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SOBOTKIN, Fred Miles, 1942-
CONTROLLER DESIGN FOR MODELING PROBLEMS WITH
APPLICATIONS TO A CLASS OF PHYSIOLOGICAL SYSTEMS.

The City University of New York, Ph.D., 1973
Engineering, electrical

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CONTROLLER DESIGN FOR MODELING PROBLEMS
WITH APPLICATIONS TO A CLASS OF PHYSIOLOGICAL SYSTEMS

by

Fred M. Sobotkin

A dissertation submitted to the Graduate
Faculty in Engineering in partial fulfillment of
the requirements for the degree of Doctor of Philosophy,
The City University of New York

1973

This manuscript has been read and accepted for the Graduate Faculty in Engineering in satisfaction of the dissertation requirement for the degree of Doctor of Philosophy.

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Abstract

CONTROLLER DESIGN FOR MODELING PROBLEMS WITH APPLICATIONS TO A CLASS OF PHYSIOLOGICAL SYSTEMS

by

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This dissertation presents an adaptive modeling technique which includes the design of a parameter controller whose control signals are utilized to identify and control a physiological system's mathematical model. Although the parameter controller can be employed as an aid toward the modeling of a variety of control systems, specific emphasis is placed on physiological control system modeling problems because the technique is well suited to some of the inherent properties of such control systems and overcomes many of the limitations encountered in the physiological control system modeling process.

The technique is based upon a model-reference system configuration and a Liapunov function formulated for this purpose. The reference system represents the actual physiological system by the stimulus-response data that is assumed to be available from experimentation. The postulated model system is constructed initially as an approximate mathematical representation of the reference system and is derived from analysis of the physiological process measurements and by study of anatomical considerations.

An important feature of the parameter controller is its "learning" capability. This feature provides a means for determining when a set of model parameters has been identified. The "learning" capability of the parameter controller has another important feature. It permits one to establish a correspondence between parameter value changes in the model and disorders or malfunctions in the system that is being modelled. If a change occurs in the system that is being modelled, the model parameters begin to fluctuate again and a new "learning" interval is initiated. The fluctuations cease when the model assumes new parameter values that correspond to the changed conditions of the reference system. This "learning" property offers a theoretically consistent approach for identifying a model's parameters from experimental data of the system being modelled. Application of the parameter controller during investigations of aberrant physiological system behavior could enhance the capability to develop automated diagnostic procedures with the aid of computers.

Several artificial modeling situations are presented in order to illustrate the design of the parameter control signals and their "learning" capabilities. For each example, some or all of the initial values assumed for the model parameters are purposely chosen to be different in value than the known reference system parameters. Using the model-reference system configuration and the parameter control signals, the model's parameters are automatically updated to assume the values of the reference system parameters. In this sense, the model is considered to have "learned" the reference system.

The modeling technique is further demonstrated by its application to the modeling of an actual physiological system, namely the human eye movement mechanism (saccadic eye movements). For this case, the parameter control signals are designed to be applied to Robinson's model of the human saccadic eye movement mechanism. Analysis of the model with its updated or "learned" parameter values revealed certain modifications in Robinson's model. The modified model then yields a saccadic response that is more representative of the measurements for a saccadic response that is obtained from eye movement experiments.

Acknowledgements

I wish to express my extreme gratitude to my mentor, Professor Ralph Mekel, for his guidance and encouragement throughout the course of this research. Our many hours of technical discussions and his numerous valuable suggestions are most appreciated.

I would like to thank all the members of my Doctoral guidance and examining committee, Professors Egon Brenner, Mansour Javid, Louis Weinberg, Hiram Hart (Physics Department), Sidney Diamond (Mount Sinai School of Medicine/CUNY) and William Blesser (Polytechnic Institute of Brooklyn), for the time and effort each has taken to read this dissertation and for their constructive criticism.

I am especially indebted to Professor Egon Brenner, Dean of the School of Engineering, for his encouragement and helpful suggestions.

I wish to particularly express my gratitude to Professor Sidney Diamond of the Mount Sinai School of Medicine for his invaluable assistance and continued interest in the preparation of this dissertation. To Dr. Adam Atkins of the Mount Sinai School of Medicine, I would like to express my sincerest thanks for our informative discussions about physiological systems and the eye movement mechanism. Thanks are also due to Professor William Blesser of the Polytechnic Institute of Brooklyn for his suggested corrections of this manuscript.

The excellent typing of this dissertation was performed by Miss Sadie Silverstein whose efforts are deeply appreciated.

Finally, I would like to acknowledge the partial support given to me by the National Aeronautics and Space Administration to carry out this research under Grant No. NGR-33-013-053.

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CHAPTER 1

INTRODUCTION

1.1 Introduction

The objective of this research effort is to develop a design method for a parameter controller to update the postulated parameters of a class of physiological control system models. This class includes those physiological systems that can be described by continuous differential equations which possess coefficients that do not fluctuate continuously with time[†].

Although this controller design can be utilized as an aid toward the modeling of a variety of control systems, specific emphasis is placed on physiological control systems because the technique is well suited to some of the inherent properties of such control systems. Furthermore, the technique overcomes many of the input restrictions encountered in the physiological control system modeling process.

The design method is mainly intended to improve upon existing models which have been formulated in previous physiological system studies, by updating their assumed parameter values to more accurately model the experimental data obtained from the actual physiological

[†] Under certain circumstances, the requirement that the coefficients do not fluctuate continuously with time can be relaxed. This is explained in Section 3.2.1.

system. After the initial model structure is determined, the model, the designed controller and the physiological system under consideration are arranged in a model-reference system configuration. Under the influence of the controller, which is designed by utilizing Liapunov's stability theory, the model's postulated parameters are continuously updated to cause the model-reference system error to approach zero.

Two types of controllers are developed. The first type requires that the model-reference system error must be always available. In the second type, the designed controller possesses "learning" capabilities in the sense that the model "learns" a set of parameter values which yields a model output response that is equivalent to the reference system response. After the "learning" process, the model can be removed from the model-reference system configuration and the model will continue to produce an equivalent reference system response provided that no further parameter changes occur within the actual reference system.

If the initial structure of the model is not truly representative of the actual physiological system under consideration, analysis of the model with the "learned" parameter values suggest some modifications and structural changes that may be needed for the model. These modifications could then lead to research studies that more clearly delineate the types of experimental data required to obtain a better understanding of the physiological system's organization and operation.

1.1.1 Possible Future Application

After establishing a model that is representative of the "normal" physiological system, the model and controller could also be used to investigate abnormal system behavior due to unusual physiological conditions and neurological disorders. By using an abnormal physiological system for a reference, the model's parameters are updated by the controller until the model's response conforms to the abnormal reference system response. The "learned" parameter values corresponding to the aberrant behavior can then be studied and an attempt may be made to establish a correspondence between certain human disorders and parameter variations or malfunctions of components in the model. Therefore the developed controller design method may be considered to be a useful tool in physiological system studies.

In the following section, the contents and contributions of each of the succeeding chapters are outlined.

1.2 Contents of Dissertation

Chapter 2 presents the historical background and the motivation behind the design method. A detailed statement of the design problem is given in Section 3.2, and the remainder of Chapter 3 is devoted to the model-reference formulation for the linear modeling problem. The formulation for a class of non-linear modeling problems is contained in Chapter 4.

Liapunov's stability theory[†] is utilized in Chapter 5 to formulate the design method. Section 5.3 contains the development of a controller without memory capabilities. Parameter control signals that possess memory ("learning") capabilities are developed in Section 5.4.

In Chapter 6, several examples representing artificial modeling situations are presented in order to illustrate the design of the model parameter control signals and their "learning" capabilities. The design method is then applied in Chapter 7 to the modeling of an actual physiological system, namely the human eye-movement mechanism (saccadic eye movements). It should be noted that Chapter 7 is not intended to be a comprehensive study of the saccadic eye movement system, but instead is mainly intended to demonstrate the procedure for applying the design method to an actual physiological system and also indicate some of the results which may be obtained. Furthermore, in a comprehensive study, actual measurements would normally be extracted from the physiological system under consideration to be used as the reference system response. In Chapter 7, the reference system response was obtained from the research studies of Cook and Stark.

The historical background and motivation for the design method are presented in the following chapter. My indebtedness to the work of many scholars in this area is indicated throughout the dissertation and in the Bibliography.

[†] Liapunov's stability theorems are given in Appendix A.

CHAPTER 2

HISTORICAL BACKGROUND AND MOTIVATION

2.1 Introduction

Adaptive control systems represent an effort to extend the operating range of conventional control systems exposed to extraordinarily broad variations of environment or parameter values. If these variations are severe, it becomes necessary to alter certain system parameters to produce a satisfactory response over the entire range of operation. Furthermore, an adaptive system also provides a means of continuously monitoring its own performance in relation to a given figure of merit or optimum condition and thereby modify its own parameters by closed loop action so as to approach this optimum. The original work in adaptive control was prompted by evolution of high performance aircraft and problems associated with their guidance and control through an ever expanding performance envelope^[25].

One of the most popular types of systems used since the conception of the adaptive control field, particularly for practical applications to devices which require rapid adaptation (i.e. autopilots), has been **MODEL-REFERENCE SYSTEMS**.

The conventional model-reference adaptive control system configuration is illustrated in Fig.2-1. In this configuration, the model, which serves as a reference, possesses the desired input-output relationship

required for the given system under investigation. A control law, represented by the controller is then designed to modify the input signal to the given system by means of an input-filter and/or adjust the given system's parameters (usually gains) to compensate for variations present in the system and thereby minimize some index of performance (I.P.). The index of performance may be a function of the model-reference error $\underline{e}$, the input $r(t)$ and the system output $\underline{C}_s$. Usually it is desired to produce a zero model-reference error and have the given system output $\underline{C}_s$ follow the reference or model output $\underline{C}_d$.

The use of model-reference adaptive control systems as aids for modeling physiological systems has not been fully explored. If the components of the configuration Fig. 2-1 are rearranged as indicated in Fig. 2-2, a modeling technique can be developed whereby the controller now generates signals which modify the model's postulated parameters to cause the model-reference error to approach zero. For this configuration, the given system to be modelled serves as a reference and the objective of this research effort is to design controllers which modify the postulated parameters of existing models which have been formulated by other researchers. The objective of the design method is accomplished by utilizing Liapunov's stability theory, and, under the influence of the controller, the model's parameters are updated to cause the model-reference error ($\underline{e}$) to approach zero.

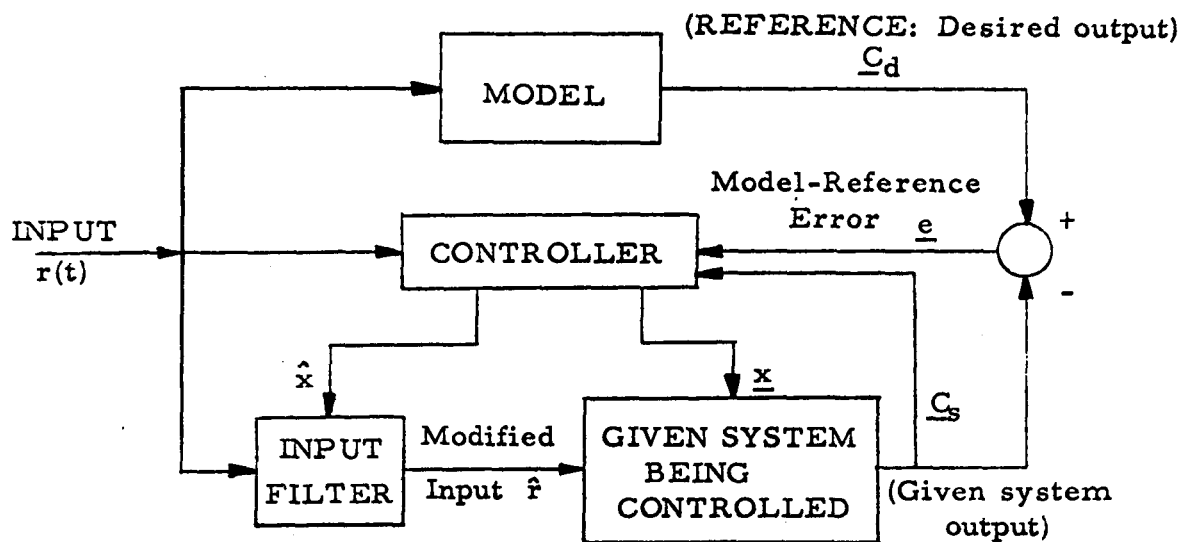


Figure 2-1 Conventional model-reference system configuration

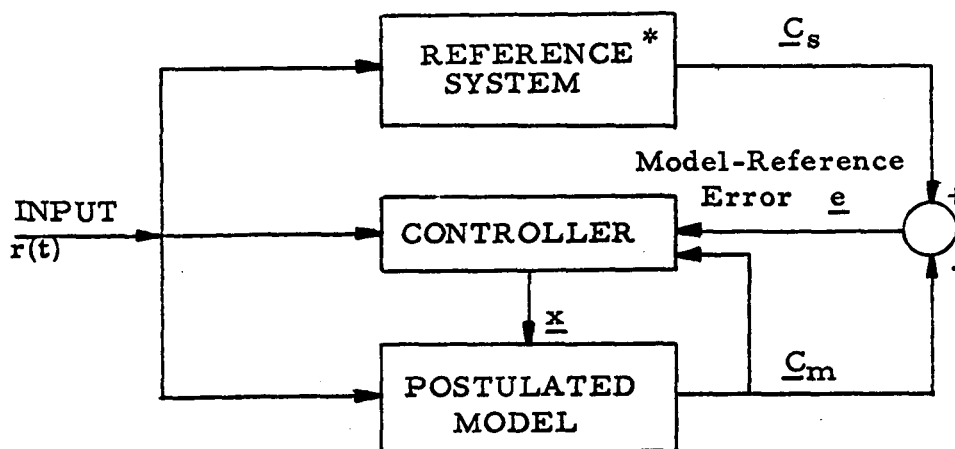


Figure 2-2 Model-reference system configuration as a modeling aid.

* The reference system represents the experimental stimulus-response data obtained from the physiological system being modelled.

The following section presents a survey of Model-Reference[†] adaptive control system research.

2.2 Survey of Literature

The concept of Model-Reference Adaptive Control Systems evolved from the work done by Whitaker et al^[83]. They considered an aircraft adaptive control system using a model-reference configuration with three adjustable system parameters and three indices of performances (IP's), one for each parameter. A major part of the research effort was needed to define the IP's leading to the adaptive control law of the three variable parameters which yield minimum interaction between the control loops. Integrals of the difference between the outputs of the model system and the actual system over a time interval chosen to produce minimum interaction between the adaptive loops for step inputs were the forms of the IP's used. Whenever the aircraft environment changes requiring corrective action, the pilot would have to apply a few step inputs into the control system activating a computer. The computer then measures the values for the IP's and feeds corrective signals to parameter or gain adjustment mechanisms. This approach was somewhat impractical because the requirement of pilot actuated "test" signals caused a disruption in the normal performance of the system.

[†] Both Figures 2-1 and 2-2 are referred to as Model-Reference systems. Throughout Section 2.2, the term Model-Reference system refers to Figure 2-1 in which the model generates the desired reference response.

Whitaker et al ^[59] later developed another technique for synthesizing the adaptive loops without requiring "test" signals. This technique is called the "MIT rule" and is based upon the minimization of the integral of the squared model-reference error. The control law obtained from the "MIT rule" leads to an overall system which is not amenable to an analytical determination of system stability and convergence properties. Analysis of quite simple systems show that instability is possible for certain inputs which cast doubts on the stability of more complex systems.

An alternate technique was developed by Bongiorno ^[8] who used a model-reference configuration, similar to Fig. 2-1, to obtain a controller for a given system with a known linear transfer function but with a time varying gain. The reference model had a constant gain and an identical transfer function as the given system. His controller operates on the system input, for which restrictions are placed on zero crossings, and using the model-reference error, counterbalances the time varying gain of the given plant. Although his controller is similar to that obtained by the "MIT rule", he also develops a Nyquist like stability criterion which he utilizes to establish an allowable range for the process gain variations so that convergence within a specified time interval is guaranteed. The bound for the varying parameter range is determined from the real frequency amplitude response of the known system transfer function. Bongiorno ^[7] also considers processes with

known fixed dynamics and with either a variable gain or variable time-constant. With the assumption that the process dynamics are known, he designs a feedback compensation network and a tandem compensation network which nullify the change in the single process parameter under consideration. The model used must be identical to the given system with the varying process parameter replaced by a constant and both convergence-time relationships and sufficient conditions for stability of the model reference system are developed. For his analysis, the varying parameters are assumed to be essentially constants and restrictions are also placed on the time between zero crossings of the input signal.

Behavior of very simple model-reference adaptive control systems with a single variable gain were investigated by Horrocks^[34]. His controller is designed as an approximation to minimize the integral of the squared model-reference error when small perturbations are applied to the system. Some of the assumptions used in his analysis are questionable and stability problems may arise depending upon both the input signal applied and the adaptive loop gains considered.

Studies for systems in which the parameter adjusting signals are derived from estimates of the partial derivatives of the model-reference error with respect to the unknown parameters, with the intention of minimizing the squared error, were conducted by White et al^[21]. Several restrictions had to be placed upon the parameter variations to effect certain calculations and this could greatly limit the scope of system

applications. Approximate differential equations are derived for the parameter adjusting signals and Liapunov's second method is then applied to these equations to deduce stability. In this analysis, stability is shown in only limited cases. Interaction between adaptive loops for different parameters can also cause serious problems by appreciably increasing the effective time constant of the adaptive loops and possibly lead to instability. White^[84] also reanalyzed the design of the system considered by Horrocks^[34] for inputs of sinusoidal and ramp functions. He assumed that the system can best adapt itself to nullify environmental changes if two adaptive parameters are used; one in the feedback loop and one in the forward path. Approximate equations for the system during adaptation are obtained by using a parameter perturbation technique and a method is presented for determining the approximate steady state model-reference error for ramp or sinusoidal inputs. It is also shown that the system under investigation may become unstable at certain frequencies.

A design technique to cancel out the effect of varying parameters in a plant for which the basic differential equations are known was developed by Dressler^[19]. By solving the differential equations of the reference model and the adaptive control system, an expression is obtained showing the explicit functional dependence of the model-reference error on the adaptive parameters. The rate of change of the unknown varying parameters are then assumed to be much slower than the controller adaptive parameters and the control law is picked such that $\Delta e \times e(t) \leq 0$, where Δe

represents the change in model-reference error $e(t)$, and is essentially due only to the adjustment of the adaptive controller in the time Δt . After the controllers are designed, a stability analysis must be performed to determine the extent of the stability regions.

All of the studies of model-reference adaptive control systems mentioned previously were based on using the "MIT rule" or some modification of that rule for synthesizing the adaptive loops. These studies usually yield systems for which theoretical analysis of the designed loops are very difficult. For even the most simple systems, the possibility of instability for the model-reference error exists for certain system conditions and inputs. Since this can easily cast doubt on the stability of more complex systems, Parks^[60] presented an alternative synthesis procedure based on Liapunov's second method and he redesigned the adaptive loops considered by other researchers in references [21], [34] and [84].

The difficult problems of stability analysis associated with the other design techniques can now be avoided because the control law is now synthesized based on Liapunov's theory to guarantee that the model-reference error approaches zero. Although Parks realized that the choice of Liapunov functions used in the design will influence the system performance, his choice of Liapunov function only guaranteed stability and not asymptotic stability. Monopoli^[54] indicates the correction needed for Parks' Liapunov function to guarantee that the model-reference system is asymptotically stable when constant inputs are applied.

Phillipson^[64,65] superimposed the input modification scheme of Grayson^[30] and the parameter adjusting feedback loops of Parks^[60] to yield a model-reference system with a greater degree of stability. Using a root locus technique, he shows that the performance of Parks' controller operating alone in the system may give unsatisfactory results for large inputs and large parameter variations.

After Parks introduced his Liapunov design technique for model-reference adaptive control system designs, a variety of other techniques using Liapunov's theory were presented^[42, 53, 55, 56, 85].

Monopoli^[53] presented a reduction of order technique for linear plants with slowly varying zeroes and poles. Since some of the plant parameters are assumed to be known, prefilters are introduced to precede the plant in the model-reference system configuration to reduce the order of the equation needed for analysis. By utilizing Liapunov's theory, the control signals are first designed for the configuration when a prefilter is included. Then, the control signals are transformed to be used in the original system without a prefilter by using integrator operators. This technique eliminated the need for generating the higher order derivatives of the plant output for the control law. Monopoli developed a similar reduction of order technique with Gitlitz^[55] to be used if there are differential operators operating on the system input.

For a class of linear-time invariant systems, Monopoli and Gilbert^[56] developed a Liapunov design technique that leads to a system

containing a set of prefilters and feedback adjustable gains as the adaptive parameters. Also, physically realizable linear filter networks must be included in both the feedback and prefilter paths. This technique also leads to a reduction of order for the system derivatives required, but was limited to systems for which the zeroes of the plant transfer function are restricted to the left half plane. In Parks^[60] development, a reduction of the needed system derivatives was only obtained if the plant transfer function was a positive real function.

Since earlier studies were mainly concerned with linear systems, Lindorff^[42] presented a method, using Liapunov's theory, for designing controllers for non-linear multi-variable systems in which the control variables are subjected to magnitude constraints, and where the system must track a non-linear model within some bound of error over a region of the plant state space. The plant is assumed to be known and furthermore it must be assumed that each output can be identified with a forcing function which are the control variables to be designed.

As it can be seen, in the past, model-reference systems have been extensively used as a tool in the development of the control signals needed for a given system to produce a desired response. This desired response is specified by the input-output relationship of a known model. The use of model-reference control systems as aids for modeling, physiological systems has not been fully explored. By adapting the model-reference system controller design technique presented by Parks^[60], while re-

arranging the basic model-reference system configuration, a modeling technique for the exploration of physiological systems can be developed.

In the next section, the motivation for the modeling design method in this dissertation is given.

2.3 Motivation for the Design Method

To date, model-reference system configurations have been used to design controllers which are used to control a given system, whether known or unknown, so that its output conforms to that of the reference model with respect to some figure of merit, when both model and system receive the same input simultaneously. In this research effort, the conventional model-reference system configuration is rearranged and it is now desired to develop a design method for a parameter controller which can be utilized as an aid in control system modeling problems with specific emphasis on physiological systems. By comparing the outputs between the given physiological system and its proposed model, the controller's function is to generate control signals which are used to modify the model's postulated parameters to cause the error between the given system under investigation and the assumed model to approach zero.

The design method is mainly intended to improve upon existing models, which have been formulated in previous physiological system studies, by updating their postulated parameter values to produce a model response that more closely matches the experimental data and measurements obtained from the actual physiological system. In other words, the

most accurate model which has been formulated to date for the particular physiological system under consideration is used as a starting point. Then the parameter control signals are designed and used to further improve upon that model.

In most physiological systems it is common for parameters to change in value due to many different conditions; e.g., subjects level of awareness or alertness, fatigue, use of certain drugs, and/or variations in the specific tasks being performed. Since these conditions alter the system response, the model's parameters will again be updated by the controller. Therefore, by the use of a model-reference system configuration and the controller design method, the model exhibits the ability of changing its parameters to account for changes in the actual system. This property is a necessity for accurate physiological system modeling.

Moreover, many researchers [10, 11, 12, 69, 70] use a building block approach for modeling physiological systems. They try to describe the individual components which comprise the physiological system under investigation by anatomical considerations and by analysis of experimental data obtained from the given system and other similar physiological systems. After incorporating the individual components in their model, they adjust certain parameter values by trial and error until the overall model behavior is compatible with experimental results. In this dissertation, the parameter control signal design, which is an adaptive automated procedure, may be used to replace the trial and error approach of adjusting the parameters.

To accomplish the design method, a Liapunov design technique, which is a modification of the techniques presented in references [30, 40, 60, and 65] is used and the control signals are constructed such that the equilibrium state of the model-reference error is asymptotically stable. Rearrangement of the model-reference configuration so that it may be used for modeling physiological systems introduces complications in the resulting model-reference error equations which are not present when the conventional model-reference design problem is considered. For the conventional case, the model is assumed to be completely known a priori and only the system being controlled contains unknown parameters. In this modeling technique situation, the given system is still unknown but now the model's parameters are also unknown a priori, due to the presence of the parameter control signals. These control signals will depend upon functions of the model-reference system error, the input $r(t)$, the model output and its derivatives. Thus the added complexity of the obtained equations places restrictions on the choice of a candidate Liapunov function and also rules out the reduction of order technique developed by Monopoli et al [53, 55, 56].

The next two chapters present a detailed statement of the design problem along with the formulation of the needed model-reference system error equations for both the linear and a class of non-linear modeling problems to be used for modeling physiological systems.

CHAPTER 3

MODEL-REFERENCE FORMULATION FOR LINEAR MODELING PROBLEM

3.1 Introduction

This chapter presents the philosophy and objective for the controller design method when applied to the linear physiological control system modeling process.

After the model is structured, the model, the controller which must be determined and the given physiological system under consideration, are arranged in a model-reference configuration for which a model-reference error equation is formulated. The objective of the design method is then accomplished by utilizing Liapunov's theory to construct the control signals such that the equilibrium state of the model-reference error equation is asymptotically stable.

Section 3.2 contains a statement of the design problem and also illustrates the type of information which is needed and must be obtained from a linear physiological system before the design procedure can be applied. The model-reference configuration used and the linear model structure is discussed in Section 3.3. In Section 3.4 the model-reference error differential equation is formulated and Chapter 5 illustrates how the objective of the design method is accomplished by utilizing Liapunov's stability theory.

3.2 Statement of Design Problem

It is desired to design a controller which can be utilized as an aid for mathematical modeling of physiological control systems.

Before the controller can be designed, the model structure must be determined and a differential equation describing the error between the model and physiological system data must be formulated. Therefore, the measurements obtained from the actual physiological system must temporarily be replaced, in the model-reference system configuration, by a mathematical representation. To accomplish this, the given physiological system under consideration must first be investigated. It is assumed that by analysis of the input-output data obtained from the physiological system, and by study of anatomical considerations, a mathematical representation with unknown parameters[†] can be found for the given system. It is further assumed that although the parameters are unknown and may change, the range in which they may vary can be determined from experimental data and/or other considerations. It should be noted that, this mathematical representation which is used to temporarily replace the actual physiological system data is essentially the form of the physiological system model.

The following section presents restrictions which are imposed on the physiological system's parameter variations. These restrictions are needed for the mathematical formulation of the controller design method.

3.2.1 Nature of Parameter Changes

It is required that the system's parameter value be time invariant during a "learning" period which is defined in Section 5.4. This requirement is satisfied if the parameter behavior can be classified by one of the following two categories:

[†] These parameters are defined in Section 3.2.2.

- 1) The system's parameter values do not fluctuate continuously with time, but instead vary by level changes. Once the parameters acquire a specific set of levels, then these level values are maintained for an extended length of time before the parameters are again permitted to change.
- 2) The parameter values are permitted to vary continuously provided that their variations are very slow with respect to time. In this case, one may subdivide the system's response into several intervals so that during each interval the parameter values may be considered to be essentially time invariant. Each of these individual intervals would then have to be associated with a separate "learning" period.

A survey of physiological system modeling studies^[50,52] reveals that many researchers use time invariant parameter values in the development of their models. Furthermore, physiological systems usually exhibit changes in their response and hence, changes in parameter values due to many different factors. Some of these changes are caused by: subjects level of awareness, fatigue, introduction of drugs,^[27, 67] neurological disorders^[4], and/or changes in the specific task being performed. In addition, there is also a variance of parameter values between different subjects (people), so that changes in parameter values will occur if the response data is obtained from different sources. Therefore, in this

dissertation, the heuristic assumption is made that there exists a class of physiological systems for which the imposed parameter variation restrictions are satisfied. The controller design method can then be utilized as an aid toward the modeling of these physiological systems.

In view of the previous considerations, it is assumed that the mathematical representation of the actual physiological system is determined to be a linear feedback system as illustrated in Fig. 3-1 and described in the following section.

3.2.2 Modeling Problem

A feedback control system block diagram is presented in Fig. 3-1. The linear plant is described by the n^{th} order-linear differential equation

$$c_d^{(n)} + \sum_{i=0}^{n-1} a_i c_d^{(i)} = \sum_{j=0}^h k_j \epsilon_d^{(j)} \quad \text{with } n > h \quad (3-1)$$

where superscripts in parentheses (n) , (i) and (j) denote the n^{th} , i^{th} and j^{th} derivative respectively. For example

$$c_d^{(i)} = \frac{d^i c_d}{dt^i} ,$$

and

$$\epsilon_d^{(j)} = \frac{d^j \epsilon_d}{dt^j} = r^{(j)} - c_d^{(j)} . \quad (3-2)$$

Coefficients $\{a_i\}$ and $\{k_j\}$ of Eq.(3-1) are unknown quantities which may

change in values as described in Section 3.2.1. Although the exact value of each coefficient is unknown because of their variational properties, the range within which each may vary is known and specified by

$$\begin{aligned} a_{i_{\min}} &< a_i < a_{i_{\max}} \\ k_{j_{\min}} &< k_j < k_{j_{\max}} \end{aligned} \quad (3-3)$$

The coefficients can therefore be represented by

$$\begin{aligned} a_i &= A_i + \alpha_i && \text{for } i = 0, 1, \dots, n-1 \\ k_j &= K_j + \gamma_j && \text{for } j = 0, 1, \dots, h \end{aligned} \quad (3-4)$$

where A_i and K_j are known nominal values which are chosen within each coefficient range, and α_i and γ_j are the unknown incremental values which represent the parameter variations.

Now that a mathematical representation has been found for the physiological system, it is desired to construct a model for the given system and to design a parameter controller for that model such that if an input $r(t)$, or an initial condition, is applied simultaneously to the given physiological system and the model, the controller will cause the error, between the reference system output $c_d(t)$ and the model output $c(t)$, to approach zero. To accomplish this task, the reference system and model are arranged in a model-reference system configuration.

3.3 Model-Reference System Configuration

The block diagram of the model-reference system configuration used is given in Fig.3-2. The reference system is temporarily represented by the

system described in Sec.3.2 with $\underline{C}_d$, an n -vector, denoting the system output $c_d(t)$ and its first $n-1$ derivatives. The model consists of two parts, a model sub-system and a parameter controller.

Since the form of the reference system has been determined, the most logical choice for the form of the model sub-system would be identical to that of the reference, and is shown in Fig.3-3. The sub-system plant can be described by the differential equation

$$c^{(n)} + \sum_{i=0}^{n-1} l_i c^{(i)} = \sum_{j=0}^h g_j \epsilon^{(j)} \quad (3-5)$$

where $c^{(i)}$ denotes $\frac{d^i c}{dt^i}$, and

$$\epsilon^{(j)} = \frac{d^j \epsilon}{dt^j} = r^{(j)} - c^{(j)} \quad (3-6)$$

The index limits " n " and " h " are the same as in the reference system Eq.(3-1). Coefficients $\{l_i\}$ and $\{g_j\}$ are defined by

$$\begin{aligned} l_i &= A_i + x_{\alpha_i} & \text{for } i = 0, 1, \dots, n-1 \\ g_j &= K_j + x_{\gamma_j} & \text{for } j = 0, 1, \dots, h \end{aligned} \quad (3-7)$$

where A_i and K_j are the same known nominal values as defined by Eq.(3-4) for the reference system, and x_{α_i} and x_{γ_j} are the parameter control signals generated by the parameter controller which must be designed. As its inputs, the parameter controller receives the input signal $r(t)$, the model output n -vector $\underline{C}$ and the model-reference error

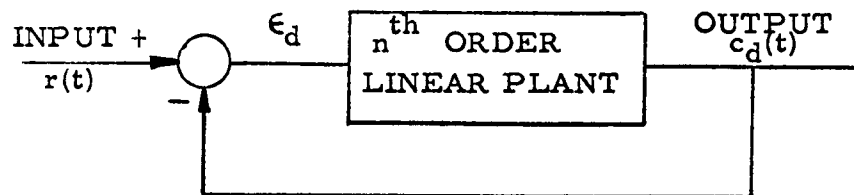


Figure 3-1 Feedback control system

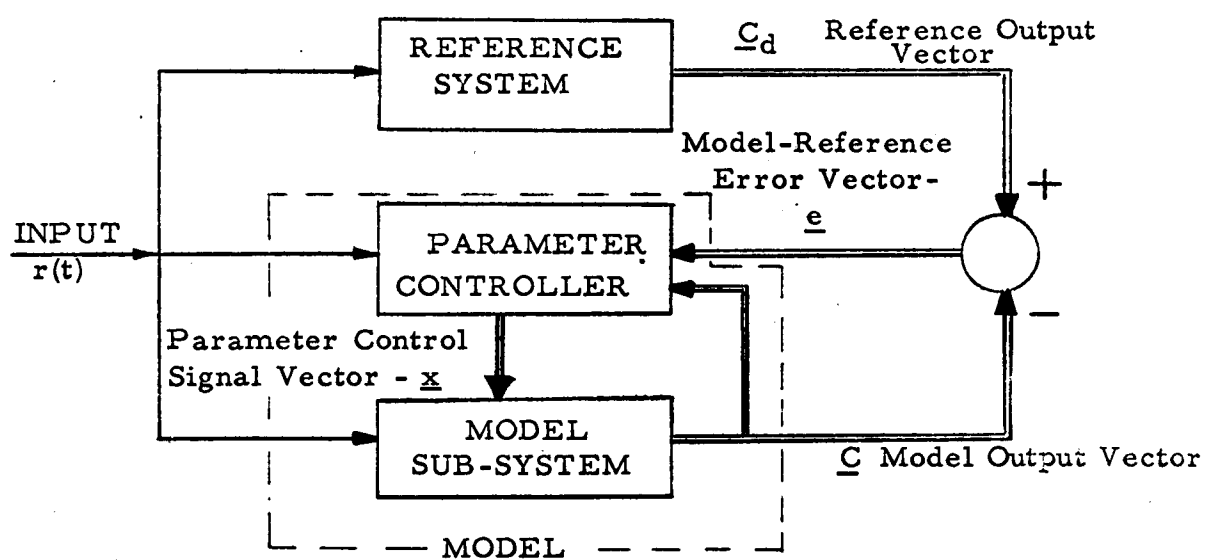


Figure 3-2 Model-reference system configuration

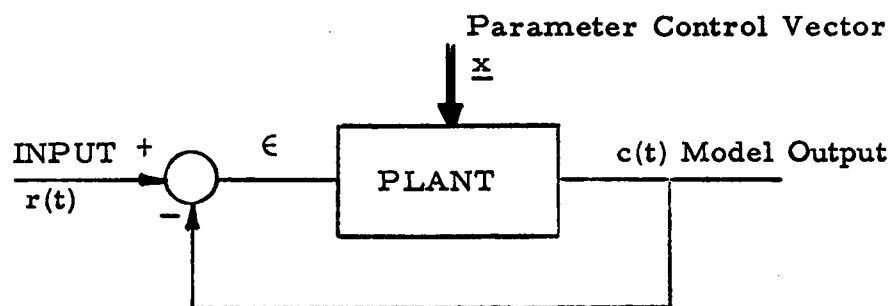


Figure 3-3 Model sub-system

vector $\underline{e}$, where

$$\underline{e} = \underline{C}_d - \underline{C} \quad . \quad (3-8)$$

The differential equation governing the model-reference error $e(t)$ can now be determined. This error equation forms the basis for the parameter controller design.

3.4 Model-Reference Error Differential Equation

The model-reference error differential equation is obtained by first subtracting Eq.(3-5) from Eq.(3-1) yielding,

$$c_d^{(n)} - c^{(n)} + \sum_{i=0}^{n-1} [a_i c_d^{(i)} - l_i c^{(i)}] = \sum_{j=0}^h [k_j \epsilon_d^{(j)} - g_j \epsilon^{(j)}] \quad . \quad (3-9)$$

When the expression $\sum_{i=0}^{n-1} a_i c^{(i)}$ is added to and subtracted from the left hand side of Eq.(3-9), the terms may be rearranged to yield

$$c_d^{(n)} - c^{(n)} + \sum_{i=0}^{n-1} [a_i c_d^{(i)} - a_i c^{(i)}] + \sum_{i=0}^{n-1} [a_i c^{(i)} - l_i c^{(i)}] = \sum_{j=0}^h [k_j \epsilon_d^{(j)} - g_j \epsilon^{(j)}] \quad . \quad (3-10)$$

Defining

$$e^{(i)} = c_d^{(i)} - c^{(i)} \quad (3-11)$$

and substituting from Eq.(3-11) along with Eq.(3-4) and Eq.(3-7) for a_i and l_i respectively into Eq.(3-10), results in

$$e^{(n)} + \sum_{i=0}^{n-1} a_i e^{(i)} + \sum_{i=0}^{n-1} [\alpha_i - x_{\alpha_i}] c^{(i)} = \sum_{j=0}^h [k_j \epsilon_d^{(j)} - g_j \epsilon^{(j)}] \quad . \quad (3-12)$$

Now using Eqs.(3-11) and (3-6), Eq.(3-2) becomes

$$\epsilon_d^{(j)} = r^{(j)} - c_d^{(j)} = r^{(j)} - c^{(j)} - e^{(j)} = \epsilon^{(j)} - e^{(j)}, \quad (3-13)$$

and substituting from Eq.(3-13) into Eq.(3-12) gives

$$e^{(n)} + \sum_{i=0}^{n-1} a_i e^{(i)} + \sum_{i=0}^{n-1} [\alpha_i - x_{\alpha_i}] c^{(i)} = \sum_{j=0}^h [k_j \epsilon^{(j)} - k_j e^{(j)} - g_j \epsilon^{(j)}]. \quad (3-14)$$

Combining all the error terms and using Eqs.(3-4) and (3-7) leads to

$$e^{(n)} + \sum_{i=h+1}^{n-1} a_i e^{(i)} + \sum_{i=0}^h (a_i + k_i) e^{(i)} = - \sum_{i=0}^{n-1} (\alpha_i - x_{\alpha_i}) c^{(i)} + \sum_{j=0}^h (\gamma_j - x_{\gamma_j}) \epsilon^{(j)} \quad (3-15)$$

which is the desired model-reference error equation.

In order to simplify Eq.(3-15), one may combine two of the summations appearing on the left hand side of the equation by defining

$$a_i^{\star} = a_i + k_i \quad \text{for } i = 0, 1, \dots, h \quad (3-16a)$$

and

$$a_i^{\star} = a_i \quad \text{for } i = h+1, \dots, n-1 \quad (3-16b)$$

Note that the coefficients for the two ranges of the indices are denoted by the same symbol $a_i^{\star}$. Equation (3-15) can now be rewritten as

$$e^{(n)} + \sum_{i=0}^{n-1} a_i^{\star} e^{(i)} = - \sum_{i=0}^{n-1} (\alpha_i - x_{\alpha_i}) c^{(i)} + \sum_{j=0}^h (\gamma_j - x_{\gamma_j}) \epsilon^{(j)} \quad (3-17)$$

To help clarify the new starred notation, Eqs.(3-16) and (3-17) are illustrated via examples for two cases; (1) when the index $n=h+1$,

and (2) when $n > h+1$.

Example No.1 $n = h+1 ; n = 3, h = 2$

The differential equation describing the linear plant of the reference system is given by

$$c_d^{(3)} + a_2 c_d^{(2)} + a_1 c_d^{(1)} + a_0 c_d = k_2 \epsilon_d^{(2)} + k_1 \epsilon_d^{(1)} + k_0 \epsilon_d$$

with $\epsilon_d = r - c_d$, $a_2 = A_2 + \alpha_2$, $a_1 = A_1 + \alpha_1$, $a_0 = A_0 + \alpha_0$, $k_2 = K_2 + \gamma_2$, $k_1 = K_1 + \gamma_1$ and $k_0 = K_0 + \gamma_0$. Therefore the equation describing the linear plant of the model sub-system is chosen as

$$c^{(3)} + l_2 c^{(2)} + l_1 c^{(1)} + l_0 c = g_2 \epsilon^{(2)} + g_1 \epsilon^{(1)} + g_0 \epsilon$$

with $\epsilon = r - c$, $l_2 = A_2 + x_{\alpha_2}$, $l_1 = A_1 + x_{\alpha_1}$, $l_0 = A_0 + x_{\alpha_0}$, $g_2 = K_2 + x_{\gamma_2}$, $g_1 = K_1 + x_{\gamma_1}$ and $g_0 = K_0 + x_{\gamma_0}$. Using Eq.(3-16), the set of coefficients $\{a_i^{\star}\}$ are

$$a_2^{\star} = a_2 + k_2, \quad a_1^{\star} = a_1 + k_1 \quad \text{and} \quad a_0^{\star} = a_0 + k_0.$$

The model-reference error Eq.(3-17) becomes

$$\begin{aligned} e^{(3)} + a_2^{\star} e^{(2)} + a_1^{\star} e^{(1)} + a_0^{\star} e = & -(\alpha_2 - x_{\alpha_2}) c^{(2)} - (\alpha_1 - x_{\alpha_1}) c^{(1)} - (\alpha_0 - x_{\alpha_0}) c \\ & + (\gamma_2 - x_{\gamma_2}) \epsilon^{(2)} + (\gamma_1 - x_{\gamma_1}) \epsilon^{(1)} + (\gamma_0 - x_{\gamma_0}) \epsilon. \end{aligned}$$

where $e = c_d - c$.

Example No. 2 $n > h + 1$; $n = 5$, $h = 2$

The differential equation describing the reference system linear plant is given by

$$c_d^{(5)} + a_4 c_d^{(4)} + a_3 c_d^{(3)} + a_2 c_d^{(2)} + a_1 c_d^{(1)} + a_0 c_d = k_2 \epsilon_d^{(2)} + k_1 \epsilon_d^{(1)} + k_0 \epsilon_d$$

with $\epsilon_d = r - c_d$, $a_4 = A_4 + \alpha_4$, $a_3 = A_3 + \alpha_3$, $a_2 = A_2 + \alpha_2$, $a_1 = A_1 + \alpha_1$,
 $a_0 = A_0 + \alpha_0$, $k_2 = K_2 + \gamma_2$, $k_1 = K_1 + \gamma_1$ and $k_0 = K_0 + \gamma_0$.

The model sub-system linear plant equation is selected as

$$c^{(5)} + l_4 c^{(4)} + l_3 c^{(3)} + l_2 c^{(2)} + l_1 c^{(1)} + l_0 c = g_2 \epsilon^{(2)} + g_1 \epsilon^{(1)} + g_0 \epsilon$$

with $\epsilon = r - c$, $l_4 = A_4 + x_{\alpha_4}$, $l_3 = A_3 + x_{\alpha_3}$, $l_2 = A_2 + x_{\alpha_2}$, $l_1 = A_1 + x_{\alpha_1}$,
 $l_0 = A_0 + x_{\alpha_0}$, $g_2 = K_2 + x_{\gamma_2}$, $g_1 = K_1 + x_{\gamma_1}$ and $g_0 = K_0 + x_{\gamma_0}$.

Using Eq. (3-16), the set of coefficients $\{a_i^{\star}\}$ become

$$a_4^{\star} = a_4, \quad a_3^{\star} = a_3, \quad a_2^{\star} = a_2 + k_2, \quad a_1^{\star} = a_1 + k_1, \quad \text{and} \quad a_0^{\star} = a_0 + k_0.$$

The model-reference error Eq. (3-17) yields

$$\begin{aligned} e^{(5)} + a_4^{\star} e^{(4)} + a_3^{\star} e^{(3)} + a_2^{\star} e^{(2)} + a_1^{\star} e^{(1)} + a_0^{\star} e = & -(\alpha_4 - x_{\alpha_4}) c^{(4)} \\ & - (\alpha_3 - x_{\alpha_3}) c^{(3)} - (\alpha_2 - x_{\alpha_2}) c^{(2)} - (\alpha_1 - x_{\alpha_1}) c^{(1)} - (\alpha_0 - x_{\alpha_0}) c \\ & + (\gamma_2 - x_{\gamma_2}) \epsilon^{(2)} + (\gamma_1 - x_{\gamma_1}) \epsilon^{(1)} + (\gamma_0 - x_{\gamma_0}) \epsilon. \end{aligned}$$

Note that $a_4^{\star} = a_4$ and $a_3^{\star} = a_3$ in Example No.2 because $n > h + 1$.

Equation (3-17) can also be written in state matrix equation form by choosing phase variables for the states.

By letting

$$\begin{aligned}
 e_1 &= e \\
 e_2 = \dot{e}_1 &= e^{(1)} \\
 e_3 = \dot{e}_2 &= e^{(2)} \\
 &\vdots \\
 e_n = \dot{e}_{n-1} &= e^{(n-1)}
 \end{aligned} \tag{3-18}$$

and defining

$$y(t) = y_p(t) = -\sum_{i=0}^{n-1} (\alpha_i - x_{\alpha_i}) c^{(i)} + \sum_{j=0}^h (\gamma_j - x_{\gamma_j}) \epsilon^{(j)} , \tag{3-19}$$

the n^{th} order differential equation given by Eq. (3-17) is equivalent to the system of n first order differential equations represented in matrix form by

$$\dot{\underline{e}} = [A] \underline{e} + \underline{y} \tag{3-20}$$

where

$$\underline{e} = \begin{bmatrix} e_1 \\ \vdots \\ e_n \end{bmatrix} , \quad \dot{\underline{e}} = \begin{bmatrix} \dot{e}_1 \\ \vdots \\ \dot{e}_n \end{bmatrix} , \quad \underline{y} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ y(t) \end{bmatrix} \tag{3-21}$$

and

$$[A] = \begin{bmatrix} 0 & 1 & 0 & \dots & 0 & 0 \\ 0 & 0 & 1 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ 0 & 0 & 0 & \dots & 0 & 1 \\ -a_0^* & -a_1^* & -a_2^* & \dots & -a_{n-2}^* & -a_{n-1}^* \end{bmatrix} \quad (3-22)$$

Liapunov's stability theorems can now be utilized to design a parameter controller, and hence the parameter control signals, such that the equilibrium state ($\underline{e} \equiv \underline{0}$) of the model-reference system error, specified by Eq. (3-20), is asymptotically stable. This procedure is shown in Chapter 5.

Before proceeding further, it should be noted that the reference system's linear plant described by Eq. (3-1), was specified for the feedback configuration in Fig. 3-1. If the reference had been an open-loop system specified by

$$c_d^{(n)} + \sum_{i=0}^{n-1} a_i^* c_d^{(i)} = \sum_{j=0}^h k_j r^{(j)} \quad (3-23)$$

where

$$a_i^* = A_i^* + \alpha_i^*$$

and

$$k_j = K_j + \gamma_j \quad ,$$

(3-24)

then, the model subsystem would have been described by

$$c^{(n)} + \sum_{i=0}^{n-1} l_i^* c^{(i)} = \sum_{j=0}^h g_j r^{(j)} \quad (3-25)$$

where

$$\ell_i^{\star} = A_i^{\star} + x_{\alpha_i}^{\star}$$

and

$$g_j = K_j + x_{\gamma_j} \quad (3-26)$$

For this case, the model-reference error equation becomes

$$e^{(n)} + \sum_{i=0}^{n-1} a_i^{\star} e^{(i)} = - \sum_{i=0}^{n-1} (\alpha_i^{\star} - x_{\alpha_i}^{\star}) c^{(i)} + \sum_{j=0}^h (\gamma_j - x_{\gamma_j}) r^{(j)} \quad (3-27)$$

When Eq. (3-27) is written in state matrix form, Eq. (3-20) is obtained

with the term $y(t)$ redefined as

$$y(t) = y_p(t) = - \sum_{i=0}^{n-1} (\alpha_i^{\star} - x_{\alpha_i}^{\star}) c^{(i)} + \sum_{j=0}^h (\gamma_j - x_{\gamma_j}) r^{(j)} \quad (3-28)$$

In the next chapter, the formulation for a class of nonlinear modeling problems is presented. This will demonstrate the modifications needed to formulate the model-reference system error equation for certain nonlinear cases.

CHAPTER 4

MODEL-REFERENCE FORMULATION FOR A CLASS OF NONLINEAR MODELING PROBLEMS

4.1 Introduction

The following nonlinear modeling problem is considered to illustrate the design of the parameter controller when it has been determined that the physiological system may be mathematically represented by the nonlinear feedback system depicted in Fig. 4-1.

4.2 Nonlinear Modeling Problem

An n^{th} order nonlinear feedback control system in block diagram is shown in Fig. 4-1.

There are two basic block elements shown for the reference system; a nonlinearity represented by $f(\epsilon_d)$, and a linear plant of n^{th} order. The nonlinearity receives as its input the actuating signal $\epsilon_d = r - c_d$ and produces the linear plant input v_d . The linear plant is governed by the n^{th} order linear differential equation[†]

[†] In some cases, a normalization procedure must be performed to modify the nonlinearity. This procedure, which is described later in this section, will also alter the linear plant equation. Equation (4-1) represents the linear plant after the normalization procedure has been applied.

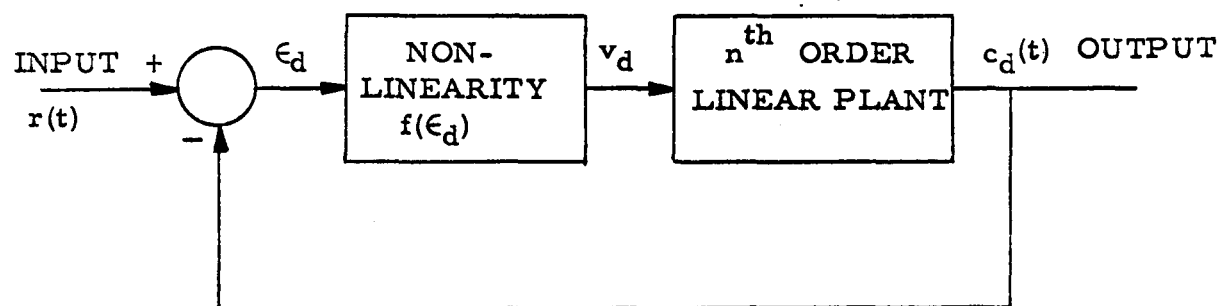


Figure 4-1 Reference system: n^{th} order non-linear feedback control system

$$c_d^{(n)} + \sum_{i=0}^{n-1} a_i c_d^{(i)} = \sum_{j=0}^h k_j v_d^{(j)} \quad , \quad n > h \quad (4-1)$$

with coefficients $\{a_i\}$ and $\{k_j\}$ being defined in a similar manner as presented for the linear problem in Chapter 3. Therefore, as in Section 3.2.2

$$\begin{aligned} a_{i_{\min}} < a_i < a_{i_{\max}} & \quad i = 0, 1, \dots, n-1 \\ k_{j_{\min}} < k_j < k_{j_{\max}} & \quad j = 0, 1, \dots, h \end{aligned} \quad (4-2)$$

and

$$\begin{aligned} a_i &= A_i + \alpha_i \\ k_j &= K_j + \gamma_j \end{aligned} \quad (4-3)$$

where $\{A_i\}$ and $\{K_j\}$ are known nominal values which are chosen within each coefficient range and $\{\alpha_i\}$ and $\{\gamma_j\}$ are the unknown incremental values which represent the level changes.

The nonlinear element, before modification by a normalization procedure described later in this section, is specified by the single valued function $\hat{f}(\epsilon_d)$ and can be represented by the power series

$$\hat{v}_d = \hat{f}(\epsilon_d) = \sum_{i=0}^p \hat{F}_i \epsilon_d^i \quad (4-4)$$

in terms of the reference system actuating signal $\epsilon_d = r - c_d$. Note that

the superscript i denotes the variable ϵ_d raised to the i^{th} power. Also note that the terms $\hat{v}_d$, $\hat{f}(\epsilon_d)$ and $\hat{F}_i$ describe the nonlinearity before the normalization modification.

It is assumed that the shape of the nonlinearity is known except for an unknown scale factor (λ) which multiplies the nonlinearity. For example, when $p = 4$, $\hat{F}_i$ can be represented by λT_i so that $\hat{F}_1 = \lambda T_1$, $\hat{F}_2 = \lambda T_2$, $\hat{F}_3 = \lambda T_3$ and $\hat{F}_4 = \lambda T_4$. Then

$$\begin{aligned}\hat{v}_d &= \lambda T_1 \epsilon_d + \lambda T_2 \epsilon_d^2 + \lambda T_3 \epsilon_d^3 + \lambda T_4 \epsilon_d^4 \\ &= \lambda [T_1 \epsilon_d + T_2 \epsilon_d^2 + T_3 \epsilon_d^3 + T_4 \epsilon_d^4]\end{aligned}$$

where the coefficients T_1 , T_2 , T_3 , and T_4 are known constants, and the variable λ represents the unknown scale factor. The requirement that the shape of the nonlinearity be known is not a very severe restriction since the types of nonlinearities most commonly encountered in physiological modeling problems are: saturation functions, limiting functions and dead zone functions. These nonlinear functions can usually be determined by analysis of the systems input-output data and anatomical considerations.

In order to simplify the formulation of the model-reference error equation while still preserving the input-output relationship of the reference system, it will now be assumed that Eq. (4-4), which specifies the nonlinearity, has been normalized so that the coefficient of the first term in the series given by Eq.(4-4) is unity. It is further assumed that the

normalization factor $\hat{F}_1$, which includes the unknown scale factor λ , has already been incorporated into the set of coefficients $\{k_j\}$ of Eq.(4-1). Hence the nonlinearity can be described by the power series

$$v_d = f(\epsilon_d) = \epsilon_d + f_1(\epsilon_d) \quad (4-5)$$

where

$$f_1(\epsilon_d) = \sum_{i=2}^p F_i \epsilon_d^i$$

and the coefficients F_i for $i = 2, \dots, p$ are known constants.

To illustrate the nonlinearity normalization procedure, the case for $p = 4$ is explained.

Before normalization, the nonlinearity is represented by

$$\hat{v}_d = \hat{f}(\epsilon_d) = \sum_{i=1}^{i=4} F_i \epsilon_d^i = \lambda [T_1 \epsilon_d + T_2 \epsilon_d^2 + T_3 \epsilon_d^3 + T_4 \epsilon_d^4]$$

and the linear plant is governed by the n^{th} order linear differential equation

$$c_d^{(n)} + \sum_{i=0}^{n-1} a_i c_d^{(i)} = \sum_{j=0}^h \hat{k}_j \hat{v}_d^{(j)} \quad (4-6)$$

In this example the normalization factor $\hat{F}_1$ is specified by

$$\hat{F}_1 = \lambda T_1 \quad .$$

Therefore, after normalization, the nonlinearity is defined by

$$v_d = \frac{\hat{v}_d}{\lambda T_1} = \epsilon_d + \frac{T_2}{T_1} \epsilon_d^2 + \frac{T_3}{T_1} \epsilon_d^3 + \frac{T_4}{T_1} \epsilon_d^4 .$$

Letting $F_2 = \frac{T_2}{T_1}$, $F_3 = \frac{T_3}{T_1}$ and $F_4 = \frac{T_4}{T_1}$, the above equation becomes

$$v_d = \epsilon_d + f_1(\epsilon_d)$$

where

$$f_1(\epsilon_d) = F_2 \epsilon_d^2 + F_3 \epsilon_d^3 + F_4 \epsilon_d^4 .$$

The plant differential equation given by Eq.(4-6) can now be rewritten as

$$c_d^{(n)} + \sum_{i=0}^{n-1} a_i c_d^{(i)} = \sum_{j=0}^h (\lambda T_1 \hat{k}_j) \frac{\hat{v}_d^{(j)}}{\lambda T_1} = \sum_{j=0}^h (\lambda T_1 \hat{k}_j) v_d^{(j)} .$$

By defining

$$k_j = \lambda T_1 \hat{k}_j ,$$

the following linear plant Eq.(4-1) is obtained:

$$c_d^{(n)} + \sum_{i=0}^{n-1} a_i c_d^{(i)} = \sum_{j=0}^h k_j v_d^{(j)} . \quad (4-1)$$

Equation (4-5) can now be combined with Eq. (4-1) yielding the reference system output equation

$$c_d^{(n)} + \sum_{i=0}^{n-1} a_i c_d^{(i)} = \sum_{j=0}^h k_j \{ \epsilon_d^{(j)} + f_1^{(j)}(\epsilon_d) \} . \quad (4-7)$$

4.3 Nonlinear Model Sub-system

As in the linear case, the same model-reference system configuration (Figure 3-2) is used. The procedure of Section 3.3 is modified as follows to produce the model sub-system shown in Figure 4.2.

Based on Eq. (4-5), one may define a nonlinearity $f(\epsilon)$ for the model subsystem by

$$v = f(\epsilon) = \epsilon + f_1(\epsilon) \quad \text{where} \quad \epsilon = r - c \quad (4-8)$$

and

$$f_1(\epsilon) = \sum_{i=2}^p F_i \epsilon^i .$$

The plant (linear part of model subsystem) equation is described by

$$c^{(n)} + \sum_{i=0}^{n-1} l_i c^{(i)} = \sum_{j=0}^h g_j v^{(j)} + x_a \quad (4-9)$$

where the coefficients $\{l_i\}$ and $\{g_j\}$ are specified by

$$\begin{aligned} l_i &= A_i + x_{\alpha i} & \text{for } i = 0, 1, \dots, n-1 \\ g_j &= K_j + x_{\gamma j} & \text{for } j = 0, 1, \dots, h . \end{aligned} \quad (4-10)$$

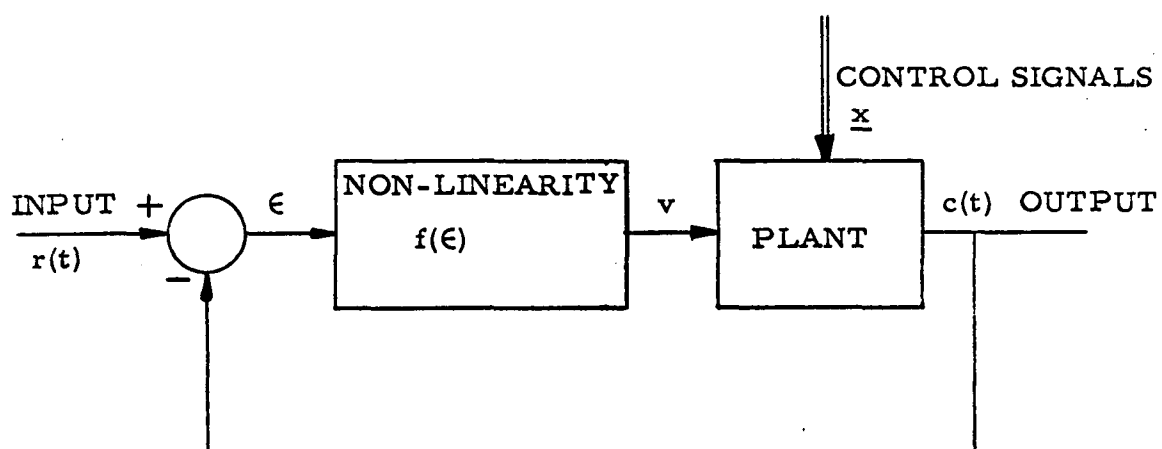


Figure 4-2 Model sub-system for non-linear problem

The terms $\{A_i\}$ and $\{K_j\}$ are the same known nominal values as defined in Eq. (4-3) and $\{x_{\alpha_i}\}$ and $\{x_{\gamma_i}\}$ are the parameter control signals generated by the parameter controller which must be designed. Equation (4-9) is similar to Eq. (3-5) which was specified for the plant in the linear case except for the term " x_a " which is an extra additive control signal to be generated by the controller. This new additive control signal is needed to compensate for some of the terms introduced by the nonlinearity.

4.4 Formulation of the Model-Reference Error Equation (Nonlinear Case)

The nonlinear model-reference error differential equation is obtained by first subtracting Eq. (4-9) from Eq. (4-1) yielding,

$$c_d^{(n)} - c^{(n)} + \sum_{i=0}^{n-1} [a_i c_d^{(i)} - l_i c^{(i)}] = \sum_{j=0}^h [k_j v_j^{(j)} - g_j v^{(j)}] - x_a. \quad (4-11)$$

When the term $\sum_{i=0}^{n-1} a_i c^{(i)}$ is added to and subtracted from the left hand side of Eq.(4-11), the terms may be rearranged to yield the following equation,

$$c_d^{(n)} - c^{(n)} + \sum_{i=0}^{n-1} [a_i c_d^{(i)} - a_i c^{(i)}] + \sum_{i=0}^{n-1} [a_i c^{(i)} - l_i c^{(i)}] = \sum_{j=0}^h [k_j v_d^{(j)} - g_j v^{(j)}] - x_a. \quad (4-12)$$

Defining

$$e^{(i)} = c_d^{(i)} - c^{(i)} \quad (4-13)$$

and substituting from Eq. (4-13), along with Eqs. (4-3) and (4-10) for a_i and ℓ_i respectively, into Eq. (4-12) results in

$$e^{(n)} + \sum_{i=0}^{n-1} a_i e^{(i)} + \sum_{i=0}^{n-1} [\alpha_i - x_{\alpha_i}] c^{(i)} = \sum_{j=0}^h [k_j v_d^{(j)} - g_j v^{(j)}] - x_a \quad (4-14)$$

Using Eqs. (4-5) and (4-8), the terms $v_d^{(j)}$ and $v^{(j)}$ in Eq. (4-14) can be replaced by

$$v_d^{(j)} = f^{(j)}(\epsilon_d) = \epsilon_d^{(j)} + f_1^{(j)}(\epsilon_d) \quad (4-15)$$

and

$$v^{(j)} = f^{(j)}(\epsilon) = \epsilon^{(j)} + f_1^{(j)}(\epsilon) \quad (4-16)$$

respectively.

Thus, one obtains the equation

$$\begin{aligned} e^{(n)} + \sum_{i=0}^{n-1} a_i e^{(i)} = & - \sum_{i=0}^{n-1} (\alpha_i - x_{\alpha_i}) c^{(i)} \\ & + \sum_{j=0}^h [k_j \epsilon_d^{(j)} - g_j \epsilon^{(j)}] + \sum_{j=0}^h [k_j f_1^{(j)}(\epsilon_d) - g_j f_1^{(j)}(\epsilon)] - x_a \end{aligned} \quad (4-17)$$

An alternate expression for $\epsilon_d^{(j)} = r^{(j)} - c_d^{(j)}$ may be found by using Eq. (4-13) and the relationship $\epsilon^{(j)} = r^{(j)} - c^{(j)}$, i.e.

$$\epsilon_d^{(j)} = r^{(j)} - c_d^{(j)} = r^{(j)} - c^{(j)} - e^{(j)} = \epsilon^{(j)} - e^{(j)} \quad (4-18)$$

Substituting from Eq. (4-18) into Eq. (4-17) produces the following equation:

$$e^{(n)} + \sum_{i=0}^{n-1} a_i e^{(i)} = - \sum_{i=0}^{n-1} [\alpha_i - x_{\alpha_i}] c^{(i)} + \sum_{i=0}^h [k_j \epsilon^{(j)} - g_j \epsilon^{(j)} - k_i e^{(j)}] \\ + \sum_{i=0}^h [k_i f_1^{(j)}(\epsilon_d) \Big|_{\epsilon_d = \epsilon - e} - g_j f_1^{(j)}(\epsilon)] - x_a . \quad (4-19)$$

Equation (4-19) can now be rearranged to yield

$$e^{(n)} + \sum_{i=h+1}^{n-1} a_i e^{(i)} + \sum_{i=0}^h (a_i + k_i) e^{(i)} = - \sum_{i=0}^{n-1} [\alpha_i - x_{\alpha_i}] c^{(i)} \\ + \sum_{j=0}^h [k_j \epsilon^{(j)} - g_j \epsilon^{(j)}] + \sum_{j=0}^h [k_j f_1^{(j)}(\epsilon_d) \Big|_{\epsilon_d = \epsilon - e} - g_j f_1^{(j)}(\epsilon)] - x_a . \quad (4-20)$$

By defining

$$a_i^{\star} = a_i + k_i \quad \text{for } i = 0, 1, \dots, h \\ \text{and} \\ a_i^{\star} = a_i \quad \text{for } i = h+1, \dots, n-1 , \quad (4-21)$$

which is identical to the definition given in Eq. (3-16) for the coefficients $\{a_i^{\star}\}$ of the linear modeling case, Eq. (4-20) becomes

$$e^{(n)} + \sum_{i=0}^{n-1} a_i^{\star} e^{(i)} = - \sum_{i=0}^{n-1} [\alpha_i - x_{\alpha_i}] c^{(i)} + \sum_{j=0}^h [k_j - g_j] \epsilon^{(j)} \\ + \sum_{j=0}^h [k_j f_1^{(j)}(\epsilon_d) \Big|_{\epsilon_d = \epsilon - e} - g_j f_1^{(j)}(\epsilon)] - x_a . \quad (4-22)$$

Equation (4-22) can be further manipulated by considering the expression

$$\sum_{j=0}^h \left\{ k_j f_1^{(j)}(\epsilon_d) \right\} \Big|_{\epsilon_d = \epsilon - e} \quad (4-23)$$

Replacing the function $f_1^{(j)}(\epsilon_d)$ in Eq. (4-23) with the polynomial expansion specified in Eq.(4-5), one obtains

$$\begin{aligned} \sum_{j=0}^h k_j f_1^{(j)}(\epsilon_d) \Big|_{\epsilon_d = \epsilon - e} &= \sum_{j=0}^h k_j \left\{ \frac{d^j}{dt^j} [f_1(\epsilon_d)] \right\} \Big|_{\epsilon_d = \epsilon - e} \\ &= \sum_{j=0}^h k_j \left\{ \frac{d^j}{dt^j} \left[\sum_{i=2}^p F_1(\epsilon - e)^i \right] \right\} . \end{aligned} \quad (4-24)$$

Now the term $(\epsilon - e)^i$ can be replaced by the binomial summation

$$\begin{aligned} (\epsilon - e)^i &= \sum_{m=0}^i (-1)^m \frac{(i)! \epsilon^{i-m} e^m}{(i-m)! m!} \\ &= \epsilon^i + \sum_{m=1}^i (-1)^m \frac{(i)! \epsilon^{i-m} e^m}{(i-m)! m!} \end{aligned} \quad (4-25)$$

yielding

$$\sum_{j=0}^h k_j f_1^{(j)}(\epsilon_d) \Big|_{\epsilon_d = \epsilon - e} = \sum_{j=0}^h k_j \left\{ \frac{d^j}{dt^j} \left[\sum_{i=2}^p F_1 \left(\epsilon^i + \sum_{m=1}^i (-1)^m \frac{(i)! \epsilon^{i-m} e^m}{(i-m)! m!} \right) \right] \right\} . \quad (4-26)$$

Since

$$\frac{d^j}{dt^j} \left[\sum_{i=2}^p F_i \epsilon^i \right] = f_1^{(j)}(\epsilon) , \quad (4-27)$$

Eq. (4-26) can be rewritten as

$$\begin{aligned} \sum_{j=0}^h k_j f_1^{(j)}(\epsilon_d) \Big|_{\epsilon_d = \epsilon - e} &= \sum_{j=0}^h \left\{ k_j f_1^{(j)}(\epsilon) + k_j \frac{d^j}{dt^j} \left[\sum_{i=2}^p F_i \left(\sum_{m=1}^i (-1)^m \frac{(i)! \epsilon^{i-m} e^m}{(i-m)! m!} \right) \right] \right\} \\ &= \sum_{j=0}^h k_j f_1^{(j)}(\epsilon) + y_a(t) + x_a \end{aligned} \quad (4-28)$$

where the function $y_a(t)$ is defined by

$$y_a(t) = \sum_{j=0}^h \left\{ k_j \frac{d^j}{dt^j} \left[\sum_{i=2}^p F_i \left(\sum_{m=1}^i (-1)^m \frac{(i)! \epsilon^{i-m} e^m}{(i-m)! m!} \right) \right] \right\} - x_a . \quad (4-29)$$

Substituting from Eq. (4-28) into Eq. (4-22), the error equation

becomes

$$\begin{aligned} e^{(n)} + \sum_{i=0}^{n-1} a_i^* e^{(i)} &= - \sum_{i=0}^{n-1} [\alpha_i - x_{\alpha_i}] c^{(i)} \\ &+ \sum_{j=0}^h [k_j - g_j] e^{(j)} + \sum_{j=0}^h [k_j - g_j] f_1^{(j)}(\epsilon) + y_a(t) . \end{aligned} \quad (4-30)$$

Using the expressions for k_j and g_j as given in Eqs. (4-3) and (4-10) respectively, in combination with Eq. (4-16), the desired model-reference

error equation for the nonlinear case is obtained as

$$e^{(n)} + \sum_{i=0}^{n-1} a_i^* e^{(i)} = - \sum_{i=0}^{n-1} [\alpha_i - x_{\alpha_i}] c^{(i)} + \sum_{j=0}^h [\gamma_j - x_{\gamma_j}] f^{(j)}(e) + y_a(t). \quad (4-31)$$

Equation (4-31) can also be written in state matrix equations form by choosing phase variables for the states. Letting

$$\begin{aligned} e_1 &= e \\ e_2 &= \dot{e}_1 = e^{(1)} \\ &\vdots \\ e_n &= \dot{e}_{n-1} = e^{(n-1)} \end{aligned} \quad (4-32)$$

and defining

$$y_p(t) = - \sum_{i=0}^{n-1} [\alpha_i - x_{\alpha_i}] c^{(i)} + \sum_{j=0}^h [\gamma_j - x_{\gamma_j}] f^{(j)}(e) \quad (4-33)$$

and

$$y(t) = y_p(t) + y_a(t) \quad , \quad (4-34)$$

the error Eq. (4-31) can be represented in matrix form by

$$\dot{\underline{e}} = [A]\underline{e} + \underline{y} \quad (4-35)$$

where

$$\underline{e} = \begin{bmatrix} e_1 \\ e_2 \\ \vdots \\ e_n \end{bmatrix}, \quad \dot{\underline{e}} = \begin{bmatrix} \dot{e}_1 \\ \dot{e}_2 \\ \vdots \\ \dot{e}_n \end{bmatrix}, \quad \underline{y} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ y(t) \end{bmatrix} \quad (4-36)$$

and

$$A = \begin{bmatrix} 0 & 1 & 0 & & 0 \\ 0 & 0 & 1 & & 0 \\ & & & \dots & \\ 0 & 0 & 0 & \dots & 0 & 1 \\ -a_0^* & -a_1^* & \dots & & -a_{n-2}^* & -a_{n-1}^* \end{bmatrix}. \quad (4-37)$$

It should be noted that $y(t)$ in Eq. (4-34) is composed of two components; the first component $y_p(t)$ contains the parameter control signals and the second component $y_a(t)$ contains the additive control signal which is needed to compensate for some of the terms introduced by the nonlinearity. In the linear modeling problem of Chapter 3, an additive control signal was not required and therefore $y(t)$ consisted of only one component $y_p(t)$ which contains the parameter control signals.

The model-reference error equations for both the linear and nonlinear modeling problems have been determined and specified in state matrix form, a form which is very convenient when using Liapunov functions. It is now desired to design a controller and hence the parameter control signals,

including the additive control signal x_a , such that the equilibrium state ($\underline{e} \equiv \underline{0}$) of the model-reference error Eqs. (3-20) and (4-35) are asymptotically stable. Liapunov's stability theory is used to accomplish the controller design, and is the basis for the design methods presented in the next chapter.

CHAPTER 5

CONTROLLER DESIGN VIA LIAPUNOV'S SECOND METHOD

5.1 Formulation of a Candidate Liapunov Function and its Time Derivative

The objective of the design method can now be accomplished by utilizing a Liapunov function $V(\underline{e})$ and its time derivative $\dot{V}(\underline{e})$. As shown in Chapters 3 and 4 for the linear and nonlinear cases respectively, the model reference error equations can be written in state vector equation form

$$\dot{\underline{e}} = [A] \underline{e} + \underline{y} \quad (5-1)$$

where $[A]$ is a square matrix as defined by Eqs. (3-22) and (4-37) for the linear and nonlinear modeling cases respectively and $\underline{y}$ is an n -dimensional vector

$$\underline{y} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ y(t) \end{bmatrix}$$

with the term $y(t)$ containing the control signals to be designed.

The quadratic form

$$V(\underline{e}) = \underline{e}^T M \underline{e} \quad (5-2)$$

is chosen as a candidate Liapunov function for the error equation, where

M is an $n \times n$ symmetric matrix of the form

$$M = \begin{bmatrix} m_{11} & m_{12} & m_{13} & . & . & . & . & . & m_{1n} \\ & m_{12} & & & & & & & \\ & \vdots & & & & & & & \\ & \vdots & & & & & & & \\ & m_{1n} & & & & & & & m_{nn} \end{bmatrix} . \quad (5-3)$$

Considering first the linear case, one obtains the time derivative of $V(\underline{e})$ along the error equation trajectory as

$$\dot{V}(\underline{e}) = \dot{\underline{e}}^T M \underline{e} + \underline{e}^T M \dot{\underline{e}} . \quad (5-4)$$

Substituting the expression for $\dot{\underline{e}}$ from Eq. (5-1) yields

$$\dot{V}(\underline{e}) = [\underline{e}^T A^T + \underline{y}^T] M \underline{e} + \underline{e}^T M [A \underline{e} + \underline{y}] \quad (5-5)$$

or

$$\dot{V}(\underline{e}) = \underline{e}^T [A^T M + M A] \underline{e} + \underline{y}^T M \underline{e} + \underline{e}^T M \underline{y} . \quad (5-6)$$

The term $\underline{y}^T M \underline{e}$ is a scalar function and M is a symmetric matrix.

Therefore

$$\underline{y}^T M \underline{e} = (\underline{y}^T M \underline{e})^T = \underline{e}^T M^T \underline{y} = \underline{e}^T M \underline{y} , \quad (5-7)$$

and

$$\underline{y}^T M \underline{e} + \underline{e}^T M \underline{y} = 2 \underline{e}^T M \underline{y} . \quad (5-8)$$

Since only the last term of the vector $\underline{y}$ is non-zero, Eq.(5-8) can be written as

$$2 \underline{e}^T M \underline{y} = 2 y(t) s(t) \quad (5-9)$$

where $s(t)$ denotes the inner product of the vector $\underline{e}$ and the last column of the M matrix, i.e.

$$s(t) = m_{1n} e_1 + m_{2n} e_2 + \dots + m_{nn} e_n \quad (5-10)$$

By defining

$$-D = A^T M + M A \quad , \quad (5-11)$$

Eq. (5-6) can now be written as

$$\dot{V}(\underline{e}) = -\underline{e}^T D \underline{e} + 2 y(t) s(t) \quad (5-12)$$

From Liapunov's theorems as stated in Appendix A, if the function $V(\underline{e})$ is positive definite and $\dot{V}(\underline{e})$ is negative definite, then the equilibrium state ($\underline{e} \equiv 0$) of the model-reference error Eq.(5-1) will be asymptotically stable. Therefore the error $e(t)$ between the model and reference system will tend toward zero if both matrices M and D are positive definite and the term $y(t)s(t)$ is either negative definite or negative semi-definite.

The controller design can now be accomplished by determining the components of the M matrix such that both M and D are positive definite, and then constructing the control signals to satisfy the inequality

$$2 y(t) s(t) \leq 0 . \quad (5-13)$$

In the next section, the procedure for determining the components of the M-matrix is presented.

5.2 Formulation of the M-Matrix for the Candidate Liapunov Function

As stated in the previous section, the components of the M-matrix must be determined such that both the M and D matrices are positive definite where the D-matrix is defined by

$$-D = A^T M + M A , \quad (5-11)$$

and the A-matrix is specified by Eqs. (3-22) and (4-37) for the linear and nonlinear modeling problems respectively.

It should be noted that the A-matrices defined for the linear and nonlinear modeling problems are identical in form and possess similar equations which define and specify their elements. In the following discussion, the A matrix developed for the linear modeling problem and its associated equations will be used without loss in generality because the A-matrix of the nonlinear case would yield equivalent results.

The elements of the A-matrix were defined by Eq. (3-16) in terms of the coefficients $\{a_i\}$ and $\{k_j\}$ which are unknown quantities that may vary by level changes within the range specified by Eq.(3-3). Some of the

elements of the M -matrix are chosen to be functions of the A -matrix components, but these M -matrix elements are constructed in such a manner that, no matter what level the coefficients $\{a_i^{\star}\}$ assume within their range, the new M and D matrices still remain positive definite. Meanwhile the relationship

$$2 y(t) s(t) \leq 0 \quad (5-13)$$

must also be satisfied where the function $s(t)$ is given by

$$s(t) = m_{1n} e_1 + m_{2n} e_2 + \dots + m_{nn} e_n \quad (5-10)$$

and where the control signals are contained in the function $y(t)$. But, in order to satisfy inequality (5-13), the control signals will have to depend upon the instantaneous values of the function $s(t)$. Therefore, since $s(t)$ equals the inner product of the error vector $\underline{e}$ and the last column of the M -matrix, these elements of the M -matrix must be known quantities. They can not be assumed to be functions of the elements $\{a_i^{\star}\}$ which would then make them dependent upon the unknown coefficients $\{a_i\}$ and $\{k_j\}$. By using Eqs. (3-16) and (3-3), the range of the A -matrix elements $\{a_i^{\star}\}$ can be established from physical considerations. Thus let

$$a_{i_{\min}}^{\star} < a_i^{\star} < a_{i_{\max}}^{\star} \quad \text{for } i = 0, 1 \dots n-1 \quad (5-14)$$

where the maximum and minimum limits are functions of the known limits given for the coefficients $\{a_i\}$ and $\{k_j\}$. It should be noted that although the parameter limits are assumed to be known, there exists the possibility that some of the maximum and minimum limits may have been erroneously determined. If this is the case and some incorrect specifications for maximum or minimum limits are used in the design method, the incorrect specifications are revealed when the memory components of the parameter control signals developed in Section 5.4 tend toward values outside their specified range. When this condition occurs, as demonstrated in the human eye-movement muscle system modeling design problem presented in Chapter 7, some of the maximum and minimum parameter limits must be updated to contain the model's "learned" parameter values.

Before proceeding with the M-matrix formulation, it is now pre-supposed that if the vector $\underline{y}$ in the model reference error Eq.(5-1) is set equal to zero for all time, the error equation would be asymptotically stable. Hence, the vector differential equation

$$\dot{\underline{e}} = A\underline{e} \quad (5-15)$$

represents a stable system for all possible values of the elements $\{a_i^*\}$ provided that the values always remain within the range specified by Eq.(5-14). In other words, it is assumed that all the A-matrix eigenvalues must always possess negative real parts, and that the elements

$\{a_i^{\star}\}$ must satisfy the Routh-Hurwitz stability criterion. In Appendix B, a set of sufficient conditions, which insure that the A-matrix eigenvalues always maintain negative real parts, are formulated by utilizing the Routh-Hurwitz stability criterion along with the range limits [Eq.(5-14)] specified for the elements $\{a_i^{\star}\}$. These sufficient conditions[†] will be referred to as the "Strong" Routh Hurwitz conditions and are illustrated in Table 5-1 for the second, third and fourth order system. The variables $a_{i\min}^{\star}$ and $a_{i\max}^{\star}$ which appear in the table are the limits defined by inequality (5-14).

System Order	"Strong" Routh-Hurwitz Conditions
second	$a_{0\min}^{\star}, a_{1\min}^{\star} > 0$
third	$a_{0\min}^{\star}, a_{1\min}^{\star}, a_{2\min}^{\star} > 0$ $a_{1\min}^{\star} a_{2\min}^{\star} - a_{0\max}^{\star} > 0$
fourth	$a_{0\min}^{\star}, a_{1\min}^{\star}, a_{2\min}^{\star}, a_{3\min}^{\star} > 0$ $a_{3\min}^{\star} a_{2\min}^{\star} - a_{1\max}^{\star} > 0$ $a_{1\min}^{\star} (a_{3\min}^{\star} a_{2\min}^{\star} - a_{1\max}^{\star}) - a_{0\max}^{\star} a_{3\min}^{\star} a_{3\max}^{\star} > 0$

Table 5-1. "Strong" Routh-Hurwitz Conditions

[†] The Routh-Hurwitz stability criterion are necessary and sufficient conditions, but the "Strong" Routh-Hurwitz conditions, being more restrictive, are only sufficient conditions for stability.

If the A-matrix elements for the modeling problem under consideration do not meet the "Strong" Routh-Hurwitz Conditions, the model-reference error equation's A-matrix can be modified so that the requirements are satisfied. This modification procedure is illustrated in Appendix C.

In view of the consideration that the A-matrix of Eq.(5-1) represents a stable system and furthermore satisfies the "Strong" Routh-Hurwitz conditions, the M and D matrices of the expression

$$-D = A^T M + M A \quad (5-11)$$

can be specified with the aid of Theorems 5-1 and 5-2 which are proven in Reference 41 and are repeated below.

Theorem 5-1

If A is a stable matrix, then the symmetric matrix M of Eq.(5-11) is uniquely determined by the symmetric matrix D.

Theorem 5-2

If A is a stable matrix, and D is a positive definite matrix, then the unique M-matrix solution to Eq.(5-11) which exists, is likewise positive definite.

The philosophy behind the M-matrix formulation can now be stated. A diagonal D-matrix is constructed with all positive elements, such that the solution of Eq.(5-11) yields an M-matrix whose last column elements

are known constants. All the other M-matrix elements will be functions of the A-matrix elements $\{a_i^{\star}\}$. Since the A matrix is stable and the D matrix is positive definite, Theorems 5-1 and 5-2 guarantee that the M-matrix exists and is positive definite.

The detailed procedure for constructing the M-matrix is presented in Appendix B and the M-matrix elements for second, third and fourth order systems are listed in Table 5-2.

Since an M-matrix can be found which insures that the candidate Liapunov function

$$V(\underline{e}) = \underline{e}^T M \underline{e} \quad (5-2)$$

is positive definite, it must be shown that the time derivative of $V(\underline{e})$ given by

$$\dot{V}(\underline{e}) = -\underline{e}^T D \underline{e} + 2 y(t) s(t) \quad (5-4)$$

is negative definite and thereby $V(e)$ will qualify as a Liapunov function. Because the D matrix was constructed to be positive definite, the requirement that the function $\dot{V}(\underline{e})$ be negative definite will be satisfied if one constrains the term $y(t) s(t)$ to be at least a negative semi-definite function. This requirement on the term $y(t) s(t)$ becomes the basis for constructing the control signals as presented in the next section.

System Order	M-Matrix Elements
Second	$m_{12} = a_{1\min}^{\star} m_{22}$ $m_{22} = \text{any positive constant}$ $m_{11} = [a_1^{\star} a_{1\min}^{\star} + a_0^{\star}] m_{22}$
Third	$m_{13} = \left[\frac{a_{1\min}^{\star} a_{2\min}^{\star} - a_0^{\star} \max}{a_{2\max}^{\star}} \right] m_{33}$ $m_{23} = a_{2\min}^{\star} m_{33}$ $m_{33} = \text{any positive constant}$ $m_{11} = a_1^{\star} m_{13} + a_0^{\star} m_{23}$ $m_{12} = a_2^{\star} m_{13} + a_0^{\star} m_{33}$ $m_{22} = a_2^{\star} m_{23} + a_1^{\star} m_{33} - m_{13}$
Fourth	$m_{14} = \left[\frac{a_{1\min}^{\star} (a_{2\min}^{\star} a_{3\min}^{\star} - a_1^{\star} \max) - a_0^{\star} \max a_{3\min}^{\star} a_{3\max}^{\star}}{a_{3\max}^{\star} a_{2\max}^{\star} - a_{1\min}^{\star}} \right] m_{44}$ $m_{24} = \frac{a_{2\max}^{\star}}{a_{1\min}^{\star}} m_{14} + \frac{a_0^{\star} \max}{a_{1\min}^{\star}} a_{3\min}^{\star} m_{44}$ $m_{34} = a_{3\min}^{\star} m_{44}$ $m_{44} = \text{any positive constant}$ $m_{11} = a_1^{\star} m_{14} + a_0^{\star} m_{24}$ $m_{12} = a_2^{\star} m_{14} + a_0^{\star} m_{34}$ $m_{13} = a_3^{\star} m_{14} + a_0^{\star} m_{44}$ $m_{22} = a_2^{\star} m_{24} + a_1^{\star} m_{34} - m_{13}$ $m_{23} = a_3^{\star} m_{24} + a_1^{\star} m_{44} - m_{14}$ $m_{33} = a_3^{\star} m_{34} + a_2^{\star} m_{44} - m_{24}$

Table 5-2 M-matrix elements for second, third and fourth order systems

5.3 Design of Controller Without Memory Capabilities

The controller design can now be accomplished by using the requirement that the expression $y(t)s(t)$ be at least negative semi-definite. In the expression $y(t)s(t)$, the term $y(t)$ includes all of the control signals. The function $s(t)$ equals the inner product of the error vector $\underline{e}$, which is measurable from the model-reference configuration, and the last column of the M-matrix, whose elements can be determined as stated in the previous section. These elements will consist of known constants.

5.3.1 Linear Modeling Case

For the linear modeling problem, the function $y(t)$ is represented by

$$y(t) = y_p(t) = - \sum_{i=0}^{n-1} (\alpha_i - x_{\alpha_i}) c^{(i)} + \sum_{j=0}^h (\gamma_j - x_{\gamma_j}) e^{(j)} . \quad (3-19)$$

Note that Eq.(3-19) contains $n+h+1$ parameter control signals $\{x_{\alpha_i}\}$ and $\{x_{\gamma_j}\}$. If the A-matrix elements $\{a_i^{\star}\}$ do not satisfy the "Strong" Routh-Hurwitz conditions, the function $y(t)$ for the linear case has to be modified to equal

$$y(t) = y_p(t) + y_{a_m}(t) , \quad (5-16)$$

where $y_{a_m}(t)$ is composed of an extra additive control signal $x_{a_m}(t)$ plus other error terms as illustrated in Appendix C. Since the evaluation

of the extra additive control signal $x_{am}(t)$ as explained in Appendix C does not in any way alter the design method for the parameter control signals, it will be assumed here that no modification of the A-matrix is needed and that $y(t) = y_p(t)$.

Therefore $y(t)$ given by Eq. (3-19) is composed of two summations

$$y(t) = y_p(t) = - \sum_{i=0}^{n-1} (\alpha_i - x_{\alpha_i}) c^{(i)} + \sum_{j=0}^h (\gamma_j - x_{\gamma_j}) \epsilon^{(j)} . \quad (3-19)$$

The first summation contains the term $c^{(i)}$ which represents the i^{th} derivative of the model subsystem output and is measurable. The second summation contains the term $\epsilon^{(j)}$ which represents the j^{th} derivative of the model subsystem's actuating error, $\epsilon = r - c$, and is also a measurable quantity. The only terms present in $y(t)$ which are both unknown and not measurable, are the incremental values $\{\alpha_i\}$ and $\{\gamma_j\}$ which represent the level changes of the unknown coefficients (Eq. (3-4)) assumed for the system being modelled.

As mentioned previously, the parameter control signals are constructed to satisfy the inequality

$$2 y(t) s(t) \leq 0 \quad (5-13)$$

or

$$- \sum_{i=0}^{n-1} (\alpha_i - x_{\alpha_i}) c^{(i)} s(t) + \sum_{j=0}^h (\gamma_j - x_{\gamma_j}) \epsilon^{(j)} s(t) \leq 0 . \quad (5-17)$$

One way to satisfy the requirement of Eq.(5-17) is to let

$$-(\alpha_i - x_{\alpha_i}) c^{(i)} s(t) \leq 0 \quad \text{for } i = 0, 1, \dots, n-1 \quad (5-18a)$$

and

$$(\gamma_j - x_{\gamma_j}) e^{(j)} s(t) \leq 0 \quad \text{for } j = 0, \dots, h. \quad (5-18b)$$

By using the above two sets of inequalities [Eq.(5-18)], each parameter control signal can be specified to be of proper magnitude and sign to insure that

$$2 y(t) s(t) \leq 0. \quad (5-13)$$

From inequality (3-3) and equation (3-4), the absolute value for the maximum level changes are determined for each member of the sets $\{\alpha_i\}$ and $\{\gamma_j\}$. These maximum level changes are denoted by

$$\alpha_{i_{\max}} = \text{Max} \{ a_{i_{\max}} - A_i, A_i - a_{i_{\min}} \} \quad \text{for } i=0, 1, \dots, n-1$$

and (5-19)

$$\gamma_{j_{\max}} = \text{Max} \{ k_{j_{\max}} - K_j, K_j - k_{j_{\min}} \} \quad \text{for } j=0, 1, \dots, h.$$

The parameter control signals are then constructed by letting

$$x_{\alpha_i} = -\alpha_{i_{\max}} \text{sgn} \{ c^{(i)} \} \text{sgn} \{ s(t) \} \quad \text{for } i=0, 1 \dots n-1 \quad (5-20)$$

and

$$x_{\gamma_j} = \gamma_{j_{\max}} \operatorname{sgn} \{ \epsilon^{(j)} \} \operatorname{sgn} \{ s(t) \} \quad \text{for } j = 0, \dots, h, \quad (5-21)$$

where the sgn function of an arbitrary time function $\omega(t)$ is defined by

$$\operatorname{sgn} \{ \omega(t) \} = \begin{cases} +1 & \text{if } \omega(t) > 0 \\ 0 & \text{if } \omega(t) = 0 \\ -1 & \text{if } \omega(t) < 0 \end{cases}$$

and $c^{(i)}$, $\epsilon^{(j)}$ and $s(t)$ are measurable time functions[†]. With this choice of parameter control signals, $y(t)$ becomes

$$\begin{aligned} y(t) = & - \sum_{i=0}^{n-1} [\alpha_i c^{(i)} + \alpha_{i_{\max}} |c^{(i)}| \operatorname{sgn} \{ s(t) \}] \\ & + \sum_{j=0}^h [\gamma_j \epsilon^{(j)} - \gamma_{j_{\max}} |\epsilon^{(j)}| \operatorname{sgn} \{ s(t) \}] \end{aligned} \quad (5-22)$$

and the expression $y(t) s(t)$ yields

$$\begin{aligned} y(t) s(t) = & - \sum_{i=0}^{n-1} [\alpha_i c^{(i)} s(t) + \alpha_{i_{\max}} |c^{(i)}| |s(t)|] \\ & + \sum_{j=0}^h [\gamma_j \epsilon^{(j)} s(t) - \gamma_{j_{\max}} |\epsilon^{(j)}| |s(t)|] . \end{aligned} \quad (5-23)$$

[†] In some cases, the sgn function may have to be replaced by a saturation function, which approximates a true sgn function, because of solution uniqueness problems associated with relay functions [40].

Since both expressions

$$\alpha_{i_{\max}} |c^{(i)}| |s(t)| \quad (5-24a)$$

and

$$\gamma_{j_{\max}} |\epsilon^{(j)}| |s(t)| \quad (5-24b)$$

are always non-negative and always greater in magnitude than $\alpha_i c^{(i)} s(t)$ and $\gamma_j \epsilon^{(j)} s(t)$ respectively, the product $y(t) s(t)$ will be a negative semi-definite function. Hence, $V(\underline{e})$ will be positive definite while its derivative $\dot{V}(\underline{e})$ will be negative definite and the equilibrium state ($\underline{e} \equiv 0$) of the model-reference error equation is asymptotically stable in the sense of Liapunov's stability theory.

Before proceeding further, it should be noted that the parameter control signals given by Eqs. (5-20) and (5-21) were designed for the model sub-system in Fig. 3-3, which is a feedback configuration. If the reference had been specified as the open-loop system defined by Eqs. (3-23) and (3-24), then the model sub-system would also have been chosen as an open loop system and described by Eqs. (3-25) and (3-26). For this situation, the model-reference error equation is specified by Eq. (3-27) and the function $y(t)$ is defined by Eq. (3-28). Using this new expression for $y(t)$ and repeating the controller design to satisfy the requirement $y(t) s(t) \leq 0$, the set of parameter control signals, defined by Eq. (3-26)

for modeling an open loop system are

$$x_{\alpha_i}^{\star} = -\alpha_{i_{\max}}^{\star} \operatorname{sgn} \{c^{(i)}\} \operatorname{sgn} \{s(t)\} \quad \text{for } i = 0, \dots, n-1 \quad (5-20a)$$

and

$$x_{\gamma_j} = \gamma_{j_{\max}} \operatorname{sgn} \{r^{(j)}\} \operatorname{sgn} \{s(t)\} \quad \text{for } j = 0 \dots h. \quad (5-21a)$$

The determination of the control signals for the nonlinear modeling case is considered in the next section.

5.3.2 Nonlinear Modeling Case

For the nonlinear modeling problem, the function $y(t)$ is specified by Eqs. (4-34), (4-33) and (4-29), which are repeated here:

$$y(t) = y_p(t) + y_a(t) \quad (4-34)$$

where

$$y_p(t) = - \sum_{i=0}^{n-1} (\alpha_i - x_{\alpha_i}) c^{(i)} + \sum_{j=0}^h (\gamma_j - x_{\gamma_j}) f^{(j)}(\epsilon) \quad (4-33)$$

and

$$y_a(t) = \sum_{j=0}^h \left\{ k_j \frac{d^j}{dt^j} \left[\sum_{i=2}^p F_i \left(\sum_{m=1}^i (-1)^m \frac{(i)! \epsilon^{i-m} e^m}{(i-m)! m!} \right) \right] \right\} - x_a. \quad (4-29)$$

As for the linear design case of Section 5.3.1, it is assumed here that the A-matrix elements satisfy the "Strong" Routh-Hurwitz conditions.

The modification otherwise required is illustrated in Appendix C. Note that the evaluation of the extra additive control signal $x_{a_m}(t)$ does not alter the design method for either the parameter control signals or the additive control signal x_a .

The control signals are now constructed so that $y(t)s(t)$ is a negative semi-definite function which is equivalent to showing that both terms $y_p(t)s(t)$ and $y_a(t)s(t)$ are each negative semi-definite functions. First, the term $y_p(t)s(t)$, in which the parameter control signals are present, will be considered. Similar to the linear design case, the only elements present in $y_p(t)$ which are both unknown and not measurable, are the variational values $\{\alpha_i\}$ and $\{\gamma_i\}$ representing the unknown coefficient level changes. By using Eq.(4-3) and inequality (4-2) the maximum level changes given by Eq.(5-19) can be obtained. The set of variables $\{c^{(i)}\}$ represents the derivatives of the model sub-system's output and the set of variables $\{f^{(j)}(\epsilon)\}$ represents the output and derivatives of the nonlinearity. All of these variables are measurable.

One way to satisfy the requirement that

$$y_p(t)s(t) = -\sum_{i=0}^{n-1} (\alpha_i - x_{\alpha_i}) c^{(i)} s(t) + \sum_{j=0}^h (\gamma_j - x_{\gamma_j}) f^{(j)}(\epsilon) s(t) \leq 0 \quad (5-25)$$

is to let

$$-(\alpha_i - x_{\alpha_i}) c^{(i)} s(t) \leq 0 \quad \text{for } i = 0, 1, \dots, n-1 \quad (5-26a)$$

and

$$(\gamma_j - x_{\gamma_j}) f^{(j)}(\epsilon) s(t) \leq 0 \quad \text{for } j = 0, 1, \dots, h \quad (5-26b)$$

Therefore, using the above two sets of inequalities, (5-26a) and (5-26b), each parameter control signal can be specified to be of proper magnitude and sign to insure that

$$y_p(t) s(t) \leq 0 \quad . \quad (5-25)$$

By choosing the parameter control signals as

$$x_{\alpha_i} = -\alpha_{i_{\max}} \operatorname{sgn}[c^{(i)}] \operatorname{sgn}[s(t)] \quad \text{for } i = 0, 1, \dots, n-1 \quad (5-27)$$

and

$$x_{\gamma_j} = \gamma_{j_{\max}} \operatorname{sgn}[f^{(j)}(\epsilon)] \operatorname{sgn}[s(t)] \quad \text{for } j = 0, \dots, h \quad , \quad (5-28)$$

Equation (5-25) becomes

$$\begin{aligned} y_p(t) s(t) = & - \sum_{i=0}^{n-1} [\alpha_i c^{(i)} s(t) + \alpha_{i_{\max}} |c^{(i)}| |s(t)|] \\ & + \sum_{j=0}^h [\gamma_j f^{(j)}(\epsilon) s(t) - \gamma_{j_{\max}} |f^{(j)}(\epsilon)| |s(t)|] \quad . \quad (5-29) \end{aligned}$$

Since both expressions

$$\alpha_{i\max} |c^{(i)}| |s(t)|$$

and

(5-30)

$$\gamma_{j\max} |f^{(j)}(\epsilon)| |s(t)|$$

are always non-negative and always greater in magnitude than

$\alpha_i c^{(i)} s(t)$ and $\gamma_j f^{(j)}(\epsilon) s(t)$ respectively, then $y(t) s(t)$ will be a negative semi-definite function.

Now that the parameter control signals are specified, the additive control signal x_a must be constructed. Defining

$$z(t) = \sum_{i=2}^p F_i \left(\sum_{m=1}^i \frac{(-1)^m (i)! \epsilon^{i-m} e^m}{(i-m)! m!} \right) \quad (5-31)$$

to simplify notations, and subdividing the additive control signal into $h+1$ components, so that it may be represented by the summation

$$x_a = \sum_{j=0}^h x_{a_j} \quad , \quad (5-32)$$

the term $y_a(t)$ can be expressed as

$$y_a(t) = \sum_{j=0}^h [k_j z^{(j)} - x_{a_j}] \quad . \quad (5-33)$$

The function $z(t)$ defined in Eq.(5-31) consists of the known nonlinearity coefficients $\{F_i\}$ and the measurable quantities $\epsilon(t)$ and $e(t)$. Since the derivatives of $z(t)$ can be generated and the maximum values of the unknown coefficients $\{k_j\}$ are specified by inequality (4-2), then one may set the components of the additive control signal equal to

$$x_{aj} = k_{j\max} |z^{(j)}| \operatorname{sgn} \{s(t)\} , \quad (5-34)$$

so that

$$x_a = \sum_{j=0}^h k_{j\max} |z^{(j)}| \operatorname{sgn} \{s(t)\} . \quad (5-35)$$

Substituting from Eq.(5-34) into Eq.(5-33), the expression $y_a(t) s(t)$ becomes

$$y_a(t) s(t) = \sum_{j=0}^h [k_j z^{(j)} s(t) - k_{j\max} |z^{(j)}| |s(t)|] . \quad (5-36)$$

Since the term $k_{j\max} |z^{(j)}| |s(t)|$ is non-negative and always greater in magnitude or equal to $k_j z^{(j)} s(t)$, expression $y_a(t) s(t)$, specified by Eq.(5-36), is a negative semi-definite function.

Because the control signals have been determined to insure that both expressions $y_p(t) s(t)$ and $y_a(t) s(t)$ are negative semi-definite functions, their sum $y(t) s(t)$ must also be negative semi-definite and

the requirement of inequality (5-13) has been satisfied. Hence $V(\underline{e})$ will be positive definite while its derivative $\dot{V}(\underline{e})$ is negative definite, insuring that the equilibrium state ($\underline{e} \equiv 0$) of the nonlinear model-reference error equation is asymptotically stable. This implies that the error between the nonlinear model output and the output of the system being modelled tends toward zero.

The parameter control signals developed for the linear modeling case and the parameter control signals plus the additive control signal developed for the nonlinear modeling case, do not possess any memory capabilities. These control signals continuously need the error information generated by the model-reference configuration, and the output of the system being modelled must continuously be available. The next section discusses two schemes which incorporate memory ("learning") capabilities in the parameter control signals.

5.4 Parameter Control Signals with Memory ("Learning") Capability

It is desirable for the parameter control signals to possess some memory ("learning") capabilities so that the model's postulated nominal parameter values are permanently updated to eventually eliminate the need of the error information being supplied by the model-reference system configuration. The interval of time required for the model to update or "learn" a set of parameter values which produces a model response that is

equivalent to the response of the given reference being modelled could be referred to as a "learning" period. This "learning" period will vary for different systems and depend upon the characteristics of the input signal $r(t)$. It should be noted that not in every situation will the "learned" model parameter values be identical to the reference system parameters. Sometimes, the set of "learned" parameters yield an "equivalent" system with respect to input-output performance only, and may also depend upon the input $r(t)$. For example, if the sinusoidal input $r(t) = \sin(\omega t)$ is applied, the model's parameters are adjusted until the steady state phasors of both the model and reference system are equal. Since it is possible that many different sets of model parameters exist for which the desired phasor is obtained, the input-output performance of both the model and reference system can be identical, and yet their parameter values may differ.

One possible means for determining when a new set of model parameters have been established is by examination of the model parameter waveforms throughout the "learning" interval. During the initial segment of a "learning" interval, the "learned" parameter waveforms usually fluctuate over a wide range of values as the model tries to conform to the reference system. In the later portion of the "learning" interval, the fluctuations die down and the parameters tend toward their desired levels. Therefore once the fluctuations have ceased, it can be assumed that a

new set of model parameter values have been established. If a new parameter level change should occur in the given reference system, the model parameter waveforms will again begin to fluctuate and a new "learning" period is initiated.

In the next section, two methods for constructing parameter control signals with memory capabilities are presented.

5.4.1 Method No.1

Sets of parameter control signals with memory ("learning") capabilities, that have been successfully tested by computer simulation are given by Eqs.(5-37) and (5-38).

$$x_{\alpha_i} = -\alpha_{i_{\max}} \operatorname{sgn}\{c^{(i)}\} \operatorname{sgn}\{s(t)\} - \alpha_{i_{\max}} \int_{\tau} \operatorname{sgn}\{c^{(i)} s(t)\} dt \quad (5-37)$$

$$x_{\gamma_j} = \gamma_{j_{\max}} \operatorname{sgn}\{e^{(j)}\} \operatorname{sgn}\{s(t)\} + \gamma_{j_{\max}} \int_{\tau} \operatorname{sgn}\{e^{(j)} s(t)\} dt$$

and

$$x_{\alpha_i} = -\alpha_{i_{\max}} \operatorname{sgn}\{c^{(i)}\} \operatorname{sgn}\{s(t)\} - \alpha_{i_{\max}} \int_{\tau} \operatorname{sgn}\{c^{(i)} s(t)\} dt \quad (5-38)$$

$$x_{\gamma_j} = \gamma_{j_{\max}} \operatorname{sgn}[f^{(j)}(e)] \operatorname{sgn}[s(t)] + \gamma_{j_{\max}} \int_{\tau} \operatorname{sgn}[f^{(j)}(e) s(t)] dt$$

The digital computer simulation results are presented in Chapter 6. Note

that Eqs.(5-37) and (5-38) are for the linear and nonlinear modeling problems respectively, and that τ represents the "learning" interval in these equations.

The above parameter control signals may violate the requirement that the term $2 y(t) s(t)$, which appears in $\dot{V}(\underline{e})$, be negative-semi-definite; but it should be noted that the conditions of Liapunov's theory, $V(\underline{e})$ be positive definite and its derivative $\dot{V}(\underline{e})$ be negative-definite, are only sufficient conditions for asymptotic stability and are not necessary.

Although no mathematical proof is presented here to verify that the control signals of Eqs.(5-37) and (5-38) yield an asymptotically stable model-reference error, a heuristic justification is presented for the case when only one parameter of the reference system is unknown. Furthermore, the simulation studies presented in Chapter 6 illustrate the model's "learning" capabilities for multiple unknown parameters and shows that the model-reference error tends toward zero when the (memory component) integral is present in the control signals, in a similar manner as the response obtained when the parameter control signals of Section 5.3 are used.

The following is a heuristic justification for the addition of a memory component, which is basically an integral, to the parameter control signal when only one reference parameter is unknown. It is based on the fact that if the parameter of the reference system differs from the model

parameter, then when the model-reference system is initialized at $t=0$, the integral initially tends in the direction to decrease the variation between the reference system parameter and the model parameter. This correction can be interpreted as updating the known nominal value of the model and reference parameter and thereby decreasing the magnitude of the unknown variational part of the reference parameter. For this discussion it is immaterial which parameter is chosen to be different because the argument remains the same no matter which one parameter is selected. For illustrative purposes, Example 2 used in Chapter 3 for the case $n=5$, $h=2$ will be used and it will be assumed that only the parameter a_1 is unknown so that the only parameter control signal needed is x_{α_1} . Therefore $a_1 = A_1 + \alpha_1$, $l_1 = A_1 + x_{\alpha_1}$, $a_4 = l_4 = A_4$, $a_3 = l_3 = A_3$, $a_2 = l_2 = A_2$, $a_0 = l_0 = A_0$, $k_2 = g_2 = K_2$, $k_1 = g_1 = K_1$ and $k_0 = g_0 = K_0$. The error equation becomes

$$\begin{aligned} e^{(5)} + A_4 e^{(4)} + A_3 e^{(3)} + (A_2 + K_2) e^{(2)} + (A_1 + \alpha_1 + K_1) e^{(1)} \\ + (A_0 + K_0) e = -(\alpha_1 - x_{\alpha_1}) c^{(1)} \end{aligned} \quad (5-39)$$

and the model equation is given by

$$c^{(5)} + A_4 c^{(4)} + A_3 c^{(3)} + A_2 c^{(2)} + (A_1 + x_{\alpha_1}) c^{(1)} + A_0 c = K_2 \epsilon^{(2)} + K_1 \epsilon^{(1)} + K_0 \epsilon. \quad (5-40)$$

Since $\epsilon = r - c$, the model equation can also be written as

$$\begin{aligned} c^{(5)} + A_4 c^{(4)} + A_3 c^{(3)} + (A_2 + K_2) c^{(2)} + (A_1 + K_1 + x_{\alpha_1}) c^{(1)} + (A_0 + K_0) c \\ = K_2 r^{(2)} + K_1 r^{(1)} + K_0 r \end{aligned} \quad (5-41)$$

Using Eq.(5-37), the control signal x_{α_1} is

$$x_{\alpha_1} = -\alpha_{1\max} \operatorname{sgn}\{c^{(1)}\} \operatorname{sgn}\{s(t)\} - \alpha_{1\max} \int_0^t \operatorname{sgn}\{c^{(1)}\} \operatorname{sgn}\{s(\tau)\} d\tau \quad (5-42)$$

where

$$s(t) = m_{14} e + m_{24} e^{(1)} + m_{34} e^{(2)} + m_{44} e^{(3)}$$

and m_{14} , m_{24} , m_{34} and m_{44} are positive constants. Initially at $t = 0$, the initial conditions on the reference system are identical to the initial conditions on the model. At present it will be assumed that only the input $r(t)$ is driving the system so that $c_d^{(i)}(0) = c^{(i)}(0) \equiv 0$ and $e^{(i)}(0) = 0$ for $i = 0, 1 \dots 4$.

After the input has been applied for a time interval Δ , which is sufficiently small with respect to the systems time constants, the signs of all the variables can be determined. If the value of the input is such that the function $R(t) = K_2 r^{(2)} + K_1 r^{(1)} + K_0 r$ goes initially in a positive direction, then from the model Eq.(5-41), it is seen that $c^{(1)}$ for

$i = 0, \dots, 5$ will be positive at $t = \Delta$. If instead $R(t)$ had gone negative after $t = 0$, then $c^{(1)}$ would have been negative. Also at $t = 0$, $x_{\alpha_1} = 0$ and the sign of the error variable can be determined by the sign of the expression $\alpha_1 c^{(1)}$ in Eq.(5-39). Therefore the value of $\text{sgn}\{s(t)\}$ can be found and the control signal sign can be determined at $t = \Delta$.

Table 5-3 tabulates the signs for the variables at $t = \Delta$, almost immediately after the system is initialized.

$R(t)$	α_1	$c^{(1)}$	$-\alpha_1 c^{(1)}$	$s(t)$	x_{α_1}	$\int$
+	+	+	-	-	+	+
-	+	-	+	+	+	+
+	-	+	+	+	-	-
-	-	-	-	-	-	-

Table 5-3 Variable Signs at $t = \Delta$

Without the integral present in the control signal, Liapunov's theory predicts that the model-reference error must tend toward zero. With the integral present, Table 5-3 shows that for a model-reference set-up with one unknown parameter, the function $-\text{sgn}\{c^{(1)} s(t)\}$ is of the same sign as α_1 at $t = \Delta$, and therefore the integral initially tends in the proper direction to correct the model parameter and update the assumed nominal value chosen within the reference parameter range.

An alternate method for constructing parameter control signals that possess memory ("learning") capabilities is next presented.

5.4.2 Method No.2

An alternate set of parameter control signals, that possess memory ("learning") capabilities, can be developed by constructing a new candidate Liapunov function

$$V_T(\underline{e}) = V(e) + V_1 \quad (5-43)$$

where $V(\underline{e})$ is the quadratic form specified by Eq.(5-2), and V_1 is an additional positive quadratic expression, proposed by Parks^[60], and specified by

$$V_1 = \sum_{i=0}^{n-1} \frac{1}{\beta_i} (\alpha_i - x_{\alpha_i})^2 + \sum_{j=0}^h \frac{1}{\Delta_j} (\gamma_j - x_{\gamma_j})^2 \quad (5-44)$$

where β_i , for $i=0, \dots, n-1$, and Δ_j , for $j=0, \dots, h$, are positive constants.

It is assumed that the variational components, $\{\alpha_i\}$ and $\{\gamma_j\}$, of the reference system parameters have currently settled at a set of values so that

$$\frac{d\alpha_i}{dt} = \frac{d\gamma_j}{dt} \equiv 0$$

for all values of the indices i and j . Therefore, the time derivative of

$V_T(e)$, given by Eq.(5-43), along the error equation trajectory is

$$\dot{V}_T(e) = \dot{V}(e) - 2 \sum_{i=0}^{n-1} \frac{1}{\beta_i} (\alpha_i - x_{\alpha_i}) \dot{x}_{\alpha_i} - 2 \sum_{j=0}^h \frac{1}{\Delta_j} (\gamma_j - x_{\gamma_j}) \dot{x}_{\gamma_j} . \quad (5-45)$$

Substituting the expression for $\dot{V}(e)$ given by Eq.(5-12), yields

$$\dot{V}_T(e) = -\underline{e}^T D \underline{e} + 2 \left\{ y(t) s(t) - \sum_{i=0}^{n-1} \frac{1}{\beta_i} (\alpha_i - x_{\alpha_i}) \dot{x}_{\alpha_i} - \sum_{j=0}^h \frac{1}{\Delta_j} (\gamma_j - x_{\gamma_j}) \dot{x}_{\gamma_j} \right\} . \quad (5-46)$$

Using the procedure explained in Sec.5.2, the components of the M-matrix are determined such that both the M and D matrices are positive definite. If the control signals are now constructed such that

$$y(t) s(t) - \sum_{i=0}^{n-1} \frac{1}{\beta_i} (\alpha_i - x_{\alpha_i}) \dot{x}_{\alpha_i} - \sum_{j=0}^h \frac{1}{\Delta_j} (\gamma_j - x_{\gamma_j}) \dot{x}_{\gamma_j} \leq 0 , \quad (5-47)$$

then $\dot{V}_T(e)$ becomes a negative definite function and $V_T(e)$, given by Eq.(5-43) will qualify as a Liapunov function. Hence, the model-reference error, Eq.(5-1), is asymptotically stable and the error $e(t)$ between the model and reference system will tend toward zero.

The parameter control signals may now be constructed by requiring inequality (5-47) to be equal to zero. It should be noted, that during the development of the control signals in Section 5.3, inequality (5-13) was not set equal to zero because the unknown incremental values $\{\alpha_i\}$ and

$\{\gamma_j\}$ are present in the term $y(t)$, and therefore the control signals would have been required to contain unknown quantities. In the present development, the unknown incremental values, which are present in the term $y(t)$, are eliminated when the equality condition is chosen in Eq.(5-47).

Substituting the expression for $y(t)$ given by Eq.(3-19), Eq.(5-47) becomes

$$\begin{aligned} & - \sum_{i=0}^{n-1} (\alpha_i - x_{\alpha_i}) c^{(i)} s(t) + \sum_{j=0}^h (\gamma_j - x_{\gamma_j}) \epsilon^{(j)} s(t) \\ & - \sum_{i=0}^{n-1} \frac{1}{\beta_i} (\alpha_i - x_{\alpha_i}) \dot{x}_{\alpha_i} - \sum_{j=0}^h \frac{1}{\Delta_j} (\gamma_j - x_{\gamma_j}) \dot{x}_{\gamma_j} = 0 \end{aligned} \quad (5-48)$$

and can then be rearranged to yield

$$\begin{aligned} & - \sum_{i=0}^{n-1} \left\{ (\alpha_i - x_{\alpha_i}) (c^{(i)} s(t) + \frac{1}{\beta_i} \dot{x}_{\alpha_i}) \right\} + \sum_{j=0}^h \left\{ (\gamma_j - x_{\gamma_j}) (\epsilon^{(j)} s(t) - \frac{1}{\Delta_j} \dot{x}_{\gamma_j}) \right\} = 0. \end{aligned} \quad (5-49)$$

Equation (5-49) consists of two summations. Letting

$$c^{(i)} s(t) + \frac{1}{\beta_i} \dot{x}_{\alpha_i} = 0 \quad (5-50)$$

for $i = 0, \dots, n-1$, and

$$\epsilon^{(j)} s(t) - \frac{1}{\Delta_j} \dot{x}_{\gamma_j} = 0 \quad (5-51)$$

for $j = 0, \dots, h$.

both summations are equal to zero, thereby satisfying Eq.(5-47). Solving Eqs.(5-50) and (5-51), the parameter control signals are

$$x_{\alpha_i} = -\beta_i \int_0^{\tau} c^{(i)} s(t) dt \quad i = 0, \dots, n-1 \quad (5-52)$$

and

$$x_{\gamma_j} = \Delta_j \int_0^{\tau} e^{(j)} s(t) dt \quad j = 0, \dots, h \quad (5-53)$$

where the integration limit τ represents the "learning" interval.

The only requirement on the constants β_i and Δ_j in Eqs.(5-52) and (5-53), for the function $V_T(e)$ of Eq. (5-43), to qualify as a Liapunov function, is that these constants be positive.

Simulation studies (Chapter 6) reveal that the model-reference error approaches zero more rapidly as the values of β_i and Δ_j are increased, but, the "learning" interval needed for the parameter control signals also increases. Different sets of values for $\{\beta_i\}$ and $\{\Delta_j\}$ may have to be tried to yield both acceptable convergence time for the model-reference error and an acceptable "learning" interval duration. One means to determine an initial set of values for the constants $\{\beta_i\}$ and $\{\Delta_j\}$ is to first examine the reference system variables $c_d^{(i)}$ and $e_d^{(j)}$ and observe their maximum magnitudes $c_{d\max}^{(i)}$ and $e_{d\max}^{(j)}$. The constants $\{\beta_i\}$ and $\{\Delta_j\}$ are initially chosen such that the products $\beta_i c_{d\max}^{(i)}$ for $i = 0, 1 \dots n-1$, and $\Delta_j e_{d\max}^{(j)}$

for $j = 0, 1 \dots h$, are within the same order of magnitude. It is then advisable to test the performance of the model and control signals, using these obtained constants, in a model-reference configuration in which the reference system is completely known. This artificial reference system should be identical in form with the model except that the parameter control signals are replaced by known constants. If the model parameter control signals yield satisfactory performance with this known reference system, then the known reference can be replaced by the actual physiological system under investigation.

The control signals given by Eqs.(5-52) and (5-53) are for the linear feedback modeling problem. For the nonlinear modeling problem, $y_p(t)$ as given by Eq.(4-33) would be substituted into Eq.(5-47) and the parameter control signals become

$$x_{\alpha_i} = -\beta_i \int_0^{\tau} c^{(i)} s(t) dt \quad \text{for } i = 0, \dots, n-1 \quad (5-54)$$

and

$$x_{\gamma_j} = \Delta_j \int_0^{\tau} f^{(j)}(\epsilon) s(t) dt \quad \text{for } j = 0, \dots, h \quad (5-55)$$

If the reference system had been specified as the open-loop system defined by Eqs.(3-23) and (3-24), then $y(t)$ is specified by Eq.(3-28) and when $y(t)$ is substituted into Eq.(5-47), the set of parameter control signals, defined by Eq.(3-26) for modeling a linear open loop system

becomes

$$x_{\alpha_i}^{\star} = -\beta_i \int_0^{\tau} c^{(i)} s(t) dt \quad \text{for } i = 0, \dots, n-1 \quad (5-56)$$

and

$$x_{\gamma_j} = \Delta_j \int_0^{\tau} r^{(j)} s(t) dt \quad \text{for } j = 0, \dots, h. \quad (5-57)$$

In the next chapter, simulation studies are presented to illustrate the controller design and test the performance of the model-reference system configuration for a variety of artificially constructed modeling problems.

CHAPTER 6

DIGITAL COMPUTER SIMULATION STUDIES

6.1 Introduction

This chapter presents several examples to illustrate the design of the model parameter control signals and display their "learning" capabilities. To create an artificial situation in order to test the performance of the designed controllers, all of the reference systems considered in this chapter are completely specified with all of their parameters known. In each example, some or all of the initial values assumed for the model's parameters are purposely chosen to be different in value than the reference system's parameters. The reference system and model are arranged in the model-reference configuration as developed in Chapter 3, where both model and reference receive the same input simultaneously. Under the influence of the parameter control signals, the model's parameters are updated and it is shown that the model-reference error approaches zero.

Plots of the controlled model parameters are shown to tend toward the chosen values of the reference system parameters and thereby illustrates the "learning" capabilities of the model. Other pertinent model characteristic quantities are continuously calculated and are graphed for comparison with those of the reference system, in order to further check on the adaptive properties of the controller.

6.2 Example No.1 - First-Order System with Two Controlled Parameters

This example illustrates the performance of the parameter control signals when applied to a first-order feedback model system in which two of the parameters are identified. Two separate simulations are presented for this example. In the first simulation, each parameter control signal consists of two components: a control signal component without memory capabilities as developed in Section 5.3 and a control signal component with memory capabilities as developed in Section 5.4.2. For the second simulation, only the control signals with memory capabilities are used.

The first order reference system considered is depicted in Fig. 6-1, and the nominal values for the parameters "a" and "k" are chosen to be $A=6$ and $K=9$. It is also assumed for this example that the maximum magnitudes of the parameter level changes, " α " and " γ ", are 5 and 8 respectively and therefore using the same terminology as in Chapter 5, we have $\alpha_{\max} = 5$ and $\gamma_{\max} = 8$. When the reference system and its model are arranged in the model-reference system configuration discussed in Chapter 3, the system shown in Figure 6-2 is obtained.

As mentioned previously, the parameter control signals x_γ and x_α each consist of two components. The components without memory capabilities will be denoted by $\hat{x}_\alpha$ and $\hat{x}_\gamma$, and are given by Eqs.(5-20) and (5-21) respectively.

Those components with memory capabilities are denoted by $(x_\alpha)_m$ and $(x_\gamma)_m$, and are given by Eqs.(5-52) and (5-53) respectively. There-

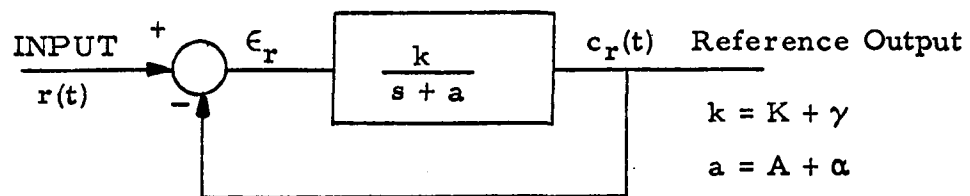


Figure 6-1 First order reference system

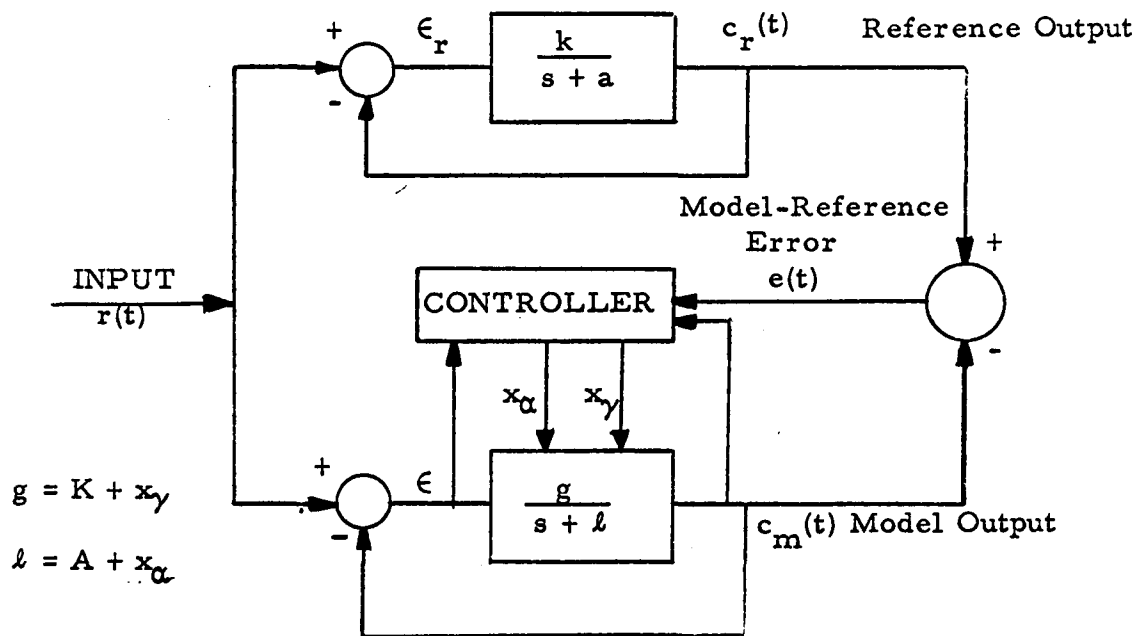


Figure 6-2 Model-reference system configuration for Example 1

fore, the parameter control signals become

$$\begin{aligned} x_{\alpha} &= \hat{x}_{\alpha} + (x_{\alpha})_m \\ x_{\gamma} &= \hat{x}_{\gamma} + (x_{\gamma})_m \end{aligned} \quad (6-1a)$$

where

$$\begin{aligned} \hat{x}_{\alpha} &= -\alpha_{\max} \operatorname{sgn} \{c_m s(t)\} \\ \hat{x}_{\gamma} &= \gamma_{\max} \operatorname{sgn} \{\epsilon s(t)\} \end{aligned}$$

and

$$\begin{aligned} (x_{\alpha})_m &= -\alpha_{\max} \int_0^T c_m s(t) dt \\ (x_{\gamma})_m &= \gamma_{\max} \int_0^T \epsilon s(t) dt . \end{aligned} \quad (6-1b)$$

For a first-order system, the M matrix of the Liapunov function

$V(\underline{e}) = \underline{e}^T \underline{M} \underline{e}$ reduces to the positive scalar constant m_{11} and

$s(t) = m_{11} e(t)$. In this example, the constant m_{11} is chosen to be 10,000, a value which yields good system performance.

To illustrate the model's "learning" capabilities, the level changes (incremental values) for the reference parameters are set equal to $\alpha = +2$, $\gamma = -5$, and the square-wave shown in Figure 6-3 is applied as the input $r(t)$ to the model-reference system configuration.

Behavior of the parameter control signal components without memory capabilities needn't be considered to demonstrate the model's "learning" ability. These components, $\hat{x}_\alpha$ and $\hat{x}_\gamma$, fluctuate back and forth between the values $\pm\alpha_{\max}$ and $\pm\gamma_{\max}$ respectively depending upon the sign of the variables $e(t)$, $c_m(t)$ and $\epsilon(t)$. Only the memory components of the model's parameter control signals need be investigated to demonstrate the model's "learning" ability. These control signal components, $(x_\alpha)_m$ and $(x_\gamma)_m$, are plotted in Figure 6-4 and it is seen that they do approach the values +2 and -5 respectively, and thereby indicating that the model has "learned" the parameter level changes present in the reference system.

Comparisons of pertinent characteristic quantities (i.e., output, time-constant, steady-state output ratio) between the reference system and model are also presented in Figure 6-5.

To further test the operation of the parameter control signal's memory components, in the second simulation of the 1st order system, the non-memory components, $\hat{x}_\alpha$ and $\hat{x}_\gamma$, of the parameter control signals in Eq.(6-1) are omitted so that

$$\begin{aligned} x_\alpha &= (x_\alpha)_m = -\alpha_{\max} \int_0^T c_m e(t) dt \\ x_\gamma &= (x_\gamma)_m = \gamma_{\max} \int_0^T \epsilon e(t) dt \end{aligned} \quad (6-2)$$

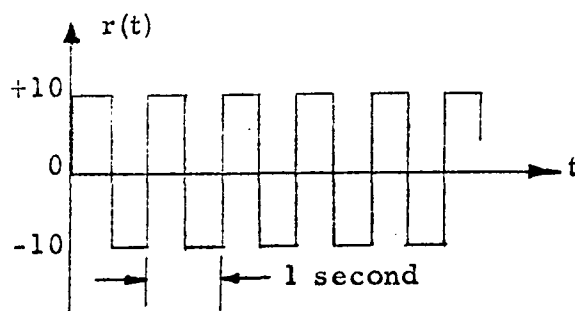


Figure 6-3 Model-reference system input: $r(t)$

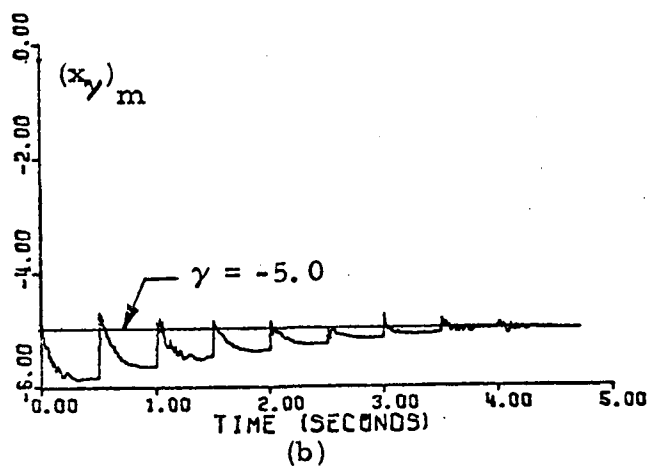
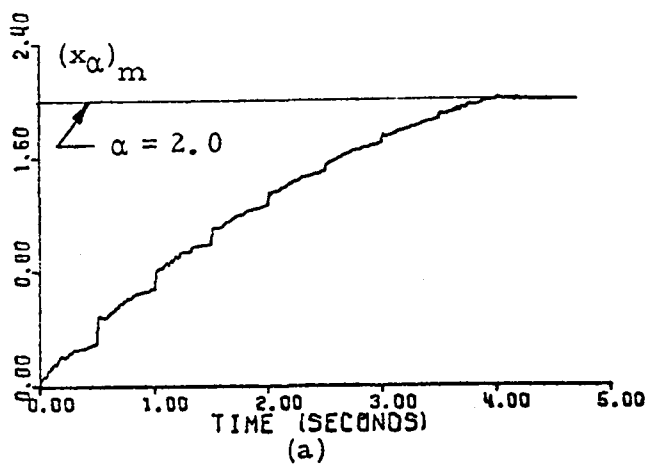


Figure 6-4 Memory components for the model's parameter control signals (Equation (6-1))
 (a) $(x_\alpha)_m$, (b) $(x_\gamma)_m$

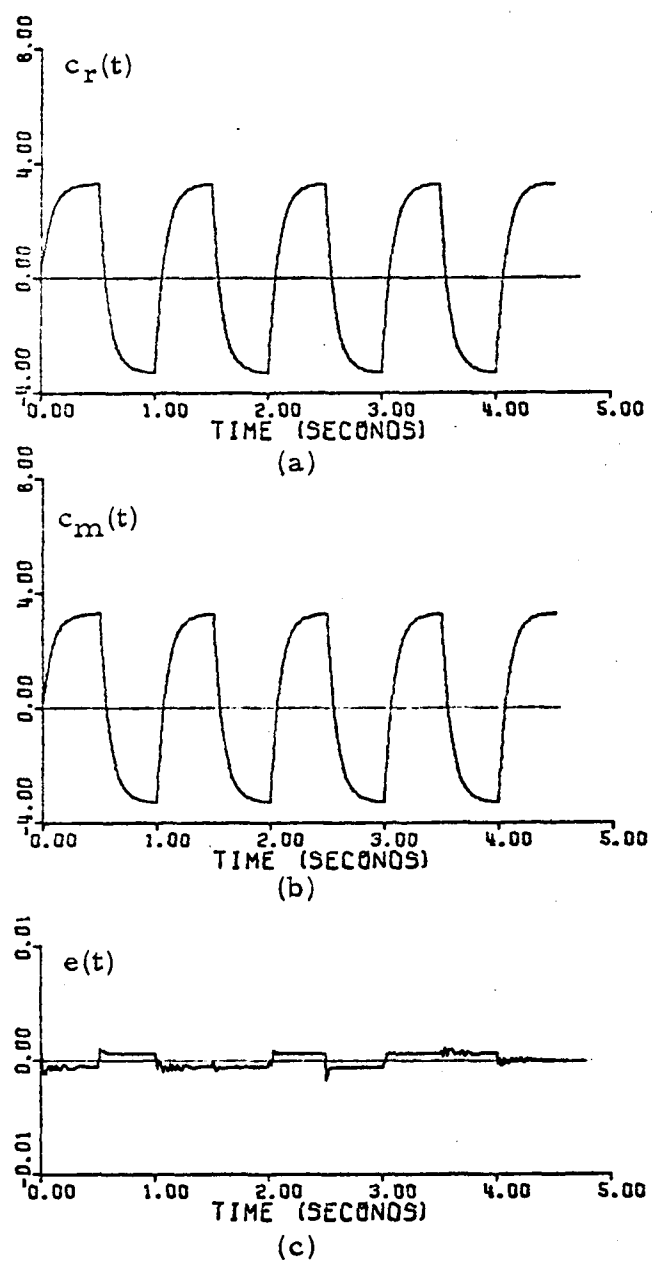


Figure 6-5 Comparisons between the first-order reference system and model using the parameter control signals of Equation (6-1)

- (a) Reference system output: $c_r(t)$
- (b) Model output: $c_m(t)$
- (c) Model-reference system error: $e(t)$

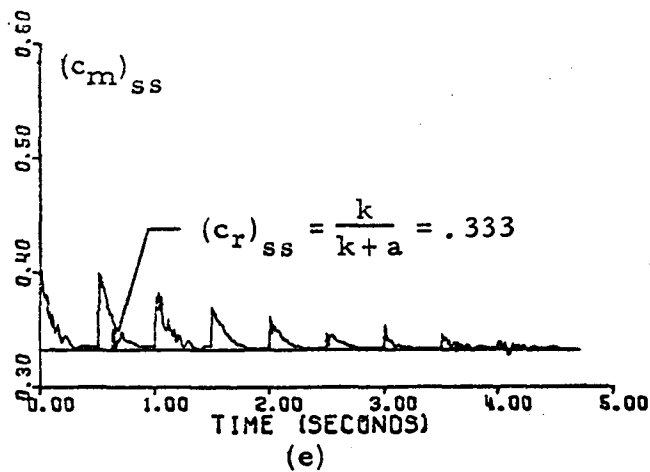
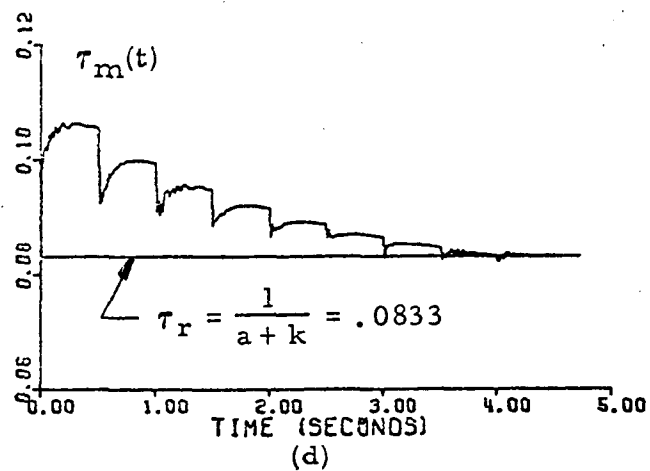


Figure 6-5 (continued)

(d) Instantaneous model time-constant: $\tau_m(t)$

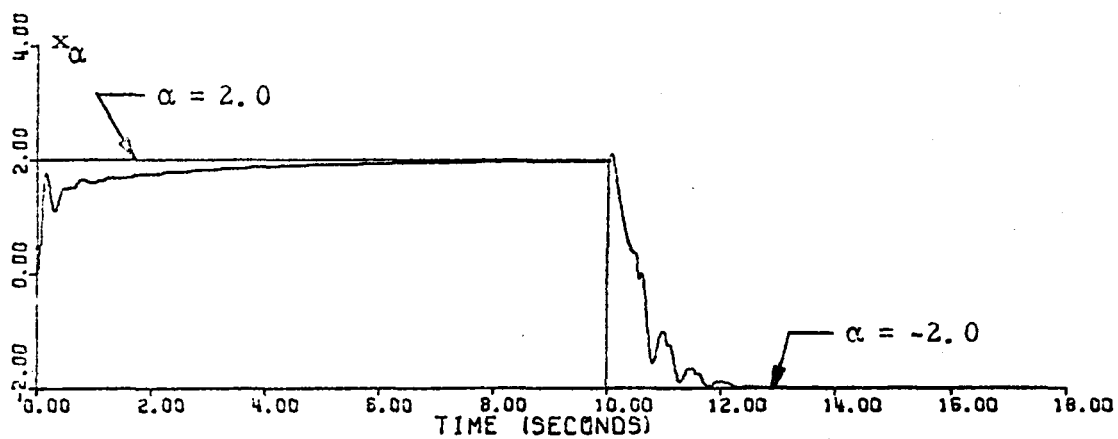
(e) Instantaneous model steady state output for a step input: $(c_m)_{ss}$

The same input $r(t)$, shown in Figure 6-3, is applied to the model-reference system and again the reference parameter level changes are initially set equal to $\alpha = 2$ and $\gamma = -5$. As an additional test to illustrate the model's "learning" capabilities, the reference parameters "a" and "k" are changed in value after $t = t_1 = 10$ seconds to 4 and 7 respectively so that the new parameter level changes α and γ become $\alpha = -2$, $\gamma = -2$. As before, the model "learns" the first set of parameters and after the level change which occurs at $t = t_1$ the model "learns" the second set of parameter values. The results of this simulation are given in Figures 6-6 and 6-7.

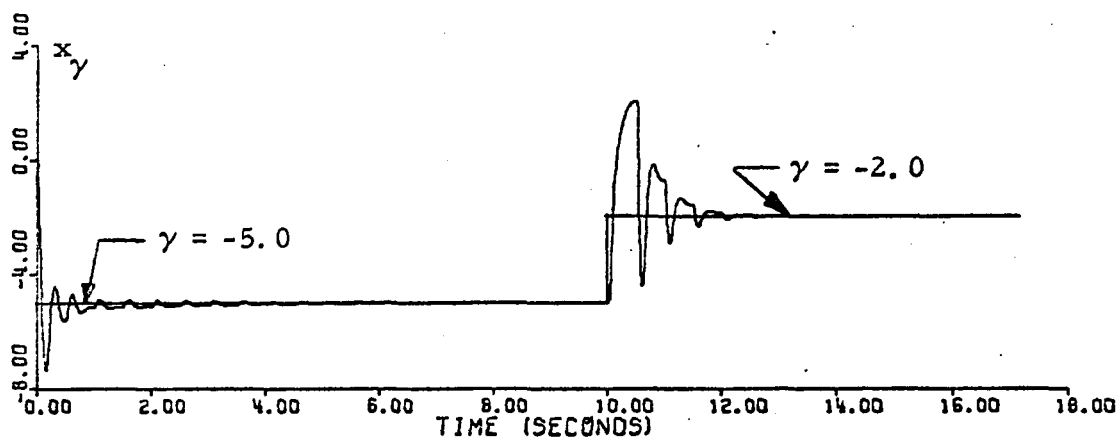
In the next section, a 2nd order system is considered with only one parameter permitted to vary at a time.

6.3 Example No.2 - Second Order System with One Controlled Parameter

For this set of examples, the reference system is shown in Figure 6-8 and the model-reference system configuration is indicated in Figure 6-9. Only one parameter is controlled in each simulation and the parameter control signals used are defined by Eq.(5-37). All of the reference parameter values and the control signals for each run are listed in Table 6-1.



(a)



(b)

Figure 6-6 Memory components for the parameter control signals defined by Equation (6-2)

(a) x_α , (b) x_γ

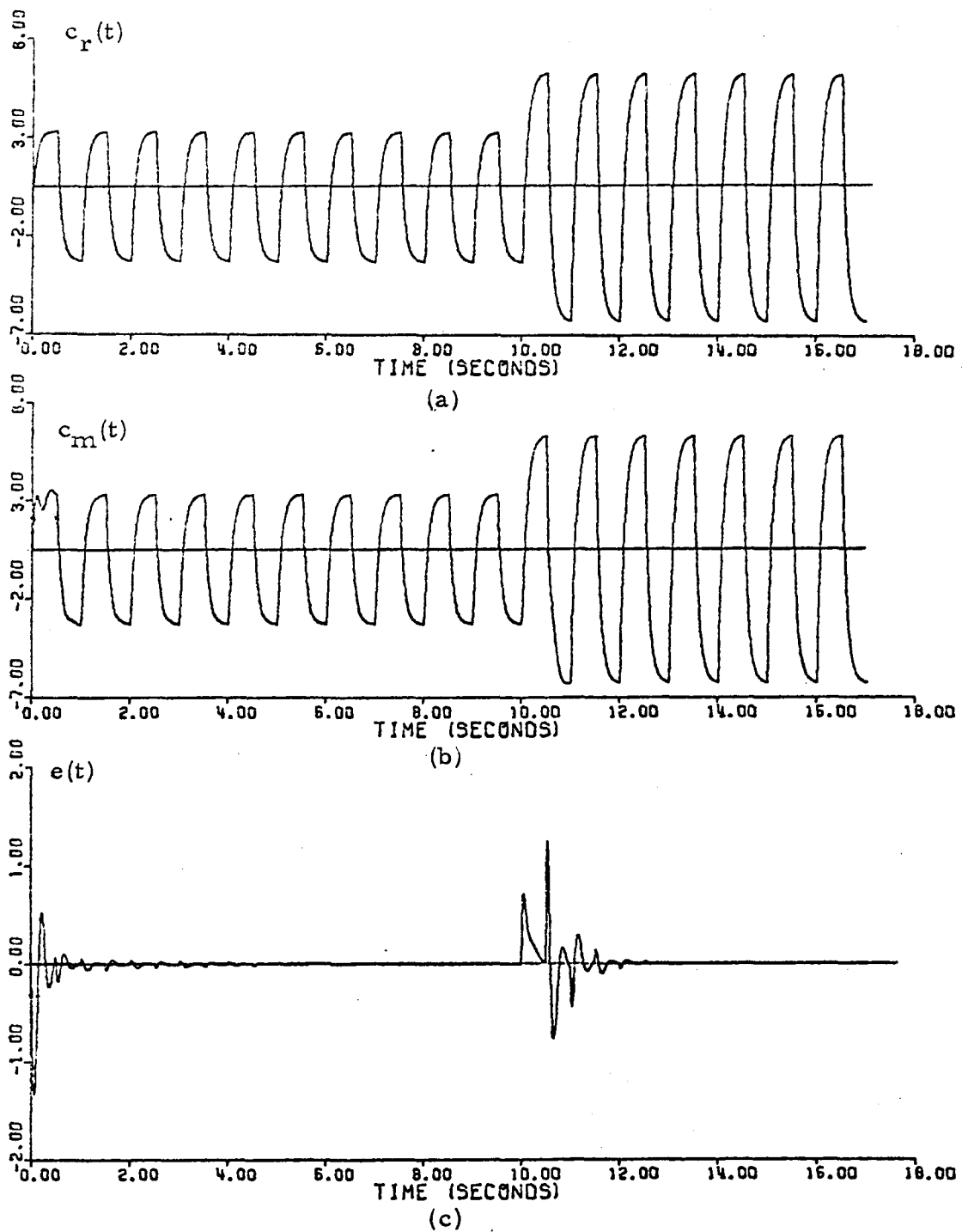
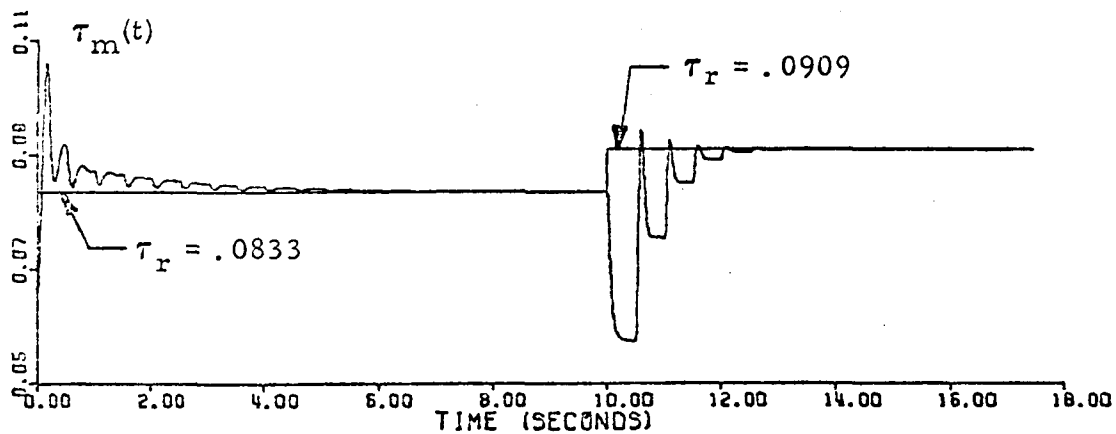
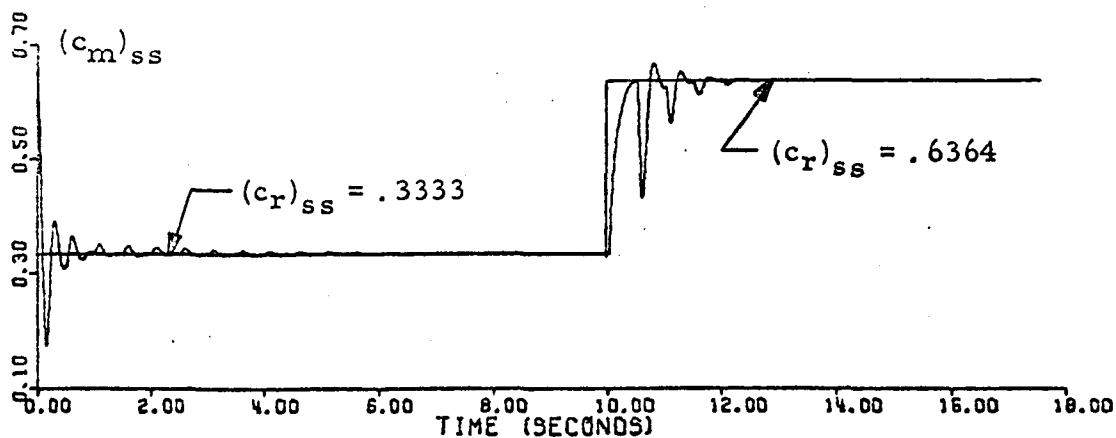


Figure 6-7 Comparison between 1st-order reference system and model using parameter control signals defined by Equation (6-2):
 (a) Reference system output - $c_r(t)$
 (b) Model output - $c_m(t)$
 (c) Model-reference error - $e(t)$



(d)



(e)

Figure 6-7 (Continued)

- (d) Instantaneous model time-constant - $\tau_m(t)$
- (e) Instantaneous model steady state output for a unity step input - $(c_m)_{ss}$

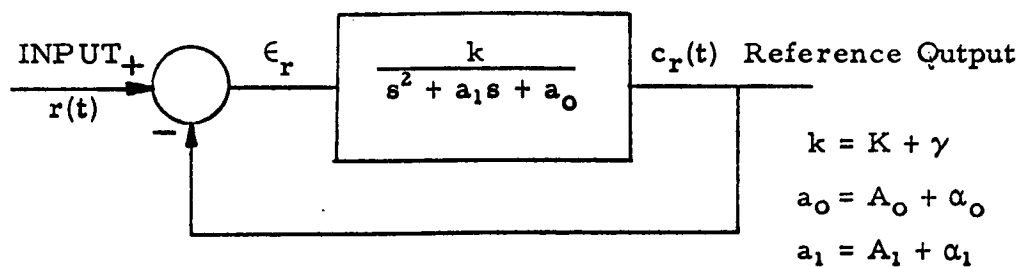
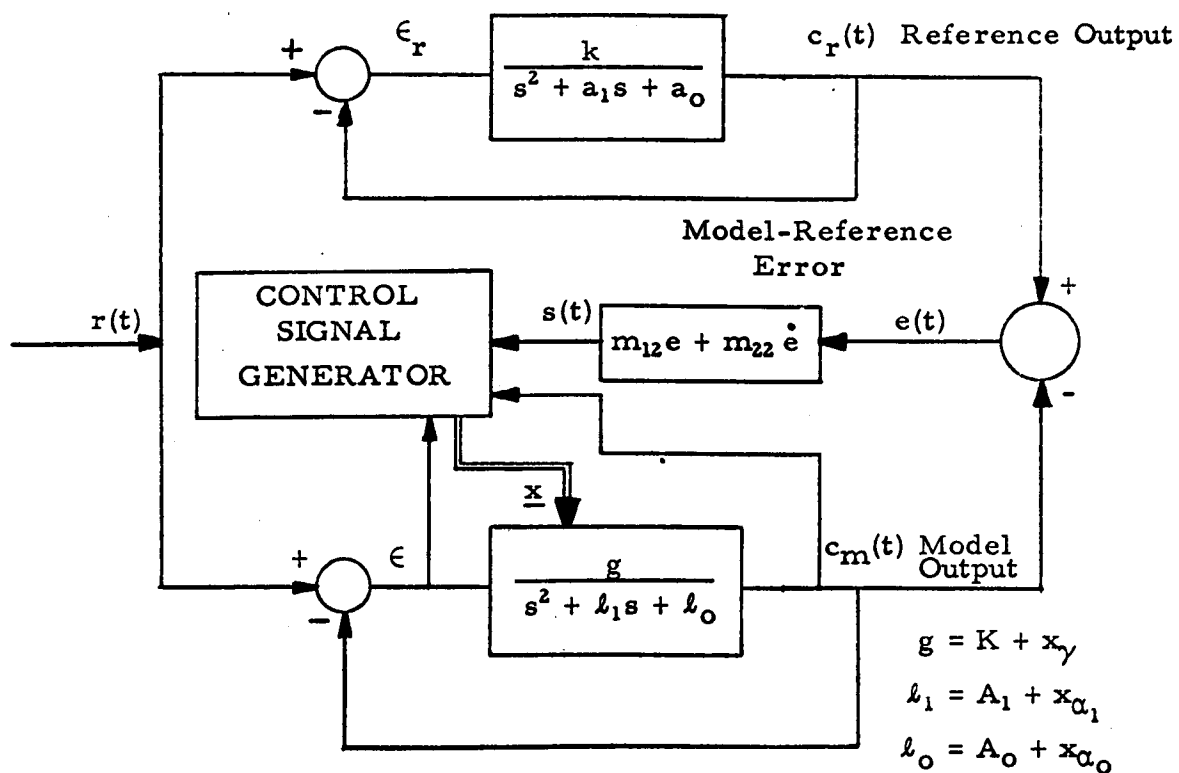


Figure 6-8 Second order reference system



	A_0, A_1, K	$\alpha_0, \alpha_1, \gamma$	$\begin{pmatrix} \alpha_0 \\ \alpha_1 \end{pmatrix} \gamma_{\max}$	Parameter Control Signals
Run No. 1	$A_0 = 4$ $A_1 = 5$ $K = 5$	$\alpha_0 \equiv 0$ $\alpha_1 \equiv 0$ $\gamma = -4$	$\alpha_{0\max} = 0$ $\alpha_{1\max} = 0$ $\gamma_{\max} = 5$	$x_{\alpha_0} = x_{\alpha_1} = 0$ $x_{\gamma} = \gamma_{\max} \operatorname{sgn} \{\epsilon s(t)\}$ $+ \gamma_{\max} \int_0^t \operatorname{sgn} \{\epsilon s(t)\} dt$
Run No. 2	$A_0 = 4$ $A_1 = 0$ $K = 5$	$\alpha_0 \equiv 0$ $\alpha_1 = 3$ $\gamma \equiv 0$	$\alpha_{0\max} = 0$ $\alpha_{1\max} = 5$ $\gamma_{\max} = 0$	$x_{\alpha_0} = x_{\gamma} = 0$ $x_{\alpha_1} = -\alpha_{1\max} \operatorname{sgn} \{\dot{c}_m s(t)\}$ $- \alpha_{1\max} \int_0^t \operatorname{sgn} \{\dot{c}_m s(t)\} dt$
Run No. 3	$A_0 = 0$ $A_1 = 0$ $K = 4$	$\alpha_0 \equiv 0$ $\alpha_1 \equiv 0$ $\gamma = +5$	$\alpha_{0\max} = 0$ $\alpha_{1\max} = 0$ $\gamma_{\max} = 5$	$x_{\alpha_0} = x_{\alpha_1} = 0$ $x_{\gamma} = \gamma_{\max} \operatorname{sgn} \{\epsilon s(t)\}$ $+ \gamma_{\max} \int_0^t \operatorname{sgn} \{\epsilon s(t)\} dt$

For all runs: $M = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$

$$s(t) = 2e(t) + \dot{e}(t)$$

input: $r(t) = 10u(t)$

Table 6-1 Reference parameter values and control signals used in Example 2.

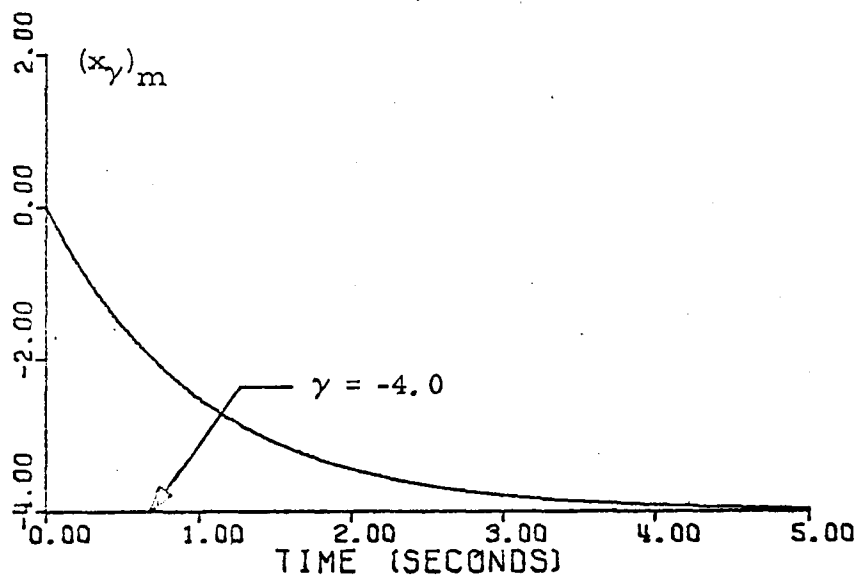
In each of the three runs, the model demonstrated its ability to "learn" the reference system parameter after a duration of approximately five seconds. This is illustrated in Figures 6-10, 6-11 and 6-12 which contain plots of the memory portion of the parameter control signals along with plots of the model-reference error $e(t)$ and its derivative $\dot{e}(t)$.

It should be noted that the parameter values in run no.3 violate the conditions for the determination of the Liapunov function M-matrix. For 2nd order systems, the term m_{12} must be positive and equal to the minimum possible value of the parameter $a_1 = A_1 + \alpha_1$, but for this case a_1 is equal to zero. The parameters a_0 , A_0 and α_0 are also equal to zero. This illustrates that even with some violations of the design procedure, satisfactory results may still be obtained due to the fact that the design method is based on Liapunov's theorems which gives only sufficient conditions for stability.

In the next example, a second order system is again investigated but this time four parameters are permitted to change in value and therefore four parameter control signals are needed.

6.4 Example No. 3 - 2nd order system with 4 parameter control signals

This example examines the performance of the parameter control signals when applied to a second order model system where four parameters are permitted to change in values. For this case the reference



(a)

Figure 6-10 Results of Example 2, Part 1

(a) Parameter control signal memory
component: $(x_\gamma)_m$

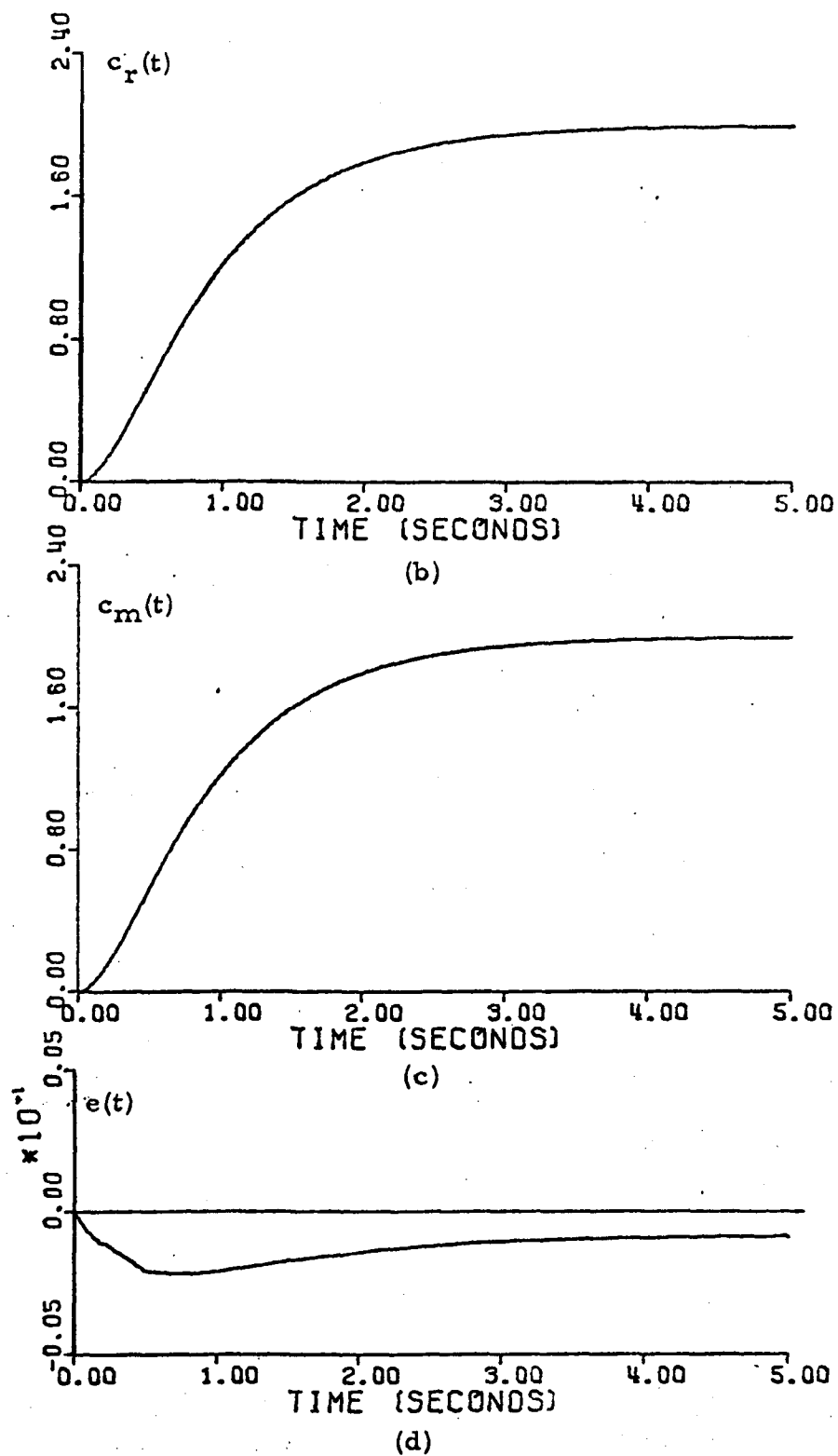


Figure 6-10 (Continued)

(b) Reference system output: $c_r(t)$ (c) Model output: $c_m(t)$ (d) Model-reference system error: $e(t)$

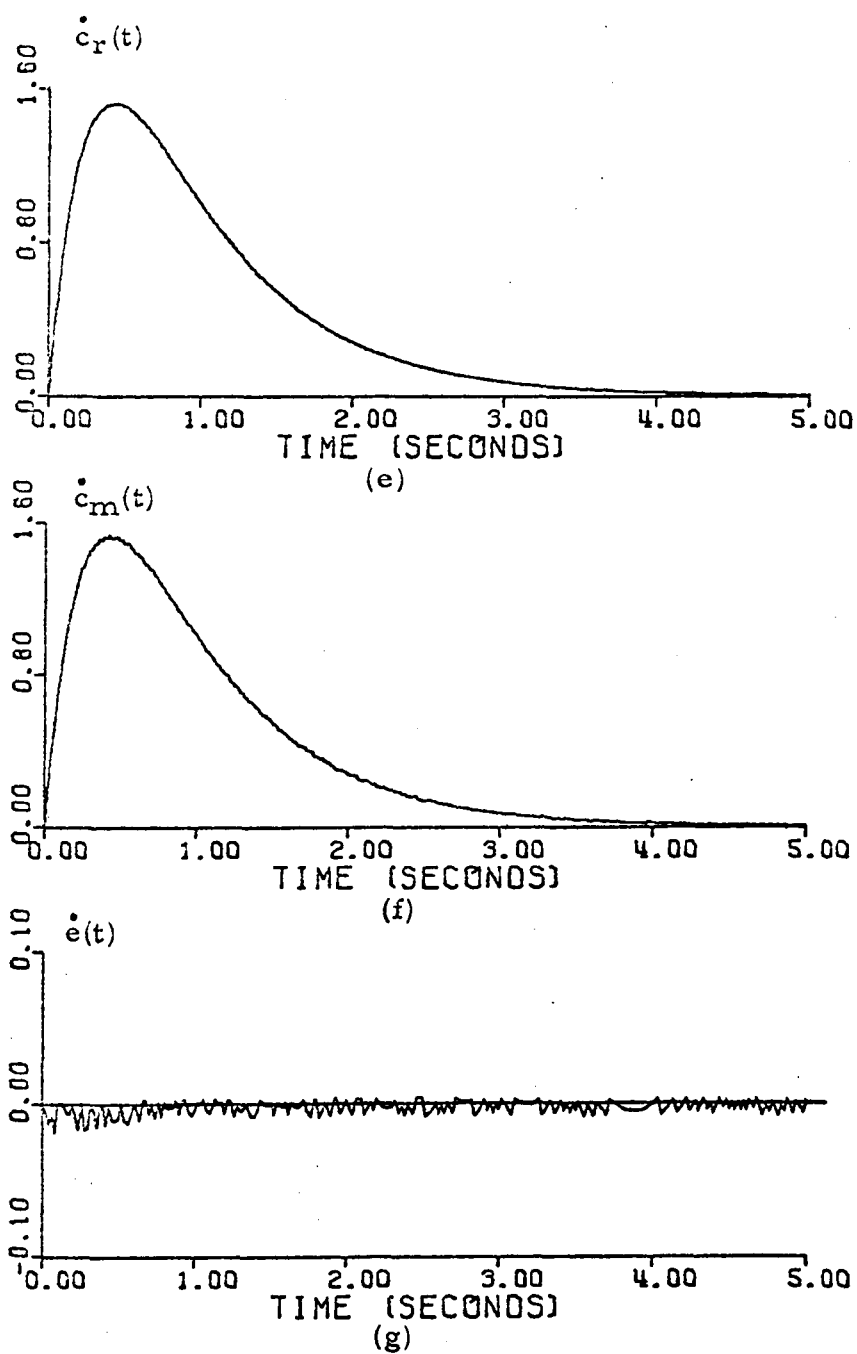


Figure 6-10 (Continued)

(e) Reference system output rate of change: $\dot{c}_r(t)$ (f) Model output rate of change: $\dot{c}_m(t)$ (g) Model-reference system error rate of change: $\dot{e}(t)$

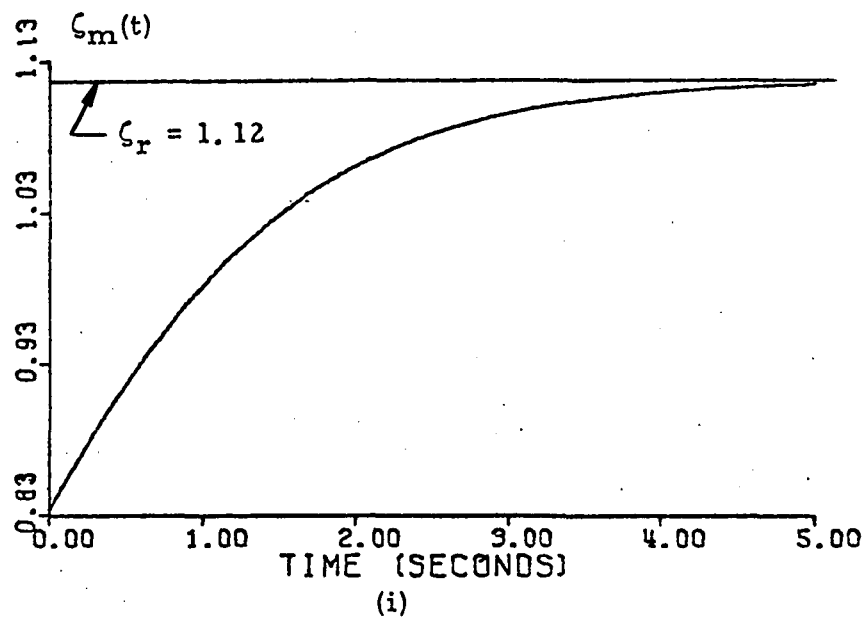
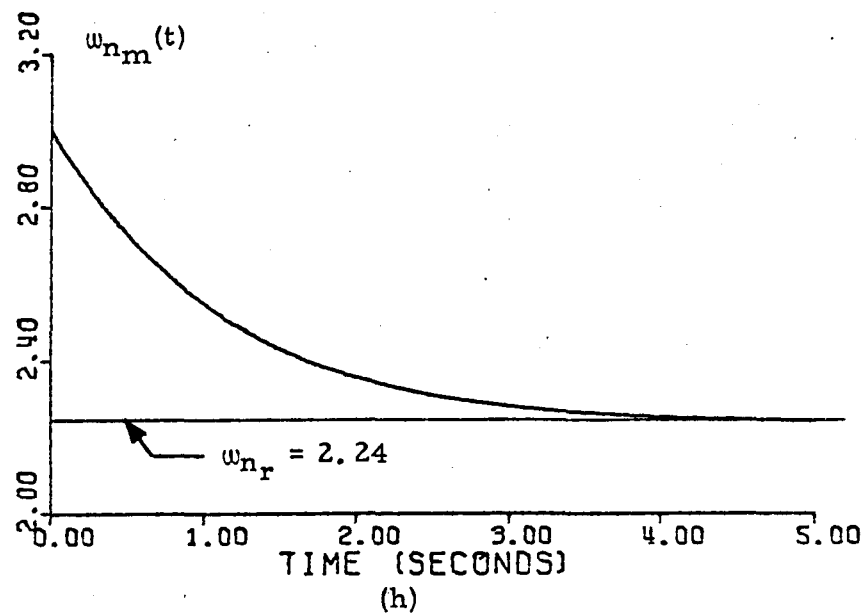


Figure 6-10 (Continued)

(h) Instantaneous model natural frequency: $\omega_{nm}(t)$ (i) Instantaneous model damping factor: $\zeta_m(t)$

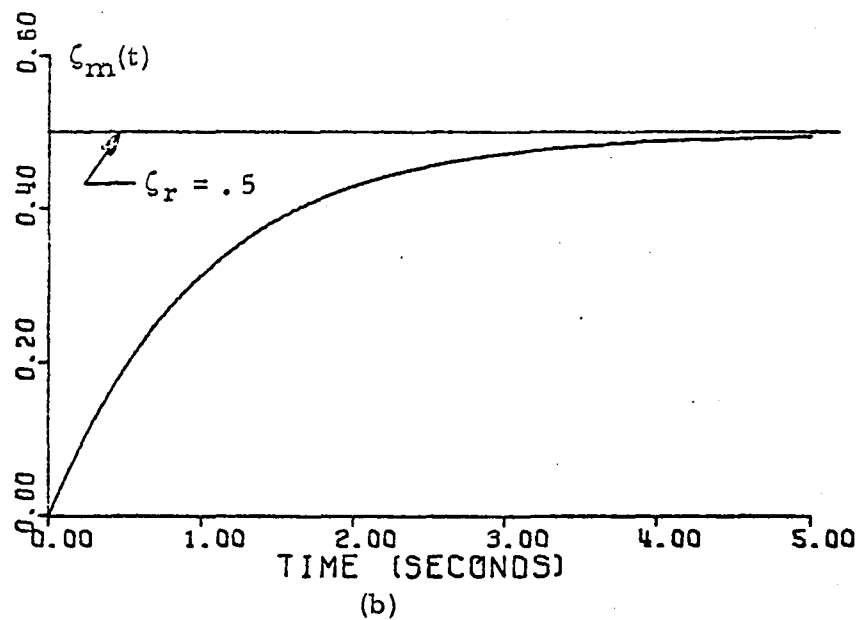
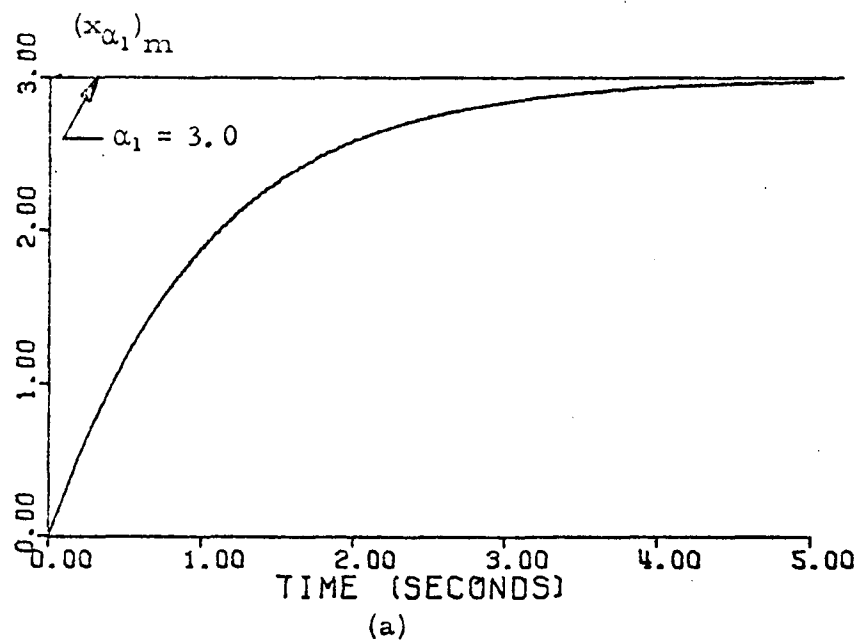


Figure 6-11 Results of Example 2, Part 2

(a) Parameter control signal memory component: $(x_{\alpha_1})_m$

(b) Instantaneous model damping factor: $\zeta_m(t)$

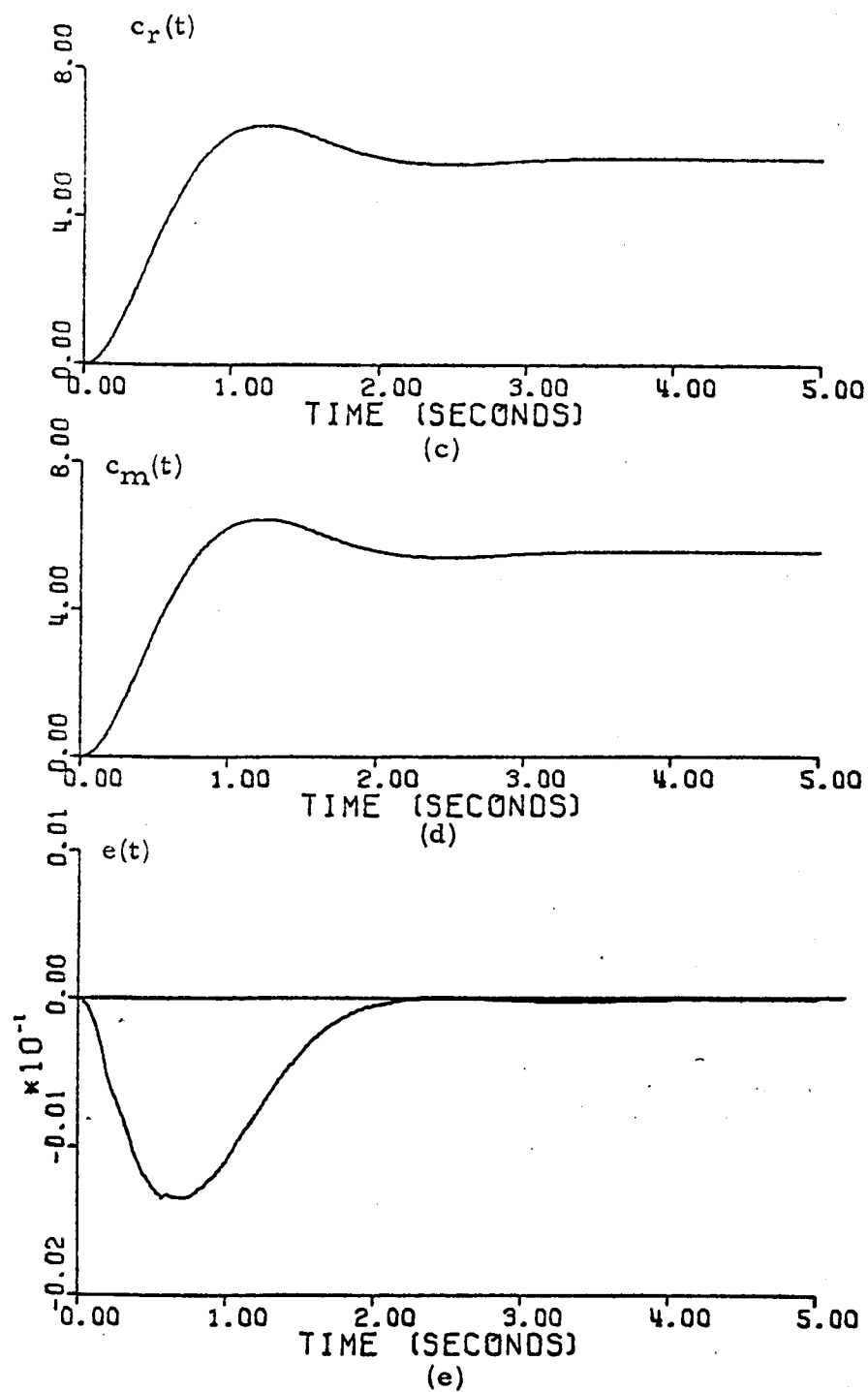


Figure 6-11 (Continued)

(c) Reference system output: $c_r(t)$

(d) Model output: $c_m(t)$

(e) Model-reference system error: $e(t)$

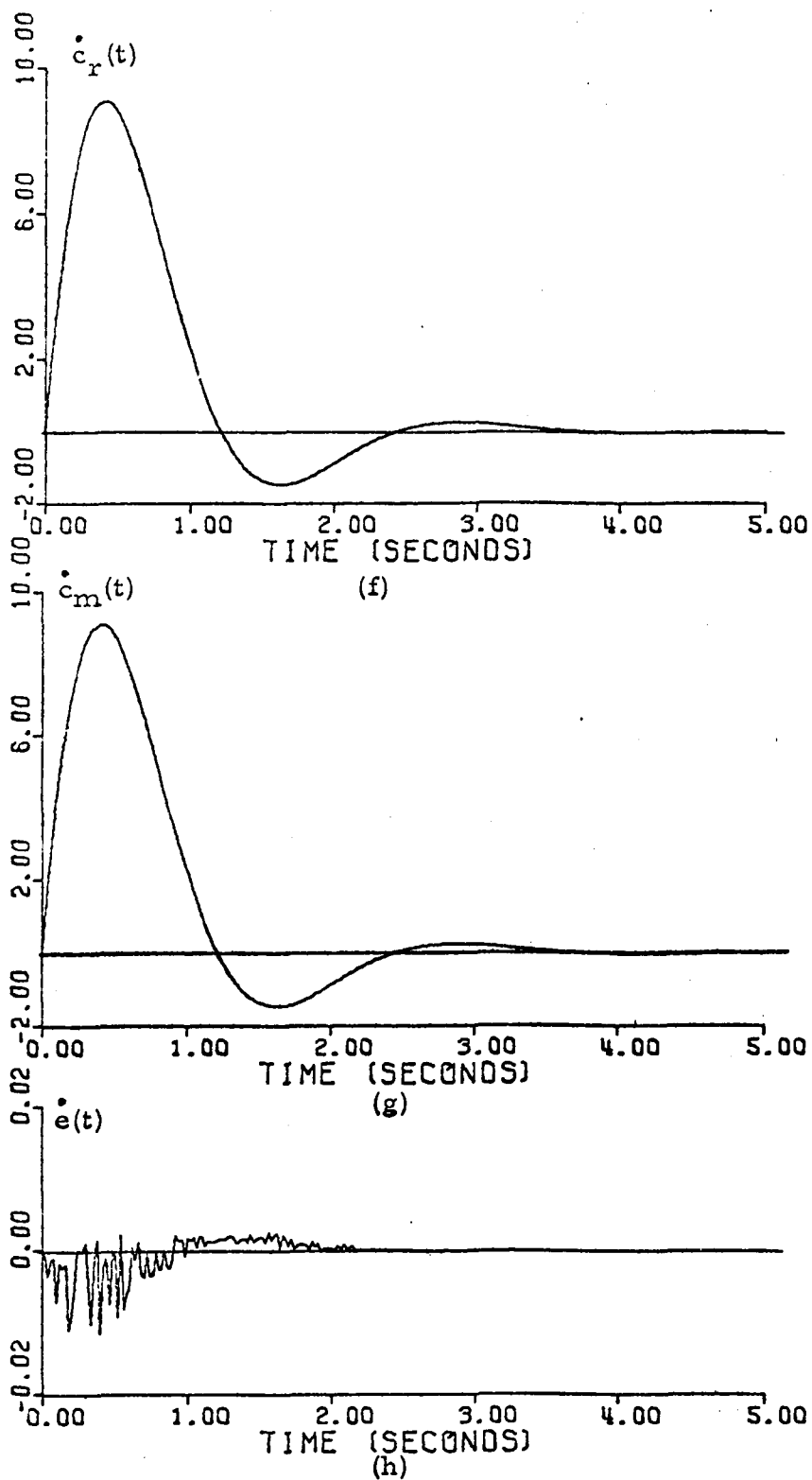
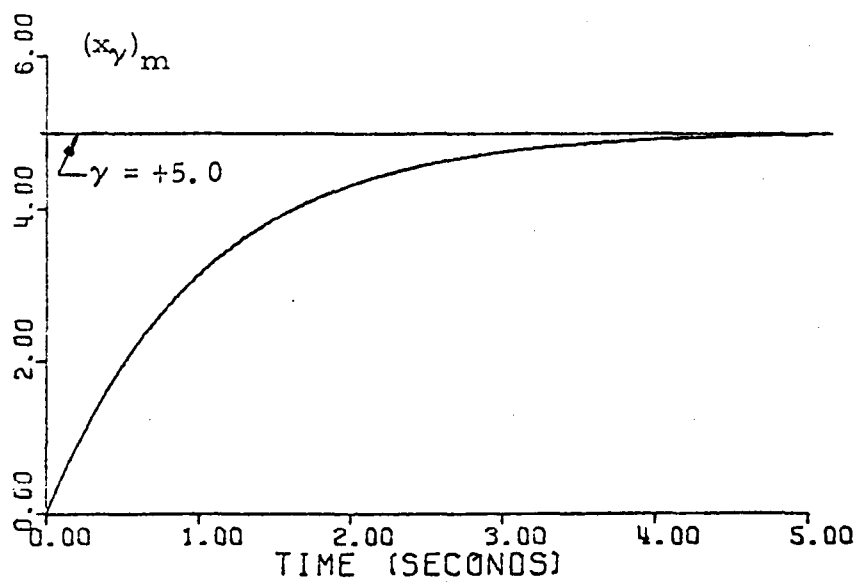
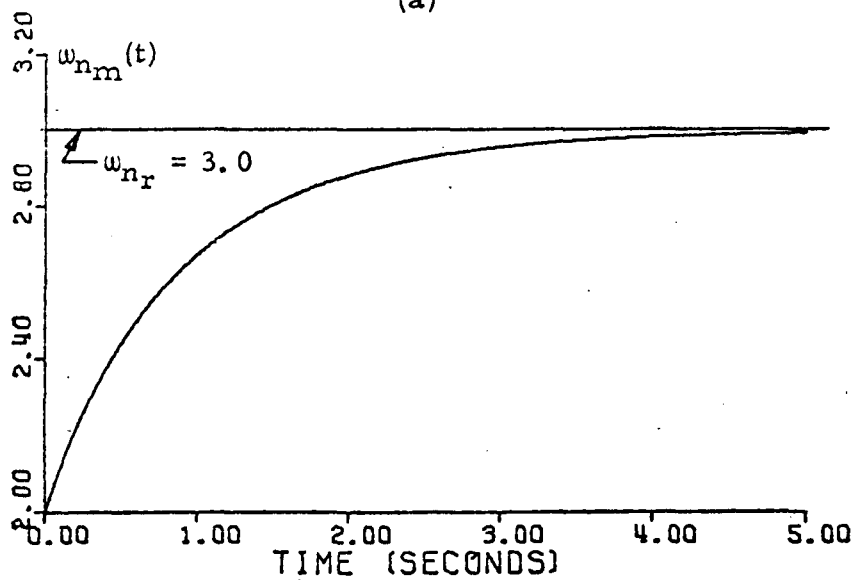


Figure 6-11 (Continued)

(f) Reference system output rate of change: $\dot{c}_r(t)$ (g) Model output rate of change: $\dot{c}_m(t)$ (h) Model-reference system error rate of change: $\dot{e}(t)$



(a)



(b)

Figure 6-12 Results of Example 2, Part 3

(a) Parameter control signal memory component: $(x_\gamma)_m$

(b) Instantaneous model natural frequency: $\omega_{nm}(t)$

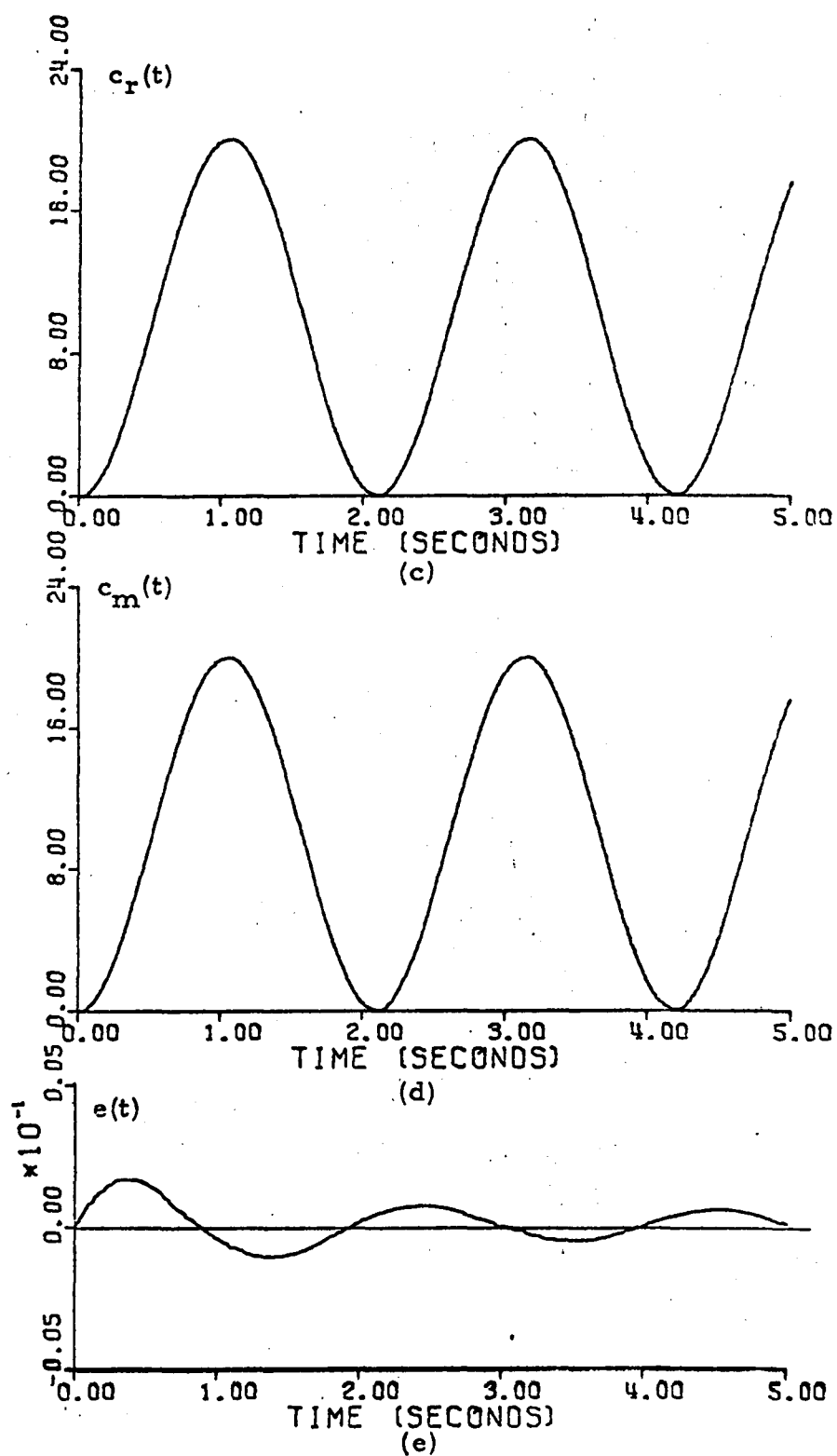


Figure 6-12 (Continued)

(c) Reference system output: $c_r(t)$ (d) Model output: $c_m(t)$ (e) Model-reference system error: $e(t)$

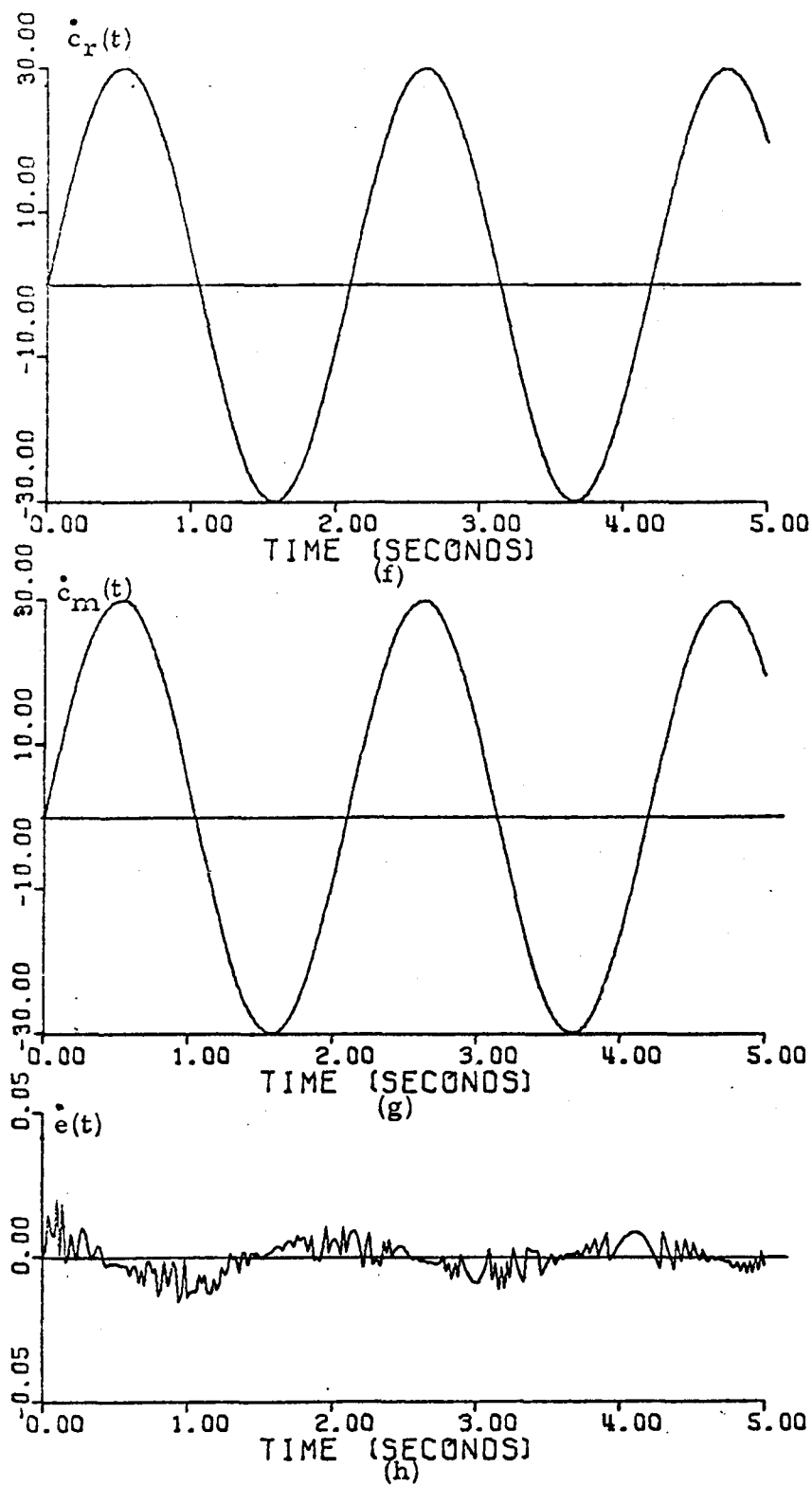


Figure 6-12 (Continued)

(f) Reference system output rate of change: $\dot{c}_r(t)$ (g) Model output rate of change: $\dot{c}_m(t)$ (h) Model-reference system error rate of change: $\dot{e}(t)$

system is shown in Fig. 6-13 and the model-reference system configuration is depicted in Fig. 6-14.

Initial values are specified for both the nominal values and level change values (incremental values) of the reference system parameters. After 42 seconds, the reference system parameters are altered and a new set of values for the level changes are specified. These parameter values along with other needed constants are listed in Table 6-2.

Nominal Values	Initial Level Change $t=0 \rightarrow 42$	Level Change $t > 42$	Max. Magnitude Level Change	M-Matrix Elements
$A_0 = 3$	$\alpha_0 = 1$	$\alpha_0 = -.5$	$\alpha_{0\max} = 3$	$m_{11} = 1$
$A_1 = 2.5$	$\alpha_1 = -.5$	$\alpha_1 = -.75$	$\alpha_{1\max} = 1$	$m_{12} = 2$
$K_0 = 3$	$\gamma_0 = 2$	$\gamma_0 = 1.0$	$\gamma_{0\max} = 2$	$m_{22} = 1$
$K_1 = 2$	$\gamma_1 = 1$	$\gamma_1 = .75$	$\gamma_{1\max} = 2$	

Table 6-2 Parameter values and constants specified for the 2nd order reference system Example 3.

The parameter control signals used in this example are given by Eqs.(5-52) and (5-53), and are indicated on the model-reference system diagram Fig. 6-14. As shown in Fig. 6-15, the input $r(t)$ applied to the model-reference system is formed by the product of two signals: a square wave and a ramp. A variety of other inputs were tried, but the product of a square wave and a ramp seems to yield the best results for this system. In order to prevent the magnitude of the input signal from growing too

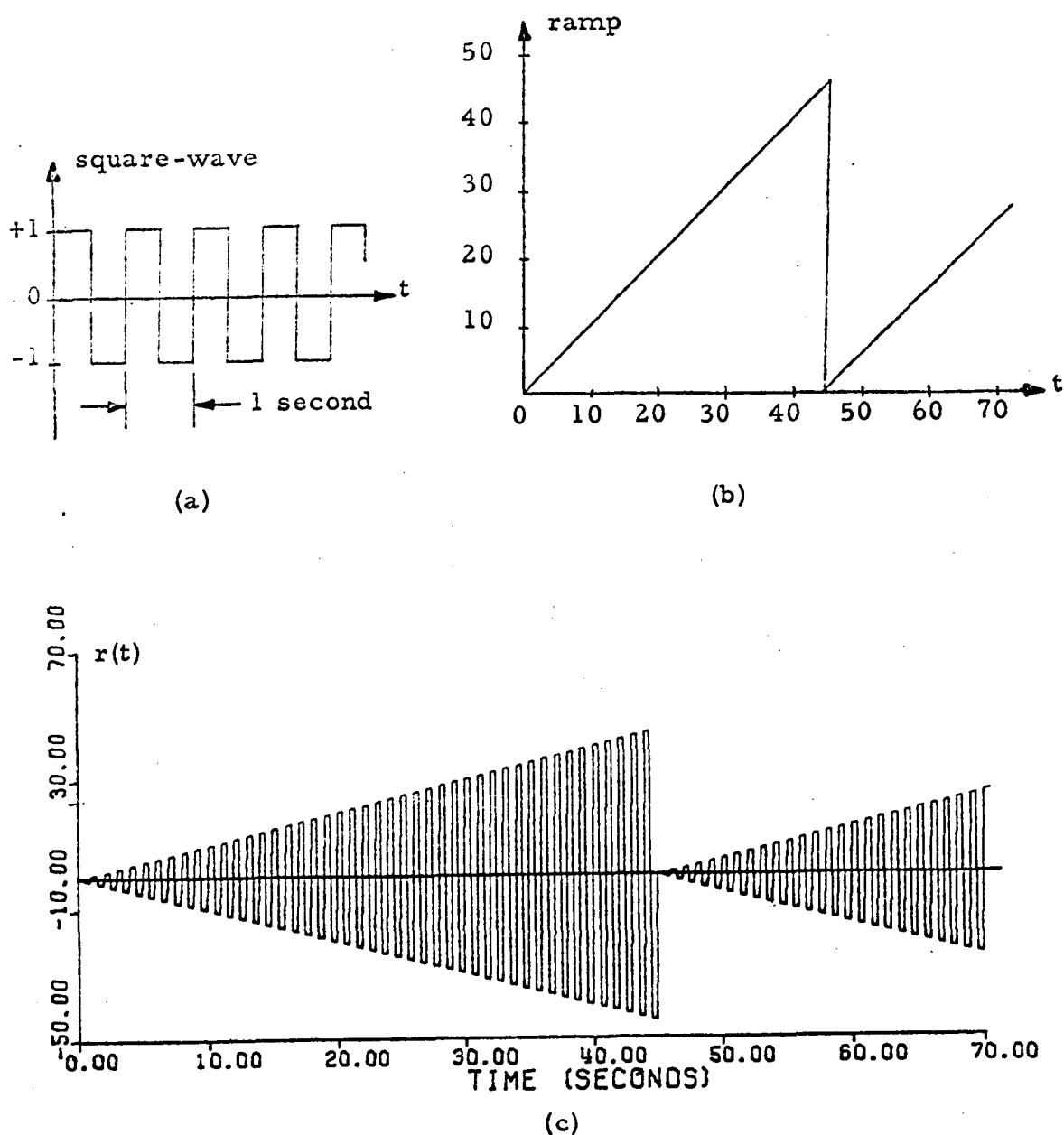


Figure 6-15 Construction of input $r(t)$ used in Example 3

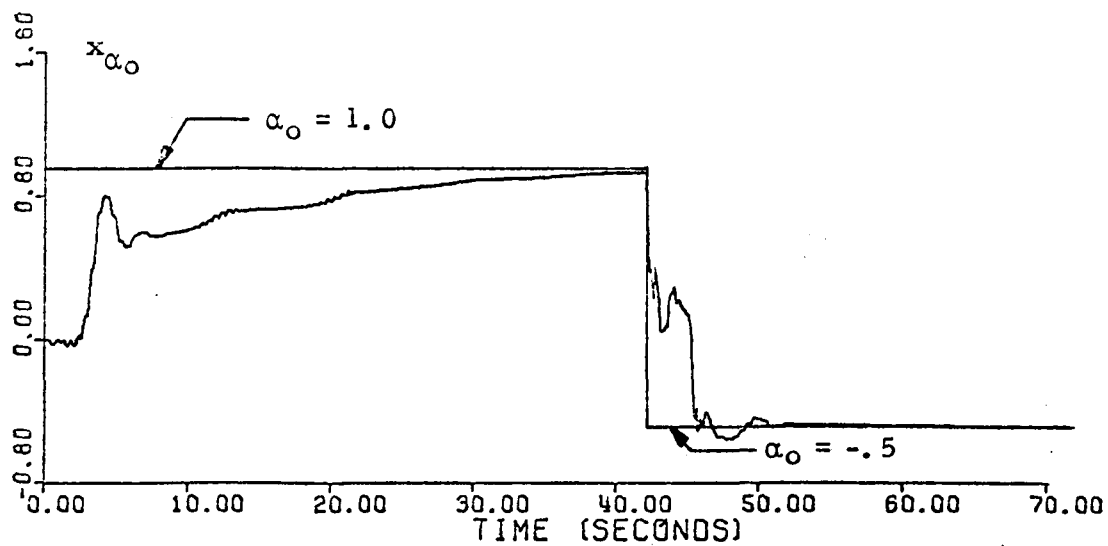
- (a) Square-wave
- (b) Ramp
- (c) Input $r(t)$; product of square-wave and ramp

large, at time $t = 45$ sec., the input was reset to zero and repeated. Various graphs are presented in Figs. 6-16 and 6-17, and they verify the model's ability to "learn" the different sets of parameter level changes that occur in the reference system.

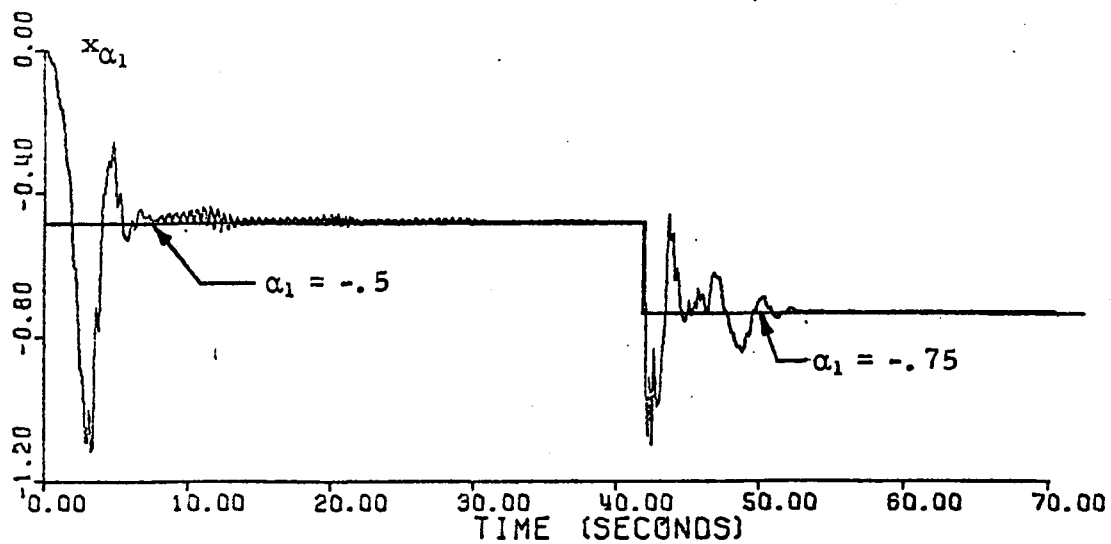
It should be noted that although in this example the time at which the reference parameters exhibit a level change is specified, this time could have been determined by inspection of the "learned" components for the model parameters plotted in Fig. 6-16. As the "learned" components approach their desired levels, their values vary very slowly. Immediately after the reference system parameters change their values, the "learned" components exhibit much larger and more rapid fluctuations. This behavior is also noticeable in the graphs of $e(t)$, $\dot{e}(t)$, $\xi_m(t)$ and $\omega_{n_m}(t)$ that are presented in Fig. 6-17.

6.5 Conclusion and Summary of Results

A number of examples have been presented in this chapter to illustrate the use of the model parameter control signals which were developed in Chapter 5. In all of these examples, the models, which include the parameter control signals, demonstrate their ability to "learn" the values of the parameters that were chosen for the reference systems, and thereby causes the model-reference error to approach zero. Graphs of some instantaneous model characteristics such as time constants, damping ratio and radian frequency are also included for comparison



(a)



(b)

Figure 6-16 Memory components for the four parameter control signals of Example 3

(a) x_{α_0}

(b) x_{α_1}

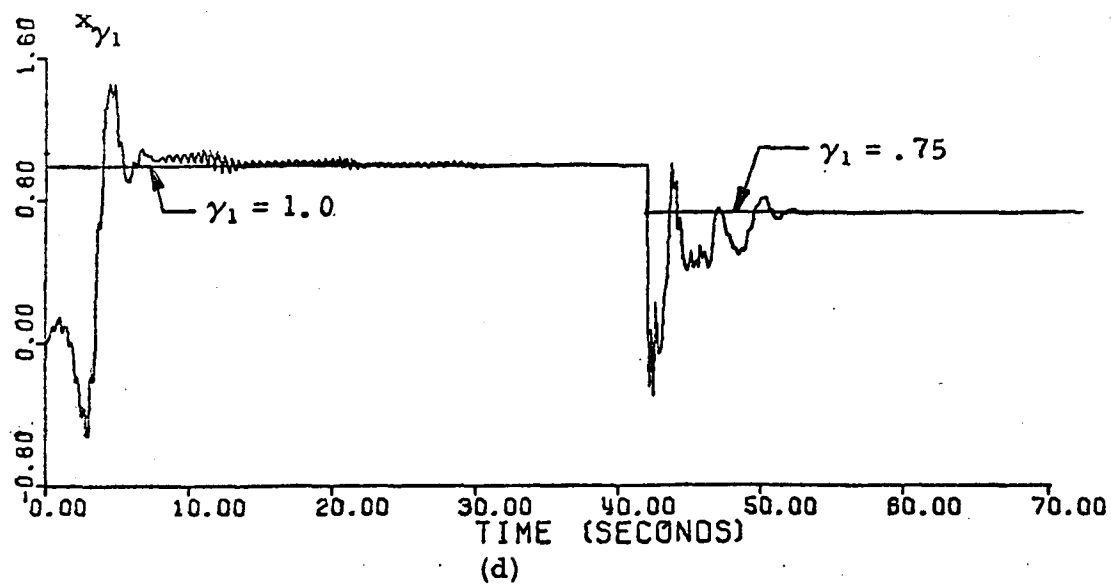
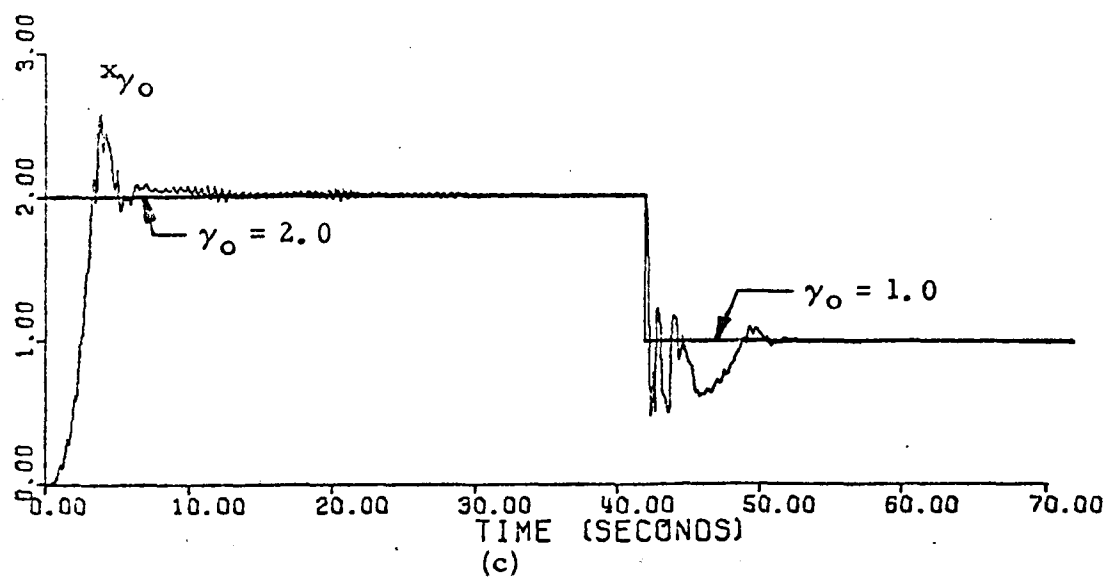
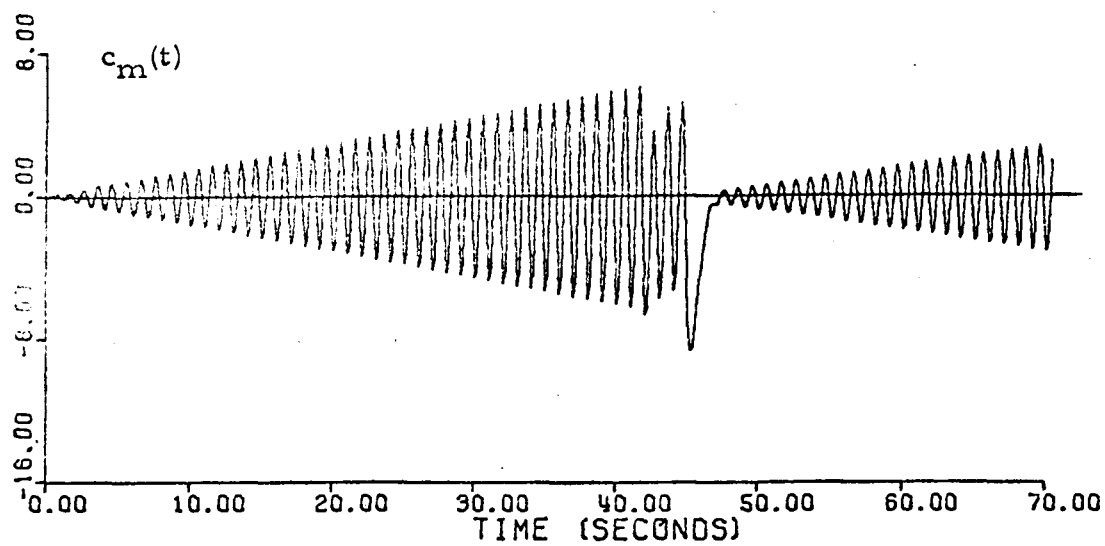
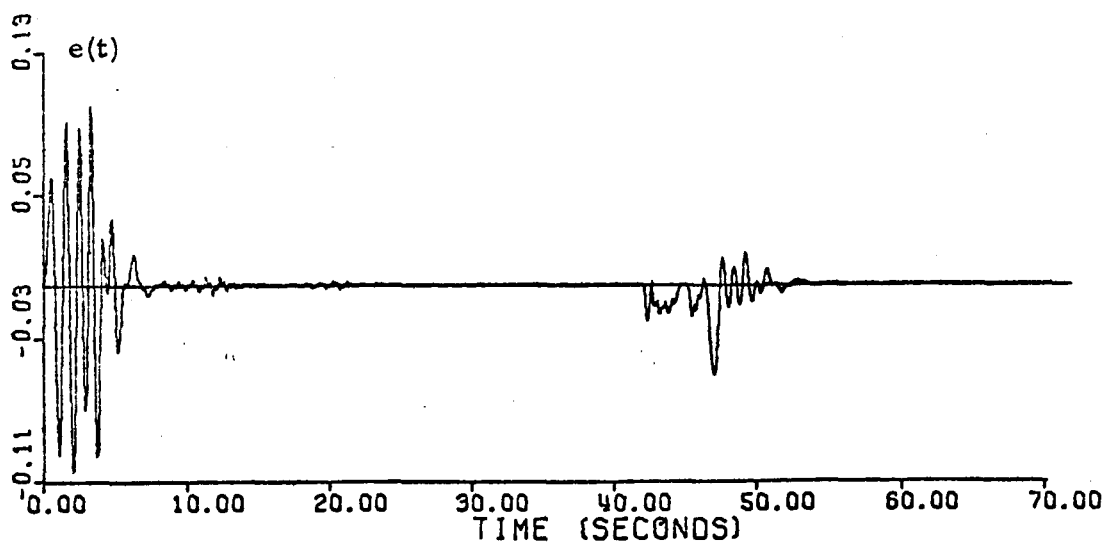


Figure 6-16 (Continued)

(c) x_{γ_0} (d) x_{γ_1}



(a)



(b)

Figure 6-17 Additional comparisons between the reference system and model of example 3

(a) Model output: $c_m(t)$

(b) Model-reference system error: $e(t)$

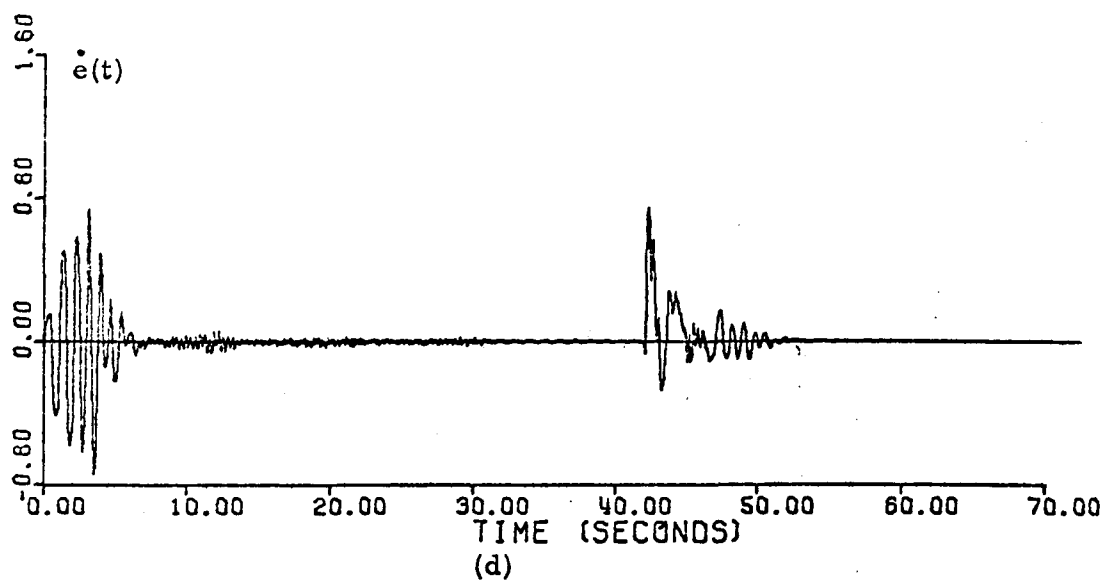
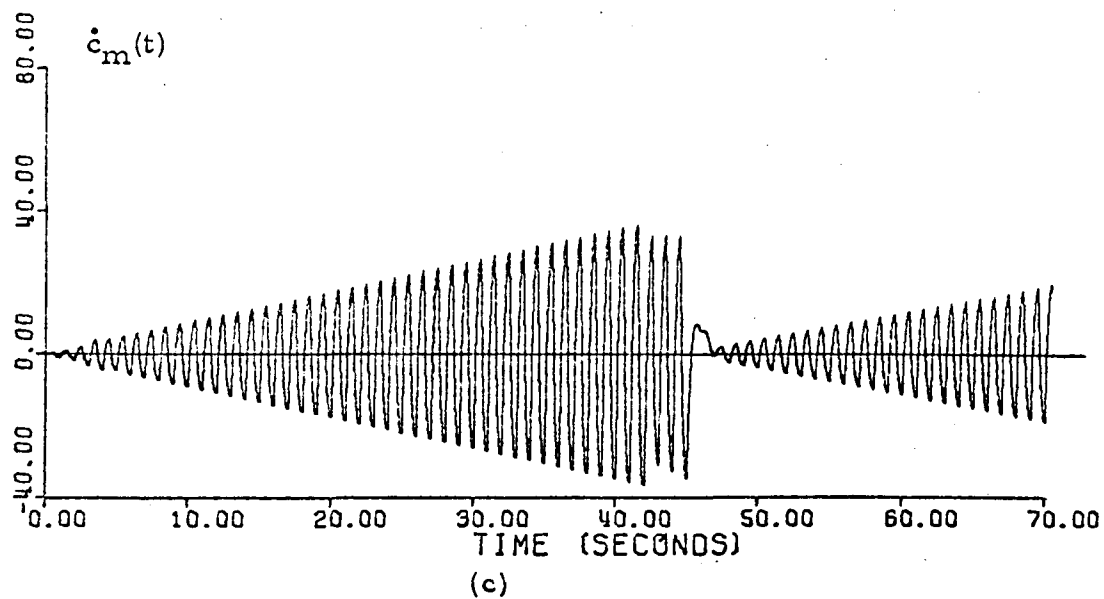


Figure 6-17 (Continued)

(c) Model output rate of change: $\dot{c}_m(t)$

(d) Model-reference error rate of change: $\dot{e}(t)$

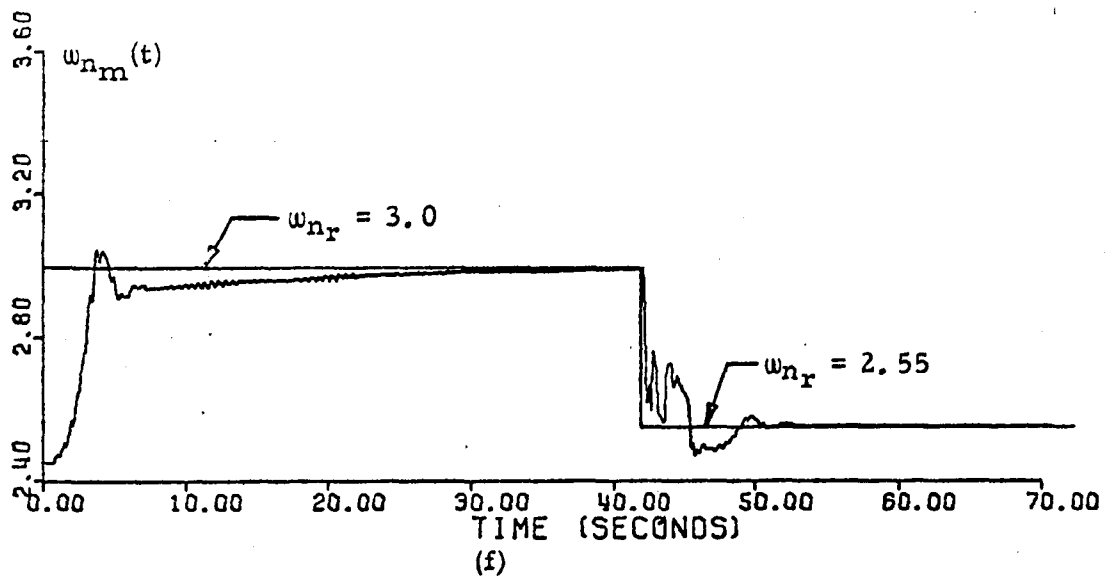
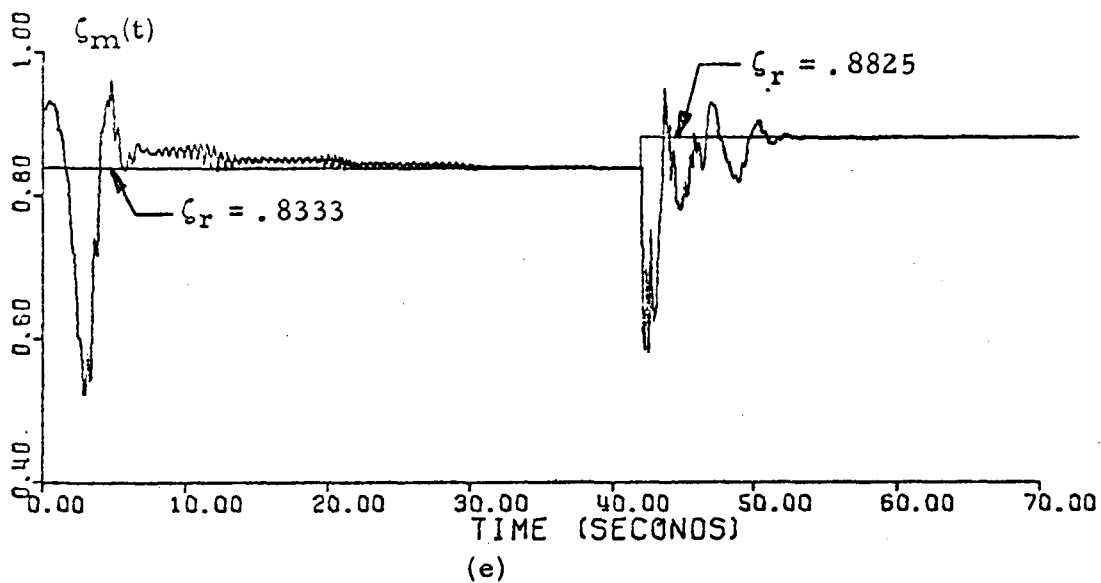


Figure 6-17 (Continued)

(e) Instantaneous model damping factor: $\zeta_m(t)$

(f) Instantaneous model natural frequency: $\omega_{nm}(t)$

with the reference system to further exhibit the model's adaptability.

In one of the examples, some of the conditions for the control signal design are purposely violated to demonstrate that even with some violations, the performance of the parameter control signals may still yield satisfactory results. This is possible because the design method is based on Liapunov's theorems which are only sufficient conditions for stability.

With the aid of simulation studies, it is seen that there are numerous factors which may influence the adaptive performance of the model and its associated parameter control signals. Examination of the control signal waveforms, reveal that when the order of the system and/or the number of required parameter control signals increases, the convergence behavior of the control signals are generally altered (see Figures 6-4, 6-6, 6-10, 6-11, 6-12, and 6-16). Furthermore, the characteristics of the model-reference system input and the choice of control signal design also affects the adaptive nature of the model. Therefore it is always advisable to test the performance and "learning" ability for the combined model and parameter control signals. This can be accomplished quite simply if both the structure and parameter values of the reference system are known. As illustrated in this chapter, the "learned" parameters of the model can be compared with the known parameters of the reference system. But when the reference system is an actual physiological system, the reference system parameters are usually

unknown and hence the comparison can not be readily made. One possible manner to perform a preliminary investigation of the model's behavior and "learning" ability is to replace the actual physiological reference system in the model-reference configuration by an artificially constructed reference system that has known parameter values. This substitute reference system should be constructed to be a duplicate of the model system, but with the parameter control signals replaced by known constants. Now the "learning" ability of the model with the parameter control signals can be tested by comparing the "learned" parameter values with the known parameter values of the substitute reference system. During the test procedure, different input signals can be applied to the artificial model-reference system and the various control signal designs of Chapter 5 may be tried. When satisfactory results are obtained, the substitute reference system can then be replaced by the actual physiological system.

The next chapter shows how the controller design method can be applied to the modeling of an actual physiological system, the human saccadic eye-movement system.

CHAPTER 7

CASE STUDY - HUMAN EYE MOVEMENT MUSCLE SYSTEM
(SACCADIC EYE MOVEMENTS)7.1 Introduction

In the previous chapter, several examples were presented in order to exhibit the design and "learning" capabilities of the model parameter control signals when applied to artificial modeling situations. In this chapter, it is demonstrated how the controller design method can be applied to the modeling of an actual physiological control system, namely the human saccadic eye-movement mechanism. It should be noted that this chapter is not intended to be a comprehensive study of the saccadic eye-movement system, but is presented to illustrate the procedure for applying the design method to a physiological system and also indicate some of the results which may be obtained.

As stated in Chapter 2, the controller design method is mainly intended to aid in the improvement of existing models, which have been formulated in previous physiological system studies, by updating their parameter values to yield an output response which is more representative of the actual system being modelled. Robinson^[69] advanced a realistic representation of the saccadic eye movement mechanism making use of data gained through experiments with external loading on the eyeball as well as through experiments under normal operating conditions. While a better representation might be obtained by assuming a more complicated neural

substrate, the Robinson model has the advantage of being the simplest system consistent with known physiology and mechanics of muscle and passive tissue, that yields reasonable quantitative agreement with experimental data. The parameter control signal design method is applied to Robinson's model with the aim of improving upon its output response while possibly obtaining a better understanding of the eye-movement system organization. For this modeling problem, the reference system's output that is needed for the model-reference system configuration, should be the actual positional measurements of a human's saccadic response to an applied change in target position. In their research studies, Cook and Stark^[10, 11, 12] performed experimental measurements of actual eye position and velocity during saccadic eye movements. They compare these measurements with the response of their proposed model of the human saccadic eye-movement mechanism and show that their model's response closely matches their experimental measurements. Therefore in this chapter, instead of repeating the actual measurements of saccadic eye-movements, Cook's proposed model of the human saccadic eye-movement mechanism will be utilized to generate the necessary response for the reference system.

Before applying the controller design method, the basic characteristics of the saccadic eye-movement system are described in order to acquaint the reader with the human saccadic eye-movement system.

7.2 Characteristics of Saccadic Eye-Movements

Saccades are the rapid voluntary eye movements (ranging to 850 deg/sec) which shift the gaze from one point to another (equivalent to tracking a step target). They are conjugate eye movements that bring the image from the periphery of the retina to the high resolution foveal region which covers an angle of approximately $1/2$ degree of the visual field.

An experiment by Westheimer^[81, 82] (1954) showed that the error correcting activities of saccadic movements are interspersed with periods of inactivity when no corrections are made regardless of how large the error might momentarily become and this lead Young and Stark^[86] (1963) to propose a sampled data model for the saccadic system. In Westheimer's experiment^[82], the target is displaced to one side and then returned to its original position after 40 msec. The eye responds by making a following saccade after approximately 150 msec - despite the fact that this movement is, at the time, wholly inappropriate and in fact takes the eye away from the target - and requires an additional 200 to 250 msec before returning to the target. This behavior suggests that, after the initiation of a sudden error, an irrevocable decision to initiate an appropriate saccade is followed by a refractory period during which additional target behavior has no effect on the occurrence or magnitude of the impending saccade.

Young and Stark^[86] assumed that a retinal position error exceeding 0.5 degrees was a sufficient stimulus to elicit a saccade after a

200 msec latency but the step-ramp stimulus of Rashbass^[67] (1961) shows this formulation to be inadequate. Robinson^[70] (1965) suggests that not only retinal error but also its rate of change is utilized by some central mechanism to make the irrevocable decision to activate a saccade and to determine its magnitude.

In 1954 Westheimer^[81] proposed that the eye moved in a saccade by the application of a step function of net muscular force, and that the mechanical system was of second order, and slightly underdamped with a natural resonant frequency of about 120 rad/sec. Robinson^[69] seeing the need for more exact knowledge of the mechanics of the eyeball, the extraocular muscles and the supporting tissues of the orbit, measured the isometric tensions developed by the eye muscles during attempted saccades, and also measured the eye movements produced by externally loading the eyeball with a step force function. His results demonstrated that the system of eyeball and orbital tissues is heavily over-damped, having an upper mechanical frequency response of only 1.1 hertz, and that it succeeds in making quick saccadic movements only under the impetus of a large, briefly applied "excess" force delivered by the extraocular muscles. The excessive force is developed from a burst of mechanically coupled bioelectric activity resulting in a rapid increase in active state tension to a maximum which is maintained until the burst abruptly ceases a few milliseconds before the globe reaches its new position. The duration of the burst determines the magnitude of the

saccade. Following the saccade, muscle tension retains a value sufficient to hold the eye in its new position. For example, in maintaining the eye 10 degrees horizontally deviated from the primary position, the horizontal rectus muscles need apply a net force of only 15 grams, but during the saccade to reach that position they apply about 43 grams during the first 40 msec of movement.

Cook and Stark^[10, 11, 12] have also studied the dynamic behavior of the human eye-positioning mechanism, and have presented a somewhat modified and refined version of the Robinson model. Both the Robinson and Cook models were primarily concerned with the actual dynamic movement behavior throughout the saccade rather than the overall eye tracking system. The delay time or latency period, which may vary from 150 to 250 msec and immediately precedes the saccadic movement, was not of interest to them and therefore was neglected. For the remainder of this chapter, the latency period will be omitted and it will be assumed that the actual dynamic movement occurs at time $t = 0$.

In the next section, a detailed description of Robinson's model is presented. This is the model for which the parameter control signals are to be developed.

7.3 Robinson's Model of the Human Saccadic Eye Movement Mechanics

A mechanical schema, developed by Robinson^[69], that describes the mechanical properties of the eyeball, restraining orbital tissues and

extraocular muscles for saccadic movements is shown in Fig. 7-1.

Robinson states that the net active-state tension (F_0) needed to drive the eye saccadically is in the form of a burst of mechanically coupled bioelectric activity resulting in a rapid increase in active state tension which abruptly ceases a few milliseconds before the globe reaches its new position but retains a tail that slowly decays to the new tonic level needed to maintain the eye in its new position. Therefore the net active state tension is composed of three components: (1) a tonic component, (2) a slow decay component, and (3) a briefly applied excess component. An increase in gaze deviation must be accompanied by a proportional increase in the tonic and slow decay component. However, the magnitude of the excess force component does not increase proportionally, but instead rises more slowly and eventually begins to decrease in magnitude for saccades greater than 15° . To compensate for this decrease in amplitude, the excessive force component is applied for a longer period of time as the saccadic magnitude increases. Thus a trading relationship is established between the strength and duration of the excessive force component.

Robinson's calculation for the moment of inertia of the eyeball as a rigid body yields a value of $.612 \times 10^{-4}$ ($\text{g} \cdot \text{sec}^2/\text{deg}$). But owing to the observation that the vitreous body of the eyeball is in greater part left behind during a saccade, he chooses the functional moment of inertia at about half of this value, namely $.297 \times 10^{-4}$ ($\text{g} \cdot \text{sec}^2/\text{deg}$), to which he

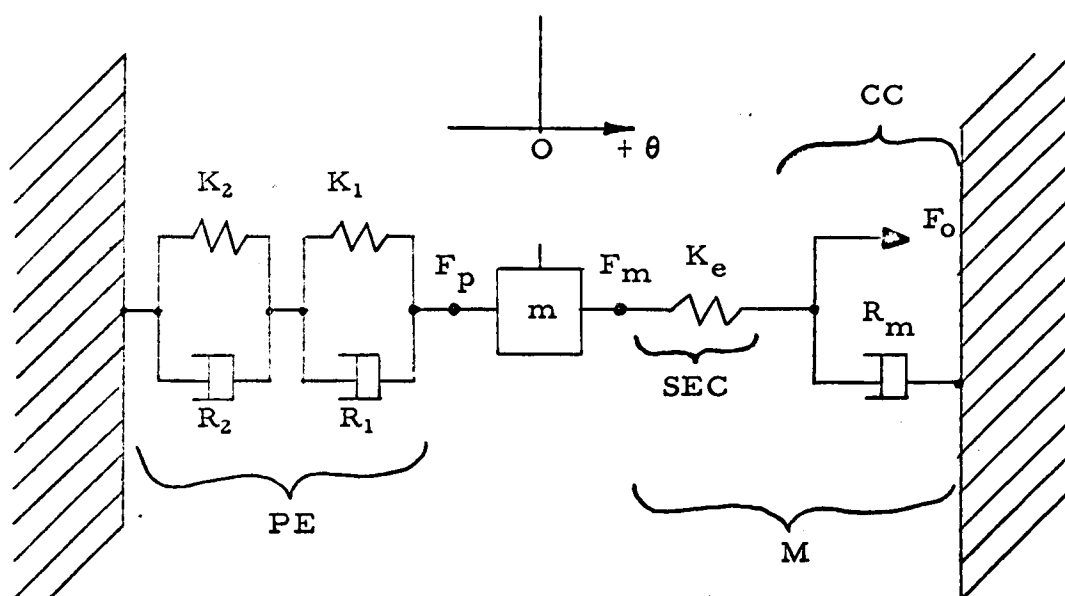


Figure 7-1 Robinson's model for the mechanical elements of the extraocular system [69]. {The active portion of the muscle (M) is divided into a contractile component (CC) and a series elastic component (SEC). All passive elements of the orbit and muscles are contained in the section (PE). }

adds the value for moment of inertia of the contact lens used in his eye movement measurements.

The active portion of the extraocular muscles are represented by a contractile component, or a dashpot of viscous damping coefficient R_m (g. sec/deg), and a series elastic component, or spring of stiffness coefficient K_e (g./deg). Initial values for both R_m and K_e are based on physiological data, and his final value is obtained, with the aid of an analog computer, by iterative trials to yield behavior compatible with his experimental results. The active muscle force (F_m) is a reflection of the net active state tension (F_o) which is filtered through the components K_e and R_m and are related by the expression

$$\frac{R_m}{K_e} \frac{dF_m}{dt} + F_m = F_o - R_m \frac{d\theta}{dt} \quad (7-1)$$

To represent the net restraining force (F_p) of all passive elements (PE) of the orbit and muscles, Robinson used two viscoelastic elements called Voigt elements. The values of R_1 , K_1 , R_2 and K_2 are again found by iterative approximation that yields the best match with his experimental results. The two Voigt elements offer a passive force F_p related to eye movement θ by

$$R_1 R_2 \frac{d^2\theta}{dt^2} + (R_1 K_2 + R_2 K_1) \frac{d\theta}{dt} + K_1 K_2 \theta = (K_1 + K_2) F_p + (R_1 + R_2) \frac{dF_p}{dt} \quad (7-2)$$

Summing the two forces F_m and F_p on the globe yields

$$m \frac{d^2\theta}{dt^2} + F_p = F_m \quad (7-3)$$

The final element values used by Robinson in his model are shown in Table 7-1.

Parameter	Symbol	Value
Net muscle series elastic stiffness	K_e	3.6 g/deg.
Net muscle force-velocity slope	R_m	.072 g. sec/deg.
Fast passive viscoelastic elements { spring stiffness viscosity	K_1 R_1	2.06 g/deg. .025 g. sec/deg.
Slow passive viscoelastic elements { spring stiffness viscosity	K_2 R_2	6.36 g/deg 1.81 g. sec/deg.
Combined passive spring stiffness	$\frac{K_1 K_2}{K_1 + K_2}$	1.556 g/deg.
Moment of inertia (eye and lens)	m	$.677 \times 10^{-4}$ g. sec ² /deg.

Table 7-1. Final element values for Robinson's model

Only saccadic eye movements of magnitude 18.75° are considered in the model-reference configuration and the three components of net-active state tension (F_0), which must be applied to Robinson's model for an 18.75° saccade, are shown in Fig. 7-2.

Now that Robinson's model has been specified, the parameter control signals can be designed. This is presented in the next section.

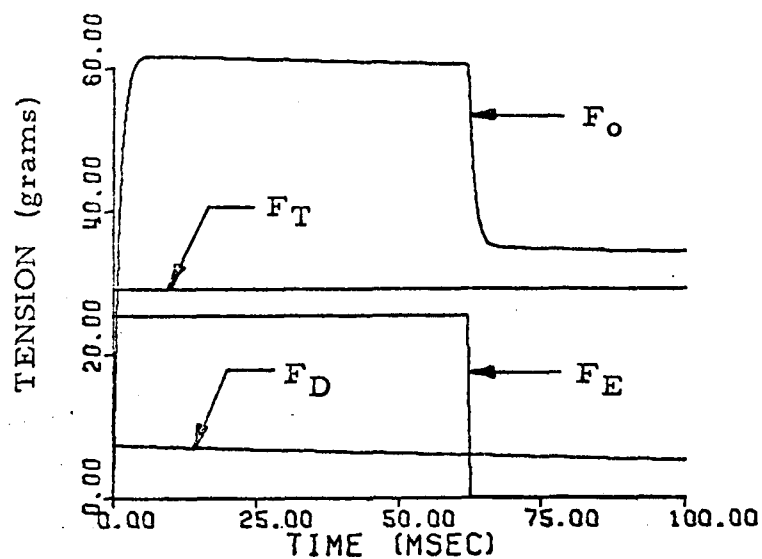


Figure 7-2 Robinson's net-active state tension for an 18.75 degree saccade

- (a) Tonic component: F_T
- (b) Slow decay component: F_D
- (c) Briefly applied excessive component: F_E
- (d) Net-active state tension: $F_O = F_T + F_D + F_E$

7.4 Parameter Control Signals for Robinson's Model

By combining Eqs. (7-1), (7-2), and (7-3), the fourth-order linear differential equation which describes the model's eye movement θ in response to the net active state tension (F_O) is obtained as

$$\frac{d^4\theta}{dt^4} + A_3^* \frac{d^3\theta}{dt^3} + A_2^* \frac{d^2\theta}{dt^2} + A_1^* \frac{d\theta}{dt} + A_0^* \theta = B_1 \frac{dF_O}{dt} + B_0 F_O \quad (7-4)$$

where

$$A_3^* = \frac{K_1 + K_2}{R_1 + R_2} + \frac{K_e}{R_m} + \frac{R_1 R_2}{m(R_1 + R_2)} = \frac{B_0}{B_1} + mB_1 + \frac{1}{m} \frac{R_1 R_2}{R_1 + R_2}, \quad (7-5a)$$

$$\begin{aligned} A_2^* &= \frac{K_e}{R_m} \frac{(K_1 + K_2)}{(R_1 + R_2)} + \frac{R_1 K_2 + R_2 K_1}{m(R_1 + R_2)} + \frac{K_e}{m R_m} \frac{R_1 R_2}{(R_1 + R_2)} + \frac{K_e}{m} \\ &= mB_0 + \frac{1}{m} \left(\frac{R_1 K_2 + R_2 K_1}{R_1 + R_2} \right) + B_1 \left(\frac{R_1 R_2}{R_1 + R_2} \right) + R_m B_1, \end{aligned} \quad (7-5b)$$

$$\begin{aligned} A_1^* &= \frac{K_1 K_2}{m(R_1 + R_2)} + \frac{K_e(R_1 K_2 + R_2 K_1)}{m R_m (R_1 + R_2)} + \frac{(K_1 + K_2) K_e}{(R_1 + R_2) m} \\ &= \frac{A_0^*}{mB_1} + B_1 \frac{(R_1 K_2 + R_2 K_1)}{R_1 + R_2} + R_m B_0, \end{aligned} \quad (7-5c)$$

$$A_0^* = \frac{K_1 K_2 K_e}{m R_m (R_1 + R_2)} = B_1 \frac{K_1 K_2}{R_1 + R_2}, \quad (7-5d)$$

$$B_1 = \frac{K_e}{m R_m} \quad (7-5e)$$

and

$$B_0 = \frac{(K_1 + K_2) K_e}{(R_1 + R_2) m R_m} = \left(\frac{K_1 + K_2}{R_1 + R_2} \right) B_1. \quad (7-5f)$$

The values for the coefficients of Eq.(7-4) can now be determined by substituting the mechanical element values from Table 7-1 into Eqs.(7-5), and the results are listed in Table 7-2. These coefficient values are chosen as the nominal parameter values discussed in Chapter 3 and defined by Eq.(3-24). Note that in previous chapters, the terms K_0 and K_1 were used to represent the nominal values B_0 and B_1 respectively. To avoid confusion, the letter "K" will be reserved in this chapter to represent spring stiffness values only.

Nominal Coefficients	Value
A_3^*	4.1883×10^2
A_2^*	1.0291×10^5
A_1^*	1.9141×10^6
A_0^*	5.2731×10^6
B_1	7.3855×10^5
B_0	3.3889×10^6

Table 7-2. Nominal-values chosen for Robinson's model

Since the system under consideration is an open loop system, the set of parameter control signals utilized are given by

$$x_{\alpha_1}^* = -\beta_1 \int_0^{\tau} c^{(1)} s(t) dt \quad (5-56)$$

and

$$x_{\gamma_j} = \Delta_j \int_0^{\tau} r^{(j)} s(t) dt \quad (5-57)$$

Here again, it should be noted that there has been some other changes in the notation used for the different variables of previous chapters. The model output is now defined by the variable θ and therefore the term $c^{(i)}$ in Eq. (5-56) should be replaced by

$$c^{(i)} = \theta^{(i)} \quad \text{for } i = 0, 1, 2, \text{ and } 3. \quad (7-6)$$

Also, the true input to the model-reference configuration, and therefore the model input, is a step change in target position of 18.75° . For Robinson's model, this 18.75° change in target position is first processed in the visual-oculomotor centers and then represented by the net-active state tension (F_0) depicted in Fig. 7-2. Therefore, the input to Robinson's model can be considered to be the net-active state tension (F_0) and the variable $r^{(j)}$ in Eq. (5-57) can be replaced by

$$r^{(j)} = F_0^{(j)} \quad \text{for } j = 0, 1. \quad (7-7)$$

The model subsystem is now represented in the form of Eq. (3-25), or

$$\theta^{(4)} + \ell_3 \star \theta^{(3)} + \ell_2 \star \theta^{(2)} + \ell_1 \star \theta^{(1)} + \ell_0 \star \theta = g_1 F_0^{(1)} + g_0 F_0 \quad (7-8)$$

where

$$\ell_i \star = A_i \star + x_{\alpha_i} \star \quad \text{for } i = 0, \dots, 3 \quad (7-9a)$$

and

$$g_j = B_j + x_{\gamma_j} \quad \text{for } j = 0, 1. \quad (7-9b)$$

Using the redefined notation, the parameter control signals of Eqs. (5-56) and (5-57) can be rewritten as

$$x_{\alpha_i}^{\star} = -\beta_i \int_0^{\tau} \theta^{(i)} s(t) dt \quad \text{for } i = 0, \dots, 3 \quad (7-10a)$$

and

$$x_{\gamma_j} = \Delta_j \int_0^{\tau} F_o^{(j)} s(t) dt \quad \text{for } j = 0, 1. \quad (7-10b)$$

The next step in the development is to determine the element values for the last column or row of the M-matrix. These values are tabulated in Table 5-2, and are needed to form the function

$$s(t) = m_{14} e_1 + m_{24} e_2 + m_{34} e_3 + m_{44} e_4 \quad (5-10)$$

for a fourth order system ($n=4$). However, before Table 5-2 can be utilized, the maximum and minimum limits must be specified. They are defined by

$$a_{i_{\min}}^{\star} < a_i^{\star} < a_{i_{\max}}^{\star} \quad \text{for } i = 0, \dots, 3. \quad (5-44)$$

It is initially assumed that

$$a_{i_{\min}}^{\star} = \frac{A_i^{\star}}{2} \quad (7-11a)$$

and

$$a_{i_{\max}}^{\star} = 2 A_i^{\star} \quad (7-11b)$$

for $i = 0, \dots, 3$, where $A_i^{\star}$ represents the chosen nominal values (Table 7-2). With these specified maximum and minimum values, the "Strong" Routh-Hurwitz Conditions (Table 5-1) are satisfied and the last column of the M-matrix can be calculated from Table 5-2. Upon examination of the waveforms plotted in Fig. 7-4 for the model sub-system

coefficients $\{\ell_i^{\star}\}$ (Eq. (7-9a)), it is seen that after a number of "learning" cycles some of the "learned" coefficients $\{\ell_i^{\star}\}$ were tending toward values outside the range specified by Eqs. (5-44) and (7-11). Therefore to correct for the initial erroneous choice of range limits, the values for $a_{i_{\min}}^{\star}$ and $a_{i_{\max}}^{\star}$ were updated after each cycle, by setting $a_{i_{\min}}^{\star}$ and $a_{i_{\max}}^{\star}$ equal to .9 and 1.1 respectively of the "learned" coefficient value $\ell_i^{\star}$ for the previous cycle. Using these changes, the values of the M-matrix elements were also updated after each "learning" cycle.

Finally, the control signal constants β_i and Δ_j were determined by using the procedure outlined in Section 5.4.2. Their values were initially chosen to insure that the products $\beta_i \theta_{\max}^{(i)}$ for $i = 0, \dots, 3$, and $\Delta_j F_0^{(j)}$ for $j = 0, 1$, are of the same order of magnitude. These values are then tested in an artificial model-reference situation similar to those described in Chapter 6. After their values are adjusted to yield satisfactory "learning" performance with a known reference system, they are then used for the control signals in the real model-reference situation.[†] Table 7-3 lists the final values obtained for the parameter control system constants.

[†] The reasons for this test procedure are discussed in Section 6.5.

Control Signal Constants	Value
β_3	8.3767×10^{-6}
β_2	2.0582
β_1	$3.8283 \times 10^{+3}$
β_0	$1.0546 \times 10^{+6}$
Δ_1	$1.4771 \times 10^{+1}$
Δ_0	$6.778 \times 10^{+4}$

Table 7-3 Parameter control signal constants
 $\{\beta_i\}$ and $\{\Delta_j\}$

7.4.1 Model-Reference System Configuration

The model-subsystem described by Eq.(7-8) along with the parameter controller which generates the designed parameter control signals are arranged in the model-reference system configuration depicted in Fig. 7-3. A step change in target position is applied as an input to the model-reference system configuration to elicit a saccadic response from both the reference system and model. For this situation the reference system's output should be the actual positional measurement of a human's saccadic eye movement in response to the applied change in target position. Instead of performing the actual measurements of saccadic eye movements to obtain the output data for the reference system, Cook's [10,11,12] model of the human saccadic eye-movement

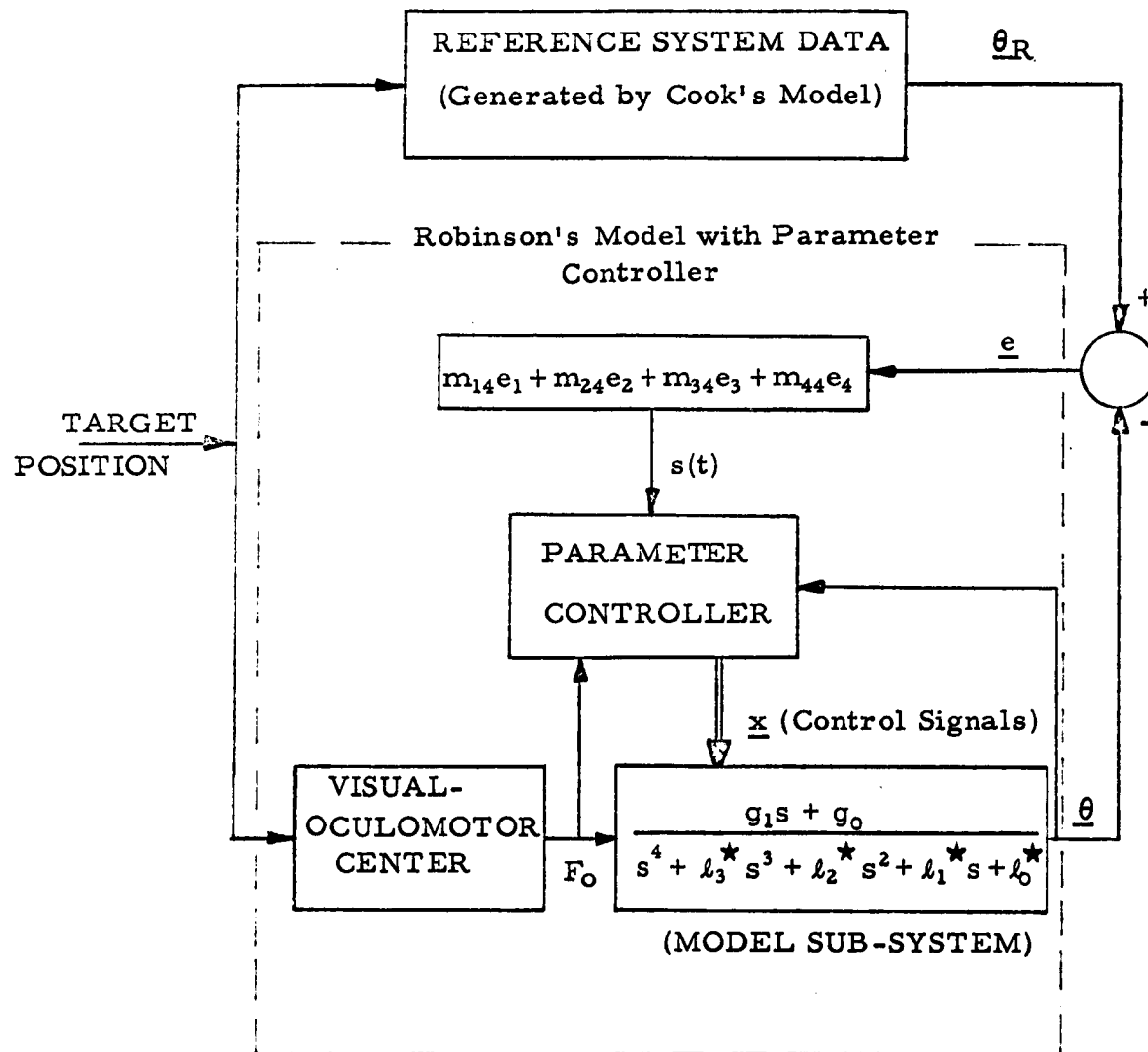


Figure 7-3 Model-reference system configuration for the human saccadic eye movement muscle system

mechanism is used to generate the necessary response (θ_R) for the reference system. A detailed description of Cook's model is presented in Appendix D.

As previously mentioned, only the dynamic movement during the saccade is considered and the initial latency or delay between the time of target position change and the onset of eye movement is neglected. Therefore, initially the response of both the model and reference system are assumed to be at their rest position $\theta = \theta_R = 0$. The input to the model-reference system is such that it permits the execution of an 18.75° saccade during a 100 msec time interval. After each 100 msec time interval, both the model and reference system are reset to their rest position. The repeated execution of these saccadic eye movements are continued until the parameter control signals cease to further update the model's parameter values during an entire saccadic movement. Upon examination of Eqs.(7-10), which describe the parameter control signals, it is seen that the control signals will only cease to further update the model's parameters during an entire saccadic movement if the function $s(t)$, defined by Eq.(5-10), approximates zero during the entire saccade. This suggests that the model-reference system error (e) must be approximately equal to zero during this saccade. Hence, in view of Fig. 7-3, the model's parameters are considered to have "learned" the reference system's parameter values and the model sub-system is assumed to have "learned" the reference system. The model with its "learned" parameter values can

be removed from the model-reference system configuration and the model's response remains equivalent to the reference system's response.

Resulting waveforms of the "learned" coefficients[†] along with the model-reference system error are plotted in Fig. 7-4. Within experimental and simulation accuracy, inspection of the control signal waveforms indicate that after approximately 100 "learning" cycles the variations of the parameter control signals are greatly reduced and for all practical purposes may be considered to have approached constant values. These constants, when added to the initially assumed nominal parameters yields the "learned" values for the model's postulated parameters and they are tabulated in Table 7-4.

Nominal Values	"Learned" Coefficients
$A_3^* = 4.1883 \times 10^2$	$l_3^* = 9.9724 \times 10^2$
$A_2^* = 1.0291 \times 10^5$	$l_2^* = 5.9068 \times 10^4$
$A_1^* = 1.9141 \times 10^6$	$l_1^* = 1.0456 \times 10^7$
$A_0^* = 5.2731 \times 10^6$	$l_0^* = 1.3667 \times 10^8$
$B_1 = 7.3855 \times 10^5$	$g_1 = 1.7324 \times 10^5$
$B_0 = 3.3889 \times 10^6$	$g_0 = 7.4604 \times 10^7$

Table 7-4 Nominal and final "learned" values
for coefficients of Eq. (7-8)

[†] The waveforms presented in Figs. 7-4(a to f) are graphs of the "learned" model coefficients. Waveforms for the corresponding parameter control signals may be obtained by subtracting the initially assumed constant nominal parameter values from the "learned" model coefficient waveforms.

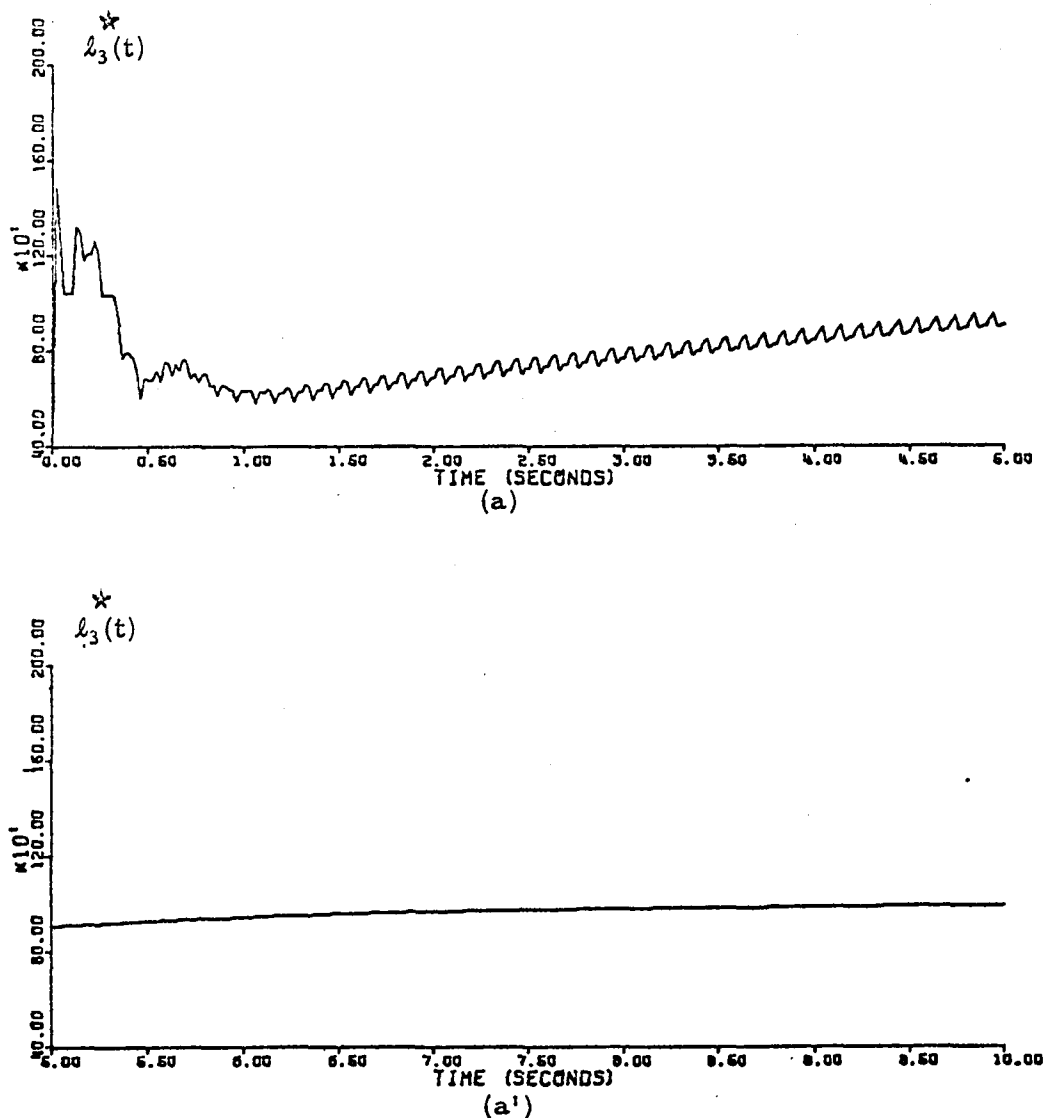


Figure 7-4 Plots of waveforms generated from the model-reference system configuration (Figure 7-3) for the human eye-movement muscle system

(a) "Learned" coefficient $l_3(t)$ for first 50 cycles

(a') "Learned" coefficient $l_3(t)$ for second 50 cycles*

* Note that the sudden drastic decrease in oscillations of the "learned" coefficients on the onset of the second 50 cycles is due to the re-specification of the values for $a_{i\max}^*$ and $a_{i\min}^*$ after the first 50 cycles. This respecification automatically updates the values of the M-matrix elements in the function $s(t)$ defined by Eq. (5-10).

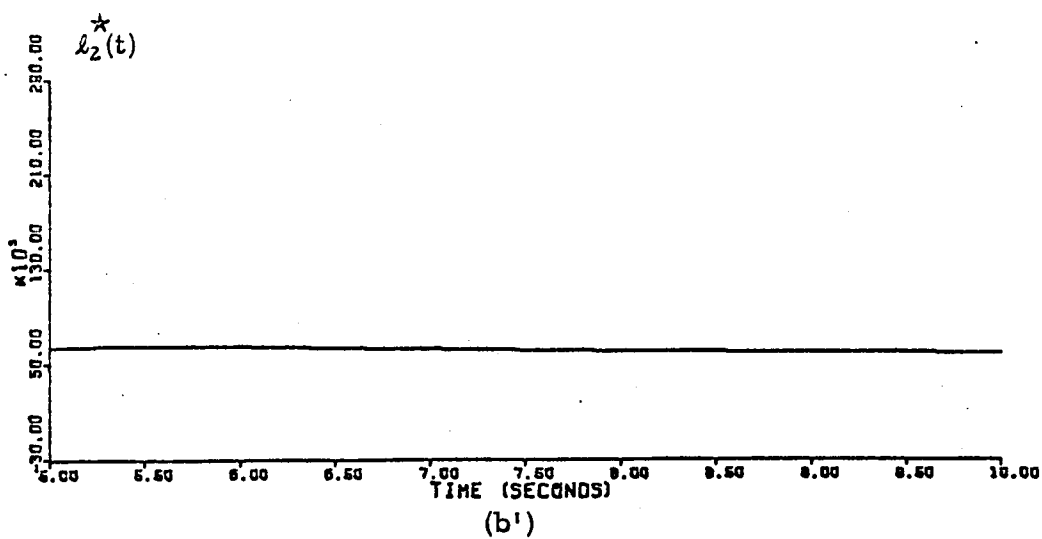
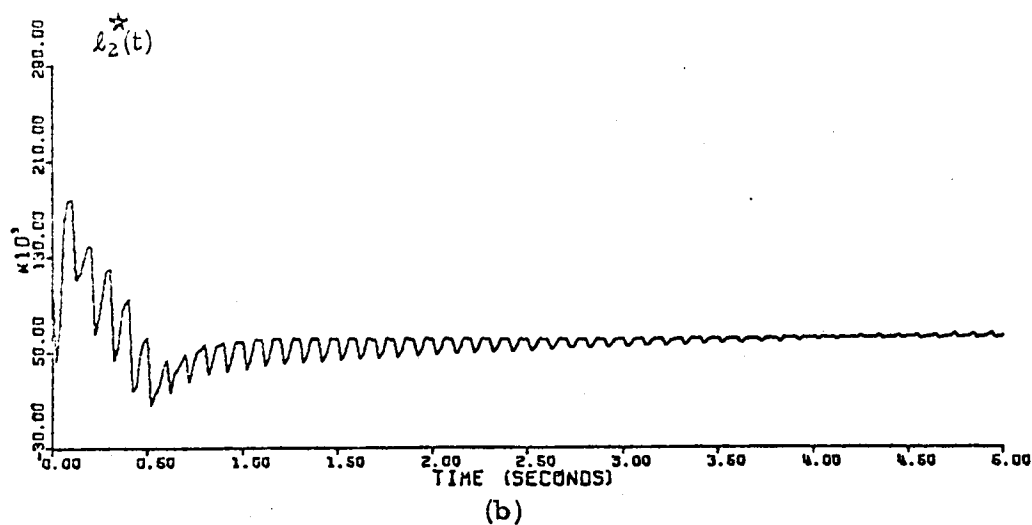


Figure 7-4 (Continued)

(b) "Learned" coefficient $l_2^*(t)$ for first 50 cycles

(b') "Learned" coefficient $l_2^*(t)$ for second 50 cycles

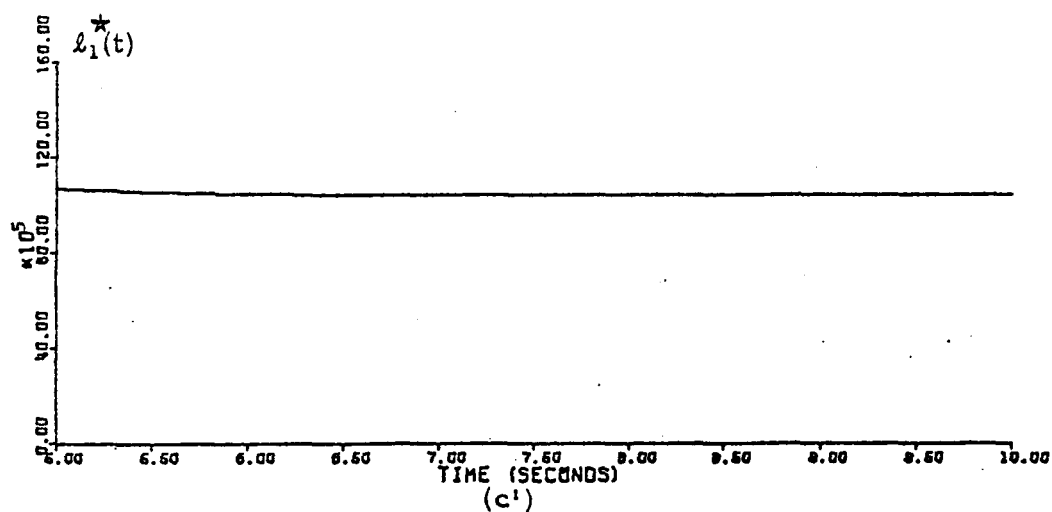
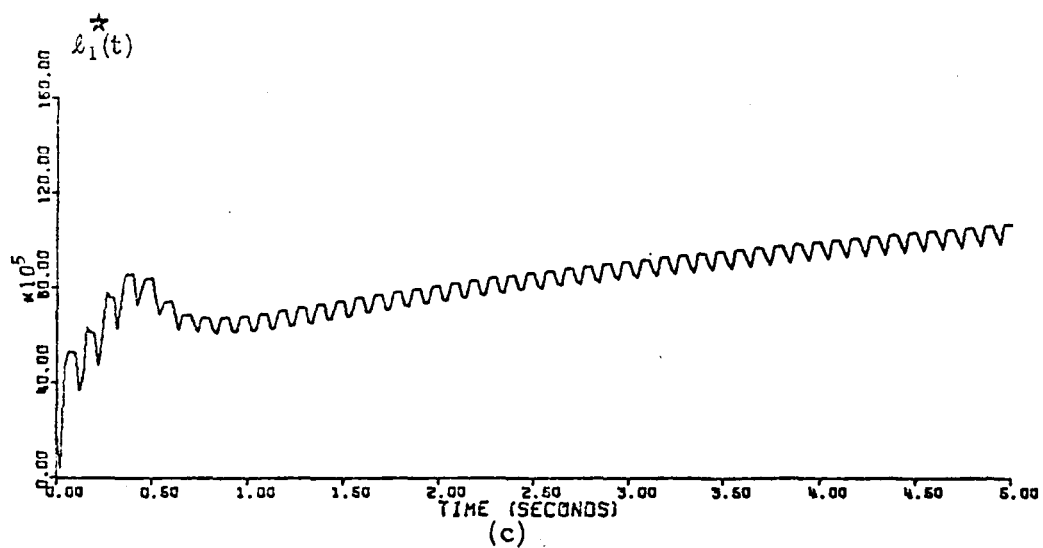


Figure 7-4 (Continued)

(c) "Learned" coefficient $l_1^*(t)$ for first 50 cycles

(c') "Learned" coefficient $l_1^*(t)$ for second 50 cycles

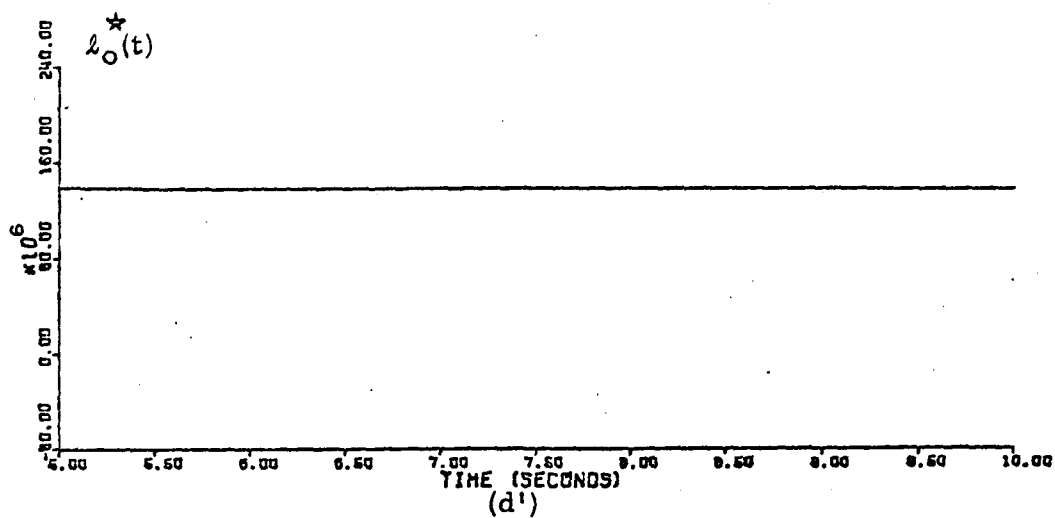
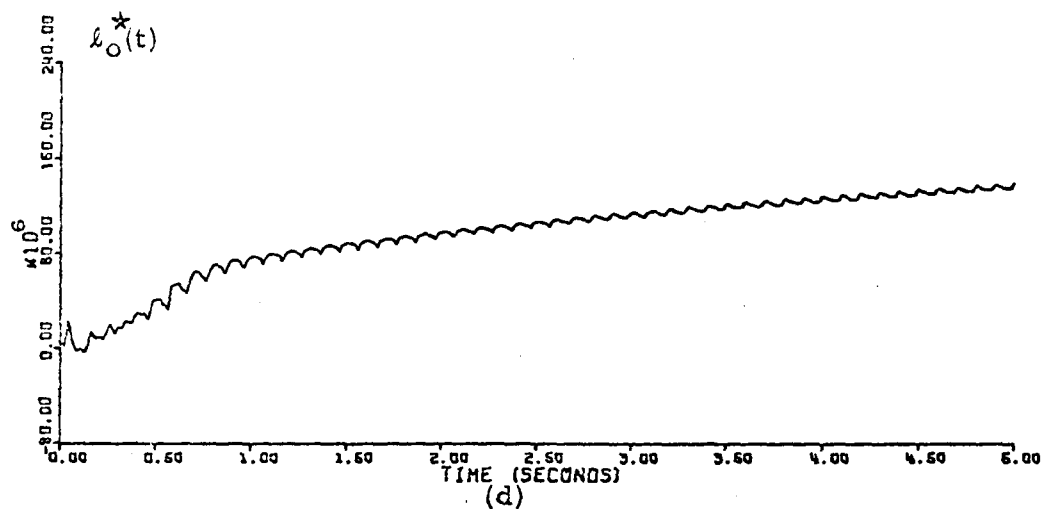


Figure 7-4 (Continued)

(d) "Learned" coefficient $l_O^*(t)$ for first 50 cycles

(d') "Learned" coefficient $l_O^*(t)$ for second 50 cycles

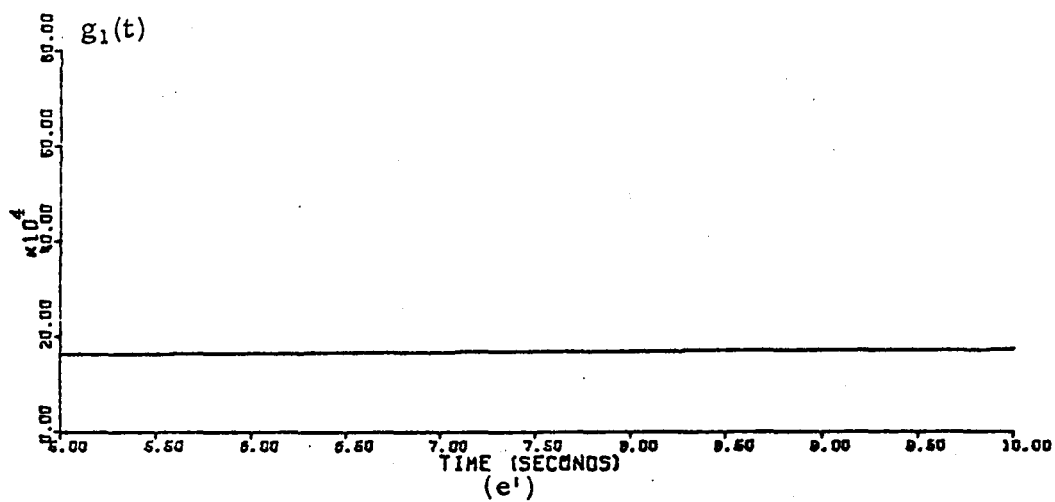
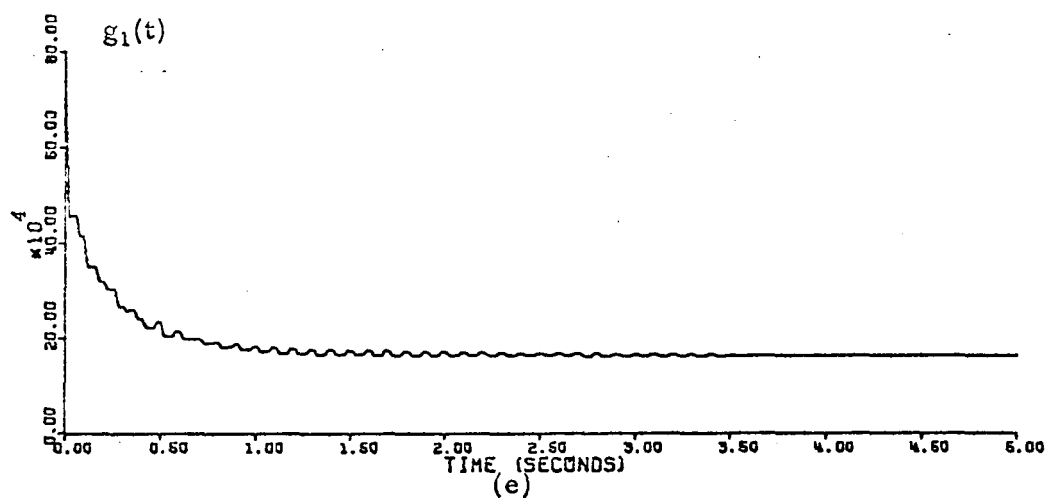


Figure 7-4 (Continued)

(e) "Learned" coefficient $g_1(t)$ for first 50 cycles

(e') "Learned" coefficient $g_1(t)$ for second 50 cycles

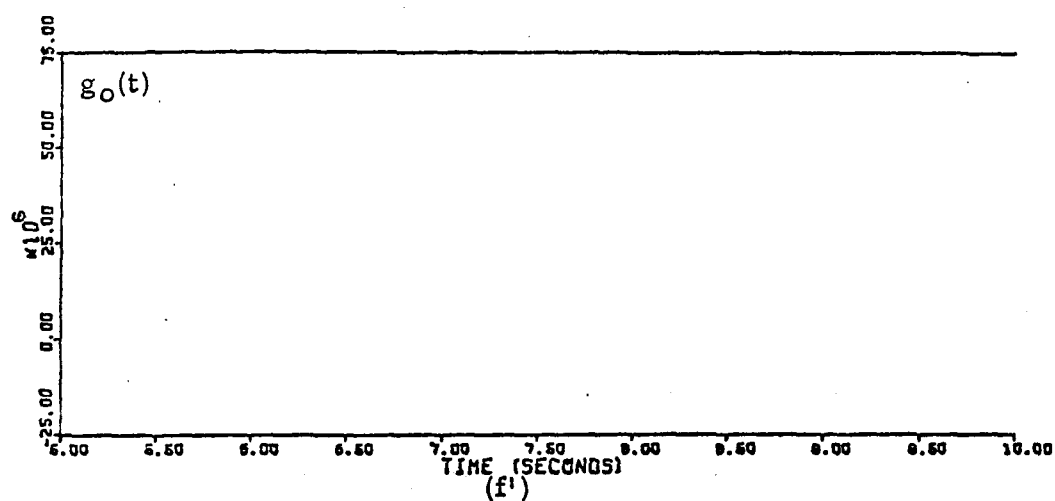
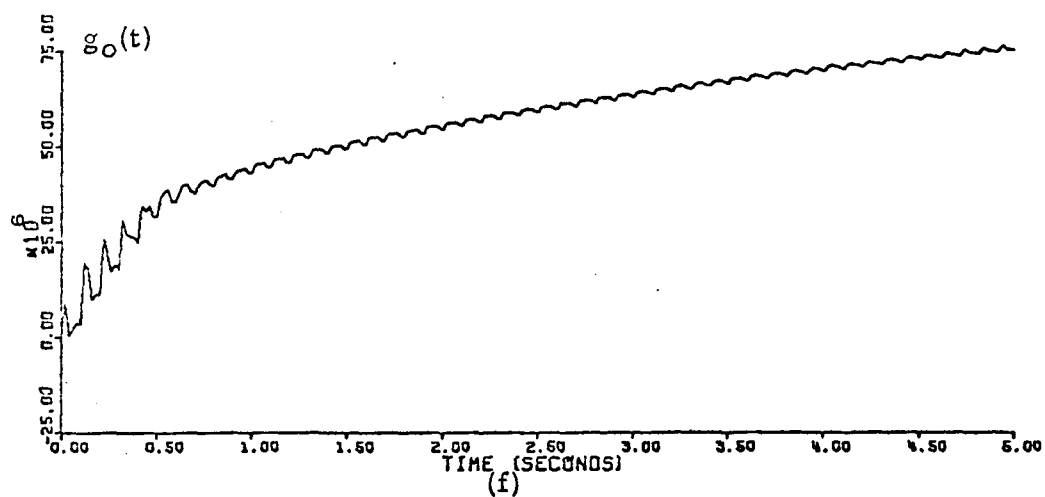


Figure 7-4 (Continued)

- (f) "Learned" coefficient $g_0(t)$ for first 50 cycles
- (f') "Learned" coefficient $g_0(t)$ for second 50 cycles

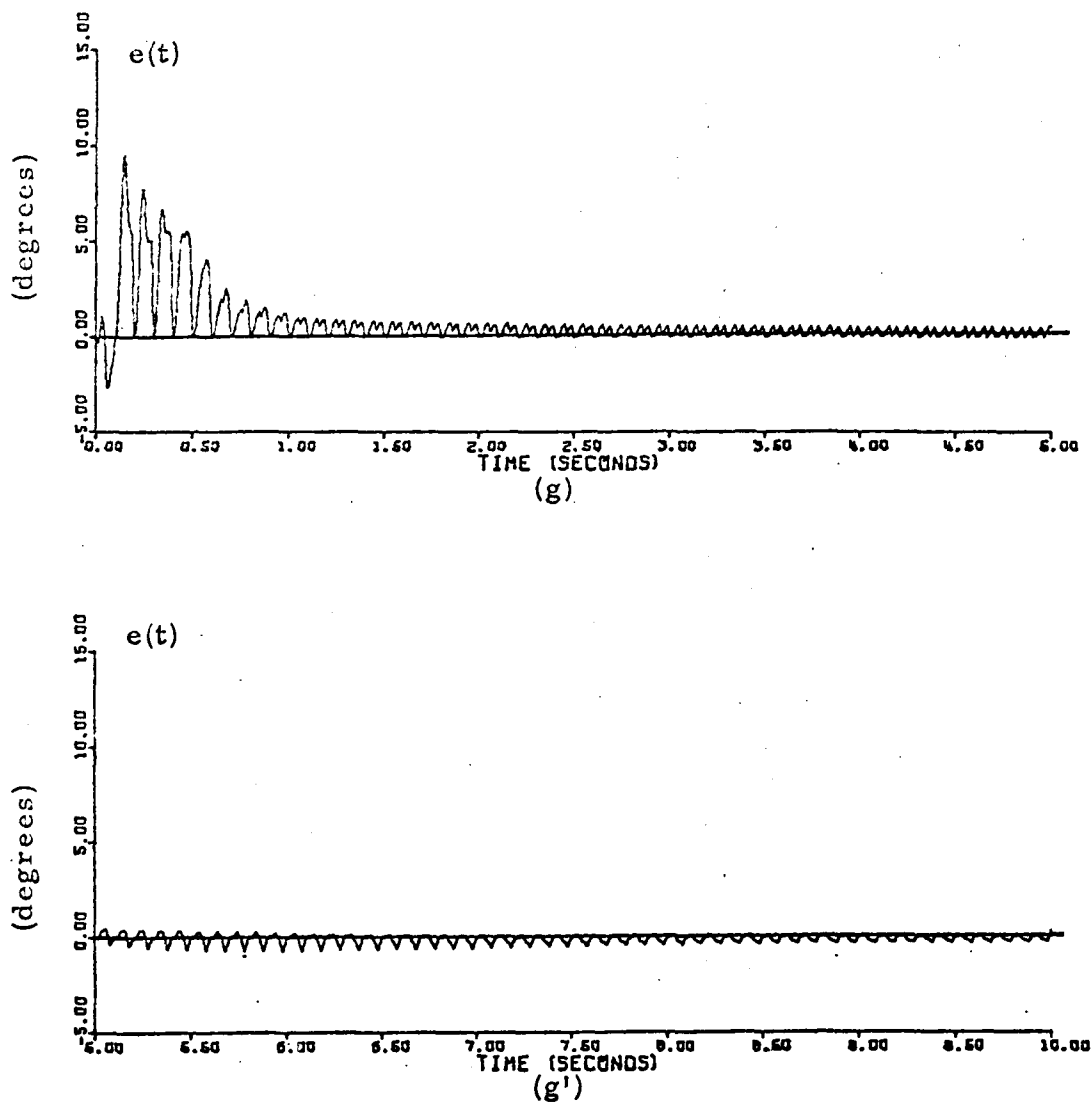


Figure 7-4 (Continued)

- (g) Model-reference system error $e(t)$ for first 50 cycles
- (g') Model-reference system error $e(t)$ for second 50 cycles

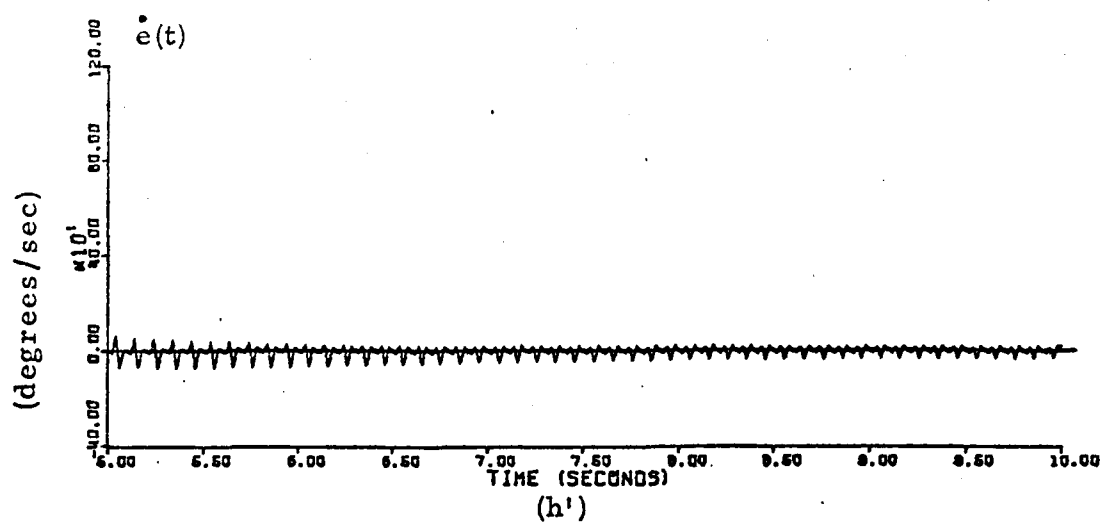
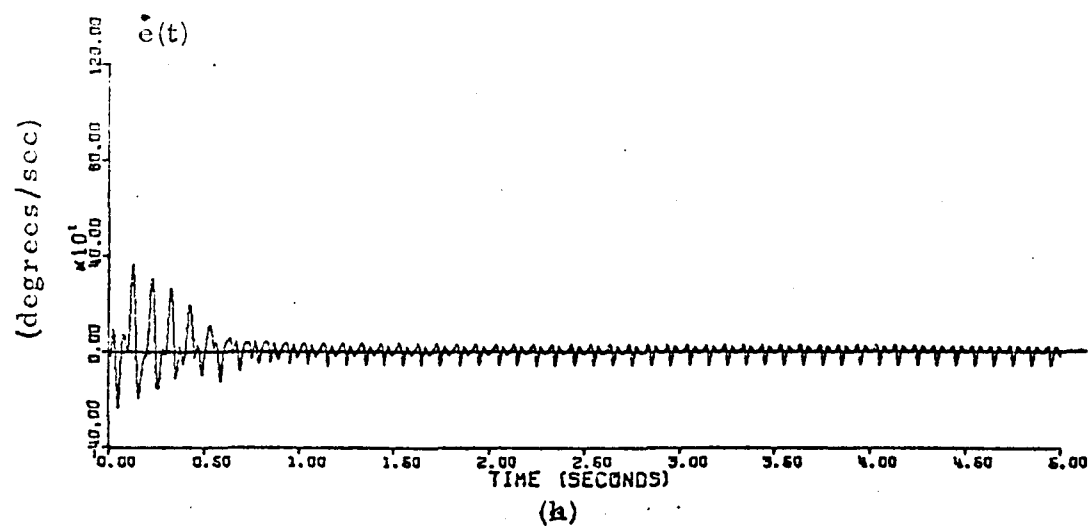


Figure 7-4 (Continued)

- (h) Model-reference system error rate of change $\dot{e}(t)$ for first 50 cycles
- (h') Model-reference system error rate of change $\dot{e}(t)$ for second 50 cycles

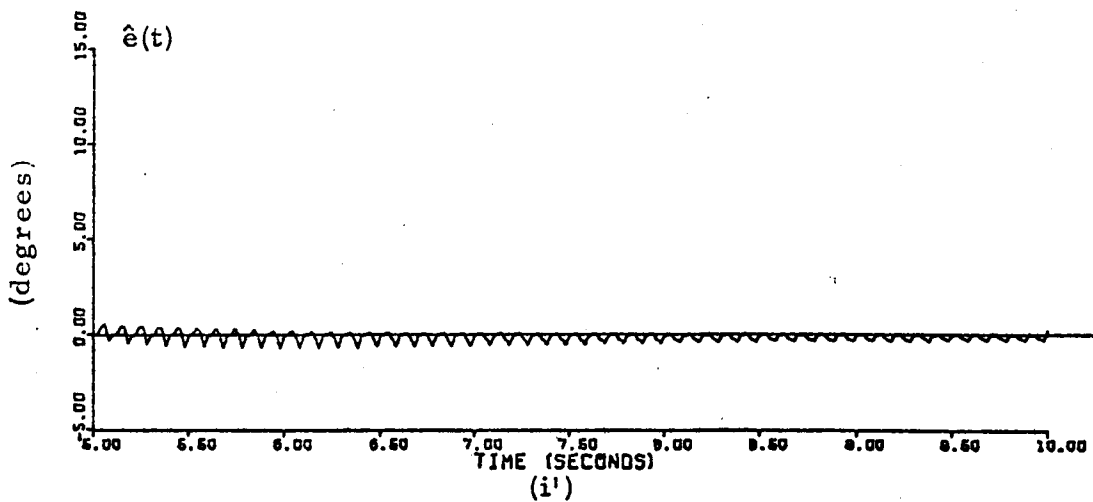
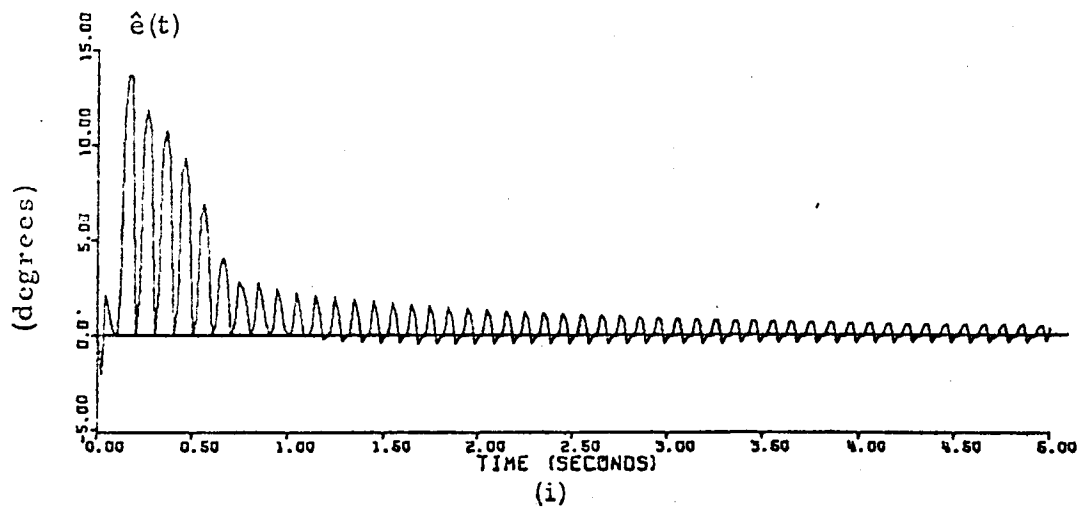


Figure 7-4 (Continued)

- (i) Error $\hat{e}(t)$ between reference system and model when model parameters are constant during each cycle and equal the "learned" values at termination of previous cycle. (first 50 cycles)
- (i') error $\hat{e}(t)$ for second 50 cycles

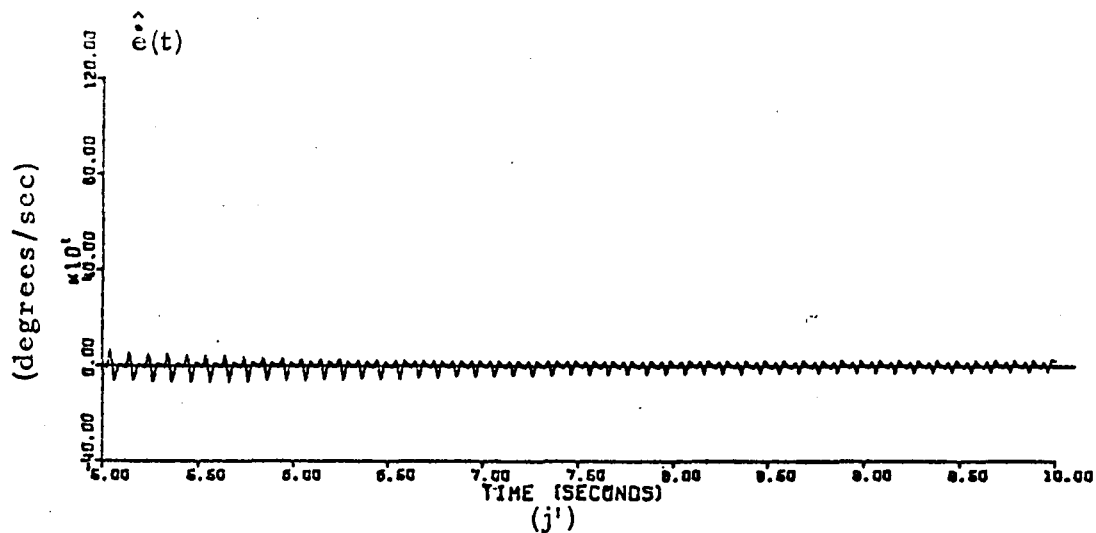
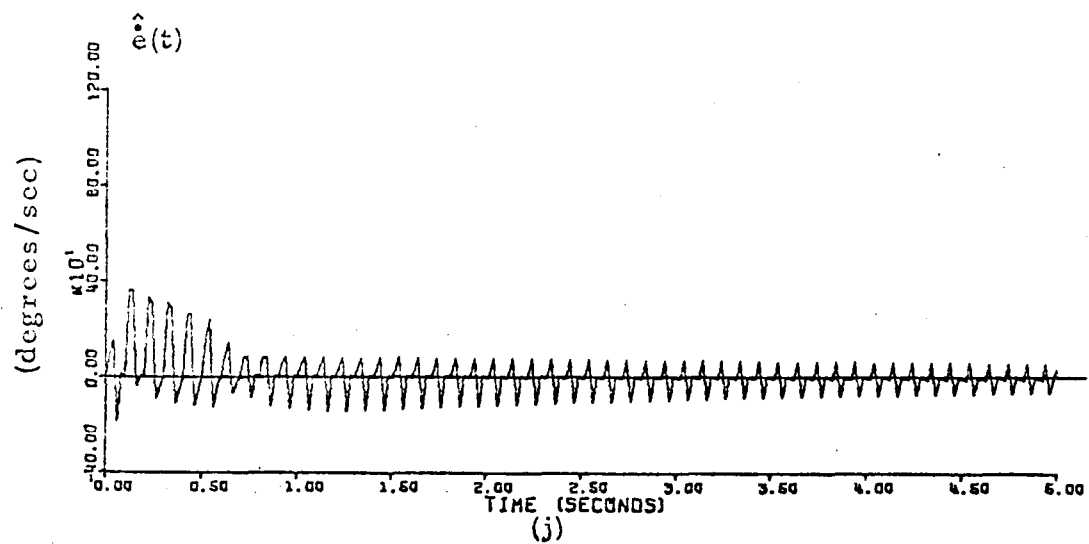


Figure 7-4 (Continued)

(j) Rate of change for error $\hat{e}(t)$: $\dot{\hat{e}}(t)$ first 50 cycles

(j') $\hat{e}(t)$ for second 50 cycles

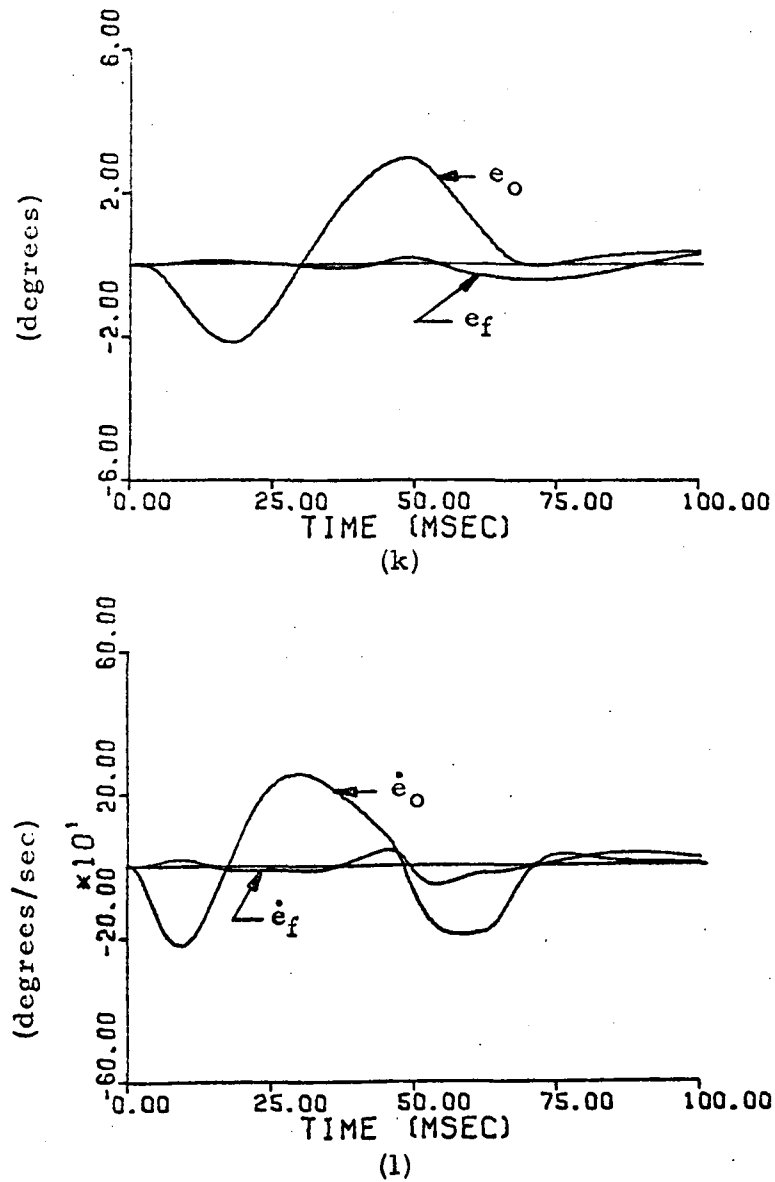


Figure 7-4 (Continued)

- (k) Eye position error between Robinson's model and reference system: (e_o)
 Eye position error between model (with "learned" parameters) and reference system: (e_f)
- (l) Eye velocity error between Robinson's model and reference system: ($\dot{e}_o$)
 Eye velocity error between model (with "learned" parameters) and reference system: ($\dot{e}_f$)

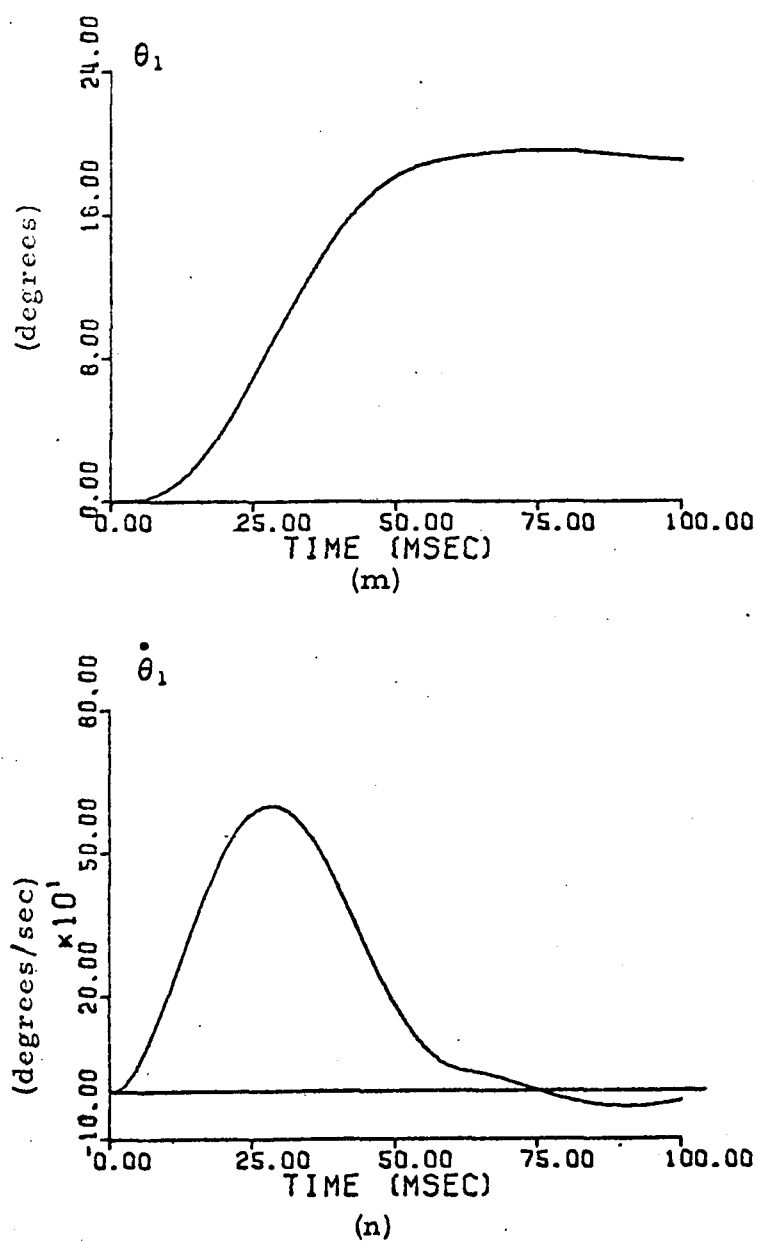


Figure 7-4 (Continued)

- (m) Eye position (θ_1) of model with "learned" parameters during an 18.75° saccade.
- (n) Eye velocity ($\dot{\theta}_1$) of model with "learned" parameters during an 18.75° saccade.

A new set of coefficients have been determined for Eq.(7-8), which yields a saccadic response conforming to the reference system's response. Now Robinson's model (Fig. 7-1) must be re-examined in order to determine the changes that are needed to obey the new differential equation. This is discussed in the next section.

7.5 Modification of Robinson's Model

It has been shown that Robinson's model may be described by the fourth order linear differential equation (Eq.(7-4)) where the nominal coefficients are defined by Eqs. (7-5) in terms of the mechanical element values. As a first approach to interpreting Robinson's model with respect to the new differential equation coefficients, one could replace the nominal values in Eqs. (7-5) by the "learned" coefficient values given in Table 7-4 and then try to solve the six simultaneous non-linear equations for the new values of mechanical elements. Since there are seven elements and only six equations, the solution is not unique. One of the element values would have to be initially chosen and then the equations can be solved for the remaining elements. But in this case, this approach will always lead to a solution where some of the element values turn out to be either negative or complex quantities. Therefore some structural changes are needed in Robinson's model to conform to the new differential equations. Hence another method must be tried to interpret the results. One possible method is to consider the active and passive

portions of the model separately. Rewriting Eq. (7-2), which relates the passive force F_p to eye movement θ , as

$$\left\{ \frac{R_1 R_2}{R_1 + R_2} s^2 + \frac{R_1 K_2 + R_2 K_1}{R_1 + R_2} s + \frac{K_1 K_2}{R_1 + R_2} \right\} \theta = \left\{ s + \frac{K_1 + K_2}{R_1 + R_2} \right\} F_p . \quad (7-12)$$

One may now define the coefficients of Eq. (7-12) by the constants

$$D_1 = \frac{R_1 R_2}{R_1 + R_2} \quad (7-13a)$$

$$D_2 = \frac{R_1 K_2 + R_2 K_1}{R_1 + R_2} \quad (7-13b)$$

$$D_3 = \frac{K_1 K_2}{R_1 + R_2} \quad (7-13c)$$

and

$$D_4 = \frac{K_1 + K_2}{R_1 + R_2} . \quad (7-13d)$$

Substituting from Eqs. (7-13) into Eqs. (7-5) and replacing the nominal coefficient values by the new "learned" values yields the following set of equations:

$$l_3^* = \frac{g_0}{g_1} + m g_1 + \frac{D_1}{m} , \quad (7-14a)$$

$$l_2^* = m g_0 + \frac{D_2}{m} + g_1 D_1 + R_m g_1 , \quad (7-14b)$$

$$l_1^* = \frac{l_0^*}{m g_1} + g_1 D_2 + R_m g_0 , \quad (7-14c)$$

$$l_0^* = g_1 D_3 , \quad (7-14d)$$

$$g_1 = \frac{K_e}{m R_m} \quad , \quad (7-14e)$$

and

$$g_0 = \frac{g_1}{D_4} \quad . \quad (7-14f)$$

Assuming that the mass (m) is the element value to be chosen, Eqs.(7-14) can be solved for the values of R_m , K_e , D_1 , D_2 , D_3 , and D_4 in terms of known quantities. Choosing Robinson's calculated value of $m = .297 \times 10^{-4}$ (g. sec²/deg) for the functional moment of inertia of the eyeball, the values for the unknown terms in Eq. (7-14) are determined. These values are listed in Table 7-5.

Parameters	Value
R_m	-.222
K_e	-1.14
D_1	.0167
D_2	2.75
D_3	788.9
D_4	430.6

Table 7-5 Calculated values for R_m , K_e , D_1 , D_2 , D_3 and D_4 when $m = .297 \times 10^{-4}$ in Eqs. (7-14).

The fact that both R_m and K_e are negative seems to indicate that the net-active state tension (F_0), which is shown in Fig. 7-2, was

inadequate and therefore did not generate a sufficient active muscle force (F_m) to drive the eye during the saccade. When negative values are substituted for both R_m and K_e in Eq. (7-1), only the right hand side of the equation is altered:

$$\left| \frac{R_m}{K_e} \right| \frac{dF_m}{dt} + F_m = F_O + |R_m| \frac{d\theta}{dt} . \quad (7-15)$$

It can be seen, that (with the requirement R_m and K_e to be positive) Eq. (7-15) could also have been obtained from Eq. (7-1) by including an eye velocity feedback term in the original net active-state tension proposed by Robinson. The inclusion of velocity feedback in eye movement models is not uncommon. Vossius^[52] included a form of velocity feedback in his model and the non-linear active damping component introduced by Cook^[10,11,12] in his model, is essentially also a form of velocity feedback. Therefore by defining the total active state tension by

$$\hat{F}_O = F_O + 2 R_m \frac{d\theta}{dt} \quad (7-16)$$

where $R_m = +.222$ (g.sec/deg.), $K_e = +1.14$ (g/deg.) and F_O is as shown in Fig. 7-2, Eq. (7-1) can be rewritten as

$$\begin{aligned} \frac{R_m}{K_e} \frac{dF_m}{dt} + F_m &= \hat{F}_O - R_m \frac{d\theta}{dt} \\ &= F_O + R_m \frac{d\theta}{dt} \end{aligned} \quad (7-17)$$

which is identical to Eq. (7-15).

The next step in the analysis is to consider the realization of the transfer function which relates the passive force F_p to eye movement θ . By combining Eqs. (7-12) and (7-13), the passive transfer function becomes

$$F_p = \frac{D_1 s^2 + D_2 s + D_3}{s + D_4} \theta \quad (7-18)$$

Utilizing the classical analogy between electrical and mechanical systems as listed in Table 7-6, the realization of the mechanical transfer function, Eq. (7-18), is equivalent to realizing the driving point impedance

$$Z(s) = \frac{D_1 s^2 + D_2 s + D_3}{s(s + D_4)} \quad (7-19)$$

Mechanical Quantities	Electrical Quantities
Inertia m	Inductance L
Viscous friction R	Resistance R
Spring stiffness K	Elastance $1/c$
Force F	Voltage e
displacement θ	Charge Q

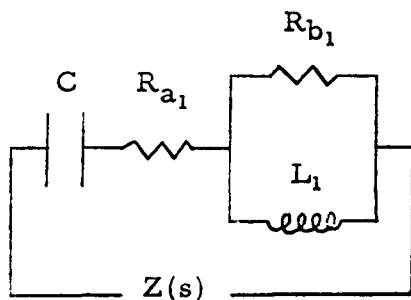
Table 7-6 Force-voltage analogy between electrical and mechanical systems^[78].

Since the zeroes of $Z(s)$ are complex conjugate quantities, the driving point impedance is not a Frcz function^[80], and therefore is not

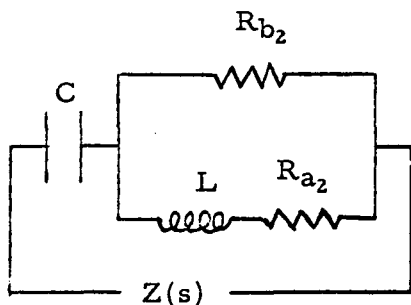
physically realizable using only resistors and capacitors. Hence, the mechanical transfer function of Eq. (7-18) can not be realized by using only springs and dashpots. Two possible realizations of the impedance $Z(s)$ using resistors, capacitors and inductors are presented in Fig. 7-5. These electrical networks can easily be transformed into their corresponding mechanical network by using the procedure outlined by Barker^[5]. First the networks of Fig. 7-5 are transformed into their dual network and then using a consistent force-current analogy between electrical and mechanical systems, the mechanical networks in Fig. 7-6 are obtained. The element values for the mechanical networks are given in Table 7-7, and are of the same order of magnitude for each network.

Mechanical Elements	Value
$K = 1/C$	1.832 gm./deg.
R_{a_1}	.00212 gm.sec./deg.
R_{b_1}	.0146 gm.sec./deg.
m_1	$.338 \times 10^{-4}$ gm.sec ² /deg.
R_{a_2}	.00243 gm.sec./deg.
R_{b_2}	.0167 gm.sec./deg.
m_2	$.443 \times 10^{-4}$ gm.sec ² /deg.

Table 7-7 Element values for the mechanical networks of Fig. 7-6.



(a)



(b)

$$C = \frac{D_4}{D_3}$$

$$R_{a1} = \frac{D_2 - 1/C}{D_4}$$

$$R_{b1} = D_1 - R_{a1}$$

$$L_1 = \frac{R_{b1}}{D_4}$$

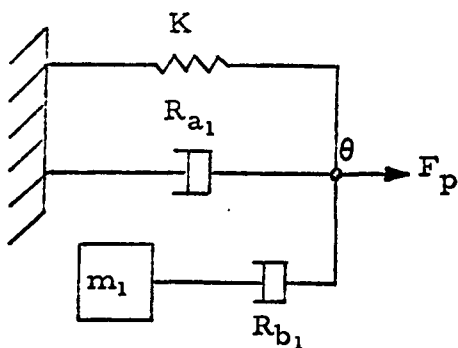
$$\Omega = D_4 - \frac{1}{D_1} (D_2 - \frac{1}{C})$$

$$R_{a2} = \frac{D_2 - 1/C}{\Omega}$$

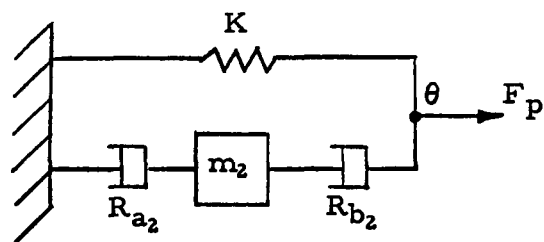
$$R_{b2} = D_1$$

$$L = \frac{D_1}{\Omega}$$

Figure 7-5 Two possible realizations of the driving point impedance $Z(s)$



(a)



(b)

Figure 7-6 Two possible realizations of the mechanical transfer function relating passive force (F_p) to eye movement (θ)

Except for an additional dashpot and an inertial element, both realizations of the mechanical transfer function relating the passive force (F_p) to eye movement (θ) resemble a Voigt muscle element. Robinson had represented the passive mechanical elements of the eye in his model by two Voigt elements. Cook used only one Voigt element in his model. The spring stiffness K is only slightly larger than the combined passive spring stiffness assumed by Robinson. The values of Ra_1 , Ra_2 , Rb_1 and Rb_2 are considerably lower in value than either of the passive viscosity elements in Robinson's original model, but Rb_1 and Rb_2 are of the same order of magnitude as the passive damping coefficient specified by Cook. Furthermore, it should be noted that both networks in Fig. 7-6 are underdamped systems. Meanwhile, Robinson's use of an overdamped system to describe the passive portion of eye muscles and orbit is more representative of the actual system. This seems to be clearly verified by the many eye-movement experiments performed by Robinson^[69]. Here again, the incorrect structure predicted by the "learned" parameter values for the passive elements of the saccadic eye mechanism may also be linked to an error in the composition of the net active state tension (F_O). Using the net-active state tension as specified in Fig. 7-2 along with an overdamped passive muscle network could possibly have resulted in an overall system which is too sluggish to describe the eye movement system. The parameter control signals may have transformed the passive network from an overdamped system into an

underdamped system in order to compensate for the possible erroneous construction of the net-active state tension. Therefore, the composition of the net-active state tension in Fig. 7-2 should be further investigated.

When one of the passive networks shown in Fig. 7-6 is substituted for the passive network proposed by Robinson for his model and a velocity feedback term is included in the determination of the net-active state tension (F_O), the output response of the model becomes more representative of the reference system. This should not be interpreted to mean that the model structure now truly corresponds to the actual structure of the human saccadic eye-movement system. But instead, the modified model structure reveals areas to be further considered in future research studies.

7.6 Summary and Findings

Applying the parameter control signal design method to Robinson's model while using Cook's model to generate the reference response for a saccadic movement has produced many interesting results. It has illustrated that the controller design method may be utilized not only to determine parameter values, but also to suggest some structural changes or modifications which may be needed for a given model.

One of the major modifications that is indicated for the Robinson model is the inclusion of some form of velocity feedback in the computation of the net-active state tension (F_O). This could correspond to the

velocity feedback that is introduced in Cook's model by his addition of the non-linear active damping element. Furthermore, the results suggest that some changes may be needed in the network of the model which represents the passive restraining forces on the eye, and/or in the composition of F_O depicted in Fig. 7-2. When the previously mentioned modifications, which are developed in this chapter, are applied to Robinson's original model, the model generates a response that is equivalent to the reference system's response. Therefore, these modifications should be considered in future research studies in order to obtain a better understanding of the human saccadic eye-movement system.

It should also be noted that both Robinson and Cook first developed the structure of their models on the basis of physiological and anatomical considerations. After making initial estimates for the values of the models' elements, they would then proceed to vary these values by trial and error until good agreement is obtained between their model's response and their experimental data. An adaptive automated procedure such as the parameter control signal design method could be used to aid them in the determination of their final model parameters.

CHAPTER 8

SUMMARY AND CONCLUSION

8.1 Summary

In this dissertation, a design method has been developed for a parameter controller which can be utilized as an aid in the exploration of physiological control system modeling problems. Existing model-reference system techniques along with new approaches were employed in the development, and the model parameter control signals are determined by utilizing Liapunov's stability theory. Although this controller can be employed as an aid toward the modeling of a variety of control systems, specific emphasis is placed on physiological control system modeling problems because the technique overcomes many of the limitations encountered in the physiological control system modeling process.

The design method is mainly intended to improve upon existing physiological system models, by automatically updating the model's postulated parameter values to more accurately conform to the experimental observations obtained from the actual physiological system. After the initial model structure has been assumed, the model, the designed controller and the reference system under consideration, are arranged in a model-reference system configuration. Under the influence of the controller, the model's postulated parameters are modified in a continuous manner to cause the model-reference system

error to approach zero and the model parameter values to approach the values of the reference system parameters. In this sense, the model is considered to have "learned" the reference system.

In Chapter 3, the development for the linear modeling problem is considered. The type of information that is needed and must be obtained for the physiological system before the design procedure can be applied is indicated. It is then shown how the model sub-system is structured and placed in the model-reference system configuration. The formulation of the model-reference error differential equation is next presented and the state matrix equation

$$\dot{\underline{e}} = [A] \underline{e} + \underline{y}$$

is derived. In this equation, $\underline{e}$ represents the model-reference error vector, Matrix $[A]$ is formed from the unknown parameters associated with the reference system and

$$\underline{y} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ y(t) \end{bmatrix},$$

where the function $y(t)$ contains the parameter control signals x_{α_i} for $i=0, \dots, n-1$, and x_{γ_j} for $j=0, \dots, h$. These control signals are designed in Chapter 5.

In Chapter 4, the model-reference formulation is presented for a class of non-linear modeling problems. In some cases, the non-linearity,

which is represented by a power series, may need to be modified by a normalization procedure. This procedure is outlined and also illustrated by an example. It is also shown that the parameter control signals needed in the non-linear case are similar to those introduced in Chapter 3, except for the fact that an extra additive control signal $x_a(t)$ must also be included for the model. The model-reference error differential equation for the non-linear modeling problem is then derived and again represented by the state matrix equation

$$\dot{\underline{e}} = [A]\underline{e} + \underline{y} .$$

For this case, the element $y(t)$, in vector $\underline{y}$, includes the extra additive control signals $x_a(t)$, the parameter control signals plus other non-linear terms.

Different methods for designing the parameter control signals utilizing Liapunov's Second Method are then presented in Chapter 5. Section 5.3 contains the development of the control signals without memory capabilities and Section 5.4 presents the formulation of control signals that possess memory or "learning" capabilities. In both cases it is required that there exists a positive definite M-matrix such that the D-matrix defined by

$$-D = A^T M + M A$$

is also a positive definite matrix. Furthermore, the elements of the A-matrix must satisfy certain requirements. These requirements are referred to as the "Strong" Routh-Hurwitz conditions and they are developed in Appendix B

for the second, third and fourth order system. It is then shown that only the last column or row of the M-matrix need be determined in order to generate the function

$$s(t) = e_1 m_{1n} + e_2 m_{2n} + \dots + e_n m_{nn}$$

which appears in all of the control signals. With the aid of Theorems 5-1 and 5-2, the M-matrix elements contained in $s(t)$ are determined and the existence of both a positive definite M and D matrix is assured. The control signals are finally determined based upon the requirement that the derivative of the Liapunov function, $\dot{V}(e)$, is a negative definite function.

In Chapter 6, several examples are presented to illustrate the design of the parameter control signals and their "learning" capabilities when applied to artificial modeling situations. For each example, some or all of the initial values assumed for the model parameters were purposely chosen to be different in value than the known reference system parameters. Using the model-reference system configuration, the model's parameters are automatically updated and they "learned" the reference system parameter values.

The application of the design method is further demonstrated in Chapter 7, where it is applied to the modeling of an actual physiological system, namely the human eye-movement mechanism (saccadic eye movements). For this situation, the parameter control signals were designed to be applied to a model proposed by Robinson^[69]. His model has the advantage of being the simplest system consistent with the known

biomechanical properties of muscle and passive tissues, and yields reasonable agreement with the results of his experimental procedures on the human eye. For this modeling problem, the reference system's response used in the model-reference system configuration would normally be the actual positional measurements of a human's saccadic response to an applied change in target position. However, in their research studies, Cook and Stark^[10, 11, 12] performed experimental measurements of eye position and velocity during saccadic eye movements, and showed that their model's response closely matches their experimental data. Therefore, instead of repeating the actual measurements of eye movements, Cook's proposed model was utilized to generate the necessary reference system response in the model-reference system configuration.

Upon analyzing the "learned" coefficients for the differential equation that describes the model response, two possible modifications are proposed for Robinson's model so that the model response will conform to the "learned" differential equations. The first modification requires the inclusion of some form of velocity feedback in the computation of the eye muscle net-active state tension (F_O). The second modification suggests network changes to represent more accurately the passive restraining forces on the eye and/or the construction of the net-active state tension (F_O).

A more detailed discussion of the results of Chapter 5, 6 and 7 is presented in the next section.

8.2 Discussion of Results

In Chapter 5, Liapunov's second method is utilized to design the parameter control signals. However, because the second method incorporates only a sufficient condition for stability, some of the requirements imposed during the controller design development may at times be violated and still yield satisfactory results. This fact is illustrated by Example 2, part 3 in Chapter 6, where the reference system parameter values are purposely chosen to violate the conditions that $a_{0\min}^*$ and $a_{1\min}^*$ must be greater than zero (Table 5-1). Another infraction of the design requirements is displayed in Chapter 7 when the initial values for $a_{i\min}^*$ and $a_{i\max}^*$ in Eq.(5-44),

$$a_{i\min}^* < a_i^* < a_{i\max}^* \quad \text{for } i = 0, \dots, 3$$

were incorrectly specified by Eqs.(7-11). This violation was made evident upon examination of the waveforms of Fig.7-4 for the model subsystem coefficients l_i^* given in Eq.(7-9a). After a number of "learning" cycles, it is observed that some of the "learned" coefficients l_i^* tended toward values outside the range specified by Eqs.(5-44) and (7-11). Therefore, the values for $a_{i\min}^*$ and $a_{i\max}^*$ had to be modified, and by use of Table 5-2, this also modifies the values calculated for the M-matrix elements.

Another important feature of Liapunov's Second Method is that Liapunov functions are not unique. This accounts for the facts that differ-

ent control signal design methods could be presented in Chapter 5 and also that certain positive constants (i.e. β_i and Δ_j) of Eqs.(5-52) and (5-53), could be arbitrarily chosen. However, the choice of controller constants can greatly affect the model-reference system behavior and particularly the duration of the parameter "learning" process. It would be of great advantage to be able to determine the constant values that yield the "optimal" "learning" process. In Section 5.4.2, a procedure was presented for determining the values of β_i and Δ_j by simulation studies, but a much better method is clearly needed and is definitely recommended for future research.

After exhibiting the "learning" capabilities of the parameter control signals in Chapter 6, the controller design method was applied to the modeling of the human saccadic eye movement mechanism. This modeling problem was not intended to be a comprehensive study of the saccadic eye movement system, but instead is mainly intended to demonstrate the suitability of this design method for dealing with an actual physiological system and also to indicate the type of results that may be obtained from the procedure. To match the response of Cook's model, used to generate the reference system data, the parameter control signals updated the coefficients of the differential equation derived from Robinson's model to the values listed in Table 7-4. Using the newly "learned" coefficient values, it is shown that some of the mechanical element values indicated for Robinson's model (Figure 7-1) would need to be either negative or

complex quantities in order for the model output response to be governed by the obtained differential equation. Therefore some structural changes of the model were considered. One change was the inclusion of a velocity feedback term in the original net-active state tension proposed by Robinson. This modification would correspond to the velocity feedback that is introduced in Cook's model by his addition of the non-linear active damping elements, B_{ag} and B_{ant} , used to describe the mechanics of the eye-muscles. Furthermore, the realization of the resulting transfer function, which relates the passive force (F_p) to eye movement, suggests that some changes are also needed in the network which represents the passive restraining forces on the eye and/or the construction of the net-active state tension (F_o) applied to the eye muscles. The modifications indicated for Robinson's model will yield an output response which is more representative of the reference system response used.

It should be emphasized that the controller design method is intended mainly as an aid to physiological control system modeling. In general, analysis of the results obtained by the controller design method will not necessarily dictate that the actual physiological system under consideration must be structured in the proposed manner. Instead, analysis of the results will suggest certain modifications which should be considered in future research studies and thereby more clearly delineate the types of experimental data required to further elucidate the organization and operation of the physiological system under consideration.

The parameter design method may also be used to investigate abnormalities in the behavior of physiological system. After establishing a model that is representative of the "normal" physiological system, the model plus controller could be used to facilitate investigations of abnormal system behavior due to aberrant physiological conditions and neurological disorders by attempting to establish a correspondence between certain human disorders and parameter variations or malfunctions of components in the model. The approach outlined in this study may lead to certain diagnostic technique improvements.

8.3 Recommendations

This dissertation presents a design method for a parameter controller that may be utilized to assist in physiological control system modeling processes. After determining the initial model structure, the model, the designed controller and the actual physiological system under consideration are arranged in a model-reference configuration. The model's assumed parameters are then updated by the controller and thereby causing the model-reference error to approach zero and also permitting the model to "learn" the reference system. The following areas of research are logical extensions of this dissertation.

- 1) Develop a criterion for the determination of the control signal constants that would yield an "optimal" time response for the parameter "learning" process.

- 2) Extension of the present design method to include different classes of non-linearities .
- 3) Investigate new forms of Liapunov functions from which different controllers may be designed.
- 4) Apply the design method in comprehensive modeling studies for a variety of physiological systems .
- 5) Investigate the usefulness of the design method to the development of diagnostic technique improvements .

APPENDIX A:

SECOND METHOD OF LIAPUNOV^[32,40,41,63]

Liapunov considered two methods for analyzing the stability of systems. The first, or indirect, method comprises all the procedures that consists of solving the system differential equations and then examining the solution for stability. His second method yields stability information without the necessity of solving the systems differential equations and is therefore called the direct method. It is Liapunov's second, or direct, method which is applied to the model-reference error equations, $\dot{\underline{e}} = \underline{F}(\underline{e}, t)$, that are formulated in Chapters 3 and 4.

Liapunov's second method is essentially a generalization of the energy concepts of classical mechanics and is based upon the existence of a positive definite scalar state function $V(\underline{e})$ of the state error vector $\underline{e}$, and must possess the following properties^[32,63]:

- (a) $V(\underline{e})$ is continuous and has continuous first partial derivatives in a region Ω about the equilibrium point $\underline{e} = 0$, where the origin is an isolated equilibrium point.
- (b) $V(\underline{0}) = 0$
- (c) In the region Ω , but excluding the origin, $V(\underline{e})$ is positive.

The derivative of $V(\underline{e})$ along the trajectory of $\underline{e}$ can now be determined. Since $V(\underline{e})$ has first partial derivatives, it has a gradient $[\nabla V]$, and one can obtain $\dot{V}(\underline{e}) = [\nabla V]^T \cdot \dot{\underline{e}}$ as its time derivative, where $[\nabla V]^T$ is the

transpose of $[\nabla V]$. With the aforementioned properties, $V(\underline{e})$ can only be considered a candidate Liapunov function. If in addition $\dot{V}(\underline{e}) \leq 0$ in the region Ω , $V(\underline{e})$ is called a Liapunov function. From these definitions, Liapunov's stability theory can be summarized by the following theorems.

(1) Stability Theorem

If there exists in some neighborhood Ω of the origin a Liapunov function $V(\underline{e})$, then the origin is stable.

(2) Asymptotic Stability Theorem

If in addition to the Theorem-1 requirement, $\dot{V}(\underline{e})$ is negative definite $[\dot{V}(\underline{e}) < 0]$ in Ω then the stability is asymptotic.

(3) Global Asymptotic Stability Theorem

If the conditions of Theorem-2 are satisfied for all $\underline{e}$ and in addition $V(\underline{e}) \rightarrow \infty$ as the Euclidean norm of $\underline{e}$ $|\underline{e}| \rightarrow \infty$, then the system is asymptotically stable in the large.

It should be noted that Liapunov's stability theorems are only sufficient conditions for stability. Furthermore, the Liapunov function $V(\underline{e})$ is not unique.

APPENDIX B:
 FORMULATION OF THE "STRONG" ROUTH-HURWITZ
 CONDITIONS AND DETERMINATION OF THE M-MATRIX

This appendix presents the formulation of the "Strong" Routh-Hurwitz conditions and the determination of the elements for the M-matrix used in the Liapunov function of Chapter 5.

Consider Eq.(5-15), $\dot{\underline{e}} = \underline{A}\underline{e}$. The A-matrix, which is assumed to be stable, is defined by Eqs. (3-22) and (4-37) for the linear and non-linear modeling cases respectively. The range of the A-matrix elements are specified by Eq.(5-14), $a_{i\min}^* < a_i^* < a_{i\max}^*$ for $i = 0, 1 \dots n$. The quadratic form $V(\underline{e}) = \underline{e}^T \underline{M} \underline{e}$ given by Eq.(5-2), is chosen as a candidate Liapunov function, where M is a symmetric matrix. Taking the time derivative of $V(\underline{e})$ along the trajectory of Eq.(5-15) results in

$$\dot{V}(\underline{e}) = \underline{e}^T [\underline{A}^T \underline{M} + \underline{M} \underline{A}] \underline{e} = -\underline{e}^T \underline{D} \underline{e}$$

where

$$-\underline{D} = \underline{A}^T \underline{M} + \underline{M} \underline{A} \quad . \quad (5-11)$$

The problem at hand is to determine the elements of the M-matrix, whose last column must consist of known quantities, such that no matter what level the elements of the A-matrix assume within their specified range, both the M and D matrices remain positive definite.

Elements of the M-matrix may be divided into two groups; the first

group, Group-I, consists of all the elements from the last column or row of the M-matrix, and the second group, Group-II, consists of all the remaining elements. It is those elements contained in Group-I which must be known constants and therefore cannot depend implicitly upon the unknown parameters (a_i^*) present in the A-matrix.

With the aid of theorems 5-1 and 5-2, the elements of the M-matrix can be determined provided certain inequality relationships, the "Strong" Routh-Hurwitz conditions, for the range limits $a_{i \max}^*$ and $a_{i \min}^*$ are satisfied. The need for these inequality relationships are made evident during the analysis and since they are very similar to the Routh-Hurwitz stability criterion, yet more restrictive, they are referred to here as the "Strong" Routh-Hurwitz conditions.

The technique for evaluating the M-matrix elements and determining the "Strong" Routh-Hurwitz condition is best presented by illustrative examples. A brief description outlining the general approach is next presented; afterward, the second, third and fourth order cases are considered in detail.

Since it has been assumed that A represents a stable matrix, Theorem (5-1) insures that the M-matrix is uniquely determined once the D-matrix has been specified. Furthermore, if the D-matrix is chosen to be positive definite, Theorem (5-2) states that the M-matrix will likewise be positive definite. Therefore the D-matrix is chosen to be a positive definite diagonal matrix so that the matrix Eq.(5-11) yields the following

set of n inequalities

$$0 < d_{pq} = -(A^T M)_{pq} - (M A)_{pq} \quad (B-1)$$

when $p = q$,

plus the following set of $\frac{n^2 - n}{2}$ equations

$$0 = d_{pq} = (A^T M)_{pq} + (M A)_{pq} \quad (B-2)$$

when $p \neq q$. Note that p and q denote rows and columns respectively and the indices $p, q = 1, 2, \dots, n$. Once the values for d_{pp} , where $p = 1, 2, \dots, n$, are determined, then both the D and M matrices are completely specified and positive definite.

The set of equations (B-2) can now be rearranged to obtain expressions for the Group-II M -matrix elements in terms of the parameters $\{a_i^\star\}$ and the Group-I matrix elements. These expressions are then substituted into the set of inequalities (B-1) resulting in n inequalities whose terms are only composed of the parameters $\{a_i^\star\}$ and Group-I matrix elements. Using the known range limits $a_{i_{\min}}^\star$ and $a_{i_{\max}}^\star$ from Eq. (5-14), boundary limits are determined for the Group-I M -matrix elements. From these boundary limits, values can be found for the elements of Group-I in terms of the known quantities $a_{i_{\min}}^\star$ and $a_{i_{\max}}^\star$. During the determination of the Group-I M -matrix elements from their boundary limits, it is seen that certain requirements must be imposed on the limits $a_{i_{\min}}^\star$ and $a_{i_{\max}}^\star$ to insure that all the inequalities relationships used in the development are

always preserved. These requirements are the "Strong" Routh-Hurwitz conditions.

After the Group-I M-matrix elements are determined, the D matrix is determined and it is not necessary to compute the remaining M-matrix elements (Group-II), because only the values for Group-I elements are needed for the parameter controller. Theorems (5-1) and (5-2) guarantee that the remaining elements exist, are unique, and produce a positive definite M-matrix.

The evaluation of the M-matrix and the "Strong" Routh-Hurwitz conditions for the second, third and fourth order system are now presented in detail.

Second-Order Case

The A and M matrices for the second-order case are

$$A = \begin{bmatrix} 0 & 1 \\ -a_0^* & -a_1^* \end{bmatrix}, \quad M = \begin{bmatrix} m_{11} & m_{12} \\ m_{12} & m_{22} \end{bmatrix}$$

with the parameters a_0^* and a_1^* confined to the ranges

$$\begin{aligned} 0 < a_{0\min}^* < a_0^* < a_{0\max}^* \\ 0 < a_{1\min}^* < a_1^* < a_{1\max}^* \end{aligned}$$

The Routh-Hurwitz stability conditions are $a_0^* > 0$ and $a_1^* > 0$.

Expanding the matrix Eq.(5-11) $-D = A^T M + M A$, yields

$$-\begin{bmatrix} d_{11} & d_{12} \\ d_{12} & d_{22} \end{bmatrix} = -\left[\begin{array}{c|c} + 2 a_0^* m_{12} & -m_{11} + a_1^* m_{12} + a_0^* m_{22} \\ \hline -m_{11} + a_1^* m_{12} + a_0^* m_{22} & 2(a_1^* m_{22} - m_{12}) \end{array} \right]$$

or

$$d_{11} = 2 a_0^* m_{12}$$

$$d_{12} = a_1^* m_{12} + a_0^* m_{22} - m_{11}$$

$$d_{22} = 2(a_1^* m_{22} - m_{12}) .$$

If the D-matrix is now constrained to be a positive definite diagonal matrix, the following set of expressions are obtained

$$0 < \frac{d_{11}}{2} = a_0^* m_{12} \quad (\text{B-3})$$

$$0 < \frac{d_{22}}{2} = a_1^* m_{22} - m_{12} \quad (\text{B-4})$$

$$0 = d_{12} = a_1^* m_{12} + a_0^* m_{22} - m_{11} . \quad (\text{B-5})$$

Since the system is assumed to be stable, both parameters a_0^* and a_1^* must always be positive. Hence, $a_{0\min}^*$ and $a_{1\min}^*$ must also be positive. Therefore the "Strong" Routh-Hurwitz conditions for the second order case are

$$a_{0\min}^* > 0 \text{ and } a_{1\min}^* > 0 .$$

From inequality (B-3), it is seen that m_{12} must be positive while inequality (B-4) indicates that $m_{12} < a_1^* m_{22}$. Both conditions will be satisfied if

$$m_{12} = a_{1\min}^{\star} m_{22} \quad ,$$

provided that m_{22} is a positive constant.

Regardless of the system order, the element in the last column and row of the M-matrix can be chosen as any positive constant because a positive common factor can be extracted from each of the M-matrix elements without altering the basic properties of the functions $V(\underline{e})$ and $\hat{V}(\underline{e})$. Therefore using Eq.(B-5)

$$m_{11} = a_1^{\star} a_{1\min}^{\star} m_{22} + a_0^{\star} m_{22} \quad (B-5a)$$

and the diagonal elements of the D matrix (from Eqs. (B-3) and (B-4)) are

$$d_{11} = 2 a_0^{\star} a_{1\min}^{\star} m_{22} \quad (B-3a)$$

and

$$d_{22} = 2 [a_1^{\star} - a_{1\min}^{\star}] m_{22} \quad . \quad (B-4a)$$

Third-order Case

The A and M matrices for the third order case are

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -a_0^{\star} & -a_1^{\star} & -a_2^{\star} \end{bmatrix} \quad , \quad M = \begin{bmatrix} m_{11} & m_{12} & m_{13} \\ m_{12} & m_{22} & m_{23} \\ m_{13} & m_{23} & m_{33} \end{bmatrix}$$

with the parameters $a_0^{\star}$, $a_1^{\star}$ and $a_2^{\star}$ confined to the ranges

$$0 < a_{0\min}^{\star} < a_0^{\star} < a_{0\max}^{\star}$$

$$0 < a_{1\min}^{\star} < a_1^{\star} < a_{1\max}^{\star}$$

$$0 < a_{2\min}^{\star} < a_2^{\star} < a_{2\max}^{\star} \quad .$$

The regular Routh-Hurwitz stability conditions are

$$a_0^*, a_1^*, a_2^* > 0 \quad \text{and}$$

$$a_1^* a_2^* - a_0^* > 0 .$$

Expanding the matrix Eq.(5-11) $-D = A^T M + M A$ yields

$$- \begin{bmatrix} d_{11} & d_{12} & d_{13} \\ d_{12} & d_{22} & d_{23} \\ d_{13} & d_{23} & d_{33} \end{bmatrix} = - \begin{bmatrix} 2 a_0^* m_{13} & a_0^* m_{23} + a_1^* m_{13} - m_{11} & a_0^* m_{33} + a_2^* m_{13} - m_{12} \\ a_0^* m_{23} + a_1^* m_{13} - m_{11} & 2(-m_{12} + a_1^* m_{23}) & a_2^* m_{23} + a_1^* m_{33} - m_{22} - m_{13} \\ a_0^* m_{33} + a_2^* m_{13} - m_{12} & a_2^* m_{23} + a_1^* m_{33} - m_{22} - m_{13} & 2(a_2^* m_{33} - m_{23}) \end{bmatrix}$$

If the D-matrix is constrained to be a positive definite diagonal matrix, the following set of inequalities are obtained from the diagonal terms;

$$a_0^* m_{13} = \frac{d_{11}}{2} > 0 \quad (B-6)$$

$$a_1^* m_{23} - m_{12} = \frac{d_{22}}{2} > 0 \quad (B-7)$$

$$a_2^* m_{33} - m_{23} = \frac{d_{33}}{2} > 0 \quad (B-8)$$

and the following set of equalities are obtained from the off diagonal terms,

$$0 = d_{12} = a_1^* m_{13} + a_0^* m_{23} - m_{11} \quad (B-9)$$

$$0 = d_{13} = a_2^* m_{13} + a_0^* m_{33} - m_{12} \quad (B-10)$$

$$0 = d_{23} = a_2^* m_{23} + a_1^* m_{33} - m_{22} - m_{13} \quad (B-11)$$

Equations (B-9), (B-10), and (B-11) can be solved for the elements m_{11} , m_{12} and m_{22} respectively, in terms of the Group-I M-matrix elements resulting

in the set of equations,

$$m_{11} = a_1^{\star} m_{13} + a_0^{\star} m_{23} \quad (B-12)$$

$$m_{12} = a_2^{\star} m_{13} + a_0^{\star} m_{33} \quad (B-13)$$

$$m_{22} = a_2^{\star} m_{23} + a_1^{\star} m_{33} - m_{13} \quad (B-14)$$

By substituting from Eq. (B-13) into Eq. (B-7) yields

$$a_1^{\star} m_{23} - a_2^{\star} m_{13} - a_0^{\star} m_{33} = \frac{d_{22}}{2} > 0 \quad (B-15)$$

Note: The elements m_{13} , m_{23} , and m_{33} must have positive values. This can be verified by solving the simultaneous set of Eqs. (B-6), (B-15) and (B-8).

The relationship of Eq. (B-8) is preserved if m_{23} is chosen equal to

$$m_{23} = a_{2\min}^{\star} m_{33} \quad (B-16)$$

where m_{33} is a positive constant. When m_{23} , defined by Eq. (B-16), is substituted into Eq. (B-15), one obtains

$$a_1^{\star} a_{2\min}^{\star} m_{33} - a_2^{\star} m_{13} - a_0^{\star} m_{33} = \frac{d_{22}}{2} > 0 \quad (B-17)$$

which can be rearranged to yield

$$m_{33} \left[\frac{a_1^{\star} a_{2\min}^{\star} - a_0^{\star}}{a_2^{\star}} \right] > m_{13} \quad (B-18)$$

By utilizing the range limits for the $a_i^{\star}$ parameters and Eq.(B-18), a lowest upper bound can be determined for m_{13} and specified by

$$m_{33} \left[\frac{a_{1\min}^{\star} a_{2\min}^{\star} - a_{0\max}^{\star}}{a_{2\max}^{\star}} \right] \geq m_{13} . \quad (\text{B-19})$$

From Eqs. (B-6) and (B-19), it is seen that m_{13} must be positive so that m_{13} may be set equal to

$$m_{13} = \left[\frac{a_{1\min}^{\star} a_{2\min}^{\star} - a_{0\max}^{\star}}{a_{2\max}^{\star}} \right] m_{33} \quad (\text{B-20})$$

provided that

$$a_{1\min}^{\star} a_{2\min}^{\star} - a_{0\max}^{\star} > 0 . \quad (\text{B-20a})$$

The above requirement is one of the "Strong" Routh-Hurwitz conditions.

Substituting the obtained values for m_{12} , m_{23} and m_{33} into Eqs. (B-6), (B-17) and (B-8) completely specifies the D-matrix as

$$0 < \frac{d_{11}}{2} = \frac{a_0^{\star}}{a_{2\max}^{\star}} (a_{1\min}^{\star} a_{2\min}^{\star} - a_{0\max}^{\star}) m_{33} \quad (\text{B-6a})$$

$$0 < \frac{d_{22}}{2} = \left\{ (a_1^{\star} a_{2\min}^{\star} - a_0^{\star}) - \frac{a_2^{\star}}{a_{2\max}^{\star}} (a_{1\min}^{\star} a_{2\min}^{\star} - a_{0\max}^{\star}) \right\} m_{33} \quad (\text{B-7a})$$

and

$$0 < \frac{d_{33}}{2} = (a_2^{\star} - a_{2\min}^{\star}) m_{33} . \quad (\text{B-8a})$$

The remaining M-matrix elements can be determined by substituting the values found for m_{13} , m_{23} and m_{33} into Eqs. (B-12), (B-13) and (B-14).

Fourth-order Case

The A matrix for the fourth order case is

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -a_0^* & -a_1^* & -a_2^* & -a_3^* \end{bmatrix}$$

where the parameters a_0^* , a_1^* , a_2^* and a_3^* are confined to the ranges

$$0 < a_{0\min}^* < a_0^* < a_{0\max}^*$$

$$0 < a_{1\min}^* < a_1^* < a_{1\max}^*$$

$$0 < a_{2\min}^* < a_2^* < a_{2\max}^*$$

$$0 < a_{3\min}^* < a_3^* < a_{3\max}^*$$

and also obey the regular Routh-Hurwitz stability conditions

$$a_0^*, a_1^*, a_2^*, a_3^* > 0$$

$$a_3^* a_2^* > a_1^*$$

$$a_1^* (a_3^* a_2^* - a_1^*) > a_0^* a_3^* .$$

Upon expanding the matrix Eq. (5-11) $-D = A^T M + M A$, where D is constrained to be a positive definite diagonal matrix, the following set of inequalities are obtained from the diagonal terms

$$0 < \frac{d_{11}}{2} = a_0^* m_{14} \quad (B-21)$$

$$0 < \frac{d_{22}}{2} = a_1^* m_{24} - m_{12} \quad (B-22)$$

$$0 < \frac{d_{33}}{2} = a_2^* m_{34} - m_{23} \quad (B-23)$$

$$0 < \frac{d_{44}}{2} = a_3^* m_{44} - m_{24} \quad (B-24)$$

and the following set of equations are obtained from the off diagonal terms .

$$0 = d_{12} = a_1^* m_{14} + a_0^* m_{24} - m_{11} \quad (B-25)$$

$$0 = d_{13} = a_2^* m_{14} + a_0^* m_{34} - m_{12} \quad (B-26)$$

$$0 = d_{14} = a_3^* m_{14} + a_0^* m_{44} - m_{13} \quad (B-27)$$

$$0 = d_{23} = a_2^* m_{24} + a_1^* m_{34} - m_{22} - m_{13} \quad (B-28)$$

$$0 = d_{24} = a_1^* m_{14} + a_3^* m_{24} - m_{14} - m_{23} \quad (B-29)$$

$$0 = d_{34} = a_2^* m_{44} + a_3^* m_{34} - m_{24} - m_{33} \quad (B-30)$$

The set of Eqs. (B-25) through (B-30) can be rearranged to yield Eqs.(B-31) through (B-36) which specify the Group-II M-matrix elements in terms of the Group-I M-matrix elements .

$$m_{11} = a_1^* m_{14} + a_0^* m_{24} \quad (B-31)$$

$$m_{12} = a_2^* m_{14} + a_0^* m_{34} \quad (B-32)$$

$$m_{13} = a_3^* m_{14} + a_0^* m_{44} \quad (B-33)$$

$$m_{22} = a_2^* m_{24} + a_1^* m_{34} - m_{13} = a_2^* m_{24} + a_1^* m_{34} - a_3^* m_{14} - a_0^* m_{44} \quad (B-34)$$

$$m_{23} = a_3^* m_{24} + a_1^* m_{44} - m_{14} \quad (B-35)$$

$$m_{33} = a_3^* m_{34} + a_2^* m_{44} - m_{24} \quad (B-36)$$

Substitutions must now be made in Eqs. (B-22) and (B-23) to eliminate the terms m_{12} and m_{23} respectively and thereby obtaining a set of 4 inequalities containing no Group-II M-matrix elements.

This is accomplished by utilizing the expressions in Eqs. (B-32) and (B-35) for m_{12} and m_{23} respectively to yield

$$0 < \frac{d_{22}}{2} = a_1^* m_{24} - a_2^* m_{14} - a_0^* m_{34} \quad (B-37)$$

$$0 < \frac{d_{33}}{2} = a_2^* m_{34} + m_{14} - a_3^* m_{24} - a_1^* m_{44} \quad (B-38)$$

Note: The elements m_{14} , m_{24} , m_{34} and m_{44} must have positive values, which can be verified by solving the simultaneous set of equations (B-21), (B-37), (B-38) and (B-24).

Examination of inequality (B-24) indicates that if

$$m_{34} = a_{3\min}^* m_{44} \quad (B-39)$$

where m_{44} is a positive constant, the relationship of Eq.(B-24) is preserved. Now, substituting from Eq. (B-39) into Eqs. (B-37) and (B-38) results in the following two inequalities

$$-a_2^* m_{14} + a_1^* m_{24} - a_0^* a_{3\min}^* m_{44} = \frac{d_{22}}{2} > 0 \quad (B-40)$$

and

$$m_{14} - a_3^* m_{24} + (a_2^* a_{3\min}^* - a_1^*) m_{44} = \frac{d_{33}}{2} > 0 \quad (B-41)$$

These two inequalities are now used to evaluate the terms m_{14} and m_{24} .

From Eq. (B-40), the relationship

$$m_{24} > \frac{a_2^*}{a_1^*} m_{14} + \frac{a_0^*}{a_1^*} a_{3\min}^* m_{44} \quad (B-42)$$

is obtained, and from Eq. (B-41), the relationship

$$\frac{m_{14}}{a_3^*} + \frac{(a_2^* a_{3\min}^* - a_1^*)}{a_3^*} m_{44} > m_{24} \quad (B-43)$$

is obtained. Using Eq. (B-42) and the limiting values of the parameters a_1^* , the greatest lower bound on m_{24} is found to be

$$m_{24} \geq \frac{a_{2\max}^*}{a_{1\min}^*} m_{14} + \frac{a_{0\max}^*}{a_{1\min}^*} a_{3\min}^* m_{44} \quad (B-44)$$

and using Eq. (B-43) the lowest upper bound on m_{24} becomes

$$\frac{m_{14}}{a_{3\max}^*} + \frac{(a_{2\min}^* a_{3\min}^* - a_{1\max}^*)}{a_{3\max}^*} m_{44} \geq m_{24} \quad (B-45)$$

Since it is known that m_{24} must be positive, the expression $(a_{2\min}^* a_{3\min}^* - a_{1\max}^*)$ in Eq. (B-45) must be constrained to be positive and becomes one of the "Strong" Routh-Hurwitz Conditions:

$$a_{2\min}^* a_{3\min}^* - a_{1\max}^* > 0 \quad (B-46)$$

The expressions in Eqs. (B-44) and (B-45) are now assumed to be satisfied by equality which leads to the following relationship

$$\frac{m_{14}}{a_{3\max}^*} + \frac{(a_{2\min}^* a_{3\min}^* - a_{1\max}^*)}{a_{3\max}^*} m_{44} = m_{24} = \frac{a_{2\max}^*}{a_{1\min}^*} m_{14} + \frac{a_{0\max}^*}{a_{1\min}^*} a_{3\min}^* m_{44}$$

and from which the value of m_{14} is found to be

$$m_{14} = \frac{a_{1\min}^{\star} (a_{2\min}^{\star} a_{3\min}^{\star} - a_{1\max}^{\star}) - a_{o\max}^{\star} a_{3\min}^{\star} a_{3\max}^{\star}}{a_{3\max}^{\star} a_{2\max}^{\star} - a_{1\min}^{\star}} m_{44} \quad . \quad (B-48)$$

Here again, since it is known that m_{14} must be positive, the numerator of Eq. (B-48) must be positive. This requirement

$$a_{1\min}^{\star} (a_{2\min}^{\star} a_{3\min}^{\star} - a_{1\max}^{\star}) - a_{o\max}^{\star} a_{3\min}^{\star} a_{3\max}^{\star} > 0 \quad . \quad (B-49)$$

is another "Strong" Routh-Hurwitz Condition. The denominator of Eq.(B-48) is already known to be positive due to Eq. (B-46). By substititng the value for m_{14} [Eq. (B-48)] into either Eqs. (B-44) or (B-45) and using the equality sign, the value of m_{24} is obtained as

$$\begin{aligned} m_{24} &= \frac{a_{2\max}^{\star}}{a_{1\min}^{\star}} m_{14} + \frac{a_{o\max}^{\star}}{a_{1\min}^{\star}} a_{3\min}^{\star} m_{44} \\ &= \frac{m_{14}}{a_{3\max}^{\star}} + \frac{(a_{2\min}^{\star} a_{3\min}^{\star} - a_{1\max}^{\star})}{a_{3\max}^{\star}} m_{44} \\ &= \frac{a_{2\max}^{\star} (a_{2\min}^{\star} a_{3\min}^{\star} - a_{1\max}^{\star}) - a_{o\max}^{\star} a_{3\min}^{\star}}{a_{3\max}^{\star} a_{2\max}^{\star} - a_{1\min}^{\star}} m_{44} \quad . \end{aligned} \quad (B-50)$$

Utilizing the two "Strong" Routh-Hurwitz conditions, Eq. (B-46) and Eq. (B-49), the numerator and denominator of Eq. (B-50) can be shown to be positive which agrees with the fact that m_{24} must be positive.

The values of the D-matrix elements are now determined by substituting the determined values for m_{14} , m_{24} , m_{34} and m_{44} into Eqs. (B-21), (B-37), (B-38) and (B-24) for d_{11} , d_{22} , d_{33} and d_{44} respectively. The values for the remaining M-matrix elements (Group-II), can be specified by

evaluating Eqs. (B-31) through (B-36) for m_{11} , m_{12} , m_{13} , m_{22} , m_{23} and m_{34} respectively.

A tabulation of the "Strong" Routh-Hurwitz Conditions and values for the M-matrix elements are given in Tables 5-1 and 5-2 respectively for the second, third and fourth order modeling problem.

APPENDIX C:

MODIFICATION OF THE A-MATRIX IF "STRONG"
ROUTH-HURWITZ CONDITIONS ARE NOT SATISFIED

If the elements of the model-reference error equation's A-matrix do not satisfy the "Strong" Routh-Hurwitz conditions, then the A-matrix must be modified. Also, an extra additive control signal, denoted by $x_{a_m}(t)$, must be introduced into the plant (linear part of model subsystem) equation in a similar manner as was presented in Eq. (4-9) for the inclusion of the extra additive control signal $x_a(t)$, needed in the nonlinear modeling case. The A-matrix elements are now redefined as the sum of two components

$$a_i^* = \hat{a}_i^* + \lambda_i \quad \text{for } i = 0, 1 \dots n-1 \quad (C-1)$$

where λ_i represents a constant to be determined.

Since the ranges of the elements $\{a_i^*\}$ are assumed to be known and given by

$$a_{i_{\min}}^* < a_i^* < a_{i_{\max}}^* \quad \text{for } i = 0, \dots n-1 \quad (5-14)$$

then the ranges of the elements $\{\hat{a}_i^*\}$ can be specified by

$$\hat{a}_{i_{\min}}^* < \hat{a}_i^* < \hat{a}_{i_{\max}}^* \quad \text{for } i = 0, \dots, n-1 \quad (C-2)$$

where the values of $\hat{a}_{i_{\min}}^*$ and $\hat{a}_{i_{\max}}^*$ are $a_{i_{\min}}^* - \lambda_i$ and $a_{i_{\max}}^* - \lambda_i$

respectively. A search must now be made to find a set of values $\{\lambda_i\}$ such that the set of newly constructed elements $\{\hat{a}_i^{\star}\}$ satisfies the "Strong" Routh-Hurwitz criterion.

Before the modification procedure is applied, Eqs. (3-17) and (3-19) can be combined to yield the model-reference error equation

$$e^{(n)} + \sum_{i=0}^{n-1} a_i^{\star} e^{(i)} = y_p(t) \quad . \quad (C-3)$$

After substituting from Eq. (C-1) into Eq. (C-3) and also including the extra additive control signal $x_{am}(t)$, the new model-reference error equation becomes

$$e^{(n)} + \sum_{i=0}^{n-1} \hat{a}_i^{\star} e^{(i)} + \sum_{i=0}^{n-1} \lambda_i e^{(i)} = y_p(t) - x_{am}(t) \quad . \quad (C-4)$$

By defining

$$y_{am}(t) = - \sum_{i=0}^{n-1} \lambda_i e^{(i)} - x_{am}(t) \quad . \quad (C-5)$$

and

$$y(t) = y_p(t) + y_{am}(t) \quad . \quad (C-6)$$

Equation (C-4) can be rewritten as

$$e^{(n)} + \sum_{i=0}^{n-1} \hat{a}_i^{\star} e^{(i)} = y(t) \quad (C-7)$$

which is identical in form to Eq.(C-3) and can therefore be represented in vector form

$$\dot{\underline{e}} = [\hat{A}] \underline{e} + \underline{y} \quad . \quad (C-8)$$

Since the elements of the $[\hat{A}]$ -matrix satisfy the "Strong" Routh-Hurwitz conditions, the parameter control signals can be determined in the normal fashion as presented in Chapter 5. The determination of the extra additive control signal $x_{am}(t)$ follows the same rules used to determine the additive control signal $x_a(t)$ needed in the nonlinear modeling case.

APPENDIX D:

COOK'S^[10,11,12] MODEL OF THE HUMAN
SACCADIC EYE-MOVEMENT MECHANISM

Cook's model of the human saccadic eye-movement mechanism which very closely matches his experimental data is used to generate the necessary 18.75° saccadic eye-movement response (θ_R) for the reference system used in Chapter 7. Following almost the same approach as Robinson in building his model, Cook introduced two important refinements. Rather than treating the net muscle force on the eye, he separately considered the forces generated by the agonist and antagonist muscles to account for the difference in muscle characteristics when they are shortening and when they are lengthening. His second refinement was the introduction of nonlinear active damping in both the agonist and antagonist muscles. Based on these physiological considerations, the composite eye movement mechanism model proposed by Cook is shown in Fig. D-1 and the element values are listed in Table D-1.

The neural activation to the agonist and antagonist muscles is represented by C_{ag} and C_{ant} respectively, and are plotted in Fig. D-2. For an 18.75° saccade when the actual eye movement begins at $t = 200$ msec, the variables indicated in Fig. D-2 will have the following values:

$$\theta_R(0) = 0, \theta_{RFINAL} = 18.75^\circ, C_{ag_{max}} = 65, C_{ant_{min}} = 2,$$

$$t_1 = 238 \text{ msec} \quad \text{and} \quad t_2 = 245 \text{ msec}.$$

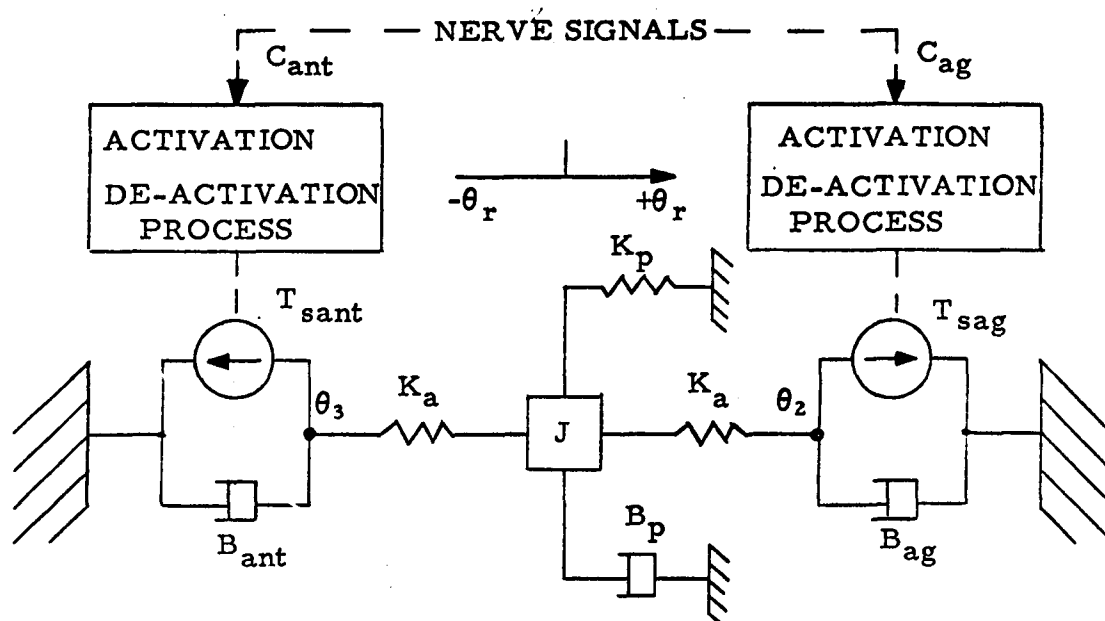


Figure D -1 Cook's composite model for the eye movement mechanism [10, 11, 12]

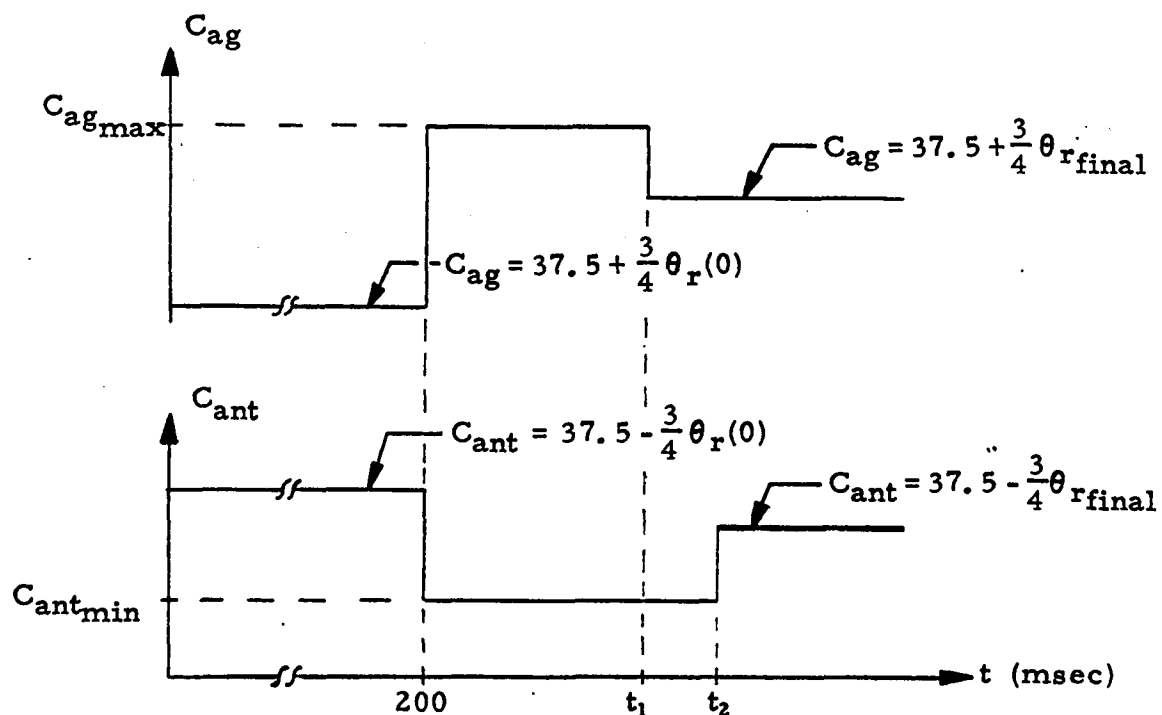


Figure D -2 Neural activation to the eye muscles during a saccade (target moves at $t=0$, eye movement begins at $t=200$ msec.)

Function	Symbol	Value
Muscle series elastic element	K_e	1.8 g. /deg
Passive damping coef.	B_p	.018 g.sec/deg
Passive elastic coef.	K_p	1.5 g./deg
Equivalent inertia	J	$.43 \times 10^{-4}$ g.sec ² /deg
Active damping in the agonist muscle	B_{ag}	$= \frac{1.25 T_{SAG}}{900 + \dot{\theta}_2} ; \dot{\theta}_2 \geq 0$ $= \frac{3 T_{SAG}}{900} ; \dot{\theta}_2 < 0$
Active damping in the antagonist muscle	B_{ant}	$= \frac{1.25 T_{SANT}}{900 - \dot{\theta}_3} ; \dot{\theta}_3 \leq 0$ $= \frac{3 T_{SANT}}{900} ; \dot{\theta}_3 > 0$
Antagonist tension source	T_{SANT}	See Fig. D-3(d)
Agonist tension source	T_{SAG}	See Fig. D-3(c)

Table D-1 Element values for Cook's model

When the response data is generated for the reference system, the actual initiation of the movement is shifted to begin at $t = 0$. The differential equations describing the motion of the eye movement (θ_R) are given by

$$T_{SAG} - B_{AG} \dot{\theta}_2 = K_a (\theta_2 - \theta_R) \quad (D-1)$$

$$K_a [\theta_2 - \theta_R] - K_p \theta_R = J \ddot{\theta}_R + B_p \dot{\theta}_R + K_a (\theta_R - \theta_3) \quad (D-2)$$

and

$$K_a [\theta_R - \theta_3] = T_{SANT} + B_{ANT} \dot{\theta}_3 \quad (D-3)$$

where the muscle tension sources are assumed to be generated from

$$\dot{T}_{SAG} + 250.0 T_{SAG} = 250.0 C_{AG} \quad (D-4)$$

and

$$\dot{T}_{SANT} + 125.0 T_{SANT} = 125.0 C_{ANT} \quad (D-5)$$

The waveforms for an 18.75° saccade are illustrated in Fig. D-3.

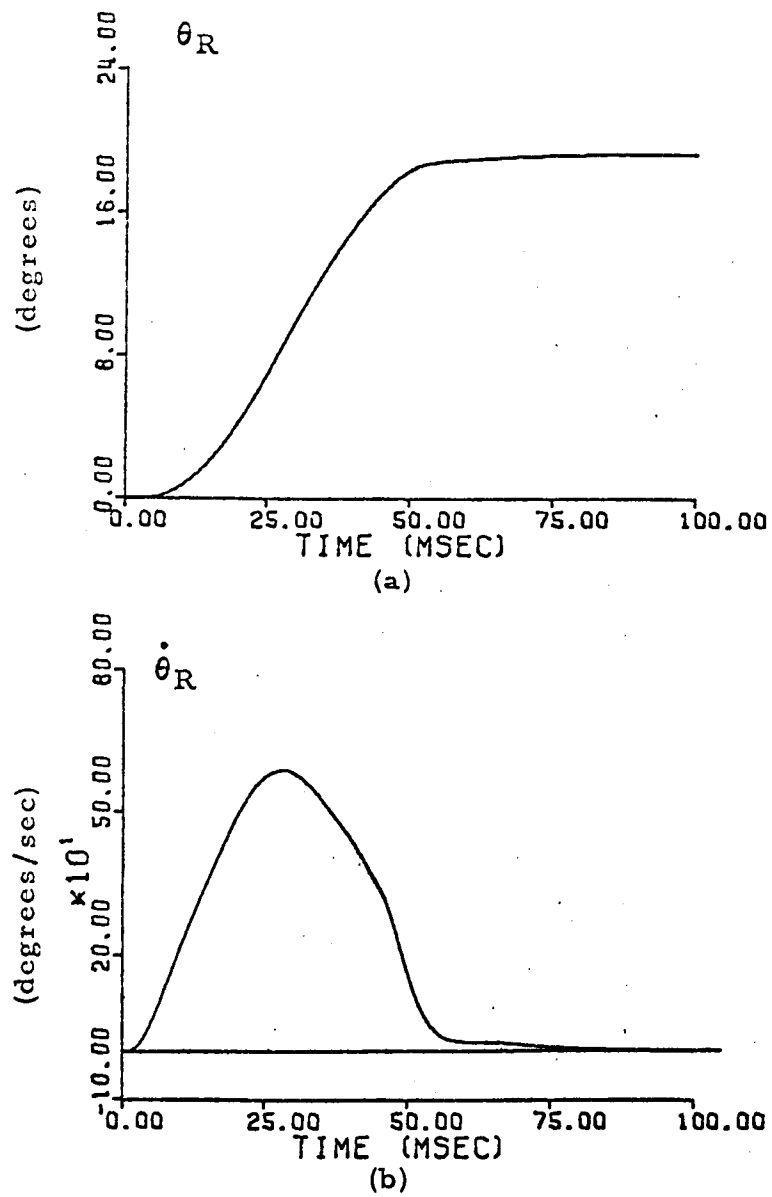


Figure D-3 Waveforms from Cook's model

(a) Eye position (θ_R) during an 18.75° saccade

(b) Eye velocity ($\dot{\theta}_R$) during an 18.75° saccade

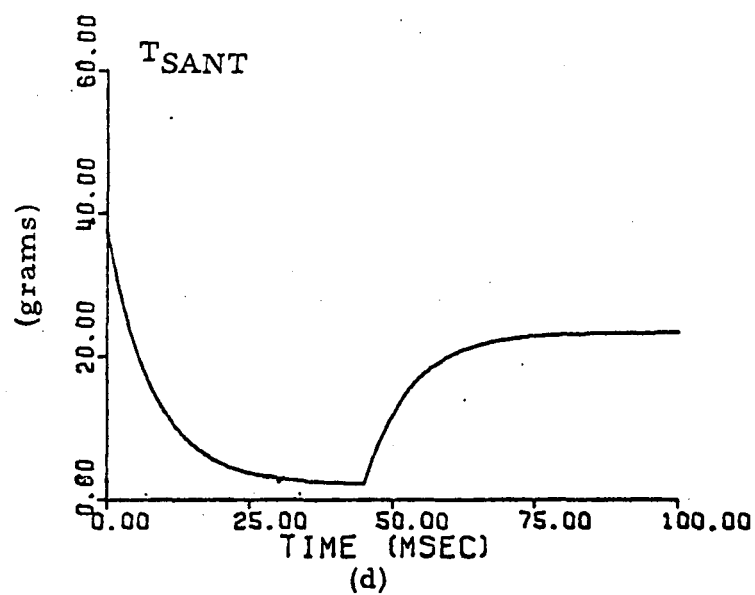
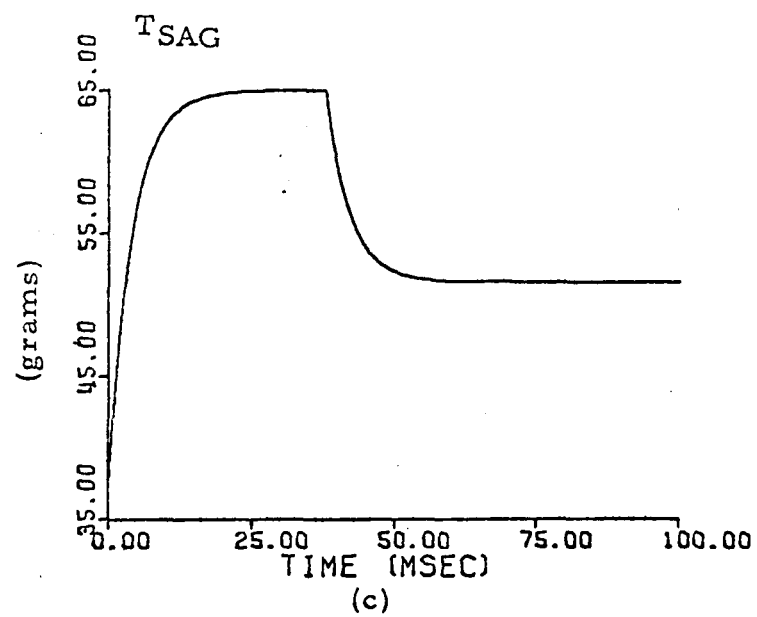


Figure D-3 (Continued)

(c) Agonist tension source (T_{SAG})

(d) Antagonist tension source (T_{SANT})

BIBLIOGRAPHY

- [1] Arbib, M.: Brains, Machines and Mathematics, McGraw Hill Book Co., 1964.
- [2] Atkin, A.: Effect of Head Movement on Gaze Movement Velocity, American Physiological Society Meeting, Providence, R.I., September 1964.
- [3] Atkin, A.: Selection of Sensory Information in Control of Pursuit Eye Movements, Psychon. Sci., Vol.8(4), pp.133-134, 1967.
- [4] Atkin, A. and Bender, M.: Ocular Stabilization During Oscillatory Head Movements; Vestibular System Dysfunction and the Relation Between Head and Eye Velocities, Archives of Neurology, Vol.19, December 1968, pp. 559-566.
- [5] Barker, J.R.: Mechanical and Electrical Vibrations, John Wiley and Sons Inc., New York, 1964.
- [6] Blesser, W.: A Systems Approach to Biomedicine, McGraw Hill Book Co., New York, 1969.
- [7] Bongiorno, J.J.: Stability of Convergence Properties of Model Reference Adaptive Control Systems, IRE Trans. A.C., April 1962, pp. 30-41.
- [8] Bongiorno, J.J.: Final Report on Computer Controlled and Model-Referenced Adaptive Feedback Control Systems, Final Report No. PIBMRI-1085-62, Polytechnic Inst. of Brooklyn, Oct.31, 1962.
- [9] Brockhurst, R.J. and Lion, K.S.: Analysis of Ocular Movements by Means of an Electrical Method, A.M.A. Arch. Ophthal, Vol. 46, 1951, pp. 311-314.
- [10] Cook, G. and Stark, L.: Dynamics of the Saccadic Eye-Movement Mechanisms, Frank J. Seiler Research Laboratory, USAF Academy, Colorado, SRL-66-0002, June 1966.
- [11] Cook, G. and Stark, L.: Derivation of a Model for the Human Eye-Positioning Mechanism, Bull. of Mathematical Biophysics, Vol. 29, 1967, pp. 153-174.
- [12] Cook, G. and Stark, L.: The Human Eye-Movement Mechanism: Experiments Modeling and Model Testing, Frank J. Seiler Research Laboratory, USAF Academy, Colorado, SRL-67-0005, June 1967.

- [13] Cook, G. and Stark, L.: The Human Eye-Movement Mechanism Experiments, Modeling and Model Testing, Arch. Ophthal, Vol. 79, April 1968, pp. 428-436.
- [14] Costello, R.G. and Higgins, T.J.: An Inclusive Classified Bibliography Pertaining to Modeling the Human Operator as an Element in an Automatic Control System, IEEE Trans. on Human Factors in Electronics, Vol. HFE-7(4), Dec. 1966, pp. 174-181.
- [15] Dallos, P.J. and Jones, R.W.: Learning Behavior of the Eye Fixation Control System, IEEE Trans. on Automatic Control, Vol. AC-8(3), July 1963, pp. 218-227.
- [16] Davson, H. (ed): The Eye; Vol.2, The Visual Process, Academic Press, 1962.
- [17] Davson, H. (ed): The Eye; Vol.3, Muscular Mechanisms, Academic Press, 1962.
- [18] Dodge, R. and Cline, T.S.: The Angule Velocity of Eye Movements, Psychol. Rev., Vol.8, 1901, pp. 145-157.
- [19] Dressler, R.V.: An Approach to Model-Referenced Adaptive Control Systems, IEEE Trans. on Automatic Control, Vol. AC-12(1), Feb. 1967, pp. 75-80.
- [20] Dutton, J.M. and Starbuck, W.H.: Computer Simulation Models of Human Behavior: A History of an Intellectual Technology, IEEE Trans. on Systems, Man and Cybernetics, Vol. SMC-1(2), April 1971, pp. 128-171.
- [21] Dymock, A.J., Meredith, J.F., Hall, A. and White, K.M.: Analysis of a Type of Model-Reference Adaptive Control Systems, Proc. IEE, Vol. 112(4), April 1965, pp. 743-753.
- [22] Fender, D.H. and Nye, P.W.: An Investigation of the Mechanisms of Eye Movement Control, Kybernetik, Vol.1, 1961, pp. 81-88.
- [23] Fender, D.H.: Control Mechanisms of the Eye, Scientific American, July 1964.
- [24] Fuchs, A.F.: Periodic Eye Tracking in the Monkey, Journ. Physiology, Vol. 193, 1967, pp. 161-171.
- [25] General Electric: A Survey of Adaptive Control Technology, General Electric, Electronics Laboratory, Syracuse, N.Y., 1964.

- [26] George, F.: Cybernetics and Biology, Freeman, 1965.
- [27] Gibbs, C.B.: The Effect of Minor Alcohol Stress on Decision Processes in a Step-Tracking Task, IEEE Trans. on Human Factors in Electronics, Vol. HFE-7(4), Dec. 1966, pp. 145-150.
- [28] Gibson, J.: Nonlinear Automatic Control, McGraw Hill, 1963.
- [29] Gottlieb, G.L. and Agarwal, G.C.: Control and Regulation of the Human Motor System, IEEE Trans. on Systems, Man and Cybernetics, Vol. SMC-1(4), Oct. 1971, pp. 379-383.
- [30] Grayson, L.P.: The Design of Intentionally Non-Linear Controllers by the Second Method of Lyapunov, Research Report PIB-MRI 884-60, Polytechnic Inst. of Brooklyn, Dec. 27, 1960.
- [31] Grodins, F.S.: Control Theory and Biological Systems, Columbia Press, 1963.
- [32] Hahn, W.: Theory and Applications of Liapunov's Direct Method, Prentice Hall, Englewood Cliffs, N.J., 1963.
- [33] Hang, C.C. and Parks, P.C.: On "Accelerated Model-Reference Adaptation via Lyapunov and Steepest Descent Design Techniques", IEEE Trans. on Automatic Control, Vol. AC-17(6), Dec. 1972, pp. 842-843.
- [34] Horrocks, T.: Investigations into Model-Reference Adaptive Control Systems, Proc. IEEE, Vol. 111(11), Nov. 1964, pp. 1894-1906.
- [35] Hyde, J.E.: Some Characteristics of Voluntary Human Ocular Movements in the Horizontal Plane, Amer. Journ. Ophthal, Vol. 48, 1959, pp. 85-94.
- [36] Johnson, L.E.: Dynamic Analysis of Visual Tracking, Systems Research Center, Case Institute of Technology, Cleveland, Ohio, SRC 28-B-63-6, April 1963.
- [37] Jones, G.M. and Milsum, J.H.: Spatial and Dynamic Aspects of Visual Tracking, IEEE Trans. Bio-Medical Engr., Vol. BME-12(2), 1965, pp. 54-62.
- [38] Jones, G.M.: The Vestibular Contribution to Stabilization of Retinal Image, Symposium on the Role of the Vestibular Organs in the Exploration of Space, U.S. Naval School of Aviation Medicine, Pensacola, Florida, NASA SP-77, Jan. 20-22, 1965, pp. 163-172.

- [39] Jury, E.I. and Pavlidis, T.: A Literature Survey of Biocontrol Systems, IEEE Trans. on Automatic Control, Vol. AC-8(3), July 1963, pp.210-217.
- [40] Kalman, R.E. and Bertram, J.E.: Control System Analysis and Design via "Second Method" of Lyapunov; Part 1, Continuous Time Systems, Journal of Basic Engineering, Trans. of the ASME, Vol. 82-D, June 1960, pp. 371-393.
- [41] LaSalle, J. and Lefschetz, S.: Stability by Liapunov's Direct Method with Applications, Academic Press, New York, 1961.
- [42] Lindorff, D.P.: Control of Non-Linear Multivariable Systems, IEEE Trans. on Automatic Control, Vol. AC-12(5), Oct.1967, pp. 506-515.
- [43] Luders, G. and Narendra, K.S.: Lyapunov Functions for Quadratic Differential Equations with Applications to Adaptive Control, IEEE Trans. on Automatic Control, Vol. AC-17(6), Dec.1972, pp.798-801.
- [44] Mekel, R. and Perujo, P.: Design of Controllers for a Class of Non-Linear Control Systems, IEEE Trans. on Automatic Control, Vol. AC-17(2), April 1972, pp. 206-213.
- [45] Mekel, R.: A Class of Liapunov Functions for High Order Non-Linear Control Systems, Asilomar Conference on Circuits and Systems, Monterey, Calif., Nov. 1-3, 1967.
- [46] Mekel, R.: Synthesis of Corrective Dynamics for Human Operator Models in Closed-Loop Control Systems, NASA Working Paper No. LWP-816, Oct. 21, 1969.
- [47] Mekel, R.: Adaptive Identification of Parameters in Models of Physiological Systems, Proc. 8th Annual Conference on Manual Control, Ann Arbor, Michigan, May 17-19, 1972.
- [48] Mekel, R.: An Adaptive Modeling Technique Using a Class of Liapunov Functions, Proc. of the 1972 International Conference on Cybernetics and Society, Oct. 9-12, 1972, Washington, D.C.
- [49] Mihram, G.A.: The Modeling Process, IEEE Trans. on Systems, Man and Cybernetics, Vol. SMC-2(5), Nov. 1972, pp.621-629.
- [50] Milhorn, H.T.: Application of Control Theory to Physiological Systems, Saunders, 1966.

- [51] Miller, J.W. and Ludvigh, E.: The Effect of Relative Motion on Visual Acuity, Survey of Ophth., Vol.7, 1962, pp. 83-116.
- [52] Milsum, J.: Biological Control Systems Analysis, McGraw Hill, 1966.
- [53] Monopoli, R.V.: Control of Linear Plants with Zeroes and Slowly Varying Parameters, IEEE Trans. on Automatic Control, Vol. AC-12(1), Feb. 1967, pp. 80-83.
- [54] Monopoli, R.V.: Liapunov's Method for Adaptive Control-System Design, IEEE Trans. on Automatic Control, Vol. AC-12, June 1967, pp. 334-335.
- [55] Monopoli, R.V. and Gitlitz, C.R.: Control of Linear Time Varying Plants with Differential Operations Acting on the Input, Fifth Annual Allerton Conf. on Circuit and System Theory, Oct. 1967, pp. 608-613.
- [56] Monopoli, R.V. and Gilbert, J.W.: Model Reference Adaptive Control Systems with Feedback and Prefilter Adjustable Gains, Proc. of the Fourth Annual Princeton Conference on Information Sciences and Systems, March 26-27, 1970, pp. 452-456.
- [57] Mueller, C.G. and Rudolph, M. (ed): Light and Vision, Life Science Library, 1966.
- [58] Nava, F. and Stark, L.: Sampling or Intermittency in Hand Control System Dynamics, Biophysical Journ., Vol.8, 1968, pp. 252-299.
- [59] Osburn, P.V., Whitaker, H.P., and Kezer, A.: New Developments in the Design of Adaptive Control Systems, Inst. of Aeronautical Sciences 1961 International Automation Symp., Paper No.61-39, 1961.
- [60] Parks, P.C.: Liapunov Redesign of Model-Reference Adaptive Control Systems, IEEE Trans. on Automatic Control, Vol. AC-11(3), July 1966, pp. 362-367.
- [61] Pazdera, J.S. and Spence, H.F.: Comments on "Accelerated Model-Reference Adaption via Lyapunov and Steepest Descent Design Techniques", IEEE Trans. on Automatic Control, Vol. AC-17(6), Dec. 1972, p. 842.
- [62] Pearson, A.E. and Vanguri, K.-S.V.R.: A Synthesis Procedure for Parameter Adaptive Control Systems, IEEE Trans. on Automatic Control, Vol. AC-16(5), Oct. 1971, pp. 440-449.

- [63] Peruo, P.: Design of Non-Linear Control Systems via a Time Domain Ratio Criterion, Doctoral Dissertation, The City University of New York, 1970.
- [64] Phillipson, P.H.: Concerning Liapunov Redesign of Model Reference Adaptive Control Systems, IEEE Trans. on Automatic Control, Vol. AC-12(5), October 1967, p.625.
- [65] Phillipson, P.H.: Design Methods for Model Reference Adaptive Systems, The Institution of Mechanical Engineering Proceedings, Vol. 183 Part 1, No.35, 1968-1969, pp. 695-706.
- [66] Pritchard, R.M.: Stabilized Retinal Images of Visual Perception, Bionics Symposium, Cornell University, Aug. 30 - Sept. 1, 1964.
- [67] Rashbass, C.: The Relationship Between Saccadic and Smooth Tracking Eye Movements, Journ. Physiology, Vol.159, 1961, pp. 326-338.
- [68] Ratliff, F. and Riggs, L.A.: Involuntary Motions of the Eye During Monocular Fixation, Journ. of Exp. Psychology, Vol.40, December 1960, pp. 687-701.
- [69] Robinson, D.A.: The Mechanics of Human Saccadic Eye Movements, Journ. Physiology, Vol.174, 1964, pp. 245-264.
- [70] Robinson, D.A.: The Mechanics of Human Smooth Pursuit Eye Movements, Journ. Physiology, Vol. 180, 1965, pp. 569-591.
- [71] Robinson, D.A.: The Oculomotor Control System: A Review, Proc. IEEE, Vol. 56(6), June 1968, pp. 1032-1049.
- [72] Rosenblith, W. (ed): Sensory Communications, MIT Press, 1961.
- [73] Schultz, D.G. and Melsa, J.L.: State Functions and Linear Control Systems, McGraw-Hill Book Co., New York, 1967.
- [74] Shahein, H.I.H. and Ghonaimy, M.A.R. and Shen, D.W.C.: Accelerated Model-Reference Adaptation via Lyapunov and Steepest Descent Design Techniques, IEEE Trans. on Automatic Control, Vol. AC-17(1), February 1972, pp. 125-128.
- [75] Snell, F.M., Shulman, S., Spencer, R.P. and Moos, C.: Biophysical Principles of Structure and Function, Addison-Wesley Publishing Co., Reading, Mass., 1965.

- [76] Staten, S.F. and Stark, L.: A Statistical Analysis of Pupil Noise, IEEE Trans. Bio-Medical Engr., Vol. BME-13-(3), July 1966, pp. 140-152.
- [77] Stark, L., Vossius, G. and Young, L.R.: Predictive Control of Eye Movements, IRE Trans. on Human Factors in Electronics, Vol. HFE-3, September 1962, pp. 52-57.
- [78] Sutherland, R.L.: Engineering Systems Analysis, Addison-Wesley Publishing Co., 1958.
- [79] Truxal, J.G.: Automatic Feedback Control Systems Synthesis, McGraw Hill Book Co., 1955.
- [80] Weinberg, L.: Network Analysis and Synthesis, McGraw Hill Book Co., 1962.
- [81] Westheimer, G.: Mechanism of Saccadic Eye Movements, A.M.A. Archives of Ophthalmology, Vol. 52, 1954, pp. 710-724.
- [82] Westheimer, G.: Eye Movement Responses to a Horizontally Moving Stimulus, A.M.A. Archives of Ophthalmology, Vol. 52, 1954, pp. 932-943.
- [83] Whitaker, H.P., Yamron, J. and Kezer, A.: Design of Model Reference Adaptive Control Systems for Air-Crafts, Report R-164, M.I.T. Instrumentation Lab., September 1958.
- [84] White, A.J.: Analysis and Design of Model-Reference Adaptive Control Systems, Proc. IEE, Vol. 113(1), Jan. 1966, pp. 175-184.
- [85] Winsor, C.A. and Roy, R.J.: Design of Model-Reference Adaptive Control Systems by Liapunov's Second Method, IEEE Trans. on Automatic Control, Vol. AC 13(2), April 1968, p. 204.
- [86] Young, L.R. and Stark, L.: Variable Feedback Experiments Testing a Sampled Data Model for Eye Tracking Movements, IEEE Trans. on Human Factors in Electronics, Vol. HFE-4, Sept. 1963, pp. 38-51.
- [87] Young, L., Zuber, B. and Stark, L.: Visual and Control Aspects of Saccadic Eye-Movements, NASA Contractor Report, NASA CR-564, Washington, D.C., September 1966.

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