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RELIABILITY ANALYSIS OF A MAINTENANCE FLOAT MODEL

by

CHRISTIAN NDUBISI MADU

.A dissertation submitted to the Graduate Faculty in Business in partial fulfillment of the requirement for the degree of Doctor of Philosophy, The City University of New York.

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ABSTRACT

RELIABILITY ANALYSIS OF A MAINTENANCE FLOAT MODEL

by

Christian N. Madu

Adviser: Professor Michael N. Chanin

The objective of this paper is to extend the concept of maintenance float modeling to include failure distributions such as: gamma, erlang-2, exponential, uniform, normal and lognormal distributions. From the maintenance float factors (MFF) derived for gamma failure distribution, the special cases of gamma such as exponential ($p = 1$), erlang-2 ($p = 2$), and constant or degenerate ($p = \infty$) were obtained. The MFF obtained for the exponential distribution using the gamma case is the same as that obtained directly from the exponential distribution. The MFF for gamma is obtained through Taylor series approximation. Using the limit theorem, it was also shown that when the number of phases $p \to \infty$, the float factor for the gamma distribution approaches 0. This implies that when failure distribution is constant, there is no need to maintain standby units. We therefore state that if the failure distribution is from a constant distribution, then the total float (F) is approaching 0. It is further shown that when $P \geq 10$, it is safe to approximate a constant or degenerate distribution.

This paper was further extended to utilize the

asymptotic property of the MFF. It was shown that as the number (N) initially in operation increases, the maintenance float factor derived for each of the distributions will approach an asymptotic value. This we refer to as the asymptotic maintenance float factor (AMFF). The AMFF shows that for $N \geq 100$, the calculation of the MFF depends only on the mean time to repair (MTTR) and the mean time between failure (MTBF) and is independent of N.

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This paper is dedicated to my grandparents
who passed away before I completed the program.

Mazi Maduko Izuogu and Oriaku Ikodie Maduko

&

Mazi Anyaogu Okonkwo and Ndi Oriaku Orieji Anyaogu, Ihuoma Anyaogu and Oduagu Anyaogu.

May your souls rest in peace.

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CHAPTER 1

INTRODUCTION

The concept of maintenance float modeling was initially introduced by Barlow, Proschan and Hunter (6) and was referred to as a 'Repairman's problem'. This problem involves a closed queueing network where there are no arrivals or depatures. It is assumed that N units are initially in operation and each of these N units have a stochastic failure rate. When a unit fails, the standby units are used to replenish the operating system in order to always maintain an average of N units in operation. The failed unit is then sent into repair and the repair is probabilistically distributed. After repair, the repaired unit returns to a standby status or goes into operation if the number of operating units is less than N.

The objective of maintenance float systems can be expressed as follows:

(1) Maximization of the reliability of the system.

or

(2) Minimization of the operating costs of the system.

A classical solution to the maintenance float problems are based on the cost minimization approach. Maintenance float problem is often difficult to model analytically. Simulation is therefore frequently used (32,91,93). The classical approach involves the use of trial and error approach to determine the number of standbys (total float) or repairmen to maintain in order to minimize the costs involved. This approach does not consider the issue of reliability. Johnson and Fernandez (32) extended the simulation work in this area by considering the effect of routine maintenance. Some other literary work on maintenance float modeling have been too restrictive in their assumptions (1,35,39,74,81,87). Such restrictive assumptions are often necessary in order to develop a mathematical model. Such assumptions may include restricting the number of units in operation or in standby to a very small number of units.

The purpose of this paper is to use reliability concepts to develop maintenance float models for different continuous failure distributions. We are interested in a maintenance float system with a large number of units in operational status and an equally large number of units maintained in standby status.

Levine (42) introduced a modeling approach where he measured reliability as the ratio of mean time between failure (MTBF) to mean time to repair (MTTR). Lowe and Lewis (44) extended Levine's study to obtain the maintenance float factors for weibull failure distribution and lognormal repair distribution. Maintenance float factors are defined as the proportion of units that have failed upto time t. Our paper extends the work of Lowe and Lewis to include the following failure distributions: exponential, gamma, erlang-2, normal, lognormal, uniform and constant failure distributions. We also use exponential and lognormal distributions as repair distributions. We further obtain some general results about the structure of the formulas developed and also show that the developed maintenance float models have asymptotic property. Utilization of this property will significantly reduce the computational efforts involved in using the model. From the asymptotic models developed, it is evident that the maintenance float factors are independent of the number of units initially in operation when that number is large. The maintenance float factors therefore depend on the mean time to repair and the mean time between failure. The maintenance float factors in this paper obey the probability axiom that is, it is a factor between 0 and 1. This factor multiplied by N gives the total float or standby.

The paper further develops cost ratios by using the cost of lost production and the cost of renting or leasing a standby as the variable costs. These cost ratios are used jointly with the total float obtained analytically, and simulation results to develop ranges over which our analytical models are applicable. Consistently, it has been shown for all failure distributions that our models give good approximations of the total float when the cost of lost production or the cost of equipment downtime is greater than the cost of renting or leasing a standby for the values of MTTR and MTBF used. Our models are also good estimators of standbys even when the cost of lost production is low. These models therefore have wide range of applications. The following assumptions are made:

(1) N units are initially in operation.

(2) The units in operation and standby are homogeneous.

(3) Repair is applied after failure and it is assumed that the unit is completely rejuvenated. It's state is as good as new after repair.

(4) The waiting time for repair is negligible. The sensitivity of this assumption is further discussed in this paper.

Graphical illustrations of the maintenance float factors were also introduced.

In the following section, we give a review of the literature in maintenance modeling. This review did not show adequate research in maintenance float modeling. This may be due to the mathematical intractability in modeling such systems.

CHAPTER 2

LITERATURE REVIEW

This section reviews recent studies in maintenance policies. Maintenance is applied to a unit in operation to prevent failure and increase the unit's reliability. This section focuses on equipment maintenance and its objective is to discuss the state of the art in maintenance policies. It includes for the most part the analytical developments in maintenance modeling. The objective of a maintenance program can be to minimize the cost of maintenance, or maximize the reliability of a unit. Through maintenance applications, components with high failure rates may be detected. The problems of some of the analytical models are stated and it is suggested that complex maintenance problems can be modeled by means of heuristic approach.

Maintenance applications in industrial and service systems are getting increasing attention due to the complexity of the systems and the increasing automation of many manufacturing systems (ie, the use of robotics, flexible manufacturing systems). These new advances are replacing the man-machine systems. Organizations relying on automated systems are therefore paying increasing attention to maintenance. The attempt is to minimize the maintenance cost which includes the cost due to equipment downtime (ie, cost of lost production), the cost of repair or maintenance etc. In order to minimize the maintenance cost, preventive maintenance (PM) as opposed to corrective maintenance (CM) is often used. White (92) defined preventive maintenance as follows:

> Maintenance operation is periodically performed on the equipment in order to prevent failure, increase equipment reliability, and detect components with high failure rates (92, p.5).

The aim of PM is to introduce a higher reliability in to the equipment by reducing the chances of machine breakdown (4,67). PM control provides corrective action based on the feedback regarding equipment performance and maintenance needs (31,49). However, since the objective of the maintenance function is to minimize cost, PM has not always been readily adopted.

Some of the most widely used maintenance policies, and replacement policies are discussed in this paper. In replacement policy, we make the decision to replace the existing equipment with an identical equipment. Replacement policy is actually not a maintenance policy but one of the end results of maintenance policy. In order to obtain the optimum or near optimum policy, mathematical models have often been used.

Several authors have utilized optimization techniques to develop maintenance models. This usually results in modeling the machine's mean time between failure (MTBF) to predict equipment failure rate (56). This approach has been quite useful in scheduling PM (55). The assumptions in the mathematical models are that there exists a cut off point where it is economically cheaper to apply PM rather than crisis maintenance. In other words, to be successful, PM has to be scheduled just prior to total machine breakdown. Such scheduling will prevent the equipment breakdown and reduce both the high cost of breakdown (ie, work stoppages, idle time costs, poor quality outputs) and the cost of frequent inspections. It is worth noting that if equipment breakdown is known, then some of the assumptions being made in these models would no longer be necessary. In fact, maintenance would be much easier to plan. Unfortunately, equipment breakdown is stochastic in nature and has thus necessitated the use of complex mathematical methods and assumptions in order to model equipment behavior.

Barlow and Hunter (5), two of the more prolific writers in the area of PM proposed two types of PM policies. These policies express the nature of future studies in PM. These two policies are discussed. As the cost of investing in new machines climb, management is forced to utilize more efficient means to better predict and control the behavior of their mechanized production systems. This has led to a change from the simple, tractable and easy to use models to the more complex mathematical models that are often more useful for academic purposes and have less utility in the practical world. Chanin and Sphicas (14) pointed out the problems with such mathematical modeling. Bullock (13) in his survey found that the utility of such models was confined only to large companies owning a significant number of machines.

The models discussed are those that involve optimal decisions to procure, inspect, repair, and/or replace equipment subject to deterioration in service. The maintenance models are classified according to the body of theories or policies they represent. Some of these include Markovian replacement models, repair limit policies, minimal repair policies and others.

Markovian Replacement Models

The Markovian replacement models are based on three types of control actions: do nothing, replace or repair (34,73). These class of problems are extensively studied in the literature (6,20,25,34,37,75).

In considering the Markov replacement model, Kao (34) assumed that the state of the system is known with certainty and that the transition from state to state follows a markov chain while the time spent in each state before transition occurs is a random variable depending on the transition. The system deterioration was assumed to be a semi-Markovian.

Thus a system starting in an initial state will go to the less desirable state in the next transition if left untouched. Subsequently the system will reach the terminal state L and will remain there. Since this process is one of deterioration, $p_{ij} = 0$ for $j \leq i$ given that $i = 0, \ldots, L - 1$.

A replace decision will change the state of the system from state L to state 0. Kao also assumed that the holding time in state i before making transition to state j is an integer-valued random variable assumed to be positive and finite. When the system is in the holding states (0,1,....,L), an occupancy cost is incurred per unit time. Using the occupancy cost of A , fixed cost of replacement R and variable cost D units per unit time needed for replacement, Kao developed a control limit policy for the longrun average cost. The model used the time the system has spent in the state as well as the state itself in determining when to replace the system.

The semi-Markovian deterioration process used here was suggested for systems that exhibit two types of failures: (1) gradual deterioration - direct transition to less desirable states and, (2) catastrophic failure - direct transition to the terminal state. Systems that exhibit this type of failure were given as electronic and mechanical equipments.

Tahara and Nishida (82) developed optimal replacement policies for models in which a system is repairable but cannot recover completely after each repair. The system state does not return to state 0 (state 0 is defined as being as good as new). This is the state for new equipment or a system that is completely rejuvenated (overhauled). In their model, they assumed that the mean life of a repairable system decreases with the number of repairs carried out. There is no preventive maintenance for this model and repairs are only carried out after failure. The system's failure rate is also of Markovian type deterioration.

The replacement models described in the literature are of a binary nature (21,34,37), i.e., a decision is made either to replace equipment or not to replace. In the two cases discussed above the action taken returns the system to a specific state. Bobos and Prontontarios (11) assuming Markovian deterioration developed a model where the state of deterioration of the equipment changes in a stochastic manner. The action taken will not bring the state of the equipment to that of a pre-determined specific state but it affects stochastically the transition chances toward "improved" states. The transition probabilities of the equipment deterioration using the model of Markov chain are assumed to satisfy the trend for monotonically increasing deterioration and rate of deterioration. Under these assumptions, an optimal policy of the control limit type was developed.

Sengupta (79) modeled a continuous time maintenance problem where deterioration is Markovian and the state of the system is observed through inspection. The system can be in any (n+1) states (0,1,2,.....,n). The system starting from state i, makes transition to the i+1 state. When the state of the system reaches state n, it remains at state n until it is replaced. The time the unit (or system) spends in states i = 0,1,.....,n-1 is exponentially distributed with mean 1/b. Assuming the inspections and replacements are instantaneous Sengupta developed a maintenance policy (a schedule of inspections and replacements) that will minimize the long run expected average cost per unit time. Sengupta's model differed from that of Luss (45) where replace action is taken immediately after an inspection. Sengupta claims that replacement need not be preceeded by an inspection (79). A control limit type of optimum policy was developed to replace the system when ever the observed state of the system is greater than some number k*.

Hayre (30) considered the preventive maintenance and inspection problem where action is required only upon failure. Two types of actions are available to the decision maker - repair or replace. Repair policies are generally less effective and less costly than replacement policies and do not bring the system's level to that of being as good as new. A repair decision brings the system to a level that is acceptable. This problem was modeled as a semi-Markov process and an explicit optimal maintenance policy for minimizing the long run average cost per unit time was obtained. The optimal policy obtained is also of control limit form.

Minimal Repair Policies

Many complex systems are made up of subsystems or subunits. A failure of the larger system may be due to a breakdown in one of the subsystems. Instead of replacing the entire system, minimal repair may be performed to restore the system back to it's state before failure. Minimal repair therefore restores the failed system to operation without affecting its failure rate (73). Minimal repair usually involves simple equipments where decisions can be made either to repair or to replace (5,13). There is usually no preventive maintenance scheduled to anticipate failure. The decision to replace is based on the age of the unit and the state of deterioration (i.e., major repair).

Barlow and Hunter (5) introduced the concept of minimal repair. From it has emerged two notable policies. Policy I involves the maintenance of simple equipments while Policy II refers to the maintenance of complex systems. In Policy I, there is no scheduled maintenance. Thus PM is carried out only at the time of failure or after t_0 hour of continuing operation without failure. In Policy II, PM is scheduled. After PM, the system's condition is upgraded and it is assumed to be "as good as new". The system is thus returned to a state that is similar to that of equipment replacement. PM involves a total system overhaul and it is scheduled in anticipation of equipment breakdown. Essentially PM is carried out after every t hour of equipment operation regardless of the number of intervening failures between PM.

If the equipment breaks down before the scheduled PM, minimal repair is carried out on the equipment and it is assumed that this will not change the system's failure rate. Minimal repair is thus defined as a repair or replacement of a minor part of the equipment, sufficient to return the equipment promptly to operational status (13).

Barlow and Hunter thus defined the optimum preventive maintenance policy derived to be the one that maximizes the "limiting efficiency" of the system. The limiting efficiency can be expressed as the fractional amount of uptime of a machine over long intervals. Cleroux, Dubuc, and Tilquin (16) considered minimal repair for an age replacement problem. The repair or replace decision in their model depends on the random cost C for repair. A replacement at failure takes place if $C > \delta c_1$, otherwise one would proceed to minimal repair of the type given by Barlow and Hunter

(5). c_1 is the constant cost of replacement at failure and the parameter which is a given percentage of the cost c_1 is given as $0 \leq \delta \leq 1$. The parameter is assumed to be known to the decision maker.

Nguyen and Murthy (67) studied a preventive maintenance problem where the life distribution of the system changes after each repair. They assumed that the failure rate function increases with the number of repairs. This assumption is similar to that made by Tahara and Nishida (82) where the mean life of the system decreases with the number of repairs. The difference between these two models is that Nguyen and Murthy considered preventive maintenance of the type mentioned by Barlow and Hunter (5) while Tahara and Nishida do not consider PM and repair is carried out only at failure. Furthermore, Nguyen and Murthy made the following assumptions; the system has two states - working or failed, state of the system is always known with certainty so that either repair or replacement decision is made immediately when the system fails, replacement and repair times are negligible.

Phelps (71) considered a case where minimal repair is performed up to time T and then replaced at the first failure after T. He assumed the time to failure to follow Weibull distribution. He compared his optimal policy with the Policy I developed by Barlow and Hunter and showed that his policy gives a lower cost. He therefore argued that "any policy which replaces between failures can be improved by waiting until the next failure before replacing, since this increases the expected cycle time while leaving cycle costs unaltered" (71). However, he acknowledged that it is more difficult to solve his policy in order to obtain a comparable optimal age-limit to Policy I of Barlow and Hunter.

In a more recent paper, Phelps (72) generalized the results obtained in his previous work by formulating the problem in the framework of semi-Markov decision processes. He showed that his previous policy which is based on the increasing failure rate of Weibull distribution is optimal in the set of all possible replacement policies under the assumption of increasing failure rate (IFR).

These sets of replacement policies are given below as

1. Perform minimal repairs up to age T and replace at age T;
2. Perform minimal repairs for the first n - 1 failures and replace at the n-th failure.
3. Perform minimal repairs up to age T and replace at the first failure after T.

Bowland and Proschan (12) extended the minimal repair concept by formulating a minimal repair-periodic replacement policy. In their model, the equipment replacement or overhaul is scheduled to take place in period T which minimizes the expected cost of repair and replacement. Two cases of this problem were considered: the first case deals with a fixed time horizon (0,T) and the second case deals with an infinite time horizon. If the equipment breaks down before the scheduled time T, 2T, 3T, ... for complete overhaul or replacement, minimal repair is performed. Any time complete overhaul or replacement is performed, the state of the system is upgraded to that of new (or as good as new) as shown by Barlow and Hunter (5). An optimal policy that will minimize the expected replacement (overhaul) cost was developed. This assumes that the number of breakdown in the interval (0,s) if no replacements (complete overhaul) occured is governed by the poisson distribution (12). This policy gives the T* at which replacement can be performed.

Eppen (25) assumed that the equipment deterioration is of Markov chain type. He argued that a decision is made after each inspection as to whether to leave an equipment in it's present state or perform some maintenance to place it in a rejuvenated state. The cost associated with this maintenance depends on the level of rejuvenation achieved. Eppen thus determined conditions for PM policy in order to achieve optimality for a Markovian deteriorating system. This he called "hysteresis repair" and leads to partial recovery of the system. Very little research has been done using the hysteresis function. Recent work by Brown, Mahoney and Sivazlian (13), extended Eppen's model by including a discount factor. Using this approach, an optimal service age was determined for an equipment. When the equipment's service age reaches a certain critical level, a decision is thus made to replace or completely overhaul the equipment. When hysteresis repair is performed, the service age immediately after repair is taken to be A(v) where v is the service age at the instant of failure. A(v) is thus called the hysteresis function and its values lies in the interval (o,v). When A(v) =v, this implies minimal repair and when A(v) = 0, it implies major repair. Hysteresis repair takes place when $0 \leq A(v) \leq v$ and the function is given as $A(v) = av$, $0 \leq a \leq 1$ where the constant a = (service age after repair)/(service age before repair). Using this, a functional equation based on cost minimization was developed in order to obtain the optimal replacement age of an equipment.

Nakagawa and Kowada (62) sought a different approach in defining minimal repair. They expressed it as failure rate which plays the most important role in reliability theory. They proceeded to derive the probability distributions of failure times and the times between failure of the system as Y_i where i = 0,1,......, n. Y_i when i = 0 equals 0 since the system starts to operate at time t = 0. The time between failures $X_n = Y_n - Y_{n-1}$. The minimal repair is defined as follows: Let $Pr(X_1 < t) = F(t)$ for $t > 0$, the system undergoes minimal repair at failure if $Pr(X_n < X/$

$X_1 + X_2 + \ldots\ldots + X_{n-1} = t) = \{F(t+x)-F(x)\}/F(t)$ where $n = 2,3,\ldots\ldots,$ for $x > 0$, $t > 0$ such that $F(t) \leq 1$, where $F(t) = 1 - F(t)$. This function known as the failure rate was originally derived by Barlow, Proschan and Hunter (5) and represents the probability that the system of age t fails in a finite interval (t, t + x). The system failure rate thus remains undisturbed after minimal repair of failures (62).

Using this failure rate definition of minimal repair, Nakagawa and Kowada derived the two replacement policies originally developed by Barlow and Hunter (5) and Morimura (54). They showed that these two policies yielded the same results. Therefore the system, if replaced at time T (> T*), should also be replaced at the n-th failure before time T, where n* policy yields the minimum cost.

Nakagawa (61) further determined the number of failures before replacement. The failed unit undergoes minimal repair between replacements. He determined the optimal number N* to minimize the mean cost rate when the scheduled replacement time T is specified. A unit is thus replaced at a scheduled time T (T>0) or at failure N (N=1,2,....) whichever occurs first otherwise minimal repair is applied at failure between replacement.

Age Replacement Models

Barlow and Proschan (6) discussed the traditional approach which is to replace a unit at failure or at time T, whichever comes first. This approach has been extended by other authors (17,27,76). The age replacement models deal with cases where the state of the system is unknown except where replacement is taking place or the cost of monitoring the state of the system is relatively high (34). The decision to replace the system thus depends only on the elapsed time since the completion of the last replacement. This type of replacement model is known as the age dependent replacement rule (34). The optimal age obtained will minimize the average cost per unit time.

Zijlstra (96) developed a renewal replacement policy for a one unit system. An expected cost rate function was introduced and used to derive the objective function.

Sivazlian and Iyer (80) formulated a dyadic age-replacement policy. The dyadic type consists of the total amount of deterioration of an equipment since acquisition (service age) and the total number of periods that has elapsed since acquisition (chronological aging). A policy is set to replace the equipment if at the end of the period, the equipment has a service age greater than a fixed service age limit S or if the equipment has been in operation for exactly N periods whichever comes first. This policy is denoted as the (S, N) policy. The service aging of the equipment per period is assumed to have a gamma distribution. The deterioration of an equipment was measured by it's actual usage and the time elapsed since acquisition. The distribution of the time interval between successive replacements was derived and using a theorem on renewal reward process, an expression for the long run expected cost per period, consisting of a a fixed replacement cost and a linear cost of operation was derived. The optimal values of S and N were obtained by minimizing the steady state expected cost.

Mehrez and Stulman (53) considered the age replacement problem where there may not exist a perfect inventory system. Two policies from the popular (s,S) family were considered.

The (1,1) policy and the (2,2) policy; In the (1,1) policy, an order of one unit is placed when s = 0 which implies that no unit is available for operation. The system thus suffers a downtime of length α since there will be no unit available for replacement. Two decision variables α, and age T were used to formulate a maintenance policy. A unit is to be replaced whenever the unit reaches age T or whenever the minimal repair cost C exceeds a predetermined value δc_1 (16,53). An assumption is made that the cost of ordering will be decreasing in α while the cost of lost production will be increasing in α . These costs are combined to obtain a cost function. The objective is thus to minimize the cost function with respect to α and T. The decision variables are thus solved by setting the derivative of the objective function equal to zero.

The (2,2) policy was similarly obtained using the two decision variables α and T. However, in this policy, no system downtime is allowed. A unit on order is not available for replacement and until it's delivery a repair is performed on the system. α is therefore the order lag time or lead time. The minimal repair policy developed by Cleroux, et al (16) is then used to determine when a repair is expensive.

Bergman (9) and Bergman and Klefsjo (10) introduced a graphical procedure based on the total-to-time-on-test (TTT) concept for the analysis of the age replacement problem. This approach was used to study age replacement problems with discounted costs.

An economic interpretation of the results derived from mathematical replacement models was given by Verheyen (89). He concludes that the moment of replacement is always determined by the equality of marginal replacement costs of postponed replacement.

Various economic life models considered factors such as units with unequal lives, the effect of accelerated cost recovery system, and an optimal replacement cycle for selling and replacing depreciable capital assets (38, 41, 43, 77).

In some of the economic life models, the decision to replace a machine is based solely on the equipment's age. This tends to ignore the cases where an equipment may require an extensive repair before the end of its economic life. The cost of repair before its economic life may exceed the benefits to be derived from the use of the equipment and it will be unprofitable to retain the unit. However, this problem has been considered in the " repair limit models ".

The analysis of these models are usually based on incomplete data and some ambiguity. For example, such models presume explicit knowledge of the salvage values and operating cost as a function of machine age (38). These data can not be easily determined given the continual changes in technology, resource costs, and market conditions.

There is also the assumption about the cash flows to be generated by the use of the equipment. Cash flows and interest rates are assumed to be known with certainty. These factors are however part of the dynamic environment.

Despite all these flaws, the economic life models offer some insight to the decision maker and probably help in reducing some of the uncertainties he is faced with.

Repair Limit Policies

The repair limit policy was first formulated by Nakagawa and Osaki (63, 66, 73). This model refers to the case where the system is allowed to operate until it fails. When the unit fails, a repair is immediately applied. If the repair is not completed after a repair limit time τ, the failed unit is replaced by a new one. However, if the repair is completed, the condition of the system is assumed to be as good as new. An optimal repair limit time ζ^* was derived to minimize the expected cost per unit time for an infinite time span. In this policy, a failed unit will be repaired if the repair time is short and will be replaced if the repair time is long. It is thus assumed that the repair cost is proportional to time. The original model has been extended by other authors. Okumoto and Osaki (69) studied the repair limit policy with lead time, while Nakagawa (59) further extended the model to derive a repair limit policy that maximizes the expected earning rate of the unit.

Nguyen and Murthy (65) proved that the model is optimal over deterministic and random repair limit policies. They further considered the case of imperfect repair (66). The imperfect repair is based on the assumption that after repair, the state of the system is not as good as new. Thus the mean life of the unit after repair is less than the mean life of a new unit.

Another version of the repair limit replacement policy was developed by Drinkwater and Hastings (24) and Hastings (28, 29). The repair limit strategy in this model is based on the cost of repair. When a unit requires a repair, it is inspected and the repair cost estimated. If the repair cost exceeds a certain amount, the unit is replaced otherwise it is repaired. The upper limit for repair costs decreases with the age of the machine.

Different variations of this model have also been studied (23, 40, 47). Di Veroli (23) considered the problem of preventive replacement and repair in the case of failure. He showed that the optimal preventive replacement policy is a function of the optimal repair limit.

The repair limit policy tends to consider the quantitative factors that affect equipment replacement decisions (28,29) while other factors such as return on alternative investments and taxation have often been neglected. Lambe (40) tried to incorporate some of these factors into the repair limit policy by using Bayes formula to combine data with previous judgement about the characteristics of the equipment. He extended the repair limit model to include past information on the machine's repair cost. He developed a model that identified the machines having more frequent or consistently higher costs than the average machine of its class. This approach deviates from the earlier models which detects the machine with very high cost for an individual repair. The characteristic pattern of the repair costs as well as the interval between these costs are random in nature. It thus becomes difficult to make a decision on when to repair or replace equipment especially when the equipment has infrequent but costly expenses. Lambe claimed that uncertainty about the cost characteristics of equipment reduces with experience.

Beichelt (7) compared the repair limit policy to the economic life time policy. In the economic life time policy attention is not given to individual deviations of repair costs of single units from the average cost. This condition is however considered by some of the repair limit policies. The repair limit policy was shown to be more cost effective than the economic life time policy. He extended the repair limit policy so that when the repair cost rate reaches or exceeds a fixed level, the unit is replaced. This approach eliminates the disadvantage of having to focus only on a single repair cost rather than the total repair cost rate (7).

Block Replacement Policy (BRP)

A decision can be made to replace all the units at an equally spaced interval independent of the failure history of the unit. The drawback with this standard approach is that new items may be replaced if one ignores the history of the units. This approach however, has been shown to be more useful and profitable when items being replaced are relatively cheap i.e, vacuum tubes, electric bulbs, and others (95). Preventive maintenance is thus applied to protect the total system from failure by replacing all the units before their actual failure and thus loosing the values of any remaining life (8). In this policy the unit is replaced any time failure is detected or at fixed points for replacement. Its simplicity relies on the fact that there is no need to record the age and failure history of the units involved. The objective is to minimize the average cost per unit of time, for an infinite time horizon.

Barlow and Hunter (5), and Barlow, Proschan and Hunter (6), studied the BRP by applying the notion of minimal repair. Minimal repair is applied to a failed unit to bring it back to an operational form. This model is restricted in application since one has to assume that not only is the system complex but also repairable. Cox (18) and Crookes (19) introduced a BRP where a failed unit remains idle until block replacement occurs, and Woodman (94) determined the optimal policy for that of Cox, and Crookes using dynamic programming.

Berg and Epstein (8) extended these policies to obtain a modified BRP. In this modified policy, failed items are replaced instantaneously after failure, but items whose age is b or less at scheduled block replacement points t, 2t, 3t, ... are not replaced by new items. An item that has an age in the range $0<b<t$ is allowed to remain in service. This policy thus incorporates the age replacement policy into the BRP. At the time of scheduled block replacement, some items

will have age x, $0<x\leq b$, these items will not be replaced. From this an optimal value for (b*,t*) was obtained. These values give minimum cost C(b*,t*). The result obtained with the modified BRP was compared to that of the standard BRP by assuming failure rate to be an erlang distribution with two phases. The policy was shown to do better than the failure replacement policy (6). The modified BRP policy tend to have much higher savings with erlang distribution having more than two phases (8).

Murthy (57) extended the failure replacement policy to take into account the initial stock of L items at the beginning of each cycle. The aim is to minimize the replacement cost as well as the penalty cost function for L. In the model introduced by Murthy, a penalty cost is incurred any time $L < [1 + n_1(T)]$ or $L > [1 + N_1(T)]$ where the former case represents shortage and the latter case surplus. The $N_1(T)$ is a random variable representing the number of items replaced due to failure in a cycle. In the notations of inventory model, we should incure both shortage and holding costs depending on the conditions that prevail. If however $L = [1 + N_1(T)]$ then there will be no such costs. Murthy obtained the optimal L* given as $L^* = 1 + M(T)$ to be the optimal stock of items at the beginning of a cycle. M(T) is the renewal function and it is related to the failure distribution F(t) by the integral renewal equation.

Murthy and Nguyen (58) further extended the BRP to include the replacement of failed items with used items. Tango (83,84) originally investigated this model. In Tango's initial model, used items of age T are used to replace a failed item while those with age less than T are discarded. This policy does not seem rational if one considers items that follow age replacement policies. In Murthy and Nguyen's approach, failed items are replaced with used items of varying age from v to T, as opposed to replacement by used items of age T only (58). The policy obtained by using all the used items created was shown to be more cost efficient over the BRP developed by Tango.

Okumoto and Elsayed (68) expanded failed replacement policy by incorporating repair cost and production loss due to the system's breakdown. They showed that optimum group maintenance policy exists for the exponential failure time distribution. This contrast from the replacement policy where there is no optimum policy for the exponential failure time distribution (6). However this is not unexpected since the exponential distribution is used to model only cases where failure follows a random process. The exponential distribution characterizes the " no wear phenomenom "(52). Age replacement policy is usually based on the wear phenomenom. This explains the use of Weibull distribution rather than the exponential distribution in modeling age replacement policies.

Further extensions of the BRP was given by Nakagawa (60). He integrated most of the models discussed above [Crooks (19), Tango (83,84) and Berg and Epstein (8)] to obtain the optimum BRP. In his model, he assumed that the failure time of a unit has a gamma distribution. The unit is replaced if a failure occurs in the interval (0, To) otherwise, it is replaced at the scheduled replacement time T. If however a unit fails in the interval (To,T) it remains failed until T or it is replaced with a used unit. The newly replaced units are not replaced when block replacement takes place. Nakagawa obtained the mean cost rate by using the results of renewal theory. The cost function is minimized to obtain the optimal To* and T* (60).

CHAPTER 3

PROBLEM DEFINITION

The preceeding section gives the current literature in the area of maintenance modeling. As is evident from this review, there have been a lot of work done in developing maintenance policies. These policies are used for cost minimization purposes to determine the optimum replacement times i.e. age replacement policy is based on determining the useful age of the unit and replacing that unit when it reaches an age that is considered unproductive. Similarly, the repair limit policy is used to make a decision on when to repair or replace a failed unit on the basis of cost of repair or time required for repair. As shown in (63), the cost of repair is proportional to the time it takes to repair the unit. All these policies thus follow a cost minimization approach. These models are unidimensional in that the maintenance decisions are made on unit by unit basis and no attempt is made to study the whole system (ie a system comprised of many units). The objective of such systems extends far from the consideration of repair or replacement of an individual unit. The aim is to minimize the system's downtime and this is often achieved by introducing standbys or redundancy, repair and maintenance policies. This consideration is of great importance in many service industries. In such service industries, the reliability of their operating system is the primary objective. Such exists in the transport industry where the transit system's reliability may be improved even at a higher cost. This type of model where reliability is increased by maintaining standbys and repair facilities is thus referred to as a "maintenance float model". It is a closed network queueing problem and neither arrivals nor depatures are allowed. We shall proceed to define the current problem and some of the work that exist in the area of maintenance float modeling.

A maintenance float is made up of an operating system consisting of independent and identically distributed (iid) units. These units operate to achieve the system's objective. When a failure occurs, the failed unit is sent into repair and it is replaced by a standby unit. At the completion of repair, the unit is sent into the reserve as standby. However, if the number of operating units is less than the fixed capacity of the operating system, the repaired unit is immediately sent into operation until this maximum capacity is attained. The objective of a maintenance float may be to maximize the reliability of the system. This is done by evaluating the tradeoffs between maintaining several identical units as standby and reducing the rate of repair service or vice versa. The optimization approach is therefore either to minimize the cost of the maintenance float system by sensitizing on the parameters (standby vs repair rate) or to maximize the reliability of the maintenance float system.

Barlow and Proschan (6) gave analytical models for various conditions of the maintenance float problem using the birth and death process. The maintenance float was therefore solved as a queueing problem. They referred to this class of problems as the "Repairman problem". In this class of problems, system reliability is increased by introducing redundancy, repair and preventive maintenance (81). Maintenance float model is also referred to as a "closed queueing network" (48). This is a network in which neither arrivals nor depatures are considered. Instead, a fixed number of customers (units) circulate through the network at all times.

This area of maintenance modeling is valuable for research due to the rapid increase in automation of production systems. As noted in (88), "a major factor in many key decisions in industry is lost production that results from the failure of crucial equipment". The downtime of the production system is often decreased by maintaining standby. However, there exist some tradeoffs.

There are various examples of maintenance float applications (ie emergency vehicles, airforce or navy fleet maintenance, life sustaining systems, and flow shops). An example is given as follows: a city may own an ambulatory service. We assume that in the time horizon under consideration, the city desire to have N number of ambulance cars in operation at any time and n number on standby. Levine (42) introduced a reliability factor that is based on the ratio of mean time to repair (MTTR) to mean time between failure (MTBF) as a variable and derived an analytical approach using an exponential function for the failure rate. Levine's method was extended by Lowe and Lewis (44) to a failure pattern which corresponds to a Weibull distribution and a repair function which follows a lognormal distribution. They obtained a total float F required to support an operational group N.

The work of Levine (42) and Lowe and Lewis (44) is extended in this paper by using the same basic assumptions and analytically obtaining the total float F for a given N for failure and repair distributions not considered. An illustration is given using graphical results. We also obtained some general results that were not mentioned in the literature. These general results reduce the computational efforts involved in deriving the model and in also using the model. Our paper further showes the range over which this model is applicable by using a marginal cost technique. This approach avoids the assignment of costs in order to test the efficiency of the model as done in (44). We further compared the failure distributions on the basis of their performance using these models. The sensitivity of our second assumption stated below was also tested through simulation study. The two assumptions made are stated as follows:

1. Perfect renewal is assumed after each repair. Thus a repair returns the unit to a completely rejuvenated state. The unit is therefore as good as new after each repair.

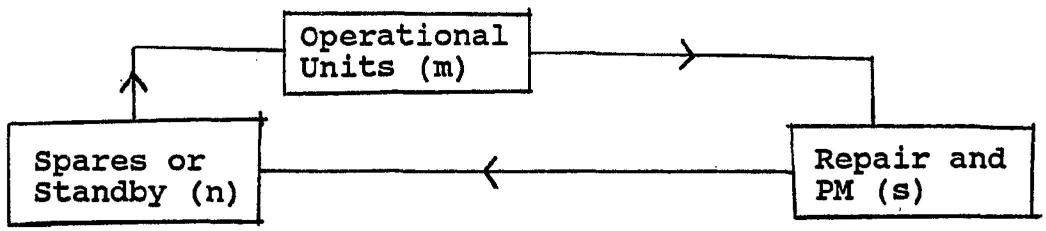
2. There is no queue for repair when a unit fails, and the unit returns immediately into service after each repair.

Lowe and Lewis showed that the result obtained by the reliability approach for the case of Weibull failure distribution and Lognormal repair distribution approximates a practical example when holding and shortage costs were the only costs considered. This result can not be replicated since it depends on the costs assigned. We use the cost ratio technique to show the applicability of these models.

In this paper we shall extend the work of Lowe and Lewis to include the following failure distributions: gamma, erlang-2, constant or degenerate, exponential, normal, lognormal and uniform distributions. We shall also obtain some general results regarding the role of MTTR in the formulas developed for the maintenance float factors, and further show that the asymptotic property of the maintenance float factors exist when the value of N (the number of units initially in operation is large). To test the efficiency of this model, a marginal cost analysis is used. This leads to developing cost ratios and the applicability of these models is determined on the basis of these cost ratios. The various failure distributions were then compared using this cost ratio analysis. One of the assumptions of this model is that the waiting time for repair is negligible. The sensitivity of this assumption was tested. Finally graphical solutions to these models are introduced for purpose of illustration and also for verifying some of the stated properties.

Other published work in the area of maintenance float modeling include the following: an analytical model was introduced for a 2-unit system with one operative unit and one spare where the spare (standby) is assumed not to fail or deteriorate (1,35,81); a 1-out-of-n standby units (74,87); an n spare system with one operational unit with n+1 units being independent and identically distributed (39). These studies were analytically based since the level of complexity involved is relatively low. The problem becomes more complex when we have a large number of identical units operating at one time and a considerable number of standbys. What we have is a complex network system that exhibits the characteristics of a queueing problem due to the sequential processes that go on and also some inventory characteristics. This production problem is difficult to model analytically. The methodology often resorted to is simulation (32,91,93). Johnson and Fernandes (32) studied the effect of routine maintenance tasks on the float level.

Description of a Maintenance float model



This model assumes that a queueing process exists when a failure occurs. The failed unit queues up for repair. The queueing process is FIFO since the units are iid. After repair the unit goes into standby until the capacity of the operational units fall below the desired level m.

The diagram above indicates that m identical units stochastically independent of one another and supported by n spares operate in the facility. If each fail according to distribution F, they are sent into the service facility where s units can be simultaneously repaired at a repair distribution G. After repair, the unit is sent in as a spare if m units are already in operation, otherwise it goes directly into operation. The combinations of exponential and erlang failure and repair distributions have been modeled using a birth and death process with m + n + 1 states (6). Note that N = m and S = n. The definitions for the notations used in this paper are given in the next page.

Definitions:

N = units initially in operation.

n = units in a float which consists of both the units in reserve plus units still undergoing repairs.

f = maintenance float factor is the proportion of units that have failed upto time t_n. $f = 1 - N_{t_n}/N$

MTBF = Mean time between failure .

MTTR = Mean time to repair.

$T(n)$ = n(MTBF + MTTR) is the expected time for the n-th renewal of a unit after an extended operating period.

t_n = the time for the n-th failure. Fix $t_n \geq t_1 + MTTR_1$, so that when the n-th unit fails, the 1st is just available. This way a total N operating units (on the average) is achieved.

N_t = the number of functioning units at time t.

N_t/N = $R(t)$ = the reliability at time t which is the fraction of functioning units at time t.

F = maintenance float to support an operational group N. $F = f.N.$

ρ = average equipment utilization

METHODOLOGY

A simulation program was written in GPSS V for the maintenance float problem described in this paper. In simulating a maintenance float, several costs such as the cost of repair, cost of leasing or renting equipments, cost of lost production and the cost of purchasing an equipment are often considered. The maintenance float problem is a classical problem in production management and the major focus have been on finding the optimum combination of both the number of standbys and the number of repairmen that will give a minimum cost. Thus the optimal solution obtained from this classical problem depends on the assignment of cost parameters, and the optimal decision rule reached changes whenever those cost functions are changed. In our study, we deviate from the cost assignment approach and try to develop a cost ratio approach to determine an optimum float requirement. We first developed the maintenance float factors for the failure distributions mentioned in the paper. These float factors are used to determine the float level needed given that N units are in operation. This float serves as our starting point for the standby requirement rather than the trial and error approach currently used. Only two variable costs are considered while the other costs are assumed fixed (i.e. cost of N units initially in operation, and the cost of repair). The cost of lost production and the cost of leasing or renting plus the cost of storage are the two variable costs. The measure of performance that is of interest in the simulation study is the average equipment utilization from which the average idle time of a unit is derived. The aim of this cost ratio approach is to develop bounds over which our model can be effectively used.

Our data for the average utilization which is used in this test was coded in order to use this test. The average utilization used in computing the cost ratios given in this paper are therefore obtained as

$$\text{Avg. Utilization} = \sum \rho_i \,/\, \text{No. of Replications}$$

We further obtained confidence intervals for the average utilization at both 95 % and 99 % confidence intervals by using the method of proportions. This is given as

$$Po \pm z_{\alpha/2} \sqrt{\frac{Po(1 - Po)}{n}}$$

where Po is the average utilization and n is the sample size which is 25 in this paper.

We used piecewise approximation in order to obtain the functions for the Erlang-2 and Lognormal distributions. These functions are needed for the simulation since we are using GPSS and it does not have built in functions for these distributions. The GPSS V manual has only the functions for the exponential and normal distributions.

CHAPTER 4

MAINTENANCE FLOAT MODELING

The time to failure or "life" of an equipment is a random variable expressed in terms of probability function. The probability function is used to define the reliability or survival function of the system. If t denotes the time to a failure random variable, then the reliability at any time t, represented by R(t) is the probability that the system will not fail by time t. R(t) is expressed as follows (26): Let T be the time to failure, then

$$R(t) = P\ (T>t) = \begin{cases} 1 - F(t), & t \geq 0 \\ 1, & t \leq 0 \end{cases}$$

which implies that $R(t) = 1 - P(T \leq t) = 1 - \int_{o}^{t} f(\tau)\, d\tau$

and F(t) is the cumulative density function while f(.) is the probability density function.

In order to determine the failure distribution of a system, it is necessary to obtain some empirical data either by reliability testings or through past experience about the system's performance. When actual observation of times to failure are taken, it is difficult to distinguish among the various nonsymmetrical probability functions. The difference between such distributions as gamma, weibull and lognormal are significant only in the tails of the distributions, but actual observations are sparse in the tails because of limited sample sizes (6). Differentiation of these distributions is based on physical considerations such as their failure rate function (hazard rate function). This function is used to determine the extreme value distributions (6). Depending on the characteristics of the equipment under consideration, they may assume different failure distributions.

In this section we derive the maintenance float factors for the continuous failure distributions stated earlier in the paper. This work as mentioned is an extension of Lowe and Lewis's paper. Lowe and Lewis (44) obtained the maintenance float factors for a system with weibull failure distribution and lognormal repair distribution. Two major assumptions are made and they are stated below:

1. The unit is completely rejuvenated after each repair and its state is returned to that of new after repair.

2. The waiting time for repair is negligible.

Exponential failure distribution

The exponential distribution is used in modeling an equipment failure rate because of the random occurrences of failure. The exponential has a constant failure rate because of the memoryless property and has been used to model complex systems made up of many subcomponents. In such systems, the failure of a subcomponent does not significantly affect the system's failure rate. This is due to the fact that the exponential distribution does not exhibit the 'wear phenomenum' present in other distributions such as weibull (6,36,52). The memoryless property restricts its use as a failure rate distribution since most systems failure rate increases with age. In other words, the failure of equipment is due to chance and not due to wear and tear.

Theorem 1:

If the failure distribution is exponential and the repair distribution is lognormal then the maintenance float factor is given as follows:

$$f = 1 - \exp[-(\ln(N/(N-1)) + \lambda \exp(u + \sigma^2/2))]$$

Proof:

We define R(t) for an exponential distribution as

$$R(t) = \exp - \lambda t \qquad (1)$$

since the failure distribution is exponential. For a lognormal repair distribution, the mean time to repair is give as follows:

$MTTR = \exp \{ u + \sigma^2/2 \}$ and we can express this mean time to repir as

$MTTR = m\exp \sigma^2/2$ where $m = \exp u$ and u is the mean of the distribution while σ is a measure of dispersion. Fix $t_n = t_1 + MTTR_1$ so that when the n-th unit fails, the 1st is just available. This way a total N operating units on the average is achieved.

$$t_n = t_1 + MTTR_1 \qquad (2)$$

from (1), we obtain that

$$\ln R(t) = - \lambda t \qquad (3)$$

and $$t = - \frac{1\ln R(t)}{\lambda} \qquad (4)$$

We now define R(t) which is the reliability at time t as the ratio of the number of units functioning at time t and the number of units initially in operation. R(t) is therefore the proportion of units functioning at time t and it is given as $R(t) = [N_t/N]$ and $t = -\frac{1}{\lambda} \ln [N_t/N]$ (5)

The time to the first failure t_1 then follows from (5) and it is expressed as

$$t_1 = -\frac{1}{\lambda} \ln [(N - 1)/N] \quad (6)$$

and can also be expressed as

$$t_1 = -\frac{1}{\lambda} [\ln (N - 1) - \ln N] \quad (7)$$

$$= \frac{1}{\lambda} [\ln N - \ln (N - 1)]$$

$$= \frac{1}{\lambda} \ln [N/(N - 1)] \quad (8)$$

Using (2) we obtain that

$$t_n = \frac{1}{\lambda} \ln [N/(N - 1)] + m\exp \sigma^2/2 \quad (9)$$

and from (1),

$$R(t_n) = N_{t_n}/N = \exp \{-\lambda t_n\} \quad (10)$$

$$= \exp \lambda[- \{ \frac{1}{\lambda} \ln (N/(N-1)) + m\exp \sigma^2/2\}] \quad (11)$$

$$= \exp\left[-\{\ln(N/(N-1)) + \lambda m \exp \sigma^2/2\}\right]$$

The maintenance float factor (f) is the proportion of units that failed upto time t_n. Thus

$$f = 1 - R(t_n) = 1 - \exp\left[-\{\ln(N/(N-1)) + \lambda m \exp \sigma^2/2\}\right] \quad (12)$$

and $0 \leq f \leq 1$. The total float F needed to support N number of operational units is then given as

$$F = fN = N\{1 - \exp[-\{\ln(N/(N-1)) + \lambda m \exp \sigma^2/2\}]\} \quad (13)$$

The only parameters needed to obtain f for the exponential failure distribution is the MTBF = $1/\lambda$, the MTTR of the repair distribution, and N the number of units initially in operation.

In Theorem 2, we assume the repair distribution to be exponential while failure distribution is still exponentially distributed. The result obtained from this allows us to make a general statement hence Corollary 1.

Theorem 2:

If the failure distribution is exponential and the repair distribution is also exponential then the maintenance float factor $f = 1 - \exp[-\{\ln(N/(N-1)) + \lambda/\lambda'\}]$ where MTTR $= 1/\lambda'$ for an exponential distribution with parameter λ'.

Proof:

Following the same line of arguements that led to equation (12), we notice from (9) that t is a function of MTTR, MTBF and N and can be expressed as

$$t = \frac{1}{\lambda}\ln(N/(N-1)) + \text{MTTR} \tag{14}$$

and from (11), equation (12) is equivalent to

$$f = 1 - \exp[-\{\ln(N/(N-1)) + \lambda\,\text{MTTR}\}] \tag{15}$$

It is observed that the MTTR factors out nicely and its presence does not affect the structure of the formula developed for the maintenance float factor. The structure of these formulas depend only on the failure distribution. This becomes more apparent with the treatment of the other failure distributions. The maintenance float factor for the exponential failure distribution can therefore be expressed as

$$f = 1 - \exp[-\{\ln(N/(N-1)) + \text{MTTR}/\text{MTBF}\}] \tag{16}$$

Corollary 1

From Theorems 1 and 2, it is observed that the structure of the maintenance float models does not depend on the repair distribution. Therefore the maintenance float factor equation developed for any given failure distribution can be used for any arbitrary repair distribution. It is therefore not necessary to derive the maintenance float factor equations for the different repair distributions. The determining factor in these models is the failure distribution. When the failure distribution is known, the formulas developed for the MFF applies irrespective of the repair distribution.

Theorem 3:

If the failure distribution is weibull and the repair distribution is lognormal, then

$$f = 1 - \exp[-\{\ln (N/(N-1))\}^{1/\beta} + \frac{m\exp \sigma^{2}/2]}{\alpha}^{\beta}$$

Proof:

The proof to this is given by Lowe and Lewis (44).

Gamma failure distribution

The gamma distribution has p number of phases and it is the sum of independent and identically distributed random variables that are exponentially distributed. When the number of phases p = 1, the gamma distribution becomes exponential. When p = 2, we obtain the erlang-2 distribution which is the sum of two iid exponential distributions. For $0<p<1$, we obtain a hyperexponential distribution and when $p = \infty$, the constant or degenerate distribution is obtained.

The failure rate for the gamma distribution is decreasing when $p < 1$ (hyperexponential), is constant when $p = 1$ (exponential), and is increasing when $p > 1$ (erlang-2, degenerate and others).

Theorem 4:

If the failure distribution is gamma with p phases and the repair distribution is lognormal then

$$f = 1 - \exp -\{[P!\{1 - (N - 1)/N\}]^{1/P} + \lambda m \exp \sigma^2/2\} \times \sum_{K=0}^{P-1} \frac{\{[P!\{1 - (N - 1)/N\}]^{1/P} + \lambda m \exp \sigma^2/2\}^K}{K!}$$

Proof:

$$R(t) = \exp -\lambda t \sum_{K=0}^{P-1} \frac{(\lambda t)^K}{K!} = N_t / N \qquad (17)$$

where P is the number of phases. When P is restricted to integer values, the gamma becomes equivalent to the erlang distribution. From (2), we obtain that

$t_n = t_1 + MTTR_1$ thus leading us to solve for t from

equation (17) as follows:

$$\exp(\lambda t) R(t) = \sum_{K=0}^{P-1} \frac{(\lambda t)^K}{K!} \qquad (18)$$

From Taylor series, $\sum_{k=0}^{P-1} \frac{(\lambda t)^k}{k!}$ converges to

$e^{\lambda t} - R$ where R is a remainder

factor and

$$|R| \leq \frac{e^{\lambda t} |\lambda t|^P}{P!} \qquad (19)$$

and $|\cdot|$ is the absolute value. We make the assumption that $\lambda t \geq 0$ since it is an absolute value and $t \geq 0$. Thus from equation (18), we obtain the following

$$e^{\lambda t} R(t) = \frac{e^{\lambda t} N_t}{N} = \sum_{K=0}^{P-1} \frac{(\lambda t)^K}{K!} = e^{\lambda t}\left[1 - \frac{(\lambda t)^P}{P!}\right] \quad (20)$$

and

$$R(t) = N_t/N = 1 - \frac{(\lambda t)^P}{P!} \quad (21)$$

It then follows that

$$\frac{(\lambda t)^P}{P!} = 1 - N_t/N \quad (22)$$

Since $N_t \leq N$, then it follows that $0 \leq \frac{(\lambda t)^P}{P!} \leq 1$.

λt is obtained by inverting (22) to 1/P. This inversion gives the following results:

$$\lambda t = \{P![1 - N_t/N]\}^{1/P} \quad (23)$$

and

$$t = \frac{1}{\lambda}\{P![1 - N_t/N]\}^{1/P} \quad (24)$$

The time to the first failure t_1 is expressed as

$$t_1 = \frac{1}{\lambda}\{P![1 - (N - 1)/N]\}^{1/P} \quad (25)$$

and t_n from equation (2) can be expressed as

$$t_n = \frac{1}{\lambda} \{p![1 - (N - 1)/N]\} + m\exp \sigma^2/2 \qquad (26)$$

and

$$R(t_n) = N_{t_n}/N = e^{-\lambda t_n} \sum_{K=0}^{P-1} \frac{(\lambda t_n)^K}{K!} \qquad (27)$$

which implies that

$$R(t) = \exp - \lambda\left\{\frac{1}{\lambda}\left[P!\left(1 - \frac{N-1}{N}\right)\right]^{1/P} + m\exp \sigma^2/2\right\} \times \sum_{K=0}^{P-1} \frac{\lambda\left\{\frac{1}{\lambda}\left[P!\left(1 - \frac{N-1}{N}\right)\right]^{1/P} + m\exp \sigma^2/2\right\}^K}{K!} \qquad (28)$$

$$= \exp - \left\{\left[P!\left(1 - \frac{N-1}{N}\right)\right]^{1/P} + \lambda m\exp \sigma^2/2\right\} \times \sum_{K=0}^{P-1} \frac{\left\{\left[P!\left(1 - \frac{N-1}{N}\right)\right]^{1/P} + \lambda m\exp \sigma^2/2\right\}^K}{K!} \qquad (29)$$

and

$$f = 1 - \exp - \left\{\left[P!\left(1 - \frac{N-1}{N}\right)\right]^{1/P} + \overset{(A)}{\lambda m\exp \sigma^2/2}\right\} \times \sum_{K=0}^{P-1} \frac{\left\{\left[P!\left(1 - \frac{N-1}{N}\right)\right]^{1/P} + \lambda m\overset{(B)}{\exp} \sigma^2/2\right\}^K}{K!} \qquad (30)$$

Corollary 2:

It is observed from equation (30) that the MTTR appeared in both (A) and (B) parts of the equation. Thus for an erlang or gamma failure distribution with a given repair distribution, the maintenance float factor (f) is given as

$$f = 1 - \exp - \left\{\left[P!\left(1 - \frac{N-1}{N}\right)\right]^{1/P} + \lambda MTTR\right\} \times \sum_{k=0}^{P-1} \frac{\left\{\left[P!\left(1 - \frac{N-1}{N}\right)\right]^{1/P} + \lambda MTTR\right\}^{K}}{K!} \tag{31}$$

It is therefore not necessary to compute the equation for the MFF for each given repair distribution since the structure of this formula depends on the failure rate distribution and not on the repair distribution. This corollary is similar to Corollary 1. All that's necessary to compute the MFF is knowledge of the failure distribution and the MTTR can be substituted into equation (31).

When P = 2 the computational effort involved in order to find the MFF is reduced since K goes from 0 to 1 (see equation 31). When P = 1, the gamma distribution becomes exponential. Equation (31) reduces to

$$f = 1 - \exp - \{1 - [(N-1)/N] + \lambda MTTR\} \tag{32}$$

where λ is the parameter of the exponential failure distribution. In Theorem 1,

$$f = 1 - \exp [-\{\ln (N/N-1) + \lambda MTTR\}] \tag{33}$$

was derived directly from the exponential distribution. These two equations (32) and (33) are identical and it is easy to show that

$$1 - (N - 1)/N = \ln (N/N - 1) \qquad (34)$$

The goodness of this approximation is illustrated by using a hypothetical example stated below:

Example: Let N = 100 then from equation (34),

$$1 - 99/100 = \ln (100/99) = 0.01$$

A special case of gamma is obtained when $0 < P < 1$ and this special form is known as the hyperexponential distribution. We did not try to solve for this special case but we believe our model also applies. Our model also apply for the case of degenerate failure distribution or constant failure distribution when P approaches infinity.

From equation (30), when P = 2, the maintenance float factor for the erlang-2 failure distribution is obtained as

$$f = 1 - [\exp - \{ \sqrt{2(1 - (N - 1)/N)} + \lambda MTTR\} \times \{\sqrt{2(1 - (N - 1)/N)} + (1 + \lambda MTTR\}] \qquad (35)$$

We are interested in the erlang-2 failure distribution since this is one of the commonly used failure distributions (74,87).

In Theorem 5, we show that the MFF for gamma approaches 0 when the number of phases (P) approaches infinity. Thus for increasing values of P, the MFF is decreasing.

Theorem 5

If the failure distribution is gamma with P-phases, the maintenance float factor (f) approaches 0 as P approaches infinity.

Proof

From the derivation for the gamma distribution, we obtained that the MFF is

$$f = 1 - \exp - \left\{ \left[P! \left(1 - \frac{N-1}{N}\right)\right]^{1/P} + \lambda MTTR \right\} \quad x \quad \sum_{K=0}^{P-1} \frac{\left\{ \left[P! \left(1 - \frac{N-1}{N}\right)\right]^{1/P} + \lambda MTTR \right\}^{K}}{K!}$$

let $\alpha = [P!(1-(N-1)/N)]^{1/P} + \lambda MTTR$

when $P \longrightarrow \infty$, f can be expresses as

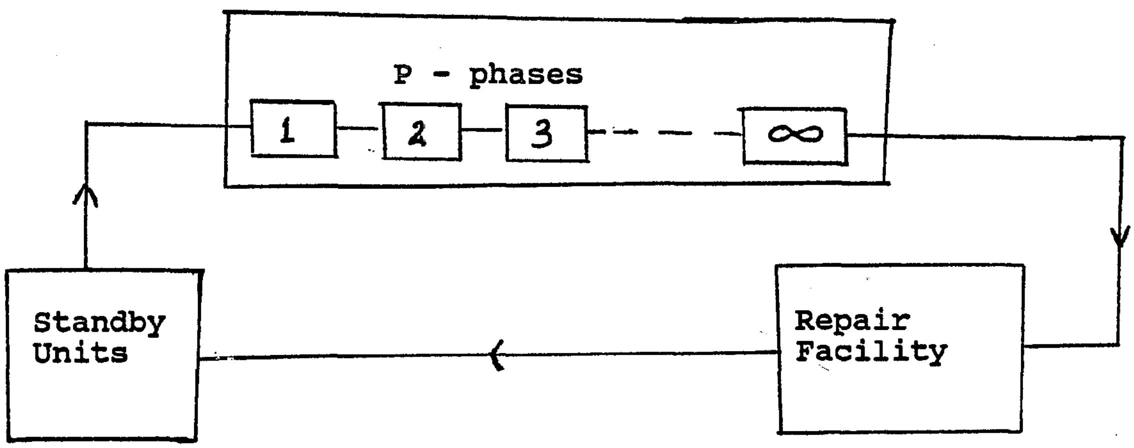
$$f = 1 - [\exp - (\alpha) \quad x \sum_{K=0}^{\infty} \frac{\alpha^{K}}{K!}] \qquad (36)$$

Notice that the series $\sum_{K=0}^{\infty} \frac{\alpha^{K}}{K!}$ converges to $\exp(\alpha)$. Thus

$$f = 1 - [\exp(-\alpha)\exp(\alpha)] \quad (37)$$

$$= 1 - \exp(-\alpha + \alpha) = 0 \quad (38)$$

Therefore the $\lim_{P \to \infty} f \to 0$. From this proof, there is no need to maintain large standby units when the failure distribution is constant or degenerate. We can intuitively explain this theorem using the sketch given below for a maintenance float model.



For a machine breakdown to occur, the unit has to go through an infinite number of failure phases. Thus the probability of failure is a function of the number of phases. If the mean time between failure is fixed for P=1, P=2, P=3, ..., P=∞ then when P = 1, the probability of failure is higher than when P =2 and the probability of failure when P =2 is higher than when P =3 and so on. We can therefore order the probabilities of failure as

$$p(1) > p(2) > p(3) > p(4) > \dots\dots\dots > p(\infty)$$

where p(.) is the probability of failure as a function of the number of phases. Let us also denote the maintenance float factor as a function of the number of phases P which is given as f(P). There is a direct relationship between p(P) and f(P). As p(P) increases, f(P) also increases. We can therefore order f(P) as follows

$$f(\infty) < \ldots\ldots < f(N) < f(N - 1) < \ldots\ldots\ldots < f(1).$$

From Theorem 5, $f(\infty)$ approaches 0. Graphical illustration to this result is given in Figure 12. This result implies that less numbers of standby's are needed when the number of phases is large. Thus the exponential failure distribution will require more number of standbys than the erlang-2 failure distribution and the degenerate failure distributions. The degenerate case as we have shown will not need any standby. It should be noted that the gamma distribution is the sum of iid exponential distributions with parameter (P/λ). In this paper, our Ps are restricted to integers. It is easily seen that the mean time between failure (MTBF) increases for increasing values of P.

Normal failure distribution

The normal distribution has a monotonically increasing function. It is used as a life time distribution when ($u > 6\sigma$) where u is the mean of the distribution and σ is the standard deviation. When $u < 6\sigma$, the truncated normal distribution is used. The normal distribution has been found useful in modeling systems where repair is applied after a unit breakdown (22). Like the weibull and gamma distributions, the normal provides reasonable estimates when failure rate increases with age. The problem with the normal distribution as an estimator for failure is that it is symmetric while the failure distributions occuring in practical applications are skewed (6).

Theorem 6

If the failure distribution is normal and the repair distribution is lognormal, then the maintenance float factor (f) is given as

$$f = \Phi\{ Z_{\alpha} + \frac{MTTR}{\sigma}\}$$

where $\Phi(.)$ is a Laplace function obtained from the normal distribution table.

$$R(t) = 1 - \Phi\left\{\frac{t - u}{\sigma}\right\} \qquad (39)$$

where u is the MTBF and σ is the standard deviation.

$$\text{Let} \quad Z_{\alpha} = \frac{t - u}{\sigma} \quad \text{and} \quad \Phi\left\{\frac{t - u}{\sigma}\right\} = \alpha \qquad (40)$$

and $\Phi(\cdot)$ is a laplace function evaluated from the normal distribution table, while Z_{α} is the z-score of normal distribution.

It is known that for a specified level of significance α,

$$t = u + Z_{\alpha}\sigma \qquad (41)$$

and

$$R(t) = 1 - \Phi\left\{\frac{t - u}{\sigma}\right\} = 1 - \alpha$$

At t_1,

$$R(t_1) = 1 - \Phi\left\{\frac{t - u}{\sigma}\right\} = 1 - \alpha = \frac{N - 1}{N} \qquad (42)$$

which gives that

$$1 - \alpha = \frac{N - 1}{N}$$

$$\text{and} \qquad \alpha = 1 - \frac{N - 1}{N} \qquad (43)$$

thus Z_{α} is evaluated as $Z_{(1 - \frac{N - 1}{N})}$

$$\text{let } t_n = t_1 + MTTR_1 \qquad (44)$$

and t_1 is obtained from equation (41) as

$$t_1 = u + Z_{\{1 - \frac{N - 1}{N}\}} \quad (45)$$

then $$t_n = u + MTTR + Z_\alpha \sigma \quad (46)$$

and

$$R(t_n) = 1 - \Phi\left\{\frac{Z_\alpha \sigma + u + MTTR - u}{\sigma}\right\} \quad (47)$$

$$= 1 - \Phi\left\{Z_\alpha + \frac{MTTR}{\sigma}\right\} \quad (48)$$

and

$$f = \Phi\left\{Z_\alpha + \frac{MTTR}{\sigma}\right\} \quad (49)$$

with a lognormal repair distribution, equation (49) becomes

$$f = \Phi\left\{Z_\alpha + \frac{m\exp \sigma_1^2/2}{\sigma}\right\}$$

Lognormal failure distribution

The lognormal distribution is often used as a repair distribution (6). Its disfavor as a failure distribution is due to the fact that its failure rate increases at first and eventually decreases to zero (6).

The lognormal has the same parameters as the normal distribution (u, σ). Its failure rate is neither always increasing nor always decreasing but takes different shapes depending on the parameters u and σ (6,26,36).

Theorem 7

If we assume a lognormal failure rate with a lognormal repair distribution then the maintenance float factor (f) can be expressed as

$$f = \Phi\left\{\frac{\ln[\exp(Z_\alpha \sigma + u) + MTTR] - u}{\sigma}\right\}$$

Proof:

$$R(t) = 1 - \Phi\left\{\frac{\ln t - u}{\sigma}\right\} \quad (50)$$

and

$$Z_\alpha = \frac{\ln t - u}{\sigma} \quad (51)$$

then

$$\ln t = Z_\alpha \sigma + u \quad (52)$$

and $t = \exp\{Z_\alpha \sigma + u\}$ (53)

α is computed exactly the same way as in the case of normal distribution.

$$\text{let } t_n = t_1 + MTTR_1 \quad (54)$$

$$= \exp\{Z_\alpha \sigma + u\} + MTTR_1 \quad (55)$$

$$\text{where } \alpha = 1 - \frac{N - 1}{N} \quad (56)$$

Substituting t_n into equation (50) gives the following result for $R(t_n)$.

$$R(t_n) = 1 - \Phi\left\{\frac{[\ln\{\exp(Z_\alpha \sigma + u) + MTTR\}] - u}{\sigma}\right\} \quad (57)$$

and

$$f = \Phi\left\{\frac{[\ln\{\exp(Z_\alpha \sigma + u) + MTTR\}] - u}{\sigma}\right\} \quad (58)$$

The asymptotic MFF for normal and lognormal failure distributions also exist as N approaches infinite size. This property implies that α approaches 0 and Z_α is evaluated as $Z_0 = -3.09$ from the normal distribution table. When this condition exists, the AMFFs for normal and lognormal failure distributions are given respectively as follows:

Normal:

$$f = \Phi\left\{-3.09 + \frac{MTTR}{\sigma}\right\}$$

Lognormal:

$$f = \Phi \left\{ \frac{[\ln \{\exp(-3.09 \quad + u) + MTTR] \quad - u}{\sigma} \right\}$$

Uniform failure distribution

Let X be a continuous random variable assuming all values in the interval [a,b], where both a and b are finite, then the probability density function of X is given by

$$f(x) = 1/(b - a), \qquad a \leq x \leq b,$$
$$= 0, \qquad \text{elsewhere}$$

X is said to be uniformly distributed over the interval [a,b].

Theorem 8

If failure distribution is uniformly distributed, then the maintenance float factor is given as

$$f = \begin{cases} (1 - (N-1)/N)(b - a) + MTTR_1, & 0 \leq MTTR_1 \leq (N-1/N)(b - a) \\ 1, & MTTR_1 \geq (N-1/N)(b - a) \end{cases}$$

Proof

$$R(t) = 1 - \frac{t - a}{b - a}, \quad a \leq t \leq b$$
$$= \frac{b - t}{b - a}, \quad a \leq t \leq b$$

and

$$t = b - (b - a) N_{t_n}/N \qquad (59)$$

and $t_1 = b - (b - a)(N - 1/N)$ (60)

thus

$$t_n = b - (b - a)(N - 1/N) + MTTR_1 \quad (61)$$

and

$$R(t_n) = \frac{(b - a)(N-1/N) - MTTR_1}{b - a} \quad (62)$$

The maintenance float factor is then given as

$$f = \frac{(b - a)\{1 - (N - 1)/N\} + MTTR_1}{b - a} \quad (63)$$

and $0 < MTTR_1 \leq \{(N - 1)/N\}(b - a)$. From equation (63), it is observed that when $MTTR_1 = \{(N - 1)/N\}(b - a)$, the maintenance float factor (f) becomes 1. Thus f is given as stated in Theorem 8.

A summary of the maintenance float factors for each of the failure distributions discussed in this paper is given in Table 1.

Asymptotic Property of f

The maintenance float factor is a function of N, MTBF, and MTTR. As shown in Table 1, the component of the MFF equation containing N is either ln {N/(N - 1)} or 1 - [N - 1]/N depending on the failure distribution. In comparing the MFF derived directly from the exponential distribution and that derived from the gamma distribution when P = 1, we stated that these two factors are approximately equal. It is further shown in Figure 11 which gives the graph of N vs ln [N/(N - 1)] that this natural logarithm approaches a constant value for increasing values of N. This suggests that the limiting property of MFF exists when N is large. This asymptotic property of MFF can be shown using the limit theorem.

Theorem 9

The limiting form of f - the maintenance float factor exists when N approaches an infinite size.

Proof

If we assume that the failure distribution is exponential, from Theorem 1, we obtain that

$$f = 1 - \exp - \{\ln (N/(N - 1)) + (MTTR/MTBF)\} \qquad (64)$$

$$\lim_{N \to \infty} f = \lim_{N \to \infty} [1 - \exp -\{\ln (N/(N - 1)) + (MTTR/MTBF)] \quad (65)$$

Since ln N/(N - 1) is the only component of equation (63) containing the N variable, the other components of f are unaffected by this limit since they are constants.

let $v = N/(N - 1)$

$$\text{and } \lim_{N \to \infty} v = \lim_{N \to \infty} N/(N - 1) \longrightarrow 1 \quad (66)$$

but

$$f = 1 - \exp - \{\ln v + MTTR/MTBF\} \quad (67)$$

and

$$\lim_{N \to \infty} f = 1 - \exp - \{\ln 1 + MTTR/MTBF\} \quad (68)$$

$$= 1 - \exp - [MTTR/MTBF] \quad (69)$$

Thus from this theorem, the limiting function of f will imply that lim ln v approaches 0 and since ln {N/(N - 1)} is approximately equal to 1 - [(N - 1)/N], their limits are also equal. This relationship can also be verified by taking the limit of 1 - [(N - 1)/N] as N approaches infinity.

Using this theorem, the asymptotic maintenance float factors were developed for all the failure distributions treated in this paper. The results are presented in Table 2.

APPLICATION OF MARGINAL COST ANALYSIS TO THE THEORETICAL MODELS

In this section we wish to develop a bound over which our theoretical models are applicable. We start by making some assumptions about the performance measures. The major performance measure in this paper is the average equipment utilization. This measure is obtained through simulation. We denote the average utilization of a unit as ρ. The mean time to repair (MTTR) and the mean time between failure (MTBF) are assumed known. The distribution of failure and the distribution of repair are also known. Using our theoretical results summarized in Tables 1 and 2, we obtain the maintenance float factors (MFF) for any fixed N units initially in operation. The MFF is used to determine the total float (F) which is used as our initial standby. This number of standbys is used with the N, MTBF for a given failure distribution and MTTR for a given repair distribution in a simulation experiment. The simulation gives the average utilization of a unit giving this initial total float. The marginal approach is introduced by incrementing our standby obtained as F. The standby is incremented and the simulation ran again to obtain the new average utilization. This process may be continued until an average utilization of 0.999 is obtained. It should be noted that the average utilization of a unit increases as the number of standby is increased provided the other factors remain unchanged. The results obtained through this approach is used to develop a marginal cost ratio test from which we make decision about the applicability of our models.

In order to develop this cost ratio test, we make the following assumptions:

1. There is a cost associated with leasing or renting a unit on standby. This cost includes the storage cost and it is denoted as Cs+h.

2. When a unit is down, there is a penalty cost due to the unit's downtime. This penalty cost is denoted as Cd = the cost of lost production and it is related to the length of downtime.

3. Let S_i refer to the total float or standby maintained and S_i is ranked in increasing order from i = 1,2, ... , n and S_n is the total float that will give the highest average utilization (ie utilization of 99.9%).

4. We use the S_i obtained from our theoretical results as the starting point and we increment the S_i value in order to achieve a higher average utiliztion. This serves two major purposes:

(1) It can be used to develop marginal cost ratios and the decision maker can determine the required number of standbys depending on the average utilization he wants to achieve.

(2) We use this to show that the cost ratio developed is a

non decreasing function. On the basis of this property, we can make a general statement about the applicability of our theoretical results.

5. Since $0 \leq f \leq 1$, the total float $F = fN \leq N$.

6. The cost of repair is assumed fixed.

Using these assumptions, we obtain the expected cost functions for any given total float S_i as follows:

$E(TC_i)$ = the expected total cost for the maintenance float given that S_i is the initial total float.

ρ_i = the average utilization for the total float S_i.

Cd = the cost of equipment downtime or the cost of lost production.

Cs+h = the cost of leasing or renting a standby unit.

i+1 = the increment of the initial total float.

N = the number of units initially in operation.

$$E(TC_i) = Cs{+}hS_i + Cd(1 - \rho_i)N \tag{70}$$

$$E(TC_{i+1}) = Cs{+}hS_{i+1} + Cd(1 - \rho_{i+1})N \tag{71}$$

and N is fixed for all i. Equating the two cost functions given in equations (70) and (71), we obtain

$$Cs{+}hS_i + Cd(1 - \rho_i)N = Cs{+}hS_{i+1} + Cd(1 - \rho_{i+1})N \tag{72}$$

and

$$Cd[\rho_{i+1} - \rho_i]N = Cs+h[S_{i+1} - S_i] \quad (73)$$

Thus from equation (73),

$$Cd/Cs+h = \frac{S_{i+1} - S_i}{N(\rho_{i+1} - \rho_i)} . \quad (74)$$

This ratio is used as a tradeoff analysis between the cost of lost production and the cost of leasing or renting a unit. The ratio given in equation (74) is a non decreasing function. This property is shown in Table 3, and we further explain this property below.

From our simulation results, it is shown that the ratio

$\frac{S_{i+1} - S_i}{N(\rho_{i+1} - \rho_i)}$ developed in equation (74) is non decreasing.

This can be explained as follows: The change in S is fixed for any fixed value of N but the change in the average utilization is increasing at a decreasing rate. Thus given that the change in S and N are fixed, then this ratio is a non decreasing function.

CHAPTER 5

RESULTS AND DISCUSSIONS

The results obtained for this paper are listed in the appendix. In Table 1 we presented the maintenance float factors for the following failure distributions: exponential, erlang-2, gamma, normal, lognormal and uniform distributions. As presented in this table, the repair distribution can be any arbitrary distribution. It is shown in Corollary 1 that the structure of the formulas obtained for the float factors is actually dependent on the failure rate distribution. Thus given an appropriate repair distribution, the structure of the formula does not change.

In Theorem 9, we proved that the limiting form of the maintenance float factors exist when N approaches an infinite size. Due to this limiting property, the asymptotic maintenance float factors for the mentioned failure distributions were obtained and presented in Table 2. From the structure of these formulas, we can conclude that if the number of units initially in operation is very large, the maintenance float factor will depend only on the failure and repair distributions and not on the number of units initailly in operation. Hence we need only the mean time to repair and the mean time between failure in order to calculate the maintenance float factors. An exception to this is the case of constant failure rate which is a special form of the gamma distribution when the number of phases p approaches infinity. It is shown in Theorem 5 that the maintenance float factor for a gamma failure distribution will approach 0 when p approaches infinity. This implies that when the failure distibution is constant or degenerate, it is not necessary to maintain a float. An intuitive arguement to this follows Theorem 5. In using our model if the failure distribution is constant, then the total float (F) will always approach 0 since F = fN, and f is approaching 0.

We have further developed ranges over which these models are applicable. The ranges were developed under the assumption that there are only two variable costs namely the cost of lost production (Cd) and the cost of leasing or renting a unit on standby or in float plus the cost of holding or storage (Cs+h). The cost of loss production is a function of the average idle time per unit. Using a marginal cost analysis approach we developed ranges or conditions over which our models are applicable.

In Table 3, we showed that the cost ratio developed as a function of average utilization is a non decreasing function of S (the number maintained in standby status). Using this property, we developed the cost ratio for the different standbys while using the total float computed from our model as the initial value (see Table 5). We increment this initial value until a 99 % average utilization was achieved.

A decision maker could therefore use this cost ratio to decide the appropriate size of float to maintain (see Table 3). Our model is shown to be efficient when the cost of equipment downtime or the cost of lost production is greater than the cost of leasing or renting a standby for the example shown. These models also apply when the inverse is true. For the distributions, we achieved the following average utilizations; 85 % for exponential failure distribution, 90 % for the erlang-2 failure, and 95 % for the uniform failure when N = 50. The average utilization for other values of N are given in Tables 4.1, 4.2, and 4.3. As shown in Table 5, if the failure distribution is exponential, N = 50, MTTR = 35 mins. and MTBF = 40 mins., and it is known that Cd/Cs+h is at least 2.041, our model can be used in that sense. If further accuracy is desired in estimating the total float, the type of analysis given in Table 3 can be used. Our model is still helpful as a starting point rather than the trial and error method often used in estimating the number of standbys. In Table 5, we also present the cost ratio for N = 75, 100, 150, and 200 for the failure distributions considered in this paper.

Tables 4.1, 4.2, & 4.3 give the average utilizations obtained for erlang-2, exponential and uniform failure distributions after 25 replications. Confidence intervals were constructed for these utilizations at both 95 % and 99 %

level of significance. These average utilizations were used in obtaining the cost ratios given in Table 5. The average utilization for normal and lognormal failure distributions are given in Table 9.

This experiment was performed for each distribution under the assumption of equal means. Given that most other distributions can be approximated to normal distribution, we used the maximum likelihood estimators of their means. For normal and lognormal distributions, a standard deviation of 10 minutes was used in both the graphs and the simulation modeling. The actual standard deviation with the means used is approximately 5 minutes but this also gives the same average utilization. A likely explanation to these high utilizations of uniform and normal failure distributions may be due to the fact that these distributions give high float values thus leading to higher total float and the average utilization increases as the number of total float is increased. The initial total float given by any of the distributions considered under the same mean is higher than the initial float given by the gamma, erlang-2 and lognormal distribution. With erlang-2 case, failure process goes through two phases thus taking twice the time it takes a unit to fail under exponential distribution. Since repair distribution is exponential, the service rate is much faster than the failure rate for the erlang-2 case and this will also contribute to its high average utilizations.

Furthermore, this implies that less number of standbys will be needed. This result is further shown in Figure 9 and Table 6.2. As the number of phases increases for the gamma distribution, the float factor approaches 0 (Theorem 5). With the gamma failure distribution, we expect to achieve a 99 % average utilization. As mentioned in the case of erlang-2, MTBF is much greater than the MTTR.

Figures 1,3,5,6,8 gave graphical illustrations of the maintenance float models given in Table 1 for the exponential, erlang-2, lognormal, normal, and gamma failure distributions respectively. In each of these cases, the maintenance float factor is plotted as the vertical axis while the mean time to repair is on the horizontal axis. The curves for N = 50, 150, and 300 were shown. In Figure 1, it was observed that for N = 150 and N = 300, the float factors seemed to converge as the mean time to repair increased. This convergence occured for the exponential case (Figure 1) when MTTR $\geq$ 5.5 hours. In Figure 10, we showed the limiting property of the float factors by plotting the ln (N/N-1) vs N. This ln is approximately equal to 1 - (N-1/N). This limiting property is further extended in Figures 2, 4, 7, and 12 where we plotted the float factors for the failure distributions against N. In Figure 9 we plotted MFF against P for the gamma distribution. We observed that the curves of MFF vs N approached horizontal asymptotes. This asymptotic property gave rise to Table 2. The MFF increases with increasing values of MTTR and decreases with increasing values of N.

This discussion would not be complete without the mention of the sensitivity of our assumption that the waiting time for repair is negligible. The curve given in Figure 11 shows that for this assumption to hold, we have to maintain large service channels or service rate has to be very high. Either case entails a high cost for repair. However, in some operations where this model may be useful minimization of repair cost may not always be of higher priority. It may therefore be necessary to assume that this cost is also small compared to the cost of lost production. An example where this situation may occur may be in life sustaining systems, defence related systems and even some flow shop operations.

In conclusion, our models tend to represent a better approach to estimating total float. For the values of MTTR and MTBF used as an example, it was shown that the cost of lost production is greater than the cost of standby plus storage in all cases. Since the MFF is a function of MTTR, MTBF and N, this result can also be reversed when these values are changed.

CHAPTER 6

SUMMARY, CONCLUSION AND FUTURE RESEARCH

The need to undertake the present study was demonstrated in Chapter 1. We defined the scope and objective of this study and a literature review of the relevant literature in maintenance policies and modeling was presented. This review showed that while there have been many maintenance models developed, little attention has been paid to maintenance float modeling. This does not negate the importance of maintenance float modeling but rather leaves a complex problem unanswered. As we pointed out in the Problem definition section, the problem with modeling a float system is that unitary approach taken in mathematical modeling can not be applied. Many of the mathematical modeling of float like systems mentioned in the same section make a number of restrictive assumptions i.e. two unit system where 1 unit is on standby and 1 unit is in operation. Our consideration has been with floats of larger sizes.

In Chapter 4 we developed the maintenance float factors for the following failure distributions: exponential, erlang-2, normal, lognormal, gamma and uniform distributions. We further showed that the structure of the formulas developed does not depend on the distribution of repair but rather depends on the failure distribution. It was also shown that the asymptotic case of the maintenance float factor exist when N approaches infinity. The graphical illustrations to these float factors were obtained and presented in the appendix. This is applied to the results obtained from our simulation. It is shown that our models give good estimates of the total float when the cost of lost production is higher than the cost of renting or leasing a standby unit for the values of MTBF and MTTR used.

Chapter 5 gives an analysis of the results obtained from both our analytical derivations, and the simulation. In this chapter it was shown that when the failure distribution is gamma, we can obtain the case of degenerate or constant distribution when p - the number of phases approach infinity. From our derivations for gamma failure distribution, it was also shown that the float for gamma gives good approximation of the float factor for the exponential distribution when p = 1. We further derived the erlang-2 failure case directly from the MFF of gamma distribution.

The asymptotic property of the maintenance float factors were graphically shown from the curves obtained when the float factors obtained for each failure distribution is plotted over N. It is shown that the maintenance float factor approaches a horizontal asymptote when N is large. It is further evident that the MFF increases with increasing values of MTTR and decreases with increasing values of N.

The MFF for normal and lognormal failure distributions were derived using the laplace function of normal distribution. For the exponential failure distribution, it is noticed that as N increases, the curves converge for increasing values of the MTTR.

The sensitivity of the assumption that waiting time for repair is negligible was tested through simulation. The result showed that the service channel has to be large in order to satisfy this assumption. It is therefore necessary to assume that the cost of unit downtime (ie cost of lost production) is much greater than the cost of repair. This assumption will make repair cost negligible and the assumption less sensitive. However, this result is obvious due to the assumption of no waiting time for repair.

Marginal cost analysis was used to develop a cost ratio. In developing this cost ratio it was assumed that there are only two variable costs (cost of equipment downtime, and the cost of leasing or renting a standby plus the storage cost). The other costs are assumed fixed. From the ratios developed for the values of MTBF and MTTR used, we were able to state conditions under which our models can be used. It is shown in this example that if the failure distribution is either uniform or normal, that Cd/Cs+h is very large compared to the other distributions. If however the failure distribution is either erlang, exponential or lognormal the ranges given for Cd/Cs+h for the different N values should be used. If further improvements in the number of total float is needed, the marginal analysis technique should be used. Our models are still useful in offering the starting point for the total float rather than the trial and error method used in the classical models. It is also shown that the float factors obtained for the gamma distribution is always lower than that obtained from the other distributions when equal means are used. In the degenerate case, it was shown through the limit theorem that the float factor is approaching 0 thus implying that the total float F is also approaching 0. We assume that for this distribution, F = 1.

We have extended the model introduced by Lowe and Lewis (44) to include other major failure distributions. We further showed that the asymptotic case of MFF exists and thus leading us to conclude that for large N, the maintenance float factor depends only on the failure and repair distribution. This property significantly reduces the formulas as shown in the case of gamma and erlang-2 failure distributions and thus makes it more attractive to use by reducing the computational efforts involved. In fact these sets of models can be used as approximate models even when N is not large enough.

Our model for gamma was shown to be an excellent approximation of the exponential case when p = 1 thus leading us to say that ln (N/N-1) = 1 -(N-1/N). We also conclude from the graphical illustrations that the plot of MFF vs N shows the existence of horizontal asymptote; that the plot of MFF vs MTTR for lognormal and gamma failures give a linear relationship; that the curves of MFF vs MTTR for exponential failure tend to converge for increasing values of MTTR; that the MFF increases with increasing values of MTTR and decreases with increasing values of N; and that the MFF for the gamma failure distribution when p is approaching infinity approaches 0.

From this study, it is concluded that these models can be applied for all cases when the objective is to maximize reliability. Our example using the ratios developed tend to show that the cost of lost production is always higher than the cost of standby plus storage. However, this result is due to the values used for the MTTR and MTBF and does not restrict the applicability of these models.

As is apparent from this paper, we restricted our scope to the consideration of continuous failure distributions. However, in practice some of the common failure distributions are discrete such as poisson, geometric and binomial (6). This study can be extended to consider these failure distributions. From our Corollary 1, the structure of our models should not be affected when a discrete failure distribution is used. In using the discrete failure distributions we can anticipate more than one unit failing at a time. Another extension of this paper may be to relax the assumption that the waiting time for repair is negligible since this assumption will be highly sensitive if the cost of repair is high.

TABLE 1

Failure distributions	Maintenance float factors
Exponential	$1 - \exp - \left\{ \ln\left(\frac{N}{N-1}\right) + \frac{MTTR}{MTBF} \right\}$
Erlang-2	$1 - \exp - \left\{ \sqrt{2\left(1 - \frac{N-1}{N}\right)} + \lambda MTTR \right\} \times \left\{ \sqrt{2\left(1 - \frac{N-1}{N}\right)} + (1 + \lambda MTTR) \right\}$
Gamma	$1 - \exp - \left\{ \left[p! \left(1 - \frac{N-1}{N}\right)\right]^{1/p} + \lambda MTTR \right\} \times \sum_{K=0}^{p-1} \frac{\left\{\left[p!\left(1-\frac{N-1}{N}\right)\right]^{1/p} + \lambda MTTR\right\}^K}{K!}$
Normal	$\Phi\left\{ Z_\alpha + \frac{MTTR}{\sigma}\right\}$
Lognormal	$\Phi\left\{ \frac{\ln\{\exp[Z_\alpha \sigma + u] + MTTR\} - u}{\sigma}\right\}$
Uniform	$\begin{cases} \left[\left(1 - \frac{N-1}{N}\right)(b-a) + MTTR\right]/(b-a) & 0 < MTTR \le \frac{N-1}{N}(b-a) \\ 1 & MTTR \ge \frac{N-1}{N}(b-a) \end{cases}$

TABLE 2

Asymptotic Maintenance float factors

Failure distributions	
Exponential	$1 - \exp - \left\{ \frac{MTTR}{MTBF} \right\}$
Erlang-2	$1 - \left\{ \left[\exp - (\lambda MTTR) \right] \times (1 + \lambda MTTR) \right\}$
Degenerate	$\longrightarrow 0$
Gamma	$1 - \exp - (\lambda MTTR) \times \sum_{K=0}^{p-1} \frac{(\lambda MTTR)^K}{K!}$
Lognormal	$\Phi \left\{ \frac{\ln\{\exp(-3.09\sigma + \mu) + MTTR\} - \mu}{\sigma} \right\}$
Normal	$\Phi \left\{ -3.09 + \frac{MTTR}{\sigma} \right\}$
Uniform	$\begin{cases} \frac{MTTR}{b-a}, & 0 < MTTR \leq b-a \\ 1 & MTTR \geq b-a \end{cases}$

Table 3

Increasing Property of Cd/Cs+h
Exponential failure distribution and
Exponential repair distribution:
N = 100; MTBF = 45 mins.; MTTR = 40 mins.

S (Standby or float)	Average Utilization	Cd/Cs+h
60	0.842	
		1.613
65	0.873	
		1.786
70	0.901	
		2.000
75	0.926	
		2.174
80	0.949	
		3.571
90	0.977	
		3.846
95	0.990	
		8.333
100	0.996	

* S_1 = 60 is obtained as f.N where f is the MFF obtained directly from the formula given on Table 1 for exponential distribution.

Table 4.1

Average Utilization
Erlang-2 failure; Exponential Repair
MTBF = 40 mins. MTTR = 35 mins.

Confidence Interval

			95%		99%	
N	S	AVG.	LL	UL	LL	UL
50	15	0.905	0.788	1	0.751	1
50	20	0.960	0.882	1	0.857	1
75	21	0.893	0.769	1	0.731	1
75	26	0.936	0.838	1	0.808	1
100	27	0.887	0.760	1	0.721	1
100	32	0.921	0.813	1	0.779	1
150	40	0.885	0.757	1	0.718	1
150	45	0.908	0.792	1	0.756	1
200	51	0.877	0.746	1	0.705	1
200	56	0.894	0.771	1	0.733	1

Average Utilization
Exponential failure; Exponential repair.
MTBF = 40 mins. MTTR = 35 mins.

Confidence Interval

N	S	AVG.	95% LL	95% UL	99% LL	99% UL
50	30	0.853	0.714	0.992	0.671	1
50	35	0.902	0.785	1	0.749	1
75	45	0.853	0.714	1	0.714	1
75	50	0.887	0.763	1	0.724	1
100	59	0.848	0.707	0.989	0.663	1
100	64	0.874	0.744	1	0.703	1
150	88	0.847	0.706	0.988	0.662	1
150	93	0.864	0.730	0.998	0.688	1
200	117	0.846	0.705	0.987	0.660	1
200	122	0.868	0.735	1	0.694	1

Average Utilization
Uniform failure distribution;
Exponential repair distribution.
MTBF = 40 mins. MTTR = 35 mins.

Confidence Interval

N	S	AVG.	95% LL	95% UL	99% LL	99% UL
50	45	0.981	0.927	1	0.911	1
50	50	0.995	0.967	1	0.959	1
75	67	0.984	0.935	1	0.920	1
75	72	0.995	0.967	1	0.959	1
100	86	0.979	0.923	1	0.905	1
100	91	0.991	0.954	1	0.942	1
150	133	0.989	0.948	1	0.935	1
150	138	0.995	0.967	1	0.959	1
200	176	0.990	0.951	1	0.939	1
200	181	0.994	0.964	1	0.954	1

Table 5

Cost Ratios

N	Failure Distributions Exponential	Erlang-2	Uniform	Normal	Lognormal
50	2.041	1.818	7.143	11.111	1.639
75	1.961	1.550	6.061	7.407	1.626
100	1.923	1.471	4.167	5.000	1.667
150	1.961	1.449	5.556	3.333	1.587
200	1.136	1.471	6.250		1.667

* S is obtained for each N by assuming that the MTBF = 40 minutes and the MTTR = 35 minutes and using the MFF formulas given in Table 1. Note that S = f.N.

TABLE 6.1

Maintenance float factors
Erlang-2 failure Distribution and
Exponential Repair Distribution:
MTBF = 4 hrs

MTTR	N					
	10	50	150	300	500	1000
3.5	0.381	0.292	0.261	0.248	0.242	0.235
4.0	0.424	0.337	0.307	0.294	0.287	0.281
4.5	0.466	0.382	0.352	0.340	0.333	0.326
5.0	0.506	0.425	0.396	0.384	0.378	0.371
5.5	0.544	0.467	0.439	0.428	0.421	0.415
6.0	0.580	0.507	0.480	0.469	0.463	0.457
6.5	0.613	0.545	0.519	0.509	0.503	0.497

Table 6.2

Maintenance float factors
Gamma failure distribution;
Exponential Repair distribution;
MTBF = 4 hours. MTTR = 3.5 hours

P	N		
	50	150	300
1	0.591	0.586	0.585
2	0.292	0.261	0.248
3	0.159	0.124	0.100
4	0.094	0.067	0.054
5	0.059	0.039	0.030
6	0.038	0.024	0.017
7	0.025	0.015	0.011
8	0.016	0.010	0.007
9	0.011	0.006	0.004
10	0.007	0.004	0.003
11	0.005	0.003	0.002
12	0.003	0.002	0.001

Table 6.3

Maintenance float factors
Exponential failure distribution;
Exponential repair distribution.
MTBF = 4 hrs.

MTTR	N					
	10	50	150	300	500	1000
3.5	0.625	0.591	0.586	0.585	0.584	0.584
4.0	0.669	0.639	0.635	0.633	0.633	0.632
4.5	0.708	0.682	0.678	0.676	0.676	0.676
5.0	0.742	0.719	0.715	0.714	0.714	0.714
5.5	0.772	0.752	0.749	0.748	0.748	0.747
6.0	0.799	0.781	0.778	0.778	0.777	0.777
6.5	0.823	0.807	0.804	0.804	0.803	0.803

Table 6.4

Maintenance float factors
Lognormal failure distribution;
Exponential repair distribution;
MTBF = 4 hours.

MTTR	N					
	50	150	300	500	1000	INF
3.5	0.0495	0.0281	0.0207	0.0150	0.0146	0.0170
4.0	0.0548	0.0322	0.0228	0.0202	0.0179	0.0166
4.5	0.0606	0.0367	0.0281	0.0238	0.0212	0.0197
5.0	0.0655	0.0409	0.0322	0.0281	0.0250	0.0233
5.5	0.0708	0.0455	0.0367	0.0322	0.0287	0.0274
6.0	0.0764	0.0505	0.0409	0.0359	0.0329	0.0314
6.5	0.0808	0.0548	0.0455	0.0409	0.0375	0.0352

Table 7

Asymptotic Maintenance float factors for the Gamma failure distribution.

MTBF = 4 hours

	P				
MTTR	1	2	3	4	5
4.00	0.632	0.264	0.123	0.082	0.067
4.50	0.675	0.310	0.148	0.093	0.072
5.00	0.713	0.355	0.174	0.106	0.077
5.50	0.747	0.400	0.202	0.120	0.082
6.00	0.776	0.442	0.231	0.135	0.087
6.50	0.806	0.483	0.262	0.151	0.094

Table 8

Maintenance float factors

Normal failure distribution
Exponential repair distribution
Standard deviation = 1hr

MTBF = 4 hours

	N					
MTTR	50	150	300	500	1000	INFINITY
3.50	0.927	0.846	0.785	0.732	0.688	0.659
4.00	0.974	0.936	0.902	0.869	0.839	0.819
4.50	0.993	0.978	0.963	0.947	0.932	0.921
5.00	0.998	0.994	0.989	0.983	0.977	0.972
5.50	1.000	0.999	0.997	0.996	0.994	0.992
6.00	1.000	1.000	1.000	1.000	0.999	0.998
6.50	1.000	1.000	1.000	1.000	1.000	1.000

TABLE 9

Average Utilization for normal & Lognormal failure distributions:

	NORMAL		LOGNORMAL	
N	Standbys	Avg. Util.	Standbys	Avg. Util.
50	47	0.988	3	0.658
	52	0.997	8	0.719
75	68	0.987	3	0.645
	73	0.996	8	0.686
100	88	0.983	4	0.645
	93	0.993	9	0.675
150	127	0.976	5	0.640
	132	0.986	10	0.661
200			5	0.635
			10	0.650

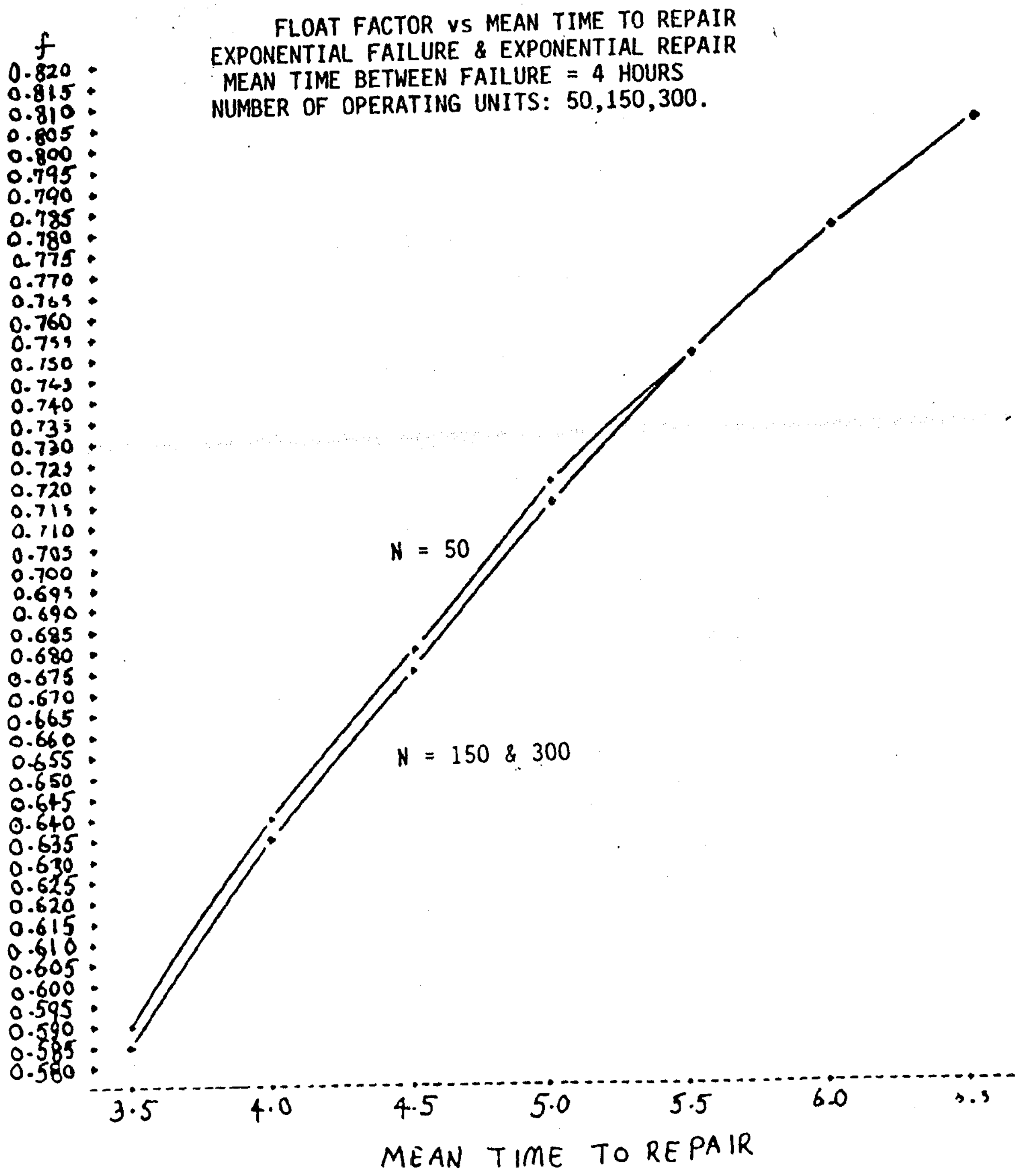

FLOAT FACTOR vs MEAN TIME TO REPAIR
EXPONENTIAL FAILURE & EXPONENTIAL REPAIR
MEAN TIME BETWEEN FAILURE = 4 HOURS
NUMBER OF OPERATING UNITS: 50,150,300.
f
0.820
0.815
0.810
0.805
0.800
0.795
0.790
0.785
0.780
0.775
0.770
0.765
0.760
0.755
0.750
0.745
0.740
0.735
0.730
0.725
0.720
0.715
0.710
0.705
0.700
0.695
0.690
0.685
0.680
0.675
0.670
0.665
0.660
0.655
0.650
0.645
0.640
0.635
0.630
0.625
0.620
0.615
0.610
0.605
0.600
0.595
0.590
0.585
0.580
N = 50
N = 150 & 300
3.5
4.0
4.5
5.0
5.5
6.0
MEAN TIME TO REPAIR

Figure 1

FLOAT FACTOR VS NUMBER OF OPERATING UNITS
EXPONENTIAL FAILURE AND EXPONENTIAL REPAIR
MEAN TIME BETWEEN FAILURE = 4 HOURS

PLOT OF S1*UNITS SYMBOL USED IS +
PLOT OF S2*UNITS SYMBOL USED IS +
PLOT OF S3*UNITS SYMBOL USED IS +
PLOT OF S4*UNITS SYMBOL USED IS +
PLOT OF S5*UNITS SYMBOL USED IS +
PLOT OF S6*UNITS SYMBOL USED IS +
PLOT OF S7*UNITS SYMBOL USED IS +

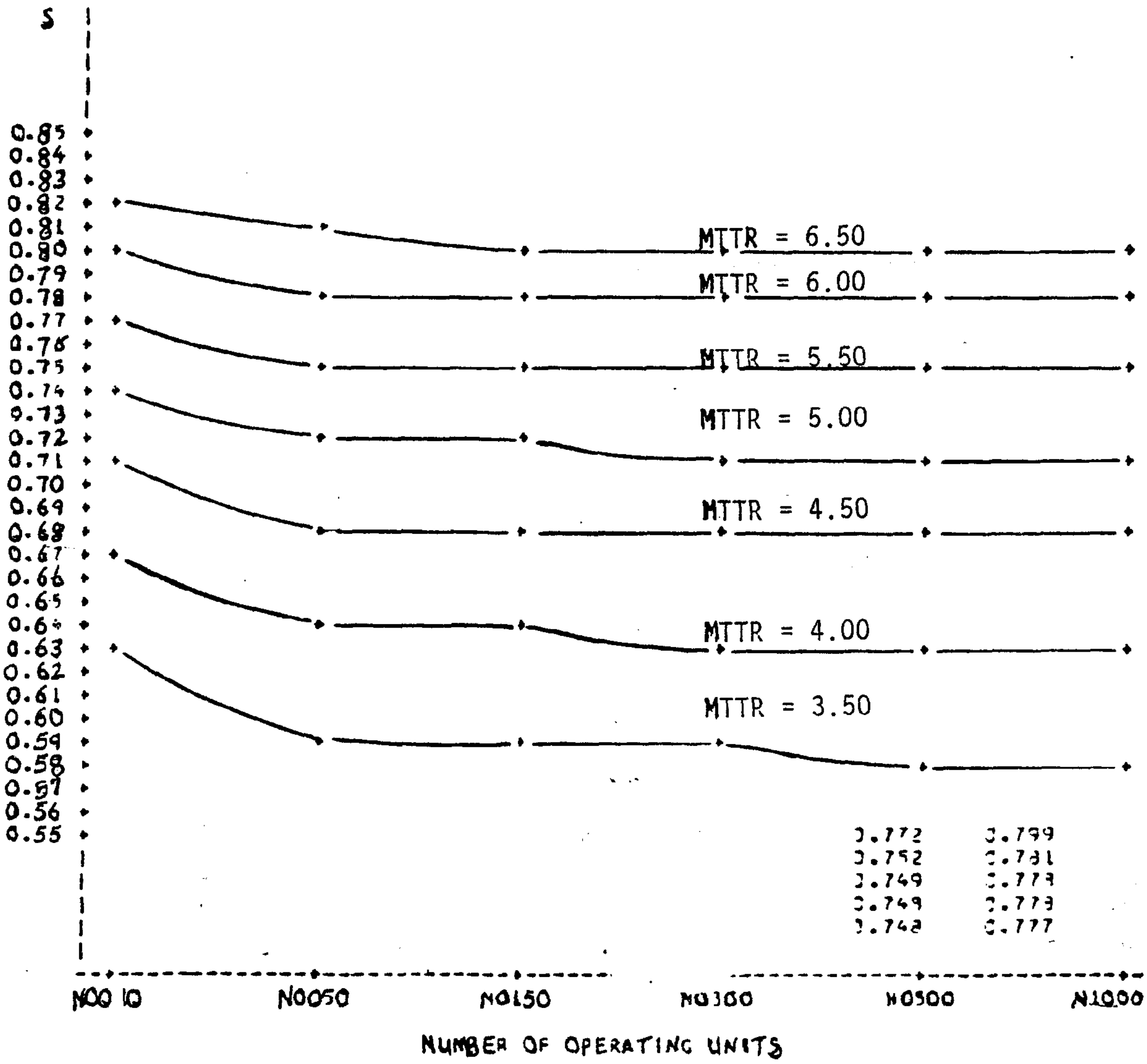


Figure 2

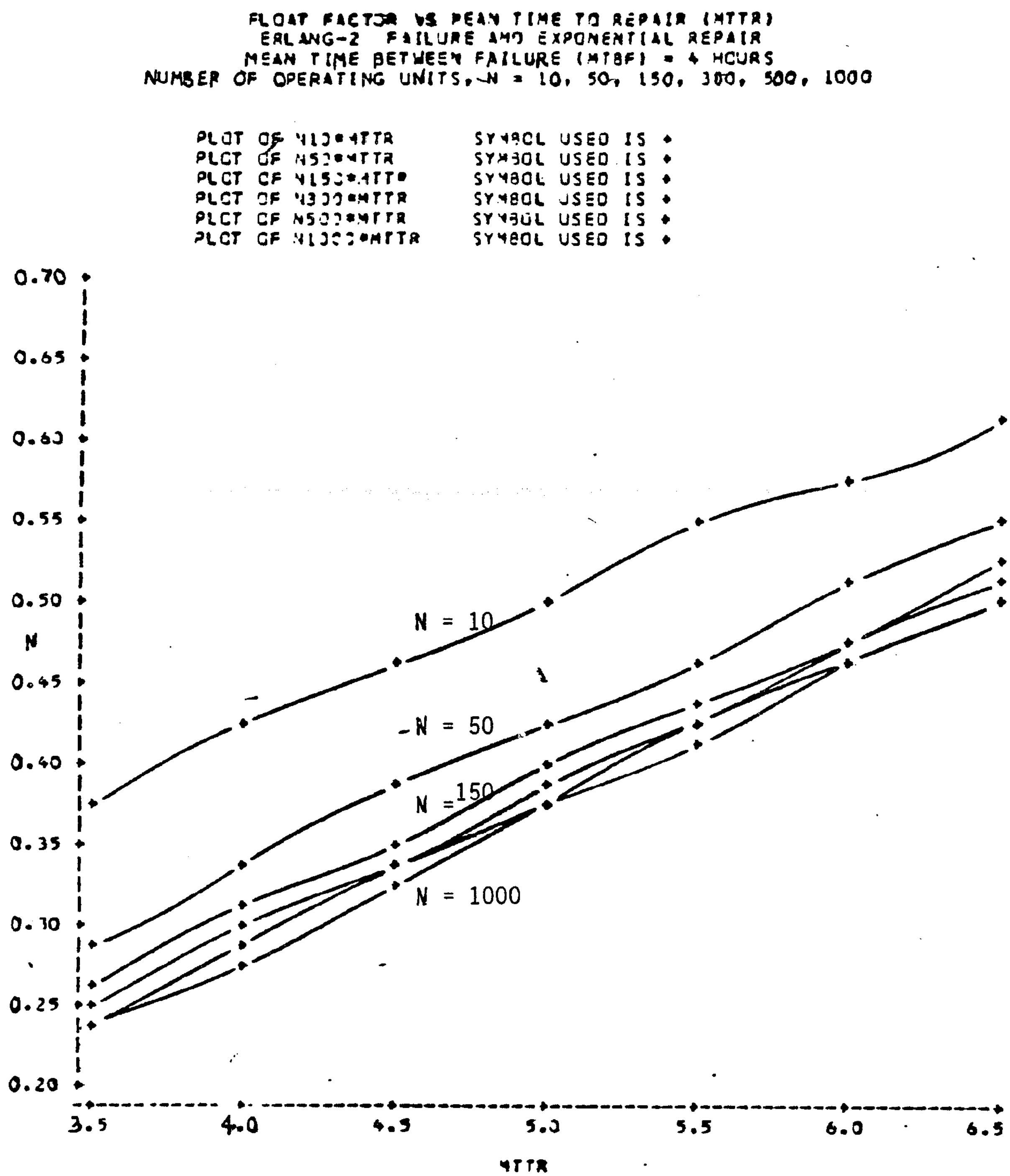

FLOAT FACTOR VS MEAN TIME TO REPAIR (MTTR)
ERLANG-2 FAILURE AND EXPONENTIAL REPAIR
MEAN TIME BETWEEN FAILURE (MTBF) = 4 HOURS
NUMBER OF OPERATING UNITS, N = 10, 50, 150, 300, 500, 1000
PLOT OF N10*MTTR SYMBOL USED IS +
PLOT OF N50*MTTR SYMBOL USED IS +
PLOT OF N150*MTTR SYMBOL USED IS +
PLOT OF N300*MTTR SYMBOL USED IS +
PLOT OF N500*MTTR SYMBOL USED IS +
PLOT OF N1000*MTTR SYMBOL USED IS +
0.70
0.65
0.60
0.55
0.50
N
0.45
0.40
0.35
0.30
0.25
0.20
N = 10
N = 50
N = 150
N = 1000
3.5
4.0
4.5
5.0
5.5
6.0
6.5
MTTR

Figure 3

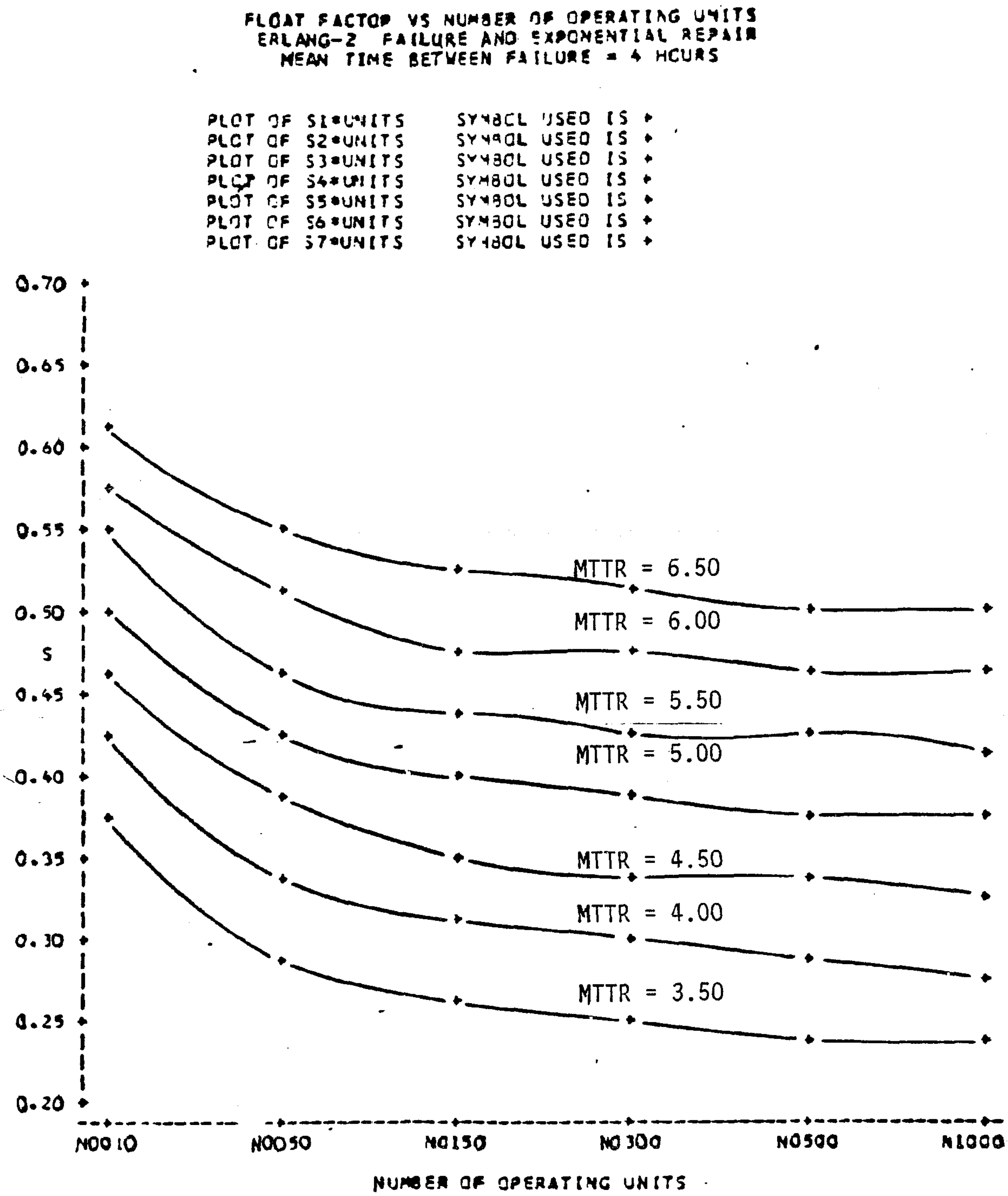

FLOAT FACTOR VS NUMBER OF OPERATING UNITS
ERLANG-2 FAILURE AND EXPONENTIAL REPAIR
MEAN TIME BETWEEN FAILURE = 4 HOURS
PLOT OF S1*UNITS SYMBOL USED IS +
PLOT OF S2*UNITS SYMBOL USED IS +
PLOT OF S3*UNITS SYMBOL USED IS +
PLOT OF S4*UNITS SYMBOL USED IS +
PLOT OF S5*UNITS SYMBOL USED IS +
PLOT OF S6*UNITS SYMBOL USED IS +
PLOT OF S7*UNITS SYMBOL USED IS +
0.70
0.65
0.60
0.55
0.50
S
0.45
0.40
0.35
0.30
0.25
0.20
MTTR = 6.50
MTTR = 6.00
MTTR = 5.50
MTTR = 5.00
MTTR = 4.50
MTTR = 4.00
MTTR = 3.50
N0010
N0050
N0150
N0300
N0500
N1000
NUMBER OF OPERATING UNITS

Figure 4

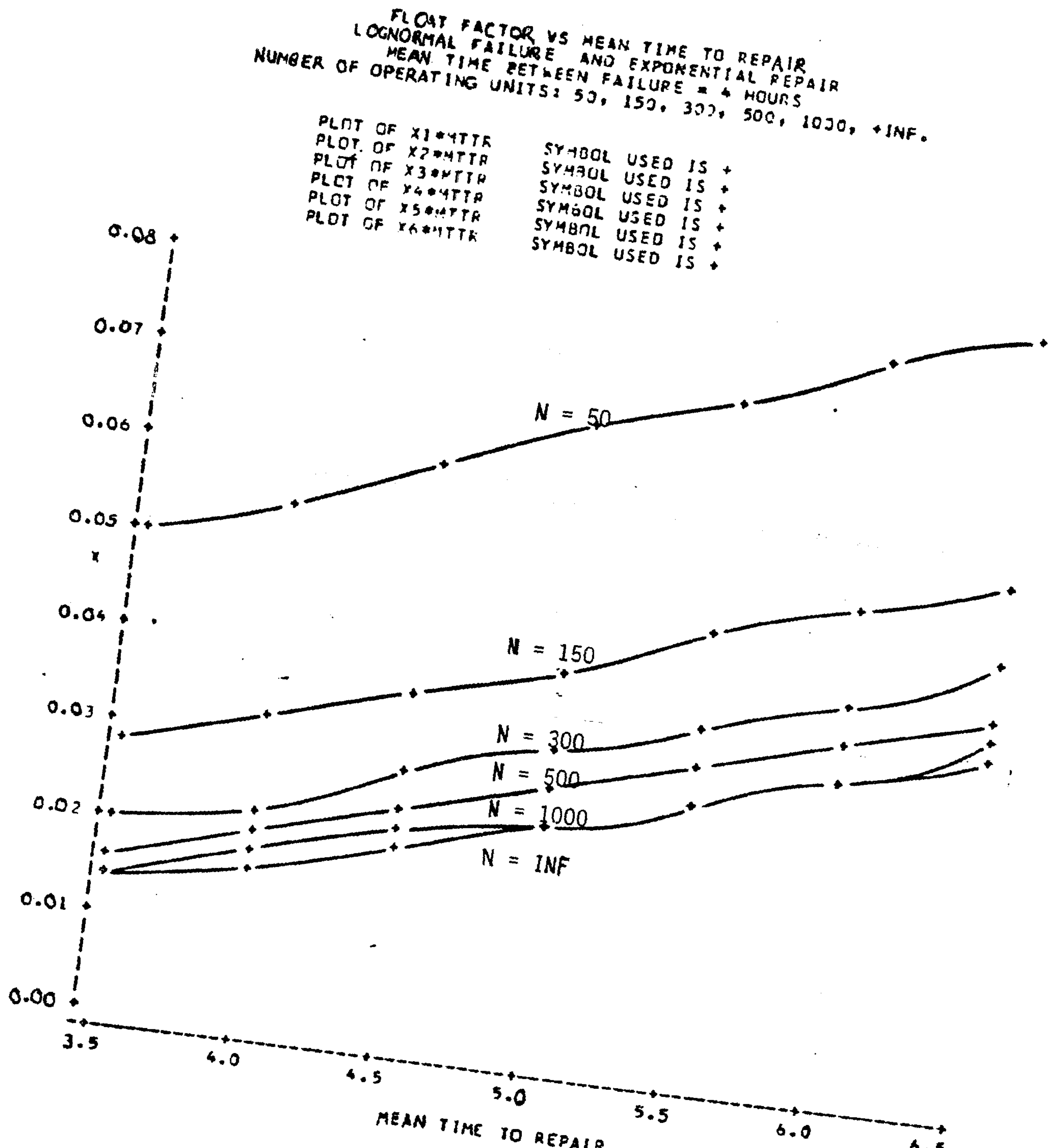

FLOAT FACTOR VS MEAN TIME TO REPAIR
LOGNORMAL FAILURE AND EXPONENTIAL REPAIR
MEAN TIME BETWEEN FAILURE = 4 HOURS
NUMBER OF OPERATING UNITS: 50, 150, 300, 500, 1000, +INF.
PLOT OF X1*MTTR SYMBOL USED IS +
PLOT OF X2*MTTR SYMBOL USED IS +
PLOT OF X3*MTTR SYMBOL USED IS +
PLOT OF X4*MTTR SYMBOL USED IS +
PLOT OF X5*MTTR SYMBOL USED IS +
PLOT OF X6*MTTR SYMBOL USED IS +
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0.00
N = 50
N = 150
N = 300
N = 500
N = 1000
N = INF
3.5
4.0
4.5
5.0
5.5
6.0
6.5
MEAN TIME TO REPAIR

Figure 5

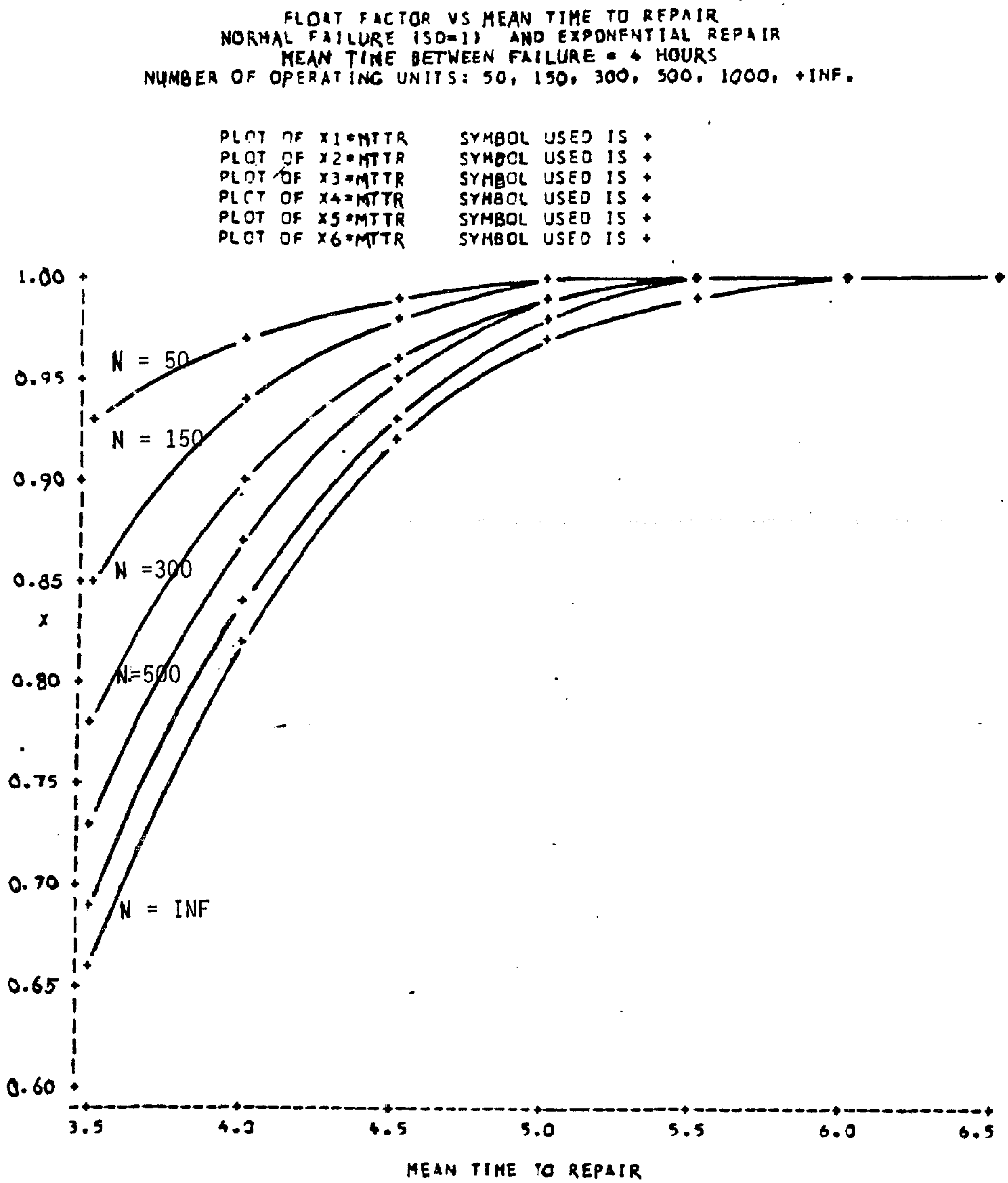

FLOAT FACTOR VS MEAN TIME TO REPAIR
NORMAL FAILURE (SD=1) AND EXPONENTIAL REPAIR
MEAN TIME BETWEEN FAILURE = 4 HOURS
NUMBER OF OPERATING UNITS: 50, 150, 300, 500, 1000, +INF.
PLOT OF X1*MTTR SYMBOL USED IS +
PLOT OF X2*MTTR SYMBOL USED IS +
PLOT OF X3*MTTR SYMBOL USED IS +
PLOT OF X4*MTTR SYMBOL USED IS +
PLOT OF X5*MTTR SYMBOL USED IS +
PLOT OF X6*MTTR SYMBOL USED IS +
1.00
0.95
0.90
0.85
X
0.80
0.75
0.70
0.65
0.60
N = 50
N = 150
N = 300
N = 500
N = INF
3.5
4.0
4.5
5.0
5.5
6.0
6.5
MEAN TIME TO REPAIR

Figure 6

FLOAT FACTOR VS NUMBER OF OPERATING UNITS
NORMAL FAILURE (SD=1) AND EXPONENTIAL REPAIR
MEAN TIME BETWEEN FAILURE = 4 HOURS
NUMBER OF OPERATING UNITS: 50, 150, 300, 500, 1000, +INF.

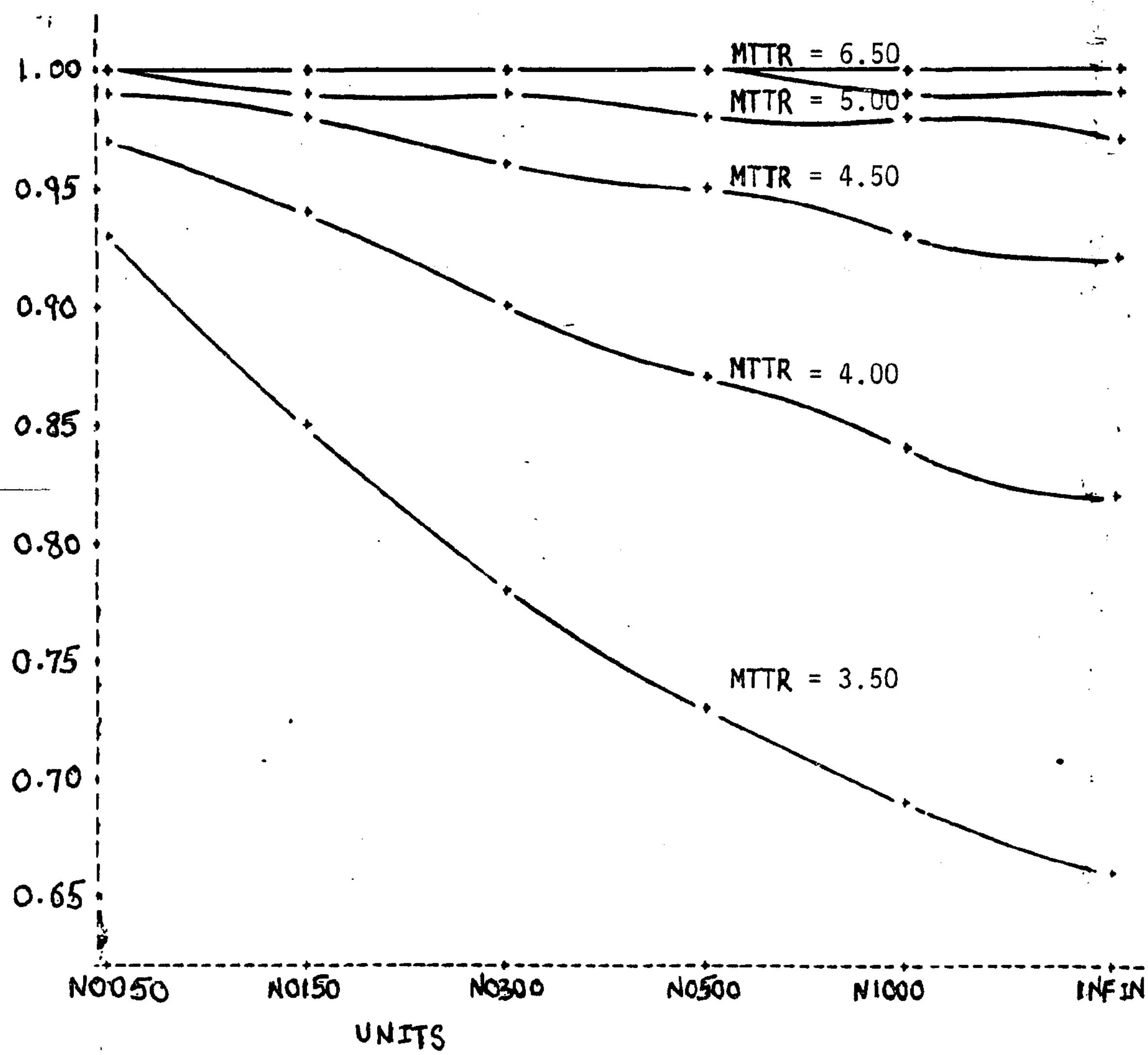


Figure 7

FLOAT FACTOR VS MEAN TIME TO REPAIR (MTTR)
GAMMA FAILURE AND EXPONENTIAL REPAIR
MEAN TIME BETWEEN FAILURE (MTBF) = 4 HOURS
AS THE NUMBER OF OPERATING UNITS, N, GOES TO INFINITY

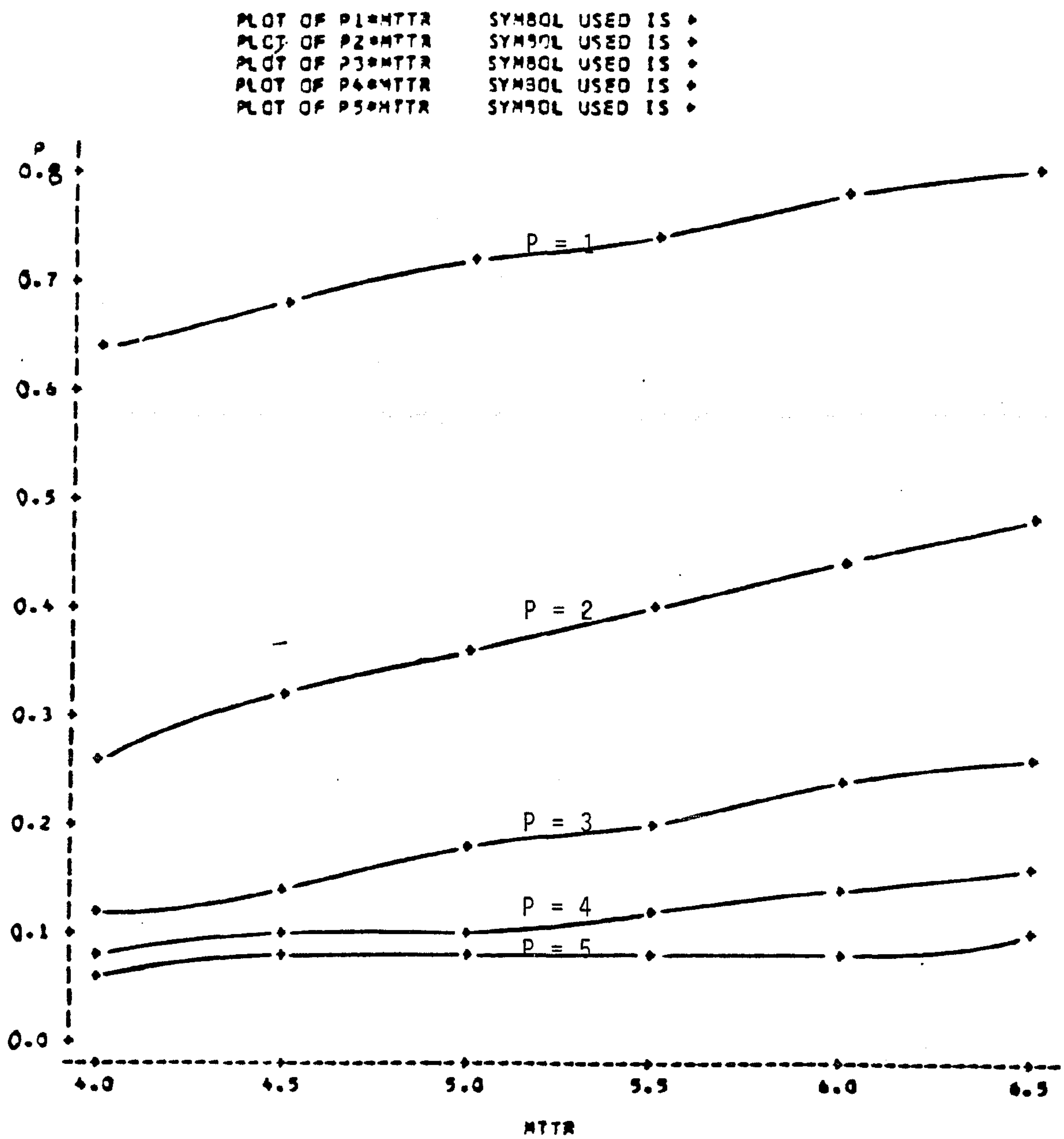


Figure 8

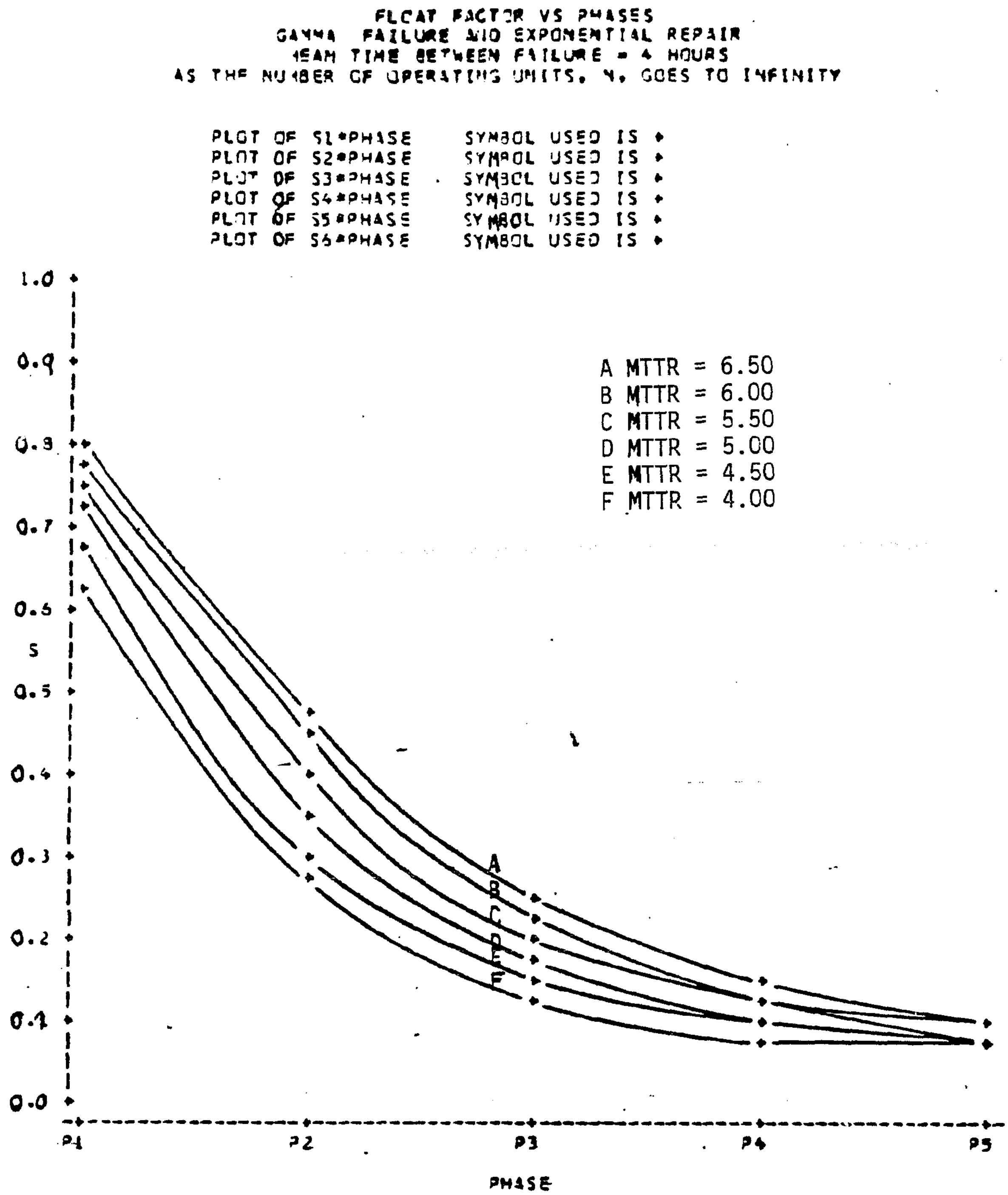

FLOAT FACTOR VS PHASES
GAMMA FAILURE AND EXPONENTIAL REPAIR
MEAN TIME BETWEEN FAILURE = 4 HOURS
AS THE NUMBER OF OPERATING UNITS, N, GOES TO INFINITY
PLOT OF S1*PHASE SYMBOL USED IS *
PLOT OF S2*PHASE SYMBOL USED IS *
PLOT OF S3*PHASE SYMBOL USED IS *
PLOT OF S4*PHASE SYMBOL USED IS *
PLOT OF S5*PHASE SYMBOL USED IS *
PLOT OF S6*PHASE SYMBOL USED IS *
A MTTR = 6.50
B MTTR = 6.00
C MTTR = 5.50
D MTTR = 5.00
E MTTR = 4.50
F MTTR = 4.00
1.0
0.9
0.8
0.7
0.6
S
0.5
0.4
0.3
0.2
0.1
0.0
A
B
C
D
E
F
P1
P2
P3
P4
P5
PHASE

Figure 9

UNITS VS LN [N/(N-1)]

PLOT OF F*N   SYMBOL USED IS +

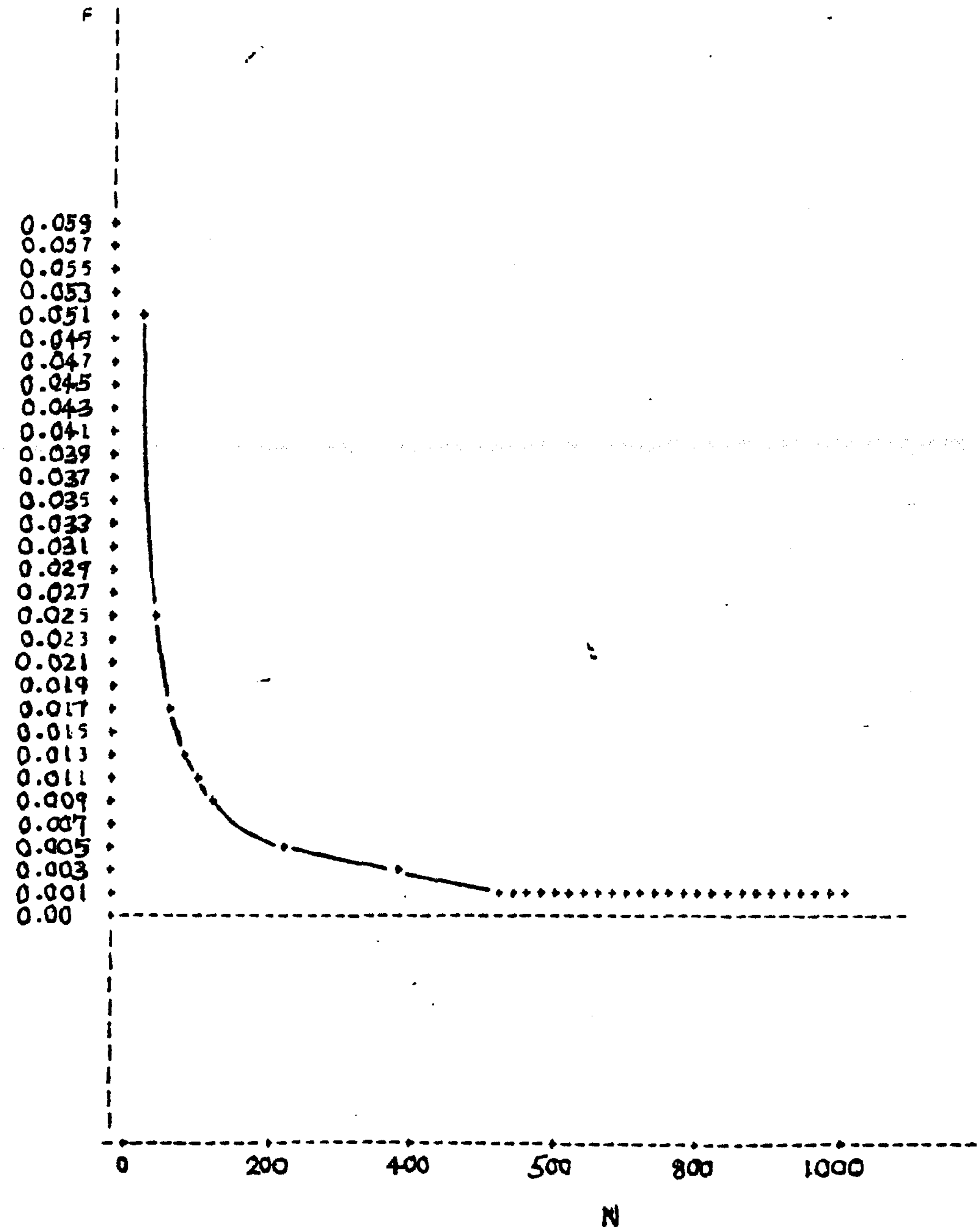


Figure 10

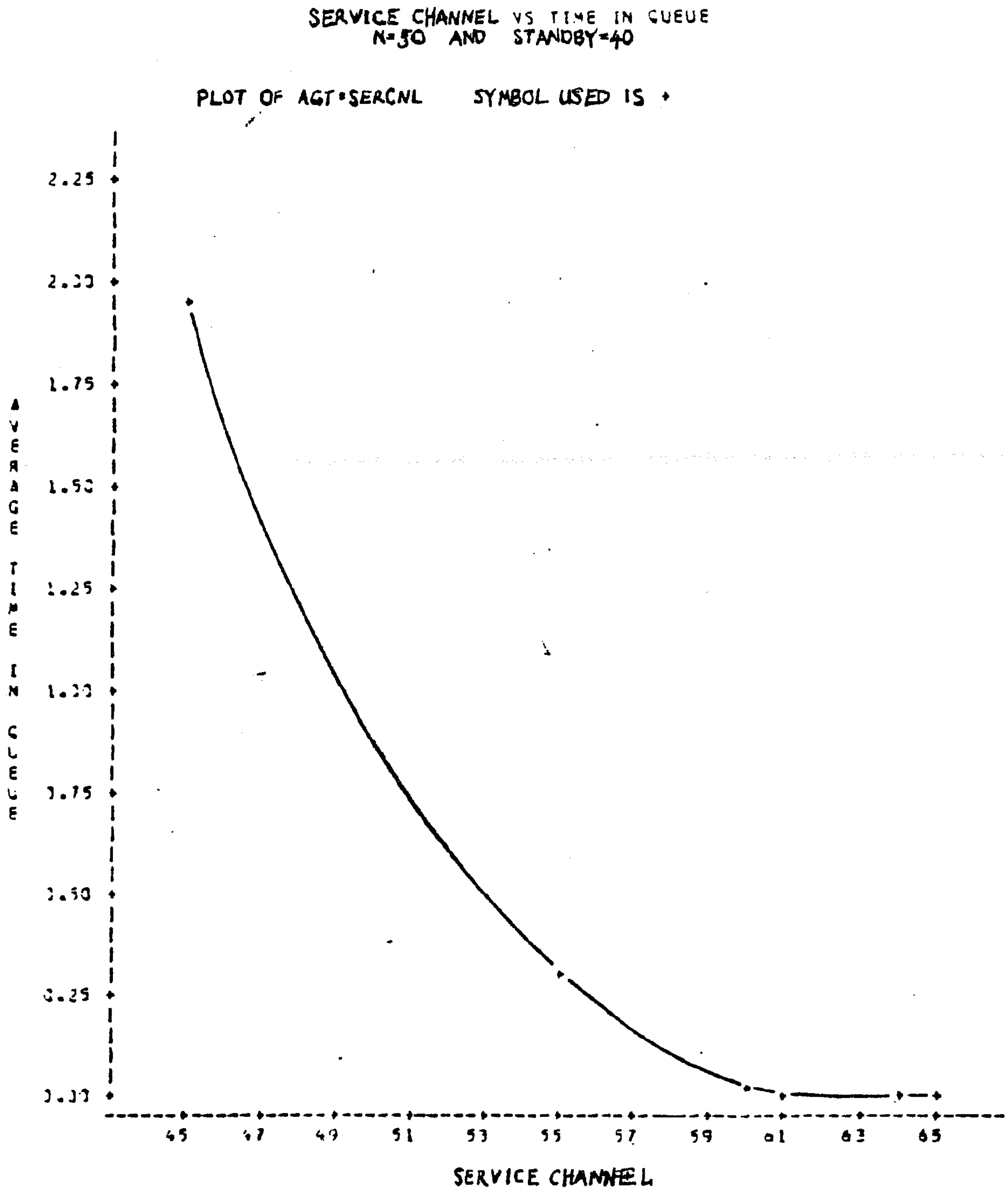

SERVICE CHANNEL VS TIME IN QUEUE
N=50 AND STANDBY=40
PLOT OF AGT*SERCNL SYMBOL USED IS +
AVERAGE TIME IN QUEUE
2.25
2.00
1.75
1.50
1.25
1.00
0.75
0.50
0.25
0.00
45
47
49
51
53
55
57
59
61
63
65
SERVICE CHANNEL

Figure 11

FLOAT FACTOR VS PHASES
GAMMA FAILURE AND EXPONENTIAL REPAIR
MEAN TIME BETWEEN FAILURE = 4 HOURS
NUMBER OF OPERATING UNITS, N = 50, 150, 300

PLOT OF S1*PHASE SYMBOL USED IS *
PLOT OF S2*PHASE SYMBOL USED IS *
PLOT OF S3*PHASE SYMBOL USED IS *

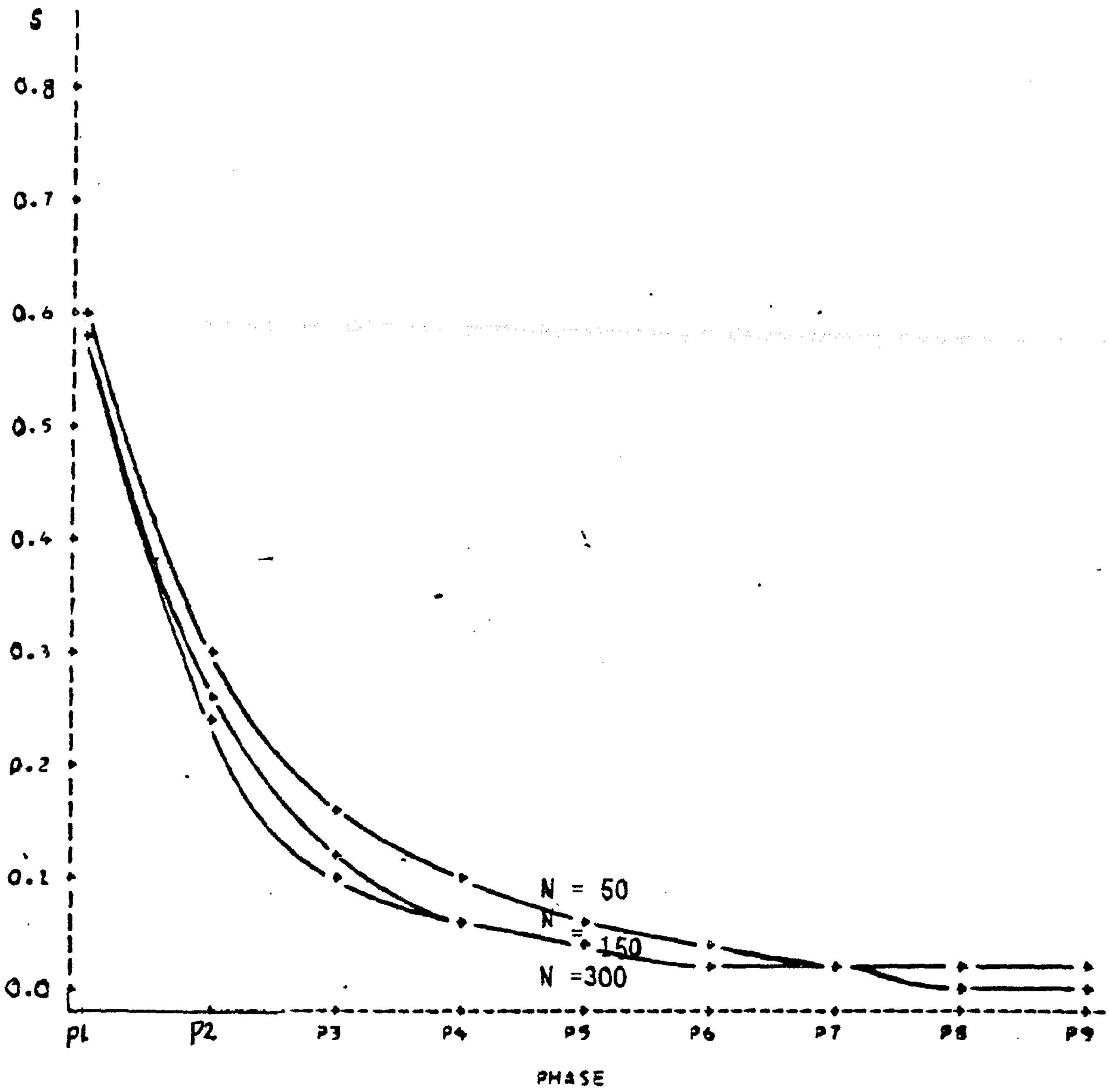


Figure 12

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