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**Li Shanlan: The impact of Western mathematics in China
during the late 19th century**

Horng, Wann-Sheng, Ph.D.

City University of New York, 1991

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A

**LI SHANLAN: THE IMPACT OF WESTERN MATHEMATICS
IN CHINA DURING THE LATE 19TH CENTURY**

by

Wann-Sheng Horng

A dissertation submitted to the Graduate
Faculty in History in partial fulfillment of
the requirements for the degree of Doctor of
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1991

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Abstract

LI SHANLAN: THE IMPACT OF WESTERN MATHEMATICS IN CHINA DURING THE LATE 19TH CENTURY

by

Wann-Sheng Horng

Li Shanlan (1811-1882) played a crucial and significant role in the transition from traditional to modern mathematics in China. His important place in the history of modern Chinese mathematics can best be appreciated in terms of his roles first as traditional mathematician, then translator of Western mathematics and science, and finally instructor at the Tong Wen Guan (the Imperial College at Beijing). These three roles were assumed by Li Shanlan during three distinct periods of his career, the "Ze Gu Xi" period (1845-1852), the "Mo Hai Shu Guan" (the London Missionary Society Press at Shanghai) period (1852-1860), and the Tong Wen Guan period (1868-1882).

In the first period, Li Shanlan's contributions to infinite series expansions for trigonometric, inverse-trigonometric and logarithmic functions served to establish him as a clear leader of the Chinese mathematical community in the middle of the 19th century. Thus prepared, Li Shanlan went on to become a very successful translator at the Mo Hai Shu Guan. His translations, in collaboration with Alexander Wylie and Joseph Edkins, of texts

on Western mathematics and science paved the way for Chinese traditional mathematics to merge into the mainstream of modern mathematics. In 1868, Li Shanlan began to teach mathematics at the Tong Wen Guan in Beijing. Here he was to play a significant role in helping to promote the institutionalization and professionalization of mathematics as a serious academic discipline in China.

Li Shanlan's life spanned two important periods of late Qing intellectual history. One is the period (1736-1820) during which the Qian-Jia school was predominant. The other is the period (1860-1890) when the Qing government was forced to Westernize or modernize in response to the Self-strengthening Movement. Each of these served to provide a favorable intellectual context and a suitable institutional basis for mathematics. In the rapid modernization of mathematics in China during the 19th century, Li Shanlan proved to be a critical figure at a crucial juncture in Chinese history.

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**Li Shanlan: The Impact of Western Mathematics
in China during the Late 19th Century**

by

Wann-Sheng Horng

**1. INTRODUCTION. The nature and significance of Chinese
mathematics in the 19th century: general orientation**

China of the nineteenth century, as in earlier centuries, is usually seen by modern historians as dominated by the old Chinese ruling-class view of the world. "In this *ancien régime*, the classical learning tolerated only change-within-tradition, the extended family system dominated the individual, a doctrine of duty eclipsed any doctrine of rights, civil administrators controlled the military and used the merchants, and the principles of moral conduct took precedence over human passions, material profit, and the letter of law," [Fairbank 1978, p. 2].¹ Under such circumstances, China's *ancien régime* sustained by an agrarian-bureaucratic system could not compete with Great Britain and other expanding empires of international trade and gunboats. Therefore a series of attempts by western powers to infiltrate and control China after 1840 proved quite successful: the Opium War of 1840-42 was followed within 15 years by the Anglo-French invasion of 1857-60, within another decade by Russia's occupation of Ili in 1871, Japan's take-over of Liu Qiu in 1874, and within yet another decade by the Sino-French war of 1883-85. Nine years later came

Japan's crashing defeat of China's forces in 1894-95, followed by the "Scramble for Concessions" of 1898 and the Boxer War of 1900.

"In retrospect, China's nineteenth-century experience therefore became a stark tragedy, an unforeseen and certainly enormous decline and fall almost without equal in history," [Fairbank 1978, p. 3]. Under such circumstances, the Qing empire was clearly ill-equipped to face such external threats to its hegemony. Indeed, late Qing intellectuals took few active measures to respond to the unprecedented challenge. Their reaction is not surprising in light of the fact that "imperial Confucianism had become a supranationalistic system which could not easily appreciate the sentiments of nationalists from outside countries." As a faithful witness to this disastrous historical process, the reformer Liang Qichao stated as late as 1896, "Here is a big mansion which has lasted a thousand years.... It is still a magnificently big thing, but when wind and rain suddenly come up, its fall is foredoomed. Yet the people in the house are still happily playing or soundly sleeping ... quite indifferent," [Quoted in Fairbank 1978, p. 6].

Comparison of the stubborn resistance of late Qing intellectuals to Western incursions with the more successful strategy adopted by Japan is instructive. Japan was also forced to open its gate to Westerners in the 1850s, but unlike China it nevertheless understood and accepted the importance of joining the international community. During the Meiji reign, Japanese

intellectuals endeavoured to learn everything from the West. Unlike China's "Zi Qiang Yun Dong"² (Self-strengthening Movement) of the 1860s, the Meiji Restoration successfully created a wholly new political and socio-economic system to promote the modern development of Japanese science and technology. The success of the Meiji Restoration was reflected directly in Japan's victory over China in 1894-95.

At the very least, comparative study of the "Zi Qiang Yun Dong" (Self-strengthening Movement) with the Meiji Restoration suggests that late Qing intellectuals failed to respond sufficiently to the Western pressure. One of the reasons might have been due in fact to "The very success which China's civilization had achieved in pre-modern times," [Fairbank 1978, p. 6]. Consequently, most Qing intellectuals never considered the possibility that China needed to make any fundamental changes in her political or socio-economic system to respond to the unexpected challenge from Western powers or even Japan.

1.1. A Short Survey of Chinese Mathematics in the Song-Yuan Period

There is no doubt, historically, about the extraordinary success of Chinese civilization. In the fields of science and technology, China's achievements were especially striking in pre-modern times, above all when compared with its western counterparts. It is by now a commonplace to say that Joseph

Needham's *Science and Civilisation in China* provides voluminous evidence to justify this assertion. In the Song and Yuan periods, Chinese scholars achieved considerable success in the study of mathematics, and in fact surpassed the level of their western counterparts. For example, the numerical solution of algebraic equations of any degree was known nearly 800 years earlier in China than was Horner's method in the West, and the method of elimination of several unknowns in systems of higher degree equations was used 500 years earlier than Bezout's corresponding theory in Europe. The method of solution of simultaneous congruences was also earlier by more than 500 years, and higher order interpolation methods were known by some 400 years earlier. Here emphasis on priority of discovery only points to the fact that "the mathematics of the Song and Yuan periods is the most splendid page in the history of Chinese mathematics in medieval times," [Li and Du 1987, p. 111].

These remarkable achievements greatly impressed the British missionary Alexander Wylie who came to take charge of the London Missionary Society Press at Shanghai in 1847 [Cf. Section 4.1]. As he put it in the 1850s, "Considering the long period that the study of mathematics had been cultivated among the Chinese, one cannot feel surprised at their having attained a respectable proficiency in the use of numbers, while yet the science had scarcely obtained a footing in England; nor would it be unreasonable to expect a much higher stage of development than actual evidence goes to prove," [Wylie 1897, p. 181]. In

addition, Wylie was also pleased to note the capabilities of his contemporary Chinese mathematicians such as Li Rui, Dai Xu and Li Shanlan. For example, in the "Preface" to his *Dui Shu Tan Yuan* ("Discovering the Source of Logarithms"), Li Shanlan was very proud of his entirely new method for computing logarithms which was based upon a geometric formula. He claimed that his method was "ten thousand times easier than the methods used by Europeans. ... Although they could only calculate the numbers, yet they are ignorant of the principle."³ To this Wylie added the following comment:

This small indication of self-satisfaction may be very well overlooked, as quite pardonable in one who has had no better aid than that afforded by the *Leuh-lih-yuen-yuen* (i.e., *Lu Li Yuan Yuan*, literally, Ocean of Calendrical Science and Astronomy), and who has here given us, as the result of four years' thought, a theorem, which in the days of Briggs and Napier, would have been sufficient to raise him to distinction [Wylie 1897 Part III "Scientific", p. 194].

One should not be surprised that Wylie regarded Li Shanlan's mathematics so highly. Wylie's esteem was entirely appropriate. Prior to his collaboration with Li Shanlan in translating Western mathematics and science into Chinese in the 1850's, what Li Shanlan had achieved in his treatises, *Fang Yuan Chan You* ("Explanation of the Square and the Circle"), *Hu Shi Qi Mi* ("Unveiling the Secrets of Arc and Sagitta") and *Dui Shu Tan Yuan*, constitute the most advanced contributions to Chinese mathematics between the second half of the 18th and the middle of the 19th century.

Thanks to his collaboration with Li Shanlan, Wylie came to be a faithful observer of 19th-century Chinese mathematics. His work on Chinese mathematics easily surpasses that of the well-known Jesuit Matteo Ricci in the late Ming Dynasty. In addition to Wylie's translation with Li Shanlan of Euclid's Elements (Books 7-15), which along with Matteo Ricci and Xu Guangqi's translation of its first six books makes a complete Chinese edition of this Greek mathematics classic, Wylie also presented his own research on Chinese calendrical science and mathematics to the Western scholarly community. His article, "Jottings on the Science of the Chinese Arithmetic" (1852), is as Joseph Needham put it, "an excellent account which can still be read with profit today," [Needham 1959, p. 1].⁴ Even more important, Qin Jiushao's "Da Yan" method (The method for solving simultaneous congruences) was explained in it for the first time. Thus Wylie's work provided a starting-point in Europe for studies of the history of Chinese mathematics, and particularly of indeterminate analysis [Libbrecht 1976, 310-327].

1.2. Western Mathematics transmitted to China in the Late Ming and Early Qing Period

In contrast, when the Italian Jesuit Matteo Ricci arrived in Macao in 1582, Chinese mathematics had suffered a long period of stagnation lasting through the 14th and 15th centuries. This was due in part to the fact that many traditional texts, including those of the Song and Yuan periods, had been lost by the time of

the early Ming Dynasty (1368-1644). Thus while correcting the Da Tong Calendar became so urgent at the end of the Ming, there were not many people who understood astronomy and the computation of the calendar well enough to understand this important revision. Even more detrimental was the fact that most of the officials in the State Observatory were conservative and apparently reluctant if not incapable of making any calendar reforms.

To higher officials of the late Ming, however, such as Xu Guangqi, reformation of the calendar was of the first priority. Since China was fundamentally an agricultural society, the calendar was the most important instrument for regulating Chinese socio-economic and cultural life. Therefore it was natural that Xu Guangqi would interpret the Da Tong Calendar's inability to predict heavenly phenomenon effectively as a symbol of the empire's decline.

At the same time, Jesuit missionaries in China clearly understood the seriousness with which certain Chinese viewed this problem, and used the issue of calendar reform to establish ties with the educated upper class. The missionaries' advisory role in revising the calendar also provided an entry point into many other areas of Chinese society, allowing them to begin proselytizing on behalf of the Catholic Church. It is against this historical background that Euclid's *Elements*, along with other mathematical / scientific material, was introduced to China. The version of Euclid's *Elements* on which Matteo Ricci and Xu Guangqi based their

Chinese translation, was the one edited by Ricci's teacher at the Jesuit Academy in Rome, the noted 16th-century Italian mathematician and reformer of the Gregorian Calendar, Christoph Clavius.⁵ The choice of translating the edition of Clavius suggests that the Jesuits also had a desire to motivate the reform of the calendar.

With such concerns in mind, Matteo Ricci wrote letters back to Rome asking that more missionaries well-versed in calendrical computation be sent to China. Among Jesuits skilled in such computations, Niccolo Longobardi, Johann Terrenz Schreck, Johann Adam Schall von Bell and Giacomo Rho were all recommended by Xu Guangqi in 1629 to serve as assistants in the Western Department of the Calendar (which Xu Guangqi was called upon to organize). Xu Guangqi's policy, with the help of his assistants, was to "gradually emend the calendar successively," and to "seek out the source [of the Western Method]." As Xu said, "To improve [on the Western Method] demands thorough understanding, and thorough understanding first requires translation," [Li and Du 1987, p. 203].

As a result, the *Chong Zhen Li Shu* containing 137 books was edited and presented to the Emperor Chong Zhen by 1634, one year after Xu Guangqi had died. This massive work involved not only the efforts of Xu Guangqi's Jesuit assistants, but the efforts of Xu Guangqi, Li Zhizao and Li Tianjing, Xu's successor at the Department [Li and Du 1987, pp. 201-203].

However, at the time three other competing schools were also re-computing the calendar, namely the Da Tong, Islamic and the Eastern Departments. The last of these was headed by Wei Wenkui. After empirically testing various theories on several occasions, emphasizing in particular their success in forecasting solar and lunar eclipses, it was clear that the Western method was the most accurate. It was therefore decided in 1643 that the calendar would be reformed, and that the Western method should be adopted.

Unfortunately, the Ming Empire did not survive long enough to see this reform of the calendar put into practice. The Manzhou's army overtook the empire in 1644, founding the Qing Dynasty. One year later, Johann Adam Schall von Bell was put in charge of the State Observatory ("Qin Tian Jian"). It was the new Qing government that officially decreed adoption of the new Western-based calendar, which was called the Shi Xian Calendar. Meanwhile, Johann Adam Schall von Bell amended the *Chong Zhen Li Shu* ("Chong Zhen Reign Treatise on Calendrical Sciences"), developing it into the *Xi Yang Xin Fa Li Shu* ("Treatise on Calendrical Science according to the New Western Methods") (a total of 100 books), which he presented to the first Qing Emperor, Shun Zhi in 1645 [Li and Du 1987, p. 204].

As far as mathematical knowledge is concerned, the *Xi Yang Xin Fa Li Shu* ("Treatise on Calendrical Science according to the New Western Methods") (or the "Chong Zhen Reign Treatise on

Calendrical Science") includes a basic survey of the methods of (Western) plane and spherical trigonometry. In addition, Napier's so-called calculation "bones", Galilean proportional dividers and several other calculating devices were also introduced in these books. On the other hand, Xue Fengzuo published a collection of books in 1656 called *Li Xue Hui Tong* ("Understanding Calendar Making") based upon techniques he had learned from the Polish missionary Nicolas Smogulescki in Nanjing. Among these, the most important were two volumes introducing logarithms: the *Bi Li Si Xian Xin Biao* ("New Tables for Four Logarithmic Trigonometric Functions") and the *Bi Li Dui Shu Biao* ("Logarithmic Tables with Explanations"). These tables, invented by John Napier (1550-1617) and refined by Henry Briggs (1556-1630), were so indispensable to the Western method of computing calendars that 18th- and 19th-century Chinese mathematicians could no longer ignore their theoretical foundations [Li and Du 1987, p. 205].

Xue Fengzuo (ca. 1620-1680), who came to Nanjing in 1653 further studied trigonometry and logarithms with Smogulescki. Earlier, Xue Fengzuo had studied traditional calendrical science with Wei Wenkui at the end of 1633, and became one of several late 17th-century scholars who converted their studies in calendrical science from traditional Chinese methods to the Western ones. Indeed, "By 1700 the best mathematical talents in China were getting their basic training in the more immediately accessible Western methods, and going on to devote their mature careers to the reconstruction and enrichment of the indigenous exact

science," [Sivin 1973, p. 72]. For example, Wang Xishan (1628-1682), who was self-taught in Western astronomy from Jesuit treatises, took as his lifelong aim the synthesis of traditional Chinese and modern Western techniques to produce a geometrical conception of the cosmos. In 1673, Wang Xishan finished his short treatise, the *Wu Xing Xing Du Jie* ("On the Angular Motions of Five Planets"), a clear geometrical description of his own model of the heavens. Instead of solely using numerical procedures for generating successive angular orientations, as was the practice of ancient Chinese astronomers, it used geometric models of successive locations in space. Even more significant, it emphasized different concepts, tools, and methods, so that geometry and trigonometry largely replaced numerical algebra. Concomitantly, the absolute sense of rotation of a planet and its relative distance from the earth became important matters for the first time. In short, Wang Xishan created a significant innovation in seventeenth-century Chinese astronomy [Sivin 1970].

According to Nathan Sivin, Wang Xishan was not the only scholar who was responsible for this Chinese "scientific revolution" [Sivin 1970]. Xue Fengzuo and Mei Wending (1633-1721) also made significant contributions. Both Mei Wending and Wang Xishan were successful in the fields of astronomy and calendrical sciences.⁶ In these areas, Xue Fengzuo's contribution was comparably minor. Mei Wending's achievements in astronomy aside, he was also the central figure in the second half of the seventeenth century to synthesize traditional Chinese and the

newly introduced Western mathematics.

1.3. Mei Wending's Mathematics

Mei Wending did not start learning calendrical computation or mathematics until he was 27 years old. But in 1669, when he was 37 and took the provincial civil examination in Nanjing, Mei Wending met Fang Zhongtong (1633-1698, another Chinese student of Nicolas Smogulescki). Apparently on Fang's recommendation, he then began to learn the methods of computation based on western computing devices. Nevertheless, it was not until 1675 that Xue Fengzuo's *Tian Xue Hui Tong*, Smogulescki's *Tian Bu Zhen Yuan* ("True Foundations of the Pacing of the Sky") and the *Chong Zhen Li Shu* ("Chong Zhen Reign Treatise on Calendrical Science") were available to Mei Wending in Nanjing where he took another civil provincial examination.

Mei Wending was easily the leading mathematician in early modern China, for his predecessors during the previous two centuries had made few advances in the study of mathematics. Given the fact that he was unable to obtain the Chinese classics of the Han to Tang Dynasties, the *Suan Jing Shi Shu* ("Ten Books of Mathematical Classics"), nor was he able to read the works of the Song and Yuan mathematicians such as Qin Jiushao, Li Ye and Zhu Shijie, what he did manage to achieve is all the more remarkable. The significance of his role as a mathematician at the time is due in part to his assimilating the newly introduced Western

mathematics. In addition to systematically compiling, editing and explicating the mathematics arriving from the West, he augmented it with innovations of his own. In short, as a self-taught expert on Western mathematics, Mei Wending not only helped to introduce subsequent Chinese mathematicians to the new Western methods, but also to his own works, including the *Ji He Tong Jie* ("Explanations in Geometry"), in which he first gave definitions, then demonstrations and finally applications [Li and Du 1987, pp. 212-216].

1.4. The Emperor Kang Xi and the *Shu Li Jing Yun*

In his lifetime, Mei Wending wrote more than 80 treatises on mathematics and calendrical science. Most of these were combined into the *Mei Shi Cong Shu Ji Yao* ("Collected Works of the Mei Family") by his grandson, Mei Juecheng, in 1761.

Half-a-century later, in 1712, Chen Houyao presented a proposal to the Emperor Kang Xi "to produce a definitive edition of the mathematical texts in order to benefit the country," [Quoted Li and Du 1987, p. 218]. Partly because of the Mei family's reputation in mathematics, the Emperor enlisted Mei Juecheng along with Chen Houyao, He Guozong and Ming Antu to compile the *Lu Li Yuan Yuan*. It contains three volumes: *Li Xiang Kao Cheng* ("Compendium of Calendrical Science and Astronomy"), *Lu Zheng Yi* ("Collected Basic Principles of Music") and *Shu Li Jing Yun* ("Collected Basic Principles of Mathematics"). The last

volume covers all aspects of mathematical knowledge of the time, and thus can be regarded as a comprehensive mathematical encyclopedia. It was widely circulated thanks to its title and the level of importance assigned to it by the Emperor Kang Xi. Needless to say its influence was very great. In fact, the *Shu Li Jing Yun* remained a compulsory text for learning mathematics and was a very important reference book for mathematical research until 1850 [Li and Du 1987, p. 209].

Mei Juecheng's compilation of *Shu Li Jing Yun* bears the deep imprint of his grandfather, Mei Wending. First of all, Mei Wending's positive view of Western astronomy and mathematics encouraged his successors to consider this foreign knowledge as worthy of study. As Western astronomy and mathematics were not overwhelmingly accepted by early Qing intellectuals⁷, their successful application (in predicting, for example, solar eclipses) by the State Observatory were not enough to justify their use. To study and utilize Western science, Mei Wending therefore faced the problem of legitimizing its status among Chinese intellectuals. Believing in the idea of progress and objectivity in science, Mei Wending argued that when a method was effective it made no sense to consider the nationality of its creator [Li Di 1984, p. 199]. Based on this argument, he went on to suggest that Western science should be considered in the same light with Chinese astronomy and mathematics. By following this recommendation, the status of Western science was eventually legitimized in China.

Viewed in this context, Mei Wending has been regarded by some historians as the creator of the "foundation myth" -- that European mathematics and astronomy actually evolved from certain techniques that originated in China and were later transplanted to the extreme fringes of other civilizations -- namely Islam and Europe as Mei seems to have believed. This idea can be found not only in Mei Wending's works on astronomy and mathematics, but also in the *Shu Li Jing Yun*, which Mei Wending's mathematics penetrates in one form or another. It is more likely, however, that the true creator of the "foundation myth" was the Ming loyalist Huang Zongxi (or Fang Yizi, Fang Zhongtong's father) [Jiang Xiaoyuan 1988]. In any case, it was the Emperor Kang Xi who enthusiastically approved this viewpoint which aroused great interest among early Qing intellectuals. Mei Wending was certainly the best example: he made every effort to encourage its acceptance.

Mei Juecheng, the grandson of Mei Wending, promoted the "foundation myth" to an even greater extent. He argued that the "Jie Gen Fang Bi Li" method (the Western algebraic method for solving quadratic equations by "completing the square") -- introduced in the *Shu Li Jing Yun* -- was nothing other than "Tian Yuan" method -- the method of traditional Chinese algebra developed in the Song-Yuan period. In his *Chi Shui Yi Zhen* ("Pearls recovered from the Red River"), compiled in the *Mei Shi Cong Shu Ji Yao* ("Collected Works of the Mei Family"), Mei

Juecheng "first realised that autochthonous Chinese mathematics had gone very far before the 17th century under unfamiliar notations," [Needham 1959, p. 53], and added that the Western "Jie Gen Fang Bi Li" method could be regarded as equivalent to the "Tian Yuan" method (in the Song and Yuan mathematical texts).

Actually, the "Jie Gen Fang Bi Li" method is nothing more than al-Khwarizmi's algebra (or its refined form as later developed by Girolamo Cardano). For example, in solving numerical quadratic equations, "completing the square" is the method fully explained by its Chinese title. However, immediately after the "Jie Gen Fang Bi Li" method was explained to the Emperor Kang Xi in 1711, the French Jesuit Jean-Francois Foucquet also introduced symbolic algebra to the Emperor. During the summer of 1712, Foucquet wrote a treatise, *A Er Re Ba La Xin Fa* ("New Method of Algebra"), for the Emperor. This introduced symbolic algebra using Descartes' notation, apparently for the first time in China [Jami 1988].

Unfortunately, the Emperor did not understand the meaning of the multiplication of symbols: "Jia multiplied by Jia, Yi multiplied by Yi, do not give any number, and one does not know the value of the result," was the comment in a note duly transmitted to Foucquet: "It seems to me that [Foucquet's] algebra is very plain and insufficient. In a word, it is laughable," [Quoted in Jami 1988]. Therefore, although compilation of the *Shu Li Jing Yun* was begun in 1713, its content included only the "Jie

Gen Fang Bi Li" method but not the symbolic algebra, apparently as a consequence of the Emperor's negative attitude towards the "new" method. This seems to be why the *A Er Re Ba La Xin Fa* was not circulated, and why it therefore had no influence on Chinese mathematics at the time.

1.5. Discourse on Mathematics: Wang Lai and Li Rui

It was not until 1859 that modern algebraic symbolism was successfully re-introduced to China by Alexander Wylie and Li Shanlan in their translation of Elias Loomis' *Elements of Analytical Geometry and the Differential and Integral Calculus*. Even so, just a few decades earlier, Wang Lai (1768-1813) and Li Rui (1773-1817) had made substantial contributions to the theory of equations wholly in the context of Chinese traditional mathematics. Compared with their Western counterparts, Wang Lai and Li Rui's achievements were not sophisticated: at most what they had attained was basically analogous to what 16th and 17th-century European algebraists had accomplished, namely, understanding the properties of roots, the relationship between roots and coefficients, and Descartes' rule of signs, etc. Even so, Wang Lai and Li Rui's results were of great significance in the history of Chinese mathematics simply because they were developed in an autonomous way, and on Chinese terms.

Equally striking is that Wang Lai and Li Rui, among others, were able to advance well beyond the algebra of the Song and Yuan

periods. Chinese mathematicians traditionally were mainly interested in the "algorithmic" aspects of algebra.⁸ They paid no attention to "qualitative" aspects of the roots (such as the relationship between roots and coefficients of the equations). This is one of the factors that led to the decline of Chinese mathematics of the Song and Yuan periods. Unfortunately, Chinese mathematicians had successfully exhausted the "algorithmic" approach to problem-solving, and only a new approach could lead to new discoveries.⁹ It was at this very point that Wang Lai and Li Rui, six hundred years later, made their own substantial contributions to the theory of equations by taking a wholly new, "qualitative" approach to traditional Chinese algebra [Horng 1989c].

1.6. Ming Antu's Mathematics

Before saying more about the context of early 19th-century Chinese mathematics in which Wang Lai and Li Rui played major roles, something needs to be said about Ming Antu (?-1765) and his *Ge Yuan Mi Lu Jie Fa* ("Quick Methods for Trigonometry and for Determining the Precise Ratio of the Circle"). This, according to Catherine Jami, "illustrates nicely the situation of Chinese mathematics of the time, both by its history and by its content," [Jami 1988].

Ming Antu was an astronomer of the State Observatory (in Beijing) which was deeply influenced by Western astronomy and

mathematics ever since the Shi Xian Calendar was officially adopted at the very beginning of the Qing Dynasty. In fact, Jesuits had been working as civil servants at the State Observatory since the 1630's. Consequently, any Chinese astronomer at the State Observatory such as Ming Antu would have been exposed to contemporary scientific knowledge of the West.

One of the Emperor's teachers in mathematics and astronomy was Pierre Jartoux (1669-1720), a French Jesuit who arrived in China in 1701. He introduced three formulas new to Chinese mathematicians: power series expansions of the sine and versed sine functions, as well as series derived from power series expansions of the arc sine function. It is possible that Ming Antu learned all of these from Jartoux himself, though the Emperor may well have been the intermediary between the Jesuits and native Chinese mathematicians. Otherwise, it is difficult to explain why Mei Juecheng also translated these three formulas in his *Chi Shui Yi Zhen*.

However, Jartoux provided no proofs or demonstrations. Indeed, the infinitesimal calculus, upon which they are based, remained unknown in China until the middle of the 19th century. It was Ming Antu who set himself the task of proving them as a challenge. His *Ge Yuan Mi Lu Jie Fa* shows that he indeed managed to do so [for a brief discussion, see Section 3.6].

The *Ge Yuan Mi Lu Jie Fa* is comprised of statements and

proofs of nine formulas: the "Du Shi San Shu" (three formulas of Master Jartoux), the power series expansions of the chord and the sagitta, and those of the four inverse functions. Although Ming Antu had worked very hard for more than 30 years in writing this treatise, he was unable to see its draft to completion. After his death, his disciple Chen Jixin finished it in 1774. Even so, it was not published until 1839. Fortunately, the first section of this treatise, i.e., the statement of the nine formulas alone, had already been circulated among Chinese mathematicians, especially among those who were closely related to the State Observatory. Thus Ming Antu's success with the nine formulas constitutes a starting-point for studies of trigonometric expansions in 19th-century China. Thanks to the early circulation of these formulas, the study of trigonometric expansions gradually came to the fore towards the end of the 18th century [Cf. Section 3.6]. Meanwhile the Qian-Jia school retained its vogue as many of the Song and Yuan mathematical texts were recovered and being reprinted.

1.7. Qian-Jia School and Qing Mathematics: Dai Zhen, Qian Daxin and Ruan Yuan

In fact, the revival of ancient Chinese mathematical texts and the assimilation of Western mathematical knowledge became major concerns of Chinese mathematicians between the second half of the 18th and the first half of the 19th century. Because this synthesis of Chinese and Western mathematics was so successful,

enabling Chinese mathematicians to make substantial contributions of their own towards the end of the 18th century (and thereafter), it is important to appreciate the advances made during this period.

From the second half of the 18th century to the first half of the 19th century, the Qian-Jia school considered mathematics as relevant to studies of the Classics. This is best explained in the works of Dai Zhen (1724-1777), a key figure of this school. Coupled with his distinguished philological studies, Dai Zhen's interests in astronomy and mathematics led him to realize that without a keen understanding of mathematics, it was impossible to understand passages in the Classics that dealt with technical, calendrical, and astronomical phenomena. For example, in his study of the *Kao Gong Ji* ("Record of Technology"), a chapter of the *Zhou Li* ("The Rituals of Zhou Dynasty"), Dai indicates clearly that scholars lacking mathematical training simply could not appreciate the texts describing engineering techniques used by the ancients.

Due in part to the need for better mathematical understanding, and in part spurred on by the challenge of European science, Dai Zhen was ecstatic when he rediscovered ancient mathematical texts such as the *Jiu Zhang Suan Shu* (only five chapters of the nine were known to Dai Zhen), or *Wu Jing Suan Shu* ("Arithmetic in the Five Classics"), etc. These he found were part of the *Yong Lo Da Dien* ("Great Compendium of the Ming Emperor

Yung Lo Era", 1403-1425), a fact he discovered while serving on the commission editing the *Si Ku Quan Shu* ("Complete Library of the Four Branches of Literature"). His accounts of these works in the section for astronomy and mathematics of the *Si Ku Quan Shu* catalog indicate how important and significant the recovery of the ancient mathematical texts was for the Qian-Jia school.

Furthermore, according to Dai Zhen, if the Classics were studied properly, they would prove to be repositories of scientific knowledge that had been lost over the years due to neglect and lack of understanding. This is also the reason why Qian Daxin, a leading contemporary scholar of the Qian-Jia school, criticized Confucian scholarship:

Confucian scholars have ... usually denigrated those who were good mathematicians as petty technicians.... In ancient times, no one could be a Confucian who did not know mathematics.¹⁰ ... Chinese methods [now] lag behind Europe's because Confucians do not know mathematics.

For otherwise, as Qian Daxin put it in the same passage:

It is not possible that the ingenuity of Europeans should surpass that of China. It is only that Europeans have transmitted [their findings] systematically from father to son and from master to disciple for generations. Hence, after long period [of progress], their knowledge has become increasingly precise [Quoted in Elman 1984, p. 83].

Consequently, "the editors [of the *Si Ku Quan Shu*] rejected placing mathematics in the 'Xiao Xue' [i.e., philology] section, which the late seventeenth-century compilers of the *Ming History* had done, in favor of what they regarded as a more logical linking of mathematics to astronomy," [Elman 1984, p. 165]. This clearly indicates that the status of mathematics had been raised in rank to a level that was at least equal to that of astronomy in 18th-century China. On the other hand, Ruan Yuan, another leading figure of the Qian-Jia school (as well as its patron), compiled and edited the *Chou Ren Zhuan* ("Biographies of Mathematical Astronomers") with Li Rui's collaboration in 1799. This work has been described by Nathan Sivin as "a programmatic synthesis of traditional and Western astronomy designed to encourage the study of the latter in order to improve the former," [Quoted in Elman 1984, p. 63]. Indeed, the theme of the *Chou Ren Zhuan* is to emphasize and expand the "foundation myth" that the roots of modern astronomy and mathematics are to be found in ancient China. With this in mind, Ruan Yuan and his close associates, Qian Daxin and Li Rui, all believed that recovery of the ancient mathematical texts would prove to be significant and worthwhile.

According to the Qian-Jia school, reconstructing antiquity meant recreating the entire range of intellectual and political domains of knowledge that had existed in traditional China. In fact, Ruan Yuan and his staff of compilers working on the *Chou Ren Zhuan* were convinced that their collection of biographies traced the consecutive changes and linear, progressive development of

technical knowledge in China and Europe. Therefore, "In the eighteenth century, the notion of the cumulative development of knowledge pervaded all the disciplines associated with evidential research [i.e., textual criticism]," [Elman 1984, p. 228].

However, although the school certainly helped promote studies of the ancient mathematical texts, its "Kao Zheng" (textual criticism)¹¹ placed important constraints on the philological scholars. Research on ancient astronomy and mathematics was valued mainly for its application to classical studies.¹² Consequently, interest in astronomy and mathematics was confined to their utility in illuminating ancient knowledge. As far as any new mathematical results might have been concerned, these were relevant. Exclusive interest in the ancient texts and their explication was so rigid that philological scholars would have ignored any mathematical innovations.

For example, the authors of the "Biography of Qian Tang", Chapter 42 of the *Chou Ren Zhuan*, offer the following rather striking and concluding remarks on Qian Tang's approximation of π = 3.16:

Since Liu Hui and Zu Chongzhi, the ratio of circumference to diameter [of a circle], i.e., π , has generally been taken as 3.14, in spite of a little discrepancy. Qian Tang [of the 18th-century Qing Dynasty] discovered the ratio 3.16 on his own, which is totally different from those of the others. In Qin Jiushao's *Shu Xue Jiu Zhang* [of the 13th century Southern Song],... I find that he also gives π = 3.16, the same as that of Qian Tang. This indicates what Qian Tang had achieved coincides implicitly with the ancients. On the other hand, Tan Tai of Jiangling [i.e., Nanjing], a noted contemporary mathematician, measured a cylindrical piece of wood (or log) with diameter 1 "zhang" (about 210 cm) to

obtain a value of 3.16 "zhang" plus [namely, $\pi = 3.16$], and thereby justified Qian Tang's value as the most appropriate [Quoted Ruan Yuan 1962, 543-545].

This passage was due to Li Rui, who also took Qin Jiushao's value for π seriously and told his master Qian Daxin the textual evidence as well. Qian Daxin in his *Shi Jia Zai Yang Xin Lu* ("Record of Self-renewal from the Ten Yokes Study") even stressed that his nephew Qian Tang's "discovery" of the value for π was not without precedent, and noted the example of Qin Jiushao's value.

What is most peculiar is that Qian Daxin cites Zu Chongzi's extremely accurate value for π : $3.1415926 < \pi < 3.1415927$, as well as two other approximations, $\pi = \frac{22}{7}$ and $\pi = \frac{355}{113}$ in the same passage. He also refers to the Western value for $\pi = 3.14159265$ as computed by Pierre Jartoux. In terms of either Zu Chongzi's or Pierre Jartoux's approximations, Qian Tang or rather Qin Jishao's $\pi = 3.16$ was a bad one, especially if one really understands how the area formula of a circle is derived and how in turn this formula can be related to the approximations of π . In his *Commentary to the Jiu Zhang Suan Shu*, Liu Hui, the greatest mathematician of third-century China, had already put forward a proof for determining the area of a circle and created the "Ge Yuan" method (method of circle measurements) to obtain the approximation of π . First, he inscribed the unit circle with regular polygons up to 96 sides, obtaining an approximation of 3.14. Secondly, he used the area formula of the circle -- $S =$

half the circumference times half the diameter ("Ban Zhou Ban Jing Xiang Cheng De Ji Bu") -- to get the approximate length of the circumference. Finally, by the definition for π , namely the ratio of the circumference to the diameter, Liu Hui attained his approximation $\pi = 3.14$. Unfortunately, the leading scholars of the Qian-Jia school, Ruan Yuan, Qin Daxin and Li Rui, among other editors of the *Chou Ren Zhuan*, all ignored this important passage. This was no doubt because the primary goal of their textual scholarship was "To Search for the truth in the actual facts" (Shi Shi Qiu Shi). Here we can see how the Qian-Jia school's narrow focus was not so conducive to mathematical studies after all. The perspective of their shared epistemology as well as methodology undoubtedly placed very tight constraints on the scholars like Wang Lai who in particular had been paying much more attention to mathematical studies proper.

1.8. Wang Lai versus Li Rui

In contrast to his contemporary scholars of the Qian-Jia school, Wang Lai was greatly concerned about innovation in his mathematical studies, although he was also well versed in the Classics.¹³ In fact, in Zhang Zhidong's *Shu Mu Da Wen* ("Questions and Answers on the Catalogue of Books", 1875), Wang Lai along with Li Rui and Jiao Xun were classified as both "mathematician" and "classical scholar" (Jing Xue Jia). Furthermore, Wang Lai was put into the "Xi Fa" (Western method) school of mathematician, while Li Rui and Jiao Xun were put into the "Zhong Fa" (Chinese method)

school. This suggests that Wang Lai was probably not well-acquainted with Ruan Yuan's circle which included most scholars of the Qian-Jia school, even though he along with Li Rui and Jiao Xun were honored as "Tan Tian San You" (three friends concentrating on astronomy and mathematics) by their contemporaries.¹⁴

But why was Wang Lai regarded as a mathematician of the "Xi Fa" school? The main reason might be that his theory of equations was based upon the "Jie Gen Fang Bi Li" method introduced in the *Shu Li Jing Yun*. In light of the fact that the "Jie Gen Fang Bi Li" method had been taken as equivalent to the "Tian Yuan" method, it seems unfair to charge that Wang Lai "had become too deeply involved with the Western method," as Luo Shilin complained [Luo Shilin 1839, pp. 669-675]. On the contrary, Li Rui created his own theory of equations in terms of Qin Jiaoshao's "Zheng Fu Kai Fang" method (Horner's method), a means of numerically solving algebraic equations of degree higher than one. Apparently it was due to this fact that Li Rui's study in the theory of equations was appreciated by mathematicians of the Qian-Jia school. Unfortunately, due attention was not paid to Wang Lai's theory of equations. In fact, his innovation received no appreciation from Ruan Yuan's circle; instead, as already noted, the scholars of the Qian-Jia school were only concerned with the revival of ancient native Chinese mathematical texts, especially those by the thirteenth-century mathematicians Qin Jiushao and Li Ye (or Li Zhi).

Nevertheless, studies of mathematical texts from the Song-Yuan period came to the fore thanks to Li Rui's collation and critical commentary of Li Ye's *Ce Yuan Hai Jing* and *Yi Gu Yan Duan*, two of the most important treatises introducing the "Tian Yuan" method. Moreover, according to Jiao Xun, it was Li Rui who discovered that the "Zheng Fu Kai Fang" method goes back to the "Shao Guang" chapter [i.e., Chapter 4] of the *Jiu Zhang Suan Shu*. This in turn added new force to the vogue of studying ancient mathematical texts.

Jiao Xun was especially impressed with the excellent Song-Yuan mathematical texts. Thus he paid a great deal of attention to them in response to Li Rui's enthusiastic work in reviving mathematical texts. However, instead of making contributions to the theory of equations as Li Rui and Wang Lai did, Jiao Xun limited his research to the mathematical structure of the *Jiu Zhang Suan Shu* itself, and in the process created a symbolic algebra of his own.¹⁵ His treatise, the *Jia Jian Cheng Chu Shi* ("On the Laws of Addition, Subtraction, Multiplication and Division"), exemplifies the extent to which native Chinese mathematical knowledge might develop in an autonomous way if only a mathematician could formulate a problem from a totally new perspective. This really was the case with Jiao Xun's studies on algebra, for he obviously grasped the significance of using symbols like the "Tian Gan" (the Heavenly Stems "Jia", "Yi" and so on) for arithmetic operations which in turn reflected general

algebraic structures [Wu Yubin 1986].

Yet Jiao Xun's algebra received no positive response or reaction from the Qian-Jia school. Wang Lai seems to have been the only mathematician who could appreciate Jiao's emphasis on the structure of algebra. Instead, scholars of the Qian-Jia school made every effort to apply the "Tian Yuan" method, as typically Li Rui did, to as many problems in the mathematical texts as they possibly could. This of course is closely related to the Qian-Jia school's primary concern with the revival of ancient texts. Thus it is no wonder that they neglected not only Jiao Xun's work on algebra, but also Li Rui's substantial contribution to the theory of equations in his *Kai Fang Shuo* ("On the Method of Extraction").

Besides, attention was unduly paid to Wang Lai's *Heng Zhai Suan Xue* ("Hengzhai's Mathematics") upon which Li Rui based his own *Kai Fang Shuo*. Wang Lai's works on mathematics deserve attention largely because he had discovered that some of the equations in Li Ye and Qin Jiushao's texts might have more than one positive root. This certainly created a new approach to traditional algebra: "The Song and Yuan mathematicians held the view that when they had found one solution of a given problem the problem was completely solved. They had never gone further and investigated whether the equation had another root or what was the nature of the roots, etc." [Li and Du 1987, p. 241] Therefore, Wang Lai's innovation was clear and significant with respect to the Qian-Jia school's criteria of "Fa Qian Ren So Wei Fa"

(discovering what our predecessors had not yet discovered).

Unfortunately, since scholars of the school were concerned mainly with the textual facts, (Wang Lai apparently did not share such narrow epistemological preconceptions), scholars of the Qian-Jia school such as Zhang Dunren and Lo Ternfeng, therefore, denigrated his mathematics.

1.3. Luo Shilin: the last faithful representative of the Qian-Jia School.

In the meantime, Wang Lai was very interested in Ming Antu's work in mathematics which, unfortunately, was not available to him. In 1804 when Wang Lai debated issues concerning the "Tian Yuan" method with Jiao Xun and Li Rui, among others, in Zhang Dunren's official residence at Yangzhou, he failed to have his host show Ming Antu's *Ge Yuan Mi Li Jie Fa*, even though Zhang had a copy of the manuscript in his possession. Again, this was probably due to Wang Lai's "getting involved too deeply with the Western method."¹⁶ Thus Zhang Dunren, a devoted scholar-official who invited Li Rui to serve as his "Mu You" (secretarial staff) at the time, apparently kept Wang Lai away from Ming Antu's work.¹⁷ It was not until 1808 that Wang Lai first read Ming Antu's work, when he obtained a copy from one of his colleagues, Zhu Hong, at the Guo Shi Guan ("The Qing Dynasty Compilation Bureau") where he was offered a post to compile the *Tian Wen Zhi* ("History of Qing Astronomy") and the *Shi Xian Zhi* ("History of the Shi Xian Calendar").

Concerning Wang Lai's mathematics, his connection with the Guo Shi Guan points out the fact once again that the State Observatory was the stronghold of the "Xi Fa" (Western Method) school, even at the time when the Qian-Jia school virtually dominated the whole scholarly community. By the same token, Chen Jie, who was recruited into the State Observatory as an astronomical student before serving as a mathematics teaching assistant at the Guo Zi Jian (Imperial Academy), had become a devoted scholar of the "Xi Fa" school. He considered Western plane trigonometry as "the so-called useful", yet the "Tian Yuan and Si Yuan" methods as "the useless". Thus he doubted if the "Tian Yuan" method could be applied to solve problems related to the triangle, and sent this question as a challenge to Luo Shilin in the 1830s. Luo Shilin (1789-1853) accepted this challenge and responded affirmatively by writing his treatise, the *San Jiao He Jiao Suan Li* ("Computing Examples on the Triangle") whose aim was to show that the "Tian Yuan" method really could be applied to solve the triangle, as was indeed the case [Guo Shirong 1987].

However, in his early years Luo Shilin was well-versed in the "Xi Fa" (Western Method) due in part to his serving as an astronomical student for seven years at the Observatory. His first treatise on mathematics, the *Bi Li Hui Tong* ("Understanding the Theory of Proportion") was little more than what he himself later criticized the "Xi Fa" school for doing -- namely "thinking stubbornly to interpret the ancient [Chinese] mathematical texts

based on the Western methods."¹⁸

Later, he changed his view dramatically, when he went to Beijing for the central civil examination in 1822, he came across Zhu Shijie's classic, the *Si Yuan Yu Jian* (1303). Luo Shilin was so impressed that without any hesitation he was converted from the "Xi Fa" to the "Tian Yuan and Si Yuan" methods. Thereafter Luo Shilin never regreted having committed himself to "reviving the ancient texts and expounding Chinese mathematics," [Quoted from Zhu Kebao 1886, 773-780; Cf. also Guo Shirong 1986].

The other reason for Luo Shilin's conversion was probably due to the influence of Ruan Yuan. Exactly when he served as Ruan Yuan's "Mu You" is unknown, yet thanks to this post Luo Shilin was able to meet many noted contemporary mathematicians. Among them were friends of Li Rui, Li Huang and Dai Dunyuan, as well as a disciple of Li Rui, Li Yunnan, all the important representative mathematicians of the Qian-Jia school in 1820-40. In addition, Luo Shilin also got along very well with the noted scholars of the "Xi Fa" school or their close associates, including Chen Jie, Dong Youcheng and Xu Youren, probably due again to his early career at the State Observatory.

Thus, in his late years Luo Shilin became a faithful scholar of the Qian-Jia school. He devoted all of his attention to the revival of Zhu Shijie's *Si Yuan Yu Jian* and *Suan Xue Qi Men*. Luo Shilin was delighted to have found the latter in a Korean edition

bought at a Beijing bookstore. While recovery of the *Si Yuan Yu Jian* provided new source materials for contemporary mathematicians, the new edition of the *Suan Xue Qi Men* of 1839 clearly illustrates how Luo Shilin was familiar with the scholarship. Of his treatises on mathematics, Luo Shilin's achievement lies primarily in showing that the "Tian Yuan" method was extraordinarily effective in dealing with various mathematical theories, and thereby constituted "the most excellent method in mathematics" [Guo Shirong 1987]. In light of subsequent mathematical studies in the 1850s (when the "Tian Yuan" method still retained its vogue despite the reintroduction of Western mathematics into China), it is no wonder that Luo Shilin came to be regarded as one of the leading mathematicians of the period 1820-40 [Horng 1989c].

It is interesting to see how Luo Shilin evaluated Ming Antu's work in mathematics. In fact, it is due to Luo Shilin's collation and edition that Ming Antu's *Ge Yuan Mi Li Jie Fa* was finally published in 1839. Luo Shilin claimed that Ming Antu's work "cannot be regarded as the Western method and thereby ignored."¹⁸ Ruan Yuan's comment in a preface to Luo Shilin's *Chou Ren Zhuan Xu Bian* ("The Amendments of the Biographies of Mathematical Astronomers"), which he had sponsored and supervised, also maintained that Ming's methods were similar to Qin Jiushao's "Da Yan" method (the method for solving simultaneous congruence). But compared with their low esteem for Wang Lai, why should Luo Shilin and Ruan Yuan have had such high regard for Ming Antu, especially

when Wang Lai and Ming Antu were both categorized as part of the "Xi Fa" school in the eyes of Luo and Ruan's contemporaries?

1.10. On Ming Antu's Mathematics: Luo Shilin versus Xiang Mingda

Ming Antu's proofs of the "nine formulas" for trigonometric expansions [See Section 3.3] are based on Euclidean geometry, as introduced in the *Elements*, using calculations with certain types of algebraic expressions which come from Liu Hui's "Ge Yuan" method (circle measurements or the method of the division of a circle). According to Catherine Jami, the link between the two is the use of continued proportion ("Lian Bi Li") first considered as a geometric entity, then as an algebraic language. Moreover, the terms of the continued proportion are called "Lu" as a "ratio" -- in exactly the sense that the term "proportion" is used in Greek mathematics. Yet the concept of "Lu" in the *Jiu Zhang Suan Shu* is different from that of ratio: there "Lu" is a term referring to any number in a series of numbers defined in relation to one another. For example, whereas Greek mathematicians like Archimedes regarded $\pi = \frac{22}{7}$, namely the "ratio" between the circumference and the diameter of a circle, Chinese mathematicians would put it this way: The "Lu" of the diameter (called "Jing Lu") is 7 and the "Lu" of the circumference (called "Zhou Lu") is 22. Likewise, while early Greek mathematicians would regard π as the ratio of 3 to 1, ancient Chinese mathematicians would refer to the "Zhou Lu" 3 and "Jing Lu" 1.²⁰ Obviously they are dealing with a similar problem, namely the value of π , but with different

explanations. Therefore, it seems that Ming Antu was familiar with the content of the Euclidean proportion theory, and successfully adopted the concept of "Lu" to designate the terms of a proportion [Jami 1988].

In addition to continued proportion, Ming Antu also used the "Jie Gen Fang Bi Li" method and transformation formulas for the trigonometric identities to derive the relationships of arc, chord and sagitta in a circle. No doubt Ming Antu was regarded as a major mathematician of the "Xi Fa" school (the school of the Western methods) when his work began to interest mathematicians posthumously in the 1830s. This was in part due to the very high praise he had received from Luo Shilin and Ruan Yuan, who at the time were the leaders of the Qian-Jia school. Yet given the fact that the "Xi Fa" school as a whole was attacked by the Qian-Jia school, why did Luo Shilin and Ruan Yuan pay such respect to Ming Antu apparently without hesitation?

One of the reasons is that by the 1830s the Qian-Jia school no longer played a central role in the scholarly community. As far as mathematical studies were concerned, their "textual criticism" -- to search for the truth in actual facts -- was by then challenged by almost all of the mathematical community. In addition to the attack by Chen Jie already mentioned, Xiang Mingda (1789-1850) also discredited "textual criticism" in mathematics, and repeated to Li Yunnan the same sort of criticism Wang Lai had already made of philological scholars. Even more detrimental was

Xu Youren's criticism of Qian Tang's value for $\pi = 3.16$. Xu actually approximated the unit circle by inscribed and circumscribed regular polygons, and thereby pointed out that the rough approximation $\pi = 3.16$ was of little significance in terms of the revival of ancient Chinese mathematical texts, since although $\pi = 3.16$ was a textual fact (in Qin Jiushao's classic, the *Shu Shu Jiu Zhang*), ancient Chinese mathematicians Liu Hui and Zu Chongzhi had already obtained better approximate values of π [Zhu Kebao 1886, pp. 782-785]. Apparently, remaining true to their epistemology and methodology in mathematical studies was very important to scholars of the Qian-Jia school. Recovery of Zhu Shijie's works was helpful, yet the assumed power of the "Tian Yuan" and "Si Yuan" methods was not sufficient to defend the stand of the Qian-Jia school as Luo Shilin had expected.²¹ Thus they had to find new wood by which to keep their fire alive. Ming Antu's mathematics was exactly what Luo Shilin and Ruan Yuan needed, since Ming's *Ge Yuan Mi Lu Jie Fa* had succeeded in provoking Chinese scholars to pay attention to the so-called "nine formulas" in the early 19th century.

With this in mind, Ruan Yuan and Luo Shilin were convinced that Ming Antu's method of "Lian Bi Li" (continued proportion) was similar to the ancient method used in Zu Chongzhi's "Zhui Shu" (method of interpolation), which had been lost since the Song Dynasty. Based on this assumption, they preferred to regard Ming Antu's method as a "new method" (Xin Fa) rather than a "Western method" (Xi Fa), yet the time now is 1839, the year Luo Shilin's

Chou Ren Zhuan Xu Bian was published.²² Thus it is not surprising that the Qian-Jia school would treat Wang Lai's mathematics unfairly. The major reason for condemning Wang Lai was the failure of his mathematics to reflect the revival of ancient Chinese methods, nor did he surpass any of the Western methods that had been introduced by then.

However, one should not blame the Qian-Jia school too much for failing to appreciate Wang Lai, for Xiang Mingda, one of the two most important mathematicians of the next generation, also overlooked Wang Lai's theory of equations, as well as Li Rui's (despite the fact that he was a close friend of Li Yunnan, a faithful disciple of Li Rui). Xiang Mingda's interest lay basically in rectification of the perimeter of an ellipse which, of course, was closely related to Ming Antu's work [See Section 3.6; Cf. also He Shaogen 1984]. In dealing with problems concerning triangles, Xiang Mingda undertook a Western approach that was totally different from Luo Shilin's "Tian Yuan" method. His treatise, the *Pien San Jiao He Jiao Shu* ("Methods of Plane Trigonometry", 1843), is in striking contrast to the methodology Luo Shilin adopted in his *San Jiao He Jiao Suan Li* (1840) [Cf Horng 1989c]. This clearly indicates that he, like Wang Lai in the immediately preceding generation, was a faithful mathematician of the "Xi Fa" school [Zu Kebao 1886, pp. 767-771].

Nevertheless, there seems to have been no real tension between Luo Shilin and Xiang Mingda, the two major, competing

mathematicians between 1820-40 [Cf. Horng 1989c]. This was undoubtedly because the Qian-Jia school no longer dominated the scholarly community. The time of the Qian-Jia school was simply over. Yet its regard for "textual criticism" had already made its imprint on subsequent mathematical study. Their defensive weapon -- the "Tian Yuan" method -- was not so dull as one might have expected. In fact, from 1840 on, the "Tian Yuan" method still played an important role in mathematical studies. As for Li Shanlan, the "Tian Yuan" method was just as significant as Western methods. However, Li Shanlan appealed to the "Tian Yuan" method in a different sense from the Qian-Jia school [Cf. Horng 1989c, 1990]. It is especially important to recognize that the "Tian Yuan" method helped Li Shanlan pave the way for transition from traditional Chinese mathematics to modern mathematics [Li 1867, "Preface"]. This is one important aspect of his career as a mathematician, translator and educator. But before we turn to his life, career and mathematics, we still need to say something about how he inherited the intellectual legacy of his predecessors. It will also be helpful to describe briefly the socio-economic background of the scholarly community in which Li Shanlan operated, in order to see him not only as a mathematician, but also as an intellectual in 19th-century China.

1.11. Social Status of Ancient Chinese Mathematicians

It is important, first of all, to remember the traditional social status of ancient Chinese mathematicians, since certain

intellectual and socio-economic factors affecting the 19th-century Qing scholarly community were closely related and relevant to Li Shanlan's career as a "professionalized" mathematician (but on a non-institutionalized basis). In general, given the Confucian view that one's moral development was the primary road to knowledge, it is understandable that ancient Chinese scholars regarded mathematics, among other fields of study, as an inferior art.²³ Although the Imperial Academy added mathematics to its curriculum in the Sui-Tang period, it was not part of the training for those destined to attain the highest ranks of the Chinese bureaucracy. Instead, it was provided for children of the lesser ranks of officials and commoners. Even though these students passed the examination, they were only entitled to enter the lowest class of the official bureaucracy (the lowest division of class nine). Therefore, when the Emperor decided to eliminate mathematics from the curriculum, no wonder the decree said that "Since mathematics ... leads only to trivial matters and everyone specializes in their own way, it is therefore decreed that it shall be abolished," [Quoted Li and Du 1987, pp. 104-105].

Under such circumstances, those interested in mathematics usually got what training they could as students of astronomy at the State Observatory, or they were taught privately by some retired country scholar who was versed in mathematics. Although they might make a living by serving at the Imperial Academy, State Observatory or local bureaucratic office, their social status was quite low. This of course was not the case with the director of

the State Observatory or the chief professor of the Imperial Academy: the reason why a mathematician was offered such a post with seemingly higher social status was largely due to the fact he was himself a scholar-official rather than a mathematician alone.

Not until the 13th century did a few noted scholars emerge to live on mathematics alone. For example, Li Ye (1192-1279), the well-known scholar who lived to retirement in his home town at Hebei (in northern China), basically taught mathematics to his disciples. Perhaps due to his assumption that mathematics mirrors the principles underlying the diversified events of natural phenomena, Li Ye clearly disregarded those who considered mathematics as inferior art.²⁴ He even went so far as to see himself as a "mathematician" in so far as his major concern at the time of his death was the survival of his treatise, the *Ce Yuan Hai Jing*, one of the most important works in explicating the "Tian Yuan" method [Mei Rongzhao 1966].

Li Ye was no exception in northern China during the Jin-Yuan period. In fact, another major mathematician, Zhu Shijie, was also taught by a retired scholar in his early days. He then travelled to the Yangzi River area in southern China to do further mathematical research by living as a mathematics tutor for wealthy merchant families. In a strict sense he was perhaps not "professionalized" completely, yet he probably was the first noted scholar in ancient China to live on mathematics alone [Li and Du 1987, pp. 115-117].

Nevertheless, such sporadic examples in Chinese history were not enough to change the social status of mathematicians as a whole. It was not until the late Qing period that the position of a mathematician, along with the evidential scholar, emerged as a "professionalized" role in its own right. Even so, how was it possible for the scholarly community to afford a mathematician acting in a "professionalized" role? To answer this question we need to take a closer look at the Qian-Jia school itself.

1.12. The Socio-economic and Intellectual Backgrounds for the Qian-Jia School

As mentioned in Section 1.7, one intellectual change made by the Qian-Jia school was especially important: it no longer considered moral development alone as the primary road to knowledge. In contrast to their predecessors, scholars of the Qian-Jia school stressed exacting research, rigorous analysis, and the collection of impartial evidence from ancient artifacts and historical documents and texts. Abstract ideas and emphasis on moral values now gave way as the primary objects of discussion among Confucian scholars to concrete facts, documented institutions, and historical events. But how could the Qian-Jia school justify this new aim of knowledge in the very context of the old Confucian tradition? According to the scholars of the Qian-Jia school, the Classics contained paradigms for social order and had an absolute claim to transhistorical truth. No wonder Dai

Zhen could claim that "The Classics provide the route to the Tao," [Quoted in Elman 1984, p. 26]. The distinguished 18th-century historian and classicist Wang Mingsheng (1722-1798) also emphasized the importance of studying the Classics for, as he put it, "The Classics are employed to understand the Tao." With this in mind, they both actually shared the opinion of Ji Yun (1724-1805), a patron of "textual criticism" scholarship when he directed the *Si Ku Quan Shu* project:

If one wants to handle the affairs of the empire, then one must weigh opinions based on moral principles. If one wants to clarify the principles of the empire, one must weigh opinions based on the Classics [Quoted in Elman 1984, p. 28].

Argued this way, the Qian-Jia school convinced themselves that study of the Classics was a legitimate alternative to the "study of the Tao" ("Tao Xue" or "Dao Xue"), which was the major concern of the Song-Ming neo-Confucian scholars. In fact, for scholars of the Qian-Jia school, "their philological presuppositions predetermined the subjects chosen for analysis by sanctioning what acceptable knowledge was and how it was to be verified," [Quoted from Elman 1984, p. 26].

It is on grounds like these that mathematics could be regarded as an acceptable, respectable and specialized field. Although mathematics, among the other five "inferior" arts, was considered as an art of "lesser learning" ("Xiao Xue", i.e., philology), in contrast to the "greater learning" ("Da Xue", i.e., moral philosophy as mentioned above), the editors of the *Si Ku Qian Shu*

did not place mathematics in the "Xiao Xue" section, which the late seventeenth-century compilers of the *Ming History* had done, in favor of what they regarded as a more logical linking of mathematics to astronomy [Elman 1984, p. 165; see Table 1 follows this paragraph]. The major late 18th-century "textual criticism" scholar Sun Xingyan (1753-1818), who was even invited by Ruan Yuan to share the directorship of the academy, Gu Jing Jing She, with Wang Chang (1725-1806), went even further to specify a new scheme of twelve major divisions of knowledge in 1800. This reflected the emergence of specialized fields that deserved independent consideration. By expanding and adjusting the four-division system of the *Si Ku Quan Shu*, Sun Xingyan raised a number of previous subdivisions to the rank of major divisions. For example, philology was separated from the Classics, and astronomy which included mathematics, was divided off from philosophy [See Table 2 follows this paragraph]. Thus as Benjamin Elman has explained, "The evidential scholars' conception of the structure of knowledge led them to propose innovations and redefinitions in order to restore what they considered the proper boundaries of knowledge," [Elman 1984, p. 167].

Table 1. Forty-four Subdivisions of the *Si Ku Chuan Shu*

<u>Classics (Jing Xue)</u>	<u>History (Shi Xue)</u>
Yi Jing (Cannon of Changes)	Dynastic Histories
Documents	Annals
Poetry	Topical Records
Rituals	Unofficial Histories
Spring & Autumn Annals	Miscellaneous Histories
Filial Piety	Official Documents
General Works	Biographies
Four Books	Historical Records
Music	Contemporary Records

Philology	Chronography
Philosophy (Zi Xhe)	Geography
Confucians	Official Registers
Military Strategies	Institutions
Legalists	Bibliographies and Epigraphy
Agriculturalists	Historical Criticism
Medicine	Literature (Ji Xue)
Astronomy & Mathematics	Elegies of Zhou
Calculating Arts	Individual Collections
Arts	General Anthologies
Repertories of Science	Literary Criticism
Miscellaneous Writers	Songs & Drama
Encyclopedias	
Novelists	
Bussshsim	
Taoism	

Source materials quoted in [Elman 1984, p. 164]

Table 2. Sun Xingyan's Division of Scholarship

1. Classics	7. History
2. Philology	8. Epigraphy
3. Philosophy	9. Encyclopedias
4. Astronomy [including mathematics]	10. Poetry
5. Geography	11. Arts
6. Medicine & Law	12. Fiction

Source materials quoted in [Elman 1984, p. 167]

By the same token, Wang Chang, when he directed the Youjiao Academy in 1789, encouraged his students to master one specialized field before pursuing knowledge extensively. Among the specialized fields, Wang Chang not only emphasized the Classics and history but also poetry, philology, epigraphy, astronomy and mathematics [Horng 1989c]. His major concern was to become a specialist in the "lesser learning." Perhaps it is against just this historical background that Jiao Xun, a well-educated scholar

of his time, worried about the constraint of such specialized scholarship: "Scholars [nowadays] master one specialty and distance themselves from others. I worry that their views will as a result not be comprehensive enough," [Quoted in Elman 1984, pp. 67-68]. Undoubtedly Jiao Xun's observation was well-fitted to the framework of traditional Confucian scholarship. Nevertheless, his comment clearly suggests that mathematics, among the "lesser learning," had already become a respectable specialized field in its own right towards the end of the 18th century.

However, only a "secularized" scholarly community could provide a living for Qing scholars committed to a single specialty. Here "secularized" is used in the sense that the community in question no longer aimed at ideals of wisdom where interests centered primarily on questions of moral philosophy. Instead, "secularized" scholars devoted themselves to the "lesser learning". They began to seek employment as "Mu You" to officials, tutors in wealthy families, academic teachers and the like to make a living. It should also be remembered that the merchant class also came to play a significant role in the scholarly world of the 18th-century Jiangnan area. Scholarship flourished due to merchant patronage, and books were printed and collected in larger numbers than ever before. The wealth of merchants, especially the salt merchants in Yangzhou, gave Jiangnan added economic and cultural importance. The imperial court granted salt monopolies to wealthy Jiangnan merchants in exchange for their services as salt-tax collectors. These

"merchant aristocrats" functioned as an economic arm of the imperial bureaucracy throughout the Qing Dynasty. No wonder many schools were reserved for sons of salt merchants, such as the Anding and Meihua academies in Yangzhou. Thus it is often difficult to distinguish literati from merchants in the academic world of 18th-century Yangzhou. For the first time, a merchant background was no impediment to a life of scholarship and research. This is really not the case with the Ming era, nor prior to it except for a few examples in the Song-Jing-Yuan period. In former times, Dai Zhen, the son of a cloth merchant, would never have been received with acclaim in Beijing society [Elman 1984, p. 95].

Official patronage was also important in creating a cultural milieu for evidential scholarship. Among the famous official patrons, Ruan Yuan was especially concerned about the study of mathematics and astronomy. While serving as director of education ("Xue Zheng") from 1793 to 1795 in Shandong province, Ruan Yuan supported and promoted men of learning -- something he did throughout his official career. Jiao Xun (1763-1820), Ruan's relative, went to Shandong to serve on Ruan Yuan's staff and later accompanied Ruan Yuan to Zhejiang. Sun Xingyan was appointed circuit intendant in Shandong where Ruan Yuan was serving there and had, as a result, frequent contact with Ruan Yuan's secretarial staffs. In 1795, when Ruan Yuan was transferred to Zhejiang as director of education, he employed more than forty Zhejiang scholars in Hangzhou to compile under his direction a

dictionary of the Classics, the *Jing Ji Zhuan Gu* ("Collected Glosses on the Classics"). This in turn served to revive pre-Tang annotations of the Confucian Canon.

In addition, Ruan Yuan invited noted scholars such as Qian Daxin, Jiao Xun, Tan Tai and Li Rui to Hangzhou to help him compile the *Chou Ren Zhuan*. In his residence in Hangzhou for about ten years either as director of education, acting governor or full governor, Ruan Yuan committed himself to promoting the study of mathematics. On the one hand, he endeavored to search for rare mathematical texts and to have them published, thereby making them available to scholars. On the other, in order to encourage scholars to undertake mathematical studies, he included mathematics as a subject in the provincial civil examinations (which usually were supervised by the director of education) and invited Jiao Xun to examine the tests. Thus Ruan Yuan acted not only as one of the great patrons of the Qian-Jia school, he also served as a key figure who created an intellectual atmosphere within which 18th-century Qing scholars could do mathematics comfortably to a certain "professionalized" extent [Qian 1983, pp. 308-309].

Now it is possible to understand why Xiang Mingda (1789-1850) retired at so early an age to his home town. In 1816, he was appointed to the "Xue Zheng" (the fifth rank of the professorship) at the Imperial Academy by passing the provincial civil-service examination. Ten years thereafter, he held a "Jin Shi" degree at

age 38 by having passed the central civil-service examination, whereby he was promoted to a magistial post. Remarkably, he not only did not take the new offer, but quit his teaching position at the Imperial Academy as well for reasons unknown to us. Back in his home town, Xiang Mingda spent his productive middle years engaged in specialized mathematical research, and doubtless he earned his living by serving as a tutor as well as a teacher at the Zhaonan Academy [Zhu Kebao 1886, pp. 767-771].

Xiang Mingda's example was no exception, since for many Qing literati, scholarship became a way of life. In fact, among his predecessors, Zhao Yi (1727-1814), Wang Mingsheng (1722-1798), and Qian Daxin (1727-1804) had already turned towards professional scholarship in the 18th century. Although each of them accepted official positions, their main interests were in academic research, and all retired from office as soon as it was practical for them to do so. For example, after retiring to private life in 1776, Qian Daxin was employed for most of his career as a teacher in prestigious Jiangnan academies. This occupation complemented his prodigious research on epigraphy, history, phonetics, calendrical science, and geography. In each case, (Zhao Yi, Wang Mingsheng, Qian Daxin and Xiang Mingda, all held the "Jin Shi" degree), official position was also a means of maintaining high social status.²⁵

Thanks to the interaction of intellectual and institutional factors, Qing literati, in their roles as scholars, were

professionalized. As Elman has argued, "the rise of k'ao-cheng studies was not only an intellectual event (as a matter of scholarly and professional choices) but also an institutional basis (for institutions supported and promoted scholars who choose to carry out such research," [Quoted Elman 1984, p. 96]. Although Qing scholars did not become "professional engineers, scientists, metallurgists, and so forth, they were experts in their fields because they possessed special knowledge unavailable to laymen. Their technical activities as researchers and teachers had social implications. Their occupations were embodied in social relationships and institutions," [Quoted Elman 1984, p. 98].

And yet for mathematicians of the time, Jiao Xun, Wang Lai, Li Rui, Luo Shilin and Xiang Mingda, such criteria of professionalization do not seem wholly applicable, for their fields of study and teaching included not only mathematics but also the Classics. However, if their common interests, activities, and goals are considered along with the development of intricate, informal social networks and feelings of solidarity among mathematicians carrying out essentially similar tasks, then the differentiation of mathematical from humanistic knowledge proclaimed by the Qian-Jia school was significant. Indeed, practitioners of the mathematical sciences now increasingly perceived their activity as distinct, autonomous and objective in contrast to other areas of study. Doubtless the self-image of mathematicians was reinforced by a sense of commitment to a common enterprise. Therefore, before institutions such as the

Tong Wen Guan, where Li Shanlan in his late years served as a mathematics instructor, could offer many teaching posts in the late 19th century, mathematicians of the Qian-Jia era had already become "professionalized" to some extent, but primarily on a non-institutionalized basis.

NOTES:

1. Here the concept "change within tradition" used by Fairbank is not without controversy, for, as Paul Cohen notes, "it reflects certain subjective preferences as to what kinds of change are more and less significant," [Cohen 1984, p. 65]. Cohen argues that several other historians like Edwin Reischauer, Albert Craig, C.Y. Hsu, Karl A. Wittfogel, and John Levenson share the same perspective on the history of 19th-century China. Even so, Cohen admits that such a formulation of 19th-century Chinese history means that such historians are still working "within a framework that virtually compels them to lay particular stress on the more stable and abiding features of Chinese cultures," [Cohen 1984, p. 65]. His example is the article by Fairbank, Alexander Eckstein and Lien-sheng Yang, "Economic Change in Early Modern China: An Analytic Framework," [Fairbank et al. 1960]. In his *Between Tradition and Modernity: Wang T'ao and Reform in Late Ch'ing China* (1974), Cohen suggests that "there existed in pre-Opium War China a number of individuals who, for one reason or another, were dissatisfied with facets of their society and believed in the possibility of its betterment. Inclined in the direction of reform before ever being exposed to influences from the West, they were in a unique position, once the Western challenge materialized, to appraise it flexibly, to see it as part opportunity and not all disaster," [Cohen 1974, p. 275]. Cohen goes on to claim in the same passage that "the Chinese culture which came into growing contact with Western civilization in the nineteenth century was anything but inert. Alive with tensions, contradictions, and problems of all kinds, it was perfectly capable of generating, from within itself, the impulse to change," (emphasis mine) [Cohen 1974, p. 275].
 In this thesis, I am not going to discuss such debatable aspects of 19th-century Chinese history. However, concerning Li Shanlan's mathematics, both that which can be regarded as a response to the impact of Western science and mathematics (including those introduced in the late Ming and early Qing as well as those after the Opium War), as well as that he developed in terms of traditional Chinese mathematics, I would argue that, like other native Chinese mathematicians at the time, Li Shanlan was perfectly capable of producing innovations within Chinese mathematics for its own sake.
2. In transliterating Chinese characters, this thesis adopts the "Chinese Phonetic Alphabet" instead of the "Wade System" for most Chinese terms, names, place names, titles for books, treatises, essays, and so on. Thus "Li Shan-lan" is transliterated here as "Li Shanlan". Nevertheless, some living Chinese authors prefer to spell their names following the Wade System. In such cases, as with Kwang-Ching Liu, I

will follow the preferences of the authors. As for the Chinese Glossary, it is given in an alphabetic order in the "Glossary" section at the end of the dissertation.

3. Li Shanlan's comment should be viewed in the context of 1840s Chinese mathematics. At the same time when his *Dui Shu Tan Yuan* was written, what he had known about infinite series expansions was only Pierre Jartoux's three formulas without demonstrations. We shall come back to this subject again in later paragraphs and sections.
4. This was because in 1856 Wylie's article was translated into German by K.L. Biernatzki, and thus became known in Europe. In addition, in 1863 O. Terquem translated Biernatzki's article into French, and in 1869 J. Bertrand made yet another French translation of the same article. Thereafter, Qin Jiushao's study of indeterminate analysis (i.e., "Da Yan" method) evoked the enthusiasm of European mathematicians like Hermann Hankel, and historians like Moritz Cantor [Cf. Libbrecht 1976, pp. 310-327].
5. See H.L.L. Busard, "Christopher Clavius," in D.C. Gillispie ed. *Dictionary of Scientific Biography* vol.3 (1971), pp. 311-312.
6. Nathan Sivin asserts that both Wang Xishan and Mei Wending contributed to an "astronomical revolution" in 17th-century China [Sivin 1970]. However, Jiang Xiaoyuan argues that there was no such "conceptual transformation" at the time. Thereafter, the Western geometric models that were introduced were regarded as instruments helping to make computations easier. Even so, Jiang Xiaoyuan agrees with Sivin that both Wang and Mei did pay great attention to the Western geometric models and even tried to explain planetary motions [Jian Xiaoyuan 1988].
7. The notorious "anti-Christian Movement" led by Yang Guangxian (or Yang Kuang-hsien, 1595-1669) in the period of Emperor Kang Xi's reign period was a good example. Huang Yi-Long has made an extensive study of this movement [Cf. Huang 1990a, 1990b & 1990c]. According to Huang Yi-long, Yang Guangxian attacked the Jesuit astronomers Adam Schall and Ferdinand Verbiest, among others, on the grounds that the Western methods used in the Shixian Calendar were inaccurate in setting of the funeral date for the fourth son of the Emperor Shun Zhi, Prince Rong (a brother of the Emperor Kang Xi). As a result, Yang Guangxian took over the State Observatory from the Jesuit astronomers in 1665. It was not until 1669 that the Jesuits led by Verbiest fought to take charge of the Observatory again, and Yang Guangxian and his Chinese staffs were subsequently all expelled or dismissed from the Observatory. In the course of this controversy, the Jesuits, like their Chinese rivals, drew upon astrology to defend their conclusions. The astrology the Jesuits had in mind was of course the Western one. It was due

largely to this fact that Chinese scholars felt a Christian threat not only to the royal astronomers, but to the civil astrologers as well.

8. Since the 1970s, historians of Chinese mathematics have begun to explain some interesting features of traditional Chinese mathematics in terms of the concept "algorithm", which has become a major theme of computer science thanks to Donald Knuth's classic in the computer science, *The Art of Computer Programming* [Knuth 1971]. Here we only cite Li Jiming's study in the *Jiu Zhang Suan Shu* [Li Jiming 1990]. Before the thirteenth century, ancient Chinese mathematical classics like the earliest *Jiu Zhang Suan Shu* all explained methods or formulas literally without demonstrations, due principally to the fact that Chinese mathematicians relied mainly on the abacus, "Suan Chou", to do the related computations. Thus it is no surprise to find that the authors of the *Jiu Zhang Suan Shu* gave an "algorithmic" procedure to determine the greatest common divisor of two given natural numbers [Cf. Chapter 1]. Similarly they provided an "algorithmic" method, essentially similar to Gaussian Elimination in terms of matrix theory, to solve simultaneous linear equations [Cf. Chapter 8]. Even as late as the 13th century when Song and Yuan mathematicians began to add demonstrations for more complicated computations, for example, solving equations of degree higher than two or solving simultaneous congruence, they still used procedures of the "algorithmic" type [Cf. Wu Wenjun 1988].
9. As to how the "algorithmic" approach to problem-solving led to the decline of Song-Yuan mathematics, I would suggest that most problems posed in the context of Chinese mathematics were solved to a large extent in terms of "algorithmic" methods. Under such circumstances, the "Si Yuan" method should be criticized not for its mere failure to solve simultaneous equations of unknowns more than four [Needham 1959], but rather in its "algorithmic" aspects interlocked with the abacus, "Suan Chou". This was because some interesting "qualitative" problems might have escaped the eyes of ancient Chinese mathematicians. For example, why a given equation or simultaneous equations was solvable by such and such a method? And if the method for the solution was justified, then what sort of relationship may have existed between the root(s) with the other given conditions, say, the coefficients of the given equation / simultaneous equations. New questions seem to have been raised by mathematicians only when they considered an alternative approach, the "qualitative" approach that seems to have been significant to the old problems. This was indeed the case with Wang Lai [Cf. Qian 1981, p. 295; Horng 1989c]. Similarly, earlier in the 1770s, Lagrange had already adopted a new, "a priori" approach to the theory of equation [Lovy 1973, p. 28], which I would like to call a "qualitative" one.
10. In fact, even in ancient times, Confucian scholars did not seem to have considered mathematics for its own sake [see the

discussion in the ongoing paragraphs], although, as Qian Daxin put it, "no one could be a Confucian who did not know mathematics," [Quoted in Elman 1984, p. 83]. For otherwise, one simply could not explain why mathematical knowledge would have existed in the ancient Classics.

11. "Kao Zheng" or "Kao Ju" (literally, the search for evidence) was one of the most important methods used by scholars of the Qian-Jia school. Why and how mid-Qing scholarship shifted from neo-Confucian philosophy to the philological tradition of evidential research has been extensively examined by Benjamin Elman [Cf. Elman 1984]. In referring to "Kao Zheng" as "evidential research", we agree with Elman that "k'ao-cheng scholars were empirically oriented because they search for an external source for the legitimation of their knowledge. Reason was regarded as a limited measure of knowledge, especially as it was used in the speculative conclusions debated by Neo-Confucian philosophers," [Elman 1984, xxiv]. In addition, Elman also observes that "Evidential scholars opposed forms of inquiry not reducible to the standards of verification used in k'ao-zheng discourse." In other words, to the "Kao Zheng" scholars or scholars of the Qian-Jia school, "valid knowledge must be corroborated by external (textual or otherwise) facts and impartial observations," [Elman 1984, xxiv]. It was here that the "evidential research" of the Qian-Jia school placed constraints on mathematical studies, for knowledge of Chinese mathematics could be valid and acceptable by demonstration of reason alone. This, however, was not enough, as exemplified by Mei Wending's *Fang Cheng Lun* ("Theory of Simultaneous Equations", 1672), which was not published until he was sure that it was corroborated by a similar method already existed in ancient mathematical texts like the *Jiu Zhang Suan Shu* [Cf. Note 12 below; Horng 1989c].
12. Since evidential scholars were interested mainly in knowledge justified by ancient textual evidence, any substantial contributions to mathematics and astronomy that had little to do with their classical studies would be ignored. For example, Mei Wending circulated his *Fang Cheng Lun* ("Theory of Solving Simultaneous Equations") privately for twenty years before he was sure that his theory could be justified by ancient mathematical texts. Although he knew there was a chapter on "Fang Cheng" in the *Jiu Zhang Suan Shu* [Chapter 8], it was not available to him at the time. Such an attitude, in which Mei Wending justified mathematical innovations in terms of examples drawn from ancient texts, undoubtedly was echoed generally by scholars of the Qian-Jia school.
13. Why Wang Lai was an apparent exception is unknown. Perhaps it was due to Wang Lai's early career with the State Observatory and the Imperial Academy where the "Xi Fa" (Western method) was the major subject of study. Thus Wang

Lai did not think the national origin of mathematical knowledge was a serious issue. Rather he seemed to have considered mathematical knowledge for its own sake -- an attitude scarcely found in the Qian-Jia school [Cf. Horng 1989c].

14. This title was given in Luo Shilin's "Biography of Li Rui," in Luo Shilin's *Chou Ren Zhuan Xu Bian*, collected in the *Chou Ren Zhuan Hui Bian* (1963), pp. 657-665.
15. Dai Zhen, Li Huang and Li Rui were interested in the study of the *Jiu Zhang Suan Shu*, yet Jiao Xun seemed to be the only scholar who went beyond what was already known in this classic. In fact, Jiao Xun thought that there were only four operations "Jia" (addition), "Jian" (subtraction), "Cheng" (multiplication) and "Chu" (division) along with their laws underlying the methods or theories of the *Jiu Zhang Suan Shu*. Thus in his *Jiu Jian Cheng Chu Shi*, Jiao Xun adopted the "Tian Gan" (the Heavenly Stems) as symbols to explain his laws of operations: If we denote the Heavenly Stems "Jia", "Yi" and "Bing" by modern symbols a , b and c , then Jiao Xun gave the following laws of operations:

$$\begin{aligned}
 a + b &= b + a; \\
 a &= a; \\
 a + a &= 2a; \\
 a - a &= 0; \\
 (a - b) + c &= a - (b - c), (a - c) + b = a + (b - c); \\
 a \cdot b &= b \cdot a; \\
 (a \cdot b) \cdot c &= (b \cdot c) \cdot a = (c \cdot a) \cdot b. \\
 (a + b)(a + b) &= aa + ab + ba + bb = a^2 + b^2 + 2ab; \\
 (a + b)^3 &= a^3 + 3a^2b + 3ab^2 + b^3,
 \end{aligned}$$

[Qian 1981, p. 287; Li Di et al., eds., 1986, pp. 420-422; Wu Yubin 1986].

16. This was Luo Shilin's comment on Wang Lai's mathematics, which also reflected the viewpoint of Luo's contemporaries in the Qian-Jia school. See Luo's "Biography of Wang Lai" in [Luo 1839, pp. 669-675].
17. The real reason why Zhang Dunren concealed Ming Antu's treatise from Wang Lai is unknown. It is highly probably that the Qian-Jia school at the time thought Ming's work to be the driving force of the "Xi Fa" (Western Method) school motivating studies of new subjects aside from those provided by the Qian-Jia school, rather than concentrating on the revival of ancient Chinese mathematical texts on their own terms.
18. This was Luo Shilin's comment given in his "Biography of Zhang Dunren" [Luo 1839, 695-699], emphasizing that Zhang's mathematics was in striking contrast to those of the "Xi Fa"

schhol.

19. Ming Antu's *Ge Yuan Mi Lu Jie Fa* was actually written in terms of the so-called "Xi Fa" school, for in this treatise one of his major concerns was to demonstrate Jartoux's three formulas on series expansions for trigonometric functions. Meanwhile, he also applied Western approach, namely the "San Fen" method [Cf. Jami 1988] to circle measurements and eventually justified the series expansions introduced by Jartoux, adding six others for the trigonometric functions [Cf. Section 3.3].
20. The concept "Lu" permeated the structure of the *Jiu Zhang Suan Shu*, and has been extensively studied by Li Jiming [Li Jiming 1990, Chapter 3]. For ancient Chinese mathematicians, a "Lu" is not a number, but something like a term of a proportion. For example, in a given circle, the "Zhou Lu" (literally the "Lu" of the circumference) to the "Jing Lu" (the "Lu" of the diameter) is just the same as the ratio of the circumference of a circle to its diameter. In this case, the "Zhou Lu" comprises a series of numbers that are to be related in the way just mentioned to the other series of numbers in the "Jing Lu". Similarly, when ancient Chinese mathematicians used such a term, they would naturally relate it to the "Fang Lu" (the "Lu" of the square that circumscribes the circle). With this in mind, the late Ming and early Qing mathematicians translated a Western proposition as follows: If, in modern notation,

$$a : b = c : d ,$$

then a , b , c and d were described by ancient Chinese mathematicians as "Yi Lu" (the first "Lu"), "Er Lu" (the second "Lu"), "San Lu" (the third "Lu") and "Si Lu" (the fourth "Lu"), respectively.

21. The "Tian Yuan" and "Si Yuan" methods were assumed by Luo Shilin to be powerful tools for reviving the ancient mathematical texts by re-solving the problems therein with these methods. This is why Luo Shilin wrote his *San Jiao He Jiao Suan Li* (1840) [Cf. Horng 1989c].
22. As mentioned above, Ming Antu's mathematics followed basically the "Xi Xue" tradition. Yet since Ming Antu apparently created his own method based on a blend of Western and Chinese traditional methods in his *Ge Yuan Mi Lu Jie Fa* and since the Qian-Jia school had lost its dominance over mathematical studies in the 1830s, by regarding Ming's methods as part of the "new method", the Qian-Jia school made a compromise in hope of retaining its academic status.
23. In his *Yan Shi Jia Xun* ("Family Instructions for the Yan Clan"), Yan Zhitui (of the Southern Dynasty, sixth century A.D.) wrote that: "Mathematics is an important subject in the

six arts. Through the ages all scholars who have participated in discussions on astronomy and calendars have to master it. However, you may consider it as a minor occupation, not a major one." In other words, mathematics should not be considered as a major occupation, although it is important among the six arts: propriety (i.e., observe the *Zhou Li* [= "The Rituals of Zhou Dynasty"] and behave properly in family as well as in civil service), music, archery, charioterring, calligraphy and mathematics. As for the social status of mathematicians, see also [Libbrecht 1973, pp. 2-12].

24. Li Ye might have been influenced by Northern Song neo-Confucian philosophers like Zhou Dunyi who, by contrast to Confucius, assumed an objective reality beyond the human mind. And it was on this assumption that Li Ye could build his philosophy of mathematics. As to how Li Ye related mathematics to neo-Confucian philosophy at the time, we know little due to the fact that research on this subject has only just begun [Cf. Kong Guoping 1988].
25. There was a great deal of privilege granted a local resident scholar who had earned a high degree in the civil examination system, or who came back from an official post [Cf. Ho Ping-Te 1962].

2. LI SHANLAN -- biographical details, early education, mathematical influence on his later career and significance

2.1. Early Life and Mathematical Studies

Li Shanlan (1811-1882) was born on January 2, 1811, in the city of Shanshi of the Hailing prefecture, Zhejiang province. "Shanlan" was the name under which he was registered at school, which must have been given to him by his father, Li Zulei. Li Zulei also called his son by the name "Xinlan," and also used the courtesy name "Jingfang" [Wang Yusheng 1983a].¹ In addition, Li Shanlan called himself "Qiuren" or "Renshu". The latter appears very often in (historical) source materials.

Li Shanlan, whose mother was Li Zulei's third wife², had two younger brothers, Xinmei and Xinkui.³ His father seemed to be versed in the Classics, and so was his grandfather, Cui Jingyuan. Thus it is understandable that both of his parents paid a great deal of attention to his early education [Wang Yusheng 1983a].

There are not enough sources, however, to provide a complete picture of Li Shanlan's early life. Yet his mathematical talent sparkled at a very early stage, as it often does for gifted mathematicians. From the time he was eight years old⁴, when he came across a copy of the *Jiu Zhang Suan Shu* shelved in his father's library, he "devoted himself to mathematical study thereafter," [Li 1867, "Preface"]. And as was especially the case



Figure 1. Li Shanlan's portrait appeared in the
Ge Zhi Hui Bian (*The Chinese Scientific
Magazine*) vol.2 (1877), no.5.

of many gifted mathematicians in traditional China, he seems to have been largely self-taught, although he did have a mathematics teacher, his fellow townsman Wu Zhaoyi [Wang Yusheng 1983a]. It is doubtful, however, that Wu Zhaoyi had any real influence over his early mathematical study, since Li Shanlan never referred to any teachers in his treatises on mathematics.

In addition to his mathematical study, Li Shanlan began writing poems when he was eleven years old. One or two years later, he also learned to write the "eight-legged" essays.⁵ Due in part to these early experiences, Li Shanlan remained interested in poetry (and philology) as well as mathematics during his entire life.⁶ However, this does not mean that he never had ambitions for an official post, which was still an important goal for the literati to pursue even as late as the 19th-century. According to the available documents, Li Shanlan presented himself for provincial civil examinations (i.e., "Xiang Shi") at least twice. The first time was in 1825.⁷ Unfortunately, he failed this examination because he had placed too much emphasis on mathematics which was not among the subjects of the examination [Li 1867, "Preface"]. Afterwards he stayed in Hangzhou (where the Zhejiang provincial examination was usually held once every three years) to search for mathematical texts. As a result, he was finally able to read Li Ye's *Ce Yuan Hai Jing* and Dai Zhen's *Gou Gu Ge Yuan Ji* [Li 1867, "Preface"].

Li Shanlan was especially impressed by the *Ce Yuan Hai Jing*,

a mathematical classic expounding the "Tian Yuan Shu" (or "Tian Yuan" method). This seemed especially significant when compared to the first six books of Euclid's *Elements*, which he had had a chance to read two years earlier [Li and Wylie 1865, "Prefaces"]. His first contact with the *Ce Yuan Hai Jing* made a vivid impression, as he recalled in 1845:

When I was fourteen or fifteen years old and just starting to read the book [i.e., the *Ce Yuan Hai Jing*], I was unable to understand its content. Yet after studying very hard for seven days and nights without taking much food or sleep, I finally comprehended all the materials concerning methods of solving equations, and thereby found out what mathematics was all about [Hai Ling Zhou Zhi Gao 1922, pp. 1775-1776].

As late as 1876, when he went to Beijing as a mathematics instructor at the Tong Wen Guan (the Imperial College), Li Shanlan still acknowledged his debt to the "Tian Yuan" method:

I began studying the *Jiu Zhang [Suan Shu]* from early childhood, but considered it to be superficially nothing. It was not until the availability of this book [i.e., the *Ce Yuan Hai Jing*] that I began to learn the comprehensiveness of mathematics. I have been devoting myself to this field since then. Thereafter [i.e., in the 1850s] I felt no difficulty in translating the Western texts on algebra and calculus due to the help of this book. This is because the theory underlying the Western methods is nothing but the theory of the "Tian Yuan Shu". Now I am teaching at the Tong Wen Guan using this book as a textbook. My major concern is to synthesize Chinese and Western methods into one method by the way of demonstrating [the Western] algebra in the "Tian Yuan Shu," [Quoted from Li Shanlan 1876, "Preface"].

On the other hand, Li Shanlan read the *Elements* (only the first six books were then available in Chinese) at the age of thirteen. He made no comments upon this Greek classic, but regretted that the Chinese edition was incomplete [Li and Wylie

1865, "Preface"]. However, the *Elements* did make a strong impression on Li Shanlan, to the extent that his *Fang Yuan Chan You* followed a format similar to that of Euclid's mathematical classic. For example, Li Shanlan gave ten principles (*Dan Zhi*) which were set out for the reader at the beginning of the treatise. He then went on to demonstrate the area formula for a circle in terms of an infinite series expansion of π [Wang Yusheng 1983b].

Li Shanlan's *Fang Yuan Chan You*, along with his other mathematical treatises, the *Hu Shi Qi Mi* and the *Dui Shu Tan Yuan* were written by 1845 [Wang Yusheng 1983b]. At about the same time, Li Shanlan served as a tutor for the Lu-Fei family in Jiaying prefecture [Li Yan 1955]. Tutoring was a job for literati, who having already gained the licentiate status ("Sheng Yuan"), could make a humble living. Li Shanlan held such a status, for otherwise he would not have been able to attend the provincial civil examination held in 1825. Thanks to this opportunity, Li Shanlan became acquainted with the famous scholars Gu Guanguang (1799-1862) and Zhang Wenhui (1813-1881) who were interested in the study of mathematics and astronomy. Through these acquaintances Li Shanlan's mathematics drew the attention of the Jiangnan academic community.

No doubt Li Shanlan had become well-versed in mathematics by the middle of the 1840s. Since he was basically self-taught, the mathematical texts available to him should be considered. Before

he went to Jiaxing in 1845, he had already read the *Jiu Zhang Suan Shu*, Euclid's *Elements* (the first six books), the *Gou Gu Ge Yuan Ji* and the *Ce Yuan Hai Jing*. The last two of these were apparently the most helpful, because he admitted that he made great progress in mathematics after he got through them. Subsequently, he tried to study the "Ge Yuan" method because he thought the theory underlying it had not been convincingly explained by Li Ye or Dai Zhen [Li 1867, "Preface"]. As a result, he was able to create the "Jian Dui" method or "Jian Zhui" method [Cf. Sections 3.2, 3.3, and 3.4]. In contrast to Ming Antu's "Ge Yuan Lian Bi Li" method, Li Shanlan used the "Jian Zhui" method to give an infinite expansion for π . In addition, based on the "Jian Zhui" method, Li Shanlan also gave infinite series expansions for logarithms as well as power series for trigonometric and inverse trigonometric functions. In this context, Li Shanlan's use of the "Jian Zhui" method suggests that he was a highly creative mathematician of the time. Since the "Jian Zhui" method was more closely related to traditional Chinese "Duo Ji" method (the method of piles summation) than to Ming Antu's "Ge Yuan Lian Bi Li" method, it is clear that after Jiao Xun, Wang Lai and Li Rui, once again Li Shanlan was the key figure among late Qing mathematicians to make substantial contributions of his own. In fact, Li Shanlan's example was no exception. This was especially the case with infinitesimal problems. Although the differential and integral calculus was not invented by the Chinese mathematicians, the 19th-century Qing mathematicians like Dong Youcheng, Xiang Mingda, Xu Youren, Dai Xu, Gu Guanguang and Li Shanlan all created

some methods equivalent in principle to those introduced by the Westerners. No wonder Alexander Wylie highly appraised these native mathematicians and suggested that "their reasoning is very close to infinitesimals," [Li and Wylie 1859, "Prefaces"]. By the same token, some historians of Chinese mathematics have been led to suggest that, had calculus not been transmitted to China in the 1850s, Chinese mathematicians might have developed it independently on their own [Li and Du 1987, p. 254].

By 1835 or so, Li Shanlan had also read Zhu Shijie's *Si Yuan Yu Jian*. In the "Preface" to his *Si Yuan Jie* ("A General Methodology in the *Si Yuan Yu Jian*", 1845), he noted specifically that "Wang Yuezhen (1813-1881) sent a hand-written copy of Zhu Shijie's *Si Yuan Yu Jian* to me in the spring of this year [i.e., 1835].⁸ Thus I began to learn that in addition to the 'Tian Yuan' (the celestial element, or the first unknown of the equation), there were also the 'Di Yuan' (the earth element, or the second unknown), 'Ren Yuan' (the human element, or the third unknown), and the 'Wu Yuan' (the material element, or the fourth unknown)." Yet Li Shanlan found it very difficult to understand any of this from the very start due to the fact that the methods of the *Si Yuan Yu Jian* lacked demonstrative procedures. Therefore, along with his student Gu Shuozeng, they together concentrated their attention day and night on the *Si Yuan Yu Jian*. After eight days of hard work they finally came to appreciate the excellence of mathematics reflected not only in the "Tian Yuan" method but in the "Si Yuan" method as well. As a result, Li

Shanlan wrote the *Si Yuan Jie*, which was an annotated commentary on the *Si Yuan Yu Jian*. This also fulfilled Gu Shuozeng's wish as well as his own -- to revive the ancient mathematics and demonstrate thereby its excellence [*Hailing Zhou Zi Guo* 1922, pp. 1775-76].

Thus it is no surprise that Li Shanlan still accepted the prevailing view that Western mathematics had its origins in China. In his "Preface" to the 1845 edition of the *Si Yuan Jie*, Li Shanlan confidently proclaimed that:

The Western methods are no better than the "Gou Gu"⁹, since the "Ba Xian" (the eight trigonometric functions) are all methods of "Gou Gu". On the other hand, Chinese methods are no better than the "Fang Cheng"¹⁰, since the "Si Yuan" methods are those of "Fang Cheng".... Both these methods are so remarkable that they would not have been created without the saints of the Three Dynasties¹¹.... At that time, the texts on "Ba Xiang" and "Fang Cheng" must also have appeared. They appeared to have been lost due to the fire set by the Emperor Qin. Yet fortunately enough, the methods of "Ba Xiang" were transmitted to the West, and thereby have survived to this day. There are, however, no contemporary scholars who understand the "Si Yuan" method, although scholars before the Yuan period could comprehend them quite well [*Hailiang Zhou Zhi Gao* 1922, pp. 1775-1776].

But when the *Si Yuan Jie* was published again, without revision, in 1867 Li Shanlan eliminated any mention of the "foundation myth" from his "Preface".¹² Even more emphatically, he also omitted the following sentence: "There are, however, no contemporary scholars who understand the 'Si Yuan' methods." Apparently in 1835 (or thereabouts) he did not know that Luo Shilin had already published the *Si Yuan Yu Jian Xi Cao* in 1834. This also means that Li

Shanlan had not as yet become acquainted with Luo Shilin. At that time his major interest in mathematics must have been the "Ge Yuan" methods, which finally led to his three classics, the *Fang Yuan Chan You*, *Hu Shi Qi Mi* and *Dui Shu Tan Yuan*.

2.2. Li Shanlan and the Scholarly community of the Jiang-Zhe Area

Li Shanlan's three mathematical treatises, as mentioned above, were greatly appreciated in the 1840s by the Jiang-Zhe scholars Zhang Wenhui and Gu Guanguang, among others. Gu Guanguang had succeeded in his family's career and became a doctor, but he was also well-trained in the Classics, astronomy and mathematics [Zhu Kebao 1886, pp. 799-802]. Li Shanlan highly praised his achievements in the fields of science and mathematics and sent his own treatises, at least the *Si Yuan Jie* and *Dui Shu Tan Yuan* to Gu Guanguang for comment. In return, Gu Guanguang wrote prefaces for both treatises. In his "Preface" to the *Dui Shu Tan Yuan*, Gu Guanguang especially showed his enthusiasm for Li Shanlan's creation of the "Jian Zhui" method. Shortly before he died, Gu Guanguang asked his son Gu Shen to invite Li Shanlan to write a preface for his mathematical treatises (and even asked that Zhang Wenhui wrote his biography as well).¹³

As for Zhang Wenhui, Li Shanlan's close association with him lasted even longer. Zhang Wenhui, by contrast to Gu Guanguang, paid less attention to the three mathematical treatises Li Shanlan wrote in the 1840s. Instead, he seemed more interested in Li

Shanlan's translations of Western mathematics and science. This might mean that Zhang Wenhu was not so well-versed in mathematics as both were Gu Guanguang and Li Shanlan. However, Zhang Wenhu and Li Shanlan shared the same aesthetic mood of the literati, such as writing poems or replying to one another in letters using the same rhyme sequence [Zhang Wenhu 1856]. When Li Shanlan visited Jiaxing in 1849, he certainly enjoyed living with Zhang Wenhu and other friends there. On the other hand, Li Shanlan also took this opportunity to learn the Classics under the direction of the master Chen Huan [Li Yan 1955]. This fact suggests that Zhang Wenhu's influence over him was considerable, since Zhang Wenhu was also a noted classical scholar of the time.

It was probably due to Chen Huan's recommendation that Li Shanlan became Zeng Guofan's "Mu You" (secretarial staff) in the early 1860s [Zeng's *Diary* volume 9]. Zhang Wenhu also served Zeng's "Mu You" at the same time [Li Yan 1955, pp. 331-361]. But even before this, Li Shanlan's career as a translator of Western mathematics and science had already begun to develop in Shanghai in the 1850's.

Before he went to Shanghai in 1852, Li Shanlan's association with Luo Shilin and Xu Youren was especially important to his career. Unfortunately, we do not know when and where Li Shanlan and Luo Shilin met, or how they got along with each other. According to his nephew, Cui Jingchang, Li Shanlan often received letters from Luo Shilin and Xu Youren discussing mathematical

problems. Li Shanlan's response was always prompt and his solutions to or explanations of the problems were as clear as possible [Cui Jingchang]. But what was their common interest in mathematics at the time? Considering Luo Shilin's relatively late interest in mathematics, the most plausible answer is a mutual interest in the "Si Yuan" method, since Luo Shilin paid almost no attention to "Xi Fa" (Western methods) in his later years [Guo Shirong 1986].

Li Shanlan's close association and friendship with Xu Youren (1800-1860) was significant to Li Shanlan's career as both mathematician and translator. It is certain that Li Shanlan met Xu Youren not later than 1857, and that the two of them got along very well [Wang Tao 1987, p. 77]. In fact, Xu Youren was interested in almost exactly the same fields of mathematics as Li Shanlan. For example, Xu Youren's treatises in mathematics suggest that he was also interested in the "Ge Yuan" method [Cf. Section 3.6], the "Duo Ji" method, the "Si Yuan" method, and so on [Xu Youren 1872]. In addition, in his *Jie Qiu Jie Yi*, Xu Youren also gave the formulas for the (surface) area of a sphere, a sector of a sphere, a spherical cap as well as the lateral surface of a cylinder with an inscribed sphere [Fu Daiwie 1988]. All these formulas had been given before without demonstrations, not only in Mei Wending's *Fang Yuan Mi Ji* ("Squares, Circles, Cubes and Spheres") but in the *Shu Li Jing Yun* as well [Fu Dawie 1988]. It was Xu Youren who took these demonstrative problems more seriously than ever before. What he did was very similar to what

Ming Antu had done with Jartoux's three formulae mentioned in Chapter 1.

Xu Youren was interested in "Xi Xue" (Western studies), despite the fact that he had no intension of converting from (Chinese) Buddhism to Christianity.¹⁴ This was perhaps one of the reasons for the close relationship between Li Shanlan and Xu Youren, since both of them seemed to have been interested in Western science and mathematics per se, wholly apart from any interest in Christianity. As Li Shanlan served in the Mo Hai Shu Guan (the London Missionary Society Press) in Shanghai as a translator of Western mathematics and science, he began to regard "Xi Xue" on its own terms.

2.3. Li Shanlan and Intellectuals in Shanghai of the 1850s

Li Shanlan went to Shanghai in 1852 to translate, in collaboration with Alexander Wylie¹⁵, the *Xu Yi Ji He Yuan Ben* (the last nine books of Euclid's *Elements*)¹⁶ and, in collaboration with Joseph Edkins¹⁷, the *Zhong Xue* (the Chinese translation of William Whewell's *Elements of Mechanics*), thanks to an invitation from Wylie. According to his "Preface" to the *Zhong Xue*, Li Shanlan started co-translating the *Elements* with Wylie on July 17, 1852 [Li and Edkins 1866]. Apparently it was at the London Missionary Society Press that Li Shanlan met Wang Tao (1823-1897), who had served as a Chinese editor there since 1849. According to Wang Tao's diary, Li Shanlan first visited his home on July 20,

1852 [Wang Tao 1852]. Immediately thereafter, Li Shanlan was urged by his friend Zhou Changqing to attend the Zhejiang provincial civil examination in Hangzhou. Consequently, he left for Hangzhou on August 28 (or thereabouts), and did not come back until the middle of November. But this does not mean that he actually was concerned about the social status provided by the civil examination system, for otherwise he should not have been urged to attend the examination by Zhou Changqing. Perhaps this time Li Shanlan was not well prepared either, since he had devoted all his effort to mathematics which again was not included among the subjects of the examination. All of this may suggest that his own perception of his role as a mathematician was clear, especially considering the late Qing intellectual and socio-economic opportunities created by the Qian-Jia school (as described in Chapter 1).

In contrast to Wang Tao, Li Shanlan was not employed full-time at the London Missionary Society Press. Instead, his pay must have depended upon how much translating he did. In fact, this was exactly how other literati who were offered similar jobs were paid.¹⁸ How they worked, apparently, was as follows. While the Westerners orally translated every sentence as best they could into Chinese, the Chinese scholar would record what he heard, improving the language and asking for further explanations if necessary. The Chinese scholars later went on to modify the translated draft into proper Chinese style. Finally, before the draft was ready for publication, both of the co-translators would

proofread the manuscript [Horng 1988b]. Under these circumstances, Li Shanlan could not have earned enough from such translations to support himself very well, although he was apparently able to make a humble living in Shanghai doing so in the 1850s. Wang Tao's salary was not very high either. Nevertheless, they both got together frequently with other literati without official positions or those who failed the civil examinations. They spent most of their leisure time, in fact, drinking wine, savoring curios, writing poems and replying to each other in rhymed sequences, visiting brothels as well as enjoying gossip discussion of current socio-political affairs [Wang Tao 1987].

The gossip must have also extended to Christianity and the Western culture they were encountering, not to mention their reactions to the foreign invasion, both in terms of trade and military forces. They were especially aware of foreign influence because they were resident in Shanghai, which had become the metropolis of China, including not only the old city but also the foreign settlements set apart in an area north of the old city wall [Fairbank 1978, pp. 239-240]. This area, among others, had been forced open to foreign settlement after 1840-42 as a result of the Sino-British Opium War. From 1843 to 1853 this initially small foreign settlement generally excluded native residents except for Chinese working as servants or in service trades. Then in March of 1853, the Taiping rebels captured Nanjing, and a flood of refugees moved south and entered this foreign settlement which

suddenly became a Sino-foreign city. Since the refugees from the Taipings also included landlords and merchant families of wealth and position, they accelerated the movement of lower Yangzi (River) landlord families into foreign contact and foreign trade. "This produced in time some community of interest between Western merchants and Chinese ruling class elements," [Quoted Fairbank 1978, p. 240].

To the late Qing literati, especially those resident in Shanghai, dominated as it was in many respects by British settlers in the 1850s [Fairbank 1978, pp. 240-241], the most serious Western threat was basically religious -- that "all missionaries, by the very nature of their calling, posed a revolutionary challenge to the traditional culture," [Cohen 1978, pp. 543-44]. On the missionaries' side, since "Confucius was regarded as the supreme enemy of China's conversion to Christianity, it was natural for the British to vent their frustrations, disappointments and anger on the most immediate embodiment of Confucius' influence, the gentry-scholar class," [Cohen 1978, p. 565].¹⁹ Yet the gentry-scholars or Chinese literati in general "were schooled from early childhood in the traditions and values of Confucianism, their social position and prestige rested, to a very considerable extent, on active identification with these traditions and values," [Cohen 1978, p. 564]. Therefore, "when missionaries' hostility towards the gentry is juxtaposed with the latter's bitter resentment against the former, one gets some sense of the depth and intensity of the cultural conflict that emerged in China

after the mid-19th century," [Cohen 1978, p. 565].

The situation got even worse when the post-1860 missionaries posed a direct threat to the gentry's traditional monopoly of social leadership in addition to challenging its fitness for cultural and moral hegemony. For example, "the missionaries were the only persons at the local level, aside from the gentry, who were permitted (by the Sino-foreign treaty) to communicate with the authorities as social equals. While missionaries operated orphanages and engaged in famine relief, the gentry felt that it alone was responsible for such social obligations. More serious, perhaps, than any of these concrete invasions of the gentry's traditional prerogatives was the missionaries' dedication to establishing schools which advocated a rival set of truths. Besides, they preached their 'truths' in public places. Especially in the Protestant case, they wrote and distributed a prodigious amount of literature. This only tended to insult the gentry, for it seemed that the foreigners assumed that China was not the 'depository of all truth and knowledge,'" [quoted Cohen 1977, p. 85].

It is no wonder, therefore, that in the 19th century, Chinese converts would do their best to conceal their Christianity. For example, in the voluminous corpus of Wang Tao's printed writings, there is not a single word indicating that he was a Christian, though he admits it quite freely in an unpublished diary of 1854 [Wang Tao 1854]. In fact, after assisting W.H. Medhurst in the

preparation of the "Delegates' Version" of the New Testament, Wang Tao was baptized in 1854 in Shanghai [Cohen 1974, pp. 19-20].

Perhaps Wang Tao's conversion was not so significant to his subsequent career as a socio-political reformer, since Wang Tao, unlike his contemporary "Christian reformers," had no intention of Christianizing China. The "Christian reformers" means, according to Paul Cohen, were "early escapees from a closed Confucian world. Christianity, for most of them, does not appear to have served (as it did for many Meiji Protestants) as a substitute world view -- a replacement for Confucianism. Rather, it served to dramatize the fact that other world views -- legitimate and respectable -- were possible. Once this became plain, Confucian society, for the first time, was placed on the defensive," [Cohen 1978, p. 585].

Even so, Wang Tao often regreted that he had worked at the Mo Hai Shu Guan. In fact, he had never been comfortable working for the Westerners. His diary entry, dated March 10, 1859, clearly shows that he felt it was too late to change things. This was also reflected in his conversation with Guan Xiaoyi who collaborated Dr. Benjamin Hobson to translate Western medical books at the Mo Hai Shu Guan for one year. When Guan Xiaoyi returned to Shanghai to apply for a similar job, unfortunately none was available. However, Elijah C. Bridgeman (1801-1861), the first American missionary to China, offered him an opportunity to help revise the Old Testament. To Wang Tao's surprise, Guan Xiaoyi refused the offer, even though it would have earned him a

humble living. Guan explained that had he done so, he would have felt guilty, since it would suggest that he had converted from Confucianism to Christianity [Wang Tao 1987, p. 107]. Wang Tao was shocked by Guan Xiaoyi's loyal commitment to Confucianism and his refusal to help circulate the *Bible* widely. In concluding his diary entry, Wang Tao indicates that he was so full of remorse that he made up his mind to go back to his home town.

Should Wang Tao have quit his job at the Mo Hai Shu Guan, had there been any other alternative? Perhaps this explains why he went to the qualifying examination for the "Xiang Shi" immediately after March 9, 1859, although he knew he was not well-prepared [Wang Tao 1987, pp. 94-100]. If he could pass the "Xiang Shi," he would then be in a better position for an official offer. Li Shanlan, however, did not take this examination that year. Perhaps he was already a little too old. Actually, after failing his last examination in the 1853, Li Shanlan was primarily concerned with his study of mathematics. Apparently, he felt more at home working at the Mo Hai Shu Guan than did Wang Tao, especially since Li Shanlan's mathematics had been highly praised by his Western colleagues.

2.4. Li Shalan and the LMS Press

As a matter of fact, Li Shanlan felt quite comfortable in collaborating with Alexander Wylie, Joseph Edkins and Alexander Williamson in translating Western mathematics and science texts,

as he said, to "the Emperor's treating the West in an equal and reasonable way." For otherwise, he "would not have dared to help translate the western texts," [Li and Wylie 1865, "Prefaces"]. In other words, had Li Shanlan not been assured that his co-translations would be well-received by the Chinese scholarly community in the 1850s, he would not have served at the Mo Hai Shu Guan. Although it is unknown whether Li Shanlan would have converted to Christianity, he clearly had interest in natural theology, a fashionable philosophy that pervaded the *Zhi Wu Xue* (*The Elements of Botany* by John Lindley), which Li Shanlan co-translated with Alexander Williamson (1829-1890) [Li and Williamson 1858, "Preface"].²⁰

Yet the main reason why Li Shanlan was happy to work at the Mo Hai Shu Guan was, as mentioned above, the fact that the knowledgeable missionaries paid high respect to his abilities in mathematics and science. Moreover, the missionaries depended upon his collaboration in translating the texts of Western mathematics and science into Chinese. But why were the missionaries so intent upon translating Western texts? Again, their strategy of evangelizing seems to have been quite similar to that of their Jesuit predecessors two hundred years earlier [Cf. Section 1.2]. To support their claim that China was not the only repository of truth and knowledge, they needed to provide other alternatives, and Western mathematics and science served to do this best [Cohen 1978]. This helps to explain why Wylie emphasized natural theology in his "Preface" to the *Xu Yi Ji He Yuan Ben*, the *Tan*

Tian, the *Dai Wei Ji Shi Ji* as well as the *Dai Shu Xue*. Once Li Shanlan saw that Western mathematics contained something new, for example, the differential and integral calculus -- and that it in fact surpassed both Chinese methods and ancient Western ones [Hua 1882] -- he was subsequently enthusiastic about making translations. Moreover, as he said in his "Preface" to the *Zhi Wu Xue* (the Chinese translation of Lindley's *Elements of Botany*), in studying its pages, "careful readers will realize the actual existence of God ('Shang Di', literally Heavenly God)," [Li and Williamson 1858, "Preface"].

The *Zhi Wu Xue* was published by the Mo Hai Shu Guan in 1858. In the same year, the *Xu Yi Ji He Yuan Ben* was also published. This must have been finished by 1855, when Zhang Wenhui was invited by the publisher Han Yunsheng to help with its proofreading [Li and Wylie 1865, "Preface"]. Moreover, by 1858 Li Shanlan must already have finished the translation of the *Zhong Xue* (Whewell's *Mechanics*), since he referred to the *Zhong Xue* in his *Huo Qi Zhen Ju* ("Remarks on the Cannons"), written in the winter of that same year. The *Zhong Xue*, also proofread by Zhang Wenhui, was published by Qian Xifu in 1859.²¹ In the same year, Li Shanlan obtained the publications of three other translated books, the *Dai Wei Ji Shi Ji* (a translation of Elias Loomis' *Elements of Analytical Geometry and the Differential and Integral Calculus*), the *Tan Tian* (a translation of John Herschel's *Outline of Astronomy*) and the *Dai Shu Xue* (a translation of Augustus De Morgan's *Elements of Algebra*). Li Shanlan was especially impressed with the *Dai Wei Ji*

Shi Ji. According to Wang Tao's diary entry dated April 20, 1859, Li Shanlan was convinced that the *Dai Wei Ji Shi Ji* was a masterpiece of mathematics. Furthermore, due to what he had contributed to its translation, he was proud, not immodestly, to claim himself as the best mathematician of his time [Wang Tao 1987, p. 109]. He also said the same thing to Hua Hengfang (1833-1902) who visited the Mo Hai Shu Guan in 1857 [Wang Tao 1987, p. 134]. At the time, although the translation of the *Dai Wei Ji Shi Ji* was unfinished, Hua Hengfang brought a handwritten copy of a few pages of the drafts back to the city Jingui (of Jiangsu province) with him. Although he studied them dilligently, he failed to grasp the significance of this work. When Li Shanlan later gave him some advice with suggestion to help him understand what he had read, Hua Hengfang eventually came to appreciate the excellence of the calculus [Hua 1882].

Hua Hengfang, in fact, was to become the leader of the Chinese mathematical community in the next generation. But this is another story, mentioned again in Chapter 4 below, in connection with his work as mathematician, translator and a key figure active in the late period of the "Self-strengthening Movement".

As for the "Self-strengthening Movement" itself, Li Shanlan was instrumental in making the movement relevant to mathematics. This was the reason why he was recruited as a mathematician to serve as Zeng Guofan's Anqing headquarter in the early 1860s.

Another episode of his later life that might have influenced the end of his career occurred in 1860. This happened during a very short stay in the official residence of Xu Youren, the then-governor of Jiangsu.

2.5. The Suzhou Episode

On May 19, 1860, Li Shanlan was invited to the governor's residence at Suzhou by Xu Youren to serve as a "Mu You" [Wang Tao 1987, p. 166]. He was accompanied by Wu Jiashan (1820-1885), a well-known scholar and mathematician of the time.²² Why Wu Jiashan accompanied Li Shanlan to visit Suzhou is unknown. However, the reason why Li Shanlan was invited by Xu Youren is quite clear. Realizing the coming threat that the Taiping rebels posed to Suzhou, Xu Youren wanted Li Shanlan to help safeguard the city. Yet how could a mathematician like Li Shanlan have been able to help enforce the defensive powers of the city? Xu Youren must have been thinking of Li Shanlan's knowledge of the cannon, since at the time Li Shanlan had already finished his *Huo Qi Zhen Jue*. Apparently, because Xu believed that mathematics was relevant to military strategy and tactics, Li Shanlan was expected to be of great help in defending Suzhou [Cui Jingchang]. Wu Jiashan may also have wanted to commit himself staying there had he been invited to serve as a "Mu You" by Xu Youren. But apparently this was not the case, for Wu Jiashan returned immediately to Shanghai in three or four days. He reported optimistically to Wang Tao that the Taiping rebels would be

stopped at Changzhou before they could go further to put pressure on Suzhou [Wang Tao 1987, p. 167].

But Wu Jiashan was overly optimistic about the defense of Changzhou. Even worse, the capture of Suzhou by the Taiping rebels shortly thereafter, on June 2, came too soon after the occupation of Changzhou on May 26 [Kuo 1976, pp. 679-84]. Xu Youren was killed and his family, including his wife and son, all jumped into a well to die with him on the same day [Wang Tao 1987, p. 174]. Unfortunately, since he left for Shanghai to ask for Western relief troops on June 1 [Cui Jingchang; Li Shanlan], Li Shanlan did not know that Suzhou had been seized by the Taiping rebels until June 3. According to Wang Tao, Li Shanlan arrived in Shanghai on June 2, and next day visited him in the early morning.

While they were discussing how to get relief troops from the Westerners, one friend of theirs brought the terrifying news that Suzhou had fallen to the rebels, as was confirmed on June 5 in Wang Tao's diary [Wang Tao 1987, pp. 173-74].

The Taiping capture of Suzhou and the death of Xu Youren came as a great shock to both Wang Tao and Li Shanlan. Both of them had placed high hopes on Xu Youren. As for Wang Tao, in the early 1860s he had presented proposals for bureaucratic reform to Xu Youren, including a request for an official post [Wang Tao 1987, p. 139]. Li Shanlan, on the other hand, seemed to have hoped primarily that mathematics would be included in the curriculum of every county academy in Jiangsu under the supervision of Xu Youren

[Wang Tao 1987, p. 86]. But he had also dreamed that Xu Youren raised funds to purchase an official post for him [Wang Tao 1987, p. 117]. Moreover, when the Taiping rebels set Suzhou ablaze, Li Shanlan's own work went up in flames as well, since all the drafts left in Suzhou of his mathematical treatises were reduced to ashes [Li 1867, "Preface"]. Fortunately, three of his books had already been published in the 1840s, namely the *Fang Yuan Chan You*, the *Dui Shu Tan Yuan* and the *Hu Shi Qi Mi* [Li 1867, "Preface"].

Li Shanlan may also have been disturbed by the Suzhou incident for another reason, for some of his close friends in Shanghai seem to have thought that in leaving Suzhou for help, Li Shanlan had abandoned his good friend Xu Youren, although they did not, perhaps, blame him for not giving up his own life for Xu Youren [Wang Tao 1987, p. 183]. There was also a rumor circulating in his home town that Li Shanlan had surrendered to the Taiping rebels, and even let his hair grow in order to look like the "Chang Mao," literally the long hair rebels. When he later went back to his hometown, his fellow townsman and good friend Jiang Shanting accompanied him as they strolled together through the streets of the town. Li Shanlan was specifically asked not to wear a hat in order to show that he had not let his hair grow as the Taiping rebels did [Li Shanlan].

This did not mean that Li Shanlan would have necessarily been hostile to the Taiping rebels. Wang Tao, for example, seems to have regarded their cause favorably. Li Shanlan, as well as his

friends in Shanghai, were apparently interested in Wang Tao's report about the Taiping rebels when he returned from a short visit, along with Joseph Edkins and Griffith John, to occupied Suzhou between June 29 to July 6, 1860. In fact, Wang Tao actually had favorable expectations of the Taiping rebels just as the missionaries did, although he apparently dared not speak out plainly. This is doubtless why he did not accept the next invitation from Joseph Edkins to visit Suzhou again [Wang Tao 1987, p. 191]. Nevertheless, through his acquaintance with Hong Rengan (1822-1864), the then "Shield King" ("Gan Wang", the title for one of the top leaders) of the Taiping rebels, Wang Tao presented a proposal to the Taiping rebels under the pseudonym Huang Wan. On February 2, 1862, he suggested that an optimal way by which the rebels might occupy Shanghai was not to make an enemy of the Westerners. Unfortunately, this proposal was discovered barely two months later, on April 4, and Wang Tao was suspected to be its author [Kuo 1976, pp. 853-55]. After hiding in the British consulate in Shanghai for almost five months, he finally fled to Hong Kong with the aid of the missionaries on October 4, 1862 [Wang Tao 1987, p. 195].

Gone also with Wang Tao was the detailed description of Li Shanlan's social life and intellectual activities in Shanghai, since after he fled to Hong Kong, Wang Tao wrote almost nothing about Li Shanlan. The real reason is unknown, but he seemed not able to obtain much information on his old friends in Shanghai very often. Fortunately, before Li Shanlan went to Anqing to

serve as a "Mu You" for the noted scholar-general Zeng Guofan (1811-1872) in 1863, there was not much about his activities in Shanghai that was of anything particular significance to his later career. Although materials about him in the period 1861-62 are quite fragmentary, no doubt his life in Shanghai was much the same as before. This means that he would still have been the center of the mathematical community of that time. Thus when Wu Jiashan again visited Shanghai in 1861, he saw Li Shanlan alone to talk about mathematics. In 1862, Li Shanlan also received a visit from Liu Yicheng, who was one of the major Chinese mathematicians towards the end of the 19th century [Li Yan 1955, pp. 331-361]. Li Shanlan seems to have enjoyed very much the usual literati way of life with other scholars who were resident in or visiting Shanghai [Li Yan 1955, pp. 331-361].

2.6. Li Shanlan and the Self-strengthening Movement

Li Shanlan went to Anqing, along with Zhang Wenhui and Zhang Siqi, to serve as of Zeng Guofan's "Mu You" in June of 1863. The "Mu You" (secretarial staff) is a member of the "Mu Fu" (the private bureaucracy). And "The mu-fu ['Mu Fu'] was a late Ming institution that had grown in importance during the Ch'ing [Qing] period." Yet, usually "the mu-yu ['Mu You'] that composed it were experts in legal, fiscal, or literary affairs who served as unofficial advisers employed by bureaucrats in the provinces," [Jones and Kuhn in Fairbank ed., 1978, p. 148] It was Zeng Guofan who for the first time in Chinese history reshaping the structure

of the "Mu Fu", namely he recruited not only general scholars but specialists as well. In a diary entry dated June 3, 1862, Zeng Guofan described Zhang Sigui as a scholar who was skillful in the methods of manufacturing Western weapons and instruments. In the same passage, Zeng Guofan also noted that Zhang Wenhui was well-versed in mathematics, the Classics and philology [Zeng's *Diary* volume 9]. In order to achieve "self-strengthening", according to Zeng Guofan, "improvement in administrative affairs and the search for virtuous talent are after all the urgent tasks; but one concrete effort should be to learn the casting of cannon and the construction of steamships and other equipment," [Quoted from Kuo and Liu in Fairbank ed., 1978, p. 496]. In fact, after the recovery of Anqing from the Taiping rebels in September of 1861, Zeng Guofan brought Hua Hengfang and Xu Shou, both of whom were well-known scientists and mathematicians, to his arsenal there. Both of them certainly knew Li Shanlan and his newly introduced Western science and mathematics quite well. Hua Hengfang, Xu Shou, Zhang Wenhui, Zhang Sigui as well as Li Shanlan all served in Zeng Guofan's "Mu Fu" from the late 1861 on. Since all of these scholars were majored in some specialized fields like mathematics and science, thus Zeng Guofan clearly contributed a new element to his "Mu Fu" at Anqing.²³

Due apparently to his experience in dealing with the Taiping rebels, Zeng Guofan came to realize the importance of the "Mu You" who basically were skillful in the methods of manufacturing. Moreover, Zeng Guofan was also impressed by Western weapons whose

great destructive power had been instrumental in suppressing the Taiping rebels [Kuo and Liu 1978, pp. 496-97]. Thus in August of 1861, while Zeng Guofan supported buying a fleet from Britain, he stressed the need to learn as well the techniques necessary to produce new military equipment in China. He went on to emphasize that "Scholars of broad intellect as well as artisans who are skillful and ingenious should be recruited, first to practise the use of [Western ships and guns], and later to make them," [Quoted from Kuo and Liu in Fairbank ed., 1978, p. 496].

In Anqing Zeng Guofan established both an arsenal and a shipyard, where he recommended that Hua Hengfang and Xu Shou, among others, should serve as officials, and not merely as part of his secretarial staff. Thus Hua Hengfang and Xu Shou were put in charge of building a steamship. They worked hard at this for three months, and successfully completed a steamengine in the summer of 1862. They went on to launch the first Chinese-made steamship (without any help from foreigners) towards the end of 1863. But because the steamengine did not function very well, the ship moved slowly -- clearly there was room for improvement.

In September of 1863, Rong Hong (more commonly known as Yung Wing), the first Chinese "returned student" who had just graduated from Yale University, was recruited to Zeng Guofan's "Mu Fu" through recommendations from Li Shanlan and Zhang Sigui. Urged by Li Shanlan, Zhang Sigui, Hua Hengfan and Xu Shou, Rong Hong presented a plan to Zeng Guofan for the introduction of Western

machinery into China. As a result, Rong Hong was commissioned by Zeng Guofan to go back to America to buy machinery for what subsequently became the Jiangnan Arsenal [Folsom 1968, p. 76].

Rong Hong left for America on December 6, 1863. Three days earlier, Zeng Guofan met with Rong Hong and Li Shanlan for a long talk. The major reason why Zeng Guofan asked Rong Hong to go abroad and purchase machinery may well have been due to Zeng's realization that "Although a small steamer had been built, its speed was so slow that it showed we had not yet learned the proper techniques," [Quoted in Chen 1961, p. 42; see also Cohen 1961, p. 43].

Nevertheless, Hua Hengfang and Xu Shou kept trying. Finally, in 1865, they succeeded in building a twenty-five ton steamer which "was able at her trial trip on the Yangtze in the year 1865 to cover 255 li or about 85 miles in fourteen hours, and to do the return journey in less than eight hours," [Quoted Chen 1961, p. 42]. By then the Arsenal had moved to Nanjing. In fact, the Anqing Arsenal moved to Nanjing in 1864, immediately after the Qing Empire regained control of the city on July 19, 1864 [Kuo 1976, pp. 1083-84]. This was due to Zeng Guofan's victory over the Taiping rebels who had occupied Nanjing as their Kingdom's capital in 1853.

In 1864, Li Shanlan and Zhang Wenhui also moved to Nanjing, where they both served as editors at the Press Bureau of the

Nanjing Arsenal [Li Yan 1955]. By then Li Shanlan had collected all of his mathematical treatises together, either in published form or in hand-written copies returned from friends. In fact, he had actually done this somewhat earlier, in February, before he moved to Nanjing [Li 1867, Liu's "Postscript"]. This collection of mathematical works, the *Ze Gu Xi Zhai Suan Xue*, was published in 1867 under the sponsorship of Zeng Guofan [Li 1867, "Preface"].

In his *Ze Gu Xi Zhai Suan Xue*, Li Shanlan included his important treatise, the *Duo Ji Bi Lei* [Cf. Section 3.5]. The exact date of its composition is unknown, although according to Li Di, it was apparently completed by 1853 [Li Di 1984, pp. 338-38]. Especially interesting is that Li Shanlan uses terms such as "Dai Shu" (algebra) and "Wei Fen" (differential) at the beginning of book one. This has led some historians to suggest that it must have been written after 1859, the year in which the *Dai Wei Ji Shi Ji* (the Chinese edition of Loomis' *Calculus*) was published. Yet in light of the content and methods of the *Duo Ji Bi Lei*, Li Di's conjecture seems more reasonable, since, according to Wang Yusheng, Li Shanlan made substantial contributions to combinatorics in terms of traditional Chinese mathematics [Wang Yusheng 1983b]. More specifically, Li Shanlan dealt with the sums of piles by linking the binomial and combinatorial coefficients via Pascal Triangle, which, mentioned in his *Duo Ji Bi Lei*, had nothing to do with the concept of infinitesimal or the Western algebraic methods. We shall come back to this subject in Chapter 3.

In 1865, Zeng Guofan approved and recommended the *Ji He Yuan Ben Shi Wu Juan* (The *Elements* in 15 books) by writing its title and a preface for it.²⁴ It includes the first six books translated by Xu Guanqi and Matteo Ricci, as well as the rest of the *Elements* translated by Li Shanlan and Alexander Wylie. In his preface, Zeng Guofan compared the different characteristics of Chinese and Western mathematics, using the *Elements* and the *Jiu Zhang Suan Shu*. He clearly pointed out the theoretical character of the *Elements*, but also reminded the reader not to ignore the *Jiu Zhang Suan Shu*, since it provided methods and examples that were excellent illustrations of the theories also covered in the *Elements*. In other words, he regarded the *Elements* and the *Jiu Zhang Suan Shu* as complementary in the sense that one was concerned basically with theories, the other with methods, and thereby readers could benefit from reading both books [Li and Wylie 1865, "Prefaces"].

Although he supported reprinting the *Ji He Yuan Ben Shi Wu Juan*, Zeng Guofan does not seem to have been concerned about using mathematics to enhance the "Zi Qiang Yun Dong" (the Self-strengthening Movement) of which he was one of the chief advocates [Cf. Zeng's "Preface" to the *Ji He Yuan Ben Shi Wu Juan*]. This was not the case with publication of the *Zhong Xue* (Whewell's *Mechanics*). In 1866, Li Hongzhang, Governor of Jiangsu, wrote the title of the reprint of the *Zhong Xue*, and his support for its publication was easily seen as related to the "Zi

Qiang Yun Dong". In a well-known letter of 1864 to Prince Gong, supervisor of the "Zi Qiang Yun Dong", he proposed a reform of imperial institutions. The most serious trouble the Empire had encountered, according to Li Hongzhang, was its exclusive emphasis on literary education in the governments' bureaucratic system. On the other hand, it left military matters to untutored men who nevertheless shared the elite's disdain for technology:

Chinese scholars and officials have been indulging in the inveterate habits of composing stanzas and verses ["Zhang Ju", i.e., the eight-legged essays] and practicing fine calligraphy; while our military officers and soldiers are, on the other hand, crude, ignorant and careless. Thus what we use is not what we have learned, and what we learned is not what we could use.... We do not realize that for several centuries the foreigners have considered the study of firearms indeed as important as that of body and mind, human nature and destiny [Quoted from Kuo and Liu in Fairbank ed., 1978, p. 498].

As this passage shows, Li Hongzhang's real reason for supporting the reprint of the *Zhong Xue* must have been to foster the policy of the "Zi Qiang Yun Dong". In his "Preface" to the *Zhong Xue*, Li Shanlan responded enthusiastically, arguing that mathematics was directly relevant to the "Self-strengthening Movement":

Alas! now the European nations have become more prosperous and stronger day by day, endangering China's frontiers. This cause lies in the fact that they possess skill in the manufacture of fire-arms, and this skill is derived from their mastery of mathematics. Both Zeng Guofan and Li Hongzhang have realized this, and so they have ordered the reprinting of this book.... In the future, if every man learns mathematics, and is more skillful in manufacturing machinery and armaments, China will be respected by all nations overseas. They will be afraid of us and will pay tribute to the Empire. The merit in reprinting this little book is indeed not small [Li and Edkins 1866, "Preface"].

Yet why were Prince Gong, Zeng Guofan and Li Hongzhang, among others, so concerned about the "Self-strengthening Movement"? According to historians Ting-Yee Kuo and Kwang-Ching Liu, self-strengthening was part of a new Qing foreign policy that emphasized conciliation with the European powers and acceptance of the treaty system in the aftermath of the Anglo-French occupation of Beijing in 1860. Yet in order to maintain peace with the foreign powers, it was regarded as necessary to build up China's own strength. "The formula," say Ting-Yee Kuo and Kwang-Ching Liu, "was all too simple: since European military power appeared to depend on technology, the adoption of this technology was regarded as the primary task," [Kuo and Liu in Fairbank 1978, p. 492].

It is against this background that the establishment of the Jiangnan Arsenal as well as the inclusion of mathematics and science in the curriculum at the Tong Wen Guan can best be understood. The Jiangnan Arsenal, known in Chinese as the "Jiangnan Ji Qi Zi Zao Zong Ju", was established by Li Hongzhang. In 1865 he approved the recommendation of Shanghai's "Daotai"²⁵, Ding Richang, to purchase the machine shop and shipyard of Thomas Hunt and Company, an American firm established in the Hongkou section of the Shanghai foreign settlement. Later that same year, the machinery Rong Hung had bought in the United States arrived and was installed at the plant at Hongkou [Kuo and Liu 1978, pp. 520-21]. In addition, the Jiangnan Arsenal also included a translation department (including a school for translators) to

produce books in Chinese on Western science and technology at the suggestion of Hua Hengfang, Xu Shou and his son, Xu Jianyin. They all came to work at the Jiangnan Arsenal in 1867 [Kuo and Liu in Fairbank 1978, p. 535].

At the time, Li Shanlan must have been in close contact with his former Anqing colleagues in Shanghai, although by then he was probably living in Nanjing. Thus when Zeng Guofan sent three hundred taels to support publication of the *Ze Gu Xi Zhai Suan Xue*, Li Shanlan was able to use former colleagues as well as the new staff at the Jiangnan Arsenal to help with the proofreading. Li Shanlan also went to Shanghai in 1868 to translate, in collaboration with John Fryer (1839-1928), Book One of Newton's *Principia*, which, unfortunately, did not survive to this day. Immediately thereafter he went to Beijing, where he served as a mathematics instructor at the Tong Wen Guan.

2.7. Li Shanlan and the Tong Wen Guan (1868-1882)

The Tong Wen Guan (the Imperial College) with which Li Shanlan was affiliated in his later years (1868-1882) was founded as a school for interpreters in 1862 under the auspices of "Zongli Yamen" [Kuo and Liu in Fairbank ed., 1978, 525].²⁶ Since the Zongli Yamen was established to handle Western relations. Thus the original purpose of the Tong Wen Guan was simply to train the translators needed to handle Sino-Western diplomacy [Kuo and Liu in Fairbank ed., 1978, p. 525]. Yet to Li Hongzhang and other

advocates of the "Zi Qiang Yun Dong" this was not enough. In April of 1863, Li Hongzhang recommended to the throne that similar foreign-language schools be established at Shanghai and Guangdong. The schools Li Hongzhang envisaged were, however, broader in their aim to teach mathematics and science as well. According to him, "That which the Westerners are good at, such as mathematical studies, the principles of science, and the techniques of manufacturing and surveying, have all been written up in books.... Can the Chinese be inferior to the Westerners in ingenuity and intelligence? If there are men well versed in the Western languages and one person can teach another, then all the skillful techniques of steamships and firearms can be gradually mastered," [Quoted from Kuo and Liu in Fairbank ed., 1978, p. 526]

Li Hongzhang's proposal was approved, and the Guang Fang Yan Guan (also known as the Shanghai Tong Wen Guan) was inaugurated in July of 1864. The number of pupils was limited to forty young men under fourteen, recommended by reputable officials and gentry. The school also included ten older students who already had a thorough Chinese literary training. As for the curriculum, English and mathematics were added to the classics, history, literary composition, and study of the writings of Zhu Xi, the noted Southern Song neo-Confucian philosopher. The English course was taught by Young J. Allen, the American missionary, and mathematics by Shi Yuechun, who had made contributions to the "Da Yan" method (the method for solving simultaneous congruences, the title "Da Yan" had been known since Qin Jiushao of the Southern

Song) in the 1860s [Zhu Kebao 1886, pp. 815-817].

By 1866 the Guang Fang Yan Guan had not as yet had enough time to become the great success Li Hungzhang had predicted. Long-range planning was still needed to produce all of the personnel anticipated for handling indigenously, within China, the new Western technology. Nevertheless, the letter Li Hongzhang sent to Prince Gong in the Spring of 1864 (as mentioned above) was sufficiently convincing that Prince Gong decided to include mathematics and astronomy in the curriculum of the (Beijing) Tong Wen Guan in 1866. As Li Hongzhang argued:

If we wish to learn about the superior weapons of foreign countries, there is nothing better than to seek machines which make machines.... If we seek machines that make machines and men who make machines, we should perhaps establish a special category of examination to select [such] scholars. If the scholars will regard passing this new category of examination as their lifelong goal by which to achieve wealth, rank and honor, it is likely that they will carry their studies through, that their techniques will be perfected, and that [then] talents will foregather [Quoted from Kuo and Liu in Fairbank ed., 1978, p. 499].

Although the civil examination system could not make such reforms immediately, by adding astronomy and mathematics to the curriculum of the Tong Wen Guan, Prince Gong hoped this might serve to meet the urgent demand of the "Zi Qiang Yun Dong". This led to the proposal made on December, 1866, that a department of "astronomy and mathematics" be added to the Tong Wen Guan and, even more significant, that scholars and officials who had the "Ju Ren" degree by passing the "Xiang Shi" (provincial civil examination)

should be encouraged to apply for admission to the new department at the Tong Wen Guan. Later, in January of 1867, Prince Gong and his colleagues further proposed that holders of the "Jin Shi" degree (and especially members of the Hanlin Academy -- including the prestigious compilers) should also be encouraged to apply. Upon completion of a three-year course they were to be given a special recommendation for official advancement [Kuo and Liu in Fairbank ed., 1978, p. 528].

By adopting such extraordinary measures as he called them, Prince Gong and his colleagues wished not merely to expand the Tong Wen Guan's curriculum. They aimed at nothing less than recognition, from the throne and such citadels of orthodoxy as the Hanlin Academy, of the legitimacy of Western learning itself [Kuo and Liu in Fairbank ed., 1978, p. 528]. In addition, before the proposal was made to the throne in early 1866, when Prince Gong authorized Rober Hart to recruit science instructors in Europe when he went back to England on furlough, having been in charge of the office of the inspector-general of customs at Beijing since 1865 [Kuo and Liu in Fairbank ed., 1978, p. 528]. Moreover, on August 24 of the same year, the throne also ordered Li Shanlan and Zou Boqi (1819-1869) to join the Tong Wen Guan at the recommendation of Guo Songtao (1818-1891), then the Governor of Guangdong who had earlier served in Li Hongzhang's "Mu Fu" in 1862 [Sun 1977, p. 170]

Yet neither Li Shanlan nor Zou Boqi acted on this official

order immediately. Their reasons were the same: both were ill at the time. Zou Boqi actually died of liver disease a few years later, in 1869. On the other hand, Li Shanlan apparently used his illness as an excuse, since at the time he was busy preparing the publication of his *Ze Gu Xi Zhai Suan Xue*. As he wrote to his former colleague of Zeng Guofan's "Mu Fu" at Anqing, Fang Yuanzheng: "because my collected works on mathematics [i.e., the *Ze Gu Xi Zhai Suan Xue*] have not been completely published yet, I firmly declined that offer. For myself, an official post is no more valuable than my life-long career -- mathematical studies," [See Figure 2; Cf. Horng 1989a].

Clearly, to Li Shanlan mathematics was the most important thing in his life. Without such an attitude, neither Li Shanlan nor any of the other contemporary mathematicians at work in China could be regarded as "professionalized" to any real extent on a non-institutionalized basis. Moreover, in the prefaces to his mathematical treatises, Li Shanlan was always very proud of his own unique, substantial contributions to mathematics [Cf. Chapter 3]. Since his treatises were written for specialists, there must have been a corresponding sense of mathematical community in China in the 1860s.²⁷

Li Shanlan joined the Tong Wen Guan basically as a professional mathematician. But Li Shanlan had been earning his "professionalized" status all along as a mathematician among the community of humanistic scholars since the 1870s. This was

元微尊先先生有道金陵一別轉三三秋相念之深
 未嘗去懷前年承
 惠木瓜至今未謝弟之疎懶又在懷生上已昨
 兄未言
 兄近患疥癩急治之餘有良醫石西人治法熟猪
 油拌麝香厚塗之乾即再塗日在無間三日可盡結
 痂仍堅不已從不輕盡疥癩而止請
 試之甚有驗也書局前月移居霞湖山色以老
 郭凡席間日憑臨吟眺神仙不覺道也惜
 賢君祥遠在徐州不能共盡此樂了
 去冬忽奉
 赴任理衙門之旨以尊學未刻竣力辭不就
 以官之禁易我千秋事也幾何原在尊學
 俱已刷印惟刻古者尊學僅刻一半大約七八
 月間方能了事了
 半生心血幸不隨劫居同
 盡今且得盡力于世丈夫志願畢矣更何求哉
 何求哉
 老兄謂之空後代我稱賀也專函敬候
 起居無任神往弟善高叩頭甲申四月

Figure 2. Li Shanlan's letter to Fang Yuanzheng

evident in a letter sent to Li Shanlan by Wang Tao from Hong Kong in 1875. In this letter Wang Tao cited a passage from Chen Li [Huang 1898, pp. 92-93], a noted classical scholar of the time, who paid high respect to Li Shanlan. More significant is the fact that Wang Tao was very pleased, as he wrote, to hear that Li Shanlan had been given a warm and respectful reception by the circle of scholar-officials in Beijing [Horng 1989a]. In fact, Li Shanlan's status as a "professional" mathematician can best be viewed as a reflection of progress in China from "an elite of the scholar to an elite of the professional" during the late 19th and early 20th centuries [Folsom 1968, p. 194].

It is likely that Li Shanlan went to Beijing in 1868, although some documents say that it was one year later, namely, 1869. There he was the only Chinese instructor who taught astronomy and mathematics. Therefore, regarding the Tong Wen Guan as one of "the earliest modern government schools in China" [Biggerstaff 1961], the role Li Shanlan played was of the utmost importance to the modernization of Chinese education in science and mathematics. Li Shanlan's contribution to this was even more significant, especially in terms of the Tong Wen Guan controversy in 1867, which clearly upset advocates of the "Zi Qiang Yun Dong".

The controversy erupted over the Tong Wen Guan's entrance examination given on June 21, 1867 [Dong 1989, p. 287]. Among the 98 original applicants, seventy-two appeared for the test. Most of these were found to be mostly unemployed middle-aged men,

"broken hacks to whom the stipend offered by the yamen proved dearer than their reputation," as one Western observer remarked [Liu 1976]. Why were scholars and officials with "Ju Ren" or "Jin Shi" degrees so reluctant to apply for the new study program? The major reason undoubtedly was an attack on the new program made by conservative higher officials in Beijing like Zhang Shengzao and Wo Ren.

The new study program of the Tong Wen Guan, including the teaching of mathematics and science was proposed in both December of 1866 and January of 1867, it was subsequently approved by the throne, then controlled by the Empress Dowager Ci Xi. Unfortunately, on March 5, 1867, Zhang Shengzao, a devoted neo-Confucian who served as a censor at Shandong, argued that the "self-strengthening" depended not on guns and ships (the manufacture of which was presumably the purpose of learning Western techniques), but on good and just government as well as "training troops and raising revenue." Zhang Shengzao was even worried that the stress on astronomy and mathematics, which to him were matters of subtle technique ("Ci Xiao"), would have an adverse effect on the spirit and moral principles of the literati. Technical knowledge and good moral character seemed to him antithetical. The throne, however, sided with Prince Gong, and responded that "astronomy and mathematics are what Confucian scholars should know and should not be regarded as matters of techniques," [Quoted from Kuo and Liu in Fairbank 1978, p. 529]. Meanwhile, a fair number of scholars did apply for the new

program.

2.8. The Tong Wen Guan Controversy and Its Intellectual Context

One may be surprised that the throne should side with Western astronomy and mathematics over Confucian scholarship. But the Qian-Jia school had already placed great emphasis on the study of astronomy and mathematics and this was apparently successful, it therefore seemed impossible for Qing scholars of the 1860s to ignore their influence, although by then the Qian-Jia school's vitality had already begun to diminish. However, since the founding of the Qing Dynasty, the Manzhou rulers had enshrined the Southern Song neo-Confucian Zhu Xi school as the moral and theoretical norm in both imperial examinations and official rhetoric [Elman 1984, p. 27]. Such an imperial Confucian ideology certainly survived even through the Empire sponsored the *Si Ku Quan Shu* project, in which the Qian-Jia School played a central role. By taking this opportunity, the Qian-Jia school came to dominate the scholarly community, especially in the Jiangnan area during the second half of the 18th century.

However, "By the early nineteenth century there were signs that this kind of scholasticism -- indeed the very consensus it represented -- was breaking down. Because the central government had ceased patronizing the compilation of vast compendia like Ch'ien-lung's *Ssu-k'u ch'uan-shu*, no longer were scholars from all over the country congregating in Peking to work on standardized

projects. Further, regional literary and philosophical schools had begun to take issue with the pedantry of textual criticism," [Quoted from Jones and Kuhn 1976, p. 146]. Therefore, to Gong Zizhen (1792-1841) and Wei Yuan (1794-1856), the two figures of the "Jing Shi Zhi Xue" (learning of statecraft), studies on textual criticism were no longer enough to stop the decline of the Empire. Statecraft and moral reform were more important than the apolitical scholarship like the textual criticism of the Qian-Jia school [Elman 1984, pp. 240-241].

Thus the philological scholarship was now fervently attacked by almost all Chinese scholars in the early 19th century. The condemnation by Fang Dongshu (1772-1851) of the Qian-Jia school was typical: "...although [the scholars of the Qian-Jia school] may be 'searching for the truth in actual facts', in reality they are performing the most extreme form of empty activity," [Quoted in Elman 1984, p. 243].

Nevertheless, "textual criticism" or philological research continued to be an important enterprise. Despite growing dissatisfaction, evidential scholarship was effectively defended by Jiangnan literati, where was originally a stronghold of the Qian-Jia school, until the Taiping rebellion [Elman 1984, p. 247]. Meanwhile, a key factor for the revival of neo-Confucian ideology at the court must also be considered. The Xian Feng Emperor (reigned 1851-1861) had genuine respect for his tutor Du Shoutian (1787-1852), a neo-Confucian scholar-statesman of the Cheng-Zhu

school. At Du Shoutian's suggestion, the Emperor promoted Qi Junzao (1793-1866), another neo-Confucian scholar, to the position of grand councillor.

Due to Qi Junzao's recommendation, Weng Xinzun (1791-1862), Zhou Zupel (1792?-1867) and Chia Chen (1798-1874), all kindred spirits, were put into high places in the bureaucracy. "An ideologically-oriented faction of officials may be said to have been formed in the process," [Quoted Liu 1976, pp. 89-90]. Members of this faction often served as examiners in the central civil examination and Hanlin Academy tests, and could claim officials as their "students". Many neo-Confucian scholars including Wo Ren (1804-1871), a famous neo-Confucian theorist who was of Mongol origin, became member of this faction. In 1862, when Wo Ren was promoted to grand secretary, he served concurrently as imperial tutor. By then the Emperor was Tong Zhi, although the court was still controlled by the Empress Dowager Ci Xi and Prince Gong. After 1865, the power of Prince Gong was diminished by Ci Xi, largely through the faction headed by Wo Ren and his partisans.

Therefore, when the higher neo-Confucian officials opposed the new study program at the Tong Wen Guan on the grounds that self-strengthening depended not on guns and ships but on good and just government as well as training troops and raising revenue, the Empress Dowager Ci Xi was prepared to take their opinions into account. On March 20, when another critical communication reached

the throne from Wo Ren, the Empress Dowager could not ignore it. In a famous passage the imperial tutor declared: "The way to uphold the foundation of the state is to emphasize propriety and a sense of duty but not expedient schemes ('Chuan Mou'). The basic policy [of the state should be to cultivate] people's morality ('Ren Xin') and not technique ('Chi-i')," [Quoted in Liu 1976, p. 93]. Here Wo Ren identified technology with despised "magical computations" ("Shu Shu"). But what was of even greater emotional appeal to literati-officials was his allegation that the Zongli Yamen wanted the Chinese to "honor the barbarians as teachers" ("Feng Yi Wei Shi") -- the very barbarians who had so recently invaded Beijing itself and were now spreading the "abominable doctrine" of Christianity [Liu 1976].

To this attack Prince Gong and his colleagues replied on April 7, arguing that it was unrealistic to expect "loyalty and faithfulness to serve as armor, or propriety and a sense of duty as a shield," [Quoted in Liu 1976]. They went on to cite the words of such loyal officials as Zeng Guofan, Li Hongzhang and Zou Zongtang, all of them advocated the introduction of Western technology. The reason for their support was simple, for these higher officials were presumably no less Confucian than Wo Ren himself. In fact, Zeng Guofan was a close associate of Wo Ren before he went back to Hunan, where he served as a "Commissioner of Local Defense" in 1852 [Kuhn 1978].

Five days later, Wo Ren sent his reply to the throne, saying

that he would not oppose the new study program at the Tong Wen Guan again. Nevertheless, he could not agree to "honor the barbarians as teachers." His major concern obviously reflected Chinese cultural pride and dignity, whereby he undoubtedly expected to excite the literati-officials' sympathy. Thus when the two memorials of Wo Ren were made public by the throne for apparent political reasons, they so stirred up literati-officials in Beijing that damaging rumors began to circulate about the Zongli Yamen's treasonous intentions. Scholars who had applied for admission to the new study program at the Tong Wen Guan were sneered at by their "fellow provincials and colleagues." An informal group of literati-officials arose to boycott the Tong Wen Guan. In late April, Prince Gong and his colleagues reported to the throne that since Wo Ren's memorials had become known, "no one had come to this yamen to apply for the entrance examination [to the new program]," [Quoted from Kuo and Liu in Fairbank 1978, p. 530]. As a result, the Zongli Yamen was forced to give up its hopes of getting the scholar-official elite to accept the study of technology. Instead, it merely requested the throne's approval to give the entrance examination for the new program as planned, to accomodate "those who at present have applied," [Kuo and Liu in Fairbank 1978, p. 530].

The outcome of this controversy, as mentioned above, was plainly a triumph for Wo Ren, who represented a characteristic cultural position as well as the self-righteousness and political interests of certain metropolitan officials. It should also be

observed that he owed this triumph partly to the Empress Dowager Ci Xi's failure to give her full support to prince Gong. Consequently, Prince Gong and his colleagues had to compromise with their conservative rivals. To save face from studying under the barbarians, the Manzhou official Chong Shi suggested that the Zhongli Yamen should recruit native Chinese mathematicians to the faculty of the Tong Wen Guan [Dong 1989, p. 283]. And so it was that on August 15, 1867, the Zongli Yamen urged Li Shanlan as well as Zou Boqi to join the Tong Wen Guan as soon as possible.

It is against this historical background that Li Shanlan's association with the "Self-strengthening Movement" can best be appreciated. Although from the very beginning of his career at the Tong Wen Guan he could not have had many able students to teach, his contributions as well as his capabilities in mathematics earned high respect not only from his Western colleagues but also from the conservative Beijing officials. Wang Tao's description of Li Shanlan's social status in Beijing in 1875, as mentioned above, rings true. In his *Shu Mu Da Wen* of the same year, Zhang Zhidong regarded Li Shanlan as one of the best mathematicians of the past fifty years. Zhang Zhidong wrote in the same passage that "this book does not include the mathematicians who are still alive. Yet due to the fact that Li Shanlan has made such great and excellent studies in astronomy and mathematics, we therefore add this one name [as an exception]." Indeed Li Shanlan was the foremost of late Qing mathematicians [Hornig 1990].

More significant to his social standing in Beijing, however, was Li Ciming, the conservative official who severely criticized the new study program at the Tong Wen Guan. Li Ciming visited Li Shanlan on February 26, 1880, a visit that was a polite gesture for Li Shanlan's having given him a copy of the *Ji He Yuan Ben* (Euclid's *Elements*) as a gift [Li Yan 1955]. In a review (of 1878) of Gu Guanguang's *Wu Ling Shan Ren Yi Shu* ("The Posthumous Treatises of Gu Guanguang"), Li Ciming cited Li Shanlan's great admiration for Gu Guanguang's achievements in astronomy and mathematics [Li Ciming 1963]. This certainly indicates that Li Shanlan had earned himself a respected place primarily as a mathematician in Beijing's scholar-official circle in the 1870s.

2.9. Last Days

In 1882 Li Shanlan was further promoted to the still higher position of an official of the third rank [Li Yan 1955, p. 360]. Nevertheless, in the summer of that same year, he still tried to write the *Ji Shu Gou Gu* ("On Pythagorean Triplets"). This treatise (which no longer survives) might have reflected his own *Kao Shu Gen Si Fa* ("Four Methods of Investigation of Prime Numbers, 1872). The latter, which he presumably wrote during the period 1867-1872, marked his most substantial contribution to mathematics in terms of number theory as introduced in Book 7 of Euclid's *Elements* [Horng 1990]. Its significance to late Qing mathematics is discussed below in Section 5.4.

Li Shanlan died on December 9, 1882. During his entire life he had never grown tired of the study of mathematics. To some Beijing scholar-officials, his being primarily a "professional" mathematician rather than a classical/traditional scholar was not enough to ensure his place in neo-Confucian sagehood. Yet his devotion to mathematics brought the high esteem of the chief instructor of the Tong Wen Guan, William Martin:

His work in mathematics is well known in China and abroad. ... the Zongli Yamen engaged him to teach mathematics at the Tong Wen Guan, where he has been working diligently for eight years without realizing his advancing years. Certainly he has devoted himself body and soul to mathematics. It has been said, however, that Li Shanlan has spent most of his life in a blind alley, and has apparently chosen a wrong subject field, namely, mathematics. What a blasphemy! This remark must have proceeded from those who do not understand the significance and the importance of mathematics. Without mathematics we cannot study astronomy, neither can we count objects in the universe. The calendar, navigation, manufacturing machinery in peace and armaments in war -- all these depend on mathematics.... Therefore Li Shanlan has made a great contribution to the world and to future generations [Martin 1877].

NOTES:

1. See Li Yan, "Li Shanlan Nian Pu," in [Li Yan 1955, 331-361]. Cf. also [Wang Yusheng 1983a]. "Courtesy name" ("Zi" in Chinese) is the one taken at the (Chinese) age of 20, by which a man is sometimes called in a polite way by his friends or colleagues [Cf. *The Pinyin Chinese-English Dictionary*, 1985].
2. In 1982, Wang Yusheng went to Hailing county to collect some important sources about Li Shanlan. Based on the *Bao Xi Li Shi Jia Cheng* ("Chronology of Li Clan of Bao Xi"), Wang reports that Li Zulei's first wife had the surname Xu, the granddaughter of the magistrate of the Hailing prefecture, Xu Jixi. After her death, Li Zulei married her younger sister. Unfortunately, Li's second wife also died soon. Neither of his first two wives had any children. Thus Li Zulei married again, this time to a daughter of the scholar Cui Jingyuan. See [Wang Yusheng 1983a].
3. According to [Wang Yusheng 1983b & 1990], Li Shanlan had two younger brothers, Xinmei and Xinqi. Li Xinmei seems to have been well-versed in mathematics too, for he even contributed a few remarks on method of series expansion for π to Li Shanlan's *Fang Yuan Chan You*.
4. [Li 1867, "Preface"] says that "Shanlan was studying under my father's direction at the age of ten." Here the "Chinese age" was counted in terms of the lunar year. Since Li Shanlan was born on the eighth day of the twelfth lunar month, his "Chinese age" was therefore usually two years ahead of his "Western age". Here all ages are based on Western reckoning.
5. Cf. [Wang Yusheng 1983a]; Li Shanlan's *Ting Xue Xuan Shi Cun* ("Poetry of Ting Xue Xuan"). The "eight-legged essay" was the literary style required for the civil examinations in the Ming-Qing period. Essays had to be written in just eight paragraphs following strict content and format. Essay also had to be written carefully in using calligraphy of small type.
6. In the "Xiao Xue" category, Li Shanlan claimed that he was versed in both philology and mathematics. See [Wang Yusheng 1983a]. As for his poetry, refer to his *Ting Xue Xuan Shi Cun*, available in a draft in the Beijing Library. I would like to thank Wang Yusheng for sending me a hand-written copy of Li Shanlan's poetry in 1988.
7. In his "Preface" to the *Ze Gu Xi Zhai Suan Xue*, Li Shanlan mentioned that Li Ye's *Ce Yuan Hai Jing* and Dai Zhen's *Gou Gu Ge Yuan Ji* were available to him when he attended the "Xiang Shi" (provincial civil examination) held at Wulin (i.e., Hangzhou). This was presumably the first time he took the "Xiang Shi", apparently in 1825. In the "Preface" to his *Si*

Yuan Jie (1845), Li Shanlan recalled that he read the *Ce Yuan Hai Jing* at the age of 14 or 15.

8. In his "Preface" (to the 1845 edition) Li Shanlan mentioned that he read the *Ce Yuan Hai Jing* ten years earlier. But as we say in Note 7 above, he read this classic in 1825 or so, therefore he must have read the *Si Yuan Yu Jian* about 1835.
9. "Gou Gu" is the title of Chapter 9 of the *Jiu Zhang Suan Shu*. It deals with the so-called Pythagorean Theorem and problems about right-angled triangles. In addition, methods of solving quadratic equations are also introduced.
10. The "Si Yuan" method was invented to solve simultaneous equations with no more than four unknowns. Cf. Section 3.1.
11. "Three Dynasties" refers to the first three dynasties in Chinese history, namely the Xia (ca. 2205-1766 B.C.), Shang (ca. 1766-1056 B.C.) and Zhou (ca. 1066-256 B.C.) Dynasties.
12. See the "Preface" to Li Shanlan's *Si Yuan Jie*, collected in [Li 1867]. See also [Wang Ping 1966, pp. 144-147].
13. See Zhang Wenhui's "Gu Guanguang Bie Zhuan (Biography of Gu Guanguang)," in the *Bie Zhuan Ji San Bian*.
14. According to William Murirhead (1822-1900), Xu Youren, who visited the Mo Hai Shu Guan (the London Missionary Society Press) in 1858, was a true believer in Buddhism. See Murirhead's *China and the Gospel* (1870), pp.193-194, quoted in [Li Yan 1955].
15. According to John Fryer, Li Shanlan visited the Mo Hai Shu Guan along with his mathematical treatises some time before 1852, and discussed mathematics with Wylie there. This was apparently the first meeting of the two key figures in the transmission of modern mathematics and science to China. Wylie was so surprised to see what Li Shanlan had actually done in mathematics that he immediately invited Li Shalan to co-translate Western texts on mathematics and science [Quoted in Li Yan 1955, p. 359]
16. The *Elements* upon which Wylie and Li Shanlan based their translation was an English edition (probably edited by Issac Barrow) including 15 books, in which the last two were added by some anonymous author. Since Matteo Ricci and Xu Guangqi had already translated the first 6 books of the edition by Clavius, Wylie and Li concentrated on the last 9 books in producing the complete Chinese edition [Cf. Section 4.6].
17. Joseph Edkins (1823-1905) came to Shanghai in 1848 and served as the chief of the London Missionary Society there. During his service in China, he paid considerable attention to Chinese literature and history. In addition to translations

- undertaken in collaboration with Li Shanlan and Wang Tao, Edkins also published many works on China, including *Religion in China* (1878), *Opium: Historical Note* (1898), *Chinese Buddhism: A Volume of Sketches, Historical, Descriptive and Critical* (1893), *China's Place in Philology* (1871), *Banking and Price in China* (1905) [Cf. Xin Ping 1990, p. 25].
18. For example, Guan Xiaoyi, a disappointed and unconventional literati of the time, also worked at the London Missionary Society Press translating Western medical books in collaboration with Dr. Benjamin Hobson. Cf. [Wang Tao 1987, p. 6].
 19. In the same passage, Cohen cites the missionary John Chalmers's observation of Chinese gentry: "Under the outward show of politeness and refinement imparted to the educated Chinese chiefly by Confucianism, there is almost nothing but cunning, ignorance, rudeness, vulgarity, arrogant assumption and inveterate hatred of everything foreign," [Cohen in Fairbank ed., 1978, p. 565].
 20. In his "Preface", Li Shanlan did not seem to separate (Western) botany from Christianity. Whether he viewed differently the relationship between Western science and Christianity elsewhere is unknown. Perhaps this "Preface" was an exception which Li Shanlan may only have reiterated the natural theology introduced by the co-translator Williamson. Williamson did not write a preface for the *Zhi Wu Xue*, yet he wrote a series of articles, the *Ge Wu Tan Yuan* for *Church News*, whose central theme was natural theology. As for the relevance of the "Preface" to our discussion, Chinese scholars of the late 19th century were concerned with the problem of separating "Xi Xue" (Western studies) from Christianity. However, this problem may already have arisen in the 1850s when Li Shanlan and Wang Tao, among others, first became acquainted with Western science and mathematics, as well as with Christianity. We shall return to this subject in Chapter 5.
 21. Just as the *Zhong Xue* was about to be published, the co-translators added a treatise written by an anonymous author on conic sections as an appendix. This treatise whose Chinese title is "Yuan Zhui Qu Xian Shuo" will be discussed in Chapter 4.
 22. Wu Jiashan already met Xu Youren at Beijing in 1852, when Wu Jiashan received the "Jin Shi" degree by passing the central civil examination -- the highest in the civil examination system during the Ming-Qing period. Subsequently he served at the Hanlin Academy [Zhu Kebao 1886, pp. 822-23]. Due to this opportunity, Wu Jiashan learned mathematics from Xu Youren, who had also received a "Jin Shi" degree earlier in 1829 and served at the "Hu Bu" (the Ministry of Revenue) in 1852. Since Xu Youren was familiar with the mathematical studies of

the "Xi Fa" school [Zhu Kebao 1886, pp. 782-785]. Therefore, he may also have influenced Wu Jiashan. This may also explain why Wu Jiashan paid so much attention to the "Xi Xue" (the Western studies) when he was in Shanghai in 1859-60 [Cf. Wang Tao 1987, pp. 156-157].

23. Zeng Guofan, a faithful Confucian, still treated his "Mu You" in a traditional way. That is, he treated them as guests who were not assigned any specific duties or responsibilities but could come and go as they chose. Even so, in order to deal with the matters of the "self-strengthening", Zeng began to recruit men of ability not just the Confucian scholars who majored no specialty in 1862 [Cf. Folsom 1968, p. 75].
24. Traditionally, when a book was put to publication, the author usually asked a noted scholar or scholar-official to write the title of the book. In so doing, the book got the approval and recommendation of the scholar who wrote the title.
25. "Dao" was an administrative unit between province and "Fu" (prefecture) in the Qing period. So "Daotai" was the official who was in charge of the "Dao".
26. The Zongli Yamen was authorized by the throne in 1861 and originally was in charge of negotiations concerning trade relations, "but it became the principal coordinating agency of all 'Western affairs', a new type of Ch'ing government activity which included the handling of diplomacy, foreign trade avenues and all matters concerning trade relations and missionaries, as well as the management of new programmes in which things Western were involved, such as language schools, army training, arsenals, shipyards, mines, merchant steamships and navy," [Kuo and Liu in Fairbank ed., 1978, p. 504].
27. According to [Ding and Zhou 1984], there appeared six editions of the *Ze Gu Xi Zhai Suan Xue* in the period 1867-1900. Yet how many copies were printed and who bought them are unknown.

3. MAJOR MATHEMATICAL DISCOVERIES DUE TO LI SHANLAN AND A CRITICAL ANALYSIS OF HIS *ZE GU XI ZHAI SUAN XUE* ("The Collected Mathematical Works of Li Shanlan", 1867)

Li Shanlan's career may be divided into three periods, namely, the *Ze Gu Xi* period (ca.1845-1852), the *Mo Hai* period (1852-1860) and the *Tong Wen Guan* period (1868-1882) [Horng 1989a]. It is due to substantial mathematical contributions of the *Ze Gu Xi* period that Li Shanlan earned himself a reputation as one of the best mathematicians of the time. This Chapter analyses and explains his *Ze Gu Xi Zhai Suan Xue* in considerable detail. Since all the historical studies of Li Shanlan have heretofore mainly been concerned with his mathematics (taking an internalistic approach), these shall be considered here. But additional questions are also relevant, namely how he inherited the legacy of early Qing mathematicians and how he influenced his successors as well. For convenience of reference, dates will be given for individual treatises (either when written or published) in the *Ze Gu Xi Zhai Suan Xue*.

The *Ze Gu Xi Zhai Suan Xue* collects the following treatises:

- 1) *Fang Yuan Chan You* ("Explanation of the Square and the Circle", 1851), proofread by Feng Junguang;
- 2) *Hu Shi Qi Mi* ("Unveiling the Secrets of Arc and Sagitta", 1851), proofread by Zhang Wenhui;
- 3) *Dui Shu Tan Yuan* ("Seeking the Source of Logarithms",

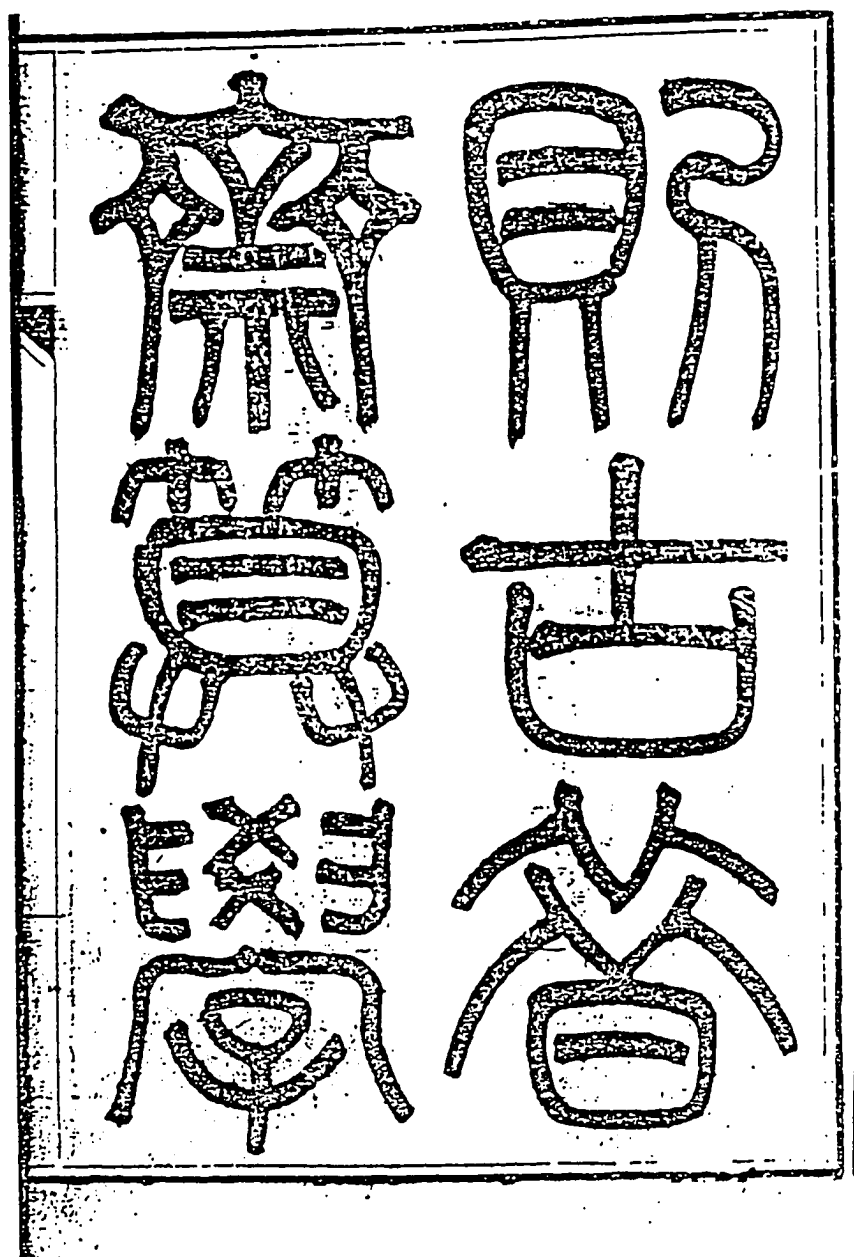


Figure 1. The Title Page of the Ze Gu Xi Zhai Suan Xue (1867)

1850), proofread by Jia Buwei;

4) *Duo Ji Bi Lei* ("Sums of Piles of Various Types", ca. 1853), proofread by Zeng Jize;

5) *Si Yuan Jie* ("On the 'Si Yuan' Method", 1845), proofread by Zeng Jihong;

6) *Lin De Shu Jie* ("Explanation of Lin De Calendar", 1848), proofread by Wang Yuezhen;

7) *Tuo Yuan Zheng Shu Jie* ("Explanation of the Methods for Ellipse"), proofread by Wang Shiduo;

8) *Tuo Yuan Xin Shu* ("New Methods for Ellipse"), proofread by Xu Shuo;

9) *Tuo Yuan Shi Yi* ("Remarks on Ellipse", ca. 1860), proofread by Hua Hengfang;

10) *Huo Qi Zheng Jue* ("On the Projectile Motion of Cannon", 1858), proofread by Ding Quzhong;

11) *Dui Shu Jian Zhui Bian Fa Shi* ("On the Equivalence of the Western and the 'Jian Zhui' Methods for Logarithms", ca. 1860), proofread by Wu Jiashan;

12) *Ji Shu Hui Qiu* ("Conversions of Infinite Series",), proofread by Xu Shuo;

13) *Tian Suan Huo Wen* ("Querries and Answers on Astronomy and Mathematics"), proofread by Ding Quzhong [Li Yan 1955].

Among these works, the *Fang Yuan Chan You*, *Hu Shi Qi Mi*, *Dui Shu Tan Yuan*, *Si Yuan Jie* and *Lin De Shu Jie* were all written by the late 1840s. From these it is quite clear that Li Shanlan's major mathematical concerns were twofold: On the one hand, he was

interested in infinite series expansions of π , logarithms and trigonometric functions, as well as related problems which were certainly the legacy of Ming Antu and Dong Youcheng's mathematics; on the other hand, he was also impressed by Zhu Shijie's *Si Yuan Yu Jian* ("Precious Mirror of the Four Elements", 1303), which led to his *Si Yuan Jie*.

Li Shanlan's *Si Yuan Jie* obviously reflected the afterglow in mathematics of the early 19th-century Qian-Jia school. Yet according to Du Shiran, Li Shanlan made no substantial contributions of his own in the *Si Yuan Jie*, nor did he do much to explain the mathematics for readers of the *Si Yuan Yu Jian* [Du Shiran 1966], although he was ever proud to claim that "no contemporary scholars understand the 'Si Yuan' method [except himself]" in 1845 [Cf. Section 2.1]. Whatever the case, Li Shanlan remained enthusiastic about the "Tian Yuan" and "Si Yuan" methods throughout his entire career. Probably this is why (as mentioned in Section 2.1) that he corresponded so often with Luo Shilin in the 1840s. More significantly, the *Si Yuan Jie* appears to have been used as a textbook in the mathematics curriculum at the Tong Wen Guan -- after 1876, the Imperial College, since at the time the mathematics course included "Xue *Si Yuan Jie*" (Learning the *Si Yuan Jie*) [Cf. Section 5.1]. However, a survey of the final mathematics examinations held at the Tong Wen Guan in 1870, 1872, 1873 and 1878's suggests that problems on the "Si Yuan" method never appeared in the tests [Horng 1990]. Therefore the "Si Yuan" method was more likely to serve as an ideological

symbol motivating the study of the "Xi Xue" (i.e., "Western studies") and thereby to save the face of the conservative officials in Beijing at the meantime.

3.1. On the "Si Yuan" Method: The *Si Yuan Jie*.

As argued in Section 2.1, the *Si Yuan Jie* was probably written in 1835. Yet it seems to have unknown to the mathematical community until 1845. By then Gu Guanguang might have known Xu Youren and Luo Shilin's study on the *Si Yuan Yu Jian* [Gu 1882a], but none of this literature was not available to Li Shanlan, for otherwise he could not have claimed that "There were no contemporary scholars who understand the 'Si Yuan' method," [Li's "Preface" to the *Si Yuan Jie* (1867)].

According to Gu Guanguang, Li Shanlan's explication of the "Si Yuan" method was more comprehensible than those by Xu Youren and Luo Shilin [Gu 1882a]. In examining this treatise carefully, Gu Guanguang was doubtless that Li Shanlan did provide some explanatory schemes for setting unknowns in the *Si Yuan Yu Jian*. Indeed, in Zhu Shijie's earlier classic, readers often found it difficult to understand how the author displayed the four unknowns, namely, the "Tian Yuan", "Di Yuan", "Ren Yuan" and "Wu Yuan" in the counting rod system. This was especially so when the readers tried to understand what the product of two non-adjacent unknowns, namely, the product of "Tian Yuan" and "Wu Yuan" or that of "Ren Yuan" and "Di Yuan". to appreciate what is involved,

consider the following schema:

Let x , y , z and u represent the four unknowns "Tian Yuan", "Di Yuan", "Ren Yuan" and "Wu Yuan" respectively, then it is easy, as Zhu Shijie did, to represent $x+y+z+u$ as the following "Si Yuan" scheme:

$$\begin{array}{ccccccc}
 & & & & (u) & & \\
 & & & & 1 & & \\
 & & & & & & \\
 (y) & 1 & \text{Tai} & 1 & (z) & & \\
 & & & & 1 & & \\
 & & & & (x) & &
 \end{array}$$

In addition, the coefficients of the multiples involving two unknowns (such as xy , zu , ...etc.) are recorded at the corresponding points of intersection of two lines. Armed with such a scheme, Zhu Shijie still had to create some places to display the coefficients of the multiples of two non-adjacent unknowns xu or yz etc. Thus for a polynomial of four unknowns like $(x+y+z+u)^2 = x^2 + y^2 + z^2 + u^2 + 2xy + 2xz + 2xu + 2yz + 2yu + 2zu$, Zhu Shijie's representation would be as follows:

$$\begin{array}{ccccccc}
 & & & & 1 & & \\
 & & & & & & \\
 & & & & 2 & 0 & 2 \\
 & & & & & & \\
 & & & & & & 2 \\
 1 & 0 & \text{Tai} & 0 & 1 & & \\
 & & & & & & \\
 & & & & 2 & 0 & 2 \\
 & & & & & & \\
 & & & & 1 & &
 \end{array}$$

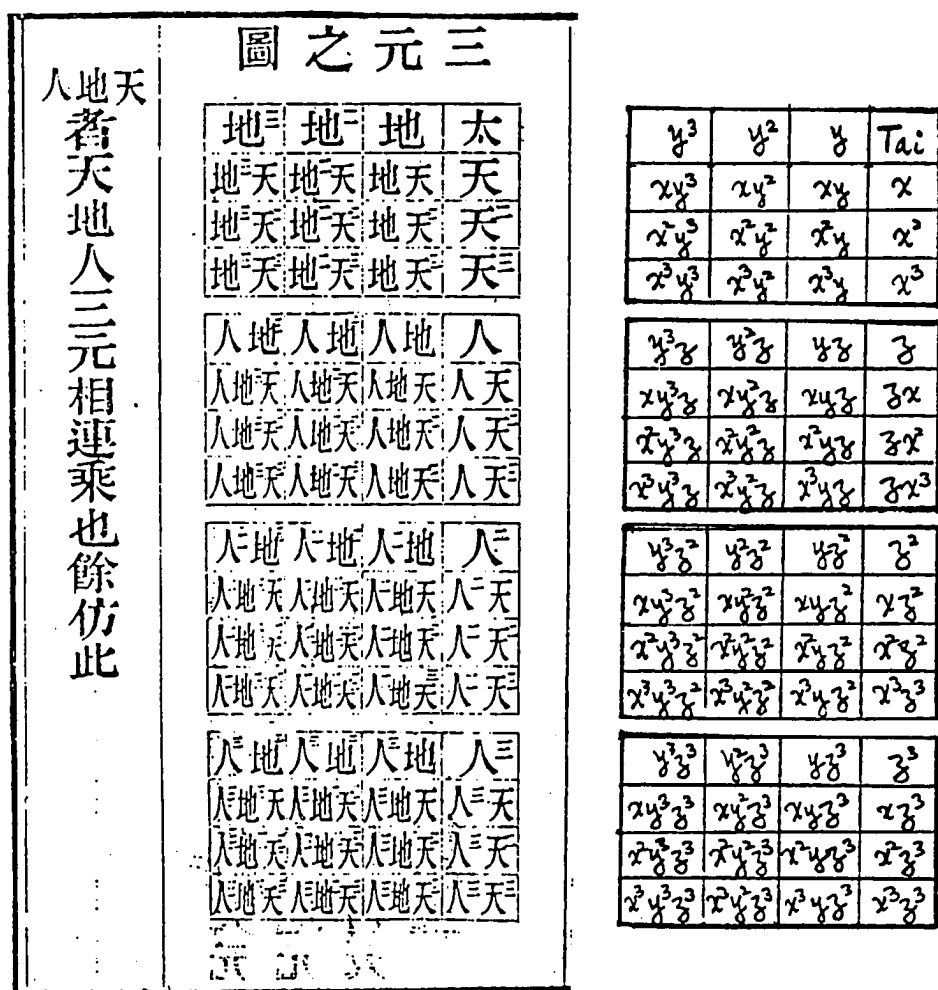


Figure 2. Li Shanlan's "San Yuan Zhi Tu"

(Schematic diagram for three unknowns).

In this scheme, the two smaller digits, namely, 2 and 2, denote the coefficients of $2xu$ and $2yz$ respectively [Li and Du 1987, 140-48]. It was against this background that Li Shanlan created a new scheme to represent the four unknowns -- "Si Yuan".

Li Shanlan's scheme was based upon four graphical representations: the "Yi Yuan Tu" (schematic diagram for one unknown), "Er Yuan Tu" (schematic diagram for two unknowns), "San Yuan Zhi Tu" (schematic diagram for three unknowns) and "Si Yuan Zhi Tu" (schematic diagram for four unknowns). The first two were similar to those given by Zhu Shijie. As for the representation of three or four unknowns, Li Shanlan's scheme provided a much more convenient way to specify the products of non-adjacent unknowns like xu or yz , etc. Here it seems worthwhile to cite his "San Yuan Zhi Tu" as evidence to show that here he was influenced by Western mathematics in the 1840s. Especially in reviving Zhu Shijie's *Si Yuan Yu Jian*, one of the most important classics in 13th-century China [Cf. Ho Ping-Yoke 1972], Li Shanlan's successful adaptation of the Western mode of representation to Chinese mathematics was quite clear. In the following citation, we translate Tian, $(\text{Tian})^=$, $(\text{Tian})^{\equiv}$ into x , x^2 , x^3 ; Di, $(\text{Di})^=$, $(\text{Di})^{\equiv}$ into y , y^2 , y^3 ; and Ren, $(\text{Ren})^=$, $(\text{Ren})^{\equiv}$ into z , z^2 , z^3 etc. Thus we have the equivalent form of the "San Yuan Zhi Tu" as shown in Figure 2.

According to Gu Guanguang, Li Shanlan had already created these representations for displaying unknowns by 1845 [Gu 1882a].

Nevertheless, it is unknown whether Li Shanlan used the algebraic symbols like $(\text{Tian})^=$, $(\text{Di})^=$, $(\text{Ren})^=(\text{Di})^=(\text{Tian})$, etc. at the time. Apparently these symbols did not appear in Chinese mathematical literature until Li Shanlan's translations of De Morgan's *Elements of Algebra* as well as Elias Loomis' *Elements of Analytical Geometry and the Differential and Integral Calculus* in the late 1850s. It seems, therefore, that the 1867 edition of the *Si Yuan Jie* must have been revised at least in this passage by Li Shanlan before being put to publication.

One feature in the *Si Yuan Jie* deeply impressed Gu Guanguang, namely Li Shanlan's geometric interpretation for the products of the unknowns [Gu 1882a]. It was entirely natural for Li Shanlan to have done this once he needed to give meaning to the products of the symbols like $(\text{Ren})^=(\text{Di})^=(\text{Tian})$, etc.. Li Shanlan interpreted a constant (term) as a point, one unknown as a line, the product of two unknowns as a rectangle in the plane, and the multiple of three unknowns as a rectangular parallelepiped. In addition, Li regarded multiples of four unknowns as "San Cheng Fang Ti" (solid of power four or solid of dimension 4), while for terms like xy^2z^3 his interpretation should have been as "solid of higher dimension," although he did not say so explicitly.

Li Shanlan should have considered the concepts of point, line, plane and solid, as well as their "products" in terms of the recently-introduced interpretation of such concepts. In the *Ji He Yuan Ben* (Euclid's *Elements*), for example, definitions for the

point, line, surface and solid had been given at the beginning of Book 1 [Xu and Ricci 1607]. Although this Greek mathematical classic gave almost no explanation for the relationships between these geometric magnitudes, the notions that a point moves continuously to generate a line or that a line sweeps continuously to generate a surface or that surfaces can be integrated into a solid were not unknown to Li Shanlan by the 1840s. In fact, as we look into the *Fang Yuan Chan You* ("Explanation of the Square and the Circle"), it seems clear that Li Shanlan was quite familiar with the ideas mentioned above. For example, in *Dan Zhi* 1 of the *Fang Yuan Chan You*, Li Shanlan claimed that "One must know that the Westerners' so-called point, line and surface are not without substance. Thus point is that which is tiny and fine enough; line is that which is long but very narrow in breadth; and surface is that which has length and breadth but is very thin."¹ In *Dan Zhi* 2, Li Shanlan explained that the "One must know that a solid can be degenerated into a surface and that a surface can be degenerated into a line." Li Shanlan also gave a very convincing illustration: "A book of one chi (ca. 23 cm) in height is comprised of piling pieces of paper; a piece of silk cloth of one zhang (i.e., 10 cm) is made of accumulating pieces of silk."²

It is quite clear, therefore, that both the *Si Yuan Jie* and *Fang Yuan Chan You* were influenced by Western mathematics to some extent. Unfortunately, we are unable to specify what sources were available to Li Shanlan at the time. Perhaps Li Shanlan really had made a synthesis of Euclid's idealized definitions of point,

line and surface with those of pre-Qin Mohists ("Mo Jia") [Wang Yusheng 1983b]. Even so, Li Shanlan apparently had created something new of his own. For example, responding to his "San Cheng Fang Yu" (in the *Si Yuan Jie*) as mentioned above, Li Shanlan went on to argue in the *Dan Zhi* 4 that "One must know that any solid of dimension greater than or equal to three can be regarded as a surface, and further as a line." Moreover, given any x either as the "Tian Yuan" of the Tian Yuan method or the "Gen" (root) of the "Jie Gen Fang Bi Li" method [Cf. Section 1.4], Li Shanlan illustrated by geometric diagrams how x^4 , x^5 and x^6 , etc., can be regarded as a line (element), a surface (element) and a solid (element) respectively, etc. under the conditions that x^1 , x^2 and x^3 have already been regarded as line, surface and solid respectively. Argued in this way, Li Shanlan asserted in *Dan Zhi* 3 that "One must know that every power of x can be regarded as line, surface or solid in a cyclic way."

Much might be said about Li Shanlan's interpretation of geometric magnitudes and their "products" (further details are given in the next section). Here it should be emphasized that the resemblance of the arguments on geometric magnitudes in the *Si Yuan Jie* as well as the *Fang Yuan Chan You* was not coincidental, since in both treatises the powers of the "Tian Yuan" were interpreted in a similar way. Such an interpretation was also reminiscent of the geometric interpretation for the "Jie Gen Fang Bi Li" method given in the *Shu Li Jing Yun* ("Collected Basic Principles of Mathematics", 1723) [Cf. Section 1.4]. In this

connection, Li Shanlan apparently was influenced by the *Shu Li Jing Yun*, although there is admittedly no direct textual evidence for this. Nevertheless, Li Shanlan certainly created his own geometric interpretations for solids of higher dimension, and thus for every power of the "Tian Yuan". For this last assertion, further evidence is given in the next section. In fact, the "Jian Zhui" method was created in the very context of Li Shanlan's geometric interpretation of the "Tian Yuan".

3.2. The "Jian Zhui" Method and the *Fang Yuan Chan You*

According to Wang Yusheng, the three methods: "Jian Zhui", "Duo Ji" and "Kao Shu Gen" (investigation of prime number), were the most significant of Li Shanlan's substantial contributions to mathematics [Wang Yusheng 1983b]. Among these, the "Jian Zhui" method was one of his earliest achievements. As mentioned above, it was probably created by the mid-1840s, since according to Gu Guanguang, Li Shanlan had already written down the *Hu Shi Qi Mi* and *Dui Shu Tan Yuan* by then [Gu 1882a]. On the other hand, his *Fang Yuan Chan You* would have been completed in the early 1840s, since this treatise the "Jian Zhui" method as ten *Dan Zhi[s]* (ten principles) which were indispensable for his *Hu Shi Qi Mi* and *Dui Shu Tan Yuan* [Wang Yusheng 1982b]. Before giving a survey of the *Fang Yuan Chan You*, it is necessary to give Li Shanlan's other six *Dan Zhi[s]* for a complete list of the ten principles.

Dan Zhi 5: "One must know what the 'Pien Jian Zhui' (two

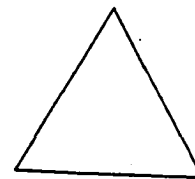
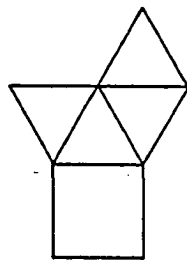
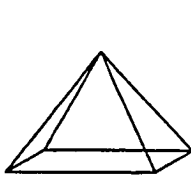
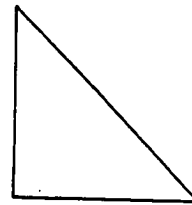
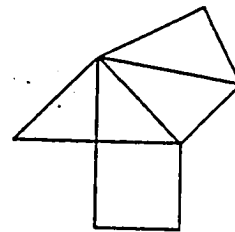
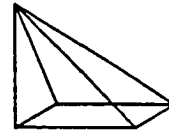
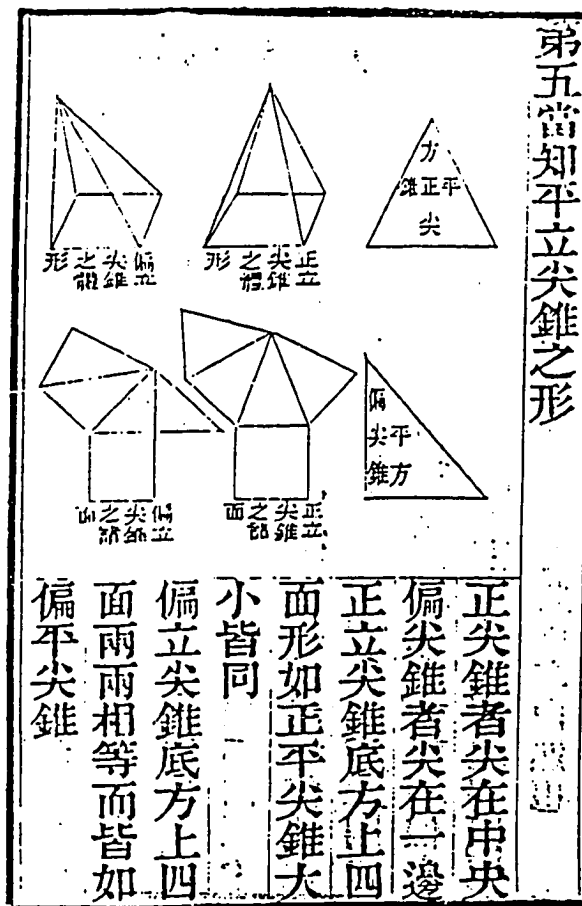


Figure 3. Illustrations for Dan Zhi 5 and their
duplicates [Cf. Fang Yuan Chan You (1867)].

dimensional pyramid) and 'Li Jian Zhui' (three dimensional pyramid) look like." As an illustration, Li Shanlan gave diagrams to show his so-called "Pien Fang Zheng Jian Zhui" (i.e., isosceles), "Pien Fang Pian Jian Zhui" (right-angled triangle), "Zheng Li Jian Zhui" (right pyramid with a square base) and "Pian Li Jian Zhui" (a slant pyramid with a square base but an edge perpendicular to the base, i.e., 'Yang Ma' in Chapter 5 of the *Jiu Zhang Suan Shu*) [Cf. Figure 3].

Dan Zhi 6: "One must know every n th power of some unknown (n greater than or equal to 3) can be interpreted as 'Jian Zhui'". To this principle, Li Shanlan also added diagrams representing "San Cheng Zheng Jian Zhui Zhi Ti" and "San Cheng Pian Jian Zhui Zhi Ti," thereby illustrating that the n th power of some unknown (n greater than or equal to 3) can be interpreted as a pyramid with a square base and inward concave lateral surface. Furthermore, the larger the power of n , the greater concavity of the lateral surface [Cf. Figure 4].

Dan Zhi 7: "One must know the principle that 'Jian Zhui' can be 'Ji Die' (piled up by accumulation)." But how? Li Shanlan argued in a way which was analogous to that of summing piles. In fact, Li Shanlan used the summation formulae of piles to approximate the volumes of the "Jian Zhui". For example, in order to find the "Li Jian Zhui", Li Shanlan approximated this pyramid, say, by a square pile with the k th layer k^2 "units" or "points" from the top [Cf. Figure 5]. Here a "point" is a geometric

magnitude that, according to Li Shanlan's interpretation, meant that "it is that which is tiny and fine enough," as described in Dan Zhi 1. It seems that in this case Li Shanlan made a leap from a sum of square pile like $\sum_{k=1}^n k^2$ (in modern notation) to a area equal to $\int_0^n x^2 dx$. Yet we should be cautious that he did not write $\int_0^n x^2 dx$ and regard it as the area of area of region under the curve $y = x^2$, although the sum formula $(\frac{1}{3!})n(n+1)(2n+1) = \sum_{k=1}^n k^2$ might have played a crucial role in his argument. Even so, Li Shanlan did point to the analogy between the sum of the finite series and the concept of the definite integral. Argued this way, he was led to bridge the gulf between finite and infinite mathematics apparently of his own.

In his Dan Zhi 8, Li Shanlan claimed that "One must know the method of computing the volume for the 'Jian Zhui' of each power."

Take the product of the height and the base area as dividend, the number of power plus one as divisor, then the quotient will be the volume of the "Jian Zhui". In other words, given a "Jian Zhui" of power p and with height h and base area a^p , its volume will be $\frac{1}{p+1} a^p h$. Yet how did Li Shanlan drive this formula? No directly relevant explanations were given in the passage. However, armed with the "Duo Ji" method (Method of piling up stacks), Li Shanlan probably derived it by an argument similar to that as rationally reconstructed by Li Di etc.

It deserves here to cite such a reconstruction in which we

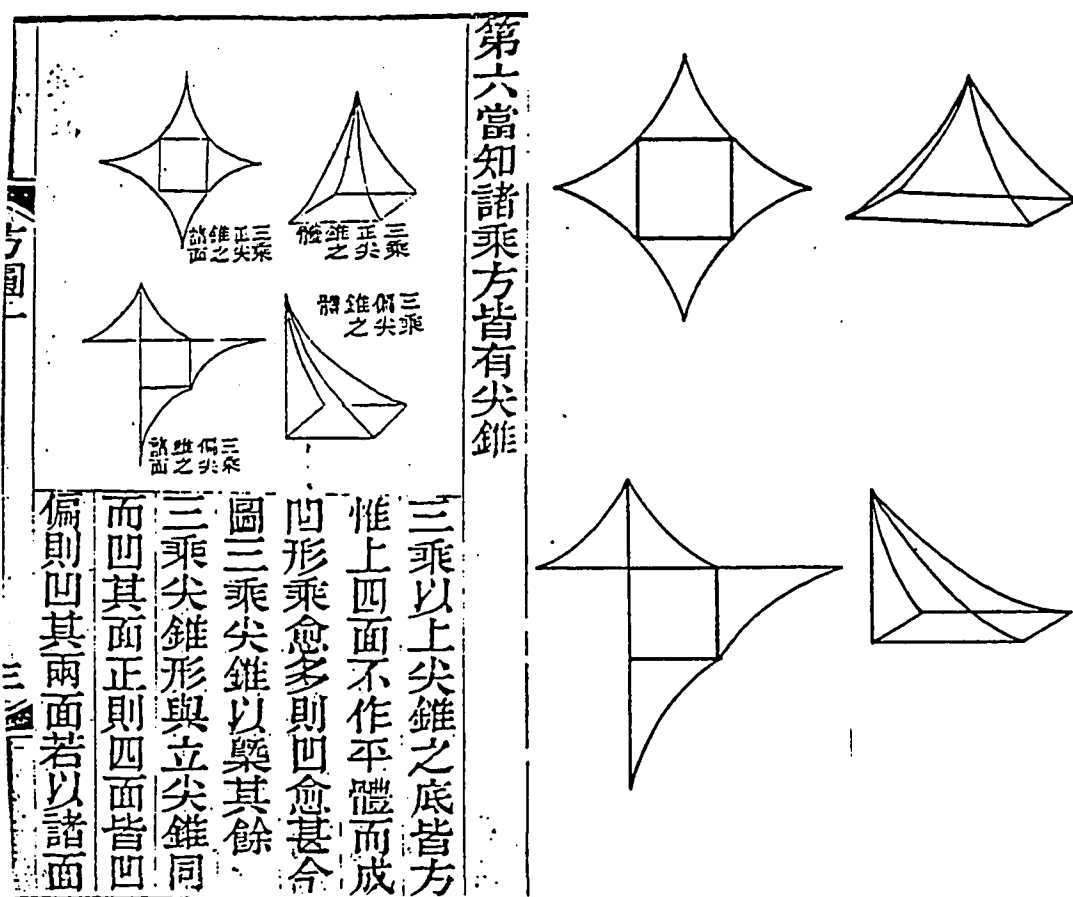


Figure 4. Illustrations for Dan Zhi 6 and their
duplicates [Cf. *Fang Yuan Chan You* (1867)]

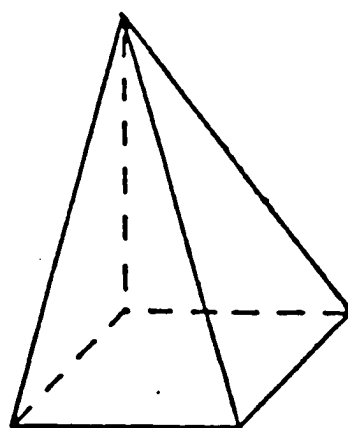
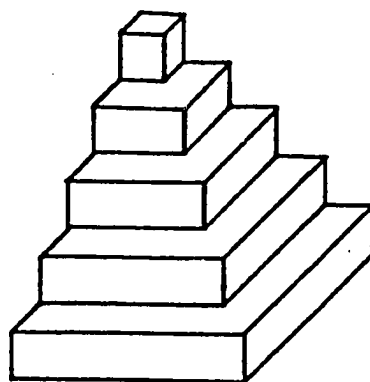


Figure 5. Li Shanlan's method of approximating a "Li Jian Zhui" (pyramid) by a square pile.

can see how Li Shanlan might have closely related his "Jian Zhui" and "Duo Ji" methods. Let us consider a "Jian Zhui" of power p (a pyramid with square base, and two inward concave lateral surfaces, see Figure 6) of height h and base area a^p . As in the "(p-1) Cheng Fang Duo" (i.e., $\sum_{k=1}^n k^p$), the area of the k th layer of the "Jian Zhui of power p " is k^p . Thus if we let x denote the distance between the k th layer and the vertex of this "Jian Zhui", and set $\alpha = \frac{k}{x}$, then the area of the k th layer is $k^p = (\alpha x)^p$. In addition, since $(\alpha h)^p = a^p$ implies $\alpha^p = (\frac{a}{h})^p$ so the area of the k th layer of the "Jian Zhui" of power p is $(\frac{a}{h})^p x^p$. Therefore, the volume of the "Jian Zhui" of power p is just the same as the following definite integral:

$$\int_0^h \left(\frac{a}{h}\right)^p x^p dx = \frac{1}{p+1} a^p h$$

which is the very content of his *Dan Zhi* 8 [Li Di et al., eds., 1986, pp. 458-469].

This rational reconstruction is of interest, for it explains what Li Shanlan might actually have done in the very context of 1840s Chinese mathematics. Yet one thing has to be clarified from the outset, namely, when did Li Shanlan obtain the summation formula for $\sum_{k=1}^n k^p$. Since this formula was included in the *Duo Ji Bi Lei* ("Sums of Piles of Various Types", 1867), at least some portion of this treatise must have been drafted in the early 1840s, even though it was not completed, as Li Di argues until

1853.

Li Shanlan's interpretation in terms of the "Jian Zhui" of power p and the "Dui Duo" of power $p-1$ as expressed in *Dan Zhi* 7 was quite amazing. Basically, Li Shanlan approximated the "Pien Jian Zhui" ("Jian Zhui" of power 2), "Li Jian Zhui" ("Jian Zhui" of power 3), and "San Cheng Jian Zhui" ("Jian Zhui" of power 4),... by the "Yuan Duo" ("Dui Duo" of power 1, for example, $\sum k$), "Yi Cheng Fang Duo" ("Dui Duo" of power 2, for example, $\sum k^2$), "Er Cheng Fang Duo" ("Dui Duo" of power 3),... respectively [Cf. Figure 7]. Such terms as "Yuan Duo," "Yi Cheng Fang Duo," and "Er Cheng Fang Duo," (used in the *Duo Ji Bi Lei*), did not appear in the *Fang Yuan Chan You* of either 1867 or 1851. Nevertheless, the *Fang Yuan Chan You* of 1851 did contain terms like "Pien Jian Dui", "Li Jian Dui", "San Cheng Jian Dui," etc. [Gu 1882a]. The term "Jian Dui" comes from the *Shu Li Jing Yun* (Book 30), where several formulas for the sums of piles were given. For example, $\sum k^2$ was called the "Si Jiao Jian Dui" ("Jian Dui" with four corners in piled square layers as shown). Yet viewed in the context of the *Fang Yuan Chan You* (1851), Li Shanlan apparently did not distinguish whether "Jian Dui" really meant "Jian Zhui" or "Dui Duo"-- an ambiguity that might explain Li Shanlan changed the term "Jian Dui" into "Jian Zhui" in the 1867 edition of the *Fang Yuan Chan You*.

Whatever the case, Li Shanlan seemed to regard the "Jian Zhui" (or "Jian Dui") of power p as accumulated by a series of

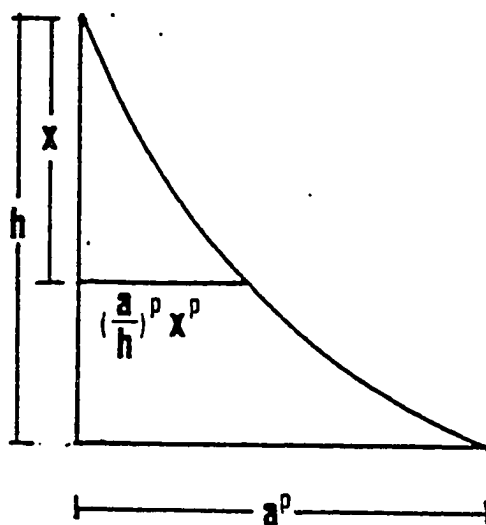
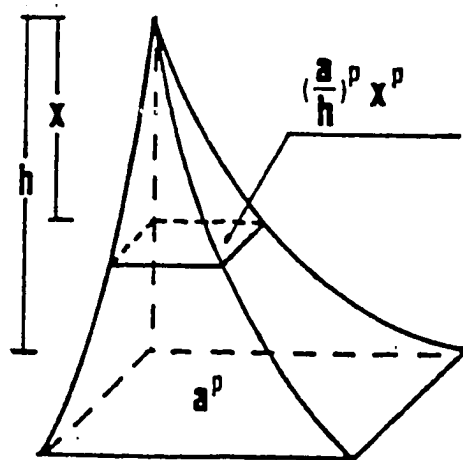
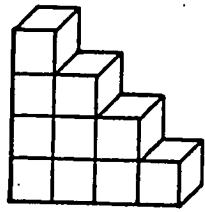
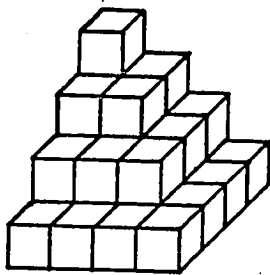


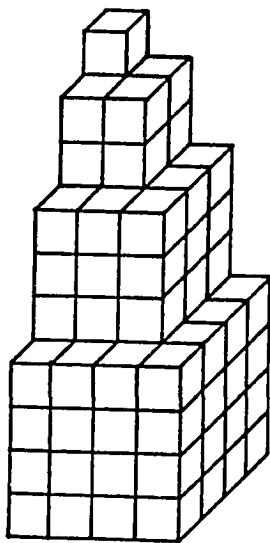
Figure 6. A "Jian Zhui" of power p with height h and base area a^p .



A)



B)



C)

Figure 7. Illustrations for
the finite series:

A) "Yuan Duo" $\sum k$.

B) "Yi Cheng Fang
Duo" $\sum k^2$.

C) "Er Cheng Fang
Duo" $\sum k^3$.

thin layers of area k^p ($k = 1, 2, \dots, n$), approximated by k^p "points" or "units". Thus the "Jian Zhui", say of power 2, can be approximated by a series of thin layers of area k^2 , then by square piles with layers of k^2 "points", $k = 1, 2, \dots, n$ [Cf. Figures 5 and 7]. Here Li Shanlan made an argument similar to that of letting n tend to infinity. If the height of the "Jian Zhui" is h , then the height of each layer, $\frac{h}{n}$, would "become thin enough to be like a piece of paper" as the amount of layers, n , "tends to be infinite". If the "Jian Zhui" of power p is approximated by a series of thin layers of area k^p , $k = 1, 2, \dots, n$, then the volume of the k th layer with height $\frac{h}{n}$ and base area $(\frac{a}{h})^p (k \frac{h}{n})^p (= a^p \cdot (\frac{k}{n})^p)$, a formula as shown above. Therefore, the volume of the latter is:

$$\begin{aligned}
 V_n &= \sum_{k=1}^n \frac{a^p h}{n^{p+1}} k^p = \frac{a^p h}{n^{p+1}} \sum_{k=1}^n k^p \\
 &= \frac{a^p h}{n^{p+1}} \sum_{i=1}^p [A_i^p \sum_{k=1}^{n-i+1} \frac{1}{p!} k (k+1) \dots (k+p-1)] \\
 &= \frac{a^p h}{n^{p+1}} \{ A_1^p \frac{1}{(p+1)!} n (n+1) (n+2) \dots (n+p) \\
 &\quad + A_2^p \frac{1}{(p+1)!} (n-1) n (n+1) \dots (n+p-1) + \dots \\
 &\quad + A_p^p \frac{1}{(p+1)!} [n-(p-1)] [n-(p-2)] [n-(p-3)] \dots [n+1] \},
 \end{aligned}$$

where A_i^p , $i = 1, 2, \dots, n$ are the so-called Eulerian Numbers or

Li Shanlan Numbers of the Second Kind, satisfying the following conditions:

$$A_i^p = 0 \quad \text{if } i = 0 \text{ or } i > p,$$

$$A_i^p = (p-i+1) A_{i-1}^{p-1} + i A_i^{p-1}$$

[Cf. Section 3.5]. Finally, let n tend to be infinite. Then one is led to the volume of the "Jian Zhui" of power p :

$$V = \frac{a^p h}{p+1}.$$

Dan Zhi 9 claims that "One must know that the 'Jian Zhui' of power greater than or equal to three can be regarded as a surface in a sense that its piled planes have been taken as a line." As an illustration, Li Shanlan noted that once the each piled plane (in a "Jian Zhui" of power $p \geq 3$) have been regarded as a line, then the "Jian Zhui" could be regarded as a planar region ("Pien Ti", literally planar body).

Finally, Dan Zhi 10 claims that "One must know that all the 'Jian Zhui' are planar, they can therefore be accumulated into one 'Jian Zhui'," [Cf. Figure 8]. Equivalently, this suggests that the definite integral of an infinite series can be computed term by term:

$$\int_a^b \left(\sum_{n=1}^{\infty} f_n(x) \right) dx = \sum_{n=1}^{\infty} \int_a^b f_n(x) dx.$$

[Wang Yusheng 1983b; Li Di et al., eds., 1986, pp. 460-461]. Yet here again we should be cautious that Li Shanlan does not seem to have any concept of definite integral applied to an integrable function, for the concept of function never came across Li Shanlan's mind.

Armed with these ten "Dan Zhi," Li Shanlan went on to give a demonstration for the area formula of unit circle. This, in fact, seems to have been one of the reasons why he wrote the *Fang Yuan Chan You*. He did not think that the theory underlying the "Ge Yuan" method was so illuminating as Li Ye's *Ce Yuan Hai Jing* ("Sea Mirror of Circle Measurements") or Dai Zhen's *Gou Gu Ge Yuan Ji* ("On the Right-angled Triangle and the Circle Measurements") [Li 1867, "Preface"]. Therefore, it was natural that he should have taken the problem seriously from the very beginning of his career.

Li Shanlan's strategy was to regard the difference between each quarter of a unit square and its inscribed circle as an accumulation of infinitely many "Jian Zhui" of power $2n$, ($n = 1, 2, 3, \dots$). According to *Dan Zhi[s]* 8, 9 and 10, one could compute the areas of each "Jian Zhui" and thereby obtain the infinite series expansion for the area of the circle.

Given the radius $AC = 1$ in Figure 9, then the "Jian Zhui" ABC, according to Li Shanlan, would be expressed as an accumulation of the infinite "Jian Zhui" of power $2n$, ($n = 1, 2, 3, \dots$), namely, "Jian Zhui" of power 2 -- ACC_1 , "Jian Zhui" of

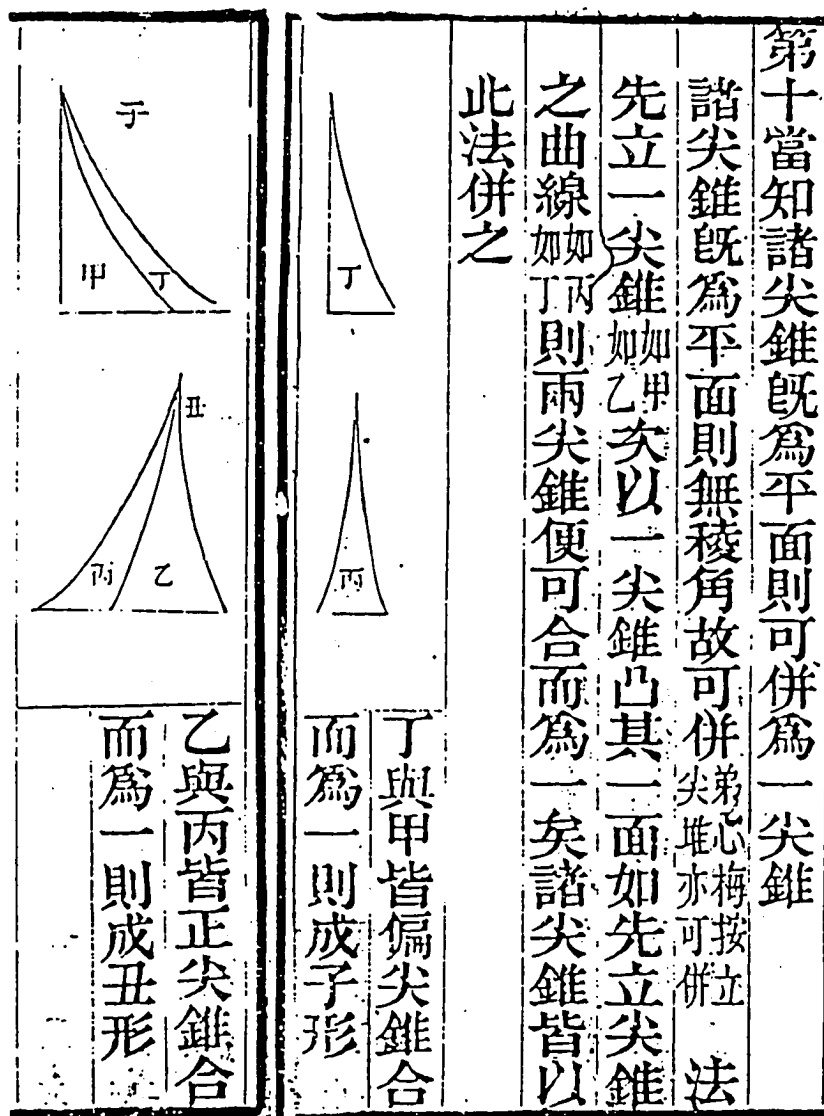


Figure 8. Xeroxcopy of Dan Zhi 10. Its illustrations show how pieces of "Jian Zhui" are accumulated into a larger "Jian Zhui".

power 4 -- AC_1C_2 , "Jian Zhui" of power 6 -- AC_2C_3 , "Jian Zhui" of power 8 -- AC_3C_4 , ... Moreover, the respective base length of these "Jian Zhui" is as follows:

$$CC_1 = \frac{1}{2} \quad BC = \frac{1}{2}$$

$$C_1C_2 = \frac{1}{4} \quad BC_1 = \frac{1}{4} \cdot \frac{1}{2}$$

$$C_2C_3 = \frac{1}{6} \quad BC_2 = \frac{1}{6} \cdot \frac{3}{4} \cdot \frac{1}{2}$$

$$C_3C_4 = \frac{1}{8} \quad BC_3 = \frac{1}{8} \cdot \frac{5}{6} \cdot \frac{3}{4} \cdot \frac{1}{2}$$

.....

Now, according to Dan Zhi 8, the areas for the "Jian Zhui" of powers 2, 4, 6, 8, ..., respectively, are $\frac{1}{2} \cdot \frac{1}{3}$, $\frac{1}{2 \cdot 4} \cdot \frac{1}{7}$, $\frac{1 \cdot 3}{2 \cdot 4 \cdot 6} \cdot \frac{1}{7}$, $\frac{1 \cdot 3 \cdot 5}{2 \cdot 4 \cdot 6 \cdot 8} \cdot \frac{1}{9}$, ... respectively [Li Di, et al., 1986, 462-463]. Therefore, according to Dan Zhi 10, the area of the "Jian Zhui" ABC equals:

$$\frac{1}{2} \cdot \frac{1}{3} + \frac{1}{2 \cdot 4} \cdot \frac{1}{5} + \frac{1 \cdot 3}{2 \cdot 4 \cdot 6} \cdot \frac{1}{7} + \frac{1 \cdot 3 \cdot 5}{2 \cdot 4 \cdot 6 \cdot 8} \cdot \frac{1}{9} + \dots$$

Finally, the area of the unit circle is:

$$\pi = 4 - 4 \left(\frac{1}{2} \cdot \frac{1}{3} + \frac{1}{2 \cdot 4} \cdot \frac{1}{5} + \frac{1 \cdot 3}{2 \cdot 4 \cdot 6} \cdot \frac{1}{7} + \frac{1 \cdot 3 \cdot 5}{2 \cdot 4 \cdot 6 \cdot 8} \cdot \frac{1}{9} + \dots \right).$$

Given the above calculations, how was the base length for the "Jian Zhui" of power $2n$ derived as shown above? Li Shanlan gave

no reasons for his computations. It was his younger brother Li Xinmei who helped to illustrate the methods.

According to Li Xinmei, Li Shanlan's method was as follows (refer to Figure 9). Set $\angle BOQ = \alpha$, construct through Q a line $\overline{EF}$ parallel to $\overline{BC}$, and set $\overline{OE} = \overline{AF} = x$. Then, "based on the theory of eight trigonometric functions" (the method of "Bao Xian"), according to Li Xinmei:

$$\cos \alpha = x$$

$$\Rightarrow \sin \alpha = EQ = (1-x^2)^{1/2}$$

$$\Rightarrow \text{covers } \alpha = QF = 1 - (1-x^2)^{1/2}.$$

Expand the function $\text{covers } \alpha = 1 - (1-x^2)^{1/2}$ as power series

$\sum_{n=1}^{\infty} a_n x^{2n}$, then the coefficients $a_1, a_2, a_3, a_4, \dots$ will be the base lengths $CC_1, C_1C_2, C_2C_3, C_3C_4, \dots$ respectively.

At this point, Li Shanlan did not explain how he obtained those $a_1, a_2, a_3, a_4, \dots$. Again, it was Li Xinmei who illustrated the method with examples. His illustration was as follows: Take $x^2 = 10^{-8}$, then

$$\begin{aligned} 1 - (1-x^2)^{1/2} &= 1 - (1-10^{-8})^{1/2} = 1 - (0.99999999)^{1/2} \\ &= 1 - 0.9999999949999999874999999374999996093749972656249 \\ &= 0.0000000050000000125000000625000003906250027343750\dots \\ &= 0.5 \cdot 10^{-8} + 0.125 \cdot 10^{-16} + 0.0625 \cdot 10^{-24} + 0.0390625 \cdot 10^{-32} \end{aligned}$$

$$\begin{aligned}
& + 0.02734375 \cdot 10^{-40} + \dots \\
& = \frac{1}{2} \cdot 10^{-8} + \frac{1}{2 \cdot 4} \cdot 10^{-16} + \frac{1 \cdot 3}{2 \cdot 4 \cdot 6} \cdot 10^{-24} + \frac{1 \cdot 3 \cdot 5}{2 \cdot 4 \cdot 6 \cdot 8} \cdot 10^{-32} \\
& \quad + \frac{1 \cdot 3 \cdot 5 \cdot 7}{2 \cdot 4 \cdot 6 \cdot 8 \cdot 10} \cdot 10^{-40} + \dots
\end{aligned}$$

In other words, Li Xinmei justified the power series expansion for $1 - (1-x^2)^{1/2}$ as follows:

$$\begin{aligned}
1 - (1-x^2)^{1/2} &= \frac{1}{2} x^2 + \frac{1}{2 \cdot 4} x^4 + \frac{1 \cdot 3}{2 \cdot 4 \cdot 6} x^6 + \frac{1 \cdot 3 \cdot 5}{2 \cdot 4 \cdot 6 \cdot 8} x^8 \\
&\quad + \frac{1 \cdot 3 \cdot 5 \cdot 7}{2 \cdot 4 \cdot 6 \cdot 8 \cdot 10} x^{10} + \dots
\end{aligned}$$

Li Xinmei went on to take into account the case $x^2 = 2 \times 10^{-8}$, justifying the last series expansion again, and finally confirming Li Shanlan's conclusion that the difference of a quarter of square and its inscribed circle was an accumulation of an infinite "Jian Zhui" of powers $2n$, $n = 1, 2, 3, \dots$

In conclusion, Li Shanlan's infinite series expansion for π and a priori demonstration of the area formula for a circle clearly went far beyond Liu Hui's "Ge Yuan" method (third century A.D.), or the method of "Ge Yuan Lian Bi Li" of Ming Antu, Dong Youcheng and Xiang Mingda. In fact, Li Shanlan was the first Chinese mathematician to advance successfully a recursive formula for the areas of "Jian Zhui" of power $2n$, $n = 1, 2, 3, \dots$. In

doing so he was able to deal with the problem of infinitesimal "Jian Zhui" algebraically. On the other hand, Li Shanlan expanded $1-(1-x^2)^{1/2}$ into a power series in a way totally different from the methods used by Dai Xu and Xiang Mingda in the same year, 1845. Li Shanlan therefore deserves no less credit for discovering the binomial expansion of $(1-x^2)^{1/2}$ than Dai Xu and Xiang Mingda [Wang Yusheng 1983b].

However, Li Shanlan was not as content as one might have expected, even though he certainly created something new of his own in this case. Since the area formula for the circle had been well-known for a long time, Li Shanlan must have been felt that merely adding a new demonstration or proof for it was not really a substantial contribution to mathematics, however how novel the new proof may have been. Li Shanlan may only have regarded his infinite series expansion for π as an illustration of the "Jian Zhui" method and little more. Similarly, he may not have been so proud of his achievement in the *Hu Shi Qi Mi* as he deserved to be. His principal concern in the 1850s, as the historian Zhang Yong has suggested, was the theory of logarithms [Zhang Yong 1939]. Yet before turning to his *Hu Shi Qi Mi* and *Dui Shi Tan Yuan*, a few last comments must be made concerning his "Jian Zhui" method.

As most historians of Chinese mathematics have suggested, Li Shanlan created the "Jian Zhui" method in the context of Chinese mathematics of the 1840s. Nevertheless, the term "Jian Dui" used in the 1851 edition of the *Fang Yuan Chan You* points to Western

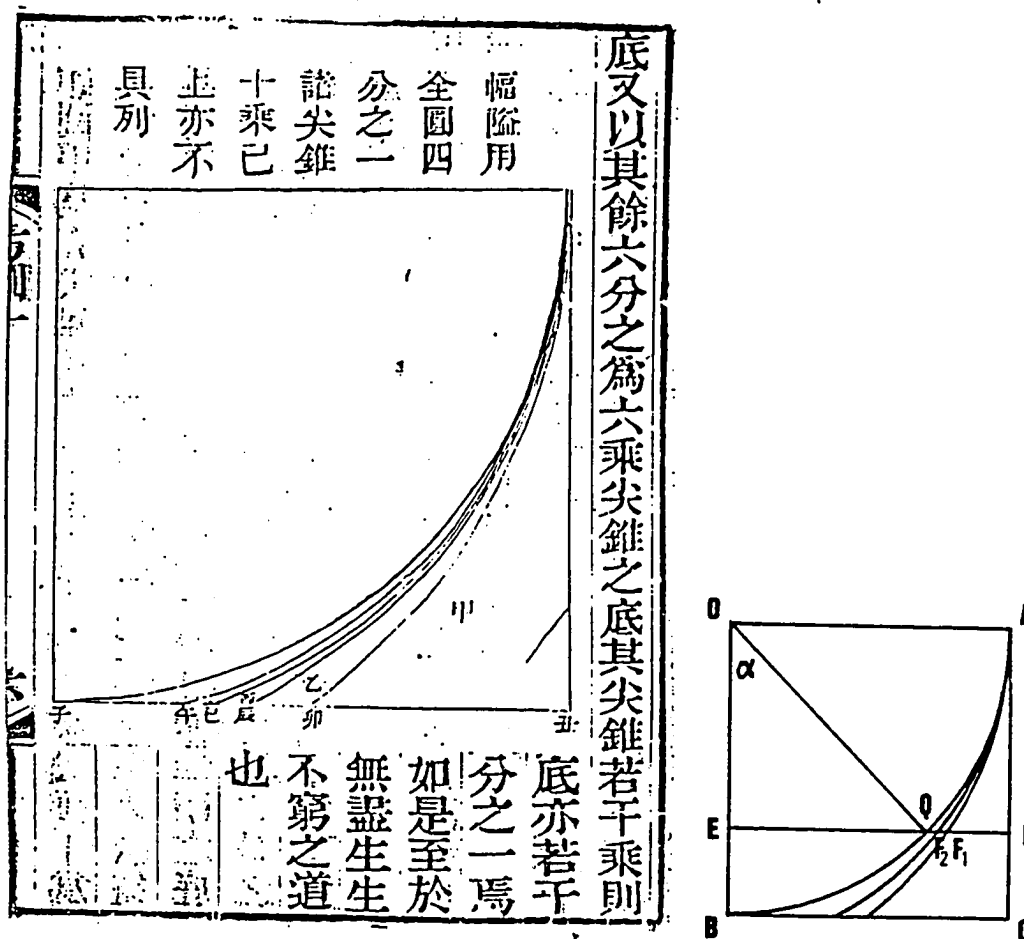


Figure 9. Li Shanlan's illustrations of how his "Jian Zhui" method can be used to approximate the difference area of unit square and one-quarter of a circle.

influence on his mathematics. We have already observed that the editors of the *Shu Li Jing Yun* (1723) coined such terms as "San Jiao Jian Dui" (triangular pile) and "Si Jiao Jian Dui" (square pile), etc. On the other hand, Chen Shiren (1676-1722) in his *Shao Guang Bu Yi* ("Supplement to the Shao Guang Chapter of the *Jiu Zhang Suan Shu*") gave the piles $\sum k$ the name "Pien Jian" (planar "Jian Dui"), $\sum \frac{k(k+1)}{2!}$ the name "Li Jian" (three dimensional "Jian Dui") respectively. Whether Li Shanlan read this work is unknown, for it was not published and was circulated only in manuscript at the time. However, Li Shanlan should have been familiar with Wang Lai's calling the triangular pile by the term "Pien San Jiao Dui" and Chen Shiren's "Li Jian" by the term "Li San Jiao Dui" etc. [Wang Lai 1798; Li Zhaohua 1986]. This is clear from the fact that both Wang Lai and Li Shanlan regarded every term k (in the above formulae) as "Gen" (literally root or radical), which certainly was the "unknown" in the method of "Jie Gen Fang Bi Li". Although Li Shanlan actually used the "Yuan" in the "Tian Yuan" method in all cases, he identifies the "Gen" with the "Yuan" in the illustration of *Dan Zhi* 3. Moreover, the number of terms in the above two formulae, i.e., n , was called "Gen Shu" (literally number of "Gen") by Wang Lai and "Yuan Shu" (literally number of "Yuan") by Li Shanlan, respectively. Therefore it is highly probable that Li Shanlan inherited this Western legacy introduced by Wang Lai's mathematics. On the one hand (as noted in Chapter 1), Wang Lai was classified into the "Xi Fa" school (School of Western Methods), and on the other hand, Li Shanlan was quite familiar with Wang Lai's collected mathematical works, *Hengzhai*

Suan Xue [Cf. Li's *Tian Suan Huo Wen* (1867); Section 3.8].

Granted that Li Shanlan's "Jian Zhui" or rather "Jian Dui" was influenced by Western mathematics, the question arises: how did Li Shanlan view the indivisible which underlay his own conceptions of point, line and surface? As mentioned above, Li Shanlan defined a point in a sense similar to that of the pre-Qin Mohists, but in a sense contradictory to that given in Euclid's *Elements*. This prompted a criticism from the early 20th-century Qing Mathematician Wu Qiqian. He wondered why Li Shanlan would ignore the definition of a point given in the very beginning of the *Elements* [Wang Yusheng 1983b]. Yet in the approximation of the "Jian Dui" of power p by the "Dui Duo" of power $p-1$, Li Shanlan must have known that the smaller the units of the "Dui Duo," the more fully it would be possible to exhaust any given volume or area. In other words, Li Shanlan could not neglect a point to be of no entity (or body). Otherwise, then Li Shanlan's "Jian Zhui" method would have been analogous to Shen Gua's "Xi Ji" method (method of crack-volumes or interstice-volumes). For a rectangular stack n layers high, with its top layer a objects and b objects long, bottom layer c objects wide and d objects long, Shen Gua (1031-1095) of the northern Song gave the following formula for the total number of the objects [Cf. Figure 10]:

$$\begin{aligned} S &= ab + (a+1)(b+1) + (a+2)(b+2) + \dots + cd \\ &= \frac{n}{6} [(2b+d)a + (2d+b)c] + \frac{n}{6} (c - a), \end{aligned}$$

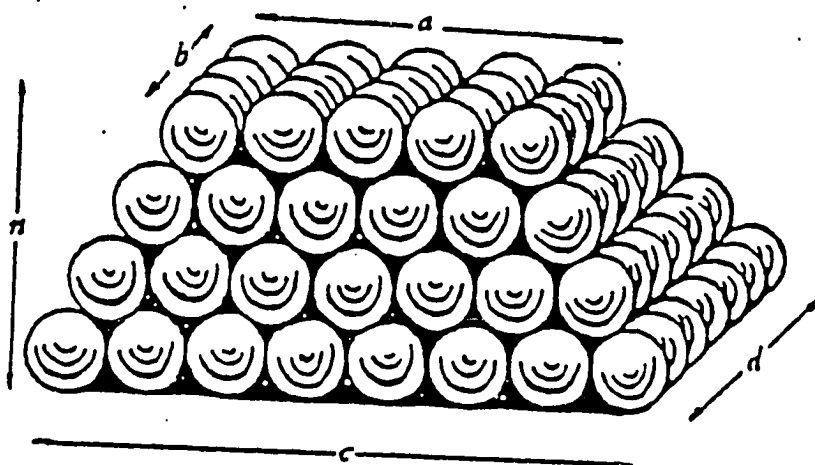


Figure 10. A rectangular stack n layers high, with its top layer a objects wide and b objects long, bottom layer c objects wide and d objects long.

where the first part of the right side in the last equality is the volume of the "rectangular platform" with a shape which can be described as follows: It has top rectangle's width and length a , and b respectively, the bottom rectangle's width and length c , and d respectively, and the height n , as given in Chapter 5 of the Chinese mathematical classic, the *Jiu Zhang Suan Shu*. In this case, Shen Gua spoke of the "Xi Ji", i.e., exactly the residual space which it was the object of the exhaustion methods to assess. Obviously, as Joseph Needham suggests, Shen Gua's "art of piling up very small things, by which he certainly had in mind something almost equivalent to the summation of indivisibles by Cavalieri 600 years later." [Needham 1959, 142]. Similarly, Li Shanlan 200 years later likewise conceived of indivisibles in a way almost analogous to that of Cavalieri. However, whether Li Shanlan ever read Shen Gua's *Meng Xi Bi Tan* ("Dream Pool Essays") is unknown, it is therefore not easy to relate Li Shanlan's "Jian Zhui" method to Shen Gua's "Xi Ji" method in any historically direct way.

Closely related to the question of infinitesimal entities is Li Shanlan's attitude towards infinity. Unfortunately, Li Shanlan said almost nothing about how infinitely many infinitesimal "Jian Zhui" were accumulated, even though he was able to deal with this problem algebraically. Perhaps this is why Li Shanlan did not think it necessary to explain what the infinite was all about. Basically Li Shanlan's mathematics were concerned primarily with how methods could be created to solve problems. On the other hand, he might not have considered the foundational problems

concerning the concept of infinity to be serious. This was in striking contrast to his predecessors Jiao Xun, Wang Lai and Li Rui's mathematics. To the "Tan Tian San You" (three friends concentrating on astronomy and mathematics) Wang Lai, Li Rui and Jiao Xun, mathematical studies seemed to have been emphasized on the structural aspects of theories, namely the theory of equations as well as the arithmetical laws of operations per se. More to the point, Ming Antu did try to explain away difficulties arising from approximating an arc by its subtending chords, infinitely many in number! For Ming Antu, a chord and an arc were not of the same category of geometrical magnitudes, even though the arc might be approximated by infinitely many chords. Yet such an approximation, according to Ming Antu, would enable mathematicians to do the computations by means of his own "Ge Yuan Mi Lian Bi Li" method, even though the infinitesimal arc was not the same as its subtending chord. Under such circumstances, Carl Boyer's comment on Cavalieri -- that his attitude towards infinity was one of "agnosticism" [Boyer 1959, p. 117], seems more applicable to Ming Antu than to Li Shanlan. This difference, however, points to a change of focus in Chinese mathematical studies during the 1840s and thereafter. In substantially a new historical context, Li Shanlan was destined to play a central role in mid-19th-century Qing mathematics.

3.3. "Jian Zhui" Method and the *Hu Shi Qi Mi*

As argued in Section 1.10, Ming Antu's mathematics came to

the attention of early nineteenth-century Qing scholars due in part to Luo Shilin, the most faithful representative of the Qian-Jia school between 1820-40, who regarded Ming Antu's work as a "new method" (Xin Fa) rather than a "Western method" (Xi Fa). Nevertheless, Ming Antu was a thorough-and-through "Xi Fa" scholar. But the attempt of the Qian-Jia school to cast Ming Antu as the creator of "Xin Fa" also reflects a change in mathematical study at this time, namely, a shift from reviving Song-Yuan mathematics to study of the three formulae introduced by Petrus Jartoux (along with the six other formulas created by Ming Antu). Likewise, the contrast between Luo Shilin and Xiang Mingda may well reflect some aspects of this new, attractive subject field as well [See Section 1.10].

Since Li Shanlan plays a central role in the history of Chinese mathematics from 1840 on, his interest in Ming Antu's mathematics is a significant factor. In fact, Li Shanlan owed as much to the "Xi Fa" tradition embodied in Ming Antu's work as he did to the revival of Song-Yuan mathematics. The latter was significant not only in terms of the transmission of Western mathematics to China in the 19th century but in terms of the general character of Chinese mathematics that emerged at the turn of the century. What, then, was the significance of the "Xi Fa" tradition as reflected in Ming Antu's mathematics?

The three formulae introduced to China by the French Jesuit Pierre Jartoux (1668-1720) can be expressed in modern notation as

follows:

$$1) \pi d = 3d + \frac{3d \cdot 1^2}{4 \cdot 3!} + \frac{3d \cdot 1^2 \cdot 3^2}{4^2 \cdot 5!} + \frac{3d \cdot 1^2 \cdot 3^2 \cdot 5^2}{4^3 \cdot 7!} + \dots;$$

or (by dividing by d on both sides)

$$\pi = 3 + \frac{3 \cdot 1^2}{4 \cdot 3!} + \frac{3 \cdot 1^2 \cdot 3^2}{4^2 \cdot 5!} + \frac{3 \cdot 1^2 \cdot 3^2 \cdot 5^2}{4^3 \cdot 7!} + \dots;$$

$$2) r \sin \frac{a}{r} = a - \frac{a^3}{3!r^2} + \frac{a^5}{5!r^4} - \frac{a^7}{7!r^6} + \dots;$$

$$3) r \text{ vers } \frac{a}{r} = \frac{a^2}{2!r} - \frac{a^4}{4!r^3} + \frac{a^6}{6!r^5} - \dots,$$

where $d = 2r$ is the diameter and an arc of the unit circle respectively [Li Di 1984, p. 285]. No demonstrations were given for any of these formulas followed, until Ming Antu justified their validity by his method of "Ge Yuan Lian Bi Li". Ming Antu also added six other formulas of his own, along with their demonstrations:

$$4) c = 2a - \frac{(2a)^3}{4 \cdot 3!r^2} + \frac{(2a)^5}{4^2 \cdot 5!r^4} - \frac{(2a)^7}{4^3 \cdot 7!r^6} + \dots;$$

$$5) h = \frac{(2a)^2}{4 \cdot 2!r} - \frac{(2a)^4}{4^2 \cdot 4!r^3} + \frac{(2a)^6}{4^3 \cdot 6!r^5} - \dots;$$

$$6) 2a = c + \frac{1^2 \cdot c^3}{4 \cdot 3! r^2} + \frac{1^2 \cdot 3^2 \cdot c^5}{4^2 \cdot 5! r^4} + \frac{1^2 \cdot 3^2 \cdot 5^2 \cdot c^7}{4^3 \cdot 7! r^6} + \dots;$$

$$7) a = \sin \alpha + \frac{1^2 \cdot \sin^3 \alpha}{3! r^2} + \frac{1^2 \cdot 3^2 \cdot \sin^5 \alpha}{5! r^4} + \dots;$$

$$8) a^2 = r \frac{2r \text{ vers} \alpha}{2!} + \frac{1^2 \cdot (2r \text{ vers} \alpha)^2}{4!} + \frac{1^2 \cdot 2^2 \cdot (2r \text{ vers} \alpha)^3}{6! r} +$$

...;

$$9) (2a)^2 = r \cdot 8h + \frac{1^2 \cdot (8h)^2}{4 \cdot 4!} + \frac{1^2 \cdot 2^2 (8h)^3}{4^2 \cdot 6! r} + \dots$$

In all of the above, c is a chord subtending the arc a or arc ACD , h is the "sagitta" perpendicular to the chord c and $\alpha = \frac{a}{r}$, the angle subtended by the arc a or arc ACD [Cf. Figure 11].

These nine formulae, collected in Book 1 of Ming Antu's *Ge Yuan Mi Lu Jie Fa* (but mistaken as "Du Shi Jiu Shu" [Nine Formulae of Jartoux]), were circulated privately in a draft manuscript from the late 18th century on. However, no demonstrations at all were included. In 1819, when Dong Youcheng (1791-1823) came across this draft copy in Beijing, he immediately began to work out the principles underlying the methods used to expand all of these

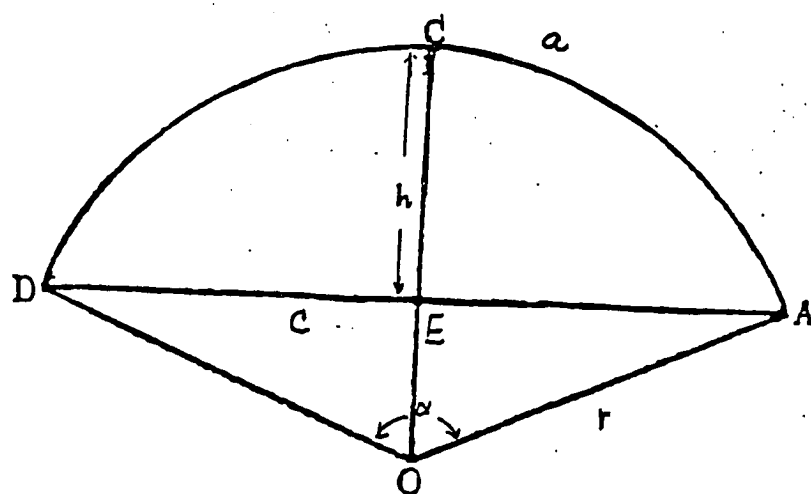


Figure 11. $a = \text{arc ACD}$, $h = CE$, $\alpha = \angle AOD$ and $r = OA$.

series. In the same year Dong Youcheng wrote his own *Ge Yuan Lian Bi Li Tu Jie* ("Graphical Illustrations on the Method of 'Lian Bi Li' in Circle Measurement"). In this treatise, Dong Youcheng used a completely different means from Ming Antu (although he also used the "Lian Bi Li" method [repeated or continued proportions]) to prove these nine formulae. In 1821, two years after this treatise was completed, Dong Youcheng saw Ming Antu's *Ge Yuan Mi Lu Jie Fa* in a complete version (four books altogether). Although Ming Antu had taken a different approaches to the subject, he found that they nevertheless had reached led to the same conclusions.

What, essentially, was Dong Youcheng's approach all about? And how did his methodology in this case ever influence his successors? To answer these questions, a few observations about Dong Youcheng's "Ge Yuan" method are necessary -- a method in marked contrast to Ming Antu's. To justify formula (4) above, Ming Antu worked formulae for the chord c subtending the given arc a , in terms of $c_{1/m}$, where $c_{1/m}$ is a smaller chord subtending $(1/m)$ th of the arc a . Using the method of "Lian Bi Li" (repeated or continued proportions), Ming Antu obtained formulae for the cases $m = 2$ and $m = 5$ respectively. He then went on to obtain the following formula:

$$c = 10 c_{1/10} - 165 \frac{c_{1/10}^3}{4r^2} + 3003 \frac{c_{1/10}^5}{4 \cdot 16r^4} - \dots$$

Based on similar arguments, Ming Antu also obtained formulae for

the cases of $m = 100$ and 1000 , respectively. Combining these together, Ming Antu eventually arrived at a formula for c in terms of $c_{1/m}$:

$$\begin{aligned}
 c = & 10000 c_{1/10000} - 166666665000 \frac{c_{1/10000}^3}{4r^2} \\
 & + 3333333000000003000 \frac{c_{1/10000}^5}{4 \cdot 16r^4} \\
 & + 31746020634921457142850000 \frac{c_{1/10000}^7}{4 \cdot 16^2 \cdot r^6} + \dots
 \end{aligned}$$

Since here $10000 c_{1/10000}$ is very close to the arc ACD subtended by the given chord c [Cf. Figure 12], thus by setting the arc ACD = $2a$, taking $10000 \cdot c_{1/10000} = 2a$, and finally substituting this in the above formula, Ming Antu concluded that:

$$\begin{aligned}
 c = & 2a - 0.166666665 \frac{(2a)^3}{4r^2} + 0.03333333 \frac{(2a)^5}{4 \cdot 16r^4} \\
 & - 0.003174602 \frac{(2a)^7}{4 \cdot 16^2 \cdot r^6} + \dots \\
 = & 2a - \frac{(2a)^3}{4 \cdot 3!r^2} + \frac{(2a)^5}{4^2 \cdot 5!r^4} - \frac{(2a)^7}{4^3 \cdot 7!r^6} + \dots
 \end{aligned}$$

Ming Antu justified the so-called last equality (which is actually

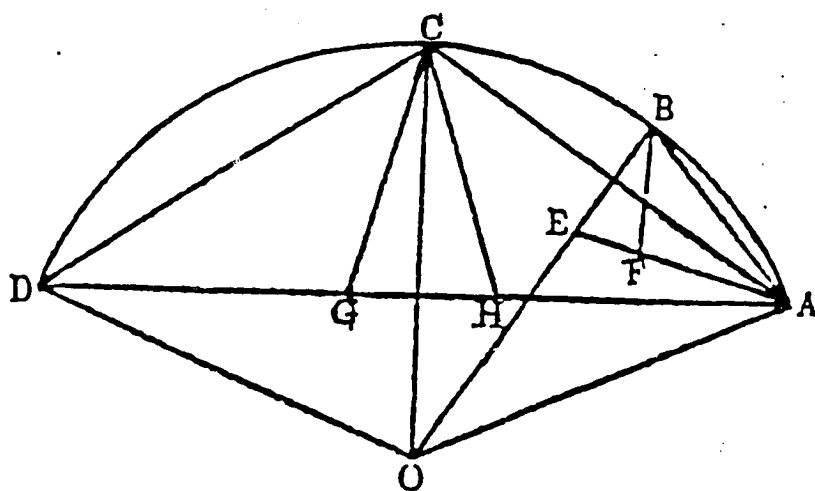


Figure 12. $2a = \text{arc ACD}$, $c = AD$, $c_{1/m} = AB$. Ming Antu's "Lian Bi Li" method based on the fact that the isosceles triangles $\triangle OAB$, $\triangle ABe$, $\triangle BEF, \dots$ are similar.

an approximation) by taking the coefficients like $1/(4 \cdot 3!)$ to be 0.16666666 ... etc. With this formula in hand, Ming Antu went on to demonstrate the other eight formulae [Cf. Li and Du 1987, pp. 234-39; Jami 1988].

Coincidentally, Dong Youcheng also applied the method of "Lian Bi Li" (repeated or continued proportions) to the line segments inside the circle [Cf. Figure 13] to arrive at the following four formulae (A), (B), (C) and (D):

$$c_n = n c - \frac{n(n^2-1)}{4 \cdot 3!} \frac{c^3}{r^2} + \frac{n(n^2-1)(n^2-9)}{4^2 \cdot 5!} \frac{c^5}{r^4} - \dots \quad (A)$$

$$b_n = n^2 b - \frac{n^2(n^2-1)}{4!} \frac{(2b)^2}{r} + \frac{n^2(n^2-1)(n^2-4)}{6!} \frac{(2b)^3}{r^2} - \dots \quad (B)$$

where c_n is the chord subtending a given arc which has been subdivided into n equal smaller arcs; and c is the chord subtending the $(1/n)$ th arc. On the other hand, the "sagitta" b_n corresponds to the chord c_n and its subtending arc, and the smaller sagitta b corresponds to the smaller chord c and its subtending smaller arc. Moreover, if $c_{1/m}$, $b_{1/m}$ denote the chord and sagitta corresponding the smaller $(1/m)$ th of the given arc, then converting the series (A) and (B) above, Dong Youcheng obtained the other two formulae relating c , $c_{1/m}$ and b , $b_{1/m}$ respectively:

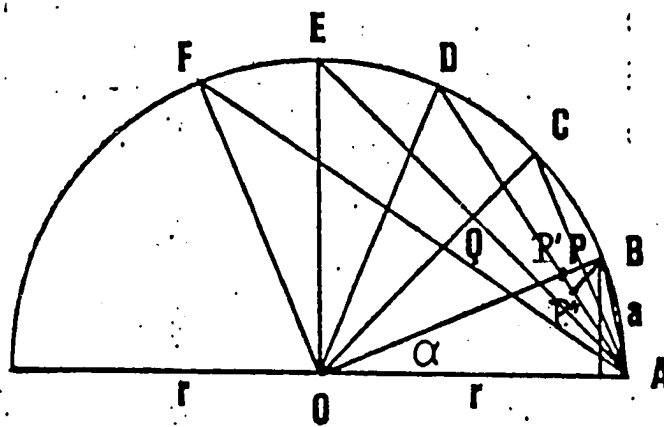


Figure 13. $r_1 = r$, the radius of the circle.

$r_2 = AB$, the chord subtending the arc $AB = a$.

$r_3 = 2 BP = BP'$, where BP is the sagitta
perpendicular to the chord AC .

$r_4 = P'P''$, where $BP' = BP''$.

....

Note that the following isosceles triangles are
all similar: $\triangle OAB$, $\triangle ABP'$, $\triangle BP'P''$, ...

$$c_{1/m} = \frac{c}{m} + \frac{(m^2-1)}{4 \cdot 3! m^3} \frac{c^3}{r^2} + \frac{(m^2-1)(9m^2-1)}{4^2 \cdot 5! m^5} \frac{c^5}{r^4} + \dots \quad (C)$$

$$b_{1/m} = \frac{b}{m^2} + \frac{(m^2-1)}{4! m^4} \frac{(2b)^2}{r} + \frac{(m^2-1)(4m^2-1)}{6! m^6} \frac{(2b)^3}{r^2} + \dots \quad (D)$$

Dong Youcheng called these four formulae "the source of the method", since by letting n tend to be infinite, the above formulae (A), (B), (C), (D) lead to Ming Antu's formulae (4), (8), (5), (9), respectively.

More important is that Dong Youcheng discovered an analogy between the laws governing the coefficients of c_n , b_n and those governing the summations of piles. For instance, Dong Youcheng noted that a general form of (A), c_{2n-1} , could be expressed as follows:

$$\begin{aligned} c_{2n-1} = & [1 \cdot r_2 - \frac{1}{2!} (n-1)n \cdot r_4 + \frac{1}{4} (n-2)(n-1)n(n+1) \cdot r_6 \\ & - \frac{1}{6!} (n-3)(n-2)(n-1)n(n+1)(n+2) \cdot r_8 + \dots] \\ & + 2 \cdot [\frac{1}{1!} (n-2) \cdot r_2 - \frac{1}{3!} (n-3)(n-2)(n-1)n(n+1) \cdot r_6 \end{aligned}$$

$$\begin{aligned}
& + \frac{1}{7!} (n-4)(n-3)(n-2)(n-1)n(n+1)(n+2) \cdot r_8 + \dots] \\
& = \left[\sum^n 1 + \sum^{n-1} 1 \right] \cdot r_2 - \left[\sum^{n-1} \frac{1}{2!} r(r+1) + \sum^{n-2} \frac{1}{2!} r(r+1) \right] \cdot r_4 \\
& + \left[\sum^{n-2} \frac{1}{4!} r(r+1)(r+2)(r+3) + \sum^{n-3} \frac{1}{4!} r(r+1)(r+2)(r+3) \right] \cdot r_6 \\
& - \left[\sum^{n-3} \frac{1}{6!} r(r+1)(r+2)(r+3)(r+4)(r+5) \right. \\
& \quad \left. + \sum^{n-4} \frac{1}{6!} r(r+1)(r+2)(r+3)(r+4)(r+5) \right] \cdot r_8 + \dots
\end{aligned}$$

While for a general term of (B), b_{2n} , Dong Youcheng gave its expansion as follows:

$$\begin{aligned}
b_{2n} &= [n \cdot r_3 - \frac{1}{3!} (n-1)n(n+1) \cdot r_5 + \frac{1}{5!} (n-2)(n-1)n(n+1)(n+2) \cdot r_7 \\
& - \frac{1}{7!} (n-3)(n-2)(n-1)n(n+1)(n+2)(n+3) \cdot r_9 + \dots] \\
& + 2 \cdot \left[\frac{1}{2!} (n-1)n \cdot r_3 - \frac{1}{4!} (n-2)(n-1)n(n+1) \cdot r_5 \right. \\
& \quad \left. + \frac{1}{6!} (n-3)(n-2)(n-1)n(n+1)(n+2) \cdot r_7 \right. \\
& \quad \left. - \frac{1}{8!} (n-4)(n-3)(n-2)(n-1)n(n+1)(n+2)(n+3) \cdot r_9 + \dots \right]
\end{aligned}$$

$$\begin{aligned}
&= \left[\sum_{k=1}^n \frac{1}{k!} r + \sum_{k=1}^{n-1} \frac{1}{k!} r \right] \cdot r_3 - \left[\sum_{k=1}^{n-1} \frac{1}{3!} r(r+1)(r+2) \right. \\
&\quad \left. + \sum_{k=1}^{n-2} \frac{1}{3!} r(r+1)(r+2) \right] \cdot r_5 + \left[\sum_{k=1}^{n-2} \frac{1}{5!} r(r+1)(r+2)(r+3)(r+4) \right. \\
&\quad \left. + \sum_{k=1}^{n-3} \frac{1}{5!} r(r+1)(r+2)(r+3)(r+4) \right] \cdot r_7 \\
&\quad + \left[\sum_{k=1}^{n-3} \frac{1}{7!} r(r+1)(r+2)(r+3)(r+4)(r+5)(r+6) \right. \\
&\quad \left. + \sum_{k=1}^{n-4} \frac{1}{7!} r(r+1)(r+2)(r+3)(r+4)(r+5)(r+6) \right] \cdot r_9 + \dots
\end{aligned}$$

Among these expressions, r_n , $n = 2, 3, 4, \dots$ was called the n -th "Lu" which is the length of the corresponding line segment inside the circle [Cf. Figure 13]. The coefficient of each "Lu" deserves special attention, since it is right here that Dong Youcheng linked the "Ge Yuan" method (method of circle measurement) with the "Duo Ji" method (method of summing piles). Each of the coefficients of $r_2, r_3, r_4, r_5 \dots$ is indeed a sum of two piles of the same type [Li Zhaohua 1987]. This represented a new approach to the study of circle measurement, and Xiang Mingda used it immediately to achieve new results [Qian 1983, 309-312]. Yet unlike Dong Youcheng, Xiang Mingda's results do not seem to have impressed Li Shanlan, who actually praised Dong Youcheng (in the "Preface" of his own *Duo Ji Bi Lei*) for his insight in linking the "Ge Yuan" method and the "Duo Ji" method. In fact, Dong Youcheng may have influenced Li Shanlan in his study of the "Duo Ji"

method, which is the main topic of Section 3.5. Nevertheless, Dong Youcheng's remarkable approach to the study of the circle measurement, especially linking it with the "Duo Ji" method, apparently had made little influence on Li Shanlan.

Instead of Ming Antu or Dong Youcheng's method of "Ge Yuan Lian Bi Li", Li Shanlan used the "Jian Zhui" method to obtain power series for trigonometric and inverse-trigonometric functions:

$$\begin{aligned} \text{i) } \alpha &= \sin \alpha + \frac{1}{3!} \sin^3 \alpha + \frac{1 \cdot 3}{2 \cdot 4 \cdot 5} \sin^5 \alpha \\ &+ \frac{1 \cdot 3 \cdot 5}{2 \cdot 4 \cdot 6 \cdot 7} \sin^7 \alpha + \dots; \end{aligned}$$

$$\text{ii) } \alpha = \tan \alpha - \frac{1}{3} \tan^3 \alpha + \frac{1}{5} \tan^5 \alpha - \frac{1}{7} \tan^7 \alpha + \dots;$$

$$\text{iii) } \alpha^2 = \sec^2 \alpha - \frac{6}{9} \sec^4 \alpha + \frac{46}{90} \sec^6 \alpha - \frac{44}{105} \sec^8 \alpha + \dots;$$

$$\text{iv) } \alpha^2 = 2 \text{ vers } \alpha + \frac{1}{12} (2 \text{ vers } \alpha)^2 + \frac{1}{90} (2 \text{ vers } \alpha)^3 + \dots;$$

$$\text{v) } \sin \alpha = \alpha - \frac{1}{3!} \alpha^3 + \frac{1}{5!} \alpha^5 - \frac{1}{7!} \alpha^7 + \dots;$$

$$\text{vi) } \text{vers } \alpha = \frac{1}{2!} \alpha^2 - \frac{1}{4!} \alpha^4 + \frac{1}{6!} \alpha^6 - \frac{1}{8!} \alpha^8 + \dots;$$

$$\text{vii) } \tan \alpha = \alpha + \frac{1}{3} \alpha^3 + \frac{2}{15} \alpha^5 + \frac{17}{315} \alpha^7 + \dots;$$

$$\text{viii) } \sec \alpha = 1 + \frac{1}{2} \alpha^2 + \frac{5}{24} \alpha^4 + \frac{61}{720} \alpha^6 + \dots,$$

where α is the arc AOQ, a quarter of a unit circle with center O
[Cf. Figure 14].

To illustrate how Li Shanlan applied his "Jian Zhui" method to obtain these expansions, it suffices to understand how his argument was applied in the case of formula (i). In Figure 14, set $\alpha = \angle AOQ$, and $x = \sin \alpha = OE = AF$, then (based on an argument similar to that in Section 3.2), the area of the "Jian Zhui" AFQ is:

$$S = \frac{1}{2} \frac{x^3}{3} + \frac{1}{2 \cdot 4} \frac{x^5}{5} + \frac{3}{2 \cdot 4 \cdot 6} \frac{x^7}{7} + \frac{3 \cdot 5}{2 \cdot 4 \cdot 6 \cdot 8} \frac{x^9}{9} + \dots$$

On the other hand, since for areas,

$$\text{sector AOQ} = \text{rectangle OAFE} - \text{triangle OQE} - \text{"Jian Zhui" AFQ},$$

it follows that

$$\frac{1}{2} \alpha^2 = x \cdot 1 - \frac{1}{2} x \sqrt{1-x^2} - S$$

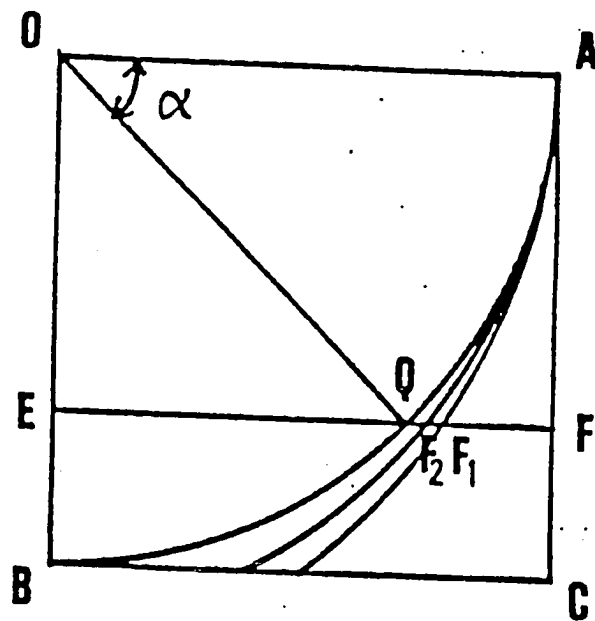


Figure 14. $\alpha = \angle AOQ$. The arc AQB is one quarter of a unit circle with $OA = OB = 1$ as radius and O as the center. $OACB$ is a square circumscribed the one-quarter of the unit circle.

$$\Rightarrow \alpha = x (2 - \sqrt{1-x}) - 2 \cdot S$$

$$= x + \frac{1}{2} x^3 + \frac{1}{2 \cdot 4} x^5 + \frac{3}{2 \cdot 4 \cdot 6} x^7 + \dots$$

$$- \left(\frac{1}{2} \frac{x^3}{3} + \frac{1}{2 \cdot 4} \frac{x^5}{5} + \frac{1 \cdot 3}{2 \cdot 4 \cdot 6} \frac{x^7}{7} + \dots \right)$$

$$\Rightarrow \alpha = x + \frac{1}{3!} x^3 + \frac{1 \cdot 3}{2 \cdot 4 \cdot 5} x^5 + \frac{1 \cdot 3 \cdot 5}{2 \cdot 4 \cdot 6 \cdot 7} x^7 + \dots$$

$$\Rightarrow \alpha = \sin \alpha + \frac{1}{3!} \sin^3 \alpha + \frac{1 \cdot 3}{2 \cdot 4 \cdot 5} \sin^5 \alpha + \frac{1 \cdot 3 \cdot 5}{2 \cdot 4 \cdot 6 \cdot 7} \sin^7 \alpha + \dots$$

[Qian 1983, p. 320; Wang Yusheng 1983].

With this formula in hand, Li Shanlan went on to demonstrate formula (iv). He replaced α by $\alpha/2$ in formula (i), whereby

$$\frac{\alpha}{2} = \sin \frac{\alpha}{2} + \frac{1}{3!} \sin^3 \frac{\alpha}{2} + \frac{1 \cdot 3}{2 \cdot 4 \cdot 5} \sin^5 \frac{\alpha}{2} + \dots$$

Multiply each side by itself respectively, and one has the following:

$$\left(\frac{\alpha}{2} \right)^2 = \left(\sin \frac{\alpha}{2} + \frac{1}{3!} \sin^3 \alpha + \frac{1 \cdot 5}{2 \cdot 4 \cdot 5} \sin^5 \alpha + \dots \right)^2$$

$$\Rightarrow \alpha^2 = 4 \sin^2 \frac{\alpha}{2} + \frac{1}{12} (4 \sin^2 \alpha)^2 + \frac{1}{90} (4 \sin^2 \alpha)^3 + \dots$$

Since $2 \text{ vers } \alpha = 4 \sin^2 \frac{\alpha}{2}$, Li Shanlan thus obtained the desired formula (iv).

As for formula (v), Li Shanlan used a straightforward conversion of series to transform formula (i) into formula (v). In fact, Li Shanlan converted the series of formula (v) back to that of formula (i) in the treatise, *Ji Shu Hui Qui* ("Conversions of Infinite Series"). Again using the conversion of series, he also derived formula (vi) based on formula (iii).

On the other hand, substituting the series of $\sin \alpha$ (as in formula (v)) and that of $\text{vers } \alpha$ (as in formula (vi)) results in the following identity:

$$\tan \alpha = \frac{\sin \alpha}{1 - \text{vers } \alpha},$$

and the method of long division, then Li Shanlan eventually got formula (vii). Similarly, based on the following trigonometric identity:

$$\sec \alpha - 1 = \frac{\text{vers } \alpha}{1 - \text{vers } \alpha},$$

Li Shanlan, again used the method of long division to arrive at

formula (viii). With formula (vii) in hand, Li Shanlan was able to demonstrate formula (ii) by the conversion of series.

Finally, for the demonstration of formula (iii), Li Shanlan created "San Shu" (literally three methods), and thereby established formula (iii) and two other variant forms. Multiplying each side of formula (ii) by itself respectively, and using the relationship that $t = 2 (\sec \alpha - 1) + (\sec \alpha - 1)^2 = \tan^2 \alpha$, Li Shanlan obtained the following series expansion for α^2 :

$$\alpha^2 = t - \frac{2}{3} t^2 + \frac{23}{45} t^3 - \frac{34}{105} t^4 + \dots$$

This was what Li Shanlan called "Yi Shu" (first method) on the ground that $\sec \alpha - 1$ was given. Multiplying each side of formula (ii) respectively by itself, substituting first α by $\alpha/2$, then using the relationship that

$$u = \frac{4(\sec \alpha - 1)}{2 + (\sec \alpha - 1)} = 4 \tan^2 \frac{\alpha}{2},$$

Li Shanlan eventually got his "You Shu" (second method):

$$\alpha^2 = u - \frac{2}{3} \frac{u^2}{4} + \frac{23}{45} \frac{u^3}{4^2} - \frac{34}{105} \frac{u^4}{4^3} + \dots$$

Now let $\sec \alpha - 1 = x$, substituting

$$\begin{aligned}
 t &= \tan^2 \frac{\alpha}{2} - 1 = \sec^2 \alpha - 1 = [2 + (\sec \alpha - 1)](\sec \alpha - 1) \\
 &= 2x + x^2
 \end{aligned}$$

into the formula for the "Yi Shu", Li Shanlan got the following expansion for α^2 :

$$\alpha^2 = 2x - \frac{5}{3}x^2 + \frac{64}{45}x^3 - \frac{26}{21}x^4 + \dots$$

Finally, replacing x by $\sec \alpha - 1$, Li Shanlan obtained the formula for his "San Shu" (third method), which was the very formula (ii) shown above [Wang Yusheng 1983; Li 1867].

Formulae (ii) and (viii) were actually Li Shanlan's original discoveries in the study of series expansions for trigonometric and inverse trigonometric functions. Alexander Wylie was greatly impressed when he read Li Shanlan's *Hu Shi Qi Mi* in the early 1850s. According to Wylie, these two formulae linking α and $\sec \alpha$ were discovered by Li Shanlan earlier than his fellow mathematicians like Dai Xu or Xu Youren [Wylie 1867]. In fact, according to Wang Yusheng, Li Shanlan's formulae concerning the relationship between $\tan \alpha$ and α appeared even earlier than Dai Xu and Xu Youren's [Wang Yusheng 1983].

In conclusion, a comparison between Ming Antu and Li Shanlan provides some interesting insights. On the one hand, both of them applied the method of converting infinite series, yet Li Shanlan

was much more concerned about this subject and actually wrote a treatise on the conversion of series, namely the *Ji Shu Hui Qiu*. Exactly when this treatise was written is unknown. Nevertheless, Li Shanlan drew upon terms and symbols from the *Dai Shu Xue* (Chinese edition of De Morgan's *Elements of Algebra*, 1859) in preparing the 1867 edition. Therefore, if Li Shanlan had written the *Ji Shu Hui Qui* ("Conversions of Infinite Series") about the same time as his completion of the *Hu Shi Qi Mi*, namely in 1845, then the 1867 edition of the *Ji Shu Hui Qui* must have been a revised version.

On the other hand, in dealing with series expansions of the trigonometric and inverse trigonometric functions, Li Shanlan's "Jian Zhui" method was a sharp contrast to Ming Antu's "Lian Bi Li" method. In other words, Li Shanlan's method was basically algebraic (despite his reliance upon geometrical analogies to interpret x^n , $n = 4, 5, 6, \dots$ [Cf. Section 3.2]). Whereas Ming Antu's concept of "Lian Bi Li" (repeated or continued proportions) was concerned with the related line segments inside a circle. Besides, Li Shanlan still used the method of long division to expand some quotients like $\frac{\sin \alpha}{1 - \text{vers } \alpha}$ into "infinite polynomials", to quote Issac Newton's term. Putting all these observations together, it seems clear certain that Li Shanlan was dealing some new methods equivalent in principle to those of the calculus. More evidences for this claim is given in the following sections, yet for now let us first turn to Li Shanlan's *Dui Shu Tan Yuan*.

3.4. "Jian Zhui" Method and the *Dui Shu Tan Yuan*

According to Zhang Yong, Li Shanlan's *Dui Shu Tan Yuan* is the major treatise among his collected mathematical works [Zhang Yong 1939]. Since it was based primarily upon the theories and methods of the *Fang Yuan Chan You* and *Hu Shi Qi Mi*, these must be considered first, apparently just as Li Shanlan Li Shanlan had done in the early 1840s. Thus he was so proud to claim that his method was "ten thousand times easier than the methods used by Europeans," [Quoted from Li Shanlan's "Preface" to the *Dui Shu Tan Yuan*]. "This small indication of self-satisfaction," as Alexander Wylie put it, "may be very well overlooked, as quite pardonable in one who has had no better aid than that afforded by the *Leuh-lih-yuen-yuen*, and who has here given us, as the result of four years' thought, a theorem, which in the days of Briggs and Napier, would have been sufficient to raise him to distinction, " [Quoted Wylie 1897, p. 194].

Wylie indeed had made a fair evaluation of Li Shanlan's study of logarithms. Since by 1840, what had been transmitted to China about Western logarithms was not only fragmentary but elementary as well. As mentioned in Section 1.2, Napier's logarithms was brought by the Polish Jesuit Nicolas Smogulescki (1611-1656) to China. However, he only introduced six-figure tables of logarithms from 1 to 10,000 (in the *Bi Li Dui Shu Biao* = "Logarithm Tables with Explanations"), but said nothing about how

they were made [Li and Du 1987, 208-209]. It was not until 1723 that the *Shu Li Jing Yun* ("Collected Basic Principles of Mathematics") reintroduced Western logarithms again. This time, however, introduced three methods in order to make a table of common logarithms from 1 to 100,000 [Book 38 of the *Shu Li Jing Yun*; Li and Du 1987, pp. 220-221].

As mentioned in Section 1.4, the *Shu Li Jing Yun* was (among the other two treatises, namely *Li Xiang Kao Cheng* and *Lu Lu Zheng Yi*) collected in the voluminous *Lu Li Yuan Yuan* (literally "Ocean of Calendrical Science and Astronomy") under the Emperor Kang Xi's supervision, beginning in 1712 (the *Lu Li Yuan Yuan* was actually the same as the *Leuh-lih-yuen-yuen* mentioned by Wylie [See the above quotation]). Thus Li Shanlan's self-satisfaction should not be regarded as exaggerated even by modern standards, since he had created a method entirely his own, namely the "Jian Zhui" method, in his study of the theories and methods of logarithms.

From the very beginning of Book 1 of his *Dui Shu Tan Yuan*, Li Shanlan claimed as a fundamental principle that "Logarithms, just like the difference of the unit square and its inscribed one-quarter of a unit circle, can be expressed an accumulation of the 'Jian Zhui'." In fact, the title of Book 1, "Ming Li" meant literally "understanding the principles". As an illustration, Li Shanlan argued with the aid of a diagram (as shown in Figure 15) that the natural logarithm of some positive number could be computed by summing the areas of a rectangle (with base b and

height h) and the following "Jian Zhui":

1-st "Jian Zhui" CBE with area $\frac{1}{2}bh$,

2-nd "Jian Zhui" CEF with area $\frac{1}{3}bh$,

3-rd "Jian Zhui" CFG with area $\frac{1}{4}bh$,

...

where $AD = h$, and $AB = BE = EF = FG = \dots = b$, as shown in Figure 15 [Cf. Wang Yusheng in Mei Rongzhao ed. 1990, p. 368]. Yet how could such a demonstration be related to natural logarithms? In fact, as Li Shanlan argued, if one were to choose an arbitrary point P on $\overline{AD}$, and then draw a line PQ parallel to AB from which P intersects the (straight or curved) lines CE , CF , CG , ..., at the points E' , F' , G' , ..., and Q , respectively, then by setting $y = DP$,

$$PE' = b, E'F' = \frac{by}{h}, F'G' = \frac{by^2}{h^2}, G'H' = \frac{by^3}{h^3}, \dots$$

[Cf. Theorem 6 of the *Dui Shu Tan Yuan*] and that the length of $\overline{PQ}$:

$$PQ = b + \frac{by}{h} + \frac{by^2}{h^2} + \frac{by^3}{h^3} + \dots$$

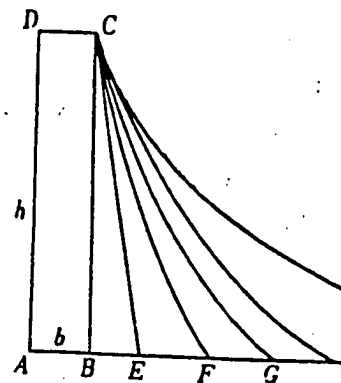
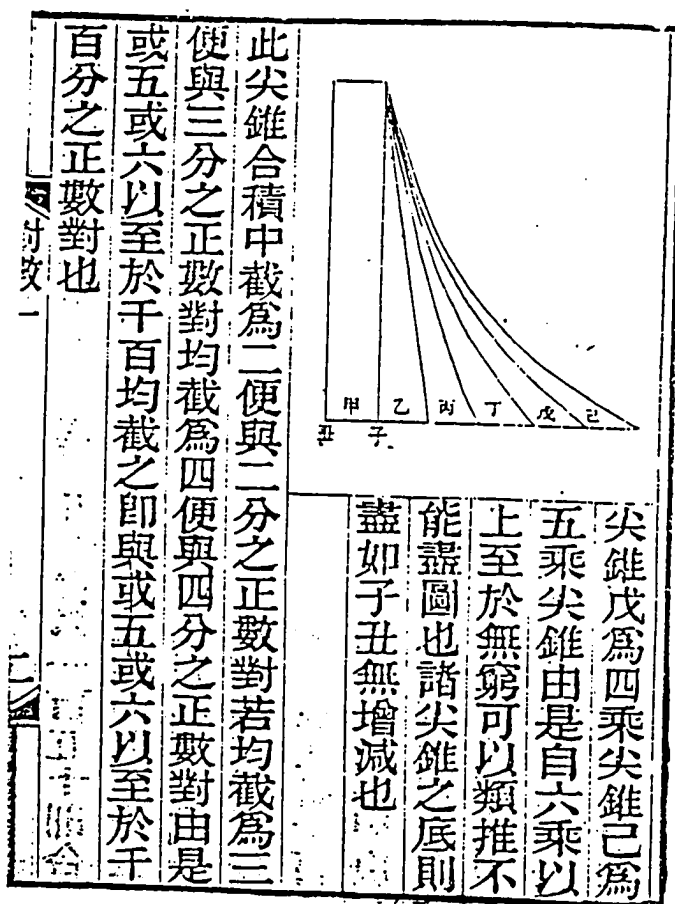


Figure 15. Li Shanlan's "Jian Zhui He Ji" and its duplicate.

$$= \frac{bh}{h-y},$$

since $0 < y < h$ [Cf. Figure 16]. Thus the point Q should lie on the hyperbolic curve $x(h-y) = bh$, an interesting fact that apparently escaped Li Shanlan's eyes [Qian Baocong 1981, pp. 321-322].

Nevertheless, based on the fact that finding the base length of the "Jian Zhui" with height y had already been discussed in the *Fang Yuan Chan You* [Cf. Section 3.2, especially Figure 5 there], Li Shanlan understood quite clearly that the area of the region DPQC (or as he called it, the "Jie Ji"), namely,

$$S(y) = by + \frac{by^2}{2h} + \frac{by^3}{3h^2} + \frac{by^4}{4h^3} + \dots = by \sum_{k=0}^{\infty} \frac{y^k}{(k+1)h^k},$$

had properties in common with those of the logarithms. Thus if $y_1, y_2, y_3, \dots$ constitute a geometric progression, then their corresponding $S(y_1), S(y_2), S(y_3), \dots$ formed an arithmetic progression [Cf. Theorem 4 of the *Dui Shu Tan Yuan*]. In fact, by setting $bh = 1$, and $y = \frac{n-1}{n} h$, then one obtains

$$S\left(\frac{n-1}{n} h\right) = \frac{n-1}{n} + \frac{1}{2} \left(\frac{n-1}{n}\right)^2 + \frac{1}{3} \left(\frac{n-1}{n}\right)^3 + \dots$$

This sum, called the "Fan Ji" of n by Li Shanlan, is the natural logarithm of n , since the above sum is nothing but:

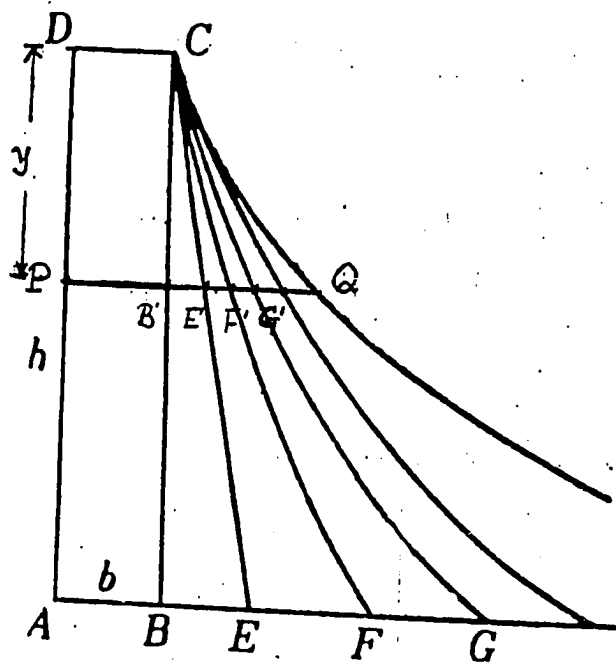


Figure 16. A "He Jian Zhui" is divided by a line
PQ parallel to the base line AB.

$$\int_0^{\frac{n-1}{n}h} \frac{1}{h-y} dy = \log_e h - \log_e \left(h - \frac{n-1}{n}h\right) = \log_e n$$

[Qian Baocong 1981, p. 322].

Based on his fundamental principle and the ten theorems listed in Book 1 of the *Dui Shu Tan Yuan*, Li Shanlan derived his series expansion for the natural logarithms. At the outset, he does not seem to have been concerned with the validity or rigor of his "Jian Zhui" method. But if the method of "Jian Zhui" proved successful in his treatise *Fang Yuan Chan You*, then why should he have hesitated to make further applications to the study of logarithms? Li Shanlan did pay considerable attention to the problem of rigor due in part to his encounter with a method for expanding logarithmic functions into infinite series introduced in the *Dai Shu Xue* (De Morgan's *Algebra*). Thus he should have been content with proving that his infinite series expansion for natural logarithms --

$$\log_e n = \log_e (n-1) + bh \left(\frac{1}{n} + \frac{1}{2n^2} + \frac{1}{3n^3} + \dots \right) \quad (*)$$

was indeed equivalent to the series expansion drawn from Chapter 12 of the *Dai Shu Xue* (De Morgan's *Algebra*):

$$\log_e n = \log_e (n-1)$$

$$+ 2bh \left[\frac{1}{2n-1} + \frac{1}{3(2n-1)^3} + \frac{1}{5(2n-1)^5} + \dots \right]. \quad (**)$$

Li Shanlan gave his proof of this in his *Dui Shu Jian Zhui Bian Fa Shi* ("On the Equivalence of the Western and the 'Jian Zhui' Methods for Logarithms") which might not appear until 1859, since in the same year in which the *Dai Shu Xue* had been published by the LMS Press at Shanghai.

On the other hand, Li Shanlan's synthesis of the Chinese and Western methods might also have been responsible for such sort of proof. Whatever the case, Li Shanlan's argument revealed how his method of "Jian Zhui" had been applied. By reading the *Dui Shu Jian Zhui Bian Fa Shi* carefully, Wang Yusheng has suggested a rational reconstruction which is described in the following paragraphs [Cf. Wang Yusheng 1983b and 1990].

According to Wang Yusheng, one key to understanding how Li Shanlan dealt with logarithms by his method of "Jian Zhui" was to be found in the three lemmas given in the *Dui Shu Jian Bian Fa Shi* [Wang Yusheng in Mei Rongzhao ed. 1990, p. 370]. In Lemma 1, Li Shanlan gave an interesting method to compute the area of the n -th (or n "Cheng") "Jian Zhui", $n = 1, 2, 3, \dots$, respectively. For the 1-st "Jian Zhui" or "Pian Jian Zhui" or a right-angled triangle with base b and height h [Cf. Figure 17a], Li Shanlan noted that its area, namely,

$$S_1 = \frac{1}{2} bh,$$

was equal to

$$S_1 = S_1' + S_1'',$$

where

$$S_1' = \frac{bh}{2 \cdot 2^2}; \quad S_1'' = \frac{bh}{2^2} + \frac{bh}{2 \cdot 2^2}.$$

Similarly, for the 2-nd "Jian Zhui" or the "Li Jian Zhui" or the "Yang Ma" (a pyramid with an edge of height h perpendicular to the rectangular base of side b and 1), Li Shanlan also discovered that its volume, namely,

$$S_2 = \frac{bh}{3},$$

could be expressed as a sum of S_2' and S_2'' , where

$$S_2' = \frac{bh}{2 \cdot 2^3}; \quad S_2'' = \frac{bh}{2^3} + 2 \cdot \frac{bh}{2 \cdot 2^3} + \frac{bh}{3 \cdot 2^3},$$

by bisecting the length b , breadth 1 and height h of the "Yang Ma" respectively [Cf. Figure 17b], a way reminiscent of Liu Hui's derivation on the volume of the "Yang Ma" [Cf. Wagner 1979].

Arguing analogously, Li Shanlan eventually was able to express the "area" of the n -th "Jian Zhui", namely, $S_n = \frac{bh}{n+1}$ [Cf. Figure 17c], as a sum of S'_n , and S''_n , where

$$S'_n = \frac{bh}{(n+1)2^{n+1}},$$

$$S''_n = \binom{n}{0} \frac{bh}{2^{n+1}} + \binom{n}{1} \frac{bh}{2 \cdot 2^{n+1}} + \binom{n}{2} \frac{bh}{3 \cdot 2^{n+1}} \\ + \dots + \binom{n}{n} \frac{bh}{(n+1)2^{n+1}},$$

and $\binom{n}{k} = \frac{n!}{(n-k)! \cdot k!}$, $k = 0, 1, 2, \dots, n$, were drawn from the $(n+1)$ -th row of the "Lian Fa Biao" -- a name which Li Shanlan called Jia Xian's "Kai Fang Zo Fa Ben Yuan Tu" (Tabulation System for Unlocking Binomial Coefficients) or the Chinese "Pascal Triangle". Li Shanlan's reasoning sounds plausible, since

$$S''_n = S_n - S'_n = \frac{bh(2^{n+1}-1)}{(n+1)2^{n+1}} \\ = \frac{bh}{2^{n+1}} \sum_{k=0}^n \frac{1}{n+1} \binom{n+1}{k+1} = \frac{bh}{2^{n+1}} \sum_{k=0}^n \frac{1}{k+1} \binom{n}{k}$$

[Wang Yusheng in MeiRongzhao 1990, pp. 370-371].

In Lemma 2, Li Shanlan dealt with the problem of how, conversely, the complementary terms of the n -th "Jian Zhui" could

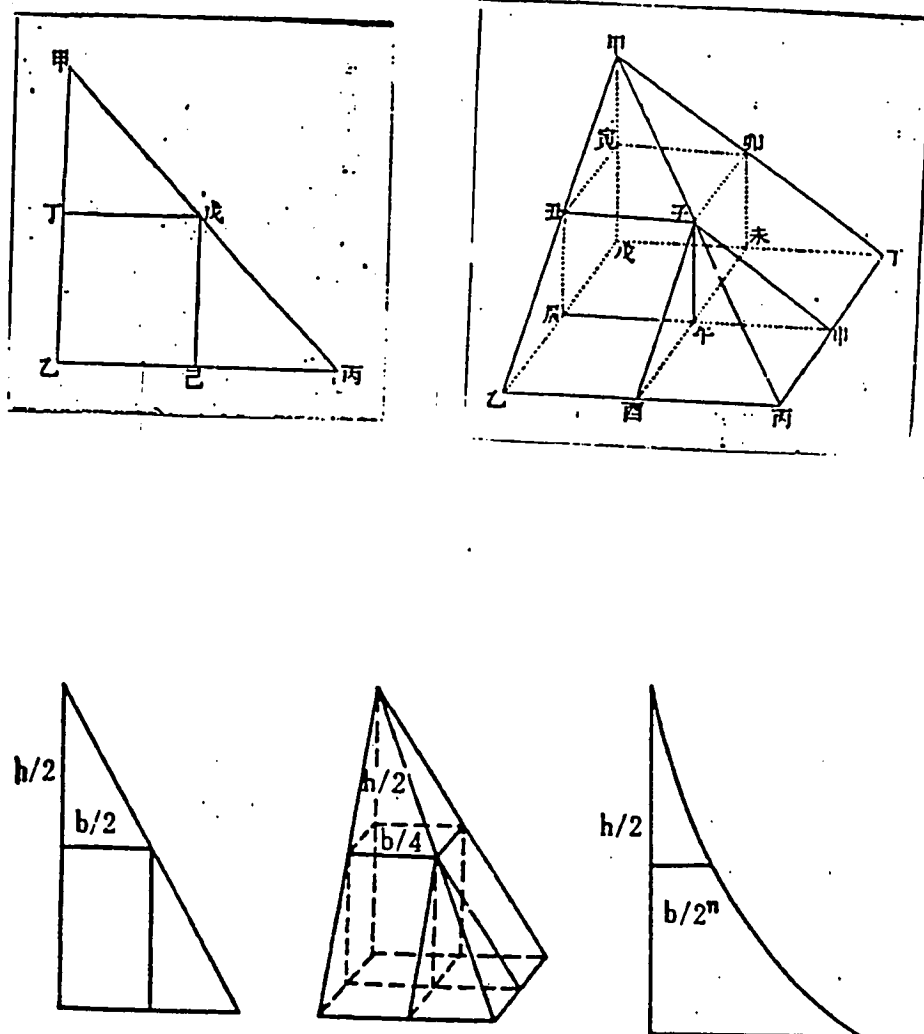


Figure 17. Two diagrams xeroxed from the *Dui Shu Jian Zhui Bian Fa Shi* and its duplicates. The last one is our illustration for the case with the n -th "Jian Zhui".

be added to make a rectangle. Argued from the cases with 1-st and 2-nd "Jian Zhui" [Cf. Figure 18], Li Shanlan noted that the area of the rectangle with base b and height h , namely, bh , could be expressed as follows:

$$bh = \frac{bh}{n+1} + \binom{n}{1} \frac{bh}{2} - \binom{n}{2} \frac{bh}{3} + \dots + (-1)^n \frac{bh}{n+1},$$

where the coefficient of each term was again drawn from the "Lian Fa Biao" along with an appropriate sign. Needless to say, Li Shanlan was just as right with in the general case, for

$$\begin{aligned} & bh \left[\frac{1}{n+1} + \sum_{k=1}^n \frac{(-1)^{k+1}}{k+1} \binom{n}{k} \right] \\ &= bh \left[\frac{1}{n+1} + \sum_{k=1}^n \frac{(-1)^{k+1}}{n+1} \binom{n+1}{k+1} \right] \\ &= \frac{bh}{n+1} \left[1 + \sum_{k=0}^{n+1} (-1)^k \binom{n+1}{k} - 1 + (n+1) \right] \\ &= bh. \end{aligned}$$

In Lemma 3 Li Shanlan first divided a "He Jian Zhui" (accumulated "Jian Zhui"), of height h and base b into two layers by a line which was, at a distance $\frac{h}{p}$ from the vertex of the "He Jian Zhui" and parallel to its base line [Cf. Figure 19]. Then the upper layer was a small "He Jian Zhui" of height $\frac{h}{p}$. The area of this small "He Jian Zhui" or the "Jie Ji" as Li Shanlan

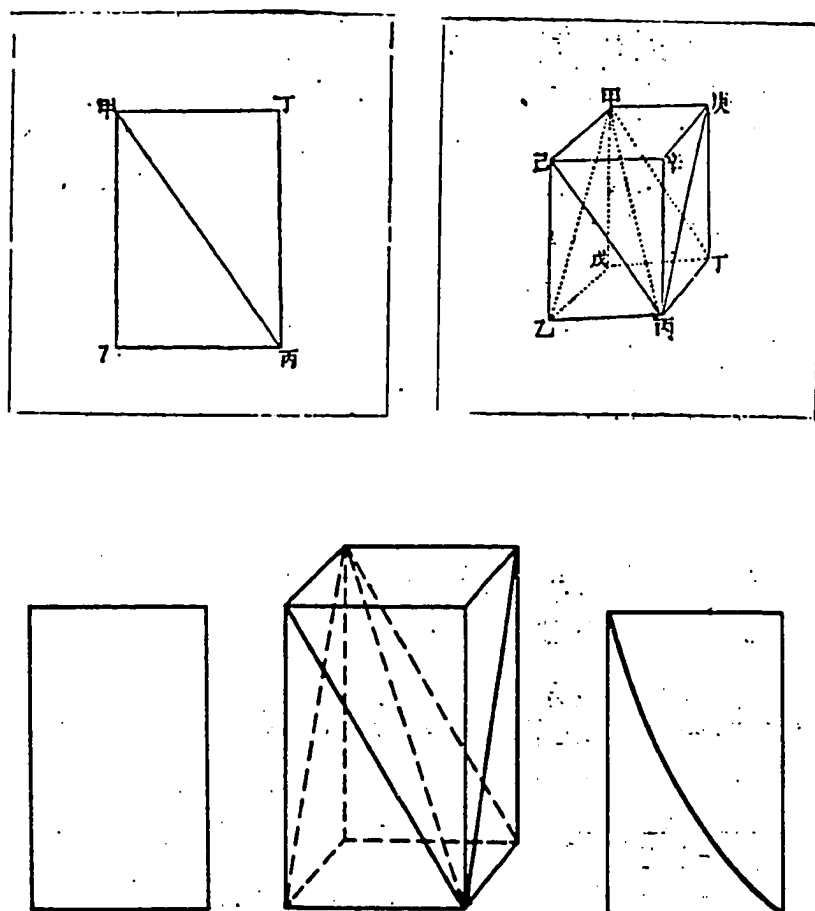


Figure 18. Two diagrams xeroxcopied from the *Dui Shu Jian Zhui Bian Fa Shi* and their duplicates. The last one is our illustration for the case with the n -th "Jian Zhui".

called it is:

$$S\left(\frac{h}{p}\right) = \sum_{k=0}^{\infty} \frac{1}{k+1} \cdot \frac{h}{p} \cdot \frac{b}{p^k} = bh \sum_{k=0}^{\infty} \frac{1}{(k+1)p^{k+1}}.$$

The above computation follows from the most important fact underlying Li Shanlan's "Jian Zhui" method (already discussed in Section 3.2) [Cf. also Figure 5], namely, that the k -th "Jian Zhui" of height $\frac{h}{p}$ has the base $\frac{b}{p^k}$. If one further divides, as Li Shanlan did, the small "He Jian Zhui" at a distance $\frac{h}{2p}$ from its vertex, then one again obtains two layers of "He Jian Zhui". Among these two layers, the upper one has the area

$$S^u = bh \sum_{k=1}^{\infty} \frac{1}{(k+1)(2p)^{k+1}},$$

while, by Lemma 1, the lower one has the area

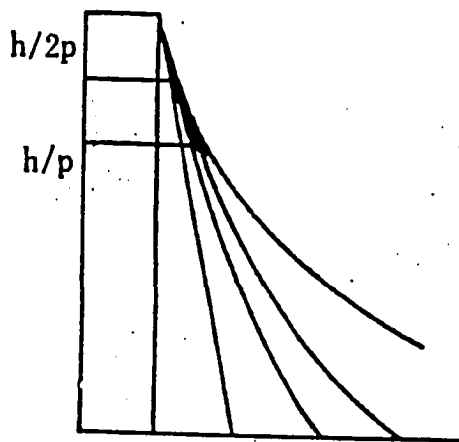


Figure 19.

$$\begin{aligned}
S^1 &= bh \sum_{j=0}^{\infty} \left[\frac{1}{(2p)^{j+1}} \cdot \sum_{k=0}^j \frac{1}{k+1} \binom{j}{k} \right] \\
&= bh \sum_{j=0}^{\infty} \frac{1}{(2p)^{j+1}} \left[\frac{1}{0+1} \binom{j}{0} + \frac{1}{1+1} \binom{j}{1} + \dots + \frac{1}{j+1} \binom{j}{j} \right] \\
&= \frac{1}{(2p)^1} \cdot \frac{1}{0+1} \binom{0}{0} + \frac{1}{(2p)^2} \left[\frac{1}{0+1} \binom{1}{0} + \frac{1}{1+1} \binom{1}{1} \right] \\
&\quad + \frac{1}{(2p)^3} \left[\frac{1}{0+1} \binom{2}{0} + \frac{1}{1+1} \binom{2}{1} + \frac{1}{2+1} \binom{2}{2} \right] \\
&\quad + \dots \\
&\quad + \frac{1}{(2p)^{k+1}} \left[\frac{1}{0+1} \binom{k}{0} + \frac{1}{1+1} \binom{k}{1} + \frac{1}{2+1} \binom{k}{2} + \dots \right. \\
&\quad \quad \left. + \frac{1}{k+1} \binom{k}{k} \right] \\
&\quad + \dots \\
&= bh \sum_{k=0}^{\infty} \frac{1}{k+1} \cdot \left[\sum_{j=k}^{\infty} \frac{1}{(2p)^{j+1}} \binom{j}{k} \right] \\
&= bh \sum_{k=0}^{\infty} \frac{1}{(k+1)(2p-1)^{k+1}} .
\end{aligned}$$

The last equality holds principally because if one sets for $k = 0, 1, 2, 3, \dots, k, \dots$,

$$T_k = \sum_{j=k}^{\infty} \frac{1}{(2p)^{k+1}} \binom{j}{k},$$

then

$$T_0 = \frac{1}{2p-1},$$

and for $k = 1, 2, 3, \dots$,

$$\begin{aligned} T_k &= \sum_{j=k}^{\infty} \frac{1}{(2p)^{j+1}} \binom{j}{k} \\ &= \frac{1}{2p} \left[\sum_{j=k}^{\infty} \frac{1}{(2p)^j} \binom{j-1}{k} + \sum_{j=k}^{\infty} \frac{1}{(2p)^j} \binom{j-1}{k-1} \right] \\ &= \frac{1}{2p} (T_k + T_{k-1}) \\ \Rightarrow T_k &= \frac{1}{2p-1} \cdot T_{k-1}. \end{aligned}$$

Therefore,

$$T_k = \frac{1}{(2p-1)^{k+1}}, \quad k = 1, 2, 3, \dots,$$

as desired.

With these lemmas in hand, Li Shanlan eventually showed that his infinite series expansion (*) for $\log_e n$ was equivalent to that

(**) given in De Morgan's *Algebra*. It was sufficient, as Li Shanlan appropriately argued, to show that

$$bh \sum_{k=0}^{\infty} \frac{1}{(k+1)(2p)^{k+1}} = 2bh \sum_{k=0}^{\infty} \frac{1}{(2j+1)(2p-1)^{2j+1}}.$$

Now just as in the argument for the proof of Lemma 3,

$$S\left(\frac{h}{p}\right) = bh \sum_{k=0}^{\infty} \frac{1}{(k+1)(2p)^{k+1}} = S^u + S^l.$$

Yet in the lower layer by which $bh \cdot T_0$ was formed, namely,

$$S_0^l = bh \cdot T_0 = bh \sum_{j=0}^{\infty} \frac{1}{(2p)^{j+1}}.$$

Now by Lemma 2,

$$S_0^l = S^{ls} = bh \left[\frac{1}{2p} + \sum_{j=1}^{\infty} \left(\frac{1}{(2p)^{j+1}} \left\{ \frac{1}{j+1} + \sum_{k=1}^j \frac{(-1)^{k+1}}{(k+1)} \binom{j}{k} \right\} \right) \right],$$

where the term S^{ls} was called "Xia Fang Tong Shu" (literally the same area as that of the sum of the rectangles in the lower layer). Finally, since

$$\begin{aligned} S\left(\frac{h}{p}\right) &= S^u + S^l + S_0^l - S^{ls} \\ &= 2bh \sum_{j=0}^{\infty} \left[\frac{1}{2j+1} \sum_{k=0}^{\infty} \frac{1}{(2p)^{2j+1+k}} \binom{2j+k}{2j} \right] \end{aligned}$$

$$= 2bh \sum_{j=0}^{\infty} \frac{1}{(2j+1)(2p-1)^{2j+1}},$$

Li Shanlan concluded that "the two series expansion are equivalent though their methods of derivation are different," [Quoted Li Shanlan's *Dui Shu Jian Zhui Bian Fa Shi*, p. 6].

Armed with the above rational reconstruction, it is now possible to discuss the ten theorems of the *Dui Shu Tan Yuan*. Here (for convenience of discussion), they are given in modern notation:

Theorem 1. Divide a "He Jian Zhui" of height h into p equal layers. For $m \geq 2$, one has the area of the region between the lines AB and A_1B_1 where A, A_1 are the points such that $AD = \frac{p-m}{p} h$, $A_1D = \frac{p-m+1}{p} h$ respectively [Cf. Figure 20],

$$s(m) = bh \sum_{k=0}^{\infty} \frac{1}{(k+1)m^{k+1}},$$

which is independent of the number p .

For the proof, Wang Yusheng suggests that Li Shanlan might have used a method similar to the following:

Proof. Consider the "Jie Jian Zhui" $DABC$ [Cf. Figure 20] that

is the upper layer determined by the line segment AB in which the point A is chosen such that $AD = \frac{p-m+1}{p} h$. Thus for the n-th "Jian Zhui" in the "Jie Jian Zhui", one has the area

$$S_n = \frac{bh}{n+1} \left(\frac{p-m+1}{p} \right)^{n+1}.$$

If one further divides the "Jie Jian Zhui" DABC into two layers by the line segment A_1B_1 in which the point A_1 is chosen such that $A_1D = \frac{p-m}{p} h$, then the n-th "Jian Zhui" S_n is also divided into two layers:

$$S'_n = \frac{bh}{n+1} \left(\frac{p-m}{p} \right)^{n+1},$$

$$S''_n = \frac{bh}{(n+1)p^{k+1}} \left[\sum_{k=0}^{n+1} \binom{n+1}{k} (p-m)^{n-k+1} - (p-m)^{n+1} \right]$$

$$= \frac{bh}{p^{n+1}} \sum_{k=0}^n \frac{1}{n+1} \binom{n+1}{k+1} (p-m)^{n-k}$$

$$= \frac{bh}{p^{n+1}} \sum_{k=0}^n \frac{1}{k+1} \binom{n}{k} (p-m)^{n-k}.$$

Therefore,

$$s(m) = \sum_{n=0}^{\infty} S''_n = \sum_{n=k}^{\infty} \left[\frac{bh}{p^{k+1}} \sum_{k=0}^n \frac{1}{k+1} \binom{n}{k} (p-m)^{n-k} \right]$$

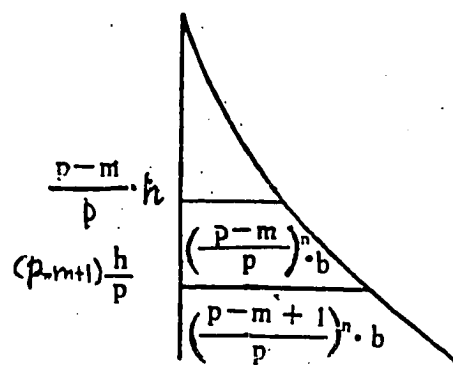
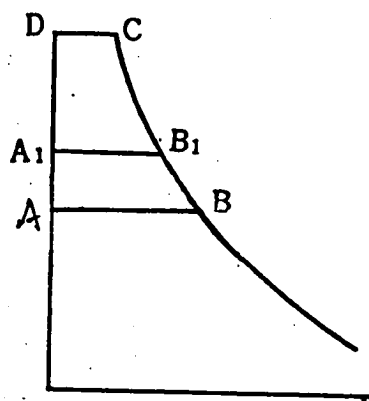


Figure 20. A_1ABB_1 is the "Jie J1" between the lines A_1B_1 and AB , where $AD = \frac{p-m}{p} h$, $A_1D = \frac{p-m+1}{p} h$.

$$\begin{aligned}
&= bh \sum_{k=0}^{\infty} \left[\frac{1}{(k+1)p^{k+1}} \sum_{n=0}^k \binom{k+n}{k} \frac{(p-m)^n}{p^n} \right] \\
&= bh \sum_{k=0}^{\infty} \frac{1}{(k+1)p^{k+1}} \frac{p^{k+1}}{m^{k+1}} \\
&= bh \sum_{k=0}^{\infty} \frac{1}{(k+1)m^{k+1}} .
\end{aligned}$$

Here the equality

$$\sum_{n=0}^k \binom{k+n}{k} \frac{(p-m)^n}{p^n} = \frac{p^{k+1}}{m^{k+1}}$$

holds because

$$\begin{aligned}
&\sum_{n=0}^k \binom{k+n}{k} \frac{(p-m)^n}{p^n} = \binom{k-1}{k-1} \\
&\quad + \sum_{n=1}^k \left[\binom{k+n-1}{k-1} + \binom{k+n-1}{k} \right] \frac{(p-m)^n}{p^n} \\
&\Rightarrow \left(1 - \frac{p-m}{p} \right) \sum_{n=0}^k \binom{k+n}{k} \frac{(p-m)^n}{p^n} \\
&= \left(1 - \frac{p-m}{p} \right) \left\{ \binom{k-1}{k-1} + \sum_{n=1}^k \left[\binom{k+n-1}{k-1} + \binom{k+n-1}{k} \right] \frac{(p-m)^n}{p^n} \right\} \\
&= \frac{p^k}{m^k}
\end{aligned}$$

$$\Rightarrow \sum_{n=0}^k \binom{k+n}{k} \frac{(p-m)^n}{p^n} = \frac{p^{k+1}}{m^{k+1}} .$$

It was by Theorem 1, the foundation of Li Shanlan's logarithms, that he was able to construct a logarithmic table of his own, which was included in Book 2 of the *Dui Shu Tan Yuan*. In fact, Book 2 was devoted to illustrating the theoretical component of his theory of logarithms (i.e., Book 1) [Wang Yusheng in Mei Rongzhao ed. 1990, p. 375]. For example, we actually rely on Book 2 in translating Theorem 1 into modern notation as given above. Thus a few words should be said about Book 2 before concluding this section. For the moment, we go on to discuss the other nine theorems of Book 1.

In connection with Theorem 2 it is necessary to explain what Li Shanlan called "Can Ji" (literally "remaining area"). It was the "remaining" portion of a given "He Jian Zhui" (called "Chuan Ji" -- literally "whole area" -- by Li Shanlan) by cutting away its portion above any line parallel to the base line of the "He Jian Zhui" [Cf. Figure 21]. In fact, Theorems 2 and 3 were meant to relate the "Chuan Ji" and its "Can Ji" with the term $s(m)$ in Theorem 1.

Theorem 2. Given a "Chuan Ji" and its "Can Ji" with height h'
 $= \frac{a}{c}$, where a, c are integers and $c > a$. If
 both the "Chuan Ji" and "Can Ji" are divided into

p equal layers respectively, and if pc is divisible by a , then for $m = 2, 3, 4, \dots, p$,

$$s(m) = s'(m),$$

where $s'(m)$ is the area between the lines $A'_1B'_1$, $A'B'$ with the conditions that

$$A'_1A = \frac{p-m}{p} h' ; A'A = \frac{p-m+1}{p} h' .$$

In his proof Li Shanlan only considered the case $h' = \frac{1}{2} h$, in which his arguments for $p = 2$ or 4 would easily verify the theorem. However, for the general case such as $h' = \frac{a}{c} h$, where a and c are positive integers and $c > a$, Li Shanlan gave no further demonstration or explanation. Nor did he say anything with the case that h, h' are incommensurable, since it is impossible to treat this case in terms of the "algorithmic" method. Here we add our proof for the theorem, aiming not at its rational reconstruction but at making the remaining theorems more convenient for discussion.

Proof of Theorem 2. Let $h' = \frac{a}{c} h$, where a and c are positive integers and $c > a$. Since $A'A'_1 = \frac{h'}{p} = \frac{1}{p} \frac{a}{c} h$, if $p \cdot c$ is divisible by a , then $h - h'$ is divisible by $\frac{h'}{p}$, namely,

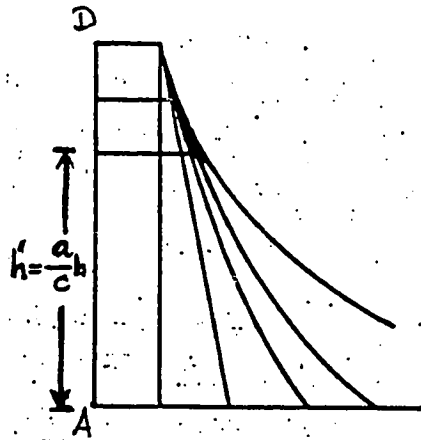


Figure 21a. $h' = \frac{a}{c} h$, where h is the height of the "He Jian Zhui" or "Chuan Ji".

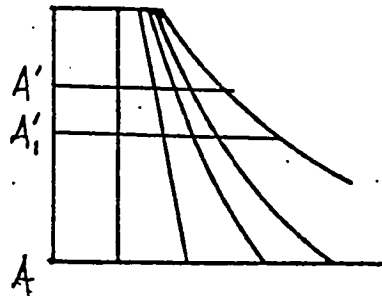


Figure 21b. The "Can Ji" has height $h' = \frac{a}{c} h$.

$$\frac{\frac{h - h'}{ah}}{pc} = \frac{pc}{a} - p .$$

Therefore, by adding the upper portion back to the "Can Ji" to form the original "Chuan Ji", and considering it to be divided into $\frac{pc}{a}$ equal layers, then for $m = 2, 3, 4, \dots, p$,

$$s'(m) = bh \sum_{k=0}^{\infty} \frac{1}{(k+1)m^{k+1}} = s(m) .$$

as desired.

If the "Chuan Ji" and its "Can Ji" are divided into p, p' equal layers respectively, then the same result holds as shown in the following theorem:

Theorem 3. Let the "Chuan Ji" and "Can Ji" have heights h, h'
 $= \frac{a}{c} h$ (a and c are positive integer and $c > a$)
 respectively. Suppose further that they are
 divided into p, p' equal layers respectively.
 Then if cp' is divisible by a , one has

$$s(m) = s'(m')$$

for $m = m' = 2, 3, \dots, \min(p, p')$.

Proof. For the "Chuan Ji" with p layers, we have the $s(m)$ as in Theorem 1. On the other hand, if the "Can Ji" of height h' is

divided into p' equal layers, then each layer has height

$$\frac{h'}{p'} = \frac{\frac{a}{c} h}{p'} = \frac{a}{cp'}.$$

Thus if cp' is divisible by a , then

$$\frac{\frac{h - h'}{h'}}{\frac{p'}{p'}} = \frac{\frac{\frac{c-a}{c} h}{\frac{a}{cp'}}}{\frac{p'}{p'}} = \frac{cp'}{a} - p'.$$

In other words, if we add the upper portion back to the "Can Ji" to form the original "Chuan Ji", then under the condition that cp' is divisible by a , the "compensated" "Chuan Ji" is divided into $\frac{cp'}{a}$ equal layers. Therefore, we have by Theorem 1 that

$$s'(m') = bh \sum_{k=0}^{\infty} \frac{1}{(k+1)m^{k+1}}$$

for $m' = 2, 3, \dots, \frac{cp'}{a}$. Consequently, for $m = m' = 2, 3, \dots, \min(p, p')$, we obtain the desired $s(m) = s'(m')$.

Now we come to the nucleus of Li Shanlan's theory of logarithms, Theorem 4. As already mentioned, the theorem was important principally because it showed quite clearly that the "Jie Ji" $S(h)$ shared a property in common with the logarithms, namely, if h_1, h_2 and h_3 were in geometric progression, then $S(h_1), S(h_2)$ and $S(h_3)$ were in arithmetic progression. More precisely:

Theorem 4. If $h_1 : h_2 = h_3 : h_4$, then

$$S'(h_2) - S'(h_1) = S'(h_4) - S'(h_3) ;$$

$$S'(h_3) - S'(h_1) = S'(h_4) - S'(h_2) .$$

where $S'(h)$ is a "Can Ji" of height h .

The proof is quite simple [Cf. Figure 22], since $h_1 : h_2 = h_3 : h_4$ implies that

$$\frac{h_2 - h_1}{h_2} = \frac{h_4 - h_3}{h_4} , \quad \frac{h_3 - h_1}{h_1} = \frac{h_4 - h_2}{h_2} ,$$

therefore **Theorem 3** leads to the desired conclusion. Yet it deserves to mention that $S'(h)$ is different from $S(h)$, for in the latter h means a distance from the vertex of the "He Jian Zhui". However, by following the argument for **Theorem 1**, we can show that **Theorem 4** holds for the case with $S(h)$ too. It seems interesting here to appreciate how Li Shanlan's demonstrations for **Theorems 1** and **2**, though only implicitly explained in Book 2 of the *Dui Shu Tan Yuan*, could be applied to this case. Here we outline a proof for the theorems that

If $h_1 : h_2 = h_3 : h_4$, then $S(h_2) - S(h_1) = S(h_4) - S(h_3)$
and $S(h_3) - S(h_1) = S(h_4) - S(h_2)$.

In Figure 23, suppose that $h = ph_1 = p'h_3$, $h_2 = mh_1$, $h_4 = mh_3$,

then

$$h_1 = \frac{h}{p}, h_2 = \frac{mh}{p}; h_3 = \frac{h}{p'}, h_4 = \frac{mh}{p'}.$$

Now since the two "Jie Ji" $A_1A_2Q_2Q_1$ and $A_3A_4Q_4Q_3$ are of heights $\frac{(m-1)h}{p}$ and $\frac{(m-1)h}{p'}$ respectively, we can divide them into $(m-1)$ equal layers of heights $\frac{h}{p}$ and $\frac{h}{p'}$ respectively. Therefore by arguments similar to those in the proofs for Theorems 1 and 2 we get the desired conclusion.

If $h_1, h_2, h_3, h_4, h_5, \dots$ assume continued proportion, that is, $h_1 : h_2 = h_2 : h_3 = h_3 : h_4 = h_4 : h_5 = \dots$, then apparently by Theorem 4 Li Shanlan obtained the result of his Theorem 5:

$$\begin{aligned} & S'(h_2) - S'(h_1) \\ &= S'(h_3) - S'(h_2) \\ &= S'(h_4) - S'(h_3) \\ &= S'(h_5) - S'(h_4) \\ &= \dots \end{aligned}$$

In his Theorem 6, Li Shanlan established a relationship between the base b of the n -th "Jian Zhui", $n = 0, 1, 2, 3, \dots$, in the "He Jian Zhui" with height and its corresponding base $b_0, b_1, b_2, b_3, \dots$, in the "Jie Ji" with height h' [Cf. Figure 24]. In fact, Theorem 6 claims that

$$\frac{b_0}{b_1} = \frac{b_1}{b_2} = \frac{b_2}{b_3} = \frac{b_3}{b_4} = \dots = \frac{b_n}{b_{n+1}} = \dots = \frac{h}{h'}$$

where $b_0 = b$. Proof for this theorem is quite easy, for as argued in Section 3.2 [Cf. especially Figure 5 there], the base of the n -th "Jian Zhui", b_n , is equal to $(\frac{h'}{h})^n b$, $n = 0, 1, 2, 3, \dots$

Clearly related to Theorem 6 are two propositions often used by Li Shanlan in his treatment of the "Jian Zhui" method. In fact, they are corollaries of Theorem 6: 1) For the base b , b' of the n -th "Jian Zhui" with height h and its "Jie Ji" with h' respectively, we have $\frac{b'}{b} = (\frac{h'}{h})^n$; 2) For the areas S , S' of the n -th "Jian Zhui" and its "Jie Ji" respectively, we have $\frac{S'}{S} = (\frac{h'}{h})^{n+1}$.

With the bases b_n of the n -th "Jie Ji" in hand, Li Shanlan took sum of these b_n , $n = 0, 1, 2, 3, \dots$ and concluded that

Theorem 7. The sum of the b_n , $n = 0, 1, 2, 3, \dots$ is convergent ("You Jin Jie", literally having an end):

$$\sum_{n=0}^{\infty} b_n = \sum_{n=0}^{\infty} (\frac{h'}{h})^n b = \frac{b}{1 - \frac{h'}{h}} = \frac{bh}{h - h'}.$$

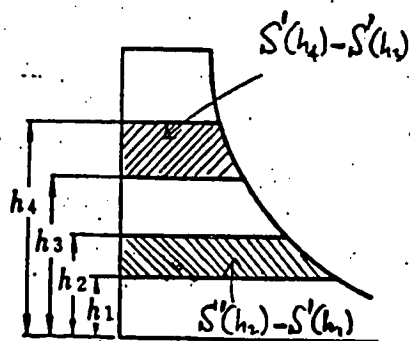


Figure 22. Illustration for Theorem 4.

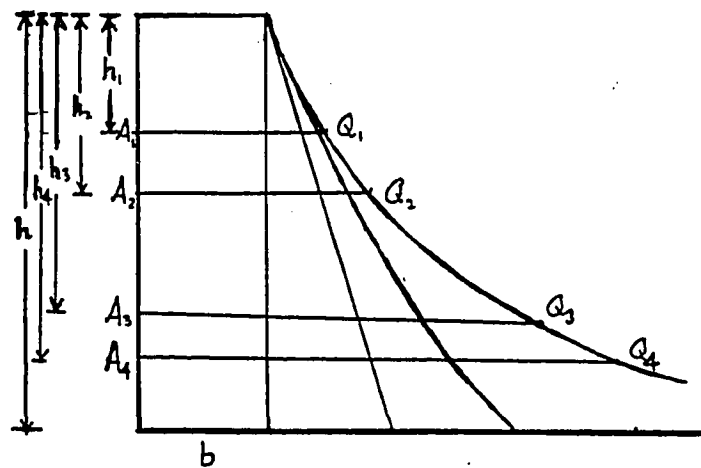


Figure 23. "Jie Ji" $A_1 A_2 Q_2 Q_1 = S(h_2) - S(h_1)$;

"Jie Ji" $A_3 A_4 Q_4 Q_3 = S(h_4) - S(h_3)$.

Besides, Li Shanlan continued to argue that $\sum_{n=0}^{\infty} b$ was divergent ("Wu Jin Jie", literally without and end) [Cf. Figure 24].

What would follow if any two given "Jie Ji" were equal? By Theorems 4 and 6, Li Shanlan his answer in the following:

Theorem 8. If two "Jie Ji" J_1 , J_2 are equal, then

$$\frac{h_1}{h_2} = \frac{b_2}{b_1} = \frac{b'_2}{b'_1}$$

or $b_1 h_1 = b_2 h_2$, $b'_1 h_1 = b'_2 h_2$ where h_1 , b_1 , b'_1 and h_2 , b_2 , b'_2 are the heights, lower bases and upper bases of the J_1 and J_2 respectively [Cf. Figure 25].

For the proof [our reconstruction!], we need only consider that

$$J_1 = S(h+h_1) - S(h) ; J_2 = S(h'+h_2) - S(h')$$

By Theorem 4, we have $h : (h+h_1) = h' : (h'+h_2)$ or

$h_1 : h_2 = h' : h$. Therefore Theorem 6 leads to the desired result.

Based on Theorem 8, Li Shanlan was able to anticipate -- in the context of 1840s Chinese mathematics -- the significant

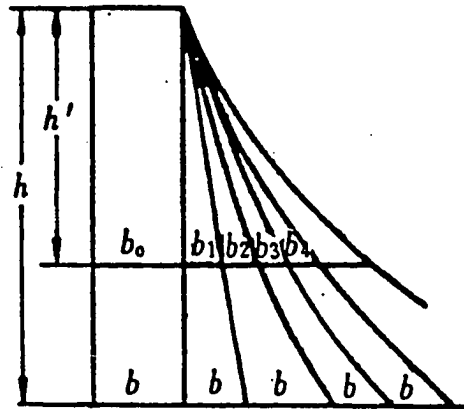


Figure 24. Illustrations for Theorems 6 and 7.

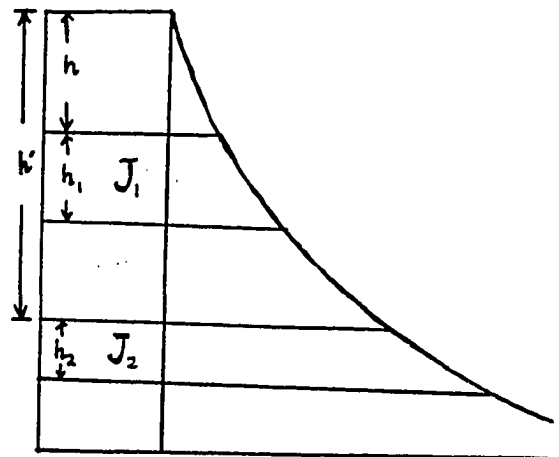


Figure 25. $J_1 = S(h+h_1) - S(h)$; $J_2 = S(h'+h_2) - S(h')$.

relationship between his theory of logarithms and the hyperbolas of the form $xy = k$. In fact, the meaning of his Theorem 9 was clearly more than a relationship between any two "Can Ji" in the "He Jian Zhui":

Theorem 9. For any "Can Ji" with bases b' , b'' and heights h' , h'' respectively, we have

$$\frac{h'}{h''} = \frac{b''}{b'}$$

or $b' h' = b'' h''$ [Cf. Figure 26].

Yet by Theorem 7, $b' h' = b'' h''$, therefore both the points (b', h') , (b'', h'') satisfy $xy = bh$ by Theorem 9. The last quadratic equation represents a hyperbola in the Cartesian coordinate system with the height and the base of the "He Jian Zhui" as the abscissa and the ordinate respectively.

Nevertheless, Li Shanlan does not seem to have known this important fact in the 1840s [Cf. Qian 1981, p. 322] nor did he ever pay due attention to the relevance of coordinate systems (introduced in De Morgan's *Algebra* or Loomis' *Calculus*) for the treatment of logarithms in his own *Dui Shu Jian Zhui Bian Fa Shi*. In other words, in the *Dui Shu Jian Zhui Bian Fa Shi*, Li Shanlan seems to have only been interested in justifying the equivalence of his own "Jian Zhui" method and the newly introduced Western method

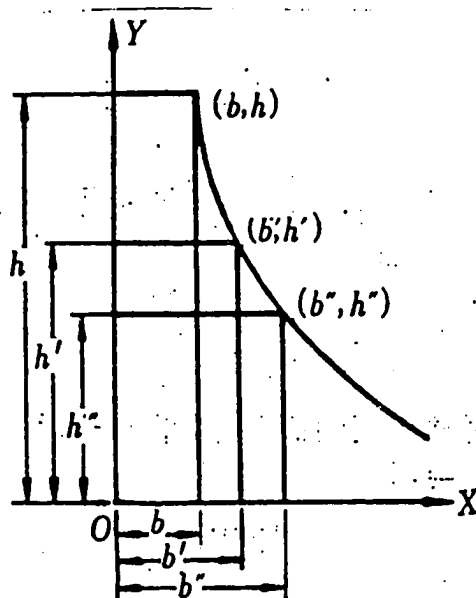


Figure 26. Illustration for Theorem 9.

of logarithm which related the area under the hyperbolic curve with the logarithms. Unfortunately he seems to have ignored some interesting aspects of his theory of logarithms, especially those relevant to the theory of analytic geometry as well as calculus.

On the other hand, Li Shanlan always relied upon demonstrations using concrete examples justifying his propositions or principles. Familiar with the deductive structure of Euclid's *Elements*, Li Shanlan adopted a similar approach to his own mathematical studies, as evident in his *Fang Yuan Chan You*, *Hu Shi Qi Mi* and *Dui Shu Tan Yuan*. Nevertheless, he failed to demonstrate a proof rigorously in these three treatises. Perhaps this was because he cared little about the problem how a proof was important for a proposition. Even for his contemporary Western mathematicians, this was still a difficult problem. Nevertheless, Western mathematicians did not lack a due appreciation of the irrational numbers, the basic idea of which the ancient Greek had understood -- in the form of incommensurable magnitudes. Had Li Shanlan taken seriously the incommensurability problems that occurred his demonstrations for **Theorems 2, 3 and 4**, he might have been able to give proofs rigorously. His failure to do so may help to explain why he almost gave no comment on Book X of the *Elements*, which he and Wylie were going to co-translate in the following decade [Cf. Section 4.6].

Even so, Li Shanlan's theory of logarithms, which was founded on his own "Jian Zhui" method, was self-explanatory. In fact, a

careful reading of the *Dui Shu Tan Yuan* would suggest that he was actually concerned about the comprehensiveness of a mathematical theory as a deductive system. This seems especially clear in light of his Theorem 10, which would have been redundant if he were merely interested in the "algorithmic" aspect of the logarithms. However, Theorem 10 certainly has more to say:

Given any "He Jian Zhui". for $m \geq 2$, we can always compute its $s(m)$. Yet it is not the case with $m = 1$, therefore we cannot compute the whole area of the "He Jian Zhui" either.

Thus, it is no exaggeration to conclude that Li Shanlan's theory of logarithms, based upon the "Jian Zhui" method, had been developed autonomously. Western influence over Li Shanlan's theory of logarithms, if any, should be traced back to the early 18th-century *Shu Li Jing Yun* in which the construction of logarithmic table was introduced. From then on, this subject began to attract the interest of Qing mathematicians, including the noted Dai Xu (1805-1860), Gu Guanguang (1799-1862) and Zou Boqi (1819-1869), in addition to Li Shanlan himself [Li Di et al., eds., 1986, pp. 442-454]. Yet compared with these contemporary mathematicians, Li Shanlan's study on logarithms was not only more systematic and elaborate in theory, but more comprehensive in computations as well. In concluding this section, a brief discussion of his "Xiang Fa" (illustrating methods) given in Book 2 of the *Dui Shu Tan Yuan*.

As mentioned above, if $bh = 1$, and $y = \frac{n-1}{n} h$, then the "Jie Ji" $S(y)$ is just the same as $\log_e n$, namely,

$$\begin{aligned}\log_e n &= S\left(\frac{n-1}{n} h\right) = \frac{n-1}{n} + \frac{1}{2} \left(\frac{n-1}{n}\right)^2 + \frac{1}{3} \left(\frac{n-1}{n}\right)^3 + \dots \\ &= \sum_{k=0}^{\infty} \frac{1}{k+1} \left(\frac{n-1}{n}\right)^{k+1}.\end{aligned}$$

Thus by taking $n = 2$ in the above infinite series, Li Shanlan obtained that

$$\log_e 2 = \sum_{k=0}^{\infty} \frac{1}{k+1} \left(\frac{1}{2}\right)^{k+1} = 0.69314713.$$

Similarly, by taking $n = \frac{5}{4}$, he obtained

$$\log_e \frac{5}{4} = \sum_{k=0}^{\infty} \frac{1}{k+1} \left(\frac{1}{5}\right)^{k+1} = 0.22314353.$$

Combining these two results, as well as one of the basic laws for logarithms, $\log(mn) = \log m + \log n$, Li Shanlan obtained the natural logarithms of 10:

$$\log_e 10 = 3 \log_e 2 + \log_e \frac{5}{4} = 2.30258492.$$

[Cf. Qian Baocong 1981, pp. 321-323; the *Dui Shu Tan Yuan* (1867) Book 2, pp. 1-5].

As for computing common base 10 logarithms, Li Shanlan first

derived the so-called "root" for the area of "Jian Zhui" -- $\mu = \log_{10} e$. He obtained this "root" by computing $\mu = \log_{10} e = (\log_e 10)^{-1} = 0.43429451$, and thereby got the common logarithms for 2, 3 and 7:

$$\log_{10} 2 = \mu \log_e 2 = 0.30310300 .$$

$$\begin{aligned} \log_{10} 3 &= \log_{10} 2 + \log_{10} \frac{3}{2} = \log_{10} 2 + \mu \sum_{k=0}^{\infty} \frac{1}{k+1} \left(\frac{1}{3}\right)^{k+1} \\ &= 0.47712126 , \end{aligned}$$

$$\begin{aligned} \log_{10} 7 &= \log_{10} 6 + \log_{10} \frac{7}{6} \\ &= \log_{10} 2 + \log_{10} 3 + \mu \sum_{k=0}^{\infty} \frac{1}{k+1} \left(\frac{1}{7}\right)^{k+1} \\ &= 0.84509805. \end{aligned}$$

By similar computations, Li Shanlan also obtained the approximate values for $\log_{10} 4$, $\log_{10} 5$, $\log_{10} 6$, $\log_{10} 8$, $\log_{10} 9$ and $\log_{10} 10$. It seems therefore clear that Li Shanlan must have known the following infinite series expansion for $\log_{10} n$:

$$\log_{10} n = \log (n-1) + \mu \sum_{k=1}^{\infty} \frac{1}{k \cdot n^k} .$$

Such a method was, according to Li Shanlan, much easier and

simpler than the old one introduced in the *Shu Li Jing Yun* (1723), since the former made use of the infinite series expansion of the logarithms while the later required taking successive square roots up to very high powers. For example, the authors of the *Shu Li Jing Yun* needed to make successive extractions of square roots up to 47 times in order to get $\log_{10} 2 = 0.3010299566$ [Cf. Qian Baocong 1981, p. 280]. However, for the same approximate value Li Shanlan was proud to report that he needed to compute only the first 20 terms of the series expansion for $\log_{10} 2$ [*Dui Shu Tan Yuan* (1851) Book 2, p. 19; or its 1867 edition Book 2, p. 16].

Facing the awkward methodology introduced in the *Shu Li Jing Yun*, Dai Xu must also have felt a need to find an alternative for computations on logarithms. In 1846, the year when Gu Guanguang wrote prefaces for Li Shanlan's *Si Yuan Jie*, *Hu Shi Qi Mi* and *Dui Shu Tan Yuan* respectively, Dai Xu wrote a preface for his own *Xu Dui Shu Jian Fa* ("Supplement for A Simple Method of Logarithms") -- in which he gave a power series expansion for $\log_{10}(1+a)$:

$$\log_{10}(1+a) = \mu \sum_{k=1}^{\infty} (-1)^{k+1} \frac{a^k}{k} .$$

where $0 < a < 1$ [Wang Yusheng in Mei Rongzhao 1990, p. 385].

Nevertheless, it should be mentioned that Dai Xu and Li Shanlan did not meet each other until 1851 [Li Yan 1955, p. 342]. This means there was little chance of any influence between them

in the study of logarithms. As one of the eight great mathematicians in the period 1840-1880³, Dai Xu had made remarkable contributions on subjects like the binomial series expansion, series expansions for trigonometric functions and power series expansion for logarithms [Qian Baocong 1981, pp. 313-317]. Parallel or even more remarkable contributions on these same subjects were doubtless had made as well by Li Shanlan. However, Li Shanlan's study of the theory of logarithms was more striking because its underlying principle -- the "Jian Zhui" method, as fully explained in the *Fang Yuan Chan You*, permeated his other notable treatises like *Hu Shi Qi Mi* and *Dui Shu Tan Yuan*. Equally significant, as argued in Section 3.2, is the fact that the "Jian Zhui" method was drawn upon from the "Duo Ji" method -- a traditional Chinese method for the summation of finite series. Thus his theory of logarithms, also shows that even in the 1840's -- a period during which Chinese mathematics was forced to re-open its gates to the West, Chinese scholars like Li Shanlan were still doing mathematics in an autonomous and independent way. A decade later, it was not much difference in the case of the *Duo Ji Bi Lei*, which was probably written in the early 1850s.

3.5. On Summation of Finite Series: The *Duo Ji Bi Lei*

The *Duo Ji Bi Lei* is the only treatise that is not only read by historians of Chinese mathematics, but by modern mathematicians as well. Since the late 1930s modern mathematicians have begun to pay attention to the marvelous Li Rensoo Identity (or the Li

Shanlan Identity) and in turn to his *Duo Ji Bi Lei* -- thanks to Zhang Yong (1911-1939), who introduced this identity to Hungarian mathematician Pal Duran in 1937. Two years later, Zhang Yong published his "*Duo Ji Bi Lei Shu Zheng* (Annotaton on *Duo Ji Bi Lei*)" (1939) in which he named the following formula the **Li Rensoo Identity**:

$$\binom{n+q}{q}^2 = \sum_{k=0}^q \binom{q}{k}^2 \binom{n+2q-k}{2q},$$

where

$$\binom{r}{m} = \begin{cases} 0, & \text{if } r < m, \\ \frac{r!}{m!(r-m)!}, & \text{if } r \geq m. \end{cases}$$

Yet this was not the only one of the great contributions that Li Shanlan had made on the summation of finite series. His *Duo Ji Bi Lei* also includes several terms equivalent to those of the combinatorics. For example, in summing the finite power series $\sum_{r=1}^n r^p$, Li Shanlan used the coefficients essentially equivalent to the Eulerian Numbers A_k^p (which the historian of Chinese mathematics Luo Jianjin prefers to call the Li Shanlan Number of the Second Kind, Cf. Section 3.2) to obtain the result:

$$\sum_{r=1}^n r^p = \sum_{k=1}^p A_k^p \left[\sum_{r=1}^{n-k+1} \binom{r+p-1}{p} \right].$$

Moreover, Li Shanlan also created a number table [See Figure 27],

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Figure 27. On the method of computing this table, Li Shanlan said, "Each entry depends on the entries immediately above to left and right, the left entry governs the number of layers along the left diagonal and the right entry governs the following layers along the right diagonal and each multiplies it according to the layer number. Combining them gives the present entry."

among several others, to display the coefficients A_k^p as follows:

				1					
				1		1			
			1		4		1		
		1		11		11		1	
	1		26		66		26		1
1		57		302		302		57	1
.....									

in which the first row, namely $A_1^1 = 1$ is the coefficient for $p = 1$, the second row, namely $A_1^2 = 1$, $A_2^2 = 1$ are the coefficients for $p = 2$, the third row, namely $A_1^3 = 1$, $A_2^3 = 4$, $A_3^3 = 1$ are the coefficients for $p = 3$, the fourth row $A_1^4 = 1$, $A_2^4 = 11$, $A_3^4 = 11$, $A_4^4 = 1$ are the coefficients for $p = 4$, the fifth row $A_1^5 = 1$, $A_2^5 = 26$, $A_3^5 = 66$, $A_4^5 = 26$, $A_5^5 = 1$ are the coefficients for $p = 5$, and so on. Li Shanlan certainly knew the general rule for these coefficients, as shown in his statement below his "Cheng Fang Duo Ge Lian Bio" (or the so-called Li Shanlan Triangle) [Cf. Figure 27], that $A_i^p = 0$, if $i = 0$ or $i > p$ and $A_i^p = (p-i+1) A_{i-1}^{p-1} + i A_{i+1}^{p-1}$ if $1 < i < p$ [Cf. Section 3.2].

Yet the *Duo Ji Bi Lei* was not written in modern notation. Nor was it possible for this treatise to have asked questions in terms of the combinatorics. Compared with Li Shanlan's other mathematical treatises, it is interesting to see that Western influence hardly had existed in the *Duo Ji Bi Lei*. Considering

the fact that it is regarded by modern mathematicians as an early classic in the combinatorics, how Li Shanlan created such an excellent mathematical work in the very context of traditional Chinese mathematics deserves our special attention.

First of all, we should say a few words about the style and format of the *Duo Ji Bi Lei*. Although the *Duo Ji Bi Lei* includes tables ("Biao") like Pascal Triangle [Cf. Figure 28], diagrams ("Tu"), methods ("Fa") and demonstrations ("Cao"), which were not so often the cases with ancient Chinese mathematical texts⁴, it was still conformal to the major features of traditional Chinese mathematics, namely using a few examples to justify the full generality of methods or formulas which were expressed literally. Considering it was written by 1853 or thereabouts, one should not expect that it would have used modern (Western) mathematical symbols or notations. Nor should we criticize Li Shanlan's failure to give rigorous arguments or demonstrations which were needed to justify his propositions in this treatise. Yet without using modern notations or symbols, how can we be certain that Li Shanlan did attain the generalities of his conclusions? In addition to the profound diagrams and tables to help illustrate his methods or propositions, Li Shanlan worked through a few examples and then noted the generality of the argument -- a way reminiscent of Blaise Pascal's studies on "arithmetic triangle". Pascal usually worked only one example and then noted the generality of the argument. If Pascal's propositions about "arithmetic triangle" look more rigorous and thus more acceptable,

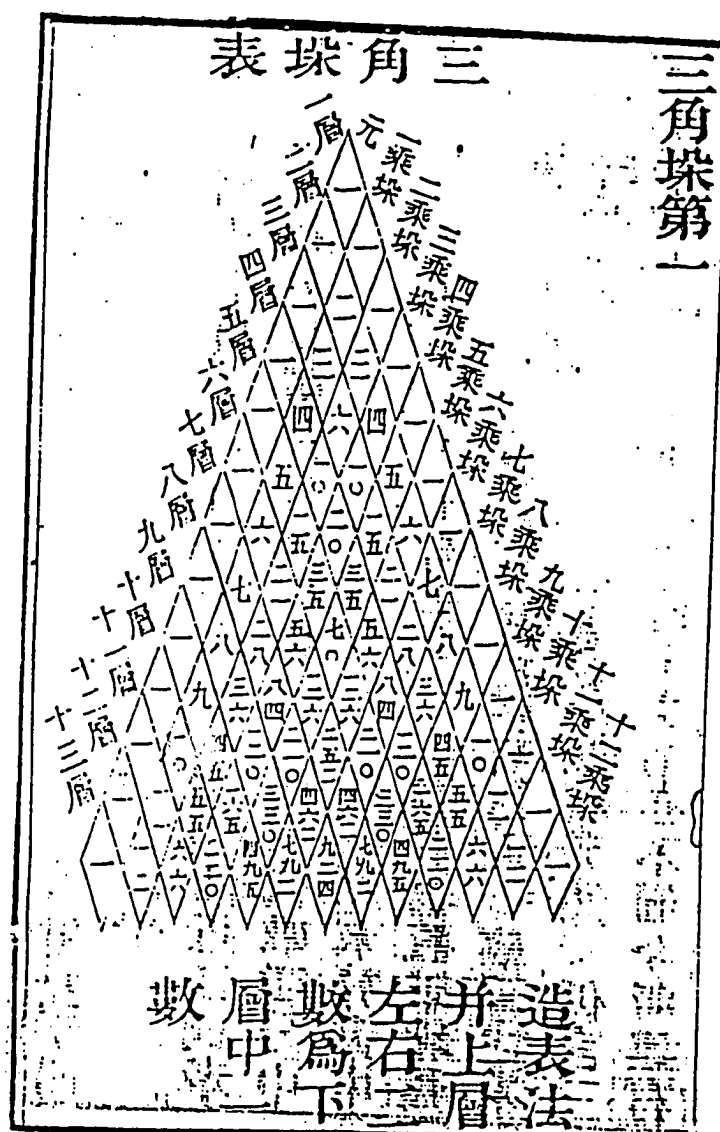


Figure 28. "Pascal Triangle" in Li Shanlan's *Duo Ji Bi Lei*.

then we would suggest that it was probably due to his use of mathematical induction, especially its explicit form used in the proof for Corollary 12 of his *A Treatise on the Arithmetical Triangle* [Cf. Edwards 1987, pp. 63-65]. Li Shanlan apparently did not make use of any argument similar to that by mathematical induction. Nevertheless, for convenience of discussion, we shall give Li Shanlan's propositions in their general forms in terms of modern notations, while in the meantime we shall keep our eyes on the rational reconstructions to avoid possible oversimplifications or overdue explanations. However, since the theme of this thesis is to trace Western impact over 19th-century Chinese mathematics through Li Shanlan's example, a brief discussion seems to be quite enough.

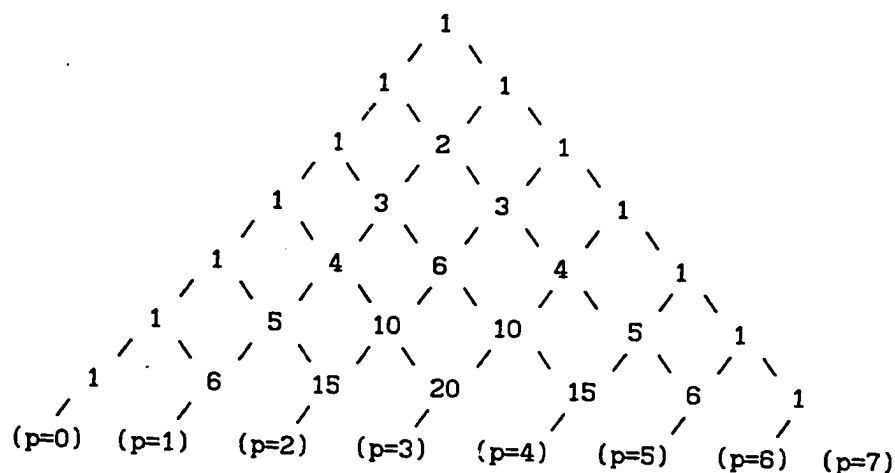
The problem of "Duo Ji" could be traced back to the Song period. As mentioned in Section 3.2, Shen Gua (1030-1094) of the northern Song (1030-1094) began to undertake such a problem. Yang Hui of the southern Song also collected several formulas for the sum of piles in his mathematical treatises like the *Xiang Jie Jiu Zhang Suan Shu* (1261) and *Suan Fa Tong Bian Ben Mo* (1274). Yet both of them did not seem to give sufficient arguments for the formulas they gave, although Shen Gua did suggest how he obtain his formula for the number of a rectangular stack n layers high, with its top layer a objects wide and b objects long, bottom layer c objects wide and d objects long [Cf. Section 3.2 and Figure 9]. An idea leading to a proof of the formula, according to Shen Gua, came from his use of given the "Duo Ji" (sum of piles) to

approximate the volume of the rectangular platform. In this process, his concept of the "Xi Ji" -- the difference between the "Duo Ji" and the volume of the rectangular platform -- came to play an significant role. In other words, Shen Gua solved the problem basically in terms of the theory of volumes [Cf. Li Di et al., eds., 1986, pp. 263-267]. However, Shen Gua's argument for his formula had nothing to do with the properties of the Pascal Triangle, by which Pascal was to make the figurative, binomial and combinatorial coefficients to be all the same thing [Edwards 1987, pp. 70-86].

On the other hand, the Chinese "Pascal Triangle" -- the "Kai Fang Zo Fa Ben Yuan Tu" (Tabulation System for Unlocking Binomial Coefficients) of Jia Xian (ca. 1100) had primarily been used in solving algebraic equations⁵, which eventually led to the Chinese "Horner's Method" -- the southerner Song Qin Jiushao's "Zheng Fu Kai Fang" method [Cf. Qing 1981, pp. 144-164].

It was not until the late 13th century that Zhu Shijie began to realize that the Pascal Triangle had something significant to do with the sum of piles. Fu Dawie, among others, is right to argue that Zhu Shijie created a new approach, in contrast to Shen Gua's, to the Chinese studies of summing the piles [Fu Dawie 1990]. In his *Si Yuan Yu Jian* ("Precious Mirror of the Four Elements", 1303), Zhu Shijie displayed a "Gu Fa Qi Cheng Fang Tu" (Binomial Coefficients up to Eighth Power) which was added not only two more rows to Jia Xian's "Kai Fang Zo Fa Ben Yuan Tu" but

parallel oblique lines drawn across the numbers as well:



With this table in hand, Zhu Shijie might have related the p -th parallel leftwards-oblique line [see the above diagram] to his formulas for the sums of piles. In fact, his "San Jiao Duo" (triangular piles) formulas:

$$\sum_{r=1}^n r = \frac{1}{2} n(n+1),$$

$$\sum_{r=1}^n \frac{1}{2!} r(r+1) = \frac{1}{3!} n(n+1)(n+2),$$

$$\sum_{r=1}^n \frac{1}{3!} r(r+1)(r+2) = \frac{1}{4!} n(n+1)(n+2)(n+3),$$

$$\sum_{r=1}^n \frac{1}{4!} r(r+1)(r+2)(r+3) = \frac{1}{5!} n(n+1)(n+2)(n+3)(n+4),$$

$$\sum_{r=1}^n \frac{1}{5!} r(r+1)(r+2)(r+3)(r+4) = \frac{1}{6!} n(n+1)(n+2)(n+3)(n+4)(n+5),$$

can be concluded in the following general form:

$$\sum_{r=1}^n \frac{1}{p!} r(r+1)(r+2) \dots (r+p-1) = \frac{1}{(p+1)!} n(n+1)(n+2) \dots (n+p)$$

or

$$\sum_{r=1}^n \binom{r+p-1}{p} = \binom{n+p}{p+1}.$$

This means that the sum of the first n terms of the p -th parallel leftwards-oblique line is equal to the n -th number of the $(p+1)$ -th parallel leftwards-oblique line.

However, Zhu Shijie did not make it clear how he actually obtained these five formulas. Nor did he explain how he justified the "Si Jiao Duo" (square piles) formulas which was already given by Yang Hui:

$$\sum_{r=1}^n r^2 = \frac{1}{3!} n(n+1)(2n+1),$$

$$\sum_{r=1}^n \frac{1}{3!} r(r+1)(2r+1) = \frac{1}{4!} n(n+1)(n+2)(2n+2).$$

Similarly, for the variant forms of the "San Jiao Duo" and "Si Jiao Duo" given in the *Si Yuan Yu Jian*, namely

$$\sum_{r=1}^n \frac{1}{2!} r(r+1) \cdot r = \frac{1}{4!} n(n+1)(n+2)(3n+1),$$

$$\sum_{r=1}^n \frac{1}{3!} r(r+1)(r+2) \cdot r = \frac{1}{5!} n(n+1)(n+2)(n+3)(4n+1),$$

as well as

$$\sum_{r=1}^n \frac{1}{3!} r(r+1)(2r) \cdot r = \frac{1}{5!} n(n+1)(n+2)(8n^2+11n+1),$$

Zhu Shijie did not give arguments or explanations for these formulas. Perhaps Zhu Shijie's major concern in the "Duo Ji" method was, as Li Shanlan suggested in his "Preface" to the *Duo Ji Bi Lei*, to develop the "Tian Yuan" and "Si Yuan" methods in terms of the "Duo Ji" problems. Nevertheless, Zhu Shijie apparently grasped the general rules underlying the "Duo Ji" formulas given in his *Si Yuan Yu Jian*, since these formulas had indeed played an important role in his study of the "Zhao Ca" method (method of finite differences). For example, in order to find the sum of the finite series (in modern notation):

$$f(n) = 1 + 4 + 10 + \dots + \frac{n(n-1)(n-2)}{3!},$$

Zhu Shijie developed a formula which can be described as follows:

$$f(n) = n\Delta + \frac{1}{2!} n(n-1)\Delta^2$$

$$+ \frac{1}{3!} n(n-1)(n-2)\Delta^3 + \frac{1}{4!} n(n-1)(n-2)(n-3)\Delta^4,$$

in which we let $f(0) = 0$, $\Delta = \Delta f(0)$, $\Delta^n = \Delta^n f(0)$, where $\Delta f(n) = f(n+1) - f(n)$, $\Delta^2 f(n) = \Delta(\Delta f(n))$, $n = 1, 2, 3, 4$. Zhu Shijie also noted that the coefficients, namely, $\frac{1}{2!} n(n-1)$, $\frac{1}{3!} n(n-1)(n-2)$, $\frac{1}{4!} n(n-1)(n-2)(n-3)$ are the sums of the "San Jiao Duo" $\sum_{r=1}^{n-1} r$, $\sum_{r=1}^{n-1} \frac{1}{2!} r(r+1)$, $\sum_{r=1}^{n-1} \frac{1}{3!} r(r+1)(r+2)$, respectively. It is due this reason that historians of Chinese mathematics have all suggested that Zhu Shijie had created a general method of finite difference [Cf. Qing 1981, pp. 197-205; Li Di *et al.*, eds., 1986, pp. 328-331].

After Zhu Shijie, Chinese mathematicians made almost no progress in the study of "Duo Ji" method. Cheng Dawei's *Suan Fa Tong Zong* ("General Survey of Mathematical Knowledge, 1593) simply repeated those formulas that had already known in Yang Hui's treatises. Although this treatise was very popular from late Ming on, Chinese mathematicians did not seem to have paid any attention to the subject. It was only after the Qian-Jia school began to revive the Song and Yuan mathematical classics that the "Duo Ji" method again drew the attentions from mathematicians. The *Shu Li Jing Yun* (1721) introduced a few formulas for the sums of piles which added nothing new to the Chinese method of "Duo Ji" method. Yet this was another then available source material that also dealt with the "Duo Ji" problems. Moreover, since the *Shu Li Jing Yun* was edited under the supervision of the Emperor Kang Xi, it may

have been more popular than the *Suan Fa Tong Zong*. Therefore, Chen Shiren (1676-1772) would have primarily referred to the *Shu Li Jing Yun* to write his *Shao Guang Shi Yi*. In this treatise, Chen Shiren adopted the terms used in the *Shu Li Jing Yun* to give a few formulas like $1 + 2 + 4 + \dots + 2^n = 2^{n+1}$ and $1 + 3 + 5 + \dots + (2n-1) = n^2$, etc., which were never known to Chinese mathematicians earlier before. Nevertheless, the *Shao Guang Shi Yi* made less influence on the subsequent studies of the subject due apparently to its lack of explanations on the formulas.

It was Wang Lai who began to undertake a serious study of the "Duo Ji" method toward the end of the 18th century [Li Zhaohua 1986]. Wang Lai innovated the study of the "Duo Ji" method by linking it to combinatorial numbers. In so doing he became the first Chinese mathematician to study the problems of the combinatorial numbers of their own sake. Whether Wang Lai was influenced by Western mathematics is unknown, although in this subject he, just as he did in his theory of equations, certainly adopted some terms translated from Western mathematics. For example, he called the "triangular piles" "San Jiao Dui" (instead of "San Jiao Duo"), a term made its first appearance in the *Shu Li Jing Yun*. As far as the Western mathematics transmitted to China (in the late Ming and early Qing period) was concerned, there was nothing concerning combinatorial numbers. On the other hand, Wang Lai also noted that the combinatorial numbers like $C_p^n = \binom{n}{p}$ never had appeared in ancient China [See Wang Lai's *Di Jian Shu Li* in *Hengzhai Suan Xue* Book 4]. Thus the relationship between the

sums of "San Jiao Dui" (triangular piles) and the combinatorial numbers C_p^n might have been a discovery due to Wang Lai.

Wang Lai's contributions to combinatorial numbers can be expressed as follows (in modern notation):

(1) $C_1^n + C_2^n + \dots + C_n^n = 2^n - 1$. Wang Lai called such a sum "Di Jian Zong Shu". He derived this formula inductively by demonstrating a number of combinatorial computations. Whether he considered the case with C_0^n is unknown. Yet he apparently understood that $C_0^n = 1$.

(2) $C_p^n = C_{n-p}^n$, where $p = 1, 2, \dots, n$. Wang Lai also explained how to determine which term among the $C_0^n, C_1^n, \dots, C_n^n$ was the central one in terms of this symmetric relationship. His method was: If $(n-1)$ is an odd number, then the central term will be the $(n/2)$ -th one. On the other hand, if $(n-1)$ is an even number, then there will be two terms, namely the $(\frac{n-1}{2})$ -th and the $(\frac{n+1}{2})$ -th ones located in the central position.

(3) $C_p^n = \frac{n!}{p!(n-p)!}$. Wang Lai called C_p^n the "Di Jian Fen Shu" (literally the part of the "Di Jian Zong Shu"). He gave graphical diagrams to illustrate the case with $n = 10$, in which he did not refer to the Pascal Triangle -- a table which was not displayed in his *Di Jian Shu Li* ("Mathematical Principle of Combinatorial Numbers"), but instead he identified C_2^{10} and C_3^{10} , etc., with the sum of the "Pien San Jiao Dui" (2-dimensional triangular piles)

and the sum of "Li San Jiao Dui" (3-dimensional triangular piles), etc., respectively. The sums of piles like the "Pien San Jiao Dui" and "Li San Jiao Dui", etc., had already been well-known facts in the history of Chinese mathematics. It was Wang Lai who created the "Di Jian Fen Shu" C_p^n and then showed that $C_p^n = \frac{n!}{p!(n-p)!}$ [Li Zhaohua 1986].

Immediately after Wang Lai, Dong Youcheng linked the sums of "Si Jiao Duo" (square piles) to the "Ge Yuan" (circle measurement) problems [Cf. Section 3.3]. Yet just as Li Shanlan's comment on Dong Youcheng's study in the "Duo Ji" method, Dong Youcheng treated the cases with sums of square piles [Cf. Li's "Preface" to the *Duo Ji Bi Lei*]. A comprehensive investigation of the "Duo Ji" method had to await Li Shanlan.

Li Shanlan's *Duo Ji Bi Lei* consists of four chapters and each chapter forms a complete system. Chapter One is about the triangular piles and its variant forms, which in modern notation can be gathered into one formula as

$$\sum_{p=0}^n \frac{1}{p!} r (r+1) (r+2) \dots [r+(p-2)] [mr+(p-m)] \\ = \frac{1}{(p+1)!} n (n+1) (n+2) \dots [n+(p-1)] [mr+(p-m+1)].$$

As mentioned above, Li Shanlan derived those formulas leading to this general formula by relying primarily on the "San Jiao Duo Biao" (Pascal Triangle) [Cf. Figure 28] which was displayed at the very beginning of Chapter One.

Chapter Two deals with the square piles. One of the major themes of this chapter is to find the sums of the finite series of the form $\sum r^p$. In dealing with the problem of summations, Li Shanlan worked through the cases with $p = 1, 2, 3$ and 4:

$$\sum_{r=1}^n r^1 = \sum_{r=1}^n \frac{1}{1!} r;$$

$$\sum_{r=1}^n r^2 = \sum_{r=1}^n \frac{1}{2!} r(r+1) + \sum_{r=1}^{n-1} \frac{1}{2!} r(r+1);$$

$$\sum_{r=1}^n r^3 = \sum_{r=1}^n \frac{1}{3!} r(r+1)(r+2) + 4 \sum_{r=1}^{n-1} \frac{1}{3!} r(r+1)(r+2)$$

$$+ \sum_{r=1}^{n-2} \frac{1}{3!} r(r+1)(r+2);$$

$$\begin{aligned} \sum_{r=1}^n r^4 &= \sum_{r=1}^n \frac{1}{4!} r(r+1)(r+2)(r+3) + 11 \sum_{r=1}^{n-1} \frac{1}{4!} r(r+1)(r+2)(r+3) \\ &+ 11 \sum_{r=1}^{n-2} \frac{1}{4!} r(r+1)(r+2)(r+3) + \sum_{r=1}^{n-3} \frac{1}{4!} r(r+1)(r+2)(r+3). \end{aligned}$$

As for the cases with $p = 5, 6, \dots$, Li Shanlan pointed out that the coefficients of each term had been displayed in his "Cheng Fang Duo Ge Liang Bio" [Cf. Figure 27]. We can re-write the terms of his formula for $p = 4$ as follows:

$$\sum_{r=1}^n r^4 = A_1^4 \binom{n+4}{4+1} + A_2^4 \binom{n+3}{4+1} + A_3^4 \binom{n+2}{4+1} + A_4^4 \binom{n+1}{4+1}$$

$$= \sum_{k=1}^4 A_k^4 \left[\sum_{r=1}^{n-k+1} \binom{r+4-1}{4} \right].$$

For any value of p , Li Shanlan's argument therefore suggests the following formula:

$$\sum_{r=1}^n r^p = \sum_{k=1}^p A_k^p \left[\sum_{r=1}^{n-k+1} \binom{r+p-1}{p} \right].$$

By mathematical induction, it is easy to show that

$$\sum_{i=1}^p A_i^p = p!.$$

Therefore, Li Shanlan may have reduced the summation of $\sum r^p$ into the problem of finding the sum of $p!$ p -th "San Jiao Duo", in which the p -th "San Jiao Duo" (triangular pile)

$$\sum_{r=1}^{n-k+1} \frac{1}{p!} r(r+1)(r+2) \dots (r+p-1)$$

($k = 1, 2, 3, \dots, p$) are the sums of the p -th parallel leftwards-oblique line in the Pascal Triangle with the respective number of terms $n, n-1, n-2, \dots, n-p+1$ [Li Zhaohua 1985].

Chapter Three of the *Duo Ji Bi Lei* deals with the problem of finding the sums of the squares of each of the terms $\binom{r+p-1}{p}$ for the triangular piles. They can be summarized by the following formula:

$$\sum_{r=1}^n \binom{r+p-1}{p}^2 = \sum_{i=1}^{p+1} [B_i^p \sum_{r=1}^{n-i+1} \binom{r+2p-1}{2p}]^2, \quad (*)$$

where B_i^p is the number shown in the "San Jiao Zi Cheng Duo Biao" (Table of the Squares of the Binomial Coefficients) [Cf. Figure 29]:

				1						$p = 0$
				1			1			$p = 1$
		1		4		1				$p = 2$
	1		9		9		1			$p = 3$
1		16		36		16		1		$p = 4$
.....										

Li Shanlan, as usually did for the other cases, gave formulas for the cases $p = 1, 2, 3$, thereby suggested that the formula for general value of p could be derived analogously. He seems to have understood the general formula, since, for example, in demonstrating the formula for $p = 3$ that

$$\begin{aligned} \sum_{r=1}^n \binom{r+3-1}{3}^2 &= \sum_{r=1}^n \binom{r+2 \cdot 3-1}{2 \cdot 3} + 9 \sum_{r=1}^{n-1} \binom{r+2 \cdot 3-1}{2 \cdot 3} \\ &\quad + 9 \sum_{r=1}^{n-2} \binom{r+2 \cdot 3-1}{2 \cdot 3} + \sum_{r=1}^{n-3} \binom{r+2 \cdot 3-1}{2 \cdot 3}, \end{aligned}$$

he actually relied on his formulas for the sums of $\sum_{r=1}^n r^6$ [Li

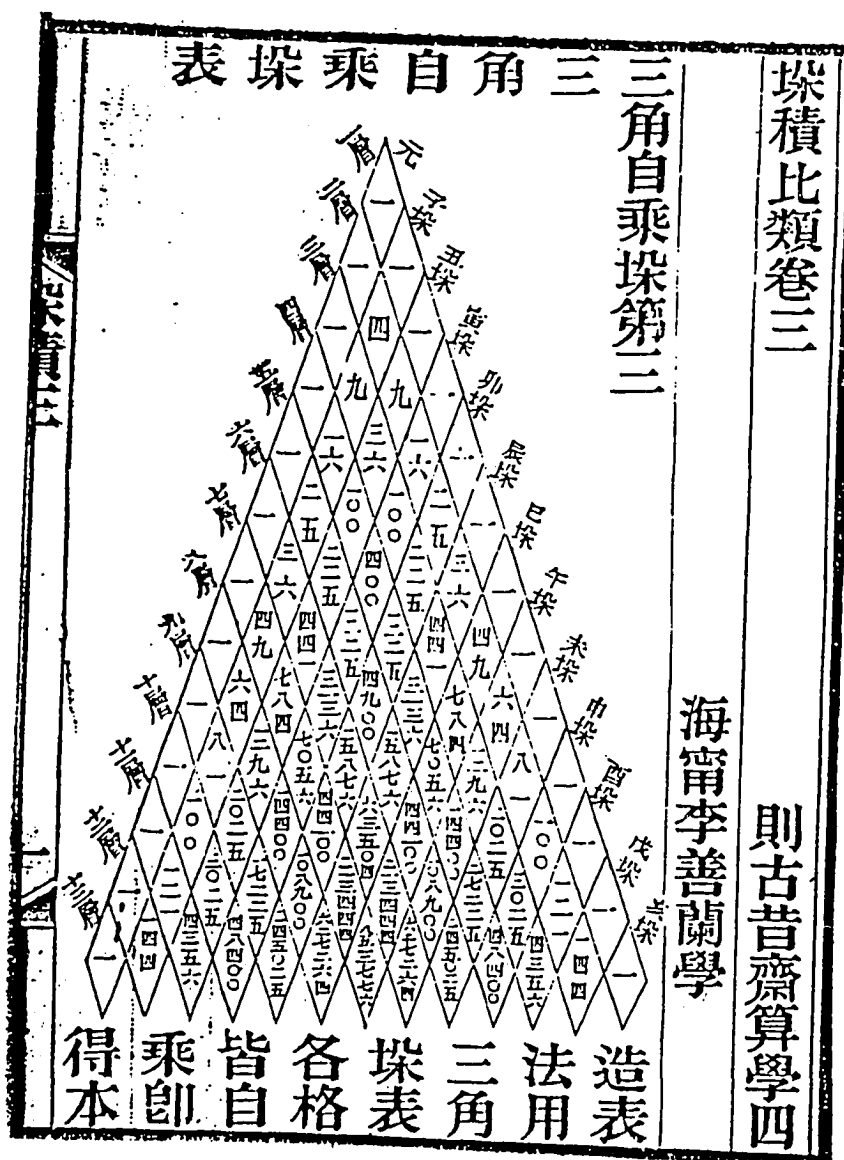


Figure 29. Li Shanlan's "San Jiao Cheng Fang Duo Biao"

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[Li Zhaohua in Cheng-Yih Chen 1987, pp. 109-128; Luo Jianjin 1982a; Martzloff 1988]. It is evident that this formula is a generalization of Zhu Shijie's "Lan Feng Xing" (peak-shaped piles) system [Cf. Li and Du 1987, p. 156, 250]. Again Li Shanlan's method of solution for this problem is to convert them into triangular piles, then multiply by predetermined coefficients and then obtain the sum [Li and Du 1987, p. 250].

In conclusion, it should be noted again that here we only give a very brief survey of the *Duo Ji Bi Lei*, one of the early masterpieces of combinatorics. Since it contains more than forty thousand Chinese characters along with several significant tables and diagrams, how Li Shanlan had actually done and how he influenced his successors Qiang Ruxun and Zhou Da in this subject deserves further investigation. Considering so many modern mathematicians have been enthusiastic about Li Rensoo Identity, Li Shanlan and his *Duo Ji Bi Lei* raise a provocative question: Was Li Shanlan the last important figure of the traditional Chinese mathematics or the first significant mathematician of modern Chinese mathematics? This question, suggested by Professor Zhang Dianzhou of Eastern China Normal University at Shanghai, is meaningful, since as we shall argue in Chapter 5 of this thesis, Li Shanlan had perceived himself as a "professionalized" role in doing mathematics since the first period of his career (ca. 1845-1852).

3.6. On Ellipse

Li Shanlan paid more attention to the ellipse than he did to the other two conics, the hyperbola and parabola. As for the hyperbola, Li Shanlan was only interested in its relevance to logarithms [Cf. Section 3.4]. For the parabola, Li Shanlan's major concern seems to have been its application to projectile motion on which he wrote the *Huo Qi Zhen Jue*, simplifying the methods introduced in the *Zhong Xue* (Whewell's *Mechanics*) [Cf. Section 3.7]. Nevertheless, in either case, Li Shanlan had no opportunity to consider the geometric properties per se as given in the texts on conics or the equivalent portion in the texts on analytic geometry. In contrast, Li Shanlan wrote three treatises on ellipse, namely, the *Tuo Yuan Zheng Shu Jie* ("Explanation of the Methods for Ellipse"), *Tuo Yuan Xin Shu* ("New Methods for Ellipse") and *Tuo Yuan Shi Yi* ("Remarks on Ellipse"), in which he did pay attention to some geometric aspects in addition to computations in terms of the ellipse. But why did he pay so much attention to this subject? How did he deal with it and what contributions did he eventually make? Once again, Li Shanlan played a key role in the late Qing mathematical community, and this time the important subject was ellipse!

The conic sections were introduced to China by the Jesuits in the late Ming and early Qing period. The knowledge they transmitted on the conics, though not comprehensive, was largely given in treatises on astronomy and calendrical sciences. For

example, conic sections were presented in the *Chong Zhen Li Shu* ("Chong Zhen Reign Treatise on Astronomy and Calendrical Sciences") supervised by Xu Guangqi and Li Tianjing, and in treatises like the *Ce Liang Quan Yi* ("Complete Theory of Surveying", 1631), which introduced definitions for ellipse, hyperbola and parabola by regarding them as sections from cutting a circular cone. In addition, the *Ce Tian Yue Shuo* also regarded an ellipse as a section of a circular cylinder cut by a slant plane. However, why this treatise used a circular cylinder instead of circular cone for a slant plane to cut is unknown. Moreover, it gave the formulae for the area of an ellipse, the volume of an ellipsoid and the volume for the revolution of an ellipse. On the other hand, the *Jiao Shi Li Shu* ("On Eclipse", 1632), *Heng Xing Li Shu* ("On the Stars", 1631) and *Ce Tian Yue Shuo* ("On the Astronomical Computation", 1633) -- the three other treatises collected in the *Chong Zhen Li Shu* -- also referred to the theory of the ellipse. For example, the *Heng Xing Li Shu* mentioned that an ellipse had two foci; the *Jiao Shi Li Shu* used the two auxiliary circles of an ellipse to determine the angle of direction for solar eclipses. And in the *Ce Tian Yue Shuo*, the editor tried to distinguish the circle and the ellipse by emphasizing that the ellipse was just a section of a circular cylinder cut by a slant plane [Niu 1989, pp. 1-11].

In 1674, the Belgian priest Ferdinand Verbiest (1623-1688) edited (in Chinese) the *Ling Tai Yi Xiang Zhi* ("On Astronomical Instruments") at the Qing State Observatory ("Qin Tian Jian").

This treatise contained 117 graphic illustrations of the astronomical instruments together with how to use them, of manipulation as well as explanations drawn from relevant knowledge not only on astronomy, mechanics and physics, but mathematics as well. More specifically, the *Ling Tai Yi Xiang Zhi* introduced the mathematical instruments for drawing an ellipse, a hyperbola and a parabola respectively -- and they worked because their underlying principles were in accordance with definitions for the ellipse, hyperbola and parabola respectively [Niu 1989, pp. 11-14].

Among all the treatises, the *Ce Liang Quan Yi*, edited by the Jesuits Jacques Rho (1593-1638) and Jean Adam Schall von Bell (1591-1666), was of the greatest importance for the introduction of mathematical knowledge closely related to astronomy. In contrast to the other treatises collected in the *Chong Zhen Li Shu*, the *Ce Liang Quan Yi* gave a more extensive treatment of the conics in general and the ellipse in particular. Even so, the *Ce Liang Quan Yi*, like the treatises mentioned above, did not explain how the methods or formulas for the conics actually worked. This did not happen until the *Shu Li Jing Yun* ("Collected Basic Principles of Mathematics", 1723) that began to emphasize theoretical aspects of the conics.

The *Shu Li Jing Yun*, which covered all aspects of Western mathematical knowledge transmitted at the time (See Section 1.4), may be regarded as a comprehensive mathematical encyclopedia of the early Qing period. For the problems under discussion, the *Shu*

Li Jing Yun included (1) the area formula of an ellipse, (2) the volume formulas for ellipsoids as solids of revolution, (3) the volume formula for the truncated cone with an ellipse as a base, and (4) the surface area formula for an ellipsoid formed by rotating the major axis. In dealing with the second formula, the editors of the *Shu Li Jing Yun* used two principles to justify their methods: One was to claim that a chord of an ellipse to the equivalent chord of its circumscribing auxiliary circle was the major axis to the minor one (the so-called Fundamental Principle of the Ellipse, Cf. Figure 30); the other is the Principle of Cavalieri or, as historians of Chinese mathematics would call it, the Liu-Zu's Principle. Armed with these two principles, the editors also treat the relationship between the volume of an ellipsoid with the volume of its circumscribing circular cylinder [Niu 1989, pp. 15-29].

Since the first two formulas had already introduced in the *Ce Liang Quan Yi*, it is not necessary to repeat here. As for the third formula, three different methods were given for its justification. In the last one, the editors circumscribed the "Shang Xia Bu Deng Tuo Yuan Mian Ti" (truncated cone with an ellipse as a base) by a "Shang Xia Bu Deng Chang Fang Mian Ti" (truncated pyramid with a rectangular base), and then made use of the Principle of Cavalieri to get the desired result. It is interesting here to note that the "Shang Xia Bu Deng Chang Fang Mian Ti" is the same as the "Chu Tong", a term used in Chapter 5 of the *Jiu Zhang Suan Shu*. Equally interesting is that Liu Hui

(3rd century A.D.) in his "Commentary" on Chapter 5 had already taken a similar approach in calculating the volume formula for a "Yuan Tai" (truncated circular cone) [Niu 1989, pp. 24-26]. In fact, Liu Hui circumscribed the "Yuan Tai" by a "Fang Tai" (truncated pyramid with square base), and used a principle similar to the Principle of Cavalieri to justify the volume formula for the "Yuan Tai" in Chapter 5 of the *Jiu Zhang Suan Shu* [Cf. Wagner 1979]. But no one knows whether the editors of the *Shu Li Jing Yun*, Mei Juecheng, Ming Antu, He Guozong and Chen Huoyao, ever had an opportunity to refer to this Chinese mathematical classic. The *Jiu Zhang Suan Shu* was unavailable to scholars from the early 14th century on. It was not until 1773 that it was reprinted and finally circulated again thanks to the edition of the *Si Ku Quan Shu* [Cf. Li and Du 1987, pp. 225-230].

It is also possible that the volume formula for the truncated cone with an ellipse as base was introduced either by the Jesuits, or in the Frenchman Pardies' text on geometry (1671), on which the portion of the *Shu Li Jing Yun* on geometry was based [Li and Du 1987, p. 218]. In any case, the treatment of these volume problems was quite successful. However, the case of the surface area of an ellipsoid as a revolution of an ellipse rotating about its major axis was not. In fact, the editors were wrong in believing that the Principle of Cavalieri could also be applied to the case of an ellipsoid inscribed in a sphere -- i.e., its radius was equal to one half of the major axis of the ellipse [Niu 1989, pp. 27-29].

The last astronomical text which transmitted Western knowledge of the ellipse to China by the 18th century was the *Li Xiang Kao Cheng Hou Bian* ("A Sequel for the *Li Xiang Kao Cheng*", 1742). The aim of this treatise was to improve the Shi Xian Calendar, decreed by the first Qing Emperor Shun Zi. The Shi Xian Calendar (which Johann Adam Schall von Bell was in charge of compiling), was based on the *Xi Yang Xin Fa Li Shu* ("Treatise on Calendrical Science according to the New Western Methods"), a re-edition of the *Chong Zhen Li Shu*, and the *Li Xiang Kao Cheng* ("Compendium of Calendrical Science and Astronomy", 1722) [Cf. Section 1.2]. For the methods of astronomical computation, however, the Jesuit Ignatius Kogler also introduced Kepler's first and second astronomical laws in the *Li Xiang Kao Cheng Hou Pian*. This raises an interesting question why didn't a contradiction arise, which should have led a refutation of the Tychonic system? Kepler's laws, which certainly helped the Chinese in their astronomical computations, assumed *a priori* the Copernican system. However, by positioning the Earth at one focus of the elliptical orbit of the "Tychonic" solar system, as did the *Li Xiang Kao Cheng Hou Pian*, the related astronomical computations were simplified but equivalent to the those manipulated solely in the Tychonic system. Furthermore, there would be no apparent contradiction between Tyconic and Keplerian systems, if the scientists like the Qing astronomers of the time were merely interested in the solar and lunar motions, and eclipses, (especially if their related computations), were done in terms of the properties of ellipses, and if for which mean time did not

arise in determining planetary motions [Jiang Xiaoyuan 1989, p. 47]. How Qing astronomers reacted to such a "compromise" between two contradictory astronomical systems is an interesting problem in the history of Chinese science. But this is an important issue in the history of astronomy, not of Chinese mathematics, no further discussion of this subject is given here [Jiang Xiaoyuan 1989].

Based on the above observations, it seems clear that the subject of ellipse was destined to become an important part of the Western science and mathematics transmitted to China in the 18th century. Doubtless the *Li Xiang Kao Cheng Hou Pian* helped to motivate studies on the ellipse, since this treatise dealt with many astronomical computations in terms of the properties of the ellipse. Apparently, the *Li Xiang Kao Cheng Hou Pian* was the most important treatise on which Qing mathematicians from the mid-18th century onward relied to study the ellipse.

The *Li Xiang Kao Cheng Hou Pian* included the following knowledge about the ellipse:

- (1) The center, foci, focal distance, semi-major and semi-minor axes as well as their relevant relationship like $a^2 - b^2 = c^2$, where a , b and c were the semi-major, semi-minor axes and one half of the focal distance respectively;
- (2) Two methods for the drawing of the ellipse;
- (3) The relationships between an ellipse and its two auxiliary circles. One was the so-called Fundamental

Principle of the Ellipse [Cf Figure 30], while the other was concerned with the relationship between the angles of an ellipse and its auxiliary circles [Cf. Figure 31];

- (4) The relationship between the areas of a sector of an ellipse and that of its auxiliary circles [Cf. Figure 32].

In addition, the *Li Xiang Kao Cheng Hou Pian* also introduced four methods of astronomical computation. For the convenience of discussion, we ignore astronomical questions and simply state the problems to which the four methods were applied.

Problem 1 -- "Yi Jiao Qiu Ji". Suppose that ABCD is an ellipse, MCNA is its auxiliary circle, O is the center of the ellipse, F_1, F_2 are the foci, semi-major axis $OA = a$, $OF_1 = OF_2 = c$ [See Figure 33]. Given $\angle AF_1P = \alpha$, find the area of the sector AF_1P of the ellipse.

Problem 2 -- "Yi Ji Qiu Jiao". Given that the area for the sector AFP of the ellipse equals 1° . (the area of circle with radius $\sqrt{ab}$), find $\angle AFP$, the point F is one focus of the ellipse [See Figure 34].

Problem 3 -- "Jie Ji Qiu Jiao". Given in that $\angle QOA = \gamma$, and the area of the sector P_1FA [Cf. Figure 35], find $\angle PFA = \alpha$ such that the area of the sector PFA equals that of the sector P_1FA , in which the point F is one focus of the ellipse.

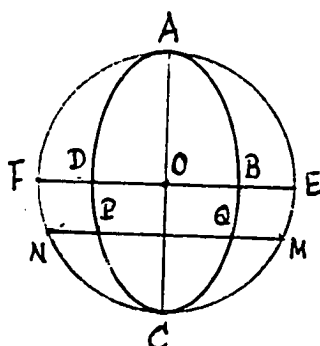


Figure 30. Fundamental Principle of Ellipse:

$$\frac{PQ}{MN} = \frac{BD}{EF} = \frac{b}{a},$$

in which a , b are the semi-major and semi-minor axes of the ellipse respectively.

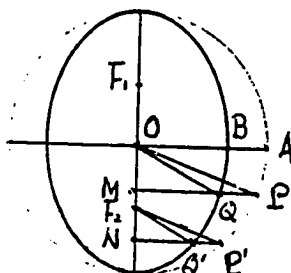


Figure 31. $\tan \angle MOQ = \frac{b}{a} \tan \angle MOP,$

$$\tan \angle NF_2P = \frac{b}{a} \tan \angle NF_2Q',$$

where F_2 is one of the foci of the ellipse whose semi-major and semi-minor axes are a , b respectively.

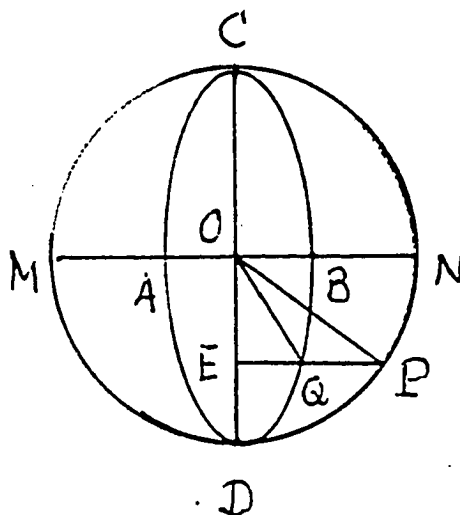


Figure 32. $S_{OQD} : S_{OPD} = b : a$, $b = OA = OB$, $a = OC = OD$.

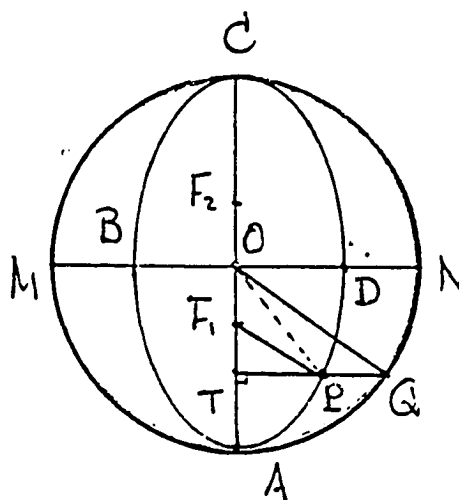


Figure 33. Given $\angle AF_1P = \alpha$, find the area of the sector S_{AF_1P} .

Problem 4 -- "Jie Jiao Qiu Jiao". Given $\angle AOQ = \gamma$ as in Figure 36, find a point P on the ellipse such that the area of the sector PFA equals that of the sector P_1OA .

Problem 4 is actually equivalent to Problem 3. The editors of the *Li Xiang Kao Cheng Hou Pian*, however, gave different methods for their solutions [Niu 1989, pp. 34-42].

The *Li Xiang Kao Cheng Hou Pian*, as mentioned above, perhaps was the last text on Western science and mathematics transmitted to China before 1840s. At least some of the basic concepts, methods and principles related to the ellipse were introduced to China through the Western treatises mentioned in this section by 1750. Nevertheless, it was not until the end of the 18th century that Qing mathematicians began to understand these materials. Jiao Xun (1763-1820) [Cf. Section 1.5] was among the first scholars who, in his *Shi Tuo* ("Explanations on the Ellipse"), described some of their properties and added some new methods to the theories of the ellipse drawn from the *Li Xiang Kao Cheng Hou Pian*. For beginners who were interested in the theory of the ellipse, the *Shi Tuo*, as Jiao Xun argued, was very helpful, yet it was no innovation in the methods for the computations in terms of the properties of the ellipse [Niu 1989, pp. 41-42].

In fact, it was not until the 1820s that native mathematicians like Dong Youcheng began to develop new interests in the theory of the ellipse. In 1821, Dong Youcheng wrote the

Tuo Yuan Qiu Zhou Shu ("On the Perimeter of the ellipse") in which he tried to compute the length of the perimeter for an ellipse. His strategy was first to regard an ellipse as a section of a circular cylinder cut by a slant plane -- an idea mentioned to him by Zhu Hong. He mistakenly thought he had reduced the problem to one similar to Problem 5 of Chapter 9 ("On Gou Gu") in the *Jiu Zhang Suan Shu*, and was therefore wrong in thinking he could solve the problem using the Pythagorean Theorem: In Figure 37, Dong Youcheng regarded the perimeter of the ellipse (with major axis a and minor axis b) as a curve spiraling upward along the cylinder -- just like a vine spiraling along a tree. Since $BC^2 = AB^2 - AC^2 = (2a)^2 - (2b)^2$, thus he obtained the length of the perimeter P as follows:

$$P = 2AB = \sqrt{(2AC)^2 + (2BC)^2} = \frac{1}{113} \sqrt{355^2(2b)^2 + 113^2 \cdot 4(a^2 - b^2)}$$

by considering a right-angled triangle with "Gou" = BC (the short leg), "Gu" = $(\frac{355}{113} 2b)$ (the long leg) and "Xuan" (the hypotenuse) = P , in which Dong Youcheng apparently adopted $\frac{355}{113}$ as an approximate for π [Niu 1989, pp. 43-44]. However, the curve with Problem 5 of Chapter 9 in the *Jiu Zhang Suan Shu* was not an ellipse but actually a spiral, therefore Dong Youcheng failed to give a correct solution for the length of the perimeter for an ellipse due basically to a false analogy. Even so, Dong Youcheng had proposed a new problem on the ellipse for Qing mathematicians later on.

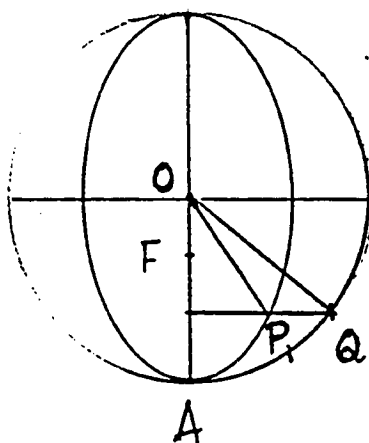


Figure 36. Given $\angle AOQ = \gamma$, find a point P on the ellipse such that

$$S_{PFA} = S_{P_1OA}.$$

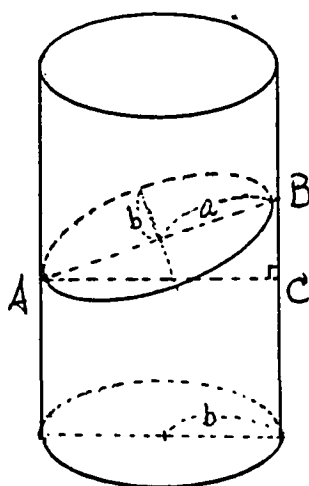


Figure 37. Illustration for Dong Youcheng's method of rectification.

About twenty years later Xiang Mingda (1789-1850) [Cf. Section 1.10] took over and successfully solved the problem. His solution or formula for the perimeter P was as follows:

$$P = 2\pi a \left(1 - \frac{1}{2^2} e^2 - \frac{1^2 \cdot 3^2}{2^2 \cdot 4^2} e^4 - \frac{1^2 \cdot 3^2 \cdot 5}{2^2 \cdot 4^2 \cdot 6^2} e^6 - \dots \right),$$

where $e^2 = \frac{a^2 - b^2}{a^2}$ [Cf. Qian 1981, pp. 71-72; Niu 1989, pp.

43-45]. Xiang Mingda seemed to have obtained the above result by 1848, since he sent a letter then to Dai Xu (1805-1860) asking his younger colleague to put the draft version in order. At the time, as Xiang Mingda wrote in his letter, he was in bad health, and was therefore afraid that he might never be able to publish his "most contented" work -- the *Tuo Yuan Qiu Zhou Shu* ("Method on the Rectification of the Perimeter of ellipse").

In 1857, Dai Xu wrote his *Tu Jie* ("Graphical Illustrations") for Xiang Mingda's *Tuo Yuan Qiu Zhou Shu*. Xiang Mingda's method of rectification, like the method of the integral calculus, approximated the small arc of the ellipse by its corresponding chord [Cf. Figure 38]. In his *Tu Jie*, Dai Xu also gave his own formula for the rectification of the perimeter of the ellipse [Cf. Figure 38]:

$$P = 2\pi b \left(1 + \frac{1}{2^2} e'^2 - \frac{1^2 \cdot 3}{2^2 \cdot 4^2} e'^4 + \frac{1^2 \cdot 3^2 \cdot 5}{2^2 \cdot 4^2 \cdot 6^2} e'^6 - \dots \right),$$

where b is the semi-minor axis of the ellipse and $e' = \frac{a^2 - b^2}{b^2}$

[Niu 1989, p. 66].

Now we come to Xu Youren (1800-1860), a close friend of Li Shanlan, whose studies on the ellipse seemed to have evoked Li Shanlan's enthusiasm. As for Xu Youren's own interest in the ellipse, it can be traced back to his early service in Beijing where he worked with Dong Youcheng and the astronomers at the State Observatory [Cf. Zhu Kebao 1886, p. 782]. In Book 1 of his *Ge Yuan Mi Lu* ("On the Methods for Circle Measurement"), Xu Youren gave formulas for the area of an ellipse, the volume of an ellipsoid as a solid of revolution, the volume of a cylinder with an ellipse as a base, the volume of a cone with an ellipse as a base, and the volume of a truncated cone with an ellipse as a base. All four formulas were valid, despite the fact that Xu Youren did not show how he got his results. Nevertheless, Xu Youren seems to have been more serious about the problem of how π should be expressed by the formulas just mentioned above than other native scholars had ever been before. In fact, in his formula for the area of an ellipse, a series expansion for π played a significant role -- such concern was reminiscent of his criticism of Qian Tang's approximation $\pi = 3.16$ [Cf. Section 1.10]. Xu Youren used the same expansion for π in the other three formulas. Here we cite his area formula for the ellipse in modern notation:

$$S = \frac{3}{4} ab \left(1 + \frac{1}{2 \cdot 3} \cdot \frac{1}{4} + \frac{1^2 \cdot 3^2}{2 \cdot 3 \cdot 4 \cdot 5} \cdot \frac{1}{4^2} + \frac{1^2 \cdot 3^2 \cdot 5^2}{2 \cdot 3 \cdot 4 \cdot 5 \cdot 6 \cdot 7} \right. \\ \left. + \frac{1}{4^3} + \dots \right),$$

in which the following series expansion was apparently adopted for

π :

$$\pi = 3 \left(1 + \frac{1}{2 \cdot 3} \cdot \frac{1}{4} + \frac{1^2 \cdot 3^2}{2 \cdot 3 \cdot 4 \cdot 5} \cdot \frac{1}{4^2} \right. \\ \left. + \frac{1^2 \cdot 3^2 \cdot 5^2}{2 \cdot 3 \cdot 4 \cdot 5 \cdot 6 \cdot 7} \cdot \frac{1}{4^3} + \dots \right)$$

[Niu 1989, pp. 68-70].

In his *Tuo Yuan Zheng Shu* ("Methods for Ellipse"), Xu Youzen mentioned at the outset that the new methods ("Xin Fa") introduced in the *Li Xiang Kao Cheng Hou Pian*, where astronomical computations were performed using the theory of ellipse, were too complicated. Therefore, he wrote the *Tuo Yuan Zheng Shu* to simplify the computations. Xu Youren was right: There was no native scholar had ever tried to improve upon these methods since they were first introduced to China in 1742.

For Problem 1 in the *Li Xiang Kao Cheng Hou Pian* -- "Yi Jiao Qiu Ji", Xu Youren gave a solution which was simpler in form than that in the *Li Xiang Kao Cheng Hou Pian*. In the latter, the

solution for the area of the sector AF_1P was given as

$$S_{AF_1P} = \frac{\pi ab}{360} \arcsin \frac{b \sin \alpha}{a + c \sin \alpha} - \frac{b^2 c \sin \alpha}{a + c \cos \alpha}$$

[Cf. Niu 1989, pp. 35-36]. Alternatively, in his *Tuo Yuan Zheng Shu*, Xu Youren argued that the solution could be expressed as

$$S_{AF_1P} = \frac{b}{a} S_{AF_1Q} ,$$

where S_{AF_1Q} was a sector of the larger auxiliary circle of the ellipse. Since Li Shanlan followed Xu Youren in simplifying the method for handling this problem, we need to explain first how Xu Youren actually solved this problem.

To reconstruct Xu Youren's method, let us first rephrase the problem. As in Figure 39, let P be any point on the ellipse with a, b as the semi-major and semi-minor axis respectively. Suppose the angle $\angle PF_1A$ is given, find S_{PF_1A} the area of the sector PF_1A [Niu 1989, pp. 70-73]. Now, to help explain his method, set $\angle PF_2F_1 = \beta$, $\angle F_2PF_1 = \gamma$. Their bisectors meet at the point M which is on the tangent to the ellipse at the point A . Thus we have

$$\tan \frac{\alpha}{2} = \frac{MA}{F_1A} = \frac{MA}{a - c} ; \quad \tan \frac{\beta}{2} = \frac{MA}{F_2A} = \frac{MA}{a + c} ,$$

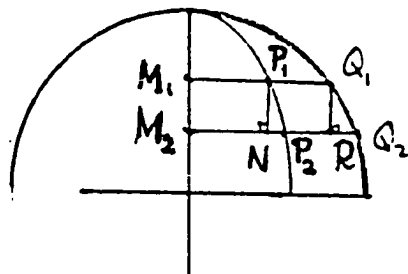


Figure 38. Dai Xu's graphical illustration for Xiang Mingda's

method of rectification: $NP_2 : RQ_2 = b : a \Rightarrow$

$$P_1P_2^2 = NP_1^2 + NP_2^2 = Q_1Q_2^2 - RQ_2^2 + \frac{b^2}{a^2} RQ_2^2$$

$$= QQ_2^2 - e^2 RQ_2^2$$

$$\Rightarrow P_1P_2 = Q_1Q_2 \sqrt{1 - e^2 s^2}, \text{ if } s = \frac{RQ_2}{Q_1Q_2}.$$

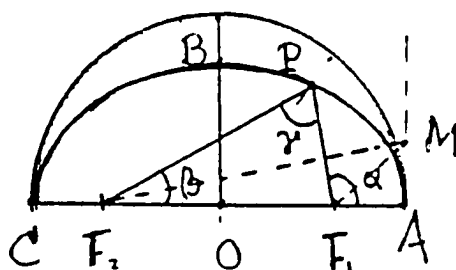


Figure 39. $OA = a$, $OB = b$, $c = OF_1 = OF_2 = \sqrt{a^2 - b^2}$. Given

$\angle AF_1P = \alpha$, find S_{AF_1P} .

where $c = OF_1 = OF_2 = a^2 - b^2$ and consequently

$$\tan \frac{\beta}{2} = \frac{a - c}{a + c} \tan \frac{\alpha}{2} . \quad (*)$$

Xu Youren called β the "Jie Jiao" (literally, borrowed angle) which could be derived by the formula (*) above.

In order to solve Problem 1, as shown in Figure 40, draw the bisector PI for the angle γ through the point P to intersect the line BO (the point O is the center of the ellipse) at the point I. Drop the perpendiculars $\overline{ID}$ and $\overline{IT}$ to the lines PF_2 and PT, respectively. On the other hand, connect the line segments $\overline{F_2I}$ and $\overline{F_1I}$. Then we can show that the right-angled triangles $\triangle PDI \cong \triangle PIT$, and thereby that the right-angled triangles $\triangle IDF_2 \cong \triangle IF_1$. Now since

$$\begin{aligned} 2a &= PF_1 + PF_2 = PF_1 + PD + PF_2 \\ &= PF_1 + F_1T + PF_1 + DF_2 = 2PF_1 + 2F_1T \end{aligned}$$

$$\Rightarrow PF_1 + F_1T = a = PD ,$$

we have

$$DI = PD \tan \frac{\gamma}{2} = a \tan \frac{\gamma}{2} .$$

Moreover,

$$\angle DF_2I = \beta + \angle OF_2I, \angle IF_1I = \alpha - \angle OF_1I,$$

$$\beta + \angle OF_2I = \alpha - \angle OF_1I$$

$$\Rightarrow \angle OF_1I = \angle OF_2I = \frac{\gamma}{2},$$

therefore,

$$S_{DPTI} = S_{PF_1IF_2} = PD \cdot DI = a^2 \tan \frac{\gamma}{2}$$

$$S_{\Delta F_1IF_2} = OI \cdot OF_1 = (c \tan \frac{\gamma}{2}) c = c^2 \tan \frac{\gamma}{2}$$

$$S_{\Delta PF_1F_2} = S_{PF_1IF_2} - S_{\Delta F_1IF_2} = (a^2 - c^2) \tan \frac{\gamma}{2} = b^2 \tan \frac{\gamma}{2}.$$

On the other hand, if we again construct $\overline{PN} \perp \overline{AC}$, extend the line PN to meet the (larger) auxiliary circle at the point Q. Set $\angle QOA = \phi$, then $PN = \frac{b}{a} NQ = \frac{b}{a} \cdot a \sin \phi = b \sin \phi$, as well as

$$S_{\Delta PF_1F_2} = \frac{1}{2} F_1F_2 \cdot PN = \frac{1}{2} \cdot 2c \cdot b \sin \phi = bc \sin \phi.$$

Therefore,

$$b^2 \tan \frac{\gamma}{2} = S_{\Delta PF_1F_2} = bc \sin \phi \Rightarrow \sin \phi = \frac{c}{b} \tan \frac{\gamma}{2} \quad (**)$$

Finally, Xu Youren linked $\sin\phi$ to the $S_{\Delta PF_1 A}$ through a relationship between $S_{\Delta PF_1 A}$ and $S_{\Delta QF_1 A}$, where the point Q is the intersection of the line PN with the (larger) auxiliary circle as shown in Figure 41. Since

$$\frac{OD}{QN} = \frac{S_{\Delta ODF_1}}{S_{\Delta OQF_1}} \quad \text{or} \quad \frac{a}{a \cdot \sin\phi} = \frac{\frac{1}{2} ac}{S_{\Delta OQF_1}} \quad (***)$$

$$\Rightarrow S_{\Delta OQF_1} = \frac{1}{2} ac \sin\phi ,$$

as well as the area of the sector $S_{QF_1 A} = S_{QOA} - S_{\Delta OQF_1}$, consequently if the following relationship holds:

$$S_{\Delta PF_1 A} = \frac{b}{a} S_{QF_1 A} ,$$

then the area of the sector $PF_1 A$, $S_{PF_1 A}$, could be determined by (*), (**) and (***) [Cf. Niu 1989, pp. 70-73]. Yet the last relationship had already been proved in the *Li Xiang Kao Cheng Hou Pian*: Since $S_{NPA} : S_{NQA} = b : a$ (by the Principle of Cavalieri) and $S_{\Delta NPF_1} : S_{\Delta NQF_1} = b : a$. Thus

$$S_{PF_1 A} : S_{QF_1 A} = b : a ,$$

as desired [Cf. Niu 1989, pp. 35-37].

As for problem 2 -- "Yi Ji Qiu Jiao", Xu Youren rephrased it as follows: Given $\angle QOA = \theta$ as in Figure 42, find the angle $\angle PF_1A$ such that the areas of the sectors $S_{PF_1A} = S_{QOA}$. This he solved the problem by a method of approximation. Here again we give our reconstruction to explain his method [Cf. Niu 1989, pp. 73-75]. Let $\angle PF_1A = \alpha$, and through the point F_2 draw the line F_2P_1 parallel to the line OQ , intersecting the ellipse at the point P_1 . The point P_1 was regarded by Xu Youren as the first approximation for the desired point P such that $S_{PF_1A} = S_{QOA}$. Now follow the method used to solve the problem -- "Yi Jiao Qiu Ji". It follows that $\angle P_1F_1A = \alpha_1$, an angle was Xu Youren called the "Jie Jiao". Now, since $\angle P_1F_1A = \angle QOA$, we have

$$\tan \frac{1}{2} \alpha_1 : \tan \frac{1}{2} \theta = (a + c) : (a - c)$$

or

$$\tan \frac{1}{2} \alpha_1 = \frac{a + c}{a - c} \tan \frac{1}{2} \theta .$$

In other words, with α_1 in hand, we can find its corresponding θ_1 and thereby the "Ji Jiao" $\Delta\theta = \theta - \theta_1$.

On the other hand, applying the Sine Rule to the triangle $\Delta F_1P_1F_2$, we have

$$\sin \angle F_2P_1F_1 : \sin \angle P_1F_2A = 2c : P_1F_1,$$

and therefore,

$$P_1F_1 = \frac{2c}{\sin \angle F_2P_1F_1} \cdot \sin \angle P_1F_2A = \frac{2c}{\sin(\alpha_1 - \theta)} \cdot \sin \theta .$$

Xu Youren called the P_1F_1 the "Jie Bian" (literally, borrowed side).

In order to get a better approximation for α , Xu Youren discovered a relationship between $\Delta\theta$ and $\Delta\alpha = \alpha - \alpha_1$. Since

$$S_{QOA} : S_{PF_1A} = a : b ;$$

by taking the point Q_1 on the (large) auxiliary circle such that $\angle Q_1OA = \theta_1$, we have

$$S_{Q_1OA} : S_{P_1F_1A} = a : b .$$

Consequently,

$$(S_{QOA} - S_{Q_1OA}) : (S_{PF_1A} - S_{P_1F_1A}) = S_{QOQ_1} : S_{PF_1P_1} = a : b .$$

But $S_{QOQ_1} = \frac{1}{2} \frac{\pi \cdot \Delta\theta}{180^\circ} \cdot a^2$, and since $\Delta\alpha$ usually is small

enough so that we can regard the arc PP_1 as a circular arc of a circle with F_1 as its center and the line segment $\overline{P_1F_1}$ as its

radius, it follows that $S_{PF_1P_1} = \frac{1}{2} \frac{\pi \cdot \Delta\alpha}{180^\circ} P_1F_1^2$. Consequently,

$$\frac{a}{b} = \frac{S_{QQQ_1}}{S_{PF_1P_1}} = \frac{\Delta\theta \cdot a^2}{\Delta\alpha \cdot P_1F_1^2}$$

$$\Rightarrow \frac{\Delta\theta}{\Delta\alpha} = \frac{P_1F_1^2}{ab} \Rightarrow \Delta\alpha = \frac{ab}{P_1F_1^2} \Delta\theta .$$

By adding $\Delta\alpha$ to α_1 or subtracting $\Delta\alpha$ from α_1 , we obtain a better approximation for α in just the same way as Xu Youren did.

Xu Youren's methods for the computations in terms of the properties of the ellipse, as shown above, impressed Li Shanlan because they were much better than those in the *Li Xiang Kao Cheng Hou Pian*. Yet Xu Youren did not give no diagrams, nor did he adequately explain his methods. It was Li Shanlan who wrote the *Tuo Yuan Zheng Shu Jie* with a major concern, as he himself said in the "Preface", for giving detailed explanations, accompanied with diagrams, of Xu Youren's methods as related the *Tuo Yuan Zheng Shu*. Since in the above reconstructions of Xu Youren's methods, which we owe to Niu Yahua [Cf. Niu 1989], we have already referred to Li Shanlan's illustrations, we shall not discuss further the *Tuo Yuan Zheng Shu Jie* here. Li Shanlan's concern with the ellipse, however, did not stop here, and so we devote the rest of this section to his studies on the subject.

In order to improve further the methods for astronomical computations involving the ellipse, Li Shanlan wrote his *Tuo Yuan Xin Shu* ("New Methods for the Ellipse"). In this treatise he gave even simpler methods for solving Problems 1 and 2 than had Xu Youren. For Problem 1 -- "Yi Jiao Qiu Ji", Li Shanlan introduced a new term, "Ping Yuan Mian Ji Jiao" (literally an angle in terms of the circle), and simplified Xu Youren's method by omitting computations of half-angle formulas, upon which Xu Youren had basically relied. Li Shanlan solved this problem by introducing a method of his own.

As in Figure 43, given $\angle AF_1P = \alpha$, Li Shanlan argued that

$$\frac{\tan \angle AF_1Q}{\tan \angle AF_1P} = \frac{a}{b} \Rightarrow \tan \angle AF_1Q = \frac{a}{b} \tan \alpha,$$

where the angle $\angle AF_1Q$ is the "Pien Yuan Mian Ji Jiao"

introduced by Li Shanlan. Therefore, in the triangle $\triangle OF_1Q$,

$$\frac{\sin \angle QOF_1}{c} = \frac{\sin (180^\circ - \angle AF_1Q)}{a},$$

and by the Sine Rule of the sides, since $\sin \angle AF_1Q$ is known, we can obtain the angle $\angle QOF_1$ and thereby the angle $\angle QOF_1 = \angle AF_1Q - \angle QOF_1$. On the other hand, construct the line F_1T such that it intersects the line OQ at the point T , so that $\overline{F_1T} \perp \overline{OQ}$. Since $\triangle ATF_1$ is similar to $\triangle ODQ$, we have $TF_1 : QD = OF_1 : OQ$. But $QD = a \sin \angle QOF_1$ and $OD = a \cos \angle QOF_1$, thus we can obtain TF_1 and

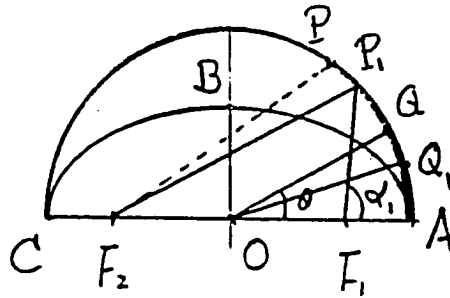


Figure 42. Given $\angle QOA = \theta$, find $\angle PF_1A$ such that

$$S_{PF_1A} = S_{QOA}.$$

thereby the area of the triangle ΔOQF_1 . now take in the arc QA a point Q_1 such that the arc length of the subarc QQ_1 is equal to TF_1 . Then the area of the sector AQ_1O is equal to $\frac{a}{b}$ times the area of the sector AF_1P . If we draw through the point Q_1 the perpendicular to $\overline{AC}$ intersecting the ellipse at the point P_1 , then the area of the sector AOP_1 is equal in area to the sector AF_1P , whereby the problem is solved.

Moreover, if the angle $\phi = \angle QOA$ and $\phi_1 = \angle Q_1OA$ are measured by radian units, then Li Shanlan's proportion $TF_1 : QD = OF_1 : OQ$,

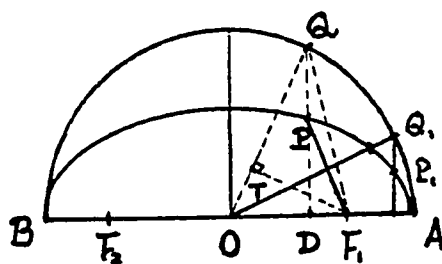
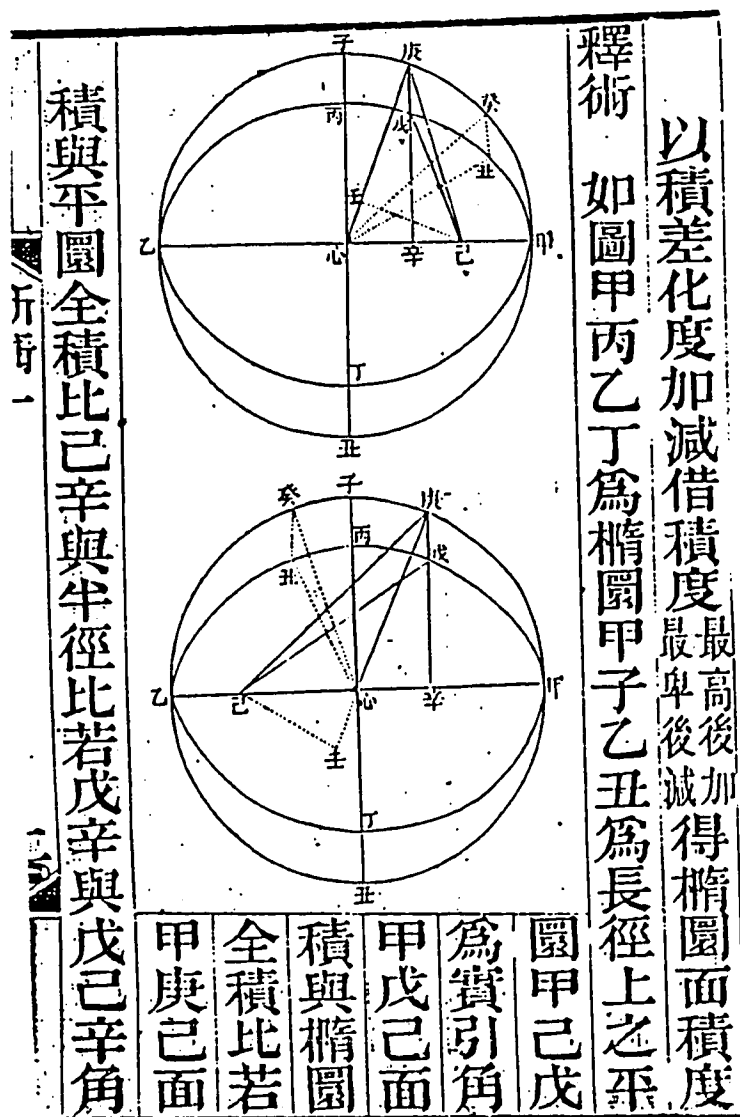


Figure 43. $\overline{F_1T} \perp \overline{OQ}$. The arc length of $QQ_1 = TF_1$.

This diagram is based on Li Shanlan's
illustrations above.

just mentioned above, leads to the proportion $a(\phi - \phi_1) : a \sin \phi = c : a$, or $\phi_1 = \phi - \frac{c}{a} \sin \phi$. The last equation is, of course, Kepler's equation [Stephenson 1987, pp. 166-167] which was introduced in the *Li Xiang Kao Cheng Hou Pian*, and in turn was derived by Xu Youren using a simpler method [Cf. Bo Shuren 1983].

For Problem 2 -- "Yi Ji Qiu Jiao", Li Shanlan gave his solution by the method of infinite series. Given the arc length for the arc AQ_1 of the (larger) auxiliary circle [Cf. Figure 43], which corresponds to the angle $\angle AOQ_1 = \phi_1$, Li Shanlan used Kepler's equation to find the angle $\angle AOQ$. In doing so, he re-wrote the equation in terms of the arc lengths q , p (corresponding to ϕ_1 , ϕ respectively), as follows:

$$q = p - \frac{c}{a} \sin \frac{p}{a} ,$$

in which $p = a\phi$, $q = a\phi_1$. Then he went on to expand the sine function and obtained the following infinite series by a method of converting series [Cf. his *Ji Shu Hui Qiu* (1867)]:

$$\begin{aligned} p = q &+ \left(\frac{c}{a} + \frac{c^2}{a^2} + \frac{c^3}{a^3} + \frac{c^4}{a^4} + \dots \right) q \\ &- \left(\frac{c}{a^3} + \frac{4c^2}{a^4} + \frac{10c^3}{a^5} + \frac{20c^4}{a^6} + \dots \right) \frac{1}{3!} q^3 \\ &+ \left(\frac{c}{a^5} + \frac{16c^2}{a^6} + \frac{91c^3}{a^7} + \frac{336c^4}{a^8} + \dots \right) \frac{1}{5!} q^5 \end{aligned}$$

$$- \left(\frac{c}{a^7} + \frac{64c^2}{a^8} + \frac{320c^3}{a^9} + \frac{5440c^4}{a^{10}} + \dots \right) \frac{1}{7!} q^7$$

$$+ \dots ,$$

whereby he found the angle $\angle QOA$. On the other hand, since $DQ = a \sin \angle QOA$, and

$$\frac{a}{b} = \frac{DQ}{DP} = \frac{a \sin \angle QOA}{DP}$$

$$\Rightarrow DP = b \sin \angle QOA ;$$

moreover, $DA = \text{versin } \angle QOA$, $DF_1 = c + DA - a$ and

$DP = DF_1 \tan \angle PF_1D$. Consequently, Li Shanlan obtained the angle $\angle PF_1D$, and concluded the angle $\angle PF_1A = 180^\circ - \angle PF_1D$.

In solving these two problems, Li Shanlan indeed gave detailed demonstrations to illustrate his methods [Cf. the *Tuo Yuan Xin Shu*]. His first method actually simplified the methods given in the *Li Xiang Kao Cheng Hou Pian*, as well as by Xu Youren. His second method was not only offered a more convenient way to obtain the desired angle than Xu Youren, it could also be regarded as solving Kepler's equation [Stephenson 1987, pp. 166-167] by a method of infinite series prevalent in modern celestial mechanics [Bo Shiren 1984]. Viewed mathematically, the method introduced infinite series for the first time in Chinese history to the computations involving the ellipse [Niu 1989, pp. 78-79].

The *Tuo Yuan Xin Shu* seems to represent a culmination of the studies on the ellipse prior to the *Dai Wei Ji Shi Ji* (Loomis' *Calculus*) [Cf. Section 4.4] and the *Yuan Zhui Qu Xian Shuo* ("On Conic Sections", by an anonymous author) [Cf. Section 4.5], which introduced more comprehensive knowledge about the ellipse, especially its theoretical aspects. Clearly, therefore, both the *Tuo Yuan Zheng Shu Jie* and the *Tuo Yuan Xin Shu*, or at least major portions of each, must have been written by the early 1850s. After his translations of the *Dai Wei Ji Shi Ji* and the *Yuan Zhui Qu Xian Shuo* were finished, Li Shanlan began to undertake further studies on the ellipse based upon those he knew from the two Western texts. According to his "Preface" to the *Tuo Yuan Shi Yi* ("Remarks on the Ellipse"), Li Shanlan explain why he wrote this treatise. Basically, he said, there was quite a lot to add to the *Yuan Zhui Qu Xian Shuo*. Moreover, the ellipse was so crucial to astronomy, that he gave the treatise as a supplement.

Nevertheless, the *Tuo Yuan Shi Yi* should not be regarded a less significant among the late 19th-century Chinese studies on the ellipse than either the *Dai Wei Ji Shi Ji* or the *Yuan Zhui Qu Xian Shuo*. In the *Tuo Yuan Shi Yi*, Li Shanlan paid considerable attention to theoretical aspects of the ellipse and related them to various construction problems about the ellipse. More significant was the fact that he actually created some interesting propositions of his own, although they have lagged behind their Western counterparts by the time Li Shanlan was writing. No matter how we evaluate his studies on this subject, the results he

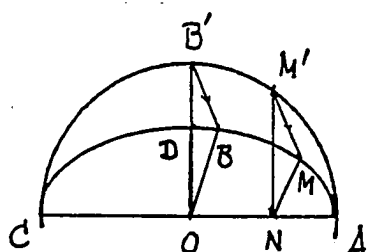
obtained once again confirm that he was able to assimilate transmitted Western mathematics in a very short time.

The *Tuo Yuan Shi Yi* is comprised of three books with a total of 44 propositions. In Book 1, Li Shanlan dealt mainly with the theory of ellipse, giving some significant contributions of his own. For example, he proved the Fundamental Principle of the Ellipse using by a method of parallel projection, by which he viewed the ellipse as a projection of its auxiliary circles [Cf. Figure 44]. The *Dai Wei Ji Shi Ji* (Loomis' *Calculus*) had already given a proof in the context of analytic geometry [Cf. Proposition 4 of Chapter 6]. Li Shanlan's proof was shorter and simpler, largely because he treated the proposition in the context of descriptive or projective geometry. The question now arises: What did he learn from Western texts on these subjects? Nothing was available from Western text on projective geometry during his service at the LMS Press. In fact, it seems that no such a text was brought to China at the time. Consequently, if the method of parallel projection was not an original creation of his own, then Li Shanlan may have been familiar with the studies on the theory of perspective by Nian Xiyao (?-1738).

Nian Xiyao learned the theory of perspective from the Italian artist Joseph Castiglione (1688-1766 or 1768), who visited China in the early 18th century. According to the historians of Chinese mathematics Shen Kangshen and Li Di, Nian Xiyao made substantial



Figure 44.
Title page
of the Tuo
Yuan Shi Yi
and its
Proposition 1.



The left is a duplicate
of the diagram shown
above. Since $\triangle OBB'$ is
similar $\triangle NMM'$, so
 $OB' : OB = NM' : NM$
 $= a : b$, where $a = OA$,
 $b = OD$.

contributions to the theory of descriptive geometry, thanks to inspiration from the Italian artist [Shen Kangshen 1982; Li Di 1984, pp. 277-285]. In fact, Nian Xiyao in his *Shi Xue* ("The Theory of Perspective", 1729 1st ed., 1735 rev. ed.), gave a comprehensive treatment of descriptive geometry in which the method of parallel projection was discussed. Nevertheless, it is unlikely that Li Shanlan ever read this treatise due to its limited circulation.

Li Shanlan also used the method of parallel projection to prove Proposition 3 which was his generalization of the Fundamental Principle of the Ellipse. It related the parallel chords of the ellipse to those of a circle with diameter equal to a diameter of the ellipse [Cf. Figure 45]. In order to explain his method, Li Shanlan even regarded the ellipse and the corresponding circle as two sections of an elliptical cylinder (a cylinder with an ellipse as base) cut by two slant planes respectively. Li Shanlan then viewed the circle as the parallel projection along the direction of the generator of the elliptical cylinder. But how did he create such an elliptical cylinder, thereby linking it with the ellipse and the corresponding circle? A rational reconstruction of his method is of interest here, because this interesting method seems to have been entirely his own.

As shown in Figure 46, given an ellipse ACBD, take any diameter, say AB, as the major axis of a new ellipse Γ . Then,

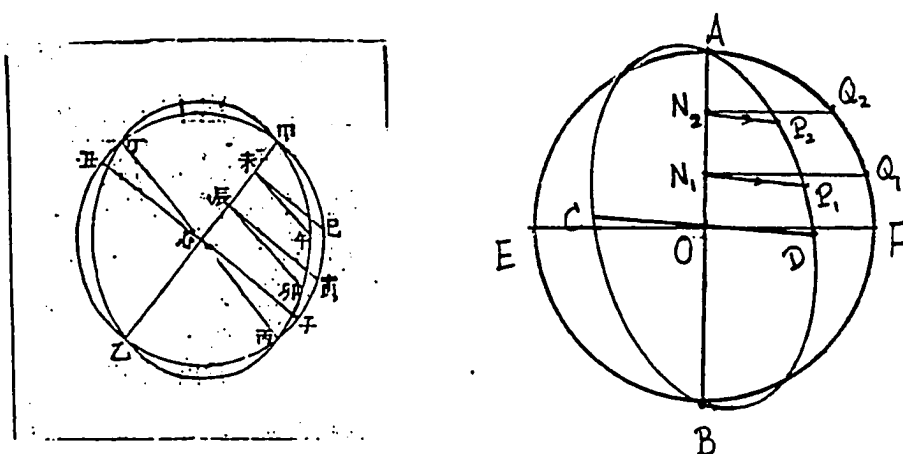


Figure 45. Li Shanlan's illustrative diagram for Proposition

3 and our duplicate in which $ACBD$ is the given ellipse and $AEBF$ is the circle drawn on the diameter $\overline{AB}$ of the ellipse. If in the ellipse, $\overline{N_1P_1} \parallel \overline{N_2P_2} \parallel \overline{CD}$, where $\overline{CD}$ is the conjugate diameter of $\overline{AB}$, and in the circle, $\overline{N_1Q_1} \parallel \overline{N_2Q_2} \parallel \overline{EF}$, where $\overline{EF}$ is a diameter perpendicular to $\overline{AB}$, then

$$\frac{N_1P_1}{N_1Q_1} = \frac{N_2P_2}{N_2Q_2} = \frac{OD}{OF} = \frac{OD}{OA}.$$

take the minor axis of the given ellipse as that of the new ellipse. Now construct an elliptical cylinder with base $A'C'B'D'$ congruent to Γ . Any one of its sections parallel to the base will also be an ellipse congruent to Γ . Let Γ' , Γ'' denote two such sections. Let a slant plane Ω , which passes through one endpoint of the minor axes of Γ' and the major axis of Γ'' , cut the cylinder such that a circular section $AEBF$ results with diameter AB . Moreover, take another plane Ω' , which passes through one endpoint of the major axis of Γ' and the minor axis of Γ'' to produce an elliptical section which is the same as the given ellipse $ACBD$. Obviously $\overline{AB}$ is the intersection of the planes Ω and Ω' . On the other hand, construct through the point B the tangents BT and BS to the ellipse $ABCD$, $AEBF$ respectively. As for the diameter $\overline{AB}$ of the ellipse $ABCD$ and of the circle $AEBF$, we have their conjugate diameter $\overline{CD}$ and another diameter $\overline{EF}$, respectively. Finally, in the ellipse $ABCD$ draw $\overline{NP}$, a line segment Li Shanlan called the "Tuo Yuan Zheng Xuan", such that it is one half the chord parallel to the (conjugate) diameter $\overline{CD}$. Similarly, in the circle $AEBF$ draw $\overline{NQ}$ such that it is one half the chord parallel to the diameter $\overline{EF}$. Thus Li Shanlan obtained the following proportion:

$$\overline{OD} : \overline{OF} = \overline{NP} : \overline{NQ} ,$$

and he even claimed that

$$\overline{OD} : \overline{OF} = \overline{N_1P_1} : \overline{N_1Q_1} = \overline{N_2P_2} : \overline{N_2Q_2} = \dots = \overline{N_1P_1} : \overline{N_1Q_1} ,$$

where $\overline{OD}$, $\overline{N_1P_1}$, $\overline{N_2P_2}$, ..., $\overline{N_1P_1}$, were the "Tuo Yuan Zheng Xuan" of the ellipse ABCD, and $\overline{OF}$, $\overline{N_1Q_1}$, $\overline{N_2Q_2}$, ..., $\overline{N_1Q_1}$, were their respective parallel projections on the circle AEBF.

Proposition 4 was a restatement of the relationship between the areas of the ellipse and those of its two auxiliary circles, which, as mentioned above, was introduced in the *Li Xiang Kao Cheng Hou Pian*. Subsequently, in Proposition 5, Li Shanlan generalized this relationship to link the area of the ellipse to that of the circle with diameter equal to a diameter of the ellipse. As shown in Figure 47, given the ellipse CEDF with $\overline{CD}$ and $\overline{EF}$ as the two conjugate diameters, and the circle ABCD with diameter CD, then by Proposition 4 and Proposition 10 of the *Yuan Zhui Qu Xiang Shuo*, Book 1, Li Shanlan concluded that $S_e : S_c = S_{IJ} : S_{MN} = (\overline{CD} \cdot \overline{GF}) : \overline{CD}^2$, where S_e , S_{IJ} were the areas of the ellipse CEDF and that of its circumscribing parallelogram, respectively, while S_c , S_{MN} were the areas of the circle ABCD and that of its circumscribing square. Consequently, Li Shanlan obtained the desired result:

$$S_e : S_c = (\overline{CD} \cdot \overline{GF}) : \overline{CD}^2 = \overline{GF} : \overline{CD},$$

in which Li Shanlan called the side $\overline{GF}$ Li Shanlan the "Shu Jin Gu" -- literally the "Gu" side (the longer leg) of the right-angled triangle EFG with diameter $\overline{EF}$ of the ellipse as the hypotenuse.

Like the preceding propositions, Propositions 6 through 9

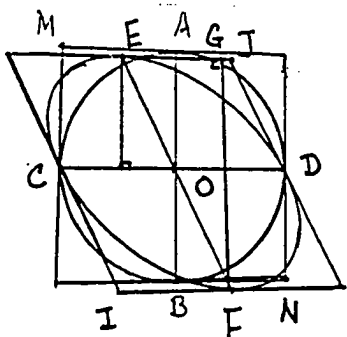
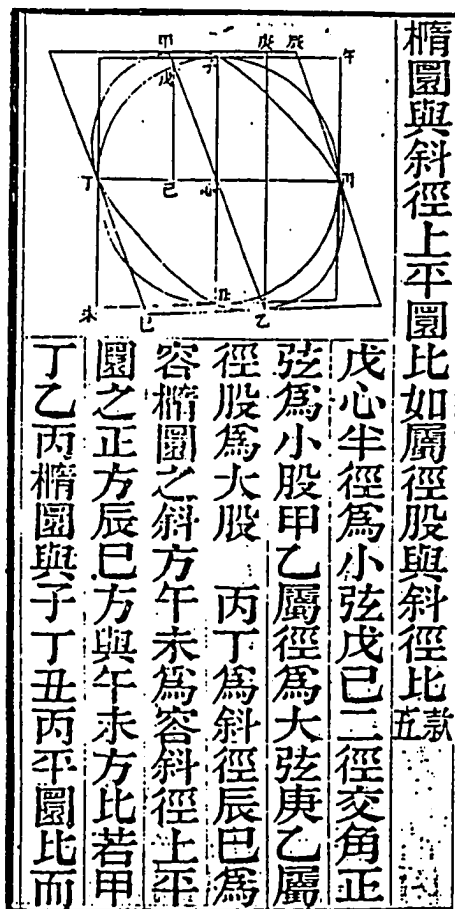


Figure 47. Proposition 5 and its graphical illustrations. Our duplicate is on the left.

were all about relationships between an ellipse and its related circles. To these propositions, Li Shanlan added something new, which marked their first appearance in the history of Chinese mathematics, although these contributions were not so significant in terms of the theory of conics *per se*. After all, the Western revival of synthetic projective geometry in the early 19th century had led French and German mathematicians to realize that the ellipse (or any other single conic section) was no longer an interesting subject for its own sake, but rather that conics as a whole were consequently a central issue in geometric research [Cf. Kline 1972, pp. 834-860].

Li Shanlan's success in handling the theory of the ellipse was especially significant in the context of Chinese mathematics of the 1860's, primarily because, as mentioned above, he was among the first of the few major Qing mathematicians who paid great attention to the subject.

As Figure 48 shows, Li Shanlan (in Proposition 6) claimed that

- (1) the areas of $\triangle ABC$ and $\triangle BDC$ were to one another as the major and minor axes of the ellipse ; similarly
- (2) the areas of hexagons were to one another as the major and minor axes of the ellipse.

Proposition 7 was a generalization of Proposition 6, just as Proposition 3 was a generalization of Proposition 2. In

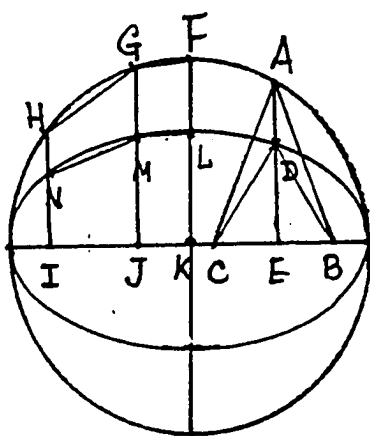
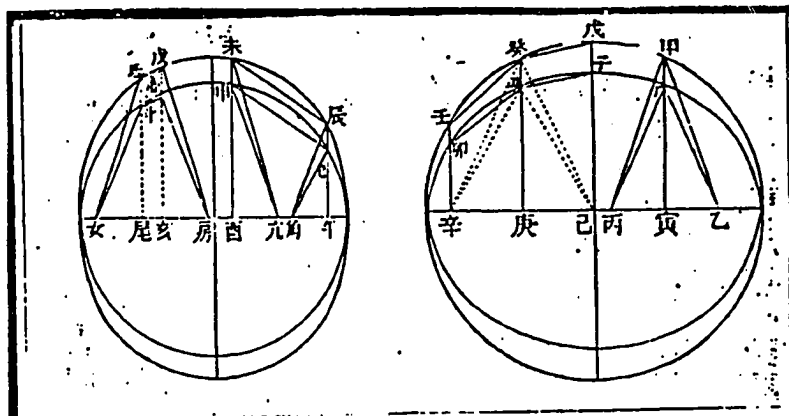


Figure 48. Graphical illustrations for Proposition 6. On the left, we only reproduce part of the figure on the right.

Proposition 7, Li Shanlan took the circle corresponding to diameter of the given ellipse. Thus as in Figure 49, Li Shanlan proved that

$$S_{\triangle ABC} : S_{\triangle ADC} = \overline{KQ} : \overline{MN} = (\overline{KL} \cdot \sin \angle KON) : \overline{MN}$$

by **Proposition 3**, where $\overline{MN}$, $\overline{KL}$ were the conjugate diameters of the ellipse and MN was the diameter of the circle; moreover, in the two triangles $\triangle ABC$ and $\triangle ADC$ with the same base $\overline{AC}$, $\overline{BE} \perp \overline{MN}$ and $\overline{DE}$ was parallel to $\overline{KL}$. Similarly, Li Shanlan also proved the case with the quadrilaterals as shown in Figure 49.

In **Proposition 8** Li Shanlan proved that, as in Figure 50,

$$\overline{PN} : \overline{QN} = \overline{LK} : \overline{PK}.$$

Here $\overline{PN}$, $\overline{QN}$ were respectively the "Tuo Yuan Zheng Xuan" drawn perpendicular to the major axis of the ellipse, and the "Zheng Xuan" (one half the chord) of the circle with radius equal to the major axis. Moreover, $\overline{LK}$, called by Li Shanlan the "Pien Yuan Yu Xuan", was the "Zheng Xuan" of the circle corresponding to the diameter of the circle, which was orthogonal to the previous one. While $\overline{PK}$, called by Li Shanlan the "Tuo Yuan Yu Xuan", was the "Tuo Yuan Zheng Xuan" perpendicular to the minor axis of the ellipse.

In **Proposition 9** Li Shanlan again considered the case of

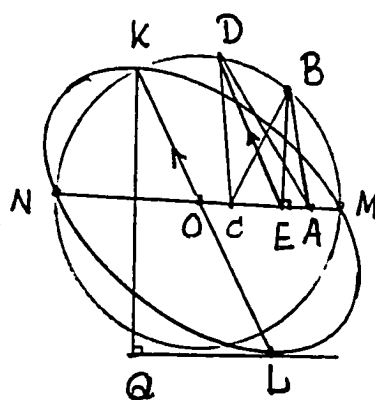


Figure 49. Proposition 7 and its graphical illustration.

Our duplicate is on the right.

the circle having the diameter equal to a diameter of the given ellipse. Thus by Proposition 3 he proved that, as in Figure 51,

$$\overline{NP} : \overline{NQ} = \overline{OD} : \overline{OA} = \overline{MP} : \overline{MR} ,$$

where $\overline{NP}$, $\overline{MP}$ were the "Tuo Yuan Zheng Xuan", "Tuo Yuan Yu Xuan" of the conjugate diameters $\overline{AB}$, $\overline{CD}$ respectively; while $\overline{NQ}$, $\overline{MR}$ were the "Pien Yuan Zheng Xuan", "Pien Yuan Yu Xuan" of the circles with diameters $\overline{AB}$, $\overline{CD}$ ($\overline{AB} > \overline{CD}$) respectively.

Discussion of the rest propositions of Book 1, namely, Propositions 10-20, will be omitted here because they were not so systematic as the preceding nine. For example, they dealt with problems like the relationship between a given ellipse (with a , b as the lengths of the two axes) and the circle (with diameter $\sqrt{ab}$), the properties of the triangle inside the ellipse and with the base on the major axis of the ellipse as well as some further problems dealing with the ellipse.

Book 2 of the *Tuo Yuan Shi Yi* is comprised of 9 propositions, namely, Propositions 21-29, all of which concerned with constructions related to the ellipse. In addition, they all shared one condition in common, that is, they all assumed one given focus. Here for brevity we choose a simple example, namely Proposition 23, to explain his method.

In Proposition 23, Li Shanlan wanted to construct the other

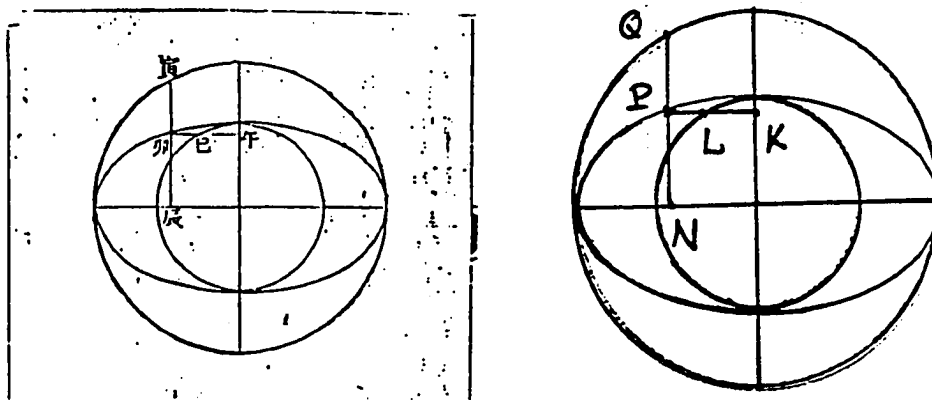


Figure 50. Graphical illustration for Proposition 8.

Our duplicate is on the right.

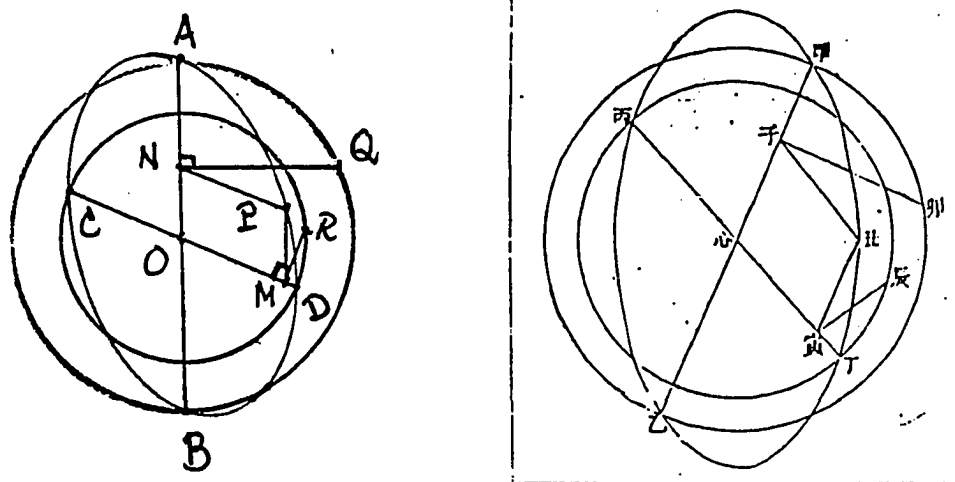


Figure 51. Graphical illustration for Proposition 9.

Our duplicate is on the left.

focus given one focus, two points on the ellipse and a tangent to the ellipse through one of the two points. As in Figure 52, if A and B are the two given points on the ellipse, CD the given tangent to the ellipse at the point B, and F_1 is the given focus, then, Li Shanlan proceeded to construct the second focus as follows: (1) connect the lines AF_1 , BF_1 respectively; construct the point E such that $\angle DBE = \angle CBF_1$. (2) extend the line EB to the point G such that $BG = BF_1 - AF_1$. (3) about each of the points A and G as centers, draw circles of equal radius greater than half the length of AG, and locate the points M, N where the two circles meet. (4) connect the line MN to meet the line BE at the point F_2 , then F_2 will be the desired focus.

Li Shanlan went on to justify his method of construction. Based on the property of the tangent to the ellipse, which he might have referred to Proposition 9 of the *Yuan Zhui Qu Xian Shuo* Book 1, the line BE goes through one of the foci. On the other hand, since $AF_1 + AF_2 = AF_1 + GF_2 = AF_1 + BG + BF_2 = AF_1 + BF_1 - AF_1 + BF_2 = BF_1 + BF_2 = 2a$ (the length of the major axis), consequently the intersection point F_2 of the perpendicular bisector MN of the line segment $\overline{GA}$ with the line BE must be the desired focus. At the end of his justification, Li Shanlan added a remarks that if the line MN were paralell to the line GE, then the given conic would be a parabola; and if the intersection point F_2 were on the other side of the tangent, then the conics would have to be a hyperbola.

In fact, Li Shanlan had become interested in constructions of conics in general, and not just the ellipse, when he was co-translating the *Yuan Zhui Qu Xian Shuo* with Edkins. Thus in Proposition 4 of Chapter 3, Li Shanlan added two methods of his own which were simpler than that given in the text for the construction of parabola [Cf. Section 4.5 and Figure 3 there].

In Book 3 of the *Tuo Yuan Shi Yi*, Li Shanlan used the methods he had just learned from the calculus to facilitate the astronomical computations in terms of the ellipse. But since he added nothing essentially new to the results he had already dealt with concerning astronomical computations in the *Tuo Yuan Xin Shu*, we conclude our discussion of Li Shanlan's treatise on the theory of ellipse here.

Li Shanlan's close friends like Gu Guanguang and Hua Hengfang were also interested in the theory of conic sections. However, Gu Guanguang's essay, "Yuan Zhui San Qu Xian Ji (On the three conic sections)", was little more than notes he had taken on the subjects from the *Dai Wei Ji Shi Ji* and *Yuan Zhui Qu Xiang Shuo* [Gu 1882a]. On the other hand, Hua Hengfang published his *Pao Wu Xiang Shuo* ("On the Parabola") in 1859, in which he was interested in how the curve could be applied to the projectile motion. According to Ding Fubao, this treatise offered detailed explanations of Li Shanlan's *Huo Qi Zhen Jue* ("On the Projectile Motion of Cannon"), therefore it was not concerned with the ellipse, nor did it have much to do with the geometrical aspects

of the theory of conics in general [Cf. Ding Fubao *et al.*, eds. 1984, p. 66]. As for Li Shanlan's successors who were interested in the theory of conics, we should mention Xia Luanxiang (1823-1864), for he, just like Li Shanlan, was concerned with the synthetic geometry of the conics. However, since his studies on the subject were mainly influenced by Western texts like the *Dai Wei Ji Shi Ji* and the *Yuan Zhui Qu Xian Shuo*, we shall return to him in Section 4.5 below.

3.7. On the Cannon: The *Huo Qi Zhen Jue*

The *Huo Qi Zhen Jue*, as mentioned in Section 2.6, was a short treatise dealing with projectile motion. In fact, its major concern was to simplify the related methods introduced in the *Zhong Xue* (Whewell's *Mechanics*), in which Book 11 was devoted to projectile motion. More specifically, the theme of the *Huo Qi Zhen Jue* was closely related to the first three propositions given in Book 11 of the *Zhong Xue* [Liu Dun 1984]. In fact, Whewell gave these three propositions to deal with the maximum ranges a projectile could reach both on the ground and against an inclined plane. Yet Li Shanlan realized the methods or demonstrations given in the *Zhong Xue* were too complicated to follow. He seems to have been concerned with providing a simpler explanation for these methods while he was still working, in collaboration with Edkins, on the translation of the *Zhong Xue* in 1858. As the *Zhong Xue* was published at the beginning of 1859, Li Shanlan began to undertake this problem enthusiastically. By working hard "over

several winter nights", he eventually discovered that the theory of circle could serve as an alternative in treating projectile motions.

For convenience of discussion we need to cite here the methods given in the *Zhong Xue*. Let v , g , and S denote respectively the initial velocity of a projectile pointing in a given direction at an elevation angle α with the (horizontal) ground, the gravitational acceleration and the range the projectile would reach. Then Proposition 1 of Book 11 could be expressed as follows:

$$S = \frac{v^2}{g} \sin 2\alpha .$$

If one further assumes that the projectile is thrown against an inclined plane at an angle β with the ground and the elevation angle α is measured from the inclined plane, then Proposition 2 asserts [Cf. Figure 53].

$$S' = \frac{2v^2 \sin(\alpha - \beta) \cdot \cos \alpha}{g \cos^2 \beta} .$$

In Proposition 3 Whewell demonstrated how the projectiles in these two cases could actually reach their respective maximum ranges:

$$S_{\max} = \frac{v^2}{g} \quad (\alpha = 45^\circ) ;$$

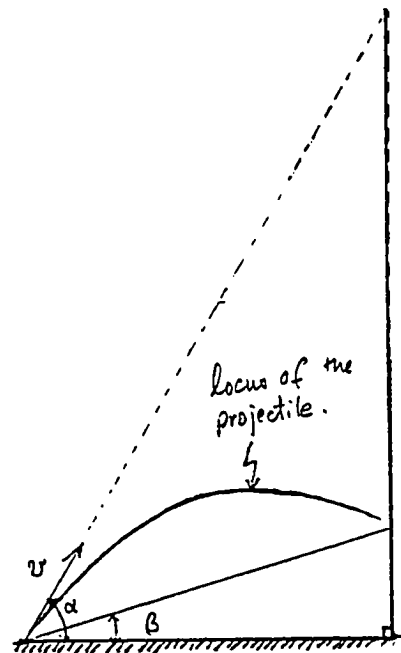
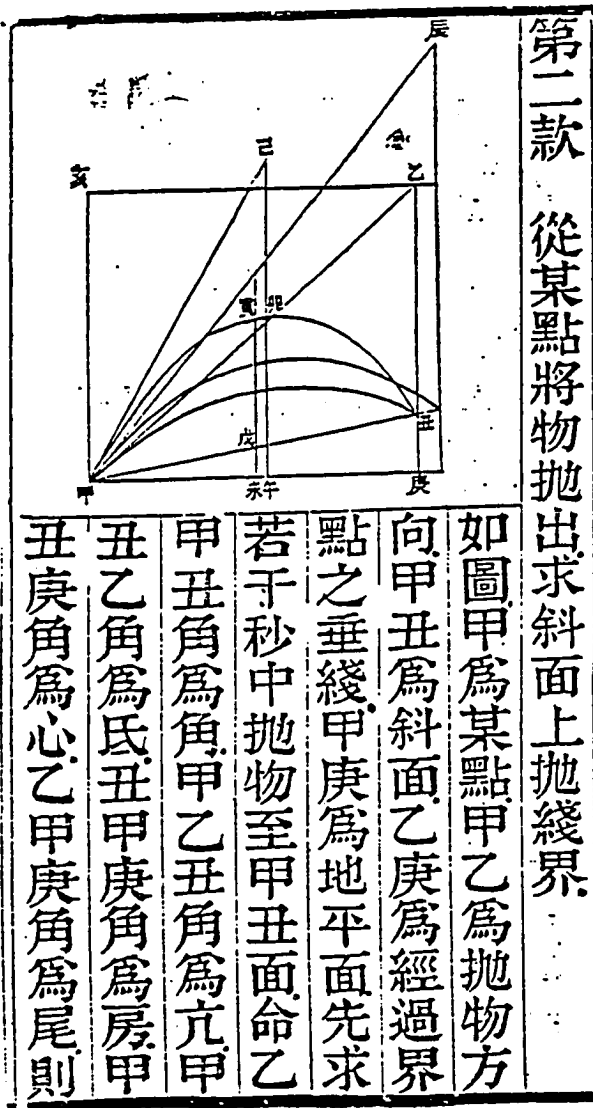


Figure 53. Proposition 2 and its illustration of
Book 11, the Zhong Xue. The diagram
downright is a duplicate of the
illustration.

$$S'_{\max} = \frac{v^2}{g(1 + \sin\beta)} \quad \left(\alpha = \frac{1}{2} (90^\circ + \beta) \right) .$$

Whewell's demonstrations for the above three propositions were quite appropriate by modern standards. For example, for the projectile to reach the maximum ranges in either case, Whewell was right to argue that $\sin 2\alpha$ and $\sin(\alpha-\beta) \cdot \cos\alpha$ ($= \sin(2\alpha-\beta) - \sin\beta$) should take their maximum values respectively. What Li Shanlan did was to determine more conveniently the elevation angles of the projectiles in question. His strategy was then to rely entirely upon the theory of the circle, which looked simpler to him perhaps, in treating the same problems. Doubtless he added nothing essentially new to the theory of projectile motion. Nevertheless, Li Shanlan did make some significant contributions to the theory for practical applications. In fact, the *Huo Qi Zhen Jue* was, according to Liu Dun, a graphic text on ballistics that was entirely his own [Liu Dun 1984].

The *Huo Qi Zhen Jue* included twelve propositions. The first two stressed the general requirements by which cannons and shells should be manufactured. Thus they actually had nothing to do with what is usually related called a "proposition". This was also true of the last so-called proposition in which Li Shanlan introduced a cannon guide rod and described its manipulation in order to fire a cannon at required elevation angle.

On the other hand, all of the nine remaining propositions

were really "propositions" since they could be justified in the theory of projectile motion. For example, Propositions 3 and 4 reasserted the conclusions (for the two cases, respectively) drawn from Proposition 3 in Book 11 of the *Zhong Xue*.

Granted that Whewell's Proposition 3, as mentioned above, was true, Li Shanlan told his readers in Proposition 3 how to determine the elevation angle 45° for $S_{\max}$, and in Proposition 4 he gave a method to determine the elevation angle $\frac{1}{2}(90^\circ + \beta)$ for $S'_{\max}$. Now if the maximum ranges $S_{\max}$ or $S'_{\max}$ were known for the cannons and the shells, then Li Shanlan showed in Proposition 5 that the shells were not able to go any further $S_{\max}$ or $S'_{\max}$. Under such circumstances he warned that cannoneers should not waste the shells.

For any target within the maximum range which a given cannon could reach, Li Shanlan showed in Proposition 6 that there were exactly two elevation angles α_1 , α_2 for the cannon to hit the target. Moreover, with the aid of two diagrams [Cf. Figure 54], he made it clear that $\alpha_1 + \alpha_2 = 2\alpha$, where α is the elevation angle for the maximum range in either case. But Li Shanlan did not show how to determine either the angle α_1 or α_2 ; he left this problem to Proposition 9.

In Proposition 7, Li Shanlan proposed a method to find the $S_{\max}$ for a newly manufactured cannon. As he argued, the $S_{\max}$, usually was too far to measure as the cannon fired a shell with

the elevation angle 45° . In resolving such a troublesome situation, he formulated the following problem:

Given a cannon fired at the inclination angle $\alpha > 45^\circ$ and its corresponding range S . Find its $S_{\max}$ [Cf. Liu Dun 1984].

Li Shanlan also explained why one had better choose α to be greater than 45° , since if one chose α to be less than 45° (by which the cannon would also hit the same target at a distance S), then the shell, due to sliding along the ground, would make an inaccurate range S after hitting the ground. Now for $\alpha > 45^\circ$ and its corresponding S , Li Shanlan constructed a diagram to show how he got the $S_{\max}$ [Cf. Figure 55]. First he constructed a right-angled triangle APB such that $\angle B = 90^\circ$, $\angle P = \alpha$, and $PB = S$. Then he took from the line AB the point C such that $PC \perp PA$, and concluded that $\frac{1}{2} (BC + AB)$ was the desired $S_{\max}$. As the illustration for Proposition 8 shows, Li Shanlan certainly knew that $\frac{1}{2} (BC + AB)$ was the radius of the circle circumscribing the right-angled triangle APC . However, he did not show this important property of the circle in his diagram [Cf. Figure 56].

Nevertheless, the "circle" began to appear in the diagrams for Propositions 8, 9, 10 and 11. Thus in Proposition 8, Li Shanlan pointed out clearly that if the $S_{\max}$ was taken as a radius to draw a circle, then, as the accompanying diagram shows [Figure 56], the chord AD subtending one quarter of the circumference of

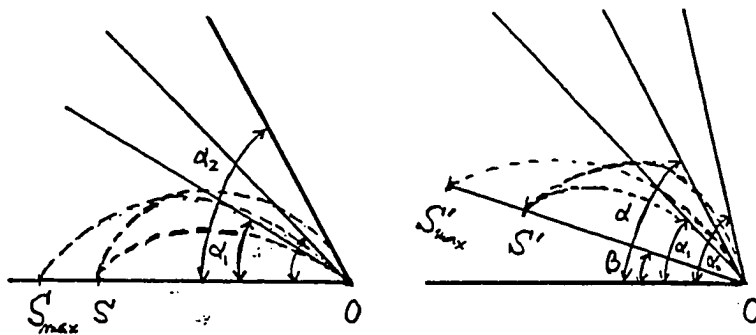
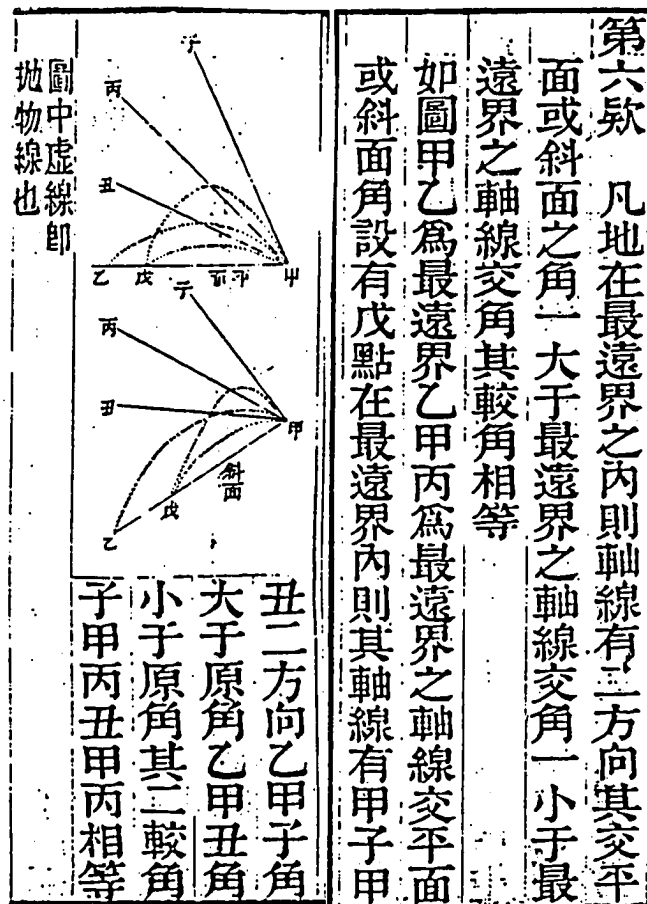


Figure 54. Proposition 6 of the *Huo Qi Zhen Jue* (1867).

Below the copy of the page on which the proposition appears are our duplicates of its illustrations.

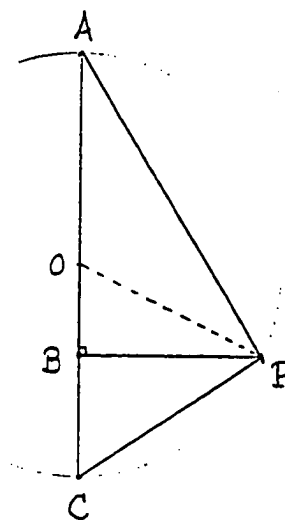
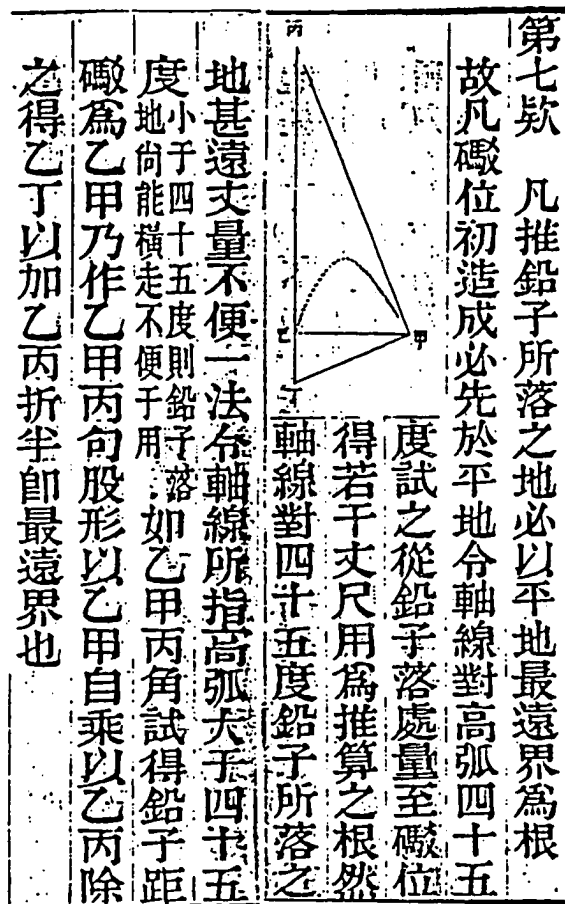


Figure 55. Proposition 7 of the *Huo Qi Zhen Jue*.

The duplicate of its illustration is
on the right.

the circle would indicate the direction for a shell to reach its $S_{\max}$ and to hit the ground at the center O. Although such a proposition looked trivial indeed, Li Shanlan did not stop there. In Proposition 9 he dealt with a general problem of how the two elevation angles could be determined for a cannon to fire so that a shell could hit the target with the required range $S < S_{\max}$. His solution is quite simple and easy to follow if one refers to his diagram for Proposition 9 [Cf. Figure 57]: Draw a circle with the center O and the radius S so that $OP = S_{\max}$ and $OM = OA = S$. Through the point M erect a perpendicular intersecting the circle at the points P_1, P_2 draw respectively the lines P_1B_1, P_2B_2 parallel to the line OP. Consequently, the directions determined by the lines P_1A (or $\angle AP_1B_1$), P_2A (or $\angle AP_2B_2$) will provide the desired results.

Now we come to Li Shanlan's diagram for the relationship between $S_{\max}$ and $S'_{\max}$, upon which he was to determine the elevation angle α for the $S'_{\max}$. This is what his Proposition 10 was all about. Following Figure 58, he constructed a right-angled triangle AOB with $\angle B = 90^\circ$, $\angle BPD = \beta$ and $AB = S_{\max}$, then draw a circle with the center O and the radius OA. Let the point P be the intersection point of the circle with the line OB. Through the point P he drew a line PC which was perpendicular to the line OA and intersected the line AB at the point D. Since $S_{\max} = AB = PD + BD$, $\sin\beta = \frac{BD}{PD}$ and

第八款 以最遠界爲半徑作平圓過圓心作地平線置礮圓周則九十度通弦爲礮軸方向圓心爲鉛子所落之處

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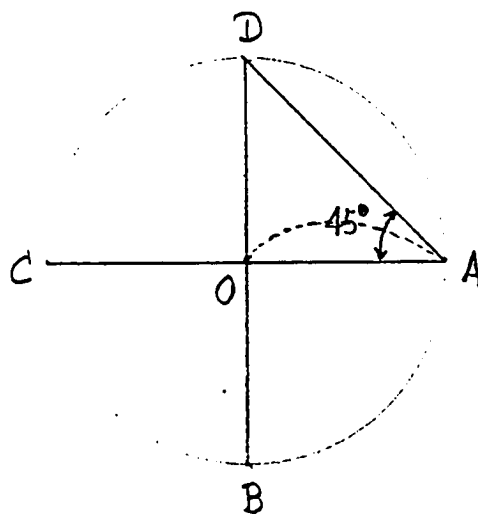
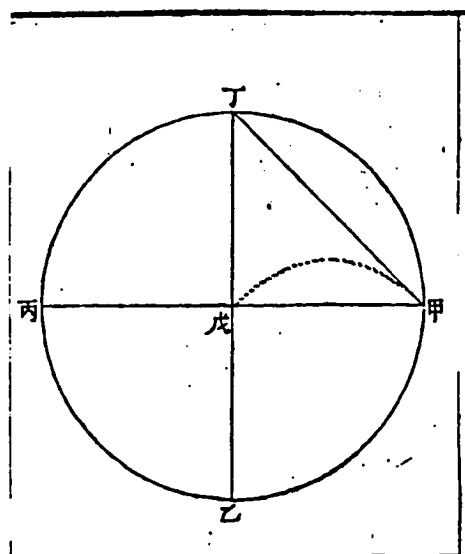


Figure 56. Proposition 8 of the *Huo Qi Zhen Jue*

and the duplicate of its illustration.

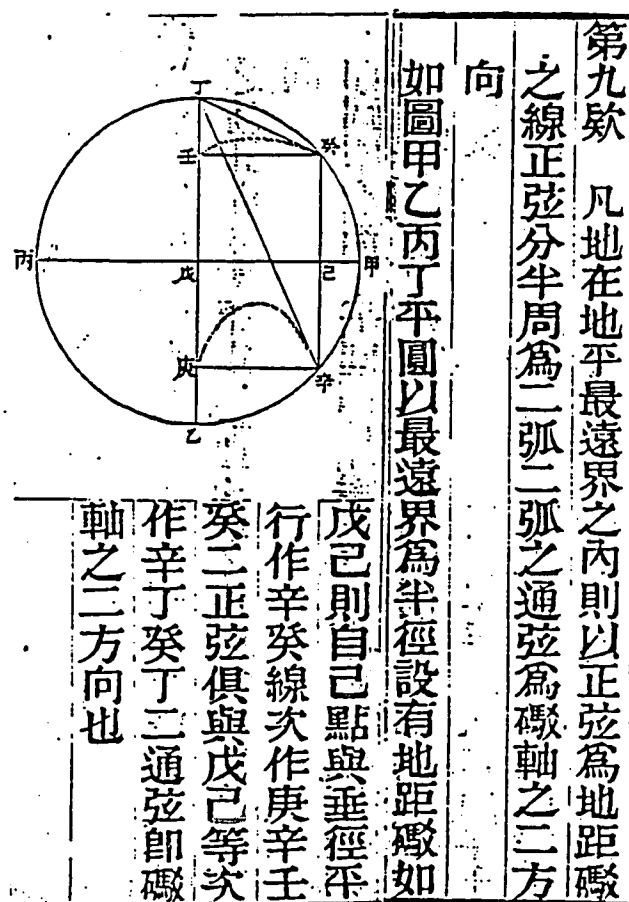


Figure 57. Proposition 9 of the *Huo Qi Zhen Jue*. On the left is our duplicate of its illustration.

$$\frac{S_{\max}}{S'_{\max}} = 1 + \sin\beta = \frac{PD + BD}{PD}.$$

Therefore $S'_{\max} = PD$ and its corresponding elevation angle with respect to the horizontal line would be $\angle APO = \frac{1}{2} (90^\circ + \beta)$.

On the other hand, Li Shanlan also considered the problem of how the elevation angle of a cannon with respect to an inclined plane at an angle β with the horizontal ground could be determined if the $S_{\max}$ were given. He gave the solution to his problem in Proposition 11. Again he drew upon the properties of the circle to construct his diagram. Given the $S_{\max} = AB$ as in Figure 59, construct $PD = S'_{\max}$ in just the same way as in Figure 58. Moreover, Li Shanlan chose a point M on the line CP such that $DM = S'$, the range by which he was going to determine the elevation angle α with respect to the inclined plane at an angle β with the horizontal line. Through the point M he then erected a perpendicular to the line OP intersecting the circle at the points P_1 and P_2 . He then drew two parallel lines through the points P_1 and P_2 respectively, both of which were parallel to the line OP and intersected the line AB at the points B_1 and B_2 , respectively. Consequently, he concluded that $\angle AP_1B_1$ and $\angle AP_2B_2$ would be the desired angles by which the range S' would be reached.

Li Shanlan's assertion for Proposition 11 certainly was correct. Based on our duplicate diagram [See Figure 59], we can

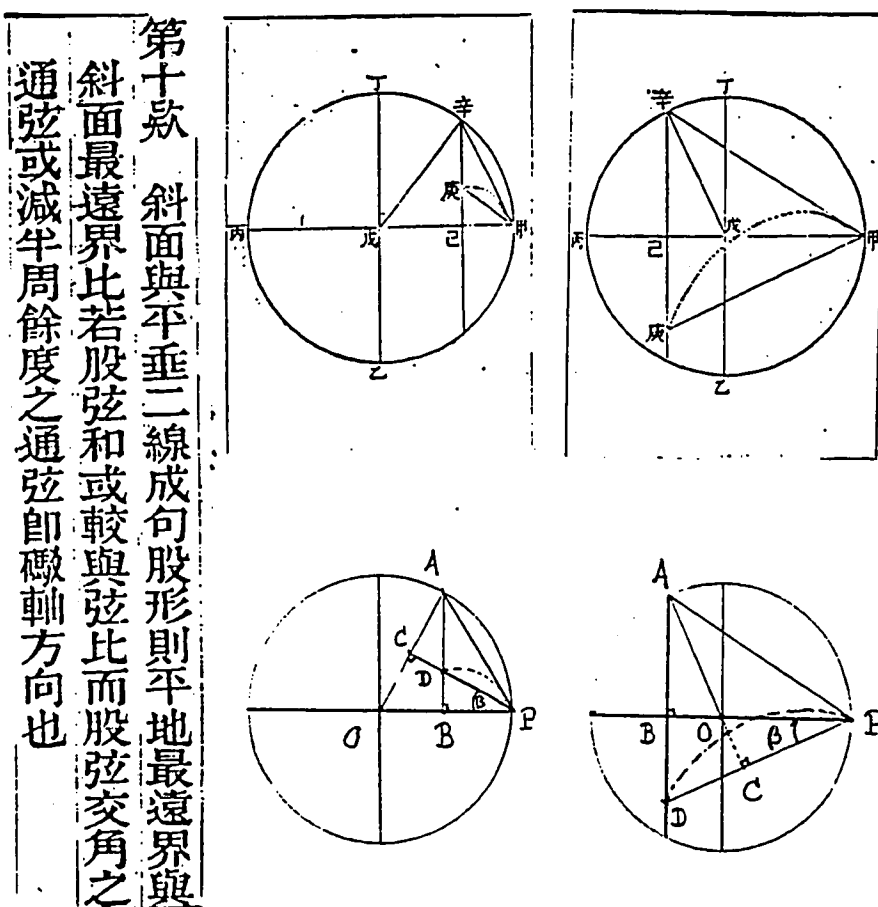


Figure 58. Proposition 10 of the *Huo Qi Zhen Jue*.

In the duplicates of its illustrations
 we can see how Li Shanlan considered
 two cases with the inclined plane.

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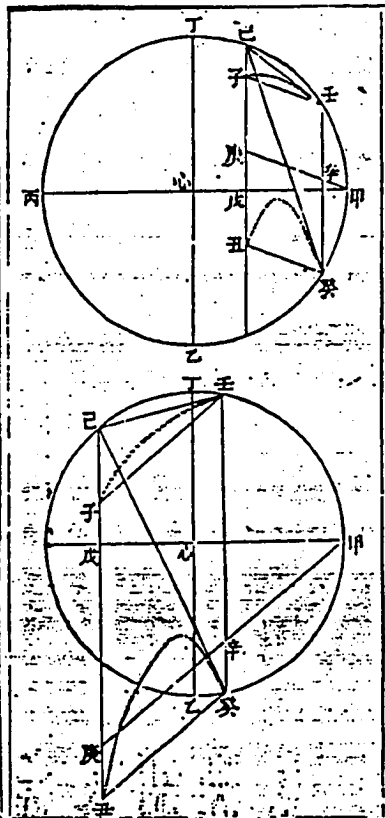
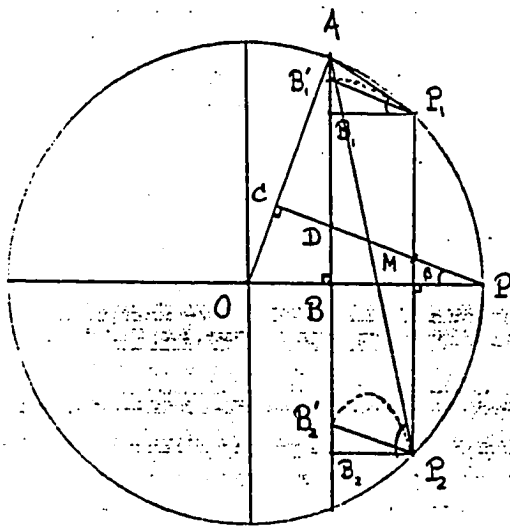


Figure 59.

Proposition 11 of the *Huo Qi Zhen Jue*. On the downleft, we duplicate one of the illustrations above.



$$AB = S_{\max} ; PD = S'_{\max} ;$$

$$MD = S'; \angle CPO = \beta.$$

reconstruct his justification as follows: Since

$$P_1A = 2 \sin \left(\frac{1}{2} \angle AOP_1 \right) = 2 \sin (\angle AP_1B_1 - \beta) ,$$

$$P_1B_1 = P_1A = P_1A \cos \angle AP_1B_1 ,$$

thus

$$\frac{DM}{PD} = \frac{P_1B_1}{PB} = \frac{2 \sin (\angle AP_1B_1 - \beta) \cdot \cos \angle AP_1B_1}{1 - \sin \beta} .$$

But on the other hand,

$$\frac{DM}{PD} = \frac{S'}{S'_{\max}} = \frac{2 \sin (\alpha - \beta) \cdot \cos \alpha}{1 - \sin \beta}$$

we have

$$\sin (\angle AP_1B_1 - \beta) \cdot \cos \angle AP_1B_1 = \sin (\alpha - \beta) \cdot \cos \alpha$$

$$\Rightarrow \sin (2\angle AP_1B_1 - \beta) - \sin \beta = \sin (2\alpha - \beta) - \sin \beta .$$

$$\Rightarrow \angle AP_1B_1 = \alpha , \text{ one of the desired elevation angle.}$$

Similarly, we can also show that $\angle AP_2B_2$ is the other desired elevation angle [Liu Dun 1984].

Here it should be mentioned that Proposition 11 was the most

comprehensive of Li Shanlan's nine propositions. This is because for $\beta = 0^\circ$, the coincidence of the points D and O would lead in this special case of Proposition 11 to Propositions 7, 8 and 9; while if $S' = S'_{\max}$, then the coincidence of the points M and P would reduce Proposition 11 to Proposition 10 [Liu Dun 1984]. In sum, the *Huo Qi Zhen Jue* was a clear, concise treatise about projectile motion and its relevance to the ballistics available to Chinese scholars in the 1850's. In style it listed propositions in an orderly fashion and offered diagrams for justification. Although it lacked rigorous proofs for the propositions, its graphic illustrations indeed suggested sufficient reasoning that could easily be translated into valid demonstrations or proofs. Therefore, the historian of Chinese mathematics Liu Dun is correct to say that the *Huo Qi Zhen Jue* the mathematical capabilities of its author, Li Shanlan. Liu Dun is also right to conclude that this treatise reflected Li Shanlan's strong interest in the blend of theory and application in mathematics [Liu Dun 1984].

In addition to the theoretical aspects of projectile motion, Li Shanlan was also interested in practical aspects of the theory, especially its relevance to artillery and gunpowder. This apparently is why he added an "Appendix" at the end of the *Huo Qi Zhen Jue*. In this "Appendix" Li Shanlan gave a table showing the amount of gunpowder that should be put in a given shell with the maximum range that the shell could reach. Although how this interesting table was produced is unknown, it once again shows that he was very concerned with the relevance of mathematics to

manufacturing as was also shown in his "Preface" to the *Zhong Xue* (Whewell's *Mechanics*). Moreover, he may have included this table at the end of the *Huo Qi Zhen Jue* in order to emphasize the relevance of mathematics to the "Self-strengthening Movement" [Cf. Section 2.6]. In fact, the primary concern of the early advocates of the "Self-strengthening Movement" was to learn from the West the techniques necessary for manufacturing machinery and armaments. In this connection, the *Huo Qi Zhen Jue* was a very important treatise. It was probably due in part to the appearance of this treatise that Li Shanlan was recruited to the Anqing headquarters of the scholar-general Zeng Guofan in the early 1860s [Cf. Section 2.6].

Whether the *Huo Qi Zhen Jue* was duly appreciated at Anqing headquarter is unknown. However, the *Huo Qi Zhen Jue*, which clearly marked an unprecedented mathematical approach to cannon and ballistics in Chinese history, did attract several late Qing scholars who apparently were concerned about the self-strengthening of the Qing Empire. For example, Wang Tao, Li Shanlan's colleague at the LMS Press, not only studied Li Shanlan's *Huo Qi Zhen Jue*, but also wrote a treatise of his own on cannon in 1863. In 1876, Xiong Fangbo, a student of Li Shanlan at the Tong Wen Guan, also wrote a treatise on cannon, the *Huo Qi Ming Zhong* ("On the Elevation Angles of Cannons"), in which he drew upon several propositions from the *Huo Qi Zhen Jue*. Xiong Fangbo certainly was familiar with the principles on which the determination of the elevation angles should be based. Thus in

1881, while he served as a mathematics instructor at the Beijing "Shen Ji Ying" Arsenal, the cannon drill of his students deeply impressed foreigners at Beijing. By the 1880s two other treatises on cannon had appeared, namely, the *Huo Qi Zhen Jue Shi Li* ("Illustrating Examples for the *Huo Qi Zhen Jue*, 1884) by Lu Jing (1858-1948), and the *Huo Qi Zhen Jue Jie Zheng* ("Illustrations on the Propositions of the *Huo Qi Zhen Jue*, 1886) by Shen Shanzheng. All of the authors of these treatises, according to Liu Dun, failed to appreciate the *Huo Qi Zhen Jue* properly due to their apparent ignorance of the theoretical aspects of the *Huo Qi Zhen Jue* [Liu Dun 1984].

Additional examples can also be given to further support Liu Dun's observation on how Li Shanlan's *Huo Qi Zhen Jue* influenced his successors. Zhang Bingsu, a distinguished student at the Wei Jing Academy [Cf. Section 5.6] in the 1880s, was enthusiastic about Li Shanlan's mathematics. At that time Lu Yinggeng gave three numerical examples showing how one could actually compute the elevation angles. For this reason Lu Yinggeng's examples were highly appreciated because they showed the "methods" for computing the elevation angles. By reading carefully both the *Huo Qi Zhen Jue* and Lu Yinggeng's three numerical examples on the elevation angles, Zhang Bingsu asserted that Lu Yinggeng's examples were nothing but illustrations of the related propositions from the *Huo Qi Zhen Jue* [see Zhang Bingsu's exercise at the Wei Jing Academy, quoted in Zhu Youxiang 1986, pp. 382-389]. However, Zhang Bingsu did not advance the study of cannon beyond what Li Shanlan had

done, either theoretically or practically.

3.8. The *Tian Suan Huo Wen*: A Notebook on Astronomy and Mathematics

The *Tian Suan Huo Wen* ("Querries and Answers on Astronomy and Mathematics") included 21 querries along with their answers by Li Shanlan himself. This treatise deserves special attention simply because it reveals some significant aspects of his studies on mathematics by 1867, the year in which he published the *Ze Gu Xi Zhai Suan Xue* which included the *Tian Suan Huo Wen*. Indeed, it collected questions / problems which were presented either by his colleagues or disciples in the period 1840-1865 [Cf. Li Shanlan's "Preface"]. But it did not include problems about infinite series expansions for π , trigonometric and inverse trigonometric functions, or logarithms upon which Li Shanlan had made great contributions. This treatise is still of interest in a historical sense, for it contains problems on elementary mathematics that apparently reflect his mathematical teaching at the Tong Wen Guan after 1868 [Cf. Sections 5.1, 5.2 and 5.3].

Basically, the problems covered in the *Tian Suan Huo Wen* may be categorized as follows:

The "Tian Yuan" method -- Querries 1, 2 and 3.

The Pythagorean Theorem -- Query 4.

Pythagorean triplets -- Querries 5, 6 and 7.

Plane Trigonometry -- Querries 8, 9, 10 and 11.

Spherical Trigonometry -- Querries 12, 13, 14, 15, 16 and 17.

Infinite periodic decimal fractions -- Query 18.

Solid Geometry -- Query 19.

Construction of the regular pentagon -- Querries 20 and 21.

Querries 1 and 2 are concerned about nine problems in Book 2 of Li Ye's *Ce Yuan Hai Jing*. In Query 3, Li Shanlan considered the difference between Li Ye's "Tian Yuan" method and Qin Jiushao's "Da Yan Qiu Yi" method. If a more detailed explanation was needed, as in this case, Li Shanlan usually gave an appropriate example to illustrate his answer.

Query 4 was especially interesting largely because it reflected Li Shanlan's attitude toward Liu Hui's *commentary* on the *Jiu Zhang Suan Shu*. In fact, Li Shanlan was asked by his disciples to give a simpler and more convenient proof of the **Pythagorean Theorem**, since, according to the citation of his disciples, he had criticized the clumsiness of Liu Hui's "Xuan Tu" Proof. Li Shanlan's answer was to give a proof which based upon Proposition 8 or 31 of Book VI of the *Ji He Yuan Ben* (Chinese edition of Euclid's *Elements*, 1607). Why did Li Shanlan fail to appreciate Liu Hui's "Xuan Tu" (hypotenuse diagram) proof which, even in comparison with Bhaskara's, is regarded by modern historians of Chinese mathematics [Cf. Lam and Shen 1984] as one of the most elegant proofs of the so-called **Pythagorean Theorem**? In light of Gu Guanguang's deep interest in Liu Hui's *Commentary*

in the 1850s, Li Shanlan's neglect of Liu Hui seems even more peculiar. Unfortunately, we are not able to give any explanation. Here we can only say that the *Jiu Zhang Suan Shu*, along with Liu Hui's *Commentary*, does not seem to have been properly appreciated by any of the noted Qing mathematicians. Jiao Xun [Cf. Section 1.5] and Gu Guanguang are, it seems, the only two exceptions. Yet Li Shanlan never referred to their studies on Liu Hui in any of his own works on mathematics -- a fact which suggests perhaps that Liu Hui's mathematics was simply not among a subject of interest to Chinese mathematicians at the time.

In Querries 5, 6 and 7, Li Shanlan dealt with problems of Pythagorean triplets. A detailed discussion of these will be given in Section 5.4, so here we only offer a few words about his commentary on Wang Lai's theory of equations. Query 6 asks for a comment on Wang Lai's solution to the problem: Find two sets of Pythagorean triples $a_1, b_1, c_1; a_2, b_2, c_2$ such that $a_1 \cdot b_1 = a_2 \cdot b_2$ and $a_1 + c_1 = a_2 + c_2$. Setting $A = a_1 \cdot b_1$ and $k = a_1 + c_1$, Wang Lai then solved the following equation (in modern notation):

$$y^3 - ky - \frac{4A^2}{k} = 0 .$$

Wang Lai obtained a positive root y and reduced the cubic equation to a quadratic one:

$$x^2 - (k - y)x + y^2 = 0 .$$

Solving this quadratic equation, Wang Lai obtained two positive roots which were $c_1 - a_1$ and $c_2 - a_2$, respectively. Combining this with the given conditions, Wang Lai eventually got the desired result [Cf. Wang Lai 1798 Book 2; Qian 1983, pp. 241-243]. Unfortunately, Li Shanlan was unable to understand why Wang Lai was so proud of the discovery that a cubic equation could have as many as three positive roots -- an unprecedented discovery in the history of Chinese mathematics that led to Wang Lai's theory of equations entirely on his own [Cf. Qian 1983, pp. 235-260; Horng 1989c]. Li Shanlan knew quite well that the "Kai Fang Shi" (numerical equation) of third degree like $ky^3 - k^2y - 4A^2 = 0$ in terms of the "Tian Yuan" method could be solved twice by the "Kai Fang" method [See Querry 6]. Even so, Li Shanlan did not grasp the significance of Wang Lai's approach to theory of equations [Cf. Qian 1983, p. 243; Horng 1989c].

As for his queries on plane and spherical trigonometry, namely, Queries 8-17, there is nothing of particular interest to be said. From the late Ming and early Qing, problems on plane and spherical trigonometry had become very popular in the mathematical community thanks to the introduction of the *Chong Zhen Li Shu* [Cf. Section 1.2; Qian 1981, pp. 240-250]. Thus it was natural for Li Shanlan's colleagues and disciples to pay considerable attention to these fields. In fact, both plane and spherical trigonometry were put into the mathematics curriculum of the Tong

Wen Guan when Li Shanlan began to serve there as a mathematics instructor [Cf. Sections 5.1, 5.2 and 5.3].

Query 18, just like Query 4, asks further explanation of the relationship between a common fraction and its infinite periodic decimal fraction as well as that between the infinite periodic decimal fraction and the infinite geometric series. Li Shanlan answered the first question with two examples: $\frac{1}{7} = 0.142857142857\dots$, $\frac{1}{13} = 0.0769230769230\dots$. It is unknown where Li Shanlan drew such interesting properties for common fractions in terms of division. On the other hand, it is peculiar that such problems never appeared on the mathematics tests in the Tong Wen Guan [Cf. Section 5.2].

Query 19 concerned solid geometry, and asked for a method to construct one section of an ellipsoid. At the end of his *Fang Yuan Mi Ji* ("On the volume and Surface Area of A Sphere"), Mei Wending remarked that his treatment of the same problem corrected an error made on this subject in the treatise on calendrical sciences. Yet Mei Wending's result was not correct either. Consequently, in his answer to this query Li Shanlan gave his own answer that he himself thought was valid.

Finally in Queries 20 and 21 Li Shanlan was asked to give a simpler method for the construction of a regular pentagon than the one given as Proposition 11 in Book IV of Euclid *Elements*. Li Shanlan indeed justified his solution by referring to Propositions

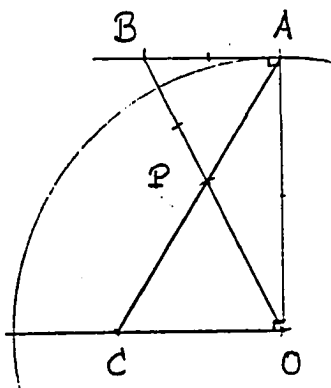


Figure 60.

Querry 20 copied from
the *Tian Suan Huo Wen*.

On the doownleft, we
duplicate its
illustration.

3, 9 and 10 in Book XIII of Euclid's *Elements* [Cf. Figure 60], which he had just translated in collaboration with Wylie. In his own geometric construction Li Shanlan showed his familiarity with Euclid. This also shows how he was able to learn Western texts on mathematics in a short time.

In conclusion, the *Tian Suan Huo Wen* clearly reflected many significant aspects of Li Shanlan's studies on mathematics as well as some major concerns of his contemporary mathematicians. Although it did not include the problems that led to his most important and significant contributions to Chinese mathematics in its decade 1840-1850, it was a faithful reflection of 19th-century Chinese mathematics, and it serves to indicate how the subject developed at the time.

3.9. Concluding Remarks

There are still other two treatises among the *Ze Gu Xi Zhai Suan Xue*, namely the *Lin De Shu Jie* ("Explanation of Lin De Calendar") and the *Ji Shu Hui Qiu* ("Conversions of Infinite Series"), which are not discussed here. The *Lin De Shu Jie* is an explication of the Lin De Calendar composed by Li Chunfeng in the Tang Dynasty, in which the author used a method of interpolation with second-order difference to deal with computations concerning problems of calendar [Qian 1981, p. 323]. Yet Li Shanlan never came back to this subject, although the *Lin De Shu Jie* drew great attentions of the Jiangnan area scholars like Wang Yuezheng in the

end of the 1840s. It should be noted that this treatise was written in order to revive ancient texts on calendrical sciences, which, just like Li Shanlan's *Si Yuan Jie*, marked an imprint of the Qian-Jia school's approach to mathematics and science. Yet since we are only concerned about Li Shanlan's mathematics in this thesis, the discussion of the *Lin De Shu Jie* therefore should be stopped here.

As for the *Ji Shu Hui Qiu*, we have already mentioned the method this treatise gives for the conversion of infinite series [See Section 3.3]. Clearly, it must have not yet been written before he finished the translation of De Morgan's *Algebra*. This is because Li Shanlan used the terms and notations given in this most newly-transmitted text on algebra. As a result, Li Shanlan's treatment of power series expansions for trigonometric and inverse trigonometric functions appeared easier than those given by Ming Antu and Dong Youcheng [Qian 1981, p. 325]. Moreover, the *Ji Shu Hui Qiu*, like Li Shanlan's other treatises *Tuo Yuan Shi Yi*, *Huo Qi Zhen Jue*, and *Dui Shu Jian Zhui Bian Fa Shi*, reflects exactly how quick and remarkable a way he learned Western mathematics introduced by western missionaries in the 1850s. Even so, the *Ji Shu Hui Qiu* was basically a book on the method of converting infinite series. In other words, it added nothing to the content of the related mathematical knowledge like the series expansions for trigonometric and inverse trigonometric functions. Therefore, it is not necessary to say anything more about this treatise here. For now, let us say a few words to conclude this chapter.

The *Ze Gu Xi Zhai Suan Xue* not only concludes Li Shanlan's mathematical studies by 1860, but Chinese mathematics in the period 1820-40 as well. This is largely because Li Shanlan's *Fang Yuan Chan You*, *Hu Shi Qi Mi* and *Dui Shu Tan Yuan* had accomplished a more comprehensive and elaborate treatment of the major issues of the "Xi Fa" school by the 1840s. On the other hand, it seems understandable that Li Shanlan's *Si Yuan Jie* made almost nothing innovative, since this treatise was primarily concerned with the revival of Zhu Shijie's classic, the *Si Yuan Yu Jian* (1303). However, his *Duo Ji Bi Lei*, which based basically on Zhu Shijie's *Si Yuan Yu Jian* too, was so original and creative that has even greatly impressed modern mathematicians since the 1930s. It should be noted that Li Shalan's *Duo Ji Bi Lei* might have been regarded as masterpiece of the so-called "Xin Fa" (new method) instead of the "Xi Fa" (Western method), if Luo Shilin (1774-1853), the last faithful representative of the Qian-Jia school, was still alive to see the composition of this treatise.

Therefore, Li Shanlan clearly was well-prepared for the third period of his career, namely serving as a mathematical instructor at the newly-founded modern college, the Tong Wen Guan. And it was at this institution that Li Shanlan played a significant role in mathematical studies in the second half of the 19th century, as clearly shown in his interest in the theory of prime numbers as well as the Pythagorean triplets from 1870 on. Besides, his teaching at the Tong Wen Guan, emphasizing in the curriculum a

synthesis of the Chinese traditional academies and the modernization of Chinese mathematical education [Cf. Sections 5.5, 5.6 and 5.7].

Notes:

1. As an illustration for Dan Zhi 1, Li Shanlan argued that "That which has color cannot be shapeless. That which has shape ("Xing") cannot be without (material) entity (or body, "Ti"). This is because the color manifests itself in the shape, and the shape is formed by entity (or body). For example, if we make a point ("Dian") by dropping a dot of (Chinese) ink on paper such that the point is fine enough like dust, then it is the finest of the shape. Yet the shape of this point(i.e., the dot of ink) cannot come from nothing. Rather it is formed by the ink. Be that as it may, is that ink not an entity? Therefore, the "Dian" (point) is the body that is tiny and fine enough ("Dian Ze Ti Zhi Xiao Er Wei Ze"). Similarly, a line ("Xian") is the body that is long but very narrow in breadth; and a surface ("Mian") is the body that has length and breadth but is very thin."
2. For this translation, Professor Joseph Dauben suggests that "it would have been more interesting to say a sequence of silk was made from the threads (lines)." I agree with his comment.
3. According to Zhu Kebao, the most important Qing mathematicians in the period 1820-40 are Xiang Mingda, Xia Luanxiang, Luo Shilin, Guanguang, Dai Xu, Xu Youren, Zou Boqi and Li Shanlan [Zhu Kebao 1886].
4. The *Jiu zhang Suan Shu* includes no diagrams or "Cao". Although Liu Hui ever added graphical diagrams in his *Commentary*, it does not survive to the Song period as shown in the Song edition of the *Jiu Zhang Suan Shu* (only the first five chapters). On the other hand, due apparently to the fact that methods became more sophisticated in 13th-century Chinese mathematics, thus it was not surprised that mathematical classics by the great mathematicians, Qin Jiushao, Yang Hui, Li Ye and Zhu Shijie, all included graphical diagrams as well as necessary demonstrations for their methods [Cf. Qian 1981, pp. 174-186; Li and Du 1987, pp. 135-148].
5. Jia Xian (ca. 1050) probably was the first Chinese mathematician to relate the Pascal's Triangle with the extraction of roots of powers higher than 3. For the square and cubic roots, the *Jiu Zhang Suan Shu* had already given treatments to their complete form [Cf. Chapter 4]. In addition, Liu Hui used graphical diagrams, which did not survive to this day, to explain the extraction methods of the square and cubic roots. However, it seems impossible to develop the methods of extracting roots of 4th power on the ground of geometrical diagrams. This fact, as many historians of Chinese mathematics have suggested, may explain why Jia Xian relied basically on the "Tabulation Systems for unlocking binomial coefficients" to develop his extraction method of powers higher than 3. In fact, the titles given for the Chinese "Pascal's Triangle", namely Jia Xian's "Kai Fang Zo Fa

Ben Yuan " (Tabulation Systems for Unlocking Binomial Coefficients) and Zhu Shijie's "Gu Fa Qi Cheng Fang Tu" (Binomial Coefficients up to 8th Power), all had the connotation that they aimed mainly at the extraction of roots.

Moreover, Jia Xian explained clearly how he related the coefficients to his "Zen Cheng Kai Fang" method which eventually led to Qin Jiushao's "Zheng Fu Kai Fang" method (Chinese Horner's method) [Cf. Qin 1981, Chapter 8; Li and Du 1987].

6. Fu Dawie gives an interesting reconstruction of Zhu's variation forms of the "San Jiao Duo" and "Si Jiao Duo" systems [Fu 1990]. Yet this does not seem to have related with our discussion of Li Shanlan's "Duo Ji" method.

4. THE TRANSMISSION OF MODERN WESTERN MATHEMATICS TO 19TH-CENTURY CHINA: THE ROLE OF TRANSLATOR AND TRANSFORMER

4.1. Alexander Wylie and the Transmission of Western Science and Mathematics to China

Having achieved great success in traditional Chinese mathematics, Li Shanlan went to the London Missionary Society Press (Mo Hai Shu Guan, abbreviated hereafter as LMS Press or Press) at Shanghai, where he worked as a translator in collaboration with Alexander Wylie. Together they published a number of Western scientific and mathematical texts into Chinese.

Neither the exact date when he began to work for the LMS Press, nor the time when he first visited the Press is known. However, it is quite certain that before 1852, the year that he began to work at the Press, Li Shanlan brought some mathematical treatises of his own to the Press among which must have been the *Fang Yuan Chan You*, *Hu Shi Qi Mi* and *Dui Shu Tan Yuan*.¹ These greatly impressed Wylie, and as a result Li Shanlan was invited to help translate Western scientific and mathematical texts at the Press.

Alexander Wylie (1815-1887), a British missionary from the National Church of Scotland, Crown Court, Covent Garden, began his service at the LMS Press in August of 1847. Wylie was invited to take charge of the printing office of the London Missionary Society at Shanghai, and at first he served basically as a printer

responsible for the producing the Delegates' Version of the *New Testament*. "From the first, therefore, Mr. Wylie was practically the employe of the Bible Society, his salary being paid from the Society's funds," [Quoted from the "Preface" by James Thomas in Wylie 1897, p. 1].

While in charge of the LMS Press at Shanghai, Wylie also took a deep interest in the conversion of Chinese to Christianity. Often he accompanied his fellow missionaries into remote regions of the country [Thomas in Wylie 1897, p. 3]. Nevertheless, Wylie's greatest contribution to China in the 1850's was the part he took in facilitating the transmission of Western science and mathematics during the 1850s. It is in this very context that his association with Li Shanlan, especially their co-translations of Western texts on geometry, algebra, calculus and astronomy, was significant for the history of science and technology in modern China.

However, nothing of Wylie's early educational background seemed to have prepared him for the important role he would soon play in this crucial period when modern Chinese science and mathematics were being created in a matter of decades. According to James Thomas, Wylie's "education was begun in the grammar school at Drumlithie, and after his return to London it was continued in Chelsea. On leaving school he was apprenticed to a cabinet-maker, and while in this business he spent some months at Hartfield, restoring Hartfield House after a fire," [Thomas in

Wyllie 1897, "Preface"]. Even so, in 1852, after his residence for only five years, Wyllie showed that Horner's method for solving equations of all orders published in 1819 was anticipated by Chinese mathematician Qin Jiushao in the 14th century. "The shortness of the time Wyllie had spent in the country when he reached this result, shows how his mind moved with freedom among the abstractions of the higher mathematics," [Edkins in Wyllie 1897, "Preface"].

Indeed, this "rare faculty" enabled Wyllie to publish his "Jottings on the Science of the Chinese Arithmetic" [Cf. Section 1.1] in 1852. It was again his rare combination of faculty in mathematics and Chinese that also enabled him "to become the translator of works on the Differential and Integral Calculus into Chinese," [Edkins in Wyllie 1897, pp. 5-6].

According to Joseph Edkins, "Wyllie studied the Sung dynasty arithmetic with the help of Li Shanlan, a native mathematical author of high attainments" during the period 1847-1852 [Edkins in Wyllie 1897, pp. 5-6]. This means that Li Shanlan must have visited Wyllie prior to July of 1852, if only occasionally [Wang Tao 1852]. Meanwhile, Li Shanlan must have reported his own research to Wyllie, along with results obtained by his colleagues like Xu Youren, Dai Xu and Gu Guanguang (among others). This information indeed stimulated Wyllie's admiration for methods obtained by Chinese mathematicians that were analogous in principle to those of the calculus [Cf. Li and Wyllie 1859,

"Preface"].

Thus came about the important collaboration of Alexander Wylie and Li Shanlan. Their co-translations served to re-open the gate for transmitting Western science and mathematics from the early 18th-century on. As mentioned in Chapter 2, the strategies of British and American Protestant missionaries (similar to that of the Jesuits two hundred years earlier), was to use science as an auxiliary to help spread of the gospel. Yet they were also aware that the Jesuit precedent was not very successful -- since too few souls had been "saved." Although the British and American missionaries also appreciated another aspect of the Jesuits' strategy -- namely, to convert Chinese leaders first, and then rely on them to spread the gospel among the common people, the 19th-century Protestants failed to convert high-ranking imperial officials. In contrast, most Christian converts in nineteenth-century China came from the lower classes. Now the Western missionaries had to face the problem seriously: If the scholarly elite could not be recruited to the Christian cause, then the status of those who were being evangelized had to be improved [Buck 1980, pp. 10-13].

It was in this connection that the missionaries seized upon the potential significance of science. Reasoning that science had "a great and continually increasing reputation," they assumed that men "well versed in it" would inevitably become "influential ... in any position in China." Therefore, the task of the missionary,

as the American Presbyterian Calvin Mateer (1836-1908) argued, was simply "to give to those converted pupils" such scientific training as would "fit them to be influential men," [Mateer 1890, quoted in Buck 1980, p. 12].

Whether Wylie looked upon science in quite this way -- as a potential factor in Christian conversion -- is unknown. Yet, as mentioned above [Cf. Chapter 2], Wylie understood science and mathematics as being directly relevant to natural theology. In fact, at the end of his short English preface to the *Xu Yi Ji He Yuan Ben* (the Li-Wylie translation of the last nine books of Euclid's *Elements*, published in 1857), Wylie remarks:

To accompany this issue with an apology would almost seem out of place. Truth is one, and while we seek to promote its advancement in science, we are but preparing the way for its development in that loftier knowledge, which as Christian men and missionaries, it is our chief desire to see consummated [Quoted in Wylie 1897, p. 12].

Moreover, a similar tone was voiced in his preface to the *Tan Tian* (the Chinese edition of John Herschel's *Outline of Astronomy*).

Astronomical phenomena "cannot fail," Wylie observes:

... to awaken in inquisitive minds of a certain order, a desire to become better acquainted with those and kindred facts in nature, which is calculated to exercise a healthful influence on the intellectual character. That such facts may lead to juster and more exalted conceptions of "Him who hath created these orbs, -- who bringeth forth their host by number and calleth them all their names; -- who hath made the earth by His power, established the world by His understanding," is the sincere desire of the translator [Quoted in Wylie 1897, p. 12].

4.2. Li Shanlan and the Transmission of Modern Mathematics to China



Figure 1. Title page of the Zhi Wu Xue (Chinese edition of John Lindley's *Elements of Botany*, 1858) written by Li Shanlan.

Li Shanlan's preface to the *Zhi Wu Xue* (the Chinese edition of John Lindley's *Elements of Botany*, issued in 1858) also seemed to stress the Christian significance of natural order. In this case the idea was to draw from the example of well-designed botanical knowledge the foundations for a new social order. Perhaps Li Shanlan did believe in such arguments, which provide circumstantial evidence at least for his possible conversion to Christianity. While working with Wylie in the LMS Press, Li Shanlan came "very near professing Christianity. Deterred by fear of prejudice to his official preferment, he retained in considerable measure the impressions he then received," [Martin 1896, pp. 368-370].

In any case, Li Shanlan seems to have worked at the LMS Press to his satisfaction. Indeed, his mathematical capabilities and training were much better than Wylie's. Thus, while translating Western texts on science and mathematics into Chinese, Wylie essentially relied on Li Shanlan. Although Li Shanlan apparently had no idea about which texts should be translated (except for the last nine books of Euclid's *Elements*, which he was eager to promote in his early days) [See Li and Wylie 1865, "Preface"], he did not assume an active role in deciding which Western texts should be published in Chinese. The reason, as historian Kiyoshi Yabutti argues, may simply be due to the fact that Li Shanlan himself was unable to read the Western texts. In contrast, intellectuals of the Japanese Meiji Restoration (1867-1868)² made a point of learning Western languages so that they could determine

what material they needed from the West in an independent and well-informed way [Yabutti 1979, pp. 175-80].

Given, Li Shanlan's lack of familiarity with English, his failure to grasp the significance of the theory of equations is understandable. This was not due merely to the fact that the *Dai Shu Xue* (the Chinese translation of Augustus De Morgan's *Elements of Algebra*) had little to do with this subject, but rather that he apparently had not understood Wang Lai and Li Rui's theory of equations [Cf. Sections 1.5 and 1.8]. In this connection, whether or not Li Shanlan could read English was of little consequence, since he was unable, it seems, to appreciate the work on the theory of equations done by Lagrange, Abel and Galois. And yet, De Morgan (and his then-contemporary British mathematicians, including the most notable William Hamilton) should perhaps also be blamed since they too failed to grasp the significance of the theory of equations.

This brief remark only points to the complexity of transmission which, as already observed, is well-illustrated in the case of the Chinese translation of De Morgan's *Elements of Algebra*. So let us begin by looking more closely at this treatise.

4.3. The Translation of De Morgan's *Elements of Algebra*: The *Dai Shu Xue*

The full title of this treatise in English reads -- "Elements of Algebra: Preliminary to the Differential Calculus, and fit for the higher class of schools in which the principles of Arithmetic are taught." The first (English) edition was published in 1835, and it was this version that was used in producing the Chinese edition, *Dai Shu Xue*. Two years later De Morgan published a revised version which he called the "Second Edition". In what follows we shall refer to this "Second Edition," since it has not been possible to locate a copy of the rare first edition. Fortunately, as De Morgan promises, the second edition "differs from the first only in verbal amendments" [De Morgan 1837, "Preface"].

In addition to De Morgan's "Preface" and "Introduction", the *Elements of Algebra* contains thirteen "Chapters":

- I. Equations of the First Degree.
- II. On the Symbols of Algebra, as distinguished from those of Arithmetic.
- III. Equations of the First Degree with more than one unknown quantity.
- IV. On Exponents, and on the Continuity of Algebraic Expressions.
- V. General Theory of Expressions of the first and second degrees; including the numerical solution of equations of the second degree.
- VI. On Limits and Variable Quantities.
- VII. Classification of Algebraic Expressions and

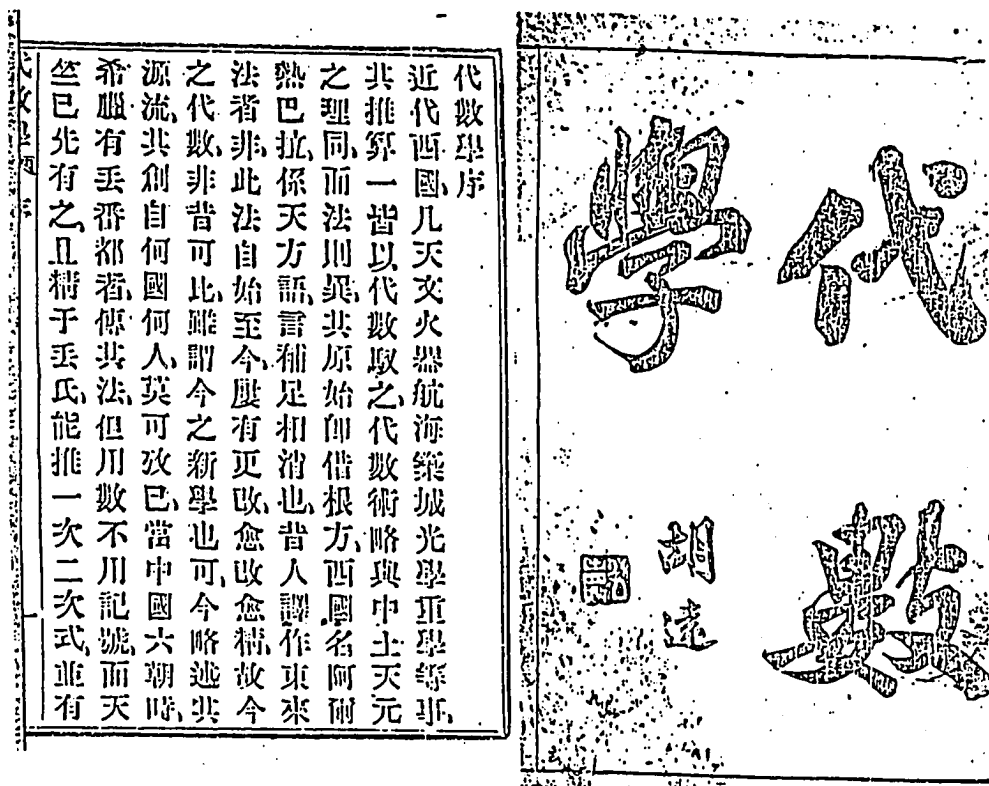


Figure 2. Title page of the *Dai Shu Xue* (Chinese edition of De Morgan's *Elements of Algebra*, 1859) and the first page of Wylie's "Preface".

Consequences. Rule of Division.

VIII. On Series and Indeterminate Coefficients.

IX. On the Meaning of Equality in Algebra, as distinguished from its meaning in Arithmetic.

X. On the Notation of Functions.

XI. On the Binomial Theorem.

XII. Exponential and Logarithmic Series.

XIII. On the Use of Logarithmic in Facilitating Computations.

From this table of contents alone one is certainly struck by the fact that this treatise includes six chapters that nowadays would be considered part of course on the calculus. For example, Chapters IV, VI, VIII, XI, XII and XIII deal with the theory of limits, continuous functions and infinite series -- all of which amply justify the book's subtitle, the *Elements of Algebra*.

Yet why was it considered necessary for a high-school-level algebra text like this to introduce the theory of limits, even if it was regarded as a preliminary to the Differential Calculus? Apparently it was due to De Morgan's teaching experience at London University, where he frequently observed that "no one knows exactly what he is doing in that science (i.e., the Differential Calculus) until he has made considerable progress in the mechanism of its operations." The reason for this, De Morgan believed, was "that the fundamental notions of the differential calculus are conventionally, and with difficulty, excluded from algebra, in which I think they ought to occupy an early and prominent place."

For otherwise, he asked, "What is $\sqrt{2}$, supposing geometry and limits both excluded?" [De Morgan 1837, "Preface"].

On the other hand, De Morgan also recommended "real algebra" as useful as "an exercise of reasoning." In fact, "De Morgan, like most of his contemporaries, saw the principal contribution of mathematics to general education as a vehicle for the enhancement of the faculty of reasoning," [Quoted Howson 1982, p. 89]. To meet such a demand, De Morgan's exposition and demonstrations looked tedious and difficult to follow. This was all the worse since "when teaching algebra the prime need is to ensure that symbolism is fully comprehended." Therefore he urged teachers to take a gradual approach:

The new symbols of algebra should not be all explained to the students at once. He should be led from the full to the abridged notation in the same manner as those were who first adopted the latter. For example, at this period he should use aa, aaa etc. and not a^2 , a^3 , and should continue to do this until there is no fear of that confusion of $2a$ and a^2 , $3a$ and a^3 which perpetually occurs [Quoted in Howson 1982, p. 92].

As for himself, De Morgan did use the symbols aa, aaa, etc., instead of a^2 , a^3 , etc., until Chapter IV of his *Elements of Algebra*. This fact help to explain why, to the modern reader, the book contains so much in the way of tedious expositions and demonstrations.

Indeed De Morgan was very concerned about how symbolic algebra was taught in schools. Elsewhere he emphasized what he took to be the best way of teaching symbolic algebra in terms that

are reminiscent of a sort of "Haeckel's Law" in mathematical pedagogy:

This refined branch of the science should not be the first taught, but ... the pupil should be led, in the track of invention, to its several parts, so as to arrive at each at the precise moment when he can understand its origin and use. A contrary course will ensure years of travelling in the dark [Quoted in Howson 1982, p. 90].

In fact, De Morgan's *Elements of Algebra* shared, in one form or another, concerns that were shared by the early 19th-century Cambridge mathematicians John Herschel, George Peacock and Charles Babbage, the founders of the Cambridge Analytical Society. These mathematicians, in contrast to their Continental European contemporaries, sought to elaborate a new approach to algebra which emphasized the arbitrariness of its symbols and the freedom of mathematicians to determine its meaning. But why was such a symbolic approach so significant for them? One part of the answer is to be found in the problem of negative and imaginary numbers [Pycior 1984].

In turn, why should the problem of negative and imaginary numbers be so disturbing and urgent for early 19th-century British mathematicians? Such numbers, of course, posed fundamental questions of long standing in the history of mathematics. And yet these numbers, introduced into Western Europe during the Renaissance, were used rather liberally in mathematics by the 18th century, despite foundational problems related to their meaning and justification were not yet settled. But there must have been some reason why the British mathematicians came to focus

their attention upon the meaning and legitimacy of these numbers towards the end of the 18th and beginning of the 19th century. According to Helena Pycior, there were mathematical, religious, philosophical and pedagogical considerations whose blend seems to have contributed to the rise and urgency of the problem of these numbers [Pycior 1984]. Here one needs only to look at both the pedagogical and more strictly mathematical aspects of the problem. Early in 1801, Robert Woodhouse, for a time the Lucasian Professor of mathematics at Cambridge, conceded "that the mind can form no idea of an abstract negative quantity", but argued for retention of the negatives in order to preserve "the chief advantages of that art [algebra]; its compendious and expeditious methods of calculation," [Quoted in Pycior 1987]. Later in the same year, however, Woodhouse told Francis Maseres, one of the key figures who waged a full-scale attack against these numbers, that: "Till the doctrines of negative and imaginary quantities are better taught than they are at present in the University of Cambridge, I agree with you that they had better not be taught," [Quoted in Pycior 1987].

This mathematical and pedagogical problem soon mushroomed into the problem of the definition of algebra, which in turn called forth the new symbolic approach. The first formulation of this new approach to algebra appeared in Peacock's *Treatise on Algebra* of 1830. According to Peacock, symbolical algebra, covering traditional algebra as well, was "the science which treats of combinations of arbitrary signs and symbols by means of

defined though arbitrary laws," [Quoted from Pycior in Grattan-Guinness 1987, p. 157]. This meant that algebraic symbols and signs stood for no particular quantities or operations on quantities, although they could eventually take on many different interpretations. The problem of the negative and imaginary numbers vanished at last: granted the new symbolic approach, their mathematicians did not need to define such numbers prior to manipulation [Pycior 1987].

There were two more features, in addition to the one just mentioned, which are also characteristic of Peacock's symbolic algebra, namely, the importance of algebraic structure determined by the laws of operations, and the principle of algebraic freedom in using arbitrary symbols and signs. Nevertheless, Peacock did not go too far: his symbolic algebra was more a science of quantity than a science of symbols. Thus in a review of 1835 of Peacock's *Treatise on Algebra*, De Morgan (who hesitated before accepting this approach in 1835, although he later became ambivalent towards it) was critical: "At first sight ... [Peacock's *Treatise on Algebra*] appeared to us something like symbols bewitched, and running about the world in search of a meaning," [Quoted from Pycior in Grattan-Guinness 1987, p. 158]. Despite his uneasiness about the symbolic approach due to its apparent abandonment of meaning where algebraic symbols and signs were concerned, De Morgan, among others, became a reluctant convert to this revolutionary approach to algebra [Pycior 1987].

Why did De Morgan take so strong and clear a stand on symbolic algebra in 1835? Prior to that year (or more specifically in the period from 1828 to 1832), De Morgan's algebraic work was primarily aimed at justifying algebra as a basic component of a liberal education by legitimating the negatives and imaginaries through appeals to examples which showed that their use led to correct (and presumably meaningful) results [Pycior 1983]. Something significant must have happened in 1835 to explain De Morgan's conversion. According to Helena Pycior, it was John Warren's *Treatise on the Geometrical Representation of the Square Roots of Negative Quantities* (1828) that provided an epistemological basis for De Morgan's turnabout. This treatise had already been cited in Peacock's *Treatise on Algebra* (1830) with which De Morgan was doubtless familiar in the early 1830s. However, De Morgan "seems to have fully recognized only in the mid-1830s that the geometrical representation of the complex numbers made a sound basis for legitimation of the imaginaries and justified the symbolical approach itself," [Pycior 1983].

Nevertheless, the *Elements of Algebra* (1835) did not reflect De Morgan's recent conversion to a new kind of algebraic thinking in 1835! For example, Warren's geometrical representation of the imaginary numbers was not drawn upon in this treatise. Instead, De Morgan introduced the imaginary numbers by a formal demonstration in order to permit a general solution of quadratic equations $ax^2 + bx + c = 0$ where $b^2 - 4ac < 0$. In this context, De Morgan's justification of the imaginaries lay mainly in

demonstrating inductively the validity of their use, although in the meantime he also reminded his readers of the fact that the problem of the imaginaries had been settled elsewhere. Perhaps it was due to his pedagogical concern that even in 1835 "De Morgan reasserted his preference for the inductive over the then-available 'rational' justifications of the negative and imaginary numbers," [Pycior 1983]. Here "rational" means apparently that these numbers could be justified "deductively" by the arguments based on a rigorous foundation like that provided by Warren's geometric representations.

De Morgan's introduction of the imaginaries in the *Elements of Algebra* was not of the cutting edge of British mathematicians in the Cambridge Analytical Society; rather, it was reminiscent of Girolamo Cardano's 200 years earlier. In his *Ars Magna* (1645), Cardano solved the equation $x^2 - 10x + 40 = 0$ to obtain two roots $5 + \sqrt{-15}$ and $5 - \sqrt{-15}$, then remarked that "Putting aside the mental tortures involved," he multiplied these two roots to obtain the product $25 - (-15)$ or 40 [Cardano 1968, 219-220]. Treating a similar case, De Morgan apparently offered no better reasons enough to justify operations with imaginaries. As for the equations with real roots, De Morgan relied entirely on an algebraic, symbolic approach without recourse to any geometrical demonstrations -- an approach that Cardano apparently thought was not enough in the *Ars Magna*. For De Morgan, symbolic approach could resolve all these problems.

In his review of Peacock's algebra, De Morgan accepted Warren's geometrical representation justifying the imaginaries. Why then did not he refer to it in his own *Elements of Algebra*? This seems especially peculiar if one considers the fact that his treatise was meant to enhance the "faculty of reasoning". Could he have thought that Warren's interpretation was too sophisticated for beginners studying algebra? On the other hand, the *Elements of Algebra* included rectangular coordinate systems as well as applications of systems of linear equations with two unknowns involving geometrical interpretations used to solve such systems of equations [Cf. Chapter III of the *Elements of Algebra*]. One might well wonder why De Morgan did not make the seemingly obvious and easy shift from the rectangular coordinate system to the complex plane with real and imaginary axes. Instead, a logical basis for the complex numbers was given by William Roan Hamilton in 1837. Hamilton introduced complex numbers in terms of real numbers, i.e., $a + \sqrt{-b}$, $c + \sqrt{-d}$ (a, b, c, d real numbers and c, d positive as well). Addition, subtraction, multiplication and division were defined in the now familiar way as operations with ordered pairs. "Under this view of complex numbers, not only are these numbers logically founded on the basis of real numbers, but hitherto somewhat mysterious $\sqrt{-1}$ is dispensed with entirely," [Kline 1972, pp. 775-76].

The rectangular coordinate system used in the *Elements of Algebra*, therefore, had nothing to do with a "rational" justification of the imaginaries. Instead, it was included in

order to provide a framework for studying the calculus. In fact, as noted above, six out of thirteen chapters in the *Elements of Algebra* dealt mainly with the calculus. For this purpose, De Morgan needed to introduce the real number system, the theory of limits and concepts of continuous functions in terms of symbolic algebra. Read in this way, his book is both illuminating and interesting. Moreover, De Morgan's pedagogical concerns also influenced his symbolic approach to algebra in so far as both "inductive" and "rational" justifications of the imaginaries were useful -- and favorable! De Morgan seemed to favor a "permanence principle" governing the relationship between arithmetic and algebra in his book of 1835 (even though in 1835 he also argued that there was no more of a necessary connection between algebra and arithmetic than there was between algebra and geometry) [Pycior 1983].

Before considering the reactions to De Morgan's algebra by Li Shanlan and his Chinese colleagues in the 1850s, it is necessary to say a few words about the algebraic contents of the book itself. Perhaps the fact of greatest surprise is that, at least to historians of Chinese mathematics, De Morgan introduced few things that were new to Chinese mathematics in the 19th century, except for its symbolic approach to algebra. This is not to say that some chapters (like I and III) were not interesting for their own sake. Yet for a contemporary Chinese scholar versed in mathematics, subjects like "equations of the first degree" [Cf. Chapter I] and "equations of the first degree with more than one

unknown quantity" [Cf. Chapter III] were nothing but the "Tian Yuan" and "Fang Cheng" methods, respectively.³ Moreover, De Morgan's symbolic approach failed to be properly appreciated by Li Shanlan and his colleagues -- another reason why its novelty to Chinese mathematicians may have failed to be apparent.

Nevertheless, the *Elements of Algebra* did contain novelties with respect to traditional Chinese algebra. Among the most interesting of these was the theory of equations up to the second degree [Cf. Chapter V]. In fact, De Morgan dealt rather extensively with the method of completing squares, by which he solved the quadratic equation $ax^2 + bx + c = 0$, where a , b , c are real numbers, to obtain two roots:

$$x_1 = \frac{-b + (b^2 - 4ac)^{1/2}}{2a} ; \quad x_2 = \frac{-b - (b^2 - 4ac)^{1/2}}{2a} .$$

De Morgan went on to discuss the various properties of the roots x_1 , x_2 based on the discriminant $b^2 - 4ac$, and showed that their sum and product:

$$x_1 + x_2 = -\frac{b}{a} ; \quad x_1 \cdot x_2 = \frac{c}{a}$$

could be derived from the identity $ax^2 + bx + c = a(x - x_1)(x - x_2)$. Finally, Descartes Rule of Signs was also applied to quadratic equations $ax^2 + bx + c = 0$ to determine the number of roots according to the signs of the coefficients a , b , and c .

Yet De Morgan did not advance further beyond quadratic equations. Nor did he stress the significance of the theory of equations (as noted above). Joseph Louis Lagrange's *Lectures on Elementary Mathematics*, a series of lectures delivered in 1795 at the Ecole Normale, provide a marked contrast. In the third lecture, Lagrange added a comment on the theory of equations to his brief historical review of the solution of equations since 1600:

The solution of equations of the third and fourth degree was quickly accomplished. But the successive efforts of mathematicians for over two centuries have not succeeded in surmounting the difficulties of the equation of the fifth degree.

Yet these efforts are far from having been in vain. They have given rise to the many beautiful theorems which we possess on the formation of equations, on the character and signs of roots, on the transformation of a given equation into others of which the roots may be formed at pleasure from the roots of the equation, and finally, to the beautiful considerations concerning the metaphysics of the resolution of equations from which the most direct method of arriving at their solution, when possible, has resulted [Lagrange 1898, pp. 64-65].

Besides, as one of the most important predecessors of Evariste Galois [Van der Waerden 1985, 76-88], Lagrange also admitted the failure to find a general formulaic solution for equations of higher degree:

We find ourselves constantly under the necessity of having recourse to other means for determining numerically the roots of a given equation, -- for to determine these roots is in the last resort the object of the solution of all problems which necessity or curiosity may offer [Lagrange 1898, p. 196].

Such a failure called for an alternative approach to problem-solving, which Lagrange probably was the first mathematician to suggest. According to Lubos Novy, Lagrange, in

contrast to his predecessors, "adopted an *a priori* approach, i.e., prior to applying these methods to equations of higher degrees he tried to decide why they yield a solution for equations of the third and fourth degrees and whether they will yield a solution for higher degrees," [Quoted Nový 1973, 28]. Indeed, as Lagrange argued, if one wants to investigate the solvability of equations "*a priori*", one must devote one's attention to the study of $f(x_1, x_2, \dots, x_n)$, a rational function of the n roots for a given equation of the n -th degree, and investigate their properties for the permutations of the roots $x_1, x_2, \dots, x_n$, i.e., how many different values they attain. Therefore, "Lagrange's success is not only in stating of the requirements for this new approach, but also that he was able to apply it extensively," [Quoted Lový 1973, p. 28]. What was De Morgan's response to this alternative approach to problem-solving? No plausible explanation has been given, as far as I know, for the reason why he failed to emphasize the significance of such a approach -- in contrast to the traditional problem-solving approach -- in his own treatise on algebra, although he was surely familiar with Lagrange's *Lectures on Elementary Mathematics* [Cf. MacCormack in Lagrange 1898, "Preface"] and meanwhile he might also have known Lagrange's *Réflexions sur la résolution algébrique des équations*, published in a local academic journal in the period 1770-1771 [Lový 1973, p. 26].

By the same token, Wylie's knowledge of Lagrange's and Continental European algebra is relevant to the transmission of

Western algebra to China in the 1850s. Whether or not he was familiar with Lagrange's algebra, the theory of equations in general and its qualitative or "a priori" aspects in particular certainly had no place in Wylie's program for introducing Western mathematics to China. According to Wylie, algebra and calculus were the major parts of Western mathematics which he was so eager to translate for Chinese readers after having published his own *A Compendium of Arithmetic* (*Shu Xue Qi Men*, in Chinese) in 1853. In so doing, his goal undoubtedly was to encourage Chinese conversions to Christian by showing that Western mathematics (as well as science), mushroomed in the Christian world, demonstrating thereby its superiority over native mathematics. In this connection, if the (newly-introduced) Western algebra was actually more convenient in "computations" than the Chinese "Tian Yuan" and "Si Yuan" methods, why should Wylie have concerned about the qualitative aspects of the theory of equations?⁴

On Li Shanlan's side, he never made any comment on De Morgan's algebra -- in fact, his notebook *Tian Suan Huo Wen* ("Querries and Answers on Astronomy ", 1867) never referred to this treatise. Nor did he ever comment on it elsewhere. This certainly was not the case with the *Dai Wei Ji Shi Ji* (Elias Loomis' *Calculus*), which he encouraged Hua Hengfang to study [Hua Hengfang 1882, Book 5]. And he was certainly familiar with the *Xu Yi Ji He Yuan Ben* (Euclid's *Elements*), whose translation Li Shanlan seems to have recommended as soon as he began to work at the LMS Press.

Li Shanlan's neglect of De Morgan's algebra therefore comes as no surprise if he did not think Western algebra was more convenient or even more powerful than the "Tian Yuan" method. Yet to explain Li Shanlan's failure to appreciate De Morgan, further reasons seemed to be needed. As noted above, Li Shanlan's failure to understand the significance of the qualitative approach which both Wang Lai and Li Rui had created in the studies of algebra also played a role. Similarly, Li Shanlan's ignorance of Jiao Xun's symbolic approach to the underlying algebraic structure of the *Jiu Zhang Suan Shu* ("Nine Chapters on Arithmetic") probably kept him from appreciating De Morgan's symbolic algebra. Nevertheless, a shift of research focus to Ming Antu's mathematics after the 1820s by Dong Youcheng, Luo Shilin and Xiang Mingda actually re-emphasized the "algorithmic" approach to mathematics which was so familiar to Chinese mathematicians of the time [Cf. Chapter 1]. Consequently, the Chinese mathematicians in the second half of the 19th century would neglect De Morgan's algebra just as Li Shanlan did, especially if they, again as Li Shanlan did, could not understand properly what Jia Xun, Wang Lai and Li Rui's innovations in traditional algebra were all about.

Additional evidence also helps to explain the limited spread of De Morgan's algebra in the late 19th-century. For example, Gu Guanguang, one of Li Shanlan's close friends who was familiar with Western science and mathematics in the period 1840-50, criticized the *Elements of Algebra* as tedious in reasoning and therefore difficult to follow. But this comment was based on Gu's reading

Chapter I of its 1857 edition which had survived from a fire [Gu Guanguang 1883]. Thus it would be more representative to consider how other scholars or mathematicians regarded De Morgan's algebra.

Consider, for example, Ding Fubao's review of the *Dai Shu Xue*, which reflected to a considerable degree how late 19th-century Chinese mathematicians regarded De Morgan's book. Ding Fubao (1874-1952), who studied mathematics under Hua Hengfang and his younger brother Hua Shifang, was a high-school mathematics teacher during the late Qing [Ding and Zhou 1984, "Preface"]. His review, which actually corroborated Gu Guanguang's negative comments, may therefore be taken as a typical response of Chinese readers to the *Dai Shu Xue* [Ding Fubao 1899].

In his review, Ding Fubao contrasted De Morgan's *Algebra* with William Wallace's *Algebra*, which Wallace had written as an item for the fourth edition of the *Encyclopedia Britannica* (1801-1810). The Chinese edition of Wallace's *Algebra*, the *Dai Shu Shu* (1873), was translated by John Fryer (1839-1913) and Hua Hengfang when they both worked in the translation department at the Jiangnan Arsenal [Cf. Section 2.6]. The *Dai Shu Shu* has twenty-five chapters including two hundred and eighty-one items. Readers could read this algebra in a way similar to that of programmed learning. Moreover, such a format was also reminiscent of the "algorithmic" approach typically used in ancient Chinese mathematical texts like the *Jiu Zhang Suan Shu*. This feature would have certainly have facilitated the wide circulation of the

Dai Shu Shu in the last three decades of the 19th-century.

Indeed, it was Wallace's *Algebra*, not De Morgan's *Elements of Algebra*, that played a central role in the late Qing high-school mathematics curriculum [Wang Yangzhong 1988, p. 57]. The only exception might have been at the Tong Wen Guan -- the Imperial College in Beijing -- where Li Shanlan served as a mathematics instructor from 1868 on [Cf. Section 5.1].⁵

4.4. The Translation of Loomis' *Calculus*: The *Dai Wei Ji Shi Ji*

Compared with the *Dai Shu Xue* (De Morgan's *Algebra*), the *Dai Wei Ji Shi Ji* (Loomis' *Calculus*) was duly appreciated and had a wider circulation in late 19th-century China [See also Chapter 5]. This is no wonder considering how its format resembled with that of Wallace's *Algebra*. Loomis' *Calculus* was written "not for mathematicians, nor for those who have a peculiar talent for fondness for the mathematics, but rather for the mass of college students of average abilities," [Loomis 1851, "Preface"]. Nevertheless, since Loomis had an attempt to explain "the philosophy of these operations" like differentiation and integration, he gave necessary arguments for the "mechanical process of differentiating and integrating" [Loomis 1851, "Preface"]. Even so, a programmed and algorithmic style seemed to be the most favorable for Loomis to write such a book. Thus the translators Li Shanlan and Wylie could follow its original format and style easily, yet translate it into a more simplified version.



Figure 3. Title page of the *Dai Wei Ji Shi Ji* (Chinese edition of Loomis' *Calculus*, 1859).

According to Gisela Kutzbach, although the American mathematician Elias Loomis' work was not always original, his research was "highly valued because of its reliability. In his own time, however, Loomis was better known for the publication of a large number of textbooks on mathematics, astronomy, and meteorology than for his scientific investigations," [Quoted Kutzbach 1972]. Apparently this was indeed the case, for his *Calculus* is still assigned as a calculus textbook nowadays for college students who majoring in engineering and business. Although Loomis did not use δ - ϵ methods, his textbook nevertheless presented its reasonings and arguments with sufficient rigor and clarity even judged by modern standards. Therefore, it deserves here to trace back the source material this text had relied on.

Loomis' *Calculus* includes eighteen sections which are divided into three parts, namely "Analytical Geometry" (including nine sections), "Differential Calculus" (including seven sections) and "Integral Calculus" (including two sections). The title of Part One, namely "Analytical Geometry" was translated into "Dai Shu Ji He" (literally algebraic geometry) in the Chinese edition, the *Dai Wei Ji Shi Ji*. In addition, the *Dai Wei Ji Shi Ji* includes 18 chapters which were just put in order with the 18 sections of the original edition. Part Two (including seven sections) discusses differential calculus with an introduction to basic concepts like functions, independent and dependent variables, limits, differentials, differential coefficients and so forth. Numerous examples are also given illustrating these concepts as well as

verifying the theorems on differentials and differential coefficients. An interesting example about the differential of x^α should be mentioned. Loomis deals with the differential of x^α (which equals $\alpha x^{\alpha-1} dx$) only when α is an integer or a fraction (positive or negative). He seems never to have bothered the case in which α is irrational. In this Loomis was at least consistent, for he never endeavoured to deal with the problem of irrationals, namely, the completeness of real numbers with which the theory of calculus was built.

By contrast, De Morgan did consider irrational numbers in his *Algebra*. He extended the binomial theorem, for example, namely the expansion of $(1+x)^\alpha$ ($-1 < x < 1$) from the case of rational α to include irrational values of α as well. This was not especially remarkable because the central theme of the *Elements of Algebra* was the rigorous treatment of its mathematics. Considering his sound symbolic approach to algebra, De Morgan's treatment of the binomial series was just as rigorous principally because he introduced the needed concepts like convergence of infinite series quite clearly [Cf. De Morgan 1837, Chapter XI].

Similarly, Loomis might have been expected to introduce the concept of convergence before going on to treat Maclaurian and Taylor's series. Yet Loomis did not give any of the relevant concepts in Chapter 11 of his *Calculus* (i.e., Section II of Part II in his 1851 edition, hereafter we refer to the Chinese edition), where it would have been appropriate to do so. In this

chapter, he deals first with the Maclaurian series of functions of one variable, i.e., with the binomial series $(1+x)^\alpha$, where α only ranges over the rationals. Nor did Loomis give a satisfactory treatment of Taylor's series for function of two variables. Under such circumstances, it is not surprising that he failed to deal with the binomial series $(x+y)^\alpha$ in general terms for all cases of α .

Still, Loomis' *Calculus* had five other chapters on differential calculus. In Chapter Twelve, he took up the theory of extrema. Given a function with higher order derivatives, Loomis used its Taylor's series and the First Derivative Test and the Second Derivative Test to determine the extrema. As illustrations of these tests, at the end of the Chapter Loomis gave several interesting examples which still appear in nowadays calculus textbooks.

Chapter Thirteen focuses on transcendental functions, including covers circular or trigonometric functions, logarithmic functions as well as exponential functions. None of these transcendental functions, by definition, is an algebraic function. At the end of this chapter, Loomis gave Maclaurian series for $\sin x$ and $\cos x$, and reminded his readers that these series were quite helpful in constructing tables for the values of sine and cosine functions.

Further applying the differential calculus, Chapter Fourteen

gives methods for finding the tangent, subtangent, normal and subnormal lines at a given point of a plane curve which is the graph of a function. Moreover, based on the fact that

$$\lim_{P \rightarrow Q} \frac{\text{the arc PQ}}{\text{the chord PQ}} = 1,$$

where the chord PQ subtends the arc PQ, Loomis went on to prove that $ds^2 = dx^2 + dy^2$, where ds is the arc element of a given curve, and dx , dy are differentials of the variables x , y respectively. Of particular interest is the theorem Loomis gave equivalent to one form of the Fundamental Theorem of Calculus:

Let $A(x)$ denote the area bounded by the curve $y = f(x)$ through the origin of the Cartesian coordinate system, the x -axis and the line $\zeta = x$, then

$$d A(x) = f(x) dx \quad \text{or} \quad \frac{dA(x)}{dx} = f(x)$$

[Cf. Figure 4].

Loomis's argument is as follows:

As $h > 0$ is sufficiently small, then

$$A(x+h) - A(x) \doteq \frac{1}{2} [f(x+h) + f(x)].$$

Expand $f(x)$ into a Taylor series, we have

$$f(x+h) = f(x) + hf'(x) + \frac{h^2}{2!} f''(x) + \dots$$

$$\Rightarrow \frac{1}{2} [f(x+h) + f(x)] = f(x) + \frac{h}{2} f'(x) + \frac{h^2}{2 \cdot 2!} f''(x) + \dots$$

$$\Rightarrow \frac{A(x+h)-A(x)}{h} = f(x) + \frac{h}{2} f'(x) + \frac{h^2}{2 \cdot 2!} f''(x) + \dots (*)$$

To find the limit:

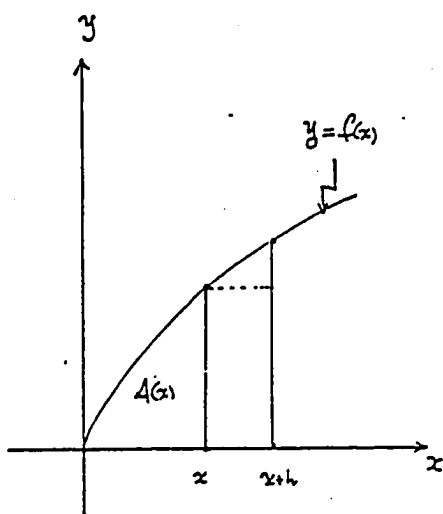
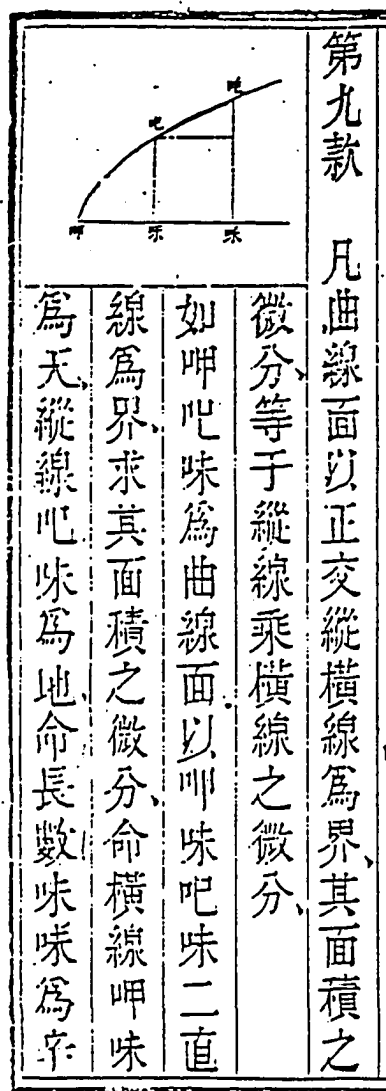


Figure 4. Duplicate of the diagram given in Loomis' *Calculus* (1859), Chapter 14, Proposition 9.

$$\lim_{h \rightarrow 0} \frac{A(x+h) - A(x)}{h} ,$$

one lets $h = 0$, and the right side of the equality (*) reduces to $f(x)$, so that by definition,

$$\frac{dA(x)}{dx} = f(x).$$

How did Loomis justify this last result by putting $h = 0$? In doing so, $\frac{A(x+h) - A(x)}{h}$ would lead to the form $\frac{0}{0}$. This was a long standing problem related to foundations of calculus and the rigor of such arguments involving infinitesimals that were not fully resolved until 1872 by Karl Weierstrass. Certainly Loomis did not supply arguments that were fully rigorous by modern standards. To find the derivative of a given function $y = f(x)$, Loomis began by computing a proportion for the differentials dy and dx , finally obtaining the result $\frac{dy}{dx}$. Instead of demonstrating this for a general function $y = f(x)$, Loomis illustrated his argument by examples. Thus in the case of $y = f(x) = x^3$ [Theorem 10 of Chapter 10 in Loomis' *Calculus*], his argument reads as follows:

If the increment of side x is h , then the corresponding increment of cube $f(x)$ is

$$f(x+h) - f(x) = (x+h)^3 - x^3 = 3x^2h + 3xh^2 + h^3.$$

Therefore,

$$\begin{aligned} &\text{the rate of change in side : the rate of change in cube} \\ &= h : 3x^2h + 3xh^2 + h^3 = 1 : 3x^2 + 3xh + h^2. \end{aligned}$$

Suppose h decreases to zero, then $3x^2 + 3xh + h^2$ approaches

to $3x^2$. Therefore by setting $h = 0$, we obtain the desired result:

the rate of change in side : the rate of change in cube
 $= 1 : 3x^2$.

Let dx , the differential of x , denote the rate of change in side, and dy , the differential of y , denote the rate of change in cube, respectively, then we rewrite the final result as

$$dx : dy = 1 : 3x^2$$

or by multiplying the inner and outer terms of the proportion respectively,

$$dy = 3x^2 dx$$

or further by making a quotient,

$$\frac{dy}{dx} = 3x^2.$$

Here it is quite clear that allowing $h = 0$ in the final step leading to the proportion $dx : dy = 1 : 3x^2$ was still problematic: Even if Loomis explained the limiting process $3x^2 + 3xh + h^2 \rightarrow 3x^2$ by substituting numerical values, he certainly was not able to eliminate the logical faults that allowed $h = 0$ in the earlier simplification of

$$h : 3x^2 + 3xh + h^3 = 1 : 3x^2 + 3xh + h^2.$$

It seems that Loomis adopted an approach to calculus similar to that of Sylvestre-Francois Lacroix (1765-1843), whose influential three-volume work of 1797-1800, *Calcul Calcul différentiel et du calcul intégral*, incorporated methods of limits and differentials as well as Taylor's series to deal with

both the differential and integral calculus [Wyllie 1859, "Preface"; I. Grattan-Guinness 1987]. So long as the rigorous foundation of analysis was not yet established, writers on calculus like Lacroix or Loomis who were concerned with pedagogy would usually present all three traditions (limits, differentials, Taylor's series) in their treatises in order to clarify as best as possible concepts or arguments about the foundations of calculus.

It is interesting here to see, on the other hand, how Li Shanlan reacted to the problematic subject of foundations. In 1861, Gu Guanguang gave his comments upon the *Dai Wei Ji Shi Ji* (Loomis' *Calculus*), but focused on the methods of differential calculus only. It is quite clear that Chinese mathematicians at the time were basically interested only in the mathematical methods and their applications. Clearly it was premature to expect Chinese mathematicians in the 1860s to think about foundational problems in the newly-introduced calculus. Nor was the case one decade later on. In 1874, Hua Hengfang, in collaboration with John Fryer, translated William Wallace's *Fluxions*, which appeared as another item in the eighth edition of the *Encyclopaedia Britannica* (1853-1860), into Chinese. This translation, *Wei Ji Suo Yuan* (literally tracing the origins of calculus) looked more fluent with respect to arguments than the *Dai Wei Ji Shi Ji* (Loomis' *Calculus*) -- reflecting Hua Hengfang's familiarity with the methods of calculus in the 1870s. Nevertheless, the *Wei Ji Suo Yuan* gave no sound definitions for differentials, derivatives or integrals. Furthermore, comparing

the concepts and methods of the *Wei Ji Shuo Yuan* with those of the *Dai Wei Ji Shi Ji* (Loomis' *Calculus*), Hua Hengfang paid much more attention to methods as such. Some explanations were given for such concepts as "differential", although his major concern still lay in the problem-solving aspects of calculus. However, before saying more about Hua Hengfang's reaction to the calculus, let us finish our brief survey of the *Dai Wei Ji Shi Ji*.

Chapter 15 deals with plane curves, including the concepts of curvature, the radius of curvature and the osculating circle or circle of curvature. In addition, Loomis also discussed the properties of involutes and cycloids, thereby showing that the involute of a cycloid is identical to the given cycloid itself.

In Chapter 16, Loomis surveys additional properties of plane curves including convexity, concavity, critical points, points of reflection and singularities. Along with these concepts, relevant examples are also given as illustrations.

Finally, the last two chapters, Chapters 17 and 18, are devoted to the integral calculus. At the beginning of Chapter 17, Loomis does not bother to define the integral. Instead, he simply points out that "the integral is a restoration of the differential, and the method of integration lies, therefore, in finding the function from which the differential is derived. For example, knowing that the function x^2 has differential $2xdx$, then given the differential $2xdx$, one knows immediately that it is

derived from the function x^2 . Consequently x^2 is the integral of $2xdx$." [Li and Wylie 1859 Chapter 17, p. 1]. Thereafter Loomis adds some basic theorems for computations of integrals. Besides, he also computes definite integrals like $\int_a^b \alpha x dx = \frac{\alpha}{2} (b^2 - a^2)$. He also notes that as integrals of $2xdx$, both $x^2 + C$ (C any constant) and x^2 lead to the value of the definite integral $\int_a^b \alpha x dx$.

Loomis also gives a method for computing the integral when its integrand (function) can be expanded into an infinite power series by the binomial theorem. For example:

Given $\frac{1}{1+x} dx$, find its [indefinite] integral.

Loomis first expands $\frac{1}{1+x}$ into the following series:

$$\frac{1}{1+x} = 1 - x^2 + x^4 - x^6 + \dots,$$

and then integrates the series to obtain the result:

$$\begin{aligned} \int \frac{1}{1+x} dx &= \int (1 - x^2 + x^4 - x^6 + \dots) dx \\ &= x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \dots \end{aligned}$$

However, Loomis does not justify this method rigorously by modern standards, for he fails to refer to any concept equivalent to uniform convergence. He does mention the convergence of the

infinite series into which $f(x)$ (the integrand of the integral) is to be expanded as a required condition, although the given examples contribute nothing to illuminate or clarify.

Chapter 18 deals basically with applications of the integral calculus. It covers computations of arc length, the area of the (planar) region bounded by the curve of a function $y = f(x)$ between the ordinates $f(a)$ and $f(b)$, the piece of the abscissa axis from $x = a$ to $x = b$, and the two straight lines $x = a$ and $x = b$, the area of a surface of revolution as well as the volume of a solid of revolution, and so on. In the case of finding the area under a curve but with the added condition that the differential of its area is $y(x)dx$, the required area, as Loomis argues, is equal to the integral of $y(x)$, i.e., $\int y(x)dx$. Here Loomis justifies this method in terms of his theorem equivalent to the Fundamental Theorem of Calculus (as already mentioned above).

How did Li Shanlan and Gu Guanguang (as well as their contemporaries) react to the most newly introduced Western mathematics -- the calculus? As for Li Shanlan himself, none of his own views on Loomis' *Calculus* are known except for a few remarks about the excellence of the methods of calculus he once made to Hua Hengfang in the 1850s, when he was still working on its translation [Hua Hengfang 1882, Book 5]. It is just as surprising that Li Shanlan made no response to Gu Guanguang's comments on the calculus. For example, in his *Queries and Answers on Astronomy and Mathematics* (1867), Li Shanlan said

nothing about either Gu Guanguang's mathematics or the calculus.

In the late 1850s, Gu Guanguang made several comments of his own about the calculus. His comparison of Loomis' *Calculus* with De Morgan's *Algebra* is suggestive and tells us something about the beginning of calculus in the 1860's China. Nevertheless, his views apparently had no influence over his contemporaries or later generations. Perhaps this was due to Gu Guanguang's critical, even harsh, evaluation of calculus with which Li Shanlan seems to have disagreed at that time. Under such circumstances, Li Shanlan naturally would ignore Gu Guanguang's views on calculus in the *Querries and Answers on Astronomy and Mathematics*.

Yet why did Gu Guanguang view calculus so negatively? This was probably due to the fact that he read calculus carefully in order to search for methods or results that had already been anticipated by the Chinese mathematicians. For example, in commenting on the binomial expansions like

$$(1 + x)^{1/2} = 1 + \frac{1}{2} x - \frac{1}{8} x^2 + \frac{1}{16} x^3 - \frac{5}{125} x^4 + \dots,$$

Gu Guanguang complained that the translators of the *Dai Wei Ji Shi Ji*, namely, Li Shanlan and Wylie, simply ignored an equivalent method that had already been invented by the Chinese mathematician Xiang Mingda. "So in this case there is no need to get help from the Westerners," [Gu Guanguang 1882a].

Likewise, in a comment on the binomial expansions dealt with

in Chapter 11 of Loomis' *Calculus*, Gu Guanguang even claimed the superiority of Xiang Mingda's four methods for binomial expansions. For example, he identified the expansions:

$$\begin{aligned} z &= (a + x)^{1/2} = a^{1/2} \left(1 + \frac{x}{a}\right)^{1/2} \\ &= a^{1/2} + \frac{1}{2} a^{1/2} \frac{x}{a} - \frac{1}{2 \cdot 4} a^{1/2} \frac{x^2}{a^2} \\ &\quad + \frac{1 \cdot 3}{2 \cdot 4 \cdot 6} a^{1/2} \frac{x^3}{a^3} - \frac{1 \cdot 3 \cdot 5}{2 \cdot 4 \cdot 6 \cdot 8} a^{1/2} \frac{x^4}{a^4} + \dots \end{aligned}$$

(if $|x| < a$);

$$\begin{aligned} z &= (a + x)^{1/2} = a^{1/2} \left(1 - \frac{x}{a+x}\right)^{-(1/2)} \\ &= a^{1/2} + \frac{1}{2} a^{1/2} \frac{x}{a+x} + \frac{1 \cdot 3}{2 \cdot 4} a^{1/2} \frac{x^2}{(a+x)^2} + \dots \\ &\quad + \frac{1 \cdot 3 \cdot 5}{2 \cdot 4 \cdot 6} a^{1/2} \frac{x^3}{(a+x)^3} + \dots \end{aligned}$$

(if $|x| < |a+x|$);

$$z = (a - x)^{1/2} = a^{1/2} \left(1 - \frac{x}{a}\right)^{1/2}$$

$$= a^{1/2} - \frac{1}{2} a^{1/2} \frac{x}{a} - \frac{1}{2 \cdot 4} a^{1/2} \frac{x^2}{a^2} - \frac{1 \cdot 3}{2 \cdot 4 \cdot 6} a^{1/2} \frac{x^3}{a^3}$$

+ ...

(if $|x| < a$);

$$z = (a - x)^{1/2} = a^{1/2} \left(1 - \frac{x}{a-x}\right)^{-(1/2)},$$

(if $|x| < |a-x|$), as respectively the "Third Method", "Second Method", "First Method" and "Fourth Method" of Xiang Mingda. Gu Guangguang justified these series expansions by substituting numerical values for x ("Tian"), a ("Jia"), respectively, which satisfied the conditions for the convergence of the series. However, he did not mention these conditions. Gu Guangguang was then pleased to suggest that "once Wylie has a chance to look at these results, he should applaud the ability of native Chinese mathematicians," [Gu Guangguang 1882a].

On the other hand, the comments Gu Guangguang made on the methods of differential calculus given in Chapter 10 of Loomis' *Calculus* were quite appropriate. For example, he claimed that the following four methods (or theorems) were actually one method (or theorem):

$$dx^n = nx^{n-1}dx, \text{ where } n \text{ is a positive integer.}$$

$$dx^\alpha = \alpha x^{\alpha-1}dx, \text{ where } \alpha \text{ is a rational.}$$

$$d(x^m)^{1/2} = \frac{mx^{m-1}}{2(x^m)^{1/2}} dx, \text{ where } m \text{ is a positive integer.}$$

$$d(a + bx + cx^2 + \dots)^\alpha = \alpha(a + bx + cx^2 + \dots)^{\alpha-1}(b + 2cx + \dots)$$

He thereby warned the readers "not to be cheated and confused by the Westerners" [Gu Guanguang 1882a].

In three examples given to illustrate Theorem 3 of Chapter 10, one finds false answers but without demonstrations. Thus, for first example:

Given that x increases (uniformly) by 2 inches per second [i.e., the increment $h = 2$ inches/sec], find the rate of change of $2x^2$ at $x = 6$ inches.

One is given the answer: 28 inches per second. And Gu Guanguang "justified" the answer as follows: Since

$$\frac{2(x+h) - 2x}{h} = \frac{4xh + 2h}{h} = 4x + 2h,$$

substituting x, h by 6, 2 respectively, one obtains the "desired" result:

$$4x + 2h = 4 \cdot 6 + 2 \cdot 2 = 28.$$

Gu Guanguang "demonstrated" the other two examples in a similar way. All these point to the fact that Gu Guanguang had

misunderstood the meaning of the concept "rate of change."⁶ The correct solution should be as follows: Since

$$\begin{aligned} &\text{The rate of change of } 2x^2 \\ &= \text{The differential of } 2x^2 \text{ (by definition)} \\ &= d(2x^2) \text{ (by convention of notation)} \\ &= 4xdx \text{ (by definition).} \end{aligned}$$

Therefore, given $dx = 2$, the rate of change of $2x^2$ at $x = 6$ equals $4 \cdot 6 \cdot 2 = 48$ [Hua Hengfang 1882, Book 10].

The last solution was given by Hua Hengfang. In fact, he also gave the correct solutions for other examples Gu had given incorrectly. Hua Hengfang also explained why the erroneous answers had been given: the author or the translators of the *Dai Wei Ji Shi Ji* mistook the rate of change of the function $y = f(x)$ as $\frac{f(x+h)-f(x)}{h}$ [Hua Hengfang 1882, Book 10]. The correct answer should be $df(x) = \frac{dy}{dx} dx$, to use the notation of Loomis' *Calculus* or Wallace's *Fluxions*. Yet why didn't Gu Guanguang avoid these errors that had been made in the *Dai Wei Ji Shi Ji*? Although we cannot answer this question, we can suggest why Hua Hengfang was able to determine the errors and give correct solution. The answer probably lies in the fact, as Hua Hengfang himself says: "The *Dai Wei Ji Shi Ji* deals with differentials to a great extent, explaining the ratio between the rate of change of a function $f(x)$ and that of its independent variable x . Readers often skip through it carelessly, so that they are not able to comprehend the

meaning of differentials," [Hua Hengfang 1882, Book 10]. Yet what is a differential? Hua Hengfang thought that the easiest way to explain differentials to readers is to illustrate the concept of differential in terms of geometrical magnitudes like points, lines, surfaces and solids. For example, a point can be taken as the differential of a line, a line can be as the differential of a surface, a surface can be as the differential of a solid.

"Therefore, formulae or expressions concerning differentials can be justified in terms of points, lines, surfaces and solids," [Hua Hengfang 1882, Book 10].

Based on these arguments, Hua Hengfang went on to justify the method that was given in Loomis' *Calculus* for finding differential coefficients of a given function, as well as that for obtaining the differential of a given function in Wallace's *Fluxions*. The method of finding the differential coefficients of a given function in Loomis' *Calculus* has already been discussed in the preceeding paragraphs. Here, for the sake of cmparison, we cite Loomis' treatment for a general function:

Given a function $y = f(x)$. Expand $f(x+h)$ into

$$f(x+h) = a_0 + a_1 h + a_2 h^2 + a_3 h^3 + \dots$$

Let $h = 0$. It then follows that

$$f(x) = a_0,$$

and thus

$$f(x+h) = f(x) + a_1 h + h^2(a_2 + a_3 h + \dots).$$

By letting

$$a_2' = a_2 + a_3 h + \dots,$$

one can rewrite the original expansion as follows:

$$f(x+h) = f(x) + a_1 h + a_2' h^2.$$

Therefore,

$$\frac{f(x+h) - f(x)}{h} = a_1 + a_1' h. \quad (*)$$

Finally, setting $h = 0$, one obtains

$$\frac{df(x)}{dx} = a_1.$$

That means a_1 is the differential coefficient of the given function [Loomis 1859, Theorem 7 of Chapter 10].

According to Hua Hengfang, the "method" given in Wallace's *Fluxions* for obtaining the differential of a given function can be described as follows:

Given a function $f(x)$, expand $f(x+h)$ into

$$f(x+h) = f(x) + f'(x)h + \frac{f''(x)}{2!} h^2 + \frac{f'''(x)}{3!} h^3 + \dots$$

Then replace h by dx in the first term containing h , namely, $f'(x)h$, and eliminate all of the terms containing higher powers of h . One thus obtains the differential of $f(x)$:

$$df(x) = f'(x)dx.$$

Hua Hengfang claimed that both methods were essentially equivalent. The question, according to Hua Hengfang, is why in

Loomis' "method" one should not put $h = 0$ until the expression (*) is to be formed, or equivalently, h has been used as a divisor, even if h is regarded as an infinitely small variable. In the meantime, why replace h by 0 on the right side of (*) but not on the left side? Instead, the denominator h of the left side of (*) is replaced by dx in order to get the final result.

This serious problem of rigor, according to Hua Hengfang, seemed to have been recognized by Wallace. Thus Wallace in his *Fluxions* preferred the argument as cited above. Yet the reason why Wallace felt justified in eliminating the terms containing powers of h higher than one is unclear, so the difficulties related to lack of rigor are still there!

Someone might argue, however, that if h is sufficiently small, then h^2 must be much less than h and h^3 much less than h^2 as well. Thus each power of h can be neglected in the computation. Yet Hua Hengfang did not accept such an argument. He reasoned that since differentiation is one of the most elaborate and precise methods in mathematics, the theories underlying the methods of differential calculus likewise had to conform to the the precision of mathemaitcal theories [Hua Hengfang 1882 Book 10, p. 6]. In this case, there must have been some justification for setting $h = 0$ in the final step of Loomis' "method". It is right here that Hua Hengfang refers to his own illustrations of the concept of differential in terms of point, line, surface and solid [Hua Hengfang 1882 Book 10, p. 6].

Based on these illustrations, Hua Hengfang asserts that "If a "Wei Chan Zhi Shu" (literally a quantity of tiny length) or infinitesimal h is regarded as a point, then each power of h should be regarded as a point too. On the other hand, if a variable x is regarded as a line, then x^2 , x^3 , x^4 should be regarded as a plane,, a solid, a four-dimensional solid respectively," [Hua Hengfang 1882 Book 10, p. 7]. Thus in the following expression:

$$(x + h)^2 - x^2 + 2xh + h^2,$$

the term h^2 could be annihilated or eliminated, for it is understood as a point added to the line, $2xh$, and contributes nothing significant to the term $2xh$. Note that $2xh$ is regarded as a line because it is a multiple of the line $2x$ and the point h . Likewise, in the following expression:

$$(x + h)^3 - x^3 = 3x^2h + 3xh^2 + h^3,$$

the terms $3xh^2$ (a line) and h^3 (a point) contribute nothing significant to the term $3x^2h$ (a surface) in the geometrical illustration. Similarly, in the expression:

$$(x + h)^4 - x^4 = 4x^3h + 6x^2h^2 + 4xh^3 + h^4,$$

the same sort of argument is also applicable. In conclusion, Hua Hengfang claims that it is not for the sake of saving computation,

but for theoretical reasons that each power of h^n ($n = 2, 3, \dots$) is either taken to be zero or eliminated as negligible. "The annihilated or eliminated terms in fact are not necessary for the computation." Hua Hengfang adds that "the theories underlying the methods of differential calculus thus have been established without any doubt. Readers can thereby learn various methods of differential calculus!" [Quoted Hua Hengfang 1882, Book 10]

As for the integral calculus as it appeared in either Loomis' *Calculus* or Wallace's *Fluxions*, Hua Hengfang admitted that it was not easy to understand after only quick reading. Moreover, they were so hard to get through that reviewing it even as dozens of times was not sufficient [Hua Hengfang 1882 Book 11, p. 7]. The reason, as Hua Hengfang observes, was that too few arguments or explanations were given in either treatise, although neither was lacking in illustrations or examples. Thus in his *Xue Suan Bi Tan* ("Notes on Learning Mathematics", 1882) Hua Hengfang quoted a few theorems and methods from each treatise, adding his own demonstrations or explications. It is not necessary to deal with these here, however, since they do not seem directly related to the transmission and influence of Loomis' *Calculus* over Hua Hengfang's subsequent mathematical studies.

According to Ding Fubao, a noted disciple of Hua Hengfang and his younger brother Hua Shifang, the *Wei Ji Suo Yuan* (Wallace's *Fluxions*) included eight chapters: the first four chapters are devoted to the differential calculus, while the last four chapter

are for the integral calculus [Ding Fubao 1899]. In comparison with Wallace's *Fluxions*, Ding Fubao also believed that Loomis' *Calculus* was difficult to follow, and did not contain enough diversified examples or demonstrations. For this reason, Ding Fubao suggested that readers should refer not only to Loomis' *Calculus* but to Wallace's *Fluxions* as well.

The *Wei Ji Suo Yuan* (Wallace's *Fluxions*) appeared in 1874. Somewhat earlier, Feng Guifeng and Chen Yang had written a treatise in 1865, the *Xi Suan Xin Fa Zhi Jie* ("Demonstrations on the New Methods of Western Mathematics"), which was basically a commentary on Loomis' *Calculus*. Both authors aimed at writing a beginner's treatise, since there were too few scholars who were prepared to get through Loomis' *Calculus*. This was indeed the case, for according to Ding Fubao, the *Xi Suan Xin Fa Zhi Jie* was especially convenient for beginners. On the other hand, this treatise was fervently attacked by the late 19th-century Chinese mathematicians for making insignificant changes in notation and symbols from those used in Loomis' *Calculus*. Nevertheless, it seems to have through more editions than the *Dai Wei Ji Shi Ji* (Loomis' *Calculus*) [Ding Fubao 1899 vol.2, p. 78].

4.5. The *Yuan Zhui Qu Xian Shuo* (On Conic Sections)

As mentioned in Section 4.4, Loomis' *Calculus* includes nine chapters on analytic geometry. Its first three chapters deal with coordinate systems and transformation formulae. In Chapters Four,

Five, Six and Seven, treatments are given for the circle, parabola, ellipse and hyperbola, respectively. Loomis dealt with all of these quadratic curves in the context of "analytic geometry". This means that he defined them "analytically". Take the parabola, for example. Loomis gave its definition analytically as follows:

The parabola is one of the conic sections. It is the locus of all points of the plane that have the same distance from a fixed point and a fixed line. The fixed point is called the focus, and the fixed line the directrix of the parabola respectively [Loomis 1859, Chapter Five].

However, Loomis did not go further to explain why the parabola defined this way is indeed a curve derived from the intersection of a plane and a cone, even though in Chapter Twelve, he did give an example as follows:

Given a cone, find the radius of the biggest parabola derived from it.

Obviously, Loomis knew, as the above example shows, that the parabola defined "analytically" is equivalent to the parabola derived from the planar "section" of the cone.

It was Pierre Dandelin (1794-1847) who was the first to use spheres touching the cone and the intersecting plane to derive properties of the conics [Gellert *et al.*, eds., 1977, p. 303]. Such an approach certainly did not appear in Loomis' *Calculus*, nor in the *Yuan Zhui Qu Xian Shuo* (*On Conic Sections*, by an anonymous author) which was included as an appendix to the Chinese

translation of Whewell's *Mechanics*, the *Zhong Xue*. However, *On Conic Sections* did show that the conic "sections," namely ellipse, hyperbola and parabola, defined geometrically as sections of cones, were indeed the same as those curves defined "analytically".

In contrast to the "analytical" method used for quadratic curves or conic sections in Loomis' *Calculus*, the *Yuan Zhui Qu Xian Shuo (On Conic Sections)* deals with these curves "synthetically". Thus readers should not expect to see any coordinate systems in this treatise. Nor did its author ever try to treat the ellipse, hyperbola or parabola as a quadratic equation with two unknowns. But this was doubtless not a theme about which its author was concerned. A more interesting question is why Joseph Edkins should have suggested that *On Conic Sections* be added as an appendix to the Chinese translation of Whewell's *Mechanics*, the *Zhong Xue*. Perhaps it was due to the fact that Whewell's *Mechanics* has one chapter, namely, Chapter Eleven, dealing with the motion of projectiles. This, of course, involved discussion of the parabola, and Edkins might have gotten the impression that the basic properties of the parabola were not discussed extensively enough in Loomis' *Calculus*, and that much more material on conic sections (including the parabola) should be added to the *Zhong Xue* in one form or another.

On Conic Sections includes three chapters dealing with the ellipse, hyperbola and parabola. At the very beginning of Chapter

One, the author gives definitions of six conic "sections": vertex (of the cone), triangle, circle, ellipse, hyperbola and parabola, depending of course upon how the planar sections intersect the cone [Cf. Figure 5]. Immediately thereafter the author gives definitions for terms like generator (of the cone), vertex of the cone or conics, diameter, conjugate diameter, principal and conjugate semi-axes, focus, and so on. Especially interesting here is that the author also gives a method of constructing the asymptotic lines for a given hyperbola, and thereby characterizes the hyperbola of equal axes like $x^2 - y^2 = a^2$ by the property that its two asymptotes are perpendicular.

On Conic Sections offers eleven theorems dealing with the ellipse in Chapter One. Compared with similar theorems given in Chapter Six of Loomis' *Calculus*, *On Conic Sections* offered nothing new on this subject except for its "synthetic" treatment. One thing, however, deserves attention: *On Conic Sections* shows that the ellipse produced by a conic section geometrically is indeed the locus of all points (of the plane) defined "analytically" by the quadratic equation $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. In other words, the "conic section" -- ellipse -- is the locus of points in the plane for which the sum of distances from the two fixed points (the foci of the ellipse) to any point on the ellipse is constant.

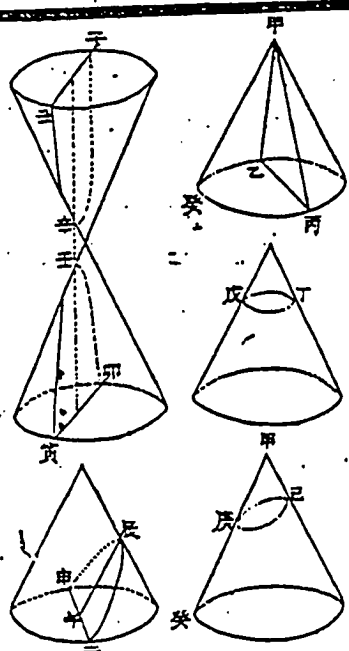
Li Shanlan must have been aware of the more limited content of *On Conic Sections*, for his *Tuo Yuan Shi Yi* ("Supplements to the Theory of the Ellipse") adds quite a lot of material which was not

圓錐曲線說卷一

英國艾約瑟口譯

海甯李善蘭筆述

圓錐任意割之其所割之面有六種界一頂點二三角形
三平圓四橢圓五雙曲線六拋物線



如圖甲爲頂點甲乙丙
爲三角形丁戊爲平圓
與底平行己庚爲橢圓
與底成角小于腰線甲
癸與底所成之角子辛

Figure 5. Title page of the Yuan Zhui Qu Xian Shuo.

covered in *On Conic Sections* or in Loomis' *Calculus*. However, as we have already observed [Cf. Section 3.6], Li Shanlan's principal interest in the ellipse lay its applications to astronomy -- a subject to which he had paid considerable attention in such treatises as the *Tuo Yuan Zheng Shu Jie* ("Explanations of the methods for Ellipse") and *Tuo Yuan Xin Shu* ("New Methods for Ellipse") [Cf. also Section 3.6]. Moreover, Li Shanlan neglected entirely the totally different approaches to ellipses taken in both *On Conic Sections* and in Loomis' *Calculus*. But then, neither his younger colleagues like Hua Hengfang, nor his disciples like Yang Zhaoyun, were able to appreciate the significance of these different approaches. In fact, Li Shanlan and Yang Zhaoyun dealt with the ellipse without reference to coordinate systems, either rectangular or polar [Yang Zhaoyun 1898]. This might suggest that the coordinate systems introduced in Loomis' *Calculus* (or in De Morgan's *Algebra*) had made no impression on Li Shanlan or his contemporaries. We shall return to this subject later; for now, let us finish our discussion of conic sections.

Chapter Two of *On Conic Sections* deals with the hyperbola in thirteen theorems which basically cover the major aspects of subject. Therefore, although armed with coordinate systems, Loomis' *Calculus* includes nothing essentially new to the subject that was not already a part of *On Conic Sections*. Instead, the last two theorems of *On Conic Sections* the last two theorems deal with the relationship between logarithms and the hyperbola. In other words, its author gives a treatment of logarithms based on

the formula that (in modern notation)

$$\int_1^x \frac{1}{t} dt = \log_e x.$$

Upon seeing this, Li Shanlan would immediately wrote his own

treatise, the *Dui Shu Jian Zhui Bian Fa Shi* ("On the Equivalence of the Western and the 'Jian Zhui' Methods for Logarithms"), to show that the Western method was equivalent to his own "Jian Zhui" method [Cf. Section 3.4].

Still, one more aspect of *On Conic Sections* deserves our attention. In Chapter Two, the author also showed that the parabola as a "conic section" was indeed the locus of points in the plane that have the same distance from some fixed point (the focus) and a fixed line (the directrix). In Chapter Three, however, *On Conic Sections* includes more material on this subject than what one finds in Loomis' *Calculus* -- a fact that may serve to justify Edkins' recommendation that *On Conic Sections* be translated into Chinese. In fact, *On Conic Sections* has sixteen theorems on the parabola compared with only nine related theorems in Loomis' *Calculus*. The additional theorems of *On Conic Sections* are quite interesting insofar as they investigate how to treat the parabola in terms of its tangent line. For example, the Corollary to Theorem Eight shows how to construct the parabola given its tangent.

Equally interesting in this connection was Li Shanlan's commentary on the geometrical construction of the parabola based on the "analytical" definition. In order to clarify the method given in the treatise [Cf. Corollary 2 to Proposition 3], Li Shanlan offered a simpler way of locating the points having the same distance to a fixed point and to a fixed line [Cf. Figure 6].

Finally, why did Li Shanlan comment upon *On Conic Sections* with Loomis' *Calculus* in hand? Li Shanlan actually referred to Loomis' *Calculus* several times in his translation of *On Conic Sections*. Thus it seems he learned nothing about conic sections in terms of "analytical" geometry from Loomis' *Calculus*. If this is indeed the case, then the treatment of conics in *On Conic Sections* must have met his needs in terms of applications. For example, in treating projectile motion, or applications of the ellipse to astronomy (as well as the relationship between logarithms and his own "Jian Zhui" method), Li Shanlan never used coordinate systems, even though they were introduced in Loomis' *Calculus*. The reason why he felt no inconvenience in so doing was that *On Conic Sections* often gives arguments or demonstrations in terms of proportions. He could therefore avoid the uses of quadratic equations of two unknowns as well as the coordinate systems in dealing with the related subjects.

In Yang Zhaoyun's treatises on mathematics, the *Xu Man Jing Lu Suan Xue* ("The Collected Mathematical Works of Yang Zhaoyun",

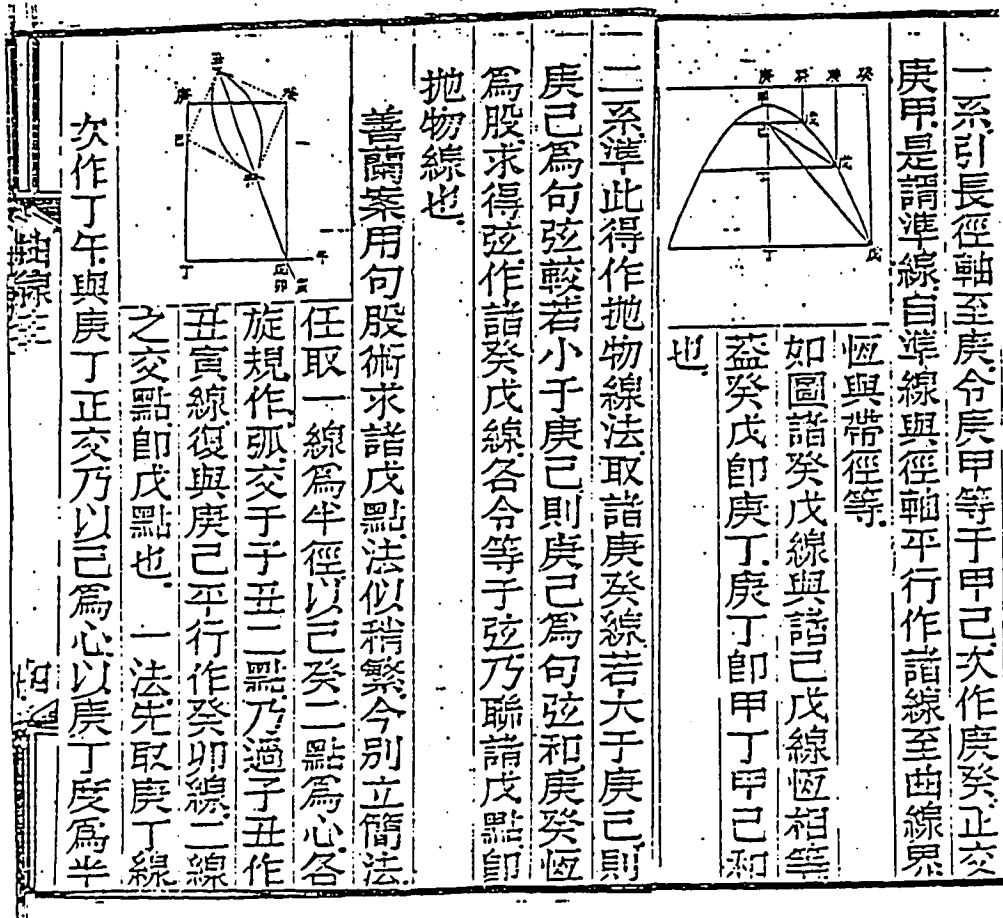


Figure 6. Corollaries to Theorem Eight of the Yuan Zhui

Qu Xian Shuo and Li Shanlan's commentary.

1898), we also find the same approach to conic sections, although he added nothing new to what was already contained in the treatise *On Conic Sections*. This fact might suggest that students like Yang Zhaoyun at the Tong Wen Guan (the Imperial College) learned the subject conic sections in a similar way. Moreover, at the Tong Wen Guan the course like calculus, in which Li Shanlan would have taught conic sections in terms of analytic geometry, probably was not so much emphasized [Cf. Sections 5.1, 5.2 and 5.3] as the other subjects, so that the bright students like Yang Zhaoyun would regard conic sections thus treated as insignificant.

Nevertheless, Yang Zhaoyun seems to have followed Li Shanlan's steps to add something, if only quite few, to the subject. It should be noted that the constructions of the so called "generalized directrices" for the ellipse as well as the hyperbola were treated in Yang Zhaoyun's *Xu Man Jing Lu Suan Xue* ("The Collected Mathematical Works of Yang Zhaoyun", 1898). Yang Zhaoyun has rightly argued that in such cases "directrices" for the ellipse and hyperbola are not really lines, as in the case of the parabola, but circles instead [Cf. Figure 7]. Besides, he also mentioned in the same passage that Chinese mathematicians had already developed these same results on their own, which apparently added something new to Western methods of construction -- but only for the directrix of the parabola. Yet Yang Zhaoyun did not refer to any Chinese mathematicians who were actually interested in the subject. Thus it is not certain whether he knew Xia Luanxiang's study of this material or not.

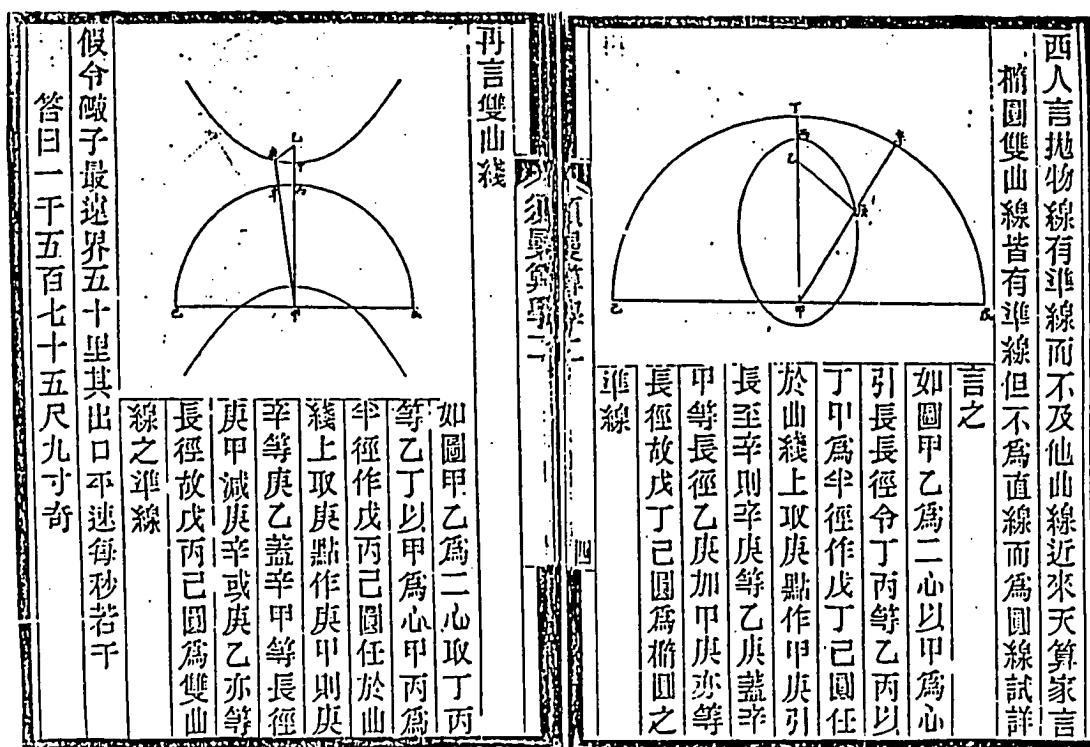


Figure 7. Two pages from Yang Zhaoyun's Xu Man Jing Lu

Suan Xue ("Collected Mathematical Works of Yang Zhaoyun") showing how to construct "directrix" for ellipse and hyperbola.

Xia Luanxiang (1823-1864) studied mathematics under Xiang Mingda (1789-1850) and then became a younger but close friend of Dai Xu (1805-1860). Probably influenced by Xiang Mingda and Dai Xu (both of whom had paid considerable attention to the theory of the ellipse), Xia Luangxiang's research was basically concerned with conics as well. However, Xia Luanxiang was able to create a new approach of his own to the subject. According to the historian of mathematics Liu Dun, Xia Luanxiang's *Zhi Qu Tu Jie* ("On the Graphic Curves", ca. 1860-1864), which deals with the intrinsic properties of conics "synthetically", is a unique work in the history of Chinese mathematics [Liu Dun 1984].

Indeed, the basic idea underlying the results in the *Zhi Qu Tu Jie* is a principle of continuity -- one concerned with continuous transformation of geometrical entities from one state to another. In this connection, Xia Luanxiang put forward four propositions, namely:

- 1) The circle is a special case of the ellipse with its two foci coincident with one another;
- 2) The Parabola is a "polar form" of ellipse or hyperbola in the sense that it can be obtained by continuously transforming the ellipse or hyperbola to the extreme situations;
- 3) Similarly, the ellipse and hyperbola can be regarded as "reciprocal forms" one another; and
- 4) The circle and hyperbola of equal axes are "reciprocal forms" one another [Liu Dun 1984].

As a whole, Xia Luanxiang created a general method of treating conics "synthetically". First, he investigated the simplest properties of the circle. Then he drew an analogy between the properties of ellipse and the circle, and finally he obtained the properties of the parabola and hyperbola by arguments related mainly to concepts of "polar form" and "reciprocal form", respectively. Xia Luanxiang boldly introduced many new concepts including the "generalized directrix," already mentioned above. He also proposed that parallel lines meet at the "point at infinity," whereby a line can be regarded as a circle with its "center at infinity". In doing so, Xia Luanxiang was eventually able to offer a unified treatment of conics in the context of "synthetic" geometry which certainly was unknown to Xia Luanxiang and his contemporary fellow-mathematicians [Liu Dun 1984; Li Di 1984, pp. 378-82].

Xia Luanxiang was certainly familiar with the *Dai Wei Ji Shi Ji* (Loomis' *Calculus*). He not only cited the classifications of algebraic curves given in Chapter 8 of the *Dai Wei Ji Shi Ji*, but based on this he even re-coined the relevant terms already used in his own *Zhi Qu Tu Jie* ("On the Graphic Curves") [Liu Dun 1984]. Furthermore, he was also able to develop some results like hyperbolic functions which had not yet transmitted to China. However, it is unknown whether the *Yuan Zhui Qu Xian Shuo* (*On Conic Sections* translated by Li Shanlan and Joseph Edkins) was available to Xia Luanxiang. Whatever the case, Xia Luanxiang apparently created a new approach to conics that was different

from that of Qing mathematicians who were mainly interested in the questions of arc length and areas related to conics in the period 1840-1860. Unfortunately, Xia Luanxiang did not seem to have exerted any influence over his successors, including even Li Shanlan's disciples like Yang Zhaoyun.

4.6. Euclid's *Elements*: The *Xu Yi Ji He Yuan Ben*

Now we turn to the translation of Euclid's *Elements*. As noted in Chapter 1, the *Xu Yi Ji He Yuan Ben* (the Chinese edition of the *Elements*, the last nine books) includes the last seven of the thirteen books of Euclid's *Elements*, as well as the two additional books ascribed to an anonymous author. Each of these nine books, according to Wylie, came from a dated English edition translated from the Greek original [Wylie and Li Shanlan 1859, "Preface"]. Yet such an edition was scarcely available in Britain at that time, so Wylie would have made every effort to obtain a copy from his homeland. Why a complete edition of Euclid was not readily available then is unclear. Perhaps the last few books were not thought of sufficiently important to general education in early 19th-century Britain. Wylie admitted that his countrymen studied Euclid's *Elements* no further than Book VI or VIII. Meanwhile, Li Shanlan also reported that:

Every country in the West has its own translation of Euclid's *Elements*. Yet Book X is sophisticated and thus cannot be understood quickly unless one reads it very carefully. Only a few Western scholars, therefore, are versed in its reasoning. Apparently this is why all Western editions for popular use omit Books VII, VIII, IX and X. Thus those abridged editions come to include the first six books and Book XI only. On the other hand, there also exist editions

including only the first six books, which are issued accompanied with the editions including 15 books [Wylie and Li Shanlan 1859, "Preface"].

No wonder that Book X of Euclid's *Elements* was become an obstacle for general readers -- as Carl Boyer has said, "Book X of the *Elements* was, before the advent of early modern algebra, the most admired -- and the most feared," [Boyer 1985, 128]. Book X not only includes as many as 115 propositions, but deals with incommensurable magnitudes to a great extent. De Morgan wrote for the *Penny Cyclopaedia* under "Irrational quantity": "Euclid investigates every possible variety of lines which can be represented by $(\sqrt{a} \pm \sqrt{b})^{1/2}$, a and b representing two commensurable lines ... This book has a completeness which none of others (not even the fifth) can boast of; and we could almost suspect that Euclid, having arranged his materials in his own mind, and having completely elaborated the 10th Book, wrote the preceding books after it and did not live to revise them thoroughly," [Quoted in Heath 1956 vol.3, p. 8].

Yet whether De Morgan's hope of popularizing mathematical knowledge through the *Penny Cyclopaedia* was ever attained or not, the teaching of Euclid's *Elements* in high schools remained very limited in scope, and usually to nothing more than the first few books or propositions of the *Elements*. For example, the geometry requirement to pass the University of London's Matriculation Examination in 1838 was an understanding of only the first book of Euclid [Howson 1982, p. 214]. On the other hand, in the "Specimen Papers for the Matriculation Examination" held in November of

1838, among "Problems in Pure Mathematics (Honours)," nothing was included about geometry beyond the first six books of Euclid's *Elements*, despite the fact that it also posed three problems on conic sections [Howson 1982, p. 216]. There was one problem on irrational numbers: "Reduce the square root of 13 to a continued fraction." However, the irrationality of the square root of 13 is not easy to deal with, although reducing it to a continued fraction is a more convenient method for obtaining its approximation than using a decimal fraction. Here the theory of limits was involved at least implicitly insofar as a continued fraction may be regarded as an infinite series. Even so, a rigorous theory of irrational numbers was not advanced until the 1870s by Europeans like Cantor and Dedekind.

The above observations, though fragmentary, point to the fact that British mathematicians at the time did not think that the irrationals, in comparison with negative and imaginary numbers, an urgent problem in their teaching or research. De Morgan apparently was one of the few exceptions. Nevertheless, he was not able to offer a rigorous treatment of irrationals -- and the arguments he advanced in his *Elements of Algebra* on the subject proved unsatisfactory. By the 1830s, the geometry taught in British public schools either did not go beyond Book VI of Euclid's *Elements* or skipped Book X, going on directly to Book XI. As Li Shanlan remarked in his "Preface" to the *Xu Yi Ji He Yuan Ben*, this was exactly what happened to the British missionaries like Wylie and Edkins when they were in schools. Therefore, how

Li Shanlan and his contemporary fellow-scholars reacted in the 1850s to this Greek classic (and to its Book X in particular) was as important as how much of Euclid's *Elements* British missionaries like Wylie and Edkins had learned in their high school days.

But before discussing Book X of the *Elements*, consider Books VII, VIII and IX. These three books deal with number theory. They are the only ones in the *Elements* that treat arithmetic as such. In them Euclid represents numbers as line segments and the product of two numbers as a rectangle. Even so, the arguments do not depend on geometry, and the statements and proofs are verbal (as opposed to modern symbolic formulations).

If we take a standard Euclid's *Elements* as edited by Thomas Heath, then *Xu Yi Ji He Yuan Ben* is a rather faithful translation for these three books. In fact, the Chinese edition only adds two propositions to the total of thirty-nine in Book VIII. (No additions are made to Books VII and IX). The first new proposition, (Proposition 20, added in the Chinese edition), is:

If three numbers are proportional, the product of the extremes is equal to the square of the mean, and conversely.

This proposition is not included in J.L. Heiberg's text, so it is not included in Heath's edition of the *Elements* (which was based upon the Heiberg edition of 1892; see Heath 1956 vol.2, p. 320]. The only possible source was the English edition that Wylie and Li Shanlan based to translate. The second new theorem, namely,

Proposition 22 in the Chinese edition (in modern notation):

If $a : b = e : f$ and $b : c = d : e$, then $a : c = d : f$.

This second theorem is also omitted by Heiberg, since, according to Heath, it "was no doubt interpolated by Theon," [Quoted Heath 1956 vol.2, p. 322].

It should be noted here that Li Shanlan (or Wylie) gave no commentary and no notes for these three books. The reason why not is unclear. Perhaps it was because Li Shanlan knew almost nothing about number theory. After all, the theory of numbers was Western tradition which was apparently unknown to Li Shanlan until the 1850s. Ancient Chinese mathematics, however, did develop some methods related to the theory of numbers. For example, in order to treat operations with fractions, the *Jiu Zhang Suan Shu* ("Nine Chapters on Arithmetic") gives a method known as "Geng Xiang Jian Sun," similar in principle to the Euclidean algorithm, to find the common divisors of two given natural numbers and thereby reduce a fraction to lowest terms [Cf. Chapter 1 of *Jiu Zhang Suan Shu*]. Nevertheless, ancient Chinese mathematicians do not seem to have been concerned about the theory of numbers as such. Li Shanlan was no exception. Although he took term "Deng Shu" ["equal number"] for the translation of "common measure" or "common divisor" in Book VII of the *Elements* from the *Jiu Zhang Suan Shu*, on the other hand he translated the method of the Euclidean algorithm as "Zhan Zhuan Xiang Chu" instead of "Geng Xian Jian

Sun" which had been used in the *Jiu Zhang Suan Shu* and the subsequent Chinese mathematical texts.⁷

However, Euclidean number theory gradually attracted the attention of Chinese mathematicians, including Li Shanlan himself (from the 1860s on). Li Shanlan's *Kao Shu Gen Fa* ("Investigations of the Properties of Prime Numbers") is only one of several examples (the *Kao Shu Gen Fa* is discussed in the next chapter). Here, the remaining books of the Chinese editions of the *Elements*, the *Xu Yi Ji He Yuan Ben*, are briefly considered.

Turning back to Book X of the *Elements*, its Chinese version includes two more propositions not found in the Greek text, namely, Propositions 13 and 117. Proposition 13:

Given two magnitudes, if one is with a third magnitude commensurable but the other is incommensurable, then the two given magnitudes are incommensurable.

This proposition is not mentioned by T.L. Heath in his critical edition with notes of the *Elements*. This might help to identify the original source upon which the Chinese edition was based. Unfortunately, this proposition has not drawn the attention of the historians of Chinese mathematics like Du Shiran, Li Di, Mei Rongzhao, Wang Yusheng and Liu Dun who all seem to accept Qian Baocong's conjecture that the Chinese edition must have come from Issac Barrow's English edition, *The Elements: the Whole Fifteen Books* [Qian 1981, pp. 324-24].

On the other hand, Chinese historians have directed their

attention to Proposition 117 in the *Xu Yi Ji He Yuan Ben*:

For any square, its side is incommensurable to its diagonal.

In the Chinese edition, three comments follow immediately, Mei Rongzhao, Wang Yusheng and Liu Dun all take them to have been written by Li Shanlan [Mei Rongzhao *et al.*, 1986]. If this was indeed the case, then Li Shanlan was the first to have gone beyond traditional Chinese mathematics and treated irrational numbers like $\sqrt{2}$ in any detail.

Here De Morgan's comment about the theory of irrational numbers is appropriate: "What is $\sqrt{2}$, supposing geometry and limits both excluded?" [De Morgan 1837, "Preface" to the 1835 edition]. As Morris Kline explains: "The Pythagorean proof tells us that $\sqrt{2}$ is not a ratio of whole numbers but it does not tell us what an irrational is," [Kline 1980, p, 105]. In other words, Greek mathematicians faced the problem but failed to solve it vigorously until Eudoxus.

On the other hand, ancient Chinese mathematicians like the anonymous authors of the *Jiu Zhang Suan Shu* ("Nine Chapters on Arithmetic") seemed to have had no concern for problems involving incommensurable magnitudes like $\sqrt{2}$. For example, in dealing with the method of extracting the square roots of non-square numbers, the *Jiu Zhang Suan Shu* concludes that "If a number cannot be extracted perfectly, then it is not extractable," [Chapter 4 of

the *Jiu Zhang Suan Shu*]. This is certainly one feature that provides a marked contrast between ancient Chinese and Greek mathematics. Liu Hui, noted mathematician of the third century A.D., does not seem to have escaped the Chinese tendency to take the "algorithmic" approach in dealing with incommensurable magnitudes like the square root of 2 [Cf. Liu Hui's *Commentary* to Chapter 1 of the *Jiu Zhang Suan Shu*]. In fact, for the case of extracting roots like $\sqrt{2}$, Liu Hui did not go beyond what the authors of the *Jiu Zhang Suan Shu* had already done approximating irrationals like $\sqrt{2}$, although he relied on a geometrical interpretation of the extraction to justify the approximation of $\sqrt{2} = 1.414\dots$

After Liu Hui, nothing further added to discussion of irrationals like $\sqrt{2}$ in the history of Chinese mathematics. For mathematicians of Song-Yuan period, the subject posed no disturbing concerns, although in some cases the approximation $\sqrt{2} = 1.4 = 7/5$ led to peculiar results. For example, in his *Yi Gu Yan Duan* ("New Steps in Computation"), Li Ye (1192-1279) used the approximation $\sqrt{2} = 1.4$ to compute the area of the square with a side equal to the diagonal of a unit square: $(1.4)^2 = 1.96 \neq (\sqrt{2})^2 = 2$. Thus in solving the quadratic equation with terms containing the square of $\sqrt{2}$, Li Ye always made the situation more complicated by computing $(1.4)^2 = 1.96$ instead of $(\sqrt{2})^2 = 2$ [Kuong Guoping 1988, p. 133]. Granted that the *Yi Gu Yan Duan* was written for common people [Lam & Ang 1984], it is understandable that Li Ye should have taken $\sqrt{2} = 1.4$. In fact, the popular

mathematical treatises of the time, such as Zhu Shijie's *Suan Xue Qi Meng* ("Introduction to Mathematical Studies") often assumed this same approximation. Yet in doing so, the mathematicians failed to handle $(\sqrt{2})^2$ in a consistent way. Although a noted 13th-century mathematician like Li Ye ignored the difficulty of approximating $\sqrt{2}$, questions like "What is $\sqrt{2}$?" remained unanswered.

Similarly, Li Shanlan, like Liu Hui, failed to see the epistemological significance of the irrational numbers like $\sqrt{2}$. There is no evidence, as far as I know, indicating that Li Shanlan was ever concerned about incommensurable magnitudes of irrational numbers, even though he should have been familiar with the infinite binomial expansions of n th roots, and the remarkable and related results obtained by Chinese mathematicians like Xiang Mingda and Dai Xu, among others [Cf. Qian Baocong 1981, 309-317]. But there can be no doubt, as discussed in Section 3.2, that some features of the hypothetico-deductive method had been considered in the *Elements* did influence his writing of the *Fang Yuan Chan You* ("Explanation of the Square and the Circle"). Moreover, his enthusiasm for the *Elements* never vanished in his entire life, and he put the *Elements* into the mathematics curriculum when he began teaching at the Tong Wen Guan, the Imperial College, in 1868 [Cf. Chapter 5]. Even so, the *Elements* does not seem to have aroused any interest on Li Shanlan's part to investigate the irrationals in any serious way.

This may explain why in the Chinese edition of *Elements*, the translators provide almost no commentaries to Book X where incommensurable magnitudes are included in terms of Eudoxus' famous theory of proportion. "The extreme complexity of the treatment of irrational magnitudes in Book X undoubtedly required more exposition and explanation to assure comprehension," [Quoted Murdoch 1972]. Compared with many other editions of the *Elements*, including Arabic ones, it may come as a surprise to find that in the Chinese edition, Book X is presented on its own, without being "dressed up" by its translators.

In fact, it is not just Book X -- the other books as well of the Chinese edition contain almost no commentaries or footnotes. The exceptions include one footnote for Proposition 15 of Book X as well as one comment accompanying an extra proposition inserted before Proposition 1 of Book XII.⁸ The extra proposition, as Li Shanlan noted, is simply a re-statement of Proposition 1 of Book X. As Li Shanlan explained in his comment, most Western abridged editions of the *Elements* omitted Books VII, VIII, IX and X. But in order to deal with propositions on areas and volumes, particularly of curvilinear figures and figures bounded by surfaces, as in Book XII (where the dominant idea is the method of exhaustion), Proposition 1 of Book X is a necessary prerequisite for a rigorous treatment of Book XII. However, the Chinese translation was not based on one of the abridged versions of the *Elements*. In the English edition upon which the Chinese translation was based, the anonymous editor repeated Proposition 1

of Book X in Book XII, presumably because he put the usually omitted four books VII - X back into an abridged edition but forgot to take out the (then unnecessary) the extra Proposition 1 of Book X.

There are still a few last peculiarities of the Chinese edition that need to be mentioned. First of all, Book XI includes one more proposition than Heath's edition, namely Proposition 38 which can be identified with the following passage cited in Heath's edition:

If a plane be at right angles to a plane, and from any one of the points in one of the planes a perpendicular be drawn to the other plane, the perpendicular so drawn will fall on the common section of planes [Quoted in Heath 1956, vol.3, p.360].

Finally, the Chinese text names an author, "Xu Xi Ge Li" (given by Li Shanlan), apparently a Chinese translation of the Alexandrian mathematician Hypsicles. Immediately after preface by the anonymous editor, a letter sent to Tui Luo Bai Xi Li Di (apparently Basilides of Tyre) is also quoted immediately after preface by the anonymous editor [Heath 1956, vol.3, p. 512]. Moreover, small difference between Book XIV and Book IV of the Chinese edition and those mentioned in Heath's edition should be mentioned here. Book XIV of the Chinese edition includes seven propositions along with four corollaries, while in the "Appendix" to Heath's edition, eight propositions with two lemmas are given [Heath 1956 vol.3, pp. 512-19]. As for Book XV, its content is identical to that given in the "Appendix" to Heath's edition [Heath 1956, vol.3, pp. 519-20].

In conclusion, we must admit that Li Shanlan seems to have learned little from the Euclidean *Elements*, the last nine books, even though it was not recommended by Wylie, but was requested by him to make the complete Chinese translation of a full text. To this Greek classic, Li Shanlan clearly added nothing essentially new. In Book X, for example, Li Shanlan should have been able to advance well beyond the traditional approach to approximating irrationals but instead, it seems that he never even considered the very heart of the matter, namely a question like "What is $\sqrt{2}$?"

On the other hand, Li Shanlan's *Kao Shu Gen Fa* ("Investigations on the Properties of Prime numbers") was certainly relevant to Books VII, VIII and IX. He probably learned the related concepts earlier from the *Shu Li Jing Yun* ("The Collected Basic Principles of Mathematics", 1723). Therefore, it is of interest to consider the influence of the *Elements* in Li Shanlan teaching at the Tong Wen Guan (the Imperial College) after 1868.

For Li Shanlan's colleagues like Gu Guanguang, successors like Hua Hengfang, and disciples like Yang Zhaoyun, the *Elements* seems to have been regarded as a remarkable tool to enhance the faculty of reason. For this purpose the first six books would have been sufficient for students studying mathematics. Consequently, Hua Hengfang would urge beginners to study nothing

more than the first six books. As for the complexity of incommensurable magnitudes, Hua said that Book X was "so sophisticated that it cannot be comprehended by the beginners," [Hua Hengfang 1882, Book 5].

For Gu Guanguang, a noted scholar well versed in Western mathematics, the *Elements* played a similar role. Above all, his interest in Book X was so unusual at the time that it deserves our special attention. In 1858, Gu Guanguang gave his illustrations of the six binomial straight lines [Cf. Heath 1956 vol.3, pp. 101-102] and six apotome straight lines [Cf. Heath 1956 vol.3, p. 177]. Here we only cite two of his passages on the first binomial and apotome straight lines. First, we quote the definitions of these terms as given in the *Elements*:

Def.X.II.I. Given a rational straight line and a binomial, divided into its terms, such that the square on the greater term is greater than the square on the lesser by the square on a straight line commensurable in length with the greater, then, if the greater term be commensurable in length with the rational straight line set out, let the whole be called a **first binomial straight line** [Quoted from Heath 1956, vol.3, p. 101].

Def.X.III.I. Given a rational straight line and an apotome, if the square on the whole be greater than the square on the annex by the square on a straight line commensurable in length with the rational straight line set out, let the apotome be called a **first apotome** [Quoted from Heath 1956, vol.3, p. 117].

With these in hand, Gu Guanguang went on to give his illustrations:

Given the greater term, 4, and lesser term, 3.464 odd [= $\sqrt{12}$] of a binomial straight line respectively. Thus the difference (square) of the squares on the greater and lesser terms, namely,

$4^2 - (\sqrt{12})^2 = 16 - 12 = 4$, has its side $\sqrt{4} = 2$ "You Deng Shu"
(commensurable) with the greater term, 4....

Add the greater and the lesser terms, one obtains the sum $4 + 3.464 \text{ odd} = 7.464 \text{ odd}$. On the other hand, subtract the lesser from the greater, one obtains $4 - 3.464 \text{ odd} = 0.535 \text{ odd}$ which is the first apotome line [Quoted Gu Guanguang 1882].

Here Gu Guanguang, like Li Shanlan, never raised the question, "What is $\sqrt{2}$?". But his computation of $\sqrt{12}$, namely $(3.464 \text{ odd})^2 = (\sqrt{12})^2 = 12$, somehow managed to avoid the error Li Ye had committed almost six hundred years earlier. Moreover, Gu Guanguang's other illustrations had nothing to do with the irrationality of numbers like $\sqrt{12}$. Therefore, the foundational questions about irrational numbers like $\sqrt{2}$ remained untouched in Chinese mathematics until the end of the 19th century. So far as I know, Wu Qiqian's *Wu Bi Li Xian Xin Jie* ("On the Irrational Lines", 1906) was probably the first Chinese treatise devoted to this subject, published at the beginning of this century. Thereafter, Chinese mathematicians trained in the West began to play an increasingly central role in mathematics research and teaching in China, consequently, problems like "What is $\sqrt{2}$?" never occurred to them, since the problem was already well treated in the recently introduced Western mathematical treatises. To raise such questions in the 20th century would have seemed irrelevant, if not purposeless or redundant.

4.7. Concluding Remarks

Considering what we know of the translation and circulation by Euclid's *Elements* (the last nine books), *On Conic Sections*, De Morgan's *Algebra* and Loomis' *Calculus*, in 1850s China, it is apparent that each of these Western texts represented different levels of difficulty. For example, De Morgan's *Algebra* and the last nine books of Euclid's *Elements* looked much harder for readers to follow principally because theoretical aspects of the subjects were the only concern of the authors. Although De Morgan, did have a pedagogical concern while he was writing the *Elements of Algebra*, yet his aim at resolving the difficulties of negatives and imaginaries as rigorous as possible apparently made his treatise too tedious for beginners [Cf. Section 4.3]. As for the *Elements*, the extensive and elaborate treatment of incommensurable magnitudes or irrational numbers given in Book X seems to have been responsible for its limited circulation in the last decades of the 19th century in China. On the other hand, *On Conic Sections* and Loomis' *Calculus* were quite popular and widely transmitted. The reason, as already suggested, might lie in their strong "algorithmic" character and the emphasis they placed on problem-solving. Their widespread use, therefore, seemed to have resulted in their exerting a clear influence on modern Chinese mathematics.

Despite the question of technical difficulty, apparent or otherwise, the strongly religious role these mathematical texts were intended to play by missionaries like Wylie and Edkins should not overlooked. The excellence of the Western mathematics meant

to demonstrate the importance, if not superiority, of Christianity proved to be an obstacle for Chinese scholars in learning the treatises like De Morgan's *Algebra*.⁹ Even so, other reasons should also be considered. Compared with Wallace's *Algebra*, the reason why De Morgan's *Algebra* failed to circulate widely, is that Wallace's *Algebra* could be approached by Chinese scholars in a way that was already familiar to them -- namely as programmed learning. This was also the case when readers contrasted Loomis' *Calculus* with Wallace's *Fluxions*, for the latter contained many diversified examples as illustrations. Therefore, to appreciate fully the complexity of transmission of Western mathematical texts to China, one should not ignore the long standing epistemology and methodology of Chinese mathematics.¹⁰ Nor could one view such a problem properly without an intellectual background of 1860-1890 China.

However, Chinese scholars or the scholar-officials of the time, the newly-introduced Western mathematics, despite the Christian overtones with which it may have been presented, was just as value-free as the Western medicine practiced at the Westerners' owned hospitals in the coastal areas. In the case of Western medicine, although native patients usually felt uncomfortable from the beginning upon entering a Western hospital, its utilitarian function could be easily understood [Turker 1982]. On the other hand, Western mathematics does not seem to have had a similar opportunity to justify its importance -- and value -- for its own sake. How the Western mathematical texts translated into Chinese

were circulated and how Chinese scholars or mathematicians responded depended largely on the individual characteristics of the text in question. Some texts like Wallace's *Algebra* were more easily appreciated than other texts like De Morgan's *Algebra*, but this had nothing to do with the their contents, than with their different styles of exposition. In general, mathematical texts with the "logarithmic" features were more easily linked to pragmatic uses. Under such circumstances, it is easy to understand why Wallace's *Algebra* was more acceptable. As the historian Hao Chang has noted: "To the extent that they (i.e, the Chinese scholar-officials) responded to the West at all, their responses reflected basically the same pragmatic approach and technically rational mode of thinking they had long applied to tackling domestic problems of statecraft," [Quoted Hao Chang 1979].

On an intellectual level, Li Shanlan's response to Western mathematics was not exceptional and it made no difference whether he played an active role in the transmission or not. The *Xu Yi Ji He Yuan Ben* (the last nine books of Euclid's *Elements*) was a text he requested for translation, from which he ironically seems to have learned much less than from the first six books. On the other hand, he certainly learned quite a lot from *On Conic Sections*, since at least one of the conics, namely the ellipse, had become a major focus of research since the 1840s, and with all of this he would have been quite familiar [Cf. Section 3.6]. Such an approach to mathematics -- which was different from that

undertaken by such mathematicians as Jiao Xun, Wang Lai and Li Rui of the Qian-Jia school -- was consolidated thanks partly to the revival of Ming Antu's mathematics [Cf. Chapter 1].¹¹ With such concerns in mind, Li Shanlan although clearly a leader of the Chinese mathematical community from the 1840s on (familiar or not with Western languages) did not have that much to do in deciding which Western texts on mathematics should be translated. However, his own translations of both Western mathematics and science, successful or not as translations, undoubtedly helped to re-open the gates for transmission of other Western mathematics and science in particular, and of Western culture in general.¹²

NOTES:

1. According to John Fryer, Li visited the Press some time before 1852 [Quoted in Li Yan 1955, p. 359].
2. The Meiji Restoration was decreed by the Meijie Emperor in 1868. Its goal was to modernize Japan by learning the West.
3. In solving simultaneous linear equations, the authors of the *Jiu Zhang Suan Shu* developed the "Fang Cheng" method (literally method of rectangular arrays) in terms of the "Chou Suan" (counting rods) system [Cf. Li and Du 1987, 46-49].
4. In his "Preface" to the *Dai Shu Xue* (De Morgan's *Algebra*), Wylie admitted that "Tian Yuan" and "Si Yuan" methods were actually excellent in solving related problems, yet he recommended the methods in the *Dai Shu Xue* due to the fact that it adopted (modern) notations and was more convenient for computations. It is quite clear from this passage that Wylie's strategy to convince Chinese readers of the *Dai Shu Xue* was to emphasize mainly the superiority of Western algebraic methods in computations.
5. Since there was a course "Dai Shu Xue" for both curricula of the Five-year and Eight-year Systems in the Tong Wen Guan [Cf. Section 5.1]. Moreover, the Chinese edition of De Morgan's *Algebra* was the only Western algebra available in China when Li Shanlan began to teach at the Tong Wen Guan. Besides, mathematics problems of the examinations, tests and exercises of the Tong Wen Guan all reflected that the *Dai Shu Xue* was the only source material about Western algebra at the time.
6. In the two English editions of Loomis' *Calculus* (1851, 1853, collected in New York City Library), the answers are all correct. That means the Chinese edition, the *Dai Wei Ji Shi Ji*, made errors here. Gu Guanguang himself seems to have been confused about some terms used in Loomis' *Calculus*. That is why he mistook several false answers to examples used in Chapter 10 as correct, and went on to demonstrate in detail those false answers.
7. There is no difference in Chinese between the term "Geng Xiang Jian Sun" and the "Zhan Zhuang Xiang Chu". Why Li Shanlan preferred the latter to the former is unknown.
8. This footnote only mentions that Proposition 22 of Book 6 (in the *Elements* translated by Xu Guanguang and Matteo Ricci) has connection with this Proposition.
9. Among the Western texts on mathematics co-translated by Li Shanlan and Wylie, the *Dai Shu Xue* is the only one in which Wylie gave an argument relating Western mathematics with Christianity. This may also help to explain why the *Dai Shu Xue* was less acceptable than the other treatises.

10. Since ancient Chinese mathematicians usually adopted an "algorithmic" approach to mathematics, some important problems like "What is the square root of 2?" were always regarded as insignificant. Therefore, how a Western text on mathematics was accepted was closely related to its format and style, as shown in the *Dai Shu Xue* and *Dai Wei Ji Shi Ji*, etc.
11. As argued in Chapter 1, Jiao Xun, Wang Lai and Li Rui all made great contributions to algebra by adopting a new, "qualitative" approach. (Note that Wang Lai was the only one among these "Tan Tian San You" [three friends concentrating on mathematics and astronomy], who was classified as a mathematician of the "Xi Fa" school]. But beginning from 1820s, Xiang Mingda's studies on ellipse and series expansions for trigonometric functions, basically following the step of Ming Antu, came to draw the attentions of the Chinese mathematicians in the second half of 19th century. At the same time, Xiang Minda or even Li Rui's disciple, Li Yingnan, were unable to appreciate the innovations made in Jiao Xun, Wang Lai and Li Rui's studies on algebra. Among the important mathematicians after 1820, Luo Shilin was the only faithful representative of the Qian-School, yet he could not grasped the significance of Li Rui's theory of equations, not to mention Wang Lai's in which the "Xi Fa" (Western method) was the very one he did not approve.
12. By the end of 19th century, all foreign texts on mathematics translated into Chinese were all concerned to have been used as textbooks. In the beginning of this movement, namely in the 1850s, Li Shanlan's translations, in collaboration with Western missionaries, of De Morgan's *Algebra*, Loomis' *Calculus*, Euclid's *Elements* (the last nine books), and *On Conic Sections* (by anonymous author) were not considered to be prepared as school textbooks, since Chinese modern school system was yet to be created. Even so, Euclid's *Elements* and Loomis' *Calculus* were among the most popular textbooks used in the Chinese modern schools toward the end of the 19th century. More significant is that many derived forms like the "careful explanations" or "short notes" of these two treatises appeared very frequently at the time [Li Yan 1955, 313-314]. In 1897, the *Dong Xi Xue Shu Lu* ("Catalogue of Western and Eastern Studies", prefaced by Cai Yuanpei) collected the school mathematics textbooks used in the period 1899-1903, in which the *Shu Xue Qi Meng* (by Wylie), Euclid's *Elements*, Wallace' *Algebra*, Loomis' *Calculus*, Wallace' *Calculus*, *On Conic Sections* (translated by Li Shanlan and Edkins) were among the 34 most popular textbooks at that time [Li Yan 1955, pp. 315-317].

5. THE PROFESSIONALIZATION OF CHINESE MATHEMATICS AND THE MODERNIZATION OF MATHEMATICS EDUCATION: Li Shanlan's teaching at the Tong Weng Guan (the Imperial College)

In 1868, Li Shanlan joined the Tong Wen Guan as a mathematics instructor or, as W.A.P. Martin has said, a professor of mathematics.¹ As mentioned in Chapter 2, Li Shanlan had not been enthusiastic about the first call from the emperor in 1866. In fact, he was reluctant then to accept such an offer because, as he himself put it, his collected mathematical works -- *Ze Gu Xi Zhai Suan Xue* -- had not yet been published completely. The appearance of the *Ze Gu Xi Zhai Suan Xue* the following year (in 1867) made it possible for Li Shanlan to respond positively to the second call from the emperor in 1868. Thus Li Shanlan -- teaching at the Tong Wen Guan, the first modern college in late 19th-century China.

Li Shanlan taught mathematics and astronomy as well as mechanics at the Tong Wen Guan until his death in 1882. Over one hundred students studied these subjects under him during the period 1868-1882. Although too few students proved successful in studying mathematical sciences towards the end of the Qing dynasty, Li Shanlan nevertheless enjoyed his teaching. Apparently, he was content with a peaceful living in Beijing at the time [Yabutti 1979, p. 180]. In fact, as his letter to Fang Yuanzhen shows, Li Shanlan clearly regarded his mathematics as the most significant concern in his life [Cf. Chapter 2]. Publication of the *Ze Gu Xi Zhai Suan Xue*, therefore, made his dream come

true. On the other hand, as he began service in his official post in Beijing, (apparently the first such position he ever held) at the age of 58 (or 59 in Chinese reckoning), he had no reason to anticipate any further official advancement for the rest of his life. In fact, from the very beginning of his third career, Li Shanlan seems to have had no ambitions for official service. Therefore he was able to act solely as a "professional" mathematics instructor, which was probably the first in the history of Chinese modern mathematics or of mathematics education that this was the case.

Li Shanlan's teaching at the Tong Wen Guan, however it may be evaluated, came to serve as a model of curriculum reform in Chinese education in the 1890's. Modern Chinese educational reform, guided basically by the Zou Ding Xue Tang Zhang Cheng ("The Reform of Educational Institutions Decreed by the Throne," 1902), was preceded by a series of events including the Reform Movement of 1898, which ended with a successful coup d'état on September 21 (1898) staged by the empress dowager Ci Xi, and the Boxer War of 1900 [Hao Chang 1980]. Underlying these political, and social cultural changes, in the 1890s was an intellectual ferment which owed a great deal to indigenous changes within the Chinese cultural tradition [Hao Chang 1980]. In the process, late Qing intellectuals eventually came to focus on questions of educational reforms. Even earlier, however, in the late 1860s, China had already had a taste of modern science education at the Tong Wen Guan thanks to its Chinese instructor for science and

mathematics, Li Shanlan.

5.1. Mathematics at the Tong Wen Guan

When Williamson Alexander Parsons Martin, who arrived in Ningpo (of the Zhejiang province) in 1850 for a Presbyterian mission, was recommended by Robert Hart to take charge of the Beijing Tong Wen Guan in 1869, it began to operate not merely as a so-called "School of Interpreters" but as a modern college or school whose curriculum included science and mathematics in addition to languages [Duus 1966]. Martin, serving as a chief instructor, had hoped to establish it as the first modern college in China. For this purpose, Martin constituted two curricula: One was for students learning foreign languages who spent their first three years in language study and then studied science and mathematics with original Western texts for five more years; The other curriculum was for older students for whom it was thought too late to learn foreign languages, and they began to study science and mathematics immediately through translated Western texts. The first curriculum was called the "Eight-Year System", while the other was called the "Five-Year System". Both programs stressed translation skills, since the translation of Western science and mathematics was a major concern of the advocates of the "Self-Strengthening Movement".

The "Eight-Year System" required the following curriculum:

The first year -- Reading, Writing, and Speaking.

The second year -- Reading, Grammar, and Translation of Sentences; Exercise in Speaking (continued through the whole course).

The third year -- Geography, History, Exercises in Translation.

The fourth year -- Arithmetic, Algebra, Translation of Dispatches.

The fifth year -- Natural Philosophy; Geometry; Plane and Spherical Trigonometry; Exercises in Translation.

The sixth year -- Theoretical and Practical Mechanics; Differential and Integral Calculus; Navigation and Surveying; Exercises in Translation.

The seventh year -- Chemistry, Astronomy, International Law, Translation of Books.

The eighth year -- Astronomy, Geology and Mineralogy, Political Economy, Translation of Books [Quoted from *Triennial Calendar of The Tungwen College* Fourth Issue 1888, p. 21].

As for the "Five-Year System", the curriculum was as follows:

The first year -- Arithmetic; Algebra and Studying the *Jiu Zhang Suan Shu*.

The second year -- Learning the *Si Yuan Jie* ("On the 'Si Yuan' Method"); Studying the *Ji He Yuan Ben*; as well as Plane and Spherical Trigonometry.

The third year -- Natural Philosophy; Chemistry;

Mathematical physics.

The fourth year -- Differential and Integral Calculus;
Navigation and Surveying; Astronomy; Theoretical and
Practical Mechanics.

The fifth year -- International Law; Political Economy;
Astronomy; Geology and Mineralogy.

These curricula were officially approved by the Zongli Yamen in 1872. Although they must have been put into practice, the results is unknown. Probably it was due to this reason that Martin permitted his instructors to change the teaching materials as they like in accordance with the responses of their students. Under such circumstances, Li Shanlan's role as an instructor of science and mathematics was of special importance simply because mathematics was such a fundamental part of the curriculum, and he was the only Chinese instructor teaching science and mathematics.

In the curriculum of the "Five-Year System", there were two courses on Chinese traditional mathematics, namely, "Studying the *Jiu Zhang Suan Shu*" and "Learning the *Si Yuan Jie*". Clearly Chinese traditional mathematics was emphasized in the mathematics taught at the Tong Wen Guan. Whether the *Si Yuan Jie* was the same one as Li Shanlan's is unknown. But a course like this at the Imperial College clearly shows that even in the 1870s Song-Yuan mathematics (and its underlying "Tian Yuan or Si Yuan" methods) was still of interest to late Qing mathematicians. Apparently in part for this reason, the Tong Wen Guan in 1876 published the Yuan

mathematician Li Ye's *Ce Yuan Hai Jing* -- a classic on Chinese algebra, more specifically the "Tian Yuan" method. Li Shanlan also used the *Ce Yuan Hai Jing* as a textbook both for his courses "Algebra" and "Learning the *Si Yuan Jie*". Consequently, in this edition of the *Ce Yuan Hai Jing*, Li Shanlan not only added his commentary but gave a preface which acknowledged his debt to the "Tian Yuan" method [Cf. Section 2.1]. Li Shanlan's major concern, as he explained in this "Preface", was "to synthesize Chinese and Western methods into one method demonstrating [elements of Western] algebra in the 'Tian Yuan Shu'," [See Citation in Section 2.1].

In teaching science Li Shanlan also applied the "Tian Yuan" method to computations of mechanics. For example, in one of the "Natural Philosophy" examinations (1873), his student Gui Rong carried out computations on mechanics by the "Tian Yuan" method. This certainly reflected the stress Li Shanlan placed on the importance of the "Tian Yuan" method in his teaching. On the other hand, he had already emphasized the role the "Tian Yuan" method played in the computations on mechanics in his short commentaries to Chapters 1 and 2 of the *Zhong Xue* (the Chinese edition of Whewell's *Mechanics*). His aim was to add the computations based upon the "Tian Yuan" method to that on algebraic method given by Whewell, since, as he put it, "considering the 'Dai Shu Fa' (i.e., algebraic method) had never been transmitted into China before and hence the readers might feel it was difficult to follow, I hereby interpolate the "Tian

Yuan" method to give additional demonstrations for one or two passages. I therefore hope that by doing so readers will be able to understand this treatise," [Quoted from Li Shanlan's "commentary" to Chapter 1 of the *Zhong Xue*]

In contrast to the "Eight-Year System," which apparently had nothing to do with traditional Chinese mathematics, the "Five-Year System" emphasized a very clear connection in that Li Shanlan sought "to synthesize Chinese and Western methods into one method by the way of demonstrating [aspects of Western] algebra in the 'Tian Yuan Shu'." Yet it should be noted that not many students enrolled in the "Five-Year System". Of 23 students who attended the 1878 annual mathematics examinations given in Chinese, 11 came from the foreign language departments. In other words, 11 of these 23 students were enrolled in the "Eight-Year System" [Cf. *Calendar of the Tungwen College* 1879].

Thus, Li Shanlan's concern for a "synthesis" concern of mathematics and science teaching depended upon the "Eight-Year System". Yet because he was the only "Professor of Mathematics" listed in the *Calendar of the Tungwen College*, he must also have been responsible for the mathematics and science tests given to the students enrolled in the "Eight-Year System." These tests, though written in English [See Section 5.2], apparently were still under his supervision, for by the aid of his assistants Xi Gan, Wang Fengzao and Gui Rong did know foreign languages, Li Shanlan was then able to give the tests written in English. Another

instructor, Samuel M. Russell, described as the "Professor of mathematics and Astronomy" in the *Calendar of the Tungwen College*, might also have been overseen these tests, yet he seems to have devoted all of his time to teaching astronomy. In collaboration with his students, Russell translated Elias Loomis' *An Introduction to Practical Astronomy* (1855) into Chinese, which was published in 1897. On the other hand, the chief instructor W.A.P. Martin must also have played a key role in these examinations -- evidences for which may be found in textbooks like the *Ge Wu Ru Meng* (*Natural Philosophy*, compiled by Martin) and *Ge Wu Ce Suan* (*Mathematical Physics*, compiled by Martin with the collaboration of his assistant Du Fameng), published by the Tong Wen Guan. Nevertheless, with respect to the standard mathematics curriculum set by the *Da Qing Hui Dian* ("Educational Archive of the Great Qing"), it is clear that Li Shanlan played a dominant role in mathematics and science teaching at the Tong Wen Guan. To achieve high standards, the students should learn mathematics starting with arithmetic, then following step by step through the *Jiu Zhang Suan Shu*, trigonometry, astronomical and mechanical surveys, and finally the Chinese "Si Yuan" method and Western algebra as well as their synthesis [Cf. *Da Qing Hui Dian* Book 100, pp. 5-11]. Here Li Shanlan was able to follow his own ideal of blending Chinese and Western mathematics together, at least in the mathematics and science taught at the Tong Wen Guan. At this very point, Li Shanlan's influence upon mathematics teaching at the late Qing traditional academies is most easily discerned [See Section 5.6].

In this connection, Li Shanlan's attitude towards "Xi Xue" (Western studies) did not bother conservative official-scholars in Beijing who waged fervent attacks on the Self-strengthening Movement in the late 1860s. Consequently, he was able to lead a detached and peaceful life in Beijing. He very much enjoyed his career teaching at the Tong Wen Guan "where [by 1877] he has been working diligently for eight years without realizing his advancing years," [Quoted from Martin 1877, pp. 368-370].

In fact, the mathematics courses were more successful than others largely because of Li Shanlan's diligent teaching, as well as that of his successor Xi Gan, which deeply impressed their contemporaries. For example, Xi Gan, who was promoted to succeed Li Shanlan in 1886, also drew the attention of students like Qi Rushan, who criticized anything about the Tong Wen Guan with but one exception: Xi Gan also earned high reputation as a mathematics instructor [Cf Su Ching 1985].

5.2. Examinations and Tests

In order to know what effect Li Shanlan had upon teaching at the Tong Wen Guan, the problems set in the "Sui Shi" ("Annual Examination"), as well as the "Da Kao" ("Great Examination" (a competitive test given every three years) provide the only available clues from which it may be possible to reconstruct a picture of his teaching activity.

Examinations at the Tong Wen Guan consisted of the "Yue Ke" (Monthly Test), "Ji Kao" (Seasonal Test), "Sui Shi" and "Da Kao". In addition, the ministers of the Zhongli Yamen ("Office of the Foreign Affairs") who superintended the Tong Wen Guan would give surprise tests of their own without prior notice. Furthermore, whereas the instructors were responsible for the "Yue Ke" and the instructors or the ministers of the Zhongli Yamen were responsible for the "Ji Kao", the ministers of the Zhongli Yamen were solely responsible for the "Sui Shi" and "Da Kao" [Su Ching 1985, 36-37].

According to the historian Su Ching, foreign languages were included as subjects of the "Sui Shi" and "Da Kao" from the very beginning. In 1870, subjects such as mathematics and physics were added. From then on, the number of subjects for the tests increased each year, but the same subjects were not necessarily included from one year to the next. However, mathematics seems to have been the only exception probably because of the significant role it had played all along in the Self-strengthening Movement. In fact, from 1879 on, the subjects for the "Da Kao" always included foreign languages, mathematics, astronomy and chemistry [Su Ching 1985, pp. 36-37].

Of all the examinations, the "Sui Shi" and "Da Kao" deserve special attention principally because these were "always conducted in the presence of Their Excellencies the Ministers for Foreign

Affairs" [Quoted from *Triennial Calendar of the Tungwen College* 1888, 22]. Students who demonstrated exceptional proficiency in their tests would be offered official positions or promoted to higher ones after serving at some official post [*Triennial Calendar of the Tungwen College* 1888, p. 15]. For example, Yang Zhaoyun, one of Li Shanlan's best students who transferred from the Shanghai Tong Wen Guan to the Beijing Tong Wen Guan in 1871, immediately gained a second place honor in the "Da Kao" held the following year, and was promptly offered a post as an interpreter at the Shanghai port office [Su Ching 1985, pp. 193-194]. Those who passed the examinations also had the privilege of continuing their studies at the Tong Wen Guan. Those who failed, however, were either expelled from the college or degraded from their official positions [*Calendar of the Tungwen College* 1898].

Among the available *Calendars of the Tungwen College*, the fourth issue (1888), fifth issue (1893) and seventh issue (1898) contained problems of the "Da Kao" held in those years. The first issue (1879) included only problems from the "Sui Shi" held the preceding year, 1878.²

It seems that Li Shanlan was asked to give the problems for the mathematics tests, since among the ministers of the Zhongli Yamen at the time, Zeng Jize, the eldest son of the scholar-general Zeng Guofang, was the only other member of the staff who knew mathematics. Moreover, Zeng Jize and his younger brother Zeng Jihong apparently learned their mathematics from Li Shanlan at

Anqing where Zeng Guofang established a headquarters in 1861 [Cf. Section 2.6]. Even so, Li Shanlan seems to have played a more significant role in these tests.

Now let us begin reviewing the mathematics problems of these tests. The "Sui Shi" problems for 1870 were as follows:

- 1) Given the three sides AB, AC, and BC of a spherical triangle, find the angle A. How about using the "Shi Jiao" method? How about using the "Kai Pien Fang De Ban Jiao" method (i.e., the half-angle formula for the sine)? Give a comparison between these two methods.
- 2) Given three angles of a spherical triangle, find the (surface) area of the triangle.
- 3) Inside a circle, any two perpendicular chords not going through the center divide the circumference into four unequal arcs. Halve the chords that subtend any two of the opposite arcs α , β ($\alpha < \beta$). At these two midpoints erect two perpendiculars to meet at the center. Consequently, among the last two perpendiculars, the one to the smaller chord subtending the arc α is half the larger chord subtending the arc β ; and conversely, the other to the larger chord subtending the arc β is half the smaller chord subtending the arc α . Can you explain ("Yan Qi Li", literally give the reason) why?
- 4) Given the values of the sine function values for any two unequal arcs α , β respectively, how many methods can you give for finding $\sin(\alpha+\beta)$ and $\sin(\alpha-\beta)$?
- 5) Suppose three equal circles are inscribed in a right-angled triangle, all with their circumferences tangent to the "Xuan" (hypotenuse). Moreover, let two of them be tangent to the "Gou" side (the smallest side of the right-angled triangle), "Gu" side respectively. Given the "Gou", "Gu" and "Xuan" (hypotenuse) of the right-angled triangle, find the radius of the circles.
- 6) Given three sides of a right-angled triangle, a ("Gou"), b ("Gu"), and c ("Xuan"), we have

$$2(c-a) \cdot (c+b) = c + (b-a).$$

Explain this identity by a geometric construction.

- 7) Given "Zhuan Xuan" c_{15} , "Ming Gou" a_{14} , find the

diameter of the inscribed circle in the right-angled triangle [Cf. Figure 1].

8) The force induced from two attracting bodies is inversely proportional to the distance between them. Can you give a reason by drawing a diagram?

9) In a right-angled triangle with sides a ("Gou"), b ("Gu"), and c ("Xuan"), we obtain c provided a , b are given; similarly we obtain b provided a , c are given; and obtain a provided b , c are given. Show ("Tui Yuan Qi Li", literally derive their reasons) each of them by geometrical diagrams.

10) Prove that a triangle with two equal sides has two equal corresponding (interior) angles. Show that the two exterior angles produced by the extended sides with the base line of the triangle are equal too.

11) Prove that the sum of the interior angles of any triangle equals two right angles.

12) A general was asked how many soldiers were in his camp? His answer: Multiply the number of soldiers by 200 and add the square of the number of soldiers, the total number is equal to 990000. How many soldiers were there?

13) Someone sells oxen and horses: He gets 800 taels for 3 horses and 5 oxen and 1120 taels for 5 horses and 3 oxen. What is the price of each ox and horse respectively?

14) A village has a population of 1500, in which the number of women is half the number of men and the number of children is double the sum of men and women. Can you calculate how many there are of each?

15) A given number is divided by 13, 11, and 7, with remainders 1, 3, and 2 respectively. Find the number.

16) Given two balls of equal weight, one of the them drops vertically and the other moves horizontally with uniform speed. After three minutes both balls have covered the same distance. At what time was the speed of the free-falling ball was equal to the one moving horizontally?

17) Two balls of weights 5 kg and 3 kg respectively have been suspended oppositively on a pulley for 3 seconds. How many meters the heavier one downfalls?

18) Given that the diameter of Jupiter is 11 times that of the Earth and that spherical balls are to one another as the triplicate ratio of their respective diameters, how large is the volume of Jupiter compared with that of the Earth?

19) Someone piles up a cube with 19683 pieces of brick. How many pieces of brick are there on each side?

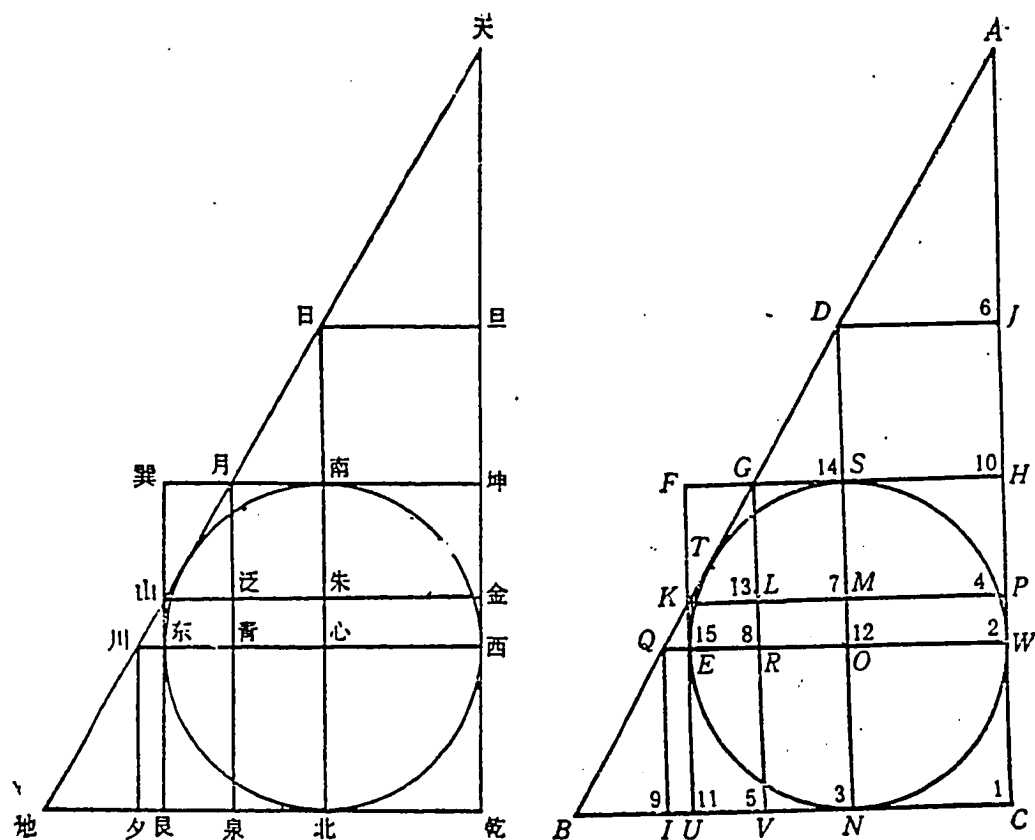


Figure 1. Duplicate of the key diagram in Li Ye's *Ce Yuan Hai*

Jing. Arabic numerals in the right diagram

indicate the right-angled triangle each refers to.

For example, number "14" refers to the right-angled

triangle DGS, thus a_{14} , b_{14} , c_{14} denote its

"Gou", "Gu" and "Xuan" sides respectively.

Similarly for the number "15" and so on.

20) A rectangular plot of land 243 "Bu" long and 27 "Bu" wide is transformed into a "Fang Tian" (i.e., square). How many "Bu" is each side of the square?

From the above list it is easy to see what these problems involved. For example, Problems 1, 2 and 4 dealt with problems from the course on "Trigonometry, Plane and Spherical." Problems 3, 10 and 11 were related to the course on "Studying the *Ji He Yuan Ben*". Problems 3, 5, 6, 9, 10 and 11 were for the course "Studying the *Jiu Zhang Suan Shu*". Problem 7, from which the *Ce Yuan Hai Jing* came, was obviously posed for the course "Learning the *Si Yuan Jie*". Questions for "Learning the *Si Yuan Jie*" probably account for Problems 12, 13 and 14 too, but they may also have been for the course "Algebra" as well. Problem 15 and its solution, which involved the Chinese Remainder Theorem, were probably covered in the course on "Arithmetic". On the other hand, Problem 8 concerned the course on "Natural Philosophy" (or "Learning the *Ge Wu Ru Meng*"). Likewise, Problems 16 and 17 were either for the course on "Natural Philosophy" or "Mathematical Physics." As for Problem 18, it was certainly for the course on "Astronomy".

These problems reflect an equal concern for both Chinese traditional mathematics as well as Western mathematics. Some of these problems (like Problems 12, 13 and 14) could be solved by methods learned from both the courses on "Learning the *Si Yuan Jie*" and "Algebra". So long as students were successful in studying their mathematics carefully, Li Shanlan's goal -- "to synthesize Chinese and Western methods into one method

demonstrating [the Western] algebra in the 'Tian Yuan Shu' -- was therefore attained.

Also of interest, some of the test problems used the terms from the ancient Chinese mathematical texts. For example, the term "Fang Tian" of Problem 20, which means "square", came from Chapter 1 of the *Jiu Zhang Suan Shu*. In addition, the terms "Zhuan Xuan" and "Ming Gou", used in Problem 7, appear in the *Ce Yuan Hai Jing*. As for the problems concerning "Plane and Spherical Trigonometry", Chinese mathematicians had been familiar with these for over two hundred years since the late Ming and early Qing. Such questions, therefore, reflected Li Shanlan's desire to synthesize Chinese and Western mathematics in his teaching. Perhaps in this regard Li Shanlan also hoped to "save" the face of the conservative official-scholars in Beijing who were not pleased to "honor the barbarians as teachers" [Cf. Section 2.8].

On the other hand, no problems were included in this test on the differential and integral calculus. Yet it seemed to have been almost the case in the history of the Tong Wen Guan. The only exception appeared in the 1878 "Sui Shi" about which we are going to talk later on. Nevertheless, the late Qing intellectuals still criticized that the Beijing Tong Wen Guan was anything but a modern college, since for them, a college could be regarded as a modern type only if the Calculus course had been taught

Moreover, the 1870 "Sui Shi" included not only the mathematics problems quoted above but the natural philosophy problems ("Ge Wu" problems) as well, although these latter problems had nothing to do with mathematical computations. Instead, the examiners asked students to explain what the theories and methods of mechanics, optics, chemistry, meteorology and steam were all about. This suggests that the 1878 "Sui Shi" should also have covered a "Test on Natural Philosophy" as a separate part of the examination. If so, then Martin must have re-edited the mathematics and physics problems when he sent a copy of them to *The Church News*.

The "Sui Shi" of 1873 and 1878 did sort out the problems concerning mathematical physics into a separate test, namely, the "Test on Natural Philosophy" ("Ge Wu" Problems). However, in the 1872 "Sui Shi", there are only two problems concerning computations on mechanics, namely, Problems 3 and 6 of the following "Test on Mathematics":

1) Given a right-angled triangle with sides a ("Gou"), b ("Gu") and c ("Xuan"), suppose that $c^2 - c = c + (b-a)$, and $b = 2 [c - (b-a)]$; find a , b and c .

2) Given a lamina of bronze shaped as a segment of a circle (radius 10000 "Chi", 1 "Chi" equals 0.30867 meters). Suppose the arc of the segment is 45 degrees, find the distance between the chord and the center of gravity.

3)³

4) In a right-angled spherical triangle ABC, suppose that $\angle A$ and $\angle B$ are given acute angles, and $\angle C$ is a right angle, find $a : b : c$, where a , b , c are the corresponding arcs of the angles A, B, C respectively.

5) Given a "Duan Gou Gu Jiao", $c_{15} - a_{15}$ ("Zhuan Gou Xuan

Jiao") in a right-angled triangle with sides a ("Gou"), b ("Gu") and c ("Xuan"), find the radius of the inscribed circle.

6) A bullet was fired upwards vertically, and then hit the ground after 20 seconds, find its maximum height and draw the appropriate diagram.

7) Given c ("Da Xuan"), and a_{15} ("Zhuan Gou") in a right-angled triangle with sides a ("Gou"), b ("Gu") and c ("Xuan"), find the diameter of the inscribed circle.

8) There is a path crossing over the summit of a mountain. Suppose each hillside section of the path has 25 "Li" (1 "Li" equals 0.5556 kilometers), and both are inclined at an angle of 45 degrees with the horizon, find the length of the straight route connecting the two extremities of the path.

9) In a right-angled triangle of sides a ("Gou"), b ("Gu") and c ("Xuan"), given that $c - (b-a) = 6$, $c - a = 8$, find a , b and c .

10) Given a "Li Fang Shi" (literally numerical equation of third degree):

$$\begin{array}{r} \tau \ 30 \\ \perp \ 9 \\ \perp \ 1 \\ \perp \ 1 \end{array}$$

Find its solution by extraction.⁴

11) Given a "San Cheng Fang Shi" (literally numerical equation of fourth degree):

$$\begin{array}{r} \perp \ 2199 \\ \tau \ 625 \\ \tau \ 75 \\ \perp \ 10 \\ \perp \ 1 \end{array}$$

Find its solution by extraction.⁵

Among these problems, the stress placed upon studying the *Ce Yuan Hai Jing* or the course "learning the *Si Yuan Jie*" is striking. For example, Problems 1, 5, 7, and 9 of this test all came from the *Ce Yuan Hai Jing*. Moreover, Problems 10 and 11 were also related to this 13th-century Chinese mathematical classic which

dealt with the theory of algebraic equations by the "Tian Yuan" method. So they were certainly part of the course "Learning the Si Yuan Jie". Perhaps this material also came up in the course on "Algebra", since the format of the equations which appear in Problems 10 and 11 as reminiscent of two sources: One was the *Dai Shu Xue* (De Morgan's *Algebra*) in which the addition and subtraction were written as τ and $\perp$ respectively; the other was Li Rui's *Kai Fang Shuo* ("On the Method of Extractions"). On the other hand, Problems 2, 4 and 8 were related to the course "Plane and Spherical Trigonometry," while Problems 3 and 6 dealt with "Mathematical Physics".

The examination of 1873, in contrast to those for 1870 and 1872, included problems on astronomical computations. In fact, the test included the following problems:

- 1) Find the time of sunrise and of sunset of the winter solstice at Beijing.
- 2) From the top of a mountain 10.5 "Li" (1 "Li" equals 0.5556 kilometers) high, a line of vision is drawn downwards with an angle $1^{\circ}6'45''$ with the horizon. Find the diameter of the Earth.
- 3) Given a cylinder of the same height and diameter the base circle, then the area of its lateral surface is equal to the surface area of a sphere of a diameter equal to the height of the cylinder, which in turn is equal to four times the area of a circle of the equal diameter. Give the reasons to explain why.
- 4) Construct three circles inscribed in a segment of a circle and justify your method.
- 5) In a right-angled triangle with sides a ("Gou"), b ("Gu") and c ("Xuan"), as well as the corresponding angles A , B and C , given angles A and B , and the "Xuan He Jiao" $a + b - c$, find a , b and c .

- 6) In a right-angled triangle, given the "Da Ca Xuan Jiao He" $c_{10} + (b_{10} - c_{10})$, and the "Xiao Ca Xuan Jiao Jiao" $c_{11} - (b_{11} - c_{11})$, find the diameter of the inscribed circle and give an algebraic expression for the diameter.
- 7) A piece of iron which is square in shape with each side 113 "Fen" (1 "Fen" equals 0.30867 centimeters), is cast into a cannon-ball. Find the diameter of the ball.
- 8) In a right-angled triangle with side a ("Gou"), b ("Gu") and c ("Xuan"), given that c_2, a_3 , find the diameter of the inscribed circle and give an algebraic expression for the diameter.
- 9) Two persons A and B stand on the same bank of a river at a distance 360 "Bu" (1 "Bu" equals 5 "Chi"; 1 "Chi" equals 0.30876 meters, so 1 "Bu" equals 1.54335 meters) viewing a tree standing on the opposite bank. Suppose A views the tree at an inclined angle $83^\circ 27'$ with the base line joining A and B, while with the same line B views the tree at an inclined angle $31^\circ 15'$. Find both the distances between the tree and A, B respectively.
- 10) In a right-angled triangle with sides a, b and c , given that $c - a = 9$ and $a + b - c = 6$, find a, b and c .
- 11) A kite with a thread of 332 "Bu" is flying in the sky. Suppose the distance between the holding finger and the point directly underneath the kite is 244 "Bu", find the height of the kite.

In this test, Problems 1 and 2 certainly involved astronomical computations. Among the others, problems came from the *Ce Yuan Hai Jing* were still predominant: it accounted for 4 out of 11 problems! On the other hand, Problem 3 is noteworthy because the theorem it concerns was proved by Archimedes. This suggests that the course "studying the *Ji He Yuan Ben* (Euclid's *Elements*)" required the students to learn more geometry than what was simply in Euclid's *Elements*.

Finally, the "Sui Shi" problems for 1878 comprised four

categories of problems, namely, "Problems on Astronomy and Mathematics," "Problems on Mathematical Physics," "Problems in Astronomy given in Foreign Languages," and "Problems in Mathematics given in Foreign Languages." One of the most significant innovations here was the appearance for the first time of problems on calculus. For example, Problem 5 of the "Problems on Astronomy and Mathematics" concerns the binomial expansion. In addition, Problem 6 of the same test was to find the volume of a surface of revolution. Similarly, among the "Problems on Astronomy given in Foreign Languages," Problem 5 asked students to derive the area formula of an ellipse. In all, the following four categories of problems were given:

"Problems on Astronomy and Mathematics":

- 1) Find Beijing time at sunrise and sunset on New Year's day, 1879.
- 2) Find Beijing time at moonrise and moonfall for the "Yuan Xiao" Festival (or the Lantern Festival, i.e., the 15th of the first lunar month of the Chinese calendar).
- 3) The speed of a free-falling body is inferred with reference to the Earth. How about its speed with respect to the Sun?
- 4) The speed of a free-falling body travelling downwards from a point at infinity could not be infinite. Explain why and find the speed.
- 5) Given $u = (x + y)^\alpha$, expand this into a binomial series.
- 6) Revolve a given circle around one of its tangents, then find the volume of the surface of revolution.
- 7) In a right-angled triangle with sides a ("Gou"), b ("Gu") and c ("Xuan"), given that $a_{15} \cdot a_2 = r^2$, where r is the radius of the inscribed circle, give a geometrical illustration.

8) Suppose a circle is inscribed in a right-angled triangle while at the same time it is circumscribed around another but similar right-angled triangle. Given only that the sum of the "Gou" and "Gu" of the outer triangle is 391, and the sum of those of the inner one is 42, find the diameter of the circle.

9) Given a "Kai Fang Shi" (literally a numerical equation):

$$\begin{array}{r} + 832320000 \\ - 30192000 \\ + 83400 \\ + 1300 \end{array}$$

Find its solution by extraction.⁶

As mentioned above, Problem 5 of this test is concerned with the binomial expansion of $(x + y)^\alpha$, where α could be any rational number. Also related to the calculus is Problem 6, which involves methods of integration. As for problems on astronomy, these comprised 4 out of the total 9 problems -- a high proportion apparently indicating that astronomy was also a focus of teaching. The most significant feature of this test was again its strong emphasis upon Chinese traditional mathematics. For instance, Problems 7 and 8 came from the *Ce Yuan Hai Jing*, while the last problem (in modern notation) which can be interpreted as "Find the numerical solution of the equation $832320000 X^3 - 30192000 X^2 + 83400 X + 1300 = 0$ " was typical of Li Rui's *Kai Fang Shuo*.

As already mentioned, the 1878 "Sui Shi" also included problems on mathematical physics as a separate category. The remaining two categories of problems given for the "Sui Shi", namely, the "Problems on Astronomy given in Foreign Languages" and "Problems on Mathematics given in Foreign Languages". It will then be possible to compare the problems in "Foreign Languages"

examinations with those which were given in Chinese. Here again the dominant role that Li Shanlan played in teaching mathematics, astronomy and mechanics will become apparent.

The "Problems on Astronomy given in Foreign Languages" included:

- 1) Find the distance between the Moon and the Earth and justify your method.
- 2) Given the diameters of some planet and the Earth respectively, and the distance between this planet and the Earth, find the formula for the angle between two lines going through the center of the planet and that of the Earth, as well as through the center of the Sun and that of the Earth.
- 3) State Kepler's three laws of planetary motions and give an explanation.
- 4) In a spherical triangle, given its sides (or arcs), we can have their corresponding angles and conversely. State the formulae.
- 5) Find the area of an ellipse and give its formula.

The "Problems on Mathematics given in Foreign Languages" included:

- 1) How many times does a clock strike over one day and night? Justify your answer by addition and multiplication.
- 2) A gardener has two pieces of a vegetable garden: One is of area $\frac{4}{5}$ "Mu" (1 "Mu" equals 240 square "Bu", so 1 "Mu" equals 571.663 square meters), while the other has $\frac{3}{7}$ "Mu". Suppose he earns $\frac{17}{9}$ taels per "Mu", then what is the total amount he earns?
- 3) San Francisco is 20654 "Li" from Shanghai (one "Li" equals to 0.5556 kilometers). Suppose a steamer travels 818 "Li" each day, how many days will it take for a round-trip?
- 4) Given that the distance between the Sun and the Earth is 274400000 "Li", and that light travels 560000 "Li" each second, how many minutes and seconds does sunlight need to reach the Earth?

5) Given that 15 seconds is needed to hear the thunder after flash of lightening, suppose the velocity of sound is 336 kilometers each second, find the place where the lightening had struck.

6) If a cannon is fired at the southern gate of a city, it takes 12 seconds to hear the rumble at the northern gate. What is the distance between these two gates?

All of the mathematics tests for 1870, 1872, 1873 and 1878 are listed above which seem to be the only source materials available about the "Sui Shi". As noted above, Li Shanlan was (presumably) responsible for all of the tests given in Chinese, and at least partly for the tests given in foreign languages. A comparison of the "Problems on Astronomy and Mathematics" (for the "Five-Year System" students) and the "Problems on Mathematics given in Foreign Languages" (basically for the "Eight-Year System" students) suggests that the students enrolled in the Five-year system had better capabilities for studying mathematics and science. In fact, among the list of students who attended the 1878 annual examination for mathematics given in Chinese ("Han Wen Sui Shi"), Xi Gan, Wang Fengzao, Du Fameng, Gui Rong, Chen Shuotian, Xiong Fangbo and Hu Yuling (all of whom were all the favorite students of Li Shanlan) occupied (in order) the first seven places. However, students took the mathematics test given in foreign languages never seem to have been highly graded by Li Shanlan [Cf. *Calendar of the Tungwen College* 1879].

5.3. Collected Mathematical Exercises of the Students: The Suan

Xue Ke Yi Yi

The exercises or test papers of eight students, Xi Gan, Wang Fengzao, Gui Rong, Chen Shuotian, Xiong Fangbo, Hu Yuling, Yang Zhaoyun and Cai Xiyong, were collected into the *Suan Xue Ke Yi* and published in 1880. This treatise, whose title Martin translated as "Mathematical Exercises," was selected and arranged by the then Li Shanlan's assistants Xi Gan and Gui Rong, and proofread by his students Chen Shuotian, Hu Yuling, Xiong Fangbo and Li Fengchun -- all under the supervision of Li Shanlan [*Triennial Calendar of the Tungwen College* 1888, p. 31]. In his "Preface" to the treatise, Martin appraised the achievements of the students in terms of their instructor in mathematics Li Shanlan: "Astronomy, geography, cannon, navigation and surveying are all the most important among the substantial learning. Through this collection, readers of this book will easily appreciate the excellent achievements of the students. Yet all of this has been due mainly to Mr. Li Shanlan's diligent labor of teaching," [Martin 1880].

The *Suan Xue Ke Yi* may indeed be regarded as concrete evidence of justifying how successful Li Shanlan was in his teaching at the Tong Wen Guan. He was actually proud of the performance of his students as shown in this collection. For example, immediately after publication of this treatise, Li Shanlan sent a copy to Liu Yueyun, who visited Li Shanlan at Beijing asking for advice in studying mathematics and natural philosophy.

In fact, by examining this treatise closely, it should be possible to determine whether Li Shanlan exaggerated his assessment of this treatise or not. For this purpose, considering the following statistical data.

The *Suan Xue Ke Yi* is comprised of four books. Book 1 basically concerns astronomy, geography, natural philosophy, navigation and surveying. The number of exercises done by each student were as follows: Gui Rong (7 exercises), Chen Shuotian (6 exercises), Wang Fengzao (6 exercises), Wang Zongfu (5 exercises) and Hu Yuling (3 exercises), and so on. Book 2 contained exercises about geometry, trigonometry, continued proportion, "Duo Ji" (summation of finite series) and infinite series. Again the number of exercises done by each student were: Wang Fengzao (9 exercises), Du Fameng (5 exercises), Gui Rong (5 exercises), Cai Xiyong (4 exercises), Zuo Bingrong (4 exercises), Chen Shuotian (3 exercises) and Yang Zhaoyun (3 exercises).

Some exercises collected in Book 2 were solved by the "Tian Yuan" method. Yet this was clearly the major concern of Book 3. This book collected exercises drawn from the *Ce Yuan Hai Jing* (which was the textbook either for the course "Learning the Si Yuan Jie" or "Algebra"). The number of exercises per student in this book were as follows: Wang Zhengxian (8 exercises), Chen Shuotian (4 exercises), Wang Zhongxiang (4 exercises) and Du Fameng (3 exercises). Finally, Book 4 concerned problems on the right-angled triangle and its inscribed circle, which was a blend

of Chapter 9 of the *Jiu Zhang Suan Shu* and the similar content of the *Ce Yuan Hai Jing*. In addition, this book also collected some additional miscellaneous problems probably drawn from the course on "Arithmetic". Yet in interestingly enough, almost all of these problems were solved by the "Tian Yuan" method. As for the number of exercises done by each student: Gui Rong (15 exercises), Chen Shuotian (4 exercises), Hu Yuling (4 exercises) and Du Fameng (4 exercises).

Among the exercises collected in the *Suan Xue Ke Yi*, Gui Rong did the most. He had as many as 29 exercises included. The following names, in decreasing order of numbers of exercises, were Chen Shuotian and Wang Fengzao (both 17 exercises), Du Fameng (14 exercises), Hu Yuling and Wang Zongfu (both 9 exercises), Wang Zhengxiang (8 exercises) and Wang Zhongxiang (7 exercises). It is evident, therefore, that Li Shanlan's favorite students -- like Gui Rong and Wang Fengzao -- did perform with greater proficiency on this scale than others. The reason why Yang Zhaoyun and Cai Xiyong had very few of their exercises included probably was due to their short stay in the Tong Wen Guan. As for Xi Gan, the future successor of Li Shanlan at the Tong Wen Guan, it is difficult to explain why only 4 of his exercises were included -- a striking contrast to Gui Rong's 29 exercises, despite the fact that both of them were assistants to Li Shanlan at the time. Perhaps it was because Xi Gan bore more responsibilities as an assistant. Whatever the case, both were students of whom Li Shanlan was most proud in his late years.

Equally significant, the *Suan Xue Ke Yi* makes it clear that Li Shanlan played a predominant role in the four "Sui Shi" (Annual Examinations) and "Da Kao" (Great Examinations) discussed above. For example, problems drawn from the *Ce Yuan Hai Jing*, as well as their solutions by the "Tian Yuan" method appeared throughout the *Suan Xue Ke Yi*. Moreover, the course on "Algebra" seems to have been equally emphasized. Thus Book 2 of the *Suan Xue Ke Yi* included an exercise of Gui Rong, which can be given in modern notation as follows:

Find the binomial expansion of $(a + x)^{1/3}$.

This problem is reminiscent of Problem 5 in the 1878 "Sui Shi", and certainly indicating that at least some students (like Gui Rong) had taken courses on "Algebra" or "Calculus" very seriously. Based on this fact, among others, it is possible to conclude that the problems about "Calculus" did appear in the tests even though Li Shanlan and his students did not seem to have paid much attention to the subject.

Finally, something more should be said about Rong again, although so far little is known about his life. Yet he certainly had attained high grades in his courses on "Mathematical Physics" and "Natural Philosophy". For example, he earned first place honors for his solutions to the "Problems on Natural Philosophy given in Chinese" ("Han Wen Ge Wu") in the "Sui Shi" for 1873. Apparently his solutions were so remarkable that the *Peking*

Magzine (No. 20) gave the first four problems along with Gui Rong's solutions. In addition, the *Peking Magazine* also published one of Gui Rong's "Yue Ke" (Monthly Tests) on "Natural Philosophy" of 1874, as well as one of his exercises on "Mathematical Physics" [Quoted in Zhu Youxian 1983 vol.1, pp. 116-125]. These can thus be no doubt that Gui Rong attained great success in his study of mathematics on the one hand, while it also justified Martin's high esteem for Li Shanlan's teaching on the other. Therefore, (censor official) Chen Jin's accusation (of 1883) that since those who taught mathematics and science did not grade students seriously, the efficacy of their teaching was doubtful. Chen Jin even alleged that half the students whose exercises were collected in the *Suan Xue Ke Yi* knew nothing about mathematics. Consequently, Chen Jin urged the throne should close the Tong Wen Guan. Nevertheless, Li Shanlan must have been satisfied with the remarkable performance (at least in examinations) of his favorite students. He certainly was very pleased to see the publication of the *Suan Xue Ke Yi* basically because it was a tribute to the success -- and accomplishments -- of his deeds of his teaching career.

5.4. Study of Prime Numbers: The *Kao Shu Gen Si Fa*

In addition to his teaching, Li Shanlan had been keeping a close eye on the late Qing mathematical community. In fact, he was the clear leader of the community whose members came basically from and were active in the Jiangnan area (i.e., those provinces

south of the Yangzi River). Although he was now far away from Jiang-Zhe centers like Hunan and Guangdong, and (especially) Shanghai, Li Shanlan and his mathematics still drew considerable attention from the mathematical community. For example, in a letter of 1875 sent to Li Shanlan, Wang Tao, then a refugee in Hong Kong [Cf. Section 2.5], reported that he was very pleased to hear of the high respect paid to Li Shanlan from scholars who had visited Li Shanlan in Beijing and had just returned to Guangdong [Cf. Horng 1989a]. Similarly, to Li Shanlan some noted mathematicians like Ding Quzhong at Hunan also asked Li Shanlan for advice by means of correspondences [Cf. Li Di *et al.*, eds., 1986, pp. 512-517].

It comes as no surprise therefore that Hua Hengfang should have asked Li Shanlan to write the preface to his own treatise, *Kai Fang Bei Shu* ("On the Other Method of Extracting Roots", 1872). In this "Preface", Li Shanlan mentioned his correspondences with Hua Hengfang about some significant problems related to prime numbers which underlay the methods of extracting roots or solving numerical equations with integral coefficients. Li Shanlan highly valued Hua Hengfang's method that "was not only creative but easier than the old ones as well," [Quoted Li 1872b]. Li Shanlan also recommended his own most recent treatise, the *Kao Shu Gen Si Fa*, as a supplement to Hua Hengfang's *Kai Fang Bei Shu*.

A few words needed to be said about the *Kai Fang Bei Shu*. The Chinese mathematical term "Kai Fang", which usually meant

"extracting square or cubic roots" (as in the *Jiu Zhang Suan Shu*, for example) was regarded as having an additional connotation, namely "solving numerical equations" in general and this seems to be common by the 11th century on [Qian 1983, pp. 144-167]. This was still the case even in the *Shu Li Jing Yun* ("Collected Basic Principles of Mathematics") of the early Qing, which introduced the Western method of "Jie Gen Fang Bi Li" [Cf. Section 1.4]. Thus Li Rui's *Kai Fang Shuo* used the same term, "Kai Fang" in its more general sense towards the end of the 18th century. At the same time Wang Lai preferred to use the method of "Jie Gen Fang Bi Li" to deal with the theory of equations. Nevertheless, he would have understood the term "Kai Fang" in terms of the connotation of "Jie Gen Fang Bi Li" [Cf. Section 1.8].

It is of interest that neither Li Rui nor Wang Lai ever touched the problem whether the constant term of an algebraic equation with integral coefficients had anything to do with the rational roots of the equation. It awaited Hua Hengfang to deal with this problem in terms of the prime factors of the constant term of the equation. In this connection, the properties of prime numbers need to be properly appreciated as well. All of this serves as background to Li Shanlan's own study of the prime numbers.

Prime numbers as such never attracted the interest of mathematicians in ancient China, unlike their counterparts in the ancient West. In Chapter 1 of the *Jiu Zhang Suan Shu*, a method

similar to the Euclidean algorithm was given to find the common divisors of two natural numbers [Cf. Section 4.6]. Yet this was aimed solely at reducing a given quotient to lowest terms. Ancient Chinese mathematicians, including Liu Hui (who made great contributions to Chinese mathematics through his *Commentary to the Jiu Zhang Suan Shu*) simply ignored the problem of factoring a given number in general. In the thirteenth century Qin Jiushao developed the "Da Yan" method to help solve simultaneous congruences and eventually created what later came to be called the Chinese Remainder Theorem. In the process, he paid attention to properties of "modulus" in the congruences in terms of number theory. However, Qin Jiushao did not study prime numbers for their own sake, even though in this context, Chinese mathematicians should not have avoided the problem, since all of the "moduli" in the simultaneous congruences needed to be relatively prime. But Chinese mathematicians after the Southern Song never came back to this problem. Needless to say that they would have neglected the concept of prime number introduced in the *Ji He Yuan Ben* (1607) or the *Shu Li Jing Yun* (1723). It was not until the 1860s that Li Shanlan began to take the related problems seriously.

Yet why did Li Shanlan become interested in the properties of prime numbers? Due to the fragmentary state of current source materials, it is impossible to give a definitive answer. Yet one plausible reason might be that in the 1860s mathematicians like Ding Quzhong, Shi Yuechen and Huang Zongxian were studying integer

solutions of indeterminate equations arising from the "One Hundred Chickens" problem (Bai Ji Shu), as well as the solutions of simultaneous congruences coming from the same subject by Qin Jiushao [Cf. Li Di et al., eds., 1986, pp. 512-516]. Since all these mathematicians were consequently aware of the significance of prime numbers, Li Shanlan may well have been led to study of prime numbers for similar reasons in the late 1860s.

If this were the case, then the draft version of Li Shanlan's *Kao Shu Gen Si Fa* may have been written as early as the late 1860s, most likely in the period of 1867-1870. On the one hand, his collected mathematical works, *Ze Gu Xi Zhai Suan Xue*, was published in 1867, and on the other, the title of the *Kao Shu Gen Si Fa* was not included in a list of the "Ze Gu Tang Suan Xue Mu Lu" ("The Content of Mathematical Works written in Ze Gu Tang"). During this period, Li Shanlan first published his *Ze Gu Xi Zhai Suan*. He then translated, in collaboration with John Fryer, Book I of Issac Newton's *Principia* (*Mathematical Principles of Natural Philosophy*) for the translation department of the Jiangnan Arsenal [Cf. Section 2.6]. Eventually, in 1868, Li Shanlan went to the Beijing Tong Wen Guan where he completed the final version of his *Kao Shu Gen Si Fa* and sent it off in 1872 to *Peking Magazine* for publication.

The first portion of the *Kao Shu Gen Si Fa* appeared in the second monthly issue of *Peking Magazine*, published in August of 1872 by the "Society for the Diffusion of Useful Knowledge in

China," founded by Westerners in Beijing. The remaining portion of this paper should have been included in the next issue of *Peking Magazine*. However, no copies of the third issue are known, and the magazine may not have survived. Thus we have to rely upon another source for a full-text of the *Kao Shu Gen Si Fa* [Li Yan 1955]. Fortunately, the *Xiang Xue Xin Bao* ("Hunan Journal", 1897) reprinted this paper, but in a somewhat abridged form.

This abridged form, however, differed only slightly (according to the historian Yan Dunjie) from the original paper. In fact, by comparing the two, Yan Dunjie found that the first portion of the *Kao Shu Gen Si Fa* in *Peking magazine* contained two more examples [Yan Dunjie 1953]. Otherwise, the *Xiang Xue Xin Bao*'s edition of the *Kao Shu Gen Si Fa* seemed reliable, and certainly sufficient for a discussion of Li Shanlan's study of prime numbers.

In the *Kao Shu Gen Si Fa*, Li Shanlan reiterated the definition of prime number ("Shu Gen", literally root of number), drawn from (Book VI) of the *Ji He Yuan Ben* (Euclid's *Elements*, the first six books, as translated into Chinese in 1607). "Given any (natural) number, we have not hitherto had any criteria to determine whether it is a prime number or not. By thinking carefully about this problem for quite a long time, I have thus obtained four methods to determine if any given (natural) number is a prime number, and thereby add a supplement to geometry," [Quoted Li Shanlan 1872].

As for the four methods, only the first one, that is, "Lu Cheng Qiu Yi Fa" (literally the method of multiplying an auxiliary number by itself and reducing the remainder to one), needs to be discussed here, since the other three methods were basically based upon this one [Cf. Li Di et al., eds., 1986, pp. 507-511].

Given a natural number N (Li Shanlan called it a "Ben Shu", literally original number), one is to determine if it is a prime number or not. Take an integer a such that $(a, N) = 1$, i.e., a is relatively prime to N , in which Li Shanlan usually takes $a = 2, 3$ or 10 (Li Shanlan called such auxiliary numbers "Yong Shu"), then there exists a natural number n such that $a^n > \frac{N}{2}$ or $a^n > N$. Thus, as Li Shanlan argued,

$$(a^n - N)^2 \equiv r_1 \pmod{N},$$

$$(a^n - N) r_1 \equiv r_2 \pmod{N},$$

$$(a^n - N) r_2 \equiv r_3 \pmod{N},$$

...

$$(a^n - N) r_{m-2} \equiv r_{m-1} = \pm a^c \pmod{N},$$

where $c = 0, 1, 2, 3, \dots$

By multiplying all the terms of both sides respectively, we obtain

$$(a^n - N)^m r_1 r_2 \dots r_{m-2} \equiv r_1 r_2 \dots r_{m-2} (\pm a^c) \pmod{N}.$$

Since all the remainders (r's) are relatively prime to N, all the r's from both sides of the last congruence may be cancelled:

$$(a^n - N)^m \equiv \pm a^c \pmod{N}$$

$$\Rightarrow a^{mn-c} \equiv \pm 1 \pmod{N}$$

$$\Rightarrow a^{mn-c} = a^d \equiv 1 \pmod{N} \quad \& \quad a^{2(mn-c)} = a^d \equiv 1 \pmod{N}.$$

Consequently, there exists a smallest exponent d such that $a^d - 1$ is divisible by N (the integer d was called "Ding Ci" by Li Shanlan). With this in hand, it is now necessary to determine if the number N is prime or not. Suppose that

$$N \equiv b \pmod{d}.$$

If $b \neq 1$, then N is a composite number. If $b = 1$, then further discussion is needed.

1) Assume that $d = pq$, where p is an even number, then

$$N \equiv 1 \pmod{d}$$

$$\Rightarrow N = 1 + \lambda pq ,$$

where λ is an integer. Take an integer r such that $0 < r < \lambda q$,
thus if

$$(1 + pr, N) = k > 1 ,$$

then N is a composite number. But if

$$(1 + pr, N) = 1 ,$$

then since $1 < 1 + pr < 1 + \lambda pq$, therefore N is a prime number.

ii) Assume that $d = p'q'$, where both p' , q' are odd numbers.
Take the number $2p'q'$ (Li Shanlan called it "Di Jia Shu"). Then,
since N couldn't be an even number (for then it is a composite
number), N must be an odd number and, by assumption, there exists
an integer μ such that

$$N = 1 + \mu p'q' .$$

But then μ should be even, for $N - 1$ is even and $p'q'$ is odd.
Thus by setting $\mu = 2\lambda'$, it follows that

$$N = 1 + 2 \lambda' p'q' .$$

Choose integers r' such that $0 < r' < 2\lambda'q'$, then similarly as in case (1), if $(1 + 2r'p', N) > 1$, N is a composite; while if $(1 + 2r'p', N) = 1$, then N is a prime number.

As illustrations, Li Shanlan immediately gave a few examples to show that his methods did work. Here we cite only four:

Example 1. Given $N = 31$. Using $a = 2$ as "Yong Shu" (auxiliary number), then since $(31 - 2^4)^2 \equiv 2^3 \pmod{31}$, one obtains $2^{8-3} = 2^5 \equiv 1 \pmod{31}$. Because $31 \equiv 1 \pmod{5}$, and $d = 5$ is odd, therefore, since $N = 31$ is not divisible by $1 + 2 \cdot 5$, it follows that $N = 31$ is a prime number.

Example 2. Given $N = 1093$. Using $a = 3$ as "Yong Shu", then since $(1093 - 3^6)^2 \equiv 3^5 \pmod{1093}$, $3^{12-5} = 3^7 \equiv 1 \pmod{1093}$. Now since $1093 \equiv 1 \pmod{7}$, and $d = 7$ is odd, therefore the fact that $N = 1093$ is not divisible by $1 + 6 \cdot 7$ leads to the conclusion that $N = 1093$ is a prime number.

Example 3. Given $N = 341$. Using $a = 2$ as "Yong Shu", then since $(2^9 - 341)^2 \equiv 2^8 \pmod{341}$, $2^{18-8} = 2^{10} \equiv 1 \pmod{341}$. Now $341 \equiv 1 \pmod{10}$, and $d = 10$, yet 341 is divisible by $1 + 10$, therefore 341 is not a prime number.

Example 4. Given $N = 14209$. Using $a = 3$ as "Yong Shu", then since $(3^9 - 14209)^3 \equiv 2^6 \pmod{14209}$, $3^{27-6} = 3^{21} \equiv 1 \pmod{14209}$. Now since $14209 \equiv 13 \pmod{21}$, the remainder 13 is

not equal to 1, therefore 14029 is not a prime number.

It is quite clear from the above discussion that Li Shanlan's "Lu Cheng Qiu Yi Fa" actually implied Fermat's Little Theorem: If N is a prime and $(a, N) = 1$, then $a^{N-1} - 1$ is divisible by N [Struik 1967, p. 108]. However, the converse is not true, that is, the proposition that

$$(a, N) = 1 \Rightarrow a^{N-1} - 1 \text{ is divisible by } N$$

could be true under the condition that N is not a prime. For this case Li Shanlan indeed offered the example $N = 341$ [Cf. Example 3 above]. Moreover, historians of Chinese mathematics have also noted that "Li Shanlan used only odd numbers $1 + rp$ (or $1 + 2r'p'$) to divide N and thereby to determine the possible factors of N . In this connection he clearly reduced the labour in determining if N is a prime number," [Quoted Li Di et al., eds., 1986, p. 509].

In fact, it is more significant that the *Kao Shu Gen Si Fa* still retained features shown in Li Shanlan's *Ze Gu Xi Zhai Suan Xue*, despite the fact that before he finished the first version of the *Kao Shu Gen Si Fa*, he seems to have already been familiar with the Western mathematics transmitted through his own translations and the work of Western missionaries at the LMS Press. This was not the case with his major treatises like the *Fang Yuan Chan You*, *Hu Shi Qi Mi*, *Dui Shu Tan Yuan*, and *Duo Ji Bi Lei* [Cf. Sections 3.2, 3.3, 3.4, and 3.5], since all of these treatises were

basically written by the time he joined the Press. In other words, like the arguments and methods he gave in the *Hu Shi Qi Mi*, *Dui Shu Tan Yuan* and *Duo Ji Bi Lei*, in the *Kao Shu Gen Si Fa* Li Shanlan was still primarily concerned with the problem-solving aspects of the subjects. Such an approach, which reflected the most important characteristic of traditional Chinese mathematics, certainly prevailed in his *Ze Gu Xi Zhai Suan Xue*. Furthermore, for the method of "Tian Yuan Qiu Yi", one of the four methods given in the *Kao Shu Gen Fa*, Li Shanlan used the very terms and algorithm from Song mathematician Qin Jiushao (1202-1261). On the other hand, Li Shanlan's method of "Tian Yuan Qiu Yi" was a response of sorts to Huang Zongxian's elaborate study of Qin Jiushao's "Da Yan Qiu Yi Shu", although his aim in determining prime numbers was different from Huang Zongxian's interest in improving Qin Jiushao's methods for solving the simultaneous congruences [Li Di *et al.*, eds., 1986, pp. 513-515; Shen Kanshen 1987].

As for the influence of the *Kao Shu Gen Si Fa* over late Qing mathematicians, consider the case of Hua Hengfang. His interest in prime numbers was apparently inspired by Li Shanlan as reflected in Li Shanlan's "Preface" to his *Kai Fang Bei Shu* ("On the Other Methods in Extracting Roots", 1872). Hua Hengfang actually wrote treatises like *Shu Gen Shu Jie*, *Qiu Cheng Shu Fa* and *Shu Gen Yan Gu* to deal with problems on prime numbers, but he failed to go beyond anything Li Shanlan had already attained in this subject [Li Di *et al.*, eds., 1986, p. 517].

Another interest of Li Shanlan's late carrer, in additional to prime numbers, were Pythagorean triplets. This subject again had stimulated the great interest among the late Qing mathematicians. Before Li Shanlan, Qing mathematicians like Luo Shilin, Luo Tengfeng, Wang Lai, Chen Jie, and Wang Yuanqi had all been enthusiastic in studying this subject [Qian 1937]. They seem to have been inspired by the early Qing mathematics treatise edited under the supervision of Emperor Kang Xi [Cf. Section 1.4], the *Shu Li Jing Yun* (1723), in which (for the first time in the history of Chinese mathematics) a general method was given for finding the "integer" sides of a right-angled triangle, or equivalently, finding the solutions in integers of the quadratic equation $x^2 + y^2 = z^2$ [*Shu Li Jing Yun* Part II, Book 12].

Although his colleagues like Gu Guanguang, Dai Xu and Xu Youren do not seem to have been interested in this subject, Li Shanlan nevertheless took it up to in the early 1860s. In his *Tian Suan Huo Wen* ("Querries and Answers on Astronomy and Mathematics"), collected in the *Ze Gu Xi Zhai Suan Xue* (1867), Li Shanlan discusses two problems (Problems 5 and 6) on the subject.

According to Problem 5: "For convenient computations with the sides of a right-angled triangle, mathematical treatises always take their ratio as 3 : 4 : 5 or 8 : 15 : 17 . Is it possible to construct the 'Gou' side (the short leg of the right angle) and the 'Gu' side (the longer leg of the right angle) of a right-angled triangle arbitrarily such that all the 'Gou' side, 'Gu' side and 'Xuan' side (the hypotenuse) are integers?" The answer, as Li Shanlan gave it, can be expressed in general as

follows: Take two integers m, n which are either both even or odd, then the two sets of solutions are

$$(i) \ a = \frac{n(m^2-1)}{2}, \quad b = mn, \quad c = \frac{n(m^2+1)}{2};$$

$$(ii) \ a = \frac{m^2-n^2}{2}, \quad b = mn, \quad c = \frac{m^2+n^2}{2}.$$

Problem 6 required finding two sets of Pythagorean triplets

$$a_1, b_1, c_1; \quad a_2, b_2, c_2,$$

such that

$$a_1 \cdot b_1 = a_2 \cdot b_2; \quad a_1 + c_1 = a_2 + c_2.$$

Based on the following identity

$$(4mn)^2 + 4mn(3m^2 - 2mn - n^2) + (3m^2 - 2mn - n^2)^2 = (3m^2 + n^2)^2,$$

where m, n are integers, Li Shanlan set

$$c_1 + a_1 = c_2 + a_2 = (3m^2 + n^2),$$

$$c_1 - a_1 = (4mn)^2,$$

$$c_2 - a_2 = (3m^2 - 2mn - n^2)^2,$$

and thereby obtained the desired two sets of Pythagorean triplets. Li Shanlan also gave two examples to illustrate this method, making clear that the method did work [Cf. Qian 1937; Li Di *et al.*, eds., 1986, p. 523]. Nevertheless, how he was able to derive the above identity is unknown. Although such formulas also were given in a Japanese mathematical treatise, the *Suan Fa Yuan Li Gua Non* (1852), the Chinese edition was not published until 1873, and there is no evidence that it was available to Li Shanlan when he was ready to publish in 1867 -- when the *Ze Gu Xi Zhai Suan Xue* was published, including, of course, the *Tian Suan Huo Wen* [Qian 1937].

Whatever the case, Liu Yicheng's interest in "Pythagorean" triplets from the 1870s on seems to have been due largely to Li Shanlan. In 1862, when Liu Yicheng asked his advice about studying mathematics from in Shanghai, Li Shanlan may well have mentioned the subject of "Pythagorean" triples [Cf. Liu Yicheng 1899, "Preface"].

In fact, Liu Yicheng played a central role in Chinese studies of number theory in the last three decades of the 19th century. This was due largely to his teaching at both the Guang Fang Yan Guan from 1873 onwards, as well as at the Qiu Zhi Academy in Shanghai two years later. Towards the end of the 19th century, his students like Zhi Baoran, Shen Shanzen, Chen Weiqi and Cui Chaoqing actually paid considerable attention to number theory. They were apparently encouraged by Liu Yicheng, since Liu Yicheng

gave large number of problems on number theory in the mathematics tests he prepared for the students, which were collected in his *Jian Yi An Suan Gao* (1899). In order to sample the kind of problems, two problems in the 1883 test were typical. In one problem, Liu Yicheng asked the student to find the Pythagorean triplets Li Shanlan had dealt with in the *Tian Suan Huo Wen*.⁷ In the Spring test that same year Liu Yicheng had already given a problem equivalent to this one, namely, "Find integers x and y such that $x^2 + xy + y^2$ is a perfect square." In other words, the student was asked to give integer solutions for the indeterminate quadratic equation

$$x^2 + xy + y^2 = z^2.$$

Clearly, if

$$x^2 = (4mn)^2, \quad y^2 = (3m^2 - 2mn - n^2)^2, \quad z^2 = (3m^2 + n^2)^2,$$

then this problem is equivalent to that of the later test or Li Shanlan's triplets mentioned above.

Such problems, according to Liu Yicheng, were new at the time [Liu Yicheng 1899 "Preface"]. Yet they were due to Li Shanlan, who created this new approach to the determination of the two right-angled triangles with equal area and equal sum of the "Gou" side and the "Xian" side. Li Shanlan had indeed opened up new territory -- Diophantine style analysis -- for Chinese

mathematicians of 1860s [Qian 1937]. No wonder Li Shanlan was the most important stimulus for the Qing mathematical community in the 1870s. In fact, he spent his last days working on a final treatise, the *Ji Shu Gou Gu*, which dealt with Pythagorean triplets. Unfortunately, it did not survive to the present. In a letter to Wang Kangnian, Hua Shifang urged Wang to publish Li Shanlan's *Gou Gu Qiu Zheng Shu Shu* ("Method of Determining Pythagorean Triplets") as soon as possible, since it had already been proofread [Cf. Wang Kangnian 1987, p. 2229]. It is highly probable that this treatise was just the *Ji Shu Gou Gu* [Cf. Li Yan 1955, p. 352]. Yet it is unknown whether the *Gou Gu Qiu Zheng Shu Shu* was published or not. Nor is known that it published but got lost eventually.

5.5. On Education

By 1875, Li Shanlan's contributions to mathematics had earned great (and well-deserved) respect not only from his Western colleagues, but even from the conservative in Beijing. Zhang Zhidong, one of the key advocates of the "Self-strengthening Movement," regarded Li Shanlan as one of the best mathematicians of the past fifty years, and rated his works highly in his own *Shu Mu Da Wen* (1975), which recommended books in several categories for students at the traditional academies. Even more remarkably, Zhang admitted that "this book does not record mathematicians who are still alive. Yet due to the fact that Li Shanlan has made such great and excellent studies in mathematics, we therefore add this one name [as an exception]." Clearly, Li Shanlan (although

he never commented on mathematical education in particular or educational system reform in general), nevertheless had a considerable effect upon mathematics education in China. In fact, he did write a "Preface" to a treatise by the German missionary Ernst Faber: *De Guo Xue Xiao Lun Lue* ("On the German School System", in Chinese, published in 1873).⁸ This preface has been overlooked by the scholars of modern Chinese history, although it was noted by the historian of Chinese mathematics Li Yan in 1953 [Li Yan 1955]. The reason, perhaps, is that Li Shanlan did not mean to address officials in particular, and it may therefore be regarded as adding nothing significant to the subject of the late Qing educational reform.

Whatever influence Li Shanlan's preface may have had, he was clearly impressed by the German educational system, which he hoped might help to reform mathematical education in China, especially at the traditional academies which he counted on Xu Youzen, then the Jiangsu Governor in 1860 [Cf. Section 2.5]. Perhaps this is why Li Shanlan was so pleased to write the "Preface" to the Chinese edition of Faber's book on German education.

At the very beginning of his "Preface," Li Shanlan reminded his readers that Germany's soldiers, as trained in German schools, had always been able to defeat their powerful neighbors. This in turn was related his observation that if soldiers were well-educated in school then they would subsequently be courageous and loyal to their country. Yet Li Shanlan admitted his surprise

that in Germany not merely soliders but everyone should be educated in the German school system. "In fact," said Li Shanlan, "the characteristic feature of the German school system is that schools are everywhere. They provided every sort of learning, and everyone is obliged to attend school," [Quoted from Li Shanlan 1873].

Li Shanlan was clearly impressed by Ernst Faber's description of the German educational system. From a Chinese perspective, it was certainly amazing to find that there were so many different kinds of schools at every level of learning, from the primary school to high schools as well as college. Specialization, and the different disciplines like technology, science, agriculture, commercial business, shipyard schools, clerks, military schools, teacher's colleges, theological seminaries, fine arts, and so on, were literally mind-boggling. There were also schools for women, the disabled, the deaf, the blind, orphans and even schools for children who were criminals. In China this must all have seemed foreign, exotic, perverse -- and yet the German success was undeniable. Equally impressive were numbers of schools which (except primary ones) amounted to a total of 1071. With 115,610 students in 1871, this must have struck Li Shanlan as remarkable (as did the fact that a year later, the total number of students has increased by more than 60,000 to 177,379 the next year. Li Shanlan was equally impressed that throughout Germany, there were as many as 10,049 college students [Cf. *The Church News* vol. 6, p. 267].

Here we are particularly interested, however, in how Li Shanlan related the German school system to the issue of professionals needed in China. Li Shanlan was especially impressed by the way Germany provided schools for virtually any sort of learning. And in every such case, specialized subjects were taught by "professionalized" teachers who were well-trained in the relevant studies at the college level. Consequently, in colleges of science, technology, agriculture, commercial business, clerks, etc, studying to become a professional was not only possible, but natural as well. Moreover, since what the students learned in the schools was designed to lead to some future practical use, in Germany "the malady of 'what we use is not what we have learned, and what we have learned is not what we could use' can be avoided," [Quoted in Li Shanlan 1873, "Preface"].

The above quoted passage, "what we use is not what we have learned, and what we have learned is not what we could use", reiterates the comment Li Shanlan made about the traditional education system, as well as the examination system in the 1850s. On the other hand, it also reflects Li Hongzhang's criticism of the learning of the Chinese scholars and officials in 1864 [See the passages quoted from Li Hongzhang above in Section 2.6]. Yet if we consider this viewpoint against the background of German educational reform of the 1870s, then we cannot escape the conclusion that Li Shanlan added something quite new to suggestions about how the traditional Chinese educational system should be reformed. He also went much further than what he had claimed in

the "Preface" he wrote for the *Zhong Xue* (a Chinese translation of Whewell's *Elements of Mechanics*): "Alas! The European nations are now becoming more prosperous and stronger day by day, endangering China's frontiers. This is due to the fact that they are skillful in the manufacture of fire-arms, and this skill is derived from their mastery of mathematics," [Quoted in Section 1.6]. With new ideas about Chinese educational reform in mind, the relevance of mathematics to manufacturing as well as its significance for the "Self-strengthening Movement" were still of importance, but Li Shanlan was probably the only late Qing intellectual who paid much attention to general education as well as to the training of professionals in the early 1870s. At the time, the major concerns of the "Self-strengthening Movement" had been re-programmed to include economic development [Kwang-Ching Liu 1988], and here key figures like Li Hongzhang and Zhang Zhidong, apparently did not think that educational reform should be given first priority. It was not until the 1890s that Qing intellectuals and officials began to show much concern for educational reform.

5.6. Mathematics and the Traditional Academies

The late Qing reform of local academies to include mathematics and science in their curricula was one of the most important advances in the transition of the Chinese education system from a traditional form to a basically modern one. In his article "On Mathematics Education Institutions of the Qing Period", Li Yan, the noted historian of Chinese mathematics,

failed to pay adequate attention to the late Qing academies [Li Yan 1955b]. Since the academies were the education base of traditional Chinese society, they would naturally become the focus of late Qing reformers. The academies underwent some educational reformations largely because a compromise seems to have been reached in the 1890s by high officials who were not adverse to radical ideas on reform where education was concerned. On the other hand, financial problems were persistent. Expanding or adjusting structure of the traditional educational institutions seemed to be more feasible than other alternatives.⁹ For example, the governor of Shanxi province, Hu Pinzhi, reformed the curriculum of the Lian De Academy, adding courses on mathematics and science in 1898. He then went on to expand the academy into a provincial school, the Jin Sheng Sheng Hui Xue Tang, in 1900. In his proposal, he stressed that: "The school is no different from the academy in so far as both they represent the educational base from which a new generation in this country will be raised. Yet my purpose is to enhance the function of the institutions but not to restructure them," [Quoted in Zhu Youxiang 1983, p. 444].

Perhaps it is therefore understandable why the reform of academies has been largely ignored by historians of Chinese modern history in the crucial transitional period of education in China. Beginning in the early 1890s, almost all of the famous academies had been reformed to include mathematics and science. Yet, this, apparently, was not for radical reformers like Kang Youwei and Liang Qichao, who led the Reform Movement of 1898 [Cf. Hao Chang

in Fairbank *et al.*, eds., 1980, pp. 274-338]. Nor was it enough for the officials and intellectuals of the post-Reform Movement of 1898. In fact, the "Qin Ding Xue Tang Zhang Cheng" [Reform of Educational Institutions], decreed by the throne in 1902, was aimed at reforming Chinese education, and therefore it signaled an end to the long history of the traditional academies.

Nevertheless, the reformers usually took the Tong Wen Guan's curriculum (especially its inclusion of mathematics and science, as well as Li Shanlan's teaching) as a model. It is against this background that Li Shanlan's influence on the late Qing mathematical education can be properly appreciated. But first it is necessary to clarify the status of the traditional academies in the Qing educational system.

As in the Ming Dynasty, Qing schools were divided into two systems: the central and the local. In the central system of the Qing dynasty, three more sorts of schools (the "Zong Xue", "Jue Luo Xue" and "Ba Qi Guan Xue") were also added to the Imperial Academy ("Guo Zi Jian"). The first two of these were founded mainly for the children of Manzhou people. For example, while the "Zong Xue" was only for royal or noble children, the "Jue Luo Xue" recruited students both from nobility and commoners. The "Ba Qi Guan Xue" was for children from the Manzhou, Mongolia or Han People who had made contributions to the founding of the Qing Dynasty. At the local level, in addition to the provincial schools ("Fu Xue"), prefecture schools ("Zhou Xue") and county

schools ("Xian Xue"), there were the community schools ("She Xue"), basically for elementary education, and the "Wei Xue" (schools for the children from the troops at the military garrison headquarters) [Yang Ronchun 1985, pp. 436-452].

In the local school system, the Qing dynasty adopted all of the Ming examples. Moreover, the Qing government also supported local schools as the Ming had, for there were 1741 state schools in 1812 and 1810 in 1886, compared with the 1200 state schools in 1398 under the Ming reign [Ho 1962, 171]. Yet seldom were these schools local centers of serious study, since the school and examination systems had been well integrated since the Ming dynasty. Under such circumstances, the Ming-Qing period state schools indeed served several important functions:

First, they supervised *sheng-yuan*'s routine work and administered government scholarships or stipends. Second, for poor and secluded districts schools were often the only places which provided library facilities. Third, through schools the state controlled the vast body of undergraduates, or *sheng-yuan*, from whom higher degree holders and officials were selected," [Quoted Ho 1962, p. 171].

In fact, sophisticated integration of the school and examination systems did help to facilitate social mobility in the Ming-Qing period [Ho 1962]. Unfortunately, since all students (at least at an early age), studied for the civil-service examinations, these products of the orthodox education system were, in most cases, stereotyped, conventional men without much capacity for creative thinking.

This tradition, in fact, is rooted in Chinese history. Since the seventh century in Sui Dynasty, Chinese rulers had set written examinations by which applicants could qualify to be high-level civil servants. This system of examination ("Ke Ju") had been an excellent institution for more than a thousand years, and had become increasingly refined. At the same time, however, it had fallen prey to formality. Most of the test questions usually asked one to choose certain phrases from the Confucian canon, to paraphrase and explain them. By the Ming-Qing period, each Confucian classic had been supplied with official annotations, and those became the only accepted interpretation for the examinations. Consequently, examination candidates tried to memorize the entire text of the classics as well as the official versions of the annotations. And thereby they might hope to write their essays in an elegant style, studded with parallel phrases and classical allusions. This was all reinforced by the requirement that an essay had to consist of eight paragraphs, written in the so-called "eight legged" ("Ba Gu") style. At the same time, the brush handwriting had to be in the "small, square style" ("Xiao Kai").

Under such circumstances, it is no wonder that the Emperor Kang Xi and his successor, Yong Zheng, should have emphasized Confucian classics for study in state schools. The principal goal of these Qing state schools thus was to prepare students for the civil-service examinations (and to well-behave themselves in accordance with the the Emperor Kang Xi's 16 rules edicted in

1700, as well as their modification by the Emperor Yong Zheng in 1724) [Yong Ronchun 1985, pp. 450-452]. In fact, as was a long tradition in China and therefore the most valuable, to become a man of virtue was the first priority in Qing state schools.

Nevertheless, one should not ignore sporadic examples of those directors of education who included problems on mathematics in the tests of the provincial examinations in order to encourage students to undertake mathematical studies. For example, Ruan Yuan included mathematics as a subject in the provincial examinations in 1795 when he served as a director of education in Zhejiang province. Ruan Yuan was destined to play a key role in the Qian-Jia school, so his stress on mathematical studies is quite understandable [Cf. Section 1.12]. As a whole, it is no exaggeration to say that mathematics (or any other "Shi Xue" -- substantial learning -- like statecraft) never really had a substantial position in the state schools curriculum, at least at a local level.

As for the central system, few Qing contributions in mathematics can be related to schools like the Imperial Academy or the "Ba Qi Guan Xue", even though the early Qing emperors did pay considerable attention to mathematical studies in these schools. During this period, the Emperor Kang Xi even founded a "Suan Xue Guan" (department of mathematics) in 1713.¹⁰ But this department was really nothing more than a compilation bureau, responsible only for the edition of the voluminous *Lu Li Yuan Yuan* ("Ocean of

Calendrical and Acoustical Calculations"). Although the editors temporarily enlisted Chen Houyao, He Guozong, Mei Juecheng and Ming Antu -- all the noted mathematicians of the time -- they were all sent back to their previous posts once the compilation (and edition) was finished. Nor can it be said that the emperor himself supervised mathematical studies at the "Suan Xue Guan". The emperor did oversee the compilation and edition of the *Lu Li Yuan Yuan*, but he did not teach the best mathematicians of the time, Mei Juecheng and Ming Antu, although he studied mathematics very hard throughout his life. The emperor's mathematics teachers were all Western missionaries like Thomas Pereyral, Jean-Francois Gerbillon, Joachim Bouvet, and Antoine Thomas, who were also asked to help edit the *Lu Li Yuan Yuan* [Li and Du 1987, p. 217]. Pierre Jartoux may also have been summoned occasionally by the emperor so that Mei Juecheng and Ming Antu could learn from him the "Du Shi San Shu" (Three Formulas of Jartoux) [Cf. Sections 1.6 and 3.3; Luo Jianjin 1988]. Unfortunately, the "Suan Xue Guan" contributed almost nothing to the institutionalization of mathematics in the early Qing, since it failed to secure a continuing basis on which mathematicians could depend to support their research and giving them official status as mathematician.

On the other hand, a series of events in the early Qing did help to promote mathematical education. In 1734, the Emperor Yong Zheng edicted to include mathematics as a subject in the curriculum of the "Ba Qi Guan Xue," apparently because he (like his father Kang Xi) paid a great deal of attention to the Manzhou

people. But only four years later, in 1738, the Emperor Qian Long, grandson of Kang Xi, stopped the teaching of mathematics. Instead, he founded a separate institute near the State Observatory for students who were interested in studying mathematics. A year later, Qian Long put the institute under the jurisdiction of the Imperial Academy in 1739. In 1745, the Emperor Qian Long even agreed that 24 astronomical students affiliated with the State Observatory should transfer to the Imperial Academy to study mathematics there [Li Yan 1955, pp. 281-320]. But the most noted mathematicians of the middle Qing like Wang Lai, Luo Shilin, Xiang Mingda, Chen Jie and Xu Youren were not trained at these institutions, although they may have some connection with these institutions early in their careers. For example, Wang Lai taught at the "Ba Qi Guan Xue"; Luo Shilin served as an astronomical student; Xiang Mingda taught at the Imperial Academy before he passed the central civil-service examination; Chen Jie was first an astronomical student and then an assistant at the Imperial Academy. Moreover, this apparently provided an opportunity for Xu Youren to meet Xiang Mingda and Chen Jie while he was serving at the Board of Revenue ("Hu Bu"), one of the six boards of the central government, by passing the central civil-service examination. No wonder they encountered interesting problems of the Western mathematics at these institutions where, apparently due to Kang Xi's enthusiasm for Western mathematics, Western learning ("Xi Xue") had been emphasized since the early Qing. Nevertheless, these mathematicians produced some remarkable results in mathematics in

their later lives and in the milieu of Jiangnan scholarly network whose knots were official academies as well as private academies influenced to some extent by the Qian-Jia school.

Manzhou policy toward private academies was at first very strict. Existing academies were watched very carefully by the early Qing emperors, principally because they knew that the late Ming academies had served as forums for political issues, causing factionalism and even leading to social instability. Therefore in 1652, the Emperor Kang Xi banned the founding of private academies, student demonstration, and gatherings of intellectuals for political purposes [Elman 1984, p. 114]. It was not until 1733 that the Emperor Yong Zheng initiated a new policy for establishing officially controlled academies in the provinces. The reason was that the Qing government recognized the need for educational facilities for a growing number of students. Therefore, the Emperor Kang Xi issued proclamations in 1713 and 1715 sanctioning the establishment of carefully supervised "charity schools" (Yi Xue) by local communities to complement the existing prefectural and county schools in each province. The remarkable growth in the number of charity schools during the 18th century reflected this attempt by the emperor to replace the private academies with officially controlled charity schools. Thus after 1733 the newly-founded academies were modeled after the officially controlled charity schools, which had already proven successful in avoiding the divisive factionalism that had permeated the late Ming academies [Elman 1984, p. 119]. Relying

on a combination of local support and official supervision, the Manzhou authorities hoped to prevent the recurrence of the activist academies that had troubled late Ming rulers. Moreover, Qing provincial officials set their own standards in choosing the directors of the academies. Consequently, the scholars who qualified to serve as directors were versed in the classics or neo-Confucian philosophy and well-behaved in moral norm. Yet to behave in moral norm was only secondary to teaching at most academies in the Qing period. The major concern of the students, similar at every level of the state schools, was to prepare for the civil-service examinations. Thus it is no wonder that the teachers of both official and private academies basically taught the students to concentrate on writing the "eight-legged" essay and to write in brushstrokes of "small, square style" [Yong Rongchun 462-466; Liu Zhaobing 1979, pp. 114-116]

There were, of course, a few exceptions where local academies did pay attention to science and mathematics as well as the classics, and history. For example, (as mentioned in Chapter 1), there was the Youjiao Academy (at Jiangxi) supervised by Wang Chang [Cf. Section 1.12], as well as the Gu Jing Jing She (at Hangzhou) and Xue Hai Tang (at Guangdong) founded by Ruan Yuan [Cf. Section 1.12]. A more interesting example is Xiang Mingda, who taught mathematics at the Zhaonan Academy (in Hangzhou) after retiring from his official post [Cf. Section 1.12]. Yet all of these examples must be considered in terms of the intellectual and socio-economic factors that had already nourished the Qian-Jia

school. Above all mathematics, now assumed to play an indispensable role in the textual criticism of the classics, was regarded as an acceptable, respectable and specialized field. Armed with such interest, mathematicians of the Qian-Jia period devoted themselves to mathematics as the "lesser learning" and began therefore to seek employment as "Mu You" (secretarial staff) to officials, as tutors in wealthy families, and as academic teachers in the Jiangnan area, the economic and cultural center of China from the 18th century on [Cf. Section 1.12].

Even so, the Qing academies had largely ignored the normal functions an educational institution should fulfill, although their continued significance for the elite was reflected in their vast numbers -- at one estimate, about 4500 in the nineteenth century [Hao Chang in Fairbank *et al.*, eds., 1980, p. 330]. Under such circumstances, the late Qing education reformers naturally came to focus on reshaping the local academies. For example, in 1896 the Jiangxi governor Hu Pingzhi presented a memorial to the throne pointing out what was being taught at the academies had nothing to do with the substantial studies like statecraft, science and mathematics. He therefore suggested that the number of students should be decreased, class hours for the "eight-legged" essay should be reduced, and new teachers should be hired to teach the courses like the classics, history, mathematics, astronomy, geography, agriculture, military strategy and tactics, along with all other disciplines having practical uses. His proposal suggested that students should study each discipline

separately so that each discipline might be explored in detail. To meet such demands, Hu Pingzhi expected scholars who were well-versed in mathematics would be appointed as teachers at the academies to encourage the students studying mathematics, science or other "sustantial studies". By such a reform of the academies Hu Pingzhi believed that the academies could work to much better effect and at the same time, their ineffective practices could be eliminated [Hu Pingzhi quoted in Zhu Youxiang 1983 vol.II, pp. 155-157].

Even before the throne approved his proposal, Hu Pingzhi had already invited Tu Renshuo, a retired censor official, to teach mathematics and science at the provincial Lian De Academy. According to Hu Pingzhi, Tu Renshuo (during his service at Beijing) had studied mathematics and science under Li Shanlan. Thus the basic courses like trigonometry, surveying, algebra and geometry which Tu Renshuo gave for students at the Lian De Academy doubtless reflected those of the Tong Wen Guan [Hu Pingzhi quoted in Zhu Youxiang 1983 vol.II, pp. 155-157]. Unfortunately, due to the unavailability of any further infomation, it is not possible to say how Li Shanlan actually influenced Tu Renshuo and his teaching at the Lian De Academy.

Nevertheless, some evidence may be gleaned from the Wei Jing Academy (at Shanxi province), the Nan Jing Academy (at Jiangsu province) and Liang Hu Academy (at Hunan province), which provide some clues about how Li Shanlan and his teaching at the Tong Wen

Guan influenced the reform of the late Qing academies.

Let us start with the Wei Jing Academy. In 1885, a study room ("Qiu You Zhai") was added to the academy to encourage primarily the study of the classics, history, neo-Confucian philosophy and politics. Yet disciplines like astronomy, geography and mathematics were also emphasized. Two years later, Liu Guangfen (1843-1903) took over the directorship of the academy and began to stress mathematics teaching. He considered mathematics to be so useful to the country that students should not look down on it as an inferior commercial art. With this in mind, he replaced the courses being offered only in preparation for the civil-service examination by mathematics courses, and required the students to answer questions on mathematics one day each month [Liu Guangfen quoted in Zhu Youxiang 1983 vol.II, pp. 380-382]. How Li Guangfen came to be proficient in mathematics is unknown, although it is known that he held a "Ju Ren" degree, by having passed the Shanxi provincial civil-service examination and served as the "Xue Zheng" (the fifth rank of the professorship) at the Imperial Academy [Li Yan 1955, pp. 321-330]. His student Zhang Bingsu was later quite successful in studying mathematics at the Wei Jing Academy.

In addition to his enthusiasm for Li Shanlan's *Huo Qi Zheng Jue* ("On the Projectile Motion of the Cannon") [Cf. Section 3.7], Zhang Bingsu also wrote a supplement of his own, the *Dai Wei Ji Shi Ji Bu Cao*, giving additional demonstrations for Li Shanlan and

Wylie's translation of Loomis' *Calculus*. "Since the *Dai Wei Ji Shi Ji* merely gives answers for its examples without adding further necessary demonstrations," as Liu Guangfen put it, "so Zhang Binggsu's supplementary demonstrations as well as errata to it would have proven that the talent of our province-fellow is not below the level of the Westerners. Those who have a noble aspiration for studying mathematics should take Zhang Bingsu as an example and thus would see no difficulty in it," [Quoted in Li Yan 1955, p. 327]. Also worth mentioning here is Liu Guangfen's "Poscript" to Zhang Bingsu's *Ying Nu Gou Gu Hu Qiu Gong Shi*, in which he praised Zhang Bingsu for so capably synthesizing Chinese and Western methods into one method in this treatise [Liu Guangfen quoted in Li Yan 1955, pp. 329-330]. Thus it is evident that Li Shanlan's teaching at the Tong Wen Guan together with his ideal of synthesizing Chinese and Western methods was clearly echoed at the Wei Jing Academy.

By contrast with the Wei Jing Academy, the founding of the Liang Hu Academy in 1897 and the reshaping of the Nan Jing Academy in 1898 were the natural outcome of the late Qing education reform policy of 1896. According to the historian Hao Chang, restructured academies and new schools began to be set up by gentry-literati, at first sporadically and then in increasing number after 1895. All these measures received due support from the provincial officials like Zhang Zhidong, Sheng Xuanhuai and Jiang Biao. Even more important than these local efforts were a number of memorials submitted to the court in 1896 asking for the

general establishment of new schools. Among these Li Duanfen's was the most convincing, for he suggested that the reshaping of traditional academies through curriculum revision was the most practical way to set up new schools. This proposal was accepted by the court and became governmental policy. In response, academies in many areas throughout the empire were reported to have made the suggested changes, and significant efforts were also made to establish new schools [Hao Chang in Fairbank *et al.*, eds., 1980, p. 330].

Apparently it was in this context that Zhang Zhidong founded the Liang Hu Academy in 1891 when he served as the governor-general of two provinces Hubei and Hunan. As for the curriculum, he set up four departments: classics, history, geography and mathematics, respectively. According to the historian Su Yin-Fong, no mathematics instructor was appointed until Hua Hengfang took charge the mathematics teaching in 1892 [Li Yan 1955, p. 374]. When revising the curriculum in 1897, Zhang Zhidong was most concerned with the teaching of mathematics, apparently because he realized that "mathematics must be made quite clear, step by step, so that the students are then able to go further. Thus to the teachers the teaching of mathematics is more laborious than other disciplines," [Zhang Zhidong 1897 quoted in Zhu Youxiang 1983, vol.II, p. 394]. With this in mind, Zhang Zhidong wrote to Hua Hengfang (1833-1905) in a letter of 1897 that: "Since a mathematics course has been required for each student of the Liang Hu Academy since its curriculum revision, ...

I intend to appoint a mathematics assistant to help with your teaching. Yet I think the right man should be versed in algebra and calculus so that he is able to share your labor. If you already have such a candidate, please let me know as soon as possible," [Quoted in Su Yun-Fong 1976]. Clearly Hua Hengfang was mainly responsible for the mathematics teaching at the Liang Hu Academy. Consequently, it was highly probable that Li Shanlan influenced the Liang Hu Academy through Hua Hengfang's teaching there, due to the strong influence Li Shanlan had upon Hua Hengfang (1833-1905), especially in his mathematics research and teaching.

Finally, there was also the Nan Jing Academy. The Nan Jing Academy was founded in 1883 and emphasized mainly the classics. During the 100 days of the Reform Movement in 1898, the Qing government reshaped each provincial academy into a modern school with courses including both Chinese and Western studies. The Nan Jing Academy was restructured into a high school ("Gao Deng Xue Tang"). Unfortunately, with the empress dowager's *coup d'etat*, educational reform, among others, was immediately stopped. Thus the reform of the Nan Jing Academy was not truly carried out. Just as many others, it was reformed in name only. Yet according to Hao Chang, in the process of modernizing Chinese education after 1896, no attempts were made to reverse changes already made [Hao Chang in Fairbank *et al.*, eds., 1980, p. 330]. Thus it is of interest to see how significant Li Shanlan's mathematics was and how his teaching played a role in the Nan Jing Academy during this

very transitional period.

In fact, mathematics teaching at the Nan Jing Academy had already begun before its reform in 1898. Based on the *Nan Jing Wen Cao* ("Archive for the Nan Jing Academy", 1901), the academy's mathematics teaching can be described from its students exercises. For example, consider the following four problems assigned to students:

- 1) Explain why calculus is complementary to algebra.
- 2) Review Li Shanlan's *Dui Shu Tan Yuan*.
- 3) In his *Fang Yuan Chan You*, Li Shanlan asserted that every power of x can be interpreted as a line, surface or solid in a cyclic way [Cf. *Dan Zhi* 3 of the *Fang Yuan Chan You*]. Yet Mr. Yan of Tongxiang (in Hunan province) has recently objected this. Is he correct or not?
- 4) Both Li Rui and Li Shanlan gave methods for constructing Pythagorean triplets. Now, if we want to construct a right-angled triangle whose "Gou", "Gu" sides and side of its inscribed square are all integers, what method you will use?

[Quoted in Zhu Youxiang 1983 vol.II, p. 423]. These problems all pointed to Li Shanlan's mathematics and his teaching, indicating that Li Shanlan's example at the Tong Wen Guan was followed faithfully by the Nan Jing Academy at this time. There is also other evidence that Hua Shifang (1854-1905) began to teach

mathematics at the Nan Jing Academy from 1897 on, and he (like his elder brother Hua Hengfang) was also enthusiastic about Li Shanlan's mathematics and translations. At the beginning of the 20th century, Li Shanlan's mathematics teaching and his underlying ideal of synthesizing Chinese and Western mathematics, served to smooth the path for the modernizing Chinese modern mathematics education in China.

It is also relevant to say something here about the mathematics textbooks used in late Qing new schools. After 1898, the need for the textbooks to teach mathematics at the primary and secondary schools was so urgent that the mathematics textbooks used in the Dengzhou Wen Hui Guan (in Shandong province), a church school founded by the American missionary Calvin Mateer, were widely adopted. These textbooks included the *Bi Suan Shu Xue* ("Pen Calculation", written jointly by Mateer and Zou Liwen in 1892), the *Dai Shu Bei Zhi* ("Outline of Algebra", written by Mateer, jointly translated by Zou Liwen and Sheng Fuwei in 1891), the *Xing Xue Bei Zhi* ("Elements of Geometry", written by Elias Loomis, freely translated by Mateer, Zou Liwen and Liu Yongxi in 1885). On the other hand, books like the *Ba Xian Bei Zhi* ("Elements of Plane Trigonometry", written by Elias Loomis, freely translated by the American missionary Peter Parker and edited by Xie Honglai in 1894) and the *Dai Xing He Can* ("The Elements of Analytical Geometry", written by Elias Loomis, freely translated by Parker and edited by Xie Honglai in 1893), were also popular mathematics textbooks of the time [Li and Du 1987, pp. 264-265].

Doubtless, a westernizing tendency in mathematics textbooks was inevitable, even though the textbooks mentioned above still represented unknowns and coefficients in algebraic equations like those expressed in the older Chinese translations of De Morgan's *Algebra* and Loomis' *Calculus*, in which Li Shanlan used the the Heavenly Stems ("Tian Gan") "Jia", "Yi", "Bing", "Ding," etc., respectively, for the coefficients a, b, c, d, etc., and for the unknowns x, y, z, u used the "Tian Yuan", "Di Yuan", "Ren Yuan", "Wu Yuan" respectively. In fact, the most popular mathematics textbooks collected in the *Dong Xi Xue Shu Lu* ("Bibliography of the Eastern and the Western Studies", 1899) were those translated from the West as well as Japan. Among the list, Li Shanlan's translations like the *Ji He Yuan Ben* (Euclid's *Elements*), the *Dai Wei Ji Shi Ji* (Loomis' *Calculus*) and the *Yuan Zhui Qu Xian Shuo* ("On Conic Sections") were still included [Li Yan 1955, pp. 312-320]. His favorite mathematical classic -- Li Ye's *Ce Yuan Hai Jing* -- as well as his own mathematical treatises like the *Fang Yuan Chan You* and *Dui Shu Tan Yuan*, nevertheless seem to have disappeared from schools. Apparently, traditional Chinese mathematics was no longer central to mathematics education in late Qing.

5.7. Professionalization of Chinese Mathematics

Along with the modernization or Westernization of Chinese mathematics education, mathematical research based on traditional subjects also faded out from the scene. For example, at the

beginning of the 20th century, Zhou Da (1878-?), one of the major mathematicians in the period 1900-1920, was still enthusiastic about Li Shanlan's "Duo Ji" method, yet later his interest changed to Western mathematics in his late career [Li Di et al., eds., 1985, p. 525]. In fact, the shift of Zhou Da's research focus is best appreciated in conjunction with the key roles mathematicians trained abroad began to play in mathematics research and education after the Xin Hai Revolution of 1911. At the same time, mathematics departments in the newly founded universities also provided an institutional base for "professional" mathematicians. For example, there were as many as 155 professors teaching in mathematics departments established in the thirty-two universities in 1932 [Li Di et al. eds., 1986, 538-540]. By then, Chinese mathematicians were completely professionalized. Consequently, a new era of modern Chinese mathematics was under way.

Even so, the professionalization of mathematics (among other disciplines), should not be regarded as simply a reaction to challenges from the West. According to historian Ssu-Yu Teng, "Nationalism, partly dictatorship, the cult of the masses, the worship of technology, the leadership of youth, and emancipation of women -- all these are new elements inspired mainly by Western contact," [Quoted in Teng 1971, p. 2]. But this was not the case with the professionalization of mathematics -- which was above all nourished in China thanks to the Qian-Jia school.

For the Qian-Jia school, emphasis upon disciplinary

specialties preceded any concern with specific academic roles or "professionalization" in China. More specifically, "The transformation of occupational criteria from possession of a broadly based range of intellectual and political concerns to possession of expertise in precise fields of evidential scholarship signals a movement toward profession," [Quoted Elman 1984, p. 99]. How did the Qian-Jia school come to appreciate the importance of specialized knowledge?

The Qian-Jia school's stress on the specialization can be traced back to its early leaders. For example, Wang Chang urged students at the Youjiao Academy to specialize in one field before they strove for broad scholarship [Cf. Section 1.12]. More specifically, he even requested the students "to study first one Classic then all the others; to study first one history then all the others." Besides, astronomy, mathematics, "Pian Ti Wen" (rhythmical prose characterized by parallelism and ornateness), poetry, "Ci" (poetry written to certain tunes with strict tonal patterns and rhyme schemes, in fixed numbers of lines and words), Philology, phonology, "Xun Gu" (critical interpretation of ancient texts), and epigraphy were, according to Wang Chang, other disciplines that required specializations. Such enthusiasm for specialization, when coupled with the strong socio-economic climate afforded in the Jiangnan area, the Qian-Jia school (for the first time in Chinese history) could appreciate mathematics, among other special disciplines, as knowledge for its own sake [Cf. Section 1.12]. Consequently, the late Qing mathematicians,

just like evidential scholars, "attained and maintained their positions within this community by virtue of the protection and patronage of broader elite segments of Confucian society that had been persuaded that there was some special value in their work," [Quoted Elman 1984, pp. 99-100].

Thanks to this legacy of the Qian-Jia school, Chen Li (1810-1882), a noted Guangdong scholar who was familiar with Zou Boqi, Wu Jiashan and Li Shanlan, still emphasized specialized study as late as the 1850s, even though the Qian-Jia school started to decline [Elman 1984, pp. 231-253]. In a letter to his friend Li Zhenbo, Chen Li maintained that "'Xun Gu', phonology, mathematics, calendrical sciences, geography, etc., are all Confucian tasks, yet one must become specialized in each of them in order to become a proficient scholar. This unfortunately is what I have never been able to do. Therefore I hope that my younger brothers and my friends will choose one discipline to specialize . . .," [Quoted in Wang Zhongyan, p. 68].

Chen Li's example, however, was not typical among Qing literati in the 1850s. It is all the more unlikely that Chen Li was a graduate from the Xue Hai Tang (in Guangdong), a noted academy founded by Ruan Yuan. But the ethos of professionalization -- committing oneself to only one single specialized field throughout one's entire life -- was something created by the Qian-Jia school [Liang Qichao 1977, pp. 77-80]. This was so clear that even in 1875, Zhang Zhidong had already

understood this. Thus in his *Shu Mu Da Wen* of the same year, Zhang Zhidong put Qing scholars into many different categories of specialists whose fields were the Classics, histories, neo-Confucian philosophy, philology, mathematics, textual criticism, epigraphy, paleography, "Pian Ti Wen", poetry, "Ci Xue" and statecraft. In addition, he also provided a special category for those scholars who were proficient simultaneous in the Classics, histories and neo-Confucian philosophy. However, he reminded his students that scholars specialized in several fields were quite rare at the time [Zhang Zhidong 1880 Book 209]. Apparently for this reason, Zhang Zhidong encouraged each student at the Zun Jing Academy (in Sichuan province) to become a specialist by choosing an appropriate discipline in accordance with his own disposition. In his "Chuang Jian Zun Jing Shu Yuan Ji (On the Founding of the Zun Jing Academy)" (1875), Zhang Zhidong recommended a list of disciplines for the students' career choices, which included the Classics, histories, philology, geography, astronomy, mathematics, statecraft, poetry and paleography. Clearly, Zhang Zhidong hoped his students would become scholars or intellectuals who were primarily good at some specialized branch of knowledge [Zhang Zhidong 1880 Book 213].

Due basically to this emphasis upon specialization, the role of late Qing mathematicians gradually came to differ from those of the Classicists and other specialists. This phenomenon again did not escape the attention of Zhang Zhidong. In his *Shu Mu Da Wen*, Zhang Zhidong had put the "Tan Tian San You" (Three Friends on

Astronomy and Mathematics) Jiao Xun, Li Rui and Wang Lai into several categories -- that of "mathematician" (Suan Xue Jia) as well as the category of the "Classicist who is proficient in Han Learning or evidential research" (Han Xue Zhuan Men Jing Xue Jia). In other words, the process of the specialization was not yet complete in the cases of Jiao Xun, Li Rui and Wang Lai by 1820. However, among scholars of the period 1820-1870 put into the category of "mathematician", Shen Qinpei, Da Xu, Li Yinnan, Dong Youcheng, Xiang Mingda, Luo Shilin, Xu Youren, Xia Luanxiang, Zou Boqi and Li Shanlan, only Dong Youcheng was given another title, namely, "Pian Ti Wen Jia" (the scholar who is proficient in the "Pian Ti Wen") [Zhang Zhidong 1880 Book 213]. Therefore, by the 1870s the role of the mathematician as a specialist had clearly been established in the Qing academic or scholarly community.

Such differentiation of mathematicians from the Classicists apparently provided an opportunity for the mathematical community to operate on its own autonomously [Porter 1982]. Yet it also means that mathematicians like Li Shanlan were not able as a result to share the academic privileges and social resources available to Classicists. Consequently, mathematicians, like other specialists, had to make their livings on their own. Fortunately, among Li Shanlan's careers, namely, he served as tutor for the wealthy family Lu-Fei at Jiaying, worked on the secretarial staffs ("Mu You") of Xu Youren and Zeng Guofan, was hired as a translator at the LMS Press in Shanghai, and taught as a mathematics instructors at the Tong Weng Guan. Only his role as

a translator was new for the scholars of the Qian-Jia school. Late Qing mathematicians, like scholars in the Qian-Jia period (1736-1820), evidently were able to make a living on their specialties. Their existence might be humble, but it was enough for a mathematician to be able to commit himself to his research. In fact, from the 19th century on, Qing mathematicians in the Jiangnan area, especially in provinces such as Jiangsu, Zhejiang and Anhui, sought out other mathematicians as mentors, acquaintances or collaborators [Wang Ping 1966, p. 108; Porter 1982]. Therefore, mathematics was gradually differentiated from humanistic knowledge, and the self-image of mathematical activity, as reflected in Li Shanlan's letter to Fang Yuanzheng [Cf. Section 2.6], was reinforced by mutual recognition and cooperation among mathematicians in their common enterprise. Something like an ethos of specialization or professionalization eventually came to the fore among mathematicians who informally constituted a mathematical community which was not yet organized by such formal institutions as the Tong Wen Guan, Guang Fang Yan Guan (the Shanghai Tong Wen Guan) or the few other academies then open in China [Lian Qichao 1977, pp. 77-80; Porter 1982].

By the early 1860's, yet another force also served to accelerate the differentiation of mathematics from humanistic knowledge. This was the importance of mathematics for the "Self-strengthening Movement", as proclaimed by its early advocates like Li Hongzhang and Zeng Guofan. This too helped to promote the social status of mathematicians. For example, since

mathematicians as well as artisans were helpful in promoting the adoption of Western technology to build shipyards and arsenals -- a major concern of the early advocates of the "Self-strengthening Movement" by the middle 1860s -- the urgent demand for such specialists including mathematicians further helped to justify their social status. This is doubtless why Li Shanlan, Zhang Siqui, Xu Shuo and Hua Hengfang were invited to join Zeng Guofan's headquarters at Anqing [Cf. Section 2.6]. By then Zeng Guofan -- a true believer in Confucian moral philosophy -- had begun to realize that Western-made gunboats alone were not enough to strengthen the power of the country. Something more had to be done. According to Zeng Guofan, "Scholars of broad intellect as well as artisans who are skillful and ingenious should be recruited, first to practise the use of [Western ships and guns] and later to make them," [Quoted from Kuo and Liu in Fairbank ed. 1978, p. 496]. Thus in 1863 among the secretaries who numbered no less than a hundred in Zeng Guofan's "Mu Fu" (the private secretarial bureaucracy) of his Anqing headquarters "there were expectant officials, learned scholars, lawyers, mathematicians, astronomers and machinists; in short, the picked and noted men of China were all drawn there," [Quoted Yung Wing 1909, p. 148].

Under such historical circumstances as these, the social status of mathematicians thus came to be assured. This was further strengthened by Zhang Zhidong's positive attitude toward mathematicians and other specialists in the 1870s. As mentioned above, Zhang Zhidong certainly approved of such specialization in

academic learning, although in his own *Shu Mu Da Wen* he did not recommend any Western texts on mathematics or science. At the same time, mathematicians too seemed to have been encouraged by new institutions like the *Tong Wen Guan* and *Guang Fang Yan Guan*, where teaching posts for mathematics had been created, if only a few. Nevertheless, a wider institutional base for the professionalization of mathematics, apart from the Imperial Academy ("*Guo Zi Jian*"), was beginning to form. Still another factor contributing to the institutionalization of mathematics in the 1890s was the reform of the traditional local academies. Although by then mathematicians were still only a small minorities among late Qing scholars, they nevertheless were aware of the significant role which they were beginning to play. But some were not entirely convinced, and among the sceptics, the late Qing scholar Qiang Ruxun's reserved criticism should be mentioned: "Mathematics is the discipline on which the calendar is to be based. Therefore, in previous generations the accuracy of calendars has been dependent of the advancement upon mathematics. Moreover, some explications of the Classics also have to rely on mathematics. On the other hand, one's daily life cannot keep going without mathematics. All these point to the fact that mathematics cannot be dispensed with. Nevertheless, scholars who are proficient in this art very often exaggerate its first priority among the learnings of the time. Would it be an adequate attitude?" [Quoted Qiang Ruxun 1898, Book 5]

Also wary of too much enthusiasm of mathematicians for

mathematics, Hua Hengfang voiced reservation as well. In his *Xue Suan Bi Tan* ("Notesbook on Mathematical Learning", 1882), Hua Hengfang warned readers not to pay too much attention to mathematics as they began to study the subject, for in working too hard they might exhaust their vitality, damage their own health or even die at an early age!¹¹ This would indeed be regrettable, especially without having left behind anything about mathematics [Hua Hengfang 1882, Book 5]. Hua Hengfang seemed to disapprove of the specialization of mathematics. He thought that "Mathematics is nothing but one of the six inferior arts, so those who undertake the study of this discipline need not pay all of their effort to it. Instead, it would be enough to take it as an enjoyable art. In the past, people played chess in their leisure time, while today we play "Ma Jiang"¹² game to kill time. If one is able to take the time otherwise play chess or cards to study mathematics, paying attention to mathematics instead of game, then doubtless one will not fail to become a noted mathematician of his time," [Quoted in Hua Hengfang 1882 Book 5, pp. 14-15].

In contrast, Li Shanlan's commitment to mathematics was of this latter sort, a serious and total commitment. "It has been said, however, that Li Shanlan has spent most of his life in a blind alley, and has apparently choosen a wrong subject field, namely, mathematics." But to such criticism of Li Shanlan's professionalized concentration on mathematics, Martin added his own comment that "This remark must have proceeded from those who do not understand the significance and the importance of

mathematics," [Quoted from Martin 1877]. Clearly Martin approved of Li Shanlan's professional attitude toward mathematics. Similarly, Li Shanlan's letter of 1867 to Fang Yuanzheng [Cf. Section 2.7], reflected the self-image of a "professionalized" role that Li Shanlan was playing in the late Qing mathematical community.

Even so, both Hua Hengfang and Martin's observations indicate that the late Qing scholarly community did not unanimously approve of "specialized" or "professionalized" mathematics. The late Qing mathematical community was informally organized. Yet the autonomy it derived from professionalization seems to have left it with only very limited resources.¹³ Under such circumstances, the mathematical community from 1860 onwards could not have operated on its own in an independent and autonomous way had it not still overlapped to some extent with the community of Classicists [Horng 1989c]. This may seem ironical but, just as Elman's explanation quoted above, late Qing mathematicians attained and maintained their social status by virtue of the protection and patronage they received from the community of the Classicists.

Nevertheless, as the curriculum controversy of the Tong Wen Guan in the 1860s demonstrates, Beijing scholar-officials (especially the neo-Confucians) apparently did not appreciate the social status already held by mathematicians of the Jiangnan area. On the other hand, the statecraft school, which emerged to dominate academic education in Hunan and Guangdong in the early

19th century, did not pay high respect to mathematics per se, although the scholars of this school regarded mathematics as closely relevant to their learning of statecraft.¹⁴ Feng Guifen's suggestion that the "Xi Xue" (Western studies) was nothing but "Suan Xue" (mathematics) seems to have emphatically pointed out the significance and relevance of mathematics to the "Self-strengthening Movement" in the early 1860s. However, for Feng Guifen and Wei Yuan, the two major figures of the statecraft school, "Western learning was given a value -- but only an instrumental and a secondary one, whereas traditional Confucian learning was still regarded as the lofty locus of intrinsic and basic value," [Quoted Hao Chang in Fairbank *et al.*, eds., 1980, p. 283]. It was not until the 1890s that late Qing scholars and intellectuals began to realize that the West was a source not only of instrumental and secondary but also of essential and central values.¹⁵ By the same token, the academic status of mathematics seems not to have been recognized by all Qing scholars and intellectuals until the late Qing reformers of education succeeded in accommodating Western learning as well as mathematics in the curriculum.

A significant part of the curriculum revision was a tendency to accept Western knowledge under the category of "Shi Xue" (substantial studies). "This had been a principal category in Confucian intellectual culture. It referred to studies with practical significance for the central and moral concerns of Confucianism. Over the centuries 'Shi Xue' had been repeatedly

invoked by Confucian scholars as a controlling category to draw the line and refute what they considered devoid of 'substance' and therefore worthless. The fact that in the new curriculum Western knowledge was given a prominent place under the category of 'Shi Xue' indicated the recognition of 'Western thought' as having a place on the traditional scale of intellectual values," [Quoted Hao Chang in Fairbank *et al.*, eds., 1980, p. 283]. In light of the fact that by the early 1890s mathematical texts still predominated among translated Western texts, it is worth asking how mathematics was thereby professionalized.

Here an important issue concerns the "Zhong Xi Hui Tong (Making the Chinese and Western learnings equivalent)" which eventually helped make mathematics professionalized partly because the rank of mathematics was raised to the level of Western studies in the late 1890s. The issue of the "Zhong Xi Hui Tong" can be traced back to the late Ming and early Qing periods. From then on, the Chinese scholars endeavoured to accommodate Western theories and methods, trying to assimilate new elements from the West. The first step, apart from coining appropriate terms for translation, was to make the (introduced) Western theories and methods "equivalent" to their Chinese counterparts.¹⁶ The most noted example was Wei Wending's remarkable achievement in mathematics, most of which was related to his claim that "Western Geometry is equivalent to Chinese 'Gou Gu' ('Ji He Ji Gou Gu')". Thereafter Mei Juecheng, a grandson of Mei Wending, provided "firm" evidence in his *Chi Shui Yi Zhen* to support the "foundation

myth" that Western mathematics and astronomy actually evolved from certain techniques that originated in China and were later transplanted to the extreme fringes of civilization. The evidence he gave in the treatise included examples illustrating how the Chinese "Tian Yuan" method was essentially equivalent to the newly introduced Western "Jie Gen Fang Bi Li" method for solving algebraic equations drawn from both Chinese and Western texts on mathematics [Cf. Section 1.4]. The "Zhong Xi Hui Tong" was still a significant task for mathematicians like Mei Juecheng in the early 18th century. In the Qian-Jia period, the major focus on mathematics was shifted to the revival of ancient texts, although scholars of the time still paid attention to the "Zhong Xi Hui Tong" due apparently to their approval of the "foundation myth" [Jiang Xiaoyuan 1988].

Consequently, mathematicians from the 1840s onwards were able to assimilate more quickly than their late Ming and early Qing predecessors new Western mathematics and science introduced to China [Li and Du 1987, pp. 255-256]. Even so, the issue of the "Zhong Xi Hui Tong" remained a disturbing problem at the time. For example, Liu Yicheng, while recalling his teaching of mathematics at the Shanghai Guang Fang Yan Guan as well as at the Qiu Zhi Academy in the period 1860-1890, regretted noting that "the students who concentrated on Chinese methods always got entangled with Chinese literary problems of the texts and were therefore unable to give concise reasonings for their solutions for the test questions. On the other hand, the students who used

the Western methods like algebra very often only demonstrated the procedures of computations and never gave any relevant reasonings," [Quoted Liu Yicheng 1899, "Preface"]. Under such circumstances, Li Shanlan's deep concern for the synthesis of Chinese and Western methods in his teaching of mathematics at the Tong Wen Guan can be best explained principally because the task of the "Zhong Xi Hui Tong" had yet to be accomplished [Cf. Section 5.1]. For Luo Shilin, the last important figure of the Qian-Jia school, the "Tian Yuan" method was basically a means of reviving the ancient Chinese mathematical texts. In contrast, Li Shanlan in the 1870s made use of the "Tian Yuan" method primarily because of the issue over "Zhong Xi Hui Tong". More significantly, Li Shanlan seems to have provided a rationale for further studies on the Western mathematics and science, expanding the possibilities for the advocates of the "Self-strengthening Movement" to run more smoothly the programs like teaching mathematics and astronomy at the Tong Wen Guan in Beijing.

By 1880's Zhang Zhidong, the last major advocate of the "Self-strengthening Movement", went far beyond the "foundation myth" to maintain that "If the 'Xi Zheng' (Western politics) and 'Xi Xue' (Western studies) would be of help to China but do nothing to promote Christianity in China, then it would be improper to refute them simply because they cannot be justified in terms of native Chinese cultural elements, let alone in the work of those who have drawn upon evidence from the ancient classical texts," [Quoted Zhang Zhidong 1880, Book 203]. Such an attitude

toward Western studies indeed surpassed that of the Qian-Jia school. However, like most scholars and intellectuals of the time, Zhang Zhidong's approach to Western studies was pragmatic and utilitarian -- they could be learned as long as they were separable from their underlying Christian culture. By the 1890s Zhang Zhidong does not seem to have taken a different approach to Western studies. Thus in his *Chuan Xue Pian* ("Exhortation to Study", 1898), he encouraged scholars and intellectuals to learn "selected" Western studies, yet at the same time he urged scholars and intellectuals to uphold orthodox Confucianism. His noted slogan -- "Zhong Ti Xi Yong" (Chinese learning for the fundamental principles, Western learning for practical applications) -- eventually took shape [Zhang Zhidong 1880, Book 202; Teng 1971, pp. 164-174].

By modern standards, or even in the eyes of his contemporary reformers, Zhang Zhidong's "Zhong Ti Xi Yong" slogan was a deliberate if conservative departure from the innovations of the leaders of the Reform Movement in 1898, Kang Youwei (1858-1927) and Liang Qichao. Even worse, as some historians have criticized, he actually betrayed the ideal of the Reform Movement in which he was somehow involved earlier [Xue Huayuan 1987, pp. 201-203]. Whatever the case, he survived after the failure of the Reform Movement to become the leader of the so-called "conservative reform" in the beginning of the 20th century. By then he was even more influential. For example, in 1903 Zhang Zhidong along with two other high officials proposed the "Zhou Ding Xue Tang Zhang

Cheng", a revision of the "Qin Ding Xue Tang Zhang Cheng (Reform of Educational Institutions)". Thus modern educational institutions were established [Li Yan 1955, pp. 307-310].

Even so, Zhang Zhidong's notorious slogan "Zhong Ti Xi Yong" -- a natural conclusion of sorts to the different lines of thinking about how the rationale of learning from the West could be established -- was interwoven with the final stage of modernization of Chinese education [Xue Huayuan 1987, p. 231]. In fact, it played a significant role as well in the reform of local traditional academies. According to the governor of Jiangxi, Hu Pingzhi, the only way to reconcile those only versed in Chinese learning with those who paid undue appreciation to Western learning was to reform the academies which, unlike the church schools founded by Western missionaries, were the strongholds of the orthodox Confucianism. In such cases, by revising their curricula to include mathematics and science, as well as Western learning, Hu Pingzhi expected the students would uphold the orthodox Confucianism and become proficient in Western learning at the same time [Hu Pingzhi in Zhu Youxian 1986 vol.II, p. 156].

Thus mathematics eventually found both intellectual justification and an institutional footing in the reformed academies in the late Qing. By the end of the late 1890s such views as "Mathematics, astronomy and science were no heterodoxy, so why would they be against the orthodox Confucianism?" of course was quite rare, yet doubtless no one would dare to criticize

mathematics education now. On the other hand, among proposals for the establishment of modern schools after the Sino-Japanese War of 1895, the training of specialists was duly emphasized. For example, in a memorial presented to the throne in 1896, Li Duanfen, a ranking official in the court and a close associate of Kang Youwei and Liang Qichao, pointed out that the new schools (founded in the period 1866-1896) like the Tong Wen Guan, Guang Fang Yan Guan, Shui Shi Xue Tang (Naval school at Tianjin), Zi Qiang Xue Tang (at Hubei), etc., failed to train specialists. And the reason, Li Duanfen continued, was that all of these schools (except for the Zi Qiang Xue Tang) did not have departments for specialized fields [Li Duanfen quoted in Zhu Youxian 1986 vol.II, pp. 484-488]. In the same year, Liang Qichao in his essay, "Xue Xiao Zong Lun (On School)", also stressed that "One would not be proficient in any specialty without differentiation of the specialized fields from one another," [Quoted Liang Qichao in Zhu Youxian 1986 vol.II, p. 482]. It was evident that all these concerns about specialized fields favored a clear tendency toward the professionalization of mathematics as well.

Liang Qichao, in the same essay, also referred to Ernst Faber's *De Guo Xue Xiao Lun Lue* ("On German School Systems"). However, he seems to have ignored Faber's motive for writing this book -- in his own "Preface", apart from that by Li Shanlan [Cf. Section 4.5], Faber was careful to remind his readers that Western technology could not be separated from Christianity. That was because, as he pointed out, "Technologies are only streams while

Christianity is the origin of water." Therefore he hoped that Chinese readers, by reading through this book, would realize that "The Western countries are not merely good at technology. ... Technology can only be developed by means of science, and science has been cultivated in the context of Christianity. No country that does not follow a similar way to develop science and technology can ever flourish," [Quoted Faber 1873, "Preface" in *The Church News*, Book 6, p. 146].

Yet as one of the major leaders of the Reform Movement of 1898, Liang Qichao should certainly have recognized that the values of Western learning were reflected not only in its technological or instrumental aspects but in some fundamental or intrinsic principles as well. Due largely to this realization, the "Self-strengthening Movement" (1860-1890) -- which had aimed primarily at innovations in technology -- realized that China had failed to pass the test of the Sino-Japanese War of 1895. Liang Qichao, among other intellectuals, understood that that a superficial copy of Western technology simply was not enough to strengthen the power of the country. By contrast, he discovered that the Meiji Restoration in Japan learned not only its technology from the West but new values in political ideals and institutions. Meanwhile, Chinese intellectuals of the time also began to note the inseparability of the "Ti" (or "Dao") and "Yong" (or "Qi"). If Western learning was found to have the instrumental value, then there must also be some "Dao" (or fundamental principle) in it as well, since, as the neo-Confucians suggested,

inherent in the "Qi" (or instrument or vessel) of anything was a "Dao". Consequently, "implicit in this line of thinking was an inclination to prize the West as the source not only of instrumental and secondary but also of essential and central values," [Quoted Hao Chang in Fairbank *et al.*, eds., 1980, p. 283].

Even so, Liang Qichao did not want a Christianized Chinese culture. Nor, apparently, did Li Shanlan, despite the fact that he, unlike Liang Qichao, might have been very near to professing Christianity [Martin 1896, pp. 368-370]. But then how could Li Shanlan isolate Western mathematics and science from Christianity? This problem in turn led to another equally urgent one: Granted that the "Dao" of orthodox Confucianism was inseparable from the "Qi" of mathematics, how did Li Shanlan then think that the study of mathematics alone could lead to the "Dao"? Such a problem or its equivalent, as argued in Sections 1.7 and 1.12, was finally settled in the evidential scholarship of the Qian-Jia school. The legacy of the Qian-Jia school indeed made it possible for Li Shanlan, among other specialists, to esteem himself as a mathematician. Li Shanlan seems to have been influenced by the primary concern of the Qian-Jia school, namely, the study of the Classics employed to understand the "Dao". Otherwise, while he was teaching at the Tong Wen Guan, he would not have written on scrolls an antithetical couplet ("Dui Lian"), saying that "I am very like a recluse living not on the mountain but in Beijing. But I am so humble to understand only mathematics and philology

among the 'Xiao Xue' (lesser learning)".

Li Shanlan's self-perception of his own role as a mathematical specialist, as reflected in his letter to Fang Yuanzheng in 1867, was related to the increasing need of the "Self-strengthening Movement" for specialists like mathematicians and scientists. Coupled with the legacy of the Qian-Jia school, the "Zhong Ti Xi Yong", and the historical environment underlying the "Self-strengthening Movement", whether or not the study of mathematics could lead to the "Dao" perhaps no longer needed to be clarified. As long as mathematical specialists found positions to teach mathematics, mathematics was inevitably going to find an institutional basis for professionalization.

After Li Shanlan, more and more professional mathematicians came on the scene. Although few students coming from the Tong Wen Guan actually served as mathematics instructors [Su Ching 1985, pp. 153-157], many students of Hua Hengfang went to teach mathematics at the newly founded provincial high schools [Wang Yusheng 1985]. In addition, Liu Yicheng's students like Zhi Baoran, Shen Shanzheng, Chen Weiqi, Cui Chaoqing, Hua Shifang, Miao Chaoquan, Tang Jinzhu and Liao Jiashuo served as mathematics instructors towards the end of the 19th century. All of these students, following the examples of Li Shanlan as well as their own instructors, continued to make contributions to the modernization of Chinese mathematics education.

On the other hand, their mathematical treatises comprise the major portion of the content of the history of Chinese mathematics in the period 1880-1920. To be specific, their research on mathematics brought to an end the transitional period during which Chinese traditional mathematics was eventually modernized. Beginning in the 1920s, the "returned students" from Europe, America and Japan began to take over mathematics teaching as well as research at the mathematics departments of the newly founded universities [Li Di et al., eds., 1986, pp. 527-560]. A new era for Chinese mathematics was beginning to dawn.

5.8. Concluding Remarks

The period 1868-1882, during which Li Shanlan taught at the Tong Wen Guan, was an important one for the modernization of mathematics and its place in education in China. It is quite clear that in this transitional period, Li Shanlan played very successful role both as a professionalized mathematician and as a mathematics instructor at the Tong Wen Guan. Even more significant, his example shows how complicated was the process by which Chinese mathematics was professionalized.

His life time (1811-1882) connected two important periods of late Qing intellectual history. One is the period (1736-1820) during which the Qian-Jia school was predominant. The other is the period (1860-1890) when the Qing government was forced to Westernize or modernize under the direction of the

Self-strengthening Movement. As argued in this Chapter, the professionalization of mathematics was partly the legacy of the Qian-Jia school, to which were added the forces of the Self-strengthening Movement, and eventually both of which served to provide favorable intellectual background and institutional basis for mathematics. Meanwhile, as mentioned several times, the self-image of Li Shanlan had of his own role as a mathematical specialist, if not wholly formed in his early life, was indispensable. Above all, helping to substantiate his status as a professionalized mathematician was the fact that Li Shanlan was the unquestioned leader of the mathematical community in China in 1870's [Cf. Section 5.4].

As for the modernization of mathematics education, Li Shanlan's role was even more crucial. His teaching, which stressed the "Tian Yuan" method and its significance in the synthesis of Chinese and Western theories and methods, helped the way for the educational reforms of the 1890s. No wonder the late Qing reformers of education were ready to follow his example.

In conclusion, Li Shanlan was highly esteemed for his own mathematical research, which easily justify his status as a mathematician in broad segments of the scholarly community. By then Li Shanlan was a "professionalized" mathematician in the non-institutionalized sense made possible by the Qian-Jia school. Eventually he was able to act as a professionalized mathematician by teaching at a formal institution, namely, the Tong Wen Guan.

Li Shanlan's example shows how completely Chinese mathematics had been professionalized by the late Qing.

NOTES:

1. "The Professor of Mathematics" was the title Martin gave to Li Shanlan in the *Triennial Calendar of the Tungwen College* (Fourth issue, 1888) which was an English translation of the (Chinese) edition, the *Tong Wen Guan Ti Ming Lu* (of the same issue). In the first issue of the *Tong Wen Guan Ti Ming Lu* (1879), Li Shanlan was called "Suan Xue Jiao Xi" (mathematical instructor).
2. On the other hand, a copy of the 1870 "Sui Shi" test sent to *The Church News* (or *Jiao Hui Xin Bao*) by Martin was also available to us. In addition, Numbers 7 and 20 of *The Peking Magazine* (or *Zhong Xi Jian Wen Lu*) collected the 1872, 1873 "Sui Shi" tests respectively.
3. This problem is missing from the source material we refer to.
4. Here the notations "⊥" and "⊖" mean "plus" and "minus" respectively. These notations made their first appearance in China in the translated *Dai Shu Xue* (De Morgan's *Algebra*). The reason why the translators used the ⊥ and ⊖ instead of the more common + and - respectively was because "+" and "-" were easily confused with the Chinese numerals "ten" ("Shi") and "one" ("Yi"). On the other hand, such an equation of third degree can be translated in modern notation as

$$x^3 + x^2 + 9x - 30 = 0.$$

The original scheme was used by Li Rui in his *Kai Fang Shuo*, who elaborated the earlier schematic form developed by the Song mathematicians Jia Xian, Liu Yi, Yang Hui and Qin Jiushao [Li and Du 1987, 117-135]. Yet why did ancient Chinese mathematicians mean "by extraction"? "In ancient China the method of solution of ordinary equations was called the 'method of opening the square' because the methods of solution for ordinary equations all evolved from the method of extracting square roots," [Li and Du 1987, p. 117].

5. The notations used are the same as Problem 10. Here the equation of fourth degree can be translated in modern notation as

$$x^4 + 10x^3 - 75x^2 - 635x + 2199 = 0.$$

6. This equation can be translated in modern notation as

$$1300x^3 + 83400x^2 - 30192000x + 832320000 = 0.$$

In the test, the notations "+" and "-" were given in Chinese "Zheng" and "Fu" respectively, exactly the way that had been used by Li Rui in his *Kai Fang Shuo*.

7. Li Yicheng called Pythagorean triplets "Zheng Shu Gou Gu Xuan" (literally the right-angled triangles with integer sides).
8. In fact, Faber published his *De Guo Xue Xiao Lun Lue* in a series of articles in *The Church News* (*Jiao Hui Xin Bao*, issued in Chinese).
9. The other alternatives include the establishment of school of modern type [Cf. Zhu Youxian 1984, p. 444].
10. This is often regarded as a significant step in the institutionalization of mathematics by historians of Chinese science and mathematics [Cf. Guo Yongfang 1983].
11. Hua Hengfang suffered a bad health in his late days due apparently to a damage that was made on his brain by working too hard in studying mathematics in his early years.
12. "Ma Jiang" is one sort of games that still prevail in China and East Asia.
13. For example, "many Ch'ing scholars who were not independently wealthy were employed on local history projects, such as compiling county, prefectural, and provincial gazetteers financed by local genrey," [Elman 1984, p. 134]. Dai Zhen, one of the leaders of the Qian-Jia school, was among the busiest compilers of local gazetteers ("Fang Zi") during the 18th century. At that time, mathematicians like Li Rui still had opportunity similar to Dai Zhen. This is why Li Rui shares two titles in Zhang Zhidong's *Shu Mu Da Wen* (1875), namely specialist in Han learning and mathematician. After 1820, Dong Youcheng was the only mathematician who earned another title, namely "Pian Ti Wen Jia" (scholar who was versed in "Pian ti Wen"). No wonder Zhang Wenhu, Li Shanlan's close friend and colleague at Anqing headquarters, was still able to make a living by serving as a compiler of gazetteer in the 1850s. Yet this did not happen to any noted Chinese mathematicians in the second half of 19th century. Thus the late Qing mathematicians hardly shared socio-economic sources with general scholars to great extent.
14. "Statecraft proposals were enunciated by pragmatically oriented scholar-officials who believed they were avoiding both the scholastic philosophy associated with Han Learning and the empty speculation of Sung-Ming Tao-hsueh, " [Elman 1984, 237]. According to the statecraft school, scholars should commit to the welfare of the country in their scholarly pursuits and should not succumbed to the bookish orientation that they felt had plagued eighteenth-century philological scholarship. This may explain why they traced this scholarship back to the noted late Ming and early Qing scholars Gu Yanwu (1613-1682) and Huang Zongxi (1610-1695).

15. Here by "instrumental and secondary values", Qing scholars meant they could and should only be applied to the practical level, while "essential and central values" were about moral philosophy which of course was dominated by the neo-Confucian philosophy. In fact, Zhang Zhidong noted slogan -- "Zhong Ti Xi Yong" (Chinese learning for the fundamental principles, Western learning for practical applications) -- shared a similar attitude toward Western studies.
16. If not, they would coin new term for the introduced concept. For example, "prime number" was translated into "Shu Gen" (literally root of number).

6. Conclusion: Li Shanlan and His Significance for the Development of Mathematics in China; Forms and Patterns of Modernization

At the turn of this century, Chinese mathematics developed rapidly, largely in response to increasing exposure to foreign influences. Not surprisingly, traditional mathematics gradually yielded to Western mathematics. Beginning from about 1920 on, as Chinese mathematicians studied in Japan, Europe, and the United States, they came to play a key role in Chinese mathematical research and in college mathematics teaching upon their return to China. By then, Chinese mathematics had been thoroughly modernized.

Such a tendency was doubtless inevitable. A decade earlier in 1912, when the Republic of China was founded, the curriculum advanced by the Education Ministry of the new government reflected a completely Western-influenced approach to college mathematics teaching.¹ This "new" curriculum, in fact, was almost a carbon copy of what had been decreed for colleges in China by the late Qing government just a decade earlier in 1903 [Li Di *et al.*, eds., 1986, pp. 535-536]. Since the turn of the century, Chinese mathematicians were not considered qualified to teach at colleges unless they were familiar with modern mathematics.

At almost this same time, mathematical curricula had also been modernized in primary and secondary schools [Li Yan 1958, p.

542; Li Di et al., eds., 1986, pp. 535-536]. As a result, the demand for qualified mathematics teachers became urgent, apparently due to the fact that all of the basic mathematical textbooks were either translated from Western examples or written (by Chinese) in modern style and format.² All this, on the one hand, can only be understood in terms of Li Shanlan's career and his mathematics, since in this transitional period, Li did play an indispensable and significant role. On the other hand, all this may as well help to explain why the traditional mathematician Zhou Da (1878-1942?), who followed Li Shanlan's steps to study traditional mathematics like the "Duo Ji" method in his early years, shifted from traditional to modern mathematics in the 1900s, although he was neither a college instructor nor a teacher at a primary or secondary school. He paid two short visits to Japan in 1902 and 1904, where he studied modern mathematics with Japanese mathematicians like Nagazawa and Kamino there [Li Di et al., eds., 1986, p. 523]³. After publication of his collected mathematical works, the *Zhou Mei Quan Suan Xue* (1909), Zhou Da never went back to traditional mathematics again, although earlier he had shown great interest in Li Shanlan's "Duo Ji" method and Hua Hengfang's "Ji Jiao" method (the method of finite difference).⁴ Not only did Zhou Da's *Duo Ji Yu Yi* ("Supplement to the 'Duo Ji' Method") make some progress what beyond Li Shanlan had achieved, but it also explained the formulas for summations of finite series (sums of piles) using the differential and integral calculus.⁵ Zhou Da's work clearly marks the transition from traditional to modern mathematics in China. Unlike Li Shanlan, he

was directly in contact with modern mathematics via Japanese mathematicians, although he, like Li Shanlan, was originally trained in the classics of traditional Chinese mathematics.

Nevertheless, as an early key figure of this transitional period, Li Shanlan's contribution to the modernization of Chinese mathematics was no less significant than those made by his successors like Zhou Da. In light of the roles Li Shanlan assumed first as traditional mathematician, then translator, and finally instructor at the Tong Wen Guan during the three periods of his career, Li Shanlan and his mathematics deserve an important place in the history of Chinese modern mathematics. Although he was not a radical reformer in the process of modernizing mathematics in China, Li Shanlan was able to advance steadily as he treated both traditional Chinese mathematics and Western mathematics together. The contrast between "traditional versus Western mathematics" does not seem to have bothered Li Shanlan. What really concerned him was mathematics *per se*.

Unlike most scholars of the Qian-Jia school, Li Shanlan justified mathematical knowledge for its own sake. To him, where mathematical knowledge came from was insignificant. Western ideas were not to be ignored or preferred simply because they came from the West. It was at the Imperial College, the Tong Wen Guan, where Li Shanlan's synthesis of traditional and Western mathematics came to be appreciated by late Qing scholars and mathematicians. Western mathematics was legitimized not as the Qian-Jia school had

argued (because it was thought to be of ancient Chinese origin), but because, as in the case of Euclid's *Elements*, it provided theoretical foundations for mathematical arguments that were not a part of traditional Chinese mathematics. Therefore, the Western approach was considered indispensable, and yet complementary to traditional Chinese mathematics which emphasized mainly the "algorithmic" aspects of mathematics, as the *Jiu Zhang Suan Shu* demonstrated. Once an institutional base for mathematics was in place at the turn of this century i.e., a modern school system including colleges as well as primary and secondary schools, a shift in curriculum from traditional to modern mathematics, made it possible for Chinese mathematicians to move gradually into the main stream of modern mathematics on an international scale [Li Di *et al.*, eds., 1986, pp. 527-582; Li and Du 1987, p. 266].

Given the fact that Western theories were at least as powerful as their Chinese counterparts, and in some cases like the differential and integral calculus even more fruitful -- why should most Chinese mathematicians at the end of the 19th century have been so reluctant to abandon their tradition? Such a question deserves attention, even though it may echo the (now controversial) "impact-response" framework adopted by some authors since 1960s to explain the history of modern China [Teng 1971; Cohen 1984].

China had developed mathematics on its own terms for over 3000 years. Indian and Islamic mathematics came to China during

the Tang (8th century) and Yuan (13th century) Dynasties respectively. However, neither seem to have made much of an impression on Chinese mathematics [Qian 1981, 108-116, 217-228; Li Di *et al.* eds., 1986, 239-246, 341-344]. It was not until the late Ming period that foreign mathematics began to exert any influence on native Chinese mathematics. This time, what was introduced (by the Jesuits) to China was Western mathematics and astronomy, including most notably the first six books of Euclid's *Elements* (1607). It is from here that our story actually begins, for Li Shanlan owed what he knew of Western mathematics to what had been introduced by the Jesuits (in the late Ming and early Qing), as is clearly reflected in his early treatises like the *Si Yuan Jie*, *Fang Yuan Chan You*, *Hu Shi Qi Mi*, *Dui Shu Tan Yuan*, *Tuo Yuan Zheng Shu Jie*, *Tuo Yuan Shi Yi* and *Ji Shu Hui Qiu* [Cf. Chapter 4].

On the other hand, Li Shanlan also developed the "Duo Ji" method entirely on his own. More specifically, his *Duo Ji Bi Lei*, in which the "Duo Ji" method played a central role, was written in terms of traditional Chinese mathematics. The subject Li Shanlan dealt with, i.e., the summations of finite series, can be traced back to Shen Gua during the northern Song and his method, i.e., the "Duo Ji" method, which may have direct links with Zhu Shijie's study of the subject in the Yuan period. To appreciate Li Shanlan's contribution to Chinese mathematics, and to determine properly his place in the historical course of Chinese science, a short survey of the history of 19th-century Chinese mathematics

may help. In particular, it is important to appreciate the most important methods and relevant basic mathematical concepts that Chinese mathematicians used to justify their arguments and demonstrations.

In the period from the late 16th to the early 18th century, the major interest of Chinese mathematics was to assimilate Western mathematics and astronomy introduced by the Jesuits. Mei Wending was a typical example. Since he was unable to obtain any of the Chinese classics of from Han to Tang Dynasties i.e., the *Suan Jing Shi Shu* ("Ten Books of Mathematical Classics") or to read works of the Song and Yuan mathematicians such as Qin Jiushao, Li Ye and Zhu Shijie, what most concerned him was a systematic compilation, edition and explication of the mathematics arriving from the West. Nevertheless, Mei Wending was able to make an important contribution of his own thanks to the newly introduced Western mathematics. His *Ji He Tong Jie* ("Explanations in Geometry"), in which he first gave definitions, then demonstrations and finally applications, reflected most clearly what he had learned from the format and style of Euclid's *Elements*.

This may help to explain Mei Wending's attitude toward mathematics. When a method was effective, it made no sense to consider the nationality of its creator. Such an attitude, clearly favorable to the newly introduced Western mathematics, was reflected in the *Shu Li Jing Yun* ("Collected Basic Principles of

Mathematics", 1723) which in turn greatly affected his grandson, Mei Juecheng, who among others, was enlisted by the Emperor Kang Xi to compile the *Lu Li Yuan Yuan* ("Ocean of Calendrical Sciences and Acoustical Calculation"). This includes, in addition to the *Shu Li Jing Yun*, two other volumes: the *Li Xiang Kao Cheng* ("Compendium of Calendrical Science and Astronomy") and *Lu Lu Zheng Yi* ("Collected Basic Principles of Music"). The *Shu Li Jing Yun* includes all aspects of mathematical knowledge of the time, and may thus be regarded as a comprehensive mathematical encyclopedia. Thanks to the supervision Emperor Kang Xi gave to production of the *Shu Li Jing Yun*, it remained a compulsory text for learning mathematics and was a very important reference book for mathematical research until 1850.

Additional knowledge of Western mathematics, especially of the ellipse, was also introduced by the Jesuits by the middle of the 18th century. Most treatises responsible for transmitting mathematics were those on Western astronomy [Cf. Section 3.6]. Needless to say, none of these managed to explain the theory of ellipse in any great detail. Even the *Shu Li Jing Yun*, which also includes several passages on the ellipse, is fragmentary. Satisfactory presentation of the mathematics of the ellipse was introduced by the *Li Xiang Kao Cheng Huo Pian* ("A Sequel for the *Li Xiang Kao Cheng*", 1742), through which 19th-century Chinese mathematicians like Xiang Mingda, Xia Luangxiang, Dai Xu, Xu Youren and Li Shanlan learned about the ellipse and went on to make interesting contributions of their own.

Unfortunately, almost no new mathematical knowledge entered China from the early 18th century until about 1850. This was a consequence of a closed-door policy adopted by the Emperor Yong Zheng (reigned 1723-1735).⁷ As a result, Chinese mathematicians began to devote greater effort to collate and revive ancient Chinese mathematical treatises, many of them forgotten or lost for over 300 years. This effort was also reflected to the popularity of the Qian-Jia school at the time. In fact, the Qian-Jia school played a dominant role in Qing scholarship during the second half of the 18th century. Because the Qian-Jia school considered mathematics as relevant to studies of the Classics, scholars of this school were enthusiastic about the recovery of ancient Chinese mathematical treatises. The intensive search for ancient texts, along with collating and studying them, was encouraged by the Emperor Qian Long (reigned 1736-1795), who sponsored composition of the *Si Ku Quan Shu* ("Complete Library of the Four Branches of Literature"). This project was patronized by the high scholar-officials Ruan Yuan and Wang Chang, and encouraged by the socio-economic conditions favoring the wealthy merchants in the Jiangnan area. Towards the end of the 18th century, the scholarship of the Qian-Jia school reached its climax. In the area of mathematics, Qian-Jia scholars recovered almost all of the ancient mathematical classics, including the *Suan Jing Shi Shu*, Qin Jiushao's *Shu Shu Jiu Zhang*, Li Ye's *Ce Yuan Hai Jing* and *Yi Gu Yan Duan*, and Zhu Shijie's *Si Yuan Yu Jian*, etc. The only exception was Zhu Shijie's *Suan Xue Qi Meng* ("introduction to

Mathematical Studies"), the Korean edition of which was discovered by Luo Shilin in 1839.

Although the Qian-Jia school avidly promoted studies of ancient mathematical texts, its "Kao Zheng" (textual criticism), nevertheless, placed important constraints on mathematical studies. Research on ancient astronomy and mathematics was valued mainly for its application in illuminating the classics. Consequently, exclusive interest in these texts solely for purposes of exegesis was so rigid that the scholars of the Qian-Jia school would have ignored any mathematical innovations. Even if they had been able to recognize these for what they were, they would have dismissed them as irrelevant.

In this context, Wang Lai (1768-1813) was unfairly treated by the Qian-Jia school because he was concerned primarily with technical innovations in his mathematical studies. Another factor may also have played a part: Luo Shilin blamed Wang Lai for becoming too deeply involved in the Western methods -- methods the Qian-Jia school avoided using in its mathematical studies. However, Wang Lai's example does serve to indicate that in this period some mathematicians at least, though only few, were able to concentrate their efforts on earlier mathematics, carefully digest it, and proceed to conduct their own research, even though the Qian-Jia school neither appreciated nor encouraged their efforts.

This minority comprised the so-called "Xi Fa" (Western

method) school which claimed the State Observatory as its stronghold, even at the time when the Qian-Jia school virtually dominated the whole scholarly community. Although all Western missionaries were deported to Macao in 1723 [See Note 7], the Jesuit astronomers working at the State Observatory were allowed to retain their posts and stay in Beijing. In fact, the Qing State Observatory continued to recruit Western missionaries to serve as astronomers until the 1820s. No wonder "Xi Fa" mathematicians like Ming Antu, Wang Lai, Chen Jie, Xiang Mingda and Xu Youren were associated with the State Observatory in one connection or another in their early careers. This was also the case with Luo Shilin. Before he converted to the "Zhong Fa" (Chinese method) school, Luo Shilin served as a student at the State Observatory, and wrote the *Bi Li Hui Tong* which clearly reflects the style of the "Xi Fa" school.

It was at the State Observatory that Ming Antu, the chief astronomer, learned the "three formulas" ("Du Shi San Shu") from Jartoux. He went on to generalize these to nine formulas, giving demonstrations. His treatise, the *Ge Yuan Mi Lu Jie Fa*, was not published until 1839, although the first section was circulated among those who were closely related to the Observatory. Thanks to the early appreciation of these nine formulas, the study of series expansions for trigonometric functions gradually came to the fore in Chinese mathematics toward the end of the 18th century.

Clearly, 1800 A.D. was a very significant year for Chinese mathematics as it was poised to enter a new phase. On the one hand, the Qian-Jia school had peaked in influence, and was on the verge of decline thereafter. The *Chou Ren Zhuan* ("Biographies of Mathematical Astronomers", 1799) seems to have brought the Qian-Jia school's dominance over Chinese mathematical studies to an end. From 1800 on, no major Chinese mathematicians devoted their efforts exclusively to the revival of ancient mathematical texts. On the other hand, Ming Antu's mathematics increasingly came to occupy the minds of most key figures of 19th-century Chinese mathematics. Consequently, the "Xi Fa" school was more powerful and popular than before. The Qian-Jia school, unable to ignore the "Xi Fa" school any longer, tried to face the challenge directly. In dealing with problems concerning triangles, Xiang Mingda's debate with Luo Shilin clearly revolved on a major mathematical issue of the day -- Was the "Tian Yuan" method (which Luo Shilin used to solve the problems concerning triangles) the best or the most appropriate in doing mathematics? Apparently Luo Shilin, the last faithful representative of the Qian-Jia school, took this challenge seriously. Nevertheless, his response was not convincing, at least to the "Xi Fa" school. This may help to explain why the Qian-Jia school eventually compromised by declaring Ming Antu's mathematics to be "Xin Fa" (New method) but not "Xi Fa" (Western method) in 1839.

From 1840 on, Chinese mathematicians seem to have been more comfortable than before in studying mathematics for its own sake,

no matter what may have inspired it. This was certainly the case for Li Shanlan. By 1840, everything in China concerning mathematics had been prepared. Doubtless, the time had come for Li Shanlan to do something significant, and the innovations he was about to make would finally sow the seeds of Western mathematics in especially receptive, Chinese soul.

Li Shanlan owed as much to the "Xi Fa" school as he did to the Qian-Jia school between 1800-1840. In fact, Li Shanlan's *Si Yuan Jie*, meant to revive Zhu Shijie's *Si Yuan Yu Jian*, was a typical work that bears the clear mark of the Qian-Jia school, even though Li Shanlan adopted the Western algebraic notation to explain the "Si Yuan" schematic diagrams. In addition, the concepts and methods used in his *Duo Ji Bi Lei* ("On the Sums of Piles") can be traced back to Zhu Shijie's *Si Yuan Yu Jian*. In this comprehensive work on the summations of finite series, Li Shanlan also referred to Dong Youcheng and Wang Lai, but he did not adopt the terms which Wang Lai and Dong Youcheng took from the *Shu Li Jing Yun* and used in their own treatises on the subject. Moreover, in dealing with problems like the following: "Given a sum S of a finite series, say, $\sum k^7$, find the number of its terms," Li Shanlan used the "Tian Yuan" method to solve the related equations [Cf. Section 3.5], even though he seems to have known Western "Dai Shu" (algebra) when he wrote the *Duo Ji Bi Lei*.

Yet this does not mean that Li Shanlan's innovation in the "Duo Ji" was only a by-product of his interest in reviving Zhu

Shijie's classic, the *Si Yuan Yu Jian*. He was especially proud that his "Duo Ji" method went beyond the long standing "nine methods" of the *Jiu Zhang Suan Shu* ("Nine Chapters on Arithmetic") [Cf. "Preface" to the *Duo Ji Bi Lei*]. Perhaps what most concerned him at the time were his mathematical studies per se. To him, revival of mathematical classics like the *Si Yuan Yu Jian* seems to have become a minor issue.

Like his predecessors of the "Xi Fa" school, Li Shanlan did not follow the Qian-Jia school's approach to mathematics. Nevertheless, they all had to operate in the intellectual and socio-economic climate that the Qian-Jia school created. The legacy of the Qian-Jia school did help Qing mathematicians from 1800 on to feel more comfortable in doing mathematics than before. For scholars of the Qian-Jia school, mathematics was closely related to their understanding of the Classics. This in turn led them to the "Dao" -- the fundamental principle of Confucianism. Consequently, even though it was a specialized field, mathematics deserved life-long commitment. On the other hand, the Qian-Jia school also established a "secularized" scholarly community that served to establish socio-economic conditions that would enable Qing scholars to commit themselves to a single specialty like mathematics. This community was "secularized" in the sense that the community in question no longer aimed at ideals of wisdom where interests centered primarily on questions of Confucian moral philosophy. Instead, "secularized" scholars devoted themselves to the "lesser learning" which included, of course, mathematics.

They began to seek employment as "Mu You" to officials, as tutors in wealthy families, or as academic teachers and the like to make a living. Most mathematicians of the time, unlike philological scholars of the Qian-Jia school, had not as yet been a "professionalized" by 1840, as the examples of Jiao Xun, Wang Lai, Li Rui, Luo Shilin and Xiang Mingda attest. However, considering their common interests, activities, and goals -- along with the development of intricate, informal social networks and feelings of solidarity among mathematicians carrying out essentially similar tasks -- the differentiation of mathematical from humanistic knowledge advocated by the Qian-Jia school was significant. By now, practitioners of the mathematical sciences increasingly perceived their activity as distinct, autonomous and objective in contrast to other areas of study [Cf. Section 1.12]. This certainly was the case with Li Shanlan. Without a closer look at the legacy of the Qian-Jia school, it is difficult to understand how Li Shanlan, among other Qing mathematicians in the second half of the 19th century, could have been "professionalized," but primarily on a non-institutionalized basis.

Li Shanlan owed many interesting problems to the "Xi Fa" school, to which made great contributions in the period 1800-1840. First of all, Li Shanlan shared the "Xi Fa" school's admiration for Euclid's *Elements*. His *Fang Yuan Chan You* ("Explanation of the Square and the Circle") reflected some features that seem to have been influenced by the format and style of the *Elements* [Cf. Section 3.2]. Moreover, his interest in undertaking the

translation of the last nine books of the *Elements* in the 1850s may also reflect the enthusiasm he had felt for this Greek classic ever since his early years. As his study of the prime numbers showed, Li Shanlan even drew upon the *Elements* as an important source for his own mathematical research [Cf. Section 5.4].

Nevertheless, Li Shanlan never seems to have commented on the striking difference in format and style between the *Elements* and the *Jiu Zhang Suan Shu*. How could he have ignored this?⁸ For Li Shanlan, his concern for mathematics lay apparently in its "algorithmic" aspects. He paid special attention to how infinite series expansions for π , trigonometric and inverse-trigonometric functions, and logarithmic function, etc., could all be expressed in just a few terms. His "Jian Zhui" method did work, since he was able to establish the (usually recursive) relationship between these few terms in such expansions.

Even so, he does not seem to have raised any special questions about the concept of infinity. For example, in order to obtain a series expansion for π or the area of the unit circle, Li Shanlan regarded the difference between each quarter of a unit square and its inscribed circle as an accumulation of infinitely many "Jian Zhui" of power $2n$, ($n = 1, 2, 3, \dots$) [See Section 3.2]. Moreover, he certainly knew something equivalent to the fact that as n tends to be sufficiently large, the area of the "Jian Zhui" of power $2n$ tends to be arbitrarily small. Unfortunately, he said almost nothing about how infinitely many

such infinitesimal "Jian Zhui" were accumulated, even though he was able to deal with this problem in terms of the algebraically "recursive" relation between the "Jian Zhui" of power $2n$. According to Li Shanlan, no "Jian Zhui" of power $2n$ would vanish even if n tended to infinity.

Ming Antu, on the other hand, did explain away difficulties arising from approximating an arc by its subtending chords, infinitely many in number. For Ming Antu, a chord and an arc were not of the same category of geometric magnitudes, even though the arc might be approximated by infinitely many chords. From the above comparison between Li Shanlan and Ming Antu, it is clear that Li Shanlan not overly concerned with the nature of such mathematical concepts as "infinitely many," "infinitesimal," etc., and their relevance to the justification of methods like the "Jian Zhui" method.⁹

Even so, Li Shanlan devoted his *Fang Yuan Chan You* to explaining what he meant by "point", "line", "surface" and "solid", as well as their relationships. Why he dealt with these is quite clear, for his "Jian Zhui" method was based on these concepts. According to Li Shanlan, "the Westerners' so-called point, line and surface are not without substance ('Wu Ti', literally "no body or entity"). Thus a point is that which is tiny ("Xiao", literally small) and fine enough ("Wei", literally sufficiently small); a line is that which is long but very narrow in breadth; and surface is that which has length and breadth but

is also very thin," [Dan Zhi 1 of the *Fang Yuan Chan You*].

Obviously, Li Shanlan gave his own explanation for Euclid's definition of a point: "A point is that which has no part." Even in the *Ji He Yuan Ben* (1607), the definition given for a point in Chinese (translated from Clavius' edition) is exactly the same as that given in Heath's edition of Euclid for a "point".

In his illustration of Dan Zhi 1, Li Shanlan went on to explain why a point cannot be without substance: "This is," he argued, "because color manifests itself in shape, and shape is formed by entity (or body). For example, if we make a point by dropping a dot of ink on paper such that the point is as fine as dust, then it is the finest of the shape. Yet the shape of this point (i.e., the dot of ink) cannot come from nothing. Rather it is formed by the ink. Be that as it may, is that ink not an entity? Therefore, the point is the body that is tiny and fine enough," [Quoted *Fang Yuan Chan You* (1867), p. 1].

However, this explanation does not help to clarify his arguments for other Dan Zhi, say Dan Zhi 7. In this Dan Zhi, Li Shanlan apparently used his explanations for "point", "line" and "surface" to deal with the problem of how a "Jian Zhui" could be piled up by its layers and in turn how each layer could be approximated by a corresponding "Duo Ji" (sum of piles) [Cf. Section 3.2]. Here, Li Shanlan seems to have thought that since a geometric magnitude like a "line" could not be "without substance", then something formed by accumulating their piles

could not be "without substance" too.¹⁰ Even if this were the case, it is unknown whether Li Shanlan equated Euclid's concept of "point" with the Chinese concept of "Duan" (literally an end or extreme part of something) as used by the Mohists in the pre-Qin period [Qian 1981, 16-21].

Although what Li Shanlan really meant by "point", "line" and "surface" is not exactly clear, why he was careful to explain these concepts to the extent that he did is understandable. His major concern was to establish or clarify his "Jian Zhui" method. In fact, he would have paid no attention to the nature of mathematical knowledge if it had had nothing to do with the methods he was trying to promote. This may also explain why he failed to appreciate Wang Lai's theory of equations, although he certainly was familiar with Wang Lai's mathematics as shown in his study of the summations of finite series [See "Preface" to the *Duo Ji Bi Lei*]. Wang Lai, the leading mathematician of the "Xi Fa" school in the early 18th century, began to treat such problems as how many positive roots a given equation could have. Such a wholly new approach to the theory of equations would doubtless have escaped Li Shanlan's eyes, since he was primarily concerned with the "algorithmic" aspects of mathematics. Similarly, it is no surprise that he did not pay much attention to Jiao Xun's *Jia Jian Cheng Chu Shi* ("On Addition, Subtraction, Multiplication and Division"), a treatise which for the first time in Chinese history, dealt mainly with the algebraic structure that underlay the *Jiu Zhang Suan Shu* in terms of the four arithmetic operations.

In fact, the studies just mentioned by Wang Lai and Jiao Xun were neglected by their successors in the next generation. Xiang Mingda (1789-1850), the leading mathematician of the "Xi Fa" school in the period 1820-1840, indicates the extent to which Chinese mathematicians at the time had gone back to the old methods of the "algorithmic" approach. Moreover, his primary concern -- series expansions for trigonometric functions and the rectification of the perimeter of the ellipse -- was echoed by Li Shanlan a generation later. It should be noted that Li Shanlan was just as enthusiastic as Luo Shilin (a rival of Xiang Mingda) about the *Si Yuan Yu Jian*. Therefore, before Li Shanlan went to Shanghai to join the Mo Hai Shu Guan (the LMS Press) in 1852, he was already been armed with both the "Xi Fa" and "Zhong Fa". Given the opportunity, Li Shanlan was thus well-prepared to play the most important role in transmitting modern mathematics and science to China.

In the Mo Hai Shu Guan, Li Shanlan, in collaboration with Wylie, Edkins and Williamson, translated Euclid's *Elements* (the last nine books), De Morgan's *Algebra*, Loomis' *Calculus*, an anonymous treatise, *On Conic Sections*, Lindley's *Botany*, Whewell's *Mechanics* and Herschel's *Astronomy*. These treatises, which cover most of the basic mathematics and science taught at the best contemporary Western colleges¹¹, paved the way for Chinese traditional mathematics to merge into the mainstream of modern mathematics. Moreover, the mathematical terms Li Shanlan coined for his translations of Western concepts were drawn from ancient

Chinese mathematics texts [Cf. Li Di *et al.*, eds., 1986, 492-493]. For example, in translating "differential and integral calculus" into "Wei Ji Fen", Li Shanlan may have taken the Chinese term from the *Shu Shu Ji Yi* ("Memoir on some Traditions of Mathematical Art", one of the "Ten Books of Mathematical Classics"). The Chinese connotation of the "Wei Ji Fen" (an abbreviation of the "Wei Fen" and "Ji Fen"), literally dividing something into sufficiently small quantities and accumulating, conversely, sufficiently small quantities into something, probably shared only part of the original meaning of the "differential and integral calculus". This, consequently, greatly aided Chinese students in the second half of the 19th century, making modern mathematics more easily accessible to them in terms of familiar Chinese expositions.

To each of the texts on mathematics that Li Shanlan helped to translate, Chinese mathematicians of the late 19th century responded in accordance with time-honored features of Chinese traditional mathematics. For example, the *Dai Shu Xue* (De Morgan's *Algebra*) drew almost no reaction -- apparently it was too theoretical, especially the rigorous arguments De Morgan gave for his propositions. These made his book difficult to follow. Chinese mathematicians were more familiar and comfortable with programmed learning -- especially well-suited to the traditional Chinese "algorithmic" approach to mathematics. It is no wonder that the *Dai Shu Xue* was less popular than the *Dai Shu Shu* (Wallace's *Algebra*). The translation by Fryer and Hua Hengfang in

1873, expressed the *Dai Shu Shu* as a form of programmed learning.

On the other hand, the *Dai Wei Ji Shi Ji* (Loomis' *calculus*) was also translated as a form of programmed learning, which was not the style of Loomis' original edition published in United States in 1851. In fact, Loomis' *Calculus* was basically written as "a course of liberal education," thus "the primary object in studying the mathematics should be the discipline of the mental powers," [Loomis 1851, "Preface"]. Loomis also complained that many textbooks on calculus published in England "are wholly occupied with the mechanical process of differentiating and integrating, without any attempt to explain the philosophy," [Loomis 1851, "Preface"]. Ironically, the Chinese edition of his book reflected exactly what Loomis had criticized in the "Preface" of his 1851 edition. However, compared with the *Dai Shu Xue*, the *Dai Wei Ji Shi Ji* was more widely circulated. In addition, Li Shanlan's response to this calculus textbook was overwhelmingly enthusiastic. Apparently this was the first time he convinced himself that Western mathematics was superior, especially because of the methods of the differential calculus, not only to ancient Chinese mathematics, but to ancient Western mathematics as well [Cf. Hua 1882 book 5].

Li Shanlan was also very impressed by the "Analytic Geometry" in Loomis' *Calculus*. Later on, in dealing with the translation of the *Yuan Zhui Qu Xian Shuo* ("On Conic Sections"), Li Shanlan referred directly to the *Dai Wei Ji Shi Ji*. In fact, the

translations of the *Dai Wei Ji Shi Ji* and the *Yuan Zhui Qu Xian Shuo* were important source materials for Li Shanlan in his studies on the ellipse. Previously, he had already written two treatises on the ellipse, namely the *Tuo Yuan Zheng Shu Jie* and *Tuo Yuan Xin Shu*. Yet it was the newly-translated *Dai Wei Ji Shi Ji* and *Yuan Zhui Qu Xian Shuo* on which he relied in making his own great contribution to the theory of the ellipse in his *Tuo Yuan Shi Yi*. Moreover, the treatments of the hyperbola given in both treatises also included materials which he used in his series expansion for logarithms given in the *Dai Shu Xue* [Cf. Section 3.4].

On the other hand, his translation of the last nine books of the *Elements* was never appreciated by the Chinese mathematicians in the 19th century. Even Li Shanlan himself did not pay much attention to this part of the *Elements*, although it was a text he had himself requested for translation. A real appreciation for this Greek classic only came later at the turn of this century as Chinese mathematicians became interested in problems related to incommensurable magnitudes. Thus the extent to which how the Western mathematical texts translated into Chinese were circulated, and how Chinese scholars or mathematicians responded to them depended largely on the individual characteristics of the texts in question.

Li Shanlan's work as a translator ended when he went to Suzhou to serve as Xu Youren's "Mu You". By then, he presumably had finished any translations on which he was working. Li Shanlan

may also have written his *Tuo Yuan Shi Yi* ("Remarks on the Ellipse") and *Dui Shu Jian Zhui Bian Fa Shi* ("On the Equivalence of the Western and the 'Jian Zhui' Methods for Logarithms") at the end of his Mo Hai Shu Guan period (1852-1860). If so, then speed of Li Shanlan's ability to appreciate -- and assimilate the newly-transmitted Western mathematics was remarkable. In any case, both of these treatises must have been completed by 1864.¹² Three years later, Li Shanlan's collected mathematical works, the *Ze Gu Xi Zhai Suan Xue*, was published thanks to Ze Guofan's financial support [Li 1867]. Shortly thereafter, in accepting the second call from the Zhongli Yamen, he began the third period of his career as a mathematical instructor at the Tong Wen Guan.

In 1868, Li Shanlan began to teach mathematics at the Imperial College at Beijing, the Tong Wen Guan. Here he was to play an important role in helping to promote the institutionalization and professionalization of mathematics as a serious academic discipline in China. The period 1868-1882, during which Li Shanlan taught at the Tong Wen Guan, was an important one for the modernization of mathematics and teaching in China. It is quite clear that during this transitional period, Li Shanlan successfully played a crucial role both as a professionalized mathematician and as a mathematics instructor at the Tong Wen Guan. Even more significant, his example shows how complicated the process was by which Chinese mathematics came to be professionalized.

Li Shanlan (1811-1882) spanned two important periods of late Qing intellectual history. One is the period (1736-1820) during which the Qian-Jia school was predominant. The other is the period (1860-1890) when the Qing government was forced to Westernize or modernize in response to the Self-strengthening Movement. Li Shanlan's case, example, the professionalization of mathematics was partly the legacy of the Qian-Jia school, to which was added the force of the Self-strengthening Movement. Each of these two served to provide a favorable intellectual context and a suitable institutional basis for mathematics. Meanwhile, the positive self-image Li Shanlan had of his own role as a mathematical specialist, if not wholly formed in his early life, was indispensable, for otherwise a sense of mathematical community would hardly have emerged in 19th-century China.

As for the modernization of Chinese mathematical education, Li Shanlan's role was especially significant. In his teaching at the Tong Wen Guan, his successful amalgamation of Chinese traditional mathematics (like the "Tian Yuan" method) and newly transmitted Western methods (like algebra) in the same curriculum must have softened the hard-line attitude against Western mathematics of the conservatives, most of them had been involved with the Tong Wen Guan controversy in the 1860s. Similarly, Li Shanlan also helped pave the way for the educational reforms of the 1890s, whereby the modernization of Chinese education, including of course mathematics, got its start.

On the other hand, Li Shanlan's position at the Tong Wen Guan can be regarded as that of a professional mathematician. Thanks to this institutional support, Li Shanlan, as a mathematician, never again had to make his living by serving as a tutor for wealthy families, as a translator at the LMS Press, or as a "Mu You" for higher scholar-officials like Zeng Guofan and Xu Youren. Moreover, at the Tong Wen Guan, Li Shanlan acted as a mathematician in the modern sense, that is to say, he not only taught mathematics but also undertook mathematical research of his own on the most interesting subjects of the time, as his study of prime numbers in the 1870s makes clear.

After Li Shanlan, more and more professional mathematicians came on the scene. Students of Li Shanlan, Hua Hengfang and Liu Yicheng, following the examples of Li Shanlan and their own instructors, continued to make contributions to the modernization of Chinese mathematics education. Their mathematical research signals an end to the transitional period during which traditional Chinese mathematics was eventually modernized. Beginning in the 1920s, mathematicians trained in Europe, the United States and Japan began to take over mathematics teaching as well as research in the mathematics departments of the newly founded universities [Li Di *et al.*, eds., 1986, 527-560]. A new era for Chinese mathematics was about to dawn.

It is, therefore, quite clear that Li Shanlan's place in the history of modern Chinese mathematics was crucial and of lasting

significance. Seen only as a late Qing intellectual, Li Shanlan's example was so commonplace that historians of modern Chinese history would not have found him of particularly interesting, despite he had a close association with Western missionaries in the 1850s. However, as a Chinese mathematician in the 19th century, Li Shanlan's life and career take on a special interest and help to illuminate the intellectual and socio-economic background of the time to such an extent that no historian of Chinese mathematics and science could ignore his importance. To historians of modern education, Li Shanlan and his mathematics also deserve attention, since he played so crucial and significant a role in modern educational reforms. In this process, Li Shanlan was hardly ever regarded as a reformer, and certainly not a radical one. Unlike Wang Tao, one of his close friends and colleagues at the Mo Hai Shu Guan, Li Shanlan does not seem to have published anything about how the empire should reform itself in order to survive, nor did he ever present any proposal on bureaucratic reform or statecraft to any higher scholar-official. Nevertheless, due apparently to his capability in mathematics, he was always able to do his best in each period of his career. The contribution he made in the process to 19th-century Chinese mathematics was of special significance. By following his steps, his successors were able to accomplish a thorough modernization of Chinese mathematics during the first two decades of this century.

NOTES:

1. The curriculum included courses like the Calculus, Differential Equations, Theory of Equations, Modern Algebra, Modern Geometry, Plane and Solid Geometry, Probability Theory and the Method of Least Squares, Theory of Functions, Calculus of Variations, Number theory, Theoretical Physics and Astronomy [Wei Genren ed., 1987, pp. 65-106]. It is not clear exactly what "Modern Geometry" included. Nor do we know to what extent these courses (like "Theory of Functions") were actually taught.
2. In other words, traditional Chinese mathematical terminology, notations and methods (like "Tian Yuan," "Duo Ji," etc.) were abandoned [Cf. Wei Genren ed., 1987, pp. 1-181]. Among books translated into Chinese, the most influential were Loomis' *Calculus* and the *Yuan Zhui Qu Xian Shuo* by anonymous author. Those written in the modern style included De Morgan's *Algebra*.
3. It should be noted that Japanese mathematics underwent its own "modernization" somewhat earlier than did China during the Meiji restoration [Cf. Mo and Xu 1987, pp. 517-523].
4. In his *Ji Jiao Shu* ("Method of Finite Differnece", 1882), Hua Hengfang used the "Ji Jiao" method, in contrast to Li Shanlan's "Duo Ji" method, to obtain several results equivalent to those in Li Shanlan's *Duo Ji Bi Lei* [Cf. Luo Jianjin 1986; Li Di et al., eds., 1986, pp. 517-523].
5. See [Li Di et al., eds., 1986, p. 525] for a very short survey of Zhou Da's mathematics. Exactly how he dealt with the sums of piles by means of calculus is not clear, and further study of his mathematics is therefore necessary to understand how Zhou Da was able to play a key role in this important transition of Chinese modern mathematics.
6. Li Shanlan was introduced to Western mathematics by British Protestants like Wylie, Edkins and Fryer, thanks to his association with the LMS Press. From the American Protestants like Martin who served as the chief instructor of the Tong Wen Guan from 1869 on, Li Shanlan must also have learned Western mathematics. However, those missionaries do not seem to have known much about mathematics before they came to China.
7. Since the beginning of the Qing Dynasty, Jesuit missionaries made every effort to maintain their influence over the Empire. The "anti-Christian Movement" led by Yang Guanxian in the early Qing may well explain the Jesuits' struggle to do so [Cf. Note 7 of Chapter 1]. For the Jesuits in China, this was not the end of troubles. A deadly blow to their mission was yet to come. In 1704, the Pope in Rome decreed that Chinese Catholics were forbidden to pay respects to their ancestors according to ancient tradition and custom. This order outraged both in the Court and the Chinese populace at large.

As a result, the Emperor Kang Xi ordered the papal legate, de Tournon, to be transferred and confined in Macao in 1707. Thereafter, the Jesuits were understandably more nervous about power struggles in the Court. They tried to help princes who were sympathetic to the Catholics succeed to the throne. Nevertheless, "when another prince, who was supported by the Buddhist lamas, succeeded to the throne and became the Emperor Yong Zheng, all foreign missionaries were immediately deported to Macao; They were not allowed to travel freely into the mainland and only a few remained to work at the State Observatory," [Li and Du 1987, p. 223].

8. Zeng Guofan's "Preface" for the *Ji He Yuan Ben Shi Wu Juan* (1864) points out quite clearly the difference in format and style between the *Elements* and the *Jiu Zhang Suan Shu*. Since the *Elements* is basically about theories, while the *Jiu Zhang Suan Shu* emphasizes methods and illustrative examples, Zeng Guofan argued that the two classics could be used together in a complementary fashion in studying mathematics [See Section 2.8]. Li Shanlan must have been aware of passage, especially as he actually knew who the real author of this "preface" was. For whatever reason, Zeng Jize (Zeng Guofan's eldest son) and Zhang Wenhui (Li Shanlan's colleague at Anqing and Nanjing) both claimed its authorship, although why they did so is unknown.
9. It should be noted that Ming Antu's mathematics was primarily "algorithmic", as shown quite clearly in his method of "Ge Yuan Lian Bi Li" (method of continued proportion for circle measurement) [Cf. also Section 3.3].
10. This was highly probable, since Li Shanlan gave his geometric interpretation for the product of unknowns in his *Si Yuan Jie*. For example, a product of two unknowns "Tian Yuan," and "Di yuan" would be interpreted as an area, since each unknown had already been interpreted as a line.
11. The only exceptions were texts on chemistry, a great portion of which were translated by Xu Shuo later at the translation department of the Jiangnan Arsenal.
12. Liu Shizhong wrote a "Postscript" for the *Ze Gu Xi Zhai Suan Xue* in 1864, and so this collected mathematical work must have been well completed by then.

GLOSSARY

A

A Er Re Ba La Xin Fa

阿爾熱巴拉新法

Anhui

安徽

Anqi

安慶

B

Ba Gu

八股

Ba Qi Guan. Xue

八旗官學

Ba Xian

八線

Ba Xian Bie Zhi

八線備旨

Bai Ji Shu

百雜術

Ban Zhou Ban Jing Xiang Cheng De Ji Bu

半周半徑相乘

得積步

Bao Xi Li Shi Jia Cheng

苞溪李氏家乘

Ben Shu

本數

Bi Li Dui Shu Biao

比例對數表

Bi Li Hui Tong

比例匯通

Bi Li Xian Xin Jie

比例線新解

Bi Suan Shu Xue

筆算數學

Biao

表

Bie Zhuan Ji San Bian

碑傳集三編

Bing

丙

Bu

步

C

Cai Xiyong

蔡錫勇

Can Ji

殘積

Cao

草

Ce Liang Quan Yi

測量全義

Ce Tian Yue Shuo

測天約說

Ce Yuan Hai Jing

測圓海鏡

Chang Mao

長毛

Changzhou

常州

Chen Huan

陳奐

Chen Huoyao	陳厚耀
Chen Jixin	陳際新
Chen Jie	陳杰
Chen Jin	陳錦
Chen Li	陳澧
Chen Shiren	陳世仁
Chen Shuotian	陳壽田
Chen Weiqi	陳維祺
Chen Yang	陳陽
Cheng Dawei	程大位
Cheng-Zhu School	程朱學派
Chi	尺
Chi-i (Ji Yi)	技藝
Chi Shui Yi Zhen	赤水遺珍
Chong Shi	崇實
Chong Zhen Li Shu	崇禎曆書
Chou Ren Zhuan	啻人傳
Chou Ren Zhuan Hui Bian	啻人傳匯編
Chou Ren Zhuan Xu Bian	啻人傳續編
Chu	除
Chuan Ji	全積
Chuan Mou	權謀
Chuang Jian Zun Jing Shu Yuan Ji	創建尊經書院記
Chuan Xue Pian	勸學篇
Ci	詞
Ci Ciao	奇巧
Ci Xue	詞學
Ci Xi	慈禧
Cui Chaoqing	崔朝慶
Cui Jingchang	崔敬昌
Cui Jingyuan	崔景遠
D	
Da Ca Xuan Jiao He	大差弦較和
Da Kao	大考
Da Qing Hui Dian	大清會典
Da Tong Calendar	大統曆

Da Xuan	大弦
Da Xue	大學
Da Yan Qiu Yi Shu	大衍求一術
Dai Dunyuan	戴敦元
Dai Shu Bie Zhi	代數備旨
Dai Shu Fa	代數法
Dai Shu Shu	代數術
Dai Shu Xue	代數學
Dai Xing He Can	代形合參
Dai Xu	戴煦
Dai Wei Ji Shi Ji	代微積拾級
Dai Zhen	戴震
Dan Zhi	當知
Dao	道
Dao Xue	道學
Daotai	道台
De Guo Xue Xiao Lun Lue	德國學校論略
Deng Shu	等數
Dengzhou Wen Hui Guan	登州文會館
Di Jia Shu	遞加數
Di Jian Fen Shu	遞兼分數
Di Jian Shu Li	遞兼數理
Di Jian Zong Shu	遞兼總數
Di Yuan	地元
Dian	點
Dian Ze Ti Zhi Xiao Er Wei Ze	點者體之小 而微者
Ding	丁
Ding Ci	定次
Ding Fubao	丁福保
Ding Quzhong	丁取忠
Ding Richang	丁日昌
Dong Xi Xue Shu Lu	東西學書錄
Du Fameng	杜法孟
Du Shi San Shu	杜氏三術
Du Shuotian	杜受田
Duan Gou Gu Jiao	端勾股較

Duan Gou Xuan Jiao	端勾弦較
Dui Lian	對聯
Dui Shu Jian Zhui Bian Fa Shi	對數尖錐變法釋
Dui Shu Tan Yuan	對數探原
Duo Ji	垛積
Duo Ji Bi Lei	垛積比類
Duo Ji Bi Lei Shu Zheng	垛積比類疏証
Duo Ji Shu	垛積術
Duo Ji Yu Yi	垛積餘義
E	
Er Chang Fang Duo	二乘方垛
Er Lu	二率
F	
Fa	法
Fa Qian Ren So Wei Fa	發前人所未發
Fang Cheng Lun	方程論
Fang Dongshu	方東樹
Fang Lu	方輿
Fang Tian	方田
Fang Yizi	方以智
Fang Yuan Chan You	方圓閑幽
Fang Yuan Mi Ji	方圓幂積
Fang Yuanzheng	方元徵
Fang Zhongtong	方中通
Feng Guifen	馮桂芬
Feng Junguang	馮峻光
Feng Yi Wei Shi	奉夷為師
Fu (prefecture)	府
Fu Xue	府學
G	
Ge Wu Ce Suan	格物測算
Ge Wu Ru Meng	格物入門
Ge Wu Tan Yuan	格物探原
Ge Wu Zhong Fa	格物中法

Ge Yuan	割圓
Ge Yuan Lian Bi Li Tu Jie	割圓連比例圖解
Ge Yuan Mi Lu	割圓密率
Ge Yuan Mi Lu Jie Fa	割圓密率捷法
Ge Yuan Shu	割圓術
Gen	根
Gen Shu	根數
Geng Xiang Jian Sun	更相減損
Gong (Prince)	恭親王
Gong Zizhen	龔自珍
Gou	勾
Gou Gu Ge Yuan Ji	勾股割圓記
Gou Gu Qiu Zheng Shu Shu	勾股求整數術
Gu	股
Gu Guanguang	顧觀光
Gu Guanguang Bie Zhuan	顧觀光別傳
Gu Jing Jing She	話經精舍
Gu Shen	顧深
Gu Shuozeng	顧說曾
Guan Xiaoyi	管小異
Guang Fang Yuan Gaun	廣方言館
Guangdong	廣東
Guangxi	廣西
Gue Rong	貴榮
Guo Shi Guan	國史館
Guo Songtao	郭嵩燾
Guo Zi Jian	國子監
H	
Hailing	海寧
Hai Ling Zhou Zhi Guo	海寧州志稿
Han Dynasty	漢朝
Han Wen Ge Wu	漢文格物
Han Wen Sui Shi	漢文歲時
Han Xue Zhuan Men Jing Xue Jia	漢學專門經學家
Han Yingsheng	韓應陞
Hangzhou	杭州

Hanlin Academy

Hebei

He Guozong

He Jian Zhui

Heng Xing Li Shu

Heng Zhai Suan Xue

Hong Rengan

Hongkou

Hu Bu

Hu Pingzhi

Hu Yulin

Hu Shi Qi Mi

Hua Hengfang

Hua Shifang

Huang Zongyi

Huang Zongxian

Hubei

Hunan

Huo Qi Ming Zhong

Huo Qi Zheng Jue

Huo Qi Zhen Jue Jie Zheng

Huo Qi Zhen Jue Shi Li

I

Ili (Yili)

J

Ji Die

Ji He Ji Gou Gu

Ji He Tong Jie

Ji He Yuan Ben

Ji He Yuan Ben Shi Wu Juan

Ji Jiao

Ji Kao

Ji Shu Gou Gu

Ji Shu Hui Qiu

Ji Xue

翰林院

河北

何國宗

合尖鐘

恆星曆書

衡齋算學

洪仁玕

虹口

戶部

胡聘之

胡玉麟

弧矢啟祕

華蘅芳

華世芳

黃宗羲

黃宗憲

湖北

湖南

火器命中

火器真訣

火器真訣解証

火器真訣釋例

伊犁

積疊

幾何即勾股

幾何通解

幾何原本

幾何原本十五卷

積較

季考

級數勾股

級數回求

集學

Ji Yun	紀昀
Jia	甲
Jia Buwei	賈步緯
Jia Jian Cheng Chu Shi	加減乘除釋
Jia Xian	賈憲
Jian Dui	尖堆
Jian Yi An Suan Gao	簡易庵算稿
Jian Zhui	尖錐
Jiang Biao	江標
Jiang Shanting	蔣杉亭
Jiangling	江寧
Jiangnan	江南
Jiangnan Ji Qi Zi Zao Zong Ju	江南機器製造 總局
Jiangsu	江蘇
Jiangxi	江西
Jiang-Zhe	江浙
Jiao Hui Xin Bao	教會新報
Jiao Shi Li Shu	交食曆書
Jiaxing	嘉興
Jie Gen Fang Bi Li	借根方比例
Jie Ji	截積
Jie Ji Qiu Jiao	借積求較
Jie Jian Zhui	截尖錐
Jie Jiao Qiu Jiao	借較求較
Jie Qiu Jie Yi	截球解義
Jingui	金匱
Jin Sheng Sheng Hui Xue Tang	晉省省會學堂
Jin Shi	進士
Jing Ji Zhuan Gu	經籍纂詁
Jing Lu	徑率
Jing Shi Zhi Xue	經世之學
Jing Xue	經學
Jing Xue Jia	經學家
Jingfang	竟芳
Jing-Yuan	金元
Jiu Zhang Suan Shu	九章算術

Ju Ren
Jue Luo Xue

舉人
覺羅學

K

Kai Fang
Kai Fang Bie Shu
Kai Fang Shi
Kai Fang Shuo
Kai Fang Zo Fa Ben Yuan Tu
Kai Pien Fang De Ban Jiao
Kang Xi
Kang Youwei
K'ao-cheng
Kao Gong Ji
Kao Ju
Kao Shu Gen
Kao Shu Gen Si Fa
Kao Zheng
Ke Ju

開方
開方別術
開方式
開方說
開方作法本源圖
開平方得半角
康熙
康有為
考証
考工記
考據
考數根
考數根四法
考証
科舉

L

Li
Li Ciming
Li Duanfen
Li Fang Shi
Li Fengchun
Li Huang
Li Hongzhang
Li Jian Dui
Li Jian Zhui
Li Rensoo
Li Rui
Li San Jiao Dui
Li Shanlan
Li Shanlan Nian Pu
Li Xiang Kao Cheng
Li Xiang Kao Cheng Huo Pian

里
李慈銘
李端棻
立方式
李達春
李潢
李鴻章
立尖堆
立尖錐
李壬叔
李銳
立三角堆
李善蘭
李善蘭年譜
曆象考成
曆象考成後編

Li Yan
 Li Ye
 Li Yunnan
 Li Zhenbo
 Li Zulei
 Lian Fa Biao
 Liang Bi Li
 Liang Hu Academy
 Liang Hu Qichao
 Liao Jiashuo
 Lin De Shu Jie
 Ling De Academy
 Ling Tai Yi Xiang Zhi
 Liu Guangfen
 Liu Hui
 Liu Yi
 Liu Yicheng
 Liu Yueyun
 Lu
 Lu Cheng Qiu Yi Fa
 Lu-Fei
 Lu Li Yuan Yuan
 Lu Lu Zheng Yi
 Luo Shilin
 Lo Ternfeng

M

Ma Jiang
 Manzhou
 Mei Juecheng
 Mei Shi Cong Shu Ji Yao
 Mei Wending
 Meng Xi Bi Tan
 Mian
 Ming Dynasty
 Ming Antu
 Miao Chaoquan

李儼
 李冶
 黎應南
 黎震伯
 李祖烈
 廉法表
 連比例
 兩湖書院
 梁啟超
 廖嘉綬
 麟德衛解
 令德書院
 靈台儀象志
 劉光寶
 劉徽
 劉益
 劉彝程
 劉徽雲
 牟
 屢乘求一法
 陸費
 律曆淵源
 律呂正義
 羅士琳
 駱騰鳳

麻將
 滿州
 梅堯成
 梅氏叢書輯要
 梅文鼎
 夢溪筆談
 面
 明朝
 明安圖
 繆朝銓

Ming Gou
Ming Li
Mo Jia
Mo Hai Shu Guan
Mu
Mu Fu
Mu You

明勾
明理
墨家
墨海書館
畝
摹府
摹友

N

Nan Jing Academy
Nan Jing Wen Cao
Nian Xiyao
Ningpo

南菁書院
南菁文鈔
年希堯
寧波

P

Pao Wu Xian Shuo
Pian Li Jian Zhui
Pian Ti Wen
Pian Ti Wen Jia
Pien Fang Pian Jian Zhui
Pien Fang Zheng Jian Zhui
Pien Jian Dui
Pien Jian Zhui
Pien Yuan Yu Xuan
Pien Yuan Zheng Xuan
Pien San Jiao Dui
Pien San Jiao He Jiao Shu

拋物線說
偏立尖錐
駢體文
駢體文家
平方偏尖錐
平方正尖錐
平尖堆
平尖錐
平圓餘弦
平圓正弦
平三角堆
平三角和較術

Q

Qi
Qian Baocong
Qian Daxin
Qian-Jia
Qian Long
Qian Tang
Qian Xifu
Qin

器
錢寶琮
錢大昕
乾嘉
乾隆
錢塘
錢熙輔
秦

Qin Ding Xue Tang Zhang Cheng	欽定學堂章程
Qin Jiushao	秦九韶
Qin Tian Jian	欽天監
Qing	清
Qiu Cheng Shu Fa	求乘數法
Qiu You Zhai	求友齋
Qiu Zhi Academy	求志書院
Qiuren	秋劬
R	
Ren Xin	仁心
Ren Yuan	人元
Rensu	壬叔
Rong Hong	容閑
Ruan Yuan	阮元
S	
San Cheng Fang Shi	三乘方式
San Cheng Fang Ti	三乘方體
San Cheng Fang Tu	三乘方圖
San Cheng Jian Dui	三乘尖堆
San Cheng Pian Jian Zhui Zhi Ti	三乘偏尖錐之體
San Cheng Zheng Jian Zhui Zhi Ti	三乘正尖錐之體
San Fen	三分
San Jiao Dui	三角堆
San Jiao Duo	三角垛
San Jiao He Jiao Suan Li	三角和較算例
San Jiao Jian Dui	三角尖堆
San Lu	三率
Shang Di	上帝
Dhang Xia Bu Deng Chang Fang Mian Ti	上下不等長方面體
Dhang Xia Bu Deng Tuo Yuan Mian Ti	上下不等橢圓面體
Shandong	山東
Shanshi	陝石
Shao Guang Shi Yi	少廣拾遺

She Xue	社學
Shen Shanzen	沈善蒸
Sheng Xuanhuai	盛宣懷
Sheng Yuan	生員
Shi	矢
Shi Jiao	矢較
Shi Shi Qiu Shi	實事求是
Shi Tuo	釋猶
Shi Xian Zhi	時憲志
Shi Xue	實學
Shi Yuechen	時日醇
Shu Gen Shu Jie	數根術解
Shu Gen Yan Gu	數根演古
Shu Li Jing Yun	數理精蘊
Shu Mu Da Wen	書目答問
Shu Shu	數術
Shu Xue Jiu Zhang	數學九章
Shui Shi Xue Tang	水師學堂
Shun Zhi	順治
Si Jiao Duo	四角垛
Si Jiao Jian Dui	四角尖堆
Si Ku Quan Shu	四庫全書
Si Lu	四率
Si Yuan Jie	四元解
Si Yuan Yu Jian	四元玉鑑
Sichuang	四川
Song	宋
Song-Jin-Yuan	宋金元
Song-Yuan	宋元
Suan Chou	算籌
Suan Fa Tong Bian Ben Mo	算法通變本末
Suan Fa Tong Zong	算法統宗
Suan Fa Yuan Li Gua Non	算法圓理括囊
Suan Jing Shi Shu	算經十書
Suan Xue Guan	算學館
Suan Xue Jia	算學家
Suan Xue Jiao Xi	算學教習

Xuan Xue Ke Yi	算學課藝
Suan Xue Qi Meng	算學啟蒙
Sui Shi	歲試
Sui-Tang	隋唐
Sun Xingyan	孫星衍
Suzhou	蘇州
T	
Tan Tai	談泰
Tan Tian	談天
Tan Tian San You	談天三友
Tang	唐
Tao	道
Ti	體
Tian Gan	天干
Tian Suan Huo Wen	天算或問
Tian Wen Zhi	天文志
Tian Yuan	天元
Tian Yuan Qiu Yi	天之求一
Tian Yuan Shu	天元術
Ting Xue Xuan Shi Cun	聽雪軒詩存
Tongxiang	桐鄉
Tong Wen Guan	同文館
Tong Wen Guan Ti Ming Lu	同文館題名錄
Tong Zhi	同治
Tu	圖
Tu Jie	圖解
Tu Renshuo	屠仁守
Tuo Yuan Qiu Zhou Shu	橢圓求周術
Tuo Yuan Shi Yi	橢圓拾遺
Tuo Yuan Xin Shu	橢圓新術
Tuo Yuan Yu Xuan	橢圓餘弦
Tuo Yuan Zheng Shu Jie	橢圓正術解
Tuo Yuan Zheng Xuan	橢圓正弦
W	
Wang Chang	王昶

Wang Lai	汪萊
Wang Fengzao	汪鳳藻
Wang Mingsheng	王鳴盛
Wang Shiduo	汪士鐸
Wang Tao	王韜
Wang Xishan	王錫闡
Wang Yuanqi	王元敍
Wang Yuezhen	汪曰楨
Wang Zhengxiang	王鎮賢
Wang Zhongxiang	王鍾祥
Wang Zongfu	王宗福
Wang Zongyan	汪宗衍
Wei Fen	微分
Wei Ji Fen	微積分
Wei Ji Suo Yuan	微積溯源
Wei jing Academy	味經書院
Wei Wenkui	魏文魁
Wei Xue	衛學
Wei Yuan	魏源
Wo Ren	倭仁
Wu Bi Li Xian Xin Jie	無比例線新解
Wu Jiashan	吳嘉善
Wu Jin Jie	無盡界
Wu Jing Suan Shu	五經算術
Wu Ling San Ren Yi Shu	武陵山人遺書
Wu Qiqian	吳起潛
Wu Ti	無體
Wu Xing Xing Du Ji	五星行度解
Wu Yuan	物元
Wu Zhaoyi	吳兆圻
Wulin	武林
X	
Xi Fa	西法
Xi Gan	席淦
Xi Ji	隙積
Xi Suan Xin Fa Zhi Jie	西算新法直解

Xi Xue	西學
Xi Yang Xin Fa Li Shu	西洋新法曆書
Xi Zheng	西政
Xia Luanxiang	夏鸞翔
Xian	線
Xian Feng (Emperor)	咸豐
Xian Xue	縣學
Xiang Fa	詳法
Xiang Jie Jiu Zhang Suan Fa	詳解九章算法
Xiang Mingda	項名達
Xiang Shi	鄉試
Xiang Xue Xin Bao	湘學新報
Xiao Ca Xuan Jiao Jiao	小差弦較較
Xiao Kai	小楷
Xiao Xue	小學
Xin Hai	辛亥
Xinlan	心蘭
Xinkui	心葵
Xinmei	心梅
Xiong Fangbo	熊方柏
Xu Dui Shu Jian Fa	續對數簡法
Xu Guangqi	徐光啟
Xu Jixi	徐季溪
Xu Jianyin	徐建寅
Xu Man Jing Lu Suan Xue	須臾精廬算學
Xu Shuo	徐壽
Xu Yi Ji He Yuan Ben	續譯幾何原本
Xu Youren	徐有壬
Xuan	弦
Xuan He Jiao	弦和較
Xue Fengzao	薛鳳祚
Xue Hai Tang	學海堂
Xue Si Yuan Jie	學四元解
Xue Suan Bi Tan	學算筆談
Xue Xiao Zong Lun	學校總論
Xue Zheng	學政
Xun Gu	訓詁

Y

Yan Dunjie	嚴敦傑
Yan Shi Jia Xun	顏氏家訓
Yan Zhitui	顏之推
Yang Guangxian	楊光先
Yang Hui	楊輝
Yang Ma	陽馬
Yang Zhaoyun	楊兆璽
Yangzhou	楊州
Yangtze River	揚子江
Yangzi River	揚子江
Yi Cheng Fang Duo	一乘方垛
Yi Gu Yan Duan	益古演段
Yi Ji Qiu Jiao	以積求較
Yi Jiao Qiu Ji	以較求積
Yi Jing	易經
Yi Lu	一率
Yi Shu	一術
Yi Xue	義學
Yin Nu Gou Gu Hu Qiu Gon Shi	盈朒勾股互求公式
Yong	用
Yong Lo Da Dian	永樂大典
Yong Shu	用數
Yong Zheng (emperor)	雍正
You Deng Shu	有等數
You Jin Jie	有盡界
You Shu	又術
Youjiao Academy	友教書院
Yuan Duo	元垛
Yuan Shu	元數
Tuan Tai	圓台
Yuan Xiao Festival	元宵節
Yuan Zhui San Qu Xian Ji	圓錐三曲線記
Yuan Zhui Qu Xian Shuo	圓錐曲線說
Yue Ke	月課
Yung Wing	容閔

Z

Ze Gu Tang Suan Xue Mu Lu

Ze Gu Xi

Ze Gu Xi Zhai Suan Xue

Zeng Guofan

Zeng Jihong

Zeng Jize

Zhan Zhuang Xiang Chu

Zhang

Zhang Bingsu

Zhang Dunren

Zhang Ju

Zhang Shengzao

Zhang Sigui

Zhang Wenhui

Zhang Zhidong

Zhao Yi

Zhaonan Academy

Zheng Fu Kai Fang Shu

Zhejiang

Zhi Baoran

Zhi Qu Tu Jie

Zhi Wu Xue

Zhongli Yamen

Zhong Ti Xi Yong

Zhong Xi Hui Tong

Zhong Xi Wen Jian Lu

Zhong Xue

Zhou Changqing

Zhou Da

Zhou Dunyi

Zhou Li

Zhou Lu

Zhou Xue

Zhou Zupei

Zhu Kebao

Zhu Shijie

則古堂算學目錄

則古昔

則古昔齋算學

曾國藩

曾紀鴻

曾紀澤

輟轉相除

丈

張秉樞

張敦仁

章句

張盛藻

張斯桂

張文虎

張之洞

趙翼

茗南書院

正負開方術

浙江

支寶楫

致曲圖解

植物學

總理衙門

中體西用

中西會通

中西聞見錄

重學

周長^訥

周達

周敦頤

周禮

周率

州學

周祖培

諸可寶

朱世傑

Zhu Xi	朱熹
Zhuan Gou	蚺勾
Zhuan Xuan	蚺弦
Zhui Shu	綴術
Zi Qiang Yun Dong	自強運動
Zi Qiang Xue Tang	自強學堂
Zi Xue	子學
Zong Xue	宗學
Zu Congzhi	祖沖之
Zu Geng	祖暅
Zunjing Academy	尊經書院
Zuo Boqi	鄒伯奇
Zou Liwen	鄒立文
Zou Ding Xue Tang Zhang Cheng	奏定學堂章程
Zuo Bingrong	左秉隆

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