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The command generator tracker approach to model reference adaptive control of multi-input multi-output plants

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THE COMMAND GENERATOR TRACKER APPROACH TO MODEL REFERENCE ADAPTIVE CONTROL OF MULTI-INPUT MULTI-OUTPUT PLANTS

by

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ABSTRACT

The Command Generator Tracker Approach

To Model Reference Adaptive Control

of Multi-input Multi-output Plants

by

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Adviser: Professor Kenneth M. Sobel

The command generator tracker approach to model reference adaptive control has become a very efficient method for controlling plants with unknown or partially known parameters. However, most of the earlier work in this area only considers linear time invariant plants which are almost strictly positive real or which can be made almost strictly positive real through feedforward augmentation.

This dissertation presents six new adaptive algorithms for the control of both linear time invariant and a class of nonlinear multi-input multi-output plants. For linear time invariant plants which are not almost strictly positive real, we use supplementary dynamics which are inserted into the adaptive controller in various locations to form

three different algorithms. Furthermore, we propose two additional algorithms in which the parameters of the supplementary dynamics are computed on-line as part of the adaptive mechanism.

For nonlinear plants, we propose an adaptive algorithm to control a plant with nonlinearities of known form, but with unknown parameters.

Design rules are described for each adaptive control algorithm. A stability analysis is provided which shows that the output tracking error will be bounded in the presence of bounded input and output disturbances, and asymptotic output tracking will be achieved under some additional conditions.

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1. INTRODUCTION

1.1 Background

Adaptive control has evolved as an attempt to implement high performance control systems when the plant dynamics are poorly known. Among numerous alternative adaptive control methods, the use of the technique known as model reference adaptive control (MRAC), developed in the later 1950's, seems to be a feasible approach for the implementation of adaptive control systems. One of the prime innovations of this technique is the presence of a reference model which specifies the desired performance. The objective of the adaptive algorithm in model reference adaptive control is to find an adaptive control law in order to make the plant output track the model output.

Model reference adaptive methods may be classified as evolving from three different strategies. First is the full state access method described by Landau [1] which requires that the state variables are measurable. Second is the input-output method which originates from Monopoli's augmented error signal concept [2] in which adaptive observers are incorporated into the controller to overcome the inaccessible plant states. Third is the command generator tracker (CGT) approach originated by Sobel, Kaufman, and Mabius [3], which does not require explicit identification of the plant.

For more than a decade, considerable research has been done in the area of the command generator tracker approach to model reference adaptive control. However, most research work has concentrated on linear time invariant (LTI) plants which satisfy some strictly positive real condition. Recently, linear-time varying (LTV) and nonlinear plants have been considered, but the conditions which the plants must satisfy are still restrictive.

In this research, several adaptive control algorithms for not almost strictly positive real (non-ASPR) LTI plants and a class of nonlinear plants are discussed. For non-ASPR LTI plants, we insert supplementary dynamics into the adaptive controller to form five different algorithms. As a significant contribution, we neither require the supplementary dynamics to be ASPR and nonsingular nor do we utilize feedforward augmentation. Therefore, a large class of plants can be controlled and asymptotic output tracking becomes possible. For nonlinear plants, we extend the adaptive algorithm to plants with nonlinearities of known form but multiplied by unknown parameters. Unlike previous work, we do not use either virtual linearization or full state feedback nor do we restrict the nonlinearities to be bounded.

1.2 Historical Review

1.2.1 Algorithms for SPR and ASPR Plants

The CGT based MRAC approach was originated by Sobel, Kaufman, and Mabius [3] for a linear time invariant (LTI) almost strictly positive real (ASPR) plant. That is, for a plant represented by the triple $\{A_p, B_p, C_p\}$, there exists a feedback gain $\tilde{K}_{pe}$ such that $\{A_p - B_p\tilde{K}_{pe}C_p,\ B_p,\ C_p\}$ is strictly positive real (SPR).

The plant described in reference 3 is as follows

$$\dot{x}_p(t) = A_p x_p(t) + B_p u_p(t) \tag{1.1}$$

$$y_p(t) = C_p x_p(t) \tag{1.2}$$

where $x_p(t) \in R^n$, $u_p(t) \in R^m$ and $y_p(t) \in R^m$. The plant matrices A_p and B_p are not explicitly known and only the output of the plant is accessible.

An asymptotically stable reference model, which generates the desired performance, is described by [3]

$$\dot{x}_m(t) = A_m x_m(t) + B_m u_m(t) \tag{1.3}$$

$$y_m(t) = C_m x_m(t) \tag{1.4}$$

where $x_m(t) \in R^{n_m}$, $u_m(t) \in R^m$ and $y_m(t) \in R^m$.

It is allowable that

$$\dim[x_p(t)] >> \dim[x_m(t)] \tag{1.5}$$

The objective of the adaptive control algorithm is to seek, with limited knowledge of the plant, the adaptive control law $u_p(t)$ such that the plant output $y_p(t)$ asymptotically tracks the model output $y_m(t)$. When perfect output tracking occurs (i.e. when $e_{yp}(t)=y_m(t)-y_p(t)=0$ and $\dot{e}_{yp}(t)=0$), the corresponding state, output, and input trajectories are defined to be the ideal plant state, ideal plant output, and ideal plant input, denoted by $x_p^*(t)$, $y_p^*(t)$, and $u_p^*(t)$ respectively. In other words, the ideal trajectories will be maintained if the plant is forced by the ideal input.

Mathematically,

$$\dot{x}_p^*(t) = A_p x_p^*(t) + B_p u_p^*(t) \tag{1.6}$$

$$y_p^*(t) = C_p x_p^*(t) = y_m(t) \tag{1.7}$$

However, for the linear model following problem, this ideal situation cannot always be obtained because it is not reasonable to assume that the plant output will follow an arbitrary model output. A simple example is that the type 1 plant may not be able to track a parabolic function. Therefore, assumptions for either the plant or the model will have to be made to ensure that the ideal plant input exists.

Sobel, Kaufman, and Mabius [3] adopted Broussard's [4] CGT assumptions which assume that the ideal trajectories are linear functions of the model state and model input. Mathematically,

$$\begin{bmatrix} x_p^*(t) \\ u_p^*(t) \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} x_m(t) \\ u_m \end{bmatrix} \tag{1.8}$$

A solution for the S_{ij} in Eq.(1.8) exists provided that (i) u_m is a constant, and (ii) the plant and model satisfy the following restrictions:

$$\begin{bmatrix} S_{11}A_m & S_{11}B_m \\ C_m & 0 \end{bmatrix} = \begin{bmatrix} A_p & B_p \\ C_p & 0 \end{bmatrix} \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \tag{1.9}$$

When $y_p(t)$ differs from $y_m(t)$ at $t=0$, we may achieve asymptotic tracking provided that a stabilizing feedback is included in the control law. Therefore, in steady state, the output error will approach zero and the plant will be excited by the ideal input described in Eq.(1.8). To see this, Sobel, Kaufman, and Mabius [3] defined the state error and output error as follows:

$$e_{xp}(t) = x_p^*(t) - x_p(t) \tag{1.10}$$

$$e_{yp}(t) = y_p^*(t) - y_p(t) \tag{1.11}$$

If the plant were known, then the control law would simply be chosen as

$$u_p(t) = \tilde{K}_{pe} e_{yp}(t) + \tilde{K}_{px} x_m(t) + \tilde{K}_{pu} u_m \qquad (1.12)$$

where $\tilde{K}_{pe}$, $\tilde{K}_{px}$, and $\tilde{K}_{pu}$ are constant matrices, and where $\tilde{K}_{pe}$ is chosen to stabilize the plant. When the output tracking error approaches zero, Eq.(1.12) becomes

$$u_p(t) = \tilde{K}_{px} x_m(t) + \tilde{K}_{pu} u_m \qquad (1.13)$$

Therefore, if a solution for the S_{ij} in Eq.(1.8) exists, then the gain matrices will be chosen as $\tilde{K}_{px}=S_{21}$ and $\tilde{K}_{pu}=S_{22}$ such that the plant will be excited by the ideal input $u_p^*(t)$. However, for most adaptive control problems, knowledge of the plant is not explicitly known. In this case, the explicit values of the constant gain matrices cannot be determined. Hence, an adaptive control law must be employed which is in the form of [3]

$$u_p(t) = K_{pe}(t)e_{yp}(t)+K_{px}(t)x_m(t)+K_{pu}(t)u_m \qquad (1.14)$$

with the gain matrices $K_{pe}(t)$, $K_{px}(t)$ and $K_{pu}(t)$ being adaptive.

Define:

$$K(t) = [K_{pe}(t) \; K_{px}(t) \; K_{pu}(t)] \qquad (1.15)$$

and

$$r^T(t) = [e^T_{yp}(t) \quad x^T_m(t) \quad u^T_m] \qquad (1.16)$$

Then, the control law becomes

$$u_p(t) = K(t)r(t) \qquad (1.17)$$

The adaptive gains are computed as a combination of an integral gain and a proportional gain as shown below [3]

$$K(t) = K^{P}(t) + K^{I}(t) \tag{1.18}$$

$$K^{P}(t) = v(t)r^{T}(t)\bar{T} \tag{1.19}$$

$$\dot{K}^{I}(t) = v(t)r^{T}(t)T \tag{1.20}$$

$$v(t) = e_{yp}(t) \tag{1.21}$$

where T is a positive definite symmetric matrix and $\bar{T}$ is a positive semi-definite symmetric matrix.

A block diagram of the adaptive algorithm for ASPR plant is shown in Figure 1.1.

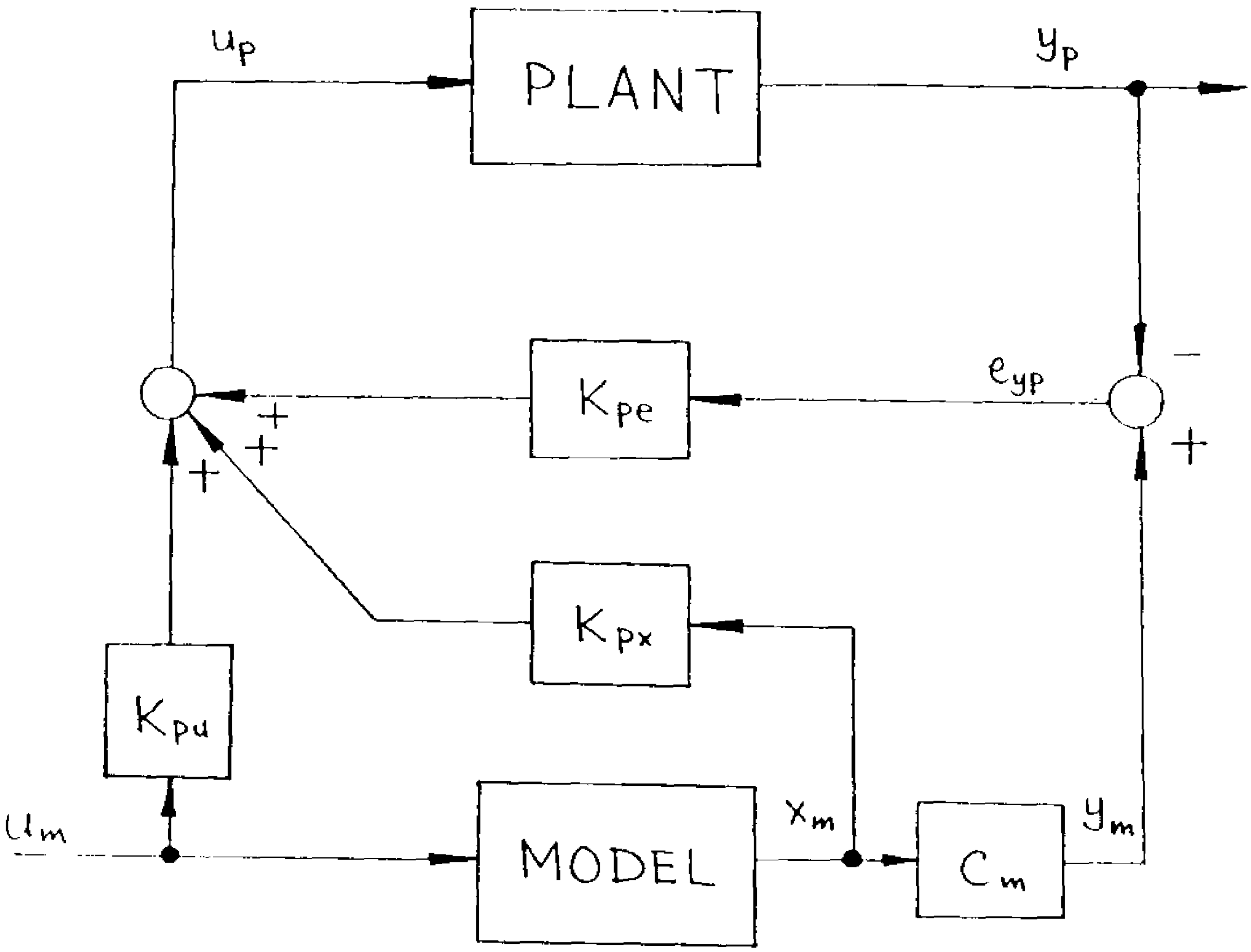

Figure 1.1 Adaptive Control for the ASPR Plant

In summary, the plant output will follow the reference model output under the assumptions that the model command is a constant signal, the ideal input exists, and the satisfaction of the sufficient conditions for stability as follows [3]:

$$(1)\ P(A_p - B_p\tilde{K}_{pe}C_p) + (A_p - B_p\tilde{K}_{pe}C_p)^T P = -LL^T \qquad (1.22)$$

$$(2)\ PB_p = C_p^T \qquad (1.23)$$

where P is a real symmetric positive definite matrix and L is a real matrix. Eqs.(1.22) and (1.23) are equivalent to requiring that the transfer matrix

$$C_p(sI - A_p + B_p\tilde{K}_{pe}C_p)^{-1}B_p \quad \text{is SPR} \qquad (1.24)$$

Later, Bar-Kana and Kaufman [5] showed that the step model command restriction is not required. Perfect output tracking may be achieved for a general class of model commands which are described by

$$u_m(t) = \sum_i \sum_j a_{ij} t^j \sin(\omega_i t + \theta_i) \tag{1.25}$$

That is, the model command is generated by unknown dynamics of the form

$$\dot{v}_m(t) = A_v v_m(t) \tag{1.26}$$

$$u_m(t) = C_v v_m(t) \tag{1.27}$$

and satisfies the rank condition as follows:

$$\dim[v_m(t)] < \dim[x_m(t)] + \dim[u_m(t)] \tag{1.28}$$

The asymptotic stability of this adaptive algorithm is ensured by using Lyapunov's second method. Although the ASPR requirement, Eq.(1.24), is restrictive, this approach has the following advantages:

(1) A multi-input multi-output plant is allowed.

(2) Perfect output tracking will be obtained.

(3) The adaptive mechanism is relatively simple compared with other adaptive algorithms.

(4) Full state access is not required.

(5) An adaptive observer is not needed.

(6) Explicit knowledge of the plant is not necessary.

(7) The order of the model could be much lower than the order of the plant.

1.2.2 Algorithms for Non-ASPR Plants

A modified approach, described by Sobel, Kaufman, and Mabius[3], extended the applicability of the CGT based MRAC algorithms to non-ASPR plants by redefining the adaptive gain computation in Eq.(1.21) as follows:

$$v(t)=Q_p[y_m(t)-y_p(t)]+G_p\{u_p^*(t)-u_p(t)+\tilde{K}_{pe}[y_m(t)-y_p(t)]\} \tag{1.29}$$

where Q_p and G_p are chosen by the designer, $u_p^*(t)$ is the ideal plant input which produces perfect tracking and $\tilde{K}_{pe}$ is an output feedback gain which stabilizes the plant. The ASPR requirement in Eq.(1.24) is modified as:

$$J + C_p(sI - A_p + B_p\tilde{K}_{pe}C_p)^{-1}B_p \qquad \text{is SPR} \tag{1.30}$$

and

$$Q_p^{-1}G_p > J \tag{1.31}$$

Eqs.(1.30) and (1.31) imply that asymptotic stability will be ensured even if the plant is not ASPR. This algorithm has not received much attention in recent years because knowledge of $u_p^*(t)$ and $\tilde{K}_{pe}$ in Eq.(1.29) requires knowledge of the plant. However, it is shown in [3] that a bounded error can be guaranteed if $u_p^*(t)$ is replaced by a nominal value $u_{pnom}^*(t)$ which is an approximation to $u_p^*(t)$.

Bar-Kana and Kaufman [5] consider the plant with disturbances which is described by

$$\dot{x}_p(t) = A_p x_p(t) + B_p u_p(t) + d_{ip}(t) \tag{1.32}$$

$$y_p(t) = C_p x_p(t) + d_{op}(t) \tag{1.33}$$

where the bounded input and output disturbances are denoted by $d_{ip} \in R^n$ and $d_{op} \in R^m$, respectively.

To remove the ASPR restriction on the plant, Bar-Kana and Kaufman[5] insert feedforward dynamics $G_a(s)$ in parallel with the non-ASPR plant as shown in Figure 1.2, such that the augmented plant satisfies the ASPR requirement. The supplementary dynamics represented by $G_a(s)$ are described by

$$\dot{x}_f(t) = A_f x_f(t) + B_f u_f(t) \tag{1.34}$$

$$y_f(t) = C_f x_f(t) \tag{1.35}$$

However, a steady state output tracking error will exist all the time because the output tracking error contains the feedforward signal $y_f(t)$.

The control law is defined as

$$u_p(t) = K_{pe}(t)e_{ypa}(t)+K_{px}(t)x_m(t)+ K_{pu}(t)u_m(t) \tag{1.36}$$

where $e_{ypa}(t)=y_m(t)-[y_p(t)+y_f(t)]$ is the difference between the model output and the augmented plant output.

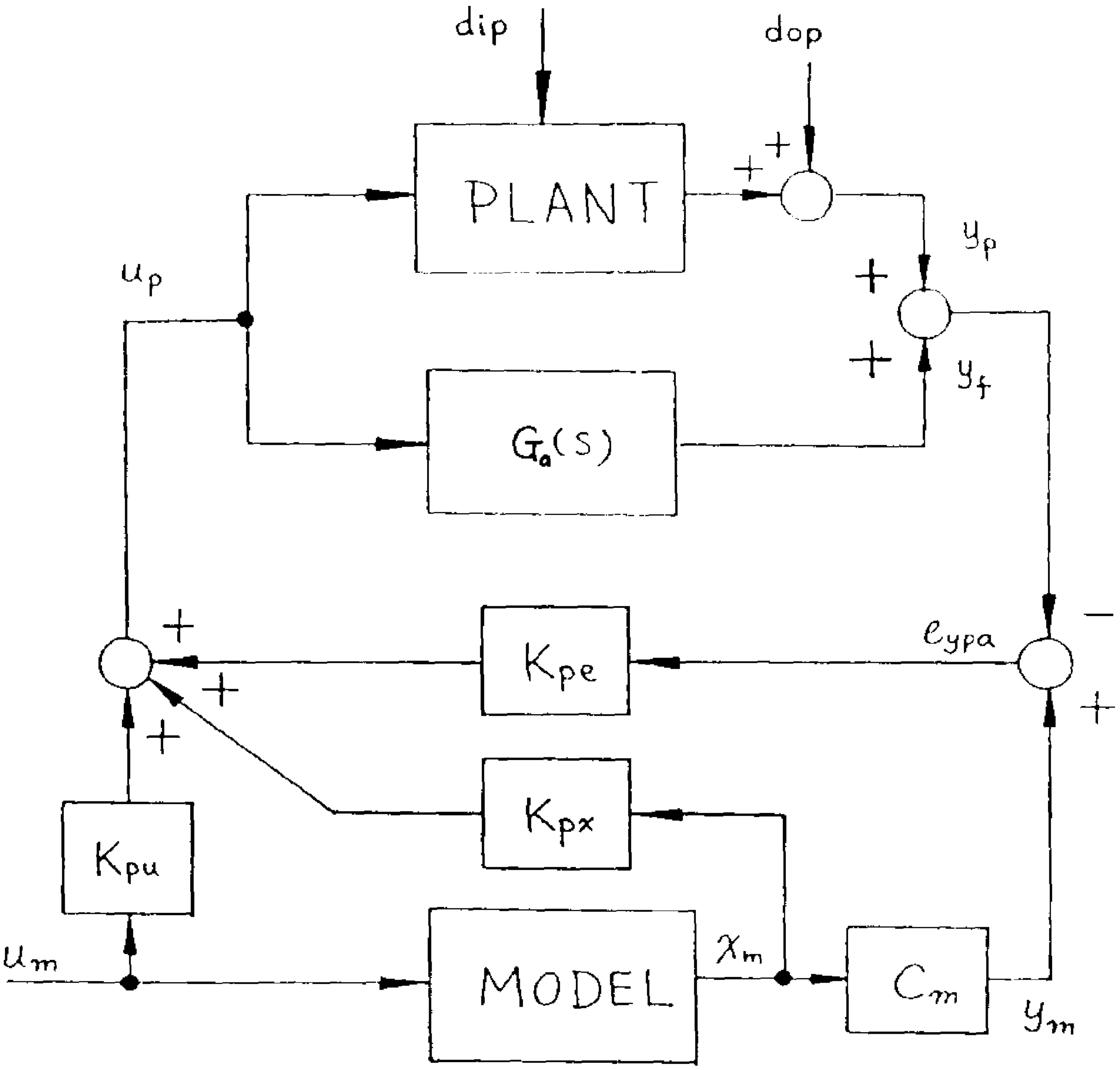

Figure 1.2 Non-ASPR Plant with Feedforward Compensation

The adaptive gains are computed as in Eqs.(1.18)-(1.20) except that the integral gain computation is modified by incorporating Ioannou's[6] fixed σ-modification. Without this modification, the integral gain in Eq.(1.20) may steadily increase and lead to practical divergence because of the existence of plant input and output disturbances. Therefore, the integral gain is now computed by using

$$\dot{K}^I(t) = v(t)r^T(t)T - \sigma K^I(t)T \tag{1.37}$$

where the signal v(t) in both the proportional gain and integral gain is replaced by $v(t)=e_{ypa}(t)$, and where σ is a positive scalar.

Since perfect output tracking is not possible for this algorithm, Bar-Kana and Kaufman [5] define the fictitious state $x_p^*(t)$, the fictitious output $y_p^*(t)$ and the fictitious input $u_p^*(t)$. These fictitious target trajectories are described as follows:

$$\dot{x}_p^*(t) = A_p^* x_p^*(t) + B_p^* u_p^*(t) \tag{1.38}$$

$$y_p^*(t) = C_p x_p^*(t) = y_m(t) \tag{1.39}$$

It is important that the reader does not confuse the fictitious target trajectories described here with the ideal trajectories described in Eqs.(1.6)-(1.7). Although the same notation will be used for both cases, it will be clear from the context which is being used.

The fictitious target trajectories do not necessarily satisfy the plant dynamics. The fictitious dynamics $\{A_p^*, B_p^*\}$ are assumed to be of the same (unknown) order as the plant dynamics $\{A_p, B_p\}$ but they may be entirely different, and only the output matrix C_p is identical.

Furthermore, reference 5 assumes that

$$x_p^*(t) = Sx_m(t) \tag{1.40}$$

where S is a real constant matrix.

The existence of a solution for S in Eq.(1.40) requires that

$$\text{rank}[\, C_p \quad C_m \,] = \text{rank}[\, C_p \,] \tag{1.41}$$

All states, errors, and gains in the adaptive algorithm will bounded provided that

i) $G_a(s)$ is known and nonsingular

ii) $G_a^{-1}(s)$ is ASPR

iii) $[\, I+G_p(s)G_a(s) \,]^{-1}G_p(s)$ is asymptotically stable

where $G_p(s)$ is the transfer function matrix of the plant.

Recently, Neat, Steinvorth, and Kaufman [8] modified Bar-Kana's[5] algorithm by incorporating feedforward dynamics $G_a(s)$, into the reference model as well as into the plant, Figure 1.3, such that the error which is expected to be bounded is the difference between the augmented plant output $z_p(t)$ and the augmented model output $z_m(t)$. That is,

$$z_p(t) = y_p(t) + g(t)*u_p(t) \tag{1.42}$$

$$z_m(t) = y_m(t)+g(t)*\{u_p(t)-K_{pe}(t)[z_m(t)-z_p(t)]\} \tag{1.43}$$

$$y_m(t)-y_p(t) = z_m(t)-z_p(t)+g(t)*\{K_{pe}(t)[z_m(t)-z_p(t)]\} \tag{1.44}$$

where $g(t)$ is the impulse response of $G_a(s)$ and the notation * is the convolution operator.

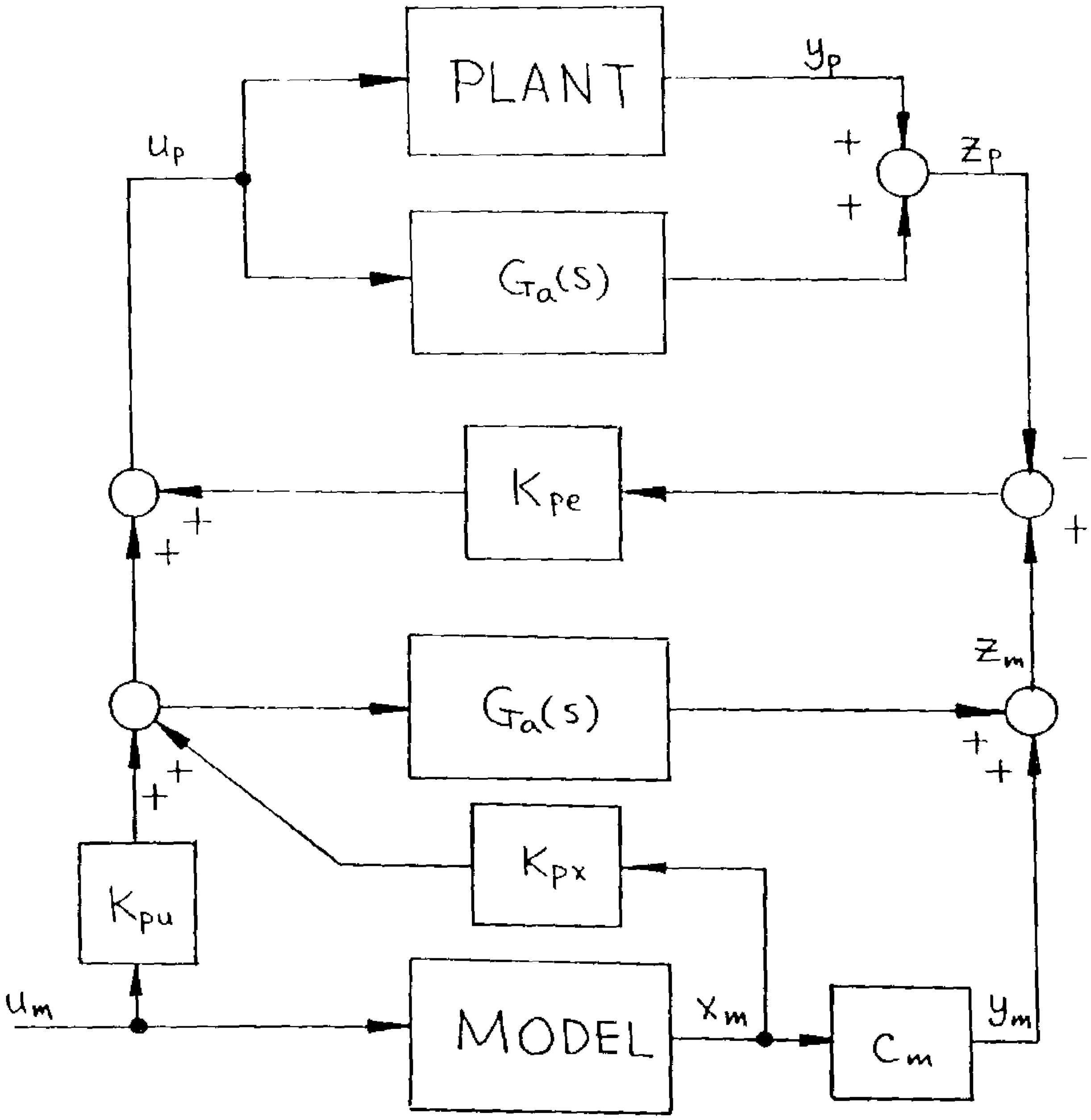

Fig. 1.3 Non-ASPR Plant with Dual-feedforward Compensation

Thus, it appears that if $z_p(t)$ approaches $z_m(t)$, then $y_p(t)$ will approach $y_m(t)$ in steady state. However, the incorporation of $G_a(s)$ into the reference model forms a time-varying model which results in a complicated stability proof.

Sobel [9] presented an algorithm for non-ASPR plants without the need for parallel feedforward by introducing constant matrices G_p and Q_p into Eq.(1.21) to obtain

$$v(t) = Q_p e_{yp}(t) + G_p K(t) r(t) \tag{1.45}$$

where Q_p and G_p are chosen by the designer to satisfy the sufficient conditions for stability.

This algorithm modified the sufficient conditions in Eqs.(1.30) and (1.31) to

$$J + (Q_p + G_p \tilde{K}_{pe}) C_p (sI - A_p - B_p \tilde{K}_{pe} C_p) B_p \quad \text{is SPR} \tag{1.46}$$

and

$$J + J^T + G + G^T < 0 \tag{1.47}$$

Eqs.(1.46) and (1.47) have more freedom than either Eq.(1.24) or Eqs.(1.30) and (1.31) because of the additional matrices Q_p and G_p. Furthermore, if the stabilizing gain $\tilde{K}_{pe}$ is known, then we can choose $Q_p = -G_p\tilde{K}_{pe}$. Then, the condition in Eq.(1.46) reduces to $J>0$, which is trivially satisfied. However, only the boundedness of the errors, states, and gains in the adaptive system are ensured.

1.2.3 Algorithms for Linear Time Varying ASPR Plants

Extension of the CGT based MRAC algorithms to linear time varying plants has been investigated by Abida and Kaufman [10]. They consider the LTV plant given by

$$\dot{x}_p(t) = A_p(t)x_p(t) + B_p(t)u_p(t) \tag{1.48}$$

$$y_p(t) = C_p x_p(t) \tag{1.49}$$

which is excited by an adaptive control signal $u_p(t)$ as described by Eqs.(1.14)- (1.21) such that the plant output tracks the output of a linear time invariant reference model described by Eqs.(1.3) and (1.4).

Abida and Kaufman[10] show that the plant output will asymptotically track the model output provided that

(1) The model command u_m is a constant

(2) The dimension of the plant input is equal to the dimension of the plant output

(3) There exist $S_{11}(t)$, $S_{12}(t)$, $S_{21}(t)$, and $S_{22}(t)$ which satisfy:

$$\dot{S}_{11}(t) = A_p(t)S_{11}(t)-S_{11}(t)A_m(t)+B_p(t)S_{21}(t) \tag{1.50}$$

$$\dot{S}_{12}(t) = A_p(t)S_{12}(t)-S_{11}(t)B_m(t)+B_p(t)S_{22}(t) \tag{1.51}$$

$$C_pS_{11}(t) = C_m(t) \tag{1.52}$$

$$C_pS_{12}(t) = 0 \tag{1.53}$$

(4) There exists a time varying positive definite matrix $P(t)$ and nonsingular matrix S such that

$$\dot{P}(t)+P(t)[A_p(t)-B_p(t)\tilde{K}_e(t)C_p]+[A_p(t)-B_p(t)\tilde{K}_eC_p]^TP(t)<0 \tag{1.54}$$

$$C=(S^TS)^{-1}B^T(t)P(t) \tag{1.55}$$

1.2.4 Algorithms for Non-linear Plants

Abida and Kaufman [10] extend the MRAC technique to nonlinear plants by using virtual linearization.

The nonlinear plant is described by

$$\dot{x}_p(t) = g(x_p(t),\ u_p(t),\ t) + p(x_p(t),\ u_p(t),\ t) \qquad (1.56)$$

$$y_p(t) = C_p x_p(t) \qquad (1.57)$$

This algorithm is used for a class of nonlinear plants which can be linearized to yield a linear time varying plant as described by Eqs.(1.48) and (1.49). In other words, if $g(x_p(t),u_p(t),t)$ and $p(x_p(t),u_p(t),t)$ in Eq.(1.56) can be written as $A_p(x_p(t),u_p(t),t)\cdot x_p(t)$ and $B_p(x_p(t),u_p(t),t)\cdot u_p(t)$, respectively, with $A_p(x_p(t),u_p(t),t)$ and $B_p(x_p(t),u_p(t),t)$ bounded, then Eq.(1.56) will be considered as a linear time varying plant as shown below:

$$\dot{x}_p(t)=A_p(x_p(t),u_p(t),t)x_p(t)+B_p(x_p(t),u_p(t),t)u_p(t) \tag{1.58}$$

$$y_p(t) = C_p x_p(t) \tag{1.59}$$

The objective is for the output of the linear time varying plant to track the output of the linear time invariant model which is described by Eqs.(1.3) and (1.4). The control law is chosen as in Eqs.(1.17)-(1.21).

Abida and Kaufman[10] show that asymptotic output tracking is obtained provided that all the conditions in section 1.2.3 are satisfied. Since the matrices $A_p(x_p(t),u_p(t),t)$ and $B_p(x_p(t),u_p(t),t)$ are functions of $x_p(t)$, $u_p(t)$, and t, the ASPR conditions, Eq.(1.54) and (1.55), must be satisfied for every possible state and input trajectory for all $t \geq 0$.

Bar-Kana [11] extends the model reference adaptive control algorithm to a plant with bounded nonlinearities described by

$$\dot{x}_p(t) = A_p(x_p(t))x_p(t) + B_p(x_p(t))u_p(t) \tag{1.60}$$

$$y_p(t) = C_p(x_p(t))x_p(t) \tag{1.61}$$

Reference 11 states that the plant will follow the linear time invariant model with bounded error if the nonlinearities $A_p(x(t))$ and $B_p(x(t))$ in Eq.(1.61) are bounded, and if the plant is almost passive via some known feedforward augmentation.

Ambrosino et. al. [12] propose an adaptive control algorithm for nonlinearities with known form. This plant is described as

$$\dot{x}_p(t) = A_p x_p(t) + A_\gamma \gamma(x_p(t),t) + B_0 R_0 u_p(t) + E_p d_{ip} \tag{1.62}$$

where $x_p(t) \in R^n$ is the accessible state, $u_p(t) \in R^m$ is the input, $d_{ip} \in R^q$ is a constant disturbance, $\gamma(x_p(t),t) \in R^\gamma$ is a vector of nonlinear functions of known form, B_0 is a known constant matrix, A_p, A_γ, and E_p are unknown constant matrices, and R_0 is a positive definite unknown constant matrix. The plant will track a linear time invariant reference model of the form described by Eqs.(1.3) and (1.4) where the full state feedback control signal is described by

$$u_p(t) = -K_x(t)x_p(t)-K_\gamma(t)\gamma(t)-K_u(t)u_m(t)-K_d(t) \qquad (1.63)$$

where $K_x(t)$, $K_\gamma(t)$, $K_u(t)$, and $K_d(t)$ are adaptive gain matrices defined by

$$K_x(t) = \int_{t_o}^{t} F_0 v(\tau)x_p^T(\tau)G_0(\tau)d\tau \qquad (1.64)$$

$$K_\gamma(t) = \int_{t_o}^{t} F_1 v(\tau)\gamma^T(\tau)G_1(\tau)d\tau \qquad (1.65)$$

$$K_{u_m}(t) = \int_{t_o}^{t} F_2 v(\tau)u_m^T(\tau)G_2(\tau)d\tau \qquad (1.66)$$

$$K_d(t) = \int_{t_o}^{t} F_3 v(\tau)d\tau \qquad (1.67)$$

where

$$F_i = F_i^T F_i, \qquad i = 0,1,2,3 \qquad (1.68)$$

$$G_i = G_i^T G_i \qquad i = 0,1,2 \tag{1.69}$$

are positive definite matrices and where

$$v(t) = C(\ x_p(t) - x_m(t)\ ) \tag{1.70}$$

$$C = B_0^T P \tag{1.71}$$

$$A_m^T P + PA_m = -Q \tag{1.72}$$

where Q is positive definite

The stability proof of this algorithm is based on hyperstability theory in which the Popov integral inequality is used. Ambrosino et. al. [12] show that asymptotic stability of the state error between the plant and model can be ensured with the satisfaction of the conditions shown below:

$$\text{rank}\ [\ A_\gamma \,\vdots\, B_0\ ] = \text{rank}\ [\ E_p \,\vdots\, B_0\ ] = \text{rank}\ [\ B_0\ ] \tag{1.73}$$

$$\text{rank}\,[\,A_p - A_m \,\vdots\, B_0\,] = \text{rank}\,[\,B_m \,\vdots\, B_0\,] = \text{rank}\,[\,B_0\,] \tag{1.74}$$

$$A_m^T P + P A_m = -Q \tag{1.75}$$

$$P B_0^{\;T} = C \tag{1.76}$$

This algorithm not only requires that the order of the plant equals the order of the model but also requires full state feedback in order to implement the control law. Furthermore, the satisfaction of the rank condition, Eq.(1.74), is not easy especially when the dimension of the plant input is much less then the dimension of the plant state. This is because A_m is square and nonsingular and hence, the rank of $[A_p - A_m\ B_0]$ is usually larger than the rank of B_0.

1.3 Contributions

The contributions of this dissertation include:

1. Incorporation of supplementary dynamics into the adaptive loop in different locations to form different algorithms for non-ASPR plants

2. Introduction of supplementary dynamics which are not restricted to be ASPR and nonsingular. Design rules are proposed for implementing the adaptive controllers.

3. Introduction of a new control law with an additional adaptive gain $K_f(t)$ which multiplies $y_f(t)$.

4. Introduction of adaptive supplementary dynamics in which all the parameters are adjusted on line as part of the adaptive computation.

5. Introduction of a larger class of plants which can be controlled by the command generator tracker approach to model reference adaptive control. The new class includes all controllable and observable plants.

6. Introduction of a metasystem representation which is a

generalized form for the CGT adaptive control algorithms. This metasystem simplifies the stability analysis.

7. Introduction of model reference adaptive control algorithms for nonlinearities of known form but multiplied by unknown parameters. Neither full state access nor the boundedness of the nonlinearities are required. The nonlinearity is treated without the use of linearization techniques.

8. Introduction of conditions for an asymptotically vanishing tracking error if no disturbances exist and the reference input is constant for $t \geq t_1$.

1.4 Outline

Chapter 2 discusses adaptive algorithms 1, 2, and 3 for linear time invariant plants which are not almost strictly positive real. Design rules are presented for the adaptive controllers, stability is proven by utilizing the metasystem representation, and conditions are given for asymptotic output tracking.

Chapter 3 presents algorithms 4 and 5 with adaptive supplementary dynamics to control not almost strictly positive real plants where the parameters of the supplementary dynamics are computed on line by the adaptive mechanism. Design rules are presented. and conditions are derived for a bounded error and for asymptotic output tracking.

Chapter 4 extends the adaptive algorithms to plants which include nonlinearities of known form with unknown parameters (algorithm 6). Conditions for bounded and asymptotic output tracking are derived.

Chapter 5 gives the conclusion of this dissertation, and discusses problems and recommendations for further research.

2. ALGORITHMS WITH FIXED SUPPLEMENTARY DYNAMICS

2.1 Introduction

Three new algorithms are proposed for controllable and observable linear time invariant multi-input multi-output plants which are not almost strictly positive real (ASPR). We insert supplementary dynamics either in feedback with the plant or in parallel with the plant or in cascade with the plant. A generalized metasystem is proposed by introducing metastates and metamatrices. This new metasystem is used to simplify the stability analysis.

The earlier non-ASPR algorithms [5,7] which only guarantee a bounded tracking error are shown to be a special case of the new parallel dynamics approach. However, our new approach (i) does not require the parallel dynamics to be ASPR and nonsingular, (ii) provides design rules for the adaptive controller, (iii) and achieves asymptotic output tracking provided that some conditions are satisfied.

A stability proof is shown with conditions for either bounded or asymptotically vanishing tracking errors. Examples are presented including the well known Rohr's example, an unstable single-input single output plant, and a two-input two-output representation of the F-8 aircraft.

2.2 Problem Formulation

A linear time-invariant plant with input and output disturbances is described by

$$\dot{x}_p(t) = A_p x_p(t) + B_p u_p(t) + E_p d_{ip}(t) \qquad (2.1)$$

$$y_p(t) = C_p x_p(t) + d_{op}(t) \qquad (2.2)$$

where $x_p(t) \in R^n$, $u_p(t) \in R^m$, and $y_p(t) \in R^\ell$. The bounded input and output disturbances are denoted by $d_{ip}(t) \in R^m$, and $d_{op}(t) \in R^\ell$, respectively. The matrices A_p and B_p are unknown, but their entries are assumed to be bounded with known bounds.

The objective is to find, without explicit knowledge of A_p and B_p, the control $u_p(t)$ such that the plant output vector $y_p(t)$ approximates "reasonably well" the output of a reference model. In particular, we would prefer that the error between the plant and model outputs is asymptotically vanishing.

The asymptotically stable reference model is described by

$$\dot{x}_m(t) = A_m x_m(t) + B_m u_m(t) \tag{2.3}$$

$$y_m(t) = C_m x_m(t) \tag{2.4}$$

where $x_m(t) \in R^{n_m}$, $y_m(t) \in R^{\ell}$, and $u_m(t) \in R^{m_m}$.

We emphasize that the order of the reference model may be much less than the order of the plant. The adaptive control algorithm will utilize supplementary dynamics which will be inserted into different locations inside the adaptive loop. In all cases, these supplementary dynamics will be internal to the adaptive controller. The state representation for these dynamics is given by

$$\dot{x}_f(t) = A_f x_f(t) + B_f u_f(t) \tag{2.5}$$

$$y_f(t) = C_f x_f(t) \tag{2.6}$$

where the form for $u_f(t)$ will depend upon whether the supplementary dynamics are inserted in a parallel path, a feedback path, or a cascade path with the plant and where $x_f(t) \in R^{n_f}$, $y_f(t) \in R^{\ell_f}$, and $u_f(t) \in R^m$. The constant matrices A_f, B_f and C_f are chosen by design rules which will be discussed in a later section.

Remark 2.1:

Unlike previous work[5], the supplementary dynamics in Eqs.(2.5) and (2.6) are not restricted to be ASPR and nonsingular. A much larger class of supplementary dynamics can be selected including some unstable dynamics.

2.3 Fictitious Target System

If the plant were known, then we could choose the control law to be

$$u_p(t) = \tilde{K}_{pe}e_{yp}(t) - \tilde{K}_{pf}y_f(t) + \tilde{K}_{px}x_m(t) + \tilde{K}_{pu}u_m(t) \tag{2.7}$$

such that if "ideal" output tracking is possible, then the plant output asymptotically tracks the reference model output. Mathematically,

$$y_p(t) = y_m(t) \tag{2.8}$$

where the plant output error is given by

$$e_{yp}(t) = y_m(t) - y_p(t) \tag{2.9}$$

It is important to note that for the linear model following problem, the "ideal" situation described by

Eq.(2.7) is not always possible. Therefore, for the adaptive control problem we utilize the approach of Bar-Kana [5] which only assumes that there exists a fictitious target system of the form

$$\dot{x}_p^*(t) = A_p^* x_p^*(t) + B_p^* u_p^*(t) \tag{2.10}$$

$$y_p^*(t) = C_p x_p^*(t) \tag{2.11}$$

which achieves $y_p^*(t) = y_m(t)$ when forced by the fictitious input $u_p^*(t)$. Note that although Eq.(2.10) is assumed to be of the same order as Eq.(2.1), these two systems may be entirely different and it is only assumed that their measurement matrices are identical.

In order to check the existence of bounded fictitious target trajectories, Bar-Kana [5] assumed a fictitious state trajectory as follows

$$x_p^*(t) = S_{11} x_m(t) \tag{2.12}$$

where S_{11} is a constant matrix satisfying

$$C_pS_{11} = C_m \tag{2.13}$$

Thus,

$$y_p^*(t) = C_px_p^*(t) = C_pS_{11}x_m(t) = C_mx_m(t) = y_m(t) \tag{2.14}$$

It is shown in Bar-Kana [5] that Eq.(2.13) has a solution for the matrix S_{11} if

$$\text{Rank } [C_p \mid C_m] = \text{Rank } [C_p] \tag{2.15}$$

which is satisfied in general, especially when $\dim[x_p(t)] \gg \dim[x_m(t)]$.

2.4 Algorithm 1 (Feedback Supplementary Dynamics)

In this new algorithm, we insert supplementary dynamics into the output feedback path as part of the adaptive mechanism. As shown in Figure 2.1, the supplementary dynamics process the output error to produce a signal which is multiplied by a new adaptive gain $K_{pf}(t)$, such that the control law is of the form,

$$u_p(t)=K_{pe}(t)e_{yp}(t)-K_{pf}(t)y_f(t)+K_{px}(t)x_m(t)+K_{pu}(t)u_m(t) \tag{2.16}$$

$$u_f(t) = y_m(t) - y_p(t)$$

$$= y_m(t) - C_p x_p(t) - d_{op}(t) \tag{2.17}$$

where $K_{pe}(t)$, $K_{pf}(t)$, $K_{px}(t)$, and $K_{pu}(t)$ are adaptive gain matrices. The plant and supplementary dynamics are concatenated to form an $(n+n_f)$ order metasystem as follows:

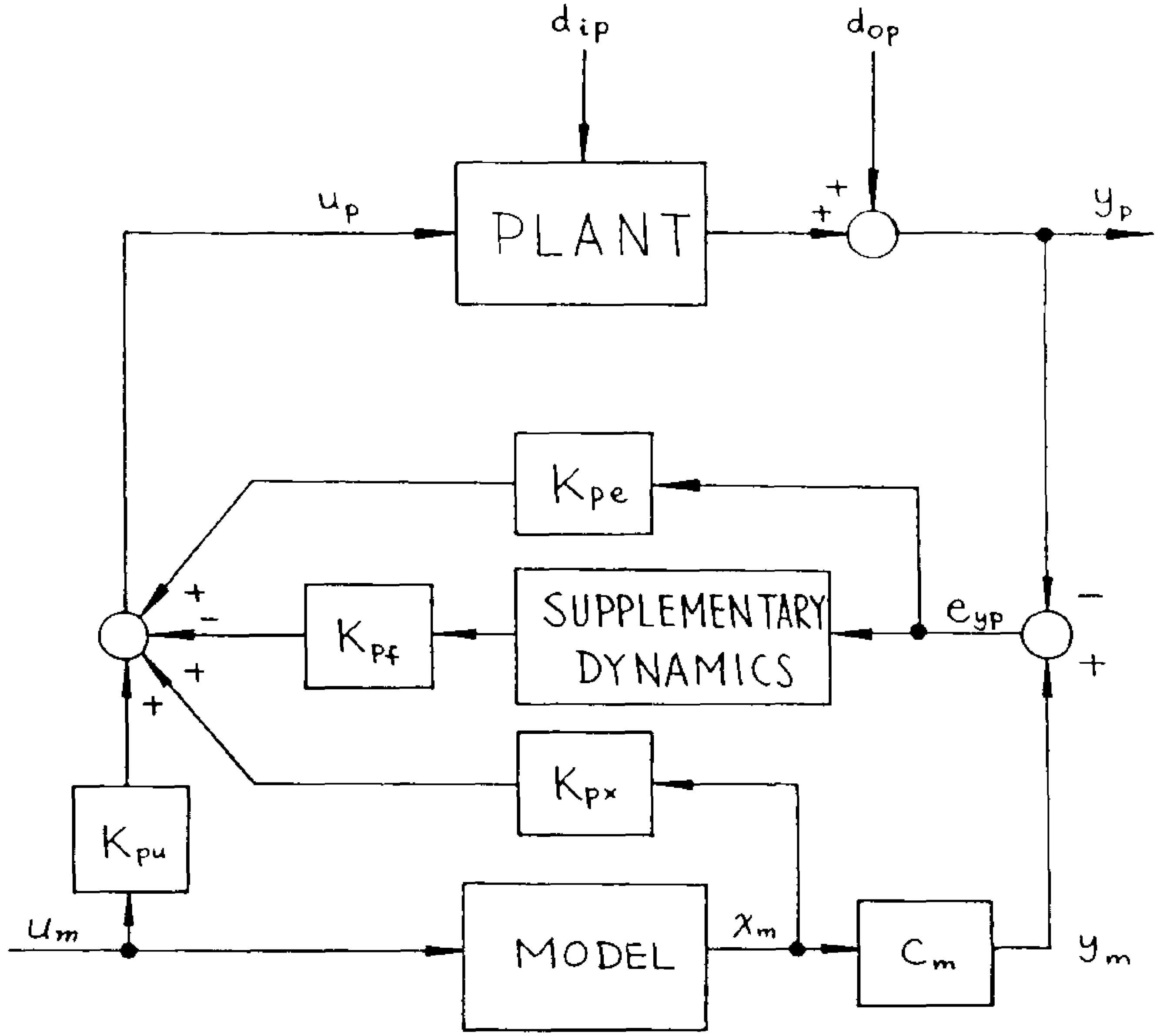

Figure 2.1 Block Diagram of Algorithm 1

$$\begin{bmatrix} \dot{x}_p(t) \\ \dot{x}_f(t) \end{bmatrix} = \begin{bmatrix} A_p & 0 \\ -B_f C_p & A_f \end{bmatrix} \begin{bmatrix} x_p(t) \\ x_f(t) \end{bmatrix} +$$

$$\begin{bmatrix} B_p \\ 0 \end{bmatrix} [K_{pe}(t) \;\; K_{pf}(t) \;\; K_{px}(t) \;\; K_{pu}(t)] \begin{bmatrix} e_{yp}(t) \\ -y_f(t) \\ x_m(t) \\ u_m(t) \end{bmatrix} + \begin{bmatrix} E_p d_{ip}(t) \\ B_f(y_m(t) - d_{op}(t)) \end{bmatrix} \tag{2.18}$$

$$\begin{bmatrix} y_p(t) \\ y_f(t) \end{bmatrix} = \begin{bmatrix} C_p & 0 \\ 0 & C_f \end{bmatrix} \begin{bmatrix} x_p(t) \\ x_f(t) \end{bmatrix} + \begin{bmatrix} d_{op}(t) \\ 0 \end{bmatrix} \tag{2.19}$$

2.5 Algorithm 2 (Parallel Supplementary Dynamics)

The configuration of Algorithm 2 is shown in Figure 2.2, where the supplementary dynamics in this case are in cascade with the plant. Therefore, we have

$$u_p(t)=u_f(t)=K_{pe}(t)e_{yp}(t)-K_{pf}(t)y_f(t)+K_{px}(t)x_m(t)+K_{pu}(t)u_m(t) \tag{2.20}$$

where $K_{pe}(t)$, $K_{pf}(t)$, $K_{px}(t)$, and $K_{pu}(t)$ are adaptive gain matrices. The plant and supplementary dynamics are concatenated to form a metasystem as follows

$$\begin{bmatrix} \dot{x}_p(t) \\ \dot{x}_f(t) \end{bmatrix} = \begin{bmatrix} A_p & 0 \\ 0 & A_f \end{bmatrix} \begin{bmatrix} x_p(t) \\ x_f(t) \end{bmatrix} + \begin{bmatrix} B_p \\ B_f \end{bmatrix} [K_{pe}(t) \ K_{pf}(t) \ K_{px}(t) \ K_{pu}(t)] \begin{bmatrix} e_{yp}(t) \\ -y_f(t) \\ x_m(t) \\ u_m(t) \end{bmatrix} + \begin{bmatrix} E_p d_{ip}(t) \\ 0 \end{bmatrix} \tag{2.21}$$

$$\begin{bmatrix} y_p(t) \\ y_f(t) \end{bmatrix} = \begin{bmatrix} C_p & 0 \\ 0 & C_f \end{bmatrix} \begin{bmatrix} x_p(t) \\ x_f(t) \end{bmatrix} + \begin{bmatrix} d_{op}(t) \\ 0 \end{bmatrix} \tag{2.22}$$

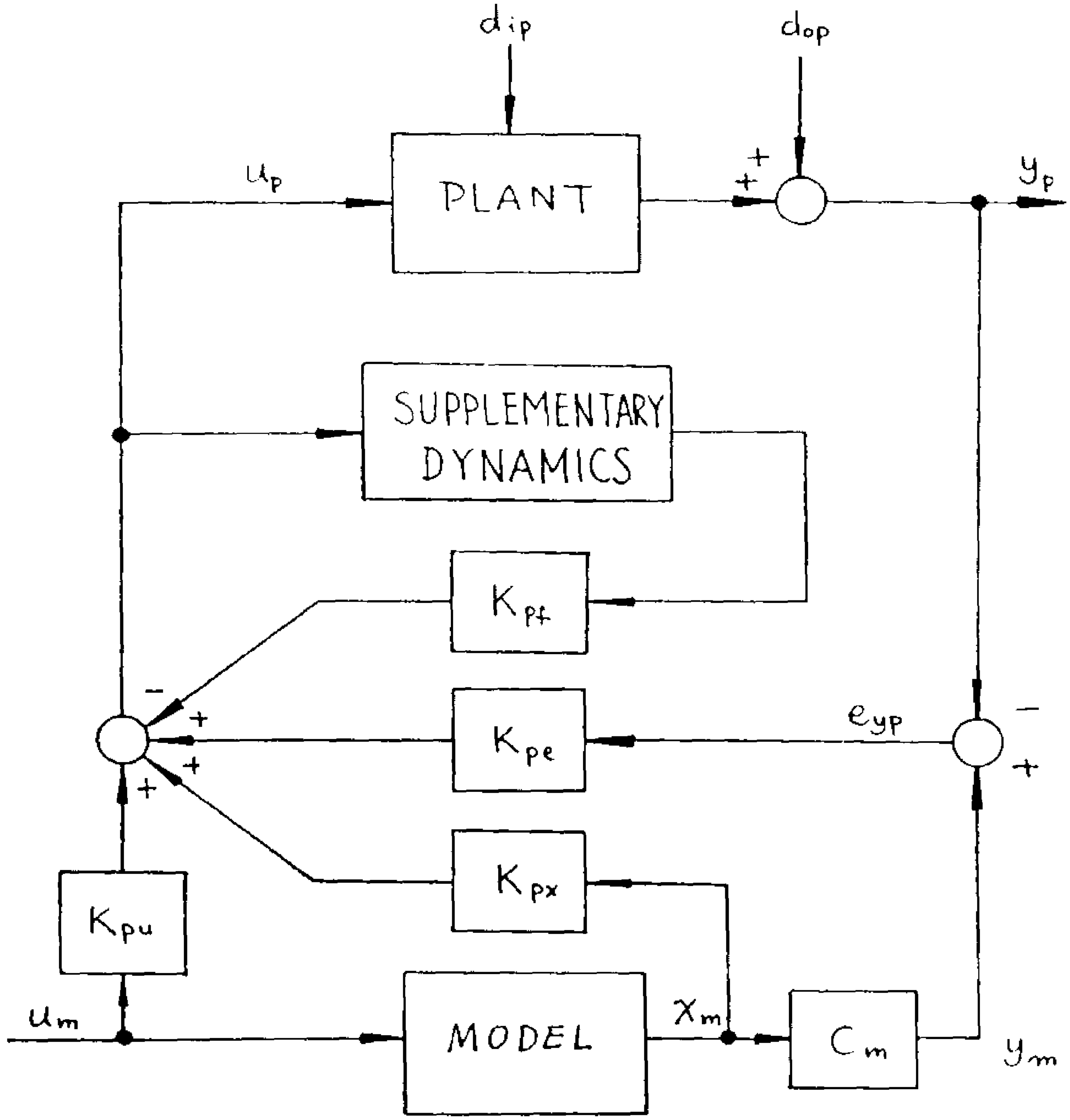

Figure 2.2 Block Diagram of Algorithm 2

2.6 Algorithm 3 (Cascade Supplementary Dynamics)

An alternative implementation is shown in Figure 2.3, where the output of the supplementary dynamics is multiplied by an adaptive gain $K_{pf}(t)$ to form an inner feedback loop for the supplementary dynamics. The input of the plant is the combination of the output of the supplementary dynamics and the control signal $u_f(t)$ via the constant matrix D_f as shown below:

$$
\begin{aligned}
u_p(t) &= D_f u_f(t) - y_f(t) \\
&= D_f u_f(t) - C_f x_f(t) \qquad (2.23)
\end{aligned}
$$

$$
u_f(t)=K_{pe}(t)e_{yp}(t)-K_{pf}(t)y_f(t)+K_{px}(t)x_m(t)+K_{pu}(t)u_m(t) \qquad (2.24)
$$

Therefore, the metasystem will be in the form of

$$
\begin{bmatrix} \dot{x}_p(t) \\ \dot{x}_f(t) \end{bmatrix} = \begin{bmatrix} A_p & -B_pC_f \\ 0 & A_f \end{bmatrix} \begin{bmatrix} x_p(t) \\ x_f(t) \end{bmatrix} +
$$

$$\begin{bmatrix} B_pD_f \\ B_f \end{bmatrix} [K_{pe}(t)\ K_{pf}(t)\ K_{px}(t)\ K_{pu}(t)] \begin{bmatrix} e_{yp}(t) \\ -y_f(t) \\ x_m(t) \\ u_m(t) \end{bmatrix} + \begin{bmatrix} E_pd_{ip}(t) \\ 0 \end{bmatrix} \tag{2.25}$$

$$\begin{bmatrix} y_p(t) \\ y_f(t) \end{bmatrix} = \begin{bmatrix} C_p & 0 \\ 0 & C_f \end{bmatrix} \begin{bmatrix} x_p(t) \\ x_f(t) \end{bmatrix} + \begin{bmatrix} d_{op}(t) \\ 0 \end{bmatrix} \tag{2.26}$$

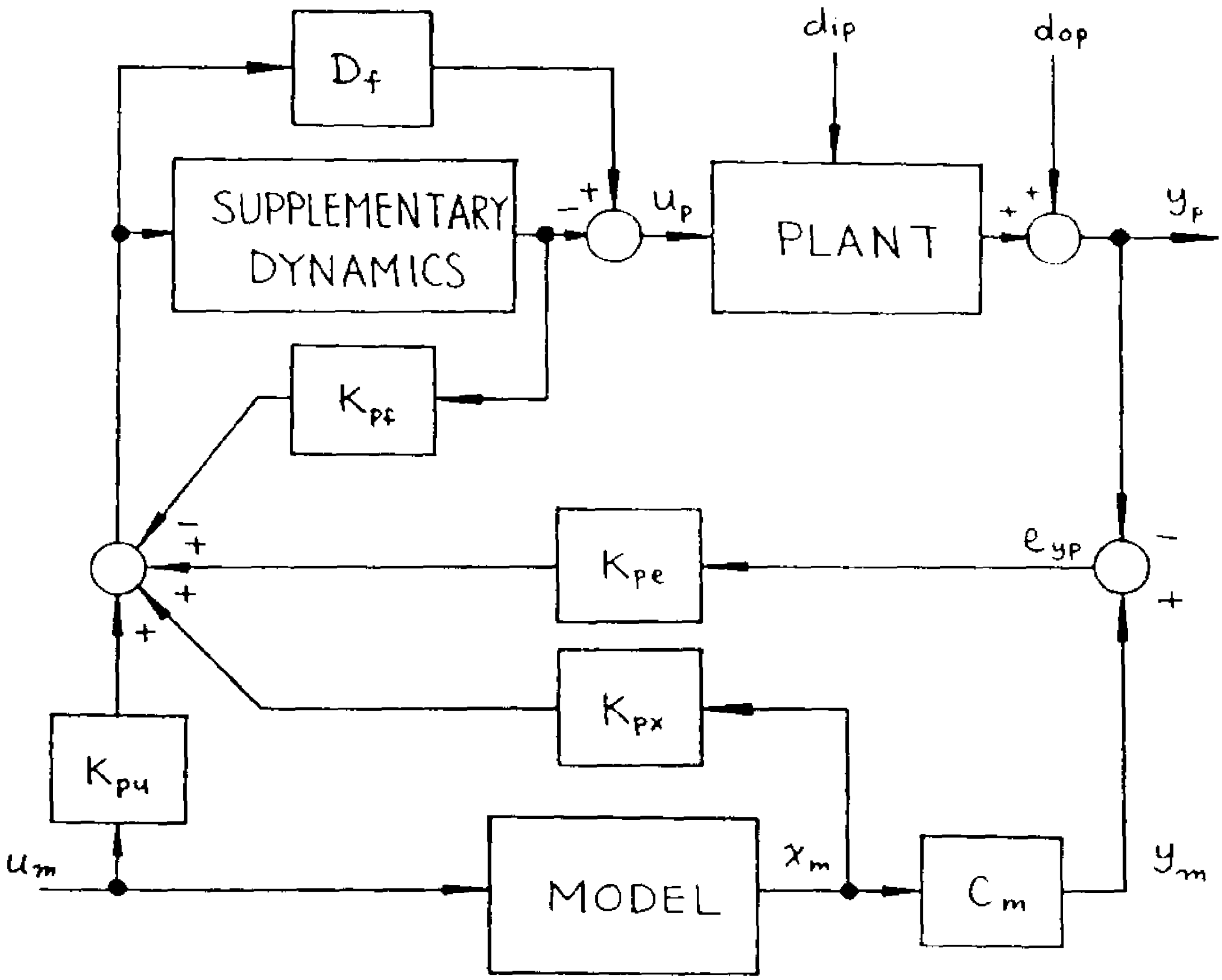

Figure 2.3 Block Diagram of Algorithm 3

2.7 Metasystem Representation

The metasystem is formed by concatenating the plant dynamics with the supplementary dynamics. Referring to the metastates and the metamatrices listed in appendix A, we may write the metasystem equations more compactly for algorithms 1, 2, and 3 as follows:

Plant Equations

$$\dot{x}(t) = Ax(t) + BK(t)r(t) + d_i(t) \tag{2.27}$$

$$= Ax(t) + Bu(t) + d_i(t)$$

$$y(t) = Cx(t) + d_o(t) \tag{2.28}$$

where

$$x(t) = \begin{bmatrix} x_p(t) \\ x_f(t) \end{bmatrix}, \qquad y(t) = \begin{bmatrix} y_p(t) \\ y_f(t) \end{bmatrix}$$

$$d_o(t) = \begin{bmatrix} d_{op}(t) \\ 0 \end{bmatrix}, \qquad C = \begin{bmatrix} C_p & 0 \\ 0 & C_f \end{bmatrix} \tag{2.29}$$

and where for Algorithm 1:

$$A = \begin{bmatrix} A_p & 0 \\ -B_fC_f & A_f \end{bmatrix}, \quad B = \begin{bmatrix} B_p \\ 0 \end{bmatrix}, \quad d_i = \begin{bmatrix} E_p d_{ip}(t) \\ B_f(y_m(t)-d_{op}(t)) \end{bmatrix} \tag{2.30}$$

Algorithm 2:

$$A = \begin{bmatrix} A_p & 0 \\ 0 & A_f \end{bmatrix}, \quad B = \begin{bmatrix} B_p \\ B_f \end{bmatrix}, \quad d_i = \begin{bmatrix} E_p d_{ip}(t) \\ 0 \end{bmatrix} \tag{2.31}$$

Algorithm 3:

$$A = \begin{bmatrix} A_p & -B_pC_f \\ 0 & A_f \end{bmatrix}, \quad B = \begin{bmatrix} B_pD_f \\ B_f \end{bmatrix}, \quad d_i = \begin{bmatrix} E_p d_{ip}(t) \\ 0 \end{bmatrix} \tag{2.32}$$

<u>Adaptive gain matrix</u>

$$K(t) = [\ K_e(t) \quad K_{px}(t) \quad K_{pu}(t)\ ] \tag{2.33}$$

where

$$K_e(t) = [\ K_{pe}(t) \quad K_{pf}(t)\ ] \tag{2.34}$$

$$r(t) = \begin{bmatrix} e_{yv}(t) \\ x_m(t) \\ u_m(t) \end{bmatrix} \tag{2.35}$$

and where

$$e_{yv}(t) = \begin{bmatrix} e_{yp}(t) \\ -y_f(t) \end{bmatrix} \tag{2.36}$$

The adaptive gains which form K(t) are chosen to be a combination of proportional and integral terms. The integral term uses Bar-Kana's [5] modification of an idea of Ioannou and Kokotovic [6] by introducing the positive scalar σ in order to guarantee robustness in the presence of parasitic disturbances. The gain matrices are shown below:

$$K(t) = K^P(t) + K^I(t) \tag{2.37}$$

where

$$K^P(t) = v(t)r^T(t)\bar{T} \tag{2.38}$$

$$\dot{K}^I(t) = [\, v(t)r^T(t) - \sigma K^I(t)\Psi]T \tag{2.39}$$

The matrix Ψ is a positive diagonal matrix which allows σ to be weighted differently in the adaptive gains K_{pe}, K_{pf}, K_{px}, and K_{pu} and where the initial integral gains are given by

$$K^I(0) = [\ K^I_{pe}(0),\ K^I_{pf}(0),\ K^I_{px}(0),\ K^I_{pu}(0)\ ] \qquad (2.40)$$

The signal v(t) is chosen based upon the Lyapunov stability analysis and is given by

$$v(t) = Qe_{yv}(t) + GK(t)r(t) \qquad (2.41)$$

where $Q = [\ Q_p\ Q_f\ ]$

and where matrix T is positive definite symmetric and $\bar{T}$ is positive semi-definite symmetric.

<u>Remark</u> <u>2.2</u>

If the plant is stabilizable through a static output feedback gain, then Eqs.(2.27) and (2.28) will reduce to an n-th order plant as discussed in reference 9. Further, in Eqs.(2.39) and (2.41), if we choose σ=0 and G=0, respectively, Eqs.(2.27) and (2.28) become the algorithm in reference 3. Therefore, this metasystem is a generalized form for earlier model reference adaptive control algorithms [3,5,7,9]. Finally, we remark that algorithms 4 and 5 which we will discuss in later sections are also special cases of the metasystem.

2.8 Error Equations

We define the plant state error as

$$e_{xp}(t) = x_p^*(t) - x_p(t) \tag{2.42}$$

and the plant output error as

$$\begin{aligned} e_{yp}(t) &= y_m(t) - y_p(t) \\ &= y_p^*(t) - y_p(t) \\ &= C_p e_{xp}(t) - d_{op}(t) \end{aligned} \tag{2.43}$$

We also define the supplementary dynamics state error as

$$e_{xf}(t) = x_f^*(t) - x_f(t) \tag{2.44}$$

where $x_f^*(t)$ is the supplementary state trajectory when the plant output $y_p(t)$ is equal to the model output $y_m(t)$. Then, the metastate error is of the form

$$e_x(t) = \begin{bmatrix} e_{xp}(t) \\ e_{xf}(t) \end{bmatrix} = x^*(t) - x(t) \tag{2.45}$$

and the error derivative will be

$$\begin{aligned} \dot{e}_x(t) &= \dot{x}^*(t) - \dot{x}(t) \\ &= \dot{x}^*(t) - Ax(t) - BK(t)r(t) - d_i(t) \end{aligned} \tag{2.46}$$

Furthermore, suppose that there exists a constant matrix $\tilde{K}$ given by

$$\tilde{K} = [\tilde{K}_e, \tilde{K}_{px}, \tilde{K}_{pu}] \tag{2.47}$$

such that

$$A_c = A - B\tilde{K}_e C \tag{2.48}$$

is a stability matrix.

Next, use Eqs.(2.45) and (2.46) to obtain

$$\dot{e}_x(t)=\dot{x}^*(t)-A[x^*(t)-e_x(t)]-B[(K(t)-\tilde{K})r(t)]-B\tilde{K}r(t)-d_i(t) \tag{2.49}$$

Define $z(t) = [K(t)-\tilde{K}]r(t)$. Then, Eq.(2.49) becomes

$$\dot{e}_x(t) = \dot{x}^*(t) - Ax^*(t) + Ae_x(t) - Bz(t) - B\tilde{K}r(t) - d_i(t) \tag{2.50}$$

Substitute Eq.(2.47) into Eq.(2.50) to obtain

$$\begin{aligned}\dot{e}_x(t)=&\dot{x}^*(t)-Ax^*(t)+Ae_x(t)-Bz(t)\\ &-B[\tilde{K}_e e_{yv}(t)+\tilde{K}_{px}x_m(t)+\tilde{K}_{pu}u_m(t)]-d_i(t)\end{aligned} \tag{2.51}$$

Define

$$e_v(t) = \begin{bmatrix} e_{xp}(t) \\ -x_f(t) \end{bmatrix} \tag{2.52}$$

and

$$\dot{x}_0(t) = \begin{bmatrix} 0 \\ \dot{x}_f(t) \end{bmatrix} \tag{2.53}$$

such that

$$e_{yv}(t) = Ce_v(t) - d_o \tag{2.54}$$

and

$$e_v(t) = e_x(t) - \dot{x}_0(t) \tag{2.55}$$

Substitute Eqs.(2.48), (2.54) and (2.55) into (2.51) to obtain

$$\dot{e}_x(t) = A_c e_x(t) - Bz(t) - F_1(t) \tag{2.56}$$

where

$$F_1(t) = -\dot{x}^*(t)+Ax^*(t)-B\tilde{K}_e C x_0^*(t)$$

$$+B[\tilde{K}_{px} x_m(t)+\tilde{K}_{pu} u_m(t)-\tilde{K}_e d_o(t)]+d_i(t) \tag{2.57}$$

<u>Remark</u> <u>2.3</u>

Note that $F_1(t)$ is bounded because $\dot{x}^*(t)$, $x^*(t)$, $x_m(t)$, $x_0^*(t)$, $u_m(t)$, $d_i(t)$, and $d_o(t)$ are bounded and $\tilde{K}_e$, $\tilde{K}_{px}$, and $\tilde{K}_{pu}$ are constant.

2.9 A Special Case of Parallel Supplementary Dynamics

We consider the adaptive control algorithm described by Bar-Kana and Kaufman [5,7] which augments the plant with a parallel transfer function matrix $G_a^{-1}(s)$ chosen such that $G_a(s)$ is a dynamic output feedback compensator which stabilizes the plant and such that the plant in parallel with $G_a^{-1}(s)$ is ASPR. We will show that the algorithm proposed by Bar-Kana [5,7] is a special case of our new parallel supplementary dynamics algorithm. The block diagram for Bar-Kana's algorithm is shown in Figure 1.2. The equations which define Bar-Kana's adaptive control law are given by [5,7]

$$u(t) = K_{pe}(t)e_{ypa}(t)+K_{px}(t)x_m(t)+K_{pu}(t)u_m(t) \qquad (2.58)$$

where

$$e_{ypa}(t) = e_{yp}(t) - y_f(t) \qquad (2.59)$$

and where the signal v(t) is defined as

$$v(t) = e_{ypa}(t) \tag{2.60}$$

<u>Lemma 2.1</u>:

The adaptive control algorithm described by Eqs.(2.58)-(2.60) is a special case of the parallel supplementary dynamics algorithm, section 2.5, with

$$Q_p = Q_f = I \tag{2.61}$$

$$G = 0 \tag{2.62}$$

$$\Psi = \begin{bmatrix} 0.5I & & & \\ & 0.5I & & \\ & & I & \\ & & & I \end{bmatrix} \tag{2.63}$$

$$T = \begin{bmatrix} T_{11} & T_{11} & & \\ T_{11} & T_{11} & & \\ & & T_{33} & \\ & & & T_{44} \end{bmatrix} \tag{2.64}$$

$$\bar{T} = \begin{bmatrix} \bar{T}_{11} & \bar{T}_{11} & & \\ \bar{T}_{11} & \bar{T}_{11} & & \\ & & \bar{T}_{33} & \\ & & & \bar{T}_{44} \end{bmatrix} \tag{2.65}$$

where T_{ij} and $\bar{T}_{ij}$ are submatrices of appropriate dimensions.

$$\dim[y_p] = \dim[y_f] = \dim[u_p] \tag{2.66}$$

$$K^I_{pe}(0) = K^I_{pf}(0) \tag{2.67}$$

<u>Proof</u>: First, it is obvious that Eq.(2.60) is a special case of Eq.(2.41) with $Q_p=Q_f=I$ and $G=0$. Next, consider the proportional gain in Eq.(2.38) given by

$$K^P(t) = v(t)r^T(t)\bar{T} \tag{2.68}$$

Using Eq.(2.60) the gain $K^P(t)$ becomes

$$K^P(t) = [e_{yp}(t)-y_f(t)][e_{yp}^T(t),\ -y_f^T(t),\ x_m^T(t), u_m^T(t)]\bar{T} \tag{2.69}$$

and using $\bar{T}$ described by Eq.(2.65) yields

$$K^P(t) = e_{ypa}(t)[e_{ypa}^T(t)\bar{T}_{11},\ e_{ypa}^T(t)\bar{T}_{11},\ x_m^T(t)\bar{T}_{33},\ u_m^T(t)\bar{T}_{44}]$$

$$= [K_{pe}^P(t),\ K_{pe}^P(t),\ K_{px}^P(t),\ K_{pu}^P(t)] \tag{2.70}$$

Next, consider the integral gain in Eq.(2.39) given by

$$\dot{K}^I(t) = [v(t)r^T(t) - \sigma K^I(t)\Psi]T \tag{2.71}$$

Using Eqs.(2.60), (2.63), and (2.64) yields

$$\dot{K}^{I}(t)=[e_{ypa}(t)e_{ypa}^{T}(t)T_{11}-0.5\sigma K_{pe}^{I}(t)T_{11}-0.5\sigma K_{pf}^{I}(t)T_{11},$$

$$e_{ypa}(t)e_{ypa}^{T}(t)T_{11}-0.5\sigma K_{pe}^{I}(t)T_{11}-0.5\sigma K_{pf}^{I}(t)T_{11},$$

$$e_{ypa}(t)x_{m}^{T}(t)T_{33}-\sigma K_{px}^{I}(t)T_{33},\ e_{ypa}(t)u_{m}^{T}(t)T_{44}-\sigma K_{pu}^{I}(t)T_{44}] \tag{2.72}$$

Let $K_{pe}^{I}(0) = K_{pf}^{I}(0)$. Then $K_{pe}^{I}(t) = K_{pf}^{I}(t)$ for all $t \geq 0$

Thus,

$$\dot{K}^{I}(t)=\Big[[e_{ypa}(t)e_{ypa}^{T}(t)-\sigma K_{pf}^{I}(t)]T_{11},$$

$$[e_{ypa}(t)e_{ypa}^{T}(t)-\sigma K_{pf}^{I}(t)]T_{11},\ [e_{ypa}(t)x_{m}^{T}(t)-\sigma K_{px}^{I}(t)]T_{33},$$

$$[e_{ypa}(t)u_{m}^{T}(t)-\sigma K_{pu}^{I}(t)]T_{44}\Big]$$

$$= [\dot{K}_{pe}^{I}(t),\ \dot{K}_{pe}^{I}(t),\ \dot{K}_{px}^{I}(t),\ \dot{K}_{pu}^{I}(t)] \tag{2.73}$$

and

$$K(t) = K^{P}(t) + K^{I}(t)$$

$$= [K_{pe}(t) \quad K_{pe}(t) \quad K_{px}(t) \quad K_{pu}(t)] \tag{2.74}$$

Finally, the adaptive control law becomes

$$u(t) = K(t)r(t)$$

$$= [K_{pe}(t) \quad K_{pe}(t) \quad K_{px}(t) \quad K_{pu}(t)] \begin{bmatrix} e_{yp}(t) \\ -y_{f}(t) \\ x_{m}(t) \\ u_{m}(t) \end{bmatrix}$$

$$= K_{pe}(t)e_{ypa}(t) + K_{px}(t)x_{m}(t) + K_{pu}(t)u_{m}(t) \tag{2.75}$$

which is Bar-Kana and Kaufman's [5] parallel feedforward algorithm.

2.10 Stability Analysis

Theorem 2.1:

Consider the metasystem representation given by Eqs.(2.27) - (2.41) with the controllable and observable LTI plant described by Eqs.(2.1) and (2.2). Further, suppose that there exists a real symmetric positive definite matrix P and real matrices J, L, W, $\tilde{K}_e$, and R, $(R+R^T)>0$ such that

$$P(A-B\tilde{K}_eC) + (A-B\tilde{K}_eC)^TP = -LL^T-R < 0 \qquad (2.76)$$

$$PB = C^T(Q^T + \tilde{K}_e^TG^T) - LW \qquad (2.77)$$

$$W^TW = J + J^T \qquad (2.78)$$

$$J + J^T + G + G^T < 0 \qquad (2.79)$$

where the matrices T and $\bar{T}$ are positive definite symmetric and positive semi-definite symmetric, respectively. Then, all states, gains and errors in the adaptive system are bounded.

Remark 2.4:

The three new MRAC algorithms presented in the previous section are special cases of the metasystem, Eq.(2.27) - (2.41), with the metastates and metamatrices listed in Appendix A. Thus, the stability for these algorithms is ensured by Theorem 2.1 with the sufficient conditions given by Eqs.(2.76)-(2.79).

Remark 2.5:

The stability is analyzed using a Lyapunov approach by forming a quadratic function which is positive definite in the state variables of the adaptive system, $e_x(t)$, and $K^I(t)$. We assume there exists positive definite symmetric matrices P and T. Then, the Lyapunov function candidate, which is positive definite, is chosen as

$$V(e_x,K^I) = e_x^T(t)Pe_x(t)+tr[(K^I(t)-\tilde{K})T^{-1}(K^I(t)-\tilde{K})^T] \tag{2.80}$$

where $\tilde{K}$ is a constant gain matrix which does not appear in the adaptive control algorithm. If the sufficient conditions, Eqs.(2.76)-(2.79), are satisfied, then the derivative of the Lyapunov function candidate, Eq.(2.80), becomes (refer to Appendix B)

$$\dot{V}(e_x,K^I) = -e_x^T(t)Re_x(t)-[L^Te_x(t)-Wz(t)]^T[L^Te_x(t)-Wz(t)]$$

$$+z(t)^T(J+J^T+G+G^T)z(t) - 2v^T(t)v(t)r^T(t)\bar{T}r(t)$$

$$-2\sigma tr[(K^I(t)-\tilde{K})\Psi(K^I(t)-\tilde{K})^T] - 2\sigma tr[(K^I(t)-\tilde{K})\Psi\tilde{K}^T]$$

$$-2e_x^T(t)PF_1(t) - 2z^T(t)F_2(t) \tag{2.81}$$

where

$$F_2=(Q+G\tilde{K}_e)Cx_0^*(t)-G[\tilde{K}_{px}x_m(t)+\tilde{K}_{pu}u_m(t)]-(Q+G\tilde{K}_e)d_o(t) \tag{2.82}$$

We observe that there exist some positive constants α_1, α_2, . . . , α_7 such that

$$\dot{V}(e_x,K^I) \le -\alpha_1\|e_x(t)\|^2-\alpha_2\|[K(t)-\tilde{K}]r(t)\|^2-\alpha_3\|K(t)-\tilde{K}\|^2$$

$$-\alpha_4\|v(t)\|^2 \ \|r(t)\|^2 + \alpha_5\|e_x(t)\| + \alpha_6\|[K(t)-\tilde{K}]r(t)\|$$

$$+\alpha_7\|[K(t)-\tilde{K}]\| \qquad (2.83)$$

If either $\|e_x(t)\|$, $\|[K(t)-\tilde{K}]r(t)\|$, or $\|K(t)-\tilde{K}\|$ increase beyond some bound, then the negative quadratic terms in Eq.(2.83) will become dominant, and thus $\dot{V}$ becomes negative. The quadratic form of the Lyapunov function $V(e_x,K^I)$ then guarantees that $e_x(t)$, $K^I(t)$ and $e_y(t)$ are bounded.

Remark 2.6:

The first three sufficient conditions, given by Eqs.(2.76) - (2.78), are equivalent to requiring that the transfer matrix given by

$$H(s) = J + (Q+G\tilde{K}_e)C(sI-A+B\tilde{K}_eC)^{-1}B \qquad (2.84)$$

is strictly positive real (SPR). The definition of strict positive realness is discussed in detail in reference 10.

Lemma 2.5:

The sufficient conditions, Eqs.(2.76)-(2.79), can be satisfied by algorithm 1 for any controllable and observable plant.

Proof:

If the plant is controllable and observable then there exists a compensator, denoted by the quadruple (A_c, B_c, C_c, D_c), which stabilizes the plant. Thus the composite plant/compensator system is represented by

$$A_{comp} = \begin{bmatrix} A_p + B_p D_c C_p & B_p C_c \\ B_c C_p & A_c \end{bmatrix} \qquad (2.85)$$

where A_{comp} is a stability matrix. Next, we observe from

Eqs.(2.18) and (2.19) that the composite system for algorithm 1 is given by

$$A_1 = A-B\tilde{K}_eC = \begin{bmatrix} A_p - B_p\tilde{K}_{pe}C_p & -B_p\tilde{K}_{pf}C_f \\ -B_fC_p & A_f \end{bmatrix} \tag{2.86}$$

which is required to be a stability matrix for some gain $\tilde{K}_e=[\tilde{K}_{pe},\tilde{K}_{pf}]$. Thus, by comparing Eqs.(2.85) and (2.86) we observe that the choice

$$\tilde{K}_{pe} = -D_c \tag{2.87}$$

$$\tilde{K}_{pf}C_f = -C_c \tag{2.88}$$

$$B_f = -B_c \tag{2.89}$$

$$A_f = A_c \tag{2.90}$$

will result in A_1 being a stability matrix. Now, we need to show that Eqs.(2.79) and (2.84) can be satisfied. Let $C_f=I$ and Let $\tilde{K}_e=[\tilde{K}_{pe},\tilde{K}_{pf}]=[-D_c,-C_c]$. Choose $G=-\gamma_1 I$ and

$J=\gamma_2 I$ where $\gamma_1<\gamma_2$ are positive scalars so that Eq.(2.79), $J+J^T+G+G^T=2\gamma_1 I-2\gamma_2 I<0$, is satisfied.. Then, let $Q=\gamma_1\tilde{K}_e$ so that Eq.(2.84) reduces to $J=\gamma_2 I>0$. Hence, there exists Q, G, $\tilde{K}_e$, and J which satisfy Eqs.(2.79) and (2.84).

<u>Lemma</u> <u>2.6</u>:

The sufficient conditions, Eqs.(2.79) and (2.84), can be satisfied by algorithm 2 for the class of plants which are output stabilizable with a proper but not strictly proper compensator, denoted by (A_c,B_c,C_c,D_c), which satisfies the mild restriction that

$$\text{Rank}\ [\ B_c\ ] = \text{Rank} \begin{bmatrix} B_c \\ D_c \end{bmatrix} \tag{2.91}$$

<u>Proof</u>:

The composite system consisting of the plant with the compensator (A_c,B_c,C_c,D_c) is given by the stability matrix A_{comp} of Eq.(2.85). We observe from Eqs.(2.21) and (2.22) that the composite system for algorithm 2 is given by

$$A_2 = A - B\tilde{K}_e C = \begin{bmatrix} A_p - B_p\tilde{K}_{pe}C_p & -B_p\tilde{K}_{pf}C_f \\ -B_f\tilde{K}_{pe}C_p & A_f - B_f\tilde{K}_{pf}C_f \end{bmatrix} \tag{2.92}$$

which is required to be a stability matrix for some gain $\tilde{K}_e = [\tilde{K}_{pe}, \tilde{K}_{pf}]$. Thus, by comparing Eqs.(2.85) and (2.92) we observe that the choice

$$\tilde{K}_{pe} = -D_c \tag{2.93}$$

$$C_f = I, \quad \tilde{K}_{pf} = -C_c \tag{2.94}$$

$$-B_f\tilde{K}_{pe} = B_c \quad (\text{ or } B_f D_c = B_c) \tag{2.95}$$

$$A_f = A_c - B_f C_c \tag{2.96}$$

will result in A_2 being a stability matrix where D_c must not be a zero matrix and where Eq.(2.95) has a solution for B_f if Eq.(2.91) is satisfied. Then, Q, G, and J can be chosen to satisfy Eqs.(2.79) and (2.84) by using the constructive method shown in the proof of Lemma 2.5.

Lemma 2.7:

The sufficient conditions, Eqs.(2.79) and (2.84), can be satisfied by algorithm 3 for the class of plants which are output stabilizable with a proper but not strictly proper compensator, denoted by (A_c, B_c, C_c, D_c), which satisfies the mild restriction that

$$\mathrm{Rank}\ [\ B_c\ ] = \mathrm{Rank}\begin{bmatrix} B_c \\ D_c \end{bmatrix} \tag{2.97}$$

Proof:

The composite system consisting of the plant with the compensator (A_c, B_c, C_c, D_c) is given by the stability matrix A_{comp} of Eq.(2.85). We observe from Eqs.(2.25) and (2.26) that the composite system for algorithm 3 is given by

$$A_3 = A - B\tilde{K}_e C = \begin{bmatrix} A_p - B_p D_f \tilde{K}_{pe} C_p & -B_p (I + D_f \tilde{K}_{pf}) C_f \\ -B_f \tilde{K}_{pe} C_p & A_f - B_f \tilde{K}_{pf} C_f \end{bmatrix} \tag{2.98}$$

which is required to be a stability matrix for some gain $\tilde{K}_e=[\tilde{K}_{pe},\tilde{K}_{pf}]$. Thus, by comparing Eqs.(2.85) and (2.98) we observe that the choice

$$D_f = I, \quad \tilde{K}_{pe} = -D_c \tag{2.99}$$

$$C_f = I, \quad \tilde{K}_{pf} = -C_c-I \tag{2.100}$$

$$-B_f\tilde{K}_{pe} = B_c \quad (\text{ or } B_fD_c = B_c) \tag{2.101}$$

$$A_f = A_c - B_f(C_c+I) \tag{2.102}$$

will result in A_3 being a stability matrix where D_c must not be a zero matrix and where Eq.(2.101) has a solution for B_f if Eq.(2.97) is satisfied. Then, Q, G, and J can be chosen to satisfy Eqs.(2.79) and (2.84) by using the constructive method shown in the proof of Lemma 2.5.

Remark 2.7:

We note that the class of plants which can be controlled by algorithms 2 and 3 is more restricted than the class of plants which can be controlled by algorithm 1. However, algorithms 2 and 3 may be the preferred controllers because we will show in section 2.11 that algorithms 2 and 3 will yield an asymptotically vanishing error provided that some additional conditions are satisfied.

2.11 Asymptotic Output Tracking

The stability analysis in the preceding section only ensures that all signals in the adaptive system are bounded. In this section, we derive conditions for algorithms 2 and 3 under which the output error is asymptotically vanishing provided that there are no disturbances and provided that the model input is constant for $t \geq t_1$. This new result is based upon extending Broussard's [4] command generator tracker (CGT) for model following control of known plants to the metasystem described by Eqs.(2.27)-(2.41).

When perfect tracking occurs (i.e., $e_{yp}(t)=\dot{e}_{yp}(t)=0$), we define the corresponding metasystem state and control trajectories to be the ideal metasystem state and ideal metasystem control trajectories, respectively. These ideal trajectories will be denoted by $x^*(t)$ and $u^*(t)$, respectively, where

$$x^*(t) = \begin{bmatrix} x_p^*(t) \\ x_f^*(t) \end{bmatrix} \tag{2.103}$$

and where $x_p^*(t)$ is the ideal plant state and $x_f^*(t)$ is the ideal supplementary dynamics state. It is important that the reader does not confuse the ideal trajectories described here with the fictitious target trajectories shown in section 2.3. Although the same notation is used for both cases, it will be clear from the context which is being used.

By definition, the ideal metasystem is such that it satisfies the same dynamics as the real metasystem. In addition, the output of the ideal plant is defined to be identically equal to the model output. Mathematically, we have

$$\dot{x}^*(t) = Ax^*(t) + Bu^*(t) \qquad \text{for all } t \geq t_0 \tag{2.104}$$

and

$$y_p^*(t) = y_m(t) \quad \text{or} \quad [C_p \quad 0]x^* = C_m x_m(t) \tag{2.105}$$

Hence, when perfect tracking occurs the real metasystem trajectories become the ideal metasystem trajectories, and the real plant output becomes the ideal plant output which is defined to be the model output.

Let us assume that the model input u_m is constant and that the ideal trajectories are linear functions of the model state $x_m(t)$ and the model input u_m. Mathematically, we have that

$$\begin{bmatrix} x^*(t) \\ u^*(t) \end{bmatrix} = \begin{bmatrix} S_1 & S_2 \\ S_{31} & S_{32} \end{bmatrix} \begin{bmatrix} x_m(t) \\ u_m \end{bmatrix} \qquad (2.106)$$

where

$$S_1 = \begin{bmatrix} S_{11} \\ S_{21} \end{bmatrix} \qquad S_2 = \begin{bmatrix} S_{12} \\ S_{22} \end{bmatrix} \qquad (2.107)$$

Thus,

$$x_p^*(t) = S_{11}x_m(t) + S_{12}u_m \qquad (2.108)$$

$$x_f^*(t) = S_{21}x_m(t) + S_{22}u_m \qquad (2.109)$$

$$u^{*}(t) = S_{31}x_m(t) + S_{32}u_m \tag{2.110}$$

Upon combining the ideal metasystem, Eq.(2.104), with the ideal plant output, Eq.(2.105), yields

$$\begin{bmatrix} \dot{x}^{*}(t) \\ y_p^{*}(t) \end{bmatrix} = \begin{bmatrix} A & B \\ [C_p \;\; 0] & 0 \end{bmatrix} \begin{bmatrix} x^{*}(t) \\ u^{*}(t) \end{bmatrix} \tag{2.111}$$

and upon substituting Eq.(2.106) into Eq.(2.111), we obtain

$$\begin{bmatrix} \dot{x}^{*}(t) \\ y_p^{*}(t) \end{bmatrix} = \begin{bmatrix} A & B \\ [C_p \;\; 0] & 0 \end{bmatrix} \begin{bmatrix} S_1 & S_2 \\ S_{31} & S_{32} \end{bmatrix} \begin{bmatrix} x_m(t) \\ u_m \end{bmatrix} \tag{2.112}$$

Now we differentiate $x^{*}(t)$ in Eq.(2.106) to obtain

$$\dot{x}^{*}(t) = S_1\dot{x}_m(t) \tag{2.113}$$

where we have used the assumption that u_m is constant.

Next, we substitute the equation of the model dynamics into Eq.(2.113) to obtain

$$\dot{x}^*(t) = S_1 A_m x_m(t) + S_1 B_m u_m \tag{2.114}$$

We concatenate Eq.(2.114) with the reference model output to obtain

$$\begin{bmatrix} \dot{x}^*(t) \\ y_m(t) \end{bmatrix} = \begin{bmatrix} S_1 A_m & S_1 B_m \\ C_m & 0 \end{bmatrix} \begin{bmatrix} x_m(t) \\ u_m(t) \end{bmatrix} \tag{2.115}$$

Comparing Eqs.(2.112) and (2.115) we obtain

$$\begin{bmatrix} S_1 A_m & S_1 B_m \\ C_m & 0 \end{bmatrix} = \begin{bmatrix} A & B \\ [C_p \;\; 0] & 0 \end{bmatrix} \begin{bmatrix} S_1 & S_2 \\ S_{31} & S_{32} \end{bmatrix} \tag{2.116}$$

If we define

$$\begin{bmatrix} \Omega_{11} & \Omega_{12} \\ \Omega_{21} & \Omega_{22} \end{bmatrix} = \begin{bmatrix} A & B \\ [C_p \;\; 0] & 0 \end{bmatrix}^{-1} \tag{2.117}$$

then

$$\begin{bmatrix} S_1 & S_2 \\ S_{31} & S_{32} \end{bmatrix} = \begin{bmatrix} \Omega_{11} & \Omega_{12} \\ \Omega_{21} & \Omega_{22} \end{bmatrix} \begin{bmatrix} S_1 A_m & S_1 B_m \\ C_m & 0 \end{bmatrix} \tag{2.118}$$

Broussard [4] has shown that an equation of the type given by Eq.(2.118) has a solution for S_1, S_2, S_{31}, and S_{32} if (i) u_m is a constant, (ii) $\dim[y_p(t)]=\dim[u_p(t)]$, and (iii) no eigenvalue of Ω_{11} is equal to the inverse of an eigenvalue of A_m.

<u>Corollary</u> <u>2.1</u>:

Let the adaptive controller be algorithm 2 or 3. The adaptive control algorithm described by Theorem 2.1 yields an asymptotically vanishing output error if the conditions of Theorem 2.1 are satisfied and if (i) u_m is constant for $t \geq t_1$, (ii) no disturbances exist and $\sigma = 0$, (iii) a solution exists for the matrices S_1, S_2, S_{31}, and S_{32} in Eq.(2.116), and (iv) there exists a real matrix E of dimension $m \times n_f$ such that

$$G = -EB_f \tag{2.119}$$

and

$$Q_f C_f = EA_f \tag{2.120}$$

Furthermore, if $\det[A_f] \neq 0$, then Eqs.(2.119) and (2.120) reduce to

$$G = -Q_f C_f A_f^{-1} B_f \tag{2.121}$$

<u>Proof</u>:

The Lyapunov derivative, Eq.(2.81), with σ=0, d_i=0, and

$d_0=0$ becomes

$$\dot{V}(e_x,K^I) = -e_x^T(t)Re_x(t) - [L^Te_x(t)-Wz(t)]^T[L^Te_x(t)-Wz(t)]$$

$$+z(t)^T(J+J^T+G+G^T)z(t)-2v^T(t)v(t)r^T(t)\bar{T}r(t)$$

$$-2e_x^T(t)PF_1 - 2z^T(t)F_2(t) \tag{2.122}$$

where

$$F_1(t) = [-\dot{x}^*(t)+Ax^*(t)] - B\tilde{K}_eCx_0^*(t) + B[\tilde{K}_{px}x_m(t)+\tilde{K}_{pu}u_m] \tag{2.123}$$

$$F_2 = (Q + G\tilde{K}_e)Cx_0^*(t) - G[\tilde{K}_{px}x_m(t) + \tilde{K}_{pu}u_m(t)] \tag{2.124}$$

Next, we obtain conditions such that $F_1(t)=0$ and $F_2(t)\to 0$.

From Eq.(2.104) and $\tilde{K}_eCx_0^*(t) = \tilde{K}_{pf}C_fx_f^*(t)$, we obtain

$$F_1(t) = -Bu_p^*(t) - B\tilde{K}_{pf}C_fx_f^*(t) + B[\tilde{K}_{px}x_m(t)+\tilde{K}_{pu}u_m] \tag{2.125}$$

$$F_2(t) = (Q_f + G\tilde{K}_{pf})C_f x_f^*(t) - G[\tilde{K}_{px}x_m(t) + \tilde{K}_{pu}u_m] \tag{2.126}$$

Then, we apply Eqs.(2.109) and (2.110) to yield

$$\begin{aligned} F_1(t) &= -B[S_{31}x_m(t)+S_{32}u_m] - B\tilde{K}_{pf}C_f[S_{21}x_m(t)+S_{22}u_m] \\ &\quad +B[\tilde{K}_{px}x_m(t) + \tilde{K}_{pu}u_m] \\ &= -B[(\tilde{K}_{px}-S_{31}-\tilde{K}_{pf}C_fS_{21})x_m(t)+(\tilde{K}_{pu}-S_{32}-\tilde{K}_{pf}C_fS_{22})u_m] \end{aligned} \tag{2.127}$$

$$\begin{aligned} F_2(t) &= (Q_f+G\tilde{K}_{pf})C_f[S_{21}x_m(t)+S_{22}u_m(t)] - G[\tilde{K}_{px}x_m(t)+\tilde{K}_{pu}u_m] \\ &= [(Q_f+G\tilde{K}_{pf})C_fS_{21}-G\tilde{K}_{px}]x_m(t)+[(Q_f+G\tilde{K}_{pf})C_fS_{22}-G\tilde{K}_{pu}]u_m \\ &= [Q_fC_fS_{21}+G(\tilde{K}_{pf}C_fS_{21}-\tilde{K}_{px})]x_m(t) \\ &\quad +[Q_fC_fS_{22}+G(\tilde{K}_{pf}C_fS_{22}-\tilde{K}_{pu})]u_m \end{aligned} \tag{2.128}$$

Next, using Eqs.(2.119) and (2.120) obtain

$$
\begin{aligned}
F_2(t) &= [EA_fS_{21} - EB_f(\tilde{K}_{pf}C_fS_{21} - \tilde{K}_{px})]x_m(t) \\
&\qquad + [EA_fS_{22} - EB_f(\tilde{K}_{pf}C_fS_{22} - \tilde{K}_{pu})]u_m \\
&= EA_fS_{21}x_m(t) - EB_f(\tilde{K}_{pf}C_fS_{21} + \tilde{K}_{px} - S_{31} + S_{31})x_m(t) \\
&\quad + EA_fS_{22}u_m - EB_f(\tilde{K}_{pf}C_fS_{22} + \tilde{K}_{pu} + S_{32} - S_{32})u_m \\
&= EA_f[S_{21}x_m(t) + S_{22}u_m] + EB_f[S_{31}x_m(t) + S_{32}u_m] \\
&\qquad + EB_f(\tilde{K}_{px} - \tilde{K}_{pf}C_fS_{21} - S_{31})x_m(t) \\
&\qquad + EB_f(\tilde{K}_{pu} - \tilde{K}_{pf}C_fS_{22} - S_{32})u_m \\
&= E[A_fx_f^*(t) + B_fu_p^*(t)] + EB_f(\tilde{K}_{px} - \tilde{K}_{pf}C_fS_{21} - S_{31})x_m(t) \\
&\qquad + EB_f(\tilde{K}_{pu} - \tilde{K}_{pf}C_fS_{22} - S_{32})u_m \\
&= ES_{21}\dot{x}_m(t) + EB_f(\tilde{K}_{px} - \tilde{K}_{pf}C_fS_{21} - S_{31})x_m(t) \\
&\qquad + EB_f(\tilde{K}_{pu} - \tilde{K}_{pf}C_fS_{22} - S_{32})u_m \qquad (2.129)
\end{aligned}
$$

We choose

$$\tilde{K}_{px}=\tilde{K}_{pf}C_fS_{21}+S_{31} \tag{2.130}$$

$$\tilde{K}_{pu}=\tilde{K}_{pf}C_fS_{22}+S_{32} \tag{2.131}$$

such that $F_1(t)=0$, and $F_2(t)=ES_{21}\dot{x}_m(t)$ which vanishes asymptotically because u_m is constant for $t\geq t_1$.

Finally, the Lyapunov derivative becomes

$$\dot{V}(e_x,K^I)=-e_x^T(t)Re_x(t)-[L^Te_x(t)-Wz(t)]^T[L^Te_x(t)-Wz(t)]$$
$$+z(t)^T(J+J^T+G+G^T)z(t)-2v^T(t)v(t)r^T(t)\bar{T}r(t)+2z^T(t)ES_{21}\dot{x}_m(t) \tag{2.132}$$

where $R>0$, $\bar{T}\geq 0$, and $J+J^T+G+G^T<0$

Note that $\dot{V}(e_x,K^I)$ is not negative definite or semi-definite due to the last term in Eq.(2.132). However, if we define

$$\dot{V}(e_x,K^I) = W_1(e_x,K^I) + W_2(e_x,K^I) \tag{2.133}$$

such that

$$W_1(e_x,K^I) \le \tilde{W}_1(e_x,K^I) \le 0 \tag{2.134}$$

where

$$W_1(e_x,K^I)=-e_x^T(t)Re_x(t)-[L^Te_x(t)-Wz(t)]^T[L^Te_x(t)-Wz(t)]$$

$$+z(t)^T(J+J^T+2G)z(t)-2v^T(t)v(t)r^T(t)\bar{T}r(t)$$

$$W_2(e_x,K^I) = 2z^T(t)ES_{21}\dot{x}_m(t) \quad \text{and}$$

$$\tilde{W}_1(e_x,K^I) = -\alpha_1\|e_x(t)\|^2-\alpha_2\|[K(t)-\tilde{K}]r(t)\|^2-\alpha_3\|v(t)\|^2 \ \|r(t)\|^2$$

and where α_1, α_2, and α_3 are some positive constants.

Then,

$$\lim_{t\to\infty} W_2(e_x,K^I) = 0 \tag{2.135}$$

which yields

$$\dot{V}(e_x,K^I) \to W_1(e_x,K^I) \quad \text{as} \quad t \to \infty \tag{2.136}$$

We call $W_1(e_x,K^I)$ the "limiting derivative of the Lyapunov function."

In Theorem 2.1, we have shown that all states, gains, and errors, in the adaptive system are bounded. Therefore, $V(e_x,K^I)$ is bounded (this is required by LaSalle's invariance set principle). Further, Eqs.(2.133)-(2.136) show that $\dot{V}(e_x,K^I)$ will approach $W_1(e_x,K^I)$ as $W_2(e_x,K^I)$ vanishes. However, Eq.(2.134) shows that $W_1(e_x,K^I)$ is bounded by $\tilde{W}_1(e_x,K^I) \le 0$. Thus, we know that $\dot{V}(e_x,K^I)$ is negative semi-definite. A modified version of LaSalle's invariance set principle [14] (Theorem C in Appendix C) is used for the stability proof of the "limiting derivative of the Lyapunov function" which shows that all signals in the adaptive control system are bounded and that the output tracking error vanishes asymptotically if Eqs(2.133)-(2.136) are satisfied.

It can be seen from Eq.(2.132) that the transient of the plant depends on the transient of the model. The asymptotically vanishing tracking error will be obtained in steady state while the bounded output tracking error will be ensured for all $t \geq 0$.

Lemma 2.8:

Let the adaptive controller be algorithm 2 or 3 and let the plant belong to the class described by corollary 2.1 with the additional restriction that $A_c=0$. Suppose that (i) u_m is constant for $t \geq t_1$, (ii) no disturbances exist and $\sigma=0$, and (3) a solution exists for the matrices S_1, S_2, S_{31}, and S_{32} in Eq.(2.116). If there exists a real matrix E of dimension $m \times n_f$ such that

$$EB_f > 0 \tag{2.137}$$

then the sufficient conditions for a bounded and asymptotically vanishing error can be satisfied.

<u>Proof</u>:

Let the supplementary dynamics and the gain $\tilde{K}_e$ be chosen as in Lemma 2.6 (algorithm 2), or Lemma 2.7 (algorithm 3). Then, choose Q_f and G to satisfy Corollary 2.1. That is,

$$Q_f = EA_f \tag{2.138}$$

$$G = -EB_f \tag{2.139}$$

We choose

$$Q_p = -EB_fD_c \tag{2.140}$$

and use Eqs.(2.138) and (2.139) to obtain

$$Q + G\tilde{K}_e = [Q_p,\ Q_f] - [\ EB_f\tilde{K}_{pe},\ EB_f\tilde{K}_{pf}]$$

$$= [-EB_fD_c-EB_f\tilde{K}_{pe},\ EA_f-EB_f\tilde{K}_{pf}]$$

$$= [-EB_f(D_c+\tilde{K}_{pe}),\ E(A_f-B_f\tilde{K}_{pf})] \tag{2.141}$$

From Lemma 2.6 or 2.7 we obtain $\tilde{K}_{pe}=-D_c$ and $A_c=A_f-B_f\tilde{K}_{pf}=0$. Thus

$$Q + G\tilde{K}_e = [-EB_f(D_c-D_c),\ EA_c] = 0 \tag{2.142}$$

and the sufficient condition given by Eqs.(2.84) and (2.79) reduces to

$$H(s) = J > 0 \tag{2.143}$$

$$J + J^T + G + G^T < 0 \tag{2.144}$$

Let us choose

$$J=\gamma_3 EB_f \text{ with } 0<\gamma_3<1 \tag{2.145}$$

Then

$$J = \gamma_3 E B_f > 0 \tag{2.146}$$

and

$$J + J^T + G + G^T = \gamma_3 (EB_f + B_f^T E^T) - (EB_f + B_f^T E^T)$$

$$= (\gamma_3 - 1)(EB_f + B_f^T E^T) < 0 \tag{2.147}$$

Hence, the sufficient conditions for a bounded error given by Eqs.(2.79) and (2.84) and the sufficient conditions for an asymptotically vanishing error given by Eqs.(2.119) and (2.120) are satisfied.

Remark 2.10:

A trivial solution for Lemma 2.8 is to choose $E=B_f^T$ such that $EB_f=B_f^T B_f>0$.

Remark 2.11:

If $\det[A_f]\neq 0$, we can choose Q_f such that $G=-Q_fA_f^{-1}B_f<0$, and $Q_p=GD_c$. This is a special case of Lemma 2.8 with $E=-Q_fA_f^{-1}$

Remark 2.12:

Lemma 2.9 shows the relationship between the sufficient conditions for a bounded error and the sufficient condition for an asymptotically vanishing error. In particular, Lemma 2.9 shows that it is possible to satisfy all of the sufficient conditions for a rather large class of plants. One example of a compensator which satisfies the assumptions of lemma 2.9 is a proportional + integral (PI) controller.

2.12 Summary of Constraints and Design Rules

Objective:

To find a control signal by means of adaptive computation such that the plant, which is not ASPR, will follow the reference model with a bounded, or an asymptotically vanishing tracking error. The plant, model, and adaptive control law employed in algorithms 1, 2, and 3 are described as follows:

Plant:

$$\dot{x}_p(t) = A_p x_p(t) + B_p u_p(t)$$

$$y_p(t) = C_p x_p(t)$$

Model:

$$\dot{x}_m(t) = A_m x_m(t) + B_m u_m(t)$$

$$y_m(t) = C_m x_m(t)$$

<u>Adaptive</u> <u>Control</u> <u>Law</u>:

$u_p(t) = K(t)r(t)$ for algorithms 1 and 2

$u_p(t) = D_f K(t)r(t) - y_f(t)$ for algorithm 3

where

$$K(t) = K^P(t) + K^I(t)$$

$$K^P(t) = v(t)r^T(t)\bar{T}$$

$$\dot{K}^I(t) = [v(t)r^T(t) - \sigma K^I(t)]T$$

$$v(t) = Qe_{yv}(t) + Gu_p(t)$$

$$r^T(t) = [\ e_{yv}^T(t) \quad x_m^T(t) \quad u_m^T(t)\ ]$$

$$e_{yv}^T(t) = [(y_m(t) - y_p(t))^T \quad -y_f^T(t)\ ]$$

$T>0$ and $T\geq 0$

<u>Assumptions</u>:

Bounded Tracking Error:

$\mathrm{Rank}[C_p \vdots C_m] = \mathrm{Rank}[C_p]$

Asymptotically Vanishing Tracking Error:

(i) u_m is constant for $t \geq t_1$.

(ii) No disturbances exist and $\sigma=0$.

(iii) A solution exists for the extended CGT condition, Eq.(2.116).

Then, the constraints and design rules for designing the supplementary dynamics and choosing parameters G and Q are as follows:

Algorithm 1: (Bounded Tracking Error)

Constraints:

(i) There exists a compensator (A_c, B_c, C_c, D_c) which stabilizes the plant

$$ \tag{2.148} $$

$$(ii)\ Q + G\tilde{K}_e = 0 \tag{2.149}$$

$$(iii)\ J + J^T + G + G^T < 0 \tag{2.150}$$

where J is a positive definite matrix.

Supplementary Dynamics:

$$\dot{x}_f(t) = A_f x_f(t) + B_f[y_m(t) - y_p(t)]$$

$$y_f(t) = C_f x_f(t)$$

Design Method:

$$(i)\ \text{Choose } C_f = I \tag{2.151}$$

(ii) Choose $\tilde{K}_e = [\tilde{K}_{pe} \quad \tilde{K}_{pf}] = [-D_c \quad -C_c]$ (2.152)

(iii) Choose $B_f = -B_c$ (2.153)

(iv) Choose $A_f = A_c$ (2.154)

(v) Choose $G<0$ (2.155)

(vi) Choose $Q = [Q_p \quad Q_f] = G[D_c \quad C_c]$ (2.156)

<u>Algorithm</u> <u>2</u>:(bounded tracking error)

<u>Constraints</u>:

(i) There exists a proper, but not strictly proper, compensator (A_c, B_c, C_c, D_c) which stabilizes the plant and which satisfies

$$\text{Rank} \; [\, B_c \,] = \text{Rank} \begin{bmatrix} B_c \\ D_c \end{bmatrix} \qquad (2.157)$$

$$\text{(ii)} \; Q + G\tilde{K}_e = 0 \qquad (2.158)$$

$$\text{(iii)} \; J + J^T + G + G^T < 0 \qquad (2.159)$$

where J is positive definite matrix

<u>Supplementary</u> <u>Dynamics</u>:

$$\dot{x}_f(t) = A_f x_f(t) + B_f u_p(t)$$

$$y_f(t) = C_f x_f(t)$$

<u>Design</u> <u>Method</u>:

(i) Choose $C_f = I$ (2.160)

(ii) Choose $\tilde{K}_e = [\tilde{K}_{pe} \quad \tilde{K}_{pf}] = [-D_c \quad -C_c]$ (2.161)

(iii) Solve $B_f D_c = B_c$ for B_f (2.162)

(iv) Choose $A_f = A_c - B_f C_c$ (2.163)

(v) Choose $G < 0$ (2.164)

(vi) Choose $Q = [Q_p \quad Q_f] = G[D_c \quad C_c]$ (2.165)

<u>Algorithm</u> <u>2</u>: (asymptotically vanishing tracking error)

<u>Constraints</u>:

(i) There exists a proper, but not strictly proper, compensator (A_c, B_c, C_c, D_c) which stabilizes the plant and which satisfies

$$\mathrm{Rank}\ [\ B_c\ ] = \mathrm{Rank} \begin{bmatrix} B_c \\ D_c \end{bmatrix} \tag{2.166}$$

$$\text{(ii)}\ Q + G\tilde{K}_e = 0 \tag{2.167}$$

$$\text{(iii)}\ J + J^T + G + G^T < 0 \tag{2.168}$$

$$\text{(iv)}\ G=-EB_f \text{ and } Q_fC_f=EA_f;\ \text{or } G=-Q_fC_fA_f^{-1}B_f \text{ if } \det[A_f]\neq 0 \tag{2.169}$$

where J is a positive definite matrix.

<u>Supplementary</u> <u>Dynamics</u>:

$$\dot{x}_f(t) = A_fx_f(t) + B_fu_p(t)$$

$$y_f(t) = C_fx_f(t)$$

<u>Design</u> <u>Method</u>:

(i) Choose $C_f = I$ (2.170)

(ii) Choose $\tilde{K}_e = [\tilde{K}_{pe} \quad \tilde{K}_{pf}] = [-D_c \quad -C_c]$ (2.171)

(iii) Solve $B_f D_c = B_c$ for B_f (2.172)

(iv) Choose $A_f = -B_f C_c$ (2.173)

(v) Choose $G = -EB_f < 0$ (2.174)

(vi) Choose $Q = [Q_p \quad Q_f] = G[D_c \quad C_c]$ (2.175)

<u>Remark</u> <u>2.13</u>:

If $\det[A_f] \neq 0$, Eqs.(2.174) and (2.175) become $G=-Q_f A_f^{-1} B_f < 0$ and $Q_p = GD_c$.

Algorithm 3: (bounded tracking error)

Constraints:

(i) There exists a proper, but not strictly proper, compensator (A_c, B_c, C_c, D_c) which stabilizes the plant and which satisfies

$$\text{Rank}\,[\,B_c\,] = \text{Rank}\begin{bmatrix} B_c \\ D_c \end{bmatrix} \tag{2.176}$$

$$\text{(ii)}\quad Q + G\tilde{K}_e = 0 \tag{2.177}$$

$$\text{(iii)}\quad J + J^T + G + G^T < 0 \tag{2.178}$$

where J is a positive definite matrix.

Supplementary Dynamics:

$$\dot{x}_f(t) = A_f x_f(t) + B_f u_p(t)$$

$$y_f(t) = C_f x_f(t)$$

<u>Design</u> <u>Method</u>:

(i) Choose $C_f = I$ and $D_f = I$ (2.179)

(ii) Choose $\tilde{K}_e = [\tilde{K}_{pe} \quad \tilde{K}_{pf}] = [-D_c \quad -C_c - I]$ (2.180)

(iii) Solve $B_f D_c = B_c$ for B_f (2.181)

(iv) Choose $A_f = A_c - B_f(C_c + I)$ (2.182)

(v) Choose $G < 0$ (2.183)

(vi) Choose $Q = [Q_p \quad Q_f] = G[D_c \quad C_c]$ (2.184)

<u>Algorithm</u> <u>3</u>: (asymptotically vanishing tracking error)

<u>Constraints</u>:

(i) There exists a proper, but not strictly proper, compensator (A_c, B_c, C_c, D_c) which stabilizes the plant and which satisfies

$$\mathrm{Rank}\ [\ B_c\ ] = \mathrm{Rank} \begin{bmatrix} B_c \\ D_c \end{bmatrix} \tag{2.185}$$

$$\text{(ii)}\quad Q + G\tilde{K}_e = 0 \tag{2.186}$$

$$\text{(iii)}\quad J + J^T + G + G^T < 0 \tag{2.187}$$

$$\text{(iv)}\quad G=-EB_f \text{ and } Q_fC_f=EA_f;\ \text{or } G=-Q_fC_fA_f^{-1}B_f \text{ if } \det[A_f]\neq 0 \tag{2.188}$$

where J is a positive definite matrix.

<u>Supplementary</u> <u>Dynamics</u>:

$$\dot{x}_f(t) = A_fx_f(t) + B_fu_p(t)$$

$$y_f(t) = C_fx_f(t)$$

Design Method:

(i) Choose $C_f = I$ and $D_f = I$ (2.189)

(ii) Choose $\tilde{K}_e = [\tilde{K}_{pe} \quad \tilde{K}_{pf}] = [-D_c \quad -C_c - I]$ (2.190)

(iii) Solve $B_f D_c = B_c$ for B_f (2.191)

(iv) Choose $A_f = -B_f(C_c + I)$ (2.192)

(v) Choose $G = -EB_f < 0$ (2.193)

(vi) Choose $Q = [Q_p \quad Q_f] = G[D_c \quad C_c]$ (2.194)

Remark 2.14:

If $\det[A_f] \neq 0$, Eqs. (2.193) and (2.194) become $G = -Q_f A_f^{-1} B_f < 0$ and $Q_p = GD_c$.

2.13 Examples

This section presents three examples to illustrate the application of the adaptive control algorithm. The examples include the so-called Rohrs' example [18], an unstable single-input single-output plant which was studied by Bar-Kana [5], and the lateral dynamics of the F-8 aircraft which was studied by Sobel et. al. [3], The adaptive gains K_e, K_x, and K_u will be initialized to zero in all of the examples.

The plant, model, and adaptive algorithm are simulated on a digital computer in order to determine the closed loop system performance. Therefore, in the simulations of the continuous systems we approximate the integrations in a discrete representation with a step size of Δt, which is chosen for each example in a manner which trades off computation time and numerical accuracy. The control algorithm equation for the integral gain update, given by Eq.(2.39), is integrated by using

$$K^I(i\Delta t+\Delta t) = K^I(i\Delta t) + \Delta t[v(i\Delta t)r^T(i\Delta t) - \sigma K^I(i\Delta t)\Psi]T \tag{2.195}$$

The plant dynamics, which are described by Eq.(2.7), are integrated by using

$$x(i\Delta t+\Delta t)=e^{A_p\Delta t}x_p(i\Delta t)+\left(\int_0^{\Delta t}e^{A\tau}\,d\tau\right)[B_pu_p(i\Delta t)+d_{1p}(i\Delta t)] \tag{2.196}$$

The model dynamics and the supplementary dynamics, described by Eqs.(2.3) and (2.5), respectively, are integrated in a manner analogous to Eq.(2.196).

<u>Example 1</u>:

(Rohrs' Example[18])

The so-called Rohrs'[18] example is a difficult problem for many other adaptive algorithms. The plant is given by

$$\frac{Y_p(s)}{U_p(s)} = \frac{2}{s+1} \; \frac{229}{s^2+30s+229} \tag{2.197}$$

The output of this plant is required to follow the output of the reference model which is shown below:

$$\frac{Y_m(s)}{U_m(s)} = \frac{1}{1+s/3} \tag{2.198}$$

Suppose we know that a stabilizing PI compensator is described by

$$G_c(s) = \frac{-(10s+35)}{s} \tag{2.199}$$

<u>Remark 2.15</u>:

Although the compensator described by Eq.(2.199) is stabilizing, it yields a closed loop system with a settling time greater than 500 seconds and a damping ratio for the dominant poles of less than 0.001. The time response of the output to a square wave input is shown in Figure 2.4 where we observe that the performance is completely unacceptable.

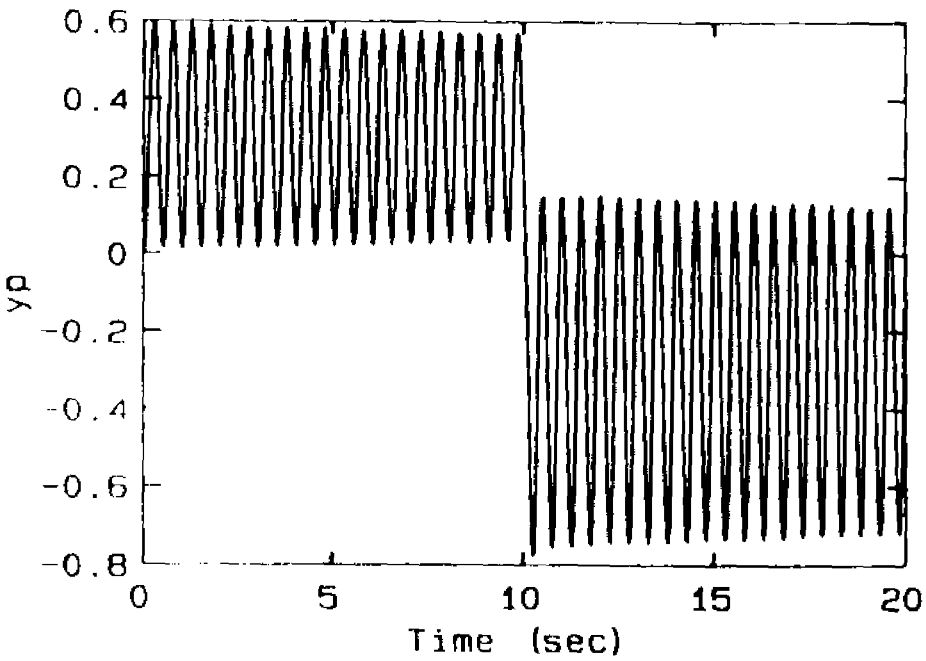

Figure 2.4 Rohrs' Example:
Output with Non-adaptive Compensator

We remark that the choice of some stabilizing compensator which is needed for computation of the matrices A_f, B_f, Q, and G is a much easier task than the computation of a compensator which satisfies stringent performance specifications for an unknown or poorly known plant.

The compensator described by Eq.(2.199) has a state space realization given by $A_c=0$, $B_c=10$, $C_c=-3.5$, and $D_c=-10$. To illustrate the difference between algorithms, we simulate Rohrs' example by using algorithm 1, 2, and 3 successively. We use a square wave reference command of magnitude 0.3 units and period of 20 second, and select $C_f=1$, $T=\bar{T}=I$, and $d_{ip}(t)=d_{op}(t)=0$.

<u>Algorithm 1</u>:

Using Eqs.(2.87)-(2.90) we obtain

$$\tilde{K}_{pe} = -D_c = 10 \tag{2.200}$$

$$\tilde{K}_{pf} = -C_c = 3.5 \tag{2.201}$$

$$B_f = -B_c = -10 \tag{2.202}$$

$$A_f = A_c = 0 \tag{2.203}$$

Therefore, the supplementary dynamics which are inserted into the output feedback path are of the form

$$\dot{x}_f(t) = -10e_{yp}(t) \tag{2.204}$$

$$y_f(t) = x_f(t) \tag{2.205}$$

Next, we choose $Q_p=57.14$, $Q_f=20$, and $G=-5.714$ such that the sufficient condition for stability $Q+G\tilde{K}_e=0$ is satisfied.

The digital computer simulation is plotted in Figure 2.5 which shows that the plant output tracks the model output with a small steady state error. This error may be reduced if the plant can be stabilized via a high gain output feedback loop.

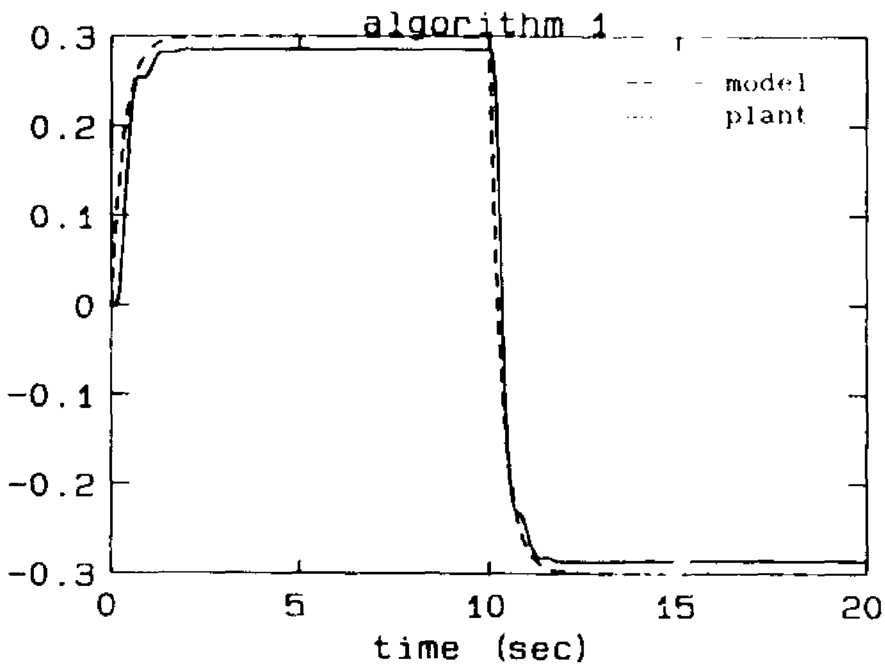

Figure 2.5 Rohrs' Example:
Plant and Model Output (Algorithm 1)

Algorithm 2:

Using Eqs.(2.93)-(2.96) we obtain

$$\tilde{K}_{pe} = -D_c = 10 \tag{2.206}$$

$$\tilde{K}_{pf} = -C_c = 3.5 \tag{2.207}$$

$$B_f D_c = B_c \Rightarrow B_f = B_c / D_c = -1 \tag{2.208}$$

$$A_f = -B_f C_c = -3.5 \tag{2.209}$$

Since $\det[A_f] \neq 0$, we can choose $Q_f = 20$ such that $G = -Q_f A_f^{-1} B_f = -5.71428 < 0$. Then, choosing $Q_p = GD_c. = 57.1428$, we obtain

$$Q = [\ 57.1428 \quad 20\ ] \tag{2.210}$$

$$G = -5.71428 \tag{2.211}$$

Therefore, the parallel supplementary dynamics are described by

$$\dot{x}_f(t) = -3.5x_f(t) - u_p(t) \tag{2.212}$$

$$y_f(t) = x_f(t) \tag{2.213}$$

The plant and model outputs are shown in Figure 2.6. We observe the excellent transient behavior with zero error in approximately 2 seconds. We remark that the condition

$G=Q_f A_f^{-1} B_f$ is necessary to achieve the asymptotically vanishing output tracking error. To demonstrate this, we simulate Rohrs' example with the same design values except G=-10. The result is plotted in Figure 2.7 which shows that asymptotic output tracking is no longer maintained.

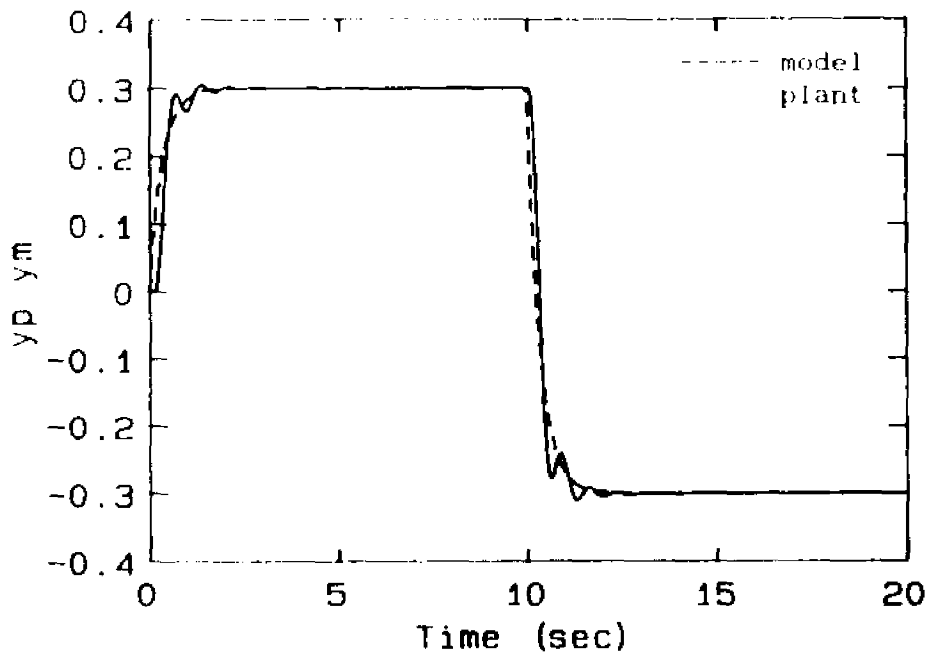

Figure 2.6 Rohrs' Example: G=-5.71428
Plant and Model Output (Algorithm 2)

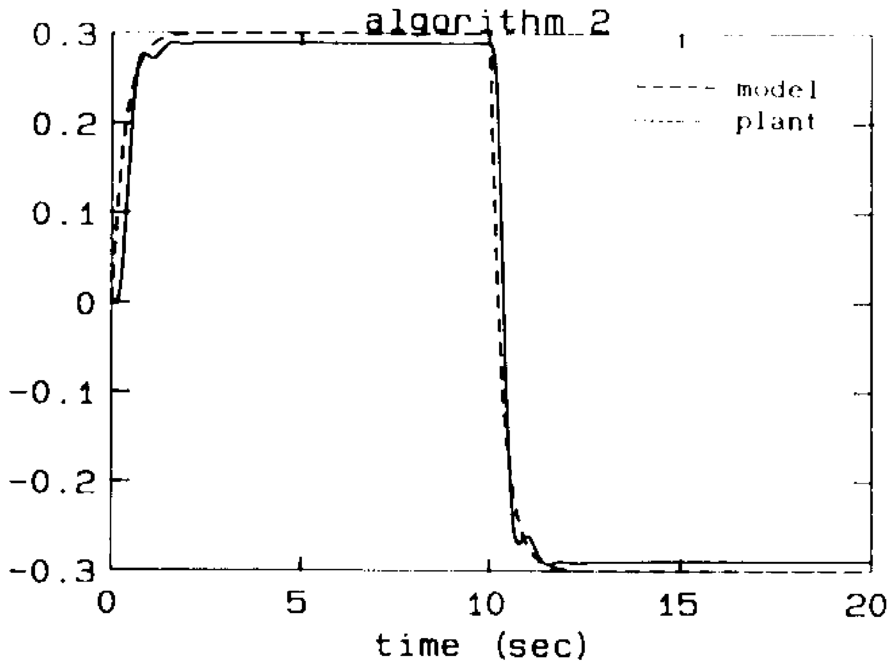

Figure 2.7 Rohrs' Example: G=-10
Output with $G^* - Q_f A_f^{-1} B_f$ (Algorithm 2)

<u>Algorithm 3</u>:

Using Eqs.(2.93)-(2.96) we obtain

$$D_f = 1; \quad \tilde{K}_{pe} = -D_c = 10 \tag{2.214}$$

$$\tilde{K}_{pf} = -C_c - 1 = 2.5 \tag{2.215}$$

$$B_f D_c = B_c \Rightarrow B_f = B_c / D_c = -1 \tag{2.216}$$

$$A_f = -B_f C_c = -3.5 \tag{2.217}$$

Since $\det[A_f]\neq 0$, we can choose $Q_f=20$ such that $G=-Q_fA_f^{-1}B_f=-8<0$. Then, we choose $Q_p=GD_c=80$ to obtain the design parameters as shown below

$$Q = [\ 80 \quad 20\ ] \tag{2.218}$$

$$G = -8 \tag{2.219}$$

The parallel supplementary dynamics are described by

$$\dot{x}_f(t) = -2.5x_f(t) - u_p(t) \tag{2.220}$$

$$y_f(t) = x_f(t) \tag{2.221}$$

The plant and model outputs are shown in Figure 2.8. We observe the excellent transient behavior with zero error in approximately 2 seconds. Finally, we simulate Rohrs' example by choosing $G=-20$ such that $G\neq -Q_fA_f^{-1}B_f$. A bounded output tracking error is observed from Figure 2.8(b).

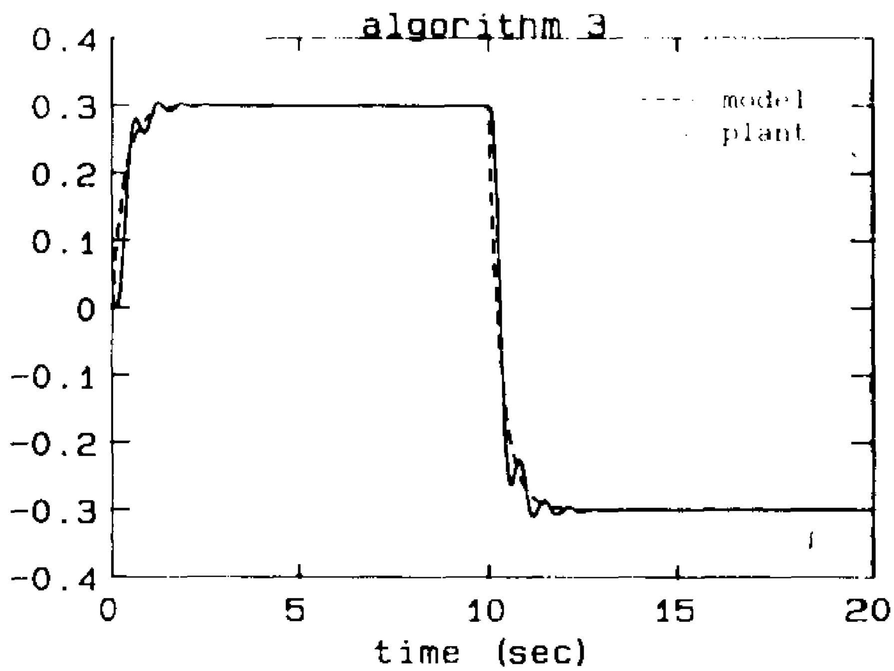

Figure 2.8 Rohrs' Example:
Plant and Model Output (Algorithm 3)

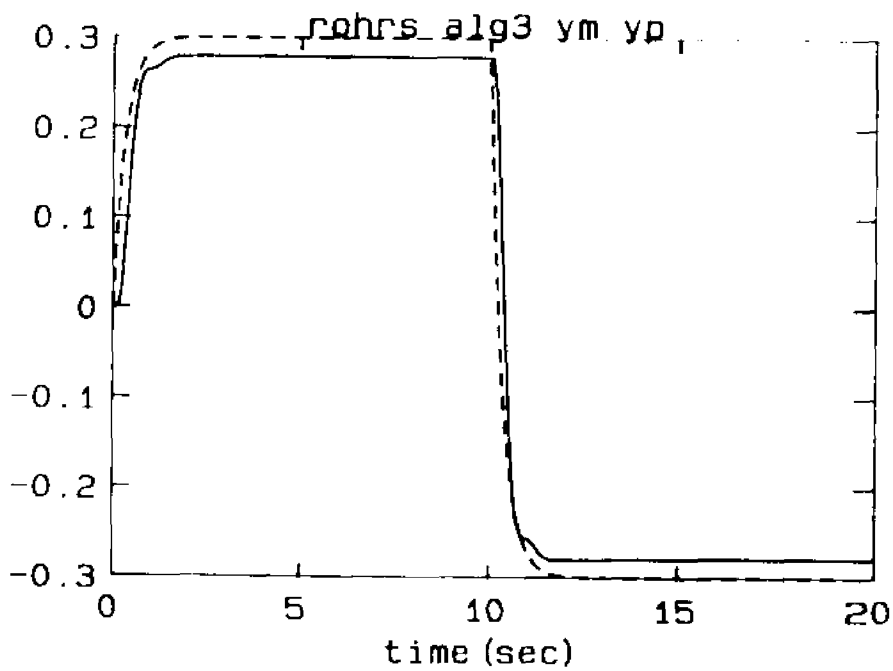

Figure 2.9 Rohrs' Example

Output with $G \neq -Q_f A_f^{-1} B_f$ (Algorithm 3)

Summary :

Comparing the simulation results for Rohrs' example shown in Figures 2.4-2.8, we conclude that the output tracking response of the plant with the non-adaptive supplementary dynamics, Figure 2.4, is not acceptable even though the closed-loop stability condition is satisfied. However, Figures 2.5-2.8(b) show that a satisfactory output tracking error can be obtained by using algorithm 1, 2, and 3 with adaptive supplementary dynamics. Furthermore, Figures 2.6 and 2.8 illustrate that an asymptotically vanishing output tracking error will be guaranteed by using algorithm 2 or 3 if the conditions in Corollary 2.1 are satisfied. However, it is hard to say which one is better between algorithms 2 and 3 because similar results are obtained in simulation.Finally, Figures 2.7 and 2.9 shows that if the condition $G=-Q_f A_f^{-1} B_f$ is violated, then the asymptotically vanishing output tracking error will not be maintained. However, the plant will still follow the model with a bounded output tracking error even though the stability condition $Q+G\tilde{K}_e=0$ is not satisfied. This indicates that a larger class of plants than those described in Theorem 2 can achieve a bounded output tracking error because $Q+G\tilde{K}_e=0$ is sufficient but not necessary.

Example 2:

(Unstable Plant using Algorithm 2 Parallel Dynamics)

Consider the unstable single-input single-output plant with transfer function given by

$$\frac{Y_p(s)}{U_p(s)} = \frac{200}{(s - 1)(s^2 + 8s + 100)} \qquad (2.208)$$

The output of the plant is required to follow the output of the reference model whose transfer function is given by Eq.(2.198).

Suppose we know that a stabilizing PI compensator is described by

$$G_c(s) = \frac{-(3s+6)}{s} \qquad (2.209)$$

The compensator described by Eq.(2.209) has a state space realization given by $A_c=0$, $B_c=3$, $C_c=-2$, and $D_c=-3$. Using Eqs.(2.93)-(2.96) we obtain

$$\tilde{K}_{pe} = -D_c = 3 \tag{2.210}$$

$$C_f = I; \quad \tilde{K}_{pf} = -C_c = 2 \tag{2.211}$$

$$B_f D_c = B_c \rightarrow B_f = B_c / D_c = -1 \tag{2.212}$$

$$A_f = -B_f C_c = -2 \tag{2.213}$$

Since $\det[A_f] \neq 0$, we can choose $Q_f=20$ such that $G=-Q_f A_f^{-1} B_f=-10<0$. Then, choose $Q_p=GD_c=30$. The design parameters are given by

$$Q = [\ 30 \quad 20\ ] \tag{2.214}$$

$$G = -10 \tag{2.215}$$

and the parallel supplementary dynamics are described by

$$\dot{x}_f(t) = -2x_f(t) - u_p(t) \tag{2.216}$$

$$y_f(t) = x_f(t) \tag{2.217}$$

The matrices T and $\bar{T}$ are chosen to be identity matrices, and the input and output disturbances are not considered. The plant and model outputs are shown in Figure 2.8 for a square wave reference command of magnitude 0.3 units and period of 40 seconds. We observe that the error is driven to zero in approximately 2 seconds albeit with a maximum overshoot of near 30 percent. The adaptive gains K_{pe} and K_{pf} are shown in figures 2.9 and 2.10, respectively. We observe that the gains become momentarily large at t=0 and t=20 seconds in order to force the error to zero when the reference command is initially applied and when the reference command changes sign.

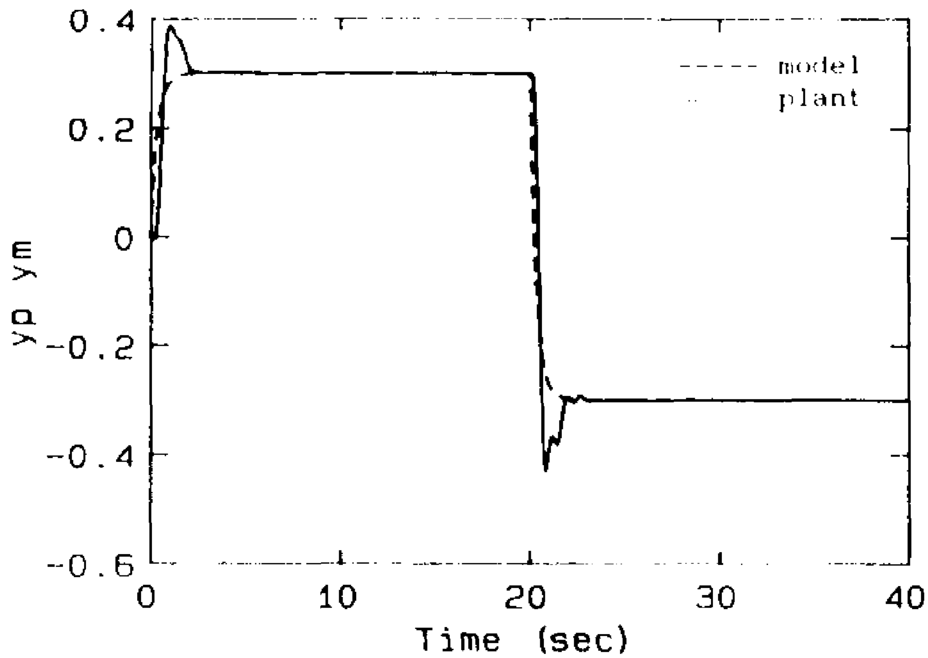

Figure 2.10 Unstable Plant:
Plant and Model Output (Algorithm 2)

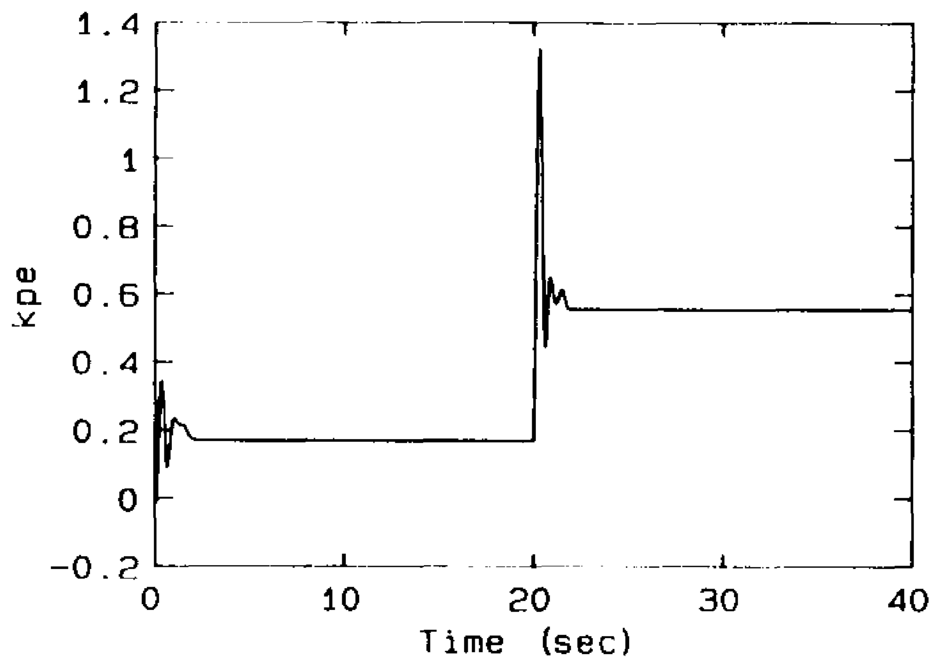

Figure 2.11 Unstable Plant:
Adaptive Gain K_{pe} (Algorithm 2)

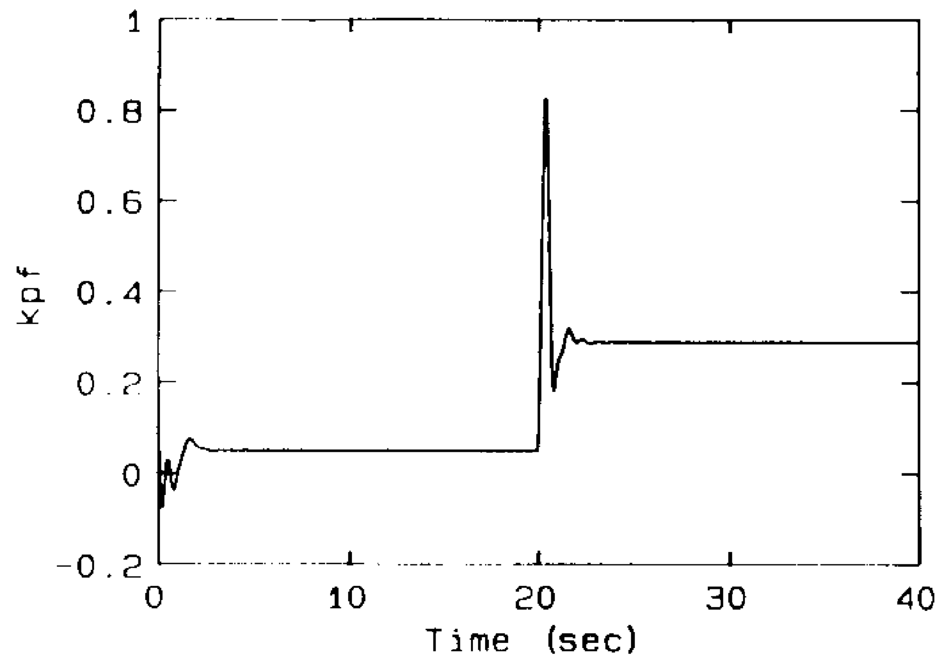

Figure 2.12 Unstable Plant:
Adaptive Gain K_{pf} (Algorithm 2)

Example 3:

(F-8 Lateral Dynamics using Algorithm 1 Feedback Dynamics)

Consider the F-8 lateral dynamics with the plant described by

$$\begin{bmatrix} \dot{p} \\ \dot{r} \\ \dot{\beta} \\ \dot{\phi} \end{bmatrix}_p = \begin{bmatrix} -3.59 & .1968 & -35.18 & 0 \\ -.0377 & -.3576 & 5.884 & 0 \\ .0688 & -.9957 & -.2163 & .0733 \\ .9947 & .1027 & 0 & 0 \end{bmatrix} \begin{bmatrix} p \\ r \\ \beta \\ \phi \end{bmatrix}_p + \begin{bmatrix} 14.65 & 6.538 \\ .2179 & -3.087 \\ -.0054 & .0516 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \delta_a \\ \delta_r \end{bmatrix}_p \tag{2.218}$$

where p is the roll rate, r the yaw rate, β the sideslip angle, ϕ the bank angle, δ_a the aileron deflection, and δ_r the rudder deflection.

The reference model is chosen as in reference 3 and is described by

$$\begin{bmatrix} \dot{p} \\ \dot{r} \\ \dot{\beta} \\ \dot{\phi} \end{bmatrix}_m = \begin{bmatrix} -10 & 0 & -10 & 0 \\ 0 & -0.7 & 0 & 0 \\ 0 & -1 & -0.7 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} p \\ r \\ \beta \\ \phi \end{bmatrix}_m + \begin{bmatrix} 20 & 2.8 \\ 0 & -3.13 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \delta_a \\ \delta_r \end{bmatrix}_m \tag{2.219}$$

This model has an eigenvalue at the origin in the complex plane which makes steady state tracking difficult.

To attempt matching of the plant and model roll rates and yaw rates, the measurements are chosen to be

$$y_p = \begin{bmatrix} p \\ r \end{bmatrix}_p \qquad y_m = \begin{bmatrix} p \\ r \end{bmatrix}_m \tag{2.220}$$

Suppose we know that a stabilizing compensator is described by the quadruple (A_c, B_c, C_c, D_c) where

$$A_c = \begin{bmatrix} -10 & 0 \\ 0 & -10 \end{bmatrix} \tag{2.221}$$

$$B_c = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \tag{2.222}$$

$$C_c = \begin{bmatrix} 1000 & 0 \\ 0 & 10 \end{bmatrix} \tag{2.223}$$

$$D_c = \begin{bmatrix} -50,000 & 0 \\ 0 & 20,000 \end{bmatrix} \tag{2.224}$$

We choose algorithm 1 for this example which uses feedback supplementary dynamics and yields a bounded error. Using Eqs.(2.87)-(2.90) from Lemma 2.5 we obtain

$$A_f = A_c \tag{2.225}$$

$$B_f = -B_c \tag{2.226}$$

$$C_f = I \tag{2.227}$$

$$[\tilde{K}_{pe} \quad \tilde{K}_{pf}] = [-C_c \quad -D_c] \tag{2.228}$$

Then, we use the constructive method from the proof of

Lemma 2.5 to choose

$$G = -\gamma_1 I = 0.001 \tag{2.229}$$

Then,

$$Q = \gamma_1 \tilde{K}_e = \gamma_1 [-D_c \quad -C_c] \tag{2.230}$$

which yields

$$Q = [Q_p \quad Q_f] = \begin{bmatrix} 50 & 0 & -1 & 0 \\ 0 & -20 & 0 & -0.01 \end{bmatrix} \tag{2.231}$$

Finally, we choose $T=0.05I$ and $\bar{T}=0.1I$.

The aileron model input is chosen to be a unit square wave $U_{sq}(t)$ with period equal to one second and the rudder model input is chosen to be $U_{sq}(t+0.25)$. A constant input disturbance is chosen to be

$$d_{ip} = \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix} \qquad \text{for} \quad t > 1 \quad \text{sec.} \tag{2.232}$$

with matrix $E_p = B_p$ and a sinusoidal output disturbance is chosen to be

$$d_{op}(t) = \begin{bmatrix} 0.2\sin 30t \\ 0.1\sin 30t \end{bmatrix} \qquad \text{for} \quad t>1 \quad \text{sec.} \tag{2.233}$$

The robustness coefficient σ is chosen to be

$$\sigma = \begin{cases} 0.0 & \text{for} \quad t < 1.6 \text{ sec} \\ 0.1 & \text{for} \quad t \geq 1.6 \text{ sec} \end{cases} \tag{2.234}$$

We observe from Figure 2.11 that the plant roll rate exhibits excellent model following with virtually no steady state error while we observe from Figure 2.12 that the plant yaw rate exhibits good model following with a small steady state error. This model following is obtained in the presence of both input and output disturbances.

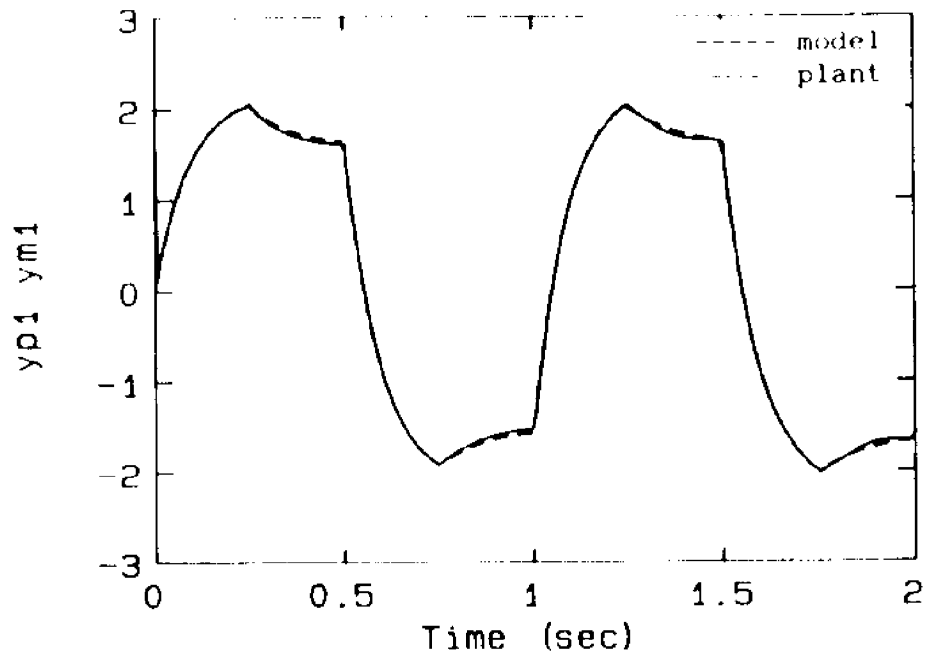

Figure 2.13 F-8
Plant and Model Roll Rate (Algorithm 1)

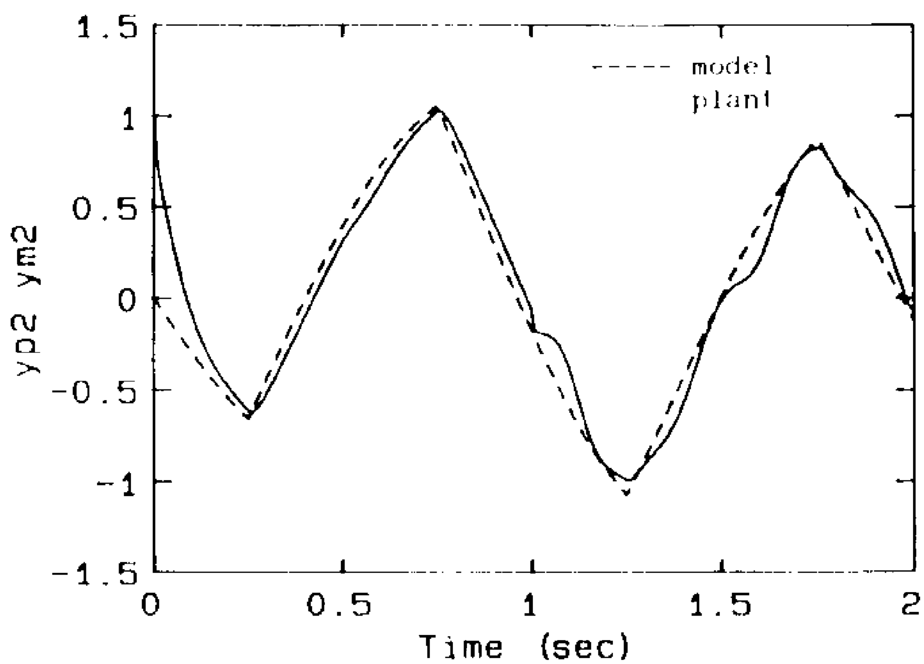

Figure 2.14 F-8
Plant and Model Yaw Rate (Algorithm 1)

3. ALGORITHMS WITH ADAPTIVE SUPPLEMENTARY DYNAMICS

3.1 Introduction

This chapter introduces algorithms 4 and 5 which use adaptive supplementary dynamics to control non-ASPR plants. Unlike algorithms 1, 2, and 3, the parameters of the supplementary dynamics in algorithms 4 and 5 are computed on-line as part of the adaptive gain computations. The metasystem representation is used for both algorithms and a Lyapunov function is used to show either a bounded or an asymptotically vanishing output tracking error.

3.2 Algorithm 4

Consider a linear time-invariant plant with input and output disturbances as described in Eqs.(2.1) and (2.2). We seek a control law which will cause the plant to track a linear time invariant reference model described by Eqs.(2.3) and (2.4). We choose the plant control signal to be

$$u_p(t) = y_f(t) \tag{3.1}$$

where $y_f(t)$ is the output of the supplementary dynamics which are shown below:

$$\dot{x}_f(t) = -A_f(t)x_f(t) + B_f(t)u_f(t) \tag{3.2}$$

$$y_f(t) = -C_f(t)x_f(t) + D_f(t)u_f(t) \tag{3.3}$$

where $x_f(t) \in R^{n_f}$ and $y_f(t) \in R^{\ell_f}$, and where

$$B_f(t) = [\ B_{f1}(t) \quad B_{f2}(t) \quad B_{f3}(t)\ ] \tag{3.4}$$

$$D_f(t) = [\ D_{f1}(t) \quad D_{f2}(t) \quad D_{f3}(t)\ ] \tag{3.5}$$

$$u_f(t) = \begin{bmatrix} e_{yp}(t) \\ x_m(t) \\ u_m(t) \end{bmatrix} \tag{3.6}$$

$$e_{yp}(t) = y_m(t) - y_p(t)$$

Although the supplementary dynamics are inserted in a output feedback path as in algorithm 1, the matrices $A_f(t)$, $B_f(t)$, $C_f(t)$, $D_f(t)$ of algorithm 4 are updated on-line as part of the adaptive control mechanism.

A block diagram of algorithm 4 is shown in Figure 3.1

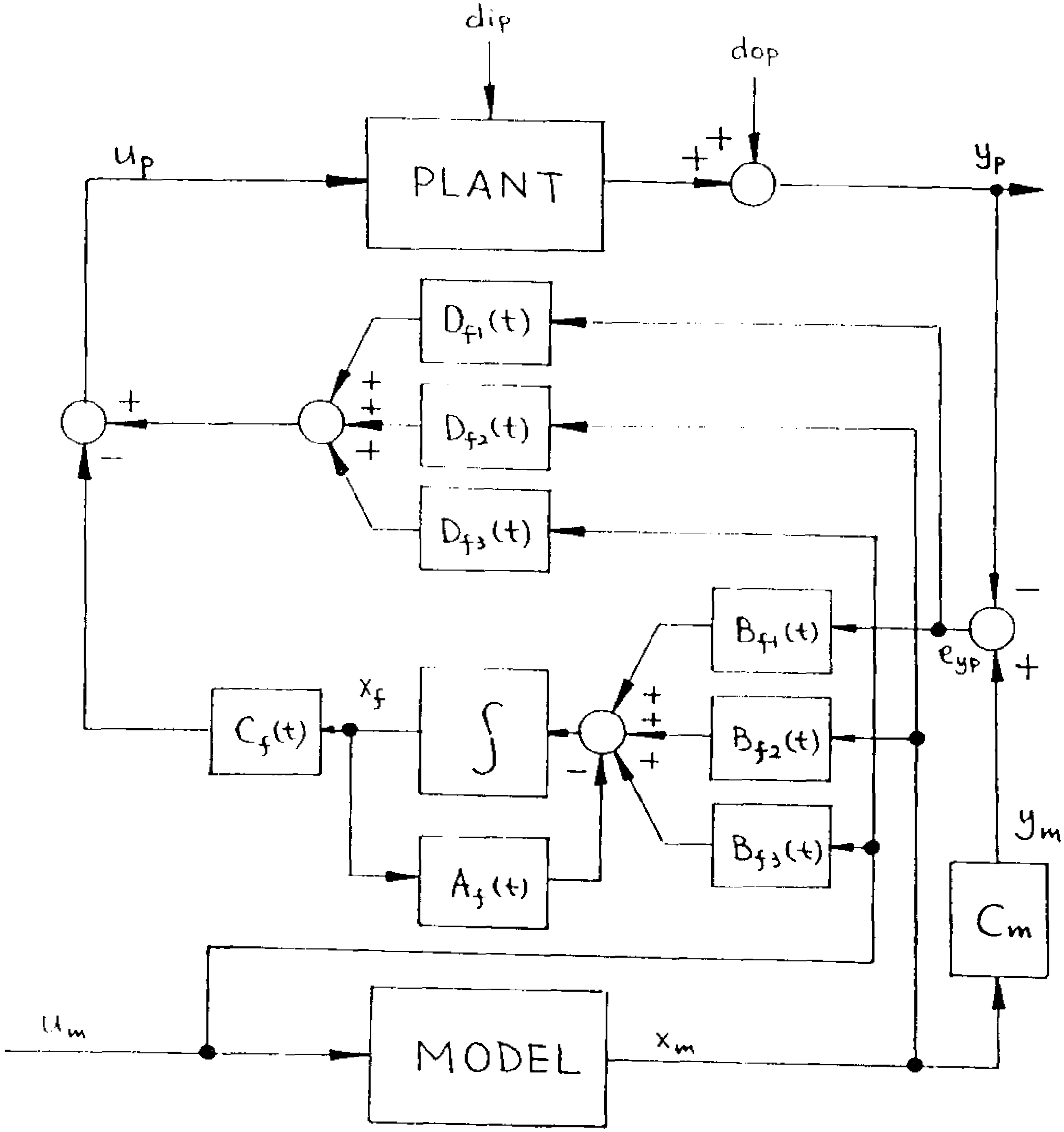

Figure 3.1 Block Diagram of Algorithm 4

Using Eq.(3.3) we can rewrite the control law as follows:

$$u_p(t) = y_f(t) = -C_f(t)x_f(t) + D_f(t)u_f(t)$$

$$= -C_f(t)x_f(t)+D_{f1}(t)e_{yp}(t)+D_{f2}(t)x_m(t)+D_{f3}(t)u_m(t) \tag{3.7}$$

Then, substitute Eq.(3.7) into Eq.(2.1) to obtain the plant dynamics which are shown below:

$$\dot{x}_p(t) = A_p(t)x_p(t) + B_p(t)D_{f1}(t)e_{yp}(t) - B_pC_fx_f(t)$$
$$+ B_pD_{f2}(t)x_m(t) + B_pD_{f3}(t)u_m(t) + E_pd_{ip}(t) \tag{3.8}$$

$$y_p(t) = C_px_p(t) + d_{op}(t) \tag{3.9}$$

Next, substitute Eqs.(3.4)-(3.6) into Eqs.(3.2)-(3.3) to obtain the supplementary dynamics which are shown below:

$$\dot{x}_f(t) = -A_f(t)x_f(t)+B_{f1}(t)e_{yp}(t)+B_{f2}(t)x_m(t)+B_{f3}(t)u_m(t) \tag{3.10}$$

$$y_f(t) = -C_f(t)x_f(t)+D_{f1}(t)e_{yp}(t)+D_{f2}(t)x_m(t)+D_{f3}(t)u_m(t) \tag{3.11}$$

We now concatenate the plant dynamics described by Eqs.(3.8) and (3.9) with the supplementary dynamics described by Eqs.(3.10) and (3.11) to form a metasystem of order $n+n_f$. The metasystem state equation is given by

$$\begin{bmatrix} \dot{x}_p(t) \\ \dot{x}_f(t) \end{bmatrix} = \begin{bmatrix} A_p & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_p(t) \\ x_f(t) \end{bmatrix} + \begin{bmatrix} B_p & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} D_{f1}(t) & C_f(t) & D_{f2}(t) & D_{f3}(t) \\ B_{f1}(t) & A_f(t) & B_{f2}(t) & B_{f3}(t) \end{bmatrix} \begin{bmatrix} e_{yp}(t) \\ -x_f(t) \\ x_m(t) \\ u_m(t) \end{bmatrix} + \begin{bmatrix} E_p d_{ip}(t) \\ 0 \end{bmatrix} \tag{3.12}$$

and the metasystem output equation is given by

$$\begin{bmatrix} y_p(t) \\ x_f(t) \end{bmatrix} = \begin{bmatrix} C_p & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} x_p(t) \\ x_f(t) \end{bmatrix} + \begin{bmatrix} d_{op}(t) \\ 0 \end{bmatrix} \tag{3.13}$$

To write the metasystem equations more compactly we define

$$x(t) = \begin{bmatrix} x_p(t) \\ x_f(t) \end{bmatrix}, \qquad y(t) = \begin{bmatrix} y_p(t) \\ x_f(t) \end{bmatrix},$$

$$d_o(t) = \begin{bmatrix} d_{op}(t) \\ 0 \end{bmatrix}, \qquad d_i = \begin{bmatrix} E_p d_{ip}(t) \\ 0 \end{bmatrix},$$

$$A = \begin{bmatrix} A_p & 0 \\ 0 & 0 \end{bmatrix}, \qquad B = \begin{bmatrix} B_p & 0 \\ 0 & I \end{bmatrix}, \qquad C = \begin{bmatrix} C_p & 0 \\ 0 & I \end{bmatrix}$$

$$r(t) = \begin{bmatrix} e_{yv}(t) \\ x_m(t) \\ u_m(t) \end{bmatrix} \qquad e_{yv}(t) = \begin{bmatrix} e_{yp}(t) \\ -x_f(t) \end{bmatrix}$$

$u(t)=K(t)r(t)$

where

$$K(t) = [\, K_e(t),\ K_x(t),\ K_u(t) \,]$$

where

$$K_e(t) = \begin{bmatrix} D_{f1}(t) & C_f(t) \\ B_{f1}(t) & A_f(t) \end{bmatrix}$$

$$K_x(t) = \begin{bmatrix} D_{f2}(t) \\ B_{f2}(t) \end{bmatrix} \qquad K_u(t) = \begin{bmatrix} D_{f3}(t) \\ B_{f3}(t) \end{bmatrix}$$

We obtain a metasystem, which is in the same form as in section 2.7, and which is repeated below:

$$\dot{x}(t) = Ax(t) + BK(t)r(t) + d_i(t)$$

$$= Ax(t) + Bu(t) + d_i(t) \qquad (3.14)$$

$$y(t) = Cx(t) + d_o(t) \qquad (3.15)$$

where K(t) is an adaptive gain matrix which is computed as show in Eqs.(2.37)-(2.41).

Remark 3.1:

Although algorithm 4 is different from the algorithms discussed in Chapter 2, their metasystem representations are identical. This is significant because the metasystem generalizes different algorithms into one form such that the previous results on system stability can be extended to the new algorithm. Also, we remark that the metasystem used here is linear time invariant and tracks a linear time invariant reference model. Therefore, our approach is different from the metasystem in reference 8 where a time varying reference model must be considered. Our approach yields a simpler stability proof with less restrictive conditions.

3.3 Algorithm 5

In this section, we consider the case of an asymptotically vanishing output tracking error. The conditions for an asymptotically vanishing error can be derived by modifying algorithm 4. The new choice for the plant control signal is given by

$$u_p(t) = y_f(t) = -C_f x_f(t) + D_f e_{yp}(t) \qquad (3.16)$$

Upon comparing Eqs.(3.1) and (3.16), we note that, in Eq.(3.16), $x_m(t)$ and $u_m(t)$, are not fed forward to the plant input and matrices C_f and D_f are constant matrices which will be selected later. A block diagram of this new adaptive control system, which is denoted as algorithm 5, is shown in Figure 3.2.

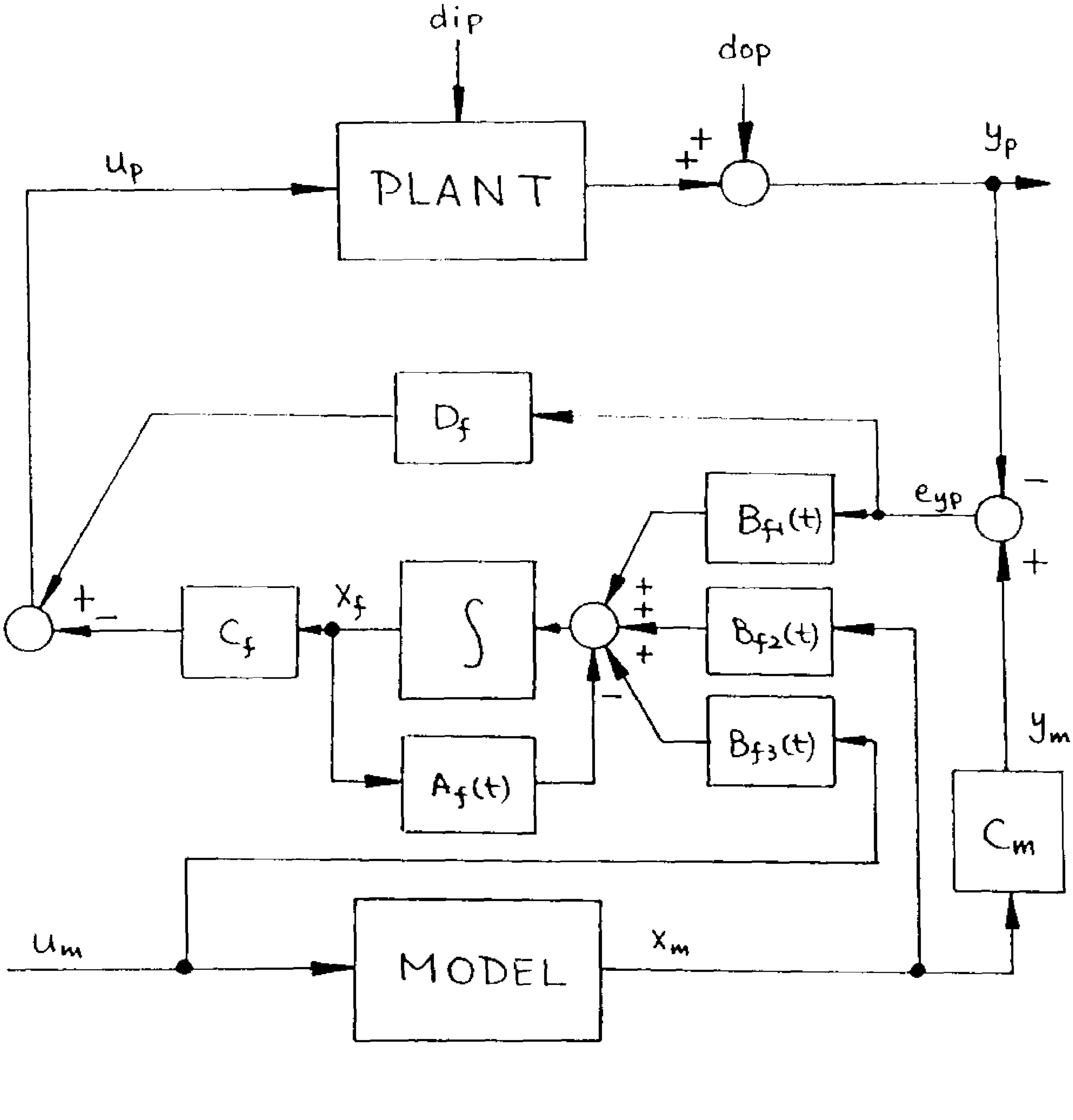

Figure 3.2 Block diagram of Algorithm 5

In Figure 3.2, $A_f(t)$ $B_{f1}(t)$, $B_{f2}(t)$, and $B_{f3}(t)$ are adaptive matrices which are updated on-line as part of the adaptive control mechanism. The plant, the reference model, and the state equation for the supplementary dynamics are the same as in algorithm 4. However, the output equation for the supplementary dynamics is different because matrices C_f and D_f are not adaptive.

We substitute Eq.(3.16) into Eq.(2.1) to obtain

$$\dot{x}_p(t) = A_p x_p(t) + B_p u_p(t) + E_p d_{ip}(t)$$

$$= A_p x_p(t) - B_p C_f x_f(t) + B_p D_f e_{yp}(t) + E_p d_{ip}(t) \tag{3.17}$$

$$y_p(t) = C_p x_p(t) + d_{op}(t) \tag{3.18}$$

Then, concatenate Eqs.(3.10), (3.17), and (3.18) to obtain the metasystem representation which is described by

$$\begin{bmatrix} \dot{x}_p(t) \\ \dot{x}_f(t) \end{bmatrix} = \begin{bmatrix} A_p & -B_p C_f \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_p(t) \\ x_f(t) \end{bmatrix}$$

$$+ \begin{bmatrix} 0 \\ I \end{bmatrix} [\, B_{f1}(t) \;\; A_f(t) \;\; B_{f2}(t) \;\; B_{f3}(t) \,] \begin{bmatrix} e_{yp}(t) \\ -x_f(t) \\ x_m(t) \\ u_m(t) \end{bmatrix}$$

$$+ \begin{bmatrix} B_p D_f C_p & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} e_{xp}(t) \\ e_{xf}(t) \end{bmatrix} + \begin{bmatrix} E_p d_{ip}(t) \\ 0 \end{bmatrix} \qquad (3.19)$$

$$\begin{bmatrix} y_p(t) \\ x_f(t) \end{bmatrix} = \begin{bmatrix} C_p & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} x_p(t) \\ x_f(t) \end{bmatrix} + \begin{bmatrix} d_{op}(t) \\ 0 \end{bmatrix} \qquad (3.20)$$

We may write the metasystem equations more compactly by defining

$$x(t) = \begin{bmatrix} x_p(t) \\ x_f(t) \end{bmatrix} \qquad y(t) = \begin{bmatrix} y_p(t) \\ x_f(t) \end{bmatrix}$$

$$A = \begin{bmatrix} A_p & -B_p C_f \\ 0 & 0 \end{bmatrix} \qquad A_0 = \begin{bmatrix} B_p D_f C_p & 0 \\ 0 & 0 \end{bmatrix} \qquad C = \begin{bmatrix} C_p & 0 \\ 0 & I \end{bmatrix}$$

$$B = \begin{bmatrix} 0 \\ I \end{bmatrix} \qquad d_i(t) = \begin{bmatrix} E_p d_{ip}(t) \\ 0 \end{bmatrix} \qquad d_o(t) = \begin{bmatrix} d_{op}(t) \\ 0 \end{bmatrix}$$

$$u(t) = K(t)r(t)$$

where

$$K(t) = [\ K_e(t),\ B_{f2}(t),\ B_{f3}(t)\ ]$$

$$K_e(t) = [\ B_{f1}(t) \quad A_f(t)]$$

$$r(t) = \begin{bmatrix} e_{yv}(t) \\ x_m(t) \\ u_m(t) \end{bmatrix} \qquad e_{yv}(t) = \begin{bmatrix} e_{yp}(t) \\ -x_f(t) \end{bmatrix}$$

Thus, we have

$$\dot{x}(t) = Ax(t) + A_0 e_x(t) + BK(t)r(t) + d_i(t) \tag{3.21}$$

$$y(t) = Cx(t) + d_o(t) \tag{3.22}$$

where the adaptive gain $K(t)$ is computed as in Eqs.(2.37)-(2.41).

Although Eq.(3.21) is not in the same form as Eq.(2.27), their metastate error derivative equations are identical as shown in the next section.

3.4 Error Equations

The metasystem representation for algorithm 4 is the same as for the algorithms in Chapter 2. Therefore, the metastate error equation for algorithm 4 is obtained from the results of section 2.8 as shown below:

$$\dot{e}_x(t) = A_c e_x(t) - Bz(t) - F_1(t) \tag{3.23}$$

where $$A_c = A - B\tilde{K}_e C \tag{3.24}$$

Then, we show that the metastate error derivative equation for algorithms 4 and 5 are in the same form by inserting the metasystem dynamics of algorithm 5, Eq.(3.21), into the the metastate error derivative equation, Eq.(2.46) to obtain

$$\dot{e}_x(t) = \dot{x}^*(t) - \dot{x}(t)$$

$$= \dot{x}^*(t) - Ax(t) - BK(t)r(t) - A_0 e_x(t) - d_1(t)$$

$$=\dot{x}^{*}(t)+Ae_x(t)-Ax^{*}(t)-Bz(t)-B\tilde{K}r(t)-A_0e_x(t)-d_1(t)$$

$$=\dot{x}^{*}(t)+Ae_x(t)-Ax^{*}(t)-Bz(t)-B\tilde{K}_e[Ce_x(t)-Cx_0^{*}(t)-d_0(t)]$$

$$-B[\tilde{K}_x x_m(t)+\tilde{K}_u u_m(t)]-A_0e_x(t)-d_1(t)$$

$$=[A-A_0-B\tilde{K}_eC]e_x(t)-Bz(t)+\dot{x}^{*}(t)-Ax^{*}(t)$$

$$+B\tilde{K}_e[Cx_0^{*}(t)+d_0(t)]-B[\tilde{K}_x x_m(t)+\tilde{K}_u u_m(t)]-d_1(t)$$

$$= A_c e_x(t) - Bz(t) - F_1(t) \qquad (3.25)$$

where

$$A_c = A-A_0-B\tilde{K}_eC \qquad (3.26)$$

<u>Remark</u> <u>3.2</u>:

The metastate error derivative equations for algorithms 4 and 5, as well as for the algorithms in Chapter 2, are identical, and can be expressed by

$$\dot{e}_x(t) = A_c e_x(t) - Bz(t) - F_1(t) \tag{3.27}$$

where the form of the matrices A_c and B are different for each algorithm as shown in Appendix A. The vector $F_1(t)$ is defined by Eq.(2.57), and is bounded as stated in Remark 2.3. Therefore, the stability proof for algorithms 4 and 5 will be the same as in section 2.10.

3.5 Stability Analysis

Theorem 3.1:

Consider algorithm 4 or 5 with a controllable and observable LTI plant. Further, suppose that there exists a real symmetric positive definite matrix P and real matrices J, L, W, $\tilde{K}_e$, and R, $(R+R^T)>0$ such that

$$P(A-B\tilde{K}_eC) + (A-B\tilde{K}_eC)^TP = -LL^T-R < 0 \qquad (3.28)$$

$$PB = C^T(Q^T + \tilde{K}_e^TG^T) - LW \qquad (3.29)$$

$$W^TW = J + J^T \qquad (3.30)$$

$$J + J^T + G + G^T < 0 \qquad (3.31)$$

where the matrices T and $\bar{T}$ are positive definite symmetric and positive semi-definite symmetric, respectively. Then, all states, gains and errors in the adaptive system are bounded.

Remark 3.3:

Consider the quadratic function which is positive definite in the state variables of the adaptive system, $e_x(t)$ and $K^I(t)$, as shown in Eq.(2.80). We see that the metasystem error $e_x(t)$, the error derivative $\dot{e}_x(t)$ and the adaptive gain $K^I(t)$ for either algorithm 4 or 5 are in the same form as in Chapter 2. Therefore, the Lyapunov derivative for algorithms 4 and 5 will be identical with the Lyapunov derivative for the algorithms in Chapter 2. Thus, the stability proof is the same as in theorem 2.1.

Remark 3.4:

The first three sufficient conditions, given by Eqs.(3.28) - (3.30), are equivalent to requiring that the transfer matrix given by

$$H(s) = J + (Q+G\tilde{K}_e)C(sI-A+B\tilde{K}_eC)^{-1}B \qquad (3.32)$$

is strictly positive real (SPR).

Lemma 3.1:

The sufficient conditions, Eqs.(3.31) and (3.32), can be satisfied by algorithm 4 for any controllable and observable plant.

Proof:

If the plant is controllable and observable then there exists a compensator, denoted by the quadruple (A_c, B_c, C_c, D_c), which stabilizes the plant. Thus the composite plant/compensator system is represented by

$$A_{comp} = \begin{bmatrix} A_p + B_p D_c C_p & B_p C_c \\ B_c C_p & A_c \end{bmatrix} \tag{3.33}$$

where A_{comp} is a stability matrix. Next, we observe from Eqs.(3.12) and (3.13) that the composite system for algorithm 4 is given by

$$A_4 = A - B\tilde{K}_e C = \begin{bmatrix} A_p - B_p\tilde{D}_{f1}C_p & -B_p\tilde{C}_f \\ -\tilde{B}_{f1}C_p & -\tilde{A}_f \end{bmatrix} \tag{3.34}$$

which is required to be a stability matrix for some gain $\tilde{K}_e$. Thus, by comparing Eqs.(3.33) and (3.34) we observe that the choice

$$\tilde{A}_f = -A_c \tag{3.35}$$

$$\tilde{B}_{f1} = -B_c \tag{3.36}$$

$$\tilde{C}_f = -C_c \tag{3.37}$$

$$\tilde{D}_{f1} = -D_c \tag{3.38}$$

will results in A_4 being a stability matrix.

$$\text{Thus} \quad \tilde{K}_e = \begin{bmatrix} \tilde{D}_f & \tilde{C}_f \\ \tilde{B}_f & \tilde{A}_f \end{bmatrix} \tag{3.39}$$

Now we need to show that the conditions in Eq.(3.31) and (3.32) can be satisfied. Let $G=-\gamma_1 I$, $J=\gamma_2 I$, where $\gamma_1>\gamma_2$ are positive scalars. Let $Q=\gamma_1\tilde{K}_e$ so that Eq.(3.32) reduces to $J>0$, and Eq.(3.31) becomes $J+J^T+G+G^T=2\gamma_2 I-2\gamma_1 I<0$. Hence, there exist Q, G, $\tilde{K}_e$, and J which satisfy the sufficient conditions for a bounded error.

Lemma 3.2:

The sufficient conditions, Eqs.(3.31) and (3.32), can be satisfied by algorithm 5 for any controllable and observable plant.

Proof:

If the plant is controllable and observable then there exists a compensator, denoted by the quadruple (A_c, B_c, C_c, D_c), which stabilizes the plant. Thus the composite plant/compensator system is represented by

$$A_{comp} = \begin{bmatrix} A_p + B_p D_c C_p & B_p C_c \\ B_c C_p & A_c \end{bmatrix} \qquad (3.40)$$

where A_{comp} is a stability matrix. Next, we observe from Eqs.(3.21) and (3.22) that the composite system for algorithm 5 is given by

$$A_5 = A - A_0 - B\tilde{K}_e C = \begin{bmatrix} A_p - B_p D_f C_p & -B_p C_f \\ -\tilde{B}_{f1} C_p & -\tilde{A}_f \end{bmatrix} \qquad (3.41)$$

which is required to be a stability matrix for some gain $\tilde{K}_e$. Thus, by comparing Eqs.(3.40) and (3.41) we observe that the choice

$$\tilde{A}_f = -A_c \qquad (3.42)$$

$$\tilde{B}_{f1} = -B_c \qquad (3.43)$$

$$C_f = -C_c \qquad (3.44)$$

$$D_f = -D_c \qquad (3.45)$$

$$\tilde{K}_e = [\ \tilde{B}_{f1}\ \tilde{A}_f] = [\ -B_c,\ -A_c\ ] \qquad (3.46)$$

will result in A_5 being a stability matrix. We note that $\tilde{A}_f$ is not required to be a stability matrix, and the dimension of $\tilde{K}_e$ in Eq.(3.46) is different with that in Eq,(3.39) because the dimension of B in Eq.(3.27) is different in algorithm 4 and 5.

Next, we need to show that Eqs.(3.31) and (3.32) can be satisfied. Let $G=-\gamma_1 I$, $J=\gamma_2 I$, where $\gamma_1<\gamma_2$ are positive scalars. Let $Q=\gamma_1\tilde{K}_e$ so that Eq.(3.32) reduces to $J>0$, and Eq.(3.31) becomes $J+J^T+G+G^T=2\gamma_2 I-2\gamma_2 I<0$. Hence, there exist Q, G, $\tilde{K}_e$, and J which satisfy the sufficient conditions for a bounded error.

3.6 Asymptotic Output Tracking

In section 2.11, we showed that the plant output will track the model output asymptotically by inserting supplementary dynamics either in parallel or in cascade with the plant. In this section, we show that asymptotic output tracking can be achieved for algorithm 5 by inserting supplementary dynamics with adaptive parameters into an output feedback loop which is inside the adaptive controller.

We extend Broussard's [4] command generator tracker (CGT) for model following control of known plants to the metasystem described by Eqs.(3.21) and (3.22) by defining $x^*(t)$, $y^*(t)$, and $u^*(t)$ as the ideal state, ideal output, and ideal input, respectively as in section 2.11 such that in the ideal situation we have

$$\dot{x}^*(t) = Ax^*(t) + Bu^*(t) \tag{3.47}$$

and

$$y_p^*(t) = C_p x_p^*(t) = y_m(t) \tag{3.48}$$

We note that Eqs.(3.47) and (3.48) are in the same form as the Eqs.(2.104) and (2.105) discussed in Chapter 2 although Eq.(3.21) is different from Eq.(2.27). Thus, there exist ideal trajectories in the form of Eq.(2.106) provided that u_m is constant and Eq.(2.116) has solutions for S_1, S_2, S_{31}, and S_{32}.

We extend Corollary 2.1 and propose a new corollary as follows

Corollary 3.1:

Algorithm 5 will yield an asymptotically vanishing output error by choosing Q_f=0 if the conditions of Theorem 3.1 are satisfied and if (i) u_m is constant for $t \geq t_1$, (ii) no disturbances exist and σ = 0. (iii) a solution exists for the matrices S_1, S_2, S_{31}, and S_{32} in Eq.(2.116).

Proof:

The proof of Corollary 3.1 is similar to the proof of Corollary 2.1 by showing that $F_1(t)=0$ and $F_2(t)\to 0$ as $t\to\infty$. However, from Remark 3.3, we know that the Lyapunov derivative in algorithm 5 is the same as in Eq.(2.122), with $F_1(t)$ and $F_2(t)$ as shown below:

$$F_1(t)=-\dot{x}^*(t)+Ax^*(t)-B\tilde{K}_eCx_0^*+B[\tilde{K}_xx_m(t)+\tilde{K}_uu_m]$$

$$= -Bu^*(t) - B\tilde{K}_fx_f^*(t) + B[\tilde{K}_xx_m(t)+\tilde{K}_uu_m]$$

$$= -B[S_{31}x_m(t)+S_{32}u_m] - B\tilde{K}_f[S_{21}x_m(t)+S_{22}u_m]$$

$$+B[\tilde{K}_xx_m(t) + \tilde{K}_uu_m]$$

$$= -B[(\tilde{K}_x-S_{31}-\tilde{K}_fS_{21})x_m(t)+(\tilde{K}_u-S_{32}-\tilde{K}_fS_{22})u_m] \qquad (3.49)$$

and

$$F_2(t) = (Q+G\tilde{K}_e)Cx_0^*(t) - G[\tilde{K}_xx_m(t)+\tilde{K}_uu_m]$$

$$= (Q_f+G\tilde{K}_f)x_f^*(t) - G[\tilde{K}_xx_m(t) + \tilde{K}_uu_m]$$

$$= (Q_f+G\tilde{K}_f)[S_{21}x_m(t)+S_{22}u_m(t)] - G[\tilde{K}_x x_m(t)+\tilde{K}_u u_m]$$

$$= [(Q_f+G\tilde{K}_f)S_{21}-G\tilde{K}_x]x_m(t)+[(Q_f+G\tilde{K}_f)S_{22}-G\tilde{K}_u]u_m$$

$$= [Q_fS_{21}+G(\tilde{K}_fS_{21}-\tilde{K}_x)]x_m(t)+[Q_fS_{22}+G(\tilde{K}_fS_{22}-\tilde{K}_u)]u_m \tag{3.50}$$

Then, we use the sufficient condition in Corollary 3.1, $Q_f=0$, to obtain

$$F_2(t) = G(\tilde{K}_fS_{21}-\tilde{K}_x)x_m(t) + G(\tilde{K}_fS_{22}-\tilde{K}_u)u_m$$

$$= G(\tilde{K}_fS_{21}-\tilde{K}_x-S_{31}+S_{31})x_m(t)+G(\tilde{K}_fS_{22}-\tilde{K}_u+S_{32}-S_{32})u_m$$

$$= G[S_{31}x_m(t)+S_{32}u_m] + G(\tilde{K}_x-\tilde{K}_fS_{21}-S_{31})x_m(t)$$

$$+G(\tilde{K}_u-\tilde{K}_fS_{22}-S_{32})u_m$$

$$= Gu^*(t) + G(\tilde{K}_x-\tilde{K}_fS_{21}-S_{31})x_m(t) + G(\tilde{K}_u-\tilde{K}_fS_{22}-S_{32})u_m$$

Further, from Eq.(3.19), we have

$$\dot{x}_f(t) = [\ B_{f1}(t)\ A_f(t)\ B_{f2}(t)\ B_{f3}(t)\ ] \begin{bmatrix} e_{yp}(t) \\ -x_f(t) \\ x_m(t) \\ u_m(t) \end{bmatrix}$$

$$= K(t)r(t) = u(t)$$

Thus, $\dot{x}_f^*(t)=u^*(t)$ which yields

$$F_2(t) = G\dot{x}_f^*(t) + G(\tilde{K}_x-\tilde{K}_fS_{21}-S_{31})x_m(t) + G(\tilde{K}_u-\tilde{K}_fS_{22}-S_{32})u_m$$

$$= GS_{21}\dot{x}_m(t)+G(\tilde{K}_x-\tilde{K}_fS_{21}-S_{31})x_m(t)+G(\tilde{K}_u-\tilde{K}_fS_{22}-S_{32})u_m \tag{3.51}$$

Finally, we choose

$$\tilde{K}_x = \tilde{K}_fS_{21}+S_{31} \tag{3.52}$$

$$\tilde{K}_u = \tilde{K}_fS_{22}+S_{32} \tag{3.53}$$

to obtain $F_1(t)=0$ and $F_2(t)=GS_{21}\dot{x}_m(t)$ which will vanish asymptotically because u_m is a constant for $t \geq t_1$ and the reference model is asymptotically stable.

Hence, Eq.(2.122) becomes

$$\dot{V}(e_x,K^I) = -e_x^T(t)Re_x(t)-[L^Te_x(t)-Wz(t)]^T[L^Te_x(t)-Wz(t)]$$

$$+z(t)^T(J+J^T+2G)z(t)-2v^T(t)v(t)r^T(t)\tilde{T}r(t)+2z^T(t)GS_{21}\dot{x}_m(t) \tag{3.54}$$

Note that Eq.(3.54) is in the same form as Eq.(2.132) where the last term vanishes asymptotically. Therefore, the remainder of the proof of asymptotic output tracking is the same as in section 2.11.

Now we show that the conditions for a bounded output tracking error, Eq.(3.31) and (3.32), and for an asymptotically vanishing output tracking error, $Q_f=0$, can be satisfied simultaneously.

<u>Lemma 3.3</u>:

Let the adaptive controller be algorithm 5 and let the plant belong to the class described by corollary 3.1 with the additional restriction that $A_c=0$. Then, the sufficient conditions for asymptotically vanishing error can be satisfied if we choose $G<0$ and $Q_p = GB_c$.

Proof:

Let $\tilde{A}_f$, $\tilde{B}_{f1}$, C_f, D_f be chosen as in Lemma 3.2, and let $Q_f=0$ as in Corollary 3.1. Then, we choose $G<0$ and $Q_p = GB_c$ to obtain

$$
\begin{aligned}
Q + G\tilde{K}_e &= [\ Q_p,\ Q_f\ ] + G[\ \tilde{B}_{f1},\ \tilde{A}_f] \\
&= [\ Q_p+G\tilde{B}_{f1},\ Q_f+G\tilde{A}_f\ ] \\
&= [\ GB_c+G\tilde{B}_{f1},\ -GA_c] \\
&= [\ GB_c-GB_c,\ -GA_c] \\
&= 0 && (3.55)
\end{aligned}
$$

Therefore, the sufficient condition in Eq.(3.32) becomes

$$J+(Q+G\tilde{K}_e)C(sI-A+B\tilde{K}_eC)^{-1}B = J \quad \text{is SPR} \tag{3.56}$$

Further, we let $J=-\gamma_4 G$ with $0<\gamma_4<1$, such that

$$J = -\gamma_4 G > 0 \tag{3.57}$$

$$J + J^T + G + G^T = (-\gamma_4 G+G) + (-\gamma_4 G+G)^T$$

$$= (1-\gamma_4)(G + G)^T < 0 \tag{3.58}$$

Therefore, the sufficient conditions for a bounded error given by Eqs.(3.31) and (3.32) and the sufficient conditions for an asymptotically vanishing error, $Q_f=0$, are satisfied.

3.7 Summary of Constraints and Design Rules

Algorithm 4: (Bounded Tracking Error)

Constraints:

(i) (A_p, B_p) controllable; (A_p, C_p) observable (3.59)

(ii) $Q + G\tilde{K}_e = 0$ (3.60)

(iii) $J + J^T + G + G^T < 0$ (3.61)

Design Method:

(i) Choose $\tilde{K}_e = -\begin{bmatrix} D_c & C_c \\ B_c & A_c \end{bmatrix}$ (3.62)

(ii) Choose $G<0$ (3.63)

(iii) Choose $Q = -G\tilde{K}_e$ (3.64)

Algorithm 5: (Bounded Tracking Error)

Constraints:

(i) (A_p, B_p) controllable; (A_p, C_p) observable (3.65)

(ii) $Q + G\tilde{K}_e = 0$ (3.66)

(iii) $J + J^T + G + G^T < 0$ (3.67)

Design Method:

(i) Choose $G < 0$ (3.68)

(ii) Choose $\tilde{K}_e = [-B_c, -A_c]$ (3.69)

(iii) choose $C_f = -C_c$ (3.70)

(iv) choose $D_f = -D_c$ (3.71)

(v) Choose $Q = -G\tilde{K}_e$ (3.72)

Algorithm 5: (Asymptotic Tracking Error)

Constraints:

(i) (A_p, B_p) controllable; (A_p, C_p) observable (3.73)

(ii) $A_c = 0$ (3.74)

(iii) $Q + G\tilde{K}_e = 0$ (3.75)

(iv) $J + J^T + G + G^T < 0$ (3.76)

Design Method:

(i) choose $C_f = -C_c$ (3.77)

(ii) choose $D_f = -D_c$ (3.78)

(iii) Choose $G < 0$ (3.79)

(v) Choose $Q_p = GB_c$; $Q_f = 0$ (3.80)

3.8 Examples

Example 1:

(Rohrs' Example using Algorithm 5)

Rohrs' example is given by Eq.(2.197) with the reference model given by Eq.(2.198). Suppose we know that a stabilizing PI compensator is described by

$$G_c(s) = \frac{-(10s+35)}{s} \tag{3.81}$$

The compensator described by Eq.(3.81) has a state space realization given by $A_c=0$, $B_c=10$, $C_c=-3.5$, and $D_c=-10$. Using Eqs.(3.77)-(3.80) we choose

$$C_f = -C_c = 3.5 \tag{3.82}$$

$$D_f = -D_c = 10 \tag{3.83}$$

$$G = -1 < 0 \tag{3.84}$$

$$Q_p = GB_c = -10; \quad Q_f = 0 \tag{3.85}$$

The matrices T and $\bar{T}$ are chosen to be $T=\bar{T}=5I$. An asymptotically vanishing output tracking error is shown in Figure 3.3 for a square wave reference command of magnitude 0.3 units and period of 20 seconds. The adaptive gains $B_{f1}(t)$ and $A_f(t)$ are shown in Figures 3.4 and 3.5, respectively. We observe that the gains become momentarily large at t=0 and t=10 seconds in order to force the error to zero when the reference command is initially applied and when the reference command changes sign.

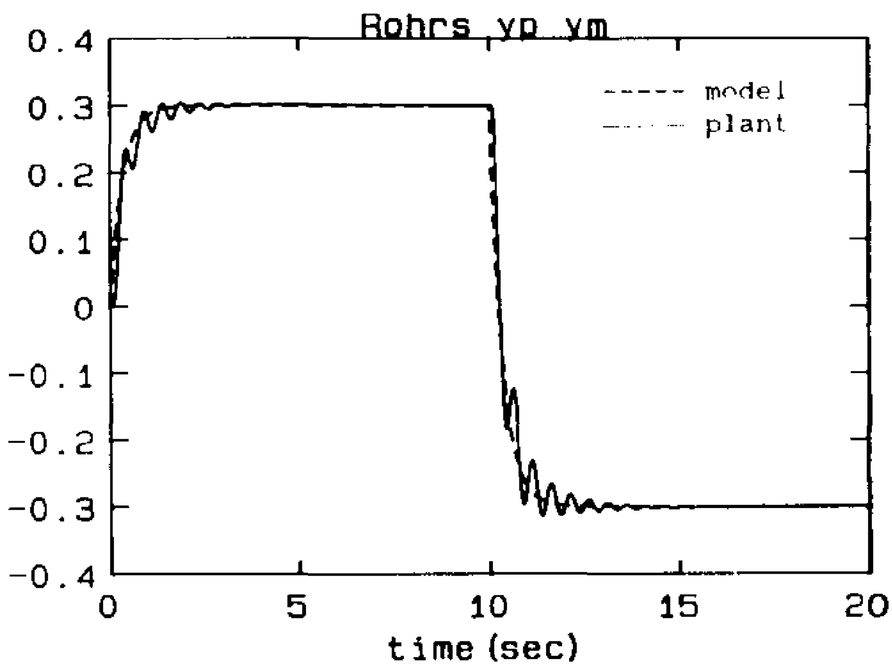

Figure 3.3 Rohrs' Example:
Plant and Model Output (Algorithm 5)

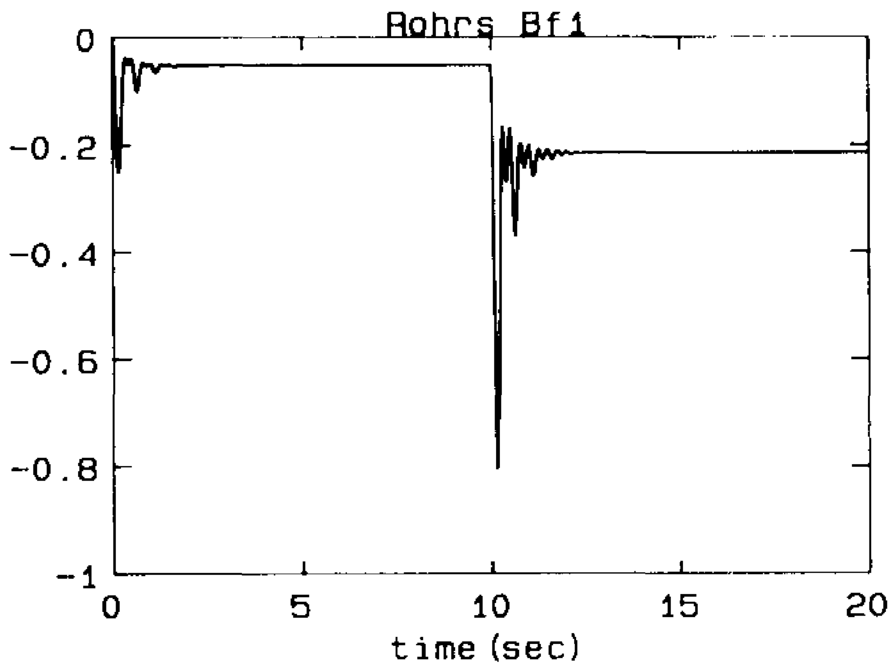

Figure 3.4 Rohrs' Example:
Adaptive Gain B_{f1} (Algorithm 5)

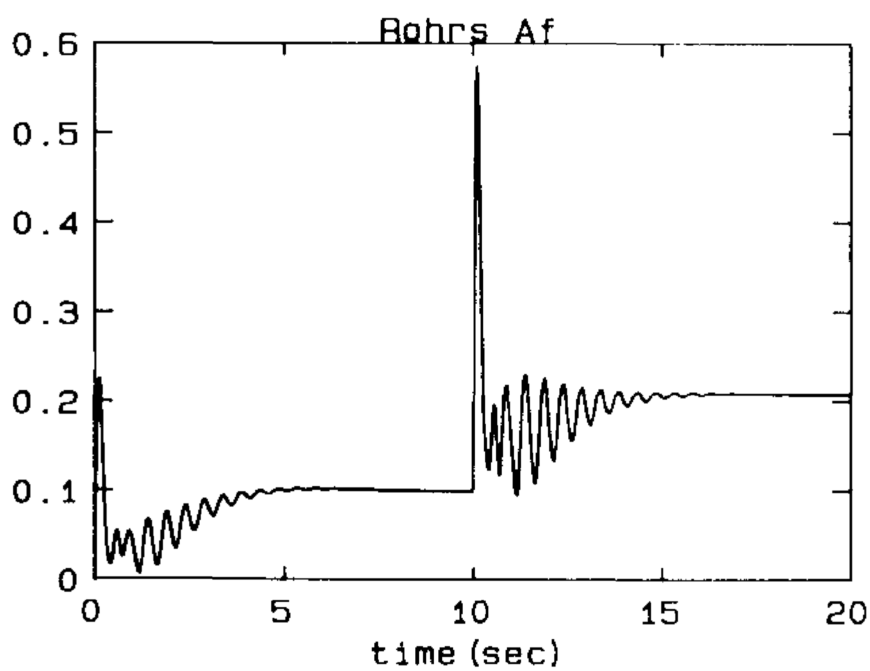

Figure 3.5 Rohrs' Example:
Adaptive Gain A_f (Algorithm 5)

Example 2:

(Unstable Plant using Algorithm 5)

Consider the unstable single-input single-output plant with transfer function given by Eq.(2.208). The output of the plant is required to follow the output of the reference model whose transfer function is given by Eq.(2.198).

Suppose we know that a stabilizing PI compensator is described by

$$G_c(s) = \frac{-(3s+6)}{s} \tag{3.86}$$

The compensator described by Eq.(3.86) has a state space realization given by $A_c=0$, $B_c=3$, $C_c=-2$, and $D_c=-3$. Using Eqs.(3.77)-(3.80) we choose

$$C_f = -C_c = 2 \tag{3.87}$$

$$D_f = -D_c = 3 \tag{3.88}$$

$$G = -1 < 0 \tag{3.89}$$

$$Q_p = GB_c = -3; \quad Q_f = 0 \tag{3.90}$$

The matrices T and $\bar{T}$ are chosen to be $T=\bar{T}=5I$. The outputs are shown in Figure 3.6 for a square wave reference command of magnitude 0.3 units and period of 40 seconds. We observe that the error is driven to zero in approximately 2 seconds albeit with a maximum overshoot of near 30 percent. The adaptive gains $B_{f1}(t)$ and $A_f(t)$ are shown in figures 3.7 and 3.8, respectively. We observe that the gains become momentarily large at t=0 and t=20 seconds in order to force the error to zero when the reference command is initially applied and when the reference command changes sign.

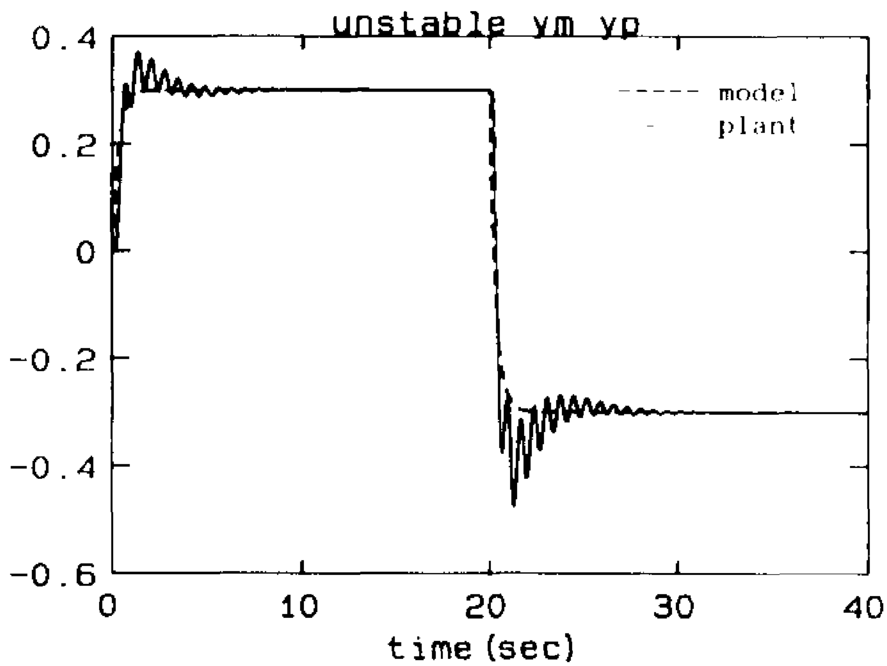

Figure 3.6 Unstable Plant:
Plant and Model Output (Algorithm 5)

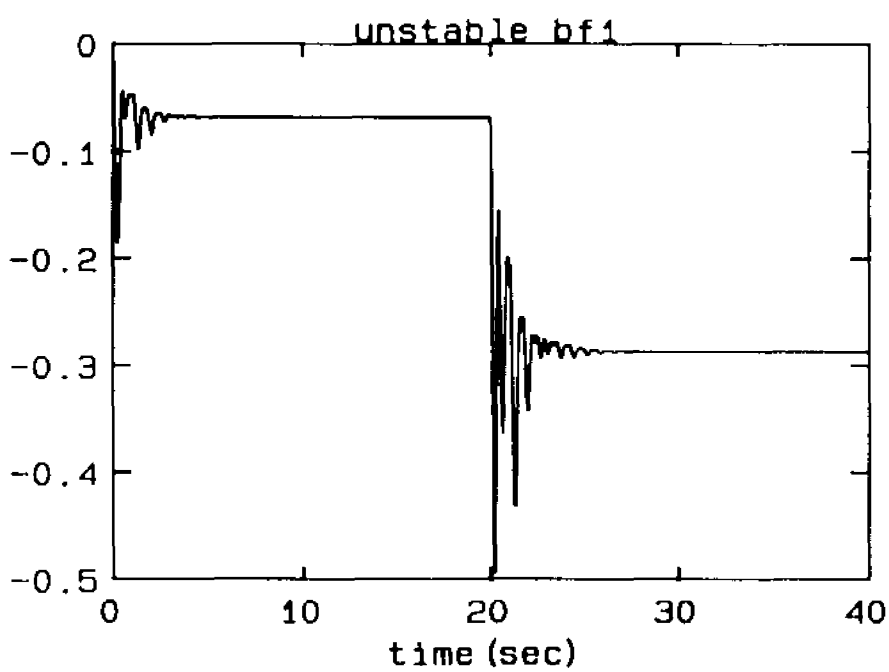

Figure 3.7 Unstable Plant:
Adaptive Gain B_{f1} (Algorithm 5)

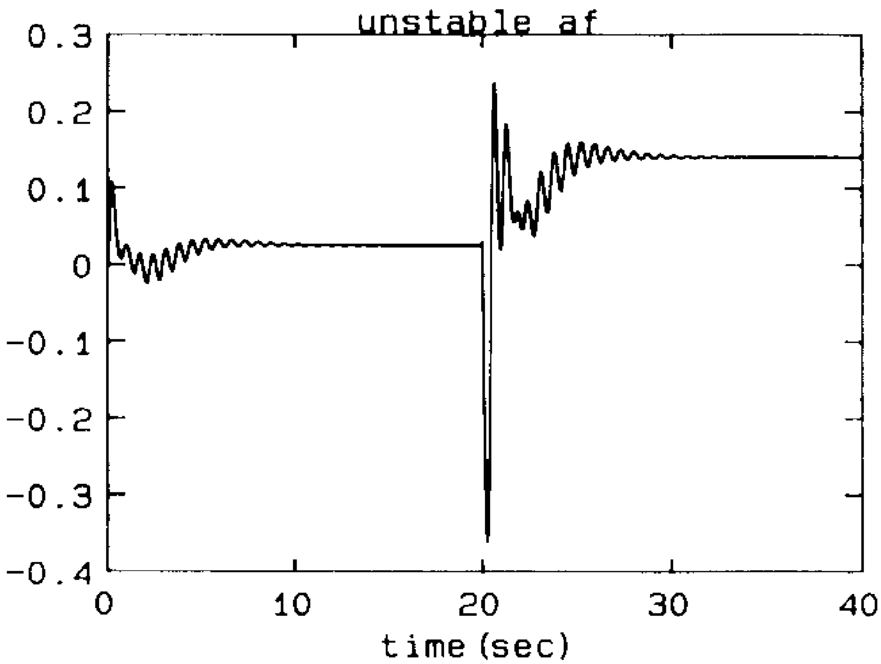

Figure 3.8 Unstable Plant:
Adaptive Gain A_f (Algorithm 5)

<u>Example 3</u>:

(F-8 Lateral Dynamics using Algorithm 4)

Consider the F-8 lateral dynamics with the plant described by Eq.(2.218) with the reference model described by Eq.(2.219). Suppose we know that a stabilizing compensator is described by the quadruple (A_c, B_c, C_c, D_c) where

$$A_c = \begin{bmatrix} -10 & 0 \\ 0 & -10 \end{bmatrix} \tag{3.91}$$

$$B_c = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \tag{3.92}$$

$$C_c = \begin{bmatrix} 1000 & 0 \\ 0 & 10 \end{bmatrix} \tag{3.93}$$

$$D_c = \begin{bmatrix} -50,000 & 0 \\ 0 & 20,000 \end{bmatrix} \tag{3.94}$$

We choose algorithm 4 for this example which uses feedback supplementary dynamics and yields a bounded error. Using Eqs.(3.62)-(3.64) we choose

$$\tilde{K}_e = -\begin{bmatrix} -50,000 & 0 & 1000 & 0 \\ 0 & 20,000 & 0 & 10 \\ -1 & 0 & -10 & 0 \\ 0 & -1 & 0 & -10 \end{bmatrix} \tag{3.95}$$

$$G = -0.001 < 0 \tag{3.96}$$

$$Q = -G\tilde{K}_e = \begin{bmatrix} 50 & 0 & -1 & 0 \\ 0 & -20 & 0 & -0.01 \\ -0.001 & 0 & 0.01 & 0 \\ 0 & -0.001 & 0 & 0.01 \end{bmatrix} \tag{3.97}$$

Then, we choose $T=0.05I$ and $\bar{T}=0.1I$ and choose u_m, $d_{ip}(t)$, $d_{op}(t)$, and σ to be the same as in example 3 of section 2.13.

The time responses for the plant and model outputs are shown in Figures 3.9 and 3.10. We observe that the plant roll rate, Figure 3.9, differs from the model roll rate at the initial time but this error is reduced quickly and vanishes at $t = 0.03$ sec. Then, the roll rate response exhibits excellent model following with virtually no steady state error. The plant yaw rate, Figure 3.10, starts with a large initial error and exhibits good model following with a small steady state error. This model following is obtained in the presence of both input and output disturbances.

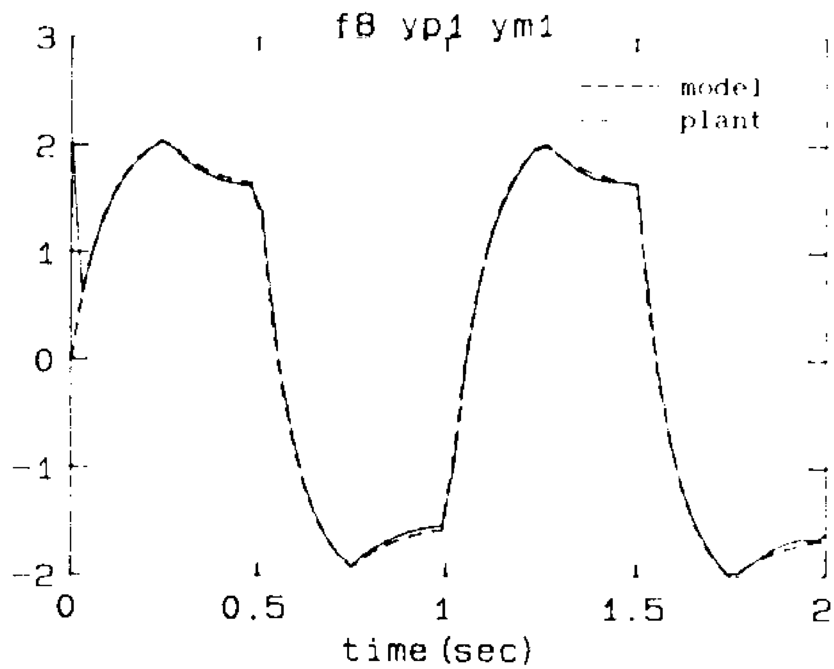

Figure 3.9 F-8: Plant and Model Roll Rate (Algorithm 4)

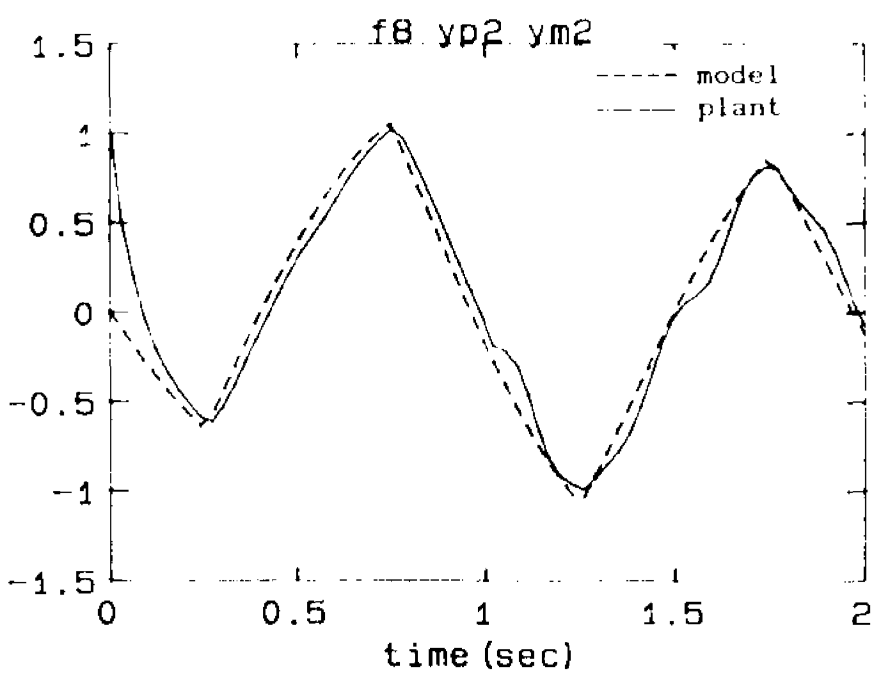

Figure 3.10 F-8: Plant and Model Yaw Rate (Algorithm 4)

4. ALGORITHM FOR NON-LINEAR PLANTS

4.1 Introduction

In the previous chapters, we have discussed the command generator tracker approach to model reference adaptive control under the assumption that the plant is linear and time invariant. However, a plant represented by linear dynamics is to some extent a mathematical abstraction that can never be encountered in the real world. It may be said that no physical system is completely linear since there are always some limits such as mechanical stops and some unexpected phenomena such as saturation or dead-zones which are inherent in the plant. When the effect of the nonlinearity is very small, linear methods of analysis can be applied to give an approximate answer which is adequate for engineering purposes. Otherwise, a nonlinear representation will be essential for an adequate description of the plant.

The use of the command generator tracker approach to model reference adaptive control for nonlinear plants has been studied by other researchers. However, some assumptions, such as the boundedness of the nonlinear functions [10], the virtual linearization of the nonlinearity [11], and full access to the plant states [12], must be used in order to prove stability.

To overcome this problem, we present an algorithm, without the assumptions which are required in references 10, 11, and 12, for plants with non-autonomous nonlinearities of known form, but with unknown parameters. We show that the output tracking error will vanish asymptotically if the linear part of the plant is ASPR. Two examples are discussed which include a pendulum oscillator and a ship autopilot.

4.2 Problem Formulation

Let the nonlinear plant be described by

$$\dot{x}_p(t)=A_px_p(t)+A_\gamma\gamma(C_px_p,u_p,t)+B_pu_p(t)+E_pd_{ip}(t) \qquad (4.1)$$

$$y_p(t) = C_px_p(t) + d_{op}(t) \qquad (4.2)$$

where $x_p(t)\in R^n$ is the plant state which is not accessible, $u_p(t)\in R^m$ is the plant input, $y_p(t)\in R^m$ is the plant output, $\gamma(C_px_p,u_p,t)\in R^r$ is a nonlinear function of known form, $d_{ip}(t)\in R^m$ and $d_{op}(t)\in R^\ell$ are bounded unknown disturbances. The matrices A_p, B_p, A_γ, C_p and E_p are unknown with appropriate dimensions.

We desire that the plant output tracks a desired trajectory which is generated by a linear time invariant reference model which is shown below:

$$\dot{x}_m(t) = A_mx_m(t) + B_mu_m(t) \qquad (4.3)$$

$$y_m(t) = C_mx_m(t) \qquad (4.4)$$

where $x_m(t) \in R^{n_m}$, $y_m(t) \in R^m$, and $u_m(t) \in R^{m_m}$

We choose a control law, which includes the nonlinear signal, as follows

$$u_p(t) = K_{pe}(t)e_{yp}(t) + K_{p\gamma}(t)\gamma(C_p x_p, u_p, t) + K_{px}(t)x_m(t)$$

$$+K_{pu}(t)u_m(t) \qquad (4.5)$$

or $u(t) = K(t)r(t)$

where

$$K(t) = [\ K_{pe}(t),\ K_{p\gamma}(t),\ K_{px}(t),\ K_{pu}(t)\ ] \qquad (4.6)$$

and where $$r(t) = \begin{bmatrix} e_{yp}(t) \\ \gamma(C_p x_p, u_p, t) \\ x_m(t) \\ u_m(t) \end{bmatrix}$$

The adaptive gain K(t) is computed as follows:

$$K(t) = K^P(t) + K^I(t) \tag{4.7}$$

$$K^P(t) = v(t)r^T(t)\bar{T} \tag{4.8}$$

$$\dot{K}^I(t) = [\ v(t)r^T(t) - \sigma K^I(t)\Psi\ ]T \tag{4.9}$$

with the initial integral gain set to

$$K^I(0) = [\ K^I_{pe}(0),\ K^I_{p\gamma}(0),\ K^I_{px}(0),\ K^I_{pu}(0)\ ] \tag{4.10}$$

and where σ is a positive scalar, T and $\bar{T}$ are positive definite and positive semi-definite, respectively, and where the signal v(t) is chosen as

$$v(t) = e_{yp}(t) \tag{4.11}$$

4.3 Error Equations

We extend Bar-Kana's[5] fictitious target system to nonlinear plants of the form described by Eqs.(4.1) and (4.2). The nonlinear fictitious target system is assumed to have the form shown below:

$$\dot{x}_p^*(t)=A_p^*x_p^*(t)+A_\gamma^*\gamma(C_px_p^*,u_p^*,t)+B_p^*u_p^*(t) \quad (4.12)$$

$$y_p^*(t) = C_px_p^*(t) = y_m(t) \quad (4.13)$$

Note that although Eq.(4.12) is assumed to be of the same order as Eq.(4.1), these two systems may be entirely different and it is only assumed that their measurement matrices are identical.

To check that the fictitious target trajectories discussed in section 2.3 will be valid for the nonlinear plant, we assume that

$$x_p^*(t) = S_{11}x_m(t) \quad (4.14)$$

Thus,

$$y_p^*(t) = C_p x_p^*(t) = C_p S_{11} x_m(t) \tag{4.15}$$

If $\operatorname{rank}[C_p]=\operatorname{rank}[C_p \vdots C_m]$, (4.16)

then Eq.(4.15) has a solution for S_{11}

and

$$y_p^*(t) = y_m(t) \tag{4.17}$$

Next, we define the state and output errors as shown below:

$$e_{xp}(t) = x_p^*(t) - x_p(t) \tag{4.18}$$

$$\begin{aligned} e_{yp}(t) &= y_m(t) - y_p(t) \\ &= y_p^*(t) - y_p(t) \\ &= C_p e_{xp}(t) - d_{op}(t) \end{aligned} \tag{4.19}$$

The state error derivative is given by

$$\dot{e}_{xp}(t) = \dot{x}_p^*(t) - \dot{x}_p(t)$$

$$= \dot{x}_p^*(t)-A_p x(t)-A_\gamma \gamma(C_p x_p, u_p, t)-B_p u_p(t)-E_p d_{1p}(t)$$

$$= \dot{x}_p^*(t) + A[x_p^*(t)-x_p(t)] - A_p x_p^*(t)$$

$$-A_\gamma \gamma(C_p x_p, u_p, t) - B_p[K(t)-\tilde{K}] - B_p[\tilde{K}_{pe} e_{yp}(t)$$

$$+\tilde{K}_\gamma \gamma(C_p x_p, u_p, t)+\tilde{K}_{px} x_m(t)+\tilde{K}_{pu} u_m(t)]-E_p d_{1p}(t)$$

$$=(A_p-B_p\tilde{K}_{pe}C_p)e_{xp}(t)-B_p z(t)+\dot{x}_p^*(t)+[-A_p S_{11}-B_p\tilde{K}_{px}]x_m(t)$$

$$+[-A_p S_{12}-B_p\tilde{K}_{pu}]u_m(t)-[A_\gamma+B_p\tilde{K}_\gamma]\gamma(C_p x_p, u_p, t)$$

$$-E_p d_{1p}(t)+B_p\tilde{K}_{pe} d_{op}(t)$$

$$=A_{pc} e_{xp}(t)-B_p z(t)-F_1(t) \tag{4.20}$$

where

$A_{pc} = A_p - B_p \tilde{K}_{pe} C_p$ is a stability matrix

$z(t) = [K(t)-\tilde{K}]r(t)$

$\tilde{K} = [\tilde{K}_{pe}, \tilde{K}_{p\gamma}, \tilde{K}_{px}, \tilde{K}_{pu}]$

and

$$F_1(t) = -\dot{x}_p^*(t) + [A_p S_{11} + B_p \tilde{K}_{px}]x_m(t) + E_p d_{ip}(t) - B_p \tilde{K}_{pe} d_{op}(t)$$
$$+ [A_p S_{12} + B_p \tilde{K}_{pu}]u_m(t) + [A_\gamma + B_p \tilde{K}_\gamma]\gamma(C_p x_p, u_p, t) \tag{4.21}$$

4.4 Stability Analysis

<u>Theorem 4.1</u>:

Consider a nonlinear plant represented by Eqs.(4.1) and (4.2). Suppose $\mathrm{rank}[B_p]=\mathrm{rank}[B_p \vdots A_\gamma]$ and suppose there exists a real symmetric positive definite matrix P and real matrices L and $\tilde{K}_e$ such that

$$P(A-B\tilde{K}_eC) + (A-B\tilde{K}_eC)^TP = -LL^T < 0 \tag{4.22}$$

$$PB = C^T \tag{4.23}$$

where the matrices T and $\bar{T}$ are positive definite symmetric and positive semi-definite symmetric, respectively. Then, all states, gains and errors in the adaptive system are bounded.

<u>Remark 4.1</u>:

The sufficient conditions, given by Eqs.(4.22) and (4.23), are equivalent to requiring that the linear part of the plant is almost strictly positive real.

Remark 4.2:

If $\mathrm{rank}[B_p]=\mathrm{rank}[B_p \vdots A_\gamma]$, then there exists an S_γ such that
$A_\gamma+B_pS_\gamma=0.$ (4.24)

Remark 4.3:

The stability is analyzed using a Lyapunov approach by forming a quadratic function which is positive definite in the state variables of the adaptive system, $e_{xp}(t)$ and $K^I(t)$ as in Eq.(2.80). The Lyapunov derivative can be obtained from Appendix B. However, the metasystem representation is no longer needed for this nonlinear case because supplementary dynamics are not used. Thus, we define $e_x(t)=e_{xp}(t)$, $e_{yv}(t)=e_{yp}(t)$, and choosing $A_c=A_{pc}$, $B=B_p$, $Q=I$, $W=0$, $G=0$, $R=0$ and $J=0$. If the sufficient conditions, Eqs.(4.22) and (4.23), are satisfied, then the derivative of the Lyapunov function in Eq.(2.81) becomes

$$\dot{V}(e_{xp},K^I) = -e_{xp}^T(t)LL^Te_{xp}(t) - 2v^T(t)v(t)r^T(t)\bar{T}r(t)$$

$$-2\sigma tr[(K^I(t)-\tilde{K})\Psi(K^I(t)-\tilde{K})^T] - 2\sigma tr[(K^I(t)-\tilde{K})\Psi\tilde{K}^T]$$

$$-2e_{xp}^{T}(t)PF_1(t)-2z^{T}(t)F_2(t) \qquad (4.25)$$

where

$$F_1(t)=-\dot{x}_p^{*}(t)+[A_pS_{11}+B_p\tilde{K}_{px}]x_m(t)+E_pd_{1p}(t)-B_p\tilde{K}_{pe}d_{op}(t)$$

$$+[A_pS_{12}+B_p\tilde{K}_{pu}]u_m(t)+[A_\gamma+B_p\tilde{K}_\gamma]\gamma(C_px_p,u_p,t) \qquad (4.26)$$

and

$$F_2(t) = d_{op}(t) \qquad (4.27)$$

Let $\tilde{K}_\gamma=S_\gamma$ and substitute Eqs.(4.14) and (4.24) into (4.26)to obtain

$$F_1(t)=-S_{11}\dot{x}_m(t)+[A_pS_{11}+B_p\tilde{K}_{px}]x_m(t)+E_pd_{1p}(t)-B_p\tilde{K}_{pe}d_{op}(t)$$

$$+[A_pS_{12}+B_p\tilde{K}_{pu}]u_m(t)+[A_\gamma+B_pS_\gamma]\gamma(C_px_p,u_p,t)$$

$$=-S_{11}\dot{x}_m(t)+[A_pS_{11}+B_p\tilde{K}_{px}]x_m(t)+E_pd_{1p}(t)-B_p\tilde{K}_{pe}d_{op}(t)$$

$$+[A_pS_{12}+B_p\tilde{K}_{pu}]u_m(t) \qquad (4.28)$$

which is bounded since $d_{1p}(t)$, $d_{op}(t)$ and $u_m(t)$ are bounded, and $\tilde{K}_{pe}$, $\tilde{K}_{px}$, and $\tilde{K}_{pu}$ are constant.

We observe that there exist some positive constants α_1, α_2, $\cdots$, α_6 such that

$$\dot{V}(e_{xp},K^I) \le -\alpha_1\|e_{xp}(t)\|^2 - \alpha_2\|K(t)-\tilde{K}\|^2 - \alpha_3\|v(t)\|^2\ \|r(t)\|^2$$

$$+\ \alpha_4\|e_{xp}(t)\| + \alpha_5\|[K(t)-\tilde{K}]r(t)\| + \alpha_6\|[K(t)-\tilde{K}]\| \tag{4.29}$$

If either $\|e_{xp}(t)\|$, $\|r(t)\|$, or $\|K(t)-\tilde{K}\|$ increase beyond some bound, then the negative quadratic terms in Eq.(4.29) will become dominant, and thus $\dot{V}$ becomes negative. The quadratic form of the Lyapunov function $V(e_{xp},K^I)$ then guarantees that $e_{xp}(t)$, $K^I(t)$ and $e_{yp}(t)$ are bounded.

4.5 Asymptotic Output Tracking for Known Plants

To show asymptotic model following of the adaptive algorithm we extend Broussard's [4] command generator tracker to the nonlinear plant by using the ideal state $x_p^*(t)$, ideal input $u_p^*(t)$, and ideal output $y_p^*(t)$. In the ideal situation, $y_p(t)=y_p^*(t)=y_m(t)$, and the ideal plant satisfies the following dynamics.

$$\dot{x}_p^*(t) = A_p x_p^*(t)+A_\gamma \gamma(C_p x_p^*, u_p^*, t) + B_p u_p^*(t) \qquad (4.30)$$

$$y_p^*(t) = C_p x_p^*(t) \qquad (4.31)$$

where $x_p^*(t)$, $y_p^*(t)$, and $u_p^*(t)$ are ideal trajectories which are different from the fictitious target trajectories used in section 4.3.

Next, we constrain the reference model command to be a constant and assume that the form of the ideal state and ideal input is as follows:

$$x_p^*(t) = S_1 x_m(t) + S_2 u_m \qquad (4.32)$$

$$u_p^*(t) = S_\gamma \gamma(C_p x_p^*, u_p^*, t) + S_{31} x_m(t) + S_{32} u_m \tag{4.33}$$

We substitute Eqs.(4.32) and (4.33) into Eqs.(4.30) and (4.31) to obtain

$$\dot{x}_p^*(t) = A_p[S_1 x_m(t)+S_2 u_m] + B_p[S_{31} x_m(t)+S_{32} u_m]$$

$$+(A_\gamma+B_p S_\gamma)\gamma(C_p x_p^*, u_p^*, t) \tag{4.34}$$

$$y_p^*(t) = C_p[S_1 x_m(t)+S_2 u_m] \tag{4.35}$$

Using $\text{rank}[B_p]=\text{rank}[B_p \vdots A_\gamma]$ and combining Eqs.(4.34) and (4.35) we obtain

$$\begin{bmatrix} \dot{x}_p^*(t) \\ y_p^*(t) \end{bmatrix} = \begin{bmatrix} A_p & B_p \\ C_p & 0 \end{bmatrix} \begin{bmatrix} S_1 & S_2 \\ S_{31} & S_{32} \end{bmatrix} \begin{bmatrix} x_m(t) \\ u_m \end{bmatrix} \tag{4.36}$$

Eq.(4.36) is the same equation as in the linear time invariant case which was discussed in Chapter 2. Thus, from section 2.11, we have

$$\begin{bmatrix} S_1 A_m & S_1 B_m \\ C_m & 0 \end{bmatrix} = \begin{bmatrix} A_p & B_p \\ C_p & 0 \end{bmatrix} \begin{bmatrix} S_1 & S_2 \\ S_{31} & S_{32} \end{bmatrix} \tag{4.37}$$

and

$$\begin{bmatrix} S_1 & S_2 \\ S_{31} & S_{32} \end{bmatrix} = \begin{bmatrix} \Omega_{11} & \Omega_{12} \\ \Omega_{21} & \Omega_{22} \end{bmatrix} \begin{bmatrix} S_1 A_m & S_1 B_m \\ C_m & 0 \end{bmatrix}$$

where

$$\begin{bmatrix} \Omega_{11} & \Omega_{12} \\ \Omega_{21} & \Omega_{22} \end{bmatrix} = \begin{bmatrix} A & B \\ [C_p, 0] & 0 \end{bmatrix}^{-1}$$

Broussard [4] has shown that an equation of the type given by Eq.(2.118) has a solution for S_1, S_2, S_{31}, and S_{32} if (i) u_m is a constant, (ii) $\dim[y_p(t)]=\dim[u_p(t)]$, and (iii) no eigenvalue of Ω_{11} is equal to the inverse of an eigenvalue of A_m.

If the plant were known, we could choose a control signal as follows:

$$u_p(t) = \tilde{K}_{pe}e_{yp}(t)+\tilde{K}_{p\gamma}\gamma(C_px_p,u_p,t)+\tilde{K}_{px}x_m(t)+\tilde{K}_{pu}u_m \tag{4.38}$$

where $\tilde{K}_{pe}$ $\tilde{K}_{\gamma}$, $\tilde{K}_{px}$, and $\tilde{K}_{pu}$ are constant gain matrices.

To illustrate that Eq.(4.37) is sufficient to yield the perfect output tracking with the control signal in Eq.(4.38), we rewrite Eq.(4.37) as follow

$$S_{11}A_m - A_pS_{11} - B_pS_{21} = 0 \tag{4.39}$$

$$S_{11}B_m - A_pS_{12} - B_pS_{22} = 0 \tag{4.40}$$

$$C_pS_{11} = C_m \tag{4.41}$$

$$C_pS_{12} = 0 \tag{4.42}$$

Under the assumption that $d_{ip}(t)=0$ and $d_{op}(t)=0$, the state error derivative equation will be

$$\dot{x}^*_p(t)-\dot{x}_p(t)=S_{11}\dot{x}_m(t)-A_px(t)-A_\gamma\gamma(C_px_p,u_p,t)-B_pu_p(t)$$

$$=S_{11}[A_mx_m(t)+B_mu_m]+A_p[x^*_p(t)-x_p(t)]-A_px^*_p(t)$$

$$-A_\gamma\gamma(C_px_p,u_p,t)-B_p[\tilde{K}_{pe}e_{yp}(t)+\tilde{K}_\gamma\gamma(C_px_p,u_p,t)$$

$$+\tilde{K}_{px}x_m(t)+\tilde{K}_{pu}u_m]$$

$$=(A_p-B_p\tilde{K}_{pe}C_p)[x_p^*(t)-x_p(t)]+[S_{11}A_m-A_pS_{11}-B_p\tilde{K}_{px}]x_m(t)$$

$$+[S_{11}B_m-A_pS_{12}-B_p\tilde{K}_{pu}]u_m-[A_\gamma+B_p\tilde{K}_\gamma]\gamma(C_px_p,u_p,t) \tag{4.43}$$

For a known plant, we choose $\tilde{K}_{p\gamma}=-S_\gamma$, $\tilde{K}_{px}=S_{21}$, $\tilde{K}_{pu}=S_{22}$. Substitute Eqs.(4.24), (4.39), and (4.40) into Eq.(4.43) to obtain

$$\dot{x}_p^*(t)-\dot{x}_p(t)=(A_p-B_p\tilde{K}_{pe}C_p)[x_p^*(t)-x_p(t)] \tag{4.44}$$

where $\tilde{K}_{pe}$ is chosen such that $A_p-B_p\tilde{K}_{pe}C_p$ is a stability matrix. Therefore, the output tracking error will vanish asymptotically. That is

$$y_m(t)-y_p(t) = y_p^*(t)-y_p(t) = C_p[x_p^*(t)-x_p(t)] \rightarrow 0 \tag{4.45}$$

4.6 Asymptotic Output Tracking for Unknown Plants

In the adaptive control problem, the plant is unknown or poorly known. Therefore, an adaptive control signal described by Eq.(4.5) is used. An asymptotically vanishing output tracking error will be obtained for the nonlinear plant described by Eqs.(4.1) and (4.2) if the conditions in the following corollary are satisfied.

Corollary 4.1:

Adaptive control algorithm 6 for the class of nonlinear plants described by Eqs.(4.1) and (4.2) will yield an asymptotically vanishing output error if the conditions of Theorem 4.1 are satisfied and if (i) u_m is constant for $t \geq t_1$, (ii) no disturbances exist and σ=0, and (iii) a solution exists for the matrices S_1, S_2, S_{31}, and S_{32} in Eq.(4.37).

Proof:

If $d_{ip}(t)=0$, $d_{op}(t)=0$, and $\sigma=0$, then the derivative of the Lyapunov function, Eq.(4.25), will become

$$\dot{V}(e_{xp},K^{I}) = -e_{xp}^{T}(t)LL^{T}e_{xp}(t) - 2v^{T}(t)v(t)r^{T}(t)\bar{T}r(t)$$

$$-2e_{xp}^{T}(t)PF_{1} \qquad (4.46)$$

where

$$F_{1}(t)=-\dot{x}_{p}^{*}(t)+[A_{p}S_{11}+B_{p}\tilde{K}_{px}]x_{m}(t)$$

$$+[A_{p}S_{12}+B_{p}\tilde{K}_{pu}]u_{m}(t)+[A_{\gamma}+B_{p}\tilde{K}_{\gamma}]\gamma(C_{p}x_{p},u_{p},t) \qquad (4.47)$$

$$= -S_{11}\dot{x}_{m}(t)+[A_{p}S_{11}+B_{p}\tilde{K}_{px}]x_{m}(t)+[A_{p}S_{12}+B_{p}\tilde{K}_{pu}]u_{m}$$

$$+[A_{\gamma}+B_{p}\tilde{K}_{\gamma}]\gamma(C_{p}x_{p},u_{p},t)$$

$$=-S_{11}[A_{m}x_{m}(t)+B_{m}u_{m}(t)]+[A_{p}S_{11}+B_{p}\tilde{K}_{px}]x_{m}(t)$$

$$+[A_{p}S_{12}+B_{p}\tilde{K}_{pu}]u_{m}(t)+[A_{\gamma}+B_{p}\tilde{K}_{\gamma}]\gamma(C_{p}x_{p},u_{p},t)$$

$$=[A_{p}S_{11}+B_{p}\tilde{K}_{px}-S_{11}A_{m}]x_{m}(t)+[A_{p}S_{12}+B_{p}\tilde{K}_{pu}-S_{11}B_{m}]u_{m}(t)$$

$$+[A_{\gamma}+B_{p}\tilde{K}_{\gamma}]\gamma(C_{p}x_{p},u_{p},t) \qquad (4.48)$$

Let $\tilde{K}_{\gamma}=S_{\gamma}$, $\tilde{K}_{px}=S_{21}$, and $\tilde{K}_{pu}=S_{22}$, we obtain

$$F_1(t) = [A_pS_{11}+B_pS_{21}-S_{11}A_m]x_m(t)+[A_pS_{12}+B_pS_{22}-S_{11}B_m]u_m(t)$$

$$+[A_\gamma+B_pS_\gamma]\gamma(C_px_p,u_p,t) \qquad (4.49)$$

Finally, using Eqs.(4.24), (4.39) and (4.40), we obtain $F_1(t)=0$, and

$$\dot{V}(e_{xp},K^I) = -e_{xp}^T(t)LL^Te_{xp}(t)-2v^T(t)v(t)r^T(t)\bar{T}r(t) \qquad (4.50)$$

which is negative definite in $e_{xp}(t)$ and $v(t)$. Thus, $e_{xp}(t) \longrightarrow 0$ which implies that

$$\begin{aligned} e_{yp}(t) &= y_m(t)-y_p(t) \\ &= C_mx_m(t)-C_px_p(t) \\ &= C_pS_{11}x_m(t)-C_px_p(t) \\ &= C_pe_{xp}(t) \longrightarrow 0 \end{aligned} \qquad (4.51)$$

Furthermore, the adaptive gain $K^I(t)$ is bounded because $V(e_{xp},K^I)$ cannot increase beyond its initial value.

4.7 Examples

Pendulum Oscillator: (Algorithm 6)

Consider the pendulum of Figure 4.1, which is discussed by Ambrosino et. al.[12]. The pendulum dynamics are shown below:

$$\ell^2\left(\frac{M_b}{3} + M_0\right)\frac{d^2\theta(t)}{d} + \phi\frac{d\theta(t)}{dt} + g\ell\left(\frac{M_b}{2} + M_0\right)\sin\theta(t) = u_p(t) \tag{4.52}$$

where $u_p(t)$ is the control torque, g is the gravitational constant, ℓ is the length of the link, M_b is the distributed mass of the link, M_o is a lumped mass, and ϕ is an unknown parameter representing the viscous friction coefficient.

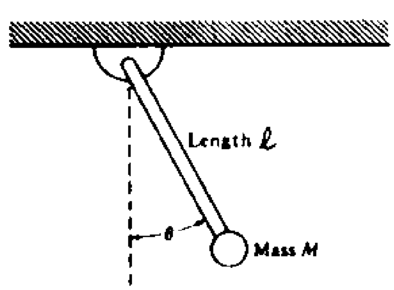

Figure 4.1 A Pendulum Oscillator

Letting $x_{p1}(t)=\theta(t)$ and $x_{p2}(t)=\frac{d\theta(t)}{dt}$, the corresponding state-space equation is

$$\dot{x}_p(t)=\begin{bmatrix} 0 & 1 \\ 0 & \dfrac{\phi}{\ell^2\left(\frac{M_b}{3}+M_0\right)} \end{bmatrix} x_p(t)+\begin{bmatrix} 0 \\ \dfrac{1}{\ell^2\left(\frac{M_b}{3}+M_0\right)} \end{bmatrix} u_p(t)$$

$$+\begin{bmatrix} 0 \\ \dfrac{g\left(\frac{M_b}{2}+M_0\right)}{\ell\left(\frac{M_b}{3}+M_0\right)} \end{bmatrix} \sin x_{p1}(t) \qquad (4.53)$$

$$y_p(t) = [\ 1\ \ 0\ ]x_p(t) \qquad (4.54)$$

Assuming M_b=1.2kg, ℓ=0.1m, ϕ=0.02, and M_0=0, we have the plant dynamics given by

$$\dot{x}_p(t)=\begin{bmatrix} 0 & 1 \\ 0 & -5 \end{bmatrix} x_p(t)+\begin{bmatrix} 0 \\ 250 \end{bmatrix} u_p(t)+\begin{bmatrix} 0 \\ -147 \end{bmatrix} \sin x_{p1}(t) \qquad (4.55)$$

$$y_p(t) = [\ 1\ \ 0\ ]x_p(t) \qquad (4.56)$$

The linear model to be tracked is [12]

$$\dot{x}_m(t)=\begin{bmatrix} 0 & 1 \\ -68 & -16 \end{bmatrix}x_m(t)+\begin{bmatrix} 0 \\ 68 \end{bmatrix}u_m(t) \tag{4.57}$$

$$y_m(t) = [\ 1\ \ 0\ ]x_m(t) \tag{4.58}$$

Digital simulations have been performed by choosing a square wave reference model input with a frequency of 0.2 Hz and an amplitude of 0.5 rad. The initial plant states and the gain weighting matrices T and $\bar{T}$ are

$$x_p^T(0) = [\ -0.1,\ \ 0\ ] \tag{4.59}$$

$$T = \bar{T} = \mathrm{diag}[2,\ .02,\ 2,\ 2,\ 2\ ] \tag{4.60}$$

The plant and model angular displacements and velocities are shown in Figure 4.2 and 4.3, respectively, with the tracking error vanishing asymptotically even though only the plant angular displacement $x_{p1}(t)$ is accessible.

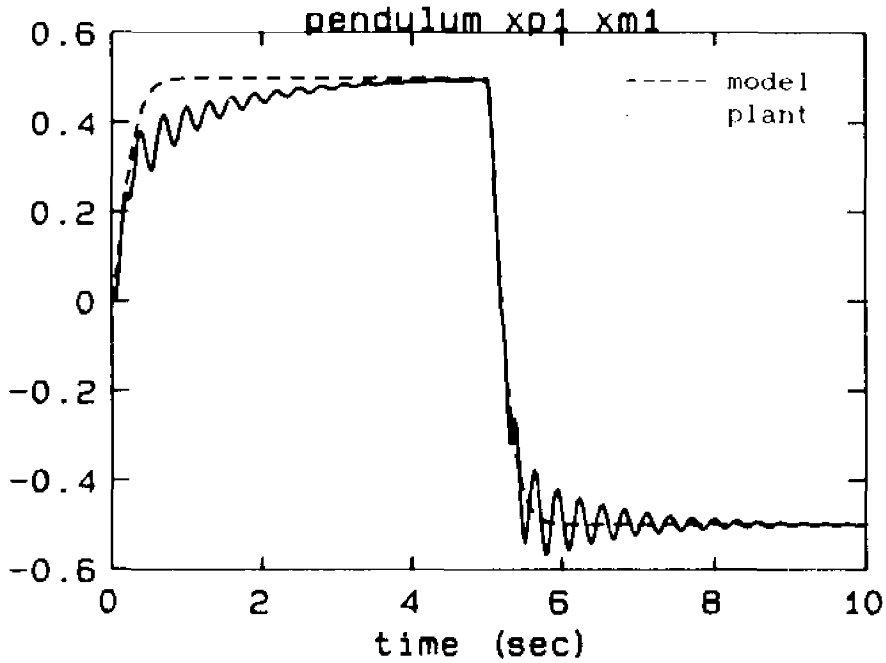

Figure 4.2 Pendulum:
Plant and Model Angular Displacements (Algorithm 6)

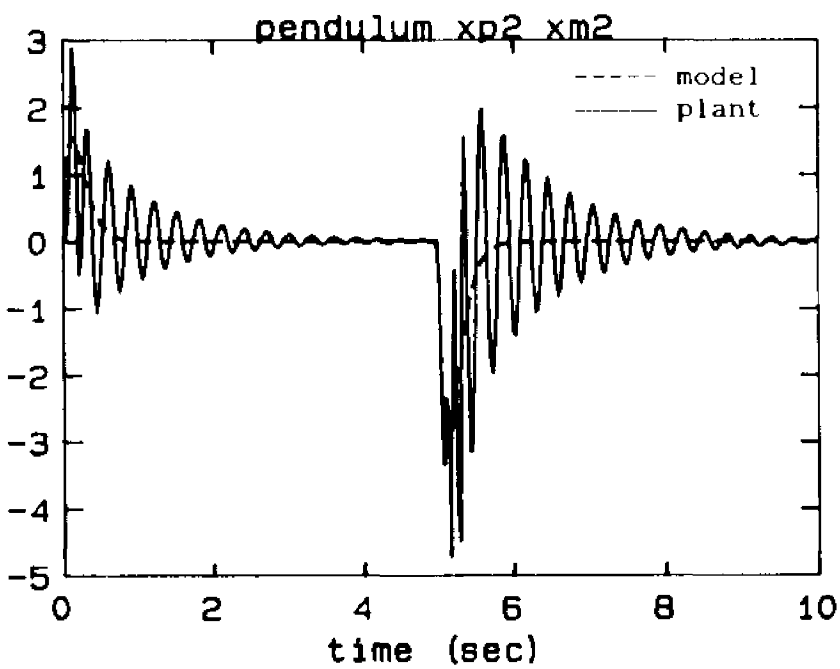

Figure 4.3 Pendulum:
Plant and Model Angular Velocities (Algorithm 6)

Ship Autopilot:

The dynamics of the ship discussed by VanAmerongen and Udink[16] are described by a third order equation as follows

$$\frac{d^3\theta(t)}{dt^3} + C\frac{d^2\theta(t)}{dt^2} + K\left[a\left(\frac{d\theta(t)}{dt}\right)^3 + b\frac{d\theta(t)}{dt} \right] = Ku_p(t) \tag{4.61}$$

where $u_p(t)$ is the rudder angle, $\theta(t)$ is the ship heading or course, $\frac{d\theta(t)}{dt}$ is the course angular velocity, C, K, a, and b are unknown parameters related to the hydrodynamic coefficients and the mass of the ship. We observe that this example cannot use Bar-Kana's approach[11] because the nonlinear term in Eq.(4.61) is not bounded.

Letting $x_{p1}(t)=\frac{d^2\theta(t)}{dt^2}$ and $x_{p2}(t)=\frac{d\theta(t)}{dt}$, we obtain

$$\dot{x}_p(t)=\begin{bmatrix} -C & -Kb \\ 1 & 0 \end{bmatrix} x_p(t)+\begin{bmatrix} K \\ 0 \end{bmatrix} u_p(t)+\begin{bmatrix} -Ka \\ 0 \end{bmatrix} x_{p2}^3(t) \tag{4.62}$$

$$y_p(t) = [\ 0 \quad 1\ ]x_p(t) \tag{4.63}$$

Assuming a=1.06, b=4, C=3.5, and K=0.05 we have

$$\dot{x}_p(t)=\begin{bmatrix} -3.5 & -0.2 \\ 1 & 0 \end{bmatrix}x_p(t)+\begin{bmatrix} 0.05 \\ 0 \end{bmatrix}u_p(t)+\begin{bmatrix} -0.053 \\ 0 \end{bmatrix}x_{p2}^3(t) \tag{4.64}$$

$$y_p(t) = [\ 0\ 1\ ]x_p(t) \tag{4.65}$$

Abida[15] controlled this plant by using virtual linearization in order to track a nonlinear reference model of the same form as in equation (4.62) except that the parameter b is different. However, in our example, we show that the output of the plant, Eq.(4.65), will asymptotically track the output of a linear reference model described by

$$\dot{x}_m(t) = \begin{bmatrix} -3 & -0.3 \\ 1 & 0 \end{bmatrix}x_m(t) + \begin{bmatrix} 0.1 \\ 0 \end{bmatrix}u_m(t) \tag{4.66}$$

$$y_m(t) = [\ 0\ 1\ ]x_m(t) \tag{4.67}$$

Since the nonlinear term in the plant dynamics does not appear in the reference model, the model following problem becomes more difficult.

Digital simulations have been performed by choosing a square wave reference model input with an amplitude of 45° and period of 20 seconds. The initial plant states are zero and the gain weighting matrices T and $\bar{T}$ are 2000I. The plant and model angular velocities are shown in Figure 4.4 where the velocity of the ship perfectly follows the output of a linear time invariant reference model. In Figure 4.5, we observe that the acceleration tracking error vanishes asymptotically even though only the plant velocity $x_{p2}(t)$ is accessible. Our simulations show better model following than the results of Abida[15] which use virtual linearization.

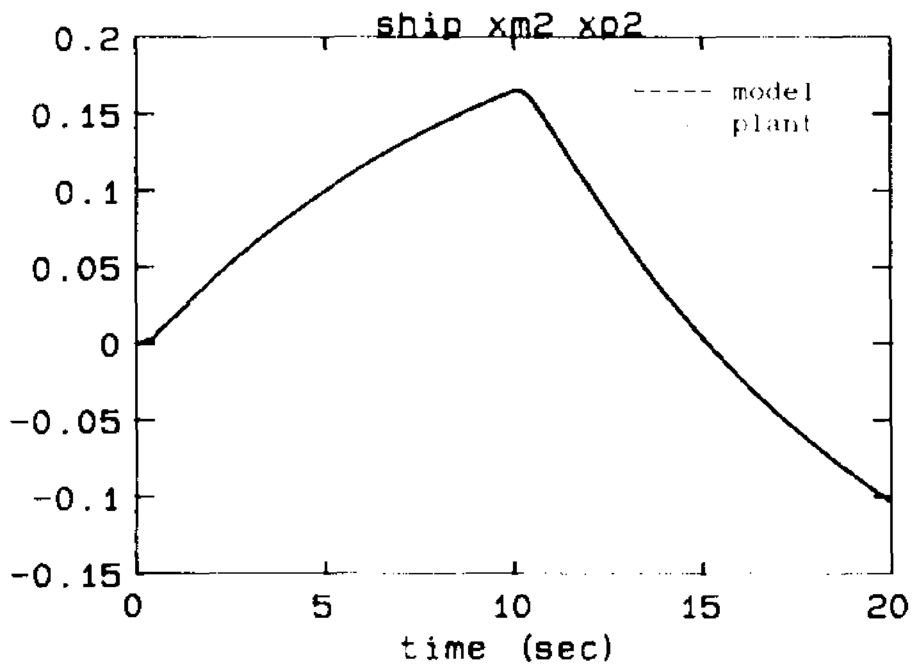

Figure 4.4 Ship Autopilot:
Plant and Model Angular Velocities (Algorithm 6)

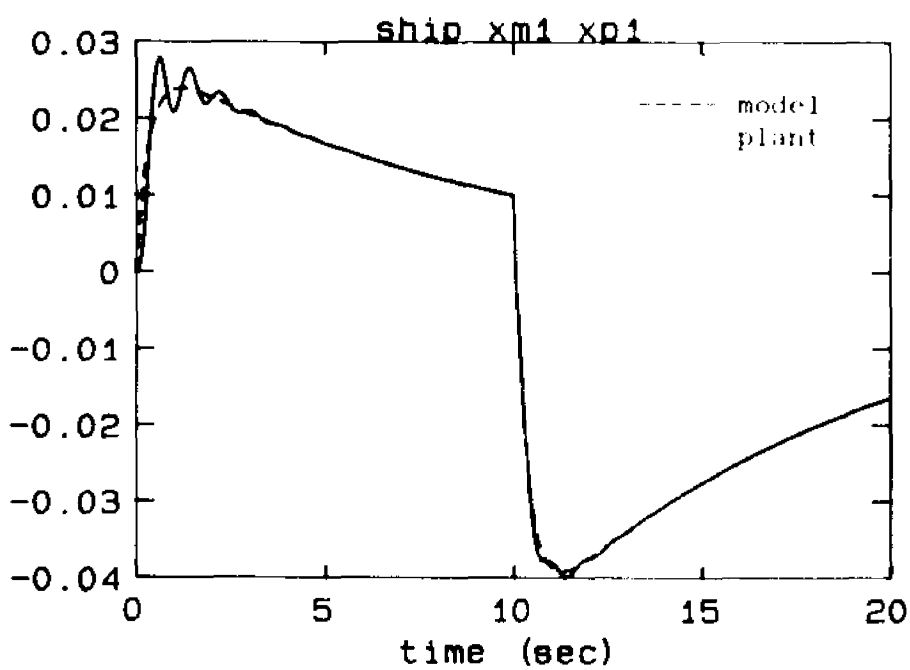

Figure 4.5 Ship Autopilot:
Plant and Model Angular Accelerations (Algorithm 6)

5. CONCLUDING REMARKS

5.1 Conclusions

This research has developed algorithms for the command generator tracker approach to model reference adaptive control of multi-input multi-output linear and nonlinear plants.

Five algorithms are presented which provide different methods for controlling a linear time invariant plant which is not almost strictly positive real. Fixed supplementary dynamics are inserted into the different locations in the adaptive loop to form algorithms 1, 2, and 3, and adaptive supplementary dynamics are used in algorithms 4 and 5 where the parameters of the supplementary dynamics are adjusted on line as part of the adaptive computation. A metasystem representation is used for system analysis and for the stability proof. A bounded output tracking error is ensured, in the presence of bounded input and output disturbances, for all controllable and observable plants while an asymptotically vanishing output tracking error will be obtained for algorithm 2, 3, and 5 with some conditions. These algorithms are especially significant because they do not need the implementation of a feedforward compensator. Therefore, the output tracking error may vanish asymptotically. Furthermore, we provide design rules for the supplementary dynamics.

For the plant with nonlinearities of known form but multiplied by unknown parameters, we propose an adaptive control algorithm without full state feedback. In contrast with previous work, neither the boundedness assumption nor the restriction on model dimension nor virtual linearization is required. Boundedness of all errors, states, and gains in the adaptive loop is guaranteed in the presence of plant input and output disturbances. Asymptotic output tracking will be achieved under some conditions.

Examples are presented using a digital computer simulation to illustrate the theoretical results.

5.2 Problems and Recommendations

Type of Model Input:

We have shown, in sections 2.11, 3.6, and 4.5, that asymptotic output tracking may be obtained for both non-ASPR LTI plants and a class of nonlinear plants, if the input and output disturbances are not present. However, the extension of Broussard's command generator tracker[4] requires that the reference model input u_m be a constant. A further study is needed to remove this restriction so that more general types of reference model inputs can be used.

Switching σ Approach:

The fixed σ-modification is used to eliminate the divergence of the integral gain in the adaptive computation in the presence of disturbances. But as a trade off, the effect of integration is lost which yields output tracking errors in steady state. To solve this problem, a switching σ-modification [17] may be considered such that the tracking error integration will be turned on (let $\sigma=0$) when the error becomes small in order to eliminate the steady state tracking errors. In other words, the scalar σ could be chosen as a function of the output error as shown below[17].

$$\sigma = \begin{cases} 0 & ; \quad \|e(t)\| < M_1 \\ \sigma_0[\|e(t)\|/M_1 - 1]; & \quad M_1 < \|e(t)\| < 2M_1 \\ \sigma_0 & ; \quad \|e(t)\| > 2M_1 \end{cases}$$

<u>Choices of the Weighting Matrices $\bar{T}$ and T</u>:

The matrices $\bar{T}$ and T are used in the adaptive control computation as part of the proportional and integral gains. Theoretically, any matrices $\bar{T}$ and T satisfying $\bar{T}\geq 0$, and $T>0$, will be acceptable for the stability requirement. However, different selections of $\bar{T}$ and T provide different model following responses, such as overshoot, rise time, setting time, etc. Therefore, an on-line adjustment of $\bar{T}$ and T may be helpful in improving the system behavior, and eliminating the need to select $\bar{T}$ and T a priori.

<u>Extension to a Larger Class of Nonlinear Plants</u>:

Although algorithm 6 achieves asymptotic model following for a class of nonlinear plants, the assumption that the linear part of the plant is ASPR will restrict the application of the algorithm. We have tried to insert the supplementary dynamics discussed in Chapters 2 and 3 into the adaptive controller of algorithm 6 in order to remove the ASPR constraint. But, when we choose the signal v(t) in the adaptive control computation as

$$v(t)=Qe_{yv}(t)+Gu_p(t)$$

where

$$u_p(t)=K_e e_{yv}(t)+K_{p\gamma}\gamma(C_p x_p,u_p,t)+K_{px}x_m(t)+K_{pu}u_m(t)$$

the nonlinear term $\gamma(C_p x_p,u_p,t)$ will show up in the Lyapunov derivative $\dot{V}_2$, Eq.(B8), which causes a problem in the stability proof. Therefore, further research is necessary to achieve asymptotic model following without the ASPR constraint on the linear part of the plant.

Furthermore, other types of nonlinear plants should be studied.

APPENDICES

A: LIST OF METASTATES AND METAMATRICES

Algorithm 1

$$x(t) = \begin{bmatrix} x_p(t) \\ x_f(t) \end{bmatrix} \qquad x^*(t) = \begin{bmatrix} x_p^*(t) \\ x_f^*(t) \end{bmatrix} \qquad x_0^*(t) = \begin{bmatrix} 0 \\ x_f^*(t) \end{bmatrix}$$

$$y(t) = \begin{bmatrix} y_p(t) \\ y_f(t) \end{bmatrix} \qquad y^*(t) = \begin{bmatrix} y_p^*(t) \\ y_f^*(t) \end{bmatrix}$$

$$u(t) = u_p(t) \qquad u^*(t) = u_p^*(t)$$

$$d_1(t) = \begin{bmatrix} E_p d_{1p}(t) \\ B_f[y_m(t)-d_{op}(t)] \end{bmatrix} \qquad d_o(t) = \begin{bmatrix} d_{op}(t) \\ 0 \end{bmatrix}$$

$$e_x(t) = \begin{bmatrix} e_{xp}(t) \\ e_{xf}(t) \end{bmatrix} \qquad e_y(t) = \begin{bmatrix} e_{yp}(t) \\ e_{yf}(t) \end{bmatrix}$$

$$e_v(t) = \begin{bmatrix} e_{xp}(t) \\ -x_f(t) \end{bmatrix} \qquad e_{yv}(t) = \begin{bmatrix} e_{yp}(t) \\ -y_f(t) \end{bmatrix}$$

$$A = \begin{bmatrix} A_p & 0 \\ -B_f C_p & A_f \end{bmatrix} \qquad B = \begin{bmatrix} B_p \\ 0 \end{bmatrix} \qquad C = \begin{bmatrix} C_p & 0 \\ 0 & C_f \end{bmatrix}$$

$$Q = [\ Q_{pp}\ Q_{pf}\ ] \qquad \Psi = I$$

$$K(t) = [\ K_e(t) \quad K_{px}(t) \quad K_{pu}(t)\ ]$$

$$K_e(t) = [\ K_{pe}(t) \quad K_{pf}(t)\ ]$$

$$K^I(0) = [\ K^I_{pe}(0) \quad K^I_{pf}(0) \quad K^I_{px}(0) \quad K^I_{pu}(0)\ ]$$

$$\tilde{K} = [\ \tilde{K}_e \quad \tilde{K}_{px} \quad \tilde{K}_{pu}\ ]$$

$$\tilde{K}_e = [\ \tilde{K}_{pe} \quad \tilde{K}_{pf}\ ]$$

$$r(t) = \begin{bmatrix} e_{yv}(t) \\ x_m(t) \\ u_m(t) \end{bmatrix}$$

Algorithm 2

$$x(t) = \begin{bmatrix} x_p(t) \\ x_f(t) \end{bmatrix} \qquad x^*(t) = \begin{bmatrix} x_p^*(t) \\ x_f^*(t) \end{bmatrix} \qquad x_0^*(t) = \begin{bmatrix} 0 \\ x_f^*(t) \end{bmatrix}$$

$$y(t) = \begin{bmatrix} y_p(t) \\ y_f(t) \end{bmatrix} \qquad y^*(t) = \begin{bmatrix} y_p^*(t) \\ y_f^*(t) \end{bmatrix}$$

$$u(t)=u_p(t)=u_f(t) \qquad u^*(t)=u_p^*(t)=u_f^*(t)$$

$$d_i(t) = \begin{bmatrix} E_p d_{ip}(t) \\ 0 \end{bmatrix} \qquad d_o(t) = \begin{bmatrix} d_{op}(t) \\ 0 \end{bmatrix}$$

$$e_x(t) = \begin{bmatrix} e_{xp}(t) \\ e_{xf}(t) \end{bmatrix} \qquad e_y(t) = \begin{bmatrix} e_{yp}(t) \\ e_{yf}(t) \end{bmatrix}$$

$$e_v(t) = \begin{bmatrix} e_{xp}(t) \\ -x_f(t) \end{bmatrix} \qquad e_{yv}(t) = \begin{bmatrix} e_{yp}(t) \\ -y_f(t) \end{bmatrix}$$

$$A = \begin{bmatrix} A_p & 0 \\ 0 & A_f \end{bmatrix} \qquad B = \begin{bmatrix} B_p \\ B_f \end{bmatrix} \qquad C = \begin{bmatrix} C_p & 0 \\ 0 & C_f \end{bmatrix}$$

$$Q = [\ Q_{pp} \quad Q_{pf}\ ] \qquad \Psi = I$$

$$K(t) = [\ K_e(t) \quad K_{px}(t) \quad K_{pu}(t)\ ]$$

$$K_e(t) = [\ K_{pe}(t) \quad K_{pf}(t)\ ]$$

$$K^I(0) = [\ K^I_{pe}(0) \quad K^I_{pf}(0) \quad K^I_{px}(0) \quad K^I_{pu}(0)\ ]$$

$$\tilde{K} = [\ \tilde{K}_e \quad \tilde{K}_{px} \quad \tilde{K}_{pu}\ ]$$

$$\tilde{K}_e = [\ \tilde{K}_{pe} \quad \tilde{K}_{pf}\ ]$$

$$r(t) = \begin{bmatrix} e_{yv}(t) \\ x_m(t) \\ u_m(t) \end{bmatrix}$$

Algorithm 2 (special case for Bar-Kana's algorithm)

$$x(t) = \begin{bmatrix} x_p(t) \\ x_f(t) \end{bmatrix} \qquad x^*(t) = \begin{bmatrix} x_p^*(t) \\ x_f^*(t) \end{bmatrix} \qquad x_0^*(t) = \begin{bmatrix} 0 \\ x_f^*(t) \end{bmatrix}$$

$$y(t) = \begin{bmatrix} y_p(t) \\ y_f(t) \end{bmatrix} \qquad y^*(t) = \begin{bmatrix} y_p^*(t) \\ y_f^*(t) \end{bmatrix}$$

$$u(t)=u_p(t)=u_f(t) \qquad u^*(t)=u_p^*(t)=u_f^*(t)$$

$$d_i(t) = \begin{bmatrix} E_p d_{ip}(t) \\ 0 \end{bmatrix} \qquad d_o(t) = \begin{bmatrix} d_{op}(t) \\ 0 \end{bmatrix}$$

$$e_x(t) = \begin{bmatrix} e_{xp}(t) \\ e_{xf}(t) \end{bmatrix} \qquad e_y(t) = \begin{bmatrix} e_{yp}(t) \\ e_{yf}(t) \end{bmatrix}$$

$$e_v(t) = \begin{bmatrix} e_{xp}(t) \\ -x_f(t) \end{bmatrix} \qquad e_{yv}(t) = \begin{bmatrix} e_{yp}(t) \\ -y_f(t) \end{bmatrix}$$

$$A = \begin{bmatrix} A_p & 0 \\ 0 & A_f \end{bmatrix} \qquad B = \begin{bmatrix} B_p \\ B_f \end{bmatrix} \qquad C = \begin{bmatrix} C_p & 0 \\ 0 & C_f \end{bmatrix}$$

$$Q_p = Q_f = I \qquad G = 0$$

$$\Psi = \begin{bmatrix} 0.5I & & & \\ & 0.5I & & \\ & & I & \\ & & & I \end{bmatrix}$$

$$T = \begin{bmatrix} T_{11} & T_{11} & & \\ T_{11} & T_{11} & & \\ & & T_{33} & \\ & & & T_{44} \end{bmatrix}$$

$$\bar{T} = \begin{bmatrix} \bar{T}_{11} & \bar{T}_{11} & & \\ \bar{T}_{11} & \bar{T}_{11} & & \\ & & \bar{T}_{33} & \\ & & & \bar{T}_{44} \end{bmatrix}$$

$$K(t) = [\ K_e(t) \quad K_{px}(t) \quad K_{pu}(t)\ ]$$

$$K_e(t) = [\ K_{pe}(t) \quad K_{pe}(t)\ ]$$

$$K^I(0) = [\ K^I_{pe}(0) \quad K^I_{pf}(0) \quad K^I_{px}(0) \quad K^I_{pu}(0)\ ]$$

$$\tilde{K} = [\ \tilde{K}_e \quad \tilde{K}_{px} \quad \tilde{K}_{pu}\ ]$$

$$\tilde{K}_e = [\ \tilde{K}_{pe} \quad \tilde{K}_{pe}\ ]$$

$$r(t) = \begin{bmatrix} e_{yv}(t) \\ x_m(t) \\ u_m(t) \end{bmatrix}$$

Algorithm 3

$$x(t) = \begin{bmatrix} x_p(t) \\ x_f(t) \end{bmatrix} \qquad x^*(t) = \begin{bmatrix} x_p^*(t) \\ x_f^*(t) \end{bmatrix} \qquad x_0^*(t) = \begin{bmatrix} 0 \\ x_f^*(t) \end{bmatrix}$$

$$y(t) = \begin{bmatrix} y_p(t) \\ y_f(t) \end{bmatrix} \qquad y^*(t) = \begin{bmatrix} y_p^*(t) \\ y_f^*(t) \end{bmatrix}$$

$$u(t) = u_f(t) \qquad u^*(t) = u_f^*(t)$$

$$d_i(t) = \begin{bmatrix} E_p d_{ip}(t) \\ 0 \end{bmatrix} \qquad d_o(t) = \begin{bmatrix} d_{op}(t) \\ 0 \end{bmatrix}$$

$$e_x(t) = \begin{bmatrix} e_{xp}(t) \\ e_{xf}(t) \end{bmatrix} \qquad e_y(t) = \begin{bmatrix} e_{yp}(t) \\ e_{yf}(t) \end{bmatrix}$$

$$e_v(t) = \begin{bmatrix} e_{xp}(t) \\ -x_f(t) \end{bmatrix} \qquad e_{yv}(t) = \begin{bmatrix} e_{yp}(t) \\ -y_f(t) \end{bmatrix}$$

$$A = \begin{bmatrix} A_p & 0 \\ 0 & A_f \end{bmatrix} \qquad B = \begin{bmatrix} B_p D_f \\ B_f \end{bmatrix} \qquad C = \begin{bmatrix} C_p & 0 \\ 0 & C_f \end{bmatrix}$$

$$Q = [\; Q_p \quad Q_f \;] \qquad\qquad \Psi = I$$

$$K(t) = [\; K_e(t) \quad K_{px}(t) \quad K_{pu}(t) \;]$$

$$K_e(t) = [\; K_{pe}(t) \quad K_{pf}(t) \;]$$

$$K^I(0) = [\; K^I_{pe}(0) \quad K^I_{pf}(0) \quad K^I_{px}(0) \quad K^I_{pu}(0) \;]$$

$$\tilde{K} = [\; \tilde{K}_e \quad \tilde{K}_{px} \quad \tilde{K}_{pu} \;]$$

$$\tilde{K}_e = [\; \tilde{K}_{pe} \quad \tilde{K}_{pf} \;]$$

$$r(t) = \begin{bmatrix} e_{yv}(t) \\ x_m(t) \\ u_m(t) \end{bmatrix}$$

Algorithm 4

$$x(t) = \begin{bmatrix} x_p(t) \\ x_f(t) \end{bmatrix} \qquad x^*(t) = \begin{bmatrix} x_p^*(t) \\ x_f^*(t) \end{bmatrix} \qquad x_0^*(t) = \begin{bmatrix} 0 \\ x_f^*(t) \end{bmatrix}$$

$$y(t) = \begin{bmatrix} y_p(t) \\ x_f(t) \end{bmatrix} \qquad y^*(t) = \begin{bmatrix} y_p^*(t) \\ x_f^*(t) \end{bmatrix}$$

$$u(t) = \begin{bmatrix} u_p(t) \\ \dot{x}_f(t) \end{bmatrix} \qquad u^*(t) = \begin{bmatrix} u_p^*(t) \\ \dot{x}_f^*(t) \end{bmatrix}$$

$$d_i(t) = \begin{bmatrix} E_p d_{ip}(t) \\ 0 \end{bmatrix} \qquad d_o(t) = \begin{bmatrix} d_{op}(t) \\ 0 \end{bmatrix}$$

$$e_x(t) = \begin{bmatrix} e_{xp}(t) \\ e_{xf}(t) \end{bmatrix} \qquad e_y(t) = \begin{bmatrix} e_{yp}(t) \\ e_{xf}(t) \end{bmatrix}$$

$$e_v(t) = \begin{bmatrix} e_{xp}(t) \\ -x_f(t) \end{bmatrix} \qquad e_{yv}(t) = \begin{bmatrix} e_{yp}(t) \\ -x_f(t) \end{bmatrix}$$

$$A = \begin{bmatrix} A_p & 0 \\ 0 & 0 \end{bmatrix} \qquad B = \begin{bmatrix} B_f & 0 \\ 0 & I \end{bmatrix} \qquad C = \begin{bmatrix} C_p & 0 \\ 0 & I \end{bmatrix}$$

$$Q = \begin{bmatrix} Q_{pp} & Q_{pf} \\ Q_{fp} & Q_{ff} \end{bmatrix} \qquad G = \begin{bmatrix} G_{pp} & G_{pf} \\ G_{fp} & G_{ff} \end{bmatrix}$$

$$K(t) = [\ K_e(t) \quad K_x(t) \quad K_u(t)\ ]$$

$$K_e(t) = \begin{bmatrix} D_f(t) & C_f(t) \\ B_f(t) & A_f(t) \end{bmatrix}$$

$$K_x(t) = \begin{bmatrix} K_{px}(t) \\ K_{fx}(t) \end{bmatrix} \qquad K_u(t) = \begin{bmatrix} K_{pu}(t) \\ K_{fu}(t) \end{bmatrix}$$

$$K^I(0) = \begin{bmatrix} D_f^I(0) & C_f^I(0) & K_{px}^I(0) & K_{pu}^I(0) \\ B_f^I(0) & A_f^I(0) & K_{fx}^I(0) & K_{fu}^I(0) \end{bmatrix}$$

$$\tilde{K} = [\ \tilde{K}_e \quad \tilde{K}_{px} \quad \tilde{K}_{pu}\ ]$$

$$\tilde{K}_e = \begin{bmatrix} \tilde{D}_f & \tilde{C}_f \\ \tilde{B}_f & \tilde{A}_f \end{bmatrix}$$

$$\tilde{K}_x = \begin{bmatrix} \tilde{K}_{px} \\ \tilde{K}_{fx} \end{bmatrix} \qquad \tilde{K}_u = \begin{bmatrix} \tilde{K}_{pu} \\ \tilde{K}_{fu} \end{bmatrix}$$

$$r(t) = \begin{bmatrix} e_{yv}(t) \\ x_m(t) \\ u_m(t) \end{bmatrix} \qquad \Psi = I$$

Algorithm 5

$$x(t) = \begin{bmatrix} x_p(t) \\ x_f(t) \end{bmatrix} \qquad x^*(t) = \begin{bmatrix} x_p^*(t) \\ x_f^*(t) \end{bmatrix} \qquad x_0^*(t) = \begin{bmatrix} 0 \\ x_f^*(t) \end{bmatrix}$$

$$y(t) = \begin{bmatrix} y_p(t) \\ x_f(t) \end{bmatrix} \qquad y^*(t) = \begin{bmatrix} y_p^*(t) \\ x_f^*(t) \end{bmatrix}$$

$$d_i(t) = \begin{bmatrix} E_p d_{ip}(t) \\ 0 \end{bmatrix} \qquad d_o(t) = \begin{bmatrix} d_{op}(t) \\ 0 \end{bmatrix}$$

$$e_x(t) = \begin{bmatrix} e_{xp}(t) \\ e_{xf}(t) \end{bmatrix} \qquad e_y(t) = \begin{bmatrix} e_{yp}(t) \\ e_{xf}(t) \end{bmatrix}$$

$$e_v(t) = \begin{bmatrix} e_{xp}(t) \\ -x_f(t) \end{bmatrix} \qquad e_{yv}(t) = \begin{bmatrix} e_{yp}(t) \\ -x_f(t) \end{bmatrix}$$

$$A = \begin{bmatrix} A_p & -B_p C_f \\ 0 & -A_{f2} \end{bmatrix} \qquad A_0 = \begin{bmatrix} B_p D_f C_p & 0 \\ 0 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 \\ I \end{bmatrix} \qquad C = \begin{bmatrix} C_p & 0 \\ 0 & I \end{bmatrix}$$

$$Q = [\ Q_p \quad Q_f\ ] \qquad \Psi = I$$

$$K(t) = [\ K_e(t) \quad K_p(t) \quad K_p(t)\ ]$$

$$K_e(t) = [B_{f1}(t) \quad A_{f1}(t)]; \quad K_x(t)=B_{f2}(t); \quad K_u(t)=B_{f3}(t)$$

$$K^I(0) = [\ B^I_{f1}(0) \quad A^I_{f1}(0) \quad B^I_{f2}(0) \quad B^I_{f3}(0)\ ]$$

$$\tilde{K} = [\ \tilde{K}_e \quad \tilde{K}_x \quad \tilde{K}_u\ ]$$

$$\tilde{K}_e(t) = [\tilde{B}_{f1}(t) \quad \tilde{A}_{f1}(t)]; \ \tilde{K}_x(t)=\tilde{B}_{f2}(t); \quad \tilde{K}_u(t)=\tilde{B}_{f3}(t)$$

$$r(t) = \begin{bmatrix} e_{yv}(t) \\ x_m(t) \\ u_m(t) \end{bmatrix}$$

B: DERIVATION OF THE LYAPUNOV DERIVATIVE

Lyapunov Function Candidate:

Let $V(e_x,K^I) = V_1(e_x,K^I)+V_2(e_x,K^I)$ (B1)

where $V_1(e_x(t))=e_x^T(t)Pe_x(t)$ (B2)

and $V_2(e_x,K^I)=tr[(K^I(t)-\tilde{K})T^{-1}(K^I(t)-\tilde{K})^T]$ (B3)

and where P and T are positive definite symmetric matrices such that the Lyapunov function candidate $V(e_x,K^I)$ is positive definite.

Stability Proof

We suppress the dependence on time for convenience. Then

$$\dot{V}_1 = \dot{e}_x^T Pe_x + e_x^T P\dot{e}_x \tag{B4}$$

substituting $\dot{e}_x$ from Eq.(2.56) yields

$$\dot{V}_1 = e_c^T(PA_c+A_c^TP)e_x - z^TB^TPe_x - e_x^TPBz - F_1^TPe_x - e_x^TPF_1 \tag{B5}$$

substitute Eq.(2.76) into Eq.(B5) to obtain

$$\dot{V}_1 = -e_x^TRe_x-e_x^TLL^Te_x - 2e_x^TPBz - 2e_x^TPF_1 \tag{B6}$$

add and subtract $-2e_x^TLWz + z^TW^TWz$ in Eq.(B6) to obtain

$$\dot{V}_1=-e_x^TRe_x-(Le_x^T-Wz)^T(Le_x^T-Wz)+2e_x^T(-LW-PB)z+z^TW^TWz-2e_x^TPF_1 \tag{B7}$$

Next, use Eq.(B3) to obtain

$$\dot{V}_2= tr[\dot{K}^IT^{-1}(K^I-\tilde{K})^T] + tr[(K^I-\tilde{K})T^{-1}(\dot{K}^I)^T] \tag{B8}$$

using $\dot{K}^I$ from Eq.(2.39) into Eq.(B8) to have

$$\dot{V}_2 = tr[(vr^T - \sigma K^I \Psi)(K^I - \tilde{K})^T] tr[(K^I - \tilde{K})(vr^T - \sigma K^I \Psi)^T)] \qquad (B9)$$

Substitute $K^I = K - K^P = K - vr^T\bar{T}$ into Eq.(B9) and use the definition $z = (K - \tilde{K})r$ to obtain

$$\dot{V}_2 = tr[vz^T] + tr[zv^T] - 2v^T vr^T \bar{T} r - 2\sigma tr[(K^I - \tilde{K})\Psi(K^I - \tilde{K})^T]$$

$$-2\sigma tr[(K^I - \tilde{K})\Psi \tilde{K}^T] \qquad (B10)$$

substitute $v = Qe_{yv} + GKr = QCe_v - Qd_o + GKr$ in Eq.(B10) to obtain

$$\dot{V}_2 = tr[(QCe_v - Qd_o + GKr)z^T] + tr[z(QCe_v - Qd_o + GKr)^T]$$

$$-2v^T vr^T \bar{T} r - 2\sigma tr[(K^I - \tilde{K})\Psi(K^I - \tilde{K})^T] - 2\sigma tr[(K^I - \tilde{K})\Psi \tilde{K}^T]$$

$$= 2e_v^T C^T Q^T z + tr[G(Kr - \tilde{K}r + \tilde{K}r)z^T] + tr[z(Kr - \tilde{K}r + \tilde{K}r)^T G^T] - 2d_o^T Q^T z$$

$$-2v^T v r^T \bar{T} r - 2\sigma tr[(K^I-\tilde{K})\Psi(K^I-\tilde{K})^T\tilde{K}^T] - 2\sigma tr[(K^I-\tilde{K})\Psi\tilde{K}^T]$$

$$= 2e_v^T C^T Q^T z + z^T(G+G^T)z + 2z^T G(\tilde{K}_e C e_v - \tilde{K}_e d_o + \tilde{K}_x x_m + \tilde{K}_u u_m) - 2d_o^T Q^T z$$

$$-2v^T v r^T \bar{T} r - 2\sigma tr[(K^I-\tilde{K})\Psi(K^I-\tilde{K})^T\tilde{K}^T] - 2\sigma tr[(K^I-\tilde{K})\Psi\tilde{K}^T]$$

$$= 2e_v^T C^T (Q^T + \tilde{K}_e^T G^T) z + z^T(G+G^T)z + 2z^T G(-\tilde{K}_e d_o + \tilde{K}_x x_m + \tilde{K}_u u_m) - 2d_o^T Q^T z$$

$$-2v^T v r^T \bar{T} r - 2\sigma tr[(K^I-\tilde{K})\Psi(K^I-\tilde{K})^T\tilde{K}^T] - 2\sigma tr[(K^I-\tilde{K})\Psi\tilde{K}^T] \tag{B11}$$

substitute Eq.(2.55) into Eq.(B11) to obtain

$$\dot{V}_2 = 2(e_x^T - x_0^{*T})C^T(Q^T + \tilde{K}_e^T G^T)z + z^T(G+G^T)z + 2z^T G(\tilde{K}_x x_m + \tilde{K}_u u_m)$$

$$-2z^T(Q+G\tilde{K}_e)d_o - 2\sigma tr[(K^T-\tilde{K})\Psi(K^I-\tilde{K})^T] - 2\sigma tr[(K^I-\tilde{K})\Psi\tilde{K}^T]$$

$$-2v^T v r^T \bar{T} r$$

$$= 2e_x^T C^T (Q^T + \tilde{K}_e^T G^T) z + z^T(G+G^T)z - 2v^T v r^T \bar{T} r - 2\sigma tr[(K^T-\tilde{K})\Psi(K^I-\tilde{K})^T]$$

$$-2\sigma tr[(K^I-\tilde{K})\Psi\tilde{K}^T] + 2z^T[G(\tilde{K}_x x_m + \tilde{K}_u u_m) - (Q+G\tilde{K}_e)d_o - (Q+G\tilde{K}_e)C x_0^*]$$

$$= 2e_x^T C^T (Q^T + \tilde{K}_e^T G^T) z + z^T(G+G^T)z - 2v^T v r^T \bar{T} r - 2\sigma tr[(K^T-\tilde{K})\Psi(K^I-\tilde{K})^T]$$

$$-2\sigma tr[(K^I-\tilde{K})\Psi\tilde{K}^T]+2z^TF_2 \tag{B12}$$

$$\text{where } F_2=G(\tilde{K}_x x_m+\tilde{K}_u u_m)-(Q+G\tilde{K}_e)d_o-(Q+G\tilde{K}_e)Cx_0^* \tag{B13}$$

combine Eqs.(B7) and (B12), to obtain

$$\begin{aligned}\dot{V} = &-e_x^TRe_x-(L^Te_x-Wz)^T(L^Te_x-Wz) + 2e_x^T[-LW-PB+C^T(Q^T+\tilde{K}_eG^T)]z \\ &+z^T(W^TW+G+G^T)z-2v^Tvr^T\bar{T}r - 2\sigma tr[(K^T-\tilde{K})\Psi(K^I-\tilde{K})^T] \\ &-2\sigma tr[(K^I-\tilde{K})\Psi\tilde{K}^T] - 2e_x^TPF_1+2z^TF_2\end{aligned} \tag{B14}$$

Finally, substitute Eqs.(2.77) and (2.78) into Eq.(B14) to obtain

$$\begin{aligned}\dot{V}(e_x,K^I) = &-e_x^T(t)Re_x(t)-[L^Te_x(t)-Wz(t)]^T[L^Te_x(t)-Wz(t)] \\ &+z(t)^T(J+J^T+G+G^T)z(t) - 2v^T(t)v(t)r^T(t)\bar{T}r(t)\end{aligned}$$

$$-2\sigma \mathrm{tr}[(K^{I}(t)-\tilde{K})\Psi(K^{I}(t)-\tilde{K})^{T}]$$

$$-2\sigma \mathrm{tr}[(K^{I}(t)-\tilde{K})\Psi\tilde{K}^{T}]-2e_{x}^{T}(t)PF_{1}-2z^{T}(t)F_{2}(t) \qquad \text{(B15)}$$

C: THE LIMITING DERIVATIVE OF THE LYAPUNOV FUNCTION

This is an extension of LaSalle's Invariance Set Principle [14], for nonautonomous differential equations when $\dot{V}(t)$ is not necessarily negative semi-definite.

Let

$$\dot{x} = f(x,t) \tag{C1}$$

be a general nonlinear nonautonomous differential equation and assume that

$$\left| \int_{\alpha}^{\beta} f(x,t)\, d\tau \right| < \mu(\beta-\alpha) \tag{C2}$$

where the function $\mu(t)$ is a modulus of continuity for the integral and $\mu(\beta-\alpha)$ is bounded for any finite interval $\beta-\alpha$ (where α and β are the limits of the integral).

Let $V(x)$ be a differentiable function, bounded from below.

Assume that the derivative $\dot{V}(x,t)$ "along the trajectories" of Eq. (C1) is

$$\dot{V}(x,t) = W_1(x,t) + W_2(x,t) \tag{C3}$$

such that

$$W_1(x,t) \le \tilde{W}_1(x) \le 0 \tag{C4}$$

where $\tilde{W}_1(x)$ is a continuous function of x and where $W_2(x,t)$ is a continuous function of x and piecewise continuous in t satisfying

$$\lim_{t\to\infty} W_2(x,t) = 0 \tag{C5}$$

Thus $V(x,t) \to W_1(x,t)$ as $t \to \infty$ and we call $W_1(x,t)$ the "limiting derivative of the Lyapunov function."

<u>Theorem</u> <u>C</u>: [14]

Under the assumptions, Eqs.(C1)-(C5), all bounded solutions of Eq.(C1) approach asymptotically the set

$$\Omega \triangleq \{\, x \mid W_1(x) \equiv 0\} \tag{C7}$$

The reader is referred to Appendix B of reference 14 for the proof of Theorem C.1.

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