THE TERM STRUCTURE OF INTEREST RATES AND ARBITRAGE FREE BOND PRICING: AN APPLICATION TO THE RUSSIAN GOVERNMENT BOND MARKET

by

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Abstract

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In this dissertation, first the term structure of interest rate is determined by describing the yield curve using cubic spline interpolation. Then, the one-factor Heath, Jarrow, and Morton (HJM) model as developed by Jarrow (1996) is used to determine arbitrage free prices for Russian Government Short Term Bonds. In this model, the initial term structure is taken as given, and the forward rate, derived from observed prices, is the driving factor. After determining the stochastic evolution of bond prices, a trading strategy is developed to construct a portfolio which gives arbitrage-free prices. The computed arbitrage-free prices are compared to the observed prices. Then the Black, Derman, and Toy model is applied. This is another arbitrage-free one-factor model. The factor driving the model is the short term interest rates. With the short term rates, the evolution of the bond prices is generated. Using the same trading strategy, a portfolio which is a synthetic bond that gives the arbitrage-free price,is created. The arbitrage free price is then compared to the observed price.

Acknowledgments

To my family

Contents

List of Tables

List of Figures

1 INTRODUCTION

There are two different approaches to modeling the term structure of interest rates. The first is to measure the term structure using numerical or statistical techniques. With these techniques the data obtained from asset prices is smoothed to describe the yield curve. In this approach no factors are assumed to be the driving force. The second approach is based on models which make assumptions about the driving factors of the asset pricing methods using either equilibrium or arbitrage arguments. (Anderson, Breedon, Deacon, Derry& Murphy, 1996)

First, different statistical and numerical techniques for the term structure of interest rate will be discussed. Then, the term structure will be constructed by using **cubic spline interpolation**. After extracting the missing data through cubic spline interpolation, arbitrage opportunities- mispricings will be searched through forward rates by using the Heath Jarrow and Morton model and through short rates by using the Black Derman and Toy model.
The first step for constructing the yield curve is to fill the gaps in the maturity spectrum; there is not a GKO (Gosudarstvenniye Kratkosrochniye Obligatsii) for every maturity.

To fill these gaps and to construct the term structure, the following methods have been used:

1.1 McCulloch(1971,1975)

McCulloch's method is widely used. The model constrains the cash flows from different bonds due at the same time to be discounted at the same rate. The method estimates a discount function from which the term structure can be derived. McCulloch uses a continuous discount function assuming continuous compounding.He imposes the restriction that the prices of the discount bonds should vary smoothly with maturity. A convenient way of doing this is to write discount function, $P(n)$, as a linear combination of certain pre specified functions:

$$P(n) = 1 + \sum_{j=1}^{J} a_j f_j(n) \tag{1}$$

The $f_j(n)$ are functions of maturity n, and the a_j are coefficients to be estimated. The discount function generated by this set of approximating functions will be a simple *kth* degree polynomial. But as the degree of polynomial increases, the estimation will be unstable. McCulloch uses the quadratic spline method to overcome the problems created by high degree polynomials. But there is a disadvantage with quadratic splines ; they have discontinuous second derivatives, leading to a forward rate curve with a discontinuous first derivative.To avoid this, McCulloch uses a cubic spline instead of a quadratic spline (McCulloch, 1975). The discount functions, $f_j(n)$, are cubic splines that are smooth around each knot point. This method can be too flexible. Since it does not constrain the discount function to be non-increasing, the forward rates calculated may be negative. In both papers, the splines are estimated using ordinary least squares.

1.2 Nelson and Siegel (1987)

They examined how the term structure of yields can be described more compactly by a few parameters. They try to model the forward rate curve rather than the term structure of interest rates. This approach avoids the problem encountered in spline-based models of choosing the best knot point specification. But it represents a less flexible model than the spline based models and fits data less well.

The drawback with flexible model is that they tend to overfit the data, giving a distorted picture of the term structure. Svensson (1994) extends Nelson-Siegel model by adding another term. The revised forward rate equation has two extra parameters compared to the original N & S model.Talk about "Term Structure of interest rate", and the Russian bond market. Later, introduce forward rates.

1.3 Fisher, Nychka, Zervos(1995)

They use smoothing splines instead of regression splines. Smoothing splines have a penalty for excess roughness and a single parameter controls the size of this penalty. An increase in penalty is a reduction in the effective number of parameters in the estimation. This method circumvents the need to exogenously supply the number and the location of the knots.The *b-spline* function consists of piecewise polynomials determined by a series of control points, (s_i, p_i), $(s_i, p_i), i = 1, 2, ..n$. The b-spline function based on cubic polynomials is named cubic b-spline. (c_i, p_i) are control points because b-spline does not pass through the points (except under special conditions). The b-spline function is useful for generating a smooth curve in design and graphics when artistic impression or a visually comfortable curve is more important than strict fitting (Nakamura, 1994).

Shea(1984) and Steeley (1991) argue that using splines as in McCulloch(1971, 1975) can generate a regressor matrix with columns that are nearly perfectly collinear (resulting in possible inaccuracies arising from the subtraction of large numbers in estimates obtained using OLS). As a solution, they advocate the use of b-splines. These are functions which are identically zero over a large portion of the approximation space (unlike those used by McCulloch) and thus avoid the loss of accuracy. By using b-splines, it is also easier to impose constraints on the spline function. By increasing the number of knots, cubic splines provide increasingly flexible functional form.

A simple, numerically stable parameterization of a cubic spline is provided by a cubic b-spline (Fisher, Nychka, and Zervos, 1995).

The above mentioned models are for smoothing the data which they had in abundance. The data that is used in this dissertation have few available bond prices for different maturities. For this reason, cubic spline interpolation is used following Jarrow (1996).

2 THE RUSSIAN GOVERNMENT BOND MARKET

When the Russian Ministry of Finance decided to find ways to finance the budget deficit without causing inflation, it chose to issue treasury securities rather than printing money. This dissertation will focus on rouble-denominated, zero-coupon, short term treasury bills or short term government obligations, GKO (Gosudarstvenniye Kratkosrochniye Obligatsii), because they account for about 90 percent of treasury securities. The maturities are three, six, and twelve months. The first issue was in May 1993, but did not grow until the end of 1994. The total volume of the GKOs outstanding was close to RUR 160,000bn in August 1996. Federal loan bonds, OFZ (Obligatsii Federalnovo Zaima) are coupon-bearing bonds and carry one to three year maturities. Coupons are floating rate and are paid every three months. The coupon rate is determined by the Central Bank of Russia which announces it seven days before payment, and is similar to GKO yield at the time of announcement.They were launched in June 1995 in an attempt to increase the maturities. Both GKOs and OFZs are traded on the Moscow Interbank Currency Exchange (MICEX), together with five other currency exchanges connected with MICEX and located in large regional cities.This enables government bonds to be traded simultaneously in remote places throughout Russia. GKOs are modeled after US T-Bills. They are zero-coupon bonds with nominal values of 1,000,000 Russian rubles. The maturities are three months, six months, and twelve months, and the main agent for the GKOs is the Central Bank of Russia.

2.1 Primary Market

Primary placements take place on Wednesdays every week in the form of primary Dutch auction. Investors can make (through their dealers) competitive and non-competitive bids. In competitive bids, a price and a volume are indicated; and in non-competitive bids only the volume is indicated, and the price is set by the Ministry of Finance. The Ministry of Finance examines all bids, and decides on a "cut-off price" and a "market price" for each issue, which are then announced by the Central Bank of Russia. The "market price" is the weighted average of all competitive bids above the cut-off price; all non-competitive bids are tendered at the "market price", and all competitive bids above the cut-off price are tendered at the bid price. Auctions are generally announced one week in advance. However, they can be and sometimes are canceled by the Ministry of Finance at shorter notice, or amended. Funds for purchasing the securities at auctions must be remitted to the MICEX one business day before the auction. Because of an inefficient clearing system, investors are usually required to transfer funds to dealers two business days before the auction.

2.2 Secondary Market

Secondary trading is done only on the MICEX and its sister exchanges. Any trading outside the MICEX is forbidden by the Central Bank of Russia. Trading takes place daily between 11:00 a.m. and 1:00 p.m. except on official holidays and auction days(Wednesdays). Turnover on the secondary market grew from RUR29 trillion in January 1996 to RUR57 trillion in July 1996. Funds for purchasing the securities during the secondary trading must also be remitted to the MICEX on the business day before the trading session, but because of the inefficient clearing system, investors are usually required to transfer funds to dealers two business days before the desired secondary trading session. Investors usually receive their proceeds only two business day after selling their securities.Sale and purchase orders of investors are presented by dealers in the form of applications to the MICEX during the trading session in real time mode. The price is a percentage of par value rounded down to the nearest $\frac{1}{100}$th of one percent. In the MICEX Trading System if there is a counter proposal among pending applications which satisfies the dealer's, the transaction is executed immediately. Otherwise, the application is entered in a list of pending applications. The Central Bank of Russia recommends that yields on GKOs be calculated in Rubles as yields to maturity, without compounding. The following formula is used:

$$Yield = \frac{N - P}{P}\frac{365}{D} \tag{2}$$

where P is the current market price, N is the par value, and D is the number of days to maturity.

The yields on GKOs have been very volatile. As the presidential election approached, annualized yields increased dramatically, after several months of slow decline. The average annualized yield on six-month GKOs was 213% in the week before the first round of election , but fell to 131% in the following week, although the outcome of the second round of voting was not certain. After the election, yields on GKOs continued to decline, falling into double digits. This decline was partly due to the entry of the non-residents into the GKO market. In early September 1996, the average annualized yield on six-month GKOs was between 50-60% range. By March 1996, there had been 60 issues of three-month GKOs, 23 issues of six-month GKOs, and six issues of 364-day GKOs.

The Central Bank of Russia is the primary force in the market. As the largest participant in the GKO market, the CBR uses its capital and information to influence the yields. It trades for its own account, and often supports certain issues that are judged to have been priced too cheaply, by buying large amounts of the issue, pushing the prices up. The CBR sometimes places bonds that were not placed in the primary market in the secondary market. The Central Bank tries its best to be fairly unpredictable, and some reports mention that many traders have given up on trying to second-guess CBR (Rinocoplus, February 1996). Selling-short is not allowed on MICEX. Russia's financial markets are not yet developed for repurchase agreements. (Repo market is under consideration.)

There are two other products in the Russian bond market which are beyond the scope of this study. The first one is the Russian Ministry of Finance (MinFin) bond. It is a dollar-denominated bond issued in May 1993 as a result of a freeze on dollar deposits placed with Vnesheconombank in 1991. The second one is Rouble promissory note. Promissory notes, sold by Russian banks and corporations, have terms with a minimum of three days. They are similar to bank deposits but have several advantages: they can be redeemed for cash roubles and there are full reendorsement rights, i.e. they can be used as a collateral or pledge.

2.3 Foreign Investors in GKO Market

Foreign investors were allowed to invest in the GKO market in February 1996. This was welcomed by foreign investors because with the inception of Ruble corridor in June 1995, the Ruble depreciated against the dollar at a rate lower than inflation so dollar investors could enjoy high real return. Reducing the Ruble depreciation rate was an attempt to de-dollarize the economy which seems to be succeeding. After the entrance of foreign investors, the yields on GKOs decreased Foreign investors are allowed to access the market, but some restrictions apply. Non-resident investors open a special ruble account (the "S-account") with designated

Russian banks. The repatriation of funds is allowed only through a three-month FX forward contract, a portion of which should be offset at the CBR at special rates. It is also possible to have options on these forward contracts (option to cancel it), so that investors can reinvest the proceeds. In other words, Ruble proceeds from bonds can only be converted into hard currency at a fixed rate set by the CBR. Due to gradual liberalization of the markets, the guaranteed yield was reduced first from 25% to 19%, then to 16%, 13%, and finally 12.5%. The proportion of forward deals which must be offset was decreased from 90% to 75%, and then to 65%.

2.4 The Interest Rates

The term structure of interest rate describes the relationship between yield to maturity of bonds and their term to maturity. These bonds (or bills in this case) should have the same risk and should be subjected to the same tax status. Spot rates are used to construct the yield curve. For zero-coupon treasury securities, yield to maturity is equal to spot rate. By this token, yields on T-bills can be used for the yield curve. But yield to maturity of the zero-coupon securities which have maturities longer than one year, namely the stripped Treasury securities or STRIPS, cannot be used as the spot rate. There are three basic problems in the U.S. term structure of interest rates regarding the use of the yield to maturity of STRIPS. First, the liquidity of the stripped market is not as great as that of the Treasury bond market. Second, specific investors may want to invest only in certain maturities due to their needs or some attractive features associated with that particular maturity. This distorts the term structure relationship. Third, the tax treatment of the stripped securities is different from that of coupon securities (McEnally, and Jordan, 1995).

Spot Rate

A t-period spot rate of interest is the per-period interest rate that is to be paid on a loan borrowed today (time 0), which would be repaid in a lump-sum with interest on a specified date in the future (time t).

Yield to Maturity

Yield to maturity is the interest rate that makes the present value of the cash flow created by the bond equal to the market price of the bond.

Forward Rate

A forward rate of interest is the interest rate, implicit in currently quoted spot rates, that would be applicable from one time point in the future to another time point in the future (Babbel&Merrill, 1996). To determine the relationship between spot rates and forward rates, there are two basic approaches:

In the first approach investors have two alternative investment opportunities:

(a) Buy a treasury bill with one year maturity.

(b) Buy a treasury bill with six month maturity and at the end of maturity buy another treasury bill with six month maturity with the proceeds of the first six month treasury bill.

Investors will be indifferent between these two investments only if they have equivalent returns. Otherwise they will choose the one with the higher return. Investors know the spot rate for one year treasury bill, s_2, and the spot rate for six month treasury bill, s_1. It is also assumed that the spot rates are quoted for six month periods; but they do not know the six month treasury bill rate six months from now. The forward rates are not a good prediction of future interest rates as demonstrated by Fama (1976) (Fabozzi, 1996). If these two alternatives give the same return, then the forward rate for the second six months implied by spot rates will be (assuming \$ 1 initial investment):

$$(1+s_2)^2 = (1+s_1)(1+f_2) \tag{3}$$

$$f_2 = \frac{(1+s_2)^2}{1+s_1} - 1 \tag{4}$$

$$f_2 = \frac{(1+s_2)^2}{1+s_1} - 1 \tag{5}$$

At the beginning of the investment horizon, $s_1 = f_1$. Forward rates are not necessarily market consensus rates, but they help investors to determine which position to take based on their perception of the future interest rates.The second approach uses the prices of securities to determine forward rates. This approach assumes a portfolio at time t:

Buy a treasury bill maturing six months from now, and sell $\frac{P_1}{P_2}$ treasury bills maturing one year from now; both securities have a face value of \$ 1. The investors hold each treasury bill until maturity. The initial cash flow of this portfolio is zero now: buying a six month treasury bill requires a cash outflow of P_1 dollars, and selling short one year treasury bill for the amount $\frac{P_1}{P_2}$ creates a cash inflow of $\left(\frac{P_1}{P_2}\right)(P_2) = P_1$ dollars. Cash inflow and cash outflow will cancel each other and result in a zero cash flow.

The first cash flow is six months from now; the maturing six month treasury bill creates a \$1. At the end of the first year there is a cash outflow of $\frac{P_1}{P_2}$ dollars. This portfolio shows that a dollar loan for a six months maturity that will be borrowed six months from now will have a cost of $\left(\frac{P_1}{P_2} - 1\right)$. This implied cost is also known as the forward rate:

$$f_1 = \frac{P_1}{P_2} - 1 \tag{6}$$

This result is in accord with the first one because the price of a bond is the present value of its future cash flow:

$$P_1 = (1 + s_1)^{-1} \tag{7}$$

and

$$P_2 = (1 + s_2)^{-2}. \tag{8}$$

If we substitute these two equations into the forward rate formula given by the prices, we will get the forward rate formula given by the spot rates.

The second approach assumes short selling but in the Russian treasury bill market, short selling is not allowed. The repo market is not yet developed so we cannot replicate short selling through reverse repo, and we cannot apply the second approach to the Russian GKO market. But as we look at spot rates and forward rates of GKOs, we see that the equation still holds. This can only be explained by the first approach and there is a market consensus on pricing GKOs.

3 THE MODELS OF THE TERM STRUCTURE OF INTEREST RATES

This section will summarize some of the many studies on Term Structure of Interest Rates.

3.1 The Black-Scholes-Merton Model

The seminal papers of Black and Scholes(1973), and Merton(1973) laid the foundations for the pricing of contingent claims. Fischer Black once said "The Black-Scholes paper should probably be called The Black-Merton-Scholes paper." (*The Economist, October 18th-24th 1997, p.75*) These papers discuss stock as underlying asset. Merton modified Black-Scholes model and showed that the stochastic diffusion process of long-term bond price is similar to the stochastic process of stock price,

$$dB = \mu B dt + \sigma B dZ(t) \tag{9}$$

where B is the discount bond price, μ is the expected rate of return or the drift term of bond, σ is the volatility of the bond, $Z(t)$ is the standard Brownian motion.

The Black-Scholes-Merton model assumes continuous trading, symmetric information, no transaction costs, no bid-ask spread, no taxes, no restrictions on short selling. In addition, we can buy and sell any number of underlying asset, and we can sell assets that we do not own. There are no arbitrage opportunities, so all risk-free portfolios must earn the same return. The lending and borrowing rates are the same and constant, and the bond price follows a geometric Brownian motion.

Using stochastic calculus rather than ordinary calculus and Ito's Lemma,

$$dC = \sigma B \frac{\partial C}{\partial B} dZ + \left(\mu B \frac{\partial C}{\partial B} + \frac{1}{2}\sigma^2 B^2 \frac{\partial^2 C}{\partial B^2} + \frac{\partial C}{\partial t} \right) dt, \tag{10}$$

and constructing a portfolio of underlying asset and derivative, it is possible to eliminate the random component. This leads to the **Black-Scholes partial differential equation,**

$$rB \frac{\partial C}{\partial B} + \frac{1}{2}\sigma^2 B^2 \frac{\partial^2 C}{\partial B^2} + \frac{\partial C}{\partial t} = rC. \tag{11}$$

The expected return, μ, is not in the Black-Scholes partial differential equation. This shows that the derivative's price is free of risk preferences. In other words this is risk-neutral valuation. Instead of using the underlying asset and the derivative, two different derivatives can be used (different in maturity dates or exercise prices), so that the market price of risk can be derived as,

$$\lambda = \frac{\mu - r}{\sigma}, \tag{12}$$

which leads to the Black-Scholes equation (3), without the random components μ and λ.

It is also possible to get the same result by using the martingale approach.The price of a European type derivative security is found by solving the partial differential equation subject to the boundary conditions defined by the payoffs. The price of a European call option on a bond, with maturity T, given by the Black-Scholes-Merton model is,

$$C = BN(d_1) - Xe^{-r(T-t)}N(d_2). \tag{13}$$

The price of a European put option on a bond is,

$$P = Xe^{-r(T-t)}N(-d_2) - BN(-d_1), \tag{14}$$

where

$$d_1 = \frac{ln(B/X) + (r + \frac{1}{2}\sigma^2)(T-t)}{\sigma\sqrt{T-t}}, \qquad d_2 = d_1 - \sigma\sqrt{T-t},$$

C is the price of a European call option on a bond with the maturity T. P is the price of a European put option on a bond with the maturity T. X is the exercise price of the option, r is the risk-free rate. σ is the volatility of the instantaneous change in the log-bond price, and $(T - t)$ is the time to expiration. $N(.)$ is the cumulative normal density function.

Neftci (1996) gives a detailed explanation on how to use martingale approach to derive the same formula.

This approach is widely used for short-dated options on long-term bonds. Although it is not a problem to use known, constant risk-free rate for pricing derivatives on stocks, it causes inconsistency for pricing derivatives on bonds. The original Black-Scholes model assumes that the short term interest rate is known and is constant through time, whereas the stochastic process is assumed for the bond price.

Another problem is that the bond ages, it has a certain maturity. The original model assumes that the price of the underlying asset follows a lognormal process, which means that the distribution of the instantaneous rate is the same through time. This assumption is appropriate for the stock but it does not hold for the bond. As the bond approaches its maturity, the variance of the interest rate decreases.

Since the instantaneous rate is normally distributed in this model, the interest rate can become negative in the one-factor model. A solution to this problem is offered by Black (1976).

3.2 Black's Model

Black's model, as described in Black(1976), is widely used for pricing interest rate derivatives, especially caps and floors, although it was originally developed for valuing commodity futures options. Since the futures price is given by $F = Se^{r(T_t)}$, the option prices on futures contracts are given by,

$$C = e^{-r(T-t)}(FN(d_1) - XN(d_2)), \tag{15}$$

and the price of a European put option on a bond is,

$$P = e^{-r(T-t)}(XN(-d_2) - FN(-d_1)), \tag{16}$$

where

$$d_1 = \frac{ln(F/X) + \frac{1}{2}\sigma^2(T-t)}{\sigma\sqrt{T-t}}, \qquad d_2 = d_1 - \sigma\sqrt{T-t},$$

C is the price of a call option on a futures contract on the underlying asset with maturity T. P is the price of a put option on a futures contract on the underlying asset with maturity T. F is the futures contract price with maturity T, and other variables are the same as before. The interest rate, r, does not appear in the d_1 and d_2 terms, but is incorporated in the futures price. This model assumes that forward prices are equal to the futures prices. But in a world where interest rates are stochastic, this is not true. When the model is used to price options on interest rates, the futures price is replaced by forward interest rate and the strike price by strike rate. Although the forward rate is stochastic, the interest rate used for discounting is constant. This is an important inconsistency of the model. This model also assumes that the volatility of F is constant.

3.3 The Vasicek Model

Vasicek (1977) developed a one-factor model in which short rate is the driving factor. The model is based on arbitrage argument similar to that of Black and Scholes (1973). It does not take the current term structure as given. The spot rate follows a continuous Markow process. The diffusion process is given by the following stochastic differential equation,

$$dr = \varphi(r,t)dt + \rho(r,t)dZ(t) \tag{17}$$

where $\varphi(r,t)$ is the instantaneous drift, $\rho(r,t)$ is the instantaneous standard deviation, and Z is a Brownian motion. The model assumes that the price of a discount bond will be determined only by the process of spot interest rate over the life of the bond. Using the dynamic of spot rate and Ito's Lemma, the stochastic differential equation of the bond is given by,

$$dB = \mu(t,s)Bdt - \sigma(t,s)BdZ(t) \tag{18}$$

where t is the current date, s is maturity, and the parameters $\mu(t,s) = \mu(t,s,r(t))$, $\sigma(t,s) = \sigma(t,s,r(t))$ are given by,

$$\mu(t,s,r) = \frac{1}{B(t,s,r)}\left(\frac{\partial B(t,s,r)}{\partial t} + \varphi\frac{\partial B(t,s,r)}{\partial r} + \frac{1}{2}\rho^2\frac{\partial^2 B(t,s,r)}{\partial r^2}\right) \tag{19}$$

$$\sigma(t,s,r) = -\frac{1}{B(t,s,r)}\left(\rho\frac{\partial B(t,s,r)}{\partial r}\right) \tag{20}$$

$\mu(t,s,r), \sigma^2(t,s,r)$ are the mean and variance of the instantaneous rate of return at time t with maturity s, given that the current spot rate is $r(t) = r$. By using the arbitrage argument, as in the Black-Scholes-Merton, it is possible to construct a hedge portfolio using bonds with different maturities. This portfolio will be instantaneously riskless and the measure of the market price of risk will be the same for bonds with different maturities,

$$\lambda(t,r) = \frac{\mu(t,s_1) - r(t)}{\sigma(t,s_1)} = \frac{\mu(t,s_2) - r(t)}{\sigma(t,s_2)} \tag{21}$$

where s_1 and s_2 are arbitrary maturity dates of two bonds.

By using the market price of risk, the partial differential equation for the bond price can be written as,

$$(\varphi + \rho\lambda)\frac{\partial B}{\partial r} + \frac{1}{2}\rho^2\frac{\partial^2 B}{\partial r^2} + \frac{\partial B}{\partial t} = rB. \tag{22}$$

Vasicek generalizes his model by assuming first, that the market price of risk $\lambda(t,r)$ is a constant, i.e., $\lambda(r,t) = \lambda$; second, that the spot rate, $r(t)$, follows the Orstein-Uhlenbeck process,

$$dr = \alpha(\gamma - r)dt + \rho dZ, \tag{23}$$

where $\alpha > 0$ is the speed of the mean reversion, also called elasticity of mean reversion. γ is the long-term mean towards which the short rate is pulled. The larger α is, the faster r is pulled back towards γ. In the previous short rate stochastic process, $\varphi(t,r)$ is replaced by $\alpha(\gamma-r)$,and $\sigma(t,r)$ is replaced by ρ. The Orstein-Uhlenbeck process has a nice property; it is stationary, whereas the previous process is unstable and after a long time it may diverge to infinite values. However the process may give negative short rates with positive probability. The counter-argument is that for appropriate values of parameters α, γ, and ρ, the probability of negative short rates is very small.

3.4 The Dothan Model

Dothan (1978) presents a valuation formula for default-free bonds where the rate of interest follows a geometric Wiener process. The model has the one factor spot rate, as the source of uncertainty. He uses Vasicek's market price of risk but diverges in the stochastic diffusion process of interest rate,

$$dr(t) = \sigma r(t)dZ(t), \tag{24}$$

where σ is a constant, and Z is a standard Brownian motion. Since a lognormal distribution is assumed for the spot rate, $r(t)$ will always be positive. Although, it has this positive spot rate property, it lacks the attractive feature of mean reversion.

3.5 The Brennan-Schwartz Model

In the previous models, prices of bonds with different maturities are perfectly correlated. To resolve this problem, Brennan and Schwarz (1977), and (1982) offered a two-factor model. In this model, the spot rate, $r(t)$, and the long term rate, $\ell(t)$, are the two driving factors. The stochastic diffusion processes are given by,

$$dr(t) = \mu_r(r, \ell, t)dt, +\sigma_r(r, \ell, t)dZ_r(t), \tag{25}$$

and

$$d\ell(t) = \mu_\ell(r, \ell, t)dt + \sigma_\ell(r, \ell, t)dZ_\ell(t), \tag{26}$$

where $dZ_r(t)$ and $dZ_\ell(t)$ are standard Brownian motions of the spot rate and the long term rate respectively. They also have the following properties,

$$E[dZ_r(t)] = E[dZ_\ell(t)] = 0, \qquad dZ_r^2(t) = dZ_\ell^2(t) = dt, \qquad dZ_r(t)dZ_\ell(t) = \rho dt$$

where, ρ is the instantaneous correlation coefficient between the two Brownian motions, $dZ_r(t)$ and $dZ_\ell(t)$. Because of the two-factor assumption, the partial differential equation for the price of the bond contains two market prices of risk. The existence of two market prices of risk complicates the model further. Brennan and Schwarz's solution is the elimination of one of these by assuming that there exists a traded consol bond, which corresponds to the second factor, the long-term interest rate.

3.6 The Cox-Ingersoll-Ross Model

Cox, Ingersol, and Ross (1985a) (CIR) developed an intertemporal general equilibrium model. In this model, anticipation, risk aversion, investment alternatives, and preferences about the timing of consumption all play a role in determining bond prices. All these factors are included in a way that is fully consistent with maximizing behavior and rational expectations. All agents are assumed to be homogenous and to maximize their utility function. The utility function has a finite time horizon and is a time-additive logarithmic function of consumption. They apply this framework to construct one and two-factor models of the term structure of interest rates. For one-factor model, the short interest rate dynamic follows an Ornstein-Uhlenbeck process,

$$dr = \alpha(\gamma - r)dt + \sigma\sqrt{r}dZ, \tag{27}$$

but the stochastic diffusion equation of CIR is different from that of Vasicek (1977), the Brownian motion in CIR is multiplied by the square root of short rate. If $\sigma^2 > 2\alpha\gamma$, r could reach zero. If $2\alpha\gamma \geq \sigma^2$, r would be strictly positive. This interest rate structure precludes the negative interest rate. If the interest rate reaches zero it could become positive again, and the interest rates would be more volatile when they are high than when they are low.

Given the diffusion process (19), the fundamental equation for the price of a zero coupon bond is,

$$\frac{1}{2}\sigma^2 r\frac{\partial^2 P}{\partial r^2} + \alpha(\gamma - r)\frac{\partial P}{\partial r} + \frac{\partial P}{\partial t} - rP = 0, \tag{28}$$

with the terminal condition of $P(\tau, 0) = 1$. Cox, Ingersoll, and Ross (1985b) showed that the price of a zero coupon bond with maturity $\tau = T - t$ is given by,

$$P(r, \tau) = A(\tau)e^{B(\tau)r} \tag{29}$$

where

$$\begin{aligned} A(\tau) &\equiv \left[\frac{2\delta e^{(\alpha-\delta)\tau/2}}{2\delta+(\alpha-\delta)(1-e^{-\delta\tau})}\right]^{2\alpha\gamma/\sigma^2} \\ B(\tau) &\equiv \frac{-2(1-e^{-\delta\tau})}{2\delta+(\alpha-\delta)(1-e^{-\delta\tau})} \\ \delta &\equiv \sqrt{\alpha^2+2\sigma^2}. \end{aligned}$$

One criticism of the CIR model is that the implication of the logarithmic utility function assumption, i.e., that portfolio decisions in the following periods are independent of wealth at the end of the current period (Subrahmanyam, 1996). Chan et.al.(1992) showed that the square root process of the CIR model does not fit the data well.

Cox, Ingersoll, and Ross extended their one-factor model to a two-factor model by using the price level, $p(t)$, as a second factor. In this extended model, following Babbel and Merrill (1996), there is a nominal unit discount bond, $N(r,p,t,T)$, which will at time T pay $\frac{1}{p(T)}$ with certainty. The stochastic diffusion process of p is given by

$$dp = \mu(p)dt + \sigma(p)dZ_p, \tag{30}$$

where dZ_p is the Brownian process for p that is uncorrelated with dZ in the interest rate process given in (19). Using the diffusion process given by (22) and Ito's Lemma, the fundamental equation for the price of this contract is,

$$\frac{1}{2}\sigma^2 r\frac{\partial^2 N}{\partial r^2} + \frac{1}{2}\sigma^2(p)\frac{\partial^2 N}{\partial p^2} + \alpha(\gamma - r)\frac{\partial N}{\partial r} + \mu(p)\frac{\partial N}{\partial p} + \frac{\partial N}{\partial t} - rN = 0. \tag{31}$$

With the boundary condition $N(r,p,T,T) = \frac{1}{p(T)}$, the solution to (23) is

$$N(r,p,t,T) = P(r,t,T)E\left[\frac{1}{p(T)}\right], \tag{32}$$

where $P(r, t, T)$ is the price of a real discount bond given in (21). For lognormally distributed prices, $\mu(p) = \mu_p p$, $\sigma(p) = \sigma_p p$, and

$$N(r, p, t, T) = e^{-(\mu_p - \sigma_p^2)(T-t)} \frac{P(r, t, T)}{p(t)}. \tag{33}$$

This model has not been used much in practice.

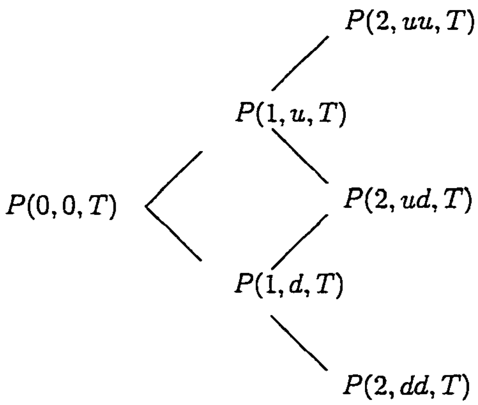
3.7 The Ho-Lee Model

The Ho-Lee Model (1986)is the first model that is based on an arbitrage-free interest rate process. It assumes that short term rate is normally distributed. This allows an analytical solution for European bond options. Although the model was presented in a binomial tree of bond prices it is possible to extend it to continuous time, in which case the stochastic process of the short rate is given by the following stochastic differential equation

$$dr = \theta(t)dt + \sigma(t)dZ, \tag{34}$$

where θ is the instantaneous drift, σ is the standard deviation, and Z is the standard Brownian motion.

The Ho-Lee model takes the term structure as given and derives the term structure movements that do not permit arbitrage profit opportunities. The model uses the full information of the term structure. It fits a binomial lattice for discount bond prices, imposing the constraint that at maturity the price of the bond will be equal to the face value. The model produces bond prices that are determined by market data. The term structure is assumed to be randomly perturbed. This perturbation is represented by the up and down movements in the binomial tree , which is time dependent:



where $P(1, u, T)$ is the price of a bond at time 1, in state u with maturity T.

To show how the term structure evolves, Ho and Lee define two functions, called perterbution functions, $h(T)$ and $h^*(T)$, such that, in the up state:

$$P(1, u, T) = \frac{P(0, 0, T)}{P(0, 0, 1)} h(T) \tag{35}$$

and, in the down state:

$$P(1, d, T) = \frac{P(0, 0, T)}{P(0, 0, 1)} h^*(T). \tag{36}$$

$h(T)$ and $h^*(T)$ should both be positive and satisfy the following condition:

$$h(T) = h^*(T) = 1. \tag{37}$$

The ratio of up state price and down state price gives

$$\frac{P(1, u, T)}{P(1, d, T)} = \frac{h(T)}{h^*(T)}. \tag{38}$$

If the probability that $h(T)$ will prevail is q and the probability that $h^*(T)$ will prevail is $(1 - q)$ then the no-arbitrage condition is given by

$$P(0, 0, T) = [qP(1, u, T) + (1 - q)P(1, d, T)]\, P(0, 0, 1), \tag{39}$$

and also satisfies the following conditions

$$qh(T)+(1-q)h^*(T)=1 \quad , \quad h(T)=\frac{1}{q+(1-q)\delta^T} \quad , \quad h^*(T)=\frac{\delta^T}{q+(1-q)\delta^T} \tag{40}$$

where δ is a constant. $0 \leq \delta \leq 1$, and also $0 \leq q \leq 1$.

In the Ho-Lee model, the price of a discount bond at time t, maturing at time T in continuous time is given by

$$P(t,T)=A(t,T)e^{-r(t)(T-T)}, \tag{41}$$

where $r(t)$ is the short rate and

$$lnA(t,T)=ln\left(\frac{P(0,T)}{P(0,t)}\right)-(T-t)\frac{\partial lnP(0,t)}{\partial t}-\frac{1}{2}\sigma^2 t(T-t)^2.$$

3.8 The Hull and White Model

Hull and White (1990) extend the Vasicek model to ensure an exact fit to the initial term structure. The model incorporates a short rate mean-reverting feature and a perfect matching of the initial term structure. The stochastic process for the short rate is given by

$$dr=(\theta(t)-ar)dt+\sigma(t)dZ \tag{42}$$

or

$$dr=a(\frac{\theta(t)}{a}-r)dt+\sigma(t)dZ \tag{43}$$

where $\frac{\theta(t)}{a}$ is the long term short-rate, a is the rate of reversion, σ is the standard deviation of the short rate, and Z is the standard Brownian motion. This model is closely related to the Vasicek model and the Ho-Lee model. The Hull and White model is a version of the Vasicek model with the time dependent reversion level. The Ho-Lee model can be treated as a particular case of the Hull and White model where the rate of reversion, a, is equal to zero. The short rates are normally distributed in the Hull and White model. This causes a problem, i.e., there is a positive probability that short rates can be negative, but for a mean reverting model it has a negative effect.

The discount bond price at time t with maturity T is given by

$$P(t,T) = A(t,T)e^{-B(t,T)r(t)}, \tag{44}$$

where

$$B(t,T) = \frac{1 - e^{a(T-t)}}{a},$$
$$lnA(t,T) = ln\left(\frac{P(0,T)}{P(0,t)}\right) - B(t,T)\frac{\partial P(0,t)}{\partial t} - \frac{v(t,T)^2}{2}, and$$
$$v(t,T)^2 = \frac{1}{2a^2}\sigma^2(e^{-aT} - e^{-at})^2(e^{2at} - 1).$$

3.9 The Black-Derman-Toy Model

In the Black-Derman-Toy (1990) model the driving factor is the short rate. The array of yields on zero coupon bonds, the yield curve, and the array of yield volatilities, the volatility curve, are inputs to the model and together they construct the term structure. The model calculates the future short rates in conformity with the inputs so that there will not be any arbitrage opportunities.

The Black-Derman-Toy model is an extended version of the Ho-Lee model. The interest rates are assumed to be lognormally distributed. The continuous time limit of the short rate dynamic can be given by

$$dr = \mu(t)rdt + \sigma(t)rdZ, \tag{45}$$

where μ is the drift, σ is the volatility, and Z is the standard Brownian motion. This geometric Brownian motion guarantees that short rates never become negative if the initial short rate is positive. Further explanation and implementation of the model are given below.

3.10 The Black-Karsinski Model

The Black-Karsinski (1991) model is an extension of the Black-Derman-Toy model in that time steps can be at varying lengths and there is mean reversion. It can also be evaluated as an extension of the Hull-White model; time steps can be at varying lengths, and short rates are normally distributed:

$$dlnr = \phi(t)(ln\mu(t) - lnr)dt + \sigma(t)dZ \quad (46)$$

where $\mu(t)$ is the long term short rate, $\phi(t)$ is the speed rate of reversion, σ is the volatility.

3.11 The Heath-Jarrow-Morton Model

As is shown in the **Interest Rates** section, given any one of the following information: the bond prices, the short rates, or the forward rates, the other two can be found by using arbitrage arguments as explained. The Heath-Jarrow-Morton (1992) model is different from the previous models for two reasons. The first reason is that, the Heath-Jarrow-Morton model uses forward rates as its driving factor, and the second reason is that it uses equivalent martingale pricing. The equivalent martingale measure ensures that the bond price is independent of the market price of risk. This result is very analogous to the Black-Scholes result. One good feature of the Heath-Jarrow-Morton model is that it can be extended to a multi-factor model, but this dissertation will focus on single-factor Heath-Jarrow-Morton model.

The forward rate at time t for date T, $f(t,T)$ is

$$f(t,T) = -\frac{\partial lnP(t,T)}{\partial T}, \quad (47)$$

and the bond price implied by the forward rate is

$$P(t,T) = e^{-\int_t^T f(t,s)ds}. \quad (48)$$

The stochastic process of the forward rate is given by

$$df(t,T) = \alpha(t,T)dt + \sigma(t,T)dZ, \quad (49)$$

where $\alpha(t,T)$ is the drift rate, $\sigma(t,T)$ is the volatility, Z is the standard Brownian motion, and T is the maturity. The spot interest rate process is a special form of the forward rate process

$$dr(t,t) = \alpha(t,t)dt + \sigma(t,t)dZ. \quad (50)$$

The forward rate is given by

$$f(t,T) = f(0,T) + \int_0^t \alpha(v,T)dv + \int_0^t \sigma(v,T)dZ, \quad (51)$$

where $f(0,T)$ is the initial forward rate. Given the forward rate, it is possible to write the spot rate as

$$r(t) = f(0,t) + \int_0^t \alpha(v,t0 + \int_0^t \sigma(v,t)dZ. \quad (52)$$

The accumulation of money is described through **money market account**. It assumes that the initial investment rolls over at the spot rate. The price of money market account is given as

$$B(t) = e^{\int_0^t r(y)dy}. \quad (53)$$

In the implementation of the model, one-period bonds are used rather than money market account, but both of them give the same result. The dynamics of the bond price is given by

$$lnP(t,T) = lnP(0,T) + \int_0^t (r(v) + b(v,T))dv - \frac{1}{2}\int_0^t a(v,T)^2 dv + \int_0^t a(v,T)dZ, \quad (54)$$

where

$$a(t,T) \equiv -\int_t^T \sigma(t,v)dv$$
$$b(t,T) \equiv -\int_t^T \alpha(t,v)dv + \frac{1}{2}a(t,T)^2.$$

The drift term $\alpha(t,T)$ in the bond price incorporates the subjective expected return of individual investors. In the implementation of the Heath-Jarrow-Morton model it is shown that it is possible to find a drift-free bond price by using equivalent martingale measure.

4 IMPLEMENTING THE MODELS

4.1 Constructing the Yield Curve

Cubic spline interpolation is used to construct the yield curve. A cubic polynomial is used between two consecutive data points. Cubic spline has desirable properties that makes it popular;

$$f(z) = a + bz + cz^2 + dz^3 \tag{55}$$

it is smooth and twice differentiable. The polynomial has four coefficients to be estimated. Four equations are needed to solve for the equations. Two of these equations come from the requirement that it has to pass through the two knot points which determine the interpolating range $x_i \leq x \leq x_{i+1}$. The other equations are derived from the condition that the first and second derivatives are continuous cross each data point, i.e.,

$$f'_{x_i}(z) = f'_{x_{i+1}}(z), and \tag{56}$$

$$f''_{x_i}(z) = f''_{x_{i+1}}(z). \tag{57}$$

4.2 Applying the Heath-Jarrow-Morton Model

This section follows Jarrow (1996). In order to determine whether the bond is fairly priced, we compare the arbitrage-free price to the market price. If they are different then arbitrage opportunities rise. First, we find the stochastic process for zero-coupon bonds.

The Stochastic Process of the Zero-Coupon Bond

The one-factor zero-coupon bond price evolution is as follows:

$$P(t+1,T;s_{t+1}) = \begin{cases} u(t,T;s_t)P(t,T;s_t) & \text{if } s_{t+1} = s_t\text{u (with probability } q_t(s_t) > 0) \\ d(t,T;s_t)P(t,T;s_t) & \text{if } s_{t+1} = s_t\text{d (with probability } 1 - q_t(s_t) > 0) \end{cases} \tag{58}$$

where

$$u(t,T;s_t) > d(t,T;s_t) \quad \text{for} \quad t < T-1, and$$

$$u(t,t+1;s_t)P(t,t+1;s_t) = d(t,t+1;s_t)P(t,t+1;s_t) = 1$$

where $P(t,T;s_t)$ is the T-maturity zero-coupon bond's price at time t under state s_t, $u(t,T;s_t)$ is the return at time t on the T-maturity zero-coupon bond in the up state, and $d(t,T;s_t)$ is the return at time t on the T-maturity bond in the down state.

The forward rate is the interest rate which is applicable from one point in time in the future to another time point in the future. It is implicit in the currently quoted spot rates. The time t-forward rate is defined as follows,

$$f(t,T;s_t) \equiv \frac{P(t,T;s_t)}{P(t,T+1;s_t)}. \tag{59}$$

The stochastic evolution of the forward rate is given by,

$$f_\Delta(t+\Delta,T;s_{t+\Delta}) = \begin{cases} \alpha_\Delta(t,T;s_t)f_\Delta(t,T;s_t) & \text{if } s_{t+\Delta} = s_t\text{u (with probability } q_t^\Delta(s_t) > 0) \\ \beta_\Delta(t,T;s_t)f_\Delta(t,T;s_t) & \text{if } s_{t+\Delta} = s_t\text{d (with probability } 1 - q_t^\Delta(s_t) > 0) \end{cases} \tag{60}$$

where $\alpha_\Delta(t,T;s_t)$ is the rate of change in the t-maturity forward rate over $[t, t+\Delta]$ in the up state; $\beta_\Delta(t,T;s_t)$ is the rate of change in the down state. These rates of changes can be defined in terms of the following stochastic processes,

$$\alpha_\Delta(t,T;s_t) \equiv \exp\left[\mu(t,T;s_t)\Delta - \sigma(t,T;s_t)\sqrt{\Delta}\right]$$

$$\beta_\Delta(t,T;s_t) \equiv \exp\left[\mu(t,T;s_t)\Delta + \sigma(t,T;s_t)\sqrt{\Delta}\right] \tag{61}$$
$$q_t^\Delta(s_t) \equiv \frac{1}{2} + \frac{1}{2}\phi(t;s_t)\sqrt{\Delta}.$$

Now the forward rate process can be written as,

$$f_\Delta(t+\Delta,T;s_{t+\Delta}) = \begin{cases} f_\Delta(t,T;s_t)\exp\left[\mu(t,T;s_t)\Delta - \sigma(t,T;s_t)\sqrt{\Delta}\right] \\ \qquad \text{if } s_{t+\Delta} = s_t\text{u (with probability } \frac{1}{2}+\frac{1}{2}\phi(t;s_t)\sqrt{\Delta}) \\ f_\Delta(t,T;s_t)\exp\left[\mu(t,T;s_t)\Delta + \sigma(t,T;s_t)\sqrt{\Delta}\right] \\ \qquad \text{if } s_{t+\Delta} = s_t\text{d (with probability } \frac{1}{2}-\frac{1}{2}\phi(t;s_t)\sqrt{\Delta}) \end{cases} \tag{62}$$

where $\mu(t,T;s_t)$ is the expected change in the logarithm of the forward rates, $\sigma(t,T;s_t)$ is the standard deviation of changes in the logarithm of the forward rates per unit time.

By using the definition of the forward rate and the stochastic evolution of bond price, the bond price process's rate-of-return parameters can be given as,

$$u(t,T;s_t) = \frac{r(t;s_t)}{\prod_{j=t+1}^{T-1}\alpha(t,j;s_t)}, \qquad d(t,T;s_t) = \frac{r(t;s_t)}{\prod_{j=t+1}^{T-1}\beta(t,j;s_t)} \tag{63}$$

using the above equations, $u(t,T;s_t)$ and $d(t,T;s_t)$, or their continuous time limits, $u_\Delta(t,T;s_t)$ and $d_\Delta(t,T;s_t)$ are given by,

$$u_\Delta(t,T;s_t) = r_\Delta(t;s_t)\exp\left\{-\sum_{j=t+\Delta}^{T-\Delta}\mu(t,j;s_t)\Delta + \sum_{j=t+\Delta}^{T-\Delta}\sigma(t,j;s_t)\sqrt{\Delta}\right\} \tag{64}$$

and

$$d_\Delta(t,T;s_t) = r_\Delta(t;s_t)\exp\left\{-\sum_{j=t+\Delta}^{T-\Delta}\mu(t,j;s_t)\Delta - \sum_{j=t+\Delta}^{T-\Delta}\sigma(t,j;s_t)\sqrt{\Delta}\right\} \tag{65}$$

where $r_\Delta(t;s_t)$ is the continuous time limit of the spot rate under state s_t, Taking pseudo probabilities, π, as $\frac{1}{2}$ the following results are found,

$$\exp\left\{\sum_{j=t+\Delta}^{T-\Delta}\mu(t,j;s_t)\Delta\right\} = \frac{1}{2}\left[\exp\left\{+\sum_{j=t+\Delta}^{T-\Delta}\sigma(t,j;s_t)\sqrt{\Delta}\right\} + \exp\left\{-\sum_{j=t+\Delta}^{T-\Delta}\sigma(t,j;s_t)\sqrt{\Delta}\right\}\right]$$

$$\equiv \cosh\left(\sum_{j=t+\Delta}^{T-\Delta} \sigma(t,j;s_t)\sqrt{\Delta}\right)$$

where $\cosh x \equiv \frac{1}{2}(e^x + e^{-x})$. Substituting this result into the above((12.8)) expression gives

$$u_\Delta(t,T;s_t) = r_\Delta(t;s_t)\left[\cosh\left(\sum_{j=t+\Delta}^{T-\Delta} \sigma(t,j;s_t)\sqrt{\Delta}\right)\right]^{-1} \exp\left\{\sum_{j=t+\Delta}^{T-\Delta} \sigma(t,j;s_t)\sqrt{\Delta}\right\} \quad (66)$$

and

$$d_\Delta(t,T;s_t) = r_\Delta(t;s_t)\left[\cosh\left(\sum_{j=t+\Delta}^{T-\Delta} \sigma(t,j;s_t)\sqrt{\Delta}\right)\right]^{-1} \exp\left\{-\sum_{j=t+\Delta}^{T-\Delta} \sigma(t,j;s_t)\sqrt{\Delta}\right\} \quad (67)$$

We substitute these in the price formula in Equation (58) using continuous time limit and get

$$P_\Delta(t+\Delta,T;s_{t+\Delta}) = \begin{cases} P_\Delta(t,T;s_t)r_\Delta(t;s_t)\left[\cosh\left(\sum_{j=t+\Delta}^{T\Delta} \sigma(t,j;s_t)\sqrt{\Delta}\right)\right]^{-1} & \\ \quad \exp\left\{\sum_{j=t+\Delta}^{T-\Delta} \sigma(t,j;s_t)\sqrt{\Delta}\right\} & \text{if} s_{t+\Delta} = s_t u \\ P_\Delta(t,T;s_t)r_\Delta(t;s_t)\left[\cosh\left(\sum_{j=t+\Delta}^{T-\Delta} \sigma(t,j;s_t)\sqrt{\Delta}\right)\right]^{-1} & \\ \quad \exp\left\{-\sum_{j=t+\Delta}^{T-\Delta} \sigma(t,j;s_t)\sqrt{\Delta}\right\} & \text{if} s_{t+\Delta} = s_t d \end{cases} \quad (68)$$

The standard deviation function, $\sigma(t,j;s_t)$, can either be calculated as **implied volatility**, or **historical volatility.**

Implied Volatilities

Given the price of traded interest rate derivatives, it is possible to estimate the volatility which equates the derivative model price equal to the observed market price. Iterative techniques are used to do this. Since derivative products of Russian Government Bonds are not traded, historical volatilities are going to be used.

Historical Volatilities

Forward rates, given by the observed past prices, are used to estimate the volatilities. Principal Component Analysis is used to approximate the volatilities.The vector stochastic process $x(t)$

is a $N \times 1$ vector

$$\begin{bmatrix} x_1(t) \\ \vdots \\ x_N(t) \end{bmatrix}$$

where, for deterministic volatility function, $x_i(t)$ is

$$x_i(t) = \frac{log f_\Delta(t+\Delta, t+i\Delta) - log f_\Delta(t, t+i\Delta)}{\Delta} \tag{69}$$

and for proportional volatility function $x_i(t)$ is

$$x_i(t) = \begin{cases} \dfrac{log f_\Delta(t+\Delta, t+i\Delta) - log f_\Delta(t, t+i\Delta)}{log f_\Delta(t, t+i\Delta)\Delta} & \text{if} f_\Delta(t, t+i\Delta) \leq M \\ \dfrac{log f_\Delta(t+\Delta, t+i\Delta) - log f_\Delta(t, t+i\Delta)}{M\Delta} & \text{if} f_\Delta(t, t+i\Delta) > M \end{cases}$$

where M is an arbitrary big number and $i = 1, \dots, N$. $x(t)$ is normally distributed random process with $N \times 1$ mean vector

$$\mu = \begin{pmatrix} \mu(\Delta) \\ \vdots \\ \mu(N\Delta) \end{pmatrix}$$

and $N \times N$ covariance matrix, Σ, whose (i, j)th element is

$$\sum_{k=1}^{N} \sigma_k(\Delta i)\sigma_k(\Delta j).$$

By using K time-series observations, the following matrix is constructed

$$X_{N\times K} \equiv [x_{N\times 1}(1) \dots x_{N\times 1}(K)].$$

Principal component analysis is used to calculate the volatilities. The volatility function is chosen to be deterministic.

4.3 Principal Component Analysis

This section mostly follows Jolliffe (1986). Principal component analysis (PCA)is a parsimonious modeling of the variation in the observed variables. Kahn(1989 and 1990) has used PCA to identify the movements in the term structure of interest rates.

PCA identifies the linear combinations of variables and orders them according to their participation in the total variance of the original data. So that the first principal component is the linear combination of variables with the highest variance, and the second principal component is the linear combination that has the second highest variance.

Principal component analysis transforms k observed variables Z_k into a linear function of K unobserved principal component.

$$Z_k = \ell_{k1}F_1 + \ell_{k2}F_2 + \ell_{k3}F_3 + \ldots + \ell_{kK}F_K \tag{70}$$

where ℓ_{k1} is the coefficient (loading) of kth variable Z_k on the first principal component, F_1. The first principal component can be written as,

$$\mathbf{F_1} = \alpha_1'\mathbf{Z} = \alpha_{11}\mathbf{Z_1} + \alpha_{12}\mathbf{Z_2} + \ldots + \alpha_{1k}\mathbf{Z_k} \tag{71}$$

where $\boldsymbol{\alpha}_1'\boldsymbol{\alpha} = 1$. It is possible to find an orthogonal matrix, $A_{N\times N}$, by principal component analysis, so that

$$\mathbf{Z'}_{K\times N} = \mathbf{F'}_{K\times N}\mathbf{A}_{N\times N}$$

The next step is to find the covariance matrix of $\mathbf{Z'}_{K\times N}$, Σ. Then covariance matrix, Σ, is decomposed into a matrix of eigenvectors, and a diagonal matrix of eigenvalues, Λ,

$$\Sigma = \mathbf{A}\Lambda\mathbf{A}'.$$

The kth column of the orthogonal matrix, $\mathbf{A}$, is α_k, which is the kth eigenvector of Σ. The kth eigenvalue of the covariance matrix, Σ, which is the kth diagonal element of the diagonal matrix Λ, is the variance of the kth principal component, F_k. In short, $\lambda_k = \text{var}(F_k) = \text{var}(\alpha_k Z)$, where λ_k is the kth eigenvalue.

The highest eigenvalue (variance) corresponds to the variance of the first principal component. By taking the variance of the first component only, it is assumed that a big portion of the total variation in the variables is accounted for. Jarrow (1996) uses the first principal component only in the one-factor model. The volatilities are computed by multiplying the eigenvector (which corresponds to the highest eigenvalue or variance) by the square root of the eigenvalue. If the kth eigenvalue is the highest eigenvalue, then the volatility vector will be

$$\alpha_k\sqrt{\lambda_k} = \begin{bmatrix} \sigma(\Delta) \\ \vdots \\ \sigma(N\Delta) \end{bmatrix} \tag{72}$$

where $\sigma(N\Delta)$ shows the volatility in Nth period.

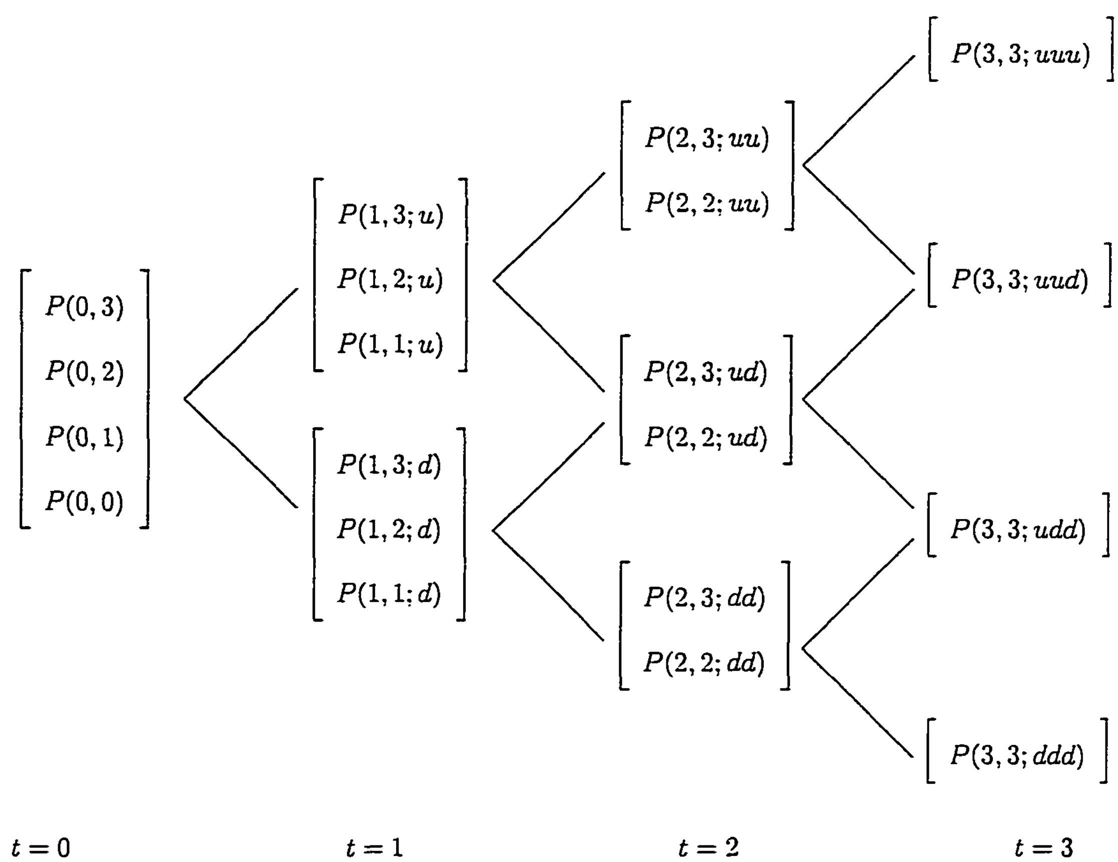
Determining the Stochastic Evolution of the Bond Prices Using Volatilities

Equation (68) gives the bond price process

$$P_\Delta(t+\Delta, T; s_{t+\Delta}) = \begin{cases} P_\Delta(t,T;s_t)r_\Delta(t;s_t)\left[\cosh\left(\sum_{j=t+\Delta}^{T-\Delta}\sigma(t,j)\sqrt{\Delta}\right)\right]^{-1}\exp\left\{\sum_{j=t+\Delta}^{T-\Delta}\sigma(t,j)\sqrt{\Delta}\right\} & \text{if } s_{t+\Delta} = s_t u \\ P_\Delta(t,T;s_t)r_\Delta(t;s_t)\left[\cosh\left(\sum_{j=t+\Delta}^{T-\Delta}\sigma(t,j)\sqrt{\Delta}\right)\right]^{-1}\exp\left\{-\sum_{j=t+\Delta}^{T-\Delta}\sigma(t,j)\sqrt{\Delta}\right\} & \text{if } s_{t+\Delta} = s_t d \end{cases}$$

The difference between this equation and equation (68) is that, the volatilities are not state dependent, i.e., there is only one volatility for every period and for all states in that period. This leads to a Markovian process, in which the nodes of the tree are combining. Different volatilities for different states in the same period would lead to a non-recombining tree.

A three-period bond process can be given by the following lattice



where

$$P(0,0) = P(1,1;u) = P(1,1;d) = P(0,1)r(0) = 100$$

$$P(2,2;uu) = P(1,2;u)r(1,u) = P(2,2;ud) = P(1,2;u)r(1,d) = P(2,2;dd) = P(1,2;d)r(1,d) = 100$$

The same holds for the bonds in the third period.

Note that the nodes will recombine in lattice,

$$P(2,2;ud) = P(2,2;du), P(3,3;uud) = P(3,3;udu) = P(3,3;duu), and P(3,3;udd) = P(3,3;dud) = P(3,3;ddu$$

$$P(1,2;u) = P(0,2)r(0)\cosh\left[\sigma(0,1)\sqrt{1}\right]^{-1}\exp\left\{\sigma(0,1)\sqrt{1}\right\}$$

$$P(1,2;d) = P(0,2)r(0)\cosh\left[\sigma(0,1)\sqrt{1}\right]^{-1}\exp\left\{-\sigma(0,1)\sqrt{1}\right\}$$

$$P(1,3;u) = P(0,3)r(0)\cosh\left[(\sigma(0,1)+\sigma(0,2))\sqrt{1}\right]^{-1}\exp\left\{(\sigma(0,1)+\sigma(0,2))\sqrt{1}\right\}$$

$$P(1,3;d) = P(0,3)r(0)\cosh\left[(\sigma(0,1)+\sigma(0,2))\sqrt{1}\right]^{-1}\exp\left\{-(\sigma(0,1)+\sigma(0,2))\sqrt{1}\right\}$$

$$P(2,3;uu) = P(1,3;u)r(1,u)\cosh\left[\sigma(0,2)\sqrt{1}\right]^{-1}\exp\left\{\sigma(0,2)\sqrt{1}\right\}$$

$$P(2,3;ud) = P(1,3;u)r(1,d)\cosh\left[\sigma(0,2)\sqrt{1}\right]^{-1}\exp\left\{-\sigma(0,2)\sqrt{1}\right\}$$

$$P(2,3;dd) = P(1,3;d)r(1,d)\cosh\left[\sigma(0,2)\sqrt{1}\right]^{-1}\exp\left\{-\sigma(0,2)\sqrt{1}\right\}.$$

Jarrow (1996) assumes that $u(t,T;s_t) > r(t;s_t) > d(t,T;s_t)$, since

$$u(0,3) = r(0)\cosh\left[(\sigma(0,1)+\sigma(0,2))\sqrt{1}\right]^{-1}\exp\left\{(\sigma(0,1)+\sigma(0,2))\sqrt{1}\right\}$$

$$d(0,3) = r(0)\cosh\left[(\sigma(0,1)+\sigma(0,2))\sqrt{1}\right]^{-1}\exp\left\{-(\sigma(0,1)+\sigma(0,2))\sqrt{1}\right\}.$$

It can be easily seen that in the case of a negative volatility, this condition can be violated. In principal component analysis, if some of the elements of the eigenvector that corresponds to the highest eigenvalue are negative then the volatilities that correspond to these negative elements will be negative. Hull (1997) mentions the negative volatility while dealing with the market price of risk. A similar analogy can be used in this case, a negative volatility shows that the forward rate of a certain maturity in a certain period is negatively related to that in the previous period. In Russian Government bond price analysis, there are negative volatilities which create anomalies in this analysis. At certain states, with negative volatilities, the bond price can exceed face value, which leads to a return smaller than 1 (remember, u is 1+ rate of return) or a negative return. With this violation, $u(t, T; s_t) \leq d(t, T : s_t)$, there is no equivalent martingale measure to prevent arbitrage.

4.4 Trading Strategies

After computing prices of bonds in different states, a trading strategy is applied to find arbitrage-free prices. Then, these arbitrage-free prices are compared to the observed prices. If there are discrepancies between the two, it is possible to have arbitrage opportunities.

In this section a replicating portfolio (or a series of them, depending on the maturity of the security we are replicating) will be constructed in order to find the arbitrage-free prices.

A one-period bond and a three-period bond will be used in order to replicate a two-period bond. At time two (end of first period) the value of this replicating portfolio, or two-period synthetic bond, equals face value, 100 in two states:

$$n_0(1; u)P(2, 2; uu) + n_1(1; u)P(2, 3; uu) = 100$$

$$n_0(1; u)P(2, 2; ud) + n_1(1; u)P(2, 3; ud) = 100,$$

where $n_0(1,u)$ is the number of one-period bonds that was invested at the beginning of first period (time one), and $n_1(1,u)$ is the number of three-period bonds that was invested at the same time. In this two-equation system, there are two unknowns, $n_0(1,u)$ and $n_1(1,u)$. Two equations can be easily solved for the two unknowns. After calculating the number of one-period and three-period bonds, the cost of the replicating portfolio can be determined and this cost should be equal to the price of the two-period bond at time one, up state, and is called arbitrage-free price:

$$n_0(1;u)P(1,2;u) + n_1(1;u)P(1,3;u) \stackrel{a}{=} P(1,2;u).$$

To find arbitrage-free price of down state at time one, the same procedure is followed, first the following two equations are solved for the unknowns

$$n_0(1;d)P(2,2;ud) + n_1(1;d)P(2,3;ud) = 100$$

$$n_0(1;d)P(2,2;dd) + n_1(1;d)P(2,3;dd) = 100.$$

Then the cost of the initial portfolio should give the arbitrage-free bond price:

$$n_0(1;d)P(1,2;d) + n_1(1;d)P(1,3;d) \stackrel{a}{=} P(1,2;d).$$

At the end of the first period, the value of the portfolio that is constructed at time zero, should be equal to the arbitrage-free prices calculated above:

$$n_0(0)P(1,1;u) + n_1(0)P(1,3;u) \stackrel{a}{=} P(1,2;u)$$

$$n_0(0)P(1,1;d) + n_1(0)P(1,3;d) \stackrel{a}{=} P(1,2;d).$$

The two equations above are solved for $n_0(0)$ and $n_1(0)$. Then the initial cost of the portfolio at time zero gives the arbitrage-free price of the two-period bond

$$n_0(0)P(0,1) + n_1(0)P(0,3) \stackrel{a}{=} P(0,2).$$

By comparing the observed price and the arbitrage free price, investment strategy will be determined.

This methodology could not be used in the Heath-Jarrow-Morton model with Russian GKOs because an equivalent martingale measure did not exist. The puzzling result is that the synthetic prices (arbitrage-free prices) are exactly the same as the observed prices despite the violation of the condition that the return in the up state should be bigger than the return in the down state.

4.5 Applying the Black-Derman-Toy Method

The model takes the short rate as driving factor. Long term rates and their volatilities are inputs to the model. The short term (spot rate) process is the output. Long term rates are the yields on zero coupon Treasury bonds. Given yields, bond prices can be found. If prices are observed then they can be used in the calculations without further manipulation.

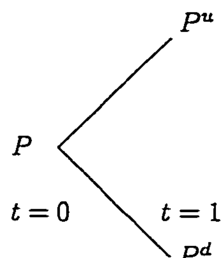
The BDT model uses local expectation hypothesis, which gives arbitrage-free prices. According to the local expectation hypothesis, all bonds should give the same expected rate of return over small holding periods

$$\frac{E_t(P(t+1,T))}{P(t,T)} = 1 + r_t$$

or

$$P(t,T) = \frac{E_t(P(t+1,T))}{1+r_t}.$$

The bond process is

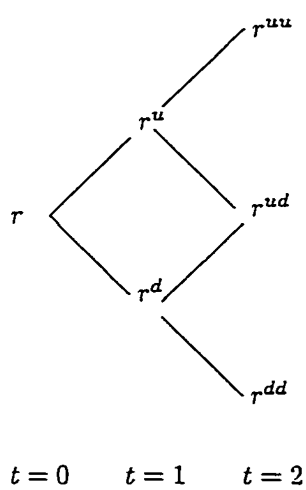


Using local expectation hypothesis and assuming equal probabilities for up and down state

$$P(1) = \frac{\frac{1}{2}(P(1)^u + P(1)^d)}{1+r}.$$

From this, the spot rate for the first period can be found by a using one-period bond which has a face value of $1.

The spot rate process is



After we find the first spot rate, we begin to work from the root forward through the tree. Using a two-period bond

$$P(2) = \frac{\frac{1}{2}(P(2)^u + P(2)^d)}{1+r}$$

and since

$$P(2)^u = \frac{P(2)^{uu}}{1+r^u}, \qquad P(2)^d = \frac{P(2)^{dd}}{1+r^d},$$

and $P(2)^{uu} = P(2)^{dd} = 1$,

$$P(2) = \frac{\frac{1}{2}\left(\frac{1}{1+r^u} + \frac{1}{1+r^d}\right)}{1+r}.$$

There are two unknowns in this equation. We also know that the volatility is a function of the log of spot rates

$$\sigma_1 = \frac{ln(r^u/r^d)}{2}.$$

The only unknowns are the two spot rates. Given these two equations, we can solve for the unknown spot rates. We proceed forward in the same way. The number of equations and unknowns increase, so in order to solve for the unknowns, the Newton-Raphson method is used.

The Newton-Raphson Method

The Newton-Raphson method is used to find the root given an initial estimate. It uses the tangential lines analytically evaluated. The method is derived by the Taylor expansion

$$f(x) = 0 = f(x_0) + f'(x_0)(x - x_0) + O(h^2). \tag{73}$$

The initial estimate is (x_0). The tangential line passing through the initial estimate is

$$g(x) = f'(x_0)(x - x_0) + f(x_0). \tag{74}$$

Let x_1 be the root of $g(x)$, then

$$f'(x_0)(x_1 - x_0) + f(x_0) = 0.$$

Manipulating the above equation,

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

and the generalized form is

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}.$$

The value of x is found by deducting the ratio of the function and its first derivative from the last value of x. The derivative of the function can be approximated by the forward difference approximation

$$f'(x_n) \simeq \frac{f(x_n + h) - f(x_n)}{h}. \tag{75}$$

As we construct the spot rate tree, we also construct the bond price tree as shown above. Using the bond price process and the spot rate process, we can apply the same **trading strategy** as described in the Jarrow method and find the arbitrage free prices.

Calculation of Volatility

Volatility is the standard deviation of the percentage changes in the yield.

$$\sigma = \sqrt{\frac{1}{1-n} \sum_{j=1}^{n} \left(ln(y_j/y_{j-1})^2 - \frac{1}{n} \sum_{j=1}^{n} ln(y_j/y_{j-1})^2 \right)} \tag{76}$$

where j is the index for time series observations, and n is the number of observations. For very short periods, the natural logarithm of the yield relative, $ln(y_j/y_{j-1})$, is almost equal to the percentage change of the yield, $\frac{y_j - y_{j-1}}{y_{j-1}}$.

5 DATA

The data used for this analysis is GKO (Gosudarstvenniye Kratkosrochniye Obligatsii) prices, which are the weighted average of transaction prices, and are available from the 29 August 1995 to the 1 October 1996. Due to the lack of data, the longest maturity is taken to be 84 days, and in the analysis the length of period for the lattice is taken to be one week. The results of the cubic spline interpolation were corrected for the anomalies, that is, the prices over one hundred roubles were set equal to one hundred roubles. For both of the models, Heath-Jarrow-Morton and Black-Derman-Toy, the volatilities are found by using the observations from the 29 August 1995 to the 1 October 1996, but the same information was used to generate the stochastic evolution of the all bond prices. In other words, for some of the synthetic bond prices, the historical volatilities that were used to find these synthetic bond prices were not technically historical. This was due to the lack of data. The trading strategies were applied by using the one week and 12-week bonds. The synthetic bond was replicating the six-week bond.

6 CONCLUSION

It is assumed that the Black-Derman-Toy model, together with the trading strategy, gives the arbitrage-free bond prices.

The observed bond prices and the Black-Derman-Toy model prices are given in Table 1. The second and fifth columns in Table 1 are the observed six-week bond prices, and the third and sixth columns are model or synthetic six-week bond prices, which are created by using one-week and 12-week bonds. The difference between the observed price and the model price implies arbitrage opportunities. If the observed bond price is lower than the model price, then it is implied that the six-week bond is cheaper relative to the one-week and 12-week bonds. In this case, one should buy the bond and short sell the synthetic bond, i.e., the combination of one-week and 12-week bonds. If the reverse is true, if the price of the actual six-week bond is higher than that of the synthetic six-week bond, then the combination of the one-week and 12-week bonds are relatively cheaper than the six-week bond. In this case, one should buy the combination of the one-week bond and 12-week bond, and sell short the six-week bond.
Since short selling is not allowed in the GKO (Gosudarstvenniye Kratkosrochniye Obligatsii) market, one can only apply one leg of the strategy: buy or not buy (although traders can create a synthetic short position, it will not be discussed here).

The descriptive statistics of the percentage difference between the model price and the actual price, $\frac{Model\ Price_t - Actual\ Price_t}{Actual\ Price_t}$ is given in Figure 1. Figure 2 is the plot of the percentage differences. If the percentage difference is positive, then the synthetic six-week bond price is higher than the actual six-week bond price, and if it is negative, then the synthetic six-week bond price is higher than the actual six-week bond price.

Assuming that the synthetic price given by the Black-Derman-Toy model is correct, trading decision can be made by using the above argument. The trade rule is that if the model price is higher than the actual price, then the six-week bond is relatively cheaper than the one-week and 12-week bonds, and the investor should buy the six-week bond. If the actual price is higher than the model price, then the six-week bond is relatively more expensive than the one-week and 12-week bonds, and the investor should buy the 12-week bond. The one-week bond is not included in this trading decision for two reasons. First, it is used as a proxy for money market account. Second, the movement of the very short term bonds are very erratic.

The results of this trading strategy can be tested by looking at the return over a certain period. At the time of the investment decision, it is assumed that there existed a mispricing and at the end of the period, it is assumed that the market will correct itself and the investor will make profits by unwinding the initial investment. In other words, if the actual six-week bond is lower than the synthetic price today, buy the actual six-week bond and sell it the next trading day. This approach is limiting in the sense that it ignores longer investment horizons and forces investment decisions to be made every day. In this analysis, bid and ask spread and transaction costs are ignored, the bonds are sold the next trading day. On the next trading day, the same investment strategy is applied, and new bonds are bought using the same investment decision rule.

Table 2 gives the daily returns of the six-week and the 12-week bonds. The third and sixth columns of Table 2 give the prices of the six-week (42-day) and 12-week (84-day) bonds. The second and fifth columns give the prices of the 41-day and 83-day bonds. For example, if an investor buys a 42-day bond on the September 4, 1995 and sells it the next trading day, September 5, 1995, the return will be, $\frac{88.7703799-88.2887404}{88.2887404} = 0.005455276$, or in a more generalized fashion, the daily return is equal to

$$Return = \frac{P(t, T-1) - P(t-1, T)}{P(t-1, T)} \tag{77}$$

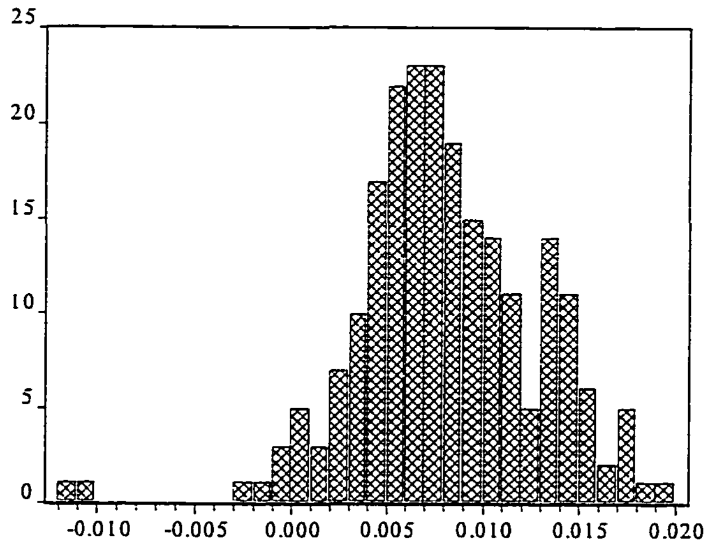
where $P(t, T)$ is the price of the T-maturity bond at time t. The fourth and seventh columns of Table 2 give the daily returns of the six-week and the 12-week bonds.

Table 3 gives the cumulative wealth of an investor, assuming one million Roubles as the initial investment. The second column of Table 3 is computed by using the returns in the fourth column of Table 2, and gives the wealth of an investor who invests only in the six-week bonds. The third column of Table 3 is computed in the same way and gives the cumulative wealth of an investor who invests only in the 12-week bonds. The fifth column of Table 3 gives the cumulative wealth of an investor who invested in either the six-week bond or the 12-week bond depending on the decision rule based on the percentage differences between the model prices and the actual prices, which are given by the fourth column, as explained above.

The investment which is based on the decision rule, for 221 consecutive trading day, yields 73.07% or a total of RUR 1,730,708, including the initial investment. By investing only in six-week bonds, the yield is 78.6% or a total of RUR 1,786,273.08. By investing only in 12-week bonds, the yield is 100.43% or a total of RUR 2,043,117. Obviously, these two naive investments are better than the decision rule. On average the percentage difference does not give a superior result. By using this result, it is possible to test whether a percentage difference is significantly different from the mean of the percentage differences. If it is not then the model does not give a superior answer.

Another approach is used by looking at the extreme values of the percentage differences. Table 4 gives the dates in which the percentage differences are at extreme points, this also can be seen in Figure 2. Out of these 14 extreme cases, in 12 of them the model gives the correct answer, i.e., the bond which was pointed by the decision rule gives a higher return than the other bond. This analysis chooses one-day periods as investment horizon by an arbitrary decision which may affect the results.

The model and the decision rule can be used when the difference between the observed price and the model price is substantial. Further analysis of other observations can help to determine a measure of difference that determines whether to use the model.



Series: SER1	
Sample 1 221	
Observations 221	
Mean	0.008123
Median	0.007810
Maximum	0.019512
Minimum	-0.011333
Std. Dev.	0.004650
Skewness	-0.371141
Kurtosis	4.542076
Jarque-Bera	26.97103
Probability	0.000001

Figure 1: The Descriptive statistics of the percentage change of the model price from the actual price.

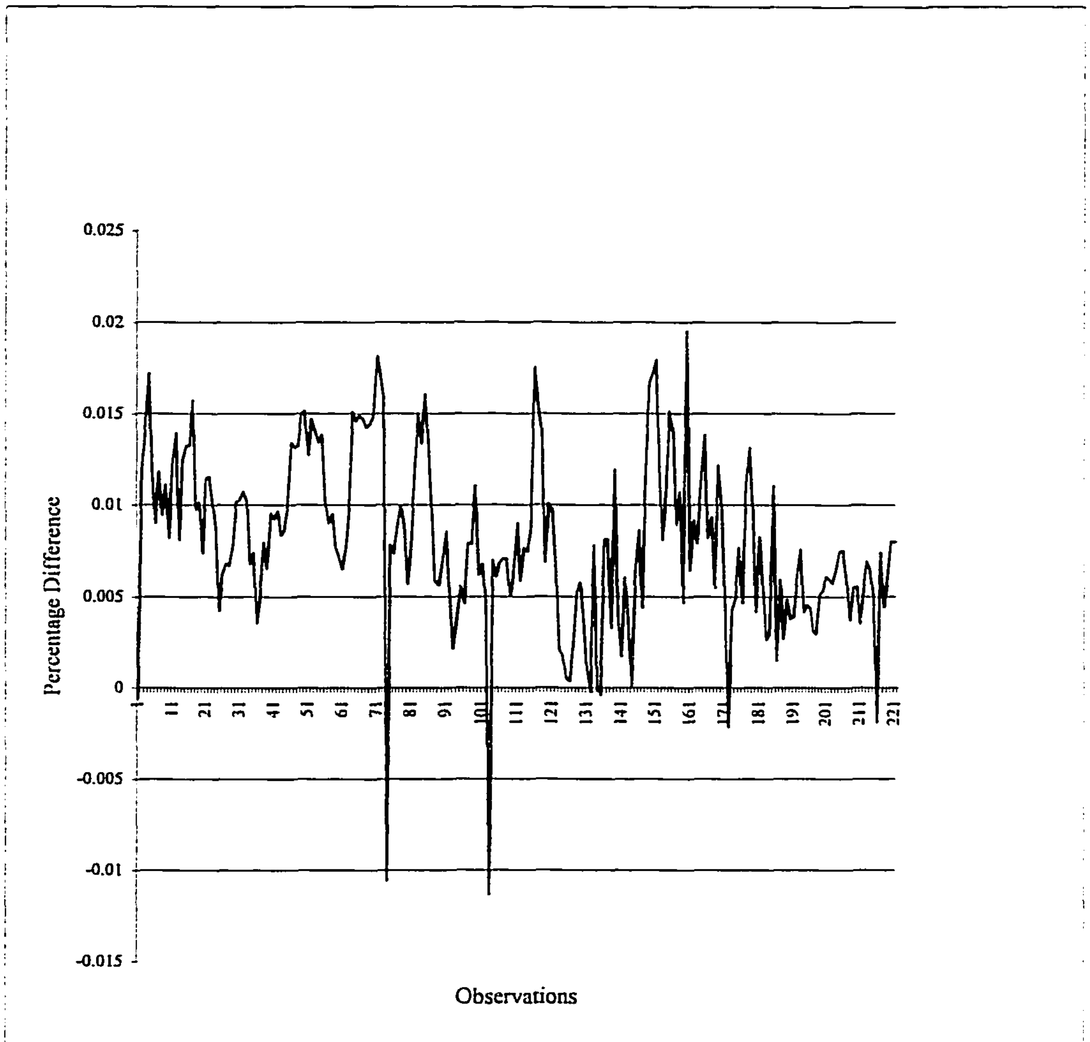


Figure 2: Comparison of the Black-Derman-Toy model prices with the actual prices.

Table 1: The actual six-week bond prices and synthetically created bond prices.

Dates	Actual Price	Synthetic Price	Dates	Actual Price	Synthetic Price
29-Aug-95	87.7450767	87.69320742	13-Nov-95	90.76419473	91.55069114
31-Aug-95	88.15460251	89.20131535	14-Nov-95	91.46783559	92.38111301
1-Sep-95	88.44882831	89.69136714	16-Nov-95	91.28752149	92.51112999
4-Sep-95	88.28874041	89.80664219	17-Nov-95	91.44385857	92.64416205
5-Sep-95	88.53736743	89.59963812	20-Nov-95	90.9956445	92.19721145
7-Sep-95	89.65296769	90.46063748	21-Nov-95	90.90756624	92.2693941
8-Sep-95	89.94149696	91.00075285	23-Nov-95	90.6941985	92.06793082
11-Sep-95	89.86369366	90.71387941	24-Nov-95	90.52464357	91.67701254
12-Sep-95	89.79329337	90.78977534	27-Nov-95	89.26694606	90.57881
14-Sep-95	90.66719754	91.41394942	28-Nov-95	88.67393801	89.91289557
15-Sep-95	91.48931898	92.63064419	30-Nov-95	90.04486725	91.25290919
18-Sep-95	90.6460225	91.90850356	1-Dec-95	90.31254936	91.56159487
19-Sep-95	90.12627501	91.47843836	4-Dec-95	90.31207922	91.23980089
21-Sep-95	90.23830538	91.36207357	5-Dec-95	90.18954347	90.99976183
22-Sep-95	90.61952482	91.8154842	7-Dec-95	91.63456306	92.50254942
25-Sep-95	89.90674514	91.09936286	8-Dec-95	92.5297666	93.24421885
26-Sep-95	88.21939861	89.60657171	9-Dec-95	93.21745793	93.88503328
28-Sep-95	89.19398748	90.06264593	13-Dec-95	92.39918943	93.00176974
29-Sep-95	89.45179754	90.35761481	14-Dec-95	92.09568777	92.80440385
2-Oct-95	88.49517171	89.14994894	15-Dec-95	92.63591444	93.56337052
3-Oct-95	88.98127232	89.99772456	18-Dec-95	91.50489818	92.88732113
5-Oct-95	90.48876727	91.53067983	19-Dec-95	90.72764234	92.05168709
6-Oct-95	90.95141368	91.88280173	21-Dec-95	88.95095545	90.2759167
9-Oct-95	91.21238521	91.9980671	22-Dec-95	87.52898206	88.81450795
10-Oct-95	91.65073706	92.03829892	25-Dec-95	86.31937725	87.54849124
12-Oct-95	93.01225607	93.58941781	26-Dec-95	86.33349412	87.57948884
13-Oct-95	92.60255291	93.23220624	28-Dec-95	86.645634	87.93123899
16-Oct-95	92.14096833	92.76155338	29-Dec-95	89.78577752	91.41465986
17-Oct-95	91.16423022	91.87331747	3-Jan-96	89.9	91.43693691
19-Oct-95	90.9903383	91.91174511	4-Jan-96	90.83634467	92.28089767
20-Oct-95	91.83504289	92.78072109	5-Jan-96	92.1270923	91.15382663
23-Oct-95	91.75814613	92.74092402	9-Jan-96	90.44575243	91.15382663
24-Oct-95	91.49242685	92.42720918	11-Jan-96	92.23280148	92.91465279
26-Oct-95	93.03312323	93.66473343	12-Jan-96	91.94962309	92.75630746
27-Oct-95	93.14010917	93.82748008	15-Jan-96	91.22948737	92.13390134
30-Oct-95	92.58745578	93.187406	16-Jan-96	91.2876405	92.09885981
31-Oct-95	92.09182381	92.54733252	18-Jan-96	93.68417316	94.2204919
2-Nov-95	92.90250192	93.64370269	19-Jan-96	93.35098627	94.08450682
3-Nov-95	93.50733135	94.12208173	22-Jan-96	92.83751538	93.96855903
4-Nov-95	93.69984027	94.59246786	23-Jan-96	93.21150576	94.60691428
8-Nov-95	93.16	94.02232308	25-Jan-96	93.24774732	94.49318385
9-Nov-95	93.75914875	94.66421628	26-Jan-96	92.85027693	94.33973264
10-Nov-95	93.37363074	94.15654987	29-Jan-96	92.71001461	93.96282889

Table 1: The actual six-week bond prices and synthetically created bond prices.

Dates	Actual Price	Synthetic Price	Dates	Actual Price	Synthetic Price
30-Jan-96	92.65606303	93.55228906	22-Apr-96	94.6943976	95.43397838
1-Feb-96	93.84098126	94.39301446	23-Apr-96	93.44207436	93.44391819
2-Feb-96	94.46660244	94.99818347	25-Apr-96	93.93468901	93.89768949
5-Feb-96	94.50122529	95.15482226	26-Apr-96	93.3350916	93.44896
6-Feb-96	93.93128276	94.73550028	29-Apr-96	93.77806062	93.43881
8-Feb-96	94.60873798	95.09102343	5-May-96	92.2460533	92.55148893
9-Feb-96	94.828012	95.03185883	6-May-96	92.33374539	93.433742
12-Feb-96	94.26161292	94.62861059	7-May-96	93.05968554	93.433742
13-Feb-96	94.19310411	94.71589527	12-May-96	92.82263342	92.98247193
15-Feb-96	94.58579581	95.02491174	13-May-96	92.45163382	93.01085148
16-Feb-96	94.89789692	95.65142869	14-May-96	92.3376345	92.66691415
19-Feb-96	94.76075388	95.51016113	16-May-96	94.08202858	94.09105276
20-Feb-96	94.85182665	95.89896553	17-May-96	93.05561169	93.64733135
22-Feb-96	95.70058147	96.29351062	20-May-96	93.0038603	93.80703739
23-Feb-96	95.58743332	96.2345681	21-May-96	93.57337112	93.9846681
26-Feb-96	95.03276501	95.48171219	23-May-96	92.00994997	93.10826874
27-Feb-96	94.40931137	93.33937784	24-May-96	91.52231208	93.05131412
29-Feb-96	94.16105792	94.81728757	27-May-96	91.16986496	92.73843069
1-Mar-96	94.22437309	94.79990129	28-May-96	90.52381544	92.1472309
4-Mar-96	94.22398445	94.86931007	30-May-96	90.95104668	92.03732874
5-Mar-96	93.76406721	94.42863295	31-May-96	91.22000833	91.74663
7-Mar-96	94.4860943	95.15587332	3-Jun-96	90.50233651	91.45593594
11-Mar-96	93.82212524	94.29584116	4-Jun-96	89.94168837	91.30187061
12-Mar-96	93.49234159	94.06031473	6-Jun-96	90.38022188	91.6366093
14-Mar-96	94.32542255	95.17512983	7-Jun-96	90.67381247	91.48698965
15-Mar-96	94.76517521	95.32228214	10-Jun-96	89.25026029	90.2056795
18-Mar-96	93.7582791	94.47634706	11-Jun-96	89.30530711	89.72420336
19-Mar-96	93.54	94.2409321	14-Jun-96	88.3918873	90.11658451
21-Mar-96	92.14759592	92.95123534	17-Jun-96	89.39087869	89.97085582
22-Mar-96	91.66144978	93.26491473	18-Jun-96	90.48571964	91.31252529
25-Mar-96	89.12791804	90.51716908	20-Jun-96	91.21689215	91.94002061
26-Mar-96	89.92045538	91.18813273	21-Jun-96	91.30174917	92.30272131
28-Mar-96	93.0026277	93.64275465	24-Jun-96	90.84088333	92.09727261
29-Mar-96	92.64851825	93.58102161	25-Jun-96	90.87660792	91.62562987
1-Apr-96	92.43898356	93.33347257	27-Jun-96	91.06484059	91.9140298
2-Apr-96	93.3418108	93.97400968	28-Jun-96	91.50976136	92.0141611
4-Apr-96	94.86491834	95.06489246	1-Jul-96	90.97780708	92.08319137
5-Apr-96	94.83513985	95.00326308	2-Jul-96	91.08779594	91.99432217
8-Apr-96	95.01993385	95.06657627	5-Jul-96	94.11806333	94.58352658
9-Apr-96	94.35050251	94.38231388	8-Jul-96	94.824237	94.62024178
11-Apr-96	92.37601859	92.59368035	9-Jul-96	93.4971652	93.89714588
12-Apr-96	92.45373414	92.940092	11-Jul-96	93.16309648	93.62223143
15-Apr-96	92.59884113	93.13358488	12-Jul-96	92.80427016	93.51768265
16-Apr-96	93.52765698	93.85991662	15-Jul-96	93.29378965	93.72868526
18-Apr-96	94.60451215	94.69502464	16-Jul-96	93.5619156	94.59172595
19-Apr-96	95.30949952	95.28822586	18-Jul-96	94.47266292	95.71113941

Table 1: The actual six-week bond prices and synthetically created bond prices.

Dates	Actual Price	Synthetic Price
19-Jul-96	94.72573907	95.6789915
22-Jul-96	94.8334168	95.2294957
23-Jul-96	93.74078503	94.51439846
25-Jul-96	94.5765187	95.09880503
26-Jul-96	94.73005839	94.98165167
29-Jul-96	94.55380279	94.83363416
30-Jul-96	93.68411455	94.71423248
1-Aug-96	95.38098071	95.52239823
2-Aug-96	95.14918129	95.71106882
5-Aug-96	94.97114672	95.22750143
6-Aug-96	94.45269813	94.9087467
8-Aug-96	95.66265083	96.02418397
9-Aug-96	96.01848617	96.3931812
12-Aug-96	95.45983045	96.0440981
13-Aug-96	94.75460672	95.47441421
15-Aug-96	95.48141342	95.87911703
16-Aug-96	95.59979694	96.03311868
19-Aug-96	95.31582936	95.72927035
20-Aug-96	94.77376482	95.06507622
22-Aug-96	95.29683593	95.5787752
23-Aug-96	95.6870712	96.1761458
26-Aug-96	95.37519012	95.88634715
27-Aug-96	95.20405807	95.78512874
29-Aug-96	95.11286669	95.68013095
30-Aug-96	95.20361102	95.75212549
2-Sep-96	94.92856908	95.54349533
3-Sep-96	94.55881008	95.26810372
5-Sep-96	95.53060818	96.2490344
6-Sep-96	95.77829595	96.34229882
9-Sep-96	95.47063382	95.82771409
10-Sep-96	95.32646294	95.85026643
12-Sep-96	95.88175848	96.41150096
13-Sep-96	96.10537586	96.44365774
16-Sep-96	95.58564711	96.09615281
17-Sep-96	95.18622825	95.84612082
19-Sep-96	95.73002976	96.34850954
20-Sep-96	95.90420246	96.3838208
23-Sep-96	94.89950556	94.72027684
24-Sep-96	93.69035834	94.38581281
26-Sep-96	94.65135954	95.07160146
27-Sep-96	95.36260002	95.92991628
30-Sep-96	94.85669189	95.61735449
1-Oct-96	94.23108258	94.98605745

Table 2: Daily returns of the 6-week and 12-week bonds.

Dates	Price of 41-day bond	Price of 42-day bond	Daily return of 6-week bond	Price of 83-day bond	Price of 84-day bond	Daily return of 12-week bond
29-Aug-95	87.9689209	87.7450767		78.1024358	77.9153999	
31-Aug-95	88.4	88.1546025	0.007463932	78.4060632	78.2260284	0.006297385
1-Sep-95	88.7225392	88.4488283	0.006442507	78.6047791	78.4122522	0.004841748
4-Sep-95	88.6038376	88.2887404	0.00175253	79.5087168	79.3202256	0.013983332
5-Sep-95	88.7703799	88.5373674	0.005455276	78.9243631	78.7357623	-0.004990688
7-Sep-95	89.88	89.6529677	0.015164587	79.9284353	79.6822341	0.015147793
8-Sep-95	90.1976496	89.941497	0.006075447	78.8710934	78.7150036	-0.010179693
11-Sep-95	90.1055657	89.8636937	0.001824172	79.3782286	79.1222968	0.008425649
12-Sep-95	90.0563372	89.7932934	0.002143731	79.5093098	79.3034392	0.004891327
14-Sep-95	90.8716173	90.6671975	0.012008959	80.9	80.6859931	0.020132303
15-Sep-95	91.7658641	91.489319	0.012117575	81.754741	81.6196028	0.013245766
18-Sep-95	90.917975	90.6460225	-0.006244925	81.5139824	81.3802728	-0.001294057
19-Sep-95	90.4062601	90.126275	-0.00264504	80.9471783	80.8044465	-0.005321861
21-Sep-95	90.5	90.2383054	0.004146682	81.2	80.9979113	0.004895195
22-Sep-95	90.9120931	90.6195248	0.00746676	81.3315138	81.102999	0.004118656
25-Sep-95	90.2051338	89.9067451	-0.004572867	80.679041	80.4039576	-0.005227402
26-Sep-95	88.4866936	88.2193986	-0.015794716	79.5319572	79.2664669	-0.010845242
28-Sep-95	89.45	89.1939875	0.013949329	79.6	79.2164888	0.004207745
29-Sep-95	89.7097161	89.4517975	0.005782101	79.6345734	79.3251949	0.005277747
2-Oct-95	88.7357566	88.4951717	-0.008004769	78.3318284	78.0432454	-0.012522711
3-Oct-95	89.2429406	88.9812723	0.008449827	78.9022153	78.6520508	0.011006332
5-Oct-95	90.8	90.4887673	0.020439443	79.9308985	79.6535593	0.016259559
6-Oct-95	91.2112892	90.9514137	0.007984659	80.7986293	80.5695384	0.014375629
9-Oct-95	91.4434056	91.2123852	0.005409393	81.3781524	81.1599366	0.010036225
10-Oct-95	91.8120366	91.6507371	0.006574232	81.7688539	81.5074537	0.007502684
12-Oct-95	93.2434477	93.0122561	0.017378045	83.9	83.624702	0.029353712
13-Oct-95	92.8148512	92.6025529	-0.002122353	83.2900334	83.0357976	-0.00400203
16-Oct-95	92.3561471	92.1409683	-0.002660896	82.2917684	82.0479047	-0.008960343
17-Oct-95	91.3768383	91.1642302	-0.008293054	81.8697488	81.6322771	-0.002171364
19-Oct-95	91.2158048	90.9903383	0.000565732	81.6554301	81.4255122	0.000283626
20-Oct-95	92.063946	91.8350429	0.01179914	81.7558694	81.5344955	0.00405717
23-Oct-95	91.9976823	91.7581461	0.001770995	81.6136024	81.4142348	0.000970227
24-Oct-95	91.7173389	91.4924269	-0.000444726	81.7255222	81.5379876	0.0038235
26-Oct-95	93.2	93.0331232	0.018663546	83.85	83.6397134	0.028355034
27-Oct-95	93.3187079	93.1401092	0.003069709	83.2852316	83.0758843	-0.004238199
30-Oct-95	92.753103	92.5874558	-0.004155097	82.4162262	82.2024468	-0.007940428
31-Oct-95	92.2638607	92.0918238	-0.003495021	82.0060262	81.8019811	-0.002389473
2-Nov-95	93.1	92.9025019	0.01094751	82.7248969	82.5059895	0.011282316
3-Nov-95	93.7252334	93.5073314	0.00885586	83.4047562	83.1869401	0.010893351
4-Nov-95	93.9135923	93.6998403	0.004344697	83.9707109	83.7519267	0.009421801
8-Nov-95	93.3642886	93.16	-0.003581134	83.6809596	83.5	-0.000847349
9-Nov-95	94	93.7591487	0.009016745	84.83	84.5964851	0.015928144

Table 2: Daily returns of the 6-week and 12-week bonds.

Dates	Price of 41-day bond	Price of 42-day bond	Daily return of 6-week bond	Price of 83-day bond	Price of 84-day bond	Daily return of 12-week bond
10-Nov-95	93.5870136	93.3736307	-0.001835929	85.2339025	84.9541087	0.007534798
13-Nov-95	90.9709647	90.7641947	-0.025731741	82.6437202	82.3913581	-0.027195725
14-Nov-95	91.7081192	91.4678356	0.010399745	83.027672	82.7965515	0.007723065
16-Nov-95	91.569843	91.2875215	0.001115228	82.48	82.2657734	-0.003823245
17-Nov-95	91.7308814	91.4438586	0.004856742	82.011576	81.8969197	-0.003089953
20-Nov-95	91.2910351	90.9956445	-0.001671227	81.3395568	81.1706091	-0.006805663
21-Nov-95	91.1945952	90.9075662	0.002186376	81.3262168	81.1807827	0.001917046
23-Nov-95	91.01	90.6941985	0.00112679	80.07	79.8486541	-0.013682828
24-Nov-95	90.8154461	90.5246436	0.001336884	79.8518282	79.6134673	3.97518E-05
27-Nov-95	89.5943189	89.2669461	-0.010277032	78.6054928	78.3607189	-0.012660854
28-Nov-95	88.9799795	88.673938	-0.003214701	78.400494	78.1694462	0.00050759
30-Nov-95	90.3204444	90.0448673	0.018568099	79.22	78.9868636	0.013439443
1-Dec-95	90.5630453	90.3125494	0.005754665	81.4720019	81.1963592	0.031462678
4-Dec-95	90.5391843	90.3120792	0.002509451	81.916443	81.7067421	0.008868424
5-Dec-95	90.3898727	90.1895435	0.000861385	81.3445842	81.1714661	-0.004432412
7-Dec-95	91.85	91.6345631	0.018410743	82.63	82.3669506	0.017968554
8-Dec-95	92.7234449	92.5297666	0.011882873	82.8105801	82.5660013	0.005386014
9-Dec-95	93.3716908	93.2174579	0.009098955	83.5879046	83.3580077	0.012376805
13-Dec-95	92.6018236	92.3991894	-0.006604282	82.4186453	82.2	-0.011269012
14-Dec-95	92.3142472	92.0956878	-0.000919296	81.32	81.1112773	-0.010705596
15-Dec-95	92.9054788	92.6359144	0.008792931	81.2466856	81.1147958	0.001669414
18-Dec-95	91.8208246	91.5048982	-0.008798854	80.5724014	80.5065859	-0.00668675
19-Dec-95	91.0410806	90.7276423	-0.005068773	80.1876524	80.0447432	-0.003961582
21-Dec-95	89.3	88.9509554	-0.015735473	77.5	77.3199081	-0.03179151
22-Dec-95	87.8117424	87.5289821	-0.012807204	77.004237	76.8146526	-0.004082664
25-Dec-95	86.6223817	86.3193773	-0.010357716	75.9523675	75.7295931	-0.011225529
26-Dec-95	86.598504	86.3334941	0.003233651	76.6264957	76.3625393	0.011843489
28-Dec-95	86.88	86.645634	0.006330172	77	76.7770273	0.00834782
29-Dec-95	90.1403364	89.7857775	0.040333277	80.9971038	80.9322792	0.054965355
3-Jan-96	90.1363961	89.9	0.003905057	80.9720232	80.85	0.000491077
4-Jan-96	91.1	90.8363447	0.013348165	82.39	82.0519276	0.019047619
5-Jan-96	92.3603312	92.1270923	0.016777277	83.0852295	82.787198	0.012593268
9-Jan-96	90.6700369	90.4457524	-0.01581571	81.2861146	81.0264115	-0.01813183
11-Jan-96	92.45	92.2328015	0.022159665	83.81	83.6344906	0.034354089
12-Jan-96	92.1535383	91.9496231	-0.000859382	83.2958224	83.1313648	-0.004049384
15-Jan-96	91.420523	91.2294874	-0.005754239	82.9548299	82.8663279	-0.002123566
16-Jan-96	91.4859998	91.2876405	0.002811727	82.8456598	82.753121	-0.000249414
18-Jan-96	93.95	93.6841732	0.029164512	85	84.8426162	0.027151592
19-Jan-96	93.7044734	93.3509863	0.000216688	84.8668037	84.6731609	0.000285086
22-Jan-96	93.1451841	92.8375154	-0.002204606	85.0295371	84.8135433	0.004208845
23-Jan-96	93.5205635	93.2115058	0.007357458	86.2479508	86.1340561	0.016912482
25-Jan-96	93.47	93.2477473	0.002773201	86.43	86.2599122	0.003435852

Table 2: Daily returns of the 6-week and 12-week bonds.

Dates	Price of 41-day bond	Price of 42-day bond	Daily return of 6-week bond	Price of 83-day bond	Price of 84-day bond	Daily return of 12-week bond
26-Jan-96	93.0639145	92.8502769	-0.001971445	86.1067592	85.9774754	-0.001775482
29-Jan-96	92.8738465	92.7100146	0.000253845	85.5226017	85.3631589	-0.005290615
30-Jan-96	92.838594	92.656063	0.001386898	85.9530052	85.7975222	0.006909846
1-Feb-96	94	93.8409813	0.014504577	86.8574711	86.7111369	0.012354073
2-Feb-96	94.6098432	94.4666024	0.008193243	87.4269841	87.2820675	0.008255539
5-Feb-96	94.6493341	94.5012253	0.001934351	87.3721457	87.2283397	0.001032035
6-Feb-96	94.1171072	93.9312828	-0.004064689	87.2191133	87.0673281	-0.000105772
8-Feb-96	94.66	94.608738	0.007757982	89.031046	88.8284997	0.022554015
9-Feb-96	94.8376742	94.828012	0.002419821	88.5511626	88.3803317	-0.003122164
12-Feb-96	94.3731332	94.2616129	-0.004796882	87.9590171	87.7814056	-0.004767063
13-Feb-96	94.3332771	94.1931041	0.000760269	88.0168177	87.8573717	0.0026818
15-Feb-96	94.79	94.5857958	0.006336938	88.03	87.8803927	0.00196487
16-Feb-96	95.0674137	94.8978969	0.005091862	88.4668435	88.3323416	0.006673284
19-Feb-96	94.9689677	94.7607539	0.000748918	88.094271	87.9393835	-0.002695169
20-Feb-96	95.1331702	94.8518266	0.003930069	88.0185123	87.8347749	0.000899811
22-Feb-96	95.84	95.7005815	0.010418074	89.6795493	89.497417	0.021002779
23-Feb-96	95.7481482	95.5874333	0.000497037	90.4252666	90.2816866	0.010367333
26-Feb-96	95.1138948	95.032765	-0.004953983	90.2040125	90.0606125	-0.000860353
27-Feb-96	94.5207667	94.4093114	-0.005387598	83.6173034	82.7682219	-0.07154414
29-Feb-96	94.32	94.1610579	-0.000946002	89.0191639	88.9115424	0.075523454
1-Mar-96	94.3953677	94.2243731	0.002488394	89.0357069	88.9282776	0.001396495
4-Mar-96	94.3862287	94.2239845	0.001717768	88.831264	88.6963924	-0.00109092
5-Mar-96	93.9293566	93.7640672	-0.003126888	88.4069232	88.2555847	-0.003263596
7-Mar-96	94.67	94.4860943	0.009661833	88.39	88.1296728	0.001523023
11-Mar-96	94.0001873	93.8221252	-0.005142629	85.8143194	85.5133741	-0.026272121
12-Mar-96	93.6893128	93.4923416	-0.001415577	85.0202749	84.7637781	-0.005766339
14-Mar-96	94.529114	94.3254225	0.011089383	86.3	86.1353857	0.018123565
15-Mar-96	94.9465077	94.7651752	0.006584494	87.4141587	87.2539593	0.014846081
18-Mar-96	93.949979	93.7582791	-0.008602276	86.5694985	86.4297667	-0.007844467
19-Mar-96	93.7425925	93.54	-0.000167309	86.32156	86.1618278	-0.001251961
21-Mar-96	92.3460347	92.1475959	-0.012764222	84	83.715953	-0.025090319
22-Mar-96	91.958923	91.6614498	-0.002047508	83.8037828	83.5184701	0.001049141
25-Mar-96	89.4649549	89.127918	-0.023963126	80.8615148	80.5417809	-0.031812787
26-Mar-96	90.1950181	89.9204554	0.011972681	82.3553348	82.1871254	0.022516933
28-Mar-96	93.2	93.0026277	0.036471619	86.22	86.0360539	0.04906942
29-Mar-96	92.871863	92.6485183	-0.001406032	85.7399716	85.5353068	-0.003441376
1-Apr-96	92.6174134	92.4389836	-0.000335729	86.1006856	85.9229593	0.006609888
2-Apr-96	93.4899034	93.3418108	0.011368795	87.0078489	86.837325	0.012626307
4-Apr-96	95	94.8649183	0.017764699	88.0978425	87.8176689	0.014515849
5-Apr-96	94.9495857	94.8351399	0.000892505	87.7097013	87.4131019	-0.001229452
8-Apr-96	95.1075808	95.0199339	0.002872785	86.9754331	86.7679143	-0.005006902
9-Apr-96	94.4833083	94.3505025	-0.005647505	86.5319597	86.3114188	-0.002719376

Table 2: Daily returns of the 6-week and 12-week bonds.

Dates	Price of 41-day bond	Price of 42-day bond	Daily return of 6-week bond	Price of 83-day bond	Price of 84-day bond	Daily return of 12-week bond
11-Apr-96	92.5753982	92.3760186	-0.018813936	82	81.8956413	-0.049951893
12-Apr-96	92.618087	92.4537341	0.002620468	83.9765634	83.7716867	0.025409435
15-Apr-96	92.7870274	92.5988411	0.003604974	83.1657987	82.8706954	-0.00723261
16-Apr-96	93.6373932	93.527657	0.011215606	82.8865025	82.588422	0.000190744
18-Apr-96	94.71	94.6045121	0.012641641	83	82.5755599	0.004983483
19-Apr-96	95.4318936	95.3094995	0.008745687	84.5774695	84.21424	0.024243367
22-Apr-96	94.9505061	94.6943976	-0.003766607	84.5805166	84.2917073	0.004349342
23-Apr-96	93.5766916	93.4420744	-0.011803296	83.6103236	83.2547913	-0.008083639
25-Apr-96	94.1	93.934689	0.007041	82.05	81.6689564	-0.014471135
26-Apr-96	93.5177596	93.3350916	-0.004438503	80.0940378	79.7090655	-0.019284178
29-Apr-96	93.9070799	93.7780606	0.006128331	78.7671563	78.4632119	-0.011816839
5-May-96	92.5564168	92.2460533	-0.013026969	73.6846508	73.3444611	-0.060901931
6-May-96	92.6894868	92.3337454	0.004807073	72.9803505	72.6597285	-0.004964392
7-May-96	93.3524185	93.0596855	0.011032511	74.5579966	74.1943005	0.026125451
12-May-96	93.0982039	92.8226334	0.00041391	76.0966342	75.618586	0.02563989
13-May-96	92.7182917	92.4516338	-0.001124098	78.3653151	77.9629067	0.036323465
14-May-96	92.5683646	92.3376345	0.001262614	80.1373178	79.7368051	0.027890329
16-May-96	94.3081761	94.0820286	0.021340611	81.35	80.8674207	0.020231497
17-May-96	93.3665529	93.0556117	-0.007604807	77.4125027	77.0479289	-0.042723237
20-May-96	93.2952582	93.0038603	0.002575304	77.4883446	77.2531368	0.005716127
21-May-96	93.8018035	93.5733711	0.008579678	77.6314089	77.4075508	0.004896528
23-May-96	92.35	92.00995	-0.013073924	74.5	74.5746642	-0.037561591
24-May-96	91.9656037	91.5223121	-0.000481973	74.759798	74.5420539	0.002482529
27-May-96	91.5631994	91.169865	0.000446747	74.288266	74.06964	-0.003404628
28-May-96	90.9069712	90.5238154	-0.00288356	74.3698525	74.1536759	0.004053112
30-May-96	91.32	90.9510467	0.008795305	74.9	74.5545302	0.01006456
31-May-96	91.5827568	91.2200083	0.006945606	75.6770486	75.4084704	0.015056339
3-Jun-96	90.8472525	90.5023365	-0.004086338	76.8396045	76.2468519	0.018978426
4-Jun-96	90.2926439	89.9416884	-0.002316986	77.0488605	76.7067523	0.01051858
6-Jun-96	90.72	90.3802219	0.008653514	76.97	76.6657217	0.003431871
7-Jun-96	90.9892451	90.6738125	0.006738456	76.0415126	75.6526417	-0.008141957
10-Jun-96	89.5570153	89.2502603	-0.012316645	74.5163499	74.3154438	-0.015019856
11-Jun-96	89.6596955	89.3053071	0.004587496	72.4953471	72.2528405	-0.024491501
14-Jun-96	88.8084258	88.3918873	-0.00556385	74.6972687	74.4037677	0.033831586
17-Jun-96	89.6796113	89.3908787	0.01456835	75.3672252	74.8965618	0.012949042
18-Jun-96	90.7378413	90.4857196	0.015068233	77.7735149	77.474479	0.038412351
20-Jun-96	91.44	91.2168921	0.010546199	80.55	80.3095387	0.039697214
21-Jun-96	91.5487497	91.3017492	0.003638115	81.1132903	80.8216896	0.010008171
24-Jun-96	91.1526817	90.8408833	-0.00163269	81.0460317	80.8032382	0.002775766
25-Jun-96	91.0603115	90.8766079	0.002415522	82.9810196	82.8504786	0.02695166
27-Jun-96	91.3	91.0648406	0.004658978	83.1	82.8498362	0.003011708
28-Jun-96	91.744614	91.5097614	0.007464719	82.8771174	82.6425368	0.000329284

Table 2: Daily returns of the 6-week and 12-week bonds.

Dates	Price of 41-day bond	Price of 42-day bond	Daily return of 6-week bond	Price of 83-day bond	Price of 84-day bond	Daily return of 12-week bond
1-Jul-96	91.3208045	90.9778071	-0.002064882	82.7141426	82.5421747	0.000866452
2-Jul-96	91.229703	91.0877959	0.002768762	84.4084299	84.190501	0.022609716
5-Jul-96	94.3092222	94.1180633	0.035366168	86.5365628	86.4198775	0.02786611
8-Jul-96	94.8202362	94.824237	0.007460554	87.8229352	87.7037536	0.016235359
9-Jul-96	93.5841616	93.4971652	-0.01307762	87.0858783	87.0378554	-0.007045027
11-Jul-96	93.4	93.1630965	-0.001039232	85.65	85.2981784	-0.015945423
12-Jul-96	93.0912132	92.8042702	-0.000771586	85.6629747	85.2833427	0.004276719
15-Jul-96	93.4378983	93.2937896	0.006827575	85.9624248	85.7466959	0.007962657
16-Jul-96	93.8335343	93.5619156	0.00578543	87.2608854	87.074252	0.017658867
18-Jul-96	94.65	94.4726629	0.011629565	88.98	88.790548	0.02188647
19-Jul-96	94.8755164	94.7257391	0.004264234	88.5301358	88.370153	-0.002932882
22-Jul-96	94.9493296	94.8334168	0.002360399	88.3949294	88.2666345	0.00028037
23-Jul-96	93.9453504	93.740785	-0.009364488	87.4387819	87.2205794	-0.009378999
25-Jul-96	94.7	94.5765187	0.010232632	88.55	88.3702655	0.015242052
26-Jul-96	94.8672412	94.7300584	0.003073939	88.46602	88.24476	0.00108356
29-Jul-96	94.6940529	94.5538028	-0.000380085	87.6261974	87.323963	-0.007009625
30-Jul-96	93.8973259	93.6841145	-0.006942892	86.8682215	86.8047964	-0.005218975
1-Aug-96	95.5	95.3809807	0.019383067	87.99	87.7063483	0.013653664
2-Aug-96	95.259046	95.1491813	-0.001278397	87.9373695	87.6846243	0.002634031
5-Aug-96	95.0919538	94.9711467	-0.000601451	86.943142	86.725024	-0.00845624
6-Aug-96	94.6246195	94.4526981	-0.003648763	86.4235348	86.2303577	-0.00347638
8-Aug-96	95.8	95.6626508	0.014264303	88.01	87.8635497	0.020638234
9-Aug-96	96.1887408	96.0184862	0.005499429	88.7595139	88.6376103	0.010197223
12-Aug-96	95.6501414	95.4598304	-0.003836186	88.8235476	88.6482788	0.002097725
13-Aug-96	94.9331838	94.7546067	-0.005516945	88.4594534	88.2871002	-0.002130051
15-Aug-96	95.57	95.4814134	0.008605315	89.35	89.1000656	0.012039129
16-Aug-96	95.712629	95.5997969	0.002421577	89.0306155	88.8249882	-0.000779461
19-Aug-96	95.4134957	95.3158294	-0.001948762	88.7731033	88.54589	-0.000584125
20-Aug-96	94.908921	94.7737648	-0.004269054	87.8556201	87.6218709	-0.007795617
22-Aug-96	95.44	95.2968359	0.007029743	88.6	88.41057	0.01116307
23-Aug-96	95.8176024	95.6870712	0.005464677	89.155329	89.0302265	0.008423867
26-Aug-96	95.5258376	95.3751901	-0.001685009	88.8420464	88.7048908	-0.002113665
27-Aug-96	95.3537723	95.2040581	-0.000224564	88.8404494	88.706716	0.001528198
29-Aug-96	95.3	95.1128667	0.00100775	88.46	88.3333708	-0.002781255
30-Aug-96	95.3829327	95.203611	0.002839426	88.7320235	88.5794098	0.004513047
2-Sep-96	95.0952591	94.9285691	-0.001138108	88.5380284	88.3904709	-0.000467168
3-Sep-96	94.7382965	94.5588101	-0.002004376	88.4257031	88.2348931	0.000398597
5-Sep-96	95.68	95.5306082	0.011857065	89.3	89.1567758	0.012071266
6-Sep-96	95.9375547	95.7782959	0.004259855	89.9895194	89.8114422	0.009340216
9-Sep-96	95.5675667	95.4706338	-0.002200177	90.3039079	90.0935899	0.00548333
10-Sep-96	95.4506077	95.3264629	-0.000209763	90.2665536	90.1572338	0.001919822
12-Sep-96	95.99	95.8817585	0.006960681	91.55	91.4361533	0.015448191

Table 2: Daily returns of the 6-week and 12-week bonds.

Dates	Price of 41-day bond	Price of 42-day bond	Daily return of 6-week bond	Price of 83-day bond	Price of 84-day bond	Daily return of 12-week bond
13-Sep-96	96.2316971	96.1053759	0.00364969	91.641267	91.5047284	0.002243245
16-Sep-96	95.7231616	95.5856471	-0.003977034	90.9475677	90.8302326	-0.006088873
17-Sep-96	95.3262935	95.1862282	-0.002713311	90.6583353	90.5385203	-0.001892512
19-Sep-96	95.85	95.7300298	0.006973401	91.0957808	90.9779799	0.006154955
20-Sep-96	96.0165599	95.9042025	0.002993106	91.0349499	90.9376693	0.000626195
23-Sep-96	94.8536706	94.8995056	-0.010953971	88.9991692	88.9368341	-0.021316801
24-Sep-96	93.8642968	93.6903583	-0.010908474	87.036499	87.0133839	-0.021367245
26-Sep-96	94.8	94.6513595	0.011843712	88.87	88.7619073	0.021337133
27-Sep-96	95.4827783	95.3626	0.008784013	90.3707105	90.3393954	0.018124929
30-Sep-96	94.9764237	94.8566919	-0.004049557	89.9404668	89.9914737	-0.004415887
1-Oct-96	94.3544969	94.2310826	-0.005294249	89.8709504	89.8057077	-0.001339274

Table3: Comparison of decision rules.

Dates	Investment in 6-week bonds	Investment in 12-week bonds	Percentage difference	Decision rule with per. diff.
29-Aug-95	1,000,000.00	1,000,000.00	-0.000591	1,000,000.00
31-Aug-95	1,007,463.93	1,006,297.39	0.0118736	1,007,463.93
1-Sep-95	1,013,954.53	1,011,169.62	0.0140481	1,013,954.53
4-Sep-95	1,015,731.51	1,025,309.14	0.0171925	1,015,731.51
5-Sep-95	1,021,272.61	1,020,192.15	0.011998	1,021,272.61
7-Sep-95	1,036,759.78	1,035,645.80	0.0090088	1,036,759.78
8-Sep-95	1,043,058.56	1,025,103.25	0.0117772	1,043,058.56
11-Sep-95	1,044,961.28	1,033,740.41	0.0094608	1,044,961.28
12-Sep-95	1,047,201.40	1,038,796.77	0.0110975	1,047,201.40
14-Sep-95	1,059,777.20	1,059,710.14	0.0082362	1,059,777.20
15-Sep-95	1,072,619.13	1,073,746.81	0.012475	1,072,619.13
18-Sep-95	1,065,920.70	1,072,357.33	0.0139276	1,065,920.70
19-Sep-95	1,063,101.30	1,066,650.39	0.0081228	1,060,248.02
21-Sep-95	1,067,509.64	1,071,871.85	0.0124533	1,064,644.53
22-Sep-95	1,075,480.48	1,076,286.52	0.0131976	1,072,593.97
25-Sep-95	1,070,562.45	1,070,660.34	0.0132651	1,067,689.14
26-Sep-95	1,053,653.22	1,059,048.77	0.0157241	1,050,825.30
28-Sep-95	1,068,350.97	1,063,504.98	0.009739	1,065,483.61
29-Sep-95	1,074,528.29	1,069,117.89	0.0101263	1,071,644.34
2-Oct-95	1,065,926.94	1,055,729.63	0.007399	1,058,224.45
3-Oct-95	1,074,933.83	1,067,349.34	0.0114232	1,067,166.26
5-Oct-95	1,096,904.88	1,084,703.97	0.0115143	1,088,978.54
6-Oct-95	1,105,663.29	1,100,297.27	0.0102405	1,097,673.67
9-Oct-95	1,111,644.26	1,111,340.10	0.0086138	1,103,611.41
10-Oct-95	1,118,952.47	1,119,678.14	0.0042287	1,111,891.46
12-Oct-95	1,138,397.68	1,152,544.85	0.0062052	1,144,529.60
13-Oct-95	1,135,981.59	1,147,932.33	0.0067995	1,139,949.16
16-Oct-95	1,132,958.87	1,137,646.46	0.0067352	1,129,734.83
17-Oct-95	1,123,563.18	1,135,176.22	0.0077781	1,127,281.76
19-Oct-95	1,124,198.81	1,135,498.18	0.0101264	1,127,919.50
20-Oct-95	1,137,463.39	1,140,105.09	0.0102976	1,141,227.98
23-Oct-95	1,139,477.83	1,141,211.25	0.0107105	1,143,249.09
24-Oct-95	1,138,971.08	1,145,574.67	0.010217	1,142,740.66
26-Oct-95	1,160,228.32	1,178,057.48	0.0067891	1,175,143.11
27-Oct-95	1,163,789.88	1,173,064.64	0.00738	1,170,162.62
30-Oct-95	1,158,954.22	1,163,750.01	0.003551	1,160,871.03
31-Oct-95	1,154,903.65	1,160,969.26	0.0049462	1,158,097.16
2-Nov-95	1,167,546.97	1,174,067.68	0.0079783	1,171,163.17
3-Nov-95	1,177,886.60	1,186,857.21	0.0065744	1,183,921.07
4-Nov-95	1,183,004.16	1,198,039.54	0.0095265	1,189,064.84
8-Nov-95	1,178,767.67	1,197,024.39	0.0092564	1,184,806.64
9-Nov-95	1,189,396.31	1,216,090.76	0.0096531	1,195,489.74

Table3: Comparison of decision rules.

Dates	Investment in 6-week bonds	Investment in 12-week bonds	Percentage difference	Decision rule with per. diff.
10-Nov-95	1,187,212.67	1,225,253.76	0.0083848	1,193,294.91
13-Nov-95	1,156,663.62	1,191,932.10	0.0086653	1,162,589.35
14-Nov-95	1,168,692.63	1,201,137.47	0.0099847	1,174,679.99
16-Nov-95	1,169,995.98	1,196,545.22	0.0134039	1,175,990.02
17-Nov-95	1,175,678.35	1,192,847.95	0.0131261	1,181,701.50
20-Nov-95	1,173,713.53	1,184,729.83	0.0132047	1,179,726.61
21-Nov-95	1,176,279.71	1,187,001.01	0.0149804	1,182,305.94
23-Nov-95	1,177,605.13	1,170,759.48	0.0151469	1,183,638.15
24-Nov-95	1,179,179.45	1,170,806.02	0.0127299	1,185,220.53
27-Nov-95	1,167,060.98	1,155,982.62	0.014696	1,173,039.99
28-Nov-95	1,163,309.23	1,156,569.39	0.0139721	1,169,269.01
30-Nov-95	1,184,909.67	1,172,113.03	0.013416	1,190,980.11
1-Dec-95	1,191,728.43	1,208,990.85	0.0138303	1,197,833.81
4-Dec-95	1,194,719.02	1,219,712.69	0.0102724	1,200,839.71
5-Dec-95	1,195,748.13	1,214,306.42	0.0089835	1,201,874.10
7-Dec-95	1,217,762.74	1,236,125.75	0.0094723	1,224,001.49
8-Dec-95	1,232,233.26	1,242,783.54	0.0077213	1,230,593.98
9-Dec-95	1,243,445.30	1,258,165.23	0.0071615	1,245,824.80
13-Dec-95	1,235,233.23	1,243,986.95	0.0065215	1,231,785.59
14-Dec-95	1,234,097.69	1,230,669.33	0.0076954	1,218,598.59
15-Dec-95	1,244,949.02	1,232,723.83	0.0100118	1,229,313.64
18-Dec-95	1,233,994.90	1,224,480.91	0.0151076	1,218,497.09
19-Dec-95	1,227,740.06	1,219,630.03	0.0145936	1,212,320.81
21-Dec-95	1,208,420.99	1,180,856.15	0.0148954	1,193,244.37
22-Dec-95	1,192,944.50	1,176,035.11	0.0146869	1,177,962.24
25-Dec-95	1,180,588.31	1,162,833.50	0.0142391	1,165,761.25
26-Dec-95	1,184,405.93	1,176,605.50	0.0144323	1,169,530.91
28-Dec-95	1,191,903.42	1,186,427.59	0.0148375	1,176,934.24
29-Dec-95	1,239,976.79	1,251,640.01	0.0181419	1,224,403.86
3-Jan-96	1,244,818.97	1,252,254.66	0.0170961	1,229,185.22
4-Jan-96	1,261,435.02	1,276,107.13	0.0159028	1,245,592.59
5-Jan-96	1,282,598.46	1,292,177.49	-0.010564	1,261,278.67
9-Jan-96	1,262,313.26	1,268,747.95	0.0078287	1,238,409.38
11-Jan-96	1,290,285.70	1,312,334.63	0.0073927	1,280,953.81
12-Jan-96	1,289,176.85	1,307,020.48	0.0087731	1,279,852.98
15-Jan-96	1,281,758.62	1,304,244.93	0.0099136	1,272,488.40
16-Jan-96	1,285,362.57	1,303,919.64	0.0088864	1,276,066.29
18-Jan-96	1,322,849.54	1,339,323.13	0.0057248	1,310,713.52
19-Jan-96	1,323,136.19	1,339,704.95	0.0078577	1,311,087.19
22-Jan-96	1,320,219.20	1,345,343.56	0.012183	1,308,196.76
23-Jan-96	1,329,932.65	1,368,096.66	0.0149703	1,317,821.76
25-Jan-96	1,333,620.82	1,372,797.24	0.0133562	1,321,476.34

Table3: Comparison of decision rules.

Dates	Investment in 6-week bonds	Investment in 12-week bonds	Percentage difference	Decision rule with per. diff.
26-Jan-96	1,330,991.66	1,370,359.86	0.0160415	1,318,871.12
29-Jan-96	1,331,329.53	1,363,109.82	0.0135133	1,319,205.91
30-Jan-96	1,333,175.95	1,372,528.69	0.0096726	1,321,035.52
1-Feb-96	1,352,513.10	1,389,485.02	0.0058826	1,337,355.69
2-Feb-96	1,363,594.57	1,400,955.96	0.0056272	1,348,396.28
5-Feb-96	1,366,232.24	1,402,401.80	0.0069163	1,349,787.87
6-Feb-96	1,360,678.93	1,402,253.46	0.0085618	1,344,301.40
8-Feb-96	1,371,235.05	1,433,879.91	0.0050977	1,374,620.80
9-Feb-96	1,374,553.20	1,429,403.10	0.0021496	1,370,329.01
12-Feb-96	1,367,959.63	1,422,589.05	0.0038934	1,363,796.56
13-Feb-96	1,368,999.64	1,426,404.15	0.0055502	1,367,453.99
15-Feb-96	1,377,674.91	1,429,206.84	0.0046425	1,370,140.86
16-Feb-96	1,384,689.84	1,438,744.35	0.0079404	1,379,284.20
19-Feb-96	1,385,726.86	1,434,866.69	0.0079084	1,375,566.80
20-Feb-96	1,391,172.86	1,436,157.80	0.0110397	1,380,972.87
22-Feb-96	1,405,666.21	1,466,321.10	0.0061957	1,409,977.14
23-Feb-96	1,406,364.87	1,481,522.94	0.0067701	1,424,594.84
26-Feb-96	1,399,397.77	1,480,248.31	0.0047241	1,423,369.18
27-Feb-96	1,391,858.37	1,374,345.22	-0.011333	1,321,535.46
29-Feb-96	1,390,541.67	1,478,140.52	0.0069692	1,421,342.38
1-Mar-96	1,394,001.89	1,480,204.73	0.0061081	1,423,327.28
4-Mar-96	1,396,396.46	1,478,589.95	0.0068488	1,421,774.55
5-Mar-96	1,392,030.08	1,473,764.43	0.0070876	1,417,134.45
7-Mar-96	1,405,479.65	1,476,009.00	0.0070887	1,419,292.78
11-Mar-96	1,398,251.79	1,437,231.12	0.0050491	1,382,004.94
12-Mar-96	1,396,272.45	1,428,943.55	0.0060751	1,374,035.83
14-Mar-96	1,411,756.25	1,454,841.11	0.0090083	1,389,273.04
15-Mar-96	1,421,051.95	1,476,439.79	0.0058788	1,409,898.30
18-Mar-96	1,408,827.67	1,464,857.91	0.0076587	1,398,838.40
19-Mar-96	1,408,591.96	1,463,023.97	0.0074934	1,397,087.11
21-Mar-96	1,390,612.38	1,426,316.23	0.0087212	1,379,254.38
22-Mar-96	1,387,765.09	1,427,812.63	0.0174933	1,376,430.35
25-Mar-96	1,354,509.90	1,382,389.94	0.0155872	1,343,446.77
26-Mar-96	1,370,727.02	1,413,517.12	0.0140978	1,359,531.43
28-Mar-96	1,420,719.65	1,482,877.58	0.0068829	1,426,242.85
29-Mar-96	1,418,722.07	1,477,774.44	0.010065	1,424,237.51
1-Apr-96	1,418,245.77	1,487,542.37	0.0096765	1,423,759.35
2-Apr-96	1,434,369.51	1,506,324.53	0.0067729	1,441,736.17
4-Apr-96	1,459,850.66	1,528,190.11	0.002108	1,462,664.20
5-Apr-96	1,461,153.58	1,526,311.28	0.0017728	1,460,865.92
8-Apr-96	1,465,351.16	1,518,669.18	0.0004909	1,453,551.51
9-Apr-96	1,457,075.58	1,514,539.35	0.0003372	1,449,598.76

Table3: Comparison of decision rules.

Dates	Investment in 6-week bonds	Investment in 12-week bonds	Percentage difference	Decision rule with per. diff.
11-Apr-96	1,429,662.26	1,438,885.24	0.0023563	1,377,188.55
12-Apr-96	1,433,408.64	1,475,446.51	0.0052606	1,412,182.14
15-Apr-96	1,438,576.04	1,464,775.18	0.0057748	1,401,968.38
16-Apr-96	1,454,710.54	1,465,054.57	0.0035525	1,402,235.79
18-Apr-96	1,473,100.47	1,472,355.65	0.0009567	1,409,223.81
19-Apr-96	1,485,983.75	1,508,050.51	-0.000223	1,443,388.14
22-Apr-96	1,480,386.63	1,514,609.53	0.0078102	1,449,665.93
23-Apr-96	1,462,913.19	1,502,365.98	1.973E-05	1,437,947.35
25-Apr-96	1,473,213.56	1,480,625.04	-0.000394	1,417,138.62
26-Apr-96	1,466,674.70	1,452,072.40	0.0081228	1,389,810.27
29-Apr-96	1,475,662.96	1,434,913.49	0.0081228	1,373,387.11
5-May-96	1,456,439.55	1,347,524.49	0.0033111	1,289,745.18
6-May-96	1,463,440.76	1,340,834.85	0.0119133	1,295,945.08
7-May-96	1,479,586.19	1,375,864.77	0.0040195	1,329,802.23
12-May-96	1,480,198.60	1,411,141.79	0.001722	1,363,898.21
13-May-96	1,478,534.71	1,462,399.35	0.0060488	1,413,439.72
14-May-96	1,480,401.53	1,503,186.15	0.003566	1,452,861.02
16-May-96	1,511,994.21	1,533,597.85	9.592E-05	1,482,254.57
17-May-96	1,500,495.78	1,468,077.59	0.0063588	1,418,927.86
20-May-96	1,504,360.02	1,476,469.31	0.008636	1,422,582.03
21-May-96	1,517,266.94	1,483,698.88	0.0043954	1,429,547.74
23-May-96	1,497,430.31	1,427,968.79	0.011937	1,410,857.95
24-May-96	1,496,708.59	1,431,513.76	0.0167063	1,410,177.95
27-May-96	1,497,377.24	1,426,639.99	0.0172049	1,410,807.94
28-May-96	1,493,059.46	1,432,422.32	0.0179336	1,406,739.79
30-May-96	1,506,191.37	1,446,839.02	0.0119436	1,419,112.50
31-May-96	1,516,652.79	1,468,623.12	0.0081228	1,440,479.14
3-Jun-96	1,510,455.23	1,496,495.28	0.0105367	1,434,592.85
4-Jun-96	1,506,955.53	1,512,236.28	0.0151229	1,431,268.92
6-Jun-96	1,519,995.99	1,517,426.08	0.0139011	1,443,654.43
7-Jun-96	1,530,238.41	1,505,071.27	0.0089682	1,453,382.43
10-Jun-96	1,511,391.01	1,482,465.31	0.0107049	1,435,481.64
11-Jun-96	1,518,324.51	1,446,157.51	0.0046906	1,400,324.54
14-Jun-96	1,509,876.78	1,495,083.31	0.0195119	1,392,533.34
17-Jun-96	1,531,873.19	1,514,443.21	0.0064881	1,410,565.31
18-Jun-96	1,554,955.82	1,572,616.54	0.0091374	1,431,820.04
20-Jun-96	1,571,354.69	1,635,045.03	0.0079276	1,488,659.31
21-Jun-96	1,577,071.46	1,651,408.84	0.0109633	1,494,075.22
24-Jun-96	1,574,496.59	1,655,992.76	0.0138307	1,491,635.86
25-Jun-96	1,578,299.82	1,700,624.52	0.0082422	1,495,238.94
27-Jun-96	1,585,653.09	1,705,746.30	0.0093251	1,502,205.22
28-Jun-96	1,597,489.54	1,706,307.98	0.005512	1,502,699.88

Table3: Comparison of decision rules.

Dates	Investment in 6-week bonds	Investment in 12-week bonds	Percentage difference	Decision rule with per. diff.
1-Jul-96	1,594,190.91	1,707,786.41	0.01215	1,499,596.98
2-Jul-96	1,598,604.85	1,746,398.98	0.0099522	1,503,749.00
5-Jul-96	1,655,141.37	1,795,064.32	0.0049455	1,545,652.64
8-Jul-96	1,667,489.65	1,824,207.84	-0.002151	1,570,746.87
9-Jul-96	1,645,682.85	1,811,356.24	0.004278	1,559,680.91
11-Jul-96	1,643,972.60	1,782,473.40	0.0049283	1,534,811.14
12-Jul-96	1,642,704.14	1,790,096.54	0.0076873	1,541,375.10
15-Jul-96	1,653,919.82	1,804,350.47	0.0046616	1,553,648.54
16-Jul-96	1,663,488.46	1,836,213.25	0.0110067	1,562,637.06
18-Jul-96	1,682,834.11	1,876,401.48	0.0131094	1,580,809.85
19-Jul-96	1,690,010.11	1,870,898.21	0.0100633	1,587,550.80
22-Jul-96	1,693,999.20	1,871,422.76	0.0041766	1,587,995.90
23-Jul-96	1,678,135.77	1,853,870.69	0.0082527	1,573,125.13
25-Jul-96	1,695,307.52	1,882,127.48	0.0055224	1,597,102.78
26-Jul-96	1,700,518.79	1,884,166.88	0.0026559	1,598,833.34
29-Jul-96	1,699,872.45	1,870,959.58	0.0029595	1,587,626.12
30-Jul-96	1,688,070.42	1,861,195.09	0.0109957	1,576,603.40
1-Aug-96	1,720,790.40	1,886,607.22	0.0014827	1,598,129.82
2-Aug-96	1,718,590.54	1,891,576.60	0.0059053	1,602,339.34
5-Aug-96	1,717,556.90	1,875,580.97	0.0026993	1,588,789.57
6-Aug-96	1,711,289.94	1,869,060.74	0.0048283	1,583,266.34
8-Aug-96	1,735,700.30	1,907,634.86	0.0037793	1,615,942.16
9-Aug-96	1,745,245.66	1,927,087.43	0.0039023	1,632,420.28
12-Aug-96	1,738,550.57	1,931,129.93	0.0061206	1,635,844.65
13-Aug-96	1,728,959.08	1,927,016.53	0.0075965	1,632,360.22
15-Aug-96	1,743,837.32	1,950,216.13	0.0041652	1,652,012.41
16-Aug-96	1,748,060.16	1,948,696.01	0.0045327	1,650,724.73
19-Aug-96	1,744,653.60	1,947,557.73	0.0043376	1,649,760.50
20-Aug-96	1,737,205.58	1,932,375.31	0.0030738	1,636,899.60
22-Aug-96	1,749,417.69	1,953,946.56	0.0029585	1,655,172.43
23-Aug-96	1,758,977.70	1,970,406.34	0.0051112	1,669,115.38
26-Aug-96	1,756,013.80	1,966,241.56	0.0053594	1,665,587.43
27-Aug-96	1,755,619.47	1,969,246.37	0.0061034	1,668,132.78
29-Aug-96	1,757,388.69	1,963,769.39	0.0059641	1,663,493.27
30-Aug-96	1,762,378.67	1,972,631.98	0.0057615	1,671,000.70
2-Sep-96	1,760,372.89	1,971,710.42	0.0064778	1,670,220.06
3-Sep-96	1,756,844.44	1,972,496.34	0.0075011	1,670,885.80
5-Sep-96	1,777,675.46	1,996,306.87	0.0075204	1,691,055.51
6-Sep-96	1,785,248.10	2,014,952.81	0.0058886	1,706,850.34
9-Sep-96	1,781,320.24	2,026,001.46	0.0037402	1,716,209.56
10-Sep-96	1,780,946.58	2,029,891.02	0.0054948	1,719,504.38
12-Sep-96	1,793,343.18	2,061,249.17	0.005525	1,746,067.61

Table3: Comparison of decision rules.

Dates	Investment in 6-week bonds	Investment in 12-week bonds	Modified per. Diff.	Decision rule with per. diff.
13-Sep-96	1,799,888.33	2,065,873.06	0.0035199	1,749,984.47
16-Sep-96	1,792,730.11	2,053,294.22	0.0053408	1,739,329.03
17-Sep-96	1,787,865.88	2,049,408.33	0.0069326	1,736,037.33
19-Sep-96	1,800,333.39	2,062,022.35	0.0064607	1,746,722.56
20-Sep-96	1,805,721.97	2,063,313.57	0.005001	1,747,816.35
23-Sep-96	1,785,942.15	2,019,330.33	-0.001889	1,710,558.50
24-Sep-96	1,766,460.24	1,976,182.80	0.0074229	1,674,008.58
26-Sep-96	1,787,381.69	2,018,348.88	0.0044399	1,709,727.12
27-Sep-96	1,803,082.08	2,054,931.31	0.005949	1,740,715.80
30-Sep-96	1,795,780.39	2,045,856.96	0.0080191	1,733,029.00
1-Oct-96	1,786,273.08	2,043,117.00		1,730,708.00

Table 4: Dates of the extreme cases.

4-Sep-95
26-Sep-95
23-Nov-95
5-Jan-96
29-Dec-95
26-Jan-96
27-Feb-96
22-Mar-96
19-Apr-96
25-Apr-96
28-May-96
14-Jun-96
8-Jul-96
23-Sep-96

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IMAGE EVALUATION TEST TARGET (QA-3)

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