STUDIES IN POST-TONAL DIATONICISM: A MOD7 PERSPECTIVE

by

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Abstract

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There is a substantial body of music written in the 20th century in which the notes of a diatonic scale predominate, but which often lacks one or more of the other basic requirements necessary to be considered tonal: 1) a centricity around a single tone perceived as tonic; 2) a harmonic organization based on triads and seventh chords; 3) a hierarchical organization of functional harmonies; and 4) a contrapuntal substructure based on the laws of species counterpoint. Such music, by the likes of Barber, Copland, Prokofiev, and Stravinsky, has always posed a problem for music theorists, since neither traditional tonal analysis nor pc-set analysis yields satisfying analytic results. This dissertation argues that the problems inherent in analyzing post-tonal diatonic music can be solved by a careful application of set theory modulo 7, in interaction with the more familiar mod12 set theory. The first chapter outlines a system of mod7 set theory designed specifically for the analysis of post-tonal diatonic music. Chapter 2 then utilizes that system to analyze a range of post-tonal diatonic works in order to demonstrate the system's validity, its flexibility, and its explanatory power. Chapter 3 rigorously examines chordal tone centers in post-tonal diatonic music, an aspect of centricity that has thus far only been discussed in the vaguest of terms. Chapter 4 deals with structural levels in post-tonal diatonic music, presenting an approach that considers both the salience of individual pitches and

their place in a work's formal and motivic structure in determining their structural weight. The final chapter explores how diatonic partitionings of the octave interact with pentatonic, whole-tone, octatonic, and chromatic partitionings in much music of the 20th century, and addresses the analytic problems posed by such interactions.

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I wish to extend my thanks to those publishers who granted me permission to include the numerous musical examples in this study. All musical examples taken from the works of Stravinsky, Bartók, or Copland are used with the permission of Boosey & Hawkes. Musical examples from the works of Schoenberg, Webern, Barber, and Talma are used with the permission of Belmont Music, European American Music Distributors Corporation, G. Schirmer, Inc., and Carl Fischer, respectively.

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Chapter 1

Set Theory, Modulo 7

Introduction

There is a substantial body of music written in the twentieth century in which the notes of a diatonic scale predominate, but which often lacks one or more of the other basic requirements necessary to be considered tonal: 1) a centricity around a single tone perceived as tonic; 2) a harmonic organization based on triads and seventh chords; 3) a hierarchical organization of functional harmonies; and 4) a contrapuntal substructure based on the laws of species counterpoint. Such music, by the likes of Barber, Copland, Prokofiev, and Stravinsky, has always posed a problem for music theorists, since neither traditional tonal analysis nor pc-set analysis yields satisfying analytic results.

This dissertation will focus specifically on post-tonal music that is diatonic, but before zooming in on these specific cases, it would be worthwhile to examine the most common approaches to post-tonal music in general. There have been three distinct analytical approaches to pitch organization in post-tonal music, which can be characterized by the terms "prolongational," "associational," and "transformational."[1] A prolongational analysis of a given post-tonal work identifies some pitches as structural and explains the others as embellishing the structural ones in various ways. The prolongational approach to the analysis of post-tonal music begins with Schenker, who used his theories of harmony and counterpoint as a club with which to beat Stravinsky.[2] Though Schenker analyzes

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[1] See Joseph N. Straus, "Voice-Leading in Atonal Music," in *Music Theory in Concept and Practice*, eds. James Baker, David Beach, and Jonathan Bernard (Rochester: University of Rochester Press, 1997), 237-274.

[2] Heinrich Schenker, *The Masterwork in Music*, vol. 2, trans. Ian Bent et al. (Cambridge: Cambridge University Press, 1994), 17-18.

sixteen measures of Stravinsky's Concerto for Piano and Wind Instruments in order to show why it is "altogether bad, inartistic, and unmusical," and why it "does not yet merit the name music," some theorists who do not share Schenker's aesthetic preferences and disagree with his criticisms, nevertheless find his analysis of the *Concerto* to hold great explanatory power.[3] Many theorists have followed Schenker's lead and have contributed their own prolongational analyses of post-tonal music; of these, perhaps the most influential was Felix Salzer, one of Schenker's students.[4] In Salzer's most provocative book, *Structural Hearing*, he

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[3] Ibid., 17-18.

[4] See Adele Katz, *Challenge to Musical Tradition: A New Concept of Tonality* (New York: Knopf, 1945; Da Capo, 1972); Felix Salzer, *Structural Hearing: Tonal Coherence in Music* (New York: Charles Boni, 1952; Dover, 1962); Allen Forte, *Contemporary Tone Structures* (New York: Teacher's College, Columbia University, 1955); Roy Travis, "Towards a New Concept of Tonality?" *Journal of Music Theory* 3 (1959), 257-284; "Directed Motion in Schoenberg and Webern," *Perspectives of New Music* 4 (1966), 84-89; and "Tonal Coherence in the First Movement of Bartók's Fourth String Quartet," *Music Forum* 2 (1970), 298-371; Joel Lester, "A Theory of Atonal Prolongations as Used in an Analysis of the Serenade, Op. 24, by Arnold Schoenberg," (Ph. D. dissertation, Princeton University, 1971); Robert Morgan, "Dissonant Prolongation: Theoretical and Compositional Precedents," *Journal of Music Theory* 20 (1976), 46-91; Edward Laufer, "Review of Schenker's *Free Composition*," *Music Theory Spectrum* 3 (1981), 154-184; Paul Wilson, "Concepts of Prolongation and Bartók's Opus 20," *Music Theory Spectrum* 6 (1984), 79-89; and *The Music of Bela Bartók*, (New Haven: Yale University Press, 1992); Steve Larson, "A Tonal Model of an 'Atonal' Piece: Schoenberg's Opus 15, Number 2," *Perspectives of New Music* 25/1-2 (1987), 418-433; Fred Lerdahl, "Atonal Prolongational Structure," *Contemporary Music Review* 4 (1989), 65-89; James Baker, "Voice-Leading in Post-Tonal Music: Suggestions for Extending Schenker's Theory," *Music Analysis* 9/2 (1990), 177-200; and "Post-Tonal Voice-Leading," in *Models of Musical Analysis: Early Twentieth Century Music*, ed. Jonathan Dunsby (Oxford: Basil Blackwell, 1993), 20-41; Charles Morrison, "Prolongation in the Final Movement of Bartók's String Quartet No. 4," *Music Theory Spectrum* 13/2 (1991), 179-196; Edward Pearsall, "Harmonic Progressions and Prolongation in Post-Tonal Music," *Music Analysis* 10/3 (1991), 345-356; and Jack Boss, "Schoenberg's Op. 22 Radio Talk and Developing Variation in Atonal Music," *Music Theory Spectrum* 14/2 (1992), 125-149; and "Schoenberg on Ornamentation and Structural Levels," *Journal of Music Theory* 38/2 (1994), 187-216.

expressed faith in "a 'new' tonality made possible through the powerful influences of Hindemith, Bartók, and Stravinsky."[5] Although some theorists may have questioned whether this music could be understood in terms of prolongation from the start, in the 1980s, theorists began to challenge prolongational analyses of post-tonal music in print.[6] In "The Problem of Prolongation in Post-Tonal Music," Joseph Straus argued that "the most profound structural determinant of common-practice tonality – prolongation – plays a negligible role in the music most characteristic of this century," and that the concept will aid analysis "only for brief, isolated moments."[7] Straus identified four necessary conditions for prolongation that post-tonal works commonly fail to meet: 1) "the consonance-dissonance condition: a consistent pitch-defined basis for determining structural weight"; 2) "the scale-degree condition: a consistent hierarchy of consonant harmonies"; 3) "the embellishment condition: a consistent set of relationships between tones of lesser and tones of greater structural weight"; and 4) "the harmony/voice leading condition: a clear distinction between the vertical and horizontal dimensions."[8] Ever since his article appeared in 1987, those theorists who have dared to speak of prolongation in this repertoire have been very careful to address Straus's arguments against doing so.[9]

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[5] Salzer, *Structural Hearing*, 7.

[6] See James Baker, "Schenkerian Analysis and Post-Tonal Music," in *Aspects of Schenkerian Theory*, ed. David Beach (New Haven: Yale University Press, 1983), 153-88; Allen Forte, "Tonality, Symbol and Structural Levels in Berg's *Wozzeck*," *The Musical Quarterly* 71 (1985), 474-99; and Joseph N. Straus, "The Problem of Prolongation in Post-Tonal Music," *Journal of Music Theory* 31/1 (1987), 1-21.

[7] Ibid., 19.

[8] Ibid., 2-5.

[9] For example, see Fred Lerdahl, "Atonal Prolongational Structure", 65-87; and Jack Boss, Schoenberg's Op. 22 Radio Talk," 125-149; and "Schoenberg on

Instead of a prolongational approach, Straus advocated that theorists take an "associational" approach to the analysis of post-tonal music.[10] An associational analysis of a post-tonal work explains pitches in the context of a motivic network without establishing a structural hierarchy of pitches. Those analyzing post-tonal works from an associational perspective have found mod12 pc-set theory to be useful in describing motivic relationships; though there is a strongly motivic/associational aspect to Schenker's theory as well, the tonal and prolongational implications that are inextricably bound up in any Schenkerian discussion of motive makes its use problematic in a post-tonal context.[11]

Ornamentation," 187-216. Straus's necessary conditions have also recently been called into question by Steve Larson, though Larson does not apply prolongation to post-tonal works. See Steve Larson, "The Problem of Prolongation in *Tonal* Music: Terminology, Perception, and Expressive Meaning," *Journal of Music Theory* 41/1 (1997), 101-136; see also Straus's response in that same issue, Joseph N. Straus, "Response to Larson," *Journal of Music Theory* 41/1 (1997), 137-139.

[10] Straus, "The Problem of Prolongation," 8-19.

[11] Associational approaches using pitch-class set theory have now been widely disseminated through four books; see Allen Forte, *The Structure of Atonal Music* (New Haven: Yale University Press, 1973); John Rahn, *Basic Atonal Theory* (New York: Longman, 1980); Robert D. Morris, *Composition with Pitch-Classes: A Theory of Compositional Design* (New Haven: Yale University Press, 1987); Joseph N. Straus, *Introduction to Post-Tonal Theory* (Englewood Cliffs, New Jersey: Prentice Hall, 1990). For more on the motivic aspect of Schenkerian analyses; see Heinrich Schenker, *Free Composition*, trans. Ernst Oster (New York: Longman, 1979), 97-101; Charles Burkhart, "Schenker's Motivic Parallelisms," *Journal of Music Theory* 22 (1978), 145-76; John Rothgeb, "Thematic Content: A Schenkerian View," in *Aspects of Schenkerian Theory*, ed. David Beach (New Haven: Yale University Press, 1983), 39-60. For a critique of the sometimes precarious balance in Schenkerian theory between prolongational and associational approaches, see Richard Cohn and Douglas Dempster, "Hierarchical Unity, Plural Unities: Toward a Reconciliation," in *Disciplining Music: Musicology and its Canons*, ed. Bergeron and Bohlman (Chicago: University of Chicago Press, 1992), 156-181; Richard Cohn, "The Autonomy of Motives in Schenkerian Accounts of Tonal Music," *Music Theory Spectrum* 14/2 (1992), 150-70; and "Schenker's Theory, Schenkerian Theory: Pure Unity or Constructive Conflict?" *Indiana Theory Review* 13/1 (1992), 1-20.

A "transformational" analysis of a post-tonal work is similar to an associational approach in that it also explains pitches in the context of motivic networks without establishing a structural hierarchy. But while an associational analysis focuses on motives that are musical objects (commonly represented as pc sets), a transformational analysis focuses on motives that are successions of operations (commonly represented as transformational networks). To understand the difference between the associational and transformational approaches, one must also understand the difference between "lines" and "voices." "Lines" are pitch-class segments that have an identical value in some musical domain; the notes of a given segment may be linked through a wide variety of means, including melodic contour, register, attack point, duration, instrumentation, articulation, and dynamics.[12] "Voices," on the other hand, represent pitch-class segments resulting from mappings within a succession of operations (e.g. a transformational motive T_2 followed by T_3 connecting three different chords might yield the voice <C D F>, though this voice might not be realized as a line in the music; i.e. there might not be a single domain such as instrumentation or register in which the pitch classes C, D, and F could be linked). The associational approach focuses on how lines express motives that are musical objects, while the transformational approach focuses on how voices express motives that are successions of operations. Like those adopting an associational approach, theorists that have taken a transformational approach to post-tonal music have done so using the language of mod12 pc-set theory almost exclusively.[13]

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[12] For a more detailed examination of lines, see Christopher Hasty, "Segmentation and Process in Post-Tonal Music," *Music Theory Spectrum* 3 (1981), 54-73.

[13] David Lewin, "Transformational Techniques in Atonal and Other Music Theories," *Perspectives of New Music* 21 (1982-83), 312-371; *Generalized Musical Intervals and Transformations* (New Haven: Yale University Press,

Diatonicism in post-tonal music presents a special problem for the associational and transformational approaches discussed above because of the inherently chromatic bias of the mod12 set theory they both employ, and it is equally frustrating for those taking a prolongational approach because, though the diatonic nature of the musical surface may serve as a hotbed for tonal allusions, the music nevertheless fails to meet Straus's necessary conditions for prolongation. One response to the problem of analyzing post-tonal diatonic music has been to focus on identifying pc collections and their tone centers. The first to respond to the problem of post-tonal diatonic music in this way was Arthur Berger, in his seminal article "Problems of Pitch Organization in Stravinsky." [14] In outlining his approach to Stravinsky's music, Berger states that "a worthwhile objective is certainly an approach that would no longer use tonality as a crutch, a new branch of theory, as it were, starting from what the music itself is, rather than what music was previously."[15] His article closely examined the intersection of diatonic and octatonic collections in Stravinsky's music, and was the spark that produced a much larger study by Pieter van den Toorn, *The Music of Igor Stravinsky*, which

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1987); "Klumpenhouwer Networks and Some Isographies that Involve them," *Music Theory Spectrum* 12/1 (1990), 83-120; "A Tutorial on Klumpenhouwer Networks, Using the Chorale in Schoenberg's Op. 11, No. 2," *Journal of Music Theory* 38/1 (1994), 79-102; and *Musical Form and Transformation: 4 Analytic Essays* (New Haven: Yale University Press, 1993); John Roeder, "A Theory of Voice Leading for Atonal Music," (Ph. D. dissertation, Yale University, 1984); "Harmonic Implications of Schoenberg's Observations of Atonal Voice Leading," *Journal of Music Theory* 33/1 (1989), 27-62; "Voice Leading as Transformation," in *Essays in Honor of David Lewin* (In *Essays in Honor of David Lewin* (Boston: Ovenbird Press, 1995), 41-58; Henry Klumpenhouwer, "A Generalized Model of Voice-Leading for Atonal Music," (Ph. D. dissertation, Harvard University, 1991).

[14] Arthur Berger, "Problems of Pitch Organization in Stravinsky," *Perspectives of New Music* 2/1 (1963), 11-42.

[15] Ibid., 11.

deftly segmented a great number of passages drawn from the composer's early, middle, and late periods into intersecting diatonic and octatonic collections, demonstrating how these collections serve as unifying elements throughout Stravinsky's *oeuvre*.[16] The Berger/Van den Toorn approach is in essence an associational one, in that it focuses on musical objects (pc collections) without a structural hierarchy of pitches.[17] Their approach does not make claims of prolongation, nor does it make use of pc sets, and thus avoids the pitfalls common to those analytic enterprises. However, by focusing on the identification of pc collections, they have consequently limited their observations to the musical surface, a limitation with which theorists since Schenker have become increasingly dissatisfied.

Allen Forte was naturally the first to apply pc-set theory to post-tonal diatonic works by Stravinsky and Bartók, though his applications in *The Structure of Atonal Music* were not intended as analyses, but merely as examples of how his associational model might be applied to music. In *The Harmonic Organization of "The Rite of Spring,"* however, Forte applied set theory to the analysis of an entire work, an application that was considered appropriate by some and

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[16] Pieter Van den Toorn, *The Music of Igor Stravinsky* (New Haven: Yale University Press, 1983). For more of Van den Toorn's work on diatonic collections in Stravinsky, see Pieter Van den Toorn, "Some Characteristics of Stravinsky's Diatonic Music," *Perspectives of New Music* 14.1 (1975), 104-138; *Stravinsky and "The Rite of Spring": The Beginnings of a Musical Language* (Berkeley and Los Angeles: University of California Press, 1987); and "Stravinsky Re-Barred," *Music Analysis* 7/2 (July 1988), 165-196.

[17] There is an implicit prolongational component, however, because it does assign centers to the collections, and in doing so establishes the most primitive kind of structural hierarchy (a hierarchy analogous to the relationship between a king and his pawns in chess, when his queen, his knights, his bishops, and his rooks are absent from the chess board).

misguided by others;[18] the success of the study was the subject of a debate in *Music Analysis*.[19] The debate is perfectly understandable, regardless of which side one chooses; though the work is only diatonic to a limited degree, there is enough diatonic material in *The Rite* to justify a dissatisfaction with any analysis of it that does not recognize the work's tonal implications, given that diatonic musical surfaces in post-tonal works are a breeding ground for tonal allusions, and are thus often heard in relation to tonality. On the other hand, *The Rite* is foreign enough to tonality to justify a dissatisfaction with any analysis that attempts to explain its pitch organization solely in terms of functional harmony and species counterpoint. Despite the misgivings many scholars had about applying set theory to post-tonal music, the approach has been widely adopted and has been the primary one for the past twenty years.

Until now, the most common response to the conflict between analyses of post-tonal diatonic music that focus on tonal allusions and those that use set theory to highlight post-tonal motivic relationships has been to present a dialectical view of the work in question, one which analyzes the music from tonal and pc-set perspectives in turn.[20] A dialectical approach aims to find the meaning behind a

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[18] See Allen Forte, *The Harmonic Organization of "The Rite of Spring,"* (New Haven: Yale University Press, 1978); Robert Moevs, "Review of *The Harmonic Organization of the Rite of Spring*, by Allen Forte," *Journal of Music Theory* 24/1 (Spring 1980), 97-107;
Susan Tepping, "A Review of Allen Forte's *The Harmonic Organization of the Rite of Spring*," *Indiana Theory Review* 4/1 (1980), 79-88.

[19] See Richard Taruskin, "Letter to the Editor," *Music Analysis* 5 (1986), 313-320; and Allen Forte, "Letter to the Editor in Reply to Richard Taruskin," *Music Analysis* 5 (1986), 321-337.

[20] For a few examples of such responses focusing on works by Stravinsky, see Arnold Whittall, "Music Analysis as Human Science? *Le Sacre du printemps* in Theory and Practice," *Music Analysis* 1 (1982), 33-53; V. Kofi Agawu, "Stravinsky's *Mass* and Stravinsky Analysis," *Music Theory Spectrum* 11 (1989),

given post-tonal diatonic work's pitch organization in the interplay between two opposing interpretations, one tonal and one post-tonal. Both interpretations are inadequate on their own because, on the one hand, the music is not tonal and thus is not suited to an analysis that explains the music in terms of its adherence to and deviation from tonal norms, and on the other hand, the music is not chromatic, and thus the kind of motivic relationships found in it are often resistant to description in terms of pc-sets, which are part of a mod12 (i. e. chromatic) system.

While approaching post-tonal diatonic works from both tonal and pc-set perspectives partially makes up for the shortcomings of each approach when applied individually, there is a lack of integration within a dialectical view that cannot be so easily dismissed. In defending his dialectical view of Stravinsky's *Mass*, Agawu writes "To say that one needs the benefit of these two essentially contradictory perspectives in order to gain the richest sense of structural procedure in the piece is not to compromise the analysis but to accept, first, that the results of either approach are genuinely fragmentary, and second, that the combined results yield irreducible conflicts."[21] Unfortunately, there is a tendency for many important musical connections to fall through the cracks in a dialectical analysis, precisely because they are part of these conflicts. Whether or not Agawu is correct in asserting that these conflicts are irreducible remains to be seen, but such conflicts in the music at any rate do not necessarily require parallel conflicts in the analytic tools we bring to bear on them. It is the goal of this dissertation to integrate the analysis of tonal allusions in post-tonal diatonic music with the

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139-163; and Chandler Carter, "Stravinsky's 'Special Sense': The Rhetorical Use of Tonality in *The Rake's Progress*," *Music Theory Spectrum* 19/1 (1997), 55-80; and "The Progress in *The Rake's* Return," (Ph. D. dissertation, City University of New York, 1995).

[21] Agawu, 161.

analysis of motivic networks by finding a common language in which to express both.

In this dissertation, I argue that the problems inherent in analyzing post-tonal diatonic music can be solved by a careful application of set theory modulo 7, in interaction with the more familiar mod12 set theory. The strength of motivic analysis made possible by mod12 set theory is often weakened considerably when applied to post-tonal diatonic music because it fails to recognize the relationship between diatonic and chromatic partitionings of the octave. In tonal theory we have terms that reflect both; interval names such as "fifth" and "sixth" refer to a diatonic partitioning, while qualities such as "major" and "minor" refer to a chromatic partitioning. But an intervallic label that only specifies the diatonic partitioning and not the chromatic one is not incomplete, as some might initially suppose; it simply refers to a broader category. In discussing tonal music, we can make meaningful statements such as "the seventh of a seventh chord must resolve down by step," which refers to both major and minor sevenths, precisely because we have established a means of describing music purely in terms of a diatonic partitioning, as well as in terms of a chromatic one.

In discussing post-tonal diatonic music, however, we do not have such an elegant language. For example, the pc sets {C D E G} and {E F G B} are representatives of the set classes (0247) and (0137), respectively, but these labels miss something. Both sets share the same diatonic pattern: moving from left to right within each set, they both move up a step twice, and then leap up a third. The mod7 set class (0124) represents both of these sets and allows us to map the first onto the second by a mod7 operation: transposition up a third. Thus, by adapting the tools of mod12 set theory to work in a mod7 diatonic universe, the analytic strength of set theory is retained.

There have been many music theorists who have already used a mod7 perspective in their investigations of seven-tone collections in general and of the diatonic collection in particular. As Jay Rahn has pointed out, there are three distinct ways in which seven-tone collections have been approached.[22] The first has been to discuss these collections purely in terms of their respective mod12 intervals (their "specific" interval sizes),[23] the second has been to discuss these collections in terms of their respective mod7 intervals (their "generic" interval sizes),[24] and the third has been to discuss these collections in terms of the connections between their respective mod7 and mod12 intervals.[25] (The terms

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[22] Jay Rahn, "Coordination of Interval Sizes in Seven-Tone Collections," *Journal of Music Theory* 35 (1991), 34.

[23] See Milton Babbitt, "The Structure and Function of Music Theory I," *College Music Symposium* 5 (1965), 49-60; Hubert Howe, "Some Combinational Properties of Pitch Structures," *Perspectives of New Music* 4/1 (1965), 45-61; Carlton Gamer, "Some Combinational Resources of Equal-Tempered Systems," *Journal of Music Theory* 11/1 (1967), 32-59; Eric Regener, "On Allen Forte's Theory of Chords," *Perspectives of New Music* 13/1 (1974), 191-212; Robert Cogan and Pozzi Escot, *Sonic Design* (New York: Prentice-Hall, 1976); Richmond Browne, "Tonal Implications of the Diatonic Set," *In Theory Only* 5/6-7 (1981), 3-21; and Robert Gauldin, "The Cycle-7 Complex: Relations of Diatonic Set Theory to the Evolution of Ancient Tonal Systems," *Music Theory Spectrum* 5 (1983), 39-55.

[24] See Eric Regener, ibid, 1974; John Clough, "Aspects of Diatonic Sets," *Journal of Music Theory* 23/1 (1979), 45-61, "Diatonic Interval Sets and Transformational Structures," *Perspectives of New Music* 18 (1979-80), 461-482; John Clough and Gerald Myerson, "Variety and Multiplicity in Diatonic Systems," *Journal of Music Theory* 29/2 (1985), 249-69; and John Clough, "Diatonic Interval Cycles and Hierarchical Structure," *Perspectives of New Music* 32/1 (1994), 228-253.

[25] See Eytan Agmon, "Diatonicism, Chromaticism, and Enharmonicism: A Study in Cognition and Perception," (Ph. D. dissertation, City University of New York, 1986), and "A Mathematical Model for the Diatonic System," *Journal of Music Theory* 33/1 (1989), 1-25; Norman Carey and David Clampett, "Aspects of Well-Formed Scales," *Music Theory Spectrum* 11/2 (1989), 187-206; Clough and Myerson, ibid; and Jay Rahn, "Constructs for Modality, ca. 1300-1550," *Canadian*

"specific" and "generic" interval sizes were coined by Clough and Myerson.)[26] However, these theorists have thus far been more concerned with revealing properties specific to seven-tone collections or to the diatonic collection than with revealing the pitch organization of individual works.[27] This dissertation will propose a system of mod7 set theory designed specifically for the analysis of post-tonal diatonic music and apply that system to a range of post-tonal diatonic works in order to demonstrate its validity.

Set Theory, Modulo 7

This chapter will present a mod7 set theory designed to be a systematic tool for analyzing those portions of the post-tonal repertoire that are primarily diatonic. Because analyzing post-tonal music mod7 is new to many theorists, a good foundation in this new system will be necessary. Most of this system can simply be borrowed and adapted from the existing mod12 system as shaped by Milton Babbitt, Allen Forte, Robert D. Morris, and David Lewin, and as codified in pedagogical texts by John Rahn and Joseph N. Straus.[28] Nevertheless, because

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Association of University School of Music Journal 8/2 (1989) 5-39, and "Coordination of Interval Sizes in Seven-Tone Collections," *Journal of Music Theory* 35 (1991), 33-60.

[26] Clough and Meyerson, 256.

[27] A notable exception to this is John Clough's analysis of Beethoven's Bagatelle, Op. 119, No. 1, in "Aspects of Diatonic Sets," *Journal of Music Theory* 23/1 (1979), 56-60.

[28] Milton Babbitt, "Some Aspects of Twelve-Tone Composition," *The Score* and *IMA Magazine* 12 (1955), 53-61, "Twelve-Tone Invariants as Compositional Determinants," *The Musical Quarterly* 46/2 (1960), 245-59, and "Set Structure as a Compositional Determinant," *Journal of Music Theory* 5/2 (1961), 72-94; Allen Forte, *The Structure of Atonal Music* (New Haven: Yale University Press, 1973); John Rahn, *Basic Atonal Theory* (New York: Longman, 1980); George Perle, *Serial Composition and Atonality*, 5th ed. (Berkeley and Los Angeles: University

there has been much fine tuning of the existing mod12 system by each of the theorists mentioned above, it is necessary to state explicitly how the mod7 system proposed here works and what nomenclature will be used. This presentation will then be followed by several analyses of post-tonal diatonic music, in which mod7 and mod12 approaches will be compared.

Now let us turn to the mod7 system that is the subject of this chapter, a system intended specifically for application in the analysis of post-tonal diatonic music. The system proposed here is meant to be used as a tool for analyzing those portions of the post-tonal repertoire that are primarily diatonic, and therefore differs from that of other theorists using a mod7 system for different purposes. Its explication will take Straus's *Introduction to Post-Tonal Theory* and Rahn's *Basic Atonal Theory* as its two main models, as both authors have already synthesized a large amount of earlier work.

Pitches, Pitch Classes, and Step Classes

It will be useful to distinguish between pitches, pitch classes, and step classes.[29] Pitches and pitch classes retain their traditional definitions in a mod7 system. A pitch is simply defined as a tone of a particular frequency. A pitch class is defined as the set of all pitches that are octave-equivalent. To these familiar definitions we add a new one which accounts for the similarities between generalized diatonic pitch structures. A "step class" in a diatonic mod7 system can

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of California Press, 1981); David Lewin, *Generalized Musical Intervals and Transformations* (New Haven: Yale University Press, 1987); Robert D. Morris, *Composition with Pitch-Classes: A Theory of Compositional Design* (New Haven: Yale University Press, 1987); and Joseph N. Straus, *Introduction to Post-Tonal Theory* (Englewood Cliffs, New Jersey: Prentice Hall, 1990).

[29] The following discussion is indebted to Morris, *Composition with Pitch-Classes*, 22-26.

be defined as the set of all notes of a particular letter name, regardless of accidentals or register (e.g. E2, E♭3, and E♯4 would all be members of the same step class).[30] One will note that the traditional notion of octave equivalence, our tendency to hear all octave-related frequencies as equivalent, is retained in our new definition, but the additional generalization of step-class membership is added, a generalization which allows us to see beyond the chromatic partitioning of the octave to larger diatonic contexts. The integer notation and labeling for the diatonic mod7 system proposed here is given as Figure 1.1; it is divided into twelve parts, one for each of the twelve diatonic collections, represented here as major scales. Each collection has one and only one representative for each step class. For the purposes of this dissertation, we will use a strictly "fixed do" notation in which 0 is always assigned to C, C♯, or C♭. However, it is also possible to adopt a "movable do" integer notation system in which 0 is assigned to the primary tone center of a given passage of music, and the remaining pitches are assigned integers relative to this contextual point of reference. As one can see, no matter what the diatonic collection, each letter name is always associated with the same integer. Thus, the step-class C (or, in integer notation, 0) would be represented by the pitches C, C♯, or C♭, depending on which diatonic collection prevailed at any given time, the step-class D (or 1) would be represented by either D, D♯, or D♭, and so on. The labeling of the twelve diatonic collections can be easily committed to memory by noting that each label's Arabic numeral represents the number of sharps in its corresponding major key signature, or the mod12 complement of the number of flats in its corresponding major key signature (the "DT" in the label is an abbreviation for diatonic). By excluding any reference to

[30] Stephen Dembski was the first to use the term "step class" in his paper "Steps and Skips from Content and Order: Aspects of a Generalized Step-Class System," (paper presented at the annual meeting of the Society of Music Theory, 1988).

each collection's center, these neutral labels will avoid the problem of creating a premature bias as to the collection's center.

Before beginning to describe how the mod7 set theory used in this dissertation works, some important questions concerning the validity of its application should be addressed. One stumbling block for many in embracing a mod7 set theory is that it requires one to ignore the placement of half steps and whole steps within a diatonic collection. But before criticizing what seems to be a lack of concern for the differences between specific interval sizes, one might do well to remember certain impressions common to all musicians. As Eytan Agmon noted, "One of the most striking facts about hearing a 'diatonic scale' is that we seem to be far less impressed by the inequality of 'half steps' and 'whole steps' than one might have supposed; our impression is rather that of a uniform progression from one 'scale step' to another."[31] Our notational system reflects this sense of uniformity. To someone who was familiar only with the notation of diatonic music, the octave would appear to be divided into seven parts.[32] Such a person would have no way of knowing that there is a distinction between half steps and whole steps within the diatonic scale. But what is to be gained from suppressing such information in an analysis? John Clough, in giving his own reasons for positing "a diatonic universe of seven pitch classes in which all intervals of the same general name are regarded as equivalent", wrote the following:

> ...musical constructs may be usefully analyzed and compared on the basis of less than complete information. I argue further...that certain pitch-structural features can be more clearly observed when intervallic information is distilled in a particular way. I believe it is a useful form of inquiry to ask: Exactly what

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[31] Eytan Agmon, "Diatonicism, Chromaticism, and Enharmonicism," 26.

[32] Ibid., 26-27.

> things are revealed through study of a limited set of pitch-interval characteristics? This posture does not advocate the permanent eschewal of information in musical analysis. We do not give up anything in making a selective inquiry; we merely set something aside.[33]

Selective inquiry is obviously not new in the world of music theory, and any theorist can remember examples in which its application in the past has produced worthwhile results.

Chromaticism often inflects music that is basically diatonic. Though mod7 analysis can still be a very powerful tool in understanding such music, we will address the problem of dealing with chromaticism in a mod7 analysis later. For the present, let us simply imagine the white notes of the piano as our referential collection, though any of the transpositions of that collection would of course work just as well.

Intervals

It is unnecessary to change the basic concepts of ordered and unordered pitch and pitch-class intervals, but again we will add new ones particular to a mod7 environment: ordered and unordered step-class intervals. A *step-class interval* is the distance in mod7 steps between two step classes; an *ordered step-class interval* measures the distance between an ordered pair of step classes, while an unordered step-class interval measures the distance between a pair of unordered step classes. There are only 6 ordered step-class intervals and only 3 unordered step-class intervals in the mod7 system. As a review, Examples 1.1a, 1.1b, 1.1c, and 1.1d illustrate ordered pitch intervals, unordered pitch intervals, ordered step-

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[33] John Clough, "Diatonic Interval Sets and Transformational Structures," *Perspectives of New Music* 18 (1979-80), 467.

class intervals, and unordered step-class intervals respectively. A pitch interval is the distance between two pitches, measured by the number of half steps between them. Ordered pitch intervals (Ex. 1.1a) indicate melodic contour, and are preceded by either a plus sign (indicating a higher pitch) or a minus sign (indicating a lower one), while unordered pitch intervals (Ex. 1.1b) do not indicate melodic contour, and are not preceded by a sign. A pitch-class interval is the distance in semitones between two pitch classes. An ordered pitch-class interval is the distance between an ordered pair of pitch classes (Ex. 1.1c) and is measured by subtracting the first pitch class from the second; if a negative number results, it must be added to 12 to yield a positive mod12 representation of the interval. An unordered pitch-class interval is the shortest possible distance between an unordered pair of pitch classes (Ex. 1.1d).

Examples 1.1e and 1.1f illustrate ordered and unordered step-class intervals respectively. As with pitch-class intervals, ordered step-class intervals (Ex. 1.1e) are always measured by subtracting the first step class from the second; if a negative number results, it must be added to 7 to yield a positive mod7 representation of the interval. Unordered step-class intervals or mod7 interval classes (Ex. 1.1f), measure the shortest possible distance between two step classes. Hence, the larger mod7 intervals of 6, 5, and 4, are represented as mod7 interval-classes by their respective complements 1, 2 and 3.

Sets, Segments, Set Classes, and Segment Classes

The term "step-class set" will be used to define any unordered collection of step classes (excluding any step-class duplications), while "step-class segment" will be used to define any ordered collection of step classes (including step-class duplications). A set class is a group of step-class sets that are all related by some specified operation. The seven step classes of the diatonic mod7 world when grouped together will be referred to as the total diatonic. Following Rahn, there will be a distinction draw between two different kinds of mod7 set classes, transpositional-types (Tn-types) and transpositional/ inversional-types (Tn/TnI-types). A mod7 Tn-type is a step-class set that represents itself and all other step-class sets that are transpositionally equivalent to itself. Example 1.2a shows all of the step-class sets that are represented by the mod7 Tn-type [023]. The step-class set [023] was selected to represent the mod7 Tn-type because it is in a standard form that will be discussed later. A mod7 Tn/TnI-type is a step-class set that represents itself and all other step-class sets that are transpositionally or inversionally equivalent to itself. Example 1.2b shows all of the step-class sets that are represented by the mod7 Tn/TnI-type (013). The step-class set (013) was selected to represent the mod7 Tn/TnI-type because it is also in a standard form which will be discussed later. A mod7 segment class is a segment that represents itself and all other mod7 segments that are transpositionally equivalent to itself (an analogue to the mod7 Tn-type of an unordered set of step classes).

In this mod7 system, curly brackets will be used to indicate step-class sets, square brackets will be used to indicate Tn-types, parentheses will be used to indicate Tn/TnI-types, angle brackets will be used to indicate segments, and angle brackets enclosed in square brackets will be used to indicate segment classes. When combining mod7 and mod12 perspectives in an analysis, one must be careful to distinguish between those set classes that are mod7 and those that are mod12. In cases where confusion may arise, superscripts denoting the modular space of the collection will be added. Thus the pc set {B C F} is an expression of $(014)^7$ and of $(016)^{12}$, the first of which is a mod7 Tn/TnI-type and the second of which is a mod12 Tn/TnI-type.

Prime forms for mod7 set classes and segment classes (the pitch-class sets and segments chosen to represent each mod7 set class and segment class) are determined in the usual way following Rahn, as described in the pedagogical texts by Rahn and Straus.[34] A segment class will always represent seven different segments, no more and no less, a fact ensured by its identity as an ordered set. Similarly, a mod7 Tn-type will always represent seven different step-class sets, no more and no less, because there are no transpositionally symmetrical sets in the mod7 universe. (This will be shown in the following section). Because segments can be of any size, there is no finite number of possible segment classes.

Example 1.3 shows a mod7 interval vector, which is a shorthand notation for representing the number of members from each mod7 interval-class in a given step-class set. An interval vector in the mod7 system is therefore limited to three places: the first place for mod7 interval-class 1, the second for 2, and the third for 3. Any step-class sets that share the same interval vector are related by either transposition or inversion; there are no Z-related step-class sets mod7.

The Mod7 Set List

A comprehensive list of mod7 Tn-types is given as Appendix A. It is arranged as follows: the first column lists the dyadic and trichordal Tn-types and the fifth column lists their complementary Tn-types (tetra- and pentachords), the second column lists the interval vectors for the Tn-types in the first column, the fourth column lists the interval vectors for the Tn-types in the fifth column. The third column lists two numbers, the first of which is the degree of transpositional symmetry for the Tn-types in the first and fifth columns and the second of which is the degree of inversional symmetry for those Tn-types (note that no mod7 Tn-types are transpositionally symmetrical beyond the trivial case of T_0). The final

[34] See Rahn, *Basic Atonal Theory*, 75-76; and Straus, *Introduction to Post-Tonal Theory*, 41-42. It should be noted that Rahn's method differs slightly from Forte's. See Allen Forte, *The Structure of Atonal Music*, 3-5.

column lists the cyclic generators for the given sets. The null set and the total diatonic are not listed. There are 18 Tn-types total. A list of Tn/TnI-types would be identical except for the fact that the [013] and [023] would be condensed to (013) and their complements [0124] and [0234] would be condensed to (0124). There are therefore 16 Tn/TnI-types.

Appendix B provides a chart that maps the diatonic mod7 Tn/TnI-types onto their mod12 equivalents. Because not all mod12 Tn/TnI-types are subsets of the total diatonic, not all of them will have equivalent mod7 Tn/TnI-types. It is worth noting that all mod12 Tn/TnI-types which contain a subset that has no diatonic equivalent will themselves have no diatonic mod7 equivalents. For example, all the mod12 Tn/TnI-types which have as a subset the mod12 Tn/TnI-type (012) -- (0123), (0124), (0125), (0127), (01234), (01235), (01236), and so on -- will have no diatonic mod7 equivalents by extension.

Transposition and Inversion

As in the mod12 universe, there are only two interval preserving operations, mod7: transposition and inversion. Transposition and inversion will both be discussed as operations that map one step class or step-class set (let us call this set A) onto another step class or step-class set (let us call this set B). Transposition (see Example 1.4a) maps set A onto set B by adding the same number of mod7 steps, n, to each pitch in set A, and this transposition is then notated T_n. Inversion (see Example 1.4b) maps set A onto set B by dividing the mod7 clock by a given axis and moving each pitch in set A from its position on one side of the axis to a similar position (an equal distance from the axis) on the other side.

The same axis of inversion can be represented in five different ways, as is shown in Ex. 1.4b. An inversion may be represented by any one of its pitch-class mappings, or by its index number, which is the mod7 sum of any pitch class in set X and its corresponding pitch class in set Y; that is, any pitch class in set Y that is an equal distance on the mod7 clock from the axis of inversion. Example 1.5 shows the seven possible axes of inversion represented on a mod7 clockface.

Retrogression and Rotation

Retrogression and rotation are both operations, which are by their nature limited to ordered groups of pitches. Retrogression is an operation that reverses the order positions of a given segment, and is notated R. Thus segment <04325>, when retrograded, becomes <52340>, or segment-class [<04562>]. Rotation is an operation that rotates the given step classes in a segment by a given number of positions to the right, *n*, and is notated R_n. Thus R_1 of <04325> is <50432>, R_2 of <04325> is <25043>, R_3 of <04325> is <32404>, and R_4 of <04325> is <43250>. As is shown by our example, the number of possible values for R_n of segment X is limited to x -1, where x represents the cardinality of segment X.

Common-Tone Relationships

Common tones between transpositionally or inversionally related step-class sets provide an important musical link in much post-tonal music. The number of common-tones that would result from a given transformation of a given step-class set can be easily found by consulting the given set's interval vector. The number of times interval class *n* occurs within the set will equal the number of common tones shared between the set and its transposition by interval *n*. The number of common tones between two sets related by inversion can be found by writing out all possible pairs of elements in the given set in integer notation, finding their mod7 sums, and then finding the mod7 sum of each element added to itself. Each sum resulting from the pairs of elements represents two common tones when inverted by the index number equaling that sum; each sum resulting from an element added to itself represents one common tone when inverted by the index number equaling that sum.

Degrees of Symmetry

As in the mod12 system, there are a number of Tn/TnI-types in the mod7 system that map onto themselves under more than one operation. All mod7 Tn-types and Tn/TnI-types map onto themselves at T_0, but none maps onto itself at any other transposition level. However, all but two of the Tn/TnI-types, (013) and (0124), map onto themselves at one level of inversion. Tn/TnI-types that map onto themselves at some level of inversion are called inversionally symmetrical. Because there are only 14 possible kinds of operations mod7 (limiting ourselves to transposition and inversion), we may find the number of distinct step-class sets within a given Tn/TnI-type by dividing fourteen by the number of operations that will produce each member of the Tn/TnI-type. For the Tn/TnI-types (013) and (0124), which are not inversionally symmetrical, there are fourteen; for all the other Tn/TnI-types, which are inversionally symmetrical, there are only seven.

Complement Relations

Two sets can be said to be complementary if their union produces the total diatonic without the duplication of any step class. Set X is the literal complement of set Y if it is comprised only of those step classes excluded from set Y. Thus, the literal complement of {C D F G} is the set {E A B}, since these pitches when added to those of the first set comprise the total diatonic. Set X is the abstract complement of set Y if, at some level of transposition or inversion, set X is comprised only of those step classes excluded from set Y. Thus, the two sets {C D F G} and {F B C} are complement-related even though they both contain the step classes C and F, because {F B C} maps onto {E A B} at T_6. Just as in the mod12 world, complement-related sets share a proportional interval vector. In the case of complement-related trichords and tetrachords, we may simply add one to each place in the trichord's interval vector to find the tetrachord's interval vector, or subtract one from each place in the tetrachord's interval vector to find the trichord's interval vector; in the case of complement-related dyads and pentachords, we may simply add three to each place in the dyad's interval vector to find the pentachord's interval vector, or subtract three from each place in the pentachord's interval vector to find the dyad's interval vector (see the list of mod7 set classes). Contrary to the mod12 universe, there are no self-complementary sets mod7, because seven elements cannot be divided evenly.

Superset and Subset Relations

Given two step-class sets, X and Y, if set X is included in set Y, then X is a subset of Y and Y is a superset of X. Because there are far fewer set classes mod7 (only 18 Tn-types and 16 Tn/TnI-types), there are naturally far fewer subset and superset relationships to be made. As in the mod12 system, a set of n elements will contain $2^n - 2$ (2 to the nth power minus 2) subsets. As Example 1.6 shows, given the four-note set class (0123), there are fourteen (16 - 2) possible subsets: four one-note sets, six two-note sets, and four three-note sets. Of course, many of these subsets belong to the same set classes, and we are left with only six different Tn/TnI-types: (0), (01), (02), (03), (012), and (013). A chart of mod7 subset and superset relations is provided as Appendix C. In Appendix C, numbers in the table represent the number of times the smaller sets, listed at the top of each column, are imbricated in the larger sets, listed to the left of each row.

Chromaticism

Although the mod7 system proposed here is meant to be used as a tool for the analysis of post-tonal diatonic music, we need not limit ourselves to *purely* diatonic music. That is, a mod7 analysis could still be a powerful tool when applied to post-tonal music that is primarily diatonic, but that is embellished with chromatic notes, provided that there is a sufficient musical reason to hear the chromatic notes as embellishments. The mod7 system described here is itself purely diatonic, and any pitch outside of the prevailing diatonic collection in a mod7 analysis will be understood as an alteration of one of its seven step classes. A mod7 analysis is therefore reductive to the same extent that the music to which it is applied is chromatic. Two important decisions must be faced in the mod7 analysis of music that is not purely diatonic: one must determine what the prevailing diatonic collection is, and then determine how the chromatic pitches embellish the diatonic ones.

To aid the analyst in determining the prevailing collections throughout a musical work, this dissertation proposes parsing the music into spans that contain at least one complete diatonic collection (i.e. all seven pitch-classes of some diatonic collection must be represented in any given span), beginning a new span whenever a pitch-class appears that is not part of the current span's pitch-class content, and closing a span once a complete diatonic collection has been stated. For example, if the current span contains the DT-0 collection (the notes of C major or one of its modes) and a B♭ appears, then a new span is begun. New spans should overlap as much as possible with the previous span; i.e. they should be extended backward up to a note with the same letter name as the new pitch class, but with a different accidental. For example, the new span should be extended backward in time from the B♭ until a B-natural occurs (because B♭ and B-natural could not fit into the same diatonic collection). If the B♭ mentioned above is preceded by an F major triad which is in turn preceded by a G major triad, then the span should be extended backward from the B♭ to include the F major triad, but not the G major one. In post-tonal diatonic music, spans will often contain more than one diatonic collection because all seven notes of any one collection will not appear before notes of another are introduced. An example of such a span is found at the beginning of Stravinsky's *Serenade in A*, given as Example 1.7. In this passage, both B♭ and B-natural appear before either collection is completed, and thus fall under one span; all seven notes of collection DT-11 are not played until m. 5, when E and G appear for the first time, and the same holds true for collection DT-0.

The analyst should create a fuzzy set for each span with more than one collection present that indicates the degree to which the notes in the span belong to each collection represented therein.[35] Each fuzzy set would therefore consist of two or more diatonic collections, each one with a value that indicates the given span's degree of membership within that collection. For example, a fuzzy set for the first six measures of *Serenade in A* would read {(DT-11, .97), (DT-0, .90)}, and would indicate that 97% of the notes in the span suggest collection DT-11 (the key signature of F major or D minor), while 90% of the notes in the span suggest

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[35] For general information on fuzzy set theory, see Lotfi Zadeh, "Fuzzy Sets," *Information and Control* 8 (1965), 338-353; and Bart Kosko, *Fuzzy Thinking* (New York: Hyperion, 1993), and *Neural Networks and Fuzzy Systems* (Englewood Cliffs: Prentice-Hall, 1992). For some applications of fuzzy sets in music theory, see Brian Robison, "Modifying Interval-Class Vectors of Large Collections to Reflect Registral Proximity Among Pitches," *Music Theory Online* 0/10 (1994); Peter Silbermann, "Pitch Class Salience and Centricity in Stravinsky's *Mass*: An Application of Fuzzy Measurement to Music Analysis," in *Proceedings of the Sixth Annual International Fuzzy Systems Association World Congress*, Vol. 1 (São Paulo, 1995), 321-324, and "A Fuzzy-Set Based Methodology for Analyzing Centricity in the Neoclassic Works by Stravinsky," (M. A. Thesis, Eastman School of Music, 1997); and Ian Quinn, "Fuzzy Extensions to the Theory of Musical Contour," *Music Theory Spectrum* 19/2 (Fall 1997), 232-263.

collection DT-0 (the key signature of C major or A minor). These membership values are determined by dividing the number of notes in the span that are part of the given diatonic collection by the total number of notes in the span. In the case of *Serenade in A*, there are 124 notes in the span (counting notes tied over the barline, but not counting notes tied within a measure), 121 of which belong to collection DT-11, and 112 of which belong to collection DT-0 (there are 3 Bs: 124 – 3 =121; and there are 12 B♭s: 124 – 12 = 112). Thus, 121 divided by 124 yields a membership value of 97% in collection DT-11, while 112 divided by 124 yields a membership value of 90% in collection DT-0.

The problems facing a mod7 analysis of music that is not strictly diatonic can be great or small depending on the particular circumstances. Let us again consider the first six measures from Stravinsky's *Serenade in A*. As has been shown above, there are two different diatonic collections that could be used to describe the structure of this passage. These two collections have six common tones: A, C, D, E, F, and G. One of the remaining two pitches that complete the mod 12 pitch-class content of this passage, namely B or B♭, must be described as a chromatic pitch in a mod7 analysis of this music. In determining what note of a given referential collection is chromatically altered, one must take into account the contextual nature of the chromatic pitch in question considered in the light of the referential collection. We might say that the referential collection here is DT-0 (A minor), and that the B♭ is a chromatic pitch employed as a passing tone to B and as an upper neighbor to A. There are good reasons for thinking this: 1) the B♭ is never more than an eighth note in length and always resolves by step either upward or downward; 2) there is an A in the bass on every strong beat in the passage (though it supports an F-major triad in the right-hand part, there are no root-position F-major chords in this passage at all); and 3) the passage ends on a root position A-minor chord. But we also might say that the referential collection here is DT-11 (F major or A Phrygian), and that the B-natural is an upper neighbor to B♭ in the left-hand part (mm. 1, 3 and 4). There are also good reasons for thinking this: 1) there are no B-naturals in the right-hand part; 2) the B-naturals in the left-hand part are never more than a sixteenth note in length; and 3) there is an F-major chord in first inversion on eight consecutive strong beats in mm. 1-5, including the first chord of the passage, which is, of course, the first chord of the piece.

In the absence of functional harmony, these kinds of observations are important in determining how tonal vestiges influence our hearing. If we hear the passage mentioned above as based on an DT-0 collection, then we will classify the B♭ as the upper neighbor to A in our mod7 analysis, and thus interpret it as a chromatic alteration of the pitch one step higher than A, namely B. But even if we hear this same passage as based on an DT-11 collection and classify the B as a chromatic alteration of B♭ (as a passing tone moving to C), our mod7 analysis would remain the same. That is, both the B's and the B♭'s would be represented as B in either reading. The conflicting accidentals would be reduced out to reveal the music's underlying diatonic frame.

In our analysis of the opening to *Serenade in A*, we have seen how one might determine the referential diatonic collection or collections in a given passage, and the degree to which the music belongs to each of those collections. We have also seen how one might determine that a given tone can be considered a chromatic alteration of a referential diatonic collection under a certain set of circumstances. The analyses here will deal with chromaticism on a case by case basis, and the treatment of chromatic pitches in each will hopefully be made clear.

Summary

This chapter began by asserting that post-tonal diatonic music has for a long time presented a problem for analysts. It is not tonal, and so it does not respond well to a tonal analysis, but at the same time, a pc-set analysis does not often work either, because the music is diatonic, and pc-set theory is inherently mod12. This chapter then suggested that adapting pc-set theory to work in a diatonic (mod7) universe would be a solution to this problem. Finally, it has defined the conventions of a particular mod7 system, a system whose features were designed specifically for the analysis of post-tonal diatonic music. The next chapter will apply this mod7 system to passages of post-tonal diatonic music, and in doing so will demonstrate how a mod7 motivic analysis can often be more revealing than one that is exclusively mod12.

Chapter 2

Motivic Analysis, Modulo 7

Having defined our mod7 system in chapter 1, chapter 2 will begin by applying that system to the analysis of five short passages from the music of Stravinsky. Although many twentieth-century composers have written post-tonal diatonic music, Stravinsky has by far received the most attention from theorists. Because of this attention, his music is an ideal starting point for illustrating the kind of insights that might be gained by adding mod7 set theory to our arsenal of analytic approaches. Following the Stravinsky analyses, this chapter will turn to the music of two other composers in order to demonstrate that the advantages inherent in adopting a mod7 perspective are not tied to any one compositional style, but can shed light on any post-tonal diatonic work.

Concerto in D

The first passage is mm. 25-35 from the first movement of *Concerto in D*. In this passage, given as Example 2.1a, the music is composed both linearly and vertically to emphasize $(013)^7$. Although it appears in only two forms, as {C♯ D F♯} or as {F♯ A B}, these two forms dominate this span of music. As a vertical sonority in mm. 25-33, it appears as {C♯ D F♯}; as a linear sonority it appears in the 1st violin part as <F♯ B A> in mm. 25-31, and in the bass part as <F♯ A B> in mm. 25-35, that is, over the entire passage in question.

$(013)^7$ is also projected as a contextual line over the entire passage. In observing the most salient pitches of the 1st violin part, one can see the same group of pitches emerge in a different ordering, as <F♯ A F♯ B F♯> (these pitches are circled in Ex. 2.1a). These pitches are arguably the most salient based on their positions within their respective phrases; all of the pitches except for the B serve as either the first or the last pitch in one of the two phrases, and the B is clearly the peak of the second phrase. (The B in m. 31 is more likely perceived as subordinate to the A which immediately follows it.) One will observe that the E♯ in these measures consistently functions as a lower neighbor to F♯, and is therefore not considered to be a structural pitch. It is almost impossible to hear this E♯ as anything other than a lower neighbor, despite the fact that functional harmony is rarely employed in the movement, because the movement is so replete with other kinds of tonal references (centricities, diatonic pitch collections, and tertian harmonies, to be specific).

The relationship between the two different forms, the vertically expressed {C♯ D F♯} and the linearly expressed {F♯ A B}, is that of inversion around an F♯ axis, as is shown in Example 2.1b. F♯ is undoubtedly the most stressed pitch in both forms of $(013)^7$, highlighting its function as both an inversional axis and a tonal center in this passage, a function it has for much of the rest of the movement (the piece starts on a *tutti* F♯, and a subtle variation of the music given as Ex. 2.1a recurs in mm. 227-47). It is played in four of the five parts (the violas have an octave D skip) in mm. 25-30; it is the only pitch played in the bass until m. 34; until m. 34 it is the only note found on the downbeats of the cello part, which has constant eighth notes throughout; and it is articulated twelve times in the theme, which is ten times more than either A or B.

The inversional relationship between the vertical and horizontal forms of $(013)^7$ would be overlooked in a mod12 analysis. Because the vertical sonority of mm. 25-33 is $(015)^{12}$ while the linear structures project $(025)^{12}$, it would be impossible for the above mentioned vertical and linear harmonies to be rendered equivalent and thus related by inversion about F♯ in a mod12 analysis.

Symphony of Psalms

Our next passage, mm. 6-18 from the first movement of *Symphony of Psalms*, can be described as a composing out of $(0135)^7$ (which represents any quality of seventh chord in major-mode tonal theory), as is shown in Example 2.2a. A highly articulated subdivision of the bars in this passage into 3 + 3 + 5 + 2 is created by the entrance of the solo cello in m. 9, the initiation of its second phrase in m. 12, and the entrance of the altos, oboes, English horn, and bassoons in m. 17, the latter of which is accompanied by a change in rhythmic texture. There are many other viable segmentations of this passage. One could, for example, interpret the bass note F in m. 9 as an upper neighbor to the E that follows, and group together the first five measures. However, the segmentation presented here is not only aurally striking, but it allows diatonic relationships to emerge that link the beginning of each segment to events on the musical surface. The beginning of each segment is reinforced by the presence of $(0135)^7$ as a vertical sonority. The transformational path of these forms of $(0135)^7$ further emphasizes the set by outlining its intervallic content through transposition. The first set form maps onto the second by T_6, mod7, the second maps onto the third by T_5, mod7, and the third maps onto the fourth by T_5, mod7; that is, the music moves from a Cma7 down a step to Bo7, then down a third to G7, and finally down another third to Em7. The pitches within the set $(0135)^7$ can be mapped onto each other via this same transformational path. As shown in Example 2.2b, in the case of {B C E G}, the C maps onto B by T_6, B maps onto G by T_5, and G maps onto E by T_5. From a modal perspective, if one views the musical surface as a Phrygian alteration of E-minor, this chord succession could be read as VI^7-V^7-III^7-I^7, but such chord labels only reveal the extent to which this passage does not conform to functional harmonic progressions, not the logic behind the chord succession.

Although this kind of organization can be succinctly explained from a mod7 perspective, one would not be able to recognize this kind of organization from a mod12 perspective at all, because a mod12 analysis is unable to relate the chords occurring on the first beat of bars 6, 9, 12, and 17 as equivalent. In a mod12 analysis, these four chords are $(0158)^{12}$, $(0258)^{12}$, $(0258)^{12}$, and $(0358)^{12}$, respectively, but in a mod7 analysis, they are all considered equivalent forms of $(0135)^{7}$. From a mod12 perspective, one could only see that all of these chords share a common subset, $(037)^{12}$; to make evident a relationship between the intervallic content of a single set and its transformational path through this music would be beyond the scope of a mod12 analysis.

Agon

Our third passage is mm. 1-20 from the first scene of *Agon*, which is shown in Example 2.3a. $(014)^{7}$ plays a significant role throughout the passage, both as a verticality in mm. 1-3, 7-13, and 20-22, and as a linear entity in mm. 5-6, and 14-19. The set {B C F} in mm. 1-3 gives way in mm. 5-6 to the set {B C F♯}, expressed as contour pitches in the two horn parts (these are circled in Ex. 2.3a). The F♯ in mm. 5-6 does not upset the listener's intuition that this opening music is primarily diatonic; the F♯ is heard as merely a chromatic coloring of F, and thus the first set can be taken to represent both in our analysis. However, the set {B C F} is unequivocally replaced in mm. 7-9 by the chord in the winds, {F G C}. In the contrasting section that follows (mm. 10-13), the strings hammer out three repetitions of {G A D}, though after the initial attack the bass note of the chord, G, is late by a sixteenth note each time. A contrapuntal section played by the brass ensues (mm. 14-19) that is strongly reminiscent of mm. 1-6. The contour pitches in the trumpet parts and the horn I part that receive the most emphasis are {C D G}, while the horn III part plays a rhythmic variant of the horn II's part in mm. 5-

6, the most-emphasized contour pitches being {B C F♯}. (The contour pitches most emphasized in these phrases were interpreted as such because they were the highest, the lowest, the first, and/or the last note in a phrase; more rigorous discussions of emphasis and structural weight will follow in Chapters 3 and 4.) This is followed in m. 20-22 by a repetition of the wind's chord from mm. 7-9, {F G C}.

Each of the various forms of $(014)^7$ mentioned above – {B C F}, {F G C}, {G A D}, {C D G}, {B C F♯}, and {F G C} – is included in one or more of three different transformation paths that replicate the intervallic structure of the set itself. Example 2.3b shows these paths and compares each to the intervallic structure of the set, represented here by the first chord, {B C F}. The first path follows sequential time, but skips the set that occurs in mm. 10-13, limiting itself to the music in which the winds and brass play a part. The second path follows sequential time beginning with the second node of the first path and includes the set in mm. 10-13. The third path, an inversion of the second (note that the transposition levels in the third path are the mod7 complements of the transposition levels in the second), follows sequential time beginning with the third node of the first and second paths.

Only the second of these transformational paths would be available to one approaching this music solely from a mod12 perspective. This is because, while {F G C}, {G A D}, and {C D G} are all members of $(027)^{12}$, {B C F} and {B C F♯} are not. {B C F} and {B C F♯} are both members of $(016)^{12}$, and thus neither could be combined with the three forms of $(027)^{12}$ mentioned above in a transformational path based on transposition. The mod7 perspective of Ex. 2.3b demonstrates a much more comprehensive understanding of this passage than would be possible through a mod12 analysis alone.

Orpheus

Our fourth passage is mm. 1-7 from the first scene of *Orpheus*. The pitch organization in the first seven measures of *Orpheus*, given as Example 2.4a, might be described best as a dialogue between the two sets $(0123)^7$ and $(0124)^7$. One can see them both in the beginning as the first four notes in the harp, <E D C B>, and the first chord in the strings, {E G A B}. Throughout this passage, $(0123)^7$ is the primary linear unit, while $(0124)^7$ is the primary harmonic one. The end of m. 7 marks the end of the scene's first section (m. 8 begins with a change in register and orchestration) and is marked by three linear forms of $(0123)^7$ – <F G A B> in the 1st violin, <C D E F> in the 2nd violin, and <F E D C♯> in the cello – as well as by a vertical form of $(0124)^7$, {B C♯ D F}. The set class $(0123)^7$ is also projected over the entire passage as the pitches of the bass part, {D E F G}. Thus, each form is not only presented in immediate succession, but also over the entire span under consideration.

In Example 2.4b, the step-class counterpoint of $(0124)^7$ in this passage is given. Its two levels show that while there is an oscillation around an E/G inversional axis followed by a descent by step on the "foreground" level, the "background" transformation is inversion around a B axis. The pitch-organization of this passage is fundamentally unlike that of the previous two analyses. The centricity of the entire passage is not reinforced through its transformational scheme as in the passage from *Concerto in D*. Nor does the transformational path of the prevailing harmony replicate its internal structure, as in the passages from *Symphony of Psalms* or *Agon*.

A third kind of global pitch-organization seems to be in evidence. The logic behind the pitch-organization of this passage is tied to the choice of its inversional axis, B. There is only one possible transformation of {E G A B} that will yield the minimum amount of step-class duplication and that is inversion around a B axis. It should be noted that, in any mapping between two mod7 tetrachords, a minimum of one step class will necessarily overlap; the overlapping step class in this case is B, which helps to make the overall transformation more audible. Example 2.4c given a spatial representation of this "almost complementary" relationship. This shift from the opening step-class set to an almost entirely new one by the end of m. 7, seems to reinforce this section's role as an antecedent, leaving the listener dependent on what is to follow. Any other step-class set at m. 7 would share more common tones with the harmony in m. 1, and thus provide a greater sense of closure than does {B C♯ D F}.

These observations are impossible to make mod12 because many of these sets are only equivalent mod7. First and foremost, the opening sonority, {E G A B}, is not equivalent to the final one, {B C♯ D F}. In a mod12 world, the first is a member of $(0247)^{12}$ and the second is a member of $(0236)^{12}$. This not only prevents the analyst from recognizing the harmony in m. 7 as part of a larger continuity, as is shown in the step-class counterpoint of Ex. 2.4b, but also prevents the analyst from recognizing this harmony's "almost-complementary" relationship with the first chord. In addition, the final 1st violin and cello phrases in m. 7, <F G A B> and <F E D C♯>, respectively, are forms of $(0246)^{12}$ and $(0134)^{12}$ and therefore cannot be tied to the opening four notes of the harp, <E D C B>, a form of $(0135)^{12}$. And the pitches of the bass line form $(0235)^{12}$, and therefore cannot be tied to the aforementioned 1st violin, cello, or harp parts.

Sonata for Two Pianos

Our final example from the music of Stravinsky is a pair of themes from the first movement of *Sonata for Two Pianos*. The first four bars of the first theme in mm. 1-4 are given as Example 2.5a. The first three notes of the first theme in m. 1, {B♭ C E}, are a form of $(013)^7$, which plays an extremely important role in the first movement. It is found in both the thematic and the accompanimental material throughout (see mm. 7-8, 10-11, 16-33, 53, 60-61, 63-64, 72-76, 80-87, and 92-93 for some of the more salient occurrences), and special emphasis is given to its initial ordering <013>^{7}. It is also projected as a contextual line several times in the movement, the first of which is in mm. 1-4, shown as the circled notes in Ex. 2.5a. These pitches, {A G E}, stand out as the peaks of the theme's melodic contour in the first four measures, and are related to the first three notes of the theme by inversion about an E axis, as is shown in Ex. 2.5b. Another interesting observation can be made about the coda theme in mm. 80-81, given as Example 2.5c. This theme is composed of two forms of (013)^{7}, <G F D> and <E♭ C B♭>, that are related by retrograde inversion about a D/E♭ axis. As is shown, the relationship is not only ordered, but also occurs in pitch space.

The coda theme is truly a summation of what has preceded it. The motive <013>^{7}, when represented as segments that are forms of (026)^{12}, can be read tonally as the seventh, root, and third of a dominant seventh chord, a possibility that Stravinsky often exploits by completing this seventh chord with the 5th of the corresponding seventh chord. In mm. 7-9, for example, the motive's appearance as <F G B> is followed each time by D, thus completing the dominant seventh chord of C, which is the melodic goal of the cadence in bar 9 (although it is questionable whether it is actually the harmonic one). Another example can be found in the linking figure that leads into the second theme in the exposition (<A♭ B♭ D F>) and the recapitulation (<D♭ E♭ G B♭>). The coda theme fuses together the most common transposition of this seventh chord, <F G B D>, (the B chromatically altered to fit in a B♭ pitch collection), and its humble beginnings <E C B♭> (the E also chromatically altered for the same reason).

Once again, a mod12 analysis of this same movement cannot see the forest for the trees. The relationship between the first three notes of the first theme and the contextual line of mm. 1-4 disappears from sight, because the mod12 analysis reads the first three notes as $(026)^{12}$, and the contextual line of mm. 1-4 as $(025)^{12}$. The relationship between the first three notes and the coda theme is also overlooked because the latter is read as two successive forms of $(025)^{12}$, and therefore cannot be tied to the beginning.

Barber, *Knoxville: Summer of 1915*

The first fourteen measures of Barber's *Knoxville: Summer of 1915* comprise a five-measure introduction followed by a setting of the first line of James Agee's poem. A piano-vocal score is given as Example 2.6a. The fact that the setting of the first line constitutes a formal unit is confirmed by the varied repetition of mm. 6-14 beginning in m. 15. Apart from two D♯s in m. 3 and three pairs of C and G-natural in mm. 12-13, the pitch content of the music is diatonic, drawn from collection DT-3. The first phrase of the melody in the bass defines its primary pitch context for the next three measures, {B C♯ E F♯}, although its high F♯ is embellished by an A and a G♯ in mm. 2-4. The primary pitch content forms one of the two primary tetrachords that govern the structure of this passage: $(0134)^7$. The continuation of the melody in mm. 4-5 transposes the primary pitch content by T_{-4}, mod7, to {E F♯ A B}, and by doing so mimics the falling-fifth motion from F♯ to B in the first phrase, a motion that is later composed out by the larger bass motion in the first line's setting from F♯ to B (interpreting the first and last pitches of the bass line in mm. 6-13 as starting point and goal, respectively).

$(0134)^7$ also plays an important harmonic role throughout this passage. On the second half of beat 4 in mm. 1-2, the melody and accompaniment together form {B C♯ E F♯}, while on beats 2 and 4 of mm. 6, 7, and 9, the accompaniment forms {C♯ D F♯ G♯}. In mm. 8, 10, and 11, the low C♯ and F♯ in the latter harmony are replaced by {A B E}, thus producing the hexachord {G♯ A B C♯ D E}. This larger harmony, however, might be better interpreted as a concatenation of the trichords {C♯ D G♯} and {A B E}, both members of the set class (014)^{7} and thus subsets of (0134)^{7}, the tetrachord they are replacing.

The other primary tetrachord of this passage is (0124), which is represented in mm. 1-2 by {F♯ G♯ A C♯} and {F♯ A B C♯}. It should be noted that these two forms of (0124)^{7} represent different mod7 Tn-types; the former represents [0234]^{7}, while the latter represents [0124]^{7}. These two forms are related by inversion around an A axis, as Example 2.6b demonstrates. The inversion about A holds an F♯ minor triad invariant, thus reinforcing the listener's intuition that F♯ is a sort of "tonic," an intuition confirmed in mm. 1-6 by F♯'s position in the melody.

(0124)^{7} is also represented by {F♯ A B C♯} in the singer's melodic line, which is composed solely of these notes with the exception of the two Es in m. 11; the Es in m. 11 complete the melodic form of (0134)^{7} that began the instrumental introduction, {B C♯ E F♯}. M. 11 makes plain the relationship between the two primary tetrachords and how they each relate to the pentatonic scale, of which they are each a subset. The pc set {F♯ A B C♯} also governs the unfolding of the bass line to this passage, though one must interpret the E, the two Ds, and the three C-naturals in mm. 11-13 as passing notes that essentially fill in a larger motion from F♯ to B that spans those measures. The important set classes represented in this passage yet to be discussed – (013)^{7} and (024)^{7} – can be understood as subsets of the primary tetrachords discussed above.

A strictly mod12 analysis of this passage would be blind to two musical connections discussed above. In the analysis above, {C♯ D F♯ G♯} was related back to the first phrase of the melody, because they both represent forms of (0134)^{7}. However, in a mod12 universe, {C♯ D F♯ G♯} is a member of (0157)^{12}, while {B C♯ E F♯} is a member of (0257)^{12}, and thus these two sets seem to have much less in common than the actual music suggests. In addition, the inversional relationship between {F♯ A B C♯} and {F♯ G♯ A C♯} in mm. 1-2 would be camouflaged to those viewing the passage from a mod12 perspective, because the former is a member of $(0247)^{12}$, while the latter is a member of $(0237)^{12}$. If one cannot recognize the two sets as equivalent, then the two sets can hardly be related by inversion.

Prokofiev's Op. 22, No. 1

Example 2.7a provides the score to Prokofiev's Op. 22, No. 1, a piece that is more chromatic than any of the previous examples, but nevertheless can be interpreted as in a quasi-E Phrygian (its melody is purely diatonic throughout). The second half of the piece, mm. 14-27, is simply a varied repetition of the first half with the addition of a middle voice that descends in eighth notes; given that the second half is varied only slightly here, the added tones in the second half are taken to be embellishing and thus to be structurally insignificant, at least from the standpoint of pitch organization.

There are many mod7 set classes that play a role in the pitch organization of this piece (triads and seventh chords are the most obvious), but it is $(014)^7$ that is most important melodically, as Example 2.7b demonstrates. In Ex. 2.7b, forms of $(014)^7$ found in mm. 1-8 of the melody have been bracketed; this melody is repeated verbatim in mm. 14-21. In addition, Example 2.7c illustrates how the melodic contour of the planing harmony in the left-hand part of mm. 1-4 highlights the transpositional path T_1 - T_3, a path that replicates the internal structure of $(014)^7$. As a result, the contour pitches for each of the lower three voices in mm. 1-4 form a representative of $(014)^7$.

$(014)^7$ governs the global organization of the piece as well. Example 2.7d shows the largest harmonic motions in the piece; all of the notes in Ex. 2.7d are the roots of root-position tertian harmonies that mark phrase beginnings and endings. The first 8-measure phrase opens with an incomplete E minor seventh chord, and ends with a minor third build on B; this is followed by two short phrases, the first of which begins on a B diminished triad and ends on an F major one, and the second of which begins on an F major seventh chord and ends on an F major triad. The second half begins on another incomplete E minor seventh chord, but this time the added line in eighth notes serves to elide the end of the long 8-measure phrase to the beginning of the next one, which begins not on a B diminished triad, but instead on a B♭ minor one. This chromatic variation of the original returns, however, to the same F major triad as the original version. The final phrase of the piece begins with a B♭ major seventh chord and ends with an E minor triad.

The notes E, F, B, and B♭ each serve as the root of a harmonic starting point/goal in the piece. Because B and B♭ are members of the same step class, B♭ is interpreted as a chromatic variant of B, and thus E, F, B, and B♭ taken together represent the trichordal mod7 set class $(014)^7$. They therefore relate to the previously mentioned representatives of that mod7 set class on the surface of the music. However, the relationship between the global organization of this piece and its surface pitch structures would be camouflaged to those approaching it from a mod12 perspective, because the surface structures are all forms of $(027)^{12}$, while the global structure would be represented as $(016)^{12}$.

Summary

This chapter has shown examples in which a mod7 perspective sheds new light on aspects of specific pitch structures that a mod12 perspective could not illuminate alone. I do not wish to advocate an eschewal of mod12 pc-set theory in the analysis of the music in question, but rather that we use both mod7 and mod12 perspectives as we use both our left and our right eyes; only with binocular vision are we are able to perceive depth, and in some sense, only with such a pairing of mod7 and mod12 perspectives will we be able to perceive the true depth of this music.

Chapter 3

Chordal Tone Centers

Introduction

The previous chapter has shown the extent to which viewing post-tonal diatonic music from a mod7 perspective can reveal aspects of its pitch organization that are impenetrable from a mod12 perspective alone. But this dissertation has yet to address one of the most frequently discussed aspects of post-tonal diatonic music: centricity. Much interesting work on the organization of pitch in post-tonal diatonic music has dealt with identifying referential pitch collections and their tone centers.[1] Despite the considerable contribution this work has made to an understanding of this music, it has at times seemed *ad hoc* because no generalized theory of centricity has yet been written that might substantiate its claims.[2] This is partially attributable to the fact that the solely mod12 perspective that has thus far

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[1] The vast majority of this work relates to the music of Stravinsky. See V. Kofi Agawu, "Stravinsky's *Mass* and Stravinsky Analysis," *Music Theory Spectrum* 11/2 (1989), 139-163; Arthur Berger, "Problems of Pitch Organization in Stravinsky," *Perspectives of New Music* 2/1 (1963), 11-42; Richard Taruskin, "Chernomor to Kaschei: Harmonic Sorcery; or, Stravinsky's 'Angle'," *Journal of the American Musicological Society* 38 (1985), 72-142, "*Chez Petrouchka*: Harmony and Tonality *chez* Stravinsky," *Nineteenth-Century Music* 10 (1987), 265-285, and *Stravinsky and the Russian Traditions* (Berkeley: University of California Press, 1996); and Pieter van den Toorn, *The Music of Igor Stravinsky* (New Haven: Yale University Press, 1983), and *Stravinsky and "The Rite of Spring": The Beginnings of a Musical Language* (Berkeley: University of California Press, 1987).

[2] Peter Silberman has already developed a theory of centricity in the neoclassical works of Stravinsky. See Peter Silberman, "A Fuzzy-Set Based Methodology for Analyzing Centricity in the Neoclassic Works by Stravinsky," (M. A. Thesis, Eastman School of Music, 1997). One significant way that the current study differs from Silberman's is that it focuses on the centricity of individual vertical sonorities, whereas Silberman's paper focused on the centricity of entire passages of music taken as a whole.

guided discussions of post-tonal diatonic works is unsuitable for such a task, given its tendency to overlook the kinds of relationships demonstrated in Chapter 2. However, by adopting the language of mod7 set theory, it is much easier to generalize about how listeners hear tone centers in music that is diatonic, but not tonal. This chapter seeks to identify factors by which we distinguish between the tone centers and the subordinate pitches of post-tonal diatonic harmonies. Secondarily, it explores tonal allusions in relation to functional harmonic progressions and to the voice leading associated with them. For the sake of simplicity, it will focus on dyads, trichords, and tetrachords that occur as vertical sonorities in post-tonal diatonic works.[3]

This chapter first considers four factors in the perception of chordal tone centers: the intervallic composition of a harmony, the doubling of a harmony, the voicing of a harmony, and the linear context in which a harmony appears. The intervallic composition of a harmony is defined here as a harmony's Tn-type in a mod7 system. The doubling of a harmony is defined here as the number of octaves in which each pitch-class of a harmony occurs. The voicing of a harmony is defined here as the registral ordering of a harmony's pitches. Finally, the linear context of a harmony is defined here as the intervals by which each pitch of a harmony is approached. The explication of these four factors will be followed by an analysis of the first movement of Stravinsky's *Cantata*. Then four additional

[3] Although examples will be drawn from Stravinsky's neoclassical music, this is not meant to suggest that he is more a diatonic than an octatonic composer, nor that the theory proposed here would be less applicable to the music of other composers writing post-tonal diatonic music (e.g. Barber or Copland). Arthur Berger and Pieter van den Toorn both argue convincingly that there is a high degree of interaction between octatonic and diatonic pitch structures in Stravinsky's music (see note 1), and we have seen evidence of such interaction in our analysis of *Concerto in D* in the previous chapter.

factors will be presented that do not effect the balance of centric forces within a harmony, but rather reflect how the particular ordering of chords and their rhythmic position in a given passage affects our hearing, and these additional factors will subsequently be applied to our *Cantata* analysis.

Example 3.1 presents six different versions of a C-major triad, and one C-major seventh chord. In the example there is a gradual shift of emphasis from the tone center C to the tone center E through subtle changes in intervallic structure, doubling, and voicing, which illustrates how each of these factors affects our perception of a harmony's tone center (linear context will be considered in later examples). In the first chord, C is emphasized most strongly by the intervallic composition of the set; it is the root of a triad. However, the first voicing also asserts C as its tone center very strongly through doubling (the C occurs twice) and voicing (the C occurs in the outer voices). In the second chord, C is a slightly weaker tone center because, although it is still the root of a triad and occurs in the bass, it is no longer doubled in so prominent a way.

The third chord doubles E instead of C, and this naturally causes the tone center C to sound still weaker, at the same time giving more emphasis to E. The chord voicing weakens C as a tone center through inversion: the bass note is E instead of C, while C is still doubled. The primary tone center of this fourth chord is still C, however: although inversion can serve to make a tertian sonority less stable, it is not strong enough by itself to mask that sonority's root.[4] In the fifth chord, the E is both doubled and in the bass, and thus E gains what C loses in strength. In the sixth chord, the priority of C as a tone center is finally challenged

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[4] This would also be true for a second-inversion triad; a second-inversion C major triad has been left out of Example 1 because such a chord would place a secondary emphasis on G, and the example is illustrating a gradual shift in emphasis from C to E (any emphasis on G would disrupt this gradual shift).

by a voicing that gives E roughly equal weight. The intervallic structure of this triad emphasizes the C as root, but the E receives comparable emphasis by virtue of its doubling, and by its placement as the highest and lowest sounding pitch.

The final chord in Example 3.1 is obtained by adding a B as an inner voice to the previous one. Doubling and voicing considerations, which emphasize E, are the same as in the sixth chord. But now, the E receives additional emphasis through the intervallic structure of the chord. The addition of B leaves us with a seventh chord; two triads are contained within this harmony, {C E G} and {E G B}, and both project their roots as tone centers. Although the intervallic structure of the chord emphasizes C more than E because C is the root of the entire sonority, E is now projected as a tone center more strongly, receiving emphasis for the same reasons it did in the previous chord--its doubling and its voicing.

In Figure 3.1, factors in the perception of chordal tone centers are outlined and the relative weight of each--the degree to which it influences our overall perception--is quantified with a value from one to four. Since we are theorizing about perception, these weights may only be judged in one way: by the degree to which they are perceptually accountable. In analysis, it will prove useful to have a formula which takes into account a pitch's total weight as a tone center relative to that of other pitches. Therefore we will define a given tone center's *relative strength* (abbreviated *rs*) as its total weight multiplied by the quotient of its total weight divided by that of all the pitches in the harmony combined. Thus, the relative strength for the chordal tone center *x* may be represented mathematically as follows: $rs(x) = T^x (T^x /(T^x + T^y + T^z + \ldots T^n))$, where T^n represents the total weight of chordal tone center *n*; for the reader's convenience, this formula is reproduced as Figure 3.2.[5] For example, the E in the final chord of Ex. 3.1 receives a total weight of 6, while the C in that chord receives a total weight of 3, and the G and B of that chord receive no weight. Therefore, the relative strength of E is equal to its total weight, 6, divided by the sum of the total weights in the chord, 9, and multiplied by its total weight, 6; thus the relative strength of E is 4 (see Figure 3.2).

Example 3.2b shows the relative strength and total weight of the tone centers C and E in each of the seven voicings in Example 3.1 (reproduced as Example 3.2a), as well as a breakdown that indicates how much weight each factor contributed to the pitch's total (the letters above the columns in the table refer to the factors given in Figure 3.1, while the numbers down the side from top to bottom correspond to the chords numbered in the example from first to last). Example 3.2c provides a summary of this information in a graphic representation. The gradual shift in emphasis from C to E in Ex. 3.1 is shown clearly in the graph. In what follows, the four factors listed in Figure 3.1 will be examined in greater depth, and then the theory proposed here will be put to the test in an analysis of the first movement from Stravinsky's *Cantata*.

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[5] Readers may wonder what rationale was behind choosing this particular formula and weighting system over other possible systems. To arrive at the formula and weighting system presented here, I listened to a great number of passages from Stravinsky's music, identified chordal tone centers in each, and asked myself what factors led to those hearings. I then began to assign weights to the different factors and to develop a formula that compared them, and adjusted those weights and the formula until the same system could explain my perception of each passage. Finally, I confirmed this system by comparing my perceptions of the passages with those of other theorists, which were in agreement each time. The formula presented here was the simplest one I could think of that would still allow for a balance between how the centric charge of a given pitch is mitigated by other centric forces within the same harmony and how that centric charge compares to that of other pitches both within and outside of its harmonic context.

Intervallic Structure

The first factor given in Figure 3.1, the intervallic composition of the set, is perhaps one of the most important because of our intimate knowledge of the triad, a knowledge that informs our hearing of centricities even in post-tonal contexts.[6] To reflect this, the first member of a given [024] – the root of a given triad – has a weight of 4 for the set's intervallic structure; the weighting of the triad is more than that of any other Tn-type member in the mod7 system. Thus the root of a triad outweighs its other members, as well as the members of other Tn-types, given a musical situation where all other factors (those of doubling, voicing, and linear context) are equal. There will be an exception, however, in the case of the diminished triad. If the [024] is represented by a diminished triad, then its first member receives no weight; this exception reflects the dissonance of the diminished triad relative to that of the major and minor triads.[7]

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[6] Hindemith's theory of chord roots is comparable to the theory of chordal tone centers presented here in several ways. Both Hindemith's theory and the one presented here seek to explain how certain pitches of post-tonal harmonies are heard as primary or central. However, there are four important differences between Hindemith's theory and the one presented here: 1) Hindemith's theory refers to these pitches as "roots," while the current study refers to them as "chordal tone centers"; 2) Hindemith's theory covers all post-tonal harmonies, while the current study is limited to those that are subsets of the diatonic collection; 3) Hindemith's theory is grounded in natural phenomena (combination tones, to be specific), while the current study is grounded in historical conditioning (the common ground shared by all musicians trained in tonal music; i.e. our "collective tonal experience"); and 4) while Hindemith's method for determining chord roots considers only the intervallic structure and voicing of the chord, the method for determining chordal tone centers proposed here considers the doubling and linear context of a chord as well as its intervallic structure and voicing. See Paul Hindemith, *The Craft of Musical Composition* (London: Schott & Co., 1942).

[7] It is unnecessary to consider the augmented triad in this study, as it is not a subset of the diatonic collection.

Example 3.3a provides the first measure from Stravinsky's *Serenade in A*, while Example 3.3b provides a table calculating the relative strength and total weight of the first chord, and Example 3.3c provides a graphic summary of the table (note that the pitches listed along the side of the table are arranged from bottom to top starting with the first member of the set; this ordering will be followed in all subsequent tables). In the very first chord of the work, F and A are presented as rival chordal tone centers. While the root of this F major triad is projected strongly by the chord's intervallic structure and doubling, this emphasis is undercut by the emphasis A receives for its position as the chord's highest and lowest pitch, and for its doubling in four octaves (including the second sixteenth note in the left hand as part of the chord). This tension in the very first chord between the rival chordal tone centers F and A reflects a conflict between these two pitches that is central to the whole first movement.[8]

The Tn-type [0135]-- represented by any quality of seventh chord in tonal music--contains two [024] subsets, and thus its 2nd and 3rd members have weights of 3 and 2 respectively; the 2nd member has more than the 3rd because it is analogous to the root of the seventh chord in tonal music, whereas the 3rd member is analogous to the root of an added-sixth chord or first-inversion seventh chord. Again an exception will be made in the case of diminished triads; if either of the triads contained within the [0135] is diminished, then the member with a diminished triad built on it receives no weight. The inversionally related tetrachords [0124] and [0234] also both contain one triadic subset. The root of this triad--the 1st member of the set--is still projected as a chordal tone center to some extent, and has a weight of 1, despite the presence of an "odd" note (the note

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[8] For an exploration of how the tension between centricities a third apart is exploited compositionally by Stravinsky, see Joseph N. Straus, "Stravinsky's Tonal Axis," *Journal of Music Theory* 26 (1982), 261-290.

a second or a fourth above it). As before, however, if the triad built on the 1st member of an [0124] or [0234] is diminished, it will receive no weight.

The Tn-type [013] is representative of a typical harmony in tonal music: the seventh chord with a missing fifth. The second member of an [013] is therefore perceived as a tone center much more strongly than its first or third members in a given musical situation where everything else is equal. In the same way its inversion, [023], can be understood tonally as a seventh chord with a missing third. This is much less common in tonal music than a seventh chord with a missing fifth, however, and therefore the 3rd member of [023] is perceived as a tone center less strongly than the 2nd member of an [013], reflecting this aspect of our collective tonal experience.

Example 3.4 is the beginning of the theme from the first movement of Stravinsky's *Concerto in D* (m. 25). The harmony in this measure, {C♯ D F♯}, is a member of [013]. Although D is emphasized because of the set's structure and its doubling, F♯ is emphasized because of the voicing of the chord (it is both the highest and the lowest pitch of the harmony), and its doubling. The sense of conflict between the potential "tonics" D and F♯ in this passage is reflected by their almost equal weighting.

There are three dyadic Tn-types in a mod7 universe: [01], [02], and [03]. The centric force of the fifth in post-tonal contexts has already been acknowledged by many theorists, and accordingly the second member of an [03] dyad--the bottom of a fifth--has been assigned a weight of 2.[9] We will make an exception to this weighting, however, in the case of the tritone. Because the tritone divides the octave evenly in half, one cannot audibly distinguish between the top of an

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[9] For example, see Berger, "Problems of Pitch Organization," 11-42.

augmented fourth and the bottom of a diminished fifth in an equal-tempered harmonic system, and therefore neither of the members in those [03] dyads that are tritones will receive weight for the dyad's intervallic structure.[10]

In the absence of fifths or fourths, thirds assert their lower notes as chordal tone centers, almost as if they were incomplete triads, and so the first member of an [02] dyad receives a weight of 2 for the dyad's intervallic structure. Another consequence of our collective tonal experience is the way we interpret thirds within a larger post-tonal diatonic harmony. Although, as subsets, thirds do not influence our perception as much as triads, the thirds found within [012] Tn-types nevertheless carry some weight. The first member of an [012], the bottom of the third, is projected as a chordal tone center and has a weight of 1.

The Tn-types we have not yet discussed - [01], [014], [0123], and [0134], do not have triads as subsets, nor do they have the strong tonal implications of [013] or [023]. In the case of [0123], there are two thirds imbricated, as well as one fifth. The thirds could be thought to project their tone centers, the 1st and 2nd members of the set, and the fifth could be thought to project its tone center, the 4th member of the set, and thus three of the four notes of the set could be potential chordal tone centers based purely on the set's intervallic structure. The strength of these members as chordal tone centers, however, seems diminished to almost nothing by their preponderance, and therefore no weights will be assigned to them. Finally, the [014] trichord and the [0134] tetrachord, both of which represent

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[10] In Hindemith's theory of chord roots, he singled out the tritone as the only interval that does not project either of its constituent tones as a root, and is therefore placed last in his series II (see Ex. 2). To determine the root of a given tritone, Hindemith abandons combination tones and takes its linear context as the principal determinant. He states that the root of a given tritone is that member which proceeds by the smallest step to the root of the following interval. See Hindemith, *The Craft of Musical Composition*, I, 81-84.

either a stack of fourths or a stack of fifths, do not seem to emphasize any one member over the others for the same reason.

Doubling

The progression of chords in Examples 3.1 and 3.2 concludes with a seventh chord very similar to the one that ends the first movement of Stravinsky's *Symphony in C*. Example 3.5 provides the last chord of that movement. Every member of the final chord in Ex. 3.5 is doubled in at least two octaves, while two of its pitches are doubled in three and four octaves, respectively. Given a musical situation such as this one, in which all of the pitches of a chord are doubled, only the doublings beyond that of the chord's least doubled member will be considered significant. Thus E has a weight of 1 for appearing in three octaves, and G has a weight of 2 for appearing in four. (Note that while the relative strengths of the pitch classes point to E as the primary tone center of the chord, in apparent contradiction to the work's title, C is the secondary tone center, and receives a great deal of emphasis throughout the work; a polarity between C and E throughout *Symphony in C* is an example of Stravinsky's proclivity for creating polarities between tonal centers a third apart in his neoclassical works).

A familiar example of the importance of octave doubling to our perception is the "Psalms chord," taken from the beginning of Stravinsky's *Symphony of Psalms* and reproduced here as Example 3.6. While E is heavily emphasized for its place in the intervallic structure of the set {E G B}, for its doubling, and for its position at the bottom of the sonority, the G is somewhat emphasized by its doubling. The G might not be perceived as the primary chordal tone center here, but it is a secondary one, and is felt as such both in the Psalms chord and over the entire span of the first movement.[11] As was noticeable in our earlier examples from *Concerto in D*, *Serenade in A*, and *Symphony in C*, Stravinsky is once again creating tension by establishing two pitch centers of almost the same strength.

Voicing

Non-tertian chords seem to project their lowest sounding pitches as chordal tone centers more strongly than tertian ones, or those with strongly tertian associations, such as [013] and [023]. Although it is true that we are able to distinguish between the bass note and the root of triads and seventh chords in inversion even in post-tonal contexts, other diatonic harmonies lacking such unambiguous tertian foundations often project their bass notes strongly, suggesting a centricity. The Tn-types [013], [023], [024], and [0135] seem to project their bass notes as tone centers less strongly than the other non-tertian diatonic trichords and tetrachords because their intervallic structure suggests a tonal hearing more strongly. And, as was mentioned before, the tonal hearing of [013] is stronger than that of [023], because of their relative frequencies within the tonal canon.

Example 3.7 provides mm. 48-49 of the second movement from Stravinsky's *Cantata*. The harmony beginning at the end of m. 48 is {E F♯ G♯}, a member of the Tn-type [012]. The third contained within this harmony, {E G♯}, asserts its lowest note, E, as a chordal tone center. The structure of this harmony alone would not be enough to project E as its tone center, because F♯ and G♯ are both doubled in the harmony and E is not. However, because of the harmony's linear context (E is scale-degree 1 of the prevailing E minor collection and is

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[11] See Milton Babbitt, "Remarks on the Recent Stravinsky," *Perspectives of New Music* 2/2 (1964), 167-168.

approached from scale degree 5, B) and because E holds a prominent position as the harmony's lowest pitch, E is heard as the harmony's chordal tone center quite strongly.[12]

Example 3.8 is m. 22 from the *Agnus Dei* of Stravinsky's *Mass*. The second chord in this measure, {D F♯ G A}, is a member of the Tn-type [0234], and thus its 1st member, D, receives some emphasis. The G is more likely heard as a chordal tone center, however, because of its position at the bottom of the chord (the voicing here particularly suggests an incomplete ninth chord), and because it is approached from a step above. Example 3.9 is mm. 37-38 from that same movement. The final chord in m. 38, {A B D♯ E}, is a member of Tn-type [0134]. The B in the bass part receives a weight of 2 because it is the lowest note, and a weight of 1 because it is approached from a step above, for a total weight of 3.

Linear Context

The linear context in which a harmony appears can strongly affect our perception of its tone center. Of all the melodic formulas borrowed from the tonal system, those involving a leading tone are perhaps the most salient in a post-tonal context. This perceptual phenomenon is reflected by the factors labeled D-1 and D-2 in Figure 3.1. *If one of the outer pitches of a given harmony is scale-degree 1 of the prevailing collection, and is approached from a half-step below, then that pitch has a weight of 3*, as is noted in D-1. The critical qualification here is "of the prevailing collection," which requires the analyst to identify the diatonic collection that prevails in a given passage, and then interpret it as either a major or a minor scale. If one diatonic collection clearly prevails in a given passage, then one may

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[12] Note that the motion from scale-degree 5 to scale-degree 1 in the cello part is accompanied by a motion from scale-degree 4 to scale-degree 3 in the second flute part; this progression is undoubtedly an allusion to a V^7 - I tonal progression.

simply translate the number and type of accidentals in the collection as a major key signature to determine the scale degrees of the notes. If there is one prevailing collection that is not purely diatonic, but is composed of the notes of a single harmonic or melodic minor scale, then one may interpret the tonic of the implied minor scale as scale-degree 1, and interpret the other scale degrees accordingly. Finally, if two or more diatonic or harmonic/melodic minor collections seem to be equally important in a given passage, then the analyst must apply the weighting criteria for linear context to all of the collections equally.

The leading tone formula not only emphasizes the chordal tone centers associated with "tonic" harmonies, but it emphasizes those associated with "dominant" harmonies as well. This phenomenon is reflected in Figure 3.1 under factor D-2: *If one of the outer pitches of a given harmony is scale-degree 7 of the prevailing collection and is approached from or moves to a half-step above, and there is a scale degree 5 present in the same harmony, then the pitch representing scale degree 5 has a weight of 3.*

Factor D-3 accounts for other patterns of melodic closure: *If one of the outer pitches of a given harmony is scale-degree 1 of the prevailing collection, and is approached from scale degrees 2, 3, or 5, then that pitch has a weight of 2.* This factor is not as heavily weighted as the other two because perceptually the tonal associations for which it accounts are not felt to be as strong as the associations of a leading-tone relationship. Nevertheless, the history of voice leading in tonal music seems strong enough that, even outside of its boundaries, one may sense a feeling of closure from such melodic cadential formulas as scale degree 2 moving to 1, 3 moving to 1, or 5 moving to 1.

Factor D-4 accounts for descending stepwise bass motions which do not involve scale degrees 1 or 7, but which nevertheless provide some centric weight: *If the bass pitch is not scale degrees 1 or 7, but is approached from a step above,* *then that pitch has a weight of 1.* This factor receives still less weight that factor D-3, because the tonal motions for which it accounts do not involve a leading-tone relationship or tonic harmony. As is suggested in Examples 3.8 and 3.9, however, descending stepwise bass lines still influence our perception of tone centers, even when they are not associated with scale-degrees 1 or 7.

The last four measures from the first movement of Stravinsky's *Cantata* are given as Example 3.10a, and a table calculating total weights and relative strengths is given as Example 3.10b. This passage illustrates how the linear context of a passage influences our perception of its chordal tone centers (note that the English horn is in F). The first chord of Ex. 3.10a is a root-position D-minor seventh chord, and is thus a member of [0135]. There are two triads contained within this harmony, {D F A} and {F A C}, and both project their lowest notes as chordal tone centers, though the D is projected more strongly than the F. D receives additional emphasis for its position as the lowest note of the chord, bringing its total weight to 4, and thus is perceived as the chordal tone center, outweighing both the F, which receives a total weight of 3 for the intervallic structure of the set and for its doubling, and the A, which receives a weight of 1 for its doubling.

The second chord, {E F G B}, offers an example of how the leading-tone relationship can influence our hearing. It is a member of [0124], and its primary tone center is heard as G, despite the fact that E receives emphasis for being the chord's lowest pitch and the root of its triadic subset. Because B is scale degree 7 of the prevailing collection, because it is approached from and moves to a half-step above in the highest voice, and because scale degree 5, G, is also present in that harmony, we hear G as the primary chordal tone center, a hearing reinforced by its doubling. The G has a total weight of 4 for its linear context and doubling.

The third chord, {G A C E}, offers another example of how the leading-tone relationship can influence our hearing. The chord is a member of [0135], and despite the fact that A receives more weight than C for the intervallic structure of the set, C is heard as the primary tone center because of the additional emphasis it receives from its linear context: C is scale-degree 1 of the prevailing collection, and it is approached from a half-step below in the highest voice.

The fourth chord is a root-position E minor seventh chord, and is thus a member of [0135]. E receives slightly more weight than G for the intervallic structure of the set, and E receives additional emphasis for its position as the lowest note of the chord, bring-ing its total weight to 4. E thus outweighs both the G, which receives a weight of 2 for the intervallic structure of the set, and the B, which receives a weight of 1 for its doubling.

The fifth chord, {E F♯ A C♯}, is a member of [0135]. Because C♯ is scale-degree 7 of the prevailing collection, because it is approached from and moves to a half-step above in the highest voice, and because scale degree 5, A, is also present in that harmony, we hear A as the primary chordal tone center, a hearing reinforced by its doubling in two octaves. Despite the fact that the F♯ is weighted more heavily than the A in the chord's intervallic structure, and despite the fact that F♯ is in the bass, the linear context which identifies the chord's top note, C♯, as a leading tone, insures that the A is heard as its primary chordal tone center. The A has a total weight of 6--3 for its linear context, 1 for its doubling, and 2 for the intervallic structure--while the F♯ has a total weight of only 4.

The sixth chord, {D E F♯ A}, is a member of [0124]. Its chordal tone center, D, receives a total weight of 12 for the intervallic structure of the set, for its doubling (D is doubled in three octaves), its voicing (it is both the highest and the lowest note of the chord), and its linear context (its top note is part of a leading-tone formula, while its lowest note is scale-degree 1 and is approached from scale-degree 3).

Post-Tonal Allusions to Functional Harmonic Cadences

The primary tone centers of each harmony in Ex. 3.10a are placed on a staff beneath the music. The cadences in the first and second endings provide examples of tonal allusions to functional harmonic cadences; the first ending is an allusion to a II^7 - V^7 - I cadence in C, while the second ending is an allusion to a II^7 - V - I cadence in D. (In a program note for the first performance by the Los Angeles Chamber Symphony Orchestra, Stravinsky himself wrote that the final cadence of the first movement was "in D major.")[13] A comparison between the first and second endings yields a number of similarities. The tonal allusion to a II - V - I cadence is reinforced by a succession of scale degrees 8 - 7 - 8 in the top voice of both endings, and in both cases, the "II^7 " harmony is found unchanged while the "dominant" harmony is obscured by adding the 3rd scale degree underneath and the "tonic" harmony is obscured through the addition of a single pitch. The sonorities of these cadences are different, but Stravinsky's mark remains the same: dissonant sonorities mask a progression of roots that has an intimate relationship with the tonal tradition.

One advantage of ordering the factors presented here into a hierarchical scheme is that the analyst may discuss the relative strengths of centricities in a roughly quantitative way. This opens up two possibilities for the analyst of post-tonal diatonic music: 1) the quantitative analysis of polarities and how they are balanced throughout a specific work; and 2) the quantitative analysis of the relative

[13] See Eric Walter White, *Stravinsky: The Composer and His Works*, 2nd edition (Berkeley: University of California Press, 1979), 469.

strengths of cadences in a specific work. As for the first possibility, Examples 3.3-3.6 have shown how Stravinsky embodied these conflicts in single sonorities, and how the conflicting centers are balanced within these sonorities. As for the possibility of analyzing the relative strengths of cadences in a specific work quantitatively, Example 3.10c provides a graphic summary of the table in Ex. 10b. One can see in the graph how the second ending of the first movement in the *Cantata* projects its "tonic," D, more strongly than the first ending projects its "tonic," C; the relative strength for C is 3.27 while the relative strength for D is 12. We can therefore argue that the cadence of the second ending is roughly four times as strong as the first.

The First Movement of Stravinsky's *Cantata*

An analysis of the first movement of the *Cantata*, the ending of which we have already examined, demonstrates how a theory of centricity might be useful, even one limited to the analysis of chordal tone centers in individual verticalities. Although we will be analyzing only the first movement of the *Cantata*, this is a significant contribution to an analysis of the work as a whole, because the first movement serves as a kind of ritornello within the seven-movement work, occurring as the first, third, fifth and seventh movements, and varied only slightly each time. Example 3.11a provides mm. 1-7, the instrumental introduction to the work as a whole.

In Ex. 3.11, chords that begin or end a phrase have been numbered with two exceptions. Only the last chords in the first and fourth phrases have been numbered. These two parallel phrases which frame the introduction begin and end with the same pitch-class set, {E G A B}, and so the final agogically accented chord in each will be taken to represent both. The remaining harmonies in these measures will be considered connecting chords, although this is not meant to imply a Schenkerian notion of prolongation, but merely reflect a basic tenet of Gestalt theory: elements at the beginning and end of a unit (whether it be a motive, a phrase, or an entire movement) are more memorable than those in the middle. When considering the tone centers of the chords, a pattern emerges that reminds us of Ex. 2 (compare Examples 3.2b and 3.2c with 3.11b and 3.11c). There we saw a gradual shift of emphasis from C to E. In the introduction to the *Cantata* we see a gradual shift of emphasis from E in mm. 1-2 to C in mm. 3-5, and then back to E in mm. 6-7. This leaves us with the impression of an E Phrygian introduction with a contrasting middle section in C that foreshadows the tonal center of the verse. (Stravinsky also mentioned in the program note cited above that the *Cantata* begins with "a short instrumental prelude in the Phrygian mode.")[14]

Example 3.12 provides the music for the first movement of the *Cantata*, while Example 3.13a provides a reduction of the two verses (mm. 8-22). Examples 3.13b and 3.13c provide a table (calculating the relative strength and total weight for each note of each chord) and a graphic summary of the table, respectively. One may use the reduction given as Ex. 3.13a to verify weights assigned based on intervallic structure, doubling, and voicing, but to verify linear context one must refer to the score (given as Ex. 3.12), because the bass line of the vocal parts (the alto II line) as well as that of the instrumental parts (the cello line) have both been taken into consideration. Metrically or textually accented chords in the verses have been numbered 1-15 in Examples 3.12 and 3.13; the remaining harmonies in these measures will be considered connecting chords. These fifteen chords can be divided into seven-plus-eight, reflecting the bipartite structure of

[14] White, *Stravinsky*, 469.

each verse. Among the first six chords, only three different pitches serve as primary chordal tone centers: C, E, and F. F occurs only once as a primary chordal tone center in this span (in chord 5), and is followed by an "E chord." Chord 5 could be interpreted as a neighboring harmony subordinate to the "E chords" of the passage, just as was the case in mm. 1-2 (see Ex. 11). This interpretation is buttressed both by the bass line, which moves from F down a half-step to E, and by the fact that the relative strength of E is twice that of F. By interpreting F as subordinate to E, we are left with just C and E as the primary chordal tone centers for this passage as a whole, reminding us specifically of their interplay in the introduction and generally of Stravinsky's fondness for creating a pair of competing chordal tone centers a third apart and utilizing the tension between them as a compositional resource.[15]

Among chords 8-15, only five different pitches serve as primary chordal tone centers: D, E, F, F♯, and G. Of these, only the G of chord 8 and the D of the chord 15 are asserted strongly as chordal tone centers. From this observation, its seems that one could view the centricities of the verse (mm. 8-22) as being based on a fifth relationship, analogous to binary forms of the common practice period, which begin the first section on the tonic, and begin the second section on the dominant. The first half of the verse begins on a "C chord," while the second half begins on a "G chord." Stravinsky, however, does not steer the second section to end on another "C chord," as one might expect, but instead moves the centricity up another fifth to end on D.

Chord 8 represents the music from m. 15 to halfway through m. 18. The prominent voice leading motion from F♯ to F♮ that occurs in these measures does

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[15] Straus, "Stravinsky's Tonal Axis," 261-290.

not affect our analysis of chordal tone centers at all, because such chromaticisms are reduced out in a mod7 analysis (see note 4). We should not ignore the initial appearance of F♯ here, however, as it foreshadows the inclusion of F♯ in chord 9 as well as in the final chord of the movement, {D E F♯ A} (see Ex. 10). Chord 10, which has F as its primary chordal tone center, can be interpreted as a neighboring harmony subordinate to chord 11, which has E as its primary chordal tone center, just as was the case in the first half of the verse. While this reading is attractive because of the parallelism with the first half of the verse, and is buttressed by the presence of a bass line moving from F down a half-step to E (the alto II part), it is a weaker relationship here because the E is not any stronger as a chordal tone center than the F, and because the instrumental bass line (the cello part) contradicts the reading by moving from E *up* a half-step to F.

This blurring of the hierarchical relationship is taken further in the next two chords, where E and F receive roughly equal emphasis and the presence of two contradictory composite bass lines (note the voice crossing in the alto parts, and between the cello and English horn parts), one instrumental and one vocal, is maintained. The E finally reasserts itself over the F in chord 14, but it is immediately followed by a harmony having a chordal tone center, D, that is more than six times as strong (comparing their relative strengths, 1.28 and 8.33, respectively). This suggests that the chordal tone centers on E in chords 4-7 are subordinate to the chordal tone centers on D in chords 9 and 15. Thus, while F is subordinate to E in the verse as a whole, E is subordinate to D in the second half of the verse. This suggested pecking order is further strengthened the second time through the verse, because of the II - V - I cadence "in D" that constitutes the second ending.

Conceptualizing a hierarchy of chordal tone centers based on their relative strengths is useful in understanding the rest of this movement as well. Reviewing the other strongly asserted chordal tone centers in the verse, we find that the "C chord" in m. 9 asserts C as its primary tone with an rs. of 5.44, and the "G chord" in mm. 15-18 asserts G as its primary chordal tone center with the same rs. of 5.44. These chords stand along with the "D chord" in m. 22 as the three most strongly asserted chordal tone centers of the verse, and thus provide support for the idea, suggested earlier, that its overall structure is based on ascending fifths and consequently provides another allusion to tonality.

Another interpretation of this movement's structure incorporates its E-centric introduction, although it also involves hearing the "G chord" in mm. 15-18 as subordinate to C. A motivic analysis of the movement is given as Example 3.14a. This analysis shows how the four-note motive E - D - C - D, presented first in mm. 3-4, can be thought of as the melodic germ that generates the top line for the rest of the movement (the soprano I and the Flute I parts).[16] The motive that is suggested as generating the top line of the second ending is missing its first note, but its prevalence elsewhere could perhaps justify interpreting it as incomplete, and we could similarly conceive of the top line in the first measure as an incomplete motive retrospectively (it is, after all, complete at the same pitch level in mm. 5-7). Example 3.14b is an interpretation of the motive in which the first D is viewed as a subordinate passing tone. It is from this vantage point that one can see the primary chordal tone centers of the whole movement, E, C, and D, as a large-scale projection of the same motive. The E-centric introduction is followed by two C-centric verses that move to a D-centric vocal cadence in m. 22. The instrumental first ending that immediately follows returns the music to a C-centric domain in

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[16] For clarity, I have shown only forms of the motive that are transpositionally related to the original <E D C D> motive in Example 3.14, though there are many other forms of the motive related by inversion, retrograde, and retrograde-inversion that overlap those shown in the example.

preparation for the repeat, while the second ending confirms the D-centric vocal cadence in m. 22.

This analysis of the *Cantata*'s first movement demonstrates how a theory of chordal tone centers can serve to explain, concretize, clarify, and validate some of our musical intuitions. It has pointed to specific reasons why the introduction sounds Phrygian, why the final cadence sounds as though it were "in D," why C and E sound like competing tone centers in both the introduction and the verse, and why the pitch organization of the whole movement sounds so unified. In addition, the analysis of each chordal tone center's relative strength has clarified and quantified the relationship of C and E at specific points in the music, as well as shown why the second ending sounds stronger than the first.

Rhythm and Ordering

In the previous analysis, we saw how a theory of chordal tone centers facilitates discussions of tonal illusions to functional harmonic progressions, as well as how it allows analysts to discuss in a roughly quantitative way the polarities in a given work and how they are balanced therein, as well as the relative strengths of cadences in a given work. By focusing on individual chords, however, one loses the sense of how the particular ordering of chords in a given passage affects our hearing. One also loses the sense of how our hearing is affected by the rhythmic character of a passage. In the next section, I will address these concerns by examining four characteristics: 1) the metric position of a harmony; 2) the duration of a harmony; 3) the order position of a harmony (whether it occurs first, last, or in the middle); and 4) the harmonic support a harmony receives from the previous harmony.

Metric, Agogic, Ordinal, and Harmonic Stress

Placing a harmony in a strong metric position is a straightforward manner in which to emphasize it; harmonies on beat 1 of a measure receive a stress that those on other beats do not. The first and second endings of Stravinsky's *Cantata* illustrate this point (see Example 3.15a). The final chord of the first ending occurs on an upbeat, serving to de-emphasize its arrival, while the final chord of the second ending occurs on beat 1, serving to emphasize its arrival. A comparison between the relative strength of the tone centers projected by these two chords would be misleading because there are so many other factors contributing to their strengths. However, a comparison between the first ending and a hypothetical first ending in which the rhythm of the chords is identical to that of the second ending will reveal the emphasis added by the final chord's metric position. Conversely, a comparison between the second ending and a hypothetical second ending in which the rhythm of the chords is identical to that of the first ending will reveal the emphasis lost by shifting the final chord to an upbeat. Example 3.15a provides the first and second endings as they occur in the *Cantata* while Example 3.15b provides the hypothetical first and second ending in which the rhythms of the first and second endings are exchanged.

Agogic accent is another common means of emphasizing a harmony. Consider how the harmony at the beginning and end of the introduction to the *Cantata* would be weakened if they were not held as long. Example 3.16a provides the introduction to the *Cantata*, while Example 3.16b provides a hypothetical introduction in which the lengths of those two chords have been truncated. In the introduction, the lengths of the two "E chords" reinforce other factors suggesting that this introduction is "Phrygian," as Stravinsky himself called it.[17] In our hypothetical introduction, the truncation of these two chords has weakened their projected tone centers enough that one might hear them as appendages to the C centric middle section in mm. 3-5, and thus interpret the introduction as being C centric rather than E centric.

Chords that occur at the beginning or end of a phrase, section, or work often seem to receive more emphasis than those that occur in the middle. This is reflected in what is retained from level to level in a prolongational graph following Lerdahl and Jackendoff's theory of tonal music; it is almost inevitable that cadences marking formal divisions will be heard at higher structural levels than the music surrounding them, and that the cadences marking the largest formal divisions of the piece will be found at the highest structural level of its graph.[18]

Those with an intimate knowledge of functional tonality cannot help but interpret a progression of tone centers in post-tonal diatonic music as they would a progression of roots in tonal music. When the root of a chord falls a fifth or ascends a fourth to the root of another chord, the root of the latter chord sounds emphasized. Whether this is because of a likeness the progression bears to the most primal and basic of tonal motions, V-I, or because of a more direct relationship it has to the overtone series is hard to say. However, it is clear that the descending fifth progression is the strongest, most goal-oriented root motion there is, and that its second chord receives an emphasis as the goal of this motion. Descending third progressions (or ascending sixth progressions) are the second strongest of the goal-oriented root motions, and likewise the second chord in such a progression receives some emphasis.

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[17] White, *Stravinsky*, 469.

[18] Fred Lerdahl and Ray Jackendoff, *A Generative Theory of Tonal Music* (Cambridge: The MIT Press, 1983).

Only descending fifth and descending third progressions will be considered here, because they are the strongest in tonal music and have two characteristics in common that are not shared by step progressions or by ascending third and fifth progressions: 1) the root of the first triad is held as a common tone in the second triad; and 2) the root of the second triad is not found within the first triad. In a post-tonal context, an association with these progressions would be made if the tone centers in the progression were also the roots of triads (either standing alone or imbricated in a larger sonority), and if the progression of tone centers was by descending third or fifth.

How each of the four factors discussed above affects the relative strengths of chordal tone centers is illustrated in Figure 3.3. For each chord, each of the four factors are considered in turn. If the chord occurs on beat 1, then it receives stress due to its metric position and 1 is added to the relative strength of each pitch class in the chord. If the chord is at least twice as long as most (i.e. more than half) of the other chords in the section, then it receives stress due to its duration and 1 is added to the relative strength of each pitch class in the chord. The order position of a chord will be considered here in relation to three structural levels: the phrase, the section, and the movement/work. If the chord occurs first or last in a phrase, then 1 is added to the relative strength of each pitch class in the chord; if it occurs first or last in a section, then 2 is added to each pitch class; if it occurs first or last in a movement or work, then 3 is added to each pitch class. The additions relating to order position are cumulative; thus, if chord *x* ends a movement, it must also end a phrase and a section, and therefore 6 (1 + 2 + 3) is added to the relative strengths of each pitch class therein. Finally, if the primary tone centers of both the chord and the chord preceding it have triads built on them, and a descending fifth motion connects the two primary tone centers, then it receives a harmonic stress and 2 is added to the relative strength of each pitch class in the chord. If the first two conditions hold, but a descending third motion connects the two primary tone centers, then the chord receives a harmonic stress and 1 is added to the relative strength of each pitch class.

This simple system of weighting might seem to give metrical emphasis short shrift. After all, metrical emphasis is felt not only on beat 1, but also to a lesser degree on every other beat, and one could also argue that hypermetrical emphasis serves to make some first beats stronger than others. And what about the role of syncopation? To begin with beat 1 and the other beats in a measure, it is true that events occurring on the beat receive more emphasis than those occurring off the beat (unless they are part of a syncope). However, one can simply reduce out chords that are on offbeats and are not part of syncopes, as was done in the first measure of the *Cantata* analysis presented earlier. Thus, in any given measure, events on beat 1 receive the most stress, followed by events on the other beats and any syncopes (which are at least recognized as structurally significant chords), with those events occurring on offbeats that are not syncopes relegated to the status of "non-harmonic."

As for hypermetrical stress, it is not really so different than what has been defined as ordinal stress here, and so adding weight for both hypermetrical and ordinal stress would most often be redundant. Both types of stress define various levels of formal articulation, though with hypermetrical stress, the analyst must make a decision between a system that is beginning-accented or end-accented. Ordinal stress as it is defined here affects both beginnings and ending, which seems appropriate given the current enterprise.

The *Cantata* Revisited

Example 3.17 charts the adjustments made to the relative strengths of tone centers in our *Cantata* analysis based on the four factors mentioned above (compare Ex. 3.17 to Exx. 3.10-3.13) . Note that while only the primary tone centers have been included, the total adjustment for each chord is in fact added to every note in that chord. The first six columns of the table chart the weight each chord receives for each of the criteria given in Example 3.17: metric stress, agogic stress, beginning/ending stress (divided into three columns that consider beginnings and endings of phrases, sections, and the movement/work, respectively), and harmonic stress.

There are two differences between the view of the *Cantata* presented here and the one presented earlier that are worth examining in greater detail. The first is the difference between chord 1 and chord 4 in the introduction. While the earlier view of this passage interpreted the C in chord 4 as its strongest tone center, the adjusted view finds this C to be dead even with the E in chord 1, and thus supports the interpretation that a polarity between C and E throughout the first movement begins in the introduction. In addition, the earlier view interpreted the introduction as E centric because it begins and ends on E, but the strength of the C in chord 4 seemed to contradict such a reading. Shouldn't the strongest chordal tone center in a passage be the tonal center of the passage as well? Though the answer to this question is "not necessarily," the case for a given tonal center is of course made stronger when it is also the strongest tone center in the given passage, or at least one of the strongest, as is the case here.

The second noteworthy difference between the view of the *Cantata* presented here and the one presented earlier is the near balance in weight between chords 1 and 2 in the verses. In the earlier view, the relative strength of the C in chord 2 was significantly stronger (5.44) than the E in chord 1 (3.77), but here the relative strength of C (6.44) is almost equal to that of E (6.77). As in the adjusted view of the introduction, the adjusted view of the verses supports the interpretation that a polarity between C and E exists throughout the first movement.

The adjusted view seems to undercut the earlier view's interpretation of C as the tonal center of the verses, because the C in chord 2 is no longer greater in strength than the E in chord 1. There are many other factors, however, that still support hearing C as the tonal center of the verses, however. Chord 1 is in fact just a {C E} dyad that is subsequently engulfed in chord 2 {B C D E F}, and could thus be interpreted as part of chord 2 in some sense (if one were to add the relative strengths of C and E in the first chord to those in the second chord, C would be projected most strongly). Chord 8 is projected very strongly and could be interpreted as the dominant of C, thus reinforcing a C-centric interpretation. Finally, the first ending is clearly "in C," and thus reinforces the C-centric interpretation of what precedes it in the section.

In this chapter, we have identified some of the factors by which we distinguish between the tone centers and the subordinate pitches of post-tonal diatonic harmonies, and have organized these factors into a hierarchical scheme. While establishing this theory, we have seen ways in which it will serve those studying centricities in post-tonal diatonic music. The theory presented here facilitates discussions of tonal allusions and how they relate to functional harmonic progressions. It also allows theorists to discuss in a roughly quantitative way the polarities in a given work and how they are balanced therein, as well as the relative strengths of cadences in a given work. Because we are listeners with a primarily tonal frame of reference, we often hear diatonic music in terms of its adherence to or deviation from tonal norms, even in post-tonal contexts. In summary, this chapter has sought to provide a more precise and systematic language with which to discuss these aural interpretations.

Chapter 4

Structural Levels

Introduction

Attempts to suggest that structural levels govern the organization of a post-tonal work prove difficult for a number of reasons. Many of these reasons stem from the associations that term has with Schenker's theories; as Joseph Straus has pointed out, atonal works very often lack the kind of pitch organization necessary to be considered prolongational, and thus to contain structural levels in the Schenkerian sense of the word.[1] Nevertheless, the concept of structural levels, when conceived more broadly, can still possess extraordinary power. Jack Boss asserts that the term is still appropriate in Schoenberg's music, because the composer himself "ranks pitches and intervals as structural and ornamental...and in some cases shows how the ornamental pitches derive from the structural pitches through ornamentation types."[2] While Boss invites those analyzing Schoenberg to think in terms of structural levels, he does not extend his invitation to those analyzing other post-tonal repertories, and limits himself to only one level beyond the musical surface, the foreground. This limitation might be consistent with Schoenberg's analyses of his own music, but if one were to analyze the music of other composers with structural levels in mind, such a limitation might be unnecessary, and to those more adventuresome analysts, unwanted. The goals of Chapter 4 are to establish a method for distinguishing between multiple structural levels in a post-tonal diatonic work, and to show three examples of post-tonal

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[1] Joseph N. Straus, "The Problem of Prolongation in Post-Tonal Music," *Journal of Music Theory* 31/1 (1987), 1-21.

[2] Jack Boss, "Schoenberg on Ornamentation and Structural Levels," *Journal of Music Theory* 38/2 (1994), 210.

diatonic music in which multiple structural levels are audible: the first scene of Stravinsky's *Agon*, the second movement of Louise Talma's Piano Sonata No. 1, and the last section of Copland's *Appalachian Spring*.

Thus far, theorists have suggested three ways in which we distinguish between structural and ornamental tones in post-tonal music, all of which are dependent upon context: 1) salience; 2) motivic significance; and 3) identification as ornament type.[3] Considering all three of these, this dissertation asserts that a given pitch is structural on some level in a post-tonal work if it is both contextually salient and is part of some form of a motive (transposed, inverted, retrograded, etc.) that regularly recurs in the music. The previous chapter established a weighted system of salience criteria that enabled the analyst to discuss in a systematic way how pitch classes receive centric weight in post-tonal diatonic music. Its goal was to find a consistent way of determining chordal tone centers, but it did not establish a parallel method for determining pitches that serve as the harmony's primary melodic tone. That is, to draw an analogy with a Schenkerian conception of structural levels, it established a consistent way of graphing a bass line, but not a top line. Figure 4.1 establishes such a parallel method by listing factors in the perception of structural top lines.

As one can see from examining Figure 4.1, two of the factors from Figure 3.1 in the previous chapter can simply be borrowed. However, two factors in the

[3] Some of these theorists include Allen Forte, *Contemporary Tone-Structures* (New York: Bureau of Publications, Teacher's College, Columbia University, 1955); Fred Lerdahl, "Atonal Prolongational Structure," *Contemporary Music Review* 4 (1989), 65-87; Joel Lester, "A Theory of Atonal Prolongations as Used in an Analysis of the Serenade, Op. 24, by Arnold Schoenberg," (Ph. D. dissertation, Princeton University, 1971); and Jack Boss, "Schoenberg on Ornamentation," 187-216; and "Schoenberg's Op. 22 Radio Talk and Developing Variation in Atonal Music," *Music Theory Spectrum* 14/2 (1992), 125-149.

perception of chordal tone centers do not come into play when considering structural, but non-centric pitches: intervallic structure and linear context. The previous chapter has shown that triads and other tertian subsets embedded in larger diatonic harmonies often project their roots as tone centers, but such projections do not affect our perception of structural pitches that are non-centric. Similarly, the previous chapter has shown that certain linear progressions – leading-tone formulas, falling fifths, and the like – carry a centric charge that can often influence our perception of chordal tone centers, but our perception of structural, but non-centric pitches is not affected by such motions. The voicing of a harmony is a factor in the perception of non-centric structural pitches, but in Chapter 3 it was defined with centric pitches in mind, and thus must be altered. In Figure 4.1, the highest pitch in a given harmony – rather than the lowest one, as is suggested in Figure 3.1 – receives a weight of 2. This factor is consistent with the perception that outer voices are more structural than inner ones, as well as with the musical intuition that pitches on the top of a texture are less centric than those on the bottom. Finally, one new factor is added: solo stress. If a pitch is part of a line that is meant to be performed at a louder dynamic level than the rest of the ensemble, or that is meant to be performed with an instrumental timbre or distinctive articulation that highlights it within the ensemble, or that is marked *Hauptstimme* (or some semantic equivalent of *Hauptstimme*), then it receives a weight of 4. Composers who place thematic material in an inner voice will usually use one of the methods considered above to ensure that it is heard as the primary melodic material.

With the list of factors in Fig. 4.1, one may calculate the relative structural weight of pitches in the topline just as was done with chordal tone centers in Chapter 3. Thus, every pitch will have two relative strengths: its relative strength as a chordal tone center, hereafter referred to as its *CRS* (its Centric Relative

Strength) and its relative strength as a topline pitch, hereafter referred to as its *TRS* (its Topline Relative Strength). In the previous chapter, we acknowledged factors that did not play a role in our perception of centricities within a chord, but that nevertheless influenced our perception of how the relative strength of pitches in different chords compared to one another. These factors are metric stress, agogic stress, ordinal stress, and harmonic stress (see Fig. 3.3); they apply to every pitch's TRS in the same way that they apply to its CRS, and weights in this chapter will adjust both the CRS and the TRS for a given pitch using Fig. 3.3, after using Fig. 3.1 and Fig. 4.1 to find the initial values for the CRS and TRS, respectively.

Pitches that are not structural must be explainable as one of three ornament types: a dividing tone, a neighbor tone, or a motive tone (all three ornament types may be followed by registral transfer).[4] Example 4.1 illustrates the first two types.[5] A dividing tone is like a passing tone, only there is no limit on the dividing interval's size, as is shown in Example 4.1a. A passing tone divides a harmonic interval evenly into mod7 steps, while a dividing tone divides some larger interval evenly or almost evenly. A post-tonal neighbor acts much like a tonal one and limits itself to intervals that are absolutely small, one or two semitones, as is shown in Example 4.1b. Finally, an ornamental motive tone is a pitch that is part of a motivic replication, but is *not* emphasized contextually.

Having summarized how one might distinguish between structural and ornamental pitches, what remains is to determine how one might distinguish between higher and lower structural levels. A logical approach to a theory of structural levels in post-tonal music would be to align those levels with the

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[4] The first two of these types comes from Lester, "A Theory of Atonal Prolongations"; the third comes from Boss, "Schoenberg on Ornamentation."

[5] Boss, "Schoenberg on Ornamentation," 195.

hierarchical organization inherent in a work's formal structure. Guided by this approach, the highest structural level of a post-tonal piece would include only those pitches present at the beginnings and endings of the largest formal divisions of that piece, the next highest level would include only those pitches present at the beginnings and endings of the next largest formal units (phrases possibly), and so on. This approach depends on the analyst's ability to find convincing formal segmentations, which is not always easy, as Christopher Hasty has pointed out: "Since post-tonal music has abandoned many of the formal resources of tonal music such as harmonic cadence, the opposition of key areas, and the various periodicities generated by the repetition of metrical units or thematic statements, we are forced to ask very elementary questions concerning the nature of musical coherence as a temporal phenomenon, questions which we often take for granted in analyzing tonal music because they are implicit presuppositions of traditional analytic concepts."[6] Thus the success or failure of the approach outlined above depends upon its adherence to or deviation from norms established in tonal music. However, post-tonal music that is diatonic tends to be conservative with respect to formal organization, and thus an approach to structural levels that is dependent upon form does not encounter many formal conundrums when dealing with such music.

One level above the musical surface, all pitches that are not some form of a motive deemed significant by the analyst will be defined as ornamental, and the remaining pitches will be taken as structural. As one progresses to higher levels, however, formal divisions will determine which members of the various motive-forms retain their structural significance. Thus, suggesting more than one

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[6] Christopher F. Hasty, "Phrase Formation in Post-Tonal Music," *Journal of Music Theory* 28/2 (1984), 168.

structural level will always involve reinterpreting the role of certain pitches as one moves from level to level. Just as a I-V-I cadence in a Schenker graph is often reinterpreted as simply a bass arpeggiation of tonic harmony on a higher level, so too can one read every pitch of a motive as structural on one level and only some of them as structural on another. Example 4.2, an analysis of the first trumpet part from m. 51 of Stravinsky's *Agon*, illustrates this point. Example 4.2a provides the motive that is the basis for this melodic fragment, while Example 4.2b provides the music itself, and Example 4.2c shows the first structural level, the foreground. All of the pitches are structural at the foreground level except for the E♭ on the first beat of m. 51, which is a passing tone, and the low B♭, which is a lower neighbor. However, at the next level of structure, shown in Example 4.2d, the entire motive becomes ornamental and only the final E♭ is regarded as structural, its structurality deriving from its stronger position in the work's formal structure: it serves to end a phrase.

Stravinsky's *Agon*, Scene 1

Example 4.3 shows three structural levels for Scene 1 from Stravinsky's *Agon*, which will be characterized as "middleground II," "middleground I," and "background." Above each graph are the measure numbers; below each graph are the labels for the two forms of the harmony serving as the primary motive of the piece, $(014)^7$, and the transformations connecting those forms of the motive. First, the reader should note that there are two distinct forms of $(014)^7$. These two forms may best be represented mod7 by Morris's figured bass (FB) equivalence class as [3 4] and [1 4], respectively, because they form the inversionally symmetrical set class (014), mod7, and therefore the distinction between its prime and inverted forms would not be recognized by the more familiar set class or

Tn/TnI-type nomenclature used in previous chapters.[7] Recognizing the inversional relationship between the two types of chords in Ex. 4.3 depends upon counting intervals from the harmony's lowest sounding pitch, and this is what Morris's FB-class does. One can see that the two forms of the motive can account for all structural harmonies in the scene until the final two measures, in which [3 4] and its inversion [1 4] are synthesized, a synthesis resulting in [1 3 4].

Before examining specific points of interest in the graphs of the first scene as shown in Ex. 4.3, it will be useful to run through the scene chronologically. Example 4.4 is an annotated copy of the score; the sets listed beneath the music refer to those given as structural in Ex. 4.3. In mm. 1-3, [3 4] is struck three times by the harp, piano, and strings together, while the trumpet plays a repeated C. This repeated note is the beginning of the primary melodic figure of the scene which, in its entirety, spans mm. 1-5; let us call it the "trumpet tune." Transpositions of the last four notes of the trumpet, <C D B A>, make up much of the melodic material to follow, including the second trumpet part, as well as the first and second horn parts in mm. 5-6. The four-note segment <C D B A> and its transpositions are all forms of <2310>^{7}, which serves as a structural motive at the "foreground" level in much the same way that (014)^7, represented by the FB-classes [3 4] and [1 4], serves as a structural motive at a "middleground" level.

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[7] Robert D. Morris, "Equivalence and Similarity in Pitch and their Interaction with Pcset Theory," *Journal of Music Theory* 39/2 (1995), 207-244. Morris explains his FB-class as follows: "the pc-ordered intervals between the non-bass pitches and the lowest, bass pitch of a pset are given as the integers 0 to B. These intervals are listed in ascending numeric order and identify the FB-class." Morris's FB-class is a mod12 revision of the traditional figured bass nomenclature, which is mod7. When Morris's nomenclature is used with mod7 step classes, the corresponding numbers between an FB-class representation and the more traditional figured bass representation will differ by one, because the mod7 FB-class interprets pitches as step classes (and thus C = 0), while the traditional figured bass interprets pitches as scale degrees (and thus, in C major, C = 1).

In m. 7, the oboes, English horn, and horns I and II sustain {F G C}, a form of [1 4], and in mm. 8-9 the cellos and double basses play a triplet figure which oscillates back and forth between G and B. One could hear this oscillation as a frustrated attempt by those instruments to achieve the union of [3 4] and [1 4]. This union, [F G B C], would be achieved if only one instrument would sustain the B along with the rest of [1 4] above, but such a union is saved for the dramatic close of the scene.

In the following four bars, the end of the trumpet tune returns at T_2 as <D E C♯ B>, accompanied by chord *y* at T_2, [G A D]. This contrasting section is followed by a contrapuntal one in mm. 14-18, in which the contour of the first and second trumpet and first horn parts outlines the structure of (014)[7]. In m. 18, the third horn part returns with almost the same variant of the trumpet tune it had in mm. 5-6; only its rhythm is different. The music of mm. 7-13 then returns in mm. 19-25.

In mm. 26-29, Stravinsky inserts an odd contrapuntal section in which three different types of symmetrical relationships are represented. The two clarinets play a melodic palindrome in rhythmic unison, only to exchange pitches on its last note; the order numbers are given above each staff. The time span of this palindrome is divided evenly into six quarter-note Bs by the piccolo and first flute (counted together), and into three dotted quarter-note Gs by the second flute. Those three parts taken together are related to the oscillating triplet figure played earlier by the cellos and double basses in two ways: 1) they share the same pitch content, B and G; and 2) the proportion of Bs to Gs is nearly two-to-one in both cases (in mm. 8-9, the cellos and double basses are only one B shy of a perfect two-to-one ratio). The trombones and the harp play a second melodic palindrome with a midpoint that occurs an eighth note later. The two midpoints of these palindromes serve to divide the movement into two roughly equal portions; they come at the end of m. 27 and the beginning of m. 28, while the movement is 60 measures long. The palindromes accentuate this near-bisymmetrical division. This section does not seem to serve the higher structural levels of the scene depicted in Ex. 4.3, but is instead linked to a structural level closer to the musical surface that still interprets the trumpet tune's last four notes, <C D B A>, as a structural motive. Through the haze of mm. 26-29, one can hear this motive at T_2, <D E C♯ B>, shared among the instruments.

The opening of the trumpet tune is played by the clarinets in the measures which follow, as the trombones sustain the fourth [D♭ G♭] above it; the total sonority is that of [3 4]. In mm. 32-35, a contrapuntal section reminiscent of mm. 15-18 occurs, bringing the music back to [1 4], this time at T_{10} of its original pitch level, and sustained in its original orchestration (by the oboes, English horn, and the first and second horn). Against [1 4], the oscillating figure of the cellos and double basses returns untransposed, serving as a kind of dissonant double pedal.[8] This figure continues as [1 4] begins an oscillation of its own between [E♭ F B♭] and [G F C] in mm. 39-42.

In mm. 43-46, the first horn plays the end of the trumpet tune, accompanied by the others horns together with the bass trombone, and they all cadence on [1 4] at T_0. This cadence is quickly answered by the beginning of the tune played by the first trumpet at T_0, then by the third horn at T_7, after which the first horn responds with the beginning of the tune at T_1. These three entries are all accompanied by a pedal F in the second horn. When the F is joined by the G♭ in mm. 48-49, the latter pitch sounds more structural because of its fifth relation to

[8] I interpret the oscillating figure in the cellos and double basses as a dissonant double pedal because it is isolated from the forms of [1 4] above it, and because it continues while those harmonies change. The interpretation of these tones as pedal points is further strengthened by the fact that the changing harmonies above it are related in a logical way: they are all related by transposition in pc-space.

the tune above it, and the F is reinterpreted as a dissonant pedal. The first horn is interrupted in mm. 49-50 by a brief contrapuntal section reminiscent of the previous one in mm.32-34, only to resume in mm. 52-54 with the tune and cadence with the other horns on [3 4] at T_0. The cadence is again answered by the trumpet, which is imitated first in the first horn, then in the third, and finally in the second trumpet, before the scene closes with the union of [3 4] and [1 4] at T_7, [C D F G].

In Ex. 4.3, the half notes represent the most fundamental structural pitches, while the eighth notes represent pitches subordinate to those represented by half notes, and the noteheads represent pitches subordinate to those represented by eighth notes. Each half note and eighth note in Ex. 4.3 marks the beginning or ending of a formal unit; though their CRS and TRS values are not given in Ex. 4.3, the pitches on the top stave of each graph are primary topline pitches (with one exception in m. 30 to be discussed below), while those on the bottom stave of each graph are primary chordal tone centers. The register of the pitches in Ex. 4.3 has been normalized in order that one may see the voice leading of the passage more clearly.

The relative distribution of the pitches in each verticality is maintained with three exceptions. The rhythmic oscillation between G and B in the cellos and basses starting in m. 8 is not interpreted as functionally being the bass voice, but as a dissonant double pedal and therefore not structural. When this figure comes back untransposed against the E♭-based sonority in mm. 35-42, it again sounds more like a dissonant double pedal than a pair of structural pitches, and this intuition is reinforced by the fact that it recurs untransposed against a transposed form of *y*, the structural harmony that occurred against the pedal in m. 8.

The low F in the second horn part of mm. 48-49 is likewise considered a pedal, simply a carry-over from the previous three measures, where that pitch was actually structural. The structural harmony in mm. 48-49, [G♭ D♭], is interpreted as an incomplete statement of [3 4] because the missing pitch that would make it complete, C, is the same pitch that was grouped with the dyad [G♭ D♭] earlier. In m. 30, the trumpet tune on C was harmonized by [G♭ D♭], and this is the only other structural occurrence of this dyad in the scene. To reflect this interpretation in Ex. 4.5, the missing C in mm. 48 and 52 has been reduced in size and placed in parentheses.

In m. 30 of the graph, the tune is now in the bass voice, and the perfect fifth that obtains between the outer voices in all of the other structural harmonies is inverted to become a perfect fourth and is placed above the tune. Because the top line is meant to represent melodic function in a structural graph, the parts have been inverted in the Middleground II graph, and the pitch C of the tune placed in the soprano (C is the primary topline pitch; the TRS values for the pitches are: G♭ = 1.66, D♭ = 1, and C = 3.66).[9] And because perfect fourths are often interpreted aurally as inversions of fifths, the G♭ has been placed at the bottom of the verticality in the graph (see Fig. 3.1a; the relatively late entrance of the tune supports interpreting this form of [1 4] as the concatenation of a harmonic fourth supporting a melodic pitch).

One could interpret the voice leading from the previous verticality to the one in m. 30 as a motion into an inner voice; the D♭ after all would continue the chain of structural parallel fifths between the outer voices that persists until the very last chord in the Middleground I and Background graphs. The D♭ is placed above the primary topline pitch, C, in the Middleground I and Background graphs

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[9] The following factors were considered in determining the TRS for each pitch: C receives a weight of 4 for its *ben marcate* marking, G♭ receives a weight of 2 because it is the highest pitch in the chord, and all three pitches received a weight of 1 because the chord begins a phrase lasting from m. 30 to m. 35. Thus: rs(C)=4(4/6)+1= 3.66; rs (G♭)=2(2/6)+1=1.66; and rs (D♭)=0(0/6)+1=1.

in order to focus on the transformational voice-leading of the passage, which is reinforced by the primary topline pitches and chordal tone centers throughout with the exception of this one spot. Thus post-tonal voice-leading graphs, as they are defined here, will show how a particular network of motives is unfolded at various levels of structure, whether these unfoldings are made manifest in associational lines, transformational voices, or both. They will show motivic unfoldings in associational lines (lines that associate all topline pitches and those that associate all chordal tone centers) at their lower levels, as these are the most salient manifestations. But at deeper levels, they will show motivic unfoldings in transformational voices, because these also play an important role in the structure of post-tonal music. It will often be the case that associational lines reinforce the transformational voices. Where they differ, one can speak of a motion into an inner voice, as this best describes how the primary topline pitch or chordal tone center is structurally subordinate to the transformational voice that, on the musical surface, seems less contextually salient.

The transformations connecting [3 4] and [1 4] reinforce hearing this scene as a structural progression of parallel fifths. Example 4.5 shows the transformational voice-leading of the "middleground Γ" graph given in Ex. 4.3. All of the connections linking [3 4] with [1 4] can be interpreted as mod7 inversions around one of three fifths: F/C, G/D, and C/G (One should remember that in a mod7 universe, accidentals are interpreted as merely colorings of the seven-note referential collection). These three axes are connected chronologically in a transformational path that is homomorphic to the internal structure of the motive itself. Example 4.6a shows this homomorphism. The initial F/C axis connecting {F B C} and {F G C} is transposed by T_1 (mod7) to become a G/D axis connecting {G D A} and {G♭ C D♭}. The axis is then transposed by T_6 back to F/C, connecting {G♭ C D♭} and {E♭ F B♭}, before it is transposed by T_4 to become C/G, governing the motion from {F G C} to {G♭ C D♭} and then back again. The first of these axial transpositions corresponds to a transposition of actual pitch-classes – {F G C} is transposed by T_1 (mod7) to become {G D A}. The remaining axial transpositions do not correspond to actual pitch-class transpositions, although the T_4 linking the F/C axis to the C/G axis is immediately followed by a transposition at T_4 linking {F G C} with a subset of the final harmony, {C D G}.

The structural verticalities represented as half notes in Ex. 4.3 – all members of [1 4] – form a transformational path that replicates the internal structural of [1 4] in a purely diatonic form. This creates another homomorphism that mirrors the first, although with a temporal distortion. As Ex. 4.6b shows, the first statement of [1 4] in Ex. 4.3, {F G C}, maps onto {G A D}by T_1 (mod7). {G A D} returns to {F G C} via T_6, and then {F G C} maps onto {C D G} by T_4. (It is hard not to think of this first scene as an auxiliary cadence piece--F, G, and C, being analogous to IV, V, and I, respectively. The structural motion in parallel fifths, however, argues strongly against a tonal reading.) Thus the deepest level of structure represented in Ex. 4.3 replicates the internal structure of [1 4] and at the same time mirrors the path from one inversional axis to the next as the scene progresses. These two homomorphisms represent the greatest analytic payoff in viewing $(014)^7$ as the most structurally important motive of the piece, and consequently in viewing the trumpet tune's motive as significant only on structural levels closer to the musical surface.

As was mentioned earlier, the noteheads in Ex. 4.3 represent neighbors to the eighth notes. The harmony beginning in m. 39 is subordinate because it does not replicate the motive, but is rather a representative of the FB-class [3 6], mod7. Even though it is a member of the same mod7 Tn/TnI-type as the motive, the E♭-based harmony, which comes both before and after it, replicates the same FB-class (a smaller family of relations), and is thus structurally superior. The harmony in m.

51 is only an incomplete statement of the motive, and does not articulate a formal division.

The motive for the piece, $(014)^7$, is represented by two mod12 set classes: $(016)^{12}$ and $(027)^{12}$. The interaction of a mod7 and a mod12 pc-set analysis is revealing. One may view $(014)^7$ as having both a dissonant and a consonant form, the first of which is represented by $(016)^{12}$ and the second of which is represented by $(027)^{12}$. These characterizations are based on our collective experience as listeners of tonal music; harmonies with tritones and semitones, such as $(016)^{12}$, have traditionally been thought of as more dissonant than harmonies without such dissonant intervals. While $(027)^{12}$ does contain the tonally dissonant interval of a whole step, in comparison to the amount of dissonance in $(016)^{12}$, it sounds fairly stable. Dissonance traditionally has had the power to propel music forward because it requires a resolution and thus sets up an expectation in the audience. While this expectation quickly vanishes in much twentieth-century music, it plays an important role here. It is established from the outset: the dissonant form of the motive appears in m. 1, and is resolved in m. 7 by the consonant form. The dissonant form appears once again in m. 30, but this time is not resolved until m. 45. Although the consonant form appears in m. 35, it is not part of the large-scale motivic replication mentioned earlier, and therefore it serves as a neighbor to the verticality following in m. 45, [F G C]. The dissonant double pedal in mm. 36-42 adds to its need for resolution. The dissonant neighboring sonority in mm. 48-54 only serves to propel the music back to [F G C] before it finds its resting place on the union of [3 4] and [1 4] in the final two measures.

Louise Talma's Piano Sonata No. 1, Second Movement

The second movement of Louise Talma's Piano Sonata No. 1 presents another example of post-tonal diatonicism in which multiple structural levels are suggested.[10] Example 4.7 provides an annotated score, while Example 4.8 provides four structural graphs of the movement. The four graphs will be characterized as "middleground I," "middleground II," "middleground III," and "background"; half notes in Ex. 4.8 represent pitches that are included at all levels of structure, while quarter notes represent pitches that are included at all structural levels but the highest, eighth notes represent pitches that are subordinate to the quarters and halves, and noteheads represent pitches that are subordinate to all of the other pitches represented in the graphs. Register has been normalized in all of the graphs to show the voice-leading more clearly. Though the CRS and TRS for each pitch are not shown in the graphs, it should be noted that all the bass pitches are primary chordal tone centers, while all of the pitches on the upper staff are primary topline pitches, with the exception of those representing mm. 28 and 39 in the middleground II, middleground I, and background graphs (these exceptions will be discussed later).

Before discussing the graphs, a pass through the annotated score (Ex. 4.7) will be helpful. The movement begins with a melody and accompaniment texture that quickly grows from three to four voices. The harmonies in mm. 1-4 make it clear that the piece is not tonal, because none are triads. Proceeding chronologically (but omitting harmonic repetitions): {B♭ F G} and {E♭ D G} are both forms of $(013)^7$, {E♭ D A} and {B♭ F C} are both forms of $(014)^7$, {E♭ D C} is a form of $(012)^7$, {C D F G} is a form of $(0134)^7$, {C D F} is a form of $(013)^7$, and {E♭ D G C} is a form of $(0124)^7$. The pitch content of mm. 1-4, the entire

[10] Though this analysis will focus on diatonic material in the movement, one will note that much of the diatonic material can be described as pentatonic material as well. For a systematic account of pentatonic structures in Louise Talma's music, see Luann Dragone, "Structural Consistency Amidst Stylistic Diversity in the Music of Talma," (Ph. D. dissertation, City University of New York, forthcoming).

DT-10 collection, establishes the diatonicism that pervades the work, though it does not establish the collection suggested by the key signature, DT-9. In mm. 5-8, the pitch content shifts to a DT-3 collection; this is followed by an oscillation between DT-8 and DT-10 in mm. 9-10, and then by DT-7 in mm. 11-14.

Mm. 15-25 constitute a varied repetition of the material in mm. 1-14. The variation technique depends heavily upon chromatic alteration, but also creatively mixes materials from mm. 1-14. For example, m. 18 occurs in a parallel position to m. 4, and resembles that measure in the following ways: the first harmonic interval is a T_{-1} transposition of the one in m. 4; the lower three voices in the second accompanimental harmony are a T_{+1} transposition of the parallel harmony in m. 4; and the topline pitches in m. 18 form (014)[7], as do the topline pitches in m. 4. However, m. 18 also bears a significant resemblance to m. 11: *both* accompanimental harmonies in m. 18 are a T_{-10} transposition of those in m. 11 (save the top note of the first harmony in m. 18); and the melodic segment in m. 18 is the T_{-5} retrograde of the first three topline pitches in m. 11. This is characteristic of Talma's variation technique throughout the movement, in cases where the varied material is not a simple transposition.

The varied repetition in mm. 15-25 leads to a five-voice transition with a slower harmonic rhythm in mm. 26-27 (m. 26 is itself a variation of m. 20, but the parallelism is somewhat masked by the new texture). The transition gives way in m. 28 to another four-voice texture marked by a continually moving bass line and repetitive inner voices that continues until another transition in mm. 37-38. The second transition is a slightly varied T_{+1} transposition of the first (mm. 26-27), and gives way in m. 39 to what initially sounds like a T_{+1} transposition of mm. 28-36, but which is significantly varied and truncated.

In mm. 43-54, the texture thins to three voices (though there are often only two), and becomes contrapuntal; though the parts here are not nearly as independent of one another as they are in a Bach fugue, it nevertheless sounds contrapuntal, because the rhythmic and melodic interest is distributed more evenly between the parts than it was previously. This section is the first major departure from melody and accompaniment texture that began the piece, and consequently sounds more like a B section than anything that came before it. It can be divided into two parts based on pc content: mm. 43-48 are composed solely of notes from the pentatonic collection {C D E G A}, while mm. 49-54 are composed solely of notes from DT-4 (which is marked by a change to the corresponding key signature in m. 49).

The latter shift to DT-4 makes for a smooth transition to the varied repetition of material from mm. 1-14 that follows in mm. 55-66, because those measures are almost a literal T_{-1} transposition of the earlier ones (cf. m. 56ff. to m. 3ff.), and thus their pc content is that of the DT-4 collection. This is followed in mm. 67-79 by a varied repetition of mm. 15-27 that begins transposed by T_{-1}. However, in m. 76 the T_{-1} transposition is replaced by a T_{-1} transposition that returns the music to the DT-9 collection of mm. 5-8 suggested by the key signature. In mm. 78-79, the varied repetition of material from the transition in mm. 26-27 adds stability to the DT-9 collection and in the following two measures a cadential gesture that ends on {E♭ G C F} closes the movement. Though the final bass note does coincide with the major key suggested by the key signature, one will note that Talma has denied the listener the E♭-major triad that would provide a stronger sense of closure in E♭., but instead ends on a form of (0124)[7].

Since the multi-leveled interpretation here depends upon the movement's formal structure, the form of the movement can be followed in the voice-leading graphs given as Ex. 4.8. The largest formal divisions of the movement are shown in the middleground I graph, and are based on texture: the melody and accompaniment texture in mm. 1-43 (the internal division at m. 39 will be discussed below) breaks in m. 43, where it is replaced by a contrapuntal texture. At m. 55, the contrapuntal texture dissolves into a varied repeat of material from the first section that continues until the end (compare mm. 55ff. with mm. 3ff.). Interpreting the harmony in m. 1 as {B♭ C D F} is based on the fact the first accompanimental dyad if the movement in m. 2, {B♭ F}, when added to notes of m. 1, <C D>, forms the larger tetrachord, and that the complete harmony occurs at a parallel position when the varied repetition begins in m. 15.

The two repeated harmonies in the graph represent sections that are varied repetitions in the music, not literal ones. In particular, one should note that the accidentals in parentheses represent the fact that one common variation technique employed in this movement is transposition by semitone, and this "chromatic sideslipping" is taken to be nonstructural throughout. Thus the key signature starting in m. 49 that represents DT-4 material is not interpreted as representing a structural change, but merely a chromatic coloring of material that belongs to DT-9.

The middleground II graph shows how the first large section (mm. 1-43) is further subdivided by a textural change in mm. 28-38 that recurs varied and transposed in mm. 39-43. While the texture in these sections is still best characterized as melody-accompaniment, the bass line in these measures now moves independently while the inner voices maintain a steady stream of eighth notes. Though m. 39 is essentially a varied repetition of m. 28, it is interpreted as structural on a higher level because it marks the end of the first large section. The four subsections of the first large section are labeled A1, A2, A3, and A4 in the music, while the contrapuntal section that follows will be labeled B, and the varied repetitions of A1 and A2 that close the movement will be labeled A1' and A2'.

The middleground III graph shows how mm. 26-27, 37-38, and 78-79 all represent contextually salient transitions that are nevertheless subordinate to the events which follow them. One will note that the harmonies in mm. 28 and 39 of the middleground III graph are voiced differently than in the middleground II graph. As was noted earlier, register has been normalized in the graphs to show the voice leading of the passage more clearly. However, in the middleground II graph (and in the middleground I and background graphs), the voicings of the harmonies have been altered as well in order to show the underlying step-class counterpoint. That is, while the top note in every chord of the middleground III graph represents a primary topline pitch, it also shows a structural motion into an inner voice that is normalized in the middleground II graph; thus the middleground II graph represents the harmonies in mm. 28 and 39 as transformational voices in order to relate it to the initial harmony, while the middleground III graph represents them as associational lines in order to clarify their relationship to the musical surface.[11]

The background graph reveals that there are two primary motives for the movement: $(014)^7$, which governs the harmonic organization of the movement with the help of its superset $(0124)^7$, and the segment <4 2 0>^{7}, which governs its linear organization. To say that $(014)^7$ governs the harmonic organization of the movement when only one of the background harmonies form that trichord requires some explanation. The reasons for interpreting $(0124)^7$ as an augmented form of $(014)^7$ lie in the extent to which $(014)^7$ governs the musical surface. The first two structural forms of $(0124)^7$ in mm. 1 and 15 very much suggest a form of $(014)^7$ to which D is added, because the D in both cases follows and displaces the C, and because the C is much longer (twice as long in m. 1, three times as long in m. 15).

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[11] For a review of the difference between transformational voices and associational lines, see Chapter 1, x-xx. It is worth noting that motions to an inner voice in post-tonal music can be understood as transformational voices that cut across registral lines.

The displacement of C by D causes the harmony to become (024)^7, and thus the (0124)^7 in mm. 1 and 15 of the graph actually represents a subtle shift in the music from (014)^7, the movement's primary harmonic motive, to (024)^7, the unordered form of the movement's primary melodic motive, <4 2 0>^{7}. This is a good example of how the two primary motives are bound together by events on the musical surface: the primary melodic motive plays a secondary role in the movement's harmonic structure, and the primary harmonic motive plays a secondary role in the movement's melodic structure. A good example of the latter case is found in mm. 1-4, in which every topline pitch is part of some form of the primary harmonic motive, (014)^7. It is worth noting that (0124)^7 is the only tetrachord that is a superset of both (014)^7 and (024)^7, and thus any synthesis of these two trichords will necessarily result in some form of (0124)^7 (the subsets and superset relationships are given in Appendix C).

Identifying (014)^7 and <4 2 0>^{7} as the primary harmonic and melodic motives is based solely on their significance in the background graph, not on the number of times they occur harmonically or melodically on the musical surface, or on any other level of musical structure. However, both are emphasized strongly on the musical surface in different ways. As was mentioned above, (014)^7 governs the topline structure of mm. 1-4; it also governs the topline structure of mm. 11-12, 15-19, 22-25, 41, 44, 46, 56, 63-64, 67-71, 74, and 76-77.[12] Almost half of the measures that are not (014)^7-governed are governed by (024)^7: mm. 5-7, 13-14, 20-21, 26-28, 37-38, 57-59, 65-66, 72-73, and 78-81. Together (014)^7 and (024)^7 explain 52 complete measures (64% of the movement's total number of

[12] By "governs the melodic structure," I mean that every pitch class in the top voice of the given span can be explained as part of some contiguous form of (014)^7.

measures, 81), and many of the remaining 29 contain pitch classes that are part of some contiguous form of $(014)^7$ and $(024)^7$, though neither trichord by itself can account for every topline pitch in the measure.

There are two other sets that are important on the musical surface: $(013)^7$ and $(0135)^7$. The former set governs the topline structure of mm. 8-10, 31-36, 39-40, 45, 48, 51-52, 54-55, 60-62, and 75; it thus accounts for 20 measures total, which is fewer than either $(014)^7$, which accounts for 29 measures, or $(024)^7$, which accounts for 23. It nevertheless plays an important role in the movement's topline structure, far greater than that of $(012)^7$, the remaining diatonic set class (there are only four). The set class $(0135)^7$ plays an role in the movement's pitch organization important enough to warrant its inclusion in the middleground III graph. It marks the beginning of the transition from the A2 section to the A3 section, as well as the transition from the A3 section to the A4 section, and the transitional material associated with it is revisited in mm. 78-79, just three measures before the end of the movement.

Three different transpositions of <4 2 0>^{7} are particularly salient because they each include a pitch that is the highest note of the movement up to that point: <D# B G#> in m. 20, <B G# E> in mm. 27-28, and <C A F> in mm. 38-39 (the C in m. 38 is also the highest note of the piece). Though the transposition of <4 2 0>^{7} that occurs in m. 70, <E C A>, does not include a new high point for the movement, its high C6 is nevertheless the highest pitch to occur since m. 39, and thus seems equally stressed. The forms in mm. 20 and 70 are both immediately followed by a repetition of the same segment, which creates still further emphasis on <4 2 0>^{7}. In addition, though $(024)^7$ does not always occur as the ordered segment <4 2 0>^{7}, the first form of $(024)^7$ in the movement, the segment <D B G> in m. 5, does in fact exemplify this particular ordering, and is repeated in mm. 6-7 before another ordering of $(024)^7$ is presented.

When moving from the background graph to the middleground I graph, one finds that the fundamental linear progression of <4 2 0>^{7} is interrupted by the return of material from earlier in the piece. This post-tonal interruption is roughly analogous to the Schenkerian concept of interruption in that a fundamental progression is allowed to proceed until the penultimate chord before being stopped and forced to start over, the second time through proceeding without hindrance to the final chord. However, this is the extent of the similarity. Besides the obvious differences between the tonal and post-tonal orientations of the two interruptions, the post-tonal interruption described here does not force the fundamental progression to retrace its steps as a Schenkerian interruption does, but it instead allows the progression to proceed from the repeat of the initial material directly to the final harmony; there is no return to the {G A D} harmony in m. 43, though in a Schenkerian interruption there is always a structural scale-degree 2 which precedes the final harmony after the interruption occurs. Schenker's concept of interruption holds greater explanatory power in terms of its applicability than the one described here, because tonality, the context in which his concept of interruption was formed, was a common language for all of the composers he studied. However, in post-tonal music, there is no such common language, and thus the post-tonal interruption asserted here will necessarily be common to far fewer pieces.

Example 4.9 represents the background as a transformational network. This network representation points out another interesting feature of the movement: though the outer voices are guided by the motive <4 2 0>^{7}, the inner voices are not. In cases like this one, a looser notion of transposition is useful; one such notion has been presented by Joseph N. Straus as near-transposition.[13]

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[13] See Joseph N. Straus, "Voice-Leading in Atonal Music," in *Music Theory in Concept and Practice*, eds. James Baker, David Beach, and Jonathan Bernard (Rochester: University of Rochester Press, 1997), 237-274.

According to Straus, "two harmonies are related by near-transposition or near-inversion if all but one of their notes are related by actual transposition or actual inversion."[14] In Ex. 4.9, the larger harmonies of the background in Ex. 4.8 have been reduced to their respective $(014)^7$ subsets and the transformational path connecting them is interpreted as T_5, mod7, followed by a near-T_5, mod7.

Copland's *Appalachian Spring*, mm. 622-end

This chapter's final example of post-tonal diatonic music that suggests multiple structural levels is the last section of Copland's *Appalachian Spring*. Example 4.10 provides a short score of the arrangement for orchestra, m. 622-end. The first question that must be addressed in this analysis is whether or not the section is indeed "post-tonal," given that the vast majority of chords therein are either triads or seventh chords, and that there is a strong sense of functional tonality in much of the section. But before addressing this important question, a chronological trip through the piece would be helpful.

The section begins with the strings playing in C major. Though the section is for the most part in C major, it borrows ♭III and ♭VI from the parallel minor. The first chord of the section is ♭III, but because ♮III occurs in the same voicing three chords later, and because the other four chords in the first two measures all are part of C major, the sense of C major is not seriously compromised. The melodic phrase in the first two measures is a form of the segment $<21021>^7$, a segment that governs the linear organization of the section's musical surface. One will note that, except for the G in m. 622, the bass part unfolds the same segment a tenth lower. The second phrase (mm. 624-625) is a varied repetition of the first;

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[14] Ibid, 268.

however, the original form of <21021>^{7} in the top voice is not varied, but repeated note for note. In the third phrase (mm. 626-630), a new form of <21021>^{7} in the melody is harmonized by an inverted form of that segment in the bass, leading the music to the dominant of C major.[15] The phrase does not end on the dominant, however, but continues from the dominant to the subdominant, the final chord of the phrase. The move from V to IV here is counterbalanced in what follows (mm. 631-639): a slightly varied repetition of the first three phrases that ends with a motion from V to VI. The V to VI motion balances the V to IV motion not only because it occurs in a parallel position, but because the VI chord is altered to be a major triad, matching the IV chord in both quality and context, if not in function.

At m. 640, the woodwinds take over, playing in a new tempo. The next phrase (mm. 640-641) is a varied repetition of the first transposed to G minor; the phrase that follows it (mm. 642-643) stays in G minor, but transposes the form of <21021>^{7} in the previous phrase down a third. The following two phrases (mm. 644-647) are varied repetitions of the two G-minor phrases which precede them, but in the latter of these, the variation reharmonizes the melodic material to cadence in F major. The strings return in m. 648 (along with a return of the original tempo and key) to join the winds in a varied repetition of the first three phrases, the last of which is recast to end with an imperfect authentic cadence in C, the first authentic cadence of the section.

The final portion of the section, marked *Andante*, features a solo flute accompanied by strings, harp, and solos in the first violin and clarinet; its material is drawn from earlier in the work (mm. 80-97; see also mm. 136-146). It is

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[15] Though the last note of the third phrase in effect makes it a form of the segment <210210>^{7}, the notes of the third phrase may be reasonably segmented into 5 + 1 given that <21021>^{7} is embedded in the longer third phrase.

essentially a coda that strives for a perfect authentic cadence with which to close the work. Its first phrase (mm. 657-659), the first melody of the section that is not a form of <21021>^{7}, ends with an imperfect authentic cadence with the third of the tonic triad in the top voice. The next phrase (mm. 660-665; actually divided into halves by the slurs in the music) is a varied repetition of the preceding phrase in which three measures of material has been inserted. It also ends on an imperfect cadence with the third of the triad in the tonic triad in the top voice. The third phrase of the coda (mm. 666-668) is another varied repetition of the coda's first phrase that also ends with an imperfect cadence, but has no additional material inserted. The only variation from the first phrase is found in its final chord, in which the fifth replaces the third in the top voice of the tonic triad. It is only with the fourth phrase of the coda (mm. 669-672) that the work reaches its definitive close: a perfect authentic cadence in C. What remains is a codetta that harkens back to the opening of the entire work over a sustained tonic pedal.

As was mentioned earlier, the first question an analyst must tackle in approaching this music is whether or not it is tonal, or perhaps put better, *to what degree* is the work tonal. Ex. 4.10 has been annotated with a traditional tonal analysis, including Roman numerals as well as the letter "p" between those harmonies that are connected through parallel fifths and octaves. Those Roman numerals offset by quotation marks represent forms of $(0124)^7$ that can be heard as simple derivations of functional harmonies. Each of the $(0124)^7$ harmonies found in mm. 622-653 contains the root, fifth, and seventh of the chord indicated by the Roman numeral, but the third of that chord has been supplanted by a pitch a step above the root. These harmonies function as either IV^7 or V^7, respectively; the "IV^7" chords all move to dominant harmonies, their sevenths resolving down by step as they should, and the "V^7" chords all move to tonic harmonies, and their sevenths also resolve down by step. Though these adherences to tonal norms might seem to obviate any post-tonal reading suggested by the intervallic structure of the chord, these adherences are undermined still further by chromatic inflections. In the case of "IV^7," the inflection is a flatting of the seventh (see mm. 627, 636, 643, and 653). Given that the seventh is already a downward-tending tone, this is no great problem and can be explained as simple mixture. But in the case of "V^7," the inflections include the flatting of both the added tone and the fifth of the chord (see mm. 624, 631, 633, and 650). The flatting of the fifth in a V^7 chord lowers the second scale degree and thus cannot be understood as derived through primary or second mixture, since scale-degree ♭2 is not part of the parallel minor mode, and it is the fifth of the chord rather than the third that is altered. Nor can it be understood in relation to the traditional Neapolitan usage of ♭2, since this chord has a dominant rather a subdominant function. The simultaneous flatting of the added tone merely reinforces the fact that this harmony is beyond the bounds of tonality. There is only one form of $(0124)^7$ in mm. 658-670, and it functions as a tonic harmony in first inversion: {C E F G}. Its foreign note, the high F in the melody (see mm. 658, 661, 667, and 670), could be understood as an incomplete upper neighbor to the E which follows in the next chord were it not for the fact that E occurs in the bass simultaneously with the F each time.

There are several other reasons to interpret this passage post-tonally, however. The relatively large number of parallel 5ths and 8vas in the passage casts suspicion over any tonal reading. What tonal piece has this many parallel fifths and octaves? The planing succession of triads in this passage reminds one of organum or of Debussy, but certainly not of any tonal work. In addition, the leading-tone is often avoided when part of a V^7 chord, either through the substitution of $(0124)^7$ discussed above, or through mixture (B is flatted in mm. 626, 635, and 652), and when it does occur, it often avoids a direct resolution to the tonic (mm. 628, 637, 659, 665, and 668). There is also a curious absence of diminished triads, both as free-standing harmonies and as subsets of dominant seventh chords. The substitution of $(0124)^7$ for V^7 mentioned above results in the avoidance of the diminished triad that would otherwise be formed between the third, fifth, and seventh of the chord, and when the II chord occurs in the G minor passage (mm. 640-645), the fifth of the chord is raised each time to form an A minor triad rather than a diminished one.

Having just argued for a post-tonal reading, it should be reiterated that there are also good reasons to interpret the section tonally; namely, there are functional harmonic progressions – if one accepts the given functional interpretations of the $(0124)^7$ – as well as several clearly presented authentic cadences. Schenkerian voice-leading graphs of the section are given as Ex. 4.11. The graphs do not show accidentals that are viewed as mere chromatic colorings of an established diatonic collection; the B♭ representing the primary topline pitch in m. 641 is included because it is part of the G minor and F major collections that span mm. 641-647. One will note that although parallel fifths and octaves are present in the Middleground III and Middleground II graphs (see mm. 626-641), they do not extend to the structural depths represented by the Middleground I and Background graphs. This seems to aptly summarize the music's place in the tonal/post-tonal continuum: it is in the middle.

While the Schenkerian perspective of Ex. 4.11 illustrates important information about the section's tonal structure, it conveys relatively little about its motives. This is because a Schenkerian graph's main function is to show the relationship between a work's musical surface and its fundamental structure, though some Schenkerians have shown motivic parallelisms in their graphs as well.[16] The fundamental structure in a voice-leading graph is a summary of the given work's most basic melodic and harmonic components. However, while the fundamental structure of a Schenkerian voice-leading graph must be a tonally functional harmonic progression, the fundamental structure of a post-tonal voice-leading graph has no tonal requirement, but rather a motivic one: each of its notes must be part of a contextually salient motive. Ex. 4.12 provides four post-tonal voice-leading graphs of mm. 622-end. In the graph, $(0124)^7$ is interpreted as the motive most fundamental to the section's structure. Though we have already noted that $\langle 21021 \rangle^7$ governs the linear organization of this section's music surface, this does not preclude another motive from governing its fundamental structure; in most post-tonal works, more than one motive plays a significant role in the work's pitch organization. Some motives govern higher structural levels, while others govern lower ones, as was the case in the first scene of *Agon*. (The motive $\langle 2310 \rangle^7$ at the end of the trumpet tune in *Agon's* first scene is important at the foreground level, as is shown in Ex. 4.2, while the motive $(014)^7$ governs the scene's background, as is shown in Ex. 4.3.)

The $(0124)^7$ motive in Ex. 4.12 does, however, play a significant role on the work's musical surface. There only three diatonic set classes presented as vertical sonorities in the last section, and $(0124)^7$ is one of them (the triad and the

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[16] For more on the relationship between motive and fundamental structure in Schenkerian analyses; see Heinrich Schenker, *Free Composition*, trans. Ernst Oster (New York: Longman, 1979), 97-101; Charles Burkhart, "Schenker's Motivic Parallelisms," *Journal of Music Theory* 22 (1978), 145-76; John Rothgeb, "Thematic Content: A Schenkerian View," in *Aspects of Schenkerian Theory*, ed. David Beach (New Haven: Yale University Press, 1983), 39-60; Richard Cohn and Douglas Dempster, "Hierarchical Unity, Plural Unities: Toward a Reconciliation," in *Disciplining Music: Musicology and its Canons*, ed. Bergeron and Bohlman (Chicago: University of Chicago Press, 1992), 156-181; Richard Cohn, "The Autonomy of Motives in Schenkerian Accounts of Tonal Music," *Music Theory Spectrum* 14/2 (1992), 150-70; and "Schenker's Theory, Schenkerian Theory: Pure Unity or Constructive Conflict?" *Indiana Theory Review* 13/1 (1992), 1-20.

seventh chord are the other two types). But $(0124)^7$ is contextually salient on the musical surface long before the last section, as Example 4.13 illustrates. Ex. 4.13a provides mm. 1-6; the first occurrence of $(0124)^7$ in the work is in m. 6, where a dominant triad is sustained over a tonic pedal in the context of A major, resulting in {E G♯ A B}. Juxtapositions of tonic and dominant harmonies, and of tonic and subdominant harmonies, are characteristic of *Appalachian Spring*. These juxtapositions are more often between two complete major triads a fifth apart, but it is noteworthy that the first nontriadic harmony of the work is not a juxtaposition of two complete tonic and dominant triads, but rather the juxtaposition of a dominant triad over a tonic pedal. Ex. 4.13b illustrates the occurrence of $(0124)^7$ in mm. 74-80, where it is the subject of an imitative section that grows in intensity until the brass and winds join together in a homophonic statement of the material that is later used in the coda (mm. 657-end). The forms of $(0124)^7$ that participate in this buildup are: {C D E G}, {F G A C}, {D E F♯ A}, and {G A B D}. Ex. 4.13c illustrates how Copland's most frequently cited use of quotation, the Shaker hymn in *Appalachian Spring* "Tis the Gift to be Simple," highlights $(0124)^7$ through its metric structure; its first sixteen measures are provided in the example, and the pitches receiving metric stress are circled.

There is only one pitch class present in the middleground I graph that is not retained at the background level: the A in mm. 639 and 647. The A, when added to the background's collection of primary topline pitches, forms $(01245)^7$. However, when the A is added to the background's collection of primary chordal tone centers, $(01235)^7$ is formed. Both of these pentachordal Tn/TnI-types play important roles on the musical surface earlier in the work, as is shown in Example 4.14. Ex. 4.14a provides the theme from mm. 62-64 and mm. 80-82, and shows how its contour pitches form $(01245)^7$. (Though not shown in Ex. 4.14a, one should also note that the metrically emphasized pitches in the theme form $(0124)^7$.)

Ex. 4.14b shows juxtapositions of major triads a fifth apart in mm. 7-10, 21-24, and 30-35; such juxtapositions will inevitably result in forms of $(01235)^7$. One of the most salient occurrences of $(01235)^7$ is not shown in Ex. 4.14, but rather in Ex. 4.10: the final harmony of the movement (marked "I + V" in the example) is another form of $(01235)^7$. It is noteworthy that the most salient occurrences of $(01245)^7$ are melodically presented, and that the most salient occurrences of $(01235)^7$ are harmonically presented, given that the pc set representing $(01245)^7$ in the voice-leading graphs of Ex. 4.12 is formed between pitches representing melodic function, and that the pc set representing $(01235)^7$ in the voice-leading graphs of Ex. 4.12 is formed between pitches representing harmonic function. The examples of $(0124)^7$, $(01245)^7$, and $(01235)^7$ provided in Exx. 4.13 and 4.14 are by no means exhaustive, but are merely given as representative examples to show that their relative importance is established early on in the work as a whole.

Regarding the structural weight of particular pitches, there are two significant differences between the Schenkerian voice-leading graphs given in Ex. 4.11 and the post-tonal voice-leading graphs given in Ex. 4.12. The first is that the initial harmonies of phrases are interpreted as structural in the post-tonal graphs, but not in the Schenkerian graphs. This is because post-tonal graphs take the beginnings and endings of formal units to be structurally superior to what occurs in between, while Schenkerian graphs take chords that function as tonics to be structurally superior to those with other harmonic functions. Thus the post-tonal voice-leading graphs interpret the first E♭-major chord of the section to be structurally superior to the section's first C-major chord, because the E♭-major chord occurs at the beginning of the first phrase, while the C-major chord is "buried" in the middle. However, the Schenkerian voice-leading graphs interpret the C-major chord in the first phrase as superior to the E♭-major chord that precedes it because the C-major chord functions as tonic harmony in the section as a whole.

The second significant difference between the relative structural weight of pitches in Exx. 4.11 and 4.12 relates to the G-minor and F-major material in mm. 640-647. The post-tonal Middleground II in Ex. 4.12 explains the structural pitches in this span as part of a linear projection of the motive $(0124)^7$, or more specifically, a projection of the same step-class segment that governs the topline of the section's post-tonal background: $<4210>^7$. The Schenkerian Middleground II in Ex. 4.11 does not interpret the initial harmony as structural (for the reason mentioned above), but understands the F and A in m. 647 as notes within the IV^7 harmony, and understands the G and B♭ in m. 641 as upper neighbors to the F and A, respectively.

One might initially describe the relationship between the tonal and post-tonal voice-leading graphs given here as one of conflict, but this would be a mistake. There is no conflict between the two kinds of graphs, because they reflect two fundamentally different aspects of the music: the tonal voice-leading graphs reflect the music's relationship to basic tonal paradigms, while the post-tonal ones reflect the music's motivic network as it unfolds on multiple structural levels. Thus, this last analysis is not a dialect (though the qualifiers "tonal" and "post-tonal" might lead one to that conclusion), because there is no inherent contradiction between the graphs in Ex. 4.11 and those in Ex. 4.12. The graphs in Ex. 4.12 are called "post-tonal," not because they represent an antithesis to the tonal voice-leading graphs of Ex. 4.11, but because they reflect aspects of the music that depend on contextual salience rather than tonal coherence.

Summary

The three ways that allow one to distinguish between structural and ornamental pitches in post-tonal diatonic music – salience criteria (as explained in Figures 3.1, 3.3, and 4.1), motivic significance, and identification as ornament type – have been used here to reveal multiple structural levels in works by Stravinsky, Talma, and Copland. As we have seen, the process of reinterpreting certain structural pitches as ornamental on a higher level makes a multi-leveled conception of pitch organization in a post-tonal diatonic work possible. While the success of this approach will be dependent upon the analyst's ability to recognize the different levels of hierarchy in a work's formal structure, the success of any comprehensive musical analysis depends upon that same ability.

Chapter 5

Beyond Mod7: Relating Diatonic and Non-Diatonic Materials

Thus far we have seen how mod7 set theory can be useful to those studying works or passages in the post-tonal repertoire that are primarily diatonic. However, it is often the case that a post-tonal work is composed of diatonic passages interwoven with passages derived from other kinds of pc collections, the most common being the pentatonic, whole-tone, octatonic, and chromatic collections. These collections can be understood as modular spaces because they each treat the octave as a modular unit and partition it in a unique way. In these works, musical ideas presented originally in a diatonic space are often transformed to occur in non-diatonic ones, and vice versa. This final chapter will focus on a transformation that maps the pentatonic, whole-tone, diatonic, octatonic, and chromatic forms of a given musical idea onto one another, and how a further generalization of the term "step-class" can lead to an equivalence class that transcends the boundaries defining individual modular spaces.

Bartók, in a lecture at Harvard University in 1943, has acknowledged his discovery and use of a transformation that maps musical entities from chromatic to diatonic modular spaces:

> [W]orking [with] chromatic degrees gave me [an] idea which led to the use of a new device. This consists of the change of the chromatic degrees into diatonic degrees. In other words, the succession of chromatic degrees is extended by leveling them over a diatonic terrain.
>
> You know very well the extension of themes in their values called augmentation, and their compression in value called diminution. These devices are very well known... Now, this new device could be called "extension in range" of a theme. For the extension we have the liberty to choose any diatonic scale or mode.[1]

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[1] Béla Bartók, "Harvard Lectures," in *Béla Bartók Essays*, ed. Benjamin Suchoff (New York: St. Martin's Press, 1976), 381. Wayne Alpern has also explored this remodularization technique in Bartók's music from a transformational perspective

Examples 5.1-5.3 show instances of this transformation in Bartók's music. Example 5.1 is taken from *Music for Strings, Percussion, and Celesta*, and shows how the beginning of the chromatic theme from the first movement maps onto the beginning of the diatonic theme in m. 204 from the fourth movement; that is, scale-degree 1 from the chromatic collection maps onto scale-degree 1 from the diatonic, scale-degree 2 maps onto scale-degree 2, etc. The numbers beneath the example provide the ordered presentation of scale degrees within the respective moduli; this nomenclature will be used in all of the following examples. Examples 5.2 and 5.3 are taken from *Mikrokosmos*. Example 5.2 shows how No. 64a maps onto No. 64b by a transformation that compresses the diatonic thematic material into a chromatic space, while Example 5.3 shows how the theme from the beginning of No. 112 maps onto the theme at the beginning of its second section by a transformation that compresses the diatonic music into a chromatic space (numerals in parentheses below the staves in Ex. 5.3 represent scale degrees which do not participate).[2]

But Bartók's transformation need not be limited to mappings between diatonic and chromatic spaces, and one can find examples of mappings to and from other modular spaces in Bartók's own music. Example 5.4 is taken from the first

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in his paper "Bartók's Compositional Process: 'Extension in Range' as a Progressive Contour Transformation," (Music Theory Midwest Conference, May 15, 1998). Examples 5.1, 5.2, 5.3, and 5.10 are taken from Alpern; Alpern's work generalizes this operation to non-modularized progressive contour transformations in Bartók as well as Berg, whereas this chapter retains an exclusively modular perspective.

[2] The terms "modular space" and modular system" will be defined here as synonyms referring to specified partitionings of the octave within the larger context of an equal-tempered harmonic system.

movement of his String Quartet No. 4, and shows how the chromatic first theme in m. 7 maps onto the octatonic segment in the first violin part of m. 158. The first part of this chapter will formalize Bartók's transformation, as well as generalize it to map musical entities to and from any one of five different modular spaces: pentatonic (mod5), whole-tone (mod6), diatonic (mod7), octatonic (mod8), and chromatic (mod12).[3]

Modular Transformations

It is possible to discuss Bartók's transformation in a more systematic way. Before doing so, however, it would be useful to replace the term "scale degree" in

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[3] This study has been limited to pentatonic, whole-tone, diatonic, octatonic, and chromatic spaces because these are the most important representatives of partitions of the octave into five, six, seven, eight, and twelve, respectively. Though many seven-note scales besides the diatonic are frequently used, musicians often understand these scales as derivations from a diatonic norm (e.g. the melodic minor is often explained as a natural minor scale with raised sixth and seventh scale degrees, and its fourth rotation, the "acoustic" scale, is often explained as a major scale with a raised fourth scale degree and a lowered seventh scale degree). It will be easy for the reader to apply much of this study to such derived spaces, but it would be overly cumbersome to include them all here. There has been much work done on the properties of the modular systems to be considered in this study. See Milton Babbitt, "The Structure and Function of Music Theory I," *College Music Symposium* 5 (1965), 49-60; Hubert Howe, "Some Combinational Properties of Pitch Structures," *Perspectives of New Music* 4/1 (1965), 45-61; Carlton Gamer, "Some Combinational Resources of Equal-Tempered Systems," *Journal of Music Theory* 11/1 (1967), 32-59; Robert Cogan and Pozzi Escot, *Sonic Design* (New York: Prentice-Hall, 1976); Jay Rahn, "Some Recurrent Features of Scales," *In Theory Only* 2/11-12 (1977), 43-52; Richmond Browne, "Tonal Implications of the Diatonic Set," *In Theory Only* 5/6-7 (1981), 3-21; Robert Gauldin, "The Cycle-7 Complex: Relations of Diatonic Set Theory to the Evolution of Ancient Tonal Systems," *Music Theory Spectrum* 5 (1983), 39-55; John Clough and Gerald Myerson, "Variety and Multiplicity in Diatonic Systems," *Journal of Music Theory* 29/2 (1985); Norman Carey and David Clampitt, "Aspects of Well-Formed Scales," *Music Theory Spectrum* 11/2 (1989), 187-206; and Jay Rahn, "Coordination of Interval Sizes in Seven-Tone Collections," *Journal of Music Theory* 35 (1991), 33-60.

Bartók's definition with the term "step class." The definition of "step class" given in chapter 1 is bound to a mod7 perspective, but it can easily be redefined here. A step class will be defined as a numbered position within a modular system, and step classes will be numbered 0 to n (n being equal to the cardinality of the modulus minus 1).[4] As before, step classes will group steps (i.e. notes in some specified scale) stretching across the entire audible spectrum according to octave equivalence, just as pitch classes group pitches stretching across the entire audible spectrum according to octave equivalence.

Let us define MODTRANS (x, y, z) as a transformation that maps each step class of a musical entity in modular system *x* onto a corresponding step class in modular system *y*, where *z* represents the "point of synchronization," the pitch class in the starting modulus that is interpreted as step-class 0. From this point of synchronization, one may then construct a table of mappings from the starting modulus to the destination modulus and use the table to map pitch classes from the first musical entity to the second one. Table 5.1 provides all of the possible rotations for the mod12, mod8, mod7, mod6, and mod5 systems considered here in integer notation, arranged so that one may easily find their corresponding step classes; it can thus be used to find the mappings for any MODTRANS operation occurring between the five systems considered. The integer notation in Table 5.1 sets the point of synchronization equal to 0, and the remaining pitch classes of that entity receive a value equal to their distance in semitones above the point of synchronization. Consequently, one can see the intervallic structure of the five systems considered here embedded in Table 1 by reading it from left to right. For

[4] Because "step-class" here is defined as an order position within a modular system, there is a danger that it might be confused with the term "order position." This chapter advocates using "order position" to refer to ordering in a context that is specific to a musical work, such as a twelve-tone row, and reserving the use of "step class" for orderings of the five modular systems discussed here.

example, find the two unique rotations of the octatonic system in Table 1; one can see that its intervallic structure is clearly embedded in the table. That is, reading the second row from left to right, it moves from 0 to 1 (half-step), from 1 to 3 (whole-step), from 3 to 4 (half-step), from 4 to 6 (whole-step), and so on. In reading the third row from left to right, it moves from 0 to 2 (whole-step), from 2 to 3 (half-step), from 3 to 5 (whole-step), from 5 to 6 (half-step), and so on. To compare similar step-classes realized in different modular spaces when the point of synchronization is step-class 0, one simply compares the integers within the same column of the table. For example, step-class 6 in a chromatic space is 6 semitones above the point of synchronization; in an octatonic space it is 9 semitones above the point of synchronization, and in a diatonic space it is either 10 or 11 semitones above the point of synchronization, depending upon which particular rotation of the collection is taken to be the referent.

Applying MODTRANS to musical entities in unevenly partitioned systems will yield different results depending on where in the system the point of synchronization lies, therefore we must consider all rotations of the unevenly partitioned systems separately, and must label them individually. Because the intervallic structure of the octatonic system is an alternating series of half steps and whole steps, it will be represented in two different ways: those beginning with a half step will be represented as 8^1, and those beginning with a whole step will be represented as 8^2. Similarly, because the intervallic structure of the diatonic system is composed of an uneven distribution of half steps and whole steps, it will be represented in seven different ways, one for each of the seven modes of the major scale. Thus the Ionian mode will be represented by 7^1, the Dorian mode will be represented by 7^2, the Phrygian mode will be represented by 7^3, and so on. The chromatic and whole-tone systems will be represented by 12 and 6 respectively, and the five rotations of the pentatonic scale will be labeled as is indicated in Table 1.

In using this chart one must keep in mind that MODTRANS often occurs in combination with transposition; of the first four examples, all but Ex. 5.2 involve transposition. Thus, in using the Table 5.1 to interpret Ex. 5.4, we set its lowest note, B♭, equal to 0 and map the first four mod12 step classes in m. 7 – (0123), or (B♭ B C D♭) –onto the first four mod8^1 step classes listed below it – (0134), or (B♭ B C♯ D) – and then transpose the result up a major third, arriving at the (D E♭ F G♭) tetrachord in m. 158. This combination of transformations would be labeled MODTRANS (12, 8^1, B♭) and T_{-4}, as is shown in Example 5.5.

Example 5.6 illustrates how this table works. In the example, the step-class segment <0124> begins in a chromatic space as <C C♯ D E>, and then is transformed to occur in each rotation of the modular spaces considered in this study (proceeding from top to bottom through the table). Thus, the first transformation is MODTRANS (12, 8^1, C) and its mappings are provided between the first two rows on the table, the second transformation is MODTRANS (8^1, 8^2, C) and its mappings are provided between the second and third rows, and so on. If the ordering and registral distribution of the pitch classes in this example were different, the mappings would nevertheless be the same: step-class 0 will always map onto step-class 0, step-class 1 will always map onto step-class 1, and so on, regardless of order position or register. C has been chosen to be the point of synchronization, but it could just as easily be any other pitch class. The table here is arranged so that the point of synchronization is interpreted as step-class 0, as this seems the most intuitive choice; however, one could produce similar tables in which other step-classes represent the point of synchronization.

While MODTRANS is commutative with respect to transposition, it is not commutative with respect to inversion; that is, the results of a compound transformation that pairs MODTRANS with inversion will differ depending upon which transformation is applied first.[5] Example 5.7 illustrates this point. If we were to transform the whole-tone segment <D E F♯ C> by MODTRANS (6, 8^2, C), we would get <D E♭ F C>. If we were to then invert <D E♭ F C> around a D axis, we would get <D C♯ B E>. On the other hand, if we were to invert <D E F♯ C> around a D axis, we would get <D C B♭ E>. If we were then to transform <D C B♭ E> by MODTRANS (6, 8^2, C), we would get <D C A♭ E♭>, not <D C♯ B E>. (Note that while transposition preserves the original succession of step classes, inversion does not.) In order to avoid problems related to the limited commutativity of the MODTRANS operation, an order of operations must be defined; thus it will be defined here that in all compound transformations involving MODTRANS, the MODTRANS operation will be applied *before* the other operations.[6]

In Ex. 5.7, some might argue that inverting <D E♭ F C> within the octatonic collection suggested by the MODTRANS operation – i.e. employing mod8 inversion – would be more appropriate, and similarly that inverting <D E F♯ C> within the whole-tone collection suggested by the MODTRANS operation would be more appropriate, rather than applying mod12 inversion to both of them. Transpositions and inversions in this chapter will be mod12 because of our

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[5] The commutativity of the MODTRANS operation with respect to transposition may be proven as follows: given the operations MODTRANS (x, y, z) and T_n, let j^1 equal the ordered pc interval from the point of synchronization to step-class j in modular system x, let j^2 equal the ordered pc interval from the point of synchronization to step-class j in modular system y, let $i = j^2 - j^1$, let pitch-class k represent step-class j in the musical entity to be transformed, let m represent the pitch-class onto which k is mapped under MODTRANS (x, y, z) followed by T_n, and let p represent the pitch-class onto which k is mapped under T_n followed by MODTRANS (x, y, z). PROOF: $m = ((k + i) + n), p = ((k + n) + i)$, therefore $m = p$.

[6] This convention will be established for consistency, though the MODTRANS operation is commutative with transposition, and thus the order of operations when combining MODTRANS and transposition is irrelevant.

familiarity with mod12 operations, and because using inversions in other modular spaces offers no analytic advantages in any of the analyses presented here, though one could relabel many of the transformations presented in this article according to the modular spaces represented. In the case of Ex. 5.7, for instance, a mod8 inversion of <D E♭ F C> around a D axis would yield <D C B E♭>, rather than <D C♯ B E>, while a mod6 inversion of <D E F♯ C> around a D axis would be the same as the mod12 inversion. Therefore the results, though different in the first case, still prove that the MODTRANS operation is not commutative with respect to inversion.

Although we have only examined pitch segments in the examples thus far, the MODTRANS operation can just as easily transform simultaneities. Example 5.8a provides m. 10 from the first movement of Bartók's Fourth String Quartet. Example 5.8b shows an interpretation of the underlying voice-leading of the passage; notes in the nonshaded boxes represent notes in the music, while notes in the shaded boxes represent notes that are the result of individual transformations leading to notes in the music. The whole-tone tetrachord, set class 4-21, is related to the chromatic tetrachord, set class 4-1, by MODTRANS (6, 12, C) and I_1. In labeling the transformations connecting these two chords, one could have substituted transposition by T_2 for the inversion by I_1; inversion has been chosen here because the voice-leading of the passage serves to make the inversional axis more audible by keeping each mapping within a single instrument, as Ex. 5.8b illustrates.

Musical entities whose intervallic structure is replicated in more than one modulus may be linked by more than one MODTRANS operation; thus, if the intervallic content of a given musical entity forms a set class that is also a subset of two or more moduli, it may be represented by any of those moduli, as is illustrated in Example 5.9. In the example, the motion from <C D E F♯> to <C C♯ D D♯>

could be labeled as either MODTRANS (6, 12, C) or as MODTRANS (7^4, 12, C), because <C D E F♯> is a subset of both the whole-tone scale and the fourth mode of the major scale. Like the analyst using set theory who is faced with the choice of labeling a transformation connecting a pair of symmetrical pc sets as either a transposition or an inversion, the analyst using MODTRANS must often choose between different interpretations of the moduli involved; in such cases, it is the interaction between the musical context and the abstract properties of the modular spaces that will guide the analyst to an appropriate representation. For instance, if <C D E F♯> were in a musical environment composed mainly of whole-tone subsets, one would prefer a whole-tone interpretation, while if it were part of a passage drawn from the notes of a G major scale, a diatonic interpretation would be more appropriate. If the musical context was constantly shifting between different modular spaces, regardless of whether they were chromatic, octatonic, diatonic, whole-tone, or pentatonic, one might interpret it as whole-tone, based on the fact that its set class, (0246), is imbricated six times in the whole-tone collection and is thus more indicative of that collection than of the diatonic one, in which it is imbricated only once.

Given that the modulus of a musical entity is often subject to more than one interpretation, the reader might wonder whether the MODTRANS operation is capable of linking any two sets of the same cardinality. This is not the case, because of the limited number of modular contexts considered here. Table 5.2 lists the number of MODTRANS mappings connecting any two of the twelve possible trichord types. As one can see from a quick study of the table, (026) is capable of mapping onto the greatest number of trichord types, nine total, while (012) is the most limited in this capacity, only capable of mapping onto three.

Table 5.1 does not account for what happens to higher step classes when moving from a larger to a smaller modulus; for instance, there are no mod6 equivalents for step classes 7-12. Two possible ways of handling such mappings will be considered here. If one can imagine the destination modulus as a clockface numbered according to its cardinality, the first mapping strategy to be considered in effect wraps the larger starting modulus around the smaller one that is its destination; let us call this strategy "modular wrap-around," or MODWRAP. Under MODWRAP, if step class x in the starting modulus is equal to or greater than the cardinality of the destination modulus, y, it maps onto step class (x - y) in the destination modulus. For instance, modular wrap-around maps step-class 8 in a mod12 space onto step-class 1 in a mod7 space. This is Bartók's own solution in *Music for Strings, Percussion, and Celesta*, as is shown in Example 5.10.[7] The high E in the chromatic theme from the first movement, circled in Ex. 5.10, maps onto the high C in the diatonic theme from the fourth movement, also circled. The table at the bottom of Ex. 5.10 shows the specific mappings, and is arranged in the same way as Table 5.1.[8]

Ex. 5.10 highlights what could be considered a flaw in this solution: there is not a separate pitch-class in the diatonic space for every step-class in the chromatic space, and thus two pitch classes in the chromatic space map onto the same pitch class in the diatonic space (C and F♯ both map onto C). Though Bartók has made the motion from one modular space to another quite audible by preserving the rhythm and melodic contour, these choices are independent of the MODWRAP transformation itself; if these characteristics of the theme were not

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[7] This example is taken from Wayne Alpern, "Bartók's Compositional Process: 'Extension in Range' as a Progressive Contour Transformation," (Music Theory Midwest Conference, May 15-16, 1998).

[8] Note that the diatonic mode in the fourth movement is not one of the seven included in Table 5.1, but is the fourth mode of the ascending melodic minor scale, also commonly known as the "acoustic scale" or "overtone scale" (see note 3).

preserved, the relationship between the two would be obscured by the fact that two different pitch classes in the original theme map onto the same pitch class when it is transformed by MODWRAP.

This situation is inevitable whenever the number of step classes actually present in a musical entity to be transformed exceeds the cardinality of its destination modulus. However, if the number of step classes used in the entity to be transformed is equal to or less than the cardinality of the destination modulus, another solution is capable of avoiding this situation; let us call this strategy "module completion mapping" or MODCOMP. Under MODCOMP, step classes in the starting modulus that are represented by a pitch class in the entity to be transformed, taken one by one in ascending order, map onto the step classes in the destination modulus in ascending order. For example, the mod12 segment <E D♯ C F F♯ G>, representing the mod12 step classes <4 3 0 5 6 7>, would map onto the mod6 segment <E D C F♯ G♯ A♯>, representing the mod6 step classes <2 1 0 3 4 5>, under module completion mappings. Step-class 0 in the mod12 space would map onto step-class 0 in the mod6 space, step-class 3 would map onto step-class 1 (because step-classes 1 and 2 are not represented in the musical entity to be transformed), step-class 4 would map onto step-class 2, etc. To put it another way, if we write the pitch classes in the starting modulus as a pc segment ordered by step class, then the order positions of these pitch classes within the segment (numbered 1 through n) will correspond to the step classes they will map onto in the destination modulus.

Example 5.11 shows how the results differ when the two mappings are applied to the same musical entity. The segment <G C♯ D E C A> under MODCOMP (12, 6, C) would map onto the segment <G♯ D E F♯ C A♯>, as is illustrated in Ex. 5.11a. When rewritten in ascending order, order position 0 of the segment (C) would map onto step-class 0; order position 1 of the segment (C♯)

would map onto step-class 1, mod6 (D); order position 2 (D) would map onto step-class 2, mod6 (E); order position 3 (E) would map onto step-class 3, mod6 (F♯); order position 4 (G) would map onto step-class 4, mod6 (G♯); and order position 5 (A) would map onto step-class 5, mod6 (A♯). The same segment would map onto <D D E G♯ C F♯> under MODWRAP (12, 6, C), as is illustrated in Ex. 5.11b; step-class 7, mod12 (G), would map onto step-class 1, mod6 (D; 7 - 6 = 1), step-class 1, mod12 (D), would also map onto step-class 1, mod6, step-class 3, mod12 (E), would map onto step-class 3, mod6 (G♯), step-class 0, mod12 (C), would map onto step-class 0, mod6 (C), and step-class 9, mod12 (A), would map onto step-class 3, mod6 (F♯; 9 - 6 = 3). Note that under MODCOMP there is a separate pitch-class in the transformed musical entity for each pitch class in the original whereas under MODWRAP both C♯ and G in a mod12 space map onto D in a mod6 space. Thus, MODCOMP provides an alternative that eliminates the mapping problem that would result from applying MODWRAP.

While Bartók is the only composer to have explicitly described the MODTRANS operation as a compositional technique, one may find examples of its use in works by others, and the operation does not seem to be linked to any one compositional style in the twentieth century. In what follows, we will see MODTRANS operations in the music of Debussy, Stravinsky, Schoenberg, and Webern.

David Lewin's recent analysis of Debussy's *Feux d'artifice* shows the thematic and motivic unity in a diverse musical surface that is constantly shifting between and layering materials that are sometimes pentatonic, sometimes diatonic, and sometimes whole-tone.[9] His Examples 4.14 and 4.15 are reproduced here as

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[9] David Lewin, *Musical Form and Transformation: 4 Analytic Essays* (New Haven: Yale University Press, 1993), 97-159.

Examples 5.12 and 5.13. As one can observe by studying Ex. 5.13, Lewin has interpreted the theme and its variations taken together as a kind of polyphonic structure that layers an ascent through a whole-tone scale over an ascent through a pentatonic scale. This interpretation allows him to draw a comparison between the theme and its variations and material from earlier in the piece (mm. 7-8). However, a more harmonic conception of the theme and its variations (i.e. one that explains all of the notes as part of one pitch class collection) would more readily match our perception of each as a unified whole, rather than suggest an interplay between two contrapuntal voices moving through different modular spaces.

Example 5.14 suggests such a harmonic conception. The first three staves in the example provide the theme and its first two variations, while the remaining two staves provide the abstracted step-by-step process by which each of the transformational pairs are linked (whole notes in the abstracted staves indicate notes in the music, while noteheads indicate notes that are the result of individual transformations leading to notes in the music). As the fourth and fifth staves show, the incipit of the theme maps onto the incipit of variation 1 by MODTRANS (5^1, 6, C) and T_{-5}, which in turn maps onto the incipit of variation 2 by MODTRANS (6, 7^1, F) and T_{-5}. Thus the incipit of the theme is presented initially in a pentatonic space, and then is varied twice to appear in first a whole-tone space and then a diatonic one. The theme's ending is similarly transformed in variation 1, while in variation 2, another transformational pattern takes hold. Example 5.15 illustrates how a pattern of two successive transpositions up a semitone generates both the middle of the theme and the middle and ending of variation 2, and how one can conceive of the transposition in the middle of variation 1 by T_{-2} as a condensation of this pattern.

Many of Lewin's trenchant observations are retained in a harmonic conception, such as the significance of T_{-5} as a pentatonic generator, guiding the lower notes of the theme and its variations along a path that defines a pentatonic collection. However, because the point of synchronization in both cases has been the lowest pitch of the theme, Lewin's observation that the T_{-4} connecting the higher notes is a whole-tone generator has been lost. This need not be the case – there is no reason why the two conceptions cannot be held side by side as complementary. But if one wished to emphasize a top-down conception of the harmonic motion, which would highlight the upper line's whole-tone ascent in Ex. 5.13, one could simply redefine the point of synchronization to be the upper note of the theme's opening interval (as well as those of its subsequent variations). To do this, we will insert a fourth argument into the label that indicates the point of synchronization's step class, if it is not considered to be step-class 0. Note, however, that the rotation of any given modular space will still be labeled based on the intervallic sequence above step-class 0; e.g., though a given octatonic musical entity might include a note a whole step above the point of synchronization, its modular space would be defined as 8^1 if the distance between step-classes 0 and 1 in the system were a half step. Thus, the transformation mapping the incipit of the theme onto variation 1 would be MODTRANS (5^1, 6, G, 3) and T_{-4}, which in turn maps onto the incipit of variation 2 by MODTRANS (6, 7^1, C♭, 3) and T_{-4}., as is shown in Example 5.16.

Example 5.17 shows how the relationship between the two primary motives in mm. 463-483 of Stravinsky's *Agon* can be understood in terms of a MODTRANS operation. In the example, the first theme is labeled **a**, with the most concentrated statement of the first motive bracketed, the second theme is labeled **b**, with the second motive bracketed, and an analysis of the MODTRANS operation is labeled **c**. The first motive, the whole-tone segment <C♯ B D♯ E♯>, accompanies a solo male dancer on stage, and is presented in stretto by the horn and piano parts of mm. 463-468. The second motive, the octatonic segment <D

C♯ E F>, accompanies a solo female dancer, and is presented by the flutes in mm. 473-479. (Although the flutes begin with the motive minus its first note, D, its subsequent occurrences include the D three out of four times.) The first motive maps onto the second under MODTRANS (6, 8^1, F, 3).[10] Note that the contour of the first motive is not preserved in the second; though composers such as Bartók and Debussy may preserve melodic contour in their modular transformations (presumably to make the transformation more audible for the listener), MODTRANS always occurs in pitch-class space and does not necessarily preserve contour.[11]

Awareness of the MODTRANS operation here throws light on one of the movement's many interesting interthematic relationships. The music accompanying the solo male dancer projects a stereotypical view of masculinity because of its athletic minor-seventh leaps, its loud dynamic level (suggesting strength), and its

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[10] I wish to thank Gerald Zaritzky for suggesting F as the point of synchronization in this passage.

[11] However, the preservation of contour and rhythm in a MODTRANS operation more than any other musical parameters greatly increases the audibility of the relationship between the two modular spaces involved. Work on contour theory includes Michael Friedman, "A Methodology for the Discussion of Musical Contour: Its Application to Schoenberg's Music," *Journal of Music Theory* 29 (1985), 223-248; Robert Morris, *Composition with Pitch-Classes: A Theory of Compositional Design* (New Haven: Yale University Press, 1987); "New Directions in the Theory and Analysis of Musical Contour," *Music Theory Spectrum* 15 (1993), 205-228; Elizabeth West Marvin and Paul A. Laprade, "Relating Musical Contours: Extensions of a Theory for Contour," *Journal of Music Theory* 31 (1987), 225-267; Elizabeth West Marvin, "The Perception of Rhythm in Non-Tonal Music: Rhythmic Contours in the Music of Edgar Varèse," *Music Theory Spectrum* 13 (1991), 61-78; "A Generalization of Contour Theory to Diverse Musical Spaces: Analytical Applications to the Music of Dallapiccola and Stockhausen," in *Concert Music, Rock, and Jazz Since 1945*, eds. Elizabeth West Marvin and Richard Hermann (Rochester: University of Rochester Press, 1995), 135-171; and Ian Quinn, "Fuzzy Extensions to the Theory of Musical Contour," *Music Theory Spectrum* 19/2 (1997), 232-263.

orchestration (the piano and horn parts together could be characterized as having a bold sound). The music accompanying the solo female dancer on the other hand is at a soft dynamic level, is chromatic, and is played by the flutes – the antithesis of the masculine stereotype projected in the A sections. The MODTRANS interpretation suggested here recognizes a commonality between two sharply contrasting themes that use stereotypes to characterize the genders of the solo male and solo female dancers. Thus that musical commonality, a similar progression of step classes realized in two different modular spaces, could be viewed as a metaphor for a human spirit that transcends gender characterizations.

The step-class segment <2310>, a step-class inversion of this motive (in which step-classes 0 and 3 map onto one another, and in which step-classes 1 and 2 map onto one another), serves to unify the diversity of modular spaces throughout *Agon*. Example 5.18 shows the interaction of <2310> and its inversion <1023> in mm. 418-427.[12] In Ex. 5.18, the step-class segments are realized only in octatonic spaces, but at the beginning of *Agon*, one may find the same motive occurring in diatonic and chromatic spaces as well. Example 5.19a provides *Agon*'s opening theme, while Example 5.19b provides the second important theme in the first movement, and Example 5.19c provides the opening theme of the second movement. In each of these examples <2310> is bracketed. In the first movement, this segment is realized in a diatonic context (see Exx. 5.19a and 5.19b), while in the second movement it is realized in a chromatic context (see Ex. 5.19c). (Although the realizations of <2310> in Exx. 5.19a and 5.19b are subsets of both the diatonic and the octatonic collections, the larger musical contexts in which they are found suggest diatonic interpretations. In the first four measures of

[12] I wish to thank Joseph Straus for bringing this particular passage to my attention.

Agon, the pc content is {F G A B C D}, which is a subset of the diatonic collection, but not of the octatonic collection, and in mm. 10-13, the pc content is {B C C♯ D E F♯ G♯ A}, which is a superset of the diatonic collection, but not of the octatonic collection.) Example 5.19d provides an analysis of the MODTRANS operation that connects the openings of the two movements, while Example 5.19e provides an analysis of the MODTRANS operation that connects the second theme of the first movement to the opening theme of the second.

Thus far, we have seen analyses in which the modular spaces suggested by the MODTRANS operation are reinforced (or at least not challenged) by the musical environments of the entities transformed – that is, the larger collection of which each entity is a part has suggested its modular context. However, as was mentioned earlier, it is the interaction between the musical context and the abstract properties of the modular spaces that will guide the analyst in MODTRANS interpretations. Smaller pc sets such as trichords or tetrachords may suggest modular representations based on how indicative they are of one of the modular spaces considered here, even when the larger musical context may not clearly project any one modular space. In the analysis which follows, textures will be marked by the interaction of motive forms presented in constantly shifting modular spaces, some of which may occur simultaneously, as if each motive form asserts its independence partially through the modular space its manifestation suggests.

The interaction between different pitch-class collections in the music of Bartók, Debussy, and Stravinsky has been well documented, and it seems fairly clear that these composers consciously used the tension created by such juxtapositions as a compositional resource. However, theorists such as Allen Forte and David Lewin have in recent years begun to shed light on the interaction of modular spaces in the music of the Second Viennese School; I am thinking specifically of Forte's recent work on Webern, and Lewin's article "Some Notes on *Pierrot*

Lunaire."[13] To my knowledge, there is no evidence to suggest that Webern or Schoenberg consciously chose to juxtapose different modular spaces in their music. However, theorists who are as interested in perception as in authorial intention may find modular transformations to be a useful tool in approaching this repertoire.

MODTRANS intersects with Schoenberg's concepts of *Grundgestalt* and developing variation in interesting ways, as is illustrated in an analysis of his *Klavierstück* Op. 11/1. Example 5.20 illustrates an interpretation of mm. 1-18 in which four incarnations of the primary motive are connected via the MODTRANS operation.[14] An analysis of a Schoenberg work that suggests interactions between different modular spaces requires some explanation, given that Schoenberg's preserial music has thus far been understood in relation to diatonic and chromatic partitionings of the octave. However, the growing interest in symmetrical partitionings of the octave (i.e. in whole-tone and octatonic collections) that began in the latter half of the nineteenth century most probably influenced Schoenberg's musical language. In addition, a significant portion of the twentieth-century concert repertoire today comprises works that unequivocally employ octatonic or whole-tone collections, and this most probably influences how its listeners perceive the rest of that repertoire, in which such references may be less overt.

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[13] Allen Forte, "An Octatonic Essay by Webern: No. 1 of the *Six Bagatelles for String Quartet*, Op. 9," *Music Theory Spectrum* 16/2 (1994), 171-195; and *The Atonal Music of Anton Webern* (New Haven: Yale University Press, 1998); and David Lewin, "Some Notes on *Pierrot Lunaire*," in *Music Theory in Concept and Practice*, eds. James Baker, David Beach, and Jonathan Bernard (Rochester: University of Rochester Press, 1997), 433-457.

[14] For two recent analyses of this work that utilizes looser notions of expansion than the one proposed here, see Elliott Antokoletz, *Twentieth-Century Music* (Englewood Cliffs, New Jersey: Prentice Hall, 1992), 10-14, and Ethan Haimo, "Atonality, Analysis, and the Intentional Fallacy," *Music Theory Spectrum* 18/2 (1996), 167-199.

One way in which modular contexts are suggested is through the use of subsets that limit the number of viable modular interpretations. Though all set classes in a pc set analysis will necessarily be subsets of the chromatic collection, this is not always the best modular interpretation, even in musical contexts that are primarily chromatic. In Schoenberg's Op. 11, No. 1, the first three notes, <B G♯ G>, form set class (014), which is not a subset of the pentatonic, whole-tone, or diatonic collections. This limits the number of viable modular contexts in which <B G♯ G> may be interpreted to two: it must be part of either an octatonic or a chromatic space. The next three notes, <A F E>, form set class (015), which is not a subset of the pentatonic, whole-tone, or octatonic collections. This similarly limits the number of viable modular contexts in which <A F E> may be interpreted to two: it must be part of either a diatonic or a chromatic space. The second trichord is similar to the first for a number of different reasons: 1) they are both presented in the same register; 2) they both have the same melodic contour; and 3) they both begin with a leap of a third and end with a motion by step.

These similarities suggest a relationship, but the trichords are not related by mod12 transposition or inversion. Chromatic interpretations of these trichords do not help us to further understand the relationship between them. However, by interpreting the first as octatonic and the second as diatonic, the two may be related via MODTRANS (8^1, 7^3, G) and T_{-3}.[15] Suggesting that there is a small-scale motion from octatonic to diatonic in the opening two melodic phrases does

[15] The third (Phrygian) rotation of the diatonic space has been chosen here rather than its seventh (Locrian) rotation because of the former's common usage and the latter's relative obscurity, though either rotation would allow the first octatonic trichord to be related to the second. The particular rotation an analyst chooses is really only significant as far as it specifies one-to-one mappings in a particular MODTRANS operation, and should not be taken to imply the kind of centricity that labels such as Phrygian and Locrian suggest.

not entail a similar interpretation of the two vertical trichords that harmonize these phrases in mm. 1-3: {F G♭ B} and {A B♭ D♭}. Indeed, the interpretation of modular spaces suggested by the compound transformation connecting these two trichords, MODTRANS (8^1, 12, F) and T_4, contradicts the interpretation of modular spaces suggested by the melody. The analyst should resist the temptation to resolve these kind of conflicts, because they may be as essential to an understanding of the music as two conflicting metrical interpretations are to an understanding of a hemiola. Those skeptical that two modular spaces may be suggested at once should be able to demonstrate the phenomenon for themselves at the piano by playing an octatonic scale in the left hand and a whole-tone scale in the right hand simultaneously.

To return to Ex. 5.20, the first three notes, <B G♯ G>, map onto the next three notes, <A F E>, under the transformations MODTRANS (8^1, 7^3, G) and T_{-3}. These next three notes in turn map onto the first three notes of the phrase beginning in m. 9, <F♯ D C>, under the transformations MODTRANS (7^3, 6, E) and T_{-4}. The phrase in mm. 9-10 is actually composed of two different modular representations of the motive joined by the common tone C; the T_{-11} relationship linking the opening form of the motive, <B G♯ G>, to the pitches in m. 10, <C G♯ A>, is obscured by the fact that in the latter form the second and third order positions have exchanged pitches and the G♯ has been displaced up one octave.[16] When the phrase in mm. 9-10 returns in mm. 17-18, this relationship is clarified by a restoration of both the ordering and the G♯'s expected register, although this reading requires interpreting the G♯4 in m. 18 an octave above as an ornament to

[16] It should be noted that a trichordal segmentation of mm. 9-11 parallel to that of mm. 1-3 would not yield <C G♯ A>, but rather <G♯ A B♭>; the segmentation presented here is meant to highlight the development of the trichordal motive, rather than the cadential importance of the dyad <A B♭>.

an embedded structural form of the motive, <C4 A3 G♯3>.[17] Within both phrases, the relationship linking the first trichord to the second is MODTRANS (6, 8^1, C) and T_{-4}.

Two interesting observations emerge upon examining these four forms of the motive and the MODTRANS operations that connect them. The first is that the MODTRANS operations might help to define the relationship between the first phrase in mm. 1-3 and the second phrase in mm. 9-10 and 17-18 as something analogous to antecedent and consequent. The MODTRANS operation that binds together the two motive forms of the first phrase moves from an octatonic space to a diatonic space, while the MODTRANS operation that binds together the two forms in the second phrase moves from a whole-tone space back to an octatonic one. This motion away from the original octatonic form of the motive in the first phrase and back again in the second phrase seems analogous to the motion typically associated with antecedent-consequent phrase pairs, in which a move away from some point of stability in the former phrase (be it tonic harmony, or simply a low point in the melodic contour) is answered by a return to that point in the latter.

In Example 5.21, a second observation is graphically illustrated. The lowest note in each of the four motive forms mentioned above is part of a transformational path that outlines the harmony {G♯ C E G}, a tetrachord whose set class, (0148), plays a significant role in the rest of the movement. Example 5.22 shows three passages in the piece where (0148) is particularly prominent. In mm. 14-17, this tetrachord receives special emphasis as a kind of "magic chord"

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[17] The idea of interpreting some pitches on the surface of Schoenberg's music as ornamental has been argued persuasively by Jack Boss in his article "Schoenberg on Ornamentation and Structural Levels," *Journal of Music Theory* 38/2 (1994).

because Schoenberg gives special instructions to hold down the notes {F A C♯ E} without playing them, so as to provide additional resonance to the same pitch classes being played two octaves below in the left hand. This is the only harmony in the movement to receive such coloristic treatment. This "magic chord," is not only a member of the same set class as the one outlined by the transformational path mentioned above, but is literally the same harmony transposed in pitch space down a minor third, the same interval of transposition that links the first two notes of the first motive form, and of course, the first two notes of the piece. The same set class also constitutes the first four notes of the five-note accompanimental line, <D F♯ A A♯ B>, that appears three times: in mm. 4-5, in m. 6, and in m. 8. This line returns at T_1 in mm. 20-21 and 22-23, and soon after returns inverted as part of a three-voice canon in mm. 25-27. Finally, the tetrachord occurs linearly in m. 28 as <C E♭ G B> and vertically in m. 58 as {E♭ G♭ B♭ D}. If the analyst were unable to perceive the motive forms as relatable by an operation such as MOD-TRANS, the transformational path carved out by the transposition levels of these motive forms would consequently be obscured.

Modular Sets and Modular Set Types

The MODTRANS operation implicitly suggests a new equivalence class. Let us define a modular set as an unordered collection of step classes. A modular set may be realized in any kind of modular space that comprises all of its step classes; for example, {01346} may be represented in a mod7^1 space as {C D F G B}, or in a mod8^1 space as {C C♯ E F♯ A}, but not in a mod6 space, because that space does not include a step-class 6.

It would be useful to have another equivalence class, each member of which represents a family of modular sets in the same way that each set class represents a family of pc sets. A modular set-type (abbreviated M-type) will be defined as an equivalence class representing all modular sets that map onto one another under mod12 transposition or inversion. To find the M-type for a given modular set, one follows the same procedure as one does for finding the prime form of a pc set, only the operations are performed on integers which represent step-classes rather than pitch-classes. The operations used to define the term "M-type" are mod12 even though the modular set can be represented in any kind of modular space, because the equal-tempered harmonic system (the mod12 system) is the background against which all of the other systems are set into relief. Thus the list of M-types duplicates exactly the list of mod12 set classes. However, while each entry in the list of set classes represents only those pc sets that are transpositionally or inversionally equivalent to itself, each entry in the list of M-types represents all of the pc sets represented by its corresponding set class, as well as all of the pc sets represented by those set classes that map onto its corresponding set class under the MODTRANS operation. For example, the M-type (013) represents not only the set class (013), but also set classes (014), (015), (025), and (026). (013) maps onto (014) under MODTRANS (12, 8^1, 0); it maps onto (015) under MODTRANS (12, 7^3, 0) or MODTRANS (12, 7^7, 0); it maps onto (025) under MODTRANS (12, 8^2, 0), MODTRANS (12, 7^1, 0), MODTRANS (12, 7^2, 0), MODTRANS (12, 7^5, 0), or MODTRANS (12, 7^6, 0); and it maps onto (026) under MODTRANS (12, 7^4, 0) or MODTRANS (12, 6, 0).

Many entries in the list of M-types represent multiple set classes. Table 5.3 shows the set classes that are represented by each of the M-types of cardinalities 3-6, excluding those set classes derived from MODTRANS mappings involving modular wrap-around. After studying the table, one can see that there are symmetries within each modular space: for instance, within the first five M-types listed there is a retrograde symmetry in each column. The mod12 set classes (013) and (016) both map onto (014) and (025) under MODTRANS (12, 8^1, 0) and

MODTRANS (12, 8^2, 0), respectively, while the mod12 set classes (014) and (015) both map onto (016) and (026) under MODTRANS (12, 8^1, 0) and MODTRANS (12, 8^2, 0), respectively. This is because the M-types (013) and (016) are related by mod8 inversion, as are the M-types (014) and (015). Similarly, the mod12 set classes (012) and (016) both map onto (013) and (024) under MODTRANS operations from chromatic to diatonic, while the mod12 set classes (013) and (015) both map onto (015), (025), and (026). This is because the M-types (012) and (016) are related by mod7 inversion, as are the M-types (013) and (015). (014), the center of this retrograde symmetry, is inversionally self-symmetrical, mod7. Finally, the M-types (012) and (015) are related by inversion mod6, as are the M-types (013) and (014).

Now let us turn to another example from the literature, the structure of which can be explained within the larger context provided by modular sets and set types. The third movement from Webern's *Five Movements for String Quartet*, Op. 5 could be viewed as an interplay between four forms of the M-type (013) – these forms are the trichordal set classes $(013)^{12}$, $(014)^{12}$, $(015)^{12}$, and $(026)^{12}$. Example 5.23 provides an annotated score of the third movement, in which various forms of these set classes are highlighted.

Example 5.24 provides an analysis of the first theme, which is first presented by the violin I and viola in m. 4, returns as a canon in all four parts in mm. 10-12, continues as a counterpoint to the third theme in mm. 12-13, and reappears in mm. 18-21 as an ostinato figure in the violin II and viola parts. The theme itself as it is initially presented consists of two three-note subphrases, <D F E> and <C F♯ B♭>, members of set classes (013) and (026) respectively, which are related by MODTRANS (12, 6, D) and T_4, as the second staff in Ex. 5.24 illustrates. Though <D F E> could be interpreted as the M-type (012) in a diatonic space or an octatonic space and <C F♯ B♭> could be interpreted as the M-type

(014) in an octatonic space, these interpretations of the subphrases cannot relate them to one another or to as much of the material that follows. On the other hand, interpreting them both as forms of the M-type (013) includes them in an all-encompassing view of the piece that seems more in keeping with the general impression of organic unity it has made on many listeners and performers. The second trichord, <C F♯ B♭>, could also be interpreted as the M-type (013) in a diatonic space, but a whole-tone representation has been chosen here for the MODTRANS label based on the fact that the set class (026) is imbricated fourteen times in the whole-tone collection and is thus more indicative of that collection than of the diatonic one, in which it is imbricated only once.

Example 5.25 provides the second theme, which is first presented by the violin I in mm. 9-10, and then returns transposed in mm. 22-23 to close the movement. The theme itself as it is initially presented consists of three three-note subphrases, <D B♭ C♯>, <B♭ C♮ B♮>, and <B♭ F A>, the first and third of which are related by MODTRANS (8^1, 7^1, B♭) and T_7, as the second staff in Ex. 5.25 illustrates (the second trichord relates to neither the first nor the third via MOD-TRANS). The three-note cello ostinato in mm. 15-21, <B G A♯> is a T_9 transposition of the second theme's first trichord.

Example 5.26 provides the third theme, which is first presented in mm. 12-14, and returns, lengthened through internal repetition, to appear in mm. 18-21. The theme itself as it is initially presented can be partitioned into four successive trichords: <E♭ D C>, <C♯ B G>, <E♭ A F>, and <E F♯ G>, which are members of set classes (013), (026), (026), and (013) respectively.[18] The first two are related

[18] A trichordal partitioning of this theme is contradicted by its rhythm and its phrasing, but is suggested by the bisymmetrical arrangement of the set classes that such a partitioning forms: (013) (026) (026) (013). It seems unlikely that this symmetry is a coincidence.

by MODTRANS (12, 6, C) and T_7, while the last two are related by MODTRANS (6, 12, E♭) and I_{10}, as the third staff of Ex. 5.26 illustrates.

The MODTRANS operations connecting the trichords of the third theme could be viewed as reinforcing the form. In both of its occurrences it first propels the music forward through an overall descending motion and then seems to slam on the brakes by reversing that direction. This impression of the line's motion is reinforced through the shortening of note values in the second half of the theme in each case, and this kind of diminution is audible even though there is a *molto ritard.* indicated in the first time (*the poco a poco accel.* indication for the theme's second appearance only strengthens our impression). The first MODTRANS operation of the theme maps the initial trichord from a chromatic to a whole-tone space, while the second one maps the third trichord from that whole-tone space back to a chromatic one. These transformations are in harmony with the impression mentioned above; one process (acceleration/moving away from the original modular space) begins the theme, and then its inverse (deceleration/returning to the original modular space) serves to end it.

As one can see in Example 5.23, nearly every pitch of the movement is part of one of the four set classes present in the themes: (013), (014), (015), and (016). All of the chords formed by the upper three parts in mm. 1-8 are members of (014), and in m. 6, the three *pizz.* chords are also connected by a transformational path that replicates that trichord: T_3 followed by T_1. On the other hand, the *pianissimo* descending gestures in the upper three parts starting at the end of m. 5 are all members of (015), as are all of the overlapping trichordal subsets of the violin I and cello lines in m. 7, and the gesture in the cello at the end of m. 8 which closes the first section.

In the second section (mm. 9-14), the second theme is accompanied by linear presentations of (014) in the violin II and viola parts in m. 9, and the second half of beat 4 in that measure is a vertical expression of that set class. The verticality on the second half of beat 1 in m. 10 is another expression of (014), and the remaining notes in the violin II and viola parts form another linear expression of that set class. In m. 14, a linear expression of (015) between the cello and viola parts, <B♭ A F>, ends the canon on the first theme (this is the same pc set that ends the second theme!), while the violin II closes the section with <G♯ A C♯>, a member of (015) and the T_4-retrograde of the gesture that closed the first section.

The third section (mm. 15-end) begins with the violin II and cello parts presenting linear forms of (014) –{G A♯ B}, {C D♭ E}, and {A C D♭} – and the viola presenting linear forms of (013) and (015) –{G♯ A B} and {E G♯ A} respectively. The remainder of the third section has already been accounted for in our previous discussion of the themes, except for the final pitch class of the movement, C♯. C♯ is projected strongly as a tone center in this movement by its position as a pedal tone in mm. 1-6, by its agogic accent in the second theme, by its position as the last note of the second section (mm. 9-14), by its position as the last note of the third section and of the movement, and by the stress it receives at that point (*sfff* and *pizz.*). The final C♯ can be tied into the network of (013) M-type forms by grouping it to the two previous pitch classes, B♭ and D, a grouping that forms the set class (014). Similarly, the C♯ that serves as a pedal throughout the first six measures can be tied into the network of (013) M-type forms in two ways. Grouping the C♯ with the C and E in the cello which follow it forms an (014). Possibly a more musical grouping would be with the lowest notes of the two ascending gestures in the cello part of the first section (mm. 1-8) – the C low in m. 7 and the low A in m. 8 – a grouping which also forms (014).

We could describe the third movement of Webern's Op. 5 as an (013) piece; that is, we could argue that its surface structure is governed by the M-type (013). We could also say that to varying degrees this M-type governs the structure of a great number of other pieces or movements. These pieces and movements constitute a family. In these pieces, the M-type (013) plays a role somewhat analogous to the role of the triad in tonal pieces. Both the triad and the M-type can be said to account for the majority of surface pitch structures in their respective repertoires, and both function differently in different contexts. In tonal music, the triad is represented as either major, minor, augmented, or diminished depending upon its musical context. For example, a triad built on C will be major in the key of C major, but minor in the key of A♭ major, and diminished in the key of D♭ major. Similarly, in a post-tonal work governed by a given M-type's structure, which pc set represents that M-type at any precise moment depends upon its musical context. For example, in an (013) piece, the M-type (013) would be represented by set classes (015), (025), or (026) in a diatonic passage, but by set classes (014) or (025) in an octatonic one.

The first movement of Stravinsky's *Concerto in D* provides another example of an (013) piece. As in virtually all of Stravinsky's neoclassical works, *Concerto in D* oscillates between diatonic and octatonic collections.[19] In *Concerto in D* these collections are expressed through the M-type (013), or by one of its supersets. The supersets used are (0124), (01245), (0134), (0135), and (01235), and among these, all but (0134) contain (024) as a subset as well (in three of these sets, (024) is imbricated twice). The imbrication of (024) in these larger sets makes possible a number of the tonal allusions with which Stravinsky endowed this work, because (024) is represented by some quality of triad in both diatonic and octatonic modular contexts.

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[19] Arthur Berger and Pieter Van den Toorn have both made strong cases for understanding Stravinsky's music in terms of the interaction between diatonic and octatonic collections. See Arthur Berger, "Problems of Pitch Organization in Stravinsky," *Perspectives of New Music* 2/1 (1963), 11-42; and Pieter van den Toorn, *The Music of Igor Stravinsky* (New Haven: Yale University Press, 1983).

An annotated short score to the introduction and first section of *Concerto in D* (mm. 1-57) is given as Example 5.27; a pc-set analysis of the music is represented on the staff at the bottom of each system, under which the M-type and set class of each pc set are listed. Each M-type is followed by the superscripted numbers 7, 8, or 7/8; these numbers indicate the modular space in which the M-type is realized (in the case of 7/8, the set could be interpreted as part of a diatonic or an octatonic space). Several of the movement's characteristics are established in the first 37 measures: 1) the M-type (013) is established as a *Grundgestalt* throughout the movement because it is heavily emphasized in this span; 2) the interpretive conflict is established between the structural F that is a part of the introduction's prevailing octatonic collection and the enharmonically equivalent but non-structural E♯ that is a chromatic lower neighbor in the context of the first theme's D-major collection, because the listener is forced to reinterpret the F, which appears first harmonically, as a nonharmonic E♯ when it appears in the first melodic gesture of the movement (see the Violin I part); 3) the larger harmonic conflict between diatonic and octatonic collections is established through the use of the pc sets (014) and (015), each of which is only a subset in <u>one</u> of these larger collections; 4) the harmonic ambiguity between the diatonic and octatonic collections in this movement is established by the melodic importance of the pc set (025), which is a subset of both collections; and 5) the conflict between D and F♯ as tonal centers is established because the F♯ receives emphasis as an axis of inversion during the first theme, while the D receives emphasis through the intervallic structure of its parent harmony, {C♯ D F♯}, which could be interpreted tonally as an incomplete seventh chord with an omitted 5th (the D thus receives weight because it is the root of this incomplete seventh chord).

A close examination of Example 5.27 reveals insights as to how (013) relates to the other M-types in the movement. The first harmonic entity that is not a realization of the M-type (013) in a diatonic or octatonic space is {B D F♯} in mm. 38-42, a representative of the M-type (024). The relationship between {B D F♯}and what has come before is clear enough: the dyad {D F♯} is preserved, while C♯ is displaced by B (note that there is a chromatic passing tone, C-natural, that serves as a transition in this displacement; the C-natural will be interpreted here as nonstructural). At the end of the first section, we have another transformation that preserves common tones. The trichord {F♯ G B} is transformed into a pentachord through the addition of E and C♯ in mm. 51-57. In each of the two cases cited, the transformation has served not only to clarify the relationship between (013) and the other sets by holding pitch classes invariant, but has also served to shift the centric weight of the sonority. In the first case, the D centricity of {C♯ D F♯}, owing to its tonal interpretation as an incomplete seventh chord, gives way to a B centricity in the sonority {B D F♯}, owing to the force with which a triadic projects its root. Similarly in the second case, the G centricity of {F♯ G B}, owing to its tonal interpretation as an incomplete seventh chord, gives way to an E centricity in the sonority {E F♯ G B C♯}, owing to the greater strength of the triad imbricated in the sonority {E G B}.

The intervallic structure of (013)[7] suggests that its second member, the root of the incomplete seventh chord it represents, is a chordal tone center (see Fig. 3.1). Though centricity in a octatonic context is beyond the scope of this dissertation, it seems reasonable to suggest that the second members of the M-type's octatonic forms could by analogy be interpreted as chordal tone centers. Tonal readings that interpret (013) M-types as incomplete seventh chords with roots that carry centric weight, and that interpret triads imbricated in larger post-tonal sonorities as subsets that project their roots as tone centers even in the context of the larger sonority, lead us to a comprehensive view of the whole first movement that shows (013) governing the structure at a higher level. Table 5.4

provides a bird's eye view of the entire movement. In the table, each section is represented by its beginning interpreted as a pitch class set, a set class, and an M-type. Those pitches in bold face print receive emphasis as a chordal tone center because they are the roots of either a triad or an incomplete seventh chord. Note that the varied repetition of the A section in mm. 90-129 begins with {C♯ D F♯ G}, a sonority that in context is clearly interpreted as {C♯ D F♯} + G, i.e. the M-type (013) and a G that does not complete the incomplete seventh chord, but actually contradicts such a reading. However, because this section is clearly a varied repetition of mm. 25-57, the G does not usurp D as the tone center here, and the A eventually appears in the melody to complete the seventh chord {C♯ D F♯ A}.

A survey of Table 5.4 reveals that only four pitch classes are projected as roots: D♭, D, F♯, and A. Example 5.28 shows how the relationship of these roots replicates the structure of the primary diatonic expression of (013) in the movement, the pitch class set {C♯ D F♯}. This interpretation does not include the A projected as the root of {A B♭ C E} because the section it begins does not exhibit the same level of independence as the others, and thus seems subordinate to the A sections the precede and follow it. However, if the A were to be included, it would provide the missing note necessary to complete the seventh chord {C♯ D F♯ A}, and also could be viewed as the dominant of the D that is the root of the first theme group. In Example 5.28, each of the (013) trichords given represents not only itself, but also the larger sets that share the same projected root. Thus {C♯ D F♯} also represents {C♯ D F♯ G}, {D F♯ A}, and {D F F♯ A B}, and {C D♭ F} also represents {C D♭ F A♭} and {D♭ F G♭ A♭}. Example 5.29 provides an analysis of the MODTRANS operation connecting the beginning of the introduction with the beginning of the first theme.

Summary

Though the focus of this dissertation has been on diatonic music, this chapter has addressed approaches to works that are not solely diatonic, because a great number of twentieth-century works include diatonic music together with music best described in terms of other modular spaces. This chapter began with a description by Bartók of a transformation that maps diatonic scale degrees onto chromatic scale degrees and vice versa. This transformation was then generalized to occur across any of five different modular spaces: pentatonic, whole-tone, diatonic, octatonic, and chromatic. Following the generalization were examples by Debussy, Stravinsky, Schoenberg, and Webern demonstrating that this transformation is not tied to any one compositional style of the 20th century. Finally, this transformation led to the recognition of a new equivalence class, the modular set type, that groups together all set classes related by MODTRANS. In summary, the generalization of Bartók's transformation and the new equivalence classes that are suggested by it allows a family of relationships that was exploited throughout the first half of the twentieth century to emerge with a new degree of clarity.

The goal of this dissertation has been to demonstrate that a mod7 perspective is the key to understanding certain aspects of post-tonal diatonic music. It has demonstrated the necessity of a mod7 perspective in its analyses of motivic connections on the musical surface of post-tonal diatonic music (chapter 2). It has proposed a theory of chordal tone centers (chapter 3) and structural levels (chapter 4) for post-tonal diatonic music, both of which depend upon a mod7 perspective for their definition and explanatory power. And finally, this chapter has suggested an even broader perspective for dealing with those works in which diatonic passages are interwoven with passages derived from other modular spaces.

Appendix A: List of Mod7 Tn-Types

The following comprehensive list of Tn-type set classes is arranged as follows: the first column lists the dyadic and trichordal set classes and the fifth column lists their complementary set classes (tetra- and pentachords), the second column lists the interval vectors for the set classes in the first column, the fourth column lists the interval vectors for the set classes in the fifth column. The third column lists two numbers, the first of which is the degree of transpositional symmetry for the given sets and the second of which is the degree of inversional symmetry of the given sets. The final column lists the cyclic generators for the given sets. The null set and the total diatonic are not listed.

[0]	000	1, 1	555	[012345]	1
[01]	100	1, 1	433	[01234]	1
[02]	010	1, 1	343	[01235]	2
[03]	001	1, 1	334	[01245]	3
[012]	210	1, 1	321	[0123]	1
[013]	111	1, 0	222	[0234]	none
[023]	111	1, 0	222	[0124]	none
[014]	102	1, 1	213	[0134]	3
[024]	021	1, 1	132	[0135]	2

18 Tn-types total

A list of Tn/TnI-types would be identical except for the fact that the [013] and [023] would be condensed to (013) and their complements [0124] and [0234] would be condensed to (0124). There are thus 16 Tn/TnI-types.

Appendix B: Diatonic Mod7 Set Classes and Their Mod12 Equivalents

Mod7 set class	Equivalent Mod12 set class
1. (01)	(01), (02)
2. (02)	(03), (04)
3. (03)	(05), (06)
NA	(012), (014), (048)
4. (012)	(013), (024)
5. (013)	(015), (025), (026)
6. (014)	(016), (027)
7. (024)	(036), (037)
NA	(0123), (0124), (0125), (0126), (0127), (0134), (0145), (0146), (0147), (0148), (0167), (0236), (0248), (0268), (0347), (0369)
8. (0123)	(0135), (0235), (0246)
9. (0124)	(0136), (0137), (0237), (0247)
10. (0134)	(0156), (0157), (0257)
11. (0135)	(0158), (0258), (0358)
NA	all pentachords not listed below
12. (01234)	(01356), (01357), (02357)
13. (01235)	(01358), (02358), (02469)
14. (01245)	(01368), (01568), (02479)
NA	all hexachords not listed below
15. (012345)	(013568), (013578), (023579), (024579)

NA = not applicable. These mod12 Tn/TnI-types are not subsets of the diatonic collection, and therefore have no equivalent in a diatonic mod7 analysis.

Appendix C: Subsets and Supersets

	(012)	(013)	(014)	(024)	(0123)	(0124)	(0134)	(0135)	(01234)	(01235)	(01245)
(0123)	2	2	0	0							
(0124)	1	1	1	1							
(0134)	0	2	2	0							
(0135)	0	2	0	2							
(01234)	3	4	2	1	2	2	1	0			
(01235)	2	4	1	3	1	2	0	2			
(01245)	1	4	3	2	0	2	2	1			
(012345)	4	8	4	4	3	5	4	3	2	2	2
(0123456)	7	14	7	7	7	14	7	7	7	7	7

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STUDIES IN POST-TONAL DIATONICISM: A MOD7 PERSPECTIVE

by

MATTHEW SANTA

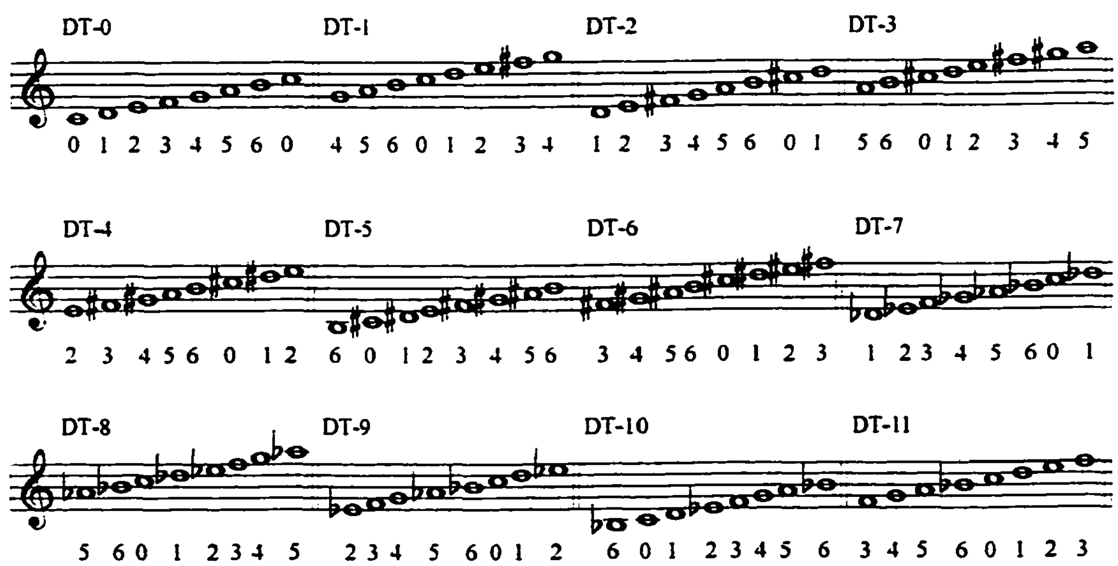
VOLUME II: FIGURES, TABLES, AND EXAMPLES

A dissertation submitted to the Graduate Faculty in
Music in partial fulfillment of the requirements
for the degree of Doctor of Philosophy,
The City University of New York

1999

Figure 1.1. Integer notation for the mod7 system.

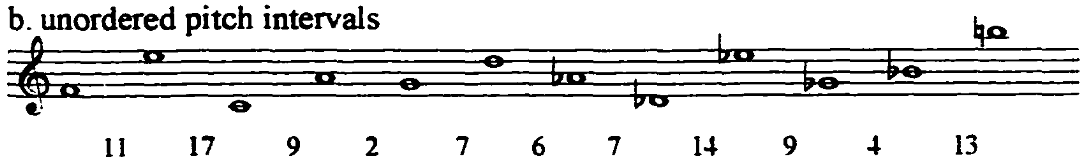
step-class	integer	represents:
C	0	C, C♯, C♭
D	1	D, D♯, D♭
E	2	E, E♯, E♭
F	3	F, F♯, F♭
G	4	G, G♯, G♭
A	5	A, A♯, A♭
B	6	B, B♯, B♭



Example 1.1. Intervals.

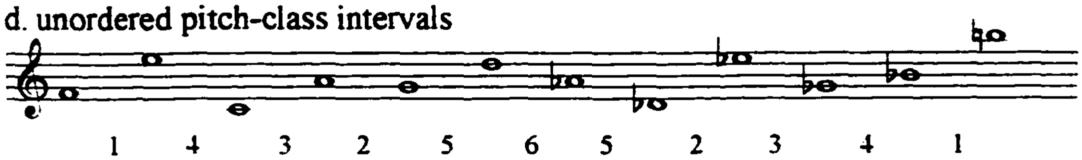
a. ordered pitch intervals

b. unordered pitch intervals

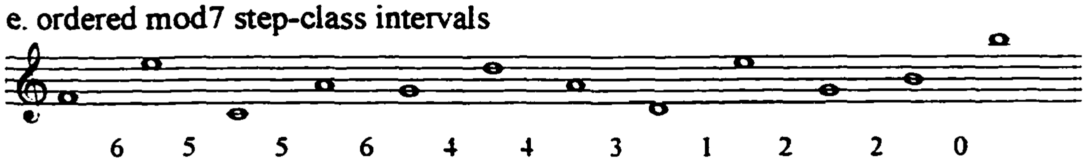


c. ordered pitch-class intervals

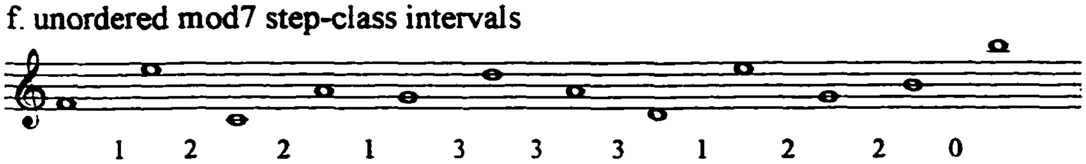
d. unordered pitch-class intervals



e. ordered mod7 step-class intervals

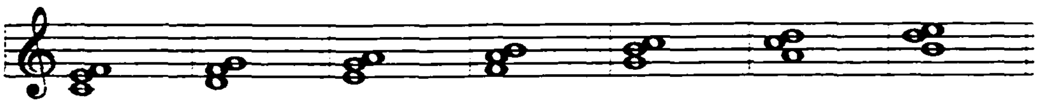


f. unordered mod7 step-class intervals



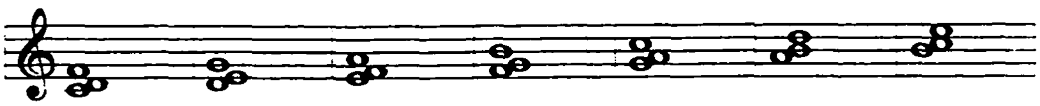
Example 1.2. Step-class sets.

a. Step-class sets of DT-0 represented by the mod7 Tn-type [023]

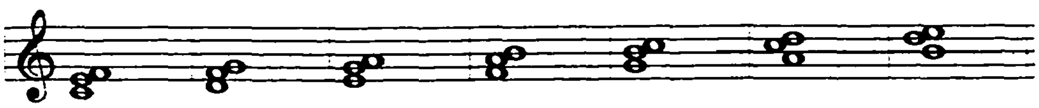


{023} {134} {245} {356} {460} {501} {612}

b. Step-class sets of DT-0 represented by the Tn/TnI-type (013)



{013} {124} {235} {346} {450} {561} {602}



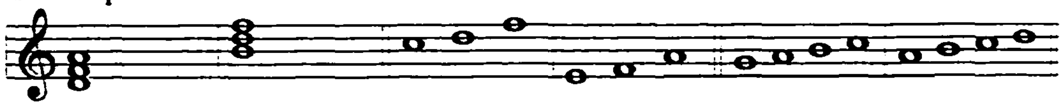
{023} {134} {245} {356} {460} {501} {612}

Example 1.3. Interval vector.

mod7 interval vector: (021)
0 members of step-class interval 1
2 members of step-class interval 2
1 member of step-class interval 3

Example 1.4. Transposition and inversion, mod7.

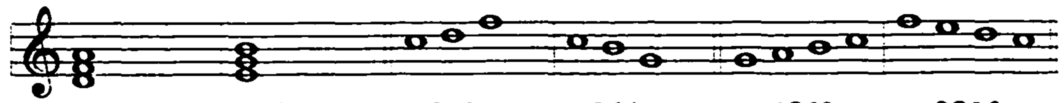
a. transposition



{135} {613} <013> <235> <4560> <5601>

T_5 T_2 T_1

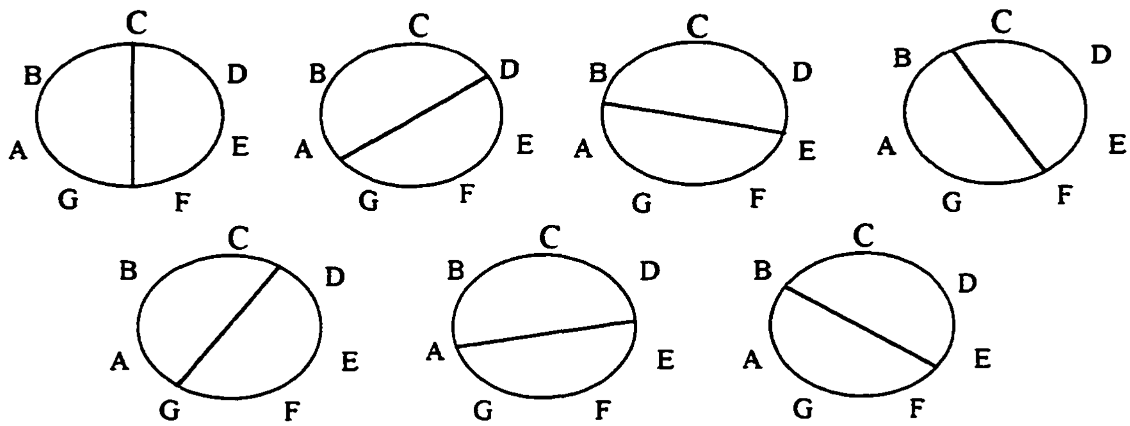
b. inversion



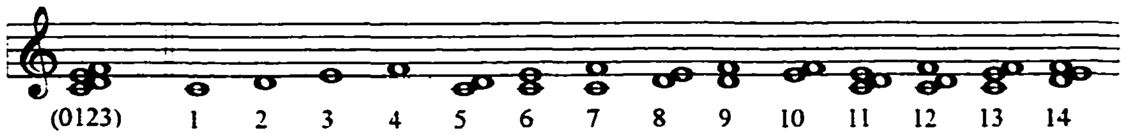
{135} {246} <013> <064> <4560> <3210>

$I_{C/C}$	$I_{C/C}$	$I_{C/C}$
$I_{D/B}$	$I_{D/B}$	$I_{D/B}$
$I_{E/A}$	$I_{E/A}$	$I_{E/A}$
$I_{F/G}$	$I_{F/G}$	$I_{F/G}$
$I_{G/F}$	$I_{G/F}$	$I_{G/F}$
$I_{A/E}$	$I_{A/E}$	$I_{A/E}$
$I_{B/D}$	$I_{B/D}$	$I_{B/D}$
I_0	I_0	I_0

Example 1.5. The seven possible axes of inversion.



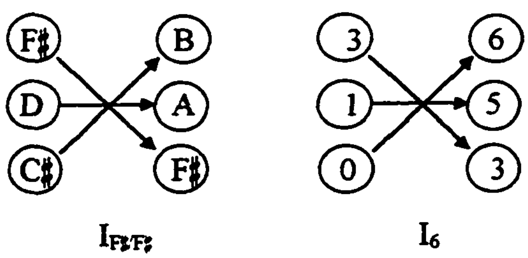
Example 1.6. Counting the subsets of (0123).



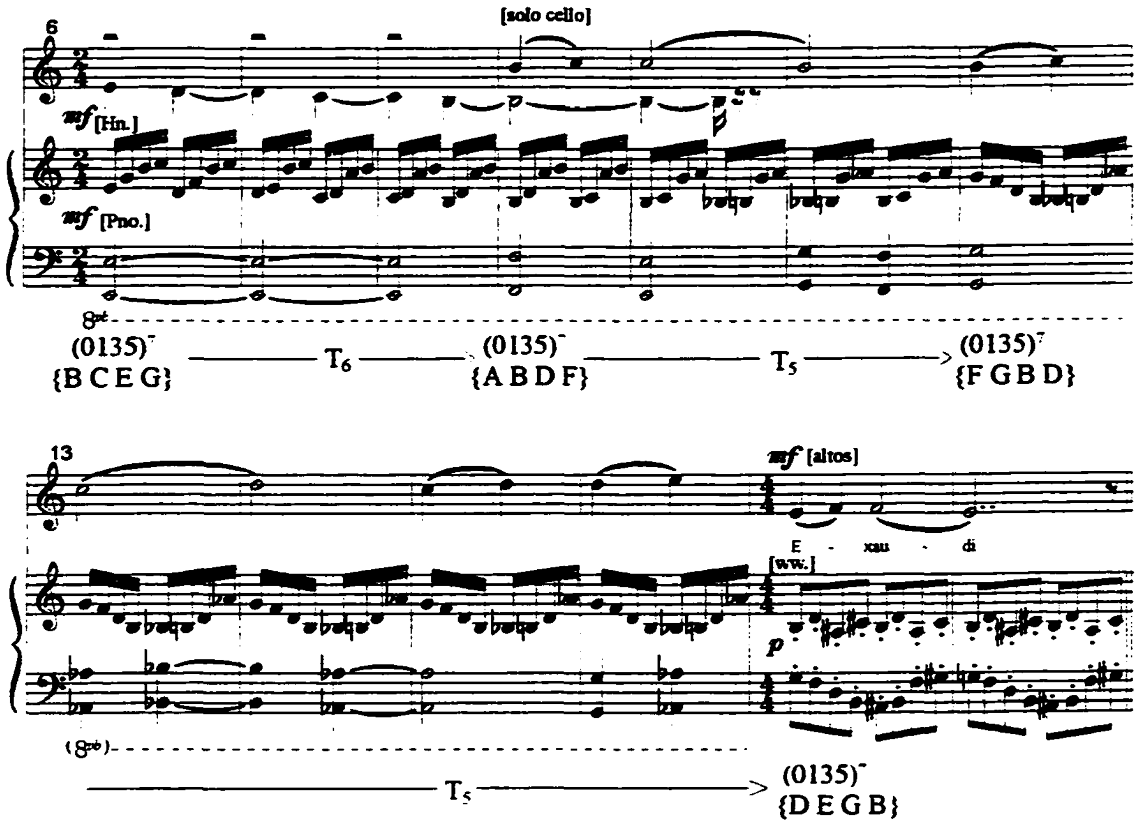
Example 1.7. Stravinsky, *Serenade in A*, I, opening.

Example 2.1a. Stravinsky, *Concerto in D*, I, mm. 25-35.

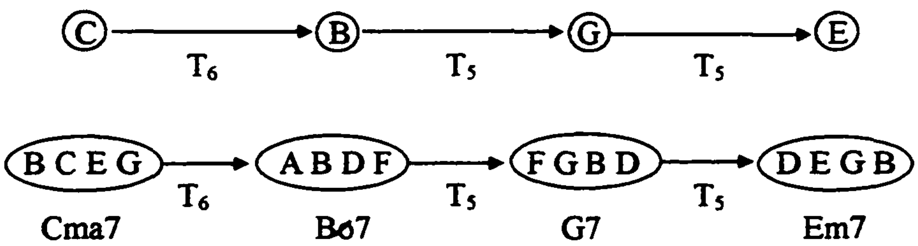
Example 2.1b. Inversion around an F♯ axis in mm. 25-35.



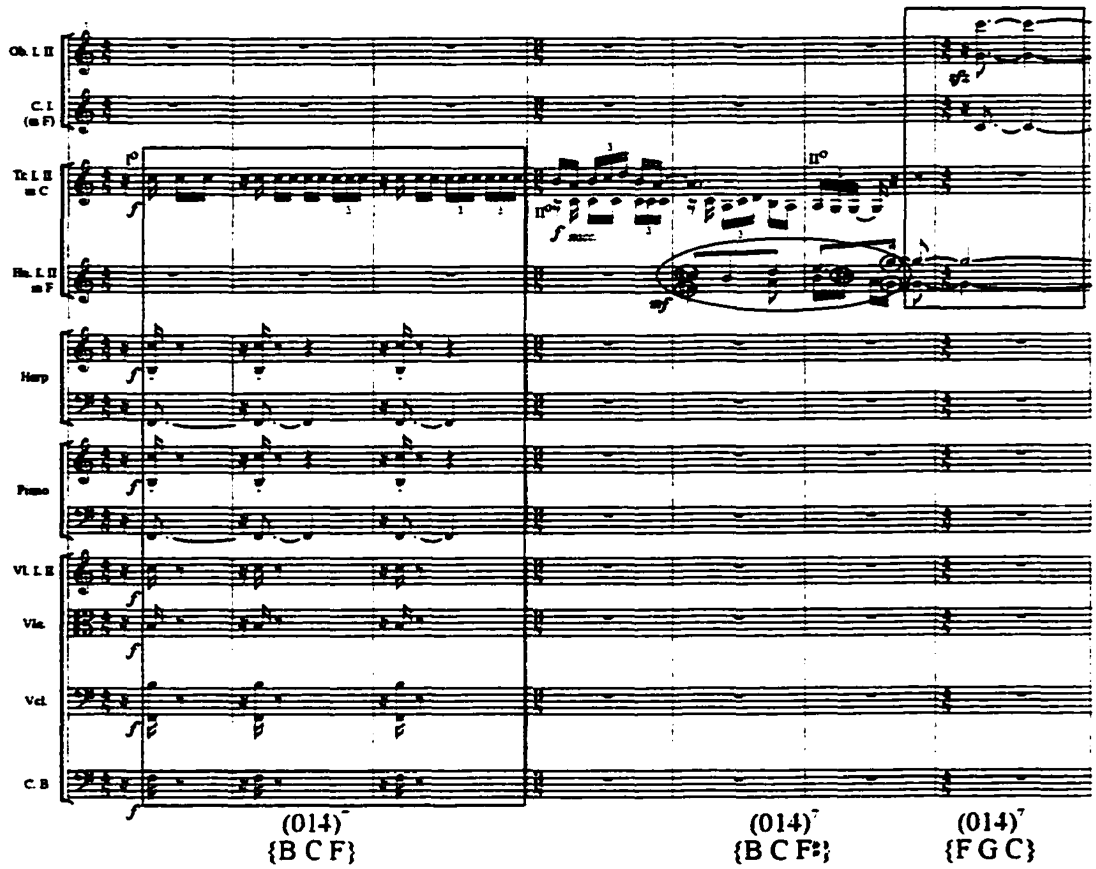
Example 2.2a. Stravinsky, *Symphony of Psalms*, I, mm. 6-18.



Example 2.2b. $(0135)^7$ composed out as a transformational path in mm. 6-18.

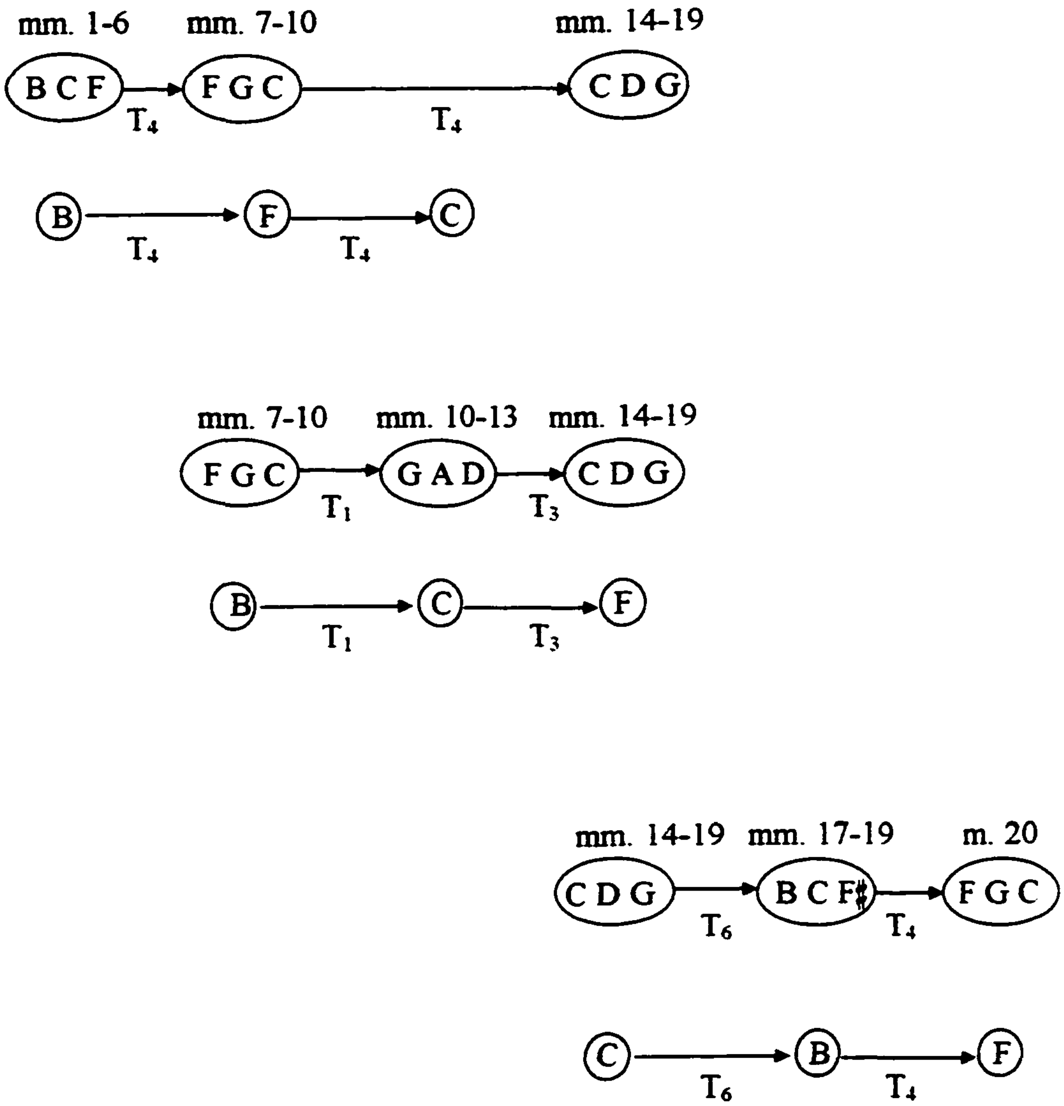


Example 2.3a. Stravinsky, *Agon*, mm. 1-20.

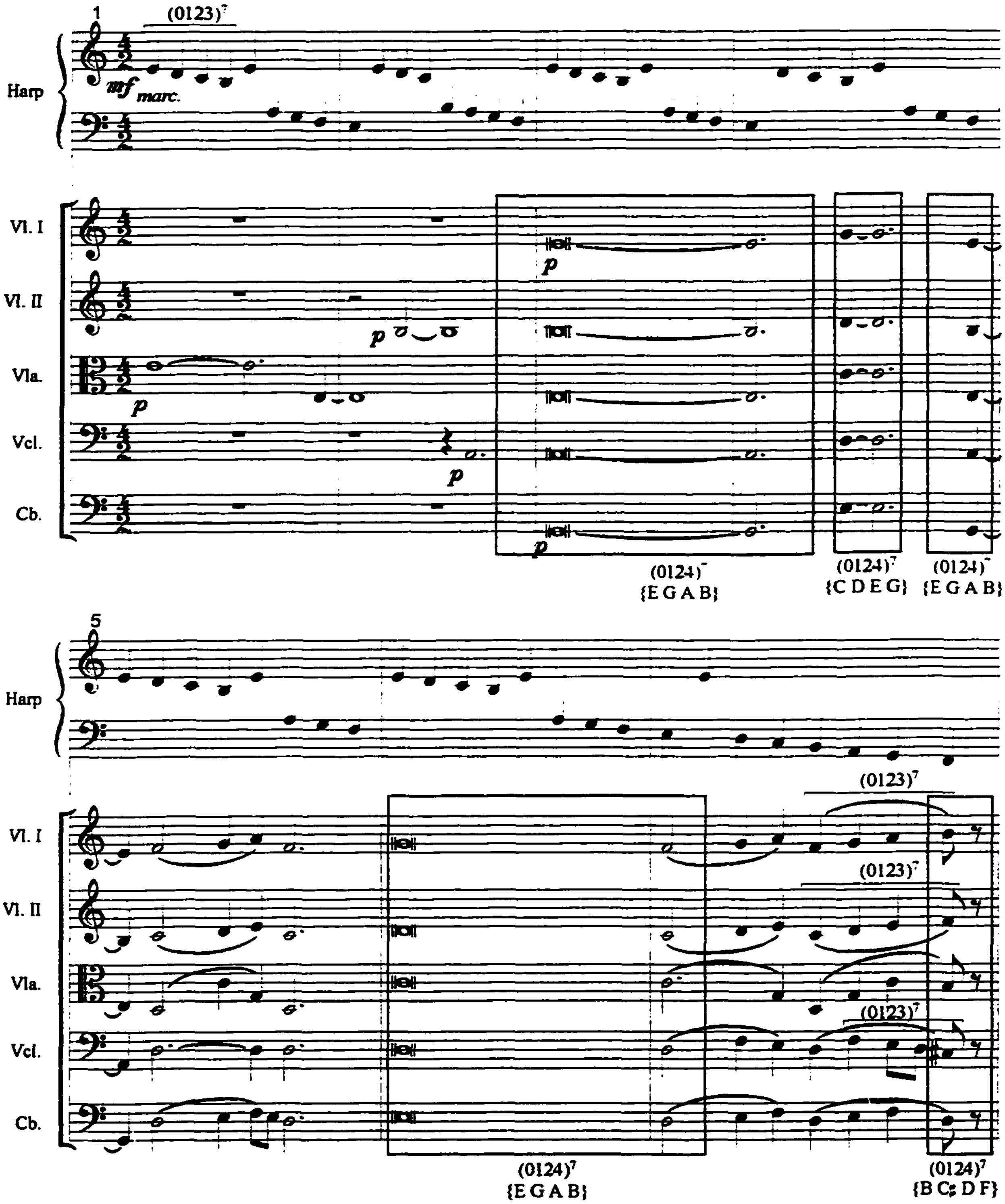


Example 2.3a (continued)

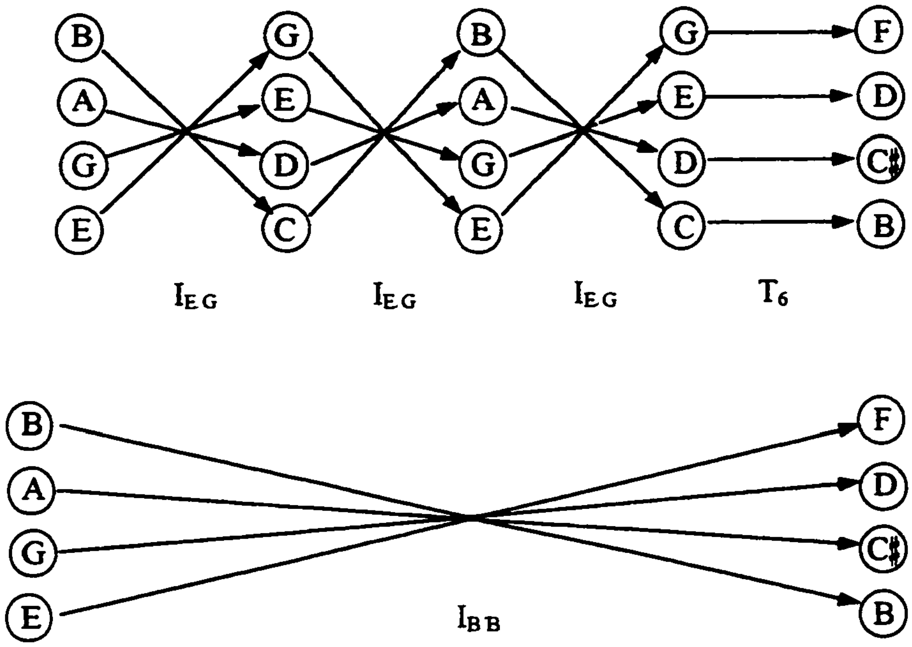
Example 2.3b. Three transformational paths replicating the intervallic structure of the opening chord.



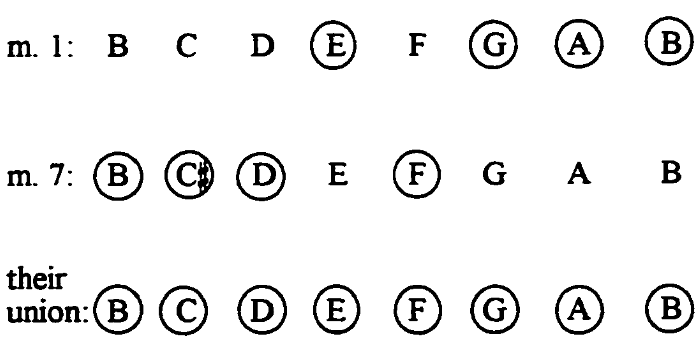
Example 2.4a. Stravinsky, *Orpheus*, mm. 1-7.



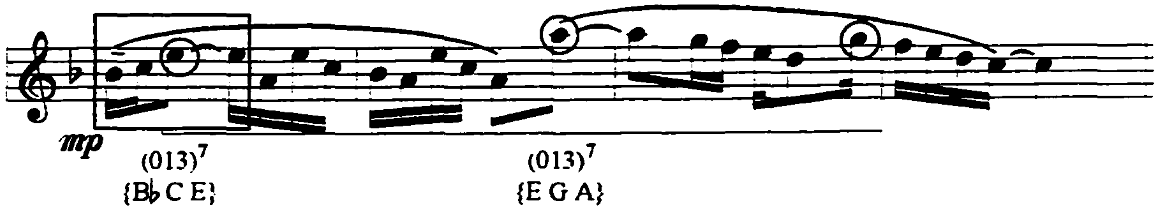
Example 2.4b. Step-class counterpoint in mm. 1-7.



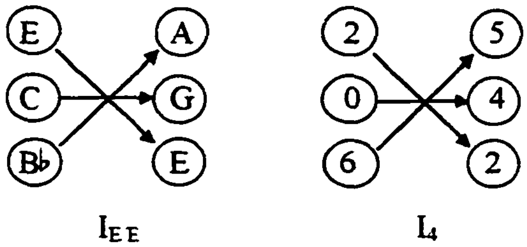
Example 2.4c. Spatial representation of the "almost complementary" relationship.



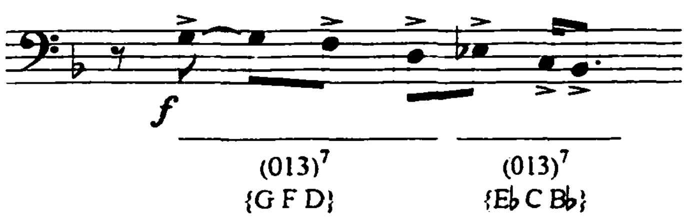
Example 2.5a. Stravinsky, *Sonata for Two Pianos*, I, mm. 1-4 (first theme).



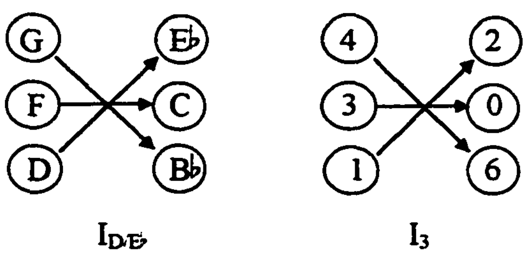
Example 2.5b. Inversion around an E axis in the first theme.



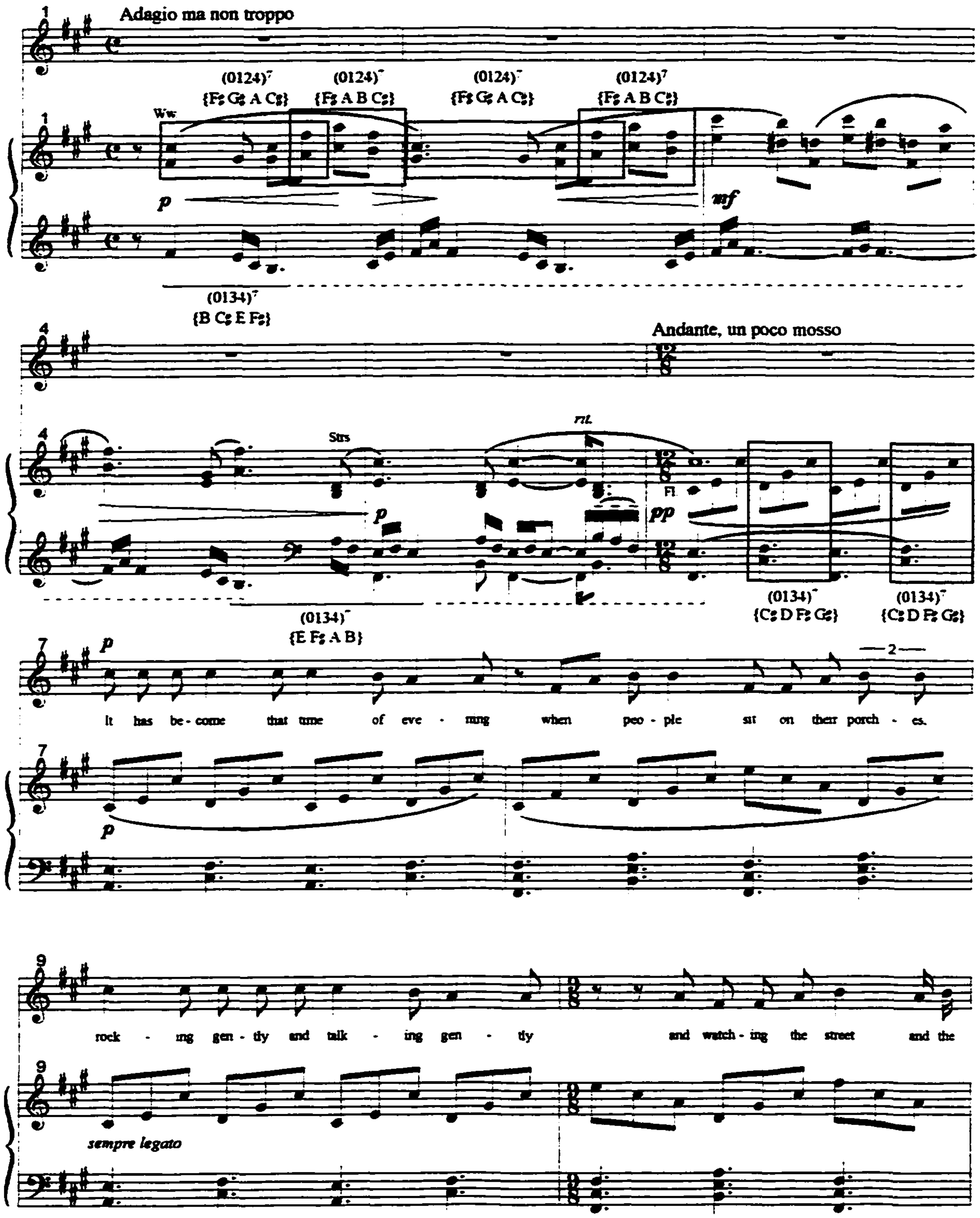
Example 2.5c. Stravinsky, *Sonata for Two Pianos*, I, mm. 80-81 (coda theme).



Example 2.5d. Inversion around a D/E♭ axis in the coda theme.

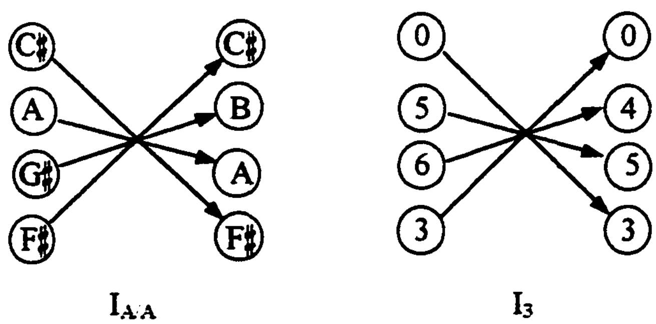


Example 2.6a. Barber, *Knoxville, Summer of 1915*, mm. 1-14. Copyright © 1949 (Renewed) by G. Schirmer, Inc. International Copyright Secured. All Rights Reserved. Used by Permission.



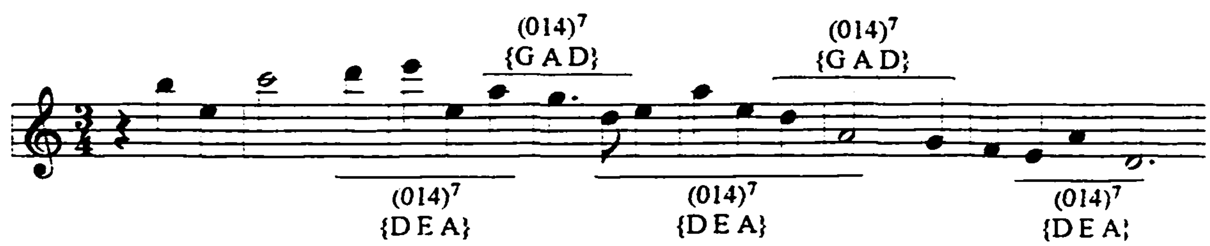
Example 2.6a continued

Example 2.6b. Inversion about A in mm. 1-2.



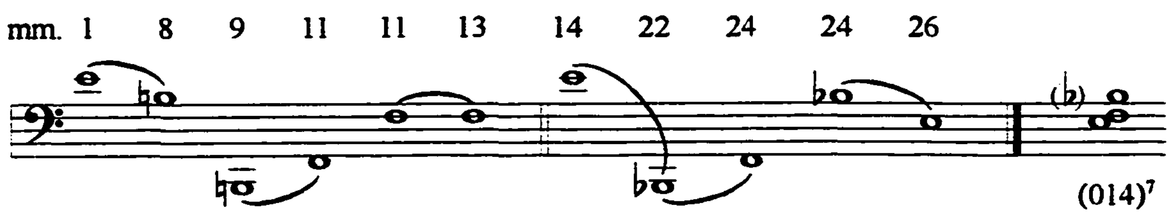
Example 2.7a. Prokofiev's Op. 22, No. 1 (1917).

Example 2.7b. $(014)^7$s in the melody of mm. 1-8 and 14-21.

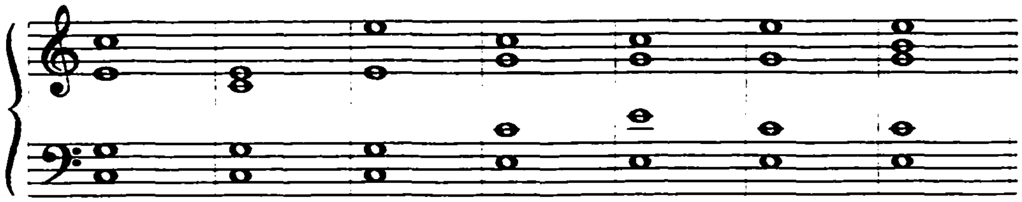


Example 2.7c. $(014)^7$ formed among the contour pitches in the lower three voices of mm. 1-4.

Example 2.7d. Largest harmonic motions in the piece.



Example 3.1. A gradual shift of emphasis from C to E.



1 2 3 4 5 6 7

Figure 3.1. Factors in the perception of chordal tone centers.

A. Intervallic Composition of the Set

Mod7 Tn-type	Weighting	Example
[024] (triad)	The 1st member has a weight of 4, except if the triad is diminished, in which case the 1st member receives no weight.	[C E G]
[0135] (7th chord)	The 2nd member has a weight of 3 and the 3rd member has a weight of 2, except if either of the triads built on those members is diminished, in which case the member with a diminished triad built on it receives no weight.	[C D F A]
[0124] or [0234]	The 1st member has a weight of 1, except if the triad built on it is diminished, in which case it receives no weight.	[C D E G], [C E F G]
[013]	The 2nd member has a weight of 3.	[C D F]
[023]	The 3rd member has a weight of 1.	[C E F]
[012]	The 1st member has a weight of 1.	[C D E]
[03] (4th/5th)	The 2nd member has a weight of 2, except in the case of the tritone. If [03] is represented by a tritone, neither member receives any weight.	[C F]
[02] (3rd/6th)	The 1st member has a weight of 2.	[C E]
[01] (2nd/7th), [014], [0123], or [0134]	No emphasis on any member.	[C D], [C D G], [C D E F], [C D F G]

(the sets above constitute the complete list of mod7 Tn-types, cardinalities 2-4)

B. Octave Doubling

Each member of the harmony has a weight of 1 for every additional octave in which it appears.

C. Voicing

1. The lowest pitch in a given [024], [0135], [013], [023] harmony has a weight of 1.
2. The lowest pitch in any other given Tn-type has a weight of 2.
3. If the lowest and highest pitches of a harmony are identical, then that pitch has a weight of 2.

D. Linear Context

1. If one of the outer pitches of a given harmony is scale degree 1 of the prevailing collection, and is approached from a half-step below, then that pitch has a weight of 3.
2. If one of the outer pitches of a given harmony is scale degree 7 of the prevailing collection and is approached from or moves to a half-step above, and there is a scale degree 5 present in the same harmony, then the pitch representing scale degree 5 has a weight of 3.
3. If one of the outer pitches of a given harmony is scale degree 1 of the prevailing collection, and is approached from scale degrees 2, 3, or 5, then that pitch has a weight of 2.
4. If the bass pitch is not scale degrees 1 or 7, but is approached from a step above, then that pitch has a weight of 1.

Figure 3.2. Formula for finding the relative strength (rs) of tone center *x*.

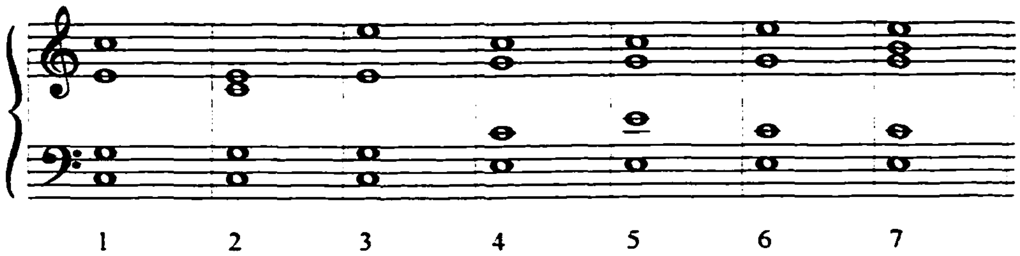
$$rs(x) = T^x (T^x / (T^x + T^y + T^z + \ldots T^n))$$

where T^x represents the total weight of tone center *x*, T^y represents the total weight of tone center *y*, T^z represents the total weight of tone center *z*, and T^n represents the total weight of tone center *n*.

E.g. $rs(E) = 6 (6/(6 + 3 + 0 + 0)) = 6 (6/9) = 36/9 = 4$

Example 3.2. Summary of Example 3.1.

a. music

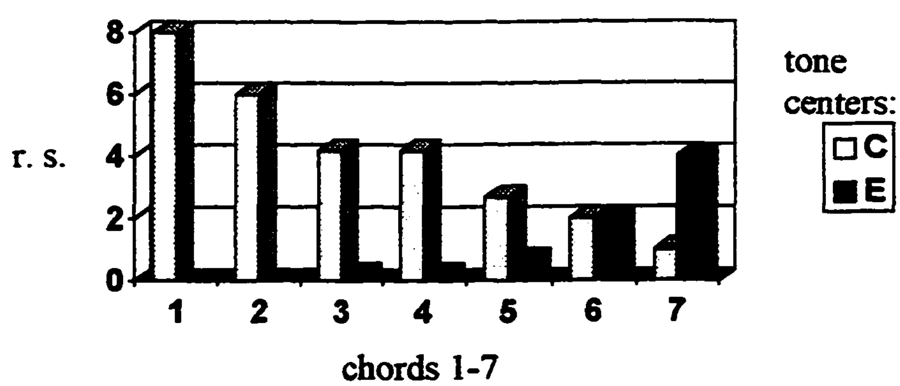


b. table

	r. s.	Total	A	B	C-1	C-2	C-3	D-1	D-2	D-3	D-4
1. C	8	8	4	1	1		2				
E	0	0									
2. C	6	6	4	1	1						
E	0	0									
3. C	4.16	5	4		1						
E	0.2	1		1							
4. C	4.16	5	4	1							
E	0.2	1			1						
5. C	2.66	4	4								
E	0.66	2		1	1						
6. C	2	4	4								
E	2	4		1	1		2				
7. C	1	3	3								
E	4	6	2	1	1		2				

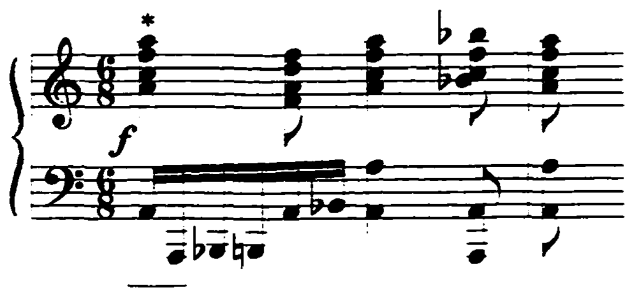
r. s. = relative strength of tone center; columns A through D-4 refer to Figure 1.

c. graph



Example 3.3. Stravinsky, *Serenade in A*, I, opening.

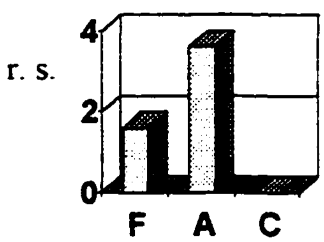
a. music



b. table

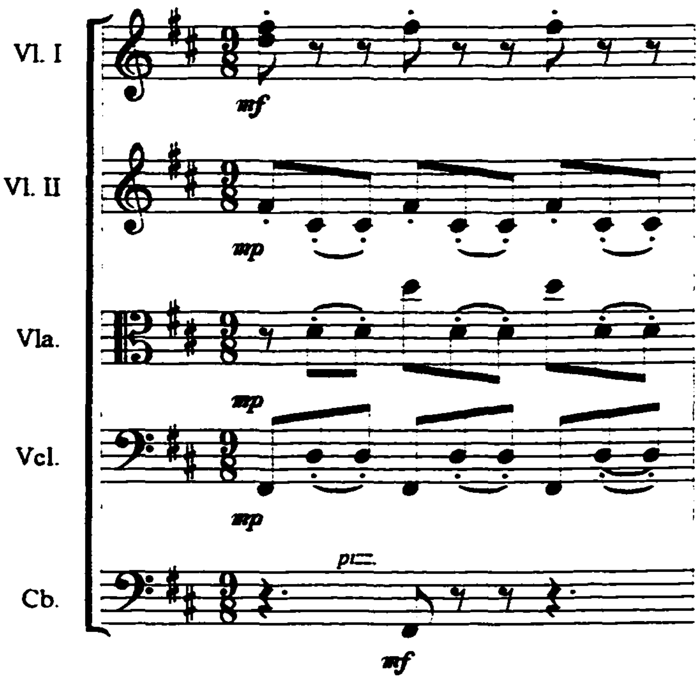
	r. s.	Total	A	B	C-1	C-2	C-3	D-1	D-2	D-3	D-4
C	0	0									
A	3.6	6		3	1		2				
F	1.6	4	4								

c. graph



Example 3.4. Stravinsky, *Concerto in D*, I, m. 25.

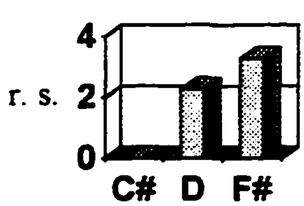
a. music



b. table

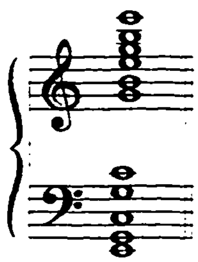
	r. s.	Total	A	B	C-1	C-2	C-3	D-1	D-2	D-3	D-4
F#	3.27	6		3	1		2				
D	2.27	5	3	2							
C#	0	0									

c. graph



Example 3.5. Reduction of Stravinsky, *Symphony in C*, I, ending.

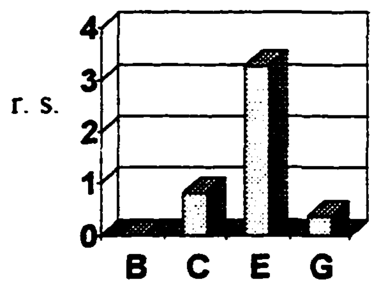
a. music



b. table

	r. s.	Total	A	B	C-1	C-2	C-3	D-1	D-2	D-3	D-4
G	0.36	2		2							
E	3.27	6	2	1	1		2				
C	0.81	3	3								
B	0	0									

c. graph



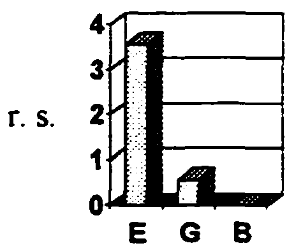
Example 3.6. Reduction of Stravinsky, *Symphony of Psalms*, I, opening chord.

a. music

b. table

	r. s.	Total	A	B	C-1	C-2	C-3	D-1	D-2	D-3	D-4
B	0	0									
G	0.57	2		2							
E	3.57	5	4		1						

c. graph



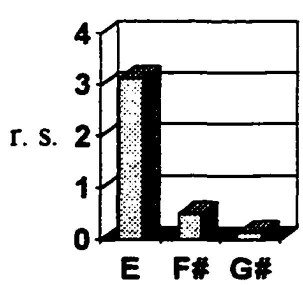
Example 3.7. Stravinsky, *Cantata*, II, mm. 48-49.

a. music

b. table

	r. s.	Total	A	B	C-1	C-2	C-3	D-1	D-2	D-3	D-4
G#	0.12	1		1							
F#	0.5	2		2							
E	3.12	5	1			2				2	

c. graph



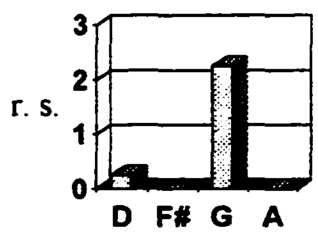
Example 3.8. Stravinsky, *Mass*, *Agnus Dei*, m. 22.

a. music

b. table

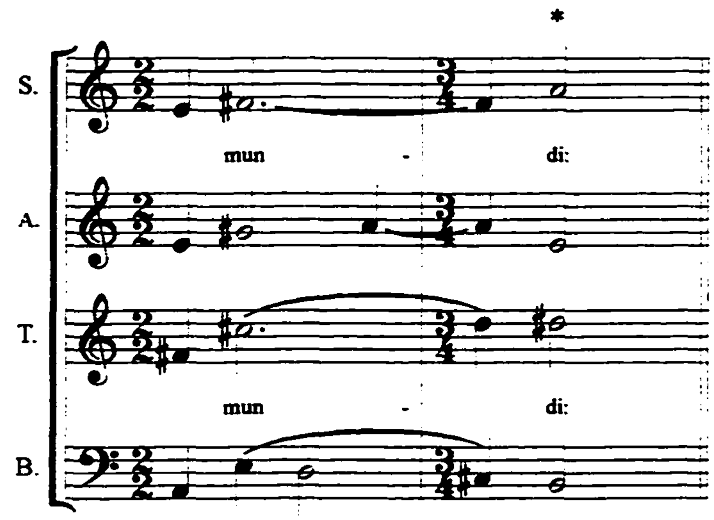
	r. s.	Total	A	B	C-1	C-2	C-3	D-1	D-2	D-3	D-4
A	0	0									
G	2.25	3				2					1
F#	0	0									
D	0.25	1	1								

c. graph



Example 3.9. Stravinsky, *Mass*, *Agnus Dei*, mm. 37-38.

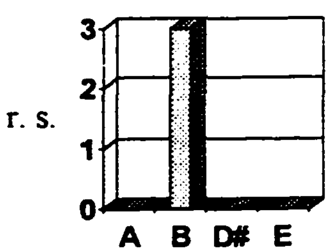
a. music



b. table

	r. s.	Total	A	B	C-1	C-2	C-3	D-1	D-2	D-3	D-4
E	0	0									
D#	0	0									
B	3	3				2					1
A	0	0									

c. graph



Example 3.10. Stravinsky, *Cantata*, I, mm. 23-24.

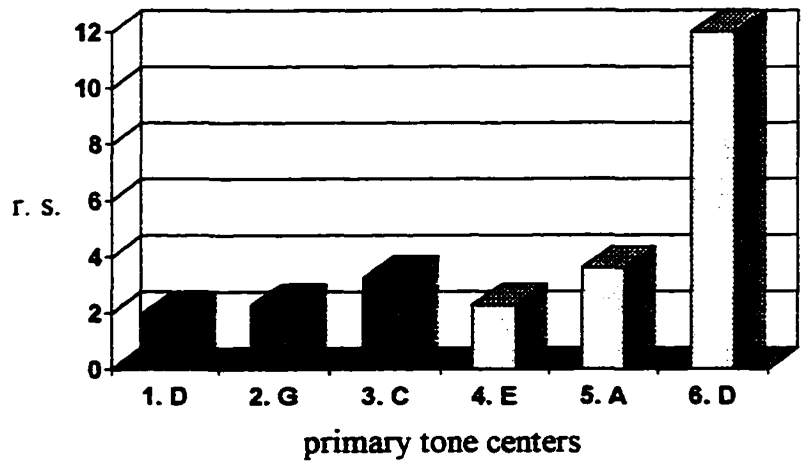
a. music

b. table

	r. s.	Total	A	B	C-1	C-2	C-3	D-1	D-2	D-3	D-4
1. A	0.12	1		1							
F	1.12	3	2	1							
D	2	4	3		1						
C	0	0									
2. B	0	0									
G	2.28	4		1					3		
F	0	0									
E	1.28	3	1			2					
3. E	0	0									
C	3.27	6	2	1				3			
A	0.81	3	3								
G	0.36	2		1	1						
4. B	0.14	1		1							
G	0.57	2	2								
E	2.28	4	3		1						
D	0	0									
5.C#	0	0									
A	3.6	6	2	1					3		
F#	1.6	4	3		1						
E	0	0									
6. A	0	0									
F#	0	0									
E	0	0									
D	12	12	1	2		2	2	3		2	

Example 3.10 continued

c. graph



Example 3.11. Stravinsky, *Cantata*, I, introduction (mm. 1-7).

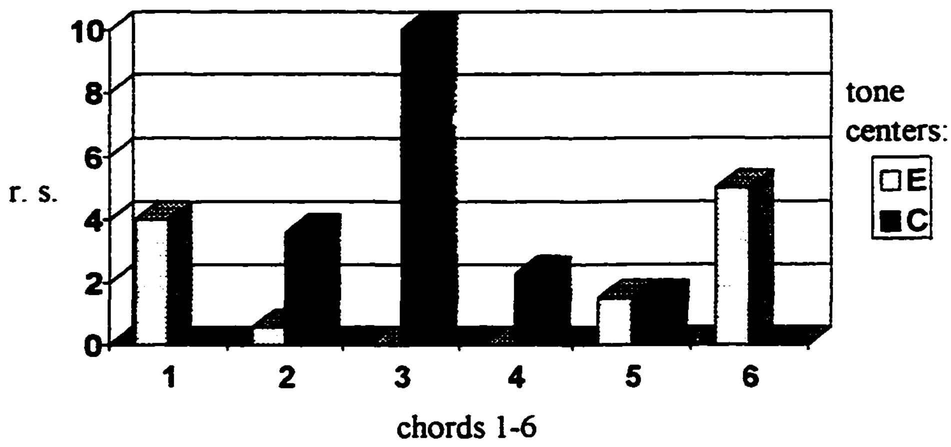
a. music

b. table

	r. s.	Total	A	B	C-1	C-2	C-3	D-1	D-2	D-3	D-4
1. B	0	0									
A	0	0									
G	0	0									
E	4	4	1	1		2					
2. G	0	0									
E	0.57	2		2							
C	3.57	5	4		1						
3. G	0	0									
F	0	0									
E	0	0									
C	10	10	1	1		2	2			2+2	
4. G	0.25	1		1							
E	0	0									
D	0	0									
C	2.25	3	1			2					
5. G	0	0									
F	0	0									
E	1.5	3		1		2					
C	1.5	3	1							2	
6. B	0	0									
A	0	0									
G	0	0									
E	5	5	1	2		2					

Example 3.11 continued

c. graph



Example 3.12. Stravinsky, *Cantata*, I.

Example 3.12 continued

Example 3.13. Reduction of *Cantata*, I, mm. 8-22.

a. reduction

b. table

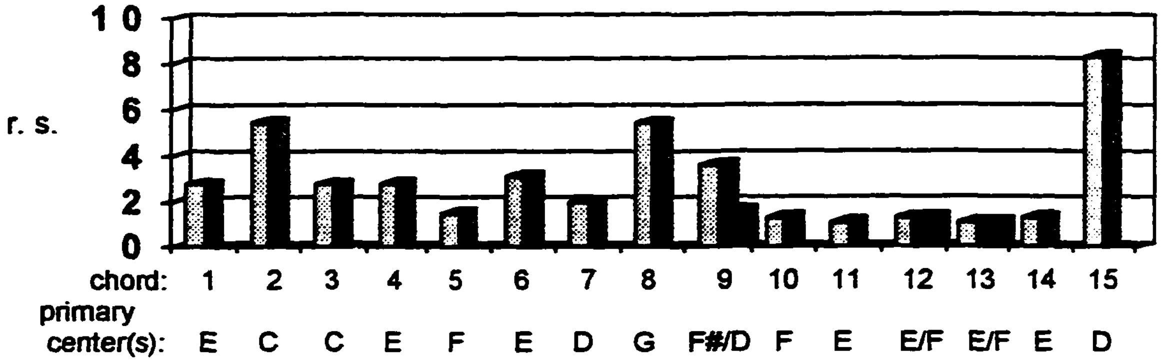
	r. s.	Total	A	B	C-1	C-2	C-3	D-1	D-2	D-3	D-4
1. E	2.77	5		1		2	2				
C	1.77	4	2			2					
2. F	0	0									
E	0.11	1		1							
D	0.11	1		1							
C	5.44	7		1		2+2				2	
B	0	0									
3. C	2.77	6		1	1					2+2	
A	0.31	2	2								
F	1.23	4	3		1						
E	0.08	1		1							
4. G	0.11	1			1						
E	2.78	5	2	1	1						1
C	1	3	3								
B	0	0									
5. C	0	0									
A	0.16	1		1							
G	0	0									
F	1.5	3		1		2					
E	0.66	2				2					
6. B	0	0									
A	0	0									
G	1	3		1		2					
E	4	5	1	1		2					1
7. E	1.12	3				2					1
D	2	4		1		2					1
C	0	0									
B	0	0									
A	0	0									
G	0.12	1		1							

Example 3.13 continued

	r. s.	Total	A	B	C-1	C-2	C-3	D-1	D-2	D-3	D-4
8. D	0	0									
B	0.44	2	2								
G	3.44	7	3	1	1		2				
F(#)	0	0									
9. A	0	0									
F#	3.6	6	2	1	1+1						1
D	1.6	4	3	1							
C#	0	0									
10.G	0.14	1		1							
F	1.28	3		1		2					
E	0.57	2				2					
C	0.14	1	1								
11.B	0.12	1		1							
A	0.5	2		2							
F	0.5	2				2					
E	1.12	3		1		2					
12.B	0	0									
G	0.14	1		1							
F	1.28	3		1		2					
E	1.28	3	1			2					
13.A	0.5	2		2							
G	0	0									
F	1.12	3		1		2					
E	1.12	3		1		2					
14.B	0	0									
A	0.14	1		1							
G	0.14	1		1							
F	0.57	2				2					
E	1.28	3		1		2					
15.A	0.33	2		2							
G	0	0									
E	0	0									
D	8.33	10		2		2+2	2				1+1

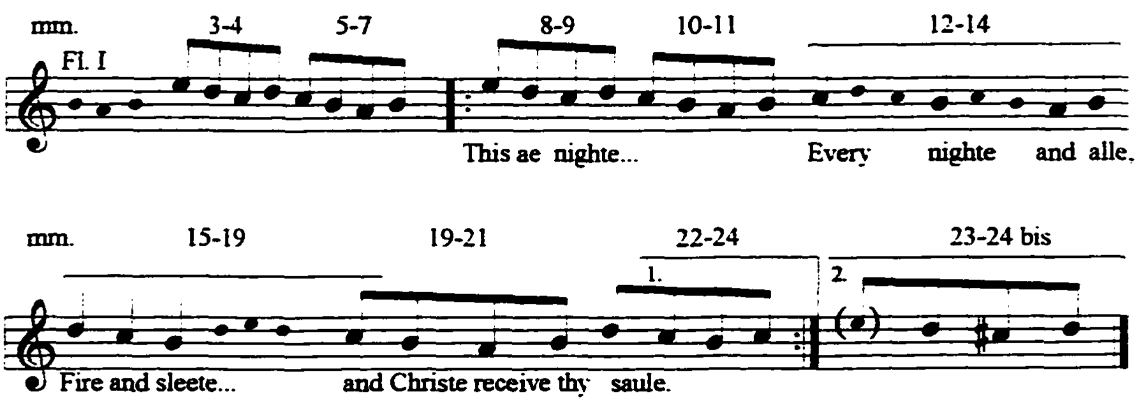
Example 3.13 continued

c. graph



Example 3.14. Motivic analysis of Stravinsky, *Cantata*, I.

a.



b.

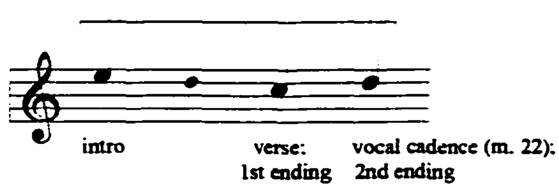


Figure 3.3. Factors that affect the relative strength of tone centers.

For each chord in a given passage, the following four factors must be considered:

Metric Stress

If the chord occurs on beat 1, then 1 is added to the relative strength of each pitch class in the chord.

Agogic Stress

If the chord is at least twice as long as most of the other chords in the section, then 1 is added to the relative strength of each pitch class in the chord.

Ordinal Stress

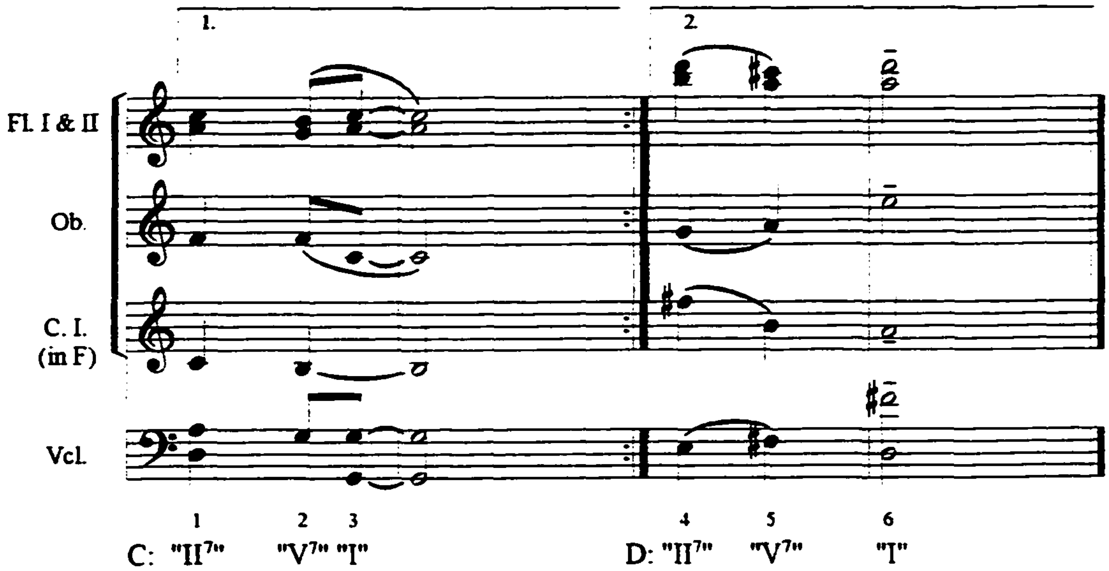
1. If the chord occurs first or last in a phrase, then 1 is added to the relative strength of each pitch class in the chord.
2. If the chord occurs first or last in a section, then 2 is added to the relative strength of each pitch class in the chord.
3. If the chord occurs first or last in a movement or work, then 3 is added to to the relative strength of each pitch class in the chord.

Harmonic Stress

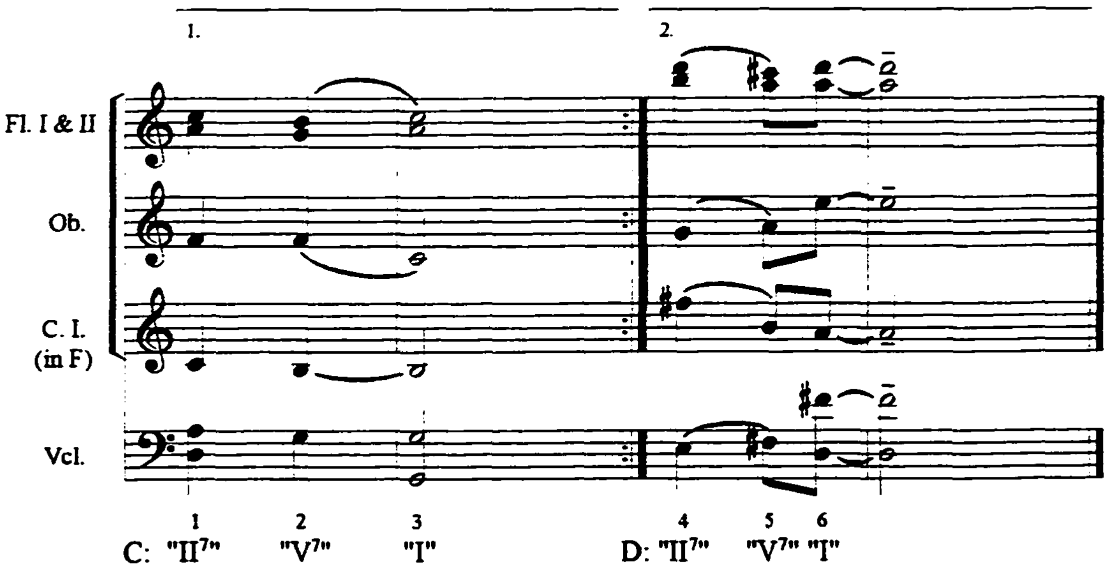
1. If the primary tone centers of both the chord and the chord preceding it have triads built on them, and a descending fifth motion connects the two primary tone centers, then 2 is added to the relative strength of each pitch class in the chord.
2. If the primary tone centers of both the chord and the chord preceding it have triads built on them, and a descending third motion connects the two primary tone centers, then 1 is added to the relative strength of each pitch class in the chord.

Example 3.15. Comparison between actual and hypothetical endings to the first movement of the *Cantata*.

a. actual endings

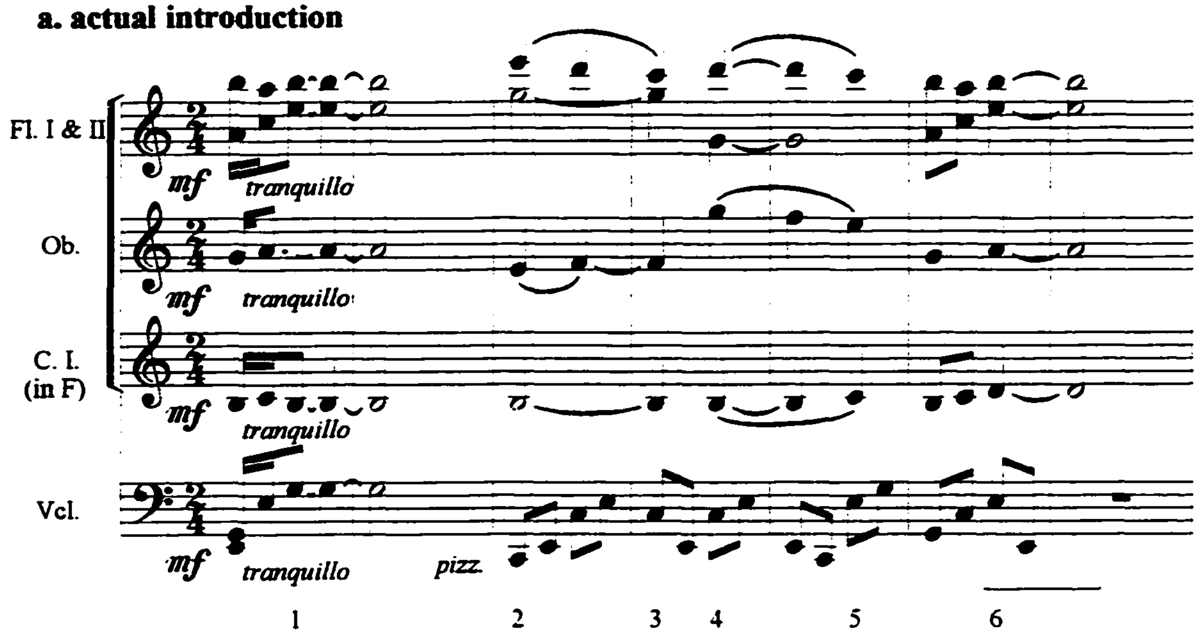


b. hypothetical endings

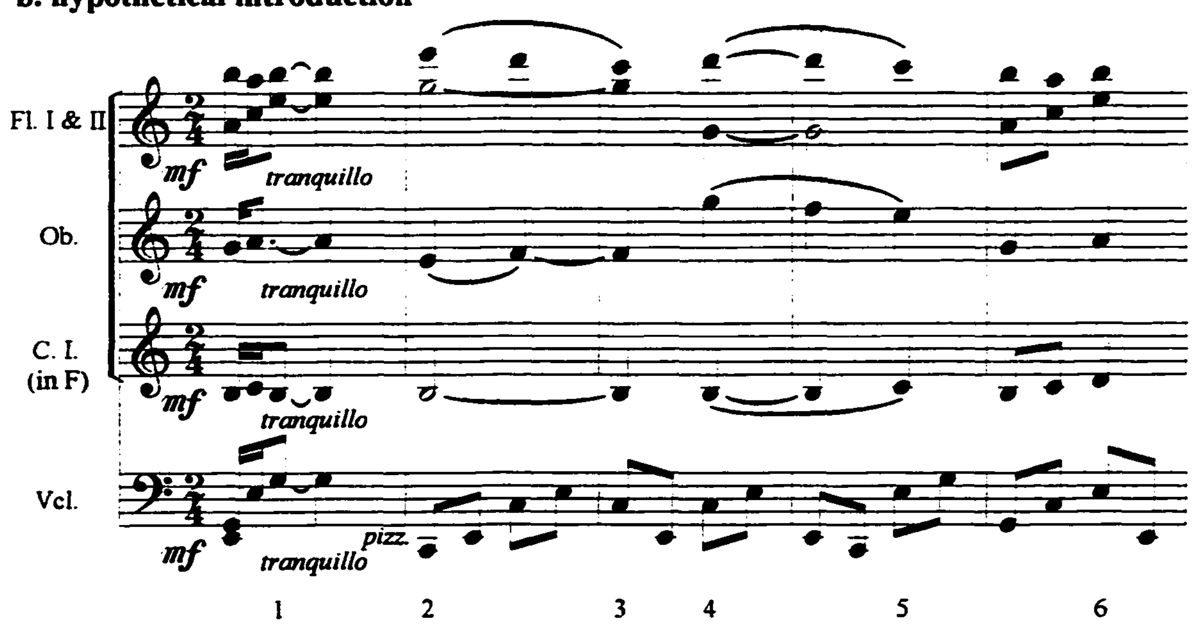


Example 3.16. Comparison between actual and hypothetical introductions to the *Cantata*.

a. actual introduction



b. hypothetical introduction



Example 3.17. Adjustments made to the *Cantata* analysis (cf. Exx. 3.10-3.13).

chord	metric stress	agogic stress	phrase init/end	section init/end	mvt. init/end	harmonic stress	total adjust.	r.s.	tone center	adjusted r.s.
intro 1	1	1	1	2	3		8	4	E	12
intro 2	1		1			1	3	3.57	C	6.57
intro 3	1		1				2	10	C	12
intro 4			1				1	2.25	C	3.25
intro 5			1				1	1.5/1.5	C/E	2.5/2.5
intro 6	1	1	1	2			5	5	E	10
1	1		1	2			4	2.77	E	6.77
2			1				1	5.44	C	6.44
3			1				1	2.77	C	3.77
4	1		1				2	2.78	E	4.78
5	1		1				2	1.5	F	3.5
6	1						1	4	E	5
7	1		1				2	2	D	4
8	1	1	1				3	5.44	G	8.44
9			1				1	3.6/1.6	F#/D	4.6/2.6
10			1				1	1.28	F	2.28
11	1						1	1.12	E	2.12
12							0	1.3/1.3	E/F	1.3/1.3
13	1						1	1.1/1.1	E/F	2.1/2.1
14							0	1.28	E	1.28
15	1		1				2	8.33	D	10.33
1st ending 1	1		1				2	2	D	4
1st ending 2							0	2.28	G	2.28
1st ending 3			1	2			3	3.27	C	6.27
2nd ending 1	1		1				2	2.28	E	4.28
2nd ending 2						2	2	3.6	A	5.6
2nd ending 3	1		1	2	3	2	9	12	D	21

Figure 4.1 Factors in the perception of structural top lines.

A. Octave Doubling

Each member of the harmony has a weight of 1 for every additional octave in which it appears (see Fig. 3.1).

B. Voicing

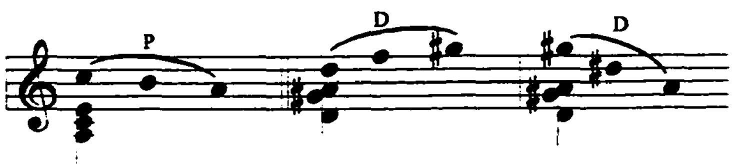
The highest pitch in any harmony has a weight of 2.

C. Solo Stress

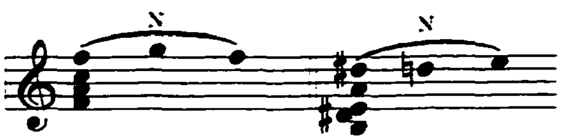
A pitch receives a weight of 4 if...

1) it is part of a line that is meant to be performed at a louder dynamic level than the rest of the ensemble.
2) it is meant to be performed with an instrumental timbre that highlights it within the ensemble.
3) it is meant to be performed with a distinctive articulation that highlights it within the the ensemble.
4) it is marked *Hauptstimme*, or some semantic equivalent of *Hauptstimme*.

Example 4.1a. Illustration of tonal passing tone and atonal division tones.



Example 4.1b. Illustration of tonal and atonal neighbor tones.



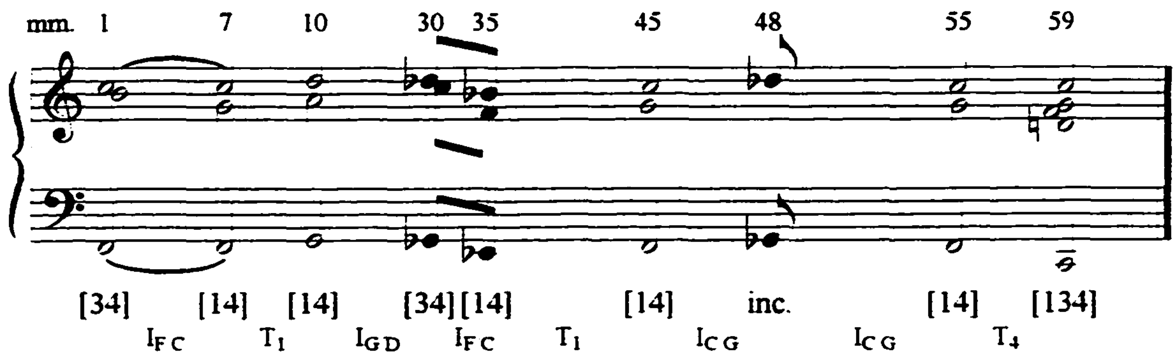
Example 4.2. Analysis of the first trumpet part in m. 51 of Stravinsky's *Agon*, Scene 1.

Example 4.3. Three voice-leading graphs of Stravinsky's *Agon*, Scene 1.

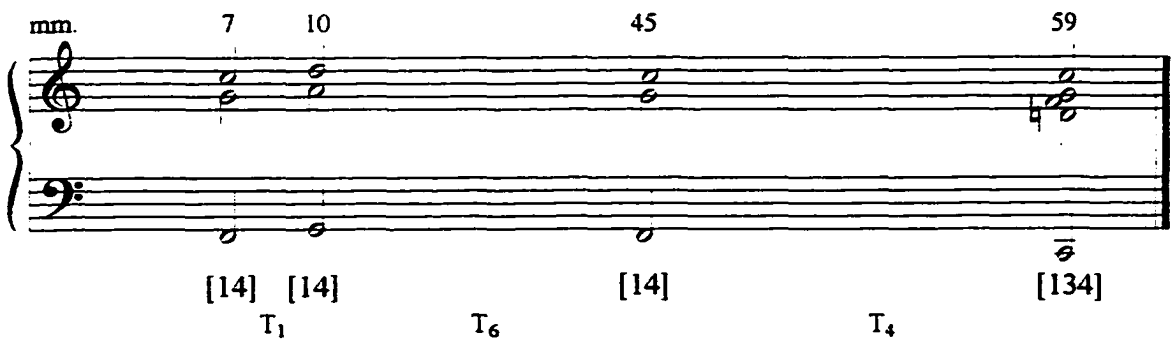
Middleground II

inc. = incomplete [34] missing C; complete sonority suggested by m. 30.

Middleground I



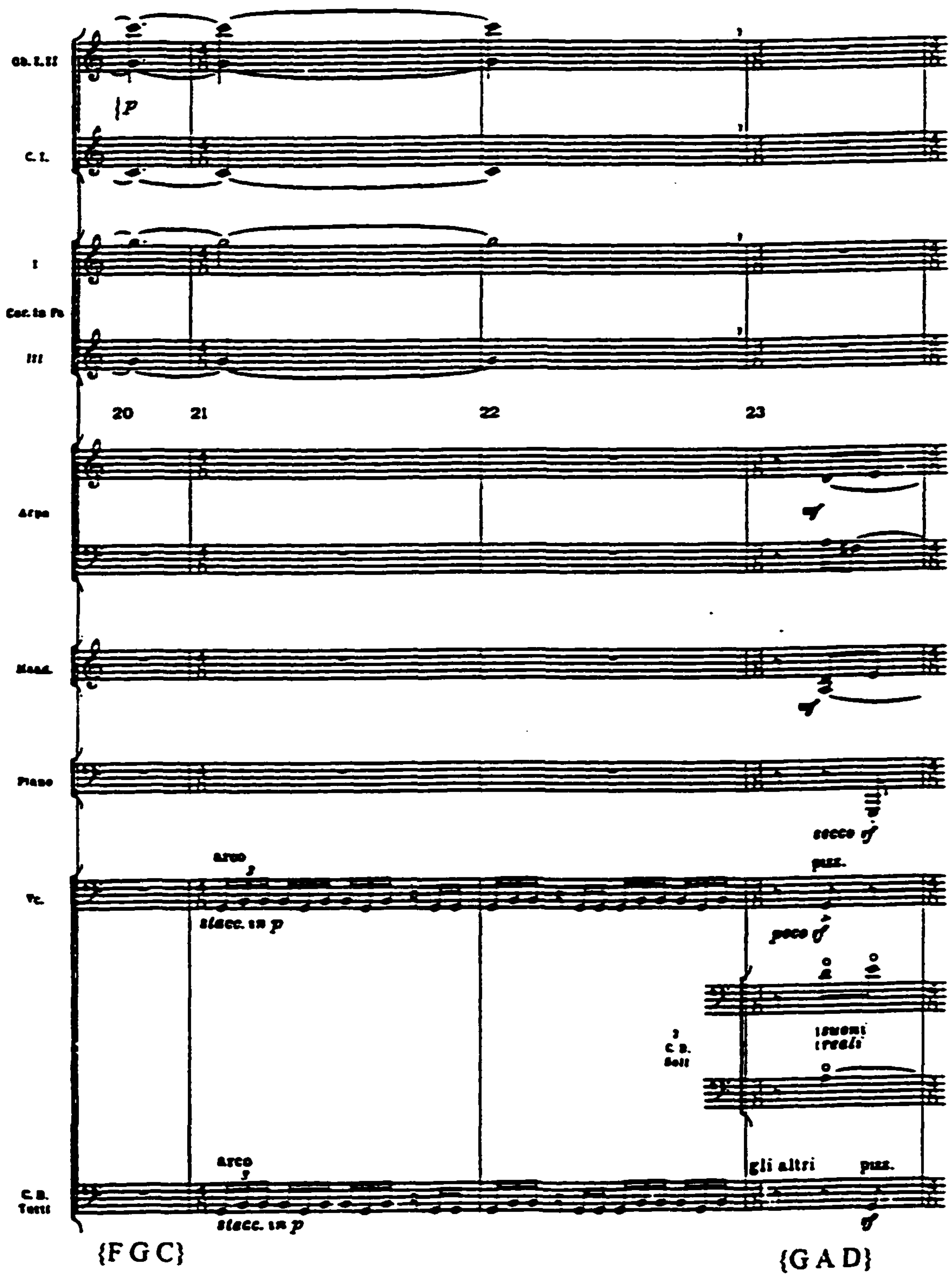
Background



Example 4.4. Annotated score to Stravinsky's *Agon*, scene 1. Copyright © 1957 by Boosey & Hawkes. All Rights Reserved. Used by Permission.

Example 4.4 continued (mm. 9-19)

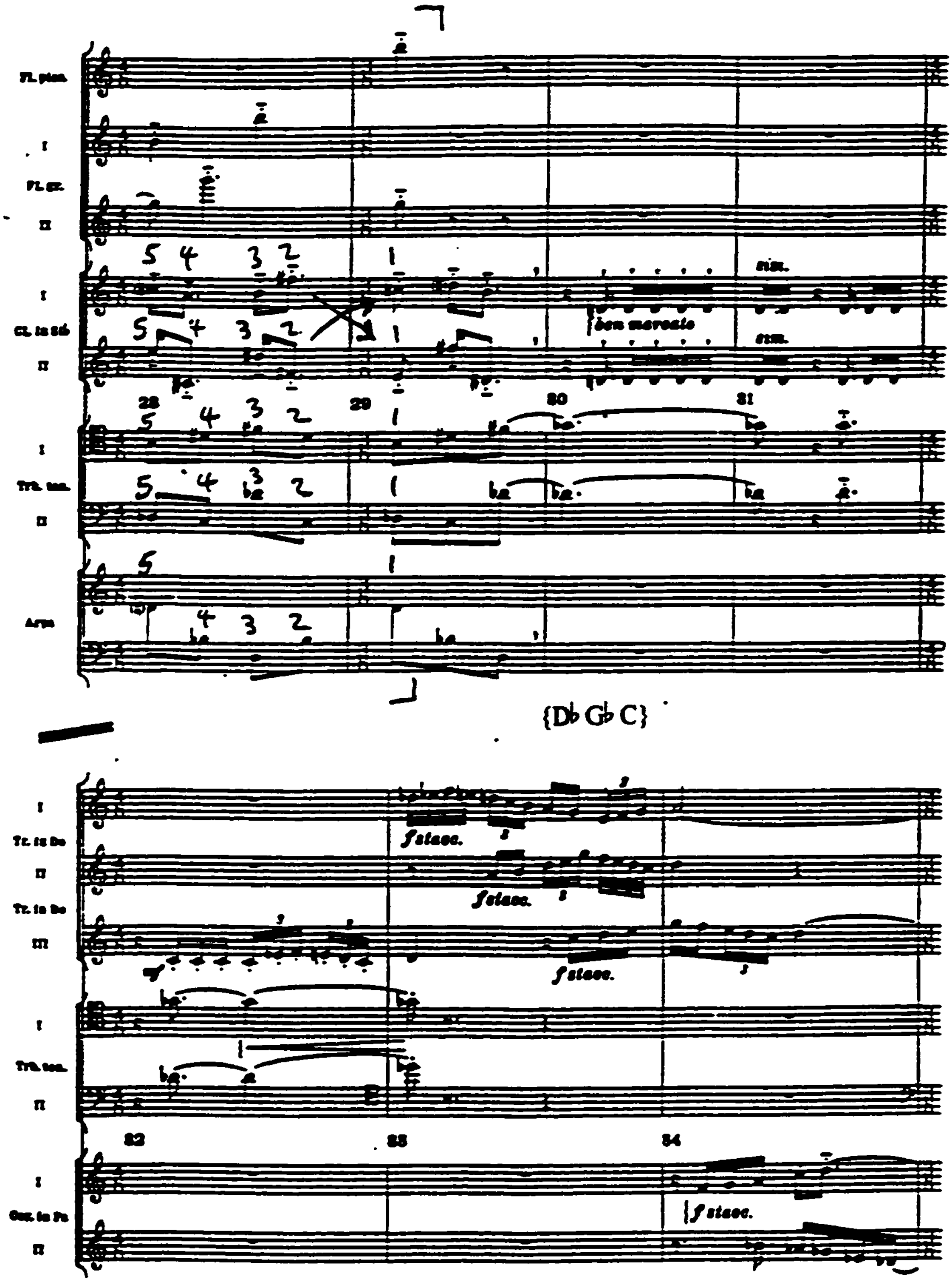
Example 4.4 continued (mm. 20-23)



Example 4.4 continued (mm. 24-27)

{G A D}

Example 4.4 continued (mm. 28-34)

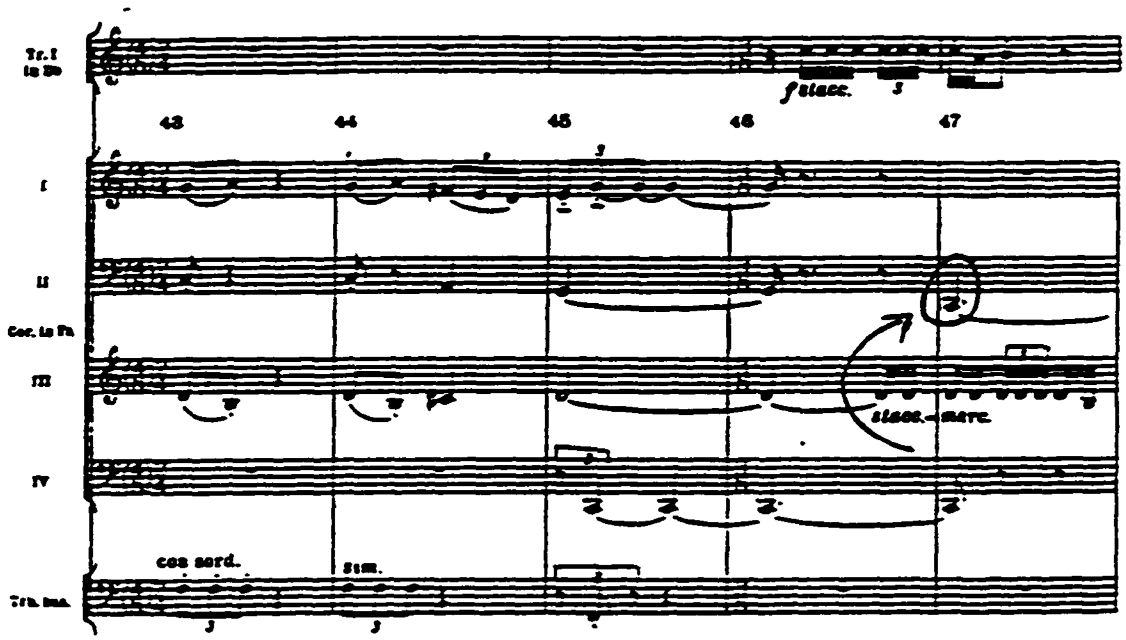


Example 4.4 continued (mm. 35–42)

{E♭ F B♭}

{F G C}   {E♭ F B♭}

Example 4.4 continued (mm. 43-51)



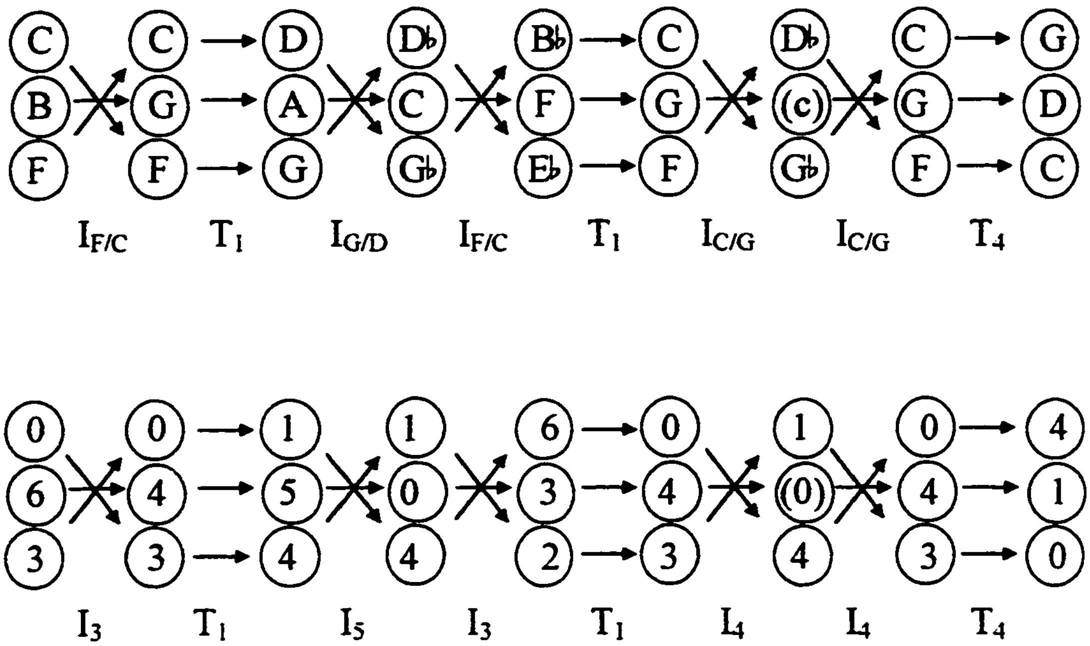
{F G C}

{E♭ B♭}

{G♭ D♭}

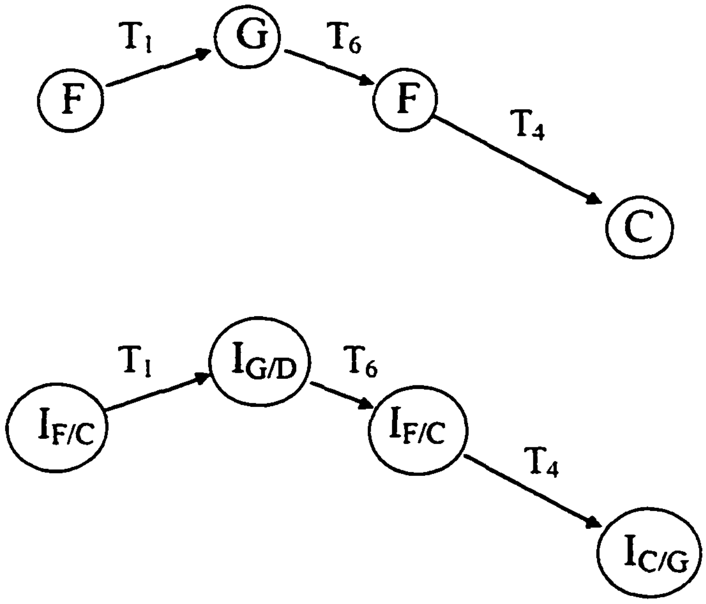
Example 4.4 continued (mm. 52-60)

Example 4.5. The transformational voice-leading of Example 4.3.

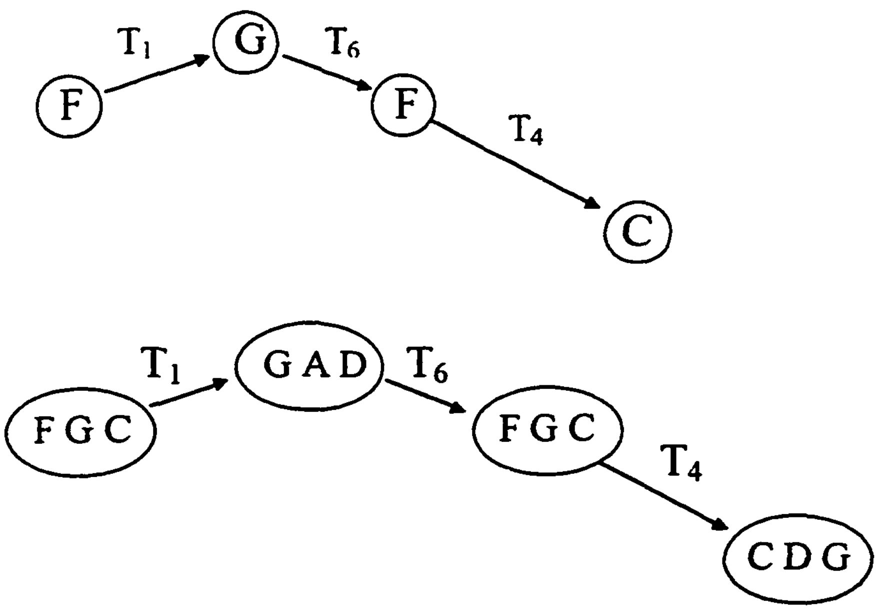


Example 4.6. Two homomorphisms.

a. Homomorphism between the chord motive and the transformational path linking the axes F/C, G/D, and C/G.



b. Homomorphism between the chord motive and the transformational path linking the most structural verticalities in Example 4.3.



Example 4.7. Annotated score to Louise Talma's Piano Sonata No. 1, II.
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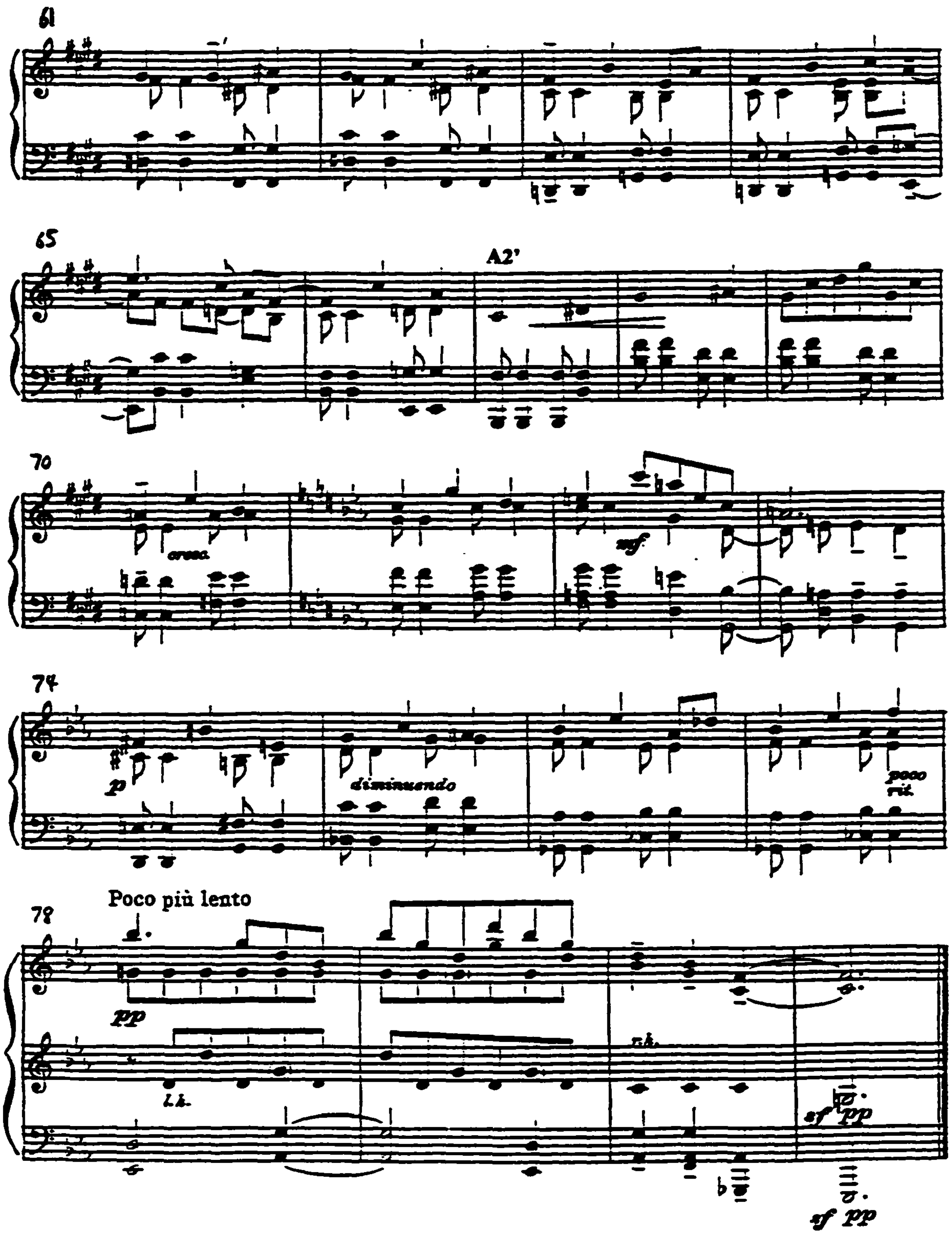
Example 4.7 continued (mm. 23-37)

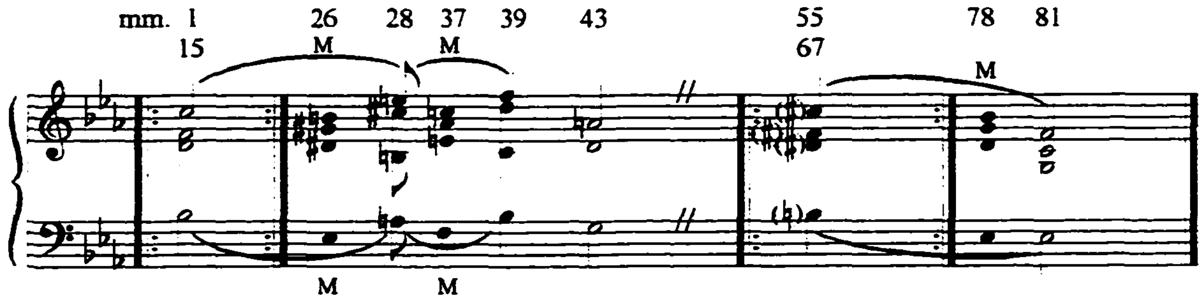
Example 4.7 continued (mm. 38-60)

Example 4.7 continued (mm. 61-81)

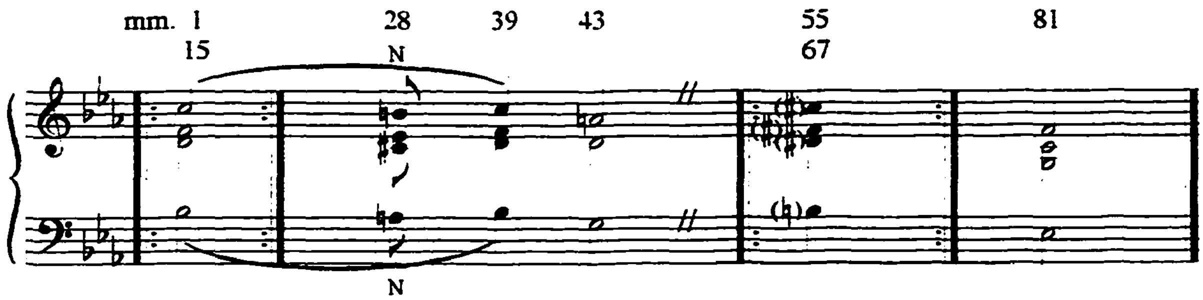


Example 4.8. Four voice-leading graphs of Talma's Piano Sonata No. 1, second movment.

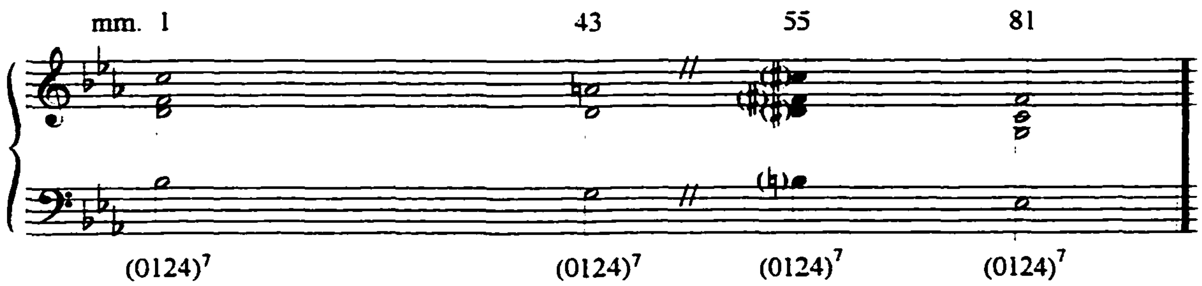
Middleground III



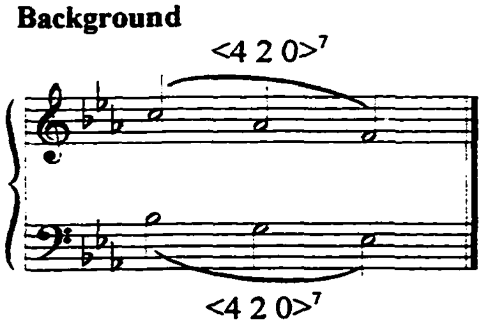
Middleground II



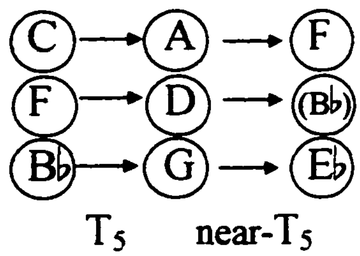
Middleground I

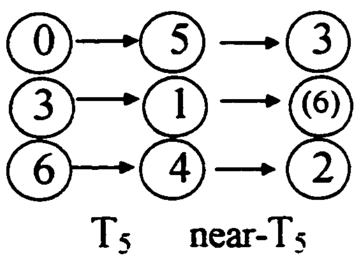


Background

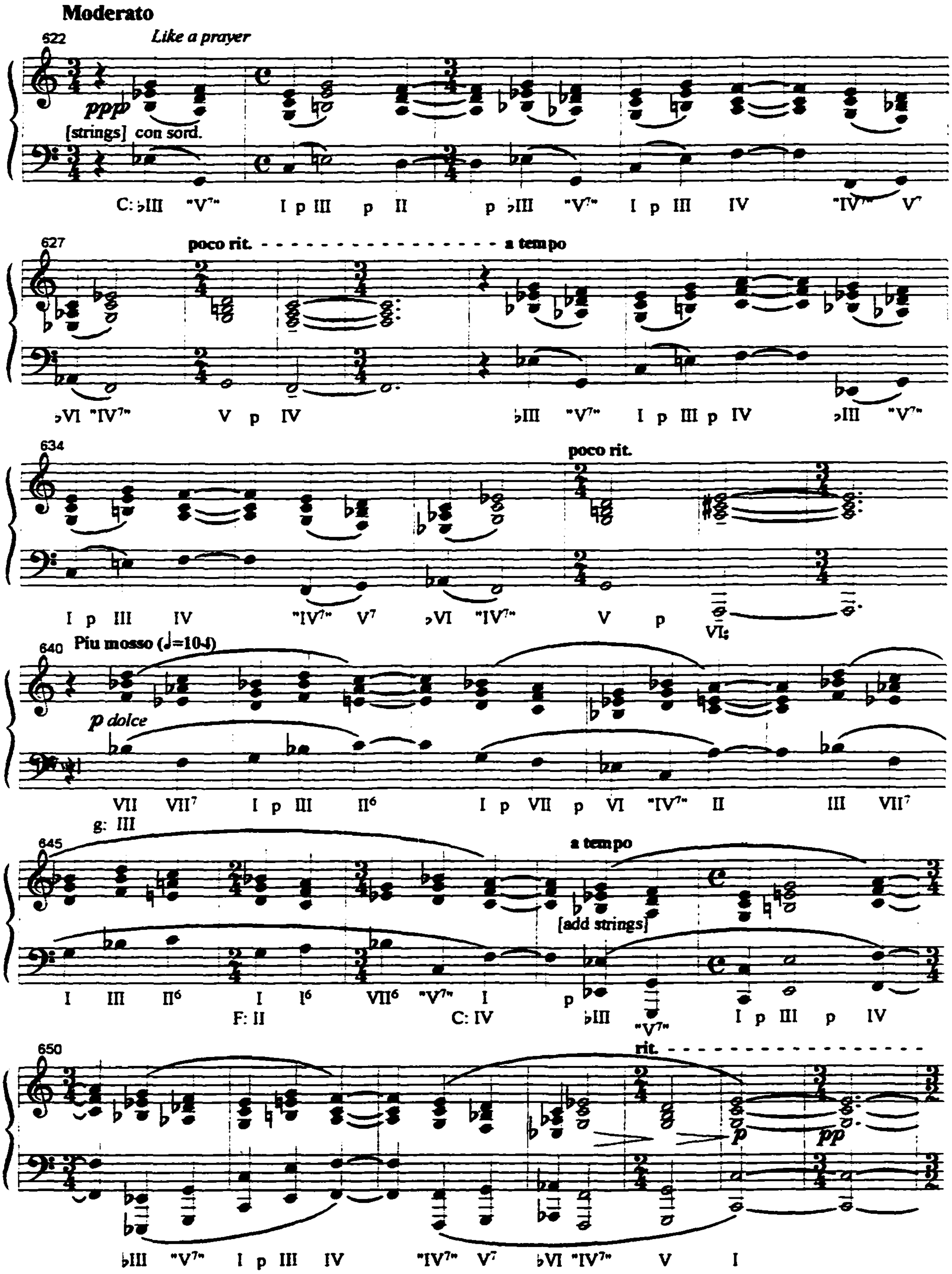


Example 4.9. The transformational voice-leading of the Background graph in Ex. 4.8.

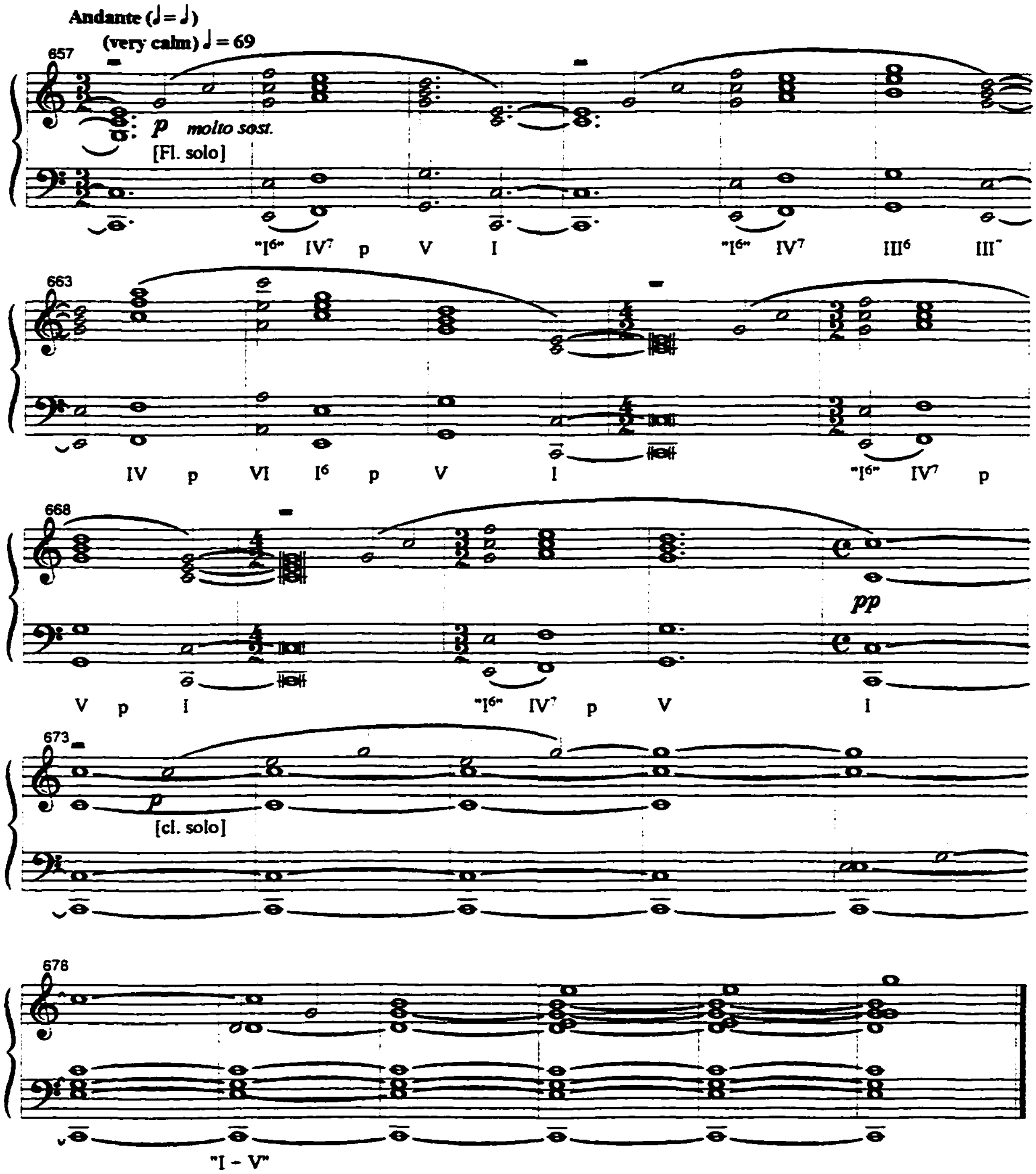




Example 4.10. Copland, *Appalachian Spring*, m. 622-end.

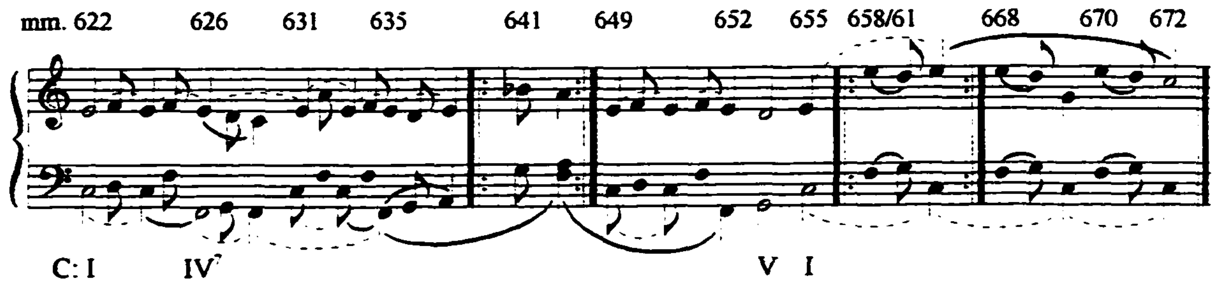


Ex. 4.10 continued

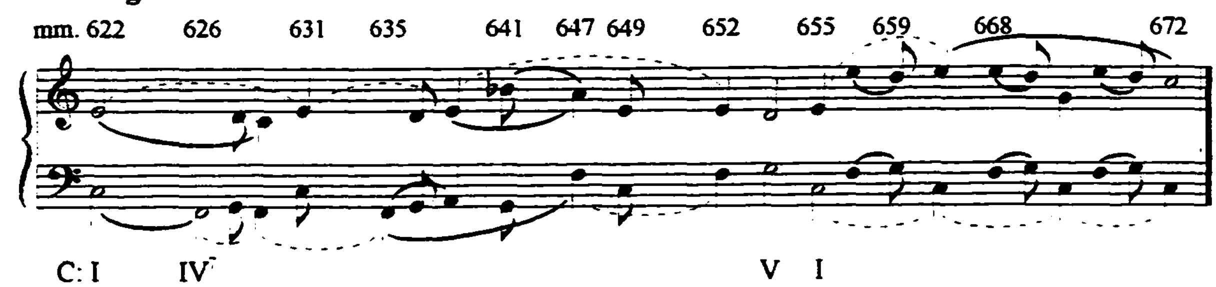


Example 4.11. A Schenkerian analysis of Copland's *Appalachian Spring*, m. 622-end.

Middleground III



Middleground II

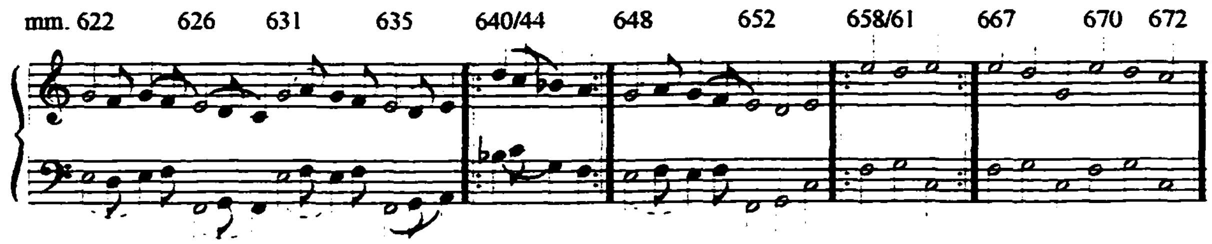


Middleground I

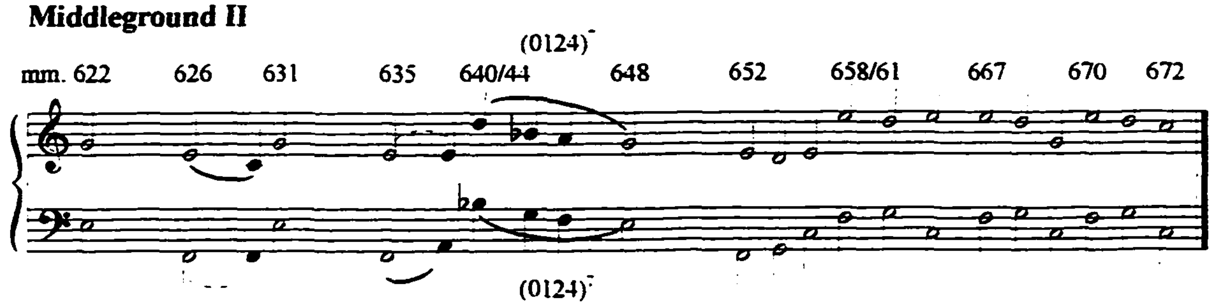
Background

Example 4.12. Four voice-leading graphs of Copland's *Appalachian Spring*, m. 622-end that take (0124)$^-$ to be structurally fundamental.

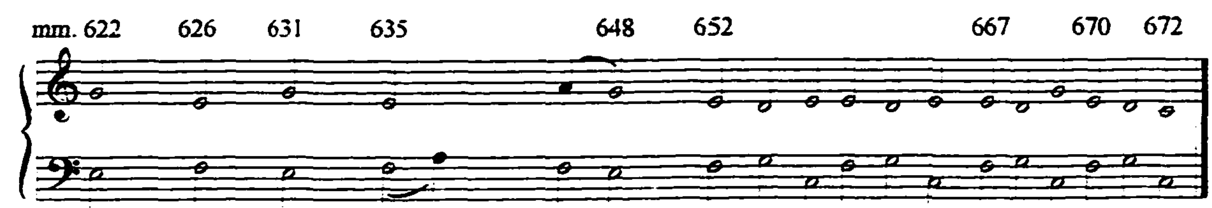
Middleground III



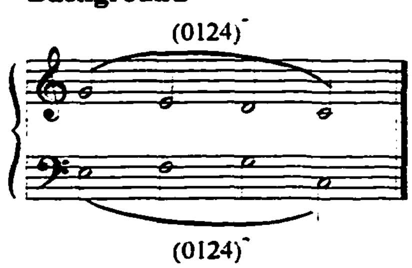
Middleground II



Middleground I

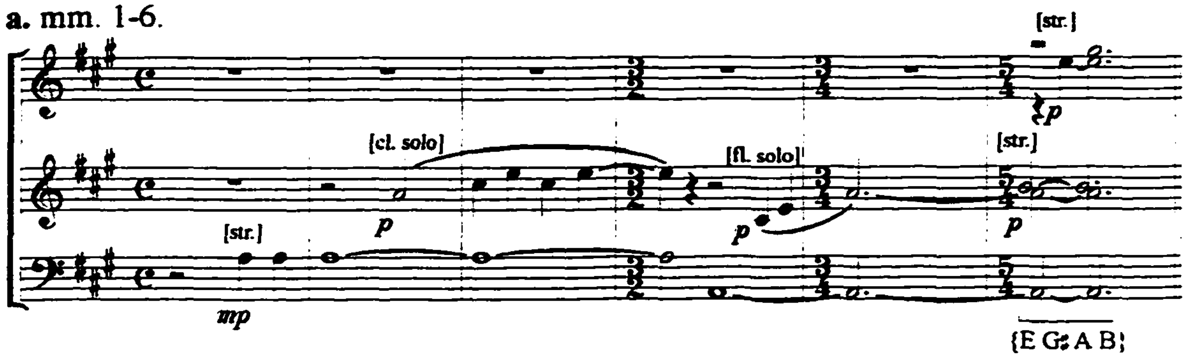


Background

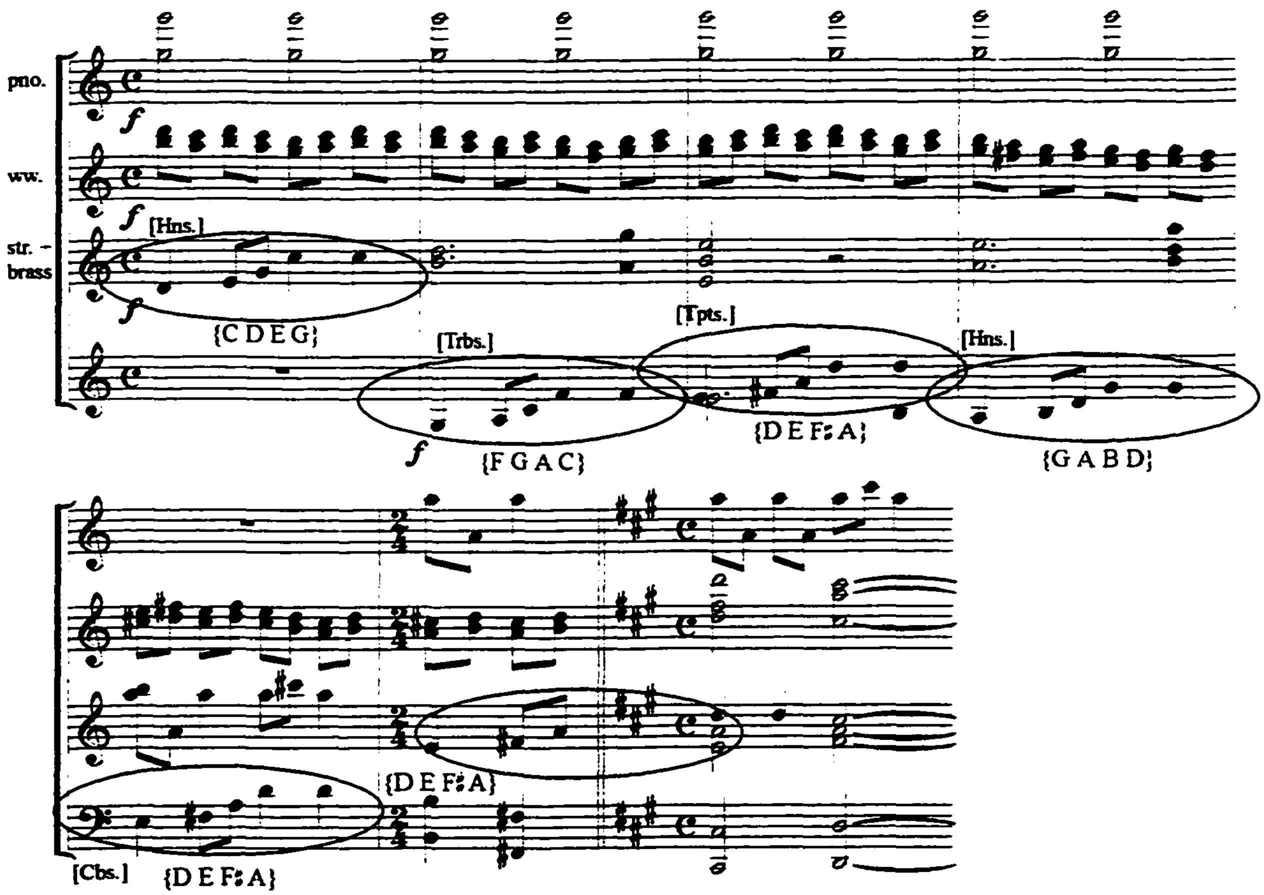


Example 4.13. Forms of (0124)[7] found earlier in *Appalachian Spring*.

a. mm. 1-6.



b. mm. 74-80.



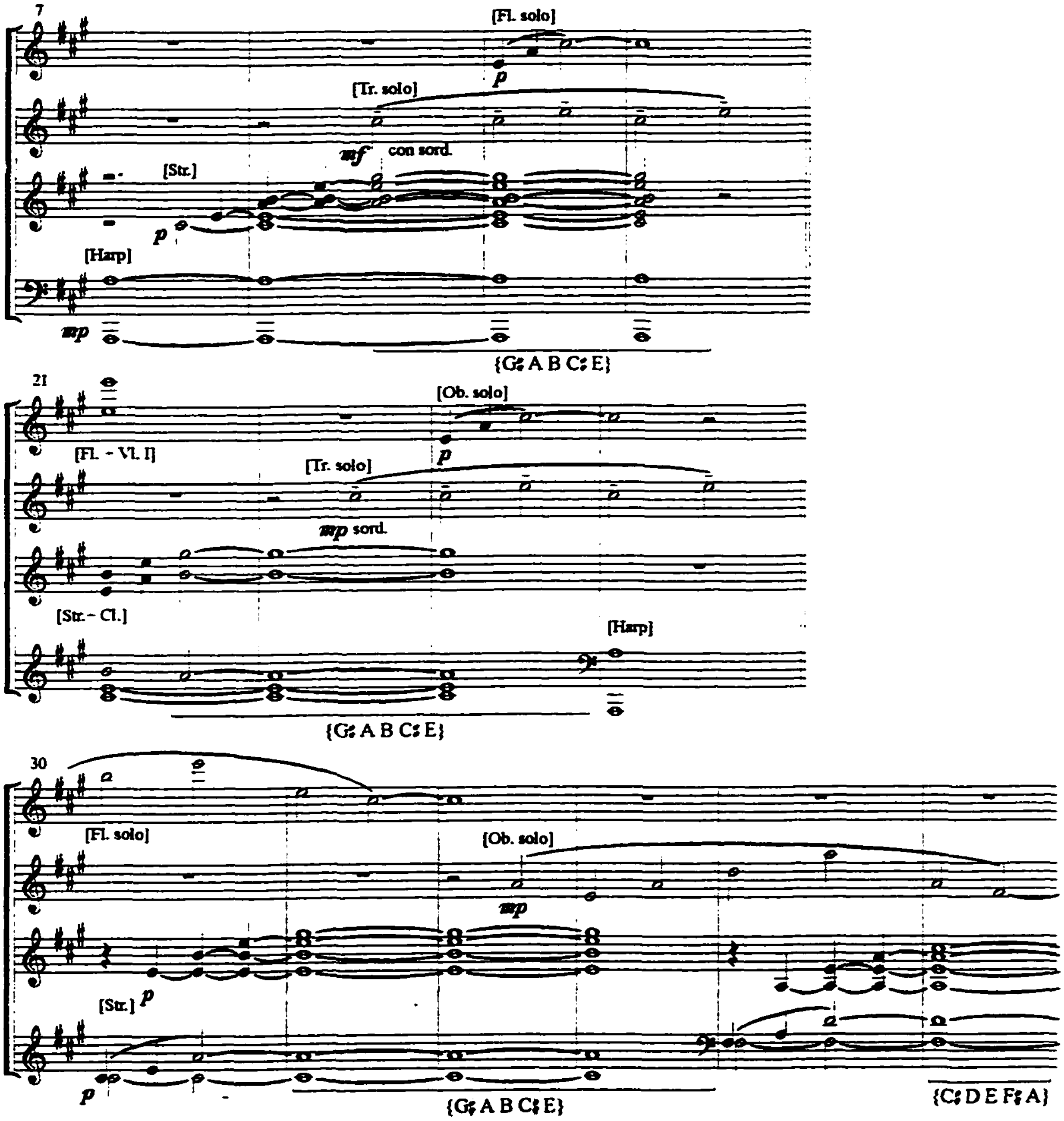
c. "Tis the Gift to be Simple," mm. 1-8; metrically emphasized pitches are circled.

metrically emphasized pitches form {C D E G}

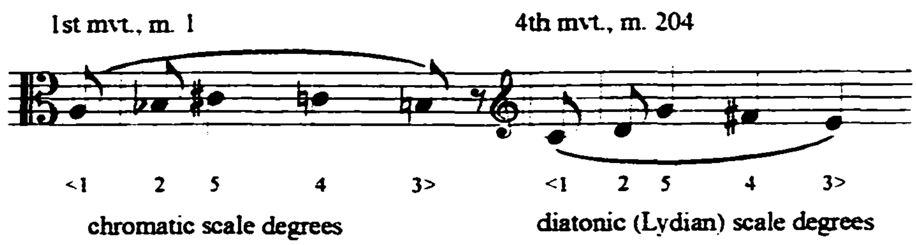
Example 4.14. Forms of (01245)7 and (01235)7 found earlier in *Appalachian Spring*.

a. Theme from mm. 62-64, and 80-82; contour pitches are circled.

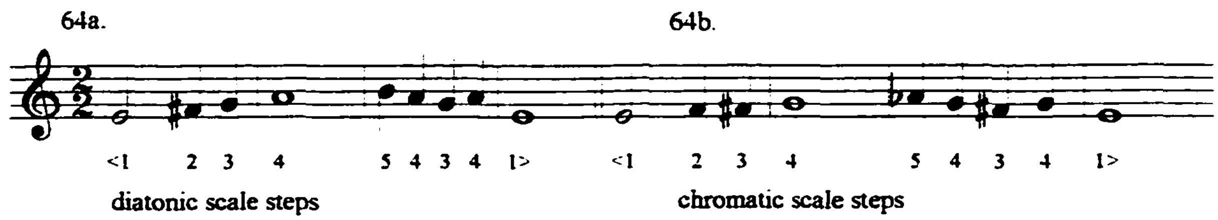
b. Forms of (01235)7 in mm. 7-10, 21-24, and 30-35.



Example 5.1. Bartók, *Music for Strings, Percussion and Celesta*, I and IV; transformation from chromatic to diatonic.

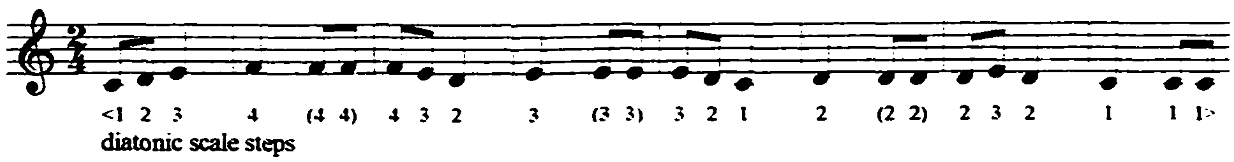


Example 5.2. Bartók, No. 64a and 64b from *Mikrokosmos*, mm. 1-4 of each; transformation from diatonic to chromatic.

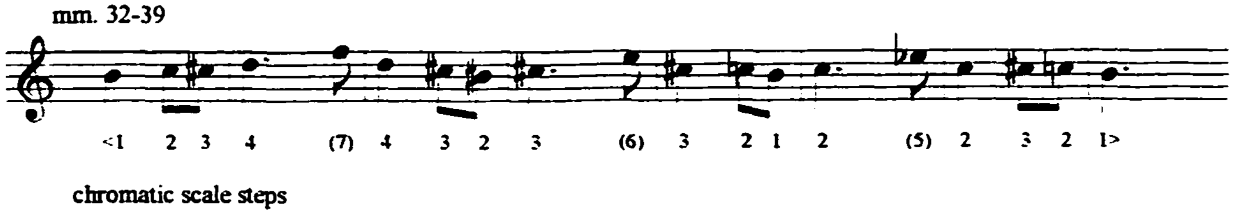


Example 5.3. Bartók, No. 112 from *Mikrokosmos*; transformation from diatonic to chromatic.

mm. 1-8



mm. 32-39



Example 5.4. Bartók, String Quartet No. 4, I; transformation from chromatic to octatonic.

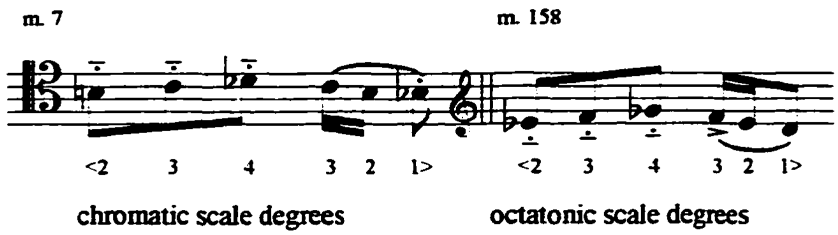
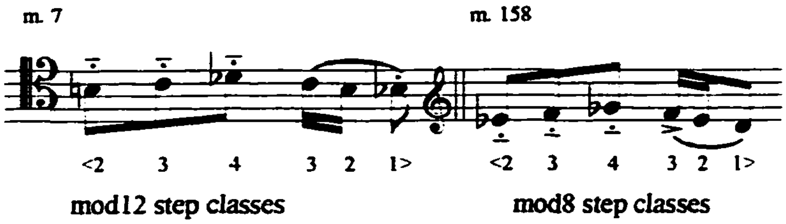


Table 5.1. Possible rotations of the mod12, mod8, mod7, mod6, and mod5 systems.

modulus	label	Step Classes											
		0	1	2	3	4	5	6	7	8	9	10	11
chromatic	**12**	0	1	2	3	4	5	6	7	8	9	10	11
octatonic	8^1	0	1	3	4	6	7	9	10				
-	8^2	0	2	3	5	6	8	9	11				
diatonic	7^1	0	2	4	5	7	9	11					
-	7^2	0	2	3	5	7	9	10					
-	7^3	0	1	3	5	7	8	10					
-	7^4	0	2	4	6	7	9	11					
-	7^5	0	2	4	5	7	9	10					
-	7^6	0	2	3	5	7	8	10					
-	7^7	0	1	3	5	6	8	10					
whole-tone	**6**	0	2	4	6	8	10						
pentatonic	5^1	0	2	4	7	9							
-	5^2	0	2	5	7	10							
-	5^3	0	3	5	8	10							
-	5^4	0	2	5	7	9							
-	5^5	0	3	5	7	10							

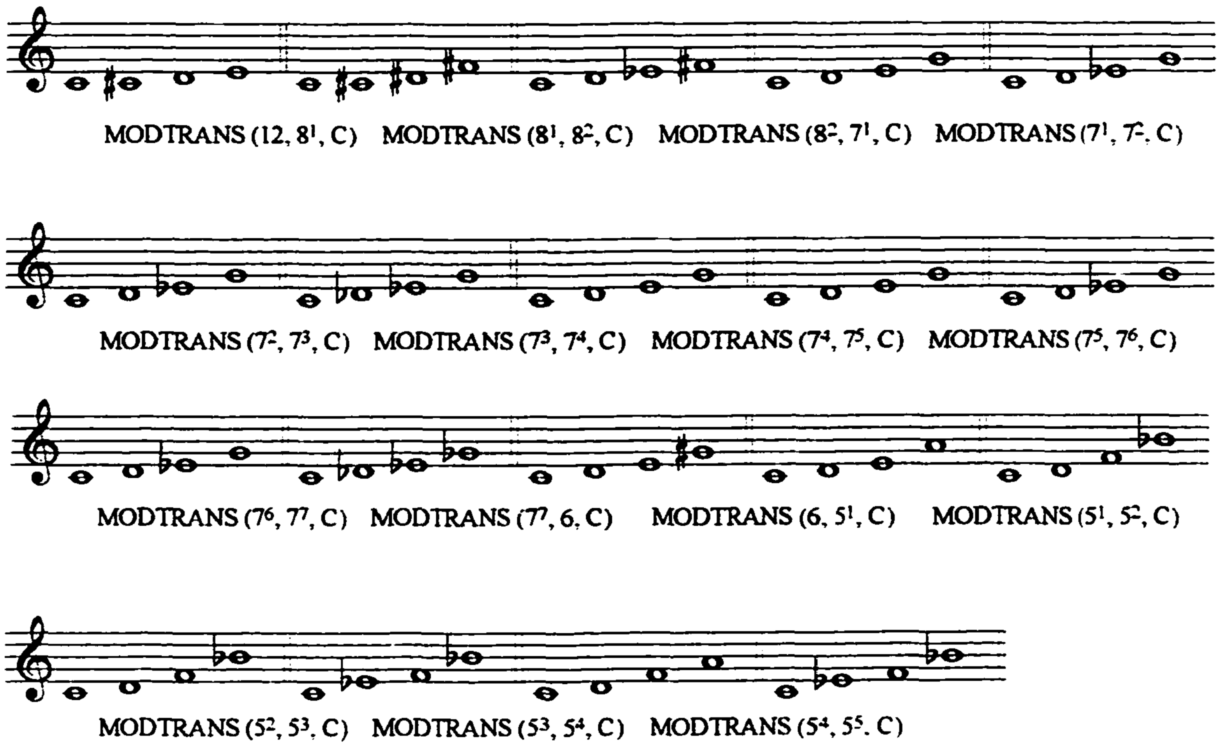
Example 5.5. Bartók, String Quartet No. 4, I; MODTRANS operation connecting mod12 and mod8 spaces.



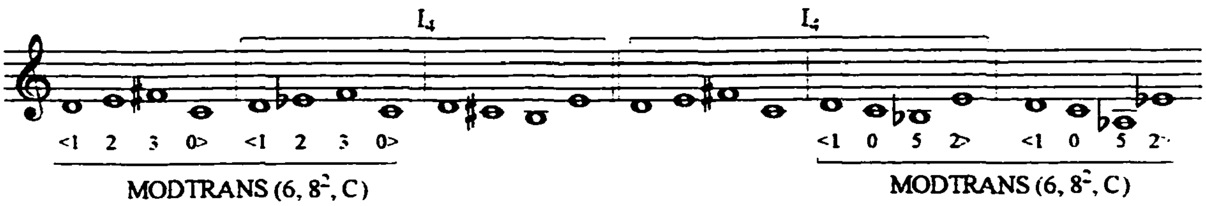
MODTRANS (12. 8[1]. B♭)

T_{-4}

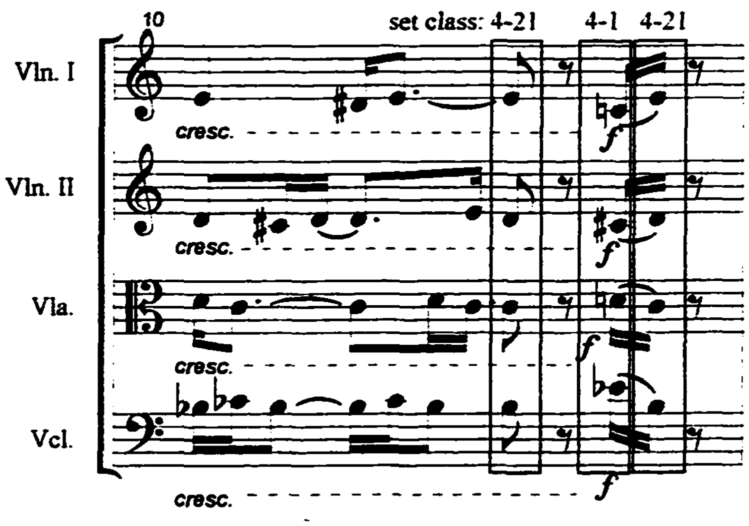
Example 5.6. Successive MODTRANS operations applied to the step-class segment <0124>.



Example 5.7. MODTRANS followed by inversion vs. inversion followed by MODTRANS.



Example 5.8a. Bartók, Fourth String Quartet, I, m. 10.



Example 5.8b. Underlying voice-leading of Bartók, Fourth String Quartet, I, m. 10.

set class:	4-21		4-1		4-21
Vl. I	E →	(C) →	C →	(C) →	E
Vl. II	D →	(D) →	C♯ →	(D) →	D
Vla.	C →	(E) →	D →	(E) →	C
Vcl.	B♭ →	(F♯) →	E♭ →	(F♯) →	B♭

L_4

MODTRANS (6, 12, C)

MODTRANS (12, 6, C)

L_4

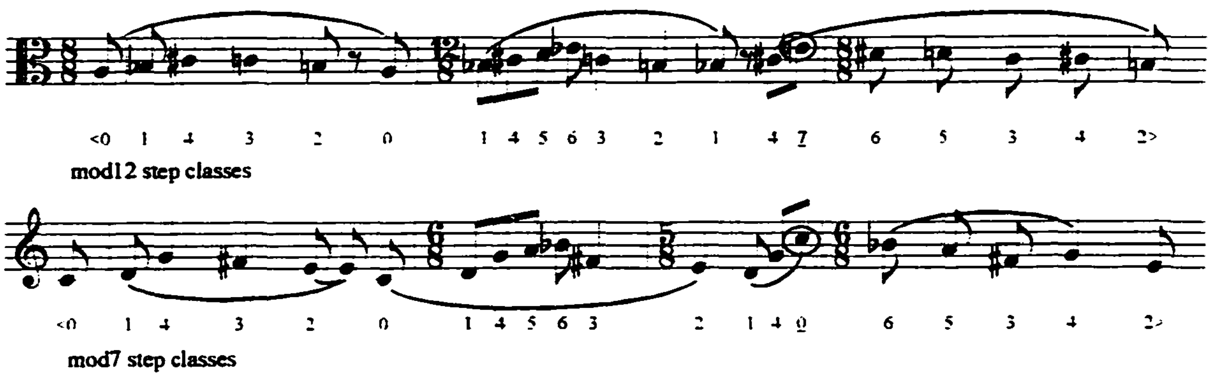
Example 5.9. Multiple MODTRANS interpretations of a single transformation.

MODTRANS (6, 12, C)
or
MODTRANS (7^4, 12, C)

Table 5.2. Number of MODTRANS mappings connecting any two trichords, s and t.

s \ t	(012)	(013)	(014)	(015)	(016)	(024)	(025)	(026)	(027)	(036)	(037)	(048)
(012)	-	6	0	0	0	5	4	0	0	0	0	0
(013)	6	-	2	4	0	34	30	4	6	0	4	0
(014)	0	2	-	4	6	2	18	8	16	0	4	0
(015)	0	4	4	-	2	2	28	12	12	0	8	0
(016)	0	8	8	0	-	9	2	12	30	0	12	0
(024)	4	24	0	0	0	-	16		3	3	8	1
(025)	0	10	10	20	0	0	-	22	24	2	32	0
(026)	0	4	6	12	6	2	28	-	16	4	12	0
(027)	0	0	6	0	15	6	1	10	-	0	24	0
(036)	0	0	0	0	2	3	0	0	14	-	26	3
(037)	0	0	0	0	2	6	2	4	18	20	-	6
(048)	0	0	0	0	0	1	0	0	3	3	8	-

Example 5.10. Bartók, *Music for Strings, Percussion, and Celesta*, themes from mvts. I and IV.



mvt.	modulus	Step Classes 0	1	2	3	4	5	6	7	8	9	10	11
I	chromatic	0	1	2	3	4	5	6	7	8	9	10	11
IV	diatonic	0	2	4	6	7	9	10	0	2	4	6	7

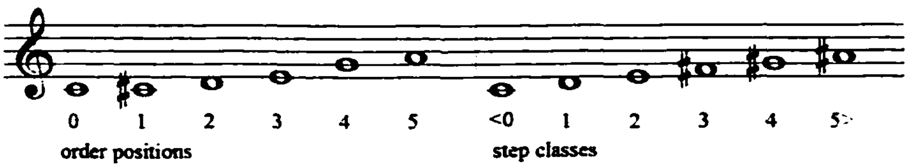
N. B. This modular space is not one of the seven diatonic modes included in Table 1, but is the fourth mode of the ascending melodic minor scale, also commonly known as the "acoustic scale" or "overtone scale" (see note 3).

Example 5.11. Step-class segment <712409> transformed by MODCOMP and MODWRAP.

a. <G C♯ D E C A> transformed by MODCOMP (12, 6, C).

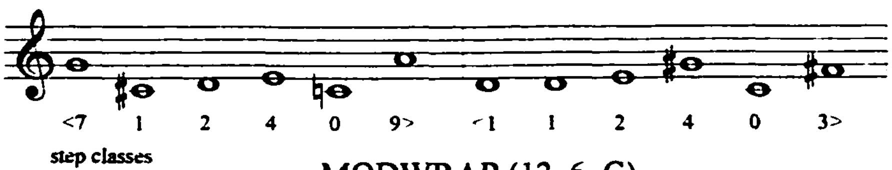
MODCOMP (12, 6, C)

rewritten in ascending order:



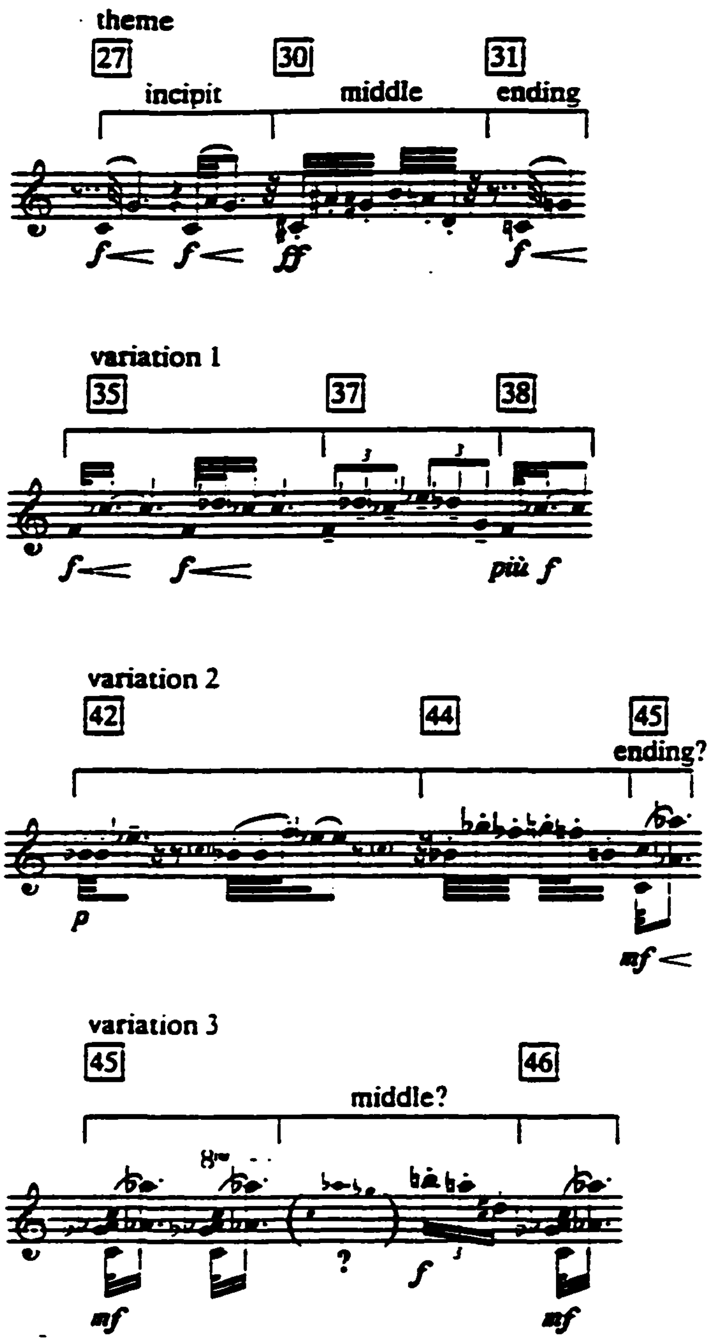
MODCOMP (12, 6, C)

b. <G C♯ D E C A> transformed by MODWRAP (12, 6, C).



MODWRAP (12, 6, C)
(7 - 6 = 1)
(9 - 6 = 3)

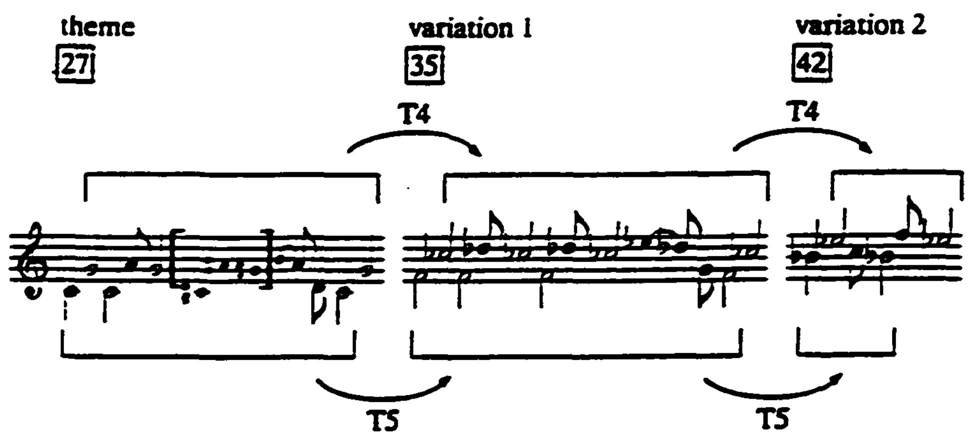
Example 5.12. Reproduction of Example 4.14 from David Lewin, *Musical Form and Transformation: 4 Analytic Essays* (New Haven: Yale Univesity Press, 1993), 122. Copyright © 1993 by Yale University. All Rights Reserved. Used by Permission.



Example 4.14. The melodies of the theme and of three variations parse into incipit, middle, and ending sections.

Example 5.13. Reproduction of Example 4.15 from David Lewin, *Musical Form and Transformation: 4 Analytic Essays* (New Haven: Yale Univesity Press, 1993), 124.

T5(WT) above (notes with stems up);
the major seconds with stems up rise by successive T4s
from the theme to variation 1 to variation 2

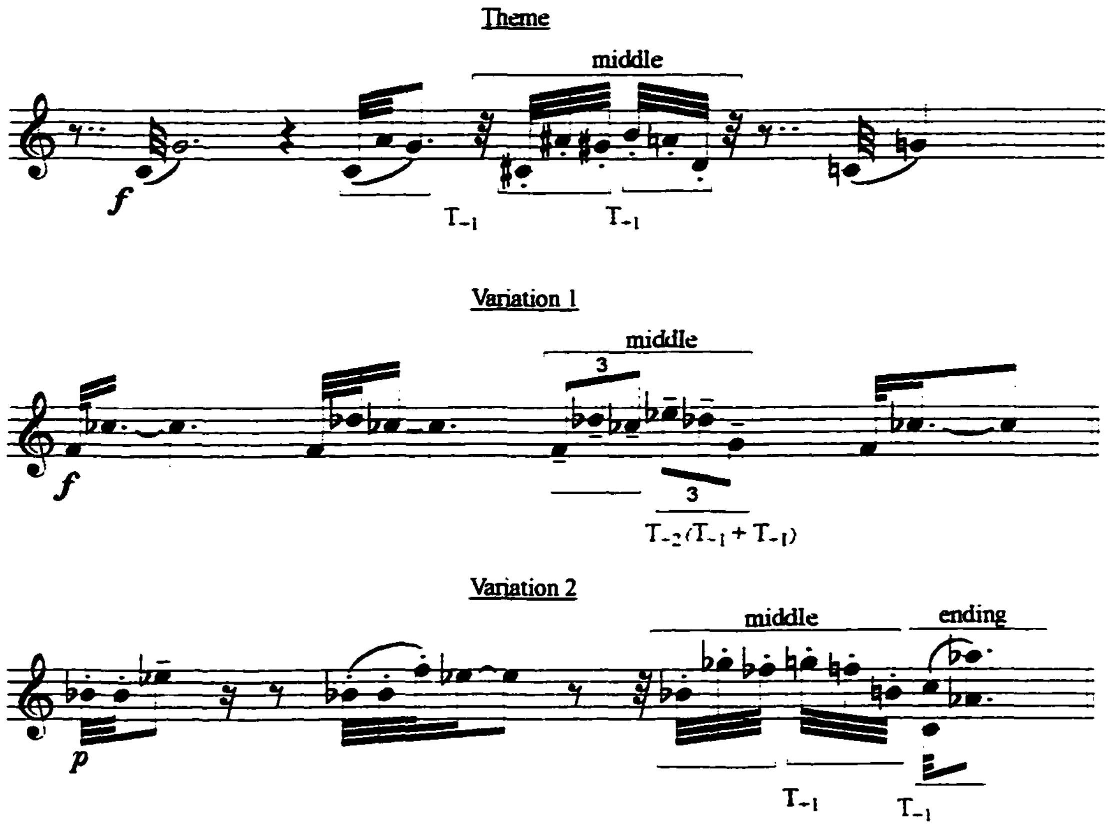


T5(PENT) below (notes with stems down);
the major seconds with stems down rise by successive T5s
from the theme to variation 1 to variation 2

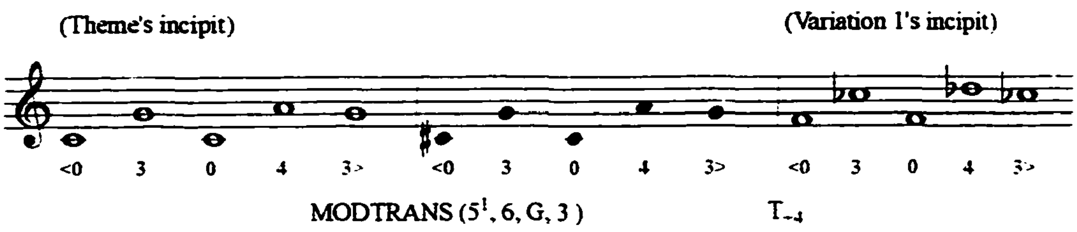
Example 4.15. Pentatonic and whole-tone structures in the melodic line of mm. 27–43.

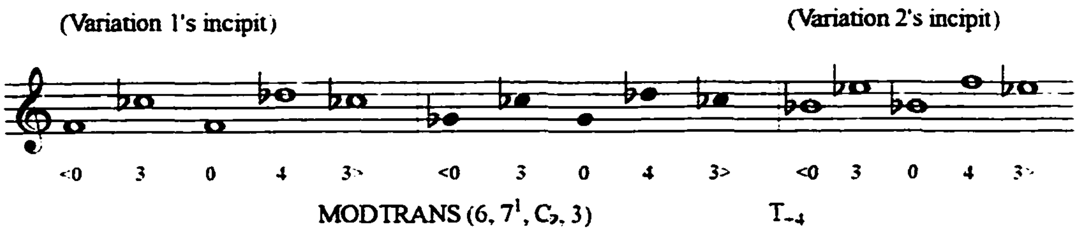
Example 5.14. MODTRANS mappings in Debussy's *Feux d'artifice*.

Example 5.15. The T_{-1}/T_{-1} pattern as generator of the middle portions of the theme and its variations.



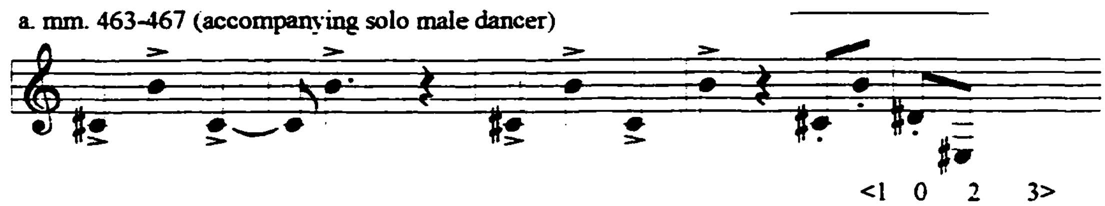
Example 5.16. MODTRANS mappings in Debussy's *Feux d'artifice* with step-class 3 as the point of synchronization.





Example 5.17. Stravinsky, *Agon*: transformation from whole-tone to octatonic in mm. 463-483.

a. mm. 463-467 (accompanying solo male dancer)



b. mm. 473-476 (accompanying solo female dancer)

c.

MODTRANS (6, 8[1], F, 3)

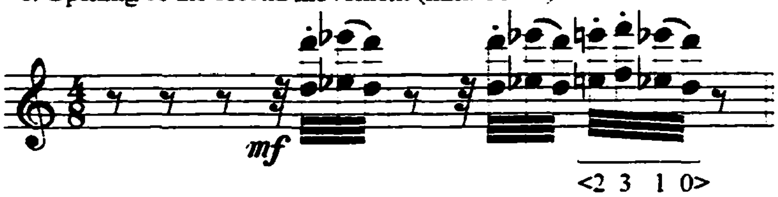
Example 5.18. Stravinsky, *Agon*, mm. 418-427; interaction between the octatonic step-class segment <2310> and its inversion <1023>.

Example 5.19. MODTRANS relationships between the first two movements of *Agon*.

a. First theme of the first movement (mm. 1-5)

b. Second theme (mm. 10-13)

c. Opening of the second movement (mm. 61-62)



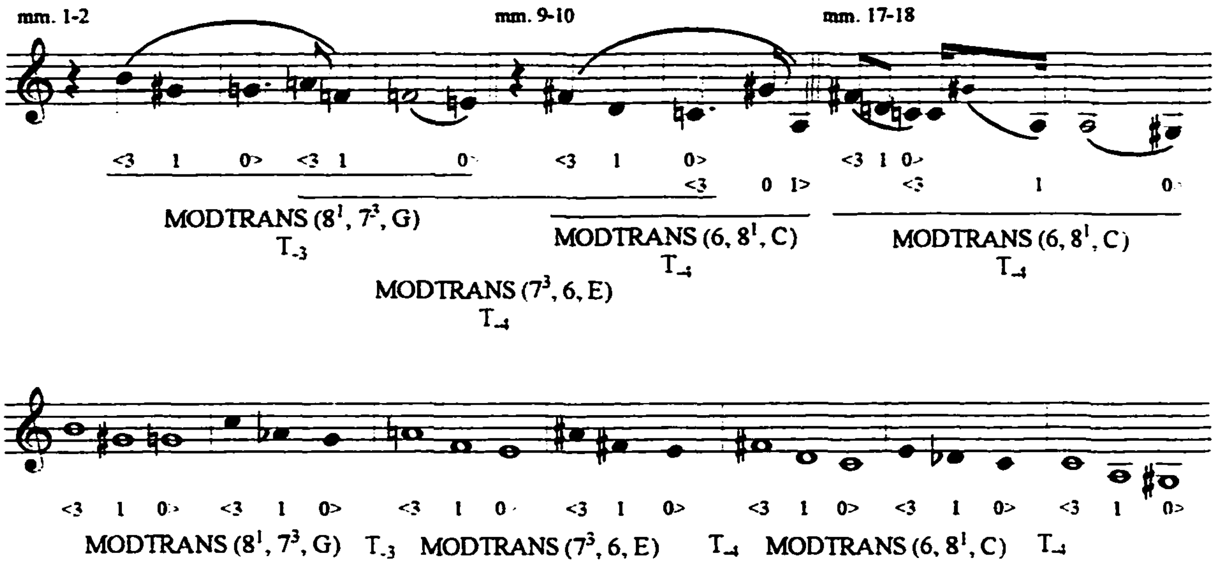
d. MODTRANS operation linking the openings of the first and second movements.

MODTRANS (7, 12, A) T_5

e. MODTRANS operation linking the second theme of the first movement to the opening of the second.

MODTRANS (7, 12, B) T_3

Example 5.20. MODTRANS mappings in Schoenberg's Op. 11, No. 1.



Example 5.21. Transformational path of (0148) in Schoenberg's Op. 11, No. 1.

Example 5.22. Motivic use of (0148) in Schoenberg's Op. 11, No. 1.

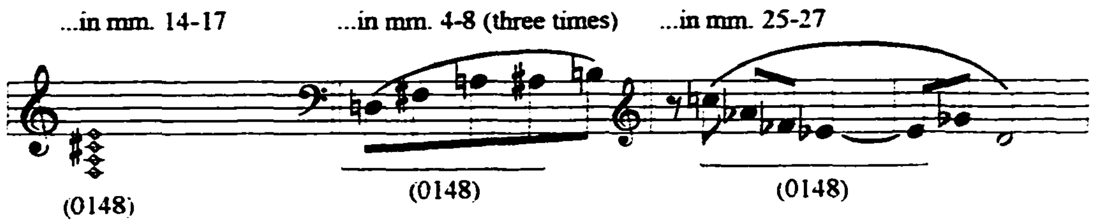


Table 5.3. Set classes represented by each of the M-types of cardinalities 3-6.

M-type	mod12	mod8	mod7	mod6	mod5
(012)	(012)	(013)	(013), (024)	(024)	(024), (025)
(013)	(013)	(014), (025)	(015), (025), (026)	(026)	(027), (037)
(014)	(014)	(016), (026)	(016), (027)	(026)	(024), (025)
(015)	(015)	(016), (026)	(015), (025), (026)	(024)	none
(016)	(016)	(014), (025)	(013), (024)	none	none
(024)	(024)	(036)	(036), (037)	(048)	(027), (037)
(025)	(025)	(037)	(036), (037)	(026)	none
(026)	(026)	(036)	(015), (025), (026)	none	none
(027)	(027)	(014), (025)	none	none	none
(036)	(036)	(037)	(016), (027)	none	none
(037)	(037)	(016), (026)	none	none	none
(048)	(048)	none	none	none	none
(0123)	(0123)	(0134), (0235)	(0135), (0235), (0246)	(0246)	(0247), (0257), (0358)
(0124)	(0124)	(0136), (0236)	(0136), (0137), (0237), (0247)	(0248)	(0247), (0257), (0358)
(0125)	(0125)	(0137)	(0136), (0137), (0237), (0247)	(0246)	none
(0126)	(0126)	(0136), (0236)	(0135), (0235), (0246)	none	none
(0127)	(0127)	(0134), (0235)	none	none	none
(0134)	(0134)	(0146)	(0156), (0157), (0257)	(0268)	(0247), (0257), (0358)
(0135)	(0135)	(0147), (0258)	(0158), (0258), (0358)	(0248)	none
(0136)	(0136)	(0347), (0358)	(0136), (0137), (0237), (0247)	none	none
(0137)	(0137)	(0136), (0236)	none	none	none
(0145)	(0145)	(0167), (0268)	(0156), (0157), (0257)	(0123)	none
(0146)	(0146)	(0147), (0258)	(0136), (0137), (0237), (0247)	none	none
(0147)	(0147)	(0137)	none	none	none
(0148)	(0148)	none	none	none	none
(0156)	(0156)	(0146)	(0135), (0235), (0246)	none	none
(0157)	(0157)	(0136), (0236)	none	none	none

Table 5.3 continued

M-type	mod12	mod8	mod7	mod6	mod5
(0158)	(0158)	none	none	none	none
(0167)	(0167)	(0123)	none	none	none
(0235)	(0235)	(0347), (0358)	(0158), (0258), (0358)	(0268)	none
(0236)	(0236)	(0147), (0258)	(0156), (0157), (0257)	none	none
(0237)	(0237)	(0146)	none	none	none
(0246)	(0246)	(0369)	(0158), (0258), (0358)	none	none
(0247)	(0247)	(0347), (0358)	none	none	none
(0248)	(0248)	none	none	none	none
(0257)	(0257)	(0347), (0358)	none	none	none
(0258)	(0258)	none	none	none	none
(0268)	(0268)	none	none	none	none
(0347)	(0347)	(0167), (0268)	none	none	none
(0358)	(0358)	none	none	none	none
(0369)	(0369)	none	none	none	none
(01234)	(01234)	(01346)	(01356), (01357), (02357)	(02468)	(02479)
(01235)	(01235)	(01347), (02358)	(01358), (02358), (02469)	(02468)	none
(01236)	(01236)	(01347), (02358)	(01356), (01357), (02357)	none	none
(01237)	(01237)	(01346)	none	none	none
(01245)	(01245)	(01367), (02368)	(01368), (01378), (02479)	none	none
(01246)	(01246)	(01369)	(01358), (02358), (02469)	none	none
(01247)	(01247)	(01347), (02358)	none	none	none
(01248)	(01248)	none	none	none	none
(01256)	(01256)	(01367), (02368)	(01356), (01357), (02357)	none	none
(01257)	(01257)	(01347), (02358)	none	none	none
(01258)	(01258)	none	none	none	none
(01267)	(01267)	(01346)	none	none	none
(01268)	(01268)	none	none	none	none

Table 5.3 continued

M-type	mod12	mod8	mod7	mod6	mod5
(01346)	(01346)	(01469)	(01368), (01378), (02479)	none	none
(01347)	(01347)	(01367), (02368)	none	none	none
(01348)	(01348)	none	none	none	none
(01356)	(01356)	(01469)	(01358), (02358), (02469)	none	none
(01357)	(01357)	(01369)	none	none	none
(01358)	(01358)	none	none	none	none
(01367)	(01367)	(01347), (02358)	none	none	none
(01368)	(01368)	none	none	none	none
(01369)	(01369)	none	none	none	none
(01457)	(01457)	(01367), (02368)	none	none	none
(01458)	(01458)	none	none	none	none
(01468)	(01468)	none	none	none	none
(01469)	(01469)	none	none	none	none
(01478)	(01478)	none	none	none	none
(01568)	(01568)	none	none	none	none
(02346)	(02346)	(01369)	(01368), (01378), (02479)	none	none
(02347)	(02347)	(01367), (02368)	none	none	none
(02357)	(02357)	(01469)	none	none	none
(02358)	(02358)	none	none	none	none
(02368)	(02368)	none	none	none	none
(02458)	(02458)	none	none	none	none
(02468)	(02468)	none	none	none	none
(02469)	(02469)	none	none	none	none
(02479)	(02479)	none	none	none	none
(03458)	(03458)	none	none	none	none
(012345)	(012345)	(013467), (023568)	(013568), (013578), (023579), (024579)	(02468T)	none
(012346)	(012346)	(013469), (023569)	(013568), (013578), (023579), (024579)	none	none

Table 5.3 continued

M-type	mod12	mod8	mod7	mod6	mod5
012347)	(012347)	(013467), (023568)	none	none	none
(012348)	(012348)	none	none	none	none
(012356)	(012356)	(013479), (014679)	(013568), (013578), (023579), (024579)	none	none
(012357)	(012357)	(013469), (023569)	none	none	none
(012358)	(012358)	none	none	none	none
(012367)	(012367)	(013467), (023568)	none	none	none
(012368)	(012368)	none	none	none	none
(012369)	(012369)	none	none	none	none
(012378)	(012378)	none	none	none	none
(012456)	(012456)	(013679)	(013568), (013578), (023579), (024579)	none	none
(012457)	(012457)	(013479), (014679)	none	none	none
(012346)	(012346)	(013469), (023569)	(013568), (013578), (023579), (024579)	none	none
(012347)	(012347)	(013467), (023568)	none	none	none
(012348)	(012348)	none	none	none	none
(012356)	(012356)	(013479), (014679)	(013568), (013578), (023579), (024579)	none	none
(012357)	(012357)	(013469), (023569)	none	none	none
(012358)	(012358)	none	none	none	none
(012367)	(012367)	(013467), (023568)	none	none	none
(012368)	(012368)	none	none	none	none
(012369)	(012369)	none	none	none	none
(012378)	(012378)	none	none	none	none
(012456)	(012456)	(013679)	(013568), (013578), (023579), (024579)	none	none
(012457)	(012457)	(013479), (014679)	none	none	none
(012458)	(012458)	none	none	none	none
(012467)	(012467)	(013469), (023569)	none	none	none
(012468)	(012468)	none	none	none	none
(012469)	(012469)	none	none	none	none

Table 5.3 continued

M-type	mod12	mod8	mod7	mod6	mod5
(012478)	(012478)	none	none	none	none
(012479)	(012479)	none	none	none	none
(012567)	(012567)	(013467), (023568)	none	none	none
(012568)	(012568)	none	none	none	none
(012569)	(012569)	none	none	none	none
(012578)	(012578)	none	none	none	none
(012579)	(012579)	none	none	none	none
(012678)	(012678)	none	none	none	none
(013457)	(013457)	(013679)	none	none	none
(013458)	(013458)	none	none	none	none
(013467)	(013467)	(013479), (014679)	none	none	none
(013468)	(013468)	none	none	none	none
(013469)	(013469)	none	none	none	none
(013478)	(013478)	none	none	none	none
(013479)	(013479)	none	none	none	none
(013568)	(013568)	none	none	none	none
(013569)	(013569)	none	none	none	none
(013578)	(013578)	none	none	none	none
(013579)	(013579)	none	none	none	none
(013679)	(013679)	none	none	none	none
(014568)	(014568)	none	none	none	none
(014579)	(014579)	none	none	none	none
(014589)	(014589)	none	none	none	none
(014679)	(014679)	none	none	none	none
(023457)	(023457)	(013479), (014679)	none	none	none
(023458)	(023458)	none	none	none	none
(023468)	(023468)	none	none	none	none

Table 5.3 continued

M-type	mod12	mod8	mod7	mod6	mod5
(023469)	(023469)	none	none	none	none
(023568)	(023568)	none	none	none	none
(023579)	(023579)	none	none	none	none
(024579)	(024579)	none	none	none	none
(02468T)	(02468T)	none	none	none	none

Example 5.23. Annotated score of Webern, Op. 5, No. 3 highlighting set class members of the M-type (013):(013), (014), (015), and (026).

Webern 5 PIECES FOR STRING QUARTET, OP. 5
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Example 5.23 continued

Webern 5 PIECES FOR STRING QUARTET, OP. 5
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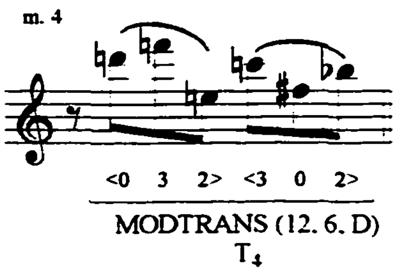
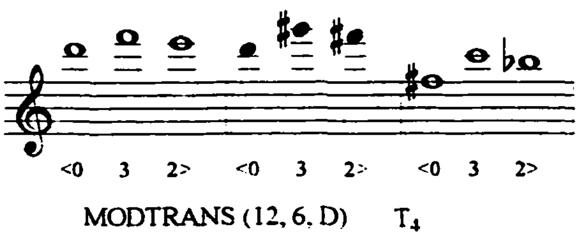
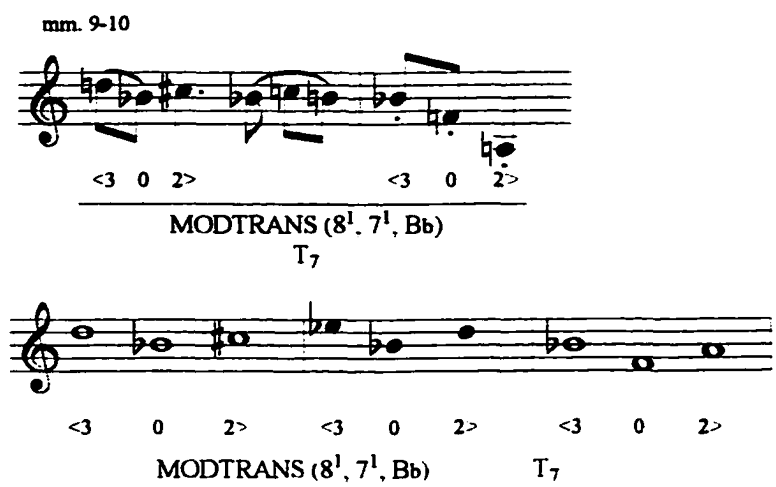
Example 5.24. Webern, Op. 5, No. 3, first theme; transformation from chromatic to whole-tone.

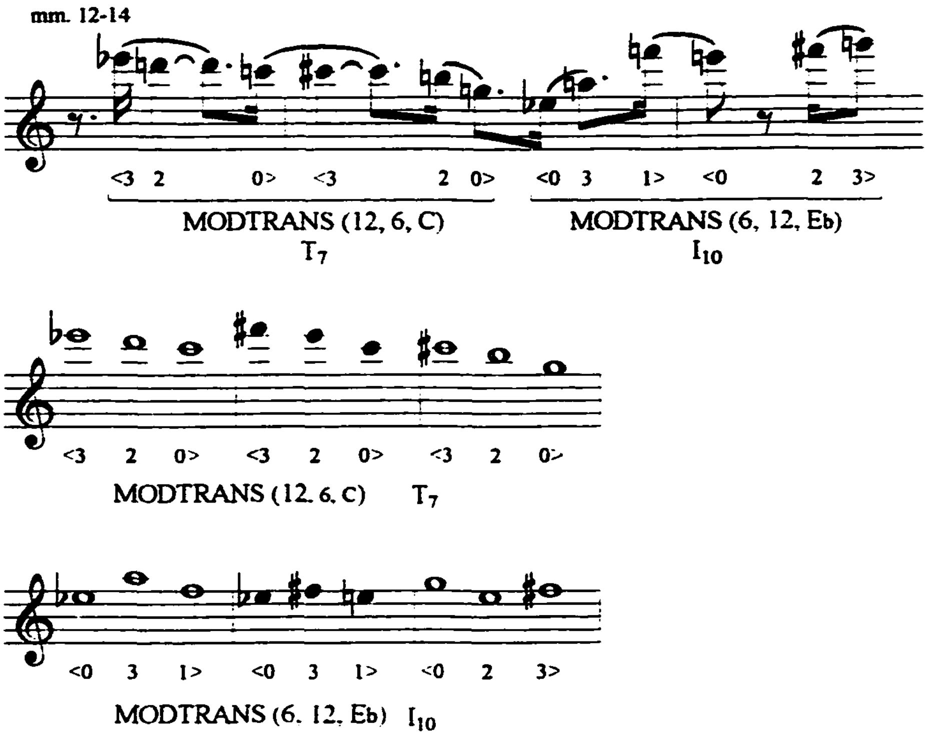




Example 5.25. Webern, Op. 5, No. 3, second theme; transformation from octatonic to diatonic.



Example 5.26. Webern. Op. 5, No. 3, third theme; transformations from chromatic to whole-tone, and from whole-tone to chromatic.



Example 5.27. Stravinsky, *Concerto in D*, analysis of mm. 1-57.

Example 5.27 continued (mm. 19-32)

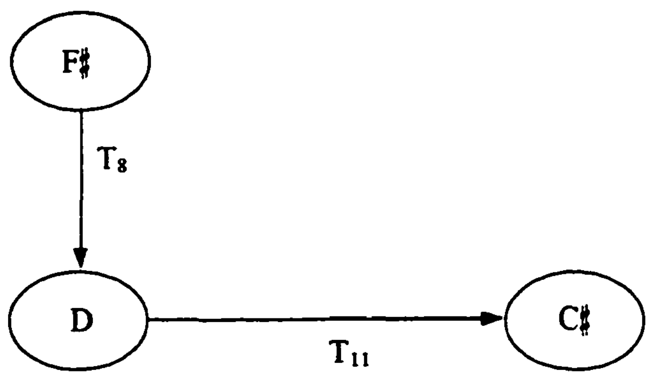
Example 5.27 continued (mm. 33-45)

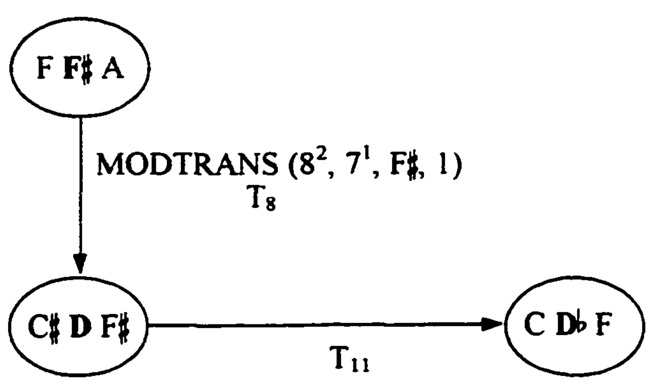
Example 5.27 continued (mm. 46-57)

Table 5.4. Bird's eye view of *Concerto in D*, I.

section	measures	pc set	set class	M-type	modulus
intro	1-24	{F F♯ A}	(014)	(013)	8
A	25-57	{C♯ D F♯}	(015)	(013)	7
B	58-61	{A B♭ C E}	(0137)	(0124)	7
A	90-129	{C♯ D F♯ G}	(0156)	(0134)	7
C	130-177	{C D♭ F A♭}	(0158)	(0135)	7
D	178-226	{D♭ F G♭ A♭}	(0237)	(0124)	7
A	227-261	{C♯ D F♯}	(015)	(013)	7
C	262-265	{D F♯ A}	(037)	(024)	7
ending	271-282	{D F F♯ A B}	(02569)	(01346)	8

Example 5.28. Relationship between the intervallic structure of {C♯ D F♯} and the transformational path of the entire first movement.





Example 5.29. Analysis of MODTRANS mapping connecting {F F♯ A} and {C♯ D F♯}.

step class	set class (014)	(015)	(015)
3	A →	(A♯) →	F♯
1	F♯ →	(F♯) →	D
0	F →	(E♯) →	C♯

MODTRANS (8^2, 7^1, F♯, 1)
and T_8

modulus	label	Step Classes 0	1	2	3	4	5	6	7
octatonic	8^2	11	0	2	3	5	6	8	9
diatonic	7^1	11	0	2	4	5	7	9	

IMAGE EVALUATION TEST TARGET (QA-3)

150mm

6"

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