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Grand Unified Theories Based on SU(N) as a Gauge Group

by

Milorad B. Popovic

A dissertation submitted to the Graduate Faculty in Physics in partial fulfillment of the requirement for the degree of Doctor of Philosophy, The City University of New York.

1982

Abstract

Grand Unified Theories Based on SU(N) as a Gauge Group

by

Milorad B. Popovic

Advisor: Professor Rabindra Nath Mohapatra

We discuss possible extensions of the minimal grand unified theory based on SU(5). We stress two basic problems of any unifing scheme: namely, the problem of generations and the possibility of having intermediate mass scales between the grand unified scale and the scale of weak interactions. We show how the principle of asymptotic freedom and the "anomaly free" condition can be used to put limits on the number of generations. In the second part of this work we study the maximal grand unification symmetry based on the SU(16) group and derive limits (using renormalization group equations) on intermediate mass scales for various possible chains of symmetry breaking. We show in particular that, in this model, the scale of grand unification can be as low as 10^5GeV without any conflict with observations. One can also have parity restoration at low energies (of order 200 GeV) provided we allow $\sin^2\theta_W = .27$. Detailed phenomenological implications of the SU(16) model for baryon non-conservation are discussed.

Acknowledgments

Most of the work described in this thesis has been done under the supervision and in collaboration with my adviser Professor Rabindra Nath Mohapatra, who made the way easier. I enjoyed our collaboration and benefited from it. For that I am and will be deeply grateful to him. I also wish to acknowledge many useful and pleasant hours spent with graduate students and professors from the High Energy Group at City College. Special thanks is due to my wife Ljiljana who provided encouragement when it was needed most.

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<u>Table Captions</u>

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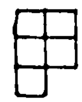

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I. Introduction[1]

At the present time high energy physics explores the structure of the elementary particles at distances of the order of 10^{-15}-10^{-16}cm. It appears that we can describe the world at those distances (or larger) rather well by two types of gauge interactions; flavor interactions[2] ($SU(2)_L \times U(1)_Y$ gauge group) and strong interactions[3] ($SU(3)_C$ gauge group).

Quarks and leptons are transformed under these gauge groups as follows:

1.) All left handed quarks and leptons are doublets under $SU(2)_L$ and all right handed fermion fields are singlets.

2.) Under $SU(3)_C$ quarks are triplet and leptons are singlet.

Comparing the masses of quarks and leptons leads one to group them into "families":

$$\begin{pmatrix} \nu_e & \vdots & u_L^R\, u_L^Y\, u_L^B \\ \vdots & & \\ e_L^- & \vdots & d_L^R\, d_L^Y\, d_L^B \end{pmatrix}, \begin{pmatrix} \nu_\mu & \vdots & c_L^R\, c_L^Y\, c_L^B \\ \vdots & & \\ \mu_L^- & \vdots & s_L^R\, s_L^Y\, s_L^B \end{pmatrix}, \begin{pmatrix} \nu_\tau & \vdots & t_L^R\, t_L^Y\, t_L^B \\ \vdots & & \\ \tau_L & \vdots & b_L^R\, b_L^Y\, b_L^B \end{pmatrix}, \ldots \qquad (I.1)$$

Each generation of fermions transforms under $SU(3)_C \times SU(2)_L$ as

$$\underset{(1,2)}{\begin{pmatrix} \nu_e \\ e_L^- \end{pmatrix}}, \quad \underset{(3,2)}{\begin{pmatrix} u_L^i \\ d_L^i \end{pmatrix}}, \quad \underset{(1,1)}{e_L^+}, \quad \underset{(\bar{3},1)}{u_L^{ic}}, \quad \underset{(\bar{3},1)}{d_L^{ic}}. \qquad (I.2)$$

The $SU(2)_L$ doublets are arranged in a column and $SU(3)_c$ triplets are indicated by superscript i. The numbers in the brackets indicate transformation properties under $SU(3)_c \times SU(2)_L$. The subscript L means that all states are left handed fields and the superscript c refers to change conjugation, defined as follows:

$$u_L^c = \frac{1}{2}(1-\gamma_5)C\gamma_0 u^* = i\gamma_2 u_R^* \qquad (I.3)$$

It is supposed that at an energy of 10^2 GeV symmetry is spontaneously broken from

$$SU(3)_c \times SU(2)_L \times U(1)_Y \ \text{to} \ SU(3)_c \times U(1)_{em} \ . \qquad (I.4)$$

The eight gauge bosons (gluons) associated with the unbroken $SU(3)_c$ gauge group remain massless.

The electro-magnetic group $U(1)_{em}$ is generated by the generator

$$Q = T_{3L} + Y \ , \qquad (I.5)$$

if T_{3L} and Y are the generators of $SU(2)_L$ and $U(1)_Y$, respectively. This breaking is realized by a doublet of Higgs scalars,

$$\Phi = \begin{pmatrix} \Phi^+ \\ \Phi^0 \end{pmatrix} \qquad \text{with V.E.V.,} \ \langle 0|\Phi|0\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ 1 \end{pmatrix} \qquad (I.6)$$

The masses M_W, M_Z of the gauge bosons $W^{\pm}$, Z^0, which are associated with three broken generators of $SU(2)_L \times U(1)_Y$, can be read from the kinetic energy term of Higgs field .

$$(D_\mu \Phi)^+ (D^\mu \Phi) = \frac{1}{2}(0, v)\left[g_L \vec{T} \vec{A}_\mu + g' Y B_\mu\right] + \cdots$$

$$= M_W^2 \, W^{+\mu} W^-_\mu + \frac{M_Z^2}{2} Z_\mu Z^\mu , \tag{I.7}$$

where $\vec{A}_\mu$ and $\vec{B}_\mu$ are gauge bosons associated with $SU(2)_L$ and $U(1)_Y$. The g_L and g' are coupling constants of $SU(2)_L$ and $U(1)_Y$, respectively. The $W^\pm$ and Z° are charged and neutral gauge fields,

$$W^\pm = \frac{1}{\sqrt{2}}\left(A^1_\mu \mp A^2_\mu\right) , \quad Z^\circ_\mu = \frac{g' B_\mu - g_L A^3_\mu}{\sqrt{g'^2 + g_L^2}} = \sin\theta_W B_\mu - \cos\theta_W A^3_\mu$$

with masses (I.8)

$$M_W^2 = \frac{1}{4} g_L^2 v^2 \quad , \quad M_Z^2 = (g_L^2 + g'^2)\frac{v^2}{4}$$

The fourth gauge boson,

$$A_\mu = \frac{g_L B_\mu + g' A^3_\mu}{\sqrt{g'^2 + g_L^2}} = \cos\theta_W B_\mu + \sin\theta_W A^3_\mu \tag{I.9}$$

which is associated with the $U(1)_{em}$, remains massless and represents the photon.

The gauge covariant kinetic energy terms for fermions of each generation ($f_{L/R}$ stands for quarks or leptons)

$$\mathcal{L}_f = \sum_f \left(\bar{f}_L i \not{D} f_L + \bar{f}_R i \not{D} f_R \right) \quad , \quad D_\mu = \partial_\mu - i g_L \vec{T}\cdot\vec{A}_\mu - i g' Y B_\mu \,, \qquad \text{(I.10)}$$

can be written now in terms of $W^\pm$, Z^0 and A_μ gauge bosons. The result is:

$$\mathcal{L} = \text{kinetic energy terms} + \frac{g_L}{2\sqrt{2}} \left(J^\mu_W W^-_\mu + J^{+\mu}_W W^+_\mu \right) +$$

$$+ \frac{g_L g'}{\sqrt{g_L^2 + g'^2}} J^\mu_{em} A_\mu - \frac{\sqrt{g_L^2 + g'^2}}{2} J^\mu_Z Z_\mu \,. \qquad \text{(I.11)}$$

Where: $J^\mu_W = \sum \bar{f}_L \gamma^\mu \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} f_L$ is the charged weak current,

$J_{em} = \sum_f \bar{f}_L \gamma^\mu Q f_L$ is the electromagnetic current,

and $J_Z = \sum_f \bar{f} \gamma^\mu \left[T_{3L}(1-\gamma_5) - 2Q \sin^2\theta_W \right] f$ is the weak neutral current.

The fermion masses are generated through Yukawa interactions,

$$\mathcal{L}_Y = h_u \bar{q}_L \tilde{\Phi} u_R + h_d \bar{q}_L \Phi d_R + h_e \bar{l}_L \Phi e_R + h.c. \quad ,$$

where $\quad q_L = \begin{pmatrix} u \\ d \end{pmatrix}_L \; , \; l_L = \begin{pmatrix} \nu \\ e \end{pmatrix}_L$ and $\tilde{\Phi} = i \sigma_2 \Phi^+ \,. \qquad$ (I.12)

While phenomenologically successful, this description is clearly incomplete and inadequate as a complete physical theory. The gauge coupling constants g_C, g_L, g' are all different in magnitude and undetermined.

There is no explanation for charge quantization. The Weinberg angle Θ_W is a free parameter. Fermion masses are arbitrary and there is not any relation between them. There is no explanation for the V-A structure of weak interactions.

In grand unified theories one tries to remove some of these defects by regarding $SU(3)_C \times SU(2)_L \times U(1)_Y$ as the low energy relic of some unified theory. The hypothesis of grand unification, then, not only removes some of the arbitrariness of the $SU(3)_C \times SU(2)_L \times U(1)_Y$ theory, but also predicts new phenomena connected with the higher local symmetry.

In this work, we study some specific aspects of the grand unification program, and arrive at specific predictions described below.

In Chapter II we review the simplest and most popular grand unified theory based on the SU(5) gauge group.

We investigate the problem of incorporating the fermion generations within the framework of a grand unified SU(N) gauge theory in Chapter III. The results of this chapter can be summarized in the following way:

1. If we assume that the light generations of fermions transform as vectors under the horizontal group, and require that the theory is anomaly free, then the requirement for asymptotically free gauge coupling will limit the number of light-fermion generations to four.

2. If we adopt the so-called survival hypothesis and

require that the theory is anomally free, then the requirement for asymptotic free gauge coupling will limit the number of light-fermion generations to eleven. We present the results of our investigation in Tables II and III.

In Chapter IV we study the constraints on the hierarchy of gauge boson mass scales from low energy physics in the maximal grand unification model based on the group SU(16). We show in particular that, in this model, the scale of grand unification can be as low as 10^5 GeV without any conflict with observations. One can also have parity restoration at low energies (of the order of 200 GeV) provided we allow $\sin^2\theta_W = .27$. For high mass unification, the dominant $\Delta B \neq 0$ process is the $\Delta(B-L)=0$ process, ($p \to e^+\pi^0$), while for low mass unification, that is, $M_U \lesssim 10^5$GeV, the only possible $\Delta b \neq 0$ process is N-$\bar{N}$ oscillation with $\tau_{N\bar{N}} > 10^7$-10^8 seconds. In either case, if $\sin^2\theta_W = .27$ and $\alpha_s = .10 - .12$, one obtains a low value for M_{W_R}, the scale of right-handed interactions.

II. Review of Grand Unified Theory Based on SU(5) Gauge Group[4]

As noted in the Introduction the different gauge groups of the Standard Model are independant of each other, that is, they commute and the total gauge group is given by the direct product

$$G = SU(3)_C \times SU(2)_L \times U(1)_Y \ . \tag{II.1}$$

It is natural to attempt to find a single group which will contain G as a subgroup. The group G has rank 4, so the unifying group must have rank $\geq$ 4. The smallest group which allows complex representations of rank 4 is SU(5)[5].

The $\bar{5}$ representation of SU(5) decomposes under $SU(3)_C \times SU(2)_L$ as

$$\bar{5} = (\bar{3},1) + (1,2) \quad , \tag{II.2}$$

while the two indices antisymmetric representation $\underline{10}$ (an antisymmetric product of two 5's) decomposes as

$$\underline{10} = (1,1) + (3,2) + (\bar{3},1) \quad . \tag{II.3}$$

The reducible representation $\bar{5} \oplus \underline{10}$ of SU(5) has the same $SU(3)_C \times SU(2)_L$ decomposition as one generation of fermions (see (I.2)). The only ambiquity we must resolve is which of the either two (3,1) in eq. (II.2) and eq. (II.3) is to be identified with d_L^c or u_L^c. Since SU(5) must break down to $SU(3)_c \times U(1)_{em}$, the electromagnetic charge has to be a generator of SU(5). Therefore, the sum of electric charges over any representation must be zero. Applying this requirement to $\bar{5}$, we see that each member of (3,1) in eq. (II.2) must have Q=1/3 and it must be identified with d_L^c.

Thus fermions of each generation are transformed as the reducible representation $\bar{5} \oplus 10$ under SU(5):

$$\bar{5} = \begin{pmatrix} d^{cR} \\ d^{cY} \\ d^{cB} \\ e^- \\ \nu \end{pmatrix}_L , \qquad \underline{10} = \begin{pmatrix} 0 & u^{cB} & -u^{cY} & u^R & d^R \\ -u^{cB} & 0 & u^{cR} & u^Y & d^Y \\ u^{cY} & -u^{cR} & 0 & u^B & d^B \\ -u^R & -u^Y & -u^B & 0 & e^+ \\ -d^R & -d^Y & -d^B & -e^+ & 0 \end{pmatrix}_L . \qquad \text{(II.4)}$$

The gauge bosons transform in the adjoint representation $\underline{24}$ of SU(5). The adjoint representation of SU(5) decomposes under $SU(3)_c \times SU(2)_L$ as

$$\underline{24} = (8,1) + (1,3) + (1,1) + (3,2) + (3,2). \qquad \text{(II.5)}$$

Beside gluons (8,1) and electroweak gauge bosons $W^{\pm}, \gamma, Z^0$, (1,3) + (1,1), the SU(5) theory contains an additional set of gauge bosons, the so called lepto-quark X,Y gauge bosons. They transform as $(3,2) + (\bar{3},2)$ under $SU(3)_c \times SU(2)_L$.

The SU(5) symmetry is broken in two stages:

$$SU(5) \xrightarrow[M_x]{} SU(3)_c \times SU(2)_L \times U(1) \xrightarrow[10^2\ \text{GeV}]{} SU(3) \times U(1)_{em}. \qquad \text{(II.6)}$$

The first stage is realized by an adjoint 24 of Higgs with the vacuum expectation value

$$\langle 0|\Phi|0\rangle = V \left(\begin{array}{ccc:cc} 1 & & & & \\ & 1 & & & \\ & & 1 & & \\ \hdashline & & & -3/2 & \\ & & & & -3/2 \end{array} \right), \qquad \text{(II.7)}$$

which clearly respects $SU(3)_c \times SU(2)_L \times U(1)$ symmetry. This vacuum expectation value gives masses to X and Y gauge bosons

$$M_x^2 = M_Y^2 = \frac{25}{8} g^2 V^2 . \qquad \text{(II.8)}$$

where g is the SU(5) gauge coupling constant.

The second stage of symmetry breaking is realized by 5 dimensional Higgs of SU(5) with vacuum expectation value

$$\langle 0|H|0\rangle = \frac{v_0}{\sqrt{2}} \left| \begin{array}{c} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{array} \right| . \qquad \text{(II.9)}$$

Having displayed the content of the representations employed, we can make the following comments about minimal grand unified theory based on the SU(5) gauge group:

1. The representations $\bar{5}$ and $\underline{10}$ have opposite anomaly numbers and the theory is anomaly free, which is a vital consistency condition for the theory to be renormalizable. The anomaly number A(R) for any representation is defined[6] as

$$A(R)\, d_{ijk} = \mathrm{Tr}\left[T_i \{T_j, T_k\}\right] , \tag{II.10}$$

where the T_i's are the fermion representation matrices. The d_{ijk} are defined in terms of generators t_i for the fundamental representation of an SU(N) group

$$d_{ijk} = \mathrm{Tr}\left[t_i \{t_j, t_k\}\right] . \tag{II.11}$$

More about anomalies will be said in Chapter III.

2. The representations $\bar{5}$ and $\underline{10}$ have $SU(3)_C \times SU(2)_L$ decomposition (see (I.2)), as does a generation of fermions, and the theory reproduces the usual strong, weak and electro-magnetic phenomenology. The representations $\bar{5}$ and $\underline{10}$ can accomodate only one generation of fermions. It is assumed that the other generations are simple repetitions of the first generation.

3. The X and Y gauge bosons couple the first 3 and the last 2 indices and mediate interactions like those in Figure 1. These interactions lead to the decay of protons

and bound neutrons into leptons and mesons in the second order of the gauge coupling. The baryon lifetime is

$$\tau_B \sim \frac{M_X^4}{M_P^5} \qquad \text{(II.12)}$$

and to prevent fast decay ($\tau_B > 10^{30}$ years), masses of X and Y gauge bosons must not be less than 10^{14} GeV.

4. As already noted, the electromagnetic charge Q is a generator of SU(5) so that the sum of the electromagnetic charges in any representation must be zero. Taking 5 as an example we have

$$3Q_{d^c} + Q_{e^-} = 0 \quad \Rightarrow \quad Q_d = -1/3 \ , \qquad \text{(II.13)}$$

therefore, we can explain charge quantization.

5. The conventional definition of $\sin^2\theta_W$ is

$$\sin^2\theta_W = \frac{g'^2}{g_L^2 + g'^2} = \frac{e^2}{g_L^2} \ , \qquad \text{(II.14)}$$

where g_L and g' are coupling constants of $SU(2)_L$ x $U(1)_Y$, respectively. Since Q is the generator of SU(5), it must be of the form

$$Q = T_3 + CT_0, \qquad \text{(II.15)}$$

where T_3 is the generator of SU(2) and T_o is a weak isosinglet normalized appropriately for a generator of SU(5). For any representation of SU(5) the following equation is satisfied

$$\sum_{5} Q^2 = (1+c^2) \sum_{5} T_3^2 \; . \qquad \text{(II.16)}$$

If we take the $\overline{5}$ representation, then,

$$\sum_{\overline{5}} Q^2 = \frac{4}{3} \quad , \quad \sum_{\overline{5}} T_3^2 = \frac{1}{2} \qquad \text{(II.17)}$$

and $c^2=5/3$. This means that the conventional $U(1)_Y$ gauge coupling g' will be equal to

$$g' = \sqrt{\frac{5}{3}} \; g_1 \; , \qquad \text{(II.18)}$$

where g_1 is the coupling constant of U(1) in (II.6). In the symmetry limit when SU(5) is good symmetry, all coupling constants become equal and from eq. (II.14) we have

$$\sin^2\theta_W = \frac{3}{8} \; . \qquad \text{(II.19)}$$

The value of the Weinberg angle is determined. Luckily enough, this prediction gets dramatically renormalized due to the large symmetry breaking. The renormalization group equations for $g_c=g_3$, $g_L=g_2$, and g_1 gauge coupling constants in the leading logaritmic variations[7] are

$$\frac{1}{g_i^2(M_W^2)} = \frac{1}{g^2(M_X^2)} + 2\, b_i \ln \frac{M_X}{M_W} \; , \qquad \text{(II.20)}$$

where

$$b_i = -\frac{1}{16\pi^2}\left\{\frac{11}{3}C_2(N) - \frac{4}{3}T_f(R) - \frac{1}{6}T_H(R)\right\} \qquad \text{(II.20)}$$

$C_2(N)$ is the quadratic Casimir operator[8] for the adjoint representation. $T_f(R)$ and $T_H(R)$ are the Casimir operators for fermion representations and Higgs representations, respectively. If one ignores the Higgs contributions, the renormalization group equations for three coupling constants g_i can be written in the following way[7]:

$$\frac{\alpha(M_W^2)}{\alpha_s(M_W^2)} = \frac{3}{8}\left\{1 - \frac{11}{\pi}\alpha \ln\frac{M_x}{M_W}\right\},$$

$$\sin^2\theta_W(M_W^2) = \frac{3}{8}\left\{1 - \frac{55}{9\pi}\alpha \ln\frac{M_x}{M_W}\right\} \qquad \text{(II.21)}$$

where $\alpha = \frac{e^2}{4\pi}$ and $\alpha_s = \frac{g_c^2}{4\pi}$. The result is independent of the number of fermions. If we put in the value of $\alpha_s \simeq 0.11$, deduced from knowing Λ_{QCD} and take for $\alpha(M_W^2) \sim \frac{1}{128}$, we find[7]

$$\sin^2\theta_W(M_W) \cong 0.20 \qquad . \qquad \text{(II.22)}$$

Marciano and Sirlin[9] have included higher order corrections and they found that

$$\sin^2\theta_W(M_W^2) = 0.216 \pm 0.004(N_H - 1) \pm 0.006 \ln\frac{100\,\text{MeV}}{\Lambda_{\overline{MS}}} \qquad \text{(II.23)}$$

where N_H is the number of light Higgs doublets with masses and $\Lambda_{\overline{MS}}$ is QCD parameter in the modified minimal subtraction (MS) renormalization prescription. For $N_H \leq 4$ and[10]

$$\Lambda_{\overline{MS}} = 0.16 \begin{array}{l} + \ 0.10 \\ - \ 0.80 \end{array} \text{GeV} \tag{II.24}$$

prediction (II.23) is in very good agreement with the experimental value[9]

$$\sin^2 \theta_W (M_W) = 0.215 \pm 0.014 \tag{II.25}$$

extracted from deep inelastic νN, $\nu \bar{N}$ scattering data.

Marciano and Sirlin have also estimated unification mass

$$M_x = (1 \text{ to } 2) \times 10^{15} \Lambda_{\overline{MS}} \tag{II.26}$$

for minimal SU(5). This estimate lead to the estimate of baryon lifetime (see J.Ellis in ref. 4)

$$\tau_B = 10^{29 \pm 2} \text{ Years} \tag{II.27}$$

The present experimental limit on baryon lifetime is[11]

$$\tau_B \geq 6 \times 10^{30} \text{ Years}$$

If we assume that the determination of Λ is correct, proton decay should be observed in the next series of experiments. If this does not happen, any theory without an intermediate mass scale in the range $M_W \lesssim \mu \lesssim M_x$ will face serious difficulties.

6. While all existing experiments appear to be consistent with the grand unified theory based on SU(5), it would be unwise to assert that there is nothing in the energy region between 10^2 GeV and 10^{14} GeV. A natural way to populate an energy "desert" is to enlarge the unification group and to try to attach some physical meaning to newly introduced intermediate mass scales. In the next chapter we will consider SU(N) groups (with $N > 5$) in an attempt to solve the problem of generations.

III. Problem of Fermion Generations in Grand Unified Theories[12]

The simplest gauge models that unify weak, electromagnetic, and strong interactions treat the various generations of quarks and leptons in a sequential fashion and do not provide any insight into the question of mixing angles and mass relations between various generations. Possible ways to make further progress have been suggested in the literature by treating the various fermion generations as representations of some discrete[13] or continuous horizontal symmetry groups[14]. With the recent surge of interest in grand unification, ways to tackle this problem in the context of SU(N) grand unified theories have been discussed by several authors[15,16].

One problem facing the unified gauge theory based on SU(N) groups is the possible occurrence of the triangular anomalies which may spoil the renormalizability of the theory. Banks and Georgi and, independently, Okubo[6] gave a general formula for calculation of the anomaly number of a general irreducible representation of the SU(N) group. Using the notations of Banks and Georgi, we state their results.

Let (q_i-1) be the number of columns with $\underline{i}$ boxes for the Young tableaux of a certain irreducible representations of SU(N). In terms of the q_i's, the anomaly number of the given irreducible representation of the SU(N) group is

$$A(q,N) = D(q,N)\sum_{i,j,k=1}^{N-1} a_{ijk}q_iq_jq_k \,, \qquad \text{(III.1)}$$

where D(q,N) is the dimension of the representation. The coefficients a_{ijk} are completely symmetric in i,j,k indices. For the case $i \leq j \leq k$,

$$a_{ijk} = \frac{2(N-3)!}{(N+2)}\, i(N-2j)(N-k). \qquad \text{(III.2)}$$

Some useful results for special cases are the following:

(a) The completely symmetric representation with m boxes in the Young tableau has the anomaly number

$$A(m,N) = \frac{(N+m)!\,(N+2m)}{(N+2)!\,(m-1)!}\,. \qquad \text{(III.3)}$$

(b) The completely antisymmetric representation has

$$A(m,N) = \frac{(N-3)!\,(N-2m)}{(N-m-1)!\,(m-1)!}\,. \qquad \text{(III.4)}$$

(c) For the general irreducible representation of the SU(4) group the anomaly number is

$$A(q,N) = \frac{D(q,4)}{60}(q_1-q_3)(q_1+q_3)(q_1+2q_2+q_3). \qquad \text{(III.5)}$$

In the case of irreducible representations of SU(N) ($N = 5$) formula (III.1) is rather cumbersome for practical evaluation. In the following we will give a prescription for obtaining the anomaly number for a given irreducible representation which, for $N = 5$, is more convenient[17] than using the general formula (III.1).

Our prescription is based on the following theorem. For an irreducible representation (q,N) of the SU(N) group, the anomaly number A(q,N) is

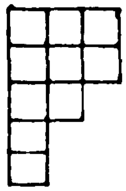
$$A(q,N) = \sum_{\alpha} C_{\alpha} \, A(q_{\alpha}, N - k). \qquad \text{(III.6)}$$

The C_{α} is the multiplicity of the representation $(q_{\alpha}, N-k)$ of SU(N-k) which is contained in the irreducible representation (q,N) of SU(N). We can prove this theorem in the following way.

Let G_i be any Non-Abelian proper subgroup of group G. Then, if the T_a are the generators of G, the T_a will clearly be block diagonal in the representation space, since T_a will not mix one irreducible representation of G_i with another. Hence eq. (III.6) follows directly from the definition of the anomaly number.

Therefore, in practice it would be much more convenient to go from SU(N) to a smaller group such as SU(4) and calculate the anomaly from formula (III.6), provided, of course, one knows the multiplicities.

We illustrate the method by example. Consider the irreducible representation,

(5,3,2) =  of SU(7).

It contains[8] the following representations of SU(6):

(5,3,2) ,	(5,3,1) ,	(5,2,2) ,	(4,3,2) ,	(5,2,1),
(a)	(b)	(c)	(d)	(e) .
	(4,3,1) ,	(4,2,2) ,	(4,2,1) .	
	(f)	(g)	(h)	

Now, notice that representations (a) and (f) of SU(6) are conjugate to each other, so they have equal and opposite anomaly numbers. Representations (b) and (d) are self-conjugate and will not contribute to the value of the anomaly number. We can proceed further, taking only representations (c), (e), (g), and (h), and finally obtain representations of SU(4).

We found the above method particularly useful in searching for anomaly-free linear combinations of representations which may be interesting as representations for fundamental fermions. Recently[18] several grand unified models have been proposed, based on SU(N) groups with N greater than five and with fermions assigned to some anomaly-free set of the totally antisymmetric representations. However, the use of only totally antisymmetric representations is a limitation because, a priori, no convincing argument seems to exist at the moment against the existence of six-dimensional, eight-dimensional, etc., irreducible representations of color group $SU(3)_C$.

The method suggested here was used to study different irreducible representations of the SU(N) groups for $N \leq 16$ with dimensions $\leq 10^3$.

In Table I we listed all representations that we studied. We assumed that all grand unified theories based on SU(N) gauge groups should contain the standard SU(5) theory[5]. Therefore, we also listed representations of SU(5) contained in the representations of the higher groups we investigated.

From this investigation we conclude that the requirement of freedom from anomalies rules out any realistic model if we consider only a single representation. From now on, we will focus our attention on simple representations of SU(N) groups.

The basic strategy proposed is to consider anomaly-free combinations of totally antisymmetric representations of SU(N) groups. The number of generations is defined as the difference of the numbers of $\underline{10}$ and $\overline{10}$ representations of SU(5), contained in the abovementioned representations. The difference between References 15 and 16 lies in their choice of anomaly-free sets. In Reference 15, Georgi requires members of the anomaly-free set (AFS) to belong to different representations of the grand unifying group. The smallest grand unification group of the above type that can yield three generations is found to be SU(11). On the other hand, in Reference 16, Frampton and Nandi allow repetition of multiplets in the anomaly-free combinations. This permits them to work with lower-rank groups such as SU(7) or SU(9), etc.

We feel that since the purpose of employing higher-rank groups is to avoid having to repeat representations, then the strategy of working with only those anomaly-free sets where each representation of the grand unifying group appears only once, ought to be preferred.[19]

We also propose to gauge the "horizontal" degree of freedom[14] that transforms various generations into each other. Therefore, if there are m generations, we propose an $SU(m)_H$ horizontal group. We consider the minimal grand unification symmetry, SU(5+m), which contains SU(5) x $SU(m)_H$, and we require light-fermion generations to transform as vectors under $SU(m)_H$ group. Let us therefore make our grand unification criteria explicit:

(a) Only totally antisymmetric representations of SU(N) are considered[15] for assignment of fermions.

(b) The set of fermion representations that is anomaly-free[6] must not contain any representation more than once.

(c) Self-conjugate representations are barred.

(d) The gauge coupling is required to be asymptotically free.

(e) The light generations of fermions are required to transform as a vector under the horizontal group.

Within this set of criteria, we find that the maximal number of light generations of fermions is four. In the above discussions, we have assumed that the very-low-energy electroweak symmetry group is $SU(2)_L \times U(1)$, with each generation transforming as a weak isodoublet.

To present our discussion, we first give our notation. For the group SU(N), the representation with m antisymmetric indices will be denoted by $[N,m]$. We then find that the only anomaly-free sets of $[N,m]$, satisfying criteria (a)-(d), are the following[20]:

$$
\begin{aligned}
SU(5) &: [5,1] + [5,3], \\
SU(7) &: [7,1] + [7,3] + [7,5], \\
SU(8) &: [8,3] + [8,6] + [8,7], \\
SU(9) &: [9,2] + [9,5], \\
&\quad [9,1] + [9,3] + [9,5] + [9,7], \\
SU(10) &: [10,3] + [10,6].
\end{aligned}
\qquad \text{(III.7)}
$$

We emphasize that this result is true for all N, provided we avoid self-conjugate representations. We also exclude the cases with two representations which are complex conjugates of each other. We note that this excludes the SU(11) group of ref. 15 from our considerations. To establish the connection between the number of generations and the grand unifying symmetry, we display the SU(5) content for each set displayed in eq.(III.7):

$$
\begin{aligned}
\text{(a)}\quad SU(7):\ [7,1] &= 2\{1\}+\{5\}\,,\\
[7,3] &= \{5\}+2\{10\}+\{\overline{10}\}\,,\\
[7,5] &= \{10\}+2\{\bar{5}\}+\{1\}\,.
\end{aligned}
\tag{III.8}
$$

$$
\begin{aligned}
\text{(b)}\quad SU(8):\ [8,3] &= \{1\}+3\{5\}+3\{10\}+\{\overline{10}\}\,,\\
[8,6] &= 3\{1\}+3\{\bar{5}\}+\{\overline{10}\}\,,\\
[8,7] &= 3\{1\}+\{\bar{5}\}\,.
\end{aligned}
\tag{III.9}
$$

$$
\begin{aligned}
\text{(c)}\quad SU(9):\ [9,2] &= 6\{1\}+4\{5\}+\{10\}\,,\\
[9,5] &= \{5\}+4\{10\}+6\{\overline{10}\}+4\{\bar{5}\}+\{1\}\,,
\end{aligned}
\tag{III.10}
$$

(a similar decomposition for the other set).

$$
\begin{aligned}
\text{(d)}\quad SU(10):\ [10,3] &= 10\{1\}+10\{5\}+\{\overline{10}\}\,,\\
[10,6] &= \{5\}+5\{10\}+10\{\overline{10}\}+10\{\bar{5}\}+5\{1\}\,.
\end{aligned}
\tag{III.11}
$$

In the above we use the notation $[N,m] = C_{mi}\{m_i\}$, where C_{mi} is the number of $\{m_i\}$-dimensional representations of SU(5) in representation $[N,m]$ of SU(N).

From eqs. (III.8)-(III.11), we can see that the SU(10) case does not satisfy our criteria for light generations belonging to the fundamental representation of SU(5). From this we conclude that the maximum number of generations is four[21].

To demonstrate our procedure, we describe the SU(8) model of eq. (III.9). The fermion multiplets are denoted by $(\Psi_{\alpha\beta\gamma})_L$, $(\Psi_{\alpha\beta\gamma\delta\lambda\tau})_L$, $(\overline{\Psi}^{\alpha})_R$. According to our prescription for identifying light fermions, we see that they are given by $(\Psi_{abA})_L$ and $(\overline{\Psi}^{aA})_R$, where lower case letters a, b, c,..., stand for SU(5) indices and upper case letters, A, B, C,..., stand for horizontal $SU(3)_H$ indices. The Greek letters, α, β,..., stand for SU(8) indices. The SU(5) indices run from 1 to 5 whereas $SU(3)_H$ indices are 6, 7, 8. The heavier fermions are then identified as $(\Psi_{ABC})_L$, $(\Psi_{aAB})_L$, $(\Psi_{abc})_L$, $(\Psi^{AB})_L$, $(\Psi^{ab})_L$, $(\Psi^{a})_L$ and $(\Psi^{A})_L$. We will now proceed to show that it is possible to choose Higgs multiplets and vacuum expectation values that yield the above mass hierarchy for the fermions. We first note that at the level of heavy-fermion masses, the horizontal symmetry $SU(3)_H$ is already broken. To simplify our discussion we give explicit particle labels. Our notation will be as follows: Lower case symbols with subscripts p (p=1,2,3) such as (u_p^i, d_p^i, e_p), will stand for light fermions of p^{th} generation. The i stands for color index; capital-letter symbols with subscripts such as (U_p^i, D_p^i, E_p, N_p) will stand for heavier fermions. The superscript c indicates charge conjugate of the state. Using these notations, representations of SU(8) group can be written as follows:

$$
\begin{aligned}
[8,3]: \quad & (\psi_{ABC})_L = N_L \\
& (\psi_{aBC})_L = (D^i_{pL}, E^+_{pL}, E^0_L) \\
& (\psi_{abC})_L = (u^i_{pL}, u^{ic}_{pL}, d^i_{pL}, e^+_{pL}) \\
& (\psi_{abc})_L = (U^i_{1L}, U^{ci}_{1L}, D^{ci}_{1L}, E^-_{1L}) \;.
\end{aligned}
$$

$$
\begin{aligned}
[8,6]: \quad & (\psi^{aA})_L = (d^{ci}_{pL}, e^-_{pL}, \nu_{pL}) \\
& (\psi^{AB})_L = M^0_{pL} \\
& (\psi^{ab})_L = (U^i_{2L}, U^{ci}_{2L}, D^{ci}_{2L}, E^-_{2L})
\end{aligned}
\qquad \text{(III.13)}
$$

$$
\begin{aligned}
[8,7]: \quad & (\psi^{A})_L = M^0_{pL} \\
& (\psi^{a})_L = (D^{ci}_{3L}, E^-_{3L}, N_{3L})
\end{aligned}
$$

In making the above identification of heavy-particle states, we have used the fact that there is negligible mixing between heavy- and light-particle states resulting from the Higgs mechanism displayed below. We first give the Higgs multiplets, Yukawa couplings, and vacuum expectation values (VEV's) that contribute to the heavy-particle masses. We choose the following Higgs multiplets: $\bar{\Phi}^{\alpha\beta}, \phi^{\alpha\beta\gamma\delta}, \bar{\Phi}^{\alpha}_{\beta\gamma}, \bar{\Phi}^{\alpha\epsilon\eta}_{\mu\nu\lambda\xi\sigma}$, where we assume antisymmetry in both super- and subindices and tracelessness (for example, $\sum \Phi^{\alpha}_{\alpha\gamma} = 0$). The heavy fermions in eq.(III.13) acquire their masses and mixings from the following gauge-invariant couplings:

$$\psi^{T\alpha\beta\gamma\delta\sigma} C^{-1} \psi_{\epsilon\eta\phi} \Phi^{\epsilon\eta\phi}_{\alpha\beta\gamma\delta\sigma} \quad , \quad \psi^{T\alpha} C^{-1} \psi_{\alpha\beta\gamma} \Phi^{\beta\gamma}$$

$$\psi^{T\alpha\beta} C^{-1} \psi_{\alpha\beta\gamma\lambda\sigma\rho} \Phi^{\gamma\lambda\sigma\rho} \quad , \quad \psi^{T\alpha\beta} C^{-1} \psi_{\alpha\mu\nu} \Phi^{\mu\nu}_{\beta} \tag{III.14}$$

where ψ's with superscripts are obtained by applying the totally antisymmetric ϵ symbol for the SU(8) group. The desired pattern of heavy-particle masses is achieved by the following:

$$\langle \Phi^{ab5}_{abABC} \rangle \neq 0 \quad , \langle \Phi^{AB} \rangle \neq 0 , \langle \Phi^{5ABC} \rangle \neq 0$$

$$\langle \Phi^{AB}_{5} \rangle \neq 0 \quad , \langle \Phi^{abc}_{abc5A\cdot} \rangle \neq 0 \tag{III.15}$$

It is clear that since three of the above VEV's break $SU(2)_L \times U(1)_Y$ symmetry, they will contribute to the W-boson mass. These VEV's are therefore constrained to be of order M_W/g 300 GeV. If we choose the coupling constants associated with the invariants in eq. (III.14) to be of order 2-3, then the heavy generation acquires its largest mass, of the order of TeV's. The important point that we stress here is that none of the couplings in eq. (III.14) upon using eq. (III.15) contribute to the light-fermion masses or light-heavy mixings. All the other Yukawa couplings are chosen smaller, compared to those in eq. (III.14), since they involve light to heavy mixings as well as light to light mixings.

The light-particle sector is completed by adjoining the following extra Higgs multiplets; $\Phi^{\hat{\alpha}}_{\mu\nu\lambda}$ with $\langle \Phi^{\hat{A}}_{5BC} \rangle \neq 0$ and $\Phi^{\alpha\beta}_{\gamma}$ with $\langle \Phi^{5A}_{B} \rangle \neq 0$. The latter contributes to the mass matrix for down quarks whereas the former contributes to the up-quark masses.

Although our choice of Higgs field is very complicated, we have shown that it is possible to get desired hierarchy of the fermion masses. Our analysis is admittedly incomplete; in discussing symmetry breaking and the fermion mass spectrum, we assumed, without proof, that certain components of scalars develop suitable vacuum expectation values. However, we must stress that it was not our aim to construct a realistic model but to show that it is possible to limit theoretically the number of light-fermion generations using criteria.(a) - (e).

Finally, we envision that the grand unification symmetry SU(5+M) (for M generations) will break down to $SU(5) \times SU(M)_H$ at some mass scale higher than 10^{15} GeV by a Higgs multiplet of type Φ^{α}_{β} . Then as we go below the mass scale 10^{15} GeV, SU(5) symmetry breaks down to $SU(3)_C \times SU(2)_L \times U(1)$. The next important mass scale is around 10^2 TeV, where the horizontal symmetry $SU(M)_H$ is completely broken down. The heavy-fermion mass (TeV) constitutes the next level in the mass hierarchy.

Before closing this section we would like to discuss our criteria (b) and (e). The question one may ask is whether the so-called survival hypothesis of Georgi[15] is compatible with our requirement (d) of asymptotic freedom for three or more generations. In this case, we have to abandon the assumption (b) of non-repetition of the member of SU(N) multiplets to get any answer[22].

We start by studying the $\beta(g)$ for SU(N) groups. If we have a set of totally antisymmetric representations $[N,m]$, with C_m denoting multiplicity for each representation, then the β for such a model (not including scalar contributions) is given by:

$$\beta(g) = -\frac{g^3}{16\pi^2}\left\{\frac{11}{3}C_2(G) - \frac{4}{3}T(R)\right\} + O(g^5), \qquad (III.16)$$

where $C_2(G) = N$ and

$$T(R) = \frac{1}{4}\sum_m C_m \binom{N-2}{m-1}. \qquad (III.17)$$

We also note at this point that the complex conjugate representation of $[N,m]$ is $[N,N-m]$ and therefore it has the same contribution to T(R) as in eq. (III.17). Therefore, in studying the asymptotic freedom constraints, it is sufficient to restrict m to be less than N/2. Of course, we will have to be careful when we start discussing the anomaly constraints since $[N,m]$ and $[N,N-m]$ have equil but opposite contributions.

Therefore we have to study the following inequality

$$11N - \sum_m C_m \binom{N-2}{m-1} > 0 . \tag{III.18}$$

Under the restriction that m is smaller than N/2, the above inequality implies that $m \leq 4$. Thus, no matter what SU(N) group we are considering, we can allow, at most, up to 4 box-column representations, resulting in the first non-trivial simplification of the discussion. Therefore, it is clear that due to the existence of inequality (III.18), coming from asymptotic freedom constraints, only the following cases must be studied:

(i) Four sets of representations denoted by

$C_i\{m_i\}$ $(i = 1, 2, 3, 4)$.

(ii) Three sets: $C_i\{m_i\}$ $(i = 1, 2, 3)$.

(iii) Two sets: $C_i\{m_i\}$ $(1 = 1, 2)$.

Each of the above cases will be further restricted by the requirement of no triangle anomalies, which we now study.

(i) Four sets of representations

Inequality (III.18) can now be written as

$$11N - \sum_{m=1}^{4} C_m \binom{N-2}{m-1} > 0 \tag{III.19}$$

With all four $C_m \neq 0$, no SU(N) groups other than SU(10) and SU(9) can satisfy the inequality (III.19). We therefore consider these two cases:

SU(10): For SU(10), inequality (III.19) can be rewritten as

$$110\,N - C_1 - 8\,C_2 - 28\,C_3 - 56\,C_4 > 0, \qquad \text{(III.20)}$$

which can be satisfied only for $C_3 = C_4 = 1$ and $C_2 \leq 3$ and $C_1 = 17$. Possible choices for C_i ($i = 1, 2$) are :

$$\begin{array}{llcl} \text{(Ai)} & C_1 = 1 & , & C_2 = 3, \\ \text{(Aii)} & C_1 \leq 9 & , & C_2 = 2, \\ \text{(Aiii)} & C_1 \leq 17 & , & C_2 = 1. \end{array} \qquad \text{(III.21)}$$

To see the possible anomaly free combinations, let us write the anomaly number $A_{[N,m]}$ for SU(N) with m antisymmetric indices:

$$A_{[10,1]} = 1\ ,\quad A_{[10,2]} = 6\ ,\quad A_{[10,3]} = A_{[10,4]} = 14\ . \qquad \text{(III.22)}$$

The only anomaly free combinations are:

$$\begin{array}{ll} \text{(Bi)} & 6\,\{9\} + \{2\} + \{7\} + \{4\}\ , \\ \text{(Bii)} & 6\,\{9\} + \{2\} + \{3\} + \{6\}\ . \end{array} \qquad \text{(III.23)}$$

The general formula for the number of generations is

$$g = C_2 - C_{N-2} + (N-6)(C_3 - C_{N-3}) + \frac{(N-8)(N-5)}{2}(C_4 - C_{N-4}) \qquad \text{(III.24)}$$

In the case of (Bi), only two light generations exist; there are none for (Bii).

<u>SU(9)</u>: The analog of eq. (III.20) in this case is :

$$99 - C_1 - 7\,C_2 - 21\,C_3 - 35\,C_4 > 0$$

and the anomaly numbers are:

$$A_{[9,1]} = 1 \quad , A_{[9,2]} = A_{[9,4]} = 5 \quad , A_{[9,3]} = 9 \tag{III.26}$$

We simply wish to point out, that one can obtain as many as seven generations in the case of SU(9). All possible combinations are listed in Table II.

(ii) <u>Three sets of representations</u>

Since $m_i \leq 4$, there exist four possible combinations for . They are: $\{1,2,3\}$, $\{1,2,4\}$, $\{1,3,4\}$ and $\{2,3,4\}$. For the first two combinations, the asymptotic freedom constraint is

$$11\,N - C_1 - (N-2)C_2 - C_3\binom{N-2}{m-1} > 0 \tag{III.27}$$

and the corresponding values of their anomaly numbers are

$$A_{[N,m]} = \frac{N-2m}{N-2}\binom{N-2}{m-1} \tag{III.28}$$

Using eq. (III.28), we find three possible constraints on C_ms:

$$C_1 - (N-4)C_2 - C_3\,\frac{N-2m}{N-2}\binom{N-2}{m-1} = 0 \tag{III.29}$$

$$C_1-(N-4)C_2+C_3\frac{N-2m}{N-2}\binom{N-2}{m-1}=0$$

$$C_1+(N-4)C_2-C_3\frac{N-2m}{N-2}\binom{N-2}{m-1}=0 \qquad \text{(III.29)}$$

In writing eq. (III.29), we have taken into account the possibility that one of the three sets must belong to a complex conjugate representation. In Table III, we list all possible values of C_1 that lead to three or more light fermion generations.

The last two possibilities, $\{1,3,4\}$ and $\{2,3,4\}$, can be analyzed similarly. In this case, the analog of eq. (III.27) and (III.28) are:

$$11N-C_1\binom{N-2}{m-1}-C_2\frac{(N-2)(N-3)}{2}-C_3\frac{(N-2)(N-3)(N-4)}{6}>0$$

and

$$\text{(Di)}\quad C_1\binom{N-2}{m-1}\frac{N-2m}{N-2}=C_2\frac{(N-6)(N-3)}{2}+C_3\frac{(N-8)(N-3)(N-4)}{6}\,,$$

$$\text{(Dii)}\quad -C_1\binom{N-2}{m-1}\frac{N-2m}{N-2}=-C_2\frac{(N-6)(N-3)}{2}+C_3\frac{(N-8)(N-3)(N-4)}{6}\,,$$

$$\text{(Diii)}\quad C_3\frac{(N-8)(N-3)(N-4)}{6}=C_1\frac{N-2m}{N-2}\binom{N-2}{m-1}+C_2\frac{(N-6)(N-3)}{2}\,.$$

(iii) Two Sets of Representations

This case is the simplest to analyze. The anomaly condition can be written as

$$C_1(N-2m_1)\binom{N-2}{m_1-1} = C_2(N-2m_2)\binom{N-2}{m_2-1}. \tag{III.30}$$

It is easy to conclude that the number of light generations is given by

$$g = C_1\left[\binom{N-5}{m_1-3} - \binom{N-5}{m_1-2}\right] + C_2\left[\binom{N-5}{m_2-2} - \binom{N-5}{m_2-3}\right] \tag{III.31}$$

Using eq. (III.30), it is possible to summarize the results in terms of the following allowed sets:

(<u>Ei</u>) For SU(11), the following combination

$$4[11,9] + [11,4]$$

has five generations.

(<u>Eii</u>) For SU(N) where N = 10,11 , the allowed combinations are

$$C_1[N,N-1] + [N,4]$$

with

$$C_1 = \frac{(N-8)(N-3)(N-4)}{6}$$

and the number of generations is five for SU(10) and nine for SU(11).

(<u>Eiii</u>) For SU(N), with 9 ≤ N ≤ 17, the allowed combinations are

$$C_1[N,N-1] + [N,3]$$

where

$$C_1 = \frac{(N-6)(N-3)}{2}$$

and there are (N - 6) generations.

We will close this part of our work with the following conclusions[23]:

1. If we assume that the light generations of fermions transform as vectors under the horizontal group, and require that the theory is anomaly free, then the requirement for asymptotically free gauge coupling will limit the number of light-fermion generations to four.

2. If we adopt the so-called survival hypothesis and require that the theory is anomaly-free, then the requirement for asymptotic free gauge coupling will limit the number of light-fermion generations to eleven. We present the results of our investigation in Tables II and III.

IV Maximal Grand Unification and Baryon Non-Conservation[24]

In the previous chapter we extended the simplest grand unified theory based on the SU(5) group, adding a horizontal family group. In this chapter we will consider one of several possible extensions of minimal unified theory, by enlarging the strong and flavor subgroups. This approach was suggested and analyzed by several authors[25] for different unification groups. In the following, we will choose the SU(16) group as the grand unification group[26]. The SU(16) is the maximal symmetry group associated with the eight fermions and eight antifermions of each generation and it is perhaps a natural candidate for the unification of electroweak and strong interactions.

Being a large group, SU(16) opens the possibility for the existence of different "partial unifications"[27] which take place between 10^2 GeV and unification energy. In order to find the constraints on possible intermediate mass scales we will use a somewhat modified form of the formula given by Dawson and Georgi[28]. This will permit us to find the hierarchy of gauge boson masses in the SU(16) model which are allowed by the present values of low energy parameters such as $\sin^2\theta_W$ and α_s. Knowing the allowed hierarchy of gauge boson masses, we will study their implications for baryon and lepton non-conservation.

To fix conventions and notations we now introduce the SU(16) model.

IV.1 SU(16) Model[26]

In grand unified theory based on the SU(16) gauge group, it is assumed that left-handed particles and antiparticles of each generation belong to the fundamental representation

$$\psi_A = \begin{pmatrix} u_i \\ \nu \\ d_i \\ e^- \\ u^c_i \\ \nu^c \\ d^c_i \\ e^+ \end{pmatrix} , \qquad i=1,2,3 . \tag{IV.1}$$

In order to cancel the anomalies, we will introduce a set of mirror fermions with right-handed chirality. They belong to the $\overline{16}$ representation of SU(16).

The fermion masses will arise by introducing a <u>136</u> dimensional symmetric Higgs multiplet $\Phi_{\{AB\}}$ which couples to ψ as

$$h\, \psi^T_A C^{-1} \psi_B\, \Phi^{\{AB\}} \tag{IV.2}$$

The mirror fermions will get masses in the same manner. Although they can be associated with a Yukawa coupling constant, h', which may be much larger than that of the basic fermions, they cannot be superheavy. This is because the mass terms for mirror fermions violate $SU(2)_L$ symmetry like those of the basic fermions. Therefore we will assume that the masses of mirror fermions are in the range of 100 to 200 GeV. To avoid a direct mass mixing between the basic and the mirror fermions in the primary Lagrangian we will impose the following discrete symmetries:

$$\psi_{L,R} \longleftrightarrow \psi_{L,R} \quad ; \quad \psi^{m}_{L,R} \longleftrightarrow -\psi^{m}_{L,R} \tag{IV.3}$$

The gauge bosons belong to the adjoint representation and there exists a distinct gauge particle for every non-diagonal transition between members of a generation. Therefore, the theory contains, besides the usual gauge bosons, an additional set of gauge particles. These include the diquark, the lepto- antiquark, the dilepton and the leptoquark which couple respectively to $\bar{q}^c_L \gamma_\mu q_L$, $\bar{q}^c \gamma^\mu \ell$, $\bar{\ell}^c \gamma^\mu \ell$ and $\bar{q} \gamma^\mu \ell$ -currents. More about the gauge boson sector will be said in Section IV.6.

One characteristic feature of the SU(16) model is that it ensures that B-baryon, L-lepton and F-fermion numbers are exact symmetries of the Lagrangian. These symmetries are violated spontaneously throughout the induced mixing between the abovementioned gauge particles. Consequently, the proton may decay via four different modes:

$$\begin{array}{ll} p \to 3\ell + q\bar{q} & p \to \ell + q\bar{q} \\ p \to 3\bar{\ell} + q\bar{q} & p \to \bar{\ell} + q\bar{q} \end{array} \tag{IV.4}$$

In addition to this, the induced mixing of gauge particles may lead to the existence of other baryon and lepton violating processes, such as $N - \overline{N}$ oscillations and neutrinoless double β-decay. Which one of these processes will be dominant depends very much upon the pattern of symmetry breaking of the SU(16) group.

Being a large symmetry group, SU(16) can break to low energy symmetry $SU(3)_c \times U(1)_{em}$ in many different ways. We consider the following three routes:

$$\text{(I)} \quad SU(16) \longrightarrow SU(12)_q \times SU(4)_l \longrightarrow SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L} \tag{IV.5}$$

In the above it is assumed that symmetry between quarks and leptons is broken at an intermediate energy scale.

$$\text{(II)} \quad SU(16) \longrightarrow SU(8)_L \times SU(8)_R \times U(1)_F \longrightarrow SU(4)_c \times SU(2)_L \times SU(2)_R \tag{IV.6}$$

This chain of symmetry breaking emphasizes the possibility that the left-handed and right-handed sectors of the theory are separate before any separation between quarks and leptons.

$$\text{(III)} \quad SU(16) \longrightarrow SO(10) \longrightarrow SU(5) \tag{IV.7}$$

This route represents a trivial extension of the SO(10) group and leads to the results already familiar for SO(10) and SU(5) models.

The Higgs multiplets necessary for the various breaking routes will be discussed later.

As indicated, the above patterns of symmetry breaking permit a hierarchy of intermediate mass-scales, filling the so-called grand desert between 10^2 GeV and unification ene gy. The assumption of the existence of intermediate mass scales raises the question of whether it is possible that any of these scales is low enough to be seen directly in high energy experiments in the near future. In this connection we remark that recently Rizzo and Senjanovic[29] have investigated the SO(10) model[30] with particular emphasis on whether the model can have parity restoration at energies as low as 100 - 300 GeV. It is therefore of interest to know whether such possibility exists in SU(16). Because Chain III is a direct extension of the SO(10) group it will not give any new result in similar analyses. Therefore in the rest of our work we will not study Chain III of symmetry breaking.

IV.2 SU(16) Breaking to $SU(12)_q \times SU(4)_L \times U(1)_{|B|-|L|}$ and the Evolution of Coupling Constants

In this section we propose to study the relation between the $SU(3)_c$, $SU(2)_L$, $U(1)$ coupling constants and the grand unified coupling g_U via the Gell-Mann-Low equations. We wish to investigate the symmetry breaking pattern (I) of the previous section, where SU(16) first breaks down to $SU(12)_q \times SU(4)_l \times U(1)_{|B|-|L|}$. The $SU(12)_q$ is the maximal symmetry of quarks and antiquarks and $SU(4)_l$ operates on the leptonic space. At the next stage, we have the two possibilities:

$$\begin{tikzcd}
SU(12)_q \times SU(4)_l \times U(1) \arrow[r] \arrow[dr] & SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L} \\
& SU(3)_c \times SU(4)_q \times SU(4)_l \times U(1)_{B-L} \arrow[u]
\end{tikzcd} \qquad \text{(IV.8)}$$

where $SU(2)_{L,R}$ operates both on quarks and leptons. The left-right symmetric group[27] is broken down in the usual manner[31] to $SU(3)_c \times SU(2)_L \times U(1)$. We will use the method of Georgi, Quinn and Weinberg[7], appropriately extended to include models with intermediate mass scales. To write down the general formula relating the gauge coupling constants at two different energies, we simply have to integrate the Gell-Mann-Low equation between successive mass scales with appropriate values for the β-function and an appropriate normalization factor for the generators. To state the general formula, let us assume the breaking of a group, G, is as follows,

$$G_N \rightarrow G_{N-1} \rightarrow \cdots\cdots , \tag{IV.9}$$

with the associated mass scales $\mu_N, \mu_{N-1}, \ldots\ldots$. Each of the intermediate symmetry groups, G_x, can be a direct product of simple groups, $G_x = \prod_\alpha G^\alpha_x$, with the associated coupling constants, g^x_α, corresponding to G^α_x. The relevant formula relating the coupling constants at the mass scales is then given by

$$\frac{1}{[g^x_\alpha(\mu_x)]^2} = \sum_\beta \frac{P^{x,x+1}}{[g^{x+1}_\beta(\mu_{x-1})} + 2 b^x_\alpha \ln \frac{\mu_{x-1}}{\mu_x} \tag{IV.10}$$

This formula is a somewhat modified form of the formula given by Dawson and Georgi[28], the difference being that in eq. (IV.10), the so-called probabilities, $P^{x,x+1}_{\alpha,\beta}$, are not normalized to add up to one. This is because all our coupling constants, g^x_α, will represent physical couplings. Therefore, the generators at each stage defined in the 16-dimensional space are not necessarily normalized to $Tr(T^x_\alpha T^x_\beta) = 1/2\, \delta_{\alpha\beta}$. The simplest way to construct the P^{xy} is to choose a convenient basis for the diagonal generators for G and find the diagonal generators for the subgroups G^y in the same basis and express it in terms of those for G^x. We will give explicit constructions for P^{xy} for the SU(16) breaking. We normalize all our SU(16) generators by

$$Tr(T_a T_b) = \frac{1}{2}\,\delta_{ab}$$

The T_a's are 16 x 16 matrices in the space of Ψ . We will now construct the diagonal generators of the SU(12) group. The matrices;

$$T_1^{wac} = \frac{1}{2}\left(\begin{array}{cccccccc:c} 1 &&&&&&&& \\ & 0 &&&&&&& \\ && 0 &&&&&& \\ &&& 0 &&&&& \\ &&&& -1 &&&& \\ &&&&& 0 &&& \\ &&&&&& 0 && \\ &&&&&&& 0 & \\ \hdashline &&&&&&&& O \end{array}\right) \qquad T_2^{wac} = \frac{1}{2}\left(\begin{array}{cccccccc:c} 0 &&&&&&&& \\ & 1 &&&&&&& \\ && 0 &&&&&& \\ &&& 0 &&&&& \\ &&&& 0 &&&& \\ &&&&& -1 &&& \\ &&&&&& 0 && \\ &&&&&&& 0 & \\ \hdashline &&&&&&&& O \end{array}\right) \,. \tag{IV.11}$$

$$T_3^{wac} = \frac{1}{2}\left(\begin{array}{cccccccc:c} 0 &&&&&&&& \\ & 0 &&&&&&& \\ && 1 &&&&&& \\ &&& 0 &&&&& \\ &&&& 0 &&&& \\ &&&&& 0 &&& \\ &&&&&& -1 && \\ &&&&&&& 0 & \\ \hdashline &&&&&&&& O \end{array}\right)$$

are the $SU(2)_{W_L}$ generators for particles. The lower 0 stands for null 8 x 8 matrix. The corresponding generators for antiparticles are obtained by interchanging the upper and lower 8 x 8 matrices, for example,

$$\bar{T}_1^{wac} = \frac{1}{2}\left(\begin{array}{c:cccccccc} O &&&&&&&& \\ \hdashline & -1 &&&&&&& \\ && 0 &&&&&& \\ &&& 0 &&&&& \\ &&&& 0 &&&& \\ &&&&& 1 &&& \\ &&&&&& 0 && \\ &&&&&&& 0 & \\ &&&&&&&& 0 \end{array}\right) \tag{IV.12}$$

and similarly for $\bar{T}_2^{wac}$ and $\bar{T}_3^{wac}$.

Next, we define the diagonal generators for chiral $SU(3)_c$ color:

$$T_4^c = \frac{1}{2\sqrt{2}}\left(\begin{array}{c|c} \mathrm{diag}(1,-1,0,0,1,-1,0,0) & \\ \hline & O \end{array}\right), \quad T_5^c = \frac{1}{2\sqrt{6}}\left(\begin{array}{c|c} \mathrm{diag}(1,1,-2,0,1,1,-2,0) & \\ \hline & O \end{array}\right),$$

$$\bar{T}_4^c = \frac{1}{2\sqrt{2}}\left(\begin{array}{c|c} O & \\ \hline & \mathrm{diag}(-1,1,0,0,-1,1,0,0) \end{array}\right), \quad \bar{T}_5^c = \frac{1}{2\sqrt{6}}\left(\begin{array}{c|c} O & \\ \hline & \mathrm{diag}(-1,-1,2,0,-1,-1,2,0) \end{array}\right). \qquad \text{(IV.13)}$$

The color and flavor singlet diagonal generator - the last diagonal generator of SU(12) is

$$T_{11} = \frac{1}{2\sqrt{6}}\,\mathrm{diag}(1,1,1,0,1,1,1,0,-1,-1,-1,0,-1,-1,-1,0) \qquad \text{(IV.14)}$$

The $SU(4)_1$ generators are given by

$$T_l^w = \frac{1}{2}\left(\begin{array}{c|c} \begin{matrix} 0 & & & & & & & \\ & 0 & & & & & & \\ & & 0 & & & & & \\ & & & 1 & & & & \\ & & & & 0 & & & \\ & & & & & 0 & & \\ & & & & & & 0 & \\ & & & & & & & -1 \end{matrix} & \\ \hline & O \end{array}\right)$$

$$\bar{T}_l^w = \frac{1}{2}\left(\begin{array}{c|c} O & \\ \hline & \begin{matrix} 0 & & & & & & & \\ & 0 & & & & & & \\ & & 0 & & & & & \\ & & & -1 & & & & \\ & & & & 0 & & & \\ & & & & & 0 & & \\ & & & & & & 0 & \\ & & & & & & & 1 \end{matrix} \end{array}\right)$$

$$T_{14} = \frac{1}{2\sqrt{2}}\begin{pmatrix} 0 & & & & & & & & & & & & & & & \\ & 0 & & & & & & & & & & & & & & \\ & & 0 & & & & & & & & & & & & & \\ & & & 1 & & & & & & & & & & & & \\ & & & & 0 & & & & & & & & & & & \\ & & & & & 0 & & & & & & & & & & \\ & & & & & & 0 & & & & & & & & & \\ & & & & & & & 1 & & & & & & & & \\ & & & & & & & & 0 & & & & & & & \\ & & & & & & & & & 0 & & & & & & \\ & & & & & & & & & & 0 & & & & & \\ & & & & & & & & & & & -1 & & & & \\ & & & & & & & & & & & & 0 & & & \\ & & & & & & & & & & & & & 0 & & \\ & & & & & & & & & & & & & & 0 & \\ & & & & & & & & & & & & & & & -1 \end{pmatrix} \tag{IV.15}$$

The B - L generator is given by

$$T_{|B|-|L|} = \frac{1}{2\sqrt{24}}\begin{pmatrix} 1 & & & & & & & & & & & & & & & \\ & 1 & & & & & & & & & & & & & & \\ & & 1 & & & & & & & & & & & & & \\ & & & -3 & & & & & & & & & & & & \\ & & & & 1 & & & & & & & & & & & \\ & & & & & 1 & & & & & & & & & & \\ & & & & & & 1 & & & & & & & & & \\ & & & & & & & -3 & & & & & & & & \\ & & & & & & & & 1 & & & & & & & \\ & & & & & & & & & 1 & & & & & & \\ & & & & & & & & & & 1 & & & & & \\ & & & & & & & & & & & -3 & & & & \\ & & & & & & & & & & & & 1 & & & \\ & & & & & & & & & & & & & 1 & & \\ & & & & & & & & & & & & & & 1 & \\ & & & & & & & & & & & & & & & -3 \end{pmatrix} \tag{IV.16}$$

The above generators are the part of the Lie algebra of both $G_o = SU(16)$ as well as $G_1 = SU(12)_q \times SU(4)_1 \times U(1)_{|B|-|L|}$. Therefore, if the grand unification mass is taken as M_U and the scale at which G_1 breaks down is taken as M_C, the relation between coupling constants in this region is simply given by:

$$\frac{1}{g_{12}^2(M_c)} = \frac{1}{g_U^2(M_U)} + 2b_{12} \ln \frac{M_U}{M_c} \qquad \text{(IV.17)}$$

$$\frac{1}{g_4^2(M_c)} = \frac{1}{g_U^2(M_U)} + 2b_4 \ln \frac{M_U}{M_c}$$

where b_{12} and b_4 stand for the 1-loop contributions to the β-function for SU(12) and SU(4).

To consider the next stage of symmetry breaking, G_1 G_2 that is,

$$SU(12)_q \times SU(4)_L \times U(1)_{B+L} \rightarrow$$
$$\rightarrow SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L},$$

we must express the diagonal generators of G_2, in terms of those of G_1. The generator of $U(1)_{B-L}$ can be expressed as the linear combination of SU(12) and SU(4) generators,

$$Y_{B-L} = (T_{11} - \sqrt{3}\ T_{14}) , \qquad \text{(IV.18)}$$

which leads to the following values for relevant "probabilities"

$$P_{B-L,SU(12)_q} = 1 \qquad P_{B-L,SU(4)_1} = 3 \qquad \text{(IV.19)}$$

Coming to the case of $SU(2)_{L,R}$, note that, at the $SU(2)_L$

level, the physical T_{3L} generator is given by

$$T_{3L} = (T_1^{W\times C} + T_2^{W\times C} + T_3^{W\times C}) + T_1^{W} . \qquad \text{(IV.20)}$$

This implies that

$$\frac{1}{g^2_{L,R}(M_c)} = \frac{3}{g^2_{12}(M_c)} + \frac{1}{g^2_4(M_c)} \qquad \text{(IV.21)}$$

or, in the language of "probabilities",

$$P_{SU(2)_L, SU(12)_q} = 3 \ , \ P_{SU(2)_L, SU(4)} = 1 \qquad \text{(IV.22)}$$

The same relation also holds for $SU(2)_R$ group.

At this point we would like to explain why, in our expressions, "probabilities" are not normalized to one. As we have already mentioned we wish to have physical couplings at each stage. Let us take as an example the transition from $SU(12)_q \times SU(4)_l$ couplings to $SU(2)_{L,R}$ couplings. The gauge boson W_{3L} (associated with T_{3L} generator) will be a linear combination of gauge bosons W_{iq} and W_l (associated with $T_i^{W\times C}$ and T_1^{W} generators). Therefore, we have to construct linear combination of W_{qi} and W_l which will permit eq.(IV.20)In the following it will be assumed that all fields are normalized to one. The part of the covariant derivative which is relevant for our discussion is

$$g_{12} \sum_{i=1}^{3} T_i^{W\times C} W_{iq} + g_4 T_l^{W} W_l \qquad \text{(IV.23)}$$

The gauge bosons W_{qi} can be expressed in terms of new fields W_q, X and Y, and eq.(IV.23) becomes,

$$\frac{g_{12}}{\sqrt{3}} T_q W_q + g_4 T_l^w W_l \,, \tag{IV.24}$$

with $T_q = \sum_{i=1}^{3} T_{qi}^{wxc}$. The gauge fields W_q and W_1 can be expressed in terms of orthonormal fields W_{3L} and W as:

$$\begin{aligned} W_q &= \frac{\sqrt{3}}{g_{12}} g_{2L} W_{3L} + a W \,, \\ W_l &= \frac{1}{g_4} g_{2L} W_{3L} + b W \,. \end{aligned} \tag{IV.25}$$

The fact that W_q and W_1 are normalized to one and are orthogonal to each other uniquely determines g_{2L}, eq.(IV.21). Similar consideration for $SU(3)_c$ leads to

$$P_{SU_c(3),\, SU(12)_q} = 4 \,. \tag{IV.26}$$

Using equations (IV.17)-(IV.22), we can write down the expressions for the gauge coupling constants at the mass scale M_R, where G_2 breaks down:

$$\begin{aligned} \frac{1}{g_3^2(M_R)} &= \frac{4}{g_U^2(M_U)} + 2 \left\{ 4 b_{12} \ln \frac{M_U}{M_C} + b_3 \ln \frac{M_C}{M_R} \right\} , \\ \frac{1}{g_{2L/R}^2(M_R)} &= \frac{4}{g_U^2(M_U)} + 2 \left\{ (3 b_{12} + b_4) \ln \frac{M_U}{M_C} + b_{2L/R} \ln \frac{M_C}{R} \right\} , \\ \frac{1}{g_{B-L}^2(M_R)} &= \frac{4}{g_U^2(M_U)} + 2 \left\{ (b_{12} + b_4) \ln \frac{M_U}{M_C} + b_{B-L} \ln \frac{M_C}{M_R} \right\} . \end{aligned} \tag{IV.27}$$

At the next stage the symmetry G_2 is broken down to $SU(3)_C \times SU(2)_L \times U(1)_Y$. The generator of $U(1)_Y$ can be expressed as the linear combination,

$$Y = \sqrt{\frac{3}{5}}\, T_{3R} + \sqrt{\frac{2}{5}}\, Y_{B-L} \tag{IV.28}$$

of the $SU(2)_R$ and $U(1)_{B-L}$ generators. Therefore, at the mass scale M_L, where the symmetry $SU(3)_C \times SU(2)_L \times U(1)_Y$ is broken down to $SU(3)_C \times U(1)_{em}$, the renormalization group equations for the gauge coupling constants are:

$$\frac{1}{g_3^2(M_L)} = \frac{4}{g_U^2(M_U)} + 2\left\{ 4 b_{12} \ln \frac{M_U}{M_C} + b_4 \ln \frac{M_C}{M_L} \right\},$$

$$\frac{1}{g_{2L}^2(M_L)} = \frac{4}{g_U^2(M_U)} + 2\left\{ (3b_{12} + b_4) \ln \frac{M_U}{M_C} + 2 b_{2L} \ln \frac{M_C}{M_L} \right\} \tag{IV.29}$$

$$\frac{1}{g_Y^2(M_L)} = \frac{4}{g_U^2(M_U)} + 2\left\{ \frac{1}{5}(11 b_{12} + 9 b_4) \ln \frac{M_U}{M_C} + \frac{1}{5}(3 b_{2R} + 2 b_B) \ln \frac{M_C}{M_R} + b_Y \ln \frac{M_R}{M_L} \right\}$$

Let us recall that the value of b_N for a group $SU(N)$ is

$$b_N = -\frac{1}{16\pi^2}\left\{ \frac{11}{3} N - \frac{4}{3} N_f - \frac{1}{3} T_N \right\} \tag{IV.30}$$

N_f is the number of fermion multiplets transforming as a fundamental representation of the group $SU(N)$. T_N is the value of the second order Casimir operator for the representations of Higgs mesons. Having the renormalization group equations for the coupling constants we are now in a position to write down the formula for the $\sin^2\theta_W(M_L)$ and $\alpha_s(M_L)$ for Chain I of symmetry breaking:

$$\sin^2\theta_W(M_L) = \frac{3}{8} - \frac{11\,\alpha(M_L)}{24\pi}\Big\{4\Big[4 - \frac{1}{22}(T_{12}-T_4)\Big]\ln\frac{M_U}{M_C} + \Big[2 - \frac{1}{22}(5T_{2L}-3T_{2R}-2T_B)\Big]\ln\frac{M_C}{M_R} + 5\Big[1 - \frac{1}{22}(T_{2L}-T_Y)\Big]\ln\frac{M_R}{M_L}\Big\} \qquad \text{(IV.31)}$$

$$\frac{1}{\alpha_s(M_L)} = \frac{3}{8\alpha(M_L)} - \frac{11}{24\pi}\Big\{4\Big[12 - \frac{3}{22}(T_{12}-T_4)\Big]\ln\frac{M_U}{M_C}$$

$$\Big[6 - \frac{1}{22}(8T_3-3T_{2L}-3T_{2R}-2T_B)\Big]\ln\frac{M_C}{M_R} + \Big[9 - \frac{1}{22}(8T_3-3T_{2L}-5T_Y)\Big]\ln\frac{M_R}{M_L}\Big\}$$

($M_L = M_W$, the conventional W-boson's mass)

In Table IV, we give the allowed values for the various intermediate mass scales for a range of values of $\sin^2\theta_W(M_L)$ from .22 to .27. The listed values are obtained without accounting for the contributions of Higgs bosons. Interestingly enough, we find that there do exist solutions with rather low mass parity restoration[29], if we accept $\sin^2\theta_W$ = .26-.27. As has been shown by Rizzo and Senjanovic[29], such "large" values of $\sin^2\theta_W$ are allowed once the right-handed current effects are taken into account. Thus, the SU(16) grand unified theory provides another class of models in which a low W_R of $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ electroweak theories[27,31] can be embedded. Phenomenological implications for such models have been discussed in references 27, 29 and 31. We note that the value of the unification mass, $M_U = M_X$, in this case can be as low as 10^5 GeV. One therefore has to be careful in introducing baryon violation in this situation.

IV.3 SU(16) Breaking via $SU(8)_L \times SU(8)_R$ Symmetry

In this section we consider the breaking of SU(16) via the chain $SU(8)_L \times SU(8)_R \times U(1)_F$. This subgroup has already been discussed in literature as a candidate for grand unification group[32]. It is clear that the physics above the $SU(8)_L \times SU(8)_R$ breaking scale is not of importance to us here. We therefore concentrate on the breaking pattern below the mass scale M_8, where $SU(8)_L \times SU(8)_R$ is broken down to $SU(2)_L \times SU(2)_R \times SU(4)_C \times U(1)_F$, etc. that is, Chain II of Section IV.1.

To obtain the evolution equations for the gauge couplings, we present the basis and the diagonal generators. The diagonal generators for the particle sector are:

$$T_1 = \frac{1}{2}\left(\begin{array}{cccccccc|c}
1 &&&&&&&& \\
& 0 &&&&&&& \\
&& 0 &&&&&& \\
&&& 0 &&&&& \\
&&&& -1 &&&& \\
&&&&& 0 &&& \\
&&&&&& 0 && \\
&&&&&&& 0 & \\
\hline
&&&&&&&& 0
\end{array}\right)
\qquad
T_2 = \frac{1}{2}\left(\begin{array}{ccccccccc|c}
0 &&&&&&&&& \\
& 1 &&&&&&&& \\
&& 0 &&&&&&& \\
&&& 0 &&&&&& \\
&&&& 0 &&&&& \\
&&&&& -1 &&&& \\
&&&&&& 0 &&& \\
&&&&&&& 0 && \\
&&&&&&&& 0 & \\
\hline
&&&&&&&&& 0
\end{array}\right)$$

$$T_3 = \frac{1}{2}\left(\begin{array}{cccccccc|c}
0 &&&&&&&& \\
& 0 &&&&&&& \\
&& 1 &&&&&& \\
&&& 0 &&&&& \\
&&&& 0 &&&& \\
&&&&& 0 &&& \\
&&&&&& -1 && \\
&&&&&&& 0 & \\
\hline
&&&&&&&& 0
\end{array}\right)
\qquad
T_4 = \frac{1}{2}\left(\begin{array}{cccccccc|c}
0 &&&&&&&& \\
& 0 &&&&&&& \\
&& 0 &&&&&& \\
&&& 1 &&&&& \\
&&&& 0 &&&& \\
&&&&& 0 &&& \\
&&&&&& 0 && \\
&&&&&&& -1 & \\
\hline
&&&&&&&& 0
\end{array}\right)$$

(IV.32)

$$T_1^c = \frac{1}{2\sqrt{2}}\left(\begin{array}{cccccccc|c}
1 &&&&&&&& \\
& 1 &&&&&&& \\
&& 0 &&&&&& \\
&&& 0 &&&&& \\
&&&& -1 &&&& \\
&&&&& -1 &&& \\
&&&&&& 0 && \\
&&&&&&& 0 & \\
\hline
&&&&&&&& 0
\end{array}\right)
\qquad
T_2^c = \frac{1}{2\sqrt{6}}\left(\begin{array}{cccccccc|c}
1 &&&&&&&& \\
& 1 &&&&&&& \\
&& -2 &&&&&& \\
&&& 0 &&&&& \\
&&&& 1 &&&& \\
&&&&& 1 &&& \\
&&&&&& -2 && \\
&&&&&&& 0 & \\
\hline
&&&&&&&& 0
\end{array}\right)$$

$$T_7^c = \frac{1}{4\sqrt{3}}\left(\begin{array}{cccccccc|c}
1 &&&&&&&& \\
& 1 &&&&&&& \\
&& 1 &&&&&& \\
&&& -3 &&&&& \\
&&&& 1 &&&& \\
&&&&& 1 &&& \\
&&&&&& 1 && \\
&&&&&&& -3 & \\
\hline
&&&&&&&& 0
\end{array}\right)
\qquad
T_{15} = \frac{1}{4\sqrt{2}}\left(\begin{array}{ccccccccc|c}
1 &&&&&&&&& \\
& 1 &&&&&&&& \\
&& 1 &&&&&&& \\
&&& 1 &&&&&& \\
&&&& 1 &&&&& \\
&&&&& 1 &&&& \\
&&&&&& 1 &&& \\
&&&&&&& 1 && \\
&&&&&&&& 1 & \\
\hline
&&&&&&&&& -I
\end{array}\right)$$

The corresponding diagonal generators for the antiparticle sector can be obtained from the T_i's by interchanging the 8 x 8 block matrices and changing the sign in the entries.

For the next stage, $SU(2)_L \times SU(2)_R \times SU(4)_C$, the "probability" functions are:

$$P_{SU(2)_L SU(8)_L} = 4 \;,\quad P_{SU(2)_R SU(8)_R} = 4 \;, \tag{IV.33}$$

$$P_{SU(4)_C SU(8)_L} = 2 \;,\quad P_{SU(4)_C SU(8)_R} = 2 \;.$$

In the following stage, we break the symmetry down to $SU(3)_C \times SU(2)_L \times U(1)_Y$, so that only two non-trivial probability functions arise, that is,

$$P_{SU(2)_R, Y} = \frac{3}{5} \;,\quad P_{SU(4)_C Y} = \frac{2}{5} \tag{IV.34}$$

Using all these results and following exactly the same procedure as in Section IV.2, we obtain the following formula for $\sin^2\theta_W(M_L)$ and (M_L):

$$\begin{aligned} \sin^2\theta_W(M_L) &= \frac{3}{8} - \frac{11\alpha(M_L)}{48\pi}\Big\{\Big[-4 - \frac{1}{11}\left(5T_{2L} - 3T_{2R} - 2T_4\right)\Big]\ln\frac{M_8}{M_R} \\ &\quad + 5\Big[2 - \frac{1}{11}\left(T_{2L} - T_Y\right)\Big]\ln\frac{M_R}{M_L}\Big\}\;, \\ \frac{1}{\alpha_S(M_L)} &= \frac{3}{8\,\alpha(M_L)} - \frac{11}{48\pi}\Big\{\Big[12 - \frac{1}{11}\left(6T_4 - 3T_{2L} - 3T_{2R}\right)\Big]\ln\frac{M_8}{M_R} \\ &\quad + \Big[18 - \frac{1}{11}\left(8T_3 + 5T_{2L} - 13T_Y\right)\Big]\ln\frac{M_R}{M_L}\Big\} \end{aligned} \tag{IV.35}$$

Again, as in Section IV.2, we will defer the inclusion of Higgs contributions to a subsequent section. In Table V we present the results of our calculations. For this chain of symmetry breaking we do not find any solutions with low energy parity restoration even for "large" values of $\sin^2\theta_W$.

IV.4 Higgs Bosons in SU(16)

We will assume that the symmetry breaking of the SU(16) model is implemented by including explicit Higgs scalar multiplets into the theory. It is important to emphasize that we will not be in a position to solve the true minimum of the full potential, which, in general, involves the interplay of several different scalar multiplets. In the following, we simply will content ourselves to spell out alternative patterns of symmetry breaking. We believe that symmetry violations have a dynamical origin (perhaps in the fermion pairing forces) and we will assume that the Higgs fields are, in fact, composites of even numbers of fermions. We first discuss the Higgs multiplets necessary for breakdown of SU(16) symmetry to $SU(3)_C \times U(1)_{em}$. We discuss their implications for neutrino massses and, in the next section, we study their impact on the gauge boson mass hierarchy.

We will consider the following Higgs multiplets:

a) 255-dimensional denoted by Φ^A_B which transforms as adjoint representation,

b) 18240 -dimensional representation denoted by $\Phi^{\{AB\}}_{\{CD\}}$,
where the curly bracket stands for symmetrization with respect to the indices within the brackets,

c) 16 -dimensional representation denoted by Φ_A ,

d) 136 -dimensional denoted by $\Phi_{\{AB\}}$.

The Symmetry breaking discussed in Reference 26 is different from ours. We therefore describe it below. The Higgs multiplet, Φ^A_B , will be used to implement the first stage of symmetry breaking in Chain II. Note that it cannot be used to implement[33] the first stage of symmetry breaking in Chain I, that is, SU(16) $SU(12)_q \times SU(4)_4 \times U(1)_{|B+L|}$ Therefore, to implement this stage of the symmetry breaking we will use the $\Phi^{\{AB\}}_{\{CD\}}$ multiplet. Let us first concentrate on Chain I. For this purpose we need the representation contents of the Higgs multiplets under the various symmetry groups involved at different stages in this chain. We use two multiplets to implement the first and second stages of the symmetry breaking. For this purpose, we first display the content of 18240 under $SU(12)_q \times SU(4)_1$:

$$\begin{aligned}\underline{18240} = &(924,4) + (78,10) + (12,4) + (12,36) + h.c.\\ &+ (5940,1) + (1,34) + (143,1) \qquad\qquad (IV.36)\\ &+ (143,15) + (1,15) + (1,1)\end{aligned}$$

where h.c. stands for conjugate representations of the ones preceding it. It is therefore clear that by giving non-zero V.E.V. to the (1,1) component of <u>18240</u>, we break the group down to $SU(12)_q \times SU(4)_l \times U(1)_{(B-L)}$. We note that in eq.(IV.36), the representations (78,10) and (143,15) contain singlets under $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$. We assume that these components develop V.E.V., which break the group down to $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ To achieve the final stages of the breaking, we use the fundamental, <u>16</u>, and the symmetric Higgs multiplet, <u>136</u>, which have the following representation content under $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_B$:

$$\underline{16} = (3,2,1)_1 + (3,1,2)_{-1} + (1,2,1)_{+3} + (1,1,2)_{-3}, \tag{IV.37}$$

$$\begin{aligned}\underline{136} = {} & (3,1,1)_2 + (6,3,1)_2 + (1+8,2,2)_0 + (3,1,1)_{-2} \\ & (6,1,3)_{-2} + (1,3,1)_{-6} + (1,2,2)_0 + (1,1,3)_6.\end{aligned}$$

We may use <u>16</u> to break the $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ symmetry down to $SU(2)_L \times U(1)$ and <u>136</u> to break down the preceding symmetry to $SU(3)c \times U(1)_{em}$.

It is necessary to point out that the neutrino mass is sensitive to the multiplets used in breaking left-right symmetry. If we use <u>16</u> to break $SU(2)_L \times SU(2)_R \times U(1)_B$ at the tree level the neutrino has only a Dirac mass. However, it can have a Majorana mass arising in higher order by a mechanism originally discussed by Witten[34]. We show the relevant graph in Figure 2. Noting that 136 gives mass to the light fermions, we can estimate the Majorana mass of the right-handed neutrino to be

$$m_{\nu_R} \simeq \frac{\alpha^2}{16\pi^2}\,\frac{m_u}{M_L}\left(\frac{M_R}{M_U}\right)^2 \lambda M\,, \qquad \text{(IV.39)}$$

where λM stands for 16 x 16 x 136, the dimensional Higgs coupling. A natural choice for M is M_U since 16 x 16 x 136 coupling is likely to be a low energy remnant of the coupling on setting $\langle \Phi^A_B \rangle \neq 0$. Thus, we expect

$$m_{\nu_R} \simeq \frac{\alpha^2}{16\pi^2}\,\lambda\left(\frac{m_u}{M_L}\right)\left(\frac{M_R}{M_U}\right) M_R\,. \qquad \text{(IV.40)}$$

For $M_R \simeq 10^8$ GeV and $M_U \simeq 10^{15}$ GeV (a choice allowed by $\sin^2\theta_W$ and α_s) we expect, $m_{\nu_R} \simeq$ 1 eV. In this case, the Dirac mass is much bigger than the Majorana mass. However, if we have $M_U \simeq M_R \simeq 10^{14}$ GeV, this leads to $m_{\nu_R} \simeq 10^4$GeV which is quite an interesting prediction. This predicts the light left-handed Majorana neutrino mass to be around 1-10 eV.

Thus, if we accept the low mass W_R-solutions, the left-right symmetry, as well as the $SU(2)_L \times U(1)_Y$ symmetry, must be broken by the 136 dimensional representation. The Majorana mass for ν_R is then given by $\sim M_R$ and $m_{\nu_L} \sim \frac{m_u^2}{M_R}$.

The first stage of the symmetry breaking for Chain II is achieved by using adjoint representation Φ^A_B. The Φ^A_B has the following representation content under $SU(8)_L \times SU(8)_R \times U(1)_F$

$$\underline{255} = (1,1)_0 + (1,63)_0 + (63,1)_0 + (8,8)_2 + (8,8)_{-2}. \qquad \text{(IV.41)}$$

Giving $(1,1)_o$ a non-zero V.E.V. breaks SU(16) to $SU(8)_L \times SU(8)_R$. Note that the vacuum expectation value of 255, $V_o = (1,1)_o$, preserves the fermion number F as well as B and L numbers. Thus, V_o gives masses only to those gauge bosons which carry the fermion number $\pm$ 2. These gauge bosons will couple to the currents,

$$\bar{f}^c_L \gamma_\mu f_L \;, \quad \bar{f} \gamma_\mu f^c_L \qquad \text{(IV.42)}$$

where f_L stands either for quarks or leptons. The rest of the breakdown is achieved by Higgs bosons, Φ_A , $\Phi_{\{AB\}}$ and $\Phi^{\{AB\}}_{\{CD\}}$, and the discussion is similar to that just given.

IV.5 Effect of Higgs Bosons on Hierarchy of Gauge Boson Masses

In this section, we will describe how to include the Higgs boson effects consistently in the renormalization group equations in grand unified theories[35]. The main problem is to find what the masses of the various components of each Higgs multiplet are likely to be. Then, it is straightforward to include their effect in the equations for the evolution of the various gauge coupling constants.

In Reference 35, a set of rules has been given to isolate which components of a given Higgs multiplet are important at a given mass scale. We summarize them here:

i) Minimal Fine Tuning: In the Higgs potential, we will do no more fine tuning than is required to obtain the hierarchy of gauge boson masses. Also, we will assume all Higgs self-couplings to be of order unity.

ii) Spontaneous Symmetry Breakdown and Intramultiplet Mass Splitting: Let the grand unification group G_o be broken in stages as follows:

$$G_0 \supset G_1 \supset \cdots\cdots . \tag{IV.43}$$

Let the associated mass scales be $\mu_0, \mu_1, \ldots$. Let Φ belong to an irreducible representation of G_o and be used to break the group G_m to G_{m+1}. Let Φ have the following representation content under G_m

$$\Phi = \sum_j \Phi_j^m \tag{IV.44}$$

If $\langle \Phi_k^m \rangle \neq 0$ is responsible for the breaking of G_m to G_{m+1}, we postulate that[35] the whole sub-multiplet Φ_j^m has a mass of order μ_m^2 and will contribute only above μ_m.

iii) Survival Hypothesis: To discuss this hypothesis, let us consider the above example. The question what is the mass of Φ_j^m $j \neq k$. We postulate that, of Φ_j^m, any set of multiplets which constitutes a full irreducible representation under any of the groups G_{m-n}, $n=0,1,2,\ldots m-1$ will acquire a mass corresponding to the mass scale breaking the group G_{m-n-1} to G_{m-n}, for example μ_{m-n-1}, unless any sub-multiplet happens to be a pseudo-Goldstone boson.

This is similar to the survival hypothesis discussed by Georgi[36] for the case of fermions.

We will now apply these criteria to the SU(16)-Higgs multiplets. Let us take each multiplet, one by one for Chain I. It is obvious that, since $\bar{\Phi}^{\{AB\}}_{\{CD\}}$ is involved in the first stage, all its sub-multiplets have a mass of order M_U and, therefore, they do not contribute to coupling constant evolution. Let us next consider the multiplet $\Phi^{\{AB\}}_{\{CD\}}$, that breaks $SU(12)_q \times SU(4)_l \times U(1)_{B-L}$. Since it is the (78,10) component which acquires a non-zero V.E.V., its mass will be of the order M_C. Therefore, it will contribute to b_{12} and b_4 above M_C. All the remaining multiplets in eq.(IV.31) are superheavy and are not relevant to us. Now, let us assume that $\underline{16}$ breaks $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ Under $SU(12)_q \times SU(4)_l$, $\underline{16}$ has the following decomposition

$$\underline{16}=(12,1) + (1,4) \qquad \text{(IV.45)}$$

where under $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$

$$(1,4)=(1,2,1) + (1,1,2) \ . \qquad \text{(IV.46)}$$

It is the (1,1,2) which breaks $SU(2)_R \times U(1)_{B-L}$, therefore, (1,1,2) has a mass of order M_R and will contribute b_{2R} and $b_{(B-L)}$. On the other hand, the multiplet (1,2,1) will also have a mass of order M_R by left-right symmetry. The (12,1) part of the multiplet will be superheavy by the survival hypothesis (iii) above.

Let us finally consider multiplet 136 that breaks $SU(2)_L \times U(1)_Y$ to $U(1)_{em}$. Under $SU(12)_q \times SU(4)_1$, the multiplet 136 has the following decomposition

$$\underline{136} = (78,1) + (12,4) + (1,10) \tag{IV.47}$$

To break $SU(2)_L \times U(1)_Y$, we use (1,10) and (78,1). Therefore, by the survival hypothesis, (12,4) will acquire a mass of order M_U. Now, under $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$, multiplets (78,1) and (1,10) have the following decompositions:

$$\begin{aligned}(78,1) = {} & (3,1,1)_2 + (3,1,1)_{-2} + (6,3,1)_2 + \\ & (6,1,3)_2 + (1{+}8,2,2)_0,\end{aligned} \tag{IV.48}$$

$$(1,10) = (1,3,1)_{-6} + (1,1,3)_6 + (1,2,2)_0 \tag{IV.49}$$

The $(1,2,2)_0$ parts in both eqs. (IV.48) and (IV.49) will acquire V.E.V.'s of order M_{W_L} and will give Dirac masses to quarks and leptons. It is clear that the multiplets (3,1,1,), (3,1,1), (6,3,1), (6,1,3), (8,2,2), (1,3,1) and (1,1,3) will acquire mass of order M_C and will therefore contribute to b_{12} and b_4. There are two left-handed doublets with mass of order M_R and two left-handed doublets with mass of order M_L. Thus, above M_R, two (1,2,2) multiplets contribute to b_{2L} and b_{2R}, whereas above M_L, two left-handed doublets contribute to b_{2L}.

We illustrate their effect only for the symmetry breaking Chain I and the resulting mass scales are given in Table VI. We have presented this simply as an illustration and, in subsequent discussions, we will not use the particular numbers reported. We will in the subsequent section discuss phenomenological implications of this model including different kinds of Higgs multiplets. The main point we wish to make, however, is that the Higgs boson effects on mass scales are not completely negligible.

IV.6 Spontaneous Symmetry Breaking and Predictions for Baryon Non-Conservation

In this section we wish to include some comments on the detailed mechanism for spontaneous breakdown via the $SU(12)_q \times SU(4)_l \times U(1)_{|B|-|L|}$ route and its phenomenological implications. We remind the reader that, in this chain we will temporarily use two symmetric-adjoint multiplets, Φ to break the group from SU(16) to $SU(12)_q \times SU(4)_l \times U(1)_{|B|-|L|}$ and then to $SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$. To implement the first stage, we assign

$$\langle \Phi^{\alpha a}_{\alpha a} \rangle = \frac{M_U}{g} \qquad \text{(IV.50)}$$

where index "a" runs over the leptons and antileptons (a =4, 8, 12, 16). The index goes over the quarks and antiquarks, that is, α =1, 2,....15 and $\alpha \neq$ a.

Now let us discuss which gauge bosons acquire mass at a given stage. For that purpose, we fix the following notation:

$V^i_j - \frac{1}{3}\Sigma V^i_i \quad i,j=1\ldots 8$	Color gluons,
$W^i_{L,R} \quad i=1,2,3$	Left and right-handed weak gauge bosons,
B	$U(1)_{B-L}$ gauge boson,
$X^a_\alpha \quad a=4,8,12,16;\ \alpha\neq a$	Lepto-quark bosons,
Y^α_β	Diquark gauge bosons,
$Y'^a_b \quad a=12,16;\ b=4,8$	Dilepton gauge bosons.

Due to (IV.50), 96 gauge bosons X^α_a acquire mass of order M_U. Note that since the gauge bosons connecting quarks and leptons of each flavor acquire mass of order $M_U \geq (10^5\text{-}10^6)$ GeV ,it is consistent with the present data on $K_L \to \mu e$.

At the second stage of symmetry breaking, we assume that a second $\Phi^{\{AB\}}_{\{CD\}}$ with the following V.E.V. exists

$$\langle \Phi^{i,i+8}_{12,4} \rangle = \langle \Phi^{4+i,12+i}_{16,8} \rangle = \frac{M_c}{g} \qquad \text{(IV.51)}$$

This gives mass to 144 gauge bosons including Y^α_β, Y'^a_b, but it does not give rise to any mixings between X,Y and Y' gauge bosons. This is because V.E.V. in eq. (IV.51) conserves baryon and lepton numbers. In fact, we find that in using multiplet $\Phi^{\{AB\}}_{\{CD\}}$, it is difficult to get any baryon non-conservation. The point is that the only interesting V.E.V. that leads to $\Delta B \neq 0$ processes and breaks the symmetry appropriately, is of type

$$\varepsilon^{ijk}\langle \Phi^{u_i d_j}_{\nu^c d^c_k} \rangle = \varepsilon^{ijk}\langle \Phi^{u_i d_j}_{e^c u^c_k} \rangle \neq 0 \qquad \text{(IV.52)}$$

where we have used explicit fermion labels instead of numerical indices. Note that we needed a multiplet with antisymmetrical indices. If we had used symmetric indices, then $\{u_i, d_i\}$ would have given rise to an $SU(2)_L$ triplet and could not have a scale more than $M_L/10$. This would conflict with our previous requirement that

$$\langle \Phi^{\{AB\}}_{\{CD\}} \rangle = \frac{M_c}{g} \qquad \text{(IV.53)}$$

A way to avoid this problem is to replace the symmetric-adjoint multiplet at the second stage by the antisymmetric-adjoint Higgs multiplet in addition to the symmetric one. This will induce (B-L) conserving proton decay via the diagram of Figure 3. The amplitude is given by

$$M(p \to e^+ \pi^0) \simeq \frac{\alpha M_c^2 \varepsilon}{M_c^2 M_u^2} \qquad \text{(IV.54)}$$

where ϵ is the mixing parameter between the diquark and lepto-quark gauge bosons. The proton lifetime constrains M_U to be larger than 10^{14} GeV unless is chosen unnaturally small. Therefore, from this point on, we will discuss physics for these values of M_U.

Let us now look for $\Delta B \neq 0$ processes with other selection rules. Note that down to the scale M_R, B-L is an exact symmetry. Once we are below the scale M_R, local B-L symmetry is broken by

$$\langle \Phi_{\nu^c \nu^c} \rangle = \frac{M_R}{g} \qquad \text{(IV.55)}$$

which obeys the selection rule Δ(B-L)=2. The combination of Δ(B-L)=2 and Δ(B-L)=0 selection rules produces both ΔB=2 processes, such as N-$\bar{N}$ oscillation[37], as well as Δ(B+L)=0 processes, such as $n \to e^- \pi^+$ decay. It appears that the dominant contribution to N-$\bar{N}$ oscillations comes from the graph in Figure 4. This graph is similar to the one noted in Reference 38 and as already noted this leads to an extremely slow N-$\bar{N}$ oscillation time, $\tau_{N\bar{N}} \geq 10^{30}$ years. Thus in this model N-$\bar{N}$ oscillation is suppressed. In fact, it appears to us that N-N oscillation will always be suppressed in simple SU(16) models for the following reasons. The only Δ(B-L)=2 V.E.V. allowed by charge conservation is that due to $\nu_L \nu_R$ condensate. This obeys ΔL=2 and ΔB=0. Therefore, to obtain ΔB=2 processes, we must insert <u>two</u> Δ(B-L)=0

V.E.V.'s that break Δ B. The only such V.E.V.'s allowed by color and electric charge conservation are $\langle\Phi^{ud}_{\nu^c d^c}\rangle$, which also gives rise to $p\to e^+\pi^0$ decay. Roughly speaking, as an order of magnitude estimate one obtains $M_{N\bar{N}} \sim |M_{p\to e^+\pi^0}|^2$ leading to $\tau_{N\bar{N}} > 10^{30}$ years. Of course, one way to avoid such a small amplitude is to introduce very complicated Higgs multiplets with six SU(16) indices, Φ_{ABCDEF} and to choose low unification mass. The allowed Higgs self-coupling should be of the form

$$\Phi^{AB}\phi^{CD}\phi^{EF}\,\Phi_{ABCDEF}$$

Setting $\langle\Phi_{uddudd}\rangle \neq 0$ would lead to large N-$\bar{N}$ oscillation amplitude.

The other Δ(B-L)=2 process which respects Δ(B+L)=0 selection rule arises from the Feynman diagram in Figure 5 and leads to decays such as $n\to e^-\pi^+$. But these appear to have a strength

$$M_{n\to e^-\pi^+} \simeq G_F\, \alpha\epsilon \frac{M_P^2}{M_u^2} \simeq 10^{-35}\,GeV^{-2} \qquad \text{(IV.56)}$$

It thus appears that in the model discussed here, the dominant proton decay mode is the B-L conserving mode $p\to e^+\pi^0$, which has a life-time 10^{30}years. Finally, we note that in the cases with low W_R mass scale, an outstanding signature will be provided by the existence of neutrinoless double β-decay transitions in a manner similar to that discussed in earlier papers [31]. Other processes in the leptonic domain will be processes like $\mu \to e\gamma$, $\mu \to 3e$, etc., as already discussed [31,39].

We close this part of our work with the following conclusions and remarks:

1. Being a large group, SU(16), in general, allows several intermediate mass scales, filling the gap between 10^2 GeV and unification energy. A single stage of symmetry breaking, that is, without intermediate mass scales, reproduces already known results for $\sin^2\theta_W$ and M_X, which we reviewed in Chapter II. This is not much of a surprise because SU(16) has a standard fermion content and $\sin^2\theta_W=3/8$ in the symmetry limit. Therefore, this result should be considered as a consistency check of our calculations rather than a conclusion.

2. Our analyses of symmetry breaking with two intermediate mass scales show that Chain II allowed a low value for the scale of right-handed interactions, if $\sin^2\theta_W=.27$ and $\alpha_s=.10$ - .12. The unification mass, in this case, can be as low as 10^5GeV.

3. For the low unification mass, that is, $M_U \sim 10^5$ GeV, the only possible baryon non-conserving process is $N - \overline{N}$ oscillation, with $\tau_{N\bar{N}} > 10^7 - 10^8$ seconds.

4. For unification mass bigger than 10^{14} GeV, the dominant proton decay mode is B-L conserving mode, with lifetime $\gtrsim 10^{30}$ years. All other baryon non-conserving processes are suppressed.

References and Footnotes

1. Reviews relevant to the material of this section include: Harrari, Phys. Reports 42C, 235 (1978); H. Fritzsch and P. Minkowsky, Phys. Reports 73C, 67 (1981) E. Abers and B. W. Lee, Phys. Reports 9C, 1 (1973).

2. S. Weinberg, Phys. Rev. Lett. 19, 1264 (1967) A. Salam, Proceedings of the Eight Nobel Symposium (John Wiley, New York, 1968)

3. W. Marciano and H. Pagels, Phys. Reports 36C, 137 (1978)

4. P. Langacker, Phys. Reports 72, 185 (1981) J. Ellis, TH. 3174 - CERN (1981) Lectures presented at the Les Houches Summer School in Physics, August 1981.

5. H. Georgi and S. L. Glashow, Phys. Rev. Lett. 32, 438 (1974)

6. J. Banks and H. Georgi, Phys. Rev. D14, 1159 (1976) S. Okubo, Phys. Rev. D16, 3528 (1977)

7. H. Georgi, H. Quinn and S. Weinberg, Phys. Rev. Lett. 33, 451 (1974).

8. D. B. Lichtenberg, Unitary Symmetry and Elementary Particles, 2nd edition (Academic Press, New York,1978)

9. W. J. Marciano and A. Sirlin, in Proc. of V.P.I Workshop on Weak Int. as Probes of Unification, ed. G. B. Collins, L. N. Chang and J. R. Ficenec, (Am. Inst. of Phys., New York, 1981)

10. A. Buras, Proc. Lepton - Hadron Int., Bonn, August, 1981.

11. J. Learned, F. Reines and A. Soni, Phys. Rev. Lett. 43, 907 (1979), M. L. Cherry et al., Phys. Rev. Lett. 47, 1511 (1981).

12. This chapter is based on ; M. Popovic, Phys. Rev. D 23, 1871 (1981) J. Chakrabarti, M. B. Popovic and R. N. Mohapatra, D 21, 3212 (1980) M. Popovic and R. N. Mohapatra, Phys. Rev. D24, 719 (1981)

13. A. De Rujula, H. Georgi, and S. L. Glashov, Ann. Phys. (N.Y.) 109, 258 (1977) F. Wilczek and A. Zee, Phys. Lett. B70, 436 (1977); S. Weinberg, Trans. N.Y. Acad. Sci. 38, (1977); H. Fritzsch, Phys. Lett. B73, 317 (1978);

R. N. Mohapatra and G. Senjanovic, Phys. Lett. B73, 70 (1978)
G. Branco, Phys. Lett. B76, 70 (1978);
D. Wyler, Phys. Rev. D19, 330 (1979);
A. Ebrahim, Phys. Lett. B76, 605 (1978);
T. Hagiwara, T. Kitazoe, G. Mainland, and K. Tanaka, Phys. Lett. 76B, 602 (1978);
M. deCrombruggie, Phys. Lett. B80, 365 (1979);
H. Harrari, H. Haut, and J. Weyers, Phys. Lett. B78, 459 (1978);
G. Segre, A. Weldon and J. Weyers, Phys. Lett. B83, 351 (1979);
R. Barbieri, R. Gatto, and F. Strocci, Phys.Lett. B74, 344 (1978);
For review see R. Gatto, Invited talk at the Einstein Symposium at Jerusalem, 1979 (unpublished).

14. R. N. Mohapatra, Phys. Rev. D9, 3461 (1974);
R. N. Mohapatra, J. C. Pati, and L. Wolfenstein, Phys. Rev. D11, 3319 (1975);
F. Wilczek and A. Zee, Phys. Rev. Lett. 42, 421 (1979);
C. L. Ong, Phys. Rev. D19, 2738 (1979);
A. Davidson, M. Koka, and K. C. Wali, Phys. Rev. 20, 1195 1975);
J. Chakrabarti, Phys. Rev. D20, 2411 (1979).

15. H. Georgi, Nucl. Phys. B156, 126 (1979).

16. P. Frampton and S. Nandi, Phys. Rev. Lett. 43, 1460 (1979).

17. See the first paper in Reference 12.

18. H. Georgi, Nucl. Phys. B156, 126 (1979);
P. Framton and S. Nandi, Phys. Rev. Lett. 43, 1460 (1979);
J. Chakabarti, M. B. Popovic and R. N. Mohapatra, Phys. Rev. D21, 3212 (1980).

19. See third paper in Reference 18

20. We have not included the effect of Higgs boson in the calculation of the   function, since we hope, in some future theory, they will arise dynamically as composites of the basic fermion fields.

21. We note that for the case of SO(10) grand unification, asymptotic-freedom constraints on the grand unifying coupling constant allow a much larger number of light generations.

22. See third paper in Reference 16.

23. For other SU(N) flavor unification models see;
T. Curtright and P. G. O. Freund, in Supergravity, ed. by D. Z. Freedman et al., North Holland;
C. W. Kim and C. Roisnel, Phys. Lett. B93, 343 (1980);
J. Ellis, et al., Phys. Lett. B93, 343 (1980);
J. E. Kim, Phys. Rev. D21, 1687 (1980);
Y. Tosa, Phys. Rev. D22, 3113 (1980).

24. This chapter is based on:
R. N. Mohapatra and M. B. Popovic, Phys. Rev. D25, 3012 (1982).

25. S. Rajpoot, Phys. Rev. D22, 2245 (1980);
Q. Shafi et al., Phys. Lett. B92, 304 (1980).

26. J. C. Pati, A. Salam and J. Strathdee, Nucl. Phys. B185, 445 (1981).

27. J. C. Pati and A. Salam, Phys. Rev. D10, 275 (1974);
R. N. Mohapatra and J. C. Pati, Phys. Rev. D11, 566 (1975)
G.Senjanovic and R.N. Mohapatra, Phys. Rev. D12, 1502 (1975).

28. S. Dawson and H. Georgi, Phys. Rev. Lett. 43, 821 (1979).

29. T. Rizzo and G. Senjanovic, Phys. Rev. Lett. 46, 1315 (1981); Phys. Rev. D24, 704 (1981).

30. H. Fritzsch and P. Minkowski, Ann. Phys. N.Y. 93, 193 (1975);
H. Georgi in: Particles and Fields, AIP, N.Y. 1975, p. 575
H. Georgi and D. V. Nanopoulos, Nucl. Phys. B155, 52 (1979).

31. R. N. Mohapatra and G. Senjanovic, Phys. Rev. Lett. 44, 912 (1980) and Phys. Rev. D23, 165 (1981).

32. N. G. Deshpande and P. Manheim, Phys. Lett. B94, 355 (1980).

33. L. F. Li, Phys. Rev. D9, 1723 (1974).

34. E. Witten, Phys. Lett. B91, 81 (1980).

35. T. Rizzo and G. Senjanovic, Brookhaven Preprint (1981);
R. N. Mohapatra and G. Senjanovic, Brookhaven Preprint (1981).

36. See first paper in Reference 23.

37. R. N. Mohapatra and R. E. Marshak, Phys. Rev. Lett. 44, 1316 (1980); Phys. Lett. B94, 183 (1980);

N. P. Chang and L. N. Chang, Phys. Lett. B92, 103 (1980); V. A. Kuzmin, Pisma Zh. Eksp. Theor. Fiz. 13, 335 (1970); S. L. Glashow, Cargese Lectures (1979).

38. T. K. Kuo and S. Love, Phys. Rev. Lett. 45, 93 (1980).

39. R. E. Marshak et al., Phys. Rev. D24, 1310 (1981).

Figure 1.

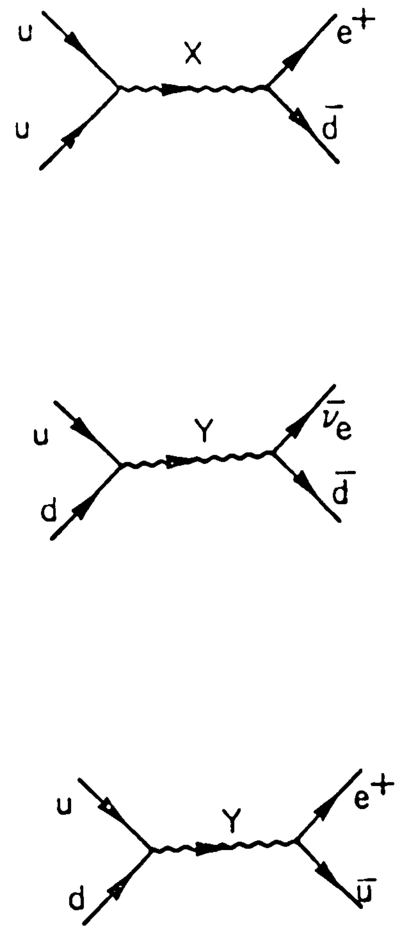


Figure 2

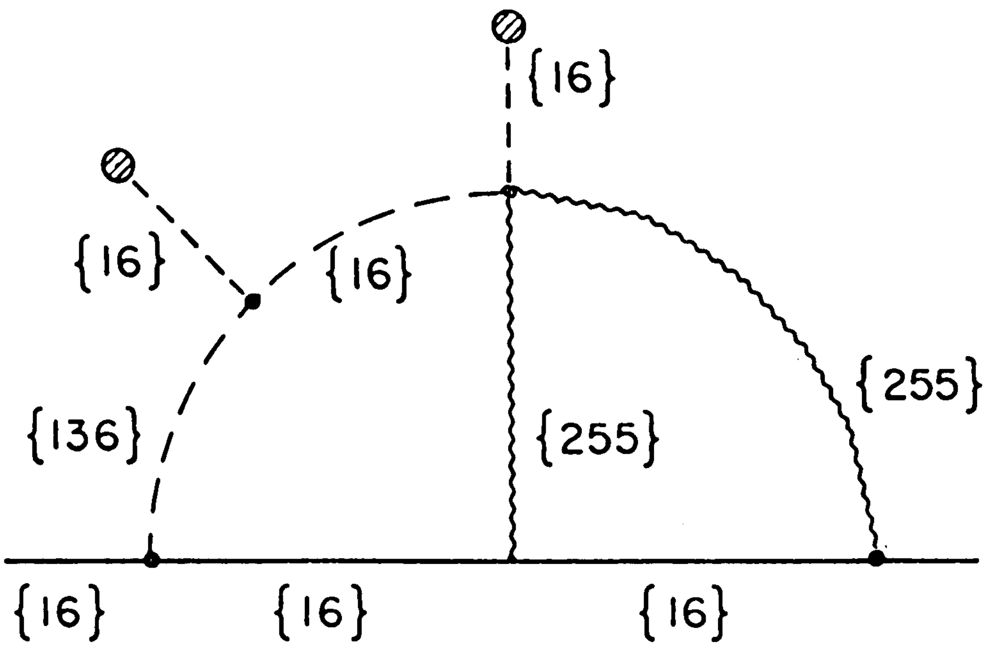


Figure 3

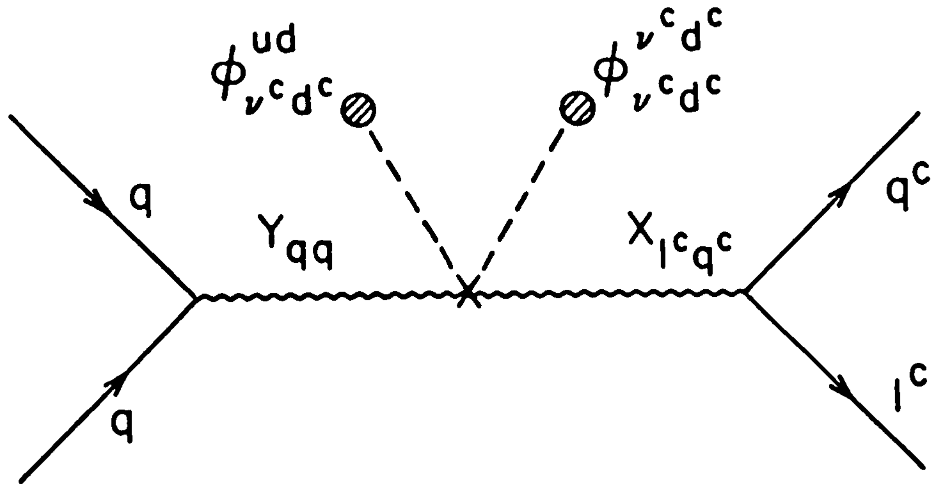


Figure 4

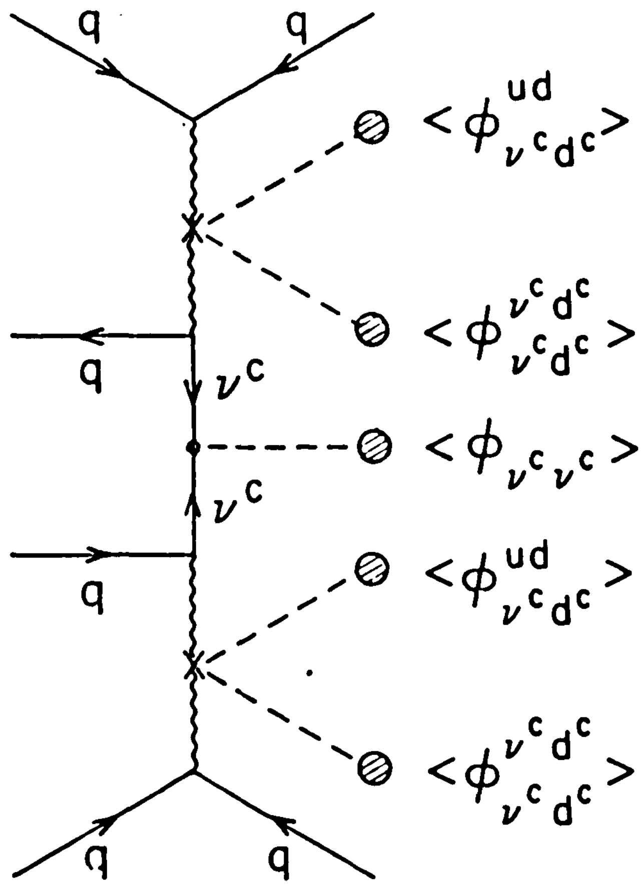
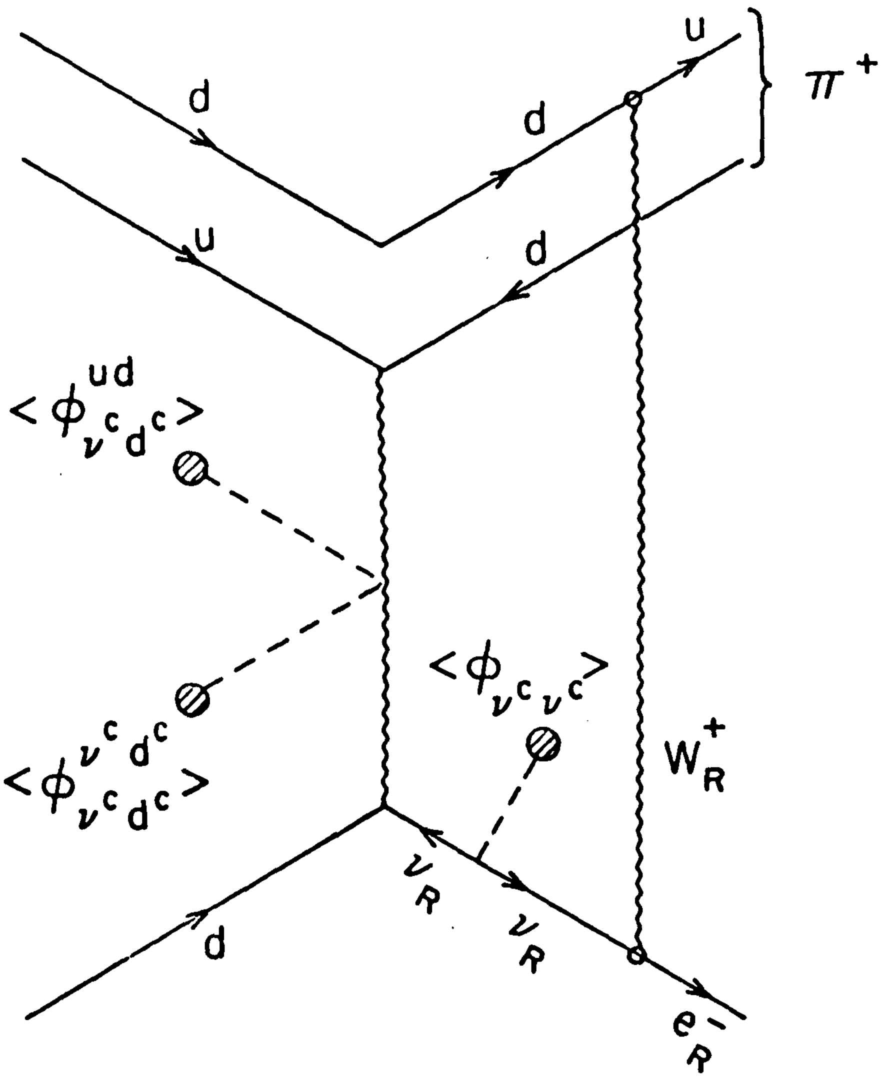

q
q
$<\phi^{ud}_{\nu^c d^c}>$
$<\phi^{\nu^c d^c}_{\nu^c d^c}>$
q
ν^c
$<\phi_{\nu^c \nu^c}>$
ν^c
q
$<\phi^{ud}_{\nu^c d^c}>$
$<\phi^{\nu^c d^c}_{\nu^c d^c}>$
q
q

Figure 5



d
u
d
d
u
d
π^+
$<\phi^{ud}_{\nu^c d^c}>$
$<\phi_{\nu^c \nu^c}>$
$<\phi^{\nu^c d^c}_{\nu^c d^c}>$
W_R^+
ν_R
ν_R
d
e_R^-

Table 1.

Representation	Dimension	Anomalies	Representations of SU(5) Representations of SU(3) contained in given representation				
(21)	40	16	(21)	(20)	(11)	(10)	
			1	2	2	4	
(22)	50	15	(22)	(21)	(11)	(20)	(10)
			1	2	3	1	2
(31)	45	6	(10)	(21)	(20)	(11)	
			3	2	4	1	
(32)	75	0	(20)	(10)	(22)	(21)	(11)
			3	3	2	4	2
(41)	24	0	(10)	(21)	(20)		
			2	1	2		

Representation	Dimension	Anomalies	Representations of SU(6) Representations of SU(5) contained in given representation			
(21)	70	27	(21)	(20)	(11)	(10)
			1	1	1	1
(22)	105	40	(22)	(21)	(11)	
			1	1	1	
(31)	105	22	(31)	(30)	(21)	(20)
			1	1	1	1
(32)	210	37	(32)	(31)	(22)	(21)
			1	1	1	1
(33)	175	0	(33)	(32)	(22)	
			1	1	1	
(41)	84	4	(41)	(40)	(31)	(30)
			1	1	1	1
(42)	189	0	(42)	(41)	(32)	(31)
			1	1	1	1
(51)	35	0	(10)	(41)	(40)	
			1	1	1	

Representation	Dimension	Anomalies	Representations of SU(7) Representations of SU(3) contained in given representation								
(21)	112	40	(21)	(20)	(11)	(10)					
			1	2	2	4					
(22)	196	77	(22)	(21)	(11)	(20)	(10)				
			1	2	3	1	2				
(31)	210	51	(31)	(30)	(21)	(20)	(11)	(10)			
			1	2	2	4	1	2			
(32)	490	126	(32)	(31)	(22)	(21)	(30)	(20)	(11)	(10)	
			1	2	2	4	1	2	2	1	
(33)	490	77	(33)	(32)	(22)	(31)	(21)	(11)			
			1	2	3	1	2	1			
(41)	224	24	(41)	(40)	(31)	(30)	(21)	(20)			
			1	2	2	4	1	2			
(42)	588	63	(42)	(41)	(32)	(31)	(40)	(30)	(22)	(21)	(20)
			1	2	2	4	1	2	1	2	1

TABLE I. *(Continued)*

Representations of SU(7)

Representation	Dimension	Anomalies	Representations of SU(5) contained in given representation							
(43)	784	0	(43)	(42)	(33)	(32)	(41)	(31)	(22)	(21)
			1	2	2	4	1	2	2	1
(51)	140	1	(10)	(41)	(40)	(31)	(30)			
			1	2	4	1	2			
(52)	392	0	(20)	(10)	(42)	(41)	(40)	(32)	(31)	(30)
			1	2	2	4	2	1	2	1
(61)	48	0	(10)	(41)	(40)					
			2	1	2					

Representations of SU(8)

Representation	Dimension	Anomalies	Representations of SU(5) contained in given representation							
(21)	168	55	(21)	(20)	(11)	(10)				
			1	3	3	9				
(22)	336	128	(22)	(21)	(11)	(20)	(10)			
			1	3	6	3	8			
(31)	378	96	(31)	(30)	(21)	(20)	(11)	(10)		
			1	3	3	9	3	9		
(32)	1008	294	(32)	(31)	(22)	(21)	(30)	(20)	(11)	(10)
			1	3	3	9	3	9	8	9
(41)	504	75	(41)	(40)	(31)	(30)	(21)	(20)	(11)	(10)
			1	3	3	9	3	9	1	3
(51)	420	20	(10)	(41)	(40)	(31)	(30)	(21)	(20)	
			1	3	9	3	9	1	3	
(61)	216	−3	(10)	(41)	(40)	(31)	(30)			
			3	3	9	1	3			
(62)	720	0	(20)	(10)	(42)	(41)	(40)	(32)	(31)	(30)
			3	9	3	9	9	1	3	3
(71)	63	0	(10)	(41)	(40)					
			3	1	3					

Representations of SU(9)

Representation	Dimension	Anomalies	Representations of SU(5) contained in given representation							
(21)	240	72	(21)	(20)	(11)	(10)				
			1	4	4	16				
(22)	540	195	(22)	(21)	(11)	(20)	(10)			
			1	4	10	6	20			
(31)	630	160	(31)	(30)	(21)	(20)	(11)	(10)		
			1	4	4	16	6	24		
(41)	1008	176	(41)	(40)	(31)	(30)	(21)	(20)	(11)	(10)
			1	4	4	16	6	24	4	16
(51)	1050	90	(10)	(41)	(40)	(31)	(30)	(21)	(20)	(11)
			5	4	16	6	24	4	16	1
(61)	720	8	(10)	(41)	(40)	(31)	(30)	(21)	(20)	
			4	6	24	4	16	1	4	
(71)	315	−8	(10)	(41)	(40)	(31)	(30)			
			6	4	16	1	4			
(81)	80	0	(10)	(41)	(40)					
			4	1	4					

TABLE I. (*Continued*)

Representations of SU(10)

Representation	Dimension	Anomalies	Representations of SU(5) contained in given representation						
(21)	330	91	(21)	(20)	(11)	(10)			
			1	5	5	25			
(22)	825	280	(22)	(21)	(11)	(20)	(10)		
			1	5	15	10	40		
(31)	990	246	(31)	(30)	(21)	(20)	(11)	(10)	
			1	5	5	25	10	50	
(71)	1155	−14	(10)	(41)	(40)	(31)	(30)	(21)	(20)
			10	10	50	5	25	1	5
(81)	440	−14	(10)	(41)	(40)	(31)	(30)		
			10	5	25	1	5		

Representations of SU(11)

Representation	Dimension	Anomalies	Representations of SU(5) contained in given representation				
(21)	440	112	(21)	(20)	(11)	(10)	
			1	6	6	36	
(91)	594	−21	(10)	(41)	(40)	(31)	(30)
			15	6	36	1	6

Representations of SU(12)

Representation	Dimension	Anomalies	Representation of SU(5) contained in given representation				
(21)	527	135	(21)	(20)	(11)	(10)	
			1	7	7	49	
(101)	780	−29	(10)	(41)	(40)	(31)	(30)
			21	7	49	1	7

Representations of SU(13)

Representation	Dimension	Anomalies	Representations of SU(5) contained in given representation				
(21)	728	160	(21)	(20)	(11)	(10)	
			1	8	8	64	
(111)	1001	−38	(10)	(41)	(40)	(31)	(30)
			28	8	64	1	8

Representation of SU(14)

Representation	Dimension	Anomalies	Representations of SU(5) contained in given representation			
(21)	910	187	(21)	(20)	(11)	(10)
			1	9	9	81

Table II.

SU(N)	$\{m_1, m_2, m_3, m_4\}$	$\{C_1, C_2, C_3, C_4\}$	g
SU(9)	$\{1, 2, 3^*, 4\}$	$\{8, 1, 2, 1\}$	3
	$\{1^*, 2, 3, 4^*\}$	$\{19, 3, 1, 1\}$	4
	$\{1^*, 2, 3, 4\}$	$\{24, 2, 1, 1\}$	7
	$\{1^*, 2^*, 3, 4\}$	$\{4, 2, 1, 1\}$	3
	$\{1^*, 2, 3, 4^*\}$	$\{14, 2, 1, 1\}$	3
	$\{1^*, 2, 3, 4\}$	$\{19, 1, 1, 1\}$	6
	$\{1^*, 2^*, 3, 4\}$	$\{9, 1, 1, 1\}$	4

Table III.

SU(N)	$\{m_1, m_2, m_3\}$	$\{C_2, C_3\}$	g
SU(7)	$\{1^*, 2, 3\}$	$\{C_2, 1\}$, $2 \leq C_2 \leq 8$	
		$\{C_2, 2\}$, $C_2 \leq 5$	
		$\{C_2, 3\}$, $C_2 \leq 5$	
		$\{3, 4\}$, $\{2, 5\}$	
		$\{1, C_3\}$, $2 \leq C_3 \leq 5$	
SU(8)		$\{1, C_3\}$ $C_3 \leq 3$	$g = C_2 + C_3(N-6)$
		$\{C_2, 1\}$, $C_2 \leq 6$	
		$\{2, 3\}$, $\{3, 2\}$	
SU(9)		$\{C_2, 1\}$, $C_2 \leq 5$	
		$\{C_2, 2\}$ $C_2 \leq 3$	
SU(10)		$\{C_2, C_3\}, C_2 + C_3 \leq 5, C_3 \leq 2$	
SU(10+i)		$\{C_2, 1\}$, $C_2 \leq 5 - i, 1 \leq i \leq 4$	
SU(15)		$\{1, 1\}$	
		$C_1 = C_2(N-4) + C_3\frac{(N-6)(N-3)}{2}$	
SU(9)	$\{1^*, 2, 4\}$	$\{C_2, C_3\}, C_2 \leq 4, C_3 \leq 2$	
SU(10)		$\{C_2, 1\}$ $, C_2 \leq 2$	$g = C_2 + C_3\frac{(N-5)(N-8)}{2}$
		$C_1 = (N-4)(C_2 + C_3\frac{(N-8)(N-3)}{6})$	

Table III. (Continued)

SU(N)	$\{m_1,m_2,m_3\}$	$\{C_1,C_2,C_3\}$	K
SU(7)	$\{1^*,2^*,3\}$	$\{5+2i,1,4+i\}, o\leq i\leq 2$	$\infty=C_3(N-6)-C_2$
		$\{4,2,5\}$	
SU(8)		$\{6+5i,1,2+i\}, o\leq i\leq 2$	
		$\{8,3,4\}$, $\{7,2,3\}$	
SU(9)		$\{7,4,3\}$, $\{13,1,2\}$	
		$\{3,3,2\}$, $\{22,1,2\}$	
		$\{17,2,3\}$	
SU(1o)		$\{8,1,1\}$, $\{22,1,2\}$	
SU(11)		$\{19,3,2\}$, $\{6,2,1\}$	
		$\{13,1,1\}$, $\{33,1,2\}$	
SU(13)		$\{8+9i,3-i,1\}$ $o\leq i\leq 2$	
SU(14)		$\{4+1oi,4-i,1\}$ $o\leq i\leq 3$	
SU(15)		$\{1o+11i,4-i,1\}$ $o\leq i\leq 3$	
SU(16)		$\{5+12i,5-i,1\}$ $o\leq i\leq 3$	
SU(17)		$\{51,2,1\}$, $\{64,1,1\}$	
SU(1o)	$\{1,2^*,3\}$	$\{2,5,2\}$	6
SU(13)		$\{1,4,1\}$	5
SU(14)		$\{6,5,1\}$	6
SU(15)		$\{1,5,1\}$	7
SU(9)	$\{1^*,2^*,4\}$	$\{5,1,2\}$	3
SU(1o)		$\{2,2,1\}$	3
		$\{8,1,1\}$	4
SU(11)		$\{7,3,1\}$	6
		$\{14,2,1\}$	7
		$\{21,1,1\}$	8
SU(9)	$\{1^*,3,4\}$	$\{14,1,1\}$	5
	$\{1,3^*,4\}$	$\{13,2,1\}$	4

Table IV.

Mass W_R	Mass M_C	Unif.Mass	Sin^2 W	
$1.5\text{x}10^{12}$	$6.7\text{x}10^{14}$	10^{15}	0.22	
$1.8\text{x}10^{10}$	$4.9\text{x}10^{14}$	10^{15}	0.23	
$1.8\text{x}10^{10}$	$9.5\text{x}10^{16}$	10^{17}	0.23	
$2.8\text{x}10^{6}$	$5.1\text{x}10^{8}$	10^{10}	0.25	
$2.8\text{x}10^{6}$	$2.6\text{x}10^{14}$	10^{15}	0.25	
$2.8\text{x}10^{6}$	$5.1\text{x}10^{16}$	10^{17}	0.25	$\alpha_S=0.11$
$4.1\text{x}10^{2}$	$5.2\text{x}10^{2}$	10^{5}	0.27	
$4.1\text{x}10^{2}$	$2.7\text{x}10^{8}$	10^{10}	0.27	
$4.1\text{x}10^{2}$	$1.4\text{x}10^{14}$	10^{15}	0.27	
$4.1\text{x}10^{2}$	$2.7\text{x}10^{16}$	10^{17}	0.27	
$6.3\text{x}10^{11}$	$5.6\text{x}10^{14}$	10^{15}	0.22	
$7.7\text{x}10^{9}$	$4.1\text{x}10^{14}$	10^{15}	0.23	
$7.7\text{x}10^{9}$	$7.9\text{x}10^{16}$	10^{17}	0.23	
$1.2\text{x}10^{6}$	$4.2\text{x}10^{8}$	10^{10}	0.25	
$1.2\text{x}10^{6}$	$2.2\text{x}10^{14}$	10^{15}	0.25	
$1.2\text{x}10^{6}$	$4.2\text{x}10^{16}$	10^{17}	0.25	$\alpha_S=0.12$
$1.7\text{x}10^{2}$	$4.3\text{x}10^{2}$	10^{5}	0.27	
$1.7\text{x}10^{2}$	$2.2\text{x}10^{8}$	10^{10}	0.27	
$1.7\text{x}10^{2}$	$1.2\text{x}10^{14}$	10^{15}	0.27	
$1.7\text{x}10^{2}$	$2.2\text{x}10^{16}$	10^{17}	0.27	

Table V.

Mass M_R	Mass M_C	$\sin^2\theta_W$	
1.2×10^{14}	6.3×10^{14}	0.22	
4.0×10^{13}	1.1×10^{15}	0.23	
1.3×10^{13}	1.9×10^{15}	0.24	
4.4×10^{12}	3.3×10^{15}	0.25	$\alpha_S=0.1$
1.5×10^{12}	5.7×10^{15}	0.26	
4.9×10^{11}	9.9×10^{15}	0.27	

<u>Table VI.</u>

Mass W_R	Mass M_C	Unif.Mass	$\sin^2\theta_W$	
4.1×10^{12}	9.6×10^{14}	10^{15}	0.23	
6.6×10^{7}	4.3×10^{9}	10^{10}	0.25	
5.3×10^{8}	7.9×10^{14}	10^{15}	0.25	
1.2×10^{9}	1.0×10^{17}	10^{17}	0.25	
7.0×10^{2}	1.7×10^{3}	10^{4}	0.27	$\alpha_S=0.11$
1.1×10^{3}	1.9×10^{4}	10^{5}	0.27	
8.6×10^{3}	3.5×10^{9}	10^{10}	0.27	
7.0×10^{4}	6.4×10^{14}	10^{15}	0.27	
1.6×10^{5}	8.2×10^{16}	10^{17}	0.27	
1.7×10^{12}	8.1×10^{14}	10^{15}	0.23	
2.7×10^{7}	4.1×10^{9}	10^{10}	0.25	
2.2×10^{8}	7.4×10^{14}	10^{15}	0.25	
5.1×10^{8}	9.4×10^{16}	10^{17}	0.25	
2.9×10^{2}	1.6×10^{3}	10^{4}	0.27	$\alpha_S=0.12$
4.4×10^{2}	1.8×10^{4}	10^{5}	0.27	
3.6×10^{3}	3.3×10^{9}	10^{10}	0.27	
2.9×10^{4}	6.0×10^{14}	10^{15}	0.27	
6.6×10^{4}	7.7×10^{16}	10^{17}	0.27	