DYNAMIC CONTRACT MODEL UNDER ASYMMETRIC INFORMATION

by

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ABSTRACT

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In this article, I propose a new approach by which incentive-compatible contract models are combined with a matching model to formulate a basic recursive contract model under asymmetric information. This model is expected to contain both incentive problems between principals and agents in "economics of incentive" and intertemporal transitory decision problems in "recursive economics." In this model I provide the traditional contract theory with a new interpretation about reservation utilities prior probability, and describe how the optimal contracts are determined in the recursive context.

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1 Introduction

This article presents and analyzes a dynamic contract model in which a long-run contract under asymmetric information is embeded into a context of recursive or searching model. In this attempt I propose a new approach by which incentive-compatible contract models are combined with recursive models.

Contract theory, a theory analyzed for incentive or mechanism design problems, has been used for many fields of industrial organization. This theoretical approach can be used to study economic structures formed by one-to-one transactions (a principal and an agent) where the traditional market theory cannot be applicable. A category of the contract theory is called adverse selection or hidden knowledge theory and it deals with the relationships between a principal and an agent where there is asymmetric information which can be observed only by the agent. It has been explained there that the principal can design an optimal contract menu offered to the agent so as to induce the agent reveal the hidden information by the revelation principle.

Laffont and Tirole (1993) Ch.9 would be a notable milestone for the adverse selection model. They provide a new framework of contract models for the long-run relationship between a principal and an agent under asymmetric information in the context of procurement. The principal and the agent in their model are assumed to be in a partnership for some economic activity for (fixed) two periods where the principal can offer different contracts in each period. However, if the principal cannot *commit* the second-period contract *ex ante*, it may cause *ratchet effect* by which the efficient type of agent can take opportunistic actions so that the principle cannot squeeze the second-period informational rent for the efficient type. Using a simple model with two types of agents, they show three types of the optimal (semi-)separating contracts by the concept of Perfect Bayesian equilibrium.

While their framework and findings are without doubt an epoch-making study, one may be able to arise some questions when turning one's eyes to more general relationships of dynamic contracts. How and where is such a fixed relationship between the principal and the agent determined? If the contract breaks, how much can the agent have benefit from outside option? Does it payoff in future? Why is the successive contract last for fixed periods? If the contract relationship with a particular principal is required to have some periods, how much does the agent need to bear staying with the contract? Then, for keeping the agent staying with the contract relationship, how much reservation utility of the agent is determined in the optimal contract? Consequently, how would the traditional adverse selection contract model be changed in such contexts to capture more dynamic economic implication?

My work in this article will show some answers for these questions by extending Laffont and Tirole's model into the recursive method. By using the illustration by Stokey and Lucas (1989), *the recursive method* can be defined as a method to solve dynamic optimization problems and have the decision rule where decision-makers must choose a sequence of actions through time to maximize the (expected) present discounted value of their payoff with either deterministic or stochastic states of economic environment. Especially, often, it is of concern where "the best future choices will depend on how much output is available at the time, and that in turn will depend on as-yet-unrealized shocks" (Stokey and Lucas, p.6). The method uses the *Principle of Optimality* by Richard Bellman to solve such problems. Such labor models as McCall (1970) have been studing the job searching problem by the recursive method. This method can be useful for other realms of economic fields as seen in several examples by Stokey and Lucas.

My strategy is as follows: I formulate Laffont and Tirole' type of dynamic contract model for more generic contexts rather than the procurement. I make it as generic as possible so that it can be applicable to non-specific markets. The dynamic contract model is supposed to be set up in a fixed two periods with a particular principal and a particular agent. Then, I formulate a recursive model *on infinite time horizon* where such dynamic contracts are repeatedly arised with different principal-agent relationships formed by matchings. With some sloppy expression, it can be said that a small dynamics model is embeded into a large dynamics model.[1)]

The main contribution which I make in this attempt is, intuitively speaking, to show how different the dynamic incentive-compatible contract in the
recursive context would be from that by Laffont and Tirole. The difference is the optimal effort level for the inefficient type of agent: As I already mentioned, Laffont and Tirole show that with the ratchet effect it is less possible that the equilibrium is full-separating in the first period of the two-period contract model since the efficient agent may not choose the assigned contract with probability one by his opportunistic behavior. This opportunistic behavior induces the principal to design the optimal contract which may bring larger informational rent to the efficient type and imposes even higher effort level on the inefficient type in the first period than in the static model. On the contrary, if the two-period contracts are repeated in different principal-agent relationships and the type of agent may vary in the relationships, the principal may not be able to impose such higher effort on the inefficient type. The inefficient agent who is offered higher effort for the wage by the current principal may think that he should reject the contract and begin to search another principal with some time and cost. As I will show it is possible in my model that the inefficient effort level in the first period of the two-period contract would be even lower than that in the static contract because of "reservation utility effect," as it were. The key point is how the reservation utility is formed by the expected present value of the utility in the future. I will carefully examine in this paper the incentive which causes this

[1)]For more precise notations I use in this article the following names to models.

Static model: the contract model in only one period.

Long-run model: the contract model in two periods at once. This is based on Laffont and Tirole's model.

Recursive model: the dynamic model where two-period contracts are repeated recursively. This model is my contribution in this article.

characteristic in the dynamic contract in the recursive model.

The structure of this article is as follows: In the next section, I will summarize a static contract model under asymmetric information. For the later discussion I will mention so called countervailing incentive problem to classify the models into two types. In Section 3, I follow the basic long-run (two-period) contract model by Laffont and Tirole (1993) Ch. 9 and show the perfect Bayesian equilibrium in my model. (While their model is about public procurement, my model is about more general field in economy to be adapted to any field.) In Section 4, I combine the static contract model with a matching model to have a basic recursive model under asymmetric information. Moreover, I extend the basic model in two directions: In Section 5, I examine countervailing incentive problem and feasibility of the optimal contracts, assuming different resevation profits in agents. In Section 7, I suggest how the results can be changed in the multi-period contract model. In Section 6, I provide some numerical calculations and simulations for the theoretical model to describe how the optimal contracts appear. Finally I summarize what I contribute for the contract theory as the conclusion.

2 The Static Contract Model: A Review

The Standard Model

First, I formulate a standard model where a *principal* contracts with an *agent* for a project for one period. An action by the agent in terms of the project, e, (we call it "effort") is assumed to determine the level of payoff of the principal. It is denoted by π, a function of e. The effort e causes cost (disutility) to the agent but the cost is dependent on a cost-parameter (or *type*) of the agent. The cost-parameter in the two-type model would be θ_L or θ_H ($\theta_H > \theta_L$) and the prior probability that it is θ_L is p and common knowledge. The cost in terms of the project is denoted by $\psi(e;\theta_i)$ ($i = L, H$) and assumed that $\psi(e;\theta_H) > \psi(e;\theta_L)$. Thus, type L is the efficient type and type H is the inefficient type. In this aritcle I ignore a fixed cost and assume $\psi(0;\theta_i) = 0$ ($i = L, H$).

In this contract model, the principal offers a "take-it-or-leave-it" contract which is composed of effort and monetary transfer (reward) from the principal to the agent which is denoted by w. If the contract is accepted by the agent, the payoff of the principal is $\pi(e) - w$ and the payoff of the agent is $w - \psi(e;\theta)$. We suppose the following assumptions.

Assumption 1 *π is continuously differentiable with respect to e, and*

$$\pi(e) \geq 0, \quad \pi'(e) > 0, \quad \pi''(e) \leq 0, \quad \forall e \geq 0. \tag{1}$$

Assumption 2 *ψ is continuously differentiable with respect to e and θ, and*

$$\psi_e(e;\theta_i) > 0, \quad \psi_{e\theta}(e;\theta_i) > 0 \quad (i = L, H). \tag{2}$$

The latter statement in Assumption 2 can be written alternatively:

Assumption 2′ If $e' > e$, then $\psi(e';\theta_H) - \psi(e;\theta_H) - \psi(e';\theta_L) + \psi(e;\theta_L) > 0$.

In the asymmetric-informational model, θ_i $(i = L, H)$ is a private information of the agent which cannot be observed by the principal. So, the objective for the principal is to design an incentive-compatible contract so as to induce the agent to reveal the private information. By the revelation principle, we can focus our attention on such contract as to make the agent to report θ directly (direct mechanism). In this article, I only examine a two-type case. In addition, we assume that the reservation utility of the agent is c for a while and that all players are risk-neutral.

The principal expects that the type of the agent is L with the probability of p $(0 \leq p \leq 1)$. So, by a usual discussion of the contract theory, the optimal *separating contract*, $\alpha = \{(e_L, w_L), (e_H, w_H)\}$, is the solution of the following problem:

Problem 0

$$\max_{\{(e_L,w_L),(e_H,w_H)\}} p[\pi(e_L) - w_L] + (1-p)[\pi(e_H) - w_H] \tag{3}$$

s.t.

$$w_H - \psi(e_H; \theta_H) = c \tag{4}$$

$$w_L - \psi(e_L; \theta_L) = c + \Phi(e_H), \quad \Phi(e) \equiv \psi(e; \theta_H) - \psi(e; \theta_L) \tag{5}$$

$$e_L \geq e_H \tag{6}$$

Here, the utility for type H is c and the utility for type L is $c + \Phi(e_H)$. $\Phi(e_H)$ is called *informational rent.* The solution is derived by the following conditions:

$$\begin{aligned}
&e_L = e_L^{SB} \quad \text{s.t.} \quad \pi'(e_L^{SB}) = \psi_e(e_L^{SB}; \theta_L) \\
&e_H = e_H^{SB} \quad \begin{cases} \text{s.t.} \quad \pi'(e_H^{SB}) = \psi_e(e_H^{SB}; \theta_H) + \frac{p}{1-p}\Phi'(e_H^{SB}) & \text{if} \quad p < 1 \\ = 0 & \text{if} \quad p = 1 \end{cases} \\
&w_L = w_L^{SB} \quad \text{s.t.} \quad w_L^{SB} - \psi(e_L^{SB}; \theta_L) = c + \Phi(e_H^{SB}) \\
&w_H = w_H^{SB} \quad \text{s.t.} \quad w_H^{SB} - \psi(e_H^{SB}; \theta_H) = c
\end{aligned} \tag{7}$$

where SB denotes the "second-best" solution.[2)] Since e_H^{SB}, w_L^{SB} and w_H^{SB} depend on the prior probability p, they are denoted by $e_H^{SB}(p)$, $w_L^{SB}(p)$ and $w_H^{SB}(p)$, respectively. The expected payoff of the principal from the contract is represented as a function with respect to p;

$$W^{AI}(p) = p[\pi(e_L^{SB}) - w_L^{SB}(p)] + (1-p)[\pi(e_H^{SB}) - w_H^{SB}(p)] \tag{8}$$

where I let AI denote "asymmetric-information," following Laffont and Tirole. Here, it turns out that $\frac{dW^{AI}}{dp} > 0$ and $\frac{d^2W^{AI}}{dp^2} > 0$.

For the benchmark case, I also formulate the first-best solution of contract where the principal does observe the agent's type. For $i = L, H$, the problem of the principal is

$$\max_{\{e_i, w_i\}} \quad \pi(e_i) - w_i \tag{9}$$

$$\text{s.t.} \quad w_i - \psi(e_i;\theta_i) = c \tag{10}$$

and the solution is derived as follows:

$$\begin{aligned} e_i = e_i^* \quad & \text{s.t.} \quad \pi'(e_i^*) = \psi_e(e_i^*;\theta_i) \\ w_i = w_i^* \quad & \text{s.t.} \quad w_i^* - \psi(e_i^*;\theta_i) = c \end{aligned} \tag{11}$$

$(i = L, H)$. Here $*$ denotes the "first-best" solution. Note that $e_L^* = e_L^{SB}$ and $e_H^* > e_H^{SB}$. The expected payoff in "full-information" is:

$$W^{FI}(p) = p[\pi(e_L^*) - w_L^*] + (1-p)[\pi(e_H^*) - w_H^*]. \tag{12}$$

Countervailing Incentive and Type-dependent Reservation Utilities

In the standard model, the agent's reservation utility is supposed to be c which is

[2)] For generality, instead of assuming the Inada condition for π, I classify the solution, e_H^{SB}, under two cases of p.

identical for type L and type H. With this assumption the principal could design an optimal contract where the profit of type L is strictly more than its reservation utility, by setting the profit of the type H equal to the reservation utility. However, if these two types' reservation utilities are different, a kind of "inversion" of contracts may happen. This happening depends on which type's individual-rationality constraint condition (or, *participation constraint condition* in more modern words) would hold with equality (*binding*) or would not hold with a strict inequality (*not binding*). This problem has been known as *countervailing incentive* problem.[3)]

This countervailing incentive problem will be quite important later in the argument of this article. As will be shown, in my recursive model the reservation utilities for type L and H are supposed to be dependent on how they evaluate the future expected discounted values. The future expected discounted values may be different in types when the type of the next match will be determined by the current type, which may cause the countervailing incentive problem.

As a preliminary matter, I classify the cases in terms of this problem in type-dependent reservation utilitiy model. First, I revise the constraint conditions for the principal's problem with assuming that the type i's reservation profit is K_i $(i = L, H)$:

$$w_L - \psi(e_L;\theta_L) - K_L \geq 0 \tag{IR_L}$$

$$w_H - \psi(e_H;\theta_H) - K_H \geq 0 \tag{IR_H}$$

$$w_L - \psi(e_L;\theta_L) - w_H + \psi(e_H;\theta_L) \geq 0 \tag{IC_L}$$

$$w_H - \psi(e_H;\theta_H) - w_L + \psi(e_L;\theta_H) \geq 0 \tag{IC_H}$$

where (IR) denotes *individual-rationality constraint condition* and (IC) denotes *incentive-compatibility constraint condition*. The problem is innately which constraints of the

[3)]In other words, it is a *type-dependent reservation utility* situation. See also Laffont and Martimort (2002) Sec. 3.3.

four would be binding. Obviously, at least two of the four constraints must be binding since the principal prefers lower wages. So I classify the cases into the following three cases with optimal conditions for individual-rationality constraint conditions:

Case (I): $K_L < K_H + \psi(e_H;\theta_H) - \psi(e_H;\theta_L)$

(IR_H) is binding but (IR_L) is not.

Case (II): $K_L > K_H + \psi(e_L;\theta_H) - \psi(e_L;\theta_L)$

(IR_L) is binding but (IR_H) is not.

Case (III): $K_H + \psi(e_H;\theta_H) - \psi(e_H;\theta_L) < K_L < K_H + \psi(e_L;\theta_H) - \psi(e_L;\theta_L)$

Both (IR_L) and (IR_H) are binding.

Throughout this article, I call *Model I* the optimal (semi-)separating contract model in *Case (I)*, *Model II* the one in *Case (II)* and *Model III* the one in *Case (III)*. On the line of K_L the ranges where each model appears can be shown in Figure 1. Thus, *Model I* corresponds to the optimal (semi-)separating contract in the usual (type-independent resrvation utility) model and *Model II* does when the countervailing incentive problem arises. I do not describe why the conditions in *Case (I)* and *Case (II)* are optimal conditions in those range. See Kusuda (2004b) or a contract theory textbook.

3 The Long-Run (Two-Period) Contract

Now, we consider a long-run contract model where the principal contracts with the agent for two successive periods. (Here, this model is based on Laffont and Tirole (1993) Ch. 9.) In this model, the principal *cannot commit* to use such a second-period contract as has been anounced in the first period. Thus, the second-period contract must be designed dependent on the contract menu chosen by the agent in the first period. The equilibrium of this model is formulated as the *perfect-Bayesian equilibrium (PBE)*.

This context is explained with regard to “ratchet effect.” The ratchet effect means here that the more efficient type of agent may choose lower effort since if the principal finds the higher effort taken by the agent in the first period and infers that it is efficient, then the principal may impose the agent on higher effort or lower rent in the second period. So, the principal must make the first-period contract so as to reduce such incentive of the agent. In my model I can explain this incentive briefly. Suppose that the principal offers the contract as explained in Section 2 in two periods. Since the incentive-compatibility constraint condition for the efficient type, L, is binding, type L may choose (e_H, w_H) instead of (e_L, w_L) in the first period. If the principal happens to believe that the agent is type H by the action in the first period, it offers (e_H^*, w_H^*) to the agent in the second period. The agent can have $\Phi(e_H^*)$ as the rent in the second period while it would have had no rents if it had chosen another equivalent contract, (e_L, w_L). The key is the *posterior belief* of the principal which is conditional on the action (or the mixed strategy) taken by the agent in the first period. The principal can design a contract such that the second period rent of the efficient type will be determined by the first-period action through the principal's belief.

The model is as follows: The contract offered by the principal in the first period is denoted by $\{(e^0, w^0), (e^1, w^1)\}$ and the contract in the second period is denoted by $\{(e_L, w_L), (e_H, w_H)\}$. Suppose that the mixed strategy played by type L for (e^0, w^0) is x^0 and that the mixed strategy played by type H for (e^0, w^0) is y^0.[4)] Then, in the second period the principal can form the posterior belief for the agent's types by updating the prior probability. Let p^0 denote the posterior belief that the agent's type is L when it observed that the agent had chosen (e^0, w^0) and let p^1 denote the posterior belief that the agent's type is L when it observed that the agent had chosen

[4)] While we may consider another mixed strategy that type L rejects the contract which is not treated by Laffont and Tirole's standard model, the probability can be zero since we can formulate a separate or semi-separating contract where type L does not have incentive to reject it, as will be seen latar. However, we may have to formulate the model so that type H is supposed to take (e^1, w^1) with y^1 and to reject the contract with $(1 - y^0 - y^1)$.

(e^1, w^1). They can be calculated by the Bayes's rule as follows:

$$p^0 = \frac{px^0}{px^0 + (1-p)y^0}, \tag{13}$$

$$p^1 = \frac{p(1-x^0)}{p(1-x^0) + (1-p)(1-y^0)}. \tag{14}$$

If p^0 or $p^1 \in (0, 1)$, the principal cannot fully separate the agent's type on the basis of the observed action taken by the agent. In this sense, the two-period contract is called the *semi-separating contract.* The game tree for this game is shown in Figure 2.

(By "semi-" separating contract, I have in mind the contract where the principal cannot separate the types of agents exactly with probability one. However, it does not rule out *full-separating contract*, i.e., the contract where in the second period the information is perfect, since it just implies $x^0 = 1$ and $y^0 = 0$, which is a special case of semi-separating contract. It should be noted that the principal cannot induce the agent to choose a particular probability as far as the incentive-compatibility constraint condition is binding. See Laffont and Tirole (1993).)

Let denote $\alpha = \{(e^0, w^0), (e^1, w^1); (e_L, w_L), (e_H, w_H)\}$ a separating contract offered to the agent. Therefore, the problem which the principal has to solve is:

Problem SS

$$\begin{aligned}\max_{\alpha} \quad & [px^0 + (1-p)y^0][\pi(e^0) - w^0] \\ & +[p(1-x^0) + (1-p)(1-y^0)][\pi(e^1) - w^1] \\ & +\delta[px^0 + (1-p)y^0]W^{AI}(p^0) \\ & +\delta[p(1-x^0) + (1-p)(1-y^0)]W^{AI}(p^1)\end{aligned} \tag{15}$$

s.t.

$$w^0 - \psi(e^0;\theta_L) + \delta u(p^0) \geq c + \delta \underline{U} \tag{LIR_L}$$

$$w^1 - \psi(e^1;\theta_H) + \delta c \geq c + \delta \underline{U} \tag{LIR_H}$$

$$w^0 - \psi(e^0;\theta_L) + \delta u(p^0) \geq w^1 - \psi(e^1;\theta_L) + \delta u(p^1) \tag{LIC_L}$$

$$w^1 - \psi(e^1;\theta_H) + \delta c \geq w^0 - \psi(e^0;\theta_H) + \delta c \tag{LIC_H}$$

$$u(\tilde{p}) \equiv c + \Phi(e_H^{SB}(\tilde{p})). \tag{16}$$

Here, (LIR_L) and (LIR_H) are *long-run individual-rationality constraint conditions* and (LIC_L) and (LIC_H) are *long-run incentive-compatibility constraint conditions.* δ denotes the discount factor $(\delta \in (0,1))$. If type L accepts the contract, it obtains $w^0 - \psi(e^0;\theta_L)$ in the first period and $u(p^0)$ in the second period, and if it rejects the contract, it obtains $c + \delta\underline{U}$ as the expected and discounted value. $u(\tilde{p})$ is the informational rent which type L obtains in the second period when the posterior belief by the principal is $\tilde{p}$. Similarly, type H obtains $w^1 - \psi(e^1;\theta_H)$ in the first period and c in the second period.

Later in this article $c + \delta\underline{U}$ will turn out quite important while Laffont and Tirole assume that the reservation utilities are zero. So they also assume that the reservation utilities for two types of agents are same and fixed. I shall explain what these values mean for *my* recursive model and make them imply a specific interpretation in the recursive context in the next section. For now I assume that $c + \delta \underline{U}$ is *exogenous* but in the next section it will be *endogenous* for my model.

Unlike the static (short-run) contract model, in the long-run contract model, we cannot figure out which incentive-compatibility constraint condition would be binding. In this article, I consider only the following *semi-separating contract.*[5)]

Type I: Only (LIR_H) and (LIC_L) are binding.

In this case, since (LIC_H) is not binding, type H has no incentive to choose (e^0, w^0). So, the solution for this problem is derived from the the following conditions:

$$\begin{aligned} &e^0 \quad \text{s.t.} \quad \pi'(e^0) = \psi_e(e^0;\theta_L) \\ &e^1 \quad \text{s.t.} \quad \pi'(e^1) = \psi_e(e^1;\theta_H) + \tfrac{px^0}{p(1-x^0)+(1-p)}\Phi'(e^1) \\ &w^0 \quad = \psi(e^0;\theta_L) + (1-\delta)c + \delta\underline{U} + \Phi(e^1) - \delta u(1) + \delta u(p^1) \\ &w^1 \quad = \psi(e^1;\theta_H) + (1-\delta)c + \delta\underline{U} \end{aligned} \tag{17}$$

Finally, we have to check that (LIC_H) is not binding. It is certified if the discount factor, δ, satisfies

$$\delta < \frac{\Phi(e^0) - \Phi(e^1)}{\Phi(e_H^{SB}(p^1)) - \Phi(e_H^{SB}(1))}. \tag{18}$$

From the above results, we have some interesting implications which corresponds to the one in Laffont and Tirole's model. First, with $x^0 < 1$, the optimal effort for the inefficient agent in the first period is larger than in the static model: $e^1 > e_H^{SB}(p)$. Thus, the principal must give the efficient agent larger rent in the first period than in the static model $(\Phi(e^1) > \Phi(e_H^{SB}(p)))$ in order to reduce $u(p^1)$, the second-period rent.

[5)] In general, I can also discuss other two types of the contracts, *Type II* and *Type III*. These types follow the category by Laffont and Tirole. Other cases are as follows.

Type II: (LIR_H) and (LIC_H) are binding.

Type III: (LIR_H) and both (LIC_L) and (LIC_H) are binding.

See Laffont and Tirole (1993), p. 391.

The efficient effort in the first period is identical with the first-best effort ($e^0 = e_L^*$). Thus, the principal must induce the inefficient type to take more effort in the first period by sacrifying the efficient'rent. Second, noting that the pooling contract in the static model must be the first-best effort for type H (say, $e^P = e_H^*$), we can generally conclude that e^1 is not larger than the pooling contract effort: $e^P \geq e^1 \geq e_H^{SB}$ where $e^1 = e^P$ with $x^0 = 0$ and $e^1 = e_H^{SB}$ with $x^0 = 1$. With lower x^0 the distortion is larger and the efficient type's effort approaches to the pooling contract effort. Later I will check these results in the recursive model.

Fixed Length of Contract Periods

Before continuing the description of the long-run contract in the recursive model, I would like to stress that throughout this article the length of the contract periods offered by the principal is assumed to be *fixed*, which follows Laffont and Tirole's model. However, since in principle the principal can design any form of contracts and offer it to the matched agent, the length of the contract period seems endogenous: she may prefer longer periods or shorter, given prior probability or other parameters. Then, it is possible that they would be no reason to set the contract period's length fixed to a number exogenously.

At least, it would be said that if the Perfect Bayesian equilibrium is more separable (more full-separating), the long-run contract may have an advantage. I will briefly check this point by comparing the two-period contract in this section to another contract where the static contract in Section 2 is repeated for two periods with different agents. For one polar case, suppose $x^0 = 1$ (full-separating). Then the principal's long-run payoff is:

$$\begin{aligned}\mathcal{W} &= p\big[\pi(e_L^*) - \psi(e_L^*;\theta_L) - c\big] \\ &\quad + (1-p)\left[\pi(e^1) - \psi(e^1;\theta_H) - c - \frac{p}{1-p}\Phi(e^1)\right] \\ &\quad + \delta W^{FI}(p) \\ &= W^{AI}(p) + \delta W^{FI}(p)\end{aligned} \tag{19}$$

since $e^1 = e_H^{SB}$ with $x^0 = 1$. On the other hand, if the principal chooses to contract with a different agent for each period in two periods, her payoff in the two periods is simply:

$$\mathcal{W} = W^{AI}(p) + \delta W^{AI}(p) \tag{20}$$

Since $W^{FI}(p) \geq W^{AI}(p)$ for any p, the two-period contract with one agent is more profitable for the principal.

For the other case suppose $x^0 = 0$. The Laffont and Tirole's type of contract brings the principal the paypff

$$\begin{aligned}\mathcal{W} &= \pi(e_H^*) - \psi(e_H^*;\theta_H) - c \\ &\quad + \delta W^{AI}(p)\end{aligned} \tag{21}$$

In this case the comparison is difficult since it depends on how large $\pi(e_L^*)$ toward $\pi(e_H^*)$. At least, it can be said that if p approaches to 0 this profit is larger than $W^{AI}(p) + \delta W^{AI}(p)$ but if p approaches to 1 it is smaller.

This result seems not strange and its implication is trivial. If the principal offers the two-period contract, at least she can use information from the observed action taken by the matched agent in the first period, which may bring her a benefit. On the other hand, if the agent with type L chooses a lower possibility to take (e^0, w^0) (i.e.,

x^0 approaches 0), the first-period payoff of the principal in the two-period contract approaches the pooling contract payoff while the second-period payoff goes to W^{AI}. Consequently, the dominance between these contracts is obvious and it highly depends on p and x^0.

Here, I would like to point out two comments. First, even if we can acknowledge that there is the possiblitiy the principle can choose the length of her optimal contract, the model set up with this assumption must be discussed as a completely different formulation. Although I admit that this argument is quite important and interesting, this setting may be an extention of the basic model by Laffont and Tirole.

Second, it is still possible for us to assume that the length of the contract is determined exogenously if we turn our eyes to real situations in economy since in some cases the relationship between a principal and an agent must continue for a fixed length in terms of technology or custom. In procurement, for example, the government may want to have a particular private partner for a fixed years when the public plan must be technically completed after a long periods of time. Or, in a labor market, an employer is supposed to offer a long-run contract to an employee customly. Some baseball players may be enrolled in a particular team and Ph.D. students are required to be registered for several years. It is not difficult to imagine such situations as the length of contracts is not a strategy of the principals.

4 Recursive Model 1 (The Random Model)

4.1 The Outline

Here, I formulate a simplest dynamic model where principals and agents exist infinitely and they repeatedly contract with each other under asymmentric information. Suppose that there are many homogeneous principals and homogeneous agents and that a principal and an agent are matched only once in a period. Now, it is a quite important assumption in my model that the cost-parameter of agents, θ, is not an innate attribute of agents but it is determined by the match between a principal and an agent. Therefore, the type of an agent cannot be observable not only by the principal who will be matched to the agent but also by the agent itself until they meet face to face. Once they are matched, the agent will immediately observe θ while the principal can never do in the whole periods of the contract. In addition, I assume in this section that θ is distributed randomly, identically and independently to eath other and it is independent of time. Thus, the model described here is called "random" model. Then, in the two-type model, an agent expects θ_L with a prior probability, p. In this situation, each agent maximizes its utility, caring about not only a current payoff but also future payoffs. I call such a kind of models "recursive model" and, in this article, I suggest what an asymmetric-informational contract designed in the recursive model would look like in the two-type model as shown below.

First, suppose that every principal and agent exist infinitely. Next, a principal offers such a two-period contract as formulated in the previous section to the matched agent at the initial moment of a period and the agent chooses whether to accept or reject it. If the agent accepts it, the two-period contract will be validated and they will transact bussiness with each other on the project for two successive periods. If the agent rejects it, it will have only c (say, "unemployment compensation") for one period and it will be matched to another principal at the initial moment in the next period.[6] This mechanism is shown in Figure 3.

In this article, I consider three types of contracts which principal can offer to the agent matched in each period; *semi-separating contract*, *pooling contract* and *shutdown contract*. By semi-separating contract I mean what I reviewed in the previous section. Again, I restrict the contract to *Type I* which is defined according to Laffont and Tirole. Thus, between the two incentive-compatibility constraint conditions, the one

[6] It is possible that c is interpreted as the minimum wage rate in the outside market where the agent can look for another employment in case the contract breaks. As far as we focus on the contract in a match in this discussion, such outside market can be generic and given.

for the efficient type of agents alone be binding. As I will show, the reservation utility for each agent is identical. So, from the argument in Section 2, it will be *Model I* where (IR_H) is binding but (IR_L) is not. In pooling contract, the principal offers a same set of effort and monetary transfer to both types of agents in the first period. Since the principal has no information from the action taken by the agent, the second period contract will be same as the separating contract of short-run version. Finally, in shutdown contract the principal offers such a contract as attractive to only the efficient agent. The second period contract will, then, be the first-best contract for type L. (I label this contract as shutdown contarct and highlight it in this article. This contract is also mentioned in Laffont and Tirole's two-period contract model. Note that this contract is not optimal in the static contract model since the principal's payoff from this contract is just a portion of the expected optimal payoff $p[\pi(e_L^{SB}) - w_L^{SB}(p)]$, in Equation (8).)

4.2 Three Types of Contracts in Dynamics

Semi-Separating Contract

This contract follows from what I explained in the previous section. The action of the agent is described as follows: If the agent observes L as its type in a match, if the agent accepts (e^0, w^0) offered by the principal, the agent will have $w^0 - \psi(e^0; \theta_L)$ in the first period and $u(p^0)$ in the second period. At the initial moment in the third period, then, the agent will be matched to a new principal and it can expect a fixed sum of expected payoffs which it is supposed to arise in the future periods since the next type will be determined randomly by assumption. Denote the discounted sum of the expected payoffs in the future periods starting in the third period by EU. Then, when the agent accepts the contract, the expected and discounted expected payoffs in the future periods from the view at the current period is:

$$w^0 - \psi(e^0; \theta_L) + \delta u(p^0) + \delta^2 EU. \tag{22}$$

Similarly, when the agent observes H and it accepts (e^1, w^1), the sum of payoffs will be evaluated as:

$$w^1 - \psi(e^1; \theta_H) + \delta c + \delta^2 EU. \tag{23}$$

If any type of the agents rejects the contract, it will have c in the first period and will be matched in the next period. Then it will have EU since the agent is on infinite time horizon and the next type is independent of the current type. Hence, the discounted expected value of payoffs in the future periods is:

$$c + \delta EU. \tag{24}$$

Therefore, the payoff of an agent is represented by the following the Bellman equation in dynamic programming. Let v denote *value function* when the agent faces payoff U from the contract. Let also $U_L \equiv w^0 - \psi(e^0; \theta_L) + \delta u(p^0)$ denote the payoff which type L gains from the second-period contract and let also $U_H \equiv w^1 - \psi(e^1; \theta_H) + \delta c$ denote the payoff which type H gains. Then, it turns out that $EU = pv(U_L) + (1-p)v(U_H)$. The Bellman equation is:

$$\begin{aligned} v(U) &= \max\left\{U,\ c + \delta(1-\delta)\left[pv(U_L) + (1-p)v(U_H)\right]\right\} \\ &= \max\left\{U,\ c + \delta(1-\delta)EU\right\} \end{aligned} \tag{25}$$

or,

$$v(U) = \max\left\{U,\ c + \delta \underline{U}\right\} \tag{26}$$

where I define a kind of average of expected utility as $\underline{U} \equiv (1-\delta)EU$. Note that we used notation $c + \delta\underline{U}$ as the reservation utility in the two-period contract in the previous section. (I also note that $\underline{U}$ depends on δ.) Thus, I was implicitly assuming that the principal designs the two-period contract so that the agent can choose its long-run optimal action by the above the Bellman equation.

In the two-period contract, $\underline{U}$ can be interpreted as a sort of "average" reservation utility for the second period in the long-run contract. What this utility implies is simple: If the agent accepts the offer by the principal, it will be virtually restricted under the two-period contact since it has no profit from leaving the contract in the second period. Then, it will have EU, the expected future value two periods after. On the contrary, if it rejects, only one period after, it will have the same EU. Thus, the profit from the future value by rejecting the contract is $(\delta - \delta^2)EU = \delta\underline{U}$, which is the present value of $(1-\delta)EU$. For only one-period gap between the two options, the acceptance has has advantage by $\underline{U}$. Later I will show that in multi-period contarcts (T-period contract), for the $(T-1)$ gaps the advantage is $\sum_{t=1}^{T-1} \delta^t \underline{U}$ and that it can be devided into $(T-1)$ terms.

Consequently, we have the following proposition:

Proposition 1 *In the two-type recursive model under asymmetric information, any type agent accepts the contract offered by the matched principal in an equilibrium where the value function is such that* $v(U_L) = U_L$ *and* $v(U_H) = U_H$.

Here, it turns out that $v(U_L) > c + \underline{U} = v(U_H)$. Furthermore, the existence and the uniqeness of the solution v can be proved mathematically. I provide a quick proof by elementary mathematics in Appendix B.

By this proposition, we can calculate the long-run reservation utility on the assumption that every principal forms the optimal contracts in the same manner, which is a common knowledge to every player. Facing the offered contract in the current period, thus, the agent will expect the same profits at the next matching in case of the break of the current contract:

$$\begin{aligned} U_L &= (1-\delta)c + \delta\underline{U} + \Phi(e^1) + \delta u(p^1) \\ &= c + R + \delta\underline{U} \end{aligned} \tag{27}$$

where $R \equiv \Phi(e^1) + \delta u(p^1) - \delta c$,[7] and

$$U_H = c + \delta\underline{U}. \tag{28}$$

Plugging these profits into $\delta\underline{U} = \delta(1-\delta)[pU_L + (1-p)U_H]$, we have

$$\delta\underline{U} = \frac{\delta(1-\delta)}{1-\delta(1-\delta)}\Big[p\Big(\Phi(e^1) + \delta u(p^1) - \delta c\Big) + c\Big] \tag{29}$$

or

$$\delta\underline{U} = \beta(pR + c) \tag{30}$$

where $\beta \equiv (\delta(1-\delta))/(1-\delta(1-\delta))$.

Here, I note that since R is dependent on e^1 through $\Phi(\cdot)$, $\delta\underline{U}$ is also dependent on e^1 and increasing with higher e^1. The reason is clear: If the principal offers higher e^1, both U_L and U_H are higher and more attractive. However, much higher U_L and U_H may induce the agent to expect higher future value with the belief that the new principal in the next matching will also offer much higher utilities, which is based on the assumption that they will meet an identical principal. This attractive future makes the reservation utility higher and so the principal will suffer. In this sense the principal can coordinate the belief of the agent for the future. The reservation utility $\underline{U}$ will be endogenous in this model and may be different from the solution in the previous section.

Noting that R varies as the optimal contract effort for the inefficient agent, e^1,

[7] Here, R denotes the "long-run informational rent."

varies, we can formulate a dynamic version of the optimal contract problem with the endogeneous reservation utility. Since (LIR_H) and (LIC_L) are binding in *Type I*, the contract wages are:

$$w^0 = \psi(e^0;\theta_L) - \delta u(p^0) + c + R + \delta\underline{U},$$
$$w^1 = \psi(e^1;\theta_H) + (1-\delta)c + \delta\underline{U}.$$

Plugging these w^0 and w^1 into *Problem SS* at Page 12, the problem becomes:

Problem SS′

$$\begin{aligned} \max_{e^0,e^1} \quad & px^0\big[\pi(e^0) - \psi(e^0;\theta_L) - c\big] \\ & +(1-px^0)\left[\pi(e^1) - \psi(e^1;\theta_H) - c - \frac{p(x^0+\beta)}{1-px^0}\Phi(e^1)\right] \\ & -p(x^0+\beta)\delta\Phi(e_H^{SB}(p^1)) + (\delta-\beta)c \\ & +px^0\delta W^{AI}(p^0) + (1-px^0)\delta W^{AI}(p^1) \end{aligned} \tag{31}$$

Here, it is helpful to define a function of e as follows:

$$V_H(e;\gamma) \equiv \pi(e) - \psi(e;\theta_H) - c - \gamma\Phi(e) \tag{32}$$

where γ is a parameter. For a $\gamma \geq 0$, the optimal e is derived as:

$$\pi'(e) - \psi_e(e;\theta_H) - \gamma\Phi'(e) = 0 \tag{FOC}$$

and

$$\pi''(e) - \psi_{ee}(e;\theta_H) - \gamma\Phi''(e) < 0 \tag{SOC}$$

where the lower condition must hold by assumptions. For the optimal e it turns out:

$$\left.\frac{de}{d\gamma}\right|_{FOC} = -\frac{1}{SOC}\big[-\Phi''(e)\big] < 0. \tag{33}$$

Now, the optimal contract effort, $\bar{e}^1$, of the above problem must be the same as:

$$\bar{e}^1 \equiv \arg\max_e \; V_H\left(e;\; \frac{p(x^0+\beta)}{1-px^0}\right) \tag{34}$$

where I put "–" (bar) on e^1 to distinguish it from the optimal solution in the exogenous reservation utility model in Section 3 which corresponds to Laffont and Tirole's model. From (SOC) the uniqeness of $\bar{e}^1$ is guaranteed. The optimal e^0 is identical with e_L^* in Equation (11). Hence, the profit of the optimal contact in two periods is:

$$\begin{aligned}\mathcal{W}^{SS} &= px^0\left[\pi(e_L^*) - \psi(e_L^*;\theta_L) - c\right]\\ &\quad + (1-px^0)\left[\pi(\bar{e}^1) - \psi(\bar{e}^1;\theta_H) - c - \frac{p(x^0+\beta)}{1-px^0}\Phi(\bar{e}^1)\right]\\ &\quad - p(x^0+\beta)\delta\Phi(e_H^{SB}(p^1)) + (\delta-\beta)c + px^0\delta W^{AI}(p^0) + (1-px^0)\delta W^{AI}(p^1)\end{aligned} \tag{35}$$

where SS denotes "semi-separating" payoff.

Pooling Contract

It is possible that the principal offers a pooling contract where a same set of (e, w) to both types of agents. The problem is, then:

<u>*Problem P*</u>

$$\max_{(e^P,w^P)} \quad \pi(e^P) - w^P + \delta W^{AI}(p)$$

$$\text{s.t.} \quad w^P - \psi(e^P;\theta_H) + \delta c = c + \delta\underline{U}.$$

Thus, (IR_H) must be binding since $\psi(e^P;\theta_H) > \psi(e^P;\theta_L)$. In this case, the wage and the utilities of agents are:

$$w^P = \psi(e^P;\theta_H) + (1-\delta)c + \delta\underline{U} \tag{36}$$

$$U_L \equiv w^P - \psi(e^P;\theta_L) + \delta u(p) = c + R^P + \delta\underline{U} \tag{37}$$

$$U_H \equiv w^P - \psi(e^P;\theta_H) + \delta c = c + \delta \underline{U} \tag{38}$$

where $R^P \equiv \Phi(e^P)+\delta u(p)-\delta c$. In the same deduction in the semi-separating contract model, the reservation utility can be derived as:

$$\delta \underline{U} = \beta(pR^P + c) \tag{39}$$

where $\beta = \delta(1-\delta)/(1-\delta(1-\delta))$.

Again, we can use $V_H(e;\gamma)$ to have the optimal solution. The problem is:

$$\max_{e^P} \quad \left[\pi(e^P) - \psi(e^P;\theta_H) - c - \beta p \Phi(e^P)\right] - \beta p \delta \Phi(e_H^{SB}) + (\delta - \beta)c + \delta W^{AI}(p) \tag{40}$$

and then the optimal effort is derived as:

$$\bar{e}^P \equiv \arg\max_e \; V_H(e;\beta p) \tag{41}$$

where $\pi'(\bar{e}^P) - \psi_e(\bar{e}^P;\theta_H) - \beta p \Phi'(\bar{e}^P) = 0$. Hence, the optimal profit of the principal is:

$$\mathcal{W}^P = \left[\pi(\bar{e}^P) - \psi(\bar{e}^P;\theta_H) - c - \beta p \Phi(\bar{e}^P)\right] - \beta p \delta \Phi(e_H^{SB}(p)) + (\delta-\beta)c + \delta W^{AI}(p). \tag{42}$$

Shutdown Contract

For the third case, suppose that p is too high and that the principal would want to shut out the inefficient type from the contract.

The problem is:

Problem SD

$$\max_{(e^S,w^S)} \quad p\left[\pi(e^S) - w^S\right] + p\delta W^{FI}(1) \tag{43}$$

$$\text{s.t.} \quad w^S - \psi(e^S;\theta_L) + \delta c = c + \delta \underline{U} \tag{44}$$

$$w^S - \psi(e^S;\theta_H) + \delta c < c + \delta \underline{U}. \tag{45}$$

Then, the contract wage and the utilities of agents from the contract are:

$$\begin{aligned}
w^S &= \psi(e^S;\theta_L) + (1-\delta)c + \delta\underline{U} \\
U_L &\equiv w^S - \psi(e^S;\theta_L) + \delta c = c + \delta\underline{U} \\
U_H &\equiv w^S - \psi(e^S;\theta_H) + \delta c = -\Phi(e^S) + c + \delta\underline{U}.
\end{aligned}$$

Here we must examine how the value function is derived from the Bellman equation carefully. The value function must be defined as:

$$v(U) = \max\{U, c + \delta\underline{U}\}.$$

From the above constraints,

$$\begin{aligned}
v(U_L) &= c + \delta\underline{U} = U_L \\
v(U_H) &= c + \delta\underline{U} > U_H
\end{aligned}$$

since if the inefficient agent rejects the contract offer she can have $c + \delta\underline{U}$ outside in the following two periods. So,

$$\begin{aligned}
\delta\underline{U} &\equiv \delta(1-\delta)EU \\
&\equiv \delta(1-\delta)\big[pv(U_L) + (1-p)v(U_H)\big] \\
&= \delta(1-\delta)(c + \delta\underline{U}).
\end{aligned}$$

Hence, $\delta\underline{U} = \beta c$. The optimal profit of the principal is:

$$\mathcal{W}^{SD} = p\big[\pi(\bar{e}^S) - \psi(\bar{e}^S;\theta_H) - c + \Phi(\bar{e}^S)\big] + p(\delta - \beta)c + p\delta W^{FI}(1) \tag{46}$$

where *SD* denotes "shutdown" contract payoff. The solution, $\bar{e}^S$, is determined as:

$$\bar{e}^S \equiv \arg\max_e \; V_H(e; -1). \tag{47}$$

Thus, $\bar{e}^S = e_L^* \equiv \arg\max_e[\pi(e) - \psi(e; \theta_L) - c]$.

4.3 Some Comparisons and Implications

The above results are summarized as the following proposition.

Proposition 2 *With $x^0 < 1$ and $0 < p < 1$,*

$$\max\left\{\bar{e}^1, e_H^{SB}\right\} < \bar{e}^P < \bar{e}^S$$

where e_H^{SB} is the static contract solution for e_H.

Proof. From (33) the optimal solution for e is decreasing in parameter γ. Here,

$$e_H^{SB} = \arg\max_e V_H\left(e; \frac{p}{1-p}\right) \tag{48}$$

from (7). Since $\min\{\frac{p(x^0+\beta)}{1-px^0}, \frac{p}{1-p}\} > \beta p > -1$ with $x^0 < 1$ and $0 < p < 1$, the proposition is derived from (34), (41), (47) and (48). □

Now we can find some implications in the recursive model, comparing with Laffont and Tirole's model. In my model, which is larger between $\bar{e}^1$ and e_H^{SB} is ambiguous while in Laffont and Tirole the optimal solution for the inefficient type's effort in the first-period contract is larger than in the static contract (i.e., $e^1 \geq e_H^{SB}$ in Section 3 in this article). It is noteworthy that with higher x^0, $\bar{e}^1$ may be even smaller than e_H^{SB} since $\beta \equiv \frac{\delta(1-\delta)}{1-\delta(1-\delta)} > 0$. I provide the order in optimal e's when $x^0 = 1$ and $0 < p < 1$ in Table 1.

e	e_L^*	$=$	$\bar{e}^{SD}$	$>$	e_H^*	$>$	$\bar{e}^P$	$>$	e_H^{SB}	$>$	$\bar{e}^1$
γ	-1	$=$	-1	$<$	0	$<$	βp	$<$	$\frac{p}{1-p}$	$<$	$\frac{p(1+\beta)}{1-p}$

$$V_H(e;\gamma) \equiv \pi(e) - \psi(e;\theta_H) - c - \gamma\Phi(e).$$

Table 1: The Optimal Efforts

The reason of this smaller $\bar{e}^1$ can be explained as a sort of "reservation utility effect." In this recursive model, the principal must pay some rent to not only type L but also type H through the reservation utility: $U_H = c + \delta\underline{U}$. Without this resrvation utility effect the principal must impose harder effort on the inefficient agent than in the static model, as I mentioned at Page 14. However, in this recursive context, the higher effort may induce the inefficient agent to leave the contract since it may expect a larger profit in future in case that it may be "reborn" as the efficient type.

I emphasize in this article that there might be possibly a counter restraint toward the participation constraint for the inefficient agent in the recursive context. It is interesting to say that in this model the principal can manipulate the agents' belief on the future payoffs by offering the utilities from the current contract while in the original Laffont and Tirole's model (and in my model, too) the agent can manipulate the principal's belief on the agent's type.

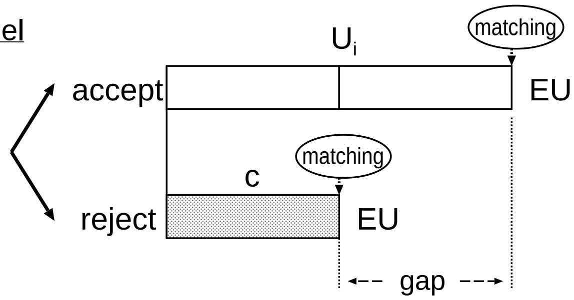
Additionally, with the existence of $\beta > 0$, the pooling contract solution for the inefficient type's effort, $\bar{e}^P$, is not identical with the first-best effort for the inefficient's type in the static model, e_H^*.

Fortunately to the principal, the maximum of β is not so high: in the two-period contract model, it cannot be larger than one fourth. So, which contract of the three types is optimal for the principal is still ambiguous and it depends on the profit and cost functions and the discout factor, δ (and so β).

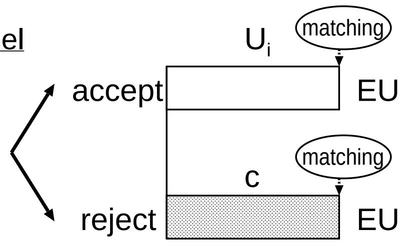
Discussion

Now, I show more verbally what this result can imply in the semi-separating contract. To make it precise, consider another model where *one-period* contract is repeated infinitely. The two mechanisms are as follows:

The 2-period Model



The 1-period Model



If the contract period is fixed to two periods, then the agent's choice and payoff is such as the top of the figure. In case the agent accepts the offer, he can have utility U_i $(i = L, H)$ for the two periods. After that he will face a new match and have EU, the maximal future value of utility when he takes the optimal choice at the moment. In case he rejects, he must wait for one period with unemployment insurance c since there is no more chance of matching for the period by the assumption. The difference is that in the latter case he can face a new match and have EU one period earlier than in the former case. So, with discount factor $\delta \in (0, 1)$, $\underline{U} = (1 - \delta)EU$ must have an important meaning. This $\underline{U}$ is interpreted as the value from being outside the contract for one period.

Let us consider next the single-period contract. Since the contract continues for only one period and so does the unemployment, whether the agent acceepts the contract offer or not, he will face a new match and have a same EU one period after in any case. Then the result is trivial. The agent's problem is:

$$\max\{U_i + \delta EU,\ c + \delta EU\}$$

which is equivalent to

$$\max\{U_i,\ c\}$$

which is just the ordinal decision rule in the static contract model since EUs (and also $\underline{U}$s) are canceled out. (Needless to say, $U_i \geq c$ means the *IR* or *participation constraint* in the static contract model.)

The key of this inference is "the gap problem" as it were. If the contract continues for two or more periods, the agent who accepts the offer will be locked in the contract for a while with seeing that EU is depreciating during the periods. On the contrary, there is no gap of the timing to get EU between in acceptance and in reject in the single-period model. If the patience of the agent being in this contract for periods is lower, the gap problem must be more serious. It must make the agent more careful for the contract and it may let him leave the principal to get another chance earlier. This point of view may possibly shed a new light on the traditional contract model under asymmetric information. In the traditional argument, the principal must give the efficient type informational rent in the optimal incentive-compatible contract, which the effort for the inefficient type must be scqueezed. However, with the gap problem, the lower utility guaranteed for the inefficient type may leave him easily if the successive contract is embeded in the recursive context. The informational rent in this case will be lower ($\Phi(\bar{e}^1)$).

In addition, this gap problem will be more serious in multi-period models. One can easily imagine that the longer the peirod of the contract would become, the bigger the effect of the gap would be on the results of the model. I will discuss it in Section 7.

5 Recursive Model 2: (The Markov Model and Countervailing Incentive)

5.1 The Countervailing Incentive Problem

In Section 5, I extend the basic recursive model designed in Section 4 by loosening the assumption of random matching types. The main discussing point is countervailing incentive problem introduced in Section 2. I examine whether the optimal contracts would be feasible or not.

First, I abandon the assumption of the identical reservation utilities of agents. Suppose instead that the long-run reservation utility of type L is $c + \delta\underline{U}(L)$ and that of type H is $c + \delta\underline{U}(H)$. Since each reservation utility may not be identical, we must check whether the possibility of the countervailing incentive problem exists. The new long-run individual-rationality constraint conditions are:

$$w^0 - \psi(e^0; \theta_L) + \delta u(p^0) \geq c + \delta\underline{U}(L) \qquad (LIR'_L)$$

$$w^1 - \psi(e^1; \theta_H) + \delta c \geq c + \delta\underline{U}(H) \qquad (LIR'_H)$$

but the long-run incentive-compatibility constraint conditions are same as in *Problem SS*. So, we can formulate as:

Model I, if

$$\delta\underline{U}(L) < \delta\underline{U}(H) + \Phi(e^1) + \delta u(p^1) - \delta c \qquad (NCV)$$

Model II, if

$$\delta \underline{U}(L) > \delta \underline{U}(H) + \Phi(e^0) + \delta u(p^0) - \delta c \qquad (CV)$$

Here, if the case is *Model I*, (LIR'_H) and (LIC_L) are binding and if the case is *Model II* (LIR'_L) and (LIC_H) are binding. We can check this fact directly: If $\delta \underline{U}(L) < \delta \underline{U}(H) + \Phi(e^1) + \delta u(p^1) - \delta c$, then, by the constraint conditions, we can say in the same argument in Section 2 that

$$\begin{aligned} w^0 - \psi(e^0;\theta_L) - (1-\delta)c - \delta \underline{U}(L) &\geq w^1 - \psi(e^1;\theta_L) - (1-\delta)c - \delta \underline{U}(L) && (LIC_L) \\ &> w^1 - \psi(e^1;\theta_H) - (1-\delta)c - \delta \underline{U}(H) && (NCV) \\ &\geq 0. && (LIR'_H) \end{aligned}$$

Thus, in this case, (LIR'_L) must hold with strict inequality. Then (LIR_H) must be binding. The principal will pay a positive rent to only type L.

Next, I introduce transition probabilities for types over two successive matchings. In the random model, I assumed that the type of agents which arises in the next matching is independent of the current state and randomly. However, if the next type depends on not only the next matching but also the innate attribute of the agent, the forthcoming type may be different in the current states. Here, I redifine the probability for the next arisen types as follows:

$p(L) =$ the probability that the type in the next matching will be L when the current type is L;

$p(H) =$ the probability that the type in the next matching will be L when the current type is H.

In the sense that the type in the next matching is detemined by the type in the current matching, I call this model the "Markov" model. In the Markov model the prior probability observed at the initial moment in each period is updated from the prior probability in the previous period. Here, it seems natural to assume that $0 <$

$p(H) < p < p(L) < 1$ but there is no reasonable reason to rule out the counterexample any time. While I show the justification for the assumption in Appendix A, in this article I do not take this relationship without notice.

Again, I define the reservation utility of type i as $c+\delta\underline{U}(i) \equiv c+\delta(1-\delta)EU(i)$ $(i = L, H)$. Also, I define the expected value function as $EU(i) \equiv p(i)v_L(U_L) + [1 - p(i)]v_H(U_H)$ $(i = L, H)$. Here, the value function, v_i, is defined as:

$$\begin{aligned} v_i(U) &= \max\Big\{U,\ c+\delta(1-\delta)\big\{p(i)v_L(U_L) + [1-p(i)]v_H(U_H)\big\}\Big\} \\ &= \max\{U,\ c+\delta(1-\delta)EU(i)\} \end{aligned} \tag{49}$$

$(i = L, H)$ Here, v_L (resp. v_H) is the maximum of expected future discounted value when the agent is type L (resp. H) at that moment. (For instance, suppose that the agent is now type L at the current matching. If the agent accepts the contract offer it would have U_L. If it rejects, then, after one-period unemployment, it would have $v_H(U_H)$ at the next matching with taking the optimal action if the type at that moment will becomes H and if it is expected that the new principal will offer U_H for type H as the current principal does.) However, as shown in Figure 4, we can think that $v_i(U_i) = U_i$ $(i = L, H)$ unless either (LIR_L) or (LIR_H) breaks. So, $\underline{U}(L) - \underline{U}(H) = (1-\delta)[p(L) - p(H)](U_L - U_H)$. Finally, let U_L and U_H denote the profits of agents as defined in the previous section. In Figure 4, I show the value functions in both the standard case (*Model I*) and the countervailing-incentive case (*Model II*).

We have a finding as summarized as below:

Proposition 3 *Model II is not feasible.*

Proof. Suppose that condition (CV) holds and Model II arises. In Model II, facing countervailing incentive, the principal must form an optimal contract so that (LIC_L)

is binding but (LIC_H) is not binding. So, we find that

$$U_L - U_H = \Phi(e^0) + \delta u(p^0) - \delta c. \tag{50}$$

Next, condition (CV) can be transformed as:

$$\begin{aligned} 0 &< \delta \underline{U}(L) - \delta \underline{U}(L) - \Phi(e^0) - \delta u(p^0) + \delta c \\ &= \delta(1-\delta)[EU(L) - EU(H)] - \Phi(e^0) - \delta u(p^0) + \delta c \\ &= \delta(1-\delta)[p(L) - p(H)](U_L - U_H) - \Phi(e^0) - \delta u(p^0) + \delta c. \end{aligned} \tag{51}$$

However, plugging equation (50) into the right-hand side of equation (51), we obtain

$$\{\delta(1-\delta)[p(L) - p(H)] - 1\} [\Phi(e^0) + \delta u(p^0) - \delta c] \tag{52}$$

which cannot be positive since $\delta(1-\delta)[p(L) - p(H)] < 1$. □

This implication seems clear. If an agent is type L in the current period, it easily becomes type L in the next period, too. And if $EU(L)$ is expected to be evaluated higher than $EU(H)$, then the reservation utility (thus, the profit in case where the agent rejects the contract) of current type L would be higher, which may cause countervailing incentive easily. However, if countervailing incentive happens, the principal has incentive to hold down the profit for type L to $c + \delta \underline{U}(L)$, which means, in turn, that the difference of the profits in type, $U_L - U_H$, cannot be beyond $\Phi(e^0) - \delta u(p^0) + \delta c$. This effect, however, prevents condition (CV) from realizing itself. Hence, countervailing incentive is consistent with asymmetric-informational contracts in our recursive model.

Recall that in the recursive model the principal can manipulate the reservation utilities for both types of agents by offering U_L and U_H. As long as (NCV) is holding, the contract which is consistent with *Model I* is feasible. Thus, the principal must design the optimal contract such that only (LIR_H) and (LIC_L) must be binding as the Type I, semi-separating contract.

5.2 The Optimal Contracts

Semi-Separating Contract

How to solve the problem is basically same as in the random model. In the similar way in the random model, it turns out:

$$U_L \equiv w^0 - \psi(e^0;\theta_L) + \delta u(p^0) = c + R + \delta \underline{U}(H)$$
$$U_H \equiv w^1 - \psi(e^1;\theta_H) + \delta c = c + \delta \underline{U}(H)$$

where $R \equiv \Phi(e^1)+\delta u(p^1)-\delta c$. Plugging these profits into $\delta\underline{U}(H) = \delta(1-\delta)\big\{p(H)U_L + [1-p(H)]U_H\big\}$, we have

$$\delta\underline{U}(H) = \frac{\delta(1-\delta)}{1-\delta(1-\delta)}\Big[p(H)\Big(\Phi(e^1)+\delta u(p^1)-\delta c\Big)+c\Big] \tag{53}$$

or

$$\delta\underline{U}(H) = \beta\big[p(H)\cdot R + c\big] \tag{54}$$

where $\beta \equiv (\delta(1-\delta))/(1-\delta(1-\delta))$, and

$$\delta\underline{U}(L) = \delta\underline{U}(H) + \delta(1-\delta)\big[p(L)-p(H)\big]R. \tag{55}$$

Here, we can check the feasibility of this contract. From (55),

$$\begin{aligned}\delta\underline{U}(L) - \delta\underline{U}(H) &= \delta(1-\delta)\big[p(L)-p(H)\big]R \\ &= \delta(1-\delta)\big[p(L)-p(H)\big]\big(\Phi(e^1)+\delta u(p^1)-\delta c\big) \\ &< \Phi(e^1)+\delta u(p^1)-\delta c\end{aligned}$$

since $\delta(1-\delta)\big[p(L)-p(H)\big]$ is smaller than unity. Hence, whatever optimal effort e^1 is, (NCV) always holds, which means that this contract model (*Model I*) is feasible.

So, the solution for the inefficient type's effort is:

$$\hat{e}^1 \equiv \arg\max_e \; V_H\left(e;\; \frac{px^0+\beta p(H)}{1-px^0}\right) \tag{56}$$

where I put "∧" (hat) on e^1 for the Markov model to distinguish it from the optimal solutions both in the exogenous reservation utility model and in the random model. The optimal payoff of the principal from the semi-separating contract in the Markov model is:

$$\begin{aligned}\mathcal{W}^{SS} &= px^0\big[\pi(e_L^*)-\psi(e_L^*;\theta_L)-c\big]\\ &+(1-px^0)\left[\pi(\hat{e}^1)-\psi(\hat{e}^1;\theta_H)-c-\frac{px^0+\beta p(H)}{1-px^0}\Phi(\hat{e}^1)\right]\\ &-\big[px^0+\beta p(H)\big]\delta\Phi(e_H^{SB}(p^1))+(\delta-\beta)c+px^0\delta W^{AI}(p^0)+(1-px^0)\delta W^{AI}(p^1).\end{aligned} \tag{57}$$

Pooling Contract

Similar to the random model, we can solve. The wage and utilities are:

$$w^P = \psi(e^P;\theta_H)+(1-\delta)c+\delta\underline{U}(H) \tag{58}$$

$$U_L \equiv w^P-\psi(e^P;\theta_L)+\delta u(p) = c+R^P+\delta\underline{U}(H) \tag{59}$$

$$U_H \equiv w^P-\psi(e^P;\theta_H)+\delta c = c+\delta\underline{U}(H) \tag{60}$$

$$\delta\underline{U}(H) = \beta\big[p(H)\cdot R^P+c\big] \tag{61}$$

$$\delta\underline{U}(L) = \delta\underline{U}(H)+\delta(1-\delta)\big[p(L)-p(H)\big]R^P \tag{62}$$

where $R^P \equiv \Phi(e^P)+\delta u(p)-\delta c$. Hence, the optimal inefficient type's effort and the optimal payoff of the principal are, respectively:

$$\hat{e}^P \equiv \arg\max_e \ V_H(e; \beta p(H)) \tag{63}$$

$$\mathcal{W}^P = [\pi(\hat{e}^P) - \psi(\hat{e}^P; \theta_H) - c - \beta p(H)\Phi(\hat{e}^P)] - \beta p(H)\delta\Phi(e_H^{SB}(p)) + (\delta - \beta)c + \delta W^{AI}(p). \tag{64}$$

Shutdown Contract

The constraint conditions are:

$$w^S - \psi(e^S; \theta_L) + \delta c = c + \delta\underline{U}(L) \tag{65}$$

$$w^S - \psi(e^S; \theta_H) + \delta c < c + \delta\underline{U}(H). \tag{66}$$

In order to hold these relationships, we must have $\delta\underline{U}(L) < \delta\underline{U}(H) + \Phi(e^S)$. The value functions are:

$$v(U_L) = c + \delta\underline{U}(L) = U_L$$
$$v(U_H) = c + \delta\underline{U}(H) > U_H.$$

By definition,

$$EU_L = p(L)\big[c + \delta\underline{U}(L)\big] + \big[1 - p(L)\big]\big[c + \delta\underline{U}(H)\big] = c + p(L)\delta\underline{U}(L) + \big[1 - p(L)\big]\delta\underline{U}(H)$$
$$EU_H = p(H)\big[c + \delta\underline{U}(L)\big] + \big[1 - p(H)\big]\big[c + \delta\underline{U}(H)\big] = c + p(H)\delta\underline{U}(L) + \big[1 - p(H)\big]\delta\underline{U}(H)$$

and $\delta\underline{U}(L) = \delta(1 - \delta)EU_L$ and $\delta\underline{U}(H) = \delta(1 - \delta)EU_H$. Obviously, it turns out $\delta\underline{U}(L) = \delta\underline{U}(H)$ since

$$\delta\underline{U}(L) - \delta\underline{U}(H) = \delta(1 - \delta)\big[p(L) - p(H)\big]\Big[\delta\underline{U}(L) - \delta\underline{U}(H)\Big]$$

and $\delta(1 - \delta)\big[p(L) - p(H)\big] > 0$. So, $\delta\underline{U}(L) = \delta\underline{U}(H) \equiv \delta\underline{U}^P = \beta c$ with holding

$\delta\underline{U}(L) < \delta\underline{U}(H) + \Phi(e^S), \;\; (e^S > 0).$

Hence, The optimal principal's payoff in the shutdown contract in the Markov model is:

$$\mathcal{W}^{SD} = p\big[\pi(\hat{e}^S) - \psi(\hat{e}^S;\theta_H) - c + \Phi(\hat{e}^S)\big] + p(\delta - \beta)c + p\delta W^{FI}(1) \tag{67}$$

with $\hat{e}^S = e_L^*$.

5.3 Implication

In the random model, the primary concern is the agent expected future's payoff. The expectation depends on the utilities from the current contract and the probability on which type it will be reborn in the next matching. I note the following observation, first.

Observation 1 *In the semi-separating contract,*

$$\frac{d\big(\delta\underline{U}(L)\big)}{dp(H)} = \delta(1-\delta)\frac{d\big(\delta\underline{U}(H)\big)}{dp(H)} < \frac{d\big(\delta\underline{U}(H)\big)}{dp(H)} = \beta R.$$

Thus, the effect of $p(H)$ *is larger on* $\delta\underline{U}(H)$ *than on* $\delta\underline{U}(L)$.

What this observation says is trivial: While both $\delta\underline{U}(L)$ and $\delta\underline{U}(H)$ depends on $p(H)$, the possibility of being type L in the next matching if type is H today, but does not depend on $p(L)$, the possibility if type is L today. Namely, the possibility from being type H to being type L is important to both types of agents. Especially, type H today might have much concern on its future since if the possibility of becoming type L tomorrow it looks better to clear up the relationship with the current principal and to "start over" with a fresh relationship with a new principal. This expectation on the future rises the reservation utility for type H toward the current principal more than for type L.

Next, if $p(H) < p$,

$$\frac{px^0}{p(1-x^0)+(1-p)} < \frac{px^0+\beta p(H)}{p(1-x^0)+(1-p)} < \frac{px^0+\beta p}{p(1-x^0)+(1-p)}$$

but if the converse holds, the middle is larger than the right-hand above. So,

Observation 2 *If $p(H) < p$, then $e^1 > \hat{e}^1 > \bar{e}^1$, but if $p(H) > p$, then $e^1 > \bar{e}^1 > \hat{e}^1$.*

In the polar case, when $x^0 = 1$ and $p(H) = 1$, $\hat{e}^1$ is at most $\arg\max_e V_H\left(e; \frac{p+\beta}{1-p}\right)$. Thus, if everyone expects that *all* of the agents with type H today will be type L tomorrow with probability one, the offered effort to type H must be the smallest one in the semi-separating contract.

Finally, note that the optimal principal's payoff in the shutdown contract is independent on $p(H)$. So, with larger $p(H)$ the shutdown contract may be possibly more attractive than the semi-separating contract or the pooling contract. I will compare the principal's payoffs from these contracts in numerical examples in Section 6.

6 Numerical Calculations

6.1 The payoffs from three contracts

Since the model which I use in this article is quite complex and such results as equilibria desparately depends on the values of parameters, it may be difficult to have clear results only from analytic study. So, one may have to use numerical calculation to obtain the characteristics which describe the implication of the model, by setting some simple and numerical example. In this section, I provide some numerical calculations to describe how the solutions of the optimal contracts look like.

In the numerical analysis, I specify functions (and so solutions) as follows:

$\pi(e) = ae$

$\psi(e;\theta) = \frac{1}{2}\theta e^2$

$\Phi(e) = \frac{1}{2}\Delta\theta e^2, \quad (\Delta\theta \equiv \theta_H - \theta_L)$

$e_L^* = \frac{a}{\theta_L}$

$e_H^* = \frac{a}{\theta_H}$

$e_H^{SB}(p) = \frac{a(1-p)}{(1-p)\theta_H - p\Delta\theta}$

$\hat{e}^1 = \frac{a(1-px^0)}{(1-px^0)\theta_H - (px^0 + \beta p(H))\Delta\theta}$

First, I check when *Type I* can be feasible. As I noted at Page 13, (LIC_H) cannot be binding if $\delta < \bar{\delta}$ where

$$\bar{\delta} \equiv \frac{\Phi(e^0) - \Phi(e^1)}{\Phi(e_H^{SB}(p^1)) - \Phi(e_H^{SB}(1))}.$$

In fact, $\bar{\delta} \geq 1.2695$ for any p and *Type I* must apper.

Next, we can draw the diagram the payoffs from the tree contracts, $\mathcal{W}^{SS}$, $\mathcal{W}^P$ and $\mathcal{W}^{SD}$, on the space of (p, δ) in the Markov model. Here, I specify parameters as: $a = 2.0$, $c = 1.0$, $\theta_L = 1.0$, $\theta_H = 1.5$, $p(H) = 0.3$ and $x^0 = 0.5$. When $\delta = 0.9$, three payoffs are shown in Figure 5. In this numerical case, the semi-separating contract usually dominates the pooling contract except on $p = 0$ where the two payoffs are same. It is because the rent in the semi-separating contract is not so high to the principal and the pooling contract has no advantage. Next, as we can expect easily, the payoff from the shutdown contract dominates other contracts' payoffs for higher p. If p is so high, then the rent to distinguish type L from the small number of type H ends up being costly to the principal. In this case, if the probability of type L is larger than about one half, the principal might wish to shut out the inefficient type of agents by the contract.

For various δ, see Figure 6. I set here $x^0 = 1$. (Thus, it is the full-separating contract.) The intersection between the separating payoff and the shutdown payoff varies in δ. As long as we can see, the lower δ is, the earlier the domination of the shutdown contract comes. Since $x^0 = 0$, this case suggests the maximum payoff from the separating contract. Even in this case, the dominant area of the shutdown contract appears on higher p.

6.2 Simulations

We can set a simulation by setting numerical parameters to capture chronological happenings. Since the situation is complex, I describe in detail a Markov-like model as follows:

First, the timing of the situation is:

1. probability p (type distribution) is announced by authority and it becomes common knowledge.
2. a principal and an agent are matched.
3. the agent alone observes the type of the matching.
4. the principal chooses the optimal contract and offers it to the matched agent.
5. the agent accepts or rejects it.

I call the market state of full-employment *Phase A* and the market state where unemployment exists *Phase B*. Noting that in the contract model discussed in this article only type H can be unemployed, we can classify the agents at that moment into several states according to types and situations. If observing one moment in the market, we can find the following five states of agents.

State L_1: type L working in the first period of a contract.

State L_2: type L working in the second period of a contract.

State H_1: type H working in the first period of a contract.

State H_2: type H working in the second period of a contract.

State H_0: type H in unemployment (outside contract).

In *Phase A* and *Phase B* these states in a period t are shown in Figure 7. Let $P_{x,y}$ denote the transition probability from state x to state y. The transition probability matrix is:

$$\begin{pmatrix} P_{L_1,L_1} & P_{L_1,L_2} & P_{L_1,H_1} & P_{L_1,H_2} & P_{L_1,H_0} \\ P_{L_2,L_1} & P_{L_2,L_2} & P_{L_2,H_1} & P_{L_2,H_2} & P_{L_2,H_0} \\ P_{H_1,L_1} & P_{H_1,L_2} & P_{H_1,H_1} & P_{H_1,H_2} & P_{H_1,H_0} \\ P_{H_2,L_1} & P_{H_2,L_2} & P_{H_2,H_1} & P_{H_2,H_2} & P_{H_2,H_0} \\ P_{H_0,L_1} & P_{H_0,L_2} & P_{H_0,H_1} & P_{H_0,H_2} & P_{H_0,H_0} \end{pmatrix}. \tag{68}$$

The matrices in *Phase A* and *Phase B* are, respectively:

$$\boldsymbol{A} = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ p(L) & 0 & 1-p(L) & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ p(H) & 0 & 1-p(H) & 0 & 0 \\ p(H) & 0 & 1-p(H) & 0 & 0 \end{pmatrix} \tag{69}$$

$$\boldsymbol{B} = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ p(L) & 0 & 0 & 0 & 1-p(H) \\ 0 & 0 & 0 & 1 & 0 \\ p(H) & 0 & 0 & 0 & 1-p(H) \\ p(H) & 0 & 0 & 0 & 1-p(H) \end{pmatrix}. \tag{70}$$

For instance, in *Phase A* state H_2 can move to state L_1 with probability $p(H)$ and H_1 with $1 - p(H)$, but in *Phase B* state H_2 can move to state L_1 with $p(H)$ and H_0 with $1 - p(H)$. Even in *Phase B* state H_1 must move to H_2 with probability 1 in the next period since the two-period contract is restricted to be continued.

Let $\boldsymbol{P}$ denote the state probability vector:

$$\boldsymbol{P} = [p_{L_1}, p_{L_2}, p_{H_1}, p_{H_2}, p_{H_0}]$$

where p_{L_1} is the probability of L_1, and so on. If $\boldsymbol{P^{(t)}}$ is the distribution in period t, the distribution in the next period, $\boldsymbol{P^{(t+1)}}$, is determined as:

$$\boldsymbol{P^{(t+1)}} = \boldsymbol{P^{(t)}}\boldsymbol{A} \quad (\textit{Phase A})$$

$$\boldsymbol{P^{(t+1)}} = \boldsymbol{P^{(t)}}\boldsymbol{B} \quad (\textit{Phase B})$$

For the first case I show a "naive" simulation where, assuming that all agents enter matchings in same timing, in each matching period the prior probability p is updated. I set parameters as; $a = 1.0$, $c = 0.0$, $\delta = 0.5$, $\theta_L = 1.0$, $\theta_H = 1.5$, $p(L) = 0.6$, $p(H) = 0.4$ and $x^0 = 0.5$. The result is shown in Figure 8. Then, both the semi-separating contract and the shutdown are increasing payoffs. At first, the semi-separating dominates the shutdown. So, all of agents are hired initially (*Phase A*). Later, the shutdown will dominate the semi-separating and agents with type H are shut out from the contracts and unemployed (*Phase B*). This happening arises since the prior probability goes up and converge to some value.

Next, I put the five states as classified above into the simulation. Furthermore, suppose initially $P_{L1} = 0.5$, $P_{L2} = 0.0$, $P_{H1} = 0.0$, $P_{H2} = 0.5$, and $P_{H0} = 0.0$. The result is shown in Figure 9 and Table 2.[8] Both the semi-separating payoff

[8] I attach the `Matlab` code for the simulation at the end of this article. However, the diagrams used in this article are made by `Gnu Octave`(2.0.17) and `Gnuplot`(4.0).

and the shutdown payoff converge but the former will dominate the latter eventually. However, in the initial periods, they display cycles. Since the degree of the cycle of the shutdown payoff is larger than of the semi-separating payoff, we have two periods when the shutdown dominates the semi-separating. In this case, I set as the working-periods for type L and type H are asymmetric in the initial state: $P_{L1} = 0.5$ and $P_{H2} = 0.5$. Namely, in the initial periods, "efficient novice-inefficient veteran" phase and "inefficient novice-efficient veteran" phase occur by turns. In even-number periods, many inefficient novices and efficient veterans arise, which induce the principal to have the incentive adopting the shutdown strategy. That is why the reversal is shown in period 2 and period 4. However, the degree of the cycle of the shutdown will be smaller later, the reversal never happen in the later periods.

7 An Extention: The Multi-Period Contract Model

In previous sections, I discussed the two-period contract model which is combined with a recursive context. In the main finding the endogenous reservation utility effect, β, lowers the rent of the efficient type with the smaller effort for the inefficient in both the random model and the Markov model. Unfortunately for us, the effect of β is not high: since the gap of timings for the agents to have the expected future value, EU (or EU_i for type i), between the acceptance and the rejection is only one period in the two-period contract, the reservation utility will not become much higher. So, it is natural to pose the following question: how the multi-period contract would be affected by the reservation utility?

In this section, without rigorous proofs I just sketch out the perspective of the multi-period contract where the principal offers a T-period contract to the matched agent. In the sketch I suggest that in the multi-period contract model the optimal solution of the semi-separating contract can be again solved by the Perfect Bayesian equilibrium concept and β can be huge in the equilibrium.

Perfect Bayesian Equilibrium and Incentive-Compatibility Constraint Conditions

First, I formulate the *Bayesian dynamic game* so as not to ruin Laffont and Tirole's notations. (For the formal setting, see Osborne and Rubinstein (1994), Fudenberg and Tirole (1991), or other textbooks.)

Since we consider only two types of contracts, 0 or 1, the *action* which the agent can take in period t is denoted by $a^{(t)} \in \{a^{0(t)}, a^{1(t)}\}$ where $a^{0(t)}$ (resp. $a^{1(t)}$) denotes that the agent chooses Contract 0 (resp. 1).[9] The agents in this game can observe *history* in period t which is $h^{(t)} \equiv (a^{(1)}, a^{(2)}, \cdots, a^{(t-1)})$. Since a Bayesian dynamic game is a sequencial game, the strategies are defined conditional on each history. Generally, a strategy of the agent with type L is a probability assigned on an action at a history: $\sigma_L(a^{(t)}|h^{(t)})$. However, by using a previous nation, I define it as $x^0(h^{(t)}) \equiv \sigma_L(a^{0(t)}|h^{(t)})$ (so, $1 - x^0(h^{(t)}) = \sigma_L(a^{1(t)}|h^{(t)})$). Hence, the strategy for type L in this T-period contract game is a set of strategies at each period, $\sigma_L = (x^0, x^0(h^{(1)}), x^0(h^2), \cdots, x^0(h^T))$. Again, we can formulate the principal's *posterior belief* on the type of the matched agent by composing the agent's strategies and the old belief and by updating the old belief. The beliefs that the agent is type L when the principal observes the agent choosing Contract 0 and 1 are, respectively:

$$\mu(\theta_L|h^{(t)}, a^{0(t)}) = \frac{\mu(\theta_L|h^{(t)})x^0(h^{(t)})}{\mu(\theta_L|h^{(t)})x^0(h^{(t)})} = 1 \tag{71}$$

$$\mu(\theta_L|h^{(t)}, a^{1(t)}) = \frac{\mu(\theta_L|h^{(t)})(1 - x^0(h^{(t)}))}{\mu(\theta_L|h^{(t)})(1 - x^0(h^{(t)})) + (1 - \mu(\theta_H|h^{(t)}))}. \tag{72}$$

The principal's strategy is as explained in the previous sections. In the first period it design the optimal contract menu conditional on the prior probability, p, but in the second period and after, it uses the posterior beliefs at that time. So, the strategy of the principal can be denoted by $\sigma_P(\mu(\theta_L|h^t))$. Finally, the agent's payoff is determined

[9] Despite some nuisance I put superscripts in brancets for indeces of "period" with leaving the Laffont and Tirole's subscripts.

by the agent's strategy at that momoent, the history, and the principal's belief. So, the payoff for type L in period t is $\tilde{u}_L^{(t)}(x^0(h^{(t)})|h^t, \sigma_P(\mu(\theta_L|h^{(t)})))$. For economizing notaions, I mean the L's strategy when the principal chooses the optimal contract properly based on the belief (say, σ_P^*), by $u_L^{(t)}(x^0(h^{(t)})|h^{(t)}) \equiv \tilde{u}_L^{(t)}(x^0(h^{(t)})|h^{(t)}, \sigma_P^*(\mu(\theta_L|h^{(t)})))$. Note that the payoff is dependent on not only the action taken in that period but also the history succeeded from the events in the previous period. Thus, the action taken by the agent in the current period will affect the next payoff through the belief updated by the principal. This payoff corresponds to $u(p^0)$ or $u(p^1)$ while in the two-period contract model the optimal strategy of the agent L was determined automatically and so it was omitted.

In the Bayesian dynamic game depicted as above I roughly define a Perfect Bayesian equilibrium as follows:

Definition 1 A Perfect Bayesian equilibrium in the T-period contract game (*Type I*) is such a set of strategies and belief, $(\sigma_L, \sigma_H, \sigma_P, \mu)$, as satisfies:

(P) The principal's contract is designed properly as described in the previous sections.

(L) For any $h^{(t)}$ and $\tilde{a}^{(t)}$,

$$\begin{aligned} & u_L^{(t)}(x^0(h^{(t)})|h^{(t)}) + \delta u_L^{(t+1)}(x^0(h^{(t+1)})|h^{(t+1)}) + \delta^2 u_L^{(t+2)}(x^0(h^{(t+2)})|h^{(t+2)}) \\ & + \cdots + \delta^{T-t} u_L^{(T)}(x^0(h^{(T)})|h^{(T)}) \\ & \geq u_L^{(t)}(\tilde{a}^{(t)}|h^{(t)}) + \delta u_L^{(t+1)}(x^0(\tilde{h}^{(t+1)})|\tilde{h}^{(t+1)}) + \delta^2 u_L^{(t+2)}(x^0(\tilde{h}^{(t+2)})|\tilde{h}^{(t+2)}) \\ & + \cdots + \delta^{T-t} u_L^{(T)}(x^0(\tilde{h}^{(T)})|\tilde{h}^{(T)}) \end{aligned} \tag{73}$$

where $h^{(s)}$ is the history when the agent takes the proper actions described in optimal strategy σ_L and $\tilde{h}^{(s)}$ is the history when the agent takes an alternative action $\tilde{a}^{(t)}$ and takes the proper actions in the following periods $(s = t+1, t+2, \cdots, T)$.

(H) Agent H takes Contract 1 with probability 1 in every periods.

(B) The principal makes the beliefs according to (71) and (72).

Here, I focus the analysis on the full-separating, i.e., $x^0 = 1$ in every period. Type L chooses Contract 0 with probability 1 in every period and the strategy is $\sigma_L^* = (x^0, x^0(h^{(2)}), x^0(h^3), \cdots, x^0(h^T)) = (a^{0(1)}, a^{0(2)}, a^{0(3)}, \cdots, a^{0(T)})$. For this strategy to be optimal, the following condition in each period must hold:

(Period 1)

$$\begin{aligned}
&u_L^{(1)}(x^0) + \delta u_L^{(2)}(x^0(a^{0(1)})) + \delta^2 u_L^{(3)}(x^0(a^{0(1)}, a^{0(2)})|a^{0(1)}, a^{0(2)}) \\
&+ \cdots + \delta^{T-1} u_L^{(T)}(x^0(a^{0(1)}, a^{0(2)}, \cdots, a^{0(T-1)})|a^{0(1)}, a^{0(2)}, \cdots, a^{0(T-1)}) \\
&\geq \\
&u_L^{(1)}(a^{1(1)}) + \delta u_L^{(2)}(x^0(a^{1(1)})) + \delta^2 u_L^{(3)}(x^0(a^{1(1)}, a^{0(2)})|a^{1(1)}, a^{0(2)}) \\
&+ \cdots + \delta^{T-1} u_L^{(T)}(x^0(a^{1(1)}, a^{0(2)}, \cdots, a^{0(T-1)})|a^{1(1)}, a^{0(2)}, \cdots, a^{0(T-1)})
\end{aligned} \tag{$LIC_L^{(1)}$}$$

$\cdots \quad \cdots \quad \cdots \quad \cdots$

(Period t) For any $h^{(t)}$,

$$u_L^{(t)}(x^0(h^{(t)})|h^{(t)}) + \delta u_L^{(t+1)}(x^0(h^{(t)}, a^{0(t)})|h^{(t)}, a^{0(t)})$$
$$+ \delta^2 u_L^{(t+2)}(x^0(h^{(t)}, a^{0(t)}, a^{0(t+1)})|h^{(t)}, a^{0(t)}, a^{0(t+1)}) + \cdots$$
$$+ \delta^{T-t} u_L^{(T)}(x^0(h^{(t)}, a^{0(t)}, a^{0(t+1)}, \cdots, a^{0(T-1)})|h^{(t)}, a^{0(t)}, a^{0(t+1)}, \cdots, a^{0(T-1)})$$
$$\geq$$
$$u_L^{(t)}(a^{1(t)}|h^{(t)}) + \delta u_L^{(t+1)}(x^0(h^{(t)}, a^{1(t)})|h^{(t)}, a^{1(t)})$$
$$+ \delta^2 u_L^{(t+2)}(x^0(h^{(t)}, a^{1(t)}, a^{0(t+1)})|h^{(t)}, a^{1(t)}, a^{0(t+1)})$$
$$+ \cdots + \delta^{T-t} u_L^{(T)}(x^0(h^{(t)}, a^{1(t)}, a^{0(t+1)}, \cdots, a^{0(T-1)})|h^{(t)}, a^{1(t)}, a^{0(t+1)}, \cdots, a^{0(T-1)})$$

$$(LIC_L^{(t)})$$

$\cdots \quad \cdots \quad \cdots \quad \cdots$

(Period $T-1$) For any $h^{(T-1)}$,

$$u_L^{(T-1)}(x^0(h^{(T-1)})|h^{(T-1)}) + \delta u_L^{(T)}(x^0(h^{(T-1)}, a^{0(T)})|h^{(T-1)}, a^{0(T)})$$
$$\geq u_L^{(T-1)}(a^{1(T-1)}|h^{(T-1)}) + \delta u_L^{(T)}(x^0(h^{(T-1)}, a^{1(T)})|h^{(T-1)}, a^{1(T)})$$

$$(LIC_L^{(T-1)})$$

(Period T) For any $h^{(T)}$,

$$u_L^{(T)}(x^0(h^{(T)})|h^{(T)}) \geq u_L^{(T)}(a^{1(T)}|h^{(T)}) \qquad (LIC_L^{(T)})$$

Thus, these conditions are *long-run incentive-compatibility constratint conditions* for type L in the multi-period contract model. Note that the above $(LIC_L^{(T-1)})$ corresponds to (LIC_L) in the two-period contract model.

The Meaning of $\underline{U}$ and Individual-Rationality Constraint Conditions

Next, consider (LIR)s. I consider here the random model, i.e., the type in the next matching is determined independent of the current type. Suppose that the principal offers a T-period contract so that the agent has no incentive to leave the contract en route. So, the agent, if accepting, will vertually tie itself with the contract in T periods. T periods after, it can have an expected discounted future value in optimization, EU (in the random model, this value is same in types). On the contrary, if the agent rejects the offer, it can have the same EU only one period after. Letting U_i denote the payoff of type i from the contract, we can compare these two options:

$$U_i + \delta^T EU,$$

$$c + \delta EU$$

to find the difference $(\delta - \delta^T)EU$ for the second term in the same way as in the two-period contract. Define as $\underline{U} \equiv (1-\delta)EU$, again. We can find that

$$\begin{aligned}
\delta\underline{U} + \delta^2\underline{U} + \cdots + \delta^{T-1}\underline{U} &= (\delta + \delta^2 + \cdots \delta^{T-1})(1-\delta)EU \\
&= \frac{\delta(1-\delta^{T-1})}{1-\delta}(1-\delta)EU \\
&= (\delta - \delta^T)EU.
\end{aligned}$$

From this result we can interepret $\underline{U}$ as follows: if the agent accepts the contract it has U_i, which implies, as it were, "the discounted value of the contract" from staying in the contract with the current principal for T periods. For the same periods, the agent would have $c + \delta\underline{U} + \delta^2\underline{U} + \cdots + \delta^{T-1}\underline{U}$ outside the contract. Thus, it would have c in the first period and $\delta\underline{U}$ (in discounted value) in the second period and $\delta^2\underline{U}$ in the third period, and so on. So, what $\underline{U}$ means is the value from being outside for one period or, as it were, the "average" reservation utility per period. I call $\delta\underline{U} + \delta^2\underline{U} + \cdots + \delta^{T-1}\underline{U}$ the *long-run reservation utility.*

With this notation we can formulate *long-run individual-rationality constraint* conditions as follows. Note that in more than two periods the contract must have constraints so as not to *quit* the contract en route while we did not consider it in the two-period contract. Rather, these "quit-proof" conditions are exactly same as the static individual-rationality constraint conditions in the two-period contract model and they have been omitted.

The long-run individual-rationality constraint conditions for type H are:

(Period 1)

$$
\begin{aligned}
& u_H^{(1)}(y^0) + \delta u_H^{(2)}(y^0(a^{1(1)})) + \delta^2 u_H^{(3)}(y^0(a^{1(1)}, a^{1(2)})|a^{1(1)}, a^{1(2)}) \\
& + \cdots + \delta^{T-1} u_H^{(T)}(y^0(a^{1(1)}, a^{1(2)}, \cdots, a^{1(T-1)})|a^{1(1)}, a^{1(2)}, \cdots, a^{1(T-1)}) \\
& \geq c + \delta \underline{U} + \delta^2 \underline{U} + \delta^3 \underline{U} + \cdots + \delta^{T-1} \underline{U}
\end{aligned}
\tag{$LIR_H^{(1)}$}
$$

$\cdots \quad \cdots \quad \cdots \quad \cdots$

(Period t) For any $h^{(t)}$,

$$
\begin{aligned}
& u_H^{(t)}(y^0(h^{(t)})|h^{(t)}) + \delta u_H^{(t+1)}(y^0(h^{(t)}, a^{1(t)})|h^{(t)}, a^{1(t)}) \\
& + \delta^2 u_H^{(t+2)}(y^0(h^{(t)}, a^{1(t)}, a^{1(t+1)})|h^{(t)}, a^{1(t)}, a^{1(t+1)}) + \cdots \\
& + \delta^{T-t} u_H^{(T)}(y^0(h^{(t)}, a^{1(t)}, a^{1(t+1)}, \cdots, a^{1(T-1)})|h^{(t)}, a^{1(t)}, a^{1(t+1)}, \cdots, a^{1(T-1)}) \\
& \geq c + \delta \underline{U} + \delta^2 \underline{U} + \delta^3 \underline{U} + \cdots + \delta^{T-t} \underline{U}
\end{aligned}
\tag{$LIR_H^{(t)}$}
$$

$\cdots \quad \cdots \quad \cdots \quad \cdots$

(Period $T-1$) For any $h^{(T-1)}$,

$$u_H^{(T-1)}(y^0(h^{(T-1)})|h^{(T-1)}) + \delta u_H^{(T)}(y^0(h^{(T-1)}, a^{1(T)})|h^{(T-1)}, a^{1(T)})$$
$$\geq c + \delta \underline{U} \qquad (LIR_H^{(T-1)})$$

(Period T) For any $h^{(T)}$,

$$u_H^{(T)}(y^0(h^{(T)})|h^{(T)}) \geq c \qquad (LIR_H^{(T)})$$

The above $(LIR_H^{(T-1)})$ corresponds to (LIR_H) in the two-period contract model.

The Suggestion from the Multi-Period Contract Model

Consider such contract as $(LIC_L^{(1)})$ and $(LIR_H^{(1)})$ are binding with $(LIC_L^{(2)}), \cdots, (LIC_L^{(T)})$ and $(LIR_H^{(2)}), \cdots, (LIR_H^{(T)})$ holding. Thus, I examine only *Type I* contract in equilibrium. For convenience, let us use these notation:

$$\delta S\underline{U} \equiv +\delta\underline{U} + \delta^2\underline{U} + \delta^3\underline{U} + \cdots + \delta^{T-1}\underline{U}$$
$$\delta S u_L \equiv \delta u_L^{(2)}(x^0(a^{0(1)})) + \delta^2 u_L^{(3)}(x^0(a^{0(1)}, a^{0(2)})|a^{0(1)}, a^{0(2)}),$$
$$+ \cdots + \delta^{T-1} u_L^{(T)}(x^0(a^{0(1)}, a^{0(2)}, \cdots, a^{0(T-1)})|a^{0(1)}, a^{0(2)}, \cdots, a^{0(T-1)}),$$
$$\delta S\tilde{u}_L \equiv \delta u_L^{(2)}(x^0(a^{1(1)})) + \delta^2 u_L^{(3)}(x^0(a^{1(1)}, a^{0(2)})|a^{1(1)}, a^{0(2)})$$
$$+ \cdots + \delta^{T-1} u_L^{(T)}(x^0(a^{1(1)}, a^{0(2)}, \cdots, a^{0(T-1)})|a^{1(1)}, a^{0(2)}, \cdots, a^{0(T-1)}),$$
$$\delta S u_H \equiv \delta u_H^{(2)}(y^0(a^{1(1)})) + \delta^2 u_H^{(3)}(y^0(a^{1(1)}, a^{1(2)})|a^{1(1)}, a^{1(2)})$$
$$+ \cdots + \delta^{T-1} u_H^{(T)}(y^0(a^{1(1)}, a^{1(2)}, \cdots, a^{1(T-1)})|a^{1(1)}, a^{1(2)}, \cdots, a^{1(T-1)})$$

So, the binding conditions are simply:

$$w^{1(1)} - \psi(e^{1(1)}; \theta_H) + \delta S u_H = c + \delta S\underline{U} \qquad (LIR_H^{(1)})$$

$$w^{0(1)} - \psi(e^{0(1)}; \theta_L) + \delta S u_L = w^{1(1)} - \psi(e^{1(1)}; \theta_L) + \delta S \tilde{u}_L \qquad (LIC_L^{(1)})$$

Without redundant explanations, the utility of type L and H are turned out:

$$\begin{aligned} U_L^{(1)} &= c + \delta S \underline{U} \\ U_H^{(1)} &= c + R + \delta S \underline{U} \end{aligned} \tag{74}$$

where $R \equiv \Phi(e^{1(1)}) + \delta S \tilde{u}_L - \delta S u_H$, and since $EU \equiv pU_L^{(1)} + (1-p)U_H^{(1)}$ and $\underline{U} = (1-\delta)EU$,

$$\begin{aligned} \delta S \underline{U} &= \sum_{t=1}^{T-1} \delta^t \underline{U} \\ &= \sum_{t=1}^{T-1} \delta^t (1-\delta) EU \\ &= \sum_{t=1}^{T-1} \delta^t (1-\delta)(pR + c) + \sum_{t=1}^{T-1} \delta^t (1-\delta) \delta S \underline{U}. \end{aligned}$$

Hence,

$$\delta S \underline{U} = \beta^{(1)} (pR + c) \tag{75}$$

where

$$\beta^{(1)} \equiv \frac{\sum_{t=1}^{T-1} \delta^t (1-\delta)}{1 - \sum_{t=1}^{T-1} \delta^t (1-\delta)} = \frac{\delta - \delta^T}{1 - (\delta - \delta^T)}.$$

In the same inference as in the two-period contract, the optimal solution for type H in the first period is determined as:

$$e^{1(1)} = \arg\max_e \left(\pi(e^{1(1)} - \psi(e^{1(1)}; \theta_H)) - c - \frac{p(x^{0(1)} + \beta^{(1)})}{1 - px^{0(1)}} \Phi(e^{1(1)}) \right). \tag{76}$$

I will not extend the discussion anymore but will only point out how $\beta^{(1)}$ differs from β in the two-period contract model. In the two-period contract model, the parameter β is not larger than 1/3. γ is composed of p or β and the effect of β on the optimal solutions are quite negligible. However, in the multi-period contract model,

$\beta^{(1)}$ may be huge with large T. Namely, if $\bar{\delta} \equiv \arg\max_{\delta} \beta^{(1)}$, then $\bar{\delta} = \sqrt[T-1]{1/T}$ and the maximum of $\beta^{(1)}$ is larger than one (see Figure 10). This large $\beta^{(1)}$ may affect the results of the optimal contracts which is discussed above. At least, the long-run reservation utility may be huge if $\beta^{(1)}$ is large.

I add another comment for the Markov model in the multi-period contract. In the same procedure, the long-run reservation utilities of type H and type L can be derived as, respectively:

$$\delta S\underline{U}_H = \beta^{(1)}(p(H) \cdot R + c) \tag{77}$$

$$\delta S\underline{U}_L = \delta S\underline{U}_H + (\delta - \delta^T)(p(L) - p(H))R. \tag{78}$$

The condition that contervailing incentive does not appear in the multi-period model (corresponding to (NCV) at Page 30) is

$$\delta S\underline{U}_L - \delta S\underline{U}_H < R \tag{79}$$

and so we verified the result that *Model I* is feasible in the multi-period contract model. Also, we can infer that *Model II* is not feasible in the multi-period contract as far as $(\delta - \delta^T)(p(L) - p(H)) < 1$. Note that this finding does not depend on the value of $\beta^{(1)}$. Conclusionally, $\beta^{(1)}$ does not affect the feasibility of *Model I* and the infeasibility of *Model II*.

8 Conclusion

In this article, I showed a new approach to extend the standard contract theory into the recursive context. It could be said that I could show a mechanism of the recursive contracts under asymmetric information in four points as summarized below.

First, I showed how the valuation for the future is built up by the agent and how it affects the reservation utility in the individual-rationality constraint conditions in the standard contract model. Proposition 1 (Page 20) and Appendix B show that the value function exists such as the agent can expect the proper future value by the Bellman equation to choose the optimal action.

Second, the principal who knows such actions by the agent must tread the reservation utility as an endogenous variable in the problem to design the optimal semi-separating contract. This constraint induces the principal to keep the utility for the agent lower and then the rent and the inefficient's effort will be suppressed more than the pooling and the shutdown contracts. (See Proposition 2 at Page 26.) It may imply that the shutdown contract will be more attractive to the principal.

Third, while the countervailing incentive problem may arise in the different reservation utility case, in the two-period contract model, the standard model where only (LIR_H) and (LIC_L) are binding (I called it *Model I*) is always feasible. On the other hand, the reverse model where (LIC_H) is binding instead of (LIC_L) (*Model II*) is absolutely infeasible, which is shown in Proposition 3 at Page 32. This result must not be affected by the parameters. However, in the multi-period contract model, the result is ambiguous. The reservation utility effect parameter, $\beta^{(1)}$, can become huge unboundly, and so the optimal contract may be changed. This is caused by the gap of the timing when the future value in the next matching is acquired.

Finally, in the Markov model, the transition probability from type H to type L (p_H) is important to determine the reservation utilities for both types of agents. Obviously, the inefficient type of agent expects for the future more than the efficient, and then if the probability of starting over well in the next matching is high, the reservation utility becomes high and the (IR) conditions becomes tight.

The standard contract model in static situations has been a "common knowledge" in microeconomics, industrial organization or other fields of economic theory. However, without the view of dynamic mechanism it possibly seems unrealistic to a certain degree. The main reason can be the reasonable interpretation for the setting of such parameters as the prior probability, the reservation utility or the types of agents. My model would shed light on the interpretaion for the setting.

First, I considered a “transitive” prior probability in my model to explain how the prior probability is determined chronologically. Setting up proper transition probabilities (p_L and p_H) makes it possible to capture the transition of the prior probability. Furthermore, when the contract market is older we can estimate which contract is optimal or not.

Second, as mentioned above, I showed how the reservation utility is determined. If the countervailing incentive problem must be concerned, the interpretation of the reservation in recursive context is to be important to evaluate optimal contracts.

Third, I formulated the static contract model into the matching game. The traditional static contract model does not tell us how the principal and the agent meet with each other. With the matching setting we can treat the contract model in dynamic context.

Consequently, my new approach would contribute various extentions for the traditional contract theory in the sense of these points.

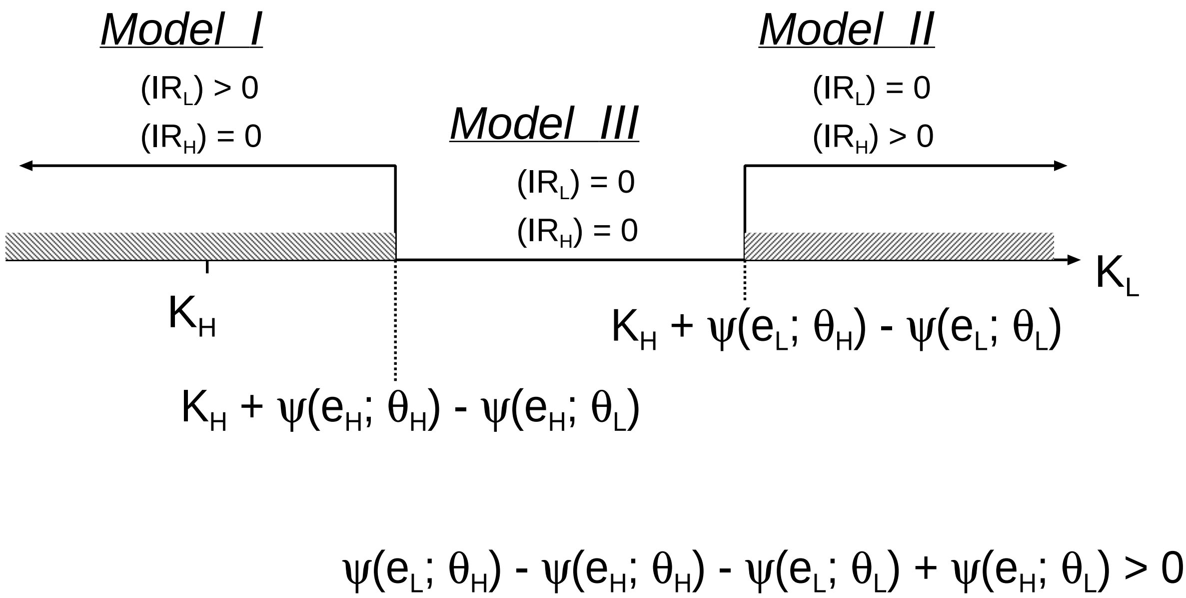


Figure 1: Different Reservation Utilities

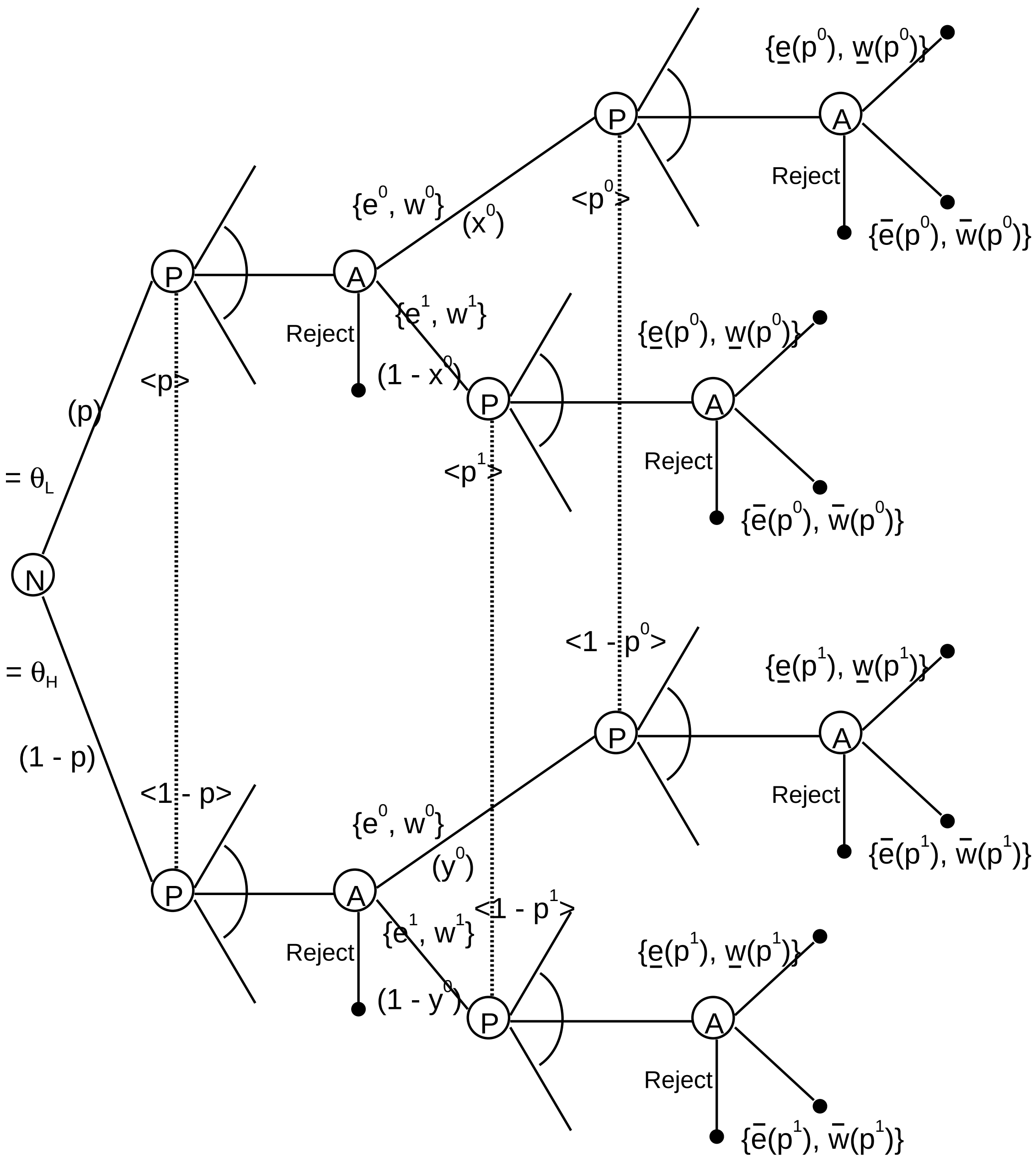


Figure 2: The Two-Period Contract Model

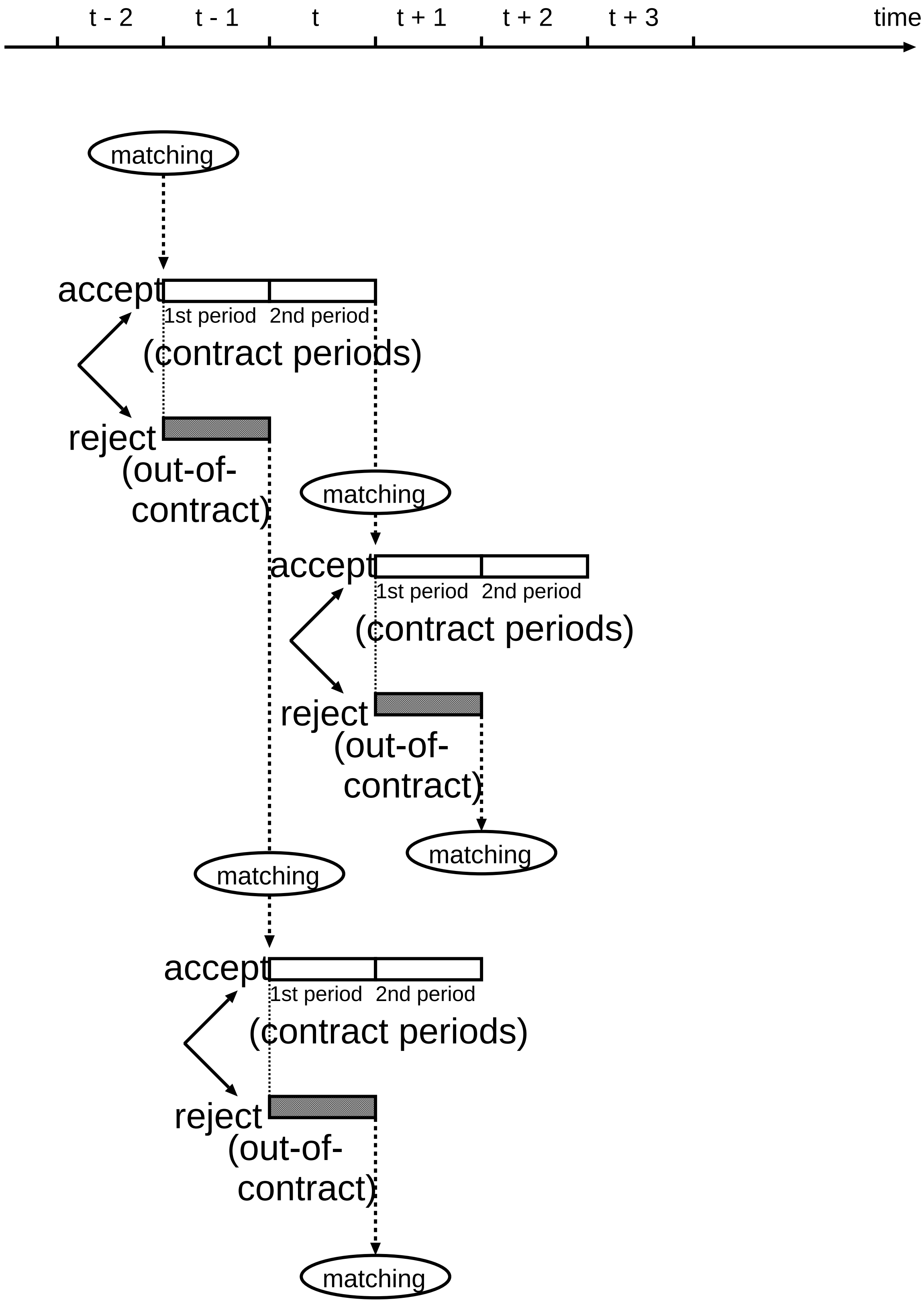


Figure 3: The Recursive Model

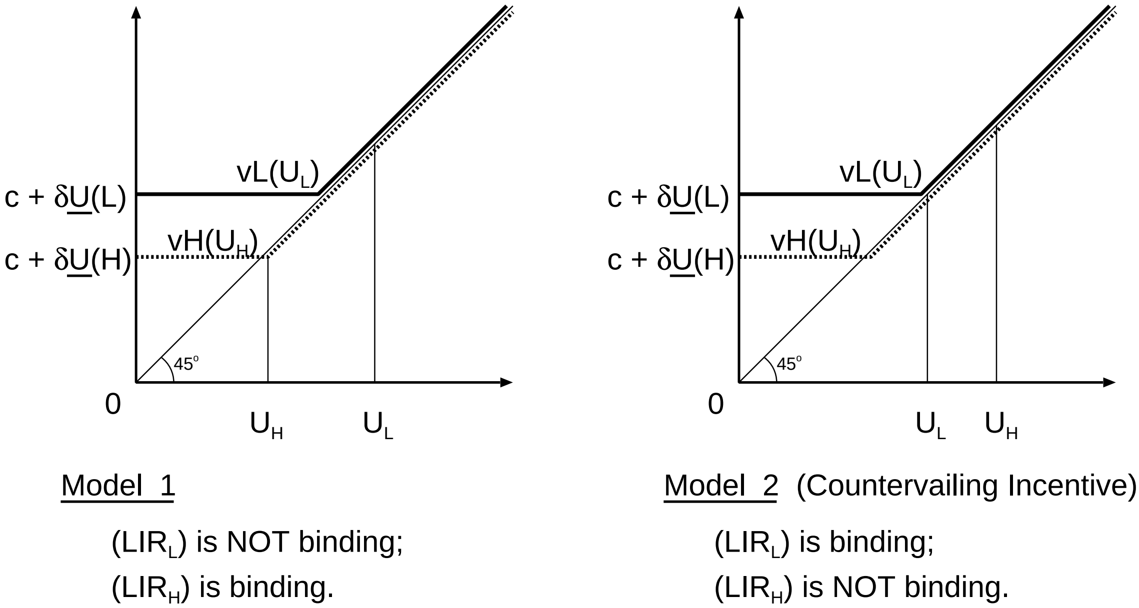


Figure 4: Countervailing Incentive

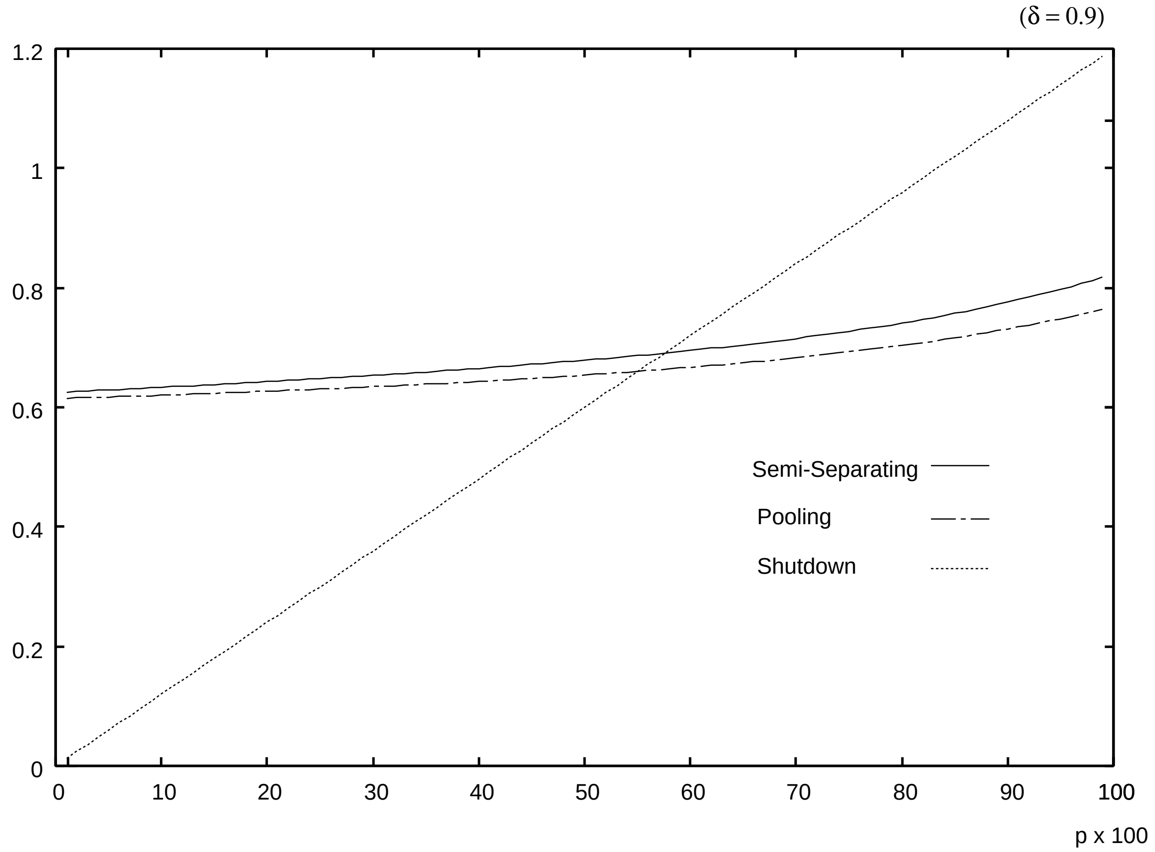


Figure 5: Principal's Payoffs in Three Contracts (1)

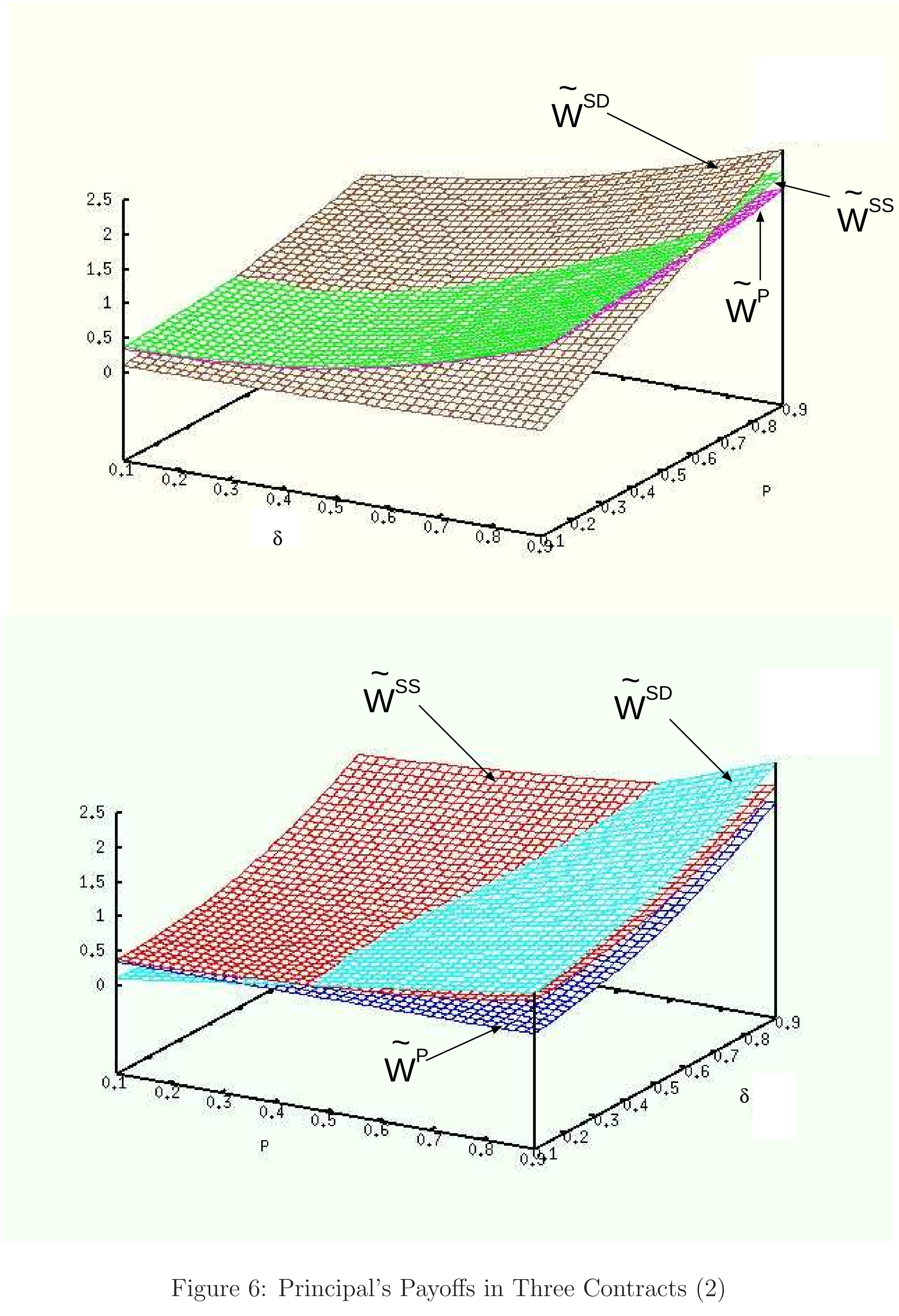


Figure 6: Principal's Payoffs in Three Contracts (2)

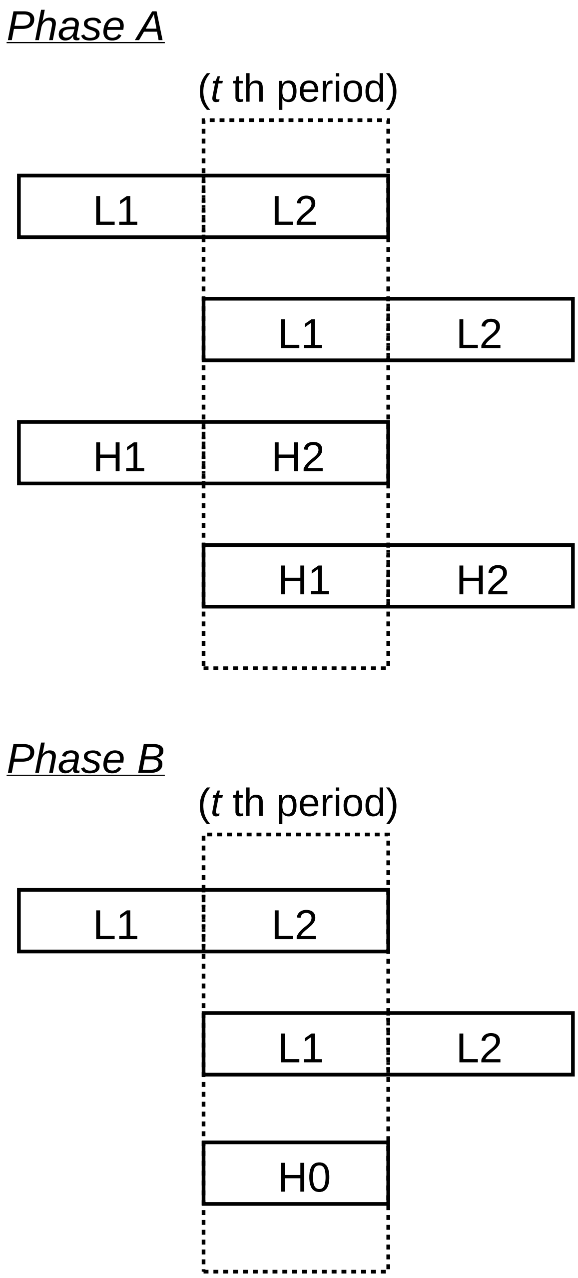


Figure 7: Full-Employment Phase and Unemployment Phase

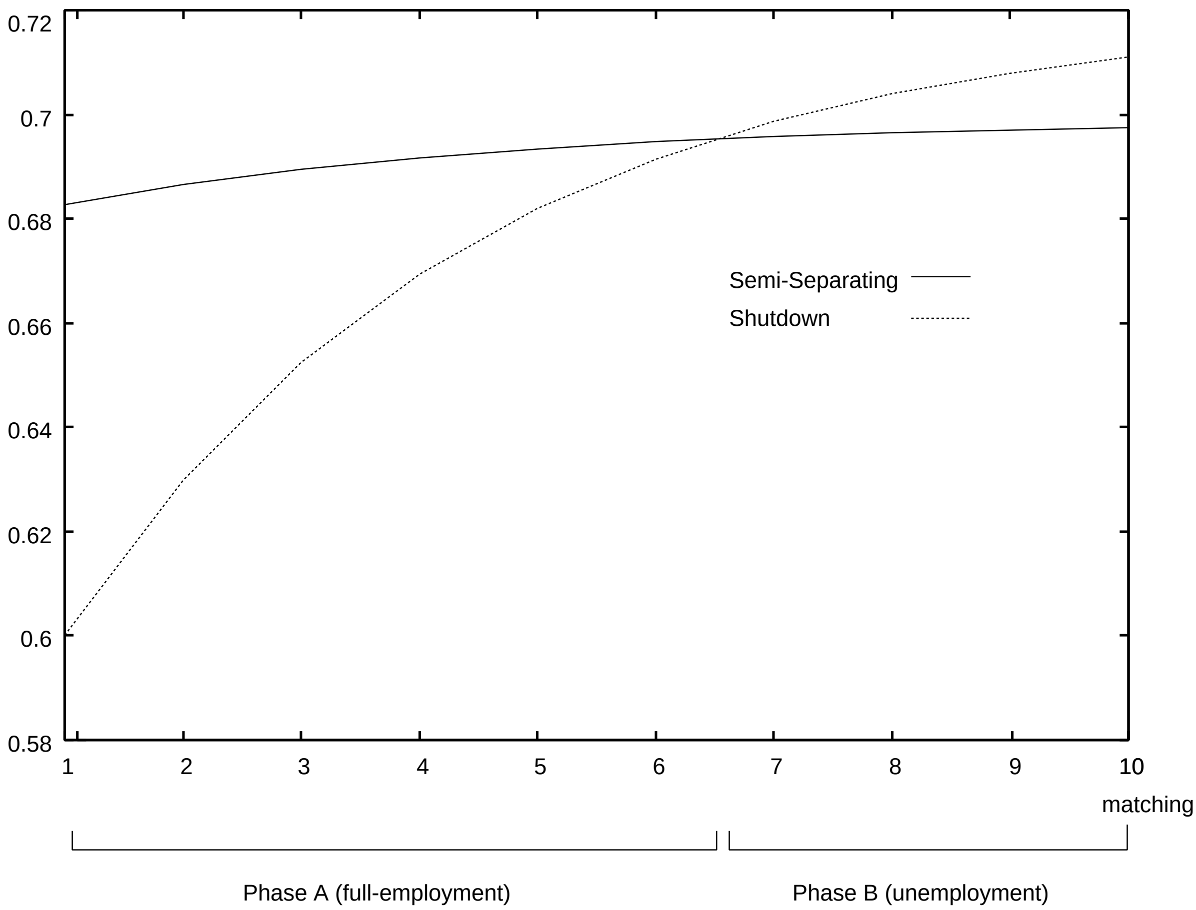


Figure 8: Simulation Result 1

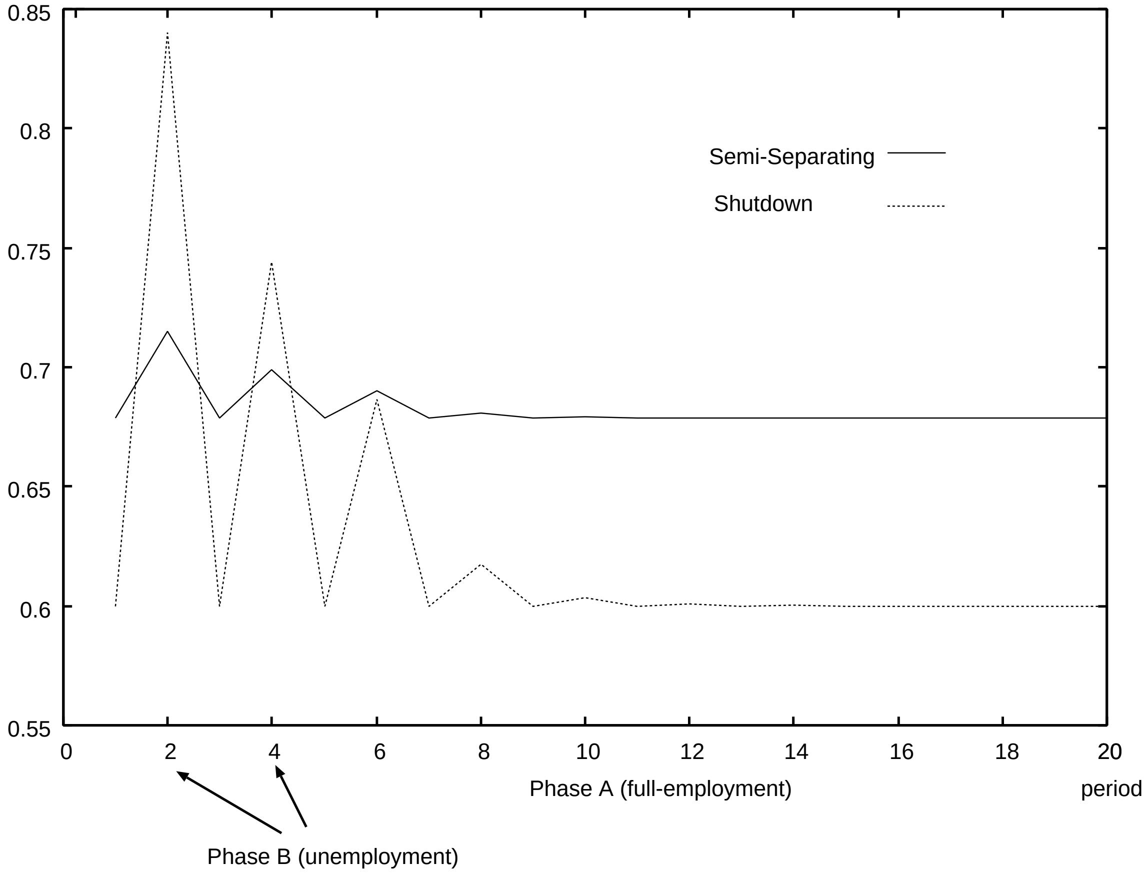


Figure 9: Simulation Result 2

t	p	p1	pL1	pL2	pH1	pH2	pH0
1	0.50000	0.33333	0.50000	0.00000	0.00000	0.50000	0.00000
2	0.70000	0.53846	0.20000	0.50000	0.30000	0.00000	0.00000
3	0.50000	0.33333	0.30000	0.20000	0.00000	0.30000	0.20000
4	0.62000	0.44928	0.32000	0.30000	0.38000	0.00000	0.00000
5	0.50000	0.33333	0.18000	0.32000	0.00000	0.38000	0.12000
6	0.57200	0.40056	0.39200	0.18000	0.42800	0.00000	0.00000
7	0.50000	0.33333	0.10800	0.39200	0.07200	0.42800	0.00000
8	0.51440	0.34626	0.40640	0.10800	0.41360	0.07200	0.00000
9	0.50000	0.33333	0.09360	0.40640	0.08640	0.41360	0.00000
10	0.50288	0.33590	0.40928	0.09360	0.41072	0.08640	0.00000
11	0.50000	0.33333	0.09072	0.40928	0.08928	0.41072	0.00000
12	0.50058	0.33385	0.40986	0.09072	0.41014	0.08928	0.00000
13	0.50000	0.33333	0.09014	0.40986	0.08986	0.41014	0.00000
14	0.50012	0.33344	0.40997	0.09014	0.41003	0.08986	0.00000
15	0.50000	0.33333	0.09003	0.40997	0.08997	0.41003	0.00000
16	0.50002	0.33335	0.40999	0.09003	0.41001	0.08997	0.00000
17	0.50000	0.33333	0.09001	0.40999	0.08999	0.41001	0.00000
18	0.50000	0.33334	0.41000	0.09001	0.41000	0.08999	0.00000
19	0.50000	0.33333	0.09000	0.41000	0.09000	0.41000	0.00000
20	0.50000	0.33333	0.41000	0.09000	0.41000	0.09000	0.00000

t	eL*	eHSB(p0)	eHSB(p1)	e1	WWSS	WWSD
1	1.00000	0.00000	0.57143	0.59065	0.67879	0.60000
2	1.00000	0.00000	0.48000	0.55566	0.71519	0.84000
3	1.00000	0.00000	0.57143	0.59065	0.67879	0.60000
4	1.00000	0.00000	0.52414	0.57035	0.69879	0.74400
5	1.00000	0.00000	0.57143	0.59065	0.67879	0.60000
6	1.00000	0.00000	0.54522	0.57871	0.69023	0.68640
7	1.00000	0.00000	0.57143	0.59065	0.67879	0.60000
8	1.00000	0.00000	0.56663	0.58832	0.68096	0.61728
9	1.00000	0.00000	0.57143	0.59065	0.67879	0.60000
10	1.00000	0.00000	0.57048	0.59019	0.67922	0.60346
11	1.00000	0.00000	0.57143	0.59065	0.67879	0.60000
12	1.00000	0.00000	0.57124	0.59056	0.67888	0.60069
13	1.00000	0.00000	0.57143	0.59065	0.67879	0.60000
14	1.00000	0.00000	0.57139	0.59063	0.67881	0.60014
15	1.00000	0.00000	0.57143	0.59065	0.67879	0.60000
16	1.00000	0.00000	0.57142	0.59065	0.67880	0.60003
17	1.00000	0.00000	0.57143	0.59065	0.67879	0.60000
18	1.00000	0.00000	0.57143	0.59065	0.67879	0.60001
19	1.00000	0.00000	0.57143	0.59065	0.67879	0.60000
20	1.00000	0.00000	0.57143	0.59065	0.67879	0.60000

Table 3: Simulation Results

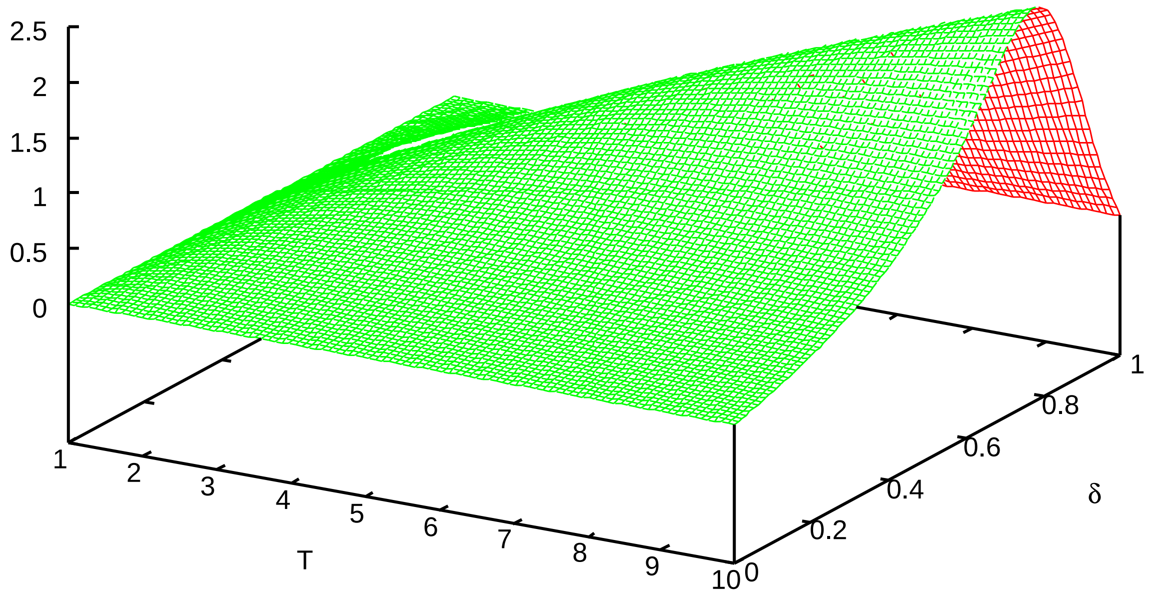


Figure 10: Reservation Parameter $\beta^{(1)}$

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%
% Simulation for Recursive Contract Model by Y.K.(11.2.2005)
%
% Matlab Code
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

clear;

a = 1.0; % parameter of a
c = 0.0; % c, reservation in one period
d = 0.9; % discount factor
dt = 0.5; % tH - tL
tL = 1.0; % theta_L
tH = tL + dt; % theta_H
pL = 0.6; % p(L)
pH = 0.4; % p(H)
x0 = 0.5; % L's strategy
b = d .* (1 - d) ./ (1 - d .* (1 - d)); % beta

% initial distribution
pL1(1) = 0.50; % probability of type L in 1st period
pL2(1) = 0.00; % probability of type L in 1st period
pH1(1) = 0.00; % probability of type H in 2nd period
pH2(1) = 0.50; % probability of type H in 2nd period
pH0(1) = 0.00; % probability of type H in unemployment
p(1) = pL1(1) + pL2(1); % probability of type L
P = [pL1, pL2, pH1, pH2, pH0]; % type distribution vector

% transition probability matrix
A = [0, 1, 0, 0, 0; pL, 0, (1 - pL), 0, 0; 0, 0, 0, 1, 0;
 pH, 0, (1 - pH), 0, 0; pH, 0, (1 - pH), 0, 0];
B = [0, 1, 0, 0, 0; pL, 0, 0, 0, (1 - pL); 0, 0, 0, 1, 0;
 pH, 0, 0, 0, (1 - pH); pH, 0, 0, 0, (1 - pH)];

for t = 1 : 20;

  % posterior probabilities
  p0(t) = (p(t) .* x0) ./ (p(t) .* x0);
  p1(t) = (p(t) .* (1 - x0)) ./ (p(t) .* (1 - x0) + (1 - p(t)));

  % e_L^{FB}, FB solution of e_L
  eLFB(t) = a ./ tL;
```

Figure 11: Matlab Code

```
 % e_H^{SB}(p^0), SB solution of e_H in 2nd period when p = p^0
 eHSBp0(t) = a .* (1 - p0(t)) ./ ((1 - p0(t)) .* tH + p0(t) .* dt);

 % e_H^{SB}(p^1), SB solution of e_H in 2nd period when p = p^1
 eHSBp1(t) = a .* (1 - p1(t)) ./ ((1 - p1(t)) .* tH + p1(t) .* dt);

 % e^{1}, SB solution of e_H in 1st period
 e1(t) = a .* (1 - p(t) .* x0) ./ ((1 - p(t) .* x0) .* tH + (p(t) .* x0
+ b .* pH) .* dt);

 % W^{AI}(p^0), static principal's payoff when posterior is p^0
 WAIp0(t) = p0(t) .* (a .* eLFB(t) - tL .* eLFB(t) .^ 2 ./ 2 - c) + (1
- p0(t)) .* (a .* eHSBp0(t) - tH .* eHSBp0(t) .^ 2 ./ 2 - c) - p0(t) .*
dt .* eHSBp0(t) .^ 2 ./ 2;

 % W^{AI}(p^1), static principal's payoff when posterior is p^1
 WAIp1(t) = p1(t) .* (a .* eLFB(t) - tL .* eLFB(t) .^ 2 ./ 2 - c) + (1
- p1(t)) .* (a .* eHSBp1(t) - tH .* eHSBp1(t) .^ 2 ./ 2 - c) - p1(t) .*
dt .* eHSBp1(t) .^ 2 ./ 2;

 % W^{FI}(1), static principal's payoff when posterior is 1
 WFI1(t) = (a .* eLFB(t) - tL .* eLFB(t) .^ 2 ./ 2 - c);

 % W^{SS}(p), dynamic principal's payoff when prior is p in semi-separating
 WWSSp(t) = p(t) .* x0 .* (a .* eLFB(t) - tL .* eLFB(t) .^ 2 ./ 2 - c) +
(1 - p(t) .* x0) .* (a .* e1(t) - tH .* e1(t) .^ 2 ./ 2 - c) - (p(t) .*
x0 + b .* pH) .* dt .* e1(t) .^ 2 ./ 2 - (p(t) .* x0 + b .* pH) .*
d .* dt .* eHSBp1(t) .^ 2 ./ 2 + (d - b) .* c + p(t) .* x0 .* d .*
WAIp0(t) + (1 - p(t) .* x0) .* d .* WAIp1(t);

 % W^{SS}(p), dynamic principal's payoff when prior is p in shutdown
 WWSDp(t) = p(t) .* (a .* eLFB(t) - tL .* eLFB(t) .^ 2 ./ 2 - c) + p(t)
.* dt .* eLFB(t) .^ 2 ./ 2 + p(t) .* (d - b) .* c + p(t) .* d .* WFI1(t);

 if WWSSp(t) > WWSDp(t) % Phase A: full-employment
   P = P * A;
 else % Phase B: unemployment
   P = P * B;
 end

 pL1(t + 1) = P(1);
 pL2(t + 1) = P(2);
 pH1(t + 1) = P(3);
 pH2(t + 1) = P(4);
 pH0(t + 1) = P(5);
```

```
  p(t + 1) = pL1(t + 1) + pL2(t + 1);

end

p0(21) = (p(20) .* x0) ./ (p(20) .* x0);
p1(21) = (p(20) .* (1 - x0)) ./ (p(20) .* (1 - x0) + (1 - p(20)));

hold off
plot (WWSSp)
hold on
plot (WWSDp)

t = linspace(1, 21, 21)';
sol1 = [t, p, p0, p1, pL1, pL2, pH1, pH2, pH0]

t = linspace(1, 20, 20)';
sol2 = [t, eLFB, eHSBp0, eHSBp1, e1, WWSSp, WWSDp]

%end
```

Appendix

A Justification for the Probabilities Believed by Agents

Why can the probability for the next matching quality be supposed to depend on the current matching quality? How can we justify $0 < p(L) < p < p(H) < 1$? Here, I provide one story for the justification on the probability of state conditional on the previous state.

Suppose that there are only two types of principals and agents, respectively:

$$\text{principal's type} = \begin{cases} G \text{ (Good)} & \text{with probability } \ p_{_P} \\ B \text{ (Bad)} & \text{with probability } \ 1 - p_{_P} \end{cases}$$

$$\text{agent's type} = \begin{cases} G \text{ (Good)} & \text{with probability } \ p_{_A} \\ B \text{ (Bad)} & \text{with probability } \ 1 - p_{_A} \end{cases}$$

where $p_{_P}, p_{_A} \in (0, 1)$. Suppose also that when one principal and one agent match, dependent on thier types, the matching quality, θ_L or θ_H, arises as follows:

		Agent	
		G	B
Principal	G	L	$\alpha L + (1-\alpha)H$
	B	$\alpha L + (1-\alpha)H$	H

where state L (H) means $\{\theta = \theta_L\}$ ($\{\theta = \theta_H\}$), $\alpha \in (0, 1)$ is a probability constant, and $\alpha L + (1-\alpha)H$ denotes that L (H) arises with probability α ($1-\alpha$). Both the principal and the agent cannot observe either type (even its own one) but only the agent can observe the arised state (L or H) before choosing its decision. The prior probability that the state is L is

$$p = p_P p_A + \alpha(1 - p_P)p_A + \alpha p_P(1 - p_A).$$

However, the agent can update the probability assigned to its own type, a, conditional on state s by Bayes's rule:

$$\text{Prob}\{a = G|\theta = \theta_L\} = \frac{p_A p(G)}{p_A p(G) + (1 - p_A)p(B)}, \qquad \text{and so on,}$$

where $p(G) \equiv \text{Prob}\{\theta = \theta_L|a = G\} = p_P + \alpha(1 - p_A)$ and $p(B) \equiv \text{Prob}\{\theta = \theta_L|a = B\} = \alpha p_P$.

Here, we assume that the agent evaluates its own type sequentially based on only the current state. Thus,

$$\text{Prob}_t\{a|\cdots, i_{t-2}, i_{t-1}, i_t\} = \text{Prob}_t\{a|i_t\}$$

where i_τ is the state in period τ.

Then, the agent who observed L as the current state can expect with $p(L)$, as follows, that the next state is L:

$$p(L) \equiv p_P \text{Prob}\{a = G|\theta = \theta_L\} + \alpha(1 - p_P)\text{Prob}\{a = G|\theta = \theta_L\} + \alpha p_P \text{Prob}\{a = B|\theta = \theta_L\}$$

Similarly, the agent who observed H as the current state can expect that the next state is L with

$$p(H) \equiv p_P \text{Prob}\{a = G|\theta = \theta_H\} + \alpha(1 - p_P)\text{Prob}\{a = G|\theta = \theta_H\} + \alpha p_P \text{Prob}\{a = B|\theta = \theta_H\}$$

We can show that $0 < p(H) < p < p(L) < 1$. Hence, it can be justified that $p(L)$ and $p(H)$ are formed in a valid inference.

Intuitively, the implication of this story can be explained as follows. If the agent observed state L in the previous period, it thinks its type to be G with high probability. Then, it will expect the next state to be L with high probability. Thus, state L may be transited to state L easier than to state H.

B The Existence of the Fixed Point

In this section I provide small preliminary for the mathematical proof of the existence of the fixed point in the dynamic model used in the main part of my model. Thus, this part is not a rigorous discussion of mathematics but an elementary "tool" for the model. The content in this section basically follows Ljungqvist and Sargent (2000), Stokey and Lucas (1989), Dieudonne (1960) and Kolmogorov and Fomin (1970).

First, I enumerate several definitions.

Definition 2 (Metric Space) A pair (X, d) of a set X and a mapping d ($d : X \times X \to R$) is called a *metric space* if $d(x, y)$ satisfies:

(I) $d(x, y) \geq 0$ for any $x, y \in X$,

(II) $d(x, y) = 0$ is equivalent to $x = y$,

(III) $d(y, x) = d(x, y)$ for any $x, y \in X$,

(IV) $d(x, z) \leq d(x, y) + d(y, z)$ for any $x, y \in X$.

d is called a *metric*.

Definition 3 (Cauchy Sequence) A sequence $\{x_n\}$ is a *Cauchy sequence* if for any $\varepsilon > 0$ there exists an integer n_0 such that $d(x_p, x_q) < \varepsilon$ for any $p, q \geq n_0$.

Definition 4 (Sequence Convergence) A sequence $\{x_n\}$ *converges to* x_0 if for any $varepsilon > 0$ there exists an integer n_0 such that $d(x_n, x_0) < \varepsilon$ for any $n \geq n_0$.

Definition 5 (Complete Metric Space) A metric space (X, d) is *complete* if any Cauchy sequence in (X, d) converges in (X, d).

Definition 6 (Operator) A mapping T from a metric space (X, d) into (X, d) is called an operator.

Usually, the term, operator, is used when it is mapping a function value, $f(x)$, into another function value, $g(x)$, for each x in the domain of these functions. In this case, it is noted as $g = Tf$.

Definition 7 (Contraction Mapping) Let (X, d) be a metric space and consider a mapping f $(f : X \to X)$. f is called a contraction mapping with modulus α if there is a real number k $(0 \leq \alpha < 1)$ such that

$$d[f(x), f(y)] \leq \alpha d(x, y)$$

for all $x, y \in X$.

Next, I summarize important results as the following proposions. For the proofs, see Kolmorogorov and Fomin (for Lemma 1) and Ljungqvist and Sargent or Stokey and Lucas (for Theorem 1 and Theorem 2).

Lemma 1 *Let $C[0, \bar{U}]$ be the set of all continuous functions mapping $[0, \bar{U}]$ into R and define a metric as:*

$$d_{\infty}(x, y) \equiv \sup_{0 \leq U \leq \bar{U}} |x(U) - y(U)| \,.$$

Then, $(C[0, \bar{U}], d_{\infty})$ is complete.

Theorem 1 (Blackwell's Sufficient Conditions) *Let T be an operator on a metric space (X, d_{∞}), where X is a space of functions. If T satisfies the following two properties;*

(Monotonicity) *For any* $x, y \in X$*, if* $x \geq y$*, then* $Tx \geq Ty$*;*

(Discounting) *Let* k *denote a function that is constant at the real value* k *for all points in the domain of the functions in* X*. For any positive real* k *and every* $x \in X$*,* $T(x+k) \leq Tx + \alpha k$ *for some* α $(0 \leq \alpha < 1)$*;*

then T *is a contraction mapping with modulus* α*.*

Theorem 2 (Contraction Mapping Theorem) *Let* (X, d) *be a complete metric space and let* $T : X \to X$ *be a contraction mapping. Then, there is a unique point* x_0 *such that* $Tx_0 = x_0$*.*

Now, we are ready to verify the Bellman equation used in the main part of the recursive model (Equation (25) at Page 19). The equation is:

$$v(U) = \max\left\{U,\ c + \delta(1-\delta)\left[pv(U_L) + (1-p)v(U_H)\right]\right\}.$$

Since $0 \leq p, \delta \leq 1$ and c, U_L and U_H are real values, $(c+\delta(1-\delta)\left[pv(U_L) + (1-p)v(U_H)\right])$ is bounded by some real value. Let $\bar{U}$ denote a value in the upper bound. Obviously v is continuous and we can consider a bounded continuous function space $C[0, \bar{U}]$ with a metrix d_∞ which is defined above. Thus, the metric space, $(C[0, \bar{U}], d_\infty)$, is complete from Lemma 1.

Consider an operator T such that:

$$Tz(U) \equiv \max\left\{U,\ c + \delta(1-\delta)\left[pz(U_L) + (1-p)z(U_H)\right]\right\}.$$

In order to verify that T has a unique solution, let us check the Blackwell's conditions for T:

(Monotonicity) Consider two mappings f and g such as $f(U) \geq g(U)$ for any $U \in$

$[0, \bar{U}]$. Then,

$$\begin{aligned} Tg(U) &= \max\{U,\ c + \delta(1-\delta)\left[pg(U_L) + (1-p)g(U_H)\right]\} \\ &\le \max\{U,\ c + \delta(1-\delta)\left[pf(U_L) + (1-p)f(U_H)\right]\} \\ &= Tf(U) \end{aligned}$$

for any U. Hence, the first condition in Theorem 1 holds.

(Discounting) Consider a mapping f and a real constant function k. Then,

$$\begin{aligned} T(f+k)(U) &= \max\{U,\ c + \delta(1-\delta)\left[p(f+k)(U_L) + (1-p)(f+k)(U_H)\right]\} \\ &= \max\{U,\ c + \delta(1-\delta)\left[p(f(U_L)+k) + (1-p)(f(U_H)+k)\right]\} \\ &= \max\{U,\ c + \delta(1-\delta)\left[pf(U_L) + (1-p)f(U_H)\right] + \delta(1-\delta)k\} \\ &\le \max\{U,\ c + \delta(1-\delta)\left[pf(U_L) + (1-p)f(U_H)\right]\} + \delta(1-\delta)k \\ &= Tf(U) + \delta(1-\delta)k. \end{aligned}$$

Since $0 \le \delta(1-\delta) < 1$, the second condition in Theorem 1 also holds.

Therefore, T is a contraction mapping with modulus $\delta(1-\delta)$.

Finally, from Theorem 2, there exists a unique solution v_0 such that $Tv_0 = v_0$. It is completed to prove the existence and the uniqueness of the solution for the Bellman equation (25).

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