

ESTIMATION OF AN EMPIRICAL FAVAR MODEL AND DSGE MODEL FOR
EVALUATING EFFECTS OF GOVERNMENT SPENDING IN JAPAN

by

KOHEI FUKAWA

A dissertation submitted to the Graduate Faculty in Economics in partial fulfillment of the
requirements for the degree of Doctor of philosophy, The City University of New York

2012

This manuscript has been read and accepted for the
Graduate Faculty in Economics in satisfaction of the
dissertation requirement for the degree of Doctor of Philosophy.

Thom Thurston

Date

Chair of Examining Committee

Merih Uctum

Date

Executive Officer

Thom Thurston

John Devereux

Terence Agbeyegbe

Supervisory Committee

THE CITY UNIVERSITY OF NEW YORK

ABSTRACT

ESTIMATION OF AN EMPIRICAL FAVAR MODEL AND DSGE MODEL FOR EVALUATING EFFECTS OF GOVERNMENT SPENDING IN JAPAN

by

KOHEI FUKAWA

Adviser: Professor Thom Thurston

This paper studies the effects of government spending on the economy through estimation of an empirical Factor Augmented Vector Autoregression (FAVAR) model and a theoretical DSGE model. We first conducted FAVAR using data for 107 time series from Japan, and found that an increase in government investment and consumption leads to an increase in private consumption and real wages. We then constructed a New Keynesian (NK) general equilibrium model with real and nominal rigidities, including Edgeworth complementarity/substitutability between private and government consumption and productive public capital. The model extends the Bouakez and Rebei (2007) model in three dimensions: constructing an NK model, including intertemporal investment adjustment cost and variable capital utilization as real rigidities, and introducing public capital stocks as an externality to the production function of intermediate goods firms. Our model succeeds in explaining private consumption and real wages increase in response to government expenditure shocks. We estimate key parameters of the model using Bayesian inference and show that private and government consumption are Edgeworth complements and marginal productivity of public capital is productive in Japan.

ACKNOWLEDGMENTS

I would like to thank Prof. Thurston, Thom B., Prof. Agbeyegbe, Terence D., Prof. Devereux, John, for taking on the defense of this paper. I especially grateful to Prof. Thom Thurston, my academic adviser, and Dr. Hiroshi Morita, my classmate, for their continuous assistance and instruction. I had much trouble when I was at NY, but I could get through the doctoral course today thanks to them. I thank Mr. Tatsuyoshi Matsumae (ESRI), Mr. Ryoichi Namba (ESRI) for many helpful discussions and comments since the outset of this dissertation. Also, I would like to thank Dr. Shin-Ichi Nishiyama (Tohoku University), Dr. Hirokuni Iiboshi (Tokyo Metropolitan University), Dr. Takayuki Ogawa (ESRI), Mr. Yasuharu Iwata (Cabinet Office), Dr. Koichiro Iwamoto (ESRI), and Mr. Ryo Hasumi (JCER) for several valuable discussions in the DSGE unit of ESRI. I thank Mr. Takashi Sakuma (Nihon University), Mr. Bunpei Nishizaki (Cabinet Office) Mr. Minoru Masujima (Cabinet Office), and other members of ESRI and the personnel division at the Cabinet Office. They gave me the opportunity and time to write this dissertation at the institute. Finally, I thank my wife and son, Yukiko and Taiyo Fukawa, for their patience and continuous support.

TABLE OF CONTENTS

ABSTRACT	iii
ACKNOWLEDGMENTS	iv
LIST OF TABLES	vi
LIST OF FIGURES	vii
CHAPTER 1. INTRODUCTION	1
CHAPTER 2. EMPIRICAL EVIDENCE	8
CHAPTER 3. THE MODEL	17
3.1 The Representative Household	17
3.2 Firms	22
3.3 The Policy Side	26
3.4 Market Clearing Condition	27
CHAPTER 4. MODEL ESTIMATION	28
4.1 Estimation Methodology	28
4.2 Priors and Calibrated Value	30
4.3 Results: Estimated Parameters	33
4.4 Impulse Response	36
CHAPTER 5. CONCLUSION	47
APPENDIX A. DATA	49
APPENDIX B. LOG LINEARIZED MODEL	54
APPENDIX C. MODEL IMPLICATIONS AND CALIBRATIONS	58
APPENDIX D. BAYESIAN INFERENCE VIA MCMC SIMULATION	77
APPENDIX E. ANOTHER SPECIFICATION FOR THE PRODUCTION FUNCTION	81
REFERENCES CITED	89

LIST OF TABLES

<u>Table</u>		<u>Page</u>
2.1	VAR and FAVAR specifications.	11
2.2	Residual Normality.	12
2.3	Serial correlation of residuals.	12
4.1	Value of calibrated parameters.	29
4.2	Prior distributions for the models estimated.	30
4.3	Posterior distributions for the models estimated	33
4.4	Impact multiplier and present value multiplier.	37
A.1	The list of informational time series for FAVAR	49
C.1	Calibrated parameters.	62
E.1	Prior and Posterior distributions for the model estimated.	81

LIST OF FIGURES

<u>Figure</u>	<u>Page</u>
2.1 Impulse response to IG impact with VAR-FAVAR comparison	14
2.2 Impulse response to CG impact with VAR-FAVAR comparison.	15
4.1 Impulse responses of estimated DSGE model (Baseline Model) (government consumption shock)	37
4.2 Impulse responses of estimated DSGE model (Baseline Model) (government investment shock).	40
4.3 Impulse responses of estimated DSGE model (Baseline Model, Public capital in PF Model) (government investment shock).	41
4.4 Impulse response comparison of empirical model and estimated DSGE model (CG shock)	44
4.5 Impulse response comparison of empirical model and estimated DSGE models (IG shock)	46
C.1 Calibrated Impulse responses of Bouakez and Rebei(2007) model. . .	66
C.2 Comparison of selected impulse responses of RBC models and NK models (CG shock)-1.	68
C.3 Image of Price movement under RBC setup and NK setup	69
C.4 Comparison of selected impulse responses of RBC models and NK models (CG shock)-2	70
C.5 Comparison of selected impulse responses of RBC model and NK model (IG shock)-1	72
C.6 Comparison of selected impulse responses of NK models (IG shock)-2	73
C.7 Comparison of selected impulse responses of different capital utilization models (CG shock)	76
E.1 Impulse responses of estimated DSGE models (Baseline Model, Model with Effective Capital in Production Function) (Government Consumption Shock)	86
E.2 Impulse responses of estimated DSGE models (Baseline Model, Model with Effective Capital in Production Function) (Government Investment Shock)	87

CHAPTER 1.

INTRODUCTION

How does government expenditure affect the economy? This is a very old question. Global financial crisis in the late 2000s precipitated renewed interest in the size of government spending multipliers. Large-scale-discretionary fiscal policies were undertaken, such as the America Recovery and Reinvestment Act of 2009 by the U.S. government,¹ and huge successive economic policy actions in 2008 and early 2009 by the Japanese government. In Japan, a short-term macro model created by ESRI (an Error Correction model) was widely used for government multiplier arguments in Japan (Sakuma et al., 2011). That model (2011 version) suggested a GDP multiplier of government investment equal to 1.07 and 1.14 in the first and second year, respectively.

To assess the effects of government spending, we first conducted an empirical study using a Factor Augmented Vector Autoregression (FAVAR) in this paper. As for empirical studies on fiscal policy effects, there have been several approaches and debates over the effects on private consumption and real wages.

The first approach is the so-called “government spending innovation approach,” which uses a Structural Vector Autoregression (SVAR) model and identifies a government spending shock using timing restrictions, which amounts to the assumption that government spending is unaffected on impact by other shocks (Blanchard and Perotti, 2002; Fatas and Mihov, 2001; Gali et al., 2007²). This approach typically finds that an

¹The act comprised approximately \$500 billion in government spending and \$290 billion in tax cuts, equating, 5.5% of GDP.

²Blanchard and Perotti (2002) assumed the government expenditure shock does not respond to the output residual within a quarter due to fiscal policy lags, and they set the elasticity of output to government spending at zero in their SVAR.

increase in public spending leads to a significant increase in private consumption and real wages.

The second approach is the “dummy variables identification approach,” or the “narrative approach” (Ramey and Shapiro, 1998; Burnside et al., 2004; Ramey, 2011). It attempts to avoid the identification problem inherent in SVAR analysis, and instead searches for fiscal episodes that can be seen as exogenous to the state of the economy. Ramey (2011) argued that large increases in military spending associated with the Korean War, the Vietnam War, the Carter-Reagan military buildup, and *9/11* exemplify such exogenous events. During the episodes of large, exogenous increases in defense spending, output rises but private consumption and real wages fall. Thus this approach produces a smaller government multiplier.

The third approach uses a large dimensional dynamic factor model (Forni and Gambetti, 2010). Several criticisms of the VAR approach to identifying policy shocks focus on the small amount of information used by low dimensional VARs. To conserve degrees of freedom, standard VARs rarely employ more than 10 variables, even though such a small number is unlikely to capture the extent of information that government actually employ. Using low-dimensional VARs means that the measurement of policy innovation is likely to be contaminated. Forni and Gambetti (2010) studied the effects of government spending through a structural dynamic factor model. They used principal components from more than 100 data series, and used sign restrictions for shock identification. They found that government spending lifts both consumption and investment with no evidence of crowding out.

This paper is of this third approach. We conducted FAVAR initiated by Bernanke,

Boivin and Elizas (2005) ³using data containing 107 time series; to make these series, I made reference to the Cabinet Office’s “Monthly Economic Reports” and “Memorandum of Recent Economic Trends,” two publications that, I believe, the government actually uses for analysis of the Japanese economy. To identify government spending shocks, we considered the timing of shocks; government spending shocks are unaffected on impact by other shocks (we used Choleski decomposition).

We compared the results with a preliminary VAR. We suggest that the factors included are Granger cause variable to most other variables included. The impulse response in the VAR and FAVAR model showed both government investment shock and government consumption shock trigger an increase in private consumption and output. Real wages also increased after both shocks, although there were sign changes during the initial periods following a government consumption shock. To the best of our knowledge, this is the first paper to apply FAVAR to analyze fiscal policy shocks in Japan. We found that an increase in government investment and consumption leads to an increase in private consumption and real wages, which is in line with previous empirical studies.

Previous efforts to reconcile this empirical evidence with theoretical modeling have taken several approaches. The first is to adopt a non-separable preference over private consumption and leisure to strengthen their substitution effects following an increase in real wages. Perotti and Monacelli (2008) employed the Greenwood, Hercowitz, and Huffman preference in an NK model, which has a feature to curtail negative wealth

³The work of Forni and Gambetti (2010) differs from FAVAR in that they tried to give the factors themselves a structural interpretation. Unobservable factors in FAVAR do not have precise meanings.

effects on labor supply as well. Their calibrated model fits SVAR impulse responses in that private consumption and real wages increase following a government spending shock. This approach, however, seems to be difficult to determine empirically.

Another approach is to embed the “deep habits” assumption of Ravn et al. (2007) into imperfectly competitive product markets. Their model assumes agents’ “addiction” to consumption goods, which brings the decrease in the elasticity of demand for goods in response to aggregate demand shocks. This decrease in elasticity induces firms to reduce markups and produce more goods, in turn sponsoring the expansionary shift in labor demand. By means of this mechanism, the model succeeded in private consumption and real wages increases in response to a government spending shock. Other papers also embed “deep habits” in their models (Zubairy, 2010).

Next, is the approach that adopts a credit-constrained agent, in which a fraction of consumers are not allowed to optimize over the life-cycle (Gali et al., 2007; Lopez-Salido and Rabanal, 2006; Iwata, 2011). A recognized drawback of this approach is its excessive reliance on the high fraction of credit-constrained agents to obtain a non-negative consumption multiplier.

Here, we adopted an alternate approach that focuses on the presence of flow of government consumption that is able to affect private consumption through modification of its marginal utility and the productive public capital. Our approach is based on the belief that government consumption and investment never deliver waste only; they enhance utility and production in the economy. By introducing Edgeworth complementarity / substitutability between private and government consumption, the latter increases marginal utility of the former and shift the labor supply curve farther outward

if they are complements. This outward shift of the labor supply curve may shift both the YS and IS curves rightward, and we may be able to replicate private consumption crowding-in. By introducing productive public capital, the capital enhance private sector production, endow households with a positive wealth effect, and increase marginal productivity of private inputs.

There have been attempts to explicitly consider Edgeworth complementarity / substitutability between private and government consumption into DSGE models, such as Bouakez and Rebei (2007), and Marattin and Marzo (2010). Bouakez and Rebei (2007) augmented a standard RBC model with complementarity between public and private consumption and habit formation to explain the crowd-in effects of private consumption, and estimated the parameters using U.S. data. They found strong complementarity between government and private consumption, and their estimated model succeeded in replicating the crowding-in effect on private consumption. But their model failed in replicating the real wage movement because their model brings only the increase in labor supply by the government spending shock. Marattin and Marzo (2010) employed an NK equilibrium model with Edgeworth complementarity / substitutability between private consumption and government spending, capital adjustment cost, and distortional fiscal policy rules. Their calibrated model of the Euro area delivered positive private consumption multipliers, but with no explicit focus on the movement in real wages.

Previous research also has attempted to incorporate productive public capital into general equilibrium models, such as Baxter and King (1993) and Leeper et al. (2009).

In addition, in Japan, many previous studies have estimated the production function with public capital, such as Mera (1973), Asako et al. (1994), Mitsui and Inoue (1995), and Kawaguchi et al. (2005). Mitsui and Inoue (1995) estimated the production function using a macroeconomic time series and found the marginal product of public capital to be around 0.25. Kawaguchi et al. (2005) estimated the marginal product of public capital using exogenous variation in the number of seats in the Diet following electoral reform in 1994 as an instrument variable. They found the elasticity to be around 0.2.

This paper builds on these contributions. Our model sections makes three key contributions as follows. First, we extend the Bouakez and Rebei (2007) model in the three respects. Ours is an NK model, adds real rigidity (variable capital utilization) and productive public capital in the production function. Because the model introduces the NK setup and allows labor demand as well shifts expansionary in response to aggregate demand shock, this model succeeds in capturing both private consumption and real wage increases in response to government expenditure shock.⁴

⁴Among studies that consider both productive government investment and Edgeworth complementarity/substitutability between private and government consumption into general equilibrium models, see Mazraani (2010) and Iwata (2012).

Mazraani (2010) augmented a standard RBC model with a distinction to government consumption and investment, using CES specifications over composite consumption and capital. She estimated the parameters using U.S. data, and found that both government consumption and capital complement private consumption and capital, and this produces a crowding-in of private consumption and private investment in response to government expenditure, although this model could not replicate the increase in real wages in initial periods.

Iwata (2012) established a medium-scale small open economy DSGE model augmented with non-separability between private and public consumption, and productive public capital. He estimated the parameters using Japanese data and found that government consumption complement private consumption and that government investment is productive in Japan (marginal productivity of public capital 0.046). His small open economy model is unsuited to display movements of real wages.

Appendix E features a model with another specification for the production function induced by Mazraani (2010).

Second, we estimated key parameters, i.e., elasticity of substitution between private consumption and government consumption or productive public capital share in the production function, jointly with other structural parameters in a general equilibrium model using Japanese data. As for Edgeworth complementarity / substitutability between private and government consumption, we found that private and government consumption are Edgeworth complements, as did Boaukez and Rebei (2007) and Okubo (2003) using U.S. and Japanese data, respectively. As for our parameter for the marginal productivity of public capital in the production function emerged at 0.30, larger than the estimate of 0.25 and 0.20 by Mitsui and Inoue (1995) and Kawaguchi et al. (2005), respectively. Also, our estimated model showed the impact output multiplier to government consumption shock amounts to 1.81, while that to government investment shock is 0.75. In addition, we compared the impulse response of government spending shocks of estimated DSGE model with VAR and FAVAR models.

Third, we show in the appendix that our NK model with investment adjustment cost succeeds in more inertial movement in private investment than under a capital adjustment cost setting in Marattin and Marzo (2010). Also, we show that variable capital utilization helps dampen the large surge in the rental rate of private capital in response to government consumption or government investment shocks, as in Christiano, Eichenbaum and Evans (2005).

The remainder of this paper is organized as follows. Chapter 2 discusses the empirical VAR and FAVAR models as benchmarks. In Chapter 3, we construct the model. Chapter 4 presents estimation results. Chapter 5 concludes the paper.

CHAPTER 2.

EMPIRICAL EVIDENCE

This chapter constructs and estimates a benchmark VAR and FAVAR to illustrate the macroeconomic effects of a government spending shock.¹

FAVAR framework: I would like to first exposit the Factor Augmented VAR.

$$\begin{pmatrix} Y_t \\ F_t \end{pmatrix} = \begin{bmatrix} \Phi_{11}(L) & \Phi_{12}(L) \\ \Phi_{21}(L) & \Phi_{22}(L) \end{bmatrix} \begin{pmatrix} Y_{t-1} \\ F_{t-1} \end{pmatrix} + \begin{pmatrix} u_t^Z \\ u_t^F \end{pmatrix} \quad (2.1)$$

Y_t is the $(M \times 1)$ vector of observable variables and assumed to have pervasive economy-wide effects, and F_t is the $(k \times 1)$ vector of unobserved factors. It captures additional economic information relevant to modeling the dynamics of Y_t .

We extracted unobserved factors from the “informational” time series included in $(N \times 1)$ vector of X_t that consists of a balanced panel of 98 quarterly macroeconomic time-series. The number of informational time series is large and must exceed the number of factors and observed variables in the FAVAR system.

We estimate FAVAR by implementing the two-step estimation (as in Bernanke et al. (2005)).

We assume that the informational time series X_t are related to the unobservable factors F_t by the following observation equation.

$$X_t = \Lambda^f F_t + \Lambda^y Y_t + e_t \quad (2.2)$$

¹To estimate VAR and FAVAR, we used Matlab codes and Eviews code provided by Carlo A. Favero in his HP (<http://didattica.unibocconi.eu/myigier/index.php?IdUte=48917&idr=3933>).

where F_t is a $k \times 1$ vector of common factors, Λ^f is a $N \times k$ matrix of factor loadings, Λ^y is $N \times M$, and e_t is a $N \times 1$ vector idiosyncratic disturbances. e_t has mean zero and is normal, and assumed to have a weak cross-correlation, that vanishes as N approaches infinity.

In the first step, factors are obtained from the observation equation (2.2) by imposing the orthogonality restriction $\widehat{\Lambda}^{f'}\widehat{\Lambda}^f/N = I$. This implies that $\widehat{F} = X'\widehat{\Lambda}^f/\sqrt{N}$, where $\widehat{\Lambda}^f$ is \sqrt{N} times the eigenvectors corresponding to the k largest eigenvalues of XX' sorted in descending order. Stock and Watson (2002) showed that the factors can be consistently estimated by the first r principles of components of X , even in the moderate changes in the loading matrix Λ . For this result to hold, it is important that the estimated number of factors, k , exceeds or equals the true number, r . Because N is sufficiently large, the factors are estimated precisely enough to be treated as data in subsequent regressions.

The second step is to estimate the FAVAR equation (2.1), replacing F_t by \widehat{F}_t .

Data: In our applications, X_t consists of a balanced panel of 98 quarterly macroeconomic time series (Please see Appendix A for details). I took all data from INDB Finder PRO (IN information center). These series are transformed to eliminate unit roots and trends, and for that we took the logarithm and detrended with a one-sided Hodrick-Prescott (H-P) filter to most series. The data span the period from 1985Q3 to 2008Q1. Government investment (\widehat{IG}), government consumption (\widehat{CG}), inflation ($\widehat{\pi}$), private consumption (\widehat{CP}), labor hours (\widehat{N}), private investment (\widehat{IP}), output (\widehat{Y}), real wages (\widehat{W}), and nominal interest rate (\widehat{i}) are the observable factors Y_t (See Appendix

A for data on each).

Estimation: We first estimated the following VAR model, and considered the extension of the baseline VAR model:

$$Y_t = \sum_{i=1}^2 A_i Y_{t-i} + u_t^Y \quad (2.3)$$

where $Y_t = (\widehat{Y}, \widehat{CP}, \widehat{IP}, \widehat{CG}, \widehat{IG}, \widehat{W}, \widehat{N}, \widehat{\pi}, \widehat{i})$ to the following FAVAR framework.

$$\begin{pmatrix} Y_t \\ F_t \end{pmatrix} = \begin{bmatrix} \Phi_{11}(L) & \Phi_{12}(L) \\ \Phi_{21}(L) & \Phi_{22}(L) \end{bmatrix} \begin{pmatrix} Y_{t-1} \\ F_{t-1} \end{pmatrix} + \begin{pmatrix} u_t^Y \\ u_t^F \end{pmatrix} \quad (2.4)$$

$$Y_t = (\widehat{Y}, \widehat{CP}, \widehat{IP}, \widehat{CG}, \widehat{IG}, \widehat{W}, \widehat{N}, \widehat{\pi}, \widehat{i})$$

$$F_t = (F_{1t}, F_{2t}, F_{3t})$$

where F_{1t} , F_{2t} , and F_{3t} are the three factors derived from the following first step.

In the first step, we regressed $X_t = \widehat{\Lambda}^Y Y_t + \varepsilon_t$ firstly, and constructed \widehat{F}_t from $X_t - \widehat{\Lambda}^Y Y_t$ by the principal component method. This is because recursive assumption (Choleski ordering) is employed in the second step and we put the unobservable factor \widehat{F}_t following the observable factors Y_t . This procedure prevents components of unobservable factors \widehat{F}_t from responding contemporaneously to the observable factors Y_t , thereby avoiding the criticism of recursive assumptions.

In the second step, we employed simple recursive frameworks. We ordered \widehat{IG} and \widehat{CG} first and treated its innovations as fiscal policy shocks in a standard way. This

Table 2.1. VAR and FAVAR specifications

Equation		IG	CG	π	CP	N	IP	Y	W	i
VAR	Adj R ²	0.53	0.44	0.16	0.46	0.68	0.85	0.75	0.54	0.95
	S.E.	3.03	0.6	0.28	0.72	0.46	2.2	0.7	0.62	0.046
FAVAR	Adj R ²	0.58	0.45	0.27	0.49	0.72	0.87	0.78	0.58	0.95
	S.E.	2.85	0.59	0.26	0.7	0.43	2.03	0.66	0.59	0.045
	$\chi^2(6)$	14.73	6.96	17.11	10.43	16.24	18.09	14.31	12.47	7.45
		0.02	0.32	0.009	0.11	0.013	0.006	0.026	0.052	0.28

In the figure, IG: Government Investment, CG: Government Consumption, π : Inflation, CP: Private Consumption, N: Labor hours, IP: Private Investment, Y: output, W: Real Wage, i: Nominal Interest rate.

ordering imposes the identifying assumption that all other observable factors ($\widehat{\pi}$, \widehat{CP} , \widehat{N} , \widehat{IP} , \widehat{Y} , \widehat{W} , \widehat{i}) and unobservable factors respond to fiscal policy shocks in the period. We estimate FAVAR by \widehat{IG} , \widehat{CG} , $\widehat{\pi}$, \widehat{CP} , \widehat{N} , \widehat{IP} , \widehat{Y} , \widehat{W} , and \widehat{i} ordering.² We used three latent factors for FAVAR, and ordered them after the observable factors. We used two lags both in VAR and FAVAR, and employing four lags led to similar impulse responses.³ Increasing the number of factors to five did not alter the qualitative nature of our results.

Empirical Result: Table 2.1 compares the VAR and FAVAR specifications for the vector $Y_t = (\widehat{IG}, \widehat{CG}, \widehat{\pi}, \widehat{CP}, \widehat{N}, \widehat{IP}, \widehat{Y}, \widehat{W}, \widehat{i})$. We tested the restriction $\Phi_{12}(L) = 0$ by F-test. The results reported in Table 2.1 illustrate that factors are jointly significant (Granger cause) in most equations; in government investment (\widehat{IG}), inflation ($\widehat{\pi}$), labor hours (\widehat{N}), private investment (\widehat{IP}), and output (\widehat{Y}) equations.

Tables 2.2 and 2.3 report the evidence on the residual analysis from the VAR and FAVAR models. Table 2.2 contains the outcome of Jarque and Bera (1980) tests of null hypothesis of normality of residuals from each equation and for the joint 9 equation

²We followed Bouakez and Rebei(2007) for this VAR ordering.

³By the Akaike info criterion and Schwarz criterion, two lags are showed preferable to four lags.

Table 2.2. Residual Normality

Equation	Jaque-Bera	IG	CG	π	CP	N	IP	Y	W	i	Joint
		$\chi^2(2)$	$\chi^2(2)$	$\chi^2(2)$	$\chi^2(2)$	$\chi^2(2)$	$\chi^2(2)$	$\chi^2(2)$	$\chi^2(2)$	$\chi^2(2)$	$\chi^2(18)$
VAR		5.99	14.82	13.72	0.28	0.39	0.41	0.65	6.67	13.77	56.7
		0.05	0.001	0.001	0.87	0.82	0.81	0.71	0.04	0.001	0.000
FAVAR		0.97	24.78	4.01	1.49	0.05	0.05	0.01	0.52	13.23	45.1
		0.62	0.000	0.13	0.47	0.98	0.98	0.99	0.77	0.001	0.000

In the figure, IG: Government Investment, CG: Government Consumption, π : Inflation, CP: Private Consumption, N: Labor hours, IP: Private Investment, Y: output, W: Real Wage, i: Nominal Interest rate.

Table 2.3. Serial correlation of residuals

LM		LAG 1	LAG 2	LAG 3	LAG 4
VAR	$\chi^2(81)$	103.8	87.32	87.65	85.85
		0.045	0.3	0.29	0.33
FAVAR	$\chi^2(144)$	152.02	163.12	148.17	162.56
		0.31	0.13	0.39	0.14

model. The null of normality is rejected for both VAR and FAVAR in the joint 9 equation model; the main cause of this rejection is the non-normality of residuals in \widehat{CG} and \widehat{i} equations.

Table 2.3 reports the Breush-Godfrey Lagrange Multiplier tests of null hypothesis of no serial correlation of residuals at all lags from one to four. The result showed in the first lag of VAR specification points toward residual autocorrelations, while the null hypothesis of absence of residual correlation at any lags cannot be rejected in FAVAR specifications.

We then proceed to a further comparative analysis of the VAR and FAVAR models by considering impulse response function to fiscal policy shock. The results are shown in Figures 2.1 and 2.2. We plot responses for a horizon of 20 periods of quarterly $Y_t = (\widehat{IG}, \widehat{CG}, \widehat{\pi}, \widehat{CP}, \widehat{N}, \widehat{IP}, \widehat{Y}, \widehat{W}, \widehat{i})$ series to a fiscal policy shock (government

investment and government consumption) of both VAR and the FAVAR model.

Figure 2.1 shows the impulse response to one standard deviation government investment shock. The red line shows the response of the FAVAR model, and the blue line is that of the VAR model. The purple and green lines show 95% confidence intervals to the shocks of the VAR.

Figure 2.1 shows there are differences in impulse response between the VAR and FAVAR models. We can see that a government investment shock triggers an increase in private consumption (2nd row, 1st column) and output (3rd row, 1st column) by both the VAR and FAVAR models. The response of consumption and output are persistent, and reach their peak in the second period, and last for more than three years in both models. Real wages (3rd row, 2nd column) also increase after a government investment shock for both models, while the increase is stronger in FAVAR specification. As for the response of private investment (2nd row, 3rd column), government investment crowds out private investment for all the time periods by VAR specification, while we can see strong crowds in for three years by the FAVAR model. We can also see the nominal interest rate increase clearly from the FAVAR model (3rd row, 3rd column).

Figure 2.2 shows the impulse response to one standard deviation government consumption shock. Figure 2.2 also shows the differences in impulse response between the VAR and FAVAR models. We can see that a government consumption shock triggers an increase in private consumption (2nd row, 1st column) and output (3rd row, 1st column) both by models here as well. As for the response of private consumption, it very persistently increases and reaches its peak in the second period by the FAVAR model. VAR impulse has a larger response than the FAVAR impulse; it reaches its peak

Figure 2.1: Impulse response to IG impact with VAR-FAVAR comparison

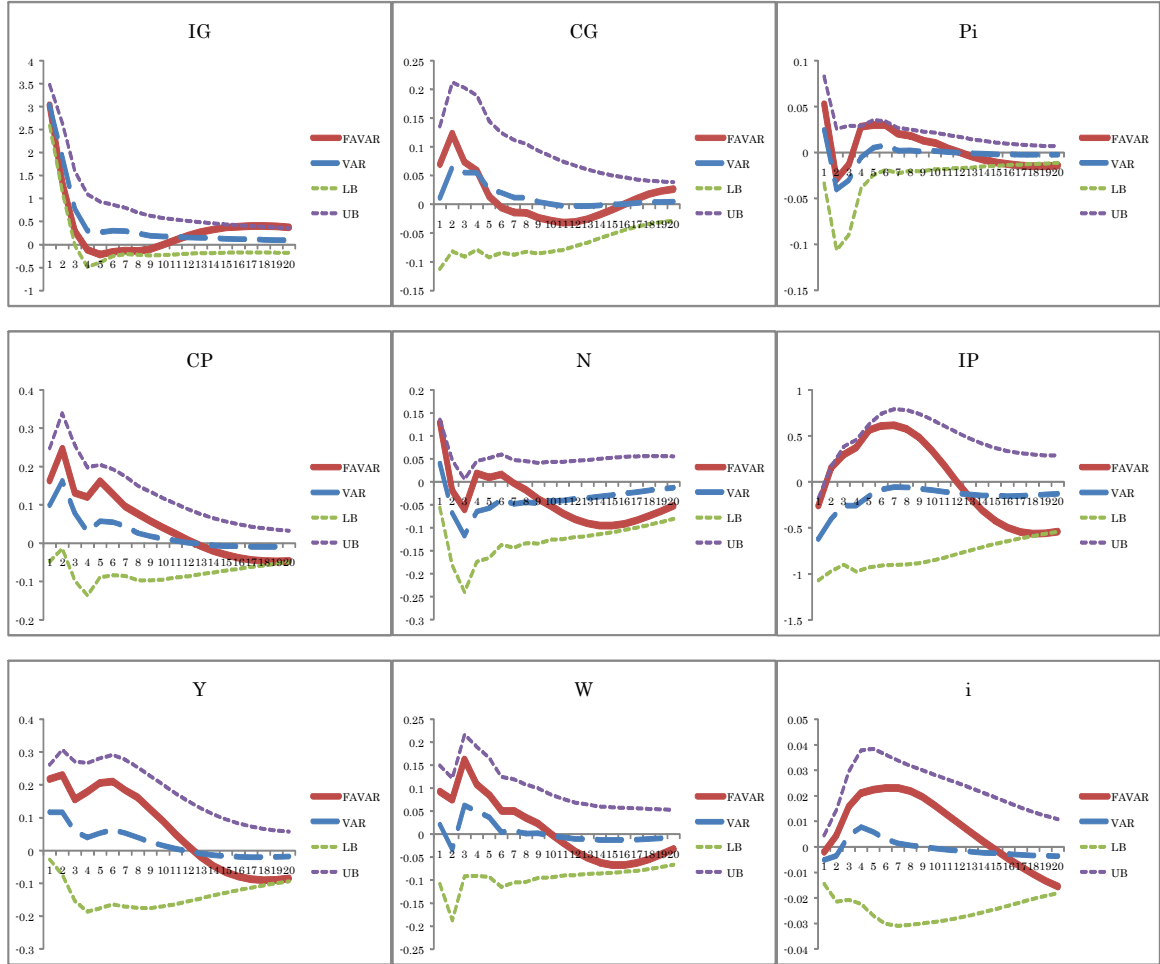


Figure 2.1 shows impulse responses of 1std. government investment shock. In the figure, IG: Government Investment, CG: Government Consumption, Pi: Inflation, CP: Private Consumption, N: Labor hours, IP: Private Investment, Y: output, W: Real Wage, i: Nominal Interest rate. FAVAR (Red solid line): Impulse response of FAVAR, VAR (Blue solid line): Impulse response of VAR, LB, UB (Green and Purple dotted Lines): Lower and Upper 95% confidence intervals of VAR.

Figure 2.2: Impulse response to CG impact with VAR-FAVAR comparison

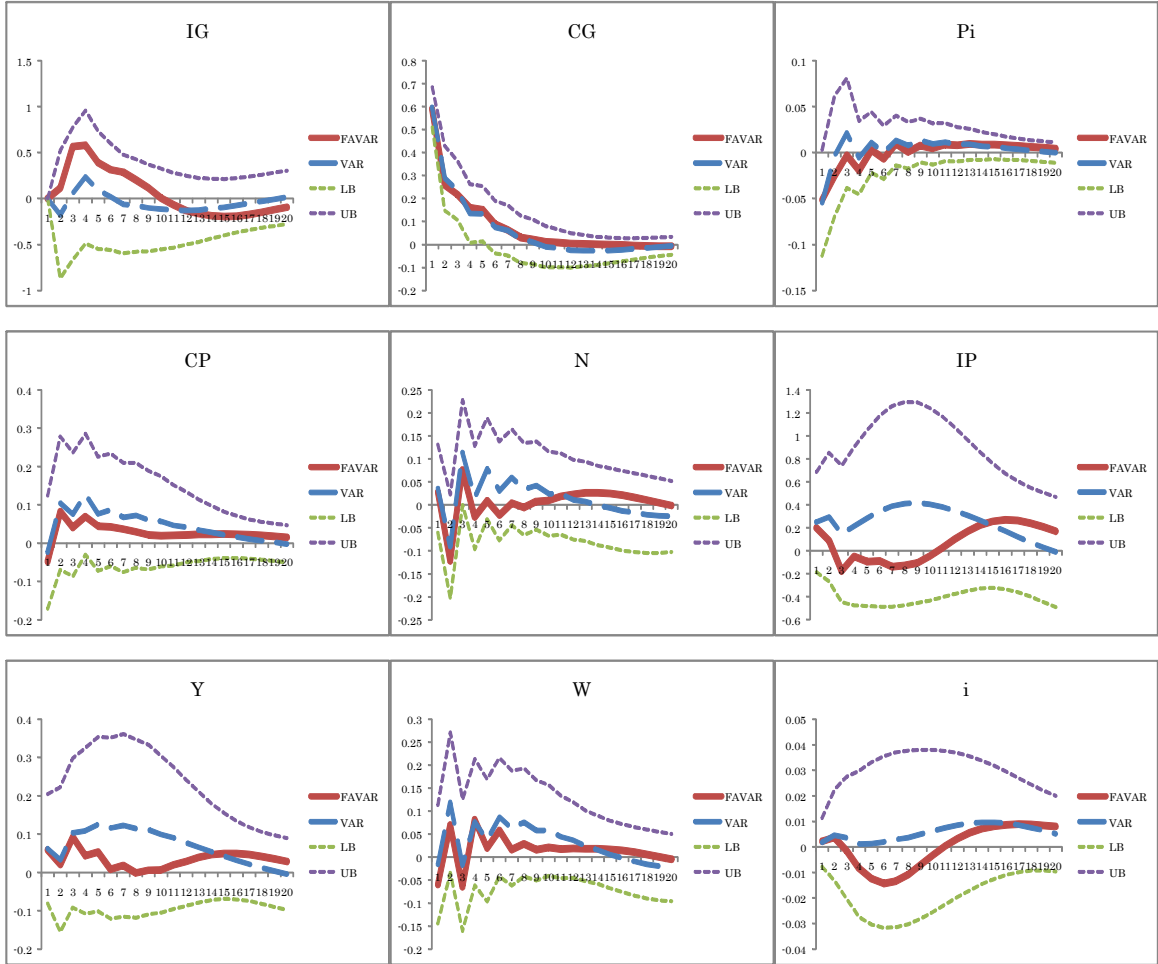


Figure 2.2 shows impulse responses of 1std. government consumption shock. In the figure, IG: Government Investment, CG: Government Consumption, Pi: Inflation, CP: Private Consumption, N: Labor hours, IP: Private Investment, Y: output, W: Real Wage, i: Nominal Interest rate. FAVAR (Red solid line): Impulse response of FAVAR, VAR (Blue solid line): Impulse response of VAR, LB, UB (Green and Purple dotted Lines): Lower and Upper 95% confidence intervals of VAR.

in fourth period. The response of output has the same tendency. It reaches its peak in 3rd period and increase for 2 years by FAVAR model. The response of real wages (3rd row, 2nd column) also seems to increase after a government consumption shock, but changes signs three times within the first four quarters for both models. As for the response of private investment (2nd row, 3rd column), a government consumption shock crowds out for the first 10 quarters by the FAVAR model, while we can see persistent crowds in movement from the VAR model.

CHAPTER 3.

THE MODEL

The model constructed extends Bouakez and Rebei (2007) model in three dimensions. First, it is an NK model. Second, we include intertemporal investment adjustment cost and variable capital utilization as real rigidities. Third, we consider both government consumption and government investment and assume that government's capital stock enters the production function.

3.1 The Representative Household

The economy is populated by a single, infinitely lived representative agent that drives utility from effective consumption (\tilde{C}_t) and leisure ($1 - N_t$). Effective consumption is assumed to be the CES index of private consumption (C^p) and government consumption (C^g).¹

$$u(\tilde{C}_t, \tilde{C}_{t-1}N_t) = \frac{1}{1-\varepsilon} \left(\frac{\tilde{C}_t}{\tilde{C}_{t-1}^\gamma} \right)^{1-\varepsilon} + \iota \ln(1 - N_t) \quad (3.1)$$

$$\tilde{C}_t = \left[\phi C_t^p \frac{v-1}{v} + (1-\phi) C_t^g \frac{v-1}{v} \right] \frac{v}{v-1} \quad (3.2)$$

where ε stands for the inverse of the long-run intertemporal elasticity of substitution, and ε and ι are positive parameters. $\gamma \in (0, 1)$ measures the degree of habit forma-

¹Previous studies that assume consumer preferences to be the CES index of private consumption and government spending include Amano and Wirjanto (1997), Okubo (2003), Linnemann and Schabert (2003), Boualez and Rebei (2007), Marrattin and Marzo (2010), and Mazraani (2010).

tion.² v is the elasticity of substitution between private consumption and government consumption and $v > 0$.

The budget constraint of the representative household is

$$C_t^p + I_t^p + \psi(U_t)K_t^p + \frac{B_t}{P_t} + Z_t \leq w_t N_t + r_t U_t K_t^p + \frac{B_{t-1}R_{t-1}}{P_t} + \frac{\Xi_t}{P_t} \quad (3.3)$$

where B_t is government debt issued at time t , which pays $B_t R_t$ units of goods at $t+1$. Z_t is the lump-sum tax paid to government. w_t is the real wage rate. N_t is the number of hours worked. Ξ_t denotes dividends from owning imperfectly competitive intermediate goods firms.

An explanation of the households' private capital holdings is necessary at this point. In this model, households make decision not only as for consumption/leisure choice but also as for private capital accumulation and capital utilization.³ Households lend their private capital K_t^p to intermediate goods firms and earn the real rental rate r_t from each effective unit of private capital $U_t K_t^p$. Thus, $r_t U_t K_t^p$ represents household earnings from supplying the effective unit of private capital, and households increase their earnings by increasing their capital utilization rate.

On the other hand, the increase of capital utilization from steady states incurs a cost, $\psi(U_t)K_t^p$. We assume the functional form for the adjustment cost of capital utilization is given by

²We assume habit formation for effective consumption following Boualez and Rebei (2007).

³We introduce variable capital utilization as in Christiano, Eichenbaum, and Evans (2005), who assumed households make capital accumulation and capital utilization decisions. This assumption was for convenience; they indicated they could work with an alternate decentralization scheme, in which firms make these decisions, albeit at the cost of a more complicated notation.

$$\psi(U_t) = \kappa \frac{U_t^{1+\Psi^{-1}} - 1}{1 + \Psi^{-1}}$$

where Ψ is the inverse of elasticity of the capital utilization with respect to the rental cost of private capital and $\Psi > 0$. Note that $\psi(U_t)$ is an increasing, convex function ($\psi'(U_t) > 0$, $\psi''(U_t) > 0$). The steady-state capital utilization rate is assumed to be $U = 1$; thus in steady states, cost is zero ($\psi(1) = 0$). Private capital evolves according to

$$K_{t+1}^p = (1 - \delta^p)K_t^p + \{1 - S(\frac{I_t^p}{I_{t-1}^p})\}I_t^p \quad (3.4)$$

where δ^p is the private capital depreciation rate, and $S(\bullet)$ denotes adjustment cost for private capital defined as a quadratic function as follows.

$$S(\frac{I_t^p}{I_{t-1}^p}) = \frac{1}{2\xi}(\frac{I_t^p}{I_{t-1}^p} - 1)^2 \quad (3.5)$$

where ξ implies the inverse of the elasticity of private investment on the price of private capital. As seen from (3.5), the greater the deviation of the current physical investment from the previous period, the more expensive the adjustment cost. Therefore, households make private capital installments as smoothly as possible to minimize the leak. Also, we note that the adjustment cost is zero in steady states, i.e., $S(1) = S'(1) = 0$.

Given the budget constraint (3.3) and capital accumulation equation (3.4), the

dynamic optimizing problem of the household is

$$\begin{aligned}
\mathcal{L} = & E_0 \sum_{t=0}^{\infty} [\beta^t u(\tilde{C}_t, \tilde{C}_{t-1} N_t) \\
& - \lambda_t \{ C_t^p + I_t^p + \psi(U_t) K_t^p + \frac{B_t}{P_t} + Z_t - w_t N_t - r_t U_t K_t^p - \frac{B_{t-1} R_{t-1}}{P_t} - \frac{\Xi}{P_t} \} \\
& - \mu_t \{ K_{t+1}^p - (1 - \delta^p) K_t^p - (1 - \frac{1}{2\xi} (\frac{I_t^p}{I_{t-1}^p} - 1)^2) I_t^p \}]
\end{aligned} \tag{3.6}$$

where λ_t stands for the Lagrange multiplier attached to the budget constraint and μ_t stands for the Lagrange multiplier attached to the capital accumulation equation at period t .

The first order conditions associated with each control variables, B_t , I_t^p , N_t , C_t^p , K_{t+1}^p , U_t are

bond holdings:

$$\lambda_t = \beta E_t [\lambda_{t+1} R_t \frac{P_t}{P_{t+1}}] \tag{3.7}$$

private investment:

$$\lambda_t = \mu_t [1 - \frac{1}{2\xi} - \frac{3}{2\xi} (\frac{I_t^p}{I_{t-1}^p})^2 + \frac{2}{\xi} (\frac{I_t^p}{I_{t-1}^p})] - \beta E_t \mu_{t+1} [\frac{1}{\xi} (\frac{I_{t+1}^p}{I_t^p})^2 (1 - \frac{I_{t+1}^p}{I_t^p})] \tag{3.8}$$

labor supply:

$$\lambda_t = \frac{l}{w_t (1 - N_t)} \tag{3.9}$$

private consumption:

$$\lambda_t = \phi (\frac{\tilde{C}_t}{C_t^p})^{\frac{1}{\nu}} \{ (\frac{1}{\tilde{C}_{t-1}^{1+\gamma}}) (\frac{\tilde{C}_t}{\tilde{C}_{t-1}^\gamma})^{-\varepsilon} - \beta \gamma E_t [(\frac{\tilde{C}_{t+1}}{\tilde{C}_{t+1+\gamma}}) (\frac{\tilde{C}_{t+1}}{\tilde{C}_t^\gamma})^{-\varepsilon}] \} \tag{3.10}$$

private capital holdings:

$$\mu_t = \beta E_t[\lambda_{t+1}\{r_{t+1}U_{t+1} - \psi(U_{t+1})\} + \mu_{t+1}(1 - \delta^p)] \quad (3.11)$$

capital utilization:

$$r_t = \psi'(U_t) \quad (3.12)$$

As for the first order condition with the private investment (3.8), the LHS represents the marginal cost of private investment. The RHS represents the marginal benefit of private investment. By investing one additional unit the household can increase the amount of private capital stock to a certain extent, but the magnitude of increase in private capital stock is reduced by the leak from private capital installments. This marginal leak is represented in the first bracket in (3.8). In addition, we set the adjustment cost function as (3.5), a marginal change in current investment also affects the next period's adjustment cost, and this effect is represented in the second bracket. Both of these effects multiplied by the shadow price of private capital form the marginal benefit from additional investment.

As for the first order condition with private consumption (3.10), the LHS is the marginal cost of private consumption. The RHS of the equation (3.10) represents the marginal utility of private consumption. $\phi(\frac{\tilde{C}_t}{C_t^p})^{\frac{1}{\nu}}$ represents the additional amount of effective consumption from one unit of additional private consumption. The second bracket represents the marginal utility from one unit of effective consumption, and the expectation term in the second bracket comes from an internal habit formation in the utility function. Notice that the second bracket reduces to $\tilde{C}_t^{-\varepsilon}$ only if current effective

consumption matters ($\gamma = 0$).

Next, equation (3.11) represents the first order condition for private capital holdings. The LHS represents the shadow price of private capital, or the marginal cost of adding one unit of private capital at time t . The RHS represents the marginal benefit of adding one unit of private capital. By adding one unit to private capital holdings at time t , households can expect to increase $(1 - \delta^p)$ units of private capital at time $t + 1$. Thus, $\beta\mu_{t+1}(1 - \delta^p)$ represents a marginal benefit of adding one unit of private capital discounted in present value. In addition, households can earn additional rental income $r_{t+1}U_{t+1}K_{t+1}^p$ by lending additional private capital, but the rental income is reduced by the additional capital utilization cost $\psi(U_{t+1})K_{t+1}^p$ brought on by the additional lending of private capital.

Finally, the first order condition for capital utilization (3.12) represents the marginal cost of capital utilization is to be equalized to the rental rate. By increasing capital utilization marginally, the household can increase its income by $r_t K_t^p$. On the other hand, an increase in capital utilization incurs a cost. By increasing the capital utilization marginally, households must sacrifice $\psi'(U_t)$ amount of consumption goods at the margin.

3.2 Firms

The final goods firms We assume that there is a continuum of intermediate goods firms

of unit mass indexed by $i \in [0, 1]$ and each firm produces an intermediate good that is different from that of other firms. The continuum of intermediate goods in period t ,

$Y_{i,t}$ gets bundled by final goods firms into final goods Y_t . The final goods production technology is

$$Y_t = \left[\int_0^1 Y_{i,t}^{\frac{\eta-1}{\eta}} di \right]^{\frac{\eta}{\eta-1}} \quad (3.13)$$

where $\eta > 1$, and η is the elasticity of substitution in production and governing the firm's markup over the marginal cost.

A profit-maximizing final goods firm chooses the amount of intermediate goods to maximize profit given aggregate price P_t and intermediate goods price $P_{i,t}$. We could have a demand function of goods i as,

$$Y_{i,t} = \left(\frac{P_{i,t}}{P_t} \right)^{-\eta} Y_t \quad (3.14)$$

Putting this demand for sector i 's output (3.14) into the bundler function (3.13),

we can have a final goods pricing rule of,

$$P_t = \left[\int_0^1 P_{i,t}^{1-\eta} di \right]^{\frac{1}{1-\eta}} \quad (3.15)$$

The intermediate goods firms The production function for intermediate goods firm i is

$$Y_{i,t} = A_t \widetilde{K}_{i,t}^{\alpha} N_{i,t}^{(1-\alpha)} (K_t^G)^{\mu} \quad (3.16)$$

where $\widetilde{K}_{i,t}^p = U_{i,t} K_{i,t}^p$ is the effective private capital stock by the firm i , $N_{i,t}$ is the aggregate labor force employed by the firm i , and all firms are subject to the same technology shock, A_t . K_t^G is the government capital stock, and the parameter μ measures the productivity of public capital. Notice that K_t^G does not have subscript i ,

thus we assume K_t^G as well has common productivity to each firm.⁴ This production function is increasing return to scale as a whole but private sector resource is a constant return to scale.^{5,6}

Under a Calvo (1983)-type sticky price setting, for any given period t , each firm has a ρ probability that it will keep the price of the previous period, and $1 - \rho$ probability that it will be able to choose its price optimally. An intermediate goods firm, i , which can choose price in period t , choose the price, $P_{i,t}^*$, to maximize profit:

$$\max_{P_{i,t}^*} E_t \sum_{j=0}^{\infty} \beta^j \rho^j [P_{i,t}^* Y_{t+j} (\frac{P_{t+j}}{P_{i,t}^*})^\eta - P_{t+j} r_{t+j} \widetilde{K}_{i,t+j}^p - P_{t+j} w_{t+j} N_{i,t+j}] \quad (3..17)$$

$$s.t. Y_{t+j} (\frac{P_{t+j}}{P_{i,t}^*})^\eta = A_t \widetilde{K}_{i,t}^p{}^\alpha N_{i,t}^{1-\alpha} (K_t^G)^\mu \quad (3..18)$$

A firm that is maximizing profit is simultaneously minimizing total cost. The cost minimization problem for the firm i can be expressed as follows.

$$\text{cost function: } \min_{\widetilde{K}_{i,t}^p, N_{i,t}} r_t \widetilde{K}_{i,t}^p + w_t N_{i,t} + mc_{i,t} (Y_{i,t} - A_t \widetilde{K}_{i,t}^p{}^\alpha N_{i,t}^{1-\alpha} (K_t^G)^\mu) \quad (3..19)$$

where the Lagrange multiplier, $mc_{i,t}$, is the marginal cost of producing $Y_{i,t}$. Solving the cost minimization, the first order condition is

$$\frac{w_t}{r_t} = \frac{(1 - \alpha) \widetilde{K}_{i,t}^p}{\alpha N_{i,t}} \quad (3..20)$$

solve (3.20) for $\widetilde{K}_{i,t}^p$ and $N_{i,t}$, and substituting these factor demands into the cost equations,

⁴This is important to make aggregation of price simple. If K_t^G is a function of i , marginal cost will be function of i .

⁵As for production function, Mitsui and Inoue (1995) showed that, in Japan, assuming a constant return to scale for the private sector is better than assuming a constant return to scale as a whole (including productive public capital) from a production function estimation using prefectural data.

⁶In Appendix E, I made another specification of the production function following the spirit of Mazraani (2010).

$$TC_{i,t} = r_t \widetilde{K}_{i,t}^p + w_t N_{i,t} = \frac{Y_{i,t}}{A_t} \frac{(1-\alpha)^{\alpha-1}}{\alpha^\alpha} r_t^\alpha w_t^{1-\alpha} \frac{1}{(K_t^G)^\mu} \quad (3..21)$$

so, the firm i 's marginal cost is

$$mc_{i,t} = \frac{TC_{i,t}}{Y_{i,t}} = \frac{1}{A_t} \frac{(1-\alpha)^{\alpha-1}}{\alpha^\alpha} r_t^\alpha w_t^{1-\alpha} \frac{1}{(K_t^G)^\mu} \quad (3..22)$$

Notice that the specification of the marginal cost (3.22) does not depend on subscript i , and this implies the marginal cost is symmetric across firms. Since the marginal cost is symmetric across firms, we simply suppress subscript i on the marginal cost.

Substituting the total costs (3.21) to profit maximization problems (3.17) yields,

$$\begin{aligned} \max_{P_{i,t}^*} E_t \sum_{j=0}^{\infty} \beta^j \rho^j [P_{i,t}^* Y_{t+j} \left(\frac{P_{t+j}}{P_{i,t}^*}\right)^\eta - P_{t+j} r_{t+j} \widetilde{K}_{i,t+j}^p - P_{t+j} w_{t+j} N_{i,t+j}] \\ = E_t \sum_{j=0}^{\infty} (\beta \rho)^j [(P_{i,t}^* - P_{t+j} mc_{t+j}) Y_{i,t+j}] \end{aligned} \quad (3..23)$$

The first order condition for the profit maximization problem yields

$$E_t \sum_{j=0}^{\infty} (\beta \rho)^j Y_{i,t+j} [1 - \eta + \eta \frac{P_{t+j}}{P_{i,t}^*} mc_{t+j}] = 0$$

Rearranging further yields the following optimal pricing rule for firm i .

$$P_{i,t}^* = \frac{\eta}{\eta - 1} \frac{E_t \sum_{j=0}^{\infty} (\beta \rho)^j (P_{t+j} Y_{i,t+j} mc_{t+j})}{E_t \sum_{j=0}^{\infty} (\beta \rho)^j Y_{i,t+j}} \quad (3..24)$$

Notice that $\eta > 1$, and the expression $\frac{\eta}{\eta-1} > 1$ is the gross markup of the intermediate goods firm i 's price over the ratio of the discounted stream of nominal total costs divided by the discounted stream of real output.

Notice that all intermediate goods firms that can fix their prices set the same markup over the same marginal cost, so in every period t , $P_{i,t}^*$ is the same for all $1 - \rho$ firms that adjust their prices, and all non-adjusting firms keep their price as it was in the previous period. Combining with the final goods pricing rule (3.15), we have the aggregate goods pricing rule,

$$P_t^{1-\eta} = \rho P_{t-1}^{1-\eta} + (1 - \rho) P_t^{*1-\eta} \quad (3..25)$$

3.3 The Policy Side

Fiscal Authority The fiscal authority purchases final goods C_t^G , I_t^G , issues bonds B_t , and levies lump-sum tax Z_t . The flow budget constraint for fiscal authority is,

$$C_t^G + I_t^G + \frac{R_{t-1}B_{t-1}}{P_t} = Z_t + \frac{B_t}{P_t} \quad (3..26)$$

Government spending C_t^G , I_t^G evolve according to an AR(1) stochastic process.

$$\widehat{C}_t^G = \rho_{CG} \widehat{C}_{t-1}^G + \varepsilon_t^{CG} \quad (3..27)$$

$$\widehat{I}_t^G = \rho_{IG} \widehat{I}_{t-1}^G + \varepsilon_t^{IG} \quad (3..28)$$

government capital stock evolve according to

$$K_{t+1}^G = I_t^G + (1 - \delta^G) K_t^G \quad (3..29)$$

where δ^G is the government capital depreciation rate.

Monetary Policy The central bank sets the nominal interest rate according to a simple feedback rule of the following rule.⁷

⁷This type of Taylor rule with interest-rate inertia can be found in Rotemberg and Woodford (1999), Clarida, Gali, and Gertler (1999), and Christiano, Eichenbaum, and Evans (2005).

$$\widehat{R}_t = \rho_r \widehat{R}_{t-1} + (1 - \rho_r) \phi_{r\pi} \widehat{\pi}_{t-1} + (1 - \rho_r) \phi_{ry} \widehat{Y}_t \quad (3..30)$$

where $\widehat{\pi}_{t-1} \equiv \log(P_{t-1}/P_{t-2})$ denote inflation rate.

3.4 Market Clearing Condition

We impose the market-clearing condition for the final goods market. We require the supply of final goods to be equal to the demand of final goods for private consumption, private investment, capital utilization, government consumption, and government investment.⁸

$$Y_t = C_t^p + I_t^p + \psi(U_t)K_t^p + C_t^G + I_t^G \quad (3..31)$$

⁸A Log-linearized version of the model could be found in Appendix B.

CHAPTER 4.

MODEL ESTIMATION

4.1 Estimation Methodology

The purpose of this chapter is to use Japanese data to obtain values for the model parameters. In particular, we are interested in measuring the extent of complementarity between government consumption and private consumption and public capital share in the production function. We describe below our estimation methodology briefly following Boaukez and Rebei (2007) and Iiboshi, Nishiyama and Watanabe (2008).

The model's solution can be written in the following recursive equilibrium law of motion:

$$s_t = G(\theta)s_{t-1} + H(\theta)\varepsilon_t \quad (4.1)$$

where s_t is a 24×1 vector of endogenous variables: $s_t = [\widehat{Y}_t, \widehat{\pi}_t, \widehat{C}_t, \widehat{C}_t^P, \widehat{I}_t^P, \widehat{\lambda}_t, \widehat{\mu}_t, \widehat{N}_t, \widehat{w}_t, \widehat{r}_t, \widehat{mc}_t, \widehat{U}_t, \widehat{K}_{t+1}^P, \widehat{K}_{t+1}^G, \widehat{R}_t, \widehat{A}_t, \widehat{I}_t^G, \widehat{C}_t^G, E_t\widehat{\pi}_{t+1}, E_t\widehat{C}_{t+1}, E_t\widehat{\lambda}_{t+1}, E_t\widehat{\mu}_{t+1}, E_t\widehat{I}_{t+1}^P, E_t\widehat{r}_{t+1}]'$, and ε_t is a vector of endogenous shocks: $\varepsilon_t = [\varepsilon_t^A, \varepsilon_t^{IG}, \varepsilon_t^{CG}]'$, and θ is the vector of deep parameters to be estimated. From equation (4.1), we set a state space model which consists of a transition equation and measurement equation as follows;

$$s_t = G(\theta)s_{t-1} + H(\theta)\varepsilon_t \quad (4.2)$$

$$y_t = Js_t \quad (4.3)$$

where y_t is the 9×1 vector of observable variables at time t and J is a 9×24 matrix that links the observable y_t vector to the unobservable s_t vector. For this state space model with Gaussian error terms, unobservable variables s_t and the likelihood of the model

are obtained using a Kalman filter. The Kalman filter is the algorithm that provides the mean and the covariance matrix of the state vector s_t ($t = 1, \dots, T$) conditional on the observations up to t , i.e., (y_1, \dots, y_T) in a linear Gaussian state space model.¹

A crucial requirement of the Kalman filter is that the number of observable variables used in the estimation does not exceed the number of shocks in the model; otherwise, the variance-covariance matrix of the residuals becomes singular. In our case, we have only three structural shocks, so we can use as little as three series. To circumvent this problem, we followed Boaukez and Rebei (2007) and add measurement errors to the variables in the measurement equation.² This yields the following empirical model:

$$s_t = G(\theta)s_{t-1} + H(\theta)\varepsilon_t \quad (4.4)$$

$$y_t = Js_t + \zeta_t \quad (4.5)$$

where $\varepsilon_t \sim i.i.d.N(0, Q(\theta))$, ζ_t is the vector of measurement errors, $\zeta_t \sim i.i.d.N(0, R)$, and $E(\varepsilon_t \zeta_t') = 0$.

For estimation, we used DYNARE, which is a convenient tool for conducting Bayesian estimations. The DYNARE toolbox derives reduced-form representation of the model and automatically checks system stability; a necessary condition are met only if the number of forward-looking variables equals the number of eigenvalues greater than one. In addition, it enables us to conduct Bayesian estimations.³ The series used

¹In Appendix D, we will briefly explain the Kalman filter.

²The addition of measurement errors to get around the singularity problem has been done by McGrattan, Rogerson and Wright (1997), Ireland (2004), and Boakez and Rebei (2007), Nishiyama et al (2011).

³DYNARE firstly evaluate likelihood using Kalman filter by maximum likelihood estimates. Then DYNARE finds the mode of posterior distribution and estimate posterior distribution of parameters of our interest. The posterior distribution is given by a nonlinear and complicated function of deep

Table 4.1 Value of Calibrated parameters

Parameter	Meanings	Value
Structural parameters		
ϕ	Weight of Private spending in effective consumption	0.8
α	Elasticity of output with respect to private capital	0.36
δ_p	Depreciation rate of Private Capital	0.025
δ_G	Depreciation rate of Public Capital	0.01
η	Price Elasticity of Demand	6
Steady-State Value		
N	Fraction of Time worked	0.31
CG/Y	Government consumption-to-output ratio	0.15
IG/Y	Government investment-to-output ratio	0.05

in the estimation are output, private consumption, private investment, government consumption, government investment, real wages, labor hours, nominal interest rate, and inflation rate. We used the common data with Chapter 2, but we limited the data period from 1985Q3 to 1998Q4 because of the zero-interest-rate policy started in February 1999 in Japan.⁴

4.2 Priors and Calibrated Value

Table 4.1 presents the values assigned to the calibrated parameters. We fix most parameters following Bouakez and Rebei (2007);⁵ $\alpha = 0.36$, which implies a labor share of 64%, $\beta = 0.99$, which implies an annual steady state interest rate of 4%, $\delta_P = 0.025$, which puts the annual steady state depreciation of private capital at 10%, $\delta_G = 0.01$, which means the annual steady state depreciation of public capital is 4%. The price

parameters, so DYNARE resorts to sampling-like methods, a Metropolis-Hastings algorithm. Again, in Appendix D, we will briefly explain the Kalman filter and the MCMC Bayesian estimation.

⁴Zero- Lower band of nominal interest rate brings model nonlinearity, which is a difficult task. We skip here this problem by limiting our data period. Doi et al. (2011) showed monetary policy response in Japan (Taylor rule coefficient on inflation or output gap) have been changed with high probability in the mid 1990s by an estimation of monetary policy function with Markov switching model.

⁵Bouakez and Rebei (2007) analyzed U.S. data, but the calibrated value used here could be thought of as common to Japan.

Table 4.2. Prior distributions for the models estimated

parameters	meanings	type	mean	s.e.
structural parameters				
σ	the Elasticity of substitution between private and public consumption	normal	0.8	0.5
ε	the inverse of the inter-temporal elasticity of substitution	normal	2	2
γ	Habit-formation parameter	beta	0.6	0.2
ξ	Inverse of adjustment-cost parameter	normal	0.25	0.75
Ψ	Capital utilization cost	normal	1	1
ρ	Calvo price no-revise probability	beta	0.7	0.2
μ	Productivity of public capital	beta	0.2	0.1
Policy parameters				
ρ_r	Interest rate smoothing coeff.	beta	0.8	0.1
ρ_π	interest rate inflation coeff.	normal	1.5	1
ρ_y	interest rate output gap coeff.	normal	0.125	0.075
Shock persistence				
ρ_a	persistence of productivity	beta	0.85	0.1
ρ_{cG}	persistence of government consumption	beta	0.85	0.1
ρ_{IG}	persistence of government investment	beta	0.85	0.1
Standard Errors of shocks				
η_a	SE of productivity shock	inv. Gamma	0.4	2
η_{cG}	SE of government consumption shock	inv. Gamma	0.2	2
η_{IG}	SE of government investment shock	inv. Gamma	0.3	2
Standard Errors for Measurement Errors				
η_Y	SE of measurement err. for output gap	inv. Gamma	0.2	2
η_{CP}	SE of measurement err. for private consumption	inv. Gamma	0.2	2
η_{IP}	SE of measurement err. for private investment	inv. Gamma	1	4
η_{cG}	SE of measurement err. for government consumption	inv. Gamma	0.1	2
η_{IG}	SE of measurement err. for government investment	inv. Gamma	0.5	4
η_N	SE of measurement err. for real wage	inv. Gamma	0.1	2
η_W	SE of measurement err. for labor hour	inv. Gamma	0.1	2
η_π	SE of measurement err. for inflation	inv. Gamma	0.1	2
η_i	SE of measurement err. for nominal interest rate	inv. Gamma	0.05	1

elasticity of demand η is set equal to 6 to be consistent with a steady state net markup of 20 percent. A preliminary attempt to estimate ϕ turned out to be poorly identifiable, so we set the same calibrated value of 0.8 as Boaukez and Rebei (2007) did, but we could estimate parameter ε because we included nominal interest rate and inflation rate as observed data while Boaukez and Rebei (2007) did not. We also set steady state value at $\bar{N} = 0.31$, $\bar{CG}/\bar{Y} = 0.15$, $\bar{IG}/\bar{Y} = 0.05$ following Boaukez and Rebei (2007).

Table 4.2 reports the prior distribution of the other estimated parameters. For the choice of prior distribution, the means were set at values that correspond with other studies, and standard errors were set so that the domain covers a reasonable range of

parameter values, including values estimated by previous studies. We primarily cited Boaukez and Rebei (2007).⁶ In addition, we made reference to Iiboshi, Nishiyama, and Watanabe (2008), Iwata (2010) and Sugo and Ueda (2008), all of whom estimated a DSGE model for the Japanese data.

We set mean of $\varepsilon = 2$, which is the value given in Boaukez and Rebei (2007), and we took a wide standard error 2 for the parameter. We set the mean at $\nu = 0.8$, which implies private and government consumption are substitutes ($\frac{1}{\nu} - \varepsilon < 0$) in the prior mean, but we took a standard error of $\nu = 0.5$, which covers the range of estimated value of ν in Boaukez and Rebei (2007) 0.3. γ, ξ, ρ could be seen in ordinary range and similar to previous studies. As for Ψ , there seems to be wide difference in prior setting in previous studies. Smets and Wouters (2003), which studies the euro zone, set its mean at 0.2 and its standard error at 0.075, while Onatski and Williams (2005), which studies U. S. data, set uniform distribution between 2.8 – 10. Here, we set prior mean and standard error of Ψ to 1.⁷ As for the parameter of productivity of public capital, μ , we made reference to Aschauer (1989) 0.36, 0.25 by Mitsui and Inoue (1995), or 0.20 by Kawaguchi et al. (2005).⁸ We made reference to Iwata (2011) for prior distribution of policy parameters and standard errors of shocks and to Sugo and Ueda (2008) for prior distribution of shock persistence.

Given the prior distributions, DYNARE calculates the posterior distributions us-

⁶To reiterate, Bouakez and Rebei (2007) estimated their model parameters using U.S data.

⁷This is the same prior mean and standard error of Ψ used by Sugo and Ueda(2008) although the specification of capital utilization is different.

⁸Mitsui and Inoue (1995) estimated production function using macroeconomic time series and found the marginal product of public capital to be around 0.25. Kawaguchi et al. (2005) estimated marginal product of public capital using prefecture-level data and exogenous variation in number of seats in the Diet by electoral reform in 1994 as an instrument variable. They found the elasticity to be around 0.2, although the effect is not precisely estimated.

ing a Metropolis-Hastings Markov chain Monte Carlo (MCMC) algorithm. We sample 1,000,000 periods each, discarding the first 500,000 periods.

4.3 Results: Estimated Parameters

Table 4.3 reports the posterior means and 90% confidence intervals (CI) for the parameters for 2 models estimated; “baseline model” means a model without public capital (the public capital productivity parameter μ is set to 0), and “public capital in PF” means public capital is in production function (μ is estimated). Several important parameters are worth commenting comparing with Bouakez and Rebei (2007). (We mostly comment on Baseline model below.)

The parameters for elasticity of substitution ν is 0.56 and the parameter of inverse of the intertemporal substitution of consumption ε is 0.95, so $\frac{1}{\nu} - \varepsilon > 0$. This means the necessary condition for government spending to increase the marginal utility of consumption is satisfied, and private and government consumption are Edgeworth complements.⁹ This result is the same with Bouakez and Rebei (2007) for U.S. data; they had $\nu = 0.33$ (and ε is calibrated value 2). An earlier study by Okubo (2003), which analyzed data in Japan and used a partial equilibrium approach based on Euler equation, estimated $\nu = 1.39$ and $\varepsilon = 0.19 \sim 0.61$ (so $\frac{1}{\nu} - \varepsilon = 0.12 \sim 0.53 > 0$) and concluded that private consumption and government consumption are complements or unrelated, so this is also in the same line.¹⁰

⁹We show model implications using calibrations in Appendix C.

¹⁰Iwata (2012) also concluded the relationship between private and public consumption may be complements using Japanese data, although the definition of effective consumption is different from us here (Iwata (2012) used a linear form, $\widehat{C}_t = C_t^P + vC_t^G$).

Table 4.3. Posterior distributions for the models estimated

parameters	Baseline Model			Public Capital in PF			(Ref.) Bouakez and Rebei (2007)
	mean	CI(Low)	CI(High)	mean	CI(Low)	CI(High)	
structural parameters							
ν	0.557	0.380	0.714	0.589	0.372	0.782	0.332
ε	0.950	0.510	1.353	0.982	0.562	1.361	2(calibrated value)
γ	0.483	0.170	0.796	0.497	0.188	0.810	0.250
ξ	1.684	1.148	2.210	1.656	1.126	2.180	1.052
Ψ	2.109	0.867	3.328	2.062	0.827	3.291	N.A
ρ	0.605	0.441	0.765	0.601	0.440	0.765	N.A
μ	N.A.	N.A.	N.A.	0.295	0.108	0.484	N.A
Policy parameters							
ρ_r	0.772	0.625	0.923	0.772	0.624	0.924	N.A
ρ_x	3.015	1.880	4.083	3.010	1.928	4.111	N.A
ρ_y	0.053	0.004	0.101	0.055	0.005	0.104	N.A
Shock persistence							
ρ_a	0.820	0.743	0.898	0.820	0.740	0.900	0.873
ρ_{ce}	0.727	0.565	0.888	0.727	0.559	0.896	0.912
ρ_{IG}	0.891	0.846	0.937	0.903	0.855	0.951	N.A
Standard Errors for Structural Shocks							
η_a	0.377	0.094	0.699	0.356	0.095	0.674	0.005
η_{CG}	0.181	0.047	0.334	0.220	0.045	0.372	0.013
η_{IG}	0.282	0.068	0.537	0.250	0.070	0.466	N.A
Standard Errors for Measurement Errors							
η_y	0.327	0.042	0.509	0.188	0.045	0.360	—
η_{CP}	0.178	0.046	0.323	0.213	0.046	0.360	—
η_{P}	0.875	0.242	1.604	0.915	0.234	1.712	—
η_{CG}	0.095	0.022	0.172	0.106	0.022	0.188	—
η_{IG}	0.484	0.111	0.920	0.503	0.111	0.935	—
η_N	0.088	0.023	0.163	0.090	0.023	0.169	—
η_w	0.086	0.023	0.158	0.091	0.024	0.164	—
η_x	0.098	0.022	0.179	0.094	0.023	0.172	—
η_i	0.051	0.011	0.097	0.056	0.011	0.103	—

The parameter for consumption habit formation γ is about 0.48, which is larger than Bouakez and Rebei (2007) 0.25. The parameter for capital utilization cost, Ψ , is 2.1 and this is similar to Onatski and Williams (2004) 2.8. The parameter of inverse of investment adjustment costs, ξ is 1.7. This implies that investment increases 1.7 percent in the long run following a 1 percent increase in Tobin's q . This estimated parameter is larger than that found in other studies, i.e. Levin et al. (2005) 0.55 or Smets and Wouters (2007) 0.14.¹¹

Regarding inflation dynamics, the Calvo price-setting parameter ρ is 0.61, so the probability that a given price can be optimized in a quarterly period ($1 - \rho$) is 0.39. This implies an average contract duration of price setting is estimated to be about 2.5 quarters.

Regarding monetary policy parameters, the coefficient on lagged interest rates in the monetary policy rule ρ_r is 0.82. This implies that monetary policy has high inertia. The response of interest rate to inflation ρ_π is 3, which is much greater than one, indicates that the monetary authority in Japan reacts very actively to inflation. On the contrary, the response to output, ρ_Y is small, at 0.05.

As for the results of “public capital in production function,” our interest is in the parameter of productivity of public capital in production function μ , and it is 0.30. This is smaller than Aschauer's (1989) 0.36 but larger than the estimate of 0.25 by Mitsui and Inoue (1995), 0.2 by Kawaguchi et al. (2005), or 0.046 by Iwata (2012). Other parameters estimated are similar to the baseline case, so we omit an explanation

¹¹Although Bouakez and Rebei (2007) estimated under capital adjustment cost setting, the parameter for adjustment cost is 1.05, and is smaller than ours.

of these results here.

4.4 Impulse Response

This section explores the impulse responses of endogenous variables to government investment and government consumption shocks for five years following each shock. The horizontal axis represents time on a quarterly scale, and the vertical axis represents the percentage deviation from equilibrium.

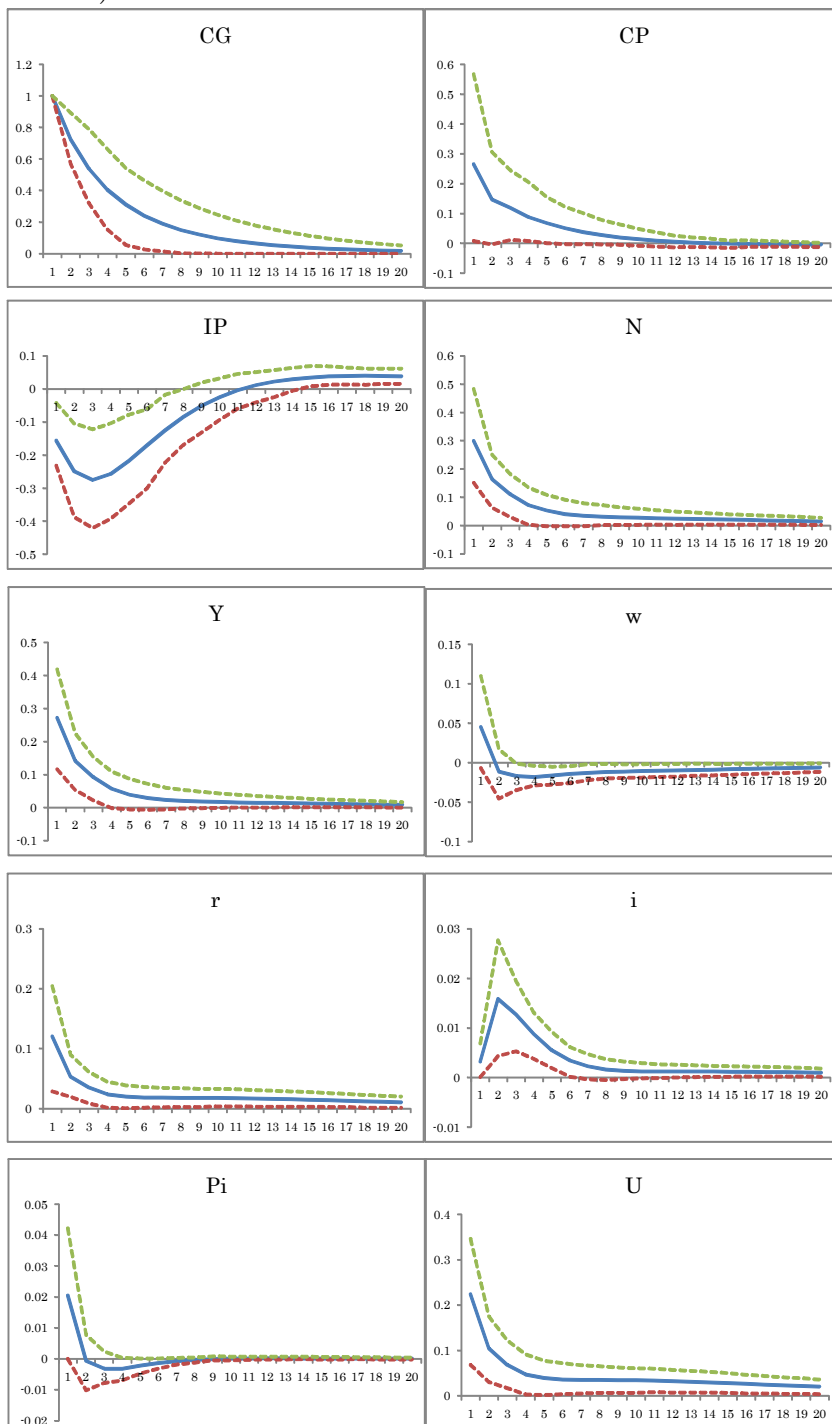
Figure 4.1 illustrates impulse responses to a government consumption shock. It captures important channels that determine the impact of a government consumption shock: crowding-out effects, wealth effects, and the effects from private and government consumption Edgeworth complementarity.

First, higher government consumption absorbs existing resources, thereby reducing the quantity of goods available to the private sector for saving (investment) or consumption. In addition, because supply should respond to this increase in aggregate demand, demand for labor and effective private capital increase. Thus the real marginal cost of intermediate goods firms increases because not all intermediate goods firms can raise their price. Real wages and the rental cost of private capital initially increase although the increase in capital utilization alleviate the surge in the rental cost of private capital.

Second, higher government consumption financed by a lump-sum tax generates a negative wealth effect, encouraging agents to work harder, thereby increasing output but reducing the marginal productivity of labor.

Third, when private and government consumption are Edgeworth complements,

Figure 4.1: Impulse responses of estimated DSGE model (Baseline Model) (government consumption shock)



CG: Government Consumption, CP: Private Consumption, IP: Private Investment, N: Labor hours, Y: output, w: Real wage, r: Rental rate of private capital, i: Nominal Interest rate, Pi: Inflation, U: Capital utilization rate.
Blue Solid line: Impulse responses, Green and Red dotted Lines: Upper and Lower bounds of 90% HPD interval.

Table 4.4 Impact Multiplier and Present Value Multiplier

Variable	1 quarter	4 quarters	8 quarters	12 quarters	20 quarters
Baseline Model					
$\Delta Y / \Delta CG$	1.81	1.41	1.28	1.28	1.34
$\Delta CP / \Delta CG$	1.04	0.91	0.88	0.85	0.80
$\Delta IP / \Delta CG$	-0.22	-0.49	-0.60	-0.58	-0.46
$\Delta Y / \Delta IG$	0.75	0.71	0.72	0.70	0.86
$\Delta CP / \Delta IG$	-0.17	-0.19	-0.21	-0.24	-0.29
$\Delta IP / \Delta IG$	-0.07	-0.16	-0.21	-0.23	-0.22
Model with Productive Public Capital in Production Function					
$\Delta Y / \Delta CG$	1.67	1.33	1.21	1.21	1.27
$\Delta CP / \Delta CG$	0.88	0.78	0.77	0.74	0.70
$\Delta IP / \Delta CG$	-0.20	-0.45	-0.56	-0.53	-0.43
$\Delta Y / \Delta IG$	0.76	0.62	0.68	0.84	1.23
$\Delta CP / \Delta IG$	-0.04	-0.03	0.04	0.11	0.26
$\Delta IP / \Delta IG$	-0.18	-0.35	-0.36	-0.27	-0.02

government consumption increases the marginal utility of private consumption, thus people consume more in the current period. On the other hand, this leads to an increase in the supply of labor and a decrease in real wage.

Let us examine the impulse responses. As for private consumption, the effect of Edgeworth complementarity dominates the crowding-out effects from our parameters estimated. As for real wages, the increase of real marginal cost dominates for short and it increases in the initial period. All in all, through these channels agents increase private consumption, decrease saving (investment), and work more. The increase in output exceeds the increase in government consumption, and we can see this point more clearly in Table 4.4.

In Table 4.4, we show fiscal multipliers for output, private consumption, and private investment. We followed Leeper, Plante and Traum (2010) and Mountford and Uhlig (2009) and report present value multipliers, which take into account the overall

dynamics associated with fiscal shocks and properly discount future macroeconomic effect;

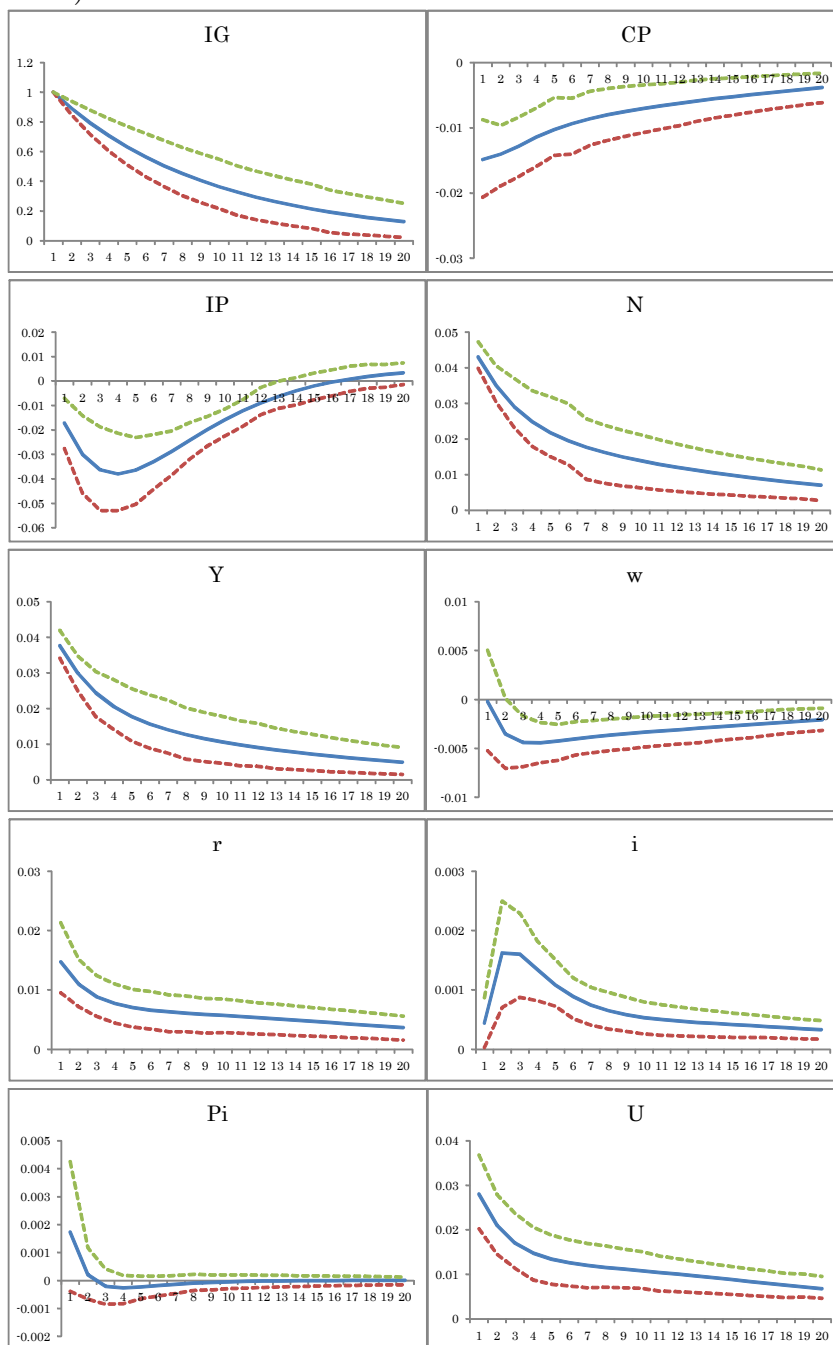
$$\text{Present-Value Multiplier}(k) = \frac{E_t \sum_{j=0}^k \left(\prod_{i=0}^j R_{t+i}^{-1} \right) \Delta Y_{t+j}}{E_t \sum_{j=0}^k \left(\prod_{i=0}^j R_{t+i}^{-1} \right) \Delta CG_{t+j}}$$

Note that the present-value multiplier at $k = 1$ is equal to impact multiplier at period 1. The impact output multiplier to CG shock amounts to 1.81.

Figures 4.2 and 4.3 illustrate impulse responses to a government investment shock. Figure 4.2 illustrates the impulse response in the Baseline model (model without public capital in the production function). Figure 4.3 compares the impulse response shown in the Baseline model with that shown in the model that includes productive public capital. These figures capture important channels, crowding-out effects, wealth effects, and the effects from productive public capital on changes in the marginal productivity of private inputs.

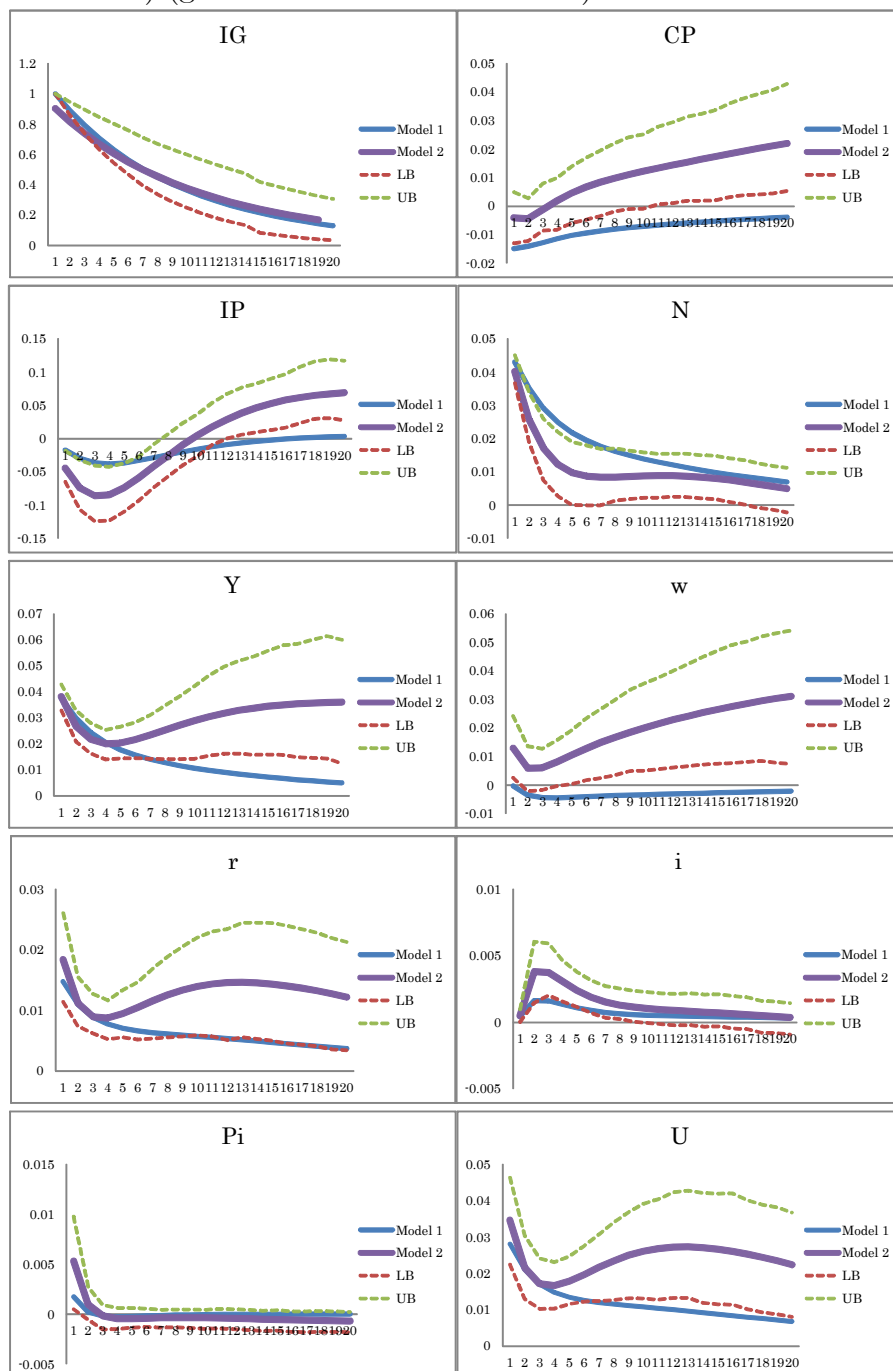
In Figure 4.2, the first two channels function as in the CG shock: greater government investment absorbs existing resources, leaving less for the private sector to save (invest) or consume. As noted above, supply should respond to this increase in aggregate demand. Demand for labor and effective private capital increase, and real marginal cost increases for intermediate goods firms because not all firms can adjust prices. Thus, real wages and the rental cost of private capital initially increase. Greater government investment financed by lump-sum tax generates a negative wealth effect, encouraging agents to work harder. This leads to an increase in output but a decrease

Figure 4.2: Impulse responses of estimated DSGE model (Baseline Model) (government investment shock)



IG: Government Investment, CP: Private Consumption, IP: Private Investment, N: Labor hours, Y: output, w: Real wage, r: Rental rate of private capital, i: Nominal Interest rate, Pi: Inflation, U: Capital utilization rate
 Blue Solid line: Impulse responses, Green and Red dotted Lines: Upper and Lower bounds of 90% HPD interval.

Figure 4.3: Impulse responses of estimated DSGE model (Baseline Model, Public capital in PF Model) (government investment shock)



IG: Government Investment, CP: Private Consumption, IP: Private Investment, N: Labor hours, Y: output, w: Real wage, r: Rental rate of private capital, i: Nominal Interest rate, Pi: Inflation, U: Capital utilization rate.

Model 1 (Blue Solid line) : Impulse response of Baseline Model, Model 2 (Purple Solid line) : Impulse response of Model with productive public capital, Green and Red dotted Lines: Upper and Lower bounds of 90% HPD interval of Model 2.

in marginal productivity of labor (real wages is affected by a decreasing effect).

Thus, Figure 4.2 shows that agents reduce consumption and saving (investment) and work more. As for real wages, the impulse response lies in the negative region because the initial effect of the increase in real marginal cost was insufficient to dominate the second effect of decreasing marginal productivity of labor, as estimated by our parameters. Output rises less than government investment. As Table 4.4 indicates, the output multiplier of the IG shock is 0.75 and never exceeds 1 during the initial period.

In Figure 4.3, however, another wealth effect arises from the opposite direction when public capital is productive, and the productivity of public capital changes the marginal productivity of private inputs. As for wealth effects, the enlarged stock of productive public capital sponsors an increase in total factor productivity and raises households' expectation that they can afford more goods in the future. This expectation discourages current saving and encourages current consumption. We see from Figure 4.3 that private investment declines more and private consumption declines less during initial periods than in the baseline model.

In the model that includes productive public capital, the marginal product of labor and private capital increases over longer horizons as the stock of public capital accumulates gradually, resulting in higher real wages and returns to private capital. Note that in this model the variable capital utilization dampens the surge in rental cost of private capital. The incentive to invest relatively cheap private capital during later periods is apparent in Figure 4.3. As for real wages and rental cost of private capital, the third effect of an increase in marginal productivity of labor or private capital dominates especially during the later periods, and the impulse responses show

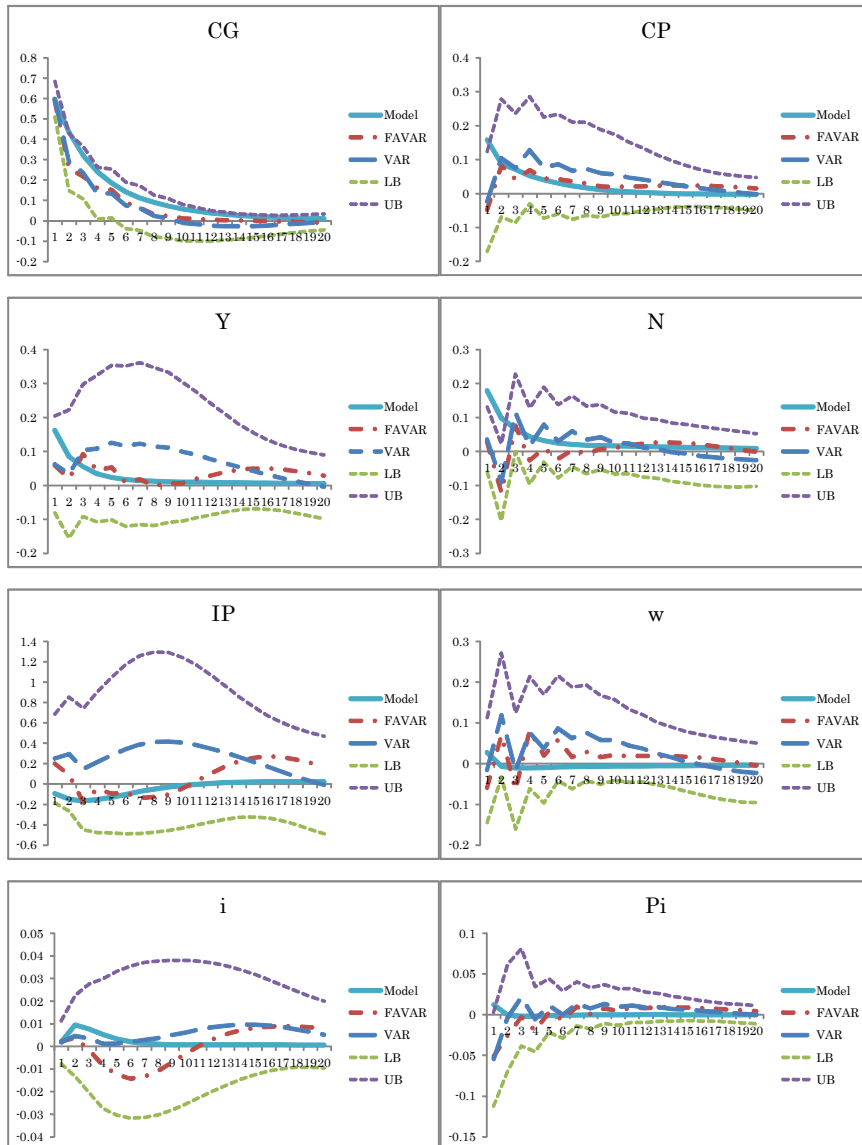
a very strong effect . We should also mention that the impulse response of real wages always come up to the positive region. Table 4.4 reveals that the output multiplier of an IG shock during the initial period is 0.76—not significantly different from the baseline case. However, after a period of 20 quarters, the output multiplier in the model featuring productive public capital reaches 1.23, whereas that in the baseline model remains at 0.86.

Finally, we would like to show the comparison of impulse responses of empirical models and estimated DSGE models.

In Figure 4.4, we showed the impulse response to government consumption shocks of the estimated DSGE model (light blue solid line), the FAVAR model (red dotted line), the VAR model (dark blue dotted line), and the confidence intervals of the VAR model (green and purple dotted line). Overall, Figure 4.4 shows the estimated model succeeds in replicating impulse response obtained in VAR and FAVAR. In particular, it generates remarkably well-matched movement of private consumption, output, and labor hours. In these cases, the model-based response is similar to the ones of empirical models both in terms of magnitude and persistence. I should also mention that this model showed better movement in real wages, while Bouakez and Rebei (2007) pointed out that this aspect was deficient in their model.

In Figure 4.5, we showed the impulse response to government investment shocks of empirical models and two estimated DSGE models; one is the baseline model (Model 1, light blue solid line) and the other is the model with productive public capital in production function (Model 2, orange solid line). Overall, Figure 4.5 shows the estimated baseline model (the model without productive public capital in production function)

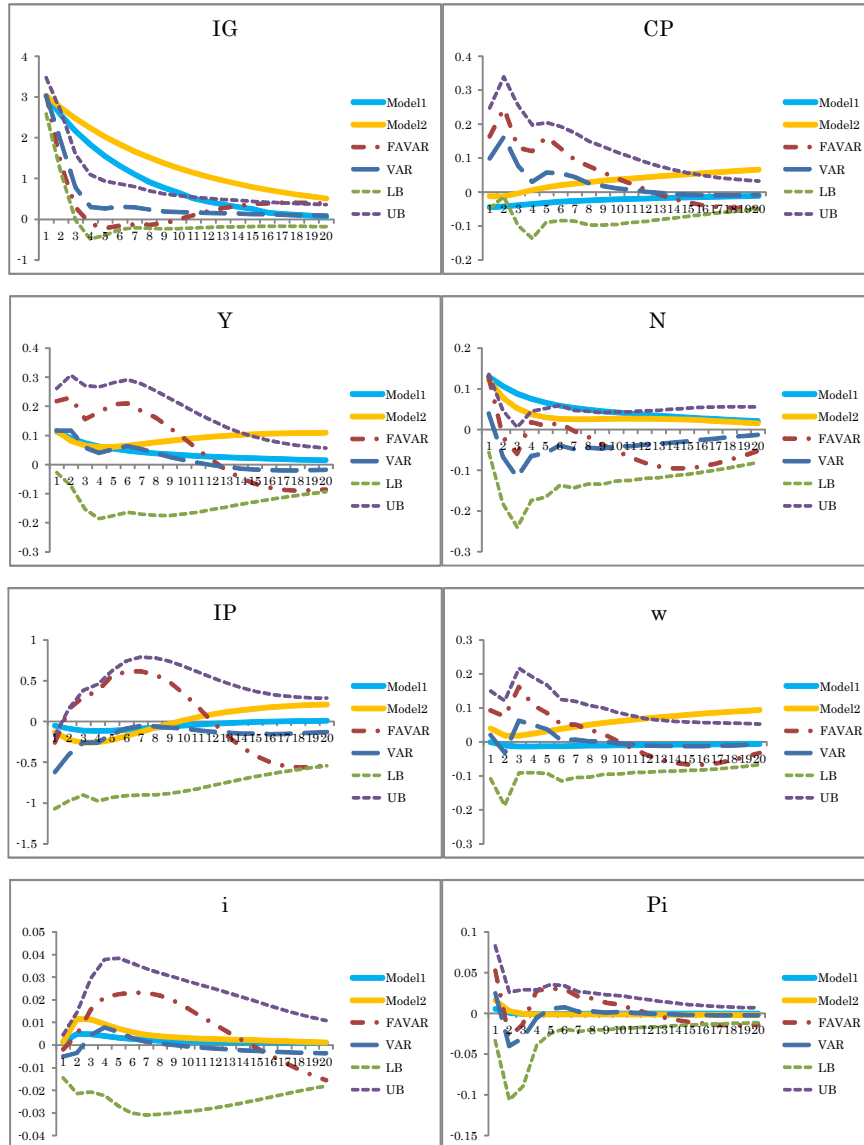
Figure 4.4: Impulse response comparison of empirical model and estimated DSGE model (CG shock)



In the figure, CG: Government Consumption, CP: Private Consumption, Y: output, N: Labor hours, IP: Private Investment, w: Real wage, i: Nominal Interest rate, Pi: Inflation.

Model (Light Blue Solid line): Impulse response of Baseline Model, FAVAR (Red dotted line): Impulse response of FAVAR, VAR (Blue dotted line): Impulse response of VAR, LB, UB (Green and Purple dotted Lines): Lower and Upper 95% confidence intervals of VAR.

Figure 4.5: Impulse response comparison of empirical model and estimated DSGE models (IG shock)



In the figure, IG: Government Investment, CP: Private Consumption, Y: output, N: Labor hours, IP: Private Investment, w: Real wage, i: Nominal Interest rate, Pi: Inflation.

Model 1(Light Blue Solid line): Impulse response of Baseline Model, Model 2 (Orange Solid line): Impulse response of Model with productive public capital, FAVAR (Red dotted line): Impulse response of FAVAR, VAR (Blue dotted line): Impulse response of VAR, LB, UB (Green and Purple dotted Lines): Lower and Upper 95% confidence intervals of VAR.

seems better at replicating impulse responses of VAR and FAVAR than the Model 2, in that we cannot see a strong increase in output or private consumption in later periods from empirical models. Another major discrepancy could be found in the response of private consumption in both models. We addressed this point in the explanation of Figure 4.2 and 4.3; neoclassical models generally decrease private consumption (in initial periods) because government investment crowds out private consumption, and productive public capital works too slowly in later periods. All in all, we should admit that our estimated model with productive public capital has deficiencies in capturing the short-run movements of government investment shock although I can say that this NK model is better in capturing the short term movement of real wages.¹² There is an area in need of more work.

¹²See also Figures C.5 and C.6 in Appendix C.

CHAPTER 5.

CONCLUSION

The purpose of this paper was to see the effect of government expenditure on the Japanese economy using an empirical Factor Augmented Vector Autoregression (FAVAR) model and to setup a theoretical DSGE model explaining the impulse response of the empirical model and to estimate the key parameters of the model.

By the empirical FAVAR model using 107 time series of Japan, we showed that an increase in government investment and consumption leads to an increase in private consumption and real wages.

We then setup an NK general equilibrium model with real and nominal rigidities, including both Edgeworth complementarity / substitutability between private and government consumption and productive public capital. In particular, we extend Bouakez and Rebei (2007) model in three dimension by (1) constructing an NK model, (2) including intertemporal investment adjustment cost and variable capital utilization as real rigidities, and (3) introducing public capital stock as an externality to the production function of intermediate goods firms. Because our model introduces an NK model and allows labor demand to expand in response to aggregate demand shocks, it succeeds in demonstrating that both private consumption and real wages increase in response to government expenditure shocks.

Then, we estimated the key parameters of the model using Bayesian inference, and provided evidence that private and government consumption are Edgeworth complements as Bouakez and Rebei (2007) found for U.S. data and Okubo (2003) and

Iwata (2012) for Japanese data, and that public capital is productive in Japan. In addition, from the estimated model, we showed the estimated impact output multiplier to government consumption shock amounts to 1.81, while that to government investment shock is 0.75.

Finally, the estimated model with Edgeworth complementarity generates remarkably well-matched movements of private consumption, output, labor hours, and real wages in response to a government consumption shock. On the other hand, the estimated model with productive public capital was deficient in capturing short-run movements of a government investment shock. This is my very first step for model development and I will take up this issue in the future work.

APPENDIX A.

DATA

All series are taken from IN information Center /INDB Finder PRO Database.

Data period is from 1985Q3-2008Q1.

Data included in the observable factors Y_t are the following.

1. Output: Real gross domestic product (billion yen) deflated by Chain-type price index (2000=100) : s.a (SNA),

2. Private consumption: Real final consumption of household (billion yen) deflated by Chain-type price index (2000=100) : s.a (SNA)

3. Private investment: Real gross capital formation of private sectors (billion yen) deflated by Chain-type price index (2000=100) : s.a (SNA)

4. Government consumption: Real government final consumption expenditure (billion yen) deflated by Chain-type price index (2000=100) : s.a (SNA)

5. Government investment: Real gross capital formation of public sectors (billion yen) deflated by Chain-type price index (2000=100) : s.a (SNA)

6. Labor hours: Index of labor hour, total hours worked, all industries, 30 or more employees (2005=100): s.a.

7. Real wages: Index of Real Wages, total amount of cash earnings in all industries, 30 or more employees (2005=100), s.a.

8. Nominal interest rate: Call rate (uncollateralized overnight, end of month)

9. Inflation: Used GDP deflator (implicit deflator), s.a. Calculated GDP Deflator (at period t)/ GDP Deflator (at period t-1).

All data except nominal interest rate and inflation are transformed to logarithms and are one-sided HP filtered. As for nominal interest rate and inflation, we use one-sided HP filtered.

The other 98 variables can be seen in Table A.1. In the table, transformation codes mean 1: one-sided HP filtered, 2: logarithm and one-sided HP filtered. The circle for seasonal adjustment means we made seasonal adjustment using X12-ARIMA because seasonally adjusted series are not provided.

Table A.1. The List of Informational time series for FAVAR

		Transf	Data	Seasonal
		Code	Frequency	adjustment
1	Real Domestic Demand (billion yen): deflated by Chain-type price index (2000=100) : s.a.	2	Q	
2	Real Private demand (billion yen) : deflated by Chain-type price index (2000=100) : s.a.	2	Q	
3	Real Private housing (billion yen): deflated by Chain-type price index (2000=100) : s.a.	2	Q	
4	Real Public demand (billion yen) : deflated by Chain-type price index (2000=100) : s.a.	2	Q	
5	Real Exports of goods and services (billion yen): deflated by Chain-type price index (2000=100) : s.a.	2	Q	
6	Real Imports of Goods and Services (billion yen): deflated by Chain-type price index (2000=100) : s.a.	2	Q	
7	National Income (at factor cost) : s.a., deflated by GDP deflator	2	Q	
8	Real Compensation of Employees (billion yen): deflated by Chain-type price index (2000=100) : s.a.	2	Q	
9	Real Consumption Expenditure of Household (billion yen): deflated by Chain-type price index (2000=100) : s.a.	2	Q	
10	Real Durable goods consumption (billion yen): deflated by Chain-type price index (2000=100) : s.a.	2	Q	
11	Real Semidurable Goods Consumption (billion yen) : deflated by Chain-type price index (2000=100) : s.a.	2	Q	
12	Real Nondurable Goods Consumption (billion yen) : deflated by Chain-type price index (2000=100) : s.a.	2	Q	
13	Real Service Consumption (billion yen): deflated by Chain-type price index (2000=100) : s.a.	2	Q	
14	Index of Household Consumption Level – General (2005=100) : s.a.	2	M	
15	Index of Household Consumption Level – Workers' household (2005=100) : s.a.	2	M	
16	Consumption Expenditure – Worker's household except Agriculture, Forestry and Fisheries household : (yen)	2	M	○
17	Real Income – Worker's household except Agriculture, Forestry and Fisheries household : (yen)	2	M	○
18	Disposable Income – Worker's household except Agriculture, Forestry and Fisheries household : (yen)	2	M	○
19	Amount of Sales in whole country : Association of Department Stores (million yen)	2	M	○
20	Sales at Large Scale Retail Store – Total (million yen)	2	M	○
21	Sales at Department Store – Total (million yen)	2	M	○
22	Sales at Supermarket –Total (million yen)	2	M	○
23	Business Investment – Total Amount, All industry except finance and insurance industry (*1) (million yen)	2	Q	○
24	Business Investment – Total Amount, Manufacturing, (*1) (million yen)	2	Q	○
25	Total Number of New Housing Construction Started – Total : s.a.	2	M	
26	Total floor Area of New Housing Construction Started – Total (1000m ²) : s.a.	2	M	
27	Total floor Area of Building Construction Started – Whole of Country, Grand Total (1000m ²)	2	M	○
28	Cost Estimates for Building Construction Started – Whole of Country, Grand Total (million yen)	2	M	○
29	Total floor Area of Building Construction Started – Whole of Country, Public Agent (1000m ²)	2	M	○
30	Total floor Area of Building Construction Started – Whole of Country, Private Agent(1000m ²)	2	M	○
31	Cost Estimates for Building Construction Started – Whole of Country, Public Agent (million yen)	2	M	○
32	Cost Estimates for Building Construction Started – Whole of Country, Private Agent (million yen)	2	M	○

33	Value of orders received for public construction (For 50 major firms) – All Public Institutions, total (million yen)	2	M	○
34	Value of orders received for public construction (For 50 major firms) – All National Government Agencies, Total (million yen)	2	M	○
35	Value of orders received for public construction (For 50 major firms) – All Municipal Agencies, Total (million yen)	2	M	○
36	Value of orders received from public institutions (5 million yen or more cases) – Total (million yen)	2	M	○
37	Contracted Value of Public construction – Total : Public Works Payment Surety Statistics (million yen)	2	M	○
38	Total Exports : Foreign Trade Statistics (million yen) : s.a.	2	M	
39	Total Imports : Foreign Trade Statistics (million yen) : s.a.	2	M	
40	Index of Inventory–shipment ratio of finished goods – Mining and Manufacturing (2005=100) : s.a.	2	M	
41	Index of Producers' Inventory – Mining and Manufacturing (2005=100) : s.a.	2	M	
42	Index of Industrial Production –Mining and Manufacturing (2005=100) :s.a.	2	M	
43	Index of Producer's shipments –Mining and Manufacturing (2005=100) : s.a.	2	M	
44	Index of Capital Utilization Rate – Manufacturing (2005=100) : s.a.	2	M	
45	Index of Capital Utilization Rate – Manufacturing, except Machinery (2005=100) : s.a.	2	M	
46	Total Amount of Electric Power Sold by 9 Major Power Companies (1000KWH)	2	M	○
47	Current Profits – All industry except finance and insurance industry, Total Amount (*1) (million yen)	2	Q	○
48	Number of Business Failure	2	M	○
49	Labor Force (10,000 persons): s.a.	2	M	
50	Number of employees (10,000 persons) : s.a.	2	M	
51	Number of Employer (10,000 persons) : s.a.	2	M	
52	Employment Index of Regular Workers (All industries, 30 Employees or more) (2005=100) :s.a.	2	M	
53	Index of Labor Hours : Extra Working Hours, all industries, 30 Employees or more (2005=100) : s.a.	2	M	
54	Real Wage Index : Contractual Cash Earnings in All Industries, 30 Employees or more (2005=100) : s.a.	2	M	
55	Unemployed (10,000 persons) : s.a.	2	M	
56	Unemployment Rate(%) : s.a.	1	M	
57	Jobs–to–Applicants Ratio : s.a.	1	Q	
58	New Job Opening–to–Applicants Ratio : s.a.	1	Q	
59	Number of Job Offers to Job Seekers : s.a.	2	Q	
60	Number of New Openings : s.a.	2	Q	
61	Domestic Wholesale Price Index – All Commodities Aaverage, yen base (2005=100)	2	M	
62	Export Price Index – All Commodities Aaverage, yen base (2005=100)	2	M	
63	Import Price Index – All Commodities Aaverage, yen base (2005=100)	2	M	
64	Corporate Services Price Index – All Services Aaverage, yen base (2005=100)	2	M	○
65	Nikkei Commodity Price Index (17 items) (End of Month)	2	M	
66	Nikkei Commodity Price Index (42 items) (End of Month)	2	M	

67	CPI :General, (2005=100)	2	M	○
68	CPI: General excluding Fresh Food, (2005=100)	2	M	○
69	CPI: General excluding Fresh Food &Energy, (2005=100)	2	M	○
70	CPI: Goods, (2005=100)	2	M	○
71	CPI: Cost of Public Services & Gas & Electricity & Water Charges, (2005=100)	2	M	○
72	CPI : Services, (2005=100)	2	M	○
73	Monetary Base, no adjustment for Reserve Requirement Changes, Average Outstanding (100 million yen) : s.a.	2	M	
74	Monetary Base, Note Issue ,Average Outstanding (100 million yen)	2	M	○
75	The Basic Discount Rate and Basic Loan Rate(official discount rate) (End of Month) (%)	1	M	
76	Long-term Prime Lending Rate (End of Month) (%)	1	M	
77	Short-term Prime Lending Rate of banks (End of Month) (%)	1	M	
78	Tokyo Stock Price Index(TOPIX): First Section of the Tokyo Securities Exchange (End of Month) (1968=100)	2	M	
79	Nikkei Stock Average 225 Selected Stocks (End of Month) (yen)	2	M	
80	Foregin Exchange Rate: Tokyo Interbank, Market Spot rate(17h), end of month (yen/\$)	2	M	
81	Foreign Effective Exchange Rate :Nominal BIS mthod (2005=100)	2	M	
82	Foreign Effective Exchange Rate : Real, BIS method (2005=100)	2	M	
83	Yield of Interest-Bearing Government Bond (10 years) (%)	1	M	
84	Yiled of Local Government Bond (10 years) (%)	1	M	
85	Yield of Government Guaranteed Bond (10 years) (%)	1	M	
86	Yiled of Interest Bearing Bank Debentures(5 years) (%)	1	M	
87	Spread bet. Long-term Prime Lending Rate and The Basic Discount Rate and Basic Loan Rate (%)	1	M	
88	Spread bet. Yield of Interest-Bearing Government Bond (10 years) and The Basic Discount Rate and Basic Loan Rate (%)	1	M	
89	Spread bet. Yiled of Interest Bearing Bank Debentures(5 years) and The Basic Discount Rate and Basic Loan Rate (%)	1	M	
90	Money Stock: M2+CD (End of Month) : s.a. (million yen)	2	M	
91	Balance on Taxation (*2) (100 million yen)	2	M	○
92	Balance on Non-Tax Revenue (*2) (100 million yen)	2	M	○
93	Balance on Defense-related Expenditure (*2) (100 million yen)	2	M	○
94	Balance on Public Works Spendings (*2) (100 million yen)	2	M	○
95	Balance on Distribution of local allocation tax (*2) (100 million yen)	2	M	○
96	Balance on Government Finantial Contribution for compulsory education (*2) (100 million yen)	2	M	○
97	Balance on Cost of Socail Security (*2) (100 million yen)	2	M	○
98	Balance on Other Spendings of General Account (*2) (100 million yen)	2	M	○

*1 Data is from 'Financial Statements Statistics of Corporations by Industry, Quartely'

*2 Data is from 'Receipts and Payments of treasury accounts with the private sector'

s.a. Series are seasonally adjusted

APPENDIX B.

LOG LINEARIZED MODEL

For the sake of Bayesian estimation, which requires the model to be in the linear state-space form, we log-linearize the model around the steady states. we simply state the result here. The hat above a variable denotes log derivation from steady state: i.e. $\widehat{x} = \ln x - \ln x^{ss}$ where x^{ss} is steady state.

Equilibrium Conditions from Housing Sector (1)Consumption Euler equation:

$$\widehat{\lambda}_t = E_t[\widehat{\lambda}_{t+1} + \widehat{R}_t - \widehat{\pi}_{t+1}] \quad (2.01)$$

where

$$\widehat{\lambda}_t = \frac{\beta\gamma(\varepsilon - 1)}{1 - \beta\gamma} E_t \widehat{\widetilde{C}}_{t+1} - \frac{\beta\gamma(\gamma(\varepsilon - 1) - 1) + \varepsilon - \frac{(1-\beta\gamma)}{\nu}}{1 - \beta\gamma} \widehat{C}_t + \frac{\gamma(\varepsilon - 1)}{1 - \beta\gamma} \widehat{\widetilde{C}}_{t-1} - \frac{1}{\nu} \widehat{C}_t^p \quad (2.02)$$

and

$$\widehat{C}_t = \phi \left(\frac{\overline{C^P}}{\widetilde{C}} \right)^{\frac{\nu-1}{\nu}} \widehat{C}_t^p + (1 - \phi) \left(\frac{\overline{C^G}}{\widetilde{C}} \right)^{\frac{\nu-1}{\nu}} \widehat{C}_t^g \quad (2.03)$$

(2)Labor Supply equation:

$$\frac{\overline{N}}{1 - \overline{N}} \widehat{N}_t = \widehat{\lambda}_t + \widehat{w}_t \quad (2.04)$$

(3) Investment Euler equation:

$$\widehat{I}_t^P = \frac{\beta}{1 + \beta} E_t \widehat{I}_{t+1}^P + \frac{1}{1 + \beta} \widehat{I}_{t-1}^P + \frac{\xi}{1 + \beta} (\widehat{\mu}_t - \widehat{\lambda}_t) \quad (2.05)$$

(4) Asset Pricing Euler equation:

$$\widehat{\mu}_t - \widehat{\lambda}_t = -(\widehat{R}_t - E_t \widehat{\pi}_{t+1}) + \frac{(1 - \delta^P) E_t (\widehat{\mu}_{t+1} - \widehat{\lambda}_{t+1}) + \bar{r} E_t (\widehat{r}_{t+1} + \widehat{U}_{t+1})}{1 - \delta^P + \bar{r}} \quad (2.06)$$

(5) Private Capital Accumulation equation

$$\widehat{K}_{t+1}^P = \delta^P \widehat{I}_t^P + (1 - \delta^P) \widehat{K}_t^P \quad (2.07)$$

(6) Capital Utilization

$$\widehat{U}_t = \Psi \widehat{r}_t \quad (2.08)$$

Equilibrium Conditions from Firm Sector (6) Cost minimization condition:

$$\widehat{N}_t = -\widehat{w}_t + \widehat{r}_t + \widehat{K}_t^P + \widehat{U}_t \quad (2.09)$$

(7) Production Function:

$$\widehat{Y}_t = \widehat{A}_t + \alpha \widehat{K}_t^P + \alpha \widehat{U}_t + (1 - \alpha) \widehat{N}_t + \mu \widehat{K}_t^G \quad (2.010)$$

(8) Inflation Low of Motion:

$$\widehat{\pi}_t = \beta E_t \widehat{\pi}_{t+1} + \frac{(1 - \rho)(1 - \beta\rho)}{\rho} \widehat{m}c_t \quad (2.011)$$

$$\widehat{m}c_t = \{(1 - \alpha) \widehat{w}_t + \alpha \widehat{r}_t - \widehat{A}_t - \mu \widehat{K}_t^G\} \quad (2.012)$$

Miscellaneous Conditions and Market Clearing Condition (9) Public Capital Accumulation equation:

$$\widehat{K}_{t+1}^G = \delta^G \widehat{I}_t^G + (1 - \delta^G) \widehat{K}_t^G \quad (2.013)$$

(10) Monetary Policy Rule:

$$\widehat{R}_t = \rho_r \widehat{R}_{t-1} + (1 - \rho_r) \phi_{r\pi} \widehat{\pi}_{t-1} + (1 - \rho_r) \phi_{ry} \widehat{Y}_t \quad (2.014)$$

(11) Market Clearing Conditions:

$$\widehat{Y}_t = \frac{\overline{C^p}}{\overline{Y}} \widehat{C}_t^p + \frac{\overline{C^G}}{\overline{Y}} \widehat{C}_t^G + \frac{\overline{I^p}}{\overline{Y}} \widehat{I}_t^p + \frac{\overline{I^G}}{\overline{Y}} \widehat{I}_t^G \quad (2.015)$$

Persistent Shocks and Forecast Errors

$$\widehat{A}_t = \rho_A \widehat{A}_{t-1} + \varepsilon_t^A \quad (2.016)$$

$$\widehat{I}_t^G = \rho_{IG} \widehat{I}_{t-1}^G + \varepsilon_t^{IG} \quad (2.017)$$

$$\widehat{C}_t^G = \rho_{CG} \widehat{C}_{t-1}^G + \varepsilon_t^{CG} \quad (2.018)$$

$$\varsigma_t^{\tilde{C}} = \widehat{\tilde{C}}_t - E_{t-1} \widehat{\tilde{C}}_t \quad (2.019)$$

$$\varsigma_t^\pi = \widehat{\pi}_t - E_{t-1} \widehat{\pi}_t \quad (2.020)$$

$$\varsigma_t^\lambda = \widehat{\lambda}_t - E_{t-1} \widehat{\lambda}_t \quad (2.021)$$

$$\varsigma_t^\mu = \widehat{\mu}_t - E_{t-1} \widehat{\mu}_t \quad (2.022)$$

$$\varsigma_t^{I^P} = \widehat{I}_t^P - E_{t-1} \widehat{I}_{t-1}^P \quad (2.023)$$

$$\varsigma_t^r = \widehat{r}_t - E_{t-1} \widehat{r}_t \quad (2.024)$$

System of Log-linearized Model From equations (2.01) through (2.024), the system of the log-linearized model are integrated as

$$\Gamma_0 s_t = \Gamma_1 s_{t-1} + \Upsilon \varepsilon_t + \Pi \varsigma_t \quad (2.025)$$

where s_t is a vector of endogenous variables: $s_t = [\widehat{Y}_t, \widehat{\pi}_t, \widehat{C}_t, \widehat{C}_t^P, \widehat{I}_t^P, \widehat{\lambda}_t, \widehat{\mu}_t, \widehat{N}_t, \widehat{w}_t, \widehat{r}_t, \widehat{mc}_t, \widehat{U}_t, \widehat{K}_{t+1}^P, \widehat{K}_{t+1}^G, \widehat{R}_t, \widehat{A}_t, \widehat{I}_t^G, \widehat{C}_t^G, E_t \widehat{\pi}_{t+1}, E_t \widehat{\lambda}_{t+1}, E_t \widehat{\mu}_{t+1}, E_t \widehat{I}_{t+1}^P, E_t \widehat{C}_{t+1}, E_t \widehat{r}_{t+1}]'$, and ε_t is a vector of endogenous shocks: $\varepsilon_t = [\varepsilon_t^A, \varepsilon_t^{IG}, \varepsilon_t^{CG}]'$. ς_t is a vector of forecast errors: $\varsigma_t = [\varsigma_t^{\widetilde{C}}, \varsigma_t^{\pi}, \varsigma_t^{\lambda}, \varsigma_t^{\mu}, \varsigma_t^{I^P}, \varsigma_t^r]'$. $\Gamma_0, \Gamma_1, \Upsilon$, and Π are the matrices of parameters.

APPENDIX C.

MODEL IMPLICATIONS AND CALIBRATIONS

This Appendix reveals several implications of our model using a static model first. Then, we compare our model with that of Boaukez and Rebei (2007) and Marzo and Marattin (2010) through the calibration of models.

In the next subsection, we set up a static model to see certain implications of the RBC model and the model featuring complementarity/substitutability between government spending and consumption in the utility function. In the following subsection, we proceed to an infinite horizon model and observe the calibration results.

Static Model

Household Preferences We first construct a static model that illustrates an RBC model implication. The economy is populated by a single representative agent that drives utility from consumption (C) and leisure ($1 - N$). Household utility function is

$$u(C, N^S) = u(C) - \Phi(N^S)$$

The representative household supplies labor, earns a wage (wN^S) and dividends (Ξ) from firms, and pays lump-sum taxes to the government (T). The budget constraint of the household is

$$C \leq wN^S + \Xi - T$$

Thus, the optimizing problem of the household is

$$\mathcal{L} = u(C) - \Phi(N^S) - \lambda(wN^S + \Xi - T - C)$$

where λ stands for the Lagrange multiplier of the budget constraint.

Households choose consumption and leisure to maximize utility. The first order conditions associated with consumption and leisure are

$$u'(C) = \lambda$$

$$\Phi'(N^S) = \lambda w$$

From the two equations, we have

$$\frac{\Phi'(N^S)}{u'(C)} = w \tag{3.01}$$

Firms We assume that there is a large number of firms, which is normalized to 1.

Firms hire labor from households to produce output, each hiring N^D amount of labor.

We assume firms require no capital to produce. The amount of output is given by

a production function $F(N^D)$. After production, the firm pays the workers wages

and shareholders dividends. The amount of dividends is equal to the firm's net profit

($\Xi = F(N^D) - wN^D$), and firms maximize profit

$$\max_{N^D} \Xi = F(N^D) - wN^D$$

so the first order condition is

$$F'(N^D) = w \tag{3.02}$$

Market Clearing

The government collects taxes and spends as much as government spending ($T = G$). In this first part, government spending does not generate any utilities to households.

Market clearing conditions are

$$C + G = F(N^S) \quad (3.03)$$

$$N^D = N^S \quad (3.04)$$

From four equations of (3.01) to (3.04) and we assume $u(C) = \log(C)$, $\Phi(N) = \frac{N^2}{2}$, $F(N) = N_t^\alpha$, $\alpha \in (0, 1)$ to make the calculation simple. Then we log-linearize equations, giving us

$$\hat{N} = \frac{\bar{G}}{\alpha \bar{N}^\alpha - (\alpha - 2)\bar{C}} \hat{G} > 0 \quad (3.05)$$

$$\hat{C} = (\alpha - 2)\hat{N} = \frac{(\alpha - 2)\bar{G}}{\alpha \bar{N}^\alpha - (\alpha - 2)\bar{C}} \hat{G} < 0 \quad (3.06)$$

$$\hat{w} = (\alpha - 1)\hat{N} = \frac{(\alpha - 1)\bar{G}}{\alpha \bar{N}^\alpha - (\alpha - 2)\bar{C}} \hat{G} < 0 \quad (3.07)$$

From the above three equations, we obtain the basic implications of government spending in the model. Notice first that $(\alpha - 2)\bar{C}$ is negative. Therefore when \hat{G} becomes positive, \hat{N} is positive and \hat{C} is negative. We see from (3.03) that the increase in G crowds out C . In addition, we see from the household budget constraint that households are taxed at an amount equaling the increase in government expenditure and they should decrease consumption of goods and leisure. In addition, note that \hat{w} becomes negative when \hat{G} increases, reflecting an increase in labor supply.

Complementarity/substitutability between government spending and private consumption

Next, we introduce complementarity / substitutability between government spending and private consumption into the model. A representative agent derives utility from effective consumption (\tilde{C}). Effective consumption is assumed to be CES index of private consumption (C) and government spending (G).

$$u(\tilde{C}, N^S) = u(\tilde{C}) - \Phi(N^S)$$

$$\tilde{C} = [\phi C^{\frac{v-1}{v}} + (1-\phi)G^{\frac{v-1}{v}}]^{\frac{v}{v-1}}$$

The first order conditions with respect to private consumption will change to

$$\frac{\partial u(\tilde{C})}{\partial C} = \lambda$$

and equation (3.01) in the first setting will change to

$$\frac{\Phi'(N^S)}{\frac{\partial u(\tilde{C})}{\partial C}} = w \quad (3.08)$$

From the four equations (3.02), (3.03), (3.04), and (3.08) and assuming $u(\tilde{C}) = \log(\tilde{C})$,

$\Phi(N) = \frac{N^2}{2}, F(N) = N_t^\alpha$, we have

$$\hat{N} = \frac{\{\psi_C + \frac{1}{\nu}(1-\psi_C)\}\bar{G} + \psi_G(\frac{1}{\nu}-1)\bar{C}}{\{\psi_C + \frac{1}{\nu}(1-\psi_C)\}\alpha\bar{N}^\alpha - (\alpha-2)\bar{C}}\hat{G} \quad (3.09)$$

$$\hat{C} = \frac{(\alpha-2)\bar{G} + \psi_G(\frac{1}{\nu}-1)\alpha\bar{N}^\alpha}{\{\psi_C + \frac{1}{\nu}(1-\psi_C)\}\alpha\bar{N}^\alpha - (\alpha-2)\bar{C}}\hat{G} \quad (3.010)$$

$$\hat{w} = (\alpha-1)\hat{N} = (\alpha-1)\frac{\{\psi_C + \frac{1}{\nu}(1-\psi_C)\}\bar{G} + \psi_G(\frac{1}{\nu}-1)\bar{C}}{\{\psi_C + \frac{1}{\nu}(1-\psi_C)\}\alpha\bar{N}^\alpha - (\alpha-2)\bar{C}}\hat{G} \quad (3.011)$$

where $\psi_C = \phi(\frac{\bar{C}}{\bar{C}})^{\frac{\nu-1}{\nu}}$, $\psi_G = (1-\phi)(\frac{\bar{G}}{\bar{C}})^{\frac{\nu-1}{\nu}}$ (, hence $\psi_C, \psi_G \in (0, 1)$). These terms comes from the derivatives of \tilde{C} .

Three equations above indicate several implications of the Bouakez and Rebei (2007) model. The denominator and numerator of (3.09) contain the term $\psi_C + \frac{1}{\nu}(1 - \psi_C)$. This term is positive, whereas the other term in the denominator $(\alpha - 2)\bar{C}$ is negative, as explained previously. Therefore, the whole denominator is positive. Next, if we see the second term of the numerator of (3.09), $\psi_G(\frac{1}{\nu} - 1)\bar{C}$ is added comparing with equation (3.05), and the sign of this term depends on the sign of $(\frac{1}{\nu} - 1)$. This term works to enhance or reduce labor supply by a government spending shock.

The same is true of equation (3.010), and the denominator is the same as in equation (3.09). $(\alpha - 2)\bar{G}$ in the numerator is the negative term, and the second term $\psi_G(\frac{1}{\nu} - 1)\alpha\bar{N}^\alpha$ is added comparing with equation (3.06). The sign of $\psi_G(\frac{1}{\nu} - 1)\alpha\bar{N}^\alpha$ depends on the sign of $(\frac{1}{\nu} - 1)$. Thus, the sign of \hat{C} depends on the magnitude of $(\alpha - 2)\bar{G}$ and $\psi_G(\frac{1}{\nu} - 1)\alpha\bar{N}^\alpha$. All in all, the term $(\frac{1}{\nu} - 1)$ is important for illustrating the effects on consumption or labor hours in response to government spending shocks. We consider these parameter values further in the following infinite horizon setting.

Bouakez and Rebei (2007) Next, we progress to the infinite horizon model. First, we

show a log-linearized Bouakez and Rebei (2007) model.

(1) Consumption Euler equation:

$$\hat{\lambda}_t = \frac{\beta\gamma(\varepsilon - 1)}{1 - \beta\gamma} E_t \widehat{\tilde{C}}_{t+1} - \frac{\beta\gamma(\gamma(\varepsilon - 1) - 1) + \varepsilon - \frac{(1-\beta\gamma)}{\nu}}{1 - \beta\gamma} \widehat{\tilde{C}}_t + \frac{\gamma(\varepsilon - 1)}{1 - \beta\gamma} \widehat{\tilde{C}}_{t-1} - \frac{1}{\nu} \widehat{C}_t^p \quad (3.012)$$

where

$$\widehat{\tilde{C}}_t = \phi \left(\frac{\bar{C}^P}{\bar{C}} \right)^{\frac{\nu-1}{\nu}} \widehat{C}_t^p + (1 - \phi) \left(\frac{\bar{C}^G}{\bar{C}} \right)^{\frac{\nu-1}{\nu}} \widehat{C}_t^g \quad (3.013)$$

(2) Labor Supply equation:

$$\frac{\bar{N}}{1 - \bar{N}} \widehat{N}_t = \widehat{\lambda}_t + \widehat{w}_t \quad (3.014)$$

(3) Asset Pricing Euler equation:

$$\widehat{\lambda}_t = E_t \widehat{\lambda}_{t+1} + \beta \bar{r} E_t \widehat{r}_{t+1} - \frac{1 + \beta \delta^P}{\xi} \widehat{K}_{t+1}^P + \frac{1}{\xi} \widehat{K}_t^P + \frac{\beta \delta^P}{\xi} \widehat{I}_{t+1}^P \quad (3.015)$$

(4) Private Capital Accumulation equation:

$$\widehat{K}_{t+1}^P = \delta^p \widehat{I}_t^P + (1 - \delta^p) \widehat{K}_t^P \quad (3.016)$$

(5) Production Function:

$$\widehat{Y}_t = \widehat{A}_t + \alpha \widehat{K}_t^P + (1 - \alpha) \widehat{N}_t \quad (3.017)$$

(6) Cost minimizing condition:

$$\widehat{N}_t = -\widehat{w}_t + \widehat{r}_t + \widehat{K}_t^P \quad (3.018)$$

$$\widehat{r}_t = \widehat{Y}_t - \widehat{K}_t^P \quad (3.019)$$

(7) Market Clearing Conditions:

$$\widehat{Y}_t = \frac{\bar{C}^p}{\bar{Y}} \widehat{C}_t^p + \frac{\bar{C}^G}{\bar{Y}} \widehat{C}_t^G + \frac{\bar{I}^p}{\bar{Y}} \widehat{I}_t^p + \frac{\bar{I}^G}{\bar{Y}} \widehat{I}_t^G \quad (3.020)$$

Persistent Shocks:

$$\widehat{A}_t = \rho_A \widehat{A}_{t-1} + \varepsilon_t^A$$

$$\widehat{I}_t^G = \rho_{IG} \widehat{I}_{t-1}^G + \varepsilon_t^{IG}$$

$$\widehat{C}_t^G = \rho_{CG} \widehat{C}_{t-1}^G + \varepsilon_t^{CG}$$

First, notice that when $\phi = 1$ and $\gamma = 0$, consumption Euler equations (3.012) and (3.013) collapse to a normal Euler equation, and this model becomes a simple RBC model.

Second, we show an implication regarding complementarity/substitutability between government consumption and private consumption following Boaukez and Rebei (2007). They showed from (3.012) and (3.013) that the effect of changes in government consumption on the marginal utility of private consumption is given by

$$\frac{\partial \widehat{\lambda}_t}{\partial \widehat{C}_t^G} = (1 - \phi) \left(\frac{\overline{C^G}}{\widetilde{C}} \right)^{\frac{v-1}{v}} \left\{ \frac{1}{\nu} - \varepsilon - \frac{\beta\gamma(\varepsilon - 1)(\gamma + 1)}{1 - \beta\gamma} \right\}$$

Next, consider the model abstracting habit formation ($\gamma = 0$), while effective consumption depends on government spending ($\phi < 1$). We can observe that the elasticity of substitution $\frac{\partial \widehat{\lambda}_t}{\partial \widehat{C}_t^G}$ has the same sign as $\frac{1}{\nu} - \varepsilon$. This indicates that government spending raises the marginal utility of consumption when the inverse of the elasticity of substitution between private consumption and government consumption $\frac{1}{\nu}$ is larger than ε . This induces positive effects on consumption, while government consumption has a negative wealth effect.¹

To clarify this point we made a calibration, as did Boaukez and Rebei (2007), using the parameter value in Table 4.1. Other parameters are in Table C.1. Figure C.1

¹In the static model, we used $\varepsilon = 1$. Therefore, we saw that $(\frac{1}{\nu} - 1)$ is important in the the response of private consumption or labor hours to government spending.

Table C.1 Calibrated parameters

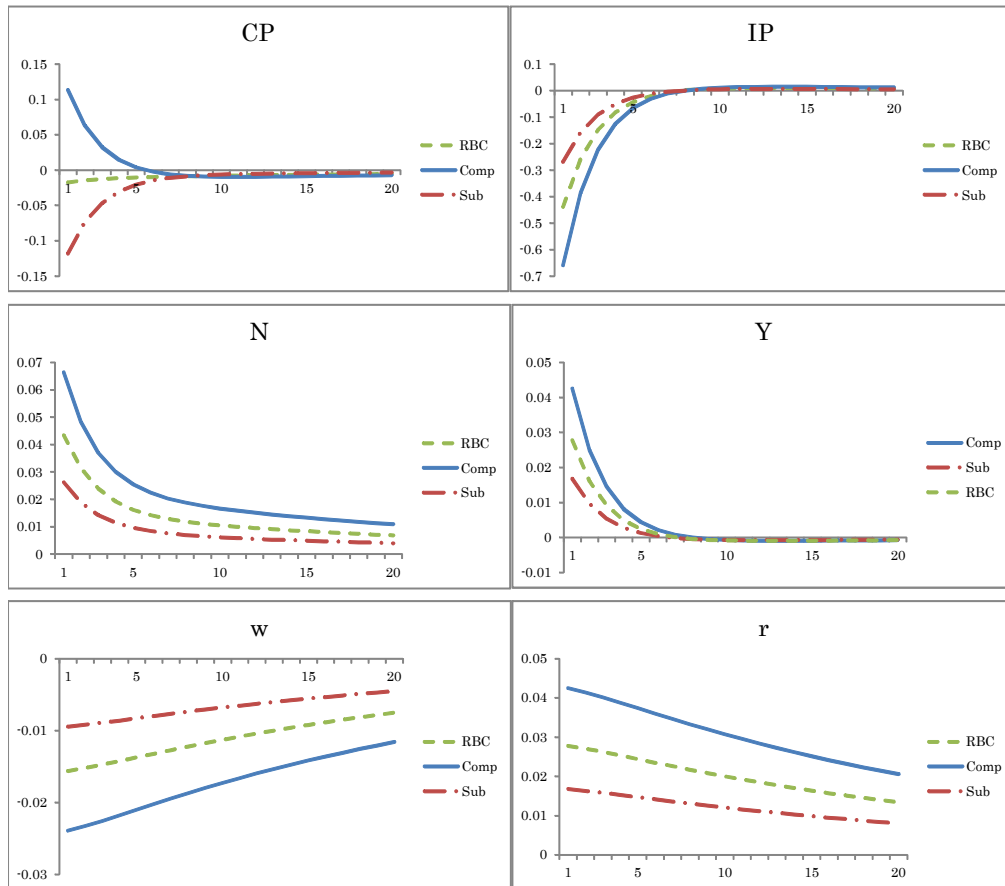
parameters	meanings	value
structural parameters		
ν	Elasticity of substitution	0.4(comp)/0.8(sub)
ε	Curvature parameter	2
γ	Habit-formation parameter	0
ξ	Adjustment-cost parameter	1
Ψ	Capital utilization cost	0
Shock persistence		
ρ_a	persistence of productivity	0.8
ρ_{cg}	persistence of government consumption	0.6
ρ_{IG}	persistence of government investment	0.6

illustrates the calibration result when $\frac{1}{\nu} - \varepsilon > 0$, indicating private consumption and government consumption are complements (in figure C.1, Comp), and when $\frac{1}{\nu} - \varepsilon < 0$, indicating they are substitutes (in figure C.1, Sub), compared with the RBC case. (We made one deviation shock to government consumption.)

Figure C.1 demonstrates that the complementary effect is strong enough to dominate the negative wealth effects under the parameter employed. On the other hand, when private consumption and government consumption are substitutes, government spending produces larger decreases in private consumption than the RBC case does. Further in this figure, labor supply and output increase more when the complementary effects are strong. Hence the decline in real wages must be larger such that marginal productivity decreases under stronger labor supply. This is the flip side of this functional form.

Constructing the NK model with investment adjustment cost, the role of capital utilization, and productive public capital We next compare features of our model in Section 3 with Boaukez and Rebei (2007) or Marzo and Marattin (2010). Our model is an NK setup,

Figure C.1: Calibrated Impulse responses of Bouakez and Rebei(2007) model



In the figure, CP: Private Consumption, IP: Private Investment, N: Labor hours, Y: output, w: Real wage
r: Rental rate of private capital.

RBC: RBC model case, Comp: Model parameter with Private consumption and Government consumption are complements ($\nu = 0.4$), Sub: Model parameter with Private consumption and Government consumption are substitutes ($\nu = 0.8$).

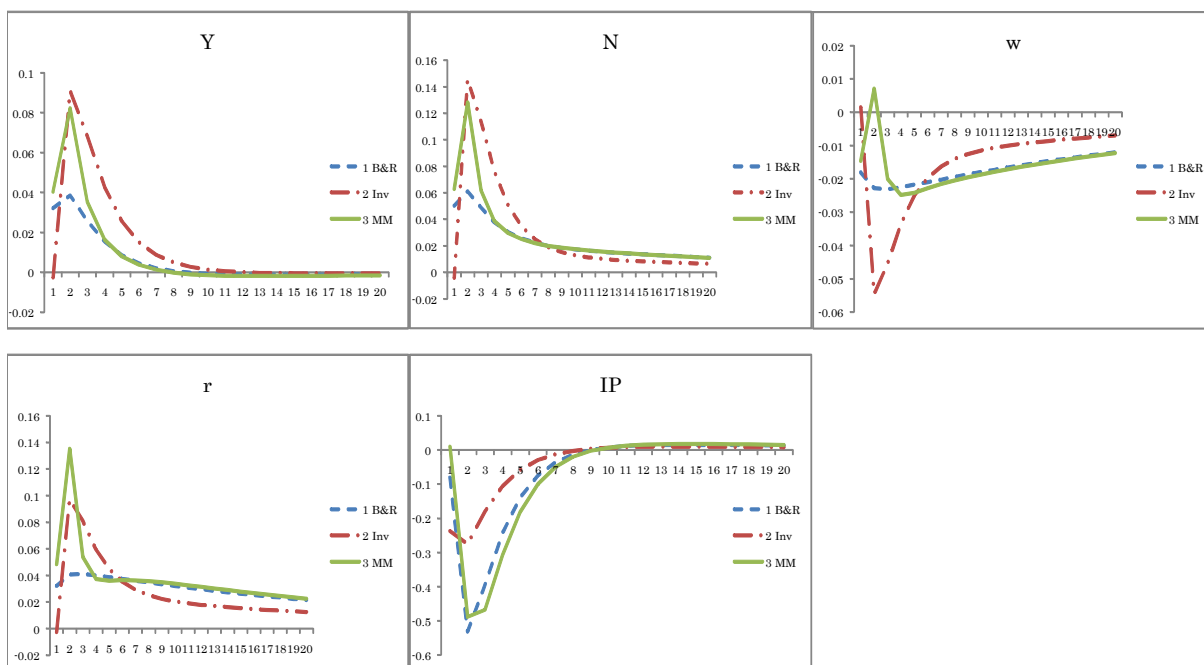
(3.019) in Boaukez and Rebei model changed to the NK Phillips curve (2.011) and (2.012). We allow government to issue bonds (2.01) and need a monetary policy rule (2.014) for setting a nominal interest rate. We switched from capital adjustment cost setting (3.015) in the Boaukez and Rebei model to investment adjustment cost setting (2.05) and (2.06). Also we added variable capital utilization, thus U_t is included in (2.06), (2.08), (2.09), and (2.010). To distinguish the differences between these, we made certain calibrations. We used a parameter of $\nu = 0.4$, $\gamma = 0.6$, and for the NK models we used price elasticity of demand $\eta = 6$, Calvo-price no price revision probability $\rho = 0.7$ and monetary policy parameters $\rho_r = 0.8$, $\rho_\pi = 1.5$, $\rho_Y = 0.05$. For a model with variable capital utilization, we used $\Psi = 2$. Other than those parameters, we used the same parameters as in Figure C.1.

Figure C.2 illustrates a comparison between the selected impulse response to a government consumption shock of the Boaukez and Rebei model (in Figure C.2, 1. B&R) and the Marzo and Marattin (2010) model (in Figure C.2, 3. MM), which extends the Boaukez and Rebei (2007) model in an NK arrangement (with capital adjustment cost). In the same figure, we also compared the Bouakez and Rebei model, which changed with the investment adjustment cost setting (2. Inv).

First, if we compare Boaukez and Rebei (2007) model with Marattin and Marzo (2010) (1. B&R with 3. MM), we notice real wages increase for a short period by 3 MM. Under the NK arrangement, not all firms can increase price with desired markup when there is excess demand for goods. As a result, real marginal costs increase, whereas price immediately adjust to clear the goods market under the RBC model.² At the

²See Figure C.3 for more detail. Under our calibrated parameters, elasticity of inflation with respect

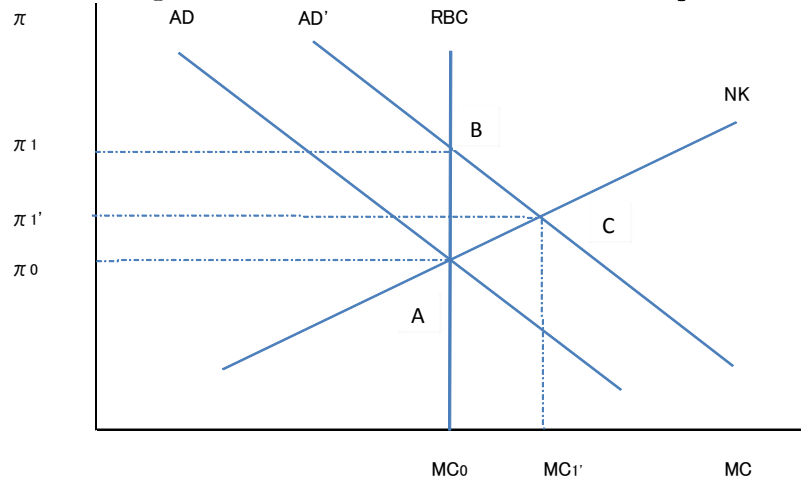
Figure C..2: Comparison of selected impulse responses of RBC models and NK models
(CG shock)-1



In the figure, Y: output, N: Labor hours, w: Real wage, r: Rental rate of private capital, IP: Private Investment.

1. B&R: Bouakez and Rebei (2007) model, 2. Inv: Changed Bouakez and Rebei model to investment adjustment cost setting,
3. MM: Marzo and Marattin (2010) model.

Figure C.3: Image of Price movement under RBC setup and NK setup



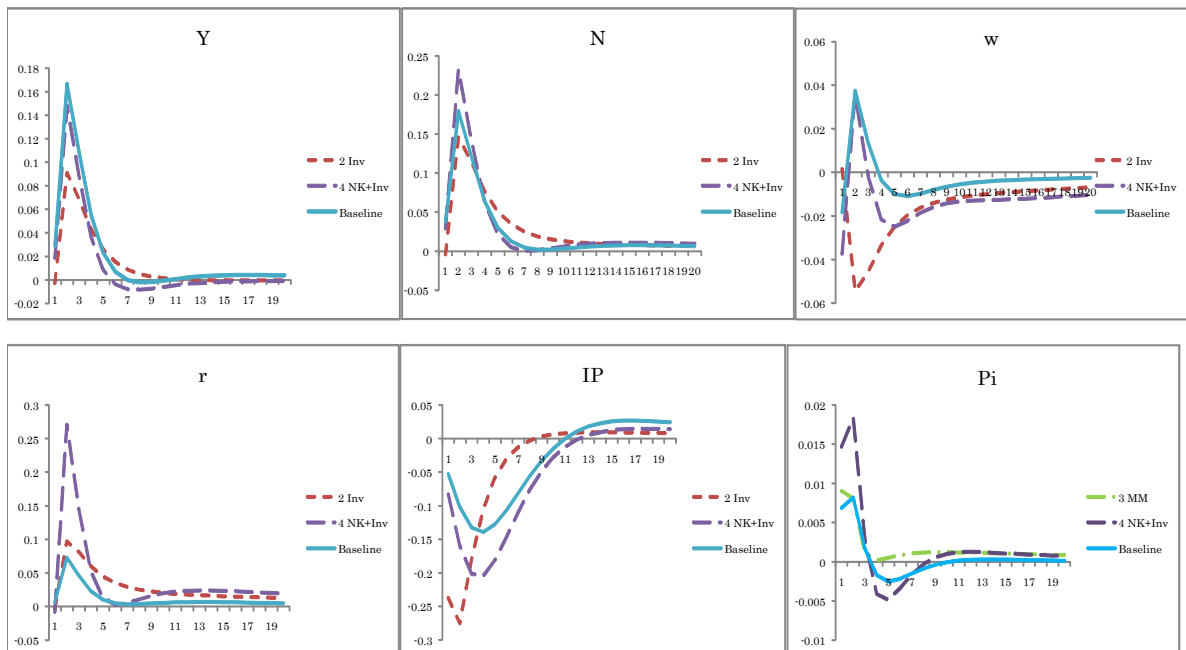
same time, the negative wealth effect prompts households to work harder, raising the number of hours worked and output. Hours, output, and the rental rate of private capital are larger in 3 MM in Figure C.2.

Second, we notice that IP movement is similar in 1 and 3, and these drops are more severe than in the investment adjustment setting (red line, 2. Inv). Under the capital adjustment cost setting, it is costly to change the *stock* of capital, whereas under the investment adjustment setting it is costly to change the *flow* of investment. Thus the latter setting generates investment inertia and limits the crowding-out effect on investment.

Next, in Figure C.4, we compare the RBC model and investment adjustment cost (2. Inv) with the NK model and investment adjustment cost (4. NK+Inv) and the NK model and investment adjustment cost and variable capital utilization (baseline).

to real marginal cost (=inverse of markup) in the NK Philips curve $(\frac{(1-\rho)(1-\beta\rho)}{\rho})$ will be 0.13, while under the RBC setting, it is ∞ . Under RBC, AD shift brings just a price surge (movement from point A to point B), while under the NK Philips curve, price cannot move enough to take the desired markup and real marginal costs increase (movement from point A to point C).

Figure C.4: Comparison of selected impulse responses of RBC models and NK models
(CG shock)-2



In the figure, Y: output, N: Labor hours, w: Real wage, r: Rental rate of private capital, IP: Private Investment, Pi: Inflation rate.

2. Inv: Changed Bouakez and Rebei (2007) model to investment adjustment cost setting (RBC setup), 4. NK+Inv: Changed 2. Inv. model with NK setup, Baseline: 4. NK+Inv model with capital utilization.

Here, we first observe the CG shock.

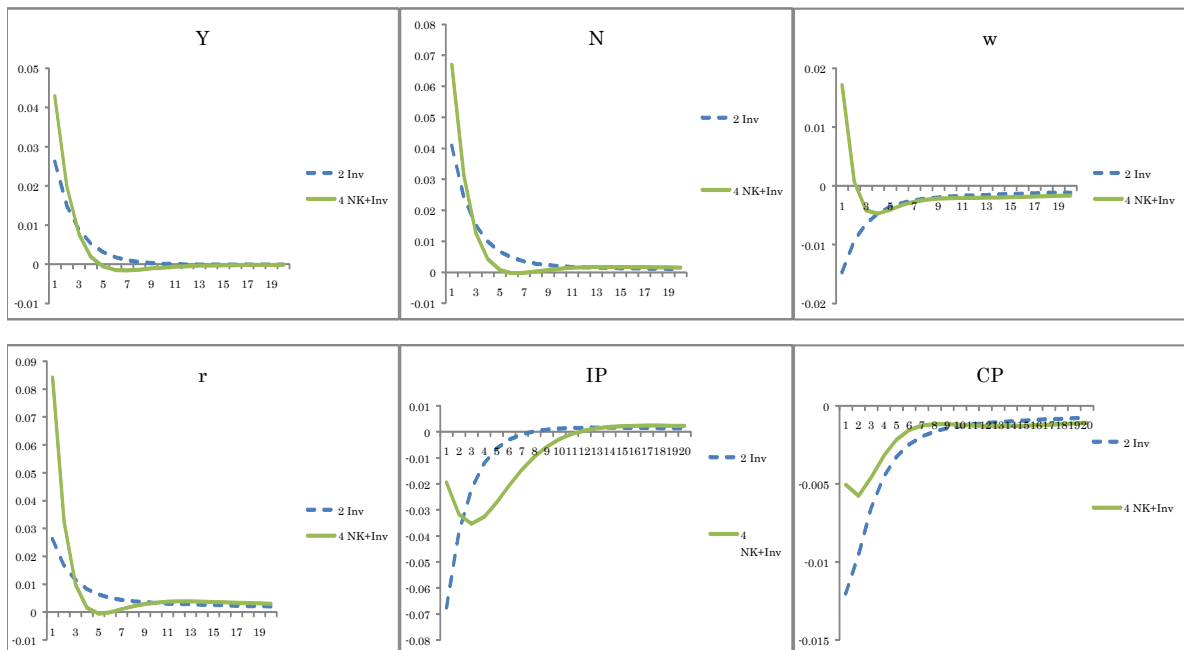
Comparing 2 with 4 produces the same statement as the comparison of 1 and 3: not all firms can increase price with desired markup when there is excess demand for goods, thus real marginal cost rises. The surge in rental cost of private capital is very sharp in 4; demand for relatively cheaper labor increases, generating a larger increase in output during initial periods. This sharp increase in rental cost of private capital in 4 brings more inertial IP movement than in 2.

Finally, if we compare 4 with the baseline, we notice variable capital utilization dampens the deficiencies in 4, the sharp rise in the rental rate of capital, with increasing the elasticity of the supply of capital services. This in turn dampens the increase in real marginal cost and prices (see equations (2.011) and (2.012)).

Then, in Figure C.5, we compare the same RBC model and investment adjustment cost (2. Inv) with the NK model and investment adjustment cost (4. NK+Inv) from an IG shock. This is also the same mechanism given in the explanation in Figure 4.2; there are a crowding-out effect and a negative wealth effect. As opposed to Figure 4.2, the impulse responses of real wages (of 4 NK+inv) in Figure C.5 rise. This is because the persistence parameter of the government investment shock ρ_{IG} is smaller (0.6) in this calibrated figure. As a result, the negative wealth effects (increase in labor supply) and the decrease of marginal productivity of labor are weaker than those in the estimated model. In this figure, the increase in real marginal cost dominates the decrease in marginal productivity of labor.

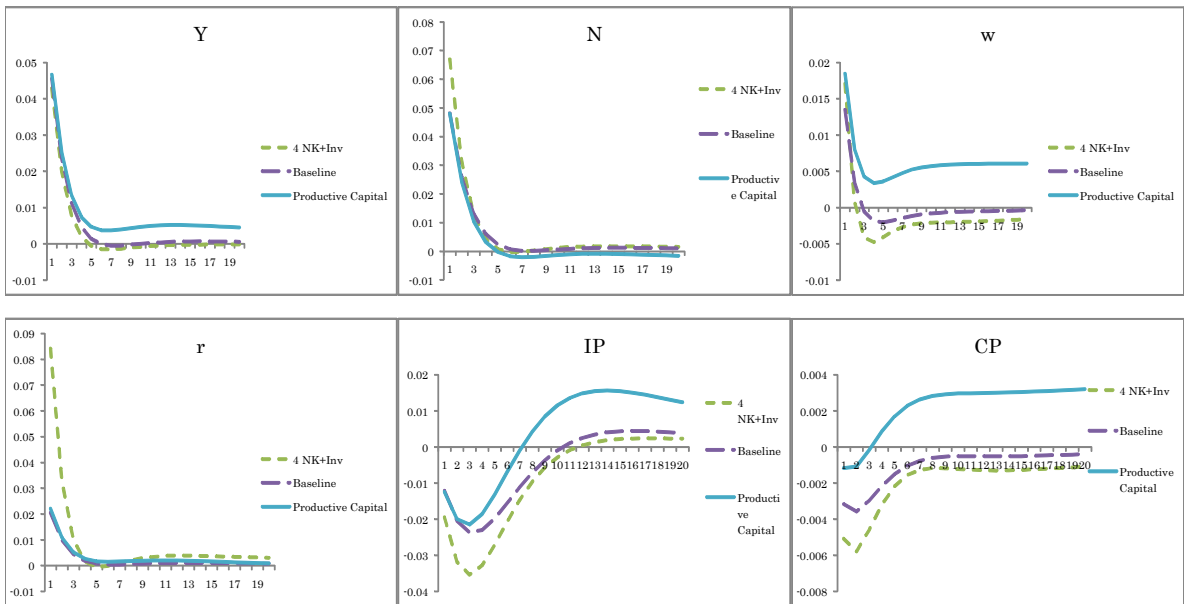
In Figure C.6, we would like to compare the NK setup and investment adjust-

Figure C..5: Comparison of selected impulse responses of RBC model and NK model
(IG shock)-1



In the figure, Y: output, N: Labor hours, w: Real wage, r: Rental rate of private capital, IP: Private Investment, CP: Private consumption. 2. Inv: Changed Bouakez and Rebei (2007) model to investment adjustment cost setting (RBC setup), 4. NK+Inv: Changed 2. Inv. model with NK setup.

Figure C.6: Comparison of selected impulse responses of NK models (IG shock)-2



In the figure, Y: output, N: Labor hours, w: Real wage, r: Rental rate of private capital, IP: Private Investment, CP: Private consumption. 4. NK+Inv: Changed 2. Inv. model with NK setup, Baseline: 4. NK+Inv model with capital utilization, Productive Capital: Baseline + Productive Public Capital in production function.

ment cost (4. NK+Inv); the NK setup and investment adjustment cost and variable capital utilization (baseline); and the NK setup and investment adjustment cost and variable capital utilization and productive public capital (Productive Capital). The last one is the model constructed in Chapter 3. As shown in Figure 4.3, we can see that greater marginal productivity caused by productive public capital produces the positive movement in real wages, especially during the later periods.

Model Comparison-Capital utilization under Greenwood and Herkoviz setting For Japanese data, Sugo and Ueda (2008) emphasized a Greenwood et al. (1998) type of capital utilization, by which higher capital utilization accelerates faster capital depreciation. We adopt CEE-type variable capital utilization in Chapter 3. According to Sugo and Ueda (2008), capital depreciation depends on capital utilization rates:

$$\delta^p(U_t) = \delta^p \psi(U_t)$$

A higher capital utilization rate leads to faster capital depreciation:

$$\psi(X) = 1 + \vartheta \frac{X^{1+\Psi^{-1}} - 1}{1 + \Psi^{-1}}$$

where Ψ is the inverse of the elasticity of capital utilization costs. In steady states, the utilization rate $U = 1$, which implies that the capital depreciation rate is equal to δ^p .

Private capital evolves according to

$$K_{t+1}^p = \{1 - \delta^p(U_t)\}K_t^p + \left\{1 - \frac{1}{2\xi} \left(\frac{I_t^p}{I_{t-1}^p} - 1\right)^2\right\}I_t^p$$

where $\frac{1}{\xi}$ controls the size of investment adjustment cost.

From this setting, we have the following log-linearized asset pricing Euler equation and private capital accumulation equation instead of (2.06), (2.07), and (2.08).

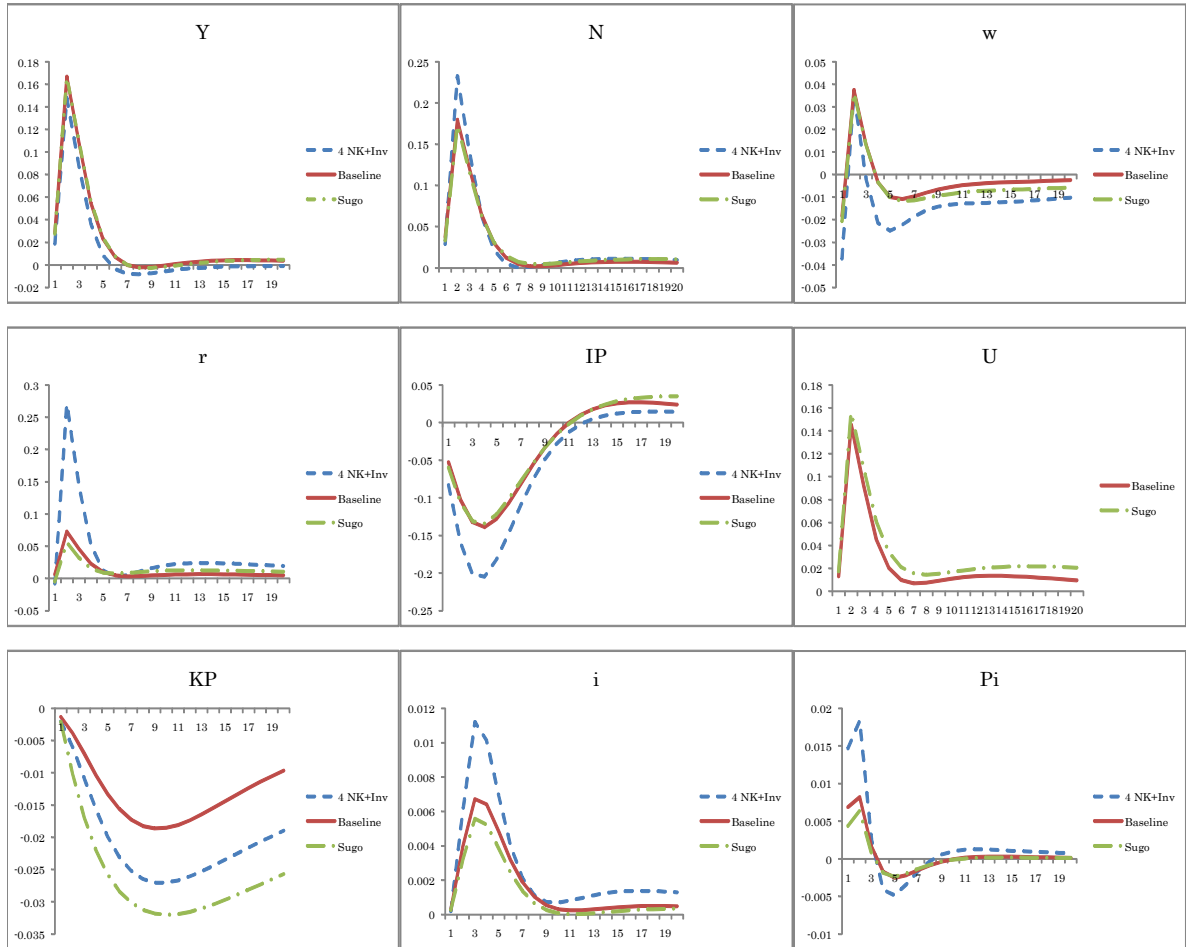
$$\widehat{\mu}_t - \widehat{\lambda}_t = -(\widehat{R}_t - E_t \widehat{\pi}_{t+1}) + \frac{(1 - \delta^P)(\widehat{\mu}_{t+1} - \widehat{\lambda}_{t+1}) + \bar{r} \widehat{r}_{t+1}}{1 - \delta^P + \bar{r}} \quad (3.021)$$

$$\widehat{K}_{t+1}^P = \delta^P \widehat{I}_t^P + (1 - \delta^P) \widehat{K}_t^P - \bar{r} \widehat{U}_t \quad (3.022)$$

$$\widehat{U}_t = \bar{r} \Psi(\widehat{r}_t + \widehat{\lambda}_t - \widehat{\mu}_t) \quad (3.023)$$

Figure C.5 illustrates the calibration result. Private capital (3rd row, 1st column) decreases more under the Greenwood type than the baseline case (CEE type) because government spending increases capital utilization, raising capital depreciation under the Greenwood type. All in all, however, there is little difference from our model in Chapter 3.

Figure C.7: Comparison of selected impulse responses of different capital utilization models (CG shock)



In the figure, Y: output, N: Labor hours, w: Real wage, r: Rental rate of private capital, IP: Private Investment, U: Capital Utilization rate, KP: Private Capital, i: Nominal Interest rate, Pi: Inflation rate. 4. NK+Inv: Changed Bouakez and Rebei model with NK setup and investment adjustment cost, Baseline: 4. NK+Inv model with CEE type capital utilization, Sugo: Changed Baseline model with Sugo and Ueda (2008) type capital utilization.

APPENDIX D.

BAYESIAN INFERENCE VIA MCMC SIMULATION

Here, we would like to explain about Bayesian inference via MCMC simulation following Iiboshi, Nishiyama, Watanabe (2008) and the DYNARE User Guide.

Likelihood of DSGE models Equations (4.4) and (4.5) constitute a linear Gaussian state space model, whose likelihood can be evaluated using the Kalman filter. The Kalman filter is the algorithm that provides the mean and covariance matrix of the state vector s_t ($t = 1, \dots, T$) conditional on the observations up to T , i.e., (y_1, \dots, y_T) in a linear Gaussian state space model.

In the model that consists of equations (4.4) and (4.5), the equations of the Kalman filter are:

One step-Ahead Prediction

$$s_{t|t-1} = Gs_{t-1|t-1} \quad (4.01)$$

$$P_{t|t-1} = GP_{t-1|t-1}G + HQ(\theta)H' \quad (4.02)$$

$$v_{t|t-1} = y_t - y_{t|t-1} = y_t - Js_{t|t-1} \quad (4.03)$$

$$F_{t|t-1} = JP_{t|t-1}J' + R \quad (4.04)$$

Updating

$$s_{t|t} = s_{t|t-1} + P_{t|t-1}J'F_{t|t-1}^{-1}v_{t|t-1} \quad (4.05)$$

$$P_{t|t} = P_{t|t-1} - P_{t|t-1}J'F_{t|t-1}^{-1}JP_{t|t-1} \quad (4.06)$$

where $s_{t|t-1}$ is the mean of s_t conditional on (y_1, \dots, y_{T-1}) , $P_{t|t-1}$ is the covariance matrix of $(s_t - s_{t|t-1})$. $v_{t|t-1}$ is the prediction error vector, and $F_{t|t-1}$ is its covariance matrix. $s_{t|t}$ is the mean of s_t conditional on (y_1, \dots, y_T) , and $P_{t|t}$ is the covariance matrix of $(s_t - s_{t|t})$.

Once the initial values of $s_{t|t-1}$ and $P_{t|t-1}$ are given, equations (4.4)-(4.5) can be solved recursively. $P_{t|t-1}J'F_{t|t-1}^{-1}$ is the Kalman gain, which determines the weight assigned to new information about s_t contained in the prediction error. The initial values are usually set equal to the unconditional mean and covariance matrix of the state vector, i.e.,

$$s_{1|0} = E(s_t) = 0 \quad (4.07)$$

$$vec(P_{1|0}) = vec(Var(s_t)) = [I - G \otimes G]^{-1}vec(HH') \quad (4.08)$$

where I is the $n \times n$ identity matrix, \otimes is the Kronecker product, and the $vec(\bullet)$ operator indicates that the columns of the matrix are being stacked one upon the other.

The log likelihood is given by

$$\ln L(\theta|data) = -\frac{nT}{2} \ln(2\pi) - \frac{1}{2} \sum_{t=1}^T \ln |F_{t|t-1}| - \frac{1}{2} \sum_{t=1}^T v'_{t|t-1} F_{t|t-1}^{-1} v_{t|t-1} \quad (4.09)$$

where T is the number of observations,¹ θ is the vector of deep parameters.

¹Maximizing this function with respect to the unknown parameters will produce their maximum likelihood. Bouakez and Rebei (2007) use this maximum likelihood estimation to estimate parameters in DSGE model.

The log posterior kernel can be expressed as

$$\ln K(\theta|data) = \ln L(\theta|data) + \ln f(\theta) \quad (4.010)$$

where first term on the right-hand side is now known after carrying out the Kalman filter recursion, and 2nd term is prior and known. To find the mode of posterior distribution, DYNARE maximizes the log posterior kernel with respect to θ . DYNARE does this using numerical methods although the likelihood function is not Gaussian with respect to θ .

MCMC Bayesian Estimation Now we are in position to find the posterior distribution of parameters. The distribution will be given by the kernel equation (4.010), but it is a nonlinear and complicated function of the deep parameters θ . Thus we cannot obtain an explicit form of it.²

In a Markov-chain Monte Carlo (MCMC) based Bayesian estimation, θ is sampled from the posterior distribution and the sampled draws are used for parameter estimation. The method used for sampling from the posterior distribution is MCMC, where sampling is not random and depends on the draws obtained in the previous sam-

²The conventional Bayesian method proceeds as follows.

(1) Set the prior distribution $f(\theta)$, which is the distribution the researcher has in mind before observing the data.

(2) Convert the prior distribution to the posterior distribution $f(\theta|data)$, which is the distribution conditional on the data using the Bayes theorem

$$f(\theta|data) = \frac{f(data|\theta)f(\theta)}{\int f(data|\theta)f(\theta)d\theta} \quad (4.011)$$

(3) Estimate the parameters θ using the posterior distribution.

Notice that the unknown parameter vector $G(\theta)$, $H(\theta)$, $Q(\theta)$ in equation (4.4) is a non-linear function of the original parameters in the DSGE model, so that the posterior distribution is non-standard even if we use normal-gamma prior distribution. When the posterior distribution is non-standard, it may be difficult to obtain the denominator of the right-hand side of the Bayes theorem (4.011) and to conduct Step (3) analytically.

pling. Since the likelihood of DSGE models can be evaluated by the Kalman filter, it is straightforward to evaluate the numerator of the right-hand side of the Bayes theorem (4.011) analytically. In such a case, we may use the Metropolis-Hastings (MH) algorithm, an MCMC method, and DYNARE uses precisely this MH algorithm.

To use the MH algorithm, we must choose a proposal density $g(\bullet|\bullet)$ from which it is possible to sample³ and an initial value θ_0 . Then, DYNARE sample $(\theta_1, \dots, \theta_N)$ from $f(\theta|data)$ by executing the following algorithm.

(1) Set $n = 1$.

(2) Draw a proposal $\theta_n^{(proposal)}$ from jumping distribution, $g(\theta|\theta_{n-1}) = N(\theta_{n-1}, cH)$

where H is the inverse of Hessian computed at the posterior mode, and c is a scalar called the adjustment coefficient or scale factor.

(3) Calculate the acceptance probability q as follows.

$$q = \min\left[\frac{f(\theta_n^{(proposal)}|data)}{f(\theta_{n-1}|data)}, 1\right] \quad (4.012)$$

(4) Accept $\theta_n^{(proposal)}$ with probability q and reject it with probability $1 - q$. Set $\theta_n = \theta_n^{(proposal)}$ when accepted and $\theta_n = \theta_{n-1}$ when rejected.

(5) if $n < N$, set $n = n + 1$ and return to (2). Otherwise, set $n = N$ and end.

³DYNARE sample $\theta_n^{(proposal)}$ from the random-walk model,

$$\theta_n^{(proposal)} = \theta_{n-1} + \nu_t, \nu_t \sim i.i.d.N(0, cH),$$

where H is the inverse of Hessian computed at the posterior mode and c is a scalar called the adjustment coefficient.

The merit of using this random walk proposal is that $g(\theta_{n-1}|\theta_n^{(proposal)}) = g(\theta_n^{(proposal)}|\theta_{n-1})$, so that the acceptance probability q collapses from

$$q = \min\left[\frac{f(\theta_n^{(proposal)}|data)g(\theta_{n-1}|\theta_n^{(proposal)})}{f(\theta_{n-1}|data)g(\theta_n^{(proposal)}|\theta_{n-1})}, 1\right]$$

to (4.012), which does not depend on the proposal density $g(\bullet|\bullet)$. Hence, we need not find a proposal density that mimics the posterior density.

APPENDIX E.

ANOTHER SPECIFICATION FOR THE PRODUCTION FUNCTION

The Model In this Appendix, I change the production function to allow the government investment shock to generate strict crowding-out effects, following the spirit of Mazraani (2010).¹ I assume effective capital in a simpler specification than Mazraani (2010), $\widetilde{K}_{i,t} = \widetilde{K}_{i,t}^P + \vartheta K_t^G$. Through this specification, the effective capital parameter with public capital ϑ should be negative if public capital complements private capital, as found by Mazraani (2010).

The intermediate goods firms The production function for intermediate goods firm i is

$$Y_{i,t} = A_t \widetilde{K}_{i,t}^\alpha N_{i,t}^{(1-\alpha)} \quad (5.01)$$

where $\widetilde{K}_{i,t} = \widetilde{K}_{i,t}^P + \vartheta K_t^G$ is the effective unit of capital, $\widetilde{K}_{i,t}^P = U_{i,t} K_{i,t}^P$ is the effective unit of private capital service, and K_t^G is the government capital stock. Notice that K_t^G does not have subscript i , thus we assume K_t^G has common productivity to each firm. All firms are subject to the same technology shock, A_t . In general, this is a constant return to scale production function.

The cost minimization problem for firm i can be expressed as follows.

$$\text{cost function: } \min_{\widetilde{K}_{i,t}^P, N_{i,t}} r_t \widetilde{K}_{i,t}^P + w_t N_{i,t} + mc_{i,t} (Y_{i,t} - A_t \widetilde{K}_{i,t}^\alpha N_{i,t}^{1-\alpha})$$

¹In Mazraani (2010), the production function as well has CES form for public and private capital, which allows public capital to substitute for private capital. As to the specification, she wrote that “here the production function has a CES form, allowing government investment shocks to generate either crowding-in or crowding-out effects on private activity.”

where the Lagrange multiplier, $mc_{i,t}$, is the marginal cost of producing $Y_{i,t}$. Solving the cost minimization problem, the first order condition is

$$\frac{w_t}{r_t} = \frac{(1-\alpha)\widetilde{K}_{i,t}}{\alpha N_{i,t}} \quad (5.02)$$

Then, under Calvo (1983) type sticky price setting, for any given period t , each firm has a ρ probability that it will keep the price of the previous period, and $1-\rho$ probability that it will be able to choose its price optimally. An intermediate goods firm, i , which can choose price in period t , choose the price, $P_{i,t}^*$, to maximize profit:

$$\begin{aligned} & \max_{P_{i,t}^*} E_t \sum_{j=0}^{\infty} \beta^j \rho^j [P_{i,t}^* Y_{t+j} (\frac{P_{t+j}}{P_{i,t}^*})^\eta - P_{t+j} r_{t+j} \widetilde{K}_{i,t+j}^p - P_{t+j} w_{t+j} N_{i,t+j}] \\ & \text{s.t. } Y_{t+j} (\frac{P_{t+j}}{P_{i,t}^*})^\eta = A_t \widetilde{K}_{i,t}^\alpha N_{i,t}^{1-\alpha} \\ & = E_t \sum_{j=0}^{\infty} (\beta\rho)^j [(P_{i,t}^* - P_{t+j}) \frac{1}{A_{t+j}} \frac{(1-\alpha)^{\alpha-1}}{\alpha^\alpha} r_{t+j}^\alpha w_{t+j}^{1-\alpha}] Y_{i,t+j} - v P_{t+j} r_{t+j} K_{t+j}^G] \end{aligned}$$

The first order condition for the profit maximization problem with respect to $P_{i,t}^*$ yields the following optimal pricing rule for firm i .

$$P_{i,t}^* = \frac{\eta}{\eta-1} \frac{E_t \sum_{j=0}^{\infty} (\beta\rho)^j (P_{t+j} Y_{i,t+j}) \frac{1}{A_{t+j}} \frac{(1-\alpha)^{\alpha-1}}{\alpha^\alpha} r_{t+j}^\alpha w_{t+j}^{1-\alpha}}{E_t \sum_{j=0}^{\infty} (\beta\rho)^j Y_{i,t+j}} \quad (5.03)$$

Notice that the optimal price choice (5.03) is not affected by K_t^G . This specification is normal and the same as the model without public capital in the production function. On the other hand, the cost minimization condition (5.02) has changed. In this specification, an increase in *effective capital* alters the opportunity cost between inputs of labor and private capital.

Log linearized Model From equations (5.01)-(5.03), the log linearized model will change from (2.09)-(2.012) in the baseline model to the following equations.

Cost minimization condition:

$$\widehat{N}_t = -\widehat{w}_t + \widehat{r}_t + \frac{\overline{K^P}}{\overline{K}}(\widehat{K}_t^P + \widehat{U}_t) + \vartheta \frac{\overline{K^G}}{\overline{K}}\widehat{K}_t^G \quad (5.04)$$

Production Function:

$$\widehat{Y}_t = \widehat{A}_t + \alpha \frac{\overline{K^P}}{\overline{K}}\widehat{K}_t^P + \alpha \frac{\overline{K^P}}{\overline{K}}\widehat{U}_t + \alpha \vartheta \frac{\overline{K^G}}{\overline{K}}\widehat{K}_t^G + (1 - \alpha)\widehat{N}_t \quad (5.05)$$

Inflation Low of Motion:

$$\widehat{\pi}_t = \beta E_t \widehat{\pi}_{t+1} + \frac{(1 - \rho)(1 - \beta\rho)}{\rho} \{(1 - \alpha)\widehat{w}_t + \alpha\widehat{r}_t - \widehat{A}_t\} \quad (5.06)$$

Estimation Results As per our estimation, all calibrated parameters except for γ used here are those shown in Table 4.1. To simplify the model, $\gamma = 0$ in Appendix E.

The left half of Table E.1 reports the prior distributions of parameters. I used the same values as almost every parameter in Table 4.2 here. As for the effective capital parameter with public capital ϑ , I set both the mean and standard error to 0.50². Given the prior distributions, DYNARE calculates posterior distributions using MCMC algorithm. We sample 300,000 periods each, discarding first 150,000 periods.

In Table E.1, I report posterior means and 90% confidence intervals (CI) for the parameters as well. The parameter for elasticity of substitution ν is 1.43, and that of the inverse of intertemporal substitution of consumption ε is 0.41. Therefore $\frac{1}{\nu} - \varepsilon$ holds above 0 (but approaches closer to 0 than in the baseline model). This means

²I also tried estimations with other prior values, such as setting the mean and standard error of ϑ at -2 and 1 , respectively. The result, however, did not change.

Table E.1 Prior and Posterior distributions for the model estimated

parameters	Prior settings				Posterior parameters		
	meanings	type	mean	s.e.	mean	CI(Low)	CI(High)
structural parameters							
ν	Elasticity of substitution	normal	0.8	0.5	1.413	0.932	1.896
ε	Curvature parameter	normal	2	0.5	0.411	0.234	0.588
ξ	Inverse of adjustment-cost parameter	normal	0.25	0.75	1.805	1.204	2.364
Ψ	Capital utilization cost	normal	1	0.5	0.951	0.169	1.698
ρ	Calvo price no-revise probability	beta	0.7	0.2	0.777	0.646	0.909
ν	Public capital complementarity/substitutability	beta	0.5	0.5	0.401	0.287	0.520
Policy parameters							
ρ_r	Interest rate smoothing coeff.	beta	0.8	0.15	0.801	0.620	0.976
ρ_π	Interest rate inflation coeff.	normal	1.5	1	2.832	1.724	3.925
ρ_y	Interest rate output gap coeff.	normal	0.125	0.075	0.042	0.003	0.079
Shock persistence							
ρ_a	Persistence of productivity	beta	0.85	0.1	0.822	0.718	0.926
ρ_{cg}	Persistence of government consumption	beta	0.85	0.1	0.728	0.547	0.915
ρ_{ig}	Persistence of government investment	beta	0.85	0.1	0.898	0.851	0.947
Standard Errors of shocks							
η_a	SE of productivity shock	inv. Gamma	0.4	2	0.334	0.095	0.627
η_{cg}	SE of government consumption shock	inv. Gamma	0.2	2	0.173	0.047	0.324
η_{ig}	SE of government investment shock	inv. Gamma	0.3	2	0.246	0.069	0.441
Standard Errors for Measurement Errors							
η_v	SE of measurement err. for output gap	inv. Gamma	0.2	2	0.173	0.049	0.322
η_{cp}	SE of measurement err. for private consumption	inv. Gamma	0.2	2	0.164	0.048	0.304
η_p	SE of measurement err. for private investment	inv. Gamma	1	4	1.164	0.214	2.260
η_{cg}	SE of measurement err. for government consumption	inv. Gamma	0.1	2	0.085	0.024	0.160
η_{ig}	SE of measurement err. for government investment	inv. Gamma	0.5	4	0.400	0.122	0.718
η_N	SE of measurement err. for real wage	inv. Gamma	0.1	2	0.109	0.021	0.234
η_w	SE of measurement err. for labor hour	inv. Gamma	0.1	2	0.091	0.022	0.168
η_π	SE of measurement err. for inflation	inv. Gamma	0.1	2	0.132	0.020	0.273
η_i	SE of measurement err. for nominal interest rate	inv. Gamma	0.05	1	0.053	0.010	0.117

the necessary condition for government spending to increase the marginal utility of consumption is satisfied in this specification as well. By chance, these parameter values coincide with those in Okubo (2003), where $\nu = 1.39$ and $\varepsilon = 0.19 \sim 0.61$.

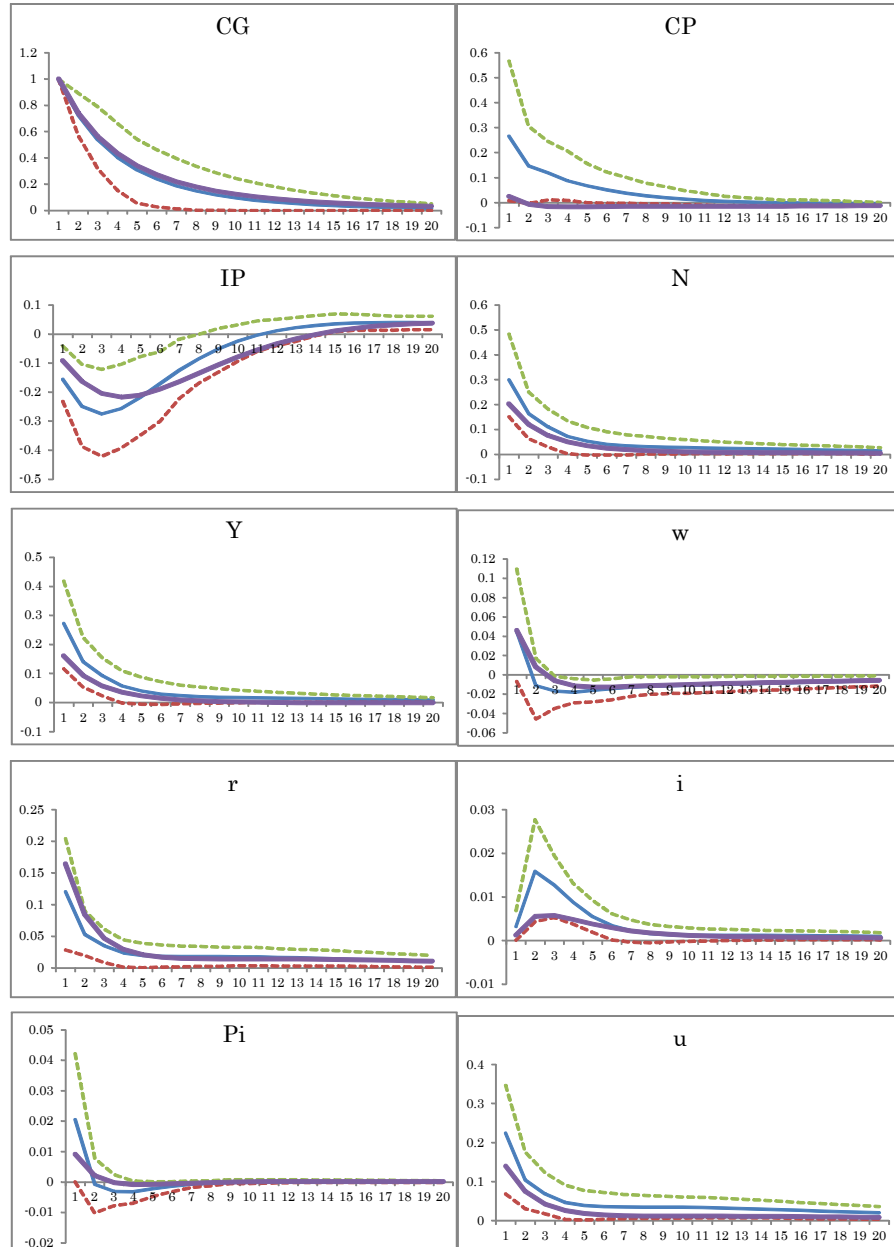
The effective capital parameter with public capital ϑ was 0.40, indicating public capital *substitutes* for private capital. This result differs from Mazraani (2010), in which the relationship is complementary.

Regarding inflation dynamics, the Calvo price-setting parameter ρ is 0.78, so an average contract duration of price setting is estimated to be about 4.5 quarters. Regarding monetary policy parameters, the coefficient on lagged interest rates in the monetary policy rule ρ_r is 0.80, The response of interest rate to inflation ρ_π and the response to output ρ_Y are 2.83 and 0.04.

Impulse response Figure E.1 illustrates the impulse responses to a government consumption shock, comparing the impulse response of the model in this Appendix with that of the baseline model in Chapters 3 and 4. From Figure E.1, we can see a similar impulse response of the baseline model with that of the model in this Appendix, other than private consumption: this is caused by parameter value differences in ν and ε (the model estimated in this appendix had the value of $\frac{1}{\nu} - \varepsilon$ much close to 0).

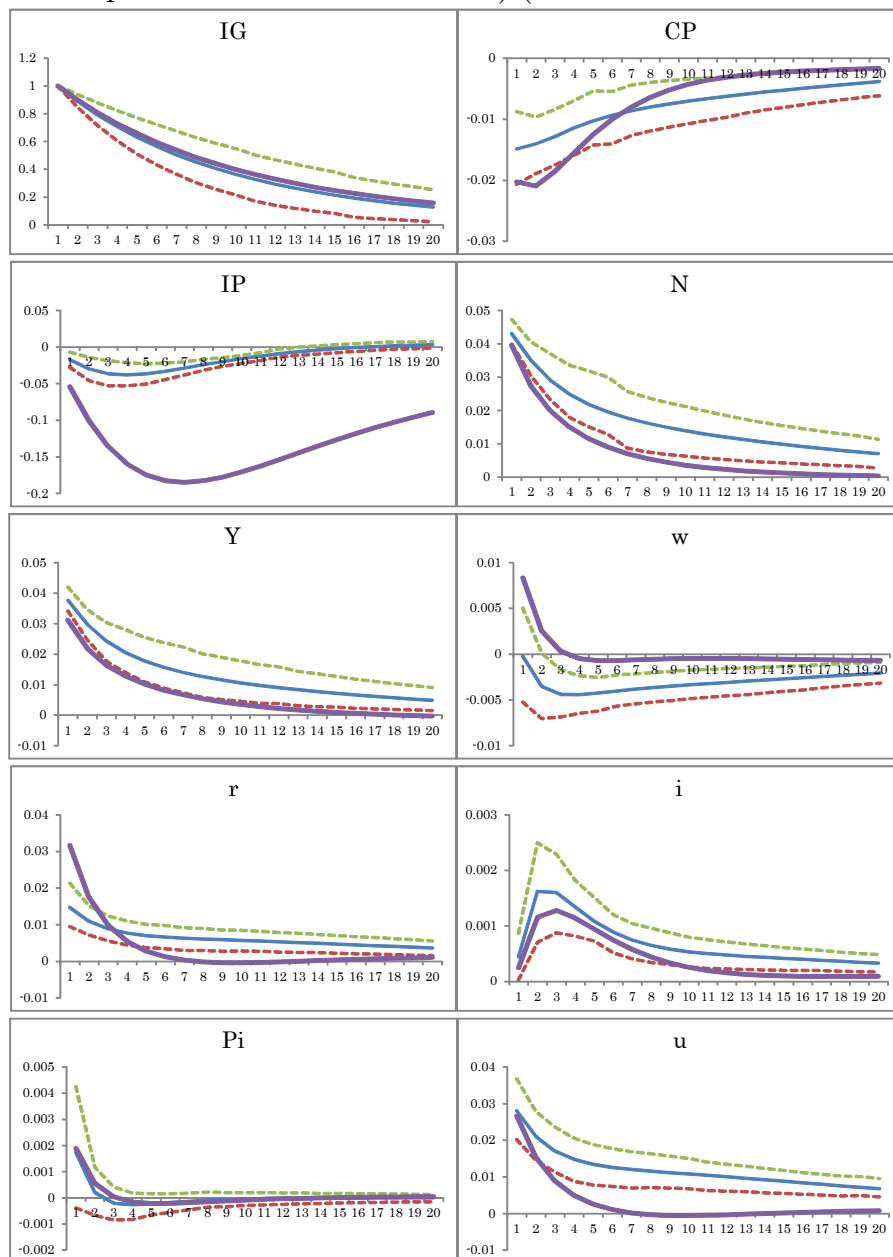
Next, Figure E.2 illustrates impulse responses to the government investment shock, comparing the impulse response of the model in this Appendix with that of the baseline model in Chapters 3 and 4. Figure E.2 indicates that the impulse response of private investment in the model in this Appendix decreases much more than that in the baseline model. This is because the increased public capital substitutes for

Figure E.1: Impulse responses of estimated DSGE models (Baseline Model, Model with Effective Capital in Production Function) (Government Consumption Shock)



CG: Government Consumption, CP: Private Consumption, IP: Private Investment, N: Labor hours, Y: output, w: Real wage, r: Rental rate of private capital, i: Nominal Interest rate, Pi: Inflation, U: Capital utilization rate. Purple Solid line: Impulse responses of model in this Appendix, Blue Solid line: Impulse responses of model in Chapter 3 (Baseline model), Green and Red dotted Lines: Upper and Lower bounds of 90% HPD intervals of Model in Chapter 3.

Figure E.2: Impulse responses of estimated DSGE models (Baseline Model, Model with Effective Capital in Production Function) (Government Investment Shock)



IG: Government Investment, CP: Private Consumption, IP: Private Investment, N: Labor hours, Y: output, w: Real wage, r: Rental rate of private capital, i: Nominal Interest rate, Pi: Inflation, U: Capital utilization rate

Purple Solid line: Impulse responses of model in this Appendix, Blue Solid line: Impulse responses of model in Chapter 3 (Baseline model), Green and Red dotted Lines: Upper and Lower bounds of 90% HPD intervals of Model in Chapter 3.

private capital in our estimation result, and public capital increases during later periods, marginal productivity of private capital (the rental rate of capital) decreases more than that in the model without public capital.

REFERENCES CITED

1. Aschauer, D. 1989. "Is Public Expenditure Productive?," *Journal of Monetary Economics*, 23 (2), 177-200.
2. Amano, R. A., Wirjanto T. S. 1998. "Government Expenditures and the Permanent-Income Model," *Review of Economic Dynamics*, 1 (3), 719-730.
3. Bai, J., Ng, S. 2006. "Confidence Intervals for Diffusion Index Forecasts and Inference for Factor-Augmented Regressions," *Econometrica*, 74 (4), 1133-1150.
4. Baxter, M., King, R. G. 1993. "Fiscal Policy in General Equilibrium," *American Economic Review* 83 (3), 315-334.
5. Bernanke, B. S., Boivin, J., Eliasch, P. 2005. "Measuring the Effects of Monetary Policy: A Factor-Augmented Vector Autoregressive (FAVAR) Approach," *The Quarterly Journal of Economics*, 120 (1), 387-422.
6. Blanchard, O., Perotti R. 2002. "An Empirical Characterization Of The Dynamic Effects Of Changes In Government Spending And Taxes On Output," *The Quarterly Journal of Economics*, 117 (4), 1329-1368.
7. Bouakez, H., Rebei, A. 2007. "Why Does Private Consumption Rise After a Government Spending Shock?," *Canadian Journal of Economics*, *Canadian Economics Association*, 40 (3), 954-979.

8. Burnside, C., Eichenbaum, M., Fisher, J. 2003. "Fiscal Shocks and Their Consequences," *Journal of Economic Theory*, 115 (1), 89-117.
9. Calvo, G. A. 1983. "Staggered Prices in a Utility-maximizing Framework," *Journal of Monetary Economics*, 12 (3), 383-398.
10. Clarida, R., Gali, J., Gertler, M. 1999. "The Science of Monetary Policy: A New Keynesian Perspective," *Journal of Economic Literature*, 37(4), 1661-1707.
11. Consolo, A., Favero, C. A., Paccagnini, A. 2009. "On the Statistical Identification of DSGE Models," *Journal of Econometrics*, 150 (1), 99-115.
12. Doi, T., Hoshi, T., Okimoto, T. 2011. "Japanese Government Debt and Sustainability of Fiscal Policy," NBER Working Paper No.17305.
13. Fatas, A., Mihov, I. 2001. "The Effects of Fiscal Policy on Consumption and Employment: Theory and Evidence," *CEPR Discussion Papers No. 2760*.
14. Forni, M., Gambetti, L. 2010. "Fiscal Foresight and the Effects of Government Spending," *CEPR Discussion Papers No. 7840*.
15. Gali, J., Lopez-Salido, J. D., Valles, J. 2007. "Understanding the Effects of Government Spending on Consumption," *Journal of the European Economic Association*, 5 (1), 227-270.
16. Greenwood, J., Hercowitz, Z., Huffman, G. W. 1988. "Investment, Capacity Utilization, and the Real Business Cycle," *American Economic Review*, 78 (3), 402-417.

17. Griffoli, T. M. "DYNARE User Guide".
18. Iiboshi, H., Nishiyama, S.-I., Watanabe, T. 2008. "An Estimated Dynamic Stochastic General Equilibrium Model of the Japanese Economy: A Bayesian Analysis," Mimeo.
19. Ireland, P. 2004. "A Method for Taking Models to the Data," *Journal of Economic Dynamics and Control*, 28 (6), 1205-1226.
20. Iwata, Y. 2011. "The Government Spending Multiplier and Fiscal Financing: Insights from Japan," *International Finance*, 14 (2), 231–264.
21. Iwata, Y. 2012. "Non-Wasteful Government Spending in an Estimated Open Economy DSGE Model: Two Fiscal Policy Puzzles Revisited," Mimeo.
22. Kawaguchi, D., Ohtake, F., Tamada, K. 2005. "The Productivity of Public Capital: Evidence from the 1994 Electoral Reform of Japan," ISEER Discussion Paper No. 627.
23. Kühn, S., Muysken, J., Van Veen, T., 2010. "The Adverse Effect of Government Spending on Private Consumption in New Keynesian Models," *Metroeconomica*, 61(4), 621-639.
24. Leeper, E. M., Plante, M., Traum, N. 2010. "Dynamics of Fiscal Financing in the United States," *Journal of Econometrics*, 156 (2), 304-321.
25. Leeper, E. M., Walker, T. B, Yang S-C. S. 2009. "Government Investment and Fiscal Stimulus in the Short and Long Runs," NBER Working Paper No.15153.

26. Linnemann, L., Schabert, A. 2004. "Can Fiscal Spending Stimulate Private Consumption?" *Economics Letters*, 82 (2), 173-179.
27. Marattin, L., Marzo, M. 2010. "The Multiplier-Effects of Non-Wasteful Government Expenditure," Working Papers 704, Dipartimento Scienze Economiche, Universita' di Bologna.
28. Mazraani, M. S., 2010. "Public Expenditures in an RBC Model: A Likelihood Evaluation of Crowding-in and Crowding-out Effects," Mimeo.
29. McGrattan, E., Rogerson, R., Wright, R. 1997. "An Equilibrium Model of the Business Cycle with Household Production and Fiscal Policy," *International Economic Review*, 38 (2), 267-290.
30. Mera, K. 1973. "II. Regional Production Functions and Social Overhead Capital: An Analysis of the Japanese Case," *Regional and Urban Economics*, 3(2), 157-185.
31. Mitsui, K., Inoue, J. 1995. "Productivity Effects of Public Capital," (Shakai Shihon no Seisanryoku Kouka) Chapter 3 of Mitsui K., Ohta K. *Productivity of Public capital and public finance*, " (Shakai Shihon no Seisansei to Kouteki Kinyu). Nihon Hyoronsha.
32. Monacelli, T., Perotti, R. 2008. "Fiscal Policy, Wealth Effects, and Markups," NBER Working Papers 14584, National Bureau of Economic Research.
33. Mountford, A., Uhlig, H. 2009. "What are the Effects of Fiscal Policy Shocks?," *Journal of Applied Econometrics*, 24(6), 960-992.

34. Nishiyama, S.-I., Iiboshi, H., Matsumae, T., Namba, R. 2011. "How Bad Was the Lehman Shock?: Estimating a DSGE Model with Firm and Bank Balance Sheets in a Data-Rich Environment," presented at ESRI-CEPREMAP joint workshop.
35. Okubo, M. 2003. "Intratemporal Substitution Between Private and Government Consumption: The Case of Japan," *Economics Letters*, 79 (1), 75-81.
36. Ramey, V. A., Shapiro, M. D. 1998. "Costly Capital Reallocation and the Effects of Government Spending," *Carnegie-Rochester Conference Series on Public Policy*, 48(1), 145-194.
37. Ramey, V. A. 2011. "Identifying Government Spending Shocks: It's All in the Timing," *The Quarterly Journal of Economics*, Oxford University Press, 126(1), 1-50.
38. Ravn, M., Schmitt-Grohe, S., Uribe, M. 2006. "Deep Habits," *Review of Economic Studies*, 73(1), 195-218.
39. Rotemberg, J., Woodford, M. 1999. "Interest Rate Rules in an Estimated Sticky Price Model," *NBER Chapters in Monetary Policy Rules*, 57-126.
40. Sakuma, T., Masujima, M., Maeda, S., Fukawa, K., Iwamoto, K. 2011. "The ESRI Short-term Macro-econometric Model of the Japanese Economy: Version 2011," ESRI Discussion Paper No. 259, Economic and Social Research Institute, Cabinet Office.

41. Shibamoto, M. 2007. "An Analysis of Monetary Policy Shocks In Japan: A Factor Augmented Vector Autoregressive Approach," *Japanese Economic Review*, 58 (4), 484-503.
42. Smets, F., Wouters, R. 2003. "An Estimated Dynamic Stochastic General Equilibrium Model of the Euro Area," *Journal of the European Economic Association*, 1 (5), 1123-1175.
43. Stock, J. H., Watson, M.,W. 2002. "Forecasting Using Principal Components From a Large Number of Predictors," *Journal of the American Statistical Association*, 97 (460), 1167-1179.
44. Sugo, T., Ueda, K. 2008. "Estimating a Dynamic Stochastic General Equilibrium Model for Japan," *Journal of the Japanese and International Economies*, 22 (4),476-502.
45. Zubairy, S. 2010. "Deep Habits, Nominal Rigidities and Interest Rate Rules," MPRA Paper No. 26053, University Library of Munich.