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NEW RENORMALIZATION PROGRAM FOR SPONTANEOUSLY BROKEN GAUGE THEORIES. APPLICATION TO PROTON DECAY.

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NEW RENORMALIZATION PROGRAM FOR SPONTANEOUSLY BROKEN GAUGE THEORIES. APPLICATION TO PROTON DECAY.

by

JUAN ANTONIO PEREZ MERCADER

A dissertation submitted to the Graduate Faculty in Physics in partial fulfillment of the requirements for the degree of Doctor of Philosophy, The City University of New York.

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ABSTRACT

NEW RENORMALIZATION PROGRAM FOR SPONTANEOUSLY BROKEN GAUGE THEORIES.
APPLICATION TO PROTON DECAY

by

J. A. PEREZ MERCADER

Adviser: Professor Ngee Pong Chang

In this work we present a new renormalization group analysis especifically designed for spontaneously broken gauge theories that takes advantage of the minimal renormalization scheme. We apply the method to a calculation of the lifetime of the proton in the context of an asymptotically free SU(5) gauge theory that we also develop in this work. The lifetime that we obtain for the current favorite value of the $\Lambda_{\overline{MS}}$ parameter, and with three generations of light fermions is 5.1×10^{30} years, corresponding to an M_x of 8.9×10^{14} GeV. We also give lifetimes as a function of the number of generations.

ACKNOWLEDGEMENTS

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C H A P T E R I

SECTION 1: INTRODUCTION

The proton (from the greek protos: first, fundamental) is the lowest lying state of the baryons, thus if we have a law for baryon number conservation the proton will be stable. Baryons are made up of quarks, and the current view is that the fundamental constituents of matter are quarks and leptons; in a grand unified gauge theory in which an attempt is made to unify the observed strong (gauge group: SU(3) of color)[Ref.1], electromagnetic and weak ("unified" by SU(2)X U(1))[Ref.2] interactions within a simple gauge group[Ref.3], quarks and leptons are treated on the same footing and are put on the same multiplet, so that there are gauge bosons (group generators) that mix quarks with leptons and gauge bosons of the same type that also mix two quarks (hence the names lepto-quark and diquark which are often used to designate these gauge bosons). It follows that in these theories proton decay can occur[Ref.4], and due to the large group used to unify, it can even be computed[Ref.5].

Proton decay is the most important feature of grand unification, and the fact that one has a model that allows to calculate the lifetime makes it even more exciting. The values of the proton lifetime that are calculated, are very close to the current experimental bound, typically between one or two orders of magnitude above it. This, coupled to the new generation of experiments[Ref.6] which are proposed or at the building stage, and which are going to be able to go about three orders of magnitude above the previous ones, makes a calculation of the lifetime a very timely subject, since this will provide an excellent test for the theory.

In this work we perform a complete calculation[Ref.7] of proton lifetime in an SU(5) gauge theory which is a modification of the Georgi-Glashow model[Ref.8]; this modification consists of adding a set of heavy fermions in the 5 and 24 of SU(5) (in every other expect the theory is identical to the Georgi and Glashow SU(5)), necessary to achieve asymptotic freedom. By imposing eigenvalue conditions[Ref.9] all the parameters of the theory are determined and all effects can be computed without any free parameters.

Without any further ado we start our work by providing the reader with a brief survey of the idea of proton stability and the experimental situation.

In the days prior to the discovery of the positron, a formulation of proton stability was given by Hermann Weyl, in an attempt to understand the Dirac theory; the year was 1929 and he wrote[Ref.10]: "It is plausible to anticipate that, of the two pairs of components of the Dirac quantity, one belongs to the electron, the other to the proton.

Further, two conservation laws of electricity will have to appear, which state (after quantization) that the number of electrons as well as of protons remains constant". Of course, the formulation had to be corrected and this was done by E.C.G. Stuckelberg[Ref.11] in 1938, who defined the "SCHWERE LADUNG" (heavy charge) more familiar to us as baryon number, and postulated its conservation.

The first mechanism for proton decay was given by E. Wigner[Ref.12] in 1949: "Without the conservation law of the number of heavy particles, the proton could disintegrate, under emission of a light quantum, into a positron, just as the electrons could disintegrate, were it not for the conservation law for the electric change, into a light quantum and a neutrino".

Many experiments were performed to test the stability of the proton and the lower bound kept increasing with the years, from[Ref.13] 10^{21} to $3x10^{30}$ years[Ref.14], which is the current lower bound.

A very simple example can show the difficulties involved in putting bounds on proton lifetime: suppose the lifetime was 10^{33} years, and consider that you had a container with 10^4 tons of H_2O i.e. 10^{10} cc. A mole of water is 18 cc and there are 18 nucleons per molecule, multiplying by Avogadro's number we get a total of $6x10^{33}$ nucleons; if the half-life were 10^{33} years, we would see an average of 3 decays per year, assuming a 100% efficiency on the part of the detectors.

The most simple way of measuring nucleon lifetime is then, to have a large container of water surrounded by photomultipliers and watch for Čerenkov light comes produced by the emitted particles in processes like

$$p \to e^{+} + \pi^{0}, \quad \pi^{0} \to 2\gamma$$

Experiments of this type[Ref.15] have recently been proposed or are already under way, and they expect to measure proton lifetimes in the range of 10^{30} to 10^{33} years (if decay does indeed occur), their sensitivity being mainly vexed by the presence of atmospheric neutrinoes. This is a very interesting range, for as we shall see by the end of this work, the lifetimes that one typically obtains in the SU(5) model[Ref.5] of Georgi and Glashow fall within this range, and therefore the proton lifetime can be an excellent test of the model.

SECTION 2: THE IDEA OF UNIFICATION. FUNDAMENTALS OF SU(5).

In this section we present a brief discussion of the idea of unification as well as a review of the most salient features of the SU(5) model of Georgi and Glashow[Ref.3], especially in what pertains to an understanding of what is necessary to calculate the proton lifetime in that model, and the mechanism through which it takes place. This model is based on the gauge group SU(5), which contains in a minimal fashion the subgroups SU(3) and SU(2) X U(1) and therefore can be used to, at the same time describe QCD[Ref.1] and the standard electro-weak theory of SU(2)xU(1)[Ref.2] in a simple group. This embedding of SU(3) X SU(2) X U(1) has immediate non-trivial consequences. Had we not taken a simple group, but just considered a "unified" theory consisting of only SU(3)XSU(2)XU(1), we would have three coupling constants, one for each of the factor groups, g_3, g_2, and g_1 and they would be parameters to be adjusted. But the embedding of SU(3) X SU(2) X U(1) in a single SU(5) and the requirement of SU(5) gauge invariance imply that all three gauge couplings be equal when the SU(5) symmetry is valid,

$$g_3 = g_2 = g_1 = \bar{g} \tag{1.1}$$

where we denote by $\bar{g}$ the symmetric, SU(5) gauge coupling.

This lies at the heart of what one means by unification. However, at the energies we live, that is, when the SU(3) X SU(2) X U(1) or actually SU(3) X $U(1)_{em}$ is valid, there is a big discrepancy between the values of the three coupling constants,

$$g_3 \sim 10^{-1} \qquad , \quad g_2 \sim g_1 \sim 10^{-2}. \tag{1.2}$$

The question arises, how can one reconciliate (1) with (2)? The answer comes in a natural way if one observes that because of the asymptotic freedom[Ref.16] of QCD and the non-asymptotically free character[Ref.17] of the electromagnetic coupling constant there will be a scale on which they will become comparable in magnitude. This is called the unification scale. To see this in more detail[Ref.4] consider the RGE (Renormalization Group Equation) for the g_i's of an effective SU(3) X SU(2) X U(1) gauge theory. To one loop and in the regime of small g_i's one has

$$\mu \frac{d}{d\mu} g_i = b_i g_i^3 \tag{1.3}$$

or

$$\frac{1}{g_i^2(\mu)} = \text{const.} - 2 b_i \log \mu \tag{1.3a}$$

where b_i are given by $b_3 = \frac{-1}{16\pi^2}\left(11 - \frac{8}{3} n_f\right)$; $b_2 = \frac{-1}{48\pi^2}(22 - 8 n_f)$; $b_1 = \frac{1}{16\pi^2} \frac{8}{3} n_f$.

Since we want (1) to be satisfied at some scale M_X, the unification scale, we just impose that boundary condition and have

$$\frac{1}{g_i^2(\mu)} = \frac{1}{\bar{g}^2(M_x)} + 2b_i \log(M_x/\mu) \qquad (1.4)$$

for a $\mu \ll M_X$. Of course, from (4) and by taking appropriate linear combinations one can determine M_X, and, for example the low energy value of the weak mixing angle, just by giving as inputs low energy parameters such as α_{em} and α_{strong}. Naturally this is a very crude estimate, but it is interesting in so far as it contains the general idea behind all the calculations of M_X. We also see how the RG plays a natural role in grand unification from the very beginning.

Thus from (4), at that unification scale, the statement (1) is satisfied and one has SU(5) invariance, at least as far as the coupling constants go.

After this little preamble we will briefly discuss the SU(5) theory itself. Call the generators of SU(5) T_α, where $\alpha = 1,\ldots,24$, and normalize them according to

$$\mathrm{Tr}\,(T_\alpha T_\beta) = \frac{1}{2}\,\delta_{\alpha\beta} \qquad (1.5)$$

They satisfy the commutation relations

$$[T_\alpha, T_\beta] = i f_{\alpha\beta\gamma} T_\gamma .$$

One can give a realization of the T_α's by defining the 24, traceless, hermitian, matrices

$$\lambda_\alpha = 2T_\alpha$$

and using for the λ_α's, $\alpha = 1,\ldots,8$ the SU(3) matrices of Gell-Mann enlarged to 5x5 dimensions. (See Appendix A for details).

The fermions were assigned by Georgi and Glashow[Ref.3] to be in generations of 15 two-component spinors. Each generation contains the L(eft) and R(ight) components of an SU(2) Cabbibo-rotated doublet of quarks each in one color (1,2,3) plus the L and R components of the positron and the R component of the $\bar{\nu}_e$. These fifteen fermions are put on a $\underline{5}$ (fundamental) and a $\underline{10}$ (second rank antisymmetric tensor), as

$$\psi_R = \begin{bmatrix} d^1 \\ d^2 \\ d^3 \\ e^+ \\ \bar{\nu}_e \end{bmatrix}_R \tag{1.6}$$

and

$$\psi_L = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 & u_3^c & -u_2^c & -u_1(\theta) & -d_1 \\ -u_3^c & 0 & u_1^c & -u_2(\theta) & -d_2 \\ u_2^c & -u_1^c & 0 & -u_3(\theta) & -d_3 \\ u_1(\theta) & u_2(\theta) & u_3(\theta) & 0 & -e^+ \\ d_1 & d_2 & d_3 & e^+ & 0 \end{bmatrix}_L \tag{1.7}$$

where 1,2,3 denote the color index, and c means charge conjugated. Also, $u_i(\theta)$ means Cabbibo rotated. In our work we will ignore this mixing. We will also refer to the set of (6) and (7) as one generation of light fermions.

The model has many nice features, it preserves the V-A structure of the weak interactions, provides a natural framework for charge quantization and for massless neutrinoes. It successfully predicts the mass of the bottom quark[Ref.18] and the prediction[Ref.19] for $\sin^2\theta_W$ is good, although at present is a little low when compared with the experimental value. The required minimal Higgs to break SU(5) down to SU(3) X SU(2) X U(1) is an adjoint of SU(5) and to further break to $SU(3)_{color} \times U(1)_{em}$, a 5-plet of SU(5).

We now show how SU(5) grand unification fixes the value of the weak mixing angle. The charge matrix, Q, can be written as a linear combination of the diagonal generators of the SU(2) and U(1) subgroups. If g and g' are respectively the SU(2) and U(1) coupling constants of the standard electroweak SU(2)XU(1) model[Refs.2,4,20], and if we identify $g=g_2$ then, because of the embedding into SU(5), we can write

$$g_1 = g' C$$

where C is a constant, determined by the SU(2) X U(1) embedding in SU(5). The charge operator is then written as

$$Q = T_3 - C\, T_0 , \qquad (1.8)$$

where T_3 is the generator of the third component of SU(2), (given by λ_{12}) and T_0 the U(1) generator, (given by λ_9). The weak mixing angle is given by

$$\sin^2\theta = \frac{g'^2}{g^2 + g'^2} = \frac{1}{1 + C^2} \tag{1.9}$$

If we square (8), take traces and remember (5), we get

$$\mathrm{Tr}(Q^2) = \mathrm{Tr}(T_3^2) - C\,\mathrm{Tr}[T_3, T_0] + C^2\,\mathrm{Tr}(T_0^2) =$$
$$= (1 + C^2)\,\mathrm{Tr}(T_3^2)$$

which of course is valid in any representation. Therefore

$$\sin^2\theta = \frac{\mathrm{Tr}(T_3^2)}{\mathrm{Tr}(Q^2)} = \frac{1}{2\,\mathrm{Tr}(Q^2)} . \tag{1.10}$$

If, for example, we apply this to the 5-plet, we get

$$\sin^2\Theta = 3/8 \tag{1.11}$$

which is the fixed value of $\sin^2\theta$ at unification energies. This value of .375 at grand unification is considerably above the low energy value that is measured, .238; however this last value is the value at low energies (O(tens of Gev's)) and there is considerable room for renormalization effects from the typical unification energies of 10^{15} GeV down to these energies. In fact, if one solves (4) for $\sin^2\theta = g'^2/(g^2 + g'^2)$ one gets (in the Georgi and Glashow SU(5))

$$\sin^2\theta(m) = \sin^2\theta_u \left[1 - \frac{10}{3} e^2(m)\,(b_1 - b_2)\,\log\left(M_x/m\right)\right] \tag{1.12}$$

(here $\sin^2\theta_u$ is the value at unification energy (11), of, 3/8, $e^2(m)$, is the electric charge of the electron at low energies O(10's GeV) and M_X is the unification mass). For typical values of $e^2(m)$ and M_X (in turn determined from α_s and α_{em} using (4)) one gets[Ref.4] values of $\sin^2\theta$ between .175 and .248. There exist[Ref.19] more sophisticated estimates on the literature which for realistic values of α_s and α_{em} typically give a low energy value of $\sin^2$ of .216 $\pm$ .008.

Finally we now turn to what is the most important consequence of SU(5), proton decay, and how it occurs within this model. From (6) one can see that there are gauge bosons, called X_i and Y_i, that give rise to the following processes (See Appendix B, for the lagrangian)

$$d_{iR} \rightarrow X_i^- + e_R^+ \tag{1.13a}$$

and

$$d_{iR} \rightarrow Y_i^- + \bar{\nu}_R \tag{1.13b}$$

(These X's correspond to linear combinations of the $\lambda_{13}\ldots\lambda_{18}$ of Appendix A and the Y_i's to linear combinations of the $\lambda_{19}\ldots\lambda_{24}$). From charge conservation we see that they have charges of -4/3 and -1/3 respectively. Similarly, from (7) one sees (cf. Appendix B) that the same objects can give rise to processes like

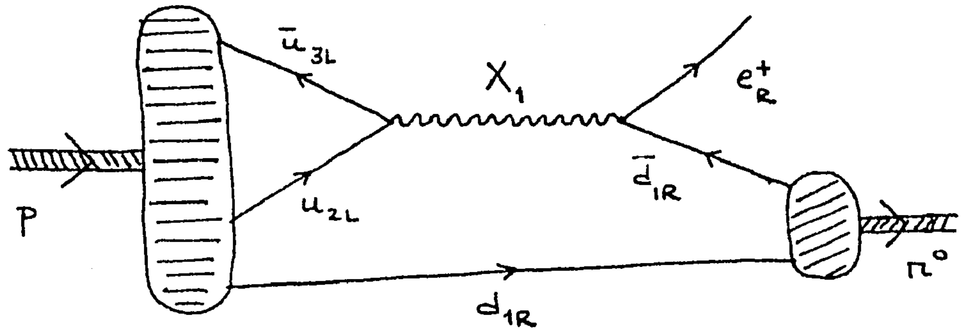
$$u_{2L} \rightarrow X_1^+ + \bar{u}_{3L} \tag{1.14a}$$

or

$$d_{2L} \rightarrow Y_1^+ + \bar{u}_{3L} \tag{1.14b}$$

Since the proton is a color singlet of two up and one down quarks, equations (13a) together with (14a) (alternatively, (13b) with (14b)) show that in this model proton decay can take place in second order in $\bar{g}$.

It can, for example, take place via a diagram such as



which represents the process

$$p \rightarrow e^+ + \pi^0. \tag{1.15}$$

(Here the blobs represent strong interaction effects, to describe the proton and the pion as a bound state of quarks). We can use this graph to show that with the present experimental data on τ_p (the proton lifetime), SU(5) is not ruled out. Let us estimate the matrix element corresponding to this graph in the case where the momentum carried by X is negligible compared with M_X. Then

$$T \simeq \frac{1}{k} \frac{\bar{g}^2}{M_x^2} \tag{1.16}$$

where k is a constant (which we will take to be of O(1)) describing the effects of the blobs in the diagram. The lifetime will roughly be given by

$$\tau_p = k \frac{1}{\bar{g}^4} \frac{M_x^4}{m_p^5}. \tag{1.17}$$

So for a $\bar{g}^2$ typically of order 0.2 and a $\tau_p \gtrsim 3\mathrm{x}10^{30}$ years one needs an $M_X \gtrsim 4\mathrm{x}10^{14}$ GeV. If, on the other hand, one solves (4) for $\log \frac{M_X}{\mu}$ in terms of g_3 and e^2,

$$\log \frac{M_X}{\mu} = \frac{4\pi^2}{33}\left[\frac{3}{e^2(\mu)} - \frac{8}{g_3^2(\mu)}\right] \qquad (1.18)$$

which for reasonable values of e^2 and g_3 at low energies, typically gives values larger than $4\mathrm{x}10^{14}$ GeV for M_X.

There are several morals to be drawn from the above fable (in the sense of Lafontaine) and they are, that if one wants to calculate τ_p (notice that τ_p is proportional to M_X^4) with any reasonable degree of confidence, one has to devise a more precise calculation of M_X than that provided by (18), and that one must, somehow, improve the formula for τ_p, (17).

Part of this work is devoted to a method of calculating M_X which takes into account a number of effects that we will discuss in the next section, and that at the same time allows one to improve (17), although not the k used in that equation.

We now give way to a discussion of previous work on proton decay.

SECTION 3: PREVIOUS WORK ON PROTON DECAY.

The first calculation of the proton lifetime within the context of SU(5), was performed by Georgi, Quinn and Weinberg[Ref.4]. In this work, (parts of which were reproduced in the previous section) by using crude renormalization group arguments and assuming that all the coupling constants for each of the three subgroups SU(3), SU(2) and U(1) would become equal at a certain energy, the unification mass, they were able to write down formulae, (12) and (18), that given α_s and α_{em} could be used to calculate the unification mass and the renormalized mixing angle. After having obtained the unification mass they simply estimated the lifetime by dimensional arguments.

Simple as this pioneering work was, its importance lies in that it presents a framework in which to perform the calculation, and put forward the ingredients for any other calculation, including this one.

The next development came with the calculation done by Buras, Ellis, Gaillard and Nanopoulos[Ref.18] (from now on referred to as BEGN) who calculated 1-loop gluon corrections to the matrix element and also took into account fermion thresholds in the integration of the RGE's for the low energy (SU(3) coupling constants) using a formula given by

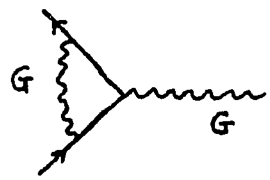
Georgi and Politzer[Ref.21]. They also gave a formula to calculate the proton lifetime

$$\tau_p^{-1}(p \to \bar{l} + any) = \frac{614\,|\psi(0)|^2}{8\pi}\left\{G_{GUM}^2\left[\frac{\alpha_s(\mu^2)}{\alpha_{GUM}}\right]^{4/(11-2f/3)}\right\}m_q^2 \quad (1.19)$$

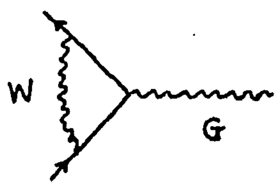
where $|\psi(0)|^2$ is the wave function for two quarks in the proton to be at he same point, m_q is the mass of the quarks, typically taken to be $(1/3)m_p$, $G_{GUM}=\frac{g_{GUM}^2}{4\sqrt{2}\,M_x^2}$, and g_{GUM} is the value of the SU(5) coupling constant at M_x; the quantity in the curly brackets comes from integrating the effective G from unification energy down to a mass scale μ. This formula allows one to calculate τ_p by calculating G at a low energy, $O(m_p)$, mass scale, and is essentially the formula we will use in our work.

After this work, Douglas Ross[Ref.22] calculated the SU(2) and U(1) corrections to the matrix element and more importantly, introduced his θ-approximation to the gauge boson thresholds in order to take into account the decoupling of the massive particles from the RGE's when one passes <u>through</u> the thresholds of the gauge bosons. However in his study he used the mass dependent RG and, as pointed out by Chang, Das and Perez-Mercader[Ref.23], there is an uncertainty as to which coupling constants are used in the different vertices; we repeat here the argument as given in the above reference. Suppose, for the sake of simplicity, that one is studying the RGE for the fermion-gluon vertex, then, among other graphs, one has to consider the graphs shown in figs. 1a, 1b and 1c.

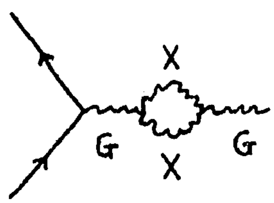
The graphs in figs. 1a and 1b can unambiguosly be said to respectively give contributions proportional to g_3^3 and $g_3g_2^2$, for energies below grand unification, since the SU(3) and SU(2) symmetries are respected; however the graph of fig. 1c is not either proportional to g_3^3 nor to $g_3g_X^2$ since, on the one hand g_X is defined as the coupling of the X-gauge bosons to the fermions, and on the other, even the space-time structure of the GXX coupling is broken, below unification energies, into a minimal "charge" interaction and a quadrupole moment coupling[Ref. 24],



1a



1b



1c

$$+ig_3\kappa_3 X^{+a}_{\nu i}\left\{\partial_\mu G_{\nu j}^{\ i} - \partial_\nu G_{\mu j}^{\ i} - ig_3[G_\mu, G_\nu]^i_j\right\} X^j_{\mu a}$$

$$+ig_2\kappa_2 X^{+a}_{\nu i}\left\{\partial_\mu W_{\nu a}^{\ b} - \partial_\nu W_{\mu a}^{\ b} - ig_2[W_\mu, W_\nu]^b_a\right\} X^i_{\mu b}$$

$$-i\frac{\sqrt{5}}{2\sqrt{3}}\, g_1\kappa_1(\partial_\mu B_\nu - \partial_\nu B_\mu) X^{+a}_{\mu i} X^i_{\nu a}$$

$$-\frac{1}{2}\left|\partial_\mu X^i_{\nu a} - ig_3 G_{\mu j}^{\ i} X^j_{\nu a} + ig_2 X^i_{\nu b} W_{\mu a}^{\ b} - \frac{i\sqrt{5}}{2\sqrt{3}} g_1 B_\mu X^i_{\nu a} - \mu\leftrightarrow\nu\right|^2$$

and in order to be consistent one would have to also include renormalization group equations for the quadrupole moments κ_3, κ_2, κ_1, which Ross did not do, (κ_3, κ_2, κ_1 would go to 1 as $Q^2 \rightarrow M_x^2$) and would complicate his analysis in a very non-trivial manner. Of course, graphs like the one in fig. 1c, do decouple[Ref.25] at low energies, but their effect becomes important as we get closer to M_x, and attempt to cross the thresholds.

The next improvements were made by Cecilia Jarlskog and Francisco Yndurain[Ref.26], who refined the BEGN lifetime formula, (19), by including decays with $\bar{\nu}_\mu$ and μ^+, (and therefore strange particles in the final state) as well as three body decays, and also computed some branching ratios; T. Goldman and D. Ross[Ref.27] did a 2-loop calculation of M_x; then came the work of Machacek[Ref.28], Marciano[Ref.29], our own[Ref.30], Din, Girardi and Sorba[Ref.30] and of Donoghue[Ref.31], all of which, except Marciano's and ours, again refined the estimates of the proton wave-function at the origin (which enters the BEGN formula, eq.(19)) by one method or another, and gave estimates of the different branching ratios.

However, the most crucial quantity in calculating proton decay (as already remarked) is the mass of M_x. As discussed before, what one does to calculate M_x is to give input values for α_s and α, the electromagnetic coupling constant, at some low energy point m; log M_x/m is essentially proportional to a linear combination of α_s^{-1} and α^{-1} at scale m (see (18)),so that M_x is very sensitive to small changes in either of the two couplings; therefore it is especially important to do a careful extrapolation of α_{em} from the Thompson limit value to the mass scale m, and here fermions thresholds are very significant. This was explicitly recognized by Goldman and Ross[Ref.33] and independently by Marciano[Ref.34], and was implicit in our work.

There are several questions in the theoretical calculation of proton decay that had not been tackled previous to our work[Ref.35], and that we now discuss.

i) In the usual method of calculating[Refs.33,34] the low energy running couplings, what one does is to define them as the value of the corresponding off-shell Green's functions evaluated at some low energy point m, and the effects of the heavy particles are neglected by using the Appelquist-Carazzone theorem[Ref.25], however, in doing the proton lifetime calculation with this method one has to integrate the proton decay matrix element from the unification energy down to energies of $O(m_p)$ and therefore the procedure is uncertain, since in choosing the initial values of the coupling constants, one is effectively working in a region where the Appelquist-Carazzone theorem cannot yet be invoked, and of course, there is also the issue of which coupling constants to use for graphs involving X-gauge-bosons in the internal loops, which becomes more important as one gets closer to their thresholds. We tackle this problem by introducing a new renormalization group approach that allows one to calculate with the symmetric theory lagrangian and "leapfrog" from the unification energies down to mass scales where one can safely use the low energy RGE's for the low energy coupling constants. The effect of this leapfrogging manifests itself in the appearance of certain constant terms in the initial values and which summarize the threshold effects, while, at the same time avoiding the ambiguity in the coupling constants.

Some work along these lines, has also been independently done by Binetrûy and Schücker[Ref.36], and lately by S. Weinberg[Ref.37] and L. Hall[Ref.38], who have used functional techniques.

ii) The scalar Higgs fields introduced for breaking the symmetry from SU(5) down to SU(3) X SU(2) X U(1) and then down to SU(3) X $U(1)_{em}$[Refs.3,18] also pose a difficulty in the calculation of proton decay; the first and most obvious difficulty, comes from the fact that since an SU(5) gauge theory with a vector and an adjoint of Higgs is not asymptotially free[Ref.39], there is no guaranty that perturbation theory is valid at high energies; in principle one could say that one could adjust the initial values of the coupling constants, λ_i, in such a way that at the energy scales where the gauge couplings unify, the λ_i's are arbitrarily small and hence insure the validity of perturbation theory in the Higgs sector. However the renormalization group equations satisfied by the λ_i's are non-linear equations and it is not clear that any set of initial values will do[Ref.40].

There is a further problem: suppose that, indeed, one has been able to solve the above problem, then, for energies much below the unification energy, M_X, the Higgs coupling constants will also split into several coupling constants[Ref.41]. For example, consider the coupling λ_1 in the SU(5) symmetric lagrangian,

$$\frac{\lambda_1}{4}(\mathrm{Tr}\ \phi^2)^2$$

where ϕ is the $\underline{24}$ of SU(5). Upon the breaking, this coupling "breaks", splits, into several quartic couplings,

$$\frac{\lambda_{133}}{4}(\varphi^i_j\varphi^j_i)^2 + \frac{\lambda_{122}}{4}(\omega^a_b\omega^b_a)^2 + \frac{\lambda_{123}}{2}(\varphi^i_j\varphi^j_i)(\omega^a_b\omega^b_a)$$

$$+\lambda_{135}(\varphi^i_j\varphi^j_i)(K^l_a K^{+l}_a) + \lambda_{125}(\omega^a_b\omega^b_a)(K^i_c K^{+c}_i)$$

$$+\lambda_{155}(K^i_a K^{+a}_i)^2$$

where φ^i_j, and ω^a_b are respectively the SU(3) and SU(2) adjoints in the 24 of SU(5) and K^i_a are the "off-diagonal" components mixing the above two. (Here i,j= 1,2..., 8 and a,b = 1,2). It is clear that making the pleiades of split couplings converge into their SU(5) values is not a simple matter and there is no a priori reason why that, if at all possible, will happen precisely at M_X or even within a few orders of magnitude; it looks more like if one had to do a very fine tuning of the initial values of the couplings in order to achieve such a "feat".

We have tackled this problem (or rather the high energy end of it) by imposing the so called eigenvalue conditions[Ref.9] on the symmetric λ_i's, i.e. we have made all the Higgs and Yukawa couplings appearing in the SU(5) symmetric lagrangian respectively proportional to $\bar{g}^2$ and $\bar{g}$, the SU(5) coupling constant. This, as is well known, has the effect of restoring the asymptotic freedom of the theory and at the same time guarantee that the quartic couplings will go to their SU(5) symmetric values at M_x. By imposing these eigenvalue conditions, all the coupling constants in the theory are determined in terms of only one coupling constant, $\bar{g}$, which in turn is determined in the SU(5) model by the strong and electromagnetic couplings, and hence one can calculate everything,(there only is one mass scale as a consequence of our renormalization program[Ref.8]; see also Appendix G) including mass renormalization effects of M_x, which we will as well include.

iii) It has been shown by Ellis, Gaillard and Nanopoulos in Ref42 ,from cosmological considerations, that the mass of the SU(3) part of the 5-plet of Higgs must be of order $\bar{g}^2 M_x$; this is substantially higher than the limit imposed by proton stability, $m_H \sim 10^{-5} M_x$. It will be shown that as a byproduct of our new RG analysis[Ref.23] and eigenvalue conditions and the requirement that the

SU(3)XSU(2)XU(1) symmetry be respected at energies immediately below grand unification, we automatically satisfy this requirement. However, the question remains as to how the SU(3) will remain unbroken as we come down in energies whereas the SU(2) breaks at energies of $O(10^2$ GeV) so as to give mass to the W-gauge bosons of the standard SU(2) X U(1) model. Research along this lines is in progress and we expect the effective mass of the SU(2) part of the 5-plet of Higgs to change sign and become negative and therefore break the symmetry down to SU(3) X U(1), while the triplet part remains with a positive mass, but we don't have as yet any final results[Ref.43]; here we will simply assume that it happens.

The remainder of this work is organized as follows. In the next chapter, chapter 2, we introduce a new renormalization group analysis[Ref.23] especially designed for gauge theories with spontaneous symmetry breakdown that takes advantage of dimensional regularization[Ref.44] and its associated minimal[Ref.45] (and variations[Ref.46]) substraction scheme, and present the results of its application to SU(5). In chapter 3 we discuss the asymptotic freedom problem and introduce an SU(5) model[Ref.8] that is asymptotically free and within which we perform our calculations. In chapter 4 we present the calculation[Ref.47] of the RGE for the proton decay matrix element. Finally, in chapter 5 we discuss how we performed the proton decay calculation and present the results.

In order to afford continuity and not to abuse the attention of the reader, we have included the most technical details in a set of appendices.

C H A P T E R II

NEW RENORMALIZATION PROGRAM.

In this chapter we introduce a new RG equation for a spontaneously broken gauge theory[Ref.23]. Before discussing the actual renormalization group equation (RGE) we want to specify the renormalization procedure that we will follow, namely the minimal scheme of 't Hooft and Veltman[Ref.45]: here one starts with a bare lagrangian $\mathcal{L}_B$, which contains all the kinetic energy and interaction parts for the spin-0, spin-1/2 and spin -1 fields, including the gauge fixing terms and the ghost lagrangian, in addition to any terms generated by shifting the scalar fields whose VEV is non-zero. Apart from these terms the lagrangian $\mathcal{L}_B$, does not contain any counterterms, furthermore all the coupling constants in the gauge sector, and in the Higgs sector have their symmetric (say SU(5)) values and they are the bare quantities.

We will call $\Gamma_u^{(n)}$ the 1PI n-point Green's function that is calculated in perturbation theory from this $\mathcal{L}_B$; and it will contain divergences that one can regularize by using a gauge invariant regularization procedure such as dimensional regularization; its dependence on the different parameters of the theory will be

$$\Gamma_u^{(n)} = \Gamma_u^{(n)}(p, g_B, M_B, \alpha_B) \tag{2.1}$$

where p denotes the external momenta, g_B is the set of bare coupling constants, M_B the set of bare masses and α_B is the bare gauge fixing parameter.

In the minimal scheme[Ref.45], one extracts the divergences of $\Gamma_u^{(n)}$ by defining the renormalized parameters as follows

$$g_B \longrightarrow \quad g_r = \hat{Z}_3^{3/2}\, \hat{Z}_1^{-1}\, g_B \tag{2.2a}$$

$$M_B \longrightarrow \quad M_r = g_r v_r \tag{2.2b}$$

$$\alpha_B \longrightarrow \quad \alpha_r = \hat{Z}_3^{-1}\, \alpha_B \tag{2.2c}$$

where $\hat{Z}_3$ and $\hat{Z}_1$ are the gauge boson wave function renormalization and the vertex renormalization constants, respectively; they only contain the typical $1/\epsilon$ part associated with divergent quantities in dimensional regularization. (Here $\epsilon = 4-n$, where n is the dimensionality of the space-time).

Then we define the function $\Gamma_r^{(n)}$ as

$$\Gamma_r^{(n)}(p, g_r, M_r, \alpha_r, \mu) = (\hat{Z}_e)^{n/2}\; \Gamma_u^{(n)}(p, g_B, M_B, \alpha_B) \tag{2.3}$$

which is a finite function of the renormalized parameters g_r, M_r, α_r, and $\hat{Z}_e$ is the $1/\epsilon$ part of the product of the wave function renormalization constants of the external legs. By construction

$$\hat{Z}_e = \hat{Z}_e\,(g_B, \alpha_B, \Lambda, \mu) \qquad (2.4)$$

where we have understood the $1/\epsilon$ as $\log \Lambda/\mu$, in terms of a regulator mass Λ Ref.48.

From now on we will always mean by $1/\epsilon$ the quantity

$$1/\epsilon + 1/2\,(-\gamma + \log 4\pi)$$

with γ the Euler-Mascheroni constant Ref.49; that is we will be using the so called "truncated minimal scheme" Ref.46.

Since we are using a mass independent regularization procedure the constants $\hat{Z}$ do not depend on the masses and, the procedure, respects any symmetries which are broken in the Lagrangian only by mass terms. Therefore, the $\hat{Z}$'s are given by the original, symmetric theory.

It is clear, that although we have succeeded in making $\Gamma_r^{(n)}$ finite, they are not as yet normalized for on-shell matrix elements, so that one needs to perform an additional, albeit finite, renormalization, through

$$\Gamma_R^{(n)}(p, g_r, M_r, \alpha_r, Q, \mu) = (Z_e)^{n/2}\,\Gamma_r^{(n)}(p, g_r, M_r, \alpha_r, \mu) \qquad (2.5)$$

where Z_e is the finite external wave function renormalization constant and Q is the substraction point; again, by construction

$$Z_e = Z_e\,(Q, g_r, M_r, \alpha_r, \mu). \qquad (2.6)$$

Then, <u>all the symmetry breaking effects are "buried" on Z_e</u>, <u>since it is Z_e that depends on the mass $M_r = g_r v_r$</u>.

We are now in position to derive the RG equation. By writing

$$\mu \frac{d}{d\mu} \Gamma_u^{(n)} = 0 = \mu \frac{d}{d\mu} \left[Z_e^{-n/2} \hat{Z}_e^{-n/2} \Gamma_R^{(n)} \right] \tag{2.7}$$

one obtains the equation

$$\begin{aligned} \Big[\mu \frac{\partial}{\partial\mu} + \beta \frac{\partial}{\partial g_r} - \gamma_M M_r \frac{\partial}{\partial M_r} \\ - 2\gamma \alpha_r \frac{\partial}{\partial \alpha_r} - n\tilde{\gamma} \Big] \Gamma_R^{(n)}(p, g_r, M_r, \alpha_r, Q, \mu) = 0 \end{aligned} \tag{2.8}$$

where we have introduced

$$\beta \equiv \mu \frac{\partial}{\partial\mu} g_r \Big|_{g_B, \Lambda} \tag{2.9a}$$

$$M_r \gamma_M \equiv -\mu \frac{\partial}{\partial\mu} M_r \Big|_{g_B, \Lambda} \tag{2.9b}$$

$$\gamma \equiv \frac{1}{2} \mu \frac{\partial}{\partial\mu} \log \hat{Z}_3 \Big|_{g_B, \Lambda} \tag{2.9c}$$

$$\tilde{\gamma} \equiv \frac{1}{2} \mu \frac{\partial}{\partial\mu} \log \hat{Z}_e \Big|_{g_B, \Lambda} + \frac{1}{2} \mu \frac{d}{d\mu} \log Z_e \tag{2.9d}$$

By scaling the momenta in $\Gamma_R^{(n)}$, and using traditional dimensional analysis, one has

$$\begin{aligned} &\Gamma_R^{(n)}(\kappa p, g_r, M_r, \alpha_r, \kappa Q, \mu) = \\ &= \mu^{d_n} \Gamma_R^{(n)}(\kappa p/\mu, g_r, M_r/\mu, \alpha_r, \kappa Q/\mu, 1) \end{aligned} \tag{2.10}$$

and applying the "dimension counting operator"

$$\kappa \frac{\partial}{\partial \kappa} + \mu \frac{\partial}{\partial \mu} + M_r \frac{\partial}{\partial M_r} \tag{2.11}$$

with eigenvalue d_n (the canonical dimension of $\Gamma_R^{(n)}$), we have

$$\left[\kappa \frac{\partial}{\partial \kappa} + \mu \frac{\partial}{\partial \mu} + M_r \frac{\partial}{\partial M_r} - \right.$$

$$\left. - d_n \right] \Gamma_R^{(n)}(\kappa p, g_r, M_r, \alpha_r, \kappa Q, \mu) = 0 \tag{2.12}$$

which combined with our previous equation gives

$$\left[-\kappa \frac{\partial}{\partial \kappa} + \beta \frac{\partial}{\partial g_r} - (1 + \gamma_M) M_r \frac{\partial}{\partial M_r} - \right.$$

$$\left. -2\gamma \alpha_r \frac{\partial}{\partial \alpha_r} + d_n - n\tilde{\gamma} \right] \Gamma_R^{(n)}(\kappa p, g_r, M_r, \alpha_r, \kappa Q, \mu) = 0 \tag{2.13}$$

which is our RGE for a spontaneously broken gauge theory.

Following standard procedures[Ref.50], we can write the solution to this RGE as ($t = \log \kappa$)

$$\Gamma_R^{(n)}(\mathring{p} e^t, g_r, M_r, \alpha_r, Q e^t, \mu) =$$

$$= \left\{ e^{d_n t - n \int^t \tilde{\gamma}\, dt'} \right\} \Gamma_R^{(n)}(\mathring{p}, \bar{g}, \bar{M}, \bar{\alpha}, Q, \mu). \tag{2.14}$$

with

$$\frac{d\bar{g}}{dt} = \beta \quad , \qquad \bar{g}(0) \equiv g_r \tag{2.15a}$$

$$\frac{d\bar{M}}{dt} = -(1 + \gamma_M)\bar{M}, \qquad \bar{M}(0) \equiv M_r \tag{2.15b}$$

$$\frac{d\bar{\alpha}}{dt} = -2\gamma\bar{\alpha}, \qquad \bar{\alpha}(0) \equiv \alpha_r. \tag{2.15c}$$

Several comments are in order here:

a) by introducing a quantity, $\tilde{M}(t)$

$$\overline{M}(t) \equiv e^{-t}\, \tilde{M}(t) \qquad (2.16a)$$

we see that

$$\frac{d\tilde{M}}{dt} = -\gamma_M\, \tilde{M}(t) \qquad (2.16b)$$

and since γ_M is a polymonial function in $\overline{g}^2$ (it is calculated in perturbation theory), it follows that M(t) can at most grow as a power of t, and hence the exponential behavior dominates in $\overline{M}(t)$. Then, for t large and positive (i.e. large energy scales) the effects of $\overline{M}(t)$ tend to disappear, whereas for t large and negative its effect is to decouple the massive loops.

b) the function $\tilde{\gamma}$ of (9d) is of $O(g^4)$ instead of $O(g^2)$.

The reason is that the μ dependences of $\hat{Z}_e$ and Z_e are identical but opposite in sign, since $\Gamma_u^{(n)}$ is independent of μ.

c) The functions β, γ, γ_M are all to be calculated in the symmetric theory. This is because they are determined by the $\hat{Z}$'s.

Finally, our result carries with it the following rules to calculate fully renormalized matrix elements with external momenta $p=e^t p^o$:

1) VERTICES, are those generated by the symmetric lagrangian with $\bar{g}(t)$ replacing the original, bare coupling constant;
2) MASSIVE particles propagate with an effective mass, $\bar{M}(t)$, instead of the bare or renormalized mass; and
3) overall MATRIX ELEMENTS are to be multiplied by an external factor which depends on the canonical dimension of the matrix element.

For convenience, we will choose $Q=\mu$ in our applications.

In order to put in perspective the above discussion let us consider the case where we have an SO(N) symmetry in the lagrangian, and let us say that by an appropriate choice of Higgs the symmetry broke down to $SO(n_1) \times SO(n_2)$ and let us pretend that we wanted to calculate the three point function for the $SO(n_2)$ sector of the resulting theory to one loop. Then, by using our result we would choose a vertex proportional to g_2 and do the calculation using the symmetric coupling constant $\bar{g}$ and whenever we found massive propagators we would use

$$\frac{1}{p^2 + \bar{M}^2}$$

and in the end we would end up with, after inserting the appropriate external wave function renormalization constants, an expression of the form

$$\Gamma_R(\mathring{p}e^t, g_r, M_r, \alpha_r, \mu) =$$

$$= \bar{g}(t) - \frac{1}{16\pi^2}\bar{g}^3(t)\left[\frac{b}{2}\log\frac{\bar{M}(t)}{\mu} + \frac{f}{2}\log\frac{\mathring{p}}{\mu} + \frac{\tilde{a}}{2}\right] + O(\bar{g}^5) \qquad (2.17)$$

where b summarizes the effects of any "heavy" loops, f the effects of any "massless" loops and $\underline{\tilde{a}}$ are the constants that appear when taking the limit $\bar{M} \gg \mathring{p}, \mu$, (in the corresponding loop integrals) which means t large and negative. We identify $\Gamma_R(\mathring{p}e^t, g_r, M_r, \alpha_r, \mu)$ with the coupling constant $g_2(t)$, take d/dt of the above expression, and, keeping terms up to $O(g^3)$, we get, remembering that $\bar{M}(t) = e^{-t}\tilde{M}(t)$,

$$\frac{dg_2}{dt} = \frac{d\bar{g}}{dt} - \frac{\bar{g}^3}{16\pi^2}\frac{b}{2}(-1) + O(\bar{g}^5) \qquad (2.18)$$

or

$$\frac{dg_2}{dt} = \frac{\bar{g}^3}{16\pi^2}\frac{1}{2}(-\bar{b} + b) + O(\bar{g}^5) \qquad (2.19)$$

where we have introduced $\bar{b}$ by $16\pi^2\frac{d\bar{g}}{dt} = -\frac{\bar{b}}{2}\bar{g}^3$. We see that the effect on the RGE for the effective coupling g_2, of our taking the low energy limit, has been to "undress" the original SO(n) symmetric coupling by the amount corresponding to the massive particles, since in $\bar{b}/2$ are included the effects of all the particles in the original SO(n)-symmetric theory; therefore

$$\frac{1}{16\pi^2}(-\bar{b} + b) \equiv -B_2$$

(2.20)

can be identified with the β-function of g_2 (since the symmetry was broken from $SO(n)$ to $SO(n_1) X SO(n_2)$, there is no coupling between the $SO(n_1)$ and $SO(n_2)$ gauge bosons), and write

$$16\pi^2 \frac{dg_2}{dt} = -\frac{1}{2} B_2 g_2^3 \tag{2.21}$$

for in B_2 only the massless contributions to g_2 renormalization are included.

In general we would have, for a theory gauge invariant under a group G , breaking down to $G_1 \times G_2 \times \ldots$, that, at "low" energies,

$$\frac{d\bar{g}^2}{dt} = -\bar{b}\,\bar{g}^4 \qquad \Longrightarrow \qquad \bar{g}^2(\tau) = \frac{c_0}{1 + c_0 \bar{b} \tau} \tag{2.22}$$

for the G-coupling constant

and

$$\frac{d\, g_i^2(t)}{dt} = -b_i\, g_i^4 \tag{2.23a}$$

which implies

$$g_i^2(\tau) = \frac{A_i}{1 + A_i B_i \tau} \tag{2.23b}$$

for the G_i-coupling constant.

Here $\tau=0$, refers to some low energy point m, related to t by

$$t = -T + \tau \quad ; \quad T = \log(M_r/m) \tag{2.24}$$

Then to one loop, our result says

$$\left[\Gamma_R(\mathring{p}e^t, g_r, M_r, \alpha_r, \mu)\right]^2 = \\ = \bar{g}^2 + \bar{g}^4(b_i t - \tilde{a}_i) + \dots \tag{2.25}$$

and substituting (23b) and (24) in (25), we have

$$\begin{aligned} g_i^2(\tau) &= \frac{A_i}{1 + A_i B_i \tau} = \\ &= \bar{g}^2\{1 + \bar{g}^2(-\tilde{a}_i - b_i T + b_i \tau) + \dots\} \equiv \\ &\equiv \bar{g}^2\{1 + \bar{g}^2(a_i + b_i\tau) + \dots\} \qquad \text{(to one loop)} \end{aligned} \tag{2.26}$$

where we have defined $a_i = -\tilde{a}_i - b_i T$.

But $g_i^2(\tau)$ has a simple pole at $\tau = -\ 1/A_i B_i$ and therefore the RHS of (26) must sum as a geometric series,

$$\frac{A_i}{1 + A_i B_i \tau} = \bar{g}^2 \; \frac{1}{1 - \bar{g}^2(a_i + b_i\tau)} = \tag{2.27a}$$

$$= \frac{c_0}{1 + c_0 \bar{b}\tau} \; \frac{1 + c_0\bar{b}\tau}{1 + c_0\bar{b}\tau - c_0(a_i + b_i\tau)} = \tag{2.27b}$$

$$= \frac{c_0/(1 - a_i c_0)}{1 + \dfrac{c_0}{1 - a_i c_0}(\bar{b} - b_i)\tau} \tag{2.27c}$$

and we can identify

$$A_i = \frac{c_0}{1-a_i c_0} \quad ; \quad B_i = \bar{b} - b_i \tag{2.28}$$

These expressions provide us with an analytic relation between the low energy and high energy parameters of the theory, in the 1-loop approximation.

We can re-express this in another, more familiar, form

$$\frac{1}{A_i} = \frac{1}{c_0} - a_i = \frac{1}{c_0} + b_i T + \tilde{a}_i \tag{2.29}$$

or

$$\frac{1}{g_i^2(m)} = \frac{1}{\bar{g}^2(m)} + b_i \log \frac{M_r}{m} + \tilde{a}_i \tag{2.30}$$

which, without the constants $\tilde{a}_i$, can be found in the original work of Georgi, Quinn and Weinberg. The difference between their result and ours, the constants $\tilde{a}_i$, summarizing the effects of crossing the threshold region where the substraction was done, ($\mathring{p} = \mu = M_r$), directly down to the low energy regions, where $\bar{M} >> p$. We shall use them, extensively in what follows.

In the SU(5) model we will consider in this work, and which will be discussed in Chapter 3, the equivalents of eq. (14) or (17) are, in the 't Hooft-Feynman gauge (see Refs.7,23,47)

$$\Gamma_R(SU(3)) \equiv g_3(t) = \bar{g}(t) - \frac{\bar{g}^3(t)}{16\pi^2}\left\{-\frac{1}{2}\log\frac{\bar{M}^2}{\mu^2} + \frac{10}{9}n_f - \frac{78}{6} + \frac{5s}{\sqrt{3}} + O(\bar{g}^5)\right\} \tag{2.31a}$$

$$\Gamma_R(SU(2)) \equiv g_2(t) = \bar{g}(t) - \frac{\bar{g}^3(t)}{16\pi^2}\left\{\frac{4}{3}\log\frac{\bar{M}^2}{\mu^2} + \frac{10}{9}n_f - \frac{75}{9} + \frac{10s}{3\sqrt{3}} + O(\bar{g}^5)\right\} \quad (2.31b)$$

$$\Gamma_R(U(1)) \equiv g_1(t) = \bar{g}(t) - \frac{\bar{g}^3(t)}{16\pi^2}\left\{5\log\frac{\bar{M}^2}{\mu^2} + \frac{10}{9}n_f - \frac{5}{6} + O(g^5)\right\} \quad (2.31c)$$

Here $\bar{g}(t)$ is the symmetric, SU(5), running coupling constant, n_f is the number of generations of light fermions as defined in chapter 2, i.e. number of sets of 5 and 10 of SU(5) and s is related to the Spence function of argument $e^{i\pi/3}$, its value is s=2.029884. Also, g_3, g_2 and g_1 are the "leap-frogged" or "undressed" coupling constants corresponding to the SU(3), SU(2) and U(1) sectors of the theory.

These expressions can be written in a very general form, equivalent to a generalization of (30) to several gauge hierarchies. If the original symmetry, G, breaks down through a series of hierarchies, into $G_1 \times G_2 \times \ldots G_i \ldots$, then the equivalent of (30) is[Ref.23], in the most convenient form for phenomenological applications, (i. e., we ignore in the T_H contribution, terms of order $\log M_a/M_{Ha}$, and $\log M_{h1}/M_{h2}$, that is ratios of masses, for a given hierarchy and within a Higgs multiplet have been assumed to be of order 1)

$$\frac{16\pi^2}{g_i^2(m)} = \frac{16\pi^2}{\bar{g}^2(m)} + c_i\left[\frac{105}{3\sqrt{3}} - \frac{76}{9}\right] + \sum_a c_i^a\left[\frac{22}{3}\log\frac{M_a}{m} - \frac{1}{3}\right]$$

$$+\sum_H T_H(R)\left[-\frac{2}{3}\log\frac{M_H}{m}\right] + \sum_h T_H(R)\left[\frac{8}{9}\right]$$

$$+\sum_F T(R)\left[-\frac{4}{3}\log\frac{M_F}{m}\right] + \sum_f T(R)\left[\frac{10}{9}\right] \tag{2.32}$$

Here, g_i is the coupling constant associated with the subgroup G_i, $\bar{g}$ is the original G coupling constant, M_a is the heavy gauge boson mass in the a-th hierarchy, M_{Ha} is the mass of the heavy scalars, and M_F is the mass of any heavy fermions; the different coefficients are group theoretical coefficients, defined below, and m is a low energy point satisfying the conditions,

$$M_a \gg m$$
$$M_{Ha},\ M_F \gg m$$
$$m \gg m_i$$
$$m \gg m_f,\ m_h$$

where m_i is the mass (if they are going to acquire it in the next hierarchy) of the G_i-gauge-bosons, and m_f and m_h the masses of the light fermions and Higgs.

The group theoretical coefficients are defined as follows

$$c_i = C_2(G_i), \quad \delta_{il} C_2(G_p) = \sum_{j,k} f_{ijk} f_{ljk},$$

$$c_i^a = \sum_{Xa,Ya} f_{iXaYa} f_{iXaYa},$$

$$c_i + \sum_a c_i^a = C_2(G),$$

$$\mathrm{Tr}(\sigma^i \sigma^j) = T(R)\,\delta_{ij},$$

where σ^i are the generators in the representation, (R), according to which the Higgs or fermions transform, and f_{ijk} are the structure constants of G; also f_{iXaYa} are the restriction of the structure constants to the "heavy" directions X_a and Y_a.

For SU(N) we have that $C_2(G) = N$, and

R	$T_H(R)$	$T(R)$
Fundamental	1/2	1/2
Adjoint	N/2	N
Antisymm. 2nd. Rank	(N-2)/2	(N-2)/2

For completeness we also include the RGE satisfied by $\bar{g}$

$$d\bar{g}^2/dt = -\bar{b}\,\bar{g}^4 \qquad (2.33)$$

where $\bar{b}$ is given by

$$16\pi^2 \bar{b} = \frac{22}{3} C_2(G) - \frac{2}{3}\sum_{\text{Higgs}} T_H(R) - \frac{4}{3}\sum_{\text{fermions}} T(R)$$

In the SU(5) model to be discussed next, the equivalent of (31) for the g_X coupling is

$$\Gamma_R(X) = \bar{g} - \frac{\bar{g}^3}{16\pi^2}\left[\frac{25}{2}\log\frac{\bar{M}}{\mu} - \frac{775}{72} + \frac{10}{9}n_f + \frac{1}{2}\zeta_X\right] + O(\bar{g}^5)$$

$$\zeta_X = \sum_{l=1}^{3} R_l\left[\frac{1+\eta_l}{2} + \frac{\eta_l}{1-\eta_l}\log\eta_l\right]\frac{1}{1-\eta_l} \tag{2.34}$$

where

$$R_l \; : \quad R_3 = 3 \quad ; \quad R_2 = 16/3 \quad ; \quad R_1 = 5/3$$

and

$$\eta_3 = M_X^2/m^2(\varphi^i_j) \quad ; \quad \eta_2 = M_X^2/m^2(\omega^a_b) \quad ; \quad \eta_1 = M_X^2/m^2(\sigma)$$

(for the definitions of φ^i_j, ω^a_b and σ, see Appendix C).

The above expressions, provide us with a connection formula from M_X region across the M_X-threshold and into a region with some intermediate $p_{inter} \ll M_X$.

C H A P T E R III

ASYMPTOTICALLY FREE SU(5) MODEL.

As pointed out by Gross and Wilczek[Ref.16], and Cheng, Eichten and Li[Ref.39], the presence of Higgs fields in the lagrangian, tends to spoil the global asymptotic freedom of the theory; that is to say, while the gauge couplings may be asymptotically free, the Higgs self-couplings are not necessarily asymptotically free. In principle there is nothing wrong with this, for if the Higgs couplings grow with increasing q^2, one can argue that in order to maintain the validity of perturbation theory at those large q^2 values, one could choose initial values of the coupling constants, small enough so that one would not have conflict with perturbation theory until one reaches absurdly high energies: in grand unified theories this would be at energy scales well beyond the unification region. This has been the point of view followed by some authors[Ref.40]. However, as shown by Chang and others[Ref.9], asymptotic freedom for all the coupling constants of the theory can be restored by imposing eigenvalue conditions on the Higgs and Yukawa couplings.

Wanting to keep the asymptotic freedom of the theory could be taken even as a matter of, something as vague as, taste; but there are a few bonuses one gets by imposing eigenvalue conditions, (apart from the obvious one of guaranteeing the validity of perturbation theory in all ranges of q^2), like for example having only one coupling constant in the theory and therefore considerably reducing the number of "free" parameters and improve the "calculability" of the theory. The other advantage one gets is that, as shown by Mahanthappa and Lemmon[Ref.51], imposing eigenvalue conditions is equivalent to vanishing wave function renormalization constants for the scalar fields, one of the conditions for compositeness[Ref.52], and therefore makes contact (albeit in an admittedly vague way), with the notion of composite Higgs that are being studied today in the form of "technicolor"[Ref.53] (according to Bjorken, this should be called "Eastmancolor™" in this part of the country..[Ref.54]), "color tumbling" and so forth.

In this chapter we present an asymptotically free SU(5)[Ref.8] theory that has asymptotic freedom and yet its low energy structure is the same as the one of the standard SU(5)[Ref.55] theory of Georgi and Glashow[Ref.3].

As shown by Cheng, Eichten and Li[Ref.39], the quartic couplings of an SU(N) gauge theory with Higgs in the vector and adjoint representations cannot be asymptotically free unless $N \geqslant 7$. One way to bypass this, is to introduce additional fermions in the theory and take advantage of the fact that the box diagrams with four external scalar legs and a fermion loop contributes negatively to the RGE of the scalar quartic couplings[Ref.56]; if, in addition, we want the low energy phenomenology of the model to be unaltered, these extra fermion degrees of freedom must be very massive. We did just that with the SU(5) model.

After trying putting the heavy fermions in the 5 and 10 of SU(5), the 5 and 15 and not being able to find any eigenvalues, we found that by putting the heavy fermions in the 5 and 24 of SU(5) we could satisfy the eigenvalue conditions.

The model[Ref.57] is described by the lagrangian

$$
\begin{aligned}
\mathcal{L} = & -\frac{1}{2}\,\mathrm{Tr}\,(\partial_\mu A_\nu - \partial_\nu A_\mu - i\bar{g}[A_\mu, A_\nu])^2 \\
& - |\partial_\mu H - i\bar{g} A_\mu H|^2 - \frac{1}{2}\,\mathrm{Tr}\,(\partial_\mu \phi - i\bar{g}[A_\mu, \phi])^2 \\
& - \sum_{\text{generations}} \{ \bar{\psi}_R \gamma_\mu D_\mu \psi_R + \bar{\psi}_L \gamma_\mu D_\mu \psi_L \} \\
& + \frac{\mu^2}{2}\,\mathrm{Tr}\,\phi^2 - \frac{\lambda_1}{4}(\mathrm{Tr}\,\phi^2)^2 - \frac{\lambda_2}{2}\,\mathrm{Tr}\,\phi^4 - \frac{\lambda_3}{4}(H^+ \cdot H)^2 \\
& + \frac{\nu^2}{2} H^+ \cdot H - \frac{\lambda_4}{2} H^+ \cdot H\,\mathrm{Tr}\,\phi^2 - \frac{\lambda_5}{2} H^+ \phi^2 H \\
& - \sqrt{2}\,h\,(\bar{\psi}_{L\alpha\beta}\, \psi_R^\alpha H^\beta + h.c.) \\
& - (h'/4)\, \epsilon_{\alpha\beta\mu\nu\lambda}\, (\tilde{\psi}_L^{\alpha\beta} C^{-1} \psi_L^{\mu\nu} H^\lambda + h.c.)
\end{aligned}
$$

$$
+ \mathcal{L}_{\text{Ghost}} + \mathcal{L}_{\text{Gauge fixing}}
$$

$$
\begin{aligned}
& - \bar{\chi} \gamma_\mu D_\mu \chi - \bar{B} \gamma_\mu D_\mu B \\
& - k_2 \bar{\chi}_\alpha \chi^\beta \phi^\alpha_\beta - k_4 (\bar{B}^\alpha_\beta \chi^\beta H^+_\alpha + h.c.) \\
& - k_5 \bar{B}^\alpha_\beta B^\beta_\gamma \phi^\gamma_\alpha - k_6 \bar{B}^\alpha_\beta B^\gamma_\alpha \phi^\beta_\gamma
\end{aligned}
$$

Here A_μ represents the set of 24 SU(5) gauge bosons;

ψ_R and ψ_L are the standard SU(5) fermion multiplets in the $\underline{5}$ and $\underline{10}$, (vector and antisymmetric second rank tensor); D_μ represents the appropiate SU(5) covariant derivatives; ϕ is the $\underline{24}$ of Higgs and H the 5-plet; C is the Dirac charge conjugaton matrix and χ and B are the extra fermion fields transforming as the $\underline{5}$ and the $\underline{24}$, respectively. (Cf. Appendix C).

Given the above lagrangian one can calculate the renormalization group equations for the different couplings. The result is given in Appendix F.

By substituting

$$\lambda_i = \Lambda_i \bar{g}^2$$

$$h_i = \Upsilon_i \bar{g}$$

$$k_i = \Omega_i \bar{g}$$

with proportionality factors Λ_i, Υ_i and Ω_i into the RGE's we can reduce this coupled system of first order quasi-linear differential equations into an algebraic system of coupled quadratic equations in Λ_i, Υ_i and Ω_i. The real solutions to this system are the eigenvalues.

If one looks at the RGE for $\bar{g}(t)$, the effective SU(5) gauge coupling constant, we have that in order for it to be asymptotically free, the condition

$$\frac{52}{3} - \frac{22}{3} n_F - \frac{4}{3} n_f > 0$$

has to be satisfied. It is interesting to notice that if $n_f > 2$, then n_F is fixed to be 1, so that asymptotic freedom for the SU(5) coupled together with the observed fact that $n_f > 2$, demands that at the most we have only one set of heavy fermion fields. The maximum n_f is 7.

To obtain the eigenvalues we put the algebraic equations in a computer and did a global search for solutions by varying n_f. The subroutine we used was ZONEJ from the BELL's SYSTEM PORT SUBROUTINE PACKAGE, which is rather fast and efficient, and very well taylored for this job. For a given n_f, we computed all the Yukawa eigenvalues, and then for each set of Yukawa eigenvalues we computed the corresponding eigenvalues for the Higgs self couplings, discarding all those eigenvalues that did not satisfy the stability conditions of the potential. (See Appendix D).

As a practical remark, we would like to point out that since in this particular subroutine one has to give initial guesses for the solutions, one needs to devise a method of giving them; essentially one has two choices: either treat all the couplings "democratically", i. e. give them all an initial guess and let the iteration procedure in the program do the "fine tuning", or treat them "hierarchycally", i. e. give a guess for one coupling and set the others equal to zero and let the machine do its homework. We found that the "democratic" method worked better and much faster, even if your initial guess is very different from the actual root. Of course we built into our program the capability of automatically giving initial guesses within rather wide ranges, so as to not miss solutions. Although we had all these refinements in our program, it still took an average of three hours of CPU time to get the eigenvalues, for both the Yukawas and the Higgs self-couplings.

The eigenvalues we found, classified according to the number of light fermion generations with which they are associated, are given in table I.

CHAPTER IV

CALCULATION OF THE RENORMALIZED PROTON DECAY MATRIX ELEMENT

In chapter 1 we discussed how proton decay takes place through the process

$$u_{2L} + d_{1R} \dashrightarrow e^+_R + u^c_{3L} \quad ; \qquad (4.1)$$

in this chapter we calculate the corrections to that matrix element using our previously discussed renormalization program. The goal is to obtain its RGE and its initial value just below unification.

By the minimal scheme[Ref.45], in the 't Hooft-Feynman gauge, one gets for the process (1),

$$\Gamma_r^{(4)}(p, g_r, M_r, \alpha_r, \mu) = -\frac{i}{2}\bar{u}(p_2)\gamma_{\mu L}u(p_1)\cdot$$
$$\cdot\bar{u}(p_4)\gamma_{\mu R}u(p_3)\left[\frac{g_r^2}{p^2+M_r^2+\Sigma} + O(g^4)\right] \qquad (4.2)$$

where Σ are the finite mass correction terms, and p_1 to p_4 are the momenta of u_{2L}, u^c_{3L}, d_{1R} and e^+_R, in that order. The factor of 1/2 appearing in front of the matrix element comes from the $1/\sqrt{2}$ that appears in the fermion lagrangian, in the couplings of X to the fermions.

Following our renormalization program, and multiplying by the external wave function renormalization constants and, for convenience evaluating at momenta $\mathring{p}e^t$, we get

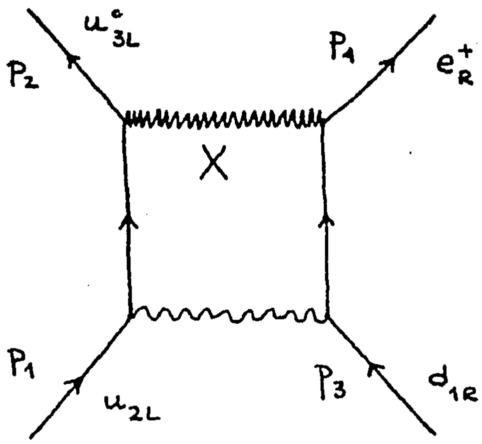
$$\Gamma_R^{(4)}(\mathring{p}e^t, g_r, M_r, \alpha_r, \mathring{p}e^t, \mu) = -\frac{i}{2}\bar{u}\gamma_{\mu L}u\,\bar{u}\gamma_{\mu R}u\cdot$$
$$\cdot\left[\frac{g_x^2}{\mathring{p}e^{2t}+m^2} + O(g^4)\right] \equiv -\frac{i}{2}\bar{u}\gamma_{\mu L}u\bar{u}\gamma_{\mu R}u\; G \qquad (4.3)$$

where g_X is the coupling constant of the X gauge bosons to the fermions and satisfies the RGE given in Appendix E; also $\mathcal{M}^2$ is defined in Appendix H and satisfies the RGE that is calculated there, and corresponds to the mass of the X gauge bosons at energies below unification, when the symmetric, SU(5) coupling constant has already split into the coupling constants for SU(3), SU(2) and U(1) and, in the case of fermions, g_X.

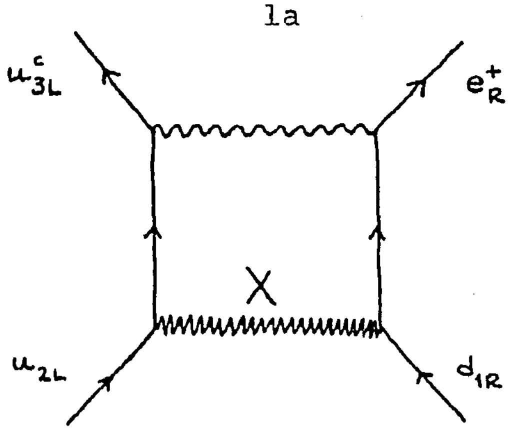
We now study the one-loop corrections, $O(g^4)$-terms in (3), to this matrix element in the region where the SU(3) X SU(2) X U(1) symmetry is valid.

The graphs to consider are shown in figures 1a, 1b, 1c and 1d, which represent the exchanges due to the SU(3), SU(2) and U(1) gauge bosons.

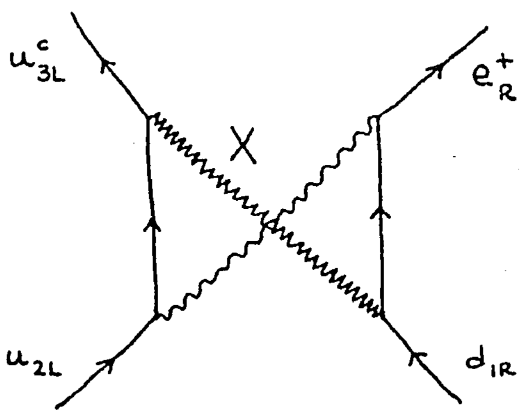
These graphs when calculated in the limit when the



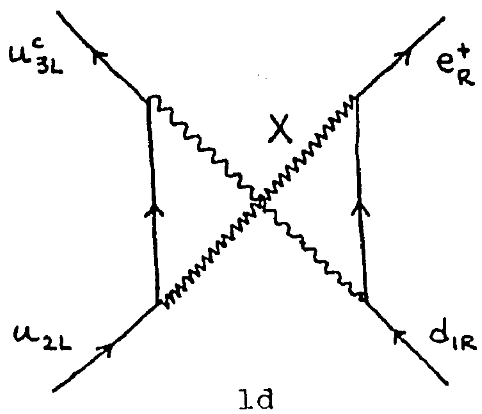
1a



1b



1c



1d

effective mass of X is very large compared to any other momenta, give the contributions, ignoring finite parts, listed in equations 4a to 4d., 4d., where as usual g_1, g_2 and g_3 denote the coupling constants of the fermions to the U(1), SU(2) and SU(3)gauge bosons.

$$\text{Fig. 1a} = \frac{1}{16\pi^2} G\left\{\frac{1}{30} g_1^2 + \frac{2}{3} g_3^2\right\} \log \frac{\bar{M}^2}{\mu^2} \qquad (4.4a)$$

$$\text{Fig. 1b} = \frac{1}{16\pi^2} G\left\{\frac{1}{5} g_1^2\right\} \log \frac{\bar{M}^2}{\mu^2} \qquad (4.4b)$$

$$\text{Fig. 1c} = \frac{1}{16\pi^2} G\left\{\frac{1}{5} g_1^2 + 3 g_2^2\right\} \log \frac{\bar{M}^2}{\mu^2} \qquad (4.4c)$$

$$\text{Fig. 1d} = \frac{1}{16\pi^2} G\left\{\frac{8}{15} g_1^2 + \frac{8}{3} g_3^2\right\} \log \frac{\bar{M}^2}{\mu^2} \qquad (4.4d)$$

With these graphs evaluated, we now proceed to obtain the RGE for G. By definition, (cf. (3)), in the low energy limit,

$$G \equiv \frac{g_x^2}{\mathcal{M}^2} + O(g^4) \quad ; t \to -\infty \qquad (4.5)$$

where $O(g^4)$ represents the above corrections. Then by taking d/dt of (5),

$$\frac{dG}{dt} = \frac{1}{\mathcal{M}^2}\frac{dg_x^2}{dt} - \frac{g_x^2}{\mathcal{M}^2}\frac{1}{\mathcal{M}^2}\frac{d\mathcal{M}^2}{dt} + \frac{d}{dt}O(g^4) \qquad (4.6)$$

and upon using the results in Appendices E and H, for dg_x/dt and $d\mathcal{M}^2/dt$, we have

$$16\pi^2 \frac{dG}{dt} = \frac{1}{\mathcal{M}^2}\, g_x^2\left[\frac{5}{6}g_1^2 + \frac{3}{2}g_2^2 + \frac{8}{3}g_3^2 + \frac{8}{3}n_f g_x^2\right] -$$

$$- \frac{g_x^2}{\mathcal{M}^2}\,\frac{8}{3}n_f g_x^2 + \left[-\frac{29}{15}g_1^2 - 6g_2^2 - \frac{20}{3}g_3^2\right]G$$

or

$$16\pi^2 \frac{dG}{dt} = G\left[-\frac{11}{10}g_1^2 - \frac{9}{2}g_2^2 - 4g_3^2\right] \tag{4.7}$$

as the low energy RGE for G when the SU(3) X SU(2) X U(1) symmetry is valid.

For the region where the SU(2) symmetry has broken down to U(1), the only contribution from the SU(2)-gauge bosons that survives is from W^0, which changes (4c) into

$$\frac{1}{16\pi^2}\, G\left\{\frac{1}{5}g_1^2 + g_2^2\right\}\log\frac{\bar{M}^2}{\mu^2} \tag{4.8}$$

and repeating the same steps that led to (7), but using the similarly modifed equation for g_x, one obtains

$$16\pi^2 \frac{dG}{dt} = G\left[-\frac{11}{10}g_1^2 - \frac{3}{2}g_2^2 - 4g_3^2\right] \tag{4.9}$$

as the RGE for G in the region where SU(3) X $U(1)_{em}$ is the valid symmetry.

For the actual calculation of the proton lifetime we will also need the high energy (symmetric) limit of $\Gamma_R^{(4)}$. This is easily obtained by noticing that in that limit $g_1 = g_2 = g_3 = \bar{g}$, so that, and using (2.34),

$$\Gamma_R^{(4)}(\mathrm{SYM}) = -\frac{i}{2}\,\bar{u}\gamma_{\mu L}u\bar{u}\gamma_{\mu R}u\left\{g_x^2/\mathcal{M}^2 + O(\bar{g}^4)\right\}\Big|_{\mathrm{SYM.}} = \tag{4.10}$$

$$= -\frac{i}{2}\,\bar{u}\gamma_{\mu L}u\bar{u}\gamma_{\mu R}u\,\frac{1}{\bar{M}^2}\left[\bar{g}^2 - \frac{\bar{g}^4}{16\pi^2}\,\frac{25}{2}\log\frac{\bar{M}^2}{\mu^2}\right] + O(\bar{g}^4)\Big|_{\mathrm{SYM}}$$

(where $\widetilde{M}^2$ was defined in (2.16); see also Appendix G),

or

$$\Gamma_R^{(4)}(\mathrm{SYM}) = -\frac{i}{2}\bar{u}\gamma_\mu u \bar{u}\gamma_{\mu R} u \, \frac{1}{\widetilde{M}^2}\left[\bar{g}^2 - \frac{\bar{g}^4}{16\pi^2}\,\frac{26}{5}\,\log\frac{\bar{M}^2}{\mu^2}\right] \tag{4.11}$$

from which we can extract the initial value (cf. (3)) for G,

$$G_{init} = \frac{1}{\widetilde{M}^2}\left[\bar{g}^2 - \frac{\bar{g}^4}{16\pi^2}\,\frac{26}{5}\,\log\frac{\bar{M}^2}{\mu^2}\right], \tag{4.12}$$

which we will use in the proton decay calculation.

C H A P T E R V

SECTION 1: CALCULATION OF THE PROTON LIFETIME.

The strategy we follow to calculate τ_p is very simple. It essentially consists[Ref.30] in the integration of the RGE for G from energies below grand unification down to energy scales of order m_p, and use the resulting value of G, at order m_p, in the formula given by BEGN[Ref.18] with the modifications of Jarlskog and Yndurain[Ref.26].

In order to carry this program through, we first need to know the unification mass, M_x, and the initial values of G, g_3, g_2 and g_1 (because the equation for G contains terms in g_3, g_2, g_1). We also need to know the value of the symmetric coupling constant at high energy (unification mass). We now show how we do each of these calculations.

To calculate M_x, we can proceed by using our RG result, established in chapter 2, and write down a formula for the log of M_x; this formula, as mentioned there, contains all the threshold effects. We will use for input parameters, α_s and α_{em} as measured at low energies, where the world is no longer SU(3) X SU(2) X U(1) symmetric but SU(3) X $U(1)_{em}$ symmetric; hence we will use the two hierarchy formula: one hierarchy for the breaking from SU(5) down to SU(3) X SU(2) X U(1) (the first hierarchy) by giving a VEV to the 24 of Higgs and another to go from

SU(3) X SU(2) X U(1) down to SU(3) X U(1)$_{em}$ (the second hierarchy) by giving a VEV to the SU(2) part of the 5-plet of Higgs. Here, of course, we will be consistent, and in accordance with the model we are working with, will, at the level of the first hierarchy, consider all the Higgs and heavy fermions, degenerate in mass with M_X, which is true up to proportionality factors of order one. For the second gauge hierarchy we will assume, as mentioned in chapter I, that the SU(2) part of the 5-plet has evolved from its $O(M_X)$ mass down to a mass of the order of the mass of the $W^{\pm}$'s which we take to be 83 GeV, and in the meantime the mass of the SU(3) part of the 5-plet has stayed at a value of, at most, $10^{-2}M_X$. We take care of these effects by introducing the parameters

$$\rho_3 \equiv \frac{M(H^i)}{M_x} \tag{5.1}$$

and

$$\rho_2 \equiv \frac{M(H^a)}{M_x} \tag{5.2}$$

By taking these remarks into account, and using (2.31), one arrives at the formula

$$\log \frac{M_x}{m} = \frac{2}{33}\left[\frac{3}{8}\;\frac{4\pi}{\alpha_{em}(m)} - \frac{4\pi}{\alpha_s(m)} - \frac{11}{2}\log\frac{M_W}{m} - \right.$$

$$\left. -\left(\frac{73}{3} - \frac{105}{\sqrt{3}} + \frac{1}{4}\log \rho_3/\rho_2\right)\right] \tag{5.3}$$

from which, by giving input values for α_{em} and α_s at m, we can determine M_x. (For typical values of ρ_2 and ρ_3, the constants and the log ρ_2/ρ_3 term have the effect of <u>reducing</u> M_x by a factor of 3.1).

To obtain the value of $\bar{g}$ at unification, we use its renormalization group equation and, for example, the equation for $1/g_3^2(m)$ given in (2.32), with, again the same input value for $\alpha_s(m)$.

We are now ready to obtain the initial values at a scale p_{inter} below unification, for G, g_3, g_2, g_1 by simply using (4.12), (G.12) and (2.31a-c).

Now we discuss the question of taking the different fermion and W-threshold effects into account in the integration of the RGE's. Let us write the system of equations,

$$16\pi^2 \frac{d}{dt}\left(\frac{1}{g_3^2}\right) = 22 - \frac{8}{3} n_f \tag{5.4a}$$

$$16\pi^2 \frac{d}{dt}\left(\frac{1}{g_2^2}\right) = \frac{44}{3}(0) - \frac{8}{3} n_f \tag{5.4b}$$

$$16\pi^2 \frac{d}{dt}\left(\frac{1}{g_1^2}\right) = -\frac{8}{3} n_f \tag{5.4c}$$

$$16\pi^2 \frac{d}{dt} \log G = -\frac{11}{10} g_1^2 - \frac{9}{2}\left(\frac{3}{2}\right) g_2^2 - 4 g_3^2 \tag{5.4d}$$

There are two remarks concerning this system of equations (5.4), as written above. One is that we have taken the derivative of g_i^{-2} instead of g_i: this is because with our integration routine (HPCG of the IBM Scientific Subroutine Package LIBrary)* they worked better when doing the numerical integration, this is related to remark three, below. Two: In the right hand side of (5.4b) and (5.4c) we have written 44(0) and 9/2(3/2) respectively: this is the value of those coefficients when the symmetry further breaks from SU(3) X SU(2) X U(1) to SU(3) X $U(1)_{em}$. And the third remark is that we will consider n_f to be a function of t, the scale parameter, as well; we explain the last two remarks in detail in what follows: they have to see with the inclussion of W- and fermion-thresholds.

Suppose, for example, that we consider the low energy RGE for g_1 in a region R_1 where $p \gg m_f$, the mass of any of the fermions counted in n_f, in this region the equation reads

$$16\pi^2 \frac{dg_1}{dt} = \frac{4}{3} n_f g_1^3 \tag{5.5}$$

Imagine that we are integrating from R_1 down to region R_2 where $p \ll m_f$ ($m_f^{(1)}$ is the most massive of the fermions in n_f), then for region R_2, (5.5) should read

$$16\pi^2 \frac{dg_1}{dt} = \frac{4}{3} (n_f - 1) g_1^3 \tag{5.5b}$$

In order to take into account this, let us say, "passing over the fermion thresholds", we might interpolate n_f with a formula such as

$$n_f(t) = n_f(\infty) - \sum_i \frac{1}{1+\beta p^2/m_i^2} \tag{5.6}$$

where $n_f(\infty)$ is the number of fermions at very high energies, and, the sum extends over all the fermions that we would consider as very massive (the i-th fermion has mass m_i) compared with $p^2=e^{2t}$.

Equation (5.6) is nothing but a poor man's way of writing one minus the Heavyside function, and it is interesting to notice that it is similar to the formula obtained by Georgi and Politzer[Ref.21] by using the mass-dependent RG, only that they gave the value .2 for β ; for reasons that will be explained later, our best value is closer to .3.

We shall treat the W-threshold problem in the integration by a similar procedure, introducing another parameter β_1 (cf. 5.7b and 5.7c below).

For the record, we write down the RGE's that we use in our integration subroutine

$$16\pi^2 \frac{d}{dt}\left(\frac{1}{g_3^2}\right) = 22 - \frac{8}{3} n_f(t) \tag{5.7a}$$

$$16\pi^2 \frac{d}{dt}\left(\frac{1}{g_2^2}\right) = \frac{44}{3}\left(1 - \frac{1}{1+\beta_1 e^{2t}/m_W^2}\right) - \frac{8}{3} n_f(t) \tag{5.7b}$$

$$16\pi^2 \frac{d}{dt}\left(\frac{1}{g_1^2}\right) = -\frac{8}{3} n_f(t) \tag{5.7c}$$

$$16\pi^2 \frac{d}{dt} \log G = -\frac{11}{10} g_1^2 - \frac{3}{2}\left(3 - \frac{2}{1+\beta_1 e^{2t}/m_W^2}\right) g_2^2 - 4 g_3^2 \tag{5.7d}$$

To determine β and β_1 we did the following. Given a certain value for $\alpha_{em}(m)$ and $\alpha_s(m)$ at a low energy scale m (cf. Appendix I), typically 6 GeV, we determined M_x, and $g(M_x)$ by the procedure described before, and also the initial values of G, g_3, g_2, g_1 at in principle, arbitrary p_{inter}. Then we gave arbitrary values to β and β_1 and set the masses of the as yet undiscovered "light" fermions (in a sense the average mass of the 3rd generation) at a value of 30 GeV, and the masses of the remaining (up to $n_f(\infty)$) generations in the sequence 300 GeV, 3000 GeV, etc. We started integrating down from unification and we compared the result obtained by integrating the system (5.7a-d) to what one would obtain if using eqs. (2.32), choosing those values of β, β_1 and p_{inter} that did the job with the best agreement between the two procedures. The agreement for our best values of β and β_1 was typically better than 98%!!!

The method is rather sensitive to the values of β and β_1, being more sensitive to β_1 than to β : for a given average value of the mass of the third generation and m_W, a change of 15% in β typically gives agreement to only 95%, whereas a similar change in β_1 gives agreement to just 90%.

For our best values β = .27 and β_1= 25, the <u>dis</u>agreement between the two procedures at some selected values of p , in the case $n_f=5$, were

$p(GeV) = 3.0 \times 10^9$........error(%) = .93

$p(GeV) = 9.6 \times 10^5$........error(%) = 1.06

$p(GeV) = 9.6 \times 10^4$........error(%) = 1.29

$p(GeV) = 9.6 \times 10^3$........error(%) = 1.20

$p(GeV) = 9.6 \times 10^2$........error(%) = 1.12

$p(GeV) = 9.6 \times 10$ ....error(%) = 1.10

After obtaining the value of G at m_p, we simply put this value in the lifetime formula of BEGN as modified by Jarlskog and Yndurain. These modifications consist of including muons and strange particles in the final state, one-quark and three-quark fusion (BEGN only included two-quark fusion) and allowing for a 4% decrease in τ_p for Cabibbo suppressed decays. The results we obtained are presented in Table II, which corresponds to a $|\psi(0)|^2$(see eq. (1.19)) of 8×10^{-3} $(GeV)^3$, as used by Jarlskog and Yndurain; however, other values of $|\psi(0)|^2$exist in the literature which place this value down to $(4.3-2.8)\times10^{-3}$ $(GeV)^3$Ref. 59 or 1.1×10^{-3} $(GeV)^3$ as in Ref. 60, and therefore give a longer lifetime.

The lifetime corresponding to a $\Lambda_{\overline{MS}}$ of .4 GeV and for three generations of light fermions (we used the same β and β_1 for all n_f from 3 to 7), is

$$M_x=8.9\times10^5 \text{ EeV}=8.9\times10^{14}\text{GeV}, \quad \tau_p=5.1\times10^{30} \text{ years.}$$

SECTION 2: CONCLUSIONS AND COMPARISONS.

We have presented a calculation of the proton lifetime in an asymptotically free SU(5) model. To avoid the problem of uncertainties in the values of the coupling constants below unification, we developed a new RG analysis which by exploiting the minimal scheme provides one with initial values for the coupling constants away from the thresholds, whose effects show up in additive constants in the initial values of the coupling constants of the effective gauge theory. Since due to the eigenvalue conditions in the Higgs sector all the parameters of the theory are calculable in terms of α_s and α_{em}, we have been able to perform a complete calculation, except for the assumption of gauge hierarchy. The value for τ_p that we obtain includes mass renormalization effects, and due to the small discrepancy between the values we obtain and those of others (see below), we conclude that the mass renormalization effects play a very small role in the value of τ_p; this is reinforced by our check on the validity of the theta-approximation of Ross.

We now do some comparison between our work and that of some other workers Refs. 33 and 34, choosing our result for a $\Lambda_{\overline{MS}}$ of .4GeV and n_f=3.

The results are,

	M_X (GeV)	τ_p (years)
Goldman and Ross	4.4×10^{14}	$(1.6-200)\times10^{30}$
Marciano	6.3×10^{14}	$(3.2-\ 60)\times10^{30}$
Ours	8.9×10^{14}	$(5.1-37)\times10^{30}$

Here our first value corresponds to $|\psi(0)|^2 = 8\times10^{-3}$ $(\mathrm{Gev})^3$ and the second to the value obtained by using Finjord's estimate for $|\psi(0)|^2$ Ref. 60 of 1.1×10^{-3} $(\mathrm{GeV})^3$, which is the value used by Goldman and Ross to obtain their first quoted τ_p of 1.6×10^{30} years.

The difference between our value of τ_p and Goldman and Ross' shows the combined effects of having asymptotic freedom and mass renormalization most clearly. Their mass value is 1/2 of ours which means their lifetime (first quoted value, the second value they give is based on an estimate of errors), should be about 1/16 of ours, however it is only 1/23 of ours, which means that the combined effect of asymptotic freedom and mass renormalization of M_X is to increase the lifetime by a factor of about 1.4.

For the case of Marciano, we can compare the masses. His value is based in a theory whose SU(2) doublet of Higgs in the original SU(5) 5-plet remains massless all the way from unification down to m_W; since he did not include threshold effects coming from the first breaking, these effects and the assumption on the Higgs doublet conspire so as to give an M_X very close to ours.

The fact that each of the calculations has different effects taken into account gives us an idea of where the proton lifetime should lie, and from the above values, and if there are only three generations of "light" fermions, and $\Lambda_{\overline{MS}}$ is .4GeV we can conclude that if SU(5) is the theory, within a few years (while data is collected), see Refs.6,15, we will see a decaying proton with a lifetime between 10^{31} and 10^{32} years.

A P P E N D I C E S

APPENDIX A

THE 24 λ'S OF SU(5).

Here we show how to write the λ's of SU(5) in their five dimensional realization.

For λ_α, α =1,...,8 we simply take the eight Gell-Mann SU(3) matrices enlarged to 5x5 dimensions:

$$\lambda_1 = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \qquad \lambda_2 = \begin{bmatrix} 0 & -i & 0 & 0 & 0 \\ i & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\lambda_3 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \qquad \lambda_4 = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\lambda_5 = \begin{bmatrix} 0 & 0 & -i & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ i & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \qquad \lambda_6 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\lambda_7 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -i & 0 & 0 \\ 0 & i & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} . \qquad \lambda_8 = \frac{1}{\sqrt{3}} \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & -2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

These 8 matrices represent the SU(3) subgroup of SU(5). For the SU(2) subgroup we just use the Pauli matrices in entries 4 and 5:

$$\lambda_{10} = \left[\begin{array}{c|cc} 0 & \multicolumn{2}{c}{0} \\ \hline 0 & 0 & 1 \\ & 1 & 0 \end{array}\right] \qquad \lambda_{11} = \left[\begin{array}{c|cc} 0 & \multicolumn{2}{c}{0} \\ \hline 0 & 0 & -i \\ & i & 0 \end{array}\right] \qquad \lambda_{12} = \left[\begin{array}{c|cc} 0 & \multicolumn{2}{c}{0} \\ \hline 0 & 1 & 0 \\ & 0 & -1 \end{array}\right]$$

We already have three diagonal matrices, λ_3, λ_8, λ_{12}, the fourth (SU(5) has rank four) can be constructed by writing

$$\lambda_9 = \left[\begin{array}{ccc|cc} a & & & & \\ & a & & \multicolumn{2}{c}{0} \\ & & a & & \\ \hline & & & b & \\ & 0 & & & b \end{array}\right]$$

so that it is proportional to the identity in the SU(3) and SU(2) subspaces. Then from $Tr(\lambda_9)=0$ and $Tr(\lambda_9^2)=2$, one gets

$$\lambda_9 = \frac{2}{\sqrt{15}} \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -3/2 & 0 \\ 0 & 0 & 0 & 0 & -3/2 \end{bmatrix}$$

The remaining twelve λ's can be written in a way similar to the way we wrote λ_4 to λ_7, i. e.:

$$\lambda_{13} = \left[\begin{array}{ccc|cc} & & & 1 & 0 \\ & O_3 & & 0 & 0 \\ & & & 0 & 0 \\ \hline 1 & 0 & 0 & \multicolumn{2}{c}{O_2} \\ 0 & 0 & 0 & & \end{array}\right] \qquad \lambda_{14} = \left[\begin{array}{ccc|cc} & & & -i & 0 \\ & O_3 & & 0 & 0 \\ & & & 0 & 0 \\ \hline i & 0 & 0 & \multicolumn{2}{c}{O_2} \\ 0 & 0 & 0 & & \end{array}\right] \qquad \lambda_{15} = \left[\begin{array}{ccc|cc} & & & 0 & 0 \\ & O_3 & & 1 & 0 \\ & & & 0 & 0 \\ \hline 0 & 1 & 0 & \multicolumn{2}{c}{O_2} \\ 0 & 0 & 0 & & \end{array}\right]$$

$$\lambda_{16}=\left[\begin{array}{ccc|cc} & & & 0 & 0 \\ & O_3 & & -i & 0 \\ & & & 0 & 0 \\ \hline 0 & i & 0 & & \\ 0 & 0 & 0 & O_2 & \end{array}\right] \quad \lambda_{17}=\left[\begin{array}{ccc|cc} & & & 0 & 0 \\ & O_3 & & 0 & 0 \\ & & & 1 & 0 \\ \hline 0 & 0 & 1 & & \\ 0 & 0 & 0 & O_2 & \end{array}\right] \quad \lambda_{18}=\left[\begin{array}{ccc|cc} & & & 0 & 0 \\ & O_3 & & 0 & 0 \\ & & & -i & 0 \\ \hline 0 & 0 & i & & \\ 0 & 0 & 0 & O_2 & \end{array}\right]$$

$$\lambda_{19}=\left[\begin{array}{ccc|cc} & & & 0 & 1 \\ & O_3 & & 0 & 0 \\ & & & 0 & 0 \\ \hline 0 & 0 & 0 & & \\ 1 & 0 & 0 & O_2 & \end{array}\right] \quad \lambda_{20}=\left[\begin{array}{ccc|cc} & & & 0 & -i \\ & O_3 & & 0 & 0 \\ & & & 0 & 0 \\ \hline 0 & 0 & 0 & & \\ i & 0 & 0 & O_2 & \end{array}\right] \quad \lambda_{21}=\left[\begin{array}{ccc|cc} & & & 0 & 0 \\ & O_3 & & 0 & 1 \\ & & & 0 & 0 \\ \hline 0 & 0 & 0 & & \\ 0 & 1 & 0 & O_2 & \end{array}\right]$$

$$\lambda_{22}=\left[\begin{array}{ccc|cc} & & & 0 & 0 \\ & O_3 & & 0 & -i \\ & & & 0 & 0 \\ \hline 0 & 0 & 0 & & \\ 0 & i & 0 & O_2 & \end{array}\right] \quad \lambda_{23}=\left[\begin{array}{ccc|cc} & & & 0 & 0 \\ & O_3 & & 0 & 0 \\ & & & 0 & 1 \\ \hline 0 & 0 & 0 & & \\ 0 & 0 & 1 & O_2 & \end{array}\right] \quad \lambda_{24}=\left[\begin{array}{ccc|cc} & & & 0 & 0 \\ & O_3 & & 0 & 0 \\ & & & 0 & -i \\ \hline 0 & 0 & 0 & & \\ 0 & 0 & i & O_2 & \end{array}\right]$$

These 24 matrices, when multiplied by 1/2 satisfy the standard commutation relations of SU(5) and are its generators in the vector realization.

APPENDIX B

THE LIGHT FERMION LAGRANGIAN

In the SU(5) model of Georgi and Glashow, the fermionic part of the lagrangian is given by

$$\mathcal{L}_{fermions} = -\bar{\psi}_R \gamma_\mu \left(\partial_\mu + \frac{i}{2} g \lambda \cdot A_\mu\right) \psi_R - \tag{B.1}$$

$$-\bar{\psi}_{L\alpha\beta}\left(\partial_\mu \delta_{\alpha\alpha'}\delta_{\beta\beta'} + \frac{i}{2} g (\lambda . A_\mu)_{\alpha\alpha'}\delta_{\beta\beta'} + \frac{i}{2} g(\lambda . A_\mu)_{\beta\beta'}\delta_{\alpha\alpha'}\right)\psi_L^{\alpha'\beta'} -$$

$$-\left(\sqrt{2}\, h\, \bar{\psi}_{L\alpha\beta}\psi_R^\alpha H^\beta + \frac{h'}{4}\epsilon_{\alpha\beta\mu\nu\lambda}\tilde{\psi}_L^{\alpha\beta} C^{-1}\psi_L{}^{\mu\nu} H^\lambda + h.c.\right)$$

where the λ 's are the matrices of appendix A, H^β is the 5-plet of Higgs, C is the Dirac charge conjugation matrix, $\gamma_2\gamma_4$, and $\epsilon_{\alpha\beta\mu\nu\lambda}$ is the totally antisymmetric tensor in 5 dimensions. For reference purposes we write in detail the fermion-gauge- boson couplings separated into the SU(3), SU(2) and U(1) parts and the X and Y interactions. We also write the explicit form of the Yukawa couplings. We will not combine terms like $\bar{d}\,\gamma_{\mu L} e + \bar{d}\,\gamma_{\mu R} e$ into $\bar{d}\gamma_\mu e$, because due to the separation in the Georgi-Glashow model of the R and L components into a <u>5</u> and a <u>10</u> , it turns out to be more convenient to have the lagrangian separated as given.

$$\begin{aligned}\mathcal{L}(SU(3)) = i g \mathring{\rho}_\mu \Big\{ & -\frac{1}{2}\bar{u}_1^c \gamma_{\mu L} u_1^c + \frac{1}{2}\bar{u}_2^c \gamma_{\mu L} u_2^c \\ & + \frac{1}{2}\bar{u}_1(\theta)\gamma_{\mu L} u_1(\theta) - \frac{1}{2}\bar{u}_2(\theta)\gamma_{\mu L} u_2(\theta) \\ & + \frac{1}{2}\bar{d}_1 \gamma_{\mu L} d_1 - \frac{1}{2}\bar{d}_2 \gamma_{\mu L} d_2 \\ & + \frac{1}{2}\bar{d}_1 \gamma_{\mu R} d_1 - \frac{1}{2}\bar{d}_2 \gamma_{\mu R} d_2 \Big\}\end{aligned} \tag{B.2}$$

$$+ig\,\rho_\mu^- \left\{ \frac{1}{\sqrt{2}}\,\bar{u}_1(\theta)\,\gamma_{\mu L}\,u_2(\theta) - \frac{1}{\sqrt{2}}\,\bar{u}_2^c\,\gamma_{\mu L}\,u_1^c + \right.$$
$$\left. + \frac{1}{\sqrt{2}}\,\bar{d}_1\,\gamma_{\mu L}\,d_2 + \frac{1}{\sqrt{2}}\,\bar{d}_1\,\gamma_{\mu R}\,d_2 \right\}$$

$$+ig\,\rho_\mu^+ \left\{ \frac{1}{\sqrt{2}}\,\bar{u}_2(\theta)\,\gamma_{\mu L}\,u_1(\theta) - \frac{1}{\sqrt{2}}\,\bar{u}_1^c\,\gamma_{\mu L}\,u_2^c + \right.$$
$$\left. + \frac{1}{\sqrt{2}}\,\bar{d}_2\,\gamma_{\mu L}\,d_1 + \frac{1}{\sqrt{2}}\,\bar{d}_2\,\gamma_{\mu L}\,d_1 \right\}$$

$$+ig\,K_\mu^{*-} \left\{ -\frac{1}{\sqrt{2}}\,\bar{u}_3^c\,\gamma_{\mu L}\,u_1^c + \frac{1}{\sqrt{2}}\,\bar{u}_1(\theta)\,\gamma_{\mu L}\,u_3(\theta) + \right.$$
$$\left. + \frac{1}{\sqrt{2}}\,\bar{d}_1\,\gamma_{\mu L}\,d_3 + \frac{1}{\sqrt{2}}\,\bar{d}_1\,\gamma_{\mu R}\,d_3 \right\}$$

$$+ig\,K_\mu^{*+} \left\{ -\frac{1}{\sqrt{2}}\,\bar{u}_1^c\,\gamma_{\mu L}\,u_3^c + \frac{1}{\sqrt{2}}\,\bar{u}_3(\theta)\,\gamma_{\mu L}\,u_1(\theta) + \right.$$
$$\left. + \frac{1}{\sqrt{2}}\,\bar{d}_3\,\gamma_{\mu L}\,d_1 + \frac{1}{\sqrt{2}}\,\bar{d}_3\,\gamma_{\mu R}\,d_1 \right\}$$

$$+ig\,\bar{K}_\mu^{*0} \left\{ -\frac{1}{\sqrt{2}}\,\bar{u}_3^c\,\gamma_{\mu L}\,u_2^c + \frac{1}{\sqrt{2}}\bar{u}_2(\theta)\,\gamma_{\mu L}\,u_3(\theta) + \right.$$
$$\left. + \frac{1}{\sqrt{2}}\,\bar{d}_2\,\gamma_{\mu L}\,d_3 + \frac{1}{\sqrt{2}}\,\bar{d}_2\,\gamma_{\mu R}\,d_3 \right\}$$

$$+ig\,K_\mu^{*0} \left\{ -\frac{1}{\sqrt{2}}\,\bar{u}_2^c\,\gamma_{\mu L}\,u_3^c + \frac{1}{\sqrt{2}}\,\bar{u}_3(\theta)\,\gamma_{\mu L}\,u_2(\theta) + \right.$$
$$\left. + \frac{1}{\sqrt{2}}\,\bar{d}_3\,\gamma_{\mu L}\,d_2 + \frac{1}{\sqrt{2}}\,\bar{d}_3\,\gamma_{\mu R}\,d_2 \right\}$$

$$+ig\,\eta_\mu^0 \left\{ \frac{1}{2\sqrt{3}} \sum_{i=1}^{2} \bar{u}_i(\theta)\,\gamma_{\mu L}\,u_i(\theta) - \frac{1}{\sqrt{3}}\,\bar{u}_3(\theta)\,\gamma_{\mu L}\,u_3(\theta) - \right.$$
$$- \frac{1}{2\sqrt{3}} \sum_{i=1}^{2} \bar{u}_i^c\,\gamma_{\mu L}\,u_i^c + \frac{1}{\sqrt{3}}\,\bar{u}_3^c\,\gamma_{\mu L}\,u_3^c +$$

$$+ \frac{1}{2\sqrt{3}} \sum_{i=1}^{2} \bar{d}_i\,\gamma_{\mu L}\,d_i - \frac{1}{\sqrt{3}}\,\bar{d}_3\,\gamma_{\mu L}\,d_3$$

$$\left. + \frac{1}{2\sqrt{3}} \sum_{i=1}^{2} \bar{d}_i\,\gamma_{\mu R}\,d_i - \frac{1}{\sqrt{3}}\,\bar{d}_3\,\gamma_{\mu R}\,d_3 \right\}$$

$$\mathcal{L}(SU(2)) = +igW_\mu^0\left\{\frac{1}{2}\sum_{i=1}^{3}\bar{u}_i(\theta)\gamma_{\mu L}u_i(\theta)-\frac{1}{2}\sum_{i=1}^{3}\bar{d}_i\gamma_{\mu L}d_i + \frac{1}{2}\bar{e}^+\gamma_{\mu R}e^+ - \frac{1}{2}\bar{\nu}_e^c\gamma_{\mu R}\nu_e^c\right\} \tag{B.3}$$

$$+igW_\mu^-\left\{\frac{1}{\sqrt{2}}\sum_{i=1}^{3}\bar{d}_i\gamma_{\mu L}u_i(\theta) + \frac{1}{\sqrt{2}}\bar{\nu}_e^c\gamma_{\mu R}e^+\right\}$$

$$+igW_\mu^+\left\{\frac{1}{\sqrt{2}}\sum_{i=1}^{3}\bar{u}_i(\theta)\gamma_{\mu L}d_i + \frac{1}{\sqrt{2}}\bar{e}^+\gamma_{\mu R}\nu_e^c\right\}$$

$$\mathcal{L}(U(1)) = +igB_\mu\left\{\frac{2}{\sqrt{15}}\sum_{i=1}^{3}\bar{u}_i^c\gamma_{\mu L}u_i^c - \frac{1}{2\sqrt{15}}\sum_{i=1}^{3}\bar{u}_i(\theta)\gamma_{\mu L}u_i(\theta) - \frac{1}{2\sqrt{15}}\sum_{i=1}^{3}\bar{d}_i\gamma_{\mu L}d_i + \frac{1}{\sqrt{15}}\sum_{i=1}^{3}\bar{d}_i\gamma_{\mu R}d_i - \frac{3}{\sqrt{15}}\bar{e}^+\gamma_{\mu L}e^+ - \frac{3}{2\sqrt{15}}\bar{e}^+\gamma_{\mu R}e^+ - \frac{3}{2\sqrt{15}}\bar{\nu}_e^c\gamma_{\mu R}\nu_e^c\right\} \tag{B.4}$$

$$\mathcal{L}(X) = +ig\sum_{i=1}^{3}X_{\mu i}^-\left\{-\frac{1}{\sqrt{2}}\epsilon_{ijk}\bar{u}_j^c\gamma_{\mu L}u_k(\theta) + \frac{1}{\sqrt{2}}\bar{d}_i\gamma_{\mu L}e^+ + \frac{1}{\sqrt{2}}\bar{d}_i\gamma_{\mu R}e^+\right\} \tag{B.5}$$

$$+ig\sum_{i=1}^{3}X_{\mu i}^+\left\{-\frac{1}{\sqrt{2}}\epsilon_{ijk}\bar{u}_k(\theta)\gamma_{\mu L}u_j^c + \frac{1}{\sqrt{2}}\bar{e}^+\gamma_{\mu L}d_i + \frac{1}{\sqrt{2}}\bar{e}^+\gamma_{\mu R}d_i\right\}$$

$$\mathcal{L}(Y) = +ig\sum_{i=1}^{3}Y_{i\mu}^-\left\{-\frac{1}{\sqrt{2}}\epsilon_{ijk}\bar{u}_j^c\gamma_{\mu L}d_k - \frac{1}{\sqrt{2}}\bar{u}_i(\theta)\gamma_{\mu L}e^+ + \frac{1}{\sqrt{2}}\bar{d}_i\gamma_{\mu R}\nu_e^c\right\} \tag{B.6}$$

$$+ig\sum_{i=1}^{3}Y_{i\mu}^+\left\{-\frac{1}{\sqrt{2}}\epsilon_{ijk}\bar{d}_k\gamma_{\mu L}u_j^c - \frac{1}{\sqrt{2}}\bar{e}^+\gamma_{\mu L}u_i(\theta) + \frac{1}{\sqrt{2}}\bar{\nu}_e^c\gamma_{\mu R}d_i\right\}$$

In eqs.(2)-(6), we have embedded the SU(3), SU(2) and U(1) as follows. Define

$$(A_\mu)^\alpha_\beta = \frac{1}{2}(\lambda^\theta)^\alpha_\beta A^\theta_\mu \tag{B.7a}$$

with $\alpha,\beta=1,\ldots,5$, and $\theta=1,\ldots,24$. Then we have taken

$$(A_\mu)^i_j \equiv G^{\ i}_{\mu j} + \frac{1}{\sqrt{15}} \delta^i_j B_\mu \tag{B.7b}$$

$$(A_\mu)^i_a \equiv \frac{1}{\sqrt{2}} \begin{pmatrix} X_{i\mu}^- \\ Y_{i\mu}^- \end{pmatrix} \tag{B.7c}$$

$$(A_\mu)^a_b \equiv \frac{1}{2}(\tau . W_\mu)^b_a - \frac{3}{2\sqrt{15}} \delta^b_a B_\mu \tag{B.7d}$$

with i,j=1,2,3 and a,b=1,2; also $G^{\ i}_{\mu j}=(1/2)(\lambda^\rho)^i_j G^\rho_\mu$, with ρ =1,...,8 and λ^ρ the eight Gell-Mann lambda matrices.

Explicitly

$$A^1_1 = \frac{1}{2}\rho^\circ + \frac{1}{2\sqrt{3}}\eta^\circ + \frac{1}{\sqrt{15}} B \tag{B.7e}$$

$$A^2_2 = -\frac{1}{2}\rho^\circ + \frac{1}{2\sqrt{3}}\eta^\circ + \frac{1}{\sqrt{15}} B \tag{B.7f}$$

$$A^3_3 = -\frac{1}{\sqrt{3}}\eta^\circ + \frac{1}{\sqrt{15}} B \tag{B.7g}$$

$$A^4_4 = -\frac{3}{2\sqrt{15}} B + \frac{1}{2} W^\circ \tag{B.7h}$$

$$A^5_5 = -\frac{3}{2\sqrt{15}} B - \frac{1}{2} W^\circ \tag{B.7i}$$

$$\sqrt{2}\,A^1_2 = \rho^- \qquad \sqrt{2}\,A^1_3 = K^{*-} \qquad \sqrt{2}\,A^1_4 = X_1^- \qquad \sqrt{2}\,A^1_5 = Y_1^- \tag{B.7j}$$

$$\sqrt{2}\,A^2_1 = \rho^+ \qquad \sqrt{2}\,A^2_3 = \bar{K}^{*0} \qquad \sqrt{2}\,A^2_4 = X_2^- \qquad \sqrt{2}\,A^2_5 = Y_2^- \tag{B.7k}$$

$$\sqrt{2}\,A^3_1 = K^{*+} \qquad \sqrt{2}\,A^3_2 = K^{*0} \qquad \sqrt{2}\,A^3_4 = X_3^- \qquad \sqrt{2}\,A^3_5 = Y_3^- \tag{B.7l}$$

$$\sqrt{2}\,A^4_1 = X_1^+ \qquad \sqrt{2}\,A^4_2 = X_2^+ \qquad \sqrt{2}\,A^4_3 = X_3^+ \qquad \sqrt{2}\,A^4_5 = W^+ \tag{B.7m}$$

$$\sqrt{2}\,A^5_1 = Y_1^+ \qquad \sqrt{2}\,A^5_2 = Y_2^+ \qquad \sqrt{2}\,A^5_3 = Y_3^+ \qquad \sqrt{2}\,A^5_4 = W^- \tag{B.7n}$$

The Yukawa couplings are explicitly given by

$$\begin{aligned}
\mathcal{L}_Y(h) = -h\{ & \epsilon_{ijk}\,\bar{u}^c_{jL}\,d_{kR}\,H^i + \epsilon_{ijk}\,\bar{d}_{kR}\,u^c_{jL}\,H_i \\
& + \bar{u}_{iL}(\theta)\,e^+_R\,H^i + \bar{e}^+_R\,u_{iL}(\theta)\,H_i \\
& + \bar{d}_{iL}\,\nu^c_R\,H^i + \bar{\nu}^c_R\,d_{iL}\,H_i \\
& + \bar{e}^+_L\,\nu^c_R\,H^4 + \bar{\nu}^c_R\,e^+_L\,H_4 \\
& - \bar{u}_{iL}(\theta)\,d_{iR}\,H^4 - \bar{d}_{iR}\,u_{iL}(\theta)\,H_4 \\
& - (\bar{d}_{iL}\,d_{iR} + \bar{e}^+_L\,e^+_R)\,H^5 - (\bar{d}_{iR}\,d_{iL} + \bar{e}^+_R\,e^+_L)\,H_5 \}
\end{aligned} \tag{B.8}$$

and

$$\mathcal{L}_Y(h') = -h' \{ \bar{u}_{iR} e_L^+ H^i + \bar{e}_L^+ u_{iR} H_i$$

$$+ \epsilon_{ijk} \bar{u}^c_{jR}(\theta) d_{kL} H^i + \epsilon_{ijk} \bar{d}_{kL} u^c_{jR}(\theta) H_i$$

$$- \bar{u}_{iR} d_{iL} H^4 - \bar{d}_{iL} u_{iR} H_4$$

$$+ \bar{u}_{iR} u_{iL}(\theta) H^5 + \bar{u}_{iL}(\theta) u_{iR} H_5 \} \tag{B.9}$$

(In (8) and (9), whenever two indices are repeated a sum over them from 1 to 3 is implied).

APPENDIX C

THE MASS TERMS FOR THE SCALARS AND HEAVY FERMIONS.

Here we list the quadratic terms in the lagrangian obtained upon shifting the 24-plet of Higgs. As usual σ denotes the singlet in Φ^α_β, φ^i_j the traceless SU(3) octect, ω^a_b the SU(2) triplet, K^i_a the "off-diagonal" Higgs in the 24-plet. Also H^i, and H^a are the SU(3)-triplet and the SU(2)-doublet part of the SU(5) 5-plet of Higgs, and c^i_a is the massive ghost in the 't Hooft-Feynman gauge. In addition we have included the mass terms for the heavy fermions necessary for asymptotic freedom.

$$\mathcal{L}_{MASS} = -\frac{1}{2}(2\mu^2)\sigma^2 - \frac{1}{2}\left(\frac{4}{5}\frac{\lambda_2}{g^2}M^2\right)\varphi^2 - \frac{1}{2}\left(\frac{16}{5}\frac{\lambda_2}{g^2}M^2\right)\omega^2 - M^2 K^i_a K^{\dagger a}_i$$

$$-\left[-\frac{\nu^2}{2} + \frac{1}{2}\left(\lambda_4 + \frac{2}{15}\lambda_5\right)\sigma_0^2\right] H_i^\dagger H^i - \left[-\frac{\nu^2}{2} + \frac{1}{2}\left(\lambda_4 + \frac{3}{10}\lambda_5\right)\sigma_0^2\right] H^\dagger_a H^a$$

$$-M^2 c^{*i}_a c_i^a - M^2 c^a_i c^{*i}_a - M^2 X^{\dagger a}_i X^i_a - M^2 Y^{\dagger a}_i Y^i_a$$

$$-\sqrt{\frac{2}{15}}(K_5 + K_6)\bar{B}^j_i B^i_j \sigma_0 - \sqrt{\frac{3}{10}}(K_5 + K_6)\bar{B}^b_a B^a_b \sigma_0 + \frac{\sqrt{6}}{\sqrt{30}}(K_5 + K_6)\bar{B}_\sigma B^\sigma$$

$$+\left(-\sqrt{\frac{2}{15}}K_5 + \sqrt{\frac{3}{10}}K_6\right)\bar{B}^i_a B^a_i \sigma_0 + \left(\sqrt{\frac{3}{10}}K_5 - \sqrt{\frac{2}{15}}K_6\right)\bar{B}^a_i B^i_a \sigma_0$$

$$-\sqrt{\frac{2}{15}}K_2 \bar{\chi}_i \chi^i \sigma_0 + \sqrt{\frac{3}{10}}K_2 \bar{\chi}_a \chi^a \sigma_0$$

Here

$$\sigma_0^2 = \frac{\mu^2}{\lambda_1 + \frac{7}{15}\lambda_2} \quad \text{and} \quad M^2 = \frac{5}{12}\bar{g}^2\sigma_0^2$$

APPENDIX D

STABILITY OF THE HIGGS POTENTIAL.

In this appendix we study the stability of the Higgs potential for the SU(5) model of Georgi and Glashow, with a 5 and a 24-plet of scalars. The potential is given by[Ref.18]

$$V(\phi, H) = -\frac{\mu^2}{2} \operatorname{Tr}\phi^2 + \frac{1}{4}\lambda_1 (\operatorname{Tr}\phi^2)^2 + \frac{1}{2}\lambda_2 \operatorname{Tr}\phi^4 \tag{D.1}$$

$$-\frac{\nu^2}{2} H^{+}\cdot H + \frac{1}{4}\lambda_3 (H^{+}\cdot H)^2 + \frac{1}{2}\lambda_4 (H^{+}\cdot H)\operatorname{Tr}\phi^2 + \frac{1}{2}\lambda_5 H^{+}\phi^2 H$$

where ϕ and H transform as the 5 and 24 of SU(5), and both μ^2 and $\nu^2 > 0$. We have also imposed the symmetry $\phi \dashrightarrow -\phi$, so that the term $\operatorname{Tr}(\phi)^3$ is not present.

A convenient way to study this potential is to define the traceless matrix

$$\phi = \sum_{i,\alpha,\beta} (\lambda^i)^\alpha_\beta \, \phi^\beta_\alpha$$

where i runs from 1 to 24 and α and β from 1 to 5. Since ϕ is a hermitian matrix it can be diagonalized by a unitary matrix, and without any loss of generality we can take it to be in diagonal form, as (SU(5) has rank 4)

$$\phi_{diag} = \begin{bmatrix} \frac{\rho}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} + \sqrt{\frac{2}{15}}\sigma & & & & \\ & -\frac{\rho}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} + \sqrt{\frac{2}{15}}\sigma & & & \\ & & -\frac{2\eta}{\sqrt{6}} + \sqrt{\frac{2}{15}}\sigma & & \\ & & & \frac{\omega}{2} - \sqrt{\frac{3}{10}}\sigma & \\ & & & & -\frac{\omega}{2} - \sqrt{\frac{3}{10}}\sigma \end{bmatrix} \tag{D.2}$$

and then

$$\mathrm{Tr}\,\phi^2 = \rho^2 + \eta^2 + \omega^2 + \sigma^2$$

and

$$\mathrm{Tr}\,\phi^4 = \frac{1}{2}\rho^4 + \frac{1}{2}\eta^4 + \frac{1}{2}\omega^4 + \frac{7}{30}\sigma^4 + \rho^2\eta^2 + \frac{4}{5}\sigma^2(\rho^2+\eta^2) + \frac{9}{5}\omega^2\sigma^2 + \frac{4}{\sqrt{5}}\rho^2\eta\sigma - \frac{4}{3\sqrt{5}}\eta^3\sigma$$

If we also allow for the fifth component of H^α to, in principle, develop a VEV, τ , the Higgs potential can then be written as

$$\begin{aligned} V = {} & -\frac{\mu^2}{2}(\rho^2+\eta^2+\omega^2+\sigma^2) + \frac{1}{4}\lambda_1(\rho^2+\eta^2+\omega^2+\sigma^2)^2 \\ & + \frac{1}{4}\lambda_2\Big(\rho^4+\eta^4+\omega^4+\frac{7}{15}\sigma^4+2\rho^2\eta^2+\frac{8}{5}\sigma^2(\rho^2+\eta^2) \\ & \qquad + \frac{18}{5}\omega^2\sigma^2 + \frac{8}{\sqrt{5}}\rho^2\eta\sigma - \frac{8}{3\sqrt{5}}\eta^3\sigma\Big) \\ & -\frac{\nu^2}{2}\tau^2 + \frac{1}{4}\lambda_3\tau^4 + \frac{1}{2}\lambda_4\tau^2(\rho^2+\eta^2+\omega^2+\sigma^2) + \\ & + \frac{1}{2}\lambda_5\tau^2\Big(\frac{\omega^2}{2}+\sqrt{\frac{3}{5}}\,\omega\sigma+\frac{3}{10}\sigma^2\Big) \end{aligned}$$

This potential has a minimum, with the desired pattern of symmetry breaking[Refs.18,58] SU(5)$\longrightarrow$ SU(3) X SU(2) X U(1), if

$$\sigma^2 = \frac{\mu^2}{\lambda_1 + \frac{7}{15}\lambda_2} \equiv v^2 \qquad \text{(D.3)}$$

and

$$\rho = \eta = \omega = \tau = 0 \qquad \text{(D.4)}$$

with

$$\lambda_2 > 0 \tag{D.5}$$

and

$$\lambda_1 + \frac{7}{15}\lambda_2 > 0 \tag{D.6}$$

In this work we will be interested in preventing the H^α from developing a VEV (see also Appendix C) at the same time as ϕ^α_β, and therefore require that the coefficient of τ^2 and τ^4 remain with the same sign, that is

$$\begin{gathered} \lambda_3 > 0 \\ -\frac{\nu^2}{2} + \frac{1}{2}\lambda_4\sigma^2 + \frac{3}{20}\lambda_5\sigma^2 > 0 \end{gathered} \tag{D.7}$$

or

$$\lambda_4 + \frac{3}{10}\lambda_5 > \frac{\nu^2}{\mu^2}\left(\lambda_1 + \frac{7}{15}\lambda_2\right) \tag{D.8}$$

With our choice of VEV,

$$\langle\sigma\rangle = v = \frac{\mu}{\left(\lambda_1 + \frac{7}{15}\lambda_2\right)^{1/2}} \tag{D.9}$$

the X_i, and Y_i with i=1,2,3, acquire a mass given by

$$m_X^2 = m_Y^2 \equiv M^2 = \frac{5}{12} g^2 v^2 \tag{D.10}$$

with (cf. eq.(2)),

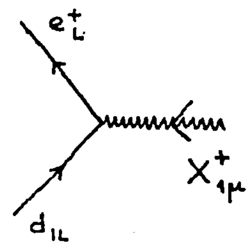
$$\langle\phi^\alpha_\beta\rangle = \begin{cases} \sqrt{\frac{2}{15}}\, v & \text{if } \alpha = \beta = \{1,2,3\} \\ -\sqrt{\frac{3}{10}}\, v & \text{if } \alpha = \beta = \{4,5\} \end{cases}$$

APPENDIX E

THE RENORMALIZATION GROUP EQUATION FOR g_X AT LOW ENERGIES

To show the general technique for the calculation of a renormalization group equation here we do the calculation for the coupling g_X at low energies, i.e. when only massless loops have to be considered.

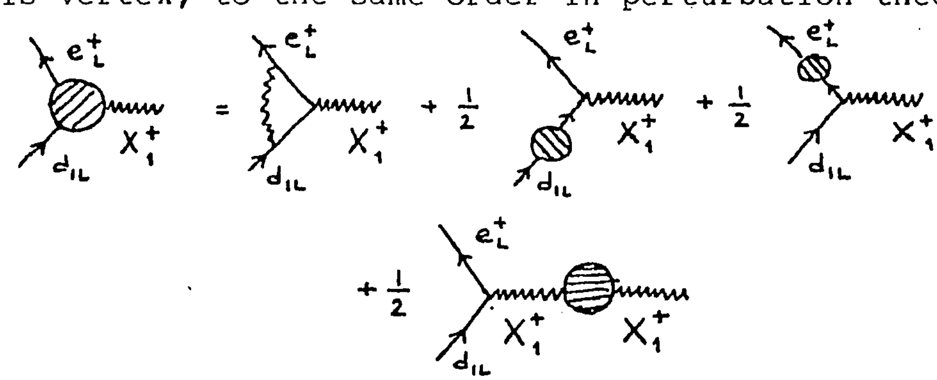
From the fermion lagrangian given in Appendix A, we see that the most convenient vertex to choose (the one that will have the least number of graphs contributuing to its renormalization) , is



given by

$$i\mathcal{L} = -g_X/\sqrt{2}\ \gamma_{\mu L}$$

In order to compute the lowest order RGE , we need to calculate the divergent parts of the the corrections to this vertex, to the same order in perturbation theory:

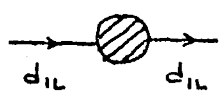


where the blobs represent the wave function renormalizations.

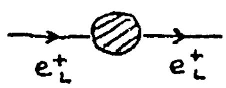
Of course this equality is the same as

$$g_{XR} = Z_X^{1/2} Z_{e_L^+}^{1/2} Z_{d_{1L}}^{1/2} Z_{Xe_L^+ d_{1L}}^{-1} g_{XB} \tag{E.1}$$

By calculating with our lagrangian and keeping only the $1/\epsilon$ part(remember that $1/\epsilon \dashrightarrow \log \Lambda/\mu$), we find

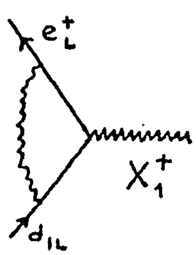


$$\dashrightarrow Z_{d_{1L}} = 1 - \frac{1}{16\pi^2}\left[\frac{8}{3} g_3^2 + \frac{3}{2} g_2^2 + \frac{1}{30} g_1^2\right] \ln\frac{\Lambda}{\mu} \tag{E.2}$$



$$\dashrightarrow Z_{e_L^+} = 1 - \frac{1}{16\pi^2}\left[\frac{6}{5} g_1^2\right] \ln\frac{\Lambda}{\mu} \tag{E.3}$$

$$\dashrightarrow Z_X = 1 - \frac{1}{16\pi^2}\left[\frac{8}{3} n_f g_X^2\right] \ln\frac{\Lambda}{\mu} \tag{E.4}$$



$$\dashrightarrow Z_{Xe_L^+ d_{1L}} = 1 - \frac{1}{16\pi^2}\left[\frac{1}{5} g_1^2 g_X\right] \ln\frac{\Lambda}{\mu} \tag{E.5}$$

So that one has, using (1) together with (2) - (5),

$$g_{xR} = g_{xB}\left\{1 + \frac{1}{16\pi^2}\left[\frac{1}{5}g_1^2 - \frac{4}{3}n_f g_x^2 - \frac{3}{5}g_1^2\right.\right.$$

$$\left.\left. - \frac{4}{3}g_3^2 - \frac{3}{4}g_2^2 - \frac{1}{60}g_1^2\right]\log\frac{\Lambda}{\mu}\right\}$$

and

$$16\pi^2\frac{dg_x}{dt} = g_1^2\left(-\frac{1}{5} + \frac{3}{5} + \frac{1}{60}\right)g_x + g_2^2\left(\frac{3}{4}\right)g_x + g_3^2\left(\frac{4}{3}\right)g_x + \frac{4}{3}n_f g_x^3 =$$

$$= \frac{5}{12}g_1^2 g_x + \frac{3}{4}g_2^2 g_x + \frac{4}{3}g_3^2 g_x + \frac{4}{3}n_f g_x^3 \qquad \text{(E.6)}$$

which is the RGE for the region where the SU(3) X SU(2) X U(1) symmetry is valid.

When the SU(2) X U(1) is broken down to $U(1)_{em}$ the equation is modified (see remark after eq.(4.7) in that the $\frac{3}{4}g_2^2 g_x$ term is reduced to $\frac{1}{4}g_2^2 g_x$, and therefore the equation is, in those regions where the symmetry SU(3) X U(1) applies

$$16\pi^2\frac{dg_x}{dt} = \frac{5}{12}g_1^2 g_x + \frac{1}{4}g_2^2 g_x + \frac{4}{3}g_3^2 g_x + \frac{4}{3}n_f g_x^3 \qquad \text{(E.7)}$$

APPENDIX F

THE RGE's FOR THE HIGGS SELF-COUPLINGS AND THE YUKAWAS.

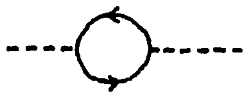
As exemplified in Appendix E in order to calculate the RGE of a coupling constant, all one needs is to calculate the divergent part of the graphs contributing to the renormalization of that coupling constant. The calculation performed there, was in the 't Hooft-Feynman gauge, but in general it is more convenient to do the calculation in the Landau gauge where a large number of graphs are finite and therefore do not contribute to the RGE; we shall do this here.

For completeness, we also list here the value of the basic integrals, i.e. their divergent parts in the Landau gauge, associated with each of the graphhs that contribute to the RGE's that we are going to calculate; all one needs to do is to put in the group theoretical coefficients that enter for each particular combination. (This is an easy but tedious task and we have just given the final answers in each case). We also list the different wave function renormalization constants.

BASIC GRAPHS (in Landau gauge).

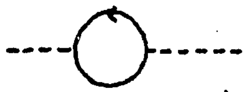
$$= -i \frac{1}{16\pi^2} \log \frac{\Lambda}{\mu} \tag{F.1}$$

$$= -i \frac{1}{16\pi^2} \log \frac{\Lambda}{\mu} \tag{F.2}$$



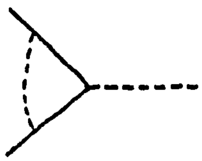
LIGHT FERMIONS

$$= +i\,\frac{1}{16\pi^2}\,2n_f \log \Lambda/\mu \qquad \text{(F.3)}$$

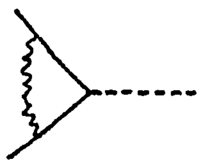


HEAVY FERMIONS

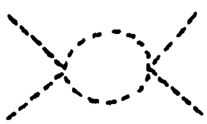
$$= +i\,\frac{1}{16\pi^2}\,4n_F \log \Lambda/\mu \qquad \text{(F.4)}$$



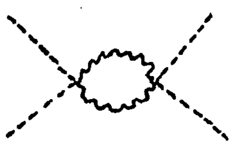
$$= -\frac{1}{16\pi^2}\,(2) \log \Lambda/\mu \qquad \text{(F.5)}$$



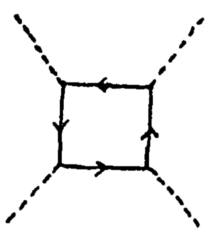
$$= +\frac{1}{16\pi^2}\,(6) \log \Lambda/\mu \qquad \text{(F.6)}$$



$$= -\frac{1}{16\pi^2}\,(2) \log \Lambda/\mu \qquad \text{(F.7)}$$

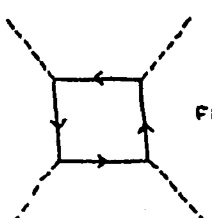


$$= -\frac{1}{16\pi^2}\,(6) \log \Lambda/\mu \qquad \text{(F.8)}$$



LIGHT FERMIONS

$$= +\frac{1}{16\pi^2}\,4n_f \log \Lambda/\mu \qquad \text{(F.9)}$$



HEAVY FERMIONS

$$= +\frac{1}{16\pi^2}\,8n_F \log \Lambda/\mu \qquad \text{(F.10)}$$

The wave function renormalization constants are:

$$Z(\psi_R) = 1 - \frac{1}{16\pi^2}\, 4h^2 \log\frac{\Lambda}{\mu} \qquad \text{(F.11)}$$

$$Z(\psi_L) = 1 - \frac{1}{16\pi^2}\,(2h^2 + 3h'^2) \log\frac{\Lambda}{\mu} \qquad \text{(F.12)}$$

$$Z(\chi) = 1 - \frac{1}{16\pi^2}\,\frac{24}{5}\,(k_2^2 + k_4^2) \log\frac{\Lambda}{\mu} \qquad \text{(F.13)}$$

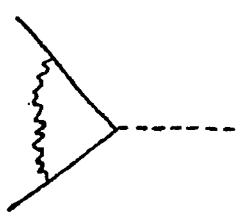
$$Z(B) = 1 - \frac{1}{16\pi^2}\left[k_4^2 + \frac{23}{5}(k_5^2 + k_6^2) - \frac{4}{5}k_5k_6\right] \log\frac{\Lambda}{\mu} \qquad \text{(F.14)}$$

$$Z(\phi) = 1 - \frac{1}{16\pi^2}\left[-30g^2 + 4k_2^2 n_F + \frac{92}{5}(k_5^2 + k_6^2)n_F - \frac{16}{5}k_5k_6 n_F\right] \log\frac{\Lambda}{\mu} \qquad \text{(F.15)}$$

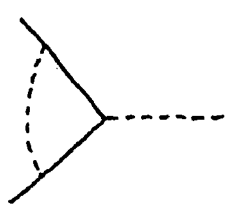
$$Z(H) = 1 - \frac{1}{16\pi^2}\left[-\frac{72}{5}g^2 + 8h^2 n_f + 6h'^2 n_f + \frac{96}{5}n_F k_4^2\right] \log\frac{\Lambda}{\mu} \qquad \text{(F.16)}$$

Next we give the contribution to the RGE's of each of the graphs.

<u>CONTRIBUTION TO</u> $16\pi^2 \frac{dh}{dt}$.

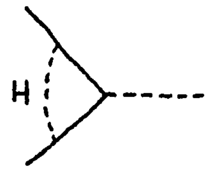


$$-\frac{54}{5}g^2 h \qquad \text{(F.17)}$$

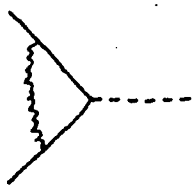


$$-6h'^2 h \qquad \text{(F.18)}$$

CONTRIBUTION TO $16\pi^2 \frac{dh'}{dt}$.

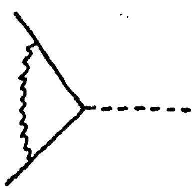


$$-8h'h^2 \tag{F.19}$$

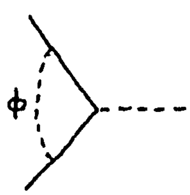


$$-\frac{72}{5}h'g^2 \tag{F.20}$$

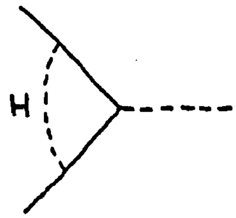
CONTRIBUTION TO $16\pi^2 \frac{dk_2}{dt}$.



$$+\frac{3}{5}k_2 g^2 \tag{F.21}$$

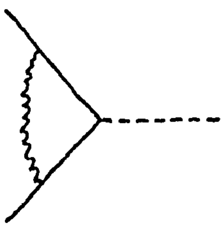


$$-\frac{2}{5}k_2^3 \tag{F.22}$$

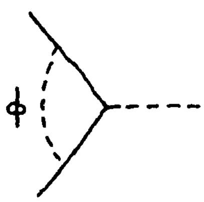


$$-\frac{4}{5}k_4^2 k_5 + \frac{46}{5}k_4^2 k_6 \tag{F.23}$$

<u>CONTRIBUTION TO</u> $16\pi^2 \frac{dk_4}{dt}$

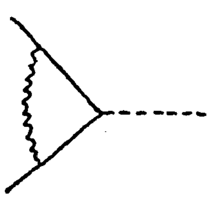


$$-15\,k_4 g^2 \tag{F.24}$$

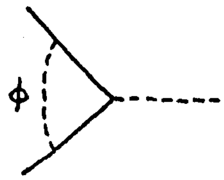


$$-\frac{4}{5}k_2 k_4 k_5 + \frac{46}{5}k_2 k_4 k_6 \tag{F.25}$$

<u>CONTRIBUTION TO</u> $16\pi^2 \frac{dk_5}{dt}$

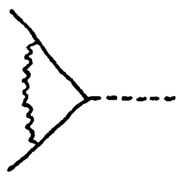


$$-15\,k_5 g^2 \tag{F.26}$$

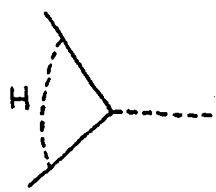


$$-\frac{2}{5}k_5^3 - \frac{8}{5}k_5^2 k_6 + 8k_5 k_6^2 - \frac{4}{5}k_6^3 \qquad \text{(F.27)}$$

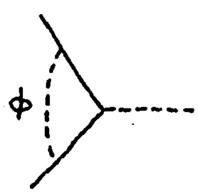
CONTRIBUTION TO $16\pi^2 \frac{dk_6}{dt}$



$$-15k_6 g^2 \qquad \text{(F.28)}$$

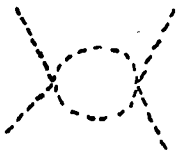


$$+2k_4^2 k_6 \qquad \text{(F.29)}$$



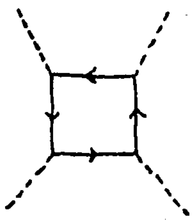
$$-\frac{4}{5}k_5^3 + \frac{42}{5}k_5^2 k_6 - \frac{6}{5}k_5 k_6^2 - \frac{2}{5}k_6^3 \qquad \text{(F.30)}$$

CONTRIBUTION TO $16\pi^2 \frac{d\lambda_1}{dt}$



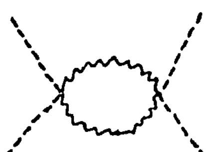
+ PERMUTAT.

$$64\lambda_1^2 + \frac{376}{5}\lambda_1\lambda_2 + \frac{672}{25}\lambda_2^2 + 5\lambda_4^2 + 2\lambda_4\lambda_5 \qquad \text{(F.31)}$$



+ PERMUTAT.

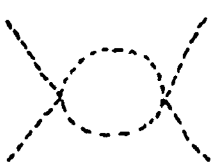
$$-\frac{16}{25} n_F \kappa_5^4 - \frac{64}{25} n_F \kappa_5^3 \kappa_6 - \frac{1296}{25} n_F \kappa_5^2 \kappa_6^2 - \frac{64}{25} n_F \kappa_5 \kappa_6^3 - \frac{16}{25} n_F \kappa_6^4 \qquad \text{(F.32)}$$



+ PERMUTAT.

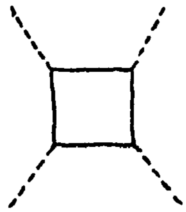
$$+9g^4 \qquad \text{(F.33)}$$

CONTRIBUTION TO $16\pi^2 \frac{d\lambda_2}{dt}$



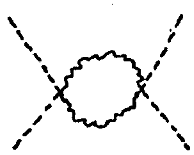
+ PERMUTAT.

$$24\lambda_1\lambda_2 + \frac{1}{2}\lambda_5^2 + \frac{128}{5}\lambda_2^2 \qquad \text{(F.34)}$$



+ PERMUTAT.

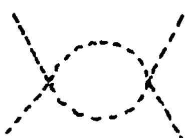
$$-\frac{84}{5}k_5^4 n_F + \frac{64}{5}k_5^3 k_6 n_F + \frac{96}{5} n_F k_5^2 k_6^2 + \frac{64}{5} n_F k_5 k_6^3 - \frac{84}{5} k_6^4 n_F - 4 k_2^4 n_F \qquad (F.35)$$



+ PERMUTAT.

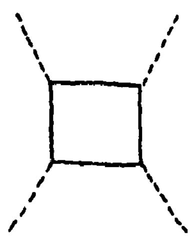
$$+\frac{15}{2} g^4 \qquad (F.36)$$

CONTRIBUTION TO $16\pi^2 \frac{d\lambda_3}{dt}$



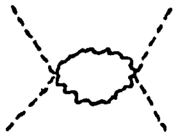
+ PERMUTAT.

$$9\lambda_3^2 + 48\lambda_4^2 + \frac{96}{5}\lambda_4\lambda_5 + \frac{132}{25}\lambda_5^2 \qquad (F.37)$$



+ PERMUTAT.

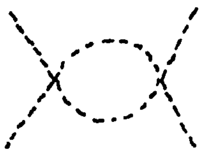
$$-32 n_f h^4 - 24 h' n_f - \frac{1856}{25} n_F k_4^4 \qquad (F.38)$$



+ PERMUTAT.

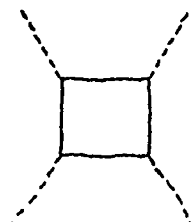
$$+\frac{396}{25}g^4 \qquad \text{(F.39)}$$

CONTRIBUTION TO $16\pi^2 \frac{d\lambda_4}{dt}$



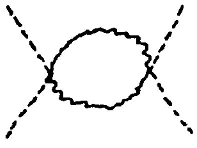
+ PERMUTAT.

$$4\lambda_4^2 + \lambda_5^2 + 6\lambda_3\lambda_4 + \lambda_3\lambda_5 + $$
$$+ 52\lambda_1\lambda_4 + \frac{188}{5}\lambda_2\lambda_4 + $$
$$+ \frac{48}{5}\lambda_1\lambda_5 + \frac{112}{25}\lambda_2\lambda_5 \qquad \text{(F.40)}$$



+ PERMUTAT.

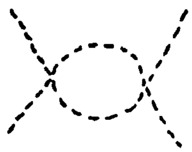
$$-16n_F k_4^2 k_2^2 - 16 n_F k_4^2 k_2 k_6 \qquad \text{(F.41)}$$
$$-\frac{16}{25}n_F k_4^2 k_5^2 - \frac{416}{25}n_F k_4^2 k_6^2$$
$$-\frac{32}{25}n_F k_4^2 k_5 k_6$$



+ PERMUTAT.

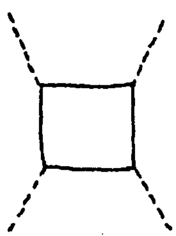
$$+3g^4 \qquad \text{(F.42)}$$

CONTRIBUTION TO $16\pi^2 \frac{d\lambda_5}{dt}$



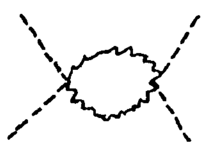
+ PERMUTAT.

$$\lambda_3\lambda_5 + 4\lambda_1\lambda_5 + 8\lambda_4\lambda_5 + \tag{F.43}$$
$$+ \frac{76}{5}\lambda_2\lambda_5 + \frac{21}{5}\lambda_5^2$$



+ PERMUTAT.

$$\frac{16}{5} n_F k_4^2 k_2^2 + \frac{32}{5} n_F k_4^2 k_2 k_5 +$$
$$+ \frac{32}{5} n_F k_4^2 k_2 k_6 + \frac{96}{5} n_F k_4^2 k_5 k_6 - \tag{F.44}$$
$$- \frac{352}{5} n_F k_4^2 k_5^2 + \frac{48}{5} n_F k_4^2 k_6^2$$



+ PERMUTAT.

$$+ 15 g^4 \tag{F.45}$$

To finally obtain the RGE's, we must add the wave function renormalization for each of the external legs. The algorithm for this is to <u>add minus a half of the wave function renormalization constant for each external leg to the RHS of</u> $16\pi^2 \frac{d\lambda}{dt}$. (This is because for each external leg, we must multiply λ_B by a factor $Z^{1/2}$(external leg), and in order to obtain the contribution to the RGE one must take $+\frac{\partial}{\partial \log\mu}$; the algorithm follows at once).

The resulting renormalization group equations for our model are:

$$16\pi^2 \, d\bar{g}/dt = -\left(\frac{55}{3} - 1 - \frac{22}{3} n_F - \frac{4}{3} n_f\right) \tag{F.46}$$

$$16\pi^2 \frac{dh}{dt} = h\left[\frac{48}{5} n_F k_4^2 + 3n_f h'^2 - \frac{9}{2} h'^2 - 18 g^2\right] + h^3 (3 + 4n_f) \tag{F.47}$$

$$16\pi^2 \frac{dh'}{dt} = h'\left[(4n_f - 6)h^2 + \frac{48}{5} n_F k_4^2 - \frac{108}{5} g^2\right] + h'^3 (3 + 3n_f) \tag{F.48}$$

$$16\pi^2 \frac{dk_2}{dt} = k_2\left[\frac{24}{5} k_4^2 + \frac{46}{5} n_F (k_5^2 + k_6^2) - \frac{8}{5} n_F k_5 k_6 - \frac{72}{5} g^2\right] + k_4^2\left[-\frac{4}{5} k_5 + \frac{46}{5} k_6\right] + k_2^3\left[\frac{22}{5} + 2n_F\right] \tag{F.49}$$

$$16\pi^2 \frac{dk_4}{dt} = k_4\left[4n_f h^2 + 3n_f h'^2 - \frac{4}{5} k_2 k_5 + \frac{46}{5} k_2 k_6 + + \frac{12}{5} k_2^2 + \frac{23}{10}(k_5^2 + k_6^2) - \frac{2}{5} k_5 k_6 - \frac{111}{5} g^2\right] + k_4^3\left(\frac{29}{10} + \frac{48}{5} n_F\right) \tag{F.50}$$

$$16\pi^2 \frac{dk_5}{dt} = k_5\left[2n_F k_2^2 + \frac{46}{5} n_F k_6^2 + \frac{63}{5} k_6^2 + k_4^2 - 30 g^2\right] + k_5^2\left[-\frac{12}{5} k_6 - \frac{8}{5} n_F k_6\right] + k_5^3\left[\frac{21}{5} + \frac{46}{5} n_F\right] - \frac{4}{5} k_6^3 \tag{F.51}$$

$$16\pi^2 \frac{dk_6}{dt} = k_6\left[2n_F k_2^2 + \frac{46}{5} n_F k_5^2 + 13 k_5^2 + k_4^2 - 30 g^2\right] + k_6^2\left[-2k_5 - \frac{8}{5} n_F k_5\right] + 2k_4^2 k_2 - \frac{4}{5} k_5^3 + k_6^3\left[\frac{21}{5} + \frac{46}{5} n_F\right] \tag{F.52}$$

$$
\begin{aligned}
16\pi^2 \frac{d\lambda_1}{dt} = {} & 64\lambda_1^2 + \lambda_1\Big[\frac{376}{5}\lambda_2 + 8n_F k_2^2 + \frac{184}{5} n_F (k_5^2 + k_6^2) - \\
& \qquad - \frac{32}{5} n_F k_5 k_6 - 60\bar{g}^2\Big] \\
& + \frac{672}{25}\lambda_2^2 + 5\lambda_4^2 + 2\lambda_4\lambda_5 + 9g^4 - \frac{16}{25} n_F k_5^4 - \frac{64}{25} n_F k_5^3 k_6 \\
& - \frac{1296}{25} n_F k_5^2 k_6^2 - \frac{64}{25} n_F k_5 k_6^3 - \frac{16}{25} n_F k_6^4
\end{aligned}
\tag{F.53}
$$

$$
\begin{aligned}
16\pi^2 \frac{d\lambda_2}{dt} = {} & \frac{128}{5}\lambda_2^2 + \lambda_2\Big[24\lambda_1 - 60g^2 + 8n_F k_2^2 + \frac{184}{5} n_F (k_5^2 + k_6^2) - \\
& \qquad - \frac{32}{5} n_F k_5 k_6\Big] \\
& + \frac{1}{2}\lambda_5^2 + \frac{15}{2} g^4 - 4n_F k_2^4 - \frac{84}{5} n_F k_5^4 + \frac{64}{5} n_F k_5^3 k_6 \\
& + \frac{96}{5} n_F k_5^2 k_6^2 + \frac{64}{5} n_F k_5 k_6^3 - \frac{84}{5} n_F k_6^4
\end{aligned}
\tag{F.54}
$$

$$
\begin{aligned}
16\pi^2 \frac{d\lambda_3}{dt} = {} & 9\lambda_3^2 + \lambda_3\Big[-\frac{144}{5} g^2 + 16n_f h^2 + 12 n_f h'^2 + \frac{192}{5} n_F k_4^2\Big] \\
& + 48\lambda_4^2 + \frac{96}{5}\lambda_4\lambda_5 + \frac{132}{25}\lambda_5^2 + \frac{396}{25} g^4 - \frac{1856}{25} n_F k_4^4 \\
& - 32 n_f h^4 - 24 n_f h'^4
\end{aligned}
\tag{F.55}
$$

$$
\begin{aligned}
16\pi^2 \frac{d\lambda_4}{dt} = {} & 4\lambda_4^2 + \lambda_4\Big[6\lambda_3 + 52\lambda_1 + \frac{188}{5}\lambda_2 - \frac{222}{5} g^2 + \\
& \qquad + 4n_F k_2^2 + \frac{92}{5} n_F (k_5^2 + k_6^2) + \frac{96}{5} n_F k_4^2 \\
& \qquad - \frac{16}{5} n_F k_5 k_6 + 8 n_f h^2 + 6 n_f h'^2\Big] \\
& + \lambda_3\lambda_5 + \lambda_5^2 + \frac{48}{5}\lambda_1\lambda_5 + \frac{112}{25}\lambda_2\lambda_5 + 3g^4 \\
& - 16 n_F k_4^2 k_2^2 - 16 n_F k_4^2 k_2 k_6 \\
& - \frac{16}{25} n_F k_4^2 k_5^2 - \frac{416}{25} n_F k_4^2 k_6^2 \\
& - \frac{32}{25} k_4^2 k_5 k_6 n_F
\end{aligned}
\tag{F.56}
$$

and finally,

$$16\pi^2 \frac{d\lambda_5}{dt} = \frac{21}{5}\lambda_5^2 + \lambda_5 \Big[\lambda_3 + 4\lambda_1 + \frac{76}{5}\lambda_2 + 8\lambda_4 - \frac{222}{5} g^2 + + 8 n_f h^2 + 6 n_f h'^2 + 4 n_F k_2^2 + + \frac{92}{5} n_F (k_5^2 + k_6^2) - \frac{16}{5} n_F k_5 k_6 + \frac{96}{5} n_F k_4^2 \Big] + 15 g^4 + \frac{16}{5} n_F k_2^2 k_4^2 + \frac{32}{5} n_F k_4^2 k_2 k_5 + \frac{32}{5} n_F k_4^2 k_2 k_6 - \frac{352}{5} n_F k_4^2 k_5^2 + \frac{96}{5} n_F k_4^2 k_5 k_6 + \frac{48}{5} n_F k_4^2 k_6^2 \tag{F.57}$$

(To obtain the RGE's for the λ's it is very convenient to work in the general tensorial SU(N) basis, (this is how these equations were obtained), where the symmetry properties are explicit and an enormous simplification in the Wick's expansions take place. For a field in the adjoint representation, the group theoretical weight associated with its propagator is given as follows:

$$\sum_{i=1}^{N} \langle \phi_k^{\ i} \phi_i^{\ k} \rangle = \frac{N^2-1}{N} \qquad (k \text{, fixed}) \tag{F.58a}$$

$$\langle \phi_k^{\ k} \phi_j^{\ j} \rangle = -\frac{1}{N} \tag{F.58b}$$

$$\langle \phi_k^{\ k} \phi_k^{\ k} \rangle = \frac{N-1}{N} \tag{F.58c}$$

$$\langle \phi_i^{\ j} \phi_j^{\ i} \rangle = 1 \tag{F.58d}$$

in the above expressions, repeated indices are not summed, unless explictly shown.)

APPENDIX G

MASS RENORMALIZATION.

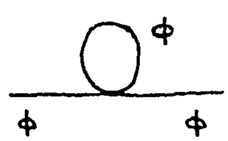
In this appendix we study the mass renormalization of M_X Ref.23. We will need the RGE for the quantity $\tilde{M}^2$ related to the effective mass $\bar{M}$ by $\bar{M}(t)=e^{-t}\tilde{M}(t)$.

The best way to calculate the RGE for M_X is to take advantage of its definition

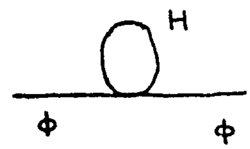
$$M_X^2 = \frac{5}{12}\bar{g}^2 v^2 = \frac{5}{12}\bar{g}^2 \frac{\mu^2}{\lambda_1 + \frac{7}{15}\lambda_2} \tag{G.1}$$

where μ^2 is the mass of the 24-plet of Higgs in the symmetric SU(5) limit, and observe that if one is dealing with a theory with eigenvalues, then M^2 is simply proportional to μ^2, and it is then sufficient to calculate the RGE for μ^2 in the symmetric, unshifted theory. This is much simpler, even though, due to the presence of λ_4 and λ_5 in the lagrangian, it involves the calculation of the RGE for ν^2 (the mass of the 5-plet of Higgs in the symmetric limit) as well. We list the contribution of the different graphs, in the symmetric unshifted theory, to the RGE's for μ^2 and ν^2.

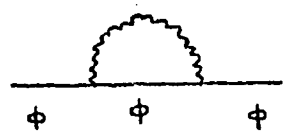
Contribution to $16\pi^2 \frac{d\mu^2}{dt}$



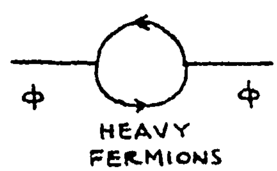
$$+52\,\mu^2\lambda_1 + \frac{188}{5}\mu^2\lambda_2$$



$$+5\lambda_4 \nu^2 + \lambda_5 \nu^2$$

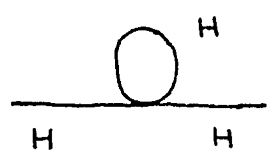


$$-30\bar{g}^2\mu^2$$

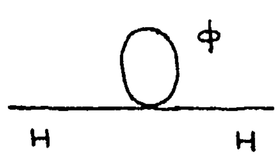


$$+\mu^2\left[\frac{92}{5}(k_5^2 + k_6^2) - \frac{16}{5}k_5 k_6 - 4k_2^2\right].$$

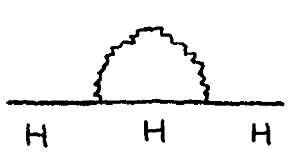
Contribution to $16\pi^2 \frac{d\nu^2}{dt}$



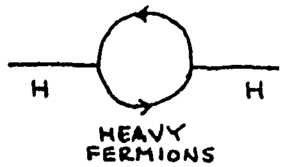
$$+6\nu^2\lambda_3$$



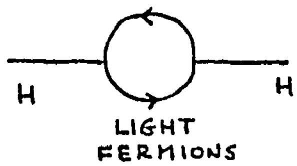
$$+48\mu^2\lambda_4 + \frac{48}{5}\mu^2\lambda_5$$



$$-\frac{72}{5}\bar{g}^2\nu^2$$



$$+ \frac{96}{5} \nu^2 k_4^2$$



$$+ (8h^2 + 6h'^2) n_f \nu^2$$

The RGE's for μ^2 and ν^2, are

$$16\pi^2 \frac{d\mu^2}{dt} = -\mu^2 \left[30\bar{g}^2 - 4k_2^2 - \frac{92}{5}(k_5^2 + k_6^2) + \frac{16}{5} k_5 k_6 - 52\lambda_1 - \frac{188}{5}\lambda_2 \right] + \nu^2 (5\lambda_4 + \lambda_5) \tag{G.2a}$$

$$16\pi^2 \frac{d\nu^2}{dt} = \frac{48}{5} \mu^2 (5\lambda_4 + \lambda_5) - \nu^2 \left[\frac{72}{5} \bar{g}^2 - \frac{96}{5} k_4^2 - 6\lambda_3 - 2n_f (4h^2 + 3h'^2) \right] \tag{G.2b}$$

In the case where $n_f=3$, upon inserting the eigenvalues, equations (2a) and (2b) can be written as

$$16\pi^2 \frac{d\mu^2}{dt} = 13.419956\, \bar{g}^2 \mu^2 + 0.847298\, \bar{g}^2 \nu^2 \tag{G.3a}$$

$$16\pi^2 \frac{d\nu^2}{dt} = 8.134063\, \bar{g}^2 \mu^2 + 21.406988\, \bar{g}^2 \nu^2 \tag{G.3b}$$

and the mass M^2 is given by

$$M^2 = 1.716121\,\mu^2 \tag{G.4}$$

so that by substituting (4) into (3a) we get

$$16\pi^2 \frac{d\tilde{M}^2}{dt} = 23.030268\,\bar{g}^2\mu^2 + 1.454066\,\bar{g}^2\nu^2 \tag{G.5}$$

We can solve the system of equations (3a-b), by introducing a new variable τ, defined by

$$\tau \equiv \frac{1}{16\pi^2}\int_0^t dt'\,\bar{g}^2(t') \quad ; \quad \frac{d\bar{g}^2}{dt} = -\frac{1}{16\pi^2}\,b\,\bar{g}^4 \tag{G.6}$$

and its solution is then given by

$$\mu^2(\tau) = a_+ e^{\lambda_+\tau} + a_- e^{\lambda_-\tau} \tag{G.7a}$$

$$\nu^2(\tau) = 10.353678\,a_+ e^{\lambda_+\tau} - 0.927207\,a_- e^{\lambda_-\tau} \tag{G.7b}$$

where a_+ and a_- are two arbitrary constants and λ_+ and λ_- are the eigenvalues of the matrix associated with system (3),

$$\lambda_+ = 22.192609 \quad , \quad \lambda_- = 12.634335 \tag{G.8}$$

Clearly one wants $\mu^2 > 0$ for $t \longrightarrow +\infty$ (because this guarantees the Higgs mechanism to break the symmetry from SU(5) down to SU(3) X SU(2) X U(1)). Suppose then, that we chose $a_+=0$, then for $t \to +\infty$, μ^2 and ν^2 become proportional ($\lambda_+ > \lambda_-$)

$$\nu^2/\mu^2 \xrightarrow{t\to+\infty} 10.353678 \tag{G.9}$$

but the masses of the parts of the Higgs 5-plet transforming as an SU(3) triplet, H^i, and an SU(2) doublet, H^a, are respectively given by

$$m^2(H^i) = -\frac{\nu^2}{2} + \frac{\mu^2}{2}\,\frac{\lambda_4 + \frac{2}{15}\lambda_5}{\lambda_1 + \frac{7}{15}\lambda_2} = -\frac{\nu^2}{2} + \frac{0.448312}{2}\mu^2 \tag{G.10a}$$

and

$$m^2(H^a) = -\frac{\nu^2}{2} + \frac{\mu^2}{2}\,\frac{\lambda_4 + \frac{3}{10}\lambda_5}{\lambda_1 + \frac{7}{15}\lambda_2} = -\frac{\nu^2}{2} + \frac{1.072410}{2}\mu^2 \tag{G.10b}$$

and then, in the $t \to +\infty$ limit, both of them would become negative and hence spontaneously break both SU(3) and SU(2).

If instead we choose $a_+=0$, we have that μ^2 and ν^2 are proportional for all t,

$$\nu^2 = -0.927207\,\mu^2 \tag{G.11}$$

and $m^2(H^i)$, $m^2(H^a)$ are positive and preserve the stability of the original vacuum at high energies.

With (11) substituted in (5) we finally get the RGE for M^2, viz.,

$$16\pi^2 \frac{d\tilde{M}^2}{dt} = 12.634335\, \tilde{g}^2 \tilde{M}^2 \tag{G.12}$$

The solution to this equation is given by

$$\tilde{M}^2(t) = \tilde{M}^2(0)\left[\frac{g^2(0)}{\bar{g}^2(t)}\right]^{\alpha} \qquad (G.13)$$

where

$$\alpha = 12.634335/\bar{b}$$

with $\bar{b}$ defined in (6) and which in our theory, with $n_f=3$, is 12.

If we recall that $M(0)=M_x$, we finally have

$$\tilde{M}(t) = M_x\left[\frac{\bar{g}(0)}{\bar{g}(t)}\right]^{1.052861} \qquad (G.14)$$

APPENDIX H

THE LOW ENERGY RENORMALIZATION GROUP EQUATION FOR THE EFFECTIVE MASS.

Here we study the RGE for M_X^2 in the low energy limit. According to our renormalization program, described in Chapter 2, one has

$$\Gamma_u^{(2)}(X|p, g_B, M_B, \alpha_B) = \hat{Z}_3^{-1}\, \Gamma_r^{(2)}(X|p, g_r, M_r, \alpha_r, \mu) \qquad \text{(H.1)}$$

where $\Gamma^{(2)}$ is the inverse two-point function for the X gauge-boson and $\hat{Z}_3$ is the pole part of the wave function renormalization constant. (As pointed out before we will always mean by $1/\epsilon$, the quantity

$$1/\epsilon \quad +\frac{1}{2}(-\gamma \quad + \log(4\pi))$$

with γ the Euler-Mascheroni constant; i.e. we use the so-called "truncated minimal scheme" Ref.46).

By explicit calculation, one finds

$$\Gamma_u^{(2)}(X|p,g,M,\alpha) = -i(p^2+M^2)\delta_{\mu\nu} + i p_\mu p_\nu (1-\alpha^{-1}) \qquad \text{(H.2)}$$

$$-i\frac{B}{\epsilon}(p^2\delta_{\mu\nu} - p_\mu p_\nu) + i\frac{1}{2}\left(B_1 \log\frac{M^2}{\mu^2} + B_2 \log\frac{p^2}{\mu^2}\right)(p^2\delta_{\mu\nu} - p_\mu p_\nu)$$

$$+\left(-i\frac{A}{\epsilon}M^2 + i\frac{A}{2}M^2 \log\frac{M^2}{\mu^2}\right)\delta_{\mu\nu}$$

$$-icp^2\delta_{\mu\nu} - i d\, p_\mu p_\nu$$

In this expression the log M^2/μ^2 terms come from massive one-loop graphs; furthermore, since all the mass renormalization graphs contributing to AM^2/ϵ involve a massive loop, the coefficient of $M^2 \log M^2/\mu^2$ is fixed as shown, if one remembers the relation between the $1/\epsilon$ and the $\log M^2/\mu^2$ terms[Ref.23] for such graphs. Wave function renormalization, on the other hand, involves both massive, B_1, and massless, B_2, loops. The finite (not log) wave function renormalization effects that arise from massive self-energy graphs are denoted by c and d.

By writing

$$\hat{Z}_3 = 1 - \frac{B_1 + B_2}{\epsilon} \tag{H.3a}$$

$$M_r^2 = M^2\left(1 + \frac{A - B_1 - B_2}{\epsilon}\right) \tag{H.3b}$$

$$\alpha_r = \hat{Z}_3 \, \alpha \tag{H.3c}$$

and using minimal renormalization, one gets, to this order,

$$\begin{aligned} \Gamma_r^{(2)}(X|p, g_r, M_r, \alpha_r, \mu) = -i\delta_{\mu\nu}\Big[& p^2 + M_r^2 - \frac{A}{2} M^2 \log\frac{M^2}{\mu^2} - \\ & - \frac{B_1}{2} p^2 \log\frac{M^2}{\mu^2} - \frac{B_2}{2} p^2 \log\frac{p^2}{\mu^2} + c p^2 \Big] + \\ & + p_\mu p_\nu \text{-TERMS} \end{aligned} \tag{H.4}$$

From the definitions of γ_M, M_r and the relation between $1/\epsilon$ and $\log \Lambda/\mu$, one has that $\gamma_M = A - B_1 - B_2$.

Multiplying by the finite external wave function renormalization constants, (i.e. removing the B_2 term from the above expression for $\Gamma_r^{(2)}(X)$ and setting $p^2 = -\bar{M}^2$) we have

$$\Gamma_R^{(2)}(X \mid p, \bar{g}, \bar{M}, \bar{\alpha}, \rho, \mu) =$$

$$= -i\delta_{\mu\nu}\left[p^2 + \bar{M}^2 - \frac{A-B_1}{2}\bar{M}^2 \log\frac{\bar{M}^2}{\mu^2} - c\bar{M}^2\right] + \tag{H.5}$$

$$+ p_\mu p_\nu - \text{TERMS}$$

Here we have introduced $\bar{M}$ (the running mass), defined by the equations

$$\frac{d\bar{M}}{dt} = -(1+\gamma_M)\bar{M} \quad , \quad \bar{M}(t=0) \equiv M_r \tag{H.6}$$

Then one writes

$$\Gamma_R^{(2)}(X \mid \mathring{\rho}, \bar{g}, \bar{M}, \bar{\alpha}, \mathring{\rho}, \mathring{\rho}) =$$

$$= -i\delta_{\mu\nu}\left[(\mathring{\rho}^2 + \bar{M}^2) - \frac{A-B_1}{2}\bar{M}^2 \log\frac{\bar{M}^2}{\mathring{\rho}^2} - c\bar{M}^2\right] +$$

$$+ p_\mu p_\nu - \text{TERMS} \tag{H.7}$$

and therefore

$$\Gamma_R^{(2)}(X \mid \mathring{\rho}e^t, \bar{g}, \bar{M}, \mathring{\rho}e^t, \mathring{\rho}) =$$

$$= -i\delta_{\mu\nu}\left[(\mathring{\rho}e^{2t} + \tilde{M}^2) - \frac{A-B_1}{2}\tilde{M}^2 \log\frac{\bar{M}^2}{\mu^2} - c\tilde{M}^2\right] + \tag{H.8}$$

$$+ p_\mu p_\nu - \text{TERMS}$$

where $\tilde{M}(t) = e^{+t}\bar{M}(t)$, and hence satisfies

$$\frac{d\tilde{M}}{dt} = -\gamma_M \tilde{M} \tag{H.9}$$

By taking the low energy limit ($t \to -\infty$) of (8), ($p^0 = \mu = M_r$) we get

$$\tilde{M}^2 - \frac{A-B_1}{2} \tilde{M}^2 \log \frac{\tilde{M}^2}{\mu^2} - c\, \tilde{M}^2 \equiv m^2 \tag{H.10}$$

defining the low energy limit of the effective mass, m^2.

The RGE satisfied by m^2 is then

$$\begin{aligned} \frac{d}{dt} m^2 &= \frac{d\tilde{M}^2}{dt} + (A - B_1)\tilde{M}^2 + O(\bar{g}^4 \tilde{M}^2) = \qquad \text{(H.11)} \\ &= -(A - B_1 - B_2)\tilde{M}^2 + (A - B_1)\tilde{M}^2 + O(\bar{g}^4 \tilde{M}^2) \\ &= + B_2 \tilde{M}^2 + O(\bar{g}^4 \tilde{M}^2) \equiv B_2\, m^2 \end{aligned}$$

to this order and in this limit.

Now, since B_2 is the contribution from massless loops, and the only one present that can couple to X, is the light fermion loop ($\underline{5}$ and $\underline{10}$), we finally have

$$16\pi^2 \frac{d\, m^2}{dt} = + \frac{8}{3} n_f\, g_x^2\, m^2 \tag{H.12}$$

where g_x is the coupling of the X-gauge-boson to the fermions.

APPENDIX I

THE VALUE OF α_s(6 GeV).

The value of α_s used as input in the M_x formula was obtained from the two loop RGE for α_s in the $\overline{MS}$ (truncated minimal) scheme[Ref.61],

$$\frac{1}{\alpha_s(Q^2)} = \frac{\beta_0 \log(Q^2/\Lambda^2_{\overline{MS}})}{4\pi} + \frac{\beta_1}{4\pi\beta_0} \log\log(Q^2/\Lambda^2_{\overline{MS}})$$

where

$$\beta_0 = 11 - \frac{4}{3} n_f \quad ; \quad \beta_1 = 102 - \frac{76}{3} n_f$$

which for $n_f=2$ and $Q^2 \ll 4m^2_{bottom}$ ($m_b \simeq 4.5$ GeV), gives

$$\frac{1}{\alpha_s(Q^2)} = \frac{25}{12\pi} \log\left(Q^2/\Lambda^2_{\overline{MS}}\right) + \frac{154}{100\pi} \log\log\left(Q^2/\Lambda^2_{\overline{MS}}\right).$$

The result for selected values of $\Lambda_{\overline{MS}}$ is

$\Lambda_{\overline{MS}}$ (GeV)	α_s
.05	.134
.1	.155
.2	.183
.3	.206
.4	.226
.5	.245
.6	.263

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T A B L E S

TABLE I.

Eigenvalues for the aymptotically free SU(5) model. The eigenvalues are classified according to the number of light generations, n_f (sets of $\underline{5}$ and $\underline{10}$), with which they are associated. For all these eigenvalues n_F=1.

n_f=7	h= -.06658g	k_2= -.90392g	k_4=-1.33840g	k_5= -.96576g	k_6= .69265g	h'= .00000g
	λ_1= .01720g^2	λ_2= .66275g^2	λ_3= 2.87232g^2	λ_4= -.06013g^2	λ_5= 2.40814g^2	
n_f=7	h= -.41914g	k_2= .27623g	k_4= -.22568g	k_5= .85456g	k_6= -.85266g	h'= -.83115g
	λ_1= .05781g^2	λ_2= .70071g^2	λ_3= 1.13386g^2	λ_4= -.01704g^2	λ_5= -.32783g^2	
n_f=6	h= .44988g	k_2= .28172g	k_4= -.28445g	k_5= .83508g	k_6= -.83186g	h'= -.85022g
	λ_1= .05322g^2	λ_2= .65002g^2	λ_3= 1.12161g^2	λ_4= -.01739g^2	λ_5= -.30189g^2	
n_f=5	h= -.48827g	k_2= -.29446g	k_4= -.34449g	k_5= -.81526g	k_6= .81006g	h'= -.87527g
	λ_1= .04844g^2	λ_2= .59725g^2	λ_3= 1.11712g^2	λ_4= -.01778g^2	λ_5= -.26466g^2	
n_f=4	h= -.07204g	k_2= -.70041g	k_4= -.99938g	k_5= -.83875g	k_6= .71595g	h'= -.69730g
	λ_1= .03039g^2	λ_2= .51184g^2	λ_3= 1.36544g^2	λ_4= -.00831g^2	λ_5= 1.08323g^2	
n_f=3	h= -.25441g	k_2= -.63549g	k_4= -.94205g	k_5= -.80956g	k_6= .70664g	h'= -.74677g
	λ_1= .02924g^2	λ_2= .45761g^2	λ_3= 1.19605g^2	λ_4= -.01237g^2	λ_5= .90917g^2	

TABLE II.

Values for M_x and proton lifetimes (in years) as function of $\Lambda_{\overline{MS}}$ and the number of light fermion generations. Here $\tau_p^{(i)}$, refers to a total of i-generations.

$\Lambda_{\overline{MS}}$	M_x	$\tau_p^{(3)}$	$\tau_p^{(4)}$	$\tau_p^{(5)}$	$\tau_p^{(6)}$	$\tau_p^{(7)}$
.05	8.8×10^{13}	8.9×10^{26}	6.0×10^{26}	3.9×10^{26}	2.6×10^{26}	1.7×10^{26}
.1	1.9×10^{14}	1.6×10^{28}	1.1×10^{28}	7.1×10^{27}	4.7×10^{27}	2.8×10^{27}
.2	4.0×10^{14}	2.8×10^{29}	1.7×10^{29}	1.2×10^{29}	7.1×10^{28}	4.2×10^{28}
.3	6.4×10^{14}	1.6×10^{30}	1.0×10^{30}	6.3×10^{29}	3.8×10^{29}	2.2×10^{29}
.4	8.9×10^{14}	5.1×10^{30}	3.2×10^{30}	2.0×10^{30}	1.2×10^{30}	6.8×10^{29}
.5	1.2×10^{15}	1.3×10^{31}	8.1×10^{30}	5.1×10^{30}	3.1×10^{30}	1.6×10^{30}
.6	1.4×10^{15}	2.8×10^{31}	1.7×10^{31}	1.1×10^{31}	6.1×10^{30}	3.3×10^{30}