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Essays on futures trading and price volatility

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ESSAYS ON FUTURES TRADING AND PRICE VOLATILITY

by

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Abstract

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One of the economic tasks of futures markets is reallocation of (spot price) risk from agents who do not wish to take them to others who are willing to do so. Consequently, if, a model which is based on rational optimizing agents and simultaneous determination of spot and futures prices arrives at a conclusion which suggests that, say, futures speculation increases spot price volatility, then, not only the mainstream Pareto improvement claim becomes ambiguous, but even the existence of futures markets can be thought to be defeating their purpose. Some regulators, professional investors and academicians voiced a similar opinion blaming computerized futures trading for the crash in 1987. The mainstream financial theory, on the other hand, supports the contrary position, namely, that futures trading decreases spot price volatility due to reductions of the price-stability disturbing effects due to demand shocks. Even though, financial theory supports a view which is being debated by a significant amount of professional traders and academicians, so far there has been no

satisfactory empirical academic study aiming at resolving this controversy. Hence, the first of the two essays in this study addresses to this problem.

Empirical observations about volatility-adjusted futures price movements exhibit significant differences in their average magnitudes depending on the type of commodity they are written on. The answer provided to the question of why we observe industry-specific differences among the traded futures contracts by the traders as well as by academicians is that there are different in nature. Despite of the fact that this answer is correct in essence, one realizes that financial theory is still lacking a comprehensive and consistent theoretical model with the help of which those differences can be explained. Hence in order to provide a theoretical framework for the observed differences and their causes a simple model is developed in the second essay and then its conclusions are tested with empirical data.

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I wish I could thank everyone in person who has been helpful to me during the research and writing process of this dissertation. However, due the limitations in space this seems to be an impossible task.

Yet, I think it would be unjust if I do not mention some names who have been most supportive to me throughout the Ph.D. program: I would like to express my deepest gratitude to Professors Ronald W. Anderson, Michael Grossman and Franklin R. Edwards, without whose help and encouragement this dissertation could not have materialized.

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I: INTRODUCTION

"As near as I can learn, and from the best information I have been able to obtain on the Chicago Board of Trade, at least 95% of the sales of that Board are of this fictitious character, where no property is actually owned, no property sold or delivered, or expected to be delivered but simply wagers or bets as to what that property may be worth at a designated time in the future....Wheat and cotton have become as much gambling tools as chips on the farobank table. The property of the wheat grower and the cotton grower is treated as though it were a "stake" put on the gambling table at Monte Carlo. The producer of wheat is compelled to see the stocks in his barn dealt with like the peas of a thimblerigger, or the cards of a three-card-monte man. Between the grain-producer and loaf eater, there has stepped in a "parasite" between them robbing them both." [1]

The paragraph quoted above reflects the thoughts of a 19th Century Senator about Futures Markets. It is a known fact, that despite the difference of 101 years, even many contemporaries of ours do not have a much better understanding of the topic than Senator Washburn. Hence, our aim in this introduction is to familiarize the readers with the fundamentals (and the jargon) of futures trading.

Futures, forward contracts and options are similar financial

[1] Senator William D. Washburn (D-Minn.) before Congress, July 11, 1892

(derivative[2]) instruments in that they all specify purchase or sale of some underlying security at some future date. The difference between the former two and the latter one is that the holder of an option to buy does not have to and will not do so if it is to his/her disadvantage. The holder of a futures/forward contract, on the other hand, is obliged to go through the agreed-on transaction. The sole difference between forward and futures trading is that futures markets have replaced informal (but yet very flexible) forward contracts with highly standardized securities which are traded in a formally organized exchange: In short, futures markets formalize and standardize[3] forward contracting.

One could define a futures contract as an agreement between a seller and a buyer that calls for the seller (called the short) to deliver to the buyer (called the long) a specified quantity and grade of an identified commodity, at a fixed time in the future, and at a price agreed to when the contract be bought and sold on designated contract markets, known as commodity or futures exchanges.

In the mainstream literature, it is widely accepted and stated that futures trading serves two economic objectives:

---

[2] A primary security offers returns based on only on the status of the issuer. In contrast, derivative securities yield returns that depend on additional factors pertaining to the prices of other assets.

[3] In terms of contract size, the acceptable grade of commodity, contract delivery dates, etc.

(i) price discovery

The economic benefits of having accurate price predictions are well-known. More accurate predictions of prices results in superior allocation of resources, and thus, in a Pareto-superior solution.

(ii) risk management (via hedging)

Hedgers use futures to shift unwanted price risk to speculators, who willingly assume the risk in the hope of making profits. It is obvious that the absence of futures markets would represent a Pareto-inferior situation, since the cost-of-risk, in that case would have to be borne not only by those who are ready and willing to do so.[4]

A major bulk of literature about futures markets is concerned with issues related to the price discovery function of futures contracts. Researchers were interested whether or not futures prices could indeed be used to accurately predict future spot prices (For an example see: Lars Peter Hansen and Robert J. Hodrick, 1980), or if that is not the case, whether the futures price should be above or below the expected

[4] Provided that futures trading will not effect the functioning of the rest of the economy the reasoning above is straightforward. However, it is common sense that, given limited resources the existence of any additional security will have some effects on other already-existing ones. Naturally, those effects could be either favorable or unfavorable. We should note that if the existence of futures markets can be shown to have some negative effects on the functioning of others, then the statement which claims that inclusion of futures markets shall lead to a socially beneficial solution (in the sense of Pareto Optimality) becomes ambiguous.

future spot price, and for what reason should that be the case. The latter issue is usually addressed by the means of two closely related concrete questions, namely:

(1) The question about who is winning/losing in Futures Markets. (i.e. speculators or hedgers)

(2) Whether or not speculators obtain a risk premium from hedgers?

One should note that the existence of a risk premium creates a downward bias in futures prices. In other words, futures price will be always below the expected terminal spot price. Hence, it is clear how the second question is linked to the relationship between spot and futures prices. Furthermore, it is known that the revenue of a speculator stems from two sources, forecasting ability and compensation for risk-taking (if there is any). Viewed from this perspective, calculation of actual revenues/losses of speculators might shed a light on whether or not there is a risk premium.[5] Obviously, since, as we mentioned before, speculators' income has two components. If the result is that speculators are making a loss, on the average, that does not mean, per se that there is no positive risk premium paid to the speculators: It could be the case that speculators might have inferior information sources (if we assume asymmetric information between hedgers and speculators) and/or worse forecasting abilities than hedgers. Note that

[5] This is the methodology of Hartzmark in his 1991, Journal of Business article.

even under the presumption of symmetric information one may still defend the existence of a positive risk premium by suggesting that speculators, on average, may be using inferior forecasting techniques than hedgers. In sum, the academic debate about the existence of a positive risk premium, and thus a downward bias in futures prices still needs further evidence to reach a satisfactory answer.

Examples for studies based on the above-mentioned issues are plentiful. As a brief list of main contributions one can mention the following: Anderson and Danthine (1981), Cootner (1960), Dusak (1973), Fort and Quirk (1988), Hartzmark (1987), Hartzmark (1991), Houthhakker (1968), Keynes (1930), Maddala and Yoo (1991), Rockwell (1967), and finally, Telser (1958).

However, eventhough there is voluminous research on these topics the answers provided are not unanimous. Namely, none of the two rival points of view seems to seize to exist. On the one hand, there are the followers of the "Normal Backwardation Theory" who argue that speculators ask for a compensation (=premium) to take the risk involved in futures trading. Anderson and Danthine, Cootner, Houthhakker and Keynes can be named as the main supporters of this position. On the other hand, there are alternative theories which postulate that the supply of speculative services is horizontal at a zero return, thus the risk premium, according to this point of view, will be bid to zero and the returns for bearing risk will disappear. The main defenders of this line of thought are Hartzmark, Rockwell and, finally, Telser.

Studies regarding the risk management issues usually concentrate on the question of "optimal hedging". The optimal hedging literature should not be pictured as a narrow one, either: It spans an entire area between the mere calculation of the optimal futures position for a given portfolio in a prtial equilibrium setting, to a question such as against what kind of factors can be hedged with the help of futures contracts. The more recent studies in this line of research are concentrating on dynamic portfolios and updating of the existing ones in a multiperiod setting. To name a few papers in this area: Anderson and Danthine (1981), Danthine (1978), Ederington, L. (1979), Holthausen (1979), Feder, Just and Schmitz (1980), Kahl, K. and Miller, S. (1989), Peters E. (1986) and Rolfo (1980).

An important issue which can be categorized under both "price discovery" as well as "risk management" headings is the simultaneous determination of spot and futures prices. Since the shapes of the relevant supply and inventory demand functions depend on the price stability, and since that varies with the introduction of futures markets, one realizes the importance of simultaneous determination of the two prices. Main articles in this area can be outlines as: Driskill, R. and McCafferty, S. (1982), Richard, S. and Sundaresan, M. (1981), Stein (1960), Stein (1980), Turnovsky (1983). This topic could be regarded as a price discovery issue since the equilibrium solution could reveal some results concerning whether or not there is a bias in futures pricing. If, on the other hand, we shift our interest from first to the second moments, we would be focusing on, not the expected prices, but, volatilities, where we should keep in mind the fact that price volatility can synonymously be interpreted as risk, and thus, can be translated into a monetary cost. As we mentioned earlier, one of the economic tasks of futures markets is reallocation of risk (or the cost of uncertainty) from agents who do not wish to take them (=hedgers) to ones who are willing to do so (=speculators). We should recall that the main reason of futures trading for hedgers is to protect themselves from the uncertainty in spot prices, i.e. spot price volatility. Consequently, if, a model based on simultaneous determination of spot and futures prices arrives at a conclusion which suggests that, say, futures speculation increases spot price volatility, then, not only the mainstream Pareto improvement claim becomes ambiguous but even the existence of futures markets could be thought of defeating their own purpose of existing. Some regulators, professional investors and academicians voiced a similar opinion blaming computerized futures trading for the crash in 1987. The consensus in financial theory about this topic, on the other hand, supported the contrary position, namely, that futures trading decreases spot price volatility due to reductions of the price-stability disturbing effects due to demand shocks. Even though, financial theory supports a view which is being debated by a significant amount of professional traders and academicians, there has been no satisfactory empirical academic study aiming at resolving this contraversy, which is making use of the contemporary mainstream futures market theory. Hence, due to the lack of research in this area, the second essay, i.e. Part III of this study shall be focusing on this very issue: With the aim of finding an empirical answer to the question of whether or not speculation in futures markets indeed does increase spot price volatility Part III of this study is devoted to a theoretical and empirical investigation of this problem.

Empirical observations about volatility-adjusted futures price movements[6] exhibit significant differences in their average magnitudes depending on the type of commodity they are written on. The answer provided to the question of why we observe industry-specific differences among the traded futures contracts by the traders as well as by academicians is that there are different in nature. Despite of the fact that this answer is correct in essence, one realizes that financial theory is still lacking a comprehensive and consistent theoretical model with the help of which those differences can be explained. The first essay, i.e. Part II, is aimed to provide a new perspective of looking at this problem just mentioned. Using the framework of a simple mainstream model, which is developed in Part II, variations among different contracts are illustrated in a systematic way.

[6] As defined in section (ii)

Part II

Industry-Specific Price Movement Patterns of Futures Contracts

i. Introduction

In 1990 about 280 million contracts were traded in Futures Markets. The corresponding figure for the year of 1960 was only 4 million contracts, translated into percentage points this is an increase by 6,900%. The types of commodities which are traded today also exhibit a dramatic change if compared with ones about thirty years ago. In 1960, agricultural products accounted for 78% of all trading, whereas more than half of all futures being traded today are contracts written on financial instruments and foreign currencies. [1]

The distinctive characteristics of futures trading have attracted many academic and professional researchers' interest to related topics, since as early as in 1930's. However, despite the fact that certain topics are explored in some depth, some others, which have emerged due to the latest evolvements in futures trading , still are looking for an elaborate investigation: The statistics we mentioned above clearly demonstrate that in the past thirty years an essential change in the character of futures trading has occurred in terms of commodities on which the contracts are written on; one commodity group does not dominate the others as it used to be the case in the past. Nowadays, we have several distinct commodity markets, i.e. agriculturals, metals & energy, financials and foreign exchange, where all of which exhibit

[1] The figures are taken from the book by Franklin Edwards and Cindy Ma with the title "Futures and Options".

very different (futures) price movement patterns on an empirical basis. Yet, the literature is lacking a comprehensive explanation for the observed (futures) price movement patterns across these markets. This precisely is the motivation of this study. In this paper our aim is to see whether we can outline and analyze the effects of (some of) the distinguishing features across markets and attempt to make a cross-market comparison of futures price movements based on a mainstream theoretical model of Futures Markets.

Section (ii) shall present the theoretical framework of our analysis. Section (iii) will derive its theoretical conclusions. In the following section we shall test the conclusions of our model using actual daily futures price data. Section (v) will summarize and conclude the essay.

ii. The Model

The model that we shall construct for the purposes of this paper is based on standard assumptions of rational agents and competitive markets with symmetric information. There are four types of agents in the model: (i) Suppliers of the raw good (that futures contracts are written on), who we will refer to as "growers" (referred to as "G" in equations), (ii) "Processors" (="P") who use the raw good which is produced by the growers as an input to produce another good, which could be either an intermediate commodity which will be used for the production of other commodities or it could be also a final commodity which will be purchased by consumers. (iii) "Inventory Holders" (="I") carry the raw commodity over time, and finally, (iv) "Speculators" (="S") who trade futures contracts for speculating purposes, only.
In the most general case, we can characterize their net revenues, i.e. profits, in spot markets as follows:

$$\begin{aligned} R^G_{T-i} &= p^G_{T-i} q^{G(G)}_{T-i} - C_G(q^{G(G)}_{T-i}) \\ R^P_{T-i} &= p^P_{T-i} q^P_{T-i} - p^G_{T-i} q^{G(P)}_{T-i} - C_P(q^P_{T-i}) \\ R^I_{T-i} &= q^{G(I)}_{T-i} [p^G_{T-i} - p^G_0 (1+\theta)^{T-i}], \qquad q^G_{T-i} \geq 0; \\ R^S_{T-i} &= 0. \end{aligned} \tag{1}$$

where T and i stand for the maturity date and time to maturity, respectively. Furthermore $q_{T-i}{}^K$ and $C_K(q_{T-i}{}^K)$ represent the amount of the agent-specific (i.e. "K") output commodity and the respective cost functions of the agents (i.e. K=G,P) which we assume to be quadratic.

$q_{T-i}^{G(I)}$ indicates the amount of the raw good that the inventory holders carry over from the initial period to T-i. P_{T-i}^{G} and P_{T-i}^{P}, on the other hand, are the discounted prices of the raw and processed commodity, and θ represents the one-period cost-of-carry of the raw good.

It is important to note that throughout our analysis we are going to normalize both the quantity and the price of the spot commodities, such that the unit of "q" will be the amount which is the amount specified for delivery by a futures contract. Similarly, "p" will be defined as the dollar value of the quantity of goods called for delivery by a futures contract. Note that since the processed good, i.e. q^P, is produced using the raw good, i.e. q^G, as an input, we can represent the former as a function of the raw good.

The agents' wealth at any given period in time can be represented as:

$$W_{T-i} = R_{T-i} + x_{T-i}^{f}\,(p_{T-i}^{f} - p_{T-i-1}^{f}) + W_{T-i-1} \tag{2}$$

where x_{T-i}^{f} and p_{T-i}^{f} stand for investors' futures (long) positions and futures prices, respectively, and R_{T-i}, once again, represents the net revenues from trading in spot markets. All agents have a utility function of the form:

$$U(W_T) = \xi - \exp(-\alpha_K W_T) \tag{3}$$

where ξ is a constant and α is the agent-specific (i.e. K=G,P,I,S) coefficient of risk aversion. Recall that by recursive substitution we can write the terminal wealth as:

$$W_T = W_0 + \sum_{i=0}^{T} R_{T-i} + \sum_{i=0}^{T} x_{T-i} (p_{T-i}^f - p_{T-i-1}^f) \tag{4}$$

Knowing that, given (3), maximizing expected utility is equivalent to maximizing:

$$E[U(W_T)] = E(W_T) - (\alpha_K/2) V(W_T) \tag{5}$$

and defining,

$$B_{T-i}^f = p_{T-i-1}^f - p_{T-i}^f \tag{6}$$

we can re-write (5) in the following fashion:

$$\begin{aligned} E[U(W_T)] &= \sum_{i=0}^{T} E(R_{T-i}) + \sum_{i=0}^{T} x_{T-i}^f (-B_{T-i}^f) \\ &- (\frac{\alpha_K}{2}) \{ \sum_{i=0}^{T} V(R_{T-i}) + \sum_{i=0}^{T} (x_{T-i}^f)^2 V(B_{T-i}^f) \\ &- 2 \sum_{i=0}^{T} \sum_{j=0}^{i} cov(R_{T-i+j}, B_{T-i}^f) x_{T-i}^f \\ &+ 2 \sum_{i=0}^{T} \sum_{j=1}^{i} cov(B_{T-i+j}^f, B_{T-i}^f) x_{T-i+j} x_{T-i} + \Omega \} \end{aligned} \tag{7}$$

where Ω stands for all other expressions which are omitted in the presentation of the equation.

Taking the derivative of $E[U(W_T)]$ with respect to x_{T-i}, setting it equal to zero and then arranging the terms, we obtain the optimal futures position of agents:

$$x_{T-i}^f = \frac{-E(B_{T-i}^f)}{\alpha_K V(B_{T-i}^f)} + [\sum_{j=0}^{i} \beta_{RB}^j - \sum_{j=1}^{i} \beta_{BB}^j x_{T-i+j}^f] \tag{8}$$

where,

$$\beta_{RB}^j = \frac{cov(R_{T-i+j}, B_{T-i}^f)}{V(B_{T-i}^f)} \; ; \quad \beta_{BB}^j = \frac{cov(B_{T-i+j}^f, B_{T-i}^f)}{V(B_{T-i}^f)} \tag{9}$$

As it can be observed $\beta_{RB}{}^{j}$ represents the relationship (in the OLS sense) between a given period's, say today's, "B" (or change in volatility-adjusted futures prices) and the j-step-ahead spot revenue of agents. Similarly, $\beta_{BB}{}^{j}$ depicts the same kind of relationship between, again, say, today's futures price change and j-step-ahead futures price changes. For the better comprehension of the dynamics behind these formulas and their implications the reader is urged to refer to Appendix A where a detailed illustration of equations (8) and (9) is provided.

Having derived the agent-specific futures positions we now can state the clearing condition for futures markets:

$$\sum_k n^k x_t^k = 0. \qquad \text{for all } t, \text{ where } k=I,G,A,S. \qquad (10)$$

Here n^K stands for the number of agents (K=G,P,I,S) participating in futures trading and once again $x_t{}^K$ stands for their respective positions. Equation (10) can simply be interpreted as a condition telling us that in futures markets the net supply of contracts should be equal to zero. Remembering the "zero-sum game" characteristic of futures markets, it is obvious that for each contract bought (= long position), there also has to be someone who is "short" one contract, hence the net supply sums up to zero.

Aggregating equation (8) under the condition which is stated in (10) we observe that, since $\beta_{BB}{}^{j}$ terms have the same values for all agents, the last term in the bracket in (8) drops out. Let us define the ratio of the number of a particular type of agents participating in futures markets ($=n^K$) to the total number of all market participants ($=\Sigma_K n^K$) as w^K where (K=G,P,I,S). In other words, w^K stands for the weight of a particular group of agents among the entire population of futures market participants. Manipulating the aggregated formula we obtain the following generic expression for futures price changes:

$$E(B^f_{T-1}) = \alpha V(B^f_{T-1}) \{ w^G \sum_j \beta^j_{RB} + w^P \sum_j \beta^j_{RB} + w^I \sum_j \beta^j_{RB} \} \quad (11)$$

In (11), α stands for an "average" risk aversion coefficient in the market, which in fact is nothing but the harmonic mean of all α_K's.[2] Having demonstrated the model in its most generic format, let us now focus on each market separately and observe how the unique characteristics of these markets affect changes in (futures) prices.

[2] The derivation can be found in Appendix B.

a. Agricultural Markets

Agricultural markets, by their very nature, impose certain specifications on agents' behavior and, thus, on their net revenue functions. A striking factor which distinguishes agricultural markets from the other ones is the existence of a significant seasonality in production. Growers of the commodity, in this case, farmers, have to wait for some time till their output commodity is actually "realized", hence there is a lag period between the date of investing and the "realization" of the output, i.e. the harvest. The length of this period in between is much shorter in other markets such as energy or metals. Obviously, this fact has an important impact on the transaction behavior of the inventory holders. Namely, they purchase the agricultural commodity on the date of the harvest and then carry it over to intermediate periods, i.e. to points in time in between the current harvest and the next. Thus, technically speaking, at the intermediate periods $q^{G(I)}$ can only take non-negative values. Consequently, we can write the net revenue functions, i.e. profits, of agents in the case of agricultural commodities as below:

$$
\begin{aligned}
R^G_{T-i} &= 0 && \textit{for all } i, \textit{ except } 0. \\
R^G_T &= p^G_T q^{G(G)}_T - C_G(q^{G(G)}_T), && \textit{for } i = 0. \\
R^P_{T-i} &= p^P_{T-i} q^P_{T-i} - p^G_{T-i} q^{G(P)}_{T-i} - C_P(q^P_{T-i}) && \textit{for all } i. \qquad (12) \\
R^I_{T-i} &= q^{G(I)}_{T-i} [p^G_{T-i} - p^G_0 (1+\theta)^{T-i}] \;; \quad q^G \geq 0, && \textit{for all } i \textit{ except } 0. \\
R^I_T &= -q^{G(I)}_T p^G_T, && \textit{for } i = 0. \\
R^S_{T-i} &= 0. && \textit{for all } i.
\end{aligned}
$$

Defining:

$$
\beta^j_{GB} = \frac{cov(p^G_{T-i+j},\ p^f_{T-i-1} - p^f_{T-i})}{V(B^f_{T-i})}
$$

$$
\beta^j_{PB} = \frac{cov(p^P_{T-i+j},\ p^f_{T-i-1} - p^f_{T-i})}{V(B^f_{T-i})} \qquad (13a)
$$

Based on the revenue functions in (12), the betas which exhibit the relationship between j-step-ahead spot revenues and (futures) price changes can be stated as follows.

$$
\begin{aligned}
\beta^{(G),j}_{RB} &= q^G_T\ \beta^i_{GB} \quad \textit{for } i{=}0; \qquad =0 \ \textit{ for all } i \textit{ except } 0 \\
\beta^{(P),j}_{RB} &= q^P_{T-i+j}\ \beta^j_{PB} - q^G_{T-i+j}\ \beta^j_{GB} \\
\beta^{(I),j}_{RB} &= q^G_{T-i+j}\ \beta^j_{GB} \quad \textit{for all } i \textit{ except } 0 \qquad (13b) \\
\beta^{(I),j}_{RB} &= -q^{G(I)}_T\ \beta^i_{GB} \quad \textit{for } i{=}0 \\
\beta^{(S),j}_{RB} &= 0.
\end{aligned}
$$

Thus, substituting the values of these betas for the more generic expressions of β_{RB}'s, by (11), we obtain:

$$
\begin{aligned}
EB^f_{T-i} = \alpha\, V(B^f_{T-i}) \cdot \{\ & (w^G - w^I)\ [q^G_T \beta^i_{GB}] + w_I [\textstyle\sum_j \beta^j_{GB} q^G_{T-i+j}] \\
& + w^P [\textstyle\sum_j (q^P_{T-i+j} \beta^j_{PB} - q^G_{T-i+j} \beta^j_{GB})]\ \}
\end{aligned} \qquad (14)
$$

Let us impose the following simplifying assumption about the relationship between q^P and q^G:

$$q^P_{T-i} = \lambda \cdot q^G_{T-i} \qquad \text{for all } i, \text{ where } \lambda > 0. \tag{15}$$

Furthermore, let us define δ as:

$$\delta_{T-i+j} = (w^G q^{G(G)}_{T-i+j} + w^I q^{G(I)}_{T-i+j}) - w^P q^{G(P)}_{T-i+j} \tag{16}$$

Inspecting the equation above we observe that it represents the net supply of the spot commodity by agents participating in futures markets; or more roughly, we could interpret it as the "excess supply" of the spot commodity by futures participants since it is defined as the quantity offered by the suppliers (denoted as $(q^G_{T-i+j})^{SU}$ below) minus the quantity absorbed by the demanders of it.[3]

In the case of agriculturals, for any intermediate period we have only one type of agent as the supplier of the good: the inventory holders. Consequently, the demanders are the processors. Hence in equation (16) in the case of agricultural markets, observe that the first term on the right hand side drops out. In the light of the two equations we just introduced, equation (14) becomes:

$$EB^f_{T-i_{Agr.}} = \alpha V(B^f_{T-i}) \cdot \{(w_G - w_I) q^G_T \beta^i_{GB} + \sum_j [\lambda \beta^j_{PB} (q^G_{T-i+j})^{su}] - \sum_j [\lambda \beta^j_{PB} - \beta^j_{GB}] \cdot \delta_{T-i+j}\} \tag{17}$$

---

[3] Equations (15) and (16) are valid for all other types of markets as well.

b. Metals and Energy Markets

Unlike in the agricultural markets, we notice three main categories of activities in metals and energy markets during the lifetime of contracts.

First, there are those agents who produce and supply the commodity, say crude oil, who, we shall refer to as producers.[4] As emphasized above, in metals/energy markets during the lifetime of contracts producers persist in their activities; there may be multiple production cycles during the lifetime of a given contract. The produced raw commodity, in our example, crude oil, is then processed and transformed into a more sophisticated, refined product, say jet fuel, by "processors".

Obviously, processors incur a cost while transforming raw commodities into relatively more refined goods. Next, as in all other markets, we observe agents who have the aim and proper facilities to store and carry over the raw commodity over time, i.e. "inventory holders". As before, the revenue of futures speculators in spot markets is zero. Once we are able to represent the functioning of metals and energy markets in this simple framework we can formulate the net revenue functions, i.e. profits, of agents in the following way:

---

[4] Producers are analogous to "growers" of agricultural markets, and will be represented with the letter "G" in the forthcoming equations.

$$R^G_{T-i} = p^G_{T-i} q^{G(G)}_{T-i} - C_G(q^{G(G)}_{T-i})$$
$$R^P_{T-i} = p^P_{T-i} q^P_{T-i} - p^G_{T-i} q^{G(P)}_{T-i} - C_P(q^P_{T-i}) \tag{18a}$$
$$R^I_{T-i} = q^{G(I)}_{T-i}\,[p^G_{T-i} - p^G_0(1+\theta)^{T-i}], \qquad q^G_{T-i} \geq 0;$$
$$R^S_{T-i} = 0.$$

Thus, the corresponding set of betas will have the following values:

$$\beta^{(G),j}_{RB} = q^G_{T-i+j}\, \beta^i_{GB}$$
$$\beta^{(P),j}_{RB} = q^P_{T-i+j}\, \beta^j_{PB} - q^G_{T-i+j}\, \beta^j_{GB} \tag{18b}$$
$$\beta^{(I),j}_{RB} = q^G_{T-i+j}\, \beta^j_{GB}$$
$$\beta^{(S),j}_{RB} = 0.$$

Recalling equations (15) and (16), i.e.

$$q^P_{T-i} = \lambda\, q^G_{T-i}\,,$$
$$\delta_{T-i+j} = (w^G q^{G(G)}_{T-i+j} + w^I q^{G(I)}_{T-i+j}) - w^P q^{G(P)}_{T-i+j} \tag{18c}$$

Hence applying equation (11) to this markets reveals:

$$EB^f_{T-i_{Met./Eng.}} = \alpha\, V(B^f_{T-i}) \cdot \{ \sum_j [\lambda\, \beta^j_{PB} (q^G_{T-i+j})^{su}] - \sum_j [\lambda\, \beta^j_{PB} - \beta^j_{GB}] \cdot \delta_{T-i+j} \} \tag{19}$$

c. Financial Markets

As in metals/energy markets, in financial markets, three basic types of activities could be described: Producers, Inventory Holders and Processors. However, unlike metals/energy markets (or agricultural markets for that matter), it is not possible to talk about physical production in financial markets. The concept of "production" of new financial securities, obviously, does not imply any "real" (in the economic sense) activity: Unlike growing, say corn or wheat, there will be no physical creation involved.[5] In other words, what is being traded is not a real commodity. Then the important question to be answered is whether or not this characteristic of financial markets alters the fact that there are actually producers of these financial goods, i.e. securities, people keeping them in their inventories, and finally others who use them to create other securities based on the primary ones. Naturally the answer is not affirmative. There are the issuers of the securities who correspond to our "growers", in the previous sections. They issue securities, make a revenue bearing a certain cost associated with it. If we take t-bill markets as an example, obviously the issuer of the security would be the Treasury. Next, we have "processors", which could be pictured as pension funds, money market funds or other types of financial intermediaries. Their main activity

[5] The material preperation of the asset itself can easily be ignored in this context, since undoubtedly the material form of the securities would be a very negligible factor among many others actually affecting the pricing of the financial securities in the market.

is purchasing the securities which were issued by the grower(s), i.e. t-bills, and produce new financial products using them as an input. Finally there are inventory holders, whose task is to carry over the underlying cash commodities (in this example, t-bills) over time. As expected futures speculators' profits in spot markets are zero at all periods. This type of setting can be characterized exactly by the set of equations as given in (18a-18c) in the previous section. Equation (11) for financials could be expressed as:

$$EB^{f}_{T-1_{Fin.}} = \alpha V(B^{f}_{T-1}) \cdot \{ \sum_{j} [\lambda \beta^{j}_{PB} (q^{G}_{T-1+j})^{su}] - \sum_{j} [\lambda \beta^{j}_{PB} - \beta^{j}_{GB}] \cdot \delta_{T-1+j} \} \quad (20)$$

iii. Comparisons

Having derived the key equations for the (futures) price movements in agricultural (equation 17), metals/energy (equation 19) and financial markets (equation 20), the next step is a relative comparison of the magnitudes of equations (17), (19) and (20). For simplicity, we introduce a new measure, Φ, which is defined as $\Phi_{T-i}=|E(B_{T-i}{}^{f})/\alpha V(B_{T-i}{}^{f})|$, then we can re-formulate our purpose and in saying that what we are after is the comparison between the Φ-values of agriculturals, metals/energy and financial markets.[6]

If we presume that the demand functions for raw and processed commodities, i.e. G and P can be written as:

$$p^G = (q^G)^{-\epsilon}, \qquad p^P = (q^P)^{-\eta}. \tag{21}$$

where ϵ and η stand for the demand elasticities of commodities G and P, respectively.

By a simple manipulation, combining (21) and (15) respectively, we can express p^P as:

$$p^P = \lambda^{-\eta}\ (p^G)^{\frac{\eta}{\epsilon}} \tag{22}$$

[6] Recalling that the coefficient of variation, C is defined as $C=\sigma/\mu$, one way of interpreting the Φ is as: $\Phi=1/(\alpha\sigma C)$. In other words, Φ and the coefficient of determination are inversely related.

Once we have an expression of p^P in terms of p^G we may re-write $\beta_{PB}{}^j$ in the following form:

$$\beta^{j}_{PB} = [\frac{1}{V(B_{T-i})}] \cdot cov[\lambda^{-\eta} (p^{G}_{T-i+j})^{\frac{\eta}{\epsilon}}, \; p^{f}_{T-i-1} - p^{f}_{T-i}] \tag{23}$$

Equation (23) shows that for a given demand elasticity for the final good (q^P), i.e. η, as the elasticity of demand for the raw commodity increases, $\beta_{PB}{}^j$ decreases and vice versa. In other terms,

[1.] When comparing two types of markets with the same η, if we observe that $\epsilon_1 > \epsilon_2$ (that the demand for the first (raw) commodity is more elastic than the other one) implies that $|\beta_{PB}|_1 < |\beta_{PB}|_2$.

It is known that spot markets clear if the quantity supplied equals quantity demanded, i.e.:

$$m^G q^G_{T-i} + m^I q^I_{T-i} - m^P q^P_{T-i} = 0. \tag{24}$$

where m^K is the number of agents of type K participating in spot markets. Dividing both sides by the total number of agents, $M = \Sigma_K m^K$, we obtain:

$$\omega^G q^G_{T-i} + \omega^I q^I_{T-i} - \omega^P q^P_{T-i} = 0. \tag{25}$$

where ω^K's represent agents' share in spot markets. Now if we make the assumption that there is a linear relationship between agents' share in spot markets, ω, and the weights of futures market participants, w, in the following form:

$$w^K = \gamma\ \omega^K, \qquad \gamma > 0 \tag{26}$$

(note that we are assuming that the coefficient gamma has the same value for all agents), then (27) holds:

$$\delta_{T-1+j} = (w^G q^{G(G)}_{T-1+j} + w^I q^{G(I)}_{T-1+j}) - w^P q^{G(P)}_{T-1+j} = 0. \tag{27}$$

Equation (27) illustrates that spot market clearing condition (24) combined with our assumption in (26) makes sure that the value of δ will be zero.

Empirical and theoretical research asserts that agricultural and metals/energy markets usually operate under inelastic demand schemes, whereas the elasticity of demand (i.e. ϵ) is much larger in the case of financial markets. Put differently, $\epsilon_{\text{Fin.}} > \epsilon_{\text{Met./Eng.}} = \epsilon_{\text{Agr.}}$ Note that these relationships by [1.] imply:

$$|\beta^j_{PB}|_{Fin.} < |\beta^j_{PB}|_{Met./Eng.} = |\beta^j_{PB}|_{Agr.} \tag{28}$$

[2.] (Spot) Market clearing condition and the assumption about the relationship between the ratio of participation in futures and spot markets assure that: $\delta = 0$, there will be no "excess supply" of the spot commodity by futures market participants.

Relaxation of the assumption about the gamma, i.e. that it has the same value for all types of agents, causes the value of delta to be different from zero. In particular,

$$\delta > 0 \quad \text{if} \quad w_G >> \omega_G \text{ and } w_P << \omega_P,$$

$$\delta < 0 \quad \text{if} \quad w_G << \omega_G \text{ and } w_P >> \omega_P.$$

It's known that participation in futures markets has a positive cost. For a given level of cost of participation we can assert that the smaller the size of agents, the less is the probability of their participating in futures trading.

Hence, it can be suggested that as the size of the firm grows so does the likelihood of participation. Then the problem reduces down to comparing the average sizes of "growers" with "processors" in a given industry, for which the use of the concentration ratios seems to be an appropriate method.

In other words, when we use concentration ratios (=CR) as a proxy for the average size of firms operating in that industry, we can make some useful observations, namely:

[3.] If CR_G:large and CR_P:small then $\delta > 0$, and
If CR_G:small and CR_P:large then $\delta < 0$.

The table on the next page exhibits the concentration ratios of some industries.

The striking conclusion that one may draw, by [3.], based on the following table is:

$$\begin{array}{llll} \textit{Agricultural Markets:} & CR_G < CR_P, & \textit{thus} & \delta < 0, \\ \textit{Metals/Energy Markets:} & CR_G > CR_P, & \textit{thus} & \delta > 0. \end{array} \qquad (29a)$$

Table I: Concentration Ratios in some industries

AGRICULTURALS:

CR_G			CR_P		
SIC	Explanation	4-Firm CR	SIC	Explanation	4-Firm CR
20110	Meat varieties: Beef, pork, turkey, etc.	0.000	20222	Processed Cheese and rel'd goods	0.600
20740	Crude Cottonseed oil, soybean oil	0.370	20323	Canned dry beans other canned prod.	0.500
20610	Sugar Cane Mill & Products	0.000	20570	Bread, Cake and related products	0.428

METALS/ENERGY:

CR_G			CR_P		
SIC	Explanation	4-Firm CR	SIC	Explanation	4-Firm CR
33310	Primary Copper	0.680	33510	Copper Products	0.493
33395	Precious Metals	0.670	39110	Jewelry	0.124
29110	Petroleum	0.503	33570	Aluminum Products	0.000
29990	Petroleum and Coal Production	0.666	33992	Other Primary Metal Products	0.290

Based on the high concentration of both "growers" and "processors" and "inventory holders", as well, in the case of financial markets, it is safe to assume that:

$$\textit{Financial Markets}: \quad CR_G \approx CR_P, \quad \textit{thus} \quad \delta = 0. \tag{29b}$$

If we inspect the table above carefully, we observe very high concentration ratios concerning some industries, such as petroleum and coal production, copper production, production of precious metals and so on. These high figures of concentration ratios might violate the assumption of perfect competition in both, growers' and processors' markets. The violation of the assumption of perfect competition in one or both of the markets could be a motivation for a further study.

Finally, if we postulate the conventional relationship between p^f and p^G, i.e.:

$$p^f_{T-i} = p^G_{T-i}\ (1+\theta-\Gamma)^{i} \tag{30}$$

(where θ stands for the average one period cost-of-carry of the commodity and Γ reflects any possible convenience yield.) Making use of (30) we can manipulate $\beta_{GB}{}^j$ such that it becomes:

$$\begin{aligned} \beta^j_{GB} &= [\frac{1}{(1+\theta-\Gamma)^{i-j}}]\ \beta^j_{FB} \\ \beta^j_{FB} &= \frac{cov(p^f_{T-i+j},\ p^f_{T-i-1} - p^f_{T-i})}{V(B^f_{T-i})} \end{aligned} \tag{31}$$

Now we can state:

[4.] At the equilibrium level of inventory holdings, an increase (decrease) in the average cost-of-carry of the commodity (i.e. θ) leads to a geometric decrease (increase) in $\beta_{GB}{}^j$.

Although it is certain that the nature and the magnitude of cost-of-carry plays an important role in across market comparisons of Φ, the effects of it is rather ambiguous. One thing which should be stated at the outset is that depending on the types of markets, the functional nature of θ may vary. Namely, in financial markets, the cost-of-carry of a commodity is mainly the opportunity cost which is nothing but an interest rate, whereas in agricultural and metals/energy markets, the cost of storage is an increasing function of the quantity stored (possibly a quadratic one). Alternatively put, in financial markets, we have a cost-of-carry function which has a linear form, whereas in agricultural and metals/energy markets its form is quadratic. Even though as essential as it seems, this piece of knowledge above does not tell us per se in which type of market the θ function would take the highest value. Hence we have to consider both cases:

$$\begin{array}{ll} (1) & \theta_{linear} < \theta_{quadratic} \\ (2) & \theta_{linear} > \theta_{quadratic} \end{array} \qquad (32)$$

a. Agriculturals vs. Financials

In the light of equations (30) - (32) we are in a position where we can compare the Φ functions of agricultural and financial markets (based on (18) and (20), respectively). We know that:

(1) By equation (31), $\delta_{Agr.} < 0$ and $\delta_{Fin.} = 0$.

(2) By equation (30), $|\beta_{PB}{}^{j}|_{Fin.} < |\beta_{PB}{}^{j}|_{Agr.}$.

Note that observation (1) in combination with the assumption of $[\lambda\beta_{PB}{}^{j} - \beta_{GB}{}^{j}] < 0$ [7], leads us to conclude that the sign of the of the last term, i.e. $-\Sigma_{j}{}^{i}\ [\lambda\beta_{PB}{}^{j} - \beta_{GB}{}^{j}]\ \delta_{T-i+j} < 0$, is negative.

Since $|\beta_{PB}{}^{j}|_{Fin.} < |\beta_{PB}{}^{j}|_{Agr.}$, as stated by (2), we infer that the absolute value of the second term in the case of agriculturals is larger than the one in the case of financials, i.e. $|[\Sigma_{j}(\lambda\beta_{PB}{}^{j}(q^{SU}{}_{T-i+j}))]_{Agr.}| > |[\Sigma_{j}(\lambda\beta_{PB}{}^{j}(q^{SU}{}_{T-i+j}))]_{Fin.}|$.

The first two points above clearly indicate that $\Phi_{Agr.} > \Phi_{Fin.}$ In order to see how to reach this conclusion, let us call:

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[7] Note that by equation (23) we can write $\beta_{PB}{}^{j} = \lambda^{-\eta}\ \xi\beta_{GB}{}^{j}$, where $\xi > 1$. Then the negativity assumption, i.e. $[\lambda\beta_{PB}{}^{j} - \beta_{GB}{}^{j}] < 0$, recalling that $\beta_{GB}{}^{j} < 0$, implies that:
$\eta < 1 + \ln(\xi)/\ln(\lambda)$, if $\ln(\lambda) > 0$, and,
$\eta > 1 + \ln(\xi)/\ln(\lambda)$, if $\ln(\lambda) < 0$,
which defines a continuous interval for η. Note that the reverse presumption, the assumption of positivity, would have implied an interval which would presuppose a discontinuous interval, i.e. one which leaves out values in the vicinity of unity and allow for either a very inelastic demand curve for the processed goods or a very elastic one.

$|[\Sigma_j(\lambda \beta_{PB}{}^j(q^{SU}{}_{T-i+j}))]_{Agr.}| > |[\Sigma_j(\lambda \beta_{PB}{}^j(q^{SU}{}_{T-i+j}))]_{Fin.}|$ as $|x_2| > |x_1|$

$(\Sigma_j{}^i\ [\lambda \beta_{PB}{}^j - \beta_{GB}{}^j]\ \delta_{T-i+j})_{Agr.,Fin.}$ as $y_{2,1}$ and,

$(w_G-w_I)q_T{}^G\beta_{GB}{}^i$ as z, where x_1, x_2, $z < 0$, $y_2 > 0$ and $y_1 = 0$.

Then our comparison of $\Phi_{Agr.}$ with $\Phi_{Fin.}$ boils down to comparing $|x_1|$ with $|x_2-y+z|$, where the second term can alternatively be expressed as $|x_2+(z-y)|$, which in turn is equal to $-(x_2+(z-y))$. Since $(z-y)$ is negative one can show that $-(x_2+(z-y)) > -x_1$, or $\Phi_{Agr.} > \Phi_{Fin}$.

However, we should note that this result could be somewhat counterbalanced if we would observe $w_G < w_I$ (i.e., if participation ratio of producers is less than the one of inventory holders)[8]. Furthermore, if one makes the alternative assumption of (32.2) instead of (32.1), once again, our conclusion about the inequality, i.e. $\Phi_{Agr.} > \Phi_{Fin}$ would be weakened.[9] But even though these factors may reduce the difference slightly, the sign of the inequality can be expected to remain unchanged. Thus, in sum, we can state that:

[i] *On the average,* $\Phi_{Agr.} > \Phi_{Fin.}$

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[8] The case where inventory holders outweigh growers, if translated to our notation in the previous footnote, can be expressed as: $z > 0$, $y > 0$, $x_2 < 0$. Then, our condition still holds as long as: $z < -(x_2-y)$.

[9] Which means that the value of θ in agricultural markets will be relatively lower than financial markets. By (29) we observe that the first derivative of $\beta_{GB}{}^j$ with respect to θ is negative. Hence as θ for the financial markets increases, so does the absolute value of $\beta_{GB}{}^j$, and a a result the absolute value of x_1 increases, as well. Thus, the difference between Φ_{Agr} and Φ_{Fin} tends to decline. However, we should note that as long as the inequality $-(x_2+(z-y_2) > -(x_1)$ is satisfied, our condition that $\Phi_{Agr} > \Phi_{Fin}$ still holds.

b. Agriculturals vs. Metals/Energy

Here is the set of information which our comparisons will be based on:

(1) By equation (31), $\delta_{Agr.} < 0$ and $\delta_{Met./Eng.} > 0$.

(2) By equation (30), $|\beta_{PB}{}^{j}|_{Met./Eng.} = |\beta_{PB}{}^{j}|_{Agr.}$.

The assumption in (32) does not enter the comparison here, since in both of the markets we observe a quadratic cost function. (1) and (2) lead to a strict negative term in the case of agricultural, whereas in the case of metals and energy the last term, i.e. $-\Sigma_j[\lambda\beta_{PB}{}^{j} - \beta_{GB}{}^{j}]\ \delta_{T-i+j} > 0$. Once again, let us call:

$|[\Sigma_j(\lambda\beta_{PB}{}^{j}(q^{SU}{}_{T-i+j}))]_{Agr.}| = |[\Sigma_j(\lambda\beta_{PB}{}^{j}(q^{SU}{}_{T-i+j}))]_{M/E.}|$ as $|x_2| = |x_1|$

$|\Sigma_j{}^{i}\ [\lambda\beta_{PB}{}^{j} - \beta_{GB}{}^{j}]\ \delta_{T-i+j}|_{Agr.} = |\Sigma_j{}^{i}\ [\lambda\beta_{PB}{}^{j} - \beta_{GB}{}^{j}]\ \delta_{T-i+j}|_{M/E.}$ as $|y_2| = |y_1|$,

and,

$(w_G-w_I)q_T{}^{G}\beta_{GB}{}^{i}$ as z,

where x_1, x_2, z are negative, y_2 is positive and y_1 is negative.

Hence, even if we assume away any differences in cost-of-carry between metals/energy and agricultural markets a simple comparison of $|x_1-y_1|$ with $|x_2-y_2+z|$, reveals the result that $y_2-z > y1$:[10] Since y_2 is equal to $-y_1$ in value and since z is negative it follows that:

$\Phi_{Agr.} > \Phi Met./Eng.$

As before, the case of $w_G < w_I$ might affect the magnitude of the

[10] Note that the inequality $|x| > |y|$ holds for all three markets.

inequality, even in the absence of cost-of-carry differences. As we introduce differences in the value of θ among the two industries, our previous conclusions will be even more fortified. Hence, it is safe to state that:

[ii] On the average, $\Phi_{Agr.} > \Phi_{Met./Eng.}$

c. Financials vs. Metals/Energy

In this case the comparison will be based on three points:

(1) By equation (31), $\delta_{Fin.} = 0$ and $\delta_{Met./Eng.} > 0$.

(2) By equation (30), $|\beta_{PB}{}^{j}|_{Fin.} < |\beta_{PB}{}^{j}|_{Met./Eng.}$.

(1) and (2), as mentioned above, lead to a small negative term in the case of financials, $[\Sigma_j(\lambda\beta_{PB}{}^{j}(q^{SU}{}_{T-i+j}))]_{Fin.}$, and the other term is zero. On the other hand, $[\Sigma_j(\lambda\beta_{PB}{}^{j}(q^{SU}{}_{T-i+j}))]_{Met./Eng.} < 0$;

Once again, let us call:

$|[\Sigma_j(\lambda\beta_{PB}{}^{j}(q^{SU}{}_{T-i+j}))]_{Fin.}| = |[\Sigma_j(\lambda\beta_{PB}{}^{j}(q^{SU}{}_{T-i+j}))]_{M/E.}|$ as

$|x_1| < |x_2|$

$|\Sigma_j{}^{i}\ [\lambda\beta_{PB}{}^{j} - \beta_{GB}{}^{j}]\ \delta_{T-i+j}|_{Fin.} < |\Sigma_j{}^{i}\ [\lambda\beta_{PB}{}^{j} - \beta_{GB}{}^{j}]\ \delta_{T-i+j}|_{M/E.}$ as $|y_1| < |y_2|$,

where x_1, x_2, are negative, y_2 is negative and y_1 is zero.

Hence, again, even if we assume away any differences in cost-of-carry between metals/energy and financial markets a simple comparison of $|x_1-y_1|$ with $|x_2-y_2|$, yields us $-x_1 < x_2 + y_2$ (See: footnote 10), which in turn can be translated into $\Phi_{Fin.} < \Phi_{Met./Eng.}$ Note that if we take differences in cost-of-carry into account, say, in the sense of (32.1), that would mean that $|\beta_{CB}|_{M/E}$ will be even higher relative to $|\beta_{GB}|_{Fin}$, causing $|x_2| >> |x_1|$, which in turn strengthens the result derived above. However, if we presume that the cost-of-carry relationship can be described by (32.2) then, depending on the magnitudes of the two Θ values, the spread between $|x_2|$ and $|x_1|$ will decrease or may even become non-positive. Thus, to conclude one may state:

[iii] *On the average,* $\Phi_{Fin.} < \Phi_{Met./Eng.}$ *if* $\Theta_{Fin.} > \Theta_{Met./Eng.}$,

and

$\Phi_{Fin.} > \Phi_{Met./Eng.}$ *if* $\Theta_{Fin.} < \Theta_{Met./Eng.}$.

So far, our analysis ignored the effect of inventory holders on δ_{T-i+j}. As we stated above, in contrast to agricultural markets where $q^{G(I)}$ was strictly non-negative at all intermediate periods, in metals/energy and financial markets, $q^{G(I)}$ could be both positive and negative, which means that at any given period inventory holders could be selling or buying inventories. Now let us investigate the effects of the relaxation of the assumption about the sign of $q^{G(I)}$.

In metals/energy markets, we observed that there is a large concentration of growers and a much smaller concentration of processors, thus concluding that $w_G >> w_P$, which in turn leads to $\delta > 0$. However, if we consider cases where $q^{G(I)}$ is negative and participation by inventory holders is significantly large, by the definition of δ, i.e. equation (18c), that would also cause the second term of the left hand side to be negative, thus decreasing the value of δ for metals/energy markets.

If we investigate financial markets in the same manner as above under the assumption of (26), we observe that equation (26) ensures that no matter what the sign of $q^{G(I)}$ is, δ still remains zero.

[5.] Continuous trading (selling/buying) of inventory holders in metals/energy markets leads to a decrease in the value of $\delta_{Met./Eng.}$, thus diminishing the magnitude of $\Phi_{Met./Eng.}$ relative to Φ_{Fin}, and hence lowering/increasing the spread between ΦFin. and $\Phi_{Met./Eng.}$ under assumption (32.1)/(32.2).

Obviously under [5.], the findings of [iii] should be interpreted keeping this possible bias in $\Phi_{Met./Eng.}$ in mind.

iv. Empirical Testing

In the following section, we shall: (1) present the graphs of the evolvement of market-specific Φ's[11] over time, (2) exhibit the parametric and non-parametric tests we have conducted in order to determine whether or not the absolute value of Φ', i.e. Φ, across the three industries are same.

<u>(1) Graphs:</u>

In this section we first picked several commodities of each market type as representatives for that group. As a second step we then calculated the "contract value" for each contract, based on the daily settlement prices and the standard amounts of each commodity which is called for delivery by a respective futures contract. Next, we calculated the value of Φ for each commodity over a life time of a given futures contract: In our case the comparisons are based on prices quoted for the December 1990 contracts. For the calculation of variances we used ten observations at any given period of time: five observations from the past and five future observations. Needless to mention: we take the ex post observations of futures prices to be identically equal to the ex ante predictions of agents, thus, reflecting our assumption of rational expectations of market participants.

A careful examination of the graphs (pp. 49-54) illustrates that there

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[11] Note that $|\Phi'| = \Phi$.

are, indeed, striking differences in the movement of futures prices across markets. It can be seen that Φ in the case of agricultural commodities has the largest magnitudes when compared with financial commodities and metals/energy. In only one case is the magnitude of price changes relatively smaller as opposed to other agriculturals, namely, in the case of soybeans. There may be several reasons leading to small price changes in soybeans. One of the reasons could be the fact that soybeans market is one of the rare markets, where futures contracts are not only traded on the raw commodity, i.e. soybeans, but also on the processed good(s) which are produced out of soybeans, e.g. contracts on soybean meal, and soybean oil. This fact may have a price stabilizing effect in the soybean market. Extensive effects of this kind of peculiarity on the (futures) price fluctuations could be the subject of a further study. Secondly, one may hypothesize that the fact that the high frequency of contracts on soybeans (as opposed to other agricultural commodities) may also reduce the magnitude of large fluctuations; a point of view, which, again, is looking for some elaborate study, to be tested empirically such that it would be theoretically confirmed/dismissed.

Overall the graphs clearly exhibit the same set of relationships between volatility-adjusted (futures) price changes across markets, as our model; it can easily be seen that magnitudes of Φ in agricultural markets is larger than metals and energy futures, and in turn, the same ratio of metals/energy, on average, is larger than the same ratio in the case of financials, which precisely was the prediction of our model.

Given bounded and rather stable variances of B, clearly these graphs could be interpreted in the following manner:

. If there exists no detectable trend in a given market, prices would follow a Martingale process, i.e. $Ep_{t+1}{}^{f} = p_{t}{}^{f}$. In such a case note that the expected price change, $B = 0$, thus $\Phi = 0$.

. On the other hand, if there is a detectable trend, then, the expected price change is not necessarily equal to zero, hence the values that Φ takes will deviate from zero. Thus, significantly non-zero values of Φ can be thought of indications of a trend under the assumptions above.

However, it is obvious, that the assumption of stable variances over time is not a realistic one, especially in markets which demonstrate obvious patterns of seasonality. Past studies have shown that the volatility of prices change during the life time of the contracts. (Anderson, 1985) Nevertheless, even with fluctuating variances we still can safely defend the first out of the points we made above, i.e. a Φ which is indistinguishable from zero, still hints at a Martingale process of futures prices; and any other value of Φ implies the existence of a trend, but the magnitude of the deviation from zero does not give us a clear idea about the strength of the trend detection, as it did in the previous case.

(2) Tests:

In order to test whether or not the (absolute) value of Φ across the three industries are same one can propose two types of empirical testing methods:

(a) Parametric Testing, i.e. in this case the test we would employ would be a "two-sample t-test". For independent random samples from two normal populations having the same unknown variance σ^2, the likelihood ratio technique yields a test based on:

$$t = \frac{\overline{x_1} - \overline{x_2} - \delta}{s_p \left(\frac{1}{n_1} + \frac{1}{n_2} \right)^{1/2}} \tag{33}$$

where,

$$s_p^2 = \frac{(n_1 - 1)\, s_1^2 + (n_2 - 1)\, s_2^2}{n_1 + n_2 - 2}$$

With the help of this test we can determine whether or not any pair of the given three industries exhibit the same price movement pattern.
In a typical case, our null hypothesis will be:

$H_0 : \mu_1 = \mu_2$

$H_1 : \mu_1$ not equal to μ_2 (i.e. Mean of, say, Financials is not equal to, say, mean of Agriculturals)

In other words, in our tests the value of delta will be equal to zero.

To test this hypothesis at the level of significance $\alpha = 0.05$, the critical region becomes:

$t \Rightarrow t_{.025} = 1.96$ and $t \Leftarrow -1.96$.

In the case of the first comparison, i.e.

financials ("1") vs. agriculturals ("2"):

the t-statistic turns out to be: -16.7064. Since $-16.7064 < -1.96$ we reject the null hypothesis in favor of the alternative one.

If on the other hand, we would like to test:

$H_0 : \mu_1 = \mu_2$

$H_1 : \mu_1 < \mu_2$ (i.e. Mean of Financials is less than mean of Agriculturals)

Then the critical region becomes $t <= t_{.05} = -1.645$. Since $-16.7064 < -1.645$ we once again reject the null hypothesis in favor of the alternative one.

In the case of the second case, i.e. in the comparison of agriculturals with metals, we find that the t-statistic becomes $t = 12.28717$. If our hypotheses are given as follows:

$H_0 : \mu_1 = \mu_2$

$H_1 : \mu_1$ not equal to μ_2 (i.e. Mean of Agriculturals is not equal to mean of Metals)

Then, the critical t-value is $t => 1.96$ and $t <= -1.96$. Thus, the calculated t-statistic exceeds 1.96, and we reject the null hypothesis.

If, on the other hand we are testing the following;

$H_0 : \mu_1 = \mu_2$

$H_1 : \mu_1 > \mu_2$ (i.e. Mean of Agriculturals is greater than the mean of Metals)

Then the critical t-value becomes 1.645. Since 12.28717 exceeds 1.645

we reject the null hypothesis in favor of the alternative hypothesis.

In the case of the third case, i.e. in the comparison of metals with financials, we find that the t-statistic becomes t = 13.02638. If our hypotheses are given as follows:

$H_0 : \mu_1 = \mu_2$

$H_1 : \mu_1$ not equal to μ_2 (i.e. Mean of Metals is not equal to mean of Financials)

Then, the critical t-value is t => 1.96 and t <= -1.96. Thus, the calculated t-statistic exceeds 1.96, and we reject the null hypothesis. If, on the other hand, we are testing the following:

$H_0 : \mu_1 = \mu_2$

$H_1 : \mu_1 > \mu_2$ (i.e. Mean of Agriculturals is greater than the mean of Metals)

Then the critical t-value becomes 1.645. Since 13.02638 exceeds 1.645 we reject the null hypothesis in favor of the alternative hypothesis.

(b) Non-parametric Testing, i.e. the test which is the non-parametric alternative to the two-sample t-test would be the so-called "U-Test", which is sometimes also referred to as "Wilcoxon Test". The advantage of the U-Test is that without having to assume that the two populations sampled have normal populations, we will be able to test the null hypothesis that we are sampling identical continuous populations against the alternative that the two populations have unequal means. According to Wilcoxon we can write the following, where W's stand for

$$U_1 = W_1 - \frac{n_1(n_1+1)}{2}$$
$$U_2 = W_2 - \frac{n_2(n_2+1)}{2} \qquad (34)$$

the sum of the ranks of the values of the first and second sample and n's stand for the two sample sizes.

Then under the null hypothesis, the means and the variances of the two U's are:

$$E(U_1) = E(U_2) = \frac{n_1 n_2}{2}$$
$$V(U_1) = V(U_2) = \frac{n_1 n_2 (n_1+n_2+1)}{12} \qquad (35)$$

Case (i): Financials vs. Agriculturals

$H_0 : \mu_1 = \mu_2$

$H_1 : \mu_1 < \mu_2$ (i.e. Mean of Financials is less than mean of Agriculturals)

In this case: $W_1 = 29{,}830$

$U_1 = 29{,}830 - 231\ (232)\ /\ 2$

$n_1 = 231,\quad n_2 = 249$

$U_1 = 3{,}034.$

$E(U_1) = 28{,}759.50$

$V(U_1) = 2{,}305{,}553.25$

Therefore, $z = -16.94$

The critical region for $z <= -z_{.05} = -1.645$

and $-16.94 < -1.645$

Hence we <u>reject</u> the null-hypothesis.

Alternatively, we could test the following null hypothesis:

$H_0 : \mu_1 = \mu_2$

$H_1 : \mu_1$ not equal to μ_2 (i.e. Means of Financials and Agriculturals are not equal to each other)

Then, the critical region for z is: $z => z_{.025} = 1.96$ or

$z <= z_{.025} = -1.96$

Hence, we once again <u>reject</u> the null-hypothesis.

<u>Case (ii) Financials vs. Metals</u>

$H_0 : \mu_1 = \mu_2$

$H_1 : \mu_1 < \mu_2$ (i.e. Mean of Financials is less than mean of Metals)

In this case, $W_1 = 37,613$

$U_1 = 37,613 - 231\ (250)\ /\ 2$

$n_1 = 231, \quad n_2 = 250$

$U_1 = 8,738.$

$E(U_1) = 28,875$

$V(U_1) = 2,319,625$

Therefore, $z = -13.22$

The critical region is: $z <= -z_{.05} = -1.645$

Since −13.22 < −1.645, we reject the null hypothesis.

Alternatively, we could test the following:

$H_0 : \mu_1 = \mu_2$

$H_1 : \mu_1$ not equal to μ_2 (i.e. Means of Financials and Metals are not equal to each other) Then the critical region is:

$z => 1.96$ or $<= -1.96$

Hence, once again we reject the null hypothesis in favor of the alternative one.

Case (iii) Metals vs. Agriculturals

$H_0 : \mu_1 = \mu_2$

$H_1 : \mu_1 < \mu_2$ (i.e. Mean of Metals is less than mean of Agriculturals)

In this case, $W_1 = 44{,}456$

$U_1 = 44{,}456 - 250\ (249)\ /\ 2$

$n_1 = 250, \quad n_2 = 249$

$U_2 = 13{,}331$

$E(U_1) = 31{,}125$

$V(U_1) = 2{,}593{,}750$

Therefore, $z = -11.05$

The critical region is: $z <= - z_{.05} = -1.645$

Since −11.05 < −1.645 we reject the null hypothesis, in favor of the alternative one.

Alternatively, we could also test:

$H_0 : \mu_1 = \mu_2$

$H_1 : \mu_1$ not equal to μ_2 (i.e. Means of Metals and Agriculturals aren't same) Then the critical region becomes:

$$z => 1.96 \text{ or } z <= -1.96$$

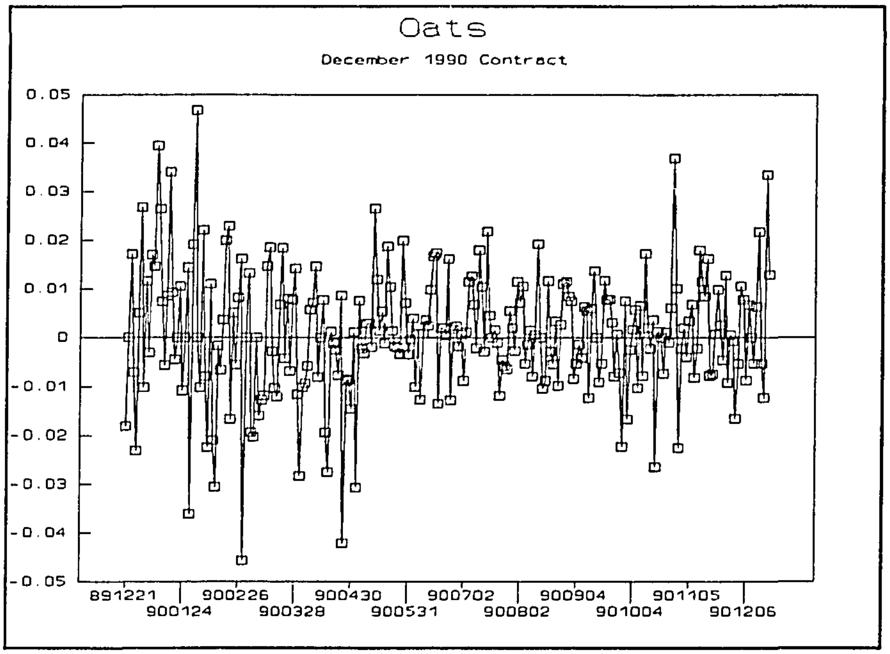
Hence, once again, we <u>reject</u> the null hypothesis.

v. Summary and Conclusions

To sum up our findings, we have found strong empirical evidence that the magnitude of volatility-adjusted futures price movement, as defined above, exhibits dependence on the industry group that the contracts are written on. Using the notation employed in the previous section, the average Φ-value is largest in the case of agriculturals. Metal contracts come second in the ordering. Financials display the smallest magnitude of Φ. The explanation of these differences is simply the fact that futures contracts can be written on a variety of commodities, which are categorized under three broad headings in this paper: agriculturals, metals/energy and financials.

In this paper we characterize (some of) the main differences between these industries, which are significant according to the economic theory. Then, we show that based on those distinctions we are able to reach important conclusions about differences in the average behavior of volatility-adjusted futures price movements among the industries.

The empirical testing performed in Section V fully supports our theoretical findings of earlier sections. Namely, as our parametric and non-parametric tests demonstrated we find that the Φ-ratio is highest in agricultural commodities, which is followed by metals and finally, the least magnitude is observed in the case of financials, among the three industry groups.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December 1990 Contract
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0.04
0.03
0.02
0.01
0
-0.01
-0.02
-0.03
-0.04
-0.05
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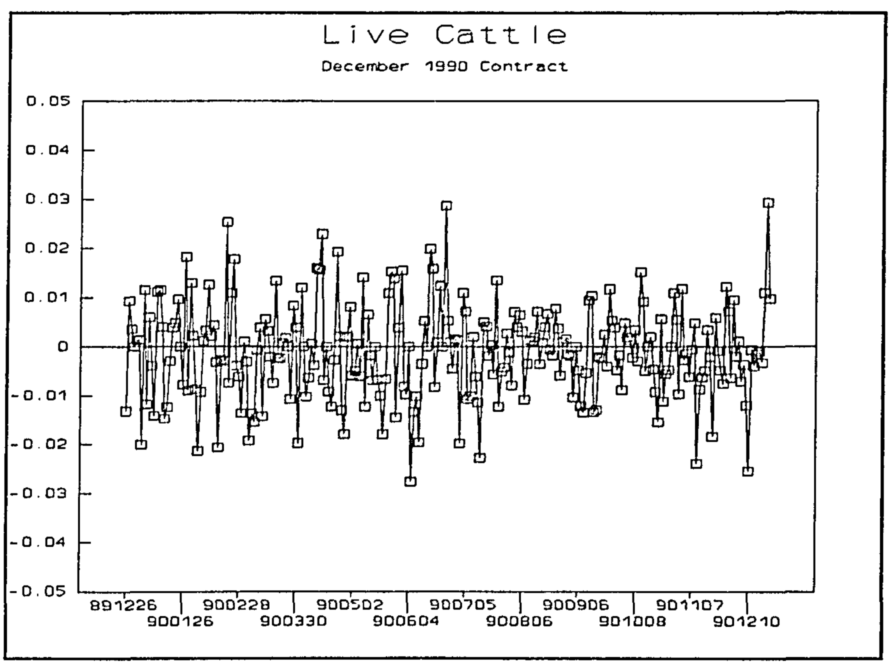
900226
900328
900430
900531
900702
900802
900904
901004
901105
901206


Live Cattle
December 1990 Contract
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0.04
0.03
0.02
0.01
0
-0.01
-0.02
-0.03
-0.04
-0.05
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900228
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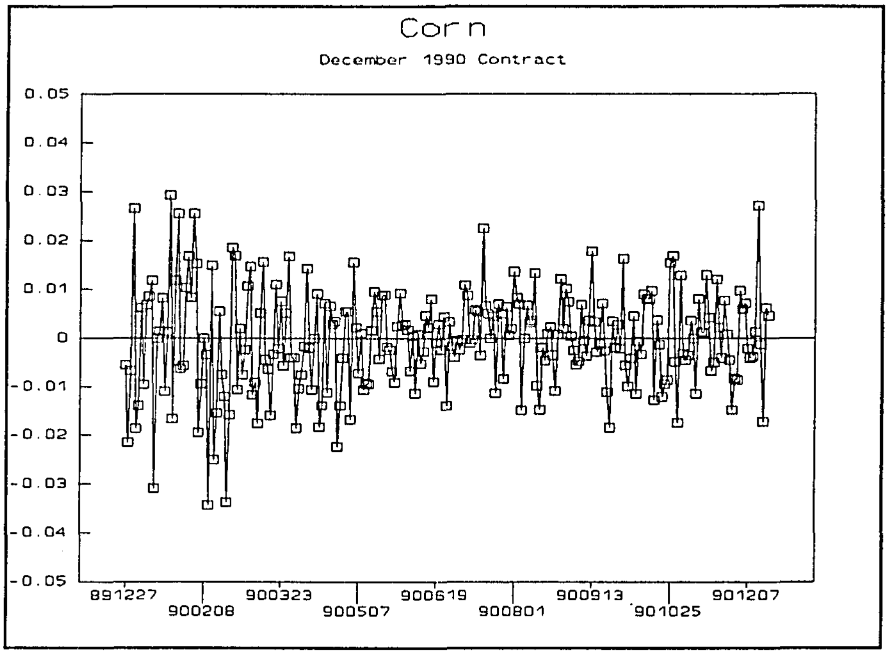

Corn
December 1990 Contract
0.05
0.04
0.03
0.02
0.01
0
-0.01
-0.02
-0.03
-0.04
-0.05
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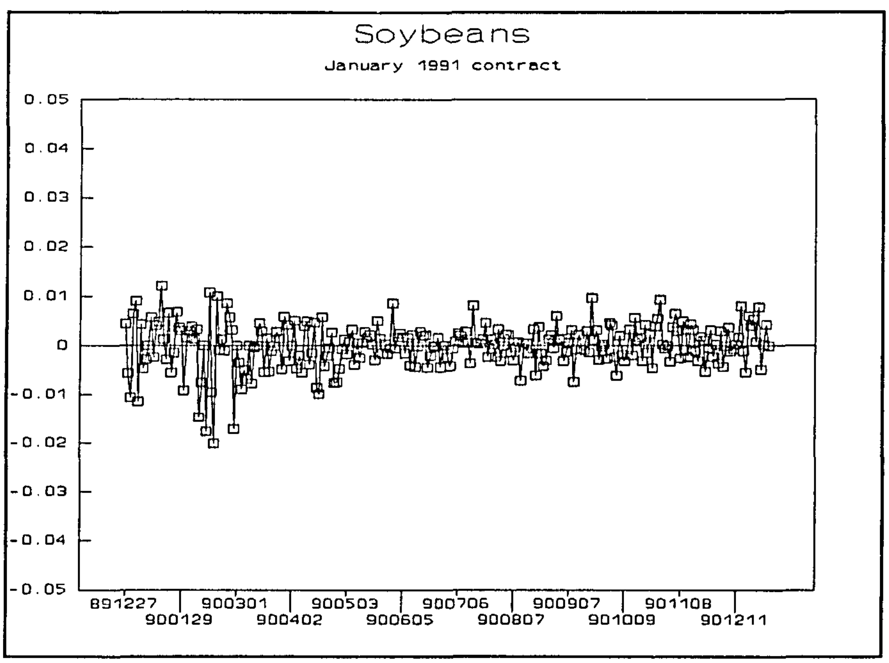

Soybeans
January 1991 contract
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0.04
0.03
0.02
0.01
0
-0.01
-0.02
-0.03
-0.04
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900907
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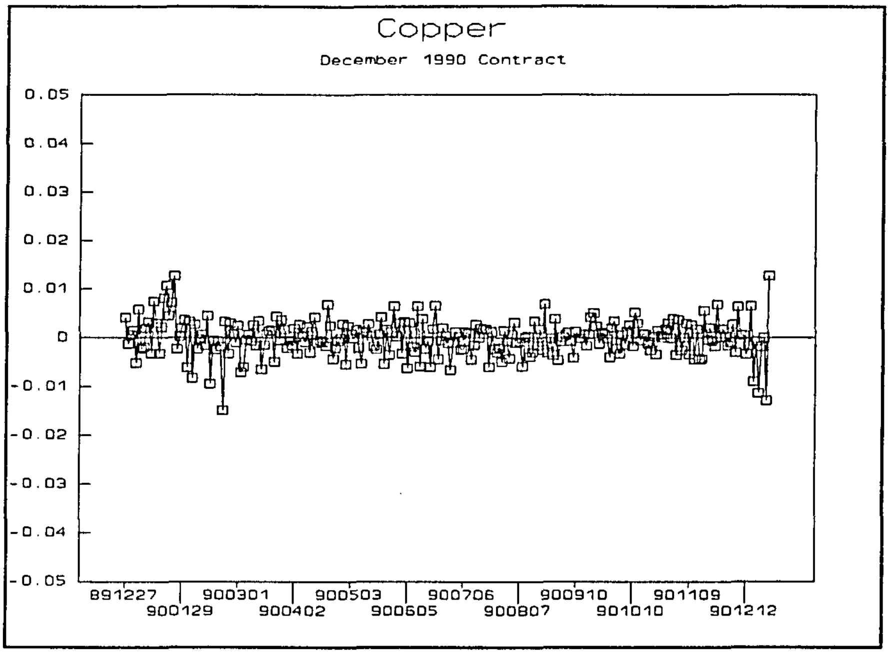
Copper
December 1990 Contract
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0.04
0.03
0.02
0.01
0
-0.01
-0.02
-0.03
-0.04
-0.05
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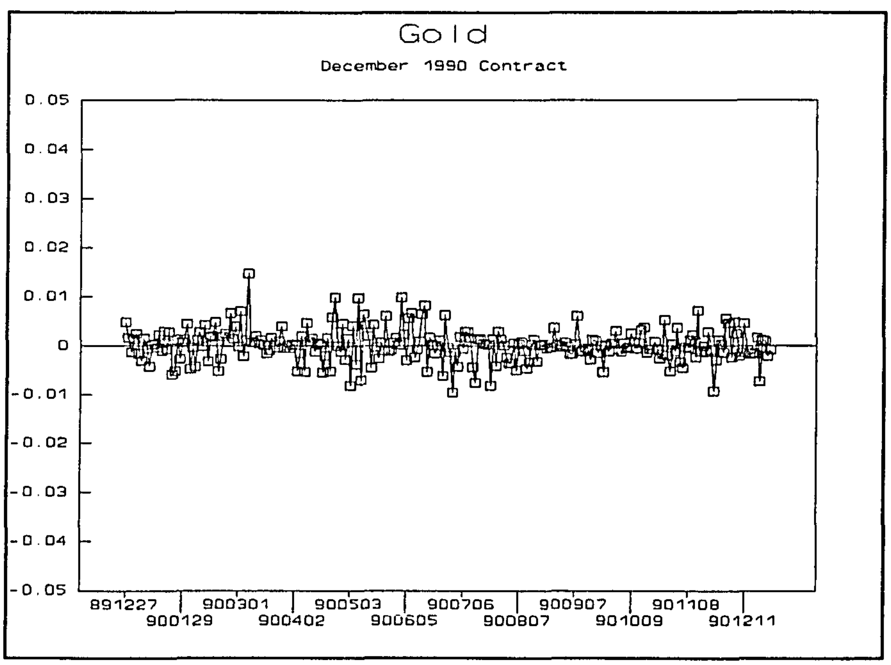
Gold
December 1990 Contract
0.05
0.04
0.03
0.02
0.01
0
-0.01
-0.02
-0.03
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-0.05
891227
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900301
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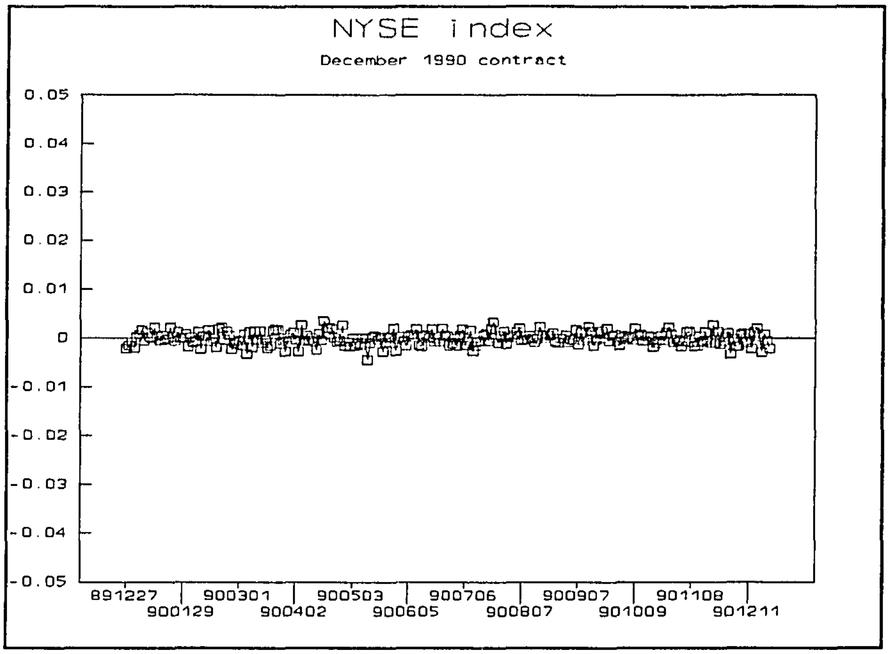
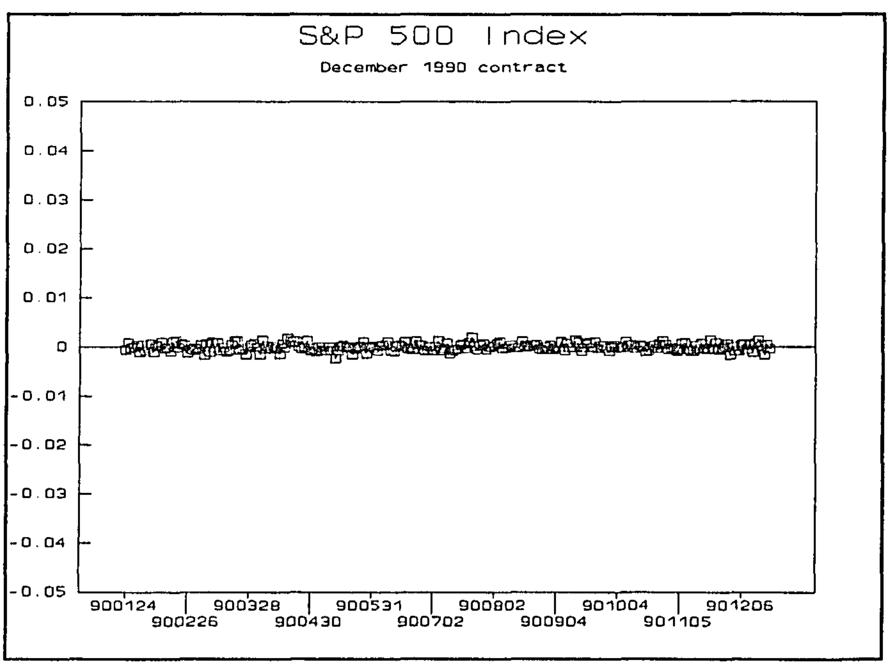

NYSE index
December 1990 contract
0.05
0.04
0.03
0.02
0.01
0
-0.01
-0.02
-0.03
-0.04
-0.05
891227
900129
900301
900402
900503
900605
900706
900807
900907
901009
901108
901211


S&P 500 Index
December 1990 contract
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0.04
0.03
0.02
0.01
0
-0.01
-0.02
-0.03
-0.04
-0.05
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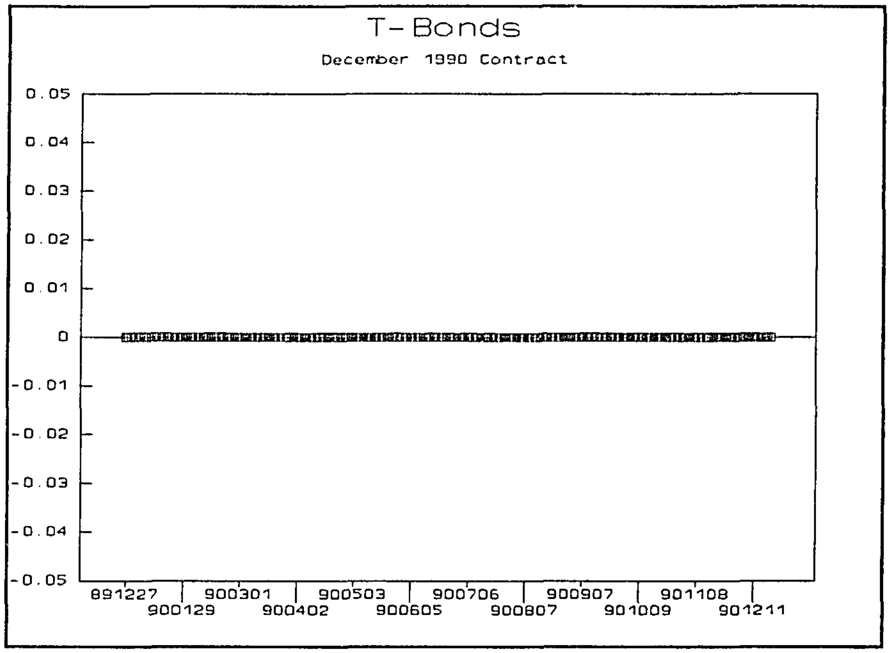
T-Bonds
December 1990 Contract
0.05
0.04
0.03
0.02
0.01
0
-0.01
-0.02
-0.03
-0.04
-0.05
891227
900129
900301
900402
900503
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900706
900807
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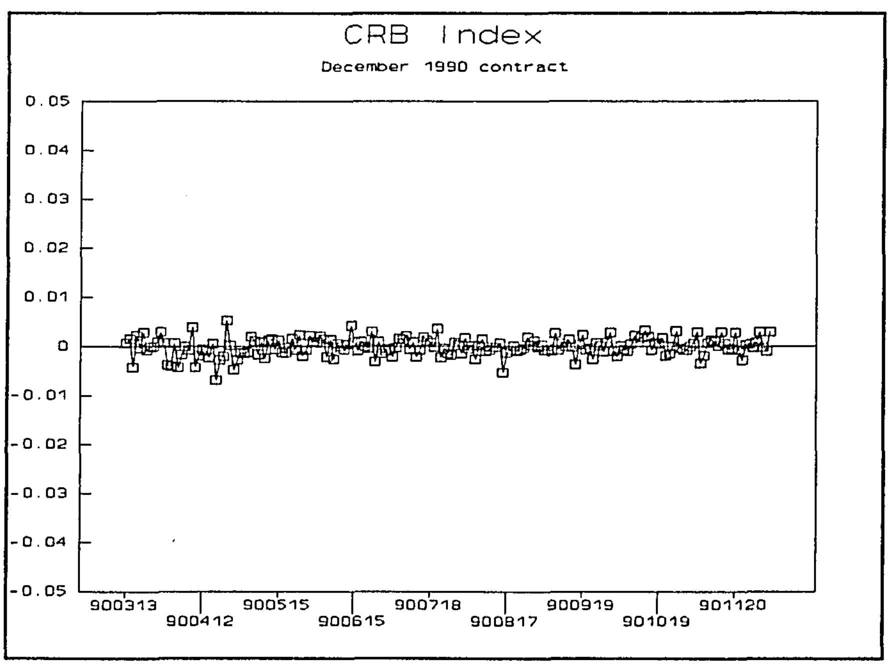
CRB Index
December 1990 contract
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0.04
0.03
0.02
0.01
0
-0.01
-0.02
-0.03
-0.04
-0.05
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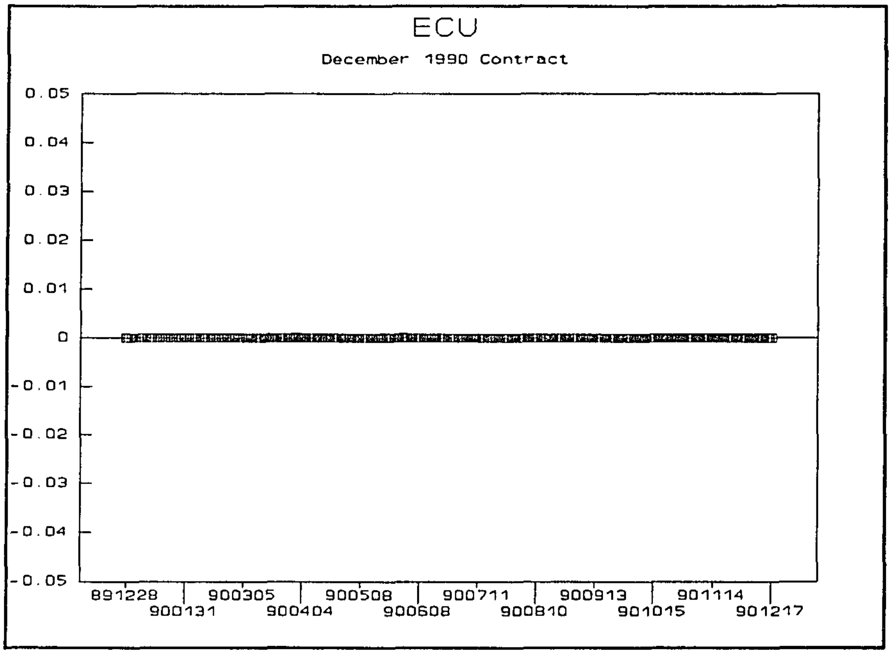
901120


ECU
December 1990 Contract
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0.04
0.03
0.02
0.01
0
-0.01
-0.02
-0.03
-0.04
-0.05
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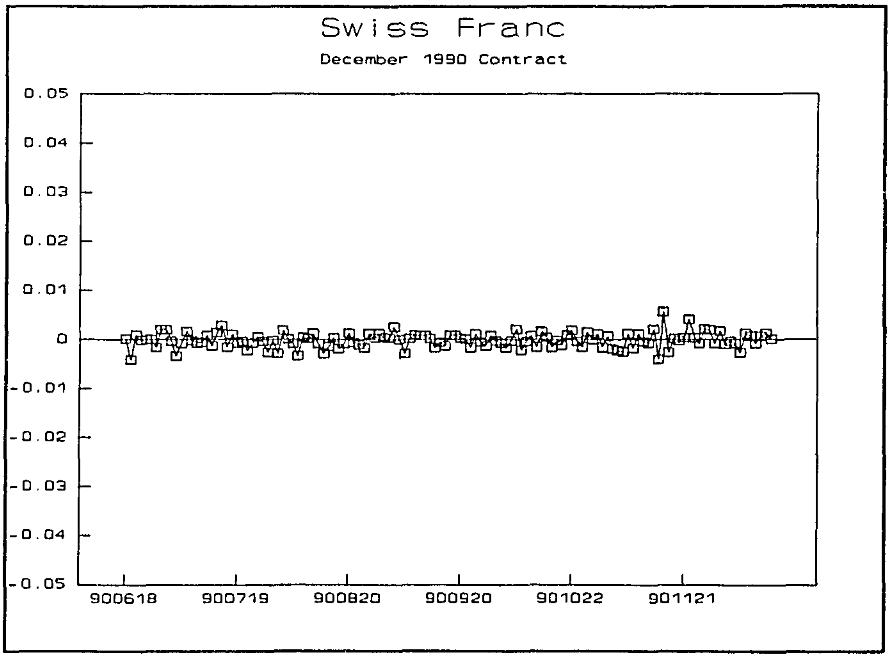
Swiss Franc
December 1990 Contract
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0.04
0.03
0.02
0.01
0
-0.01
-0.02
-0.03
-0.04
-0.05
900618
900719
900820
900920
901022
901121

Part III

Effect of Speculation in Futures Markets on Spot Price Variability

i. Introduction

Does speculation in futures markets tend to stabilize or destabilize spot prices, in particular, what can we say about the effects of speculation on metal prices? The effects of speculation in Futures Markets have been widely discussed both empirically and theoretically at least for the last four decades by now.

One of the earlier positions about this issue was one of Milton Friedman's (1953), where, basically he was suggesting that speculators were making profits by pushing prices towards the equilibrium level, and, thus reducing the fluctuations of the futures price around the market's expectation of the future spot price.

Anne Peck argued in 1976 also in favor of this view. She distinguished among two distinct roles a futures market can perform, (i.e. facilitating the storage decision and facilitating the production decision) and she concluded that spot prices are more stable with futures trading (and speculation) than without it, at least in the long run.

In addition to that Cox's empirical study in 1976 also supports the view that futures trading does not increase spot price variability.

On the other hand, in 1977 O.D. Hart devised a model in which he showed that a sophisticated speculator could make money by exploiting the naive forecasting rules of less sophisticated agents and thus destabilizing futures price whereby realizing a profit out of these transactions.

Among the more recent contributions one can mention Newbery (1984, 1987), Kawai (1983), Sarris (1983), Turnovsky and Campbell (1985), McCafferty and Driskill (1980), Driskill and McCafferty (1982), and finally Driskill, McCafferty and Sheffrin (1991).

Sarris (1983), using a dynamic model that integrates micro behavior of commodity and future speculators into the simultaneous determination of cash and futures prices in a storable commodity market, demonstrates that futures markets tend to stabilize the period-to-period fluctuations in the cash-market commodity prices, in both, the short run and the long run if all speculators are risk averse and inventory holders, do not change their risk attitudes after the introduction of futures trading. Turnovsky and Campbell (1985) also exhibit similar results using their simulation approach, i.e. futures markets always improve the stability of the spot market.

Newbery (1984 and 1987) makes a point that if a producer has market power, then, even under rational expectations, it may be in the producer's advantage to destabilize the cash market through speculation.

Kawai (1983), however, shows that even if one assumes common knowledge and rational expectations and no market power, still the result need not to be that speculation will be stabilizing the cash prices. Kawai's findings are also in accordance with Driskill and McCafferty (1982) and Driskill, McCafferty and Sheffrin (1991). Namely, as Kawai puts it: " the identification of the source of random disturbances is crucial (...) if the demand disturbance is the primary random element in the commodity market, then the introduction of a futures market tends to stabilize spot prices; whereas if the inventory demand disturbance is preponderant, a futures market tends to be price-destabilizing. (...) The role of production disturbances is generally ambiguous."

Obviously, if producers and speculators are risk-averse, the slopes of the relevant supply and inventory demand functions depend upon the degree of price stability, and this, in turn, varies with the introduction of the futures markets. To incorporate these important aspects adequately, it is necessary to derive the behavioral relationships for agents from underlying optimizing considerations. Despite all this progress in this line of research about futures markets, we observe that the existing literature, while properly assigning a role for growers, does not consider the possible existence intermediate processors of the commodity[1], when modelling the issue at

[1] Introduction of processors into the theory in some cases reveals very different predictions from those of the classical literature and some of the more recent papers. A good example of this case is Hirshleifer's

hand.

Hence, in this study we would like to incorporate the case of existence of a second production stage into the existing body of literature about cash-price volatility and observe its effects. After having seen the theoretical implication of the model, as a next step, using our data we would like to determine, empirically, whether or not speculation increases cash price volatility for the commodity(ies) we will be focusing on.

(1988) paper where in contrast with Hicks's theory he shows that processors tend to take long positions and that existence of a second production stage promotes downward price bias.

ii. Model

The model which will be employed in this paper is a Rational Expectations model, in the sense of Muth (1961), which assumes that spot and futures prices are determined simultaneously in an environment where uncertainty exists due to mainly three causes:[2]

(i) Consumer demand for the raw good as well as the processed good is uncertain.

(ii) Storage costs fluctuate over time.

(iii) The (spot) price spread between the raw and processed good is subject to uncertainty.

There are four principal actors in the model: (i) producers, (ii) inventory holders, (iii) processors and, finally (iv) consumers of the final commodity.

Producers

The producers in this model produce the raw commodity and sell it in the spot markets. In addition to that they also sell short futures contracts. If we denote q_{t+1} as the amount of raw commodity produced and p_{t+1} as its spot price and if we let x_{t+1} stand for the amount sold forward in the futures market, and f_t for the futures price, we can

[2] Our model is very similar to the one of D-M-S, however ours allows for the existence of a second stage in production, thus, increasing the flexibility of the model and also allowing for a source of uncertainty stemming from the price spread.

write producer profits in the spot market and profits due to futures trading as follows:

$$\begin{aligned} R^{p}_{t+1} &= p_{t+1}q_{t+1} - C(q_{t+1}) \\ R^{p,f}_{t+1} &= x^{p}_{t+1}(f_t - p_{t+1}) \\ &\textit{where,} \\ C(q_{t+1}) &= w_t L_t \end{aligned} \tag{1}$$

In the equations above the variable "C" stands for the cost function of producers, where "L" indicates quantities of inputs and "w" stands for the respective input price. Then the total profit function becomes:

$$\pi^{p}_{t+1} = P_{t+1}(q_{t+1} - x_{t+1}) + f_t x_{t+1} - C(q_{t+1}) \tag{2}$$

If we assume that all agents (including producers) have a utility function of the form:

$$U = \psi - \exp(-\alpha\pi) \tag{3}$$

where ψ and α are constants; α is the coefficient of risk aversion. Then maximizing expected utility is equivalent to maximizing:

$$E_t U_{t+1} = E_t \pi_{t+1} - \left(\frac{\alpha}{2}\right) V(\pi_{t+1}) \tag{4}$$

Then as it has been noted by Holthausen (1979), and Feder, Just and Schmitz (1980), the first-order condition (F.O.C.) for producers implies that production is a function of current input prices and current futures price, which could be stated as:

$$Q_{t+1} = \kappa f_t \quad \textit{where } \kappa > 0 \tag{5}$$

In addition to that, the second equation of the F.O.C.'s state that:

$$x^{p}_{t+1} - Q_{t+1} = \frac{f_t - E_t P_{t+1}}{\alpha\sigma^2_p} \tag{6}$$

where $\sigma_p{}^2$ stands for the estimate of the variance of spot prices, and E stands for the expectations operator.

Inventory Holders

Inventory holders carry the raw commodity over time and take speculative positions in the futures markets. Their net revenues, i.e. profits, in the spot markets and in the futures market are respectively,

$$R^{I}_{t+1} = I_t(p_{t+1} - p_t) - C_t I_t$$
$$R^{I,f}_{t+1} = F^{I}_t(f_t - p_{t+1}) \tag{7}$$

In [7] "I_t" and "C_t" stand for the amount of inventory carried over time, and its cost, respectively. Furthermore "F" indicates the magnitude of inventory holders' futures position. Given $I_t \geq 0$, then solving the Lagrangian the F.O.C.'s are

$$F^{I}_t - I_t = \frac{f_t - E_t p_{t+1}}{\alpha \sigma^2_p}$$
$$p_t + C_t - f_t = 0 \quad if\ I_t > 0$$
$$p_t + C_t - f_t \geq 0 \quad if\ I_t \geq 0 \tag{8}$$

The linear cost structure generates a perfectly elastic inventory demand. It has been shown that as long as $C_t = f_t - p_t$ and $I_t \geq 0$, we find that $C_t = f_t - p_t$ always emerges as a market equilibrium conditions. (See: D-M-S 1991 and M-D 1980). Hence in the following parts we shall assume that $C_t = f_t - P_t$ and $I_t > 0$.

Processors

The processors in our model purchase the raw commodity to process and transform it into a more sophisticated commodity, thereby incurring a cost.

Their profits in the spot markets can be shown to be equal to:

$$R_{t+1}^{proc} = (p_{t+1}^{A} - p_{t+1})\, q_{t+1}^{A} - C(q_{t+1}^{A})$$
$$\text{where,} \tag{9}$$
$$C(q^{A}) = \gamma_0 q^{A} + \frac{\gamma_1}{2}(q^{A})^2$$

where their net revenue from futures trading has the same form as of the producers or the inventory holder, i.e.

$$R_{t+1}^{proc,f} = x_t^{proc}(f_t - P_{t+1}) \tag{10}$$

In equations [9] and [10] γ_0, γ_1 are positive scalars and x_t^{proc} stands for the futures position of processors.

Since futures position can be written as:

$$x^{proc} = -\frac{cov[R_{t+1}^{proc}, (P_{t+1} - f_t)] + \frac{f_t - P_{t+1}}{\alpha}}{var(P)} \tag{11}$$

or:

$$x^{proc} = -\frac{1}{\sigma_p^2} \cdot \{ q^{A} cov(p^{A}, p) - q^{A}\sigma_p^2 + (\frac{1}{\alpha})(f_t - E_t p_{t+1}) \} \tag{12}$$

alternatively:

$$x^{proc} = -q^{A}[\frac{cov(p^{A}, p)}{\sigma_p^2} - 1] + (\frac{1}{\alpha\sigma_p^2})(f_t - E_t p_{t+1}) \tag{13}$$

<u>Final Consumers</u>

We pose a downward-sloping demand curve for the processed commodity, which we shall denote as q^A:

$$q^{A} = a - bp^{A} + v \tag{14}$$

where p^A is the price of the processed good and ν is a disturbance term, which is assumed to be white noise.

Now, making use of [9], and the perfectly competitive pricing condition, i.e. p = MC, we obtain:

$$p^A - p = \gamma_0 + \gamma_1 q^A + \zeta \tag{15}$$

where ζ is a white noise disturbance term representing the uncertainty in the price spread.

Note that [14] can be written as:

$$p^A = (\frac{1}{b})\ [a - q^A + \nu] \tag{16}$$

to obtain the inverse demand function, which, because of the relationship posed in [15], could be expressed as:

$$p = (\frac{a}{b} - \gamma_0) - q^A(\frac{1}{b} + \gamma_1) + \frac{\nu}{b} - \zeta \tag{17}$$

If we can postulate a linear relationship between the output of the raw material and the one of the processed commodity, of the form $q = \Phi q^A$, then we obtain:

$$q = \frac{\phi}{(\frac{1}{b} + \gamma_1)}\ [(\frac{a}{b} - \gamma_0) - p + \frac{\nu}{b} - \zeta] \tag{18}$$

which can be thought of:

$$q = a^* - b^* p + \nu^* \tag{19}$$

where;

$$a^{*} = (a - b\gamma_0) \cdot \phi(\frac{1}{1 + b\gamma_1})$$
$$b^{*} = b \cdot \phi(\frac{1}{1 + b\gamma_1}) \qquad (20)$$
$$v^{*} = (v - b\zeta) \cdot \phi(\frac{1}{1 + b\gamma_1})$$

We observe that x^{proc}, i.e. [13] could alternatively be written as

$$x^{proc} = -q^{A}[(\frac{\sigma_v^2}{\sigma_v^2 + b^2\sigma_\zeta^2}) - 1] + \frac{1}{\alpha\,\sigma_p^2}(f_t - E_t p_{t+1}) \qquad (21)$$

Note that if $b^2\sigma^2_\zeta \rightarrow 0$, then $x^{proc} = x^{speculator}$. In other words, the behavior of processors cannot be distinguished from the behavior of speculators, who have no physical involvement in the industry, necessarily.

This condition stated above can be fulfilled, if, either:

(i) $\sigma^2_\zeta \rightarrow 0$, i.e. if the uncertainty in the price spread vanishes.

(ii) $b^2 \rightarrow 0$, i.e. if the demand curve for the processed good is very inelastic.

It can easily be shown that for the futures market to clear the following condition should be satisfied:

$$X^{P}_{t+1} + F^{I}_{t} + X^{proc}_{t+1} = 0 \qquad (22)$$

Defining the speculative intensity[3], i.e. η as:

$$\eta = \frac{1}{\sigma_p^2}\left(\frac{1}{\alpha_p} + \frac{1}{\alpha_{proc}} + \frac{1}{\alpha_I}\right) \tag{23}$$

We can restate [22], using the F.O.C.'s for producers and inventory holders and equations for the futures positions of processors, producers, and inventory holders, and finally [5], to obtain:

$$\begin{aligned} &\eta\,(f_t - E_t p_{t+1}) + I_t + \frac{\kappa f_t}{\phi}[\phi - \omega] = 0 \\ &\textit{where,} \\ &\omega = \left(\frac{\sigma_v^2}{\sigma_v^2 + b^2\sigma_\zeta^2}\right) - 1 \end{aligned} \tag{24}$$

The spot market equilibrium is reached, on the other hand, when the demand for additional inventory holding equals the difference between the supply and demand of the raw commodity.

Thus:

$$\begin{aligned} &D_t + (I_t - I_{t-1}) - Q_t = 0, \\ &\textit{where;} \\ &I_{t-1} = F_{t-1}^I - \frac{\kappa^I}{2\sigma_p^2}(f_{t-1} - E_{t-1}p_t) \\ &Q_t = X_t - \frac{\kappa^P}{2\sigma_p^2}(f_{t-1} - E_{t-1}p_t) \end{aligned} \tag{25}$$

or,

$$D_t + I_t - F_{t-1}^I - X_t^P + \frac{\kappa^I + \kappa^P}{2\sigma_p^2}(f_{t-1} - E_{t-1}p_t) = 0 \tag{26}$$

---

[3] Note that in equations (21), (8) and (6) η can be thought as a measure of sensitivity of agents futures' positions with respect to any spread between the futures price and expected spot price. Thus, we may call an increase in η as an increase in speculative intensity

which can be written as:

$$(a^{*}-b^{*}p_t+v_t^{*}) + I_t - \omega q_t^{A} + \eta(f_{t-1} - E_{t-1}p_t) = 0 \tag{27}$$

Manipulating the equation above we get:

$$(a^{*}-b^{*}p_t+ v_t^{*})\cdot[1 - \frac{\omega}{\phi}] + I_t + \eta(f_{t-1} - E_{t-1}p_t) = 0 \tag{28}$$

Defining,

$$\theta = \phi\cdot(\frac{1}{1+b\gamma_1}) \tag{29}$$

where,

(*i*) $(\cdot) \in [0, 1]$
(*ii*) $\phi > 0$

we can write this relationship as:

$$[1-\frac{\omega}{\phi}]\cdot\{\theta[(a-b\gamma_0) - bp_t + (\nu-b\zeta)]\} + I_t + \eta(f_{t-1}-E_{t-1}p_t) = 0$$

which, alternatively, can be written as:

$$(\mu-\beta p_t+u_t) + I_t + \eta(f_{t-1}-E_{t-1}p_t) = 0, \tag{30}$$

where:

$$\mu = [1-\frac{\omega}{\phi}]\cdot\theta(a-b\gamma_0),$$
$$\beta = [1-\frac{\omega}{\phi}]\cdot\theta b,$$
$$u_t = [1-\frac{\omega}{\phi}]\cdot\theta(\nu_t-b\zeta_t).$$

Taking the difference between [24] and [30], we reach the following:

$$\kappa f_t\cdot[1-\frac{\omega}{\phi}] - \mu + \beta p_t - u_t+ \eta(f_t-f_{t-1}) - \eta(E_tp_{t+1}-E_{t-1}p_t) = 0 \tag{31}$$

If we make use of the relationship between the spot and futures prices and solve the equation for the spot price we obtain:

$$\textit{Since:}\quad f_t = p_t + c_t \tag{32}$$

$$p_t = [\frac{1}{\kappa'+\beta+\eta}]\cdot\{\eta p_{t-1} - c_t(\kappa'+\eta) + c_{t-1}\eta + \mu + u_t + \eta(E_t p_{t+1} - E_{t-1}p_t)\}$$

Now, the above equation reveals,

$$\textit{Defining}\quad c_t = \overline{c} + e_t\,,\qquad \kappa' = \kappa\cdot[1-\frac{\omega}{\phi}]\,;$$

$$p_t = [\frac{1}{\kappa'+\beta+\eta}] \tag{33}$$

$$\cdot\{(\mu-\kappa'\overline{c}) + \eta p_{t-1} + \kappa' e_t + \eta e_t - \eta e_{t-1} + u_t + \eta(E_t p_{t+1} - E_{t-1}p_t)\}$$

which we can represent as:

$$p_t = \pi_0(\mu-\kappa'\overline{c}) + \pi_1 p_{t-1} + \pi_2 e_t + \pi_3 e_{t-1} + \pi_4 u_t \tag{34}$$

where π_i's can be solved by the well-known technique of Muth (1961). Following D-M-S, the solutions for the π_i's are:

$$\begin{aligned}
\pi_0 &= \frac{\kappa'}{\kappa'+\beta}\\
\pi_1 &= \frac{\eta(1-\pi_1)}{\eta(1-\pi_1)+\kappa'+\beta}\\
\pi_2 &= \frac{\eta(1-\pi_1)+\kappa'}{\eta(1-\pi_1)+\kappa'+\beta}\\
\pi_3 &= \frac{-\eta(1+\pi_3)}{\eta(1-\pi_1)+\kappa'+\beta}\\
\pi_4 &= \frac{1}{\eta(1-\pi_1)+\kappa'+\beta}
\end{aligned} \tag{35}$$

and

$$\pi_1 = 1 + \frac{\alpha+\beta}{2\eta} - \frac{\alpha+\beta}{2\eta}[\frac{1+4\eta}{\alpha+\beta}]^{\frac{1}{2}} \tag{36}$$

which again by D-M (1982) varies monotonically from zero to one as η

varies from zero to infinity.

Once π_1 is known other π_i's can all be solved as functions of π_1:

$$
\begin{aligned}
&[\eta(1-\pi_1) + (\kappa'+\beta)] = \frac{\eta(1-\pi_1)}{\pi_1} \\
&\pi_1(\kappa'+\beta) = \eta(1-\pi_1) - \eta\pi_1(1-\pi_1) \\
&\pi_1(\kappa'+\beta) = \eta - 2\eta\pi_1 + \eta\pi_1^2 = \eta(1-\pi_1)^2 \\
&\therefore \quad \frac{\eta(1-\pi_1)}{\pi_1} = \frac{\kappa'+\beta}{1-\pi_1}
\end{aligned}
\tag{37}
$$

$$
\begin{aligned}
&[\eta(1-\pi_1) + \kappa'+\beta]\pi_3 = -\eta(1+\pi_3) \\
&\frac{\eta(1-\pi_1)}{\pi_1}\pi_3 = -\eta(1+\pi_3) \\
&\therefore \quad \pi_3 = -\pi_1
\end{aligned}
\tag{38}
$$

$$
\begin{aligned}
&\pi_2 = \pi_1 + \frac{\kappa'}{\eta(1-\pi_1) + \kappa'+\beta} = \pi_1 + \frac{\kappa'\pi_1}{\eta(1-\pi_1)} \\
&\pi_2 = \pi_1 + \frac{\kappa'(1-\pi_1)}{\kappa'+\beta} \\
&\therefore \quad \pi_2 = \frac{\kappa'+\beta\pi_1}{\kappa'+\beta}
\end{aligned}
\tag{39}
$$

$$
\begin{aligned}
&\because \quad [\eta(1-\pi_1) + (\kappa'+\beta)] = \frac{\eta(1-\pi_1)}{\pi_1} \quad \textit{and} \quad \frac{\eta(1-\pi_1)}{\pi_1} = \frac{\kappa'+\beta}{1-\pi_1} \\
&\therefore \quad \pi_4 = \frac{(1-\pi_1)}{\kappa'+\beta}
\end{aligned}
\tag{40}
$$

The link between the futures price and the spot price is given by the F.O.C of inventory holders, i.e. $C_t = f_t - p_t$, thus the stochastic process of the futures price is given by:

$$\because \quad e_t = \bar{c} - c_t \quad \text{and} \quad f_t = p_t + c_t$$

$$f_t = \pi_0'\bar{c} + \pi_1 f_{t-1} + (1-\pi_2)\, c_t + \pi_4 u_t \tag{41}$$

$$\text{where:} \quad \pi_0' = \pi_0 + \pi_2 + \pi_3$$

This relationship above provides us with a testable equation which shall be employed to verify the suggestion that the behavior of actual futures price data over time can be represented using the setup described so far. In other words, if the estimated coefficients of lagged futures price and cost–of–carry do –indeed– obey the theoretical relationship derived from the model presented above, then, we can safely conclude that our theoretical framework is able to correctly represent the stochastic movements of futures prices over time. Thus, if that is the case, we can make use of the results of our theoretical model to make inferences about the effects of futures speculation on cash price volatility.

Let us leave the empirical testing of the model aside for the moment, and try to reach theoretical conclusions based on our setup. As a next step, we notice that we can write the spot price as a function of all current and past errors, i.e.

$$\begin{aligned} p_t = {} & \pi_2 e_t + \pi_1(\pi_2-1)\, e_{t-1} + \pi_1^2(\pi_2-1)\, e_{t-2} + \ldots + \pi_1^n(\pi_2-1)\, e_{t-n} \\ & + \pi_4 u_t + \pi_1\pi_4 u_{t-1} + \pi_1^2\pi_4 u_{t-2} + \ldots + \pi_1^n\pi_4 u_{t-n} + \ldots \end{aligned} \tag{42}$$

Hence, the variance of the spot price is:

$$\sigma_p^2 = \{ \pi_2^2 + \frac{(1-\pi_2)^2\pi_1^2}{1-\pi_1^2} \}\sigma_\epsilon^2 + \{\frac{\pi_4^2}{1-\pi_1^2}\}\sigma_u^2 \tag{43}$$

assuming that $\mathrm{Cov}(\epsilon,u) = 0$.

Substituting the values of the π_i's , we obtain:

$$\sigma_p^2 = \{ (\frac{\kappa'+\beta\pi_1}{\kappa'+\beta}) + \frac{[1 - \frac{\kappa'+\beta\pi_1}{\kappa'+\beta}]^2 \pi_1^2}{1-\pi_1^2} \} \sigma_\epsilon^2 + \{ [\frac{(1-\pi_1)}{\kappa'+\beta}]^2 \frac{1}{1-\pi_1^2} \} \sigma_u^2 \tag{44}$$

where the (i)'st and (ii)nd terms on the RHS can be written as:

$$\begin{aligned} \{i\} &= \frac{1}{(\kappa'+\beta)^2} \cdot [(\kappa')^2 + 2\kappa'\beta\pi_1 + \frac{2\beta^2\pi_1^2}{1+\pi_1}] \\ \{ii\} &= \frac{1}{(\kappa'+\beta)^2} \cdot [\frac{(1-\pi_1)}{(1+\pi_1)}] \end{aligned} \tag{45}$$

We notice that the derivative of π_1 w.r.t. η is positive, i.e.

$$\frac{\partial\pi_1}{\partial\eta} = \frac{(1-\pi_1)^2}{\kappa'+\beta+2\eta(1-\pi_1)} > 0 \tag{46}$$

Substituting for the variables we get:

$$\begin{aligned} \sigma_p^2 = {} & [\frac{1}{(\kappa'+\beta)^2}] \cdot \\ & \{ [\kappa'[1-\frac{\omega}{\phi}]^2 + 2\kappa'[1-\frac{\omega}{\phi}]^2\theta b\pi_1 + \frac{2[1-\frac{\omega}{\phi}]^2\theta^2 b^2\pi_1^2}{1+\pi_1}] \sigma_\epsilon^2 \\ & + [\frac{1-\pi_1}{1+\pi_1}] \sigma_u^2 \} \end{aligned} \tag{47}$$

which gives us the steady-state variance of the spot prices. Taking its derivative w.r.t. π_1, we obtain:

$$\frac{\partial \sigma_p^2}{\partial \pi_1} = [\cdot]$$

$$\cdot \{ [2\kappa [1-\frac{\omega}{\phi}]^2 \theta b + \frac{4[1-\frac{\omega}{\phi}]^2 \theta^2 b^2 \pi_1}{1+\pi_1} - \frac{2[1-\frac{\omega}{\phi}]^2 \theta^2 b^2 \pi_1^2}{(1+\pi_1)^2}] \sigma_\epsilon^2 \quad (48)$$

$$+ [\frac{-1}{1+\pi_1} - \frac{1-\pi_1}{(1+\pi_1)^2}] \sigma_u^2 \}$$

or

$$\frac{\partial \sigma_p^2}{\partial \pi_1} = \frac{1}{(1+\pi_1)^2} [\cdot] \cdot \quad (49)$$

$$\{[1-\frac{\omega}{\phi}]^2 (2\kappa\theta b (1+\pi_1)^2 + 4\theta^2 b^2 \pi_1 (1+\pi_1) - 2\theta^2 b^2 \pi_1^2] \sigma_\epsilon^2 - 2\sigma_u^2 \}$$

Defining,

$$\sigma_\epsilon^2 = \lambda \cdot \sigma_u^2 \quad (50)$$

We can write the equation above as:

$$\frac{\partial \sigma_p^2}{\partial \pi_1} = [\cdot] \frac{2\sigma_u^2}{(1+\pi_1)^2} \quad (51)$$

$$\cdot \{ [1-\frac{\omega}{\phi}]^2 \cdot [\kappa\theta b (1+\pi_1)^2 + 2\theta^2 b^2 \pi_1 + \theta^2 b^2 \pi_1^2] \lambda - 1 \}$$

and observe that:

$$\textit{In order } \frac{\partial \sigma_p^2}{\partial \eta} \geq 0: \quad \frac{\partial \sigma_p^2}{\partial \pi_1} \geq 0 \text{ , } \textit{and thus } \{\cdot\} \geq 0 \quad (52)$$

which alternatively could be stated as:

$$\lambda [1-\frac{\omega}{\phi}]^2 \theta b \cdot [\kappa(1+\pi_1)^2 + 2\theta b \pi_1 + \theta b \pi_1^2] \geq 1 \quad (53)$$

Recalling the definition of ß, this condition becomes:[4]

$$\lambda\beta\cdot[\,\kappa[1-\frac{\omega}{\phi}](1+\pi_1)^2 + 2\beta\pi_1 + \beta\pi_1^2\,] \geq 1 \qquad (54)$$

where: $\lambda\beta > 0$

which by using equations [33] and [39] could alternatively be expressed as:

$$\lambda\beta^2\cdot[\,(\frac{\pi_1-\pi_2}{\pi_2-1})\cdot(1+\pi_1)^2 + 2\pi_1 + \pi_1^2\,] \geq 1 \qquad (55)$$

For simplicity, we will refer to the term on the left-hand-side of [55] as Ω. Now, since by definition:

$$\lambda = \frac{\sigma_\epsilon^2}{\sigma_u^2}\,,\ or\quad \lambda = \frac{\sigma_\epsilon^2(1-\pi_1)^2}{V(\pi_4 u_t)\,(\kappa'+\beta)^2} \qquad (56)$$

We are able to re-write the first term on the left-hand-side in equation [55] as:

$$\lambda\beta^2 = \frac{\sigma_\epsilon^2(1-\pi_1)^2}{V(\pi_4 u_t)\cdot[\frac{\pi_2-\pi_1}{1-\pi_2}+1]^2} \qquad (57)$$

Combining [55] with [57] we obtain a testable condition which is based on the estimates of only three parameters, i.e. π_1, $\pi2$ and the variance of $\pi4u_t$, which is nothing but the variance of the residuals in our regression equation [41]. (Note that the LHS of the condition, i.e. Ω,

---

[4] where:

$$(1-\frac{\omega}{\phi}) = (\frac{1}{\phi})\cdot[(1+\phi)-(\frac{\sigma_v^2}{\sigma_v^2+b^2\sigma_\zeta^2})]$$

is <u>independent</u> from β). In other words, the coefficients we obtain from [41] are sufficient for the proposition under investigation to be empirically tested.

One may be curious about the possible effects of the inclusion of processors to the model on the critical condition. Indeed, inclusion of processors to the model and, in addition to that allowing for uncertainty in the price spread (between the raw and processed good) ought to have an impact on the results.

By recalling equations [20], [29] and [30], i.e.

$$\nu^* = (\nu - b\zeta)\,\phi\,\left(\frac{1}{1+b\gamma_1}\right)$$
$$\theta = \phi\,\left(\frac{1}{1+b\gamma_1}\right) \qquad (58)$$
$$u = \left[1-\frac{\omega}{\phi}\right]\theta(\nu - b\zeta)$$

we can state that:

$$u_t = \left[1-\frac{\omega}{\phi}\right]\cdot\nu_t^* \qquad (59)$$

and conclude that:

$$\sigma_u^2 = \left[1-\frac{\omega}{\phi}\right]^2\cdot\sigma_{\nu^*}^2. \qquad \textit{where} \quad \left[1-\frac{\omega}{\phi}\right] \geq 1. \qquad (60)$$

Making note of the fact that in the absence of processors the u_t term in equation [31] is nothing but what we refer to as ν_t^* in [19]. In other words, the variance of u in the presence of processors is larger than the variance of u in the absence of them; so that by including processors into our model we tend to increase the magnitude of σ_u^2. In our model, based on (47) one can show that increases in speculative intensity increase that portion of spot price variance due to storage-cost shocks, σ_ϵ^2, and decrease that portion due to demand shocks, σ_u^2. Thus, an increase in the magnitude of σ_u^2 creates a theoretical _bias_ towards accepting the hypothesis that an increase in futures speculation, actually decreases spot price volatility. In other words, we are more likely to conclude that speculative activity of producers and inventory holders smoothes out price disturbances due to demand shocks.

iii. Empirical Testing

In order to test the empirical validity and the implications of the model we chose four of the most heavily traded metals in COMEX, i.e. copper, gold, silver and aluminum. As a first step, we constructed the daily near-futures series for each of them for the 1980's. (and, in some cases, depending on the availability of the data, we included 1990, as well)[5] Then we aggregated our data based on calendar weeks, to obtain weekly price series. In doing so, we took all market closings equal to or exceeding two days into account, in order to avoid any possible "market closing effects". As a third step, by eliminating one out of two observations we reduced down the number of observations of our weekly price series by half, and got a price series consisting of bi-weekly observations of weekly averages. For convenience we shall refer to that series as the bi-weekly price series, and we shall use them (See Appendix 2) to check the stability of the coefficients between the two sub-samples.

Since the model is based on steady-state variances, the application of equation [41] to daily data could lead us to erroneous conclusions due

[5] If we call the daily near-futures series that we generate as f_t, then calculating the values of f_{t-1} as mere lags of f_t would lead to theoretical inconsistencies: As an example, the lag of the first March-contract price in the series will not be the quote on the March-contract on the preceding day, but it would be the closing of the February contract on its maturity date. Thus, one has to adjust the lagged futures price series, f_{t-1}, for switching dates, (such as the one given in the example above) so that the lag of the March-futures price will still be the previous day's quote on the March-contract.

to the high amount of noise in daily prices. Therefore in order to avoid that problem we chose our unit of observation as a "week".

To be able to estimate (41) requires that all variables in the equation need to be stationary. Since Nelson and Plosser's influential article (1982) the importance of detecting the nature of the trend of a time series is widely recognized. Some series may have deterministic trends, i.e. the level of the series increases by some fixed amount each period. In this case the trend can easily be eliminated by regressing the series on some polynomial of time trend. On the other hand, some others may exhibit stochastic trends, i.e. the level of the series increases each period by some amount, however the rate of increase deviates from its average by some unpredictable random amount. A test developed by Dickey and Fuller (1981) can be used to test the hypothesis whether a particular series demonstrates stochastic or deterministic trend.

To implement the Dickey-Fuller test one has to estimate:

$$\Delta x_t = \alpha_0 + \beta_0 t + \beta_0' t^2 + \beta_1 x_{t-1} + \sum_{j=1}^{n} \delta_j \Delta x_{t-j} + e_t \qquad (61)$$

where x is in natural logs and Δ is the difference operator. The variable x has a random growth component if $\beta_0=\beta_0'=\beta_1=0$. In order to test the null hypothesis, which claims that $\beta_0=\beta_0'=\beta_1=0$, we first had to estimate the equation above for all four markets individually, i.e. copper, gold, silver and aluminum. In order to check whether or not the estimated coefficients of β_0, β_0', β_1 are indeed equal to zero or not one simply has to conduct a joint test of those coefficients and calculate the corresponding F-ratios under the null-hypothesis. According to Dickey-Fuller[6] the critical F-values (at: $\alpha=0.05$) are 6.34 for $n=250$, 6.30 for $n=500$ and 6.25 for $n=\infty$. Accordingly, if the calculated F-value is smaller than the critical value, then we cannot reject the null-hypothesis, and thus conclude that the series has a stochastic trend. On the other hand, if the estimated F-value exceeds the critical value, then we reject the null-hypothesis and conclude that the series exhibits a deterministic trend. The following table summarizes our results. The figures in the boxes are the F-values estimated under the null hypothesis of: $\beta_0=\beta_0'=\beta_1=0$.

DICKEY-FULLER TEST RESULTS

Variable \ Metal	COPPER n = 452	GOLD n = 568	SILVER n = 541	ALUMINUM n =366
(1) f_t	6.58357	3.19911	6.48256	3.95892
(2) f_{t-1}	6.67349	3.61493	7.71500	3.92289
(3) c_t	3.46759	43.10663	62.78325	3.85301

As the table clearly demonstrates:

. In the case of copper variables (1) and (2) have deterministic trends whereas variable (3) has a stochastic one.

[6] The critical values are taken from Table (vi) of their 1981 Econometrica article.

. In the case of gold variables (1) and (2) have stochastic trends where (3) has a deterministic one.

. In the case of silver all three, i.e variables (1), (2) and (3), exhibit a deterministic trend

. In the case of aluminum all of the three variables exhibit a stochastic trend.

It has been widely recognized that the conventional techniques of linear regression analysis can result in highly misleading conclusions when the variables contain stochastic trends[7], the most important among them being the overestimation of the t-values, if the levels of variables are used instead of the first differences. Obviously, the threat that this kind of mistake would be posing would be the acceptance of some variables as "significant", whereas they really are not.[8]

Therefore, in order to avoid any technical problems of this nature, whenever we fail to reject the null hypothesis of a stochastic trend, the trend elimination should be done by taking the first differences. On the other hand, in those cases where we observe that a series exhibits a deterministic trend, instead of using that variable in its levels, we ought to use the residual of a regression of that particular

---

[7] Stock and Watson, Econometrica, 1988

[8] Usually, spurious results of this nature can be detected by low Durbin-Watson statistics. (Granger, 1989)

variable on time and time-square. After the necessary trend elimination procedures, we estimate [41] for all four markets: copper, gold, silver and aluminum. The following table summarizes the results of these estimations:

REGRESSION RESULTS[9]

	COPPER	GOLD	SILVER	ALUMINUM
π_1	0.999252	0.920000	0.998000	0.909000
S.E. of π_1	0.002987	0.009946	0.001936	0.016360
π_2	0.999973	0.999885	0.999922	0.999990
S.E. of π_2	0.000002	0.000008	0.000004	0.000001
R-Square	0.996004	0.928602	0.997502	0.928755
D-W Statistic	1.979552	2.193629	1.935861	2.224948
$V(\pi_4 u_t)$	0.000049	0.000052	0.000124	0.000050
κ'	> 0	> 0	> 0	> 0
Ω	41.28416	761.8371	76.02689	4999.440

[9] For those readers who are suspicious/not fully convinced about the correctness/validity of the above-mentioned approach, (i.e. a Dickey-Fuller test, determining whether a given series exhibits a deterministic or stochastic trend) in Appendix 1, we also provide a series of alternative tables in the appendix, where the trend elimination is performed simply by taking the first differences. The first table reports the values of the coefficients and the second and third ones report Monte Carlo simulation results.

Note that with the slight exception of aluminum markets, the other three regressions for copper, gold and silver exhibit healthy statistical properties in terms of their Durbin-Watson statistics. Aluminum is the only case where the D-W statistic lies outside of the acceptable interval.

Subsequent to the estimation of [41], we check for:

(i) For the consistency of the estimated coefficients with the theory

(ii) For accuracy of the estimated coefficients

(iii) For accuracy of the numerical estimate of Ω in [55]

(i) Consistency with theory:

Estimates of π_1, π_2 and the variance of $\pi_4 u_t$ are not inconsistent with the model as long as the value of κ' exceeds zero. A non-positive value of the κ would imply a downward-sloping supply curve, or a vertical supply curve which is not sensitive to the futures prices. A positive relationship between the supply and futures prices would suggest that κ' has to have a positive sign. Thus, in other words, as a primary check we examine whether or not the coefficients are in harmony with the theory.

In this spirit we calculate the interval of values that κ can take as the magnitude of β varies between 0 and ∞. Obviously, if for a possible positive value of β, κ becomes negative, then the conclusions, which are drawn based on the regression coefficients, would be meaningless. They would have no significance, since, in such a case, one of the parameters, i.e. κ, which by theory is positive, is estimated to be non-positive, thus causing a contradiction between the presumptions and findings of the model. Hence, the advantage of calculating the sign of κ is that it actually serves as a check on the model and the coherence between the theory and empirical data. Eventhough we keep the standards of passing the test fairly high[10], our model successfully demonstrates its power of correctly replicating empirical phenomena: As it can be seen in the previous table the κ-values in all four cases turn out to be positive.

(ii) Accuracy of coefficients:

In order to check for the accuracy of the regression coefficients a Monte Carlo sampling experiment was performed based on the following statistical model:

$$y = X\beta + e, \quad \textit{or more specifically;} \qquad (62)$$
$$f_t = \hat{\pi}_0 + \hat{\pi}_1 f_{t-1} + (1-\hat{\pi}_2)\, c_t + e$$

where e is a normal random vector with mean vector zero and variance $\sigma^2 = s^2$. Note that the model in [62] is of standard OLS nature, whereas in three out of four cases (all three metals except copper) we estimated our coefficients using Cochrane-Orcutt's method for correcting the estimates for the presence of autocorrelation, we had to perform a transformation of the variables, in order to be able to make

[10] They are "high" in the sense that we are ready to dismiss the model under the condition of only a single occurance of a non-positive κ-value.

use of [62]. In other words, following a textbook approach[11] in the presence of autocorrelation what is being minimized is:

$$e'\Psi^{-1}e = e'P'Pe, \text{ or alternatively:} \\ = (1-\rho^2)(y_1 - x_1'\beta)^2 + \Sigma_{t=2}^{T}[y_t - \rho y_{t-1} - (x_t - \rho x_{t-1})'\beta]^2 \tag{63}$$

where ρ is the autocorrelation coefficient in:

$$e_t = \rho e_{t-1} + v_t, \text{ where } E(v) = 0 \text{ and } V(vv') = \sigma_v^2 I \tag{64}$$

Applying P to y and X leads to the following set of transformations:

$$y^* = \begin{bmatrix} \sqrt{1-\rho^2}y_1 \\ y_2 - \rho y_1 \\ y_3 - \rho y_2 \\ \dots \\ y_T - \rho y_{T-1} \end{bmatrix} \text{ and }$$

$$X^* = \begin{bmatrix} \sqrt{1-\rho^2} & \sqrt{1-\rho^2}x_{12} & \dots & \sqrt{1-\rho^2}x_{1K} \\ 1-\rho & x_{22} - \rho x_{12} & \dots & x_{2K} - \rho x_{1K} \\ 1-\rho & x_{32} - \rho x_{32} & \dots & x_{3K} - \rho x_{2K} \\ \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots \\ 1-\rho & x_{T2} - \rho x_{T-1,2} & \dots & x_{TK} - \rho x_{T-1,K} \end{bmatrix} \tag{65}$$

Note that the first observation is treated differently from the remainder.[12]

The transformed model for the first observation is given by:

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[11] As an example one may refer to "Introduction To The Theory and Practice of Econometrics", by G. G. Judge, R. C. Hill, W. E. Griffiths, H. Luethepohl, T-C. Lee or "Econometric Methods" by J. Johnston.

[12] K and T stand for the number of explanatory variables and for the number of observations, respectively.

$$\sqrt{1-\rho^2}\,y_1 = \sqrt{1-\rho^2}\,x_1'\beta + e_1^* \quad \text{where: } e_1^* = \sqrt{1-\rho^2}\,e_1 \tag{66}$$

Whereas for the other observations we have:

$$y_t - \rho y_{t-1} = (x_t - \rho x_{t-1})'\beta + v_t \quad \text{where: } v_t = e_t - \rho e_{t-1} \tag{67}$$

Thus utilizing the estimated ρ-coefficients we transform our variables as described by [66] and [67], and obtain a new linear model on which we can apply standard OLS estimation methods.

Using the transformed variables in [62] we "shock" our model by keeping it subject to a random error vector which is normally distributed and we repeat this process for 1,000 times. Hence at the end of the simulation process, 1,000 "y" (in our case "f_t^*") sets of size n were generated.[13]

Based on the simulated series and our pre-determined series, i.e. lagged futures price series and basis series, we re-estimate the values of the estimated β-vector. Finally, we test the null-hypothesis of no difference between individual β-coefficients which are estimated based on the simulation and our initial β-coefficients. Our findings are

[13] Here n stands for the number of observations available for the variables c_t and f_{t-1}; n takes the values of 452, 568, 541 and 366 for the cases of copper, gold, silver and aluminum, respectively.

summarized in the following table:

MONTE CARLO SIMULATION RESULTS

	COPPER	GOLD	SILVER	ALUMINUM
Constant	0.0000073 (0.686)	0.0000195 (1.939)	-.0000147 (0.955)	0.0000336 (2.829)
π_1	-.0000537 (0.565)	0.0002499 (0.814)	-.0000011 (0.017)	0.0002618 (0.511)
π_2	0.0000000 (0.432)	-.0000003 (1.210)	-.0000001 (0.850)	0.0000000 (0.106)
$Var(\pi_4 u_t)$[14,15]	-.0000002 0.000049	-.0000003 0.000052	-.0000004 0.000124	0.0000001 0.000050
Dn of 95% C.I.	0.000046	0.000049	0.000116	0.000047
Up of 95% C.I.	0.000052	0.000055	0.000130	0.000055

[14] Recalling that the estimate s^2 follows a chi-square distribution, it is clear that use of t-distribution would not be appropriate. Hence, we calculate the 95% confidence interval for the variance, which is defined as: $s^2/C_{.025}^2 < \sigma^2 < s^2/C_{.975}^2$. Where C^2 is the ratio of chi-square to respective degrees of freedom, and $C_{.025}^2$ and $C_{.975}^2$ are the upper and lower critical values of C^2.

[15] The first and second rows report the mean value of the difference between the Monte Carlo estimate and the initial variance and the value of the initial estimate of the variance term, respectively.

In table above the figures reported stand for the mean values of the difference between the Monte Carlo estimates and the coefficients of the initial regression equations. Values in the brackets are absolute of t-values, under the null hypothesis of no difference.

(iii) Accuracy of the Ω-estimate

The last step is about verifying the accuracy of our estimate of Ω. To that end we shall employ a similar methodology as the one outlined above. In particular, this last step can be thought of as an extension of the Monte Carlo experiment performed to verify the accuracy of the estimated coefficients in the model. Namely, in this section, the value of Ω is calculated for each of the 1,000 draws and is based on the estimated π_1 π_2 and $V(\pi_4 u_t)$ values. Then the following step is to compare the simulation-generated Ω values with the one which was induced by our initial regression.

Once again, the null-hypothesis will be that there is no statistically significant difference between those two figures; and the alternative hypothesis will suggest that there is indeed a statistically significant difference. However, eventhough the setup of the test is very similar to the previous one, unlike the previous test, one is not allowed to apply a t-test, in order to determine whether or not the null hypothesis cannot be rejected in this case. Recall the definition of a t-distribution:

If $\overline{x}$ and s^2 are the mean and the variance of a random sample of size n from a normal population with the mean μ and the variance σ^2, then

$$t = \frac{\overline{x} - \mu}{s/\sqrt{n}}$$

has the t distribution with n-1 degrees of freedom.

As the definition above[16] emphasizes the underlying distribution for a random variable has to be normal such that the t-value will indeed be the correct statistic to use.

Inspecting the definition of Ω one can easily determine that Ω is not distributed normally. Rather, the distribution function ought to be a fairly complex function, since Ω can be described as the division of the product of some variables (each being a normally distributed variable) by the product of some others (again, each being a normally distributed variable). Hence, under this circumstance it is evident that the t-statistics becomes meaningless, and thus, we have to search for another measure to replace the t-test, such that we are able to check whether or not our null-hypothesis holds.

One may argue that as the sample size increases and approaches infinity by Central Limit Theorem we can state that the distribution of the random variable will converge to the standard normal distribution. Or precisely worded:

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[16] Definition taken from "Mathematical Statistics" by J. E. Freund and R. E. Walpole

If x_1, x_2, ..., and x_n constitute a random sample from an infinite popula having the mean μ, the variance σ^2, and the moment generatingfunction $M_x(t)$, then the limiting distribution of

$$z = \frac{\overline{x} - \mu}{\sigma/\sqrt{n}}$$

as $n \to \infty$, is the standard normal distribution.

In this definition[17] the emphasis is on the word "infinite"; in a context like ours one can only suggest to be "approximating" to infinity, since by nature our sample is a finite one. As the sample size grows say from 500 to 1,000, for some that can already be seen as an "infinite" population, whereas for others even a sample size of 40,000 or 50,000 can be ordinary. Hence, the question of whether or not a given finite sample can be said to be approximating an infinite one is an issue which is left to the researcher, and the magnitude of the error committed by assuming that the sample at hand exhibits standard normal distribution is difficult to quantify.

An alternative way of tackling this type of question, where the assumption of population normality is not satisfied, is by turning to a techniques that are free of this distribution requirement. Such statistics are called distribution free or non-parametric, and are preferred for two reasons:[18]

[17] Definition taken from "Mathematical Statistics" by Freund and Walpole

[18] Quote from "Introductory Statistics for Business and Economics" by Wonnacott and Wonnacott

(i) The corresponding classical statistic may be invalid. (That is, its confidence level may not actually be as high as 95%.)

(ii) But even in applications where the classical (parametric) statistic is reasonably valid, a non-parametric statistic may be much more efficient (have smaller variance) and hence have a narrower confidence interval.

The non-parametric test which corresponds to the one-sample t-test is the so-called "sign test". The one-sample sign test applies when we sample a continuous symmetrical population, so that the probability of getting a sample value exceeding the mean and the probability of getting a sample value less than the mean are both 0.50.[19] To test the null hypothesis $\mu=\mu_0$ against an appropriate alternative on the basis of a random sample of size n, we replace each sample value exceeding μ_0 with a minus sign, and then we test the null hypothesis that the number of plus signs is the value of a random variable having a binomial distribution with the parameters n and $\theta=0.5$.

To perform a one-sample sign test, when the sample is large, we use the normal approximation to the binomial distribution. To generalize, consider a sample of n ordered observations x_1, x_2, ...,x_n from a population (which is not necessary symmetric) with unknown median ν. Then the confidence interval can be defined by counting off q

[19] If we relax the assumption of symmetry, then the null hypothesis will be $\mu^*=\mu^*_0$, instead of $\mu=\mu_0$, where μ^* is the population median.

observations from each end:

$$X_q \leq \nu \leq X_r, \qquad where: \; r = n - q + 1 \tag{68}$$

If the confidence level is set at 95%, (i.e. α=0.05) then $P(S \geq r)$=0.025 (P and S stand for probability and number of successes in n trials, respectively). By dividing both of the sides by n (sample size) we obtain the normal approximation to the binomial distribution. Since the sample size is large, the sample proportion, P=S/n, has an approximately normal distribution. The first two moments can be shown to be equal to:

$$\begin{aligned} \mu_P &= \pi = 0.5 \\ \sigma_P &= \sqrt{\frac{\pi(1-\pi)}{n}} \quad or: \\ &= \frac{1}{2\sqrt{n}} \end{aligned} \tag{69}$$

By manipulation it can be shown that:

$$r = \frac{n}{2} + 0.98\sqrt{n} \tag{70}$$

Once r is found q can be obtained by [68] and hence the confidence interval is formed, which again was defined in [68] and thus we can test our null hypothesis of zero mean. (or median, depending on our assumption about symmetry: if we keep our criteria for symmetry high, the test below could be interpreted as one which is testing the median value of the distribution.) In particular, for n=1,000 and α=0.05, equation [70] yields that r=531. In turn, by [68] we find that q=470.

In other words, once the series -which are simply the differences between the Monte Carlo estimates of Ω and the estimates of the initial regression equation in the cases of Copper, Gold, Silver and Aluminum- are sorted by ascending order, the 95% confidence interval can be found by looking at the spread which is located between the values of 470-th and 531-st observations of the series.

The following table summarizes the results:

ACCURACY OF Ω: Confidence Intervals for zero-mean

H_0: μ=0	COPPER	GOLD	SILVER	ALUMINUM
α = 0.05				
Lower Value	-10.2968	-15.1105	-3.01013	-199.576
Upper Value	17.18753	4.532440	4.361407	138.9581
α = 0.01				
Lower Value	-14.7645	-16.6288	-4.47425	-236.507
Upper Value	20.30855	8.456820	5.839086	194.0343

As the table above clearly demonstrates according to the calculated confidence intervals, we are not able to reject the null-hypothesis of a zero mean. In other words, we can conclude that the mean value of the deviation of the MC-estimate of Ω from the initial value of Ω is statistically not different from zero. Thus, in sum, from our results we can deduct that all our coefficients, π_1, π_2, $V(\pi_4 u_t)$ and Ω are accurate estimates of their corresponding population parameters.

iv. Summary and Conclusions

This study is an attempt to empirically test the relationship between speculation in futures markets and spot price volatility. The contribution of this study is two-fold: (1) The theoretical framework employed in this paper is a more generalized version of an earlier model developed by Driskill, McCafferty and Sheffrin (1991). (2) This study actually is attempting to apply the theoretical findings to the empirical data, which is a task which is crucial in testing the significance and usefulness of the model and which was lacking in previous works in this line of literature.

In the current model spot and futures prices are determined simultaneously in an environment where uncertainty exists due to three causes: (i) Consumer demand for the raw good, as well as the processed good is uncertain, (ii) Storage costs fluctuate over time, and, finally, (iii) The price spread between the raw and processed goods is subject to unceratinty. In this framework there are four principal actors in the model: producers, inventory holders, processors and consumers of the final commodity. As we showed earlier, the inclusion of processors creates a bias towards accepting the hypothesis that speculation in futures markets actually decreases spot price variability. However, despite that bias, the empirical results in all four metals exhibit a strong rejection of the above-mentioned hypothesis in favor of the opposite one, i.e. that speculation in futures markets tend to exacerbate the effects of storage-cost shocks on spot price variability.

APPENDIX A:

In order to illustrate the theoretical properties of equation (8), it may be useful to observe the implications of this relationship by going over a simple example.

Let us assume that a farmer whose net revenue function can be characterized as:

$$R^G_{T-i} = p^G_{T-i} q^G_{T-i} - C_G(q^G_{T-i}), \quad \text{for } i = 0$$
$$R^G_{T-i} = 0, \quad \text{otherwise.} \qquad (i)$$

would like to hedge his/her crop in a two-date setting, (i.e. i=0,1.), via futures trading. Thus, based on (8) his/her futures position when entering the last period (when i=0, i.e. on the maturity date) is:

$$x_T = \frac{-E(B^f_T)}{\alpha V(B^f_T)} + \beta^0_{RB} \qquad (ii)$$

Making use of (i) we can decompose $\beta_{RB}{}^0$:

$$\text{Since:} \quad \beta^j_{RB} = q^G_T \beta^j_{GB}$$
$$\text{we can write it as:} \quad \beta^0_{RB} = q^G_T \beta^0_{GB} \qquad (iii)\text{-}(iv)$$

where,

$$\beta^0_{GB} = \frac{cov(p^G_T, p^f_{T-1} - p^f_T)}{V(B^f_{T-1})} \qquad (v)$$

Knowing that the spot and futures prices are equal to each other on the maturity date because of arbitrage laws, we can state that:

$$\beta^0_{GB} = \frac{cov(p^f_T, p^f_{T-1})}{V(B^f_{T-1})} - \frac{V(p^f_T)}{V(B^f_{T-1})} < 0. \qquad (vi)$$

assuming that the variance of futures prices is constant. Now, based on (vi), equation (ii) becomes,

$$x_T = \frac{-E\,(B_T^f)}{\alpha\, V(B_T^f)} + q_T^G\, \beta_{GB}^0, \qquad \text{where } \beta_{GB}^0 < 0. \tag{vii}$$

Similarly, his/her position one period before the maturity date is given by (8) as:

$$x_{T-1} = \frac{-E\,(B_{T-1}^f)}{\alpha\, V(B_{T-1}^f)} + \beta_{RB}^0 + \beta_{RB}^1 - \beta_{BB}^1 x_T \tag{viii}$$

which boils down to:

$$x_{T-1} = \frac{-E\,(B_{T-1}^f)}{\alpha\, V(B_{T-1}^f)} + q_T^G \beta_{GB}^1 - \beta_{BB}^1 x_T \tag{ix}$$

since $\beta_{RB}{}^0 = 0$ when i = 1 due to (i).

Alternatively, (ix) can also be re-stated as:

$$x_{T-1} = \frac{-E\,(B_{T-1}^f)}{\alpha\, V(B_{T-1}^f)} + q_T^G \left[\frac{cov(p_T^f,\ p_{T-2}^f - p_{T-1}^f)}{V(B_{T-1}^f} \right] - \beta_{BB}^1 x_T \tag{x}$$

Now, let us focus on the implications of (vii) and (x) under two different cases:

Case 1: Assume that $\{B_t{}^f\}$ is a Martingale process. As an example let us take: $B_T{}^f = p_{T-1}{}^f - p_T{}^f = \epsilon_T$, where ϵ_T is distributed normally with a mean of zero and a bounded variance of $\sigma_B{}^2$.

Under this assumption it can easily be shown that $\beta_{GB}{}^1$, $\beta_{GB}{}^0 = -1$ and $\beta_{BB}{}^1 = 0$. Hence,

$$x_T = \frac{-E(B_T^f)}{\alpha V(B_T^f)} - q_T^f \qquad \textit{(xi)}$$

which states that, in the final period farmers will have their entire crop hedged and if the expected futures price movement is not equal to zero, then they also will have a small speculative position which is nothing but the first term on the right-hand-side of (xi). Consequently, (x) takes the following form:

$$x_{T-1} = \frac{-E(B_{T-1}^f)}{\alpha V(B_{T-1}^f)} - q_T^G \qquad \textit{(xii)}$$

since $\beta_{GB}{}^1 = -1$ and $\beta_{BB}{}^1 = 0$. Equations (xi) and (xii) tell us that if $(B_t{}^f)$ follows a Martingale process farmers' hedging positions will not change over time, i.e. they will always remain fully hedged. As is obvious from the two equations there might be intertemporal differences in farmers' total positions which are entirely due to their speculative activities. This result is consistent with Anderson and Danthine's (1983) (Hereafter A–D) findings where they show that initially farmers underhedge and, as the maturity date approaches, their hedging positions increase. On the maturity date they reach a full hedge. The results, in this case are basically the same; since all prices in the current model are in terms of present values whereas they were defined as nominal prices in A–D's 1983 study. It is precisely this discounting mechanism which leads to this seemingly difference between the two results.

Case 2: In contrast to Case 1 where we have $\beta_{BB}{}^1 = 0$, now let us presume that $\beta_{BB}{}^1 > 0$, i.e. we are postulating a positive trend in $(B_t{}^f)$.

Obviously with an assumption of $\beta_{BB}{}^1 > 0$ we have to consider to sub-cases:

(2.a) $\beta_{BB}{}^1 > 0$ and $p_{T-2}{}^f > p_{T-1}{}^f > p_T{}^f$

(2.b) $\beta_{BB}{}^1 > 0$ and $p_{T-2}{}^f < p_{T-1}{}^f < p_T{}^f$.

Let us illustrate these with the following example. Take: $B_t{}^f = \epsilon_T + \mu_T$; or $p_T{}^f = p_{T-1}{}^f + \mu_T + \epsilon_T$.
Note that in (2.a) $\mu_T < 0$ where $\mu \sim (\mu_L, \sigma_\mu)$, and in (2.b) $\mu_T > 0$ with $\mu \sim (\mu_U, \sigma_\mu)$; $(\mu_L < \mu_U)$.

Hence;

$$\beta^0_{GB} = \frac{V(B^f_{T-1}) + cov(\mu_T,\ p^f_{T-1}) - V(p^f_T)}{V(B^f_T)}, \quad alternatively,$$

$$\beta^0_{GB} = -\frac{(\sigma^2_B + \sigma^2_\mu)}{\sigma^2_B}, \quad i.e. \quad |\beta^0_{GB}| > 1, \quad (where\ \sigma^2_B = \sigma^2_\mu + \sigma^2_\epsilon)$$

So that equation (vii) translates into:

$$x_T = \frac{-E(B^f_T)}{\alpha V(B^f_T)} - \left(\frac{\sigma^2_B + \sigma^2_\mu}{\sigma^2_B}\right) q^G_T \qquad \textbf{(xiii)}$$

In other words, as opposed to Case 1, in the case with a trend in the market farmers would be overhedging in the last period. (Both in 2.a and 2.b)

In order to demonstrate the farmers' position at one period before the maturity we need to calculate the values for $\beta_{GB}{}^1$ and $\beta_{BB}{}^1$. It can easily be shown that under the assumption above that:

$$\beta^1_{BB} = \frac{cov(\mu_T, \mu_{T-1})}{\sigma^2_B}, \qquad \beta^1_{GB} = -\left(\frac{\sigma^2_B + cov(\mu_T, \mu_{T-1})}{\sigma^2_B}\right) \qquad \textit{(xiv)}$$

where $|\beta^0_{GB}| > |\beta_{GB}| > 1$

Thus,

$$x_{T-1} =$$

$$-\frac{E(B^f_{T-1})}{\alpha V(B^f_{T-1})} - q^G_T\left[\frac{\sigma^2_B + cov(\mu_T, \mu_{T-1})}{\sigma^2_B}\right] - \left[\frac{cov(\mu_T, \mu_{T-1})}{\sigma^2_B}\right] x_T \qquad \textit{(xv)}$$

where $x_T < 0$.

APPENDIX B:

Given the equation for agent-specific futures positions, i.e. (8):

$$x^{f}_{T-i} = \frac{-E(B^{f}_{T-i})}{\alpha_K V(B^{f}_{T-i})} + [\sum^{i}_{j=0} \beta^{j}_{RB} - \sum^{i}_{j=1} \beta^{j}_{BB} x^{f}_{T-i+j}] , \qquad K = G,\ P,\ I,\ S.$$

Using the market clearing condition equation (10) and aggregating over the market, we obtain:

$$0 = [\frac{-E(B^{f}_{T-1})}{V(B^{f}_{T-1})}]\cdot[\frac{n^G}{\alpha^G} + \frac{n^I}{\alpha^I} + \frac{n^P}{\alpha^P} + \frac{n^S}{\alpha^S}] + \{n^G(\sum_j \beta^{G,j}_{RB}) + n^I(\sum_j \beta^{I,j}_{RB}) + n^P(\sum$$

or if we define $w^K = n_K / \Sigma_K n^K$ and solve for $E(B_{T-1})$ we obtain:

$$E(B^{f}_{T-i}) = \frac{V(B^{f}_{T-i})}{[\frac{w^G}{\alpha^G} + \frac{w^I}{\alpha^I} + \frac{w^P}{\alpha^P} + \frac{w^S}{\alpha^S}]}\cdot\{ w^G(\sum_j \beta^{G,j}_{RB}) + w^I(\sum_j \beta^{I,j}_{RB}) + w^P(\sum_j \beta^{P,j}_{RB})$$

From equation (11) onwards we define:

$$(\frac{1}{\alpha}) = [\frac{w^G}{\alpha^G} + \frac{w^I}{\alpha^I} + \frac{w^P}{\alpha^P} + \frac{w^S}{\alpha^S}]$$

In other words, α represents the harmonic mean of all futures market participants' risk aversions.

Theoretically, one could also assume only two different levels of risk aversion prevailing in the market, i.e. one of hedgers ($=\alpha_S$), and one of speculators ($=\alpha_S$). Then we would define α simply as:

$$(\frac{1}{\alpha}) = [\frac{w^H}{\alpha^H} + \frac{w^S}{\alpha^S}]$$

APPENDIX C:

In this section we shall report the results for copper, gold and silver, which are derived under the assumption that first-differencing is the method of trend-elimination. We shall not be reporting the results for the aluminum market since in our preceding analyses, following the Dickey-Fuller approach, we had already used first-differencing in order to eliminate the trend in that market. The following table summarizes the estimation results of equation [41]

REGRESSION RESULTS (First Differences)

	COPPER	GOLD	SILVER
π_1	0.914369	0.979000	0.980000
S.E. of π_1	0.011650	0.008540	0.010490
π_2	0.999972	0.999883	0.999921
S.E. of π_2	0.000003	0.000006	0.000005
R-Square	0.945216	0.958785	0.942041
D-W Statistic	2.20	2.34	2.32
$V(\pi_4 u_t)$	0.000079	0.000043	0.000195
κ'	> 0	> 0	> 0
Ω	3199.173	401.2622	494.5386

Even though, at the first glance, the nature of our previous results, i.e. $\Omega > 1$, seems to be reconfirmed by this new set of regressions, one might feel uneasy to depend on the accuracy of the current set of regression results. A careful inspection of the Durbin-Watson statistics shows us that in all three cases the D.W. values are equal to and larger than 2.20 which happens to be the borderline between "the neutral zone" and the zone of "negative first-order autocorrelation". Recalling that these Durbin-Watson values were obtained based on the Cochrane-Orcutt estimation method, the very existence of high D.W. values should serve as a warning signal about the complexity of the problem. Thus, we can safely conclude that strict-first-difference-trend-elimination not only is incapable of eliminating the trend correctly, but it also transforms our series into a complex form which cannot be eliminated by conventional methods. This result once again reassures our belief in the correctness of our methodology of choosing the Dickey-Fuller test as a guiding tool in determining the characteristics of the series.

APPENDIX D:

It may be of importance to the reader to find out, whether or not the markets we have been working on experienced any significant structural change over our sample period, and if they did, whether or not those changes indeed lead to any modifications of our conclusions. In order to check for the existence of a structural change, we divide our sample into two subsamples in order to obtain two statistically independent (and non-overlapping) samples.

Consequent to the creation of the smaller data sets, we ran the same regression equation, i.e. [41], to get the new set of coefficients based on those two new estimation samples. Then, based on the estimated coefficients we calculated the value of Ω in each case and checked whether or not there was any sign change. As a next step, using a two-sample t-test we tested the null hypothesis that $\pi_1^{i}=\pi_1^{ii}$ and later on the hypothesis that $\pi_2^{i}=\pi_2^{ii}$, where i and ii refer to the two subsamples. The last step was about testing of the "robustness" of the third coefficient, i.e. $\pi_4 u_t$. Since we test that coefficient residually, we can deduce that the distribution of that estimate will follow a chi-square distribution, thus in order to test whether or not the samples reveal statistically different coefficients we create the 95% confidence intervals[20] using, say, the first-sample coefficient and try to see whether the second-sample estimate happens to be in that

[20] For this end we use C^2 distribution which is simply defined as the ratio of Chi-square to the respective degrees of freedom.

interval, or not.

The following six tables summarize our findings about the coefficients (and the significance of the difference between them) and the corresponding values of κ' and Ω. As a close inspection of the findings would indicate only in the case of gold the problem of robustness seem to be existing since the two π values, π_1 and π_2, are significantly different from each other in the two samples. $\pi_4 u_t$, on the other hand, demonstrates different behavior in the cases of copper and silver. However, eventhough the magnitudes of some of the coefficients may not be stable, at 95% confidence level, as we change the sample size, this has a negligible effect on our final results, i.e. the magnitude of Ω. As it can be seen, estimates of Ω remain to be significantly above one. The following tables present our results:

TABLE I

	ALUMINUM (i)	ALUMINUM (ii)	SILVER (i)	SILVER (ii)
π_1	0.969000	0.950000	0.999000	0.996000
π_2	0.999994	0.999994	0.999927	0.999916
$\pi_4 u_t$	0.000058	0.000055	0.000072	0.000167
κ'	> 0	> 0	> 0	> 0
Ω	> 1	> 1	> 1	> 1

TABLE II

	ALUMINUM (i) STD. ERR.	ALUMINUM (ii) STD. ERR.	POOLED VARIANCE	2-SAMPLE t-RATIO
π_1	0.014910	0.016010	0.043555	0.868469
π_2	0.000001	0.000001	0.000000	0.253078
	Lower Limit	Upper Limit	Estimate	Result:
$\pi_4 u_t$	0.000046	0.000066	0.000058	O.K.

TABLE III

	SILVER (i) STD. ERR.	SILVER (ii) STD. ERR.	POOLED VARIANCE	2-SAMPLE t-RATIO
π_1	0.002086	0.003421	0.002167	-0.74808
π_2	0.000005	0.000006	0.000000	-1.28282
	Lower Limit	Upper Limit	Estimate	Result:
$\pi_4 u_t$	0.000064	0.000080	0.000167	no

TABLE IV

	GOLD (i)	GOLD (ii)	COPPER (i)	COPPER (ii)
π_1	0.916831	0.987997	1.000150	0.994857
π_2	0.999676	0.999978	0.999986	0.999985
$\pi_4 u_t$	0.000049	0.000050	0.000067	0.000047
κ'	> 0	> 0	< 0	> 0
Ω	> 1	> 1	< 1	> 1

TABLE V

	GOLD (i) STD. ERR.	GOLD (ii) STD. ERR.	POOLED VARIANCE	2-SAMPLE t-RATIO
π_1	0.010969	0.0096520	0.030213	4.870319
π_2	0.000019	0.000010	0.000000	13.48391
	Lower Limit	Upper Limit	Estimate	Result:
$\pi_4 u_t$	0.000044	0.000054	0.000050	O.K.

TABLE VI

	COPPER (i) STD. ERR.	COPPER (ii) STD. ERR.	POOLED VARIANCE	2-SAMPLE t-RATIO
π_1	0.004133	0.004907	0.004631	-0.82496
π_2	0.000002	0.000002	0.000000	-0.42658
	Lower Limit	Upper Limit	Estimate	Result:
$\pi_4 u_t$	0.000059	0.000077	0.000047	no

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