YaSin, Allaine

THE ROLE OF COHERENCE EFFECTS IN THE MEASUREMENT PROCESS

City University of New York PH.D. 1986

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by

Allaine YaSin

A dissertation submitted to the Graduate Faculty in Physics in partial fulfillment of the requirements for the degree of Doctor of Philosophy, The City University of New York.

1986

This manuscript has been read and accepted for the Graduate Faculty in Physics in satisfaction of the dissertation requirement for the degree of Doctor of Philosophy.

8-7-86
date

Daniel Greenberger
Chairman of the Examining Committee

8/7/86
date

Executive Officer

Prof. Daniel M. Greenberger

Prof. Joel I. Gersten

Prof. Kenneth Rubin

Prof. Aage Peterson

Prof. Edward P. Tryon

THE CITY UNIVERSITY OF NEW YORK

ABSTRACT

The Role of Coherence Effects
in the Measurement Process

by

Allaine YaSin

Adviser: Professor Daniel M. Greenberger

Information measures corresponding to the development of a system's wave and particle properties are derived. The formulism is extended to the spin $^1/_2$ operators. For both cases pure states are shown to be the maximal information states.

Coherence which plays a crucial role in the information measures' definition is also found to be of paramount importance in the measurement process. With the aid of the "haunted measurement" gedankenexperiment it is shown that the incoherence introduced by the interaction between the measuring apparatus and the observed system can be removed thereby reversing the measurement process, obliterating any trace of its occurrence. This is a direct consequence of the quantum mechanical correlations between the two systems. Thus, such correlations previously thought to be significant only in the microscopic domain must be taken into account in the macroscopic realm for a proper description of the measuring process.

Haunted measurement versions of Wheeler's delayed choice experiment, Schrodinger's cat experiment and the EPR paradox are also discussed.

ACKNOWLEDGEMENTS

I extend my gratitude to the Northeast Regional Resource Center and the National Science Foundation for providing financial support thereby lessening some of my mundane concerns.

The City College Physics Department has been extremely supportive throughout this endeavor. Professor Michael Arons, Mrs. Ernestine Thomas, with her extremely kind and encouraging words, Mrs. Bertha Danziger, Professor Joseph F. Aschner, Juan Pajuelo, and all of the other staff and faculty were notably gracious. For this I offer my heartfelt thanks.

The concept for the first half of this dissertation evolved during discussions involving Michael A. Horne, Anton Zeilinger and myself while we were visiting the Neutron Diffraction Laboratory at M.I.T., the guests of the director Clifford Shull. I am fortunate to be one of many physicists whose work was bolstered by a collaboration with Professor Shull's laboratory. The congenial, humane atmosphere was made more so by Ken Finkelstein's efforts to make me feel comfortable. I am forever indebted to Michael and Carol Horne for their magnanimity, helping to make the entire experience a memorable one.

I must also express my appreciation to Peter Delfyett of the Institute for Ultrafast Spectroscopy and Lasers for the many productive conversations concerning optical phase conjugation.

There is a group of very special people - Stanley P. Davis, Najah Haqiqah, Emily and Mulzim Fida-I, Ghaniyyah Hafeez, and Marie Celeste Barreiro - who have made an immeasurable impact on my life. They were always there irrespective of the circumstances. They have my deepest love and respect.

And finally, it is most difficult to find words to express the profound contribution Professor Daniel M. Greenberger has made to my maturation as a physicist and human being. He personified commitment and integrity, remaining in touch with my vision, even when I was not. To him, I give my sincerest respect and admiration.

It is impossible to acknowledge all of the persons who have empowered me to realize this work. To those persons not mentioned, I thank you too.

DEDICATION

I dedicate this thesis to my parents - Alexander and Evelyn - whose love have always been an inspiration, to my sisters and their children - who loved me in spite of me - and to my brothers.

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CHAPTER I

INTRODUCTION

A. The Importance of the Complementarity Principle to the Formulation and Interpretation of Quantum Theory

The interpretation of quantum theory's mathematical formalism as put forth by Neils Bohr was based on the complementarity principle:

Any given application of classical concepts precludes the simultaneous use of other classical concepts which in a different connection are equally necessary for the elucidation of the phenomenon.[1]

The complementarity principle has its mathematical expression in Heisenberg's indeterminacy relations[2]

$$\Delta p \Delta q \sim \hbar \qquad (1.1)$$
$$\Delta E \Delta t \sim \hbar$$

with Δp the latitude in the momentum measuremənt and Δq the uncertainty in the canonically conjugate position observable. ΔE and Δt are similar ranges for the energy and the corresponding spread in time. The conjugate variables for which these relations hold are associated with the classical mutually exclusive particle and wave idealizations. Both concepts corpuscular and wave are however equally indispensable for a complete microscopic description. The momentum and energy exchanges between atomic systems are described by the particle picture irrespective of space-time specifications. The matter wave indicates the localization of atomic systems by yielding the probability distribution of the system in space and time. The Heisenberg relations are therefore essential to ensure the consistency of quantum theory by assigning the limits within which the use of classical concepts belonging to the 2 extreme pictures may be applied without contradiction.[3]

Einstein could not accept quantum theory's apparent statistical nature and its renunciation of classical causality. This led to an ongoing dialogue between he and Bohr lasting approximately 28 years during which time Einstein offered a number of gedankenexperiments to highlight what he considered to be paradoxes inherent in the theory.[4] Although he did not succeed in proving the theory incongruous, he managed to crystalize the epistemological problems raised by the theory's development in ingenious fashion.

B. Discussion of Einstein's Double Slit Gedankenexperiment

In an attempt to provide a counterexample to the complementarity principle and thereby demonstrate the inconsistency of quantum mechanics, Einstein proposed a modification of the double slit experiment (Fig. 1.1).[5,6] The first screen is free to move along the x axis and will recoil with each photon's deflection towards a particular slit in screen no. 2. Employing momentum conservation, Einstein endeavored to prove one could determine the photon's path and simultaneously preserve the interference pattern. Bohr, in his historic rebuttal, cited the uncertainty principle to show a successful trajectory measurement necessitates a specification of the screen's momentum with an accuracy which would cause the interference pattern to be washed out. He thus succeeded in defending the consistency of quantum mechanics.

In 1979 W.K. Wootters and W.H. Zurek[7] examined Bohr's defense in detail and conclusively confirmed the possibility of retaining an amazingly strong interference pattern while ascertaining knowledge of the particle's path. They do this by utilizing Shannon's definition of information* to quantitatively define the wave and particle knowledge available to the experimenter. The authors, however, do not demand as did Einstein, a 100% reliable determination of the photon's route; since, for this situation Bohr's assertions are valid.

C. Review of Wootters and Zurek Analysis.

See Fig. 1.1. Upon impact the photon and plate 1 constitute a single quantum mechanical system, inducing one to study the effect of the measurement of the plate on the photon's wavefunction as displayed by the interference pattern. Plate 1 is denoted by a harmonic oscillator wave function $\psi(x)$ which is a minimum uncertainty representation. The cost of obtaining information about the photon's momentum - as evidenced in the disturbance of the photon's phase - is therefore diminished.

$$\psi(x) = \pi^{-1/4} a^{-1/2} e^{-x^2/2a^2} \qquad \Delta x^2 = a^2/2 \tag{1.2}$$

In momemtum space:

$$\phi(k) = \pi^{-1/4} a^{1/2} e^{-a^2 k^2/2} \qquad \Delta k^2 = 1/2a^2$$

To first approximation the wave functions for the photon's passing through slits A and B respectively are:

$$\begin{aligned} U_A(\xi) &= f(0)\, e^{i\alpha(\xi)} e^{ik_0\xi} \\ U_B(\xi) &= f(0)\, e^{i\alpha(\xi)} e^{-ik_0\xi} \end{aligned} \tag{1.3}$$

with

$$f(0) \propto \text{constant}$$
$$\alpha(\xi) = \frac{2\pi}{L}\left[L^2 + \frac{S^2}{8} + \frac{\xi^2}{2}\right] \tag{1.4}$$
$$K_0 = \pi s / L\lambda$$

See Fig. 1.1 for parameters' definition.

Screen no. 1's final momemtum is experimentally ascertained. If its initial wavevector is k_i then its final wavevector k_f satisfies $k_f = k_i \pm k_0$ in as much as the photons can only impart the momenta, $\pm k_0 = \pm \pi s/L\lambda$. The total recorded distribution $D(k_f)$ of the plate's wave numbers will be the sum of all partial distributions $D_{k_i}(k_f)$ weighted by $|\phi(K_i)|^2$:

$$D_{k_i}(k_f) = \tfrac{1}{2}\left[\delta(k_f - k_i + k_0) + \delta(k_f - k_i - k_0)\right. \tag{1.5}$$

$$D(k_f) = \int dk_i \, |\phi(k_i)|^2 \, D_{k_i}(k_f)$$
$$D(k_f) = \left[a/2\pi^{1/2}\right]\left\{e^{-a^2(k_f + k_0)^2} + e^{-a^2(k_f - k_0)^2}\right\} \tag{1.6}$$

The scintillations at plate 3 signal the photons' passage through the apertures in plate 2. Plate 1's momemtum and the position of the scintillations are recorded for each event. The interference pattern is acquired by counting all photons landing within the interval $\xi \to \xi + d\xi$. The authors constructed the distribution function $D(k_f)$ in a similar fashion.

Wootters and Zurek, however, demonstrate the interference pattern $F(\xi)$ can also be realized from a weighted sum of the partial interference patterns. The partial interference pattern - "the distribution of scintillations arising only from those photons which have been associated with a definite measured momentum k_f of plate 1" - is central to establishing the self consistency of the authors' analysis and their subsequent information calculation. In analogy to the double slit experiment involving two slits of unequal areas, they postulate the partial interference pattern to be

$$i_{k_f}(\xi) = 1 + 2P_A^{1/2}(k_f)P_B^{1/2}(k_f)\cos 2k_0\xi \qquad (1.7)$$

where $P_A(k_f)$, $P_B(k_f)$ are the probabilities of the photon passing through slits A and B respectively, and are defined by the ratio of the 2 contributions to the final distribution function (see 1.6):

$$\begin{aligned} f(k_f) &= P_A(k_f)/P_B(k_f) \\ &= e^{-\alpha^2(k_f+k_0)^2} / e^{-\alpha^2(k_f-k_0)^2} \end{aligned} \qquad (1.8)$$

in conjunction with the requirement

$$P_A(k_f) + P_B(k_f) = 1$$

Adding all the partial interference patterns, find

$$\begin{aligned} F(\xi) &= \int D(k_f)\, i_{k_f}(\xi)\, dk_f \\ &= 1 + e^{-\alpha^2/k_0^2}\cos 2k_0\xi \end{aligned} \qquad (1.9)$$

Even at this point in the calculation one is able to illustrate the simultaneous manifestation of the photon's wave and particle nature. "Let $f(k_f)=\frac{99}{1}$, that is, out of 100 photons, we expect 99 to traverse the more likely slit.'[7] One would expect the interference pattern to be destroyed by a measurement of such accuracy; yet the pattern's contrast is rather substantial:

$$C = \frac{I_{max} - I_{min}}{I_{max} + I_{min}} \simeq 1/5 \qquad (1.10)$$

Thus, despite knowing the photon's path with 99% certainty, the photon's wave character is still highly developed. We procure the same result for F(ξ) if we measure plate 1's position instead of its momentum in order to monitor the photon's trajectory, indicating the equivalence of the 2 procedures and therefore the consistency of the author's reasoning.

Note, the two limiting cases of Einstein's version of the double slit experiment do not surprise us at all (see 1.9):

when

1) there is no determination of the photon's path, $ak_o = 0$ and the interference pattern is perfect.

2) each photon's path is resolved completedly, $ak_o = \infty$ and the interference pattern is thoroughly nullified.

As Wootters and Zurek assert, "the intermediate situation, in which one obtains <u>some</u> information about the photon's paths and still retains an interference pattern having <u>some</u> degree of clarity appears paradoxical because .... we lack a good way of talking about such a situation and we have no simple rule which tells us what to expect." The authors use information theory to develop a rule which defines the extent to which the two complementary aspects of light may be simultaneously manisfested. C.E. Shannon defined the information measure in the following manner: if it's possible for a system to be in any of N states, with the probability

P_k to be in state k, then the amount of information we lack concerning the system is the positive number

$$H(p) = -\sum_{k=1}^{N} P_k \ln P_k \qquad \sum_{k=1}^{N} P_k = 1 \tag{1.11}$$

To investigate the theoretical limit on the amount of extractable information, they determined the method of generating the partial interference pattern pertaining to a particular value of the contrast such that the information regarding the photons' path is maximized. They discovered, in order to obtain the interference pattern

$$F(\xi) = 1 + e^{-a^2 k_0^2} \cos 2k_0 \xi$$

with ak_0 = .4769 (the value used throughout the work), one must sacrifice a minimum of 71.7% of the available information regarding the photon path.

How much particle information does one forfeit when one performs Einstein's experiment? According to equation (1.8)

$$P_A(k_f) = \frac{e^{-a^2(k_f+k_0)^2}}{e^{-a^2(k_f+k_0)^2} + e^{-a^2(k_f-k_0)^2}} \tag{1.12}$$

and

$$P_B(k_f) = 1 - P_A(k_f)$$

The average information we lack per photon using (1.11) and (1.6) is

$$H = \int_{-\infty}^{\infty} dk_f\, D(k_f) [P_A(k_f) \ln P_A(k_f) + P_B(k_f) \ln P_B(k_f)] \tag{1.13}$$

Wootters and Zurek evaluate this integral numerically and find H=72.8%, indicating 72.8% of the usable information is dissipated. This is only slightly greater than the minimum value, 71.7%, associated with this interference pattern. The Einstein experiment consequently renders almost as much information as one could possibly secure - 1 - 72.8% = 27.2% compared to 1 - 71.7% = 28.3%.

D. Reformulation of the Problem

The previous computations are somewhat involved due to the adaptation of an information measure derived for a classical probabilistic theory - which quantum theory is not. In Chapter III a gedankenexperiment incorporating the neutron interferometer will be employed to derive particle and wave information measures, and it will be shown, again, that we can simultaneously access information pertaining to these 2 classically mutually exclusive pictures.[9,10]

Using density matrix techniques to encompass the partially coherent case, we verify Fano's referral to pure states as states of "maximum knowledge".[11]

The spin 1/2 operators are also studied and the polarization vector is shown to be the "natural" information measure for these complementary quantities. First, however, the apparatus will be introduced in Chapter II.

E. The Measurement Process in Quantum Theory

The quantum theoretical description requires a partitioning of the universe into two parts, one consisting of the observed system, and the other the observer. The theory then describes events which occur in the observed sector (so long as there are no interactions with the observing portion), with Schrodinger's equation. As soon as an interaction - that is measurement - ensues one must apply another method (method 2 given below) to properly denote the final system. Since this requisite partition procedure is not a priori evident, the inability to incorporate the measurement process within the general time evolution prescription lends an ad-hoc quality to the theory. The measurement description is unique due to its apparent non-causal and irreversible aspects. In his treatise, Von Neumann[12] rigorously formulated quantum theory's mathematical foundations and lucidly highlighted this "peculiar duality." Recapitulating the pertinent conclusions: if a system is in a state ϕ_0 with the Hamiltonian H at time t=0, its time development is given by Schrodinger's equation

$$i\hbar \frac{\partial}{\partial t}\phi(t) = H\phi(t) \tag{1.14}$$

$\phi(t)$ thus satisfying

$$\rho(t) = \sum_i W_i |\phi_i(t)\rangle\langle \phi_i(t)$$

$$\phi(t) = e^{-iHt/\hbar}\,\phi_0 \tag{1.15}$$

provided H is time independent. Similarly, a mixture ρ_0 (see Appendix A)

$$\rho_0 = \sum_i W_i |\phi_i\rangle\langle\phi_i|$$

$$\rho_0 = \sum_i W_i |\phi_i\rangle\langle\phi_i| \tag{1.16}$$

is transformed to $\rho(t)$

$$\rho(t) = e^{-iHt/\hbar}\,\rho_0\,e^{iHt/\hbar}$$
$$\rho(t) = \sum_i W_i |\phi_i(t)\rangle\langle \phi_i(t)| \tag{1.17}$$

Call this tranformation method 1.

On the other hand, if a measurement of a quantity A possessing a discrete spectrum, distinct eigenvalues and eigenfunctions $\psi_1, \psi_2, \ldots$ is performed on the aforementioned system, a non-causal change transpires from which any state ψ_i can result with the probability $|(\psi_i,\phi_O)|^2$. That is, a mixture

$$\rho = \sum_{i=1}^{\infty} |(\psi_i, \phi_o)|^2 |\psi_i\rangle\langle\psi_i| \tag{1.18}$$

is generated.

Analogously, a system whose initial state is given by the statistical operator ρ_O suffers a change of state to ρ'.

$$\rho' = \sum_{i=1}^{\infty} |(\psi_i, \rho_o \psi_i)|^2 |\psi_i\rangle\langle\psi_i| \tag{1.19}$$

Equations (1.18) and (1.19) exemplify method 2. The two procedures are fundamentally different. Method 1 is causal since if we know the state at some time t_1, we can predict it absolutely for time t_2. On the other hand, method 2 is non-causal. The initial state is mapped into any one of the orthonormal states with a corresponding probability. Our ability to predict the evolved state is subsequently reduced. Furthermore it can be shown the transition $\phi_O \to \rho$ in principle cannot be achieved by a unitary transformation. The proof is simple and involves the invariance of the trace with respect to unitary transformations. The process typifyed by $\phi_O \to \rho$ therefore cannot be executed by method 1 which is a unitary operation.

Von Neumann also demonstrates the processes' thermodynamic dissimilarities. By associating a gaseous ensemble with the properly normalized statistical mixture ρ he derives the ensemble's entropy to be

$$S = - Nk\ Tr\rho \ell_n \rho$$

N = number of systems constituting the ensemble

K = Boltzmann constant (1.20)

As is discussed in the Appendix, ρ's eigenvalues w_i lie within the range $0 \leq w_i \leq 1$. The mixture's entropy is therefore ≥ 0. For a pure state, S=0. A system in a pure state evolving via method 1 exhibits no entropy change while method 2 induces an entropy increase. According to the second law of thermodynamics, method 1 is reversible and method 2 is not.

In principle method 1 describes a microsystem's evolution proceeding from any and all microscopic interactions while the measurement process depicts the system's transformation after a macroscopic intervention and manifestation of its properties. There is no a priori reason for method 2's presence in quantum mechanics as a unique evolutionary mechanism; after all, a macrosystem in theory is simply the large number limit of many microsystems. If we can characterize the microsystems utilizing a basic (time) development scheme, the same scheme (perhaps with some internally consistent alterations) should be applicable in principle on a macroscopic scale.

In an effort to further investigate method 2's peculiar nature we will examine its irreversibility feature in detail via the exploitation of a gedankenexperiment, the "haunted measurement."[13] Essentially, a neutron traversing the neutron interferometer will interact with a macroscopic structure for a path determination measurement. It will appear the system's coherence is destroyed because the interference pattern (which would result if no measurement was performed) is obliterated. This is in accord with method 2's prediction (1.18) whereby the resulting system can at most exhibit partial coherence (here it is totally incoherent). In contradiction to this conclusion the existence of the system's "latent coherence" will be demonstrated, permitting the measurement's reversal and consequent reintroduction of the interference pattern. The interference pattern is an illustration of the system's "overt coherence." The subjective character of coherence is therefore made evident, since the interaction with the macroscopic system was not enough to remove all coherence. The relationship between simultaneous knowledge of a system's complementary properties and overt coherence developed in Chapter III will be used to postulate the connection between information obtained via the measurement procedure and the two types of coherence.

These conclusions will be applied to the delayed choice experiment, the EPR paradox and to the Schrodinger's Cat experiment.

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13. A. Peres, Phys. Rev. D. 22, 879 (1980) proposed a similar gedankenexperiment in an attempt to reveal the "undoing of a quantum measurement." He nevertheless only included the equivalent of phase one of our experiment which is not sufficient to reverse - undo - the interaction.

CHAPTER II

THE NEUTRON INTERFEROMETER

In this chapter the neutron interferometer is featured and its unique properties highlighted.

A. Early Development

Prior to 1965 interferometry based on the coherent splitting of the incident wave's amplitude was limited to electromagnetic waves in the radiofrequency to ultraviolet range and was accomplished with beam splitters. Using significantly different techniques and principles, Ulrich Bonse and Michael Hart[1] in 1965 succeeded in devising an interferometer capable of coherently dividing and recombining x-rays. The Bragg reflection capability of a virtually flawless silicon crystal was employed to partition the 1A^{o} wavelength waves which then traversed paths spatially separated by a distance of approximately 1 cm before merging via additional Bragg reflections. The pair discovered they could produce oscillations in the output beam by varying the optical pathlength of 1 of the beams; hence, the beam's relative phasing was experimentally accessible.

In 1974 at the Austrian Atomic Institute in Vienna, Helmut Rauch, Wolfgang Treimer and Bonse[2] conducted the first interferometry experiment with thermal neutrons.

B. Description and Operation

The apparatus is depicted in Fig. 2.1. Since its inception many variants, for example 2 earred devices, have been proposed and fabricated; however, throughout this work the discussions will be confined to the discussions to the arrangement indicated in Fig. 2.1 and to idealizations thereof.

The 3 slabs, each perpendicular to a set of strongly reflecting planes, are machined from a single nearly perfect silicon crystal. The crystal is generally from 5 cm to 10 cm in length with the distance between the ears-which are equal to within 1 um-, a few centimeters.[3]

In Fig. 2.2 a schematic diagram of the interferometer is given. The collimated, nominally monochromatic neutron beam penetrates the instrument at A and is Laue scattered (a form of Bragg scattering where the reflected and incident wave traverse the crystal), thereby producing beams I and II. The details of the neutron-crystal interaction will not be elaborated upon since it is not pertinent to our analysis. There are, however, extensive accounts in the literature[4-7]. In the regions B and C Laue scattering again occurs, causing the 2 innermost beams to constructively overlap at D. The beams G and O entering the counters are the results of a further reflection at D of the coherently interfering beams I and II.

If an experiment is performed which induces a phase shift β in beam II relative to beam I, the intensities appearing in detectors C2 and C3 will be of the form

$$I_2 = \gamma - \alpha \cos \beta$$
$$I_3 = \alpha (1 + \cos\beta) \qquad (2.1)$$

where α and γ are constants which depend upon the incident flux, the crystal structure and the neutron-nucleus scattering length of silicon.[8] The intensities thus alternate between the counters C2 and C3 as a function of β. The non-interfering beam I' serving as a reference is monitored by the counter C1.

Typically the incoming neutrons possess an energy on the order of 10^{-2} eV and a wavelength approximately equal to 1A°. For an interferometer 10 cm long, each beam will undergo 10^{9} oscillations. The monochromaticity required to maintain coherence is produced by the Laue scattering process and not by extra-crystal collimation of the incident beam. In fact, the entering particles have a fractional spread in wavelength[3] $\frac{\delta\lambda}{\lambda} = .01$.

The Bragg angle θ is characteristically 20°-30°. Along the interferometer's axis, a typical neutron's momentum K_z is given by

$$K_z = K\cos\theta \tag{2.2}$$

The fractional spread in K_z is therefore

$$\frac{\delta K_z}{K} \simeq (\sin\theta)\,\delta\theta \tag{2.3}$$

From the uncertainty principle wave packets with a coherence length = δz where

$$\delta z \simeq .03\,\text{mm} \tag{2.4}$$

can theoretically be defined by the interferometer.[4] C. Shull[9] in 1968 and more recently Kaiser et.al.[10] have obtained experimental results supporting this inference.

The flux output of the research reactors utilized for these experiments are such that 1 neutron traverses the apparatus per second, minimizing the possibility of interaction between particles.[5]

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CHAPTER III

A CONTINUOUS FORMULATION OF THE COMPLEMENTARITY PRINCIPLE

Here we present a model for the interferometer from which the internal structure of the complementarity principle for the canonically conjugate kinematic variables and the spin 1/2 observables is developed.

A. Formularization of the Complementarity Principle for the Position and Momentum Operators

1. The Idealized Apparatus

Fig. 3.1 depicts the interferometer model central to the discussion. The half-silvered mirrors M and M' each bisects the incoming wave and imparts a $\pi/_2$ phase shift to the reflected waves.[1,2] M1 and M2 are specularly reflecting mirrors. The 2 subbeams constituting the deviated beam, unlike those composing the forward beam are thus 180° out of phase. The counting rate of the line of detectors located at the output is a measure of the relative phase existing between the two beams. By inserting an aluminum wedge (E) and varying x which alters beam I's optical path length, the dependence of the beams' relative phasing on the distance x is acquired. See Fig. 3.2. The effects of the phase shifting absorber positioned at F' (Fig. 3.1) is easily probed as a function of the wedge's position. No device is situated at F during this portion of the treatment. For convenience the analysis will be confined to the forward disturbance.

2. The Coherent Case

Our ability to manipulate the interference pattern affords a degree of predictability, that is, information regarding the neutrons' wave properties which will be incorporated into an analytically simple and intuitively satisfying wave information measure. See Fig. 3.1. We can represent the incoming beam by a plane wave propagating in the x-z plane.

$$\Psi_{inc}(\vec{r},t) = a\, e^{ik_x x + ik_z z - i\omega t} \tag{3.1}$$

Our argument does <u>not</u> depend on the variable z. It will subsequently be suppressed. The disturbance at the forward counter array is characterized by

$$\Psi(x) = A e^{ikx} + B e^{i\phi} e^{-ikx} \qquad k = k_x \tag{3.2}$$

A,B real; B < A

where A and B are the amplitudes of beams I and II respectively.

The interference pattern's constrast will be utilized to detail the neutrons' wave features as well as to provide the measure of wave knowledge. The contrast is given by

$$C = \frac{I_{max} - I_{min}}{I_{max} + I_{min}}$$

$$C = \frac{2AB}{A^2 + B^2} \tag{3.3}$$

The probability the neutron traverses the instrument via beam I is

$$P_I = \frac{A^2}{A^2+B^2} \tag{3.4}$$

If by "particle knowledge" we mean the ability to probabilistically predetermine particle path, when

$$P_I = P_{II}$$

with

$$P_{II} = \frac{B^2}{A^2+B^2} \tag{3.5}$$

we have no knowledge regarding the neutron's trajectory. Consistently, from Eq. (3.3), the contrast is a maximum. Define

$$\mathcal{P} \equiv P_I - P_{II} \tag{3.6}$$

to be the measure of particle knowledge. The amount by which P_I exceeds P_{II} determines the accuracy of our predictions regarding the outcome of a measurement; that is, the difference gauges the degree to which our predictive capability exceeds that of ordinary chance.

Parameterizing the amplitudes,

$$\begin{aligned} A &= R\cos\gamma \\ B &= R\sin\gamma \end{aligned} \tag{3.7}$$

Our continuous form of the complementarity principle satisfy

$$\mathcal{P}^2 + C^2 = 1 \tag{3.8}$$

since

$$P = \cos 2\gamma \tag{3.9a}$$

$$C = \sin 2\gamma \tag{3.9b}$$

For example: if the absorber is such that it absorbs 99% of the impinging beam one can assert all the particles in the forward beam will arrive via path I, and be correct 99% of the time. The contrast will be discernible at 20% indicating the simultaneous preservation of wave information.

We obtain markedly similar results with Shannon's information measure. Setting

$$P_I = P_1 \qquad P_{II} = 1 - P_I = P_2 \tag{3.10}$$

and substituting Eq. (3.10) into Eq. (1.11) gives the lack of information regarding the particle path. H is a maximum for

$$H(1/2) = H_0 \tag{3.11}$$

The ratio $H(p)/H_0$ provides the relative amount of unknown information concerning the system's corpuscular development.

In Fig. 3.3 the information theoretical measure $H(p)/H_0$ is compared to C^2.

3. The Density Matrix Representation

We had from 3.2 for the exiting particles' wave function

$$\Psi = Ae^{ikx} + Be^{i\phi}e^{-ikx}$$
$$\Psi = A\Psi_1 \quad + Be^{i\phi}\Psi_2 \tag{3.12}$$

The density operator (see Appendix) for the system is

$$\rho = \frac{1}{I}|\Psi\rangle\langle\Psi| \tag{3.13a}$$

$$I = A^2 + B^2 \tag{3.13b}$$

Expanding ρ in terms of the basis states $|\psi_i\rangle$ i=1,2 furnished in Eq. (3.12) determines the density matrix [ρ]:

$$[\rho] = \frac{1}{I}\begin{pmatrix} A^2 & ABe^{-i\phi} \\ ABe^{i\phi} & B^2 \end{pmatrix} \tag{3.14}$$

The previous information measures are readily reconstructed in terms of the elements of [ρ]:

$$P = \rho_{11} - \rho_{22} \tag{3.15a}$$

and

$$C = \sum_{i \neq j} |\rho_{ij}| \tag{3.15b}$$

In general, one would say the system is a <u>coherent</u> <u>superposition</u> of the basis states $|\psi_i\rangle$ because the off diagonal terms are nonzero. In particular, a pure state is <u>completely</u> <u>coherent</u>.[3] From Eq. (3.15) it is evident the off-diagonal terms contain valuable information regarding the system's complementary picture. Hence we have managed to interpret the information incorporated in the off-diagonal elements of the density matrix.

4. Generalization to the Partially Coherent Case

Suppose incoherence is induced in the recombined beam by the performance of a measurement which determines absolutely the neutrons' path. In Fig. 3.1 replace M2 with a mirror m2 which can slide freely (when unhinged) along the x axis. m2's mass is such that if it is unhinged, the neutron can upon collision produce a measurable change in m2's position. Assume N' out of a total of N neutrons leave the interferometer under these conditions, N_b via path 1, N_c by way of path 2. The other N-N'=Na neutrons leave the device when m2 is held stationary; that is, no trajectory measurement is conducted. What are the particle and information measures for this system?

ρ for the outgoing beam is (see Appendix) given by

$$\rho = I_a|\psi_a\rangle\langle\psi_a| + I_b|\psi_b\rangle\langle\psi_b| + I_c|\psi_c\rangle\langle\psi_c| \quad (3.16a)$$

where

$$I_i = \frac{N_i}{N} = \text{fraction of particles in } |\psi_i\rangle \text{ portion} \quad (3.16b)$$

of the beam

and

$$|\psi_a\rangle = \sum_{k=1}^{2} a_k|\phi_k\rangle \quad (3.16c)$$

$$|\psi_b\rangle = |\phi_1\rangle$$
$$|\psi_c\rangle = |\phi_2\rangle \quad (3.16d)$$

$$|\phi_1\rangle = e^{ikx} \text{ for path I}$$
$$|\phi_2\rangle = e^{-ikx} \text{ for path II} \qquad (3.16e)$$

Using $|\phi_\ell\rangle$ $\ell=1,2$ as the basis states:

$$[\rho] = \begin{pmatrix} I_a|a_1|^2 + I_b & a_1 a_2^* I_a \\ a_2^* a_1 I_a & I_a|a_2|^2 + I_c \end{pmatrix} \qquad (3.17)$$

The contrast satisfies

$$C = \frac{2 I_a |a_1 a_2|}{I_a(|a_1|^2 + |a_2|^2) + I_b + I_c} \qquad (3.18)$$

Similarly,

$$P = P_I - P_{II}$$
$$= \frac{I_a(|a_1|^2 - |a_2|^2) + I_b - I_c}{I_a(|a_1|^2 + |a_2|^2) + I_b + I_c} \qquad (3.19)$$

The analysis is simplified by defining the relative degree of coherence of beams I and II to be respectively $\frac{I_b}{I_a|a_1|^2}$ and $\frac{I_c}{I_a|a_2|^2}$. It is apparent these quantities are equal. To see this, let N_c' be the number of absorbed particles. If t is the absorber's transmission coefficient then

$$t = \frac{N_c}{N_c + N_c'} \qquad (3.20)$$

Employing probability arguments

$$N_c + N_c' = N_b \qquad (3.21)$$

If $\frac{1}{2}$ is the intensity of beam II prior to entering the absorber (the beams are equally split by M- Fig. 3.1),

$$I_a |a_2|^2 = \tfrac{1}{2} t \tag{3.22a}$$

and

$$I_a |a_1|^2 = \tfrac{1}{2} \tag{3.22b}$$

Using Eqs. (3.16b), (3.20), (3.21) and (3.22) it immediately follows that

$$I_b / I_a |a_1|^2 = I_c / I_a |a_2|^2 \tag{3.23}$$

or

$$\frac{I_c}{I_b} = \frac{|a_2|^2}{|a_1|^2}$$

Define as in the pure case

$$I_a^{1/2} |a_1| = R \cos \gamma \tag{3.24a}$$

$$I_a^{1/2} |a_2| = R \sin \gamma \tag{3.24b}$$

The degree of total coherence is given by

$$\frac{R}{I^{1/2}} = \cos \beta \tag{3.25}$$

Upon substituting Eqs. (3.23), (3.24) and (3.25) into Eq. (3.18), the contrast is determined to be

$$C = \cos^2 \beta \sin 2\gamma \tag{3.26}$$

From Eq. (3.19)

$$P = \cos 2\gamma \tag{3.27}$$

Comparing the above with Eq. (3.9) - the coherent measures - we see the wave measure is reduced by the factor corresponding to the degree of coherence. Also

$$\mathcal{P}^2 + \mathcal{C}^2 < 1 \tag{3.28}$$

Our total knowledge is accordingly diminished. The pure, that is, completely coherent state gives the maximum information possible about the simultaneous development of the complementary properties of the system.

B. Application to the Spin 1/2 Operators

1. The Proposed Experiment

A. Zeilinger advanced a class of interferometric experiments[4] in 1979 which would directly test the superposition principle for the spin 1/2 states. Four years later he, J. Summhammer, G. Badurek et al. succeeded in coherently superimposing oppositely polarized neutron beams of equal amplitude.[5] The final beam was polarized perpendicular to the polarization of each initial beam, in agreement with quantum theoretical predictions. According to the classical view one would suspect a mixture to result, which clearly was not the case.

One of the experiments proposed by Zeilinger will be adopted here in order to develop the information measures for the spinor space. In Fig. 3.1 a spin flipper is situated at F and a phase shifting absorber at F'. Stern Gerlach filters are the output measuring devices.

The incoming beam is polarized in the +z direction and can be represented by the unit spinor $|+_z\rangle$,

$$\Psi_{inc} = | \uparrow_z \rangle \tag{3.29}$$

After entering the spin flip coil, beam II's spin is rotated 180° about the y axis. The unitary operator $U_R(\omega)$ describes spin rotations:

$$U_R(\omega) = e^{-i\frac{\vec{\sigma}\cdot\vec{\omega}}{2}}$$

$$= \cos\frac{\omega}{2} - i\frac{\vec{\sigma}\cdot\vec{\omega}}{|\vec{\omega}|}\sin\frac{\omega}{2} \tag{3.30}$$

ω is the rotation angle and $\frac{\vec{\omega}}{|\vec{\omega}|}$ a unit vector parallel to the rotation axis. For $\omega = 180°$ and $\frac{\vec{\omega}}{|\vec{\omega}|} = \hat{y}$

$$U_R(180°) = -i\sigma_y \tag{3.31}$$

2. <u>The Coherent Case</u>

The exiting beam in the forward direction is

$$|\Psi'\rangle = A|\uparrow_z\rangle + Be^{i\phi}e^{-i\sigma_y\pi/2}|\uparrow_z\rangle \tag{3.32a}$$

$$|\Psi'\rangle = A|\uparrow_z\rangle + Be^{i\phi}|\downarrow_z\rangle \tag{3.32b}$$

A and B real; B < A

We are concerned with the simultaneous development of the neutron's complementary spin properties in the z,x and y directions. To what extent can one predict the neutron's spin state in the z direction and simultaneously predict the state in the xy plane?

Although the density matrix for the beam can be secured at this point in a straight forward manner it will instead be rendered in terms of the polarization vector (Appendix). The information measures' physical significance is then readily perceivable.

Normalizing $|\psi'\rangle$ defines $|\psi\rangle$:

$$|\psi\rangle = \frac{1}{\sqrt{N}}\,|\psi'\rangle \tag{3.33a}$$

$$N = A^2 + B^2 \tag{3.33b}$$

In matrix representation

$$|\uparrow_z\rangle = \begin{pmatrix}1\\0\end{pmatrix} \tag{3.34a}$$

$$|\downarrow_z\rangle = \begin{pmatrix}0\\1\end{pmatrix} \tag{3.34b}$$

$$|\psi\rangle = \frac{1}{\sqrt{N}}\begin{pmatrix}A\\Be^{i\phi}\end{pmatrix} \tag{3.35}$$

Parameterizing, let

$$\cos\eta/2 = \frac{A}{\sqrt{N}} \tag{3.36a}$$

$$\sin\eta/2 = \frac{B}{\sqrt{N}} \tag{3.36b}$$

$|\psi\rangle$ can consequently be written:

$$|\psi\rangle = \begin{pmatrix}\cos\eta/2\\e^{i\phi}\sin\eta/2\end{pmatrix} \tag{3.37}$$

The i^{th} component of the polarization vector satisfies

$$P_i = \langle \sigma_i \rangle \tag{3.38}$$

The outgoing beam's polarization vector is therefore

$$P_x = \sin\eta \cos\phi \tag{3.39a}$$

$$P_y = \sin\eta \sin\phi \tag{3.39b}$$

$$P_z = \cos\eta \tag{3.39c}$$

with

$$|\vec{P}| = 1 \tag{3.39d}$$

See Fig. 3.4.

The density matrix

$$[\rho] = \frac{1}{2}\begin{pmatrix} 1+\cos\eta & \sin\eta(\cos\phi - i\sin\phi) \\ \sin\eta(\cos\phi + i\sin\phi) & 1-\cos\eta \end{pmatrix} \tag{3.40a}$$

$$[\rho] = \frac{1}{2}\begin{pmatrix} 1+P_z & P_x - iP_y \\ P_x + iP_y & 1-P_z \end{pmatrix} \tag{3.40b}$$

is procured from the density operator

$$\rho = |\psi\rangle\langle\psi| \tag{3.41}$$

by choosing σ_z's eigenstates $\begin{pmatrix}1\\0\end{pmatrix}$ and $\begin{pmatrix}0\\1\end{pmatrix}$ as basis states.

In analogy with our previous particle measure, the measure of our knowledge of the departing neutrons' spin state in the z direction, K_z, is defined to be (the probability the spin state is $|\uparrow_z\rangle$) = $P_I(z)$ less (the probability the state is $|\downarrow_z\rangle$) = $P_{II}(z)$. (See Eqs. (3.35) and (3.37)):

$$K_z = \frac{A^2 - B^2}{A^2 + B^2} \tag{3.42a}$$

$$K_z = \cos\eta \tag{3.42b}$$

In terms of [ρ]'s elements,

$$K_z = \rho_{11} - \rho_{22} \tag{3.42c}$$

If the forward beam enters a Stern Gerlach filter whose magnetic field $\vec{H}$ is oriented at an angle ϕ with respect to the x axis the entire beam will pass through (Fig. 3.5). One can show, if $K_{xy}(\phi)$ is the accuracy with which we can predict the spin state of the neutrons entering the filter; that is, whether it is parallel or antiparallel with respect to the magnetic field then

$$K_{xy}(\phi) = \frac{A'^2 - B'^2}{A'^2 + B'^2} \qquad \begin{matrix} A' > B' \\ A', B' \text{ real} \end{matrix} \tag{3.43}$$

where $|\psi'\rangle$ is expanded in terms of the eigenvectors $|\uparrow_\phi\rangle$, and $|\downarrow_\phi\rangle$ of $\vec{\sigma}\cdot\hat{H}(\phi)$:

$$|\psi'\rangle = A'|\uparrow_\phi\rangle + B'|\downarrow_\phi\rangle \tag{3.44}$$

$$\hat{H}(\phi) = \vec{H}/|\vec{H}| \tag{3.45a}$$

$$[\vec{\sigma}\cdot\hat{n}(\phi)]|\uparrow_\phi\rangle = |\uparrow_\phi\rangle \tag{3.45b}$$

$$|\uparrow_\phi\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\ e^{i\phi}\end{pmatrix} \tag{3.45c}$$

$$[\vec{\sigma}\cdot\hat{n}(\phi)]|\downarrow_\phi\rangle = -|\downarrow_\phi\rangle \tag{3.45d}$$

$$|\downarrow_\phi\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\ -e^{i\phi}\end{pmatrix} \tag{3.45e}$$

Kxy(ϕ)'s equivalence to the sum of [ρ]'s off-diagonal terms can also be demonstrated:

$$K_{xy}(\phi) = \sum_{i\neq j} |\rho_{ij}| \tag{3.46a}$$

Hence the off-diagonal quantities convey information about the spin state in the xy plane. From Eq. (3.40)

$$K_{xy}(\phi) = \sin\eta \tag{3.46b}$$

Complete knowledge concerning the beam's spin state consist of information regarding the spin in the z direction as well as in the xy plane. The corresponding measures are related in an intuitively appealing and sound manner:

$$K_z^2 + K_{xy}^2(\phi) = \cos^2\eta + \sin^2\eta = 1 \tag{3.47}$$

Can we further decompose the planar spin measure into "components" along the x and y axis? Specifically, what is the probability $P(x^+)$ the exiting neutrons are polarized in the $|\uparrow_x\rangle$ state?

$$P(x^+) = \langle \uparrow_x | \rho | \uparrow_x \rangle = \frac{1}{2}(1 + \sin\eta\cos\phi) \tag{3.48a}$$

$$|\uparrow_x\rangle = \frac{1}{2}(|\uparrow_z\rangle + |\downarrow_z\rangle) \tag{3.48b}$$

Similarly, the probability the neutrons are polarized in the $|+_x\rangle$ state, $P(x^-)$, is

$$P(x^-) = \langle \downarrow_x | \rho | \downarrow_x \rangle = \frac{1}{2}(1 - \sin\eta\cos\phi) \tag{3.49a}$$

Our measure for the neutron's spin in the x direction, K_x, is therefore,

$$K_x = |P(x^+) - P(x^-)| = |\sin\eta\cos\phi| \tag{3.50}$$

Eq. (3.39a) indicates the equivalence between K_x and the polarization component P_x:

$$K_x = |P_x| \tag{3.51}$$

Likewise, K_y is given by

$$K_y = |P(y^+) - P(y^-)| = |\sin\eta\sin\phi| \tag{3.52a}$$

$$K_y = |P_y| \tag{3.52b}$$

Our total knowledge about the system's spin is detailed in its polarization vector and

$$K_z^2 + K_x^2 + K_y^2 = P_z^2 + P_x^2 + P_y^2 = 1 \tag{3.53}$$

For a pure state the polarization vector's magnitude is always 1. It will be demonstrated that this is in fact its maximum value (the information available is maximized) and as depicted in Section A.4 a reduction in the system's coherence prompts a corresponding decrease in our knowledge of the system's state.

3. The Partially Coherent Case

Let the input beam be composed of two independently prepared subbeams in states $|\chi_a'\rangle$ and $|\chi_b'\rangle$. The intensity of the output subbeams characterized by $|\chi_a\rangle$ and $|\chi_b\rangle$ are W_a and W_b respectively. Also

$$W_a + W_b = 1 \tag{3.54}$$

The polarization vector $\vec{P}$ associated with the final total beam is[3]

$$\vec{P} = W_a \vec{P}^{(a)} + W_b \vec{P}^{(b)} \tag{3.55}$$

Stated in component form,

$$P_i = W_a \langle \chi_a | \sigma_i | \chi_a \rangle + W_b \langle \chi_b | \sigma_i | \chi_b \rangle \tag{3.56a}$$

$$P_i = W_a P_i^{(a)} + W_b P_i^{(b)} \qquad i = 1, 2, 3 \tag{3.56b}$$

Since the polarization vectors for the constituent beams describe pure states,

$$|\vec{P}^{(a)}| = |\vec{P}^{(b)}| = 1 \tag{3.57}$$

The total polarization vector has the magnitude:

$$|\vec{P}|^2 = W_a^2 |\vec{P}^{(a)}|^2 + W_b^2 |\vec{P}^{(b)}|^2 + 2W_aW_b\,(\vec{P}^{(a)}\cdot\vec{P}^{(b)}) \tag{3.58a}$$

and

$$|\vec{P}|^2 < (W_a + W_b)^2 = 1 \qquad \vec{P}^{(a)} \neq \vec{P}^{(b)} \tag{3.58b}$$

The total information available is therefore <1. Some knowledge as determined by $\vec{P}(a)\cdot\vec{P}(b)$ is not available to us. Because of the way incoherence was introduced into the system in the preceding calculation detailed in part A, the relative degrees of coherence of the 2 beams were equal; and, we discovered that our decreased ability to manifest the contrast (the wave property of the system) resulted in a corresponding information loss. How the loss is manifested in the present example is also prescribed by the system's preparation.

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CHAPTER IV

THE HAUNTED MEASUREMENT

In the following we propose and examine the haunted measurement gedankenexperiment in order to investigate the measurement process in quantum theory.

A. The Model

Displayed in Fig. 4.1 is the optical analog of the neutron interferometer employed in this discussion. The output counters of Fig. 3.1 are replaced by a screen on which the interference pattern will materialize. Two identical sets of double mirrors, subsystem MS1 and MS2 are inserted in beam II. The two mirrors composing each unit are rigidly attached and can glide unhindered in the x direction. Although each subsystem's total mass, M_2 is much greater than the neutron's mass m_1, the neutron will still be able to discernibly displace either unit upon impact.

During phase one of the treatment we will position MS1, exclusively, in beam II in order to determine the neutron's trajectory. The "overtly-incoherent" outcome of the neutron - MS1 scattering experiments is then considered. MS2 will be included for phase two wherein it will be rigidly attached to MS1 permitting the reintroduction of the system's overt-coherence and thus evincing the system's latent coherence.

B. Phase One - The Measurement Process and the Destruction of Overt Coherence

Denote the neutron's coordinates by x_1, z_1 and those of MS1 by x_2, z_2. The origin of the coordinate system will be taken to be the center of the equilibrium position of the first mirror struck in MS1-MS11. The neutron has a well defined momentum. It therefore possesses a very narrow packet in momentum space, and a broad packet $U_1(X_1)$ with the spread "a", in position space:

$$\delta K_1 \simeq \frac{1}{a} << K_0 \qquad (4.1)$$

$$a >> \lambda_0 = 2\pi / K_0$$

$$U_1(x_1) = A e^{iK_0 x_1} e^{-x_1^2/a^2} \qquad (4.2)$$

The neutron is essentially a free particle in the z direction; hence, the z coordinate will be suppressed.

One of the ways in which the evolutionary mechanisms, methods 1 and 2 introduced in Chapter 1 differ is method 1 involves microsystems only and method 2, one or more macrosystems. By macroscopic, one usually means a system of many degrees of freedom, large size, and high mass.[1] Here, however, we will accurately describe a macrosystem - the mirror set(s) - quantum mechanically in the limit of one degree of freedom and demonstrate the induced coherence loss during the measurement process. To this end, L, the distance between the mirrors constituting each subsystem (Fig. 4.2) will serve as a macroscopic parameter representing the order of magnitude of the size of the entire apparatus. The mirror is macroscopic if

$$L \gg a \tag{4.3}$$

It is essential for the neutron's wave packet to maintain its integrity, that is, the relative phasing of the constituent eigenfunctions for the duration of the experiment. The group velocity approximation provides the requirements which must be satisfied in order for this to be accomplished. The inequalities given in Eq. (4.1) and fulfilled by the neutron's wavefunction state the criteria for the application of the expansion. For a free particle in this approximation the width of the packet is the coherence length since (see Fig. 4.3) at time t=0:

$$\text{at point A: } \psi(x) = e^{ikx} \tag{4.4a}$$

$$\text{at point B: } \psi(x+\delta x) = e^{i(k+\delta k)(x+\delta x)} \tag{4.4b}$$

Due to phase randomization, the phase at A is incoherent with the phase at B, a distance "a" away:

$$\delta k\, \delta x \simeq 2\pi \tag{4.5}$$

Now at time t:

$$\psi(x,t) = e^{i(k_0+\delta k')x - i\omega(k_0+\delta k')t} \tag{4.6a}$$

$$0 \le \delta k' \le \delta k \tag{4.6b}$$

$$\omega(k) = \hbar k^2/2m_1 \tag{4.6c}$$

$$\psi(x,t) = e^{i(k_0x-\hbar k_0^2t/2m)}\, e^{i\delta k'(x-\hbar k_0t/m)}\, e^{-i\hbar(\delta k'^2)t/2m_1} \tag{4.6d}$$

With this expansion first order effects do not cause irreversible incoherence. To see this, consider time $t_1 = \frac{a}{V_g}$ wherein the packet travels a distance "a". V_g is the group velocity:

$$V_g = \left.\frac{\partial \omega(K)}{\partial K}\right|_{K=K_0} \tag{4.7}$$

(For this situation, assume second order - δk^2 - effects are negligible). See Fig. (4.3). The phase at A is completely coherent with that at A', and, that at B with B'. This is true even through the phases at points A and B; and, A' and B' are incoherent.

The coherence persists until the term involving $(\delta k)^2$ becomes significant, over some distance "d" which must be much greater than L in order for the neutron pulse to maintain its integrity during its traversal through the apparatus.

$$\frac{\hbar(\delta K)^2 t_2}{2m_1} \simeq \frac{\hbar}{2m_1 a^2}\frac{d}{V_g} \simeq 2\pi \tag{4.8a}$$

$$\frac{a^2}{L\lambda} \gg 1 \tag{4.8b}$$

The incoherence thus generated is irreversible. The reversible property of the expansion's first order term will be utilized to probe the measurement process. It can be seen with the aid of Eq. (4.1) the condition indicated by Eq. (4.8b) is met, verifying the theoretical consistency of this approach.

D.M. Greenberger[1] considered the problem of a neutron of momentum hk_0 scattering off of a "heavy" movable mirror of momentum ≈ 0. Solving the 2 body Schrodinger equation

$$\left(\frac{P_1^2}{2m_1} + \frac{P_2^2}{2m_2}\right)\Psi(x_1,x_2) = E\Psi(x_1,x_2) \tag{4.9}$$

he showed the system's wave function to be

$$\Psi(x_1,x_2,t) = \left\{e^{ik_0x_1}U(x_1-v_{g_1}t)V(x_2) - e^{-ik_0x_1''}U(x_1''-v_{g_1}t)V(x_2'')\right\} \times e^{-i(\omega_{10}+\omega_{20})t} \tag{4.10}$$

x_1 denoting the neutron's variables, x_2 those of the mirror. The second term pertains to the recoiled system. x_1'' and x_2'' are linear functions of the neutron and mirror post-collision positions:

$$x_1'' = \frac{m_1 - M_2}{M}x_1' + \frac{2M_2}{M}x_2'$$

$$x_1'' \simeq -(1-2\epsilon)x_1' + 2x_2' \tag{4.11a}$$

$$x_2'' = \frac{2m_1}{M}x_1' + \frac{M_2 - m_1}{M}x_2'$$

$$x_2'' \simeq x_2' + 2\epsilon x_1' \tag{4.11b}$$

$$M = m_1 + M_2$$
$$\epsilon = m_1/M_2 \tag{4.11c}$$

This result can easily be applied to the haunted measurement analysis with

x_2 simply corresponding to MS11 as stated previously.

To choose $V(x_2)$'s functional representation we first note momentum conservation demands the transfer of any momentum uncertainty to the neutron. To not upset the neutron's monochromaticity, need

$$\delta K_2 \leq \delta K_1 \tag{4.12a}$$

In as much as Eq. (4.12a) entails

$$\delta x_2 \geq \delta x_1 \tag{4.12b}$$

we will assume

$$\delta K_2 \simeq \delta K_1 \tag{4.12c}$$

and let

$$V(x_2) \simeq B e^{-x_2^2/a^2} \tag{4.13}$$

B = normalization constant

The mirror's position is only approximately known, accordingly, the pulse's relative phasing will be virtually annihilated on impact. D.M. Greenberger[1] calculated the resulting loss in contrast which would occur if beams I and II were superimposed following the collision and showed how this was due to the x_1', x_2' coupling in the neutron's recoil function Eq. (4.10). This is an example of an element from the universe's observer sector interacting with a member from the observed sector. The subsequent incoherence is predicted by Von Neumann. But this is a case of latent order because the collision with the second mirror composing subsystem MS1, MS12, will restore the packet's original coherence. This will occur even though MS1's initial position is uncertain. After the collision with MS11 the neutron will rebound and move towards MS12. The entire subsystem MS1 will also recoil, with MS11 moving away from the neutron and MS12 approaching (Fig. 4.2b). MS1's motion must be sufficient to indicate a collision has taken place, that is to say,

$$V_2' \gg \delta V_2' \tag{4.14a}$$

$$K_2' \gg \delta K_2' \tag{4.14b}$$

This guarantees the mirror's displacement would be so large one could not reasonably attribute it to quantum mechanical uncertainty. This is also how we define a "macroscopic effect". Equations (4.14) are satisfied due to momentum conservation and to the mass relationship $M_2 \gg m_1$ which causes the neutron to reverse itself:

$$K_2' \simeq 2K_1 \tag{4.15a}$$

$$\delta K_2' \simeq 2\delta K_1 \ll K_1 \simeq K_2'/2 \tag{4.15b}$$

$$V_2' \simeq \frac{2m_1}{M_2} v_1 \tag{4.15c}$$

The relative speed of the neutron - MS1 system, v_1, is preserved - a property of elastic collisions. The neutron and MS12 will thus collide at time $T = L/v_1$. In the relative coordinate reference frame at the time of impact MS12 would have traveled a distance d_R resulting from the neutron's initial impact with MS11. (See Fig. 4b). To first order in $\frac{m_1}{M_2}$, d_R is found to be

$$d_R \simeq V_2' T \simeq \frac{2m_1}{M_2} v_1 T \simeq \frac{2m_1}{M_2} L \tag{4.16}$$

The distance is independent of v_1, allowing all momentum components in the neutron packet to strike MS12 at the same location in space, thereby reversing all previously imparted first order uncertainties.

MS1 has endured a net shift in position of d_R which must be larger than "a" to insure the displacement's macroscopicity. This requirement is fulfilled and sets an experimental upper limit on M_2, for,

$$V_2' > \delta v_1 \tag{4.17a}$$

Using Eq. (4.15c)

$$\frac{V_2'}{\delta v_1} = \frac{2m_1}{M_2} \frac{v_1}{\delta v_1}$$

$$= \frac{2m_1}{M_2} \frac{K_1}{\delta K_1}$$

$$\frac{V_3'}{\delta v_1} \simeq \frac{2m_1}{M_2} \frac{a}{\lambda} > 1 \tag{4.17b}$$

By Eqs. (4.1) and (4.3)

$$\frac{2m_1}{M_2} \frac{L}{a} \gg \frac{2m_1}{M_2} \frac{a}{\lambda} > 1$$

$$\frac{2m_1}{M_2} \frac{L}{a} \simeq \frac{d_R}{a} \gg 1 \tag{4.17c}$$

For the actual neutron interferometer the Bragg window = $\frac{\delta K_1}{K_1} \sim 10^{-6}$. M_2 is subsequently limited to $<10^6$ neutron masses.

To obtain the final wave function $\psi_{II\ REC}$ characterizing beam II after the neutron rebounds from MS12 one performs a calculation analogous to that leading to Eq. (4.10):

$$\psi'_{II\ REC} = e^{i(Kx+\phi)} U(x_1 - v_g t - \ell)\, v(x_2 + d_R)\, e^{-i(\omega_{10}+\omega_{20})t} \qquad (4.18)$$

Note the decoupling of the neutron and mirror variables and the maintenance of the original functional form indicating the restoration of the pulse's relative phasing. Nevertheless, the neutron wave packet is delayed by ℓ from the position it would occupy were the mirrors infinitely massive where

$$\ell = \frac{4m_1}{M_2} L > a \qquad (4.19)$$

At this juncture beam II is incoherent with respect to beam I (Fig. 4.1)

$$\psi_I = e^{-iK_0 x} U(-x + v_{g_1} t)\, v(x_2)\, e^{-i(\omega_{10}+\omega_{20})t} \qquad (4.20)$$

due to 2 effects; the neutron and mirrors' wave functions for $\psi_{II\ REC}$ each deviates from its counterpart in ψ_I by a distance greater than "a". Beam I can be delayed by the appropriate amount if the proper device is positioned in the path ACD. This, however, would still leave the macroscopic path difference for the mirrors' wave functions. Specifically, the system's wave function would be

$$\Psi(x_1,x_2,t) = \Psi_I' + \Psi_{II\,REC}'$$
$$= e^{-i(\omega_{10}+\omega_{2})t}\{e^{i\phi}e^{ik\alpha}U(x_1-v_{g_1}t+\ell)\,V(x_2+d_R) + e^{i\phi'}e^{-ik\alpha}U(-x+v_g t+\ell)\,V(x_2)\} \quad (4.21)$$

with point A the new coordinate origin and t measured from the time the neutron strikes point A. It is evident the manifestation of an interference pattern at D requires

$$\langle V(x_2+d_R)\,|\,V(x_2)\rangle \neq 0 \quad (4.22)$$

which is not the case.

To summarize, a measurement of the neutron path has been conducted. The trace left by the neutron during its passage is permanent. One could observe MS1's position at any later time to ascertain the neutron's trajectory through the interferometer. In essence, a macroscopic record equivalent to a pointer reading in its information content is secured with this experimental arrangement. In accordance with the uncertainty principle the interference pattern - overt coherence - is destroyed. Von Neumann cites the irreversible production of a mixture during the measurement procedure as the cause for the experimental results:

$\psi_{initial}$ = incoming beam at point A, a pure state $\longrightarrow$ $\rho=\sum_{i=1}^{2} w_i|\phi_i\rangle\langle\phi_i|$ = mixture at D. (4.23)

$|\phi_1\rangle$, $|\phi_2\rangle$ designates state of beam I, II respectively

w_i = i'th eigenvalue of ρ

We shall demonstrate with the introduction of MS2 into beam II that the procedure is in fact reversible, owing to the presence of latent order in the system. This suggests the reversibility question has a subjective component. Measurements may appear irreversible and incoherence producing because we do not know how to reintroduce coherence - the experimentalist simply does not keep track of the phases introduced by the devices interacting with the system under investigation.

C. Phase Two-Measurement Reversal and the Reintroduction of Overt Coherence

The way to reintroduce coherence between beams I and II and thereby reconstruct the interference pattern is to undo MS1's macroscopic displacement. Rigidly appending MS2 to MS1 (Fig. 4.1), would produce the requisite nullification. To see this: the MS1-MS2 megasystem, call it MS, is restricted to motion along the x axis. The neutron displaces MS by d_R when it collides with the subsystem MS1. The particle subsequently rebounds from point B and enters MS2 causing the megasystem to return to its original location. The measurement - the interaction with MS1 - is "haunted" by the later impact with MS2. If we designate the time the pulse interacts with MS1 as t_1 and the time the pulse enters MS2 as t_2 we see the existence of a window in time $\Delta t = t_2 - t_1$ during which a record of the particle path exits. Superficially this procedure may appear to be inconsistent with the uncertainty principle: one might be able to observe the motion of MS, thereby establishing particle path exactly and also be able to retain the interference pattern. This, however, is not the case. For one to actually observe MS's position during the interval Δt, one must for example interact with MS with a laser and record the reflected beam's Doppler shift. This act will affect the phases of the neutron-MS wave function and in accord with the uncertainty principle, destroy the final interference pattern. Nevertheless, it is in principle evident, if the experimentalist painstakingly keeps track of all phases injected into the observed system, the conducted measurement can be reversed. The overt incoherence which masks information regarding the system's complementary properties can thus be undone. Using ρ to characterize the observed system as in Eq. (4.23) is not a fundamental necessity as von Neumann stipulates but rather due to a lack of knowledge regarding the states of the various interacting systems. Two excellent examples to support this assertion are the spin echo experiment[2] and optical phase conjugation.[3,4,5] In the first case an apparently disordered spin ensemble regains its coherence with its exposure to a magnetic field and produces a strong nuclear induction signal. Optical phase conjugation incorporates the time reversal property of electromagnetic waves to produce an antidistorted wave from a distorted one obtained when a laser beam is transmitted through an amplifying medium. The antidistorted wave (phase conjugated beam) is retransmitted through the inhomogeneous amplifying medium, propagating backward with respect to the original beam. The output beam is very powerful and undistorted. It is thus clear that amplification, an essential component of the measurement process, does not a priori produce incoherence.

D. Applications to the Delayed Choice, Schrodinger's Cat, and EPR Experiments

The issues surrounding the various "measurement paradoxes" in quantum mechanics are brought into exceptionally sharp focus with the proposed gedankenexperiment. The haunted measurement is an example of Wheeler's "delayed choice" experiment.[6] Classically, the neutron must "choose" to traverse the interferometer via path I or II at point A (Fig. 4.1). One can discover which by observing MS during the interval Δt. We have seen previously this "choice" is in truth observer dependent. If one does not observe MS an interference pattern is exhibited indicating passage by means of both trajectories. From a classical viewpoint the measurement, conducted after the neutron has passed point A, affects the "particle's" earlier behaviour. Quantum theory contrarily demands coherence to exist between both possible results, both sets of complementary properties, until the system is disturbed.

Schrodinger's cat experiment[7] focuses on the superposition aspect of ψ. Here, a cat is either dead or alive depending on the outcome of a quantum event. His state is characterized by a wave function ψ, a linear combination of the alive and dead states; and, only after an observation will we know the cat to be in a definite state. There are two quantum theoretical peculiarities exposed by this gedankenexperiment. The first, the suspended animation aspect, is essentially the same feature highlighted by the delayed choice experiment. In terms of the haunted measurement, the neutron - our cat - is in suspended animation in regard to its path designation. Only after looking at MS within Δt will we know what state the neutron is in. There is an additional bonus obtained with the haunted measurement. For Schrodinger's experiment the superposition postulate cannot be demonstrated prior to observation whereas for the haunted measurement if one does not look at MS, the coherence between states is displayed by the existence of the interference pattern.

Due to this supplemental property our "cat" can even be resurrected. By observing MS one will discern the neutron took path ABD 50% of the time, corresponding to the cat being alive. The other half of the time the neutron followed path ACD, that is, the cat is dead. If no measurement is performed until after Δt MS will no longer be displaced, coherence is maintained. This is equivalent to the cat being resurrected in 50% of the cases!!! We, therefore, have a "haunted" version of the cat experiment.

The second peculiarity involves the cat's macroscopic nature. The unique treatment of macroscopic systems in quantum theory has already been examined. Our megasystem MS is macroscopic with one degree of freedom. It is therefore not representative of the most general type of macrosystem. Nevertheless, we have shown a macroscopic system can be represented by a coherent linear state combination in the indicated limit. In principle then we can maintain the superposition principle is valid for the cat with its many degrees of freedom.

Our gedankenexperiment is also pertinent to the EPR paradox. In 1935 Einstein, Podolsky and Rosen suggested the following "minimal" criterion of an element of physical reality:

If, without in any way disturbing a system, we can predict with certainty (i.e., with probability equal to unity) the value of a physical quantity, then there exists an element of physical reality corresponding to this physical quantity.[8]

They conclude the system must have had this property prior to the observation procedure. Quantum theory's depiction of physical reality of course violates this simplistic and common-sense rule. It denies such properties exist prior to measurement even if this measurement is remote.

In terms of the haunted measurement the particle's trajectory is an element of reality since by judiciously looking at MS we can determine whether the neutron is in path I or II. This is a remote measurement on beam I in as much as MS is located in beam II. EPR would assert the neutron consequently has a definite trajectory at all times, whether or not a measurement is carried out. But as discussed previously, to associate a path with the neutron is impossible in principle without a measurement. Thus an element of reality apparently is developed by beam I when the neutron interacts with MS1 and is lost if no observation is conducted when the neutron strikes MS2. Apparently the very criterion used to delineate an element of reality whose existence EPR assert is assured for all time in the absence of an interaction, also permits us to destroy an element of reality in like fashion. EPR's common-sense criterion is simply inconsistent.

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CHAPTER V

CONCLUSION

We have shown it is theoretically possible within the domain of quantum mechanics to perform simultaneous measurements in order to acquire information regarding two or more noncommuting variables. The accessible information can be represented in an intuitively satisfying form without appealing to artificial constructions. The complementary properties of a system in a pure state can be simultaneously developed to a larger degree than those for a system in a mixed state; this development being determined by the measurement procedure.

The measurement process, within the limits analyzed, introduces overt incoherence derivable from the uncertainty principle. One, therefore, need not introduce another evolutionary mechanism (method 2) to adequately describe it; Schrodinger's equation is sufficient. Method 2 is an approximation indicating the observer has not properly followed and accounted for all macroscopic interactions involving the observed system.

Once a measurement to absolutely determine one of the system's properties is performed, the property is manifested and information is obtained. The complementary property, nevertheless is washed out. Latent coherence - the quantum mechanical correlations existing between the observed system and the measuring apparatus - permits one to reverse the procedure, reintroducing the system's complementary properties and the corresponding (latent) information. For example, we can combine the experiments depicted in Chapters III and IV by placing an absorber in beam II at some location (x,z) before the megasystem MS positioned at (x_2,z_2):

$$x<x_2 \qquad\qquad (5.1)$$

$$z<z_2$$

If no observation is performed during Δt we have some knowledge regarding the path the exiting neutrons followed and an interference pattern is produced. Specifically, once the pulse in beam II leaves MS1 there is a macroscopic trace indicating particle path. The overt coherence is thereby masked, the neutron's wave feature hidden. Once the pulse collides with MS2, overt coherence is restored and the initial information regarding the neutron's trajectory and wave nature holds.

With this study of coherence effects in the measurement process we have been able to illustrate the subtle role quantum mechanical correlations play in the macroscopic realm. Unless such correlations are properly treated, a truly general and complete measurement theory analysis does not appear feasible.

APPENDIX

THE DENSITY OPERATOR

Several excellent references are available on density matrix theory.[1,2,3] Below the basic theory is summarized.

1. The Pure State

A pure state is characterized by the existence of an experiment that gives a result predictable with certainty when performed on a system in that state and in that state only. Such an experiment is called "complete."[1] For example, if its possible to find an orientation of the Stern-Gerlach device for which a given beam is completely transmitted, then we say the beam is in a pure spin state specified by the magnetic field's direction and the joint state of all the particles can be represented in terms of one and the same state vector[3] $|X\rangle$.

One can also identify a pure state by particularizing the operator corresponding to the experiment. For measurements of a neutron beam's spin direction, the Pauli matrices are the appropriate operators and its state is given in terms of the polarization vector[1] $\vec{P}$, the i'th component being

$$P_i = \langle \sigma_i \rangle \qquad i = x, y, z \tag{A.1}$$

and

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \tag{A.2}$$

Physically $\vec{P}$ can be procured by performing spin determinations on subensembles of the beam.

When it is not convenient to identify a pure state by either of the above methods the state may be identified as a linear superposition of eigenstates of any suitable complete set of operators.[1] A general pure spin state can be represented by a linear superposition of the eigenstates of σ_z:

$$|X\rangle = a_1\begin{pmatrix}1\\0\end{pmatrix} + a_2\begin{pmatrix}0\\1\end{pmatrix} = \begin{pmatrix}a_1\\a_2\end{pmatrix} \qquad (A.3)$$

The coefficients a_1 and a_2 can be parameterized in terms of 2 angles whose physical significance is evinced with the aid of the polarization vector. Let

$$a_1 = \cos\frac{\theta}{2}, \qquad a_2 = e^{i\delta}\sin\frac{\theta}{2} \qquad (A.4)$$

A system in the state $|X\rangle$ will have a polarization vector whose components are

$$P_x = \sin\theta\cos\delta \qquad (A.5a)$$

$$P_y = \sin\theta\sin\delta \qquad (A.5b)$$

$$P_z = \cos\theta \qquad (A.5c)$$

$$\text{and } |\vec{P}| = 1 \qquad (A.5d)$$

θ and δ can be interpreted as the polar angles of $\vec{P}$. See Fig. A.1. If a beam is sent through a Stern-Gerlach apparatus oriented parallel to $\vec{P}$, the entire beam will pass through. The direction of the polarization vetor is therefore the direction in which the spins are pointing.[3]

2. The Mixed State

Quantum mechanical systems exist for which an experiment which would otherwise be complete, does not give a unique result predictable with certainty. Such systems are called mixtures, equivalently, we can say any one of the systems is in a mixed state. For example, if a beam's polarization state is investigated by sending it through a Stern-Gerlach filter in various orientations and it is found that it's impossible to find an orientation which permits the entire beam to be transmitted, then the beam is in a mixed state.

A mixture is described by indicating the way in which it was prepared.[3] To acquire the polarization vector for a beam consisting of 2 independently prepared subbeams with N_A and N_B particles in states $|X_A\rangle$ and $|X_B\rangle$ respectively, one must statistically average the separate beams:

$$\vec{P} = \frac{N_A}{N}\langle X_A|\vec{\sigma}|X_A\rangle + \frac{N_B}{N}\langle X_B|\vec{\sigma}|X_B\rangle \qquad N = N_A + N_B \tag{A.6}$$

$|\vec{P}|$ will be less than 1 since if

$$\vec{P}^{(i)} = \langle X_i|\vec{\sigma}|X_i\rangle \qquad i = A, B \tag{A.7}$$

then from A.5d $|\vec{\beta}(i)| = 1$. Hence,

$$|\vec{P}|^2 = \left(\frac{N_A}{N}\right)^2 + \left(\frac{N_B}{N}\right)^2 + \frac{2N_A N_B}{N^2}\vec{P}^{(A)}\cdot\vec{P}^{(B)}$$

$$|\vec{P}|^2 < \left(\frac{N_A + N_B}{N}\right)^2 = 1 \qquad \text{(A.8)}$$

3. The Density Operator and Matrix

The density (statistical) operator for a system consisting of N independently prepared subsystems each in a state $|X_i\rangle$, i=1,2,..N is given by

$$\rho = \sum_{i=1}^{N} W_i |X_i\rangle\langle X_i| \qquad \text{(A.9a)}$$

Here, $|x_i\rangle$ normalized: $\langle X_i|X_i\rangle = 1$ (A.9b)

W_i = the statistical weight of the i'th state: $\sum_{i=1}^{N} W_i = 1$ (A.9c)

The pure state is the case N=1.

To construct the density matrix [ρ], a complete set of orthonormal states is chosen for the basis and $|X_i\rangle$ is expanded accordingly:

$$|X_i\rangle = \sum_{\ell=1}^{M} a_\ell^{(i)} |\phi_\ell\rangle \qquad \text{(A.10)}$$

Substituting A.10 into A.9 will yield the matrix in the designated representation:

$$[\rho] = \sum_{i l l'} W_i a_l^{(i)} a_{l'}^{(i)*} |\phi_l\rangle\langle\phi_{l'}| \tag{A.11}$$

If the matrix is not diagonal the system is said to be a coherent superposition of the basis states. A diagonal density matrix with more than one nonzero element denotes a system which is an incoherent superposition of the basis states.[3]

4. Properties of the Statistical Operator

a. ρ is Hermitean

$$\rho = \rho^{+} \tag{A.12}$$

b. The probability of finding a system characterized by the density operator ρ in the state $|\psi\rangle$ is given by

$$\langle\Psi|\rho|\Psi\rangle \tag{A.13}$$

c. The trace of [ρ] is a constant independent of the representation. From A.9b,c and A.10

$$\mathrm{Tr}[\rho] = \sum_i \rho_{ii} = 1 \tag{A.14}$$

d. The expectation value of any operator $Q=\langle Q\rangle$ satisfies

$$\langle Q\rangle = \mathrm{Tr}(\rho Q) \tag{A.15}$$

If $|X_i\rangle$ is not normalized (see A.9, A.14) then

$$\langle Q\rangle = \frac{\mathrm{Tr}(\rho Q)}{\mathrm{Tr}\,\rho} \tag{A.16}$$

e. $(T_r\rho^2) = (T_r\rho)^2$ for pure state (A.17a)

$(T_r\rho^2) < (T_r\rho)^2$ for a mixture (A.17b)

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FIGURE CAPTIONS

Fig. 1.1 Einstein's version of the double slit experiment. The photon's path and the interference pattern were to be determined simultaneously. S(n) denotes the n'th screen. ξ indicates the interference pattern's extension along the x axis. The distance between adjacent screens is given by L. Slits A and B are separated by a distance S which is much larger than each slit's width.

Fig. 2.1 Diagram of the LLL interferometer and the 3 counters (usually ^{3}He detectors). The dimensions a and d are machined to optical precision. Reprinted from Rep. Prog. Phys. 46, 301 (1983).

Fig. 2.2 The "top view" of the interferometer. The lattice planes, usually the 220 planes, are continuous from slab to slab. At F a device - a gas cell, a solid material sample or a magnetic field producing system - is inserted. Such a device will cause a phase shift β. From Physics Today 33, no. 9, 24 (1980).

Fig. 3.1 Model of the neutron interferometer. M and M' are half silvered mirrors. M1 and M2 are fully reflecting mirrors. The aluminum phase shifting device at E is used to produce the "reference" interference pattern with repect to which the phase of the altered pattern resulting from the presence of the measuring devices or state preparation instruments at F and F' is compared. The output monitoring devices C1 and C2 will be either a line of counters or a pair of Stern Gerlach apparatuses. C1 receives the forward beam, whereas C2 samples the deviated disturbance.

Fig. 3.2 The solid curve is the counting rate of either counter as a function of x- the position of E- in the absence of a measuring apparatus at F or F' in Fig. 3.1. The dotted curve is obtained when a device is placed at F and/or F'. ϕ indicates the phase shift induced by the experiment.

Fig. 3.3 Graph of the information measures, $H(p)/H_0$ and C^2 vs. probability. The solid curve is C^2. The dashed curve is $H(p)/H_0$.

Fig. 3.4 The polarization vector $\vec{P}$.

Fig. 3.5 Graph of the magnetic field's orientation in the xy plane. The field is produced by a Stern Gerlach device.

Fig. 4.1 Optical analog of the neutron interferometer. At point A the incoming beam is bisected by the semitransparent mirror with each component specularly reflected by the mirrors at points B and C. MS1 and MS2 are identical sets of rigidly attached double mirrors. The output beams interfere on the screen located at D. Only MS1 will be present during phase one. MS2 will be inserted and rigidly attached to MS1 for phase two. The device at F will delay beam ACD the extra distance ℓ travelled by the beam ABD.

Fig. 4.2a The MS1 subsystem. The elements of MS1 are the mirrors MS11 and MS12.

Fig. 4.2b The MS1 subsystem after the interaction with the neutron pulse. MS1 moves along the x axis with a velocity v_2' after the neutron-MS11 collision. The neutron later strikes MS12 bringing MS1 to a halt. MS1's net displacement from equilibrium is d_R.

Fig. 4.3 Diagram of the neutron pulse at 3 different times. The packet's mean wavelength λ is much less than "a". Points A and B in the packet are incoherent. At time t_1 corresponding to the pulse's propagation a distance "a", points A and A' are coherent as are B and B'. After some time t_2, the pulse travels a distance d over which the second order effects become significant. Such terms initiate incoherence.

Fig. A.1 Diagram of the polarization vector.

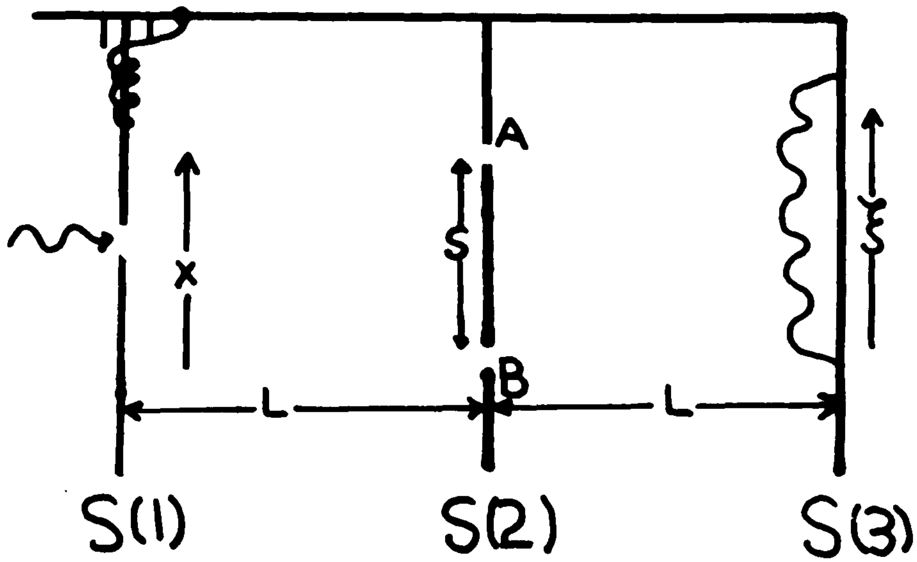


FIG. 1.1

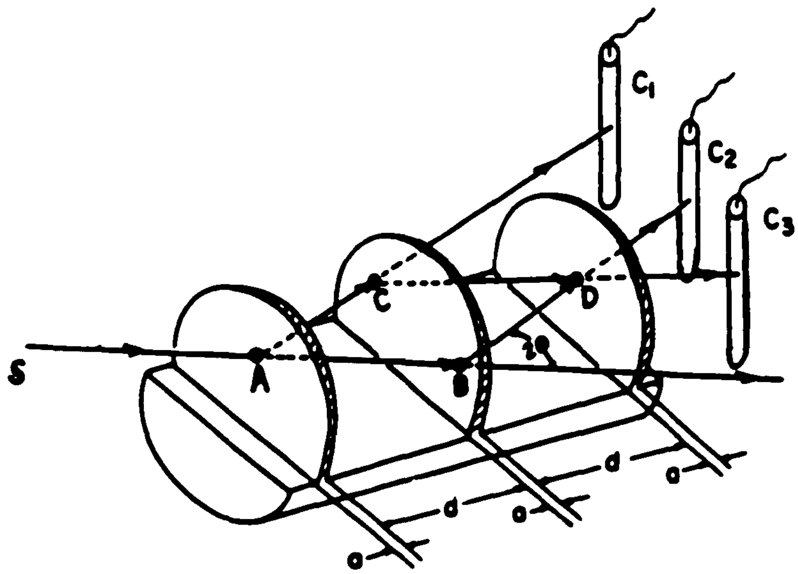
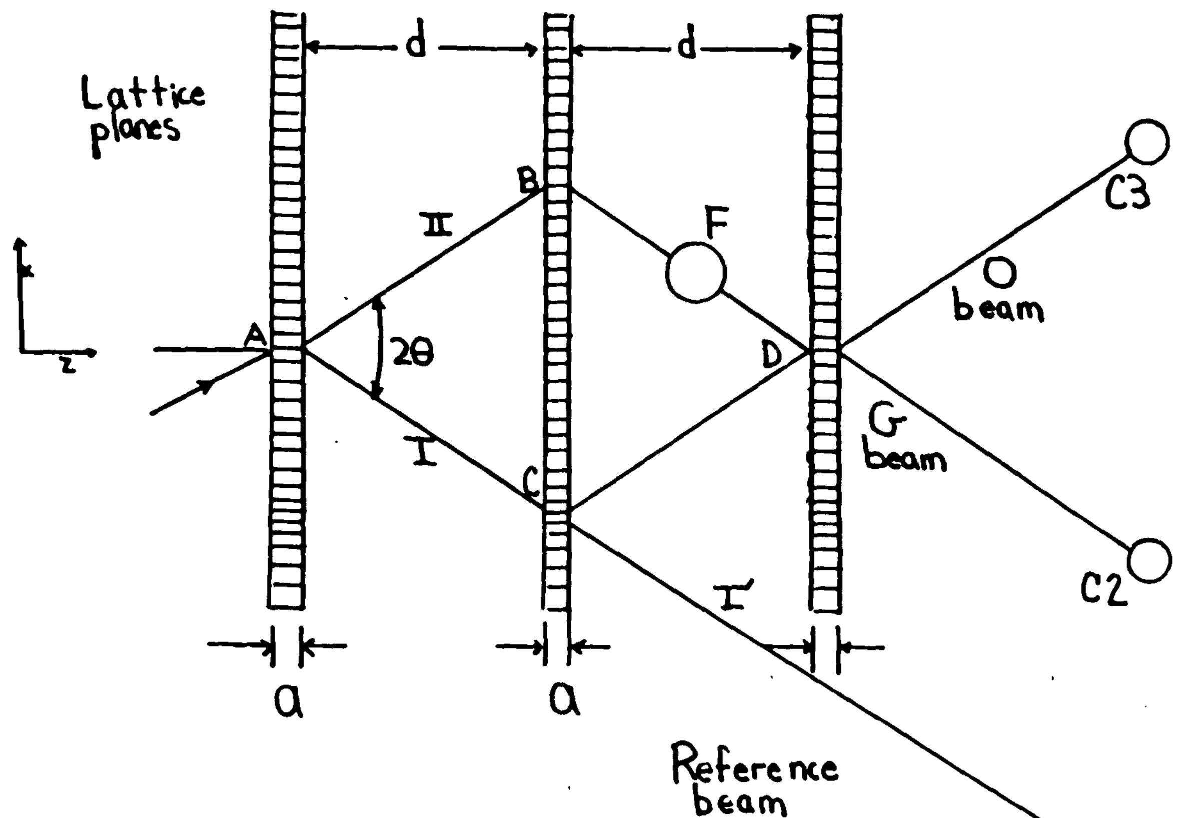


FIG. 2.1



Lattice planes
d
d
A
B
C
D
F
II
I
I'
2θ
a
a
C3
C2
C1
O beam
G beam
Reference beam
x
z

FIG. 2.2

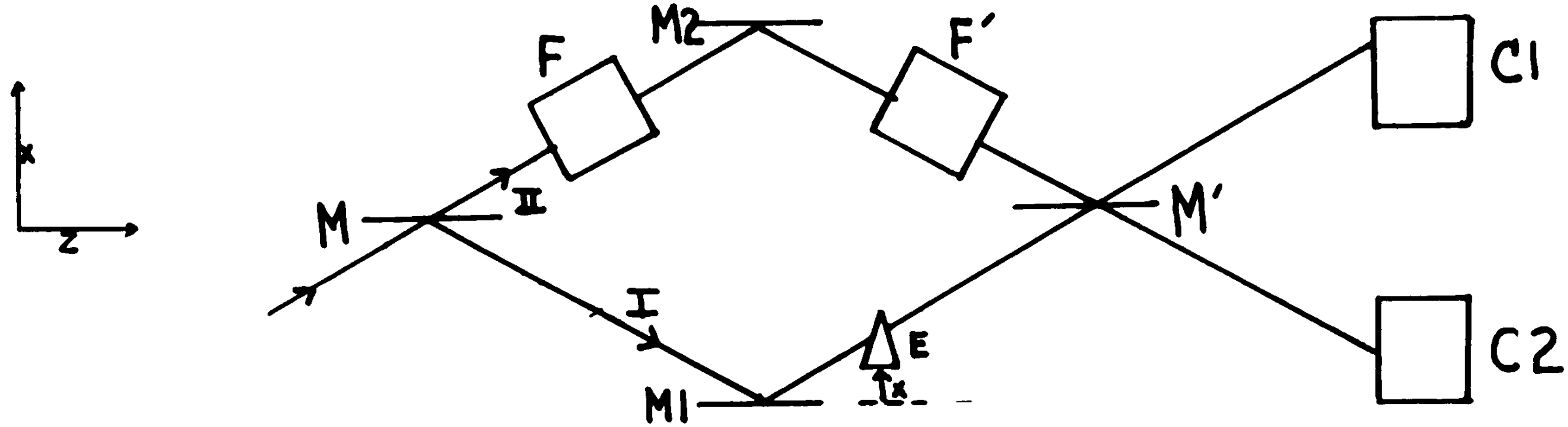


FIG. 3.1

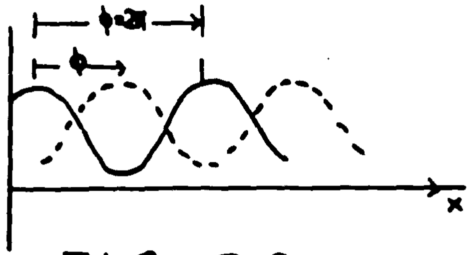


FIG 3.2

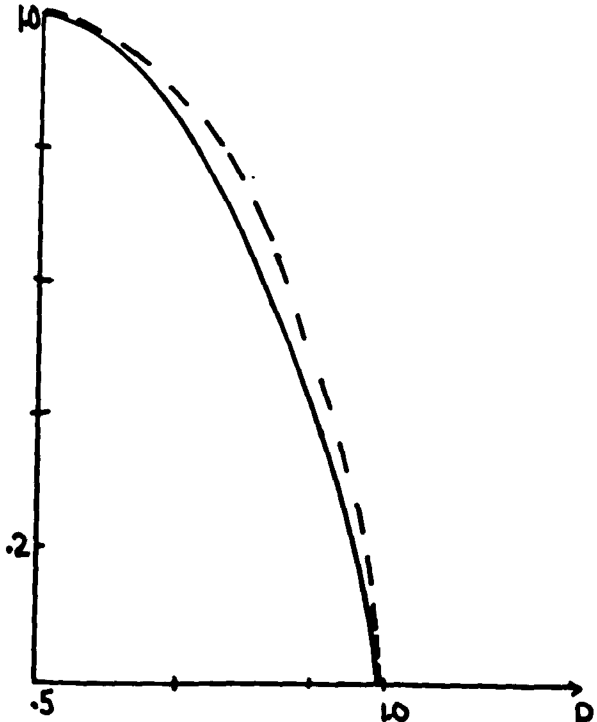


FIG. 3.3

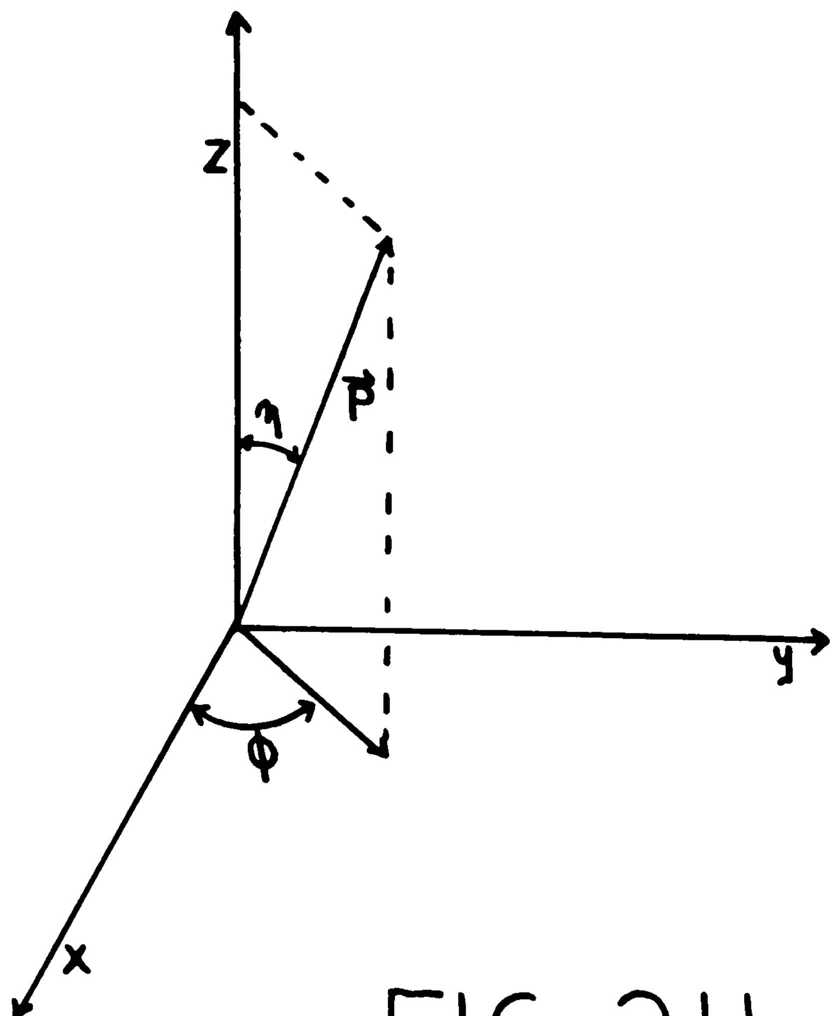


FIG. 3.4

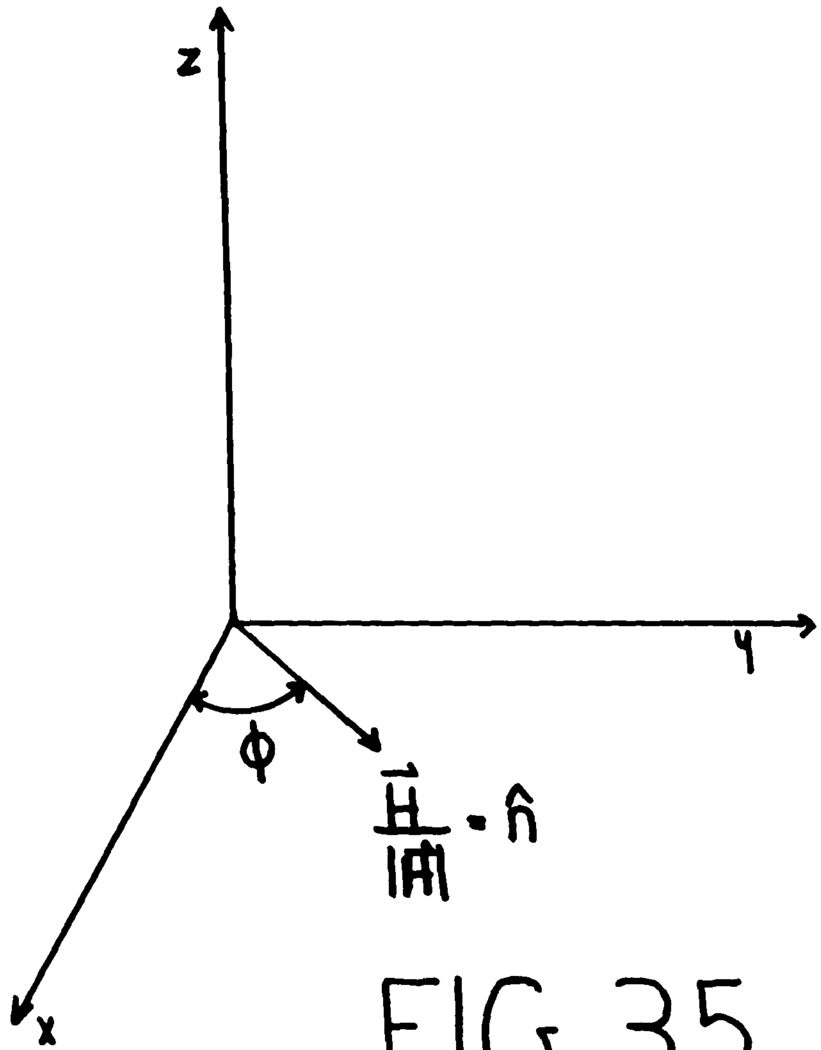


FIG. 3.5

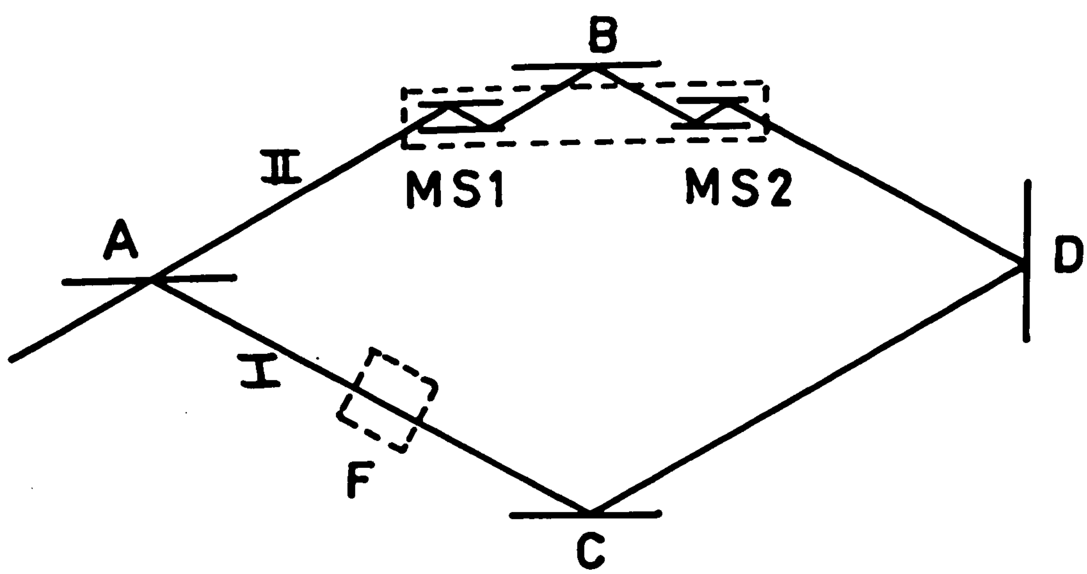


FIG. 4.1

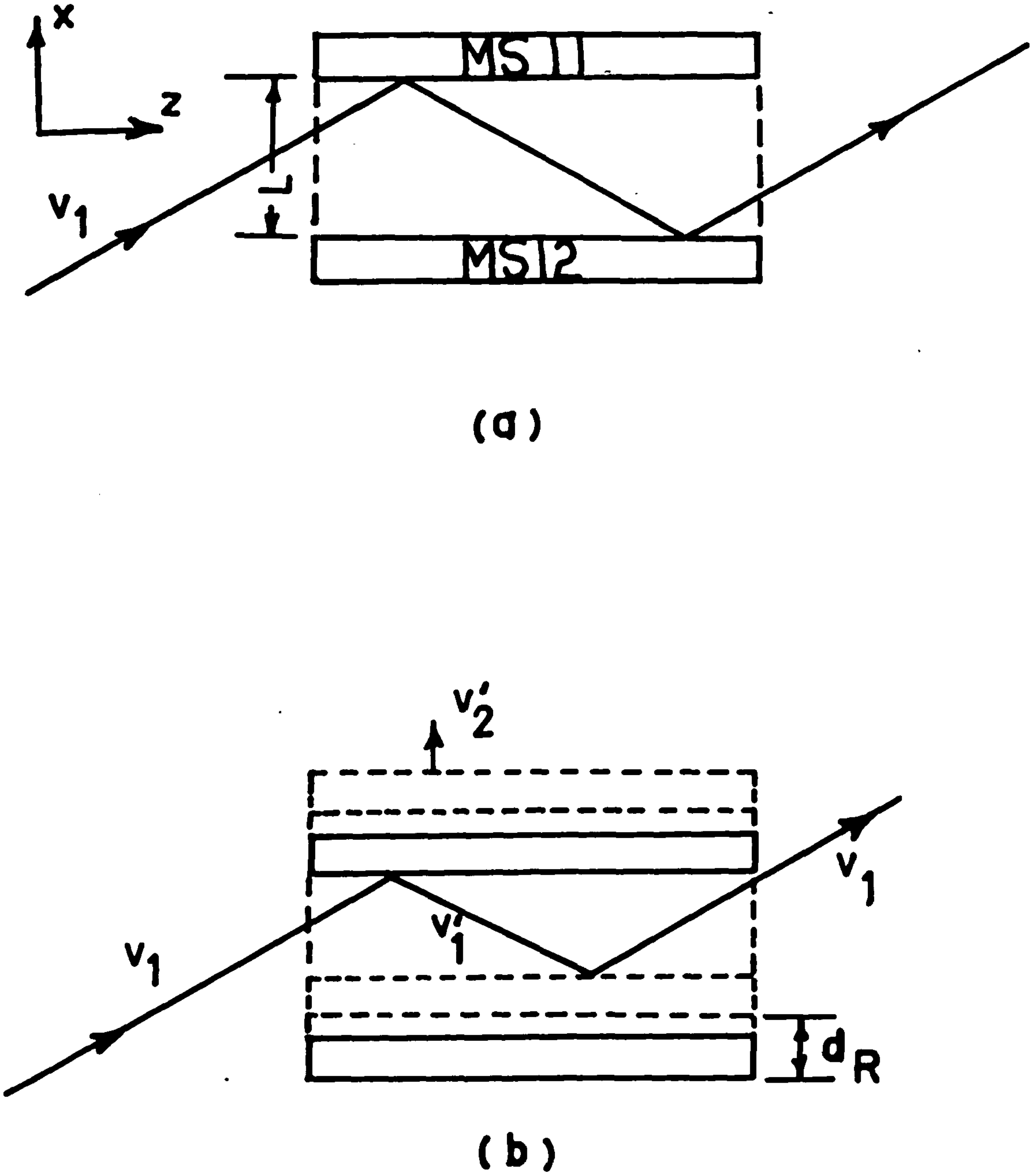


FIG. 4.2

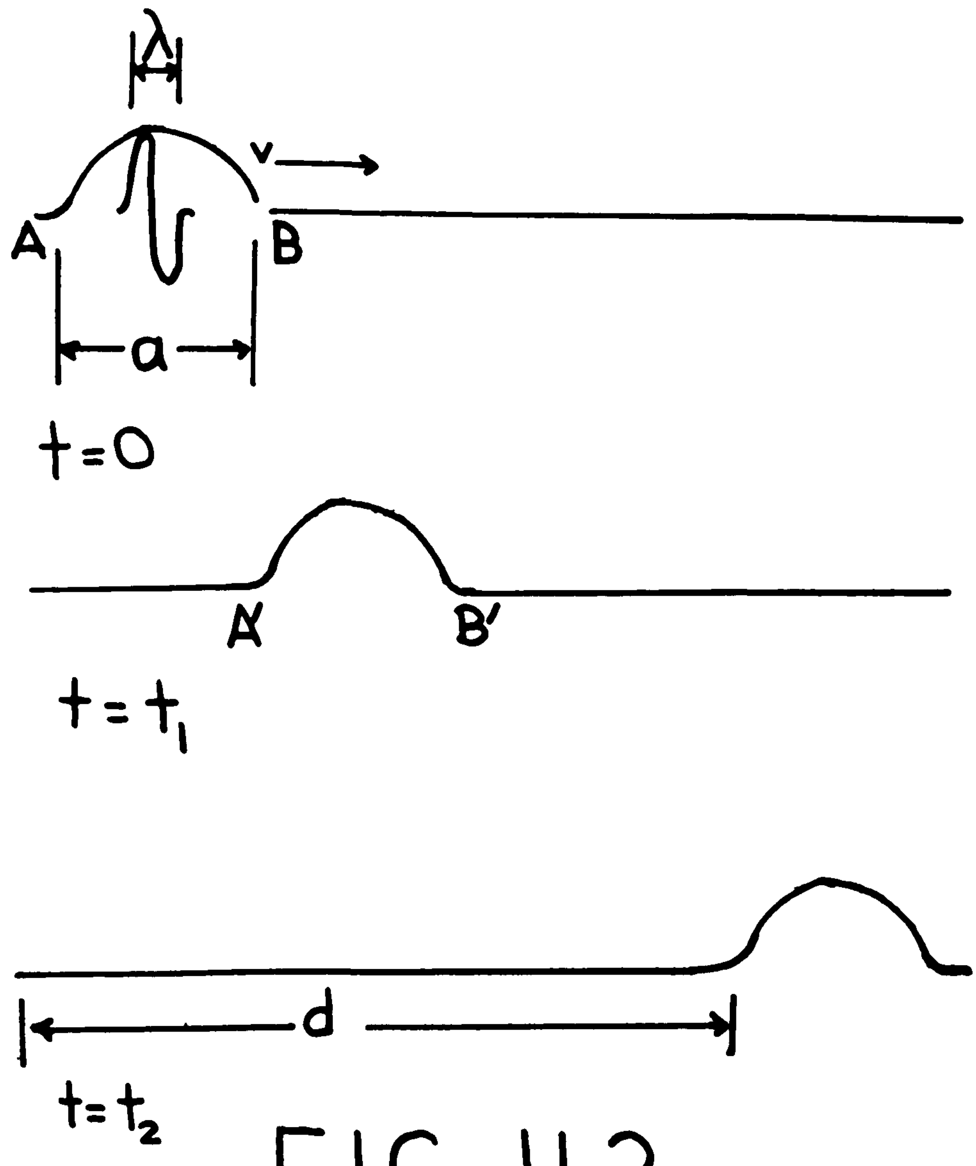


FIG. 4.3

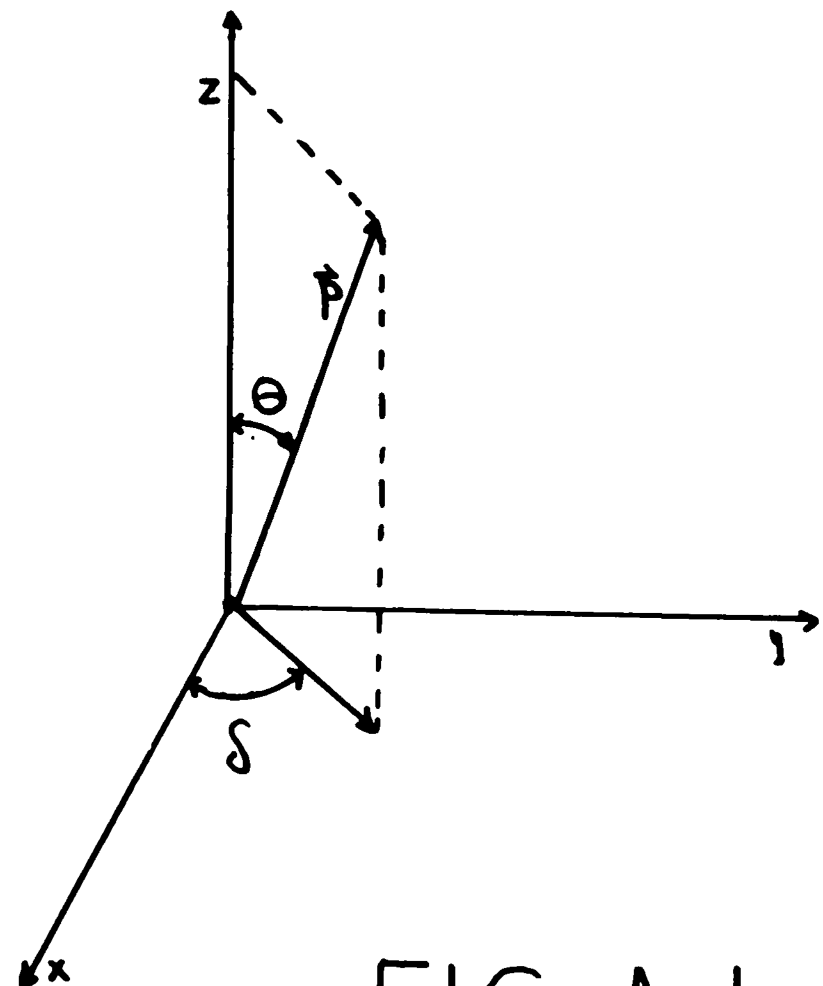


FIG. A.1

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