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N=1 Supergravity with a Non-minimal Coupling :
A Class of Models

by

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ABSTRACT

N=1 Supergravity with a Non-minimal Coupling :
A Class of Models

by

Xizeng Wu

Advisor: Professor Ngee Pong Chang

In this work a class of locally SUSY GUTs with a nonminimal coupling is proposed. It is observed that from the superspace point of view the Wess-Zumino supergravity action is much simpler than the supergravity with the minimal coupling. So the overall scalar potential is derived and a new class of superpotential for the hidden sector is found. The effective potential thus derived turns out to be a generalization and improvement of the Ovrut-Wess mechanism. The above formalism is applied to a SU(5) SUSY GUT model and it is found that there is no anti de-Sitter problem any more, and the SU(2)xU(1) symmetry is broken at the scale $M_{2/3}$. The squarks, sleptons, gluinos and photinos will get their masses radiatively. Besides, it is also discussed how to get a stable hierarchy.

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I. Introduction

Since two years ago the interest of people on supersymmetry has been enhanced very much. The motivation of this enhancement is rooted in the attempts to solve the so-called gauge hierarchy problem in the Grand Unified theories (GUTs). GUTs are the theories that unify weak,electromagnetic and strong interactions and seem to be a major part of the physics culture nowadays, and very justifiably so[1]. They do explain some long-standing mysteries such as charge quantization and the order of magnitude of the fine structure constant and also make some detailed predictions for low energy phenomenology ($\sin\theta_w$, m_b/m_τ etc.) which compare quite successfully with experiment[1,2]. The most exciting prediction of GUTs is that nucleus should decay with a lifetime of 10^{31} years or so,which is accessible to the present round of experiments being conducted now worldwide[1]. However, GUTs are certainly not without defects. Most of these are associated with the scalar Higgs sector and the most serious one is the so-called gauge hierarchy problem[1].

Briefly, the gauge hierarchy problem is as follows. In GUTs one has at least two stages of the spontaneous symmetry breaking: one that provides the grand unification scale (M_X) and one that gives the electroweak unification scale (M_W). What seems very bizarre is the gross difference between the two scales

$$\frac{M_W}{M_X} \sim 10^{-12}$$

Such an incredibly huge difference in scales is very difficult first to be produced and then be maintained in conventional field theories. This is because the quantum loop correction will destroy the hierarchy set up at the tree level. Actually in conventional field theories the spontaneous symmetry breaking occurs thanks to the Higgs mechanism and then the scale of the breaking is more or less the same as the mass of the Higgs particle. But the mass of Higgs particle will get radiative corrections. In general the radiative corrections are quadratically divergent

$$\delta m^2 \sim \Lambda^2 \sim M_X^2$$

where is the cut-off and the natural scale for it is M_X. These divergent contributions come from the diagrams involving fermion, gauge boson and scalar loops (Fig. 1). So ,unless there is some mechanism, some kind of symmetry to make the sum of diagrams in Fig. 1 vanish, the hierarchy will be destroyed

$$\frac{M_W}{M_X} \sim O(1)$$

and we will be in trouble. Unfortunately in conventional field theories there is no such kind of symmetry to avoid the divergent radiative correction to the masses of scalars. So one has to finetune parameters at each order of the loops to keep the hierarchy intact. This situation is quite unnatural[1].

In order to solve this difficulty we should either somehow throw away altogether scalar particles or find some kind of symmetry to make the contributions from fermion loops and boson loops cancel each other. The first attempt leads to the techni-hypercolour theories which encounter some serious difficulties[3]. And the second idea is just the point at which supersymmetry comes in.

Supersymmetry is a symmetry between fermions and bosons, which was originally introduced by Gel'fand, Likhtam, Volkov, Akulov, Wess and Zumino about ten years ago. It was shown that supersymmetry is the unique fermion-boson symmetry which is compatible with the Poincare symmetry group[4]. In the conventional field theory symmetries of the theory always transform fermion to fermion and boson to boson, acting, in general, on the internal quantum numbers carried by these particles. And according to the Coleman-Mandula theorem[5] the only possible conserved quantities are those which transform as tensors under the Lorentz group. So we may have the energy-momentum operator P_μ , the Lorentz group generator $M_{\mu\nu}$ and some Lorentz invariant conserved quantum numbers (electric charge, baryon number, etc.) as the conserved quantities. But in supersymmetric theories one introduces a new kind of conserved quantities: conserved charges Q_α that transform as spinors[4]. The Q_α's have spin 1/2, so acting on any particle they change its spin by $\pm$ 1/2

$$Q_\alpha \,|\, \mathrm{Spin}\ S > \;\sim\; |\, \mathrm{Spin}\ S \pm \tfrac{1}{2} >$$

Since the Q_α's are conserved and commute with the Hamiltonian , they do not change the mass of the particle on which they act. This means that fermions and bosons are degenerate in the mass. A unique and highly important feature of supersymmetric field theories is their mild and rather soft ultra-violet behaviour. The root of this remarkable property lies in the existence of the high degree of fermion-boson symmetry. In supersymmetric gauge theories a miracle occurs, and the net result of adding up the diagrams of Fig. 1 is zero. Most remarkably, the result holds to any order in perturbation theory. The magic reason is the mutual cancellation of boson and fermion loops. These cancellation are due to the restriction on couplings dictated by supersymmetry. In general, such a kind of cancellation leads to the famous non-renormalization theorems[6]. These theorems imply non-renormalization (finite or infinite) of certain parameters or of certain relations between parameters---some thing unimaginable in ordinary theories.

For N=1 globally supersymmetric theories with a gauge group G commuting with supersymmetry, the non-renormalization theorem states that there are no quadratic mass renormalization and no renormalizations for the Yukawa and scalar couplings among chiral multiplets. The first result can be intuitively understood from the fact that scalars are supersymmetric partners of chiral fermions and fermions can at most have a logarithmically divergent self-mass. So the scalar masses, through supersymmetry, are protected by chiral invariance from having a quadratic divergence. This argument however is only heuristic because it does not explain why even an independent fermionic mass renormalization does not occur, i.e., why $Z_m=1$ and the mass and coupling constant renormalization for matter chiral multiplets are only induced by their common wave function renormalization. In supersymmetric theories, one typically has[6]

$$\frac{M_{ren.}}{M_{bare}} = \left[\frac{G_{ren.}}{G_{bare}}\right]^{\frac{1}{2}} Z \text{ (wave function ren.)}$$

where $M_{ren.}$ and M_{bare} stand for the renormalized mass and bare mass , $G_{ren.}$ and G_{bare} stand for the renormalized coupling constant and bare one, respectively. This result can be understood from a superspace power counting rule[6] and a supergraph technique which will be used in this thesis extensively. Using the technique, it has been shown that to any order of perturbation theory, a supersymmetric renormalisable theory in 1 or more loops will never give an F-term effective Lagrangian. This means that some allowed supersymmetric F-term effective couplings and mass terms among chiral multiplets are never generated in perturbation theory if they are not present in the original bare Lagrangian. Conversely, if the original Lagrangian contained some F-term couplings these couplings will not be corrected by higher loop one-particle irreducible corrections. This non-renormalization property is often referred to as the so-called supernaturalness.

Now it is obvious that the unnaturalness with the gauge hierarchy problem in GUTs can be overcome ,at least partly, by the supernaturalness in the supersymmetric gauge unified theories(SUSY GUTs)[7]. Before going into detail let's reconsider the whole implication of the gauge hierarchy problem. Strictly speaking,a natural satisfactory solution of the difficulty should be able to answer the following three questions:

1) Why is $M_W/ M_X= 10^{-12}$

2) how can one separate, at tree level, the masses of the Higgs doublet and its GUT partner, the coloured Higgs triplet ?

3) how can one make this mass hierachy stable against radiative corrections ?

Clearly SUSY GUTs give a natural answer to the problem 3). In general, in SUSY GUTs, the mass hierachy is set up at tree level through a fine tuning of parameters in theories, but the fine tuning is fixed once and for all due to the non-renormalization property of SUSY GUTs. In some models[8] the need for fine tuning is avoided by using the so-called sliding singlet or a complicated combination of group representations of Higgs fields. As for answering the question 1), some attempts have been made [9] in SUSY GUTs. So although the cure is far from satisfactory, the gauge hierarchy problem has at any rate been improved.

Of course on the way to building a realistic SUSY GUT there are still many pitfalls. One of the basic problems of supersymmetry is its strong resistance against breaking. If supersymmetry plays any role in nature it must be somehow broken, because we do not observe in nature the degeneracies among different spin that would be predicted by supersymmetry. But in the global supersymmetry the vacuum energy is positive or zero and zero vacuum energy correponds to a supersymmetric invariant vacuum[4]. So in contrast with ordinary gauge theories, supersymmetric states are energetically favored. So if we want to spontaneously break supersymmetry, we had better exclude the supersymmetric states from our physically accessible ones.

This turns out to be a formidable task. Even though one can spontaneously break supersymmetry by using the O'Raifeartaigh mechanism[10] or the Fayet-Illiopoulos mechanism[10], it has been realized that it is difficult to build a phenomenologically acceptable model of SUSY GUT.

In fact there are three principal difficulties yet to be solved[11]. Take the SU(5) SUSY GUTs [7, 11] as an example. In this kind of model supersymmetry is spontaneously broken at a scale $M_s \sim 10^{10}$—10^{12} Gev. First of all we want to have a large SU(5) breaking at the tree level in order to get predictions like those of the non-supersymmetric SU(5) GUT. But it turns out to be very difficult to construct a model in which the SU(3)xSU(2)xU(1) invariant vacuum is unique. In globally SUSY GUTs one always has the degenerate vacua with different symmetry breaking patterns[7]. In the models where supergravity is not introduced, even if supersymmetry is spontaneously broken it still is difficult to lift the degeneracy of vacua at the tree level. Usually one has to either just simply assume the realization of the SU(3)xSU(2)xU(1) vacuum without any explanation or appeal to quantum loop corrections to lift the degeneracy[11].

The second difficulty is how to break the SU(2)XU(1) symmetry with a correct mass hierarchy M_W/M_X. In this case, more often than not one has to appeal to quantum loop corrections again to get a negative mass-squared for the Higgs doublet, and one needs some fine tuning of parameters in models or some constraints on models[11]. This makes model building very complicated and unnatural.

The third difficulty is finding a mechanism which leads to a phenomenologically reasonable spectrum for the squarks, sleptons and gauginos[11, 12].

In the foregoing, the supersymmetry we have discussed is the N=1 global supersymmetry. On the other hand, there are locally supersymmetric theories which are called supergravity[13]. In supergravity, supersymmetry is gauged and the corresponding gauge field is called the gravitino. Gravitinos along with gravitons mediate gravitation. So supergravity is gravity with a local symmetry between bosons and fermions. It seems more and more evident that the correct way to include gravitation is through supergravity. Since the Planck mass $M_P \sim 10^{19}$ Gev, the scale involved in gravitation is much larger than M_W and M_X, so it might naively appear that there would be no effect of gravitation on low energy particle physics. More careful studies revealed that such is not the case.

What are the residual effects of gravitation on the particle world at large? This interesting problem is drawing intensive attention recently. Ovrut and Wess first proposed a mechanism to introduce the effects of gravitation into SUSY GUTs. They coupled a Yang-Mills system to Wess-Zumino supergravity with a large M term[14]. They proved that if the graviton and gravitino interactions are turned off and the cosmological constant cancelled, the global supersymmetry is broken by explicit softly breaking operator with dimension 2 or less. It has been shown by us that through this mechanism the degeneracy of vacua can be lifted, the SU(2)xU(1) symmetry can be broken, and the $SU(3)xU_{em}(1)$ invariat vacuum with a correct mass hierarchy can be obtained just at the tree level[16]. So the first two difficulties mentioned before have been resolved.

On the other hand, Cremmer et al [15] and others[14] gave a more general component Lagrangian of the N=1 supergravity coupled to an arbitrary Yang-Mills system by using tensor calculus techniques. Since then many authors investigated locally SUSY GUTs or their low energy effective theories[17] based on the so-called minimal coupling of matter and supergravity. In these models supersymmetry is broken at a large scale $M_s \sim 10^{10}-10^{12}$ Gev by a hidden sector of gauge singlets. Through the superHiggs effect,[17] the gravitino gets a mass

$$M_{3/2} \sim kM_s^2 \sim M_W.$$

It was found that supergavity can lift the degeneracy, trigger the SU(2)xU(1) symmetry breaking at the scale $M_{3/2} \sim M_W$, and give all scalar fields masses bounded below by the mass of the gravitino. So these investigations reveal that the gravitino may play a important role in particle interaction.

Of course, the existing locally supersymmetric models are far from perfect. The models of locally SUSY GUTs with the minimal coupling may face difficulties with the anti-de Sitter vacua, the SU(2)xU(1) symmetry breaking in 'realistic' models[17] , generating the light fermion masses[29] and the unstability of the hierarchy[30]. In these models the physically wanted $SU(3)xU_{em}(1)$ invariant vacuum has higher energy than other vacua (say,SU(5) or SU(4) etc.). It is hardly acceptable in spite of attempts to try to justify it [17]. Due to the loss of correspondence of the absolute minimum of energy to the physical vacuum, the analysis of the symmetry breaking is not transparent. In the effective thoeries, a more complicated superpotential still needs to be found in order to break the SU(2)XU(1) symmetry[17], because the popular Polony potential can not trigger this breaking. Besides, generating the light fermion masses through the Yukawa couplings will cause breaking of the color and E.M. symmetries[29]. Furthermore, it has been shown[30] that the tree level hierarchy may be destroyed by the loop corrections.

In this thesis a class of non-minimal locally SUSY GUTs is proposed. We observed that from the superspace point of view the Wess-Zumino supergravity action is much simpler than the minimal coupling of supergravity. So in Chapter II we derive the overall scalar potential through integration of θ. Using this formula and a new class of superpotential for the hidden sector, we give an effective theory which turns out to be a generalization and improvement of the Ovrut - Wess mechanism. In Chapter III the above result is applied to a SU(5) SUSY GUT model, and it is found that there is no anti de-Sitter problem any more, and the SU(2)xU(1) symmetry is broken at the scale $M_{2/3}$. The squarks, sleptons, gluinos and photinos will get their masses radiatively. In chapter IV more models will be discussed to show how matter fermions get their masses through the Yukawa couplings without causing troubles and how to get a stable hierarchy. Chapter V presents the conclusions of this thesis.

II. General Formulation

Supergravity has two kinds of different formulations. One is the component formulation and the other one is the superspace formulation. Superspace is an enlargement of ordinary space-time to spinning space[19]. The base-manifold of superspace has points parametrized by coordinates

$$z^M = (x^m, \theta^\mu, \bar{\theta}_{\dot{\mu}})$$

where the capital letter M represents the four -vector index m as well the spinor indicdes μ and $\dot{\mu}$. M, m and μ are all upper indices, while $\dot{\mu}$ is a lower index. x^m are space -time coordinates and θ^μ, $\bar{\theta}_{\dot{\mu}}$ are Grassman variables whose Lorentz characters are those of the Weyl spinor. From a group theoretical point of view superspace may be regarded as the quotient space G/L in which G is the graded (14-dimensional) Poincare group[4] and L is the Lorentz group. The energy-momentum P_m and the supercharge Q_α , $\bar{Q}^{\dot{\alpha}}$ act as translations and supertranslations in superspace. A supertranslation with infinitesimal parameters , shifts the supercoordinates z^M as follows

$$x^m \dashrightarrow x^m + i(\theta\sigma^m\bar{\xi} - \xi\sigma^m\bar{\theta} \quad)$$

$$\theta^\mu \dashrightarrow \theta^\mu + \xi^\mu \qquad (1)$$

$$\bar\theta_{\dot\mu} \dashrightarrow \bar\theta_{\dot\mu} + \bar\xi_{\dot\mu}$$

This supertranslation is called the global supersymmetry transformation. The composition rule of supersymmetry transformations is obtained by performing the commutator of two infinitesimal changes of parameters ξ_1, ξ_2

$$[\,\delta_1\,,\ \delta_2\,] = (-2i(\xi_2\sigma^m\bar\xi_1 - \xi_1\sigma^m\bar\xi_2\ ),\ 0,\ 0\ )$$

which is a manifestation of the fact that an infinitesimal translation of the space-time point x^m can be obtained in superspace by performing two infinitesimal supersymmetry transformations. And this means that the supercharges Q_α, $\bar Q_{\dot\alpha}$ satisfy the following algebra

$$\{\,Q_\mu\,,\ \bar Q_{\dot\mu}\,\} = 2i\,\sigma^m_{\mu\dot\mu}\,\partial_m$$

$$\{\,Q_\mu\,,\ Q_\nu\,\} = \{\,\bar Q_{\dot\mu}\,,\ \bar Q_{\dot\nu}\,\} = 0 \tag{2}$$

A superfield V(z) is defined as a function of z in superspace. Because of eq. (2), V(z) is equivalent to a finite collection of ordinary fields carrying different spins in Minkowski space-time. In fact V(z) can be expanded as

$$\begin{aligned} V(z) = f(x) &+ \theta\,\phi(x) + \bar\theta\,\chi(x) + \theta^2\,m(x) + \bar\theta^2\,n(x) \\ &+ \theta\sigma^m\bar\theta\ \ v_m(x) + \bar\theta^2\theta\,\psi(x) + \theta^2\bar\theta\,\lambda(x) \\ &+ \theta^2\bar\theta^2\,d(x)\ , \end{aligned} \tag{3}$$

so V(z) unifies in a single object (a superscalar) 8 bosonic (f, m, n, v_m and d) and 8 fermionic fields (ϕ, χ, ψ and λ). Under the global supersymmetry transformation V(z) transforms as a scalar

$$V'(z') = V'(\ x^m + i(\theta\sigma^m\bar\xi - \xi\sigma^m\bar\theta),\ \ \theta+\xi\ ,\ \ \bar\theta+\bar\xi\ \ \ )$$

$$V'(z') = V(z) \tag{4}$$

and in this way transformation rules of components of V(z) are determined[4]. From eqs. (1) and (4) it is easy to deduce covariant derivatives

$$D_M \equiv \left(D_m = \partial_m \,,\ D_\mu = \frac{\partial}{\partial\theta^\mu} + i\sigma^m_{\mu\dot\mu}\bar\theta^{\dot\mu}\partial_m \,,\ \bar D^{\dot\mu} = \frac{\partial}{\partial\bar\theta_{\dot\mu}} + i\theta^\mu\sigma^m_{\mu\dot\lambda}\epsilon^{\dot\lambda\dot\mu}\partial_m \right), \tag{5}$$

$$[\partial_m \,,\ D_\mu] = 0 \,,$$

$$\{D_\mu, Q_\nu\} = \{D_\mu \,,\ \bar Q^{\dot\mu}\} = \{\bar D^{\dot\mu} \,,\ Q_\mu\} = 0,$$

$$\{D_\mu \,,\ \bar D^{\dot\mu}\} = 2i\,\sigma^m_{\mu\dot\lambda}\,\epsilon^{\dot\lambda\dot\mu}\,\partial_m \ . \tag{6}$$

Eq. (6) shows that the flat superspace has supertorsion even though its curvature vanishes. The covariant derivatives D_M play a crucial role in the derivation of globally supersymmetric Lagrangians and invariant constraints. Using them one can define the chiral superfields which are "smaller" than general superfields by the condition

$$\bar D^{\dot\mu}\phi = 0 \ .$$

Global supersymmetry and flat superspace techniques have been extensively used in order to perform a systematic investigation of supersymmetric field theories [4]. Furthermore, the supergraph technique has been developed[6] recently. It has proven to be especially useful in the manifestly covariant analyses of quantum corrections and divergences of supersymmetric quantum field theories. In Chapter III we will use this technique to calculate the radiative corrections to masses of squarks , sleptons and gauginos.

In the foregoing, supersymmetry is a global symmetry. But it can be gauged and in this way we are led to supergravity. In this case we would like to construct theories invariant under x-dependent supersymmetry transformations. As in Eq. (1), such transformations induce motions in superspace

$$x_m \dashrightarrow x_m + i\,\theta\sigma^m\bar{\xi}(x) - \xi(x)\sigma^m\bar{\theta}$$
$$\theta^\mu \dashrightarrow \theta^\mu + \xi^\mu(x)$$
$$\bar{\theta}_{\dot{\mu}} \dashrightarrow \bar{\theta}_{\dot{\mu}} + \bar{\xi}_{\dot{\mu}}(x).$$

These motions generate certain coordinate transformations

$$z^M \dashrightarrow z'^M = z^M + \xi^M(z),$$

where $\xi^M(z)$ is an arbitrary function on superspace. In general, these motions contain standard general coordinate transformations with parameters $\xi^m(x, \theta=0)$ and local supersymmetry transformations with parameters $\xi^\mu(x, \theta=0)$ and $\bar{\xi}_{\dot{\mu}}(x, \theta=0)$. Here M refers to a superworld index. To formulate supergravity in a curved superspace one has to develop the geometry of superspace. The first proposal of a curved superspace was made by Arnowitt and Nath [20] who used a super Riemannian geometry in superspace. However, the flat superspace of global supersymmetry does not fit naturally as a special case of a super-Riemannian geometry[21]. Wess and Zumino proposed an alternative approach based on an affine (non-metric) geometry[19]. In this formulation, at each point in superspace one erects a locally flat reference frame and defines the vielbein fields

$$E_M^A(z),$$
$$A = (a, \alpha, \dot{\alpha}), \quad M = (m, \mu, \dot{\mu})$$

and their inverse,where the early capital letters (A-L) refer to tangent space indices, the late capital letters (M-Z) to world indices, respectively. As in any affine space we can define the covariant derivatives

$$\mathcal{D}_M = \frac{\partial}{\partial z^M} - \Phi_M,$$
$$\Phi_M = \Phi_M^{ab} X_{ab},$$

where Φ_M is the Lie-algebra valued superconnection and X_{ab} are the Lorentz generators. It means that the Lorentz group is the tangent space group (structure group). The vielbein and the connection are the basic dynamic variables. The covariant derivatives of vielbein are called supertorsion, and those of the connection are called supercurvature. Covariant derivatives with tangent space indices

$$\mathcal{D}_A = E_A^M \mathcal{D}_M \tag{7}$$

satisfy the graded commutation relations[19]

$$[\mathcal{D}_A, \mathcal{D}_B \} = -R_{AB}^{ab} X_{ab} - 2\, T_{AB}^C \mathcal{D}_C \tag{8}$$

where T_{AB}^C is the supertorsion and R_{AB} is the supercurvature. Torsion and curvature satisfy the Bianchi identities[19]. Moreover some additional kinematical constraints have to be further added in order to completely specify the geometry and to reduce the number of component fields as much as possible. In this formulation there are constraints on the supertorsion. Actually it is impossible to set all torsion components to zero as seen from eq. (6), for that would exclude supersymmetric flat space as a solution of supergravity. It turns out that

$$T_{\underline{\alpha}\underline{\beta}}^{\ \ \underline{\gamma}} = T_{\dot\beta\dot\alpha}^{\ \ c} = T_{\alpha\beta}^{\ \ c} = T_{\underline{\alpha} b}^{\ \ c} = T_{a\underline{\beta}}^{\ \ c} = T_{ab}^{\ \ c} = 0,$$
$$T_{\alpha\dot\beta}^{\ \ c} = T_{\dot\beta\alpha}^{\ \ c} = 2i\,\sigma^{c}_{\alpha\dot\beta} \tag{9}$$

are the proper constraints. Here denotes either or . One has to solve the Bianchi identities subject to these constraints, and it was shown that [19] all the components of the torsion and curvature may be expressed in terms of the superfields

$$R\ ,\quad G_{\alpha\dot\alpha}\quad \text{and}\quad W_{\alpha\beta\gamma}\ .$$

These three multiplets are the supersymmetric extension of the curvature scalar, the Einstein tensor and the Weyl tensor[22]. For instance, R contains the contracted (scalar) curvature tensor at θ^2 level, $G_{\alpha\dot\alpha}$ contains a tensor which includes the Einstein tensor at $\theta\bar\theta$ level and the Rarita-Schwinger operator at the $\theta^2\bar\theta$ level. For $\theta = 0$, $W_{\alpha\beta\gamma}$ contains the Rarita-Schwinger field strength. But as far as independent fields are concerned, they are the graviton $e_m^{\ a}(x)$, gravitino $\psi_m^{\ \alpha}(x)$, $\bar\psi_{m\dot\alpha}(x)$, and auxiliary fields $M(x)$ and $b_a(x)$. It was found that[19]

$$E_M^{\ A} = \begin{pmatrix} e_m^{\ a} & 1/2\,\psi_m^{\ \alpha} & 1/2\,\bar\psi_m^{\ \dot\alpha} \\ 0 & \delta_\mu^{\ \alpha} & 0 \\ 0 & 0 & \delta^{\dot\mu}_{\ \dot\alpha} \end{pmatrix} +$$

$+$ dependent terms,

$$R(Z) = -1/6 \quad M(x) + \theta \text{ dependent terms,}$$

$$G_a(Z) = -\frac{1}{3} b_a(x) + \quad \text{dependent terms .} \qquad (10)$$

The spin-2 graviton couples to the energy momentum tensor, while the spin-3/2 gravitino couples to the spin-3/2 supercurrent. The auxiliary fields are just enough to equalize the number of bosonic and fermionic degrees of freedom off mass shell.

Now the geometry has been completely specified. It is also referred to as the kinematics. It is the analog of the specification of geometry in Einstein's theory as being Riemannian. Just as in Einstein's theory the dynamics are given by giving an action, the dynamics of the pure supergravity are given by the action

$$A = \int d^4x d^4\theta \quad E \qquad (11)$$

where $\quad E = \det E_M^A,$

and E is called the vector density superfield. So the pure supergravity action is simply the measure of superspace but subject to the kinematic constraints Eq. (9).

Of course what interests us is the coupling of supergravity and the Yang-Mills systems. The action of the globally supersymmetric Yang-Mills theory is well known[4] as

$$A = \int d^4x d^4\theta \quad S^+ e^{gV} S$$
$$+ \int d^4x d^2\theta \quad (1/2C_2(G)\tilde{g}^{\,2}) \operatorname{Tr} W^\mu W_\mu$$

$$+ \int d^4x d^2\theta \quad (g(S_i) + \text{h. c.}) \quad , \qquad (12)$$

where S_i are the chiral fields transforming as a representation of a gauge group G, V is the gauge vector superfield, W_α is the gauge covariant chiral superfield which contains the vector field strength and is defined as

$$W_\alpha = -1/4 \quad \bar{D}^2 e^{-gV} D_\alpha \, e^{gV} \quad , \qquad (13)$$

$g(S_i)$ is the superpotential which is gauge invariant, $\tilde{g}$ is the gauge coupling constant and

$$C_2(G) = \mathrm{Tr}(\; T^\alpha \; T^\beta \;)_{adj} .$$

So the most general action of the coupled Yang-Mills and supergravity system is given by[15]

$$A = \int d^4x d^4\theta \quad E \; (\; \Phi \, (S^+ e^{gV} S) + \mathrm{Re}\; (1/R \quad g(S))$$
$$+ \mathrm{Re}(1/R \quad f_{\alpha\beta}\,(S_i) W^\alpha \; W^\beta \;) \quad) \; , \qquad (14)$$

where Φ is a real function of S and S, $f_{\alpha\beta}$ is a chiral superfield transforming as the symmetric product of the adjoint representation of G. Using Eq.(14), Cremmer et al found a general Lagrangian in component formalism[15]. The overall scalar potential is

$$e^{-1} \; V = - \frac{1}{k^4} \exp(-\mathcal{G}) \, [\, 3 + (\mathcal{G}''^{-1})^j_i \, \mathcal{G}'^i \mathcal{G}_{,j}] + \frac{D^2}{2} \; , \qquad (15)$$

where
$$e = \sqrt{-\det |g_{\mu\nu}|} \; ,$$
$$\mathcal{G} = 3 \log\left(\frac{-k^2}{3} \, \Phi(z, z^*) \right) - \log \frac{k^6}{6} |g(z)|^2 \qquad (16)$$
$$= J - \log \frac{k^6}{4} |g|^4 \; ,$$
$$D^2 = \frac{1}{k^4} \, \tilde{g}^2 \, \mathrm{Re} \, f^{-1}_{\alpha\beta} \, (\mathcal{G}'^i T^{\alpha}{}_i{}^j z_j)(\mathcal{G}'^k T^{\beta}{}_k{}^l z_l) \; ,$$
$$k = 4 \pi G \; .$$

So in order to specify V completely, Φ and $f_{\alpha\beta}$ should be chosen. In the popular minimal coupling one chooses

$$\Phi = -\frac{3}{\kappa^2}\exp\left(-\frac{\kappa^2}{6}\, z_i z^{*i}\right), \tag{17}$$

$$f_{\alpha\beta} = \delta_{\alpha\beta},$$

and one has

$$e^{-1}V = \frac{1}{2}\exp\left(\frac{\kappa^2}{2} z_i z^{*i}\right)\left\{\left|\frac{\partial g}{\partial z_i} + \frac{\kappa^2}{2} z^{*i} g\right|^2 - \frac{3}{2}\kappa^2 |g|^2\right\} + \frac{1}{2}(D^\alpha)^2, \tag{18}$$

where $D^\alpha = \frac{1}{2}\,\tilde{g}\, z^{*i}\, T^{\alpha}{}_i{}^j z_j$. (19)

From Eq.(14) with Φ given by Eq.(17), it is clear that the so-called minimal coupling looks complicated in superspace. Since we do not yet know what kind of choice of Φ corresponds to the physics of the world, it definitely appears worthwhile to try an other choice. We should try a choice of Φ which makes the action look more simple and natural. Before doing so, it is worthwhile to review quickly the history of this supergravity-Yang-Mills coupling.

Ovrut and Wess first proposed a mechanism to introduce the effect of supergravity into the globally supersymmetric Yang-Mills theory[14]. They couple the Wess-Zumino supergravity to Yang-Mills matter and get an action

$$A = \int d^4x\, d^2\theta\; \mathcal{E}\Big\{ \Xi S + \frac{1}{2\tilde{g}^2 C_2(A)} T_r\, m^\alpha m_\alpha + [-\tfrac{5}{4}(\bar{\mathcal{D}}^{\dot\alpha}\bar{\mathcal{D}}_{\dot\alpha} - 8R)S^+ + g(S)) + (\tfrac{-6}{\kappa^2})R + (-\tfrac{\Delta^+}{\kappa^2}) + h.c.\;] \Big\}, \tag{20}$$

where

$$\Xi = -\frac{1}{4}(\bar{\mathcal{D}}^2 - 8R)S^+ e^{\tilde{g}V},$$

$$m_\alpha = -\frac{1}{4}(\bar{\mathcal{D}}^2 - 8R)(e^{-\tilde{g}V}\mathcal{D}_\alpha e^{\tilde{g}V}),$$

$$g(S) = f_i S_i + 1/2\ m_{ij} S_i S_j + 1/3\ \lambda_{ijk} S_i S_j S_k$$

$\mathcal{E}$ denotes the chiral density superfield, R denotes the curvature chiral field, $\bar{\mathcal{D}}_{\dot{\alpha}}$ is the covariant derivative and g is the superpotential. The second from last term

$$\int d^4x\, d^2\theta \ \left(\left(\frac{-6}{\kappa^2}\right) \mathcal{E} R + h.c. \right) \tag{21}$$

is the action for pure supergravity. It is the same action as that given by Eq.(11), but in Eq.(21) the Lagrangian is written in F-type density form by using the chiral density $\mathcal{E}$ [19], while in Eq.(11) the Lagrangian is written in D-type form by using the vector density E. In the superspace formulation it is more convenient to use the chiral density . A large so-called M term

$$\int d^4x\, d^2\theta \ \left(\left(\frac{-\Delta^{*}}{\kappa^2}\right) \mathcal{E} + h.c. \right)$$

is put in Eq.(20), and in the limit of $k \to 0$ it introduces a residual effect into the global SUSY action. Actually in that limit supersymmetry is broken through discarding an infinite cosmological constant ($-\frac{|\Delta|^2}{\kappa^4}$). Then the action becomes

$$A = A_{susy} + A_{sb},$$

where A_{susy} is globally supersymmetric and

$$A_{susy} = \int d^4x\, d^4\theta \ S^{+} e^{gV} S - \left[\int d^4x\, d^2\theta \ g(S) + h.c. \right] + \frac{1}{2 g^2 C_2(A)} \ T_r \int d^4x\, d^2\theta \ W^{\alpha} W_{\alpha} \ .$$

A_{sb} is an explicit SUSY softly breaking term

$$A_{sb} = \int d^4x \ \left(\frac{2}{3} \Delta f A_s + \frac{\Delta m}{6} A_s^2 + h.c. \right), \tag{22}$$

where A_s is the scalar partner of S.

This term is just a remnant of supergravity. This action is renormalizable to any order of perturbation[14]. We have applied this mechanism to a SU(5) SUSY GUT model[16] and found that the A_{sb} term lifts the degeneracy of vacua and breaks SU(2)xU(1) symmetry. So in this model a unique $SU(3)xU_{em}(1)$ invariant vacuum with the correct mass hierarchy $M_W/M_X \dashrightarrow 10^{-12}$ is obtained simply at the tree level[16], and M_W is related to the parameter Δ which is the strength of the residual effect of supergravity.

But the Ovrut-Wess mechanism has some shortcomings. They are rooted in the fact that supergravity breaking in that mechanism is not a spontaneous breaking. So the strength of this breaking cannot be related to the mass of the gravitino. This is why the parameter in the action is put in by hand. Therefore the theory loses some of its predictive power. For the same reason this theory cannot justify itself for dealing with a infinite negative cosmological constant while the experimental facts indicate a zero cosmological constant. Besides, an explicit breaking of supergravity also causes breaking of the local Lorentz invariance.

Motivated by our desire for a simpler choice of the function Φ (Eq.(14)) and for an improvement Of the Ovrut-Wess mechanism, in this thesis we derive the overall scalar potential corresponding to the Wess-Zumino supergravity, we investigate the way to break it spontaneously, and we analyse the physical implications of the effective theory extracted from it.

In order to look for a simple choice of Φ , the strategy we adopted is the following. We take the Wess-Zumino supergravity, formulated by them in F-term form rather than in D-term form . That is , they expressed their action as a chiral density to be integrated over θ , while our general action is a vector density to be integrated over $\theta,\bar\theta$. In order to compare and analyse the Wess-Zuminosupergravity, we have carried out the θ integration of their density. For this we make use of the expansion of $\mathcal{E}$ and Ξ [19]

$$\mathcal{E} = e\left\{ 1 + i\,\theta\sigma^a\bar\psi_a - \theta^2[\,M^* - \bar\psi_a\bar\sigma^{ab}\bar\psi_b\,]\right\},$$

$$\begin{aligned}
\Xi = {} & -4F^* + \tfrac{4}{3}MA^* \\
& + \theta\left\{ -4\sqrt{2}\; i\sigma^c\hat{\mathcal{D}}_c\bar\chi - \tfrac{2}{3}\sqrt{2}\,\sigma^a b_a\bar\chi + \tfrac{4}{3}A^*[\,2\sigma^{ab}\psi_{ab} \right. \\
& \qquad \left. - i\sigma^a\bar\psi_a M + i\psi_a b^a\,]\right\} \\
& + \theta^2\left\{ -4\,e_a{}^m\mathcal{D}_m\hat{\mathcal{D}}^a A^* - \tfrac{8}{3}i b_a\hat{\mathcal{D}}^a A^* - \tfrac{2}{3}\sqrt{2}\,\bar\psi_{ab}\bar\sigma^{ab}\bar\chi \right. \\
& \qquad + 2\sqrt{2}\,\bar\psi_a\hat{\mathcal{D}}^a\bar\chi - \tfrac{8}{3}M^*F^* - i\tfrac{2}{3}\sqrt{2}\,\bar\psi_a\bar\chi b^a + i\tfrac{\sqrt{2}}{3}\bar\psi_a\bar\sigma^a\sigma^b\bar\chi b_b \\
& \qquad + \tfrac{4}{3}A^*[-\tfrac{1}{2}R_{ab}{}^{ab} + i\bar\psi^a\bar\sigma^b\psi_{ab} - i\,e_a{}^m\mathcal{D}_m b^a + \tfrac{2}{3}M^*M \\
& \qquad + \tfrac{1}{3}b_a b^a + \tfrac{1}{2}\bar\psi\bar\psi M - \tfrac{1}{2}\psi_a\sigma^a\bar\psi_b b^b + \tfrac{1}{8}\epsilon^{abcd}(\\
& \qquad \left. \bar\psi_a\bar\sigma_b\psi_{cd} + \psi_a\sigma_b\bar\psi_{cd}\,)\,]\right\},
\end{aligned}$$

$$\begin{aligned}
R = {} & -\tfrac{1}{6}\{\,M \\
& \qquad + \theta[\,\sigma^a\bar\sigma^b\psi_{ab} - i\sigma^a\bar\psi_a M + i\psi_a b^a\,]
\end{aligned}$$

$$+ \theta^2 [-\tfrac{1}{2} R_{ab}{}^{ab} + i \bar{\psi}^a \bar{\sigma}^b \psi_{ab} + \tfrac{2}{3} M^* M + \tfrac{1}{3} b^a b_a$$
$$- i e^m_a \mathcal{D}_m b^a + \tfrac{1}{2} \bar{\psi} \bar{\psi} M - \tfrac{1}{2} \psi_a \sigma^a \bar{\psi}_b b^b$$
$$+ \tfrac{1}{8} \epsilon^{abcd} (\bar{\psi}_a \bar{\sigma}_b \psi_{cd} + \psi_a \sigma_b \bar{\psi}_{cd})] \} .$$

Substituting these expressions back into eq.(20)and completing the integral over θ, we get

$$A = \int d^4x \quad \mathcal{L} \qquad ,$$
$$e^{-1} \mathcal{L} = - \frac{1}{2\kappa^2} R \{ [1 - \frac{\kappa^2}{6} (|z_i|^2 + |y_a|^2) - \frac{\kappa^2}{6} (c^{*i} z_i + c_i z^{*i})] \}$$
$$- \tfrac{1}{2} (\partial_\mu z_i \partial^\mu z^* + \partial_\mu y_a \partial^\mu y^{*a})$$
$$+ \tfrac{1}{3} [1 - \frac{\kappa^2}{6} (|z_i|^2 + |y_a|^2) - \frac{\kappa^2}{6} (c^{*i} z_i + c_i z^{*i})] b^a b^a$$
$$- \tfrac{1}{3} [1 - \frac{\kappa^2}{6} (|z_i|^2 + |y_a|^2) - \frac{\kappa^2}{6} (c^{*i} z_i + c_i z^{*i})] M M^*$$
$$+ \cdots , \qquad (23)$$

where $e = \sqrt{- \det | g_{ab} |}$

and z is the scalar component field of the gauge singlet chiral superfield S_i and y^a is that of the gauge multiplet S^a. b_a and M are defined by Eq.(10). Now we may compare Eq.(31) with the result of Cremmer et al obtained before the Weyl rescaling and elimination of the auxiliary fields[15], and we find

$$\Phi = - \frac{3}{\kappa^2} [1 - \frac{\kappa^2}{6} (|z_i|^2 + |y_a|^2) - \frac{\kappa^2}{6} (c^{*i} z_i + c_i z^{*i})] , \qquad (24)$$

so

$$J = 3 \log [1 - \frac{\kappa^2}{6} (|z_i|^2 + |y_a|^2) - \frac{\kappa^2}{6} (c^{*i} z_i + c_i z^{*i})] \qquad (25)$$

Comparing this result with that given by Eq.(17) for the minimal coupling, now it is evident that this Φ is much simpler than that of the minimal coupling. It is simpler in the sense that in superspace the action is a straightforward generalisation of the usual kinetic energy term for the chiral fields, with the constant term $-3/k^2$ simply the pure supergravity term.

Using this Φ , the overall scalar potetial is found[18] to be

$$e^{-1}\, V = \frac{1}{2I^2}\Big[\, \hat{g}^i \hat{g}_j \Big(\delta^i_j - \frac{\frac{\kappa^2}{6}(z_i + c_i)(z^{*j} + c^{*j})}{1 + \frac{\kappa^2}{6} c_i c^{*i}} \Big)$$

$$+ \hat{g}^a \hat{g}_b \Big(\delta^b_a - \frac{\frac{\kappa^2}{6} y_a y^{*b}}{1 + \frac{\kappa^2}{6} c_i c^{*i}} \Big) - \Big(\frac{\kappa^2}{6} \hat{g}^i \hat{g}_b \frac{(z_i + c_i)\, y^{*b}}{1 + \frac{\kappa^2}{6} c_i c^{*i}} + h.c. \Big)$$

$$- \frac{3}{2} \kappa^2 |g|^2 \Big] + \frac{1}{2}\, \tilde{D}^2 \quad , \qquad (26)$$

where

$$I = - \frac{\kappa^2}{3}\, \Phi(z, y)$$

and the z's and y's refer to the scalar component of the chiral superfields S and T, they are gauge singlets and multiplets, respectively. Here

$$\hat{g}^i = \frac{\partial g}{\partial z_i} - \frac{\partial I}{\partial z_i}\, g \qquad (27)$$

and $$(\tilde{D})^2 = g^2 \frac{1}{\kappa^4} \big(\mathcal{G}^{\prime i}\, T^{\alpha}{}_i{}^j z_j \big)\big(\mathcal{G}^{\prime k}\, T^{\alpha}_k{}^l z_l \big)$$

The coupling between matter and supergravity is a nonminimal one because the kinetic energy terms for the scalar fields are noncanonical ones

$$\frac{-e}{2} \Big(\frac{\delta^{\alpha}_{\beta}}{I} + \frac{\kappa^2}{6}\, \frac{Z^{*\alpha} Z_\beta}{I^2} \Big) D_\mu Z_\alpha D^\mu Z^{*\beta} \quad , \qquad (28)$$

where $Z_\alpha = (z_i + c_i\ ,\ y_a)$. The theory here thus makes sense only if z_i satisfies the constraint

$$\kappa^2\, |\langle z_i \rangle|^2 < 6 \quad , \qquad (29)$$

Before we put in explicit GUT fields for model building (which will be done in next chapter), it is instructive to consider a class of superpotentials with only one singlet complex field, z:[18]

$$g_2 = \sum_{n=0}^{\infty} a_n z^n$$

where the a_n are complex c-numbers. Note that we do not assume <u>a priori</u> that g_2 is a cubic polynomial since supergravity theory is any case nonrenormalisable. Also we should note that the Polony superpotential and the O'Raifertaigh mechanism, which are used in all existing minmal coupling models for breaking SUSY and giving a zero cosmological constant, will lead to scalar potentials that are unbounded from below in this nonminimal coupling model. So one really needs to find a g_2 to break SUSY and give a zero cosmological constant.

A very interesting feature of the potential as given in Eq.(26) is that, in contrast with the minimal case, it can be made invariant under the reflection of the complex field $z \rightarrow -z$ if we set c_i to be equal to zero, and restrict the form of $g_2(z)$. For this analysis it is convenient to rewrite Eq.(26) as

$$e^{-1} V = \frac{1}{1-\frac{\kappa^2}{6}|z_i|^2} \left[\frac{1}{2} \left| \frac{\partial g}{\partial z_i} \right|^2 - \frac{3}{4} \kappa^2 \left| g - \frac{1}{3} z_i \frac{\partial g}{\partial z_i} \right|^2 \right] , \quad (30)$$

Namely, the following theorem is derived :

<u>Theorem</u> If $V(z) = V(-z)$ for z in the complex plane, then

$$g_2(z) = a_0 \left(1+ \frac{\kappa^2}{2} z^2\right) + a_1 z \left(1+ \frac{\kappa^2}{18} z^2\right)$$

If, furthermore, $V = 0$ at the minimum then

$$V(z) = 0$$

and

$$g_2(z) = a_0 \left(1 \pm \frac{\kappa}{\sqrt{6}} z\right)^3 \quad (31)$$

Proof By direct inspection from Eq.(14), it is easy to show first that invariance under $z \to -z$ in the complex z plane requires

$$a_n = 0 \qquad (\text{If } n > 3),$$

$$a_1 a_2 = \frac{\kappa^2}{2} a_0 a_1 ,$$

$$a_2 a_3 = \frac{\kappa^2}{18} a_1 a_2 .$$

Since the overall phase of g_2 is irrelevant, a_o can always be chosen real. Next the relative phase of a_1 may be absorbed by the phase of z, and then the constraint equations above completely fix the phases of $a_{2,3}$ to be real. The scalar potential then reads

$$e^{-1} V = \frac{1}{2}\left(a_1^2 - \frac{3}{2}\kappa^2 a_0\right)\left(1 - \frac{2}{3}\kappa^2 (\mathrm{Im}\, z)^2 \frac{1}{(1-\frac{\kappa^2}{6}|z|^2)^2}\right),$$

If $\rho = a_1^2 - \frac{3}{2}\kappa^2 a_0^2 < 0$, the minimum is obtained for $\mathrm{Im}\ z = 0$. However, at the minimum, V is then negative. If $\rho > 0$, the potential is not bounded from below. For $\rho = 0$, i.e.

$$a_1^2 = \frac{3}{2}\kappa^2 a_0^2$$

V vanishes identically and $g_2(z)$ is given by Eq.(31).

Thus if we require invariance under reflection in the complex z plane and a zero cosmological constant, the resulting V is completely flat. Supersymmetry is actually broken by g_2, as may be demonstrated by evaluating

$$\hat{g}_2(z) = a_1 \left(1 \pm \frac{\kappa z}{\sqrt{6}}\right)^2 \left(1 \pm \frac{\kappa z^*}{\sqrt{6}}\right) \frac{1}{(1-\frac{\kappa^2}{6}|z|^2)} .$$

For all z satisfying the constraint Eq.(29), $g_2(z)$ is not zero. The flat V implies that in the effective theory in the limit as $k \to 0$ the z field is absolutely massless. This feature of the theory has however no direct phenomenological implication as long as the coupling of the scalar singlet with matter is small. In the conventional treatment where g is set equal to

$$g = g_1(y) + g_2(z) ,$$

the GUT fields contribute to g_1 while the singlet fields contribute only to g_2, and induced couplings between z and y are indeed small. The total potential $V(g_1+g_2)$ is then given by

$$e^{-1} V(g_1+g_2) = \frac{1}{2} \frac{1}{\left(1-\frac{\kappa^2}{6}(|y_a|^2+|z|^2)\right)^2}\Big[\left|\frac{\partial g_1}{\partial y_a}\right|^2 - \frac{3}{2}\kappa^2 |G_1(y_a)|^2 - \frac{3}{2}\kappa^2 \left(G_1(y_a)\, G_2^*(z) + h.c.\right)\Big] + \frac{\tilde{D}^2}{2} , \qquad (32)$$

where

$$G_1(y) = g_1 - \frac{1}{3} y_a \frac{\partial g_1}{\partial y_a} ,$$

$$G_2(z) = g_2 - \frac{1}{3} z \frac{\partial g_2}{\partial z} = a_0\left(1 \pm \frac{\kappa}{\sqrt{6}} z\right)^2 .$$

From Eq.(32) an effective theory for gauge matter can be derived by letting $k \to 0$. Actually only in this limit will the theory be renormalisable. This is equivalent to having neglected all interactions containing k explicitly. A deeper understanding of supersymmetry corrections, containing powers of k, should however be undertaken so that we could trust the present approach as a consistent mathematical truncation of a more fundamental theory encompassing quantum gravity. For there to be a residual effect as $k \to 0$, it is necessary to suppose that

$$a_0 = m / k^2 \qquad (33)$$

where m is a mass scale. Because the z field is responsible for SUSY breaking, the fermionic partner of the z field is the goldstino, and it is eaten by the massive gravitino through the super-Higgs mechanism[15]. It then follows that the gravitino mass is given by

$$m_{3/2} = \frac{1}{k^2} e^{-\mathcal{G}}$$
$$= \frac{m^2}{4} (1+a)^3 (1+a^*)^3 \frac{1}{(1-|a|^2)^3} \,. \tag{34}$$

In the limit $k \dashrightarrow 0$, the z field decouples, and for y_a fields the effective theory finally reads

$$V(y_a) = \frac{1}{2(1-|a|^2)^2} \Big[\Big| \frac{\partial g_1}{\partial y_a} \Big|^2 \mp 3 m_{3/2} \Big(\frac{(1-|a|^2)^{3/2} (1+a^*)^{3/2}}{(1+a)^{3/2}} G_1(y_a) + $$
$$+ h.c. \Big) \Big] + \frac{1}{2} (D^\alpha)^2 \tag{35}$$

where
$$D^\alpha = \tilde{g}\, y^{*a} T_a^{\alpha b}\, y_b \tag{36}$$

and we have reparametrised

$$z = a\sqrt{6} / k\,, \quad \text{with} \quad |a| < 1. \tag{37}$$

The effective potential for y's looks like a potential of global SUSY plus explicit SUSY breaking terms. But to identify the whole effective theory as exactly that form one has to check the whole Lagrangian carefully. It turns out that the kinetic energy terms of the y_a and their fermionic partners are not in canonical forms yet, and in limit $k \to 0$ they become

$$-\frac{1}{2(1-|a|^2)} D_\mu y^{*a} D^\mu y_a$$

and
$$-\frac{1}{2(1-|a|^2)} \bar{\chi}_{La} \not{\partial} \chi_R^a + h.c. \tag{38}$$

respectively. Also, there are some terms here which are absent in global SUSY theory:

$$\Big\{ -\sqrt{\frac{g^*(z,y)}{g(z,y)}} \frac{1}{2(1-|a|^2)^{3/2}} g''^{ab} \bar{\chi}_{La} \chi_b - \frac{1}{4(1-|a|^2)} \frac{g'^b}{g} \bar{\chi}_{La} \not{\partial} y_b \chi_R^a$$

$$+ \frac{1}{4} \bar{\lambda}^{\alpha}_{L} \gamma_{\mu} \lambda^{\alpha}_{R} \frac{g'^{a}}{g} D^{\mu} y_{a} \Big\} + h.c. \tag{39}$$

We found that under the following rescalings of y_a and g_1 and chiral rotations of χ^a and λ^α

$$\begin{aligned} y^{a} &\longrightarrow (1-|a|^{2})^{\frac{1}{2}} y^{a} , \\ \chi^{a} &\longrightarrow \left(\frac{g^{*}(z,y)}{g(z,y)}\right)^{1/4} (1-|a|^{2})^{\frac{1}{2}} \chi^{a} , \\ \lambda^{\alpha} &\longrightarrow \left(\frac{g(z,y)}{g^{*}(z,y)}\right)^{1/4} \lambda^{\alpha} , \\ g &\longrightarrow \sqrt{2}\,(1-|a|^{2})^{3/2} g , \end{aligned} \tag{40}$$

the whole Lagrangian can be put in exactly the form of a global SUSY Lagrangian plus explicit SUSY breaking interactions, i.e.

$$A = A_{gl.susy} + A_{sb} , \tag{41}$$

where $A_{gl.susy}$ is given by eq. (22) and

$$A_{sb} = \int d^{4}x \left\{ z\, \Delta \left(g_{1} - \frac{1}{3} y_{a} \frac{\partial g}{\partial y_{a}} \right) \right\} + h.c.$$

where $$\Delta = -\frac{3}{\sqrt{2}} \frac{(1-|a|^{2})(1+a^{2})^{\frac{1}{2}}}{(1+a)^{3/2}} m_{3/2} \tag{42}$$

In order for the theory to be renormalisable g_1 can contain at most cubic terms of y_a so it can be generally written as

$$g_{1} = g_{1}^{(0)} + g_{1}^{(1)} + g_{1}^{(2)} + g_{1}^{(3)} ,$$

where the superindices 0, 1, 2, 3 refer to the powers of fields. So finally we get

$$A_{sb} = \int d^{4}x \left\{ z\, \Delta \left(g_{1}^{(0)} + \frac{2}{3} g_{1}^{(1)} + \frac{1}{3} g_{1}^{(2)} \right) + h.c. \right\} . \tag{43}$$

Comparing Eqs. (41) and (43) with Eq. (22), it is clear that with this approach from a simple choice of superspace action we have derived a natural generalization of the Ovrut-Wess mechanism. What is more, the theory given here improves upon that theory in two respects: instead of a negative infinite cosmologial constant, we have a zero cosmological constant and the SUSY breaking strength in our effective theory is just the mass of the gravitino rather than an arbitrary parameter put in by hand [14]. Besides, this theory is renormalisable to any order in perturbation.

Now some remarks are in order. Firstly, our effective theory is different from that derived from the minimal coupling. Unlike the latter, there are no cubic terms and no common mass terms for all scalars among the SUSY breaking interactions in our theory. This is a significant difference. It will make the SU(2)xU(1) symmetry breaking easier to be done than in the case of minimal coupling[17]. On the other hand, for our theory the globally SUSY sum rule[14] still holds at the tree level

$$\sum_{J=0}^{1} (2J+1)(-1)^{2J} M^2 = 0$$

This is because the contribution to the trace of the scalar mass-squared matrix only come from terms in

$$4\, \mathrm{Tr} \frac{\partial^2 V}{\partial y_a \partial y^{*b}}$$

but Eq. (43) gives no contribution to that. So at the tree level it is impossible to make all squarks and sleptons more massive than their fermionic partners. Squarks and sleptons will get their mass through radiative corrections. In the next chapter we will discuss the physical implications of these points in more detail.

Secondly, the residual effect of supergravity is entirely in the parameters m and a. $\sqrt{\frac{m}{\kappa}}$ sets the scale for SUSY breaking and m sets the scale for the gravitino mass. In the next chapter we will see how to determine m from phenomenology. The parameter a is the analog of A in the effective theory derived from the minimal coupling. Its exact value may depend on the model, but we always have $|a| < 1$. So the magnitude of the gravitino mass is independent of the exact value of a.

III. A SU(5) SUSY GUT MODEL

In oder to see the physical implication of this non-minimal coupling let's examine a specific model. We choose the superpotential g_1 as

$$g_1(y_a) = \lambda_1\left(\tfrac{1}{3}\, T_r \Sigma^3 + \tfrac{1}{2} m\, T_r \Sigma^2\right) + \lambda_2 (H'\Sigma H + 3m' H'H) + \lambda_3 U H'H + C \qquad (44)$$

Here Σ , H, H' and U are chiral superfields in the $\underline{24}$, $\underline{5}$, $\underline{5}^*$, and singlet representations of SU(5) gauge group, respectively. C is a constant, and we shall denote the GUT mass by m and $m' \sim m$. Of course there are gauge fields and matter fields also. This potential has been used by many authors[7] in globally SUSY GUT models and in locally SUSY GUT models with the minimal coupling. Now we apply our theory to this potential in this chapter, and in the next chapter we will discuss models with other potentials.

In the case of global SUSY this g_1 gives the following scalar potential:

$$V_{g1.susy} = T_r \left|\frac{\partial g}{\partial \Sigma}\right|^2 + \left|\frac{\partial g}{\partial H}\right|^2 + \left|\frac{\partial g}{\partial H'}\right|^2 + \left|\frac{\partial g}{\partial U}\right|^2 + \frac{1}{2} D^2 \qquad (45)$$

where

$$\frac{\partial g}{\partial \Sigma} = \lambda_1(\Sigma^2 + m\Sigma) + \lambda_2 H^T H'^T - \tfrac{1}{5} I_5 (\lambda_1 T_r \Sigma^2 + \lambda_2 H'H),$$

$$\frac{\partial g}{\partial H} = \lambda_2 (H'\Sigma + 3m' H'^T) + \lambda_3 U H'^T ,$$

$$\frac{\partial g}{\partial H'} = \lambda_2 (\Sigma H + 3m' H^T) + \lambda_3 U H^T ,$$

$$\frac{\partial g}{\partial U} = \lambda_3 H'H ,$$

$$D^2 = D^x_y \, D^y_x \; ,$$

$$D^x_y = \tilde{g} \left\{ [\Sigma, \Sigma^\dagger]^x_y + [H^x H^\dagger_y - \tfrac{1}{5} \delta^x_y H^\dagger H] - [H'^{\dagger x} H'_y - \tfrac{1}{5} \delta^x_y H' H'^\dagger] \right\} \qquad (46)$$

where g is the gauge coupling constant. In this case the following supersymmetric vacua with $\langle V \rangle = 0$ exist:

(1) SU(5) invariant vacuum

$$\langle \Sigma \rangle = \langle H \rangle = \langle H' \rangle = 0, \quad \langle U \rangle \text{ arbitrary}. \qquad (47)$$

(2) SU(4)xU(1) invariant vacuum

$$\langle \Sigma \rangle = \frac{m}{3} \begin{pmatrix} 1 & & & & \\ & 1 & & & \\ & & 1 & & \\ & & & 1 & \\ & & & & -4 \end{pmatrix} \qquad (48)$$

$$\langle H \rangle = \langle H' \rangle = 0, \quad \langle U \rangle \text{ arbitrary}.$$

(3) SU(3)xSU(2)xU(1) invariant vacuum

$$\langle \Sigma \rangle = m \begin{pmatrix} 2 & & & & \\ & 2 & & & \\ & & 2 & & \\ & & & -3 & \\ & & & & -3 \end{pmatrix} \qquad (49)$$

$$\langle H \rangle = \langle H' \rangle = \langle M \rangle = \langle M' \rangle = 0, \quad \langle U \rangle \text{ arbitrary}.$$

So it is obvious that, in the globally SUSY GUT, vacua are degenerate and SU(2)xU(1) symmetry is not broken yet. Even in the case that SUSY is somehow broken (but without introducing the effect of supergravity), the situation is the same at the tree level[11]. But in our theory the scalar potential is given as

$$V = V_{gl.susy} + V_{sb} ,$$

where $V_{gl.susy}$ is given by Eq.(45) and

$$V_{sb} = \Delta \left(C + \frac{\lambda_1}{2} Tr\Sigma^2 + \lambda_2 m' H'H \right) + h.c. , \tag{50}$$

where Δ is given by Eq.(42). It should be noted that here a minus sign in Eq.(43) has been chosen. It is clear that V_{sb} is the remnant of supergravity. This breaking is soft in the sense of not generating unwanted quadratic divergences[24]. But it can generate a new logarithmic divergence, as we will see later. The strength of this breaking is determined by Δ, which has the same order of magnitude as $m_{3/2}$. Without loss of generality we take as real, less than zero and $\frac{|\Delta|}{m} \ll 1$. In order to find the vacuum corresponding to the potential V, we expand V in powers of Δ and then search for minima. The results are given as follows:

(1) SU(5) invariant VEV's

$$\langle \Sigma \rangle = \langle H \rangle = \langle H' \rangle = 0,$$
$$\langle U \rangle : \text{arbitrary}. \tag{51}$$

The value of the energy at these VEV's is

$$V_0 = 2 \Delta C . \tag{52}$$

(2) SU(4)xU(1) invariant VEV's

$$\langle H \rangle = \langle H' \rangle = 0 ,$$
$$\langle U \rangle : \text{arbitrary} ,$$

$$\langle \Sigma \rangle = d\, m \begin{pmatrix} 1 & & & & \\ & 1 & & & \\ & & 1 & & \\ & & & 1 & \\ & & & & -4 \end{pmatrix} \tag{53}$$

In this case

$$V = \lambda_1 m^4 d^2 [180\lambda_1 d^2 + 20(\lambda_1 + \frac{\Delta}{3m}) - 120\lambda_1 d] + 2\Delta C. \quad (54)$$

For

$$d = \frac{3 + \sqrt{1 - 8\Delta/3m\lambda_1}}{12}, \quad (55)$$

V reaches a local minimum

$$V_0 = 2C\Delta + \frac{20}{27}\lambda_1 m^4 (\frac{\Delta}{m}) + o((\frac{\Delta}{m})^2). \quad (56)$$

(3) SU(3)xSU(2)XU(1) invariant VEV's

$$\langle H \rangle = \langle H' \rangle = 0$$

$\langle U \rangle$: arbitrary

$$\langle \Sigma \rangle = bm \begin{pmatrix} 2 & & & & \\ & 2 & & & \\ & & 2 & & \\ & & & -3 & \\ & & & & -3 \end{pmatrix} \quad (57)$$

In this case

$$V = 30\lambda_1^2 m^4 b^2 [(b-1)^2 + \frac{\Delta}{3m\lambda_1}] + 2\Delta C \quad (58)$$

For

$$b = \frac{1}{4}(3 + \sqrt{1 - \frac{3\Delta}{8\lambda_1 m}}), \quad (59)$$

V reaches a local minimum

$$V_0 = m^4 (10\lambda_1 \frac{\Delta}{m} - \frac{10}{3}\frac{\Delta^2}{m^2}) + 2\Delta C + o(m^4 (\frac{\Delta}{m})^3). \quad (60)$$

(4) SU(3)xU_{em}(1) invariant VEV's

$$\langle U \rangle = \frac{\lambda_2}{\lambda_3}(S_0 - 3m'),$$

$$\langle H^x \rangle = \langle H'_x \rangle = \frac{1}{\sqrt{2}} A_5 \delta^x_5,$$

$$\langle \Sigma \rangle = \begin{pmatrix} 2ma & & & & \\ & 2ma & & & \\ & & 2ma & & \\ & & & -3ma+\epsilon_2 & \\ & & & & -3ma-\epsilon_2 \end{pmatrix}. \quad (61)$$

In this case

$$
\begin{aligned}
V = {} & \frac{6}{5}\lambda_1^2\epsilon_2^4 + \frac{2}{5}\lambda_1\lambda_2\epsilon_2^2 A_5^2 - \lambda_1\lambda_2(m-6ma)\epsilon_2 A_5^2 \\
& + \frac{1}{5}\lambda_2^2 A_5^4 + 2\lambda_1^2(m-6ma)^2\epsilon_2^2 + 30m^4\lambda_1^2(a-a^2)^2 \\
& - 12\lambda_1^2 m^2(a-a^2)\epsilon_2^2 - 3\lambda_1\lambda_2 m^2(a-a^2)A_5^2 + \lambda_2^2 A_5^2\epsilon_2^2 \\
& + 9\lambda_2^2 A_5^2 m^2 a^2 + \lambda_2^2 A_5^2 S_0^2 - 2\lambda_2^2 A_5^2\epsilon_2 S_0 \\
& - 6m\lambda_2^2 a A_5^2 S_0 + 6\lambda_2^2 m A_5^2 a\epsilon_2 + \frac{1}{4}\lambda_3^2 A_5^4 + 10\lambda_1\Delta a^2 m^3 \\
& + \frac{2}{3}\lambda_1 m\Delta\epsilon_2^2 + m'\Delta\lambda_2 A_5^2 + 2c\Delta \quad . \qquad (62)
\end{aligned}
$$

After a quite tedious calculation (which is given in Appendix I), it is found that for

$$a = 1 + \frac{\lambda_2^2}{30\lambda_1\lambda_3^2}\left[3\left(\frac{m'}{m}-1\right) - 10\frac{\lambda_3^2}{\lambda_2^2}\right]\frac{\Delta}{m} + O\left(\left(\frac{\Delta}{m}\right)^2\right), \qquad (63)$$

$$s_0 = 3m + O(\Delta), \qquad (64)$$

$$\epsilon_2 = \frac{\lambda_2^2}{10\lambda_1\lambda_3^2}\,\frac{m'-m}{m}\,\Delta\ , \qquad (65)$$

$$A_5^2 = \frac{-\lambda_2}{\lambda_3^2}(m'-m)\,\Delta\ , \qquad (66)$$

V reaches the minimum

$$V_0 = 2c\Delta + m^4\left\{10\lambda_1\frac{\Delta}{m} + \left[-\frac{10}{3} - \frac{\lambda_2^2}{\lambda_3^2}\left(\frac{m'-m}{m}\right)^2\right]\frac{\Delta^2}{m^2}\right\}. \qquad (67)$$

So if one chooses

$$C = -\frac{m^4}{2\Delta}\left\{10\lambda_1\frac{\Delta}{m} + \left[-\frac{10}{3} - \frac{\lambda_2^2}{\lambda_3^2}\left(\frac{m'-m}{m}\right)^2\frac{\Delta^2}{m^2}\right.\right\} \qquad (68)$$

and $$m' > m\ , \qquad (69)$$

then it is evident that

$$(V_0)_{SU(5)} > (V_0)_{SU(4)\times U(1)} > (V_0)_{SU(3)\times SU(2)\times U(1)} > (V_0)_{SU(3)\times U_{em}(1)} = 0. \qquad (70)$$

This is a remarkable result. Firstly, as expected, in this model the vacuum is unique. There is no generacy of vacua any more. So one of the principal difficulties of global SUSY is resolved. Secondly, the physical $SU(3)XU_{em}(1)$ vacuum is located at the absolute minimum of energy and with a zero cosmological constant. As mentioned before, one feature of models of locally SUSY GUT with the minimal coupling is that if the cosmological constant of any one of the vacua is adjusted to be zero then other vacua are at lower energy. It means that the physical vacuum with a zero cosmological constant , which corresponds to the Minkowski space, has higher energy than that of unphysical vacua , which correspond to the anti de-Sitter space. It is very hard to explain why the physical vacuum is stable agaist the decay into the anti-de Sitter vacua. This is a quite unattractive feature of these models with minimal coupling. Besides, if the physical vacuum is not the energy ground state any more, then we lose an important criterion for judging which vacuum is the physical one, and this makes the analysis of gauge symmetry breaking pattern very ambiguous[17]. As seen above, our model is free of these difficulties.

Now let us return to the hierarchy problem. To show that the vacuum given by Eq. (61) can give a correct mass hierarchy, it suffice to calculate masses of gauge bosons. The masses are given by

$$(M^2)_{\alpha\beta} = \tilde{g}^2 \langle S^\dagger \rangle T^\alpha T^\beta \langle S \rangle \quad . \tag{71}$$

From Eq.(61) it is found that

$$\begin{aligned} M^2_{X_\mu^i} &= M^2_{Y_\mu^i} = \frac{25}{2}\tilde{g}^2 m^2 \\ M^2_{W_\mu^\pm} &= \frac{5}{2}\tilde{g}^2 \frac{\lambda_2}{\lambda_8}(m'-m)|\Delta|, \\ M^2_{Z_\mu} &= 4\tilde{g}^2 \frac{\lambda_2}{\lambda_3}(m'-m)|\Delta| \end{aligned} \tag{72}$$

and

$$\frac{M_W}{M_X} = \sqrt{\frac{\lambda_2}{5\lambda_3^2}}\ \frac{\sqrt{(m'-m)|\Delta|}}{m} \quad . \tag{73}$$

So if one takes

$$m = M_X \ , \qquad [\ (m'-m)\ |\Delta|\ \ ]^{\frac{1}{2}} = \frac{\lambda_3}{\sqrt{\lambda_2}} M_W , \tag{74}$$

then the correct gauge hierarchy can be obtained. Of course we do not determine (m'-m) and separately yet. They will be determined later.

At the end of the last chapter we mentioned that in this effective theory, the mass sum rule holds at the tree level. In this model, at the tree level, all the matter fermions are massless. Due to the mass sum rule, one cannot have all scalar partners of quarks and leptons heavier than their fermionic partners, respectively. But the phenomenology teaches us that this kind of tree level mass spectrum of the scalars cannot be realistic. Actually, according to an analysis of existing experimental facts, some lower bounds of selectron and squark masses have been set[25]:

$$m_{\tilde{e}} > 16 \text{ Gev} \quad , \tag{75}$$

$$m_{\tilde{q}} > 15 \text{ Gev} \quad , \tag{76}$$

$$m_{\tilde{\nu}_e} + m_{\tilde{\nu}_\mu} > m_\tau \ , \tag{77}$$

where $\tilde{e}$, $\tilde{q}$ and $\tilde{\nu}$ refer to scalar electron, scalar quark and scalar neutrino, respectively. Otherwise they would have been detected through the following processes:

$$e^+ e^- \longrightarrow \tilde{e}\,\tilde{e} \ ,$$

$$e^+ e^- \longrightarrow \tilde{q}\,\tilde{q} \ ,$$

$$\tau \longrightarrow \tilde{\nu}_e\,\tilde{\nu}_\mu\,\lambda \quad .$$

So from Eqn's. (75),(76) and (77), $\tilde{e}$ and $\tilde{q}$ should be much heavier than their fermionic partners.

To satisfy the phenomenology, one has to generate masses radiatively for $\tilde{e}$ and $\tilde{q}$[12]. In the following we calculate radiative mass corrections for squarks, sleptons and gauginos.

Using the conventional Feynman graphs in component field formalism to calculate is a formidable task for such a semi-'realistic' model . We believe that the most convenient and natural setting of this calculation is in a superfield formalism. In the following calculations we will use the superfield Feynman rules given by Grisaru, Siegel and Rocek in Ref. [25], and its extention in the supersymmetric R_ξ gauge given recently by Ovrut and Wess in Ref.[26].

In Ref. 12 a detailed discussion about calculating the radiative corrections in spontaneously broken SUSY theory was given. Of course in our theory, there are non-supersymmmetric interactions given by Eq.(43). But it is crucial to notice that these interactions can be rewritten in a superfield formalism as

$$A_{sb} = -\left\{\int d^4x\, d^2\theta \left[\frac{\lambda'}{6} \eta_1 T_r \Sigma^2 + \lambda_2 \eta_2 H' H \right] + h.c. \right\}, \quad (78)$$

where $\eta_1 = m\Delta\theta^2 \quad , \quad \eta_2 = m'\Delta\theta^2 \quad , \quad (79)$

and Σ , H and H' are superfields. can be taken as spurion superfields and their appearance breaks supersymmetry. The reason is that in a superfield

Lagrangian, supersymmetry is equivalent to a translation invariance in superspace, the space of x's and θ's. Giving a superfield $\Phi(x, \theta, \bar{\theta})$ a fixed, x-independent, θ-dependent value is equivalent to breaking this translation invariance. So as written in the form of Eq.(75), the softly breaking interactions are taken as a coupling, in a manifestly supersymmetric fashion, between the quantum fields and spurion fields. In this way superfield Feynman rules still can be used.

To calculate graphs, one also needs a suitable regularization technique. In the past, conventional dimensional regularization has been conjectured to give problems for SUSY theories[28]. The reason is that the relative number of bosonic and fermionic degrees of freedom varies with dimension n, so an action which is supersymmetric in four dimensions is not necessarily so in n dimensions. It is proven that a so-called regularization by dimensional reduction preserves the Slavov-Taylor identities of both supersymmetric and gauge invariance[28]. The dimensional reduction technique consists of continuing in the number of space-time dimensions from 4 to n, where n is less than 4, but keeping the numbers of components of all other tensors fixed. In superfield language it can be seen that this procedure is equivalent to performing all the $\mathcal{D}$ algebra in supergraphs in four dimensions, and evaluating the resulting scalar Feynman integrals in n dimensions. In this thesis, this regularization technique is used to calculate supergraphs.

In the following calculations, we use the R_ξ gauge and set $\xi = 1$. Before calculating graphs, one needs to diagonalize the matrices $|\tilde{m}'|^2$ and $\tilde{m}'^2$ simultaneously[27], where the definitions of $|\tilde{m}'|^2$ and $\tilde{m}'^2$ are given in Table I. This is necessary because only in this way will the propagators be diagonal. After a straightforward but tedious calculation, the eigenstates of $|\tilde{m}'|^2$ and $\tilde{m}'^2$ are found and listed in Table I.

Now we are in position to calculate graphs. First consider the radiative mass corrections to the right-handed scalar electron. The main contribution comes from graphs shown in Fig. 2 and Fig.3. Figure 2 gives a contribution

$$-\frac{1}{32\pi^2}\,\frac{e^2\tilde{g}_2^{\,*}}{m_z^4}\,tg^2\theta_W \int d^4x\, d^4\theta\ \langle A_{H5}\rangle\langle A_{H5}^*\rangle\langle F_{H5}\rangle\langle F_{H5}^*\rangle\theta^2\bar{\theta}^2 E_R^* E_R\,, \tag{80}$$

where $\langle A_{H5}\rangle = \frac{1}{\sqrt{2}}\ A_5$, and A_5 is given by Eq. (66), m_z is the mass of the weak Z_μ boson and is given by Eq. (72), $\langle F_{H5}\rangle$ is the VEV of the F component of H_5 field, and

$$\begin{aligned}\langle F_{H5}\rangle &= \left\langle \left(\frac{\partial W}{\partial H_5}\right)^* \right\rangle \\ &= \frac{\lambda_2^{1/2}}{\lambda_1\lambda_2}\sqrt{(m_1 - m_2)|\Delta|}\ \Delta\,.\end{aligned} \tag{81}$$

So after intergation over θ, one gets a contribution to the effective action

$$\int d^4x\ (-\delta\, m^2_{\tilde{e}_R})\ A^*_{E_R} A_{E_R}\,, \tag{82}$$

where

$$\delta m^2_{\tilde{e}_R} = \frac{8}{25\pi^2}\frac{\lambda_3^2\lambda_2^2}{\lambda_1^2}\sin^4\theta_W\cos^2\theta_W m_W^2\frac{|\Delta|}{m'-m} . \quad (83)$$

Here m is the mass of the weak W boson and is given by Eq.(72). In the calculation the relation

$$\tilde{g}_2 = \frac{e}{\sin\theta_W}$$

has been used. If one chooses

$$\frac{|\Delta|}{m'-m} \sim 1 \quad (84)$$

and $\lambda_1 \sim 10^{-2}$, $\lambda_2 \sim 10^{-1}$, $\lambda_3 \sim 1$ (85)

then $\delta m^2_{\tilde{e}_R} \sim 10^{-1} m_W^2$ (86)

Another contribution to the e mass comes from Fig.3. It contributes

$$\int d^4x\, d^4\theta \; (-\delta m^2_{\tilde{e}_R})\; \theta^2\bar{\theta}^2 E_R E_R^+ ,$$

where $\delta m^2_{\tilde{e}_R} = \frac{e^2\tilde{g}_2^2}{16\pi^2} tg^2\theta_W \frac{\langle F_{HS}\rangle^2}{m_Z^2}$

$$= \frac{4}{\pi}\sin^4\theta_W\frac{\lambda_3^2}{\lambda_1^2} m_W^2 \sim 10^{-1} m_W^2 . \quad (87)$$

In addition, Fig. 4 gives a contribution

$$\frac{1}{16\pi^2} e^2 tg^2\theta_W \tilde{g}_2^2 \frac{\langle A^*_{HS}\rangle\langle F_{HS}\rangle}{m_Z^2}\int d^4x\, A_{E_R} F^*_{E_R} .$$

But because

$$F^*_{E_R} = \frac{\partial W}{\partial E_R} = 0$$

in this model, this term really contributes nothing to the $\tilde{e}_R$ mass. It should be noted that Fig.5 is zero, because the integration over θ is zero. On the other hand , because $\tilde{e}_R$ is massless at the tree level, the wave function renormalisation does not contribute to mass correction. Putting all contributions together, one obtains the conclusion that the right-handed scalar electron gets a mass of the order of m_W. What is more, from Eq.(84) and Eq.(72) one finds

$$|\Delta| \sim m_W ,$$

from which it follows that $\quad m_{3/2} \sim m_W \quad . \qquad (88)$

Equation (88) shows that gravitino mass sets the weak interaction scale and the mass scale for the scalar partner of matter fermions. We arrive at the same conclusion as the theories with the minimal coupling do but through a quite different route.

Evidently, the similar graphs will give similar contributions to left handed scalar electron, scalar neutrino and scalar quarks.

At the tree level there are other kinds of massless fermions. They are fermionic partners of gluons and photons: gluino $\tilde{g}$ and photino $\tilde{\gamma}$. In our model, $\tilde{g}$ and $\tilde{\gamma}$ also get their masses radiatively. The main contributions to gluino masses come from the graphs shown in Fig.6-a and Fig.6-b. To do the D's algebra, one can use the identities

$$D^2\bar{D}^2D^2(q) = -q^2D^2$$

and

$$D^2\bar{D}^2 + 2q^{\alpha\dot{\alpha}} D_\alpha \bar{D}_{\dot{\alpha}} = D^\alpha \bar{D}^2 D_\alpha \quad . \qquad (89)$$

Then one gets contributions from Fig.6

$$\int d^4x\, d^4\theta \left(-\frac{\alpha_s}{16}\right)|\Delta|\left(V_j^i D^\alpha \bar{D}^2 D_\alpha V_i^j \right)\theta^2 = \frac{-\alpha_s}{16}|\Delta|\lambda_j^i \lambda_i^j , \qquad (90)$$

where $\alpha_s = \tilde{g}_3^2/4\pi$. So the gluino gets a mass

$$\delta m_{\tilde{g}} = \frac{1}{16}\alpha_s|\Delta| \sim 10^{-1}\alpha_s m_W . \qquad (91)$$

In addition to Fig.6, Fig.7 gives a similar contribution

$$\delta m_{\tilde{g}} \sim 10^{-1}\alpha_s|\Delta| \sim 10^{-1}\alpha_s m_W \qquad (92)$$

As for the photino, the main contributions come from Fig.8 and Fig.9 and a similar caculation leads to a mass

$$\delta m_{\tilde{\gamma}} \sim \frac{\alpha}{10} m_W \tag{93}$$

where $\alpha = e^2/4\pi$ .Because gluinos and photinos are massless at the tree level, their wave function renormalisations make no contribution to the mass corrections.

IV. MORE MODELS

The SU(5) SUSY GUT model discussed in preceding chapter is not realistic yet in two respects: firstly, quark and lepton masses are not explained. Secondly, as we will show that the hierarchy is not stable agaist loop corrections. In this chapter some models will be proposed in an effort to solve these problems. It should be noted that in the effective theories derived from the minimal coupling, these problems also exist and have been recognized recently[29][30]. We will see that our effective theory is no worse off, if no better off, than that with minimal coupling in these respects.

In general, in non-supersymmetric or globally SUSY models, quarks and leptons pick up their masses from Yukawa couplings, through VEV's of Higgs fields. Although this way is far from perfect for explaining matter fermion mass patterns, we can nevertheless try it in our model. So we add Yukawa coupling terms

$$f_1^{ij}\,\epsilon_{uvwxy}\,H^u M_i^{vw} M_j^{xy} + f_{2\,j}^{\;i}\,H'_x M_i^{xy} M'^{j}_y \qquad (95)$$

$$(x,y = 1,2,\ldots 5\;; \qquad i = 1,2,\ldots n_f\,.\,)$$

to the action given by Eq.(44), where M_i^{xy}, M'^{j}_y are the $\underline{10}$ and $\underline{5}^*$ quark-lepton superfields. n_f is the number of generations. Since quarks and leptons are in complex representations, it follows that if they have non-zero VEV's, the D terms may not be zero and this is not energetically favorable for the vacuum. So we suppose

$$\langle M^{xy}_{i} \rangle = \langle M'^{3}_{y} \rangle = 0 \quad . \tag{96}$$

Under this assumption, it is evident that the analysis about the vacuum searching made in the preceding chapter is still valid and leads us to the $SU(3) \times U_{em}(1)$ vacuum given by Eq's. (63)-(66). Then at the tree level the mass of the electron is

$$m_e = \frac{f_1}{\sqrt{2}} A_5 \quad , \tag{97}$$

that of the d quark is

$$m_d = m_e = \frac{f_1}{\sqrt{2}} A_5 \quad , \tag{98}$$

that of the u quark is

$$m_u = \frac{8}{\sqrt{5}} f_2 A_5 \quad , \tag{99}$$

and that of the neutrino

$$m_\nu = 0 \tag{100}$$

As we have shown in the preceding chapter

$$A_5 \sim m_W$$

so $f_1 \sim f_2 \sim 10^{-6}$.

On the other hand, one has at the same time

$$\langle F_{M^5} \rangle = \langle F_{M'^5_x} \rangle = \frac{1}{\sqrt{2}} \frac{\lambda_2}{\lambda_1} A_5 \Delta \delta^5_x \quad , \tag{101}$$

and therefore one has the following terms in the action:

$$\int d^4x \, d^2\theta \left\{ f_1 \langle F_{M5} \rangle \theta^2 (E_R E_L + D^i \bar{D}_i) + 8 f_2 \langle F_{M5} \rangle \theta^2 \bar{U}_i U^i + h.c. \right\} , \tag{102}$$

where E and E are left-handed and right-handed electron superfield, respectively. D^i is the d^i quark superfield and U^i is the u^i quark superfield. These terms are not supersymmetric and cause mass splittings (supergaps) among fermionic, scalar and psudoscalar partners. If the supergaps created by these terms are larger than m_e, m_d and m_u, then it means that at the tree level, some linear combinations of scalar partners of quarks and leptons will have negative masses. This suggests that the assumption

$$\langle M^{xy} \rangle = \langle M'_y \rangle = 0$$

does not hold any more, for it causes color or E.M. symmetry breaking which is unacceptable.

To avoid this situation one should require

$$\left(\tfrac{1}{\sqrt{2}} f_1 A_s\right)^2 > f_1 \langle F_{HS} \rangle$$

and
$$\left(\tfrac{1}{\sqrt{2}} 8 f_2 A_s\right)^2 > 8 f_2 \langle F_{HS} \rangle ,$$

i.e.
$$\frac{1}{m'-m} |\Delta| < \frac{\lambda_1^2}{2\lambda_3^2} f_1^2 \qquad (103)$$

and
$$\frac{1}{m'-m} |\Delta| < \frac{32\lambda_1^2}{\lambda_3} f_2^2 \qquad (104)$$

Once Eq's. (103) and (104) are satisfied, the mass supergaps are less than the supersymmetric mass, and therefore all scalar partners have positive masses. In this case VEV's of scalar quarks and leptons must be zero. But since f_1 and f_2 are extremely small, meeting Eq's. (103) and (104) means an extremely small value of $|\Delta|/(m'-m)$. Then Eq's.(72) and (84) mean very small radiative masses for squarks and sleptons, and a very small mass for the gravitino. Surprisingly, the situation here is very much like that in the minimal coupling[29], although the mechanism from which the problem arises is here very different. In the minimal case, there seems to be no solution of this problem if one insists upon breaking SU(2)xU(1) at the tree level. In our case there may be a way out.

This hope comes from the radiative masses of squarks and sleptons. If the radiative masses are much larger than supergaps, then there will be no trouble with color or E.M. symmetry breaking.

In this model the contributing graphs are not only Fig's. 2 and 3, but also graphs shown in Fig's (10) and (11) involving Yukawa vertices. It should also be noted that because squarks and sleptons have the tree level masses , one also has to calculate the corrections from wave function renormalisations. The graphs for wave function renormalisations are shown in Fig.12. Putting all the contributions together, it is found (in Appendix II) that if one chooses

$$\frac{|\Delta|}{m'-m} \sim 1, \quad \lambda_1 \sim 10^{-2}, \quad \lambda_2 \sim 10^{-1}, \quad \lambda_3 \sim 1,$$

then the main contributions still come from Fig.2 and Fig.3 and

$$\delta\, m^2_{\tilde{e}_R} \sim 10^{-1}\, m^2_W \qquad (105)$$

Therefore the radiative maasses ($\frac{1}{\sqrt{10}} M_W$) are much larger than the supergap at the tree level ($\sim\sqrt{f_1}\ M_W$). So all scalar quarks and leptons have positive masses ($\sim\frac{1}{\sqrt{10}}M_W$). Then one gets

$$\langle M^{x\bar{y}} \rangle = \langle M'_{\bar{y}} \rangle = 0$$

and there is no trouble with color or E.M. breaking any more.

Of course, other ways out are possible. One possibility is that one does not put the Yukawa interactions of eq.(95) in the action like we did in preceding chapter. The quark and lepton masses will be produced radiatively[31]. A way out along this line is under study.

Now we turn to the problem of stability of the hierarchy set at the tree level by Eq's. (73) and (74). One has to make sure that loop corrections will not destroy this hierarchy. Especially one has to watch out for generation of the non-diagonal mass terms

$$\int d^4x \ \mu^2 A_{Ha} A_{H'_a} \qquad (a = 4, 5) \tag{106}$$

in the action, where A_{H^a} and $A_{H'_a}$ are the scalar partners of the Higgs doublets H^a and H'_a, respectively. If these terms are generated with masses much larger than M_W, then the mass hierarchy will be destroyed.

In our model the troublesome contribution comes from Fig.13. It gives a contribution

$$\frac{1}{16\pi^2} \left(\ln \frac{\Lambda^2}{\mu^2} + \ln \frac{m^2_{A'}}{\mu^2} \right) \frac{5}{3} m \Delta \int d^4x \, d^2\theta \, U \ , \tag{107}$$

where $m^2_{A'} = 25 m^2 \lambda_2^2$ and Λ is a cutoff. So this means that in the bare Lagrangian one should have a term

$$\lambda_5 \int d^4x \, d^2\theta \ m \Delta U \ , \tag{108}$$

and Eq.(106) means a renormalisation for λ_5. Here the important point is that

$$\lambda_5 m|\Delta| \sim m|\Delta| \gg m_w^2 . \tag{109}$$

This generated term will cause a non-diagonal mass term

$$\sim m\Delta \int d^4x \; A_{Ha} A_{H'a} .$$

Since $m|\Delta| \gg M_W$, therefore the hierarchy set at the tree level will be destroyed. Besides, Fig.4 gives a contribution

$$\frac{1}{96\pi^2} \frac{m\Delta^2}{\lambda_2} \int d^4x \; A_U . \tag{110}$$

This will cause a shift of A_U and F_U and therefore generate a term

$$\int d^4x \; (m\Delta^2)^{\frac{2}{3}} A_{Ha} A_{H'a} \tag{111}$$

which destroy the hierachy, because

$$(m\Delta^2)^{2/3} \gg \Delta^2 .$$

At first sight , it seems that we can discard the term $\lambda_3 UH'H$ and then there are no graphs like Figs. 13 and 14. But this case, as we have proven in Ref.(16), gives $\sqrt{m|\Delta|} \sim m_w$, so $|\Delta|$ is too small to give radiative masses to squarks and sleptons .

As mentioned before, the models with the minimal coupling also have an unstable hierarchy[30]. It has been recognized that the dimension 3 SUSY breaking operator plays an important role in upsetting the hierarchy[30]. It has also been demonstrated that in order to get a stable hierarchy, a SU(5) SUSY GUT model with the minimal coupling must contain at least $\underline{24}$, $\underline{75}$, $\underline{50}$ and $\underline{\overline{50}}$ representations in the heavy sector of the theory[30].

Thus the Higgs structure in such a model is very complicated.

In our theory there is no dimension 3 SUSY breaking operator. So simpler solutions are expected. Firstly, if one can somehow eliminate Figs. 13 and 14, then hierarchy will be stable. To do so we revised our action to be

$$\lambda_1\left(\frac{m}{2}T_r\Sigma^2+\frac{1}{3}T_r\Sigma^3\right)+\lambda_2(H'\Sigma H+3m'H'H)+\lambda_3(\tilde{H}'\Sigma H + 3m''\tilde{H}'\tilde{H})+\lambda_4 U(\tilde{H}'H+H'\tilde{H})+f_1 M'_x M^{xy}H'_y + f_2\,\epsilon^{uvwxy}M^{uv}M^{wx}H^y, \tag{112}$$

where $\tilde{H}'$ and $\tilde{H}$ are a new pair of $\underline{5}$ and $\underline{5}^*$ fields, $m''\sim m$, and other symbols are the same as previously. Evidently, the vacuum is still $SU(3)xU_{em}(1)$ invariant. But since there are no propagators for $(\tilde{H}'^{\alpha})^{\dagger}H'^{\alpha}_{\;}$, $(\tilde{H}'_4)^{\dagger}H'_4$, $\tilde{H}'_4$ H^4 and H'_4 $\tilde{H}'_4$ the graphs like Figs.13 and 14 cannot be drawn and a stable hierarchy is obtained.

The second way out is to make use of a heavy SU(3)XSU(2)XU(1) singlet. For example , we can use a new superpotential

$$g=\frac{1}{3}\lambda_1\left(T_r\Sigma^3+\frac{m}{2}T_r\Sigma^2\right)+\lambda_2(H'\Sigma H+3m'H'H)+\lambda_3 T_r(\Sigma\Omega^2)+\lambda_4 H'\Omega H+f_1 M^{xy}M'_y H'_x+f_2\,\epsilon^{uvwxy}M^{uv}M^{wx}H^y, \tag{113}$$

where is a $\underline{24}$ representation and other previous notations are kept. Using a perturbation expansion of the scalar potential in powers of , as in the calculation done in the previous chapter, it is found that the vacuum is still $SU(3)xU_{em}(1)$ invariant and

$$\langle \Sigma \rangle = \begin{pmatrix} 2ma & & & & \\ & 2ma & & & \\ & & 2ma & & \\ & & & -3ma+\epsilon_2 & \\ & & & & -3ma-\epsilon_2 \end{pmatrix}, \tag{114}$$

$$\langle H^5 \rangle = \langle H'_x \rangle = \frac{1}{\sqrt{2}} A_5 \delta_x^5 \tag{115}$$

$$\langle \Omega \rangle = \langle M_X \rangle = \langle M^{XY} \rangle = 0 \quad , \tag{116}$$

where $a = 1 - \frac{1}{3\lambda_1}\frac{\Delta}{m} + \left\{ -\frac{2}{3\lambda_1} + \frac{3\lambda_2^2}{8\lambda_1\lambda_4^2}\left[\lambda_1\lambda_2\left(\frac{e}{m}+\frac{1}{\lambda_1}\right)^2 + \frac{e\lambda_1}{3m} - \frac{1}{3}\right]\right\}\frac{\Delta^2}{m^2}$, (117)

$$\epsilon_2 = \frac{\lambda_2^2}{8\lambda_1^2\lambda_4^2}\left[\lambda_1\lambda_2\left(\frac{e}{m}+\frac{1}{\lambda_1}\right)^2 + \frac{e}{3m}\lambda_1 - \frac{1}{3}\right] m\left(\frac{\Delta}{m}\right)^2 , \tag{118}$$

$$A_5^2 = \frac{-5\lambda_2}{2\lambda_1\lambda_4^2}\left[\lambda_1\lambda_2\left(\frac{e}{m}+\frac{1}{\lambda}\right)^2 + \frac{e}{3m}\lambda_1 - \frac{1}{3}\right]\Delta^2 , \tag{119}$$

and e is defined as

$$m' - m = e\frac{\Delta}{m} . \tag{120}$$

This vacuum gives the correct hierachy at the tree level, if one chooses

$$m = M_X , \quad \text{and} \quad |\Delta| \sim M_W$$

On the other hand , all Ω^α_β get masses $\sim$ m from the coupling Tr($\Sigma\,\Omega^2$). In fact, if one parametrizes Ω as

$$\Omega = \begin{pmatrix} \omega_j^i + \sqrt{\frac{1}{15}}\,\Omega_0 & \Omega_{\vec{x}}\ ,\ \Omega_{\vec{y}} & \\ \Omega^{\dagger}_{\vec{x}}\ ,\ \Omega^{\dagger}_{\vec{y}} & \frac{1}{\sqrt{2}}\Omega_3 - \sqrt{\frac{3}{20}}\,\Omega_0 & \Omega^{+} \\ & \Omega^{-} & -\frac{1}{\sqrt{2}}\Omega_3 - \sqrt{\frac{3}{20}}\,\Omega_0 \end{pmatrix} ,$$

then $m^2_{\omega_j^i} = 4\lambda_3^2 m^2 ,$ $\qquad m^2_{\Omega_{\vec{x},\vec{y}}} = \lambda_3^2 m^2$ (121)

$$m^2_{\Omega^\pm} = m^2_{\Omega_3} = 9\lambda_3^2 m^2 \quad \text{and} \quad m^2_{\Omega_0} = \lambda_3^2 m^2 . \tag{122}$$

In this model there are graphs shown in Figs.15 and 16 . They contribute terms like that given by eqs.(107) and (108) :

$$m\Delta\int d^4x\, d^2\theta\, \Omega_0 \quad \text{and} \quad m\Delta^2\int d^4x\, A_{\Omega_0} \tag{123}$$

respectively. But due to the huge mass of $\Omega_0$and it turns out that these terms only cause small shifts of $A_{\Omega_0}$and F_{Ω_0}

$$
\begin{aligned}
A_{\Omega_0} &\longrightarrow A_{\Omega_0} + \Delta \\
F_{\Omega_0} &\longrightarrow F_{\Omega_0} + \Delta^2
\end{aligned}
\tag{124}
$$

Since $|\Delta| \sim M_W$, these small shifts of A_{Ω_0} and F_{Ω_0} only cause changes in the VEV's of A_{HG} and $A_{H'_G}$ at most of the order of m_W, so the tree level hierarchy is preserved.

V. Conclusions

To conclude, we would like to stress again the main points discussed so far. In this thesis an effective theory is derived from the Wess-Zumino non-minimal supergravity which is simpler from the point of view of the superspace action. This effective theory turns out to be a globally SUSY theory with explicit softly breaking interactions. Unlike the effective theory derived from the minimal coupling, there are no dimension 3 operators here and no common mass terms. Some SU(5) models based on this effective theory are investigated, and it is shown that

(i) a unique $SU(3)\times U_{em}(1)$ vacuum with zero cosmological constant can be obtained simply at the tree level. This vacuum is really the energy ground state, and no anti-de Sitter vacuum exists. The weak interaction scale is set by the gravitino mass;

(ii) the gauge hierarchy is set at the tree level, and it can be preserved against loop corrections;

(iii) scalar partners of quarks and leptons, gluinos and photinos get their masses $\sim \frac{1}{\sqrt{10}} M_W$, $\frac{\alpha_s}{10} M_W$ and $\frac{\alpha}{10} M_W$, respectively, from radiative corrections.

All these features show that supergravity may play a significant role in particle physics at presently accessible energies.

Of course, like the models based on the minimal coupling, the models discussed here are not perfect. There are many problems to solve. For example, what is the origin of the gauge hierarchy ? What is the origin of light matter fermion masses? Furthermore, one should also try to examine the full theory of coupled Yang-Mills matter and supergravity, rather than the effective theory only. To do so the number one obstacle is the non-renormalisability of the N=1 supergravity. Some ideas have been proposed to deal with the renormalisation of supergravity[32], but more investigations are needed to see if they really work. In brief, things just began and there is a long way ahead to go.

Appendix I : THE SU(3)xU_{em}(1) INVARIANT VEV'S AND THE VACUUM

The SU(3)xU_{em}(1) invariant VEV's of fields are parametrized by Eq.(61) and the vacuum energy V is given by Eq.(62) in terms of these parameters. Comparing Eq.(61) with Eq.(49), we can expand these parameters in terms of $\xi \equiv \Delta/m$ ($|\xi| \ll 1$) as follows

$$a = 1 + x\,\xi + y\,\xi^2 + \ldots\quad, \tag{A-1}$$

$$\epsilon_2 = r\,\xi + s\,\xi^2 + \cdots\,, \tag{A-2}$$

$$A_5^2 = u\,\xi + v\,\xi^2 + \cdots\,, \tag{A-3}$$

$$S = md + e\,\xi + f\,\xi^2 + \cdots\,, \tag{A-4}$$

where e, d, r, s, u, v, x and y are to be determined from the minimum condition of V. Substituting Eq's.(A-1) through (A-4) back into Eq.(62), we get an expansion for V

$$V = V_1 + V_2 + V_3 + \ldots\quad, \tag{A-5}$$

where V_n is the term propotional to and

$$V_0 = 0\,, \tag{A-6}$$

$$V_1 = \xi\,[\,10\lambda_1 m^4 + \lambda_2^2\, m\, u\,(3-d)^2\,]\,, \tag{A-7}$$

$$\begin{aligned} V_2 = \xi^2[\,&5\lambda_1\lambda_2\, m r u + \tfrac{1}{5}\lambda_2^2 u + 50\lambda_1^2 m^2 r^2 \\ &+ 30\lambda_1^2 m^4 x^2 + 3\lambda_1\lambda_2 m^2 x u + \tfrac{1}{4}\lambda_2^2 u^2 \\ &+ 20\lambda_1 m^4 x + \lambda_2\, m m' u\,]\,. \end{aligned} \tag{A-8}$$

Because $u\xi > 0$, V_1 reaches the minimum at

$$d = 3\,. \tag{A-9}$$

From the conditions

$$\frac{\partial V_2}{\partial x} = \frac{\partial V_2}{\partial u} = \frac{\partial V_2}{\partial r} = 0\,,$$

it follows that

$$60\lambda_1^2 m^4 x + 3\lambda_1\lambda_2 m^2 u + 20\lambda_1 m^4 = 0 , \qquad \text{(A-12)}$$

$$5m\lambda_1\lambda_2 r + (\tfrac{3}{5}\lambda_2^2 + \tfrac{1}{2}\lambda_3^2)u + 3\lambda_1\lambda_2 m^2 x + mm'\lambda_3 = 0 , \qquad \text{(A-13)}$$

$$5\lambda_1\lambda_2 mu + 100\lambda_1^2 m^2 r = 0 \qquad \text{(A-14)}$$

These equations have a solution, which is given by Eqs.(63)-(65). It is easy to verify for this solution that

$$\frac{\partial^2 V_2}{\partial x^2} , \frac{\partial^2 V_2}{\partial r^2} , \frac{\partial^2 V_2}{\partial u^2} , \frac{\partial^2 V_2}{\partial x \partial r} , \frac{\partial^2 V_2}{\partial u \partial x} , \frac{\partial^2 V_2}{\partial u \partial r} \geqslant 0 \qquad \text{(A-15)}$$

So at these values of r, u and x, V_2 reaches its minimum, and this minimum is calculated through Eq.(A-8) yielding the result given by Eq.(67).

Appedix II : THE RADIATIVE MASS OF e_R IN THE CASE WITH YUKAWA COUPLINGS

If there are Yukawa couplings of quarks and leptons with H' and H as given by Eq. (95), then there are new graphs which might contribute to the $\tilde{e}_R$ mass. (Such new graphs do not include those shown in Fig's. 3 and 4.) For example, Fig. 10 (a)gives a contribution

$$-\frac{1}{256\pi^2}\lambda_2^2 f_1^2 \frac{1}{m_{S_2}^2}\int d^4x\, d^4\theta\, \eta^\dagger\eta\, E_R^\dagger E_R$$

$$= \frac{\lambda_2^2 f_1^2}{256\pi^2}(\lambda_2 m'\Delta)^2 \frac{1}{m_{S_2}^2}\int d^4x\, A_{E_R}^* A_{E_R} \quad , \qquad (A\text{-}16)$$

where $\eta = \lambda_2 m'\Delta\theta^2$ is a spurion field, and S_2 is a light eigenvector of $|\tilde{m}'|^2$ and $\tilde{m}^2$ matrices defined in Table I. m_{S_2} is the mass of S_2 and

$$m_{S_2}^2 = \frac{\lambda_2^2}{\lambda_1^2}\Delta^2 + \frac{3}{5}\tilde{g}^2 A_5\Delta \, .$$

So Eq. (A-16) gives a contribution

$$\sim f_1^2\lambda_2^2 m'^2 \, . \qquad (A\text{-}17)$$

This mass is much larger than the contribution from Fig's.3 and 4. But on the other hand, since the F components of Σ have VEV's

$$\langle F_\Sigma\rangle = \langle (\frac{\partial W}{\partial \Sigma})^*\rangle = m\Delta \begin{pmatrix} \frac{2}{3}I_3 & 0 \\ 0 & -I_2 \end{pmatrix} , \qquad (A\text{-}18)$$

there are other contributing graphs shown in Figs. 10(b)--(d). These graphs give similar contributions like that of Fig.10(a) but with the opposite sign. The important point is that all the contributions from Fig's. 10(a)--(d) largely cancel each other, and give a net contribution

$$-\lambda_2^2 f_1^2 |\Delta| (m'-m) A^*_{E_R} A_{E_R} \qquad \text{(A-19)}$$

i.e. $$\delta m^2 \sim \lambda_2^2 f_1^2 |\Delta| (m'-m) \sim \lambda_2^2 m_e^2 \ll m_W^2 . \qquad \text{(A-20)}$$

Evidently this net contribution is negligible. In addition,, the internal S_2 lines can be substituted by R_1 and R_4, which are the other light eigenvectors of the matrices $|\tilde{m}'|^2$ and $\tilde{m}^2$. In this way new graphs are formed and again they give negligible contributions to the mass of $\tilde{e}_R$.

To discuss the mass of $\tilde{e}_R$ we also should consider the possible non-diagonal mass terms. For example, Figs.11(a) can give a contribution

$$\frac{1}{16\pi^2} f_1^2 \frac{\lambda_2^2 m' \Delta^2}{\lambda_1 m^2_{h'_4}} \int d^4x \; A^*_{E_R} F_{E_R} \quad , \qquad \text{(A-20)}$$

where h'_4 is defined in TableI and a non-diagonal propagator

$$\Delta^{h^{4\dagger} h_4'^{\dagger}} = \frac{m'_{h^{4\dagger} h_4'^{\dagger}}}{q^2 (q^2 + |\tilde{m}'|^2_{h^4})} \bar{D}^2 \delta^4(\theta - \theta')$$

is used for the calculating this graph. Since

$$F_{E_R} = \left(\frac{\partial W}{\partial E_R}\right)^* = m_e A^*_{E_L} + \cdots$$

so eq.(A-20) generate a non-diagonal mass term

$$\sim \int d^4x \; m_e^3 \frac{m'}{m_W^2} A^*_{E_L} A^*_{E_R} \qquad \text{(A-21)}$$

This is a large contribution. But again there is a compensating contribution, this time from Fig.11(b) and the net result of Fig's. 11(a) and (b) is

$$\sim \frac{m_e}{m_w} m_{\tilde{e}}^2 \int d^4x\, A^*_{E_L} A^*_{E_R} \tag{A-22}$$

Evidently this is a negligibly small contribution.

Furthermore, because $\tilde{e}_R$ has a tree level mass m_e, then the wave function renormalisation will give a contribution

$$m_{\tilde{e}}^2 \ (Z_3' - 1) \tag{A-23}$$

to the radiative mass. Here Z_3' is the finite wave function renormalisation constant. The graphs contributing to the wave function renormalisation of E_R are shown in Fig's. 12(a--c), and they give

$$\frac{f_1^2}{32\pi^2}\left[\frac{2}{\epsilon} - \gamma_E - \ln 4\pi - \ln \frac{m^2_{S_2, R_1, R_4, h_4'}}{\mu^2} - 1\right] \int d^4x\, d^4\theta\, E_R^+ E_R \,,$$

$$\frac{6 f_2^2}{\pi^2}\left[\frac{2}{\epsilon} - \gamma_E - \ln 4\pi - 1 - \ln \frac{m^2 h_i'}{\mu^2}\right] \int d^4x\, d^4\theta\, E_R^+ E_R \,,$$

$$\frac{-e^2}{16\pi^2}\left[\frac{2}{\epsilon} - \gamma_E - \ln 4\pi - 1 - \ln \frac{m_{\tilde{e}}^2}{\mu^2}\right] \int d^4x\, d^4\theta\, E_R^+ E_R \,, \tag{A-24}$$

respectively, where $m^2_{S_2, R_1, R_4, h_4', h_i'}$ are the masses of S_2, R_1, R_4, h'_4, h'_i given in Table I. Thus it is easy to see that

$$m_{\tilde{e}}^2 \ (Z_3' - 1) \sim \frac{m_{\tilde{e}}^2}{\pi^2} \left(-e^2 \ln \frac{m_{\tilde{e}}^2}{\mu^2} + f_1^2 \ln \frac{m_w^2}{\mu^2} + f_2^2 \ln \frac{m_s^2}{\mu^2} \right) \ll m_w^2 . \tag{A-25}$$

In summary, in this appendix we have proven that the contributions to the radiative mass of e from Fig's. 10, 11 and 12 are negligible, and therefore the main contributions come from Fig's. 3 and 4 .

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Table I

EIGEN-VECTOR	DEFINITION ($\lvert\Delta\rvert \ll m$)	$\lvert\tilde m'\rvert^2$	$\tilde m^2$	$\lvert\tilde m'\rvert^2 + \tilde m^2$
Σ^i_j	Σ^i_j	$25 m^2 \lambda_1^2$	0	$25 m^2 \lambda_1^2$
Σ^i_4, Σ^4_i	$\Sigma^i_4\ ,\ \Sigma^4_i$	$\frac{4}{9}\ \Delta^2$	$\frac{25}{2}\tilde g^2 m^2$	$\frac{25}{2}\ \tilde g^2 m^2$
h^i	$\frac{1}{A}\left\{H^i + \frac{\sqrt{2}A_5}{10m}\ \Sigma^i_5\right\}$	$25\lambda_2^2 m^2$	0	$25\lambda_2^2 m^2$
h'_i	$\frac{1}{A}\left\{H'_i + \frac{\sqrt{2}A_5}{10m}\ \Sigma^5_i\right\}$	$25\lambda_2^2 m^2$	0	$25\lambda_2^2 m^2$
σ^i_5	$\frac{1}{A}\left\{\frac{\sqrt{2}A_5}{10m}H^i - \Sigma^i_5\right\}$	$\frac{\Delta^2}{9}$	$\frac{25}{2}\tilde g^2 m^2$	$\frac{25}{2}\ \tilde g^2 m^2$
σ^5_i	$\frac{1}{A}\left\{\frac{\sqrt{2}A_5}{10m}H'_i - \Sigma^5_i\right\}$	$\frac{\Delta^2}{9}$	$\frac{25}{2}\tilde g^2 m^2$	$\frac{25}{2}\ \tilde g^2 m^2$
h^4	$\frac{1}{B}\left\{H^4 + \frac{\sqrt{2}}{10}\frac{\lambda_2 A_5}{\lambda_1 m}\Sigma^4_5\right\}$	$\frac{\lambda_2^2}{\lambda_1^2}\Delta^2$	$\frac{\tilde g^2}{4}A_5^2$	$\frac{\lambda_2^2}{\lambda_1^2}\Delta^2 + \frac{\tilde g^2}{4}A_5^2$
h'_4	$\frac{1}{B}\left\{H'_4 + \frac{\sqrt{2}}{10}\frac{\lambda_2 A_5}{\lambda_1 m}\Sigma^5_4\right\}$	$\frac{\lambda_2^2}{\lambda_1^2}\Delta^2$	$\frac{\tilde g^2}{4}A_5^2$	$\frac{\lambda_2^2}{\lambda_1^2}\Delta^2 + \frac{\tilde g^2 A_5^2}{4}$
σ^4_5	$\frac{1}{B}\left\{\frac{-\sqrt{2}}{10}\frac{\lambda_2 A_5}{\lambda_1 m}H^4 + \Sigma^4_5\right\}$	$25\lambda_1^2 m^2$	0	$25 m^2\lambda_1^2$
σ^5_4	$\frac{1}{B}\left\{\frac{-\sqrt{2}}{10}\frac{\lambda_2 A_5}{\lambda_1 m}H'_4 + \Sigma^5_4\right\}$	$25\lambda_1^2 m^2$	0	$25\lambda_1^2 m^2$
S_2	$\frac{1}{\sqrt{2}}\left\{H^5 - H'_5\right\}$	$\frac{\lambda_2^2}{\lambda_1^2}\ \Delta^2$	$\frac{2}{5}\tilde g^2 A_5^2$	$\frac{\lambda_2^2}{\lambda_1^2}\Delta^2 + \frac{2}{5}\tilde g^2 A_5^2$
R_1	$\frac{1}{C}\left\{-\frac{\lambda_2 A_5}{10\lambda_1 m}(\sqrt{30}\varphi_0 + \varphi_3) - U - \frac{H^5 + H'_5}{\sqrt{2}}\right\}$	$\lambda_3 A_5^2 - \Delta\frac{\lambda_2\lambda_3 A_5}{\lambda_1}$	0	$\lambda_3^2 A_5^2 - \frac{\lambda_2\lambda_3}{\lambda_1}A_5\Delta$
R_2	$\frac{1}{B}\left\{-\frac{\lambda_2 A_5}{10\lambda_1 m}(H^5 + H'_5) + \varphi_3\right\}$	$25\lambda_1^2 m^2$	0	$25\lambda_1^2 m^2$
R_3	$\frac{1}{D}\left\{\sqrt{\frac{3}{20}}\frac{\lambda_2 A_5}{m\lambda_1}(H^5 + H'_5) + \varphi_0\right\}$	$m^2\lambda_1^2$	0	$m^2\lambda_1^2$
R_4	$\frac{1}{C}\left\{-\frac{\lambda_2 A_5}{10\lambda_1 m}(\sqrt{30}\varphi_0 + \varphi_3) + U + \frac{H^5 + H'_5}{\sqrt{2}}\right\}$	$\lambda_3^2 A_5^2 - \frac{\lambda_2\lambda_3 A_5\Delta}{\lambda_1}$	0	$\lambda_3 A_5^2 + \frac{\lambda_2\lambda_3 A_5\Delta}{\lambda_1}$

where[27]

$$\tilde m'_{ij} = m_{ij} + 2\lambda_{ijk}\langle\phi_k\rangle ,$$

$$\tilde m^2_{ij} = \frac{1}{2}\tilde g^2\, T^\alpha_{ik} T^\alpha_{\ell j}\langle\phi_k\rangle\langle\phi^\dagger_\ell\rangle ,$$

$$A = \sqrt{1 + A_5^2/(50m^2)} ,$$

$$B = \sqrt{1 + \lambda_2 A_5^2/(50\lambda_1^2 m^2)} ,$$

$$C = \sqrt{2 + 8\lambda_2^2 A_5^2/(25\lambda_1^2 m^2)} ,$$

$$D = \sqrt{1 + 3\lambda_2^2 A_5^2/(10\lambda_1^2 m^2)} .$$

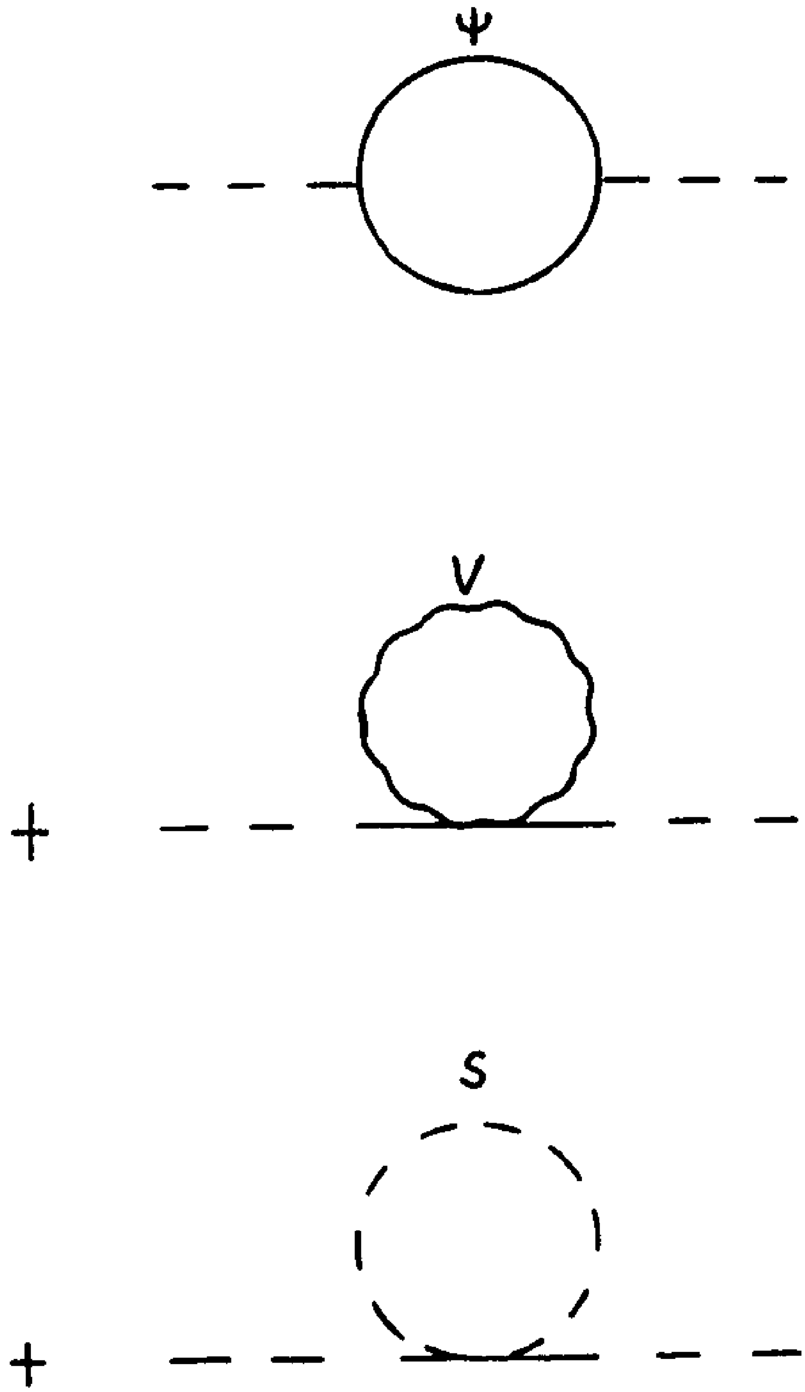
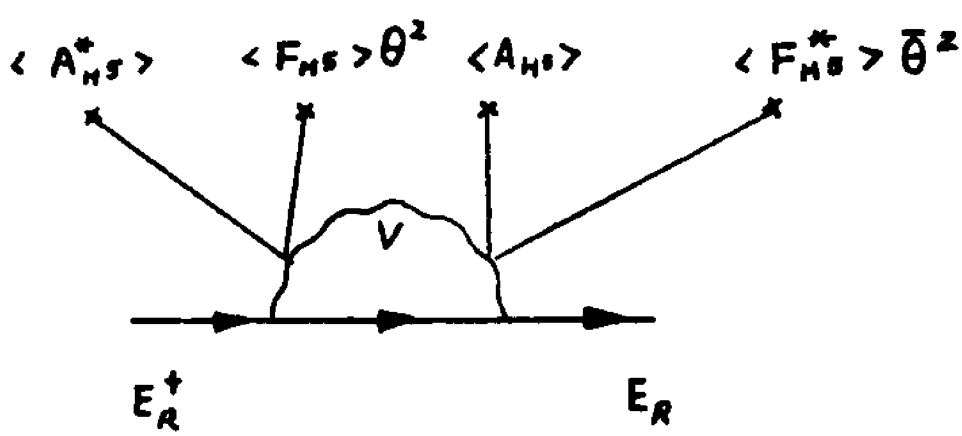
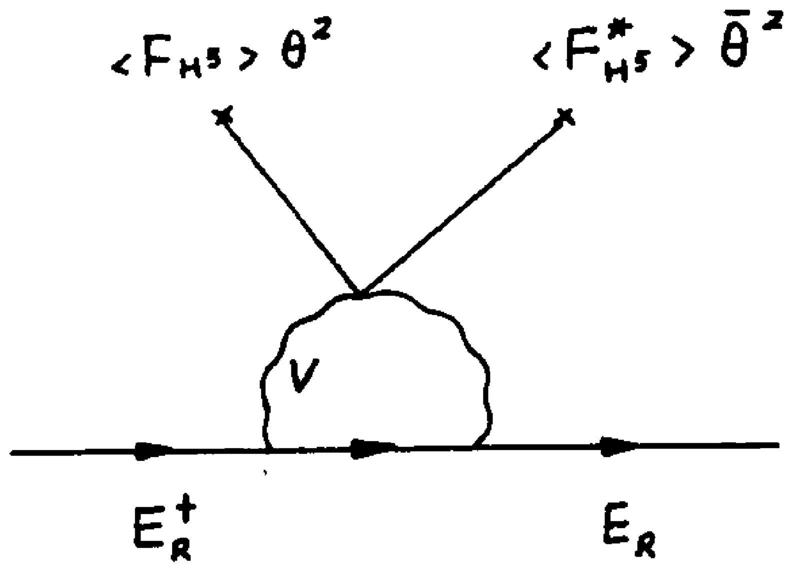


FIG. 1



$\langle A^*_{HS} \rangle$
$\langle F_{HS} \rangle \theta^2$
$\langle A_{HS} \rangle$
$\langle F^*_{HS} \rangle \bar{\theta}^2$
V
$E^\dagger_R$
E_R

FIG. 2



$\langle F_{H^5} \rangle \theta^2$
$\langle F^*_{H^5} \rangle \bar{\theta}^2$
V
$E_R^\dagger$
E_R

FIG. 3

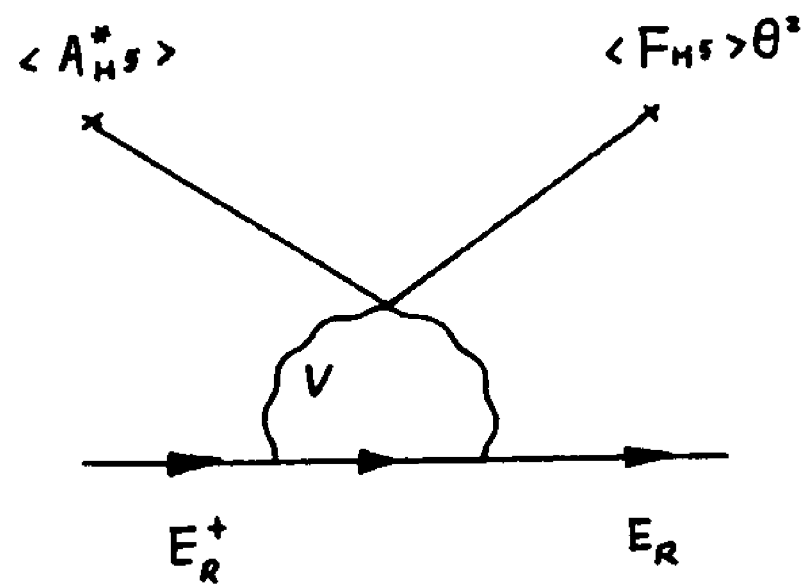


FIG. 4

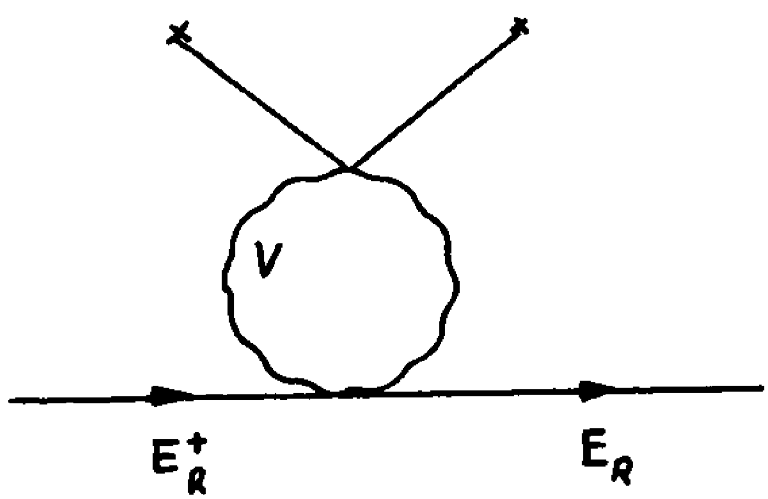


FIG. 5

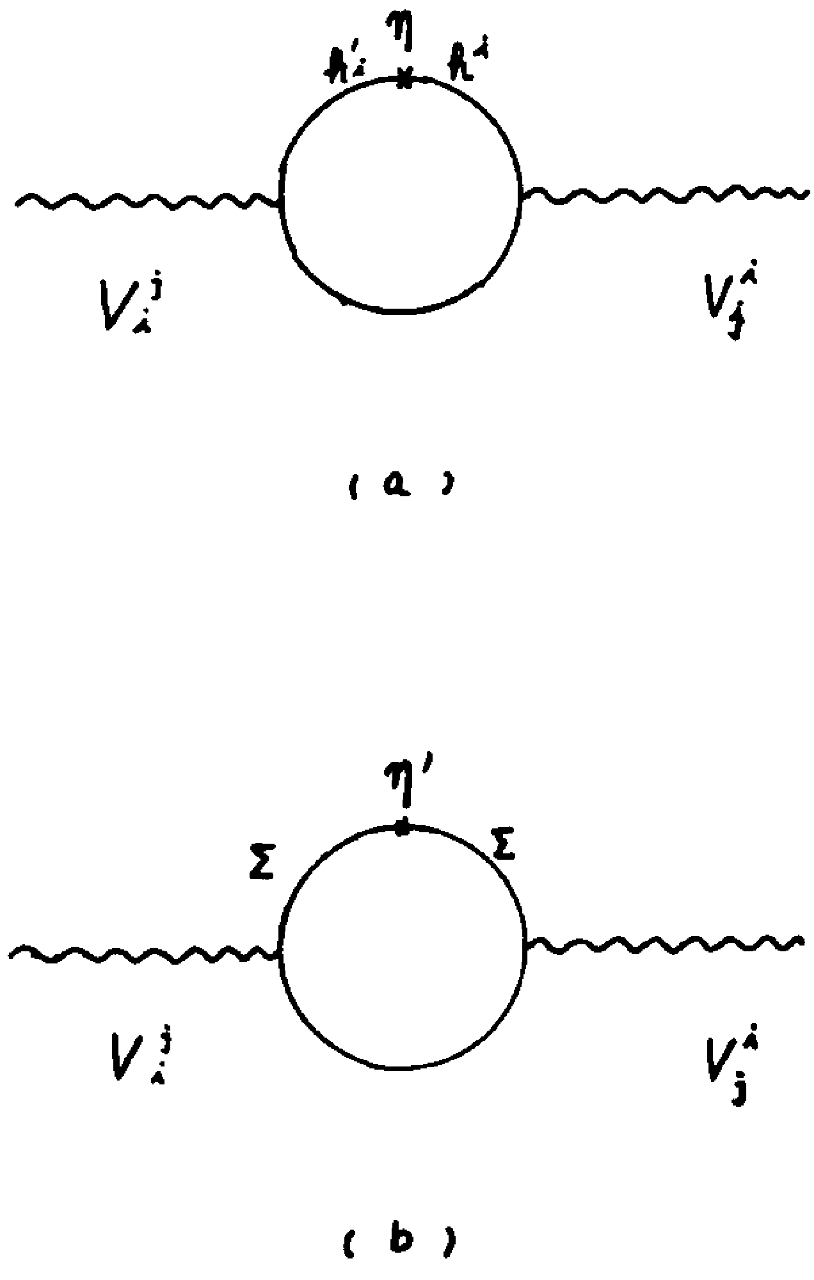
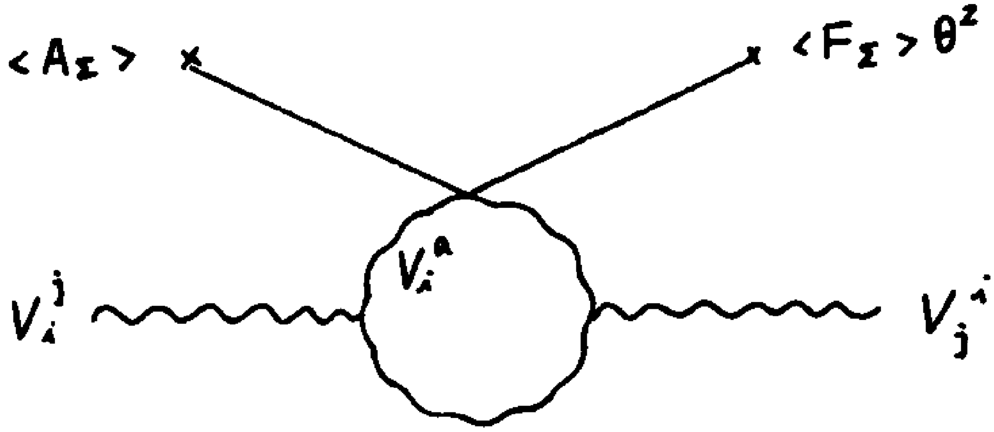


FIG. 6



$\langle A_\Sigma \rangle$
$\langle F_\Sigma \rangle \theta^2$
V_i^a
V_i^j
V_j^i

FIG. 7

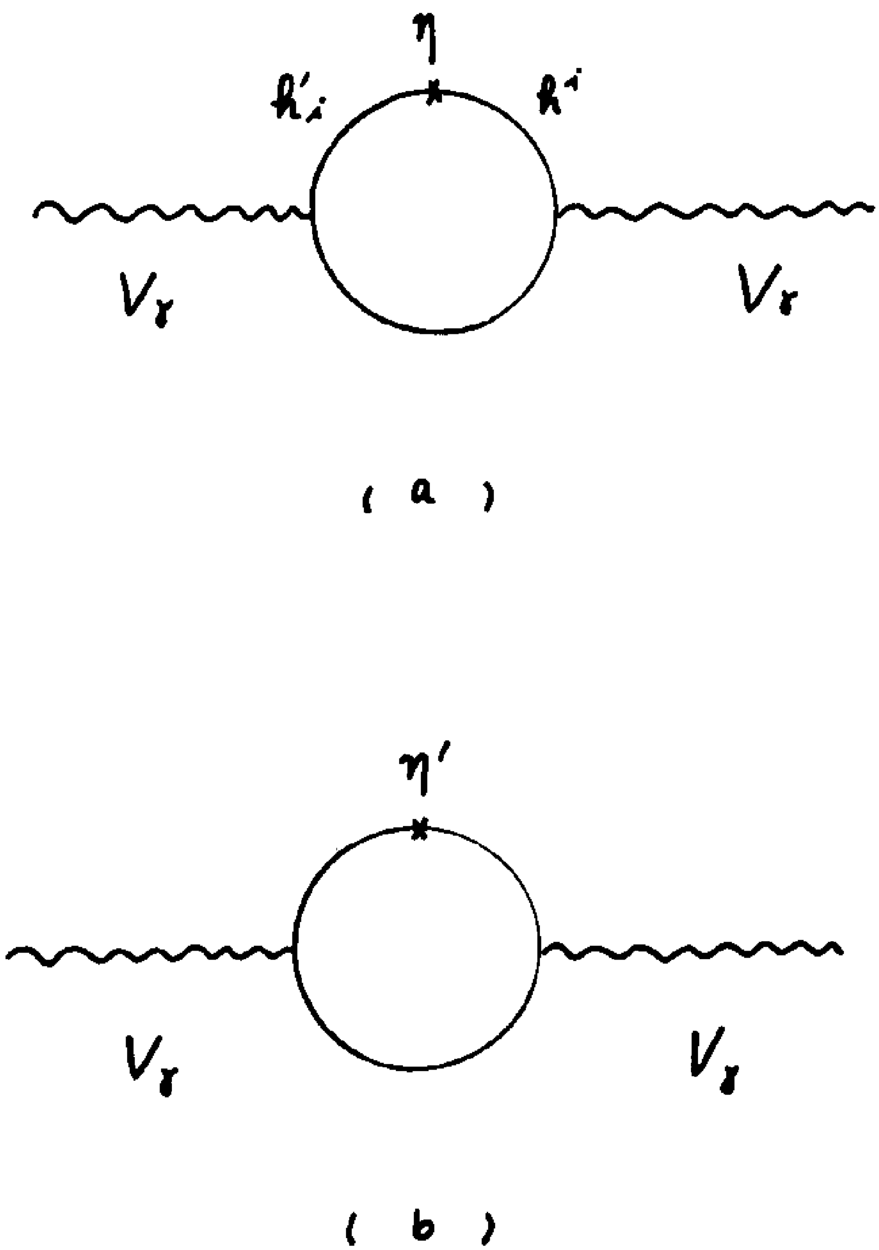


FIG. 8

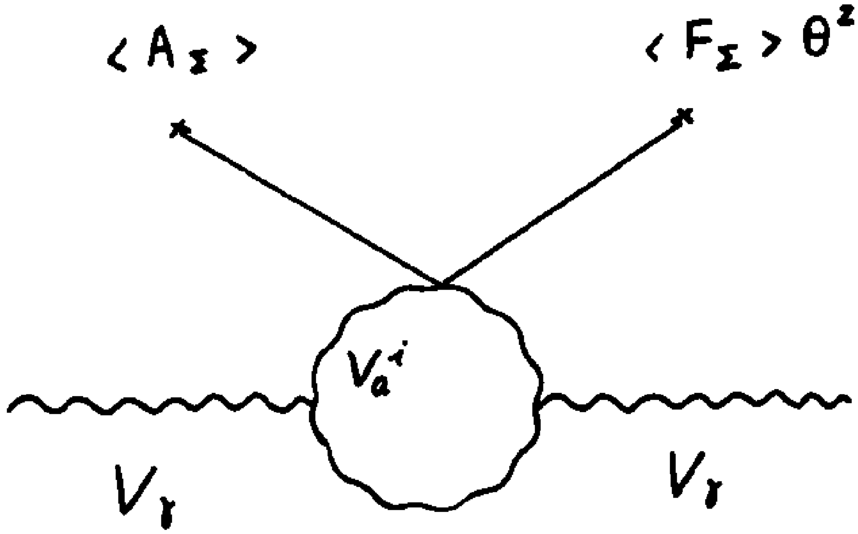
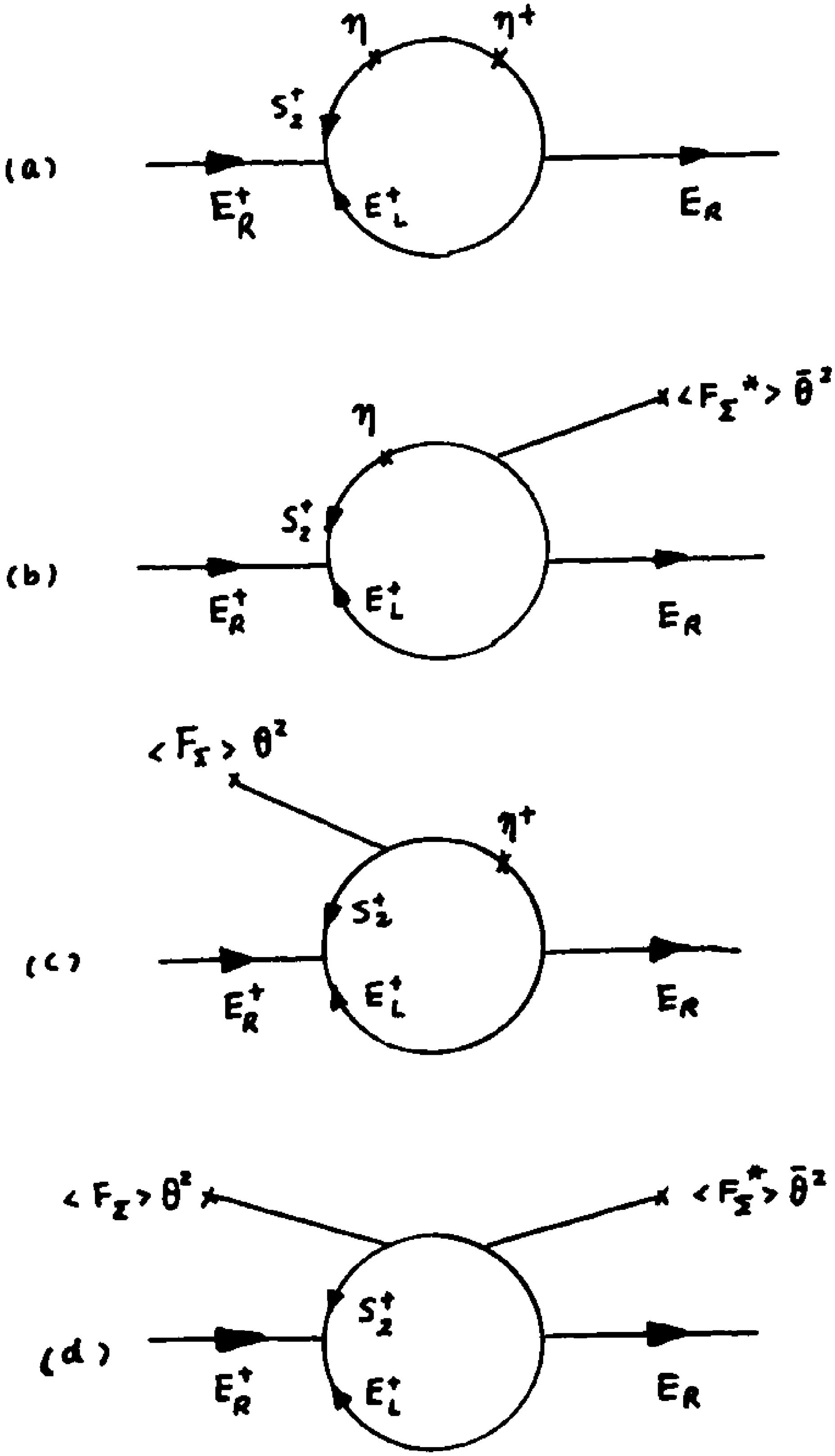

$\langle A_\Sigma \rangle$
$\langle F_\Sigma \rangle \theta^2$
V_a^i
V_γ
V_γ

FIG. 9



(a)
η
η^+
S_2^+
E_R^+
E_L^+
E_R
(b)
η
$\langle F_\Sigma{}^* \rangle \bar\theta^2$
S_2^+
E_R^+
E_L^+
E_R
(c)
$\langle F_\Sigma \rangle \theta^2$
η^+
S_2^+
E_R^+
E_L^+
E_R
(d)
$\langle F_\Sigma \rangle \theta^2$
$\langle F_\Sigma^* \rangle \bar\theta^2$
S_2^+
E_R^+
E_L^+
E_R

FIG. 10

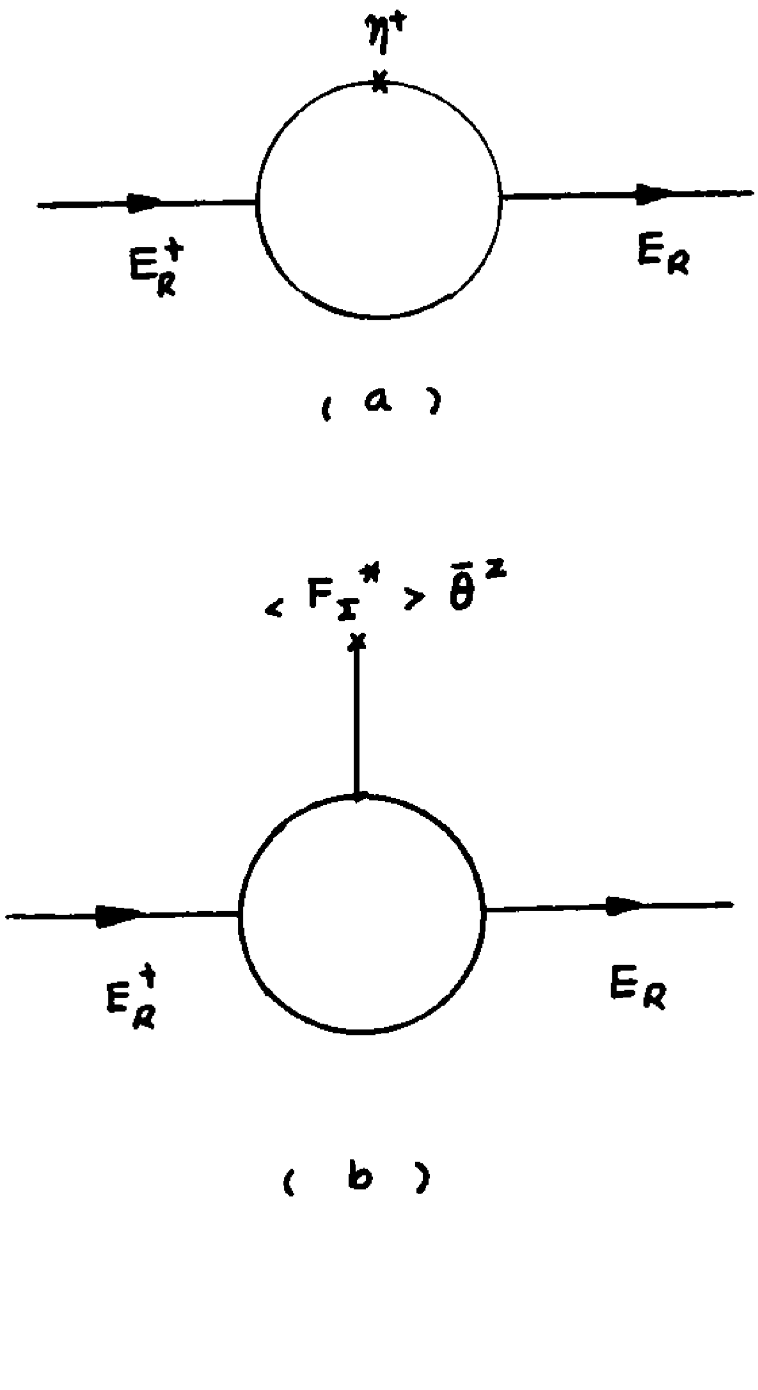


FIG. 11

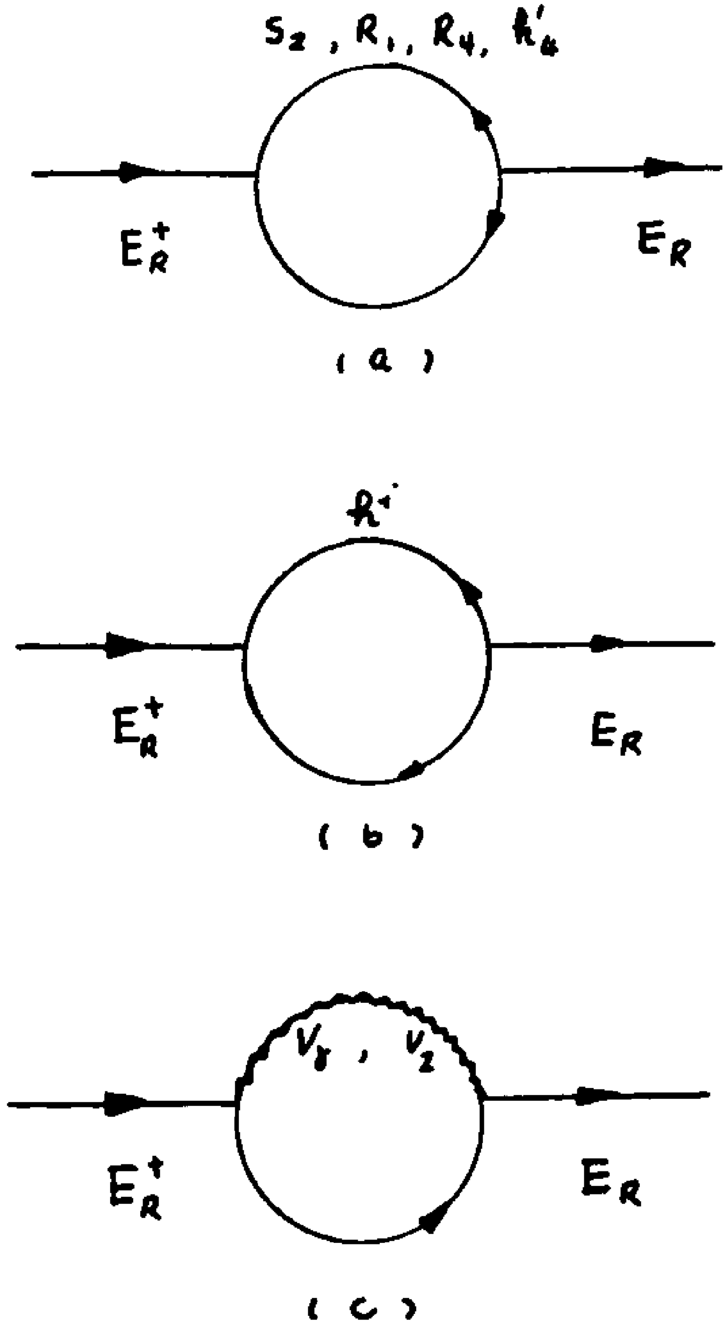
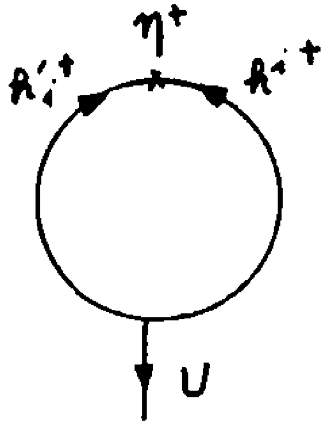
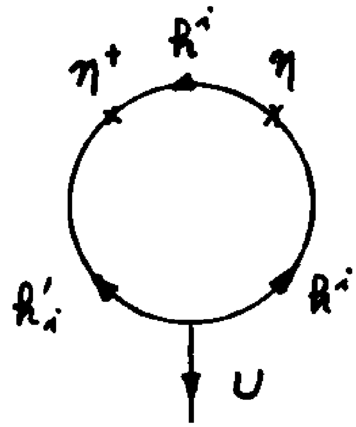


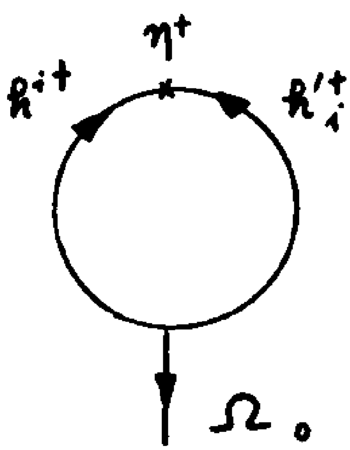
FIG. 12



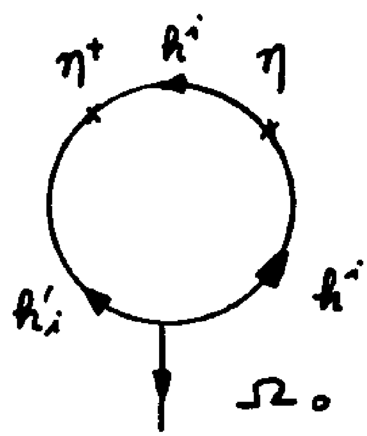
(13)



(14)



(15)



(16)

FIG'S. 13-16