Order Number 9405536

Stochastic interest rate and stochastic volatility models of currency futures options

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City University of New York, 1993

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STOCHASTIC INTEREST RATE AND STOCHASTIC VOLATILITY MODELS OF CURRENCY FUTURES OPTIONS

by

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A dissertation submitted to the Graduate Faculty in Business in partial fulfillment of the requirements for the degree of Doctor of Philosophy, The City University of New York.

1993

This manuscript has been read and accepted for the Graduate Faculty in Business in satisfaction of the dissertation requirement for the degree of Doctor of Philosophy.

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Abstract

STOCHASTIC INTEREST RATE AND STOCHASTIC VOLATILITY MODELS OF CURRENCY FUTURES OPTIONS

by

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Currency futures options are used by multinational corporations and investors to hedge the transaction exposure in currency translation. In order to estimate the cost of hedging, investors have to price these futures options; mispricing of the options can lead to substantial losses. In this respect, developing pricing models for these options and empirically testing their pricing accuracy is of prime importance.

While the value of a currency futures option contract depends on the price and volatility of the underlying futures contract, domestic and foreign interest rates, and time to expiration, existing theoretical pricing models are based on the simplifying assumption that the volatility of the underlying asset and the interest rates are non-stochastic. In this study, we derive two different pricing models for options on currency futures which allow domestic and foreign interest rates and the underlying asset variability to be stochastic. Closed form solutions are obtained for European options, and using quadratic approximations American option prices are derived. Pricing efficiency tests of these two models, relative to the constant interest rate and constant underlying asset volatility model, are conducted using transactions data from the International Monetary Market for the following major currencies, i.e. British pound, Deutsche mark, Japanese yen, and Swiss franc. The data contain a total of 6.5 million trades over a period of four years between 1986 and 1989.

Results of the efficiency tests indicate that none of the three models tested is superior to the others in all maturity classes and the at-, in-, and out-of-the money categories. The stochastic interest rate model and stochastic volatility model give better pricing efficiency for some in-the-money options. The constant interest and constant asset volatility model gives better pricing efficiency for at-the-money options. All three models are generally inefficient in pricing out-of-money options.

Pricing efficiency tests are joint tests of model validity and market efficiency. Mispricing by models can be due to one of these two factors, and one needed area of future research will be the use of these models to identify market inefficiency and profit opportunities.

Acknowledgements

I am extremely grateful to Professor Kishore Tandon for his expertise and wisdom in guiding me through this study and its successful completion. I thank him for his valuable and probing critique at the various stages of development of this dissertation. I am thankful to Professors Joel Rentzler, Giora Harpaz and Abraham Simon, who gave me excellent insights during the course of this study.

I have the deepest appreciation for my wife Shyamala, who stood by me during some of the most difficult times in my life, and encouraged me to complete this arduous work.

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CHAPTER I

INTRODUCTION

Currency futures options started trading at the International Monetary Market (IMM) of the Chicago Mercantile Exchange in 1985, and have since attracted considerable investor interest, as evidenced by the volume of the options traded. Multinational corporations and investors, who face exchange risk in terms of both uncertain cash flows and exchange rates, can use the currency futures options to hedge their risk. In order to estimate the cost of hedging, investors have to price these futures options; mispricing of the options can lead to substantial losses. In this respect, developing pricing models for these options and empirically testing their pricing accuracy is of prime importance. This study undertakes both the theoretical model building as well as the testing of pricing efficiency of these models with actual market data.

While the value of a currency futures option contract depends on the price and volatility of the underlying futures contract, domestic and foreign interest rates, and time to expiration, existing theoretical pricing models are based on the simplifying assumption that the volatility of the underlying asset and the interest rates are non-stochastic. Compared to the studies on the pricing of spot currency option contracts, empirical studies on the pricing efficiency of theoretical futures options models are few. The purpose of this study is to derive pricing models for currency futures options which allow interest rates and the underlying asset variability to be stochastic, and to test the efficiency of these models relative to the constant interest rate and constant underlying asset volatility model, using transactions data from the IMM.

One of the major innovations in futures market has been the introduction of financial futures in the 1970s. IMM started trading foreign exchange futures contracts in 1972, and currently trades futures on Australian dollar, British pound, Canadian dollar, Deutsche mark, French franc, Japanese yen and Swiss franc. The introduction of foreign exchange futures coincided with the floating exchange rate system, and the accompanying high volatility in exchange rates, simultaneously contributing to the large increase in trading volume of these contracts.

Later, the Philadelphia Stock Exchange (PHLX) introduced options on spot exchange rate in 1982 and currently trades options on British pound, Canadian dollar, Deutsche mark, Japanese yen and Swiss franc. In 1982 the Commodities Futures Trading Commission (CFTC) authorized the futures exchanges to introduce options on futures contracts on a trial basis. CME initially introduced British pound futures options and later extended the trading to Canadian dollar, Deutsche mark, Japanese yen and Swiss franc.

The existence of a large market for derivative securities itself shows that these securities fulfill certain needs that cannot be met by holding the underlying primary asset alone. The primary reason for the existence and growth of derivative securities is their usefulness in managing the risk associated with holding a position in the underlying security. The existence of derivative securities market depends on the variability of underlying asset prices and their size itself. Large variations in the underlying asset prices make it important to manage the risk of a position on that asset.

In the early stages of its development, futures market primarily functioned as a vehicle for managing price uncertainty. Later developments in this market provided a venue for speculators to speculate on movements in the price of the underlying asset. Futures markets also provide a mechanism for discovering the price of spot asset at a future date. Huang and Litzenberger (1988) provide the theoretical justification for the existence of options markets, in that they allow the completion of the market. A complete market is defined as one in which investors get a positive return in any one state of the world and zero in all other states. Grossman (1988) showed that options lead to informational efficiency of the market by providing information about future price volatility. Options can also play the role of insurance.

For most of the currency futures options contracts traded, there exists a corresponding option on the underlying spot security and hence the question remains for the need of futures options. Some of the reasons attributed for the success of futures options are: i) futures options have lower capital requirements compared to spot options[1]; ii) they do not have liquidity problems, at times associated with spot options; and iii) since futures contracts and the options on them are traded on the same floor, the transaction costs are lower for the traders.

Until the inception of organized options markets in the 1970's, pricing of option contracts attracted relatively limited interest among the finance theorists. Sprenkle (1961), Samuelson (1969), and Thorp and Kassouf (1967), developed theoretical and empirical models to price warrants, but these models were investor preference dependent, and hence had limited theoretical appeal.

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[1]For hedging in currency options market investor has to actually take a position in spot currency, which involves a higher capital outlay, compared to taking a position in futures market, where the margin requirements are much smaller.

In their seminal paper, Black and Scholes (1973), using continuous time mathematics, derived a preference free valuation formula for European type options. The basic assumptions underlying their model were that the markets operate continuously, the underlying asset returns follow a geometric Brownian motion, that interest rates and the volatility of the underlying asset are constant, and the underlying asset pays no dividend. Merton (1973) extended the model to include dividend payments and derived a closed form solution for options with the underlying asset paying a continuous stream of dividends.

Since most of the options traded in the market are American type, allowing the option holder to exercise it prior to maturity, it is necessary to derive valuation models which allow early exercise. Merton (1973) showed that it is not optimal for a call option on a non-dividend paying asset to be exercised prior to maturity, and hence both European and American type options should have identical values. However, for an American call option on a dividend paying asset, Merton showed that it may be rational to exercise the option prior to maturity. American put options on both dividend and non-dividend paying assets can be rationally exercised prior to maturity. Given that it can be optimally exercised prior to maturity, an American option carries a premium over a European one.

Black (1975) suggested that for assets paying deterministic dividends at discrete intervals, the American option value can be found by first estimating a series of European option prices just before each payment of dividend, and the greatest of these option prices will give the value of the American option. Roll (1977), Geske (1979), and

Whaley (1981) developed these models further to derive American option prices for discrete-leakage assets. Finally, Barone-Adesi and Whaley (1987) derived an analytical approximation for American option prices for assets with continuous leakages.

Numerous models have been developed by relaxing the assumptions in the Black-Scholes option pricing model (BS model). Cox, Ross, and Rubinstein (1979) suggest an alternate discrete-time model. In their model, the underlying asset returns follow a multiplicative binomial process. Even though asymptotically this model converges to BS model, the underlying economic principles are easier to understand in this model. Other models with different distribution assumptions are the jump process model by Cox and Ross (1976), which assumes that the asset return takes discrete jumps based on certain informational events. Hull and White (1987), Scott (1987), and Wiggins (1987), relaxing the assumption of constant interest rates, derive option pricing models with stochastic asset volatilities.

For futures option contracts, also known as commodity option contracts, the asset on which the option is written is a futures contract. With the additional assumption that there is no other cost of carry, except the interest rate differential, on the futures contract, Black (1976) derived a valuation model for European futures options. If the option and the underlying futures contract expire on the same date, Black's model for futures option has the same value as the option on the spot asset, on which the futures contract is written. Since futures contracts are similar to assets which pay continuous dividend where the leakage is the interest rate, it may be optimal to exercise American call and put options on futures prior to maturity. Brenner, Courtadon, and

Subrahmanyam (1985) develop a model for American futures options and calculate the early exercise premium for these options. Assuming interest rates are stochastic, Ramaswamy and Sundaresan (1985) derive a numerical solution for the pricing of American futures options. Their simulation studies show that stochastic interest rate option values can differ substantially from corresponding values for the constant interest option model.

Since the publication of the classical work on option pricing by Black and Scholes, a large number of studies have been conducted to test the empirical validity of the model. As Galai (1983) explains, the empirical tests of option pricing model are joint tests of model validity, market efficiency, market synchronization, and data accuracy. In an efficient market it is not possible to have a consistently abnormal, risk-adjusted, after tax profit. In a synchronous market, both option prices and underlying asset prices are available simultaneously. If data is accurate, and markets are efficient and synchronous, then the model accuracy implies: i) that model is able to predict the future prices accurately, and ii) no abnormal profits can be made by using the model for pricing the options.

Black and Scholes (1972) tested their model for over-the-counter stock options for a period between May 1966 to July 1969. Their results indicated that the options on high-volatility stocks were underpriced, while options on low-volatility stocks were overpriced. Part of this volatility bias in option pricing can be eliminated by using implied volatility estimates from the model itself. MacBeth and Merville (1979) used implied volatility in their study and found that the BS model overpriced out-of-the-money call options and underpriced the in-the-money call options. Rubinstein (1985) used transactions data between August 1976 and August 1978, and estimated implied volatility using BS model. His results indicated that implied volatility estimates are not stationary over time.

Some of the above biases displayed by the BS model may be due to the assumption that the underlying asset pays no dividend, and that the options are of "European" type. Whaley (1981) derived an American option pricing model and found that the model priced options better compared to the BS model. Other studies make different assumptions about the stock price process, e.g.the constant elasticity of variance model, but none of these models were able to substantially improve the pricing accuracy compared to the BS model.

Ritchken (1987) summarizes the results from the empirical studies of stock option models as:

i) BS model gives fairly accurate pricing of at-the-money options with durations longer than two months, and assets paying no dividend.

ii) There is a considerable mispricing of deep in-the-money and deep out-of-money options.

iii) Short term options as well as the high and low volatility options are often mispriced.

Currency options are similar to options with underlying asset paying continuous dividend, where the dividend paid is the foreign interest rate. To price European currency options, Beiger and Hull (1983), Garman and Kohlhagen (1983), and Grabbe (1983) developed a modified BS model. For pricing American currency options, the

Barone-Adesi and Whaley (1987) model can be used.

Currency options come in variety of forms, traded in both organized as well as over the counter markets. PHLX currently lists options on Australian dollar, British pound, Canadian dollar, Deutsche mark, French franc, Japanese yen, Swiss franc and European Currency Units (ECU). In 1985, CBOE began selling European currency options, which were taken over by the PHLX in August 1987. Existence of both European and American options on the same currency give the researchers an opportunity to study the early exercise premium.

Over the counter currency options, usually European, are non-standardized, written for large amounts, and are sold on the interbank market in London and New York. American futures style currency options, which require the buyer and writer of the option to post a margin with the Exchange and are marked to the market, are sold on London International Financial Exchange (LIFE). Credit Suisse First Boston (CSFB) trades European currency options in Geneva.

Using a modified BS model, with stochastic interest rates or stochastic volatility, researchers have examined the efficiency of the currency spot option markets, and their pricing accuracy, and measures of early exercise premium on American currency options. Shastri and Tandon (1986) using a modified BS model, find evidence of *ex post* inefficiency in the currency option market, which disappear when *ex ante* hedging strategies are employed. Studies by Tucker (1986) and Bodurtha and Courtadon (1986) also indicate that markets are efficient, after controlling for transaction costs and execution lags.

Bodurtha and Courtadon (1988) use Barone-Adesi and Whaley (1987) approximation of American option pricing and test the efficiency of the model. Results from their study indicate that the model prices at the money options fairly accurately, but overprices in-the-money options and underprices out-of-the-money options.

With the availability of simultaneous prices of American and European currency options since 1986, Jorion and Stoughton (1989) calculate early exercise premium and find that it is high for options on currencies which have a higher interest rate than the domestic currency (i.e. the U.S. dollar), and low for options on currencies with a lower interest rate than the domestic one. Comparison of actual early exercise premiums with estimated premiums using the Barone-Adesi and Whaley model, show that the latter model predicted the premiums fairly accurately.

Foreign exchange rates are highly sensitive to interest rate changes. Hilliard, Madura, and Tucker (1991) use a stochastic interest rate model to price currency options and find that their model prices currency spot options more accurately. On the other hand, Melino and Turnbull (1990) and Chesney and Scott (1989) use stochastic volatility models and find that such models price currency options more accurately than the constant volatility model.

The purpose of this study is to use these stochastic models to test the pricing of futures options. A pricing model that allows for the domestic and foreign interest rates to be stochastic, is derived in Chapter 2. This model assumes that the interest rates follow an arithmetic Brownian motion and at first derives a closed form solution for a European currency futures option. Later, using a modified Barone-Adesi and Whaley model, American call and put option models are derived. In Chapter 3, we derive a second pricing model, which allows for the underlying currency exchange rate volatility to be stochastic and derive a semi-closed form European option pricing model. To estimate the early exercise premium, an approximation, based on Barone-Adesi and Whaley model, is derived.

We then test the pricing efficiency of the two models in Chapter 4, using transactions data for futures options on four major currencies traded on the IMM over a four year period between 1986 and 1989. We compare the pricing efficiency of these stochastic interest rate model and the stochastic volatility model to the constant interest and constant volatility model of currency futures options. Chapter 5 concludes the findings of this study.

CHAPTER 2

CURRENCY FUTURES OPTION MODEL WITH STOCHASTIC INTEREST RATES

In this Chapter we first develop a European currency futures option model with stochastic interest rates and constant volatility, with the underlying spot currency as the state variable. This model is a little similar to the European option pricing model for currency spot options developed by Hilliard, Madura, and Tucker (1991), the main difference between the two being the specification of the interest rate process and the subsequent bond valuation model. Since the currency futures options traded are American, our valuation model has to be modified to incorporate the early exercise premium associated with the American options. Next, using the quadratic approximation method used by Barone-Adesi and Whaley (1987), we derive an American options model for futures options.

2.1. Notation

$S(t)$ = spot exchange rate at time t (dollars per unit of foreign currency)

$F(t,T)$ = futures contract on the foreign currency at time t with expiration at time T.

$B_1(t,T)$ = domestic currency pure discount bond maturing at time T.

$B_2(t,T)$ = foreign currency pure discount bond maturing at time T.

$c(t,T)$ = price of an European call option on $F(t,T)$ with exercise price K.

$C(t,T)$ = price of an American call option on $F(t,T)$ with exercise price K.

r(t) = instantaneous domestic short term interest rate at time t.

f(t) = instantaneous foreign short term interest rate at time t.

2.2 Assumptions

A1. Investors are risk neutral and there are no taxes or transaction costs.

Assumption of perfect markets is fairly standard in option pricing literature, while the risk neutrality assumption needs some explanation. In the BS model, an option can be instantaneously replicated by a riskless hedge, and the option price depends on the underlying asset price, strike price, time to expiration, volatility of the underlying stock, and the riskless interest rate, but not on expected return from the stock. Since the instantaneous hedge is risk free, Cox and Ross (1976) used the risk neutral valuation to derive the option price. When interest rates are stochastic, it is not possible to form an instantaneous riskless hedge. If the forward rates are unbiased predictors of the future spot rates, then the expected return on the forward contract is zero. Empirical studies, however, generally reject the unbiasedness of the forward rates[1]. Cumby (1988), and Mark (1988) attempt to explain the forward rate forecast error in terms of time varying risk premia. With the risk neutrality assumption, we avoid the risk premia on forward contracts. Cox, Ingersoll and Ross (1981) show that in the presence of stochastic interest rates, futures prices differ from the forward prices. But empirical investigations by Cornell and Reinganum (1981), and Chang and Chang (1990), show that the difference

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[1] See Hansen and Hodrick (1980), Cumby and Obstfeld (1981), and Hodrick and Srivastava (1984).

between forward and futures prices is statistically insignificant. With the risk neutrality assumption, we allow for the risk premium on futures contract to be zero.

A2. Spot exchange rate changes are normally distributed:

$$\frac{dS}{S}=\mu_s dt+\sigma_s dZ_s$$

where μ_s and σ_s are constant.

Empirical investigations by Westerfield (1977), Rogalski and Vinso (1978), Calderon-Rossell and Ben-Horim (1982), Tucker and Scott (1987), and Boothe and Glassman (1987) indicate that daily and weekly exchange rate changes exhibit kurtosis in excess of a normal distribution. Alternate distributions for exchange rate changes suggested are the stable Paretian, Student t, and a mixture of two normals. The study by Boothe and Glassman indicates that the mixture of two normals gives a more accurate representation of high frequency exchange rate changes, whereas the quarterly exchange rate changes are better represented by the normal distribution.

A3. Short term interest rates are described by the following processes[2]:

[2]Hsieh (1988) used a similar interest rate model to derive a European currency option pricing model.

$$dr=\mu_r dt+\sigma_r dZ_r$$

$$df=\mu_f dt+\sigma_f dZ_f$$

where μ_r, μ_f, σ_r and σ_f are functions that depend solely on the current level of the domestic interest rates (r) and foreign rates (f), respectively, and are defined such that the rate cannot be negative. The assumption that interest rates follow an arithmetic Brownian motion has a major drawback, since it allows for a negative interest rate with a positive probability. Rabinovitch (1989) and Hilliard, Madura, and Tucker (1991) use the Ornstein-Uhlenbeck process to describe the interest rates process. Even though this process is theoretically appealing in modelling interest rates, empirical estimation of the parameters of the process is difficult. Theoretically, the most appealing description is the locally lognormal diffusion process described by Cox, Ingersoll, and Ross (1985). Locally lognormal means that small changes in the interest rates are proportional to the current interest rate itself, which precludes the possibility of negative interest rates. But the use of this interest rate model does not result in a closed form option pricing model, and hence is precluded here.

A4. Bond returns are normally distributed as follows:

$$\frac{dB_1}{B_1}=\mu_1 dt+\sigma_1(t,T)\,dZ_1$$

$$\frac{dB_2}{B_2}=\mu_2 dt+\sigma_2(t,T)\,dZ_2$$

The above description of the bond prices is a single factor model in which bond prices are determined by one state variable, namely, the short term interest rate. As Merton (1973) explains, the only requirement of this model is that unanticipated returns on the bond are not serially correlated. The means and variances of these two processes are described in greater detail in section 2.4.

dZ_S, dZ_1, dZ_2, dZ_r and dZ_f are standard Wiener processes.

2.3 Derivation of Partial Differential Equation

In the absence of carrying cost other than the interest rate differential between the two currencies, the foreign currency futures price is:

$$F(t,T)=\frac{S(t)\,B_2(t,T)}{B_1(t,T)} \tag{1}$$

From eqn. (1), it is possible to replicate the futures contract by borrowing in the domestic currency by issuing domestic risk free bonds, converting the proceeds at current exchange rate into the foreign currency and investing the proceeds in the foreign risk free bond. Consider a portfolio comprising of call option on futures contract and a combination of domestic and foreign bonds. Let the value of this portfolio be

$$P=C+\alpha G+\beta B_1 \tag{2}$$

where $G = S(t)B_2$, and $dG = (.)dt + G(\sigma_S dZ_S + \sigma_2 dZ_2)$[3]. Call option is a function of G,B_1, and time to expiration of the option contract and can be written as $c = c(G,B_1,\tau)$, where $\tau=(T-t)$. Merton (1973) has shown that call prices are homogeneous in G and B_1[4]. Using Euler's theorem, it is possible to show that

$$c=c_G+c_{B_1}B_1 \tag{3}$$

Substituting for c in eqn. (2) and setting $\alpha = -c_G$ and $\beta = -c_{B1}$, it can be shown that the value of this portfolio $P = 0$. Applying Ito's lemma to eqn. (2), it can be shown that

$$dP=c_G dG+c_{B_1}d_{B_1}-c_\tau dt+\frac{1}{2}c_{B_1B_1}B_1^2\sigma_{B_1}^2+\frac{1}{2}c_{GG}G^2\sigma_G^2+2c_{B_1G}Cov(dG,dB_1)$$

Since $dP=0$, and setting $\alpha = -c_G$, and $\beta = -c_{B1}$,

$$\frac{1}{2}c_{B_1B_1}B_1^2\sigma_{B_1}^2+\frac{1}{2}c_{GG}G^2\sigma_G^2+2c_{B_1G}Cov(dG,dB_1)=0 \tag{4}$$

The above equation is non-stochastic and using risk-neutral valuation of Cox and Ross (1976), the value of the European call option can be found, subject to the following boundary condition

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[3] $G = G(S,B_2,t)$ and $G_S = B_2$ and $G_{B2} = S$

$$dG=G_S ds+G_{B_2}dB_2+G_t dt+\frac{1}{2}[G_{SS}\sigma_S^2+G_{B_2B_2}\sigma_{B_2}^2+2G_{SB_2}\rho_{SB_2}]dt$$

Substituting for G_S and G_{B2} and simplifying

$$dG=(.)dt+G(\sigma_S dZ_S+\sigma_2 dZ_2)$$

[4] See Feiger and Jacquillat (1979), Magrabe (1978), and Hilliard, Madura, and Tucker (1991) for similar derivations.

$$c(F,T,t)=B_1(t,T)\hat{E}[\max(0,F_T-K)] \qquad (5)$$

where $\hat{E}$ is expectation operator.

2.4. Estimation of Instantaneous Variance of the Bonds

The bond pricing model for domestic currency is derived as follows:

$B_1(t,T)$ is the value of a bond at time t which matures at time T. Using Ito's lemma, we have

$$dB_1=B_r dr+B_t dt+\frac{1}{2}\sigma_1 B_{rr}dt$$

$$=B_{1r}[\mu_r dt+\sigma_r dZ_r]+B_{1t}dt+\frac{1}{2}\sigma_r^2 B_{1rr}dt$$

$$=[B_{1t}+B_{1r}\mu_r+\frac{1}{2}\sigma_r^2 B_{1rr}]dt+[\sigma_r B_{1r}]dZ_r$$

Hence,

$$\frac{dB_1}{B_1}=\mu_{B_1}dt+\sigma_1 dZ_r$$

where

$$\mu_{B_1}=\frac{[B_{1t}+B_{1r}\mu_r+\frac{1}{2}\sigma_r^2 B_{1rr}]}{B_1}$$

and

$$\sigma_1 = \frac{\sigma_r B_{1r}}{B_1}$$

Since $B_1(t,T)$ is a pure discount bond,

$$B_1(t,T) = e^{-r(T-t)}$$

The partial derivative and standard deviation are

$$B_{1r} = -(T-t)B_1 \quad and \quad \sigma_1 = -(T-t)\sigma_r \tag{6}$$

Similarly for the foreign bond, the standard deviation is

$$\sigma_2 = -(T-t)\sigma_f \tag{7}$$

2.5. European Currency Futures Call Option Model

To derive the closed form solution for the call option price it is necessary to first derive the risk-neutral distribution of the currency futures returns. From eqn. (1) , the futures price can be expressed as $F(t,T) = F(B_1,B_2,S)$. Applying Ito's lemma

$$dF = F_{B_1}dB_1 + dF_{B_2}dB_2 + dF_S dS + dF_t dt + \frac{1}{2}[F_{B_1B_1}dB_1^2 + F_{B_2B_2}dB_2^2 + F_{SS}dS^2$$

$$+2F_{B_1B_2}dB_1dB_2 + 2F_{B_1S}dB_1dS + 2F_{B_2S}dB_2dS]$$

But,

$$dB_1 dB_2=\rho_{12}dt,\quad dB_1 dS_1=\rho_{1S},\quad dB_2 dS=\rho_{2S},\quad (dB_1)^2=B_1^2\sigma_1^2 dt,$$

$$(dB_2)^2=B_2^2\sigma_2^2 dt \text{ and } (dS)^2=S^2\sigma_S^2 dt$$

and

$$dF=F_{B_1B_1}dB_1+F_{B_2B_2}dB_2+F_{SS}dS+\frac{1}{2}[F_{B_1B_1}B_1^2\sigma_{B_1}^2+F_{B_2B_2}B_2^2\sigma_{B_2}^2+F_{SS}S^2\sigma_{SS}^2+2F_{B_1B_2}\rho_{B_1B_2}$$

$$+F_{B_1S}\rho_{B_1S}+F_{B_2S}\rho_{B_2S}]\,dt$$

Partial derivatives of F from eqn. (1) are

$$F_S=B_2B_1^{-1},\quad F_{B_2}=SB_1^{-1},\quad \text{and } F_{B_1}=-SB_2B_1^{-2}$$

Substituting these values,

$$dF=-\frac{SB_2}{B_1^2}dB_1+\frac{S}{B_1}+\frac{B_2}{B_1}+(\,.\,)\,dt$$

The futures return process is

$$\frac{dF}{F}=(\,.\,)\,dt+\frac{dS}{S}-\frac{dB_1}{B_1}+\frac{dB_2}{B_2}$$

$$=(\,.\,)\,dt+\sigma_S dZ_S-\sigma_1 dZ_1+\sigma_2 dZ_2$$

In a risk neutral world with no transactions cost, the instantaneous rate of return of the futures contract is zero. Hence the risk adjusted rate of return of the futures contract can be written as

$$\frac{d\hat{F}}{\hat{F}}=\sigma_S dZ_S-\sigma_1 dZ_1+\sigma_2 dZ_2 \qquad (8)$$

The conditional variance of $\log(F_T/F_t)$ is

$$v^2 = Var(\log(\frac{F_T}{F_t})\,|F_t) = \int_t^T (\sigma_s\ -\sigma_1\ \sigma_2)'\ cov(\underline{dZ}\ \underline{dZ}')\ \ (\sigma_s\ \sigma_1\ \sigma_2)$$

But

$$Cov(\underline{dZ}\ \underline{dZ}') = \begin{bmatrix} 1 & \rho_{S1} & \rho_{S2} \\ \rho_{S1} & 1 & \rho_{12} \\ \rho_{S2} & \rho_{12} & 1 \end{bmatrix}$$

Simplifying,

$$v^2 = \int_t^T (\sigma_S^2 + \sigma_1^2 + \sigma_2^2 - 2\rho_{S1}\sigma_S\sigma_1 + 2\rho_{S2}\sigma_S\sigma_2 - 2\rho_{12}\sigma_1\sigma_2)\,dt$$

Substituting for the values of σ_1 and σ_2 from equations (6) and (7) respectively,

$$v^2 = \int_t^T \sigma_S^2 dt + \int_t^T \tau^2 (\sigma_r^2 + \sigma_f^2 - 2\rho_{12}\sigma_r\sigma_f)\,dt + 2\int_t^T \tau(\rho_{S1}\sigma_S\sigma_1 - \rho_{S2}\sigma_S\sigma_2)\,dt$$

$$v^2 = \sigma_S^2\tau + \frac{\tau^3}{3}(\sigma_r^2 + \sigma_f^2 - 2\sigma_{rf}) + \tau^2(\sigma_{Sr} - \sigma_{Sf}) \tag{9}$$

Under the assumption of risk neutrality,

$$F(T) = F(t,T)\,e^{-\frac{1}{2}v^2 + vZ}$$

$$\hat{E}[F(T_1,T_2)] = \int_{-\infty}^{\infty} (F(t,T_2)\,e^{-\frac{1}{2}v^2+vZ} - K)\,\frac{1}{\sqrt{2}\pi}\,e^{-\frac{1}{2}Z^2}dZ$$

$$F(T_1,T_2) \geq K \Rightarrow F(t,T_2) \geq K$$

Taking logs on both sides of the above equation

$$-\frac{1}{2}v^2 + v\tilde{Z} \geq \ln\frac{K}{F(t,T)}$$

$$\tilde{Z} \geq \frac{\ln\frac{K}{F(t,T)} + \frac{1}{2}v^2}{v}$$

$$Let \ \frac{\ln\frac{F(t,T)}{K} - \frac{1}{2}v^2}{v} = d_2$$

The call option can be written as

$$c = B(t,T)\int_{Z=-d_2}^{\infty}(F(t,T)\,e{-}\frac{1}{2}v^2+vZ-K)\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}Z^2}dZ$$

$$= B(t,T)\int_{Z=-d_2}^{\infty}F(t,T)\,e^{-\frac{1}{2}v^2+vZ}\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}Z^2}dZ - B(t,T)\int_{Z=-d_2}^{\infty}K\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}Z^2}dZ$$

First term can be written as

$$B_1(t,T)\,F(t,T)\frac{1}{\sqrt{2\pi}}\int_{Z=-d_2}^{\infty}e^{-\frac{1}{2}v^2+vZ-\frac{1}{2}Z^2}dZ$$

Let

$$y = Z - v; \ \ and \ \ dy = dZ$$

Again the first term can be written as

$$B_1(t,T)\,F(t,T)\int_{Z=-d_2}^{\infty}e^{-\frac{1}{2}(v^2-2vZ-\frac{1}{2}Z^2)}dZ$$

$$=B_1(t,T)F(t,T)\int_{Z=-d_2}^{\infty} e^{-\frac{1}{2}(Z-v)^2}dZ$$

$$=B_1(t,T)F(t,T)\int_{y=-d_2-v}^{\infty} \frac{1}{2}e^{-\frac{1}{2}y^2}dZ$$

Let

$$d_1=\frac{\ln\frac{F(t,T)}{K}+\frac{1}{2}v^2}{v}$$

From this the call option price can be written as

$$c=B_1(t,T)\left[F(t,T)N(d_1)-K\ N(d_2)\right] \tag{10}$$

When interest rates are non stochastic, the variance of the futures return will become

$$v^2=\sigma_S^2\tau$$

and this model will be the same as the modified Black's model.

Using put-call parity, the European put option value can be written as,

$$p=B_1(t,T)\{K[1-N(d_2)]-F(t,T)[1-N(d_1)]\} \tag{11}$$

2.6. American Currency Futures Option Model

An American option can be exercised any time up to the maturity of the option. For an American call option with continuous leakages, like the currency futures option, it may be rational to exercise the option prior to maturity. This early exercise option has a value and this can be written as:

$$\varepsilon_c(F,\tau) = C(F,\tau) - c(F,\tau)$$

where $C(F,\tau)$ is the American call option with time to maturity τ. Barone-Adesi and Whaley (1986) (B-A W model) use quadratic approximation to estimate the early exercise premium for option contracts on underlying assets with continuous leakages. The key insight of B-A W model is that the early exercise premium has the same stochastic characteristics of the underlying assets and a partial differential equation similar to the option itself. Brenner, Courtadon, and Subrahmanyam (1985) showed that for a call option on a futures contract the partial differential equation is

$$\frac{1}{2} c_{FF} F^2 \sigma_{FF} - rc - c_\tau = 0 \qquad (12)$$

In a world with constant interest rates, the volatility of the futures contract is the same as the underlying asset on which the futures contract is written. But, as shown in section 2.5, the volatility of a futures contract will be different from that of the underlying asset, when interest rates are stochastic. With the additional risk neutrality assumption, it was shown in the same section that the only difference between the foreign currency futures contract and the underlying asset, in a world with stochastic interest rates, is its volatility estimate. Hence the B-A W model can be adapted to estimate the early exercise premium in a world where the interest rates are stochastic. The partial differential equation for the early exercise premium then is:

$$\frac{1}{2} v^2 F^2 \varepsilon_{FF} - r\varepsilon - \varepsilon_\tau = 0 \qquad (13)$$

where ϵ_τ is the early exercise premium.

Using the quadratic approximation, the following American call option model is derived. (For detailed derivations, see Appendix 2A).

$$C(F,\tau)=c(F^*,\tau)+A_2\left(\frac{F}{F^*}\right)^{q_2} \qquad if\ F<F^*$$

$$or, \quad C(F,\tau)=F-K \qquad if\ F\geq F^* \qquad (14)$$

where,

F^* is the critical futures price below which there will be no early exercise of the call option,

$$A_2=\frac{F^*}{q_2}\{1-B_1(t,T)n[d_1(F^*)]\}$$

$$q_2=\frac{-(N-1)+\sqrt{(N-1)^2+4\frac{M}{K}}}{2}, \quad N=\frac{2}{v^2} \quad and \quad M=\frac{2r(t)}{v^2}$$

The American put option is

$$P(F,\tau)=p(F,\tau)+A_1\left(\frac{F}{F^{**}}\right)^{q_1} \qquad if\ F>F^{**}$$

$$or, \quad P(F,\tau)=K-F \qquad if\ F\geq F^{**} \qquad (15)$$

where,

F^{**} is the critical futures price below which there will be no early exercise of the put option,

$$A_1 = \frac{F^{**}}{q_1}\{1 - B_1(t,T)N[-d1(F^{**})]\}$$

$$q_1 = \frac{-(N-1) - \sqrt{(N-1)^2 + 4\frac{M}{K}}}{2}$$

In this chapter, we develop a model which assumes that the dollar as well as foreign interest rates are stochastic. For an interest sensitive instrument like foreign exchange rates, the random shocks in interest rates can result in level shifts in exchange rates. The difference between this model of stochastic interest rates and the constant interest rate model is the volatility parameter, which in this model includes variances and covariances of dollar and foreign interest rates.

Appendix 2A

Early exercise premium for an American call option can be written as

$$\epsilon_c(F,\tau)=C(F,\tau)-c(F,\tau)$$

where τ = T-t, the time left for expiration of the option, and C(F,τ) and c(F,τ) are the American and European call options respectively. Using Ito's lemma the partial differential equation for the early exercise premium can be written as

$$\frac{1}{2}v^2F^2\epsilon_{FF}-r\epsilon-bF\epsilon_F-\epsilon_\tau=0 \qquad \text{(A1)}$$

where b is the cost of carry of the futures contract.
Multiplying equation (A1) with $2/v^2$ and then substituting M = $2r/v^2$ and N = $2b/v^2$, it can then be written as

$$F^2\epsilon_{FF}-M\epsilon+NF\epsilon_F-\left(\frac{M}{r}\right)\epsilon_\tau=0 \qquad \text{(A2)}$$

Let

$$\epsilon(F,K)=X(\tau)f(F,X) \qquad \text{(A3)}$$

$$where\ X(\tau)=1-e^{-r\tau}$$

The partial derivatives are,

$$\epsilon_{FF}=Xf_{FF}\ and\ \epsilon_\tau=X_\tau f(F,X)+X(\tau)X_\tau f_X$$

Substituting these values in equation (A2) and simplifying,

$$F^2f_{FF}+NFf_F-\left(\frac{M}{X}\right)f-(1-X)Mf_X=0 \qquad \text{(A4)}$$

As an approximation of this, the last term is dropped and the equation is rewritten as

$$F^2 f_{FF}+NFf_F-(\frac{M}{X})f=0 \qquad \text{(A5)}$$

This a second degree ordinary differential equation. Let $f=aF^q$,

$$aF^q[q^2+(N-1)q-\frac{M}{X}]=0 \qquad \text{(A6)}$$

The roots for the above equation are,

$$q_1=\frac{-(N-1)-\sqrt{(N-1)^2+4\frac{M}{X}}}{2} \quad and \quad q_2=\frac{-(N-1)+\sqrt{(N-1)^2+4\frac{M}{X}}}{2}$$

The general solution can be written as

$$f(F)=a_1F^{q_1}+a_2F^{q_2} \qquad \text{(A7)}$$

Since $q_1<0$, and if a_1 not equal to 0, the function f(F,X) approaches ∞ as the futures price F approaches 0. This is unacceptable and then a_1 has to be equal to zero. The American call option is then

$$C(F,\tau)=c(F,\tau)+Xa_2F^{q_2} \qquad \text{(A8)}$$

where F=0, $C(F,\tau) = c(F,\tau) = 0$. The boundary condition imposed for early exercise of an American option is $C(F,\tau) <$ F-K. Let the critical futures price below which the American call will not be exercised be F^*. Let

$$F^*-K=c(F^*,\tau)+Xa_2F^{q_2} \qquad \text{(A9)}$$

Let the slope of the exercisable call $C(F^*,\tau)$ be equal to one. Then

$$1=B_1(t,T)N[d_1(F^*)]+Xq_2a_2F^{*q_2-1} \qquad \text{(A10)}$$

where $B_1(t,T)N[d_1(F^*)]$ is the partial derivative of $c(F^*,\tau)$ with respect to F^*. Solving for

a_2 from equations (A9) and (A10)

$$a_2=\frac{\{1-B_1(t,T)N[d_1(F^*)]\}}{Xq_2'F^{*q_2-1}} \tag{A11}$$

Substituting equation (A11) into (A9)

$$F^*-K=c(F^*,\tau)+\frac{\{1-B_1(t,T)N[d_1(F^*)]\}F^*}{q_2} \tag{A12}$$

Since F^* is the only unknown in the above equation, the value of F^* can be obtained using the algorithm given by Barone-Adesi and Whaley. Substituting equation (A12) into (A8) and simplifying, the solution can be obtained as

$$C(F,\tau)=c(F,\tau)+A_2\left(\frac{F}{F^*}\right)^{q_2} \quad if \ F<F^*$$

$$C(F,\tau)=F-K \qquad\qquad if \ F\geq F^* \tag{A13}$$

where

$$A_2=\frac{F^*}{q_2}\{1-B_1(t,T)N[d_1(F^*)]\}$$

Similarly for the value of the American put option, the critical futures price F^{**} can be shown to be

$$K-F^*=p(F^{**},\tau)-\frac{\{1-B_1(t,T)N[-d_1(F^{**})]\}F^{**}}{q_1} \tag{A14}$$

By solving for F^{**} using the algorithm mentioned above, the American put option price can be written as

$$P(F,\tau)=p(F,\tau)+A_1\left(\frac{F}{F^{**}}\right)^{q_1} \quad if \ F>F^{**}$$

$$P(F,\tau)=K-F \qquad\qquad if \ F\leq F^{**} \qquad\qquad (A15)$$

where,

$$A_1=-\left(\frac{F^{**}}{q_1}\right)\{1-B_1(t,T)N[-d_1(F^{**})]\}$$

CHAPTER 3

Currency Futures Option Model with Stochastic Volatility

Another key assumption in the Black-Scholes model is that the variance of the underlying asset is constant. With the additional assumption that interest rates are constant, it can be shown that, for an option contract written on a currency futures contract, the underlying asset is the currency exchange rate itself. Studies by Melino and Turnbull (1987), and Boothe and Glassman (1987) found that volatility parameter in exchange rates is highly unstable over time. Hull and White (1987), Scott (1987) and Wiggins (1987) studied the effect of stochastic volatility on equity option prices and found that at the money European options with stochastic volatility are less than Black-Scholes model prices. In this chapter a stochastic volatility model for currency futures is developed.

3.1. Notation

This section uses the same notations as in 2.1.

3.2. Assumptions

A5. The dynamics of the spot price is given by the following stochastic differential equation

$$dS=(r_d-r_f)\,Sdt+\sigma_S SdZ_S \qquad (1)$$

In a risk neutral world the expected return from holding foreign exchange will be the difference between the interest rates in foreign and domestic economies.

A6. The variance of the spot exchange return follows a diffusion process

$$d\sigma_S=\mu(\sigma_S)\,dt+v(\sigma_S)\,dZ_\sigma \tag{2}$$

where dZ_S and dZ_σ are standard Wiener process with $E(dZ_S dZ_\sigma) = \delta dt$. $\mu(\sigma_S)$ is the instantaneous mean and $v^2(\sigma_S)$ is the instantaneous variance of the volatility process. Though the above specification of the volatility process is general, for empirical testing purposes it is necessary to have a more tractable specification. In this paper, we consider the following ad hoc specification, which is same as the one described by Chesney and Scott (1989), to represent the volatility process:

$$d\ln\sigma_S=\beta(\alpha-\ln\sigma_S)\,dt+\gamma\,dZ_\sigma \tag{3}$$

The above specification implies that log of the variance follows a mean reverting process following dynamics: (See Appendix 3A for detailed derivation)

$$\frac{d\sigma_S}{\sigma_S}=[\frac{1}{2}\gamma^2+\beta(\alpha-\ln\sigma_S)]\,dt+\gamma\,dZ_\sigma \tag{4}$$

A7. Both domestic and foreign interest rates are non-stochastic[1].

3.3. Derivation of Partial Differential Equation

Considering that interest rates are non-stochastic and that there are no other transactions and carrying costs, except the interest rate differential, the futures price can be expressed as

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[1]By allowing for both stochastic interest rate and stochastic volatility, a theoretically a more appealing model can be created. But solving the model form a closed for solution is extremely difficult.

$$F(t,T)=S(t)\,e^{(r_d-r_f)\tau} \tag{5}$$

Applying Ito's lemma to the above equation,

$$dF=F_S dS+F_t dt+\frac{1}{2}F_{SS}(dS)^2$$

Simplifying the above equation by substituting the values for the partials, we get

$$dF=\sigma_S F dZ_S \tag{6}$$

Cox, Ingersoll, and Ross (1985) have shown that in a continuous-time, continuous-state economy, the value of any contingent claim C($\underline{X}$,t) must satisfy the following fundamental differential equation

$$\frac{1}{2}\sum_i\sum_j (CovX_i,X_j)\,C_{X_iX_j}+\sum_i(\mu_i-\phi_i)\,C_{X_i}+C_t-r(X,t)\,C+d(X,t)=0 \tag{7}$$

where ϕ_i is the factor risk premium associated with X_i, and d(X,t) is the continuous payment flow received by the claim. Since there are no dividend payments for a call option, d(X,t) = 0.

The call option on the futures contract depends on two state variables, viz., futures contract and the standard deviation of the spot exchange rate. In order to form a riskless hedge one must use these two securities; but, of these two, only futures contract is traded in the market place. Thus there is a possible risk premium associated with the standard deviation of spot exchange rate and this is denoted by ϕ_S. The above general differential equation (7) can then be rewritten for the call as follows:

$$\frac{1}{2}C_{FF}\sigma_S^2F^2+C_{F\sigma}\delta\gamma\sigma_S^2F+\frac{1}{2}C_{\sigma\sigma}\gamma^2\sigma_s^2+C_\sigma[\sigma_S(\frac{1}{2}\gamma^2+\beta(\alpha-\ln\sigma_S)-\phi_\sigma]+C_t-r_dC=0 \quad (8)$$

3.4. European Currency Futures Option Model

The above partial differential equation can be solved using the method adopted by Cox, Ingersoll, and Ross (1985). The call option price can be written as

$$c(F,v,t)=\hat{E}_t\{e^{-r(T-t)}\max[0,F_T-K]\} \quad (9)$$

where $\hat{E}$ is the risk-adjusted expectations operator. Assuming for the time being that risk premiums $\phi_\sigma = 0$ and $\delta = 0$, using the results in Karlin and Taylor (1981, p.374-375), the futures price can be written as:

$$F_T=F_te^{\{\int_t^T-\frac{1}{2}\sigma_S^2ds+\int_t^T\sigma_SdZ_S\}}$$

Let

$$V=\int_t^T\sigma_S^2ds$$

Then the value of the European call option can be shown to be

$$c(F,\sigma_S,\tau)=B_1(t,T)\,[F(t,T)N(d_3)-KN(d_4)] \quad (10)$$

where,

$$d_3=\frac{\ln\frac{F(t,T)}{K}+\frac{1}{2}V^2}{V} \quad and \quad d_4=\frac{\ln\frac{F(t,T)}{K}-\frac{1}{2}V^2}{V}$$

This is similar to Black's model with V replacing σ^2_t. Scott (1987) has shown that the value of the European call option can be derived from the above by integrating this equation for all possible values of V as follows,

$$c(F,\sigma_S,t)=\int_0^\infty [B_1(t,T)F(t,T)N(d_3)-KN(d_4)]\,d\hat{F}(V) \qquad (11)$$

where F is the distribution function of V, using risk adjusted $d\sigma_S$ process from equation (2). The solution to the above integral can be obtained by the Monte Carlo simulation described in section 3.7. The put option price is

$$p(F,\sigma_S,t)=\int_0^\infty B_1(t,T)\{K[(1-N(d_3)]-F(t,T)[1-N(d_4)]\}\,d\hat{F}(V) \quad (12)$$

3.5. American Currency Futures Model

The American Currency Futures Option model can be derived using the Barone-Adesi and Whaley (1987) method of quadratic approximation. Since the European option itself does not have a closed form solution, the American option is derived using the following two step procedure. First, the European option price is derived using the above model, and then using the option price and equation (3.1), the implied volatility $\underline{V}$ of the option is estimated. This implied volatility $\underline{V}$ is the average value of the volatility of the option, and this average value is then used in the Barone-Adesi and Whaley model to derive the American option prices. (See Appendix 3B for detailed derivation).

$$C'(F,\tau)=c'(F,\tau)+A_4(F/F'^{*})^{q_4} \quad if \ F<F'^{*}$$

$$C'(F,\tau)=F-K \qquad\qquad if \ F\geq F'^{*} \tag{13}$$

where

$$A_4=\frac{F'^{*}}{q_4}\{1-B_1(t,T)N[d_3(F'^{*}]\}$$

$$d_3(F'^{*})=\frac{\ln(F'^{*}/K)+\frac{1}{2}V^2}{V}$$

$$q_4=[-(N'-1)+\sqrt{(N'-1)^2+4M'/X]}/2$$

$$M'=\frac{2r_d}{V^2} \ and \ N'=\frac{2b}{V^2}$$

and the American put option is

$$P'(F,\tau)=p'(F,\tau)+A_3(F/F'^{**})^{q_3} \quad if\ F>F'^{**}$$

$$P'(F,\tau)=K-F \quad if\ F\leq F'^{**} \tag{14}$$

where

$$A_3=-(F'^{**}/q_3)\{1-B_1(t,T)N[d_3(F'^{**}]\}$$

and

$$q_3=[-(N'-1)-\sqrt{(N'-1)^2+4M'/X}]/2$$

3.6. Estimating the Parameters of the Volatility Process

For empirical estimation the parameters for the volatility process are calculated using a discrete approximation of the stochastic process. Cox and Miller (1965) has shown that a discrete-time model for a variable x which has in each period an expected change μ and a variance ϕ^2 can be written as:

$$\Delta_{X_t}=X_{(t+1)}-X_t=\mu+\phi\xi_t$$

where,

$$E[\xi]=0 \ and \ Var[\xi]=1$$

Since $\ln\sigma_S$ follows a Ornstein - Uhlenbeck process, the instantaneous mean and variance can be written as, [Cox and Miller (1965)]

$$\mu=\alpha+(\ln\sigma_{st})-\alpha)e^{-\beta} \quad and \quad \phi=\frac{\gamma^2}{2\beta}(1-e^{-2\beta})$$

Substituting these values in the discrete approximation,

$$\ln\sigma_{s(t+1)}-\ln\sigma_{st}=\alpha+(\ln\sigma_{st}-\alpha)e^{-\beta}+\frac{\gamma^2}{2\beta}(1-e^{-2\beta})\xi_t$$

$$i.e., \quad \ln\sigma_{s(t+1)}=\alpha(1-e^{-\beta})+e^{-\beta}\ln\sigma_{st}+e_t \tag{15}$$

where,

$$e\sim N[0,\frac{\gamma^2}{2\beta}(1-e^{-2\beta})] \quad ; \quad \sigma_e^2 = \frac{\gamma^2}{2\beta}(1-e^{-2\beta})$$

The above equation can be rewritten as a linear regression equation as follows,

$$\ln\sigma_{s(t+1)}=a+b\ \ln\sigma_{st}+e_t \tag{16}$$

From the regression estimates the values of the parameters are estimated.

3.7 Numerical Estimation of the Option Price

The option prices are estimated from equations (11) and (12) using Monte Carlo simulation. Following Hull and White (1987), the time interval T-t for the option to expire is divided into n equal sub-intervals. Variance V_i at time t+i(T-t)/n is generated using the following formula:

$$V_i \;=\; =V_{i-1}e^{[(\mu-\xi^2/2)\Delta t+v_i\xi\sqrt{\Delta t}]} \tag{17}$$

where, μ is the mean, and is the mean of the variance process, and v_i is an independent standard normal variate. For the model estimation purposes, n is set equal to 90, and using an antithetic normal variate method, 1000 simulations were run for each option. Using the values of estimated variance, option prices are calculated. This procedure is then repeated using negative standard normal variable $-v_i$, and the price of each option is calculated. The price of the option is the average of these two estimates.

Appendix 3A

Let $x=\ln\sigma_S$ and $\sigma_S = e^x$. Since $\sigma_S = F(x,t)$ applying Ito's lemma

$$d\sigma_S=F_x dx+F_t dt+\frac{1}{2}F_{xx}(dx)^2$$

But,

$$F_x=e^x, \text{ and } F_{xx}=e^x$$

and,

$$dx=\beta(\alpha-\ln\sigma)dt+\gamma dZ_\sigma$$

$$(dx)^2=\gamma^2; \quad dZ_\sigma^2=\gamma^2 dt$$

Substituting,

$$d\sigma_S=\sigma_S[\beta(\alpha-\ln\sigma_S)dt+\gamma dZ_\sigma]+\sigma_S\gamma^2 dt$$

$$\frac{d\sigma_s}{\sigma_s}=[\beta(\alpha-\ln\sigma_s)+\frac{1}{2}\gamma^2]dt+\gamma dZ_\sigma$$

Appendix 3B

American call option is,

$$C'(F,\tau)=c'(F,\tau)+A_4\left(\frac{F}{F'^*}\right)^{q_4} \quad if \ F<F'^*$$

$$C'(F,\tau)=F-K \qquad\qquad if \ F\geq F'^*$$

where F'^* is estimated from the following equation,

$$F'^*-K=c'(F'^*,\tau)+\frac{\{1-B_1(t,T)N[d_3(F'^*)]\}}{q_4}$$

and

$$A_4=\frac{F'^*}{q_4}\{1-B_1(t,T)N[d_3(F'^*]\}$$

$$d_3(F'^*)=\frac{\ln(F'^*/K)+\frac{1}{2}V^2}{V}$$

$$q_4=[-(N'-1)+\sqrt{(N'-1)^2+4M'/X}]/2$$

$$M'=\frac{2r_d}{V^2} \ and \ N'=\frac{2b}{V^2}$$

Similarly, the American put option is

$$P'(F,\tau)=p'(F,\tau)+A_3(F/F'^{**})^{q_3} \quad if \ F>F'^{**}$$

$$P'(F,\tau)=K-F \qquad\qquad if \ F\leq F'^{**}$$

where

$$A_3=-(F'^{**}/q_3)\{1-B_1(t,T)N[d_3(F'^{**}]\}$$

and

$$q_3=[-(N'-1)-\sqrt{(N'-1)^2+4M'/X}]/2$$

CHAPTER 4

PRICING EFFICIENCY TESTS

Using transactions data for four currencies, empirical tests of pricing efficiency of the Stochastic Interest Rate Model, Stochastic Volatility Model, and the Implied Volatility Model are conducted. In the following sections, we discuss the structure of the tests and the results.

4.1 Data

The data on currency futures options for the following major currencies i.e. Deutsche Mark, Swiss Franc, Japanese Yen, and British Pound are obtained from the transactions data tapes maintained by the Chicago Mercantile Exchange (CME) during the four year period between Jan. '86 to Dec. '89. Since there was a change in the contract size of British Pound option, in July 1988, only the contracts up to June 1988 are used in our study for this option. The total transactions for the options and futures combined during this period are in excess of 6.5 million trades. CME trades four futures contracts each year, and twelve monthly options contracts written on the nearest expiring futures contract. Futures contracts expire on the third Wednesday of the contract month, where as the futures options expire two Fridays before the third Wednesday of the contract month. Since the models assume that the futures contract and the option contract expire at the same time, only the March, June, September, and December options contracts are considered for this study[1]. The sample thus comprises of sixteen quarter ending options, beginning with the March '86 contract and ending with the Dec. '89 contract. The transactions tape data contains the details of date and time of the trade, contract month, bid and ask quotes (not transactions), strike price, and options price. CME transactions tape also provide the prices of currency futures contracts. Futures and options data are merged to synchronize the futures price closest to the options transaction. While merging simultaneous options and futures data, a time window of one minute is selected to get the futures price closest to the option transaction. Since this study tests the pricing efficiency of the models, bid and ask quotes are dropped from the data set. If there is more than one transaction within one minute, only the last transaction is kept in the sample.

Other model parameters required are the spot exchange rate, domestic interest rate, and the foreign interest rate. Daily currency spot exchange rates for each of the four currencies are obtained from the International Macro Data Tape of the Federal Reserve Bank. All the three models require domestic (U.S.) interest rates to price the option. Depending on the term structure, the interest rates can vary for different time periods. If the option has less than one month to expire, the one month Eurodollar interest rate is used; if the option expiration is between 1 and 3 months, the three month Eurodollar interest rate is used and even if the option has between 3 to 6 month to expiration, we

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[1]Even for quarter ending options, the futures contract expires seven business days after the option expires. Violation of the convergence assumption will lead to the mis-specification of the model for options which are closer to expiration. In order to avoid this problem, options which expire within one week are excluded from the sample.

continue using the three month Eurodollar interest rate. For options with more than 6 months to expiration, the six month Eurodollar interest rate is used. Eurodollar interest rates for one, three, six, and twelve months, as well as three month interest rates in Germany, Japan, Switzerland, and Britain are all taken from the Fed Tape.

4.2 Structure of the Tests

4.2.1 Constant Interest and Constant Volatility Model (Implied Volatility Model)

Based on Black's model for futures option, the following currency futures option model is derived, under the assumption of constant foreign and domestic interest rates, and constant volatility of the underlying asset.

$$C(F,\tau)=c(F^*,\tau)+A_2\left(\frac{F}{F^*}\right)^{q_2} \qquad \text{if } F<F^*$$

$$\text{or,} \qquad C(F,\tau)=F-K \qquad \text{if } F\geq F^* \qquad (1)$$

where $c(F,\tau)$ is the European call option price, and F^* is the critical futures price below which there will be no early exercise of the call option,

$$A_2=\frac{F^*}{q_2}\{1-B_1(t,T)n[d_1(F^*)]\}$$

$$q_2=\frac{-(N-1)+\sqrt{(N-1)^2+4\frac{M}{K}}}{2}, \quad N=\frac{2}{\sigma^2} \quad \text{and} \quad M=\frac{2r(t)}{\sigma^2}$$

$$d_1 = \frac{\ln\frac{F(t,T)}{K} + \frac{1}{2}\sigma^2\tau}{\sigma\sqrt{\tau}}$$

The American put option is

$$P(F,\tau) = p(F,\tau) + A_1\left(\frac{F}{F^{**}}\right)^{q_1} \qquad if\ F > F^{**}$$

$$or, \quad P(F,\tau) = K - F \qquad if\ F \geq F^{**} \qquad (2)$$

where $p(F,\tau)$ is the European put option price, F^{**} is the critical futures price below which there will be no early exercise of the put option, and

$$A_1 = \frac{F^{**}}{q_1}\{1 - B_1(t,T)N[-d_1(F^{**})]\}$$

$$q_1 = \frac{-(N-1) - \sqrt{(N-1)^2 + 4\frac{M}{K}}}{2}$$

The inputs for the model are the futures price, Eurodollar interest rates, strike price, and the estimate of the underlying futures volatility. All the inputs, except the volatility are observable, and taken directly from the data tapes. Given the other input parameters and the market price of the option, the volatility implied by the model can be estimated. The implied volatility thus estimated indicates the option market's estimation of the underlying spot asset volatility, as well as the model bias.

Using an initial seed value for volatility, the option price is estimated using the above American option model. This model price is then compared with the actual market price of the option, and if the difference between the actual and the estimated price is more than 0.1%, the volatility is adjusted using an iterative algorithm, until the difference is less than 0.1%. This is the implied volatility for that particular option. Since there are options with different strike prices traded at the same time, the average implied volatility can be estimated by first estimating the implied volatilities of options with different strike prices and then averaging them using appropriate weights. Latane and Rendleman (1976) use partial derivative of the model with respect to the volatility as the weight. This technique weights at-the-money options more heavily than in-the-money and out-of-money options. Chiras and Manaster (1977) use the option price elasticity with respect to volatility as the weight. This method gives more weight to at-the-money options. Another method of implied volatility estimation is provided by Whaley (1982), in which the implied volatility is estimated by minimizing the squared pricing errors. In this study, the implied variance of the option which has a strike to asset price ratio closest to one is selected as the implied volatility for all options with different strike prices , written on a particular futures contract. Since the iterative procedure for estimating the implied volatility is computationally time consuming, only one implied volatility estimation per contract is made each day, using the last available transaction for that day.

4.2.2 Stochastic Interest Rate Model

Next we test the pricing efficiency of the option pricing model with stochastic interest rate, described in section 2.6. The volatility parameter in this model is a function of the variances and covariances of the spot exchange rate, domestic interest rate, and the foreign interest rate. For the estimation of the variances and covariances of the interest rates, we use three month interest rates[2]. The following sections describe the estimation procedures used in estimating the variances and covariances.

4.2.2.1 Cross-correlations between interest rates and exchange rates

Daily exchanges rates and interest rates are used to estimate the historical variances. Historical daily mean and variance are estimated using 20 observations of exchange rates and interest rates. The variance is then annualized by multiplying the daily variance by 252, which is approximately the number of business days in a year, to obtain the annual variance. The square root of annual variance gives the estimation of volatility (standard deviation) used in the model. These observations are also used to estimate the covariances between the exchange rate and interest rate. Tables 1a - 1d present the mean, variance and covariance of the exchange rate and interest rate for the British pound, Deutsche mark, Japanese yen, and Swiss franc respectively.

Except for the British pound, there is a negative covariance between the exchange rate and the U.S. Eurodollar interest rate and between the exchange rate and that country's Eurocurrency interest rates. Tests also indicate a statistically significant correlation between the exchange rates and interest rates. This substantiates the earlier assumption that, random shocks to interest rates can affect the level of exchange rate.

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[2]We estimated the variances and covariances using interest rates for 1 month, 3 months, 6 months, and one year. The result indicate that there is no significant difference in the variances and covariances for different interest rates, and hence we selected 3 month interest rate as the representative sample.

The covariance between the Eurodollar and foreign interest rate is positive for Germany and Switzerland, but statistically insignificant for Japan and Britain.

4.2.3 Stochastic Volatility Model

Pricing efficiency of the stochastic volatility model described in Chapter 3 is tested using the last transaction for the day for British pound, Deutsche mark, Japanese yen, and Swiss franc futures options. Since the estimation procedure involves Monte Carlo simulation, which is computationally time consuming, daily data is used for testing this model, instead of the transactions data. Parameters of the volatility process are estimated using the following equation described earlier in 3.6.

$$\ln\sigma_{s(t+1)} = a + b\ln\sigma_{st} + \epsilon_t$$

The above equation is a discrete approximation of the Ornstein-Uhlenbeck process. Using the estimates of the parameters of the above regression equation, the parameters of the stochastic process is estimated using the following equalities.

$$a=\alpha(1-e^{-\beta}); \qquad \rho=e^{-\beta}; \quad and \quad \sigma_\epsilon^2=\frac{\gamma^2}{2\beta}(1-e^{-2\beta})$$

Natural logarithm of daily historical volatility is used in the above equation as the dependent variable, and the first lag of the historical volatility as the independent variable. The parameters of this equation are estimated using the ARIMA procedure in SAS statistical package. The above equation is a discrete time approximation of the Ornstein-Uhlenbeck process. Ornstein-Uhlenbeck process is a mean reverting process, in which random shocks move the asset volatility from the long term mean, but in absence of any further random shocks, the volatility reverts back to the long term mean. The parameters of the process give the speed at which the volatility settles back to the long term mean. If the Ornstein-Uhlenbeck process is the true representation of the volatility process, then the parameter estimates from the above equation should remain constant across time. Our tests indicate that, for the time period under study, there is a shift in the long term mean and the adjustment parameters of the volatility process for all the four currencies studied. In order to incorporate the changes in the parameters of the stochastic process over time, we estimate four separate yearly estimations of the parameters. For estimation procedure, daily data for the past five years is used. The parameter estimates are summarized in Tables 2a - 2d.

The mean daily volatility is the mean reverting level of daily volatility of the underlying asset. This mean-reverting daily volatility is annualized by multiplying it with the square root of 252, which is approximately the number of trading days in a year. Since the tests indicate that there is change in the mean daily volatility of the volatility process over time, four yearly estimations were made for each of the four currencies. Mean-reverting level of volatility for all four currencies increased over time, which is an indication of the increase in volatilities in the currency markets. Instead of taking one single set of volatility estimate, we have used four different sets of parameters (one for each year[3]), which in our opinion will incorporate the changes in the parameters during the study period.

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[3]This does not imply that there is only one variance estimate per year. Variances for each option is simulated using these parameters.

4.3 Test Results

Results of pricing efficiency tests for the three models are summarized in Table 3 to 34. Each table separates the options into three categories, viz., at the money, in the money and out of money. Each cell in the table gives the average pricing error, the absolute pricing error, the number of positive errors, and the number of options in each cell. Average pricing error is equal to the difference of actual market option price and the model option price divided by the actual option price. If the average pricing error is positive, it indicates that the model underprices the option, and a negative pricing error indicates that the model overprices the option. Absolute pricing error indicates the size of the errors. If there are large positive and negative errors, these will cancel each other, resulting in a small average pricing error. Absolute pricing error can show whether there are such large positive and negative errors. The number of positive errors indicate the percentage cases in which the model underprices the options. Using a sign test, the systematic bias in pricing is tested by estimating the number of positive errors that are statistically different from 50 percent.

Call options are considered in-the-money if the ratio of strike to futures price (K/F) is less than 0.98, and out-of-money if this ratio is more than 1.02. Rest of the options are considered at-the-money. For put options if the ratio of futures price to strike (F/K) is less than 0.98, the option is considered in-the-money, and if this ratio is more than 1.02, the option is out-of-money. All other put options are considered at-the-money. Options are also broken down into three maturity classes, which are 7 days to 1 month, 1 month to 3 months, and more than 3 months. As a result of difficulty in estimating the implied volatility of options with maturity less than a week, such short term options are excluded from this study. In the following sections we discuss the results of the pricing efficiency tests.

4.3.1 Call Option Pricing (Tables 3 -18)

Tables 3 through 6 present results of pricing efficiency for the British pound call options for the three different models for all maturities, short-term maturities (one week to one month), medium term maturities (one to three months) and maturities exceeding three months. Similarly Tables 7 through 10 present results for the Deutsche mark, Tables 11 through 14 for Japanese yen and Tables 15 through 18 for Swiss franc.

Test results indicate that none of the models can consistently price options of all classes and maturity ranges better than others. For all the four options tested, implied volatility model has the best overall pricing efficiency. Pricing efficiency results for all the call options on the four currency futures contracts are given in Tables 3 (British pound), 7 (Deutsche mark), 11 (Japanese yen), and 15 (Swiss franc). The pricing error is lowest for Swiss franc (over pricing of 0.29 percent), and the highest for British pound (over pricing of 1.28 percent). Implied volatility model also has better pricing efficiency for at-the-money options, which can be explained by the fact that the implied volatilities are estimated using at-the-money options. For at-the-money options of British pound and Japanese yen, the implied volatility model gives a slight over-pricing (0.36 percent and 0.07 percent respectively), and for Deutsche mark and Swiss francs there is a slight under-pricing (0.41 percent and 1.08 percent respectively). For British pound, Japanese

Yen, and Swiss franc, in-the-money options are priced better by the stochastic volatility model. The pricing errors for in-the-money options by the stochastic volatility model range from an over-pricing of 6.31 percent for Japanese yen to an under-pricing of 1.41 percent for the British pound. Stochastic interest rate model prices Deutsche mark in-the-money options with the least error of -0.09 percent. None of the three models perform particularly well for out-of-money options, with all models showing a significant under-pricing. Here implied volatility model has the lowest pricing error for all currencies vis-a-vis the other two models, ranging from +7.77 percent for Japanese yen to +10.48 percent for Deutsche mark.

Implied volatility model has the lowest absolute pricing error, which indicates that this model has few large errors. The range for absolute pricing error is 9.02 percent for British pound, and 10.26 for Japanese yen. Both stochastic interest rate and stochastic volatility model have large absolute errors, except for some in-the-money options.

The percentage of positive errors generally go along with the pricing errors. If the pricing error indicates under-pricing and if the number of positive errors are greater than 50 percent, then it is an indication of a bias in the model towards under-pricing. Sign test of positive errors show that they are significantly different from the 50 percent level, which indicates the bias of the models towards under-pricing of the options.

Further tests are conducted for call options of different maturities. The results for British pound call options for different maturities are given in Tables 4, 5, and 6. In overall pricing efficiency, the implied volatility model out-performs the other two models in 7 to 30 day and 31 to 90 day maturity ranges. For call options with more than 90 days to expiration (Table 6), the stochastic interest rate model has the lowest pricing error (under-pricing of 3.43 percent), but a large absolute pricing error (30.55 percent), indicating that there are large positive and negative errors which cancel each other out, resulting in a small mean pricing error. For in-the-money call options on British pound, stochastic interest rate model gives better pricing efficiency for the maturity class of 31 to 90 day (Table 5), where as the stochastic volatility model prices in-the-money options better in other maturity classes (Tables 4 and 6).

Pricing efficiency tests for Deutsche mark call options of various maturity ranges are analyzed in Tables 8, 9 and 10. For options that expire in 7 to 30 days, as well as for those options with more than 90 days to expiration, implied volatility model slightly under-prices the options. For Deutsche mark options with 31 to 90 days to expiration, the implied volatility model shows a small over-pricing. Overall, prices estimated by the implied volatility model have smaller pricing errors for Deutsche mark call options. Stochastic interest rate model prices the in-the-money options of maturities greater than 30 day better. Implied volatility model gives better pricing efficiency for at-the-money and out-of-money options.

The results of Japanese yen call options pricing results are given in Tables 12, 13, and 14. Implied volatility model has better pricing efficiency for longer maturity options with more than 30 days to expiration. Contrary to currency spot options, implied volatility model gives a better pricing of in-the-money options with more than 90 days to expiration. For in-the-money options with 7 to 90 days to expiration, stochastic volatility model has the lowest pricing error. Stochastic interest rate model outperforms the other two models in pricing at-the-money short maturity (7 to 30 days) call options on yen futures.

Swiss franc call option pricing results of different maturity classes are summarized in Tables 16, 17, and 18. At-the-money and out-of-money short maturity options (less than 30 days) are priced better by stochastic interest model while stochastic volatility model is most efficient in pricing in-the-money short maturity options. Implied volatility model has a better pricing efficiency for all options with more than 90 days to expiration, and at-the-money and out-of-money options with 31 to 90 day maturities. In-the-money options for 31 to 90 day maturities to expiration are priced better by stochastic interest rate model.

4.3.2 Put option pricing

Tables 19 through 22 present results of pricing efficiency for the British pound put options for the three models for all maturities, short-term maturities (one week to one month), medium term maturities (one to three months) and maturities exceeding three months. Similarly Tables 23 through 26 present results for the Deutsche mark, Tables 27 through 30 for the Japanese yen and Tables 31 through 34 for the Swiss franc.

Efficiency tests of put option pricing by various models indicate that all models are less efficient in pricing the put options compared to pricing call options. Similar to the call option pricing results, the implied volatility model has a better overall pricing efficiency than the other two models. The results of overall pricing efficiency tests for all the four currency futures options are given in Tables 19, 23, 27, and 31. Implied volatility model slightly under prices all the options (2.58 percent for Deutsche mark options to 5.58 percent for Swiss franc options). In different categorizations based on the strike to underlying asset ratio, the implied volatility model out performs the other two models in all categories for British pound, Deutsche mark, and Swiss franc options, and in two categories for Japanese yen. The only exception is the in-the-money Japanese yen options, where pricing based on stochastic interest rate is the most efficient.

Implied volatility model priced all the options with the least absolute errors. The range of absolute errors varied from 17.00 percent for British pound to 21.13 percent for Swiss franc. Comparing these numbers with the absolute pricing errors of call options, we find that all the models are less efficient in pricing the put options. The sign test on the percentage positive errors shows a statistically significant model bias towards under pricing of the put options.

The results of the British pound put option pricing efficiency test for various maturity classes are summarized in Tables 20, 21, and 22. The put options with less than 30 days to expiration are underpriced by all models, with implied volatility model giving the least underpricing of 24.97 percent. Out-of-money and at-the-money options of all maturity classes for British pound put options are priced more efficiently by the implied volatility model, with less underpricing compared to the call options pricing by the implied volatility model. In-the-money options with 7 to 30 day and more than 90 day maturities are equally well priced by all the three models considered.

Efficiency test results of Deutsche mark put options of various maturity classes are given in Tables 24, 25, and 26. Implied volatility model prices the options with the least error for all option categories for all maturities. Except for out-of-money options with more than 90 days to expiration, all other put options with more than 30 days to expiration are slightly overpriced by the implied volatility model. For options with less than 30 days for expiration, stochastic volatility model has the lowest pricing error for the sample of all options.

Pricing efficiency tests for Japanese yen put options of various maturity classes are summarized in Tables 28, 29, and 30. None of the three models tested yielded efficient pricing for the sample of all yen put options with less than 30 days to expiration, with the stochastic volatility model outperforming others with the lowest pricing error of +10.31 percent. In the category of options with less than 30 days to expiration, in-the-money options are priced equally well by all the three models with the implied volatility model yielding the smallest underpricing of 3.29 percent. In maturity classes with more than 90 days to expiration, the implied volatility model has the best overall pricing, while the stochastic interest rate model has the lowest error for in-the-money and out-of-money options with 31 to 90 days to expiration.

Results for the Swiss franc put option pricing results are presented in Tables 32, 33, and 34. Stochastic volatility model has the best overall pricing efficiency for samples of all options, and in-the-money options with less than 30 days to expiration. The implied volatility model give the best pricing for all categories of options with 31 to 90 days to expiration (Table 33). For option maturities greater than 90 days (Table 34), the stochastic interest rate model yields the least error and the best pricing efficiency.

The results of pricing efficiency tests by various models as discussed above lead to the conclusion that the implied volatility models gives the best overall pricing efficiency. But detailed analysis of various in, at, and out-of-money option categories, as well as various maturity classes indicate that the stochstic interest rate model and the stochastic volatility model can yield better pricing efficiency for certain classes of options. For call options, the implied volatility model performs poorly relative to the other two models for in-the-money options. There is no clear cut superiority for any particular model for pricing put options, with the stochastic interest rate model giving better pricing efficiency for options with less than 30 days to expiration.

4.4 Comparison of the Pricing Efficiency Test Results with Other Spot Currency Option Model Test Results

Hilliard, Madura, and Tucker (1991) use a stochastic interest rate model to test the pricing efficiency of spot currency options. They compare the pricing efficiency of their stochastic interest rate model with the traditional model of constant interest rate and constant historic volatility, and find that the stochastic interest rate model give better pricing efficiency. They also find that the pricing efficiency of the stochastic interest rate model over historic volatility model increase as the time to maturity increases. Melino and Turnbull (1987), use a stochastic volatility model to test the pricing efficiency on spot currency options of British pound, Canadian dollar, Deutsche mark, Japanese yen, and Swiss franc. They compare the volatility estimates using the implied volatility model with historic volatility estimates and find that the implied volatility is always greater than the historic volatility of the underlying asset. Chesney and Scott (1987) use a different stochastic volatility model to price European currency spot options traded by Credit Suisse First Boston in Geneva. Results of their study indicate small gains in hedged positions using the stochastic volatility model, compared to constant volatility model.

CHAPTER 5

CONCLUSIONS

Currency futures options in the last few years have emerged as an important tool for hedging the currency exchange risk. The cost of hedging the exchange risk depends on the accuracy of the pricing models used in estimating the option price. In this respect, it is important to develop pricing models which can accurately price the options. This study deals with both the theoretical modeling as well as the empirical testing of existing and newly developed options models, using actual CME transactions data for a four year period, 1985-89.

Analysis of exchange rates and foreign and dollar interest rates indicate a statistically significant correlation between the exchange rates and interest rates. The study shows that the interest rates are stochastic in nature, even for relatively short time periods. The theoretical implication of this is that stochastic interest rates should be incorporated within the framework of option pricing models. This research develops such a pricing model allowing the domestic and foreign interest rates to be stochastic. Generally speaking, a closed form solution to a pricing model is more attractive than a partial differential equation, which has to be solved numerically. But finding a closed form solution can put constraints on the type of stochastic process that can be used in describing the interest rate process. In this respect, the stochastic interest rate model developed here is constrained by the use of arithmetic Brownian motion to describe the interest rate process. But this allows for a closed form solution for the option pricing model.

Of all the parameters in an option pricing model, only the volatility parameter is not directly observable. The traditional implied volatility model tries to overcome this problem by computing the volatilities using the theoretical model and the actual option prices. Various studies indicate that these implied volatilities are not constant over time, as envisaged by the option pricing model. Chapter 3 of this research develops a pricing model which allows for the underlying asset volatilities to be stochastic, and derives a semi-closed form solution for pricing the options. In our empirical estimations, we use historical volatilities to estimate the parameters of the stochastic process, and find that there is a shift in the mean reverting level of volatilities. It may be useful to test the model using the implied volatilities as input in the estimation of the parameters of the stochastic process, and then using these parameters in a stochastic volatility model.

Empirical testing of the efficiency of the pricing models show that none of the three models tested is superior relative to the others in all maturity classes and the at-, in-, and out-of-money categories. Implied volatility models give better pricing efficiency for all at-the-money options. This may be due to the fact that the implied volatilities are computed using at-the-money options. The stochastic interest rate model and the stochastic volatility model give better pricing efficiency for some in-the-money options. Holding everything else constant, it is found that the volatility estimates are higher for out-of-money options, as compared to in-the-money options.

Since pricing efficiency tests are joint tests of model bias, market efficiency, market synchronization, and data accuracy, mispricing of the options can be due to any one of these factors. In this study we use synchronized data from the Exchange, which eliminates the possibility of data inaccuracy and non-synchronization. An important area for future research will be to use these models to identify market inefficiency and profit opportunities in terms of mispricing of futures options.

TABLE 1

Panel A

Means, Standard Deviations, and Covariances of
British Pound Exchange Rates and Interest Rates
From Jan. 1, 1986 to Dec. 31, 1989.

	Means	s.d	Exchange Rate	Euro $ 3 month Interest Rate
Exchange Rate@	1.6317	0.1511		
Euro $ 3 month Interest Rate	0.0769	0.0332	0.00642	
British Pound 3 month Interest Rate	0.1116	0.0513	0.05572	0.00000

@ Dollars per 1 British Pound.

TABLE 1

Panel B

Means, Standard Deviations, and Covariances of
Deutsche Mark Exchange Rates and Interest Rates
From Jan. 1, 1986 to Dec. 31, 1989.

	Means	s.d	Exchange Rate	Euro $ 3 month Interest Rate
Exchange Rate@	0.5306	0.1470		
Euro $ 3 month Interest Rate	0.0769	0.03324	-0.02994	
Deutsche Mark 3 month Interest Rate	0.0497	0.02811	-0.04055	0.01734

@ Dollars per 1 DM.

TABLE 1

Panel C

Means, Standard Deviations, and Covariances of
Japanese Yen Exchange Rates and Interest Rates
From Jan. 1, 1986 to Dec. 31, 1989.

	Means	s.d	Exchange Rate	Euro $ 3 month Interest Rate
Exchange Rate@	0.6995	0.18846		
Euro $ 3 month Interest Rate	0.0769	0.03324	-0.01109	
Japanese Yen 3 month Interest Rate	0.0370	0.01656	-0.024790	0.00000

@ Dollars per 100 Yen.

TABLE 1

Panel D

Means, Standard Deviations, and Covariances of
Swiss Franc Exchange Rates and Interest Rates
From Jan. 1, 1986 to Dec. 31, 1989.

	Means	s.d	Exchange Rate	Euro $ 3 month Interest Rate
Exchange Rate@	0.6320	0.1913		
Euro $ 3 month Interest Rate	0.0769	0.0332	-0.03378	
Swiss Franc 3 month Interest Rate	0.0440	0.0389	-0.04494	0.02360

@ Dollars per 1 Swiss Franc.

TABLE 2 - PANEL A
Parameter Estimates for the Volatility Process
British Pound

$$ln\sigma_{t+1} = a + \rho ln\sigma_t + \epsilon_t$$

Period	a	ρ	σ^2_ϵ	β	γ	α	Mean Daily Volatility	Mean Annual Volatility
1981 - 85	-0.07232	0.984411	0.011293	0.015711	0.107108	-4.63953	0.009662	0.153382
1982 - 86	-0.07275	0.984582	0.012702	0.015537	0.113582	-4.71902	0.008923	0.141661
1983 - 87	-0.08274	0.982167	0.013561	0.017993	0.117502	-4.63977	0.009659	0.153344
1984 - 88	-0.08654	0.981397	0.013914	0.018777	0.119067	-4.65217	0.009540	0.151456

Notes: σ^2_ϵ is the variance of ϵ_t.
β, γ, and a are the parameters of the discrete approximation of the Ornstein-Uhlenbeck process given below.

$$\ln\sigma_{S(t+1)} = a(1-e^{-\beta})+e^{-\beta}\ln\sigma_{St}+\epsilon_t$$

TABLE 2 - PANEL B
Parameter Estimates for the Volatility Process
Deutsche Mark

$$ln\sigma_{t+1} = a + \rho ln\sigma_t + \epsilon_t$$

Period	a	ρ	σ^2_ϵ	β	γ	α	Mean Daily Volatility	Mean Annual Volatility
1981 - 85	-0.10805	0.979140	0.011274	0.021079	0.107304	-5.18007	0.005627	0.089334
1982 - 86	-0.11463	0.977818	0.011139	0.022431	0.106729	-5.16816	0.005695	0.090405
1983 - 87	-0.10773	0.978921	0.011909	0.021303	0.110294	-5.11096	0.006030	0.095726
1984 - 88	-0.11509	0.977005	0.011770	0.023262	0.109755	-5.00532	0.006702	0.106393

Notes: σ^2_ϵ is the variance of ϵ_t.

TABLE 2 - PANEL C
Parameter Estimates for the Volatility Process
Japanese Yen

$$ln\sigma_{t+1} = a + \rho ln\sigma_t + \epsilon_t$$

Period	a	ρ	σ^2_ϵ	β	γ	α	Mean Daily Volatility	Mean Annual Volatility
1981 - 85	-0.09539	0.982221	0.016621	0.017938	0.130081	-5.36605	0.004672	0.074174
1982 - 86	-0.08455	0.983945	0.019276	0.016185	0.139963	-5.26655	0.005161	0.081933
1983 - 87	-0.07134	0.986129	0.022077	0.013967	0.149624	-5.14398	0.005834	0.092618
1984 - 88	-0.08156	0.983582	0.023309	0.016553	0.153941	-4.96814	0.006956	0.110423

Notes: σ^2_ϵ is the variance of ϵ_t.

TABLE 2 - PANEL D
Parameter Estimates for the Volatility Process
Swiss Franc

$$ln\sigma_{t+1} = a + \rho ln\sigma_t + \epsilon_t$$

Period	a	ρ	σ^2_ϵ	β	γ	α	Mean Daily Volatility	Mean Annual Volatility
1981 - 85	-0.08859	0.982206	0.010653	0.017953	0.104141	-4.97934	0.006878	0.109193
1982 - 86	-0.09496	0.980756	0.010797	0.019431	0.104921	-4.93464	0.007193	0.114185
1983 - 87	-0.08553	0.982498	0.011813	0.017656	0.109653	-4.88734	0.007541	0.119716
1984 - 88	-0.09825	0.979271	0.011472	0.020946	0.108235	-4.74005	0.008738	0.138714

Notes: σ^2_ϵ is the variance of ϵ_t.

Table 3

Pricing Errors of British Pound Call Options - All Maturities

	Implied Volatility Model	Stochastic Interest Model	Stochastic Volatility Model
<u>All options</u>			
Error	- 1.28*	14.57*	10.64*
Absolute error	9.02*	25.45*	15.90*
% of Positive errors	53.43♀	73.91♀	73.41♀
Number of observations	8,624	8,762	2,681
<u>At the money</u>			
Error	- 0.36*	17.05*	12.56*
Absolute error	8.57*	28.23*	17.87*
% of Positive errors	54.56♀	76.89♀	74.78♀
Number of observations	5,271	5,374	1,380
<u>In the money</u>			
Error	- 7.57*	- 2.43*	- 1.41*
Absolute error	7.98*	7.79*	4.73*
% of Positive errors	1.91♀	38.84♀	35.19♀
Number of observations	995	999	412
<u>Out of money</u>			
Error	8.35*	40.34*	37.38*
Absolute error	13.42*	51.21*	39.89*
% of Positive errors	72.65♀	81.88♀	88.98♀
Number of observations	2,358	2,389	889

* significantly different from 0.0% at 1 percent.
♦ significantly different from 0.0% at 5 percent.
♀ significantly different from 50.0% at 1 percent.
♣ significantly different from 50.0% at 5 percent.

Notes:- 1. Error = (Market price - model price)/market price.

2. For Implied Volatility Model and Stochastic Interest Rate Model, transaction data is used. For Stochastic Volatility Model, only the last transaction for the day is used. The difference in number of observations between Implied Volatility Model and Stochastic Interest Rate Model is because of the elimination of the out-layers in estimation of the implied volatility.

Table 4

Pricing Errors of British Pound Call Options

7 to 30 days to Expiration

	Implied Volatility Model	Stochastic Interest Model	Stochastic Volatility Model
All options			
Error	1.64*	7.99*	10.00*
Absolute error	10.13*	17.70*	14.52*
% of Positive errors	65.27♀	73.02♀	71.85♀
Number of observations	1,843	1,927	547
At the money			
Error	6.08*	16.55*	17.74*
Absolute error	11.77*	23.70*	22.99*
% of Positive errors	74.24♀	82.09♀	78.01♀
Number of observations	1,308	1,379	332
In the money			
Error	- 6.20*	- 6.00*	- 0.62
Absolute error	6.53*	6.84*	3.39*
% of Positive errors	4.17♀	17.60♀	37.11♣
Number of observations	264	267	97
Out of money			
Error	22.00*	30.83*	55.63*
Absolute error	25.74*	45.45*	58.83*
% of Positive errors	81.55♀	81.14♀	83.05♀
Number of observations	271	281	118

* significantly different from 0.0% at 1 percent.
♦ significantly different from 0.0% at 5 percent.
♀ significantly different from 50.0% at 1 percent.
♣ significantly different from 50.0% at 5 percent.

Table 5

Pricing Errors of British Pound Call Options

31 to 90 days to Expiration

	Implied Volatility Model	Stochastic Interest Model	Stochastic Volatility Model
<u>All options</u>			
Error	- 2.74*	17.75*	11.72*
Absolute error	8.99*	26.47*	16.44*
% of Positive errors	47.46♀	75.77♀	75.34♀
Number of observations	6,073	6,103	1,715
<u>At the money</u>			
Error	- 2.33*	19.44*	13.91*
Absolute error	8.33*	28.75*	18.19*
% of Positive errors	45.00♀	76.54♀	76.36♀
Number of observations	3,558	3,581	825
<u>In the money</u>			
Error	- 8.30*	- 0.27	- 1.30♦
Absolute error	8.76*	7.42*	5.06*
% of Positive errors	5.94♀	49.03	35.40♀
Number of observations	673	673	274
<u>Out of money</u>			
Error	6.33*	45.39*	42.14*
Absolute error	12.29*	53.58*	43.66*
% of Positive errors	69.32♀	83.99♀	91.72♀
Number of observations	1,842	1,849	616

* significantly different from 0.0% at 1 percent.
♦ significantly different from 0.0% at 5 percent.
♀ significantly different from 50.0% at 1 percent.
♣ significantly different from 50.0% at 5 percent.

Table 6

Pricing Errors of British Pound Call Options

More than 90 days to Expiration

	Implied Volatility Model	Stochastic Interest Model	Stochastic Volatility Model
All options			
Error	4.18*	3.43*	7.54*
Absolute error	7.54*	30.55*	15.14*
% of Positive errors	73.87	60.79♀	67.54♀
Number of observations	708	732	419
At the money			
Error	3.68*	2.21*	5.84*
Absolute error	5.72*	31.26*	14.04*
% of Positive errors	75.06	62.56♀	64.13♀
Number of observations	405	414	223
In the money			
Error	- 4.97*	-11.63*	- 3.59*
Absolute error	5.17*	14.80*	5.27*
% of Positive errors	6.90♀	18.64	29.27♀
Number of observations	58	59	41
Out of money			
Error	14.07*	20.27*	23.77*
Absolute error	15.33*	42.15*	28.36*
% of Positive errors	87.76♀	67.57♀	82.58♀
Number of observations	245	259	155

* significantly different from 0.0% at 1 percent.
♦ significantly different from 0.0% at 5 percent.
♀ significantly different from 50.0% at 1 percent.
♣ significantly different from 50.0% at 5 percent.

Table 7

Pricing Errors of German Mark Call Options - All Maturities

	Implied Volatility Model	Stochastic Interest Model	Stochastic Volatility Model
All options			
Error	- 0.69*	20.18*	22.11*
Absolute error	9.21*	27.82*	27.61*
% of Positive errors	57.15♀	81.47♀	90.55♀
Number of observations	91,201	92,655	9,006
At the money			
Error	0.41*	27.19*	38.51*
Absolute error	8.37*	33.25*	38.53*
% of Positive errors	59.02♀	85.85♀	99.51♀
Number of observations	45,905	46,872	2,861
In the money			
Error	- 7.01*	- 0.09*	5.90*
Absolute error	7.06*	6.84*	5.96*
% of Positive errors	1.24♀	55.38♀	96.10♀
Number of observations	12,219	12,161	1,951
Out of money			
Error	10.48*	44.45*	52.03*
Absolute error	16.81*	58.71*	92.38*
% of Positive errors	75.22♀	84.81♀	81.85♀
Number of observations	33,077	33,622	4,194

* significantly different from 0.0% at 1 percent.
♦ significantly different from 0.0% at 5 percent.
♀ significantly different from 50.0% at 1 percent.
♣ significantly different from 50.0% at 5 percent.

Table 8

Pricing Errors of German Mark Call Options

7 to 30 days to Expiration

	Implied Volatility Model	Stochastic Interest Model	Stochastic Volatility Model
All options			
Error	2.01*	14.74*	3.53*
Absolute error	9.76*	21.34*	22.98*
% of Positive errors	66.79♀	80.47♀	76.51♀
Number of observations	20,342	21,926	1,592
At the money			
Error	5.99*	23.99*	3.60*
Absolute error	10.73*	27.62*	36.18*
% of Positive errors	76.68♀	87.81♀	98.02♀
Number of observations	12,614	13,619	555
In the money			
Error	- 6.02*	- 2.72*	2.09*
Absolute error	6.08*	4.06*	2.14*
% of Positive errors	2.24♀	41.89♀	93.65♀
Number of observations	2,370	2,411	362
Out of money			
Error	14.03*	30.76*	-185.36*
Absolute error	22.29*	60.31*	317.45*
% of Positive errors	72.06♀	79.27♀	49.63
Number of observations	5,358	5,896	675

* significantly different from 0.0% at 1 percent.
♦ significantly different from 0.0% at 5 percent.
♀ significantly different from 50.0% at 1 percent.
♣ significantly different from 50.0% at 5 percent.

Table 9

Pricing Errors of German Mark Call Options

31 to 90 days to Expiration

	Implied Volatility Model	Stochastic Interest Model	Stochastic Volatility Model
All options			
Error	- 2.39*	24.78*	19.50*
Absolute error	9.21*	29.43*	24.77*
% of Positive errors	51.55♀	83.76♀	91.06♀
Number of observations	60,068	59,946	4,720
At the money			
Error	- 1.76*	32.10*	39.76*
Absolute error	8.28*	35.69*	39.77*
% of Positive errors	49.21♣	87.00♀	99.85♀
Number of observations	28,554	28,523	1,349
In the money			
Error	- 7.65*	1.85*	5.35*
Absolute error	7.67*	6.60*	5.41*
% of Positive errors	2.80♀	61.70	96.16♀
Number of observations	8,219	8,121	1,121
Out of money			
Error	7.50*	53.85*	51.14*
Absolute error	15.39*	61.49*	98.83*
% of Positive errors	72.52♀	87.49♀	83.24♀
Number of observations	23,295	23,302	2,250

* significantly different from 0.0% at 1 percent.
♦ significantly different from 0.0% at 5 percent.
♀ significantly different from 50.0% at 1 percent.
♣ significantly different from 50.0% at 5 percent.

Table 10

Pricing Errors of German Mark Call Options

More than 90 days to Expiration

	Implied Volatility Model	Stochastic Interest Model	Stochastic Volatility Model
All options			
Error	3.54*	7.35*	32.81*
Absolute error	8.69*	27.79*	33.44*
% of Positive errors	70.14♀	70.81♀	97.96♀
Number of observations	10,791	10,783	2,694
At the money			
Error	3.11*	10.49*	37.88*
Absolute error	6.04*	29.89*	37.88*
% of Positive errors	71.08♀	73.28♀	99.90♀
Number of observations	4,737	4,730	957
In the money			
Error	- 5.32*	- 5.51*	9.57*
Absolute error	5.46*	10.85*	9.63*
% of Positive errors	4.66♀	18.64	97.86♀
Number of observations	1,630	1,629	468
Out of money			
Error	18.38*	20.77*	69.07*
Absolute error	19.43*	49.90*	71.98*
% of Positive errors	93.26♀	78.10♀	96.53♀
Number of observations	4,424	4,424	1,269

* significantly different from 0.0% at 1 percent.
♦ significantly different from 0.0% at 5 percent.
♀ significantly different from 50.0% at 1 percent.
♣ significantly different from 50.0% at 5 percent.

Table 11

Pricing Errors of Japanese Yen Call Options - All Maturities

	Implied Volatility Model	Stochastic Interest Model	Stochastic Volatility Model
All options			
Error	- 0.71*	-13.47*	25.81*
Absolute error	10.26*	38.24*	31.99*
% of Positive errors	58.25♀	48.63♀	88.21♀
Number of observations	65,147	67,182	8,555
At the money			
Error	- 0.07*	- 9.80*	37.91*
Absolute error	9.34*	38.51*	38.18*
% of Positive errors	58.98♀	52.76♀	96.73♀
Number of observations	36,191	37,637	3,240
In the money			
Error	- 7.64*	-10.92*	6.31*
Absolute error	7.71*	13.35*	6.55*
% of Positive errors	1.01♀	24.46♀	91.79♀
Number of observations	5,331	5,355	1,254♀
Out of money			
Error	7.77*	-30.75*	49.87*
Absolute error	17.52*	75.83*	90.22*
% of Positive errors	70.04♀	47.56♀	80.30♀
Number of observations	23,625	24,190	4,061

* significantly different from 0.0% at 1 percent.
♦ significantly different from 0.0% at 5 percent.
♀ significantly different from 50.0% at 1 percent.
♣ significantly different from 50.0% at 5 percent.

Table 12

Pricing Errors of Japanese Yen Call Options

7 to 30 days to Expiration

	Implied Volatility Model	Stochastic Interest Model	Stochastic Volatility Model
All options			
Error	4.38*	- 0.01*	6.72*
Absolute error	11.28*	26.21*	27.96*
% of Positive errors	72.43♀	55.35♀	73.58♀
Number of observations	15,026	16,547	1,658
At the money			
Error	8.18*	4.73*	33.76*
Absolute error	11.87*	29.33*	34.25*
% of Positive errors	82.55♀	60.51♀	94.44♀
Number of observations	10,220	11,341	683
In the money			
Error	- 6.35*	- 8.42*	2.81*
Absolute error	6.58*	9.30*	3.07*
% of Positive errors	2.84♀	19.85♀	86.56♀
Number of observations	1,018	1,033	253
Out of money			
Error	12.69*	-13.86*	-156.95*
Absolute error	25.29*	62.75*	283.10*
% of Positive errors	63.83♀	50.08	49.31
Number of observations	3,788	4,173	722

* significantly different from 0.0% at 1 percent.
♦ significantly different from 0.0% at 5 percent.
♀ significantly different from 50.0% at 1 percent.
♣ significantly different from 50.0% at 5 percent.

Table 13

Pricing Errors of Japanese Yen Call Options

31 to 90 days to Expiration

	Implied Volatility Model	Stochastic Interest Model	Stochastic Volatility Model
All options			
Error	- 2.46*	-13.53*	25.13*
Absolute error	10.24*	38.10*	30.21*
% of Positive errors	51.99♀	47.16♀	89.89♀
Number of observations	45,211	45,725	4,904
At the money			
Error	- 2.42*	10.51*	39.76*
Absolute error	9.21*	38.54*	39.97*
% of Positive errors	47.76♀	50.45	97.75♀
Number of observations	23,520	23,845	1,686
In the money			
Error	- 8.16*	-10.10*	6.23*
Absolute error	8.20*	12.80*	6.42*
% of Positive errors	5.34♀	25.75	86.55♀
Number of observations	3,936	3,945	803
Out of money			
Error	5.79*	-28.45*	55.25*
Absolute error	16.67*	73.72*	92.21*
% of Positive errors	69.01♀	47.50♀	83.27♀
Number of observations	17,755	17,935	2,415

* significantly different from 0.0% at 1 percent.
♦ significantly different from 0.0% at 5 percent:
♀ significantly different from 50.0% at 1 percent.
♣ significantly different from 50.0% at 5 percent.

Table 14

Pricing Errors of Japanese Yen Call Options

More than 90 days to Expiration

	Implied Volatility Model	Stochastic Interest Model	Stochastic Volatility Model
All options			
Error	3.95*	-33.77*	36.24*
Absolute error	8.83*	57.82*	37.48*
% of Positive errors	72.44♀	39.74♀	96.24♀
Number of observations	4,910	4,910	1,993
At the money			
Error	2.42*	-30.48*	36.69*
Absolute error	6.07*	54.40*	36.96*
% of Positive errors	68.38♀	39.41♀	96.56♀
Number of observations	2,451	2,451	871
In the money			
Error	- 5.66*	-23.71*	10.47*
Absolute error	5.68*	27.07*	10.90*
% of Positive errors	1.06♀	23.61	92.42♀
Number of observations	377	377	198
Out of money			
Error	16.26	-50.99*	64.16*
Absolute error	18.72*	93.34*	68.55*
% of Positive errors	90.15♀	43.04♀	96.75♀
Number of observations	2,082	2,082	924

* significantly different from 0.0% at 1 percent.
♦ significantly different from 0.0% at 5 percent.
♀ significantly different from 50.0% at 1 percent.
♣ significantly different from 50.0% at 5 percent.

Table 15

Pricing Errors of Swiss Franc Call Options - All Maturities

	Implied Volatility Model	Stochastic Interest Model	Stochastic Volatility Model
<u>All options</u>			
Error	- 0.29*	4.96*	19.99*
Absolute error	9.45*	25.63*	23.62*
% of Positive errors	57.79♀	62.22♀	87.46♀
Number of observations	67,577	68,800	8,857
<u>At the money</u>			
Error	1.08*	7.97*	29.88*
Absolute error	8.74*	28.18*	30.46*
% of Positive errors	61.11♀	65.24♀	94.68♀
Number of observations	35,716	36,491	3,064
<u>In the money</u>			
Error	- 6.96*	- 5.72*	4.41*
Absolute error	7.06*	9.39*	4.87*
% of Positive errors	1.35♀	32.17♀	83.76♀
Number of observations	7,870	7,853	1,656
<u>Out of money</u>			
Error	8.87*	16.64*	49.25*
Absolute error	16.67*	50.12*	70.87*
% of Positive errors	71.37♀	67.38♀	83.59♀
Number of observations	23,991	24,456	4,137

* significantly different from 0.0% at 1 percent.
♦ significantly different from 0.0% at 5 percent.
♀ significantly different from 50.0% at 1 percent.
♣ significantly different from 50.0% at 5 percent.

Table 16

Pricing Errors of Swiss Franc Call Options

7 to 30 days to Expiration

	Implied Volatility Model	Stochastic Interest Model	Stochastic Volatility Model
All options			
Error	3.98*	2.06*	8.45*
Absolute error	10.97*	22.42*	21.73*
% of Positive errors	71.56♀	59.35♀	76.08♀
Number of observations	16,595	16,456	1,710
At the money			
Error	8.28*	7.92*	33.61*
Absolute error	11.83*	25.67*	33.97*
% of Positive errors	81.49♀	65.48♀	95.04♀
Number of observations	10,654	10,673	645
In the money			
Error	- 6.26*	- 7.84*	1.67*
Absolute error	6.55*	8.71*	2.26*
% of Positive errors	2.52♀	19.77♀	72.92♀
Number of observations	1,589	1,452	336
Out of money			
Error	15.74*	- 3.21*	-59.62*
Absolute error	23.87*	54.80*	191.91*
% of Positive errors	71.47♀	57.49♀	60.77♀
Number of observations	4,352	4,331	729

* significantly different from 0.0% at 1 percent.
♦ significantly different from 0.0% at 5 percent.
♀ significantly different from 50.0% at 1 percent.
♣ significantly different from 50.0% at 5 percent.

Table 17

Pricing Errors of Swiss Franc Call Options

31 to 90 days to Expiration

	Implied Volatility Model	Stochastic Interest Model	Stochastic Volatility Model
All options			
Error	- 2.19*	10.89*	20.09*
Absolute error	9.30*	24.33*	22.93*
% of Positive errors	50.60♣	67.40♀	88.72♀
Number of observations	44,471	44,536	4,993
At the money			
Error	- 1.46*	13.75*	31.20*
Absolute error	8.43*	27.31*	31.63*
% of Positive errors	49.49	69.53♀	95.52♀
Number of observations	21,998	21,998	1.585
In the money			
Error	- 7.48*	- 2.79*	4.27*
Absolute error	7.53*	7.50*	4.67*
% of Positive errors	0.63♀	38.53♀	84.66♀
Number of observations	5,439	5,412	965
Out of money			
Error	6.01*	28.97*	55.97*
Absolute error	15.43*	48.22*	74.03*
% of Positive errors	67.98♀	73.78♀	85.92♀
Number of observations	17,034	17,126	2,443

* significantly different from 0.0% at 1 percent.
♦ significantly different from 0.0% at 5 percent.
♀ significantly different from 50.0% at 1 percent.
♣ significantly different from 50.0% at 5 percent.

Table 18

Pricing Errors of Swiss Franc Call Options

More than 90 days to Expiration

	Implied Volatility Model	Stochastic Interest Model	Stochastic Volatility Model
All options			
Error	3.98*	-20.41*	24.84*
Absolute error	8.37*	36.93*	25.66*
% of Positive errors	71.83♀	36.92♀	93.55♀
Number of observations	6,511	6,506	2,154
At the money			
Error	3.55*	-20.10*	26.91*
Absolute error	5.93*	37.42*	27.75*
% of Positive errors	73.63♀	37.43♀	92.81♀
Number of observations	3,064	3,062	834
In the money			
Error	- 5.04*	-19.32*	6.68*
Absolute error	5.15*	21.05*	7.33*
% of Positive errors	3.80♀	15.73	91.55♀
Number of observations	842	839	355
Out of money			
Error	16.94*	-22.56*	52.95*
Absolute error	18.33*	56.79*	54.27*
% of Positive errors	91.70♀	43.14♀	94.92♀
Number of observations	2,605	2,605	965

* significantly different from 0.0% at 1 percent.
♦ significantly different from 0.0% at 5 percent.
♀ significantly different from 50.0% at 1 percent.
♣ significantly different from 50.0% at 5 percent.

Table 19

Pricing Errors of British Pound Put Options - All Maturities

	Implied Volatility Model	Stochastic Interest Model	Stochastic Volatility Model
All options			
Error	4.50*	36.46*	28.43
Absolute error	17.00*	42.83*	37.15
% of Positive errors	65.22♀	87.20♀	88.43♀
Number of observations	6,861	7,053	2,109
At the money			
Error	5.83*	41.82*	34.35*
Absolute error	16.74*	43.80*	34.63*
% of Positive errors	66.93♀	83.13♀	97.42♀
Number of observations	4,221	4,366	1,124
In the money			
Error	- 1.57*	13.01*	9.39*
Absolute error	6.13*	13.47*	9.50*
% of Positive errors	55.07♀	96.19♀	98.37♀
Number of observations	434	444	184
Out of money			
Error	5.06*	37.43*	32.89*
Absolute error	28.86*	67.97*	80.45*
% of Positive errors	63.96♀	73.87♀	73.53♀
Number of observations	2,206	2,243	801

* significantly different from 0.0% at 1 percent.
♦ significantly different from 0.0% at 5 percent.
♀ significantly different from 50.0% at 1 percent.
♣ significantly different from 50.0% at 5 percent.

Table 20

Pricing Errors of British Pound Put Options

7 to 30 days to Expiration

	Implied Volatility Model	Stochastic Interest Model	Stochastic Volatility Model
All options			
Error	24.97*	37.86*	27.19*
Absolute error	31.20*	45.38*	39.80*
% of Positive errors	90.49♀	84.23♀	80.00
Number of observations	1,472	1,528	445
At the money			
Error	32.86*	51.59*	45.54*
Absolute error	33.22*	52.03*	46.38*
% of Positive errors	98.59♀	96.41♀	95.61♀
Number of observations	1,067	1,113	296
In the money			
Error	4.98*	6.43*	6.97*
Absolute error	4.98*	6.43*	6.97*
% of Positive errors	100.00♀	98.02♀	100.00♀
Number of observations	95	99	43
Out of money			
Error	1.04*	-15.48*	-67.24*
Absolute error	97.45*	104.50*	145.84*
% of Positive errors	59.68♀	36.62♀	28.30♀
Number of observations	310	314	106

* significantly different from 0.0% at 1 percent.
♦ significantly different from 0.0% at 5 percent.
♀ significantly different from 50.0% at 1 percent.
♣ significantly different from 50.0% at 5 percent.

Table 21

Pricing Errors of British Pound Put Options

31 to 90 days to Expiration

	Implied Volatility Model	Stochastic Interest Model	Stochastic Volatility Model
All options			
Error	1.19*	38.64*	30.42*
Absolute error	15.22*	44.03*	40.53*
% of Positive errors	58.13♀	89.36♀	90.24♀
Number of observations	4,939	4,953	1,353
At the money			
Error	1.22*	43.07*	36.66*
Absolute error	14.45*	44.05*	36.67*
% of Positive errors	56.52♀	93.93♀	99.24♀
Number of observations	2,905	2,917	661
In the money			
Error	- 2.58*	15.08*	10.37*
Absolute error	6.19*	15.29*	10.45*
% of Positive errors	45.25	97.47	99.16♀
Number of observations	316	316	119
Out of money			
Error	4.56*	43.23*	36.21*
Absolute error	26.53*	70.35*	88.25*
% of Positive errors	63.21♀	80.12♀	78.01♀
Number of observations	1,718	1,720	311

* significantly different from 0.0% at 1 percent.
♦ significantly different from 0.0% at 5 percent.
♀ significantly different from 50.0% at 1 percent.
♣ significantly different from 50.0% at 5 percent.

Table 22

Pricing Errors of British Pound Put Options

More than 90 days to Expiration

	Implied Volatility Model	Stochastic Interest Model	Stochastic Volatility Model
<u>All options</u>			
Error	0.66*	20.78*	23.10*
Absolute error	9.91*	32.03*	24.95*
% of Positive errors	60.44♀	76.48♀	92.60♀
Number of observations	450	574	311
<u>At the money</u>			
Error	- 1.11*	21.22*	20.46*
Absolute error	7.96*	31.52*	21.09*
% of Positive errors	52.61♀	75.30♀	93.41♀
Number of observations	249	336	167
<u>In the money</u>			
Error	- 8.61*	8.90*	7.97*
Absolute error	8.72*	12.90*	8.39*
% of Positive errors	4.35♀	75.86♀	90.91♀
Number of observations	23	29	22
<u>Out of money</u>			
Error	9.73*	25.33*	41.14*
Absolute error	15.70*	43.21*	47.54*
% of Positive errors	78.65♀	78.47♀	91.80♀
Number of observations	178	209	122

* significantly different from 0.0% at 1 percent.
♦ significantly different from 0.0% at 5 percent.
♀ significantly different from 50.0% at 1 percent.
♣ significantly different from 50.0% at 5 percent.

Table 23

Pricing Errors of German Mark Put Options - All Maturities

	Implied Volatility Model	Stochastic Interest Model	Stochastic Volatility Model
All options			
Error	2.58*	32.03*	18.60*
Absolute error	20.78*	49.45*	47.65*
% of Positive errors	64.75♀	79.95♀	66.85♀
Number of observations	63,162	64,933	8,369
At the money			
Error	6.52*	46.22*	49.12*
Absolute error	18.46*	47.96*	49.20*
% of Positive errors	69.33♀	96.10♀	99.15♀
Number of observations	29,537	30,951	2,583
In the money			
Error	- 1.35*	8.18*	9.61*
Absolute error	5.63*	9.05*	9.61*
% of Positive errors	59.16♀	93.28♀	98.67♀
Number of observations	3,822	4,016	1,189
Out of money			
Error	- 3.43*	18.73*	-22.28*
Absolute error	39.84*	90.49*	138.45*
% of Positive errors	60.93♀	61.60♀	40.85♀
Number of observations	29,803	29,966	4,597

* significantly different from 0.0% at 1 percent.
♦ significantly different from 0.0% at 5 percent.
♀ significantly different from 50.0% at 1 percent.
♣ significantly different from 50.0% at 5 percent.

Table 24

Pricing Errors of German Mark Put Options

7 to 30 days to Expiration

	Implied Volatility Model	Stochastic Interest Model	Stochastic Volatility Model
All options			
Error	18.70*	32.21*	11.76*
Absolute error	34.15*	47.09*	29.11*
% of Positive errors	86.53♀	77.78♀	55.09♀
Number of observations	16,262	17,784	1,428
At the money			
Error	33.19*	54.72*	48.22*
Absolute error	33.66*	55.16*	48.80*
% of Positive errors	98.36♀	97.06♀	95.99♀
Number of observations	9,892	11,094	549
In the money			
Error	4.46*	6.65*	4.80*
Absolute error	4.46*	6.65*	4.80*
% of Positive errors	99.85♀	98.95♀	94.93♀
Number of observations	1,305	1,414	276
Out of money			
Error	-33.80*	-45.98*	-174.52*
Absolute error	138.09*	129.28*	203.85*
% of Positive errors	60.01♀	32.44♀	7.16♀
Number of observations	5,065	5,276	617

* significantly different from 0.0% at 1 percent.
♦ significantly different from 0.0% at 5 percent.
♀ significantly different from 50.0% at 1 percent.
♣ significantly different from 50.0% at 5 percent.

Table 25

Pricing Errors of German Mark Put Options

31 to 90 days to Expiration

	Implied Volatility Model	Stochastic Interest Model	Stochastic Volatility Model
All options			
Error	- 1.67*	34.16*	11.87*
Absolute error	18.89*	51.84*	48.12*
% of Positive errors	56.47♀	81.49♀	62.43♀
Number of observations	40,151	40,268	4,580
At the money			
Error	- 0.51*	47.37*	53.53*
Absolute error	15.52*	47.71*	52.53*
% of Positive errors	56.08♀	97.37♀	100.00♀
Number of observations	17,312	17,476	1,318
In the money			
Error	- 3.49*	9.33*	9.83*
Absolute error	6.17*	9.81*	9.83*
% of Positive errors	41.34♀	91.90	99.86♀
Number of observations	2,228	2,308	731
Out of money			
Error	- 2.88*	23.91*	-80.15*
Absolute error	36.50*	95.40*	179.56*
% of Positive errors	58.43♀	66.82♀	32.10♀
Number of observations	20,611	20,484	2,531

* significantly different from 0.0% at 1 percent.
♦ significantly different from 0.0% at 5 percent.
♀ significantly different from 50.0% at 1 percent.
♣ significantly different from 50.0% at 5 percent.

Table 26

Pricing Errors of German Mark Put Options

More than 90 days to Expiration

	Implied Volatility Model	Stochastic Interest Model	Stochastic Volatility Model
All options			
Error	- 0.43*	21.91*	32.82*
Absolute error	10.65*	41.94*	54.36*
% of Positive errors	61.54♀	76.57♀	83.02♀
Number of observations	6,749	6,881	2,361
At the money			
Error	- 3.25*	23.97*	44.86*
Absolute error	7.25*	35.43*	44.86*
% of Positive errors	44.53♀	82.28♀	100.00♀
Number of observations	2,333	2,381	716
In the money			
Error	- 5.21*	5.47*	13.87*
Absolute error	5.68*	11.97*	13.88*
% of Positive errors	12.80♀	76.53♀	99.49♀
Number of observations	289	294	196
Out of money			
Error	5.10*	25.14*	31.38*
Absolute error	17.01*	61.60*	99.99*
% of Positive errors	74.56♀	73.35♀	72.39♀
Number of observations	4,127	4,206	1,449

* significantly different from 0.0% at 1 percent.
♦ significantly different from 0.0% at 5 percent.
♀ significantly different from 50.0% at 1 percent.
♣ significantly different from 50.0% at 5 percent.

Table 27

Pricing Errors of Japanese Yen Put Options - All Maturities

	Implied Volatility Model	Stochastic Interest Model	Stochastic Volatility Model
<u>All options</u>			
Error	3.14*	9.46*	19.14*
Absolute error	20.81*	41.57*	47.94*
% of Positive errors	64.81♀	68.82♀	66.84♀
Number of observations	71,252	73,802	8,558
<u>At the money</u>			
Error	6.16*	20.77*	48.06*
Absolute error	19.21*	39.94*	48.31*
% of Positive errors	67.41♀	80.60♀	97.92♀
Number of observations	34,037	35,889	2,938
<u>In the money</u>			
Error	- 1.74*	- 1.19*	8.01*
Absolute error	5.15*	8.61*	8.06*
% of Positive errors	56.21♀	71.54♀	96.65♀
Number of observations	4,196	4,340	990
<u>Out of money</u>			
Error	0.27*	- 8.51*	-22.80*
Absolute error	38.77*	75.74*	131.24*
% of Positive errors	63.23♀	55.74♀	41.00♀
Number of observations	33,012	24,190	4,630

* significantly different from 0.0% at 1 percent.
♦ significantly different from 0.0% at 5 percent.
♀ significantly different from 50.0% at 1 percent.
♣ significantly different from 50.0% at 5 percent.

Table 28

Pricing Errors of Japanese Yen Put Options

7 to 30 days to Expiration

	Implied Volatility Model	Stochastic Interest Model	Stochastic Volatility Model
All options			
Error	20.28*	28.01*	10.31*
Absolute error	38.03*	46.15*	30.40*
% of Positive errors	86.05♀	79.04♀	53.61♣
Number of observations	16,570	18,467	1,549
At the money			
Error	34.89*	47.39*	46.46*
Absolute error	35.52*	48.47*	47.84*
% of Positive errors	98.33♀	95.07♀	92.47♀
Number of observations	10,393	11,834	651
In the money			
Error	3.29*	4.01*	3.50*
Absolute error	3.30*	4.01*	3.50*
% of Positive errors	99.33♀	98.41♀	89.47♀
Number of observations	898	994	221
Out of money			
Error	-29.01*	-39.02*	-176.78*
Absolute error	133.75*	134.19*	201.35*
% of Positive errors	59.67♀	41.87	8.01♀
Number of observations	5,283	5,589	4,917

* significantly different from 0.0% at 1 percent.
♦ significantly different from 0.0% at 5 percent.
♀ significantly different from 50.0% at 1 percent.
♣ significantly different from 50.0% at 5 percent.

Table 29

Pricing Errors of Japanese Yen Put Options

31 to 90 days to Expiration

	Implied Volatility Model	Stochastic Interest Model	Stochastic Volatility Model
<u>All options</u>			
Error	- 0.36*	8.08*	15.50*
Absolute error	18.27*	38.66*	51.75*
% of Positive errors	57.89♀	67.28♀	63.88♀
Number of observations	49,250	49,796	4,917
<u>At the money</u>			
Error	- 0.46*	15.98*	52.15*
Absolute error	16.27*	36.44*	52.15*
% of Positive errors	54.33♀	75.33♀	100.00♀
Number of observations	21,594	21,938	1,608
<u>In the money</u>			
Error	- 2.78*	- 1.27*	9.01*
Absolute error	5.67*	8.79*	9.01*
% of Positive errors	47.42♣	66.49♀	99.83♀
Number of observations	3,037	3,059	595
<u>Out of money</u>			
Error	1.89*	- 1.53*	-62.23*
Absolute error	33.24*	68.71*	157.79*
% of Positive errors	62.30♀	60.26♀	34.63♀
Number of observations	24,619	24,799	2,714

* significantly different from 0.0% at 1 percent.
♦ significantly different from 0.0% at 5 percent.
♀ significantly different from 50.0% at 1 percent.
♣ significantly different from 50.0% at 5 percent.

Table 30

Pricing Errors of Japanese Yen Put Options

More than 90 days to Expiration

	Implied Volatility Model	Stochastic Interest Model	Stochastic Volatility Model
<u>All options</u>			
Error	0.10*	-13.32*	30.32*
Absolute error	10.09*	53.78*	49.07*
% of Positive errors	62.75♀	48.72♀	84.23♀
Number of observations	5,425	5,589	2,092
<u>At the money</u>			
Error	- 2.87*	- 3.07*	41.24*
Absolute error	7.82*	47.01*	41.46*
% of Positive errors	48.39	54.41♀	98.23♀
Number of observations	2,050	2,117	679
<u>In the money</u>			
Error	- 4.65*	-14.28*	9.80*
Absolute error	5.03*	19.19*	10.04*
% of Positive errors	1.09♀	28.94	95.98♀
Number of observations	265	273	174
<u>Out of money</u>			
Error	6.94	-28.57*	32.27*
Absolute error	18.46*	80.94*	94.29*
% of Positive errors	76.62♀	46.64♀	74.90♀
Number of observations	3,110	3,199	1,239

* significantly different from 0.0% at 1 percent.
♦ significantly different from 0.0% at 5 percent.
♀ significantly different from 50.0% at 1 percent.
♣ significantly different from 50.0% at 5 percent.

Table 31

Pricing Errors of Swiss Franc Put Options - All Maturities

	Implied Volatility Model	Stochastic Interest Model	Stochastic Volatility Model
All options			
Error	5.58*	25.46*	19.63*
Absolute error	21.13*	41.64*	44.43*
% of Positive errors	68.39♀	81.11♀	72.75♀
Number of observations	50,061	51,785	8,095
At the money			
Error	8.86*	34.72*	43.19*
Absolute error	19.46*	38.89*	43.41*
% of Positive errors	72.50♀	91.83♀	98.53♀
Number of observations	24,127	25,349	2,727
In the money			
Error	- 0.78*	5.47*	7.69*
Absolute error	5.14*	6.78*	7.74*
% of Positive errors	60.60♀	87.27♀	98.50♀
Number of observations	2,731	2,908	991
Out of money			
Error	2.73*	19.36*	- 4.49*
Absolute error	37.52*	77.04*	118.13*
% of Positive errors	65.04♀	68.81♀	50.94
Number of observations	23,203	23,528	4,377

* significantly different from 0.0% at 1 percent.
♦ significantly different from 0.0% at 5 percent.
♀ significantly different from 50.0% at 1 percent.
♣ significantly different from 50.0% at 5 percent.

Table 32

Pricing Errors of Swiss Franc Put Options

7 to 30 days to Expiration

	Implied Volatility Model	Stochastic Interest Model	Stochastic Volatility Model
All options			
Error	23.29*	31.69*	11.30*
Absolute error	35.94*	44.65*	30.85*
% of Positive errors	88.66♀	82.56♀	58.15♀
Number of observations	13,827	15,239	1,589
At the money			
Error	34.39*	48.42*	49.75*
Absolute error	34.67*	48.81*	50.16*
% of Positive errors	98.97♀	97.81♀	96.42♀
Number of observations	8,672	9,722	642
In the money			
Error	4.58*	5.71*	4.28*
Absolute error	4.62*	5.72*	4.28*
% of Positive errors	99.75♀	99.03♀	97.44♀
Number of observations	813	923	266
Out of money			
Error	- 5.90*	-17.93*	-167.78*
Absolute error	113.42*	112.19*	200.01*
% of Positive errors	66.00♀	47.20♀	8.09♀
Number of observations	4,344	4,594	681

* significantly different from 0.0% at 1 percent.
♦ significantly different from 0.0% at 5 percent.
♀ significantly different from 50.0% at 1 percent.
♣ significantly different from 50.0% at 5 percent.

Table 33

Pricing Errors of Swiss Franc Put Options

31 to 90 days to Expiration

	Implied Volatility Model	Stochastic Interest Model	Stochastic Volatility Model
<u>All options</u>			
Error	0.98*	27.88*	16.58*
Absolute error	18.28*	41.13*	48.32*
% of Positive errors	60.33♀	83.57♀	71.48♀
Number of observations	31,976	32,172	4,646
<u>At the money</u>			
Error	1.27*	34.09*	46.82*
Absolute error	15.52*	36.01*	46.82*
% of Positive errors	59.22♀	91.00♀	100.00♀
Number of observations	14,042	14,174	1,495
<u>In the money</u>			
Error	- 2.44*	5.93*	8.76*
Absolute error	5.34*	6.90*	8.76*
% of Positive errors	45.58♣	83.71♀	99.83♀
Number of observations	1,771	1,835	605
<u>Out of money</u>			
Error	2.81*	30.79*	-33.35*
Absolute error	33.66*	78.12*	146.91*
% of Positive errors	62.80♀	77.04♀	48.00♣
Number of observations	16,163	16,163	2,546

* significantly different from 0.0% at 1 percent.
♦ significantly different from 0.0% at 5 percent.
♀ significantly different from 50.0% at 1 percent.
♣ significantly different from 50.0% at 5 percent.

Table 34

Pricing Errors of Swiss Franc Put Options

More than 90 days to Expiration

	Implied Volatility Model	Stochastic Interest Model	Stochastic Volatility Model
All options			
Error	0.19*	- 2.02*	31.57*
Absolute error	10.50*	38.48*	44.59*
% of Positive errors	63.13♀	57.93♀	88.66♀
Number of observations	4,258	4,374	1,860
At the money			
Error	- 4.11*	2.59*	33.02*
Absolute error	7.64*	30.76*	33.56*
% of Positive errors	41.97♀	59.88♀	97.12♀
Number of observations	1,413	1,453	590
In the money			
Error	- 4.82*	- 0.40*	9.32*
Absolute error	5.24*	10.12*	9.71*
% of Positive errors	12.93♀	58.00♣	94.17♀
Number of observations	147	150	120
Out of money			
Error	6.66*	- 5.14*	41.76*
Absolute error	15.36*	55.32*	77.89*
% of Positive errors	76.95♀	56.91♀	83.74♀
Number of observations	2,698	2,771	1,150

* significantly different from 0.0% at 1 percent.
♦ significantly different from 0.0% at 5 percent.
♀ significantly different from 50.0% at 1 percent.
♣ significantly different from 50.0% at 5 percent.

References

Barone-Adesi, G., and R.E. Whaley. "Efficient Analytic Approximation of American Option Values." *Journal of Finance* 42, (June 1987), 301-320.

Biger, N., and J. Hull. "The Valuation of Currency Options." *Financial Management* 12, (Spring 1983), 24-28.

Black, F., "Fact and Fantasy in the Use of Options." *Financial Analysts Journal* 31, (July-August 1975), 36-72.

Black, F., "The Pricing of Commodity Contracts." *Journal of Financial Economics*, 3 (Jan./March 1976), 167-179.

Black, F., and M. Scholes. "The Valuation of Option Contracts and a Test of Market Efficiency." *Journal of Finance*, 27 (May 1972), 399-418.

Black, F., and M. Scholes. "The Pricing of Options and Corporate Liabilities." *Journal of Political Economy*, 81 (May 1973), 637-659.

Bodurtha, J., and G. Courtadon. "Efficiency Tests of the Foreign Currency Options Market." *Journal of Finance* 41 (March 1986), 151-162.

Bodurtha, J., and G. Courtadon. "The Pricing of Foreign Currency Options." *Monograph Series in Finance and Economics*, Salomon Brothers Center for the Study of Financial Institutions, New York, 1988.

Boothe, P., and D. Glassman. "The Statistical Distribution of Exchange Rates." *Journal of International Economics*, 10 (Dec. 1987), 297-319.

Brenner, M., G. Courtadon, and Subrahmanyam. " Options on the Spot and Options on Futures." *Journal of Finance*, 40 (Dec. 1985), 1303-1317.

Calderon-Rossell, J.R., and M. Ben-Horim. "The Behavior of Foreign Exchange Rates." *Journal of International Business Studies*, 13 (Fall 1982), 99-111.

Chesney, M., and L. Scott. "Pricing European Currency Options: A Comparison of Modified Black-Scholes Model and a Random Variance Model." *Journal of Financial and Quantitative Analysis*, 24 (Sep. 1989), 267-284.

Chang, C., and J. Chang, "Forward and Futures Prices: Evidence from the Foreign Exchange Market." *Journal of Finance*, 45 (1990), 1333-1336.

Cornell, B., and M. Reinganum. "Forward Prices and Futures Prices: Evidence from the Foreign Exchange Market." *Journal of Finance*, 36 (1981), 321-346.

Cox, D.R., and H.D. Miller. *The Theory of Stochastic Processess*. John Wiley & Sons, New York, 1965.

Cox, J.C., and S.A. Ross. "The Valuation of Options for Alternative Stochastic Processes." *Journal of Financial Economics* 3 (1976), 145-166.

Cox, J.C., J. Ingersoll, and S.A. Ross. "The Relationship Between Forward Prices and Futures Price." *Journal of Financial Economics*, 9 (1981), 321-346.

Cox, J.C., J. Ingersoll, and S.A. Ross. "An Intertemporal General Equilibrium Model of Asset Prices." *Econometrica*, 53 (March 1985), 385-408.

Cox, J.C., S.A. Ross, and M. Rubinstein. "Option Pricing: A Simplified Approach." *Journal of Financial Economics* 7 (1979), 229-263.

Cumby, R.E. "Is it Risk? Explaining Deviations from Uncovered Interest Rate Parity." *Journal of Monetary Economics*, 22, (1988), 279-300.

Cumby, R.E., and M. Obstfeld. "A Note on Exchange-Rate Expectations and Nominal Interest Rate differentials: A Test of Fisher Hypothesis." *Journal of Finance*, 36, (June 1981), 697-704.

Feiger, G., and B. Jacquillat. "Currency Option Bonds, Puts and Calls on Spot Exchange and the Hedging of Contingent Foreign Earnings." *Journal of Finance*, 34 (Dec. 1979), 1129-1139.

Galai, D. "A Survey of Empirical Tests of Option Pricing Models." In *Option Pricing*, ed., M. Brenner. D.C. Heath, Lexington, Mass., 1983, 45-80.

Garman, M., and S. Kohlhagen. "Foreign Currency Option Values." *Journal of International Money and Finance* 2 (December 1983), 231-238.

Geske, R. "A Note on An Analytic Formula for Unprotected American Call Options on Stocks with Known Dividends." *Journal of Financial Economics* 7 (December 1979), 375-380.

Grabbe, J. O. "The Pricing of Call and Put Options on Foreign Exchange." *Journal of International Money and Finance* 2 (December 1983), 239-253.

Grossman, S. "An Analysis of the Implications for Stock and Futures Price Volatility of Program Trading and Dynamic Hedging Strategies." *Journal of Business*, 61, (1988), 275-298.

Hansen, L.P., and R.J. Hodrick. "Forward Exchange Rates as Optimal Predictors of Future Spot Rates: An Econometric Analysis." *Journal of Political Economy , 88 (Oct. 1980), 829-853.*

Hseih, D.A. "A Model of Foreign Currency Options with Random Interest Rates". Working Paper, University of Chicago, March 1988.

Hilliard, J., J. Madura, and A. Tucker. "Currency Option Pricing with Stochastic Domestic and Foreign Interest Rates." *Journal of Financial and Quantitative Analysis*, 26 (June 1991), 139-151.

Hodrick, R.J., and S. Srivastava. "An Investigation of Risk and Return in Forward Foreign Exchange." *Journal of International Money and Finance*, 3, (April 1984), 5-29.

Huang, C., and R.H. Litzenberger. *Foundations for Financial Economics*. North-Holland, New York, 1988.

Hull, J., and A. White. "The Pricing of Options on Assets with Stochastic Volatilities." *Journal of Finance*, 2 (June 1987), 281-300.

Jorion, P., and N.M. Stoughton. "An Empirical Investigation of the Early Exercise Premium of Foreign Currency Options." *Journal of Futures Markets* 9 (1989), 365-375.

Karlin, S., and H.M. Taylor. *A Second Course in Stochastic Processess*. Academic Press, Inc., New Yrok, 1981.

MacBeth, J., and L. Merville. "An Empirical Examination of the Black-Scholes Call Option Pricing Model." *Journal of Finance*, 34 (December 1979): 1173-1186.

Margrabe, W. "The Value of an Option to Exchange One Asset for Another." *Journal of Finance*, 33 (March 1978), 177-186.

Mark, N.C. "Time-Varying Betas and Risk Premia in the Pricing of Forward Foreign Exchange Contracts." *Journal of Financial Economics*, 22 (1988), 335-354.

Melino, A., and S.M. Turnbull. "Pricing of Foreign Currency Options with Stochastic Volatility." *Journal of Econometrics* 45 (1990), 239-265.

Merton, R. "Theory of Rational Option Pricing." *Bell Journal of Economics*, 4 (Spring 1973), 141-183.

Rabinovitch, R. "Pricing Stock and Bond Options when the Default-Free Rate is Stochastic." *Journal of Financial and Quantitative Analysis*, 23 (Dec. 1989), 447-457.

Ramaswamy, K., and S.M.Sundaresan. "The Valuation of Options on Futures Contracts." *Journal of Finance*, 40 (Dec. 1985),1319-1340.

Ritchken, P. *Options - Theory, Strategy,and Applications*, Scott, Foresman and Company, Glenview, Illinois, 1987.

Rogalski, R.J., and J.D. Vinso, "Empirical Properties of Foreign Exchange Rates." *Journal of International Business Studies*, (Fall 1968), 69-79.

Roll, R. " An Analytic Valuation Formula for Unprotected American Call Options on Stocks with Known Dividends." *Journal of Financial Economics* 5 (November 1977), 251-258.

Rubinstein, M. "Nonparametric Tests of Alternative Option Pricing Models Using All Reported Trades and Quotes on the 30 Most Active CBOE Option Classes From August 23, 1976 Through August 31, 1978." *Journal of Finance*, 40 (June 1985), 455-480.

Samuelson, P.A. "Rational Theory of Warrant Pricing." *Industrial Management Review* 10 (Winter 1969), 17-46.

Scott, L. O. "Option Pricing when the Variance Changes Randomly: Theory, Estimation, and an Application." *Journal of Financial and Quantitative Analysis*, 22 (Dec. 1987), 419-438.

Shastri, K., and K.Tandon. "Valuation of Foreign Currency Options: Some Empirical Tests." *Journal of Financial and Quantitative Analysis*, 21 (June 1986), 145-160.

Sprenkle, C. "Warrant Prices as Indications of Expectations." *Yale Econ. Essays 1*, (1961), 179-232.

Thorp, E.O., and S.T. Kassouf. *Beat the Market*. Random House, New York, 1967.

Tucker, A.L. "Empirical Tests of the Efficiency of Currency Option Market." *Journal of Financial Research* 7 (Winter 1986), 275-285.

Tucker, A.L. *Financial Futures, Options, & Swaps*. West Publishing Company, St. Paul, 1991.

Tucker, A.L., and E. Scott. "A Study of Diffusion Processes for Foreign Exchange Rates." *Journal of International Money and Finance*, 6 (1987), 465-478.

Westerfield, J. "An Examination of Foreign Exchange Risk Under Fixed and Floating Regimes." *Journal of International Economics*, 7 (May 1977), 181-200.

Whaley, R. "On the Valuation of American Call Options on Stocks with Known Dividends." *Journal of Financial Economics* 9 (June 1981), 207-212.

Wiggins, J. B. "Option Values under Stochastic Volatility: Theory and Empirical Estimates." *Journal of Financial Economics*, 19 (Dec. 1987), 351-372.