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DYNAMICAL GENERATION OF FERMION MASS

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ABSTRACT

The problem of dynamical generation of fermion masses and chiral symmetry breaking is studied. Another formulation for the Chang-Chang approach, which is based on the Nambu-Jona-Lasinio (NJL) mechanism, is presented. It is showed that the solution $m=\Lambda_c e^{1/6}$ is independent of the two-loop constant. The gauge independence of the solution of the self-consistency condition in two loop accuracy is showed through an explicit calculation in arbitary gauge and renormalization group analysis. The up-down quark mass difference is calculated by including QED interaction as perturbation to the QCD calculation and in two-loop accuracy the result is of the right sign and also of right order of magnitude. The supersymmetric extension of NJL model (SNJL) is studied. It is showed that there is a nontrivial solution of self consistency condition for the SNJL model. Using the bifurcation theory, the conditions of the stable nontrivial solution of the NJL and SNJL models are found and the critical temperature is calculated. It is showed that in supersymmetric gauge field theories chiral symmetry is broken via NJL mechanism and the dynamically generated mass is $M = \Lambda_{c(susy)} e^{1/2}$.

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NOTATION

Our spacetime metric is $\delta_{\mu\nu}$. The momentum 4-vector is $k = (\vec{k}, ik_0)$ with norm squared

$$k^2 = k_\mu k_\mu = \vec{k}^2 - k_0^2 = -m^2.$$

Summation over repeated indices will always be understood. The Dirac γ_μ's, $(\mu = 1, \cdots, 4)$ are all hermitian with square equal to one. $\not{p}$ is defined as

$$\not{p} = -i\gamma \cdot p$$

We will work with natural units in which $\hbar = c = 1$.

But, in chapter 4, 5 and 6, in which we study the supersymmetric theories, we follow the notations in the book "Supersymmetry and Supergravity" by J. Wess and J. Baggers, as do most of the other recent work in this field.

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CHAPTER 1: GENERAL INTRODUCTION

In the lexicon of renormalizable field theory, the fermion mass is destined to play a subtle but very important role. In the old days, we simply put in the fermion mass by hand. Unlike the vector boson, putting in a fermion mass does not disturb or destroy the manifest renormalizability of the theory. The successful QED is such a theory where m_e is put in by hand. It has, in fact, met the experimental test to such an embarrassing degree that, indeed, it could in turn be used to probe the hadronic content of the theory.

But in our drive to grand unification[1], it is natural to assume the fermions to be fundamentally massless and to seek for the theory to generate the masses dynamically. Setting the fermion masses to zero also gains for us some chiral symmetry. This is useful in protecting the fermions from developing large mass due to higher-order radiative corrections in a grand unified theory. But this chiral invariance may only be formal. It is well known that fermion field theories can encounter the Adler-Bell-Jackiw anomaly[2] which destroys the chiral invariance. For non-Abelian gauge theories, the non-abelian chiral anomalies even threaten the renormalizability of the theory, unless the fermion content has been arranged so that the total non-Abelian chiral anomaly vanishes.

In grand unified theories (GUT's), this is usually arranged. The fermion mass is then finally generated through their Yukawa coupling to Higgs fields,

$$- h\bar{\Psi}\Psi\Phi$$

which upon symmetry breaking becomes($\Phi \rightarrow \Phi + v$)

$$- h\ v\bar{\Psi}\Psi - h\bar{\Psi}\Psi\Phi$$

The mass, so obtained, is the "current" mass with renormalization-group transformation properties that any tree Lagrangian mass parameter should have.

Such a mechanism has often been used to study and analyze the observed fermion-mass spectrum. In order to fit the spectrum, it is found that the Yukawa coupling constant for the electron is of order 10^{-5}, while the h for the u and d quarks is of order 10^{-4}. These extremely small coupling constants are to be compared with the gauge coupling constants in the theory, being of order 10^{-1}. Perhaps in the fundamental Lagrangian these h's should really be zero. In that case, we must attribute the fermion masses (at least of the first generation) to dynamical mass generation.

The pioneering work on dynamical fermion mass generation was by Nambu Jona-Lasinio (NJL)[3] in the early 60's. The idea of NJL was motivated by the observation of an interesting analogy between the properties of Dirac particles and the quasi-particle excitations that appear in the theory with great success by Bardeen, Cooper, and Schrieffer (BCS)[4]. In BCS theory, an energy gap between the ground state and the excited states of superconductor is due to the fact that the attractive phonon mediated interaction between electrons produces correlated pairs of electrons, and it takes a finite amount of energy to break this correlation. In NJL theory, the fermion was originally massless so that the spectrum was continuous. As a result of the direct field-theoretic generalization of BCS pairing interaction, a gap in the energy spectrum arises, giving the fermion a non zero mass.

The Lagrangian of NJL model is

$$\mathcal{L} = -\bar{\psi}\gamma\cdot\partial\psi + \mathcal{L}_I \tag{1,1}$$

where $\mathcal{L}_I$ is four fermion interaction

$$\mathcal{L}_I = \frac{1}{4} g \left[(\bar{\Psi}\Psi)^2 - (\bar{\Psi}\gamma_5\Psi)^2 \right] \qquad (1.2)$$

This Lagrangian, $\mathcal{L}$, is invariant under the chiral transformation

$$\Psi \rightarrow e^{i\gamma_5\alpha}\,\Psi \qquad (1.3)$$

In the usual perturbation theory the fermion will remain massless in any finite order due to this chiral symmetry. The crucial assumption of NJL is that due to the nonperturbative effect, the physical mass of fermion could be nonzero. To look for the physics of such a "paired" vacuum, they proposed to do a perturbative expansion around the massive free Lagrangian rather than the naive one. That is they choose

$$\mathcal{L}_o' = -\bar{\Psi}\gamma\cdot\partial\Psi - m\bar{\Psi}\Psi \qquad (1.4)$$

as the free Lagrangian and write

$$\mathcal{L}'_I = \mathcal{L}_I + m\bar{\Psi}\Psi \qquad (1.5)$$

as the new interaction Lagrangian. $\mathcal{L}'_I$ is now the new interaction that perturbs around this massive vacuum. In order that it be the same theory as the original

Lagrangian (1.1) there has to be a self consistency condition that fixes this paramater m. The self consistency condition they chose is that radiactive corrections due to $\mathcal{L}'_I$ must not change the mass m.

$$m - \Sigma(p) \Big|_{\not p = m} = 0 \qquad (1.6)$$

where Σ is calculated from the self-energy graphs. At the 1-loop level, they find

$$\Sigma(p) \Big|_{\not p = m} = - \frac{2ig}{(2\pi)^4} \int \frac{d^4p}{p^2 + m^2} \qquad (1.7)$$

and their self consistency condition gives

$$m = - \frac{2ig}{(2\pi)^4} \int \frac{d^4p}{p^2 + m^2} \qquad (1.8)$$

The NJL self-consistency condition admits two solutions. One is the (trivial) solution m = 0. The other is the nontrivial broken symmetry solution $m \neq 0$, where m is given by

$$1 = - \frac{2ig}{(2\pi)^4} \int \frac{d^4p}{p^2 + m^2} \qquad (1.9)$$

Chiral symmetry is broken in this nontrivial case. From (1.9) we can in principle determine the dynamically generated mass m. But unfortunately the NJL model is non renormalizable in the sence of power counting, we have to introduce a momentum cutoff Λ in (1.9) and m is determined as a function of g and Λ by (1.9).

The main problem of NJL theory is that four fermion interaction is not renormalizable. We can not really calculate the dynamically generated fermion mass without introducing cutoff. The reliability of the results of the calculation is spoiled by the cutoff procedure.

After NJL's pioneering work, a lot of work followed in this direction. Many authors try to apply the NJL approach in renormalizable theories and calculate the dynamically generated fermion masses. Much of the work in this direction can be classified into the following categories:

1) Study of four-fermion interactions.[5-7]

This line of work is a direct continuation of the NJL effort to study the physics of relativistic generalization of BCS. Of particular interest in this direction is the work of Gross and Neveu[5] who studied it as a field theory in two dimensions, where the theory is both renormalizable and asymptotically free.

2) Schwinger-Dyson equation[8-13]

Here the idea is to study the Schwinger-Dyson equation for complete fermion propogator and look for chiral symmetry non-invariant solution of the Schwinger-Dyson equation. Because the Schwinger-Dyson equation for gauge theories can be solved so far only under the so-called truncated approximation, this approach arbitarily picks out for study a subset of the full perturbative series. It thus suffers a lack of gauge invariance of the theory. The main feature of this approach is that they are able to study numerically the so-called regular or irregurar solutions to the dynamical symmetry breaking.

3) Study of chiral symmetry breaking due to confinement[14-19]

That instantons can lead to chiral symmetry breaking has already been noted by 't Hooft[14] since 70's. However no precise numerical estimate is yet available although the feeling is that its magnitude is too small. Recently, Adler and Davis[17] have made a new effort in this direction, making use of confinement potencial in an effective 4-fermion Hamiltonian approach. Their numerical results confirm the feeling that confinement contribution to chiral symmetry breaking is too small.

4) renormalization group (short distance) effects in chiral symmetry breaking.[20-25]

We will give a brief review on these studies and try to point what we should study further. For more detail review and references, please see ref. 26.

In category 1, Gross and Neveu[5] studied two dimensional massless fermion field theories with quartic interactions, which is essentially equivalent to the NJL models, save the fact that in two dimensions they are renormalizable. These models are also asymptotically free. The models are expanded in powers of 1/N, where N is the number of components of the fermion field. In such an expansion one can explicitly sum to all orders in the coupling constant. It is found that dynamical chiral symmetry breaking occurs, fermion mass is generated dynamically, $\bar{\Psi}\Psi$ develops a nonvanishing vacuum expectation value. They also argued that infrared-stable theories, such as QED, can not produce masses dynamically. Even though two dimension is not a realistic world, Gross and Neveu did throw some light on the four dimensions theories in the problem of dynamical mass generation.

Eguchi and Sugawara[6] studied the modified NJL model by adding a interation term $g'((\bar{\Psi}\gamma_\mu\Psi)^2 + (\bar{\Psi}\gamma_\mu\gamma_5\Psi)^2)$ to the NJL Lagrangian, that is most general form of four-fermion interactions invariant under the chiral transformation. They found the more complicated gap equation. Since the model is also not renormalizable they did not ease the difficulties of the original NJL model, but the study of the implication of the new gap equation suggested to them a stringlike picture for the extended hadrons.

Turning next to the general category of the Schwinger-Dyson approach, we discuss the work of Jackiw and Johnson[8], Cornwall and Norton.[9] They showed that it is consistent for a gauge theory of fermions interacting with massless Abelian vector mesons to have symmetry breaking solutions such that there is a finite physical mass for the fermion as well as for the vector meson. Jackiw and Johnson considered a theory described by the Lagrangian

$$L = i\bar{\Psi}\not{\partial}\Psi - \frac{1}{4}F_{\mu\nu}\ F_{\mu\nu} + gJ_5^{\mu}A_{\mu} \qquad (1.10)$$

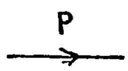
$$J_5^{\mu} = i\bar{\Psi}\gamma_5\gamma_{\mu}\Psi \qquad (1.11)$$

This theory is chirally invariant and renormalizable. They assumed that the Schwinger-Dyson equation for the fermion mass operator $\Sigma(p)$ has a symmetry-breaking solution, $\{\gamma^5, \Sigma(p)\} \neq 0$. This can happen if there is a massless, bound excitation in the fermion antifermion channel. The proper vertex function $\Gamma_5^{\mu}(p,p)$

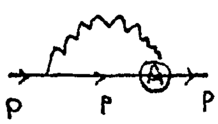
associated with J_5^μ satisfies a Ward-Takahashi identity.

$$q_\mu \Gamma_5^\mu (p, p+q) = \gamma^5 G^{-1}(p+q) + G^{-1}(p)\gamma^5 \quad (1.12)$$

where G(p) is the complete fermion two point Green's function. This Green's function is given by the following Schwinger-Dyson equation, shown diagrammatically as eq. (1.13)

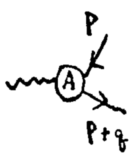


$= G(p) = \dfrac{i}{\not{p} - \Sigma(p)}$

$\Sigma(p) = -ig^2$ 

(1.13)

$= i\gamma^\mu \gamma^5$



$= \Gamma_5^\mu(p, p+q)$

To lowest order in the coupling they showed that there is a symmetry breaking solution to eq. (1.13). They relate this solution to the residue of the pole in Γ_5^μ. This in term induces in the two point function of the axial vector meson a mass term for the axial vector meson

(Schwinger mechanism). They thus derived a dynamical relation between the axial vector meson mass and the fermion mass. But, they could not calculate the dynamically generated fermion mass. And this is in general true of all the Schwinger-Dyson equation analyses of dynamical chiral symmetry breaking.

Many early work[8-11] used Abelian gauge theory instead of non Abelian theory, largely to avoid the tedious complications of ghost scalars which accompany closed loops of vector mesons. There have also been quite a few works to study the dynamical generation of fermion mass in non-Abelian theory.

Eichten and Feinberg[12] extended the work of Jackiw-Johnson and Cornwall-Norton to the non-Abelian case. Using Schwinger mechanism, they even could generate a dynamical breakdown of non-Abelian gauge symmetry without fermions. But for the chiral symmetry breaking part, in their approximation, the result and mechanism is very similar to that of Jakiw et al. They too were unable to calculate the dynamically generated fermion mass.

Kleinert[13] studied the quark mass generation problem in QCD by simplifying the QCD Lagrangian to one which neglects color and contains only the basic mesons as bound states. This is achieved by using singlet gluons of very large mass μ. Using the NJL approach, then he found a gap equation which contains logarithmically divergent factor. The dynamical mass M thus cannot determined by the gap equation. Only by studying the properties of bound states, can M be determined in terms of observable quantities.

Some authors tried to associate the origin of chiral symmetry breaking with confinement, another necessary property of QCD. It has been shown, in some lattice gauge theory calculation, within the mean field approximation, that confinement and the chiral symmetry breaking are appearing or disappearing together[15].

Cornwall[16] in 1980 illustrated how a field theory of confinement automatically lead to spontaneous breakdown of chiral symmetry, with accompanying massless pions. Yaouanc et al[16] showed, using the Bogoliubov-Valatin variational method, that the chiral-invariant vacuum is unstable for a color, Fourth-component vector powerlike potencial r^{α} ($\alpha < 3$) independently of the strength of coupling constant. They computed the vacuum expectation value $\langle\bar{\Psi}\Psi\rangle$ and the mass gap for the new vacuum in the case of the harmonic oscillator $\alpha=2$. They found the mass gap A(0) is negative. Of course, that is not a very attractive feature without meantioning the limitation of the Bogoliubov-Valatin variational method and the confining potencial they chose $V \sim r^2$. Adler and Davis[17] analyzed chiral symmetry breaking in QCD in Coulomb gauge. Using the Ward identities, they derived the renormalized gap equation. Working within the ladder approximation, they presented the results of numerical solution in the case of an infrared-singular confining potential $V_c \sim q^{-4}$. Their results of dynamical quark mass, f_π and $\langle \bar{\Psi}\Psi \rangle$ are all much smaller than the experimental results. That indicated that the contribution of short distance effect are very important and should be included.

It seems to me that it is very hard to get definite result by connecting the chiral symmetry breaking in QCD to confinement because we do not understand well confinement and especially a confining potencial V(r) like r^α $(\alpha > 0)$, being positive everywhere, is very hard to give a negative energy bound state.

In fact, the quark-quark potential generated by short-distance one-gluon exchange may also lead to chiral symmetry breaking. If the gluon coupling is bigger than some critical value α_s^{crit}, there is indeed instability of the chiral-invariant vacuum[19]. Computer simulations in lattice gauge theory suggest that the range of force responsible for chiral symmetry breaking is relatively short, independent of confiment[20]. This result seems to be quite natural, since we expect intuitively a very small size for such a tightly bound state as a pion.

There are also quite a few authors who tried to study the chiral symmetry breaking in QCD due to the short distance effect.

Lane[21] tried to find the solutions to the homogeneous Bethe-Salpeter equation for the symmetry breaking part G of the quark propagator in the limit $p^2 \to -\infty$ in an asymptotically free gauge field theory. He found a "regular" solution $G(p) \sim (\ln p)^A/p^2$, which corresponds to the Goldstone mode, and an "irregular solution" corresponding to $m_0 \neq 0$. Politzer[22] extended the analysis in operator product expansion and showed that the contribution of dynamical chiral symmetry breaking to the effective fermion mass goes like $\langle \bar{\psi}\psi \rangle (\log p^2)^A/p^2$ for large p. Scadron et al[24] also studied the properties of the running dynamical fermion mass. But none of them were able to really calculate the dynamically generated fermion mass.

K. Higashijima[25] studied the solution of Schwinger-Dyson equation for the quark propagator in QCD. He simplified the Schwinger-Dyson equation by assuming that the short-range force rather than the confining force is responsible for the chiral-symmetry breaking in QCD and the kernel of Schwinger-Dyson equation may be approximated by the one-gluon-exchange contribution since the coupling constant becomes smaller at short distance. He used the so called renormalization group improved Schwinger-Dyson equation by using the running coupling constant g(t) instead of g(0) in the Schwinger-Dyson equation. But in order to avoid the divergent region of g(t) he had to define g(t) as

$$g^2(t) = \begin{cases} A/t & (t>t_c) \\ A/t_c & (t<t_c) \end{cases}$$

with $t = \ln(p/\mu)$.Such a definition of g(t) seems too arbitary. From numerical calculation, he found that the dynamical generated fermion mass is very sensitively dependent on the critical point t_c which is put in by hand.

Summarizing the works mentioned above, we find that much progress in different directions of the problem of dynamical generation of fermion mass has been made, but it is still far away from the satisfactory final answer. The main problem is that there is not a systematical method to really calculate the dynamically generated fermion mass in QCD. There is also one thing which should be point out that in all the above studies of chiral symmetry breaking in gauge field theory, because of the complicacy of the Schwinger-Dyson equations the ladder approximation has to be applied, the dynamical fermion mass calculated in this approximation is gauge dependent. The argue of which gauge is the best gauge is very hard to find a answer.

Recently, progress in the problem of dynamical generation of fermion mass has been made by L.N. Chang and N.P. Chang.[27] Chang and Chang developed a new approach to calculate the dynamically generated fermion mass in QCD based on the NJL mechanism. They start from a massless QCD Lagrangian. Because the fundamental fields are massless, the Lagrangian possesses a chiral symmetry. In dynamical symmetry breaking, it is conjectured that in analogy with the ferromagnet the ground state does not respect this global symmetry. A sea of fermion-antifermion pairs condense in the ground state so that

$$(\bar{\Psi}\Psi) \neq 0 \tag{1.15}$$

and the system spontaneously breaks the chiral symmetry Whereas the original NJL mechanism was proposed in the context of an unrenormalizable field theory, Chang and Chang made a renormalization-group analysis of the NJL gap equation (i.e., the self consistency condition) and found that it is indeed a renormalization-group invariant for QCD. They have found that the mass of the quark so generated is, to two-loop renormalization-group accuracy,

$$M = \Lambda_c^{(2)} \, e^{\frac{1}{b}} \quad , \tag{1.16}$$

where $\Lambda_c^{(2)}$ is the two-loop-invariant cutoff in QCD. This mass M is a renormalization-group (RG) invariant.

It is first time in QCD that a dynamically generated fermion mass is really calculated from the first principle of field theory. The result is accurate to two-loop renormalization group accuracy and agree with the experimental value. In the calculation, they did not follow the other authors in this field to make the ladder approximation, but included all the graphs in two-loop level. This fact may indicate that the dynamically generated fermion mass calculated in this way is gauge invariant. Indeed, in chapter II we will show that it is true by explicit calculation in arbitary gauge and using the renormalization group analysis.[28] This gauge independent will make the physical meaning of the dynamical fermion mass clearer.

The use of the renormalization group technique enables them to correctly count the non-perturbative effects by summing up the all leading and next to leading logarithm contribution up to infinite order in coupling constant. During the perturbative calculation in the Chang-Chang approach, counterterms propotional to M for canceling the infinities are neccasary, but after we impose the self-consistency condition, these counter terms all disappear automatically. This remarkable fact again shows that the approach is consistent, the renormalization group analysis correctly counts the nonperturbative effects.

In a recent development,[29] Chang and Chang also showed that the NJL gap equation can be understood in terms of the critical limit point from above of a massive QCD renormalizatiom field theory. The theory exhibit bifurcation in the zero bare mass limit, and the chiral symmetry remains broken. They also calculated the $\langle\bar{\psi}\psi\rangle$ in this limit. The results are also agree with the experimetal results.

Since the problem of dynamical generation of fermion masses is so important and the Chang-Chang approach is so powerful in studying this problem, in this thesis, we will further study the Chang-Chang approach and the NJL models. Because of the importance of the supersymmetry in the particle physics theory we will also study the supersymmetric extension of the NJL model[30] and the application of Chang-Chang approach in supersymmetric gauge field theories[31] in this thesis.

In chapter II, we will review the Chang-Chang approach and give another formulation for the Chang-Chang approach. We will extend their study to include an investigation of the role that two-loop renormalization could play in their mass determination. We show that the solution $m = \Lambda_c e^{\frac{1}{b}}$, which is independent of the two-loop constant, continuous to be a solution without any modification, and point out that exists another solution that is dependent on the value of two loop constant. We also study the Chang-Chang approach in arbitary gauge, and show that, even though the gap equation obtained in perturbation calculation is explicitly dependent on gauge, the solution of the self-consistency condition is gauge independent.

In chapter III, we apply the formalism to the notorious proton neutron mass difference problem. By including QED interaction as perturbation to the QCD calculation then calculating the up-down quark mass difference, we show that in two-loop accuracy the result is of the right sign and also of right order of magnitude.

In chapter IV, we study the supersymmetric extension of NJL model(SNJL) and show that there is a nontrivial solution of self consistency condition as in ordinary NJL model and clarify some misclaims in the supersymmetric NJL model.

In chapter V, we study the finite temperature effect in the NJL and SNJL models. Using the bifurcation theory, we find the condition of the stable nontrivial solution of the SNJL models and calculate the critical temperature above which chiral symmetry is restored.[32] The critical temperature in the SNJL model is much lower than that in the NJL model.

In chapter VI, we study the dynamical chiral symmetry breaking and mass generation in supersymmetric gauge field theories and show that the chiral symmetry is broken and the dynamically generated mass is

$$M = \Lambda_{c(susy)} e^{1/2} \qquad (1.17)$$

In chapter VII, we summarize the conclusions and make some remarks on the further study directions. Parts of chapter II and Chapter III of this thesis have been published in Physical Review D30,790 (1984) with Prof. N.P. Chang as coauthor.

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CHAPTER 2 DYNAMICAL FERMION MASS GENERATION IN QCD

2.1 QCD AND RENORMALIZATION GROUP

QCD is a renormalizable quantum field theory of the strong interactions. It appeared in the early 70's through a combined effort of many people.[1] Its fundamental fields are Dirac spinor fields describing quarks with color and flavor and gauge fields corresponding to chargeless and massless particle of spin 1, called gluons, which interact with the quarks and themselves.

The Lagrangian density of QCD is

$$L = -\frac{1}{4} \mathrm{Tr}\, F_a^{\mu\nu} F_{\mu\nu}^a + \bar{\Psi}_\alpha^A \left(-\gamma_\mu D_{\alpha\beta}^\mu - m\, \delta_{\alpha\beta}\right) \Psi_\beta^A \qquad (2.1)$$

where the index $A = 1,2,\ldots N_f$ refers to flavor, the index $\alpha = 1,2,\ldots N$ refers to the color degrees freedom and

$$D_{\alpha\beta}^\mu = \delta_{\alpha\beta} \partial^\mu - igT'^a_{\alpha\beta} A_a^\mu(x) \qquad (2.2).$$

where T'_a are the generators of SU(N) in fermion's representation, $T'_a T'_a = C_f I$

L is invariant under local gauge transformation

$$\psi_\alpha^A(x) \longrightarrow \psi_\alpha'^A(x) = \left[e^{igT^a\theta_a(x)} \right]_{\alpha\beta} \psi_\beta^A(x)$$

$$T^a_{\alpha\delta} A^\mu_a(x) \longrightarrow T^a_{\alpha\delta} A'^\mu_a(x) = \left[e^{igT^a\theta_a(x)} \right]_{\alpha\beta} T^a_{\beta\gamma} A^\mu_a(x) \cdot \left[e^{-igT^a\theta_a(x)} \right]_{\gamma\delta}$$
$$- \frac{i}{g} \left[\partial^\mu e^{igT^a\theta_a(x)} \right]_{\alpha\beta} \left[e^{-igT^a\theta_a(x)} \right]_{\beta\delta} \quad (2.3)$$

where T_a are the generators of SU(N) in its fundamental representation. Because the kinetic part of gluon is not invertible, we should add a gauge fixing term L_{gf} and the Faddeev-Popov ghost term L_{fp} to the Lagrangian (2.1)[2], then

$$L = L_{QCD} + L_{gf} + L_{pf}$$

where

$$L_{gf} = -\frac{1}{2a} \left[\partial_\mu A^\mu_a(x) \right]^2 \quad (2.4)$$

$$L_{pf} = - D^\mu \bar{c}^a \partial_\mu c_a$$

When we calculate the diagrams with loops we find divergent intergrals due to the behavior of the integrands at high virtual momenta. Before we can manipulate safely the divergent integrals, we must regularize them. We will use dimensional regularization[3], which preserves manifestly the gauge invariance of the theory and can be used to prove the renormalizability of QCD. The crucial point of the method consists in giving a meaning to the divergent integrals by changing the dimension of space-time from 4 to D, where $D=4-\epsilon$. In the limit of $\epsilon \to 0$, the divergence appears as pole $1/\epsilon$. For the useful formulae in dimensional regularization, see Appendix A.1.

To renormalize the theory is to give a well defined prescription to eliminate all divergent parts in such a way that we obtain finite results for the Green's functions, in the limit $\epsilon \to 0$, to any order in perturbation theory. Formally, we can add counterterms L_c to the QCD Lagrangian density corresponding to each divergent diagram appearing in the theory,

$$
\begin{aligned}
L_c = {} & \hat{C}_3 \cdot \frac{1}{2} \partial_\mu A_\nu^a \left(\partial^\mu A_a^\nu - \partial^\nu A_a^\mu \right) + \hat{C}_6 \frac{a}{2} \partial_\mu A_a^\mu \partial_\mu A_a^\nu \\
& + \hat{C}_2 \bar{\Psi}_\alpha^A \gamma^\mu \partial_\mu \Psi_\alpha^A + \hat{C}_4 m_A \bar{\Psi}_\alpha^A \Psi_\alpha^A \\
& - i \hat{C}_{1f} \bar{g} \bar{\Psi}_\alpha^A T'^a_{\alpha\beta} \gamma^\mu A_\mu^a \Psi_\beta + \hat{C}_{1YM} \cdot \frac{\bar{g}}{2} f_{abc} \left(\partial_\mu A_\nu^a - \partial_\mu A_\nu^a \right) A_b^\mu A_c^\nu \\
& + \hat{C}_5 \cdot \frac{\bar{g}^2}{4} f_{abc} f_{ade} A_\mu^b A_\nu^c A^{e\mu} A^{d\nu}
\end{aligned}
\tag{2.4}
$$

where

$$\bar{g} = \mu^{\frac{\epsilon}{2}} g$$

$$\hat{C}_i = \sum_{n=1}^{\infty} \hat{C}_i^{(n)} \bar{g}^{2n} \tag{2.5}$$

If all the extra terms need to remove the ultraviolet divergences are the terms in (2.4), then the theory is renormalizable.

We define the renormalization constant Z_i as

$$Z_i = 1 - \hat{C}_i \tag{2.6}$$

Then the Lagrangian is

$$L_R = L + L_C \tag{2.7}$$

If we define the bare fields, bare coupling constants, bare mass and bare gauge parameter as in the following,

$$A^a_{\mu_0} = Z_3^{\frac{1}{2}} A^a_\mu, \qquad \psi_0 = Z_2^{\frac{1}{2}} \psi$$

$$g_{Y0} = Z_1 Z_3^{-\frac{3}{2}} \bar{g}, \qquad g_{F0} = Z_{1F} Z_3^{-\frac{1}{2}} Z_2^{-1} \bar{g} \tag{2.8}$$

$$g_{40} = Z_4^{\frac{1}{2}} Z_3^{-1} \bar{g}, \qquad \tilde{g}_0 = \tilde{Z}_1 \tilde{Z}_3^{-1} Z_3^{-\frac{1}{2}} \bar{g}$$

$$m_0 = Z_m Z_2^{-1} m, \quad C_0 = Z_3^{\frac{1}{2}} C$$

Then the Lagrangian L_R can be written in terms of bare quantities. L_R is in the same form as L only if

$$g_{Y0} = g_{F0} = g_{40} = \tilde{g}_o = g_o \tag{2.9}$$

which means the renormalization constants can not be arbitrary. (2.9) can be satisfied only if the renormalization procedure preserve gauge invariance.

The minimal subtraction renormalization scheme (MS) of 't Hooft and Veltman[4] is defined as follows. We choose the renormalization constants C_i in such a way that they cancel exactly the poles in the Green's functions.

For example, in one loop level, the fermion two point function can be calculated from the diagram in Fig.2.1, and the result is:

$$\Gamma_F^{(2)} = \gamma\cdot p - im + \frac{\bar{g}^2}{16\pi^2} C_f \{(\gamma\cdot p - im)\, a \,[\tfrac{2}{\epsilon} - \ln 4\pi + \gamma_E$$

$$+ \ln\frac{m^2}{\mu^2} - 1 - \frac{m^2}{p^2} + (1 - \frac{m^4}{p^4}) \ln (1 + \frac{p^2}{m^2})]$$

$$- im\left[3\left(\frac{2}{\epsilon} - \ln 4\pi + \gamma_E\right) - 4 - a\left(1+\frac{m^2}{p^2}\right) + \left(3 + \frac{am^2}{p^2}\right)\left(1+\frac{m^2}{p^2}\right)\ln\left(1+\frac{p^2}{m^2}\right)\right] \Big\}$$

$$-\hat{C}_2 \gamma\cdot p + i\hat{C}_4 m \tag{2.10}$$

then in MS scheme in one loop level

$$\hat{C}_2 = \frac{g^2}{(4\pi)^2} C_f a \cdot \frac{2}{\epsilon} \tag{2.11}$$

$$\hat{C}_4 = \frac{g^2 C_f}{(4\pi)^2} \frac{6+2a}{\epsilon} \tag{2.12}$$

Bardeen, et al[5] realized that these poles always appear in the combination

$$\frac{1}{\hat{\epsilon}} = \frac{1}{\epsilon} - \frac{\gamma_E}{2} + \frac{1}{2}\log 4\pi \tag{2.13}$$

$$= \frac{1}{2}(4\pi)^{\frac{\epsilon}{2}}\Gamma\left(\frac{\epsilon}{2}\right) + O(\epsilon)$$

It is natural to eliminate $\frac{1}{\hat{\epsilon}}$ instead of $\frac{1}{\epsilon}$ by introducing the modified minimal subtraction scheme ($\overline{\text{MS}}$).

2.2 The Renormalization Group

Physical observables must be independent of the renormalization scheme, that is renormalization invariance under the renormalization group.[6] Callan-Symanzik[7] equation is powerful method for studying the behaviour at small distance of the Green's functions. t' Hooft and Weinberg[8] formulated more general methods. The central idea in these methods is to treat in a similar way the coupling constant, mass and gauge parameter renormalization constants.

The relation between the renormalized and bare Green's functions with n_{YM}, n_F gluon and quark lines, respectively, is

$$\Gamma(p_1, p_2, \ldots p_n, \lambda, a, m, \mu) = \lim_{\epsilon \to 0} Z_\Gamma(\mu, \epsilon) \cdot$$

$$\cdot \Gamma_0(p_1, p_2, \ldots\ldots p_n, \lambda_0, a_0, \epsilon) \quad ,$$

$$Z_\Gamma(\mu, \epsilon) = Z_3^{-n_Y/2} Z_2^{-n_F/2} \quad , \quad \lambda = \frac{g^2}{16\pi^2} \qquad (2.14)$$

Because the bare Green's function is obviously μ-independent,

$$\mu \frac{d}{d\mu} \Gamma_0(p_1, p_2, \ldots, p_n; g_0, a_0, m_0; \epsilon) = 0 \qquad (2.15)$$

and using (2.14),

$$\left[\mu \frac{\partial}{\partial \mu} + \mu \frac{d\lambda}{d\mu} \frac{\partial}{\partial \lambda} + \frac{\mu}{m} \frac{dm}{d\mu} m \frac{\partial}{\partial m} + \mu \frac{da}{d\mu} \frac{\partial}{\partial a} \right] \cdot$$

$$\cdot \Gamma_R(p_1,\ldots,p_n;\lambda,a,m,\mu)$$

$$= \frac{1}{Z_\Gamma}\frac{dZ_\Gamma}{d\mu}\cdot \Gamma_R(p_1,\ldots,p_n;\lambda,a,m,\mu) \tag{2.16}$$

let

$$\mu\frac{d}{d\mu}\lambda \equiv \beta(\lambda,a)\,\lambda$$

$$\frac{\mu}{m}\frac{dm}{d\mu} \equiv -\gamma_m(\lambda,a)$$

$$\frac{\mu}{Z_3}\frac{d}{d\mu}Z_3 \equiv \gamma(\lambda,a)$$

$$\frac{\mu}{Z_2}\frac{d}{d\mu}Z_2 \equiv \gamma_F(\lambda,a) \tag{2.17}$$

$$\mu\frac{d}{d\mu}a \equiv a\,\delta(\lambda,a)$$

in the MS, $\overline{\text{MS}}$ scheme, β, γ_m is gauge independent and accurate to two loop[9],

$$\beta = -b_1\lambda - b_2\lambda^2$$

$$\gamma_m = h_1\lambda + h_2\lambda^2$$

where,

$$b_1 = \frac{22}{3}C_2 - \frac{4}{3}f$$

$$b_2 = \frac{68}{3} C_2^2 - 4 C_f f - \frac{20}{3} C_f^2$$

$$h_1 = 6\ C_f$$

$$h_2 = 3\ C_f^2 + \frac{97}{3} C_f\ C_2 - \frac{10}{3} C_f\ f, \qquad (2.18)$$

then

$$\left(\mu \frac{\partial}{\partial \mu} + \beta \lambda \frac{\partial}{\partial \lambda} + \delta\, a \frac{\partial}{\partial a} - \gamma_m m \frac{\partial}{\partial m} - \gamma_\Gamma \right) \Gamma_R = 0 \qquad (2.19)$$

with

$$\gamma_\Gamma = -\frac{1}{2} \left[n_{YM} \gamma + n_F \gamma_F \right]$$

From dimension analysis we find

$$\Gamma_R(p_1, \ldots, p_n; \lambda, a, m; \mu) \equiv \mu^{d_\Gamma} \cdot$$

$$\cdot \Psi(p_1/\mu, \ldots, p_n/\mu; \lambda, a, m/\mu) \qquad (2.20)$$

where d_Γ is the dimension of Γ_R and Ψ is a dimensionless function of its dimensionless arguments.

If we rescale p_i, we can find

$$\Gamma_R(\xi p_1, \xi p_2, \ldots \xi p_n; \lambda, a, m, \mu) = \xi^d \, \Gamma_R(p_1, \ldots, p_n; \lambda, a, \frac{m}{\xi}, \frac{\mu}{\xi})$$

let $\xi = e^t$, we can find

$$\left[-\frac{\partial}{\partial t} + \beta\lambda\frac{\partial}{\partial\lambda} + \delta a \frac{\partial}{\partial a} - (1+\gamma_m) m\frac{\partial}{\partial m} + d_p - \gamma_p \right] \cdot$$

$$\cdot \, \Gamma_R(e^t p_1, \ldots, e^t p_n; \lambda, a, m, \mu) = 0 \tag{2.21}$$

That is the fundamental equation of renormalization group equation.

2.2 Formalism of Chang-Chang Approach

In the QCD Lagrangian (2.1), the quark mass is put by hand, but the natural way, as mentioned in chapter 1, is that in the fundamental Lagrangian the quark mass is zero and the effective quark mass is generated dynamically.

If m=0 in the QCD Lagrangian (2.1), then this Lagrangian is invariant under a chiral transformation.

$$\psi \rightarrow \psi' = e^{i\gamma_5 \alpha} \psi \tag{2.22}$$

This chiral invariance protects the fermion from having a mass in any finite order of perterbation calculation. However, the physical ground state may not respect this chiral symmetry. The fermions then acquire a mass associated with this dynamical symmetry breaking.

Chang and Chang[10] have developed a new approach to calculate the dynamically generated fermion mass in QCD based on the Nambu-Jona-Lasinio[11] (NJL) mechanism. Whereas the original NJL mechanism was proposed in the context of an unrenormalizable field theory, Chang and Chang made a renormalization-group analysis of the NJL gap equation (i.e., the self-consistency condition)and found that it is indeed a renormalization-group invariant for QCD. They have found that the mass of the quark so generated is, to two-loop renormalization-group accuracy,

$$M = \Lambda_c^{(2)} \; e^{\frac{1}{\zeta}} \quad , \tag{2.23}$$

where $\Lambda_c^{(2)}$ is the two-loop-invariant cutoff in QCD. This mass M is a renormalization-group (RG) invariant.

They start from the QCD Lagrangian

$$L = L_0 + L_I \tag{2.24}$$

where L_0 is the massless kinetic terms for the fermion and gauge fields. Following NJL, the Lagrangian (2.24) is rewritten as

$$L = (L_0 - M\bar{\Psi}\psi) + (L_I + M\bar{\Psi}\psi) \qquad (2.25)$$

and the perturbation theory is to be taken around the nonperturbative vacuum ($M\neq 0$). To do this new perturbation theory, Chang and Chang took the crucial step in introducing the intermediate Lagrangian

$$L' = (L_0 - M\bar{\Psi}\Psi) + (L_I + \delta M\bar{\Psi}\psi) \qquad (2.26)$$

The δM is treated just like a counterterm in the usual Lagrangian field theory, except that it is used to fix the renormalized two-point proper Green's function to be

$$\Gamma_r^{(2)}(p) = (\gamma\cdot p - iM)\,\tilde{Z}_2^{-1} \qquad (2.27)$$

for $p^2 \ll M^2$. The M in (2.27) is the same M as in (2.26). To one loop in λ they found

$$\delta M = -3\,C_f\,M\,(\log M^2/\mu^2 - 1/3) \qquad (2.28)$$

and to all orders in λ, but accurate to one loop RG accuracy, by using the renormalization group analysis, they found

$$M - \delta M = M \left(\frac{\lambda}{\lambda_0} \right)^{6C_f/b_1} \tag{2.29}$$

where ($\log \mu = t$)

$$\frac{d}{dt} \lambda = -b_1 \lambda^2 \tag{2.30}$$

$$\frac{1}{\lambda_0} = \frac{1}{\lambda} + b_1 (\log M/\mu - 1/6) \tag{2.31}$$

Imposing the self-consistency condition $\delta M=M$, the NJL self-consistency condition can be solved by $\frac{1}{\lambda_0}=0$, or

$$1 + \lambda b_1 (\log M/\mu - 1/6) = 0 \tag{2.32}$$

In terms of $\Lambda_c^{(1)}$, the one-loop cutoff, defined by

$$\frac{1}{\lambda} + b_1 \log \frac{\Lambda_c^{(1)}}{\mu} = 0 \tag{2.33}$$,

the NJL dynamically generated mass accurate to one loop RG accuracy, is then given by

$$M = \Lambda_c^{(1)} e^{1/6} \tag{2.34}$$

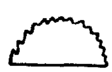
Here asymptotic freedom is very important, because if the theory is not asymptotically free, b_1 is negative, as is true for example in QED. Then $\frac{1}{\lambda_o}=0$ is not the solution of $M-\delta M=0$. The only solution in that case is

$$M = 0 \tag{3.35}$$

2.4 Two-loop Analysis

To find how two loop perturbation affect the result of dynamiccally generated fermion mass, we should calculate the diagrams in Fig.2.2. We find

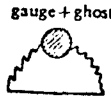
$$\Gamma_r = \gamma\cdot p - iM$$



$$+\left[\frac{M^2}{\mu^2}\right]^{-\epsilon/2}\left[\gamma\cdot p(\lambda C_f)\left[\frac{2\alpha}{\epsilon}+\frac{\alpha}{2}\right]-iM(\lambda C_f)\left[\frac{6+2\alpha}{\epsilon}+1+\alpha\right]\right]$$

$$+\lambda^2\left[\frac{M^2}{\mu^2}\right]^{-\epsilon}\left[1+\frac{\epsilon^2\pi^2}{24}\right]\left[\gamma\cdot p(C_f f)\left[-\frac{1}{\epsilon}+A_f\right]-iM(C_f f)\left[-\frac{4}{\epsilon^2}-\frac{4}{\epsilon}+B_f\right]\right.$$

gauge + ghost



$$+\gamma\cdot p(C_f C_2)\left[\frac{13-3\alpha}{4\epsilon}+\frac{27}{8}+\frac{\alpha}{16}+\frac{3\alpha^2}{16}\right]$$

$$-iM(C_f C_2)\left[\frac{13-3\alpha}{\epsilon^2}+\frac{41+4\alpha+3\alpha^2}{4\epsilon}+\frac{85}{8}-3\alpha+\frac{3\alpha^2}{2}+\frac{13}{12}\pi^2-\frac{\pi^2\alpha}{4}\right]$$

$$+\gamma\cdot p(C_f C_2)\left[\frac{3\alpha(\alpha+1)}{\epsilon^2}+\frac{9+17\alpha+8\alpha^2}{4\epsilon}+A_i+\alpha A_{i1}+\alpha^2 A_{i2}\right]$$

$$-iM(C_fC_2)\left[\frac{9+12\alpha+3\alpha^2}{\epsilon^2}+\frac{15+5\alpha+2\alpha^2}{\epsilon}+B_t+\alpha B_{t1}+\alpha^2B_{t2}\right]$$

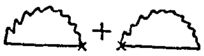
$$+\gamma\cdot p(C_f{}^2)\left[-\frac{2\alpha^2}{\epsilon^2}-\alpha\frac{(24+3\alpha)}{2\epsilon}+A_s+\alpha A_{s1}+\alpha^2A_{s2}\right]$$

$$-iM(C_f{}^2)\left[\frac{18-2\alpha^2}{\epsilon^2}-\frac{21+6\alpha+\alpha^2}{\epsilon}+B_s+\alpha B_{s1}+\alpha^2B_{s2}\right]$$

$$+\gamma\cdot p(C_f{}^2-\tfrac{1}{2}C_fC_2)\left[\frac{4\alpha^2}{\epsilon^2}-\frac{3-5\alpha^2}{2\epsilon}+A_c+\alpha A_{c1}+\alpha^2A_{c2}\right]$$

$$-iM(C_f{}^2-\tfrac{1}{2}C_fC_2)\left[\frac{4\alpha(\alpha+3)}{\epsilon^2}+\frac{(-9+2\alpha+3\alpha^2)}{\epsilon}+B_c+\alpha B_{c1}+\alpha^2B_{c2}\right]\Bigg]$$

$$+\lambda^2\left[\frac{M^2}{\mu^2}\right]^{-\epsilon/2}\left[1+\frac{\epsilon\pi^2}{48}\right]$$



$$\times\Bigg[\gamma\cdot p[(\alpha+3)C_fC_2+4\alpha C_f{}^2]\left[-\frac{2\alpha}{\epsilon^2}-\frac{\alpha}{2\epsilon}-\frac{\alpha}{8}\right]$$

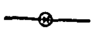
$$-iM[(\alpha+3)C_fC_2+4\alpha C_f{}^2]\left[\frac{-6-2\alpha}{\epsilon^2}-\frac{1+\alpha}{\epsilon}-\frac{1+\alpha}{2}\right]$$

$$-iM[(\alpha-\tfrac{13}{3})C_fC_2+\tfrac{4}{3}C_ff]\left[\frac{6}{\epsilon^2}+\frac{1}{\epsilon}+\frac{1}{2}\right]$$

$$+\gamma\cdot pC_f{}^2\left[\frac{4\alpha^2}{\epsilon^2}+\frac{\alpha(\alpha+12)}{\epsilon}+\frac{\alpha(\alpha+3)}{4}\right]$$

$$-iMC_f{}^2\left[\frac{-36+4\alpha}{\epsilon^2}+\frac{30+8\alpha+2\alpha^2}{\epsilon}+3+4\alpha+3\alpha^2\right]\Bigg]$$

$$+i(Z_mZ_2-1)(M-\delta M)$$



$$+i\delta M$$

$$+\gamma\cdot p(\lambda C_f)\left[-2\alpha\frac{\delta M}{M}\right]-iM\lambda C_f\left[\frac{M^2}{\mu^2}\right]^{-\epsilon/2}\left[\frac{-6-2\alpha}{\epsilon}+5+\alpha\right]\frac{\delta M}{M},$$

where all the two-loop constants can found in table 2.1.

In Fig.2.2, the new mass insertions due to the δM term are represented by a cross on a circle. In Fig. 2.2 (g), we are to take the order λ piece of δM, as determined in (2.28), because in this perturbation calculation we keep only terms up to λ^2. In a = 0 gauge, we find

$$\Sigma = M - \delta M$$

$$= M\left[1 + C_f(3L-1) + \lambda^2 D(3L-1)^2 + \lambda^2 E(3L-1) + \lambda^2 F + O(\lambda^3)\right] \qquad (2.42)$$

where

$$D = \tfrac{1}{2}C_f^2 - \tfrac{1}{12}b_1 C_f$$

$$E = \tfrac{1}{2}C_f^2 + \tfrac{97}{18}C_f C_2 - \tfrac{5}{9}C_f f \qquad (2.43)$$

$$F = 32.0675\, C_f^2 - 6.84185 C_f C_2 + 1.73209 C_f f$$

Then we find

$$\left[\mu\frac{\partial}{\partial\mu} + \beta\lambda\frac{\partial}{\partial\lambda}\right]\Sigma = -(h_1\lambda + h_2\lambda^2)\,\Sigma \qquad (2.44)$$

or

$$\left[-\ (b_1\lambda^2+b_2\lambda^3)\frac{\partial}{\partial\lambda} + (h_1\lambda+h_2\lambda^2)\right]\Sigma \doteq 0 \tag{2.45}$$

where

$$\frac{d\lambda}{dt} = -b_1\lambda^2-b_2\lambda^3 \tag{2.46}$$

and $b_1, b_2; h_1, h_2$ are as in (2.18). We can use the renormalization group analysis to sum up the leading logarithm and next to leading logarithm contributions. To see that clearly, let us write

$$\Sigma = \sum_{n=0}^{\infty} \sum_{m=0}^{n} a_{n,m} L^m \lambda^n$$

$$\beta = -\sum_{n=1}^{\infty} b_n \lambda^n$$

$$h = \sum_{n=1}^{\infty} h_n \lambda^n \tag{2.47}$$

then from the renormalization group equation

$$\left[\frac{\partial}{\partial t} + \beta\lambda\frac{\partial}{\partial\lambda} + h\right]\Sigma = 0 \tag{2.48}$$

we find

$$\sum_{m=0}^{\infty} -m a_{n,m} L^{m-1} + \sum_{n'=1}^{n} \sum_{m=0}^{n-n'} \left[-(n-n') b_{n'} a_{n-n',m} L^m\right.$$

$$+h_{n'}a_{n-n',m}\, L^m \Big] = 0 \tag{2.49}$$

If we only consider the leading logarithm terms of $\sum$, i.e. $\sum_{n=0}^{\infty} a_{n,n}\lambda^n L^n$, then

$$-na_{n,n} - (n-1)b_1 a_{n-1,n-1} + h_1 a_{n-1,n-1} = 0 \tag{2.50}$$

$$a_{n,n} = \frac{1}{n!} \prod_{m=0}^{n-1} (h_1 - mb_1) \tag{2.51}$$

So, we can see by using renormalization group analysis we need only to know b_1 and h_1 from one loop calculation to find the coeffcients of $\lambda^n L^n$ which can come directly from the n loop calculation. Summing up the leading logarithm contribution

$$\begin{aligned}\sum &= M \sum_{n=0}^{\infty} a_{n,n}\lambda^n L^n + O(\sum_{n=1}^{\infty} \lambda^n L^{n-1}) \\ &= M \sum_{n=0}^{\infty} \frac{1}{n!}\lambda^n L^n \prod_{m=0}^{n-1} (h_1 - mb_1) \quad + O(\sum_{n=1}^{\infty} \lambda^n L^{n-1}) \\ &= M(1+b_1\lambda)^{h_1/b_1} + O(\sum_{n=1}^{\infty} \lambda^n L^{n-1})\end{aligned} \tag{2.52}$$

In a one loop calculation, besides $a_{1,1} = h_1/2$, we also can calculate directly $a_{1,0}$, then we can write

$$\sum = M\left[1+b_1\lambda(L+a_{1,0}/h_1)\right]^{h_1/b_1} + O(\sum_{n=2}^{\infty} \lambda^n L^{n-1}) \tag{2.53}$$

which has summed up the leading logarithm contribution as well as the one loop constant contribution.

We can also count the next-to-leading logarithm contributions by calculating two loop diagrams only. We find from (2.49)

$$-(n-1)a_{n,n-1}+\left[h_1-(n-1)b_1\right]a_{n-1,n-2}$$
$$+\left[h_2-(n-2)b_2\right]a_{n-2,n-2}=0 \qquad (2.54)$$

Therefore,

$$a_{n,n-1}=\frac{1}{n-1}\left\{\left[h_2-(n-2)b_2\right]a_{n-2,n-2}\right.$$
$$\left.+\left[h_1-(n-1)b_1\right]a_{n-1,n-2}\right\}$$
$$=\frac{1}{(n-1)!}\sum_{m=0}^{n-2} m!(h_2-mb_2)a_{m,m}\prod_{k=m+2}^{n-1}(h_1-kb_1)$$

$$=\frac{1}{(n-1)!}\sum_{m=0}^{n-2}\frac{m!(h_2-mb_2)}{(h_1-mb_1)(h_1-mb_1-b_1)}\prod_{k=0}^{n-1}(h_1-kb_1) \qquad (2.55)$$

summing to next-to-leading logarithm contributions, we find

$$\Sigma=M\sum_{n=0}^{\infty}(a_{n,n}\lambda^nL^n+a_{n,n-1}\lambda^nL^{n-1})+O(\sum_{n=2}^{\infty}\lambda^nL^{n-2})$$

$$= M \left(\frac{\lambda}{\Lambda_{01}}\right)^{h_1/b_1} \left(\frac{b_1+b_2\lambda}{b_1+b_2\Lambda_{01}}\right)^{-h_1/b_1+h_2/b_2} + O\left(\sum_{n=2}^{\infty} \lambda^n L^{n-2}\right)$$

where

$$\frac{1}{\Lambda_{01}} = \frac{1}{\lambda} + \frac{b_2}{b_1} \log\left(\frac{b_1+b_2\lambda}{b_1+b_2\Lambda_{01}} \frac{\Lambda_{01}}{\lambda}\right) + \frac{b_1}{2} \left(L + \frac{a_{1,0}}{h_1}\right) \qquad (2.57)$$

If we count the contribution of the two-loop constant $a_{2,0}$, then

$$\sum = M\left(\frac{\lambda}{\Lambda_0}\right)^{\frac{h_1}{b_1}} \left(\frac{b_1+b_2\lambda}{b_1+b_2\Lambda_{02}}\right)^{-\frac{h_1}{b_1}+\frac{h_2}{b_2}} + O\left(\sum_{n=3}^{\infty} \lambda^n L^{n-2}\right) \qquad (2.58)$$

where

$$\frac{1}{\Lambda_{02}} = \frac{1}{\Lambda_{01}} - b_1 a' \qquad (2.59)$$

$$a' = \frac{b_1 a_{2,0}}{b_2 h_1 - b_1 h_2} \qquad (2.60)$$

We can get the nontrivial solution of self consistency condition $\sum = 0$ by solving

$$\frac{1}{\Lambda_{01}} = 0 \qquad (2.61)$$

Substituting the definition of QCD two-loop cutoff $\Lambda_c^{(2)}$

$$\frac{1}{\lambda} + \frac{b_2}{b_1} \log \frac{b_1+b_2\lambda}{\lambda} + b_1 \log \frac{\Lambda_c^{(2)}}{\mu} = 0 \qquad (2.62)$$

into (2.61), we find $M = \Lambda_c^{(2)} e^{1/6}$. The solution (2.35) remains valid even in the presence of two-loop constants. In fact we can prove that to any loop the solution $M = \Lambda_c e^{\frac{1}{6}}$ remains valid if the theory is truly asymptotically free (i.e. $\beta < 0$ for all μ). The RG equation for Σ to n-loop is

$$\frac{d}{dt}\Sigma = -(h_1\lambda + h_2\lambda^2 + \cdots h_n\lambda^n)\Sigma$$

$$\frac{d}{dt}\lambda = -(b_1\lambda^2 + b_2\lambda^3 + \cdots b_n\lambda^{n+1}) = \beta(\lambda)$$

We write

$$\Sigma = M\left(\frac{\lambda}{\lambda_{01}}\right)^{\frac{h_1}{b_1}}\tilde{\Sigma} \tag{2.63}$$

where $\frac{1}{\lambda_{01}}$ is a RG invariant and is determined by

$$\int_{\lambda_{01}}^{\lambda}\frac{d\lambda}{\beta(\lambda)} = \log\mu + \bar{a} \tag{2.63a}$$

$\bar{a}$ is a constant to be deternmed. We can find

$$\frac{d\tilde{\Sigma}}{dt} = -(\tilde{h}_2\lambda^2 + \tilde{h}_3\lambda^3 + \ldots)\tilde{\Sigma}, \quad \tilde{h}_2 = h_2 - \frac{h_1}{b_1}b_2, \ldots \tag{2.64}$$

We can solve the RG equation for $\tilde{\Sigma}$. It is easy to see that in a power series expansion

$$\tilde{\Sigma} = 1 + \frac{\lambda^2}{2}\tilde{h}_2 L + \lambda^2 f + \cdots \tag{2.64a}$$

It means that $\tilde{\Sigma}$ has no effect to the one loop constants in power series expansion. We can determine the constant $\bar{a}$ in $\frac{1}{\lambda_{01}}$ by the perturbative seriers for Σ , and find that it is given completely by the one loop constants in Σ , $\bar{a}$ = -log M + 1/6. Using the definition of QCD cutoff

$$\int_{\lambda}^{\infty} \frac{dz}{\beta(z)} = \log \frac{\Lambda_c^{(n)}}{\mu} \tag{2.64b}$$

we find finally

$$\frac{1}{\lambda_{01}} = \log \frac{\Lambda_c^{(n)}}{\mu} \tag{2.65}$$

The solution of the self-consistency condition Σ=0 can be satisfied by $\frac{1}{\lambda_{01}}$=0, then to any loop

$$M = \Lambda_c^{(n)} e^{\frac{1}{6}} \tag{2.66}$$

It is interesting to note, nevertheless, that there is an alternate solution with $\frac{1}{\lambda_{02}}$ = 0, which leads to the complementary solution

$$M_2 = \Lambda_c^{(2)} e^{a'+1/6} \tag{2.67}$$

For the case of three generations and SU(3),

$$a' = -\ 0.435355. \tag{2.68}$$

2.5 ANOTHER FORMALISM

Another way to look at the Chang-Chang approach is to look at (2.26) as defining a massive QCD with m_r =

$M - \delta M$ as the mass parameter. Therefore, the RG equation for m_r is fixed by the usual MS renormalization to be (to two-loop accuracy)

$$\frac{d}{dt} m_r = - (h_1 \lambda + h_2 \lambda^2) m_r \qquad (2.69)$$

and the corresponding equation for λ reads

$$\frac{d\lambda}{dt} = - b_1 \lambda^2 - b_2 \lambda^3 \qquad (2.70)$$

In general, to two-loop RG accuracy, Eq. (2.69) may be solved by

$$m_r = m_0 \left(\frac{\lambda}{\lambda_{01}} \right)^{\frac{h_1}{b_1}} \left(\frac{b_1 + b_2 \lambda}{b_1 + b_2 \lambda_{02}} \right)^{\frac{h_2}{b_2} - \frac{h_1}{b_1}} \qquad (2.71)$$

where m_0, λ_{01}, and λ_{02} are RG invariants. In particular, we choose to write λ_{01}, λ_{02} as in (2.57), (2.59).

So far this appears to be a formal exercise in the renormalization-group theory. In claiming that (2.71) is a solution of (2.69), what we mean is that λ is the function of t as given by (2.70) and m_r is a function of t through its dependence on λ. λ_{01}, λ_{02}, and m_0 are independent of t.

Suppose now we proceed to calculate the two-point Green's function in massive QCD (which we shall refer to as the old theory). In general, we will find (at $p \to 0$)

$$\Gamma_r^{(2)}(\text{old}) = \tilde{Z}_2^{-1}\,(\gamma\cdot p - i\mathcal{M}) \qquad (2.74)$$

with $\mathcal{M}$ a RG invariant. By explicit calculation it can be verified that, indeed, $\mathcal{M}$ is given by

$$\mathcal{M} = m_r\,\left(\frac{\lambda}{\lambda_{01}}\right)^{-\frac{h_1}{b_1}}\left(\frac{b_1+b_2\lambda}{b_1+b_2\lambda_{02}}\right)^{\frac{h_1}{b_1}-\frac{h_2}{b_2}} \qquad (2.75)$$

where, in using the right-hand side, we are to continually re-express λ_{01} in terms of λ, μ, m_r, achieving finally a complete perturbative expansion for $\mathcal{M}$ in λ, μ, m_r.

Now what about the limit $m_r \to 0$? Equation (2.71) tells us that there is a trivial way to achieve the limit, viz. by taking $m_0 = 0$. Perturbatively then, the full two-point function is also zero. But there is a non-trivial way to achieve $m_r = 0$. That is for $m_0 \neq 0$, we look for

$$\frac{1}{\lambda_{01}} = 0 \qquad (2.76)$$

In that case, the $\mathcal{M}$ of the two-point function is nonvanishing. In fact, it is exactly m_0. Contact is finally made with the earlier approach when we realize that $m_0 = M$.

2.5 GAUGE INDEPENDENCE

So far every thing has been analysed in the a=0 gauge. In dynamical symmetry breaking, a particularly difficult problem has been to establish the gauge independence of the dynamical mass so generated. In this section we will now exhibit the problem in a general $a \neq 0$ gauge and show how the dynamically generated mass can indeed be gauge invariant.

Consider the Lagrangian (2.26), and continue to treat δM as a counterterm, used to fix $\Gamma_r^{(2)}$ to be (2.27) even when $a \neq 0$. To one-loop, we then find

$$\delta M = - 3\lambda c_f M(L-1/3) + \frac{1}{2} aC_f, \qquad (2.77)$$

and

$$\tilde{Z}_2 = 1+ \lambda aC_f(L-1/2) \qquad (2.78)$$

At this level, it is hard to see how the a-dependent terms will disappear upon inclusion of higher-order terms.

But, upon including two-loop terms we find

$$\sum = M \left[1+\lambda C_f(3L-1-a/2) + D\lambda^2(3L-1)^2 + \lambda^2.\right.$$

$$\cdot(E + E_1 a + E_2 a^2)(3L-1) + \lambda^2 (F + aF_1$$

$$+ a^2 F_2)\Big] + O(\lambda^3) \tag{2.79}$$

where D, E, F are given in (2.43), and

$$E_1 = \frac{1}{2} C_f^2 + \frac{1}{4} C_f C_2,$$

$$E_2 = \frac{1}{12} C_f C_2, \tag{2.80}$$

$$F_1 = 7.36953 C_f^2 + 8.25602 C_f C_2,$$

$$F_2 = - 16.3273 C_f^2 - 3.88538 C_f C_2,$$

We now check on the RG properties of Σ, using again the fact that M is an RG invariant. Following our remark which follows Eg.(2.68), it is no surprise that we find

$$\frac{1}{\Sigma}\frac{d}{dt}\Sigma = -(h_1\lambda + h_2\lambda^2), \tag{2.81}$$

and the right-hand side of (2.81) is in fact gauge invariant.

Based on this fact, we find that the solution to (2.81) must be of the form

$$\sum(a\neq 0,\lambda,\ M,\mu)=Z_a(a,\lambda,\frac{M}{\mu})\sum(a=0,\lambda,M,\mu) \qquad (2.82)$$

where $\sum(a=0,\lambda,M,\mu)$ is the series we had before. Here Z_a is given by the series expansion

$$Z_a= 1-\tfrac{1}{2}\lambda aC_f +\lambda^2C_fC_2\ \frac{a(3+a)}{12}(3L-1)$$

$$+\lambda^2C_fC^2(8.25602a - 3.88538a^2)$$

$$+\lambda^2C_f^2(7.36953a - 16.3273a^2)+O(\lambda^3) \qquad (2.83)$$

and

$$\frac{d}{dt}Z_a=0 \qquad (2.84)$$

WE can express Z_a in a renormalization-group invariant form

$$Z_a = e^{-(\frac{a_0}{2})\lambda_a C_f} \qquad (2.85)$$

where

$$\frac{1}{\lambda_a} = -b_1(t-\bar{a}) \qquad (2.86)$$

$$\bar{a} = \ln M + \frac{16.5120C_2 + 14.7391C_f}{b_1} +$$

$$\frac{(\frac{26}{3}C_2 - 2f)(3.88538C_2 + 16.2032C_f)}{b_1 C_2} \qquad (2.87)$$

Unlike $\frac{1}{\lambda_\partial}$, $\frac{1}{\lambda_a}$ is not zero.

Equation (2.82) exhibits the gauge dependence of the self-consistency condition. We need to obtain the solution

$$\sum(a,\lambda,M,\mu) = 0 \qquad (2.88)$$

in general for $a \neq 0$. But (2.82) tells us that it is sufficient to solve for the a=0 gauge, since if $\sum(a=0,\lambda,M,\mu)$ vanishes, $\sum(a,\lambda,M,\mu)$ will also vanish, at least for a range of a close enough to zero. Therefore, in (2.26), when M is given by (2.23) the two-point proper Green's function will still be given by

$$\Gamma_r^{(2)}(p) = (\gamma \cdot p - iM)\,\tilde{Z}_2^{-1} \qquad (2.89)$$

with, of couse, a gauge-dependent $\tilde{Z}_2$.

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CHAPTER 3 UP DOWN QUARKS MASS DIFFERENCE[1]

The proton-neutron mass difference problem,[2] probably the oldest puzzle in hadron physics, has challenged and frustrated generations of theorists. According to the quark model of hadrons, if we can find the right answer of up and down quark mass difference then we can answer the proton-neutron mass difference problem. In this chapter we will try to calculate the mass difference of up and down quarks in the scheme of dynamical quark masses generation. We will show that our result is of the right sign and also of the right order of magnitude.

In QCD, we can only generate the same mass for the up and down quarks. In order to study the origin of of the mass difference between the up and down quarks, we should consider including the effects of QED. The theory that we study thus is SU(3)*U(1). The Lagrangian is

$$L = -\bar{\psi}_u \gamma^\mu (\partial_\mu - ig_3 T_a A^a_\mu - iQ_u e B_\mu) \psi$$

$$-\bar{\psi}_d \gamma^\mu (\partial_\mu - ig_3 T_a A^a_\mu - iQ_d e B_\mu) \psi_d - \frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} - \frac{1}{4} B_{\mu\nu} B_{\mu\nu}$$

$$+ L_{gf} + L_{ghost} \tag{3.1}$$

Here we have ignored the second and third generations of the quarks. This is justified since the masses of the second and third generations are much bigger than that of the first generation and by the decoupling theorem[3] their contribution at low energies will be suppressed. All the calculation will be down in this decoupling scheme.

The procedure for mass generation will be the same as in Chapter 2. For simplicity, we will from now on work exclusively in a = 0 gauge.

First, consider the mass determination to one-loop RG accuracy. We should calculate the diagrams in Fig. 3.1. We find

$$\delta M_Q = -\frac{h}{2}{}_1\lambda_3 (L - \frac{1}{3}) - \frac{3}{2} Q^2 (L - \frac{1}{3}) \tag{3.2}$$

where

$$\lambda_3 = \frac{g^2}{16\pi^2} ,$$

$$\lambda_1 = \frac{e2}{16\pi^2} \tag{3.3}$$

let

$$\Sigma_Q = M_Q - \delta M_Q \tag{3.4}$$

then the renormalization group equation is

$$\frac{d}{dt}\Sigma_Q = -(h_1\lambda_3 + 6\lambda_1 Q^2)\Sigma_Q \tag{3.5}$$

with

$$\frac{d}{dt}\lambda_3 = -b_1\lambda_3^2 \tag{3.6}$$

$$\frac{d}{dt}\lambda_1 = b_1'\lambda_1^2 \tag{3.7}$$

where

$$b_1' = \frac{4}{3} f \tag{3.8}$$

We have the solution to (3.5) as

$$\Sigma_Q = M_Q \left(\frac{\lambda_3}{\lambda_{30}}\right)^{\frac{h_1}{b_1}} \left(\frac{\lambda_1}{\lambda_{10}}\right)^{-\frac{6}{b_1'}} \tag{3.9}$$

where

$$\frac{1}{\lambda_{30}} = \frac{1}{\lambda} + b_1(L-1/3)/2 \tag{3.10}$$

$$\frac{1}{\lambda_{10}} = \frac{1}{\lambda_1} - b_1'(L-\tfrac{1}{3})/2 \qquad (3.11)$$

and the self consistency condition can be satisfied by

$$\frac{1}{\lambda_{30}} = 0 \qquad (3.12)$$

we find the solution is

$$M_Q = \Lambda_c \, e^{\frac{1}{6}} \qquad (3.13)$$

the situation is the same as when QED was turned off. To one-loop RG accuracy then, the degeneracy between up and down quark is not lifted.

We proceed now to two-loops. Here, because λ_1 is so much smaller than λ_3, we have kept only terms to first order in λ_1, as a perturbation to order λ_1 of the previous SU(3) result. We should calculate the diagrams in Fig. 3.2, the self-consistency condition now reads

$$\Sigma_Q = M_Q\Big\{1+\lambda_3 C_f(3L-1) + \lambda_3^2(3L-1)^2(\tfrac{1}{2}C_f^2-\tfrac{b_1}{12}C_f)$$

$$+ \lambda_3^2(3L-1)\frac{h_2}{6} + Q^2\lambda_1(3L-1) + Q^2\lambda_1\lambda_3 \cdot$$

$$\cdot C_f(3L-1)^2 + \frac{\lambda_1\lambda_3 Q^2}{6}h_{13}(3L-1) + Q^2\lambda_1\lambda_3 C_f d_{13}$$

$$+ O(\lambda_1^2 \ \lambda_3^3) \qquad (3.14)$$

To perform the RG-summation, we are to take[4]

$$\frac{d}{dt}\lambda_3 = -b_1\lambda_3^2 - b_2\lambda_3^3 - b_{31}\lambda_3^2\lambda_1 \qquad (3.15)$$

$$\frac{d}{dt}\lambda_1 = 0 \qquad (3.16)$$

This last approximation is to make the summation a straight forward one and is entirely consistent with keeping only order $\lambda_1 \lambda_3^n$ terms in the perturbation series in (3.14). The equation for Σ_Q then reads

$$\frac{d}{dt}\Sigma_Q = -\left(h_1\lambda_3 + Q^2 h_{13}\lambda_1\lambda_3 + h_2\lambda_3^2 + 6Q^2\lambda_1\right)\Sigma_Q \qquad (3.18)$$

The solution is

$$\Sigma_Q = M_Q \left(\frac{\lambda_3}{\lambda_{03}}\right)^{(h_1 + h_{13}\lambda_1 Q^2)/b_1} \cdot \left(\frac{b_1 + b_2\lambda}{b_1 + b_2\lambda_{03}}\right)^{\frac{h_2}{b_2} - (h_1 + h_{13}\lambda_1 Q^2)/b_1} \cdot e^{6Q^2\lambda_1(t - t_0)} \qquad (3.19)$$

with

$$\frac{1}{\lambda_{03}} = \frac{1}{\lambda_3} + \frac{b_2}{b_1}\log\left(\frac{b_1 + b_2\lambda_3}{\lambda_3}\cdot\frac{\lambda_{03}}{b_1 + b_2\lambda_{03}}\right) - b_1 t + a_3 \qquad (3.20)$$

Upon expanding Σ_Q as a power series and comparing with

(3.14), we find

$$a_3 = b(\log M_Q - \frac{1}{6} + Q^2 \lambda_1 \frac{d_{13}}{6}) \tag{3.21}$$

in (3.14) d_{13} represents the genuine two-loop constant of the self-energy graphs involving both gluon and photon exchange. Explicit calculation gives

$$d_{13} = 64.1350 \tag{3.22}$$

From the solution

$$\frac{1}{\lambda_{03}} = 0 \tag{3.23}$$

we find

$$M_Q = \Lambda_c^{(2)} e^{(1 - Q^2 \lambda_1 d_{13})/6} \tag{3.24}$$

$$= M_0 \; e^{-Q^2 d_{13} \lambda_1 / 6}$$

$$= M_0 \; (1 - Q^2 \lambda_1 d_{13}/6) \tag{3.25}$$

where M_0 is the mass generated in QCD. From here, we can immediately find

$$M_u - M_d = - M_0 \cdot d_{13} \, e^2/288\pi^2$$

$$= -(.2)\%\ M_0 \qquad (3.26)$$

which is of the right sign and also of the right order of magnitude. But we hesitate to claim any physical significance since the meaning of M_u, M_d itself is not completely clear. Also, until we have understood the generation problem, it would be dangerous to apply this to the higher generations where $m_c > m_s$ and $m_t > m_b$. If we argue that the heavier generations decouple, then this result is of significance.

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CHAPTER 4 : SUPERSYMMETRIC-NJL MODEL

In recent years, the problem of dynamical symmetry breaking in supersymmetric theories[1] has received considerable attention. The connection between chiral symmetry and supersymmetry has been investigated.[2,3] Besides curing the gauge hierarchy problem in the grand unified theory, supersymmetry was expected to play a crucial role in attempts to construct a dynamical theory of composite quark and lepton theory.[4] Such a theory has to explain why the composite fermion's size is so small compared with their Compton wave length, $\langle r \rangle_f \ll 1/m_f$. In other words, to bound the preons to such small size we need very strong interaction, the scale of this interaction, $\bar{\Lambda}$, must be much biger than Λ_c in QCD. Following the mechanism of fermion mass generation in QCD, the preons are expected to get mass of order $\bar{\Lambda}$ that is much bigger than m_f. In a supersymmetric composite fermion theory, there may be some ways out of this problem. One possibility has been that, supersymmetry protects chiral symmetry, as claimed by some authors,[3] this chiral symmetry will then prevent preons from obtaining a dynamical mass of order $\bar{\Lambda}$. It is thus very important to study the dynamical generation of fermion mass in supersymmetry theory and see if chiral symmetry breaking occurs in the supersymmetric theories.

In this chapter, we will introduce the supersymmetric NJL model and study the supersymmetric gap equation to test whether or not supersymmetry protect chiral symmetry. Our study[5] shows that the claim in ref. 2, that supersymmetry protects chiral symmetry, is not true. The authors in ref. 2 did not correctlly generalize the supersymmetric gap equation. In fact, chiral symmetry is broken in supersymmetric NJL model and the dynamically generated fermion mass is given by a similar condition as in ordinary NJL model. We should try to find some other way to explain the smallness of the composite fermion. One possible way way out would be that, since the chiral symmetry is broken, the massless Goldstone boson must appear, and the light composite fermions may be generated as supersymmetric partners of the Goldstone bosons.

4.1 INTRODUCTION TO SUPERSYMMETRY

Supersymmetry is an extension of the Poincare group obtaind by introducing a spinorial charge Q. Under quite general assumptions it can be showed to be the largest possible symmetry of the S-matrix. This symmetry can be realized on ordinary fields by transformations that mix bosons and fermions (component field approach). Superspace - superfield approach is much more compact. Superspace is an extension of ordinary space-time to include extra anticommuting coordinates in the form of N two-component Weyl spinors θ. Superfields $S(x, \theta)$ are functions defined over this space. They can be expanded in a Taylor series with respect to the anticommuting coordinates θ, $\bar{\theta}$. Because the square of an anticommuting quantity vanishes, this series has only a finite number of terms. The coefficients obtained in this way are the ordinary component fields. In superspace, the supersymmetry algebra is represented by translation and relations involving both the spacetime and the anticommuting coordinates. The transformations of the component fields follow from the Taylor expansion of translated and rotated superfields.

To work in superfield form, we should introduce our notations and conventions. We will use the notations and conventions as in ref.6 in this chapter and chapter V and VI.

Our index conventions are as follows: the two-component complex (Weyl) spinor representation $(\frac{1}{2}, 0)$ is labeled by $\psi^{\alpha} = (\psi^1, \psi^2)$. The complex-conjugate represention $(0, \frac{1}{2})$ is labeled by doted index $\bar{\psi}^{\dot{\alpha}} = (\bar{\psi}^1, \bar{\psi}^2)$. A four-component Dirac spinor is the combination of an undotted spinor with a dotted one $(\frac{1}{2}, 0) \oplus (0, \frac{1}{2})$,

$$\psi = \begin{pmatrix} \psi_{+\alpha} \\ \bar{\psi}_{-\dot{\alpha}} \end{pmatrix} \tag{4.1}$$

A bispinor $(\frac{1}{2}, \frac{1}{2})$ is labeled with one dotted and one undotted index, e.g. $V^{\alpha\dot{\alpha}}$,

$$\begin{aligned} V^{\alpha\dot{\alpha}} &= \sigma_m^{\alpha\dot{\alpha}} \quad V^m \\ V^m &= -\frac{1}{2} \bar{\sigma}^{m\dot{\alpha}\alpha} V_{\alpha\dot{\alpha}} \end{aligned} \tag{4.2}$$

where σ^m is the Pauli matrix,

$$\begin{aligned} \sigma^1 &= \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \qquad \sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \\ \sigma^3 &= \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \qquad \sigma^0 = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}, \end{aligned} \tag{4.3}$$

Spinor with upper and lower indices are related through the ϵ- tensor.

$$\psi^{\alpha} = \epsilon^{\alpha\beta}\psi_{\beta} \quad , \qquad \psi_{\alpha} = \epsilon_{\alpha\beta}\psi^{\beta} \tag{4.4}$$

where

$$\epsilon_{\alpha\beta} = -\epsilon_{\beta\alpha} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \tag{4.5}$$

The ϵ- tensor may also be used to raise the indices on the σ- matries :

$$\bar{\sigma}^{m\dot{\alpha}\alpha} = \epsilon^{\dot{\alpha}\dot{\beta}}\epsilon^{\alpha\beta}\sigma^{m}_{\beta\dot{\beta}} \tag{4.6}$$

The Dirac matrices are

$$\gamma^{m} = \begin{pmatrix} 0 & \sigma^{m} \\ \bar{\sigma}^{m} & 0 \end{pmatrix} \tag{4.7}$$

$$\gamma^{5} = i\gamma^{0}\gamma^{1}\gamma^{2}\gamma^{3}$$

or explicitly

$$\gamma^{5} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \tag{4.8}$$

The superalgebra is:

$$\{Q_\alpha , \bar{Q}_{\dot\alpha}\} = 2\sigma^m_{\alpha\dot\alpha} \; P_m$$

$$\{Q_\alpha , Q_\beta\} = \{\bar{Q}_{\dot\alpha} , \bar{Q}_{\dot\beta}\} = 0 \qquad (4.9)$$

$$[P_m , Q_\alpha] = [P_m , \bar{Q}_{\dot\alpha}] = 0$$

$$[P_m , P_n] = 0$$

The supergroup element is defined as ,

$$h(x^m, \theta^\alpha, \bar\theta_{\dot\alpha}) = e^{i(-x^m P_m + \theta^\alpha Q_\alpha + \bar\theta_{\dot\alpha}\bar{Q}^{\dot\alpha})} \qquad (4.10)$$

where $x^m (m = 0, 1, 2, 3)$ are space-time coordinates and θ^α, $\bar\theta_{\dot\alpha}$ are anti-commuting parameters. $(x^m, \theta^\alpha, \bar\theta_{\dot\alpha})$ are the coordinates of superspace.

The most general form for superfield $S(x, \theta, \bar\theta)$ is:

$$S(x, \theta, \bar\theta) = f(x) + \theta\phi(x) + \bar\theta\bar\chi(x)$$

$$+ \theta^2 m(x) + \bar\theta^2 n(x) + \theta\sigma^m\bar\theta V_m(x)$$

$$+ \bar\theta\theta^2\bar\lambda(x) + \bar\theta^2\theta\psi(x) + \bar\theta^2\theta^2 d(x) \qquad (4.11)$$

where $f(x)$, $\phi(x)$, $\bar{\chi}(x)$, $m(x)$, $n(x)$, $V_m(x)$, $\bar{\lambda}(x)$, $\psi(x)$ and $d(x)$ are the component fields of S. In general, $S(x, \theta, \bar{\theta})$ transforms as a reducible multiplet under a supersymmetric transformation.

We call a superfield $\phi(x, \theta, \bar{\theta})$ as chiral field, if

$$\bar{D}_{\dot{\alpha}} \phi = 0 \quad , \tag{4.12}$$

where

$$\bar{D}_{\dot{\alpha}} = -\frac{\partial}{\partial \bar{\theta}^{\dot{\alpha}}} - i\, \theta^{\alpha} \sigma^{m}_{\alpha\dot{\alpha}} \partial_m$$
$$D_{\alpha} = \frac{\partial}{\partial \theta^{\alpha}} + i\, \sigma^{m}_{\alpha\dot{\alpha}} \bar{\theta}^{\dot{\alpha}} \partial_m \tag{4.13}$$

The component fields of a chiral field transform irreducibly under supersymmetric transformation. The most general form of chiral superfield $\phi(x, \theta, \bar{\theta})$ can be written as

$$\phi(x, \theta, \bar{\theta}) = A(x) + \sqrt{2}\,\theta\psi(x) + \theta^2 F(x) + i\,\theta\sigma^m\bar{\theta}\partial_m A$$
$$- \frac{i}{\sqrt{2}} \theta^2 \partial_m \psi \sigma^m \bar{\theta} + \frac{1}{4} \theta^2 \bar{\theta}^2 \partial^2 A \tag{4.14}$$

We can write $\phi(x, \theta, \bar{\theta})$ in terms of

$$y = x + i\,\theta\sigma\bar{\theta}, \tag{4.15}$$

$$\phi(x,\theta,\bar{\theta}) = A(y) + \sqrt{2}\,\theta\,\psi(y) + \theta^2 F(y) \tag{4.16}$$

For a chiral field ϕ, a simplest supersymmetry invariant Lagrangian is

$$\mathcal{L} = \int d^4\theta\,\bar{\phi}\phi - \left[\int d^2\theta\left(\frac{m}{2}\phi^2 + \frac{\lambda}{3}\phi^3\right) + \text{h.c.}\right] \tag{4.17}$$

where

$$\int d^4\theta\,\bar{\phi}\phi = -(\partial^n A^*)(\partial_n A) + i\,(\partial_n\bar{\psi})\bar{\sigma}^n\psi + F^* F \tag{4.18}$$

provides the kinetic energy part of the Lagrangian, while

$$\int d^2\theta\,\frac{m}{2}\phi^2 = \frac{m}{2}(2AF - \psi^2) \tag{4.19}$$

provides the mass term and

$$\int d^2\theta\,\frac{\lambda}{3}\phi^3 = \lambda(A^2 F + 2A\psi^2) \tag{4.20}$$

provides the interaction part. When we calculate the radiactive correction of the theory, we can use the component field approach and write the Feynman rules just the same way as in ordinary field theory. But the so-called supergraph approach will greatly simplify the calculation. We will use the supergraph approach in supersymmetry calculation and component field approach in the theory with a supersymmetry soft breaking term.

4.2 NJL MODEL IN TWO COMPONENT FORM

In order to study the supersymmetric extension of NJL model, it is better to express the NJL Lagrangian[7] in terms of two component Weyl spinors $\psi_{\pm}(x)$. We can write $\mathcal{L}_I$ as :

$$\mathcal{L}_I = g\,\bar{\psi}_L \psi_R \bar{\psi}_R \psi_L \tag{4.21}$$

where

$$\psi_L = \frac{1}{2}(1-\gamma_5)\psi\,, \quad \psi_R = \frac{1}{2}(1+\gamma_5)\psi \tag{4.22}$$

$$\psi = \psi_L + \psi_R \tag{4.23}$$

and

$$\begin{aligned} \psi_L &= \begin{pmatrix} \psi_{+\alpha} \\ 0 \end{pmatrix}, & \bar{\psi}_L &= -(0,\ \bar{\psi}_{+\dot{\alpha}}) \\ \psi_R &= \begin{pmatrix} 0 \\ \bar{\psi}_-^{\dot{\alpha}} \end{pmatrix}, & \bar{\psi}_R &= -(\psi_-^{\alpha},\ 0) \end{aligned} \tag{4.24}$$

then

$$\mathcal{L}_0 = (\psi_-^{\alpha}, \bar{\psi}_{+\dot{\alpha}}) \begin{pmatrix} 0 & i\sigma^{\mu}\partial_{\mu} \\ i\bar{\sigma}^{\mu}\partial_{\mu} & 0 \end{pmatrix} \begin{pmatrix} \psi_{+\alpha} \\ \bar{\psi}_-^{\dot{\alpha}} \end{pmatrix}$$

$$= i(\partial_{\mu}\bar{\psi}_+)\,\bar{\sigma}^{\mu}\psi_+ + i(\partial_{\mu}\bar{\psi}_-)\,\bar{\sigma}^{\mu}\psi_-$$

$$L_I = g(\bar{\psi}_+\bar{\psi}_-)(\psi_-\psi_+) \qquad (4.25)$$

The mass term $m\bar{\psi}\psi$ can be written as

$$L_m = -m\psi_+\psi_- - m\bar{\psi}_+\bar{\psi}_- \qquad (4.26)$$

But before we go to the supersymmetric extension of NJL model, we should express the self-consistency condition (1.6) in this two component Weyl spiors form.

In general, we can write the self-energy as :

$$\Sigma = gA(p^2)\not{p} - gB(p^2)m$$

here $gA(p^2)$ give the contribution to the wave function renormalization. The self-consistency condition

$$m = \Sigma(p)\Big|_{\not{p}=m} \qquad (4.27)$$

becomes

$$m = gm\left[A(p^2) - B(p^2)\right]\Big|_{p^2 = -m^2} \qquad (4.28)$$

We also can carry out the self-consistency condition in another way through the renormalized two-point function of fermion.

The unrenormalized two-point function $\Gamma^{(2)}$ is

$$i\Gamma^{(2)} = \not{p} - m + m - gA(p^2)\not{p} + gB(p^2)m \qquad (4.29)$$

If we define the renormalized fermion fields ψ_r as

$$\psi_r = Z_2^{-\frac{1}{2}}\psi_0 \qquad (4.30)$$

where

$$Z_2 = 1 + gA(-m^2) \qquad (4.31)$$

then, in general, the free Lagrangian $\mathcal{L}_0'$ can be written in terms of renormalized fields

$$\mathcal{L}_0' = Z_2\bar{\psi}_r i\not{\partial}\psi_r - Z_m Z_2 m \bar{\psi}_r \psi_r \qquad (4.32)$$

because of the self-consistency condition, we do not need the renormalization of m, so $Z_m = 1$ and

$$\mathcal{L}_0' = Z_2 \bar{\psi}_r \left(i\partial\!\!\!/ - m \right) \psi_r \qquad (4.33)$$

The renormalized two-point function is

$$i\Gamma_r^{(2)} = (p\!\!\!/ - m)\left[1 + gA(-m^2)\right] - gA(p^2)p\!\!\!/ + gB(p^2)m + m \qquad (4.34)$$

The self-consistency condition can be written as

$$i\,\Gamma_r^{(2)} = p\!\!\!/ - m \qquad (4.35)$$

then (4.35) yields,

$$gm\left[A(-m^2) - \beta(-m^2)\right] - m = 0$$

or

$$m = gm\left[A(-m^2) - \beta(-m^2)\right] \qquad (4.36)$$

That is just the same condition as (4.28). In this procedure first we should carry out the wave fuction renormalization, write the free Lagrangian in terms of renormalized fields, then calculate the renormalized two point function. The self-consistency condition is just equation (4.35) under the condition $p^2 = -m^2$ instead of equation (4.27) under the more complicated condition $\not{p} = m$. In this way we can easily get the self-consistency condition in two-component Weyl form.

The two-point function of two-ponent fields are

$$\Gamma^{++} = \Gamma^{--} = \left[1 - gA(p^2)\right]\sigma \cdot p$$
$$\Gamma^{+-} = \left[1 - gB(p^2)\right] m \qquad (4.37)$$

the four-component two-point function $\Gamma^{(2)}$ can written as

$$\Gamma^{(2)} = \begin{pmatrix} \Gamma^{+-} & \Gamma^{++} \\ \bar{\Gamma}^{--} & \bar{\Gamma}^{+-} \end{pmatrix} \qquad (4.38)$$

The free Lagrangian in terms of renormalized two-component fielfs is

$$\mathcal{L}_0' = Z_2 \left\{ i(\partial_\mu \bar{\psi}_+)\bar{\sigma}_\mu \psi_+ + i(\partial_\mu \bar{\psi}_-)\bar{\sigma}^\mu \psi_- - m\,\psi_- \psi_+ - m\bar{\psi}_- \bar{\psi}_+ \right\} \qquad (4.39)$$

Then the renormalized two-point functions of the two-component fields are :

$$\Gamma_r^{++}\Big|_{p^2=-m^2} = \Gamma_r^{--}\Big|_{p^2=-m^2} = \sigma \cdot p$$
$$\Gamma_r^{+-}\Big|_{p^2=-m^2} = m - m + gm\left[A(-m^2) - B(-m^2)\right] \qquad (4.40)$$

The self-consistency condition is

$$\Gamma_{\Upsilon}^{+-}\Big|_{p^2=-m^2} = m$$

therefore,

$$m = gm\Big[A(-m^2) - B(-m^2)\Big] \tag{4.41}$$

We have the same condition as (4.28). Now we are ready to go to the supersymmetric extension of NJL model.

4.3 SUPERSYMMETRIC GENERATION OF NJL MODEL

To generalize the NJL to supersymmetric form, we introduce two scalar superfields ϕ_+ and ϕ_- ,

$$\phi_{\pm} = A_{\pm}(y) + \sqrt{2}\,\theta\psi_{\pm}(y) + \theta\theta F_{\pm}(y) \tag{4.42}$$

with

$$y = x^{\mu} + i\,\theta\sigma^{\mu}\bar{\theta}$$

Let

$$\mathcal{L}_0 = \int d^2\theta d^2\bar{\theta}(\bar{\phi}_+\phi_+ + \bar{\phi}_-\phi_-) \qquad 4.43)$$

if we write $\mathcal{L}_0$ in terms of component fields, we can find $\mathcal{L}_0$ generates the kinetic terms for ψ_+, ψ_- plus the terms of $\psi_\pm$'s scalar partners $A_\pm$, $F_\pm$. Let

$$\mathcal{L}_I = g \int d^2\theta\, d^2\bar{\theta}\;\; \bar{\Phi}_+ \bar{\Phi}_- \Phi_+ \Phi_- \tag{4.44}$$

In terms of component fields,

$$\mathcal{L}_I = -g\;(A_-^* A_- \partial_\mu A_+^* \partial_\mu A_+ + A_-^* A_+^* \partial_\mu A_+^* \partial^\mu A_- + A_+^* A_- \partial_\mu A_-^* A_+ + A_+^* A_+ \partial_\mu A_-^* \partial^\mu A_-)$$

$$+ ig(A_+^* A_+ \partial_\mu \bar{\psi}_- \bar{\sigma}^\mu \psi_\pm \ldots) + g| F_+ A_- + F_- A_+ - \psi_+ \psi_- |^2 \tag{4.45}$$

We can find $\mathcal{L}_I$ generates the interaction part of $g\bar{\psi}_+\bar{\psi}_-\psi_+\psi_-$ plus the terms of the interaction included $A_\pm$, $F_\pm$. The mass term $m\psi_+\psi_-$ can come from ,

$$\mathcal{L}_m = -\int d^2\theta\, m \Phi_+ \Phi_- - \int d^2\bar{\theta}\, m\, \bar{\Phi}_+ \bar{\Phi}_- \tag{4.46}$$

According to NJL we can adopt

$$\mathcal{L}_0' = \mathcal{L}_0 + \mathcal{L}_m \tag{4.47}$$

as free Lagrangian and

$$\mathcal{L}_I' = \mathcal{L}_I - \mathcal{L}_m \tag{4.48}$$

as interaction Lagrangian.

We can introduce four-component super fields $\Psi, \bar{\Psi}$, so that we can represent the Dirac fermion and write the Lagrangian of the supersymmetric extension of NJL model in a form similar to the original NJL model. Let

$$\Psi = \begin{pmatrix} \Phi_+ \\ \bar{\Phi}_- \end{pmatrix}$$

$$\bar{\Psi} = \Psi^+ \gamma_4 = -(\Phi_-, \bar{\Phi}_+) \tag{4.49}$$

then the Lagrangian can be written as

$$\mathcal{L} = \int d^2\theta \int d^2\bar{\theta} \left\{ -\bar{\Psi} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \Psi \right.$$

$$\left. + \frac{g}{4} \left[(\bar{\Psi}\Psi)^2 - (\bar{\Psi}\gamma_5\Psi)^2 \right] \right. \tag{4.50}$$

This Lagrangian is invariant under the chiral transformation

$$\Psi \longrightarrow e^{i\gamma_5 \alpha} \Psi \tag{4.51}$$

According to NJL's idea, $\mathcal{L}_0'$ and $\mathcal{L}_I'$ can be written as following,

$$\mathcal{L}_0' = \int d^2\theta d^2\bar{\theta} \left\{ -\bar{\Psi} \begin{pmatrix} m\delta(\bar{\theta}) & 1 \\ 1 & m\delta(\theta) \end{pmatrix} \Psi \right\} \tag{4.52}$$

$$\mathcal{L}_I' = \int d^2\theta d^2\bar{\theta} \left\{ \bar{\Psi} \begin{pmatrix} m\delta(\bar{\theta}) & 0 \\ 0 & m\delta(\theta) \end{pmatrix} \Psi + \frac{g}{4} \left[(\bar{\Psi}\Psi)^2 - (\bar{\Psi}\gamma_5\Psi)^2 \right] \right\} \tag{4.53}$$

The four component two point function is

$$\Gamma^{(2)} = \begin{pmatrix} \Gamma^{+-} & \Gamma^{++} \\ \bar{\Gamma}^{--} & \bar{\Gamma}^{+-} \end{pmatrix} \tag{4.54}$$

in tree level

$$\Gamma_0^{(2)} = \begin{pmatrix} \delta(\bar{\theta})\, m & 1 \\ 1 & \delta(\theta)\, m \end{pmatrix} \tag{4.55}$$

In one loop approximation, after the wave function renormalization, we can write $\mathcal{L}_0'$ in terms of renormalized fields Φ_{+r}, Φ_{-r} etc.,

$$\mathcal{L}_0' = Z_2 \int d^2\theta \int d^2\bar{\theta} \left[\bar{\Phi}_{+r}\Phi_{+r} + \bar{\Phi}_{-r}\Phi_{-r} - m\delta(\bar{\theta})\Phi_{+r}\Phi_{-r} - m\delta(\theta)\bar{\Phi}_{+r}\bar{\Phi}_{-r} \right] \tag{4.56}$$

with

$$Z_2 = 1 + gA(-m^2)$$

where $gA(p^2)$ comes from the contribution of Fig.4.1

The renormalized two point function

$$\Gamma_r^{++}\big|_{p^2=-m^2} = \Gamma_r^{--}\big|_{p^2=-m^2} = 1$$
$$\Gamma_r^{+-}\big|_{p^2=-m^2} = m + gmA(-m^2) - gmB(-m^2) - m \tag{4.57}$$

where $gB(p^2)$ comes from the contribution of Fig.4.2.

4.4 SUPERGRAPH CALCULATION OF SNJL MODEL

In order to calculate A and B , we should calculate the supergraph Fig.4.1 and Fig.4.2. Firstly, we should derive the Feynman rules.

The kinetic action with chiral sources $j_\pm$, $\bar{j}_\pm$ is

$$S^{(2)} = \int d^4x d^4\theta \, (\bar{\Phi}_+\Phi_+ + \bar{\Phi}_-\Phi_-)$$

$$+\left[\int d^4x \, d^2\theta \, (-m\Phi_+\Phi_- + j_+\Phi_+ + j_-\Phi_-) + h.c.\right]$$

$$= \int d^4x d^4\theta \left[(\Phi_+, \bar{\Phi}_-) \, O \begin{pmatrix} \Phi_- \\ \bar{\Phi}_+ \end{pmatrix} + (\Phi_+, \bar{\Phi}_-) \begin{pmatrix} \Box^{-1} D^2 j_+ \\ \Box^{-1} \bar{D}^2 \bar{j}_- \end{pmatrix} \right] \tag{4.58}$$

where

$$O = \begin{pmatrix} \dfrac{-mD^2}{\Box} & 1 \\ 1 & -\dfrac{m\bar{D}^2}{\Box} \end{pmatrix} \tag{4.59}$$

We have used $\Box^{-1}\bar{D}^2 D^2\phi = \phi$ and $\int d^4\theta = \int d^2\theta\,\bar{D}^2$

We can find that

$$O^{-1} = \begin{pmatrix} \dfrac{m\bar{D}^2}{\Box - m^2} & 1 + \dfrac{m^2\bar{D}^2 D^2}{\Box(\Box - m^2)} \\ 1 + \dfrac{m^2 D^2\bar{D}^2}{\Box(\Box - m^2)} & \dfrac{m D^2}{\Box - m^2} \end{pmatrix} \qquad (4.60)$$

$W_o(j)$ is defined by

$$e^{W_o(j)} = \int \mathcal{D}\phi_+ \mathcal{D}\bar{\phi}_+ \mathcal{D}\phi_- \mathcal{D}\bar{\phi}_- e^{S^{(2)}} \qquad (4.61)$$

After carrying out the funtional integral, we find

$$W_o(j) = \int d^4x\, d^4\theta \Big[-\bar{j}_+ \frac{1}{\Box - m^2} j_+ - j_- \frac{1}{\Box - m^2} j_- - \frac{1}{2}\Big(j_+ \frac{mD^2}{\Box(\Box - m^2)} j_- + h.c.\Big)\Big] \qquad (4.62)$$

For a general interaction Lagrangian,

$L_{int}(\phi_+, \bar{\phi}_+, \phi_-, \bar{\phi}_-)$, we can write

$$Z(j) = e^{\int d^4x\, d^4\theta\, L_{int}\left(\frac{\delta}{\delta j_+}, \frac{\delta}{\delta \bar{j}_+}, \frac{\delta}{\delta j_-}, \frac{\delta}{\delta \bar{j}_-}\right)} e^{W_o(j)} \qquad (4.63)$$

then the Feynman rules for the Lagrangian are as following

Propagators: $\quad \bar{\Phi}_\pm \Phi_\pm \quad : \quad -\frac{1}{p^2+m^2}\,\delta^4(\theta - \theta')$

$$\Phi_{\pm}\Phi \quad : \quad -\frac{mD^2}{p^2(p^2+m^2)}\,\delta^4(\theta-\theta')$$

$$\bar{\Phi}_{\pm}\bar{\Phi}_{\mp} \quad : \quad -\frac{m\bar{D}^2}{p^2(p^2+m^2)}\,\delta^4(\theta-\theta')$$

Vertex:

$$\Phi_{+}\Phi_{-}\bar{\Phi}_{+}\bar{\Phi}_{-} \quad : \quad ig \tag{4.64}$$

Using the Feynman rules, we can calculate gA from Fig.4.1. The result is

$$gA(p^2) = g\,m\int\frac{d^4q}{(2\pi)^4}\,\frac{-i}{q^2+m^2} \tag{4.65}$$

Fig.4.2 contains a closed loop containing only the propagator, this propagator is proportional to $\delta(\theta-\theta') = (\theta-\theta')^2$. It follows that when $\theta=\theta'$, as is the case for a closed loop, the propagator vanishes.

$$B = 0 \tag{4.66}$$

The self consistency condition yields

$$m = gm\left\{A(-m^2) - B(-m^2)\right\} \tag{4.67}$$

After we substitute (4.65) and (4.66) into (4.67) we find the nontrivial solution is

$$1 = g \int \frac{d^4 q}{(2\pi)^4} \frac{-i}{q^2+m^2} \tag{4.68}$$

We have the equation as in the ordinary NJL model. If (4.68) is satisfied fermion can generate a non zero Dirac mass. The chiral symmetry is broken. The supersymmetry does not protect chiral symmetry. Since the supersymmetry is unbroken in the model, the vacuum energy density is zero in both m=0 or m≠0 cases. We can not tell which phase is enegetically favored. But supersymmetry does not rule out the m≠0 solution in the supersymmetric NJL model.

4.5 SNJL MODEL WITH A SOFT SUSY BREAKING TERM

It is also very interesting to study the effect of a soft supersymmetry breaking term to the supersymmetry NJL model. If we add a scale mass term

$$\mathcal{L}_a = - a^2 (A_+^* A_+ + A_-^* A_-) , \tag{4.69}$$

to the Lagrangian (4.50), then the new Lagrangian perserves the invariances (4.51). The propagators of the component fields are as following:

$$A(x)\ A^{*}(x') \ :\ i\ F\ (x-x',\ m^2 + a^2\ )$$

$$F\ (x)\ F^{*}(x')\ :\ (\Box - a^2)\, iF\ (x-x';\ m^2+a^2)$$

$$A\ (x)\ F\ (x')\ :\ -imF(x-x';m^2+a^2)$$

$$A^{*}(x)F^{*}(x')\ :\ -imF(x-x';m^2+a^2)$$

(4.70)

$$\psi_{\pm\alpha}(x)\ \psi^{\beta}_{\mp}(x')\ :\ i\,\delta^{\beta}_{\alpha}\ m\ F(x-x';m^2)$$

$$\bar{\psi}_{\pm\dot\alpha}(x)\ \bar{\psi}_{\mp\dot\beta}(x')\ :\ i\,\delta^{\dot\alpha}_{\dot\beta}\ m\ F(x-X';m^2)$$

$$\psi_{\pm\alpha}(x)\ \bar{\psi}_{\pm\dot\beta}(x')\ :\ \sigma^{m}_{\alpha\dot\beta}\,\frac{\partial}{\partial x^{m}}\,F(x-X';m^2)$$

with

$$F(x,m^2) = -\int \frac{d^4p}{(2\pi)^4}\ \frac{e^{ipx}}{p^2+m^2-i\epsilon} \qquad (4.71)$$

We can write the interaction part in terms of component fields as following :

$$\begin{aligned}\mathcal{L}_I = \; & ig\,[\,A_+^* A_+ \partial_\mu \bar{\Psi}_- \bar{\sigma}^\mu \Psi_- + A_+^* A_- \partial_\mu \bar{\Psi}_- \bar{\sigma}^\mu \Psi_+ + A_-^* A_+ \partial_\mu \bar{\Psi}_+ \bar{\sigma}^\mu \Psi_- \\ & + A_-^* A_- \partial_\mu \bar{\Psi}_+ \bar{\sigma}^\mu \Psi_+ + \partial_\mu A_+^* A_+ \bar{\Psi}_- \bar{\sigma}^\mu \Psi_- + \partial_\mu A_+^* A_- \bar{\Psi}_- \bar{\sigma}^\mu \Psi_+ \\ & + \partial_\mu A_-^* A_+ \bar{\Psi}_+ \bar{\sigma}^\mu \Psi_- + \partial_\mu A_-^* A_- \bar{\Psi}_+ \bar{\sigma}^\mu \Psi_+\,] + \\ & + g\,|F_+ A_- + F_- A_+ - \Psi_+ \Psi_-|^2 \end{aligned} \tag{4.72}$$

After we calculate the self energy of fermion, we find that

$$A(p^2) = F(0,\ m^2+a^2)$$

$$B(p^2) = 2\left[F(0,\ m^2+a^2) - F(0,m^2)\right] \tag{4.73}$$

Then the self-consistency condition is

$$1 = 2gF(\ 0\ ,\ m^2\ ) - gF(\ 0,\ m^2+a^2) \tag{4.74}$$

In the limit of $a^2 \longrightarrow 0$, i.e. the supersymmetric case, it is as

$$1 = gF(\ 0,\ M^2\ )\ , \tag{4.75}$$

which agree with the result of supergragh calculation.

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Chapter 5: BIFURCATION AND FINITE TEMPERATURE EFFECT IN SNJL MODEL

5.1 BIFURCATION THEORY AND STABILITY OF SOLUTION OF GAP EQUATION

It has been shown in last chapter that in supersymmetric extension of NJL model, besides the trivial solution m = 0, there is a nontrivial solution $m \neq 0$ which satisfies the condition of the SNJL gap equation,

$$m = m\bar{\lambda}\left[1 - \frac{m^2}{\Lambda^2}\log\left(\frac{\Lambda^2}{m^2}+1\right)\right] \tag{5.1}$$

where

$$\bar{\lambda} = \frac{g\Lambda^2}{16\pi^2} \tag{5.2}$$

One may ask which solution is stable against the small thermal perturbation? In ordinary NJL model people may argue that the massive vacuum has lower energy than the massless one, so the nontrivial solution is stable[1]. But in the supersymmetric extension of NJL model, due to supersymmetry, the vacuum energy is zero in both cases.

We can not tell which phase is energetically favored. In a recent paper[2], L.N. Chang and N.P. Chang have used bifurcation theory[3] to study the problem of chiral symmetry breaking and calculate the vacuum expectation value of $\bar{\psi}\psi$ in QCD. Our study also shows bifurcation theory is very useful in the study of stability of the solutions of gap equation in NJL models.

Firstly, we will introduce some definitions and theorems in bifurcation theory.

We can write the gap eqution of NJL models as[1,4]

$$m = f(m,\bar{\lambda}) \qquad (5.3).$$

Then we can solve the gap equation iteratively by equation

$$m_{n+1} = f(m_n,\bar{\lambda}) \qquad (5.4).$$

Given a initial value m_0, then we can have a sequence $(m_0, m_1, m_2, \ldots, m_n)$. Such a sequence will be called as a solution of the equation (5.4). If $a = f(a,\bar{\lambda})$, then a is the fixed point of f. The definition of stability of the constant solution is as following[5],

(1) There is some neighbourhood U of a such that if $m_0 \in U$ then $m_n \to a$ as $n \to \infty$.

(2) Given any neighbourhood U of a then we can find a neighbourhood V of a such that if $m_0 \in V$ then $m_n \in U$ for all $n > 0$.

If both these conditions hold then the constant solution will be called uniformly asymptotically stable.

A very important theorem as the criterion of uniformly asymtotic stability is as following:

Let a be a fixed point of f and A the dXd ($d \geq 1$) matrix of f'(a). If all the eigenvalues u of A are less then one in modulus then a is uniformly asymptotically stable. In this paper, we will only concern with the case of d=1. The idea of the proof is to write

$$f(x) = f(a) + f'(a)(x-a) + O(x-a) \tag{5.5}$$

and reduce the problem to one of linear algebra by showing that the O-term have no effect on stability. for complete proof see ref. 5.

Now let us test the stability of the solutions of gap equation in SNJL models. In this case,

$$f(m,\bar{\lambda}) = m\bar{\lambda}\left[1 - \frac{m^2}{\Lambda^2}\log\left(\frac{\Lambda^2}{m^2}+1\right)\right] \tag{5.6}.$$

There are two fixed points: 0 and a , where a satisfies the equation

$$1 = \bar{\lambda}\left[1 - \frac{a^2}{\Lambda^2}\log\left(\frac{\Lambda^2}{a^2}+1\right)\right] \qquad (5.7).$$

To test the stability of the trivial solution m = 0, we found

$$f'(0,\bar{\lambda}) = \bar{\lambda} \qquad (5.8)$$

Therefore, if $\bar{\lambda} < 1$ then the trivial solution m=0 is stable, if $\bar{\lambda} > 1$, the trivial solution m=0 is unstable.

For the nontrivial solution m = a, we found

$$f'(a,\bar{\lambda}) = 3 - \frac{2\bar{\lambda}\Lambda^2}{a^2+\Lambda^2} \qquad (5.9)$$

It can be shown that $f'(a,\bar{\lambda}) < 1$ for $1 < \bar{\lambda} < \infty$, therefore the nontrivial solution m = a is stable if $1 < \bar{\lambda} < \infty$.

Thus we have a bifurcation at $\bar{\lambda} = 1$, which may be indicated on the bifurcation diagram in Fig. 5.1. The solid and broken lines indicate stable and unstable fixed points respectively. Thus at $\bar{\lambda} = 1$ the fixed point m = 0 loses its stability and a new stable fixed point m = a is created as $\bar{\lambda}$ passes 1 and the chiral symmetry is broken.

5.2 FINITE TEMPERATURE EFFECT

It is natural to ask whether the stability of the nontrival solution is affected by the temperature effect and is there a critical temperature β_c^{-1} above β_c^{-1} the broken chiral symmetry would be restored.

In order to study the temperature effect to the solution of the gap equation, we should calculate the Green's function in finite temperature. I will briefly describe the formalism for perturbative calculations at general finite temperature[4].

In general, the physical quantities with which we will be concerned here are the partition function

$$Q = Tr \ (e^{-H\beta}) \qquad (5.10)$$

and its variational derivatives with respect to external perturbation, the temperature Green's functions

$$Q \langle T_r \{ A(\vec{x}_1, \tau_1) B(\vec{x}_2, \tau_2) \dots \} \rangle$$

$$= T_r \{ T_\tau \{ A(\vec{x}_1, \tau_1) B(\vec{x}_2, \tau_2) \dots \} e^{-H\beta} \qquad (5.11)$$

where H is the Hamiltonian, β^{-1} is the temperature (times Boltzmann's constant), $A(\vec{x}, \tau)$ is an operator defined in terms of the Schrodinger-representation operator A(x) by

$$A(x,\tau)=e^{H\tau}A(x)e^{-H\tau}, \tag{5.12}$$

and T_τ denotes ordering according to the values of τ, with τ values decreasing from left to right, and with an extra minus sign for odd permutations of fermion operators. τ value is in the range

$$0 \lesssim \tau \lesssim \beta \tag{5.13}$$

It is therefore convenient to express these Green's fuctions as a Fourier integral over momenta and a Fourier sum over discrete energies. We can write

$$\langle T_\tau\{A(\vec{x}_1,\tau_1)B(\vec{x}_2,\tau_2)\ldots\}\rangle = \int d^3p_1 d^3p_2 \ldots \sum_{\omega_1}\sum_{\omega_2}$$

$$\ldots G(\vec{p}_1,\omega_1,\vec{p}_2,\omega_2,\ldots)$$

$$\cdot \exp(i\vec{p}_1\cdot\vec{x}_1 - i\omega_1\tau_1 + i\vec{p}_2\cdot\vec{x}_2 - i\omega_2\tau_2 + \ldots) \tag{5.14}$$

where

$$\omega = \frac{\pi}{\beta} \begin{cases} \text{even integer (bosons)} \\ \\ \text{odd integer (fermion)} \end{cases} \tag{5.15}$$

We follow the well-known diagrammatic procedure to calculate the G's evaluate the Feynman diagrams as usual in field theory except that every internal energy p_o is replaced with a quantity $i\omega$ satisfying the "quantization" condition (5.15) and all energy integrals replaced with ω sums:

$$\begin{aligned} p^0 &\rightarrow i\omega \\ \int d^4p &\rightarrow \frac{i2\pi}{\beta} \int d^3\vec{p} \sum_{\omega} \\ \delta^4(p-p') &\rightarrow \left(\frac{i2\pi}{\beta}\right)^{-1} \delta_{\omega\omega'} \delta^3(\vec{p}-\vec{p}') \end{aligned} \tag{5.16}$$

That is so called imaginary time formalism.[6]

5.3 CRITICAL TEMPERATURE

The self consistency condition of NJL models at zero temperature is

$$m = -2gm \int \frac{d^4p}{(2\pi)^4} \frac{i}{p^2+m^2} \quad (5.17).$$

We can write the self consistency condition of NJL models in temperature β^{-1} as

$$m = 2gmI(\beta^{-1}) \quad (5.18).$$

At zero temperature,

$$I(0) = \int \frac{d^4p}{(2\pi)^4} \frac{-i}{p^2+m^2} \quad (5.19)$$

then at temperature β^{-1}, it becomes

$$I(\beta^{-1}) = \frac{1}{\beta} \sum_{n=-\infty}^{+\infty} \int \frac{d^3\vec{p}}{(2\pi)^3} \frac{1}{\vec{p}^{\,2} + (2n+1)^2 \frac{\pi^2}{\beta^2} + m^2} \quad (5.20).$$

After carring out the sum,

$$I(\beta^{-1}) = \int \frac{d^3\vec{p}}{(2\pi)^3} \frac{1}{E} \left(\frac{1}{2} - \frac{1}{e^{E\beta}+1} \right) \quad (5.21)$$

where,

$$E = \sqrt{p^2 + m^2} \quad (5.22)$$

therefore

$$I(\beta^{-1}) = I(0) - \int \frac{d^3\vec{p}}{(2\pi)^3} \frac{1}{E(e^{E\beta}+1)}$$

$$= I(0) - \frac{1}{(2\pi)^2} \int dp \frac{p^2}{(p^2+m^2)[e^{(p^2+m^2)^{\frac{1}{2}}\beta}+1]} \qquad (5.23).$$

Let

$$x = p\beta , \quad a = m\beta \qquad (5.24),$$

then

$$I(\beta^{-1}) - I(0) = -\frac{1}{2\pi^2\beta^2} W(a) \qquad (5.25)$$

where

$$W(a) = \int_0^\infty dx \frac{x^2}{(x^2+a^2)^{\frac{1}{2}}\left[e^{(x^2+a^2)^{\frac{1}{2}}}+1\right]} \qquad (5.26).$$

After integral we can find

$$W(a) = \frac{1}{4}(\gamma - \log\pi - \frac{1}{2})a^2 + \frac{1}{8}a^2\log a^2 + \frac{\pi^2}{12} + J(a) \qquad (5.27)$$

where γ is Euler constant and

$$J(a) = \sum_{n=0}^{\infty} (2n+1)\pi^2 \left[\left(1 + \frac{a^2}{(2n+1)^2\pi^2}\right)^{\frac{1}{2}} - 1 - \frac{a^2}{2\pi^2(2n+1)^2} \right] \qquad (5.28)$$

For small a, in other words, at high temperature,

$$W(a) = \frac{1}{4}\left(\gamma - \log\pi - \frac{1}{2}\right)a^2 + \frac{1}{8}a^2\log a^2 + \frac{\pi^2}{12} + O(a^4) \qquad (5.29)$$

Therefore

$$I(\beta^{-1}) = I(0) - \frac{1}{24\beta^2} - \frac{(1/2 + \log\pi - \gamma)}{8\pi^2}$$

$$- \frac{m^2\log(m^2\beta^2)}{16\pi^2} + O(m^4\beta^2) \qquad (5.30)$$

The self-consistency condition is

$$m = 2mg\left[I(0) - \frac{1}{24\beta^2} - \frac{(\frac{1}{2} + \log\pi - \gamma)m^2}{8\pi^2} - \frac{m^2\log m^2\beta^2}{16\pi^2}\right] \qquad (5.31)$$

and function

$$f(m,\bar{\lambda}) = 2m\bar{\lambda}\left[1 - \frac{m^2}{\Lambda^2}\log\left(\frac{\Lambda^2}{m^2}+1\right) - \frac{2\pi^2}{3\beta^2\Lambda^2}\right.$$

$$\left. + \frac{2(\frac{1}{2} + \log\pi - \gamma)m^2}{\Lambda^2} - \frac{m^2\log m^2\beta^2}{\Lambda^2}\right] \qquad (5.32).$$

In order to check the stability of the trivial solution $m=0$, we should calculate $f'(0,\lambda)$ and we found

$$f'(0,\bar{\lambda}) = 2\bar{\lambda} - \frac{9}{12\beta^2} \qquad (5.33).$$

The bifurcation point is

$$\bar{\lambda} = \frac{1}{2} + \frac{g}{24\beta^2} \qquad (5.34)\ .$$

If $2\bar{\lambda} - \frac{g}{12\beta^2} < 1$, then the solution m=0 is stable for all the value of $\bar{\lambda}$, the chiral symmtry is not broken. The critical temperature is

$$\beta_c^{-1} = \sqrt{\frac{12(2\bar{\lambda}-1)}{g}} \qquad (5.35),$$

above β_c^{-1}, the chiral symmetry is restored.[5]

For SNJL model, in finete temperature, we should treat the fermion loop and boson loop in different ways because of the different "quantization" condition (5.15). Supersymmetry is broken in finite temperature.

For the fermion loop, as already being expressed in (5.20)

$$I_f\,(\beta^{-1}) = \frac{1}{\beta}\sum_{n=-\infty}^{\infty}\int\frac{d^3\vec{p}}{(2\pi)^3}\,\frac{1}{\vec{p}^{\,2} + (2n+1)^2\frac{\pi^2}{\beta^2} + m^2} \qquad (5.36)$$

from (5.30), we can found for high temperature

$$I_f(\beta^{-1}) = I(0) - \frac{1}{24\beta^2} + \frac{(\frac{1}{2} + \log\pi - \gamma)}{8\pi^2} + \frac{m^2\log(m^2\beta^2)}{16\pi^2}$$

$$+ O(m^4 \beta^2) \qquad (5.37)$$

For boson loop,

$$I_b(\beta^{-1}) = \frac{1}{\beta} \sum_{n=-\infty}^{\infty} \int \frac{d^3\vec{p}}{(2\pi)^3} \frac{1}{\vec{p}^{\,2} + \frac{2n\pi^2}{\beta^2} + m^2} \qquad (5.38)$$

After carry out the sum,

$$I_b(\beta^{-1}) = \int \frac{d^3\vec{p}}{(2\pi)^3} \frac{1}{E} \left(\frac{1}{2} + \frac{1}{e^{E\beta} - 1} \right)$$

$$= I(0) + \frac{1}{(2\pi)^2} \int \frac{dp \quad p^2}{(p^2+m^2)^{\frac{1}{2}} \left[e^{(p^2+m^2)^{\frac{1}{2}}\beta} - 1 \right]}$$

$$= I(0) + \frac{1}{2\pi^2\beta^2} W_b(a) \qquad (5.39)$$

where

$$W_b(a) = \int_0^{\infty} d\, x \frac{x^2}{(x^2+a^2)^{\frac{1}{2}} \left[e^{(x^2+a^2)^{\frac{1}{2}}} - 1 \right]}$$

$$= \frac{\pi^2}{6} - \frac{1}{4}\left(\gamma - \log 4\pi - \frac{1}{2} \right) a^2 - \frac{1}{8} a^2 \log a^2 - \frac{\pi a}{2} + O(a^3) \qquad (5.40)$$

Then in finite temperature β^{-1},

$$B(p^2) = -2 \left[I_f(\beta^{-1}) - I_b(\beta^{-1}) \right]$$

$$= -\frac{1}{\pi^2\beta^2} \left[-W\ (a) - W_b(a) \right]$$

$$= \frac{1}{\pi^2\beta^2}\left[\frac{\pi^2}{4} - \frac{\pi}{2}a + \frac{1}{4}\log 4a^2\right] \qquad (5.41)$$

$$A(p^2) = I_b(\beta^{-1}) = I(0) + \frac{1}{2\pi^2\beta^2}W_b(a) \qquad (5.42)$$

Then the self consistency condition is

$$m = gm\left[I(0) + \frac{1}{2\pi^2\beta^2}(-2W(a) - W_b(a)\right] \qquad (5.43)$$

function $f(m,\lambda)$ is

$$f(m,\bar\lambda) = m\bar\lambda\left[1 - \frac{m^2}{\Lambda^2}\log\left(\frac{\Lambda^2}{m^2}+1\right) - \frac{8\pi^2}{3\beta^2\Lambda^2}\right] \qquad (5.44)$$

In order to check the stability of the trivial solution m=0, we found

$$f'(0,\bar\lambda) = \bar\lambda - \frac{g}{6\beta^2} \qquad (5.45)$$

the bifurcation point is

$$\bar\lambda = 1 + \frac{g}{6\beta^2} \qquad (5.46)$$

The critical temperature is

$$\beta_c^{-1} = \sqrt{\frac{6(\bar{\lambda}-1)}{g}} \tag{5.47}$$

Above this temperature the chiral symmetry is restored.

The conclusion of this chapter is that at zero temperature if $\frac{g\Lambda^2}{16\pi^2} > 1$, the trivial solution m=0 of SNJL model is unstable and the chiral symmetry broken phase is stable. The temperature effect changes the bifurcation point and the effective mass m. There is a critical temperature β_c^{-1} above that the chiral symmetry is restored,

$$\beta_c^{-1} = \sqrt{\frac{6(\bar{\lambda}-1)}{g}}$$

which is smaller than that in the ordinary NJL model.

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CHAPTER 6 DYNAMICAL MASS GENERATION IN SUPERSYMMETRIC GAUGE FIELD THEORIES

In chapter 2, we have applied NJL approach[1] in renormalizable QCD and, by using the renormalization group analysis, found that the dynamically generated fermion mass in QCD is $M = \Lambda_c e^{1/6}$ where Λ_c is QCD cut off. In chapter 4, we have shown that there is a nontrivial solution $M \neq 0$ for the self-consistency condition of the supersymmetric NJL model. It is natural to ask whether or not chiral symmetry is broken in supersymmetric gauge theory via NJL approach. In this chapter we will show that there is a nontrivial solution $M = \Lambda_c \; e^{1/2}$ for the self-consistency condition of supersymmetric gauge field theory. Chiral symmetry is broken via the NJL mechanism.

This chapter is organized as following, first, we will give a brief review of supersymmetric Yang-Mills gauge theory with massless matter fields, then solve the self-consistency condition in one loop accuracy by using the renormalization group analysis and show in two loop accuracy that the one loop solution remains valid.

In order to describe a supersymmetric gauge theory, we should introduce vector superfields V, which satisfies the constrain $V = V^{+}$, the component field expansion of V is

$$V(x,\theta,\bar{\theta}) = c + i\theta\chi - i\bar{\theta}\bar{\chi} + \frac{i}{2}\bar{\theta}^2(M - iN) - \theta\sigma^m\bar{\theta}\, V_m$$

$$+ i\theta^2\bar{\theta}(\bar{\lambda} + \frac{i}{2}\bar{\sigma}^m\partial_m\chi) - i\bar{\theta}^2\theta(\lambda - \frac{i}{2}\sigma^m\partial_m\bar{\chi})$$

$$+ \frac{1}{2}\theta^2\bar{\theta}^2(D + \frac{1}{2}\partial^2 c) \qquad (6.1)$$

We will consider an internal symmetry group G = SU(n). If Φ is chiral, then the action of this group on Φ is defined as:

$$\Phi \rightarrow \Phi' = e^{-i\Lambda}\Phi \qquad (6.2)$$

where $\Lambda = \Lambda^A T_A$, $\bar{D}_{\dot{\alpha}}\Lambda = 0$, T_A is the generator of group G. It can be shown that Φ' is chiral too, because

$$\bar{D}_{\dot{\alpha}}\Phi' = \bar{D}_{\dot{\alpha}}(\Phi - i\Lambda\Phi + \cdots)$$

$$= \bar{D}_{\dot{\alpha}}\Phi - i\left[(\bar{D}_{\dot{\alpha}}\Lambda)\Phi + \Lambda(\bar{D}_{\dot{\alpha}}\Phi)\right] + \cdots$$

$$= 0 \qquad (6.3)$$

Correspondingly, the antichiral field transform as

$$\bar{\Phi} \rightarrow \bar{\Phi}' = \bar{\Phi}\, e^{i\bar{\Lambda}} \qquad (6.4)$$

$$\bar{\Lambda} = \bar{\Lambda}^A T_A \qquad (6.5)$$

$$D_\alpha \bar{\Lambda} = 0 \qquad (6.6)$$

Then under local transformation $\int d^4x d^4\theta \bar{\Phi} \Phi$ is not longer invariant, we should introuduce a gauge field to covarianlize the action.

We find that if

$$e^{V'} = e^{i\bar{\Lambda}}\, e^V\, e^{-i\Lambda} \qquad (6.7)$$

where

$$V = V^A\, T^a \qquad (6.8)$$

then

$$\int d^4\, x\, d^4\theta\, \bar{\Phi}\, e^V \Phi \qquad (6.9)$$

is invariant under local internal transformation (6.2) and

$$v \longrightarrow v' = v - \frac{i}{g}(\bar{\Lambda} - \Lambda) \tag{6.10}$$

We may view the vector field v as the supersymmetric generation of the Yang-Mills potential. To construct the corresponding supersymmetric field strength, we find that the superfields

$$W_\alpha = -\frac{1}{4}\bar{D}\,\bar{D}\,(e^{-gV}\,D_\alpha e^{gV}) \tag{6.11}$$

are chiral and gauge invariant and the component fields of W_α contain

$$f_m^n = \partial_m V^n - \partial_n V^m \,, \tag{6.12}$$

which is corresponding to the gauge field strength.

Then we can write the Lagrangian of supersymmetric Yang-Mills fielfs as

$$L_{YM} = \frac{1}{64g^2}\mathrm{Tr}\int d^2\theta\; W_\alpha W^\alpha \tag{6.13}$$

As in ordinary Yang-Mills field theory, we find the kinetic operator in (6.13) is not invertable, we must choose gauge-fixing functions.

The gauge variant quantity $F = \bar{D}^2 V$ is a suitable gauge-fixing function becauce it has the same spin and superspin as the gauge parameter Λ and can be made to vanish by a gauge transformation. The gauge-fixing term is

$$L_{g.f} = -\frac{1}{g^2} \mathrm{Tr} \int d^4\theta \, (D^2 V)(\bar{D}^2 V) \qquad (6.14)$$

Then we can find the supersymmetric Fedeev-Popov ghost Lagrangian L_{FP} as following. We define the functional determinant.

$$\Delta(V) = \int \mathcal{D}\Lambda \mathcal{D}\bar{\Lambda} \, \delta(F(V,\Lambda,\bar{\Lambda}) - f) \, \delta(\bar{F}(V,\Lambda,\bar{\Lambda}) - \bar{f}) \qquad (6.15)$$

where

$$F(V,0,0) = f. \qquad (6.16)$$

We can write

$$Z = \int \mathcal{D} V \Delta^{-1} e^{S_{kin.} + S_{gf}} \qquad (6.17)$$

$$\begin{aligned} \Delta(V) = & \int \mathcal{D}\Lambda \mathcal{D}\bar{\Lambda} \, \delta\left[\frac{\delta F(V,0,0)}{\delta\Lambda}\Lambda + \frac{\delta F(V,0,0)}{\delta\bar{\Lambda}}\bar{\Lambda}\right] \cdot \\ & \cdot \delta\left[\frac{\delta \bar{F}(V,0,0)}{\delta\Lambda}\Lambda + \frac{\delta \bar{F}(V,0,0)}{\delta\bar{\Lambda}}\bar{\Lambda}\right] \\ = & \int \mathcal{D}\Lambda \mathcal{D}\bar{\Lambda} \mathcal{D}\Lambda' \mathcal{D}\bar{\Lambda}' \cdot e^{\{\int d^4x d^2\theta \, \Lambda' \left[\frac{\delta F(0,0,V)}{\delta\Lambda}\Lambda + \frac{\delta F(0,0,V)}{\delta\bar{\Lambda}}\bar{\Lambda}\right]} \\ & + \int d^4x d^2\bar{\theta} \, \bar{\Lambda}' \left[\frac{\delta \bar{F}(0,0,V)}{\delta\Lambda}\Lambda + \frac{\delta \bar{F}(0,0,V)}{\delta\bar{\Lambda}}\right] \end{aligned} \qquad (6.18)$$

We can write

$$\delta F(V,0,0) = \delta \bar{D}^2 V = \bar{D}^2 \delta V \tag{6.19}$$

under a gauge transformation

$$e^{V'} = e^{i\bar{\Lambda}} e^{V} e^{-i\Lambda}, \tag{6.20}$$

for infinitesimal Λ

$$\delta V = L_{\frac{1}{2}V}\left[-i(\Lambda+\bar{\Lambda}) + \coth L_{\frac{1}{2}V}\, i(\bar{\Lambda}-\Lambda)\right] \tag{6.21}$$

where

$$L_X Y \equiv [X,Y] \tag{6.22}$$

To find $\Delta^{-1}(V)$, we need only replace the parameters Λ, Λ' by anticommuting chiral ghost fields c, c'. Finally, we find

$$S_{FP} = iT_r \int d^4x\, d^2\theta\; c'\, \bar{D}^2 (\delta V)$$

$$+ iT_r \int d^4x\, d^2\bar{\theta}\; \bar{c}' D^2 (\delta V)$$

$$= T_r \int d^4x\, d^4\theta\; (c'+\bar{c}')\; L_{\frac{1}{2}V}\left[(c+\bar{c})\right.$$

$$+ \coth L_{\frac{1}{2}V}(c-\bar{c}) \Big] \qquad (6.23)$$

and

$$Z = \int \mathcal{D}V\mathcal{D}c\mathcal{D}c'\mathcal{D}\bar{c}\mathcal{D}\bar{c}'\, e^{S_{YM}+S_{matter}+S_{gf}+S_{FP}} \qquad (6.24)$$

Now, the quadratic part of the gauge field has the form

$$\int d^4x d^4\theta \; -\frac{1}{g^2}T_r\left[\frac{1}{2}V\; D^\alpha\bar{D}^2D_\alpha V + (D^2V)(\bar{D}^2V)\right] \qquad (6.25)$$

Integrating by parts, we can write

$$(D^2V)(\bar{D}^2V) = \frac{1}{2}V\;(D^2\bar{D}^2+\bar{D}^2D^2)\;V \qquad (6.26)$$

Because

$$D^\alpha D^2 D_\alpha + D^2\bar{D}^2 + \bar{D}^2D^2 = \Box^2 \qquad (6.27)$$

The kinetic part of gauge field V is :

$$L^{(2)}_{YM} = \int d^4\theta \; - \frac{1}{2g^2}V\,\Box\, V \qquad (6.28)$$

The quadratic part of the ghost action has the form

$$L^{(2)}_{FP} = T_r\int d^4\theta\;\; (c'+\bar{c}')\;\; (c-\bar{c})$$

$$= T_r \int d^4\theta \, (\bar{c}'c - c'\bar{c}) \tag{6.29}$$

then we can find the propagators are as following:

$$\begin{array}{ll} VV & -\frac{1}{p^2}\,\delta^4(\theta-\theta') \\ \bar{c}'c & \frac{1}{p^2}\,\delta^4(\theta-\theta') \\ c'\bar{c} & \frac{1}{p^2}\,\delta^4(\theta-\theta') \end{array} \tag{6.30}$$

In supersymmetric gauge theory we need two chiral superfields Φ_+ and Φ_- to represent a four components Dirac field,

$$\Psi \simeq \begin{pmatrix} \Phi_+ \\ \bar{\Phi}_- \end{pmatrix} \tag{6.31}$$

The supersymmetric gauge theory Lagrangian can be written as

$$L = L_{gauge} + L_{gf} + L_{FP} + L_{matter} \tag{6.32}$$

where

$$L_{gauge} = \frac{1}{64g^2} Tr \int d^2\theta \, W_\alpha W^\alpha \quad . \tag{6.33}$$

$$L_{matter} = \int d^4\theta \left(\bar{\Phi}_+ e^{gV} \Phi_+ + \bar{\Phi}_- e^{gV} \Phi_- \right) \tag{6.34}$$

where

$$V = V^i G_i, \qquad [G_i, G_j] = i f_{ij}{}^k G_k \ ,$$

$$W_\alpha = \bar{D}^2 (e^{-gV} D_\alpha e^{gV}), \qquad L_X Y = [X, Y] \ ,$$

$$f_{ijk} f_{jkl} = C_2 \delta_{il}, \qquad Tr \ G_i G_j = T_A \delta_{ij},$$

$$G_i G_i = C_f \ I, \tag{6.35}$$

where, G^i is the generator of the representation A and $\Phi_\pm$ transform according to the representation A of the gauge group G. We can write L as

$$L = L_o + L_I$$

where L_o is the free Lagrangian,

$$L_0 = L^{(2)}_{gauge} + L^{(2)}_{FP} + L^{(2)}_{matter} \tag{6.37}$$

The Feynman rules are as in (6.30). We can add a mass term L_{mass} to L_{matter},

$$L_{mass} = -\int d^2\theta\, M\phi_+\phi_- - \int d^2\bar{\theta}\, M\bar{\phi}_+\bar{\phi}_- \tag{6.38}$$

Then the propagators become as in Chapter 4. We can write $L_{matter} + L_{mass}$ in terms of Ψ, $\bar{\Psi}$

$$L_{matter}+L_{mass} = -\int d^4\theta\, \bar{\Psi} \begin{pmatrix} M\delta(\bar{\theta}) & e^{gV} \\ e^{gV} & M\delta(\theta) \end{pmatrix} \Psi \tag{6.39}$$

If M=0, L is invariant under chiral transformation ,

$$\Psi \to e^{i\alpha\gamma_5}\Psi \tag{6.40}$$

or,

$$\begin{aligned} \phi_+ &\to e^{i\alpha}\phi_+ \\ \phi_- &\to e^{i\alpha}\phi_- \end{aligned} \tag{6.41}$$

Then in the usual perturbation theory due to the chiral symmetry the scale superfields will remain massless in any finite order. But if we following NJL and Chang-Chang, we can adopt

$$L_0{}' = L_0 + L_{mass} \tag{6.42}$$

as free Lagrangian and

$$L_I{}' = L_I - \delta M/M\, L_{mass} \tag{6.43}$$

as interaction Lagrangian and perturbate around this massive vacuum. Then in tree level, the two point function of ψ is

$$\Gamma^{(2)}(\bar{\Psi},\Psi) = \int d^4\theta\, \bar{\Psi}(p) \begin{pmatrix} \delta(\bar{\theta})M & 1 \\ 1 & \delta(\theta)M \end{pmatrix} \Psi(-p) \tag{6.44}$$

As in ref. 2, δM is treated just like a counterterm and used to fix the renormalized two point function to be

$$\Gamma^{(2)}_r(\bar{\Psi},\Psi) = \int d^4\theta\, \bar{\Psi}_r(p) \begin{pmatrix} \delta(\bar{\theta})M & 1 \\ 1 & \delta(\theta)M \end{pmatrix} \Psi_r(-p) \tag{6.45}$$

for $p^2 \ll M^2$. We should determine δM perturbatively loop by loop and then use the renormalization group analysis to sum up all the leading logarithm, next to leading logarithm contributions. After summing up the high order contributions, we should impose the self consistency condition $\delta M = M$, and solve M. To one loop accuracy, following the procedure described in Chapter 4, in order to determine δM, we should first carry out the wave function renormalization. The contribution to the wave function renormalization of Ψ comes from the graph in Fig. 6.1. We find that the unrenormalized superfields Ψ can be presented by renormalized superfields Ψ_γ for $p^2 \ll M^2$ by

$$\Psi = \hat{Z}_2 \tilde{Z}_2 \Psi_\gamma \tag{6.46}$$

where

$$\hat{Z}_2 = 1 + 2\lambda C_f \frac{1}{\bar{\epsilon}} \tag{6.47}$$

and $\tilde{Z}_2$ is finite wave function renormalization factor,

$$\tilde{Z}_2 = 1 - \lambda A^{(1)} \tag{6.48}$$

$$A^{(1)} = 2C_f \;(\log M/\mu \; -1/2\;) \tag{6.49}$$

In the above calculation, we have used modified minimum subtraction scheme ($\overline{MS}$), n =4 - ϵ ,

$$\frac{1}{\bar{\epsilon}} = \frac{1}{\epsilon} - \frac{1}{2}\gamma_E - \frac{1}{2}\log 4\pi \tag{6.50}$$,

and the two step renormalization procedure[8]. Now we can seperate δM to two parts

$$\delta M = M\,(-\lambda A^{(1)} + \lambda B^{(1)}) \qquad (6.51)$$

where $B^{(1)}$ comes from the contribution of the graph in Fig. 6.2. Using the background field method, the calculation shows

$$B^{(1)} = 0 \qquad (6.52)$$

In order to sum up the leading logarithm contribution, we need the β function of the gauge coupling constant λ

$$\frac{d}{dt}\lambda = -b\lambda^2 \qquad (6.53)$$

From the calculation of ref. 9, we found

$$b = 6C_2 - 8T_A \qquad (6.54)$$

The one loop supersymmetric gauge theory renormalization group invariant cutoff $\Lambda^{(1)}_{c(SUSY)}$ can be defined by

$$0 = \frac{1}{\lambda} + b \log \frac{\Lambda^{(1)}_{c(SUSY)}}{\mu} \qquad (6.55).$$

We can write δM as

$$\delta M = M - \sum \qquad (6.56),$$

where,

$$\Sigma = M \left[1 + \lambda C_f (\log \frac{M^2}{\mu^2} - 1) \right] \quad (6.57).$$

Using the RG analysis to sum up the leading logarithm contribution, Σ becomes

$$\Sigma = M \left[1 + \lambda b (\log \frac{M}{\mu} - 1/2) \right]^{2C_f/b} \quad (6.58)$$

The self consistency condition $M = \delta M$ implies

$$\Sigma = 0 \quad (6.59).$$

Since $C_f/b > 0$, it can be satisfied by

$$1 + b \lambda \left(\log \frac{M}{\mu} - \frac{1}{2} \right) = 0 \quad (6.60).$$

Solving the equation by substituting (6.55) into (6.60), we find

$$M = \Lambda^{(1)}_{c(SUSY)} e^{\frac{1}{2}} \quad (6.61).$$

To two loop accuracy, we should calculate the graphs in Fig.6.3. We find for $p^2 << M^2$

$$A^{(2)} = \frac{1}{4} h_2 \left(\log^2 \frac{M^2}{\mu^2} - 4 \log \frac{M^3}{\mu^2} \right) \tag{6.62}$$

$$B^{(2)} = 0 \tag{6.63}$$

where

$$h_2 = - C_f \ (2 C_f - b) \tag{6.64}$$

Then

$$\begin{aligned} \Sigma &= M(1 - \lambda A^{(1)} - \lambda^2 A^{(2)} + \lambda B^{(1)} + \lambda^2 B^{(2)}) \\ &= M \left[1 + \lambda C_f (\log M^2/\mu^2 - 1) + \frac{1}{4} \lambda^2 h_2 \left(\log^2 \frac{M^2}{\mu^2} - 4 \log \frac{M^2}{\mu^2} \right) \right] \end{aligned} \tag{6.65}$$

It is easy to find the RG equation for Σ ,

$$\frac{d\Sigma}{dt} = - (2 C_f \lambda + h_2 \lambda^2) \Sigma \tag{6.66}$$

To solve (6.66), we still need the β function of gauge coupling constant λ ,

$$\frac{d}{dt} \lambda = - b \lambda^2 - c \lambda^3 \tag{6.67}$$

where[10]

$$c = 12\ C_2^2 - 8T_A(2C_f + C_2\ ) \qquad (6.68)$$

Solving equation (6.66),

$$\Sigma = M\left(\frac{\lambda}{\lambda_0}\right)^{2C_f/b}\left(\frac{b\lambda+c}{b\lambda_0+c}\right)^{-2C_f/b + h_2/c} \qquad (6.69)$$

where

$$\frac{1}{\lambda_0} = \frac{1}{\lambda} - \frac{c}{b}\log\left(1+\frac{b}{c\lambda}\right) + b\left(\log\frac{M}{\mu} - \frac{1}{2}\right) \qquad (6.70)$$

We can satisfy the self consistency condition by

$$\frac{1}{\lambda} - \frac{c}{b}\log\left(1+\frac{b}{c\lambda}\right) + b\left(\log\frac{M}{\mu} - \frac{1}{2}\right) = 0 \qquad (6.71)$$

or in terms of the two loop supersymmetric gauge theory cutoff defined by

$$\frac{1}{\lambda} - \frac{c}{b}\log\left(1+\frac{b}{c\lambda}\right) + b\left(\log\frac{\Lambda^{(2)}_{c(SUSY)}}{\mu}\right) = 0 \qquad (6.72)$$

then

$$M = \Lambda^{(2)}_{c(SUSY)}e^{1/2} \qquad (6.73)$$

The one loop solution remains valid in the two-loop level.

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CHAPTER 7 CONCLUSIONS AND REMARKS

7.1 CONCLUSIONS

In this thesis under the guidence of Prof. N.P. Chang, we study the problem of dynamical fermion mass generation and chiral symmetry breaking. We have calculated the two-loop diagrams in arbitary gauge and proved the two-loop constant does not affect the solution of the self-consistency condition. We have also proved the mass parameter which is the solution of self-consistency condition is independent of gauge.[1]

We have calculated the difference of up and down quarks by introducing the QED interaction as a perturbation to QCD. The result is $M = -(\ .2\%)M$, which is of the right sign and of right order of magnitude. We also study the supersymmetric extension of NJL model[2]. We have found that chiral symmtry is broken in supersymmtry NJL model, fermion acquires dynamical mass M, which satisfies the similar gap equation of ordinary NJL model. We have also found, by using the bifurcation theory[3], that in supersymmetric NJL model the nontrivial solution $m \neq 0$ is stable if $\frac{g\Lambda^2}{16\pi^2} > 1$. We have also studied the finite temperature effect in supersymmtric NJL model[4] and calculated the critical temperature β_c^{-1} above which the chiral symmtry is restored,

$$\beta_c^{-1} = \sqrt{\frac{3(\lambda-1)}{8}} \qquad (7.1)$$

We also study the dynamical fermion mass generation in supersymmtric gauge field theory[5] via Chang-Chang approach, the solution of self consistency condition is

$$M = \Lambda_{c(SUSY)} e^{\frac{1}{2}} \qquad (7.2)$$

Therefore chiral symmetry is broken in supersymmetric gauge field theory via NJL approach.

7.2 THE PROBLEMS TO BE FURTHER STUDIED

Since how to dynamically generate fermion masses is one of the most important problems in particle physics, and the NJL approach[6] and the Chang-Chang approach[7] are very useful in this study, there are many problems that we should study further. There are three main problems to be studied.

(1) CACULATION OF DYNAMICALLY GENERATED LEPTON MASSES AND MASS RATIO OF FAMILIES

A preliminary study has shown that it is possible to dynamically generate electron mass in grand unified theory via the diagrams in Fig. 7.1.[8] But how to properly count the renormalization group effect and the implication of bifurcation theory[5] in this model are necessary to study further.

Probably, the family problem is the more challenging one. We are going to use the decoupling scheme[9] and the renormalization group analysis to find the possible second solution of the gap equation. In order to get the large mass ratio of the families we are going to use the supersymmetric model if it is necessary. We have found[6] the solution of the self consistency condition for supersymmetric gauge field theory is

$$M = \Lambda_{c(susy)} e^{\frac{1}{2}} \tag{7.3}$$

Maybe, this can provide us another mass scale to serve as the mass scale of the third generation.

(2) PROPERTY OF THE GOLDSTONE PARTICLE

Since chiral symmetry is spontaneously broken, there must be a Goldstone boson. From our preliminary investigation, we found that the Goldstone boson appears as a pole in the effective vertex function of the quark fields coupled to a external pseudo-scalar source. The singularity arises as the result of the self consistency condition. We are going to do the two loop calculation of the vertex function and find out the residue of the pole, then calculate the constant f_{π}. At the same time we also can check the ward identity of this chiral symmetry breaking theory. We may also find the property of the Goldstone sector of the supersymmetric gauge field theory is useful in constructing composite models, in such model one has to answer the question of how to make the composite quark light.

(3) DYNAMICAL SUPERSYMMETRY BREAKING VIA NJL APPROACH AND ANOMALY

In a preliminary study we have found that in a pure supersymmetric Yang-Mills field theory in Wess-Zumino gauge, the gaugino can dynamically generate a mass via NJL approach and the gauge boson remains massless. In the point of view of spectrum, the boson-fermion symmetry is broken. However, since the Wess-Zumino gauge explicitly breaks supersymmetry, we can not claim too much so far. We are going to study the case in a supersymmetric gauge, then the conclusion will be more convincing. If our calculation shows that supersymmetry is dynamically broken via NJL approach, does this contradict with Witten's index theorem[10]? Is there some kind of anomaly in such theory? All these should be studied carefully.

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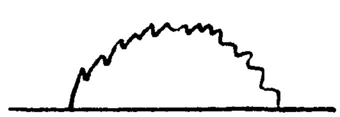


Fig. 2.1 Diagrams contributed to fermion two-point function in one-loop level

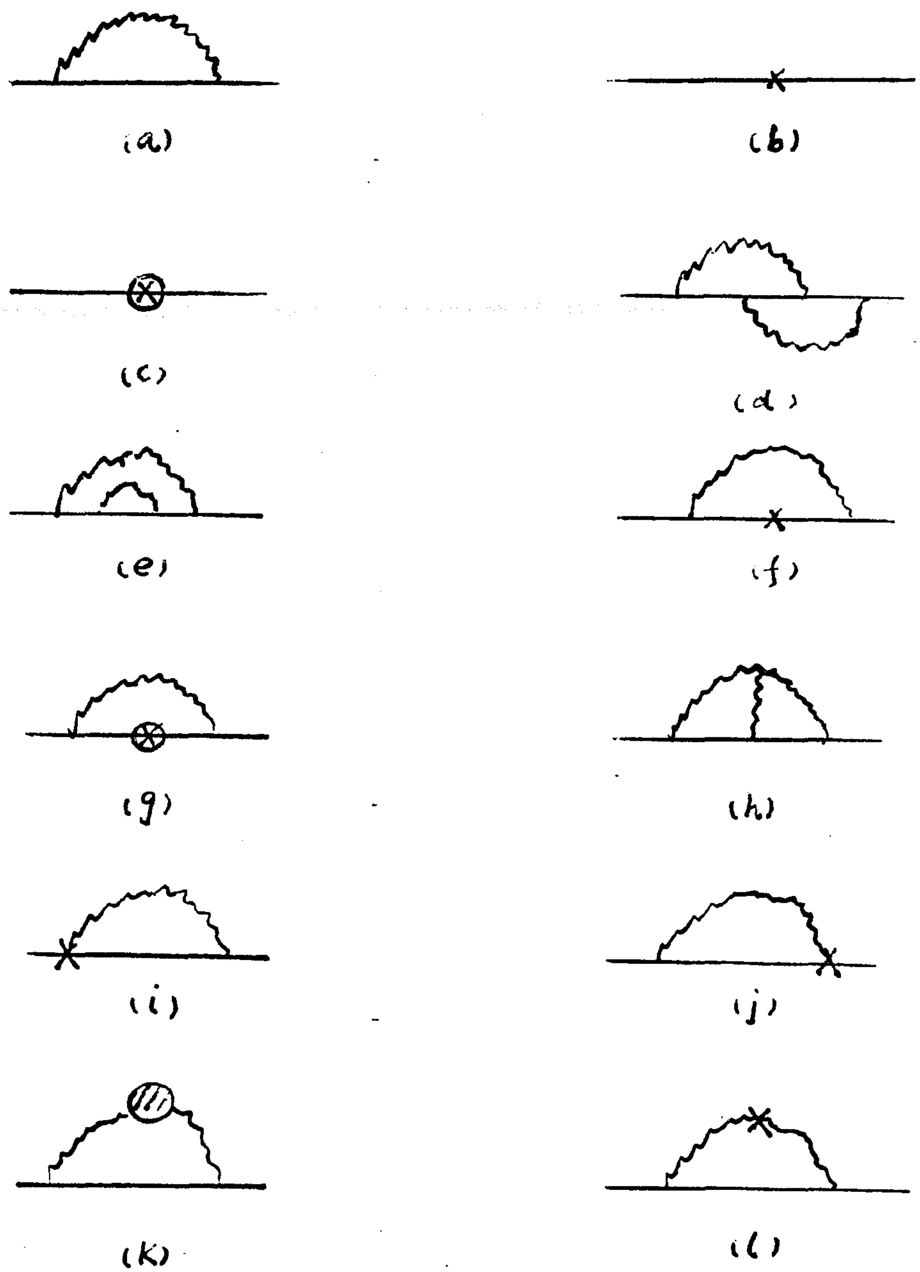


Fig. 2.2 Diagrams contributed to fermion two-point function in two-loop level

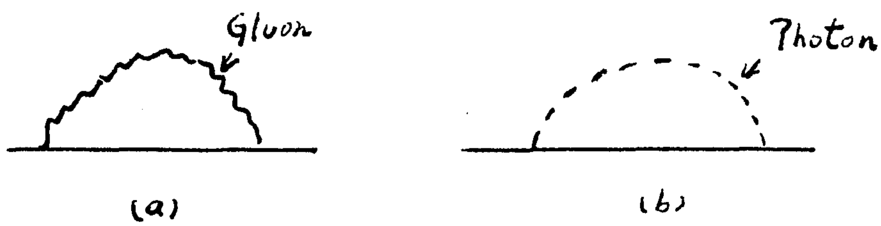


Fig. 3.1 One-loop diagram of quark two-point function in QCD+QED

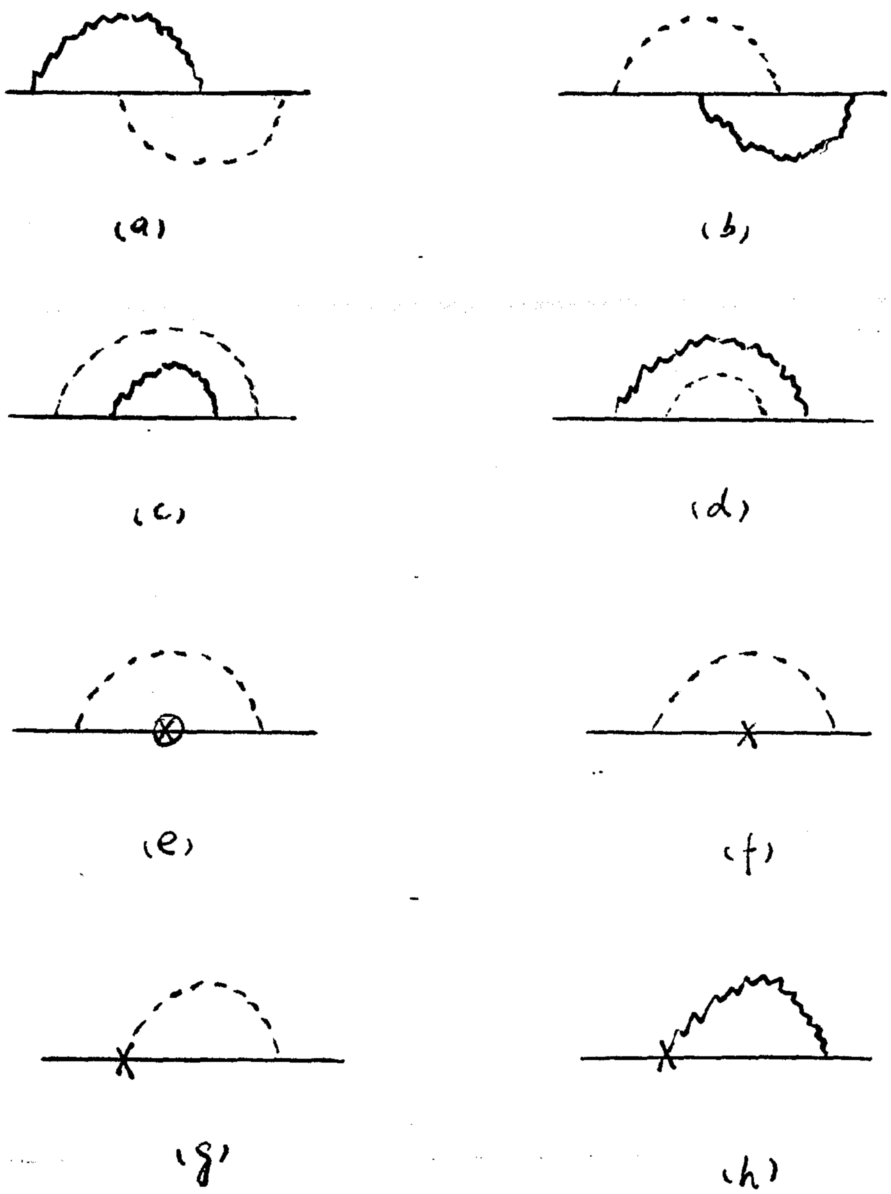


Fig. 3.2 Diagrams contributed to quark two-point function in two-loop level due to QED

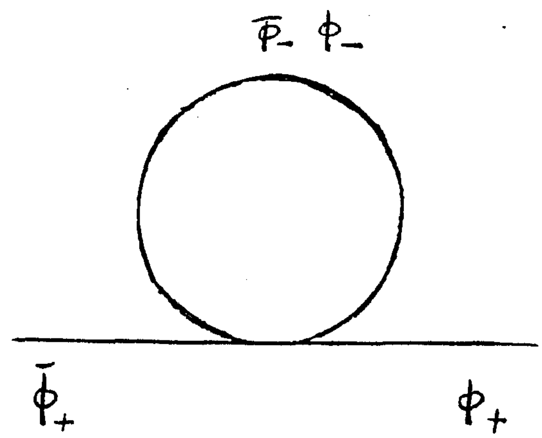


Fig. 4.1 Contribution to A(P)

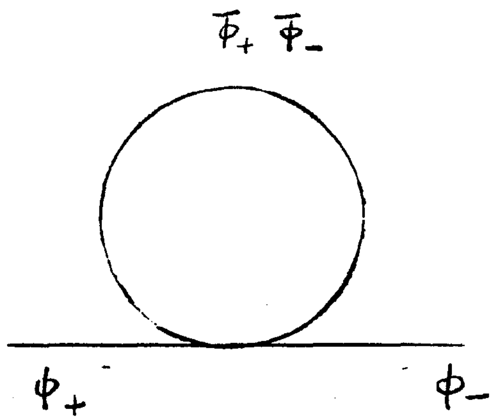


Fig. 4.2 Contribution to B(P)

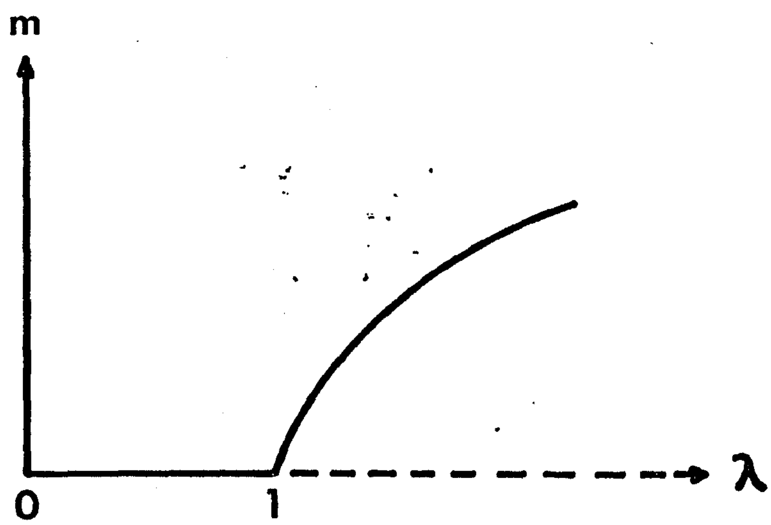


Fig. 5.1 Bifurcation diagram

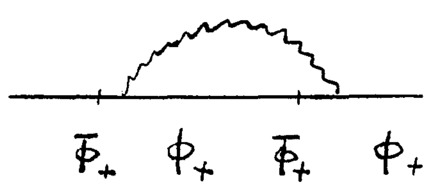


Fig. 6.1 Contribution to A(P) in one-loop

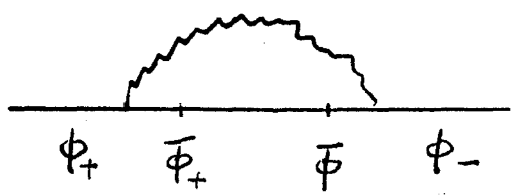


Fig. 6.2 Contribution to B(P) in one-loop

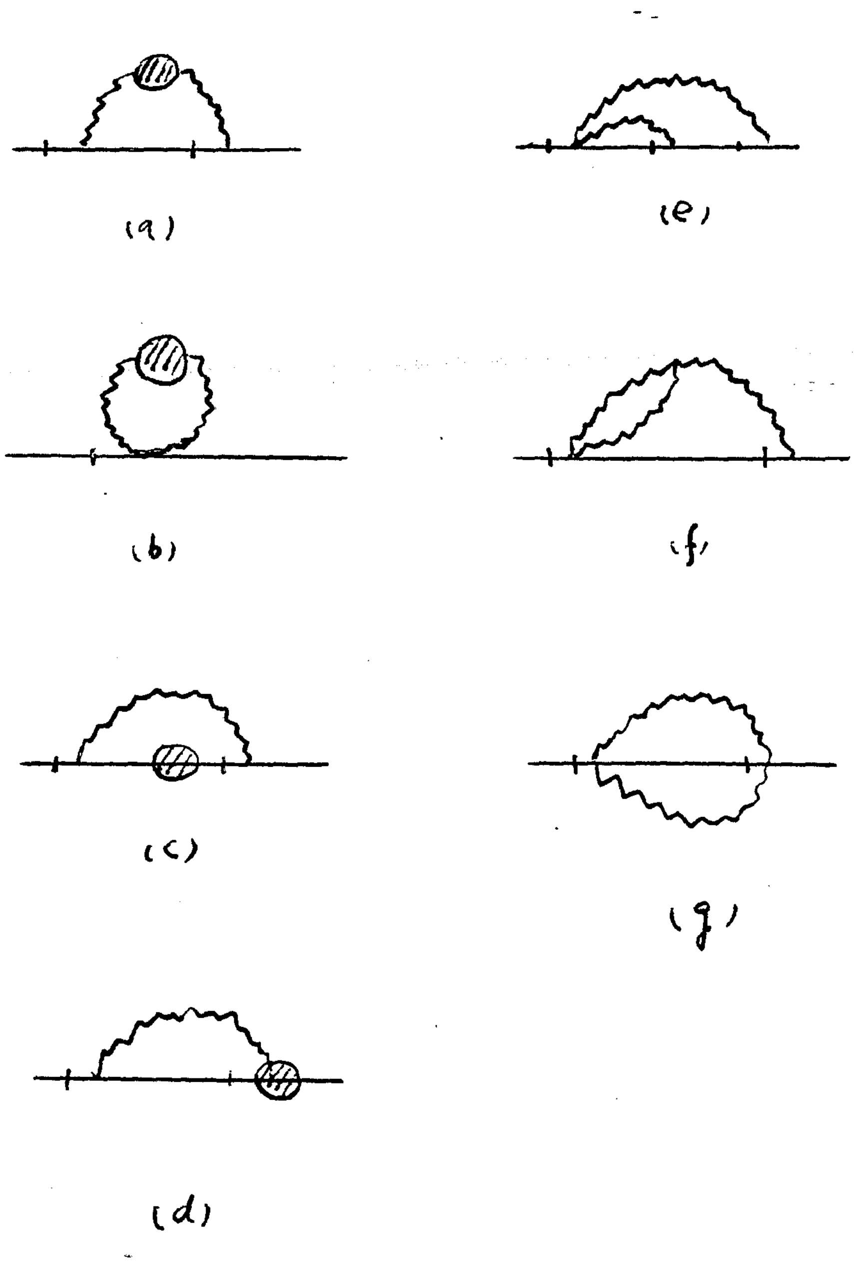


Fig. 6.3 Two-loop contribution

Table 2.1 Two-loop constants

Label	A	A_1	A_2	B	B_1	B_2
 s	0	$-11+\pi^2$	$-\frac{3}{8}-\pi^2$	$-\frac{29}{2}+\frac{3\pi^2}{2}$	-7	$-\frac{1}{2}-\frac{\pi^2}{6}$
 t	12.2653	3.85533	-3.52358	-12.3542	0.379222	2.74148
 c	16.5551	-2.05047	0.603112	-23.8168	-6.30044	7.07079
 f	4.13427	0	0	1.06885	0	0

APPENDIX CALCULATION OF TWO LOOP DIAGRAMS

A.1 USEFUL FORMULAE FOR TWO LOOP CALCULATION

A1.1 D-dimensional integrals

Let write $\int \frac{d^D k}{(2\pi)^D}$ as $\int_{''}$, $(k^2+M^2-i\epsilon)$ as Mk, $(M^2-i\epsilon)$ as M^2.

$$\int_{''} \frac{1}{(Mk)^N} = \frac{i}{(4\pi)^{D/2}} \frac{\Gamma(N-D/2)}{\Gamma(N)} \cdot \frac{1}{(M^2)^{N-D/2}} \tag{A1}$$

$$\int_{''} \frac{k^2}{(Mk)^N} = \frac{iD}{2(4\pi)^{D/2}} \frac{\Gamma(N-1-D/2)}{\Gamma(N)} \frac{1}{(M^2)^{N-D/2}} \tag{A2}$$

$$\int_{''} \frac{(k^2)^a}{(Mk)^N} = \frac{i}{(4\pi)^{D/2}} \frac{\Gamma(a+D/2)}{\Gamma(D/2)} \frac{\Gamma(N-a-D/2)}{\Gamma(N)(M^2)^{N-a-D/2}} \tag{A3}$$

$$\int_{''} \frac{k_\mu k_\nu}{(Mk)^N} = \frac{i}{(4\pi)^{D/2}} \cdot \frac{\delta_{\mu\nu}}{2} \frac{\Gamma(N-1-D/2)}{\Gamma(N)\,(M^2)^{N-D/2}} \tag{A4}$$

$$\int_{''} k_\mu k_\nu f(k^2) = \frac{\delta_{\mu\nu}}{D} \int_{''} k^2 f(k^2) \tag{A5}$$

$$\int_{''} k_\mu k_\nu k_\alpha k_\beta f(k^2) = \frac{\delta_{\mu\nu}\delta_{\alpha\beta} + \delta_{\mu\alpha}\delta_{\nu\beta} + \delta_{\mu\beta}\delta_{\nu\alpha}}{D^2+2D} \int_{''} k^4 f(k^2) \tag{A6}$$

A2.2 Euler-gamma and Euler-beta functions

Euler-gamma function is defined as

$$\Gamma(a) = \int_0^\infty dt\ t^{a-1} e^{-t} \tag{A7}$$

$$\Gamma(a+1) = a\,\Gamma(a) \tag{A8}$$

$$\Gamma(1+\epsilon) = 1-\gamma_E\,\epsilon + \sum_{n=2}^{\infty} \frac{(-\epsilon)^n}{n!}\ \zeta(n) \tag{A9}$$

where $\zeta(n)$ is Riemann's function

$$\zeta(s) = \sum_{n=1}^{\infty} n^{-s} \tag{A10}$$

and $\gamma_E \sim 0.5772$ is the Euler constant.

$$(R^2)^{\frac{\epsilon}{2}} = 1 + \frac{\epsilon}{2} \log R^2 + \frac{\epsilon^2}{8} \log^2 R^2 + O(\epsilon^3) \tag{A11}$$

Euler-beta function B(a,b) is defined as

$$B(a,b) = \int_0^1 dx\ x^{a-1} (1-x)^{b-1} \tag{A12}$$

$$B(a,b) = \frac{\Gamma(a)\,\Gamma(b)}{\Gamma(a+b)} \tag{A13}$$

A1.3 Feynman parametrization

$$\frac{1}{a^{\alpha}b^{\beta}} = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} \int_0^1 dx \, \frac{x^{\alpha-1}(1-x)^{\beta-1}}{[ax+b(1-x)]^{\alpha+\beta}} \tag{A14}$$

A1.4 Dirac algebra in D dimensions

$$\delta_{\mu\mu} = D \tag{A15}$$

$$\{\gamma_\mu, \gamma_\nu\} = 2\delta_{\mu\nu} \mathbb{1} \tag{A16}$$

$$\gamma_\mu \gamma_\mu = D \mathbb{1} \tag{A17}$$

$$\mathrm{Tr}\,(\gamma_\mu \gamma_\nu) = 4\delta_{\mu\nu} \tag{A18}$$

$$\gamma_\mu \not{p} \gamma_\mu = (2-D)\not{p} \tag{A19}$$

$$\gamma_\mu \not{p}_1 \not{p}_2 \gamma_\mu = 4 p_1 \cdot p_2 + (D-4)\not{p}_1 \not{p}_2 \tag{A20}$$

$$\gamma_\mu \not{p}_1 \not{p}_2 \not{p}_3 \gamma_\mu = -2\not{p}_3 \not{p}_2 \not{p}_1 - (D-4)\not{p}_1 \not{p}_2 \not{p}_3 \tag{A21}$$

A1.5 Useful formulae in loop calculation

If $\int_0^1 dz\, z^{-1} f(z,0) \mathrm{Log}\, M(z)$ convergent, then

$$\int_0^1 dz \frac{z^{-1+\epsilon} f(z,\epsilon)}{[M(z)]^{\epsilon}} = \int_0^1 dz\, z^{-1+\epsilon} f(z,\epsilon)\left[1-\epsilon \ln M\right] + O(\epsilon^2)$$

$$= \int_0^1 dz\, z^{-1+\epsilon} f(z,\epsilon) - \epsilon \int_0^1 dz\, z^{-1} f(z,0) \ln M(z) + O(\epsilon^2) \qquad \text{(A22)}$$

A.2 Calculation of two loop diagrams with the computer program REDUCE.

We need to calculate two loop diagrams including the two loop constants. This is very tidious job. The computer program "REDUCE" can dramatically reduce the algebra and matries calculation. But "REDUCE" does not know how to calculate the momenta integrals. We should reduce the integrals to a few fundamental momentum integrals in D dimention and give computer a integral table about those foundamental integrals. Then the computer can devide the diagram integral into many fundamental parts, and use the integral table to find the results of those fundamental parts. Then it can sum up these preliminary results to give us a final result including the logarithm terms and constant terms. Our computer programs in A.3 give the examples of such procedure. In this section, we are going to give some explaination about the programs. We will analyse the culculation of diagram in Fig. 2.2 (h), that is the most complicated one among all diagrams in Fig. 2.2.

Because we are only interested in the low energy case (p 0), this fact can simplify our calculation quite a lot. To see that, we can write two loop part of the fermion two point function as

$$\Gamma^{(2)}_{(p)} = \int_{\cdot\cdot} f(k,q,p)$$

$$=A(p^2)\gamma\cdot p + B(p^2) \quad \text{(A23)}$$

where $\int \frac{d^n k}{(2\pi)^n} \frac{d^n q}{(2\pi)^n}$ is written as $\int_{\cdot\cdot}$. In the limit of $p \to 0$

$$\Gamma^{(2)}(p) = A(0)\cdot\gamma\cdot P + B(0) \quad \text{(A24)}$$

$$B(0) = \Gamma^{(2)}(0)$$

$$= \int_{\cdot\cdot} f(k,q,0) \quad \text{(A25)}$$

To calculate A(0),we can see

$$\frac{\partial}{\partial p_\mu} \Gamma^{(0)}_{(p)} \Big|_{p_\mu=0} = A(0)\,\gamma_\mu \quad \text{(A26)}$$

thus

$$A(0) = \frac{1}{n}\,\gamma_\mu \frac{\partial}{\partial p_\mu} \Gamma^{(2)}_{(p)} \Big|_{p_\mu=0}$$

$$= \frac{1}{n} \int_{\cdot\cdot} \gamma_\mu \frac{\partial}{\partial p_\mu} f(k,q,p) \Big|_{p=0}$$

$$= \int '' f_1(k,q) \tag{A27}$$

where n is the dimension of space-time and

$$f_1(k,q) = \frac{1}{n}\gamma_\mu \frac{\partial}{\partial p_\mu} f(k,q,p)\Big|_{p_\mu=0} \tag{A28}$$

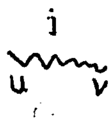
Then in the integrals of A(0) and B(0) the only dimensionful quantities in the integrands are k, q and M. This is much easer to integrate.

For the convence of writting a program, let us write the Feynman rules as following:

$$=V_e(j_1,j_2,j_3)$$

$$=\delta_{v_1v_2}(j_1-j_2)_{v_3}+\delta_{v_2v_3}(j_2-j_3)_{v_1}$$

$$+\delta_{v_3v_1}(j_3-j_1)_{v_2} \tag{A29}$$



$$= \frac{G_p(j,u,v)}{i\, j^4} \tag{A30}$$

where

$$G_p(j,u,v)=\delta_{uv}j^2-xj_uj_v \qquad \text{(A31)}$$

and

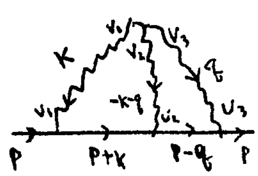
$$x = 1-a \qquad \text{(A32)}$$

a is the gauge parameter.

$$= \frac{iF_p(j)}{j^2+M^2} \qquad \text{(A33)}$$

$$F_p(j)=\not{j} + M \qquad \text{(A34)}$$

Now we calculate the diagram in Fig. 2.2(h).



$$= E_e(p)$$

$$=\int_{11} \frac{(-i)V_p\{\gamma_{v_1}F_p(p+k)\gamma_{v_2}F_p(p-q)\gamma_{v_3}\}}{D(k,q,p)} \qquad \text{(A35)}$$

where

$$V_p=V_e(k,-k-q,q)G_p(k,u_1,v_1)G_p(-k-q,u_2,v_2)G_p(q,u_3,v_3) \qquad \text{(A36)}$$

$$D(k,q,p)=k^4q^4(k+q)^4((p+k)^2+M^2)((p-q)^2+M^2) \qquad (A37)$$

let

$$DM = \int_{''} \frac{V_p\gamma_{u_1}\gamma_{u_2}\cdot(-q)\gamma_{u_3}}{D(k,q,0)} \qquad (A38)$$

then

$$E_e(0)=-2iM\ DM2 \qquad (A39)$$

We need also calculate

$$\gamma_\mu \frac{\partial E_e}{\partial P_\mu}\Big|_{P_\mu=0} = -i\gamma_\mu \int_{''} \frac{V_p\ \gamma_{v_1}}{D(k,q,0)}\Big\{ \gamma_\mu\gamma_{v_2}(-\not{q}) + \not{k}\gamma_{v_2}\gamma_\mu$$

$$+ \not{k}\gamma_{v_2}(-\not{q})\left(-\frac{2k_\mu}{k^2+M^2} + \frac{2q_\mu}{q^2-M^2}\right)$$

$$+ M^2\gamma_{v_2}\left(-\frac{2k_\mu}{k^2+M^2} + \frac{2q_\mu}{k^2+M^2}\right)\Big\}\gamma_{v_3} \qquad (A40)$$

Let

$$DM_1 = -\int_{''} \frac{\gamma_\mu V_p\gamma_{v_1}\gamma_\mu\gamma_{v_2}\not{q}\gamma_{v_3}}{D(k,q,0)(k^2+M^2)} \qquad (A41)$$

$$D_1 = \int_{''} \frac{2\gamma_\mu V_p\gamma_{v_1}\not{k}\gamma_{v_2}\not{q}k_\mu\gamma_{v_3}}{D(k,q,0)(k^2+M^2)} \qquad (A42)$$

$$D_2 = \int_{''} \frac{-2M^2 V_p \gamma_\mu \gamma_{\nu_1} \gamma_{\nu_2} K_\mu \gamma_{\nu_3}}{D(k,q,0)\,(K^2+M^2)} \tag{A43}$$

then

$$A(0) = \frac{-2i}{n}\,(\,DM1+D1+D2\,) \tag{A44}$$

We can write DM_i, where i=1,2 as

$$DMi = \int_{''} \frac{q^2\, f_i(k,q)}{q^2 K^4 q^2 (k+q)^4 (k^2+M^2)(q^2+M^2)}$$

$$= \int_{''} \int_0^1 dx\,dy \frac{f_i(k,q)\, y(1-y)\, \Gamma(4)}{K^4\left[(q+yk)^2 + y(1-y)k^2 + x(1-y)M^2\right]^4} \tag{A45}$$

let

$$q' = q + yk \tag{A46}$$

then

$$DMi = \int_{''} \int_0^1 dx\,dy \frac{y(1-y)\,\Gamma(4)\, f_i(k, q'-ky)}{K^4 (k^2+M^2)\left[q'^2 + y(1-y)k^2 + x(1-y)M^2\right]^4} \tag{A47}$$

Since the integral of odd power of q'is zero,

$$DMi = \int_{''} \int_0^1 dx\,dy\, D' \cdot \tfrac{1}{2}\left[f_i(k, q'-yk) + f_i(k, -q'-yk)\right] \tag{A48}$$

where

$$D' = \frac{\Gamma(4)\, y\,(1-y)}{K^4 (K^2+M^2) \left[q'^2 + y(1-y)K^2 + x(1-y)M^2 \right]^4} \tag{A49}$$

Because

$$\int '' \, (k \cdot q')^2 f(k^2, q'^2) \tag{A50}$$

$$= \frac{1}{n} \int '' \, k^2 q^2 f(k^2, q^2)$$

we can write

$$f_i(k, q' - yk) + f_i(k, -q' - yk)$$

as

$$\sum_S \left(a^i_{0s}\, y^s K^6 + a^i_{1s}\, y^s K^4 q'^2 + a^i_2(y) k^2 q'^4 \right) \tag{A51}$$

The "REDUCE" Program can calculate the coefficients a^i_{0s}, a^i_{1s}, a^i_{2s}. Then the fundamental integrals that we should calculate for the computer are

$$T_{12}(s) = \int '' \int dx\,dy \; y^s K^2 q'^4 D' \tag{A52}$$

$$T_{11}(s) = \int '' \int dx\,dy \; y^s K^4 q'^2 D' \tag{A53}$$

$$T_{10}(s) = \int '' \int_0^1 dx dy \; y^s K^6 \; D' \qquad \text{(A54)}$$

then

$$DM_i = \sum_s \left[a^i_{0s} T_{10}(s) + a^i_{1s} T_{11}(s) + a_{2s}{}^i T_{12}(s) \right] \qquad \text{(A55)}$$

We should integral out $T_{12}(s)$, $T_{11}(s)$, $T_{10}(s)$.

$$T_{12}(s) = \int '' \int dx dy \; \frac{y^s (q'^2)^2 K^2 \Gamma(4) \; y(1-y)}{K^4 (k^2+M^2) \left[q'^2 + y(1-y)K^2 + x(1-y)M^2 \right]^4}$$

$$= \frac{n(n+2)\, i}{4 (4\pi)^{n/2}} \int \frac{d^n k}{(2\pi)^n} \int_0^1 dx dy dt \frac{y^s \, y(1-y) \Gamma(\frac{\epsilon}{2})}{(K^2+tM^2)^2 \left[y(1-y)K^2 + x(1-y)M^2 \right]^{\frac{\epsilon}{2}}}$$

$$= \frac{n(n+2)\, i}{4 (4\pi)^{n/2}} \int \frac{d^n k}{(2\pi)^n} \int_0^1 dx dy dz dt \; \frac{y^s [y(1-y)]^{1-\epsilon/2} (1-z) z^{-1+\frac{\epsilon}{2}} \Gamma(2+\frac{\epsilon}{2})}{\left[K^2 + \frac{xz}{y} M^2 + t(1-z) M^2 \right]^{2+\frac{\epsilon}{2}}}$$

$$= \frac{n(n+2)(-1)\Gamma(\epsilon)}{4 (4\pi)^n} \int_0^1 dx dy dz dt \; \frac{y^{s+1-\frac{\epsilon}{2}} (1-y)^{1-\frac{\epsilon}{2}} (1-z) z^{-1+\frac{\epsilon}{2}} t^{-\epsilon}}{\left[1-z + \frac{zx}{yt} \right]^{\epsilon}} \qquad \text{(A56)}$$

Using the formula (A22),

$$\int_0^1 dz \frac{z^{-1+\frac{\epsilon}{2}} f(z,x,y,\epsilon)}{\left[1 - z u(x,y) \right]^{\epsilon}}$$

$$= \int_0^1 dz \left\{ z^{-1+\frac{\epsilon}{2}} f(z,x,y,\epsilon) - \epsilon z^{-1} f(z,x,y,0) \log\left[1 - zu(x,y) \right] \right\} + O(\epsilon^2)$$

we find

$$T_{12} = \frac{n(n+2)(-1)\Gamma(\epsilon)}{4(4\pi)^n}\int_0^1 dxdydzdt \left\{ y^{s+1-\frac{\epsilon}{2}}(1-y)^{1-\frac{\epsilon}{2}}(1-z)z^{-1+\frac{\epsilon}{2}}t^{-\epsilon} \right.$$

$$\left. - \epsilon\left[\log\left(1-z+\frac{zx}{yt}\right)\right] y^{s+1}(1-y)(1-z)z^{-1} \right\}$$

$$= \frac{n(n+2)(-1)\Gamma(\epsilon)}{4(4\pi)^n}\int_0^1 dxdydzdt \left\{ B\left(s+2-\frac{\epsilon}{2}, 2-\frac{\epsilon}{2}\right)B(1-\epsilon,1)B\left(\frac{\epsilon}{2},2\right) \right.$$

$$\left. + \epsilon\, Tc12(s) \right\} \qquad \text{(A57)}$$

where

$$Tc12(s) = -\int dxdydz\ y^{s+1}(1-y)(1-z)z^{-1}\ln\left(1-z+\frac{zx}{yt}\right) \qquad \text{(A58)}$$

We can calculate Tc12(s) by a numerical integral program. Using the properties of B(a,b), from A(13), A(9), and A(10), we can let REDUCE program expand B(a,b) in the power of ϵ, and keep only the first a fer terms. Then computer can expand T12(s) in the power of ϵ.

$$T_{12}(s) = \frac{t_0^s}{\epsilon^2} + \frac{t_1^s}{\epsilon} + t_2^s \qquad \text{(A59)}$$

In the same way, we can find

$$T_{11}(s) = \int_{11} \int dx\,dy\; y^s K^4 q'^2\, D'$$

$$= \frac{n(-1)\Gamma(\epsilon)}{2(4\pi)^n} \int dx\,dy\,dz\; \frac{y^s[y(1-y)]^{-\frac{\epsilon}{2}} z^{\frac{\epsilon}{2}}}{[1-z+\frac{zx}{y}]^{\epsilon}}$$

$$= \frac{n(-1)\Gamma(\epsilon)}{2(4\pi)^n} \Big\{ B(s+1-\tfrac{\epsilon}{2}, 1-\tfrac{\epsilon}{2})\, B(1+\tfrac{\epsilon}{2}, 1)$$

$$+ \epsilon\, Tc11(s) \Big\} \qquad \text{(A60)}$$

where

$$Tc11(s) = -\int dx\,dy\,dz\; y^s \log(1-z + zx/y) \qquad \text{(A61)}$$

$$T10(s) = \int_{11} \int dx\,dy\; y^s(1-y)\, K^6\, D'$$

$$= \frac{-n\,\Gamma(\epsilon)}{2(4\pi)^n} \int dx\,dy\,dz\; \frac{y^{s-1-\frac{\epsilon}{2}}(1-y)^{-\frac{\epsilon}{2}} z^{1+\frac{\epsilon}{2}}}{(1-z+\frac{zx}{y})^{\epsilon}}$$

$$= \frac{-n\,\Gamma(\epsilon)}{2(4\pi)^n} \int dx\,dy\,dz\; \frac{y^{s-1-\frac{\epsilon}{2}}(1-y)^{-\frac{\epsilon}{2}} z^{1-\frac{\epsilon}{2}} x^{-\epsilon}}{(1-\frac{y}{xz}-\frac{y}{x})^{\epsilon}}$$

$$= -\frac{n\,\Gamma(\epsilon)}{2(4\pi)^n} \Big\{ B(s+\tfrac{\epsilon}{2}, 1-\tfrac{\epsilon}{2})\, B(2-\tfrac{\epsilon}{2}, 0)\, B(1-\epsilon, 0) \qquad \text{(A62)}$$

$$+ \epsilon\, Tc11(s) \Big\}$$

where

$$Tc10(s) = -\int dxdydz\ y^{s-1} z \ln\left[1+ y(1-z)/x\right] \qquad (A63)$$

Now we are ready to compute DM1 and DM2. We give the computer a integral table with the fundamental integrals T10(S), T11(s), T12(s). Computer calculate a^i_{0s}, a^i_{1s}, a^i_{2s}, and expand T10(s), T11(s), T12(s) in the power of ϵ, finally, calculate the result of DMi from equation (A55).

In the same way, we can find

$$D1=\sum_S\left[b_{0s}T0(s)+b_{1s}T1(s)+b_{2s}T2(s)\right] \qquad (A64)$$

$$D2=\sum_S\left[b_{20s}T20(s)+b_{21s}T21(s)+b_{22s}T_{22}(s)\right] \qquad (A65)$$

where b_{0s}, b_{1s}, b_{2s}, b_{3s}, b_{20s}, b_{21s}, b_{22s} can be calculated by computer.

We can also calculate all the $T_i(s)$ and $T_{2i}(s)$ in the same way as do for T12(s).

After all the DM1, DM2, D1, D2 are calculated, computer can give us the final result of the diagram. The other diagrams can be calculated in the same way.

A3. Computer programs for two-loop diagrams calculation

```
                                                   TIME-12.29.40 DATE-84-01-21  *
***********************************************************************************
//LIST IN.DATA(EX35);

COMMENT        FERMION'S TWO POINT DIAG. WITH  3-GLUONS     ( E )
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               ***********************************************
               ***********************************************;

OPERATOR TC1,TC2,D1,D2,D3,D4,D5,DM,DM1,DM2,D1S,D2S,D3S,DMS,DM1S,BE1,
TC11,TC12,TC21,TC22,T11,T12,T1,T2,TT1,TT2,TT11,TT12,TT21,TT22;
VECTOR P,K,Q,K1,K2,K3,K4,Q1,Q2,Q3,Q4,R,R1,R2,R3;
INDEX U,U1,U2,U3,V,V1,V2,V3,W;
VECDIM N;
LET N=4-Z,Z**3=0,N1=(1+Z/4+Z*Z/16)/4;
LET Q2.Q2**2*K2.K2=T2,Q2.Q2*K2.K2**2=T1,K2.K2**3=T0;
LET Q2.K2**2=K2.K2*Q2.Q2*N1;

FOR ALL J,J1,J2,J3,U,V  LET GP(J,U,V)=U.V*J.J-X*J.U*J.V,
 VE(J1,J2,J3)=V1.V2*V3.(J1-J2)+V2.V3*V1.(J2-J3)+V3.V1*V2.(J3-J1),
 FP(J)=G(L,J)+M;
 FOR ALL K,Q LET VP(K,Q)=VE(K,-K-Q,Q)*GP(K,U1,V1)*GP(-K-Q,U2,V2)
 *GP(Q,U3,V3);

FOR ALL K,Q LET D1(K,Q)=VP(K,Q)*G(L,U1,K,U2,-Q,U3,-K)*2/(K.K*Q.Q);
FOR ALL K,Q LET D2(K,Q)=VP(K,Q)*G(L,U1,U2,U3,Q)*2/Q.Q;
FOR ALL K,Q LET DM1(K,Q)=VP(K,Q)*G(L,U1,U,U2,-Q,U3,U)/Q.Q;
FOR ALL K,Q LET DM2(K,Q)=VP(K,Q)*G(L,U1,U2,-Q,U3)/Q.Q;
FOR ALL D4,K,Q LET D5(D4,K,Q)=(D4(K,Q-Y*K)+D4(K,-Q-Y*K))/2;

FOR ALL X1,X2 LET
GA1(X1,X2)=(X1+X2*Z)*GA1(X1-1,X2),GA1(0,X2)=1;
FOR ALL X2 LET GA1(-1,X2)=1/(X2*Z);
FOR ALL X1,X2 LET
GA2(X1,X2)=(1-X2*Z/X1+(X2*Z/X1)**2)*GA2(X1-1,X2)/X1,GA2(0,X2)=1;
FOR ALL X1,X2,X3,X4 LET
BE1(X1,X2,X3,X4)=GA1(X1,X2)*GA1(X3,X4)*GA2(X1+X3+1,X2+X4)*(1+
X2*X4*RE2*Z**2);

FOR ALL Y1 LET Y**Y1*TT0=(BE1(0,-1,0,0)*
BE1(Y1-3,+1/2,0,-1/2)*BE1(1,-1/2,1,0)+TC0(Y1-2)*Z)*N*(N+2)*Z/4;

FOR ALL Y1 LET Y**Y1*TT10=(BE1(0,-1,0,0)*
BE1(Y1-3,+1/2,0,-1/2)*BE1(0,-1/2,1,0)+TC10(Y1-2)*Z)*N*Z/2;

FOR ALL Y1 LET Y**Y1*TT1= -
(BE1(Y1-2,-1/2,0,-1/2)*BE1(0,1/2,1,0)+TC1(Y1-2)*Z)*N*N*Z/4;
FOR ALL Y1 LET Y**Y1*TT2=
(BE1(Y1-1,-1/2,1,-1/2)*BE1(-1,1/2,1,0)+TC2(Y1-2)*Z)*N*(2+N)*Z/4;
D1(K1,Q1)*Y*Y;
FOR ALL K1,Q1 SAVEAS D1S(K1,Q1);

FOR ALL Y1 LET Y**Y1*TT20=
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TC20(Y1-2)*Z*N*Z/2;

FOR ALL Y1 LET Y**Y1*TT21=
TC21(Y1-2)*Z*N*Z/2;
FOR ALL Y1 LET Y**Y1*TT22=(BE1(0,-1,0,0)*Z*
BE1(Y1-1,-1/2,1,-1/2)*BE1(-1,1/2,1,0)+TC22(Y1-2)*Z)*N*(2+N)*Z/4;

FOR ALL Y1 LET Y**Y1*TT11=
(BE1(Y1-2,+1/2,0,-1/2)*BE1(0,1/2,0,0)+TC11(Y1-2)*Z)*N*Z/2;
FOR ALL Y1 LET Y**Y1*TT12=(BE1(0,-1,0,0)*
BE1(Y1-1,-1/2,1,-1/2)*BE1(-1,1/2,1,0)+TC12(Y1-2)*Z)*N*(2+N)*Z/4;
D2(K1,Q1)*Y*Y;
FOR ALL K1,Q1 SAVEAS D2S(K1,Q1);


D5(D2S,K2,Q2);
FOR ALL T0, T1,T2 SAVEAS D2D(T0,T1,T2);

D5(D1S,K2,Q2);
FOR ALL  T0,T1,T2 SAVEAS D1D(T0,T1,T2);

DM1(K1,Q1)*Y*Y;
FOR ALL K1,Q1 SAVEAS DM1S(K1,Q1);

DM2(K1,Q1)*Y*Y;
FOR ALL K1,Q1 SAVEAS DM2S(K1,Q1);
D5(DM2S,K2,Q2);
FOR ALL  T1,T0,T2 SAVEAS DM2D(T0,T1,T2);

D5(DM1S,K2,Q2);
FOR ALL T0,T1,T2 SAVEAS DM1D(T0,T1,T2);


FOR ALL T0,T10,T20
LET DPI(T0,T10,T20)=2*(D1D(T0,0,0)+D2D(T20,0,0)+DM1D(T10,0,0)
)*N1,
 DMI(T10)=2*DM2D(T10,0,0);

DMI(T10)/(1-Y);
FOR ALL T10 SAVEAS DMII(T10);

DPI(T0,T10,T20)/(1-Y);
FOR ALL T0,T10,T20 SAVEAS DPII(T0,T10,T20);

LET DPX0=DPII(TT0,TT10,TT20),
    DMX0=DMII(TT10);

LET DPX=2*(D1D(0,TT1,TT2)+D2D(0,TT21,TT22)+DM1D(0,TT11,TT12)
)*N1;
LET DMX=2*DM2D(0,TT11,TT12);

 DPX+DPX0;
FOR ALL X,Z SAVEAS PX(X,Z);
DMX+DMX0;
FOR ALL X,Z SAVEAS MX(X,Z);

 LET TC10(0) =  -0.31843001;
 LET TC10(1) =  -0.32194602;
 LET TC10(2) =  -0.19720495;
 LET TC10(3) =  -0.14173663;
 LET TC10(4) =  -0.11053032;
 LET TC10(5) =  -0.09055650;
 LET TC10(6) =  -0.07668513;
 LET TC11(0) =  -0.14389080;
 LET TC11(1) =   0.07004255;
 LET TC11(2) =   0.07336587;
```

```
 LET TC11(3) =   0.06424826;
 LET TC11(4) =   0.05562361;
 LET TC11(5) =   0.04862976;
 LET TC11(6) =   0.04304707;
 LET TC12(0) =  -0.18724072;
 LET TC12(1) =  -0.06971884;
 LET TC12(2) =  -0.03527203;
 LET TC12(3) =  -0.02101165;
 LET TC12(4) =  -0.01385062;
 LET TC12(5) =  -0.00977994;
 LET TC12(6) =  -0.00725698;
 LET TC0(0) =  -0.43920797;
 LET TC0(1) =  -0.16649228;
 LET TC0(2) =  -0.10075688;
 LET TC0(3) =  -0.07200074;
 LET TC0(4) =  -0.05596148;
 LET TC0(5) =  -0.04574972;
 LET TC0(6) =  -0.03868320;
 LET TC1(0) =  -0.07194537;
 LET TC1(1) =   0.01751069;
 LET TC1(2) =   0.02083335;
 LET TC1(3) =   0.01866008;
 LET TC1(4) =   0.01628531;
 LET TC1(5) =   0.01429142;
 LET TC1(6) =   0.01267697;
 LET TC2(0) =   0.50000000;
 LET TC2(1) =   0.23502136;
 LET TC2(2) =   0.19836599;
 LET TC2(3) =   0.15192825;
 LET TC2(4) =   0.12305301;
 LET TC2(5) =   0.10338026;
 LET TC2(6) =   0.08912176;
 LET TC20(0) =  -0.01751392;
 LET TC20(1) =   0.00440907;
 LET TC20(2) =   0.00605936;
 LET TC20(3) =   0.00530150;
 LET TC20(4) =   0.00435865;
 LET TC20(5) =   0.00356509;
 LET TC20(6) =   0.00294135;
 LET TC21(0) =   0.49918497;
 LET TC21(1) =   0.17752987;
 LET TC21(2) =   0.10506535;
 LET TC21(3) =   0.07426530;
 LET TC21(4) =   0.05735393;
 LET TC21(5) =   0.04669249;
 LET TC21(6) =   0.03936438;
 LET TC22(0) =  -0.58001012;
 LET TC22(1) =  -0.17578429;
 LET TC22(2) =  -0.08362466;
 LET TC22(3) =  -0.04866054;
 LET TC22(4) =  -0.03177320;
 LET TC22(5) =  -0.02235669;
 LET TC22(6) =  -0.01657797;
//LIST OUT.DATA(EX35);

MM(1-A,Z);
                  2  2                   2  2            2              2
( - 2250000*A *Z *RE2 - 8224427*A *Z  - 6000000*A *Z - 9000000*A  -

                   2                        2
 9000000*A*Z *RE2 - 11376674*A*Z  - 15000000*A*Z - 36000000*A - 6750000*

  2                       2
 Z *RE2 + 37062525*Z  - 45000000*Z - 27000000)/1500000
```

```
PP(1-A,Z);
             2  2                   2  2              2               2
(4500000*A *Z *RE2 + 21141470*A *Z  + 12000000*A *Z + 18000000*A  +

              2                    2
 4500000*A*Z *RE2 - 23131987*A*Z  + 16500000*A*Z + 18000000*A - 73591977

   2
 *Z  - 13500000*Z)/3000000

//LIST IN.DATA(EX24);

COMMENT     FERMION'S TWO POINT DIAG. WITH OVERLAPED GLUONS  ( C )
```

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            ************************************************
            ************************************************

;
OPERATOR TC1,TC2,D1,D2,D3,D4,D5,DM,DM1,DM2,D1S,D2S,D3S,DMS,DM1S,BE1,
TC11,TC12,TC21,TC22,T11,T12,T1,T2,TT1,TT2,TT11,TT12,TT21,TT22,D1D,
TC0,TC00,TC01,TC30,TC31,TC32,TC40,TC41,TC10,TC20,TC33;
VECTOR P,K,Q,K1,K2,K3,K4,Q1,Q2,Q3,Q4,R,R1,R2,R3;
INDEX U,U1,U2,U3,V,V1,V2,V3,W,U4,V4,U5,V5;
VECDIM N;
LET N=4-Z,Z**3=0,N1=(1+Z/4+Z*Z/16)/4;
LET Q2.Q2**2*K2.K2=T2,Q2.Q2*K2.K2**2=T1,K2.K2**3=T0;
LET Q2.Q2**2*M**2=TM2,Q2.Q2*K2.K2*M**2=TM1,K2.K2**2*M**2=TM0;
LET Q2.K2**2=K2.K2*Q2.Q2*N1;

FOR ALL J,J1,J2,J3,U,V,V1  LET GT(J,V1,U,V)=2*U.V*J.V1-X*(V1.U*J.V+
V1.V*J.U);

FOR ALL J,J1,J2,J3,U,V  LET GP(J,U,V)=U.V*J.J-X*J.U*J.V,
 VE(J1,J2,J3)=V1.V2*V3.(J1-J2)+V2.V3*V1.(J2-J3)+V3.V1*V2.(J3-J1),
 FP(J)=G(L,J)+M;
 FOR ALL K,Q LET VP(K,Q)=VE(K,-K-Q,Q)*GP(K,U1,V1)*GP(-K-Q,U2,V2)
 *GP(Q,U3,V3);

FOR ALL K,Q LET DM2(K,Q)=G(L,U1)*FP(K)*G(L,U2)*FP(K+Q)*
G(L,U3)*FP(Q)*G(L,U4)*GP(K,U1,U3)*GP(-Q,U2,U4);

FOR ALL K,Q LET DM1(K,Q)=G(L,U,U1,U,U2)*FP(K+Q)*
G(L,U3)*FP(Q)*G(L,U4)*GP(K,U1,U3)*GP(-Q,U2,U4);

FOR ALL K,Q LET D2(K,Q)=G(L,U1)*FP(K)*G(L,U2)*FP(K+Q)*
G(L,U3)*FP(Q)*G(L,U4,U)*GT(-Q,U,U2,U4)*GP(K,U1,U3);


FOR ALL K,Q LET D1P(K,Q)=G(L,U1)*FP(K)*G(L,U2)*FP(K+Q)*
G(L,U3)*FP(Q)*G(L,U4,U)*GP(K,U1,U3)*GP(-Q,U2,U4)*4*Q.U/Q.Q;


FOR ALL K,Q LET D1(K,Q)=-G(L,U1)*FP(K)*G(L,U2)*FP(K+Q)*
G(L,U3)*FP(Q)*G(L,U4,U)*GP(K,U1,U3)*GP(-Q,U2,U4)*2*K.U/K.K;

FOR ALL D4,K,Q LET D5(D4,K,Q)=(D4(K,Q-Y*K)+D4(K,-Q-Y*K))/2;


FOR ALL X1,X2 LET
GA1(X1,X2)=(X1+X2*Z)*GA1(X1-1,X2),GA1(0,X2)=1;
FOR ALL X2 LET GA1(-1,X2)=1/(X2*Z);
FOR ALL X1,X2 LET
GA2(X1,X2)=(1-X2*Z/X1+(X2*Z/X1)**2)*GA2(X1-1,X2)/X1,GA2(0,X2)=1;
```

```
FOR ALL X1,X2,X3,X4 LET
BE1(X1,X2,X3,X4)=GA1(X1,X2)*GA1(X3,X4)*GA2(X1+X3+1,X2+X4)*(1+
X2*X4*RE2*Z**2);

D1(K1,Q1)*Y*Y;
FOR ALL K1,Q1 SAVEAS D1S(K1,Q1);

FOR ALL Y1 LET Y**Y1*TT41=
-TC41(Y1-2)*Z*N*Z/2;
FOR ALL Y1 LET Y**Y1*TT40=
-(TC40(Y1-2)*Z)*Z;

FOR ALL Y1 LET Y**Y1*TT30=(BE1(0,-1,0,0)*
BE1(Y1-3,+1/2,0,+1/2)*BE1(0,-1/2,0,0)+TC30(Y1-2)*Z)*Z;
FOR ALL Y1 LET Y**Y1*TT31=(BE1(0,-1,0,0)*
BE1(Y1-2,-1/2,1,-1/2)*BE1(-1,1/2,1,0)+TC31(Y1-2)*Z)*N*Z/2;

FOR ALL Y1 LET Y**Y1*TT21=
-TC21(Y1-2)*Z*N*Z/2;
FOR ALL Y1 LET Y**Y1*TT20=
-(TC20(Y1-2)*Z)*Z;

FOR ALL Y1 LET Y**Y1*TT01=
-TC01(Y1-2)*Z*N*Z/2;
FOR ALL Y1 LET Y**Y1*TT00=
-(TC00(Y1-2)*Z)*Z*N/2;

FOR ALL Y1 LET Y**Y1*TT0=(BE1(0,-1,1,0)*
BE1(Y1-4,+1/2,0,+1/2)*BE1(1,-1/2,1,0)+TC0(Y1-2)*Z)*N*(N+2)*Z/4;

FOR ALL Y1 LET Y**Y1*TT1=(BE1(0,-1,1,0)*
BE1(Y1-3,+1/2,1,+1/2)*BE1(0,-1/2,1,0)+TC1(Y1-2)*Z)*N*N*Z/4;
FOR ALL Y1 LET Y**Y1*TT2=(BE1(0,0,1,0)*
BE1(Y1-2,-1/2,2,-1/2)*BE1(-1,1/2,1,0)+TC2(Y1-2)*Z)*N*(2+N)*Z/4;

FOR ALL Y1 LET Y**Y1*TT10=(BE1(0,-1,1,0)*
BE1(Y1-4,+1/2,0,+1/2)*BE1(1,-1/2,0,0)+TC11(Y1-2)*Z)*N*Z/2;

FOR ALL Y1 LET Y**Y1*TT11=(BE1(0,-1,1,0)*
BE1(Y1-3,+1/2,1,+1/2)*BE1(0,-1/2,0,0)+TC11(Y1-2)*Z)*N*Z/2;
FOR ALL Y1 LET Y**Y1*TT12=(BE1(0,-1,0,0)*
BE1(Y1-2,-1/2,2,-1/2)*BE1(-1,1/2,1,0)/2+TC12(Y1-2)*Z)*N*(2+N)*Z/4;
D2(K1,Q1)*Y*Y;
FOR ALL K1,Q1 SAVEAS D2S(K1,Q1);

D5(D2M,K2,Q2);
D5(D1M,K2,Q2);
D5(D2,K2,Q2);
D5(D1,K2,Q2);

D5(D2S,K2,Q2);
FOR ALL  T1,T2,T0,TM0,TM1 SAVEAS  D2D(T0,T1,T2,TM0,TM1);

D5(D1S,K2,Q2);
FOR ALL  T1,T2,T0,TM0,TM1 SAVEAS  D1D(T0,T1,T2,TM0,TM1);

DM1(K1,Q1)*Y*Y;
FOR ALL K1,Q1 SAVEAS DM1S(K1,Q1);

DM2(K1,Q1)*Y*Y;
FOR ALL K1,Q1 SAVEAS DM2S(K1,Q1);
D5(DM2S,K2,Q2);
FOR ALL  T1,T2,T0,TM0,TM1 SAVEAS  DM2D(T0,T1,T2,TM0,TM1);
```

```
D5(DM1S,K2,Q2);
FOR ALL  T1,T2,T0,TM0,TM1 SAVEAS  DM1D(T0,T1,T2,TM0,TM1);

D1P(K1,Q1)*Y*Y;
FOR ALL K1,Q1 SAVEAS D1PS(K1,Q1);

D5(D1PS,K2,Q2);
FOR ALL  T1,T2,T0,TM0,TM1 SAVEAS  D1PD(T0,T1,T2,TM0,TM1);

(D1D(TT0,TT1,TT2,TT00,TT01)+D1PD(TT10,TT11,TT12,TT20,TT21)
+DM1D(TT10,TT11,TT12,TT20,TT21)+D2D(TT10,TT11,TT12,TT20,TT21))*N1;
FOR ALL X,Z SAVEAS PP(X,Z);
DM2D(TT10,TT11,TT12,TT20,TT21)      ;
FOR ALL X,Z SAVEAS MM(X,Z);
 LET TC10(0) = -79.7705383 ;
 LET TC10(1) =  -2.42692947;
 LET TC10(2) =  -0.73636150;
 LET TC10(3) =  -0.40860325;
 LET TC10(4) =  -0.27805996;
 LET TC10(5) =  -0.20921624;
 LET TC10(6) =  -0.16705358;
 LET TC11(0) =  -1.69056797;
 LET TC11(1) =  -0.32775807;
 LET TC11(2) =  -0.13054323;
 LET TC11(3) =  -0.06884366;
 LET TC11(4) =  -0.04216264;
 LET TC11(5) =  -0.02833970;
 LET TC11(6) =  -0.02029697;
 LET TC12(0) =  -0.74174410;
 LET TC12(1) =  -0.22501737;
 LET TC12(2) =  -0.12200063;
 LET TC12(3) =  -0.08009076;
 LET TC12(4) =  -0.05790978;
 LET TC12(5) =  -0.04441978;
 LET TC12(6) =  -0.03546899;
 LET TC0(0) =  -9.33107872;
 LET TC0(1) =  -0.34846568;
 LET TC0(2) =  -0.11181736;
 LET TC0(3) =  -0.06294769;
 LET TC0(4) =  -0.04315818;
 LET TC0(5) =  -0.03263984;
 LET TC0(6) =  -0.02616565;
 LET TC1(0) =  -1.11437798;
 LET TC1(1) =  -0.20209319;
 LET TC1(2) =  -0.07827842;
 LET TC1(3) =  -0.04059086;
 LET TC1(4) =  -0.02456112;
 LET TC1(5) =  -0.01635404;
 LET TC1(6) =  -0.01162270;
 LET TC2(0) =  -0.20971990;
 LET TC2(1) =  -0.03438689;
 LET TC2(2) =  -0.01412360;
 LET TC2(3) =  -0.00780835;
 LET TC2(4) =  -0.00496517;
 LET TC2(5) =  -0.00342736;
 LET TC2(6) =  -0.00249896;
 LET TC20(0) =  16.8261414 ;
 LET TC20(1) =   0.43433994;
 LET TC20(2) =   0.09662318;
 LET TC20(3) =   0.04223532;
 LET TC20(4) =   0.02388619;
 LET TC20(5) =   0.01545431;
 LET TC20(6) =   0.01086011;
 LET TC21(0) =   0.87590826;
 LET TC21(1) =   0.13081652;
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```
LET TC21(2) =   0.04436287;
LET TC21(3) =   0.02076996;
LET TC21(4) =   0.01155382;
LET TC21(5) =   0.00715778;
LET TC21(6) =   0.00477395;
LET TC00(0) =   7.24172878;
LET TC00(1) =   0.17141664;
LET TC00(2) =   0.03766574;
LET TC00(3) =   0.01672191;
LET TC00(4) =   0.00961819;
LET TC00(5) =   0.00631602;
LET TC00(6) =   0.00449501;
LET TC01(0) =   0.54160661;
LET TC01(1) =   0.07669175;
LET TC01(2) =   0.02610146;
LET TC01(3) =   0.01238195;
LET TC01(4) =   0.00698594;
LET TC01(5) =   0.00438595;
LET TC01(6) =   0.00296098;

//LIST OUT.DATA(EX24);


MM1(1-A,Z);
           2    2               2                  2                  2
(63637091*A *M*Z  + 27000000*A *M*Z + 36000000*A *M - 56703954*A*M*Z  +

                                               2
18000000*A*M*Z + 103000000*A*M - 226648575*M*Z  - 81000000*M*Z)/9000000

PP1(1-A,Z);
              2  2              2                 2                 2
( - 5427101*A *Z  - 22500000*A *Z - 36000000*A  + 18454203*A*Z  -

            2
148995630*Z  + 13500000*Z)/9000000

//LIST IN.DATA(EX46);


COMMENT    FERMION'S TWO POINT DIAG. WITH TWO NONOVERLAPED GLUONS (C)
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*************************************************
*************************************************;


OPERATOR TC1,TC2,D1,D2,D3,D4,D5,DM,DM1,DM2,D1S,D2S,D3S,DMS,DM1S,BE1,
TC11,TC12,TC21,TC22,T11,T12,T1,T2,TT1,TT2,TT11,TT12,TT21,TT22,D1D,
TC0,TC00,TC01,TC30,TC31,TC32,TC40,TC41,TC10,TC20,TC33;
VECTOR P,K,Q,K1,K2,K3,K4,Q1,Q2,Q3,Q4,R,R1,R2,R3;
INDEX U,U1,U2,U3,V,V1,V2,V3,W,U4,V4,U5,V5;
VECDIM N;
LET N=4-Z,Z**3=0,N1=(1+Z/4+Z*Z/16)/4;
LET Q2.Q2**2*K2.K2=T2,Q2.Q2*K2.K2**2=T1,K2.K2**3=T0;
LET Q2.Q2**2*M**2=TM2,Q2.Q2*K2.K2*M**2=TM1,K2.K2**2*M**2=TM0;
LET Q2.K2**2=K2.K2*Q2.Q2*N1;

FOR ALL J,J1,J2,J3,U,V,V1  LET GT(J,V1,U,V)=2*U.V*J.V1-X*(V1.U*J.V+
V1.V*J.U);

FOR ALL J,J1,J2,J3,U,V  LET GP(J,U,V)=U.V*J.J-X*J.U*J.V,
 VE(J1,J2,J3)=V1.V2*V3.(J1-J2)+V2.V3*V1.(J2-J3)+V3.V1*V2.(J3-J1),
 FP(J)=G(L,J)+M;
 FOR ALL K,Q LET VP(K,Q)=VE(K,-K-Q,Q)*GP(K,U1,V1)*GP(-K-Q,U2,V2)
 *GP(Q,U3,V3);
```

```
FOR ALL K,Q LET DM2(K,Q)=G(L,U1)*FP(K)*G(L,U2)*FP(K+Q)*
G(L,U3)*FP(K)*G(L,U4)*GP(K,U1,U4)*GP(-Q,U2,U3);

FOR ALL K,Q LET D2(K,Q)=G(L,U1)*FP(K)*G(L,U2)*FP(K+Q)*
G(L,U3)*FP(K)*G(L,U4,U)*GT(-K,U,U1,U4)*GP(-Q,U2,U3);

FOR ALL K,Q LET D1P(K,Q)=G(L,U1)*FP(K)*G(L,U2)*FP(K+Q)*
G(L,U3)*FP(K)*G(L,U4,U)*GP(K,U1,U4)*GP(-Q,U2,U3)*4*K.U/K.K;

FOR ALL D4,K,Q LET D5(D4,K,Q)=(D4(K,Q-Y*K)+D4(K,-Q-Y*K))/2;

FOR ALL X1,X2 LET
GA1(X1,X2)=(X1+X2*Z)*GA1(X1-1,X2),GA1(0,X2)=1;
FOR ALL X2 LET GA1(-1,X2)=1/(X2*Z);
FOR ALL X1,X2 LET
GA2(X1,X2)=(1-X2*Z/X1+(X2*Z/X1)**2)*GA2(X1-1,X2)/X1,GA2(0,X2)=1;
FOR ALL X1,X2,X3,X4 LET
BE1(X1,X2,X3,X4)=GA1(X1,X2)*GA1(X3,X4)*GA2(X1+X3+1,X2+X4)*(1+
X2*X4*RE2*Z**2);

FOR ALL Y1 LET Y**Y1*TT21=
-(BE1(Y1-2,-1/2,1,-1/2)*BE1(-1,1/2,1,0)*BE1(0,-1,0,0)+
TC21(Y1-2))*Z*N*Z/2;
FOR ALL Y1 LET Y**Y1*TT20=
-(BE1(Y1-3,-1/2,0,1/2)*BE1(0,1/2,1,0)+
TC20(Y1-2))*Z*Z;


FOR ALL Y1 LET Y**Y1*TT11=(
BE1(Y1-2,-1/2,1,-1/2)*BE1(-1,1/2,1,0)+TC11(Y1-2)*Z)*N*Z/2;

FOR ALL Y1 LET Y**Y1*TT10=(
BE1(Y1-3,-1/2,0,+1/2)*BE1(0,1/2,1,0)+TC10(Y1-2)*Z)*N*Z/2;
D2(K1,Q1)*Y*Y;
FOR ALL K1,Q1 SAVEAS D2S(K1,Q1);

D5(D2S,K2,Q2);
FOR ALL  T1,T2,T0,TM0,TM1 SAVEAS  D2D(T0,T1,T2,TM0,TM1);

DM2(K1,Q1)*Y*Y;
FOR ALL K1,Q1 SAVEAS DM2S(K1,Q1);
D5(DM2S,K2,Q2);
FOR ALL  T1,T2,T0,TM0,TM1 SAVEAS  DM2D(T0,T1,T2,TM0,TM1);

D1P(K1,Q1)*Y*Y;
FOR ALL K1,Q1 SAVEAS D1PS(K1,Q1);

D5(D1PS,K2,Q2);
FOR ALL  T1,T2,T0,TM0,TM1 SAVEAS  D1PD(T0,T1,T2,TM0,TM1);

(D1PD(TT10,TT11,TT12,TT20,TT21)
+D2D(TT10,TT11,TT12,TT20,TT21))*N1;
FOR ALL X,Z SAVEAS PP(X,Z);
```

```
DM2D(TT10,TT11,TT12,TT20,TT21)      ;
FOR ALL X,Z SAVEAS MM(X,Z);
 LET TC10(0) =   Z2/2-3/8  ;
 LET TC10(1) =   1/4       ;
 LET TC10(2) =   1-Z2/2    ;
 LET TC10(3) =   5/36      ;
 LET TC10(4) =   0.11458218;
 LET TC10(5) =   0.09777659;
 LET TC10(6) =   0.08541548;
 LET TC11(0) =  -1+Z2/2    ;
 LET TC11(1) =  -Z2/6+1/6  ;
 LET TC11(2) =  -0.07458121;
 LET TC11(3) =  -0.05586131;
 LET TC11(4) =  -0.04395507;
 LET TC11(5) =  -0.03580229;
 LET TC11(6) =  -0.02991812;
 LET TC12(0) =  -0.74174410;
 LET TC12(1) =  -0.22501737;
 LET TC12(2) =  -0.12200063;
 LET TC12(3) =  -0.08009076;
 LET TC12(4) =  -0.05790978;
 LET TC12(5) =  -0.04441978;
 LET TC12(6) =  -0.03546899;
 LET TC20(0) =  -1.33099556;
 LET TC20(1) =  -0.06614083;
 LET TC20(2) =  -21/12+Z2  ;
 LET TC20(3) =  -1/12      ;
 LET TC20(4) =   0.06164534;
 LET TC20(5) =   0.05076971;
 LET TC20(6) =   0.03916211;
 LET TC21(0) =  -Z2+1    ;
 LET TC21(1) =  -1/4       ;
 LET TC21(2) =  -0.13853419;
 LET TC21(3) =  -0.09001374;
 LET TC21(4) =  -0.06395632;
 LET TC21(5) =  -0.04815853;
 LET TC21(6) =  -0.03777563;
 LET TC35(0) =   0.42460525;
 LET TC35(1) =   0.42460525;
 LET TC35(2) =   0.42460525;
 LET TC35(3) =   0.42460525;
 LET TC35(4) =   0.42460525;
 LET TC35(5) =   0.42460525;
 LET TC35(6) =   0.42460525;
 LET TC36(0) =  -0.65609080;
 LET TC36(1) =  -0.65609080;
 LET TC36(2) =  -0.65609080;
 LET TC36(3) =  -0.65609080;
 LET TC36(4) =  -0.65609080;
 LET TC36(5) =  -0.65609080;
 LET TC36(6) =  -0.65609080;
 COMMENT  FUNC
 COMMENT  FUNC
      FUNCTION FUNC(MT,NS1,X,Y,Z)
      GOTO  ( 10,11,12,13,14,15,16,17,18,19),MT
   10 FUNC =-LOG(1-Y*(1-Z))*(1-Z)*Y**(NS1-1)
      RETURN
   11 FUNC =-LOG((1-Z)+Z/(1-Y))*(1-Y)*(1-Z)*Y**NS1/Z
      RETURN
   13 FUNC =(1-Z)*Y**(NS1-1)*(1-Y)/(1-Y*Z-Y)
      RETURN
   14 FUNC =-Y**NS1*(1-Z)/(X*(1-Z)+Z/(1-Y)) ;
//LIST OUT.DATA(EX46);

MM(1-A,Z);
       2    2        2    2       2          2              2
```

```
( - 2*A *M*Z *Z2 - A *M*Z  - 2*A *M*Z - 4*A *M - 14*A*M*Z  - 12*A*M*Z +

         2                 2
 18*M*Z *Z2 - 29*M*Z  - 42*M*Z + 36*M)/2

PP(1-A,Z);
     2  2           2  2          2           2           2             2
(8*A *Z *Z2 + 3*A *Z  + 12*A *Z + 16*A  - 48*A*Z *Z2 + 88*A*Z  + 96*A*Z)

/9

//LIST IN.DATA(EX21);
COMMENT    FERMION'S TWO POINT DIAG. WITH FOUR  GLUONS (B)
```

```
                   ************************************************
                   ************************************************;

OPERATOR TC1,TC2,D1,D2,D3,D4,D5,DM,DM1,DM2,D1S,D2S,D3S,DMS,DM1S,BE1,
TC11,TC12,TC21,TC22,T11,T12,T1,T2,TT1,TT2,TT11,TT12,TT21,TT22,D1D,
TC0,TC00,TC01,TC30,TC31,TC32,TC40,TC41,TC10,TC20,TC33;
VECTOR P,K,Q,K1,K2,K3,K4,Q1,Q2,Q3,Q4,R,R1,R2,R3;
INDEX U,U1,U2,U3,V,V1,V2,V3,W,U4,V4,U5,V5;
VECDIM N;
LET N=4-Z,Z**3=0,N1=(1+Z/4+Z*Z/16)/4;
LET Q2.Q2**2*K2.K2=T2,Q2.Q2*K2.K2**2=T1,K2.K2**3=T0;
LET Q2.Q2**2*M**2=TM2,Q2.Q2*K2.K2*M**2=TM1,K2.K2**2*M**2=TM0;
LET Q2.K2**2=K2.K2*Q2.Q2*N1;


FOR ALL X1,X2 LET
GA1(X1,X2)=(X1+X2*Z)*GA1(X1-1,X2),GA1(0,X2)=1;
FOR ALL X2 LET GA1(-1,X2)=1/(X2*Z);
FOR ALL X1,X2 LET
GA2(X1,X2)=(1-X2*Z/X1+(X2*Z/X1)**2)*GA2(X1-1,X2)/X1,GA2(0,X2)=1;
FOR ALL X1,X2,X3,X4 LET
BE1(X1,X2,X3,X4)=GA1(X1,X2)*GA1(X3,X4)*GA2(X1+X3+1,X2+X4)*(1+
X2*X4*RE2*Z**2);

LET CN=(2*BE1(1,-1/2,1,-1/2)-2*BE1(0,-1/2,0,-1/2)
+X*(2*Z*BE1(0,-1/2,1,-1/2)-1*BE1(0,-1/2,1,-1/2))
-X*X*Z*BE1(0,-1/2,0,-1/2)/8)*2;
CN*(N-1)*(1+Z/2+Z*Z/4);
FOR ALL X,Z SAVEAS MM(X,Z);
CN*(1+Z/2+Z*Z/4)*(2-N-(1-N)*(1-Z)*(1+Z/4+Z*Z/16)/2-(2-N)*(1
+Z/4+Z*Z/16)/4);
FOR ALL X,Z SAVEAS PP(X,Z);

//LIST OUT.DATA(EX21)


MM(1-A,Z);
       2  2        2           2                          2
( - 7*A *Z  - 6*A *Z - 12*A*Z  - 8*A*Z + 24*A - 85*Z  - 82*Z - 104)/8

PP(1-A,Z);
    2  2       2             2
(3*A *Z  + A*Z  - 12*A*Z + 54*Z  + 52*Z)/16

//LIST IN.DATA(EX21)
COMMENT    FERMION'S TWO POINT DIAG. WITH FOUR  GLUONS (B)
```

```
        ******************************************************
        ******************************************************;

OPERATOR TC1,TC2,D1,D2,D3,D4,D5,DM,DM1,DM2,D1S,D2S,D3S,DMS,DM1S,BE1,
TC11,TC12,TC21,TC22,T11,T12,T1,T2,TT1,TT2,TT11,TT12,TT21,TT22,D1D,
TC0,TC00,TC01,TC30,TC31,TC32,TC40,TC41,TC10,TC20,TC33;
VECTOR P,K,Q,K1,K2,K3,K4,Q1,Q2,Q3,Q4,R,R1,R2,R3;
INDEX U,U1,U2,U3,V,V1,V2,V3,W,U4,V4,U5,V5;
VECDIM N;
LET N=4-Z,Z**3=0,N1=(1+Z/4+Z*Z/16)/4;
LET Q2.Q2**2*K2.K2=T2,Q2.Q2*K2.K2**2=T1,K2.K2**3=T0;
LET Q2.Q2**2*M**2=TM2,Q2.Q2*K2.K2*M**2=TM1,K2.K2**2*M**2=TM0;
LET Q2.K2**2=K2.K2*Q2.Q2*N1;


FOR ALL X1,X2 LET
GA1(X1,X2)=(X1+X2*Z)*GA1(X1-1,X2),GA1(0,X2)=1;
FOR ALL X2 LET GA1(-1,X2)=1/(X2*Z);
FOR ALL X1,X2 LET
GA2(X1,X2)=(1-X2*Z/X1+(X2*Z/X1)**2)*GA2(X1-1,X2)/X1,GA2(0,X2)=1;
FOR ALL X1,X2,X3,X4 LET
BE1(X1,X2,X3,X4)=GA1(X1,X2)*GA1(X3,X4)*GA2(X1+X3+1,X2+X4)*(1+
X2*X4*RE2*Z**2);

LET CN=(2*BE1(1,-1/2,1,-1/2)-2*BE1(0,-1/2,0,-1/2)
+X*(2*Z*BE1(0,-1/2,1,-1/2)-1*BE1(0,-1/2,1,-1/2))
-X*X*Z*BE1(0,-1/2,0,-1/2)/8)*2;
CN*(N-1)*(1+Z/2+Z*Z/4);
FOR ALL X,Z SAVEAS MM(X,Z);
CN*(1+Z/2+Z*Z/4)*(2-N-(1-N)*(1-Z)*(1+Z/4+Z*Z/16)/2-(2-N)*(1
+Z/4+Z*Z/16)/4);
FOR ALL X,Z SAVEAS PP(X,Z);

//
//LIST IN.DATA(EX22)
COMMENT    FERMION'S TWO POINT DIAG. WITH FERMION LOOP (A)
```

```
        ******************************************************
        ******************************************************;

OPERATOR TC1,TC2,D1,D2,D3,D4,D5,DM,DM1,DM2,D1S,D2S,D3S,DMS,DM1S,BE1,
TC11,TC12,TC21,TC22,T11,T12,T1,T2,TT1,TT2,TT11,TT12,TT21,TT22,D1D,
TC0,TC00,TC01,TC30,TC31,TC32,TC40,TC41,TC10,TC20,TC33;
VECTOR P,K,Q,K1,K2,K3,K4,Q1,Q2,Q3,Q4,R,R1,R2,R3;
INDEX U,U1,U2,U3,V,V1,V2,V3,W,U4,V4,U5,V5;
VECDIM N;
LET N=4-Z,Z**3=0,N1=(1+Z/4+Z*Z/16)/4;
LET Q2.Q2**2*K2.K2=T2,Q2.Q2*K2.K2**2=T1,K2.K2**3=T0;
LET Q2.Q2**2*M**2=TM2,Q2.Q2*K2.K2*M**2=TM1,K2.K2**2*M**2=TM0;
LET Q2.K2**2=K2.K2*Q2.Q2*N1;


FOR ALL X1,X2 LET
GA1(X1,X2)=(X1+X2*Z)*GA1(X1-1,X2),GA1(0,X2)=1;
FOR ALL X2 LET GA1(-1,X2)=1/(X2*Z);
FOR ALL X1,X2 LET
GA2(X1,X2)=(1-X2*Z/X1+(X2*Z/X1)**2)*GA2(X1-1,X2)/X1,GA2(0,X2)=1;
FOR ALL X1,X2,X3,X4 LET
BE1(X1,X2,X3,X4)=GA1(X1,X2)*GA1(X3,X4)*GA2(X1+X3+1,X2+X4)*(1+
X2*X4*RE2*Z**2);
```

```
 LET TC10(0) =  -0.58907169;
 LET TC10(1) =  -0.19288909;
 LET TC10(2) =  -0.11092412;
 LET TC10(3) =  -0.07723749;
 LET TC10(4) =  -0.05909539;
 LET TC10(5) =  -0.04780448;
 LET TC10(6) =  -0.04011536;
 LET TC11(0) =  -0.45887107;
 LET TC11(1) =  -0.14451098;
 LET TC11(2) =  -0.08210766;
 LET TC11(3) =  -0.05686041;
 LET TC11(4) =  -0.04337161;
 LET TC11(5) =  -0.03501690;
 LET TC11(6) =  -0.02934541;

LET CN=8*BE1(1,-1/2,1,-1/2)*BE1(-1,1/2,1,0);

LET CN1=8*BE1(1,-1/2,1,-1/2)*BE1(-1,1/2,2,0);
CN*Z*BE1(0,-1,0,0)*(N-1)+24*TC10(0)*Z*Z;
FOR ALL X,Z SAVEAS MM(X,Z);
Z*(CN*BE1(0,-1,0,0)*(2-N)*(1-N1)-CN1*BE1(1,-1,0,0)*(1-N))-12*TC10(0)
*Z*Z+24*TC11(1)*Z*Z;
FOR ALL X,Z SAVEAS PP(X,Z);

//LIST OUT.DATA(EX22);

MM(X,Z);
          2                   2
(250000*Z *RE2 - 267213*Z  + 1000000*Z + 1000000)/125000

PP(X,Z);
           2
(2134273*Z  - 500000*Z)/250000

//LIST C2.FORT(C2)
          IMPLICIT REAL*8(A-H,O-Z)
          EXTERNAL FUNC
          A=0.
          B=1.
          C=0.
          D=1.
          M=6
          N=6
     101   FORMAT( ' COMMENT  FUNC')
           DO 23 IT=10,11
           MT=IT
           MW=MT
           DO 23 NS1=0,6
           MT1=MT-9
           CALL ¥DQDGM(FUNC,MT1,NS1,A,B,M,C,D,N,S,IER)
            WRITE(6,100) MW,NS1,S
       100 FORMAT(  ' LET TC',I2,'(',I1,') = ',F12.8,';')
        23 CONTINUE
            WRITE(6,101)
           STOP
           END
           FUNCTION FUNC(MT,NS1,X,Y,Z)
           IMPLICIT REAL*8(A-H,O-Z)
           GOTO (10,11),MT
        10 FUNC =-LOG(1-Z+Z/(Y*X*(1-X)))*Y**NS1*X*(1-X)*(1-Z)/Z
           RETURN
        11 FUNC =-LOG(1-Z+Z/(Y*X*(1-X)))*Y**NS1*X*(1-X)*(1-Z)**2/Z
           RETURN
           END
           SUBROUTINE ¥DQDGM(FUNC,MTS,NS1,A,B,M,C,D,N,S,IER)
```

```
C
C     S-1511-1 COPYRIGHT HITACHI,LTD. 1980
C
C     STATUS                 - S-1511-1 01-00
C
C     HISTORY                - DATE.    1980.3
C
        IMPLICIT REAL*8(A-H,O-Z)
        DIMENSION Y(6,12),X(6,12),AA(24),U(24),BB(24),V(24)
       1,A1(19),A2(19),A3(19),A4(15)
       2,X1(19),X2(19),X3(19),X4(15)
        DIMENSION NAME¥(2)
        EQUIVALENCE (Y(1),A1(1)),(Y(20),A2(1)),(Y(39),A3(1)),(Y(58),A4(1))
       1,            (X(1),X1(1)),(X(20),X2(1)),(X(39),X3(1)),(X(58),X4(1))
        DATA  X1    /                  0.86113631159405260D+00
       1,  0.96028985649753620D+00, 0.93156063424671920D+00
       2,  0.98940093499164990D+00, 0.99312859918509490D+00
       3,  0.99518721999702140D+00, 0.33998104358485630D+00
       4,  0.79666647741362670D+00, 0.90411725637047490D+00
       5,  0.94457502307323260D+00, 0.96397192727791380D+00
       6,  0.97472855597130950D+00, 0.00000000000000000D-40
       7,  0.52553240991632900D+00, 0.76990267419430470D+00
       8,  0.86563120238733170D+00, 0.91223442825132590D+00
       9,  0.93827455200273280D+00, 0.00000000000000000D-40 /
        DATA  X2    /                  0.18343464249564980D+00
       1,  0.53731795428661740D+00, 0.75540440335500300D+00
       2,  0.83911697182221880D+00, 0.88641552700440100D+00
       3,  0.00000000000000000D-40, 0.00000000000000000D-40
       4,  0.36783149899818020D+00, 0.61787624440264370D+00
       5,  0.74633190646015080D+00, 0.82000198597390290D+00
       6,  0.00000000000000000D-40, 0.00000000000000000D-40
       7,  0.12523340851146890D+00, 0.45801677765722740D+00
       8,  0.63605368072651500D+00, 0.74012419157855440D+00
       9,  0.00000000000000000D-40, 0.00000000000000000D-40 /
        DATA  X3    /                  0.00000000000000000D-40
       1,  0.28160355077925890D+00, 0.51086700195082710D+00
       2,  0.64809365193697560D+00, 0.00000000000000000D-40
       3,  0.00000000000000000D-40, 0.00000000000000000D-40
       4,  0.95012509837637440D-01, 0.37370608871541960D+00
       5,  0.54542147138883950D+00, 0.00000000000000000D-40
       6,  0.00000000000000000D-40, 0.00000000000000000D-40
       7,  0.00000000000000000D-40, 0.22778585114164510D+00
       8,  0.43379350762604510D+00, 0.00000000000000000D-40
       9,  0.00000000000000000D-40, 0.00000000000000000D-40 /
        DATA  X4    /                  0.51086700195082710D+00
       1,  0.76526521133497330D-01, 0.31504267969616340D+00
       2,  0.00000000000000000D-40, 0.00000000000000000D-40
       3,  0.00000000000000000D-40, 0.00000000000000000D-40
       4,  0.00000000000000000D-40, 0.19111886747361630D+00
       5,  0.00000000000000000D-40, 0.00000000000000000D-40
       6,  0.00000000000000000D-40, 0.00000000000000000D-40
       7,  0.00000000000000000D-40, 0.64056892862605620D-01 /
        DATA  A1    /                  0.34785484513745390D+00
       1,  0.10122853629037630D+00, 0.47175336386511830D-01
       2,  0.27152459411754100D-01, 0.17614007139152120D-01
       3,  0.12341229799987200D-01, 0.65214515486254620D+00
       4,  0.22238103445337450D+00, 0.10693932599531840D+00
       5,  0.62253523938647890D-01, 0.40601429800386940D-01
       6,  0.28531388628933660D-01, 0.00000000000000000D-40
       7,  0.31370664587788730D+00, 0.16007832854334620D+00
       8,  0.95153511682492780D-01, 0.62672048334109070D-01
       9,  0.44277438817419810D-01, 0.00000000000000000D-40 /
        DATA  A2    /                  0.36268378337836200D+00
       1,  0.20316742672306590D+00, 0.12462897125553390D+00
       2,  0.83276741576704750D-01, 0.59298584915436780D-01
       3,  0.00000000000000000D-40, 0.00000000000000000D-40
```

```
     4,  0.23349253653835480D+00, 0.14959598981657670D+00
     5,  0.10193011981724040D+00, 0.73346491411030310D-01
     6,  0.00000000000000000D-40, 0.00000000000000000D-40
     7,  0.24914704581340280D+00, 0.16915651939500250D+00
     8,  0.11819453196151840D+00, 0.86190161531953270D-01
     9,  0.00000000000000000D-40, 0.00000000000000000D-40 /
      DATA  A3      /            0.00000000000000000D-40
     1,  0.18260341504492360D+00, 0.13168863844917660D+00
     2,  0.97618652104113890D-01, 0.00000000000000000D-40
     3,  0.00000000000000000D-40, 0.00000000000000000D-40
     4,  0.18945061045506350D+00, 0.14209610931833210D+00
     5,  0.10744427011596560D+00, 0.00000000000000000D-40
     6,  0.00000000000000000D-40, 0.00000000000000000D-40
     7,  0.00000000000000000D-40, 0.14917293647260380D+00
     8,  0.11550566805372560D+00, 0.00000000000000000D-40
     9,  0.00000000000000000D-40, 0.00000000000000000D-40 /
      DATA  A4      /            0.13168863844917660D+00
     1,  0.15275338713072580D+00, 0.12167047292780340D+00
     2,  0.00000000000000000D-40, 0.00000000000000000D-40
     3,  0.00000000000000000D-40, 0.00000000000000000D-40
     4,  0.00000000000000000D-40, 0.12583745634682830D+00
     5,  0.00000000000000000D-40, 0.00000000000000000D-40
     6,  0.00000000000000000D-40, 0.00000000000000000D-40
     7,  0.00000000000000000D-40, 0.12793819534675220D+00 /
      DATA NAME¥ /4H¥DQD,4HGM  /
      IER=0
      I=M
      L=N
      IF(M-1)  600,100,100
  100 IF(M-6)  200,200,600
  200 IF(N-1)  600,300,300
  300 IF(N-6)  400,400,600
  400 NN=4*M
      C1=(B+A)/2.0D0
      C2=(B-A)/2.0D0
      N2=M+M
      DO 1 J=1,N2
      U(J)=C1-C2*X(M,J)
      U(NN-J+1)  = C1+C2*X(M,J)
      AA(J)=Y(M,J)
    1 AA(NN-J+1) = Y(M,J)
      MM=4*N
      D1=(D+C)/2.0D0
      D2=(D-C)/2.0D0
      M2=N+N
      DO 2 K=1,M2
      V(K)=D1-D2*X(N,K)
      V(MM-K+1)  = D1+D2*X(N,K)
      BB(K)=Y(N,K)
    2 BB(MM-K+1) = Y(N,K)
      S=0.0D0
      DO 3 J=1,NN
      DO 3 K=1,MM
      DO 3 K3=1,MM
    3 S=S+AA(J)*BB(K3)*BB(K)*FUNC(MTS,NS1,U(J),V(K),V(K3))
      S=C2*D2*S*D2
      M=I
      N=L
      RETURN
      CALL ¥SURTM(NAME¥,IER)
      RETURN
  600 CALL ¥SUERM(NAME¥,1200,IER)
      M=6
      N=6
      GO TO 400
      END
```

```
      SUBROUTINE ¥SUERM (NAME, ISET, IER)
C
C   S-1511-1 COPYRIGHT HITACHI,LTD. 1980
C
C   STATUS                 - S-1511-1 01-00
C
C   HISTORY                - DATE.    1980.3
C
      DIMENSION NAME(2)
      COMMON /¥SUCOM/ MLEVEL, NOUT
      IER = ISET
      ILEVEL = MLEVEL
      IF (ILEVEL .GE. 11) ILEVEL = ILEVEL-10
      IF (IER.LT.1000 .OR. ILEVEL.EQ.4) GO TO 999
      IF (ILEVEL .NE. 2) GO TO 10
        IF (IER .LT. 2000) GO TO 999
        GO TO 30
C
   10 IF (ILEVEL .NE. 3) GO TO 20
        IF (IER .LT. 3000) GO TO 999
        GO TO 40
C
   20 IF (IER .GE. 2000) GO TO 30
        WRITE(NOUT,100) NAME, IER
        GO TO 999
C
   30 IF (IER .GE. 3000) GO TO 40
        WRITE(NOUT,200) NAME, IER
        GO TO 999
C
   40 WRITE(NOUT,300) NAME, IER
      GO TO 999
C
      ENTRY ¥SUSTM (LEVEL, NFILE)
C
      MLEVEL = 1
      IF (LEVEL.GE.1 .AND. LEVEL.LE.4) MLEVEL = LEVEL
      IF (LEVEL.GE.11 .AND. LEVEL.LE.14) MLEVEL = LEVEL
      NOUT = NFILE
      GO TO 999
C
      ENTRY ¥SURTM (NAME, IER)
C
      IF (MLEVEL .GE. 11) WRITE(NOUT,400) NAME, IER
  999 RETURN
C
  100 FORMAT(1H0,'*** WARNING ERROR   FROM MSLII ROUTINE ',2A4,
     1             ' ( IER = ',I6,' ) ***')
  200 FORMAT(1H0,'*** PARAMETER ERROR FROM MSLII ROUTINE ',2A4,
     1             ' ( IER = ',I6,' ) ***')
  300 FORMAT(1H0,'*** TERMINAL ERROR  FROM MSLII ROUTINE ',2A4,
     1             ' ( IER = ',I6,' ) ***')
  400 FORMAT(1H0,'*** RETURN MESSAGE  FROM MSLII ROUTINE ',2A4,
     1             ' ( IER = ',I6,' ) ***')
      END
      BLOCK DATA
      COMMON /¥SUCOM/ MLEVEL, NOUT
      DATA MLEVEL, NOUT / 1, 6/
      END
//LIST C2.FORT(C12)
      IMPLICIT REAL*8(A-H,O-Z)
      EXTERNAL FUNC
      A=0.
      B=1.
      C=0.
      D=1.
```

```
      M=6
      N=6
  101 FORMAT( ' COMMENT  FUNC')
      DO 23 IT=10,11
      MT=IT
      MW=MT
      DO 23 NS1=0,6
      MT1=MT-9
   33 CALL ¥DQDGM(FUNC,MT1,NS1,A,B,M,C,D,N,S,IER)
        WRITE(6,100) MW,NS1,S
  100 FORMAT(  ' LET TC',I2,'(',I1,') = ',F12.8,';')
   23 CONTINUE
       WRITE(6,101)
      STOP
      END
      FUNCTION FUNC(MT,NS1,T,U,V)
      IMPLICIT REAL*8(A-H,O-Z)
      X=T*T
      Y=U*U
      Z=V*V
      GOTO (10,11),MT
   10 F1 =LOG(1-Z+Z/(Y*X*(1-X)))
      F2=1/(1-Z+Z/(Y*X*(1-X)))*(-1+1/(Y*X*(1-X)))*LOG(Z)
      FUNC =(F1+F2)*Y**NS1*X*(1-X)*T*U*V*8
      RETURN
   11 F1 =LOG(1-Z+Z/(Y*X*(1-X)))*(2-Z)
      F2=1/(1-Z+Z/(Y*X*(1-X)))*(-1+1/(Y*X*(1-X)))*LOG(Z)
      FUNC =(F1+F2)*Y**NS1*X*(1-X)*T*U*V*8
      RETURN
      END
      SUBROUTINE ¥DQDGM(FUNC,MTS,NS1,A,B,M,C,D,N,S,IER)
C
C   S-1511-1 COPYRIGHT HITACHI,LTD. 1980
C
C   STATUS              - S-1511-1 01-00
C
C   HISTORY             - DATE.    1980.3
C
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION Y(6,12),X(6,12),AA(24),U(24),BB(24),V(24)
     1,A1(19),A2(19),A3(19),A4(15)
     2,X1(19),X2(19),X3(19),X4(15)
      DIMENSION NAME¥(2)
      EQUIVALENCE (Y(1),A1(1)),(Y(20),A2(1)),(Y(39),A3(1)),(Y(58),A4(1))
     1,           (X(1),X1(1)),(X(20),X2(1)),(X(39),X3(1)),(X(58),X4(1))
      DATA  X1    /                      0.86113631159405260D+00
     1,  0.96028985649753620D+00, 0.98156063424671920D+00
     2,  0.93940093499164990D+00, 0.99312359913509490D+00
     3,  0.99518721999702140D+00, 0.33998104358485630D+00
     4,  0.79666647741362670D+00, 0.90411725637047490D+00
     5,  0.94457502307323260D+00, 0.96397192727791380D+00
     6,  0.97472855597130950D+00, 0.00000000000000000D-40
     7,  0.52553240991632900D+00, 0.76990267419430470D+00
     8,  0.86563120238783170D+00, 0.91223442825132590D+00
     9,  0.93827455200273280D+00, 0.00000000000000000D-40 /
      DATA  X2    /                      0.18343464249564980D+00
     1,  0.58731795428661740D+00, 0.75540440835500300D+00
     2,  0.83911697182221880D+00, 0.88641552700440100D+00
     3,  0.00000000000000000D-40, 0.00000000000000000D-40
     4,  0.36783149899818020D+00, 0.61787624440264370D+00
     5,  0.74633190646015030D+00, 0.82000198597390290D+00
     6,  0.00000000000000000D-40, 0.00000000000000000D-40
     7,  0.12523340851146890D+00, 0.45801677765722740D+00
     8,  0.63605368072651500D+00, 0.74012419157855440D+00
     9,  0.00000000000000000D-40, 0.00000000000000000D-40 /
      DATA  X3    /                      0.00000000000000000D-40
```

```
     1,  0.28160355077925890D+00, 0.51086700195082710D+00
     2,  0.64809365193697560D+00, 0.00000000000000000D-40
     3,  0.00000000000000000D-40, 0.00000000000000000D-40
     4,  0.95012509837637440D-01, 0.37370608871541960D+00
     5,  0.54542147138883950D+00, 0.00000000000000000D-40
     6,  0.00000000000000000D-40, 0.00000000000000000D-40
     7,  0.00000000000000000D-40, 0.22778585114164510D+00
     8,  0.43379350762604510D+00, 0.00000000000000000D-40
     9,  0.00000000000000000D-40, 0.00000000000000000D-40 /
      DATA   X4       /             0.51086700195082710D+00
     1,  0.76526521133497330D-01, 0.31504267969616340D+00
     2,  0.00000000000000000D-40, 0.00000000000000000D-40
     3,  0.00000000000000000D-40, 0.00000000000000000D-40
     4,  0.00000000000000000D-40, 0.19111886747361630D+00
     5,  0.00000000000000000D-40, 0.00000000000000000D-40
     6,  0.00000000000000000D-40, 0.00000000000000000D-40
     7,  0.00000000000000000D-40, 0.64056892862605620D-01 /
      DATA   A1       /             0.34785484513745390D+00
     1,  0.10122853629037630D+00, 0.47175336386511830D-01
     2,  0.27152459411754100D-01, 0.17614007139152120D-01
     3,  0.12341229799987200D-01, 0.65214515486254620D+00
     4,  0.22238103445337450D+00, 0.10693932599531840D+00
     5,  0.62253523938647890D-01, 0.40601429300386940D-01
     6,  0.23531383628933660D-01, 0.00000000000000000D-40
     7,  0.31370664587783730D+00, 0.16007832854334620D+00
     8,  0.95158511632492730D-01, 0.62672048334109070D-01
     9,  0.44277438817419810D-01, 0.00000000000000000D-40 /
      DATA   A2       /             0.36268378337836200D+00
     1,  0.20316742672306590D+00, 0.12462897125553390D+00
     2,  0.83276741576704750D-01, 0.59298584915436780D-01
     3,  0.00000000000000000D-40, 0.00000000000000000D-40
     4,  0.23349253653835480D+00, 0.14959598881657670D+00
     5,  0.10193011981724040D+00, 0.73346481411080310D-01
     6,  0.00000000000000000D-40, 0.00000000000000000D-40
     7,  0.24914704581340280D+00, 0.16915651939500250D+00
     8,  0.11819453196151840D+00, 0.86190161531953270D-01
     9,  0.00000000000000000D-40, 0.00000000000000000D-40 /
      DATA   A3       /             0.00000000000000000D-40
     1,  0.18260341504492360D+00, 0.13168863844917660D+00
     2,  0.97618652104113890D-01, 0.00000000000000000D-40
     3,  0.00000000000000000D-40, 0.00000000000000000D-40
     4,  0.18945061045506850D+00, 0.14209610931838210D+00
     5,  0.10744427011596560D+00, 0.00000000000000000D-40
     6,  0.00000000000000000D-40, 0.00000000000000000D-40
     7,  0.00000000000000000D-40, 0.14917298647260380D+00
     8,  0.11550566805372560D+00, 0.00000000000000000D-40
     9,  0.00000000000000000D-40, 0.00000000000000000D-40 /
      DATA   A4       /             0.13168863844917660D+00
     1,  0.15275338713072580D+00, 0.12167047292780340D+00
     2,  0.00000000000000000D-40, 0.00000000000000000D-40
     3,  0.00000000000000000D-40, 0.00000000000000000D-40
     4,  0.00000000000000000D-40, 0.12583745634682830D+00
     5,  0.00000000000000000D-40, 0.00000000000000000D-40
     6,  0.00000000000000000D-40, 0.00000000000000000D-40
     7,  0.00000000000000000D-40, 0.12793819534675220D+00 /
      DATA NAME¥ /4H¥D99,4HGM  /
      IER=0
      I=M
      L=N
      IF(M-1)  600,100,100
  100 IF(M-6)  200,200,600
  200 IF(N-1)  600,300,300
  300 IF(N-6)  400,400,600
  400 NN=4*M
      C1=(B+A)/2.0D0
      C2=(B-A)/2.0D0
```

```
      MLEVEL = 1
      IF (LEVEL.GE.1 .AND. LEVEL.LE.4) MLEVEL = LEVEL
      IF (LEVEL.GE.11 .AND. LEVEL.LE.14) MLEVEL = LEVEL
      NOUT = NFILE
      GO TO 999
C
      ENTRY ¥SURTM (NAME, IER)
C
      IF (MLEVEL .GE. 11) WRITE(NOUT,400) NAME, IER
  999 RETURN
C
  100 FORMAT(1H0,'*** WARNING ERROR   FROM MSLII ROUTINE ',2A4,
     1               ' ( IER = ',I6,' ) ***')
  200 FORMAT(1H0,'*** PARAMETER ERROR FROM MSLII ROUTINE ',2A4,
     1               ' ( IER = ',I6,' ) ***')
  300 FORMAT(1H0,'*** TERMINAL ERROR  FROM MSLII ROUTINE ',2A4,
     1               ' ( IER = ',I6,' ) ***')
  400 FORMAT(1H0,'*** RETURN MESSAGE  FROM MSLII ROUTINE ',2A4,
     1               ' ( IER = ',I6,' ) ***')
      END
      BLOCK DATA
      COMMON /¥SUCOM/ MLEVEL, NOUT
      DATA MLEVEL, NOUT / 1, 6/
      END
//LIST C2.FORT(C4)
      EXTERNAL FUNC
      A=0.
      B=1.
      C=0.
      D=1.
      M=6
      N=6
 101  FORMAT( ' COMMENT  FUNC')
      DO 23 IT=10,19
       MT=IT
       MW=MT
       IF (MT.GT.12) MW=MT-13
       IF (MT.GT.15) MW=MT+4
       IF (MT.GT.17) MW=MT-18
       DO 23 NS1=0,6
       MT1=MT-9
       CALL ¥DQDGM(FUNC,MT1,NS1,A,B,M,C,D,N,S,IER)
         WRITE(6,100) MW,NS1,S
  100 FORMAT(  ' LET TC',I2,'(',I1,') = ',F12.8,';')
   23 CONTINUE
       WRITE(6,101)
       STOP
       END
       FUNCTION FUNC(MT,NS1,X,Y,Z)
       GOTO  ( 10,11,12,13,14,15,16,17,18,19),MT
   10 FUNC =-LOG((1-Z)*(1-Y)*Y/(Z*X)+1-Y+Y/X)*(1-X)*Y**(NS1-2)
       RETURN
   11 FUNC =-LOG((1-Z)*(1-Y)*Y/(Z*X)+1-Y+Y/X)*(1-Y)*(1-X)*Y**(NS1-1)
       RETURN
   12 DZ =1-Z+Z*(X/Y+1./(1-Y))
       DY =Z*(X/Y+1./(1-Y))
       FUNC =-(LOG(DZ)*DZ-DY*LOG(DY))*(1-Y)*Y**(NS1)*(1-X)/Z
       RETURN
   15 FUNC =-LOG(1-Z+Z*X/Y+Z/(1-Y))*Y**NS1*(1-Z)*(1-Y)**2*(1-X)/Z
       RETURN
   13 FUNC =-LOG(1-Y+Y/X+(1-Z)*Y*(1-Y)/Z/X)*Z*(1-Z)*(1-X)*Y**(NS1-2)
       RETURN
   14 FUNC =-LOG(1-Y+Y/X+(1-Z)*Y*(1-Y)/Z/X)*(1-Y)*(1-Z)*(1-X)*Y**(NS1
     1-1)
       RETURN
   16 FUNC =Y**(NS1-1)*Z*(1-Y)*(1-X)/((1-Z)*Y*(1-Y)+Z*X*(1-Y)+Z*Y)
```

```
      BLOCK DATA
      COMMON /¥SUCOM/ MLEVEL, NOUT
      DATA MLEVEL, NOUT / 1, 6/
      END
//
//LIST C2.FORT(C5)
      EXTERNAL FUNC
      EXTERNAL FUNC
      A=0.
      B=1.
      C=0.
      D=1.
      M=6
      N=6
 101  FORMAT( ' COMMENT  FUNC')
      DO 23 IT=18,18
      MT=IT
      MW=MT
      IF (MT.GT.12) MW=MT-10
      IF (MT.GT.15) MW=MT+4
      IF (MT.GT.18) MW=MT-18
      DO 23 NS1=0,6
      MT1=MT-9
      CALL ¥DQDGM(FUNC,MT1,NS1,A,B,M,C,D,N,S,IER)
        WRITE(6,100) MW,NS1,S
  100 FORMAT(  ' LET TC',I2,'(',I1,') = ',F12.8,';')
   23 CONTINUE
       WRITE(6,101)
      STOP
      END
      FUNCTION FUNC(MT,NS1,X,Y,Z)
      GOTO  ( 10,11,12,13,14,17,15,16,18),MT
   10 FUNC =-LOG(1+Y*(1-Z)/(Z*X))*Z*Y**(NS1-1)
      RETURN
   11 FUNC =-LOG(1-Z+Z*X/Y)*Y**NS1
      RETURN
   12 DZ=1-Z+Z*X/Y
      FUNC =-(LOG(DZ)*DZ-Z*X*LOG(X*Z/Y)/Y)*(1-Y)*Y**(NS1+1)/Z
      RETURN
   15 FUNC =-LOG(1-Z+Z*X/Y)*Y**NS1*(1-Z)*(1-Y)*Y/Z
      RETURN
   13 FUNC =-LOG(1+Y*(1-Z)/(Z*X))*Z*Y**(NS1-1)*(1-Z)
      RETURN
   14 FUNC =-LOG(1-Z+Z*X/Y)*Y**NS1*(1-Z)
      RETURN
   17 FUNC =1./(1-Z+Z*X/Y)*Y**NS1*(1-Z)
      RETURN
   16 FUNC =1./(1-Z+Z*X/Y)*Y**NS1*(1-Z)*Z/Y
      RETURN
   18 DZ=Y*(1-Z)+Z*X
      FUNC =LOG(DZ)*X*(1-Y)*Y**NS1+(-1./(NS1+1)+1./(NS1+2))*.75
      RETURN
      END
      SUBROUTINE ¥DQDGM(FUNC,MTS,NS1,A,B,M,C,D,N,S,IER)
C
C    S-1511-1 COPYRIGHT HITACHI,LTD. 1980
C
C    STATUS                 - S-1511-1 01-00
C
C    HISTORY                - DATE.    1980.3
C
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION Y(6,12),X(6,12),AA(24),U(24),BB(24),V(24)
     1,A1(19),A2(19),A3(19),A4(15)
     2,X1(19),X2(19),X3(19),X4(15)
      DIMENSION NAME¥(2)
      EQUIVALENCE (Y(1),A1(1)),(Y(20),A2(1)),(Y(39),A3(1)),(Y(58),A4(1))
     *            (Y(1),X1(1)),(Y(20),X2(1)),(X(39),X3(1)),(X(58),X4(1))
```

```
      S=C2*U2*S*U2
      M=I
      N=L
      RETURN
      CALL ¥SURTM(NAME¥,IER)
      RETURN
  600 CALL ¥SUERM(NAME¥,1200,IER)
      M=6
      N=6
      GO TO 400
      END
      SUBROUTINE ¥SUERM (NAME, ISET, IER)
C
C   S-1511-1 COPYRIGHT HITACHI,LTD. 1980
C
C   STATUS                 - S-1511-1 01-00
C
C   HISTORY                - DATE.     1980.3
C
      DIMENSION NAME(2)
      COMMON /¥SUCOM/ MLEVEL, NOUT
      IER = ISET
      ILEVEL = MLEVEL
      IF (ILEVEL .GE. 11) ILEVEL = ILEVEL-10
      IF (IER.LT.1000 .OR. ILEVEL.EQ.4) GO TO 999
      IF (ILEVEL .NE. 2) GO TO 10
        IF (IER .LT. 2000) GO TO 999
        GO TO 30
C
   10 IF (ILEVEL .NE. 3) GO TO 20
        IF (IER .LT. 3000) GO TO 999
        GO TO 40
C
   20 IF (IER .GE. 2000) GO TO 30
        WRITE(NOUT,100) NAME, IER
        GO TO 999
C
   30 IF (IER .GE. 3000) GO TO 40
        WRITE(NOUT,200) NAME, IER
        GO TO 999
C
   40 WRITE(NOUT,300) NAME, IER
      GO TO 999
C
      ENTRY ¥SUSTM (LEVEL, NFILE)
C
      MLEVEL = 1
      IF (LEVEL.GE.1 .AND. LEVEL.LE.4) MLEVEL = LEVEL
      IF (LEVEL.GE.11 .AND. LEVEL.LE.14) MLEVEL = LEVEL
      NOUT = NFILE
      GO TO 999
C
      ENTRY ¥SURTM (NAME, IER)
C
      IF (MLEVEL .GE. 11) WRITE(NOUT,400) NAME, IER
  999 RETURN
C
  100 FORMAT(1H0,'*** WARNING ERROR   FROM MSLII ROUTINE ',2A4,
     1               ' ( IER = ',I6,' ) ***')
  200 FORMAT(1H0,'*** PARAMETER ERROR FROM MSLII ROUTINE ',2A4,
     1               ' ( IER = ',I6,' ) ***')
  300 FORMAT(1H0,'*** TERMINAL ERROR  FROM MSLII ROUTINE ',2A4,
     1               ' ( IER = ',I6,' ) ***')
  400 FORMAT(1H0,'*** RETURN MESSAGE  FROM MSLII ROUTINE ',2A4,
     1               ' ( IER = ',I6,' ) ***')
      END
      BLOCK DATA
```

```
     3,  0.12341229799987200D-01, 0.65214515486254620D+00
     4,  0.22238103445337450D+00, 0.10693932599531840D+00
     5,  0.62253523938647890D-01, 0.40601429800386940D-01
     6,  0.28531338628933660D-01, 0.00000000000000000D-40
     7,  0.31370664587788730D+00, 0.16007832854334620D+00
     8,  0.35158511682492780D-01, 0.62672048334109070D-01
     9,  0.44277438817419810D-01, 0.00000000000000000D-40 /
      DATA  A2       /            0.36268378337836200D+00
     1,  0.20316742672306590D+00, 0.12462897125553390D+00
     2,  0.93276741576704750D-01, 0.59298584915436780D-01
     3,  0.00000000000000000D-40, 0.00000000000000000D-40
     4,  0.23349253653835480D+00, 0.14959598816576700D+00
     5,  0.10193011981724040D+00, 0.73346481411080310D-01
     6,  0.00000000000000000D-40, 0.00000000000000000D-40
     7,  0.24914704581340280D+00, 0.16915651939500250D+00
     8,  0.11819453196151840D+00, 0.86190161531953270D-01
     9,  0.00000000000000000D-40, 0.00000000000000000D-40 /
      DATA  A3       /            0.00000000000000000D-40
     1,  0.18260341504492360D+00, 0.13168863844917660D+00
     2,  0.97618652104113890D-01, 0.00000000000000000D-40
     3,  0.00000000000000000D-40, 0.00000000000000000D-40
     4,  0.18945061045968500D+00, 0.14209610931838210D+00
     5,  0.10744427011596560D+00, 0.00000000000000000D-40
     6,  0.00000000000000000D-40, 0.00000000000000000D-40
     7,  0.00000000000000000D-40, 0.14917298647260380D+00
     8,  0.11550566805372560D+00, 0.00000000000000000D-40
     9,  0.00000000000000000D-40, 0.00000000000000000D-40 /
      DATA  A4       /            0.13168863844917660D+00
     1,  0.15275338713072580D+00, 0.12167047292780340D+00
     2,  0.00000000000000000D-40, 0.00000000000000000D-40
     3,  0.00000000000000000D-40, 0.00000000000000000D-40
     4,  0.00000000000000000D-40, 0.12583745634682830D+00
     5,  0.00000000000000000D-40, 0.00000000000000000D-40
     6,  0.00000000000000000D-40, 0.00000000000000000D-40
     7,  0.00000000000000000D-40, 0.12793319534675220D+00 /
      DATA NAME¥ /4H¥DJD,4HGM  /
      IER=0
      I=M
      L=N
      IF(M-1)  600,100,100
  100 IF(M-5)  200,200,600
  200 IF(N-1)  600,300,300
  300 IF(N-6)  400,400,600
  400 NN=4*M
      C1=(B+A)/2.0D0
      C2=(B-A)/2.0D0
      N2=M+M
      DO 1 J=1,N2
      U(J)=C1-C2*X(M,J)
      U(NN-J+1)  = C1+C2*X(M,J)
      AA(J)=Y(M,J)
    1 AA(NN-J+1) = Y(M,J)
      MM=4*N
      D1=(D+C)/2.0D0
      D2=(D-C)/2.0D0
      M2=N+N
      DO 2 K=1,M2
      V(K)=D1-D2*X(N,K)
      V(MM-K+1)  = D1+D2*X(N,K)
      BB(K)=Y(N,K)
    2 BB(MM-K+1) = Y(N,K)
      S=0.0D0
      DO 3 J=1,NN
      DO 3 K=1,MM
      DO 3 K3=1,MM
    3 S=S+AA(J)*BB(K3)*BB(K)*FUNC(MTS,NS1,U(J),V(K),V(K3))
```

```
     1/(Z*X*(1-Y)+Z*Y))
      GOTO 100
      RETURN
   18 FUNC =Y**NS1*(1-Z)*Z*(1-Y)*(1-X)/((1-Z)*Y*(1-Y)+Z*X*(1-Y)+Z*Y)/Y
      GOTO 100
      RETURN
   19 FUNC =Y**NS1*(1-Z)*(1-Y)**2*(1-X)/((1-Z)*Y*(1-Y)+Z*X*(1-Y)+Z*Y)
  100 FUNC =FUNC*U*T*V*8
      RETURN
      END
      SUBROUTINE ¥DQDGM(FUNC,MTS,NS1,A,B,M,C,D,N,S,IER)
C
C     S-1511-1 COPYRIGHT HITACHI,LTD. 1980
C
C     STATUS              - S-1511-1 01-00
C
C     HISTORY             - DATE.    1980.3
C
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION Y(6,12),X(6,12),AA(24),U(24),BB(24),V(24)
     1,A1(19),A2(19),A3(19),A4(15)
     2,X1(19),X2(19),X3(19),X4(15)
      DIMENSION NAME¥(2)
      EQUIVALENCE (Y(1),A1(1)),(Y(20),A2(1)),(Y(39),A3(1)),(Y(58),A4(1))
     1,            (X(1),X1(1)),(X(20),X2(1)),(X(39),X3(1)),(X(58),X4(1))
      DATA  X1    /                      0.86113631159405260D+00
     1,  0.96028985649753620D+00, 0.93156063424671920D+00
     2,  0.93940093499164990D+00, 0.99312859918509490D+00
     3,  0.99518721999702140D+00, 0.33998104358485630D+00
     4,  0.79666647741362670D+00, 0.90411725637047490D+00
     5,  0.94457502307323260D+00, 0.96397192727791380D+00
     6,  0.97472855597130950D+00, 0.00000000000000000D-40
     7,  0.52553240991632900D+00, 0.76990267419430470D+00
     8,  0.86563120238783170D+00, 0.91223442825132590D+00
     9,  0.93827455200273280D+00, 0.00000000000000000D-40 /
      DATA  X2    /                      0.18343464249564980D+00
     1,  0.58731795428661740D+00, 0.75540440835500300D+00
      2,  0.83911697182221880D+00, 0.88641552700440100D+00
      3,  0.00000000000000000D-40, 0.00000000000000000D-40
      4,  0.36783149899818020D+00, 0.61787624440264370D+00
      5,  0.74633190646015080D+00, 0.82000198597390290D+00
      6,  0.00000000000000000D-40, 0.00000000000000000D-40
      7,  0.12523340851146890D+00, 0.45801677765722740D+00
      8,  0.63605368072651500D+00, 0.74012419157855440D+00
      9,  0.00000000000000000D-40, 0.00000000000000000D-40 /
       DATA  X3    /                      0.00000000000000000D-40
      1,  0.28160355077925890D+00, 0.51086700195082710D+00
      2,  0.64809365193697560D+00, 0.00000000000000000D-40
      3,  0.00000000000000000D-40, 0.00000000000000000D-40
      4,  0.95012509837637440D-01, 0.37370608871541960D+00
      5,  0.54542147138883950D+00, 0.00000000000000000D-40
      6,  0.00000000000000000D-40, 0.00000000000000000D-40
      7,  0.00000000000000000D-40, 0.22778585114164510D+00
      8,  0.43379350762604510D+00, 0.00000000000000000D-40
      9,  0.00000000000000000D-40, 0.00000000000000000D-40 /
       DATA  X4    /                      0.51086700195082710D+00
      1,  0.76526521133497330D-01, 0.31504267969616340D+00
      2,  0.00000000000000000D-40, 0.00000000000000000D-40
      3,  0.00000000000000000D-40, 0.00000000000000000D-40
      4,  0.00000000000000000D-40, 0.19111886747361630D+00
      5,  0.00000000000000000D-40, 0.00000000000000000D-40
      6,  0.00000000000000000D-40, 0.00000000000000000D-40
      7,  0.00000000000000000D-40, 0.64056892862605620D-01 /
      DATA  A1    /                      0.34785484513745390D+00
      1,  0.10122853629037630D+00, 0.47175336386511830D-01
      2,  0.27152459411754100D-01, 0.17614007139152120D-01
```

```
      COMMON /*SUCUM/ MLEVEL, NOUT
      DATA MLEVEL, NOUT / 1, 6/
      END
//LIST C2.FORT(C14)
      IMPLICIT REAL*8(A-H,O-Z)
      EXTERNAL FUNC
      A=0.
      B=1.
      C=0.
      D=1.
      M=6
      N=6
  101 FORMAT( ' COMMENT  FUNC')
      DO 23 IT=10,19
      MT=IT
      MW=MT
      IF (MT.GT.12) MW=MT-13
      IF (MT.GT.15) MW=MT+4
      IF (MT.GT.17) MW=MT-18
      DO 23 NS1=0,6
      MT1=MT-9
      CALL ¥DQDGM(FUNC,MT1,NS1,A,B,M,C,D,N,S,IER)
        WRITE(6,100) MW,NS1,S
  100 FORMAT(  ' LET TC',I2,'(',I1,') = ',F12.8,';')
   23 CONTINUE
        WRITE(6,101)
      STOP
      END
      FUNCTION FUNC(MT,NS1,U,V,T)
      IMPLICIT REAL*8(A-H,O-Z)
      X=T*T
      Y=U*U
      Z=V*V
      GOTO  ( 10,11,12,13,14,15,16,17,18,19),MT
   10 FUNC =-LOG((1-Z)*(1-Y)*Y/(Z*X)+1-Y+Y/X)*(1-X)*Y**(NS1-2)
      GOTO 100
      RETURN
   11 FUNC =-LOG((1-Z)*(1-Y)*Y/(Z*X)+1-Y+Y/X)*(1-Y)*(1-X)*Y**(NS1-1)
      GOTO 100
      RETURN
   12 DZ =(1-Z)*Y*(1-Y)+(1-Y)*Z*X+Z*Y
      DX =-Y*(1-Y)+X*(1-Y)+Y
      DY =X*(1-Y)+Y
      F1 =-LOG(DZ/Y/(1-Y))*DX+DY*LOG(DY*Z/Y/(1-Y))
      F2 =LOG(Z)*DX/DZ*Y*(1-Y)
      FUNC =(F1+F2)*(1-Y)*Y**(NS1-1)*(1-X)
      GOTO 100
      RETURN
   15 DZ =(1-Z)*Y*(1-Y)+(1-Y)*Z*X+Z*Y
      DX =-Y*(1-Y)+X*(1-Y)+Y
      F1 = LOG(1-Z+Z*X/Y+Z/(1-Y))
      F2 = LOG(Z)*DX/DZ
      FUNC = (F1+F2)*Y**NS1*(1-Y)**2*(1-X)
      GOTO 100
      RETURN
   13 FUNC =-LOG(1-Y+Y/X+(1-Z)*Y*(1-Y)/Z/X)*Z*(1-Z)*(1-X)*Y**(NS1-2)
      GOTO 100
      RETURN
   14 FUNC =-LOG(1-Y+Y/X+(1-Z)*Y*(1-Y)/Z/X)*(1-Y)*(1-Z)*(1-X)*Y**(NS1
     1-1)
      GOTO 100
      RETURN
   16 FUNC =Y**(NS1-1)*Z*(1-Y)*(1-X)/((1-Z)*Y*(1-Y)+Z*X*(1-Y)+Z*Y)
      GOTO 100
      RETURN
   17 FUNC =Y**(NS1-1)*(1-Y)*(1-X)*LOG(((1-Z)*Y*(1-Y)  X*(1-Y)+Z*Y)
```

```
      M=I
      N=L
      RETURN
      CALL ¥SURTM(NAME¥,IER)
      RETURN
  600 CALL ¥SUERM(NAME¥,1200,IER)
      M=6
      N=6
      GO TO 400
      END
      SUBROUTINE ¥SUERM (NAME, ISET, IER)
C
C   S-1511-1 COPYRIGHT HITACHI,LTD. 1980
C
C   STATUS              - S-1511-1 01-00
C
C   HISTORY             - DATE.    1980.3
C
      DIMENSION NAME(2)
      COMMON /¥SUCOM/ MLEVEL, NOUT
      IER = ISET
      ILEVEL = MLEVEL
      IF (ILEVEL .GE. 11) ILEVEL = ILEVEL-10
      IF (IER.LT.1000 .OR. ILEVEL.EQ.4) GO TO 999
      IF (ILEVEL .NE. 2) GO TO 10
        IF (IER .LT. 2000) GO TO 999
        GO TO 30
C
   10 IF (ILEVEL .NE. 3) GO TO 20
        IF (IER .LT. 3000) GO TO 999
        GO TO 40
C
   20 IF (IER .GE. 2000) GO TO 30
        WRITE(NOUT,100) NAME, IER
        GO TO 999
C
   30 IF (IER .GE. 3000) GO TO 40
        WRITE(NOUT,200) NAME, IER
        GO TO 999
C
   40 WRITE(NOUT,300) NAME, IER
      GO TO 999
C
      ENTRY ¥SUSTM (LEVEL, NFILE)
C
      MLEVEL = 1
      IF (LEVEL.GE.1 .AND. LEVEL.LE.4) MLEVEL = LEVEL
      IF (LEVEL.GE.11 .AND. LEVEL.LE.14) MLEVEL = LEVEL
      NOUT = NFILE
      GO TO 999
C
      ENTRY ¥SURTM (NAME, IER)
C
      IF (MLEVEL .GE. 11) WRITE(NOUT,400) NAME, IER
  999 RETURN
C
  100 FORMAT(1H0,'*** WARNING ERROR   FROM MSLII ROUTINE ',2A4,
     1             ' ( IER = ',I6,' ) ***')
  200 FORMAT(1H0,'*** PARAMETER ERROR FROM MSLII ROUTINE ',2A4,
     1             ' ( IER = ',I6,' ) ***')
  300 FORMAT(1H0,'*** TERMINAL ERROR  FROM MSLII ROUTINE ',2A4,
     1             ' ( IER = ',I6,' ) ***')
  400 FORMAT(1H0,'*** RETURN MESSAGE  FROM MSLII ROUTINE ',2A4,
     1             ' ( IER = ',I6,' ) ***')
      END
      BLOCK DATA
```

```
      RETURN
   17 FUNC =Y**(NS1-1)*(1-Y)*(1-X)*LOG(((1-Z)*Y*(1-Y)+Z*X*(1-Y)+Z*Y)
     1/(Z*X*(1-Y)+Z*Y))
      RETURN
   18 FUNC =Y**NS1*(1-Z)*Z*(1-Y)*(1-X)/((1-Z)*Y*(1-Y)+Z*X*(1-Y)+Z*Y)/Y
      RETURN
   19 FUNC =Y**NS1*(1-Z)*(1-Y)**2*(1-X)/((1-Z)*Y*(1-Y)+Z*X*(1-Y)+Z*Y)
      RETURN
      END
```