ESSAYS ON INTERNATIONAL CAPITAL ASSET PRICING

by

HENGYONG MO

A dissertation submitted to the Graduate Faculty in Economics

in partial fulfilment of the requirements for the degree of

Doctor of Philosophy, The City University of New York

2007

UMI Number: 3245045

UMI Microform 3245045

ProQuest Information and Learning Company
300 North Zeeb Road
P.O. Box 1346
Ann Arbor, MI 48106-1346

This manuscript has been read and accepted for the Graduate Faculty in Economics in satisfaction of the dissertation requirement for the degree of Doctor of Philosophy.

12-20-2006 — Professor Liuren Wu

Date — Chair of Examining Commitee

12-20-2006 — Professor Thom Thurston

Date — Executive Officier

Professor Liuren Wu

Professor Michael Grossman

Professor Salih Neftci

Professor Turan Bali

Supervisory Committee

THE CITY UNIVERSITY OF NEW YORK

ABSTRACT

ESSAYS ON INTERNATIONAL CAPITAL ASSET PRICING

by

Hengyong Mo

Advisor: Professor Liuren Wu

Essay1: We study the risk dynamics and pricing in international economies through a joint analysis of the time-series returns and option prices on the S&P 500 Index of the United States, the FTSE 100 Index of the United Kingdom, and the Nikkei-225 Stock Average of Japan. We develop an international capital asset pricing model, under which the return on each equity index is decomposed into a global component and a country-specific component. Both components are controlled by separate volatility processes. For each economy, separate market prices are assigned for the two return risk components and the two volatility risk components. Model estimation reveals several interesting insights. First, global and country-specific return and volatility risks show different dynamics. Global return movements contain a larger discontinuous component, and global return volatility is more persistent than the country-specific counterparts. Second, investors charge positive prices for global return risk and negative prices for volatility risk, suggesting that investors are willing to pay positive premiums to hedge against downside global return movements and upside volatility movements. Third, the three economies contain different risk profiles and also price risks differently. Japan contains the largest idiosyncratic risk component and smallest global risk component. Investors in the Japanese market also price more heavily against future volatility increases than against future market downfalls.

Essay2: We study the pricing of illiquid cross exchange rate options by identifying stochastic discount factors embedded in currency triangles. We develop dynamic models of stochastic discount factors, under which the stochastic discount factor in each economy is decomposed into a global diffusion risk component and a country-specific jump-diffusion risk component. Separate stochastic time changes are further applied to the two com-

ponents so that stochastic volatilities can come separately from both global and country-specific risks. We propose to identify both the global and the country-specific risks for three economies using options on the three currency pairs that form a currency triangle. Then, by incorporating options on any other currency pair that links to one of the three economies, we can also identify the country-specific risk dynamics in the fourth economy and thus price options on currency pairs that involve the fourth economy.

ACKNOWLEDGEMENTS

Numerous individuals helped me to make this dissertation possible and make my study such an unforgettable experience. The list is long, but I cherish each contribution to my development as a researcher.

My first debt of gratitude must go to my advisor, Dr. Liuren Wu, a gracious mentor who instilled in me the qualities of being a productive researcher. I shall never forget his willingness to take a chance with me. I deeply appreciate how much time he devoted to providing perceptive comment and helpful advice in every phase of my dissertation.

Special thanks to Dr. Michael Grossman who lends me wholehearted support and assistance. He is one of the best people I have ever met in my life. I am extremely grateful to him for introducing me various job opportunities. For everything you've done for me, Dr. Grossman, I thank you.

I extend my gratitude to Dr. Salih Neftci for his kindness, support, and encouragement over the past five years. He is the person who first introduced me to the fascinating field of financial economics. I also thank his generosity in providing the valuable opportunity to interact with the world outside the university.

I wish also to thank Dr. Turan Bali for his invaluable advice and encouragement, and most importantly, for opening my view to a systematic perspective on risk management and empirical asset pricing. It is my honor to have the opportunity to work with him.

I am fortunate to have Dr. Thom Thurston as my mentor and chairman of the Economics department. He always keeps his door open, and is willing to listen to his student. All his kind help, especially the essential assistance before I came to the U.S., will never be forgotten.

Thanks also go out to my supervisor at the NBER, Dr. Robert Lipsey, who generously had me involved in many interesting projects and also provided ample room for me to explore many different new things. I also thank all research associates and staffs at the NBER New York Office for their helpful advice and support.

While working on my dissertation, I profited a lot from the opportunities to present and discuss my work-in-progress in the seminars at Baruch College, the Graduate Center, and the 2006 FMA Annual Meeting. My special gratitude goes to the faculty and seminar participants, especially Craig Lewis who discussed my paper at the FMA meeting,

I am also very grateful to all the people who provide feedback at various phases of the dissertation: Thep J. Vermaelen (the editor), two anonymous referees, Gurdip Bakshi, Peter Carr, and Harvey Stein.

I am particularly thankful to Yi Tang, my good friend, classmate, and co-author. We not only studied, relaxed, and traveled well together, but also encouraged each other from time to time. This made the writing of a dissertation a less lonely and isolating experience. His excellent cooking also adds a flavor to my life in Flushing.

Finally, and most importantly, I would like to thank my wife Vanessa. Her support, encouragement, quiet patience and unwavering love were undeniably the bedrock upon which the past five years of my life have been built. I thank my sister, Annie, who taught me how to survive and strive in this world. Also, I thank my parents for their love and understanding during the long years of my education.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

Part I

International Capital Asset Pricing: Evidence form Options

1.1. Introduction

In a world with different degrees of integration, how risks are priced? Do investors price global risks the same as they price country-specific risks? Given the predominant evidence on stochastic volatility, how are the volatility risks priced? Are the global and country-specific return risks driven by different volatility dynamics? Are the volatility risks associated with different return risks priced differently? How do investors in different economies price these risks differently? In this paper, we address these questions through an integrated analysis of the time-series returns and option prices on three equity indexes underlying three economies: the S&P 500 Index of the United States (US), the FTSE 100 Index of the United Kingdom (UK), and the Nikkei-225 Stock Average of Japan.

Traditional literature on international capital asset pricing studies the behavior of risks and risk premiums using mainly regression analysis on stock portfolio returns. More recently, accompanying the rapid expansion of the derivatives market and the theoretical progress in option pricing is the realization that options contain important information about the underlying security return dynamics and about the pricing of various sources of risks. Each index level represents a point estimate of its mean value, but the prices of options across the whole spectrum of strikes and maturities provide a complete picture of the risk-neutral distribution of the underlying equity index across all possible realizations and conditioning horizons. A joint analysis of equity index returns and index options reveals important information about how various sources of risks are priced on the index. In this paper, we exploit the rich information content in option prices underlying three equity indexes and study the risk dynamics and market pricing in the three economies under a dynamically consistent international asset pricing framework.

We build an international capital asset pricing model, under which the return on each equity index is decomposed into a global risk component and a country-specific risk component, with each component modeled as a jump-diffusion process to capture both the continuous and discontinuous movements in the equity index. Furthermore, we apply separate stochastic time changes to the two jump-diffusion return risk components so that stochastic volatility can come separately from both global and country-specific risks. For each economy, we assign separate market prices for the two return risk components and the two volatility risk components in each index. Under this model specification, we derive tractable solutions for the index option values and the likelihood of the index returns.

We estimate the model using six and a half years of data from January 1996 to August 2002 on the time-series returns and option prices on the three equity indexes. At each date and on each index, we have 20 over-the-counter options quotes at four maturities and five strike levels. For the model estimation, we cast the option pricing model into a state-space form, define the state-propagation equation according to the dynamics of the four variance rates (one global and three country-specific) underlying the three economies, and define the measurement equations based on the 60 option series underlying the three indexes. We use an extended version of the Kalman filter to obtain the ex-ante forecasts and ex-post updates on the conditional mean and variance of the four state variables and 60 measurement series. We then construct the likelihood function on the 60 option series assuming normal forecasting errors. Furthermore, given the variance rates filtered from the option series, we construct the likelihood on the three return series using fast Fourier inversion of the conditional characteristic function of the index returns. We estimate the model parameters by maximizing the sum of the likelihood values from the options and the index returns.

Model estimation reveals several interesting results on the risk dynamics and market pricing among the three economies. First, global and country-specific return and volatility risks show different dynamics. The global return risk component contains a larger proportion of discontinuous movements, especially large downside jump movements, than does the country-specific return risk component, suggesting that large downside movements are more likely to exhort a global impact whereas small diffusive movements are more likely to be local fluctuations. Furthermore, the global and country-specific risks are driven by different volatility dynamics. The variance rate underlying the global risk component is more persistent than the variance rates underlying the country-specific risks, suggesting that it is more difficult to predict global risk variations than country-specific risk variations.

Second, investors price return risk and volatility risk differently. The market prices of the global return risk are positive, but the market prices on both global and country-specific volatility risks are highly negative, suggesting that investors are willing to pay positive premiums to hedge against downside global return movements and upside volatility movements. The market prices of country-specific return risks vary across the three economies.

Finally, cross-sectional comparison shows that the three economies contain different risk profiles and also price risks differently. Japan contains the largest idiosyncratic risk component and smallest global risk component. Investors in the Japanese market also price more heavily against future volatility increases than against future market downfalls.

In related literature, there is a long list of theoretical and empirical works on international capital asset pricing.[1] The theoretical literature often specifies a linear relation between the expected excess returns on a stock or a stock portfolio and its exposures to various risk factors in the international economies. Empirical works focus on estimating these linear relations using stock portfolio returns. In contrast, we develop an international capital asset pricing model that accommodates both continuous and discontinuous movements, and generates separate stochastic volatilities from two risk sources. We relate these different sources of risks and their market prices to option prices on stock indexes. The relations are highly nonlinear, but nevertheless analytically tractable. Relying on the rich information in a large cross section of option prices, we can achieve a better distinction and identification on different sources of risks and their market pricing. Therefore, our key contribution to the literature is to advocate the use of the large cross sections of options data in answering questions in international capital asset pricing and to propose a dynamically

[1]Prominent examples include Solnik (1974), Subrahmanyam (1975), Grauer, Litzenberger, and Stehle (1976), Adler and Dumas (1983), Errunza and Losq (1985), Harvey (1991), Chan, Karolyi, and Stulz (1992), Ferson and Harvey (1993), Bekaert and Harvey (1995), Dumas and Solnik (1995), Basak (1996), De Santis and Gerard (1997), Fama and French (1998), De Santis and Gerard (1998), Cavaglia, Hodrick, Vadim, and Zhang (2002), Carrieri, Errunza, and Majerbi (2004), Zhang (2006), and Bali and Wu (2005). Also refer to Karolyi and Stulz (2003) for an excellent survey.

consistent theoretical framework for doing so.

In a related working paper, Driessen and Maenhout (2004) also use options on the three equity indexes to study the volatility and jump risks in an international setting. Similar to us, they also emphasize the importance of the information content in options. Unlike us, they adopt a more traditional approach in performing the risk-return analysis. They use returns on at-the-money straddles to proxy what they refer to as the "crash-neutral volatility" and returns on out-of-the-money put options with a 0.96 strike-to-spot ratio to proxy jump risks. Then, they use regression analysis to study how these proxy risk factors are priced. Our structural modeling approach is complementary to their linear approximation. In particular, by building and estimating a structural model, we explicitly exclude arbitrage opportunities and hence guarantee the dynamic and cross-sectional consistency among all option prices. As a result, we can make full use of the large cross sections of options data and include all of them in our estimation, rather than choosing only one or two of them as risk proxies while discarding the others. Furthermore, our risk factors have more explicit economic meanings and their nonlinear impacts on index returns and option prices are fully revealed through the estimated model parameters.

The important information content in the options market has also been exploited in a single economy context. Several recent papers study the stock index return dynamics and the market price of risks using time-series returns and options on the S&P 500 index, e.g., Jackwerth and Rubinstein (1996), Bakshi, Cao, and Chen (1997), Bates (2000), Pan (2002), Engle and Rosenberg (2002), Bakshi, Kapadia, and Madan (2003), Bakshi and Kapadia (2003), Eraker (2004), and Bliss and Panigirtzoglou (2004). More recently, Bakshi, Carr, and Wu (2005) study the pricing of risks in international economies using currency returns and currency options.

The paper is organized as follows. Section 1.2 describes the data and summarizes the stylized evidence. Section 1.3 proposes an international asset pricing model and derives its implications on moment conditions on index returns, risk premiums, exchange rate dynamics, option pricing, as well as the conditional likelihood of time-series returns. Section 1.4

outlines the estimation strategy based on the time series of both equity index returns and index option prices. Section 1.5 discusses the estimation results. Section 1.6 performs specification analysis. Section 1.7 concludes.

1.2. Stylized evidence on the international equity index options markets

In this section, we describe the data set and summarize the stylized features of the international equity index options market.

1.2.1. Data description and summary statistics

We obtain over-the-counter quotes from a major investment bank on options underlying three major equity indexes: the S&P 500 Index of the United States, the FTSE 100 Index of the United Kingdom, and the Nikkei-225 Stock Average of Japan. Henceforth, we label the three indexes as SPX, FTS, and NKY, respectively. The quotes are available daily from January 1996 to August 2002. We sample the data weekly on every Wednesday to avoid weekday effect. Each series contains 346 weekly observations.

The same data source also provides the matching equity index level at each date. For reference and comparison with the options behavior, we report the summary statistics of the weekly log returns in Table 1.1 on the three equity indexes during our sample period. The average annualized returns are fairly low during our sample period, 5.91% for SPX, 1.74% for FTS, and even negative at -10.9% for NKY. The annualized standard deviation estimates are 18.19%, 17.24%, and 22.08% for SPX, FTS, and NKY, respectively. The skewness estimate on SPX is significantly negative. The estimate on FTS is negative but not statistically different from zero. The skewness estimate on NKY is positive but insignificant. The excess kurtosis estimates on all three indexes are positive and statistically significant, indicating that all three return distributions have thicker tails than a normal distribution.

The options quotes are in terms of the Black and Scholes (1973) implied volatilities. For each index and at each date, we have 20 implied volatility quotes at four fixed time-to-maturities and five strike levels. The time-to-maturities are at one, three, six, and 12 months, and the five strike levels are at 80, 90, 100, 110, and 120% of the spot index level. Across the three economies, we have 60 implied volatility series. To convert the implied volatility quotes into option prices using the Black-Scholes formula, we need information on dividend yields and interest rates. Dividend yields are provided by the same bank, which estimates the dividend yields from the relative pricing of the spot and futures on the equity index. For interest rates, we download from Bloomberg the LIBOR rates of the corresponding maturities and currencies and convert them into continuously compounded interest rates.

Table 1.2 reports the sample statistics of the implied volatility quotes. The first panel reports the sample average of the implied volatilities at each fixed maturity and strike price level. Across all maturities and all three indexes, the sample averages of the at-the-money implied volatility quotes (striking at 100% spot level) are all higher than the corresponding annualized standard deviation estimates from the index returns (Table 1.1). Since the at-the-money implied volatility approximates the risk-neutral expected value of the return volatility (Carr and Lee (2003)), the positive difference between the average implied volatility and the return standard deviation reflects negative volatility risk premia. Similar to our statistics, several recent studies have documented negative variance risk premia on stock indexes (Bondarenko (2004), and Carr and Wu (2004b)).

The first panel of Table 1.2 also shows that the average implied volatility at a fixed maturity increases as the strike price decreases, generating the well-documented implied volatility smirk pattern. The Black-Scholes model assumes a normal distribution for the index return. The smirk shape along the strike price dimension has long been regarded as evidence for the return distribution to be negatively skewed under the risk-neutral measure, with the slope of the smirk reflecting the degree of skewness. Table 1.1 shows that the skewness estimates from the time-series returns are significantly negative only for SPX, and even become positive for NKY. In contrast, the average implied volatility smirks on all three indexes and across all maturities are strongly negatively sloped, suggesting that the return distribution is highly negatively skewed under the risk-neutral measure, even through the distribution is not as highly skewed under the statistical measure. We can also attribute the skewness differences under the two measures to risk premia.[2] For example, Polimenis (2006) shows that under a representative agent model with constant relative risk aversion on terminal wealth, the risk-neutral return distribution can be negatively skewed even if the statistical return distribution is symmetric, as long as the return shows positive excess kurtosis. Consistent with his argument, the excess kurtosis estimates in Table 1.1 are all significantly positive.

The implied volatility smirk pattern observed in Table 1.2 indicates that investors worldwide pay higher premium for out-of-the-money put options than for the corresponding out-of-the-money call options regardless of the underlying index or its geographical location. One potential explanation for this global phenomenon is that downside movements in any index are likely to be highly correlated with those in other markets due to worldwide contagion. For example, Das and Uppal (2004) argue that large downside jumps in international equity markets tend to occur at the same time. Since downside risk cannot be readily diversified by investing in different equity markets, and in aggregation the markets are long in these stock indexes, insurance for downside movements on these indexes commands a high premium.

For SPX and FTS, the average implied volatility level at 80 percent strike is about twice as high as the average implied volatility level at 120 percent strike, generating a steep smirk shape. The average implied volatility smirk on NKY is flatter. The difference in

[2]Part of the differences between the statistical and risk-neutral estimates can also come from measurement errors. For example, options could have priced in some extreme events such as market crashes, but these events may not have happened during our sample period. In this case, the impact of these extreme events in both volatility and higher moments shows up in the risk-neutral distribution estimated from the option prices, but does not show up in the statistical distribution estimated from the time series sample.

slope shows up more vividly in Fig. 1.1, where we plot and contrast the average implied volatility smirks of the three indexes at each maturity. The four panels in Fig. 1.1 represent the four different maturities. In each panel, the dash-dotted line denotes the average implied volatility smirk on NKY, which is visibly flatter than the average implied volatility smirk on SPX (solid line) and FTS (dashed line). The flatter smirk on NKY implies that the risk-neutral return distribution on NKY is less negatively skewed than the risk-neutral return distributions on SPX and FTS.

The second panel of Table 1.2 reports the standard deviation estimates of the implied volatility quotes, which are about 3-5% for SPX, 5-10% for FTS, and 4-7% for NKY. The third and fourth panels report the skewness and excess kurtosis estimates of the implied volatility quotes. These estimates are mostly small, indicating that the quotes do not experience large discontinuous updates. The last panel reports the weekly autocorrelation estimates, which range from 0.86 to 0.98. At a fixed moneyness level, the autocorrelation estimates tend to increase with increasing option maturities.

1.2.2. International equity index volatility co-movements

Fig. 1.2 plots the time series of the at-the-money implied volatilities. Each panel represents one maturity. The three lines in each panel denote the three indexes. The implied volatilities on the three indexes all show large time-variation over our sample period, from as low as 10% to as high as 50%. The strong time-variation suggests that a reasonable model for the stock indexes should allow for stochastic volatility.

The implied volatilities all started at relatively low levels, but they all spiked up during the hedge fund crisis in late 1998. The time series show visible co-movements between the implied volatilities on the three indexes. Table 1.3 reports the cross-correlation estimates on weekly changes in implied volatilities between different pairs of the three underlying equity index at each fixed time to maturity and moneyness level. Across all maturities and strikes, the correlations between SPX and FTS are higher than their correlations with

NKY, indicating that the Japanese equity index options implied volatilities contain a larger proportion of idiosyncratic movements. When we estimate the cross-correlation between the three weekly index return series during the same sample period, we obtain an estimate of 0.7 between SPX and FTS returns, 0.38 between SPX and NKY returns, and 0.37 between FTS and NKY returns. Hence, the Japan equity index return also contains a larger proportion of idiosyncratic movements.

For a given pair of economies, the correlations at low strikes are stronger than those at high strikes. Longin and Solnik (2001) document that correlations between international equity market returns are higher in bear markets than in bull markets. Our evidence points to similar correlation patterns in return volatilities.

To understand the factor structure in volatility co-movements, we perform principal component analysis on weekly changes in the implied volatilities. First, we estimate the correlation matrix between the 60 implied volatility series underlying the three indexes. Then, we compute the eigenvalues of the correlation matrix. The normalized eigenvalues can be interpreted as the percentage variation explained by each principal component. Fig. 1.3 uses the bar chart to show the percentages of variation explained by each of the first ten principle components on the weekly implied volatility changes. The first component accounts for over 50% of the variation in the weekly volatility changes. The contribution from the second principal component is much lower at about 20% percent. When we link the bars with a line to highlight the speed of decay for the explained variation, we find an obvious slope change after the first principal component.

The literature has not arrived at a consensus criterion in determining the optimal number of common factors in principal component analysis. Although some researchers have proposed statistical tests, e.g., Connor and Korajczyk (1993), a common practice is to plot the eigenvalues and visually inspect for slope changes. Principal components with eigenvalues falling on different slopes represent different levels of commonality. The slope changes in Fig. 1.3 indicate that the first principal component possesses a different level of commonality from the other principal components. We regard this component as a global volatility component. The next three principal components fall on one slope and can be regarded as country-specific volatility components. The contribution from the remaining components is negligible. Overall, the evidence suggests that we can use four volatility factors to model the implied volatility movements, with one global factor and three country-specific factors.

1.3. An international capital asset pricing model

With the stylized evidence in mind, we propose an international asset pricing model, under which each equity index return is decomposed into two orthogonal return risk components: a global component and a country-specific component. Each component contains both continuous and discontinuous movements. The volatilities underlying the two return risk components are stochastic and follow separate dynamics.

Formally, we consider N economies by fixing a probability space $(\Omega, \mathcal{F}, \mathbb{P})$ endowed with a standard complete filtration $\mathbf{F} = \{\mathcal{F}_t | t \geq 0\}$. We let S_t^i denote the time-t level of the stock index in economy i, for $i = 1, 2, \cdots, N$, under the local currency of economy i, and let $s_t^i \equiv \ln S_t^i / S_0^i$ denote the log index return over the time horizon $[0, t]$. We specify the index return dynamics as,

$$\begin{aligned} s_t^i &= \mu_t^i + \left(r^i - q^i\right) t \\ &\quad + \beta^i \left(W^g + J^g\right)_{\mathcal{T}_t^g} - \left(\tfrac{1}{2}(\beta^i)^2 + k_{J^g}\left[\beta^i\right]\right) \mathcal{T}_t^g \\ &\quad + \xi^i \left(W^c + J^c\right)_{\mathcal{T}_t^c} - \left(\tfrac{1}{2}(\xi^i)^2 + k_{J^c}\left[\xi^i\right]\right) \mathcal{T}_t^c, \quad i = 1, 2, \cdots, N, \end{aligned} \tag{1.1}$$

where μ_t^i denotes the conditional expected excess return (risk premium) on stock index i over horizon $[0, t]$ that is commensurate with the underlying risks and their market pricing , r^i denotes the instantaneous country-i interest rate and q^i the dividend yield of the relevant maturity, both of which we assume deterministic, $(W^g + J^g)$ denotes a global jump-diffusion return risk factor, and $(W^c + J^c)$ denotes an orthogonal country-specific jump-diffusion return risk component.

To capture the stochastic variation in the index return variance and higher moments, we apply separate stochastic time changes $\mathcal{T}_t^g$ and $\mathcal{T}_t^c$ to the global and country-specific return risk components, respectively. We assume locally predictable and continuous time changes and specify them via the *instantaneous activity rates* v_t^g and v_t^c:

$$\mathcal{T}_t^g \equiv \int_0^t v_s^g ds, \quad \mathcal{T}_t^c \equiv \int_0^t v_s^c ds. \tag{1.2}$$

Under our specification in (1.2), a time-changed Brownian motion $W_{\mathcal{T}_t}$ is equivalent in probability to the classic representation $\int_0^t \sqrt{v_s} dW_s$. The instantaneous activity rate v_t is equivalent to the instantaneous variance rate of the Brownian motion. When we apply the stochastic time change to a jump-diffusion Lévy process $(W+J)$, the activity rate is also proportional to the jump arrival rate. Thus, applying the stochastic time change is a convenient way of generating stochastic volatility from both Brownian motions and jumps. We refer interested readers to Carr and Wu (2004a) for the mathematical treatment of time-changed Lévy processes and Wu (2006b) for a review on modeling financial security returns using time-changed Lévy processes.

In equation (1.1), we use a set of scaling coefficients $\left\{\beta^i \in \mathbb{R}^+\right\}_{i=1}^N$ to capture the loading of each stock index on the global risk factor. For parsimony, we also use a set of scaling coefficients $\left\{\xi^i \in \mathbb{R}^+\right\}_{i=1}^N$ to capture the average difference in the country-specific risk levels across the N different economies. With these scaling coefficients, we assume that the country-specific risk component $(W^c + J^c)_{\mathcal{T}_t^c}$ is independent but identical across the N economies. For identification, we normalize the scaling on the SPX index in the US economy to unity: $\beta^{SPX} = \xi^{SPX} = 1$. Then, the scaling coefficient estimates on other economies capture their relative differences from the US economy.

The terms following each risk component, $\left(\frac{1}{2}(\beta^i)^2 + k_{J^g}\left[\beta^i\right]\right)\mathcal{T}_t^g$ and $\left(\frac{1}{2}(\xi^i)^2 + k_{J^c}\left[\xi^i\right]\right)\mathcal{T}_t^c$, are concavity-adjustment terms, with $k_J[s]$ denoting the cumulant exponent of the Lévy jump component J, defined as

$$k_J[s] \equiv \frac{1}{t}\ln \mathbb{E}\left(e^{sJ_t}\right), \; s \in \mathcal{D} \subset \mathbb{C}.$$

It is the property of Lévy processes that the cumulant exponent does not depend on the time horizon. Traditionally, the cumulant exponent is defined on the positive real half-line, but it is convenient to extend the definition to the subsect of the complex plane $\mathcal{D}$ where the expectation is well-defined.

The general model formulation in (1.1) decomposes the return of each equity index into four risk components: a jump-diffusion global return risk component, a jump-diffusion country-specific return risk component, and stochastic volatility risks underlying each of the two return risk components. Through model design and estimation, we can study how investors respond to different sources of risks in international economies. We can also study the degree of international integration in financial markets by estimating the relative proportion of variation in the global risk component versus the country-specific risk component.

1.3.1. Jump structure and activity rate dynamics

We specify the dynamics of both v_t^g and v_t^c as following square-root processes,

$$dv_t^g = \kappa_g \left(\theta_g - v_t^g\right) dt + \omega_g \sqrt{v_t^g} dZ_t^g, \quad dv_t^c = \kappa_c \left(\theta_c - v_t^c\right) dt + \omega_c \sqrt{v_t^c} dZ_t^c, \quad (1.3)$$

where (κ_g, κ_c) control the mean-reversion speeds of the activity rate processes, (θ_g, θ_c) denote their long-run means, and (Z^g, Z^c) are standard Brownian motions that are correlated with the corresponding Brownian motions in the index return by: $\rho_g dt = E(dW_t^g dZ_t^g)$ and $\rho_c dt = E(dW_t^c dZ_t^c)$. There is one global activity rate process v_t^g that drives the volatility of the global return component, but it is important to realize that there are N independent processes for v_t^c, one for each economy, although they share identical dynamics.

We further assume that the jump components J^g and J^c are governed by double-exponential specifications (Kou (2002)), the Lévy densities of which are given by:

$$\pi^g[x] \;=\; \begin{cases} \lambda^g e^{-\zeta_+^g x}, & x>0 \\ \lambda^g e^{-\zeta_-^g |x|}, & x<0 \end{cases}, \quad \pi^c[x] = \begin{cases} \lambda^c e^{-\zeta_+^c x}, & x>0 \\ \lambda^c e^{-\zeta_-^c |x|}, & x<0 \end{cases}, \tag{1.4}$$

where the proportional coefficients $(\lambda^g, \lambda^c) \in \mathbb{R}^+$ control the average arrival rate of the jumps and the exponential coefficients $(\zeta_\pm^g, \zeta_\pm^c) \in \mathbb{R}^+$ determine the relative asymmetry of the jump size distribution.

Economically, incorporating the jump components is important in capturing large discontinuous movements in economic fundamentals and financial security prices as shown in Almeida, Goodhart, and Payne (1998), Andersen, Bollerslev, Diebold, and Vega (2003), Andersen, Bollerslev, Diebold, and Vega (2005), Beber and Brandt (2005), and Pasquariello and Vega (2005). Statistically, it also helps to generate index return non-normality and realistic index option behaviors. Under the jump specification in (1.4), the cumulant exponent is,

$$k_J[s] = \lambda\left((\zeta_+ - s)^{-1} - (\zeta_+)^{-1} + (\zeta_- + s)^{-1} - (\zeta_-)^{-1}\right), \quad \mathbb{R}[s] \in (-\zeta_-, \zeta_+), \tag{1.5}$$

where we drop the superscripts (g,c) on the coefficients to reduce notation clustering when no confusion shall occur. For the cumulant exponent to be well-defined, the real part of the cumulant coefficient s must be within $(-\zeta_-, \zeta_+)$. Thus, for the concavity terms to be well-defined in (1.1), we need $\zeta_+^g > \beta^i$ and $\zeta_+^c > \xi^i$ for all i.

The cumulant exponent of the Brownian motion component is $k_W[s] = \frac{1}{2}s^2$. By taking successive derivatives on the cumulant exponents against the exponent coefficient s and evaluating the derivatives at $s=0$, we can derive the variance (m_2) and higher-order cumulants ($m_n, n = 3, 4, \cdots$) per unit time generated from each jump-diffusion return risk component ($W_t + J_t$):

$$\begin{aligned} m_2 &= 1 + 2\lambda\left(\zeta_+^{-3} + \zeta_-^{-3}\right), \\ m_n &= \lambda n!\left(\zeta_+^{-(n+1)} + (-1)^n \zeta_-^{-(n+1)}\right), \quad n = 3, 4, \cdots. \end{aligned} \tag{1.6}$$

The diffusion component contributes one unit to the variance, but zero to all higher-order cumulants. When dampening coefficients are the same for upside and downside jumps $\zeta_+ = \zeta_-$, all odd-order cumulants become zero. A negatively skewed return distribution under the statistical measure $\mathbb{P}$ can be generated with $\zeta_+ > \zeta_-$.

In the absence of stochastic time change, the return variance and higher cumulants generated from the global and country-specific jump-diffusion return components are all constant. Such a model would generate an implied volatility surface that is constant over time. To capture the strong time-variation in index option implied volatilities, we apply stochastic time changes to the two jump-diffusion return components as in (1.2). The instantaneous return variance and higher-order cumulants per unit calendar time in economy i becomes:

$$\begin{aligned} m_{2t} &= v_t^g(\beta^i)^2\left(1+2\lambda^g\left((\zeta_+^g)^{-3}+(\zeta_-^g)^{-3}\right)\right)+v_t^c(\xi^i)^2\left(1+2\lambda^c\left((\zeta_+^c)^{-3}+(\zeta_-^c)^{-3}\right)\right), \\ m_{nt} &= v_t^g(\beta^i)^n\lambda^g n!\left((\zeta_+^g)^{-(n+1)}+(-1)^n(\zeta_-^g)^{-(n+1)}\right) \\ &+ v_t^c(\xi^i)^n\lambda^c n!\left((\zeta_+^c)^{-(n+1)}+(-1)^n(\zeta_-^c)^{-(n+1)}\right), \quad n=3,4,\cdots. \end{aligned} \tag{1.7}$$

Time variation in return variance comes from two sources: The activity rate variation from the global return component and the activity rate variation from the country-specific return component. The higher return cumulants also become stochastic due to the variation of the activity rates.

1.3.2. Market price of global and country-specific return and volatility risks

To study the pricing of different sources of risks in each economy, we specify a flexible structure for the nominal pricing kernel of each economy i:

$$\mathcal{M}_t^i = \exp\left(-r^i t\right)\mathcal{E}\left(-\gamma_g^i\left(W^g+J^g\right)_{\mathcal{T}_t^g}-\gamma_c^i\left(W^c+J^c\right)_{\mathcal{T}_t^c}-\gamma_{v^g}^i Z_{\mathcal{T}_t^g}^g-\gamma_{v^c}^i Z_{\mathcal{T}_t^c}^i\right), i=1,\cdots,N, \tag{1.8}$$

where $\mathcal{E}(\cdot)$ denotes the stochastic exponential martingale operator, which defines the measure change from the statistical measure $\mathbb{P}$ to the risk-neutral measure $\mathbb{Q}^i$ in currency i. The pricing kernel assigns separate market prices for the global and the country-specific return risk components. Furthermore, the kernel allows separate pricing for the two sources of volatility risks. The subscripts on γ refer to the different sources of risk. The superscript i on γ refers to the country i-specific nature of the pricing. Thus, our specification allows different economies to have different market prices on the four sources of risks.

To gain more economic intuition on the pricing kernel, we appeal to the classic Lucas (1982) exchange economy, in which the pricing kernel can be interpreted as the ratio of the marginal utilities of aggregate wealth over two time horizons. More specifically, if we use X_t to denote the return innovation over horizon $[0,t]$ on the aggregate wealth and use γ to denote the relative risk aversion of a constant-relative-risk-aversion (CRRA) utility representative agent, the martingale component of the pricing kernel can be written as $\mathcal{E}(-\gamma X_t)$. Our pricing kernel specification in equation (1.8) goes far beyond this prototype economy. We decompose the index return in each economy into a global and a country-specific jump-diffusion risk component. Through stochastic time change, we allow both components to generate stochastic volatility. Most importantly, we allow the two return risk components and the two stochastic volatility risk components to have separate market prices.

In an ideal world with both financial and goods market integration, all goods should be priced the same in real terms across the world and the market price of the same source of risk should be the same across all economies. In this ideal case, and when the economies are fully symmetric ($\beta^i = \xi^i = 1$ for all i), the market price coefficients $(\gamma_g^i, \gamma_c^i, \gamma_{gv}^i, \gamma_{cv}^i)$ should be the same across different economies. Nevertheless, in reality, even with growing evidence of global financial market integration, deviation from purchasing power parity remains persistent, with convergence to the parity condition extremely slow. The short-run deviations are often large in magnitude, and volatile in their fluctuations (Rogoff (1996)). These deviations suggest that the international goods market remains quite segmented, even though the financial market is more integrated. Furthermore, in nominal terms, the gross violation of the uncovered interest rate parity is repeatedly documented.[3] Fama (1984) attributes this gross violation to time-varying risk premium. Sarkissian (2003) further attributes the time-varying risk premium to incomplete consumption risk sharing. In accordance with the above evidence, we do not constrain the market pricing of the same source of risk across different economies to be the same. Instead, we allow them to be different and estimate their difference using the time-series returns and option prices on the three equity indexes.

Given the pricing kernel specification in (1.8), the measure change from the statistical measure $\mathbb{P}$ to the risk-neutral measure $\mathbb{Q}^i$ in currency i is governed by the following exponential martingale,

$$\left.\frac{d\mathbb{Q}^i}{d\mathbb{P}}\right|_t \equiv \mathcal{E}\left(-\gamma_g^i \left(W^g + J^g\right)_{\mathcal{T}_t^g} - \gamma_c^i \left(W^c + J^c\right)_{\mathcal{T}_t^c} - \gamma_{v^g}^i Z^g_{\mathcal{T}_t^g} - \gamma_{v^c}^i Z^i_{\mathcal{T}_t^c}\right). \tag{1.9}$$

The equity index return dynamics under measure $\mathbb{Q}^i$ becomes,

$$\begin{aligned} s_t^i &= \left(r^i - q^i\right)t + \beta^i (W^{g\mathbb{Q}} + J^{g\mathbb{Q}})_{\mathcal{T}_t^g} - (\tfrac{1}{2}(\beta^i)^2 + k_{J^g}^{\mathbb{Q}}[\beta^i])\mathcal{T}_t^g \\ &\quad + \xi^i (W^{c\mathbb{Q}} + J^{c\mathbb{Q}})_{\mathcal{T}_t^c} - (\tfrac{1}{2}(\xi^i)^2 + k_{J^c}^{\mathbb{Q}}[\xi^i])\mathcal{T}_t^c, \end{aligned} \tag{1.10}$$

where the Lévy densities of the jump components under the risk-neutral measure $\mathbb{Q}$ change as follows,

$$\pi^{g\mathbb{Q}}[x] = e^{-\gamma_g^i x}\nu^g[x] = \begin{cases} \lambda^g e^{-\left(\zeta_+^g + \gamma_g^i\right)x}, & x > 0 \\ \lambda^g e^{-\left(\zeta_-^g - \gamma_g^i\right)|x|}, & x < 0 \end{cases}, \tag{1.11}$$

$$\pi^{c\mathbb{Q}}[x] = e^{-\gamma_c^i x}\nu^c[x] = \begin{cases} \lambda^c e^{-\left(\zeta_+^c + \gamma_c^i\right)x}, & x > 0 \\ \lambda^c e^{-\left(\zeta_-^c - \gamma_c^i\right)|x|}, & x < 0 \end{cases}. \tag{1.12}$$

If we define $\zeta_+^{\mathbb{Q}} \equiv \zeta_+ + \gamma$ and $\zeta_-^{\mathbb{Q}} \equiv \zeta_- - \gamma$, it is obvious that the jumps also obey double-

[3] Prominent examples include Hsieh (1984), Mishkin (1984), Hodrick (1987), Bekaert and Hodrick (1992), Backus, Gregory, and Telmer (1993), Dumas and Solnik (1995), Engel (1996), Bansal (1997), Bansal and Dahlquist (2000), and Backus, Foresi, and Telmer (2001).

exponential specifications under the risk-neutral measure $\mathbb{Q}^i$. The cumulant exponent under measure $\mathbb{Q}$ becomes,

$$k_J^{\mathbb{Q}}[s] = \lambda \left(\left(\zeta_+^{\mathbb{Q}} - s \right)^{-1} - \left(\zeta_+^{\mathbb{Q}} \right)^{-1} + \left(\zeta_-^{\mathbb{Q}} + s \right)^{-1} - \left(\zeta_-^{\mathbb{Q}} \right)^{-1} \right). \qquad (1.13)$$

For the model to be well-defined, we need that $\zeta_\pm^{\mathbb{Q}} > 0$, $\zeta_+^{g\mathbb{Q}} > \beta^i$, and $\zeta_+^{c\mathbb{Q}} > \xi^i$ for all i.

Equations (1.11) and (1.12) reveal how the market prices of return risks can generate a difference in the return skewness under the two measures $\mathbb{P}$ and $\mathbb{Q}$. In particular, a positive market price increases the dampening on positive jumps and reduces the dampening on negative jumps, and thus making the return distribution more negatively skewed under the risk-neutral measure. Thus, a positive market price on either or both of the two return jump components can explain our observation that the option implied risk-neutral skewness on the index returns is more negative than the skewness estimated from the index returns. Our model specification also reveals the reasons behind Polimenis (2006)'s result that positive excess kurtosis and risk aversion leads to a negatively skewed return distribution under the risk-neutral measure even if the distribution is symmetric under the statistical measure. Under our setup, the prerequisite for positive excess kurtosis becomes a prerequisite for a jump component, and risk aversion leads to a positive market price of return risk.

From the pricing kernel specification, we can derive the instantaneous return risk premium on stock index i generated from each unit of global return activity rate risk ($v_t^g = 1$):

$$\eta_g^i \quad = \quad \gamma_g^i \beta^i + \gamma_{vg}^i \beta^i \rho_g + \left(k_{J^g}[\beta^i] - k_{J^g}^{\mathbb{Q}}[\beta^i] \right), \qquad (1.14)$$

where the first component $\gamma_g^i \beta^i$ is due to the stock index's exposure to the global diffusion risk W_t^g, the second component $\gamma_{vg}^i \beta^i \rho_c$ is due to the index return's indirect exposure to the variance risk Z_t^g resultant from the correlation $\rho_g = \mathbb{E}[dW_t^g dZ_t^g]/dt$, and the last component is the premium due to the exposure to the global jump risk component J_t^g. Positive market price on the global return risk ($\gamma_g^i > 0$) and negative market price on the global variance risk ($\gamma_{vg}^i < 0$), together with negative correlation between return and variance, both lead to positive return risk premium. It can be shown that a positive market price on the return risk also generates a positive premium from the jump component: $k_{J^g}[\beta^i] - k^{\mathbb{Q}}_{J^g}[\beta^i] > 0$ if and only if $\gamma^i_g > 0$ (Appendix C).

Analogously, the instantaneous return risk premium generated from each unit of the country-specific return activity rate risk ($v^c_t = 1$) is,

$$\eta^i_c \;=\; \gamma^i_c \xi^i + \gamma^i_{v^c} \xi^i \rho_c + \left(k_{J^c}[\xi^i] - k^{\mathbb{Q}}_{J^c}[\xi^i] \right). \tag{1.15}$$

Taking together, we can write the total expected excess return μ^i_t on stock index i over horizon $[0,t]$ as the sum of the two unit risk premiums multiplied by their respective risk levels:

$$\mu^i_t = \int_0^t \eta^i_g v^g_s ds + \int_0^t \eta^i_c v^c_s ds = \eta^i_g \mathcal{T}^g_t + \eta^i_c \mathcal{T}^c_t. \tag{1.16}$$

The cumulant exponent in equation (1.13) implies that the risk-neutral variance ($m^{\mathbb{Q}}_2$) and higher-order cumulants ($m^{\mathbb{Q}}_n, n = 3, 4, \cdots$) per unit time generated from each jump-diffusion return risk component take on analogous forms to their statistical counterparts in equation (1.6):

$$\begin{aligned} m^{\mathbb{Q}}_2 &= 1 + 2\lambda \left((\zeta_+ + \gamma)^{-3} + (\zeta_- - \gamma)^{-3} \right), \\ m^{\mathbb{Q}}_n &= \lambda n! \left((\zeta_+ + \gamma)^{-(n+1)} + (-1)^n (\zeta_- - \gamma)^{-(n+1)} \right), \quad n = 3, 4, \cdots. \end{aligned} \tag{1.17}$$

Comparing the moment conditions under the two measures, we can show that the risk-neutral variance and higher even-order moments are larger than their statistical counterparts as long as (i) $\xi_+ \geq \xi_-$ (the statistical distribution is symmetric or negatively skewed) and (ii) $\gamma > 0$ (the market price of return risk component is positive). Polimenis (2006) draws a similar conclusion under a Lévy setup.

From the pricing kernel in (1.8), we can also derive the $\mathbb{Q}^i$-dynamics for the two activity rates:

$$dv_t^g = \left(\kappa_g\theta_g - \left(\kappa_g + \gamma_{vg}^i\omega_g + \gamma_g^i\omega_g\rho_g\right)v_t^g\right)dt + \omega_g\sqrt{v_t^g}dZ_t^g, \quad (1.18)$$

$$dv_t^c = \left(\kappa_c\theta_c - \left(\kappa_c + \gamma_{vc}^i\omega_c + \gamma_c^i\omega_c\rho_c\right)v_t^c\right)dt + \omega_c\sqrt{v_t^c}dZ_t^c. \quad (1.19)$$

For both activity rates, the measure change from $\mathbb{P}$ to $\mathbb{Q}^i$ induces a change in the mean-reversion speed. For the global activity rate v_t^g, the first adjustment term $\gamma_{vg}^i\omega_g$ is induced by the market pricing (γ_{vg}^i) of the global volatility risk (Z_t^c). A negative market pricing for the global variance risk would make the mean-reversion speed slower and hence the process more persistent under the risk-neutral measure. The second adjustment term $\gamma_g^i\omega_g\rho_g$ is induced by the market pricing (γ_g^i) of the global return diffusion risk (W_t^g) and its correlation with the diffusion variance risk (Z_t^g). A positive market pricing for the global return risk and a negative correlation between return and variance risks will make this second term negative as well and will make the global activity rate even more persistent under the risk-neutral measure. The adjustments for the country-specific activity rate v_t^c are defined analogously.

The market prices not only alter the activity rate persistence under the risk-neutral measure, but also affect the risk-neutral means of the activity rates:

$$\theta_g^{\mathbb{Q}} = \theta_g \frac{\kappa_g}{\kappa_g + \gamma_{vg}^i\omega_g + \gamma_g^i\omega_g\rho_g}, \quad \theta_c^{\mathbb{Q}} = \theta_c \frac{\kappa_c}{\kappa_c + \gamma_{vc}^i\omega_c + \gamma_c^i\omega_c\rho_c}. \quad (1.20)$$

Therefore, market prices that make the risk-neutral processes more persistent also make the risk-neutral long run means higher than their statistical counterparts. From the data, we have observed that the return standard deviation is lower than the sample averages of the risk-neutral expected value of the return volatility. Under our model, this evidence can be accounted for by (i) a negatively skewed return jump component and positive market price of return risk, and (ii) a combination of negative market price of variance risk and positive market price of return risk that makes the risk-neutral mean activity rates higher than their statistical counterparts.

Overall, our model specification uses 30 parameters to control the return and volatility dynamics for the three equity indexes, as well as the market prices of various sources of risks from each economy:

$$\Theta \equiv [\{\beta^i, \xi^i\}_{i=1}^2, \{\gamma_g^i, \gamma_c^i, \gamma_{vg}^i, \gamma_{vc}^i\}_{i=1}^3, \lambda^g, \beta_+^g, \beta_-^g, \kappa_g, \theta_g, \omega_g, \rho_g, \lambda^c, \beta_+^c, \beta_-^c, \kappa_c, \theta_c, \omega_c, \rho_c],$$

where (β^i, ξ^i) control the average scaling of the global and country-specific risk level for each country, with their magnitudes normalized to one for the US economy, $(\gamma_g^i, \gamma_c^i, \gamma_{vg}^i, \gamma_{vc}^i)$ control the economy i's market prices for the four sources of risks, $(\lambda^g, \beta_+^g, \beta_-^g)$ control the jump structure of the global jump component, $(\kappa_g, \theta_g, \omega_g, \rho_g)$ control the global activity rate dynamics, $(\lambda^c, \beta_+^c, \beta_-^c)$ control the jump structure of the country-specific jump component in each economy, and $(\kappa_c, \theta_c, \omega_c, \rho_c)$ control the country-specific activity rate dynamics for each economy.

1.3.3. Currency denomination and exchange rate dynamics

We specify the dynamics of each stock index in its own local currency. The index options that we use in our model estimation are also settled in the local currency of the index. While we do not incorporate currency returns and currency options into our model estimation, our pricing kernel specification in (1.8) has direct implications on the exchange rate dynamics between two currencies i and j. Let P^{ji} denote the price of currency j in currency i (e.g., P^{ji} denotes the dollar price of one pound if j denotes pound and i denotes dollar, with dollar being the home currency), our pricing kernel specification in (1.8) imply the following dynamics for the currency return,[4]

[4]It is possible that a fully specified pricing kernel also include risk components that are orthogonal to the stock index dynamics. These risk components do not affect the pricing of the stock index or stock index options, but can induce additional variation in the currency dynamics. See Bakshi, Carr, and Wu (2005) for a more detailed discussion.

$$\begin{aligned}\ln P_t^{ji}/P_0^{ji} &= \ln \mathcal{M}_t^f/\mathcal{M}_t^i \\ &= (r^i - r^j)t + (\gamma_g^i - \gamma_g^j)(W^g + J^g)_{\mathcal{T}_t^g} + (\gamma_{v^g}^i - \gamma_{v^g}^j)Z^g_{\mathcal{T}_t^g} \\ &\quad + \gamma_c^i (W^c + J^c)_{\mathcal{T}_t^c} - \gamma_c^j (W^c + J^c)_{\mathcal{T}_t^c} + \gamma_{v^c}^i Z^c_{\mathcal{T}_t^c} - \gamma_{v^c}^j Z^c_{\mathcal{T}_t^c},\end{aligned} \tag{1.21}$$

where the currency return is driven by six sources of risks: the global index return risk $(W^g + J^g)$, the global activity rate risk Z^g, the two independent country-specific risks from the two countries $(W^c + J^c)$ from countries j and i, respectively, and the two independent country-specific activity rate risks Z^c from countries j and i, respectively. When the economies are fully symmetric in terms of both risk and pricing, the global risk components $(W^g + J^g)$ and Z^g will drop out of the currency dynamics:

$$\ln P_t^{ji}/P_0^{ji} = (r^i - r^j)t + \gamma_c^i (W^c + J^c)_{\mathcal{T}_t^c} - \gamma_c^j (W^c + J^c)_{\mathcal{T}_t^c} + \gamma_{v^c}^i Z^c_{\mathcal{T}_t^c} - \gamma_{v^c}^j Z^c_{\mathcal{T}_t^c}. \tag{1.22}$$

In a similar but more stylized setup, Brandt, Cochrane, and Santa-Clara (2006) compare the stock portfolio return variance to the variance of the exchange rate to analyze the degree of international risk sharing between two economies. Bakshi, Carr, and Wu (2005) use time-series returns and option prices on three currency pairs that form a triangular relation to study the pricing kernel dynamics in the three economies. In this paper, we exploit the information in stock index returns and index options to identify the risk and pricing in three economies.

1.3.4. Option pricing

To price options on each stock index i, we first derive the generalized Fourier transform of the log index return under the risk-neutral measure $\mathbb{Q}^i$ defined in currency i,

$$\phi_{s_t^i}(u) \equiv \mathbb{E}^{\mathbb{Q}^i}\left(e^{ius_t^i}\right), \; u \in \mathcal{D} \in \mathbb{C}.$$

Then, we compute option prices numerically via fast Fourier inversion (Carr and Madan (1999)).

Under our model specification, the generalized Fourier transform can be derived in analytical form:

$$\phi_{s_t}(u) = e^{iu\left(r^i - q^i\right)t - a_g(t) - b_g(t)v_0^g - a_c(t) - b_c(t)v_0^c}, \tag{1.23}$$

where $\left(v_0^g, v_0^c\right)$ denote the current levels of the two activity rates and the coefficients on each activity rate are given by,

$$\begin{aligned}
a_g(t) &= \frac{\kappa_g\theta_g}{\omega_g^2}\left[2\ln\left(1 - \frac{h_g - \kappa_g^{\mathbb{N}}}{2h_g}\left(1 - e^{-h_g t}\right)\right) + (h_g - \kappa_g^{\mathbb{N}})t\right], \\
b_g(t) &= \frac{2\psi_g\left(1 - e^{-h_g t}\right)}{2h_g - \left(h_g - \kappa_g^{\mathbb{N}}\right)\left(1 - e^{-h_g t}\right)}, \\
a_c(t) &= \frac{\kappa_c\theta_c}{\omega_c^2}\left[2\ln\left(1 - \frac{h_c - \kappa_c^{\mathbb{N}}}{2h_c}\left(1 - e^{-h_c t}\right)\right) + (h_c - \kappa_c^{\mathbb{N}})t\right], \\
b_c(t) &= \frac{2\psi_c\left(1 - e^{-h_c t}\right)}{2h_c - \left(h_c - \kappa_c^{\mathbb{N}}\right)\left(1 - e^{-h_c t}\right)},
\end{aligned} \tag{1.24}$$

where

$$\begin{aligned}
h_g &= \sqrt{\left(\kappa_g^{\mathbf{N}}\right)^2 + 2\omega_g^2\psi_g}, \\
\kappa_g^{\mathbb{N}} &= \kappa_g + \gamma_{vg}^i\omega_g + \gamma_g^i\omega_g\rho_g - iu\beta^i\omega_g\rho_g, \\
\psi_g &= \frac{1}{2}(\beta^i)^2\left(u^2 + iu\right) - k_{Jg}^{\mathbb{Q}}\left[iu\beta^i\right] + iuk_{Jg}^{\mathbb{Q}}\left[\beta^i\right], \\
h_c &= \sqrt{(\kappa_c^{\mathbf{N}})^2 + 2\omega_c^2\psi_c}, \\
\kappa_c^{\mathbb{N}} &= \kappa_c + \gamma_{vc}^i\omega_c + \gamma_c^i\omega_c\rho_g - iu\xi^i\omega_c\rho_c, \\
\psi_c &= \frac{1}{2}(\xi^i)^2\left(u^2 + iu\right) - k_{Jc}^{\mathbb{Q}}\left[iu\xi^i\right] + iuk_{Jc}^{\mathbb{Q}}\left[\xi^i\right],
\end{aligned}$$

and the risk-neutral cumulant exponents of the jump components $k_J^{\mathbb{Q}}[\cdot]$ are given by equation (1.13). Appendix A provides the derivation for the transform.

1.3.5. Characteristic functions and the likelihood of index returns

We estimate our model using a maximum likelihood method. To construct the likelihood of the index returns, we first derive the characteristic function of the index return under the statistical measure $\mathbb{P}$ and apply fast Fourier inversions to obtain the conditional density of the index return.

Plugging the expected excess return in (1.16) into (1.1), we can rewrite the return process under $\mathbb{P}$ as,

$$\begin{aligned} s_t^i &= \left(r^i - q^i\right) t + \beta^i \left(W^g + J^g\right)_{\mathcal{T}_t^g} - \left(\frac{1}{2}(\beta^i)^2 + k_{J^g}^{\mathbb{Q}}\left[\beta^i\right] - \left(\gamma_g^i + \gamma_{v^g}^i \rho_g\right) \beta^i\right) \mathcal{T}_t^g \\ &\quad + \xi^i \left(W^c + J^c\right)_{\mathcal{T}_t^c} - \left(\frac{1}{2}(\xi^i)^2 + k_{J^c}^{\mathbb{Q}}[\xi^i] - \left(\gamma_c^i + \gamma_{v^c}^i \rho_c\right) \xi^i\right) \mathcal{T}_t^c. \end{aligned} \tag{1.25}$$

The characteristic function of the index return is defined as,

$$\varphi_{s_t^i}(u) \equiv \mathbb{E}^{\mathbb{P}}\left[e^{ius_t^i}\right], \; u \in \mathbb{R}$$

which we can solve as exponential affine functions of the activity rates,

$$\varphi_{s_t^i}(u) = e^{iu\left(r^i - q^i\right)t - a_g(t) - b_g(t) v_0^g - a_c(t) - b_c(t) v_0^c}, \tag{1.26}$$

where the coefficients take the same form as in (1.24), with the following re-definitions

$$\begin{aligned} \kappa_g^{\mathbb{N}} &= \kappa_g - iu\beta^i \omega_g \rho_g, \quad \kappa_c^{\mathbb{N}} = \kappa_c - iu\xi^i \omega_c \rho_c, \\ \psi_g &= \tfrac{1}{2}(\beta^i)^2 u^2 - k_{J^g}\left[iu\beta^i\right] + iu\left(\tfrac{1}{2}(\beta^i)^2 + k_{J^g}^{\mathbb{Q}}[\beta^i] - \left(\gamma_g^i + \gamma_{v^g}^i \rho_g\right)\beta^i\right), \\ \psi_c &= \tfrac{1}{2}(\xi^i)^2 u^2 - k_{J^c}\left[iu\xi^i\right] + iu\left(\tfrac{1}{2}(\xi^i)^2 + k_{J^c}^{\mathbb{Q}}[\xi^i] - \left(\gamma_c^i + \gamma_{v^c}^i \rho_c\right)\xi^i\right). \end{aligned} \tag{1.27}$$

Given the characteristic function, we can use fast Fourier transform to obtain the density functions of the log index return as a function of the current activity rate levels.

1.4. Estimation strategy

We estimate the model using the time-series of both the index returns and option prices on the three indexes. Since the activity rates are not directly observable, we cast the model into a state-space form and infer the activity rates at each date using an efficient filtering technique. We estimate the model parameters by maximizing the sum of weekly likelihood values on options and index returns.

In the state-space form, we regard the global activity rate and the country-specific activity rates in the three economies as the unobservable states and use $v_t \equiv [v_t^g, v_t^{c,SPX}, v_t^{c,FTS}, v_t^{c,NKY}]^\top$ to denote the (4×1) state vector. The state propagation equation is specified as an Euler approximation of the activity rate dynamics:

$$v_t = A + \Phi v_{t-1} + \sqrt{\mathcal{G}_t}\,\varepsilon_t, \quad v_t \in \mathbb{R}^{4+} \tag{1.28}$$

where ε_t denotes an i.i.d. standard normal innovation vector and

$$\begin{aligned} \Phi &= \exp(-\kappa\Delta t), \quad \kappa = \operatorname{diag}[\kappa_g, \kappa_c \mathbf{e}_3], \\ A &= (I_4 - \Phi)\theta, \quad \theta = [\theta_g, \theta_c \mathbf{e}_3^\top]^\top, \\ \mathcal{G}_t &= \operatorname{diag}\left[\omega_g^2 v_{t-1}^g, \omega_c^2 v_{t-1}^{c,SPX}, \omega_c^2 v_{t-1}^{c,FTS}, \omega_c^2 v_{t-1}^{c,NKY}\right]\Delta t, \end{aligned} \tag{1.29}$$

where $\Delta t = 7/365$ corresponds to the weekly frequency of the data, diag[] denotes a diagonal matrix with diagonal elements given in the bracket, I_d denotes an identity matrix of dimension d, and $\mathbf{e}_d$ denotes a vector of ones of dimension d.

Measurement equations are based on the observed option prices, assuming additive, normally-distributed measurement errors:

$$y_t = O[v_t;\Theta] + e_t, \quad E(e_t e_t^\top) = \mathcal{I}, \tag{1.30}$$

where y_t denotes the out-of-the-money option prices computed from the implied volatility quotes, scaled by Black-Scholes vega of the option at time t, $O[v_t;\Theta]$ denotes the corresponding model-implied values as a function of the parameter set Θ and the state vector v_t. We assume that the scaled pricing errors are i.i.d. normal with zero mean and constant variance. Hence, we can write the covariance matrix as, $\mathcal{I} = \sigma_e I_{60}$, with σ_e being a scalar. The dimension of the measurement equations correspond to the 60 implied volatility series across the three economies.

The objective function (1.30) deserves some explanation. One may choose to define the pricing error as the difference between the Black-Scholes implied volatility quote and its model-implied fair value. However, since our algorithm generates option prices from the return Fourier transform, converting the option prices into Black-Scholes implied volatility involves an additional minimization routine that can be inefficient when embedded in the global optimization procedure. By dividing the out-of-the-money option prices by its Black-Scholes vega, we are essentially converting the option price into the implied volatility space via a linear approximation.

When the state propagation and measurement equations are Gaussian linear, the Kalman filter provides efficient forecasts and updates on the mean and covariance of the state vector and observations. In our application, the state propagation equation is Gaussian linear, but the measurement equations are nonlinear functions of the state vector. We use an extended version of the Kalman filter, the unscented Kalman filter, to handle the nonlinearity of the measurement equation. Specifically, let $\overline{v}_t, \overline{P}_t, \overline{y}_t, \overline{V}_t$ denote the time-$(t-1)$ ex ante forecasts of time-t values of the state vector, the covariance of the state vector, the measurement series, and the covariance of the measurement series, respectively. Let $\widehat{v_t}$ and $\widehat{P_t}$ denote the ex post update, or filtering, on the state vector and its covariance at the time t based on observations (y_t) at time t. The unscented Kalman filter uses a set of deterministically chosen (sigma) points to approximate the state distribution. If we let d denote the number of states (four in our model) and let $\delta > 0$ denote a control parameter, we generate a set of $2d+1$ sigma vectors χ_i according to the following equations,

$$\begin{aligned} \chi_{t,0} &= \widehat{v}_t, \\ \chi_{t,i} &= \widehat{v_t} \pm \sqrt{(d+\delta)(\widehat{P_t} + \mathcal{G}_t)_j}, \quad j = 1, \ldots, d; \quad i = 1, \ldots, 2d, \end{aligned}$$

with the corresponding weights w_i given by,

$$w_0 = \delta/(d+\delta), \quad w_i = 1/[2(d+\delta)], \quad i = 1, \ldots, 2d.$$

These sigma vectors form a discrete distribution with w_i being the corresponding probabilities. We can verify that the mean, covariance, skewness, and kurtosis of this distribution are $\widehat{v}_t$, $\widehat{P}_t + \mathcal{G}_t$, 0, and $d+\delta$, respectively. Given the sigma points, the prediction steps are given by:

$$\begin{aligned}
\overline{\chi}_{t,i} &= A + \Phi \chi_{t,i}; \\
\overline{v}_{t+1} &= \sum_{i=0}^{2d} w_i \overline{\chi}_{t,i}; \\
\overline{P}_{t+1} &= \sum_{i=0}^{2d} w_i (\overline{\chi}_{t,i} - \overline{v}_{t+1})(\overline{\chi}_{t,i} - \overline{v}_{t+1})^\top; \\
\overline{y}_{t+1} &= \sum_{i=0}^{2d} w_i O\left[\overline{\chi}_{t,i};\Theta\right]; \\
\overline{V}_{t+1} &= \sum_{i=0}^{2d} w_i \left(O\left[\overline{\chi}_{t,i};\Theta\right] - \overline{y}_{t+1}\right)\left(O\left[\overline{\chi}_{t,i};\Theta\right] - \overline{y}_{t+1}\right)^\top + \mathcal{I},
\end{aligned} \tag{1.31}$$

and the filtering updates are given by

$$\begin{aligned}
\widehat{v}_{t+1} &= \overline{v}_{t+1} + \mathcal{K}_{t+1}(y_{t+1} - \overline{y}_{t+1}); \\
\widehat{P}_{t+1} &= \overline{P}_{t+1} - \mathcal{K}_{t+1}\overline{V}_{t+1}\mathcal{K}_{t+1}^\top,
\end{aligned} \tag{1.32}$$

with

$$\mathcal{K}_{t+1} = \left[\sum_{i=0}^{2d} w_i \left(\overline{\chi}_{t,i} - \overline{v}_{t+1}\right)\left(O\left[\overline{\chi}_{t,i};\Theta\right] - \overline{y}_{t+1}\right)^\top\right]\left(\overline{V}_{t+1}\right)^{-1}. \tag{1.33}$$

We refer the reader to Wan and van der Merwe (2001) for general treatments of the unscented Kalman filter.

Given the forecasted option prices $\overline{y}$ and their conditional covariance matrix $\overline{V}$ obtained from the filtering technique, we compute the log likelihood value for each week's observations on the option prices assuming normally distributed forecasting errors,

$$l_{t+1}[\Theta]^O = -\frac{1}{2}\log\left|\overline{V}_{t+1}\right| - \frac{1}{2}\left((y_{t+1} - \overline{y}_{t+1})^\top \left(\overline{V}_{t+1}\right)^{-1}(y_{t+1} - \overline{y}_{t+1})\right). \tag{1.34}$$

Furthermore, given the extracted activity rates from the options data, we compute the statistical return density for each equity index by applying fast Fourier inversion to the characteristic function in (1.26). Let $l_{t+1}[\Theta]^s$ denote the sum of the weekly log likelihood on the three equity indexes computed from this fast Fourier inversion. We choose model parameters to maximize the sum of the weekly log likelihood values on both options and time-series returns,

$$\Theta \equiv \arg\max_{\Theta} \mathcal{L}[\Theta, \{y_t\}_{t=1}^T], \quad \text{with} \quad \mathcal{L}[\Theta, \{y_t\}_{t=1}^T] = \sum_{t=0}^{T-1} \left(l_{t+1}[\Theta]^O + l_{t+1}[\Theta]^s \right), \quad (1.35)$$

where $T = 346$ denotes the number of weeks in our sample.

The pricing errors on options represent option variations that are not captured by the international capital asset pricing model. For model estimation, we make the additional assumption on the distribution of the forecasting and pricing errors as being normal and independent of the return dynamics. Thus, we can write the joint log likelihood of options and returns as the sum of their marginal log likelihood values. In constructing the log likelihood of the three return series, we replace the joint log likelihood of the three series with the sum of the three marginal log likelihood of each series. This practice is purely for computational feasibility because deriving the joint density of the three return series would involve multi-dimensional integrations that are computationally intensive for our model estimation. Using the product of marginal densities incurs some theoretical information loss, but provides significant gains in numerical stability and feasibility. Hence, we can regard our estimation method as a limited information maximum likelihood method (Greene (1999)).

1.5. Results and discussion

Based on the estimates of the structural parameters of the international capital asset pricing model, we first gauge the model performance and then analyze the risk and pricing characteristics in the three economies.

1.5.1. The model captures the overall index options behavior well

Our model simultaneously and consistently prices index options across the three economies. To gauge the model's performance in option pricing, we first compute the model's pricing errors, defined as the difference in Black-Scholes implied volatility between the market quotes and the model-generated values. Then, we compute the explained variation on each implied volatility series, defined as one minus the ratio of the pricing error variance to the variance of the original implied volatility series. Table 1.4 reports the model's explained variation for each series, with the last row reporting the aggregate explained variation for all options underlying each equity index.

The model generates satisfactory overall performance on all three equity indexes. The aggregate explained variation is over 90% for all three indexes. Comparing the performance across different series, we find that the model performs better on long-term options than on short-term options, and better on at-the-money options than on out-of-the-money options. For out-of-the-money options, the model performs better at low strikes than at high strikes.

The model's largest deviations from the market are at 120% strikes and one- and three-month maturities for SPX, where the explained variations are as low as 30% and 25.1%, respectively. When we inspect the data in more detail, we find that the quotes on these two series are not updated as frequently as data on the other series, especially during the first three years of our sample. We observe prolonged periods when the quotes on these two series stay flat while quotes on the other series move significantly over time. When we compute the model performance excluding the first three years of data, the explained variations on the two series increase to 65% and 75%, respectively. Thus, we regard the poor model performance at high strikes and short maturities as partly driven by data issues. Our experience with both over-the-counter and exchanged-listed index options data also tells us that out-of-the-money index puts (at lower strikes) are more actively traded than out-of-the-money calls (at higher strikes). Accordingly, quotes on out-of-money calls are often more reliable and updated more frequently than quotes on out-of-the-money calls.

1.5.2. The Japanese market contains the largest portion of country-specific movements

Table 1.5 reports the estimates and the absolute magnitudes of the t-statistics (in parentheses) for the structural parameters of the model. The first panel reports the estimates on the loading coefficients, which capture the average difference in the risk levels across the three economies. For identification reasons, we normalize the loading to the US economy (SPX) to one. Thus, deviations from one on the coefficient estimates of other economies capture their relative difference in risk levels from the US economy. The t-statistics reported in the first panel are calculated against the null hypothesis of one.

The β coefficients capture the loading of the global return risk factor on each index. The estimate on FTS is greater than one, but the estimate on NKY is significantly smaller than one. On the other hand, the ξ estimates capture the loading of the country-specific return risk on each index. The estimate on FTS is smaller than one, but the estimate on NKY is greater than one. Therefore, compared to the US economy, the UK economy contains a larger proportion of global movements and a smaller proportion of country-specific movements. In contrast, the Japanese economy contains the smallest portion of the global movements but the largest portion of the country-specific movements. These results are consistent with the sample correlation statistics on the index returns and implied volatilities. Both index returns and option implied volatilities have stronger cross-correlations between SPX and FTS than between NKY and the other two indexes, indicating that NKY has a larger proportion of country-specific movements.

1.5.3. Large downside index movements are more likely to be global

The second panel of Table 1.5 reports the estimates that control the jump structures in the global and the country-specific return components. For each component, the proportional coefficient λ controls the overall mean arrival rate of jumps. The larger λ is, the more frequent jumps of either direction arrive. The dampening coefficients ζ_+ and ζ_- control the arrival rate of large positive and negative jumps, respectively. The larger the coefficient, the less likely to observe a large jump arrival. The difference between ζ_+ and ζ_- generates asymmetry in the return innovation distribution.

The parameter estimates show that for both jump components, the dampening coefficient estimates on the positive jumps, β_+, are larger than the corresponding dampening coefficient estimates on the negative jumps, β_-. Hence, for both return components, down jumps are more likely to arrive than up jumps. The distributions for the two return components are both negatively skewed.

Comparing the jump structures in the two return components, we find that the dampening coefficients on both negative and positive jumps are larger for the country-specific component than for the global component, suggesting that global movements are more likely to contain large jumps. Furthermore, the proportional coefficient λ is also larger for the global component than for the country-specific component, indicating that the global component contains more discontinuous movements overall. These results suggest that large discontinuous movements in the equity indexes, especially large downside movements, are more likely to exhort a global impact whereas small movements are more likely to be local noises.

1.5.4. Global stochastic volatility is more persistent than local stochastic volatility

The third panel of Table 1.5 reports the parameter estimates that control the activity rates underlying the global and country-specific return risk components, respectively. These stochastic activity rates are responsible for generating stochastic volatility and stochastic higher moments. The parameter κ measures the mean-reversion speed of the underlying activity rate. Its estimate for the country-specific activity rate κ_c is at 3.0393, which corresponds to a weekly autocorrelation of 0.9432, and a relatively short half life of about 13 weeks.[5] In contrast, the mean-reversion speed estimate for the global activity rate κ_g is not significantly different from zero, implying near non-stationary behaviors. These estimates suggest that the global activity rate is much more persistent than the country-specific activity rates. The difference in persistence implies that it is more difficult to predict global risk variations than to predict country-specific risk variations.

The parameter ρ measures the instantaneous correlation between the return diffusion component and the corresponding activity rate innovation. The correlation estimates for both return components ρ_g and ρ_c are strongly negative. Thus, a fall in equity index prices is usually associated with an increase in volatility, consistent with the well-known leverage effect (Black (1976)). Comparing ρ_g with ρ_c, we find that the negative correlation is stronger between the global return component and its activity rate than between the country-specific return component and its activity rate. Therefore, a global fall in the equity index is more likely to be associated with a hike in the global volatility than a local fall in the index is to the local volatility.

The negative correlation also generates negative skewness in the return distribution over longer horizons. The different estimates for the correlations and the jump structures between the global and local return components suggest that the global return component is more negatively skewed than the country-specific return component at both short and long horizons. A more negatively skewed global return component can be caused by contagion (Claessens and Forbes (2002)). The volatility of the global factor increases more following bad news regarding the world market. Holding the volatility of the country-specific return component the same, the increased volatility of the global factor also generates increased

[5] The half life is defined as the number of weeks (n) that it takes for the autocorrelation to decay by half of its first-order level $\varphi_1 = 0.9432$. It is solved by $n = \ln(\varphi_1/2)/\ln(\varphi_1)$.

correlation in the international stock market. These results are consistent with the findings of Longin and Solnik (2001), Ang and Bekaert (2002), and Das and Uppal (2004), who document higher correlations in bear markets.

1.5.5. Investors in the US and UK markets price return risks more heavily than do investors in the Japanese market

The fourth panel in Table 1.5 reports the parameter estimates that define the market prices of global and country-specific return risks. Most empirical studies in the international asset pricing literature find that the global risk factor is positively priced, but the results on the pricing of country-specific risk factors are mixed. Our estimation results generate similar qualitative conclusions. All three economies charge a positive market price for the global return risk, but their pricing on the country-specific risk show different signs.

The positive market price of the global risk indicates that investors are willing to pay a positive premium to hedge against global downturns in the market. Statistically, the positive market price increases the dampening on up jumps and reduces the dampening on down jumps under the risk-neutral measure $\mathbb{Q}$. As a result, the distribution of the global return component becomes more negatively skewed under the risk-neutral measure. As we have discussed in the theory section, the negative statistical skewness and positive market price of return risk also makes the risk-neutral return variance higher than its statistical counterpart.

Of the three economies, the UK stock market not only has the largest portion of global return movements, but it also charges the highest price for the global return risk. On the other hand, the Japanese stock market contains the least proportion of global movements. Investors in the Japanese stock market also charge the lowest price for the global return risk. The behavior of the US market is in the middle, but closer to the UK than to Japan. These different market prices on the global return risk across different economies lead to different levels of skewness in the risk-neutral distribution.

Investors in the US and UK market also charge large and positive prices for their country-specific return risks. In contrast, the market price estimates in Japan are small and even negative on the country-specific return risk component. The positive market prices in US and UK make the risk-neutral distribution on SPX and FTS even more negatively skewed. The slightly negative estimate on Japanese country-specific return risk reduces the negative skewness of the risk-neutral return distribution.

These differences in the market price estimates are consistent with the differences in the slopes of the average implied volatility smirk (Fig. 1.1). The smaller market price for global return risk and the negative market price for country-specific risk in Japan contribute to the flatter implied volatility smirk on NKY. The average differences in risk compositions across the three economies also lead to different levels of skewness in the risk-neutral distribution. The global loading coefficient estimate on NKY is the smallest and the country-specific loading estimate on NKY is the largest among the three indexes. Since the global return component is more negatively skewed than the country-specific return component, the smaller loading on the global return risk factor and the larger loading on the country-specific return risk factor in Japan also contribute to the flatter implied volatility smirk on NKY.

Fig. 1.4 plots the time series of the three indexes. Both SPX and FTS experienced dramatic gains during the first half of our sample. The momentum was broken only after the burst of the internet bubble in 2000. In contrast, the NKY index has been declining over the whole sample period, with only a temporary rebound in 1999 that was quickly reversed. Indeed, the NKY level at the start of our sample is merely half of its peak at 38,915 on December 29, 1989. The index level went down by another half by the end of our sample. The different time-series pattern further illustrates the unique feature of the Japanese stock market and explains the estimated dominance of country-specific movements. The flatter implied volatility smirk on NKY suggests that investors are willing to pay relatively higher price for out-of-the-money call options on NKY than those on SPX or FTS. With the prolonged recession, investors in the Japanese market expect a recovery in the future.

This expectation makes them willing to take on the stock market and bear the return risks without charging much of a risk premium.

1.5.6. Market prices of volatility risks are negative, more so for investors in the Japanese market

Using mainly regression analysis on stock portfolio returns, the traditional literature on international asset pricing focuses mostly on the pricing of return risks. Using options, which are natural instruments for volatility risk trading, we can learn more about the volatility risk dynamics and the market pricing of volatility risks. The last panel of Table 1.5 reports the estimates on the market prices of the global and country-specific activity rate risks, which are the drivers for stochastic volatility and stochastic higher moments. The estimates suggest that investors in the three markets treat the activity rate risks similarly. All the estimates are significantly negative, for both the global and country-specific risks. The negative market price for the activity rate risk suggests that investors are willing to pay a positive premium to hedge against future volatility increases.

Bondarenko (2004), Carr and Wu (2004b), and Carr and Wu (2006) have found that variance swap rates, which can be regarded as the risk-neutral expected value of the realized variance, are higher than the realized variance on average. They attribute the difference to negative variance risk premia. Our evidence and estimation results confirm their findings in an international context. Furthermore, we decompose the negative variance risk premia into four sources: (1) global return jump risk and positive market price of global return risk, (2) global stochastic volatility and negative market price of global volatility risk, (3) country-specific return jump risk and positive market price of country-specific return risk, and (4) country-specific stochastic volatility and negative market price of country-specific volatility risk. Combining a negatively skewed return jump innovation with positive market price of risk makes the return innovation variance higher under the risk-neutral measure than its statistical counterpart, as shown in equation (1.17). The negative market prices on the activity rates also make the risk-neutral mean of the activity rates higher than their statistical mean values.

The negative market price on the activity rate risk and positive market price on the return risk dictate that the return volatility is more persistent under the risk-neutral measure than under the statistical measure. While the statistical persistence controls the time-series behavior of return volatility, the risk-neutral persistence controls the cross-sectional behavior of the option implied volatility. Hence, the persistence difference under the two measures imply that although we observe strong mean reversion on return volatility or option implied volatility, a shock on the instantaneous activity rate has a long-lasting effect on options of long maturities.

Across the three economies, we find that investors in the Japanese market charge the most negative price for the volatility risk whereas investors in the US market charge the least negative price, whether global or country-specific. These findings are intriguing and new to the literature. Intuitively, these results suggest that compared to investors in the US and the UK markets, investors in the Japanese market are more concerned about future increase in volatility than about future market crashes. As a result, investors in Japan price more heavily on the volatility risk than the return risk.

To gain economic intuition and make more sense out of the market price estimates, we use equations (1.14)-(1.16) to compute the annualized return risk premiums induced by each risk component. Stochastic activity rates induce stochastic return risk premiums as shown in equation (1.16). We compute the average risk premiums evaluated at the long-run mean of the activity rates. Table 1.6 reports the estimates. For global risks, the risk premiums induced by the diffusion return risk are fairly low and less than 1% for all three economies. The return jump risk induces a return risk premium of 0.92% for SPX, 1.65% for FTS, and merely 0.01% for NKY. In both cases, the close to zero market price of return risk on NKY leads to close to zero risk premiums. In contrast, the global activity rate risk induces a more significant return risk premium for all three economies through its correlation with the return diffusion risk. The estimates are 2.46% for SPX, 3.31% for FTS, and 3.19% for NKY. Although the market price of global activity rate risk is high for NKY, the index's lower loading on the global factor lowers the induced risk premium. Overall, the three sources of global risks induce an average risk premium of 4.07% for SPX, 5.91% for FTS, and 3.21% for NKY.

For the country-specific risks, the return diffusion risk induces a return premium of 3.75% for SPX, and 3.13% for FTS. However, the slightly negative market price on NKY implies a negative but close to zero risk premium. Furthermore, since the country-specific jumps have more significant dampening on the tails and hence are not as significant as the global jumps, they also induce smaller return risk premiums, which are less than 1% for all three indexes. Nevertheless, the country-specific activity rate induces large risk premiums on all three indexes, ranging from 4.41% to 6.44%.

1.6. Specification analysis

We have designed and estimated an international capital asset pricing model that is flexible enough to allow different market pricing from different economies and to deliver overall satisfactory pricing performance. In the meantime, we strive to maintain parsimony in the specification to enhance the model identification and stability. For example, by assuming identical but independent country-specific jump structures and activity rate dynamics after allowing for an average scaling difference, we are able to reduce the number of model parameters significantly. In this section, we perform specification analysis to gauge the validity and robustness of our assumptions.

1.6.1. Do investors in different economies price the risks differently?

In specifying the pricing kernels for each economy, we observe that even with growing evidence of global financial market integration, the international goods market remains segmented, and deviations from purchasing power parity remain persistent, with convergence to the parity condition slow. We further observe that in nominal terms, the literature has repeated documented the gross violation of the uncovered interest rate parity. In accordance with these observations, we allow each economy to have different market pricing on the global and country-specific risks.

The parameter estimates in Table 1.5 are largely consistent with our conjecture that the market prices vary across economies. For example, we find that the market prices of return risks in Japan are much lower than the corresponding market prices in the other two economies. When we construct a Wald test on the null hypothesis that the market prices are identical across the three economies, the hypothesis is rejected beyond any reasonable confidence level.

To further test the hypothesis, we estimate a constrained version of our model by setting $\gamma_g^{SPX} = \gamma_g^{FTS} = \gamma_g^{NKY}$, $\gamma_c^{SPX} = \gamma_c^{FTS} = \gamma_c^{NKY}$, $\gamma_{vg}^{SPX} = \gamma_{vg}^{FTS} = \gamma_{vg}^{NKY}$, and $\gamma_{vc}^{SPX} = \gamma_{vc}^{FTS} = \gamma_{vc}^{NKY}$. With these constraints, the model has eight fewer parameters. Based on the maximized likelihoods on the two models, we construct a likelihood ratio statistic between the two models $\mathcal{X} = 2(\mathcal{L}_{unconstraint} - \mathcal{L}_{constrained})$. Our benchmark model has a maximized likelihood of 27224. The constrained model generates a maximized likelihood of 26209, leading to an estimate of 2030 for the likelihood ratio statistic. The statistic has a chi-square distribution with eight degrees of freedom. Our estimate leads to a p-value that is virtually zero. The test strongly rejects the null hypothesis that the market prices are identical across all three economies.

1.6.2. Do different economies have different jump structures and activity rate dynamics?

For parsimony, we apply an average scale (ξ^i) on the country-specific risks and then assume that the scaled country-specific risks have identical but independent jump structures and activity rate dynamics across the three economies. To gauge how this parsimony assumption costs us in performance, we also estimate a more general version of the model, under which we allow the jump structure and activity rate dynamics for each economy to be different. Specifically, we allow $(\lambda^c, \zeta_+^c, \zeta_-^c, \kappa_c, \omega_c, \rho_c)$ to be different for the three economies. Nevertheless, we need to keep θ_c to be the same for model identification if we keep the different scaling coefficients ξ^i for the three economies. This extended model has 12 more model parameters than the benchmark model.

This more flexible specification generates a maximized likelihood value of 27502. The likelihood ratio statistic against the benchmark model is computed at 558, which has a chi-square distribution with 12 degrees of freedom. The chi-square test rejects the benchmark model for this more flexible specification, indicating that the parsimony assumption does induce a cost in performance.

Table 1.7 reports the explained variation on the option quotes from this more flexible specification. The explained variation does not improve much on options at 120% strike and one- and three-month maturities, partly confirming our conjecture that the bad performance at these two series is mainly driven by data issues. Nevertheless, this more flexible model does improve moderately the explained variation on some other series.

Table 1.8 reports the parameter estimates on this more flexible model. The qualitative conclusions from these estimates are largely consistent with those from our benchmark model in Table 1.5. When allowed to have different jump structures and activity rate dynamics, we find that the jump arrival rate λ is the smallest for NKY. The estimate for the correlation ρ_c between the return and activity rate is also the least negative for NKY. Both components allow the NKY return distribution to have a lower skewness than that for SPX and FTS, consistent with the data observation.

1.7. Conclusion

In this paper, we exploit the rich information in option prices underlying three major equity indexes and study the risk dynamics and market pricing in the three underlying economies within a dynamically consistent international capital asset pricing model framework. We first develop an international capital asset pricing model, under which the return on each equity index is decomposed into two orthogonal jump-diffusion components: a global component and a country-specific component. Separate stochastic time changes are further applied to both components so that stochastic variance and higher moments can come from both global and country-specific risks. For each economy, we assign separate market prices for the two return risk components and the two volatility risk components. Under this specification, we obtain analytically tractable solutions to the option prices on the three indexes and likelihood functions for the index returns. Through a joint estimation on the index returns and option prices on the three economies, we investigate how investors respond to different sources of risks in international economies.

Our estimation reveals several interesting features about the risk and pricing in the three economies. First, we find that global and country-specific return and volatility risks show different dynamics. Global return movements contain a larger proportion of discontinuous movements, especially large downside movements. Global return volatility is more persistent than the country-specific return volatility. Second, we find that investors charge positive prices for global return risk and negative prices for both global and country-specific volatility risks, showing that investors are willing to pay positive premiums to hedge against downside return movements and upside volatility movements. Third, the three economies contain different risk profiles and also price risks differently. Japan contains the largest idiosyncratic risk component and smallest global risk component. Investors in the Japanese market price more heavily against future volatility increases than against future market downfalls.

Appendix A. Generalized Fourier transforms

The index return dynamics in equation (1.10) under the home currency risk-neutral measure $\mathbb{Q}^i$ can be regarded as an affine combination of time-changed Lévy processes. To derive its generalized Fourier transform of the index return, we apply the theorem in Carr and Wu (2004a) and convert the Fourier transform of each time-changed Lévy component into the Laplace transform of the underlying stochastic time change,

$$\begin{aligned}\phi_{s_t^i}(u) &\equiv \mathbb{E}^{\mathbb{Q}^i}\left(e^{ius_t^i}\right),\ u \in \mathcal{D} \in \mathbb{C} \\ &= e^{iu\left(r^i-q^i\right)t}\mathbb{E}^{\mathbb{N}^g}\left[e^{-\psi_g \mathcal{T}_t^g}\right]\mathbb{E}^{\mathbb{N}^c}\left[e^{-\psi_c \mathcal{T}_t^c}\right],\end{aligned}$$

where ψ_g and ψ_c denote the characteristic exponents of the concavity-adjusted global and country-specific return risk components, respectively,

$$\begin{aligned}\psi_g &= \tfrac{1}{2}(\beta^i)^2\left(u^2+iu\right) - k_{J^g}^{\mathbb{Q}}\left[iu\beta^i\right] + iuk_{J^g}^{\mathbb{Q}}\left[\beta^i\right], \\ \psi_c &= \tfrac{1}{2}(\xi^i)^2\left(u^2+iu\right) - k_{J^c}^{\mathbb{Q}}\left[iu\xi^i\right] + iuk_{J^c}^{\mathbb{Q}}\left[\xi^i\right],\end{aligned} \tag{A1}$$

and $\mathbb{N}^g$ and $\mathbb{N}^c$ denote two new measures defined by the following complex-valued exponential martingales:

$$\left.\frac{d\mathbb{N}^g}{d\mathbb{Q}}\right|_t = \exp\left(iu\beta^i\left(W^g+J^g\right)_{\mathcal{T}_t^g} + \left(\frac{1}{2}u^2(\beta^i)^2 - k_{J^g}^{\mathbb{Q}}\left[iu\beta^i\right]\right)\mathcal{T}_t^g\right), \tag{A2}$$

$$\left.\frac{d\mathbb{N}^c}{d\mathbb{Q}}\right|_t = \exp\left(iu\xi^i\left(W^c+J^c\right)_{\mathcal{T}_t^c} + \left(\frac{1}{2}u^2(\xi^i)^2 - k_{J^c}^{\mathbb{Q}}\left[iu\xi^i\right]\right)\mathcal{T}_t^c\right). \tag{A3}$$

By Girsanov's theorem, the $\mathbb{N}^g$-dynamics of v_t^g and the $\mathbb{N}^c$-dynamics of v_t^c become,

$$dv_t^g = \left(\kappa_g\theta_g - \kappa_g^{\mathbb{N}}v_t^g\right)dt + \omega_g\sqrt{v_t^g}dZ_t^g, \quad dv_t^c = \left(\kappa_c\theta_c - \kappa_c^{\mathbb{N}}v_t^c\right)dt + \omega_c\sqrt{v_t^c}dZ_t^c, \tag{A4}$$

where $\kappa_g^{\mathbb{N}} = \kappa_g + \gamma_{v^g}^i\omega_g + \gamma_g^i\omega_g\rho_g - iu\beta^i\omega_g\rho_g$ and $\kappa_c^{\mathbb{N}} = \kappa_c + \gamma_{v^c}^i\omega_c + \gamma_c^i\omega_c\rho_c - iu\xi^i\omega_c\rho_c$. Since the activity rate dynamics remain affine under the new measures, we follow Carr and Wu (2004a) and solve the Laplace transforms in exponential affine forms as given in (1.23).

Appendix B. Characteristic functions

The index return dynamics under the statistical measure $\mathbb{P}$ is given in (1.25), which can again be regarded as an affine combination of time-changed Lévy processes. We treat the characteristic function as a special case of the generalized Fourier transform, in which the characteristic coefficient is defined on the real line instead of the complex plane. Then, we again apply the theorem in Carr and Wu (2004a) and convert the characteristic function of each time-changed Lévy component into the Laplace transform of the underlying stochastic time change,

$$\begin{aligned} \varphi_{s_t^i}(u) &\equiv \mathbb{E}^{\mathbb{P}}\left(e^{ius_t^i}\right),\ u\in\mathbb{R} \\ &= e^{iu\left(r^i-q^i\right)t}\mathbb{E}^{\mathbb{N}^g}\left[e^{-\psi_g\mathcal{T}_t^g}\right]\mathbb{E}^{\mathbb{N}^c}\left[e^{-\psi_c\mathcal{T}_t^c}\right], \end{aligned}$$

where ψ_g and ψ_c denote the characteristic exponents of the drift-adjusted global and country-specific return risk components, respectively,

$$\begin{aligned} \psi_g &= \tfrac{1}{2}(\beta^i)^2u^2 - k_{J^g}\left[iu\beta^i\right] + iu\left(\tfrac{1}{2}(\beta^i)^2 + k_{J^g}^{\mathbb{Q}}[\beta^i] - (\gamma_g^i + \gamma_{v^g}^i\rho_g)\beta^i\right), \\ \psi_c &= \tfrac{1}{2}(\xi^i)^2u^2 - k_{J^c}\left[iu\xi^i\right] + iu\left(\tfrac{1}{2}(\xi^i)^2 + k_{J^c}^{\mathbb{Q}}[\xi^i] - (\gamma_c^i + \gamma_{v^c}^i\rho_c)\xi^i\right), \end{aligned} \quad \text{(B5)}$$

and $\mathbb{N}^g$ and $\mathbb{N}^c$ denote two new measures defined by the following exponential martingales:

$$\left.\frac{d\mathbb{N}^g}{d\mathbb{P}}\right|_t = \exp\left(iu\beta^i\left(W^g+J^g\right)_{\mathcal{T}_t^g} + \left(\frac{1}{2}(\beta^i)^2u^2 - k_{J^g}\left[iu\beta^i\right]\right)\mathcal{T}_t^g\right), \quad \text{(B6)}$$

$$\left.\frac{d\mathbb{N}^c}{d\mathbb{P}}\right|_t = \exp\left(iu\xi^i\left(W^c+J^c\right)_{\mathcal{T}_t^c} + \left(\frac{1}{2}(\xi^i)^2u^2 - k_{J^c}\left[iu\xi^i\right]\right)\mathcal{T}_t^c\right). \quad \text{(B7)}$$

By Girsanov's theorem, the $\mathbb{N}^g$-dynamics of v_t^g and the $\mathbb{N}^c$-dynamics of v_t^c become,

$$dv_t^g = \left(\kappa_g\theta_g - \kappa_g^{\mathbb{N}}v_t^g\right)dt + \omega_g\sqrt{v_t^g}dZ_t^g, \quad dv_t^c = \left(\kappa_c\theta_c - \kappa_c^{\mathbb{N}}v_t^c\right)dt + \omega_c\sqrt{v_t^c}dZ_t^c, \quad \text{(B8)}$$

with $\kappa_g^{\mathbb{N}} = \kappa_g - iu\beta^i\omega_g\rho_g$ and $\kappa_c^{\mathbb{N}} = \kappa_c - iu\xi^i\omega_c\rho_c$. Again, the affine structure of the activity rate dynamics guarantees that we have an exponential affine solution for the Laplace transform as in (1.26).

Appendix C. Risk premiums from the jump components

The return risk premiums from the diffusion components can be derived straightforwardly as the covariance between the log pricing kernel and the index return dynamics. The risk premiums from the jump components are given by the difference between the cumulant exponents under the two measures. We take the global jump risk component J^g as an example. Its contribution to the return risk premium per unit activity rate risk is given by,

$$\begin{aligned}
\eta^i_{J^g} &= k_{J^g}[\beta^i] - k^{\mathbb{Q}}_{J^g}[\beta^i] \\
&= \lambda^g\left(\frac{1}{\zeta^g_+-\beta^i} - \frac{1}{\zeta^g_+} + \frac{1}{\zeta^g_-+\beta^i} - \frac{1}{\zeta^g_-}\right) - \lambda^g\left(\frac{1}{\zeta^g_++\gamma^i_g-\beta^i} - \frac{1}{\zeta^g_++\gamma^i_g} + \frac{1}{\zeta^g_--\gamma^i_g+\beta^i} - \frac{1}{\zeta^g_--\gamma^i_g}\right) \\
&= \left(\frac{\lambda^g\gamma^i_g}{(\zeta^g_++\gamma^i_g-\beta^i)(\zeta^g_+-\beta^i)} - \frac{\lambda^g\gamma^i_g}{(\zeta^g_++\gamma^i_g)\zeta^g_+}\right) + \left(\frac{\lambda^g\gamma^i_g}{(\zeta^g_--\gamma^i_g)\zeta^g_-} - \frac{\lambda^g\gamma^i_g}{(\zeta^g_--\gamma^i_g+\beta^i)(\zeta^g_-+\beta^i)}\right) \\
&= \lambda^g\gamma^i_g\left[\frac{(\zeta^g_++\gamma^i_g)\zeta^g_+ - (\zeta^g_++\gamma^i_g-\beta^i)(\zeta^g_+-\beta^i)}{(\zeta^g_++\gamma^i_g-\beta^i)(\zeta^g_+-\beta^i)(\zeta^g_++\gamma^i_g)\zeta^g_+} + \frac{(\zeta^g_--\gamma^i_g+\beta^i)(\zeta^g_-+\beta^i) - (\zeta^g_--\gamma^i_g)\zeta^g_-}{(\zeta^g_--\gamma^i_g)\zeta^g_-(\zeta^g_--\gamma^i_g+\beta^i)(\zeta^g_-+\beta^i)}\right].
\end{aligned} \tag{C9}$$

We need $\zeta^g_+ > \max(0, \beta^i, -\gamma^i_g, \beta^i - \gamma^i_g)$ and $\zeta^g_- > \max(0, -\beta^i, \gamma^i_g, -\beta^i + \gamma^i_g)$ for the index return dynamics to be well-defined under both measure. Under these conditions, the terms in the denominators in the bracket are both positive. It is easy to verify that the numerators in the bracket are also positive. Therefore, given that $\lambda^g > 0$, the risk premium induced by the jump component is positive if and only if the market price γ^i_g is positive. An analogous argument applies to the country-specific jump component.

Table 1.1
Summary statistics of stock index returns

Statistics	SPX	FTS	NKY
Mean	0.0591	0.0174	-0.1090
Standard deviation	0.1819	0.1724	0.2208
Skewness	-0.2625	-0.0585	0.1335
	(1.98)	(0.44)	(1.01)
Excess kurtosis	0.8007	2.3575	1.0668
	(3.04)	(8.95)	(4.05)
Autocorrelation	-0.0849	-0.0995	-0.0286

Entries are the descriptive statistics of weekly log returns on the S&P 500 Index (SPX), the FTSE 100 Index (FTS), and the Nikkei-225 Stock Average (NKY). Absolute t-statistics of the skewness and excess kurtosis statistics are given in parentheses. The mean and standard deviation estimates are annualized. The sample starts on January 3, 1996 and ends on August 14, 2002, with 346 weekly observations for index.

Table 1.2
Summary statistics of the implied volatility quotes

Index	SPX				FTS				NKY			
Strike\Maturity	1m	3m	6m	12m	1m	3m	6m	12m	1m	3m	6m	12m
						Mean						
80	0.35	0.31	0.28	0.27	0.34	0.30	0.29	0.27	0.34	0.31	0.29	0.27
90	0.28	0.26	0.25	0.24	0.27	0.26	0.25	0.24	0.29	0.27	0.26	0.26
100	0.21	0.21	0.21	0.22	0.21	0.21	0.22	0.22	0.25	0.24	0.24	0.24
110	0.16	0.18	0.19	0.20	0.17	0.18	0.19	0.20	0.23	0.23	0.23	0.23
120	0.15	0.16	0.16	0.18	0.15	0.16	0.17	0.18	0.22	0.22	0.22	0.22
						Standard deviation						
80	0.06	0.05	0.05	0.05	0.10	0.08	0.07	0.07	0.07	0.06	0.06	0.05
90	0.05	0.05	0.04	0.04	0.08	0.07	0.07	0.06	0.07	0.06	0.05	0.05
100	0.05	0.04	0.04	0.04	0.07	0.06	0.06	0.06	0.06	0.05	0.05	0.04
110	0.04	0.04	0.04	0.04	0.06	0.06	0.05	0.05	0.06	0.05	0.04	0.04
120	0.04	0.03	0.04	0.03	0.05	0.05	0.05	0.05	0.05	0.04	0.04	0.04
						Skewness						
80	0.64	0.84	0.66	0.36	0.38	0.29	0.22	0.31	0.70	0.42	0.26	0.08
90	0.91	0.76	0.55	0.29	0.62	0.43	0.35	0.41	0.81	0.44	0.23	0.00
100	0.91	0.53	0.33	0.18	0.84	0.59	0.53	0.52	0.73	0.39	0.16	-0.07
110	0.64	0.35	0.07	0.09	1.02	0.74	0.73	0.73	0.76	0.37	0.12	-0.12
120	0.42	-0.11	-0.34	-0.11	1.17	0.82	0.84	0.93	0.68	0.29	0.02	-0.19
						Excess kurtosis						
80	0.62	1.18	0.90	0.15	-0.16	-0.52	-0.62	-0.33	0.20	-0.05	-0.17	-0.28
90	1.54	1.40	1.06	0.32	0.22	-0.20	-0.30	-0.07	0.54	0.11	-0.12	-0.32
100	1.82	1.39	1.15	0.49	0.92	0.22	0.13	0.30	0.65	0.14	-0.15	-0.40
110	1.71	1.44	1.25	0.71	1.83	0.68	0.61	0.84	0.87	0.05	-0.26	-0.50
120	1.43	0.92	0.81	0.74	2.86	1.10	1.04	1.51	0.83	-0.26	-0.51	-0.61
						Weekly autocorrelation						
80	0.91	0.94	0.96	0.97	0.94	0.96	0.97	0.98	0.92	0.95	0.97	0.98
90	0.88	0.93	0.96	0.97	0.93	0.95	0.96	0.97	0.91	0.95	0.96	0.97
100	0.86	0.92	0.95	0.97	0.91	0.94	0.96	0.97	0.91	0.94	0.96	0.97
110	0.88	0.91	0.94	0.96	0.90	0.94	0.95	0.97	0.90	0.93	0.95	0.97
120	0.92	0.93	0.94	0.95	0.90	0.93	0.95	0.96	0.87	0.92	0.95	0.97

The five panels report the mean, standard deviation, skewness, excess kurtosis, and weekly autocorrelation estimates on implied volatility series at different maturities and strike levels for the three stock indexes.

Table 1.3
Cross-index correlation between implied volatilities

Index	SPX-FTS				SPX-NKY				FTS-NKY			
Strike\Maturity	1m	3m	6m	12m	1m	3m	6m	12m	1m	3m	6m	12m
80	0.57	0.60	0.59	0.57	0.29	0.33	0.34	0.35	0.39	0.40	0.37	0.37
90	0.56	0.60	0.59	0.56	0.31	0.34	0.35	0.36	0.40	0.42	0.40	0.39
100	0.53	0.57	0.56	0.55	0.30	0.32	0.33	0.35	0.36	0.40	0.39	0.38
110	0.47	0.52	0.53	0.53	0.26	0.28	0.29	0.31	0.29	0.35	0.36	0.35
120	0.36	0.39	0.47	0.49	0.15	0.21	0.26	0.27	0.27	0.30	0.32	0.31

Entries report the cross-correlation estimates of the weekly changes in the implied volatilities between different pairs of the underlying indexes at each strike and maturity level.

Table 1.4
Model performance in pricing options

Index	SPX				FTS				NKY			
Strike\Maturity	1m	3m	6m	12m	1m	3m	6m	12m	1m	3m	6m	12m
80	0.774	0.866	0.930	0.933	0.761	0.946	0.970	0.959	0.761	0.850	0.893	0.898
90	0.827	0.927	0.970	0.952	0.923	0.981	0.989	0.973	0.858	0.942	0.965	0.942
100	0.874	0.967	0.984	0.953	0.966	0.995	0.994	0.978	0.946	0.987	0.988	0.941
110	0.740	0.963	0.977	0.950	0.908	0.989	0.988	0.980	0.908	0.946	0.958	0.931
120	0.300	0.251	0.944	0.927	0.649	0.753	0.976	0.972	0.828	0.885	0.894	0.894
Aggregate	0.914				0.938				0.911			

Entries report the model's explained variation on each implied volatility series, defined as one minus the ratio of the pricing error variance to the variance of the original implied volatility series. The pricing error is defined as the difference in implied volatilities between the market quotes and the model-generated values. The last row reports the aggregate explained variation for all option series underlying each of the three equity indexes.

Table 1.5
Maximum likelihood estimates of model parameters

Global risk			Country-specific risk		
Average risk levels					
β^{FTS}	1.1737	(20.0)	ξ^{FTS}	0.9683	(20.0)
β^{NKY}	0.7052	(20.0)	ξ^{NKY}	1.1381	(20.0)
Jump structures					
λ^{g}	0.1630	(6.39)	λ^{c}	0.0299	(20.0)
ζ^{g}_{-}	0.5250	(7.63)	ζ^{c}_{-}	1.1231	(20.0)
ζ^{g}_{+}	1.4583	(7.65)	ζ^{c}_{+}	1.9004	(20.0)
Activity rate dynamics					
κ_g	0.0079	(0.02)	κ_c	3.0393	(18.2)
θ_g	0.0361	(0.02)	θ_c	0.0394	(20.0)
ω_g	1.0179	(20.0)	ω_c	0.5262	(20.0)
ρ_g	-0.6335	(20.0)	ρ_c	-0.4309	(20.0)
Market prices of return risks					
γ_g^{SPX}	0.1913	(4.13)	γ_c^{SPX}	0.9533	(20.0)
γ_g^{FTS}	0.2257	(4.30)	γ_c^{FTS}	0.8210	(20.0)
γ_g^{NKY}	0.0047	(0.08)	γ_c^{NKY}	-0.0008	(3.90)
Market prices of variance risks					
$\gamma_{v^g}^{SPX}$	-1.0776	(2.58)	$\gamma_{v^c}^{SPX}$	-2.7817	(9.30)
$\gamma_{v^g}^{FTS}$	-1.2336	(2.96)	$\gamma_{v^c}^{FTS}$	-2.6865	(8.78)
$\gamma_{v^g}^{NKY}$	-1.9793	(4.61)	$\gamma_{v^c}^{NKY}$	-3.3337	(11.0)

Entries report the maximum likelihood estimates of the model parameters and the absolute magnitudes of the t-statistics (in parentheses). The t-statistics reported in the first panel are calculated against the null hypothesis of one, whereas those in other panels are against the null hypothesis of zero. For brevity, we truncate the t-values to a maximum of 20. Estimation is based on weekly index returns and index options data from January 3, 1996 to August 14, 2002.

Table 1.6
Mean return risk premiums induced by each risk component

Economy	Global risks				Country-specific risks			
	Diffusion	Jump	Volatility	Total	Diffusion	Jump	Volatility	Total
SPX	0.0069	0.0092	0.0246	0.0407	0.0375	0.0055	0.0472	0.0903
FTS	0.0096	0.0165	0.0331	0.0591	0.0313	0.0026	0.0441	0.0780
NKY	0.0001	0.0001	0.0319	0.0321	-0.0000	-0.0000	0.0644	0.0644

Entries report the mean annualized return risk premium on the stock indexes induced by each risk component: The return diffusion risk, the return jump risk, and the activity rate risk in both the global and country-specific risk components.

Table 1.7
Option pricing performance when country-specific jump structures and activity rates are allowed to differ

Index	SPX				FTS				NKY			
Strike\Maturity	1m	3m	6m	12m	1m	3m	6m	12m	1m	3m	6m	12m
80	0.768	0.882	0.944	0.942	0.771	0.950	0.970	0.955	0.754	0.849	0.894	0.899
90	0.826	0.937	0.972	0.952	0.935	0.983	0.988	0.970	0.851	0.940	0.963	0.936
100	0.884	0.971	0.983	0.951	0.972	0.995	0.994	0.978	0.945	0.988	0.987	0.933
110	0.792	0.960	0.978	0.950	0.933	0.989	0.991	0.983	0.913	0.952	0.963	0.926
120	0.300	0.372	0.940	0.930	0.674	0.840	0.976	0.975	0.833	0.890	0.901	0.895
Aggregate	0.917				0.944				0.911			

Entries report the more flexible model's explained variation on each implied volatility series, defined as one minus the ratio of the pricing error variance to the variance of the original implied volatility series. The pricing error is defined as the difference in implied volatilities between the market quotes and the model-generated values. The last row reports the aggregate explained variation for all option series underlying each of the three equity indexes. The model allows different jump structures and activity rate dynamics across different economies.

Table 1.8
Allowing countries to have different jump structures and activity rate dynamics

Parameters\Index	SPX		FTS		NKY		Global	
			Average risk levels					
β	1	(–)	1.2059	(20.0)	0.9791	(20.0)		
ξ	1	(–)	0.8301	(20.0)	1.1755	(20.0)		
			Jump structures					
λ	0.0187	(1.52)	0.0179	(2.13)	0.0160	(0.30)	0.1964	(5.29)
ζ_-	0.8550	(8.74)	0.9482	(6.37)	0.8527	(1.56)	0.3421	(6.61)
ζ_+	3.3185	(20.0)	3.4321	(20.0)	3.6115	(2.25)	1.4045	(13.9)
			Activity rate dynamics					
κ_c	3.2474	(20.0)	3.2474	(20.0)	4.0902	(19.9)	0.0063	(0.01)
θ_c	0.0404	(20.0)	0.0404	(20.0)	0.0404	(20.0)	0.0281	(0.01)
ω_c	0.4775	(20.0)	0.6471	(20.0)	0.4911	(20.0)	0.9559	(20.0)
ρ_c	-0.5058	(20.0)	-0.4653	(20.0)	-0.4074	(20.0)	-0.6374	(20.0)
			Market prices					
γ_g	0.0717	(2.35)	0.1310	(4.25)	-0.0476	(1.45)		
γ_c	0.6732	(20.0)	0.4695	(20.0)	-0.2915	(20.0)		
γ_{v^g}	-1.2068	(2.14)	-1.3818	(2.45)	-1.9103	(3.37)		
γ_{v^c}	-2.8376	(9.39)	-2.9318	(7.70)	-3.2165	(8.47)		

Entries report the maximum likelihood estimates of the model parameters and the absolute magnitudes of the t-statistics (in parentheses, truncated to a maximum of 20) on a model that allows the country-specific risk jump structures and activity rate dynamics to differ across different economies. The t-statistics reported in the first panel are calculated against the null hypothesis of one, whereas those in other panels are against the null hypothesis of zero. Estimation is based on weekly index returns and index options data from January 3, 1996 to August 14, 2002.

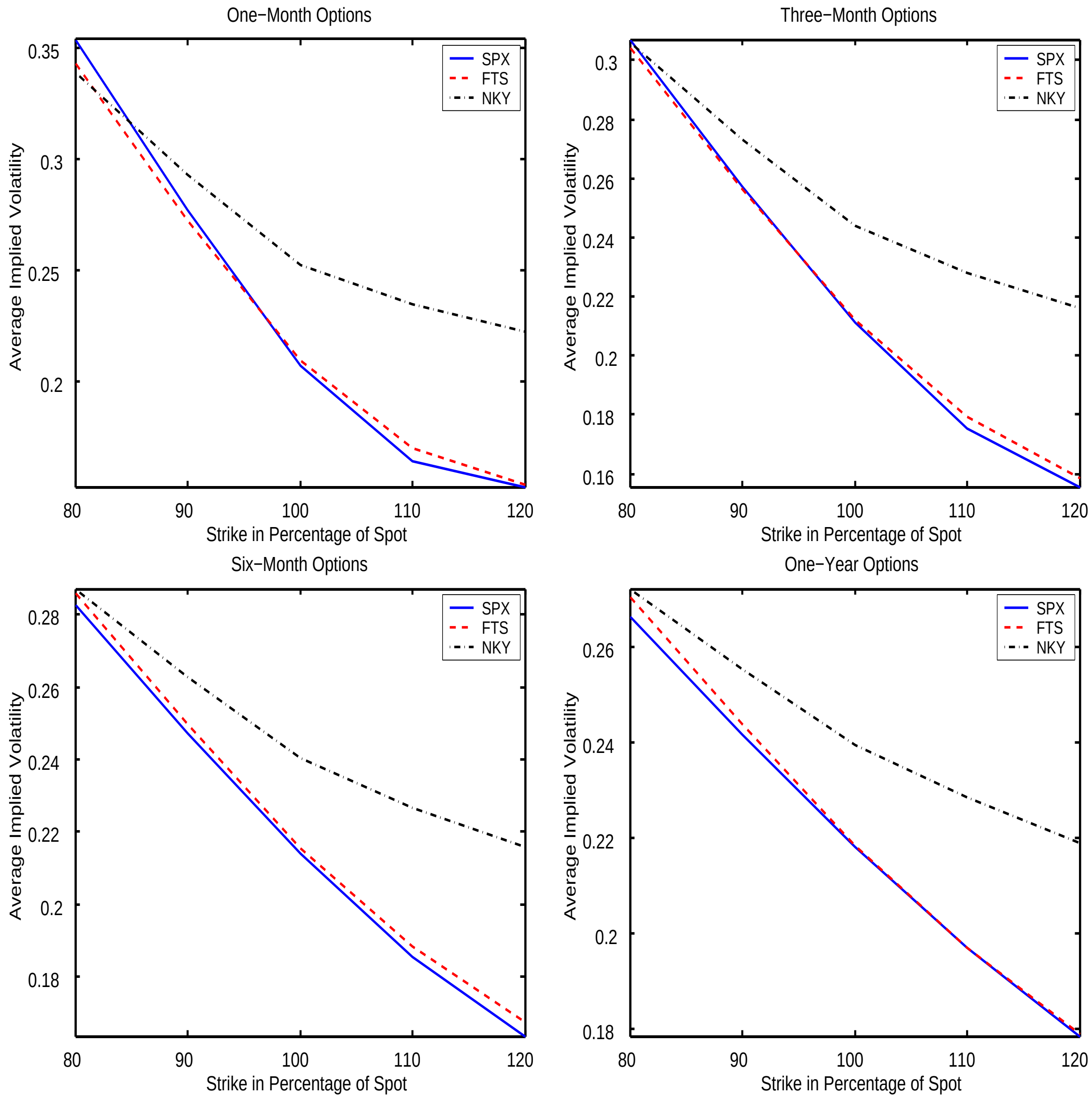

Fig. 1.1. Average implied volatility smirks on stock indexes. Lines are the sample averages of the implied volatility quotes plotted against the fixed moneyness, defined as strike prices in percentages of the underlying spot level. The four panels represent four different maturities. The three lines in each panel represent three stock indexes: SPX (solid line), FTS (dashed line), and NKY (dash-dotted line).

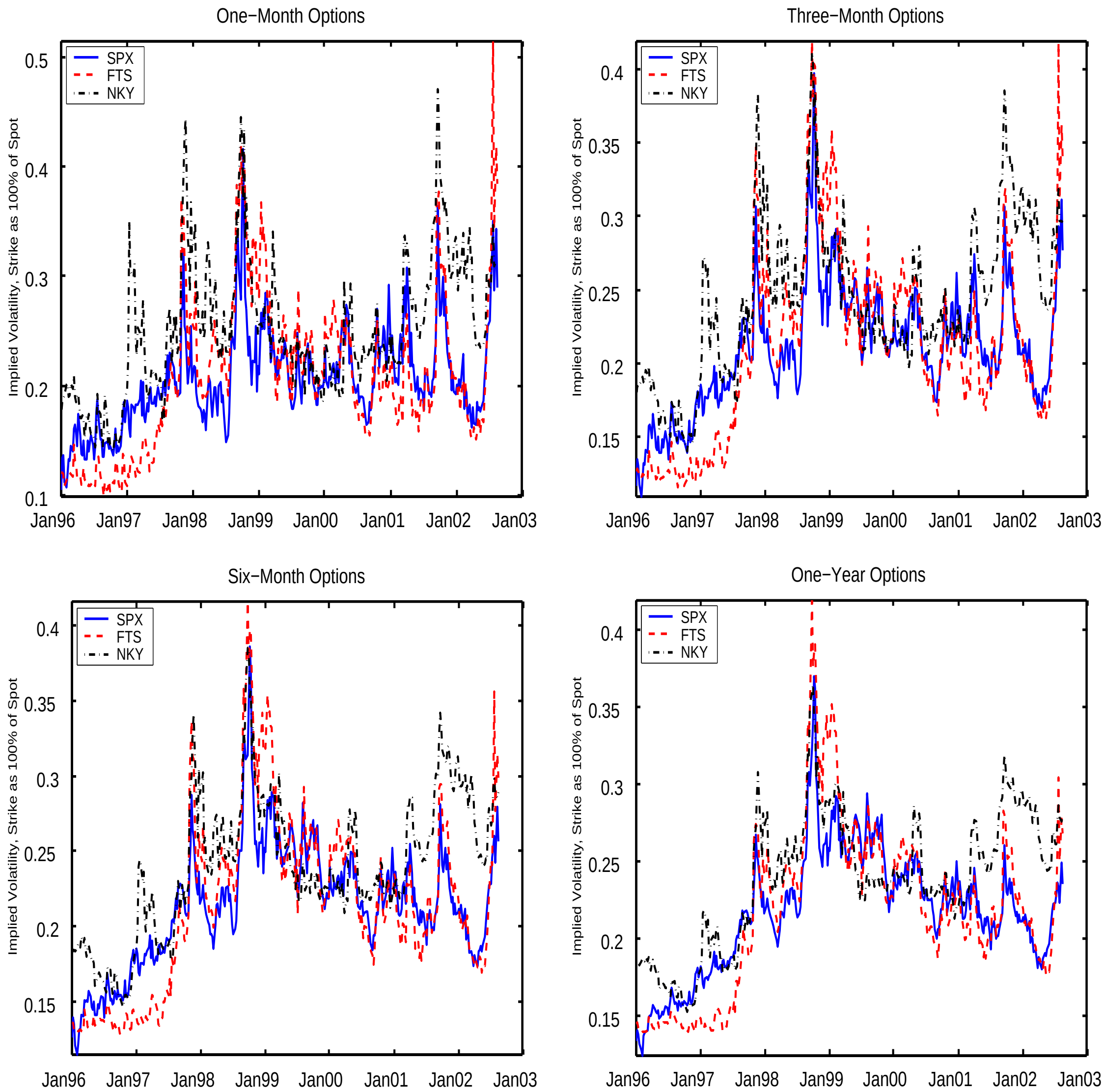

Fig. 1.2. Time series of at-the-money implied volatilities. Lines are the time series of at-the-money spot implied volatility quotes for the S&P 500 Index (SPX, solid line), the FTSE 100 index (FTS, dashed line), and the Nikkei-225 Stock Average (NKY, dash-dotted line). Each panel represents one maturity. Data are weekly from January 3, 1996 August 14, 2002.

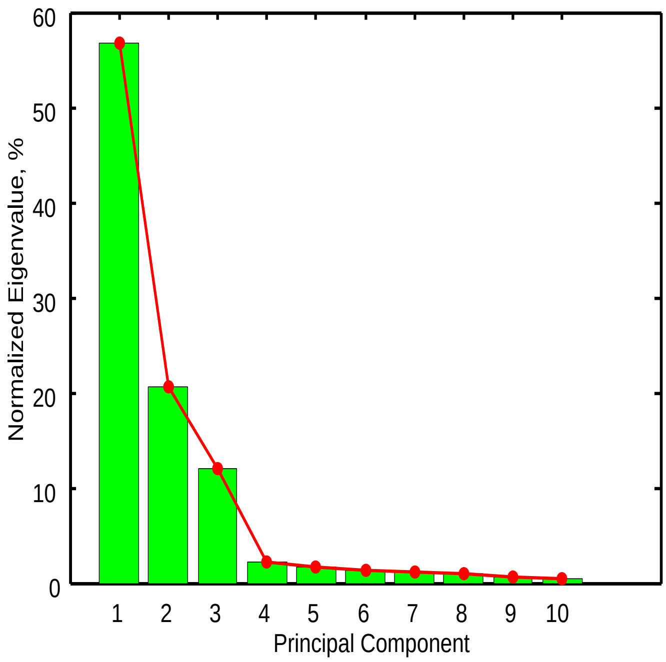

Fig. 1.3. Percentage variance explained by each principle component. The bar chart plots the percentage of aggregate variation explained by each of the first ten principle components on the 60 series of weekly implied volatility differences on the three equity indexes. We first estimate the correlation matrix of the 60 series of weekly changes of implied volatilities. We then compute the eignevalues of the correlation matrix. We use the normalized eigenvalues to represent the percentage explained variation from each principal component.

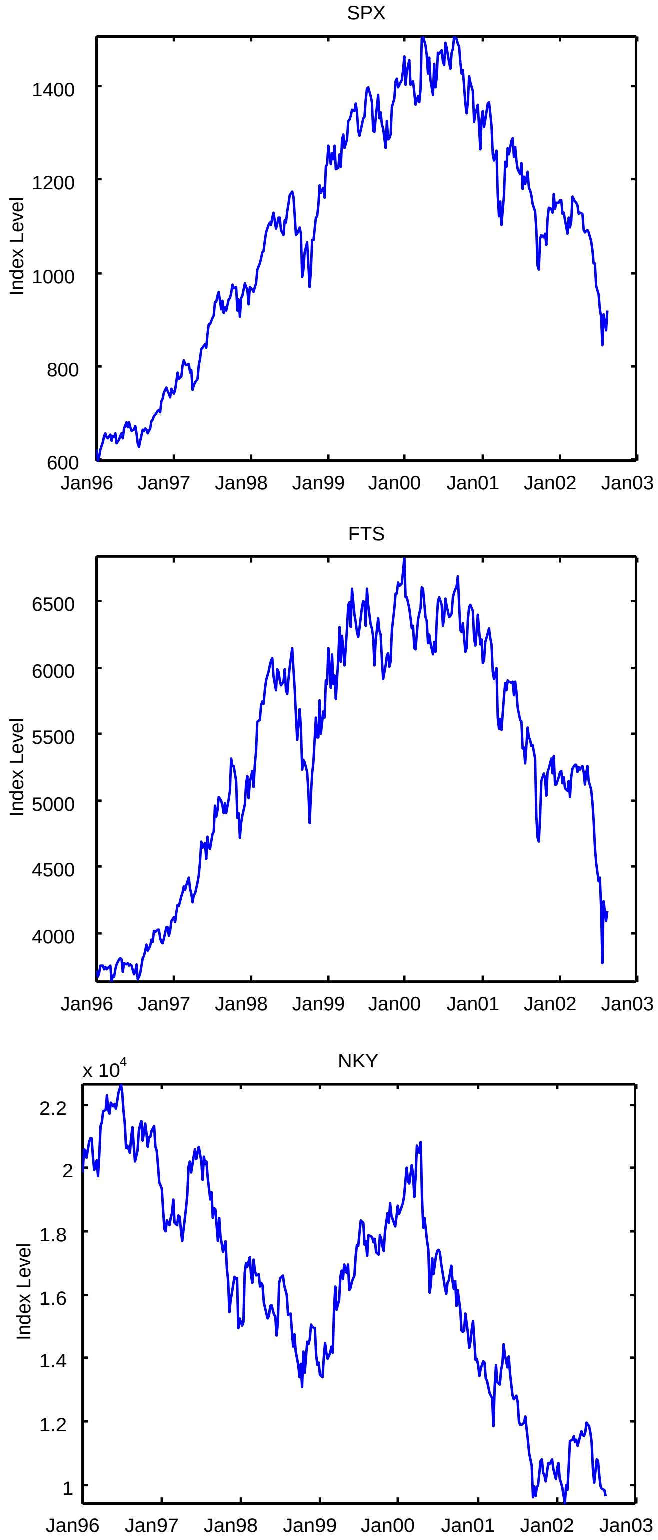

Fig. 1.4. Time series of stock indexes. The three indexes are the S&P 500 Index, the FTSE 100 index, and the Nikkei-225 Stock Average. The sample starts on January 3, 1996 and ends on August 14, 2002, with 346 weekly observations for each series.

Part II

Consistent Pricing of Illiquid Cross Exchange Rate Options via Stochastic Discount Factors Embedded in Currency Triangles

2.1. Introduction

The currency options market has experienced tremendous growth in recent years. According to the statistics provided by the Bank of International Settlement, the average daily turnover on global over-the-counter (OTC) foreign exchange options was about $117 billion in 2004, which almost tripled the estimate in 1995. Currency options are now widely used as basic instruments to hedge currency risk exposures. Central banks also use the forward-looking information in currency options to assess market expectation about the future course of foreign exchange rates (See, for example, Mylonas and Schich (1999), International Monetary Fund (2002), Bank of International Settlements (2004), and Bank of England (2004)).

However, the options trading activity on different currency pairs is far from uniform. Transactions on exchange rates that include the US dollar on one side of the currency pair has been dominating the trading volume and account for about 79% of the average daily turnover. Contracts involving euro, yen, and British pound, the next three most liquid contracts in global OTC foreign exchange options markets, account for about 44%, 32%, and 10% of global daily turnover, respectively.[6] On the other hand, options on exchange rates that do not involve the above four currencies, especially those with emerging market currencies, generally have much lower trading activities. Given the dominance of the dollar in global currency and its derivatives trading activities, the market often refer to dollar-linked currency pairs as *primary* exchange rates and refer to currency pairs that do not involve the dollar as *cross* exchange rate. Given the low trading activity of many cross exchange rates, the OTC quotes on their options are often rare and unreliable. How to price options on cross rates when reliable market quotes are not available has become an important topic among practitioners.

Pricing cross rate options also constitutes an important and interesting issue for the academia. Via the currency triangular relation, the cross exchange rate can be completely determined by the two relevant primary exchange rates. For example, the yen price of pound is simply equal to the product of the yen price of dollar and the dollar price of pound. However, the distribution of the cross rate is not completely determined by the marginal distributions of the two primary exchange

[6]Since each transaction involves two currencies, the sum of weighted percentages in individual currencies is equal to 200%. The statistics are from Bank of International Settlements (2005).

rates. We must also know the correlation structure between the primary rates in order to obtain the cross rate distribution from the marginal distributions of the two primary rates. Accordingly, option prices on the two primary rates do not uniquely determine the option values on the cross rate, unless under some parametric assumptions.

The extant literature often prices cross rate options by first inferring the risk-neutral densities of the primary rates from primary rate options and then building the risk-neutral density of the cross rate via some copula assumptions. Examples include Bikos (2000), Rosenberg (2003), Taylor and Wang (2005), and Bennett and Kennedy (2004). These studies determine the dependence pattern either using prices of at-the-money options on the cross exchange rate (assuming that they are available) or using historical exchange rate series. In this paper, instead of separately specifying the risk-neutral process for each exchange rate, we propose to model the stochastic discount factor of each economy and derive the exchange rate dynamics as the ratio of the stochastic discount factors of the two economies. In any economy precluding arbitrage, there always exists a stochastic discount factor that links future payoffs to their present values (Harrison and Kreps (1979)). In complete markets, the stochastic discount factor is unique, and the ratios of the stochastic discount factors between two economies determine the exchange rate between them.

We develop dynamic models of stochastic discount factors. Similar to the specification in Bakshi, Carr, and Wu (2005), we decompose the stochastic discount factor in each economy into a global diffusion risk component and a country-specific jump-diffusion risk component. Separate stochastic time changes are further applied to the two components so that stochastic volatilities can come separately from both global and country-specific risks. In principle, we can identify both the global and the country-specific risks for three economies using options on the three currency pairs that form a currency triangle. Then, by incorporating options on any other primary exchange rate (the dollar price of a fourth currency), we can also identify the country-specific risk dynamics in the fourth economy, and thus price options on currency pairs that involve this fourth economy.

We illustrate our method by applying the model to four years of options data on six currency pairs that span four economies. The options data are obtained from the BBA-Reuters FX Option Volatility Benchmark. The six currency pairs include three dollar-lined primary exchange rates: euro-dollar (EURUSD), pound-dollar (GBPUSD), and dollar-yen (USDJPY), as well as three cross exchange rates: euro-yen (EURJPY), euro-pound (EURGBP), and pound-yen (GBPJPY). The options are quoted at six fixed time-to-maturities at one week, one, three, six, 12 months, and two years for delta-neutral straddles. In addition, 25-delta risk reversals and butterfly spreads are also available at three fixed time-to-maturities at one, three, and 12 months.

The six currency pairs constitute our full sample. To analyze the out-of-sample performance, we partition the full sample into three subsamples. Each subsample includes the three dollar-linked primary rates and one of the three cross rates. Thus, in each subsample, the four currency pairs contain a currency triangle and an additional primary exchange rate. Through the currency triangle, we can identify both the global and the country-specific risks for the three economies. With the additional primary exchange rate, we can also identify the country-specific risk dynamics for the fourth economy. Then, we can use the estimated model to price options on the remaining two cross rates that are not included in the model estimation.

For estimation, we cast the option pricing model into a state-space form, define the state-propagation equation according to the dynamics of the five variance rates (one global and four country-specific) underlying the four economies, and define the measurement equations based on the 72 (48 for the subsample estimation) option series underlying the six (four) currency pairs. We use an extended version of the Kalman filter to obtain the ex-ante forecasts and ex-post updates on the conditional mean and variance of the five state variables and 72 (48) measurement series. We then construct the likelihood function on the option series assuming normal forecasting errors and estimate the model parameters by maximizing the sum of the likelihood values.

The estimation results show that the average in-sample root mean squared pricing errors are from 0.36 to 0.48 implied volatility percentage point. The out-of-sample performance is also satisfactory. When options on the two cross rates are excluded from model estimation but included in updating the underlying variance rates, the average root mean squared pricing errors range from 0.60 to 0.69 volatility point. When the cross-rate information is excluded from both model estimation and variance rate updating, the average root mean squared errors are from 1.11 to 1.46 volatility points.

Our work represents a direct extension of the work by Bakshi, Carr, and Wu (2005), although with a different objective. Bakshi, Carr, and Wu show that one can identify the multi-dimensional structure of the stochastic discount factors in international economies using options on currencies that form a currency triangle. They apply this idea to identify the risk and pricing behavior in the

US, the UK, and Japan using options on the three relevant currency pairs. In this paper, we also use currency options to identify stochastic discount factors. However, our work goes beyond a currency triangle, but relies more on a chain role that links many economies and hence many currency pairs. Furthermore, identifying the stochastic discount factor is not our final objective, but an intermediate step through which we price options where reliable market quotes are not available using options with reliable market quotes.

The paper is organized as follows. Section 2.2 propose the model for stochastic discount factors and derives its implications for option pricing. Section 2.3 describes the data and summarizes the stylized evidence. Section 2.4 demonstrates the sample construction and outlines the estimation strategy based on time series of currency option prices. Section 2.5 discusses the estimation results and assesses both the in-sample and out-of-sample model performance. Section 2.6 concludes.

2.2. The Model Framework

In this section, we propose a dynamic model of stochastic discount factors in international economies and derive its implications for currency option pricing.

2.2.1. Modeling stochastic discount factors

We describe a set of N economies by fixing a filtered complete probability space $\{\Omega, \mathcal{F},$ $\mathbb{P}, (\mathcal{F}_t)_{0 \leq t \leq \mathcal{T}}\}$, with some fixed horizon $\mathcal{T}$. We assume no arbitrage in each economy. Therefore, for each economy, we can identify at least one strictly positive process, $\mathcal{M}_t^h$ $(h = 1, \ldots, N)$, which we call the state-price deflator, such that the deflated gains process associated with any admissible trading strategy is a martingale (Duffie (1992)). We further assume that $\mathcal{M}_t^h$ itself is a semimartingale. The ratio of $\mathcal{M}_t^h$ at two time horizons will henceforth be referred as the stochastic discount factor.

We propose a dynamic model of stochastic discount factors in international economies, under which the stochastic discount factor is decomposed into three orthogonal components: an interest rate component, a global risk component and a country-specific risk component. The volatilities underlying the global and country-specific components are both stochastic and follow separate dynamics. Using the language of time-changed Lévy processes (Carr and Wu (2004a)), we can write our model for the stochastic discount factor in the home economy as (with $\mathcal{M}_0 = 1$),

$$\begin{aligned} \ln \mathcal{M}_t^h &= \left[-r^h t\right]\left[-\gamma^h W^g_{\Xi^g_t} - \frac{1}{2}\left(\gamma^h\right)^2 \Xi^g_t\right] \\ &\quad \left[-\xi^h\left(W^h_{\Xi^h_t} + J^h_{\Xi^h_t}\right) - \left(\frac{1}{2}\left(\xi^h\right)^2 + k_J[-\xi^h]\right)\Xi^h_t\right]. \end{aligned} \tag{2.1}$$

The first bracket captures the contribution from interest rates with r^h denoting the continuously compounded spot interest rate of the relevant maturity, which we assume deterministic. The second bracket specifies global diffusion risk factor, with W^g denoting a standard Brownian motion, and Ξ^g_t denoting a stochastic time change that captures the stochastic volatility associated with the global risk factor. γ^h is the loading of economy h on the global risk factor. The second term in the bracket compensates the first term to make it an exponential martingale.

The third bracket specifies a country-specific jump-diffusion risk component with W^h denoting another standard Brownian motion independent of the global risk component W^g, and J^h denotes a pure jump Lévy component. Here we add a jump component to the country-specific component to capture short-term currency return non-normalities. Separate stochastic time change Ξ^h_t are applied to this country-specific risk component to capture the stochastic volatility associated with the idiosyncratic risk. To maintain parsimony, we assume that $\left(W^h + J^h\right)_{\Xi^h}$ are independent but identical across all economies and ξ^h captures the average scale difference of the idiosyncratic risk across different economies. For identification, we normalize the loadings to the first economy to unity: $\gamma^1 = \xi^1 = 1$.

We model the jump component J^h_t by the exponentially dampened power law, the Levy density of which is given by:

$$\pi^h[x] = \begin{cases} 0, & x > 0 \\ \lambda e^{-\beta|x|}|x|^{-\alpha-1}, & x < 0 \end{cases}, \tag{2.2}$$

with $\alpha \in [-1, 2]$ and $\lambda, \beta > 0$.We adopt this specification from Carr, Geman, Madan, and Yor (2002) as Wu (2006a) shows that the DPL jump specification can match evidence in stocks and currencies. In equation (2.2), we set the arrival rate of positive jumps to zero, given previous evidence that only negative jumps are priced. In this specification, the power coefficient α controls the jump type. The model generates finite-activity jumps when $\alpha < 0$, and infinite-activity jumps when $\alpha \geq 0$. Since we only have option quotes up to 25-delta, it is unlikely we can distinguish different jump types. Hence, we fix α at 1. The cumulant exponent of the jump component with $\alpha = 1$ is:

$$k[s] \equiv \frac{1}{t} \ln \mathbb{E}\left(e^{sJ_t^h}\right) = \lambda(\beta + s)\ln(1 + s/\beta), \quad s \in \mathcal{D} \subseteq \mathcal{C}, \tag{2.3}$$

where $\mathbb{E}(\cdot)$ denotes the expectation operator under measure $\mathbb{P}$. A cumulant exponent is normally defined on the positive real line, but it is convenient to extend the definition to the complex plane, $s \in \mathcal{D} \subseteq \mathcal{C}$, where the exponent is well-defined.

To generate stochastic volatility on the stochastic discount factor, we apply stochastic time changes Ξ_t^g and Ξ_t^h to the global and country-specific risk components, respectively. We assume locally predictable and continuous time changes and specify them via the instantaneous activity rates v_t^g and v_t^h:

$$\Xi_t^g \equiv \int_0^t v_s^g ds, \quad \Xi_t^h \equiv \int_0^t v_s^h ds. \tag{2.4}$$

We model the $\mathbb{P}$-dynamics of the two activity rates using square-root processes,

$$dv_t^g = \kappa^g\left(\theta^g - v_t^g\right)dt + \omega^g \sqrt{v_t^g} dZ_t^g, \tag{2.5}$$

$$dv_t^h = \kappa^h\left(\theta^h - v_t^h\right)dt + \omega^h \sqrt{v_t^h} dZ_t^h, \tag{2.6}$$

where (κ^g, κ^h) control the mean-reversion speeds of the two activity rate processes, (θ^g, θ^h) denote their long-run means, and (Z^g, Z^h) are two Brownian motions that are correlated with the two Brownian motions in the stochastic discount factor:

$$\rho^g dt = \mathbb{E}(dW_t^g dZ_t^g), \quad \rho^h dt = \mathbb{E}(dW_t^h dZ_t^h).$$

The correlations between other pairs of Brownian motions are assumed to be zero.

Our specifications on the stochastic discount factors follow largely from Bakshi, Carr, and Wu (2005). Our choice of the jump structure (with only downside jumps and $\alpha = 1$) is also motivated by the empirical evidence in that paper. On the other hand, we allow a more flexible risk structure among different economies. They use the same scaling coefficient ξ^h on both the global risk component and the country-specific risk component. Thus, although the aggregate risk level can differ across economies, the relative proportional of global versus country-specific risks are fixed. In our specification, we allow the coefficients to be different so that the relative proportion of the two sources of risks can also differ across economies.

2.2.2. Currency return dynamics

In complete markets, the stochastic discount factor is unique, and the ratios of the stochastic discount factors between two economies determine the exchange rate between them (Lucas (1982), Dumas (1992), Bakshi and Chen (1997), Basak and Gallmeyer (1999), and Backus, Foresi, and Telmer (2001)). Let S_t^{fh} denote the time-t currency-h price of currency f, with h being the home economy, we have,

$$\frac{S_{t+\tau}^{fh}}{S_t^{fh}} = \frac{\mathcal{M}_{t+\tau}^{f}/\mathcal{M}_t^{f}}{\mathcal{M}_{t+\tau}^{h}/\mathcal{M}_t^{h}}, \qquad h,f = 1,2,\ldots,N. \tag{2.7}$$

When the market is not completed by domestic securities such as stocks and bonds, the currencies and currency options can help complete the market by requiring the stochastic discount factors to price the observed currency and currency options via the above no-arbitrage relation.

Under the stochastic discount factor specification in (2.1), the log return on the exchange rate $s_t^{hf} \equiv \ln S_t^{hf}/S_0^{hf}$ over the horizon $[0,t]$ is,

$$\begin{aligned} s_t^{hf} \;=\; & \left(r^h - r^f\right) + \left(\gamma^h - \gamma^f\right) W^g_{\Xi_t^g} + \frac{1}{2}\left(\left(\gamma^h\right)^2 - \left(\gamma^f\right)^2\right)\Xi_t^g \\ & + \left(\xi^h\left(W^h_{\Xi_t^h} + J^h_{\Xi_t^h}\right) + \left(\frac{1}{2}\left(\xi^h\right)^2 + k_J[-\xi^h]\right)\Xi_t^h\right) \\ & - \left(\xi^f\left(W^f_{\Xi_t^f} + J^f_{\Xi_t^f}\right) + \left(\frac{1}{2}\left(\xi^f\right)^2 + k_J[-\xi^f]\right)\Xi_t^f\right). \end{aligned} \tag{2.8}$$

In this specification, the exchange rate dynamics between the two economies (h and f) are governed by one diffusion global risk component (W^g), two country-specific jump-diffusion risk components ($W^h + J^h$ and $W^f + J^f$), and three activity rates (v_t^g, v_t^h, v_t^f) that define the three stochastic time changes ($\Xi_t^g, \Xi_t^h, \Xi_t^f$).

To price options, we need to derive the risk-neutral dynamics on the currency return. The measure change from the statistical measure $\mathbb{P}$ to the home-country risk-neutral $\mathbb{Q}^h$ is defined by the exponential martingale:

$$\left.\frac{d\mathbb{Q}^h}{d\mathbb{P}}\right|_t \equiv \exp\left(-\gamma^h W^g_{\Xi^g_t} - \frac{1}{2}\left(\xi^{hg}\right)^2 \Xi^g_t\right)$$
$$\exp\left(-\xi^h\left(W^h + J^h\right)_{\Xi^h_t} - \left(\frac{1}{2}\left(\xi^h\right)^2 + k_J\left[-\xi^h\right]\right)\Xi^h_t\right). \quad (2.9)$$

The currency return dynamics under the home-economy risk-neutral measure $\mathbb{Q}^h$ becomes,

$$\begin{aligned} s^{hf}_t &= \left(r^h - r^f\right) + \left(\gamma^h - \gamma^f\right) W^g_{\Xi^g_t} - \frac{1}{2}\left(\gamma^h - \gamma^f\right)^2 \Xi^g_t \\ &+ \left(\xi^h\left(W^h_{\Xi^h_t} + J^h_{\Xi^h_t}\right) - \left(\frac{1}{2}\left(\xi^h\right)^2 + k^{\mathbb{Q}}_J[\xi^h]\right)\Xi^h_t\right) \\ &- \left(\xi^f\left(W^f_{\Xi^f_t} + J^f_{\Xi^f_t}\right) + \left(\frac{1}{2}\left(\xi^f\right)^2 + k^{\mathbb{Q}}_J[-\xi^f]\right)\Xi^f_t\right). \end{aligned} \quad (2.10)$$

Since J^f is independent of J^h, J^f remains unchanged under $\mathbb{Q}^h$. The home-economy jump component changes from $J^h_{\Xi^h_t}$ to $J^{h\mathbb{Q}}_{\Xi^h_t}$ under $\mathbb{Q}^h$, where the Lévy density for $J^{h\mathbb{Q}}_t$ becomes

$$\pi^{\mathbb{Q}h}[x] = e^{-\xi^h x}\pi^h[x] = \begin{cases} 0, & x > 0 \\ \lambda e^{-\left(\beta-\xi^h\right)|x|}|x|^{-\alpha-1} & x < 0 \end{cases}. \quad (2.11)$$

Hence, $\pi^{\mathbb{Q}h}[x]$ and $\pi^h[x]$ share the same parametric form with $\beta^{\mathbb{Q}} = \beta - \xi^h$. For $\beta^{\mathbb{Q}} > 0$, we need $\beta > \xi^h$. Under measure $\mathbb{Q}^h$, the global and country-specific variance rates processes change to

$$\begin{aligned} dv^g_t &= \left(\kappa^g\theta^g - \kappa^{g\mathbb{Q}} v^g_t\right) dt + \omega^g \sqrt{v^g_t}\, dZ^g_t, \\ dv^h_t &= \left(\kappa^h\theta^h - \kappa^{h\mathbb{Q}} v^h_t\right) dt + \omega^h \sqrt{v^h_t}\, dZ^h_t. \end{aligned}$$

with $\kappa^{g\mathbb{Q}} = \kappa^g + \gamma^h\omega^g\rho^g$ and $\kappa^{h\mathbb{Q}} = \kappa^h + \xi^h\omega^h\rho^h$. The process for v^f_t does not change under measure $\mathbb{Q}^h$ since Z^f is independent of Z^g and Z^h.

2.2.3. Currency option pricing

Given the $\mathbb{Q}^h$-dynamics of the log currency return in (2.10), we can derive its generalized Fourier transform as in Carr and Wu (2004a),

$$\begin{aligned}
\phi_s^{\mathbb{Q}} &\equiv \mathbb{E}^{\mathbb{Q}}\left(e^{ius_t^{hf}}\right) \\
&= \mathbb{E}^{\mathbb{Q}}\left(e^{iu\left(\left(r^h-r^f\right)t+\left(\left(\gamma^h-\gamma^f\right)W^g_{\Xi^g_t}-\frac{1}{2}\left(\gamma^h-\gamma^f\right)^2\Xi^g_t\right)\right)}\right) \\
&\quad \times\mathbb{E}^{\mathbb{Q}}\left(e^{iu\left(\left(\xi^h\left(W^h_{\Xi^h_t}+J^h_{\Xi^h_t}\right)-\left(\frac{1}{2}\left(\xi^h\right)^2+k_J^{\mathbb{Q}}\left[\xi^h\right]\right)\Xi^h_t\right)+\left(-\xi^f\left(W^f_{\Xi^f_t}+J^f_{\Xi^f_t}\right)-\left(\frac{1}{2}\left(\xi^f\right)^2+k_J\left[-\xi^f\right]\right)\Xi^f_t\right)\right)}\right) \\
&= e^{iu\left(r^h-r^f\right)t}\mathbb{E}^{\mathbb{N}^g}\left(e^{-\psi^g[u]\,\Xi^g_t}\right)\mathbb{E}^{\mathbb{N}^h}\left(e^{-\psi^h[u]\,\Xi^h_t}\right)\mathbb{E}^{\mathbb{N}^f}\left(e^{-\psi^f[u]\,\Xi^f_t}\right), \qquad (2.12)
\end{aligned}$$

where $\left(\psi^g[u],\psi^h[u],\psi^f[u]\right)$ denote the characteristic exponents of the three Lévy components prior to time-change that are due to the global risk component, home country-specific risk component, and foreign country-specific risk component, respectively:

$$\begin{aligned}
\psi^g[u] &= \frac{1}{2}\left(\gamma^h-\gamma^f\right)^2\left(iu+u^2\right), \\
\psi^h[u] &= iu\left(\frac{1}{2}\left(\xi^h\right)^2+k_J^{\mathbb{Q}}\left[\xi^h\right]\right)+\frac{1}{2}u^2\left(\xi^h\right)^2-k_J^{\mathbb{Q}}\left[iu\xi^h\right], \\
\psi^f[u] &= iu\left(\frac{1}{2}\left(\xi^f\right)^2+k_J\left[-\xi^f\right]\right)+\frac{1}{2}u^2\left(\xi^f\right)^2-k_J\left[-iu\xi^f\right].
\end{aligned}$$

$\left[\mathbb{N}^g,\mathbb{N}^h,\mathbb{N}^f\right]$ denote three new measures defined by the following complex-valued exponential martingales,

$$\begin{aligned}
\left.\frac{d\mathbb{N}^g}{d\mathbb{Q}^h}\right|_t &= \exp\left(iu\left(\left(\gamma^h-\gamma^f\right)W^g_{\Xi^g_t}-\tfrac{1}{2}\left(\gamma^h-\gamma^f\right)^2\Xi^g_t\right)+\psi^g[u]\,\Xi^g_t\right), \\
\left.\frac{d\mathbb{N}^h}{d\mathbb{Q}^h}\right|_t &= \exp\left(iu\left(\xi^h\left(W^h_{\Xi^h_t}+J^h_{\Xi^h_t}\right)-\left(\tfrac{1}{2}\left(\xi^h\right)^2+k_J^{\mathbb{Q}}\left[\xi^h\right]\right)\Xi^h_t\right)+\psi^h[u]\,\Xi^h_t\right), \qquad (2.13) \\
\left.\frac{d\mathbb{N}^f}{d\mathbb{Q}^h}\right|_t &= \exp\left(iu\left(-\xi^f\left(W^f_{\Xi^f_t}+J^f_{\Xi^f_t}\right)-\left(\tfrac{1}{2}\left(\xi^f\right)^2+k_J\left[-\xi^f\right]\right)\Xi^f_t\right)+\psi^f[u]\,\Xi^f_t\right).
\end{aligned}$$

To take the expectation, we need the dynamics for the variance rates under their respective new measures:

$$dv_t^g = \left(\kappa^g\theta^g - \kappa^{\mathbb{N}^g} v_t^g\right) dt + \omega^g \sqrt{v_t^g} dZ_t^g, \tag{2.14}$$
$$dv_t^h = \left(\kappa^h\theta^h - \kappa^{\mathbb{N}^h} v_t^h\right) dt + \omega^h \sqrt{v_t^h} dZ_t^h, \tag{2.15}$$
$$dv_t^f = \left(\kappa^h\theta^h - \kappa^{\mathbb{N}^f} v_t^f\right) dt + \omega^h \sqrt{v_t^f} dZ_t^f, \tag{2.16}$$

with

$$\begin{aligned} \kappa^{\mathbb{N}^g} &= \kappa^g + \gamma^h\omega^g\rho^g - iu\left(\gamma^h - \gamma^f\right)\omega^g\rho^g, \\ \kappa^{\mathbb{N}^h} &= \kappa^h + (1-iu)\xi^h\omega^h\rho^h, \\ \kappa^{\mathbb{N}^f} &= \kappa^h + iu\xi^f\omega^h\rho^h. \end{aligned} \tag{2.17}$$

Since the three activity rates follow affine dynamics under their relevant measures $\mathbb{N}^g$, $\mathbb{N}^h$, and $\mathbb{N}^f$, the expectations in (2.12) generate exponential-affine solutions:

$$\phi_s^{\mathbb{Q}} = e^{iu\left(r^h - r^f\right)t}\mathbb{E}^{\mathbb{N}^g}\left(e^{-\psi^g[u]\Xi_t^g}\right)\mathbb{E}^{\mathbb{N}^h}\left(e^{-\psi^h[u]\Xi_t^h}\right)\mathbb{E}^{\mathbb{N}^f}\left(e^{-\psi^f[u]\Xi_t^f}\right) \tag{2.18}$$
$$= e^{iu\left(r^h - r^f\right)t} e^{-b_g(t)v_0^g - c_g(t) - b_h(t)v_0^h - c_h(t) - b_f(t)v_0^f - c_f(t)}, \tag{2.19}$$

where $\left(v_0^g, v_0^h, v_0^f\right)$ are the time-0 levels of the three activity rates and the coefficients $[b(t), c(t)]$ on each activity rate take the same functional forms:

$$\begin{aligned} b_c(t) &= \frac{2\psi^c\left(1-e^{-\eta^c t}\right)}{2\eta^c - \left(\eta^c - \kappa^{\mathbb{N}_c}\right)\left(1 - e^{-\eta^{h}t}\right)}, \\ c_c(t) &= \frac{\kappa^c\theta^c}{\omega^2}\left[2\ln\left(1 - \frac{\eta^c - \kappa^{\mathbb{N}_c}}{2\eta^c}\left(1 - e^{-\eta^c t}\right)\right) + (\eta^c - \kappa^{\mathbb{N}_c})t\right], \end{aligned} \tag{2.20}$$

with $\eta^c = \sqrt{\left(\kappa^{\mathbb{N}_c}\right)^2 + 2\omega^2\psi^c}$ and for $c = g, h, f$, respectively. Given the generalized Fourier transform, we can now follow Carr and Madan (1999) and use fast Fourier inversion to obtain option prices.

2.3. Data Description

We obtain the currency options data from the British Bankers' Association (BBA). The BBA and Reuters jointly introduced the FX options volatility benchmark to improve market transparency. They have been publishing the historical currency options data through their websites. From this data set, we choose options on six currency pairs that span four economies. Three of the currency pairs are dollar-linked primary exchange rates: dollar price of euro (EURUSD), dollar price of pound (GBPUSD), and yen price of dollar (USDJPY). The other three are cross exchange rates that do not involve the dollar: yen price of euro (EURJPY), pound price of euro (EURGBP), and yen price of pound (GBPJPY). Thus, the spanned four economies are the US, the UK, the Euro Area, and Japan.

The BBA currency options quotes are in terms of delta-neutral straddle implied volatility (ATMV), 25-delta risk reversals (RR25), and 25-delta strangle margins (SG25, also known as butterfly spreads). A straddle is a portfolio of a call option and a put option with the same strike. The market quote of a delta-neutral straddle corresponds to a near-the-money implied volatility (ATMV) that makes the Black-Scholes delta of the straddle portfolio zero.

The 25-delta risk reversals (RR25) measure the difference between Black-Scholes implied volatilities of a 25-delta out-of-the-money call option and a 25-delta out-of-the-money put option: $RR25 = IV(25c) - IV(25p)$. Thus, risk reversal quotes capture the slope of the implied volatility curve, which measures the skewness of the risk-neutral return distribution.

The 25-delta strangles (SG25) are quoted as a premium of the average volatility of the 25-delta out-of-the-money call and 25-delta out-of-the-money put options over the at-the-money volatility: $SG25 = (IV(25c) + IV(25p))/2 - ATMV$. Strangle quotes capture the average curvature of the smile, which reflects the kurtosis of the risk-neutral currency return distribution.

The quotes are available daily from October 3, 2001 to September 28, 2005. For model estimation, we sample the data weekly on every Wednesday to avoid weekday effect. Each series contains 209 weekly observations. The straddle implied volatility quotes are available at six fixed maturities at one week, and one, three, six, 12 months, and two years. The 25-delta risk reversals and strangle quotes are available at three fixed maturities at three, six, and 12months.

At maturities where risk reversal and strangle quotes are available, we can combine them with the straddle implied volatility to recover the implied volatilities for the 25-delta call option and 25-delta put option:

$$IV(25c) = SG25 + ATMV + RR25/2, \quad IV(25p) = SG25 + ATMV - RR25/2. \tag{2.21}$$

To convert the implied volatilities into option prices, we need information on domestic and foreign interest rates as well as spot exchange rates. We download spot exchange rates from Bloomberg. We also download LIBOR and swap rates from Bloomberg and use them to construct the spot rate curve for each economy.

Table 2.1 reports the summary statistics on the straddle, risk reversal, and strangle quotes for each of the six currency pairs. The first panel reports the sample average of each series. The average straddle implied volatilities for EURGBP are about 6.74 volatility points across all maturities. Those for the other five currency pairs are higher, from 8.22 to 10.38 percentage points. The average strangles are positive for all currency pairs and across all maturities, implying that out-of-the-money option implied volatilities are generally higher than at-the-money implied volatilities. This feature of the data shows that excess kurtosis in the currency return distribution is a robust and persistent feature of the OTC currency options market. On the other hand, the average risk reversals differ in signs across different currency pairs, indicating that the risk-neutral currency return can on average be either positively or negatively skewed, depending on the currency pair.

The second panel reports the standard deviation of each series. The standard deviations of the straddle implied volatilities range from 0.67 to 1.79 percentage points. The standard deviations of risk reversals are in general larger than their mean magnitudes, and are much larger than the standard deviations of the strangles. As have been documented in Carr and Wu (2006), the high standard deviation on the risk reversal indicates that the risk-neutral currency return skewness is strongly time varying.

The third and fourth panels report the skewness and excess kurtosis of the implied volatility quotes. These estimates are mostly small, indicating that the quotes do not experience large discontinuous updates. The last panel reports the weekly autocorrelation estimates, which range from 0.60 to 0.98. These serial dependence reflects the time-series dynamics of the return volatility. For delta-neutral straddles and risk reversals, the autocorrelation estimates tend to increase with increasing option maturities.

Figure 2.1 plots the time series of delta-neutral straddle implied volatility at three selected maturities: one month (solid lines), three months (dashed lines), and 12 months (dash-dotted lines). Each panel represents one currency pair. Over the sample period, the plots show significant variation in implied volatilities, more so at shorter maturities, consistent with their larger standard deviations observed in Table 2.1.

Figure 2.2 plots the time series of 25-delta risk reversals (solid lines) and strangles (dashed lines) for each currency pair. The multiple lines for both risk reversals and strangles represent the three different option maturities at one, three, and 12 months, which we do not distinguish in the plot. For all currency pairs, the 25-delta strangles (dashed lines) fluctuate little during the four-year span at all three option maturities. In contrast, the risk reversals vary strongly over time, so much so that the sign also switches over time.

2.4. The Estimation Strategy

In this section, we first describe how we use options on one currency triangle plus another exchange rate to identify the risk dynamics in the four economies and to price options on other cross exchange rates between the four economies that are not included in the model estimation.

2.4.1. Subsample construction

We have options on three dollar-linked primary rates (EURUSD, GBPUSD, and USDJPY), and three cross exchange rates that do not involve the dollar (EURJPY, EURGBP, and GBPJPY). These six currency pairs span four economies: the US, the Euro area, the UK, and Japan. To show how we can price options on cross exchange rates based on options quotes on other related currency pairs, we construct three subsamples from the full sample of the six currency pairs. As shown in Table 2.2, each subsample contains four currency pairs, with three of them forming a currency triangle and the other being a primary exchange rate that links dollar to the fourth currency. Options on these four currency pairs are used for model estimation and for in-sample pricing performance analysis. Options on the two excluded cross exchange rate are used for out-of-sample performance analysis. The underlying idea is that using options on the currency triangle, we can identify both the global risk dynamics and the dynamics on the country-specific risks in the three economies. Then, incorporating another primary exchange rate that links the dollar to a fourth currency, we can also learn the risk dynamics for the fourth economy. Thus, we can price options on the two excluded cross exchange rates even though their options are not included in the model estimation. The two excluded cross rates are formed by linking the fourth economy to the two non-US economies in the currency triangle.

Figure 2.3 further illustrates the idea using schematic graphs. Each panel represents one subsample construction. Currency pairs linked with solid lines are included in model estimation, whereas currency pairs linked with dashed lines are excluded from model estimation and are used for out-of-sample analysis. The schematic graphs show that the relation we rely on for identification is built upon the currency triangle, but goes beyond it as it is more of a chain rule that links all the economies and all exchange rates together. We estimate our models using both the full sample that includes options on all six currency pairs and the three subsamples that include options on four currency pairs each.

Our purpose is to use options on actively traded exchange rates to identify the stochastic discount factors of the involved economies, from which we can price options on the less actively traded exchange rates. In general, we can add any liquid option to the triangular pair in order to infer the cross-rate option prices of our interest. For example, if our interest is to price the illiquid cross-rate options between euro and Brazilian real, we can use the dollar-real options to replace options on dollar-yen in the first subsample. Then, from the estimated model, we can price options on euro-real and pound-real. To better assess the model performance, we illustrate our method using liquid exchange rates involving the four of the most actively traded currencies.

2.4.2. Maximum likelihood estimation with unscented Kalman filter

We estimate the model using the time-series of option prices. Since the activity rates are not directly observable, we cast the model into a state-space form and infer the activity rates at each date using an efficient filtering technique. Then, we estimate the model parameters by maximizing the sum of weekly likelihood values on option prices.

In the state-space form, we regard the global activity rate and the country-specific activity rates in the four economies as the unobservable states and use $v_t \equiv [v_t^g, v_t^{US}, v_t^{EU}, v_t^{UK}, v_t^{JP}]^\top$ to denote the (5×1) state vector, with the superscript US, EU, UK, JP denoting the four economies, respectively. The state propagation equation is specified as an Euler approximation of the activity rate dynamics:

$$v_t = A + \Phi v_{t-1} + \sqrt{\mathcal{G}_t}\,\varepsilon_t, \quad v_t \in \mathbb{R}^{5+} \tag{2.22}$$

where ε_t denotes an i.i.d. standard normal innovation vector and

$$\begin{aligned}
\Phi &= \exp(-\kappa\Delta t), \quad \kappa = \begin{bmatrix} \kappa^g & 0 \\ 0 & \kappa^h I_4 \end{bmatrix}, \\
A &= (I_5 - \Phi)\theta, \quad \theta = [\theta^g, \theta^h \mathbf{e}_4^\top]^\top, \\
\mathcal{G}_t &= \left\langle (\omega^g)^2 v_{t-1}^g, (\omega^h)^2 v_{t-1}^{US}, (\omega^h)^2 v_{t-1}^{EU}, (\omega^h)^2 v_{t-1}^{UK}, (\omega^h)^2 v_{t-1}^{JP} \right\rangle \Delta t,
\end{aligned} \tag{2.23}$$

where $\Delta t = 7/365$ corresponds to the weekly frequency of the data, I_d denotes an identity matrix of dimension d, $\mathbf{e}_d$ denotes a vector of ones of dimension d, and $\langle\cdot\rangle$ denotes a diagonal matrix with the diagonal elements given in the bracket.

Measurement equations are based on the observed option prices, assuming additive, normally-distributed measurement errors:

$$y_t = O[v_t;\Theta] + e_t, \quad E(e_t e_t^\top) = \mathcal{J}, \tag{2.24}$$

where y_t denotes the out-of-the-money option prices computed from the implied volatility quotes, scaled by Black-Scholes vega of the option at time t, $O[v_t;\Theta]$ denotes the corresponding model-implied values as a function of the parameter set Θ and the state vector v_t. We assume that the scaled pricing errors are i.i.d. normal with zero mean and constant variance. Hence, we can write the covariance matrix as, $\mathcal{J} = \sigma_e I_m$, where the dimension ($m$) of the measurement equations corresponds to 72 option series for the full sample estimation and 48 option series for each of the three subsample estimations.

When the state propagation and measurement equations are Gaussian linear, the Kalman filter provides efficient forecasts and updates on the mean and covariance of the state vector and observations. In our application, the state propagation equation is Gaussian linear, but the measurement equations are nonlinear functions of the state vector. We use an extended version of the Kalman filter, the unscented Kalman filter, to handle the nonlinearity of the measurement equation. Specifically, let $\overline{v}_t, \overline{P}_t, \overline{y}_t, \overline{V}_t$ denote the time-$(t-1)$ ex ante forecasts of time-t values of the state vector, the covariance of the state vector, the measurement series, and the covariance of the measurement series, respectively. Let $\widehat{v}_t$ and $\widehat{P}_t$ denote the ex post update, or filtering, on the state vector and its covariance at the time t based on observations (y_t) at time t. The unscented Kalman filter uses a set of deterministically chosen (sigma) points to approximate the state distribution. If we let d denote the number of states ($d = 5$ in our application) and let $\zeta > 0$ denote a control parameter, we generate a set of $2d+1$ sigma vectors χ_i according to the following equations,

$$
\begin{aligned}
\chi_{t,0} &= \widehat{v}_t, \\
\chi_{t,i} &= \widehat{v}_t \pm \sqrt{(d+\zeta)(\widehat{P}_t + \mathcal{G}_t)_j}, \quad j = 1,\ldots,d; \quad i = 1,\ldots,2d,
\end{aligned}
$$

with the corresponding weights w_i given by,

$$
w_0 = \zeta/(d+\zeta), \quad w_i = 1/[2(d+\zeta)], \quad i = 1,\ldots,2d.
$$

These sigma vectors form a discrete distribution with w_i being the corresponding probabilities. We can verify that the mean, covariance, skewness, and kurtosis of this distribution are $\widehat{v}_t$, $\widehat{P}_t + \mathcal{G}_t$, 0, and $d+\zeta$, respectively. Given the sigma points, the prediction steps are given by:

$$
\begin{aligned}
\overline{\chi}_{t,i} &= A + \Phi\chi_{t,i}; \\
\overline{v}_{t+1} &= \sum_{i=0}^{2d} w_i \overline{\chi}_{t,i}; \\
\overline{P}_{t+1} &= \sum_{i=0}^{2d} w_i (\overline{\chi}_{t,i} - \overline{v}_{t+1})(\overline{\chi}_{t,i} - \overline{v}_{t+1})^\top; \\
\overline{y}_{t+1} &= \sum_{i=0}^{2d} w_i O\left[\overline{\chi}_{t,i};\Theta\right]; \\
\overline{V}_{t+1} &= \sum_{i=0}^{2d} w_i \left(O\left[\overline{\chi}_{t,i};\Theta\right] - \overline{y}_{t+1}\right)\left(O\left[\overline{\chi}_{t,i};\Theta\right] - \overline{y}_{t+1}\right)^\top + \mathcal{J},
\end{aligned}
\tag{2.25}
$$

and the filtering updates are given by

$$\begin{aligned} \widehat{v}_{t+1} &= \bar{v}_{t+1} + \mathcal{K}_{t+1}\left(y_{t+1} - \bar{y}_{t+1}\right); \\ \widehat{P}_{t+1} &= \overline{P}_{t+1} - \mathcal{K}_{t+1}\overline{V}_{t+1}\mathcal{K}_{t+1}^{\top}, \end{aligned} \tag{2.26}$$

with

$$\mathcal{K}_{t+1} = \left[\sum_{i=0}^{2d} w_i \left(\overline{\chi}_{t,i} - \bar{v}_{t+1}\right)\left(O\left[\overline{\chi}_{t,i};\Theta\right] - \bar{y}_{t+1}\right)^{\top}\right]\left(\overline{V}_{t+1}\right)^{-1}. \tag{2.27}$$

We refer the reader to Wan and van der Merwe (2001) for general treatments of the unscented Kalman filter.

Given the forecasted option prices $\bar{y}$ and their conditional covariance matrix $\overline{V}$ obtained from the filtering technique, we compute the log likelihood value for each week's observations on the option prices assuming normally distributed forecasting errors,

$$l_{t+1}[\Theta] = -\frac{1}{2}\log\left|\overline{V}_{t+1}\right| - \frac{1}{2}\left(\left(y_{t+1} - \bar{y}_{t+1}\right)^{\top}\left(\overline{V}_{t+1}\right)^{-1}\left(y_{t+1} - \bar{y}_{t+1}\right)\right). \tag{2.28}$$

Then, we choose model parameters to maximize the aggregate log likelihood values:

$$\Theta \equiv \arg\max_{\Theta} \mathcal{L}[\Theta, \{y_t\}_{t=1}^{N}], \quad \text{with} \quad \mathcal{L}[\Theta, \{y_t\}_{t=1}^{N}] = \sum_{t=0}^{N-1} l_{t+1}[\Theta], \tag{2.29}$$

where $N = 209$ denotes the number of weeks in our sample. The model has 17 parameters: $\Theta \equiv [\gamma^{EU}, \gamma^{UK}, \gamma^{JP}, \kappa^g, \theta^g, \omega^g, \rho^g, \xi^{EU}, \xi^{UK}, \xi^{JP}, \kappa^h, \theta^h, \omega^h, \rho^h, \lambda, \beta, \sigma_e]$. We normalize the loading coefficients on the US to unity: $\gamma^{US} = \xi^{US} = 1$. Altogether, 15,048 option quotes are used in the full sample estimation and 10,032 option quotes are used in each of the three subsample estimations.

2.5. Empirical Results

Through model estimation, we can identify the stochastic discount factors for the four economies, from which we can construct exchange rate dynamics between any two economies. In this section, we first assess the model's in-sample and out-of-sample performance. Then, we discuss the stochastic discount factor dynamics implied by the structural parameter estimates.

2.5.1. In-sample performance

Table 2.3 reports the in-sample root mean squared pricing error (rmse) for the full sample estimation and the three subsample estimations, respectively. The pricing error is defined as the difference in implied volatilities between the market quotes and the model-generated values. The average root mean squared pricing errors range from 0.36 to 0.48 volatility point across different estimations. Comparing the performance across different maturities, we find that the model performs better on long-term options than on short-term options. The larger pricing errors for short-term options are consistent with their wider bid-ask spread observed in the market. For instance, the average bid-ask spreads for EURJPY and GBPJPY with maturity of one week are about 1.20 and 2.10 percentage points, respectively. But the bid-ask spreads on longer maturity options are normally within one percentage point.

Another way to investigate the model performance is to check where there are the remaining structures in the pricing errors. The mean pricing error of a good model should be close to zero and show no obvious structures along both the moneyness and the maturity dimensions. Figure 2.4 plots the mean pricing error in volatility points along the moneyness dimension at three selected maturities at one month (solid lines), three months (dashed lines), and 12 months (dash-dotted lines). Each panel represents on currency pair. Since in-sample performances for the three subsample estimations are similar to that for the full sample estimation, we only report results on the full-sample estimation to save space. For all currency pairs, the mean pricing errors are well within half a percentage point, the average bid-ask spread for the implied volatility quotes. The mean pricing errors do not show discernable systematic structures along either the moneyness or maturity dimension.

Figure 2.5 plots the mean absolute pricing error on the six currency pairs. The mean absolute pricing errors are also small, and do not show any systematic patterns. Therefore, overall our estimated model generates good pricing performance on options across all six currency pairs.

2.5.2. Out-of-sample performance

For out-of-sample performance, we analyze how the model estimated using options on four currency pairs performs in pricing options on the other two currency pairs that are excluded from the model estimation. We consider two possible scenarios. In the first scenario, we do not have historical options quotes on the two cross rates and hence we need to exclude them from the model estimation. Nevertheless, we start to have quotes on them now. Therefore, although we cannot use their historical quotes for model estimation, we can use their current quotes, together with quotes on the other four currency pairs, to extract the variance rates using the unscented Kalman filter. In the second scenario, options quotes on the two cross rates are not available. Hence, we need to both estimate the model parameters and extract the variance rates only using options on the four currency pairs.

Table 2.4 computes the out-of-sample root mean squared pricing error for options on the two cross rates based on the first scenario. Although options on the two cross rates are excluded from model estimation in each subsample, they are included in extracting the variance rates. Compared with the corresponding in-sample performance reported in the first panel of Table 2.3, the out-of-sample pricing errors are slightly larger, with average root mean squared pricing errors ranging from 0.60 to 0.69 volatility point. The pricing errors at very short and long maturity are larger than those at moderate maturities.

Figures 2.6 reports the out-of-sample mean pricing errors along the moneyness dimension at selected maturities of one (solid lines), three (dashed lines), and 12 (dash-dotted lines) months. The mean pricing errors are mostly within half a percentage point, the average bid-ask spread for liquid currency options. Again, we observe no obvious structures from the pricing error plots.

Figure 2.7 plots the out-of-sample mean absolute pricing errors along the moneyness dimension at selected maturities of one (solid lines), three (dashed lines), and 12 (dash-dotted lines) months. The mean absolute pricing errors are all within one percentage point, with no obvious structures.

To test the statistical significance of the difference between in-sample and out-of-sample pricing performances, we adopt the likelihood ratio statistic constructed by Vuong (1989). The test statistic is initially developed to test the null hypothesis that two non-nested models are equivalent in terms of likelihood. We can think of the statistic as a test of whether the out-of-sample likelihoods for options on the the two excluded cross rates are the same as their corresponding in-sample likelihoods. Formally, we let $LR(\Theta_{Full}, \Theta_{Sub})$ denote the log likelihood ratio between the likelihoods of the options on the two excluded cross rates computed from the full-sample estimation and the subsample estimation, respectively,

$$LR(\Theta_{Full}, \Theta_{Sub}) \equiv \mathcal{L}(\Theta_{Full}) - \mathcal{L}(\Theta_{Sub}), \tag{2.30}$$

Vuong constructs a test statistic based on this log likelihood ratio,

$$\mathcal{R} = LR(\Theta_{Full}, \Theta_{Sub}) / (\widehat{s}\sqrt{N}), \tag{2.31}$$

where N denotes the number of weeks in the time series and $\widehat{s}$ denotes the standard deviation estimate of the weekly log likelihood ratio $(l_{Full} - l_{Sub})$. Vuong proves that $\mathcal{R}$ is asymptotically normally distributed $N(0,1)$. Based on the weekly log likelihood estimates, we compute the sample mean and standard deviation of the likelihood ratio and then construct the test statistic in equation (2.31). In estimating $\widehat{s}$, we adjust serial dependence in the weekly log likelihood ratios based on Newey and West (1987) with the lags optimally chosen following Andrews (1991) under an AR(1) specification.

Table 2.4 reports the likelihood ratio test statistic $\mathcal{R}$ in the first row of each panel. The statistic for the first subsample where cross-rates involving Japanese Yen are excluded is about 1.38, which is smaller than 1.96, the critical value at 95% level. On the other hand, the statistics for the other two subsamples are both larger than 1.96. Therefore, when the two excluded cross-rates are used to update the variance rates, the null hypothesis of similar in-sample and out-of-sample performance cannot be rejected for the first subsample, but the in-sample and out-of-sample performances for the other two subsamples are significantly different.

Table 2.5 reports the out-of-sample root mean squared pricing errors based on the second scenarios, where options on the two cross rates are excluded both from model estimation and from variance rate extraction. Since the options information are excluded completely, the out-of-sample pricing errors become larger than the corresponding in-sample performance reported in the first panel of Table 2.3 and also larger than the out-of-sample errors computed based on the first scenario in Table 2.4. The average root mean squared pricing errors are similar for the first two subsamples at about 1.11 volatility points. The average root mean squared pricing error is larger at about 1.46 volatility points for the third subsample, where cross-rates involving euro are excluded.

Figure 2.8 plots the out-of-sample mean pricing error along the moneyness dimension at selected maturities of one (solid lines), three (dashed lines), and 12 (dash-dotted lines) months. The mean pricing errors are mostly within one percentage point, the average bid-ask spread for illiquid cross-rate options. The mean pricing errors also show no obvious remaining structure. Figure 2.9 plots the corresponding out-of-sample mean absolute pricing error. Here, we observe that short-term options generate larger out-of-sample pricing errors than long-term options, showing that short-term options exhibit more supply-demand idiosyncratic movements that cannot be as readily inferred from other option price quotes.

When we perform the likelihood ratio test, the results on $\mathcal{R}$-statistics in Table 2.5 show that, when the two excluded cross-rates are not used in either parameter estimation or state updates, the null hypothesis of similar in-sample and out-of-sample performance are rejected for all three subsamples. Therefore, options on the cross rates contain useful information and should be used for both model estimation and states extraction when they are available. Neverthless, when they are not available, our proposed pricing method generates satisfactory out-of-sample pricing performance.

2.5.3. Stochastic discount factor dynamics

Table 2.6 reports parameter estimates and their absolute magnitudes of the t-statistics (in parentheses) for different sample estimations. These structural parameters determine the stochastic discount factor dynamics in the four economies. We report parameter estimates both from the full-sample estimation and from the subsample estimations. Nevertheless, since the full sample is by construction more symmetric, we focus our discussion about parameter estimates from the full sample estimation.

The estimates on the loading coefficients γ^h and ξ^h capture the average scale difference of the global and country-specific risks across the four economies. For identification reasons, we normalize the loading to the US economy to one. Thus, deviations from one on the coefficient estimates capture their relative difference in risk levels from the US economy. Their corresponding t-statistics are calculated against the null hypothesis of one.

The γ^h coefficients capture the loading of the global risk factor on each economy. All the estimates are larger than one, with the values being 1.660, 1.396, and 1.275 for the Euro area, the UK, and Japan, respectively. On the other hand, the ξ^h estimates capture the loading of the country-specific risk components on each economy. Different from their rankings on the global risk factor, the loading factor estimates on the country-specific risk components are 0.723, 1.382, and 1.908 in ascending order for the Euro area, the UK, and Japan respectively. Taken together, the different coefficient estimates suggest that compared to the US economy, the Euro area contains a larger proportion of global risk components and a smaller proportion of country-specific risk components. In contrast, the Japanese economy contains the largest portion of the country-specific risk component and a smaller proportion of the global risk component. The loading coefficients on the UK are relative symmetric, 1.396 on the global risk and 1.382 on the country-specific risk, indicating that the UK and the US share similar risk structures, but the risk (or risk premium) level in the UK is higher.

The parameter κ measures the mean-reversion speed of the underlying variance rate. The estimate for the country-specific variance rate κ^h is 1.775, corresponding to a weekly autocorrelation of 0.9661, and a half life of about half an year. In contrast, the mean-reversion speed estimate for the global variance rate κ^g is insignificantly different from zero, implying near non-stationary behaviors. These estimates suggest that the global variance rate is much more persistent than the country-specific variance rates. The difference in persistence implies that it is more difficult to predict changes in global risk than to predict changes in country-specific risk.

The correlation parameters ρ^g and ρ^h capture how the variance rates change following global and country-specific shocks, respectively. The correlation estimates for both return components ρ^g and ρ^h are strongly negative. The negative correlations imply that the variance rates increase when the economy receives a negative shock.

The coefficient λ controls the overall arrival rate of jumps. The larger λ is, the more frequent jumps arrive. The dampening coefficient β control the arrival rate of large negative jumps. A larger dampening coefficient β implies a smaller arrival rate of large negative jumps. Nevertheless, the estimates for the two parameters have low t-values, indicating that out-of-money options up to 25-delta are probably not deep enough to accurately identify the tail behaviors of currency returns.

2.6. Conclusion

We study the pricing of illiquid cross exchange rate options by identifying stochastic discount factors embedded in currency triangles. We develop dynamic models of stochastic discount factors, under which the stochastic discount factor in each economy is decomposed into a global diffusion risk component and a country-specific jump-diffusion risk component. Separate stochastic time changes are further applied to the two components so that stochastic volatilities can come separately from both global and country-specific risks. We propose to identify the stochastic discount factors using options on actively traded currency pairs that include a currency triangle. Then, using the estimated model, we can price options on the less actively quoted cross currency pairs.

We illustrate our method by applying the model to six currency pairs that span four economies (US, Euro area, UK, and Japan). The six currency pairs include three U.S. dollar-linked primary exchange rates (euro-dollar, pound-dollar, and dollar-yen), and three cross exchange rates that do not include the dollar (euro-yen, euro-pound, and pound-yen). For out-of-sample analysis, we partition the six currency pairs into three subsamples, each of which includes the three dollar-linked primary rates and one of the three cross rates. Thus, in each subsample, the four currency pairs contain a currency triangle and an additional primary exchange rate. Through the currency triangle, we can identify both the global and the country-specific risks for the three economies. With the additional primary exchange rate, we can also identify the country-specific risk dynamics for the fourth economy. Then, we can use the estimated model to price options on the remaining two cross rates that are excluded from the model estimation. Our estimation results show that the out-of-sample performance is satisfactory. The root mean squared errors on the excluded options average from 1.11 to 1.46 volatility points.

Table 2.1
Summary statistics of currency options quotes
Entries in the five panels report, respectively, the mean, standard deviation, skewness, excess kurtosis, and weekly autocorrelation of the straddle (ATMV), risk reversal (RR25), and strangle (SG25) quotes. Data are weekly from October 3, 2001 to September 28, 2005, 209 observations for each series.

Contract	ATMV						RR25			SG25		
Maturity	1w	1m	3m	6m	12m	2y	1m	3m	12m	1m	3m	12m
						Mean						
EURUSD	9.72	9.77	10.00	10.22	10.35	10.38	0.34	0.42	0.49	0.21	0.23	0.25
GBPUSD	8.22	8.25	8.44	8.62	8.73	8.88	0.12	0.15	0.18	0.18	0.20	0.21
USDJPY	9.64	9.44	9.34	9.33	9.33	9.30	-0.68	-0.88	-1.11	0.30	0.33	0.37
EURGBP	6.53	6.53	6.69	6.86	6.96	6.88	0.20	0.19	0.17	0.17	0.18	0.20
EURJPY	9.37	9.37	9.46	9.60	9.68	9.71	-0.35	-0.38	-0.42	0.27	0.29	0.32
GBPJPY	9.26	9.26	9.36	9.50	9.58	9.62	-0.40	-0.44	-0.47	0.27	0.29	0.32
						Standard Deviation						
EURUSD	1.49	1.15	0.91	0.81	0.75	0.71	0.41	0.37	0.32	0.04	0.04	0.04
GBPUSD	1.54	1.22	0.96	0.79	0.71	0.67	0.31	0.25	0.22	0.03	0.02	0.02
USDJPY	1.59	1.13	0.84	0.76	0.75	0.76	0.64	0.72	0.92	0.06	0.06	0.06
EURGBP	1.27	1.00	0.82	0.78	0.76	0.79	0.21	0.16	0.16	0.03	0.03	0.04
EURJPY	1.74	1.34	1.07	1.00	1.00	1.00	0.55	0.58	0.62	0.05	0.05	0.06
GBPJPY	1.79	1.37	1.05	0.94	0.89	0.87	0.51	0.53	0.58	0.05	0.05	0.06
						Skewness						
EURUSD	0.40	0.19	0.02	0.05	0.21	0.27	0.07	-0.21	-0.27	-0.15	-0.21	0.03
GBPUSD	0.57	0.23	0.04	0.10	0.06	0.07	0.10	-0.00	-0.20	0.08	-0.07	0.16
USDJPY	0.88	0.53	0.34	0.55	0.89	1.01	-0.83	-0.18	-0.14	-0.12	-0.06	-0.16
EURGBP	0.72	0.41	0.11	0.21	0.35	0.03	0.17	0.16	0.09	0.51	0.52	0.14
EURJPY	0.95	0.79	0.74	0.82	0.97	1.00	-0.18	-0.11	-0.10	0.66	0.74	0.50
GBPJPY	0.95	0.74	0.49	0.43	0.57	0.67	-0.37	-0.14	-0.09	0.87	0.86	0.63
						Kurtosis						
EURUSD	-0.19	-0.59	-0.79	-0.76	-0.47	-0.36	-0.42	-0.37	-0.59	-0.93	-0.71	-0.39
GBPUSD	0.88	0.63	0.65	0.16	-0.26	-0.31	-0.48	-0.62	-0.67	-0.81	-0.75	-0.94
USDJPY	1.47	-0.33	-0.31	0.62	1.51	1.67	3.62	0.98	-0.37	-0.32	-0.43	-0.50
EURGBP	-0.10	-0.85	-1.17	-0.75	-0.34	-0.68	-0.16	-0.51	-0.32	-0.64	-0.62	-1.36
EURJPY	0.74	0.08	0.19	0.73	1.19	1.28	0.78	-0.28	-1.06	0.70	0.31	-0.25
GBPJPY	0.93	0.24	0.09	0.40	0.85	1.03	0.86	-0.27	-0.95	0.50	0.05	-0.34
						Weekly Autocorrelation						
EURUSD	0.70	0.87	0.90	0.91	0.92	0.92	0.87	0.92	0.95	0.96	0.97	0.96
GBPUSD	0.82	0.93	0.95	0.96	0.97	0.97	0.86	0.89	0.94	0.96	0.96	0.94
USDJPY	0.60	0.78	0.84	0.89	0.92	0.93	0.82	0.92	0.98	0.95	0.95	0.94
EURGBP	0.85	0.92	0.95	0.96	0.96	0.97	0.82	0.88	0.95	0.96	0.97	0.97
EURJPY	0.79	0.89	0.91	0.93	0.94	0.95	0.91	0.95	0.98	0.94	0.95	0.95
GBPJPY	0.82	0.90	0.92	0.93	0.94	0.94	0.89	0.94	0.98	0.93	0.94	0.93

Table 2.2
Sub-sample construction for out-of-sample performance analysis
This table shows how we construct subsamples to test the out-of-sample performance of the estimated model on currency pairs that are not included in the model estimation. The full sample contains options on three dollar-linked primary exchange rates and three cross exchange rates that do not involve the dollar. Each subsample includes a currency triangle plus another dollar-linked primary exchange rate that links to another (the fourth) currency. Options on the remaining two cross exchange rates are excluded from the model estimation and are used for out-of-sample performance analysis.

Sub-	In sample				Out of sample	
samples	Currency triangle			Primary rate	Excluded cross rates	
1	EURUSD	GBPUSD	EURGBP	USDJPY	EURJPY	GBPJPY
2	EURUSD	USDJPY	EURJPY	GBPUSD	EURGBP	GBPJPY
3	GBPUSD	USDJPY	GBPJPY	EURUSD	EURGBP	EURJPY

Table 2.3
In-Sample pricing performance
Entries report the in-sample root mean squared pricing error (rmse) for the full sample estimation and the three subsample estimations, respectively. For each estimation, we also report the grand average rmse. The pricing error is defined as the difference in implied volatilities between the market quotes and the model-generated values.

Contract	ATMV						25-delta call			25-delta put		
Maturity	1w	1m	3m	6m	12m	2y	1m	3m	12m	1m	3m	12m
	Full Sample: average rmse=0.48											
EURUSD	0.88	0.46	0.18	0.22	0.30	0.43	0.46	0.38	0.41	0.45	0.20	0.43
GBPUSD	0.75	0.41	0.25	0.29	0.31	0.35	0.35	0.21	0.29	0.42	0.36	0.42
USDJPY	1.04	0.58	0.31	0.32	0.41	0.62	0.35	0.44	0.82	0.79	0.55	0.44
EURGBP	0.73	0.46	0.29	0.35	0.39	0.62	0.44	0.38	0.43	0.54	0.41	0.45
EURJPY	1.30	0.85	0.39	0.22	0.34	0.56	0.55	0.26	0.60	0.98	0.59	0.32
GBPJPY	1.21	0.74	0.31	0.28	0.35	0.52	0.52	0.32	0.57	0.84	0.48	0.32
	Subsample 1: average rmse=0.36											
EURUSD	0.64	0.28	0.15	0.15	0.16	0.30	0.28	0.27	0.22	0.27	0.22	0.26
GBPUSD	0.56	0.26	0.14	0.18	0.22	0.30	0.32	0.18	0.21	0.19	0.25	0.28
USDJPY	1.05	0.59	0.23	0.21	0.35	0.54	0.33	0.45	0.82	0.81	0.56	0.48
EURGBP	0.61	0.31	0.14	0.34	0.43	0.61	0.20	0.24	0.44	0.43	0.26	0.42
	Subsample 2: average rmse=0.42											
EURUSD	0.66	0.31	0.16	0.12	0.14	0.33	0.33	0.33	0.22	0.34	0.23	0.22
GBPUSD	0.55	0.27	0.18	0.21	0.25	0.35	0.33	0.26	0.24	0.28	0.32	0.34
USDJPY	1.15	0.71	0.35	0.20	0.26	0.44	0.44	0.37	0.71	0.92	0.65	0.47
EURJPY	1.13	0.70	0.28	0.18	0.34	0.57	0.35	0.35	0.66	0.90	0.54	0.29
	Subsample 3: average rmse=0.39											
EURUSD	0.58	0.26	0.16	0.16	0.16	0.34	0.25	0.31	0.25	0.30	0.22	0.21
GBPUSD	0.49	0.22	0.15	0.16	0.21	0.33	0.27	0.24	0.22	0.20	0.20	0.22
USDJPY	1.12	0.68	0.30	0.19	0.29	0.47	0.40	0.34	0.73	0.86	0.58	0.46
GBPJPY	1.10	0.68	0.27	0.19	0.33	0.55	0.35	0.35	0.63	0.86	0.53	0.33

Table 2.4
Out-of-sample performance when options on the two cross rates are excluded from model estimation but included in state extraction
Entries report the out-of-sample root mean squared pricing error (rmse) for options on the two cross rates in each of the three subsamples. Options on these cross rates are excluded from model estimation but included in state variable extraction. The pricing error is defined as the difference in implied volatilities between the market quotes and the model-generated values. Each panel represents one subsample, for which we also report the grand average rmse and a likelihood ratio test statistic $\mathcal{R}$ against the null hypothesis of similar in-sample and out-of-sample performance.

Contract	ATMV						25-delta call			25-delta put		
Maturity	1w	1m	3m	6m	12m	2y	1m	3m	12m	1m	3m	12m
	Subsample 1: average rmse=0.67, $\mathcal{R} = 1.38$											
EURJPY	1.44	1.00	0.47	0.20	0.36	0.57	0.64	0.28	0.64	1.13	0.69	0.32
GBPJPY	1.34	0.84	0.33	0.48	0.72	0.79	0.57	0.35	0.87	0.96	0.54	0.61
	Subsample 2: average rmse=0.60, $\mathcal{R} = 2.94$											
EURGBP	0.67	0.42	0.29	0.38	0.64	1.19	0.45	0.31	0.60	0.52	0.45	0.77
GBPJPY	1.31	0.81	0.37	0.45	0.58	0.60	0.55	0.34	0.67	0.95	0.55	0.46
	Subsample 3: average rmse=0.69, $\mathcal{R} = 4.08$											
EURJPY	1.21	0.81	0.45	0.44	0.82	1.31	0.52	0.35	1.00	0.95	0.61	0.67
EURGBP	0.64	0.42	0.29	0.42	0.86	1.54	0.43	0.24	0.94	0.40	0.30	0.86

Table 2.5
Out-of-sample performance when options on the two cross rates are excluded from both model estimation and state extraction
Entries report the out-of-sample root mean squared pricing error (rmse) for options on the two cross rates in each of the three subsamples. Options on these cross rates are excluded from both model estimation and state variable extraction. The pricing error is defined as the difference in implied volatilities between the market quotes and the model-generated values. Each panel represents one subsample, for which we also report the grand average rmse and a likelihood ratio test statistic $\mathcal{R}$ against the null hypothesis of similar in-sample and out-of-sample performance.

Contract	ATMV						25-delta call			25-delta put		
Maturity	1w	1m	3m	6m	12m	2y	1m	3m	12m	1m	3m	12m
	Subsample 1: average rmse=1.11, $\mathcal{R} = 2.47$											
EURJPY	2.19	1.72	1.11	0.71	0.57	0.66	1.41	0.88	0.69	1.82	1.28	0.62
GBPJPY	1.95	1.41	0.88	0.77	0.85	0.86	1.14	0.73	0.94	1.56	1.05	0.79
	Subsample 2: average rmse=1.12,$\mathcal{R} = 4.88$											
EURGBP	1.97	1.72	1.34	1.06	0.92	1.24	1.70	1.32	0.90	1.72	1.38	1.01
GBPJPY	1.42	1.03	0.81	0.81	0.77	0.68	1.01	0.92	0.89	1.05	0.70	0.58
	Subsample 3: average rmse=1.46, $\mathcal{R} = 2.62$											
EURJPY	1.90	1.63	1.30	1.13	1.23	1.54	1.42	1.25	1.37	1.59	1.29	1.07
EURGBP	1.93	1.78	1.48	1.28	1.26	1.68	1.71	1.44	1.33	1.71	1.43	1.22

Table 2.6
Maximum likelihood estimates of model parameters
Entries report the maximum likelihood estimates of the structural parameters and the absolute magnitudes of the t-statistics (in parentheses). The t-statistics for loading coefficients γ and ξ are calculated against the null hypothesis of one, whereas those for other parameters are against the null hypothesis of zero. Estimation is on weekly currency options data from October 3, 2001 to September 28, 2005, 209 observations for each series.

Θ	Full sample		Subsamples excluding options on cross rates with					
			JPY		GBP		EUR	
Global risk								
γ^{EU}	1.660	(2.649)	1.838	(2.291)	2.442	(2.115)	1.407	(24.032)
γ^{UK}	1.396	(2.647)	1.569	(2.330)	1.925	(2.116)	1.341	(24.142)
γ^{JP}	1.275	(2.648)	1.334	(2.339)	1.656	(2.122)	1.173	(23.970)
κ^g	0.022	(0.956)	0.011	(0.257)	0.001	(0.038)	0.014	(0.485)
θ^g	0.088	(1.462)	0.008	(0.279)	0.000	(0.000)	0.030	(0.398)
ω^g	0.071	(2.604)	0.083	(2.241)	0.052	(2.166)	0.217	(8.791)
ρ^g	-0.583	(7.722)	-0.413	(5.212)	-0.355	(7.765)	-0.128	(5.701)
Country-specific risk								
ξ^{EU}	0.723	(6.121)	0.878	(7.985)	1.092	(4.680)	1.582	(16.427)
ξ^{UK}	1.382	(10.536)	0.893	(6.150)	1.155	(5.156)	1.237	(9.469)
ξ^{JP}	1.908	(20.748)	1.316	(9.882)	1.476	(15.759)	1.797	(19.798)
κ^h	1.775	(42.481)	2.427	(37.969)	2.132	(53.294)	2.331	(53.494)
θ^h	0.002	(19.159)	0.003	(21.256)	0.003	(23.304)	0.002	(19.787)
ω^h	0.188	(43.549)	0.268	(38.521)	0.250	(41.682)	0.193	(39.759)
ρ^h	-0.315	(34.183)	-0.428	(31.939)	-0.408	(48.800)	-0.380	(38.894)
λ	0.009	(0.112)	0.007	(0.133)	0.003	(0.067)	0.009	(0.154)
β	2.929	(0.116)	1.623	(0.196)	2.215	(0.072)	2.764	(0.170)

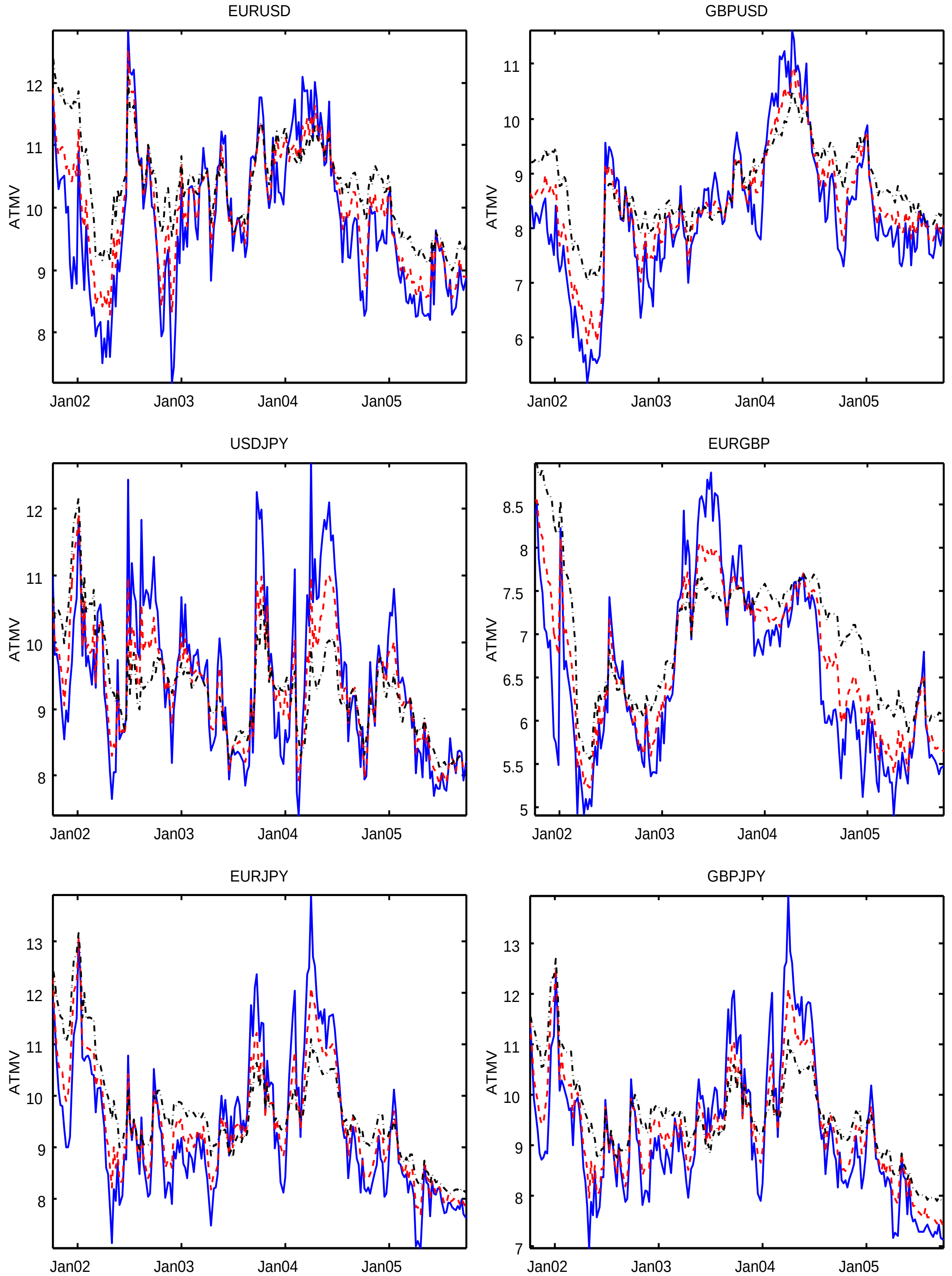

Fig. 2.1. **Delta-neutral straddle implied volatilities over calendar time.** Lines denote the time series of the delta-neutral straddle implied volatility quotes at three selected maturities: one month (solid lines), three months (dashed lines), and 12 months (dash-dotted lines). Each panel represents one currency pair. Data are weekly from October 3, 2001 to September 28, 2005, 209 weekly observations for each series.

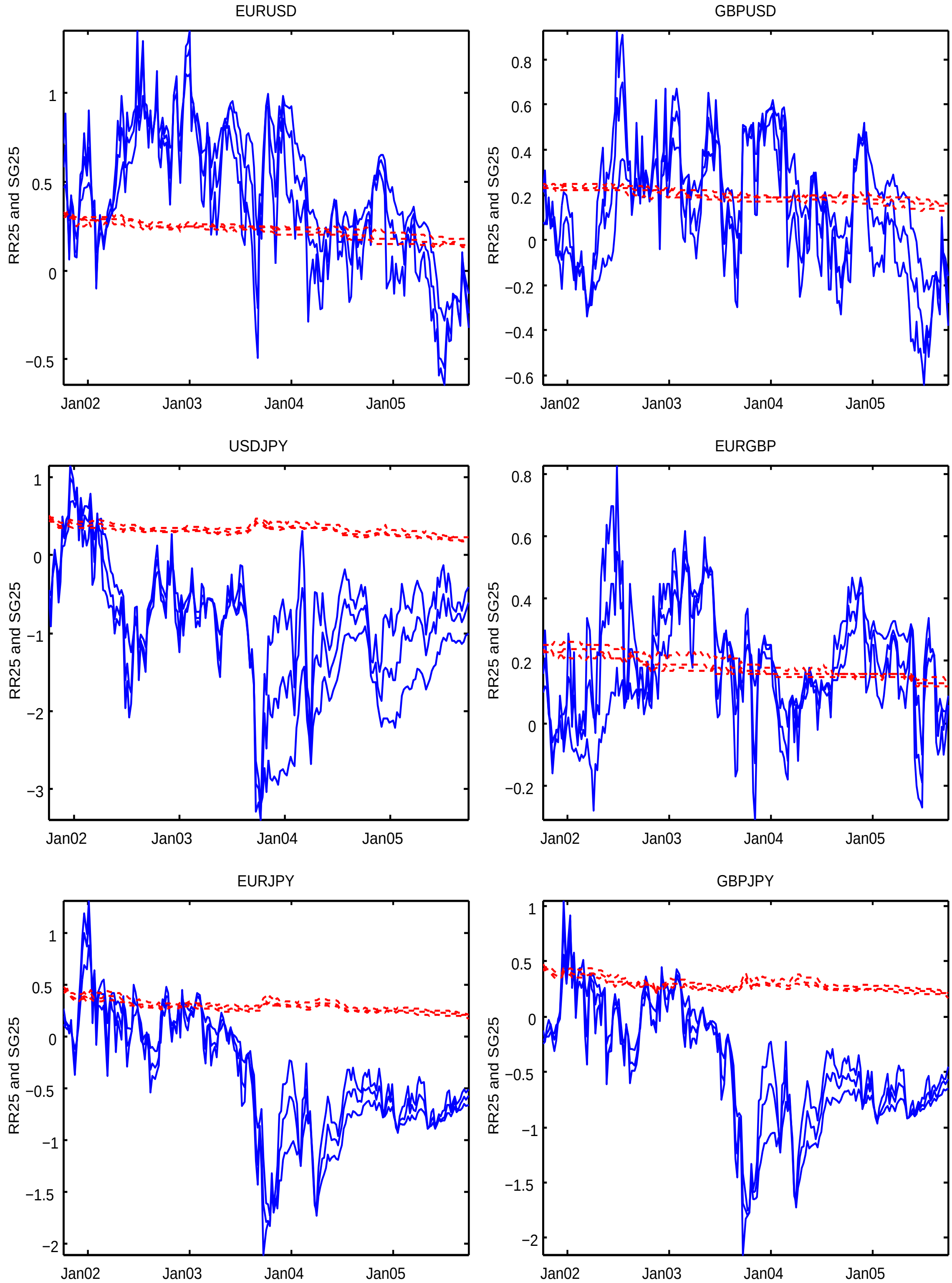

Fig. 2.2. **Risk reversals and strangles over calendar time.** Lines denote the time series of 25-delta risk reversals (solid lines) and 25-delta strangles (dashed line), both with maturities of one, three, and twelve months, respectively. Each panel represents one currency pair. Data are weekly from October 3, 2001 to September 28, 2005, 209 weekly observations for each series.

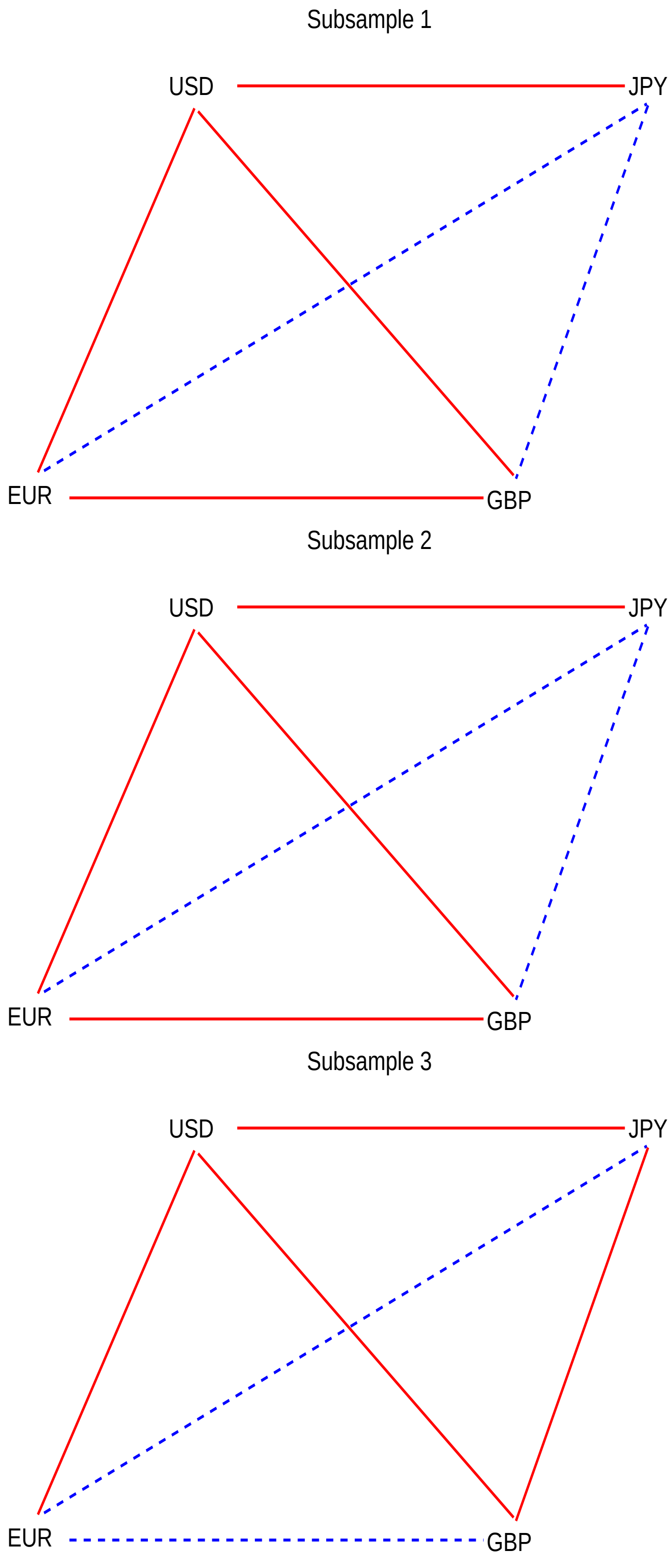

Fig. 2.3. **Illustration of the subsample construction.** Currency pairs linked with solid lines are included in model estimation. Currency pairs linked with dashed lines are excluded from model estimation and are used for out-of-sample analysis.

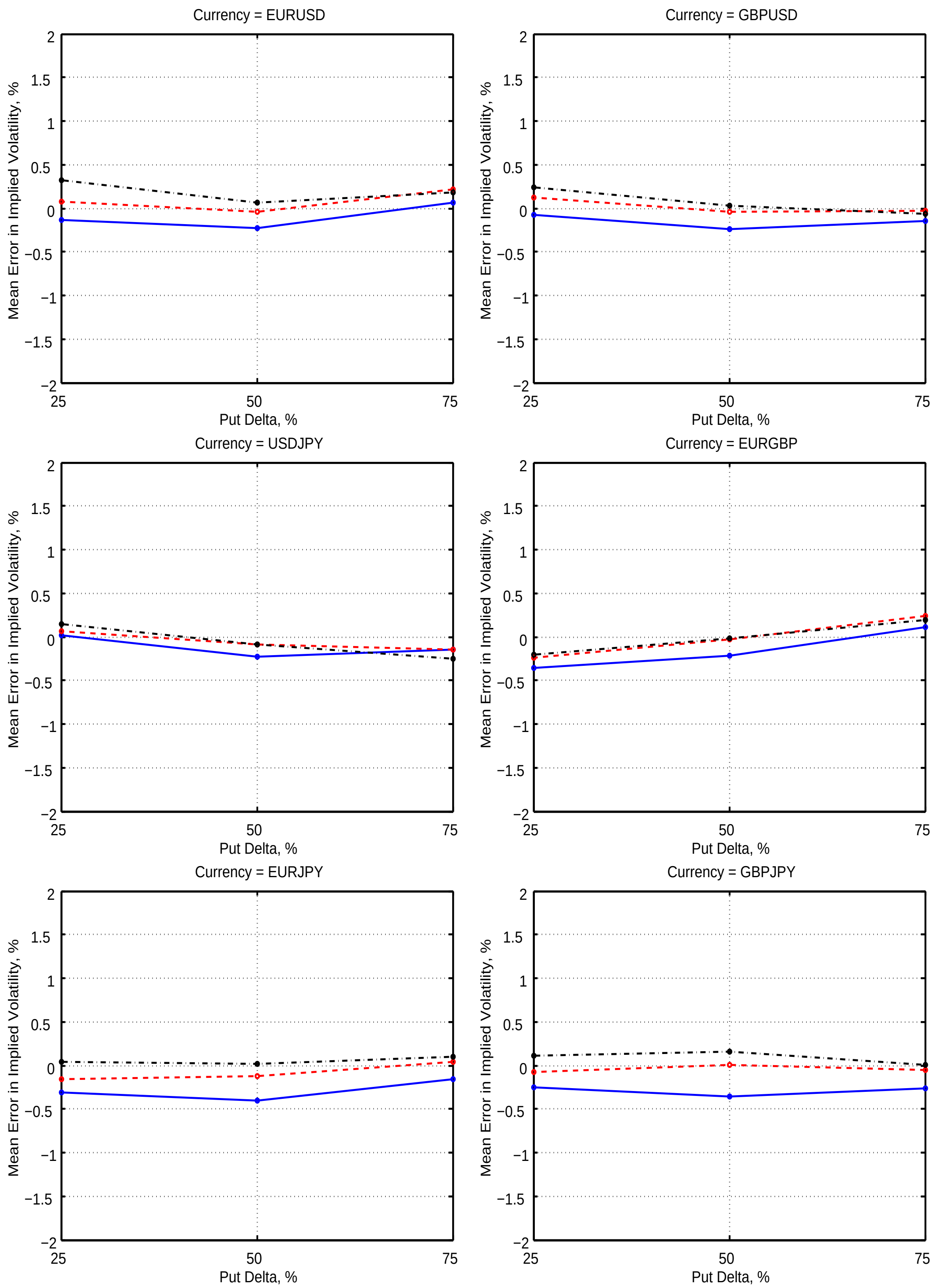

Fig. 2.4. **In-sample mean pricing error in implied volatility.** The pricing error is defined as the difference in implied volatilities between the market quotes and the model-generated values from the full sample estimation. We compute the mean pricing error at each moneyness and maturity. The three lines represent three chosen maturities at one (solid lines), three (dashed lines), and 12 (dash-dotted lines) months.

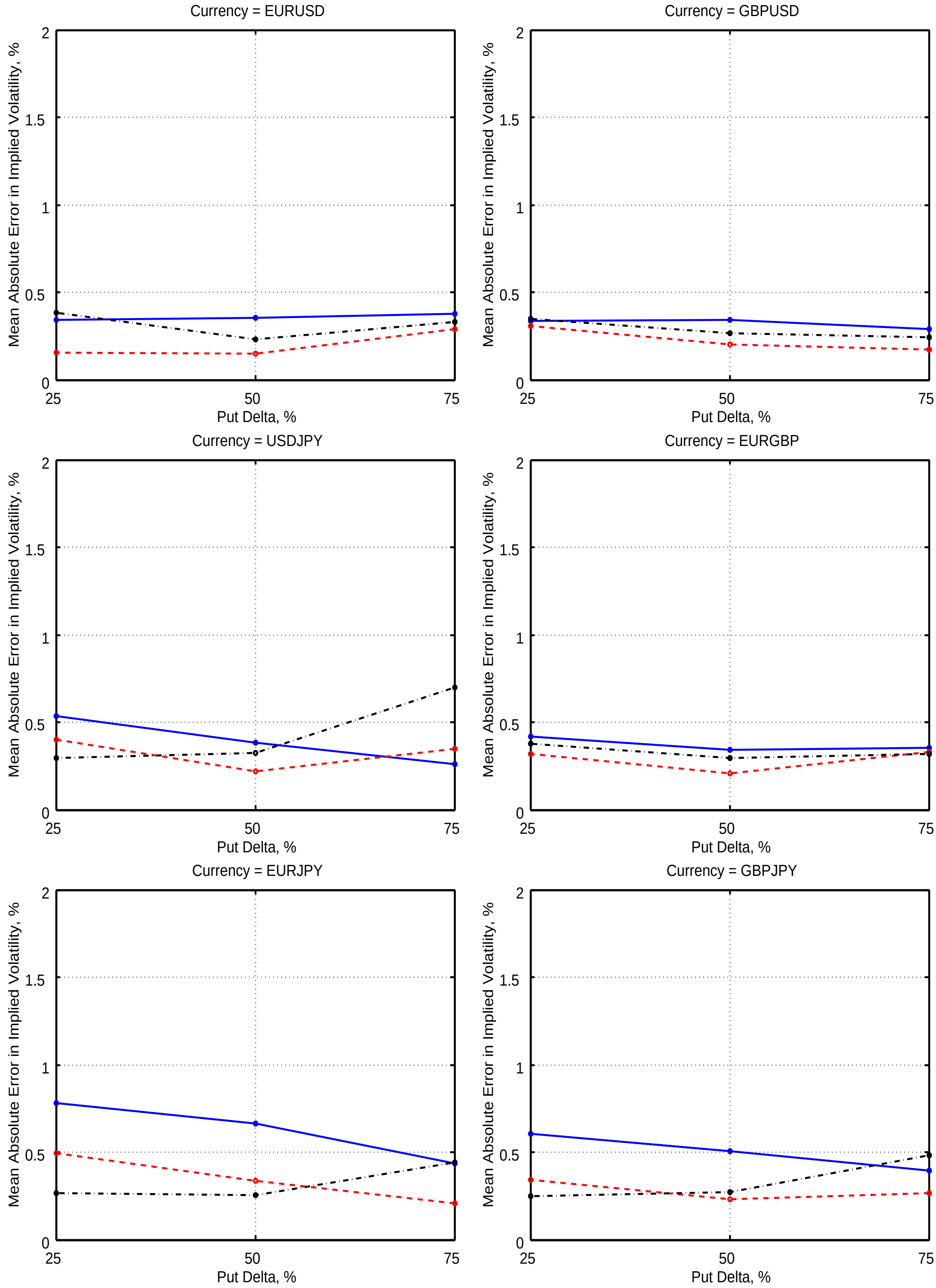

Fig. 2.5. **In-sample mean absolute pricing error in implied volatility.** The pricing error is defined as the difference in implied volatilities between the market quotes and the model-generated values from the full sample estimation. We compute the mean absolute value of the pricing error at each moneyness and maturity. The three lines represent three chosen maturities at one (solid lines), three (dashed lines), and 12(dash-dotted lines) months.

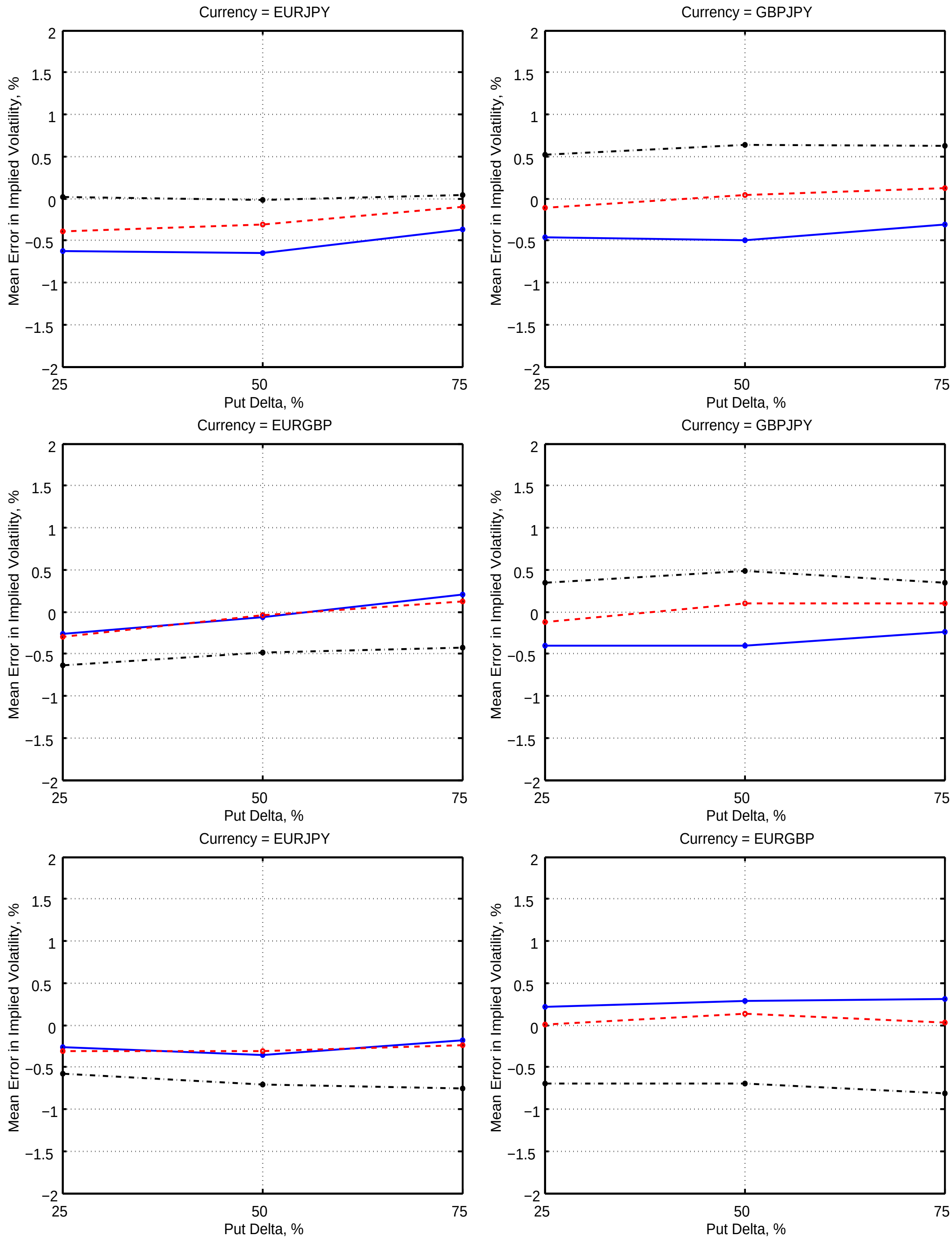

Fig. 2.6. **Out-of-sample mean pricing error when options on the two cross rates are excluded from model estimation but included in state extraction.** The pricing error is defined as the difference in implied volatilities between the market quotes and the model-implied values, which are computed based on models estimated excluding options on the two cross rates. Nevertheless, these options are included in extracting the variance rates. We compute the mean pricing error at each moneyness and maturity. The three lines represent three chosen maturities at one (solid lines), three (dashed lines), and 12 (dash-dotted lines) months.

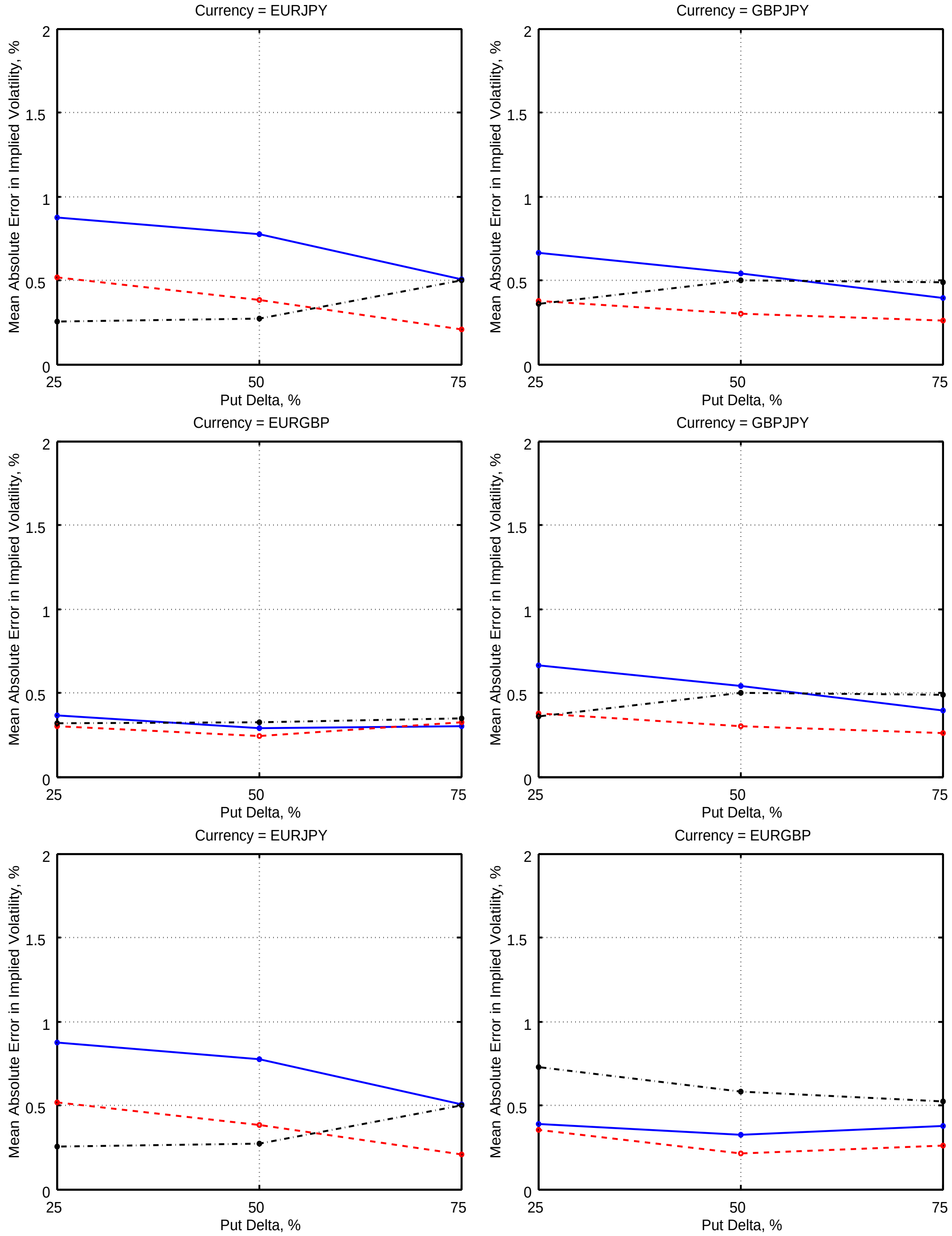

Fig. 2.7. **Out-of-sample mean absolute pricing error on the two cross rates are excluded from model estimation but included in state extraction.** The pricing error is defined as the difference in implied volatilities between the market quotes and the model-implied values, which are computed based on models estimated excluding options on the two cross rates. Nevertheless, these options are included in extracting the variance rates. We compute the mean absolute value of the pricing error at each moneyness and maturity. The three lines represent three chosen maturities at one (solid lines), three (dashed lines), and twelve (dash-dotted lines) months.

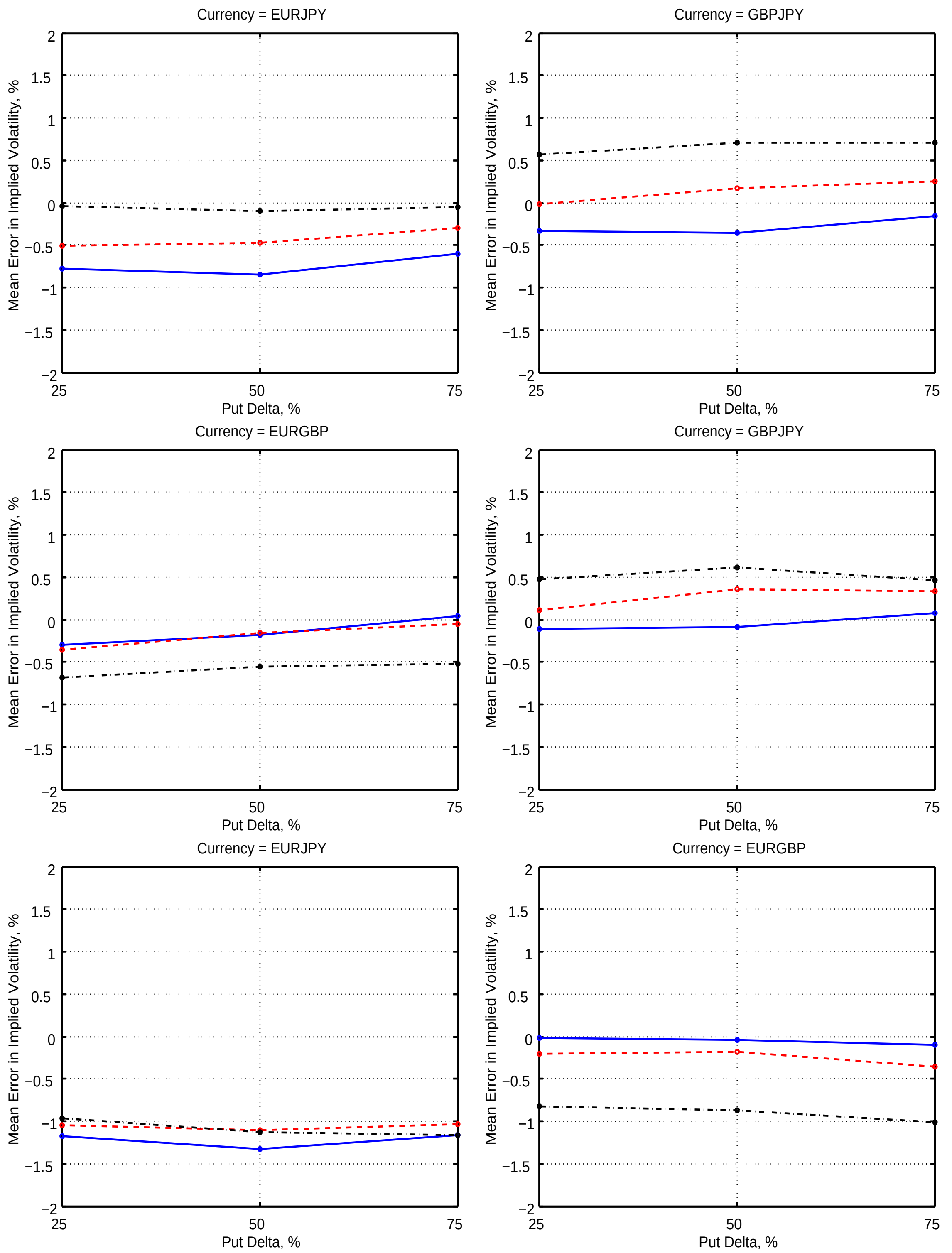

Fig. 2.8. **Out-of-sample mean pricing error when options on the two cross rates are excluded from both model estimation and state extraction.** The pricing error is defined as the difference in implied volatilities between the market quotes and the model-implied values, where the model is estimated and states are extracted without using options on the two cross rates. We compute the mean pricing error at each moneyness and maturity. The three lines represent three chosen maturities at one (solid lines), three (dashed lines), and 12 (dash-dotted lines) months.

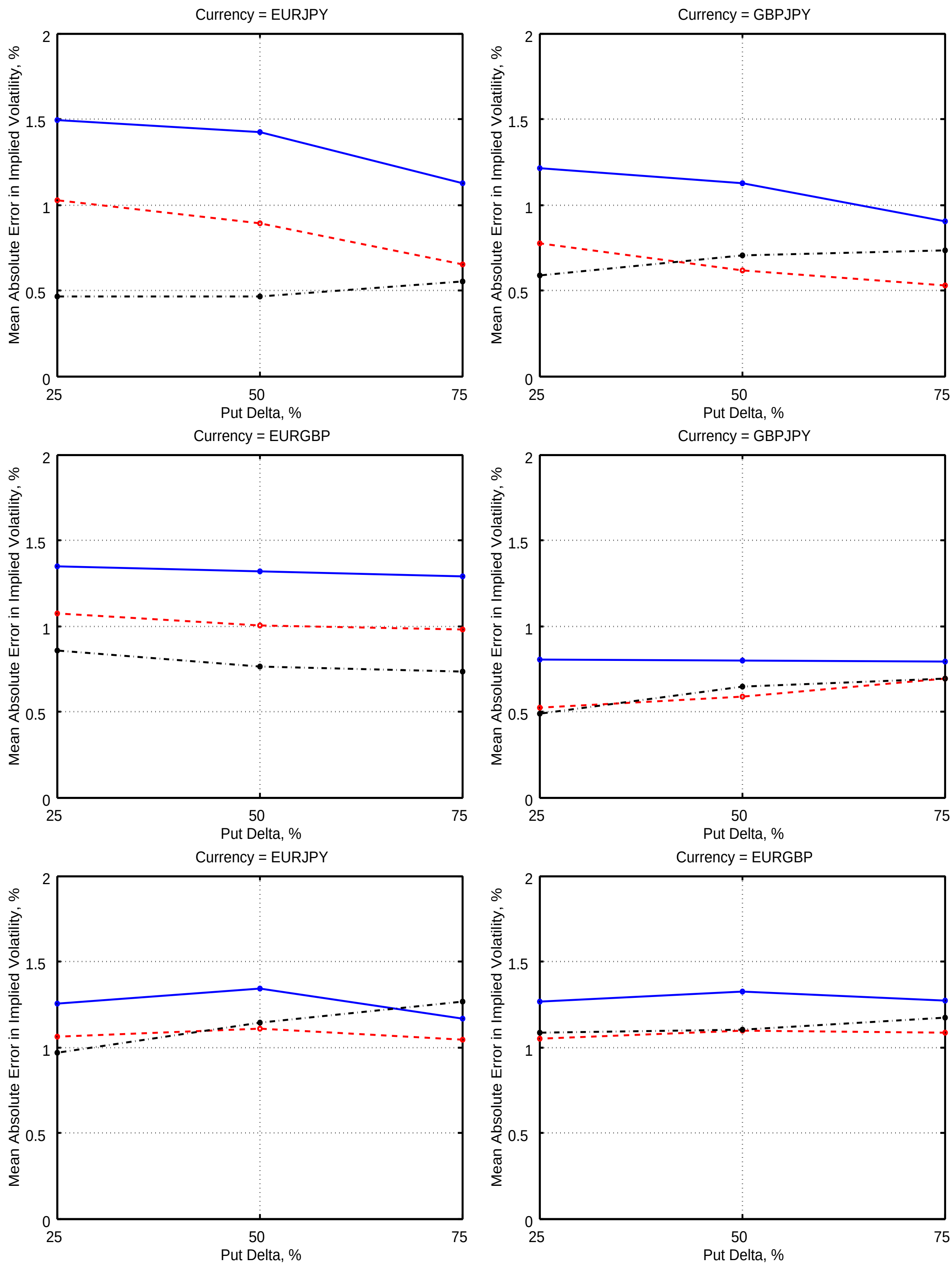

Fig. 2.9. **Out-of-sample mean absolute pricing error when options on the two cross rates are excluded from both model estimation and state extraction.** The pricing error is defined as the difference in implied volatilities between the market quotes and the model-implied values, where the model is estimated and states are extracted without using options on the two cross rates. We compute the mean absolute value of the pricing error at each moneyness and maturity. The three lines represent three chosen maturities at one (solid lines), three (dashed lines), and 12 (dash-dotted lines) months.

Bibliography

Adler, M., Dumas, B., 1983. International portfolio choice and corporation finance: A synthesis. Journal of Finance 38, 925–984.

Almeida, A., Goodhart, C., Payne, R., 1998. The effects of macroeconomic news on high frequency exchange rate behavior. Journal of Financial and Quantitative Analysis 33, 383–408.

Andersen, T., Bollerslev, T., Diebold, F., Vega, C., 2003. Micro effects of macro announcements: Real-time price discovery in foreign exchange. American Economic Review 93, 38–62.

Andersen, T., Bollerslev, T., Diebold, F., Vega, C., 2005. Real-time price discovery in stock, bond and foreign exchange markets. NBER Working Paper # 11312 .

Andrews, D., 1991. Heteroskedasticity and autocorrelation consistent covariance matrix estimation. Econometrica 59, 817–858.

Ang, A., Bekaert, G., 2002. International asset allocation under regime switching. Review of Financial Studies 15, 1137–1187.

Backus, D., Foresi, S., Telmer, C., 2001. Affine term structure models and the forward premium anomaly. Journal of Finance 56, 279–304.

Backus, D. K., Gregory, A. W., Telmer, C. I., 1993. Accounting for forward rates in markets for foreign currency. Journal of Finance 48, 1887–1908.

Bakshi, G., Cao, C., Chen, Z., 1997. Empirical performance of alternative option pricing models. Journal of Finance 52, 2003–2049.

Bakshi, G., Carr, P., Wu, L., 2005. Stochastic risk premiums, stochastic skewness in currency options, and stochastic discount factors in international economies. Working paper. University of Maryland, New York University, and Baruch College.

Bakshi, G., Kapadia, N., 2003. Delta-hedged gains and the negative market volatility risk premium. Review of Financial Studies 16, 527–566.

Bakshi, G., Kapadia, N., Madan, D., 2003. Stock return characteristics, skew laws, and the differential pricing of individual equity options. Review of Financial Studies 16, 101–143.

Bakshi, G. S., Chen, Z., 1997. Equilibrium valuation of foreign exchange claims. Journal of Finance 52, 799–826.

Bali, T., Wu, L., 2005. The role of exchange rates in the intertemporal risk-return relation in international economies. Working paper. Baruch College.

Bank of England, 2004. Quarterly bulletin, February. London.

Bank of International Settlements, 2004. 74th annual report: 1 April 2003 - 31 March 2004. Basel, Switzerland.

Bank of International Settlements, 2005. Triennial central bank survey: Foreign exchange and derivatives market activity in 2004. Basel, Switzerland.

Bansal, R., 1997. An exploration of the forward premium puzzle in currency markets. Review of Financial Studies 10, 369–403.

Bansal, R., Dahlquist, M., 2000. The forward premium puzzle: Different tales from developed and emerging economies. Journal of International Economics 51, 115–144.

Basak, S., 1996. An intertemporal model of international capital market segmentation. Journal of Financial and Quantitative Analysis 31, 161–188.

Basak, S., Gallmeyer, M., 1999. Currency prices, the nominal exchange rate, and security prices in a two country dynamic monetary equilibrium. Mathematical Finance 9, 1–30.

Bates, D., 2000. Post-'87 crash fears in the S&P 500 futures option market. Journal of Econometrics 94, 181–238.

Beber, A., Brandt, M., 2005. The effect of macroeconomic news on beliefs and preferences: Evidence from the options market. Journal of Monetary Economics (forthcoming).

Bekaert, G., Harvey, C., 1995. Time-varying world market integration. Journal of Finance 50, 403–444.

Bekaert, G., Hodrick, R. J., 1992. Characterizing predictable components in excess returns on equity and foreign exchange markets. Journal of Finance 47, 467–509.

Bennett, M. N., Kennedy, J. E., 2004. Quanto pricing with copulas. Journal of Derivatives 12, 26–45.

Bikos, A., 2000. Bivariate FX PDFs: A stering ERI application. Working paper. Bank of England.

Black, F., 1976. Studies of stock price volatility changes. In: Proceedings of the 1976 American Statistical Association, Business and Economical Statistics Section American Statistical Association, Alexandria, VA.

Black, F., Scholes, M., 1973. The pricing of options and corporate liabilities. Journal of Political Economy 81, 637–654.

Bliss, R. R., Panigirtzoglou, N., 2004. Option-implied risk aversion estimates. Journal of Finance 59, 407–446.

Bondarenko, O., 2004. Market price of variance risk and performance of hedge funds. Working paper. University of Illinois at Chicago.

Brandt, M., Cochrane, J., Santa-Clara, P., 2006. International risk sharing is better than you think, or exchange rates are too smooth. Journal of Monetary Economics 53, 671–698.

Carr, P., Geman, H., Madan, D., Yor, M., 2002. The fine structure of asset returns: An empirical investigation. Journal of Business 75, 305–332.

Carr, P., Lee, R., 2003. At-the-money implied as a robust approximation of the volatility swap rate. Working paper. Bloomberg LP.

Carr, P., Madan, D., 1999. Option valuation using the fast Fourier transform. Journal of Computational Finance 2, 61–73.

Carr, P., Wu, L., 2004a. Time-changed Lévy processes and option pricing. Journal of Financial Economics 71, 113–141.

Carr, P., Wu, L., 2004b. Variance risk premia. Working paper. Bloomberg and Baruch College.

Carr, P., Wu, L., 2006. A tale of two indices. Journal of Derivatives 13, 13–29.

Carrieri, F., Errunza, V., Majerbi, B., 2004. Does emerging market exchange risk affect global equity prices?. Journal of Financial and Quantitative Analysis forthcoming.

Cavaglia, S., Hodrick, R., Vadim, M., Zhang, X., 2002. Pricing the global industry portfolios. NBER Working Paper 9344 Columbia University.

Chan, K. C., Karolyi, G. A., Stulz, R. M., 1992. Global financial markets and the risk premium on U.S. equity. Journal of Financial Economics 32, 137–167.

Claessens, S., Forbes, K. J., 2002. International Financial Contagion. Kluwer Academic Publishers, Boston.

Connor, G., Korajczyk, R. A., 1993. A test for the number of factors in an approximate factor model. Journal of Finance 48, 1263–1291.

Das, S. R., Uppal, R., 2004. Systemic risks and international portofio choice. Journal of Finance 59, 2809–2834.

De Santis, G., Gerard, B., 1997. International asset pricing and portfolio diversification with time-varying risk. Journal of Finance 52, 1881–1912.

De Santis, G., Gerard, B., 1998. How big is the premium for currency risk?. Journal of Financial Economics 49, 375–412.

Driessen, J., Maenhout, P., 2004. The world price of jump and volatiltiy risk. Working paper. University of Amsterdam and INSEAD.

Duffie, D., 1992. Dynamic Asset Pricing Theory. Princeton University Press, Princeton, New Jersey.

Dumas, B., 1992. Dynamic equilibrium and the real exchange rate in a spatially separated world. Review of Financial Studies 5, 153–180.

Dumas, B., Solnik, B., 1995. The world price of foreign exchange risk. Journal of Finance 50, 445–479.

Engel, C., 1996. The forward discount anomaly and the risk premium: A survey of recent evidence. Journal of Empirical Finance 3, 123–191.

Engle, R. F., Rosenberg, J. V., 2002. Empirical pricing kernels. Journal of Financial Economics 64, 341–372.

Eraker, B., 2004. Do stock prices and volatility jump? Reconciling evidence from spot and option prices. Journal of Finance 59, 1367–1404.

Errunza, V., Losq, E., 1985. International asset pricing under mild segmentation: Theory and test. Journal of Finance 40, 105–124.

Fama, E. F., 1984. Forward and spot exchange rates. Journal of Monetary Economics 14, 319–338.

Fama, E. F., French, K. R., 1998. Value versus growth: The international evidence. Journal of Finance 53, 1975–1999.

Ferson, W. E., Harvey, C. R., 1993. The risk and predictability of international equity returns. Review of Financial Studies 6, 527–566.

Grauer, F. L. A., Litzenberger, R. H., Stehle, R., 1976. Sharing rules and equilibrium in an international capital market under uncertainty. Journal of Financial Economics 3, 233–256.

Greene, W. H., 1999. Econometric Analysis. Prentice Hall, New York.

Harrison, J. M., Kreps, D. M., 1979. Martingales and arbitrage in multiperiod securities markets. Journal of Economic Theory 20, 381–408.

Harvey, C. R., 1991. The world price of covariance risk. Journal of Finance 46, 111–157.

Hodrick, R., 1987. The Empirical Evidence on the Efficiency of Forward and Futures Foreign Exchange Markets. Harwood Academic Publishers, New York.

Hsieh, D. A., 1984. Tests of rational expectations and no risk premium in forward exchange markets. Journal of International Economics 17, 173–184.

International Monetary Fund, 2002. Global financial stability report: Market developments and issues, September. Washington DC.

Jackwerth, J. C., Rubinstein, M., 1996. Recovering probability distributions from contemporary security prices. Journal of Finance 51, 347–369.

Karolyi, G. A., Stulz, R. M., 2003. Are financial assets priced locally or globally?. In: Constantinides, G. M., Harris, M., Stulz, R. M. (Eds.), Handbook of the Economics of Finance. North Holland, Amsterdam, Netherlands.

Kou, S. G., 2002. A jump-diffusion model for option pricing. Management Science 48, 1086–1101.

Longin, F., Solnik, B., 2001. Extreme correlation of international equity markets. Journal of Finance 56, 649–676.

Lucas, R. E., 1982. Interest rates and currency prices in a two-country world. Journal of Monetary Economics 10, 335–359.

Mishkin, F. S., 1984. Are real interest rates equal across countries? an empirical investigation of international parity conditions. Journal of Finance 39, 1345–1357.

Mylonas, P., Schich, S., 1999. The use of financial market indicators by monetary authorities. Working paper. Organisation for Economic Co-operation and Development, Paris.

Newey, W. K., West, K. D., 1987. A simple, positive semi-definite, heteroskedasticity and autocorrelation consistent covariance matrix. Econometrica 55, 703–708.

Pan, J., 2002. The jump-risk premia implicit in options: Evidence from an integrated time-series study. Journal of Financial Economics 63, 3–50.

Pasquariello, P., Vega, C., 2005. Informed and strategic order flow in the bond markets. Working paper. University of Michigan and University of Rochester.

Polimenis, V., 2006. Skewness correction for asset pricing. Working paper. University of California at Riverside.

Rogoff, K., 1996. The purchasing power parity puzzle. Journal of Economic Literature 34, 647–668.

Rosenberg, J. V., 2003. Non-parametric pricing of multivariate contingent claims. Journal of Derivatives 10, 9–26.

Sarkissian, S., 2003. Incomplete consumption risk sharing and currency risk premiums. Review of Financial Studies 16, 983–1005.

Solnik, B. H., 1974. An equilibrium model of the international captial market. Journal of Economic Theory 8, 500–524.

Subrahmanyam, M. G., 1975. On the optimality of international capital market integration. Journal of Financial Economics 2, 3–28.

Taylor, S. J., Wang, Y., 2005. Option prices and risk-neutral densities for currency cross-rates. Working paper. Lancaster University.

Vuong, Q. H., 1989. Likelihood ratio tests for model selection and non-nested hypotheses. Econometrica 57, 307–333.

Wan, E. A., van der Merwe, R., 2001. The unscented Kalman filter. In: Haykin, S. (Eds.), Kalman Filtering and Neural Networks. Wiley & Sons Publishing, New York.

Wu, L., 2006a. Dampened power law: Reconciling the tail behavior of financial asset returns. Journal of Business 79, 1445–1474.

Wu, L., 2006b. Modeling financial security returns using lévy processes. In: Birge, J., Linetsky, V. (Eds.), Handbook of Financial Engineering. Elsevier.

Zhang, X., 2006. Specification tests of international asset pricing models. Journal of International Money and Finance 25, 275–307.