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Spontaneous Breakdown of Parity in a Class of Gauge Theories

by

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Abstract

Spontaneous Breakdown of Parity in a Class of Gauge Theories

by

Goran Senjanović

Advisor: Professor Rabindra Nath Mohapatra

We discuss $SU(2)_L \times SU(2)_R \times U(1)$ based left-right symmetric gauge theory in which parity is broken spontaneously. If the number of Higgs scalars is kept minimal (such that it can provide symmetry breaking to $U_{em}(1)$ and give the mass to the fermions), then that theory at low energies agrees with the standard theory in the realm of both charged and neutral weak interactions. Important experimental consequences of that result are stressed. The analysis of both weak and strong CP-violation in such theories is presented. We show how in these theories both Cabibbo angle and CP violating phase can be related to quark mass ratios. Finally, the problem of calculability of neutrino mass and its possible remedies are given.

Acknowledgments

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I. Introduction

In the last decade we have witnessed tremendous experimental and theoretical advances in the realm of the weak interactions. In 1967 for the first time a consistent theory with unambiguous and finite predictions has been presented.[1] By the analogy with quantum electrodynamics and using the principle of spontaneous symmetry breaking Weinberg and Salam have independently constructed[2] a unified gauge theory of weak and electromagnetic interactions, based on the gauge group $SU(2)_L \times U(1)$, subscript L on SU(2) denoting that all the right-handed fermions are singlets under the group SU(2). In 1971 'tHooft, in a series of brilliant papers,[3] has shown that such theories are indeed renormalizable as was conjectured but not proved by original creators of the theory. That has started the flow of theoretical and experimental work in the coming years. In 1973 the existence of neutral currents which are the necessary outcome of the Weinberg-Salam or the standard theory (hereafter, the standard model will denote the original Weinberg-Salam model) was experimentally confirmed. That result can in turn be translated as the fact that the standard model is the minimal unified gauge theory of weak and electromagnetic interactions, and as the experiments seem to suggest it is probably a phenomenologically correct theory, at least at present energies. As far as neutrino induced neutral interactions are concerned it is well established that for the value of $\sin^2\theta_W \simeq 0.25$ ($\sin\theta_W$ is the theoretically free parameter, a function of gauge coupling constants -- see section 1. in chapter II) the standard theory is in excellent agreement with the experiment which in view of its simplicity is a remarkable result. In the last year or so, however, somewhat disturbing findings have been reported: although standard theory predicts a sizeable amount of parity violation in atoms, experimental results[4] were consistent with parity conservings atomic processes. But all the measurements were performed for heavy atoms where the atomic calculations behind the experiments cannot be taken without caution due to the crude approximations necessitated by the large number of nucleons. Also, it should be mentioned that the Novosibirsk group[5] has obtained contradicting results: their findings agree with the predictions of the standard theory. This discrepancy can and should be resolved by performing experiments on light atoms such as hydrogen and deuterium. However in this work we will ignore it and simply assume that the standard theory will survive the mentioned conflicting situation in atomic experiments. The reason is the following: very recently a group of experimentalists at SLAC[6] has measured polarized electron-nucleus scattering amplitudes where the theory behind is well established parton model of hadrons and therefore the results are much more unambiguous. The results are in favor of the standard model: it gives excellent agreement with the experiment for $\sin^2\theta_W \simeq 0.22$. At this stage it is premature to say whether the small discrepancy between above value of $\sin^2\theta_W$ and the one determined by neutrino scattering ($\sin^2\theta_W \simeq 0.25$) should be taken seriously; we believe it is only fair to say that the standard theory is definitely in good standing.

However, there is an important link missing in the standard theory: it does not explain V-A character of β and μ decay, it is put in by hand, as we mentioned, by allowing only the left-handed components of fermions to couple to charged gauge mesons. A few years ago an attempt has been made to construct a theory in which both left-handed and right-handed fermions participate in β decay, but the V-A character of the observed interactions emerges as a result of a natural suppression of right-handed gauge currents. Namely, Pati, Salam and Mohapatra have discussed a gauge theory[7,8] based on the group $SU(2)_L \times SU(2)_R \times U(1)$ which was completely left-right symmetric, except for the masses of Higgs scalars. This has in turn enabled them to make the right-handed charge gauge meson W_R^+ much heavier than W_L^+ and therefore obviously to suppress its contribution at low energies. It was subsequently shown by Mohapatra and author[9] that such a picture can emerge as a result of spontaneous symmetry breaking; namely a completely left-right symmetric theory allows for an asymmetric (spontaneously broken) solution which violates parity. In these theories the observed V-A structure of weak interactions is only a low energy phenomenon which should disappear when one reaches the energies of order 10^3 GeV or higher. In such a picture, all interactions above these energies are supposed to be parity conserving (the large mass of W_R^+ would not count at such energies) and describable by a single gauge coupling constant g ($g^2/4\pi \approx \alpha$). The enlargement of the gauge group and increase in the number of Higgs scalars seems to be the necessary price to be paid in order to bring parity violation on the same (respectable) footing as the violation of other, continuous symmetries. Namely, one could try (as people did) to make a vector-like SU(2) x U(1) theory in which both the left-handed and right-handed fermions are in doublets. However, these theories are definitely in contradiction to neutrino experiments and are ruled out. Therefore, we are led naturally to the $SU(2)_L \times SU(2)_R \times U(1)$ gauge theory which predicts the doubled number of charged gauge mesons (4-$W_L^{\pm}$ and $W_R^{\pm}$) as opposed to the standard theory (2-$W^{\pm}$) and also the doubled number of massive neutral gauge mesons (two, against one in the standard model). In this paper we analyze in detail some of the most important features of such a theory, led through most of our work by the requirement of the minimal number of the particles. Namely, we concentrate on a theory with a minimal Higgs assignment which is necessary to break down the symmetry to the electromagnetic gauge invariance and to provide the masses for fermions. As we shall see it turns out similar to the fact that the standard model is a minimal gauge theory with V-A changed currents, that this minimal left-right symmetric theory seems to be the correct one. It, of course has to agree with the standard model in the interactions of neutrinos (that is dictated by experiment), but more than that it happens that in the limit of infinitely heavy $W_R^{\pm}$ the predictions of both theories are identical both in the realm of charged and neutral currents. In other words, at the tree level (order G_F) up to the small corrections due to the finite mass of $W_R^{\pm}$ these theories are otherwise phenomenologically indistinguishable. With the increased precision of the present experiments and by going to higher energies they will, of course, be distinguishable -- we discuss that in chapter I.

This paper is mainly a review of some well known facts about left-right symmetric theories concentrated mainly on the work done by the author in collaboration mostly with R. N. Mohapatra and also G. C. Branco. The original result is the proof of the stability of the parity broken solution of the theory presented in Appendix A. Also, somewhat different derivation and explanation of the SU(2) x U(1) limit of the theory given by Pati, Rajpoot and Salam[10] is given in chapter II and Appendix D. The paper is organized in the following manner:

Chapter II contains basic material and is necessary to follow the rest of the work. It is here that we discuss the choice of Higgs assignment, the pattern of symmetry breaking and the predictions of the resulting theory. In Appendixes A, B, C and D we offer some detailed proofs relevant for this chapter. We should mention that we completely concentrate on the minimal left-right symmetric gauge theory characterized also by the so-called manifest[11] left-right symmetry (that is the same left and right charged currents). At the beginning of chapter II, we offer a very brief review of some of the basic properties of the standard theory (which will probably bore the reader, but is helpful for the comparison with the $SU(2)_L \times SU(2)_R \times U(1)$ theory).

In chapter III we discuss both the weak and strong CP-violation in these kind of theories. Namely, left-right gauge theories allow for a particularly interesting theory of CP-violation where the amount of CP-nonconservation is linked to parity violation in nature. In section 2. of chapter III we examine the question of strong CP-violation due to recently discovered instanton effects and show how they can be naturally suppressed in $SU(2)_L \times SU(2)_R \times U(1)$ theories with resulting CP-violation of superweak nature mediated by the exchange of heavy Higgs particles in the four quark case or the left-right symmetric generalization of Kobayashi-Maskawa[12] theory where the fermionic currents violate CP through the interactions with gauge mesons in the case of six quarks.

Recently, a number of people have discussed the possibility of determining Cabibbo angle as the function of light quark masses, by imposing the discrete symmetry on the left-right symmetric gauge theories. In chapter IV we review their work and also present the model suggested by Mohapatra and the author[13] in which both Cabibbo angle and CP-violating phase are expressed through the masses of light quark flavors. The discussion is again in the context of left-right gauge theories.

One of the important features of these theories is that due to left-right symmetry these theories predict both left and right-handed neutrinos which are therefore in general massive. Although their masses can be taken to be as small as we wish, the question remains whether it is possible to account naturally for their small or vanishing masses (natural here, in the sense of the technical language of gauge theories, means without adjusting the parameter of the Langrangian). In section V we present some pioneering work done in this direction by G. C. Branco and author.[14] We have derived various constraints on left-right symmetric theories which they must satisfy in order to have calculable neutrino masses. We have also constructed a realistic model in which neutrinos are massless to all orders in perturbation theory.

Finally, in section VI we offer a summary and analysis of the presented material. We comment on possible differentiation between the Weinberg-Salam theory and the minimal left-right symmetric theory. In particular, we stress the importance of the possible discovery of the light neutral Higgs scalar which would serve to refute most of the left-right symmetric theories.

It is fair to emphasize once again that this work is by no means a complete analysis of the work done on left-right symmetric theories. For excellent review and thorough discussions of many different aspects of these kinds of theories we refer the reader to the articles by Mohapatra[15] and Pati.[16]

II. <u>The minimal left-right symmetric $SU(2)_L \times SU(2)_R \times U(1)$ gauge theory and it's relation to the standard theory of weak and electromagnetic interactions</u>

This is the basic section of our work. Here we analyze in detail (including appendixes A, B, C and D) the symmetry breaking in left-right symmetric theories, with the special emphais on the spontaneous violation of parity. The theory we have in mind is the minimal theory in the terms of the Higgs sector, which manifestly preserves parity prior to symmetry breaking. As we shall see, it is characterized by the same left and right charged currents; so that the dominant V-A character of β and μ decay is then attributed to the large mass of $W_R^{\pm}$, the right-handed charged gauge meson. We also present somewhat original derivation and explanantion of the phenomenologically very important fact: <u>In the limit of very large mass of right-handed charge gauge meson, the effective neutral current Hamiltonian in this theory is the same as in the standard theory</u>. At the expense of maybe boring the reader, we start section II by reviewing some of the basic properties of the standard model of weak and electromagnetic interactions. Next, we analyze in detail the problem of symmetry breaking which manifestly breaks left-right symmetry showing that the asymmetric solution is the absolute minimum of the classical Higgs potential, in a range of parameters of the Langrangian. We then turn out attention to the charged sector of the theory, by discussing the β and μ decay as seen in this theory and we present some speculations about the possible parity conserving character of weak interactions at high energies. Finally, we discuss the neutral gauge meson sector where we show the already mentioned equivalence with the standard model.

II.1. The standard theory: a review

The gauge group is $SU(2)_L \times U(1)$, with coupling constants g and g', respectively. The 2N quarks and 2N leptons (N was 2 in the original cormulation of Weinberg and Salam,[2] but today we know that N must be larger) in left doublets with right-handed components being singlets:

$$Q_{1L} = \begin{pmatrix} u^o \\ d^o \end{pmatrix}_L , \quad Q_{2L} = \begin{pmatrix} c^o \\ s^o \end{pmatrix}_L , \cdots$$

$$u_R^o,\ d_R^o,\ c_R^o,\ s_R^o,\ \cdots \tag{2.1}$$

where the superscript zero denotes that they are not eigenstates of the mass matrix and:

$$\psi_{1L} = \begin{pmatrix} \nu \\ e \end{pmatrix}_L , \quad \psi_{2L} = \begin{pmatrix} \nu' \\ \mu \end{pmatrix}_L , \cdots$$

$$e_R,\ \mu_R,\ \cdots \ . \tag{2.2}$$

The electric charge operator is defined as:

$$Q = T_3 + \frac{Y}{2} \tag{2.3}$$

where $\vec{T}$ and Y are generators of SU(2) and U(1) respectively. The minimal Higgs assignment that breaks the symmetry down to $U_{em}(1)$ is the doublet under SU(2):

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^o \end{pmatrix} \tag{2.4}$$

with the U(1) quantum number 1:

$$Y\phi = \phi \ . \tag{2.5}$$

By minimizing the classical potential with a wrong sign for the mass term, one obtains:

$$\langle\phi\rangle = \begin{pmatrix} o \\ v \end{pmatrix}, \quad (v \text{ is real}) \tag{2.6}$$

The gauge meson eigenstates in turn become:

$$\begin{aligned} W^{\pm}_{\mu} &= \frac{W_{1\mu} \mp i\, W_{2\mu}}{\sqrt{2}} \quad , \quad m_W^2 = \frac{1}{4}\, g^2 v^2 \\ A_{\mu} &= (g' W_{3\mu} + g B_{\mu}) \frac{1}{\sqrt{g^2+g'^2}} \quad , \quad m_A^2 = 0 \\ Z_{\mu} &= (g W_{3\mu} - g' B_{\mu}) \frac{1}{\sqrt{g^2+g'^2}} \quad , \quad m_Z^2 = \frac{1}{4}\, v^2 (g^2+g'^2) \end{aligned} \tag{2.7}$$

where $W^{\pm}$ are two massive charged gauge mesons, A is the photon and Z massive neutral gauge meson ($\vec{W}$ and B are gauge fields associated with SU(2) and U(1) groups, respectively). We can also write:

$$\begin{aligned} A_{\mu} &= \sin\theta_W\, W_{3\mu} + \cos\theta_W\, B\mu \\ Z_{\mu} &= \cos\theta_W\, W_{3\mu} - \sin\theta_W\, B\mu \end{aligned} \tag{2.8}$$

with

$$\tan\theta_W \equiv g'/g\ . \tag{2.9}$$

Then we have the well-known relation between M_W and M_Z

$$M_Z = \frac{M_W}{\cos\theta_W} \tag{2.10}$$

which implies the heavier neutral gauge meson than the charged ones.

Once we know the physical states we can display the currents. The charged fermionic currents are obviously purely V-A in character (since only left-handed fermionic components are coupled to $W^{\pm}$). The neutral current interaction will have the usual photonic piece and the piece due to the exchange of the massive neutral gauge meson Z. In order to complete the review of the standard model, we display below the neutral current associated with the Z particle

$$J^Z_\mu = \frac{g}{\cos\theta_W} \bar{f} \gamma_\mu (T_3 L - \sin^2\theta Q_{e\ell}) f \tag{2.11}$$

where f denotes fermions and $L \equiv \frac{1-\gamma_5}{2}$.

The neutrino induced neutral current interactions suggest $\sin^2\theta_W \approx 0.25$[17] It seems also that the agreement with the recent SLAC experiment on the polarized electron-nucleon scattering is obtained for the similar value of $\sin^2\theta_W$. In view of its simplicity, we can say that Weinberg-Salam theory has spectacular agreement with the experiment.

II.2. The minimal left-right symmetric theory and the spontaneous violation of parity

We have mentioned at the end of the previous section that Weinberg-Salam theory has very good chances to turn out to be the correct theory of weak and electromagnetic interactions. But isn't then any attempt to construct a different gauge theory just a leisure game for an idle theoretical physicist? We shall try to show that the answer to this question lies in the negative. First, we wish to emphasize that the original motivation for $SU(2)_L \times SU(2)_R \times U(1)$ gauge theories was not phenomenological, but "philosophical" in a sense: it has been suggested as a possible explanation of the V-A structure of the β and μ decay. Therefore, the fact that it agrees with the standard theory in most aspects at the present energies is only welcome; however, they will certainly differ at higher energies at which the mass of $W_R^\pm$ will play an important role.

The theory we have in mind is based on gauge groups $SU(2)_L \times SU(2)_R \times U(1)$ with coupling constants g_L, g_R and g' respectively. The immediate consequence of the left-right symmetry which we will impose on the Lagrangian is that left and right gauge coupling constants are the same: $g_L = g_R = g$. The 2N quarks and 2N leptons are placed in doublets (the nature will choose N)

$$Q_{1L} = \begin{pmatrix} u^o \\ d^o \end{pmatrix}_L , \quad Q_{2L} = \begin{pmatrix} c^o \\ s^o \end{pmatrix}_L , \ \ldots + L \leftrightarrow R$$

$$\psi_{1L} = \begin{pmatrix} \nu \\ e \end{pmatrix}_L , \quad \psi_{2L} = \begin{pmatrix} \nu' \\ \mu \end{pmatrix}_L , \ \ldots + L \leftrightarrow R \tag{2.12}$$

The representation content of fermionic multiplets is

$$Q_{iL} \equiv (\tfrac{1}{2},0,\tfrac{1}{3}), \quad Q_{iR} \equiv (0,\tfrac{1}{2},\tfrac{1}{3})$$

$$\psi_{iL} \equiv (\tfrac{1}{2},0,-1), \quad \psi_{iR} \equiv (0,\tfrac{1}{2},-1) \tag{2.13}$$

The electric charge operator is defined as

$$Q_{e\ell} \equiv T_{3L} + T_{3R} + \frac{Y}{2} \tag{2.14}$$

where $\vec{T}_L$, and $\vec{T}_R$ and Y are the generators of the $SU(2)_L$, $SU(2)_R$ and $U(1)$ respectively. In order to produce fermionic mass matrices <u>we are led</u> to the following Higgs multiplets

$$\Phi = \begin{pmatrix} \phi_1^o & \phi_1^+ \\ \phi_2^- & \phi_2^o \end{pmatrix} , \qquad \tilde{\Phi} \equiv \tau_2 \Phi^* \tau_2 \tag{2.15}$$

with transformation properties

$$\Phi \equiv (\tfrac{1}{2},\tfrac{1}{2}^*,0), \quad \tilde{\Phi} \equiv (\tfrac{1}{2},\tfrac{1}{2}^*,0) \tag{2.16}$$

As is obvious from (2.16) Φ and $\tilde{\Phi}$ serve to connect left and right fermionic multiplets and eventually after spontaneous symmetry breaking to produce the mass matrices, as we shall see later. Now, the most general form of $\langle\Phi\rangle$ which preserves electromagnetic gauge invariance is

$$\langle\Phi\rangle = \begin{pmatrix} k & o \\ o & k' \end{pmatrix} \tag{2.17}$$

But, then

$$(T_{3L} + T_{3R})\langle\Phi\rangle = 0$$

$$Y\langle\Phi\rangle = 0 \tag{2.18}$$

so that symmetry is broken down to U(1) x U(1). Therefore, in order to complete the symmetry breaking we obviously need more Higgs multiplets. The simplest choice (and the minimal one) is to introduce two Higgs doublets

$$\chi_L = \begin{pmatrix} \chi_L^+ \\ \chi_L^o \end{pmatrix}, \quad \chi_R = \begin{pmatrix} \chi_R^+ \\ \chi_R^o \end{pmatrix} \tag{2.19}$$

with the following transformation properties

$$\chi_L(\tfrac{1}{2},0,1), \quad \chi_R = (0,\tfrac{1}{2},1) \tag{2.20}$$

Under the parity operation the fields transform as follows

$$\begin{aligned} &\vec{W}_L \leftrightarrow \vec{W}_R \\ &f_L^o \leftrightarrow f_R^o \\ &\chi_L \leftrightarrow \chi_R \\ &\Phi \leftrightarrow \Phi^+, \quad \tilde{\Phi} \leftrightarrow \tilde{\Phi}^+ \end{aligned} \tag{2.21}$$

where $\vec{W}_{L,R}$ stands for left and right gauge mesons, respectively and f^o denotes quark and lepton doublets Q_i and ψ_i.

The transformation property of $\Phi:\Phi\leftrightarrow\Phi^+$ under left-right symmetry leads to symmetric fermionic matrices, as we show now. Namely, the most general Yukawa couplings are given by

$$L_y = \overline{f^o_{iL}}\,(a_{ij}\Phi + b_{ij}\tilde{\Phi})f^o_{jR} + \bar{f}^o_{jR}(a^*_{ij}\Phi^+ + b^*_{ij}\tilde{\Phi}^+)f^o_{iL} \tag{2.22}$$

where $f^o = (Q,\psi)$.

Now, left-right symmetry leads to the condition

$$a_{ij} = a^*_{ji},\ b_{ij} = b^*_{ji}\ . \tag{2.23}$$

From eq. (2.22) and using (2.17) we obtain the following fermionic mass matrices

$$M_{1ij} = a_{ij}k + b_{ij}k'^*$$

$$M_{2ij} = a_{ij}k' + b_{ij}k^* \tag{2.24}$$

where subscript $i = 1,2$ denotes $T_3 = \frac{1}{2}$ and $T_3 = -\frac{1}{2}$ fermions. If we ignore for the time being the question of CP-violation, we can work with real Yukawa couplings and also assume that k and k' are real (in Appendix A we prove that for a range of parameters in the classical potential it is possible to achieve that). In that case obviously M_1 and M_2 are real and symmetric, which means that we can diagonalize them by orthogonal transformations. In other words, we can find O_1 and O_2 such that

$$f^o_{iL,R} = O_i f_{iL,R} \tag{2.25}$$

and

$$O_i^T M_i O_i = D_i \tag{2.26}$$

where D_i denote diagonal up and down fermionic mass matrices (i = 1,2 stands, as before, for $T_3 = \frac{1}{2}$ and $T_3 = -\frac{1}{2}$ fermions). Since that means that left and right Cabibbo like angles are the same, we have an example of a manifest left-right symmetry characterized by the same left and right charged currents. That in turn implies that W_R^+ must be very heavy. We wish to remark that although in the case of two or more Φ's it is possible to avoid manifest left-right symmetry (that is one can have different left and right charged currents), in the case of 4 quarks we are forced to the choice of heavy W_R^+ independently of the details of the Higgs sector.

It is remarkable that although the classical Higgs potential is symmetric under $\chi_L \leftrightarrow \chi_R$, the following pattern of symmetry breaking emerges as the minimum of the potential for a range of parameters (for the proof we refer the reader to Appendix B)

$$\langle\chi_L\rangle = 0, \qquad \langle\chi_R\rangle = \begin{pmatrix} o \\ v \end{pmatrix}$$

$$\langle\Phi\rangle = \begin{pmatrix} k & o \\ o & k' \end{pmatrix} \tag{2.27}$$

The unbroken symmetry is $U_{em}(1)$ generated by the charge operator as defined in eq. (2.14).

i. charged currents

The charged gauge meson mass matrix is then given by

$$
\begin{array}{ccc}
 & W_L^+ & W_R^+ \\
\begin{array}{c} W_L^- \\ W_R^- \end{array} & \multicolumn{2}{c}{\begin{pmatrix} \frac{g^2}{4}(k^2+k'^2) & -\frac{g^2}{2}kk' \\ -\frac{g^2}{2}kk' & \frac{g^2}{4}(k^2+k'^2+v^2) \end{pmatrix}}
\end{array}
\tag{2.28}
$$

The discussion of physical eigenstates is left for Appendix C. There we also show how the tiny mixing between W_L^+ and W_R^+ can be essentially ignored in most of the analysis; in other words we can work with W_L^+ and W_R^+ as approximate physical states. Well, that in turn implies the usual Cabibbo like form of the charged currents with left and right mixing angles exactly the same as the product of manifest left-right symmetry. In other words, the form of the charged currents is

$$
\begin{aligned}
J_{\mu L}^+ &= \bar{\nu}_L\gamma_\mu e_L + \bar{\nu}_L'\gamma_\mu\mu_L + (\overline{u}\overline{c})_L\gamma_\mu O\binom{d}{s}_L \\
J_{\mu R}^+ &= \bar{\nu}_R\gamma_\mu e_R + \bar{\nu}_R'\gamma_\mu\mu_R + (\overline{u}\overline{c})_R\gamma_\mu O\binom{d}{s}_R
\end{aligned}
\tag{2.29}
$$

where O is an orthogonal matrix. In the case of 4 quarks it is given by:

$$
O = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \tag{2.30}
$$

where θ is the Cabibbo angle.

The fact that left and right charged currents are the same is, as we stated earlier, crucial. It implies, that no matter what the number of quarks is, the right-handed charged gauge meson must be very heavy (see appendix C).

ii. Neutral currents

From (2.27) we obtain the following mass matrix for the neutral gauge mesons

$$\begin{array}{c|ccc} & W_L^3 & W_R^3 & B \\ \hline W_L^3 & & & \\ W_R^3 & & & \\ B & & & \end{array}\quad \begin{pmatrix} \frac{1}{4}g^2(k^2+k'^2) & -\frac{1}{4}g^2(k^2+k'^2) & 0 \\ -\frac{1}{4}g^2(k^2+k'^2) & \frac{1}{4}g^2(k^2+k'^2+v^2) & -\frac{1}{4}gg'v^2 \\ 0 & -\frac{1}{4}gg'v^2 & \frac{1}{4}g'^2v^2 \end{pmatrix} \tag{2.31}$$

The eigenvalues are (in the approximation $(k^2 + k'^2)/v^2 << 1$)

$$M_Z \cong \frac{M_{W_L}}{\cos\theta}$$

$$M_X \cong \frac{M_{W_R}\cos\theta}{\sqrt{\cos 2\theta}} \tag{2.32}$$

with the angle θ defined as

$$\sin^2\theta \equiv \frac{g'^2}{g^2+2g'^2} \tag{2.33}$$

(Notice that our definition of angle θ is different from the usual one.)[18]

The neutral gauge meson eigenstates are given by

$$A_\mu = \sin\theta(W^3_{L\mu} + W^3_{R_\mu}) + \sqrt{\cos 2\theta}\, B_\mu,\ M_A = 0$$

$$Z_\mu \cong \cos\theta\, W^3_{L\mu} - \sin\theta\tan\theta W^3_{R\mu} - \tan\theta\sqrt{\cos 2\theta}\, B_\mu$$

$$X_\mu \cong \frac{\sqrt{\cos 2\theta}}{\cos\theta}\, W^3_{R\mu} - \tan\theta B_\mu \tag{2.34}$$

From the above eigenstates it is a simple exercise to find the neutral current generated by the light massive gauge meson (the only one relevant at low energies)

$$J_\mu^Z = \frac{g}{\cos\theta} \bar{f} \gamma_\mu [T_3 L - Q\sin^2\theta] f$$

where

$$L \equiv \frac{1-\gamma_5}{2} \quad . \tag{2.35}$$

Since, from (2.32) $\cos\theta \, M_Z = M_{W_L}$ we obtain the following important result: in the limit of the large mass of W_R^+ and with identification $\sin\theta = \sin\theta_W$ the minimal left-right symmetric gauge theory predicts all the lowest order neutral current interactions to be exactly the same as in the Weinberg-Salam model.

More than that since we have $e = g\sin\theta$ like in the Weinberg-Salam model, if $\sin\theta = \sin\theta_W$ we also predict the same masses for the light gauge meson as in the Weinberg-Salam theory:

$$\begin{aligned} M_{W_L}(LR) &\cong M_{W_L}(WS) \\ M_Z(LR) &\cong M_Z(WS) \end{aligned} \tag{2.36}$$

This result is not new; it has been known to Pati, Rajpoot and Salam.[10] In Appendix D we offer a more elegant and simpler derivation based on the work of Georgi and Weinberg.[19] We should perhaps emphasize that the result does not follow from Georgi-Weinberg theorem which concerns only neutrino interactions and assumes different pattern of symmetry breaking. It is rather the explanation which we now present that leads to our results.

II.3. The $SU(2)_L \times U(1)$ limit of a minimal left-right symmetric theory

Here we analyze the hierarchy of symmetry breaking consistent with our assumption $v >> k, k'$. The first stage of symmetry breaking occurs through $\langle\chi_R\rangle$. It breaks $SU(2)_R \times U(1)$ down to $U'(1)$, since

$$(T_{3R} + \frac{Y}{2}) \langle\chi_R\rangle = 0 \tag{2.37}$$

so that the generator of $U'(1)$ is $\frac{Y'}{2} = T_{3R} + \frac{Y}{2}$. The coupling constant that corresponds to $U'(1)$ is then given by g''

$$\frac{1}{g''^2} = \frac{1}{g^2} + \frac{1}{g'^2} \quad \text{or } g'' = \frac{gg'}{\sqrt{g^2+g'^2}} \tag{2.38}$$

Therefore

$$\begin{aligned} G &= SU(2)_L \times SU(2)_R \times U(1) \\ &\quad \big\downarrow \langle\chi_R\rangle \\ G' &= SU(2)_L \times U'(1) \end{aligned} \tag{2.39}$$

The effect of the first step of symmetry breaking through $\langle\chi_R\rangle$ is to give the mass to heavy particles: the charged gauge meson $W_R^{\pm}$ and the neutral gauge meson X. By the analogy with the Weinberg-Salam model (the reader should notice that χ_R in the $SU(2)_R \times U(1)$ group is like the usual doublet ϕ in the standard model

$$B'_\mu = (g'W^3_{R\mu} + gB_\mu)\frac{1}{\sqrt{g^2+g'^2}} \;, \quad M^2_{B'} = 0$$

$$X_\mu = (gW^3_{R\mu} - g'B_\mu)\frac{1}{\sqrt{g^2+g'^2}} \;, \quad M^2_X = \frac{1}{4}(g^2+g'^2)v^2 \tag{2.40}$$

Since we are assuming $v >> k, k'$ we can now forget about χ_R and the initial group and for the purpose of low energy physics start with $G' = SU(2)_L \times U(1)$ and a Higgs scalar Φ. Now it is easy to see that:

$$T_{3L}\begin{pmatrix} k & o \\ o & k' \end{pmatrix} = \frac{1}{2}\begin{pmatrix} k & o \\ o & -k' \end{pmatrix}, \quad \frac{Y'}{2}\begin{pmatrix} k & o \\ o & k' \end{pmatrix} = \frac{1}{2}\begin{pmatrix} -k & o \\ o & k' \end{pmatrix} \tag{2.41}$$

situation completely analogous to the Weinberg-Salam model. There, $T_{3L}\langle\phi\rangle_{WS} = -\frac{1}{2}\langle\phi\rangle_{WS}$, $\frac{Y}{2}\langle\phi\rangle_{WS} = \frac{1}{2}\langle\phi\rangle_{WS}$. It is not difficult to see that we have the following correspondence between the two theories at low energies

WS model	LR model
$g \quad\quad g$	$g \quad\quad g''$
$SU(2)_L \times U(1)$	$SU(2)_L \times U'(1)$
g	g
g'	$g'' = \frac{gg'}{\sqrt{g^2 + g'^2}}$
$\langle\phi\rangle^2_{WS}$	$k^2 + k'^2$
$\tan\theta_W \equiv g'/g$	$\tan\theta \equiv g''/g = \frac{g'}{\sqrt{g^2 + g'^2}}$ (2.42)

But then using well-known results from the Weinberg-Salam theory given in (2.7) we obtain the following gauge meson eigenstates:

$$A_\mu = g''W^3_{L\mu} + gB'_\mu, \qquad M^2_A = 0$$

$$Z_\mu = gW^3_{L\mu} - g''B'_\mu, \qquad M^2_Z = \frac{1}{4}(g^2 + g''^2)(k^2 + k'^2) \tag{2.43}$$

where A denotes photon and Z light massive neutral gauge meson. From the formula for g" (Eq. (2.3)) and using Eqs. (2.40) and (2.41) we easily obtain the formulas for gauge meson eigenstates as given previously. However, eq. (2.43) is more transparent for our purposes. It is obvious now that with identification $g''_{LR} = g'_{WS}$ we have a complete equivalence between the two theories. But that leads to $\tan\theta = \tan\theta_W$, where once again: $\tan\theta = g''/g = g'/\sqrt{g^2+g'^2}$, and θ_W is the usual Weinberg angle as defined in eq. (2.9).

By now, it is generally accepted that the standard model has excellent agreement with all the neutrino induced neutral current data. Namely, it conforms to all the experiments with $\sin^2\theta_W$ determined to be approximately 0.25. The shadow of doubt has been cast on it by the atomic parity violation results. The situation is still confusing, since Oxford and Washington groups[4] have observed parity violation on a level much smaller than the standard model prediction (they are consistent with parity conserving atomic processes), on the other hand the Novosibirsk group has obtained results consistent with Weinberg-Salam theory. In view of that and very crude approximations used in the atomic calculations behind these experiments (all the results have been found for heavy atoms), one should take these results with caution. Obviously, we need precise results for parity violation in hydrogen and deuterium. Recently, a group at SLAC has measured the amount of parity violation in polarized electron-nucleon scattering as a test of neutral current effects which do not involve neutrinos. The results agree with the predictions of the standard theory, however for slightly smaller value $\sin^2\theta_W \simeq 0.22$ than the neutrino interactions suggest. At this stage it is maybe premature to conclude anything decisively, but the standard theory is definitely in good standing. We are reaching the stage in which radiative corrections will play a more and more important role. The question is: Where does it leave the left-right symmetric theory analyzed in this paper. Obviously, in the limit of infinite mass of $W_R^{\pm}$ the $SU(2)_L \times SU(2)_R \times U(1)$ and the $SU(2)_L \times U(1)$ theories are equivalent, so that they are equally in good (or bad) shape in that limit.[20] We cannot say anything definitely about the predictions with the finite M_{W_R} for the simple reason that we do not know it (we know only its lower limit) and also at that stage one should include the radiative corrections. One comment is worth mentioning: The parity violation is maximal when $M_{W_R} \to \infty$, so that effects due to finite M_{W_R} cannot solve the possible discrepancy between slightly different predictions for $\sin\theta_W$ in the standard theory as given by neutrino and polarized electron-nucleon scattering; it could make that discrepancy only bigger[21] (in other words, we need more, not less parity violation in electron-nucleon scattering). However, the radiative corrections in this theory may have interesting and large contributions for heavy Higgs scalars.[21] Namely, it has been observed by A. Šokorac and the author[22] in the context of neutral current parity conserving left-right symmetric theories that the Higgs scalar contributions may be nontrivial in the form $m_H^2/M_{W_L}^2$, therefore providing much larger contributions than naively expected, in the case of heavy Higgs scalars. Of course, it remains to be seen whether these effects would appear in the case of the minimal left-right symmetric theory. If they do appear and the mentioned discrepancy in predictions for $\sin\theta_W$ persists in the future, it may provide an interesting case for the left-right symmetric theories. We hope that we have successfully argued in favor of left-right symmetry in nature being spontaneously broken. First, it puts on the same footing the breaking of gauge symmetry and discrete symmetry such as parity. Secondly, it is still in good shape in view of old and new experiments (more or less as the standard theory). Moreover, it may be the necessary enlargement if the standard model does not conform completely to the experiments. If the reader is still not convinced, we offer chapters III and IV, in which we discuss the appealing properties of these theories that both Cabibbo angle and CP violating phases may be determined through the light quark mass ratios and also as an interesting way out of the axion problem.

In passing, we would like to emphasize once again that we were forced to the described choice of Higgs sector on the account of what we know about charged weak currents and also by the requirement to have a minimal particle spectrum. The approximate equivalence at low energies of the standard model and the minimal left-right symmetric model in the realm of neutral currents turned out ot be a welcome consequence of such a pattern of symmetry breaking.

We finish this chapter by comparing the minimal left-right symmetric theory with the standard model in view of their complexities (or rather simplicity in the case of the standard model); that is we display in Table 1 the price which may be necessary to be paid in order to understand the V-A theory of β and μ decay. From Table 1 we see that from 5 particles (excluding fermions whose number obviously can be taken the same in both theories) in the standard model (3 massive gauge mesons, a photon and a physical Higgs scalar) we are led to 17 physical particles (6 gauge mesons, a photon and <u>10</u> physical Higgs scalars), a definitely large increase. Especially disturbing is the increase of the number of Higgs particles: from 1 to 10. It is interesting to observe that in order to construct a theory of weak and electromagnetic interactions Weinberg and Salam postulated the existence of two new particles (neutral gauge meson[22] and the neutral Higgs scalar; charged gauge mesons were suspected much earlier) and in order to understand the violation of parity it seems that we need a minimum of 12 new particles.

III. Weak and strong CP violation

1. Weak CP violation

The question of CP violation in four quark $SU(2)_L \times SU(2)_R \times U(1)$ gauge theories has been discussed originally by Mohapatra and Pati.[24] For details we refer the reader to their paper; here we only briefly recall some of their main results:

a) via the so called "isoconjugate" relations one has experimentally desirable relation

$$\eta_{+-} = \eta_{oo} \tag{3.1}$$

where

$$\eta_{ij} \equiv \frac{M(K_L \to \pi_i \pi_j)}{M(K_S \to \pi_i \pi_j)} \tag{3.2}$$

b) secondly, CP and parity violation are linked since η_{+-} is proportional to $M^2_{W_L}/M^2_{W_R}$ which is a measure of parity violation in charged weak currents

$$\eta_{+-} \simeq \sin(\delta_L - \delta_R)(m^2_{W_L}/m^2_{W_R}) \frac{\sin 2\theta_R}{\sin 2\theta_L} \tag{3.3}$$

where angles θ_L, θ_R and phases δ_L and δ_R are defined by Cabibbo rotations

$$U_L = \begin{pmatrix} \cos\theta_L & \sin\theta_L e^{i\delta_L} \\ -\sin\theta_L e^{-i\delta_L} & \cos\theta_L \end{pmatrix}, \quad U_R = \begin{pmatrix} \cos\theta_R & \sin\theta_R e^{i\delta_R} \\ -\sin\theta_R e^{-i\delta_R} & \cos\theta_R \end{pmatrix} \tag{3.4}$$

To illustrate their meaning we write the general form of charged currents

$$J^+_{\mu L} = (\bar{u}\ \bar{c})_L \gamma_\mu U_L \binom{d}{s}_L$$

$$J^+_{\mu R} = (\bar{u}\ \bar{c})_R \gamma_\mu U_R \binom{d}{s}_R \tag{3.5}$$

which couple to W_L^+ and W_R^+, respectively. The question can be raised of how to achieve complex rotations. Now, from our analysis in chapter II we have learned that quark mass matrices are given by (see chapter II)

$$M_{1ij} = a_{ij}k + b_{ij}k'^*$$
$$M_{2ij} = a_{ij}k' + b_{ij}k^* \tag{3.6}$$

with

$$a_{ij} = a^*_{ji}$$
$$b_{ij} = b^*_{ji} \tag{3.7}$$

We assume the theory to be CP invariant prior to symmetry breaking, that is we take a_{ij} and b_{ij} as real numbers. In eq. (3.6) k and k' are the vacuum expectation values of the Higgs multiplet Φ

$$\langle\Phi\rangle = \begin{pmatrix} k & o \\ o & k' \end{pmatrix} \tag{3.8}$$

In Appendix A we have shown how one can even with a single ϕ multiplet achieve complex k and k' (or at least one of them). That in turn implies that quark mass matrices in this case are symmetric and complex. Without much further ado we use the fact that such matrices can be diagonalized without the loss of generality by the following unitary transformations

$$U_R^i = U_L^{i*} \ , \ i = 1,2 \tag{3.9}$$

But from the form of charged currents in terms of unphysical weak eigenstates

$$J^{+}_{\mu L} = (\bar{u}\ \bar{c})_{oL}\ \gamma_\mu \begin{pmatrix} d \\ s \end{pmatrix}_{oL}$$

$$J^{+}_{\mu R} = (\bar{u}\ \bar{c})_{oR}\ \gamma_\mu \begin{pmatrix} d \\ s \end{pmatrix}_{oR} \tag{3.10}$$

and using

$$\begin{pmatrix} u \\ c \end{pmatrix}_{oL} = U_{1L} \begin{pmatrix} u \\ c \end{pmatrix}_{L}, \quad \begin{pmatrix} u \\ c \end{pmatrix}_{oR} = U_{1R} \begin{pmatrix} u \\ c \end{pmatrix}_{R}$$

$$\begin{pmatrix} d \\ s \end{pmatrix}_{oL} = U_{2L} \begin{pmatrix} d \\ s \end{pmatrix}_{L}, \quad \begin{pmatrix} d \\ s \end{pmatrix}_{oR} = U_{2R} \begin{pmatrix} d \\ s \end{pmatrix}_{R} \tag{3.11}$$

we obtain easily (see eq. (3.5))

$$U_{L,R} = U^{+}_{1L,R}\ U_{2L,R}\ . \tag{3.12}$$

In terms of left and right Cabibbo rotations it means that

$$U_R = U^{*}_L \tag{3.13}$$

which leads to the relations (see eq. (3.4)

$$\theta_R = \theta_L$$

$$\delta_R = -\delta_L \tag{3.14}$$

In other words, we have the same left and right mixing angles and opposite phases. In the terminology of ambidextrous theories such theories have been baptized as pseudo-manifest left-right symmetric models (as to denote the difference between them and manifest models characterized by exactly the same left and right charged currents in the tree approximation).

We can then express CP violating amplitude as a function of only two parameters

$$\eta_{+-} \simeq \sin 2\delta \, \frac{m_{W_L}^2}{m_{W_R}^2} \tag{3.15}$$

The above relation offers an interesting possibility: If we can calculate or at least determine δ as a function of say light quark masses, we will have the complete link between the P and CP nonconservation in nature. Actually, in collaboration with R. N. Mohapatra we have achieved that. We wil describe our results in section IV.

Another interesting relation emerges from (3.15)

$$\frac{m_R^2}{m_{W_L}^2} \cong \frac{\sin 2\delta}{\eta_{+-}} \tag{3.16}$$

From $\eta_{+-} \simeq 10^{-3}$; it then provides a possible upper bound on the mass of the heavy right-handed charged gauge meson

$$\frac{m_{W_R}^2}{m_{W_L}^2} \lesssim 10^{-3} \tag{3.17}$$

However, as we shall see in section 2 of this chapter this kind of theory (a pseudomanifest left-right symmetric theory) may be in trouble in a sense that it cannot naturally account for the absence of strong CP violation induced by instanton effects. Therefore, eq. (3.17) must be taken with some caution (as the whole theory of CP violation which we just presented) and it certainly cannot be used as an experimentally confirmed upper limit for a mass of W_R^+.

Finally, before closing this section we discuss the magnitude of electric dipole moment of the neutron in this kind of a theory. Namely the prediction (eq. (3.1)) $\eta_{+-} = \eta_{oo}$ that comes out naturally in this theory is the same as in superweak theories of CP violation. It is important, therefore to determine the order of magnitude of dn (the electric dipole moment of a neutron) to be able to compare this theory with a superweak one. It is easy to see that the graph shown in Fig. 1 will give dominant contribution to d_n since the mixing between W_L^+ and W_R^+ which is proportional to k'k* becomes now complex. It will give rise to the CP violating amplitude of the form

$$d_n F_{\mu\nu} \bar{d} \gamma_5 \sigma^{\mu\nu} d \tag{3.18}$$

It is easy to give order of magnitude estimate of d_n[25]

$$d_n \simeq eG_F \, m_q \tan\xi \; \frac{\mathrm{Im}\, m_{W_L-W_R}}{\mathrm{Re}\, m_{W_L-W_R}} \tag{3.19}$$

where m_q is typical hadronic mass of order of 1 GeV (since we work with quarks it will be the mass of the charmed quark), ξ is the mixing angle between W_L^+ and W_R^+ and $m_{W_L-W_R}$ denotes the mixing matrix element. Since $\tan\xi$ is constrained experimentally to be not larger than approximately few percent, the present upper bounds on the electric dipole moment of the neutron imply[26]

$$\frac{\mathrm{Im}\, m_{W_L-W_R}}{\mathrm{Re}\, m_{W_L-W_R}} \lesssim 10^{-2} \tag{3.20}$$

Unless a natural way which would suppress either ξ or $\frac{\mathrm{Im}\, m_{W_L-W_R}}{\mathrm{Re}\, m_{W_L-W_R}}$ is found, we can say that pseudomanifest left-right symmetric theories predict large dipole moment of neutron, at the level of the upper experimental limits. In that respect they are easily distinguished from the superweak theory which predicts extremely small dn. If in the future more precise experiments push down the value of the electric dipole moment of the neutron, the safest way to save left-right symmetric theories would be to make W_L-W_R mixing real. That also serves to suppress naturally the strong CP violating effects, as we shall see in the following section.

2. Strong CP violation

Within the framework of Quantum Chromodynamics it has been established[27] that multiple vacuum structure of non-abelian gauge theories leads to strong P and T- non-invariant effects. The effective Lagrangian describing this effect can be written as

$$L_{eff} = c[\mathrm{Det}\ e^{i\theta}\ \bar{q}_L^o q_R^o + \mathrm{Det}\ e^{-i\theta}\ \bar{q}_R^o q_L^o] \quad . \tag{3.21}$$

Present upper limits on the electric dipole moment of neutron imply that $\theta \lesssim 10^{-9}$. In the absence of a convincing mechanism to understand such a small value of θ, it has been suggested by Peccei and Quinn[28] that it is best to introduce an extra global axial U(1) symmetry into the theory of weak, electromagnetic and strong interactions and remove θ altogether by a U(1) symmetry transformation. In such theories strong CP non-conservation is absent to all orders in the coupling constant. However, as has been noted by Weinberg[29] and Wilczek[30] in realistic models, the extra U(1) global symmetry is spontaneously broken, thus leading to a zero mass pseudo-Goldstone particle, the axion which acquires a tiny mass due to instanton effects. Given a gauge model of weak and electromagnetic interactions the coupling of the axion to matter can be predicted.[29,30,31] Present estimates of the strength of these couplings in $SU(2)_L \times U(1)$ models appear to be inconsistent[32] with the results from reactor and beam dump experiments. Thus, one must search for credible alternative to the idea of axions. One alternative is to have the up quark to be massless. This however, appears to be in conflict with our understanding of chiral symmetry breaking and therefore may be unacceptable.

Recently, Bég and Tsao[33] and R. N. Mohapatra and the author[34] have shown that in left-right symmetric gauge models that conserve P and T prior to spontaneous symmetry breakdown the problem of strong CP noninvariance can be solved without the need for axions or massless quarks. We will describe that work in this section. First let us notice that requirement of left-right symmetry of the entire Langrangian prior to symmetry breaking implies that there is no strong CP violation at this level.[33] Namely, as is obvious from eq. (3.21) under left-right conjugation: $\mathrm{Det}|\bar{q}^o_L q^o_R| \to \mathrm{Det}|\bar{q}^o_R q^o_L|$, therefore θ_o (the bare value of θ) must be equal to zero. We could expect naively that the problem is therefore solved. However, the story is not complete. Since we want to construct a theory which has weak CP violation, the unitary matrices that diagonalize the quark mass matrices will be complex, and unless they are unimodular, may induce θ. Namely, from eq. (3.11) we can easily obtain:

$$\mathrm{Det}|\bar{q}^o_L q^o_R| = \mathrm{Det}(U^+_{1L} U^+_{2L} U_{1R} U_{2R}) \times \mathrm{Det}|\bar{q}_L q_R| \tag{3.22}$$

where q_L and q_R denote the physical quark fields (eigenstates of quark mass matrices). On the other hand, we have:

$$U^{+}_{iL} M_i U_{iR} = D_i \tag{3.23}$$

where $M_i (i=1,2)$ are $T_3 = +\frac{1}{2}$ and $T_3 = -\frac{1}{2}$ quark mass matrices, and D_i denote their mass eigenvalues:

$$D_1 = \begin{pmatrix} m_u & o \\ o & m_c \end{pmatrix}, \qquad D_2 = \begin{pmatrix} m_d & o \\ o & m_s \end{pmatrix} \tag{3.24}$$

From eqs. (3.22), (3.23) we then obtain:

$$\mathrm{Det}\,|\bar{q}^o_L q^o_R| = \frac{\mathrm{Det}(M_1 \cdot M_2)^*}{|\mathrm{Det}\ M_1 M_2|}\ \mathrm{Det}\,|\bar{q}_L q_R| \tag{3.25}$$

It is now evident that the condition which sets $\theta=0$ at the tree level is equivalent to:

$$\frac{\mathrm{Det}\ M_1}{|\mathrm{Det}\ M_1|} = \frac{(\mathrm{Det}\ M_2)^*}{|\mathrm{Det}\ M_2|} \tag{3.26}$$

Although it can be arranged that thephase due to up quarks cancels the down quarks phase,[35] in this work we choose the simpler condition:

$$(\mathrm{Det}\ M_i)^* = \mathrm{Det}\ M_i \tag{3.27}$$

which automatically satisfies eq. (3.26). As a result, after the diagonalization of the fermion mass matrices, no strong CP-violating phase is introduced. Thus at the tree level $\theta=0$ <u>naturally</u>. We then compute the one loop contribution to the mass matrix. We show that, if the W^+_L-W^+_R mixing is real, the mass matrix including the one loop effects continues to satisfy the relation in eq. (3.27). Therefore, any nonzero contribution to θ can only arise at the two or higher loop level, <u>thus providing a natural suppression of strong CP non-invariance</u>.

We work first within the framework of four quarks and four leptons assigned as usual in a left-right symmetric manner to doublet representation of SU(2)'s. (Generalization to the case of six quarks is dealt with at the end.) To discuss spontaneous breakdown of P and T symmetries, we must specify the Higgs sector of the theory. We chose two Higgs multiplets that couple to the fermions: $\Phi_i = (\frac{1}{2},\frac{1}{2},0)$ $i = 1,2$; the rest of spontaneous symmetry breaking is achieved in the standard manner as described in chapter II. Underleft-right symmetry, the Φ's transform as:

$$\Phi_1 \to \Phi_1^+ , \quad \Phi_2 \to \tilde{\Phi}_2^+ \tag{3.28}$$

We now introduce additional discrete symmetry D, under which the fields transform as follows

$$\Phi_1 \to i\Phi_1, \; \Phi_2 \to \Phi_2$$

$$Q_{1L} \to Q_{1L}, \; Q_{1R} \to -iQ_{1R}$$

$$Q_{2L} \to -iQ_{2L}, \; Q_{2R} \to Q_{2R}$$

We do not discuss the leptons, since an arbitrary leptonic mass matrix can be generated in our model consistent with all symmetries. The invariant Yukawa couplings of Φ_i's to Q_i's are given by

$$
\begin{aligned}
L_Y = {} & a\bar{Q}_{1L}\Phi_1 Q_{1R} + b\bar{Q}_{2L}\tilde{\Phi}_1 Q_{2R} \\
& + c(\bar{Q}_{1L}\Phi_2 Q_{2R} + \bar{Q}_{2L}\tilde{\Phi}_2 Q_{1R}) \\
& + d(\bar{Q}_{1L}\tilde{\Phi}_2 Q_{2R} + \bar{Q}_{2L}\Phi_2 Q_{1R}) + \text{h.c.}
\end{aligned}
\tag{3.29}
$$

For a range of parameters in the Higgs potential, there exists a stable minimum energy configuration for which $\langle\Phi_1\rangle_{ij} = k_i\delta_{ij}$ and $\langle\Phi_2\rangle_{ij} = ke^{i\alpha}\delta_{ij}$ (k,k_i are real). Eq. (3.29) leads to the following quark mass matrices

$$
M_1 = \begin{pmatrix} ak_1 & k(ce^{i\alpha}+de^{-i\alpha}) \\ k(ce^{-i\alpha}+de^{i\alpha}) & bk_2 \end{pmatrix}
$$

$$
M_2 = \begin{pmatrix} ak_2 & k(ce^{i\alpha}+de^{-i\alpha} \\ k(ce^{-i\alpha}+de^{i\alpha}) & bk_1 \end{pmatrix}
\tag{3.30}
$$

These mass matrices are hermitean and therefore satisfy eq. (3.27). As a result $\theta=0$ even after spontaneous breakdown. The resulting gauge interactions have manifest left-right symmetry for charged currents. Incidentally these mass matrices lead to an interesting relation between quark masses i.e. $m_u m_c = m_d m_s$ and also the following relation between mixing angles for up and down quarks: $(\sin^2\theta_u/\sin^2\theta_d) = (m_s-m_d)/(m_c-m_u)$.

We should mention at this point that we have tried hard to construct a pseudo-manifestly left-right symmetric theory (as described in section III.) that has natural absence of strong CP violation up to one loop. Namely, a manifest theory characterized by hermitian mass matrices has equal left and right charged currents, since $U_L=U_R$. But that means $\theta_L=\theta_R$, $\delta_L=\delta_R$ so that

$\eta_{+-} = 0$ (see the previous section) and the only way to have a theory of CP violation is to achieve that CP violating amplitudes reside in the Higgs sector (unless one is willing to sacrifice oneself to enlarge a gauge group, which we are not). After many trials we failed in such an attempt. Actually, we believe that we will be able to prove that the theory that can account for the suppression of strong CP violation has to be a manifest left-right symmetric theory. That would be a somewhat discouraging result since we have seen in section III.1 that the pseudomanifest theory has many attractive features, especially that it links P and CP violation in nature and that it would provide an urgently needed upper bound on the mass of the right-handed charged gauge meson. Meanwhile, the problem is still open.

To discuss the one-loop effects, we let $\Delta e^{i\sigma} \equiv c\, e^{+i\alpha} + d\, e^{-i\alpha}$. Then by redefining $Q_2 \to e^{-i\sigma} Q_2$, $Q_1 \to Q_1$, we see that the mass matrix becomes real and symmetric and there is no CP-violation in the gauge interactions. Therefore, in our model any graph that involves only gauge-bosons and fermions will never induce θ. In particular, at the one-loop level, gauge boson contributions to θ vanish. To study the contribution of Higgs bosons to $\theta^{1\text{-loop}}$ as well as the nature of weak CP-violation, we write the Yukawa couplings in terms of the redefined quark fields:

$$\begin{aligned} L_Y = \; & a\bar{Q}_{1L}\Phi_1 Q_{1R} + b\bar{Q}_{2L}\tilde{\Phi}_1 Q_{2R} \\ & + c(e^{-i\sigma}\bar{Q}_{1L}\Phi_2 Q_{2R} + e^{i\sigma}\bar{Q}_{2L}\tilde{\Phi}_2 Q_{1R}) \\ & + d(e^{-i\sigma}\bar{Q}_{1L}\tilde{\Phi}_2 Q_{2R} + e^{i\sigma}\bar{Q}_{2L}\Phi_2 Q_{1R}) + \text{h.c.} \end{aligned} \tag{3.31}$$

It is then easily checked that the Yukawa interactions induce effective CP-violating four quark operators. Using eq. (3.31), we will now demonstrate that at the one-loop level even the Higgs-boson contributions do not induce strong CP-violation. We first note that all the divergent one loop graphs preserve the hermiticity of the quark mass matrices and thus do not induce θ. We then study the one loop graphs that are finite (see Fig. 2 for a typical graph). They involve mixing between the various Higgs bosons so we have to study the ϕ^4-type terms from the Higgs potential that mix the various Higgs bosons. For a typical term of this type $(\mathrm{Tr}(\Phi_1\Phi_j^\dagger\Phi_k\Phi_\ell^\dagger))$, the Higgs boson mixing looks like $\mathrm{Tr}\ \langle\Phi_i\rangle\ \langle\Phi_j^\dagger\rangle\ \Phi_k\ \Phi_\ell^\dagger$ + other possibilities. Since $\langle\Phi_2\rangle_{\mathrm{vac.}}$ is complex, such graph could add a finite complex contribution to the quark mass matrix. A detailed calculation, to be presented in a subsequent communication shows that all such graphs are real, or add complex contributions in such a way that quark mass matrices still satisfy eq. (3.27). There is actually a simple reason why it happens. The reason in that, under left-right conjugation, $\Phi_1 \to \Phi_1^\dagger$ and $\Phi_2 \to \tilde{\Phi}_2^\dagger$. Since $\langle\Phi_1\rangle = \langle\Phi_1^\dagger\rangle$ and $\langle\Phi_2\rangle = \langle\tilde{\Phi}_2^\dagger\rangle$, the left-right symmetry is not at all broken by $\langle\Phi_1\rangle$ and $\langle\Phi_2\rangle$. (It is of course broken by other Higgs bosons.) Thus, the one loop contributions we described respect manifest left-right symmetry. Therefore, they lead only to hermitean mass matrices. We, thus, prove that $\theta^{\text{1-loop}} = 0$. Thus, θ may only be induced at the two or higher loop level and is likely to be $\lesssim 10^{-8}$. Even at the two loop level, it must involve either a gauge meson plus a Higgs boson or two Higgs bosons. Since Yukawa couplings are usually $\sim g\,\frac{m_q}{m_W}$, the biggest contribution to θ at two loop level could be

$\sim g^2(G_F\, m_q^2)\sin\delta \approx 10^{-10}$, since the CP-violating phase δ must be $\sim 10^{-3}$ to account for CP-violation in K^o-decay.

Our theory of CP-violation is superweak[36] type. The reason is that, after diagonalizing the fermion mass matrix, we obtain effective $\Delta S=2$ CP-conserving $H_W^{(+)}$ and CP-violating $H_W^{(-)}$ Hamiltonians. The effective coupling for $H_W^{(+)}$ is $\frac{G_F m_q^2}{m_\phi^2}$, where m_ϕ is the typical mass of neutral Higgs bosons. We have assumed $m_\phi \approx 300$ GeV to suppress its contribution to K_1-K_2 mass difference. The strength of $H_W^{(-)}$ is $G_F \frac{m_q^2}{m_\phi^2} \sin\delta$. Thus, if we assume $\delta \approx 10^{-3}$, the effective $H_W^{(-)}$ is of superweak type.[37]

A six quark extension: To extend our considerations to the case of six quarks, we add a third quark doublet $Q_3 = (p_3,n_3)$ and define the action of the extra discrete symmetry on Q_3 as follows: $Q_{3L} \to -iQ_{3L}$ and $Q_{3R} \to Q_{3R}$. The invariant Yukawa coupling is given by

$$\begin{aligned} L_Y = {} & a\bar{Q}_{1L}\Phi_1 Q_{1R} + b\bar{Q}_{2L}\tilde{\Phi}_1 Q_{2R} + h\bar{Q}_{3L}\tilde{\Phi}_1 Q_{3R} \\ & + c(\bar{Q}_{1L}\Phi_2 Q_{2R} + \bar{Q}_{2L}\tilde{\Phi}_2 Q_{1R}) + d(\bar{Q}_{1L}\tilde{\Phi}_2 Q_{2R} + \bar{Q}_{2L}\Phi_2 Q_{1R}) \\ & + c'(\bar{Q}_{1L}\Phi_2 Q_{3R} + \bar{Q}_{3L}\tilde{\Phi}_2 Q_{1R}) + d'(\bar{Q}_{1L}\tilde{\Phi}_2 Q_{3R} + \bar{Q}_{3L}\Phi_2 Q_{1R}) \\ & + f(\bar{Q}_{2L}\tilde{\Phi}_1 Q_{3R} + \bar{Q}_{3L}\tilde{\Phi}_1 Q_{2R}) + \text{h.c.} \end{aligned} \tag{3.32}$$

It produces the following quark mass matrices

$$M_1 = \begin{pmatrix} ak_1 & k(ce^{i\alpha}+de^{-i\alpha}) & k(c'e^{i\alpha}+d'e^{-i\alpha}) \\ k(ce^{-i\alpha}+de^{i\alpha}) & bk_2 & fk_2 \\ k(c'e^{-i\alpha}+d'e^{i\alpha}) & fk_2 & hk_2 \end{pmatrix}$$

$$M_2 = \begin{pmatrix} ak_2 & k(ce^{i\alpha}+de^{-i\alpha}) & k(c'e^{i\alpha}+d'e^{-i\alpha}) \\ k(ce^{-i\alpha}+de^{i\alpha}) & bk_1 & fk_1 \\ k(c'e^{-i\alpha}+d'e^{+i\alpha}) & fk_1 & hk_1 \end{pmatrix} \tag{3.33}$$

Since they are hermitian they lead to $\theta=0$ naturally at the tree level. Moreover, each mass matrix has two complex off-diagonal elements with different phases. Therefore, unlike the four quark case, the CP-violation cannot be removed from the theory by redefinition of the phase of the quark fields. Thus, in this model, CP-violation resides in the gauge boson interaction. This provides a left-right symmetric generalization of the Kobayashi-Maskawa[12] model. Again in this case, we verify that the $\theta^{1\text{-loop}}=0$. In this model CP-violating phase need not be very small; to account correctly for $K_L^0 \to 2\pi$ decay it is predicted[38] to be of order 10^{-1}.

IV. Cabibbo angle, CP-violation and quark masses

One of the fundamental problems in the theory of weak-interactions is to understand the origin of Cabibbo angle, the mixing angle between strange and non-strange quark flavors. In most unified gauge theories of weak and electromagnetic interactions, there exist one or more such flavor mixing angles, which remain as arbitrary and undetermined parameters. Often these mixing angles are taken to be complex in order to account for the phenomenon of CP-violation.[39] The magnitude of CP-violation remains arbitrary. Recently, a number of theorists[40] have discussed one possible approach to the problem, in which extra discrete symmetries are imposed on the gauge models of weak and electromagnetic interactions to restrict the number of arbitrary parameters that appear in the fermion mass matrix thereby yielding zeroth order relations between Cabibbo angle and quark-masses.[41] Reasonable values for the Cabibbo angle can be obtained by appropriate choice of the quark masses. The CP-violating phase, however, remains a free parameter in these theories.

We review here briefly the approach of Weinberg.[40] He has succeeded by the use of discrete symmetries to achieve the following form of the quark mass matrices

$$M_i = \begin{pmatrix} o & a_i \\ a_i & b_i \end{pmatrix} \tag{4.1}$$

where a_i and b_i are real numbers, and $i = 1,2$ denotes $T_3 = \frac{1}{2}$ and $T_3 = -\frac{1}{2}$ quarks, respectively.

The apparent trouble of the above mass matrices is that their determinants are negative: det $M_i < 0$. But Weinberg notices that by a chiral transformation one can always change a sign of a fermionic mass term, so that its sign has no physical meaning. The form of (4.1) is particularly interesting since it is through the mixings a_i that the light quark flavors u and d pick up their masses; if not for the mixing they would be massless. If we diagonalize above mass matrices by orthogonal transformations

$$O_i^T M_i O_i = D_i \tag{4.2}$$

where

$$D_i = \begin{pmatrix} m_u & o \\ o & m_c \end{pmatrix}, \quad D_2 = \begin{pmatrix} m_d & o \\ o & m_s \end{pmatrix}$$

$$O_i = \begin{pmatrix} \cos\theta_i & \sin\theta_i \\ -\sin\theta_i & \cos\theta_i \end{pmatrix} \tag{4.3}$$

we easily obtain

$$\tan^2\theta_1 = \frac{m_u}{m_c}, \quad \tan^2\theta_2 = \frac{m_d}{m_s} \tag{4.4}$$

That completely determines Cabibbo angles as a function of light quark masses

$$\theta_c = \theta_2 - \theta_1 = \tan^{-1}\left(\frac{m_d}{m_s}\right)^{1/2} - \tan^{-1}\left(\frac{m_u}{m_c}\right)^{1/2} \tag{4.5}$$

The usual current algebra estimates given then good agreement with the experimentally determined Cabibbo angle: $\theta_c \simeq 13°$.

Here we will describe the attempt by Mohapatra and the author[13] to extend the above approach in such a way as to determine both the real mixing angles as well as the CP-violating phases in terms of the quark masses. We work within an $SU(2)_L \times SU(2)_R \times U(1)$ gauge model of weak and electromagnetic interactions based on four (light) quark-flavors. The extra (heavy) quarks to be included will be chosen as not to mix with the light quarks by imposing extra global symmetries on the theory.

We recall the important relation given in section III.4

$$\eta_{+-} \simeq \sin 2\delta \frac{M^2_{W_L}}{M^2_{W_R}} \tag{4.6}$$

which holds true in a pseudomanifestly symmetric left-right symmetric theories characterized by: $\theta_R = \theta_L$, $\delta \equiv \delta_R = -\delta_L$. A determination of δ which we seek would then lead to a direct relation between P and CO violation.

Our main results are the following: In an $SU(2)_L \times SU(2)_R \times U(1)$ theory with four quark flavors, there are in general two real mixing angles and three phases for each quark charge thus leading to ten arbitrary parameters. As we have shown in section III.1, if we choose the Higgs mesons $\Phi(\frac{1}{2},\frac{1}{2},0)$ to transform under left-right symmetry as $\Phi \leftrightarrow \Phi^+$ and ignore W_L-W_R mixing effects then the number reduces to a total of two real angles and two phases. We will display discrete flavor symmetries, that enable us to obtain four "natural" zeroth order equations relating them to the quark-masses, thus determining all the parameters in terms of the quark masses. Higgs multiplets necessary to give the desired pattern of symmetry breaking are chosen as follows

$$\Phi(\tfrac{1}{2},\tfrac{1}{2},0)\,, \quad \tilde{\phi} \equiv \tau_2\phi^*\tau_2$$

$$\chi_L^i = (\tfrac{1}{2},0,1),\ \chi_R^i = (0,\tfrac{1}{2},1),\ i = 1,2\ .$$

The following discrete symmetries are imposed on the Langrangian

(i) Left-right ($L\leftrightarrow R$) symmetry defined in the usual manner

$$f_{iL} \leftrightarrow f_{iR}$$

$$\Phi \leftrightarrow \Phi^+, \quad \tilde{\Phi} \leftrightarrow \tilde{\Phi}^+, \quad \chi_L^i \leftrightarrow \chi_R^i \tag{4.7}$$

(f_i are fermionic multiplets)

(ii) Discrete symmetry D

$$Q_{1L} \to i\, Q_{1L}, \quad Q_{1R} \to Q_{1R}$$

$$Q_{2L} \to -i\, Q_{2L}, \quad Q_{2R} \to -Q_{2R}$$

$$\Phi \to i\Phi, \quad \tilde{\Phi} \to -i\tilde{\Phi}$$

$$\chi_L^1 \to i\chi_L^1\,, \quad \chi_R^1 \to \chi_R^1$$

$$\chi_L^2 \to i\chi_L^2\,, \quad \chi_R^2 \to -\chi_R^2 \tag{4.8}$$

other fields remain unchanged (Q_i are quark doublets, as usual). Consistent with the above discrete symmetries, the following vacuum expectation values for the Higgs potential emerges for a range of values of the parameters in the Higgs potential

$$\langle\Phi\rangle = \begin{pmatrix} k_1 e^{i\alpha_1} & 0 \\ 0 & k_2 e^{i\alpha_2} \end{pmatrix} ;$$

$$\langle\chi_L^i\rangle = \begin{pmatrix} 0 \\ \lambda_L^i \end{pmatrix} \quad \text{and} \quad \langle\chi_R^i\rangle = \begin{pmatrix} 0 \\ \lambda_R^i \end{pmatrix} \tag{4.9}$$

Our main results follow from considerations of the quark mass matrix, which is derived from the most general Yukawa Coupling (L_Y) involving quarks[43] invariant under the gauge symmetry and the discrete symmetries given in eq. (4.7) and (4.8)

$$L_Y = a\bar{Q}_{1L}\Phi Q_{1R} + b\bar{Q}_{2L}\Phi Q_{2R} + c(\bar{Q}_{1L}\tilde{\Phi}Q_{2R} + \bar{Q}_{2L}\tilde{\Phi}Q_{1R}) \tag{4.10}$$

The quark mass matrices for the quark flavors of charge $+\frac{2}{3}$ (called M_1) and charge $-\frac{1}{3}$ (called M_2) can be written down using eq. (4.9) and (4.10)

$$M_1 = \begin{pmatrix} ak_1 d^{i\alpha_1} & ck_2 e^{-i\alpha_2} \\ ck_2 e^{-i\alpha_2} & bk_1 e^{i\alpha_1} \end{pmatrix}$$

$$M_2 = \begin{pmatrix} ak_2 e^{i\alpha_2} & ck_1 e^{-i\alpha_1} \\ ck_1 e^{-i\alpha_1} & bk_2 e^{i\alpha_2} \end{pmatrix} \tag{4.11}$$

It is easily seen that these matrices can be diagonalized by the following bi-unitary transformation[43] (D_1 and D_2 are the diagonal mass matrices for the up and down quark sector)

$$U_{iL}^{+} M_i U_{iR} = D_i \tag{4.12}$$

where

$$U_{iL} = e^{i\phi_i} \begin{pmatrix} \cos\theta_i & e^{i\delta_i} \sin\theta_i \\ -e^{-i\delta_i} \sin\theta_i & \cos\theta_i \end{pmatrix} \tag{4.13}$$

and

$$U_{iR} = U_{iL}^{+} \tag{4.14}$$

Eq. (4.12)-(4.14) lead to the following relations between the quark masses m_u, m_d, m_s and m_c and the mixing angles θ_i and phases δ_i (i = 1,2)

$$\delta_1 = \delta_2 \equiv \delta \tag{4.15}$$

$$2m_d m_s \sin^2\theta_2 \cos 2\delta + (m_s^2 + m_d^2)\cos^2\theta_2 = 0 \tag{4.16a}$$

$$2m_u m_c \sin^2\theta_1 \cos 2\delta + (m_c^2 + m_u^2)\cos^2\theta_1 = 0 \tag{4.16b}$$

$$\frac{\sin 2\theta_1}{\sin 2\theta_2} = \frac{m_s^2 - m_d^2}{m_c^2 - m_u^2} \tag{4.16c}$$

Eq. (4.15), (4.16a) - (4.16c) allow us to determine θ_1, θ_2 and δ in terms of the quark masses

$$\cos^2\theta_1 = \frac{1 - A}{1 - B} \; ; \qquad \cos^2\theta_2 = \frac{B}{A} \left(\frac{1 - A}{1 - B}\right) \tag{4.17}$$

where

$$B = \frac{m_s m_d}{m_c m_u} \frac{m_c^2 + m_u^2}{m_s^2 + m_d^2} \; ; \qquad A = \sqrt{B} \; \frac{m_s^2 - m_d^2}{m_c^2 - m_u^2} \tag{4.18}$$

$$\cos 2\delta = - \frac{m_s^2 + m_d^2}{2m_d m_s} \cot^2\theta_2 \tag{4.19}$$

The relation between Cabibbo angle θ_c and the angles θ_1 and θ_2 is obtained by writing the left and right-handed charged currents $J^+_{\mu,L} = \bar{P}\gamma_\mu(1-\gamma_5)U_L N$ and $J^+_{\mu,R} = \bar{P}\gamma_\mu(1+\gamma_5)U_R N$ with $P = (u,c)$ and $N = (d,s)$

$$U_L = U^+_{1L}\ U_{2L} = e^{i(\phi_2-\phi_1)} \begin{pmatrix} \cos(\theta_2-\theta_1) & \sin(\theta_2-\theta_1)e^{i\delta} \\ -\sin(\theta_2-\theta_1)e^{-i\delta} & \cos(\theta_2-\theta_1) \end{pmatrix} \tag{4.20}$$

with $U^*_L = U_R$. It thus follows that $\theta_c = \theta_2-\theta_1$. The phase $\phi_1-\phi_2$ can be gotten rid of by redefining the fields $W^{\pm}_{L,R}$ and becomes insignificant in the limit of small W_L-W_R mixing.

We now proceed to present estimates of Cabibbo angle θ_c and the CP-violating phase δ using plausible values of quark masses. We first remark that, within the framework of Quantum Chromodynamics, there is a considerable amount of uncertainty in the values of the light quark masses that enter the weak interaction Lagrangian. It is, however, generally believed that, a hierarchy of the type $m_c > m_s >> m_d > m_u$, where u and d quark masses close to a few MeV's and c and s quark masses of the order of a few hundred MeV's is probably close to the truth. Working in this spirit, we show in fig. 3. the values of the mass ratios (m_c/m_s) and (m_d/m_u) which yield $\theta_c \simeq .23$. It turns out that, to satisfy eq. (4.17), we must have $A \geqslant 1$ which leads to the following inequality among quark masses in order for our considerations to make sense

$$(m_d/m_u) \geqslant (m_c/m_s)^3 \tag{4.21}$$

Despite its appearance, for $m_d/m_s \simeq 0$ and $m_u/m_s \simeq 0$, there is no contradiction between current algebra results for πN scattering length, pseudoscalar meson masses and eq. (4.21). For example, if $\langle 0|\bar{u}u|0\rangle = \langle 0|\bar{d}d|0\rangle$, iso-spin relation $\mu^2(\pi^+) = \mu^2(\pi^o)$ holds, regardless of the value of (m_d/m_u). Similarly, Weinberg estimates[40] that for πN scattering lengths, the only reliable prediction of theory is

$$a(\pi^o n \to \pi^o n) - a(\pi^o p \to \pi^o p) \simeq \frac{m_N(m_\Xi - m_\Sigma)}{\pi(m_N + m_\pi)f_\pi^2} \left(\frac{m_d - m_u}{m_s}\right) \tag{4.22}$$

This prediction also does not put severe constraints on (m_d/m_u), thereby allowing us the freedom to satisfy eq. (4.21). We have a range of values for $m_d/m_u \simeq 2.6$, $m_c/m_s \simeq 1.2$ to $m_d/m_u \simeq 9.5$, $m_c/m_s \simeq 2$ which yield $\theta_c \simeq .23$ (see fig. 3). The value of m_c is clearly smaller than the effective charm quark mass m_c^{eff} that enters the charmonium claculations but, the relation between m_c and m_c^{eff} is far from clear at this stage of the theory. We, therefore, feel that the range of quark mass ratios given above (fig. 3) is quite acceptable. The predcitions for $\sin 2\delta$ depend on the choice of the mass ratio m_s/m_d. In Fig. 4, we present $\sin 2\delta$ as a function of m_d/m_u for the choice of $m_s/m_d = 10$.

To conclude, we would like to emphasize that, within left-right symmetric $SU(2)_L \times SU(2)_R \times U(1)$ theories, it is possible to predict the Cabibbo angle and the CP-violating amplitude using only the values of light quark mass ratios. Finally, we comment that there are ways to accommodate heavier quark flavors within our framework without altering the results; one way is to decouple the heavier quark flavors from the lighter ones.[44] Another way is to go to higher groups.[45]

V. The question of neutrino mass in left-right symmetric gauge theories

One of the outstanding problems in the theory of weak interactions is the understanding of the neutrino mass. All present experimental evidence is consistent with massless neutrinos and yet we lack a deep theoretical understanding of why this is the case. There is a simple theoretical principle which prevents the neutrinos from acquiring mass, namely γ_5 invariance. That is to say, the masslessness of the neutrinos is guaranteed in an *ad hoc* manner by only introducing their left-handed components. This is exactly the situation one encounters in the standard model. On the other hand, in left-right symmetric models, one has neutrinos with both left-handed and right-handed helicities and the challenge is then to understand in a natural manner the bounds on neutrino masses. Usually in these theories the neutrino masses are kept small by arbitrary choice of the free parameters of the underlying Lagrangian.

With G. C. Branco we have addressed ourselves to the problem of the neutrino mass in theories where both left-handed and right-handed neutrinos are present.[14] Our aim was twofold: (i) to derive the constraints on these models which follow from the requirement of calculability of the neutrino mass; (ii) to construct a realistic left-right symmetric gauge theory in which the neutrinos are massless (to all orders in perturbation theory). We have succeeded in both and our results can be most simply stated as follows: calculability of the neutrino mass requires an "orthogonal" structure for the left-handed and right-handed charged currents. We now describe our work.

As we know from section II, we must introduce fields $\Phi_i = (\frac{1}{2}, \frac{1}{2}^*, 0)$ $(i=1,2,\ldots)$ in order not to have all the fermions massless. We remind the reader that we need a minimum of two Φ's in order to have different left and right Cabibbo like angles (see from example ref. 15). It is clear that we should choose a pttern of symmetry breaking such that ν_e, ν_μ have vanishing mass at the tree level. Since only Φ_i can couple to fermions, this can be achieved in two different ways:[46]

(i) By forbidding any coupling of Φ_i to the lepton doublets ℓ_1, ℓ_2 (this could be arranged, for example, by introducing an extra discrete symmetry).

(ii) By requiring that the neutral Higgs which couple to the neutrinos have zero vacuum expectation values, i.e.

$$\langle \Phi_j \rangle_{vac.} = \begin{pmatrix} o & o \\ o & k_j \end{pmatrix} \tag{5.1}$$

where Φ_j stand for Higgs multiplets with non-vanishing coupling to ℓ_1, ℓ_2.

Case (i) would lead to a situation in which not only the neutrinos but also the electron and muon are massless in tree approximation. Here, we will concentrate on case <u>(ii) in which the neutrinos have vanishing mass in tree approximation, while the electron and muon acquire mass through the Yukawa couplings to Higgs fields with non-zero vacuum expectation values</u>. Once we have obtained vanishing neutrino masses at the zeroth order level, the next step will be to investigate if higher order corrections can give neutrino masses. It is clear that this question is intimately related to the problem of W_L-W_R mixing. To illustrate the connection, let us assume that there is at least one Higgs field Φ_i such that $\langle \mathrm{tr}\, \Phi_i^{\dagger}\ \tau_2\ \Phi_i^{*}\ \tau_2 \rangle_{vac} \neq 0$. This would imply a non-vanishing W_L-W_R mixing at the tree level, and one would then expect the neutrino mass to arise at one loop level through the diagram of Fig. 5. The evaluation of this diagram gives

$$\frac{m_\nu}{m_e} = \frac{\alpha}{\pi} \sin 2\xi\ \ell n \frac{M^2_{W_R}}{M^2_{W_L}} \tag{5.2}$$

where ξ is the mising angle between $W_L^{\pm}$ and $W_R^{\pm}$. One could then naively think that such a scheme would lead to a calculable and naturally small neutrino mass, the extra suppression factor (besides α) being given by the necessarily small mixing angle. However, the pioneering work of Georgi and Glashow[46] tells us that this attempt is doomed to failure, since the diagram of Fig. 5 cannot lead to a calculable neutrino mass. The reason can be easily seen from the diagram of Fig. 6 which contributes to $\langle \Phi_j^o \rangle_{vac.}$, where Φ_j^o is a neutral Higgs with non-vanishing coupling to $\bar{\nu}_L\ \nu_R$. The contribution of this diagram to $\langle \Phi_j^o \rangle_{vac.}$ is infinite and therefore it must be cancelled by a counter term. This in turn implies that we should have $\langle \Phi_j^o \rangle_{vac.} \neq 0$ at the tree level and therefore the neutrino mass becomes a free parameter of the theory. Since this is exactly the situation one wants to avoid, one obtains the following constraint on the pattern of symmetry breaking

$$\langle 0 | \mathrm{tr}\ \Phi_i^{+}\ \tau_2\ \Phi_i^{*}\ \tau_2 | 0 \rangle_{tree} = 0 \tag{5.3}$$

where subscript i refers to all Φ's. (Summation over i is not assumed). Eq. (5.3) implies that there is no W_L-W_R mixing at the tree level. It turns out that this constraint can only be satisfied in a rather restricted class of left-right symmetric models. This can be easily seen from the diagram of Fig. 7. This diagram contributes to W_L-W_R mixing and since it is infinite, it requires a counter term. One is then again led to the situation in which there is W_L-W_R mixing at the tree level. Since we have shown that this is incompatible with calculability of the neutrino mass, we are naturally led to the following constraint: the left-handed and right-handed charged weak currents should be "orthogonal" to each other, when written in terms of the physical particles. In other words, for any given up quark (Q = 2/3) the left- and right-handed charged currents should couple it to different down quarks (Q = - 1/3). Since the currents $(\bar{u}\, d(\theta_c))_L$, $(\bar{c}\, s(\theta_c))_L$ are known to be present, it is clear that this constraint requires the introduction of a minimum of 8 quark flavors. It is then possible to arrange the flavor mixing in such a way that the diagram of Fig. 7 is not present, and therefore to avoid the previous difficulties. We now present our model.

The fermions are assigned in the following manner

Quarks

$$\begin{pmatrix} u \\ d(\theta) \end{pmatrix}_L , \begin{pmatrix} c \\ s(\theta) \end{pmatrix}_L , \begin{pmatrix} t_1 \\ b_1(\phi) \end{pmatrix} , \begin{pmatrix} t_2 \\ b_2(\phi) \end{pmatrix}$$

$$\begin{pmatrix} u \\ b_1(\alpha) \end{pmatrix}_R , \begin{pmatrix} c \\ b_2(\alpha) \end{pmatrix}_R , \begin{pmatrix} t_1 \\ d(\beta) \end{pmatrix}_R , \begin{pmatrix} t_2 \\ s(\beta) \end{pmatrix}_R \qquad (5.4)$$

Leptons

$$\begin{pmatrix}\nu_e\\ e\end{pmatrix}_L , \quad \begin{pmatrix}\nu_\mu\\ \mu\end{pmatrix}_L , \quad \begin{pmatrix}M^o\\ M^-(\phi')\end{pmatrix}_L , \quad \begin{pmatrix}N^o\\ N^-(\phi')\end{pmatrix}_L$$

$$\begin{pmatrix}\nu_e\\ M^-\end{pmatrix}_R , \quad \begin{pmatrix}\nu_\mu\\ N^-\end{pmatrix}_R , \quad \begin{pmatrix}M^o\\ e(\beta')\end{pmatrix}_R , \quad \begin{pmatrix}N^o\\ \mu(\beta')\end{pmatrix}_R \tag{5.5}$$

where $\theta,\phi,\alpha,\beta,\phi'$ and β' are mixing angles for the pairs (d_L, s_L), (b_{1L}, b_{2L}), (b_{1R}, b_{2R}), (u_R, c_R), (M_R^-, N_R^-) and (e_R, μ_R), respectively.

The charged currents in left and right sectors are then manifestly "orthogonal"[47] to each other.

The mechanism of obtaining the above fermionic charged currents "naturally" was given in ref. 47. One first notices that it requires the following quark mass matrices (we discuss leptons later)

$$M^u = \begin{pmatrix}M^u_{ij} & o\\ o & M^u_{\alpha\beta}\end{pmatrix} , \quad M^d = \begin{pmatrix}o & M^d_{i\alpha}\\ M^d_{\beta i} & o\end{pmatrix} \tag{5.6}$$

where the superscripts u and d denote up $(Q = \frac{2}{3})$ and down $(Q = -\frac{1}{3})$ quarks respectively; $i = 1,2$ counts the familiar light quark flavors u, d, s and c and $\alpha = 3,4$ stands for heavy quarks t_1, t_2, b_1 and b_2. Now the above matrices are in turn diagonalized by bi-orthogonal transformations (we assume M^u and M^d to be real; this is we ignore the question of CP-violation)

$$\begin{aligned} O_L^{uT} \; M^u \; O_R^u &= D^u \\ O_L^{dT} \; M^d \; O_R^d &= D^d \end{aligned} \tag{5.7}$$

where

$$O_L^{u,d} = \begin{pmatrix} V(\theta_L^{u,d}) & o \\ o & V(\phi_L^{u,d}) \end{pmatrix} \; ; \quad O_R^u = \begin{pmatrix} V(\theta_R^u) & o \\ o & V(\phi_R^u) \end{pmatrix} \; ; \; O_R^d = \begin{pmatrix} o & V(\theta_R^d) \\ V(\phi_R^d) & o \end{pmatrix} \tag{5.8}$$

and V are orthogonal 2x2 matrices. Comparison with eq. (5.4) tells us that we have achieved our talk with the identification: $\theta_c = \theta_L^d - \theta_L^u$, $\phi = \phi_L^d - \phi_L^u$, $\alpha = \theta_R^d - \theta_R^u$ and $\beta = \phi_R^d - \phi_R^u$. Using a discrete symmetry of the type of ref. 50 one can consistently with the vanishing W_L-W_R mixing naturally obtain the above mass matrices. However, such discrete symmetry is spontaneously broken (as we shall see later) by the vacuum expectation values of Higgs scalars and there is no way to guarantee that W_L-W_R mixing does not arise as a result of higher order effects. On the other hand, we failed to find any graphs that could induce W_L-W_R mixing or equivalently neutrino mass. In other words, the question of neutrino mass would remain unanswered in such a model. In view of that, we have tried and succeeded to find <u>a discrete symmetry that would not be spontaneously broken by the tree level vacuum expectation values of the Higgs scalars and would imply a vanishing neutrino mass to all orders in perturbation theory</u>. We now discuss our model in detail. A minimal number of 6Φ multiplets is needed, three of them which couple to quarks and three of them that couple to leptons. We impose a discrete symmetry upon the Lagrangian

D

$$q_{iL} \to q_{iL} \; ; \quad \ell_{iL} \to \ell_{iL} \; ; \quad \Phi_1^q \to \Phi_1^q \; ; \quad \Phi_1^\ell \to e^{-i\frac{3\pi}{4}} \Phi_1^\ell$$

$$q_{\alpha L} \to e^{i\frac{\pi}{2}} q_{\alpha L}; \; \ell_{\alpha L} \to e^{-i\pi} \ell_{\alpha L}; \; \Phi_2^q \to e^{-i\frac{\pi}{2}} \Phi_2^q; \; \Phi_2^\ell \to e^{i\frac{\pi}{4}} \Phi_2^\ell$$

$$q_{iR}\to q_{iR};\ \ell_{iR}\to e^{-i\frac{3\pi}{4}}\ell_{iR};\ \Phi_3^q\to e^{i\frac{\pi}{2}}\Phi_3^q\ ;\ \Phi_3^\ell\to e^{-i\frac{\pi}{4}}\Phi_3^\ell$$

$$q_{\alpha R}\to e^{i\frac{\pi}{2}}q_{\alpha R};\ \ell_{\alpha R}\to e^{-i\frac{\pi}{4}}\ell_{\alpha R}; \tag{5.9}$$

where q and ℓ denote quark and leptonic doublets; i = 1,2 refers to light fermionic flavors and α =3,4 to heavy fermions and the subscripts q or ℓ on Φ, stand for Higgs scalars that couple to quarks and leptons, respectively.

The relevant fields have the following transformation properties under left-right conjugation

$$q_L\leftrightarrow q_R\ ;\ \ \ell_L\leftrightarrow\ell_R$$

$$\Phi_1^f\leftrightarrow\Phi_1^{f^\dagger};\ \ \Phi_2^f\leftrightarrow\Phi_3^{f^\dagger}\ ;\ \ \Phi_3^f\leftrightarrow\Phi_2^{f^\dagger};\ f\equiv\ell,q\ . \tag{5.10}$$

The above discrete symmetries restrict the Yukawa couplings to be of the form[48]

$$\begin{aligned}L_Y = {}& a_{ij}\bar{q}_{iL}\Phi_1^q q_{jR} + b_{\alpha\beta}\bar{q}_{\alpha L}\Phi_1^q q_{\beta R} + c_{i\alpha}(\bar{q}_{iL}\Phi_2^q q_{\alpha R} + \bar{q}_{\alpha L}\Phi_3^q q_{iR}) + \\ & + b'_{\alpha\beta}\bar{\ell}_{\alpha L}\Phi_1^\ell\ell_{\beta R} + c'_{i\alpha}(\bar{\ell}_{iL}\Phi_2^\ell\ell_{\alpha R} + \bar{\ell}_{\alpha L}\Phi_3^\ell\ell_{iR}) + \text{h.c.}\end{aligned} \tag{5.11}$$

It can be shown that the following pattern of symmetry breaking emerges as a result of the minimization of the Higgs potential (for a range of the parameters)

$$\langle\Phi_1^f\rangle_{\text{vac.}} = \begin{pmatrix} k_1^f & o \\ o & o\end{pmatrix}\ ;\quad \langle\Phi_{2,3}^f\rangle_{\text{vac.}} = \begin{pmatrix} o & o \\ o & k_{2,3}^f\end{pmatrix} \tag{5.12}$$

which in turn implies the desired form of the fermion mass matrices. Notice that ν_e, ν_μ are massless in tree approximation, since there are no couplings of the type $\bar{\ell}_{iL}\ \Phi\ell_{jR}$. However, as we mentioned before the discrete symmetry D is spontaneously broken and therefore it is not clear at this point whether this appealing scenario remains unaltered in higher order perturbation theory. Therefore, we further restrict the Higgs potential so that the Lagrangian is invariant under the additional discrete symmetry (the rest of the Lagrangian is automatically invariant)

R

$$f_{iL} \to f_{iL}\ ;\quad f_{iR} \to \tau_3 f_{iR}\ ;\ f_{\alpha L} \to -f_{\alpha L}\ ;\ f_{\alpha R} \to -\tau_3 f_{\alpha R}$$

$$\Phi_1^f \to \Phi_1^f \tau_3$$

$$\Phi_{2,3}^f \to -\Phi_{2,3}^f \tau_3 \tag{5.13}$$

where

$$\tau_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \tag{5.14}$$

In order to complete our program we have checked that the pattern of symmetry breaking shown in (5.12) is consistent with the introduction of the new symmetry. <u>We emphasize again that since the symmetry R is not broken by the vacuum expectation values (5.12) (and only by those), the pattern of symmetry breaking will be unchanged to all orders in perturbation theory</u>. This follows from the Georgi-Pais theorem:[49] If a Lagrangian is invariant under a discrete symmetry R and if the vacuum expectation values in tree approximation do not violate R, then the vacuum expectation values in higher order will still respect R (provided that there is no accidental symmetry present).[50] In conclusion, we have succeeded to construct a realistic gauge theory in which, in conformity with left-right symmetry, ν_e and ν_μ are four component Dirac particles, and yet they are massless. Before closing this section, we remark upon an interesting feature of our theory. Although the general analysis of Glashow and Weinberg,[51] tells us that the neutral Higgs exchange cannot possibly conserve all the flavors, it is remarkable that the light fermionic flavors are exactly conserved (we will not present the rather simple proof of this fact). In other words, despite the fact that our Higgs sector is rather complicated, in lowest order Higgs exchanges naturally conserve both strangeness and charm.

In conclusion, we have presented here the various conditions that a left-right symmetric theory must satisfy in order to have a calculable neutrino mass and we have exhibited a minimal model where neutrinos are massless. One of the conclusions of our analysis is that manifest left-right symmetric theories are incompatible with a calculable neutrino mass. Our conclusion was based on the rather reasonable assumption that the electron and muon acquire mass through the coupling with neutral Higgs fields with non-zero vacuum expectation value. Since manifestly left-right symmetric theories have great aesthetical appeal, we would like to offer some speculations on the possibility (i) mentioned on page 47. That is, we will consider the case in which the muon and electron (as well as their neutrinos) are massless in tree approximation. In this case there is no obvious conflict between equal left and right flavor mixing angles and calculability of the neutrino mass. However, one faces the problem of finding a mechanism which can generate the muon and electron masses in higher orders. We have not fully analyzed this question but it is clear that in order to give mass to μ and e one would probably have to enlarge the gauge group beyond $SU(2)_L \times SU(2)_R \times U(1)$. Neverthelsss, once this non-trivial problem is solved, the rather interesting relation between m_e and m_{ν_e} (similarly for m_μ, ν_μ) expressed by eq. (5.2), will then be valid, since one would no longer face the danger of graph in fig. 6 (this graph would not be present).

V. Summary and conclusions

In this work we have discussed and emphasized some of the basic properties of left-right symmetric theories:

(i) The initially completely left-right symmetric theory allows the pattern of symmetry breaking that violates parity at the level which is dictated by V-A structure of β decay.

(ii) The minimal such theory (minimal in the sense of the choice of the gauge group and the Higgs assignment within it) in the limit of pure V-A charged weak currents, agrees completely with the standard model both in the realm of charged and neutral currents. The corrections due to finite amount of V+A currents in this model are small and are in agreement with the experiment. That has important consequence that Weinberg-Salam theory cannot be singled out at present energies as a candidate for a unified gauge theory of weak and electromagnetic interactions.

(iii) The same minimal theory can account for the CP-violation in neutral kaon system. More than that, it achieves that in such a way as to link the amounts of CP and parity violations in nature. Although similar to superweak theories of CP violations in the sense that it predicts $\eta_{+-} = \eta_{oo}$ exactly, it differs from them in the prediction of the magnitude of electric dipole moment of neutron: whereas it is negligible in superweak theories, here it is predicted to be generally at the level of the upper experimental bound. More precise experiments are obviously urged for.

(iv) Both Cabibbo angle and the CP violating phase can in such theories be expressed as functions of light quark mass ratios only. Current algebra estimates lead then to acceptable numerical results.

(v) The strong CP-violation due to instanton effects can be naturally suppressed in left-right symmetric theories without involving the existence of a light scalar particle, axions or a massless quark. It is essentially due to the fact that P and CP are softly broken, so that potentially dangerous instanton effects which violate both parity and time reversal can be controlled and therefore avoided.

The burning question is then: How can we distinguish left-right symmetric and the standard theory? Now, they certainly differ at high energies, but what about present energies? The possible discovery of the charged Higgs scalar (or scalars) would not help (they are necessarily present even in the minimal left-right symmetric theories), since they can be accounted for in the $SU(2)_L \times U(1)$ gauge theory, if we increase the number of Higgs doublets. In our opinion, most clear signature against manifest left-right symmetric theory would be the discovery of a light neutral Higgs boson (its mass being of order of 10-100 GeV). Namely, these theories violate strangeness through the exchange of neutral Higgs scalars and since we know that neutral currents must conserve strangeness up to extremely large extent, it could only be explained by assuming neutral Higgs scalars to be very heavy. Therefore, the existence of light neutral Higgs scalars would obviously embarass any proponent of manifest left-right symmetry. However, it would not rule out all the $SU(2)_L \times SU(2)_R \times U(1)$ theories that conserve parity prior to symmetry breaking. Namely, as we pointed in section V, the left-right symmetric theory with "orthogonal" structure of left and right charged currents does conserve strangeness and charm, while violating heavy quark flavors. It is worth emphasizing once again, that such theories can predict calculable neutrino masses. In particular, we discussed a model that leads naturally to the massless neutrinos to all orders in perturbation theory.

One could (naively) hope that the discrepancy between the Oxford and Washington atomic parity violation results which are consistent with the parity conserving theory and the SLAC polarized electron-nucleon scattering findings which favor parity violating theory would persist in the future. Obviously then, the standard theory would be ruled out with its unambiguous predictions which are the product of its appealing simplicity. On the other hand, one would expect left-right symmetric theories to be able to conform to such a situation, once sufficiently changed by enlarging the Higgs sector. However, as we have shown in Appendix D it is an impossible task, since <u>all the parity violating effects in any left-right symmetric theory have the same structure as in the standard model</u>[52] (the difference only being in a possible different strength of these effects). We then have a situation in which, although we may not be able to differentiate between the two types of the theories at the present energies, it could happen (we believe it is not likely) that both theories may be rejected if the discrepancy persists.

The models that would benefit from the mentioned discrepancy are already being suggested. We deeply hope that they will not be called for. Our (prejudiced) belief is that the standard theory will turn to be correct at present energies and that eventually the heavy right-handed charged gauge meson will manifest themselves.

Appendix A

The most general renormalizable potential that can be constructed with fields $\Phi_1=\Phi$ and $\Phi_2=\tilde{\Phi}$ ($\tilde{\Phi}\equiv\tau_2\Phi^*\tau_2$) is the following

$$V = -\sum_{ij=1,2} \mu_{ij}^2 \, \mathrm{tr}\, \Phi_i^\dagger \Phi_j + \sum_{ijk\ell=1,2} \lambda_{ijk\ell}\, \mathrm{Tr}\Phi_i^\dagger\Phi_j \cdot \mathrm{Tr}\Phi_k^\dagger\Phi_\ell$$

$$+ \sum_{ijk\ell=1,2} \lambda'_{ijk\ell}\, \mathrm{Tr}\, \Phi_i^\dagger\Phi_j\Phi_k^\dagger\Phi_\ell \tag{A.1}$$

Left-right symmetry under which the fields Φ_1 and Φ_2 transform as: $\Phi_i \leftrightarrow \Phi_2^\dagger$ (i=1,2) dictates

$$\mu_{ij} = \mu_{ji}$$

$$\lambda_{1212} = \lambda_{2121},\ \lambda_{iijk} = \lambda_{iikj},\ \lambda_{ijkk} = \lambda_{jikk}$$

$$\lambda'_{ijk\ell} = \lambda'_{\ell ijk} = \lambda'_{k\ell ij} = \lambda'_{jk\ell i} \tag{A.2}$$

and $\lambda_{ijk\ell}$ and $\lambda_{k\ell ij}$ describe the same terms. We look for the vacuum expectation value of Φ in the form

$$\langle\Phi\rangle = \begin{pmatrix} ke^{i\alpha} & o \\ o & k'e^{i\alpha'} \end{pmatrix} \tag{A.3}$$

where k and k' are real numbers. First, we will show that one of the phases can be always eliminated. Namely, under gauge symmetry the Φ transforms in the following manner

$$\Phi \to \Phi' = U_L \Phi U_R^\dagger \tag{A.4}$$

where U_L and U_R are unitary unimodular 2x2 matrices. So by performing rotations around the z axis, that is by choosing

$$U_L = \begin{pmatrix} e^{i\rho_L} & o \\ o & e^{-i\rho_L} \end{pmatrix}, \quad U_R = \begin{pmatrix} e^{i\rho_R} & o \\ o & e^{-i\rho_R} \end{pmatrix} \tag{A.5}$$

the $<\Phi>$ gets rotated away into

$$<\Phi'> = \begin{pmatrix} ke^{i(\alpha+\rho_L-\rho_R)} & o \\ o & k'e^{i(\alpha'+\rho_R-\rho_L)} \end{pmatrix} \tag{A.6}$$

Therefore, by choosing $\alpha' + \rho_R - \rho_L = 0$ we can always put the vacuum expectation value of Φ in the form

$$<\Phi> = \begin{pmatrix} ke^{i\delta} & o \\ o & k' \end{pmatrix} \tag{A.7}$$

We now analyze the behavior of the potential as a function of δ. From (A.1), (A.2) and (A.7) we obtain

$$V = A + B\cos\delta + C\cos 2\delta \tag{A.8}$$

where

$$A = [-(\mu_{11}^2 + \mu_{22}^2) + (\lambda_{1111} + \lambda_{1122} + \lambda_{2211} + \lambda_{2222})(k^2+k'^2)](k^2+k'^2)$$

$$+ 4(\lambda_{1221} + \lambda_{2112} + 2\lambda'_{1221})k^2k'^2 + (\lambda'_{1111} + \lambda'_{2222})(k^4+k'^4)$$

$$B = [-\mu_{12}^2 + 4(\lambda_{1112} + \lambda_{1211} + \lambda_{2221} + \lambda_{2122} + \lambda'_{1112} + \lambda'_{2221}) \times$$

$$\times (k^2+k'^2)]\ kk'$$

$$C = 4(2\lambda_{1212} + \lambda'_{1212})k^2k'^2 \tag{A.9}$$

The reader should notice that both B and C are proportional to kk'. Therefore, if k'=0 or k=0 there can be no solution with $\delta \neq 0$. Assuming that both k, $k' \neq 0$ (as it must be in reality to achieve realistic quark mass matrices) we get the following condition for the extremum of the potential

$$0 = \frac{\partial \langle V \rangle}{\partial \delta} = - (B + 2C \cos\delta)\sin\delta \tag{A.10}$$

which implies two possible solutions

$$\delta_1 = 0$$

$$\delta_2 = \cos^{-1} (- B/2C) \tag{A.11}$$

Now, the second derivative of $\langle V \rangle$ is then

$$\frac{\partial^2 \langle V \rangle}{\partial \delta^2} = - B \cos\delta - 2C \cos 2\delta \tag{A.12}$$

Therefore

$$\left.\frac{\partial^2 \langle V \rangle}{\partial \delta^2}\right|_{\delta_1=0} = - B - 2C$$

$$\left.\frac{\partial^2 \langle V \rangle}{\partial \delta^2}\right|_{\delta_2 \neq 0} = 2C \tag{A.13}$$

We can see from (A.13) that both solutions are perfectly acceptable in different ranges of parameters. In other words

1) if $C < 0$, and $-B+2|C|>0$ the minimum of the potential occurs at the vanishing phase: $\delta_1=0$ (the other solution becomes local maximum of the potential);

2) if $C > 0$, $-B-2C<0$, the minimum is at the nonvanishing phase: $\cos\delta_2 = -B/2C$ ($|\cos\delta_2|<1$). ($\delta_1=0$ solution is now a local maximum).

Also from the form of the potential given in (A.8), one has

$$\langle V\rangle \Big|_{\delta_1=0} = A$$

$$\langle V\rangle \Big|_{\delta_2\neq 0} = A-C \tag{A.14}$$

so we see again that for $C > 0$, the solution with nonvanishing phase has a lower energy than a solution with a vanishing phase as we concluded previously (and vice versa in the case $C<0$).

We have then proved that even with a single Φ we can have complex expectation values, result which then has a profound consequence that we do not have necessarily to increase the number of Higgs multiplets in order to explain the phenomenon of CP-violation in the neutral kaon system.

Appendix B. Symmetry breaking in the minimal left-right symmetric theory $SU(2)_L \times SU(2)_R \times U(1)$

As described in section II, the theory consists of the following Higgs multiplets

$$\chi_L(\tfrac{1}{2},0,1), \quad \chi_R(0,\tfrac{1}{2},1)$$

$$\phi(\tfrac{1}{2},\tfrac{1}{2}^*,0) \tag{B.1}$$

and $\tilde{\chi}_L \equiv i\,\tau_2\,\chi_L{}^*$, $\tilde{\chi}_R \equiv i\,\tau_2\,\chi_R{}^*$, $\tilde{\Phi} \equiv \tau_2\,\Phi^*\,\tau_2$.

We want to prove that the minimum of the potential will be in the form

$$\langle\chi_L\rangle = 0, \quad \langle\chi_R\rangle = \binom{o}{v}, \quad \langle\Phi\rangle = \begin{pmatrix} k & o \\ o & k' \end{pmatrix} \tag{B.2}$$

Now, to simplify the calculation we will analyze the theory in the limit k'=0. Our results should be taken only approximately; but however they illustrate theimportant fact: parity can be broken spontaneously. In order to ensure the existence of such a solution (k'=0) we require the Higgs potential be invariant under the following additional discrete symmetry D

$$\chi_L \to \chi_L, \quad \chi_R \to \chi_R, \quad \Phi \to e^{i\frac{\pi}{2}}\,\Phi \tag{B.3}$$

The most general Higgs potential consistent with the left-right symmetry and the symmetry D is then given by

$$
\begin{aligned}
V = & -\mu_1^2 \, \mathrm{tr}\Phi^{+}\Phi + \lambda_1(\mathrm{tr}\Phi^{+}\Phi)^2 + \lambda_2 \, \mathrm{tr}\Phi^{+}\Phi \, \Phi^{+}\Phi \\
& + \frac{1}{2}\lambda_3(\mathrm{tr}\Phi^{+}\tilde{\Phi} + \mathrm{tr}\tilde{\Phi}^{+}\Phi)^2 + \frac{1}{2}\lambda_4(\mathrm{tr}\,\Phi^{+}\tilde{\Phi} - \mathrm{tr}\,\tilde{\Phi}^{+}\Phi)^2 \\
& + \lambda_5 \, \mathrm{tr}\Phi^{+}\Phi \, \tilde{\Phi}^{+}\tilde{\Phi} + \frac{1}{2}\lambda_6[\mathrm{tr}\,\Phi^{+}\tilde{\Phi} \, \Phi^{+}\tilde{\Phi} + \mathrm{h.c.}] \\
& - \mu_2^2(\chi_L^{+}\chi_L + \chi_R^{+}\chi_R) + \rho_1[(\chi_L^{+}\chi_L)^2 + (\chi_R^{+}\chi_R)^2 \\
& + \rho_2 \, \chi_L^{+} \, \chi_L \, \chi_R^{+} \, \chi_R + \\
& + \alpha_1 \, \mathrm{tr}\,\Phi^{+}\Phi(\chi_L^{+}\chi_L + \chi_R^{+}\chi_R) + \\
& + \alpha_2(\chi_L^{+}\Phi\Phi^{+}\,\chi_L + \chi_R^{+}\,\Phi^{+}\Phi\,\chi_R) + \alpha_2'(\chi_L^{+}\tilde{\Phi}\tilde{\Phi}^{+}\chi_L + \chi_R^{+}\tilde{\Phi}^{+}\tilde{\Phi}\,\chi_R) \\
& + \beta_2(\tilde{\chi}_L^{+}\Phi\Phi^{+}\tilde{\chi}_L + \tilde{\chi}_R^{+}\Phi^{+}\Phi\,\tilde{\chi}_R) + \beta_2'(\tilde{\chi}_L^{+}\tilde{\Phi}\tilde{\Phi}^{+}\tilde{\chi}_L + \tilde{\chi}_R^{+}\tilde{\Phi}^{+}\tilde{\Phi}\,\chi_R)
\end{aligned}
\qquad (B.4)
$$

We are looking for the minimum of the potential that preserves the $U(1)_{em}$ gauge invariance

$$
\langle\Phi\rangle = \begin{pmatrix} k & o \\ o & k' \end{pmatrix}, \quad \langle\chi_L\rangle = \begin{pmatrix} o \\ v_L \end{pmatrix}, \quad \langle\chi_R\rangle = \begin{pmatrix} o \\ V_R \end{pmatrix} \qquad (B.5)
$$

From the conditions for the extremum we obtain

$$
\begin{aligned}
\frac{\partial V}{\partial k} &= k[-\mu_1^2 + 2(\lambda_1+\lambda_2)k^2 + (4\lambda_3+\lambda_5+\lambda_6)k'^2 + (\alpha_1+\alpha_2'+\beta_2)(v_L^2+v_R^2)] = 0 \\
\frac{\partial V}{\partial k'} &= k'[-\mu_1^2+2(\lambda_1+\lambda_2)k'^2 + (4\lambda_3+\lambda_5+\lambda_6)k^2 + (\alpha_1+\alpha_2+\beta_2')(v_L^2+v_R^2)] = 0 \\
\frac{\partial V}{\partial v_L} &= v_L[-\mu_2^2 + 2\rho_1 v_L^2 + \rho_2 v_R^2 + \alpha_1(k^2+k'^2) + (\alpha_2+\beta_2')k'^2 + (\alpha_2'+\beta_2)k^2] = 0 \\
\frac{\partial V}{\partial v_R} &= v_R[-\mu_2^2+2\rho_1 v_R^2 + \rho_2 v_L^2 + \alpha_1(k^2+k'^2)+(\alpha_2+\beta_2')k'^2+(\alpha_2'+\beta_2)k^2]=0
\end{aligned}
\qquad (B.6)
$$

Obviously, there is a solution that extremizes the potential shown in eq. (B.5)

$$k' = v_L = 0, \quad v_R \equiv v \neq 0, \; k \neq 0 \tag{B.7}$$

where v and k satisfy simpler equations

$$\mu_1^2 = 2(\lambda_1+\lambda_2)k^2 + (\alpha_1+\alpha_2'+\beta_2)v^2$$

$$\mu_2^2 = 2\rho_1 v^2 + (\alpha_1+\alpha_2'+\beta_2)k^2 \tag{B.8}$$

We will now show that our solution also minimizes the potential, that is we will show that for a range of parameters of the potential the Higgs mass matrix is positive-definite. To introduce our notation, we write

$$\Phi = \begin{pmatrix} \phi_1^o & \phi_1^+ \\ \phi_2^- & \phi_2^o \end{pmatrix}, \quad \chi_L = \begin{pmatrix} \chi_L^+ \\ \chi_L^o \end{pmatrix}, \quad \chi_R = \begin{pmatrix} \chi_R^+ \\ \chi_R^o \end{pmatrix} \tag{B.9}$$

i. <u>charged Higgs sector</u>

After some straightforward and tedious algebra, we obtain the following mass matrix for the charged Higgs scalar particles

$$\begin{array}{c} \phi_2^+ \\ \chi_L^+ \\ \phi_1^+ \\ \chi_R^+ \end{array} \begin{vmatrix} 0 & 0 & 0 & 0 \\ 0 & (\rho_2-2\rho_1)v^2+(\Delta\alpha-\Delta\beta)k^2 & 0 & 0 \\ 0 & 0 & (\Delta\alpha-\Delta\beta)v^2 & (\Delta\alpha-\Delta\beta)kv \\ 0 & 0 & (\Delta\alpha-\Delta\beta)kv & (\Delta\alpha-\Delta\beta)k^2 \end{vmatrix} \tag{B.10}$$

where

$$\Delta\alpha \equiv \alpha_2-\alpha_2' \ , \quad \Delta\beta \equiv \beta_2-\beta_2' \tag{B.11}$$

Now, since in the limit $k'=0$ W_L^+ and W_R^+ are exact gauge meson eigenstates, we find the following two charged goldstone bosons that eventually become longitudinal components of W_L^+ and W_R^+, respectively

$$G_L^+ = \phi_2^2 \ , \quad m_{G_L}^2 = 0$$

$$G_R^+ = \frac{1}{(k^2+v^2)^{1/2}}(k\phi_1^+ - v\chi_R^+) \ , \quad m_{G_R}^2 = 0 \tag{B.12}$$

The rest of diagonalization is then trivial; we also have two charged physical Higgs scalars

$$H_L^+ = \chi_L^+ \ , \quad m_{H_L}^2 = (\rho_2-2\rho_1)v^2 + (\Delta\alpha-\Delta\beta)k^2$$

$$H_R^+ = \frac{1}{(k^2+v^2)^{1/2}}(v\phi_1^+ + k\chi_R^+), \ m_{H_R}^2 = (\Delta\alpha-\Delta\beta)(k^2+v^2) \tag{B.13}$$

Obviously, the conditions for minimum become the positivity of Higgs scalar masses

$$\Delta\alpha-\Delta\beta = \alpha_2-\alpha_2' + \beta_2'-\beta_2 > 0$$

$$\rho_2-2\rho_1 > 0 \tag{B.14}$$

ii. <u>neutral Higgs scalars</u>

We will work with real and imaginary components of neutral Higgs scalars, since they do not couple to each other in the mass matrix.

Following the order of increasing complexity we first concentrate on the <u>fields represented by imaginary components</u> (in our notation denoted as ϕ^o_{1i}, ϕ^o_{2i}, χ^o_{Li}, χ^o_{Ri})

$$\begin{array}{c} \phi^o_{1i} \\ \phi^o_{2i} \\ \chi^o_{Li} \\ \chi^o_{Ri} \end{array} \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & A & 0 & 0 \\ 0 & 0 & B & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

with

$$A = 2k^2(4\lambda_3 + \lambda_5 + \lambda_6 - \lambda_2) + v^2(\Delta\alpha - \Delta\beta)$$

$$B = (\rho_2 - 2\rho_1)v^2 \tag{B.15}$$

Again, we have two real neutral goldstone bosons which correspond to two massive neutral gauge mesons. They are easily found to be (in the limit $k^2 << v^2$, since only in this limit we know neutral gauge meson eigenstates -- see section III)

$$G_Z \simeq \phi^o_{1i} , \qquad m^2_{G_Z} = 0$$

$$G_Z \simeq \chi^o_{Ri} , \qquad m^2_{G_X} = 0 \tag{B.16}$$

We have also two neutral massive scalar particles

$$H^o_1 = \phi^o_{2i} , \qquad m^2_{H^o_1} = A$$

$$H^o_2 = \chi^o_{Li} , \qquad m^2_{H^o_2} = B \tag{B.17}$$

The positivity of their masses leads to the condition

$$A > 0 \, , \quad B > 0 \tag{B.18}$$

or more precisely

$$\Delta\alpha-\Delta\beta > 0, \ \rho_2-2\rho_1 > 0, \ 4\lambda_3+\lambda_5+\lambda_6-\lambda_2 > 0 \tag{B.19}$$

which obviously does not contradict previously obtained condition, eq. (B.14).

Finally, we display the <u>mass matrix of the real components of the neutral Higgs scalars</u>

$$\begin{array}{c|cccc|}
\phi^o_{2r} & A & 0 & 0 & 0 \\
\chi^o_{Lr} & 0 & B & 0 & 0 \\
\phi^o_{1r} & 0 & 0 & 4k^2(\lambda_1+\lambda_2) & 2vk(\alpha_1+\alpha_2'+\beta_2) \\
\chi^o_{Rr} & 0 & 0 & 2vk(\alpha_1+\alpha_2'+\beta_2) & 4\rho_1 v^2
\end{array} \tag{B.20}$$

Obviously, we have four more massive Higgs particles: ϕ^o_{2r}, χ^o_{Lr} and two orthogonal linear combinations of ϕ^o_{1r} and χ^o_{Rr}. The positivity of their masses requires additional constraints

$$\begin{aligned}
&4k^2(\lambda_1 + \lambda_2) + 4\rho_1 v^2 > 0 \\
&4\rho_1(\lambda_1 + \lambda_2) - (\alpha_1 + \alpha_2' + \beta_2)^2 > 0
\end{aligned} \tag{B.21}$$

which does not contradict any of the previous conclusions. In other words, in the range of parameters

$$\rho_2 - 2\rho_1 > 0$$

$$\alpha_2 - \alpha_2' + \beta_2' - \beta_2 > 0$$

$$4\lambda_3 + \lambda_5 + \lambda_6 - \lambda_2 > 0$$

$$\lambda_1 + \lambda_2 > 0, \ \rho_1 > 0$$

$$4\rho_1(\lambda_1 + \lambda_2) - (\alpha_1 + \alpha_2' + \beta_2)^2 > 0 \tag{B.22}$$

our solution is the local minimum of the classical potential. However, in order that our solution represents a stable particle configuration, it must also be an absolute minimum of the potential. There are two other possible symmetry breaking patterns which preserve the electromagnetic gauge invariance:

a) the case of unbroken gauge symmetry

$$\langle\chi_L\rangle = \langle\chi_R\rangle = \langle\phi\rangle = 0 \tag{B.23}$$

b) the case of broken gauge symmetry, but conserved left-right symmetry

$$\langle\chi_L\rangle = \langle\chi_R\rangle \neq 0 \text{ with } \begin{array}{c}\langle\phi\rangle = 0 \\ \text{or} \\ \langle\phi\rangle \neq 0\end{array} \tag{B.24}$$

Now, the case a) is always a local maximum, since the second derivatives of the potential with respect to fields at the extremum are negative or zero

$$\frac{\partial^2 V}{\partial \chi_L^+ \partial \chi_L}\bigg|_{\langle\chi_L\rangle=0} = \frac{\partial^2 V}{\partial \chi_R^+ \partial \chi_R}\bigg|_{\langle\chi_R\rangle=0} = -\mu_2^2$$

$$\frac{\partial^2 V}{\partial \phi^+ \partial \phi}\bigg|_{\langle\phi\rangle=0} = -\mu_1^2 \tag{B.25}$$

the rest vanish.

Similarly, in case b) independently of the value of $\langle\phi\rangle$, at least one of the Higgs masses is proportional to $2\rho_1-\rho_2$ with a positive coefficient and therefore becomes negative for the condition $\rho_2-2\rho_1 > 0$ (see eq. (B.22)). We show that in the simplest case when Higgs multiplet ϕ is not present (generalization is trivial). From (B.6) taking $k'=k=0$ and $v_L=v_R=v\neq0$ we get

$$-\mu_2^2 + (2\rho_1+\rho_2)v^2 = 0 \tag{B.26}$$

We have only two charged Higgs scalars χ_L^+ and χ_R^+ and they both become longitudinal components of W_L^+ and W_R^+, respectively (now $\langle\chi_L\rangle \neq 0$. Similarly, the two imaginary components of neutral Higgs scalars χ_L^o and χ_R^o become neutral Goldstone bosons. The real components mix among themselves and lead to the following mass matrix

$$\begin{array}{c} \chi_{Lr}^o \\ \chi_{Rr}^o \end{array} \begin{vmatrix} 4\rho_1 v^2 & 2\rho_2 v^2 \\ 2\rho_2 v^2 & 4\rho_1 v^2 \end{vmatrix} \tag{B.27}$$

whose eigenvalues in turn become

$$m_1{}^2 = 2(2\rho_1 + \rho_2)v^2$$

$$m_2{}^2 = 2(2\rho_1 - \rho_2)v^2 \tag{B.28}$$

But for $\rho_2 - 2\rho_1 > 0$, which was one of our minimum conditions -- see eq. (B.22), $m_2{}^2 < 0$ so that this solution becomes a local maximum (or more precisely a saddle point).

Since the symmetric equations for $\langle\chi_L\rangle$ and $\langle\chi_R\rangle$ allow only

$$\langle\chi_L\rangle = \langle\chi_R\rangle = 0$$

$$\langle\chi_L\rangle = \langle\chi_R\rangle \neq 0$$

$$\langle\chi_L\rangle = 0, \ \langle\chi_R\rangle \neq 0 \text{ (or vice versa)} \tag{B.29}$$

It is then evident that our asymmetric solution is not only a local minimum, but also an absolute minimum of the Higgs potential in the range at parameters as given in eq. (B.22).

We end up this discussion by an amusing comment. It is easy to see that all of the masses (except one of them) of Higgs particles are proportional to v^2, the large vacuum expectation value; in other words most of the masses can be written as

$$m_H{}^2 \simeq \lambda v^2 \tag{B.30}$$

where λ is a typical ϕ^4 coupling constant. It can be written as:

$$m_H{}^2 \simeq \frac{\lambda}{g^2} \ M_{W_R}^2 \simeq 100\lambda \ M_{W_R}^2 \tag{B.31}$$

Since λ should not exceed one (in order to have possible perturbation theory) we obtain the usual upper limit[35] for the Higgs scalar masses:

$$m_H{}^2 \lesssim 100 \ M_{W_R}^2$$

relation which is the standard model is obtained by substituting $M^2_{W_R} \rightarrow M^2_W$. In other words, except for one particle, the rest of physical Higgs scalars could be extremely heavy (much heavier than 1 TeV).

Finally we comment on our approximation k' = 0. Although physically unacceptable (it would lead to a vanishing Cabbibo angle, since up and down quark mass matrices would be proportional to each other - see Chapter II), as we have shown in Appendix C k' must be much smaller than k, so that results can be taken as a leading approximation for small k'. Also, we should mention that in order to achieve k' = 0 we had to implement a discrete symmetry D which has substantially simplified the Higgs potential by eliminating many possible terms. Since even with that reduced number of coupling constants we had a consistent set of constraints which followed from the requirement of a stable particle configuration, it is even more likely to be so in the real case of nonvanishing k' when we have much more many coupling constants, and therefore less chance at running into a contradiction.

Appendix C. Gauge meson eigenstates in the minimal left-right symmetric theory

For the sake of completeness in this appendix we derive physical gauge meson eigenstates (eigenstates of mass matrices) and their eigenvalue. We have the following Higgs scalars (see section II):

$$\chi_L = \begin{pmatrix} \chi_L^+ \\ \chi_L^o \end{pmatrix} \qquad \chi_R = \begin{pmatrix} \chi_R^+ \\ \chi_R^o \end{pmatrix}$$

$$\Phi = \begin{pmatrix} \phi_1^o & \phi_1^+ \\ \phi_2^- & \phi_2^o \end{pmatrix} \tag{C.1}$$

with the pattern at symmetry breaking:

$$<\chi_L> = 0, \quad <\chi_R> = \binom{0}{v}, \quad <\Phi> = \begin{pmatrix} k & 0 \\ o & k' \end{pmatrix}, \; v > k,k' \tag{C.2}$$

The piece of the Langrangian containing their covariant derivatives is:

$$L_D = (D_\mu \chi_L)^\dagger D^\mu \chi_L + (D_\mu \chi_R)^\dagger D^\mu \chi_R + Tr(D_\mu \Phi)^\dagger D^\mu \Phi \tag{C.3}$$

where:

$$D_\mu \chi_L = \partial_\mu \chi_L - \frac{ig}{2} \vec{\tau}\cdot\vec{W}_L \chi_L$$

$$D_\mu \chi_R = \partial_\mu \chi_R - \frac{ig}{2} \vec{\tau}\cdot\vec{W}_R \chi_R$$

$$D_\mu \Phi = \partial_\mu \Phi - \frac{ig}{2} (\vec{\tau}\cdot\vec{W}_L \Phi - \Phi\vec{\tau}\cdot\vec{W}_R) \tag{C.4}$$

Gauge meson mass matrices are then obtained by substituting (C.2) into (C.3) and using (C.4). We do not go through that simple exercise; we just give the resulting mass matrices.

a) <u>Charged gauge mesons</u>

$$
M^{(+)} = \begin{array}{c|cc} & W_L^+ & W_R^+ \\ \hline W_L^- & \frac{g^2}{4}(k^2+k'^2) & -\frac{g^2}{2}\,k\,k' \\ W_R^- & -\frac{g^2}{2}\,kk' & \frac{g^2}{4}(k^2+k'^2+v^2) \end{array} \tag{C.5}
$$

The eigenstates and eigenvalues of this matrix are then:

$$
\begin{aligned}
W_1^+ &= \cos\xi \; W_L^+ + \sin\xi \; W_R^+ \\
W_2^+ &= -\sin\xi \; W_L^+ + \cos\xi \; W_R^+
\end{aligned} \tag{C.6}
$$

where:

$$
\tan 2\xi = -\frac{4kk'}{v^2} \tag{C.7}
$$

and:

$$
\begin{aligned}
M_{W_1}^2 &= \frac{g^2}{4}[k^2 + k'^2 + \frac{v^2}{2} - ((\frac{v}{2})^2 + 4k^2k'^2)^{1/2}] \\
M_{W_2}^2 &= \frac{g^2}{4}[k^2 + k'^2 + \frac{v^2}{2} + ((\frac{v}{2})^2 + 4k^2k'^2)^{1/2}]
\end{aligned} \tag{C.8}
$$

In other words, W_1 is a light charged gauge meson with predominantly V-A couplings (notice that $|\tan\xi| << 1$), whereas W_2 is a heavy particle with mostly V+A couplings to the fermions. To display that more clearly, we give the approximate eigenvalues in the limit $v^2 >> k^2, k'^2$:

$$
\begin{aligned}
M_{W_1}^2 &\simeq \frac{g^2}{4}(k^2 + k'^2) \\
M_{W_2} &\simeq \frac{g^2}{4}(v^2 + k^2 + k'^2)
\end{aligned} \tag{C.9}
$$

Obviously: $M_{W_2}^2 >> M_{W_1}^2$.

Experiment dictates:

$$
\frac{M_{W_2}^2}{M_{W_1}^2} \gtrsim 9 \tag{C.10}
$$

or in other words: $v^2 \geq 8(k^2 + k'^2)$ (C.11)

which in turn implies

$$|\tan 2\xi| \leq \frac{1}{4} \tag{C.12}$$

However, experiment puts a more stringent limit on ξ[11]:

$$|\tan \xi| \leq 0.06 \tag{C.13}$$

Now, above condition can be easily met by taking k' to be sufficiently small:

$$\frac{k'}{k} \sim \frac{1}{10} \tag{C.14}$$

That justifies some of our analysis (see Appendix B, for example) where, in order to reach some definite results we work in the limit of very small or negligible k'. We wish to warn the reader that although relation (C.14) is perfectly acceptable, we cannot take $k' = 0$ since that would lead to vanishing Cabbibo angle. Since $\tan\alpha$ is so small, we can safely ignore the mixing between W_L^+ and W_R^+ in most of the material discussed in this paper and work with W_L^+ and W_R^+ as appropriate physical states, having in mind $M^2_{W_R} \gg M^2_{W_L}$.

(b) neutral gauge mesons

In the similar manner in which we obtained the mass matrix of charged gauge mesons, we have the following mass matrix for the neutral gauge mesons:

$$M^2_{W_{3L}, W_{3R}, B} = \begin{pmatrix} \frac{g^2}{4}(k^2+k'^2) & -\frac{g^2}{4}(k^2+k'^2) & 0 \\ -\frac{g^2}{4}(k^2+k'^2) & \frac{g^2}{4}(k^2+k'^2+v^2) & -\frac{gg'}{4}v^2 \\ 0 & -\frac{gg'}{4}v^2 & \frac{g'^2}{4}v^2 \end{pmatrix} \tag{C.15}$$

Since det $M^2 = 0$ (as it should be; we must have a photon even after the symmetry breaking) we then obtain easily the eigenvalues of the above matrix:

(we work in the approximation $v^2 >> k^2, k'^2$)

$$M_Z^2 \simeq \frac{g^2}{4} \frac{g^2 + 2g'^2}{g^2 + g'^2} (k^2 + k'^2)$$

$$M_X^2 \simeq \frac{g^2}{4} (g^2 + g'^2)v^2$$

$$M_A^2 \simeq 0$$

If we define: $\tan\theta \equiv \frac{g'}{\sqrt{g^2+g'^2}}$, we can write above eigenvalues in the form:

$$M_Z = \frac{M_{W_L}}{\cos\theta}$$

$$M_X = \frac{M_{W_R} \cos\theta}{\sqrt{\cos 2\theta}} \qquad (C.17)$$

Once we have eigenvalues it is a trivial exercise to obtain the physical states. First, we have a photon which is given by an exact relation:

$$A_\mu = \sin\theta(W^3_{L\mu} + W^3_{R\mu}) + \sqrt{\cos 2\theta}\; B\mu \qquad (C.18)$$

Besides the photon, we have the "light" and the heavy neutral gauge mesons:

$$Z_\mu \simeq \cos\theta \; W^3_{L_\mu} - \sin\theta \tan\theta \; W^3_{R_\mu} - \tan\theta\sqrt{\cos 2\theta}\; B_\mu$$

$$X_\mu \simeq \frac{\sqrt{\cos 2\theta}}{\cos\theta} W^3_{R_\mu} - \tan\theta \; B_\mu \qquad (C.19)$$

As a check of our computation, the reader can be easily convinced that the above eigenstates are orthonormal as they should be.

Appendix D: Neutral currents: minimal left-right symmetric theory versus the standard theory

In this appendix we prove that, in the limit $M_{W_R} \to \infty$, the neutral current Hamiltonian in the minimal $SU(2)_L \times SU(2)_R \times U(1)$ gauge theory is equal to the $SU(2)_L \times U(1)$ neutral Hamiltonian. We employ the method of Georgi and Weinberg.[19] We review it here briefly.

Consider a gauge theory based on an arbitrary group G. Let the gauge symmetry be broken down to $U(1)_{em}$ via Higgs mechanism. We can write the electric charge operator as

$$Q_{e\ell} = \sum_\alpha C_\alpha T_\alpha \tag{D.1}$$

where T_α are Hermition, electrically neutral generators of G. As we said, we have a photon $\underset{\sim}{p}$, or in other words a massless eigenvector of a gauge meson mass matrix M^2:

$$M^2 \underset{\sim}{p} = 0, \quad \underset{\sim}{p}^2 = 1 \tag{D.2}$$

The $\underset{\sim}{p}$ is given by

$$\underset{\sim}{p} = \{\rho_\alpha\}, \quad p_\alpha \equiv (\sum_\alpha c_\alpha^2/g_\alpha^2)^{-1/2} \; C_\alpha/g_\alpha \tag{D.3}$$

where g_α is the coupling constant of the gauge field A_μ^α coupled to the generator T_α. The photon is then

$$A_\mu = \sum_\alpha p_\alpha A_\alpha^\mu \tag{D.4}$$

Georgi and Weinberg then derive the very useful formula for the effective neutral current interaction in the following manner: Consider any U(1) subgroup of G with a generator To and let $\{T_\alpha\} = \{T_o, T_i\}$. They show that the neutral current Hamiltonian is given by

$$H_N = \frac{1}{2} \sum_{ij} (\bar{f} \gamma_\mu n_i f) \bar{f} \gamma_\mu n_j f M_{ij}^{-2} \tag{D.5}$$

where f denotes fermions;

$$n_i \equiv \frac{g}{C_i} (C_i T_i - \frac{e^2}{g_i^2} C_i^2 Q_{e\ell}) \tag{D.6}$$

and M_{ij}^{-2} is the inverse of M_{ij}^2 (M_{ij}^2 is, in the obvious notation, a mass matrix of a subset of gauge mesons which consists of all of the gauge mesons, but the one that corresponds to generator T_o). In the case of our interest, $G = SU(2)_L \times SU(2)_R \times U(1)$. We take T_o to be Y, the generator of U(1). Using $C_i = 1$, $g_i = g$ we have

$$n_L = g(T_3 \frac{1-\gamma_5}{2} - \frac{e^2}{g^2} Q)$$
$$n_R = g(T_3 \frac{1+\gamma_5}{2} - \frac{e^2}{g^2} Q) \tag{D.7}$$

The neutral current Hamiltonian is as given in eq. (D.5) with i = L,R. From

$$M^2 = \begin{pmatrix} \frac{1}{4} g^2(k^2+k'^2) & -\frac{1}{4} g^2(k^2+k'^2) \\ -\frac{1}{4} q^2(k^2+k'^2) & \frac{1}{4} g^2(v^2+k^2+k'^2) \end{pmatrix}$$

The inverse of M^2 is found to be

$$M^{-2} = \begin{pmatrix} \frac{4}{g^2v^2} \cdot \frac{k^2+k'^2+v^2}{k^2+k'^2} & \frac{4}{g^2v^2} \\ \frac{4}{g^2v^2} & \frac{4}{g^2v^2} \end{pmatrix} \tag{D.9}$$

Obviously for v^2 very large only M_{11}^{-2} matrix elements will contribute at low energies. Therefore, we get the leading expression for the effective Hamiltonian

$$H_N \simeq \frac{1}{2} \bar{f} \gamma_\mu n_L f \bar{f} \gamma^\mu n_L f \frac{1}{\frac{1}{4} g^2 (k^2+k'^2)} \tag{D.10}$$

But this is obviously the result that we would obtain in the standard model in a trivial manner, with a substitution $k^2+k'^2 \rightarrow \langle\phi\rangle^2_{WS}$, where ϕ_{WS} is the doublet under $SU(2)_L$ as defined in eqs. (2.3) and also with the equality of $\sin^2\theta_W = \frac{e^2}{g^2_{WS}}$ and $\sin^2\theta_{LR} = \frac{e^2}{g^2_{LR}}$, which is exactly the condition we have derived in Chapter II using different methods. The method used here is obviously very powerful and elegant, since it saves the labor of the usual diagonalization of a gauge meson mass matrix and the derivation of the physical currents. As an illustration of the power of this method we give two interesting applications:

i. general ℓ-R symmetric theory versus standard model-conditions for equivalence

Let us take $T_o = Y$ as before. The most general submatrix M^2_{ij} of gauge meson masses is

$$M^2_{ij} = \begin{pmatrix} a & b \\ b & c \end{pmatrix} \tag{D.11)(D.11}$$

The inverse M^{-2}_{ij} is easily found to be

$$M^{-2}_{ij} = \begin{pmatrix} \frac{c}{ac-b^2} & \frac{-b}{ac-b^2} \\ -\frac{b}{ac-b^2} & \frac{a}{ac-b^2} \end{pmatrix} \tag{D.12}$$

The condition for the equivalence of the two theories is, from our previous analysis, easily recognized as

$$M_{12}^{-2} << M_{11}^{-2}$$
$$M_{22}^{-2} << M_{11}^{-2} \tag{D.13}$$

so that only n_L contributes as in the standard theory. But that condition leads then to

$$c >> a,b \tag{D.14}$$

We conclude quite generally that as long as (D.14) is satisfied (independently of the details of the Higgs sector), the left-right symmetric theory and the standard theory will predict the same neutral current phenomena. What we called a minimal ambidextrous theory is only a simplest example which led to this desired result.

ii. Parity violation (in neutral currents) in left-right symmetric theories

From eqs. (D.5), (D.6) and (D.7) we write explicitly the neutral current effective Hamiltonian in left-right symmetric theories based on $SU(2)_L \times SU(2)_R \times U(1)$ gauge group

$$H_N = \frac{1}{2}[M_{LL}^{-2}(\bar{f}\gamma_\mu n_L f)^2 + M_{LR}^{-2}\,\bar{f}\gamma_\mu n_L f\;\bar{f}\gamma^\mu n_R f$$
$$+ M_{RL}^{-2}\,\bar{f}\gamma_\mu n_R f\;\bar{f}\gamma^\mu n_L f + M_{RR}^{-2}\,(\bar{f}\gamma_\mu n_R)^2] \tag{D.15}$$

where $(\bar{f}\gamma_\mu n_i f)^2 = \bar{f}\gamma_\mu n_i f\;\bar{f}\gamma^\mu n_i f;\quad i = L,R.$

We can draw some important conclusions from the above form. Since $n_{L/R} = T_3\ L,R - Q\sin^2\theta$ it is obvious that the parity violating piece of the neutral current Hamiltonian in left-right symmetric theories (independently of the Higgs sector) is of the same form as in the standard theory.

The reason for that is very simple: The parity violating Hamiltonian in an $SU(2)_L \times SU(2)_R \times U(1)$ gauge theory is obviously proportional to:

$$H_N^{pv} \propto \bar{f}\, \gamma_\mu\, t_3\, \gamma_5\, f\, \bar{f}\, \gamma_\mu\, (\tfrac{1}{2}\, t_3 - Q \sin^2\theta) f \tag{D.16}$$

which is exactly the expression in the standard theory. In other words, the structures are the same; only their respective strength can differ. Of course, as we have stated many times the minimal theory will have completely equal (for $M_{W_R}^2 \to \infty$) both parity conserving and parity violating neutral currents as the standard model. However, for parity violating processes we find amazingly close predictions for both theories, even for finite M_{W_R}. Namely, from eq. (D.15) we can easily obtain the parity violating amplitude (remember that $M_{LR}^{-2} = M_{RL}^{-2}$)

$$H_N = (M_{LL}^{-2} - M_{RR}^{-2}) \bar{f}\, \gamma_\mu\, (\frac{T_3}{2} - Q \sin^2\theta) f\, \bar{f}\, \gamma_\mu\, (- \frac{T_3\gamma_5}{2}) f \tag{D.17}$$

But from eq. (D.12)

$$M_{LL}^{-2} - M_{RR}^{-2} = \frac{c-a}{ac-b^2} \tag{D.18}$$

In the minimal model (see eq.(2.31))

$$a = \tfrac{1}{4}(k^2+k'^2),\quad c = \tfrac{1}{4}(k^2+k'^2+v^2),\quad b = -\tfrac{1}{4}\,(k^2+k'^2) \tag{D.19}$$

so that

$$M_{LL}^{-2} - M_{RR}^{-2} = \frac{1}{\frac{1}{4}(k^2+k'^2)} \tag{D.20}$$

Notice that eq. (D.20) is an exact result — we did not assume $v^2 >> k^2, k'^2$. But if we neglect the mixing between W_L and W_R then obviously $M_{LL}^{-2} - M_{RR}^{-2} \simeq \frac{1}{M_{W_L}^2}$; so that in this case minimal left-right symmetric theory and the standard model agree exactly in their predictions for the parity violating scattering amplitudes. But since we know experimentally that the W_L-W_R mixing is very small (~ 5%), we can conclude that to the great extent tree level amplitudes relevant for the SLAC experiment are the same in both theories. Of course, the radiative corrections in these theories will differ as we discussed in chapter II.

Finally, we conclude with a remark concerning the parity conserving neutral interactions with a class of left-right theories.[55] It is easy, using the Georgi-Weinberg method, to read off the condition for parity conservation in neutral currents from eq. (D.15)

$$
\begin{aligned}
M_{LL}^{-2} &= M_{RR}^{-2} \\
M_{LR}^{-2} &= M_{RL}^{-2}
\end{aligned}
\tag{D.21}
$$

The second condition is actually automatically satisfied since M_{ij}^{-2} is a symmetric matrix (see eq. (D.12). Therefore, eq. (D.21) ammounts to

$$M_{LL}^{2} = M_{RR}^{2} \tag{D.22}$$

This condition can be easily understood if we recall the fact that the matrix

$$M_{ij}^{2} = \begin{pmatrix} a & b \\ b & a \end{pmatrix} \tag{D.23}$$

has eigenstates $W_L^3 - W_R^3$ and $W_L^3 + W_R^3$, which have definite parity transformations.

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Figure captions

Fig. 1. One loop graph that contributes to electric dipole moment of neutron through W_L-W_R mixing.

Fig. 2. A typical finite one-loop graph involving the Higgs bosons that contributes to quark mass matrices.

Fig. 3. The curve which represents the values of m_d/m_u and m_c/m_s that yield the correct Cabibbo angle (θ_c=13°). This is independent of the ratio m_s/m_d.

Fig. 4. sin 2δ versus m_d/m_u shown for values of m_d/m_u taken from fig. 3, for m_s/m_d equal to ten.

Fig. 5. The diagram that induces neutrino mass through the W_L-W_R mixing.

Fig. 6. The graph that contributes to $<\phi_j^o>$, where ϕ_j° is a neutral Higgs boxon with nonvanishing coupling to $\bar{\nu}_L\nu_R$.

Fig. 7. The one-loop diagram that contributes to W_L-W_R mixing through the fermionic exchange.

Table I. Comparison between the standard Weinberg-Salam model and the minimal left-right symmetric theory (for relevant discussion see chapter II and Appendixes B and D).

Gauge Group	Weinberg-Salam SU(2) x U(1)	Minimal L-R $SU(2)_L \times SU(2)_R \times U(1)$
Gauge Mesons	$W^{\pm}$, A, Z $M_Z = \frac{M_W}{\cos\theta_W}$, $M_A = 0$	$W_L^{\pm}$, $W_R^{\pm}$, A, Z, X $M_Z = \frac{M_{W_L}}{\cos\theta}$, $M_X = \frac{\cos\theta M_{W_R}}{\sqrt{\cos 2\theta}}$, $M_A = 0$
Fermionic Assignment	$\binom{\nu}{e}_L$, $\binom{\nu'}{\mu}_L$; $\binom{u}{d(\theta)}_L$, $\binom{c}{s(\theta)}_L$ e_R, μ_R; u_R, d_R, c_R, s_R	$\binom{\nu}{e}_L$, $\binom{\nu'}{\mu}_L$; $\binom{u}{d(\theta)}_L$, $\binom{c}{s(\theta)}_L$ $\binom{\nu}{e}_R$, $\binom{\nu'}{\mu}_R$, $\binom{u}{d(\theta)}_R$, $\binom{c}{s(\theta)}_R$
Higgs sector and Symmetry Breaking	$\phi = \binom{\phi^+}{\phi^o}$, $\langle\phi\rangle \neq 0$ Physical Higgs: Re ϕ^o	$\Phi = \begin{pmatrix} \phi_1^o & \phi_1^+ \\ \phi_2^- & \phi_2^o \end{pmatrix}$, $\chi_L = \begin{pmatrix} \chi_L^+ \\ \chi_L^o \end{pmatrix}$, $\chi_R = \begin{pmatrix} \chi_R^+ \\ \chi_R^o \end{pmatrix}$ $\langle\Phi\rangle = \begin{pmatrix} k & o \\ o & k' \end{pmatrix}$, $\langle\chi_L\rangle = 0$, $\langle\chi_R\rangle = \binom{o}{v}$ Physical Higgs: 4 charged, 6 real neutral particles (as given in Appendix B)
Basic Difference	V-A charged currents by hand	V-A currents as effect of spontaneous symmetry breaking
Basic Similarity	The same neutral current (when $M_{W_R} \to \infty$)	

Table I

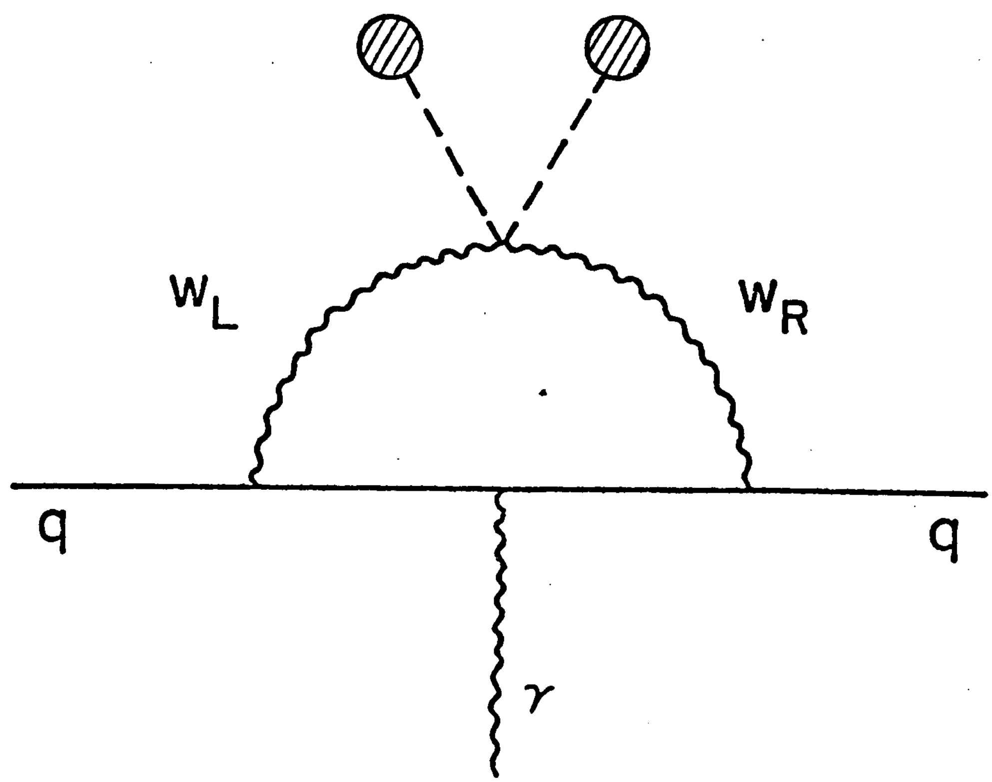

Fig. 1

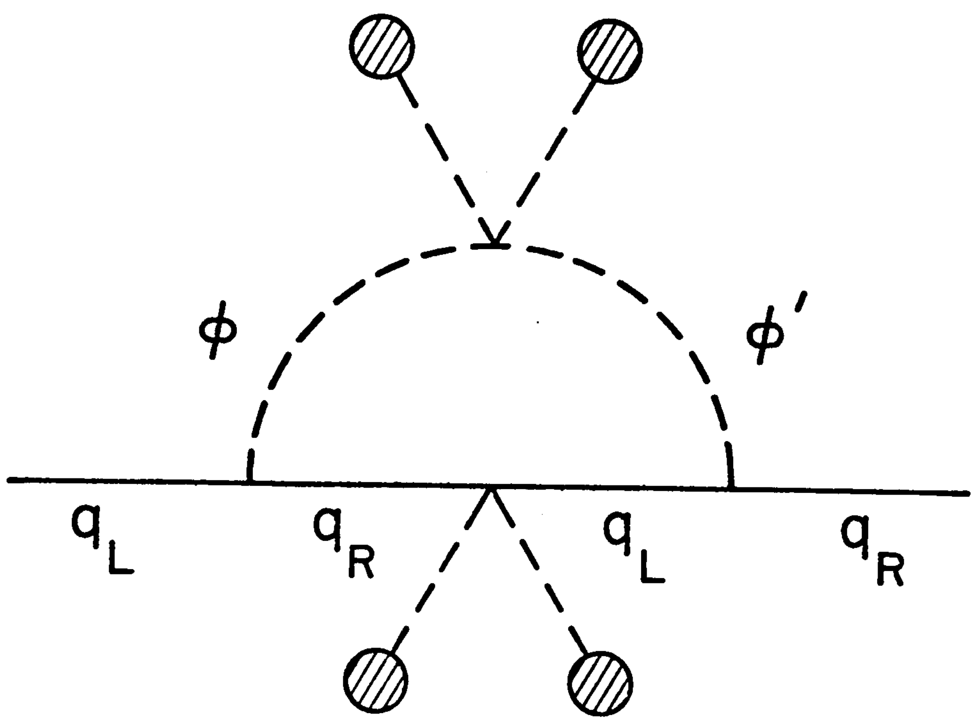

Fig. 2

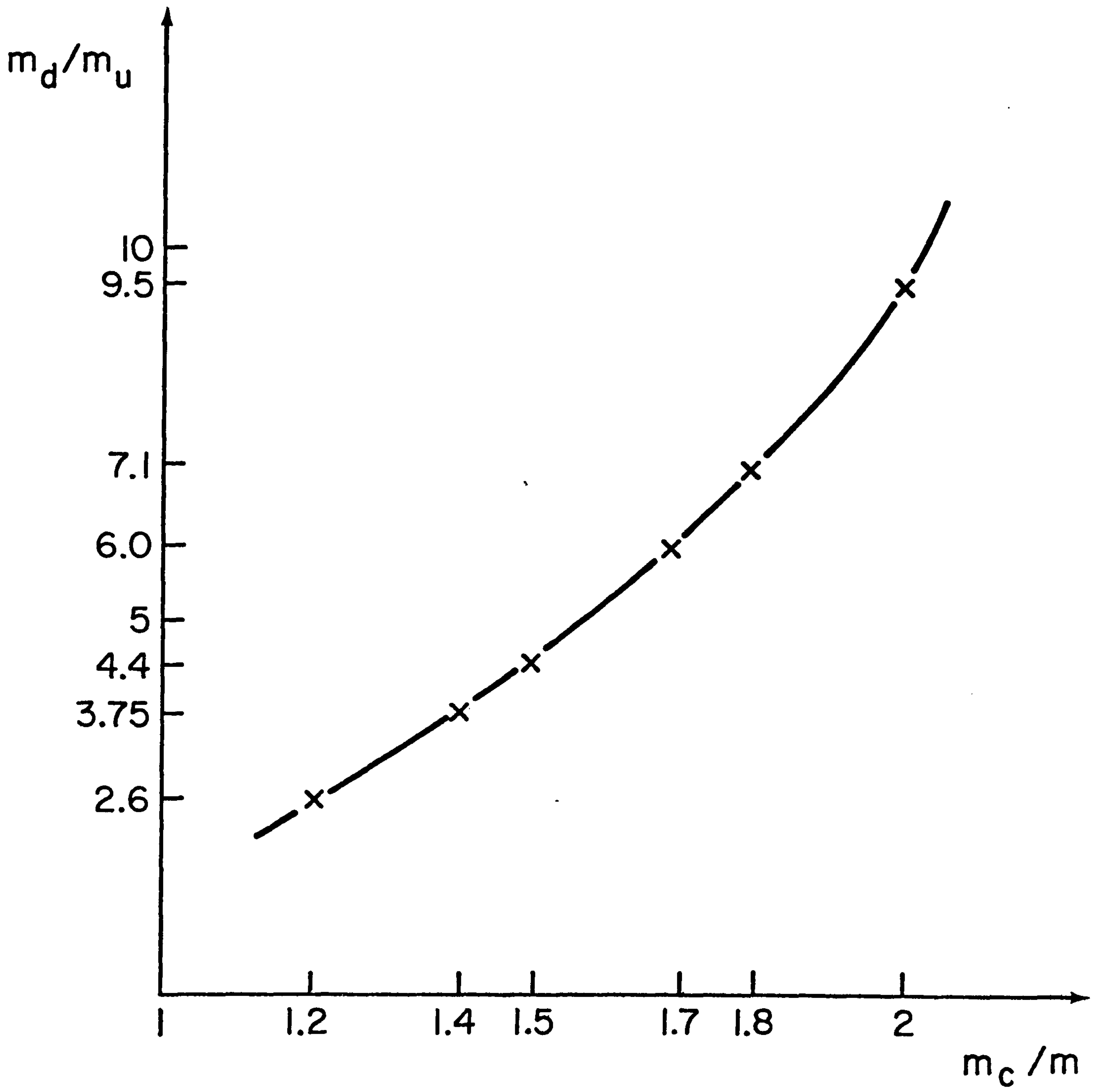

Fig. 3

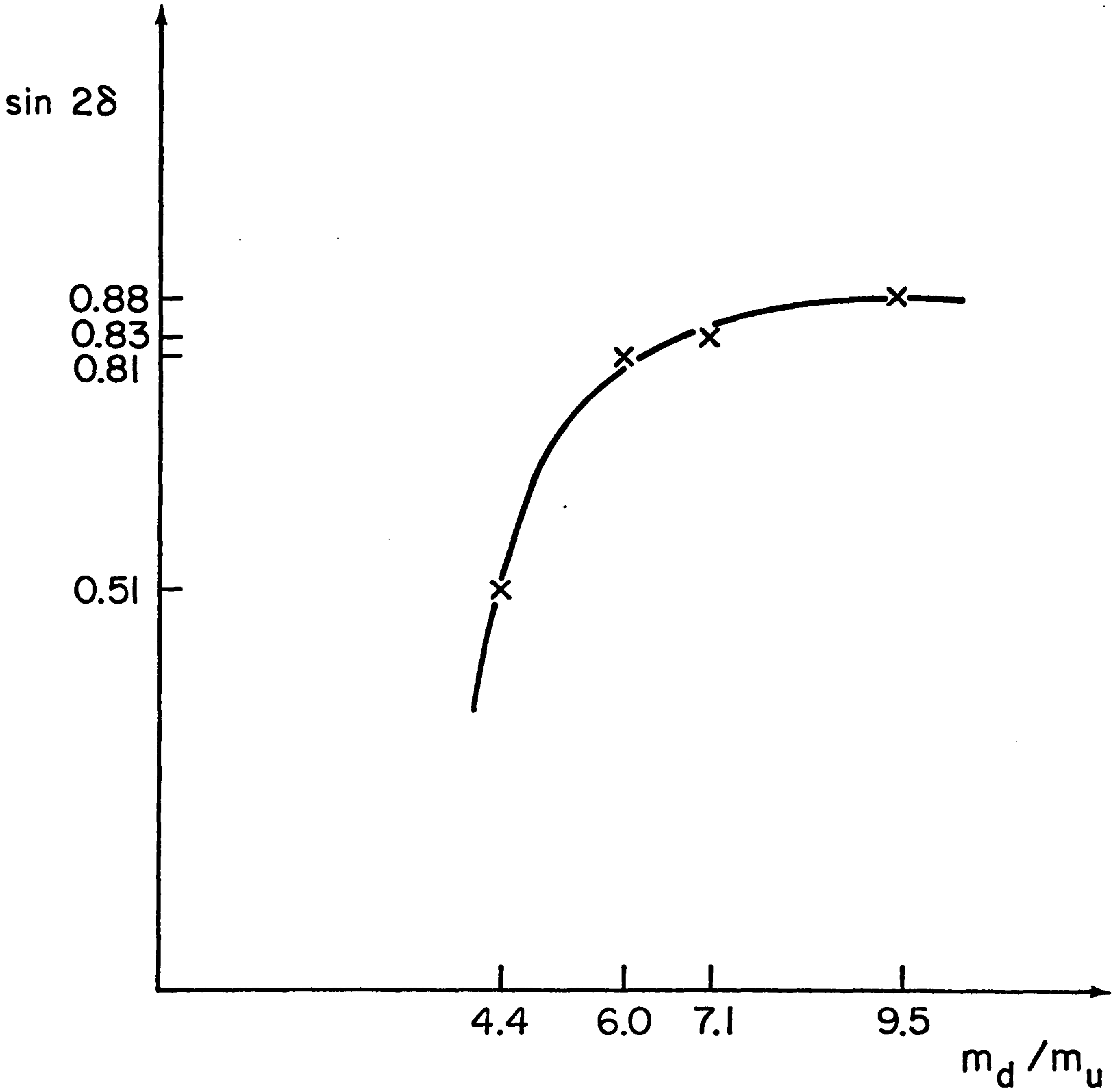

Fig. 4

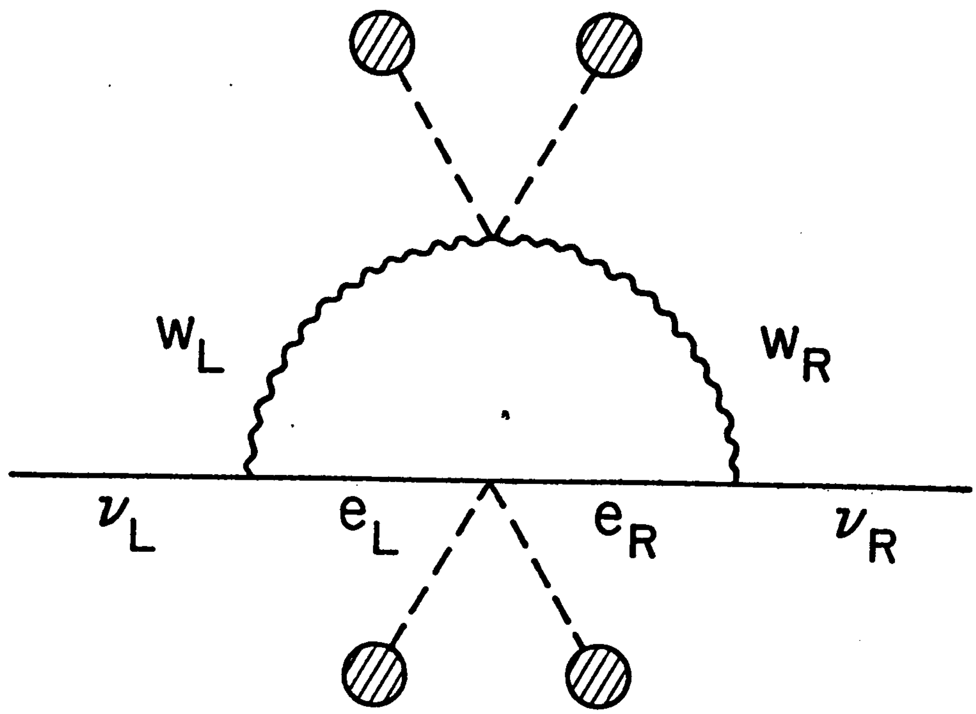

Fig. 5

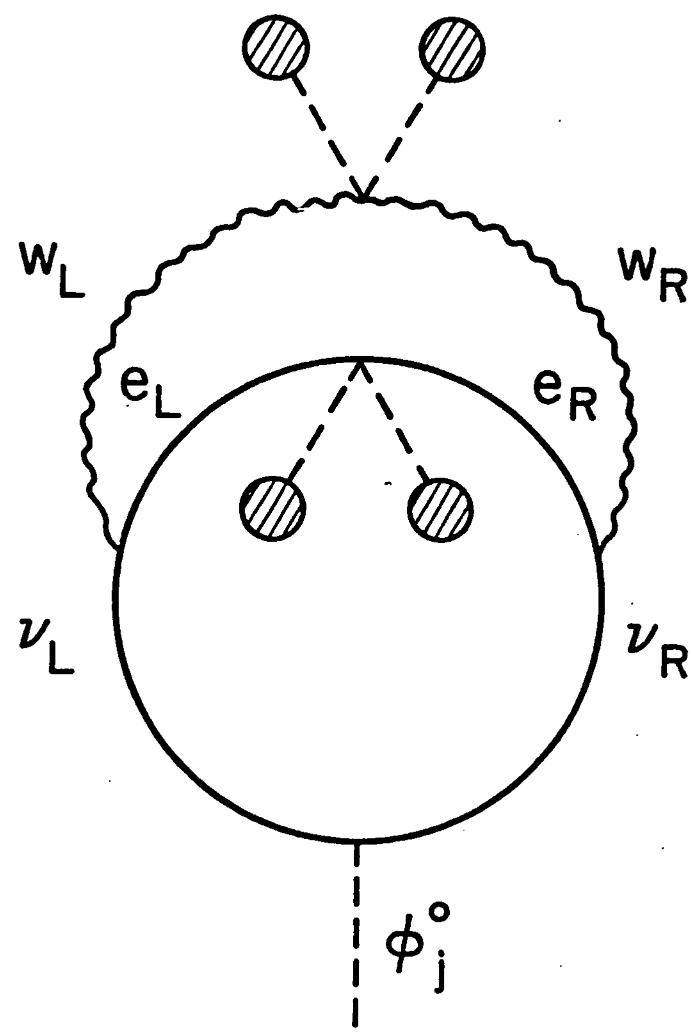

Fig. 6

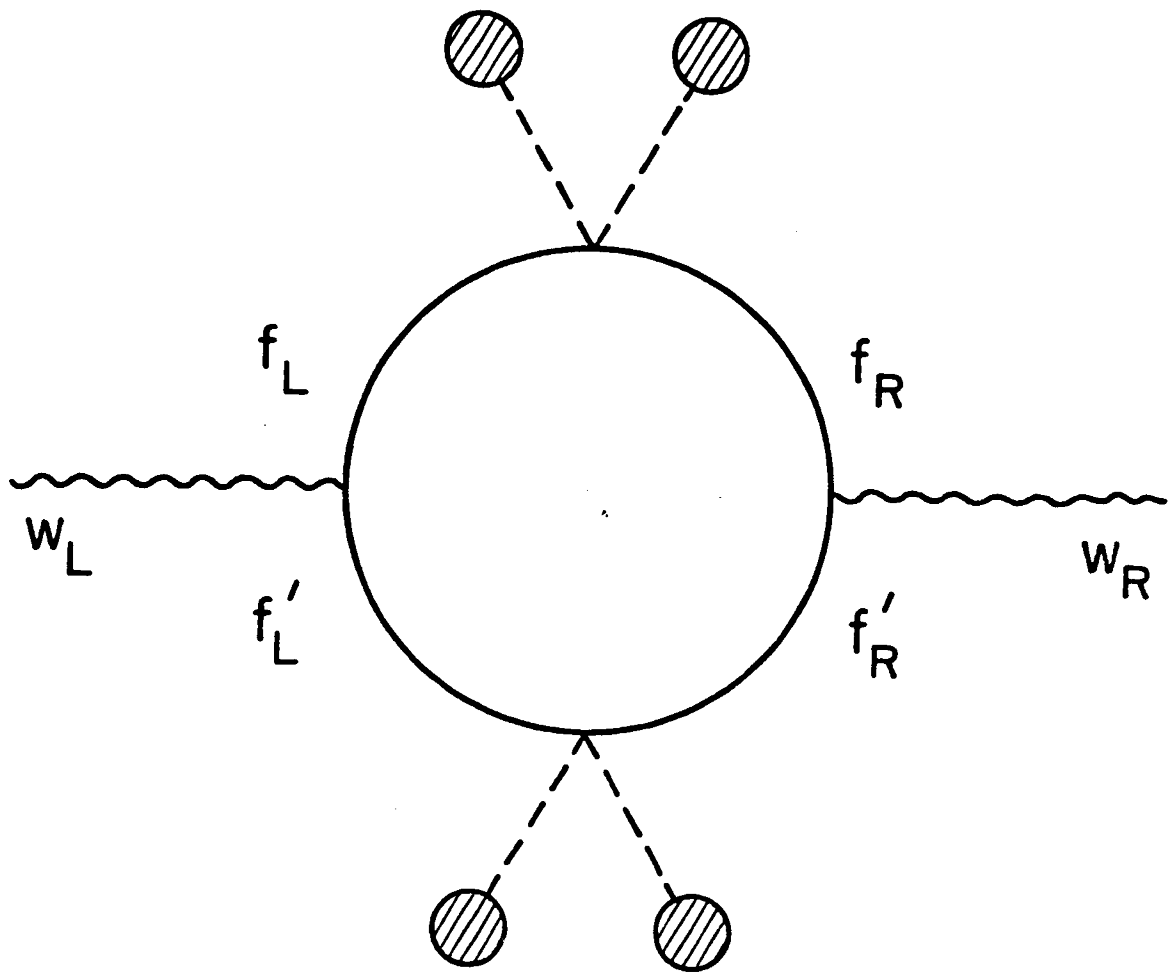

Fig. 7