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Jang, Deogjin, Ph.D.

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U·M·I
300 N. Zeeb Rd.
Ann Arbor, MI 48106

FORMAL EDUCATION AND ECONOMIC GROWTH

by

DEOGJIN JANG

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4/22/93 — Salih Neftci

Date — Chair of Examining Committee

4/22/93 — Michael Grossman

Date — Executive Officer

Professor Salih Neftci

Professor Michael Grossman

Professor Mitchell Kellman

Supervisory Committee

THE CITY UNIVERSITY OF NEW YORK

Abstract

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Deogjin Jang

Adviser: Professor Salih Neftci

Roles of formal education on the long-run growth rate are examined. Individuals specialize in accumulating human capital for multiple time periods in childhood and produce output as well as help their children learn knowledge in adulthood. It is shown, first, that constant returns to scale and human capital of adults as an input in the educational production function are necessary and sufficient conditions for the economy to grow positively in the steady state. That is, human capital of children doesn't have to be an input in the educational production function in order to show the growth of the economy in the steady state.

Second, if they face a probability of death in adulthood, individuals would emphasize current consumption relative to future consumptions, which may imply that the economy invests less in the output sector and thus grows at a slower rate.

Third, if children's learning is not affected by their previous learning, the steady-state growth rate falls as periods of schooling increase. When students' learning only depends on the efforts of adults independent of their previous amount of learning, the increase in periods of schooling would just scale down the productive parameter of the educational production function through adult-student ratio, which has the same effect as once and for all decrease in the level of technology in the education sector.

once and for all decrease in the level of technology in the education sector.

Fourth, if the educational production function depends on children's as well as adults' human capital, and the share of children's human capital in the production function is sufficiently high but less than one, the growth rate of the economy might be positively related to periods of schooling.

Those findings may imply that, without knowing the true educational production function, the effect of periods of schooling on the growth rate is inconclusive: it depends on the functional form of the schooling technology.

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Chapter I. Introduction

According to Denison(1961, 1979, and 1985), over the period 1929-1982, the amount of education in nonresidential sector rose 0.63 percent per annum on potential basis and 0.65 percent per annum on actual basis and education contributed 16 percent of the long-term growth of total potential output and 14 percent of total actual output. Based on Denison's estimate on education sector, Lucas(1988) estimated that about 64 percent of the growth of human capital could be explained by formal education. This may be downward biased in that Denison's education index only measures the level of human capital accumulated due to the increase in the quantity of education. Denison (1979, p. 45) acknowledges

> "The education index takes account only of the quantitative aspects of formal education. ... If changes occurred in the 'quality' of education, other than by extension of the school year and improved attendance, this is not reflected."

Recently some researchers confirmed that the quality of school was a factor which improved student learning. Finn and Achilles(1990) conducted a large-scale experiment for kindergarten and 1st year students in Tennessee. They found that reductions in the class-size significantly increase test scores on reading and math exams.

Card and Krueger(1992) found that school quality measured by pupil/teacher ratio, term length, and relative teacher pay had a significant effect on returns to education. They found that rates of return are higher for individuals who attended schools with lower pupil-teacher ratio and higher relative teacher salaries. Their findings are robust in the sense that the introduction of a wide variety of other variables does not change their basic results.

Hanushek, Gomes-Nato, and Harbison(1992) find that, among other things, teacher quality enhances student learning after examining Brazilian rural data. Given these empirical findings on micro data, it is interesting to examine whether schooling is a factor for the economy to grow.

Even though there are many endogenous growth models published recently, most of them are unable to explain the peculiar behavior of a schooling sector, which requires multiple time periods to complete. Rather they are capturing the effect of on-the-job training on the growth of the economy. For example, Lucas(1988), Rebelo(1991) and Tamura(1991) assume that individuals can accumulate their own human capital throughout their life-time and they decide, at each period of time, to choose how much time to spend on producing output and the remaining time on accumulating their own human capital.

Having the neoclassical setup in the accumulation of human capital, Ben-Porath(1967) identified two meaningful phases in life-time. The first phase is the period which people allocate all their available time to accumulate human capital. So people don't work in the output sector in the first phase. The second phase is the period which people divide their time to both sectors. Wallace and Ihnen(1975) calculated the period of schooling, the period of the first phase, of the Ben-Porath model with an additional assumption that education loans are not available. The maximum length of schooling in their model was approximately 6.3 years given a set of structural parameters.

Becker, Murphy, and Tamura(1990) discussed the role of formal education in the aggregate economy in the life-cycle context with endogenous fertility. They introduced the altruism of parents toward their children running as follows: parents maximize a dynastic utility function with respect to fertility and time spent on investing in human capital and producing output. They also assumed perfect substitutability of human capital of children and their teacher-parents and two periods of life-time, childhood and adulthood. They were able to explain the existence of multiple steady states, which are locally stable, and the requirement of sufficiently big favorable productivity or other shocks for the transition of the economy to the higher steady state.

Stokey(1991) analyzed the effect of formal education on the growth of the economy using optimal control theory. In her paper, people live one unit of time with certainty and determine a period of schooling endogenously so that the present discounted value of life-time earning is maximized. She assumes that the effectiveness of investment in human capital depends upon the stock of knowledge in society while the investment is undertaken. Using her notation, the human capital of an individual who is born at date t and spends $\beta(t)$ units of time on learning is $G(t)\phi[\beta(t)]$, where $G(t)$, $t>=0$, denotes the stock of knowledge of the society at date t, and $\phi:[0,1] \rightarrow \mathbb{R}$ is strictly increasing, strictly concave, and twice continuously differentiable with $\phi(0)=1$. Further, she assumes that the growth rate of the stock of knowledge, $G'(t)/G(t) = g[\beta(t\text{-}1)]$, $t>=1$, where $g(\cdot)$ is continuous and strictly increasing with $g(0)=0$, which means that the growth rate of knowledge at date t depends only on the investment decision of members of cohort t-1.

Let's take an example to see how knowledge evolves over time in Stokey(1991). Suppose that all individuals spend one half of their life on accumulating human capital and the other half on working in the output sector. Then an individual born at any time t will attend school until time $t+0.5$. As of time t, all cohorts who have been born since time t-0.5 are attending schools. Normalizing the size of each cohort to 1, total number of students as of time t is 0.5.

The stock of knowledge G(t) at time t is:

$$G(t) = G(0)\exp\left(\int_0^t g[\beta(s-1)]\,ds\right)$$

where

$G(t)$ is the stock of knowledge at time t in the economy

$G(0)$ is the initial stock of knowledge in the economy

$\beta(s\text{-}1)$ is the amount of time cohort s-1 spent on learning

Hence, the stock $G(t)$ of knowledge at time t does not depend on the learning efforts of cohorts born after time t-0.5, who are students at time t, meaning that knowledge of students is not included in the aggregation of knowledge of the society. Furthermore, this does not include the stock of knowledge of workers born between t-1 and t-0.5 even though they finished learning and are working in the output sector. Note that people live only one unit of time with certainty. Therefore, the stock of knowledge of all cohorts who are living currently is not included in the stock of knowledge of the society whether they are students or workers. The stock of knowledge of the society at time t is the aggregation of the stock of knowledge of cohorts who are already dead as of time t. If I consider knowledge as human capital embodied in people, it is very unlikely that the knowledge remains alive even after people who own the knowledge were dead.

One other point I would like to note on her paper is the learning technology. Her learning technology, $G(t)\phi[\beta(t)]$, means that the level of knowledge of individuals depends on the length of schooling, $\beta(t)$, and on the external effect, $G(t)$. It is independent of student's ability. That is, how much students know does not matter for how much they can learn. But I fully agree with Becker, Murphy, Tamura(1990)'s proposition that the benefit from embodying additional knowledge in a person may depend positively rather than negatively on the knowledge he or she already has. As a matter of fact, I show, in chapter III, that this property of an educational technology would be crucial for a theoretical model to be able to explain the effect of schooling on the growth rate of the economy which is consistent with the empirical findings: a positive relationship between the period of schooling and the growth rate of the economy.

In this paper, I discuss an endogenous growth model with a formal education as a means of accumulating human capital. I define childhood as periods individuals attend schools and adulthood as periods individuals work in the output sector. I assume, throughout the paper, that there is no accumulation of human capital in the output sector.[1] Therefore, the only way individuals can increase human capital is by attending schools. So the model in this paper differs from Lucas(1988) or Tamura(1991) in that I exclude the post-schooling accumulation of human capital and allow some members of household, children, to specialize in the accumulation of human capital.

In chapter II, I use a schooling technology which depends on the ability of children, a fraction of adults' time allocated to their children, and on externality of the average level of human capital of adults in a society. Also I assume that individuals live infinitely but face a constant probability of death throughout their adulthood. With this framework, I find that all we require for the economy to grow positively in the steady state is constant social returns to scale in the schooling technology, which is a well-known property in the literature. Then the steady state of the model is discussed.

In chapter III, I assume that individuals live infinitely and an educational production function exhibits constant returns to scale in human capital of children and adults. I find that the effect of years of schooling on the growth of the economy is inconclusive: if the schooling technology only depends on the efforts of adults, the growth rate of the economy is negatively related to the period of schooling while, if the technology depends on human capital of children as well as human capital of adults, the growth rate of the economy could be positively related to the period of schooling.

In chapter IV, empirical results are summarized. The long-run growth rates of

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[1]We can add on-the-job training sector in the model, which makes it a three-sector economy.

countries are regressed on variables measuring the quality of schooling as well as the quantity of schooling in addition to other variables. Chapter V concludes the paper.

Chapter II. A Model with Finitely-lived Agents

A. Specification of the Model

The economy consists of many identical households, each of which consists of a continuum of adults and children. At a point in time, new children are born in a household at a constant rate, p_b, defined as the ratio of new-born children over the total number of children in the household, and every child is endowed with the same level H_{c0} of human capital. Before working in the output sector, they go to schools, where they accumulate knowledge for multiple time periods s. When they finish schooling, they start to work in the output sector. It is assumed that individuals can not accumulate human capital in the output sector, which means there is not any kind of on-the-job training. Knowing that there is no opportunity to accumulate human capital while they work, adults who take care of their children's well-being might have their children educated because more educated people tend to receive higher income. Further, adults could be better off in the future as children with higher human capital start to work because children with higher human capital would contribute more to the household income. If the welfare of the household depends on household consumption, adults would be benefitted from their children once the children start to work.

Only adults face a constant probability of death p_d, independent of ages, which is a crucial assumption for the aggregation of human capital in the society.[2] Even though individuals are uncertain about their death, it is assumed that there is no uncertainty in the aggregate, that is, the constant number of adults die each period. If

[2]See Blanchard(1985).

the probability of death for adults, p_d, is equal to one, then it is an overlapping generation model while it is a Ramsey model when the probability of death is zero. For simplicity, I assume that the rate of birth is equal to the rate of death at p $(= p_b = p_d)$.[3] If we normalize the initial size of a cohort to p, then the size of the cohort declines over time and is equal to $p*\exp[-p(t-z)]$ for any time $t>=z$, where z is the time in which the cohort enters a labor market. Thus the size of adults at any time is equal to one.

Further, I assume that when p number of adults die in the household each period, the same number of the eldest children leave schools and start to work to support for their household so that the size of the children in the household is equal to one. Then the period s of schooling is equal to $1/p$.

At every point in time, the constant number p of children are born in the household and are endowed with the same level H_{c0} of human capital. New-born children attend schools to accumulate human capital. If we assume that leisure is not valued and either schooling is compulsory or the human capital which children accumulate in schools can not be used in the production of output until they finish schooling, which could be considered as time-to-build-like human capital, then children would allocate all their available time to learn knowledge in schools during childhood. After finishing schooling, children start to work in the output sector. Hence, the level of human capital of people is determined in their childhood.

It is assumed that, whether he or she is a child or an adult, everybody is endowed

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[3]I made this assumption so that population is constant over time. If the rate of birth differs from that of death, we have a model that population increases at a rate of (p_b-p_d) over time.

with one unit of time each period of time. Children spend the time on attending schools while adults spend a fraction u_t of their time on teaching their children and the other fraction $(1-u_t)$ on producing output.

Suppose that the amount of learning of a child at any time t depends on his or her ability to learn, a fraction u_t of time which adults devote to teach their children, and externally on the average level of the human capital of adults in a society. Specifically, the schooling technology is given by

$$\dot{H}_{ct} = \delta H_{ct}^{\alpha_1} (u_t H_{at})^{\alpha_2} \bar{H}_t^{\gamma} \qquad \text{(II-1)}$$

given that the endowment of human capital of a child is H_{c0}

where

H_{ct} is the stock of human capital of a child at time t

u_t is a fraction of time adults allocate to their children at time t

H_{at} is the per capita adult stock of human capital in the household at time t

$\bar{H}_t$ is the average stock of human capital of adults in the economy at time t

The technology implies that the amount of learning of a child depends on its level of knowledge H_{ct} and a fraction u_t (or $u_t H_{at}$ in effective unit) of time which adults allocate to the education sector:[4] the higher the ability of children (higher H_{ct}), the greater the amount of learning. If adults in the household have higher human capital than other households, the children in the household learn more knowledge than the children of the household with less human capital even though both households devote the same amount of labor time to teach their children. Furthermore, the amount of learning of a child depends externally on the average level of human capital of adults $\bar{H}_t$ in the society. Children will benefit more from the society if the average level of human capital of adults in a society is higher.

Note that physical capital is not in the production function of human capital. Even if I include physical capital in the human capital technology as an input, it wouldn't change fundamental results of the paper as long as human capital sector uses human capital of adults more intensively than the output sector does.

As I assumed above, adults can not accumulate their own human capital. Therefore, the only way to increase the average stock of the human capital of adults in a household is through the entrance of new work-force into a labor market. If the level of the human capital of new work-force at time t is higher than that of adults who die at time t, the level of the human capital of adults in the household increases, and thus the economy grows.

Eq. (II-1) shows how the human capital of each child evolves over time while

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[4] I assume that there is no joint production of the output and the education.

he/she attends schools. Solution of Eq. (II-1), which is a form of a Bernoulli differential equation, gives us the level of the human capital of a child who had been born at time *t-s* and finished *s* periods of schooling.[5] That is

$$H_{ct,t-s} = \left(H_{ct-s,t-s}^{1-\alpha_1} + (1-\alpha_1)\delta\int_{t-s}^{t}(u_z H_{az})^{\alpha_2}\bar{H}_z^{\gamma}\,dz\right)^{\frac{1}{1-\alpha_1}} \qquad \text{(II-2)}$$

where

$H_{ct,t-s}$ is the level of human capital of a child, as of time *t*, born at time *t-s*

$H_{ct-s,t-s}$ is the level of human capital of a child, as of time *t-s*, born at time *t-s*

Note that every child is endowed with the same level H_{c0} of human capital and attends schools for *s* periods. Therefore, $H_{ct-s,t-s}$ is equal to H_{c0} for every child born at any time. Letting $H_{ct,t-s}$ be H_{gt} to simplify the notation, we have

$$H_{gt} = \left(H_{c0}^{1-\alpha_1} + (1-\alpha_1)\delta\int_{t-s}^{t}(u_z H_{az})^{\alpha_2}\bar{H}_z^{\gamma}\,dz\right)^{\frac{1}{1-\alpha_1}} \qquad \text{(II-3)}$$

Eq. (II-3) is the level of human capital at any time *t* which children will attain at the end

[5] A differential equation is called a Bernoulli differential equation if it is the form $\dot{y}+P_t y_t=Q_t y_t^n$. A Bernoulli differential equation could be solved by the change of variables.

of schooling. Then the level of human capital of adults, which is the engine of growth in the paper, could be calculated as follows:

$$H_{at} = \int_{-\infty}^{t} H_{gz} p\, e^{-p(t-z)} dz \qquad \text{(II-4)}$$

where

H_{gz} is defined as Eq. (II-3).

H_{gz} also could be interpreted as the level of the human capital of adults who started to work at time z and p*exp[-p(t-z)] is the number of people in a household who became adults at time z and still are alive as of time t. Therefore, $H_{gz}p$*exp[-p(t-z)] is the amount of the human capital, as of time t, of the adults in the household who had started to work at time z and are still alive as of time t. Integrating it from $-\infty$ to t gives us the level of the human capital of adults in the household at time t.

Differentiation of Eq. (II-4) with respect to time t gives us a law of motion for the level of the human capital H_{at} of adults in the household. It is given by

$$\dot{H}_{at} = p(H_{gt} - H_{at}) \qquad \text{(II-5)}$$

The first term in the right hand side pH_{gt} is the amount of the knowledge added to the household due to the entrance of new work-force and the second term pH_{at} is the amount of the knowledge reduced from the household due to the death of adults. Note that the size of adults in the household is equal to one. Therefore, the equation implies that the average level of the human capital of adults in the household increases as long as the level of the human capital of new work-force at time t is greater than that of adults who die at time t. Human capital is assumed not to be depreciated.

Production of output depends on the level of physical capital K_t and the effective unit $(1-u_t)H_{at}$ of labor allocated to the output sector. Further, it is assumed that the production function exhibits constant returns to scale in physical capital K_t and the effective labor $(1-u_t)H_{at}$. Unlike the production function in the education sector, it is assumed that there is no externality in the production of output. At any moment of time, output produced is either consumed or invested in the form of physical capital. Physical capital is assumed to be never depreciated. Then assuming a Cobb-Douglas technology, a household income equation is

$$A K_t^{\beta} ((1-u_t)H_{at})^{1-\beta} = C_t + \dot{K}_t \qquad \text{(II-6)}$$

where

A is the productivity parameter in the output sector

K_t is the per capita adult level of physical capital at time t

H_{at} is the per capita adult level of human capital at time t

C_t is total consumption of the household at time t

It is assumed that there are no technology shocks in the economy so that A is constant over time. Note that C_t is total consumption of the household, not per capita consumption while output and physical capital are represented in terms of per capita adult(labor). The reason is that the size of a household is normalized to 2: half of them are children and the other half adults, and only adults work in the output sector while children as well as adults consume final output. If I define C_t as the per capita consumption, then $2C_t$ is used instead of C_t on the right hand side of Eq. (II-14) with a minor modification on the welfare function, which I describe later. But it would not change basic properties of the model. It is just a matter of scale.

In Becker, Murphy, and Tamura(1990), it is assumed that fixed amounts of hours and goods are required to rear each child. Instead of following their assumptions, I assume that the utility function of children is identical to that of adults. Therefore, in this paper, children and adults are identical agents except that children specialize in the accumulation of human capital while adults produce final output as well as help their children accumulate human capital. Specifically, the household maximizes the following welfare U_z of the household:[6]

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[6]Rebelo(1991) justifies use of these preferences in the endogenous growth model. If per capita consumption is used in Eq. (14), then the welfare of household is

$$U_z = E\int_z^{\infty} e^{-\varrho(t-z)} \frac{2C_t^{1-\sigma}}{1-\sigma} dt \quad .$$

$$U_z = E\int_z^{\infty} e^{-\varrho(t-z)} \frac{C_t^{1-\sigma}}{1-\sigma} dt \qquad \text{(II-7)}$$

An expectation operator is included in the equation because adults face uncertain lifetime. Therefore, adults in the household maximize the present discounted utilities getting from family consumptions during their expected life-time. Adults have the same expected life-time regardless of their ages. This is so because we assume that a probability of death is independent of ages. The utility function is a class of a constant elasticity of substitution where the elasticity is equal to $1/\sigma$. It reduces to a log utility function when $\sigma=1$.

Under the assumption that there is no other kind of uncertainty, we can convert Eq. (II-7) to the following:

$$U_z = \int_z^{\infty} e^{-(\varrho+p)(t-z)} \frac{C_t^{1-\sigma}}{1-\sigma} dt \qquad \text{(II-8)}$$

Note that a probability of death is added to the rate of time preference, which implies adults facing uncertain lifetime discount future utilities more heavily than otherwise. We may interpret Eq. (II-8) as the present discounted sum of future utilities of the household, where the effective discounted rate is the sum of the pure rate of time preference and the probability of death.

Denoting $\rho=\varrho+p$ to simplify notation, we have

$$U_z = \int_z^{\infty} e^{-\rho(t-z)} \frac{C_t^{1-\sigma}}{1-\sigma} dt \qquad \text{(II-9)}$$

B. Solution of the Economy

Therefore, the representative household maximizes

$$U_z = \int_z^{\infty} e^{-\rho(t-z)} \frac{C_t^{1-\sigma}}{1-\sigma} dt \qquad \text{(II-9)}$$

subject to

$$A K_t^{\beta} ((1-u_t)H_{at})^{1-\beta} = C_t + \dot{K}_t \qquad \text{(II-6)}$$

$$\dot{H}_{at} = p(H_{gt} - H_{at}) \qquad \text{(II-5)}$$

where

$$H_{gt} = \left(H_{c0}^{1-\alpha_1} + (1-\alpha_1)\delta \int_{t-s}^{t} (u_z H_{az})^{\alpha_2} \bar{H}_z^{\gamma} dz\right)^{\frac{1}{1-\alpha_1}} \qquad \text{(II-3)}$$

Because of the existence of externality in the educational technology, the solution for the perfectly competitive economy diverges from that of the planned or social economy, and the perfectly competitive solution is Pareto suboptimal compared to the planned economy solution. But the existence theorem of the perfectly competitive solution with externality has been well-known since the work of Romer(1986). The argument runs as follows: representative agents will solve the optimal control problem assuming that the path for the average stock of human capital $\bar{H}_t$ of a society is exogenously determined. If the solution path for the average stock of human capital H_{at} of adults in the household is equal to the exogenous path of $\bar{H}_t$, then the perfectly competitive solution exists. On the other hand, in the planned economy, the social planner will determine the average stock of human capital $\bar{H}_t$ of a society endogenously too.

Let's discuss the planned economy first. In the planned economy, the social planner will determine the paths for C_t, u_t, K_t, H_{at}, and $\bar{H}_t$ so that the representative household maximizes Eq. (II-9), subject to Eqs. (II-3), (II-5), and (II-6). The average level of human capital H_{at} of adults in the household and that $\bar{H}_t$ of a society are endogenous variables and are identical to each other because it is assumed that all households are identical in every respect. Hence, I am going to use H_{at} in place of $\bar{H}_t$ to get the solution for the planned economy.

The Hamiltonian is

$$\begin{aligned} \mathrm{H} = {} & \frac{C_t^{1-\sigma}}{1-\sigma} + \theta_{1t}\left(AK_t^{\beta}((1-u_t)H_{as})^{1-\beta} - C_t\right) \\ & + \theta_{2t}p\left(\left(H_{c0}^{1-\alpha_1} + (1-\alpha_1)\delta\int_{t-s}^{t}(u_z H_{az})^{\alpha_2} H_{az}^{\gamma}\,dz\right)^{\frac{1}{1-\alpha_1}} - H_{at}\right) \end{aligned} \qquad \text{(II-10)}$$

Note that C_t and u_t are control variables and K_t and H_{at} are state variables. Even though adults in the household can not accumulate their own human capital, they can help their children accumulate it so that the average level of human capital of adults in the household can be increased.

The first-order conditions for the problem are:

C_t:

$$C_t^{-\sigma} = \theta_{1t} \tag{II-11}$$

u_t:

$$\theta_{1t} A K_t^{\beta} H_{at}^{1-\beta}(1-\beta)(1-u_t)^{-\beta} = \theta_{2t} p \delta \alpha_2 u_t^{\alpha_2 - 1} H_{at}^{\alpha_2+\gamma}\left(H_{c0}^{1-\alpha_1} + (1-\alpha_1)\delta \int_{t-s}^{t} (u_z H_{az})^{\alpha_2} H_{az}^{\gamma}\, dz\right)^{\frac{\alpha_1}{1-\alpha_1}} \tag{II-12}$$

K_t:

$$\dot{\theta}_{1t} = \rho\theta_{1t} - \theta_{1t} A \beta K_t^{\beta-1}((1-u_t)H_{at})^{1-\beta} \tag{II-13}$$

H_{at}:

$$\dot{\theta}_{2t} = \rho\,\theta_{2t} - \theta_{1t} A K_t^{\beta}(1-\beta)(1-u_t)^{1-\beta} H_{at}^{-\beta} - \theta_{2t} p \left(\delta(\alpha_2+\gamma)u_t^{\alpha_2} H_{at}^{\alpha_2+\gamma-1}\left(H_{c0}^{1-\alpha_1} + (1-\alpha_1)\delta\int_{t-s}^{t}(u_z H_{az}^{\alpha_2+\gamma}\, dz\right)^{\frac{\alpha_1}{1-\alpha_1}} - 1\right) \tag{II-14}$$

Eqs. (II-11) - (II-14) and two transversality conditions for K_t and H_{at} are necessary and sufficient conditions for the problem. Eq. (II-11) implies that the marginal utility of consumption should be equal to the shadow price of output. Eq. (II-12) implies that the marginal product of time in physical unit should be the same in both sectors. Those two first order conditions describe the static optimality conditions. Eqs. (II-13) and (II-14) give you laws of motion for shadow prices for physical and human capital, respectively.

In order to get the rates of change for C_t, u_t, K_t, and H_{at} in the planned economy, let's eliminate θ_{1t} and θ_{2t} from the first-order conditions above. First, using Eqs. (II-11) and (II-13)

$$\rho + \sigma\frac{\dot{C}_t}{C_t} = A\,\beta K_t^{\beta-1}((1-u_t)H_{at})^{1-\beta} \qquad \text{(II-15)}$$

The right hand side of the equation is the marginal product of physical capital. So Eq. (II-15) implies that the return on physical capital should be equal to the sum of the effective discount rate and the growth rate of consumption divided by the intertemporal elasticity of substitution, and it is constant at the steady state. This implies that the real interest rate is constant over time at the steady state.

Taking the natural logarithm of Eq. (II-12), differentiating it with respect to t, and rearranging it, we have

$$\frac{\dot{\theta}_{2t}}{\theta_{2t}} = \frac{\dot{\theta}_{1t}}{\theta_{1t}} + \beta\frac{\dot{K}_t}{K_t} + (1-\beta-\alpha_2-\gamma)\frac{\dot{H}_{at}}{H_{at}} + \left(1-\alpha_2+\beta\frac{u_t}{1-u_t}\right)\frac{\dot{u}_t}{u_t} - \alpha_1\frac{\dot{H}_{gt}}{H_{gt}} \qquad \text{(II-16)}$$

where

$$\frac{\dot{H}_{gt}}{H_{gt}} = \frac{\delta\left((u_t H_{at})^{\alpha_2} H_{at}^{\gamma} - (u_{t-s} H_{at-s})^{\alpha_2} H_{at-s}^{\gamma}\right)}{H_{c0}^{1-\alpha_1} + (1-\alpha_1)\delta \int_{t-s}^{t} (u_z H_{az})^{\alpha_2} H_{az}^{\gamma}\, dz} \qquad \text{(II-17)}$$

Using Eqs. (II-11), (II-12), and (II-16), we can rewrite Eq. (II-14) as follows:

$$-\sigma\frac{\dot{C}_t}{C_t} + \beta\frac{\dot{K}_t}{K_t} + (1-\beta-\alpha_2-\gamma)\frac{\dot{H}_{at}}{H_{at}} + (1-\alpha_2+\beta\frac{u_t}{1-u_t})\frac{\dot{u}_t}{u_t} - \alpha_1\frac{\dot{H}_{gt}}{H_{gt}}$$

$$= \rho + p\left(1 - \delta u_t^{\alpha_2-1}(\alpha_2+\gamma u_t H_{at}^{\alpha_2+\gamma-1} H_{gt}^{\alpha_1}\right) \qquad \text{(II-18)}$$

In summary, laws of motion for the planned economy are Eqs. (II-3), (II-5), (II-6), (II-15), and (II-18).

On the other hand, the solution for the perfectly competitive equilibrium diverges from the social optimum because of externality in the educational production function. The representative household maximizes the following Hamiltonian, taking $\bar{H}_t$ exogenously.

$$H = \frac{C_t^{1-\sigma}}{1-\sigma} + \theta_{1t}\left(A K_t^{\beta}((1-u_t)H_{as})^{1-\beta} - C_t\right)$$

$$+ \theta_{2t} p\left(\left(H_{c0}^{1-\alpha_1} + (1-\alpha_1)\delta\int_{t-s}^{t}(u_z H_{az})^{\alpha_2}\bar{H}_z^{\gamma}\,dz\right)^{\frac{1}{1-\alpha_1}} - H_{at}\right) \qquad \text{(II-10')}$$

In this case, the first order conditions for the perfect competition are the same as those of the planned economy except that Eq. (II-14) changes to the following:

$$\dot{\theta}_{2t} = \rho\theta_{2t} - \theta_{1t} A K_t^{\beta}(1-\beta)(1-u_t)^{1-\beta}$$

$$- \theta_{2t} p\left(\delta\alpha_2 u_t^{\alpha_2} H_{at}^{\alpha_2+\gamma-1}\left(H_{c0}^{1-\alpha_1} + (1-\alpha_1)\delta\int_{t-s}^{t}(u_z H_{az})^{\alpha_2} H_{az}^{\gamma}\,dz\right)^{\frac{\alpha_1}{1-\alpha_1}} - 1\right) \qquad \text{(II-14')}$$

Then Eq. (II-18) becomes

$$-\sigma\frac{\dot{C}_t}{C_t} + \beta\frac{\dot{K}_t}{K_t} + (1-\beta-\alpha_2-\gamma)\frac{\dot{H}_{at}}{H_{at}} + (1-\alpha_2+\beta\frac{u_t}{1-u_t})\frac{\dot{u}_t}{u_t} - \alpha_1\frac{\dot{H}_{gt}}{H_{gt}} \qquad \text{(II-18')}$$

$$= \rho + p\left(1 - \delta\alpha_2 u_t^{\alpha_2-1} H_{at}^{\alpha_2+\gamma-1} H_{gt}^{\alpha_1}\right)$$

Therefore, laws of motion for the competitive economy consist of Eqs. (II-3), (II-5), (II-6), (II-15) and (II-18').

C. Steady State Analysis

First of all, I discuss the steady-state solution for the planned economy. Suppose that the steady state of the economy exists such that u_t is constant over time and C_t, K_t, H_{at}, and H_{gt} grow at a constant rate. Taking the natural logarithm of Eq. (II-15) and differentiating it with respect to t, it is shown that the growth rate of H_{at} is the same as that of K_t along the steady state path. Then we can prove, from Eq. (II-5) and Eq. (II-6) respectively, that the growth rate of C_t also equals that of K_t, and H_{at} grows at the same rate as H_{gt}. Therefore, H_{gt}, H_{at}, C_t, and K_t grow at the same constant rate if the steady state exists.

In order to see whether the steady state exists, let's work on Eq. (II-3). Suppose that the growth rate of H_{at} is constant at λ along the steady state path. Taking the natural logarithm of Eq. (II-3) and differentiating it with respect to t, we get

$$\frac{\dot{H}_{gt}}{H_{gt}} = \frac{\delta u^{\alpha_2}\left(1 - e^{-\lambda s(\alpha_2+\gamma)}\right)}{\dfrac{H_{c0}^{1-\alpha_1}}{H_{at}^{\alpha_2+\gamma}} + (1-\alpha_1)\delta\lambda^{-1}(\alpha_2+\gamma)^{-1}u^{\alpha_2}\left(1 - e^{-\lambda s(\alpha_2+\gamma)}\right)} \qquad \text{(II-19)}$$

Assuming the initial endowment H_{c0} of human capital of children is trivial relative to H_{at}, then we have:

$$\frac{\dot{H}_{gt}}{H_{gt}} = \left(\frac{\alpha_2+\gamma}{1-\alpha_1}\right)\lambda \tag{II-20}$$

Therefore, as long as $\alpha_1+\alpha_2+\gamma=1$ and $\alpha_1 \neq 1$, the growth rate of H_{gt} is equal to that of H_{at}, which may imply that the steady state with a positive growth rate exists. The former condition implies that the production function in the education sector should be constant returns to scale, and the second condition implies that the marginal product of human capital of children in their learning diminishes ($\alpha_1 < 1$) or the educational production function is independent of human capital of children ($\alpha_1=0$). Those two conditions together imply that somehow children need to be helped from the society whether internally (by deliberate actions of adults) or externally for the economy to grow positively in the steady state. Even though the amount of learning of children is independent of the previous learning ($\alpha_1=0$), the economy could grow positively in the steady state as long as the education technology exhibits constant returns to scale on the human capital of adults ($\alpha_1+\gamma=1$).

In the case where the amount of children's learning depends only on their human capital, then Eq. (II-3) reduces to the followings:

When $0 < \alpha_1 < 1$,

$$H_{gt} = \left(H_{c0}^{1-\alpha_1} + (1-\alpha_1)\delta s\right)^{\frac{1}{1-\alpha_1}}$$

When $\alpha_1 = 1$,

$$H_{gt} = H_{c0}e^{\delta s}$$

In either case, the level of human capital H_{gt} of the new work-force depends on the initial endowment H_{c0}, the period s of schooling, and other structural parameters, meaning that H_{gt} is constant over time. Then the average stock of human capital of adults in the household is constant over time too. The new work-force just replaces adults who die at the instant of time. The engine of growth, human capital of adults, remains at the same level over time at the steady state, and thus the economy does not grow.

In order to calculate the growth rate λ of the planned economy and the optimal fraction u_t of time adults allocate to the education sector in the planned economy under assumptions that $\alpha_1+\alpha_2+\gamma=1$ and $\alpha_1 \neq 1$, let's substitute Eq. (II-3) into Eq. (II-5). Then we get

$$\frac{\dot{H}_{at}}{H_{at}} = p\left(\frac{\left(H_{c0}^{1-\alpha_1} + (1-\alpha_1)\delta\int_{t-s}^{t} u_z^{\alpha_2} H_{az}^{\alpha_2+\gamma}\, dz\right)^{\frac{1}{1-\alpha_1}}}{H_{az}} - 1\right) \tag{II-21}$$

$$= p\left(\left(\left(\frac{H_{c0}}{H_{at}}\right)^{1-\alpha_1} + (1-\alpha_1)\delta\, u_t^{\alpha_2}\, \lambda^{-1}(\alpha_2+\gamma)^{-1}(1-e^{-\lambda s(\alpha_2+\gamma)})\right)^{\frac{1}{1-\alpha_1}} - 1\right)$$

Assuming the endowment of human capital H_{c0} of children is trivial relative to human capital H_{at} of adults along the steady state path, we have

$$\frac{\dot{H}_{at}}{H_{at}} = p\left((\delta\, u^{\alpha_2}\, \lambda^{-1}(1-e^{-\lambda s(\alpha_2+\gamma)}))^{\frac{1}{1-\alpha_1}} - 1\right) \tag{II-22}$$

Rearranging this,

$$\left(\frac{\lambda+p}{p}\right)^{1-\alpha_1} = \delta u^{\alpha_2}\left(\frac{1-e^{-\lambda s(\alpha_2+\gamma)}}{\lambda}\right) \tag{II-23}$$

This gives you the growth rate of H_{at} in terms of u_t and other structural parameters. In the steady state, Eq. (II-18) reduces to

$$\rho + \sigma\lambda = p\left(\delta\, u^{\alpha_2-1}(\alpha_2+\gamma u_t)H_{at}^{\alpha_2+\gamma-1}\, H_{gt}^{\alpha_1} - 1\right) \tag{II-24}$$

Using Eqs. (II-3) and (II-24), we have

$$\left(\frac{\rho+\sigma\lambda}{p}+1\right)^{1-\alpha_1} = \delta u^{-\gamma}(\alpha_2+\gamma u)^{1-\alpha_1}\left(\frac{1-e^{-\lambda s(\alpha_2+\gamma)}}{\lambda}\right)^{\alpha_1} \tag{II-25}$$

Solving Eqs. (II-23) and (II-25) simultaneously, we can get the optimal fraction u_t of time which adults allocate to the education sector and the growth rate λ of the planned economy.

On the other hand, to get the steady state solution for the competitive economy, we use Eq. (II-18') instead of Eq. (II-18). In the steady state, Eq. (II-18') reduces to

$$\rho + \sigma\lambda = p\left(\delta\, \alpha_2 u^{\alpha_2 -1} H_{at}^{\alpha_2+\gamma-1} H_{gt}^{\alpha_1} - 1\right) \tag{II-24'}$$

Using Eqs. (II-3) and Eq. (II-24'), we have

$$\left(\frac{\rho+\sigma\lambda}{p}+1\right)^{1-\alpha_1} = \delta u^{-\gamma}\alpha_2^{\;1-\alpha_1}\left(\frac{1-e^{-\lambda s(\alpha_2+\gamma)}}{\lambda}\right)^{\alpha_1} \tag{II-25'}$$

Solving Eqs. (II-23) and (II-25') simultaneously, we can get the growth rate λ and the optimal fraction u_t of time of adults allocated to the education sector for the competitive economy.

In the case where the amount of learning of children doesn't depend on their ability to learn ($\alpha_1=0$), we can compare the rate of growth for the planned economy to that of the competitive economy. Let's denote λ_s as the growth rate of the planned economy and λ_c as the growth rate of the competitive economy. Then

$$\lambda_s = \frac{1}{\sigma}\left(p(\delta u^{-\gamma}(\alpha_2+\gamma u)-1) - \rho\right) \qquad \text{(II-26)}$$

$$\lambda_c = \frac{1}{\sigma}\left(p(\delta \alpha_2 u^{-\gamma}-1) - \rho\right) \qquad \text{(II-27)}$$

Then the difference of the growth rates is

$$\lambda_s - \lambda_c = \frac{p}{\sigma}(\gamma \delta u^{1-\gamma}) \qquad \text{(II-28)}$$

Therefore, the planned economy grows faster than the competitive economy as long as externality is present in the production function of the education sector ($\gamma>0$).

Even though laws of the economy are so complicated that we could not get a closed form solution for the economy, we learned, from the previous discussion, that neither externality nor human capital of children are not required for the economy to grow positively at the steady state. All I required is that the educational production function exhibits constant returns to scale and includes human capital of adults as an input.

For a moment, let's assume that the production function in the education sector only depends on human capital of adults internally with constant returns, that is, $\alpha_1=\gamma=0$ and $\alpha_2=1$. Those conditions imply that the production function in the education sector depends only on a fraction of time adults allocate to the education sector and exhibits constant returns.[7] Hence, the amount of learning of children is not affected by the previous learning of the children. Then the growth rate λ of the economy in the steady state can be obtained from Eq. (II-25) as follows:

$$\lambda = \frac{1}{\sigma}(p\delta - \rho - p) \qquad \text{(II-25'')}$$

On the other hand, from Eq. (II-23), the optimal fraction u of time adults allocate to their children is

$$u = \frac{1}{\delta}\left(\frac{\rho+\sigma\lambda}{p}\right)\left(\frac{\lambda}{1-e^{-\lambda n}}\right) \qquad \text{(II-23')}$$

Once λ and u are determined, the ratios of physical capital over human capital of labor (K_t/H_{at}), consumption over physical capital (C_t/K_t), and human capital of children over that of adults(H_{gt}/H_{at}) are uniquely determined from Eqs. (II-15), (II-6) and (II-3),

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[7]That is, the schooling technology is $\delta u_t H_{at}$, which is similar to Lucas(1988).

respectively:

$$\frac{K_t}{H_{at}} = \left(\frac{A\beta}{\rho+\sigma\lambda}\right)^{\frac{1}{1-\beta}}(1-u) \qquad \text{(II-15')}$$

$$\frac{C_t}{K_t} = \frac{1}{\beta}(\rho+\lambda(\sigma-\beta)) \qquad \text{(II-6')}$$

$$\frac{H_{gt}}{H_{at}} = \delta u(1-e^{-\lambda n}) \qquad \text{(II-3')}$$

Even though C_t, K_t, and H_{at} are nonstationary over time at the steady state, those ratios and u are stationary.

From the equations above, we can derive the following properties of the economy. First, Eq. (II-25'') implies that the economy with a lower death rate will grow faster than the economy with a higher death rate. Notice that the effective discount rate is the sum of the rate of time preference and the probability of death. If people face a higher probability of death, they will emphasize the current consumption relative to the future consumption, which means less investment and thus lower growth. Therefore, even though two economies have the same rate of time preference, the economy with a lower death rate grows faster than the economy with a higher death rate. In addition, the economy with a higher death rate experiences relatively large decrease in the human capital of adults due to the death, which will reduce the growth rate of the economy further.

Second, the quality of schooling captured by the productivity parameter δ in the education sector has a positive effect on the growth of the economy.[8] Once and for all increase in δ will increase the steady-state growth rate of the economy permanently.

Third, the quantity of schooling represented by the period s of schooling is not a growth effect-a variable which changes the steady state growth rate of the economy, but a level effect-a variable which only changes the levels of economic variables at the steady state. According to Eq. (II-25''), the growth rate λ of the economy is independent of the period s of schooling. On the other hand, Eq. (II-23') implies that adults will reduce time they allocate to the education sector as the period s of schooling increases. Then the output increases, so do consumption and physical capital but consumption-physical capital ratio remains the same. Therefore, once and for all increase in the quantity of schooling just increases the steady-state level of output, consumption, and physical capital without affecting the growth rate of the economy.

This seems to be contradictory to what Denison(1985) found. According to him, over the period 1929-1982, the quantity of schooling contributes 14 percent of the growth of the U.S. economy. But the independence of a growth rate on the quantity of schooling is model-specific in that, according to Eq. (II-25), the growth rate λ of the economy depends on the period s of schooling as long as α_1 is positive, that is, as long as the amount of learning of a child is affected by his human capital as well as human

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[8]See Glomm and Ravikumar(1992).

capital of adults. Even though there is no clear roles for human capital of children in the education sector to show the growth of the economy in the steady state, it might be worth exploring for possible roles for human capital of children in other perspectives. In fact, Bloom(1976, p. 11) describes

> "The amount of learning depends on the previous learning of the students, the extent to which the student is motivated to engage in the learning process, and the extent to which the instruction to be given is appropriate to the learner."

In the next chapter, I show that, when the amount of learning of children depends on their previous amount of learning as well as human capital of adults, which captures 1st and 3rd factors of Bloom, not only the economy would grow positively in the steady state, but also the growth rate of the economy may be positively related to the average years of schooling.[9] Also I assume, in the next chapter, that there is no externality in the schooling technology and individuals live infinitely.[10]

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[9]I am not able to quantify the motivation of students to study, which is a Bloom's 2nd factor, and therefore, I exclude this from factors which could influence students' learning.

[10]Tamura(1991) introduces externality in the human capital sector to show the convergence in levels of income as well as in growth rates.

Chapter III. A Model with Infinitely-lived Agents

A. Specification of the Model

The economy consists of many identical households, each of which consists of a continuum of children and adults. At a point in time, new children are born in households at a constant rate n and every child attends schools for s periods before entering a labor market where the length s of schooling is exogenously determined.[11] If I normalize the size N_0 of a household at time 0 to one, then the size N_t of the household at time t is equal to e^{nt} and the sizes of labor(adults) L_t and students S_t in the household at time t are $e^{n(t-s)}$ and $e^{n(t-s)}(e^{ns}-1)$, respectively. Because, at time t, $n*e^{n(t-s)}$ number of children enter a labor market after completing s periods of schooling, the total stock H_{at} of human capital of adults in the household could be calculated by

$$H_{at} = \int_{-\infty}^{t} n e^{n(v-s)} h_{gv} dv \qquad \text{(III-1)}$$

where h_{gv} is the level of human capital of an individual who starts to work in the

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[11]In this paper human capital of workers is constant throughout their working-life, and so are their wages assuming no externality. Therefore, rather than take schooling exogenous, I could assume that children, if not their parents, would determine the optimal schooling, at the time they are born, by maximizing the present discounted value of future income streams taken future wages and a discount rate as given. It would be easy to determine the optimal schooling if the economy is also in the steady state, where the interest rate is constant over time. But I assume that the exogenous schooling just happens to be the same as the optimal period of schooling. This would not be harmful when we restrict our discussion only to the steady state, where the period of schooling is constant over time. Because of the mathematically complicated nature of the problem, I am not able to study the economy in the transition path.

output sector at time v.

There are $n^*e^{n(v-s)}$ number of people with human capital of h_{gv} at time t, all of whom had been born at time v-s and completed s periods of schooling. It is assumed that all new children are endowed with the same level h_{c0} of human capital. If we assume that leisure is not valued, and either school is compulsory or the human capital which children accumulate in schools can not be used in the production of output until they finish schooling (time-to-build-like human capital), children would allocate all their available time on learning knowledge in schools. After finishing schooling, children start to work in the output sector. With an assumption that there is no on-the-job training, the stock of human capital of individuals is determined in their childhood.

It is assumed that, whether he or she is a child or an adult, everybody is endowed with one unit of time each period. Children spend the time on learning knowledge in schools while adults spend a fraction u_t of their time to teach their children and the other fraction $(1-u_t)$ on producing output.

Suppose that the amount of learning of a child at time t is determined by the following production function:[12]

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[12] If all adults in the household have the same level of human capital at h_{at}, then the production function becomes $\delta h_{ct}^{\alpha}(u_tL_t/S_t)^{1-\alpha}h_{at}^{1-\alpha}$, where u_tL_t is the amount of time which the household allocate to their children, which could be interpreted as the total hours of teaching in the household. Therefore, the amount of learning of a child at time t depends positively on his level human capital h_{ct}, teacher-student ratio u_tL_t/S_t, and the average level of human capital h_{at} of adults(teachers).

$$\dot{h}_{ct} = \delta h_{ct}^{\alpha}\left(\frac{u_t H_{at}}{S_t}\right)^{1-\alpha}$$

given that the endowment of human capital of children is h_{c0}

where

δ is a parameter representing productive efficiency in the education sector

h_{ct} is the level of human capital of a student at time t

u_t is a fraction of human capital of adults allocated to their children at time t

H_{at} is the total stock of human capital of adults in the household at time t

S_t is the number of students in the household at time t

The technology implies that, if adults in the household have higher human capital, children in the household would learn more knowledge than the children of the household with less human capital even though both households devote the same amount of labor time to teach their children. On the other hand, children in the household with higher adult-student ratio would learn more than the children in the household with lower adults-student ratio even if the levels of human capital of those two households are the same.

Denoting h_{at} as the average level of human capital H_{at}/L_t of adults in the household and noting that the sizes of adults and students are $e^{n(t-s)}$ and $e^{n(t-s)}(e^{ns}-1)$, respectively, we can rewrite the equation as

$$\dot{h}_{ct} = \Delta h_{ct}^{\alpha}\left(u_t h_{at}\right)^{1-\alpha} \qquad \textbf{(III-2)}$$

where

$\Delta(= \delta/(e^{ns}-1)^{1-a})$ could be interpreted as a new productive parameter in the educational production function.

Noting that $1/(e^{ns}-1)$ is adult-student ratio, the increase in the period s of schooling scales down the productive parameter in the educational production function through adult-student ratio.[13]

Eq. (III-2) shows how human capital of a child evolves over time. The solution of the equation becomes

$$h_{ct,t-s} = \left(h_{ct-s,t-s}^{1-\alpha} + \Delta(1-\alpha)\int_{t-s}^{t}(u_v h_{av})^{1-\alpha}\,dv\right)^{\frac{1}{1-\alpha}} \qquad \text{(III-3)}$$

where

$h_{ct,t-s}$ is the level of human capital of a child, as of time t, born at time t-s.

$h_{ct-s,t-s}$ is the level of human capital of a child, as of time t-s, born at time t-s.

Because every child is endowed with the same level of human capital of h_{c0}, $h_{ct-s,t-s}$ is equal to h_{c0}. Letting human capital $h_{ct,t-s}$ be h_{gt} for simplicity, we have

[13]It is shown later that, because of this effect, the growth rate of the economy is negatively related to the period of schooling when $\alpha=0$.

$$h_{gt} = \left(h_{c0}^{1-\alpha} + \Delta(1-\alpha)\int_{t-s}^{t}(u_v h_{av})^{1-\alpha}\, dv\right)^{\frac{1}{1-\alpha}} \qquad \text{(III-4)}$$

Eq. (III-4) represents the level of human capital of a child, who is born at time t-s, acquired at the end of schooling. Noting that $ne^{n(t-s)}h_{gt}$ is the amount of human capital of new work-force at time t, the total stock of human capital of labor in the household evolves over time as follows:

$$\dot{H}_{at} = ne^{n(t-s)}\left(h_{c0}^{1-\alpha} + \Delta(1-\alpha)\int_{t-s}^{t}(u_v h_{av})^{1-\alpha}\, dv\right)^{\frac{1}{1-\alpha}} \qquad \text{(III-5)}$$

Production of output is assumed to be the same as in the previous chapter. That is,

$$\dot{K}_t = AK_t^{\beta}((1-u_t)H_{at})^{1-\beta} - N_t c_t \qquad \text{(III-6)}$$

A representative household maximizes the welfare of the family:

$$\int_0^{\infty} e^{-\rho t} \frac{N_t c_t^{1-\sigma}}{1-\sigma} dt \qquad \text{(III-7)}$$

B. Solution of the Economy

The representative household maximizes the welfare of the family, Eq. (III-7), with the constraints of Eqs. (III-5) and (III-6). After converting aggregate variables into per capita adult variables, the Hamiltonian of the problem is

$$\begin{aligned} \mathrm{H} = \frac{e^{ns} c_t^{1-\sigma}}{1-\sigma} &+ \theta_{1t}\left(A\, k_t^{\beta}\left((1-u_t) h_{at}\right)^{1-\beta} - e^{ns} c_t - n k_t\right) \\ &+ \theta_{2t}\left(n\left(h_{c0}^{1-\alpha} + \Delta(1-\alpha)\int_{t-s}^{t} (u_v\, h_{av})^{1-\alpha}\, dv\right)^{\frac{1}{1-\alpha}} - n h_{at}\right) \end{aligned} \tag{III-8}$$

Note that e^{ns} is the population-adult(labor) ratio, c_t is per capita consumption, k_t and h_{at} are per capita adult physical and human capital, respectively.

The first-order conditions for the problem are:

c_t:

$$c_t^{-\sigma} = \theta_{1t} \tag{III-9}$$

u_t:

$$\begin{aligned} &\theta_{1t}\, A(1-\beta) k_t^{\beta}\, h_{at}^{1-\beta} (1-u_t)^{-\beta} \\ &= \theta_{2t}\, n\, \Delta\, (1-\alpha) u_t^{-\alpha} h_{at}^{1-\alpha} \left(h_{c0}^{1-\alpha} + \Delta(1-\alpha)\int_{t-s}^{t} (u_v h_{av})^{1-\alpha} dv\right)^{\frac{\alpha}{1-\alpha}} \end{aligned} \tag{III-10}$$

k_t:

$$\dot{\theta}_{1t} = \rho\theta_{1t} - \theta_{1t}\left(A\,\beta\,k_t^{\beta-1}((1-u_t)h_{at})^{1-\beta} - n\right) \qquad \text{(III-11)}$$

h_{at}:

$$\begin{aligned}\dot{\theta}_{2t} &= \rho\,\theta_{2t} - \theta_{1t}A(1-\beta)k_t^{\beta}(1-u_t)^{1-\beta}\,h_{at}^{-\beta} \\ &- \theta_{2t}\left(n\,\Delta\,(1-\alpha)u_t^{1-\alpha}h_{at}^{-\alpha}\left(h_{c0}^{1-\alpha}+\Delta(1-\alpha)\int_{t-s}^{t}(u_v h_{av})^{1-\alpha}dv\right)^{\frac{\alpha}{1-\alpha}} - n\right)\end{aligned} \qquad \text{(III-12)}$$

Eq. (III-9) - (III-12) and two transversality conditions for k_t and h_{at} are necessary and sufficient conditions for the problem. Eq. (III-9) implies that the marginal utility of consumption should be equal to the shadow price of output. Eq. (III-10) implies that the marginal product of adults' time in physical unit should be the same in both sectors. Those two first order conditions describe the static optimality conditions for the economy. Eqs. (III-11) and (III-12) are laws of motion for shadow prices of physical and human capital, respectively.

In order to get the rates of change for real variables, c_t, u_t, k_t, and h_{at}, let's eliminate θ_{1t} and θ_{2t} from the first-order conditions described above. Using Eqs. (III-9) and (III-11)

$$\rho + n + \sigma\frac{\dot{c}_t}{c_t} = A\,\beta\,k_t^{\beta-1}((1-u_t)h_{at})^{1-\beta} \qquad \text{(III-13)}$$

The right hand side of the equation is the marginal product of physical capital. So Eq. (III-13) implies that the marginal product of physical capital in the output sector should be equal to the sum of the rate of time preference, population growth rate, and the growth rate of consumption divided by the intertemporal elasticity of substitution. If the growth rate of consumption is constant over time, which would be true in the steady state, the return on physical capital is constant over time in the steady state.

Taking the natural logarithm of Eq. (III-10), differentiating it with respect to t, and rearranging it, we have

$$\frac{\dot{\theta}_{2t}}{\theta_{2t}} = \frac{\dot{\theta}_{1t}}{\theta_{1t}} + \beta\frac{\dot{k}_t}{k_t} + (\alpha-\beta)\frac{\dot{h}_{at}}{h_{at}} + \left(\alpha+\beta\frac{u_t}{1-u_t}\right)\frac{\dot{u}_t}{u_t} - \alpha\frac{\dot{h}_{gt}}{h_{gt}} \qquad \text{(III-14)}$$

After rearranging Eqs. (III-9), (III-10), and (III-14) and substituting them into Eq. (III-12), we get

$$-\sigma\frac{\dot{c}_t}{c_t} + \beta\frac{\dot{k}_t}{k_t} - (\alpha-\beta)\frac{\dot{h}_{at}}{h_{at}} + \left(\alpha + \beta\frac{u_t}{1-u_t}\right)\frac{\dot{u}_t}{u_t} - \alpha\frac{\dot{h}_{gt}}{h_{gt}} \qquad \text{(III-15)}$$

$$= \rho + n - n\Delta(1-\alpha)(u_t h_{at})^{-\alpha}\left(h_{c0}^{1-\alpha} + \Delta(1-\alpha)\int_{t-s}^{t}(u_v h_{av})^{1-\alpha}\right)^{\frac{\alpha}{1-\alpha}}$$

Therefore, the evolution of the economy could be summarized by Eqs. (III-5), (III-6), (III-13), and (III-15) and two transversality conditions for state variables, k_t and h_{at}.

C. Steady State Solution

In the steady state, u_t is constant over time and c_t, h_{gt}, k_t, and h_{at} grow at a constant rate. Human capital h_{at} of adults(labor) is the engine of growth and can be increased only through the entrance of new work-force in a labor market. Hence, for the economy to grow positively in the steady state, h_{gt} should grow at a constant rate, meaning that human capital of the cohort who enters a labor market at time t is greater than that of the immediate previous cohort and the percentage change of human capital of immediate two cohorts is constant over time.

Suppose that h_{gt} grows at a constant rate λ along the steady state path. Then Eq. (III-1) implies that average level of human capital of adults in the household also grows at a rate of λ. Then Eq. (III-13) implies that the growth rate of physical capital k_t is the same as that of the average level of human capital h_{at} of adults in the household, Eq. (III-6) means that per capita consumption c_t grows at a rate of λ, and Eq. (III-15) indicates that a fraction u_t of time which adults allocate to the schooling sector is constant over time in the steady state. Consequently, per capita consumption c_t, per capita adult human capital h_{at}, and per capita adult physical capital k_t grow at the same rate as h_{gt} and u_t is constant in the steady state.

Noting that $\Delta = \delta/(e^{ns}-1)^{1-\alpha}$, the growth rate λ of the economy would be obtained from Eq. (III-15) as follows:

$$\left(\frac{\sigma\lambda+\rho+n}{n}\right)^{1-\alpha} = \delta\left(\frac{1-\alpha}{e^{ns}-1}\right)^{1-\alpha}\left(\frac{1-e^{-\lambda(1-\alpha)s}}{\lambda}\right)^{\alpha} \qquad \text{(III-15')}$$

On the other hand, the optimal fraction u_t of time which adults allocate to the education sector in the steady state could be calculated from Eq. (III-5) as follows:

$$u = \delta^{\frac{1}{1-\alpha}}\left(\frac{e^{ns}-1}{n}\right)(\lambda+n)\left(\frac{\lambda}{1-e^{-\lambda(1-\alpha)s}}\right)^{\frac{1}{1-\alpha}} \qquad \text{(III-5')}$$

Solving Eqs. (III-15') and (III-5') simultaneously, the effect of the period s of schooling on the growth rate λ of the economy and on the optimal fraction u_t of time of adults allocated to their children could be discussed.

For a moment, let's assume that children's human capital doesn't play any role on their learning, that is, $\alpha=0$. Then Eq. (15') is reduced to the following:[14]

$$\lambda = \frac{1}{\sigma}\left(\frac{n\delta}{e^{ns}-1} - \rho - n\right) \qquad \text{(III-15'')}$$

[14]When $\alpha=0$, the educational production function becomes $\Delta u_t h_{at}$.

The period s of schooling is only appeared to the term $1/(e^{ns}-1)$, adult-student ratio, in the equation, implying that the period s of schooling has negative effect on the growth of the economy only up to the extent that it reduces adult-student ratio.[15]

Figure 1(a) shows this negative relationship diagrammatically. To draw Figure 1(a), I assumed that the rate of time preference was 0.04, which is the value in Kydland and Prescott(1982). I choose δ to be 0.88 so that it resembles the U.S. economy, whose real GDP per capita labor grew 1.34 percent a year over the period 1960-1985. Over that period, population in the U.S. grew at 1.13 percent per year and mean years of schooling of labor force was about 12 years. The figure indicates that increase in the period s of schooling would reduce the growth rate of the economy in the steady state and finally would have a negligible effect on the rate of growth of the economy.

Then it may be argued that, if the growth rate of the economy is negatively related to the period of schooling, the economy could grow at a higher rate if the economy lowers the periods of schooling. But if years of schooling is significantly low, then it might not be feasible for the economy to grow positively. This point would be explained by Figure 1(b) plotting the relationship between the period s of schooling and the optimal fraction u of time which adults allocate to the education sector.

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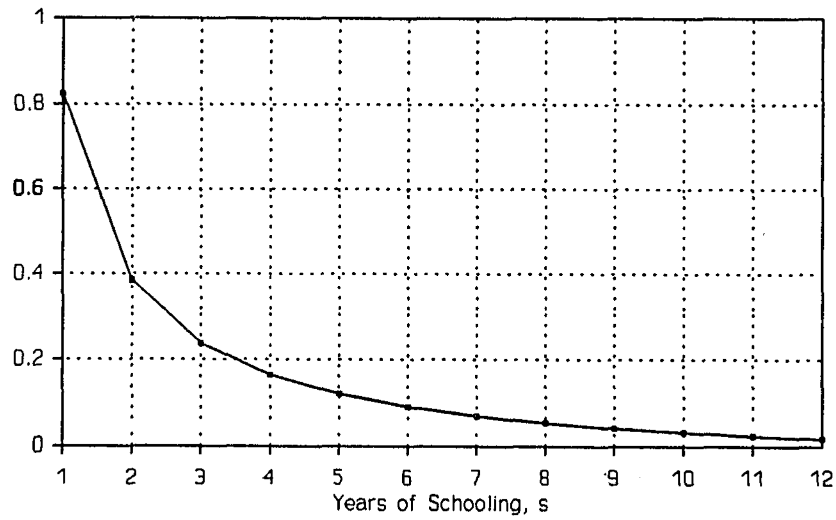
[15]The corresponding steady-state growth equation in Lucas(1988) is $\lambda=(1/\sigma)(\delta-\rho-n)$, implying that $\delta n/(e^{ns}-1)=\delta$ in his paper. Adult-student ratio, $e^{ns}-1$, could be interpreted as the ratio of people who help students learn to the students, is normalized to one in Lucas(1988) because he assumes on-the-job training human capital accumulation, where workers(adults) learn for themselves. Also n, which is a population growth rate, could be interpreted as the ratio of workers who are accumulating human capital to total workers, which is normalized to one in Lucas(1988).

Fig (1a). Growth rate

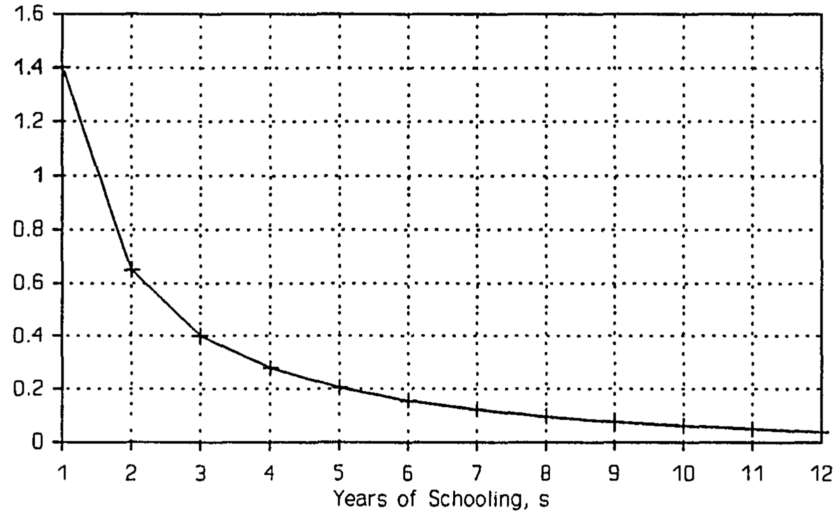


When $\alpha = 0$, Eq. (III-5') reduces to

$$u = \frac{\lambda(\lambda+n)}{n\delta}\frac{e^{ns}-1}{1-e^{-\lambda s}} \tag{III-5''}$$

Substituting Eq. (III-15'') into Eq. (III-5''), the optimal fraction u of time adults allocate to the schooling sector would be represented as a function of structural parameters.

Fig (1b). u



As you see, for some lower years s of schooling, the optimal fraction u of human capital of adults allocated to the education sector is very high and sometimes greater than one, meaning that, in order to sustain high growth rates, the economy should allocate most of its effort to accumulate human capital, produces less output, and thus sacrifices current consumption. When years of schooling is 12, the optimal fraction of time of adults allocated to the education sector is less than 10 percent. Then the economy would grow at a rate of 1.34 percent a year.

The reason that the growth rate is negatively related to the period of schooling would be explained by the production function in the education sector, Eq. (III-2):

$$\dot{h}_{ct} = \Delta h_{ct}^{\alpha} (u_t h_{at})^{1-\alpha} \tag{III-2}$$

where

$\Delta(= \delta/(e^{ns}-1)^{1-\alpha})$ is a new productive parameter in the education sector

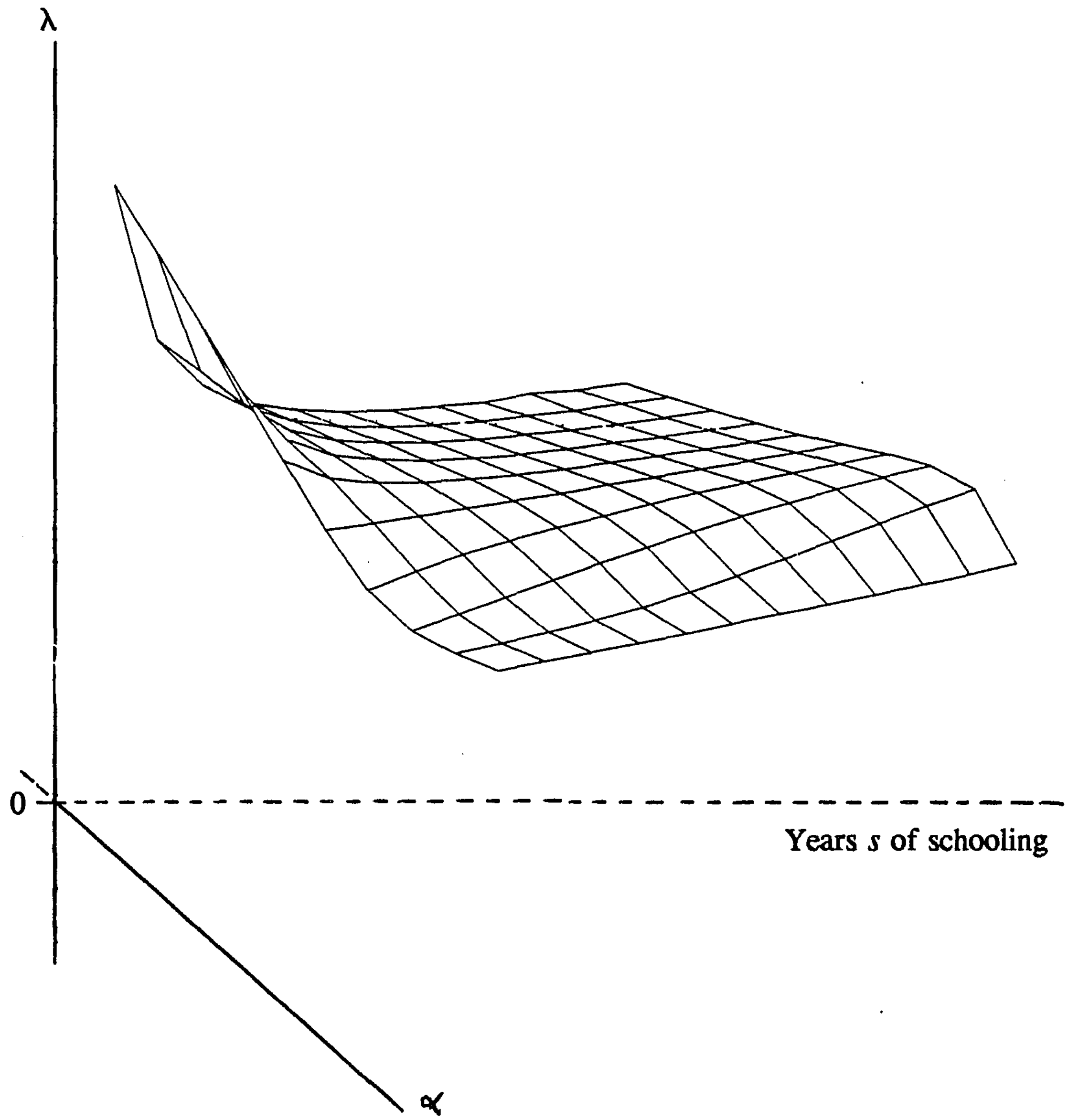
As the period s of schooling increases, the new production parameter, Δ, which is equal to $\delta/(e^{ns}-1)$, falls, which has the same effect as the once and for all decrease in the level of technology in the educational production function. Therefore, if the educational production function is independent of human capital of students, $\alpha=0$, only this effect of the period of schooling prevails and thus the economy grows at a lower rate due to the once and for all increase in the period of schooling.[16]

However, this negative relationship between years of schooling and the growth rate of the economy seems to be inconsistent with the empirical findings. Denison(1985) estimated that the quantity of schooling after adjusting annual average school days contributed 14 percent of the growth of the U.S. economy over the period 1929-1982. The negative relationship between the growth rate and the period of schooling is due to the assumption that learning of children does not depend on their previous learning, which is the assumption of most of endogenous growth models with schooling as a way

[16]If adults-student ratio is equal to one, the growth rate of the economy is independent of the period of schooling, which was shown in chapter II.

of accumulating human capital (See Lucas(1988), Becker and et al.(1990), Becker and Murphy(1992), and Glomm and Ravikumar(1992)). Therefore, we may have to be careful about choosing a functional form on the schooling technology in order for a model to be able to explain the empirical findings, specially when we don't know the true educational production function.

Fig 2. Growth rate of the economy



In general, as you see from Eqs. (III-15') and (III-5'), the growth rate of the economy is highly nonlinear in periods s of schooling and so we would not be able to solve them analytically. Therefore, I use matlab M-file (fsolve.m) to solve them numerically. Using the same parameter values as Figure 1(a), I solved the equations for λ and u by changing years s of schooling and the share α of children's human capital in the educational production function. Figure 2 shows what happens to the steady-state growth rate as s and α change. When α is less than about 0.53, the growth rate is negatively related to the period s of schooling while it is positively related to s when α is higher than 0.53 and less than 1.

In order to see why it happens, let's rewrite Eq. (III-15'), the equation for the steady-state growth rate

$$\left(\frac{\sigma\lambda+\rho+n}{n}\right)^{1-\alpha} = \delta\left(\frac{1-\alpha}{e^{ns}-1}\right)^{1-\alpha}\left(\frac{1-e^{-\lambda(1-\alpha)s}}{\lambda}\right)^{\alpha} \qquad \textbf{(III-15')}$$

In the equation, the period s of schooling appears in two terms, $1/(e^{ns}-1)$ and $((1-e^{-n(1-\alpha)s})/\lambda^{\alpha})$ The first term is adult-student ratio, and the second term is nonzero as long as $0<\alpha<1$, that is, as long as children's as well as adults' human capital are inputs of the schooling technology. Therefore, the increase in the period s of schooling has two channels to affect the growth rate of the economy: negative effects through adult-student ratio and positive effects through the second term. For lower values of α, the effect of the first term dominates that of the second term, and thus the net effect of the period of schooling on the growth is negative.

On the other hand, for higher values of α, the positive effect of schooling on the growth rate of the economy dominates the negative effect of schooling so that net effect of the period of schooling on the growth of the economy is positive.

Fig 3. The fraction u of adults' time allocated to the education sector

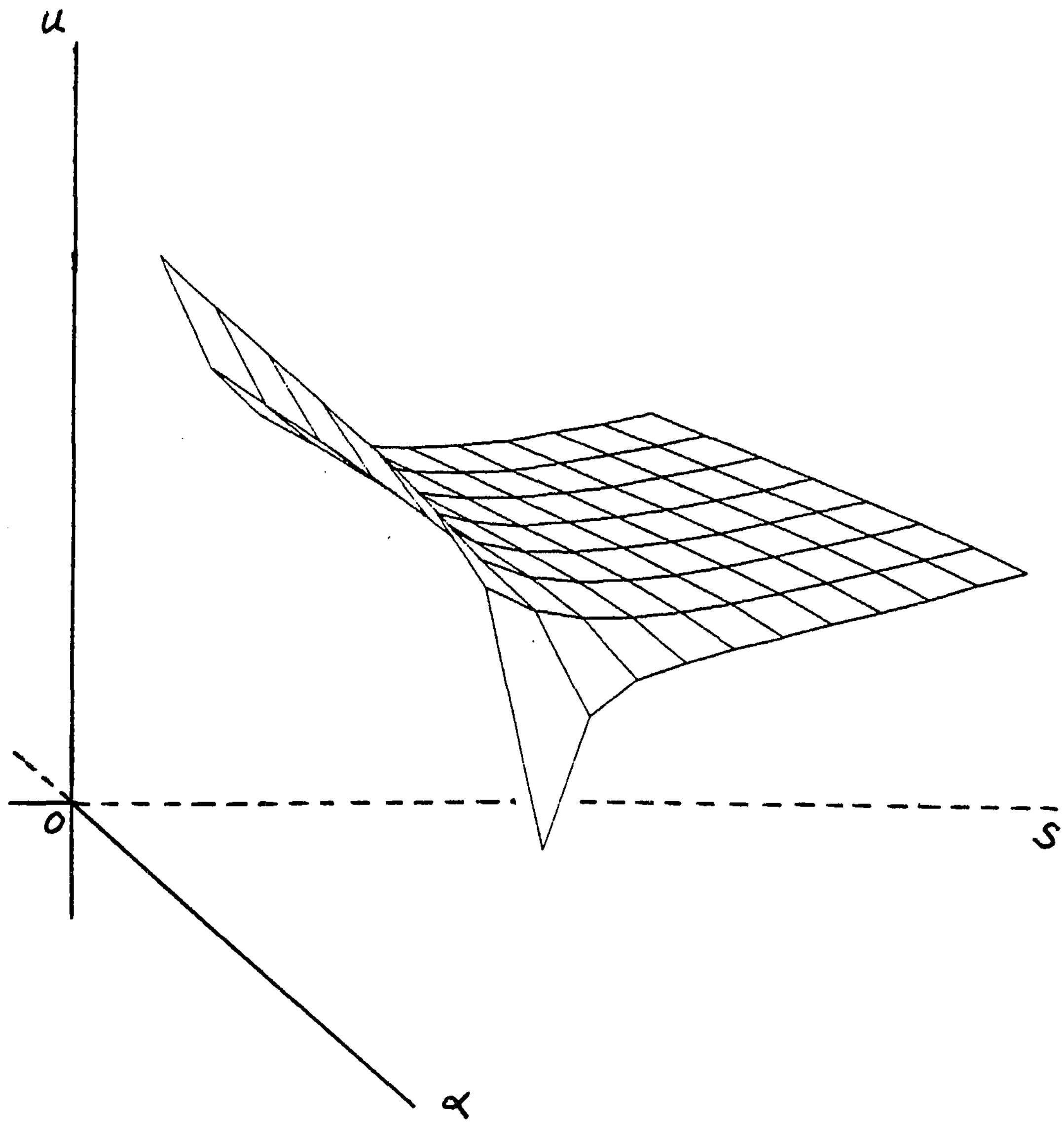


Figure 3 shows what happens to the optimal fraction u of time adults allocate to the education sector as α and s change. At given level of α, u falls as s increases, implying that adults spend more time on producing output and enjoy current consumption as the period of schooling increases. In Appendix A, I present figures for λ and u for various values of s and α.

Chapter IV. Empirical Results

A. Survey

There are many articles which tested endogenous growth models with human capital as the engine of growth. Barro(1991) regressed growth rates of real per capita GDP over the period 1960-1985 on primary enrollment rate in 1960(PRIM60), secondary enrollment rate in 1960(SEC60), and other variables. The coefficients of PRIM60 and SEC60 are individually significantly different from zero at 1% significance level. In order to measure differences in the quality of education across countries, he used data on student-teacher ratios at the initial year, 1960, and find a significant negative relationship (t-value=1.7) between primary school student-teacher ratio in 1960 and growth rates of real per capita GDP over 1960-1985 and insignificant positive relationship between secondary school student-teacher ratio and growth rates. Also he finds that the initial adult literacy rate is significantly negatively related(t-value=2) to the growth rates of real GDP when the regression includes enrollment ratios additionally and significant positive relationship when the regression only includes the adult literacy ratio as a proxy for human capital.

But, as Levine and Renelt(1992) indicate, primary enrollment and adult literacy ratio may not be preferable as proxies for human capital because many countries have reached upper bounds for these variables and most of endogenous growth models require reproducible human capital which can be accumulated without bound.

The "base" regression of Levine and Renelt(1992) is

$$GYP = 0.83 - 0.35\ RGDP60 - 0.38\ GPO + 3.17\ SEC + 17.5\ INV$$
$$(-0.98)\qquad (-2.5)\qquad (-1.73)\qquad (2.46)\qquad (6.53)$$

(R^2 = 0.46, number of observations = 101; t-values in parentheses)

where

GYP : Growth rate of per capita real GDP

INV : Investment share of GDP

GPO : Growth of population

SEC : Secondary-school enrollment rate in 1960

RGDP60: Real GDP per capita in 1960

Secondary-school enrollment rate in 1960 is significantly positively related to the per capita growth rate. But they find that the enrollment rate becomes insignificant when other variables are incorporated in the regression. Further, using the same data set as Barro(1991), they find that primary and secondary school enrollment rates become insignificant when different variables from Barro(1991) are added to the regression, concluding that the relationship between per capita growth rate and enrollment rates is not robust.

On the other hand, Mankiw, Romer, and Weil(1992) use a different variable, which they call SCHOOL, as a proxy for human capital. The variable SCHOOL is the average percentage of the working-age population in secondary school for the period 1960-1985 and calculated by multiplying secondary school enrollment rate by the fraction of working-age population that is of school age (aged 15 to 19), where the enrollment is based on children whose ages are between 12 and 17. Their result shows the significant positive relationship between log of SCHOOL and log difference in GDP per working age person between 1960-1985.

Kyriacou(1991) uses the estimated average school years in labor force as an index for human capital and finds an insignificant negative relationship between the growth rate of per capita GDP and the rate of change in average years of schooling. Benhabib and Spiegel(1992) include other variables in addition to Kyriacou's estimate of average school years of labor force and find that log difference in average years of schooling is always insignificant and almost always with a negative coefficient.

The articles testing endogenous growth models summarized above have the following similarities in their empirical methodologies. First, they run ordinary least squares, using cross-country data, to estimate an aggregate accounting equation with a long-run growth rate as a dependent variable.

Second, they use data on the education sector such as enrollment rates or mean years of schooling of labor force as a proxy for human capital. But enrollment rates and mean years of schooling are variables just measuring the quantitative effect of schooling. Even though years of schooling may be a better proxy for human capital than enrollment rates, it is one of many factors which affect the accumulation of human capital in the education sector. Further it was shown, in chapter III, that mean years of schooling would be negatively correlated to the steady-state growth rate under certain circumstances. In this sense, Kyriacou(1991) and Benhabib and Spiegel's(1992) assumption that mean years of schooling is an index of the level of human capital may be inappropriate.

B. Data Description

In chapter III, it was shown that the steady-state growth rate was

$$\left(\frac{\sigma\lambda+\rho+n}{n}\right)^{1-\alpha} = \delta\left(\frac{1-\alpha}{e^{ns}-1}\right)^{1-\alpha}\left(\frac{1-e^{-\lambda(1-\alpha)s}}{\lambda}\right)^{\alpha} \qquad \text{(III-15')}$$

where δ is a productive parameter in the educational production function and s is a period of schooling.

The economy with higher δ would grow faster than the economy with lower δ because, even if the two economies put the same amount of effort to the education sector, students in the economy with higher δ learns more than other economies. In this sense, δ may be capturing the quality of schooling.

The literature in other fields showed that the quality of schooling was indeed a factor which improved student learning in micro data. For example, Finn and Achilles(1990) found that student-teacher ratio is negatively related to the amount of learning. Card and Krueger(1992) found that rate of return are higher for students who attended schools with lower student-teacher ratio and higher teacher salaries. In order to capture the effect of δ on growth rates, I use the share of educational expenditure to GNP in year 1960 (EDX60) as well as student-teacher ratios in primary schools(STRPRI) and secondary schools (STRSEC). EDX60 is from Human Development Report 1991 and STRPRI and STRSEC are from Barro(1991). It is assumed, in the literature, that as the amount of resources allocated to the education sector and/or student-teacher ratio falls, student are learning better. Therefore, I predict a positive relationship between growth rates and EDX60 and negative relationship between growth rates and student-teacher ratio (STRPRI and STRSEC).

On the other hand, it was shown, in chapter III, that the effect of the quantity s of schooling on the growth rate was inconclusive: when the educational production function depends on children's as well as adults' human capital, mean years of schooling may have a positive effect on the steady-state growth rate if share of human capital of students in the educational production function is high enough but less than one. Otherwise, it is negatively related to the growth rate. But the literature predicts a positive relationship between the period of schooling and growth rates. Mean years of schooling of labor force in 1980(SQN80), which comes from Human Development Report 1991, is used as a proxy for s.

Investment share to GDP(I/GDP) is also added in the regression. I/GDP, the annual average of share of investment to GDP over the period 1960-1985, which comes from Summers and Heston(1991), is assumed to be positively related to the growth rate.

The definitions and sources of variables for regressions are:

DEATH: Average crude death rate over the period 1960-1985: World Population Prospects, UN, 1986

EDX60: Share of public expenditures on education to GNP in 1960. It includes the expenditures on the provision, management, inspection and support of pre-primary, primary and secondary schools; universities and colleges; vocational, technical and other training institutions; and general administration and subsidiary services: Human Development Report 1991, UNDP

G6085: Annual average growth rate of per capita labor real GDP over the period 1960-1985: Summers and Heston(1991)

GDP60: Per capital labor real GDP in 1960, the initial year: Summers and Heston(1991)

I/GDP: Annual average share of investment to GDP over the period 1960-1985(%): Summers and Heston(1991)

LIT60: Adult literacy rate in 1960: Barro(1991)

PRIM60: Enrollment ratio for primary education in 1960. It is constructed as ratio of total students enrolled in primary education to estimated number of individuals in the age bracket 6-11 years: Barro(1991)

SEC60: Enrollment ratio for secondary education in 1960. It is constructed as ratio of total students enrolled in secondary education to estimated number of individuals in the age bracket 12-17 years: Barro(1991)

SQN80: Average number of years of schooling received per person aged 25 and over in 1980: Human Development Report 1991, UNDP

STRPRI: Student teacher ratio in primary schools 1960: Barro(1991)

STRSEC: Student teacher ratio in secondary schools 1960: Barro(1991)

Table 1 summarizes basic statistics and a correlation matrix for the variables. The sample consists of all countries in Summers and Heston(1991) which I could collect data on the variables used in the regressions, excluding socialist and oil-exporting countries.

Table 1(a). Summary statistics

Variables	MEAN	S.D.	MIN.	MAX.
G6085	0.021	0.016	-0.014	0.07
GDP60	6.028	5.429	0.494	24.65
I/GDP	17.994	8.099	1.820	34.73
LIT60	0.535	0.334	0.015	0.99
PRIM60	0.744	0.339	0.050	1.44
SEC60	0.216	0.224	0.004	0.86
SQN80	4.425	3.152	0.200	12.20
EDX60	2.610	1.272	0.300	5.90
STRPRI	35.994	9.638	19.500	69.88
STRSEC	19.383	7.019	6.181	37.73
DEATH	13.890	5.505	5.660	25.76

Table 1(b). Correlation Matrix

	G6085	I/GDP	LIT60	PRIM60	SEC60	SQN80	EDX60	STRPRI	STRSEC	GDP60
I/GDP	0.45									
LIT60	0.26	0.61								
PRIM60	0.39	0.61	0.89							
SEC60	0.25	0.57	0.82	0.71						
SQN80	0.25	0.62	0.91	0.80	0.91					
EDX60	0.22	0.37	0.47	0.43	0.62	0.57				
STRPRI	-0.11	-0.28	-0.38	-0.28	-0.39	-0.40	-0.30			
STRSEC	0.13	0.06	-0.04	-0.02	0.03	-0.04	0.02	0.29		
GDP60	0.03	0.48	0.76	0.66	0.80	0.84	0.57	-0.44	-0.10	
DEATH	-0.40	-0.59	-0.84	-0.85	-0.63	-0.72	-0.40	0.26	-0.02	-0.60

The correlation matrix indicates that growth rates of real per capita labor(G6085) is positively correlated to all variables except student-teacher ratio in primary schools(STRPRI) and a death rate(DEATH). Variables measuring the quantity of schooling such as SQN80, PRIM60, and SEC60 are positively correlated the educational expenditures(EDX60) and negatively correlated to student-teacher ratio in primary schools, indicating that countries which have higher quantity of schooling also tend to have higher quality of schooling. On the other hand, the growth rate of the economy is negatively correlated to the rate of death(DEATH) as predicted in chapter II.

C. Empirical Results

Table 2 summarizes a set of regression results. A dependent variable is annual growth rates(G6085) of per capita adult(labor) real GDP over the period 1960-1985, which is calculated from Summers and Heston(1991). Because they are cross-section regressions, I use White(1980) method to control heteroskedasticity.

Investment ratio(I/GDP) is significantly positively related to the long-run growth rates. Educational expenditures-GNP ratio(EDX60), a variable measuring the quality of education, is positively related to the growth rate and marginally significant. But the other variables which measure the quality of schooling, student-teacher ratio in primary schools(STRPRI) and student-teacher ratio in secondary schools(STRSEC), are not significant and STRSEC has even an incorrect sign. It is assumed, in the literature, that, as student-teacher ratio is lower, students learn more, and the economy grows faster.

Table 2. Regression results (time period: 1960-1985)
Heteroskedasticity consistent covariance matrix

Independent variables	Dependent variable: growth rate of per capita labor real GDP (1)	(2)	(3)	(4)	(5)
CONSTANT	0.00208	-0.00003	-0.00366	-0.00889	-0.00666
	(0.582)	(-0.003)	(-0.931)	(-0.96)	(-0.669)
I/GDP	0.00083**	0.00084*	0.00062*	0.00061	0.00066*
	(2.546)	(2.358)	(1.736)	(1.583)	(1.795)
EDX60	0.00244	0.00237	0.00224	0.0024	0.00241
	(1.577)	(1.393)	(1.468)	(1.437)	(1.487)
STRPRI		-0.00010	-0.00011		-0.00013
		(-0.428)	(-0.532)		(-0.681)
STRSEC		0.00028	0.00035		0.0003
		(1.046)	(1.388)		(1.110)
SQN80	-0.00065	-0.00054			-0.00241*
	(-0.869)	(-0.613)			(-1.786)
PRIM60			0.01647**	0.02023**	0.02674**
			(2.458)	(2.767)	(3.212)
SEC60			-0.02314	-0.02618*	-0.00396
			(-1.968)	(-2.051)	(-0.206)
# of OBS.	92	80	92	80	80
R^2	0.17	0.21	0.22	0.28	0.30
F	5.98	3.86	6.06	4.73	4.41

t-values in parentheses
* represents significance at 5%
** represents significance at 1%

On the other hand, mean years of schooling of adults in 1980(SQN80), a variable which measures the quantity of schooling, is negatively correlated to the growth rate even though it is not significant. It is assumed, in the literature, that mean years of schooling is positively related to the growth rate while, in this paper, the effect depends on the educational production function: if the educational production function does not depend on human capital of students, the steady-state growth rate is negatively related to mean years of schooling.

But primary-school enrollment ratio(PRIM60) and secondary-school enrollment ratio(SEC60) are positively correlated to the long-run growth rate and only PRIM60 is significant at 1% significance level(Eqs. (3) and (4)). When SQN80 and enrollment ratios, PRIM60 and SEC60, are included in the same regression(Eq. (5)), primary-school enrollment ratio(PRIM60) continues to be significant while mean years of schooling(SQN80) is significantly negatively correlated to the growth rate. This may be due to high correlation between SQN80 and PRIM60. Table 1(b) shows the correlation coefficient between SQN80 and PRIM60 is 0.80. In fact, SQN80 and PRIM60 measure the same thing, the quantity of schooling.

In order to control the initial income differences among the countries, I added the initial real per capita labor GDP(GDP60) to each regression in Table 2. Table 3 shows the results. When the initial level of GDP(GDP60) is added, GDP60 is significantly negatively related to the long-run growth rate, and I/GDP continues to be positively related to the growth rate.

Educational expenditures(EDX60) is significantly positively related to the growth rate at 5% significance level, which is expected to be the case, and mean years of schooling(SQN80) is also positively related to the growth rate at 5% significance level. But primary school student-teacher ratio(STRPRI) is not significant even though the sign is as expected and secondary-school student-teacher ratio(STRSEC) has a wrong sign and is insignificant.

Table 3. Regression results (time period: 1960-1985)
Heteroskedasticity consistent covariance matrix

Independent variables	Dependent variable: growth rate of per capita real GDP (6)	(7)	(8)	(9)	(10)
CONSTANT	0.00073	0.00292	-0.00421	-0.00237	-0.00234
	(0.222)	(0.338)	(-1.167)	(-0.285)	(-0.279)
GDP60	-0.00209**	-0.00219**	-0.0021**	-0.00213**	-0.00212**
	(-4.750)	(-4.805)	(-5.238)	(-5.268)	(-4.836)
I/GDP	0.00077**	0.00078*	0.00061*	0.00061*	0.00061*
	(2.455)	(2.279)	(1.880)	(1.734)	(1.755)
EDX60	0.00345*	0.00337*	0.00304*	0.00318*	0.00317*
	(2.223)	(1.995)	(2.074)	(1.990)	(1.983)
STRPRI	-0.00017	-0.00020	-0.00020		
	(-0.857)	(-1.143)	(-1.149)		
STRSEC	0.00020	0.00020	0.00020		
	(0.727)	(0.738)	(0.727)		
SQN80	0.00219*	0.00236*	-0.00006		
	(2.512)	(2.566)	(-0.045)		
PRIM60		0.02250**	0.02560**	0.02573**	
		(3.740)	(3.977)	(3.540)	
SEC60		0.00833	0.00488	0.00531	
		(0.748)	(0.434)	(0.321)	
# of OBS.	92	80	92	80	80
R^2	0.28	0.33	0.35	0.42	0.42
F	8.65	6.06	9.39	7.32	6.32

t-values in parentheses
* represents significance at 5%
** represents significance at 1%

Table 4. Regression results (time period: 1960-1985)
Heteroskedasticity consistent covariance matrix

Independent variables	Dependent variable: growth rate of per capita real GDP (11)	(12)	(13)	(14)	(15)
CONSTANT	0.00399	-0.00078	-0.00154	-0.00314	-0.00265
	(1.290)	(-0.073)	(-0.465)	(-0.331)	(-0.268)
LIT60	-0.00275	0.00472	-0.03521**	-0.02839**	-0.02393*
	(-0.262)	(0.415)	(-3.510)	(-2.574)	(-1.974)
I/GDP	0.00093**	0.00092**	0.00077*	0.00076*	0.00077*
	(2.626)	(2.390)	(2.091)	(1.864)	(1.970)
EDX60	0.00126	0.00128	0.00056	0.00076	0.00084
	(0.842)	(0.799)	(0.403)	(0.511)	(0.569)
STRPRI		-0.00005		-0.00014	-0.00014
		(-0.192)		(-0.657)	(-0.654)
STRSEC		0.00026		0.00030	0.00028
		(1.010)		(1.178)	(1.071)
SQN80	-0.0004	-0.00094			-0.00102
	(-0.408)	(-0.931)			(-0.706)
PRIM60			0.03256**	0.03113**	0.03121**
			(3.812)	(3.346)	(3.327)
SEC60			0.00648	-0.00113	0.00573
			(0.502)	(-0.088)	(0.0326)
# of OBS.	89	77	89	77	77
R^2	0.18	0.22	0.27	0.30	0.30
F	4.72	3.36	6.16	4.20	3.68

t-values in parentheses
* represents significance at 5%
** represents significance at 1%

When mean years of schooling(SQN80) and enrollment ratios(PRIM60 and SEC60) are included in the same regression(Eq. 9), SQN80 becomes insignificant while

PRIM60 is significant at 1% significance level. This may be due to high correlation, 0.80, between those two variables.

Table 4 shows regression results when literacy rate in 1960(LIT60) is included instead of GDP60. When PRIM60 is included along with LIT60(Eqs. (13), (14), and (15)), primary school enrollment rate(PRIM60) is significantly positively related to the growth rate at 1% significance level, and LIT60 is negatively related to the growth rate at 1% significance level, which may be interpreted as follows: if literacy rate in 1960(LIT60) is proportional to the level of human capital in 1960, then countries which have lower level of initial human capital grew faster than other countries over the period 1960-1986. But when mean years of schooling(SQN80) is added in the regression along with literacy rate(LIT60)(Eqs. (11) and (12)), neither of the two are significant.

Further, when LIT60 is incorporated instead of GDP60, none of variables which measure the quality of schooling are not significant. But investment ratio(I/GDP) is still significantly positively related to the growth rate.

In Table 5, I examine whether the growth rate estimation for countries located in Africa and Latin America are significantly different from the rest of the world. Eq. (16) incorporates intercept dummies for African(AFRICA) and Latin American(LATIN) countries, Eq. (17) incorporates slope dummies for EDX60 and SQN80, and Eq. (18) includes slope dummies as well as intercept dummies. ASQN80 is a variable which has mean years of schooling for countries in Africa and zero for other countries and LSQN80 is a variable which has mean years for Latin American countries and zero for other countries. AEDX60 and LEDX60 are slope-dummies for educational expenditures, defined as the similar manner to ASQN80 and LSQN80. If coefficients of those slope dummies are significantly different from zero, we conclude that coefficients of the variables for each region are significantly different from the coefficients of the variables for the rest of the world.

When only intercept dummies, LATIN and AFRICA, are added to the regression(Eq. (16)), LATIN is significantly negatively related to the growth rate at 1% significance level, implying that Latin American countries' growth rates were lower than the rest of the world even though they would have had the same conditions as the rest of the world. When variables which estimate slope differences, ASQN80, LSQN80, AEDX60, and LEDX60, are added to Eq. (6)(Eq. (17)), LEDX60 is significantly negatively related to the growth rate at 1% significance level.

When slope dummies as well as intercept dummies are included(Eq. (18)), LEDX is significantly negatively related to the growth rate at 1% significance level. In Eq. (18), coefficients for EDX60 and LEDX60 are 0.00214 and -0.00458, respectively, implying that, even though Latin American countries would have allocated the same percentage of GNP to the educational sector, the marginal effect of the resources on the growth rate for Latin American countries would be lower by 0.00458 or 0.458 percent, than the rest of the world. In other words, Latin American countries have lower productivity in the education sector than the rest of the world. On the other hand, African countries are not significantly different from the rest of the world.

Table 5. Regression results (time period: 1960-1985)
Heteroskedasticity consistent covariance matrix

Independent variables	Dependent variable: growth rate of per capita real GDP (16)	(17)	(18)
CONSTANT	0.04105**	0.0419**	0.0410
	(4.613)	(4.532)	(0.355)
AFRICA	-0.00820		-0.01121
	(-1.465)		(-0.385)
LATIN	-0.01346**		-0.00727
	(-4.148)		(-0.946)
GDP60	-0.00194**	-0.00200**	-0.00188**
	(-4.323)	(-4.714)	(-4.460)
I/GDP	0.00039	0.00039	0.00040
	(1.313)	(1.267)	(1.317)
SQN80	0.00047	0.00051	0.00037
	(0.571)	(0.637)	(0.465)
ASQN80		-0.00168	-0.00023
		(-0.546)	(-0.066)
LSQN80		0.00022	0.00091
		(0.205)	(0.561)
EDX60	0.002201*	0.00301*	0.00214
	(1.813)	(1.890)	(1.471)
AEDX60		-0.00033	0.00174
		(-0.120)	(0.568)
LEDX60		-0.00588**	-0.00458**
		(-2.562)	(-2.073)
DEATH	-0.00125**	-0.00158**	-0.00126**
	(-3.506)	(-4.395)	(-2.835)
# of OBS.	92	92	92
R^2 =	0.41	0.41	0.42
F =	8.27	6.34	5.27

t-values in parentheses
* represents significance at 5%
** represents significance at 1%

Note that, in Table 5, average death rate over the period 1960-1985(DEATH) is included in all equations. The coefficient for DEATH is negatively correlated to the growth rate of the economy as predicted in chapter II.[17]

Table 6. Mean statistic

Variables	Group A Countries with initial GDP below average	Group B Countries with initial GDP above average
GDP60	2.60	11.62
G6085	0.019	0.022
I/GDP	15.04	22.63
EDX60	2.11	3.38
SQN80	2.73	7.10
STRPRI	40.21	30.89
STRSEC	19.89	18.58
DEATH	16.70	9.95

Next I divide countries into two groups: countries with the initial GDP below the average and countries with the initial GDP above the average. Average level of per capital labor real GDP in 1960 was $5,865. Appendix B lists countries for each group. Countries which have the initial level of per capita labor real GDP(GDP60) lower than

[17]Note that annual crude death rate, DEATH, is included in all equations in Table 5. When DEATH is not included in the regression, African countries have an intercept dummy significantly negatively related to the growth rate and a slope coefficient which is not significant. But the dummies for Latin American countries continue to be significant without DEATH in the regression.

the average, Group A, grew 3 percentage slower and invested less(lower I/GDP ratio), on average, than countries with higher initial real GDP, Group B. Their educational statistics are worse than Group B in 1960: educational expenditures-GNP ratio(EDX60) for Group A was 2.11%, compared to 3.38% for Group B and primary-school student-teacher ratio was 40.21, compared to 30.89 for Group B. Mean years of schooling for countries with lower initial GDP was 2.73 years in 1980 while it was 7.10 years for countries with higher initial GDP. Also countries with lower initial GDP has a crude death rate almost twice as high as countries with higher initial GDP.

Apparently poor countries had worse educational statistics and grew slower than rich countries, on average. However, there are some exceptions. For example, Botswana and Lesotho in Africa had per capita labor real GDP of $1,081 and $637 in 1960, respectively, and grew 7.86% and 5.81% per year over the period 1960-1985. Japan and Korea had the initial GDP of $5,684 and $2,730 and grew 5.52% and 5.09% a year, respectively.

Table 7 summarizes regression results for both groups separately. For countries with lower initial GDP, SQN80 is significantly positively related to the growth rate at 1% significance level. EDX60 is significantly positively related to the growth rate at 5% significance level when student-teacher ratios in primary and secondary schools are not included and marginally positively related to the growth rate when the student-teacher ratios are included. The student-teacher ratios have expected signs, but only student-teacher ratio in primary schools(STRPRI) is significant marginally.

Table 7. Regression results (time period: 1960-1985)
Heteroskedasticity consistent covariance matrix

Dependent variable: growth rate of per capita real GDP				
Independent Variables	Group A Countries with initial GDP below average		Group B Countries with initial GDP above average	
	(19)	(20)	(21)	(22)
CONSTANT	0.00149	0.01605*	0.00288	-0.00889
	(0.318)	(1.685)	(0.369)	(-0.683)
GDP60	-0.00395*	-0.00598**	-0.00089*	-0.00109**
	(-2.043)	(-2.562)	(-2.348)	(-2.466)
I/GDP	0.00053	0.00065	0.00121**	0.00103**
	(1.158)	(1.383)	(5.356)	(4.727)
SQN80	0.00413**	0.00547**	-0.00011	0.00038
	(2.779)	(3.482)	(-0.135)	(0.430)
EDX60	0.00452*	0.00430	0.00067	0.00108
	(1.852)	(1.561)	(0.681)	(1.080)
STRPRI		-0.00030		0.00021
		(-1.665)		(0.900)
STRSEC		-0.00016		0.00042
		(-0.506)		(1.336)
# of OBS.	60	51	32	29
R^2	0.31	0.41	0.47	0.56
F	6.20	5.19	6.08	4.72

t-values in parentheses
* represents significance at 5%
** represents significance at 1%

On the other hand, for countries with higher initial GDP, none of educational statistics are significant. In summary, for countries with lower initial GDP, the education sector played an important role in the growth rate while, for countries with higher initial GDP, the education sector played little role.

Finally, I divided countries into another two groups: countries with lower growth rate and countries with higher growth rate over the period 1960-1985. Average growth rate of countries over the period 1960-1985 was 2.01% a year. Table 8 summarizes average values for the variables for each group.

Table 8. Mean statistic

	Group C	Group D
Variables	Countries with growth rate below average	Countries with growth rate above average
GDP60	5.55	6.22
G6085	0.008	0.035
I/GDP	13.77	22.23
EDX60	2.23	2.94
SQN80	3.50	5.21
STRPRI	37.86	35.70
STRSEC	18.51	20.42
DEATH	16.35	11.93

Comparing school statistics for both groups, countries with higher growth rates(Group D), are better at EDX60, STRPRI, and SQN80 and worse at STRSEC, student-teacher ratio in secondary schools than countries with lower growth rate(Group C).

Table 9. Regression results (time period: 1960-1985)
Heteroskedasticity consistent covariance matrix

Dependent variable: growth rate of per capita real GDP				
Independent Variables	Countries with growth rate below average		Countries with growth rate above average	
	(23)	(24)	(25)	26)
CONSTANT	0.00546 (1.563)	0.01950** (3.605)	0.02752 (4.825)	0.02216 (2.085)
GDP60	-0.00043 (-1.240)	-0.00055 (-1.487)	-0.00264** (-3.951)	-0.00254** (-3.060)
I/GDP	-0.00016 (-0.670)	-0.00018 (-0.786)	0.00031 (0.804)	0.00037 (0.860)
SQN80	0.00208** (3.055)	0.00203** (2.974)	0.00114 (1.185)	0.00094 (0.942)
EDX60	-0.00009 (-0.070)	0.00036 (0.227)	0.00355** (2.379)	0.00378** (2.537)
STRPRI		-0.00026** (-3.339)		0.00010 (0.423)
STRSEC		-0.00024 (-1.066)		0.00003 (0.083)
# of OBS.	50	42	42	38
R^2	0.18	0.40	0.28	0.32
F	2.52	3.81	3.63	2.39

t-values in parentheses
* represents significance at 5%
** represents significance at 1%

Table 9 summarizes regression results for each group. For countries grew slower(Group C), mean years of schooling(SQN80) and student-teacher ratio in primary schools(STRPRI) were important factors for the growth of the economy while educational expenditure-GNP ratio(EDX60) is not significant. On the other hand, for countries grew faster(Group D), EDX60 was an important factor for the growth of the economy. Based on the results, it may be argued that faster growth countries mostly relied on the quality(EDX60) of schooling for their growth while slower growth countries mostly relied on the quantity(SQN80) of schooling for their growth.

Chapter V. Conclusions

I examined roles of formal schooling sector on the growth rate of the economy. I assumed, throughout the paper, that people specialize to accumulate their own human capital for multiple time periods before they work in the output sector. When they become adults, people produce output as well as help their children learn knowledge. Adults and their children are tied together by the family welfare function in the paper, which depends on the household consumption. Because they can't accumulate their own human capital in adulthood, adults support their children and help them accumulate human capital. By doing so, the household could increase total stock of human capital and earn higher income in the future.

In chapter II, I assume that individuals face uncertain life-time throughout their adulthood. It was shown, first, that the economy could grow at a positive rate as long as the educational production function as well as output production function exhibit constant returns to scale, which is a well-known property of endogenous growth models. The presence of human capital of children in the educational production function is not required for the economy to grow positively. But it is shown that, if human capital of children is not an input in the educational production function and there is no externality, steady-state growth rate is independent of the period of schooling.

Second, that the steady-state growth rate is negatively related to the rate of death: economies with higher rates of death become more impatient and thus emphasize current consumption relative to future consumption, implying that economies with higher death rates grower slower. Further, economies with higher death rates would experience large decrease in adults' human capital due to the death. Therefore, growth rates of those economies are lower than those with lower death rates.

In chapter III, I assume that individuals live infinitely and the educational production function exhibits constant returns to scale in human capital of children and adults. I found that the effect of years of schooling on the growth of the economy depends on the specification of the educational production function. If the schooling technology only depends on the efforts of adults, the growth rate of the economy is negatively related to the period of schooling. This is so because, if the production function only depends on human capital of adults, the decrease in adult-student ratio due to the increase in the period of schooling would just scale down the productive parameter in the education sector, and thus would reduce the growth of the economy in the steady state. The reason that the growth rate is independent of the period of schooling in chapter II was that adult-student ratio was normalized to one by an assumption.

On the other hand, if the technology depends on human capital of children as well as that of adults, the growth rate of the economy could be positively related to the period of schooling as long as share of children's human capital in the educational production function is high enough, higher than about 0.53, but less than one. When the production function depends on children's human capital in addition to adults', there is a channel which has a positive effect on the growth of the economy. As share of human capital of children increases, this positive effect becomes larger, and finally dominates the negative effect of the period of schooling on the growth of the economy.

It has been little known empirically about the true educational production function. One reason may be that it is almost impossible to measure output in the sector, the level of human capital. This paper indicates that, unless we know the educational production function, we may not be able to draw conclusions about the effect of the period of schooling on the growth rate of the economy. In this sense, an assumption that the period of schooling is a proxy for the level of human capital does not seem to be appropriate. Even though the amount of learning of students would be increased as they attend schools for longer periods, the rate of change of human capital of two consecutive new work-force would fall and so does the growth of the economy when students' learning does not depend on their level of human capital, which is a usual assumption made in the literature.

On the other hand, the quality of schooling, which is measured by the productivity parameter in the education sector, is positively related to the growth rate of the economy.

In chapter IV, I ran several sets of regressions to estimate the effect of the quality of schooling as well as the quantity of schooling on the growth rate of the economy. The quality of schooling, which is measured by the amount of resources allocated to the education sector, had a positive effect on the growth rate of the economy for most of cases. On the other hand, the period of schooling is negatively related to the growth of the economy when the initial level of GDP is not included while it is positively related to the growth rate of the economy once the initial level of GDP is included.

Appendix

Appendix A. Figures for growth rates and *u*

1. When $\alpha=0$

Fig (1a). Growth rate

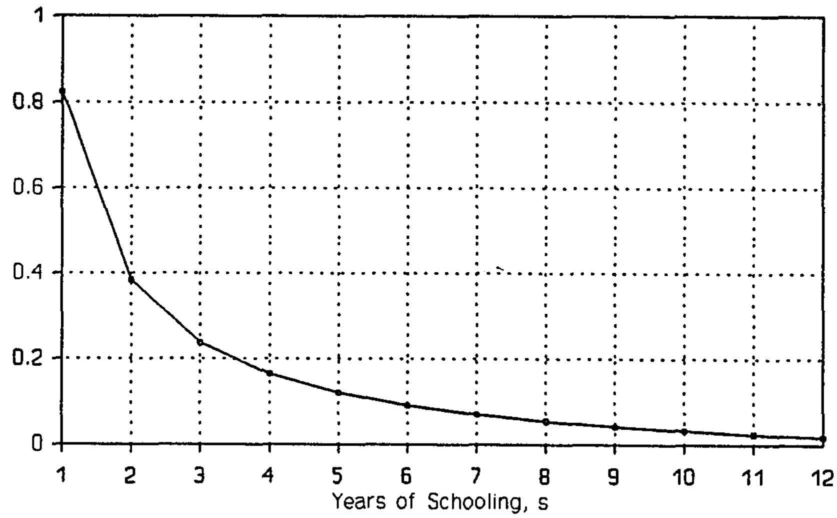
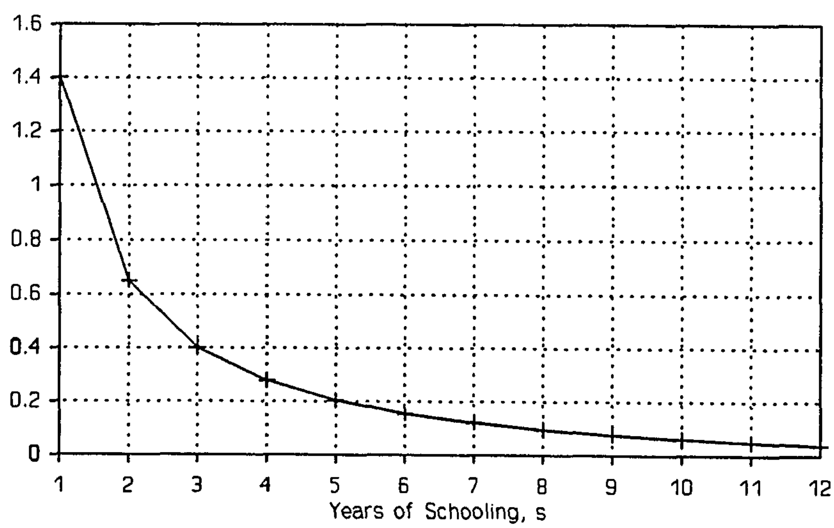


Fig (1b). u



2. When $\alpha=0.1$

Fig (2a). Growth rate

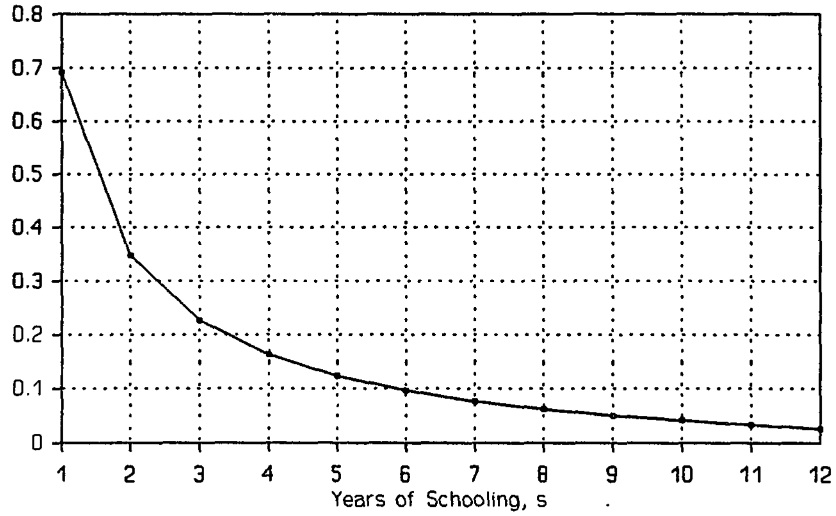
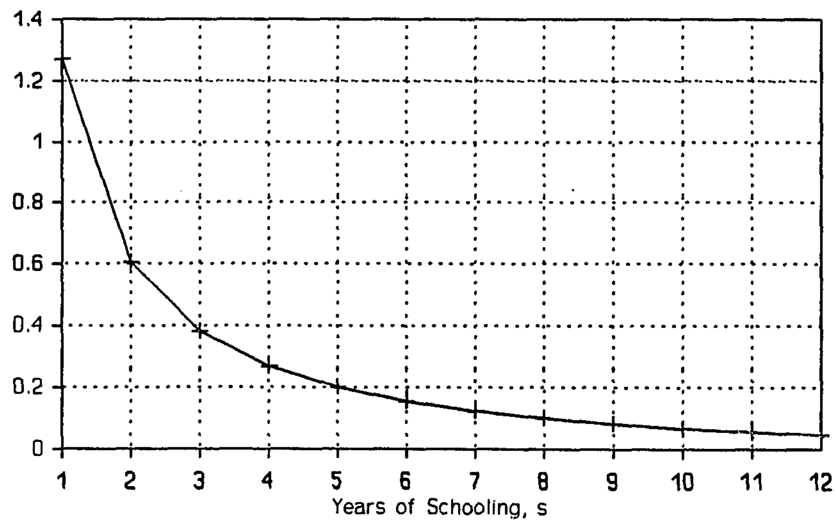


Fig (2b). u



3. When α=0.2

Fig (3a). Growth rate

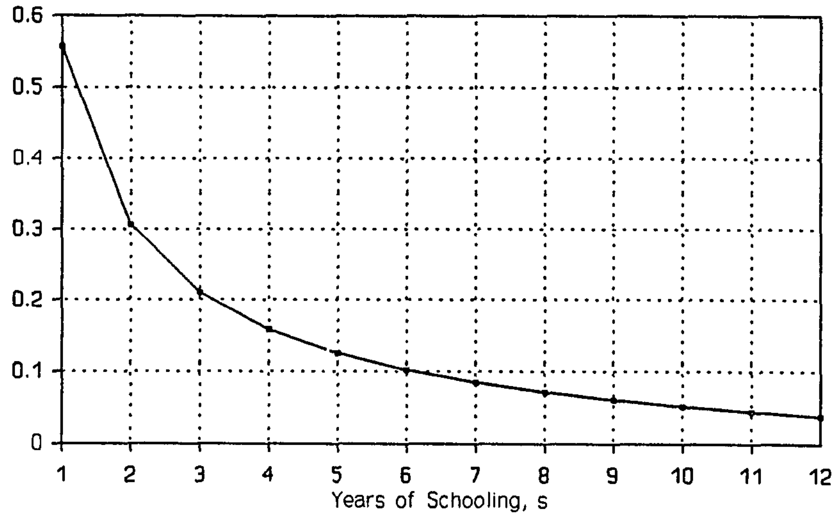
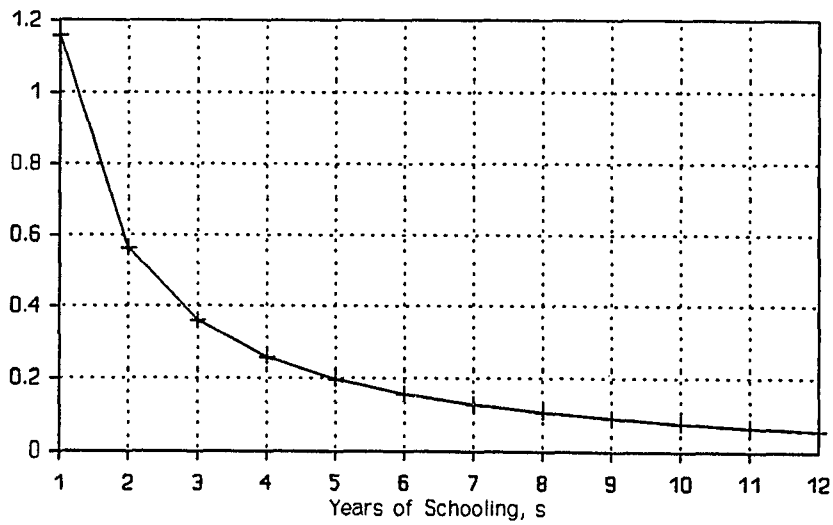


Fig (3b). u



4. When $\alpha=0.3$

Fig (4a) Growth rate

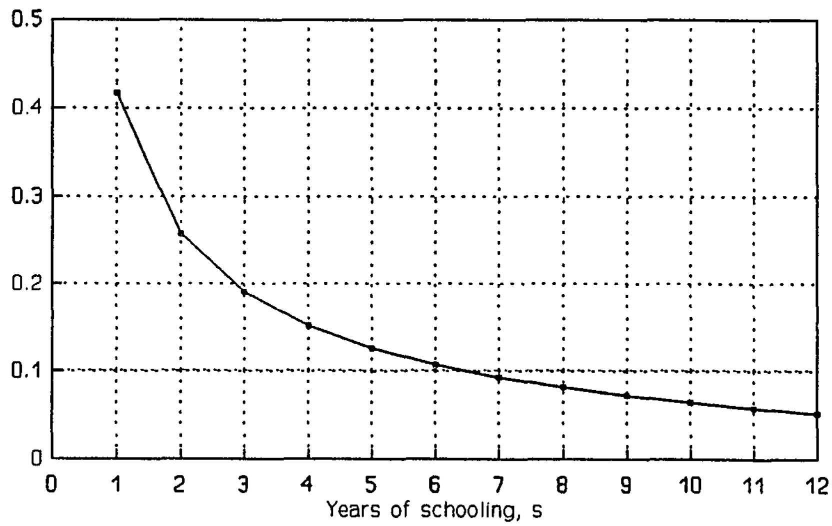
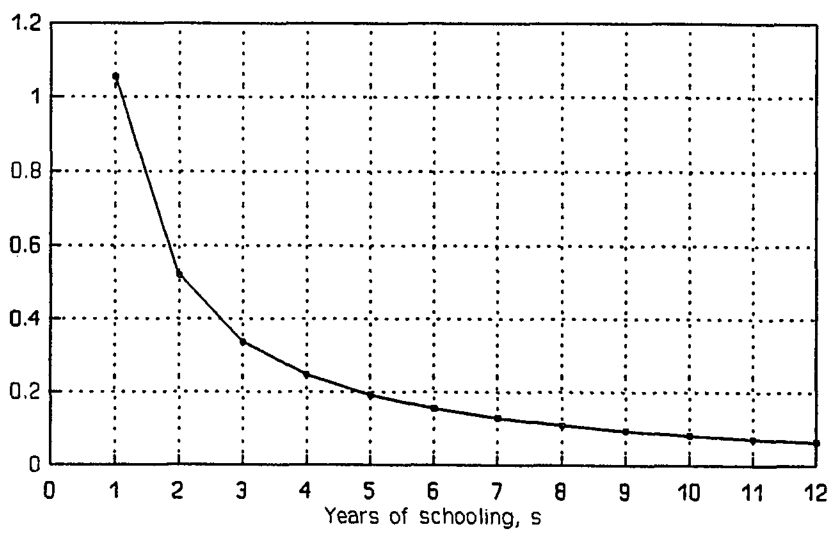


Fig (4b). u



5. When $\alpha=0.4$

Fig (5a). Growth rate

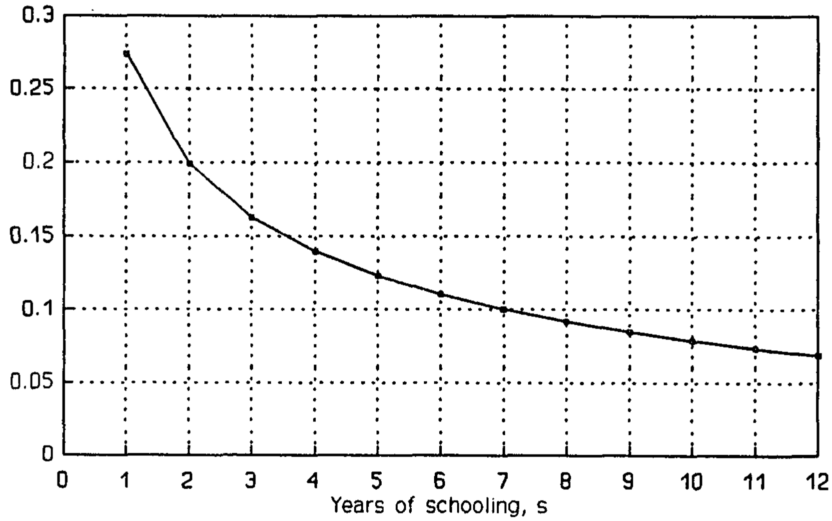
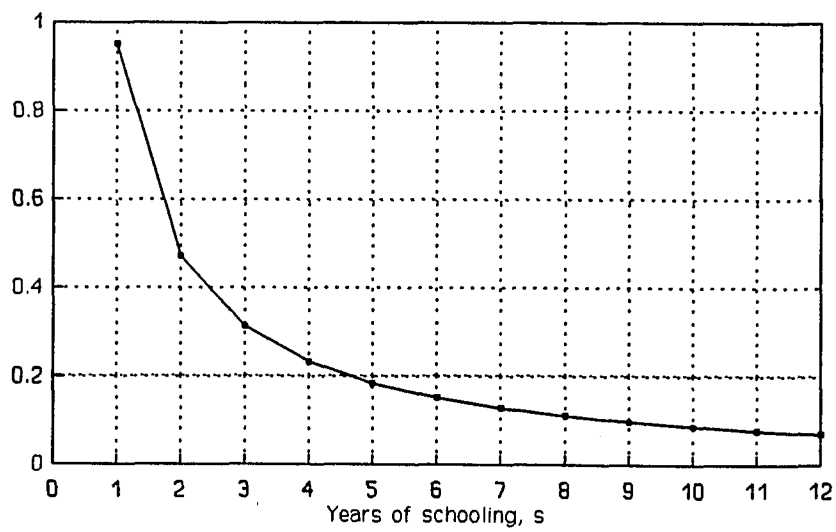
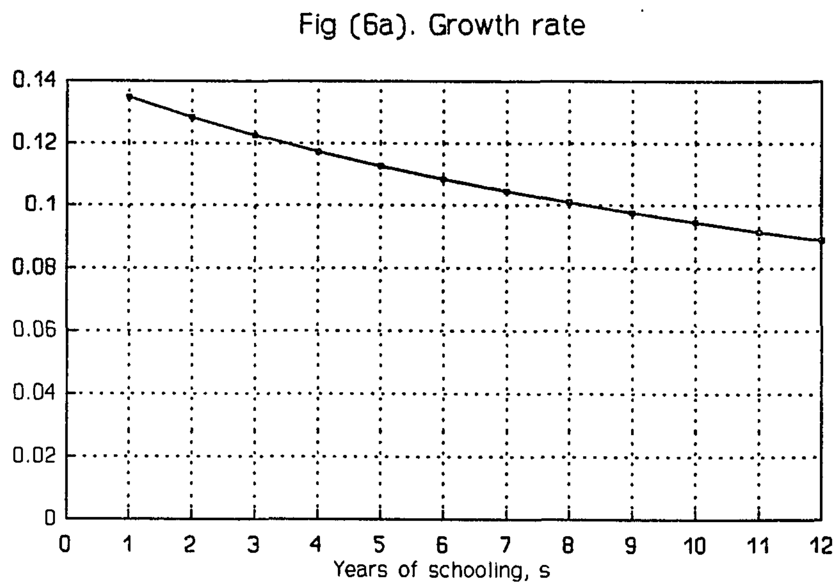
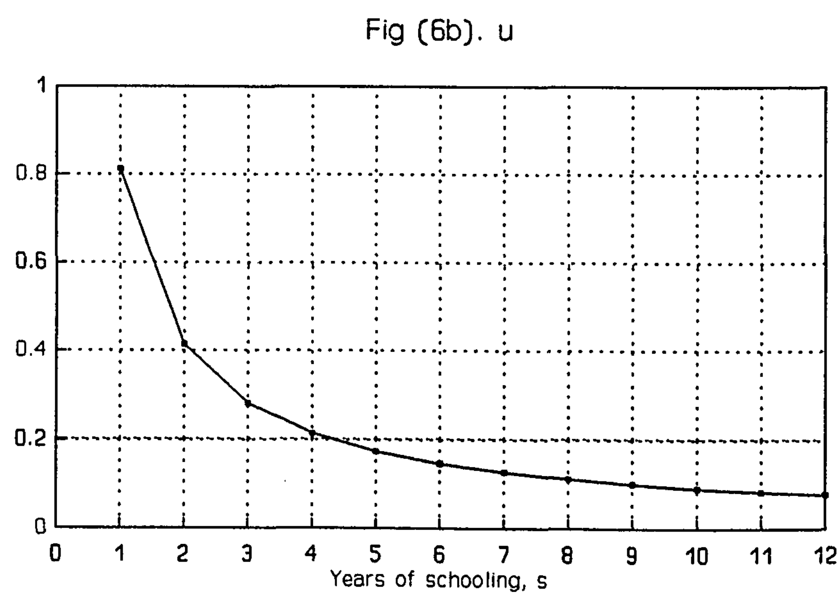


Fig (5b). u



6. When α=0.5





7. When $\alpha=0.6$

Fig (7a). Growth rate

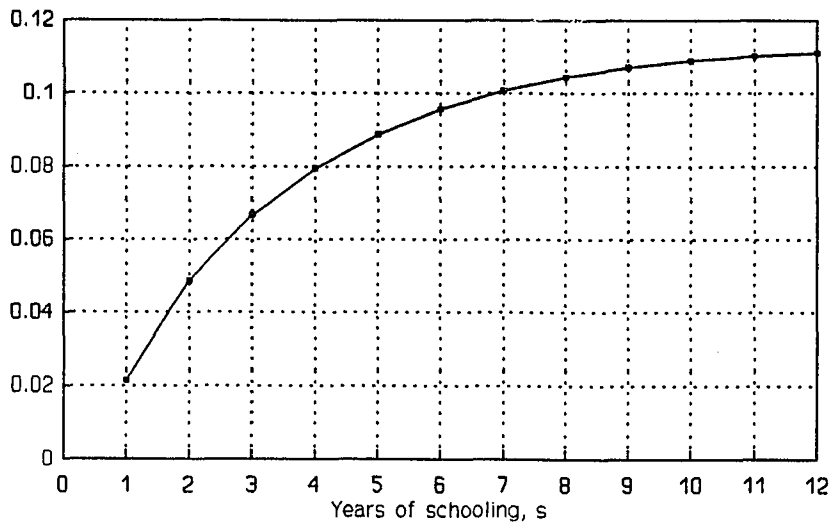
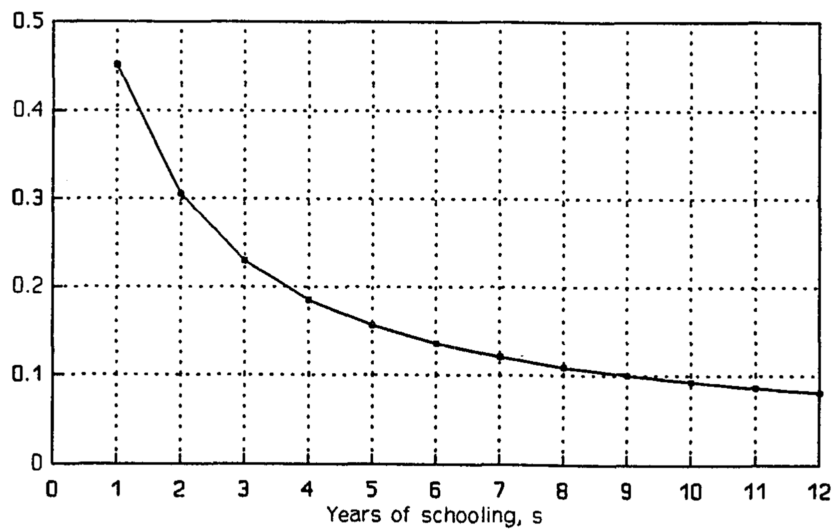
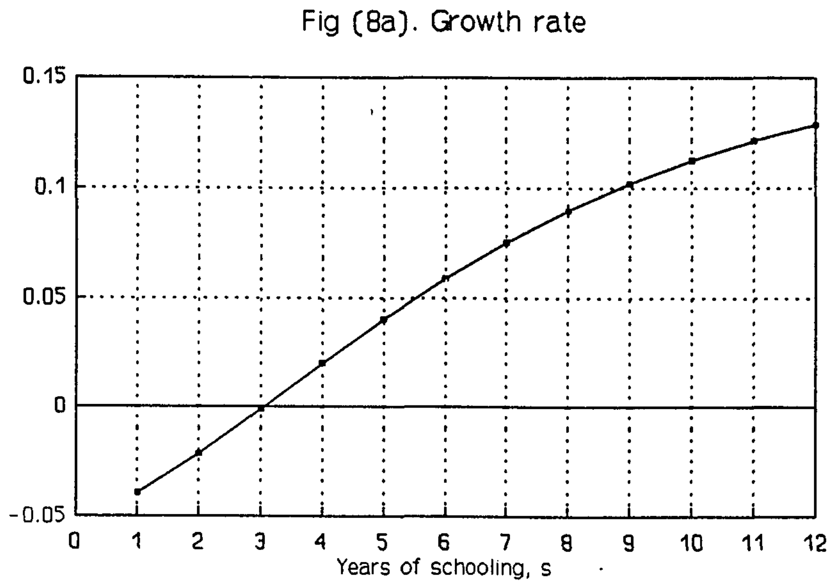
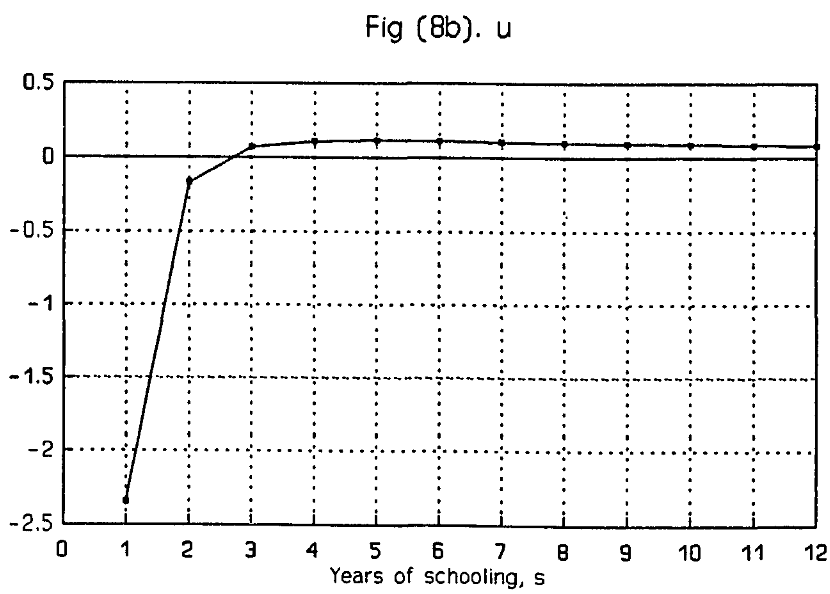


Fig (7b). u



8. When $\alpha=0.7$





9. When $\alpha=0.8$

Fig (9a). Growth rate

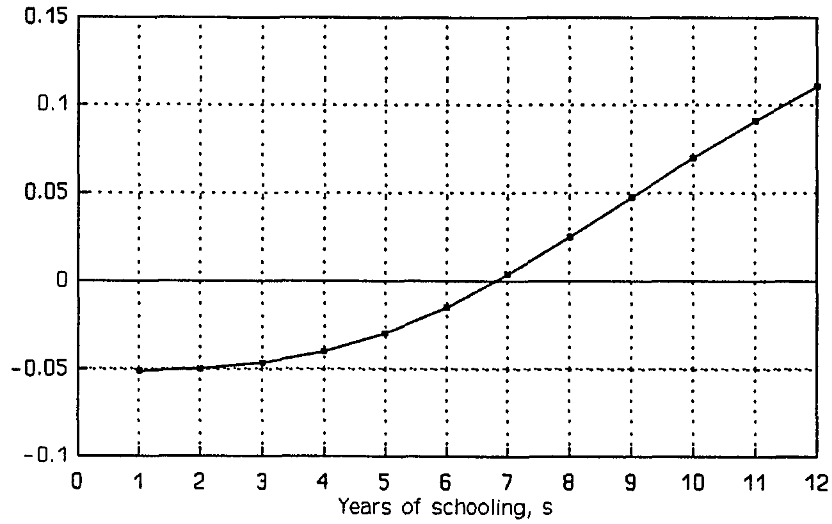
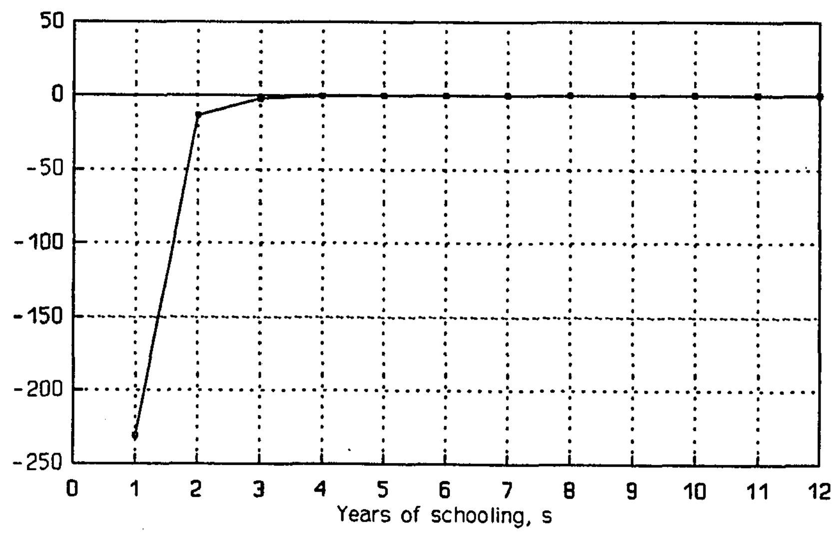


Fig (9b). u



10. When α=0.9

Fig (10a). Growth rate

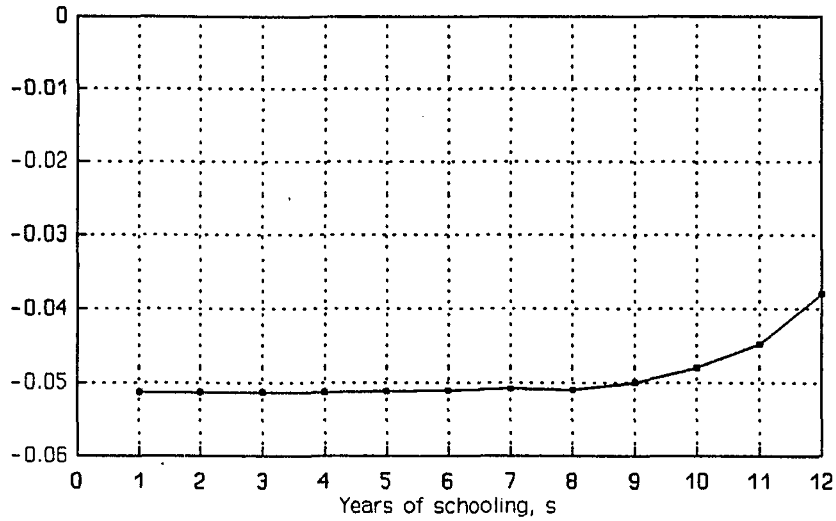
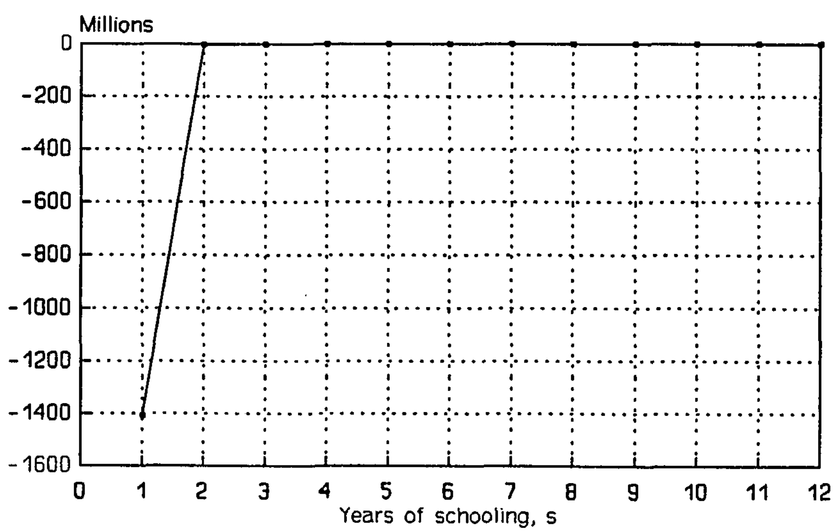


Fig (10b). u



Appendix B: Lists of Countries by Various Groups

1. Group A: Countries with per capita labor real GDP in 1960 below average

Country	Continent	GDP60($1,000)
Angola	AF	2.716
Benin	AF	1.752
Botswana	AF	1.081
Burundi	AF	0.785
Cameroon	AF	1.507
Central Afr. R	AF	1.335
Chad	AF	1.999
Congo	AF	2.368
Egypt	AF	1.917
Ethiopia	AF	0.512
Gambia	AF	0.776
Ghana	AF	2.416
Guinea	AF	0.905
Ivory Coast	AF	1.890
Kenya	AF	1.425
Lesotho	AF	0.637
Liberia	AF	2.308
Madagascar	AF	1.981
Malawi	AF	0.850
Mali	AF	1.371
Mauritania	AF	2.519
Morocco	AF	3.040
Mozambique	AF	2.607
Niger	AF	1.022
Nigeria	AF	3.212
Rwanda	AF	0.963
Senegal	AF	2.700
Sierra Leone	AF	1.878
Somalia	AF	2.151
Sudan	AF	2.771
Swaziland	AF	2.166
Tanzania	AF	0.494
Togo	AF	0.886
Tunisia	AF	4.982
Uganda	AF	0.749
Zaire	AF	0.797
Zambia	AF	3.235

1. Group A: (Continued)

Country	Continent	GDP60($1,000)
Zimbabwe	AF	2.122
Bolivia	AM	3.306
Brazil	AM	4.367
Dominican Rep.	AM	4.373
Ecuador	AM	4.614
El Salvador	AM	4.000
Guatemala	AM	5.213
Guyana	AM	5.413
Haiti	AM	1.728
Honduras	AM	2.820
Jamaica	AM	4.461
Panama	AM	4.595
Paraguay	AM	3.744
Bangladesh	AS	1.831
Burma(Myanmar)	AS	0.743
Cyprus	AS	4.972
India	AS	1.394
Japan	AS	5.684
Jordan	AS	5.127
Korea, South	AS	2.730
Malaysia	AS	5.152
Nepal	AS	1.190
Pakistan	AS	2.389
Philippines	AS	3.100
Sri Lanka	AS	3.869
Thailand	AS	1.934
Turkey	AS	3.288
Greece	EU	4.647
Malta	EU	5.250
Portugal	EU	4.257
Papua N. Guinea	OC	2.104

2. Group B: Countries with per capita labor real GDP in 1960 above average

Country	Continent	GDP60($1,000)
Algeria	AF	6.340
Mauritius	AF	7.115
South Africa	AF	8.477
Argentina	AM	8.595
Barbados	AM	8.740
Canada	AM	20.816
Chile	AM	9.524
Colombia	AM	6.192
Costa Rica	AM	7.147
Mexico	AM	9.923
Nicaragua	AM	5.921
Peru	AM	6.659
Suriname	AM	7.998
Trinidad & Tobag	AM	13.128
U.S.A.	AM	24.650
Uruguay	AM	10.855
Venezuela	AM	12.252
Hong Kong	AS	5.966
Israel	AS	11.024
Singapore	AS	7.334
Syria	AS	6.448
Austria	EU	9.322
Belgium	EU	13.516
Denmark	EU	12.920
Finland	EU	10.326
France	EU	12.367
Germany, West	EU	12.786
Iceland	EU	13.651
Ireland	EU	8.141
Italy	EU	10.598
Luxembourg	EU	16.633
Netherlands	EU	15.745
Norway	EU	13.863
Spain	EU	7.113
Sweden	EU	14.821
Switzerland	EU	19.935
U.K.	EU	13.766
Australia	OC	17.753
Fiji	OC	8.509
New Zealand	OC	19.378

3. Group C: Countries with growth rates below average

Country	Continent	GDP60($1,000)	Growth Rate
Angola	AF	2.716	-1.33%
Benin	AF	1.752	1.04
Burundi	AF	0.785	0.99
Central Afr. R.	AF	1.335	0.21
Chad	AF	1.999	-1.26
Ethiopia	AF	0.512	1.35
Ghana	AF	2.416	-0.44
Guinea	AF	0.905	0.67
Kenya	AF	1.425	1.47
Liberia	AF	2.308	0.39
Madagascar	AF	1.981	-1.02
Malawi	AF	0.850	1.95
Mali	AF	1.371	0.03
Mauritania	AF	2.519	0.38
Mauritius	AF	7.115	1.30
Mozambique	AF	2.607	-2.20
Niger	AF	1.022	0.80
Nigeria	AF	3.212	-0.40
Rwanda	AF	0.963	1.62
Senegal	AF	2.700	-0.12
Sierra Leone	AF	1.878	1.54
Somalia	AF	2.151	0.22
South Africa	AF	8.477	1.68
Sudan	AF	2.771	0.25
Uganda	AF	0.749	0.71
Zaire	AF	0.797	0.67
Zambia	AF	3.235	-1.39
Argentina	AM	8.595	1.03
Barbados	AM	8.740	1.36
Bolivia	AM	3.306	1.69
Canada	AM	20.816	1.47
Chile	AM	9.524	0.45
Costa Rica	AM	7.147	1.33
El Salvador	AM	4.000	0.56
Guatemala	AM	5.213	1.60
Guyana	AM	5.413	-2.38
Haiti	AM	1.728	0.46
Honduras	AM	2.820	1.58
Jamaica	AM	4.461	0.52
Mexico	AM	9.923	1.94

3. Group C: (Continued)

Country	Continent	GDP60($1,000)	Growth Rate
Nicaragua	AM	5.921	0.20%
Peru	AM	6.659	0.99
Trinidad& Tobag	AM	13.128	1.64
U.S.A.	AM	24.650	1.34
Uruguay	AM	10.855	0.18
Venezuela	AM	12.252	1.25
Bangladesh	AS	1.831	1.16
India	AS	1.394	1.06
Nepal	AS	1.190	1.60
Philippines	AS	3.100	1.86
Sri Lanka	AS	3.869	1.23
Switzerland	EU	19.935	1.56
U.K.	EU	13.766	1.90
Australia	OC	17.753	1.67
Fiji	OC	8.509	0.90
New Zealand	OC	19.378	0.61

4. Group D: Countries with growth rates above average

Country	Continent	GDP60($1,000)	Growth Rate
Algeria	AF	6.340	3.34%
Botswana	AF	1.081	7.86
Cameroon	AF	1.507	4.58
Congo	AF	2.368	4.45
Egypt	AF	1.917	5.41
Gambia	AF	0.776	3.34
Ivory Coast	AF	1.890	2.68
Lesotho	AF	0.637	5.81
Morocco	AF	3.040	3.19
Swaziland	AF	2.166	3.52
Tanzania	AF	0.494	2.77
Togo	AF	0.886	2.45
Tunisia	AF	4.982	2.88
Zimbabwe	AF	2.122	2.06
Brazil	AM	4.367	3.73
Colombia	AM	6.192	2.02
Dominican Rep.	AM	4.373	2.04
Ecuador	AM	4.614	2.78
Panama	AM	4.595	3.35
Paraguay	AM	3.744	2.58
Suriname	AM	7.998	2.22
Burma(Myanmar	AS	0.743	2.73
Cyprus	AS	4.972	4.44
Hong Kong	AS	5.966	4.83
Israel	AS	11.024	3.23
Japan	AS	5.684	5.52
Jordan	AS	5.127	3.45
Korea, South	AS	2.730	5.09
Malaysia	AS	5.152	3.47
Pakistan	AS	2.389	2.73
Singapore	AS	7.334	4.44
Syria	AS	6.448	4.68
Thailand	AS	1.934	3.77
Turkey	AS	3.288	3.37
Austria	EU	9.322	3.53
Belgium	EU	13.516	2.52
Denmark	EU	12.920	2.15
Finland	EU	10.326	3.10
France	EU	12.367	2.93
Germany, West	EU	12.786	2.58

4. Group D: (Continued)

Country	Continent	GDP60($1,000)	Growth Rate
Greece	EU	4.647	4.80%
Iceland	EU	13.651	2.03
Ireland	EU	8.141	2.60
Italy	EU	10.598	3.74
Luxembourg	EU	16.633	2.28
Malta	EU	5.250	4.09
Netherlands	EU	15.745	2.19
Norway	EU	13.863	2.78
Portugal	EU	4.257	3.51
Spain	EU	7.113	3.80
Sweden	EU	14.821	2.01
Papua N. Guinea	OC	2.104	2.03

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