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# Taxes, discount points, and the intrinsic value of the prepayment option

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# TAXES, DISCOUNT POINTS, AND THE INTRINSIC VALUE OF THE PREPAYMENT OPTION

by

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This dissertation is dedicated to my father Allan Unger Stone.
He taught me how to think.

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# CHAPTER I

## Introduction

This dissertation is an addition to the current body of research on mortgage debt. The thesis of this dissertation is that the individual mortgagor's tax bracket, expectations about his future tenure in the property being mortgaged, and the future movements in the cost of mortgage capital, can explain the array of thirty year conventional mortgage terms that exist in the market place.

The specific mortgage terms examined in this dissertation are the contractual interest rate and the initial number of discount points that were paid by the mortgagor to the mortgagee when the mortgage was issued.

Conventional mortgage debt is the most common type of mortgage issued in the U.S.A. The interest rate on conventional mortgage debt is fixed for the life of the loan. It is characterized by the equal monthly payments which are determined such that the original principal is amortized over the stated maturity of the mortgage. Conventional mortgage debt gives the mortgagor the right to call the market value of the mortgage from the owner of the mortgage, at any time prior to the maturity of the mortgage, for a price equal to the current mortgage balance plus any transaction costs. Conventional mortgage debt typically gives the owner of the mortgage the right to call the outstanding mortgage balance from the mortgagor when the property collateralizing the mortgage is sold.

A mortgagor is managing three assets (human capital,real estate capital, and a prepayment option). Due to economic constraints it is often impossible to maximize the value of all three assets. If a job opportunity arises in a different region, a mortgagor might sell his home at a loss and be forced to exercise a prepayment option which has no intrinsic value. If we observe a mortgagor making this choice we would have to assume the benefits from the new job offset his capital loss on the real estate and the call option (see Hall, and Chari & Jagannathan[1])

Chapter (II) of the dissertation illustrates that the amount by which a mortgagor can increase his wealth by exercising his prepayment option is a function of the mortgagor's tax bracket. The major implication of this argument is that since structural changes in the tax system will affect the intrinsic value of all currently outstanding prepayment options, the value of a mortgage or portfolio of mortgages should reflect the possibility of income tax rates

[1]"Adverse Selection in a Model of Real Estate Lending" Journal of Finance, Vol. XLIV, No.2, June 1989.

being adjusted and or the tax treatment of mortgage interest being altered. The effect of taxes on the intrinsic value of the prepayment option is compared across three different mortgage structures, at three points in time. Each mortgage structure is conventional in its contractual terms. The three mortgage structures we analyze differ with respect to the initial discount/contract rate combination. The larger discounts are associated with lower contract rates. The effective cost of mortgage capital is held constant across the three structures for a common holding period.

Chapter (II) analyzes two variables that measure the degree of prepayment risk to which a mortgagee is exposed. We measure standard deviation of the intrinsic value of the option relative to the expected intrinsic value of the option (coefficient of variation).

This ratio measures the percentage by which the actual intrinsic value of the option is expected to deviate from the expected intrinsic value. The coefficient of variation differs across structures. Although the expected value of the intrinsic value of the prepayment option decreases as the number of discount points paid increases, the coefficient of variation increases. This implies that the value of principal a mortgagee can expect to be refinanced at any time is less for those mortgages which were originally issued at a discount, but the actual value is more likely to deviate from this expected value than it would for mortgages issued at par. The second measurement of risk is the probability that the mortgagor can increase his wealth by exercising the prepayment option (the probability that the option is in the money). The results of the simulations that were conducted conclude, that the probability of an option being in the money is a function of the original discount at which the mortgage was issued.

The mortgage market offers an array of contract rate/discount point combinations for conventional mortgages at a single point in time. A mortgagor can choose between issuing a mortgage at a discount (paying discount points) or issuing the mortgage at par (paying no discount points.) Often a single lender will offer a mortgagor a choice of at least three different combinations of points and contract rates. There is a trade off between discount points and contract rate. Higher points are associated with a lower contract rate. The formula for this trade off is not consistent across lenders.

Chapter (III) models a mortgagor's choice from a menu of original discount point/contract rate combinations which are offered by a lender. The mortgagor makes his choice with respect to the criteria of maximizing his wealth. The results of our analysis are derived by applying the expected monetary algorithm to the mortgagor's choice problem. Our results show that for a given set of expectations regarding the future cost of mortgage capital, a mortgagor's choice of a discount point/contract rate combination depends on his income tax bracket. Our analysis also implies that within a single tax bracket the choice of the contract rate/discount point combination chosen by a mortgagor depends on his expectations about when and by what magnitude interest rates will change. Taxes and expectations are sufficient to explain the supply of mortgage contracts that have high, medium, and low original discount points.

The integration of the mortgage market with the capital and money markets of the world, link the movements in mortgage rates with the movements in interest rates on other fixed income securities. Chapter (IV) tests the hypothesis that the effective cost of conventional mortgage capital for a given holding period differs significantly across regions. We employ the method of "dummy" variables to test for a state effect on the effective cost of mortgage capital.

We reduce various discount point/contract rate combinations, which are available at a point in time within a specific geographic area, to an average par equivalent effective rate/par equivalent holding period combination. The innovation involved in this reduction is that we do not specify the holding period. We estimate the holding period and the effective rate simultaneously. The diversity of mortgage terms (contract rate /discount point combinations) offered in a single geographic area necessitates reducing the terms in the area to a standard measurement in order to make inter-regional comparisons. We use the estimated average par equivalent effective rate/par equivalent holding period for eight states to construct a term structure of mortgage interest rates.

We find evidence that the mortgage yield curve for the country is not unique. One possible explanation of multiple term structures in the mortgage market is that mortgage yields reflect the idiosyncratic risks of a specific region. An efficient secondary mortgage market should reduce idiosyncratic regional risks via inter-regional diversification.

As of the fourth quarter of 1988 there was 2 billion one hundred and fifteen million dollars of mortgage debt secured by 1 to 4 family residential real estate[2].

[2]June 1989 Federal Reserve Bulletin.

CHAPTER II

# THE VALUE OF THE MORTGAGOR'S PREPAYMENT OPTION ACROSS DIFFERENT TAX BRACKETS AND DISCOUNT POINT/CONTRACT RATE COMBINATIONS

This paper illustrates that the value of a mortgagor's prepayment option at expiration is a decreasing function of the mortgagor's tax bracket and the discount at which the mortgage was issued. A mortgagee can obtain identical yields on fixed rate mortgages over a common holding period via different contract rate/discount point combinations. Although the yield of two combinations for a given holding period may be identical, the expected intrinsic value of the prepayment option, the standard deviation of the intrinsic value of the prepayment option, and the probability that the option is in the money at expiration, will differ across equivalent yield contract rate/discount point combinations. We simulate the intrinsic value of the mortgagor's prepayment option across three discount point/contract rate combinations and five tax brackets at three points in time. Our results indicate that prepayment data should vary across the tax bracket of the mortgagor, and the number of discount points that were paid when the mortgage was issued.

## I. Introduction

Models of a mortgagor's behavior with respect to refinancing a current mortgage liability are presented by Green & Shoven [7], (G&S), Navratil [15], Hall [8], Curley and Guttentag [5], (C&G), Jacob,Lord,& Tilley[12], (J&L&T), and Carron & Hogan [3],(C&H). All the authors model the likelihood of a mortgagor exercising his prepayment option at any point in time as a function of the difference between the mortgage contract rate and the current cost of mortgage capital. Jacob, Lord and Tilley model the mortgagor's refinancing decision as depending upon the current difference between the contract rate of interest and market rate of interest as well as the path of past differences between the contract and market cost of mortgage capital. Askin and Lowell [1], (A&L), summarize the reasons a mortgagor will exercise his prepayment option. They say "prepayments result from such basic situations as, 1) changed housing requirements, 2) favorable refinancing opportunities, 3) job relocation and 4) loss of income, natural disaster, death, divorce and other involuntary circumstances."

Boyle [2], discusses the valuation of Canadian mortgage backed securities. It can be inferred from his analysis that a major cause of the difference between the incentive for Canadian mortgagors to refinance their mortgages and American mortgagors to refinance their mortgages stems from the fact that interest paid to finance a mortgage in the U.S. is tax deductible while mortgage interest is not tax deductible in Canada[1]. Our paper

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[1]Qualified residence interest.- (A) In general,-- The term "qualified residence interest" means any interest which is paid or accrued during the taxable year on-

(i) acquisition indebtedness with respect to any qualified residence of the taxpayer, or

(ii) home equity indebtedness with respect to any qualified residence of the taxpayer.

For purposes of the preceding sentence, the determination of whether any property is a qualified residence of the taxpayer shall be made as of the time the interest is accrued.

illustrates how the tax treatment of interest affects the intrinsic value of the mortgagor's prepayment option. The simulations we run for the zero percent tax bracket can be thought of as the case when interest on mortgage debt is not tax deductible.

According to Chinloy [4], mortgagees charge mortgagors an interest rate premia as compensation for the prepayment risk associated with conventional mortgage assets. When mortgagors issue a conventional mortgage they are financing the purchase of two assets, real estate and a "complex financial package containing a current call option and a compound call option"[2]. The options Chinloy refers to are the option to call the mortgage liability from the mortgagee at any time prior to the maturity of the mortgage. If the mortgagor forgoes the opportunity to exercise his option at

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(B) Acquisition indebtedness.-

(i) In general.- The term "acquisition indebtedness" means any indebtedness which-

(I) is incurred in acquiring, constructing, or substantially improving any qualified residence of the taxpayer, and

(II) is secured by such residence.

Such term also includes any indebtedness secured by such residence resulting from the refinancing of indebtedness meeting the requirements of the preceding sentence (or this sentence); but only to the extent the amount of the indebtedness resulting from such refinancing does not exceed the amount of the refinanced indebtedness.

(ii) $1,000,000 limitation.- The aggregate amount treated as acquisition indebtedness for any period shall not exceed $1,000,000 ($5000,000 in the case of a married individual filing a separate return).

[2]A compound option is, "an option on an option" (see Cox and Rubinstein)

time (t) he has exercised his option to call another prepayment option from the lender, which in turn is an option to prepay the current mortgage or an option on a future option. Chinloy argues that because interest on mortgage debt is tax deductible, the federal government is subsidizing the purchase of options on mortgage debt.

An implication of our analysis is that the tax structure affects the value of the prepayment options issued by the lender. As a mortgagor's tax bracket increases, the opportunity cost of not exercising a prepayment option decreases. This would imply that a mortgagor in a relatively high tax bracket would value a prepayment option less than a mortgagor in a relatively low tax bracket. The price of the prepayment option is reflected in the contract interest rate and the number of discount points that must be paid. The tax treatment of points and interest lowers the cost of the prepayment option. The decrease in the cost of the option is greatest for those mortgagors in the highest tax brackets[3].

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[3]Points are currently deductible if paid in respect of any indebtedness incurred in connection with the purchase or improvement of, and secured by, the principal residence of the taxpayer to the extent that,under regulations prescribed by the Secretary,such payment of points is an established business practice in the area in which such indebtedness is incurred, and the amount of such payment does not exceed the amount generally charged in such area. Code Sec. 462(g)(2)

"Although IRS had not issued regulations in connection with the above Section, it announced on May 13, 1986, that points paid to refinance a mortgage would have to be amortized over the life of the loan. On September 22,1986, Hall says that "What the borrower gains by exercising the prepayment option is only and exactly what the lender loses". Our results point out that taxes drive a wedge between the value called from the lender and the value called by the mortgagor.

This paper focuses on the relationship between the expected intrinsic value of a mortgagor's prepayment option and, 1) the mortgagor's tax bracket and, 2) the number of discount points he paid when writing the mortgage. The implications of this relationship are that, 1) for a given effective mortgage interest rate the original contract rate/discount point combination of the mortgage affects the value that a mortgagor can call away from the mortgagee. 2) changes in the current income tax laws and or income tax structure will change the intrinsic value of outstanding prepayment options. and 3) The mortgagee can reduce his exposure to reinvestment risk by purchasing mortgages at a discount.

We discuss two measures of reinvestment risk. The first measurement of reinvestment risk is the standard deviation of the value that a mortgagee expects to be refinanced at

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three Congressmen with the support of approximately 200 other members of the House, urged Treasury Secretary James A. Baker to withdraw the announcement and pending ruling. Legislation has been proposed to clarify what the proponents insist has been the congressional intent for years, i.e., points whether paid for the purchase of a home, a home improvement or in a refinancing, should be treated the same, i.e., deductible in the year paid." [CPA Journal, May 1987] time (t). The second measure of risk is the probability that the intrinsic value of the prepayment option at time (t) is in the money. We assume that mortgagor's only exercise their prepayment option when it is in the money. The strength of our analysis relies on the empirical fact that the depth to which the option is in the money is positively correlated with the likelihood of a mortgagor refinancing the mortgage, see (G&S), Hendershott, Hu, and Villani [10], (H&H&V), and Schwartz and Torous [17], (S&T).

Part II of the paper describes the economic model. Part III of the paper is an interpretation of the simulated data. Part IV is the conclusion.

## II. Model

We assume that the cost of mortgage capital follows a lognormal probability distribution*. We assume that interest rates can change only at the beginning of the period while all interest and principal payments are scheduled to be made at the end of the period. The

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* A lognormal assumption implies that the percentage change in interest rates is a random variable that is normally distributed around a constant mean. Using the lognormal distribution to generate the cost of mortgage capital insures that the expected absolute change in the interest rate will be higher when the current interest rate is higher and the distribution guarantees that rates will never be negative. Equal relative changes in the cost of mortgage capital about the mean are equally likely.

Carron and Hogan say that for simulation "Lognormality of interest rates has become an industry standard".

prepayment options are assumed to be European options with expiration dates at the end of each year[5]. Consistent with United States Federal tax law interest is 100% tax deductible (see footnote #1). Discount points associated with the original mortgage are tax deductible. Points which are paid when the original mortgage is refinanced, must be amortized over the life of the new mortgage. The percentage of the points amortized each year are tax deductible in that year. In this paper we assume that the points which must be paid when the mortgage is refinanced are zero. We make this assumption in order to focus on the effect the original mortgage structure has on the expected intrinsic value of the option and the associated risk.

The call option is evaluated at three expiration dates, across five tax brackets and, three different mortgage contracts. Prepayment options expire at the beginning of each period, i.e. one year before the next payment is due.

The three mortgage contracts we examine are all thirty year conventional mortgages. The before tax effective yield to the bank is 10% if the mortgages are held until maturity. The 10% yield is obtained with three different contract rate/discount point combinations[6].

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[5]A European option can only be exercised at its expiration date.

[6] The formula we use to find the contract rate for a given number of discount points, a given effective rate and a stated maturity is:

The thirty year conventional mortgages considered in this paper are structured so that there are thirty equal payments made at the end of each year.

The three mortgage contracts analyzed in the paper are:

Structure (a): contract rate=10%, discount points=0
Structure (b): contract rate=9.8833%, discount points=1
Structure (c): contract rate=9.7666%, discount points=2

Each payment consists of interest and principal. At the end of thirty years the original principal will have been amortized. Typically conventional mortgages in the U.S. require monthly payments. The magnitude of our results will only be slightly altered if we use monthly compounding. The direction of our results will not be affected if we use monthly compounding instead of annual compounding.

Each mortgage contract gives the mortgagor the right to call the market value of his mortgage ($PV_t$) from the mortgagee at an exercise price equal to the outstanding principal of the mortgage ($B_t$) at time (t). In our analysis (t) is the expiration date of the prepayment option. The value of the prepayment option at time (t) is the $MAX[(PV_t-B_t),0]$. When the present value to time (t), of all future principal and interest payments exceeds the outstanding principal at time (t), the mortgagor can increase his wealth by exercising the option. If the value of the outstanding principal exceeds the present value of the mortgage

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$r_c = [r_e(1-p)-p/2n]$ where $r_c$=contract rate, $r_e$=effective rate, p=discount points and n=number of years over which the mortgage amortizes.

liability, the mortgagor can not increase his wealth by exercising the option.

The intrinsic value of the mortgagors' prepayment option at time (t) is:

$$OP_t = \mathrm{Max}[(PV_t - B_t/(1-p_t)), 0] \qquad (1)$$

where

$$PV_t = \frac{B_0}{1-p} \frac{r_c}{[(1+r_c)^n - 1]} \sum_{i}^{n} \frac{(1-T)\{(1+r_c)^n - (1+r_c)^{i-1}] + (1+r_c)^{i-1}}{[1+(1-T)r_e]^i}$$

$i=t+1,...,n$

$B_0$ = purchase price of the house assuming zero down payment

$p$ = initial discount points

$T$ = marginal tax rate

$r_c$ = contract rate

$n$ = maturity of the mortgage (in our paper $n=30$)

$B_t$=outstanding balance at time t

$r_e$ = effective rate

$r_e$ is generated by a lognormal probability distribution[7].

We simulated the rate at which the remaining interest and principal payments are discounted back to time (t)[8].

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[7]The density function of a lognormal distribution is:

$$f(x) = \frac{1}{x\sqrt{2\pi\sigma_1^2}} \exp\frac{-(\ln x - \mu_1)^2}{2\sigma_1^2} \quad \text{where } \mu_1 = \ln[\mu^2/(\sigma^2+\mu^2)^{1/2}], \quad \sigma_1 = \ln[(\sigma^2+\mu^2)/\mu^2]$$

[8] We used the software program @RISK produced by the Palisade Corporation, 2189 Elmira Rd. Newfield, N.Y. U.S.A The cost of mortgage capital was simulated by the sampling method of Latin Hypercube. Latin Hypercube

The discount rate we simulated is the mortgagor's before tax opportunity cost of mortgage capital*. We simulated the expected discount rate for time t=1,12,and 24. The mean of the lognormal distribution was 10% (the effective cost of mortgage capital over thirty years for all original mortgage contracts). The standard deviation for the distribution was 5%. We ran one hundred iterations for each tax bracket at time t=1,12 and 24 for mortgage structures a,b and c. By substituting the simulated value of the expected discount rate into $PV_t$ we were able to simulate the expected intrinsic value of the mortgagor's prepayment option at time (t=1,12,and 24), across five tax brackets and three mortgage contracts. The before tax discount rate is adjusted for taxes by multiplying it by one minus the mortgagor's marginal income tax rate. The outstanding principal at time (t) is not affected by the simulated data. $B_t$ is a proportion of $B_o/(1-p)$.

We estimate the probability that the intrinsic value of the option will be in the money at time t=1,12,and 24. For each of the one hundred iterations which are performed for a single simulation of the current cost of mortgage capital we

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sampling stratifies the cumulative distribution into a number of intervals equal to the number of simulations to be performed.

*The mortgagor's before tax opportunity cost of capital is the current market contract rate on thirty year mortgage capital for mortgages written at par. (zero discount points) assign the value 1 if the intrinsic value of the option evaluated at the outcome of the iteration is positive and we assign a value of zero to the iteration otherwise.

$I_i=1$ if $Max[(PV_t-B_t),0] > 0$

$I_j=0$ otherwise

(j) stands for the iteration number: j=1,2,3.....100

The probability of the intrinsic value of the option being in the money at time (t) is approximated[10] by $\Sigma I_j/100$.

## III. Results

The expected intrinsic values of the mortgagor's prepayment option for years (1,12&24) are presented in exhibit (I,II,and III). For each structure and tax bracket there are two entries. The first number is the expected intrinsic value of the option. The value in parenthesis is the coefficient of variation[11].

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[10]The Law of Large Numbers (Bernoulli's form)

[11]The coefficient of variation is defined as the standard deviation relative to the expected value. In our context the coefficient of variation is the standard deviation of the intrinsic value of the prepayment option relative to the option's expected intrinsic value.

Exhibit (I)

Expected intrinsic value of prepayment option at Time (t=1)

| | (a) | (b) | (c) |
|---|---|---|---|
| contract rate | 10% | 9.8833% | 9.7666% |
| discount points | 0 | 1 | 2 |
| TAX | | | |
| 0% | $40,464 | $39,672 | $38,127 |
| | (1.251) | (1.275) | (1.283) |
| 11% | $38,164 | $37,213 | $36,142 |
| | (1.246) | (1.266) | (1.279) |
| 28% | $33,683 | $32,941 | $31,909 |
| | (1.228) | (1.254) | (1.271) |
| 33% | $32,234 | $31,387 | $30,633 |
| | (1.223) | (1.244) | (1.266) |
| 50% | $26,535 | $25,774 | $25,047 |
| | (1.198) | (1.219) | (1.232) |
| Frequency | 59-60% | 58-59% | 57-58% |

Exhibit (II)

Expected intrinsic value of prepayment option at Time (t=12)

| | (a) | (b) | (c) |
|---|---|---|---|
| contract rate | 10% | 9.8833% | 9.7666% |
| discount points | 0 | 1 | 2 |
| Tax | | | |
| 0% | $24,934 | $24,268 | $23,630 |
| | (1.215) | (1.250) | (1.262) |
| 11% | $22,906 | $22,332 | $21,606 |
| | (1.201) | (1.232) | (1.240) |
| 28% | $19,863 | $19,042 | $18,513 |
| | (1.200) | (1.207) | (1.223) |
| 33% | $18,678 | $18,133 | $17,550 |
| | (1.186) | (1.206) | (1.225) |
| 50% | $14,846 | $14,391 | $14,003 |
| | (1.177) | (1.183) | (1.211) |
| Frequency | 59%-60% | 58%-59% | 57%-58% |

Exhibit (III)

Expected Intrinsic Value of Prepayment Option at Time (t=24)

| | (a) | (b) | (c) |
|---|---|---|---|
| contract rate | 10% | 9.8833% | 9.7666% |
| discount points | 0 | 1 | 2 |
| Tax | | | |
| 0% | $5,250 | $5,059 | $4,193 |
| | (1.165) | (1.178) | (1.405) |
| 11% | $4,751 | $4,474 | $4,469 |
| | (1.159) | (1.170) | (1.211) |
| 28% | $3,930 | $3,825 | $3,683 |
| | (1.156) | (1.183) | (1.203) |
| 33% | $3,678 | $3,562 | $3,425 |
| | (1.149) | (1.164) | (1.186) |
| 50% | $2,820 | $2,712 | $2,261 |
| | (1.142) | (1.160) | (1.180) |
| Frequency | 59%-60% | 58%-59% | 57-58% |

The results illustrate that: 1) the expected intrinsic value of the mortgagor's prepayment option within a single mortgage structure declines as the mortgagor's tax bracket increases, 2) for an equivalent effective rate over a common holding period the expected intrinsic value of the mortgagor's prepayment option decreases within a single tax bracket as the number of discount points increases, 3) The coefficient of variation increases within a tax bracket as the mortgage discount increases, 4) The coefficient of variation decreases within a given discount point/contract combination as the mortgagor's tax bracket increases, 5)The coefficient of variation decreases over time across all tax brackets and all mortgage structures, 6) The probability of the intrinsic value of the option being in the money (reported as "Frequency" in the exhibits) decreases as the original mortgage discount increases, and is not affected across tax brackets or time.

Our results imply that the opportunity cost of not exercising a prepayment option is greater for a mortgagor in the 11% tax bracket than for a mortgagor in the 28% tax bracket.

If the cost of mortgage capital falls by (x%) then the mortgagor in the 28% tax will be less likely to exercise his prepayment option than the mortgagor in the 11% tax bracket. If the mortgagee charges both mortgagors the same rate of interest, the mortgage purchased from the mortgagor in the 11% tax bracket will have a higher degree of reinvestment risk than the mortgage issued by the mortgagor in the 28% tax bracket.

Interest the mortgagor pays to the mortgagee is tax deductible. As the mortgagor's tax bracket increases, his mortgage interest liability decreases. The rate at which the mortgagor's liabilities are discounted to time (t) is $r_e(1-T)$. The derivative of the after tax discount rate with respect to the effective rate ($r_e$) is (1-T). As (T) increases the derivative decreases. For a given decrease in the cost of mortgage capital (effective rate) the intrinsic value of the mortgagor's prepayment option will increase by a smaller amount the higher is his tax bracket. When the mortgagor must pay points to refinance $B_t$ the present value of his mortgage liability may not increase by enough to place the option in the money at time (t).

Examination of row (1) in exhibit I,II and III, reveals that structure (a) is preferable to the mortgagor. The yield for this mortgage is 10% regardless of the time the mortgage is outstanding because the mortgage was issued at par. The yield of structure (b) will be greater than 10% if the mortgage is refinanced prior to its maturity because it was issued at discount. The lender is concerned with the value of the principal which may be refinanced. The lender's potential loss does not vary across the mortgagor's tax bracket. The value that reveals the mortgagee's potential loss are the entries in the row corresponding to the 0% tax bracket. The intrinsic value of the option associated with structure (a) is greater than the intrinsic value of the prepayment option associated with structure (b) or (c). The mortgagee would prefer a mortgagor to choose structure (c). Ex post the mortgagee will only be indifferent between the two structures if the mortgagor does not exercise the prepayment option associated with either structure. Stone & Zissu [16], Kau & Keenan [13], illustrate that the tax treatment of discount points and the mortgagor's uncertainty with respect to the future cost of mortgage capital can make structure (b) the mortgagor's preferable choice.

Although the duration of structure (c) exceeds the duration of structure (a), the difference is not great enough to offset the fact that the prepayment option of mortgage structure (c) is expected to be out of the money. When the mean of the lognormal distribution used to generate the cost of mortgage capital (10%) is realized the prepayment options of mortgages (b)&(c) will be out of the money. This is simply because the contract rates on mortgages (b&c) are less than 10%. It is for this reason the intrinsic value of the option associated with structure (c) is greater than the intrinsic value of the option associated with structure (b)[12]. The duration difference can be observed in the higher coefficient of variation for structure (b).

A lender who owned a portfolio consisting of only structure (a) mortgages would be exposed a higher degree of reinvestment risk than a lender who owned a portfolio of structure (c) mortgages. At any point in time the lender who owns mortgages of structure (a) will expect a greater amount of principal to be called away more frequently. The frequency statistic we calculate measures the likelihood of

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[12]The duration calculation for a mortgage is given by:

$\text{Duration}=[(1+r_c)/r_c] - [n/(1+r_c)^{n}-1$

(r) is the periodic contract interest rate specified in the conventional mortgage contract. (n) is the number of periods until the mortgage matures.

a mortgage being called away from the mortgagor[13]. There is a 60% probability that the option of structure (a) will be in the money at any time (t). The prepayment option of structure (c) has a probability of 57% of being in the money.

From the point of view of the lender, mortgage structure (c) is the riskiest structure in the sense that there is a higher probability that the actual intrinsic value of mortgage (c)'s prepayment option will deviate from its expected value. Mortgage structure (c) is the least risky in the sense that the probability of it being called is less than for structures (a & b).

As the tax bracket increases to the extreme 100% rate the prepayment option would be pushed to "at the money" independent of all future interest rate changes across all discount point/contract rate combinations. The present value of the mortgagor's interest liability would be totally insulated from changes in the cost of mortgage capital if he were in the 100% bracket.

In 1986 a person with a gross income of $100,000 who was filing his income tax as a "head of household" would

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[13]The probability of a prepayment option being in the money is sensitive to the variance of the lognormal probability distribution that was used to simulate $r_m$. An increase in the variance of the distribution will increase the probability of all structures being in the money. The probability of being in the money will always be larger for structure (a), but increases in the variance of the interest rate process will decrease this difference.

have been in the 50% federal income tax bracket. In 1987 this person providing his gross income hadn't declined would be in the 38.5% tax bracket and in 1988 he would be in the 33% tax bracket.

If this person had written a thirty year mortgage for $187,000 in 1985 (t=0) with a contract rate of 9.8% and paid one discount point, his prepayment option would be worth $20,905 one year later, if the cost of mortgage capital fell to 8% at time (t=1) and he had to pay 1 point to refinance his current mortgage. If he had written this mortgage at the beginning of 1987 (t=0) and the cost of mortgage capital fell to 8% plus 1 point, his prepayment option would be worth $23,808 at the beginning of 1988 (t=1). If he had written the mortgage at the beginning of 1988 (t=0), then at the beginning of 1989, (t=1) the intrinsic value of his prepayment option would be worth $25,005. The tax reform act of 1986 changed the intrinsic value of the outstanding prepayment options by altering the present value of the mortgagor's liabilities.

## IV. Conclusion

Our analysis implies that the likelihood of a mortgagor exercising his prepayment option is affected by the mortgage structure and the mortgagor's tax bracket. The Federal Income Tax structure drives a wedge between, the amount a mortgagor can increase his wealth by exercising his option and the value of the assets called away from the owner of the mortgage[14].

The reinvestment risk to which owners of mortgages and mortgaged backed securities are exposed, will depend on the tax brackets of the mortgagors and the original discount at which the mortgage was sold.

We have shown the lender is not indifferent between a mortgage issued by the mortgagor in the 11% tax bracket and a mortgage issued by a mortgagor in the 33% tax bracket. If a mortgagee were given the mutually exclusive choice of buying identical mortgages from a mortgagor in the 11% tax bracket or a mortgagor in the 33% tax bracket at identical terms the mortgagee would choose to lend to the mortgagor in the 33% tax bracket. In fact the two mortgagors can borrow mortgage capital at the same terms.

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[14]The tax effect on the value of the prepayment option increases when state taxes are considered. The top income tax bracket in California is 9.3%. The income tax is based on the Federal Tax Base. The after tax cost of mortgage capital in California is $r_c(1-T^s)(1-T^f)$. $T^s$=State tax bracket. $T^f$=Federal tax bracket

Prepayment options which are marginally in the money when refinancing costs are zero may be out of the money when the refinancing costs are positive. The prepayment options which are marginal are those owned by mortgagor's in relatively high tax brackets, and those options associated with mortgages which were issued at relatively large discounts.

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CHAPTER III

# CHOOSING A DISCOUNT POINT/CONTRACT RATE COMBINATION

## Abstract

Confronted with an array of contract rate/discount point combinations from which he may choose, a mortgagor must evaluate his wealth with respect to each choice. The future path of the cost of mortgage capital will affect the wealth of the mortgagor to a different degree depending on which contract rate/discount point combination he chooses. Using the Expected Monetary Value algorithm, we show that variation, of marginal income tax rates and expectations about the future path of the cost of mortgage capital across mortgagors, can explain the demand for different contract rate/discount point combinations. Our results provide a rational for a mortgagor choosing an intermediate point/contract rate combination.

## I. Introduction

The purpose of this paper is to show that the mortgagor's choice of the discount at which he sells his mortgage contract and the contract rate associated with the discount is dependent on the mortgagor's marginal tax rate and the mortgagor's ex ante expectations of what the cost of mortgage capital will be at some future date.

Kau and Keenan [5] (K&K) analyze the raison d'etre of discount points in the mortgage market. They abstract from an uncertain world and show that existence of points does not rely on arguments of risk shifting between the mortgagor and mortgagee. (K&K) illustrate that in competitive capital markets the current tax treatment of discount points is sufficient to explain the existence of discount points. They demonstrate that if the marginal tax rate of the lender is less than the marginal tax rate of the mortgagor, discount points are a means by which the lender can increase his after tax yield without increasing, and possibly even decreasing the after tax yield paid by the mortgagor. This is the tax arbitrage referred to by (K&K).

Harris and Sirmans [4] (H&S) argue that mortgagees use points as a means of lowering the value of the mortgagor's prepayment option. In return for the reduction in the mortgagees' exposure to prepayment risk the mortgagee is willing to buy a mortgage with a lower contract rate. (H&S) show that the reduction in the contract rate due to the reduction in the value of the mortgagor's prepayment option vis a vis a higher exercise price is a reduction in addition to the bond effect. The bond effect refers to the mortgagees' ability to receive the same effective yield with different combinations of points and contract rates.

It can be inferred from [H&S]'s argument that a mortgagor who does not plan on refinancing his original mortgage for twenty years will choose to buy a relatively less valuable prepayment option from the mortgagee than a mortgagor who plans on exercising the option in five years. Discount points raise the exercise price of the prepayment option and raise the value of the mortgage contract to the mortgagee (lower the yield)[1]. The implications of (H&S)'s analysis is that when the level of points can be negotiated between the mortgagor and the mortgagee, the mortgagor can in effect sell his call option back to the mortgagee in return for a lower contract rate. A mortgagor who is not planning on exercising his prepayment option will be better off paying the points required to reduce the value of the option to zero.

Dunn and Spatt [3] (D&S) comment on the relationship between a mortgagor's willingness to exchange discount points for a lower contract rate. According to (D&S) the discount at which a utility maximizing mortgagor is willing to sell his mortgage increases as the mortgagor's expectations about his tenure in the mortgaged property increase. (D&S) state that knowing the discount at which the mortgagor sold the mortgage exposes, ex ante, a mortgagor's tenure decision. This is valuable information to the lender and the ultimate owner of the mortgage or assets secured by the mortgage.

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[1]The exercise price of the mortgagor's prepayment option is $(B_t/(1-p_t))$ where $B_t$ is the outstanding balance at time (t) and $p_t$ are the refinancing points at time (t). The original balance of the mortgage along with the contract rate determines $B_t$. The original mortgage balance is $B_o/(1-p)$ where p is the percent of $B_o$ that must be paid to the lender at time (0). Ceteris paribus $B_t$ is an increasing function of $B_o/(1-p)$.

The literature on discount points focuses, on the reason that discount points exist and, the information a mortgagor's choice of a discount point/contract rate combination reveals about the ex ante value he places on the option to prepay. The existing literature does not explicitly model the mortgagor's choice of a discount point/contract combination. [K&K] show that the preference for points is an increasing function of the mortgagor's tax bracket. They do not explain why an intermediate discount point/contract rate combination would be chosen rather than a zero point or high point combination. The article by [D&S] makes a link between the mortgagor's ex ante belief about his tenure in the mortgaged property and the choice of a discount point contract combination but they do not explicitly model the mortgagor's choice.

In this paper we combine the tax explanation of points [K&K], the self selection explanation of points [D&S] and the option effect of points [H&S], to construct a model that explains the mortgagor's choice of a specific contract rate/discount point combination. We illustrate that the specific contract rate/discount point combination chosen by a mortgagor is a function of his tax bracket and of his expectations about future interest rates. The contract rate/discount point combination which the mortgagor chooses affects his future prepayment decision by changing the value of the difference between the present value of his mortgage liabilities and the outstanding mortgage balance at every time (t).

In part II of the paper we define the mortgagor's wealth function and model his choice of contract terms. Part III of the paper is an interpretation of the results generated by the model. Part IV is the conclusion of the paper.

## II. Model

We assume that the cost of mortgage capital follows a very simple stochastic process[2]. At the beginning of a known year the current market interest rate on fixed rate conventional mortgages either falls by (x) basis points (state 2) or does not change (state 1). The cost of mortgage capital can change once every five years. Five years is the original maturity of all mortgages analyzed in this paper. By restricting the number of changes in the cost of mortgage capital to a maximum of one per five year period, and by assuming the mortgagor knows when the cost of mortgage capital can fall, we reduce the time value of the prepayment option to zero. We show that the time (t) at which the cost of mortgage capital can change and the probability the mortgagor attaches to each state of the world can affect his

[2]The cost of mortgage capital is the zero point equivalent interest rate that mortgagors must pay on newly issued five year conventional mortgages. Given a discount point contract rate combination the zero point equivalent rate can be computed. (see footnote 7)

Bayes decision[3]. The assumption of this simplified interest rate process is not inconsistent with our objective of modelling the choice of mortgage contract terms across tax brackets and expectations. The other assumptions we make are: 2) a mortgagor can borrow and lend at the same effective interest rate, 3) the income of the mortgagor is exogenous, 4) 100% of the value of the house is mortgaged at time (0), and 5) at the time of refinancing, the mortgagor is offered only one contract rate/discount point combination.

Our hypothetical mortgagor's wealth at time (t) is the time (t) value of his assets minus the time (t) value of his liabilities. The value of the mortgagor's assets at time (t) is the sum of the value of his primary residence ($B_0$), the tax shield created by the tax deductibility of the original discount points paid, and the intrinsic value of his option to call his mortgage. The value of the mortgagor's liability at time (t) is the value of his mortgage obligation on his primary residence discounted to time (t)[4].

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[3] A decision $d^*$ which minimizes loss function L(d) or maximizes pay-off function R(d) over d (d is a subset of D, the space of all viable decisions) is commonly called a Bayes decision. A Bayes decision will be optimal if the expected monetary algorithm relates to the decision makers true objective. (Smith pp 3-4).

[4]The mortgagor's income is exogenous. It is not explicitly included in the wealth function because his income would not be affected by his choice of mortgage structure or his prepayment decision. The wealth function in the model can be thought of as the fraction of individuals' wealth devoted to the financing of his primary residence.

The value of his mortgaged property is assumed to appreciate at the constant rate of ($\alpha r_e$%) per year with ($-1<\alpha<1$)[5]. ($r_e$) is equal to the before tax effective yield to maturity on mortgage capital.

A mortgagor has preferences which can be described by a utility function that is linear with respect to monetary pay offs[6]. This is the assumption of risk neutrality. At the time the mortgagor issues his mortgage he must choose from an array of discount point/contract rate combinations. The objective of the mortgagor when he makes his choice at t=0 is to maximize his expected wealth for the time at which the state of the world can change.

Refinancing opportunities are not known with certainty when the original contract is written. Mortgagors may choose to prepay their mortgage before it matures. Their choice of a contract rate/discount point combination depends on the length of time the mortgagor expects to service the original

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[5]The rate at which the mortgaged property appreciates is relevant to the individual deciding between buying property and renting property.

Once the mortgagor has decided to buy the property the expected rate of appreciation is not a relevant decision variable with respect to the discount point/contract rate combination. Changing $\alpha$ increases or decreases $W_{11}$ and $W_{12}$ by the same amount across all tax brackets. An increase or decrease in $\alpha$ will have a larger effect on the mortgagor's wealth in later periods because of compounding. For simplicity we assume $\alpha$=1.

[6]When an individuals's utility function is linear in pay-off the expected monetary value algorithm is appropriate, utilities being equal to pay-offs (Smith pp. 43, [8]).

mortgage. All mortgages we consider are conventional fixed rate mortgages that give the mortgagor the option of calling the market value of the outstanding mortgage balance from the lender for an amount equal to the book value of the mortgage.

The three mortgage contracts from which the mortgagor may choose are constructed such that the effective yield to the mortgagee is the same for each structure if the mortgage is not prepaid prior to its maturity[7]. Market data reveals that the discount point/contract rate combinations offered by lenders on a specific date have different effective yields if the mortgage is completely amortized. If the three mortgage structures have the same effective rate over five years, the mortgage with the highest points lowest contract rate will have the highest effective rate for any time less than the five year maturity. The mortgage with the lowest points and highest contract rate will have the lowest effective rate over any time less than the five year maturity. The

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[7]In order to obtain a desired periodic effective yield, $r_e$, the loan points to be charged, p, on a fixed rate mortgage of %$B_0$ with contractual annual rate $r_c$ are given by:

$p=1-[r_c/(1-(1+r_c)^{-n})][(1-(1+r_e)_{-n})/r_e]$

When p=0 the effective rate equals the contractual rate. ($r_e=r_c$)

Given $r_c$ and p, the effective yield cannot be explicitly obtained because the equation that must be solved is a polynomial of an order higher than 2 in $r_e$. An approximation formula, which achieves yields very close to the exact yield, is given by:

$r_e = (2r_c + p/n)/2(1-p)$ [Prakash, Karels, and Fernandez] intermediate point/contract rate combination will have an intermediate effective yield for times less than the five year maturity. If the mortgagor simply chose that mortgage structure which had the lowest effective cost of mortgage capital, they would not necessarily make the Bayes decision, (that decision that maximizes their expected wealth). In a certain world the mortgagor would either choose the high point combination or the low point combination. The introduction of uncertainty gives value to the intermediate choice.

Capital gains tax on the sale of the property is assumed to be perpetually deferred.[8] Discount points which are paid

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[8]If property (in this section called "old residence") used by the taxpayer as his principal residence is sold by him, and, within a period beginning 2 years before the date of such sale and ending 2 years after such date, property (in this section called "new residence") is purchased and used by the taxpayer as his principal residence, gain (if any) from such sale shall be recognized only to the extent that the taxpayer's adjusted sales price (as defined in subsection (b)) of the old residence exceeds the taxpayer's cost of purchasing the new residence. (section 1034 rollover of gain on sale of principal residence, Internal Revenue Code 22,521)

(a) At the election of the taxpayer, gross income does not include gain from the sale or exchange of property if

1) the taxpayer has attained the age of 55 before the date of such sale or exchange, and

2) during the 5-year period ending on the date of the sale or exchange, such property has been owned and used by the taxpayer as his principal residence for periods aggregating 3 years or more.

Limitations: Dollar limitation.- The amount of the gain excluded from gross income under subsection (a) shall not exceed $125,000 ($62,500 in the case of a separate return by a married individual.) [section 121. One-time exclusion on gain from sale of principal residence by individual who has attained age 55. Internal Revenue Code 120 (d)(6)] when purchasing the property are considered interest and are treated as such for tax purposes. When the mortgagor refinances his current mortgage obligation and is required to pay discount points, the points are not tax deductible. Interest paid to service the mortgage liability is tax deductible.

(t) stands for the end of the year. Interest rates are assumed to change only at time t=1,2,3 or 4. Mortgage payments are due at the end of the year. We assume that the call option is of the European style in that it can only be exercised at time $t=1+\delta,2+\delta,3+\delta$,or $4+\delta$. The interval of time $t-\delta$ is small such that the time value of money between t and $\delta$ is zero.

The model we use to analyze the mortgagor's decision of a discount point/contract rate combination is the expected monetary value (EMV) algorithm. The formal model follows.

**The Expected Monetary Value (EMV) Algorithm**

If the current effective rate falls (state $\Theta=2$) to ($r_e$-x%) within n years (n=life of the contracted mortgage), the mortgagor refinances his existing mortgage. The new mortgage contract is written with a contract rate/discount point combination that corresponds to the new effective cost of mortgage capital. If the effective rate doesn't change, the contract rate remains the same (state $\Theta=1$).

Let $\pi$ denote the probability that $\Theta=2$, and $(1-\pi)$ the probability that $\Theta=1$, at time t. The mortgagor's wealth is a function of decision d and the state of the world $\Theta$ which is realized at time t. Equation (1) defines the wealth of the mortgagor:

$$W_{tij} = B_0(1+r_e)^t + TpB_0[1+(1-T)r_e]^t - PV_t(i,j) + Max\{[PV_t(i,j) - B_t(i)/(1-p)], 0\} \quad (1)$$

$W_{tij}$ = mortgagor's wealth at time (t) for decision i and outcome j.

$B_0(1+r_e)^t$ = value at time t of the property being mortgaged

$TpB_0[1+(1-T)r_e]^t$ = time t value of the tax shield created by the tax deductibility of the originated discount points.

$PV_t(i,j)$ = present value at time t of the mortgage for decision i and state j (see appendix B)

$B_t(i)$=outstanding mortgage balance at time (t) for decision i

p = number of points the mortgagor must pay per $ borrowed

$Max\{[PV_t(i,j)-B_t(i)/(1-p)],0\}$ = intrinsic value of the option at time (t) for decision $i$ and outcome $j$, where $B_t(i)/(1-p)$ is the strike price at time t. The value of the option is always zero in state $\Theta=1$ (interest does not change).

In this paper the mortgagor must choose from three possible point-contract combinations. The three combinations are:

$d_1$: $r = 9.8\%$ $p = 1\%$

$d_2$: $r = 9.6\%$ $p = 2\%$

$d_3$: $r = 9.4\%$ $p = 3\%$

Each of the combination has a before tax effective rate of 10% over a period of five years. In this paper all mortgages are five year conventional mortgage[9]. The possible states of the world are:

$\Theta_1$ = the effective rate remains at 10%.

$\Theta_2$ = the effective rate drops to 8.18% (new contract rate is 8% with 1 point to refinance).

We assume that the mortgagor's utility is linear in pay-off, utility being equal to pay-offs (see footnote #6).

The wealth functions for $d_i$ and $\Theta_j$ (i=1,2,3 and j=1,2) are:

$$W(d_1,\Theta_1)=B_o(1+r_m)^t+TpB_o[1+(1-T)r_m](1,1)-PV_t(1,1)= W_{11} \quad (2)$$

$$W(d_1,\Theta_2)=B_o(1+r_m)^t+TpB_o[1+(1-T)r_m]^t(1,2)-$$
$$PV_t(1,2)+Max\{[PV_t(1,2)-B_t(1)/(1-p)],0\} = W_{12} \quad (3)$$

$$W(d_2,\Theta_1)=B_o(1+r_m)^t+TpB_o[1+(1-T)r_m]^t(2,1)-PV_t(2,1)= W_{21} \quad (4)$$

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[9]A loan structured in such a way that the total payment at the end of each period (the sum of the principal and interest) was equal, or level. 100% of the mortgage principal is amortized over the life of the mortgage (see Senft [7]).

$W(d_2, \Theta_2) = B_o(1+r_m)^t + TpB_o[1+(1-T)r_m]^t(2,2) -$

$PV_t(2,2) + Max\{[PV_t(2,2) - B_t(2)/(1-p)], 0\} = W_{22}$ (5)

$W(d_3, \Theta_1) = B_o(1+r_m)^t + TpB_o[1+(1-T)r_m]^t(3,1) - PV_t(3,1) = W_{31}$ (6)

$W(d_3, \Theta_2) = B_o(1+r_m)^t + TpB_o[1+(1-T)r_m]^t(3,2) - PV_t(3,2) +$
$Max\{[PV_t(3,2) - B_t(3)/(1-p)], 0\} = W_{32}$ (7)

$\bar{W}(i)$ = expected wealth associated with d = i

$$\bar{W}(i) = \sum_{j=1}^{2} W_{ij}\pi_j$$

$\bar{W}(1) = W_{11}(1-\pi) + W_{12}(\pi) = \pi(W_{12}-W_{11})+W_{11}$ (8)

$\bar{W}(2) = W_{21}(1-\pi) + W_{22}(\pi) = \pi(W_{22}-W_{21})+W_{21}$ (9)

$\bar{W}(3) = W_{31}(1-\pi) + W_{32}(\pi) = \pi(W_{32}-W_{31})+W_{31}$ (10)

The mortgagor evaluates his preferences at a subjective probability ($\pi_m$). It is possible that for a given ordering of expected wealth functions, three mortgagors who only differ in their expectations ($\pi_m$), will each choose a different discount point contract rate combination. For certain $\pi_m$ the mortgagor may not have an unambiguous Bayes decision.

$\bar{W}(1) > \bar{W}(2) > W(3)$ across all decisions when;

$\pi(W_{12}-W_{11})+W_{11} > \pi(W_{22}-W_{21})+W_{21} > \pi(W_{32}-W_{31})+W_{31}$ (11)

equation (11) corresponds to: $d_1 > d_2 > d_3$

In order for inequality (11) to hold, the following inequalities must be true:

$\pi > (W_{21}-W_{11})/[W_{12}+W_{21}-W_{11}-W_{22}]$ for $\bar{W}(1) > \bar{W}(2)$ (12)

$\pi > (W_{31}-W_{11})/[W_{12}-W_{11}-W_{32}+W_{31}]$ for $\bar{W}(1) > \bar{W}(3)$ (13)

$\pi > (W_{21}-W_{31})/[W_{22}-W_{21}-W_{32}+W_{31}]$ for $\bar{W}(2) > \bar{W}(3)$ (14)

## III. Results

We graph the expected wealth function for each decision across time (t) and across personal income tax brackets. There are three series of graphs (see appendix A for series a,b,c). Series (a&b) illustrate how the ordering of the expected wealth functions change across tax brackets and time. The graphs in series (c) are an illustration of the case where the expected wealth functions intersect. The intercept on the left vertical axis is the expected wealth of a mortgagor in state one of the world ($\Theta=1$), given that the mortgagor has made decision $d_i$, and the mortgagor is certain that state $\Theta=1$ will occur,($\pi=0$). The intercept of the expected wealth function with the right vertical axis is the expected wealth of a mortgagor in state two of the world ($\Theta=2$), given that the mortgagor has made decision $d_i$, and the mortgagor is certain state $\Theta=2$ will be realized,($\pi=1$). The expected wealth of the mortgagor is a linear function of the probability ($\pi$) of the state of the world changing. Once the two intercepts for each decision have been calculated the expected wealth function can be plotted. Points on the expected wealth function are weighted averages of the wealth in each state of the world. The weights ($\pi$ and ($1-\pi$)) are the probabilities of each state of the world being realized at time (t). The horizontal axis of each graph measures the ($\pi$) associated with each level of expected wealth.

For a given $\pi_s$ (the subjective probability) the mortgagor will choose the discount point/contract rate combination which generates the highest expected wealth. The expected wealth associated with two or more choices may be equal for some ($\pi$). To arrive at the Bayes decision when this is the case the mortgagor must compare his beliefs about the likelihood of $\Theta=1$, and $\Theta=2$ occurring with the probability ($\pi$), at which the expected wealth functions are equal. If the mortgagor's subjective probability ($\pi_s$) equals the probability at which the decisions are equal ($\pi^*$) he will be indifferent between the intersecting decisions. If $(\pi_s)<(\pi^*)$ he will evaluate the choices at ($\pi_s$), and choose the decision that maximizes his expected wealth for this subjective probability.

Graph (Ic) illustrates that a mortgagor's choice of a discount point/contract rate combination depends on his expectations about the future level of the cost of mortgage capital. If the mortgagor attaches a probability ($\pi_s$), to the event that the cost of mortgage capital does not fall at time t, which is less than ($\pi^{*2/3}$), {($\pi^{*2/3}$) refers to the probability at which decision 2 equals decision 3}, decision 3 will maximize the mortgagor's expected wealth. If the mortgagor's assessment of the probability that interest rates do not fall at time t, is less than ($\pi^{*1/2}$) and greater than ($\pi^{2/3}$), decision 2 will maximize the mortgagor's wealth. If ($\pi_s$) is greater than ($\pi^{*1/2}$) the mortgagor's Bayes decision will be decision 3. Graph (IIc) plots the expected wealth functions for a mortgagor in a higher tax bracket than the mortgagor represented in graph (Ic). Both graphs (Ic&IIc) correspond to the same point in time. The most striking difference between graphs (Ic&IIc) is that in graph (IIc) decision two never maximizes the mortgagor's wealth. The range of probabilities ($\pi^*$) over which decision 1 maximizes the mortgagor's wealth increases as the mortgagor's tax bracket increases. As the mortgagor's tax bracket continues to increase the ordering of the decisions becomes completely independent of the mortgagor's expectations. In high tax brackets (see T=75% in series a) and low tax brackets (see T=11% and T=28% in series a), the mortgagor's decision of a discount point/contract rate combination is driven completely by tax considerations. When the mortgagor is in an intermediate tax bracket his expectations motivate his decision.

Our initial proposition that a mortgagor's choice of a contract rate/discount point combination depends on his tax bracket and his expectations about the future cost of mortgage capital is confirmed by the graphs presented in series (a). As a mortgagor's tax bracket increases ceteris paribus from 11% to 75% the ordering of the decisions from highest expected wealth to lowest expected wealth switches from $(d_1,d_2,d_3)$ to $(d_3,d_2,d_1)$. The tax bracket in which the reordering is actually realized depends on the point/contract rate combinations, and the time at which the state of the world is expected to change. Comparison of the graphs in series (a) with the graphs in series (b), illustrates that the reordering takes place within a lower tax bracket the longer is the expected time before a state change. In other words the longer someone expects to have the same mortgage liability the lower will be the income tax bracket at which a high point low contract rate decision dominates. In series (a), the complete reordering takes place between the 65% and 72% tax brackets. In series b (t=4), the switch takes place between the 12% and 15% tax brackets.

**IV. Conclusion**

We have shown that the contract rate/discount point combination that maximizes the expected wealth of a mortgagor depends on the tax bracket of the mortgagor, the time over which he expects the effective cost of mortgage capital to remain constant, and the probability the mortgagor attaches to each state of the world for every time (t).

Our analysis demonstrates that within a range of tax brackets and expectations, the intermediate point/contract rate combination is the preferable choice. It is possible that the mortgagor is indifferent between two of the choices, which both in turn dominate the third choice. If this is the case, a change in his tax bracket and or a revision of his expectations about the future cost of mortgage capital will be sufficient to provide a Bayes decision.

One of Dunn and Spatt's conclusions is that knowing the array of combinations from which a mortgagor chose his contract rate/discount point combination, is valuable information for the institution or individual who ultimately owns the mortgage. This information may reveal the mortgagor's expected tenure in the mortgaged property. Our analysis illustrates that knowing mortgagor (A) chose the high point/low contract rate combination and that mortgagor (B) chose the low point/high contract rate combination, does not necessarily mean that mortgagor (A) plans on a longer tenure in his house than mortgagor (B). The time they expect to have their original mortgage liability may be the same. Their choices may have been determined by their marginal income tax rates and or subjective beliefs about the future cost of mortgage capital. Mortgagors in the same tax bracket and with the same expected tenure in the mortgaged property may choose different combinations if they have different expectations about the future cost of mortgage capital.

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# CHAPTER IV

# The EFFECTIVE COST OF MORTGAGE CAPITAL ACROSS TIME AND SPACE

This paper tests the hypothesis that differences across state mortgage markets are significant enough to make the required rate of return on mortgage capital significantly different across states. In order to examine the required rate of return on mortgage capital within a single region, the large number of different contract rate/discount point combinations that exist within a single region are reduced to an average effective rate over an implied holding period. A mortgage yield curve is constructed from the combinations of holding periods and effective rates implied by the contract rate/discount point combinations within each state. A regression model is employed to test for significant differences in the twelve year effective mortgage interest rate across regions. The results support the hypothesis that investors require significantly different returns from mortgage capital across states.

## I. Introduction

The objective of this paper is to test the hypothesis that the effective cost of mortgage capital for a given holding period is statistically different across regions. Our hypothesis is confirmed. Confirmation of our hypothesis is insufficient to confirm the hypothesis of regional segmentation in the mortgage market.

The secondary mortgage market makes it possible to hold mortgage portfolios that are diversified with respect to regions. If the idiosyncratic risks of specific regions can be identified and the secondary market is efficient, mortgage yields should only reflect risks that are systematic across all regions, (see Corgel&Gray [4], Clauretie [3]). Financial assets which are held in a diversified portfolio are priced according to their marginal risk with respect to the portfolio, (Sharpe [13]). This does not imply that there should be a unique mortgage yield curve. Mortgage markets in certain regions may have less overall systematic risk than other mortgage markets, (see Clauretie).

The presence of market segmentation suggests that securities can be subdivided into distinct groups according to some characteristic and that there exists a lack of substitution between these groups on the part of borrowers and or investors, (Kidwell & Koch[9],Van Horne[15]).

In a segmented mortgage market the supply curve of mortgage capital will be relatively inelastic within restricted regions. When the mortgage market is integrated with the money market and capital markets, the supply of mortgage capital will be relatively elastic across all regions.

The dominant type of mortgage in the U.S. is the conventional mortgage. The maturity of the conventional mortgage is typically thirty years although the fifteen year mortgage has recently become more popular. Conventional mortgages in the U.S. give the mortgagor the right to call the mortgage from the lender. Mortgagors can span the maturities from 1-30 years by issuing long term mortgages and calling them prior to maturity. The time at which the mortgage is called is unknown to the lender when the mortgage is issued.

We employ two economic models to test for significant differences in the effective mortgage rates that were being offered across eight states on June 16,1989. The first model (model I) calculates the average holding period and effective rate implied by the contract rate/discount point combinations in a specific region. The implied holding period/effective rate combination is the combination that equates the present value of a $187,000 mortgage issued at the average discount point/contract rate combination for the region to the present value of a $187,000 mortgage issued at par. The implied holding period/effective rate combination is calculated for each of the eight states. To construct a mortgage yield curve we plot the implied holding period/effective rate combinations in yield/maturity space.

Our second model (model II) tests for the existence of risk differentials across regional mortgage markets by regressing the average effective mortgage rates for a given holding period which are implied by the contract terms on a specific day within a specific state, against dummy variables representing each state. Comparison of the estimated coefficients on each geographic dummy variable with a base coefficient, will reveal statistically significant differences between the effective rates offered in each state.

Rudolph,Zumpano, and Karson [12] (R&Z&K) measure the impact of the secondary mortgage market on the efficiency of the mortgage market. They hypothesize that a well functioning secondary market in mortgage instruments should enable mortgage funds to move with virtual freedom from areas of capital surplus to areas of capital shortage. Arbitrage should reduce the differences in the price of mortgage funds across regions. They find that the secondary market has not significantly reduced differences in local markets. Their data on mortgage terms were compared across the years 1968 and 1978. Their results must be approached with caution because during the 1980's there has been significant deregulation of the banking industry[1]. (R,Z,&,K) comment that the efficiency of the secondary market may have been outweighed by legal prohibitions on branch banking, lending radius restrictions, resource immobility and monopoly power.[2]

Buckley and Gross [2] separate regional mortgage markets into "segmented and potentially integrated markets". They test the hypothesis that the introduction of money market instruments (MMC's) by thrifts affected mortgage rates in

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[1]The Depository Institutions Deregulation and Monetary Control Act of 1980, and The Garn-St Germain Depository Institutions Act of 1982.

[2]In 1980 the Depository Institutions Deregulation and Monetary Control Act legislation overrode state usury ceilings.[Cooper & Fraser]

segmented markets but not in integrated markets. They conclude that (MMC's) increased mortgage interest rates in states where usury ceilings on mortgage rates were below the national average.

Merriken [11] examines the ability of financial institutions to vary rates of return on mortgages by segmenting mortgage loan markets. Merriken finds evidence of product segmentation in the mortgage market across the three main groups of lenders (Banks, Thrifts, and Mortgage Bankers). He finds that within each segment the price of the mortgage instrument reflects its riskiness relative to default free bonds. The interpretation of Merriken's results are that segmentation across mortgage products only exists on the supply side.

Kidwell and Koch [9] argue that for the market of a financial security to be segmented, both the supply side and the demand side must be unable to and arbitrage between the financial instrument in question, and other financial instruments. If asset substitution is possible on one side of the market, yield discrepancies between assets will disappear. Their study examines the role of market segmentation in influencing yields on municipal securities relative to yields on U.S.Treasury securities. They find statistical evidence that the municipal market is segmented by maturity.

Corgel and Gay [4] use mean-variance portfolio analysis to measure the gains from geographic diversification. Benefits from regional diversification can be derived from the reduction of nonsystematic prepayment risk. Nonsystematic prepayment risk in this context is prepayment risk which is specific to a local economy.

If investors can manage a regionally diversified portfolio of mortgages with relatively low transaction costs, the terms of mortgages across geographic areas should only reflect a premia for the mortgage's contribution to the systematic risk of a diversified portfolio.

We test for the presence of significant differences in the effective cost of mortgage capital across regions. Significant differences should not be interpreted as a symptom of market segmentation. The differences in the cost of mortgage capital across the country may reflect a "premia for systematic risk".

Section II of the paper describes the data. In section III we develop the two economic models (model I and model II). Section IV of the paper is an interpretation of the results generated by the two models. Section V is the conclusion.

## II. Data

Our sample includes the contract rate/discount point combinations offered by lenders in eight states (AZ, CA, FL, IL, MA, NY, OH, TX). The lenders in the sample are banks, thrifts, and mortgage companies. For each combination, the minimum initial equity a mortgagor must invest to secure the offered terms, is known. There are an average of six observations per state (52 observations). The rates are those that were being offered during the week of June 16, 1989. The terms offered are for FNMA conforming loans[3]. The maximum loan amount for which the terms are applicable is $187,000. The data was provided by HSH Associates[4].

## III. Model

### Model (I)

We find the average holding period and effective interest rate combination for each state that is the closest to a par loan. The holding period that satisfies this equation will be referred to as the Par Equivalent Holding Period (PEHP). The effective interest rate that satisfies this equation will be referred to as the Par Equivalent Effective Rate (PEER).

The values for (PEER) and, (PEHP) were calculated as follows:

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[3]The mortgage terms conform to Federal National Mortgage Association (FNMA) underwriting standards.

[4]Twelve Hundred Route 23, Butler, NJ 07405 / (201) 838-3330.

| | Martrix 1 | | Matrix 2 | | |
|---|---|---|---|---|---|
| | Mortgage 1 (s) | | Mortgage 2 (s) | | Matrix T (s) |
| | 1 2.......30 | | 1 2.......30 | | 1 2 ......30 |
| .095 | $A_{1s}$ | + | $A_{2s}$ | +...= | $A_{Ts}$ |
| . | | | | | |
| . | | | | | |
| . | | | | | |
| .14 | | | | | |

$A_{1s}$=is a cell for mortgage 1 in state s with an effective rate of .095 computed as follows:

$A_{1s}=[p_{1s}B_o + \Sigma\ PMT_{1s}/(1+r_s)^i + B_{1st}/(1+r_s)^h - B_o]^2$

where

$p_1$ = initial points charged when mortgage 1 in state s is issued,

s = AZ, CA, FL, Il, MA, NY, OH, TX.

$B_o$ ($187,000)=the present value of a mortgage which is issued at par assuming the yield curve is flat and has not shifted since the mortgage was issued,

$PMT_1$=periodic payment for mortgage 1 in state s,

$r_s$ = effective rate ($r_s$=.095,........,.14) ,

i = period (i=1,...,h),

h= holding period,

$B_{1sh}$= outstanding balance for mortgage 1 in state s for a holding period h.

$A_T = A_{1s} + A_{2s} + ...+ A_{ms}$

where m = number of contract rate/discount point combinations offered within a specific state.

Matrix T = Matrix 1 + Matrix 2 +....+ Matrix m.

We solve for the holding period and effective interest rate combination that minimizes the sum of the squared deviations of the present value of mortgages from the par value ($187,000) within a single state (this corresponds to the minimum value in Matrix T). The numerical technique used to solve this equation is polynomial least squares approximation[5]. We used a nonsequential factorial search method to solve for the values $h^*_s$ and $r^*_{ss}$[6]. The possible values of $h^*_s$ and, $r^*_s$ are located within a two dimensional grid. The columns of the grid are possible holding periods. The rows of the grid are possible effective rates. There is a single grid for each mortgage contract being offered within the state. The minimum within the final grid for each state corresponds to the measurements of, (PEER) and, (PEHP). Our result are reported in exhibit (I).

The result of these computations reveals that on average within a specific state, mortgagors must issue mortgage debt at a discount if they expect to call it prior to ($h^*_s$) and mortgages may be issued at a premium if mortgagors expect to

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[5]Atkinson provides a clear explanation of this method.

[6]Nonsequential factorial search is a search which is conducted over evenly spaced points in a simply connected region of a Euclidean space. Each of the $x_i$ coordinates is assigned a set of evenly spaced points, called grid points, and only the values of $x_i$ at these grid points are used. The function f(x) is evaluated for all possible combinations of the grid points, and the grid value X which yields the "best" f(x) is deemed the winner. (Donald A. Pierre)

call it at a time greater than the implied holding period. The discount and premiums are measured relative to the (PEER).

Exhibit (I)

| State | $r^*_{[illegible]}$ | $h^*_{[illegible]}$ |
|---|---|---|
| Arizona | 11.65% | 1 year |
| California | 12.15% | 1 year |
| Florida | 12.55% | 1 year |
| Illinois | 11.95% | 1 year |
| Massachusetts | 12.25% | 1 year |
| New York | 13.25% | 1 year |
| Ohio | 11.15% | 2 years |
| Texas | 10.25% | 5 years |

We plot (PEER)/(PEHP) combinations in yield/holding period space. In the same space we plot the constant maturity yields on treasury securities for the date of June 16,1989. Investors require a higher yield from mortgage debt than they do for debt issued by the Federal government. Mortgage securities have specific risks which do not affect government debt. The most important of these risks are, 1) prepayment risk, and 2) default risk. Debt issued by the government can not be called prior to its maturity and there is virtually no possibility that the government will default on its obligations.

Graph I shows the yield curve for government debt on July 16,1989, (the data is reported in exhibit II), and the mortgage yield curve implied by the data in exhibit (I).

Exhibit II

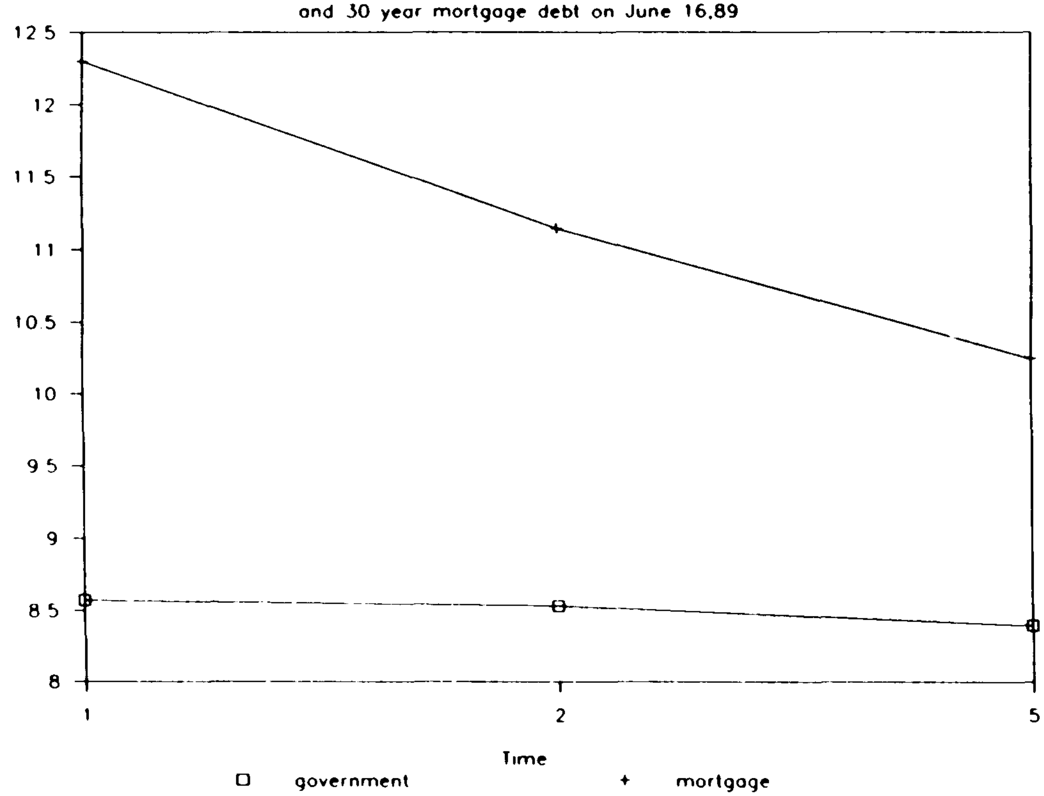
| | | |
|---|---|---|
| Government Debt | N | R |
| | 1 Year | 8.57% |
| | 2 years | 8.53% |
| | 3 years | 8.48% |
| | 5 years | 8.40% |

| | | |
|---|---|---|
| Mortgage Debt | $h^*$ | $r_m$ average[7] |
| | 1 year | 12.3% |
| | 2 years | 11.15% |
| | 5 years | 10.25% |

[7] $r_m$ is the average of the (PEER) for a specific (PEHP), across all states.

Graph I

The term structure for treasury debt
and 30 year mortgage debt on June 16,89



One indication of systematic regional differences in prepayment and default risks would be different (PEER) across states for the same (PEHP). We in fact observe a number of different (PEER) for a (PEHP) of one year.

According to the expectations theory of the term structure of interest rates, a downward sloping yield curve is caused by the anticipation that short term interest rates will fall (see Van Horne [15] and Bierwag [1]). If the cost of short term funds are correlated with mortgage interest rates, the downward sloping yield curve would lower the value of mortgages, by increasing the value of the mortgagor's prepayment option. The current (PEER) and (PEHP) may reflect the anticipation of a decline in short term rates of interest.

All of the (PEER) correspond to relatively short maturities.

All states except for Texas and Ohio have (PEHP) of one year. The spread between treasury debt and mortgage debt narrows as the (PEHP) increases.

Curley and Guttentag[5] calculate the internal yields on discounted mortgages. Rather than base the yield measurement on an assumed average life, they estimate the termination probabilities relevant to a mortgage of given characteristics (maturity,policy year, and the relationship between the contract rate on the mortgage and market yield in the specified policy year). Curley and Guttentag [5] note that the original discount at which the mortgage is issued raises the yield above the contract rate by a larger amount the shorter the face maturity (n), the higher the contract rate ($r_c$), and the shorter the prepayment period. They illustrate that larger initial discounts have a much larger impact on yield when the time to prepayment is relatively short.

Hendershott, Shilling and Villani [7] calculate the yield spread between new mortgage commitments and treasury securities. They adjust for the expected differences in the timing of the cash flows generated by the mortgage instrument and the treasury instrument in order to isolate the impact the following five factors have on the spread.

1) expected losses due to the probability and cost of adverse refinancing, assumptions and defaults. 2) aversion to the possibility of these occurring, 3) any additional tax burden from investment in mortgages (e.g. interest is taxable at the state and local level), 4) origination costs, if the yield measures the cost to the borrower rather than the yield to the ultimate investor and 5) market inefficiencies if any.

A conclusion we can draw from model (I) is that lenders in different states require different rates of return on mortgage capital of the same maturity and, the term structure for the cost of conventional mortgage capital has the same slope as the term structure for Treasury debt.

**Model (II)**

We test for geographical differences in the effective cost of mortgage capital across regions by regressing the effective rate implied by a chosen holding period, for each lender against the lender's state. In this model we do not simultaneously solve for the implied holding period (PEHP) and the implied effective rate of interest (PEER). We assume a holding period for all mortgagor's across all regions. A dummy variable is used to link the rate with the state. We run the regression model for a holding period of 12 years. The econometric model is presented in equation (1).

$$r_{ei}=\alpha_1+\alpha_2 AZ_i+\alpha_3 CA_i+\alpha_4 FL_i+\alpha_5 IL_i+\alpha_6 MA_i+\alpha_7 NY_i+\alpha_8 OH_i+\alpha_9 \%DWN+u_i \quad (1)$$

where $r_{ei}$ = effective rate for mortgage i estimated as follows:

$$\$187,000 = p(187,000) + \Sigma\ PMT/(1+r_e)^j + B_t/(1+r_e)^{12} \quad (2)$$

where \$187,000=par value

p=discount points

PMT=payments

$B_t$=outstanding balance at time t

j=1,...,12

$AZ_i$ = 1 if mortgage i is offered by a lender in Arizona

= 0 otherwise

$OH_i$ = 1 if mortgage i is offered by a lender in Ohio

= 0 otherwise

%DNW = percent down payment.

We do not assign a dummy variable to the state TX (Texas) to avoid the "dummy-variable trap"[*]. We arbitrarily chose the state of Texas as the base. The intercept $\alpha_1$ will reflect the coefficient of Texas. The remaining coefficients of the dummy variables ($\alpha_2 \ldots \alpha_8$) measure by how much the intercepts differ from the base state as follows:

the mean effective rate for Arizona is

$$E(r_e/AZ_i=1, CA_i=0, \ldots, OH_i=0, \%Dwn_i) = (\alpha_1+\alpha_2) + \alpha_9\%DWN_i \quad (3)$$

the mean effective rate for California is

$$E(r_e/AZ_i=0, CA_i=1, FL_i=0, \ldots, OH_i=0, \%DWN_i) = (\alpha_1+\alpha_3) + \alpha_9\%DWN_i \quad (4)$$

the mean effective rate for Texas is

$$E(r_e/AZ_i=0, CA_i=0, \ldots, OH_i=0, \%DWN) = \alpha_1 + \alpha_9\%DWN_i \quad (5)$$

[*]If a qualitative variable has m categories, introduce only (m-1) dummy variables. If this rule is not followed there may be perfect multicollinearity across the columns of the independent variable matrix.

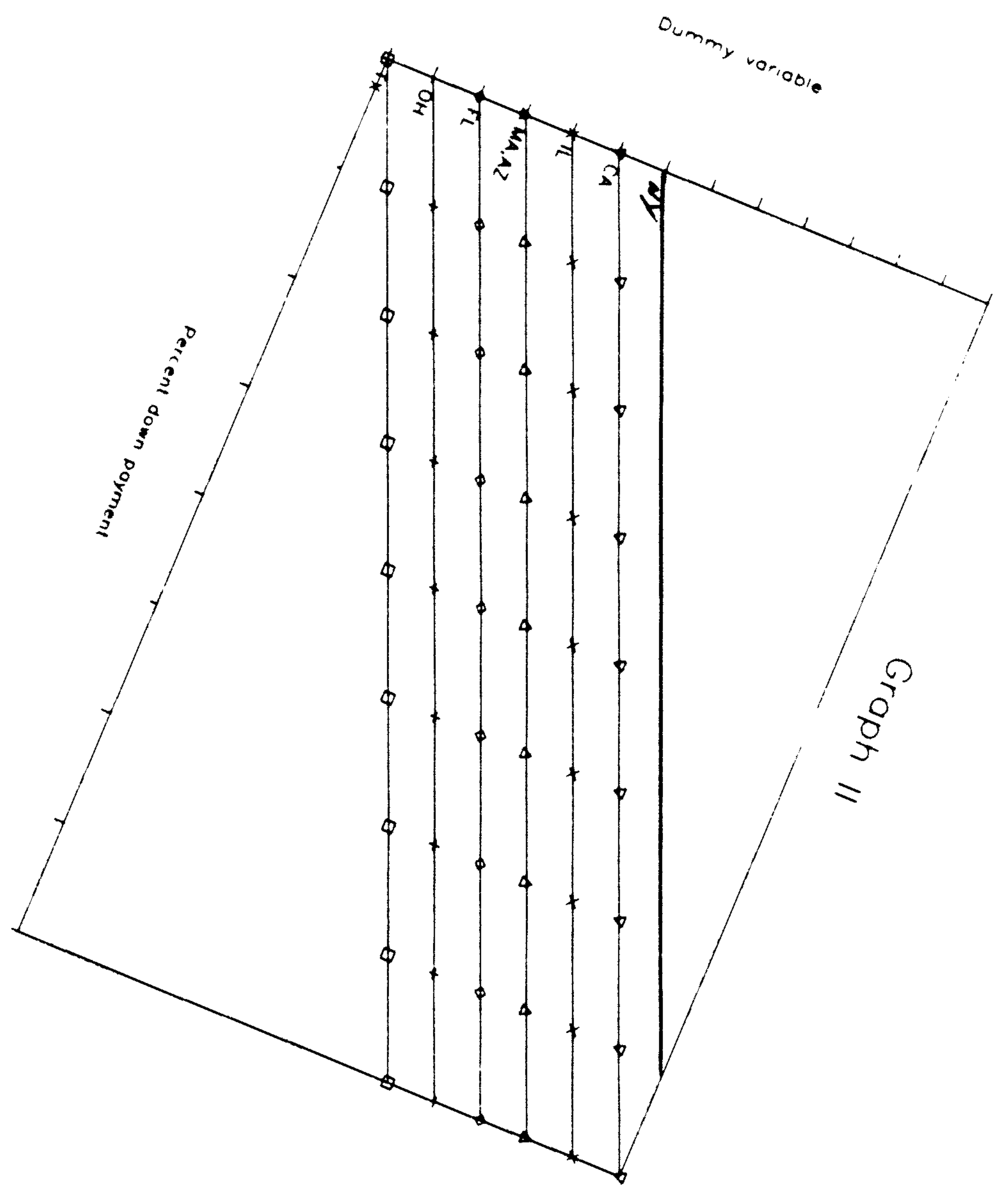
## IV. Results

Regression results of equation (1):

Exhibit III

| Independent Variables | Coefficients | T-statistics | |
|---|---|---|---|
| CONST | .098 | 79.49 | R=.53 |
| AZ | .0025 | 1.6 | |
| CA | .0034 | 2.0 | |
| FL | .002 | 1.28 | |
| IL | .0031 | 2.02 | |
| MA | .0025 | 1.66 | |
| NY | .0056 | 3.82 | |
| OH | .0014 | 0.089 | |
| %DNW | .0228 | 3.32 | |

## Graph II


Dummy variable
Percent down payment
OH
FL
MA,AZ
IL
CA
Graph II

The coefficients on the state dummy variables measure the amount by which the mean effective rate of a specific state differs from the mean effective rate of the base state Texas. The fact that all of the coefficient are positive illustrates that for a twelve year holding period Texas had the lowest effective rate.

We classify the states as those where the 12 year effective rate is statically significant and different from the base state (at the 5% level), and those states where the 12 year effective rate is not statistically different from the effective rate in the base state. (greater than the 5% level) (exhibit IV).

Meador[10] examines the effects that regional mortgage laws have on inter regional mortgage rate differentials. He finds that the geographical diversity in foreclosure laws does explain part of the observed regional rate differential.

Exhibit IV

12 YEAR EFFECTIVE RATE: BASE STATE IS TEXAS

| Statistically Different | Not Statistically Different |
|---|---|
| California | Arizona |
| Illinois | Florida |
| New York | Massachusetts |
| | Ohio |

The positive and significant coefficient on the variable representing the down payment as a percent of the home price is somewhat puzzling. It is taken for granted that ceteris paribus mortgagors who finance the purchase of a home with a low loan to value ratio are less likely to default, (see Meador). Mortgagors who are less likely to default are compensated by issuing their mortgages at better terms than mortgagors who are poorer credit risks. The contract rate/discount point combinations which we used to calculate the (PEER),(PEHP) and effective rates for a given holding period did not include charges for private mortgage insurance*. We use the variable ($EX_1$) as a proxy for the

*In order for a conventional mortgage to conform to FNMA underwriting standards, mortgages with initial loan to value ratios greater than 80% must be insured to a level such that FNMA is not exposed to the possibility of loosing more than 75% of the value of the mortgaged property. The value of the property for insurance purposes is defined as the lower of the properties sales price, or appraised value.
To calculate the amount to be privately insured, FNMA's requirement for a given down payment is obtained with the following formula:
[(Unpaid mortgage balance-75% of property's value)/mortgage balance]= required mortgage insurance as a percent of the initial mortgage balance. For a 5% down payment and a home value of $187,000, FNMA would require 22% of the original mortgage principal to be insured.
The lender's exposure to loss due to a mortgagor defaulting can be computed by subtracting the down payment plus the amount of the mortgage principal which is insured from the original value of the property:
Exposure=value of the property-(down payment+insured principal). In the above example the exposure would equal $138,567.
The lender's exposure is equal to 74.1% of the property's value.

default risk associated with each mortgage. We substitute ($EX_i$) for the variable (%DOWN) in regression (1). We make this substitution to correct for the bias caused by the interest rates which do not reflect the cost of private mortgage insurance. The results of this regression (regression 2) are presented in exhibit (V) and graph (III).

Exhibit V

| | |
|---|---|
| Log-Likelihood= | 237.58 |
| Restricted (Slopes=0)= | 219.3 |
| Chi-Squared (8) = | 36.56 |
| Significance Level = | .14495E-05 |
| R-Squared = | .51 |

| Independent Variables | Coefficients | T-statistics |
|---|---|---|
| Const | .103303 | 69.6 |
| AZ | .30E-02 | 1.92 |
| CA | .39E-02 | 2.38 |
| FL | .20E-02 | 1.26 |
| IL | .35E-02 | 2.23 |
| MA | .31E-02 | 2.01 |
| NY | .61E-02 | 4.24 |
| OH | .18E-02 | 1.15 |
| EX | -.29E-03 | -3.04 |

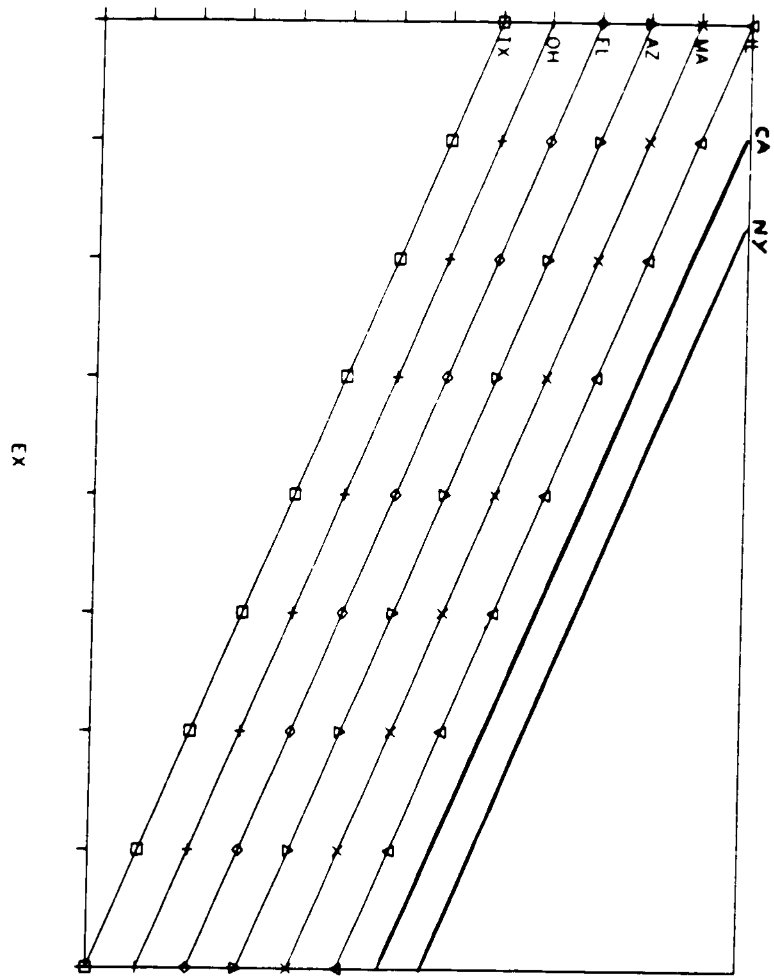
---

To proxy for the default risk to which a mortgagee is exposed we create the variable ($EX_i$). ($EX_i$) replaces %DNW in the "corrected regression".(exhibit V).

$EX_i$= (Exposure/Down payment) When the down payment equals the value of the property, $EX_i$=0. When the down payment equals zero, $EX_i$ is infinite.

## Graph III

Dummy variable



The sign of the variable which proxies for the down payment now conforms to what is observed in the market. Lenders require a premium for lending to mortgagor's who invest relatively low amounts of equity in the property which

is being financed. When the loan to value ration is high, the probability that the future value of the property will fall below the debt value and that default will occur, rises, (see Meador). The coefficients on all the states accept Florida increase relative to model (I). The coefficient on Florida is approximately unchanged. Massachusetts is significantly different from Texas in model (II).

We test for the significance of the state effect with the F-test. The restricted model is model (IIr) In model (IIr) all the dummy variables are set to zero simultaneously.

$$r_m = \beta_0 + \beta_1 EXP + e \qquad \text{(IIr)}$$

The unrestricted model is model (II).

The following formulation of the F-test is employed:

$$F(r,N-K-1) = \frac{(R^2_u - R^2_r)/r}{R^2_u/(N-K-1)}$$

where $R^2_u$ is the R-Squared for the unrestricted model (II),

$R^2_r$ is the R-squared for the restricted model (IIr),

r=number of restrictions (7): $\alpha_2 = \alpha_3 = \ldots = \alpha_8 = 0$

N-K-1=52-8-1

We obtain an F statistic equal to 3.06. The result of the F test implies that the restricted variables are simultaneously significant at the 5% level. We can conclude that there is a significant statistical relationship between the effective cost of mortgage capital and the state in which the mortgage capital is being supplied.

## V. Conclusion

We have constructed a mortgage yield curve for eight states by calculating the average, par equivalent yields, and par equivalent holding periods for a sample of mortgage terms which were being offered the week of June 16,1989. The term structure of mortgage rates has the same slope as the treasury yield curve. The treasury yield curve was downward sloping on June 16. As expected the mortgage yield curve is above the treasury yield curve. The (PEHP) were on the short side of the yield curve (from 1 to five years). We conjecture that this is a function of the negatively sloped treasury yield curve. Further research will examine the (PEHP) which exist when the yield curve is upward sloping. If the slope of the yield curve is a reasonable predictor of short term interest rates, we would expect the (PEHP) to be longer when the yield curve is upward sloping.

We do not find a unique mortgage yield curve. For a (PEHP) of one year there are multiple (PEER). This does not imply that the mortgage market is segmented. The implication of the multiple (PEER) for a specific (PEHP) is that the required rate of return on mortgage capital differs across regions. If two regions existed, one populated by mortgagors in the 10% tax bracket and the other inhabited by mortgagors in the 33% tax bracket, the prepayment risk in each region would be positively correlated but not equal. [Stone & Zissu]

Our regression results imply that the effective cost of mortgage capital between some states are significantly different and between other states statistically insignificant.

We can not claim to have proved that mortgage markets are segmented across regions because we have not controlled for regional differences in prepayment risk and default risk. It is interesting to note that the week of June 16, 1989, Citibank and Citicorp Savings (both subsidies of the Citicorp Holding Company) were offering at least eight different contract rate/discount point combinations across eight states for loans that required an initial 80% loan to value ratio.

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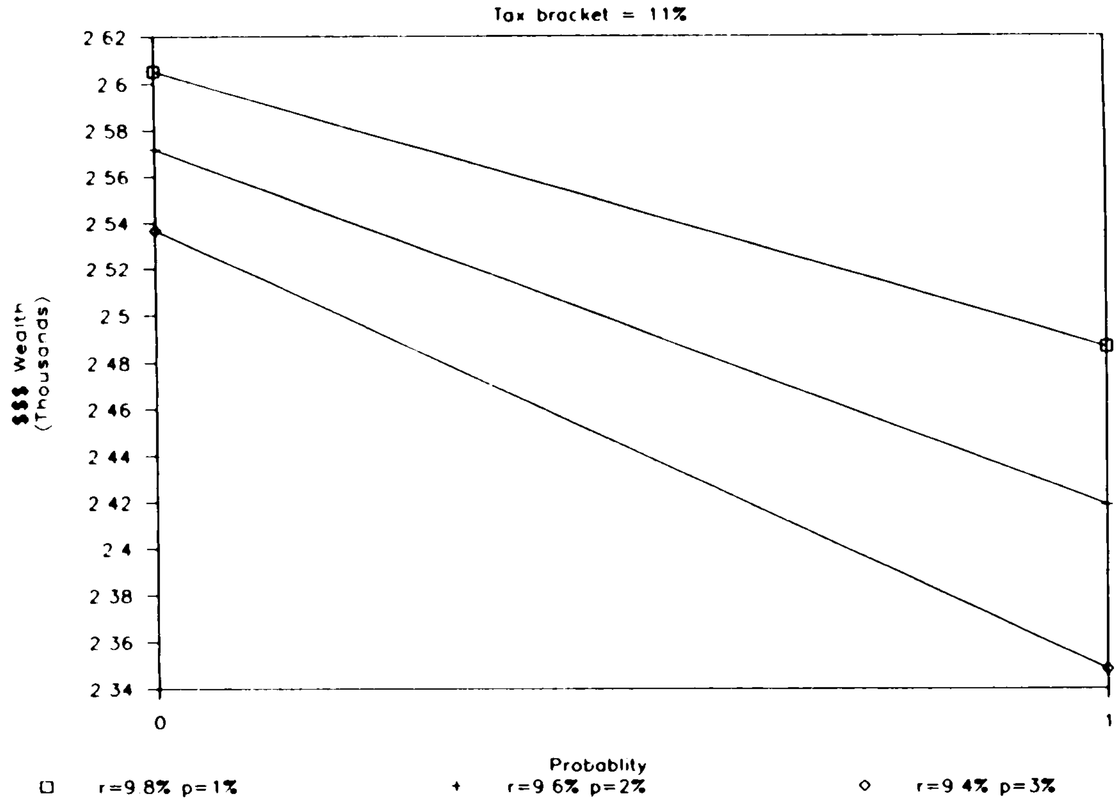
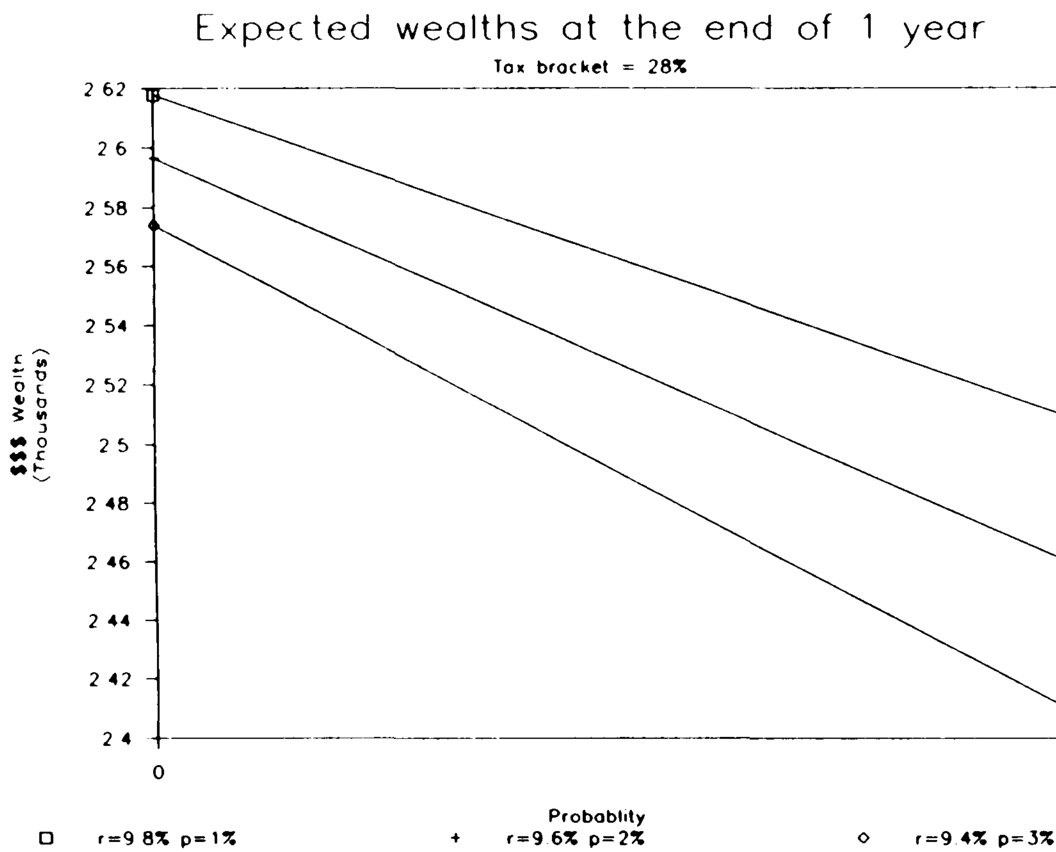
APPENDIX A

Series A

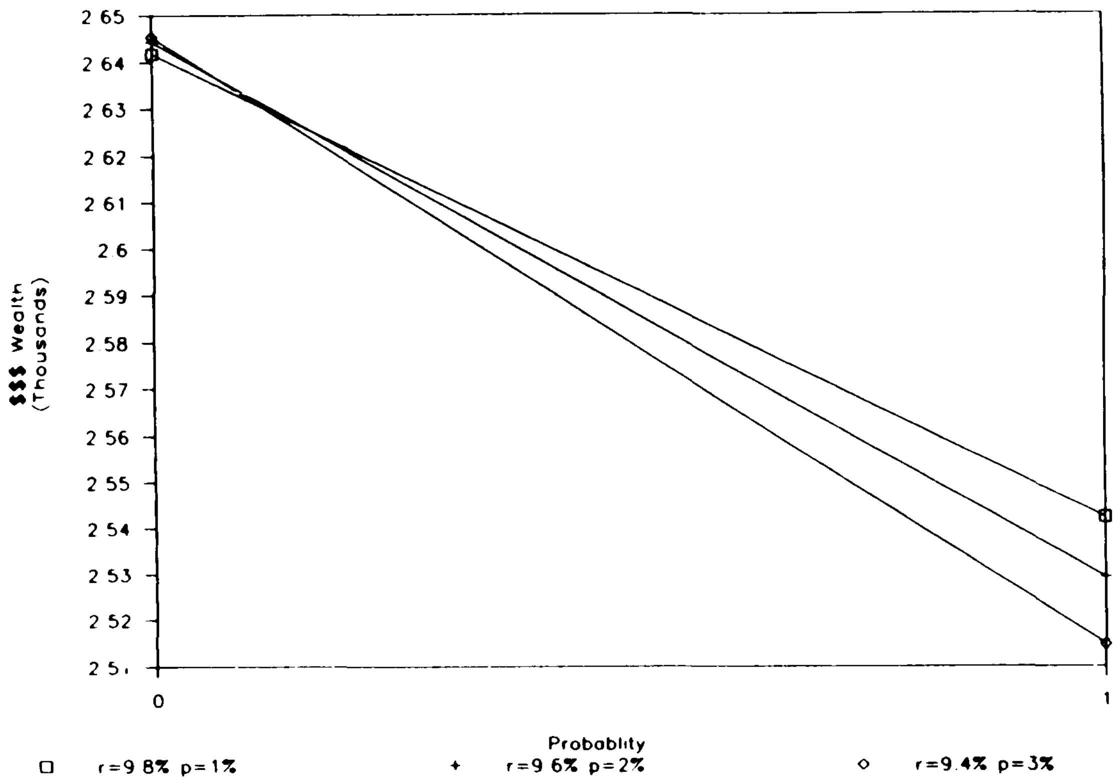
Expected wealths at the end of 1 year

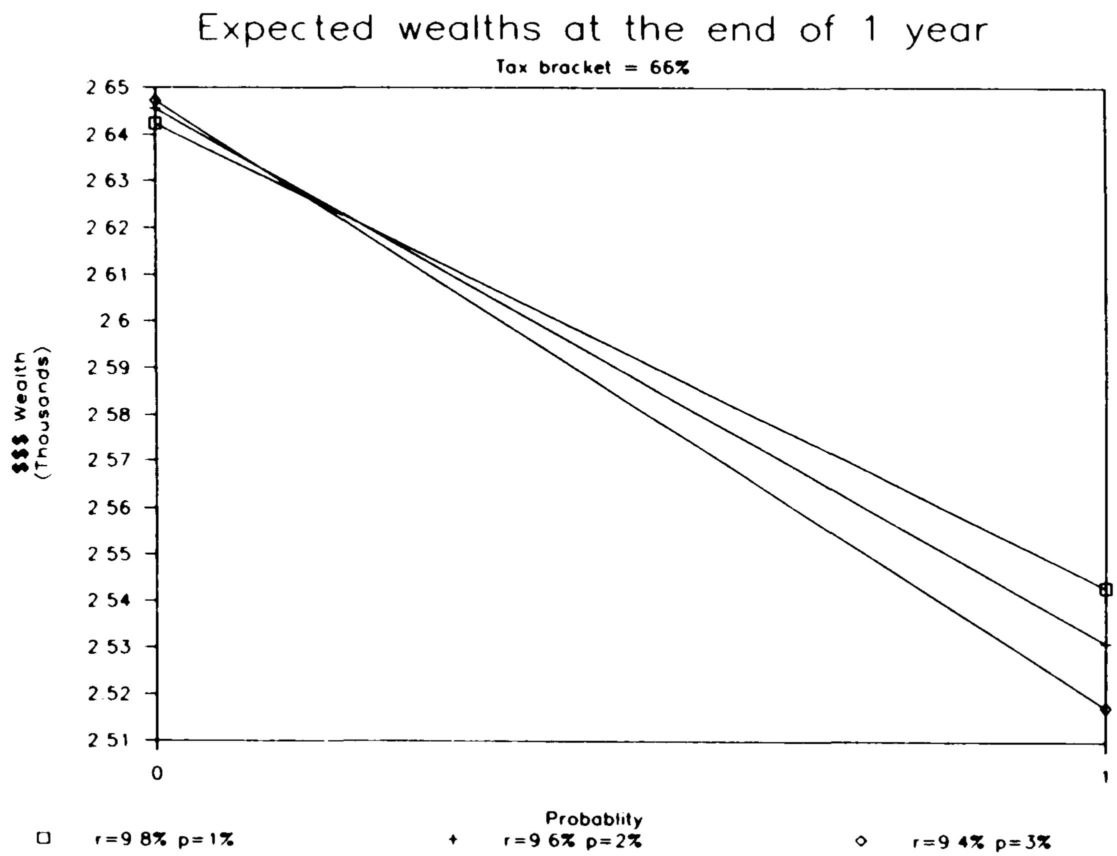


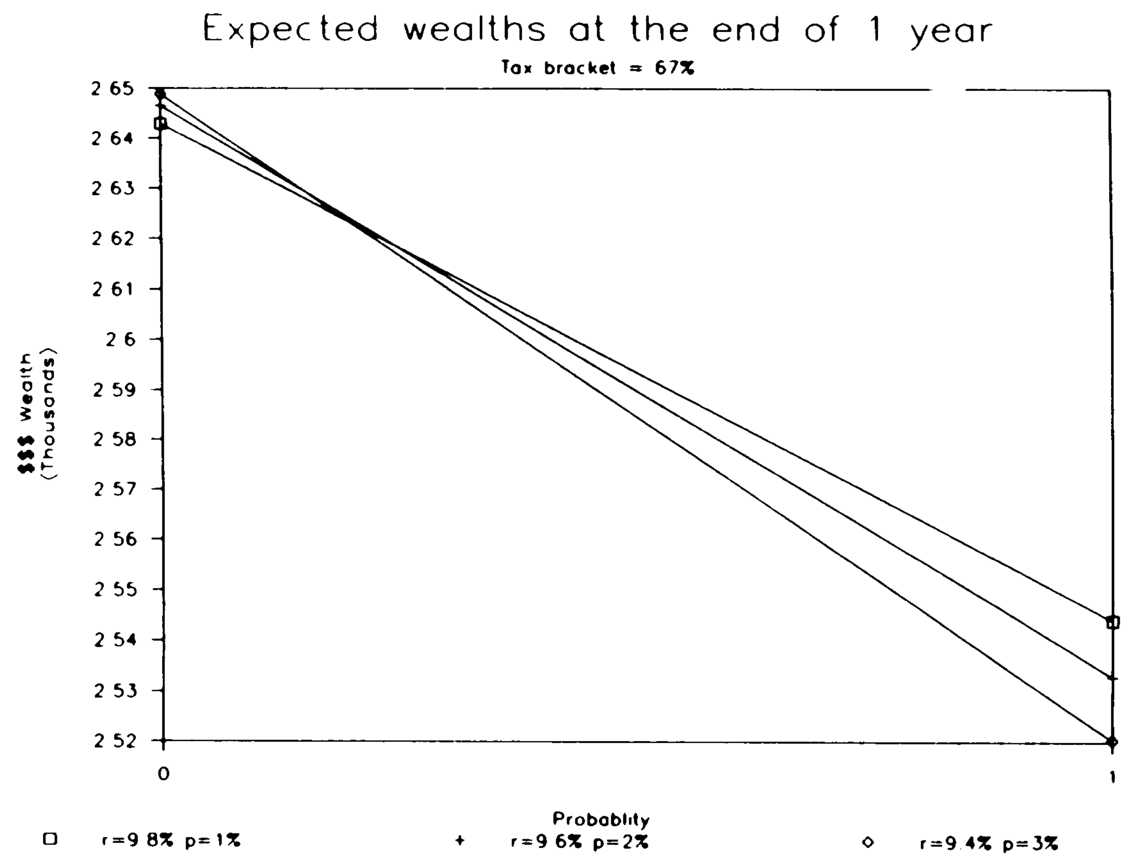

Expected wealths at the end of 1 year
Tax bracket = 28%
2.62
2.6
2.58
2.56
2.54
2.52
2.5
2.48
2.46
2.44
2.42
2.4
$$$ Wealth
(Thousands)
0
1
Probablity
r=9.8% p=1%
r=9.6% p=2%
r=9.4% p=3%

Expected wealths at the end of 1 year

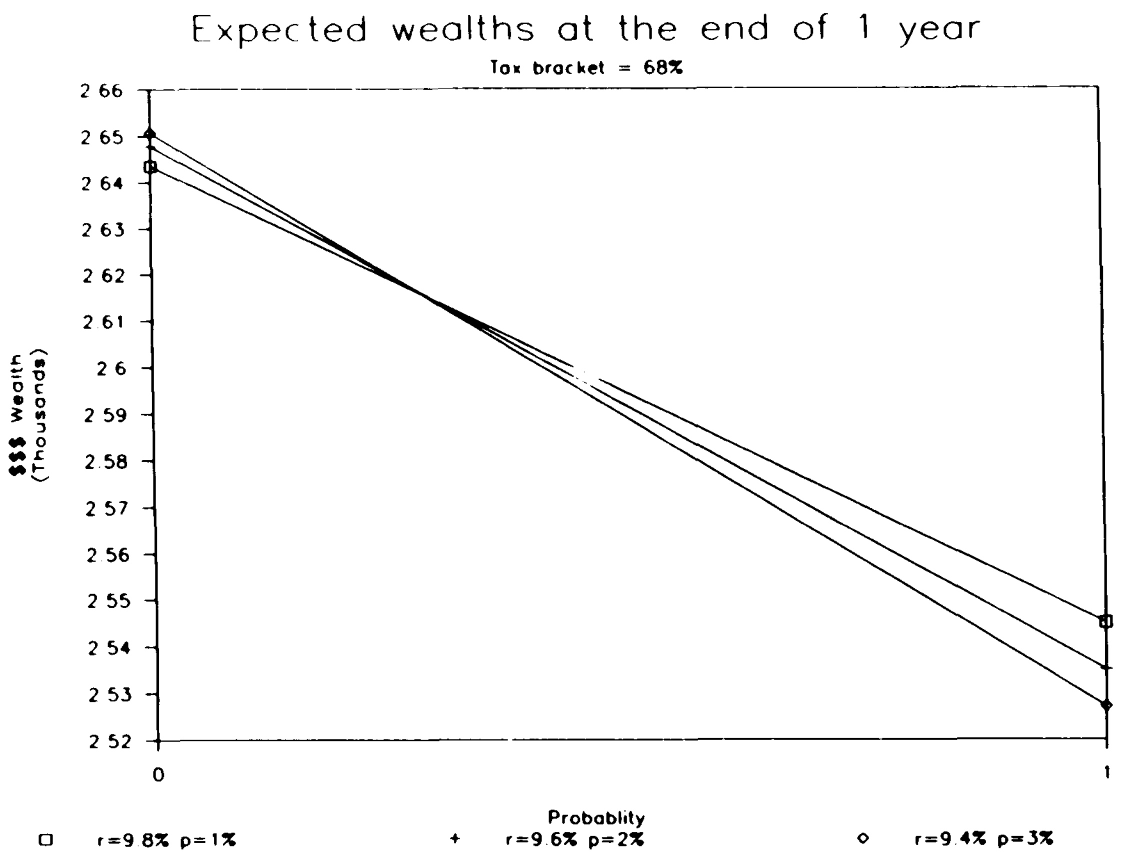
Tax bracket = 65%




Expected wealths at the end of 1 year
Tax bracket = 66%
$$$ Wealth
(Thousands)
2 65
2 64
2 63
2 62
2 61
2 6
2 59
2 58
2 57
2 56
2 55
2 54
2 53
2 52
2 51
0
1
Probablity
r=9 8% p=1%
r=9 6% p=2%
r=9 4% p=3%

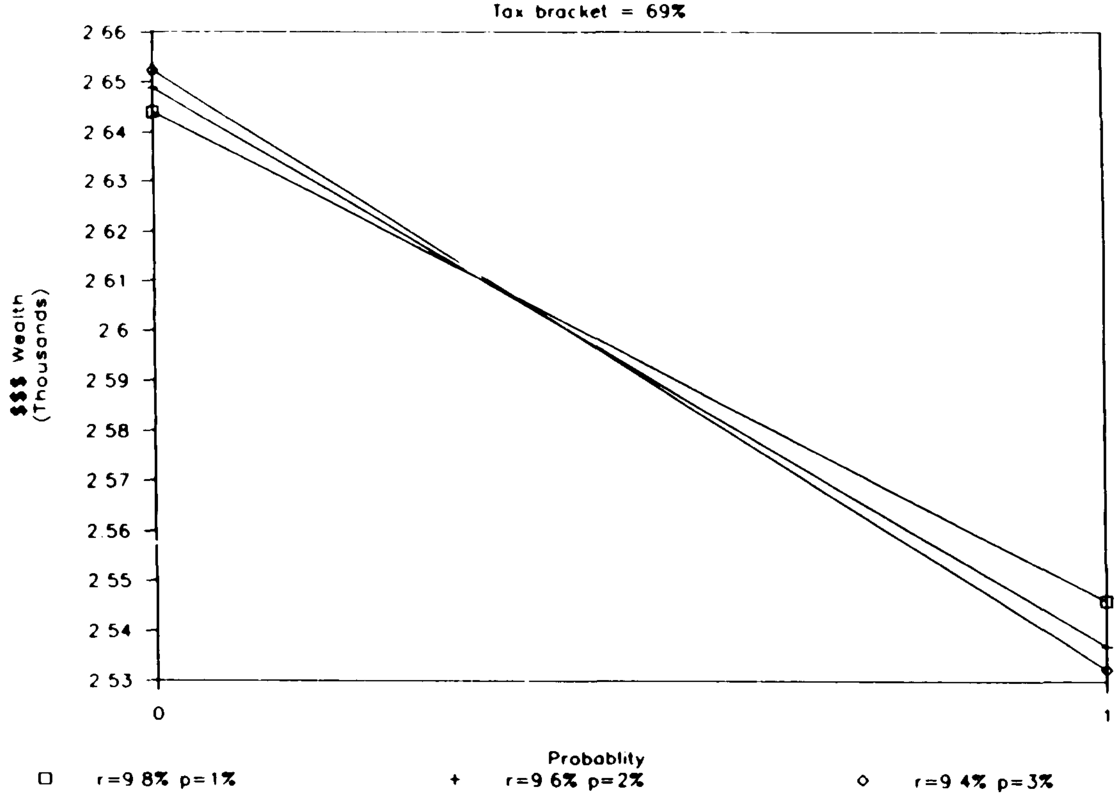


Expected wealths at the end of 1 year
Tax bracket = 67%
$$$ Wealth (Thousands)
2 65
2 64
2 63
2 62
2 61
2 6
2 59
2 58
2 57
2 56
2 55
2 54
2 53
2 52
0
1
Probablity
r=9 8% p=1%
r=9 6% p=2%
r=9.4% p=3%

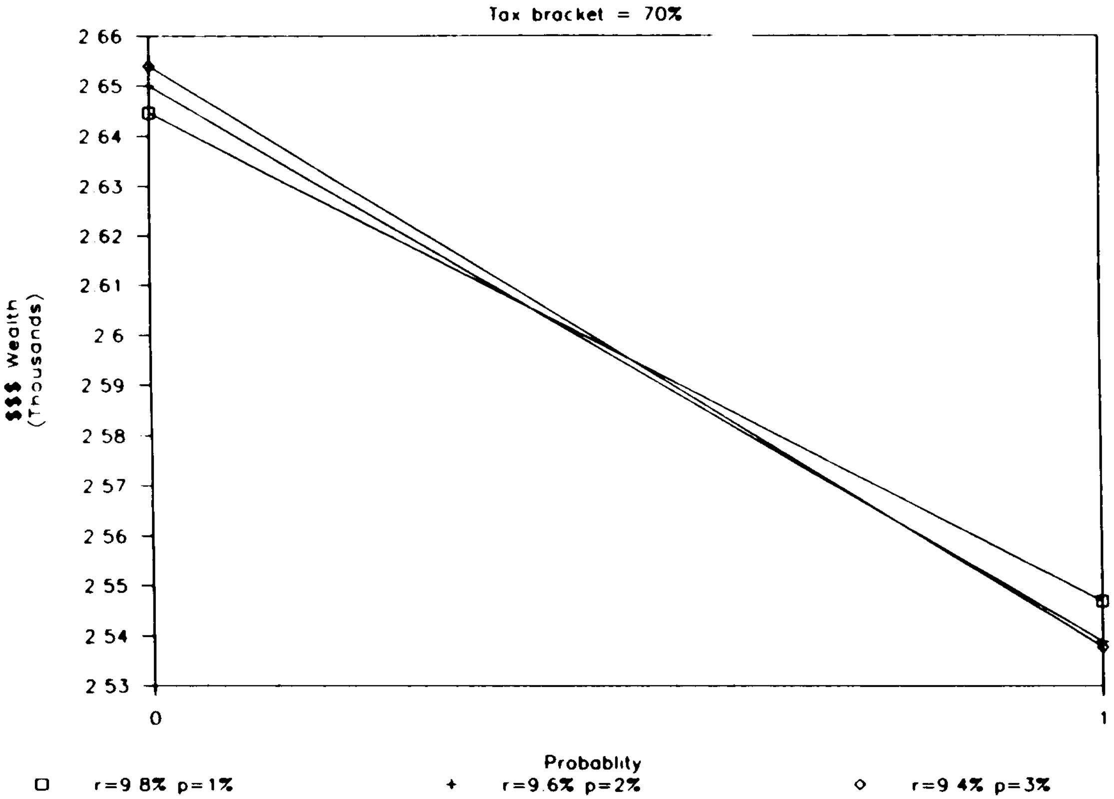

Expected wealths at the end of 1 year
Tax bracket = 68%
2 66
2 65
2 64
2 63
2 62
2 61
2 6
2 59
2 58
2 57
2 56
2 55
2 54
2 53
2 52
$$$ Wealth
(Thousands)
0
1
Probablity
r=9 8% p=1%
r=9 6% p=2%
r=9 4% p=3%

Expected wealths at the end of 1 year

Tax bracket = 69%

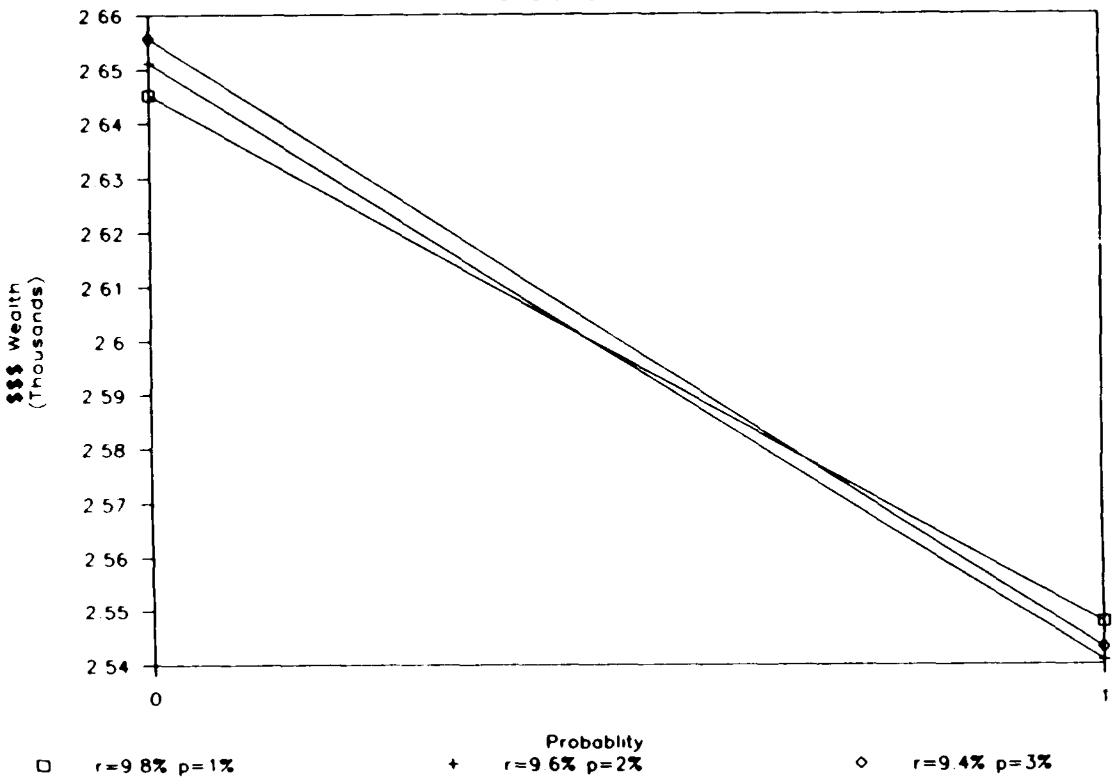


# Expected wealths at the end of 1 year



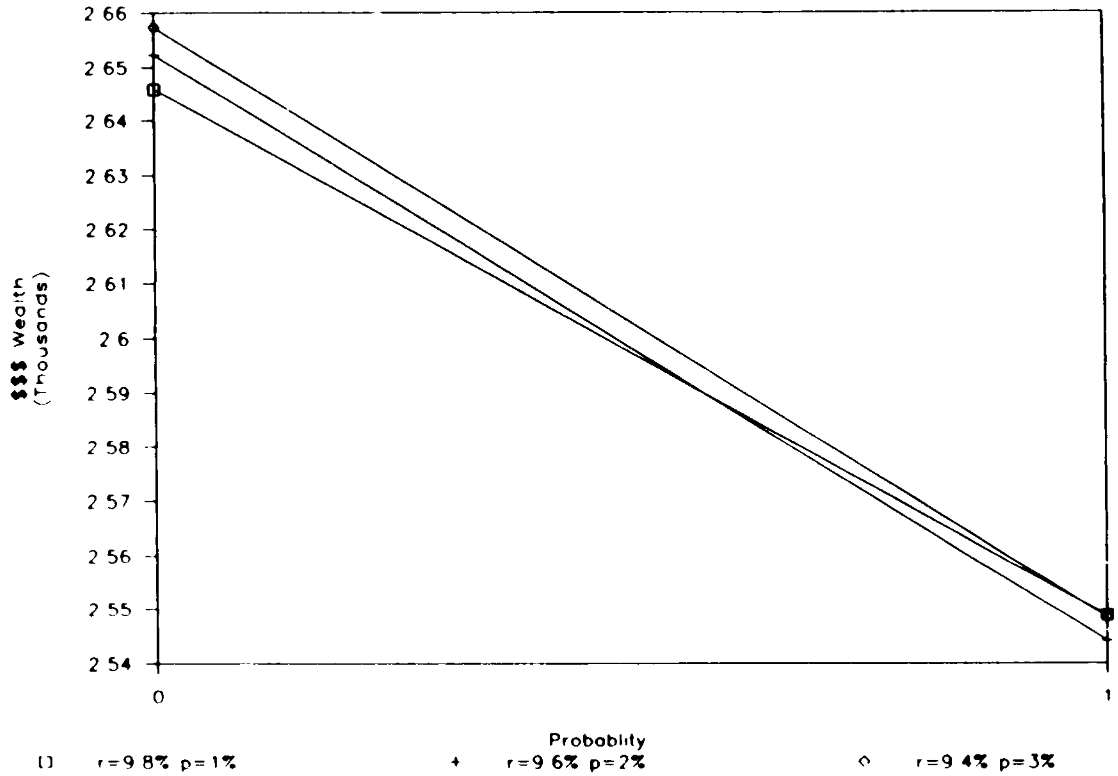
Expected wealths at the end of 1 year

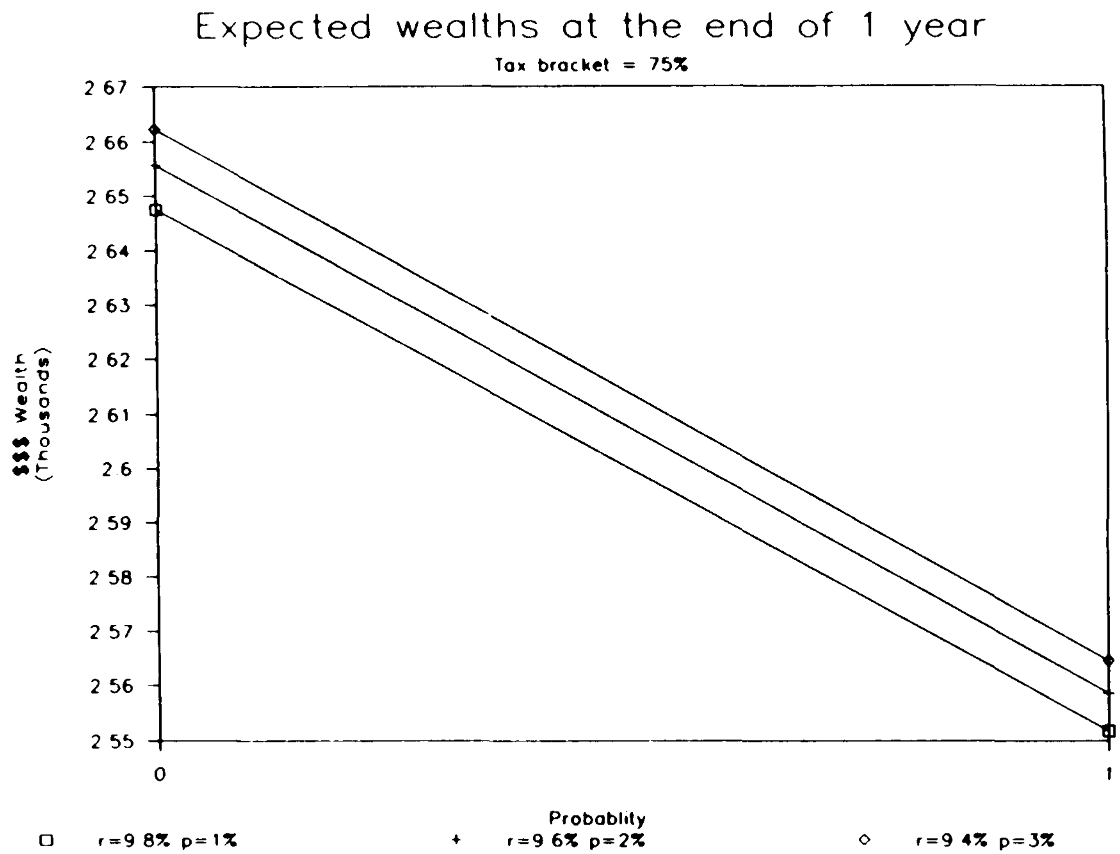
Tax bracket = 71%



Expected wealths at the end of 1 year

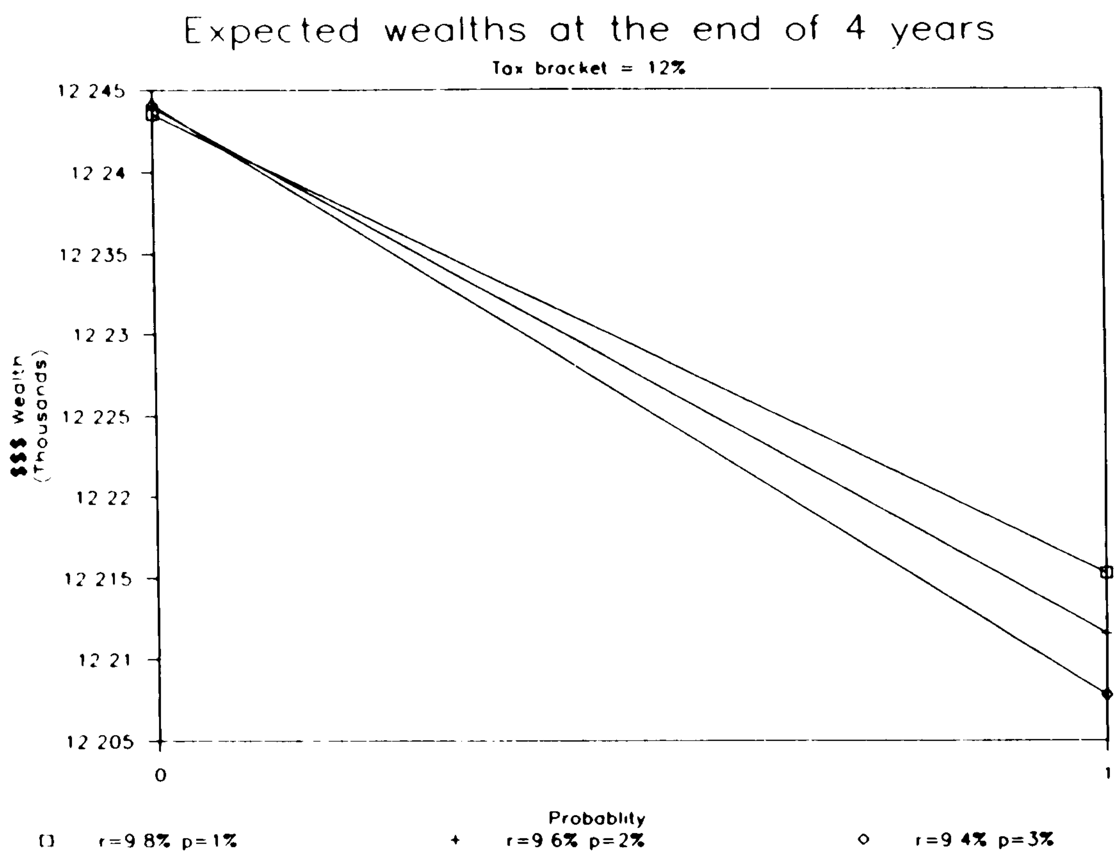
Tax bracket = 72%



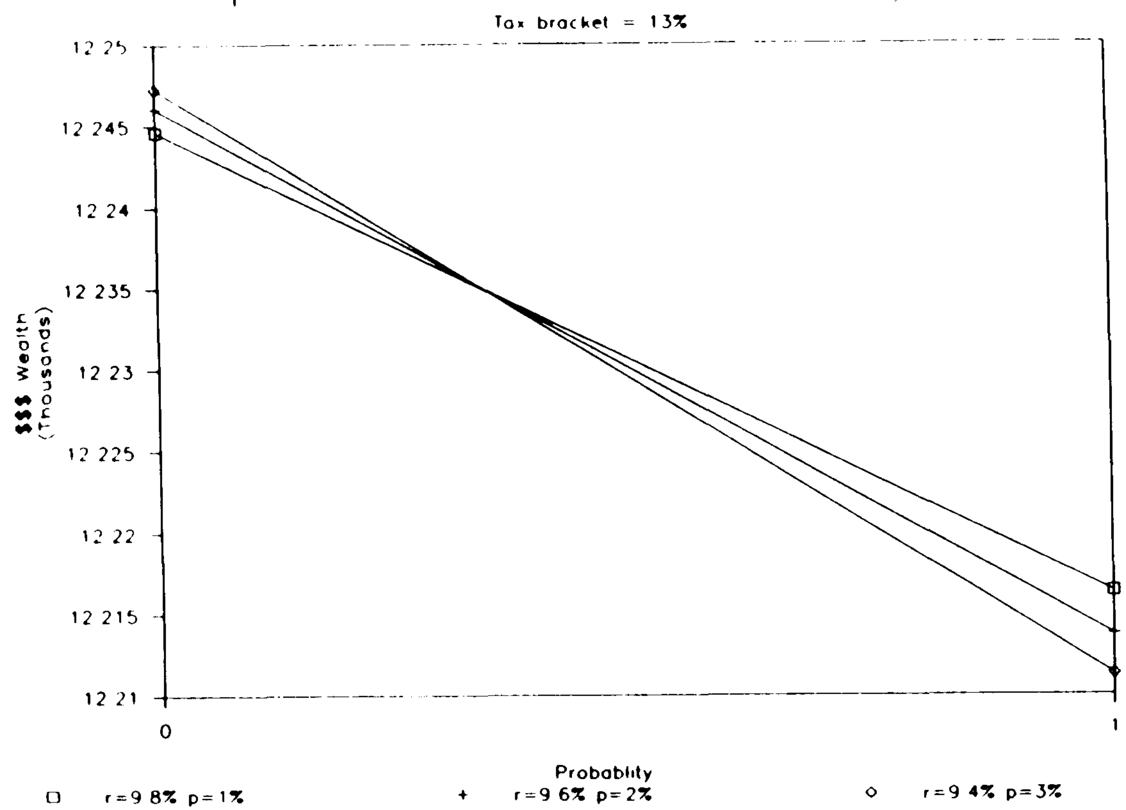

Expected wealths at the end of 1 year
Tax bracket = 75%
$$$ Wealth (Thousands)
2 67
2 66
2 65
2 64
2 63
2 62
2 61
2 6
2 59
2 58
2 57
2 56
2 55
0
1
Probablity
r=9 8% p=1%
r=9 6% p=2%
r=9 4% p=3%

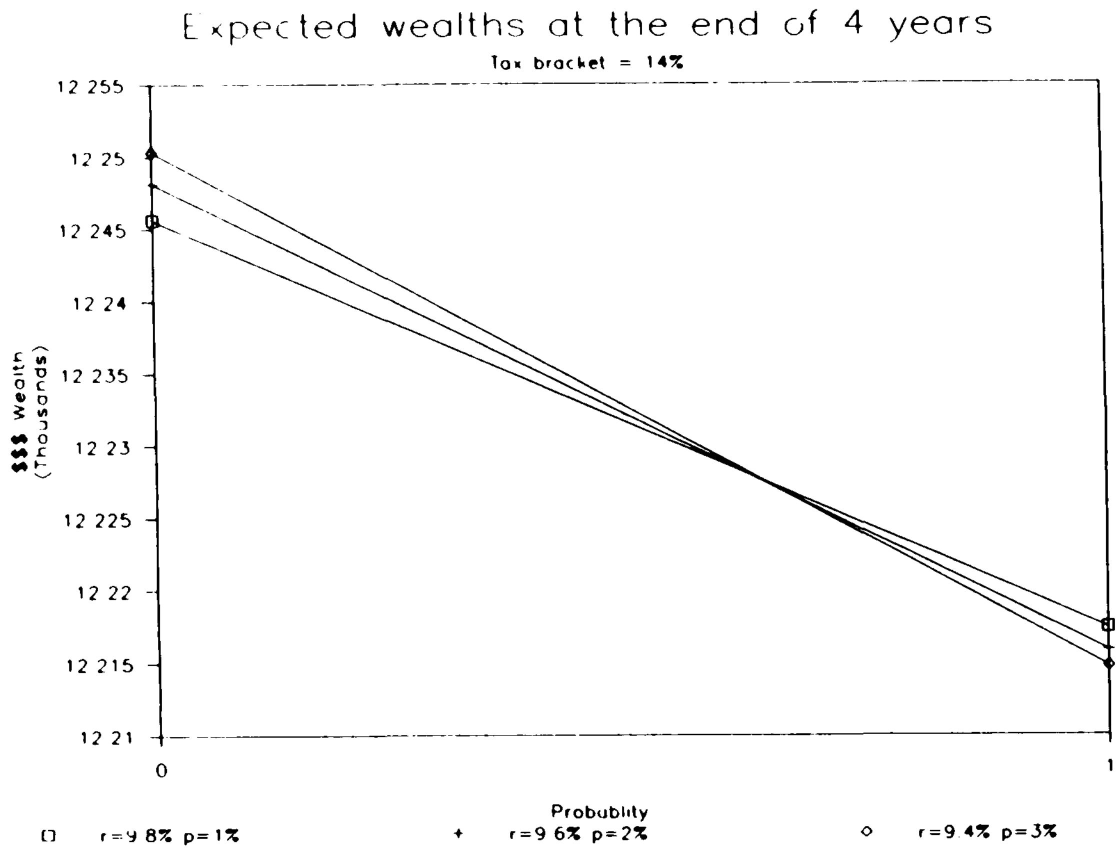
APPENDIX A

Series B

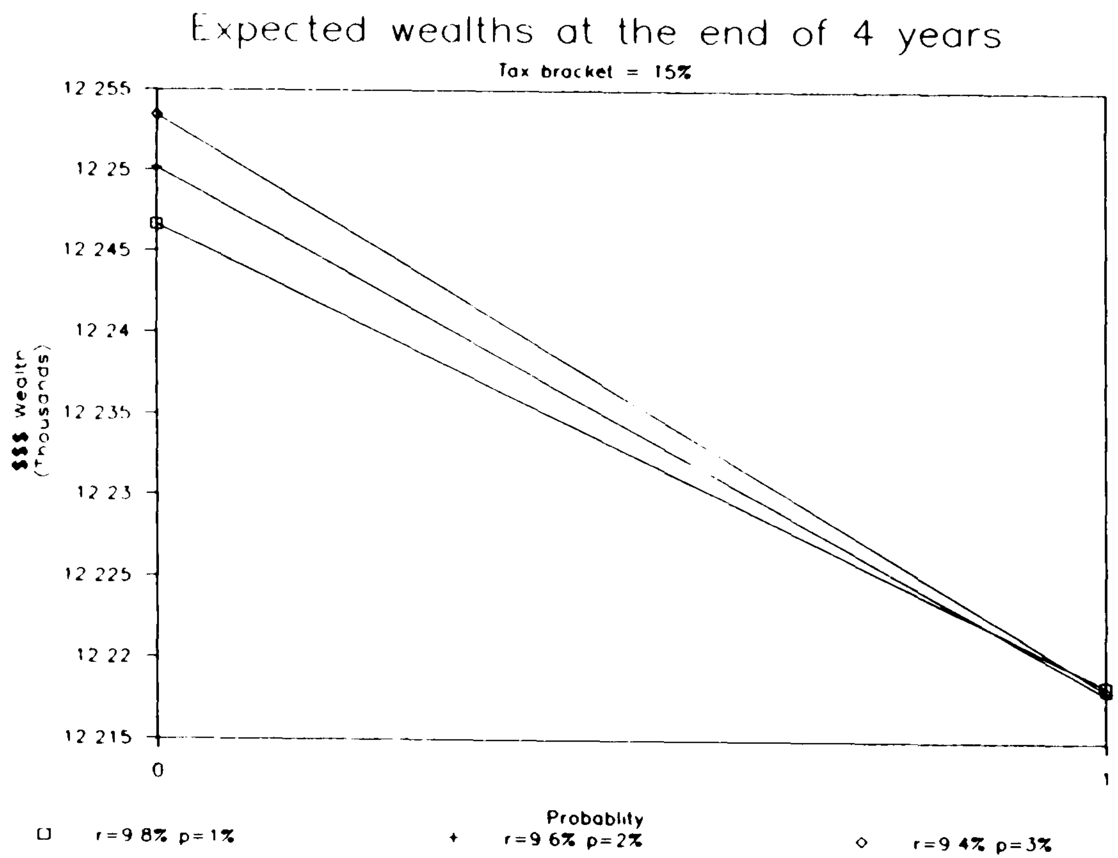

Expected wealths at the end of 4 years
Tax bracket = 12%
12 245
12 24
12 235
12 23
12 225
12 22
12 215
12 21
12 205
$$$ Wealth
(Thousands)
0
1
Probablity
r=9 8% p=1%
r=9 6% p=2%
r=9 4% p=3%

Expected wealths at the end of 4 years





Expected wealths at the end of 4 years
Tax bracket = 14%
12 255
12 25
12 245
12 24
12 235
12 23
12 225
12 22
12 215
12 21
$$$ Wealth
(Thousands)
0
1
Probublity
r=9 8% p=1%
r=9 6% p=2%
r=9.4% p=3%

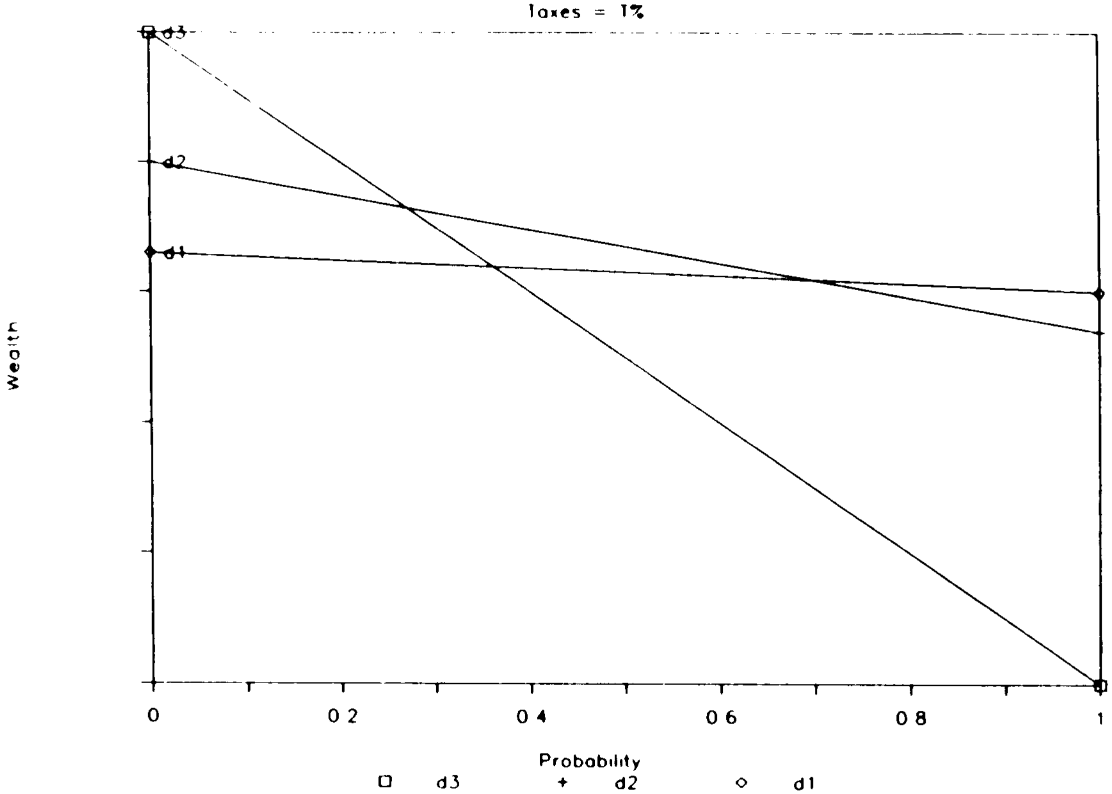

Expected wealths at the end of 4 years
Tax bracket = 15%
$$$ wealth
(Thousands)
12 255
12 25
12 245
12 24
12 235
12 23
12 225
12 22
12 215
0
1
Probablity
r=9 8% p=1%
r=9 6% p=2%
r=9 4% p=3%

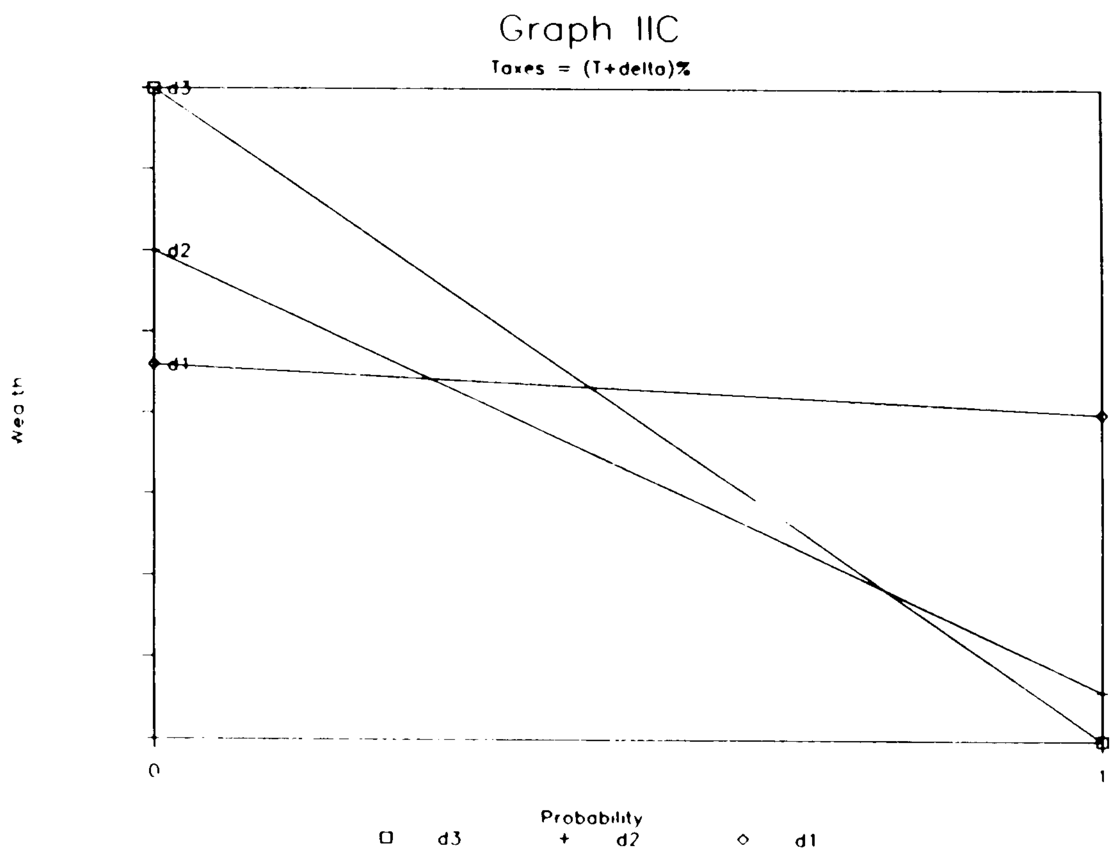
# APPENDIX A

Series C

## Graph Ic

Taxes = 1%




Graph IIC
Taxes = (T+delta)%
d3
d2
d1
Wealth
0
1
Probability
d3
d2
d1

Appendix B

The yearly payment of a 30 year conventional mortgage is the following:

$YP= B_o r(1+r)^n/[(1+r)^n-1]$ $(a_1)$,

which is obtained from the present value formula of an annuity:

$B_o = YP \sum_{t=1}^{n} [1/(1+r)^t]$ $(b_2)$

$YP_c$ = yearly payment for a conventional mortgage.

$B_o$ = initial balance.

r = before tax yearly interest rate on a mortgage.

n = number of years for a conventional mortgage (usually 30).

Equation (b1) can be separated into two components: the principal payment at time t, Princ(t), and the interest payment at time t, Int(t), as follows:

$Princ(t) = rB_o(1+r)^{t-1}/[(1+r)^n-1]$ $(b_3)$

$Int(t) = rB_o[(1+r)^n-(1+r)^{t-1}]/[(1+r)^n-1]$ $(b_4)$

The derivation of equation $(b_1)$,$(b_3)$, and $(b_4)$ can be found in the appendix C.

The after tax present value of the conventional mortgage is then obtained using equation $(b_3)$ and $(b_4)$ as follows:

$PV=rB_o[(1+r)^n-1]^{-1}\{\sum_{t=1}^{n}[1+(1-T)r_m]^{-t}\{(1-T)[(1+r)^n-(1+r)^{t-1}]+(1+r)^{t-1}\}\}$ $(b_5)$

Appendix C

Derivation of equation ($b_1$):

$B_0 = YP\ \Sigma\ [1/(1+r)^t]$ ($c_1$)

after simplifying ($c_1$) we have:

$B_0 = YP[(1+r)^n - 1]/r(1+r)^n$ ($c_1$)

from ($c_1$) we can derive YP as follows:

$YP = B_0 r(1+r)^n/[(1+r)^n - 1]$ ($c_2$)

Derivation of equation ($b_3$):

Each principal payment is equal to the yearly payment YP minus the interest rate (r) times the remaining balance.

$Princ(1) = \{rB_0(1+r)^n/[(1+r)^n - 1]\} - rB_0$ ($c_3$)

after simplifying ($c_3$) we have:

$Princ(1) = rB_0/[(1+r)^n - 1]$ ($c_3'$)

$Princ(2) = \{rB_0(1+r)^n/[(1+r)^n - 1]\} - r\{B_0 - rB_0/[(1+r)^n - 1]\}$ ($c_4$)

after simplifying ($c_4$) we have:

$Princ(2) = rB_0(1+r)/[(1+r)^n - 1]$ ($c_4'$)

$Princ(3) = \{rB_0(1+r)^n/[(1+r)^n - 1]\} -$

$r\{B_0 - rB_0/[(1+r)^n - 1] - rB_0(1+r)/[(1+r)^n - 1]\}$ ($c_5$)

after simplifying ($c_5$) we have:

$Princ(3) = rB_0(1+r)^2/[(1+r)^n - 1]$ ($c_5'$)

We can generalize ($c_3'$), ($c_4'$), and ($c_5'$) as follows:

$Princ(t) = rB_0(1+r)^{t-1}/[(1+r)^n - 1]$ ($c_6$)

Derivation of equation ($b_4$):

Each interest payment is equal to the yearly payment at time (t) minus the principal payment at time (t).

$$Int(t)=YP_c(t)-Princ(t) \qquad (c_7)$$

eq.($b_4$)=eq.($b_1$)-eq.($b_3$) as follows:

$$Int(t)=rB_o(1+r)^n/[(1+r)^n-1]-rB_o[(1+r)^{t-1}/[(1+r)^n-1] \qquad (c_8)$$

simplifying ($c_8$) we have:

$$Int(t)=rB_o[(1+r)^n-(1+r)^{t-1}]/[(1+r)^n-1] \qquad (b_4)$$