yielding approximately 75% correct responses) a number of times during each determination. After at least 20 reversals (approximately 110 trials), the median value of attenuation in the signal channel (ΔV) was calculated and then converted to increment power (ΔI), taking into account the in-phase addition of the standard and signal waveforms.

At least three DLs were obtained at each of a number of intensity levels for each stimulus condition. Wherever the range of the three Weber fractions exceeded 3 dB, a fourth determination was made.

Chapter III

INTENSITY DISCRIMINATION WITH REPRODUCIBLE NOISE BURSTS

Average Weber fractions (ΔI/I) for the two observers are presented in Tables I and II. For each table, the first entry (reading down) for 5000-Hz, 100-msec noise bursts is the Weber fraction (in dB) for standard bursts that were 20 dB above absolute threshold. For observer IG, absolute threshold was at 10 dB SPL; for observer PJ, threshold was at 8 dB SPL. (Sound pressure levels are all referred to 0.0002 μbar)

The tables contain the measured values of ΔI/I for the various combinations of intensity, duration and bandwidth that were studied. Adjacent rows represent 5 dB changes in <u>energy</u>. Within each row, therefore, are the Weber fractions obtained for standard stimuli that were equal in energy but which differed in intensity, duration and bandwidth. For example (see Table I), a 100-msec noise burst with W = 5000 Hz and spectrum level equal to 3 dB SPL has the same energy as one with W = 500 Hz, T = 10 msec, and a spectrum level of 23 dB.

It is clear from the data in Tables I and II that reproducible noise bursts which are equal in energy do not all yield equal discriminabilities. Rather, as will be seen in the next sections of this chapter, discriminability for these stimuli depends somewhat on burst duration, but is relatively independent of bandwidth.

Table I

Weber Fractions (ΔI/I in dB) for Reproducible Noise Bursts

Observer IG

W = 5000 Hz						W = 500 Hz			
T = 100 msec		T = 31.6 msec		T = 10 msec		T = 100 msec		T = 10 msec	
I[a]	ΔI/I[b]	I	ΔI/I	I	ΔI/I	I	ΔI/I	I	ΔI/I
								30(3)	-3.3
30(-7)	-7.0	35(-2)	-4.1	40(3)	-3.3	30(3)	-5.6	40(13)	-3.2
		40(3)	-6.4						
40(3)	-6.1	45(8)	-5.5	50(13)	-5.4	40(13)	-6.6	50(23)	-3.4
		50(13)	-5.1						
50(13)	-7.4	55(18)	-5.4	60(23)	-4.3	50(23)	-6.3	60(33)	-4.8
		60(23)	-3.9						
60(23)	-6.3	65(28)	-5.2	70(33)	-3.5	60(33)	-6.4	70(43)	-3.3
		70(33)	-5.1						
70(33)	-5.8	75(38)	-4.1	80(43)	-4.4	70(43)	-7.3		
		80(43)	-5.6						
80(43)	-7.6	85(48)	-7.2	90(53)	-7.1	80(53)	-6.5		
Mean	-6.7		-5.2		-4.7		-6.4		-3.6

[a] Intensity of standard burst. The first value is the overall SPL in dB; the second value, in parentheses, is the spectrum level (in dB) at the center of the passband.

[b] Weber fraction in dB.

Table II

Weber Fractions (ΔI/I in dB) for Reproducible Noise Bursts

Observer PJ

W = 5000 Hz						W = 500 Hz			
T = 100 msec		T = 31.6 msec		T = 10 msec		T = 100 msec		T = 10 msec	
I[a]	ΔI/I[b]	I	ΔI/I	I	ΔI/I	I	ΔI/I	I	ΔI/I
								28(1)	-1.2
28(-9)	-6.2	33(-4)	-5.8	38(1)	-2.9	28(1)	-3.6	38(11)	-0.8
		38(1)	-5.5						
38(1)	-6.0	43(6)	-5.2	48(11)	-3.6	38(11)	-5.4	48(21)	-2.1
		48(11)	-3.6						
48(11)	-5.6	53(16)	-2.9	58(21)	-3.4	48(21)	-6.3	58(31)	-1.3
		58(21)	-3.6						
58(21)	-5.7	63(26)	-4.3	68(31)	-1.8	58(31)	-5.3	68(41)	-4.5
		68(31)	-3.1						
68(31)	-3.8	73(36)	-3.0	78(41)	-1.7	68(41)	-5.4		
		78(41)	-3.8						
78(41)	-3.9	83(46)	-3.4	88(51)	-4.5	78(51)	-7.0		
Mean	-5.2		-4.0		-3.0		-5.5		-2.0

[a] Intensity of standard burst. The first value is the overall SPL in dB; the second value, in parentheses, is the spectrum level (in dB) at the center of the passband.

[b] Weber fraction in dB.

The data in Tables I and II are plotted as Weber functions in Figures 4 through 7. Since the plotted functions are more or less horizontal, it can be seen that Weber's law obtained in almost all cases. Weber's law also predicts a slope of unity for the intensity-discrimination function which plots log ΔI against log I. The data obtained with broad-band, long-duration bursts are plotted in this fashion in Figure 8. The theoretical line, which the data points approximate closely, has a slope of 1.0 and an intercept equal to the average Weber fraction for the two listeners (-6.0 dB). The calculated slopes for the lines that best fit the data points are 1.09 for IG and 1.05 for PJ. Slopes of the intensity-discrimination functions for all conditions are presented in Table III. There do not appear to be any serious departures from Weber's law, except, perhaps, for observer PJ's data obtained with narrow-band noises.

Effects of Stimulus Duration

The effects of noise-burst duration on intensity discrimination can be seen in Figures 4 and 5 where 10 log $\Delta I/I$ is plotted as a function of the spectrum level of the standard burst. The parameter is the duration of the bursts. Comparison of the two Weber ratios at each intensity level indicates that in all instances discriminability was improved by increasing the duration of the noise bursts by one log-unit. For the broad-band noises (Figure 4), a decade increase in duration resulted in an average improvement of approximately 2 dB in $\Delta I/I$. For the narrow-band stimuli (Figure 5), $\Delta I/I$ changed by approximately 3 dB as the result of a ten-fold change in duration. Since

Figure 4

Weber functions for bursts of reproducible noise (W = 5000 Hz). The parameter is duration: ▲—▲, T = 100 msec; ●—●, T = 10 msec. The upper graph, (a), is for observer IG; the lower graph (b), is for observer PJ.

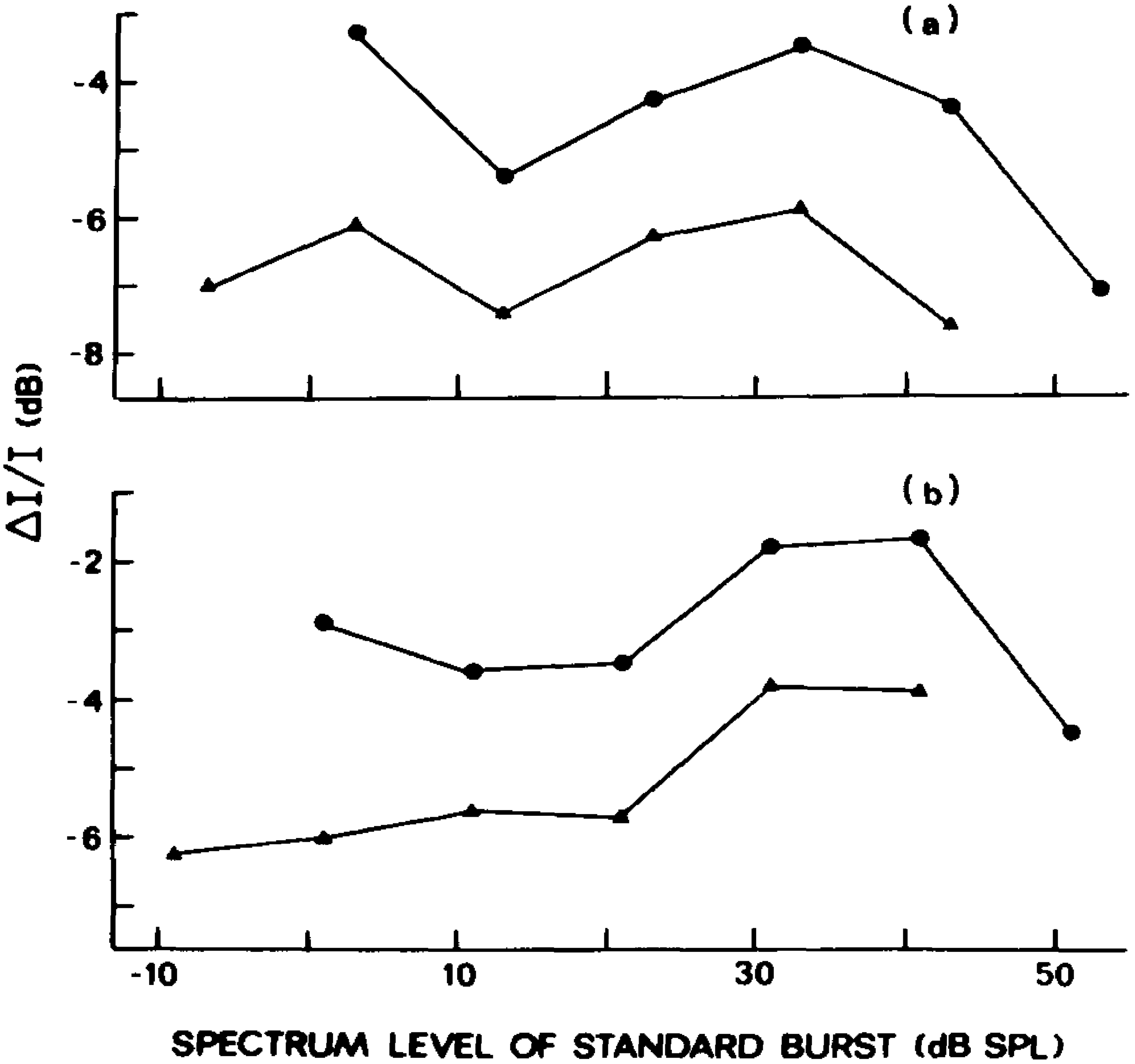

(a)
(b)
-4
-6
-8
-2
-4
-6
ΔI/I (dB)
-10
10
30
50
SPECTRUM LEVEL OF STANDARD BURST (dB SPL)

Figure 5

Weber functions for bursts of reproducible noise (W = 500 Hz). The parameter is duration: ▼—▼, T = 100 msec; ■—■, T = 10 msec. (a) observer IG, (b) observer PJ.

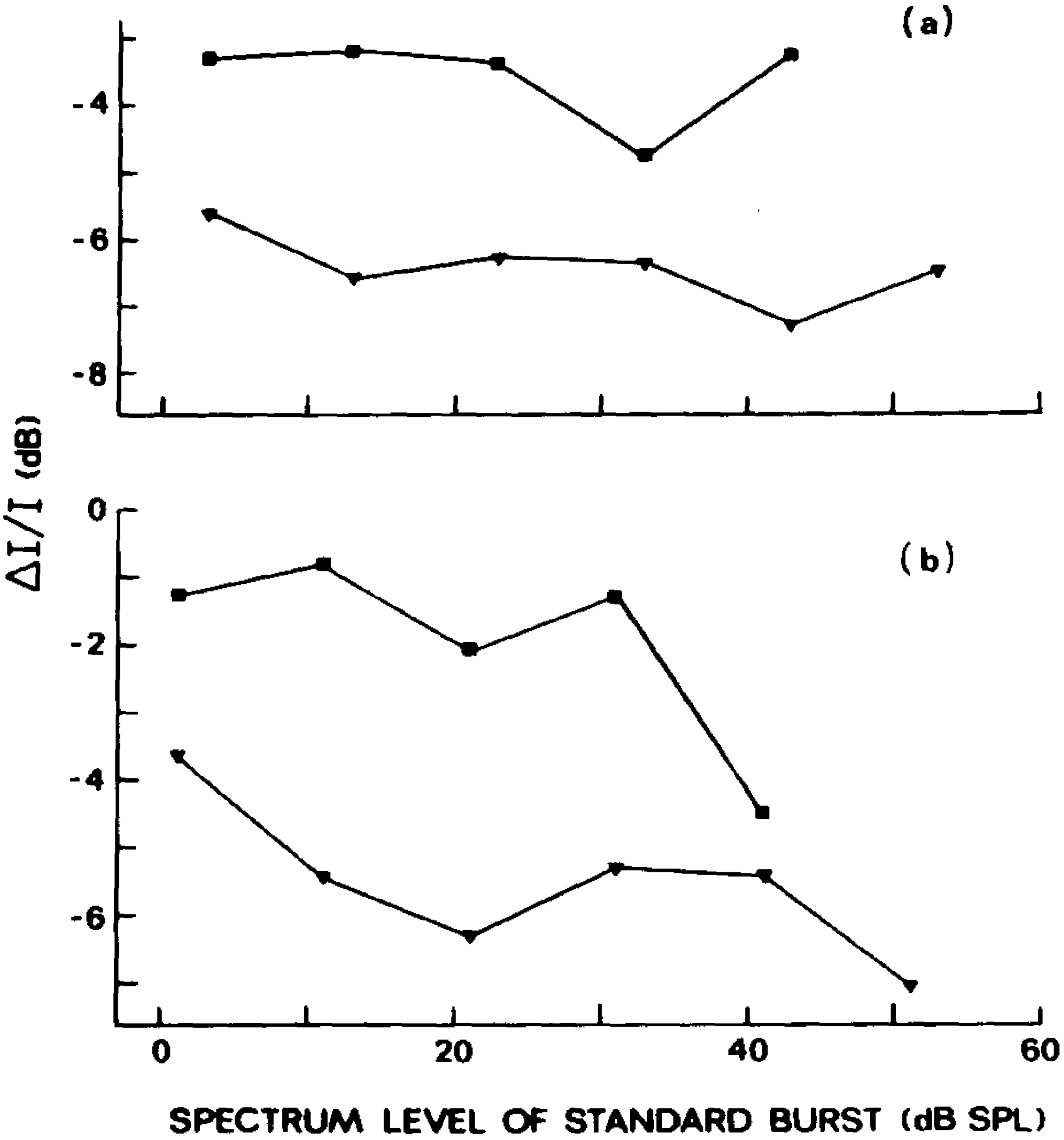

(a)
-4
-6
-8
ΔI/I (dB)
0
(b)
-2
-4
-6
0
20
40
60
SPECTRUM LEVEL OF STANDARD BURST (dB SPL)

Figure 6

Weber functions for reproducible noise bursts (T = 100 msec). The parameter is bandwidth: ▲—▲, W = 5000 Hz; ▼—▼, W = 500 Hz. (a) observer IG, (b) observer PJ.

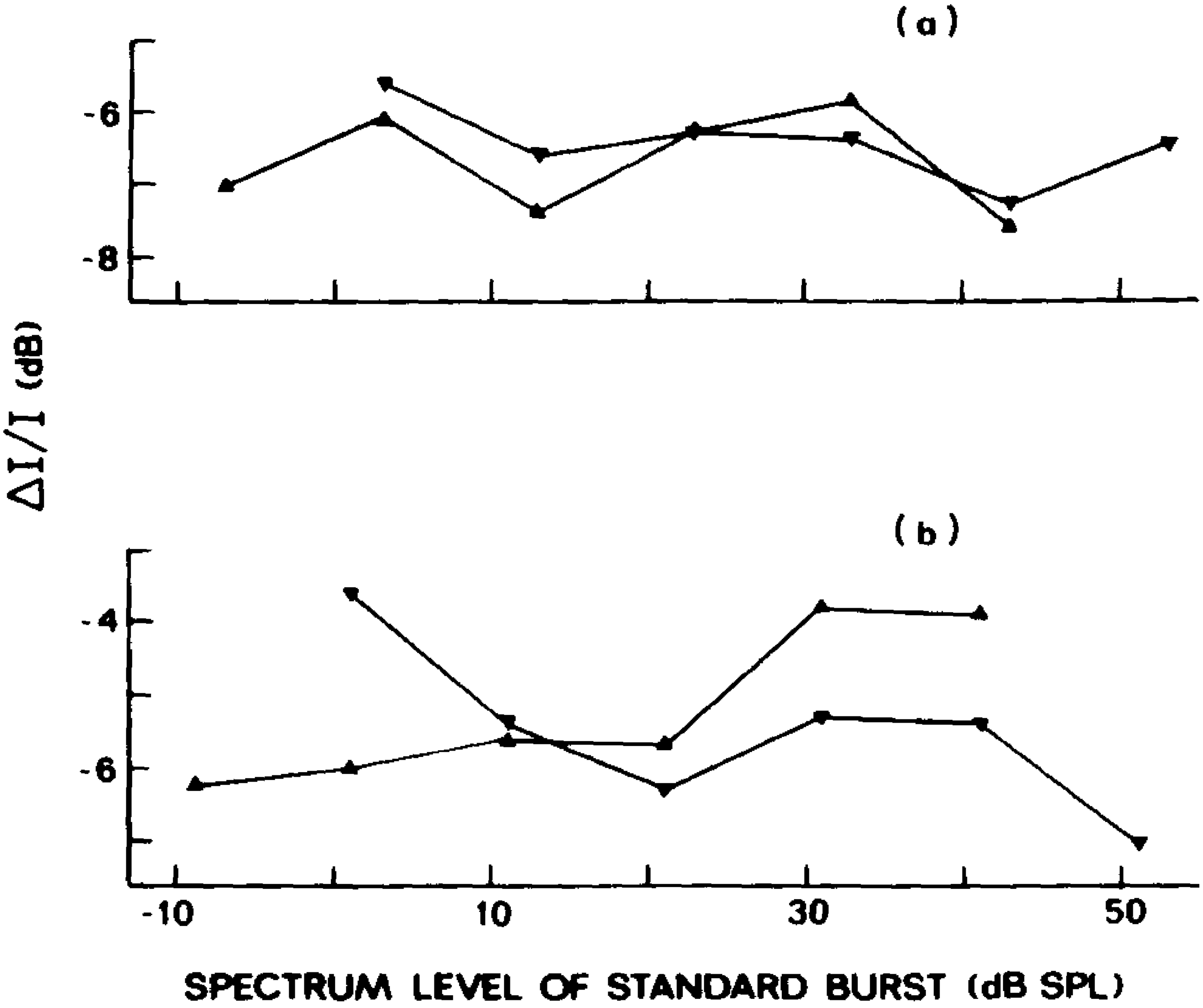

(a)
-6
-8
ΔI/I (dB)
(b)
-4
-6
-10
10
30
50
SPECTRUM LEVEL OF STANDARD BURST (dB SPL)

Figure 7

Weber functions for reproducible noise bursts (T = 10 msec). The parameter is bandwidth: ●—●, W = 5000 Hz; ■—■, W = 500 Hz. (a) observer IG (b) observer PJ.

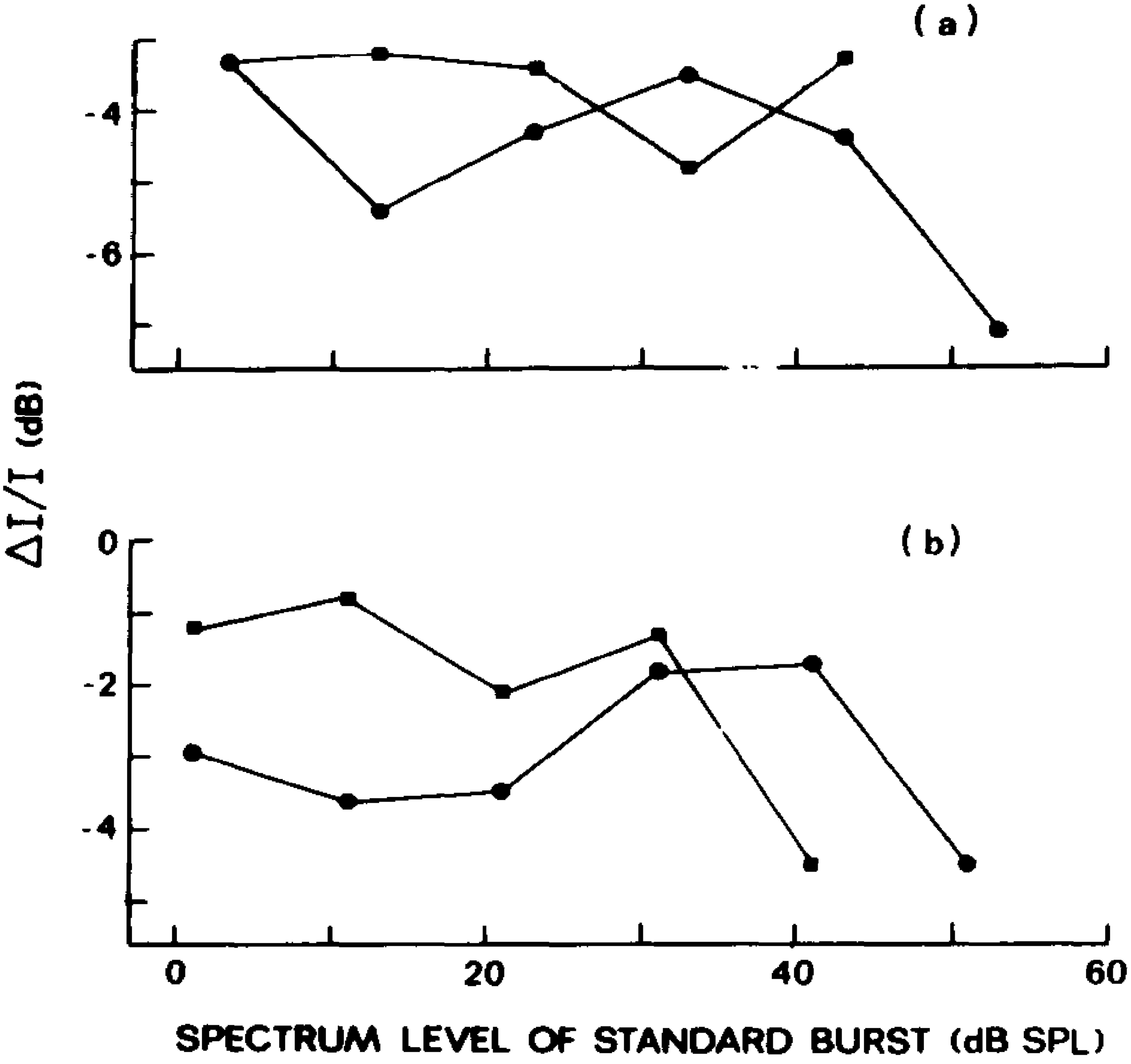

(a)
-4
-6
ΔI/I (dB)
(b)
0
-2
-4
0
20
40
60
SPECTRUM LEVEL OF STANDARD BURST (dB SPL)

Figure 8

Intensity-discrimination function for reproducible noise bursts with $W = 5000$ Hz and $T = 100$ msec. The line is theoretical and has a slope of 1.0. Its intercept is equal to the average Weber fraction (in dB) for the two listeners: IG (■) and PJ (●).

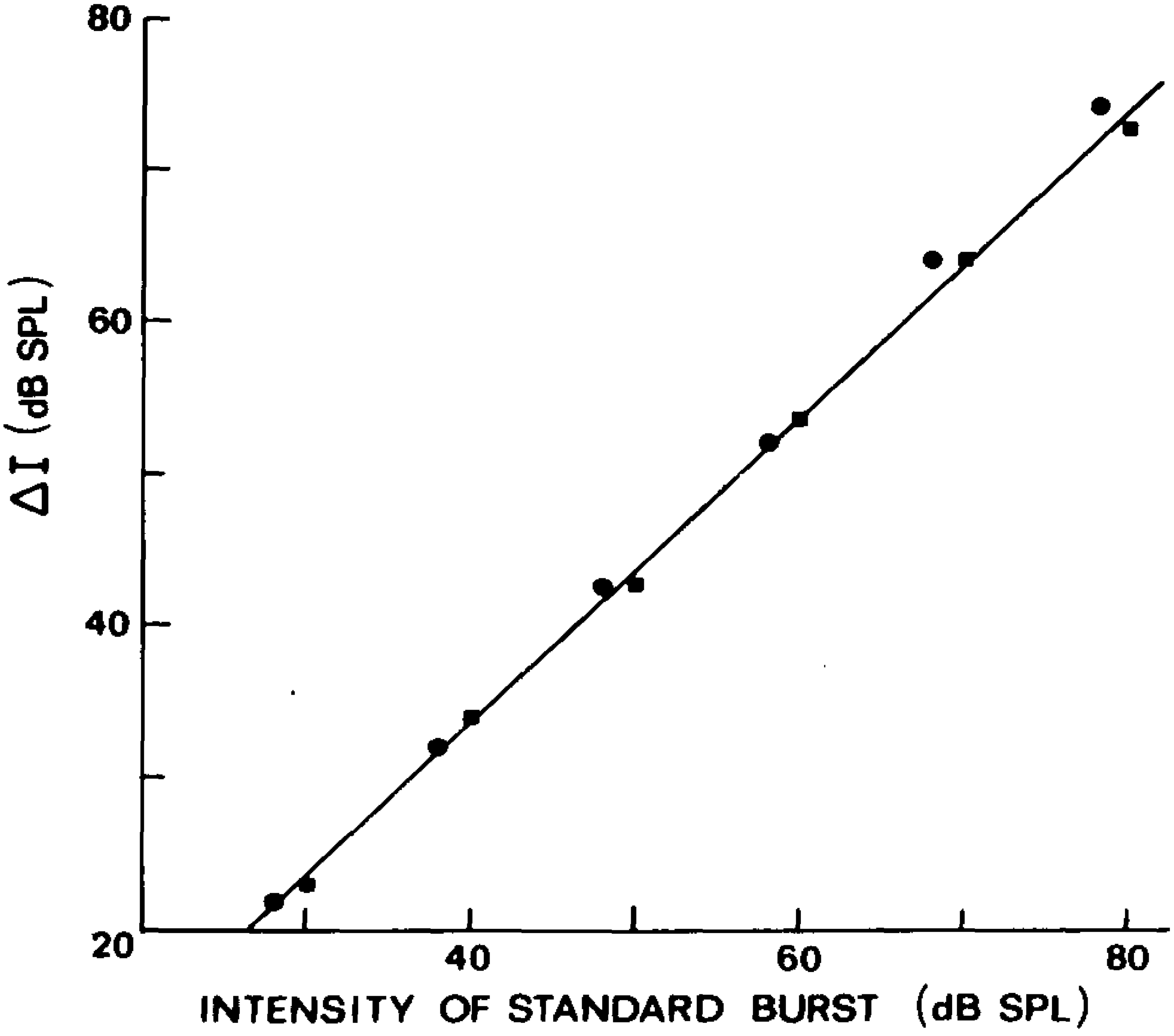

80
60
40
20
ΔI (dB SPL)
40
60
80
INTENSITY OF STANDARD BURST (dB SPL)

Table III

Slopes of Lines Best Fitting the Plots of Log ΔI against Log I (Reproducible Noise Bursts)

	W = 5000 Hz			W = 500 Hz	
Observer	100[a]	31.6	10	100	10
IG	1.09	1.00	0.96	0.98	0.97
PJ	1.05	1.04	1.00	0.94	0.93

[a] Burst duration in msec.

Weber's law obtained, this duration-effect implies that stimuli which are equal in energy do not yield equal discriminability. This result can be seen by comparing the Weber fraction obtained for 100-msec bursts at a given spectrum level with the Weber fraction obtained with 10-msec bursts which are 10 dB more intense. For example, (see Figure 4a), with the long-duration bursts at a spectrum level of 23 dB, the obtained $\Delta I/I$ for observer IG was -6.3 dB. With a 10-msec burst of equal energy (i.e., at 33 dB), the Weber fraction was -3.5 dB.

Effects of Noise Bandwidth

The effects of bandwidth on intensity discrimination can be seen in Figures 6 and 7 where 10 log $\Delta I/I$ is plotted as a function of the spectrum level of the standard burst. The parameter here is the bandwidth of the noise. Comparison of the two Weber functions, for each figure, indicates that bandwidth has little effect on discriminability with bursts of pseudo-random noise. For the long-duration noise bursts, a decade change in bandwidth resulted, on the average, in little or no change in $\Delta I/I$. For the short-duration noise bursts, a ten-fold increase in bandwidth yielded an average improvement of 1 dB in $\Delta I/I$. Since the functions are more or less horizontal and intertwine with one another, it follows that equal-energy bursts (generated by trading intensity for bandwidth) are equally discriminable.

Bandwidth-Duration Reciprocity

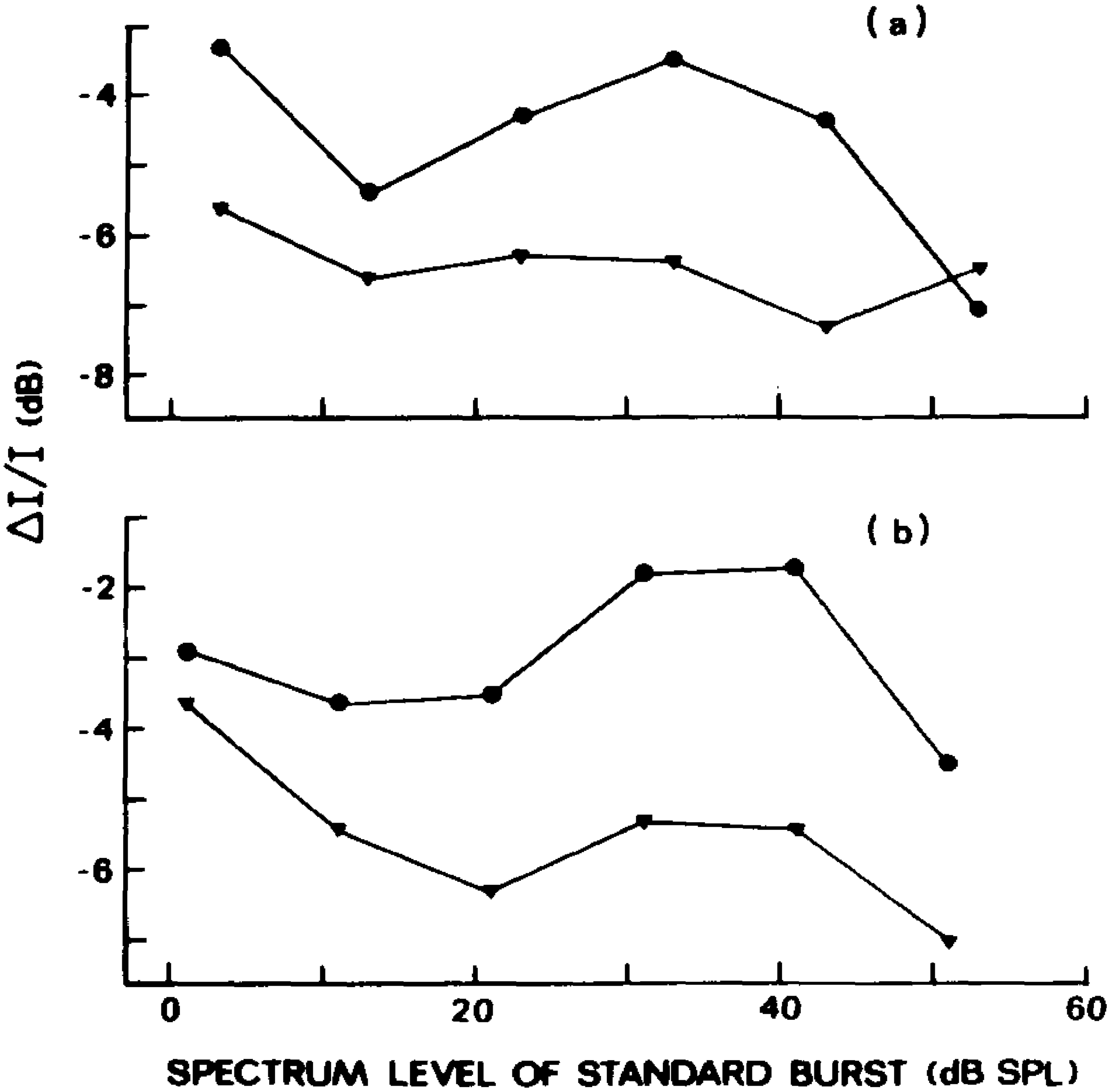
Taken together, the finding of a duration-effect and the absence of a bandwidth-effect indicate that discriminability must change when these two variables are traded for each other. This result -- the failure of bandwidth-duration reciprocity -- is presented in Figure 9. In this figure, each pair of data points at a given spectrum level plots the Weber fractions for one pair of equal-energy stimulus bursts. This follows from the fact that energy is given by the product of noise-power density, duration and bandwidth, and that spectrum level is monotonic with noise-power density. The results are that discriminability is better (by approximately 2 dB) for 100-msec, 500-Hz noise bursts than for 10-msec, 5000-Hz stimuli. The magnitude of this effect is greatest at intermediate levels of stimulus energy.

Discussion

Two of the findings with reproducible bursts of noise will now be discussed in relation to an extension of McGill's neural theory of auditory intensity discrimination (McGill, 1967; McGill & Goldberg 1968a, 1968b). The theory states: (1) When there are no fluctuations in stimulus energy from presentation to presentation, internal processes become sole source of variability; (2) The number of neural counts evoked by a fixed-energy stimulus varies in Poisson fashion from trial to trial; (3) The mean number of counts is related to stimulus energy by a power function; specifically, $\bar{N} = bE^c$. As discussed above (see p. 19), McGill & Goldberg (1968a, 1968b) show that the mapping exponent, c, can be reclaimed from the empirically determined

Figure 9

Weber functions for reproducible noise bursts with W = 5000 Hz, T = 10 msec (●—●) and with W = 500 Hz, T = 100 msec (▼—▼). (a) observer IG, (b) observer PJ.


(a)
-4
-6
-8
ΔI/I (dB)
(b)
-2
-4
-6
0
20
40
60
SPECTRUM LEVEL OF STANDARD BURST (dB SPL)

slope of the intensity-discrimination function ($\log \Delta I = m \log I + \log k$). Furthermore, their model predicts that m must lie between 0.5 ("square-root law") and 1.0 (Weber's law). It will be recalled, that other considerations dictate a value of m which is closer to 1.0 than to 0.5.

Support for the model comes from several studies of pure-tone intensity discrimination in which values of m were found to be approximately 0.9 (see p. 20). In order to apply the model to intensity discrimination with noises, the following assumption -- similar to one originally presented by McGill (1967, p. 362) -- is introduced: the energy in each frequency region of the noise stimulus is mapped onto neural counts via the same two transfer constants, b and c. With this simplification, the predictions for intensity discrimination with reproducible bursts of white noise are similar to those for tones. Thus: (1) equal stimulus energies yield equal discriminabilities, and (2) the slope of the intensity-discrimination function will be slightly less than 1.0.

With respect to the first prediction, we have seen that equal energies do yield equal discriminabilities when changes in noise bandwidth compensate for changes in burst intensity. By itself, this result suggests that the simplification introduced in order to apply McGill's model to noise stimuli is not unreasonable. On the other hand, maintaining equal stimulus energies by exchanging intensity for duration, or duration for bandwidth, does not result in equal discriminabilities. Apparently, the mapping of energy onto neural counts is dependent on burst duration.

The second prediction, too, was not borne out: intensity-discrimination functions with approximately unit slope were found for almost all of the conditions tested. That is, Weber's law, and not a "near-miss" to Weber's law, obtains for intensity discrimination with reproducible noise bursts. This result raises further difficulties for the McGill model. It is possible that the discrepancy between our observations and those of McGill and Goldberg (1968a, 1968b) is the result of differences between the stimuli, viz., reproducible noise bursts vs. tones. Another possibility is that the form of the Weber function (and, therefore, of the intensity-discrimination function) depends upon testing procedures. We used a two-interval forced-choice procedure, while McGill and Goldberg tested with a Yes-No psychophysical method. This latter possibility is ruled out in the next chapter.

Chapter IV

PURE-TONE INTENSITY DISCRIMINATION

It is conceivable that disparities between our observations with reproducible noise bursts (Chapter III) and those of McGill and Goldberg (1968a, 1968b) with 1000-Hz tones are due to procedural differences. McGill and Goldberg employed a Yes-No detection task while a two-interval forced-choice procedure was utilized in Experiment 1. We therefore decided to examine intensity discrimination for 1000-Hz tones using the same apparatus, procedure and listeners as in Experiment 1.

Since the results of Experiment 1 indicate that discriminability depends on noise burst duration, we decided, in addition, to examine the effects of burst duration with tonal stimuli. That the slope of the intensity-discrimination function is affected by tone-burst duration is suggested by the fact that McGill and Goldberg obtained a slope of 0.94 with 150-msec stimuli (1968b) and a slope of 0.90 with tone-bursts that lasted for 15-20 msec (1968a).

Method

The magnitude of the DL as a function of intensity was determined for 1000-Hz tonal stimuli with burst durations of 10 and 100 msec. The general procedures were identical to those described for Experiment 1. For the present purpose a Clarke-Hess (743) function generator replaced the noise generator in the system represented in Figure 2 (p. 29). The output of the function generator was monitored as to frequency by means of a General Radio (1192) counter. The mercury relay, used for gating the noise stimuli of Experiment 1, was also removed. When appropriately enabled, the function generator produced tone bursts which started at a zero-crossing and contained an integral number of cycles. The output of the function generator was fed to the Krohn-Hite filter whose low-pass and high-pass sections were now set to have the same cut-off frequency (1000 Hz). As a result, the tone burst envelopes were shaped to have rise- and fall-times of approximately 2.5 msec. The durations of the "10 msec" and "100 msec" tone bursts were 9.4 and 99.4 msec, respectively, between the half-power points.

The two listeners of Experiment 1 also served in this experiment. For observer IG, absolute threshold for 100-msec tone bursts was 4 dB SPL; for observer PJ, threshold was at -6 dB SPL.

Results

Average Weber fractions ($\Delta I/I$ in dB) for the two observers are presented in Table IV. Adjacent rows in the table represent 10 dB changes in stimulus energy. Within each row, for each observer, are data obtained for standard stimuli that were equal in energy but differed in intensity and duration; a 100-msec burst of tone at a given SPL is equal in energy to a 10-msec stimulus which is 10 dB more intense. It is clear from the results that bursts of tone which are equal in energy do not yield equal discriminabilities.

Table IV

Weber Fractions (ΔI/I in dB) for 1000-Hz Tone Bursts

Observer IG				Observer PJ			
T = 100 msec		T = 10 msec		T = 100 msec		T = 10 msec	
I[a]	ΔI/I[b]	I	ΔI/I	I	ΔI/I	I	ΔI/I
30	-3.8	40	+0.4	28	-3.5	38	+0.1
40	-1.3	50	+1.0	38	-4.8	48	-2.7
50	-3.1	60	-1.6	48	-5.6	58	-3.6
60	-4.0	70	-5.0	58	-4.1	68	-5.1
70	-6.8	80	-4.8	68	-7.0	78	-6.7
80	-5.7	90	-7.9	78	-8.1	88	-7.4

a SPL of standard burst (in dB).

b Weber fraction in dB.

Rather, as can be further seen in the Weber functions (Figure 10), differential sensitivity for these stimuli depends on burst duration as well as on the intensity level of the standard stimulus.

The Weber functions indicate that Weber's law does not obtain at either duration. Rather, discriminability improves with intensity level. Furthermore, the rate of improvement is greater for 10-msec tone bursts than for 100-msec tone bursts. Comparison of the two Weber functions, for either listener, shows that discriminability at low intensity levels is better with the longer stimulus bursts. This difference diminishes with level as the two functions approach each other.

The difference in slope between the Weber functions implies that the slopes of the intensity-discrimination functions, where log ΔI is plotted against log I, are also dependent on duration. This can be seen in Figures 11 and 12 and in Table V. The theoretical lines in Figures 11 and 12 have slopes of 1.0 and intercepts equal to the average Weber fractions for the two listeners. It is obvious that the data points in both cases define lines whose slopes are less than one and that the departure from the theoretical line is greater for the 10-msec plots.

Discussion

In one respect, the present findings confirm previous observations: the slopes of the intensity-discrimination functions with tonal stimuli were slightly less than 1.0. That is, the "near-miss" to Weber's law was obtained.

Figure 10

Weber functions for 1000-Hz tone bursts. The parameter is duration: ■—■, T = 100 msec; ▲—▲, T = 10 msec. (a) observer IG, (b) observer PJ.

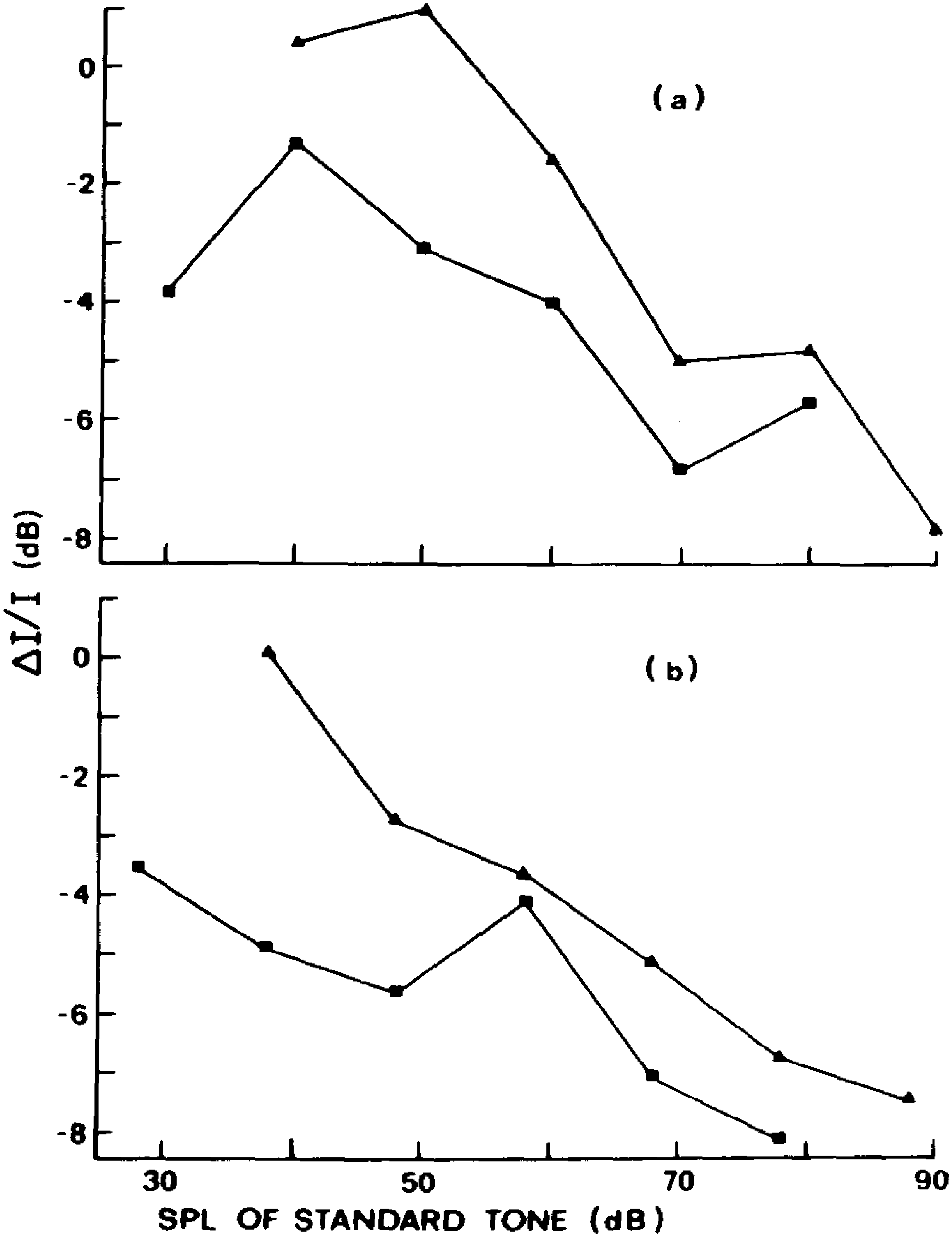

(a)
(b)
0
-2
-4
-6
-8
ΔI/I (dB)
30
50
70
90
SPL OF STANDARD TONE (dB)

Figure 11

Intensity-discrimination function for 100 msec tone bursts (f = 1000 Hz). The line is theoretical and has a slope of 1.0. The intercept is equal to the average Weber fraction (in dB) for the two listeners: IG (■) and PJ (●).

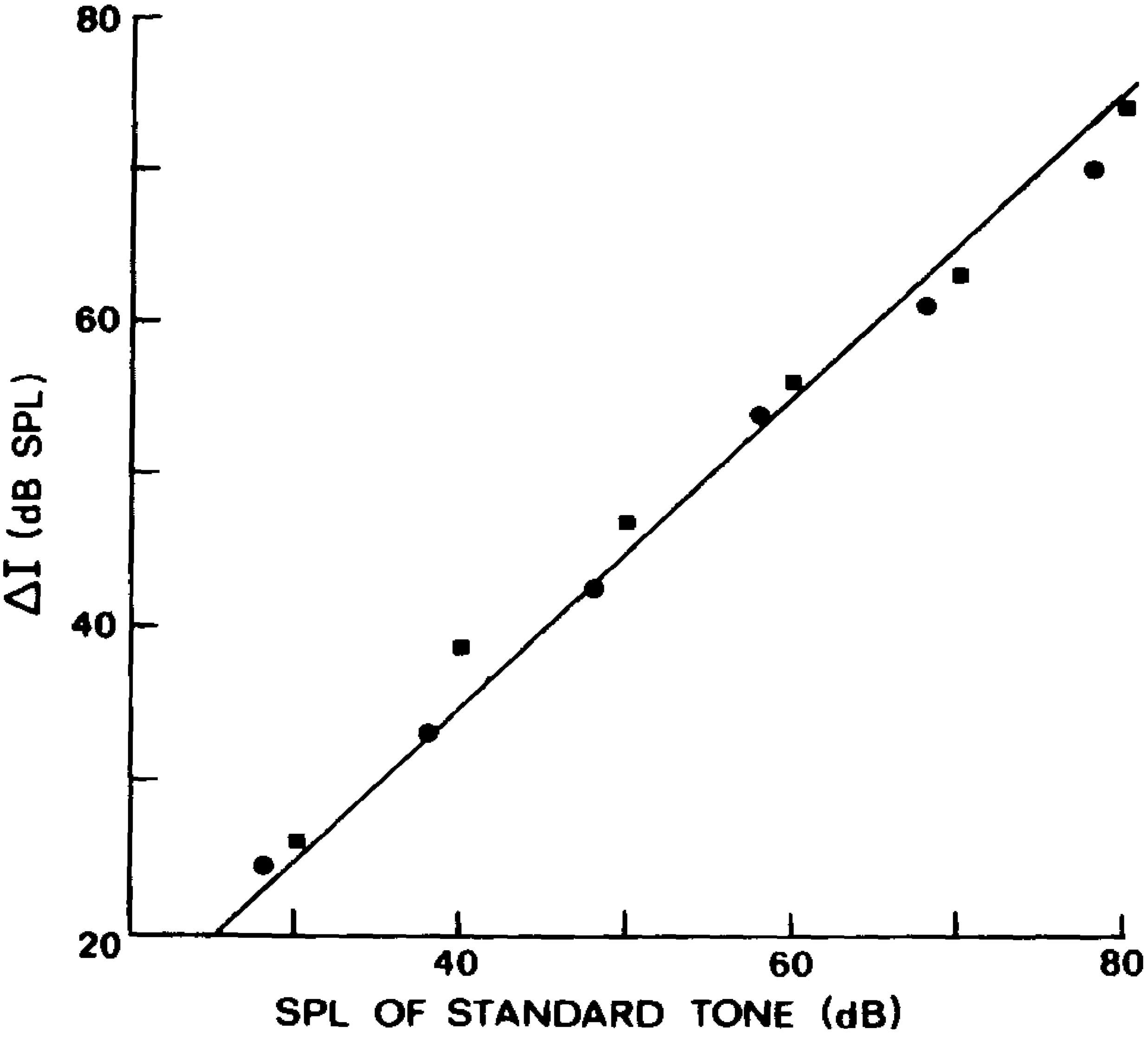

80
60
40
20
ΔI (dB SPL)
40
60
80
SPL OF STANDARD TONE (dB)

Figure 12

Intensity-discrimination function for 10-msec tone bursts (f = 1000 Hz). The line is theoretical and has a slope of 1.0. The intercept is equal to the average Weber fraction (in dB) for the two listeners: IG (■) and PJ (●).

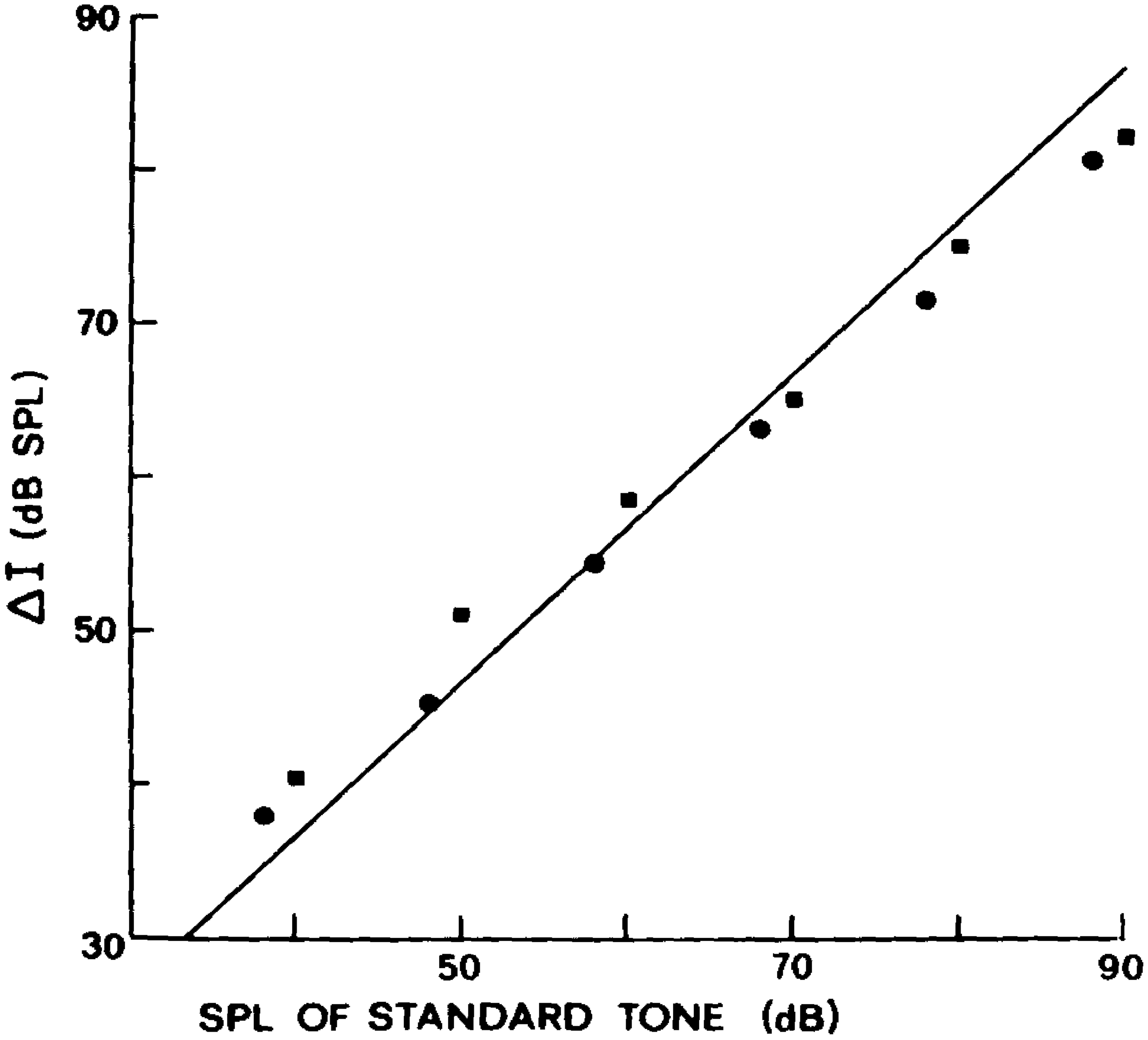

90
70
50
30
ΔI (dB SPL)
50
70
90
SPL OF STANDARD TONE (dB)

Table V

Slopes of Lines Best Fitting the Plots

of Log ΔI against Log I

(1000-Hz Tone Bursts)

Observer	100[a]	10
IG	0.92	0.82
PJ	0.92	0.85

[a] Burst duration in msec.

On the other hand, the results do present two serious problems for the McGill model: (1) The model maps energy onto neural counts. Presumably, equal-energy stimuli evoke equal mean counts and yield equal discriminabilities. The data in Table IV and in Figure 10 show that this is not so. (2) It can be shown (see Appendix I) that the formulation presented by McGill and Goldberg predicts an improvement of approximately 1 dB in the Weber fraction for a 10 dB increase in signal duration. This is also not so: the effect of duration is greater than 1 dB at low intensity levels and is virtually absent at higher levels.

Even when it deals with pure-tone intensity discrimination, therefore, the model requires modification. It is clear that the mapping of acoustic energy onto neural counts is somehow duration-dependent. This point is discussed further in the next chapter.

Chapter V

GENERAL DISCUSSION

The results of Experiments 1 (Chapter III) and 2 (Chapter IV) indicate, respectively, that intensity discrimination with pseudo-random bursts of noise is described by Weber's law and that a "near-miss" to Weber's law describes discrimination with pulsed tones. In the first sections of this chapter, we present a model of auditory neural coding which is based on these findings. Our observations with respect to the effects of duration and bandwidth are explicitly incorporated into the model.

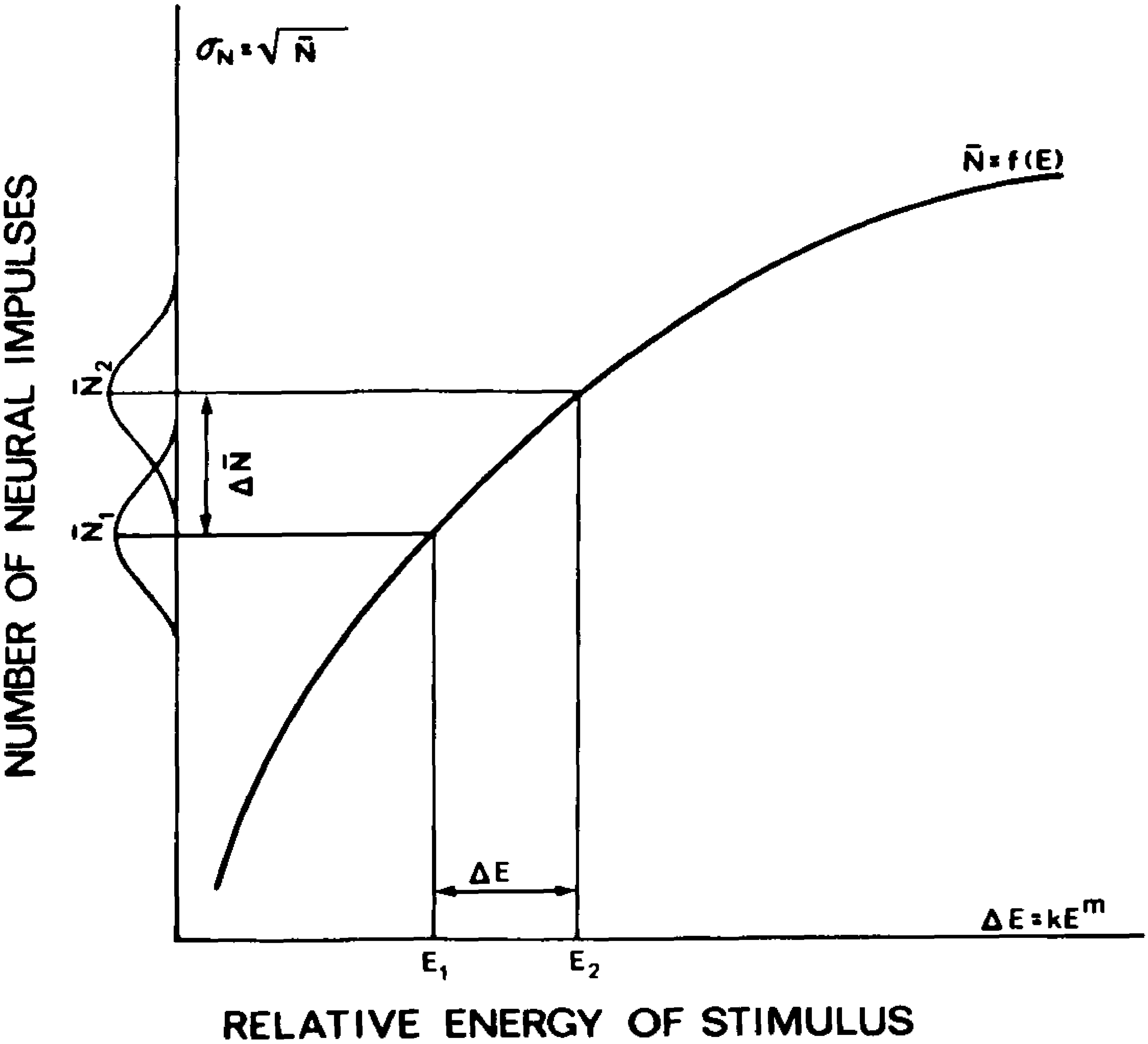
With reproducible noises, these latter effects did not parallel those predicted for real noises by the stimulus-oriented, energy-detector model (Green, 1960a): we did not obtain 5 dB improvements in the Weber fraction associated with tenfold increases in either duration or bandwidth. In a later section of this chapter, we present a third experiment whose results further indicate that the relationship between stimulus fluctuations and internal noise is not a simple additive one.

Neural Counting Model

The model, whose principal features are portrayed in Figure 13, is in the tradition of McGill (see pp. 18ff) and is concerned with the mapping of stimulus energies onto neural effects (neural impulses).

Figure 13

Neural counting model of energy discrimination


$\sigma_N = \sqrt{\bar{N}}$
$\bar{N} = f(E)$
NUMBER OF NEURAL IMPULSES
$\bar{N}_2$
$\bar{N}_1$
$\Delta\bar{N}$
ΔE
$\Delta E = kE^m$
E_1
E_2
RELATIVE ENERGY OF STIMULUS

A stimulus of fixed energy E_1 evokes -- on a single presentation -- a number of neural impulses. The number of impulses evoked by repeated presentations of E_1 is assumed to be distributed in Poisson fashion with mean equal to $\overline{N}_1$ and standard deviation equal to $\sqrt{\overline{N}_1}$. This "Poisson assumption" is based on "mass-flow" considerations (McGill, 1967; McGill & Goldberg, 1968a, 1968b; Perkel & Bullock, 1969; Siebert, 1968). In similar fashion, a stimulus of greater energy (E_2) evokes a second Poisson distribution of neural impulses. A mapping function describes the relationship between E and $\overline{N}$.

In a forced-choice discrimination trial, the observer counts impulses in each interval and selects the interval with the greater number of impulses as the one containing the increment. Thus, the probability of a correct response is the probability that $(N_2 - N_1)$ is greater than zero. This probability can be determined from the distribution of differences. Since the difference between two Poisson variables is not Poisson, we follow Mueller (1950) in assuming that $\overline{N}_1$ is always greater than five. As a result, both count distributions, as well as the difference distribution, are approximately normal. We assume that the standard deviations of the two distributions are equal in the region defined by the psychometric function. It follows that the probability that $(N_2 - N_1)$ is greater than zero can be determined from the magnitude of $\overline{N}_2 - \overline{N}_1$ ($= \Delta\overline{N}$) in standard deviation units;

$$d' = \frac{\Delta\overline{N}}{\sqrt{\overline{N}_1}} \quad . \tag{8a}$$

At a level of approximately 75% correct discriminations ($d' = 1$),

$$\Delta\bar{N} = \sqrt{\bar{N}_1} \quad . \qquad (8b)$$

With standard stimulus E_1 evoking $\bar{N}_1$ impulses on the average, $\Delta\bar{N}$ is specified and can be reflected onto the abscissa (via the mapping function) to give ΔE ($= E_2 - E_1$).

We now proceed to define the mapping function on the basis of our results. Following the approach of McGill and Goldberg, the discriminability data in both Experiments 1 and 2 are described by a modified form of the Weber equation:

$$\frac{\Delta E}{E_1^{\,m}} = k, \qquad (9)$$

where E_1 (and ΔE) are arbitrarily normalized to the energy at absolute threshold and the value of the "Weber constant", k, depends on stimulus parameters. With reproducible noise bursts, Weber's law obtains and m, in equation (9), is equal to 1. With tonal stimuli, m is less than 1.

Since ΔE is associated with $\Delta\bar{N}$, equation (8b) and equation (9) can be considered jointly. Passing from differences to differentials (see Appendix II), we have

$$\frac{d\bar{N}}{\sqrt{\bar{N}}} = \frac{dE}{kE^m}, \qquad m < 1. \qquad (10)\ (II\text{-}3)$$

In Appendix II, we show two solutions of equation (10). These depend upon the parameter m:

(a) When $m < 1$,

$$\bar{N} = bE^{c}, \qquad c > 0. \qquad (11)\ (II\text{-}5)$$

(b) When $m = 1$,

$$\bar{N} = h(\log E)^{2}. \qquad (12)\ (II\text{-}8)$$

Equations (11) and (12) define two families of mapping functions: the power law for tones and the log-square for reproducible noises. Again, in both equations, stimulus energies (E) are arbitrarily normalized to the energy at absolute threshold. It is important to note that initial conditions which are specified by the constants of integration are not the same in the two mapping equations. This matter is treated in Appendix II. Finally, intensity discrimination near absolute threshold is not considered.

Discrimination With Pulsed Tones

McGill and Goldberg suggest (see pp. 19 f, above) that the mapping of stimulus energy onto neural counts is via the power law

$$\bar{N} = bE^{c}, \qquad 0 < c \leq 1, \qquad (5)$$

where c can be reclaimed from the slope of the intensity-discrimination function. The proportionality constant b is unspecified. The present model generates an identical law for pure-tone intensity discrimination (see equation 11). However, in the present case both parameters, the proportionality constant b and the mapping exponent c, can be reclaimed empirically. Specifically, (see Appendix II):

$$c = 2(1 - m), \qquad (13)\ (7)\ (\text{II-6})$$

where m is the slope of the line that best fits the plots of log ΔE against log E; and

$$b = \frac{1}{c^2 k^2}, \qquad (14)\ (\text{II-7})$$

where

$$k = \frac{\Delta E}{E_1^{\,m}} . \qquad (9)$$

Matters would be simple if the effect of duration which is predicted by equation (9) (see Appendix I) were borne out in the data. As we have seen, however, (p. 67) the Weber functions in Figure 10 are sufficiently different that they cannot be fitted by equation 9 unless m and k are permitted to be different at the two durations.

Table VI contains, therefore, seperate values of m and k for the two observers at durations of 10 and 100 msec. Vaules of m are taken from Table V. Each value of k is the mean ($\Delta E/E^m$) calculated from the data in Table IV. Each mean is based on six Weber fractions determined over a range of 50 dB. Mapping functions are given in the last column of Table VI. Clearly, they are dependent on duration; c must decrease and b increase, as duration is lengthened from 10 to 100 msec. Segments of the functions for observer IG are plotted in Figure 14.

Table VI

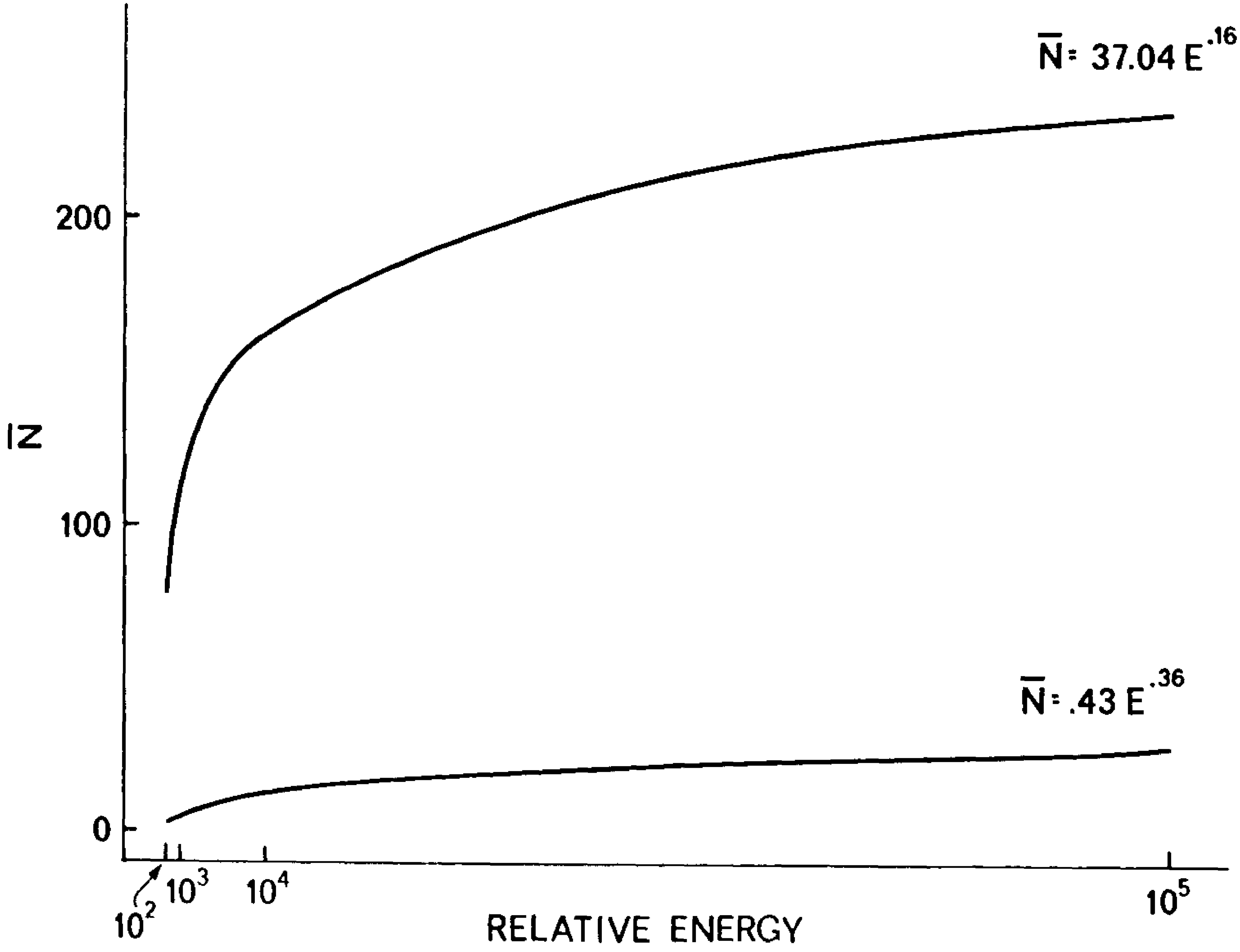
Values of m and k (Empirically Determined) and

Estimates of c and b (from Equations 13 and 14)

Duration	Observer	m	k	c	b	Equation (11)
10 msec	IG	0.82	4.26	0.36	0.43	$\bar{N} = .43\ E^{.36}$
	PJ	0.85	2.91	0.30	1.31	$\bar{N} = 1.31\ E^{.30}$
100 msec	IG	0.92	1.03	0.16	37.04	$\bar{N} = 37.04\ E^{.16}$
	PJ	0.92	0.85	0.16	54.35	$\bar{N} = 54.35\ E^{.16}$

Figure 14

Mapping functions for bursts of 1000-Hz tone. The curves are based on psychophysical data from observer IG. Upper curve, T = 100 msec; lower curve, T = 10 msec.


$\bar{N} = 37.04\,E^{.16}$
$\bar{N} = .43\,E^{.36}$
$\bar{N}$
200
100
0
10^2
10^3
10^4
10^5
RELATIVE ENERGY

In Table VII, we present estimated mean numbers of counts ($\overline{N}_1$) evoked by standard tones at the sensation levels employed in Experiment 2. Estimated values of $\overline{N}_2$ at discrimination threshold are also given. These results are calculated from the mapping functions (see Table VI). At all levels, 100-msec bursts of tone yield more neural impulses than do 10-msec bursts of equal energy. The table displays the "volume compression" implied by the mapping exponent in equation (11) (see McGill and Goldberg, 1968a). With stimulus duration equal to 100-msec, 5 log units of stimulus energy are compressed into 0.8 log units of impulses. It is clear that there is less compression for 10-msec tone bursts; the same energy range yields 1.8 log units of counts for one observer (IG) and 1.5 for the other (PJ).

The model generates Weber functions which converge and eventually cross at high stimulus levels -- much the same as are seen in the data. The mapping functions do not themselves have to intersect to yield this result. Discriminability for a given E_1, depends on the mean number of evoked counts <u>and</u> on the slope of the mapping function. Equal discriminabilities at the two durations, do not require equal $\overline{N}$s. Weber fractions are given by the interaction of $\Delta\overline{N}$ ($=\sqrt{\overline{N}_1}$) and the slope in the region of $\overline{N}_1$.

It should be noted that mean neural counts in Table VII are generated by values of E which are arbitrarily referred to absolute threshold. Stimulus energies are not given in physical units. That is to say, equation (11), like the McGill equation, does not provide a direct

Table VII

Estimates of $\bar{N}_1$, $\bar{N}_2$ and $\Delta E/E_1$ (dB) for Pulsed Tones

Duration	Observer IG				Observer PJ			
	E_1 (dB SL)	$\bar{N}_1$	$\bar{N}_2$	$\Delta E/E_1$ (dB)	E_1 (dB SL)	$\bar{N}_1$	$\bar{N}_2$	$\Delta E/E_1$ (dB)
10 msec	26	3.7	5.6	+3.4	34	13.7	17.4	+0.8
	36	8.4	11.3	+1.0	44	27.4	32.6	-1.1
	46	19.2	23.6	-1.2	54	54.7	62.1	-2.9
	56	44.1	50.7	-3.3	64	109	119	-4.6
	66	101	111	-5.2	74	218	233	-6.2
	76	231	246	-7.2	84	434	455	-7.7
100 msec	26	96.4	106	-0.8	34	190	204	-2.8
	36	139	151	-2.0	44	275	292	-3.7
	46	201	215	-3.0	54	397	417	-4.6
	56	291	308	-3.7	64	574	598	-5.3
	66	421	442	-4.6	74	830	859	-6.1
	76	608	633	-5.5	84	1200	1235	-6.9

transformation from ergs, for example, to mean counts. A rearrangement of equation (11) which is instructive, is as follows (see also Appendix II):

$$\bar{N} = \frac{1}{4(1 - m)^2} \left(\frac{\Delta E}{E}\right)^{-2} . \qquad \text{(15) (II-4)}$$

Here, too, mean counts are not given by E measured in ergs; i.e., both sides of equation (15) are dimensionless. Instead, the independent variable may be regarded as E normalized to ΔE.

Equation (15) summarizes the operation of the neural counting model for pure-tone intensity discrimination. Given m, the slope of the intensity discrimination function, and given a Weber fraction $\Delta E/E$, the mean number of counts evoked by E is specified.

Discrimination With Reproducible Bursts of Noise

In McGill and Goldberg's model, the finding of Weber's law is treated as a degenerate case where $c = 0$. In contrast, the present model generates a log-square mapping law for cases where m equals 1:

$$\bar{N} = h\ (\log E)^2 . \qquad \text{(12) (II-8)}$$

In Appendix II, we show that h can be reclaimed from the Weber constant k. Specifically,

$$h = 1.32/k^2 \qquad \text{(16) (II-9)}$$

The results of Experiment 1 indicate: (1) the Weber constant (k) depends on noise-burst duration. (2) To a first approximation, k does not depend on bandwidth. Thus, in terms of the model, noise bursts map onto mean counts via the same mapping function (i.e., h does not change) when bandwidth and intensity are traded, or when either alone is changed. However, when stimuli differ in duration -- even if they be equal in energy -- they are mapped onto $\bar{N}$ via two distinct mapping functions (which differ in h). Segments of the function for observer IG are plotted in Figure 15.

Table VIII contains estimates of the mapping constant (equation 16) for the two observers at two durations. Results generated by the model are presented in Table IX. As it should, the theory generates Weber's law, with better discriminability for 100-msec stimuli than for 10-msec stimuli.

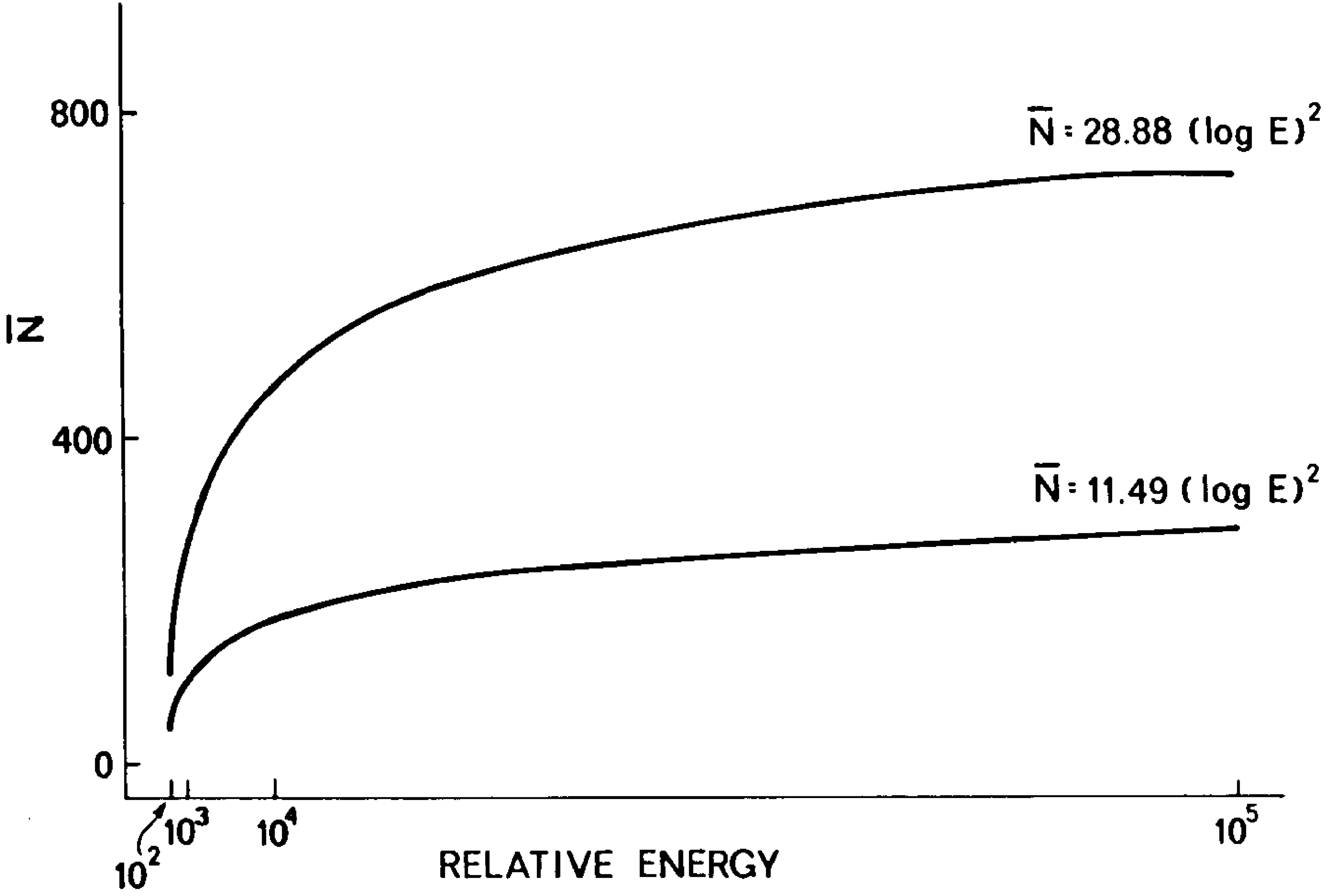
Mean neural counts in Table IX are generated by values of E which are arbitrarily referred to absolute threshold. If a lower reference level were used, the estimated mean at each SL would be correspondingly larger. It is clear that the mean counts evoked by noise stimuli are given only relatively.

Ten-millisecond stimuli consistently map onto lower mean neural counts than do 100-msec stimuli that are of equal energy. Furthermore, at each intensity level, 100-msec stimuli map onto higher mean neural counts than do 10-msec stimuli. In both cases, changes in discriminability result from the shift from one mapping function to the other.

As with tones, the neural coding of noise energy exhibits compression. Neural counts increase by 22 dB as energy is raised from threshold to 120 dB SL. The log-square formulation in equation (12) shows that this is independent of duration.

Figure 15

Mapping functions for reproducible bursts of noise. The curves are based on psychophysical data from observer IG. W = 5000Hz. Upper curve, T = 100 msec; lower curve, T = 10 msec.



800
400
0
IZ
$\bar{N} = 28.88\ (\log E)^2$
$\bar{N} = 11.49\ (\log E)^2$
10^2
10^3
10^4
10^5
RELATIVE ENERGY

Table VIII

Estimates of the Mapping Constant

$$h = 1.32/k^2$$

Duration	Observer	k^*	h
10 msec	IG	0.34	11.49
	PJ	0.50	5.26
100 msec	IG	0.21	28.88
	PJ	0.30	14.47

* Values of k are mean $\Delta I/I$ obtained with broad-band stimuli (see Tables I and II).

Table IX

Estimates of $\bar{N}_1$, $\bar{N}_2$ and $\Delta E/E_1$ (dB) for Reproducible Noise Bursts

Duration	E_1 (dB SL)	Observer IG			Observer PJ		
		$\bar{N}_1$	$\bar{N}_2$	$\Delta E/E_1$ (dB)	$\bar{N}_1$	$\bar{N}_2$	$\Delta E/E_1$ (dB)
10 msec	20	46	53	-4.0	21	26	-2.0
	30	103	114	-4.0	47	54	-2.0
	40	184	197	-4.0	84	93	-2.0
	50	287	304	-4.0	132	143	-2.0
	60	414	434	-4.0	189	203	-2.0
	70	563	587	-4.0	258	274	-2.0
100 msec	20	116	126	-6.4	58	65	-4.5
	30	260	276	-6.4	130	142	-4.5
	40	462	484	-6.4	232	247	-4.5
	50	722	749	-6.4	362	381	-4.5
	60	1040	1072	-6.4	521	544	-4.5
	70	1415	1453	-6.4	709	736	-4.5

Justification of the Model

Whether auditory coding mechanism really behave in the manner specified by our model remains an open question. While there is statistical justification for the "Poisson-assumption" concerning fluctuations in neural coding, and while there are many studies of firing patterns in single auditory units, there is no direct physiological evidence concerning the mapping of energy onto counts in the whole auditory nerve.

The single unit data collected by Kiang and his associates (1965) yield results which agree, in a general way, with our notions about the effects of stimulus duration and intensity level on number of impulses. These investigators present post-stimulus-time histograms which indicate that the rate of discharge in individual eighth-nerve fibers peaks during the first 10 msec of stimulation and then levels off. For both tones and noises, the finding is that the rate of firing is nearly constant after the first 10 msec. The height of the initial peak increases with level of stimulation. It would appear from their findings (see, for example, their Figures 6.2 and 6.3), however, that the increase in number of impulses is greater when duration is extended from 10 msec to 100 msec than when intensity is increased by 10 dB.

These data are for coding in single units. To describe discriminability, information about neural firings in the entire auditory nerve is needed; it is highly unlikely that all elements in the nerve contribute equally to the total evoked discharge. We have already mentioned the "auditory nerve" reconstructed by Siebert from the data of Kiang et al.

(see Chapter I). Unfortunately, for our purposes, predictions with respect to the effects of duration on discriminability have not been formulated.

Siebert (1965, 1968), it should be noted, includes spontaneous activity in his analysis of eighth-nerve firings. We, like McGill, have elected to ignore background counts, We note, however, that each mapping equation includes a constant of integration which specifies initial conditions, i.e., the number of counts at absolute threshold ($E = 1$). These constants do not appear in equations (11) and (12); for the sake of simplicity, they were both *independently* set to zero (see Appendix II).

Reference to equation (12), which maps noise energies, shows that, when E equals one, $\bar{N}$ equals zero; threshold noise bursts are considered to evoke no impulses. For the mapping of tonal energies, on the other hand, setting the constant of integration to zero yields a different initial condition: specifically, when E equals one in equation (11), $\bar{N}$ equals b ($= 1/(\Delta E)^2 C^2$. Values of b are given in Table VI. With 10-msec tones at threshold, $\bar{N}$ is only about one count. At 100 msec, the mean number of background counts is about 50. The constants of integration are obviously parameters which can be adjusted. Lacking an estimate of spontaneous activity in the whole nerve, we deal with the mapping equations in their simplest form.

The model we have described is based on psychophysical data obtained with reproducible stimuli. It is not within the scope of this thesis to consider -- as did McGill (1967) -- the behavior of the model when varying stimuli are employed. This remains a problem for the future.

On the Addition of Internal and Stimulus Variabilities

Stimulus-oriented theories characteristically assert that discrepancies between optimal behavior and the behavior of real observers can be attributed to "internal noise". "Attenuation factors" are often introduced to account for these differences. For example, Green (1960a) has applied the energy-detector model to the detection of Gaussian-noise signals against noise backgrounds. He found, as predicted by the model, that tenfold increases in either duration or bandwidth yield half log-unit improvements in the Weber fraction. Since the discrepancy between real and ideal behavior was consistently 5 to 6 dB, Green was able to reconcile the two by means of an attenuation constant.

As we have noted earlier (see p. 13), an attenuation constant of 6 dB implies that there is three times as much internal noise as there is stimulus noise -- at all intensities, durations and bandwidths. Why the larger neural variabilities "follow" the smaller stimulus fluctuations so that they mirror the latter with respect to intensity, duration and bandwidth, is not altogether clear.

In Experiment 1, where stimulus fluctuations were absent, the Weber fractions improved by only 2 to 3 dB when noise-burst durations were increased from 10 to 100 msec. Tenfold increases in bandwidth, furthermore, resulted in virtually no improvement in discrimination. These two findings imply different things about neural fluctuations than are implied by Green's results: i.e., that internal noise varies only slightly with duration and hardly at all with bandwidth. Since Green's results were obtained with continuous rather than gated backgrounds, we conducted a third experiment in which gated bursts of real noise were employed.

The apparatus and procedures were identical to those described in Chapter II (see Figure 2). In this experiment, however, the output sequence-length of the Hewlett Packard 3722A noise generator was set to "infinite." As a result, bursts of real noise were generated by the apparatus and the noise waveforms varied within trials as well as between trials.

Weber fractions ($\Delta I/I$ in dB) for both real (random) and reproducible (pseudo-random) noise bursts are presented in Table X. The Weber fractions for the reproducible stimuli are from Experiment 1. Two levels of I were used, and the effects of tenfold changes in duration and bandwidth are presented. Each Weber fraction in the table is the mean of at least six determinations of the DL.

In general, discriminability with real noises is poorer than that with reproducible noises. This result is not surprising; in the

Table X

Weber Fractions (ΔI/I in dB) Obtained with Real and with Reproducible Noise Bursts

		Real Noise			Reproducible Noise		
I[a]	Duration	500[b]	5000	Difference[c]	500	5000	Difference
				Observer IG			
	10 msec	-1.4	-3.2	1.8	-3.2	-5.4	2.2
13	100 msec	-4.7	-6.9	2.2	-6.6	-7.4	0.8
	Difference[d]	3.3	3.7		3.4	2.0	
	10 msec	-1.7	-1.6	-0.1	-4.8	-3.5	-1.3
33	100 msec	-3.1	-6.2	3.1	-6.4	-5.9	-0.5
	Difference	1.4	4.6		1.6	2.4	
				Observer PJ			
	10 msec	-0.6	-2.1	1.5	-0.8	-3.6	2.8
11	100 msec	-4.9	-6.2	1.3	-5.4	-5.6	0.2
	Difference	4.3	4.1		4.6	2.0	
	10 msec	-0.6	-1.2	0.6	-1.3	-1.8	0.5
31	100 msec	-5.5	-3.4	-2.1	-5.3	-3.8	-1.5
	Difference	4.9	2.2		4.0	2.0	
				Means Across Observers and Levels			
	10 msec	-1.1	-2.0	0.9	-2.5	-3.6	1.1
	100 msec	-4.6	-5.7	1.1	-5.9	-5.7	-0.2
	Difference	3.5	3.7		3.4	2.1	

a Spectrum level of standard burst

b Bandwidth in Hz

c Weber fraction (in dB) at W = 500 Hz minus Weber fraction at W = 5000 Hz

d Weber fraction (in dB) at T = 10 msec minus Weber fraction at T = 100 msec former case there is more variability than in the latter. Furthermore, it is obvious throughout Table X that the effects of removing stimulus fluctuations are related to the relative amount of variability removed. For example: (1) For real noises with $W = 5000$ Hz and $T = 100$ msec, relative stimulus variability -- $\sigma/M \approx 1/(2WT)^{\frac{1}{2}}$, see p. 13 -- is only about 3%. The summary data at the bottom of Table X show most clearly that discriminability does not change when this small amount of variability is removed. (2) In contrast, the sigma-to-mean ratio for the short-duration, narrow-band, noises is approximately 30%. For these stimuli, discriminability improves by roughly 1.5dB when reproducible noises are employed.

Our results with real noises confirm the findings of Experiment 1 with respect to the effects of duration and bandwidth on internal noise. The data differ from those predicted and obtained by Green; tenfold increases in either duration or bandwidth did not yield 5 dB improvements in the Weber fraction. The summary data in the lower left of Table X refer to the real noise stimuli. They show that increasing burst duration by a log unit results in an average improvement of approximately 3.5 dB. Increasing bandwidth by a log unit yields an average improvement of only 1 dB. When both duration and bandwidth were increased, discriminabilty improved by only 4.5 dB. This is much less than the 10 dB improvement suggested by Green. Our results show that internal noise is independent of bandwidth and only somewhat dependent on duration.

These assertions about internal fluctuations follow from what are two independent studies. Results with reproducible noises (Experiment 1) and with real noises (Experiment 3) provide comparable estimates of internal variability. That is the amounts of internal noise needed to reconcile the differences between real and optimal observers in the latter experiment approximate the estimates of internal variability obtained in the first experiment. For example (see Table XI), consider the data for observer PJ which were obtained with noise bursts whose spectrum level equalled 31 dB, and whose bandwidth and duration were 5000 Hz and 10 msec, respectively. With real noises, $\Delta I/I$ equalled 0.76. Subtracting 0.10 $\left[= 1/(2WT)^{\frac{1}{2}}\right]$

real noises, $\Delta I/I$ equalled 0.76. Subtracting 0.10 $\left[= 1/(2WT)^{\frac{1}{2}}\right]$

which exactly matches the Weber fraction obtained with pseudo-random stimuli. Table XI, which contains all such comparisons, indicates that arithmetic subtraction of stimulus fluctuations -- where they do exist -- yields estimates of internal noise which are similar to those obtained with stimuli from which energy fluctuations have been physically removed.

The results in Table XI allow us to estimate the proportion of total noise (internal plus stimulus-produced) that is internal noise. These proportions were calculated after combining the grand means at the bottom of Table XI. They are presented in Table XII. It is clear that the relative contribution of internal noise to total noise depends

Table XI

Estimates of Internal Variability Obtained with Real and Reproducible Noise Bursts

		Real Noise		Reproducible Noise	
		$\sigma/M = (\Delta I/I) - 1/(2WT)^{\frac{1}{2}}$		$\sigma/M = \Delta I/I$	
I^a	T^b	500^c	5000^c	500^c	5000^c
		Observer IG			
13	10	0.40	0.38	0.48	0.29
	100	0.24	0.17	0.22	0.18
33	10	0.36	0.59	0.33	0.45
	100	0.39	0.21	0.23	0.26
		Observer PJ			
11	10	0.55	0.52	0.83	0.44
	100	0.22	0.21	0.29	0.28
31	10	0.55	0.66	0.74	0.66
	100	0.18	0.43	0.30	0.42
		Means Across Observers and Levels			
	10	0.46	0.54	0.59	0.46
	100	0.26	0.26	0.26	0.28

a Spectrum level of standard burst

b Duration in msec

c Bandwidth in Hz

Table XII

Proportion of Total Noise That Is Internal Noise

Duration	Bandwidth	
	500 Hz	5000 Hz
10 msec	0.62	0.83
100 msec	0.72	0.90

on stimulus conditions. Neural fluctuations account for almost all of the variability with broad-band, long-duration stimuli and for roughly 60% of the variability with the narrow-band, short-duration stimuli. Once again, we see why it is that tenfold changes in stimulus bandwidth and duration do not produce half log-unit changes in discrimination.

In making the above comparisons and calculations, we assumed that the listener behaves "in the same way" (with respect to strategy, criterion, etc.) when listening to random and to pseudo-random noises. This assumption was tested in the following experiment with observer PJ. Three listening conditions were employed: (1) Blocks of 100 trials in which 50 trials with real noises were randomly interspersed with 50 trials containing reproducible noises. (2) Blocks of 100 trials with reproducible noises only. (3) Blocks of 100 trials with real noises.

The results are given in Table XIII. Each entry in the table is bases on 200 trials with 5000-HZ, 10-msec bursts at 31 DB spectrum level. The increment (ΔI) was fixed to yield approximately 75% correct responses for real noises. Once again, discriminability is better for reproducible stimuli than for real noises. More important is the fact that the proportion of correct responses, for each of the two types of stimuli, is independent of the method of presentation. It is clear that the observer does not adopt different strategies, etc., when listening to the two kinds of noise.

Table XIII

Effect of Method of Presentation on Proportion of Correct Responses for Real and Reproducible Noise Bursts

Method of Presentation	Real	Reproducible
Interspersed	0.73	0.83
Separate	0.74	0.82

Pulsed Versus Continous Backgrounds

In Table X we show that discriminability with real noises improves by approximately 3.5 dB when the durations of the bursts are increased by a log unit, and that the improvement is approximately 1 dB when noise bandwidth is increased by a decade. These results are at odds with the 5 dB improvements reported by Green (1960a) for both cases. It is important to note that there are two procedural differences between the present study and that of Green: In our experiment, the background noises were pulsed for the durations of the observation intervals and the bandwidths of the background waveforms equalled those of the signal. In contrast, Green's noise signals were presented against backgrounds that were continuous and broadband. Unpublished data recently collected in this laboratory by C. Berner, confirm the duration-and bandwidth-effects of Experiment 3, and show that matters are different when signal and background are not matched with respect to duration and bandwidth.

Concluding Remarks

We have made a number of statements concerning the nature of internal noise -- suggesting both what it is and what it is not. In the immediately preceding sections of this chapter (see pp.88ff), we have attacked the notion that real listeners are like ideal mechanisms to which a second source of noise is added. We found that matters were not simplified by considering internal noise as if it were additional stimulus noise; problems arose when we came to specify relative variabilities contributed by the two.

The only simple relationship we encountered was the fact of Weber's law with noise stimuli. Here, internal variability parallels stimulus noise. On the other hand, matters were different in all of the other situations studied in this thesis:

1. Internal variability is not as dependent on noise-burst duration as is stimulus variability.
2. Internal variability is even less dependent (hardly at all) on bandwidth than is stimulus variability.
3. The "near-miss" to Weber's law observed with tones, implies -- for these stimuli -- that internal variability is not proportional to stimulus level.
4. The "nearness of the miss" is a function of duration. This complicates matters even more.

All of these complications, however, can be incorporated into a neural counting model in which Poisson-distributed neural events are combined with non-linear mapping functions. This model, which was described at the beginning of this chapter (see pp.68ff), is also an "ideal" mechanism. It seems, however, to have more heuristic value. How "realistic" it is, remains a problem for the future.

Appendix I

DERIVATIONS FROM McGILL'S COUNTING MODEL

McGill and Goldberg (1968a) demonstrate that the exponent of the mapping function

$$\bar{N} = bE^{c}, \qquad 0 < c \leqslant 1, \qquad \text{(I-1)}$$

can be reclaimed from the slope of the intensity-discrimination function

$$\log \Delta I = m \log I + \log k.$$

Specifically, they show that

$$c = 2(1 - m).$$

An alternate proof is as follows: Consider two Poisson distributions of counts evoked by standard energy E_1 and by an incremented stimulus energy E_2 ($= E_1 + \Delta E$), with means $\bar{N}_1$ and $\bar{N}_2$ ($= \bar{N}_1 + \Delta \bar{N}$), respectively. When $\bar{N}_1$ is greater than 5, the Poisson distributions are approximately normal. If we assume that the two distributions have equal variances, then when d' equals 1 (probability of a correct response in 2AFC equals approximately 0.75),

$$\bar{N}_2 = \bar{N}_1 + \sqrt{\bar{N}_1} ,$$

since the variance of a Poisson distribution equals the mean.

Let

$$\bar{N}_1 = bE_1{}^{2(1 - m)} \qquad \text{(I-1a)}$$

and

$$\bar{N}_1 + \sqrt{\bar{N}_1} = bE_2{}^{2(1 - m)} . \qquad \text{(I-1b)}$$

Dividing,

$$\left(\frac{E_2}{E_1}\right)^{2(1 - m)} = 1 + \frac{\sqrt{\bar{N}_1}}{\bar{N}_1} = 1 + \frac{1}{\sqrt{\bar{N}_1}} .$$

Taking the square root,

$$\left(\frac{E_2}{E_1}\right)^{1 - m} = \left(1 + \frac{1}{\sqrt{\overline{N}_1}}\right)^{\frac{1}{2}} ,$$

and taking logarithms

$$(1 - m) \log \left(\frac{E_2}{E_1}\right) = \frac{1}{2} \log \left(1 + \frac{1}{\sqrt{\overline{N}_1}}\right) .$$

Since

$$\frac{E_2}{E_1} = \frac{E_1 + \Delta E}{E_1} = 1 + \frac{\Delta E}{E_1} ,$$

$$(1 - m) \log \left(1 + \frac{\Delta E}{E_1}\right) = \frac{1}{2} \log \left(1 + \frac{1}{\sqrt{\overline{N}_1}}\right) .$$

The approximation

$$\log (1 + x) \approx x , \qquad x \ll 1,$$

can now be applied.

Thus,

$$(1 - m) \frac{\Delta E}{E_1} \approx \frac{1}{2\sqrt{\overline{N}_1}} .$$

From equation (I-1a),

$$\sqrt{\overline{N}_1} = b^{\frac{1}{2}} E_1^{1 - m} .$$

Substituting,

$$(1 - m) \frac{\Delta E}{E_1} \approx \frac{1}{2b^{\frac{1}{2}} E_1^{1 - m}} .$$

Thus,

$$\Delta E \approx \frac{E_1}{2b^{\frac{1}{2}}\ (1 - m)\ E_1^{\,1 - m}} \ ,$$

$$\Delta E \approx \frac{E_1^{\,m}}{2b^{\frac{1}{2}}\ (1 - m)} \ .$$

Taking logs,

$$\log \ \Delta E \approx m \log E_1 + \log k, \qquad \text{(I-2a)}$$

where k absorbs all remaining constants.

The effects of changing stimulus duration can now be demonstrated. Transposing and taking antilogarithms,

$$k = \frac{\Delta E}{E_1^{\,m}} \ . \qquad \text{(I-2b)}$$

Since,

$$\Delta E = \Delta I \cdot T \quad \text{and} \quad E_1^{\,m} = I_1^{\,m} \cdot T^m \ ,$$

$$k = \frac{\Delta I \cdot T}{I_1^{\,m} \cdot T^m} \ .$$

Thus,

$$k = \frac{\Delta I \cdot T^{1 - m}}{I_1^{\,m}} \ . \qquad \text{(I-3)}$$

The intensity-duration reciprocity is specified by equation (I-3). When, for example, $m = 0.9$, $(1 - m) = 0.1$, and

$$k = \frac{\Delta I \cdot T^{0.1}}{I_1^{\,0.9}} \ .$$

Thus, for fixed I_1, a decade increase in T results in a 1 dB decrease in ΔI.

Appendix II

DERIVATION OF MAPPING FUNCTIONS

Given: (1) That discrimination of energy differences is described by

$$d'\cdot k = \frac{\Delta E}{E^m}, \qquad E \geqslant 1, \qquad \text{(II-1)}$$

where E is stimulus energy relative to absolute threshold and k is the "Weber ratio."

(2) The energy increment ΔE is associated with the increment in mean neural effect, $\Delta\bar{N}$, whose relative magnitude

$$\frac{\Delta\bar{N}}{\sigma_N} = \frac{\Delta\bar{N}}{\sqrt{\bar{N}}} = d' \qquad \text{(II-2)}$$

determines discriminability.

We recall (see p. 71) the "Poisson assumption" concerning the distributions of neural counts and the approximation of the Poisson distributions by normal density functions.

We wish to derive the function $\bar{N} = f(E)$ relating stimulus energy, E, to mean evoked impulses, $\bar{N}$. If we assume $\bar{N} = f(E)$ to be monotonic, continuous and differentiable, we may combine equations (II-1) and (II-2) and pass from differences to differentials by letting d' approach zero. This yields a differential equation

$$\frac{d\bar{N}}{\sqrt{\bar{N}}} = \frac{dE}{kE^m} \qquad \text{(II-3)}$$

whose solution takes one of two forms, depending on the value of m.

When $m \neq 1$

$$2\bar{N}^{\frac{1}{2}} = \frac{1}{k}\,\frac{E^{1-m}}{(1-m)} + C$$

$$\bar{N} = \left[\frac{1}{2k}\,\frac{E^{1-m}}{(1-m)} + \frac{C}{2}\right]^{2}$$

When $m = 1$

$$2\bar{N}^{\frac{1}{2}} = \frac{1}{k}\ln E + C$$

$$\bar{N} = \left(\frac{1}{2k}\ln E + \frac{C}{2}\right)^{2}$$

In both solutions, the constants of integration, C, are arbitrarily set equal to zero. As a result, the initial conditions (background counts) are not the same for the two solutions. This is justifiable if we consider the two mapping situations to be sufficiently distinct that direct comparisons of the variables (Es and $\bar{N}$s) cannot be made.

$$\bar{N} = \frac{E^{2(1-m)}}{k^{2}\left(2(1-m)\right)^{2}}$$

$$= \frac{1}{4(1-m)^{2}}\left(\frac{\Delta E}{E}\right)^{-2}, \qquad \text{(II-4)}$$

since $k = \Delta E/E^{m}$.

Alternately,

$$\bar{N} = b\,E^{c}, \qquad \text{(II-5)}$$

where $c = 2(1-m)$ (II-6)

and $b = \frac{1}{k^{2}c^{2}}$. (II-7)

$$\bar{N} = \frac{1}{4k^{2}}(\ln E)^{2}$$

$$= \frac{1.32}{k^{2}}(\log_{10} E)^{2}$$

$$\bar{N} = h\,(\log E)^{2}, \qquad \text{(II-8)}$$

where $h = \frac{1.32}{k^{2}}$. (II-9)

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