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INTERMEDIATE MASS SCALES IN SUPERSYMMETRIC GAUGE THEORIES

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INTERMEDIATE MASS SCALES IN SUPERSYMMETRIC GAUGE THEORIES

by

CHARANJIT.S.AULAKH.

A dissertation submitted to the Graduate Faculty in
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This manuscript has been read and accepted by the Graduate Faculty in Physics in satisfaction of the dissertation requirement for the degree of Doctor of Philosophy.

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Abstract

INTERMEDIATE MASS SCALES IN SUPERSYMMETRIC GAUGE THEORIES

by

Charanjit S. Aulakh

Adviser: Professor R. N. Mohapatra

The basic lore adequate to the building of Supersymmetric models is introduced . An $SO(10)$ model is constructed and an analysis of constraints on the Supersymmetry(SUSY) breaking mass scale is carried out. One finds that $M_S \sim 10^{12}$ GeV if M_R and M_S are identified. Other motivations for M_S to be of this 'intermediate' magnitude are discussed. The potentials in different Poincare Supergravities (SUGRYS) are computed and shown to select the Old-Minimal(OM) version as best suited to model building. The embedding of the Dimopoulos-Raby model in OM SUGRY is studied and the induction of the Grand unified scale from M_S demonstrated. A general analysis of decoupling is given along with a discussion of the extension to $SO(10)$ models. In an appendix, we derive the 'complete set' of soft SUSY breaking terms for the supersymmetric Standard model .

येन चेतयते ह्रीदं चित्तं चेतयते न यम् ।

जागृत्स्वप्नसुषुप्त्यादि तस्मै श्रीगुरवे नमः ॥

यस्य ज्ञानादितं विश्वं न दृश्यं भिन्नभेदतः ।

सदेकरूपरूपाय तस्मै श्रीगुरवे नमः ॥

यत्सत्येन जगत्सत्यं यत्प्रकाशेन भातितम् ।

यदानन्देन नन्दन्ति तस्मै श्रीगुरवे नमः ॥

अखण्डमण्डलाकारं व्याप्तं येन चराचरम् ।

वत्पदं दर्शितं येन तस्मै श्रीगुरवे नमः ॥

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Chapter I

Introduction

The last two years have seen an explosion of activity directed toward the implementation of Supersymmetry(SUSY) in the context of Grand Unified gauge Theories(GUTs) of the Elementary particle interactions. The developments traced in this thesis have led to the conclusion that embedding global supersymmetry in N=1 Supergravity(SUGRA) is required if the characteristic features of globally SUSY models are to be reconciled with what is known about the mass spectra and interactions of the Elementary particles. Moreover, it has been realized that a scale $O(10^{10}-10^{12}\text{GeV})$ for the breaking of SUSY is inexorably selected by quite diverse considerations. This scale is the 'intermediate mass scale' of the title of this theses.

In chapter 2 we have given an account of the basic features of globally SUSY models. We begin with the principles of the construction of Supersymmetric gauge invariant Lagrangians describing the interaction of scalar and fermionic fields with gauge (boson and fermion) fields. After reviewing spontaneous breaking of SUSY and tree level mass spectra we go on to discuss the Gauge Hierarchy problem which is the primary motivation for the introduction of SUSY. Following a review of the 'Non-renormalization' theorems which allow supersymmetric theories to allay this

disease we describe the initial Supersymmetric GUT proposed by Dimopoulos and Georgi. We conclude the chapter with a description of constraints on the mass spectrum of the superpartners of the fermionic matter of the standard Weinberg-Salam model. We also discuss dimension 5 Baryon number violating operators present in SUSY GUTs.

In Appendix I associated with Chapter 2 we introduce one notation and the basic concepts regarding Superfields and supersymmetry transformations. Thus chapter 2 and Appendix I may be omitted by the reader adept in the techniques of SUSY model building.

In chapters 3 and 4 we describe, in detail, a supersymmetric $SO(10)$ model of the Dimopoulos-Georgi type. This work was done in collaboration with Professor R.N. Mohapatra⁽¹⁷⁾. After demonstrating the feasibility of associating $M_R \sim M_S$ we show that this identification leads, via low energy constraints, to a value for $M_S \sim 10^{10} - 10^{12} \text{ GeV}$.

In chapter 5 we review the other lines of development which converge in the realization that realistic theories require $M_S \sim 10^{11} \text{ GeV}$. These include cosmological constraints on the Gravitino mass, the Witten geometric hierarchy model, embedding global SUSY in $N=1$ Old Minimal Supergravity, the low energy potential in $N=1$ Supergravity theories and the induction of the weak scale by

radiative corrections in such theories .

In chapter 6 we describe in detail investigations done in collaboration with Professors M. Kaku and R.N. Mohapatra concerning the structure of the potential in various alternative Poincare Supergravities that can be derived from the $N=1$ Superconformal theory. We conclude that with possibly one exception not yet investigated, such theories do not offer the attractive features of $N=1$ Old Minimal(OM) SUGRA.

Finally, in chapter 7 we recount our own work on the induction of the GUT scale, at tree level, via a Witten superpotential, in $N=1$ OM SUGRA models. We first consider an $SU(5)$ model and demonstrate how the GUT scale can be induced. We then present a derivation of the low energy potential in such theories which does not make the assumptions current in the literature⁽⁴⁴⁾. Finally we present a discussion of $SO(10)$ versions of this mechanism and discuss the difficulties associated with the construction of a realistic model.

In Appendix II we present a determination of the complete set of soft breaking terms required by the renormalizability of the low energy Lagrangian in SUSY theories. This work was done in collaboration with Professor R.N. Mohapatra⁽¹¹⁾.

The reader who wishes to acquaint himself with the original matter in this thesis, need peruse only chapters

3,4,6,7 and Appendix II. Chapters 2 and 5 along with Appendix I are provided to ensure continuity of concept and notation and will be superfluous for the expert reader.

Chapter 2

BASICS OF MODEL BUILDING

In this chapter we introduce the basic lore of SUSY model building so as to provide continuity with our own work. The relevant notation and properties of supermultiplets may be found in Appendix I. The adept reader is advised to spare himself the trouble of covering this familiar ground. The material in this chapter draws heavily on ref.2-10.

Section 1:

i) Lagrangians: The construction of SUSY invariant Lagrangians rests on the simple observation that the SUSY variation of the F term of a chiral multiplet and the D term of a vector multiplet are four divergences

$$\delta D = -\bar{\epsilon} i \not{\partial} \lambda(x) \quad (2.1)$$

$$\delta F = -\bar{\epsilon} i \not{\partial} \psi(x). \quad (2.2)$$

Since there are no other densities with this property it is vector and chiral superfields that find the most application in model building.

One supersymmetrizes the matter multiplets by providing them with SUSY partners. Since the matter fields of the standard model (Quarks, Leptons) are fermions and no vector particles with quark/lepton quantum numbers exist, their partners are necessarily scalars. We shall adopt the prevalent (somewhat whimsical) terminological device of

prefixing the usual names by an s (thus sleptons, squarks, etc.). The other scalar field (F) in the chiral multiplet turns out to be auxiliary both in the sense that it was originally introduced only for the purpose of linearizing the SUSY transformation laws and because the construction of the SUSY invariant Lagrangians is such as to render them non-propagating by denying them the motility of double derivative terms.

The kinetic terms for the matter and s-matter fields are provided by using

$$[\Sigma \Sigma^\dagger]_D$$

or its gauge invariant generalization. (see below)

$$[\Sigma^\dagger e^{gG} \Sigma]_D$$

As for gauge fields they are embedded in gauge vector multiplets (which carry the adjoint representation index). Thus they acquire fermionic partners (gauginos). This multiplet is then used in two ways - in the matter kinetic terms to provide covariant derivatives and in the form of chiral field strengths to form the gauge kinetic terms:

$$[W^\alpha W_\alpha]_F$$

Before going on to present the general form of the SUSY invariant gauge - matter Lagrangian (with the Wess-zumino condition imposed), we briefly discuss gauge transformations in the superfield context. The usual gauge transformation

of matter fields generalizes chiral multiplets as

$$\Phi_+ \longrightarrow e^{i\Lambda_+^k(x,\theta)Q^k} \Phi_+(x,\theta) \quad (2.3a)$$

$$\Phi_- \longrightarrow e^{i\Lambda_-^k(x,\theta)Q^k} \Phi_-(x,\theta) \quad (2.3b)$$

where Λ^k are chiral parameter superfields and Q^k are Hermitian generators.

The gauge potentials are then chosen to transform as

$$e^{G^k Q_k} \longrightarrow e^{i\Lambda_{+k}^* Q^k} e^{G^l Q_l} e^{-i\Lambda^m Q_m}.$$

The most general renormalizable, gauge invariant, supersymmetric Lagrangian for the interaction of matter fields (embedded in chiral multiplets) and gauge fields is then a sum of three separately SUSY invariant pieces:

$$\mathcal{L} = \mathcal{L}_{\text{Gauge kinetic}} + \mathcal{L}_{\text{kinetic}} + \mathcal{L}_g. \quad (2.4a)$$

$$\mathcal{L}_{G.K} = -\frac{1}{4} F_{\mu\nu}^{\alpha 2} + \bar{\lambda}^\alpha (i \not{D} \lambda_-)^\alpha + \frac{1}{2} D^{\alpha 2} \quad (2.4b)$$

$$\begin{aligned} \mathcal{L}_K = & |\not{D}_\mu A_\pm|^2 + (\bar{\Psi} i \not{D} \Psi_\pm + F_\pm^\dagger F_\pm) \\ & + i g \sqrt{2} (A_\pm^\dagger \bar{\lambda}^\alpha Q^\alpha \Psi_\pm - \bar{\Psi}_\pm \lambda_\mp^\alpha Q^\alpha A_\pm) \\ & \pm g A_\pm^\dagger D^\alpha Q_\pm^\alpha A_\pm + \int_{(A)}^k D^k. \end{aligned} \quad (2.4c)$$

where

$$\nabla_\mu A_\pm = (\partial_\mu - ig V_\mu^\alpha Q^\alpha) A_\pm \quad (2.5)$$

$$\nabla_\mu \psi_\pm = (\partial_\mu - ig V_\mu^\alpha Q_\pm^\alpha) \psi_\pm \quad (2.6)$$

$$\nabla_\mu \lambda^\alpha Q^\alpha = \partial_\mu \lambda^\alpha Q^\alpha - ig [V^\alpha Q^\alpha, \lambda^\beta Q^\beta]$$

Q^k are the gauge group generators in the representation formed by the chiral multiplet. $\sum_k$ are mass² parameters and the sum runs over $U(1)$ factor groups of the gauge group only. The last term is the so called Fayet-Illiopoulos term. Finally

$$\begin{aligned} \mathcal{L}_g &= [g(\Phi_\pm)]_F + h.c. \\ &= F_i^\dagger g^i + \frac{1}{2} g^{ij} \psi_i^T C^{-1} \psi_j + h.c. \end{aligned} \quad (2.7)$$

where g is a gauge invariant polynomial of degree less than three (the restriction comes from the requirement of renormalizability). Note that the F and D fields have no kinetic terms and therefore must be eliminated via their equations of motion:

$$-F_i^\dagger = g_i^* \quad D^\alpha = g(A_+^\dagger Q^\alpha A_+ - A_-^\dagger Q^\alpha A_-) \quad (2.8)$$

so that the potential

$$V = F_i^\dagger F_i + \frac{1}{2} D^{\alpha 2} \quad (2.9)$$

becomes

$$V = |g'(A)|^2 + g^2/2 (A_+^\dagger Q^\alpha A_+ - A_-^\dagger Q^\alpha A_-)^2 \quad (2.10)$$

which is clearly positive-semi definite.

This Lagrangian is invariant under

$$\begin{aligned} \delta V_\nu &= -\frac{i}{\sqrt{2}} \bar{E} \gamma_\nu \lambda \\ \delta \lambda^\alpha &= \frac{i}{\sqrt{2}} (i \gamma_5 D^\alpha + \frac{1}{2} \sigma_{\mu\nu} V_{\mu\nu}^\alpha) \\ \delta D^\alpha &= -\frac{i}{\sqrt{2}} \bar{E} \not{\gamma}_5 \lambda \end{aligned} \quad (2.11)$$

$$\delta A_\pm = \bar{E} \psi_\pm \quad \delta \psi_\pm = F_\pm \epsilon_\pm - i \not{D} A_\pm \epsilon_\mp$$

$$\delta F_\pm = \bar{E} (-i \not{D} \psi_\pm + i \sqrt{2} \lambda_\mp^\alpha Q^\alpha A_\pm)$$

ii) Spontaneous symmetry breaking

We here present the characteristic features of spontaneous breaking of global supersymmetry. That is to say while the dynamics represented by the Lagrangian is globally supersymmetric the ground state is not, so that the supersymmetry is spontaneously broken and a Goldstone fermion (the Goldstino) appears.

Assuming that the vacuum state is Lorentz and translation invariant we have from the v.e.v. of the equations of infinitesimal SUSY transformations

$$\langle \delta \psi_\pm \rangle = \langle F_\pm \rangle \epsilon_\pm \quad (2.12)$$

$$\langle \delta \lambda_\pm^\alpha \rangle = -\frac{i}{\sqrt{2}} \langle D^\alpha \rangle \epsilon_\pm$$

while all others are zero because

$$\langle \lambda \rangle = \langle \psi \rangle = \langle \partial_\mu A \rangle = 0 \quad (2.13)$$

from Lorentz and translation invariance. Since the Goldstone phenomenon is characterized by the non-vanishing of the v.e.v's of that infinitesimal vacuations of field under the action of generators of the broken symmetry and since

$$\delta \psi_\pm(0) = -i [\psi_\pm(0), \bar{\epsilon} Q] \quad (2.14)$$

where Q is the generator of SUSY transformations

$$\delta \lambda^\alpha_\pm(0) = -i [\lambda_\pm(0), \bar{\epsilon} Q] \quad (2.15)$$

we recognize that $\langle F \rangle$, $\langle D^\alpha \rangle$ are the order pararmeters of SUSY breaking. It is now easy to show that if either is non-zero then a massless Goldstone fermion must appear in the excitation spectrum of the supersymmetric Lagrangian.

Thus,

$$\begin{aligned} \delta \psi_\pm(0) &= -i [\psi_\pm(0), \bar{\epsilon} Q] \\ &= -i \int d^3 \vec{x} [\psi_\pm(0), \bar{\epsilon} S_0(x)] \end{aligned} \quad (2.16)$$

where S_0 is the fourth component of the supersymmetry current which may be derived by the usual methods or by

'boosting' of other symmetries to be

$$\begin{aligned}
 i S_{\mu\pm} = & F_{\mp}^{\dagger} \gamma_{\mu} \psi_{\mp} + \gamma_{\mu} \psi_{\mp}^c F_{\pm} + i \nabla_{\nu} A_{\pm}^{\dagger} \gamma_{\nu} \gamma_{\mu} \psi_{\pm} \\
 & + \gamma_{\nu} \gamma_{\mu} \psi_{\pm}^c i \nabla_{\nu} A_{\mp} + \frac{i}{\sqrt{2}} (D^{\alpha} - \frac{1}{2} \sigma_{\kappa\lambda} W_{\kappa\lambda}^{\alpha}) \gamma_{\mu} \lambda_{\pm}^{\alpha}
 \end{aligned}
 \quad (2.17)$$

so one may write, following the usual method,

$$S^4(x) \langle \psi_{\pm}(0) \rangle = i \partial_{\mu} \langle T (S_{\mu}(x) \psi_{\pm}(0)) \rangle \quad (2.18)$$

Using equation (2.16) for $\delta \psi_{\pm}$ and the conservation of S_{μ} one finds

$$\begin{aligned}
 i \partial_{\mu} \langle T (\psi_{\pm}(0) S_{\mu}(x)) \rangle &= P_{\pm} \langle F_{\pm} \rangle S^4(x) \\
 P_{\pm} &= (1 \pm i \gamma_5) / 2.
 \end{aligned}
 \quad (2.19)$$

Since in momentum space one can write the spectral representation

$$\begin{aligned}
 & \int d^4x e^{ik \cdot x} \langle T (\psi_{\pm}(x) S_{\mu}(x)) \rangle \\
 &= P_{\pm} [M_1^{\pm} k_{\mu} + (M_2^{\pm} \eta_{\mu\nu} + M_3^{\pm} k_{\mu} k_{\nu}) \gamma_{\nu} \\
 & \quad + M_4^{\pm} \sigma_{\mu\nu} k_{\nu}]
 \end{aligned}
 \quad (2.20)$$

where $M_i = M_i(k^2)$ so that one has

$$\begin{aligned}
 k^2 M_1^{\pm} &= \langle F_{\pm} \rangle \\
 M_2^{\pm} + k^2 M_3^{\pm} &= 0
 \end{aligned}
 \quad (2.21)$$

Thus it is clear that M_1 has a zero mass pole of residue $\langle F_+ \rangle$ indicating the presence of a zero mass 'goldstino' in the fermion spectrum. Similar arguments may be applied to SUSY breaking due to $\langle D^a \rangle$. Thus in general the goldstino is given by a mixture of the fermionic fields with the relative proportions governed by the v.e.v.'s of F and D . On linearizing the expression for the supercurrent about the vacuum solution one has

$$\begin{aligned} i S_{\mu+} &= \langle F_-^{\dagger} \rangle \gamma_{\mu} \psi_- + \gamma_{\mu} \psi_-^c \langle F_+ \rangle + i \langle D^a \rangle \gamma_{\mu} \lambda_-^a \quad (2.22) \\ &= f \gamma_{\mu} \nu_- + \text{bilinear terms.} \end{aligned}$$

Thus the Goldstone spinor ν_- is defined by

$$\nu_- = \frac{1}{f} \left(\langle F_-^{\dagger} \rangle \psi_- + \psi_-^c \langle F_+ \rangle + i \langle D^a \rangle \lambda_-^a \right)$$

from this expression and equation (2.17) one can show that⁽³⁾ for any chiral multiplet the super gap (ie. the bose-fermi mass splitting) depends linearly on the scale of supersymmetry breaking f and on its coupling to the goldstino, that is

$$m_B^2 - m_f^2 = e f \quad (2.23)$$

This relation will prove important to us later.

iii) Fermionic mass generation: Whereas the masses of scalar bosons are found as usual from the second derivative

of the potential, it is worth mentioning the special form of the fermion mass matrix in supersymmetric theories.

There are two sources of mass generation: the first is from the superpotential which yields Yukawa interactions

$$\frac{1}{2} g^{ij} \psi_{\pm i}^T C^{-1} \psi_{\pm j} + h.c. \quad (2.24)$$

So that the v.e.v. of g^{ij} form one set of contributions to the mass matrices. The other set of contributions comes from the terms

$$i g \sqrt{2} (A_{\pm}^{\dagger} \lambda^{\alpha} Q^{\alpha} \psi_{\pm}) + h.c. \quad (2.25)$$

In this case a Dirac mass term arises when A acquires a v.e.v. When supersymmetry is unbroken one finds that the gauge fermion partners with a chiral fermion to form a massive Dirac particle of mass equal to the gauge boson mass. the scalar partner if the pseudo-scalar which is eaten by the corresponding gauge boson also becomes massive with that mass. Thus a gauge vector supermultiplet with one massless vector and one Majorana fermion fuses with a chiral supermultiplet with one scalar one pseudo scalar and one Weyl fermion to form a massive vector, one massive Dirac particle and one (real) scalar particle.

More on tree level mass relations

From the above considerations and the form of the

scalar potential it is easy to derive^(4,6) the following tree level mass² matrices.

spin zero M_0^2

$$M_0^2 = \begin{pmatrix} g^{ab} g_{bc} + D_c^{\alpha a} D_c^{\alpha} + D_c^{\alpha a} D^{\alpha} & g^{abc} g_b + D^{\alpha a} D^{\alpha c} \\ g_{abc} g^b + D_a^{\alpha} D_c^{\alpha} & g_{ab} g^{bc} + D_a^{\alpha} D^{\alpha c} + D_a^{\alpha c} D^{\alpha} \end{pmatrix} \quad (2.26)$$

spin 1/2 $M_{1/2}^2$

$$M_{1/2}^2 = \begin{pmatrix} g^{ab} g_{bc} + 2 D_c^{\alpha a} D_c^{\alpha} & -\sqrt{2} g^{ab} D_b^{\beta} \\ -\sqrt{2} D^{\alpha b} f_{bc} & 2 D^{\alpha c} D_c^{\beta} \end{pmatrix} \quad (2.27)$$

spin 1 M_1^2

$$M_1^2 = D_a^{\alpha} D^{\beta a} + D^{\alpha a} D_a^{\beta} \quad (2.28)$$

where derivatives $\frac{\partial}{\partial z_a}$ ($\frac{\partial}{\partial \bar{z}^a}$) are denoted by super(sub)-scripts.

Then we immediately find the Ferrara-Girardello-Palumbo⁽⁴⁾

mass relation

$$STr M^2 = \sum_J (-1)^J (2J+1) m_J^2 = 2 g_{\alpha} D^{\alpha} Tr T^{\alpha} \quad (2.29)$$

Since it is well known that $\text{Str } M^2$ controls the quadratic divergences in the scalar mass matrix (at the one loop level) this equation shows that such divergences do not occur at one loop - provided there are no Abelian gauge groups whose generators have non-zero trace.

Section 2

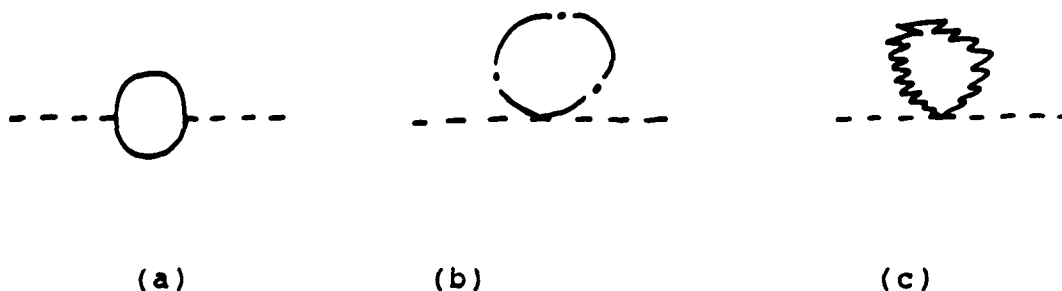
i) The Gauge Hierarchy problem

Apart from aesthetic considerations the main motivation for the introduction of supersymmetry as an invariance of Lagrangians used to describe the world of particle physics is that SUSY appears to be the only symmetry which can automatically prevent the appearance of quadratic mass renormalizations of scalar masses in theories with fundamental (non-composite) scalar fields. The presence of these quadratic renormalizations of scalar mass squared parameters in theories with widely disparate mass scales - such as the current Grand Unified Theories (GUTs), is called the Gauge Hierarchy Problem(GHP). Let us briefly review this phenomenon. The Weinberg-Salam model of Electroweak interactions now widely accepted as the 'standard' model contains as an intrinsic ingredient scalars which transform as $(2, 1)$ under the Electroweak gauge group $SU(2) \times U(1)$. In order that the spontaneous breaking of gauge symmetry that is a central element of the Weinberg-Salam model be possible and compatible with the experimental datum (G_{fermi}) which places the characteristic mass scale of the weak

interactions at approximately 10^2 Gev, the mass parameters associated with such scalars must necessarily be of the same order of magnitude. On the other hand, once the Electroweak theory had proven its viability it was natural to attempt a further (Grand) Unification of the $SU(2) \times U(1)$ gauge group with the gauge group $SU(3)$ which is thought to describe the strong interactions. Since the gauge coupling constants differ by a factor of 10 or so, it was realized by Georgi, Quinn and Weinberg that the scheme of embedding the low energy gauge group in a larger semi-simple gauge group such as $SU(5)$ or $SO(10)$, E_6 , etc. could be feasible only if the differences between the various gauge couplings were understood to arise from renormalization group effects which changed the (equal) values of the various couplings at high energies where the Unifying symmetry held good (or had 'just' been broken) to their observed (and widely disparate) values at low energies. We shall review the GQW calculation in a later section. At this point it is sufficient to note that all GUT schemes yield a unification scale $M_U \gg M_W$ the scale of weak interactions.

For instance in the original GQW calculation for the 'minimal' $SU(5)$ GUT $M \sim 10^{15}$ Gev. Thus GUTs always have two widely disparate mass scales M_W and M_U . Since this is so one must ask what mechanism, if any, is operating to prevent these mass scales from being 'scrambled' by the effects of radiative corrections. To be more precise one

knows that the scalar doublets necessary for Electroweak symmetry breaking must have a mass $\sim 10^2 \text{ GeV}$. On the other hand in any GUT these doublets are a subrepresentation of an irreducible representation of the Grand Unifying gauge group. For instance in the $SU(5)$ models they are embedded in the fundamental 5 or $\bar{5}$ of $SU(5)$. Since this is so these scalars which belong to the 'light' sector necessarily have interactions with the heavy sector of the theory which is comprised of the gauge bosons corresponding to the broken generators (ie. those in G/G_{123}), the (colored) parts of the representations containing the electroweak Higgs scalars etc. Thus one immediately sees, even at the one loop level that graphs such as



_____ . _____ .	Heavy scalar
~~~~~	Heavy gauge boson
-----	Light scalar
_____	Fermion

will contribute terms of  $O(M_U^2)$  to the light scalar mass matrix. By means of a 'fine-tuning' of parameters (that is, a choice of values accurate to  $\log(M_U/M_W)$  decimal

places) it is possible to adjust the tree level Lagrangian so that the large contributions have very small coefficients and the net contribution is  $O(M_W^2)$ . However, one finds (as one would expect on dimensional grounds) that the large contributions reappear at each succeeding loop level. Thus it is necessary to perform fine tunings order-by order. Clearly this is an extremely unappealing feature of the theory. On aesthetic grounds a theory in which a scale separation, albeit one put in by hand at the tree level, was preserved by the quantum dynamics of the fields would evidently be much preferable to one in which each succeeding level of dynamic interaction required its own adjustment of parameters. Of course one would prefer the initial parameter adjustment also to be unnecessary. In fact, as we shall see SUSY GUTs have opened up the possibility of this as well.

ii) Non renormalization theorems

In this section we briefly review the so called 'non-renormalization' theorems of exactly globally supersymmetric theories.

These results are most conveniently derived using the so called 'super graph' methods. For reasons of space we omit a detailed presentation of these methods and confine ourselves to a concise account of the central results in the language of Super-Graphs. The central result⁽⁵⁾ consists of the simple statement that

Theorem: Every (one particle irreducible) term in the effective action of a supersymmetric theory can be written as the D component of a product of superfields.

In other words, every term in the integrand of the effective action although possibly non-local in the space-time argument is always local in the anti-commuting variables  $\theta$  so that each term can be reduced to

$$\int d^4\theta d^4x_1 \dots d^4x_n F(x_1, \dots, x_n, \theta) = [F(\theta, x_1, \dots, x_n)]_D (2.30)$$

It also follows that the effective action is manifestly supersymmetric, that is to say that  $F$  is a product of superfields and covariant derivatives.

From this theorem it follows that there is no independent renormalization of the parameters massive, or otherwise, that control the strength of the interactions represented by the superpotential. This is so because from the renormalizability of the Lagrangian it follows that all divergent terms in the effective action must be representable in the same form as the tree level Lagrangian, that is, as a sum of F and D type densities. However, we saw above that no F-type density arises in perturbation theory. Hence, it follows that the only renormalization suffered by F type parameters is that induced by the (common) wave function renormalization engendered by the logarithmic divergences present in the D-type terms. In

particular, this means that an F-coupling set to zero at tree level stays zero even if there is no other symmetry which forbids its appearance at loop level. The above features of SUSY theories are employed under the names of technical naturalness and supernaturalness. Thus, it is technically natural to fine-tune parameters since the fine-tuning is maintained and one may set allowed couplings to zero since the supernaturalness of the SUSY Lagrangian prevents their resurrection via radiative mechanisms.

As is well known, the various miracles mentioned above occur due to cancellations between the divergent contributions of fermionic and bosonic loops due to the difference in sign between them and are exact due to the SUSY imposed relations between the parameters of the Lagrangian.

iii) Soft breaking of SUSY

Although SUSY theories have various convenient and remarkable properties, supersymmetry imposes such tight restrictions on the Lagrangian that obtaining a model that accords with the known phenomenology is a non-trivial problem. Specifically, it is well known that the 'observed' fundamental particle spectrum of the current standard model (by which we shall throughout mean

$G_{123} = U(1) \times SU(2)_L \times SU(3)_C$  with quarks and leptons in the standard assignment) exhibits no trace of supersymmetry. Thus the scalar partners (sQuarks, sLeptons) predicted by

SUSY⁽¹⁾ must have masses  $> 20$  GeV in order to have escaped detection. Whereas the partners of the W will have masses  $O(M_W)$  due to the supersymmetric Higgs mechanism . The predictions of the current models as well as experimental constraints on gluino, photino and Higgsino masses are much more ambiguous. Thus the photino might be very light⁽¹⁾ and yet have escaped detection since its interactions with fermionic matter are suppressed by s-matter masses and since it is neutral. Similiar considerations apply to gluinos, though in their case a limit  $m_{\text{gluino}} > 5 \text{ GeV}$

has been suggested⁽¹⁾.

It is thus natural to attempt to build SUSY models with explicit SUSY breaking mass terms for the unobserved partners provided such mass terms can be shown to be soft, that is, they should not destroy the absence of quadratic divergences which is the basic feature motivating the introduction of SUSY. In this section we summarize the results of L. Girardello and M.T. Grisaru⁽⁶⁾ on the various terms which satisfy this criterion and some which do not.

a)  $\mathcal{L}_{\text{break}} = \mu^2 (A^2 + B^2) = \mu^2 |Z|^2$  (scalar mass) where  $Z = A + iB$  is the scalar field of a chiral multiplet. The following divergences arise:

- i) A logarithmic divergence in the coefficient of  $(A^2 + B^2)$ .

ii) If A is a singlet then a log divergent term proportional to A may arise.

iii) If Z couples to a U(1) gauge field then a log divergent D term may arise.

b)  $\mathcal{L}_{\text{break}} = \mu^2 (A^2 - B^2)$

a log divergent F term may be generated.

c)  $\mathcal{L}_{\text{break}} = \mu \bar{\lambda} \lambda$  (gaugino mass)

i)  $\mu$  is logarithmically renormalized.

ii) Log divergent A, F, FA-GB,  $A^2+B^2$  terms may arise

d)  $\mathcal{L}_{\text{break}} = \gamma \text{Re} z^3 = \gamma (A^3 - 3AB^2)$

Log divergent A, F, FA-GB,  $A^2+B^2$ ,  $\bar{\lambda} \lambda$  terms arise

There are no other soft terms. Specifically  $\mu \bar{\psi} \psi$  and  $A^3$  are not soft and induce quadratic divergences. The reason why  $\bar{\psi} \psi$  is not soft is simply that its presence destroys the relation between couplings which contain the mass parameters of the Superpotential, e.g.,

$$g(z) = m z^2 + z^3/3 \quad (2.31)$$

then

$$V = |m + \lambda z^2|^2 = \lambda^2 |z|^4 + 2m\lambda \text{Re} z^3 + m^2 |z|^2 \quad (2.32)$$

so that the trilinear couplings depend on m, whereas the fermionic mass term (if  $\langle z \rangle = 0$ ) is  $m \bar{\psi} \psi$  and if we add  $\mu \bar{\psi} \psi$  as an explicit breaking the requisite relation is destroyed.

### Section 3: The Dimopoulos - Georgi Model

In this section we review the Dimopoulos-Georgi⁽⁷⁾ SU(5) SUSY model which pioneered the building of supersymmetric Gut models. We confine ourselves to a concise exposition of the route followed by them and a summary of the problems facing the model.

i) Quarks and leptons of each 'standard family' along with their scalar partners were assigned to chiral multiplets transforming as reducible  $\bar{5}+10$  representations  $\chi_k^i, \phi_{kij}$  of SU(5) where k is a generation and (i,j) are SU(5) indices.

The GU symmetry is broken from SU(5) to  $G_{123}$  by the scalars contained in a 24plet chiral multiplet.

The scalar partner of the  $\begin{pmatrix} \nu \\ e \end{pmatrix}_L$  cannot be used as Electro-Weak Higgs doublets since interactions such as  $[\phi\chi\chi]_F$  would then lead to ultrafast baryon decay via the color triplets in the  $\chi$ . Many other problems such as the electron acquiring a Dirac mass with charged gauginos would also occur.

Thus they introduced a pair H+H transforming as  $5+\bar{5}$  under SU(5), in which SU(2) doublets are embedded. A pair of Higgs is necessary for anomaly freedom and  $5+\bar{5}$  is the simplest choice. More over since the superpotential only admits fields of the same chirality it would not be possible

to use the same Higgs to give mass to both types of quarks.

ii) The superpotential was chosen to be

$$g = \lambda_1 \text{Tr} \left( \frac{1}{3} \Sigma^3 + \frac{m}{2} \Sigma^2 \right) + \lambda_2 \bar{H} (\Sigma + 3m' \mathbb{1}_5) H \quad (2.33) \\ + f_{ab} H \varphi_a \varphi_b + g_{ab} \bar{H} \chi_b \varphi_a$$

SU(5) indices have been suppressed and a, b are generation indices. Certain couplings (e.g.  $\chi \varphi \varphi$ ) have been omitted so as to avoid ultra fast proton decay via light color triplets. This restriction can be understood as arising from a discrete symmetry under which only  $\chi, \varphi$  change sign).

iii) The potential then has three degenerate minimal at

Little Group

- a)  $\langle \Sigma \rangle = 0$  SU(5)
- b)  $\langle \Sigma \rangle = m/3 \text{diag}(\mathbb{1}_4, -4)$  SU(4) x U(1) (2.34)
- c)  $\langle \Sigma \rangle = 2m \text{diag}(\mathbb{1}_3, -\frac{3}{2}\mathbb{1}_2)$  G₁₂₃

With all other v.e.v.'s = 0.

The degeneracy of GUT minimae is a characteristic feature of global SUSY GUTs. We shall discuss this problem at a later stage.

iv) The scalar Higgs doublets have a mass² (from  $|F_H|^2 + |F_{H'}|^2$ ) proportional to  $(m-m')^2$ . If one performs the fine tuning

$$m = m' + \epsilon m \quad (2.35) \\ \epsilon \sim O\left(\frac{m_W}{M_U}\right)$$

one finds that the doublets are massless on the scale of  $M_U$ . The ratio  $m/m'$  is preserved by radiative corrections since both suffer a (common) wave function renormalization. Hence there is no need to fine tune at each order of perturbation theory. This is the 'technical solution' of the gauge hierarchy problem. It begs the question of the fine tuning itself as also of the existence of the two scales. We shall discuss these problems in subsequent chapters.

v) The effective low energy Lagrangian is simply a supersymmetric  $G_{123}$  invariant gauge theory with gauge bosons, matter fermions and Higgs scalars along with their supersymmetric partners. All these fields are massless and have zero v.e.v. Thus apart from the problem of degenerate GUT vacua the following questions must be answered:

1) What is the mechanism of supersymmetry breaking and at what scale does it operate? In other words what lifts the degeneracy between matter (fermions) and s-matter (scalars).

2) How do the doublets acquire v.e.v.'s so as to break the Electroweak symmetry?

If one attempts to answer these questions by constructing a model in which SUSY is broken spontaneously at low energies ( $\sim \text{TeV}$ ) one runs into the following theorem due to Dimopoulos and Georgi:

Theorem: In a  $G_{123}$  invariant theory of quarks and leptons

with spontaneously broken SUSY the tree approximation mass matrix describes a color triplet spinless meson of one of the following kinds: a charge  $2/3$  triplet not heavier than the lightest charge  $2/3$  quark or a charge  $-1/3$  triplet not heavier than the lightest charge  $-1/3$  quark.

The basic reason for this phenomenon is that the spontaneous supersymmetry breaking leaves the fermion mass matrix uncharged but splits scalars and pseudo scalars in opposite directions. Thus if supersymmetry is spontaneously broken then in order that the experimental limits on the masses of s-matter ( $\sim 20$  GeV) be respected radiative mechanisms must generate a supergap larger than  $10^1$  GeV. Various models have been proposed to accomplish this but all invoke a complicated set of interactions among a large number of new fields introduced for just this purpose.

The other alternative is to add 'soft' terms which break supersymmetry explicitly but do not destroy the non-renormalization properties crucial to this class of model. This route was followed by Dimopoulos and Georgi. Furthermore, as we shall see the promotion of global to local supersymmetry, in a class of models, results in precisely the set of soft terms necessary to solve the two problems mentioned above.

Dimopoulos and Georgi thus proposed adding the following soft terms to the Lagrangian:

$$a) \mu_m^2 [ |A_\phi|^2 + |A_\chi|^2 ] \quad (2.36a)$$

$$b) \tilde{\mu} \bar{\lambda} \lambda \quad (2.36b)$$

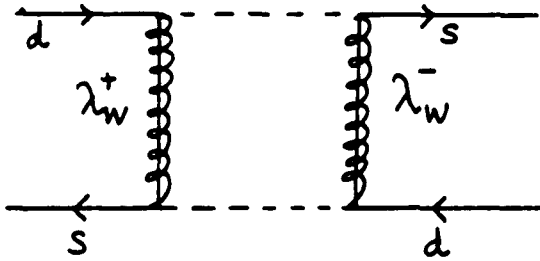
c) An appropriate  $\text{mass}^2 \sim O(m_W)$  matrix for the  $5+\bar{5}$  Higgs scalars

$$d) -\mu_\Sigma^2 |A_\Sigma|^2$$

e) A mass term for the Higgsinos.

As we have seen, e) is not in fact 'soft' but this was not well known when the DG model was first proposed. All mass parameters are  $\leq 1\text{TeV}$ , in order that the 'super protection theorems be effective down to  $M_W$ . We note at this point that it has been shown by Mohapatra and the author⁽¹¹⁾ (see Appendix II) that the renormalizability of the low energy Lagrangian requires that the soft breaking terms be introduced as a 'complete set' consisting of s-matter masses, cubic couplings and gaugino masses. Dimopoulos and Georgi also noted that a super-GIM mechanism

which suppressed dangerous flavor-changing box graphs



S=2 box graph

operates provided smatters with identical  $G_{123}$  quantum numbers have (almost) equal masses. Thus  $\mu_m$  should not depend on the generation index. This restriction was made quantitative in the work of Ellis et al⁽⁹⁾ who found that the following constraints should hold

$$\Delta m_{\text{septon}}^2 / m_{\text{septon}}^2 < 10^{-5} (\text{GeV})^{-2} m_{\lambda_W}^2 \quad (2.37)$$

$$\Delta m_{\text{squark}}^2 / m_{\text{sq}}^2 < 10^{-3}$$

where  $\Delta m^2$  is the splitting between different s-matter generations.

vi) The scale of super unification may be deduced by modifying the coefficients  $b_N$  in the formula

$$\frac{\partial g}{\partial t} = \frac{b_N}{16\pi^2} g^3 \quad (2.38)$$

to account for the presence of superpartners. The general

formula is

$$b_N = -11/3 T(\text{Adj}) + 4/3 T(\text{Weyl/Majoranafermions}) + 1/3 T(\text{scalars}) \quad (2.39)$$

Where  $T$  is the Casimir operator.

Since we have a set of Majorana fermions in the adjoint representation and each chiral multiplet is accompanied by a complex scalar

$$b_N^{\text{SUSY}} = -3T(\text{Adj}) + T(\text{Chiral}) \quad (2.40)$$

Many authors have discussed the constraints placed on mass scales and representations in a SUSY theory. We confine ourselves to quoting the results for  $SU(5)$  since we shall discuss the  $SO(10)$  case in some detail in ensuing chapters. All estimates depend crucially on the value of the QCD scale parameter QCD estimates of which range between 100 and 300 MeV. Ellis et al⁽⁸⁾ give

	Number of light Higgs doublets		
$\sum \theta_w^2(M_w)$	$.233 + .005 \ln(.3 \text{ GeV} / \Lambda_{\overline{MS}})$	2	
$\sum \theta_w^2(M_w)$	$.259 + .002$	4	$(\Lambda_{\overline{MS}} = 140 \text{ GeV})$
$M_U$	$= 6 \times 10^{16} \Lambda_{\overline{MS}} \text{ GeV}$	2	(2.41)
$M_U$	$= 3 \times 10^{15} \Lambda_{\overline{MS}} \text{ GeV}$	4	

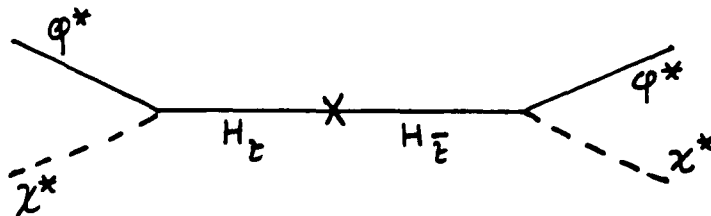
## Section 4

### Proton decay in SUSY GUTs

Weinberg, Sakai and Yanagida⁽¹⁰⁾ pointed out that there exist supersymmetric  $G_{123}$  invariant Baryon number violating operators of dimension 5 in the low energy sector of SUSY models. For instance the operator

$$O_5 = \{ d_{R\alpha}^a U_{R\beta}^b U_{R\gamma}^c e_R \}_F \epsilon^{\alpha\beta\gamma} \quad (2.42)$$

( $\alpha, \beta, \gamma$  are color indices and  $a, b, c$  are generation indices) violates baryon number. Note that although LPI terms in the effective action must necessarily be D type densities the same is not true of one particle reducible terms, specifically for heavy particle reducible terms. Thus the above operator can arise as via exchange of Higgsino color triplets

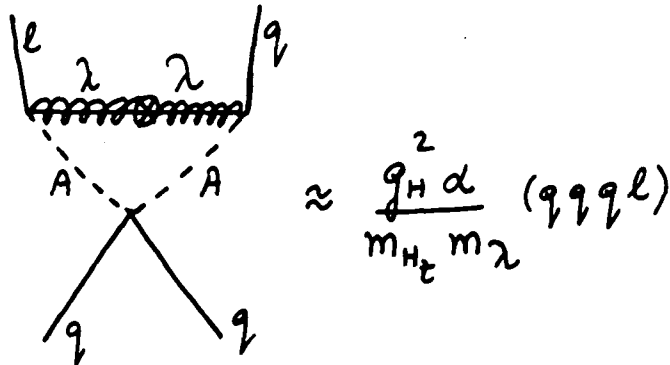


and at low energies appears as a non-renormalizable term

$$\text{Diagram} \times \left( \frac{1}{M_{H_t}} \right)$$

By dressing the scalar legs with a gaugino exchange one gets

a 4 fermi operator



which violates baryon number with the indicated strength. Ellis et al⁽⁹⁾ have estimated these graphs and renormalization group corrections to their coefficients and arrive at the constraints

$$m_\lambda m_{H_t} \gtrsim 10^{18} \text{ GeV}^2 \quad (2.43)$$

$$m_{H_t}^2 / m_\lambda \gtrsim 10^{19} \text{ GeV}^2$$

in order that there be no conflict with the current bounds on the proton lifetime.

In summary, the DG model has the following features

- i) It demonstrates the feasibility of supersymmetrizing the standard SU(5) model and removes the necessity of order by order fine tunings to prevent mass scale scrambling
- ii) The  $G_{123}$  invariant vacuum is degenerate with the SU(5) and SU(4) x U(1) invariant vacua.
- iii) One must introduce ad-hoc soft breakings of SUSY in order to lift Bose-Fermi degeneracy. Smaller masses² must be equal to one part in  $10^3$ .

### Chapter 3

#### AN SO(10) SUSY GUT

We saw in the last chapter that it is possible to supersymmetrize the SU(5) GUT model provided one is willing to live with ad-hoc soft supersymmetry breaking terms, necessary to achieve a mass spectrum compatible with experiment. Furthermore, in naive models of the DG type a workable mechanism to lift the GUT vacuum degeneracy is lacking although there exist speculations⁽¹²⁾ that the Universe may choose the correct vacuum in the course of its thermal evolution. However, detailed calculations indicate that this mechanism, is not, in fact, viable. As we shall see in later chapters the promotion of SUSY to a local invariance leads to a class of models in which the degeneracy of GUT vacua is lifted and the soft terms necessary to provide (equal) masses for the s-matter fields arise naturally.

In this chapter we focus on an earlier stage of model building. Namely, we explore the possible connection of the SUSY breaking scale with scales associated with the breakdown of gauge symmetries. As is well known, the minimal SU(5) models predict a Grand Desert stretching over the energy interval  $10^2$  GeV to  $M_U$  ( $10^{15}$  GeV). However, there are other GUTs with oases, that is to say, intermediate mass scales associated with the breaking of gauge symmetries. It is therefore natural to enquire whether the SUSY breaking scale is associated with the scale

at which an intermediate gauge symmetry breakdown occurs.

The best known alternatives to SU(5) grand unification is the SO(10) model of Fritsch and Minkowski.⁽¹³⁾ Although the SU(5) model is embedded able in the SO(10) one with the additional virtues of irreducible Grand Unifying matter fermion representations, the 'see-saw' mechanism for neutrino mass⁽¹⁴⁾ suppression, etc., there exists a more interesting class of SO(10) models. In this latter class SO(10) breaks down to the standard group along a chain which includes the left-right symmetric group  $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ . Recall that the Weinberg-Salam model exhibits a puzzling difference in its treatment of different chiralities of the same Dirac fermion. Since observation rules out identical weak interactions for both chiralities it is natural to consider whether the difference between the two was not the result of the spontaneous breaking of a gauge group in which the left handed and right handed sectors were interchangeable.

Moreover, Marshak and Mohapatra⁽¹⁵⁾ observed that in theories with a L-R symmetric gauge group the following expression for the electric charge operator holds

$$Q = T_{3L} + T_{3R} + (B-L)/2 \quad (3.1)$$

where  $T_3$  is the diagonal generator of SU(2) and B-L is Baryon number-Lepton number. Since Q is conserved this immediately yields the relation

$$\Delta(B-L) \simeq 2 \Delta T_{3R}$$

at energies  $\gg M_W$ . It is well known that the minimal SU(5) theory automatically conserves B-L and thus does not permit phenomena such as neutron-anti-neutron oscillations which do not conserve B-L. Thus it is exciting to realize that the existence of such oscillations could be tied to a right handed symmetry in such models. Moreover, computation of the nn oscillation time  $\tau_{n\bar{n}} \sim (M_{\text{Nucleon}}^4/M_R^5)$  yields the constraint (from the observed stability of matter) that the scale of B-L violation  $M_R \gtrsim 10^6 - 10^7$  GeV. On the other hand Marciano and Senjanovic⁽¹⁶⁾ have shown that in non-SUSY SO(10) models M can be as low as 200 GeV. It was therefore natural for us to enquire as to any possible link between the L-R breaking scale  $M_R$  and the scale of supersymmetry breaking  $M_S$ . Of course there exist other groups such as SU(16) which can also break via the L-R symmetric group. These are presently under investigation by the author and collaborators. However, we shall present only the SO(10) case in detail and confine ourselves to a few remarks on the situation in SU(16). The present chapter recounts work done in collaboration with Prof. R.N. Mohapatra.⁽¹⁷⁾

Section 2 Group theory: In this section we review some of the gauge group  $SO(10)$  and of GUT models based upon it.

The following facts will prove convenient⁽¹⁸⁾.

One introduces

$$\{ \chi_i ; i = 1 \dots 5 \}$$

such that

$$[\chi_i, \chi_j]_+ = 0 \quad [\chi_i, \chi_j^\dagger]_+ = \delta_{ij} \quad (3.2)$$

Then if one defines

$$\begin{aligned} \mathcal{P}_{2j-1} &= -i (\chi_j - \chi_j^\dagger) \\ \mathcal{P}_{2j} &= (\chi_j + \chi_j^\dagger) \end{aligned} \quad (3.3)$$

one finds

$$[\mathcal{P}_\mu, \mathcal{P}_\nu]_+ = 2\delta_{\mu\nu} \quad \mu, \nu = 1 \dots 10 \quad (3.4)$$

ie. the  $\mathcal{P}_\mu$  form a Clifford algebra of rank 10. Thus the 45 generators of  $SO(10)$  can be constructed as

$$\Sigma_{\mu\nu} = \frac{1}{2i} [\mathcal{P}_\mu, \mathcal{P}_\nu] \quad (3.5)$$

The spinor representation  $\psi$  of  $SO(10)$  is 32 dimensional but can be split into invariant subspaces (in analogy to the

Lorentz group) using the chiral projectors  $P_{\pm} = (1 \pm \bar{P}_0)/2$

$$\psi_{\pm} = P_{\pm} \psi \quad (3.6)$$

where

$$\begin{aligned} \bar{P}_0 &= i \bar{P}_1 \dots \bar{P}_{10} = \prod_j (1 - 2\eta_j) = (-1)^n \\ \eta_j &= \chi_j^\dagger \chi_j \quad n = \sum_j \eta_j \end{aligned} \quad (3.7)$$

One then has  $(\chi_j | 0 \rangle = 0)$  the representation

$$\begin{aligned} \psi_+ = |0 \rangle \psi_0 &+ \frac{1}{2} \chi_j^\dagger \chi_k^\dagger |0 \rangle \psi^{jk} \\ &+ \frac{1}{4!} \epsilon^{jklmn} \chi_k^\dagger \chi_l^\dagger \chi_m^\dagger \chi_n^\dagger |0 \rangle \bar{\psi}_j \end{aligned} \quad (3.8)$$

and  $\psi_0, \psi^{jk}, \bar{\psi}_j$  transform as  $1, 10, \underline{5}$  under  $SU(5)$ . Thus one can embed a standard family in the 16 dimensional 'chiral' spinor of  $SO(10)$  along with an  $SU(5)$  singlet particle.

On the other hand if we look at the decomposition of the 16 under the subgroups

$G_{LR} = SU(2)_L \times SU(2)_R \times SU(3)_C \times U(1)_{B-L}$  and  $G_{123}$  we have

$$\begin{aligned} 16 = (2, 1, 3, 1/3) &+ (2, 1, 1, -1) + (1, 2, \underline{3}, -1/3) \\ &+ (1, 2, 1, 1) \end{aligned} \quad (3.9)$$

under  $G_{LR}$

and

$$16 = (2, 3, 1/3) + (2, 1, -1) + (1, 1, 2) + (1, \underline{3}, -4/3) \quad (3.10) \\ + (1, \underline{3}, 2/3) + (1, 1, 0)$$

under  $G_{123}$ .

This makes it clear that the extra singlet particle is the right-handed neutrino since it is a part of the  $SU(2)_R$  doublet containing  $e_L^c$ .

Thus the matter fermions are embedded in 16 dimensional representations of  $SO(10)$ . In addition to this we need Higgs scalars to break gauge symmetries and of course the gauge multiplet itself. One indispensable multiplet is the fundamental 10plet which contains a two  $SU(2)_L$  doublets. It is well known that the dominant breaking of the Electroweak symmetry must come from v.e.v. of  $SU(2)_L$  doublets rather than, for instance triplets, since the  $e$  parameter of the weak interactions is strongly affected by the representation responsible for the breaking.

The two principal chains via which  $SO(10)$  can break down to  $G_{123}$  are:

a)

$$SO(10) \longrightarrow SU(5) \longrightarrow G_{123} \longrightarrow SU(3) \times U(1)_{em} \quad (3.11a)$$

$$\begin{array}{c}
 \text{b) } SO(10) \longrightarrow G_{PS} \left( SU(2)_L \times SU(2)_R \times SU(4) \right) \\
 \downarrow M_C \\
 G_{LR} \xrightarrow{M_R} G_{123} \\
 \downarrow M_W \\
 SU(3) \times U(1)_{em}
 \end{array} \quad (3.11b)$$

As mentioned we shall be interested only in chain b). In order to construct GUT models following this chain we need to find representations which can accomodate Higgs scalars whose v.e.v. can accomplish the required breaking.

The following table of subreps of  $SO(10)$  irreps under the maximal subgroup  $G_{PS}$  is of use in determining combinations of irreps useful in constructing a GU model.

$$\begin{aligned}
 10 &= c(1,1,6) + d(2,2,1) \\
 16 &= q_L(2,1,4) + q_L^c(1,2,\underline{4}) \\
 45 &= l(3,1,1) + r(1,3,1) + f(1,1,15) + g(2,2,6) \\
 54 &= (1,1,1) + (3,3,1) + (1,1,20') + (2,2,6) \\
 126 &= \sigma(1,1,6) + \Delta_L(3,1,10) + \Delta_R(1,3,\underline{10}) + \alpha(2,2,15) \\
 210 &= s(1,1,1) + a(1,1,15) + \omega_L(3,1,15) + \omega_R(1,3,15) \\
 &\quad + t(2,2,10) + t(2,2,\underline{10}) + b(2,2,6)
 \end{aligned} \quad (3.12)$$

From this table it is evident that the breaking of  $G_{PS}$  can be achieved via either the 54 or the 210. However, since we

wish to investigate the possibility of neutron oscillations in  $SO(10)$  we need to include the 126 representation which includes the  $(1,3,10)$  subrep necessary for the classic Marshak-Mohapatra nn oscillation diagram⁽¹⁵⁾. Since the 54 has no cubic couplings with the  $126 + 126$  (the real combination is necessary for anomaly freedom) we settled upon the 210 dimensional representation as the one appropriate for symmetry breaking;. The 210 has the added advantage that its  $(1,1,15)$  subrep can be used to break  $SO(10)$  directly to  $G_{LR}$ .

Thus the set of subreps utilized by us were:

$$\underline{\phi}(210) , \underline{\Sigma}(126) , \underline{\bar{\Sigma}}(126) , \underline{\chi}(16) , H(10) , \underline{\bar{\psi}}(45) \quad (3.13)$$

of which  $\underline{\phi}, \underline{\Sigma}, \underline{\bar{\Sigma}}$  and  $H$  are Higgs multiplets.  $\underline{\chi}^a$  contains matter fields and  $\underline{\bar{\psi}}$  is the gauge multiplet.

### Section 3 Breaking Chains and Superpotentials:

We consider two distinct breaking chains on the pattern of (3.11b) In the first we identify  $M_U = M_C$  and break  $SO(10)$  down to  $G_{LR}$  directly at  $M_U$ . In the second we identify  $M_C = M_R$  and break  $G_{PS}$  down to  $G_{123}$  at  $M_R$ . Of course it is possible to retain more distinct scales and even to introduce more scales by breaking  $G_{LR}$  to  $G_{123}$  in one or more stages but since this would merely introduce more free parameters and require further Higgs representations we have confined ourselves to these simple choices. We mention that in the second of the above two

chains it is necessary to introduce a singlet field in order to accomplish the symmetry breaking.

We now write down the superpotential for the two chains. Before doing so we note the following facts about the representations used.

$$\begin{aligned} 16 \times 16 &= 10 + 120 + 126 \\ 16 \times \underline{16} &= 1 + 210 + 45 \end{aligned} \quad (3.14)$$

$$\begin{aligned} \begin{Bmatrix} 10 \\ 120 \\ 126 \end{Bmatrix} &\sim \chi^T B \begin{Bmatrix} T_\mu T_\nu T_\lambda \\ T_\mu T_\nu T_\lambda T_\sigma T_\kappa \end{Bmatrix} \chi \\ \begin{Bmatrix} 1 \\ 45 \\ 210 \end{Bmatrix} &\sim \bar{\chi} \begin{Bmatrix} T_\mu T_\nu \\ T_\mu T_\nu T_\sigma T_\lambda \end{Bmatrix} \chi \end{aligned}$$

thus 210 is the 4 index anti-symmetric tensor contained in the product of 16x16 while 126 is the 5 index object in 16x16.

Note, however, that  $^{10}C_5=252$  hence there is a further constraint

$$\begin{aligned} \Sigma_{\mu_1 \dots \mu_5} &= + \Sigma_{\bar{\mu}_1 \dots \bar{\mu}_5} \\ \bar{\Sigma}_{\mu_1 \dots \mu_5} &= - \Sigma_{\bar{\mu}_1 \dots \bar{\mu}_5} \end{aligned} \quad (3.15)$$

amongst the components of the 126 and 126 besides anti-symmetry. Here the conjugate index set  $\{\bar{\mu}\}$  is defined by requiring that

$$T_{\mu_1 \dots \mu_5} T_0 = T_{\bar{\mu}_1 \dots \bar{\mu}_5} \quad (3.16)$$

or equivalently that

$$e^{\mu_1 \dots \mu_5} \bar{\mu}_1 \dots \bar{\mu}_5 = 1 \quad (3.17)$$

From this it follows that

$$\Sigma_{\mu_1 \dots \mu_5} \bar{\Sigma}_{\mu_1 \dots \mu_5} = 0 \quad (3.18)$$

$$\Phi_{\mu_1 \dots \mu_4} \Sigma_{\mu_1 \mu_2} \alpha_1 \alpha_2 \alpha_3 \bar{\Sigma}_{\mu_3 \mu_4} \alpha_1 \alpha_2 \alpha_3 = 0$$

The superpotential in the two cases studied by us is :

$$a) \quad SO(10) \xrightarrow{M_U} G_{LR} \xrightarrow{M_R} G_{123} \xrightarrow{M_W} SU(3) \times U(1)_{em}.$$

$$g = M_1 \Phi^2 + \alpha \Phi^3 + M_2^{ij} \bar{\Sigma}^i \Sigma^j + \sum_k b_k^{ij} (\Phi \Sigma^i \bar{\Sigma}^j)_k \\ + M_3 H^2 + c_1^i \Phi H \Sigma^i + c_2^i \Phi H \bar{\Sigma}^i \quad (3.19)$$

$$+ d_1 \chi^T \gamma_5 \gamma_\mu \chi H_\mu + d_2 \chi^T \gamma_5 \gamma_\mu \gamma_\nu \gamma_\sigma \gamma_\lambda \gamma_\kappa \chi \bar{\Sigma}_{\mu\nu\sigma\lambda\kappa}$$

Here

$$\Phi^2 = \Phi_{\mu\nu\rho\sigma} \Phi_{\mu\nu\rho\sigma} \quad \Phi^3 = \Phi_{\mu\nu\rho\sigma} \Phi_{\rho\sigma\kappa\lambda} \Phi_{\kappa\lambda\mu\nu} \quad (3.19)$$

$$\Sigma \bar{\Sigma} = \Sigma_{\mu\nu\rho\sigma\lambda} \bar{\Sigma}_{\mu\nu\rho\sigma\lambda} \\ \Phi H \Sigma = \Phi_{\alpha\beta\gamma\delta} H_\lambda \Sigma_{\alpha\beta\gamma\delta\lambda} \\ H^2 = H_\mu H_\mu$$

B is the conjugation matrix for SO(10) and k runs over various SO(10) invariants that can be formed from (We have provided for upto 3  $\Sigma, \bar{\Sigma}$  pairs using indices i, j).

For example,

$$(\Phi \Sigma \bar{\Sigma})_i = \Phi_{\mu\nu\alpha\beta} \Sigma_{\mu\nu\gamma\delta\lambda} \bar{\Sigma}_{\alpha\beta\gamma\delta\lambda} \quad (3.20)$$

$$b) \quad SO(10) \xrightarrow{M_U} G_{PS} \xrightarrow{M_R} G_{123} \xrightarrow{M_W} SU(3)_c \times U(1)_{em}$$

In this case we replace the first two terms in 3.19a by

$$\lambda_1 X (\Phi^2 - 24 M_1^2) + \alpha \Phi^3 \quad (3.21)$$

where X is a singlet and set  $M_3=0$ .

In addition to the supersymmetric SO(10) gauge invariant Lagrangian derived from the above superpotential and the usual gauge invariant kinetic terms we add soft terms of dimension 2 and 3 with mass parameters  $\mu \sim M_S$  the supersymmetry breaking scale. The scalar terms consist of positive and negative mass squared terms for the matter fields as well as trilinear interactions required by renormalizability (see Appendix II). In addition, we add mass terms for the gauge fermions. Thus,

$$\begin{aligned} \mathcal{L}_{soft} = & \mu_\Sigma^2 (A^\Sigma{}^\dagger A^\Sigma + A^{\bar{\Sigma}}{}^\dagger A^{\bar{\Sigma}}) \\ & + \mu_H^2 (A^H{}^\dagger A^H) - \mu_\chi^2 A^\chi{}^\dagger A^\chi \\ & + \mu_\chi A_\chi{}^T B \not{P}_\mu A_\chi \not{H}_\mu + \mu_{\bar{\Sigma}} (A_\chi A_\chi A_{\bar{\Sigma}}) \\ & + \mu_\lambda \lambda_{\mu\nu}^T C^{-1} \lambda_{\mu\nu}. \end{aligned} \quad (3.22)$$

#### Section 4: Symmetry breaking.

Before proceeding with an analysis of the symmetry breaking we note a few general theorems about the patterns of symmetry breaking in Globally SUSY theories. As we saw earlier the potential in globally SUSY theories has the form (2.8)

$$V = |F_i|^2 + \frac{1}{2} D_\alpha^2$$

where

$$F_i = -g_i(A) \quad D_\alpha = g \left[ (A_{i+}^\dagger T_\alpha A_{+i}) - (A_{-i}^\dagger T_\alpha A_{-i}) \right] \quad (2.8)$$

The absolute minimum of  $V$  preserves SUSY only if  $\langle F_i \rangle = 0$  and  $\langle D^\alpha \rangle = 0$ . Then at the minimum  $V=0$ . The possible v.e.v.'s of the scalar fields⁽¹⁹⁾  $A_i$  can be characterized by their little group,  $G_i$ . By definition for the generators  $T$  of  $G_i$

$$T^\alpha \langle A_i \rangle = 0$$

The remaining generators of  $G$  lie in the coset  $G/G_i = C$  and  $T^\alpha \langle A_i \rangle \neq 0$  for  $T \in C$ . If, however,  $T^\alpha \in C$  is not a  $G_i$  singlet then since  $\langle A_i \rangle$  is a  $G_i$  singlet

$$\langle A_i^\dagger T^\alpha A_i \rangle = 0 \quad (3.23)$$

by the Wigner-Eckhart theorem. So one has⁽¹⁹⁾

Lemma: If  $G/G_i$  has no generator which is a singlet under  $G$  then all  $\langle D^\alpha \rangle = 0$ .

For real representations of  $G$  the generators  $T^\alpha$  can be

represented by antisymmetric matrices so that

$$\begin{aligned} & (\text{Re } A_a^i - i \text{Im } A_a^i) T_{ab}^\alpha (\text{Re } A_b^i + i \text{Im } A_b^i) \\ &= i (\text{Re } A_a^i \text{Im } A_b^i - \text{Im } A_a^i \text{Re } A_b^i) T_{ab}^\alpha \end{aligned} \quad (3.24)$$

So it is sufficient for such representations to have

$$\text{Re } A^i \sim \text{Im } A^i$$

to get  $\langle D^\alpha \rangle = 0$  for all  $\alpha$ .

Thus in particular if one has a reducible real representation  $R+R$  ( $R+R$  are conjugate irreps) then all  $D$  vanish provided  $|\langle R \rangle| = |\langle \bar{R} \rangle|$ .

We also note the following formula for the  $D^\alpha$  in  $SO(10)$

$$\begin{aligned} D_{\mu\nu} = & ig [ 4 A_{\mu\gamma\alpha\beta}^\dagger A_{\nu\gamma\alpha\beta} + 5 A_{\mu\gamma\alpha\beta\lambda}^\Sigma A_{\nu\gamma\alpha\beta\lambda}^\Sigma \quad (3.25) \\ & + A_\mu^\dagger A_\nu + i A^\chi{}^\dagger \Sigma_{\mu\nu} A^\chi - 5 A_{\mu\gamma\alpha\beta\lambda}^{\Sigma\dagger} A_{\nu\gamma\alpha\beta\lambda}^\Sigma ] \\ & - (\mu \leftrightarrow \nu). \end{aligned}$$

here  $\Sigma_{\mu\nu}$  are the  $SO(10)$  generators in the spinorial representation of  $SO(10)$ . We also have the following gauge yukawa couplings

$$\begin{aligned} \mathcal{L}_{gY} = & \sqrt{2} g [ 4 A_{\mu\alpha\beta\gamma} \bar{\lambda}_{\mu\nu} \psi_{\nu\alpha\beta\gamma} + A_\mu^\dagger \bar{\lambda}_{\mu\nu} \psi_\nu \quad (3.26) \\ & + (5 A_{\mu\gamma\alpha\beta\sigma}^\Sigma \bar{\lambda}_{\mu\nu} \psi_{\nu\gamma\alpha\beta\sigma}^\Sigma - (Z \rightarrow \bar{Z})) \\ & + i A^\chi{}^\dagger \bar{\lambda}_{\mu\nu} \Sigma_{\mu\nu} A^\chi ] + h.c. \end{aligned}$$

We now give the v.e.v. which accomplish the patterns of symmetry breaking that we have focussed our attention upon. In arriving at the assignment of v.e.v. indicated it is useful to realize that one may divide the 10 vectorial indices of  $SO(10)$  into two sets A: A,B: (1-6) and a,b,...(7-10). The indices A,B and a,b can then be taken to be the vectorial indices of the  $SO(6)$  and  $SO(4)$  subgroups of  $SO(10)$  respectively. Furthermore, one can check that various v.e.v.'s actually have the little groups claimed for them by utilizing the  $SU(5)$  basis for  $SO(10)$  described in Section 2. Thus we know that the following assignments of quantum numbers hold from our embedding of the  $5 + 10$  reps of  $SU(5)$  in the 16  $SO(10)$

	B-L	$I_{3L}$	$I_{3R}$	Color	Charge
$ 0\rangle$	0	0	-1/2	0	0
$\chi_1^\dagger  0\rangle$	-2/3	0	0	3	-1/3
$\chi_4^\dagger  0\rangle$	0	0	1/2	0	1
$\chi_5^\dagger  0\rangle$	0	0	1/2	0	0

$\langle 0|$  and  $\chi_j$  carry opposite labels. If we extract the 10 dimensional irreps  $T(\varphi_j, \tilde{\varphi}_j)$  where  $\varphi_j, \tilde{\varphi}_j$  couple to  $16 \times 16$  as

$$(16^T B \chi_j (16) \varphi_j + 16^T B \chi_j^\dagger 16 \tilde{\varphi}_j) \quad (3.27)$$

Then if we define

$$T_{2j-1} = \varphi_j \quad T_{2j} = \tilde{\varphi}_j \quad (3.28)$$

$\varphi_j, \tilde{\varphi}_j$  have the quantum numbers of  $\chi_j^\dagger, \chi_j$ . Now one can easily check that the following v.e.v. achieve

$$a) \quad SO(10) \xrightarrow{M_U} G_{LR} \xrightarrow{M_R} G_{123}$$

$$i) \quad \langle A^{\bar{1}} \rangle = \langle (1, 1, 1, 0) \rangle \neq 0$$

where  $(1, 1, 1, 0)$  lies within the  $(1, 1, 15)$  subrep under  $G_{PS}$  of 210. The appropriate index values are

$$\langle A_{1234} \rangle = \langle A_{1256} \rangle = \langle A_{3456} \rangle = V_0 \quad (3.29)$$

ii) To avoid a proliferation of terms we label the  $\Sigma, \bar{\Sigma}$  v.e.v. simply by the fermionic labels carrying the appropriate quantum numbers. Thus the L-R symmetry is thus broken by

$$\langle A_{\nu_e \nu_e}^{\Sigma} \rangle = \langle \Delta_R (1, 3, 1, +2) \rangle = V_R \quad (3.30)$$

$$\langle A_{\nu_e \nu}^{\Sigma} \rangle = \langle \alpha (2, 2, 1, 0) \rangle = \kappa \quad (3.31)$$

and the same v.e.v. for  $\bar{\Sigma} (\Delta_R \rightarrow \bar{\Delta}_R, \alpha \rightarrow \bar{\alpha})$ .

iii) Finally, the Electroweak symmetry also breaks via

$$\langle A_{q,10}^H \rangle = \langle d (2, 2, 1, 0) \rangle = \kappa \quad (3.32)$$

Note that we have chosen equal values for the doublets with a view to making SUSY preserving gauge breaking possible.

$$b) \quad SO(10) \xrightarrow{M_U} G_{PS} \xrightarrow{M_R} G_{123} \xrightarrow{M_W} SU(3)_C \times U(1)_{em}$$

i) In this case the (1,1,1) subrep of the 210 must be utilized. Thus we set

$$\langle A_{\overline{28910}}^{\overline{\Phi}} \rangle = \langle S(1,1,1) \rangle = V_0$$

ii)

$$\begin{aligned} \langle A_{\nu_c \nu_c}^{\Sigma} \rangle &= \langle \Delta_R(1,3,1,+2) \rangle = V_R \\ \langle A_{\overline{\nu}_c \overline{\nu}_c}^{\Xi} \rangle &= \langle \overline{\Delta}_R(1,3,1,-2) \rangle = V_R \end{aligned}$$

iii)

$$\langle A_{9,10}^H \rangle = \langle d(2,2,1,0) \rangle = \kappa$$

This completes our prescription of the pattern of v.e.v.

### Section 5: Preservation of SUSY

In order that the v.e.v.'s assigned above with  $V_0 \sim M_U$ ,  $V_R \sim M_R$ ,  $\kappa \sim M_W$  actually preserve supersymmetry down to scales  $O(M_R)$  it is necessary to show that no F or D auxiliary field receives contributions larger than  $O(M_R^2)$ . In order to achieve this it is necessary to introduce extra fine tuning conditions over and above those necessary to keep the Higgs doublets light.

The derivation of these conditions is fairly

tedious and we shall try to be concise within the bounds of clarity.

i) : We first consider the D terms.

$SO(10), G_{PS}, G_{LR}$  are all rank 5 groups and thus have equal numbers of diagonal generators. Hence

$SO(10)/G_{\text{intermediate}}$  contains no diagonal generators. The rotation properties of the coset generators  $G$  under the little group are determined by their commutators with the generators of  $G_{\text{int}}$ . Thus it is obvious that in this case (as well as any other where the number of diagonal generators of the unifying and little groups is the same)

$G/G_{\text{int}}$  has no generator which is a singlet under  $G_{\text{int}}$ .

Hence by the Lemma of (Section 3.4)  $\langle \Phi \rangle$  as assigned in (3.4) gives no contribution to any  $\langle D^a \rangle$ .

Furthermore, since  $\langle \Sigma \rangle = \langle \bar{\Sigma} \rangle$  it is clear from (3.4) that there are no contributions of  $O(M_R^2)$  or  $O(M_R M_W)$

either. This is easy to check explicitly using (3.25).

ii) F Terms

Case A:

We have, in schematic notation,

$$-F_{\Phi}^{\dagger} = 2M_1 \Phi + 3\alpha \Phi^2 + \sum_k b_k (Z \bar{Z})_k + c_1 (H \Sigma) + c_2 (H \bar{\Sigma}) \quad (3.33a)$$

$$-F_{\Sigma}^{\dagger} = M_2 \bar{\Sigma} + \sum_k b_k (\Phi \bar{Z})_k + c_1 (\Phi H) \quad (3.33b)$$

$$-F_{\bar{\Sigma}}^{\dagger} = M_2 \Sigma + \sum_k b_k (\Phi Z)_k + c_2 (\Phi H) + d_2 \chi \chi \quad (3.33c)$$

$$-F_H^\dagger = 2M_3 H + c_1(\Phi \Sigma) + c_2(\Phi \bar{\Sigma}) + d_1 \chi \chi \quad (3.33d)$$

$$-F_\chi^\dagger = d_2(\chi \bar{\Sigma}) + d_2(\bar{\chi} H) \quad (3.33e)$$

And the following pattern of v.e.v. (under  $G_{LR}$ ).

$$\begin{aligned} \langle \Phi \rangle &\sim \langle (1, 1, 1, 0) \rangle \in \langle (1, 1, 15) \rangle \\ \langle \Sigma \rangle &\sim \langle \bar{\Sigma} \rangle \sim \langle (1, 3, 1, \pm 2) \rangle \oplus \langle (2, 2, 1, 0) \rangle \\ \langle H \rangle &\sim \langle (2, 2, 1, 0) \rangle \end{aligned}$$

These  $F$  terms must all vanish down to  $O(M_R^2)$ . Even if we allow ourselves the freedom to fine tune the constraints on the representations involved are non-trivial. Thus other proposed models⁽²⁰⁾ are in fact not viable due to unnoticed contributions  $O(M_R M_U)$  etc. to  $\langle F_i \rangle$ .

i) To  $O(M_U^2)$  we have only  $\langle F_{ABCD}^\Phi \rangle \sim \langle F_{(1,1,1,0)}^\Phi \rangle$  possibly non-zero but if  $V_0 \neq 0$  it will automatically minimize at

$$V_0 = -M_1/a \quad (3.34)$$

No other  $F$  terms have any possible contributions of this order.

We must now check that the  $(\Phi \Sigma), (\Phi H)$  terms in  $F_\Sigma, F_{\bar{\Sigma}}, F_H$  do not contribute to the v.e.v. of these auxiliary fields. Crucial to eliminating these is the term  $M_2 \bar{\Sigma} \Sigma$  in  $g$ . We see that since  $\langle \Phi \rangle \sim (1, 1, 1, 0)$

hence  $\langle \bar{\Phi} \bar{\Sigma} \rangle, \langle \bar{\Phi} H \rangle$  transform like  $\langle \Sigma \rangle, \langle H \rangle$  respectively.

Two cancellations of terms  $O(M_U M_R)$  and  $O(M_U M_W)$

in  $F_{\Sigma}, F_{\bar{\Sigma}}$  can therefore be simultaneously achieved by fine tuning

$$M_2 - 3 V_0 B_{\Delta} = 0 + O(M_R) \quad (3.35a)$$

$$5M_2 - 3 V_0 B_{\alpha} + 6 c_1 V_0 = 0 + O(M_R) \quad (3.35b)$$

provided one also sets

$$c_1 = c_2 = c \quad (3.35c)$$

This still leaves us with  $F_H$ . There one needs to tune

$$M_3 + 12 c V_0 = 0 + O(M_R) \quad (3.35d)$$

Note that since the contraction between  $\bar{\Phi} + \Sigma$  in  $F_H$  is

$$\langle \bar{\Phi}_{\mu\nu\lambda\sigma} \rangle \langle \Sigma_{\mu\nu\lambda\sigma\kappa} \rangle = \langle \bar{\Phi}_{ABCD} \Sigma_{ABCD\kappa} \rangle \quad (3.36)$$

the  $(1,3,1,\pm 2)$  v.e.v. of  $\Sigma, \bar{\Sigma}$  do not contribute since they have index pattern ABCab. This set of fine tunings is thus enough to remove all contributions to  $V$  which could possibly be  $> O(M_R^4)$ .

#### Case B:

In this case  $A_{78910} = V_0$  induces the breaking  $SO(10) \rightarrow G_{PS}$ . We now have (with obvious contractions

suppressed).

$$-F_X^\dagger = \lambda (\bar{\Phi}^2 - 24 M^2) \quad (3.37a)$$

$$-F_{\bar{\Phi}}^\dagger = 2X \bar{\Phi} \lambda + 3a \bar{\Phi}^2 + \sum_k b_k (\bar{Z} \bar{Z})_k + c_1 (H \bar{Z}) + c_2 (H \bar{Z}) \quad (3.37b)$$

thus we see that

$$\langle \bar{\Phi}^2 \rangle = 24 V_0^2 = 24 M^2 \quad (3.38)$$

but further notice that

$$\frac{\partial (\bar{A}^\dagger)^3}{\partial A_{\mu\nu\lambda\sigma}} \sim A_{\lambda\sigma\rho\omega} A_{\rho\omega\mu\nu} \quad (3.39)$$

now if  $(\mu\nu\lambda\sigma) = (78910)$  then it is obvious that the antisymmetry of indices implies  $\rho\omega \neq (78910)$ . Thus we see that for the only index pattern for which the first term in (3.37a) is non-zero the second term vanishes. Hence,

$$\langle A^X \rangle = 0 \quad (3.40)$$

The other F equations for the rest of the  $F_i$  are the same as given in (3.33). Terms  $O(M_U M_R)$  are now removed by the same types of tunings used in Case A. Specifically, one finds

$$M_2 + B_0 V_0 = O(M_R) \quad (3.41)$$

Since  $H_A = 0$  ( $A=1\dots 6$ ) it is obvious that  $\langle \bar{\Phi} H \rangle$  coming from the  $C_i$  invariants cannot contribute to  $F_{\bar{Z}}$  or  $F_{\bar{Z}}$ .

Further, since  $M_3=0$

$$\langle -F_{H\mu}^\dagger \rangle = C_1 \langle \Phi_{abcd} \rangle \langle \Sigma_{abcd\mu} \rangle + C_2 \langle \bar{\Phi}_{abcd} \rangle \langle \bar{\Sigma}_{abcd\mu} \rangle \quad (3.42)$$

but in this case  $\langle \Sigma_{abcd\mu} \rangle = 0$  so there is no problem at all. The only tuning conditions are thus

$$M_2 + B \quad V_0 = 0 + O(M_R) \quad (3.43)$$

We have now shown that it is indeed possible to preserve SUSY down to  $M_R$  provided one is willing to fine tune the parameters in the potential.

It is now clear that the potential at the classical minimum has the form

$$\sum_n a_n V_R^n \kappa^{4-n} - \mu_\Sigma^2 V_R^2 - \mu_H^2 \kappa^2 \quad (3.44)$$

and for appropriate ranges of parameters we obtain the required hierarchy  $V_R \gg \kappa$ . However, since SUSY is already broken at this level the fine-tuning conditions must be imposed order by order in perturbation theory.

## Chapter 4

### Mass spectra and constraints on mass scales

#### Introduction

In the previous chapter we set up an  $SO(10)$  model which allowed for the association of  $M_R$  and  $M_S$  as well as the possibility of  $n$ - $n$  oscillations via the inclusion of the  $\Delta$  subreps of the  $126 + \underline{126}$  reps of  $SO(10)$  in the intermediate scale field spectrum. We saw that a set of fine tuning conditions was necessary to achieve this. These fine tuning conditions have definite consequences for the mass spectrum of the theory. Specifically they result in the occurrence of pseudo-Goldstone (PG) supermultiplets which can acquire mass only via radiative corrections. Since this is forbidden as long as SUSY is unbroken it follows that they receive masses of  $O(M_S)$  only. Since they have large numbers of colored particles they result in tight constraints on the size of the separation of the scales  $M_S$  and  $M_U$  as long as one accepts - as one must if one is using one loop results - perturbative unification. On the other hand one could regard these PG multiplets as characteristic of the particular representations chosen by us rather than of the  $SO(10)$  gauge group in general. For instance, by including more representations with v.e.v.  $O(M_U)$  we could break the symmetries responsible for the presence of PG set. Therefore, while analyzing the low energy structure we have also considered the case when the PG multiplets of our particular model are absent and the

mass spectrum displays only the features one would naively expect from a scenario with intermediate L-R symmetry and SUSY breaking scales identified plus the constraints arising from the complexity of the representations.

In this chapter we first explore the mass spectrum and then present a discussion of the constraints on mass scales imposed by the low energy parameters  $\sin^2 \theta_W(M_W)$  and  $\alpha_3(M_W)$

### Section 1: Mass spectra

Supersymmetry equalizes fermion and boson masses above  $M_S$  and, on the other hand, radiative corrections give masses  $O(M_S)$  to PG supermultiplets. Thus we need only perform the (simpler) determination of fermion masses to arrive at the mass spectrum above  $M_S$ . Recall (chap 2) that the fermion masses have just two sources at tree level

a) From the superpotential

$$\frac{1}{2} \psi_i^T C^{-1} \psi_j g^{ij} + h.c.$$

b) From the gauge-Yukawa couplings

$$ig\sqrt{2} \langle A_i^T \rangle \bar{\lambda}^\alpha T_\alpha \psi_i + h.c.$$

We use the notation of (3.12) in referring to subreps of various irreps.

#### Case A: i) Masses in the 210

The super heavy  $O(M_U)$  gauge fermion masses come

from

$$ig\sqrt{2} \langle A_{ABCD} \rangle \bar{\lambda}_{D\mu} \psi_{\mu ABC} + h.c. \quad (4.1)$$

Since  $\lambda_{DE}$  is the  $f(1,1,15)$  subrep of and  $SU(3) \times U(1)_{B-L}$  is preserved only the  $(1,1,3 (4/3) + \underline{3} (-4/3))$  subreps get mass against  $\psi_{EABC}$ .

The index pattern aABC corresponds to the  $t(2,2,10) + \bar{t}(2,2,\underline{10})$  subreps of  $(210)$ . The 10 decomposes under  $SU(3) \times U(1)_{B-L}$  as

$$10 = 3(-2/3) + 6(2/3) 1(-2) \quad (4.2)$$

Thus the  $(2,2,3(2/3) + 3(-2/3))$  submultiplets corresponding to  $\lambda_{\alpha\beta}$  get a mass against  $(2,2,\underline{3}(-2/3) + 3(2/3))$  in the  $t + \bar{t}$  subreps.

The other subreps get mass from the superpotential if at all. Since the cubic term gives masses of form  $\langle A_{ABCD} \rangle \psi_{A\beta\mu\nu} \psi_{C\delta\mu\nu}$  it is clear that  $s(1,1,1) (\psi_{a\mu c\delta})$  and  $b(2,2,6) (\psi_{a\mu cA})$  get no mass from it. Hence they are superheavy due to the  $\phi^2$  term. The fermionic mass terms for  $(210)$  subreps coming

from the superpotential are

$$\begin{aligned}
 24 \{ & M \Psi_{\mu_1 \dots \mu_4}^T C^{-1} \Psi_{\mu_1 \dots \mu_4} \\
 & + \alpha V_0 [ \Psi_{34\mu\mu'} \Psi_{\mu\mu'12} + (34 \leftrightarrow 12) \\
 & \quad + \Psi_{56\mu\mu'} \Psi_{\mu\mu'12} + (12 \leftrightarrow 56) \\
 & \quad + \Psi_{56\mu\mu'} \Psi_{\mu\mu'34} + (56 \leftrightarrow 34) ] \\
 & - \alpha V_0 [ \Psi_{24\mu\mu'} \Psi_{\mu\mu'13} + (24 \leftrightarrow 13) \\
 & \quad + \text{similar terms from the } (14)(32), (15)(26), \\
 & \quad (16)(25), (35)(26) \text{ and } (36)(25) \text{ index} \\
 & \quad \text{combinations} ] \}
 \end{aligned} \tag{4.3}$$

The sum is over distinct pairs  $\mu\mu'$  and 4-tuples  $(\mu_1, \mu_4)$  only. Using  $M_1 = -\alpha V_0$  we can find the mass matrices for various index combinations. It is now easy to see that

$$M_{(1,1,15)} = \text{Diag} ( \xi_3, \eta_2 \otimes \mathbb{I}_6, 24M ) \tag{4.4}$$

where  $\xi_n (\eta_n)$  are  $n \times n$  matrices with 1 on the diagonal and  $-1(+1)$  on off diagonal elements. Thus one finds that the  $(1,1,8,0)$  and  $(1,1,1,0)$  subreps have masses of  $48M$  and  $24M$  respectively. The  $(1,1,3(4/3)+\underline{3}(-4/3))$  get no mass. However, these are precisely the submultiplets that are eaten by the corresponding gauge fermions in the  $f(1,1,15)$  of  $\tilde{\Psi}(45)$  and hence the whole of the  $(1,1,15)$  is superheavy. The subreps  $w_L(3,1,15)$ ,  $w_R(1,3,15)$  have index structure ABab. By the same reasoning as for the  $(1,1,15)$  one finds that while the  $(1,3,8,0) + (3,1,8,0)$  and  $(3,1,1,0) + (1,3,1,0)$  subreps have masses  $48M$  and  $24M$

respectively the triplets  $(3, 1, 3(4/3) + \underline{3}(-4/3)) + (1, 3, 3(4/3) + \underline{3}(-4/3))$  do not get such a mass. Since, in addition they cannot be eaten by the gauge fermions in the course of the superheavy breaking they remain massless.

Finally one may also compute the masses for the  $t + \bar{t}$  subreps. One finds that while the color triplets and singlets get masses of  $48M$  and  $96M$  the color sextets  $(2, 2, 6(2/3) + 6(-2/3))$  remain light. Thus the pseudo-Goldstone supermultiplet set in Case A is

$$P_A : [ w_L^3 (3, 1, 3(4/3) + \underline{3}(-4/3)) + w_R (4.5) \\ + t(2, 2, 6(2/3)) + t(2, 2, \underline{6}(-2/3)) ]$$

ii) Masses in  $\Sigma(126)$  and  $\bar{\Sigma}(126)$

a) subreps: The linearity of the superpotential in makes computation of the masses simple. The tuning condition (3.35b) used to remove contributions  $O(M_U M_R)$  to  $F$  also remove contributions  $O(M_U)$  to the color singlets in the  $\Delta$  subreps. By left-right symmetry this removal is true for  $\Delta_L$  and  $\Delta_R$ . It is convenient to work in an  $SU(4)$  notation and suppress  $SU(2)_{L/R}$  indices. Then if we write the couplings of the subreps  $\bar{\sigma}, \bar{\Delta}, \bar{\alpha}$  of  $\Sigma$  and  $\bar{\Sigma}$  with the  $a(1, 1, 15)$  as

$$\sum_k b_k (\Sigma \bar{\Sigma})_k = B_\sigma^{ij} (a \sigma_i \sigma_j) + B_\Delta^{ij} (a \Delta_L^i \bar{\Delta}_L^j + L \rightarrow R) \\ + B_\alpha^{ij} a \alpha_i \bar{\alpha}_j \quad (4.6)$$

with obvious SU(4) index contractions, it is easy to see that

$$\begin{aligned}
 -\langle F_{\Delta_i} \rangle &= \langle M_2^{ij} \bar{\Delta}^j + B_{\Delta}^{ij} a \bar{\Delta}^j \rangle \\
 &= \langle M_2^{ij} + v_0 \tilde{B}_{\Delta}^{ij} \rangle \langle \bar{\Delta}_{a4}^j \rangle + (M_2^{ij} - 2v_0 \tilde{B}_{\Delta}^{ij}) \langle \bar{\Delta}_{a4}^j \rangle \\
 &\quad + (M_2^{ij} - 3v_0 \tilde{B}_{\Delta}^{ij}) \langle \bar{\Delta}_{44}^j \rangle
 \end{aligned} \tag{4.7}$$

where  $a$  runs over the color indices. This leads to the tuning condition (3.35a). (Similar considerations for  $F_H$  give 3.35d.

Furthermore, the fermion masses for the  $\Delta$ 's are now given by the terms of the above expression contracted with  $\Delta^{a4}, \Delta^{a4}, \Delta^{44}$  respectively (with  $\Delta$  now standing for the fermion fields). Thus if only  $\Delta_{44}^i$  has a v.e.v. it is obvious that sufficient freedom exists given two (three) pairs to tune the sextet or (and) triplet subreps to have zero mass. We note that the constraints of L-R symmetry complexity of the 126 and 126 representations and the requirement that superpotential be cubic means that the subreps always come in sets of four. Some authors have considered lesser numbers of  $\Delta$ s above  $M_R$  thus not merely avoiding the question of how to achieve such a spectrum but also violating L-R symmetry above  $M_R^{(17b)}$ .

b) The colored subrep  $\alpha(2,2,15)$  also get superheavy masses since the tuning conditions do not remove superheavy contributions from their mass matrices. The  $\alpha(2,2,1(0)) + \bar{\alpha}(2,2,1(0))$  subreps mix with  $d(2,2,1,0)$  from  $H$  in a

fermionic mass matrix

$$\begin{bmatrix} M_3 & 6cV_0 & 6cV_0 \\ 6cV_0 & 0 & 5M_2 - 13V_0 B_\alpha \\ 6cV_0 & 5M_2 - 13V_0 B_\alpha & 0 \end{bmatrix} \quad (4.8)$$

Using the tuning condition (3.35) we get two superheavy masses  $M_3/2, 3M_3/2$  and one mass zero to  $O(M_U)$ .

d) The color triplets in  $\sigma(1,1,6)$  and  $\bar{\sigma}(1,1,6)$  mix with  $c(1,1,6)$  giving a mass matrix

$$\begin{bmatrix} 2M_3 & cV_0 & cV_0 \\ cV_0 & 0 & M_2 + V_0 B_\sigma \\ cV_0 & M_2 V_0 B_\sigma & 0 \end{bmatrix} \quad (4.9)$$

and are thus all superheavy.

Case B: In this case, by reasoning similar to that followed above, the superheavy v.e.v.

$$\langle A_{7,8,9,10}^T \rangle \sim O(M_U)$$

along with the superheavy masses  $M_1$  and  $M_2$  lead to the following mass spectrum

a) Pseudo-Goldstone supermultiplets

$$P_B : [t(2,2,10) + t(2,2,\underline{10}) \quad a(1,1,15)] \quad (4.10)$$

b) All the  $\Delta$  submultiplets remain at mass  $M_R$  since  $SU(4)$

is unbroken at  $M_U$ . The rest of the supermultiplets in  $\Sigma$  and  $\bar{\Sigma}$  are superheavy.

c)  $b(2,2,6)$  in  $\bar{\phi}$  is eaten by  $g(2,2,6)$  in  $\bar{\psi}$

d)  $s(1,1,1)$ ,  $w_L(3,1,15)$ ,  $w_R(1,3,15)$  get masses from the cubic term.

e) The  $c(1,1,6)$  in  $H(10)$  becomes superheavy due to the mass term

$$C \langle A_{abcd} \rangle [C_A \sigma_{Aabcd} + C_A \bar{\sigma}_{Aabcd}] \quad (4.11)$$

while the  $d(2,2,1,0)$  in  $H(10)$  cannot get a superheavy mass because it has index pattern  $H_a$  and so  $\langle \bar{\phi} \rangle \Sigma H$  does not give  $H_a$  a mass to  $O(M_U)$ . This is an example of an 'incomplete multiplet' type of mechanism. Recall that in Case B one does not need the tuning condition  $M_3 + 12cV_0$  of Case (A). as explained in section (4.5).

## Section 2 Renormalization Group Analysis Preliminaries:

In this section we shall investigate the constraints imposed by the low energy parameters  $\sin^2 \theta(M_W)$  and  $\alpha_3(M_W)$  on the mass scales of our model. We therefore need to display the mass spectra used in the various cases we studied.

### Mass spectra

The mass spectra used as input for the renormalization group analyses are displayed below. Details are given later. We display multiplets with mass  $M_R$  and use  $(A, \psi)$  generically for (scalar, Weyl fermion) and  $(\lambda, V_\mu)$  generically for gauge field (fermions, vector potentials).

<u>Multiplet</u>	Type of fields	SO(10) Index structure	Order of mass
$r(3,1,1)$	$\lambda, V$	ab	$M_R$
$l(3,1,1)$	$\lambda$	ab	$M_R$
$f(1,1,8,0)$	$\lambda$	AB	$M_R$
$f(1,1,1,0)$	$\lambda$	AB	$M_R$
$q(1,2,4)+q_L^c(2,1,4)$	$A$	A	$M_R$
$q(2,1,1,+1)$	$A, \psi$	A	$M_R$
One $SU(2)_L$ doublet	$A, \psi$	a	$M_R$
One (Higgs) $SU(2)_L$ doublet	$A$	a	$M_W$
$q(2,1,4)+q_L^c(1,2,\underline{4})$	$\psi$	Fermion masses	
$z_0, w_\pm$	$V$		
			$M_W$

Subcases:

$$\begin{aligned}
 \text{A i)} \quad \Delta_R^{1+\bar{1}}(1,3,1,\underline{+2}) \quad A, \psi \quad abAbC \quad M_R \\
 \Delta_L^{1+\bar{1}}(3,1,1,\underline{+2}) \quad A, \psi \quad abABC \quad M_R
 \end{aligned}$$

ii) In addition to A_i) we have the following intermediate mass supermultiplets:

$$\begin{aligned}
 \Delta_R^{6+\bar{6}}(1,3,6,\underline{\pm\frac{2}{3}}) \quad A, \quad abABC. \quad M_R \\
 \Delta_L^{6+\bar{6}}(3,1,6,\underline{\pm\frac{2}{3}}) \quad A, \quad abABC. \quad M_R
 \end{aligned}$$

iii) In addition to ii) we have

$$\begin{aligned}
 \Delta_R^{3+\bar{3}}(1,3,3, \ 2/3) \quad A, \quad abABC. \quad M_R \\
 \Delta_L^{3+\bar{3}}(3,1,3, \ 2/3) \quad A, \quad abABC. \quad M_R
 \end{aligned}$$

Finally for all the cases described above, there are pseudo Goldstone supermultiplets

$$P_A : [ w_L^3 + w_R^3 + t^6 + t^6 ] \quad (4.5)$$

Their masses are, at most, of order  $M_S$  and thus they must be reckoned into the renormalization group equations above  $M_R$ . We call this case Aa). When  $P_A$  is included in the super-heavy set we call it case Ab). The point of including them in the superheavy set is that if the Higgs sector is further complicated by including 45 and 54 dimensional irreps, the set  $P_A$  may be made superheavy, by destroying the accidental symmetry responsible for their small mass.

Case B:

This corresponds to case A iii) with gauge group  $G_{PS}$

above  $M_R$ . The pseudo Goldstone supermultiplets are

$$P_B: a(1,1,15) + t(2,2,10) + \bar{t}(2,2,\underline{10}) \quad (4.12)$$

subcases a) and b) correspond to intermediate and superheavy masses for  $P_C$ , respectively.

### Case C

In this case we assume  $M_S \gg M_R$ . We display the results of the renormalization group analysis under this assumption for cases A a) i as table 4.4. In the other subcases the constraint of perturbative unification, ie. that  $\alpha_{\text{strong}}(M_U)$  remain bounded by a number less than 1, does not allow any significant lowering of the value of  $M_R$ .

### Section 3: Renormalization Group Analysis

In this section, we will write down the  $\sin^2 \theta_w(M_w)$  and the QCD coupling  $\alpha_3(M_w)$  in terms of the intermediate mass scales. The formulae are identical to these in the non-supersymmetric case, except that, the coefficients in the evolution of coupling constants will be different for the supersymmetric case. For generality, let us consider, the following symmetry breaking chain:

$$SO(10) \xrightarrow{M_U} G_{PS} \xrightarrow{M_C} G_{LR} \xrightarrow{M_R} G_{123}$$

Let us recall the important steps in the derivation of the equations for  $\sin^2 \theta_w$  and  $\alpha_3(M_w)$ : First we note that, all generators of the  $SO(10)$  group defined in the 16 dimensional spinor representation, for example, must be

normalized in the same way and if we take  $I_{3L}$ ,  $I_{3R}$ ,  $I_{3C}$ , etc., defined as physical  $SU(2)_L$ ,  $SU(2)_R$  and  $SU(3)_C$  generators respectively, we obtain

$$\text{Tr} I_{SO(10)}^2 = \text{Tr} I_{3L}^2 + \text{Tr} I_{3R}^2 = \text{Tr} (B-L)^2 = 2 \quad (4.13)$$

We note, however, that the physical  $B-L$  and  $Y$  generators over the whole 16-dimensional representation do not satisfy equation(4.13). Taking  $\text{Tr} (B-L)^2_{16}$  and  $\text{Tr} Y^2_{16}$ ,

we conclude that

$$\frac{B-L}{2} = \sqrt{\frac{2}{3}} I_{B-L} \quad \frac{Y}{2} = \sqrt{\frac{5}{3}} I_Y \quad (4.14)$$

Since  $Y/2 = I_{3R} + (B-L)/2$ , we obtain,

$$\frac{Y}{2} = I_{3R} + \sqrt{\frac{2}{3}} I_{B-L} \quad (4.15a)$$

$$I_Y = \sqrt{\frac{3}{5}} I_{3R} + \sqrt{\frac{2}{5}} I_{B-L} \quad (4.15b)$$

From equation (4.15) we conclude that

$$\frac{1}{g_Y^2} = \frac{3}{5} \frac{1}{g_{2R}^2} (M_R) + \frac{2}{5} \frac{1}{g_{B-L}^2} (M_R) \quad (4.16a)$$

and

$$g'(M_W) = \sqrt{\frac{3}{5}} g_Y(M_W) \quad (4.16b)$$

Now, let us write:

$$\frac{\partial g_i}{\partial t} = \frac{A_i}{16\pi^2} g_i^3 \quad (4.17)$$

where  $g_i$  goes over  $g_{2L}$  for  $SU(2)_L$ ,  $g_{2R}$  for  $SU(2)_R$ ,  $g_{BL}$  for  $I_{BL}$  and  $g_Y$  for  $I_Y$  coupling respectively.

Now, we remember the following additional boundary conditions representing the Grand Unification of coupling constants:

$$g_i(M_U) = g_{SO(10)}(M_U) ; i = 2L, 2R, B-L, 3 \quad (4.18)$$

Integrating eqn. (4.17) between various mass scales and using boundary condition eqns. (4.13), (4.14) and (4.15), we obtain the following equations (we assume  $M_X = M_U$  and use the fact that,  $\sin^2 \theta_W = e^2/g^2_{2L}$ ):

$$\begin{aligned} \sin^2 \theta_W(M_W) = & \frac{3}{8} + \frac{5\alpha(M_W)}{16\pi} \left[ (A_{2L} - A_Y) \ln \frac{M_R}{M_W} \right. \\ & \left. + (A_{2L}^S - \frac{3}{5} A_{2R}^S - \frac{2}{5} A_{B-L}^S) \ln \frac{M_U}{M_R} \right] \quad (4.19) \end{aligned}$$

and

$$\begin{aligned} 1 - \frac{8\alpha(M_W)}{3\alpha_3(M_W)} = & \frac{5\alpha(M_W)}{6\pi} \left[ \left( \frac{3}{5} A_{2L} + A_Y - \frac{8}{5} A_3 \right) \ln \frac{M_R}{M_W} \right. \\ & \left. + \left( \frac{3}{5} (A_{2L}^S + A_{2R}^S) + \frac{2}{5} A_{B-L}^S - \frac{8}{5} A_3^S \right) \ln \frac{M_U}{M_R} \right] \quad (4.20) \end{aligned}$$

where  $A_{2L}$  and  $A_3$  stand for  $SU(2)_L$  and  $SU(3)_C$  contributions to eqn.(4.17) for  $M_W < \mu < M_R$  and  $A_{2L}^S$  and  $A_3^S$  etc. denote the same contributions for the mass range  $M_S \sim M_R < \mu < M_U$ . For case B the relevant formulae are obtained by replacing  $A_{BL}^S$  and  $A_3^S$  everywhere by  $A_4^S$ .

For case (A), the formulae for  $A_{2L}$ ,  $A_Y$  and  $A_3$  are:

$$\begin{aligned} A_{2L} &= -22/3 + 4/3 N_G + 1/6 \\ A_Y &= (4/3) N_G + 1/10. \\ A_3 &= -11 + (4/3) N_G \end{aligned} \quad (4.21)$$

We have assumed that only one complex scalar  $SU(2)_L$  doublet Higgs contributes between  $M_W$  and  $M_R$ .

$N_G$  is the number of fermion generations: we point out that to calculate  $A_Y$  we use the formula:

$$A_Y = \frac{3}{20} \sum_i Y_i^2 \quad (4.22)$$

where  $i$  runs over fermions and scalar bosons. The rest of the coefficients appearing in eqn.(4.19) and (4.20), i.e.

$A_{2L}^S$ ,  $A_{2R}^S$ ,  $A_3^S$ ,  $A_{BL}^S$  receive contributions from whole supermultiplets since they were in the region of energy  $\mu \gtrsim M_S \gtrsim M_R$ . Above  $M_S$  we have the supersymmetric coefficients for the non-Abelian factors

$$A_N^S = -3N + 2N_G + \sum_i T_N(R_i)$$

where  $T_N$  is the quadratic Casimir for the appropriate irrep and  $N_G$  the number of generations. The sum runs over all particles with mass  $\sim M_R$  except, of course, the matter supermultiplets. So we get:

$$\begin{aligned}
 A_{2L}^S &= -6 + 2N_G + \sum_i T_L(R_i) \\
 A_{2R}^S &= -6 + 2N_G + \sum_i T_R(R_i) \\
 A_{B-L}^S &= 2N_G + \sum_i T_{B-L}(R_i) = 2N_G + \frac{3}{8} \sum_i (B-L)^2 \\
 A_3^S &= -9 + 2N_G + \sum_i T_3(R_i) \\
 A_4^S &= -12 + 2N_G + \sum_i T_4(R_i)
 \end{aligned} \tag{4.23}$$

where  $i$  goes over supermultiplets other than those coming from  $\chi^a(16)$  and the gauge vector supermultiplets  $\bar{\Psi}_3(1,1,8,0)$ ,  $\bar{\Psi}_4(1,1,15)$ ,  $\bar{\Psi}_R(1,3,1)$ ,  $\bar{\Psi}_L(3,1,1)$ . The values of  $T$  for various subreps are given below as Table 4.1a and 4.1b.

For Case C ( $M_S \gg M_R$ )

$$\begin{aligned}
 \text{Sum}^2 \theta_W(M_W) &= \frac{3}{8} + \frac{5\alpha(M_W)}{16\pi} \left[ (A_{2L} - A_Y) \ln \frac{M_R}{M_W} \right. \\
 &\quad + \left( A_{2L}' - \frac{3}{5} A_{2R} - \frac{2}{5} A_{B-L} \right) \ln \frac{M_S}{M_R} \\
 &\quad \left. + \left( A_{2L}^S - \frac{3}{5} A_{2R}^S - \frac{2}{5} A_{B-L}^S \right) \ln \frac{M_U}{M_S} \right].
 \end{aligned} \tag{4.24}$$

$$1 - \frac{8\alpha(M_W)}{3\alpha_3(M_W)} = \frac{5\alpha(M_W)}{6\pi} (\ln 10) \left[ \left( \frac{3}{5} A_{2L} + A_Y - \frac{8}{5} A_3 \right) (\mu - \omega) \right. \\ \left. + \left( \frac{3}{5} A'_{2L} + \frac{3}{5} A_{2R} + \frac{2}{5} A_{B-L} - \frac{8}{5} A'_3 \right) (s - \lambda) \right. \\ \left. + \left( \frac{3}{5} A^S_{2L} + \frac{3}{5} A^S_{2R} + \frac{2}{5} A^S_{B-L} - \frac{8}{5} A^S_3 \right) (v - s) \right] \quad (4.25)$$

where the coefficients  $A^S_{2L}$ ,  $A^S_{2R}$ ,  $A^S_{BL}$  and  $A^S_3$  include the effect of the whole chiral superfield (i.e. both  $\Phi$  and  $A$ ) whereas the rest of the coefficients are the same as in non-supersymmetric theories. Primes refer to the energy interval  $M_R < \mu < M_S$ . We examine the consequences of Higgs multiplet spectrum of Case Aa)i. The results of the renormalization group analysis for cases A, B, C are listed as Tables 4.2a, 4.2b, 4.3, 4.4.

#### Section 4: Results and Discussion

We display our results as tables 4.2-4.4.

The constraint of perturbative unification alone that is the requirement that  $\alpha_3(M_U) < 1$  is sufficient to force  $r = \log_{10} M_R(\text{Gev})$  to be greater than 9 in all cases except case Abi). For instance, in Case Aa) the large number of colored particles gives

$$A^S_3 = 23 + \frac{15}{2} n_6 + \frac{3}{2} n_3 \quad (4.26)$$

where  $n_6$  ( $n_3$ ) is the number of  $SU(2)$  triplet,  $SU(3)$

sextet (triplet) submultiplets. Since

$$2g_3 / \partial \ln \mu = \frac{A_3}{16\pi^2} g_3^3 \quad (4.27)$$

it is easy to show that with  $u > 14$  ( a constraint imposed by current limits on the proton lifetime)

$$\frac{1}{g_3^2(M_U)} = (.87 r - .81) - .03(15/2n_6 + 3/2 n_3)(u-r) \quad (4.28)$$

Thus even for  $n_6=n_3=0$ ,  $r > 9$  to accord with the perturbative unification assumption. For  $n_6=4$ ,  $n_3=0$ , 4 the situation is even worse. Similar conclusions apply to cases Ab)ii) and iii) as well as to cases Ba) and Bb). Hence even without performing a detailed analysis it is apparent that low energy parity restoration cannot occur in any scenario projected excepting possibly case Ab).

If we survey the results of tables 4.2-4.4 we see that  $M_S \sim M_R$  then both scales must be  $\gtrsim 0(10^{12} \text{GeV})$ . Furthermore, these constraints will be even tighter if we had taken the superpartners of low energy matter and gauge fields to have masses  $0(M_W)$ . We do not expect our conclusion to be any different in other types of  $SO(10)$  models since even the 'minimal' mass spectrum of Case Aai) has  $M_R \sim 10^{12} \text{GeV}$ . On the other hand we see that if  $\sin^2 \theta_W(M_W)$  is taken to be large enough (assuming that

its actual value is different from the currently quoted one due to ambiguities resulting from non-standard values of the parameter  $M_W^2/M_Z^2 \cos^2 \theta_W$ , then for  $M_S \gg M_R$ ,  $M_R$  can be quite low.

We thus expect that if phenomena such as neutrinoless double  $\beta$  decay,  $N-\bar{N}$  oscillations or neutrino masses in the electron volt region are observed then they would rule out supersymmetric  $SO(10)$  GUTs (as well as minimal  $SU(5)$  grand unification!). This is because these phenomena are intimately connected to the scale of B-L symmetry breaking and, in general, cannot occur for  $M_{B-L} \sim M_R \sim 10^7$  GeV.

On the other hand it is possible that in a different grand unifying group such as  $SU(16)$  where the unification scale can be as low as  $10^8$  GeV (Non SUSY version) the stringent constraints that are imposed due to the large numbers of colored particles may not arise. This is being investigated by A. Mohanty and the author. Preliminary considerations indicate that in  $SU(16)$  there exist breaking chains where the unification mass is low. Even though large numbers of colored supermultiplets may contribute between  $M_R$  and  $M_U$  the low value of  $M_U$  permits  $M_R$  to be small enough to allow observable B-L violating phenomena.

We note that the Weinberg-Sakai criterion for suppressing dimension 5 operators leading to proton decay

is well satisfied since

$$m_{\psi\lambda} \ m_{\psi H_3} \gg 10^{18} \text{ GeV}^2$$

As we shall see in the ensuing chapters, supersymmetry breaking scales  $O(10^{10}-10^{12} \text{ GeV})$  are of great current interest. After surveying current models and our own contributions to SU(5) type models with  $M_S \sim 10^{11} \text{ GeV}$  we shall return to the question of merging such models with an SO(10) one.

TABLE 4.1a

$SU(2)_L \times SU(2)_R$ $\times U(1)_{B-L}$ $\times SU(3)_C$ subrep					
	$T_{2L}$	$T_{2R}$	$T_{BL}$	$T_Y$	$T_3$
$(2, 2, 0, 1)$	1	1	0	3/5	0
$(3, 1, 2, 1)$	2	0	9/2		0
$(1, 3, 2, 1)$	0	2	9/2		0
$(3, 1, 2/3, 6)$	12	0	3		15/2
$(3, 1, -2/3, 3)$	6	0	3/2		3/2
$(1, 3, -2/3, 3)$	0	6	3/2		3/2
$(3, 1, 4/3, 3)$	6	0	6		3/2
$(1, 3, 4/3, 3)$	0	6	6		3/2
$(2, 2, 2/3, 6)$	6	6	4		10
$(1, 1, 3, 2/3)$	0	0	1		1/2

TABLE 4.1b  
Values of T for  $G_{PS}$  multiplets

$H_1$ Multiplet	$T_{2L}$	$T_{2R}$	$T_4$
gauge	-6	-6	-12
(3,1,10)	20	0	9
(1,3,10)	0	20	9
(2,2,10)	10	10	12
(1,1,15)	0	0	4

TABLE 4.2a

<u>Case Aa)</u>				
	$\alpha_3(M_W)$	$\sin^2 \theta_w(M_W)$	r	u
i)	.100	.214	13.5	14.1
	.110	.21	12.7	14.1
	.120	.2075	12.3	14.1
ii)	.10	.205	13.6	15.5
	.11	.203	14.1	15.6
	.12	.205	14.8	15.35
iii)	.10	.224	13.1	14.5
	.11	.221	13.24	14.8
	.12	.219	13.5	15.1
$r = \log_{10} M_R$ $u = \log_{10} M_U$				

Table 4.2a

Representative values of  $M_R$  and  $M_U$  for case Aa)

(Pseudo-Goldstone supermultiplets included)

Case Ab)

	$\alpha_3(M_W)$	$\sin^2 \theta_W(M_W)$	r	u
i)	.10	.219	13.5	14.1
	.11	.219	12.5	14.0
	.12	.219	12.0	14.0
ii)	.10	.216	13.3	16.1
	.11	.212	13.7	16.3
	.121	.209	13.9	16.6
iii)	.100	.245	11.9	14.1
	.110	.242	12.2	14.8
	.121	.239	12.4	15.1

$$r = \log_{10} M_R \quad u = \log_{10} M_U$$

Table 4.2b

Representative values of  $M_R$  and  $M_U$  for case Ab).

Case Ba)	$\alpha_3(M_W)$	$\sin^2 \theta_w(M_W)$	r	u
	.1	.222	14.1	15.8
	.110	.219	14.4	16.1
	.120	.215	14.6	16.3

Bb)	.100	.25	12.8	14.5
	.110	.25	12.9	14.8
	.118	.25	13.0	15.0

Table 4.3

Values of  $M_R$  and  $M_U$  for breaking to  $G_{PS}$ .

Case C

$\alpha_3(M_W)$	$\sin^2 \theta_W(M_W)$	r	s	u
.100	.248	6.5	12.9	14
.109	.257	4	12.3	14
.12	.244	6.5	11.8	14
.121	.261	2.5	11.7	14

$$s = \log_{10} M_S$$

Table 4.4

Values of  $M_R, M_S, M_U$  for the mass spectrum A(b) when

$$M_S > M_R.$$

## Chapter 5

### Introduction to Supergravity models

Introduction: In the preceding chapters we discussed the construction of globally SUSY models and the introduction of intermediate mass scales into supersymmetric scenarios. Naive SUSY extension of usual GUT models were found to suffer from two structural problems:

- i) The different GUT vacua were found to be degenerate. Hence it was not clear what mechanism could be responsible for selecting the  $G_{123}$  invariant vacuum over the others.
- ii) From the non observation of s-matter particles and flavor changing neutral currents we find (see section 2.3) that not only must the s-matter fields have masses  $> 20$  GeV but these masses must be equal between s-matters of different generations (but of otherwise identical quantum numbers). Of course, one can add ad-hoc soft masses to comply with these constraints. However, this increases the number of free parameters, is aesthetically displeasing, and thus adds nothing to our understanding. One would prefer to achieve a mass spectrum of the above type by spontaneous SUSY breaking. In that case, as explained in section 2.1 the matter sector must be coupled to the goldstino without any suppression and  $M_S$  can then be  $O(M_W)$ . Else, the matter-smatter-goldstino coupling could be highly suppressed but  $M_S \approx 10^{10}-10^{12}$  GeV again resulting in a supergap  $O(M_W)$ .

Explicit construction has shown that the first alternative leads to highly complex models with many new fields introduced with restricted couplings purely to achieve the required end. No uniqueness seems to operate and the models are quite ugly. We shall not consider the alternative  $M_S \approx M_W$  any further.

The second alternative ie.,  $M_S \sim 10^{11} \text{ GeV}$  is, on the other hand, a truly fascinating one. Several streams of argument flow together into the scenario in which supersymmetry is broken in the intermediate sector. The first of these, as discussed in the chapters 3-4, was that resulting from the identification of  $M_R$  and  $M_S$ . In this chapter, we shall discuss the so called '(globally SUSY) geometric hierarchy' ⁽²²⁾ (GH), cosmological constraints on the gravitino mass (both of which put  $M_S \approx 10^{11} \text{ GeV}$ ) and review Supergravity models.

Gravitinos (spin 3/2 partners of the graviton) occur within the context of Supergravity(SUGRA) models. As we shall see, the introduction of SUGRA lifts the degeneracy between different GUT vacua (as do the GH models). Moreover, both these types of models can also induce a supergap  $O(M_W)$  in the light sector. Indeed, in the Old Minimal (OM) SUGRA with canonical kinetic terms (see below) all scalars get equal contributions  $O(M_{3/2}^2)$  to their mass squared matrices at tree level, while fermions do not. In globally SUSY GH models

approximately equal smatter masses are induced via radiative corrections.

Moreover, GH models hold out the possibility of inducing the GUT scale from the intermediate scale where SUSY is broken. Conversely, locally SUSY extensions of the usual GUT models hold out the possibility that the weak scale (ie. the SU(2) doublet v.e.v.'s) is induced by SUGRY effects.

Clearly, the description of models of such sweeping import would occupy too much space if carried out in detail. Moreover, we wish to present this material here largely to serve as an introduction to a discussion of our own contributions to this area which we discuss in chapter 7. Hence we present our material in the following form. In section 2 we discuss the Witten Hierarchy⁽²³⁾ generating mechanism and constraints on its fundamental mass scale from cosmological considerations. In section 3 we (very briefly) review the Dimopoulos-Raby geometric hierarchy. In section 4 we present the form of the low energy Lagrangian in OM SUGRY models. ( In chapter 6 we derive the potential in the OM case as well as present some of our own results on the potential in alternative SUGRYs).⁽²⁴⁾ We then discuss (section 5) Weinberg's mechanism for lifting GUT vacuum degeneracy using the potential of OM SUGRY. We also present a brief review of his arguments leading to the conclusion that  $M_{3/2} \gtrsim 10^4$

GeV (or is very light  $M_{3/2} \lesssim 1$  KeV). Finally, in Section 6 we present a brief review of attempts to induce the weak scale from  $M_S$  via SUGRA effects at tree level. After showing why this is not feasible in minimal models we summarize the successful induction of the weak scale via renormalization effects, in the context of OM SUGRA, in section 7. This will then leave us fully prepared for the presentation of our proposal for geometric hierarchy based on an O'Raifeartaigh potential, in Chapter 7.

## Section 2: The Witten Mechanism

The Witten mechanism which generates hierarchies in globally SUSY gauge theories is based on the O'Raifeartaigh (OR) superpotential. The OR superpotential has the generic form

$$g = X f_1(\dot{A}^i, M) + Z f_2(A^i, M) \quad (5.1)$$

where  $A^i$  are chiral scalar superfields and  $M$  is a mass parameter. Then we see that

$$-F_X^\dagger = f_1(A^i, M) \quad (5.2a)$$

$$-F_Z^\dagger = f_2(A^i, M) \quad (5.2b)$$

$$-F_A^\dagger = X f_{1,i} + Z f_{2,i} \quad (5.2c)$$

field symbols now stand for scalar components. Here  $f_1$  and  $f_2$  are chosen so that 5.2a and 5.2b are inconsistent, hence  $F_X$  and/or  $F_Z$  are non-zero at the minimum of the

potential. Therefore SUSY is broken. for instance,

$$f_1 = A^2 - M^2 \quad f_2 = A^2 \quad (5.3)$$

will achieve this end. Note, moreover, that for  $F_A = 0$  to hold

$$(X + Z) A = 0 \quad (5.4)$$

Thus,  $\langle X \rangle, \langle Z \rangle$  are not determined at tree level. It is only necessary that

$$\langle X \rangle = -\langle Z \rangle \quad (5.5)$$

Huq⁽²⁵⁾ asked the question whether radiative corrections fix  $X$  and  $Z$ . Operating in the context of a Wess-Zumino type model (ie. without gauge interactions) he found

$$\langle X \rangle = \langle Z \rangle = 0 \quad (5.6)$$

Witten further enquired whether the presence of gauge interactions could modify this result. He found that  $\langle X \rangle, \langle Z \rangle$  could be different from zero and in fact could be  $M$ . He thus proposed a hierarchy generating mechanism based on the  $SU(5)$  gauge group in which  $M$  is identified with  $M_W$  and the GUT scale is generated by radiative corrections. He took  $X$  to be a  $SU(5)$  singlet while  $A$  and  $Z$  were 24plets of  $SU(5)$ .

Then

$$g = \alpha_1 X (\bar{t} A^2 - M^2) + \alpha_2 \bar{t} A^2 Z \quad (5.7)$$

and at the tree level

$$V = |g_X|^2 + \bar{t} |g_Z|^2 + \bar{t} |g_A|^2 + \frac{e^2}{2} \bar{t} \{ [A, A^\dagger] + [Z, Z^\dagger] \}^2 \quad (5.8)$$

This potential may be shown to be minimized at

$$\langle A \rangle = \frac{\alpha_2 M}{(\alpha_2^2 + 30\alpha_1^2)^{1/2}} \text{Diag} (2 \underline{1}_3, -3 \underline{1}_2) \quad (5.9)$$

$$\langle Z \rangle = \frac{\alpha_1}{\alpha_2} \langle X \rangle \text{Diag} (2 \underline{1}_3, -3 \underline{1}_2) \quad (5.10)$$

note that the  $SU(3) \times SU(3) \times U(1)$  vacuum is chosen in contrast to the Dimopoulos-Georgi model.  $\langle X \rangle$  is undetermined at tree level. At the one loop level, one obtains contributions proportional to  $\ln \langle X \rangle$  from loops containing the  $(8,1)(1,3)(3,2)(3,2)$  subreps of  $A, Z$ . He found,

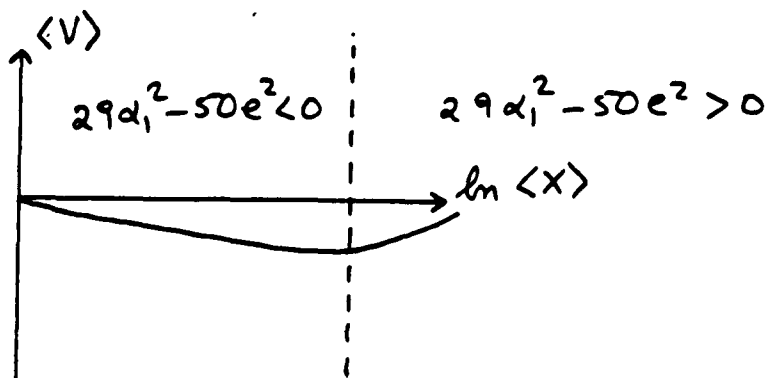
$$V'^{\text{loop}} = \frac{M^4 \alpha_1^2 \alpha_2^2}{\alpha_2^2 + 30\alpha_1^2} \left( 1 + \frac{30\alpha_2^2}{30\alpha_1^2 + \alpha_2^2} (29\alpha_1^2 - 50e^2) \ln \langle X \rangle \right) \quad (5.11)$$

for  $\langle X \rangle \gg M$ .

If the coefficient of  $\ln \langle X \rangle$  is negative ie.,

$$29\alpha_1^2 - 50e^2 < 0 \quad (5.12)$$

then clearly  $V$  decreases with  $X$ . On the other hand, from renormalization group arguments we know that one must use running coupling constants defined at a scale  $\mu \sim \ln \langle X \rangle$ . Thus provided the model is asymptotically free  $\lambda$  will increase and  $e$  decrease as  $\langle X \rangle$  increases. Thus at some point the expression in (5.12) could switch sign even if it started out as negative. Hence the potential would then increase with  $\ln \langle X \rangle$  and would therefore develop a minimum at the scale where the sign switched from negative to positive



Thus  $\langle Z \rangle \sim \langle X \rangle$  could be  $\gg M$ . Since  $Z$  breaks  $SU(5)$  to  $G_{123}$  this means that we have a candidate mechanism for generating the GUT scale from a lower scale.

We shall discuss attempts to build realistic GUT models based on this mechanism in the next section. In this chapter we merely note some arguments⁽²⁶⁾ against the identification of  $M_W$  and  $M$ .

The first of these is simply that if  $M = M_W$  then since  $F_X$  and  $F_Z$  cannot couple directly to the matter sector a supergap  $O(M_W)$  is not possible via radiative

corrections, at least with a 'minimal' set of fields.

The other arguments against  $M_S \sim M \sim M_W$  come from cosmological considerations. Let us work out the mass spectrum of various  $G_{123}$  subreps of A and Z. Under  $SU(3) \times SU(2)$

$$24 = (1,1) + (3,2) + (\underline{3},2) + (8,1) + (1,3) \quad (5.13)$$

The  $(3,2) + (\underline{3},2)$  subreps of Z are eaten in the  $SU(5)$  breaking at the GUT scale. Furthermore the masses of the fermions in  $\Lambda(24)$  will be given by writing

$$\begin{aligned} A &= \lambda^i A_i & \epsilon \lambda_i \lambda_j &= 2 \delta_{ij} \\ \langle Z \rangle &= \sqrt{15} \lambda_{24} & Z_0 &= \sqrt{15} (\alpha_1 / \alpha_2) \times \lambda_{24} \end{aligned} \quad (5.14)$$

then one has

$$2\alpha_1 \times \psi_{A_i} \psi_{A_i} + \alpha_2 (\epsilon \lambda_i \lambda_j \lambda_{24}) Z_0 \psi_{A_i} \psi_{A_j} \quad (5.15)$$

i) for the  $(3,2)$  components which have purely off diagonal components it is easy to see that only the first term contributes and gives a superheavy mass to the  $(3,2)$  fermions and therefore also to their superpartners.

ii) for the  $(8,1)$  component consider for example

$$\lambda_i = \lambda_j = \lambda_3 = (\sigma_3 \circ_3)$$

$$\text{We get } \epsilon \lambda_3^2 \lambda_{24} = 4 / \sqrt{15}$$

So now the coefficient of  $\psi_{A_3} \psi_{A_3}$  is  $(2\alpha_1 \times + 4\alpha_1 \times)$ . So by  $SU(3)$  symmetry the  $(8,1)$  is superheavy. Similarly, so is

the (3,1) in A. However, for the singlet in A one has

$$2 \alpha_1 X \psi_3 \psi_3 + \sqrt{15} z_0 \alpha_2 T_1 \lambda_{24}^3 \psi_3 \psi_3 \quad (5.16)$$

So the singlet fermion gets no mass. This is also true for its scalar partner.

iii) The multiplets in Z not eaten by gauge supermultiplets have mass terms only from

$$2 \alpha_2 \langle A_i \rangle \epsilon \lambda_i \lambda_j \lambda_k \psi_{A_j}^T C^{-1} \psi_{Z_k} + h.c. \quad (5.17)$$

and hence mix with A in a mass matrix of form

Hence, they are light with a mass  $O(M^2/M_U)$ . This does not apply to the singlet (1,1) since A(1,1) is light.

iv) The goldstino is the fermionic partner of

$$\xi = (\alpha_2^2 + 3\alpha_1^2)^{-1/2} (\alpha_1 \sqrt{30} z_{24} + \alpha_2 X) \quad (5.18)$$

and is orthogonal to the mixture

$$\eta = (\alpha_2^2 + 3\alpha_1^2)^{-1/2} (-\alpha_2 z_{24} + \alpha_1 \sqrt{30} X) \quad (5.19)$$

which along with  $A_{24}$  has a mass  $O(M)$ .  $\text{Im } \xi$  is the Goldstone boson associated with the R invariance of the potential (Provided the R symmetry has a QCD anomaly it becomes an invisible axion).  $\text{Re } \xi$  gets masses  $O(M^2/M_U)$  through loops. Thus a light complex scalar transforming as (1,1) usually appears in this model. This turns out to be a general feature of such models. (26)

These particles do not have direct couplings to those in the standard low energy spectrum. Thus since they are light they are long-lived and hence behave as free particles. They can acquire couplings to light particles via one loop effects with  $\langle F_{X,Z} \rangle$  insertions. These necessarily have suppressions due to superheavies. One has couplings to light fermions and gauge particles like⁽²⁶⁾

$$\mathcal{L}_\xi = \frac{e\alpha}{4\pi} \left( \frac{M}{\langle X \rangle} \right)^2 \bar{\psi} \psi \xi + \frac{\alpha}{24} \frac{1}{\langle X \rangle} \xi G_{\mu\nu}^2 \quad (5.20)$$

thus the decay rate is  $O(M^6 / X^5)$  while  $m_\xi \approx O(M^2 / \langle X \rangle)$ . The scattering cross-sections are  $O(\alpha^3 / \langle X \rangle^2)$  for temperatures  $T \ll M$ , which are the relevant ones since  $\xi$  is long-lived. By considering the effects of such particles on the matter density of the universe, on Helium synthesis and of their decay on the entropy density, Hung and Suzuki showed that the range  $10^5 < M < 10^{10} \text{ GeV}$  is rigorously excluded.

Moreover, Pi and Ginsparg⁽²⁶⁾ have examined the temperature behaviour of the Witten Model. They find that SU(5) remains unbroken until temperatures fall below  $M$  the SUSY breaking scale so that  $A$  can get a v.e.v. and trigger large breaking. If the breaking occurs at  $O(M_W)$  then it will not be possible to produce even light ( $O(10^{10} \text{ GeV})$ )

triplet Higgs leave alone superheavy gauge particles. Thus baryon number violating interactions will be highly suppressed. Moreover, the Universe will remain in equilibrium with respect to these interactions since at such late times its expansion rate ( $\dot{R}/R \approx T^2/M_P$ ) is very slow. Hence the 'successful' prediction of  $n_B/n$  will not be available.

We conclude that  $M > 10^{11}$  GeV is required in such hierarchy generating models. We further note here that a detailed renormalization group analysis of the Witten model⁽²⁶⁾ shows that a fine tuning of parameters is necessary to achieve a hierarchy via one loop radiative effects.

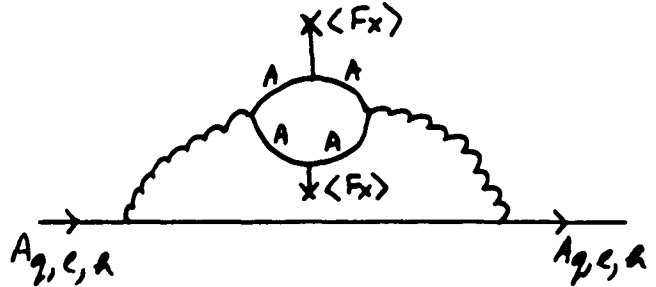
### Section 3: The Dimopoulos-Raby Model

In this section we briefly survey the above mentioned model⁽²²⁾ which incorporates the Witten mechanism in a scenario with  $M_S \approx O(10^{12} \text{ GeV})$ ,  $M_U \approx 10^{19} \text{ GeV}$  and the weak scale together with (equal) s-matter masses are generated via radiative corrections to be  $O(M_S^2/M_U)$ .

We first note the origin of the SUSY decoupling theorem in such models which protects light states from receiving large non-SUSY radiative contributions to their masses even though SUSY is broken at high scale. The reason for this is that the v.e.v.  $\langle F_{X,Z} \rangle$  couple only to superheavy states hence the light mass scale generated by

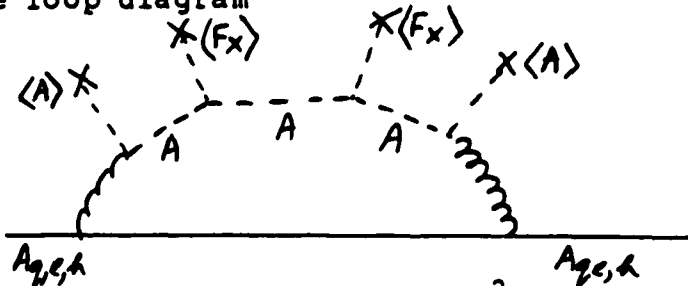
these v.e.v. will be  $O(F_{X,2}/M_U)$ . Since  $M_U \sim M_P$  it follows that these masses are  $O(10^3-10^5 \text{ GeV})$ .

S-matter fields thus get masses from loop diagrams and these are positive. The dominant diagrams occur at two loops,



are  $O(M_S^2 / 2 M_U)$  and may be shown to be positive.

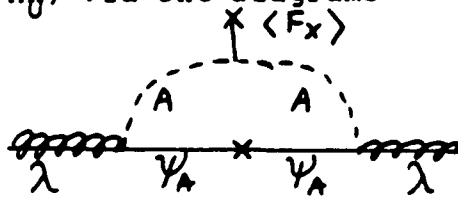
The one loop diagram



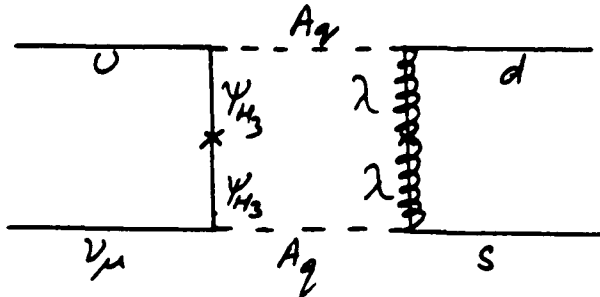
is suppressed by an additional  $(M_S^2/M_U)$ . Since the Yukawa couplings for the first two generations are small it follows that upto small corrections the s-matter in the first two generations has equal mass, as required (see section 2.3). For the third generation there are other contributions to s-matter masses.

It is also possible to get negative contributions to the scalar Higgs doublet masses at two loop level since they have additional couplings to superheavy multiplets. We will not belabor this point but refer the reader to the literature. The gauge fermions also get masses

$O(\alpha M^2/M_U)$  via the diagrams



The complete superpotential in the D.G. model has  $H(5)$  and  $\bar{H}(5)$  coupled to  $(10)(10)$  and  $(\bar{5})(10)$ . However,  $H$  and  $\bar{H}$  are not given any direct couplings. Rather, they introduce additional multiplets  $H_1, \bar{H}_1$  and couple these to  $H$  and  $\bar{H}$ . The reason for these is that only  $A$  (not  $Z$ ) can be used to split color triplets from singlets in  $H$  and  $\bar{H}$  because coupling  $Z$  to  $\bar{H}H$  destroys the SUSY breaking property of  $g_{\text{Witten}}$ . Thus these triplets have mass  $O(10^{12} \text{ GeV})$  only. Hence, in order to suppress the dimension 5 operators (which involve fermionic  $\bar{H}H$  mass insertions) it is necessary to ensure that  $H, \bar{H}$  do not have any direct coupling. The relevant diagram is



Since the radiatively induced coupling is  $O(M_S^2/M_U) \bar{\psi}_H \psi_H$  this graph will be suppressed by  $O(M_S/M_U)$  relative to the one in the case where  $H, \bar{H}$  have a direct mass  $O(M)$ .

We note that the strong CP problem can be solved in such a model by using the anomalous R invariance of the Lagrangian resulting in an invisible axion with mass  $O(10^{-12} \text{ eV})$  and light particle couplings  $O(10^{-21})$ .

Finally note that in such models all subreps in the  $Z$  field not eaten in the course of the gauge breaking remain light. Hence they exert a marked influence on the evolution of coupling constants. Even optimistic estimates give  $\sin^2 \theta_W \approx .25$  for  $M_U \approx 10^{20} \text{ GeV}$ ! (see 7.9) Hence such models tend to lose predictive power for  $\sin$ .

An  $SO(10)$  model using 54 dimensional representations has also been proposed.⁽²⁷⁾ It shares the virtues mentioned above and also associates  $M_R = M_S$ . Unfortunately, in this case the light multiplets in  $Z(54)$  include a  $(1,1,20)$  dimensional representation of  $G_{PS}$ . This leads to an unacceptable situation with respect to  $\sin^2 \theta_W(M_W)$  and  $\alpha_3(M_U)$  (See 7.9)

#### Section 4: The OM SUGRY Lagrangian

Various Lagrangians invariant under the Super Poincare group are known. All of them may be conveniently derived from the Lagrangian invariant under the superconformal group  $SU(2,2,1)$  which couples arbitrary sets of chiral multiplets to a gauge multiplet in a gauge invariant manner. Of these, the best known is the Old Minimal (OM) version mentioned earlier. As we show in chapter 6 the other versions, taken literally, do not offer

the attractive features of OM SUGRA. Moreover, considered from another point of view involving point transformation redefinitions of multiplets, several of the other supergravities can be shown to be subcases of the OM version. (28)

In this section we merely present the result of the calculation performed by the authors of ref. (29). It represents the OM SUGRA Lagrangian for the most general coupling of an arbitrary gauge multiplet in a locally gauge invariant way. We also note certain features of the mass matrices and symmetry breaking.

The Lagrangian is completely specified by two functions  $\mathcal{G}(\Phi, \Phi^*)$  and  $f_{\alpha\beta}(\Phi)$  of the chiral superfields. Here  $\mathcal{G}$  is real and  $f_{\alpha\beta}$  is an analytic function while  $\alpha, \beta$  are a symmetrized pair of indices of the adjoint representation of the gauge group. The bosonic part of the Lagrangian is

$$\begin{aligned}
 e^{-1} \mathcal{L}_B = & \kappa^{-4} e^{-\mathcal{G}} (3 + g_i' g''^{-1j} g'^j) \\
 & - \frac{1}{2\kappa^2} \tilde{g}^2 \operatorname{Re} f_{\alpha\beta}^{-1} (g'^k T_{\alpha}^j z_j) (g'^k T_{\beta}^l z_l) \\
 & - \frac{1}{2\kappa^2} R + g''^{-1j} D_{\mu} z_i D_{\mu} z^{*j} \\
 & - \frac{1}{4} \operatorname{Re} f_{\alpha\beta} F_{\mu\nu}^{\alpha} F_{\mu\nu}^{\beta} + \frac{i}{4} \operatorname{Im} f_{\alpha\beta} F_{\mu\nu}^{\alpha} F_{\mu\nu}^{\beta}
 \end{aligned} \quad (5.21)$$

We have used  $z_i$  instead of  $A_i$  for scalar fields. Where  $e$  is the determinant of the vierbein,  $R$  the scalar curvature and  $F_{\mu\nu}$  are the gauge field strengths.  $D_{\mu}$  are

gauge covariant derivatives. The derivative dependent fermionic terms are

$$e^{-1} \mathcal{L}_{F,K} = \frac{1}{2} \text{Re} f_{\alpha\beta} \left( -\frac{1}{2} \bar{\lambda}^\alpha \not{D} \lambda^\beta \right) - \frac{1}{4} \bar{\psi}_\mu \gamma_5 \chi_\nu D_\rho \psi_\sigma \epsilon^{\mu\nu\rho\sigma} e^{-1} \\ + g_j''^i \bar{\chi}_i \not{D} \chi_{Li} + \text{h.c.}$$

+derivative dependent  $O(\kappa^2)$  interaction

terms.

(5.21)

Finally there is  $\mathcal{L}_{F,M}$  the fermionic Lagrangian with no derivatives. We again quote only the part bilinear in fermionic terms since the other terms are suppressed by powers of  $\kappa$ .  $\psi_\mu$  is the gravitino (the partner of the graviton).

$$e^{-1} \mathcal{L}_{F,M} = \frac{1}{\kappa} e^{-g/2} \bar{\psi}_{\mu R} \sigma^{\mu\nu} \psi_{\nu L} + \frac{1}{2\kappa} e^{-g/2} g_e' g_e''^k f_{\alpha\beta k} \bar{\lambda}^\alpha \lambda^\beta \\ + \frac{e^{-g/2}}{\kappa} [g''^{ij} - g''^i g''^j - g_e' g_e''^k g_k'''^{ij}] \bar{\chi}_{Ri} \chi_{Lj} \\ - \frac{i}{2} \tilde{g} g''^i T_i^{\alpha j} z_j \bar{\psi}_i \gamma \lambda^\alpha - e^{-g/2} g''^i \bar{\psi}_i \gamma \chi_{Li} \\ + \frac{i}{2} \text{Re} f_{\alpha\beta} f_{\alpha\beta\gamma} \tilde{g} g''^i T_i^{\alpha j} z_j \bar{\chi}_R^k \lambda^\gamma \\ + 2i \tilde{g} g_k''^{ij} T_j^{\alpha k} z^j \bar{\lambda}^\alpha \chi_{Li}$$

(5.22)

+ (4 fermi contact terms).

In the above derivatives with respect to  $(\kappa z_i)$ ,  $(\kappa z^{*i})$  are denoted by super (sub) scripts.

It is clear that for canonical kinetic terms for the chiral multiplets we should take

$$y''_{ij} = -\frac{1}{2} \delta_{ij}$$

This can be achieved by taking  $y = -z_i z^{*i/2} + (F(Z) + \text{hc})$ .

In fact, it may be shown that the function  $y$  is given in this case by

$$y = -\frac{\kappa^2}{2} z^{*i} z_i - \log \frac{|g|^2 \kappa^6}{4} \quad (5.23)$$

where  $g$  is the superpotential used to specify the interactions of chiral supermultiplets in the original superconformal Lagrangian. As for  $f_{\alpha\beta}$  one can choose  $f_{\alpha\beta} = \delta_{\alpha\beta}$  for minimal gauge kinetic terms. However, it is possible to obtain these by a rescaling of  $F_{\mu\nu}^\alpha$  and  $\lambda^\alpha$  even if  $f$  is not unity. This is because one may rescale  $F$  and  $\lambda$  simultaneously since they are part of the same supermultiplet. The difference between the two options lies principally in the mass terms for gauginos coming from

$$\frac{1}{2\kappa} e^{-\langle y \rangle / 2} \langle f'_{\alpha\beta k} \rangle y'^k y''^{-k} \bar{\lambda}^\alpha \lambda^\beta + \text{h.c.} \quad (5.24)$$

Unless explicitly mentioned we take  $f^{\alpha\beta} = \delta^{\alpha\beta}$  (See also

Appendix II). With (5.23) for  $\mathcal{L}$  one obtains the potential

$$V = \frac{1}{2} e^{\kappa^2 z^* z / 2} \left[ |g|^2 + \frac{\kappa^2 z^* z}{2} |g|^2 - \frac{3}{2} \kappa^2 |g|^2 \right] + \frac{1}{2} D_\alpha^2 \quad (5.25)$$

where

$$D^\alpha = -\frac{1}{\kappa} \tilde{g} g'' T_i^{\alpha j} z_j = \frac{1}{2} \tilde{g} z^* T_i^{\alpha j} z_j$$

The second equality is the global expression and follows from the gauge invariance of  $g$ .

We refer the reader to ref.29 for the SUGRA transformations under which  $\mathcal{L}$  is invariant. However, we shall need to know that

$$\delta \chi_{Li} = \frac{1}{\kappa^2} \epsilon_L g'' T_i^{\alpha j} z_j e^{-g/2} + \text{fermionic and} \quad (5.26)$$

derivative dependent terms.

$$\begin{aligned} \delta \lambda_R^\alpha &= -i \frac{\tilde{g}}{2} \epsilon_R D^\alpha + \dots \\ &= (i \tilde{g} / 2 \kappa) \epsilon_R \text{Re } f_{\alpha\beta} g'' T_i^{\beta j} z_j \end{aligned} \quad (5.27)$$

So that we recognize  $\langle g' \rangle$  and  $\langle D^\alpha \rangle$  as the order parameters of SUSY breaking.

We now discuss the 'super-Higgs' effect whereby the massless fermion resulting from the breakdown of SUSY is 'eaten' to form a massive gravitino. The fermionic mass

terms are

$$\mathcal{L}_{F,M}^{(2)} = \frac{e^{-g/2}}{\kappa} \left[ \bar{\psi}_\mu \sigma^{\mu\nu} \psi_\nu - \bar{\psi} \cdot \gamma g'' \chi_{Li} + \chi_{Li}^T C^{-1} (g''^{ij} - g' g'^{ij}) \chi_{Lj} \right] - \frac{i\kappa}{2} \bar{\psi} \cdot \gamma \lambda^\alpha D^\alpha + 2i \bar{\lambda}^\alpha \chi_{Li} D^\alpha \quad (5.28)$$

The Goldstino is defined as the massless combination which couples to  $\psi_\mu$

$$\begin{aligned} \eta_L &= e^{-g/2} (g'' \chi_{Li} - \frac{i e^{g/2}}{2} \tilde{g} g'^{T\alpha\beta} \tilde{g} \lambda^\alpha) \\ &= e^{-g/2} (g' \chi_{Li} + \frac{i D^\alpha}{2} \lambda_L^\alpha) \\ &= e^{-g/2} \tilde{\eta}_L \end{aligned} \quad (5.29)$$

Here the scalar field dependent coefficients are to be computed at the minimum of the scalar potential where

$$\left\langle \frac{\partial V}{\partial z} \right\rangle = 0$$

Furthermore, since the interpretation of masses in de-Sitter space is problematic and since from 'experiment' (cosmology) one knows that the cosmological constant is very small one also requires that

$V=0$  at the minimum.

So one has

$$\begin{aligned} |g'_i|^2 &= -\frac{1}{4} D_\alpha^2 e^g + \frac{3}{2} \\ g_i D_\alpha^2 + 2 D^\alpha D^\alpha_i &= 0 \end{aligned} \quad (5.30)$$

One can show that since the fermionic mass terms are

$$\mathcal{L}_{F,M}^{(2)} = \frac{e^{-g/2}}{\kappa} \left[ \bar{\psi} \cdot \sigma \cdot \psi - \bar{\psi} \cdot \gamma \tilde{\eta}_L - \frac{2}{3} \tilde{\eta}_L \cdot \eta_L \right. \\ \left. + (\chi_{Li}^T C^{-1}, \lambda_{L\alpha}^T C^{-1}) \begin{pmatrix} M^{ij} & M^{i\beta} \\ M^{\alpha j} & M^{\alpha\beta} \end{pmatrix} \begin{pmatrix} \chi_{Lj} \\ \lambda_{L\beta} \end{pmatrix} \right] \quad (5.31)$$

where

$$M^{ij} = \frac{e^{-g/2}}{\kappa} \left[ g''^{ij} - \frac{1}{3} g''^i g'^j \right] \quad (5.32)$$

$$M^{\alpha i} = \frac{i\kappa}{3} (g'^i D^\alpha + 3 D^{\alpha i})$$

$$M^{\alpha\beta} = (-\kappa^3/6) e^{g/2} D^\alpha D^\beta$$

is indeed the massless eigenstate of M above.

One may also compute the supertrace of the mass matrix. One finds

$$\begin{aligned} \text{Tr } M_{3/2}^2 &= (4/\kappa^2) e^{-g} = \kappa^2 |g|^2 e^{\kappa^2 2 \cdot 2^{N-1}/2} \\ \text{STr } M^2 &= \sum_{j=0}^{3/2} (-1)^j (2j+1) M_j^2 \\ &= (N-1) (2m_{3/2}^2 - \kappa^2 D^\alpha{}^2) + 2\tilde{g}_\alpha D^\alpha \text{Tr } T^\alpha \end{aligned} \quad (5.33)$$

or

$$\begin{aligned} \text{Tr } (M_0^2 - M_{1/2}^2 + 3M_1^2) &= 2(N+1) m_{3/2}^2 - \kappa^2 (N-1) D_\alpha{}^2 \\ &\quad + 2\tilde{g}_\alpha D^\alpha \text{Tr } T^\alpha \end{aligned}$$

here N is the number of chiral multiplets. Thus we see that the supergap in the light sector tends to be  $O(m_{3/2})$ . This can also be seen from the fact that all

scalar fields possess a contribution

$$\frac{1}{4} e^{\kappa^2 |z^i|^2} | \kappa^2 \langle g \rangle |^2 \quad (5.34)$$

to their mass matrix.

### Section 5 SUGRA GUTS:

We have seen that in OM SUGRA with canonical kinetic terms the potential is given by

$$V = e^{\kappa^2 z_i z^{*i}} \left[ |g^i + \kappa^2 z^{*i} g|^2 - 3 \kappa^2 |g|^2 \right] + \frac{1}{2} D_\alpha^2 \quad (5.35)$$

where we have redefined  $\frac{\kappa^2}{2} \rightarrow \kappa^2$ .

Thus besides the global potential  $|g^i|^2 + \frac{1}{2} D_\alpha^2$  we have additional contributions and the potential is no longer positive definite. Assuming that no gauge (D) field gets v.e.v.  $O(M_P)$  the dominant correction comes from terms proportional to  $g$ . Moreover, we note that unlike the global case the inclusion of a field independent constant  $C$  in  $g$  changes the potential.

Let us focus on the  $SU(5)$  model with

$$g = C + \frac{M}{2} \text{Tr } Z^2 + \frac{\lambda}{3} \text{Tr } \Sigma^3 \quad (5.36)$$

for clarity.  $\Sigma$  is a 24plet of  $SU(5)$  and  $M \sim M_{\text{Gut}}$ . We saw earlier that in the global case there are three degenerate solutions with vacuum symmetry  $SU(5), SU(4) \times U(1)$  and  $G_{123}$  respectively and each preserves SUSY

since  $\langle F_2 \rangle = 0$  in all cases. This preservation of invariance will in general carry over to the local case so that  $\langle g_i \rangle = 0$ .

The v.e.v. which satisfy this equation will be the global ones plus small corrections  $O(\kappa^2 g)$ . Thus the vacuum energy will be proportional to  $-\langle g \rangle^2 \kappa^2$ . Since  $\langle g \rangle$  varies with the little group the degeneracy is lifted. To leading order in  $\kappa$  one has,

$$\Sigma_m = \Sigma_m^0 \left( \underline{1}_n, -\frac{n}{m} \underline{1}_m \right)$$

$$\langle g \rangle_m = C + (M^3/\lambda) \frac{mn(m+n)}{6(n-m)^2} \quad (5.37)$$

If we fine tune  $C$  such that  $\langle g \rangle = 0$  for  $m=2$  we find that the vacuum is flat for  $G_{123}$  but has negative energies for the  $SU(5)$  and  $SU(4) \times U(1)$  vacua. However, it has been shown that in the case where the vacuum has several local minima differing in energy by an amount  $\epsilon$  small compared to the overall depth of the potential, gravitational effects can stabilize a vacuum of higher energy against decay via tunneling to a lower energy state.^(30,31) Bubbles of the lower energy cannot form if

$$\epsilon \leq \epsilon_c = \frac{3}{4} \kappa^2 S_1^2 \quad (5.38)$$

where  $S_1$  is the bubble surface tension. This is basically because when gravitational effects are included there is

contribution  $-\kappa^2 \pi S_1 R^3$  to  $\langle V \rangle$  over and above usual dominant surface tension terms. Hence, the volume contribution

$$-\left(\frac{4\pi}{3} \epsilon + \kappa^2 S_1^2 \pi\right) R^3 \quad (5.39)$$

cannot overcome the positive contribution to  $V$  coming from the usual surface term  $4\pi R^2 S_1$  if

$$\epsilon \leq \epsilon_c = \frac{3}{4} \kappa^2 S_1^2 \quad (5.40)$$

The energy then remains positive and the higher vacuum is stable.

The 'surface tension'  $S_1$  can be computed and in the SUSY case one finds⁽³¹⁾

$$S_1 \geq 2 | \langle f(z_1) \rangle | \quad (5.41)$$

where  $z_1$  is the v.e.v. characterizing the lower vacuum.

Thus one finds

$$\epsilon = 3\kappa^2 |f(z_1)|^2 \leq \frac{3}{4} \kappa^2 S_1^2 = \epsilon_c \quad (5.42)$$

In general, the bound can be saturated but very general theorems^(31,32) on the positivity of the energy spectrum of SUGRA theories with asymptotically flat space times indicate that the result will persist when checked to higher orders in  $\epsilon$  even if  $\epsilon = \epsilon_c$  to lowest order.

Thus one concludes that at the expense of a fine tuning it is possible to lift vacuum degeneracy in SUGRA

models.

Before closing this section we review the bound on the gravitino mass derived by Weinberg⁽³⁴⁾ from cosmological considerations. Pagels and Primack⁽³³⁾ have shown that the gravitino mass is bounded by 1 KeV if they have survived to the present epoch due to the unacceptable contribution to the matter energy density that would arise if they had a greater mass. Since  $m_{3/2}^2 \sim \kappa^2 |g|^2$  and in typical models which break SUGRA spontaneously  $\langle g \rangle \sim \kappa^{-1} M_S^2$  it follows that the SUSY breaking scale  $M_S$  is then bounded above by  $10^6$  GeV.

On the other hand, if the gravitino is very heavy, ie.  $M_S$  is very large then even though its' interactions with light matter are suppressed by powers of  $\kappa$  it will be able to decay before the present time.

By estimating the gravitino decay rate  $\Gamma_{3/2}$  as  $\kappa^2 m_{3/2}^3$  and noting that at  $T \sim m_{3/2}$  the expansion rate  $H$  of the Universe is  $\kappa T^2 \approx m_{3/2}^2 \kappa$  one sees that  $(\Gamma_{3/2}/H) \sim \kappa m_{3/2}$  hence the gravitinos will survive till temperatures  $T \ll m_{3/2}$ . At these temperatures they will be non-relativistic and the Universe will be gravitino dominated. Hence when they finally decay at a temperature  $T_{\text{decay}}$  (obtained by equating the expansion rate of the gravitino dominated Universe to  $H = T^2 \kappa$ ) they reheat the universe to temperatures  $T \sim (m_{3/2}^3 \kappa)^{1/2}$  and raise the entropy density by a

factor  $O(K m_{3/2})^{-1/2}$ . If  $T_{\text{decay}} < .4 \text{ MeV}$  (ie.  $M_S < 10^{13} \text{ GeV}$ ) then the present  $n_B/n_\gamma$  ratio of  $O(10^{-9})$  must be the diluted remnant of an earlier larger one. Hence the He/De ratio observed would be the much larger one corresponding to a higher  $n_B/n_\gamma$  unless the reheating is sufficient to break up the previously formed nuclei and set the  $n_B/n_\gamma$  to its presently inferred value. Weinberg then obtained the bound

$$\begin{aligned} m_{3/2} &> 10^4 \text{ GeV} \\ M_S &> 10^{11} \text{ GeV} \end{aligned} \quad (5.43)$$

He also noted that if  $M_S$  were very large (eg.  $O(10^{15} \text{ GeV})$ ) then the entropy increase must be  $> 10^4$  thus requiring the current baryon asymmetry generation scenarios to account for  $n_B/n_\gamma \sim 10^{-6}$  or lower.

#### Section 6 Low Energy Behaviour of SUGRA GUTS:

In this section we briefly sketch proposals to generate the weak scale via SUGRA effects at the tree level. This idea was pioneered in the work of R. Arnowitt, A. Chamseddine and P. Nath (ACN)⁽³⁵⁾ within the context of an  $SU(5)$  SUGRA model with intermediate scale SUSY breaking.

a) ACN proposed the superpotential

$$\begin{aligned} g &= g_0 + g_1 \\ g_0 &= m_0^2 z + c \\ g_1 &= \lambda_1 \left( \frac{1}{3} T_2 z^3 + \frac{1}{2} T_1 z^2 \right) + \lambda_2 \bar{H} (\Sigma + 3 M' \mathbb{1}) H \\ &\quad + \lambda_3 U \bar{H} H + g_{ij}^U H(10)_i (10)_j + g_{ij}^d \bar{H}(\bar{5})_i (10)_j \end{aligned} \quad (5.44)$$

where  $Z$  and  $U$  are singlet fields  $m_0 \sim 10^{11} \text{ GeV}$  and the rest of the notation has been introduced in earlier chapters.

The piece  $g_0$  is called the Polonyi superpotential. It can be shown (see chap. 7 for the minimization of the Polonyi potential) that the use of such a superpotential leads to the breaking of SUSY at a scale  $O(m_0)$  and the appearance of a gravitino of mass  $O(\kappa m_0^2)$ . The idea of ACN was that the mixing of the  $g_0$  and  $g_1$  sectors due to SUGRA effects could result in a situation whereby the low energy effective potential contained terms capable of inducing  $SU(2) \times U(1)$  breakdown at a scale set by the gravitino mass. The presence of the low mass 'sliding singlet'  $U$  was crucial to this proposal for the induction of the weak scale at tree level. The model required the fine tuning  $M=M'$ . The sliding singlet mechanism does not generalize to the SUGRA case (see chapter 7). They were also able to show that the hierarchy demonstrated by expanding around the global solutions to the superpotential (5.44) above could be preserved to all orders in  $\kappa$ .

Unfortunately the presence of light (Heavy) type couplings (e.g.  $U H_3 H_3$ ) has been shown to destabilize the hierarchy of such models. We refer the reader to the original work⁽³⁵⁾ for the details of these models.

We now summarize the results of recent

investigations on the form of the effective potential at low energies in SUGRA theories. Nilles, Srednicki and Wyler (NSW)⁽³⁶⁾ considered a 'minimal' tree level scenario where the superpotential was taken to have the separable form

$$W = h(z_i) + g(y_a) \quad (5.45)$$

$z_i$  are the scalar fields which break local SUSY while  $y_a$  are the light s-matter and scalar Higgs fields. The  $z_i$  are taken to be  $G_{123}$  singlets and do not appear in the low energy sector. By making the ansatz

$$\begin{aligned} \langle z_i \rangle &= h_i / \kappa \\ \langle h_i \rangle &= m a_i^* \kappa^{-1} \\ \langle h \rangle &= m \kappa^{-1} \end{aligned} \quad (5.46) \quad m \sim m_{3/2}$$

in accordance with the paradigm of the Polonyi superpotential and by setting  $V=0$  at the minimum of the potential they showed that the low energy effective potential in the limit

$$\begin{aligned} \kappa &\rightarrow 0 \\ m_{3/2} &\sim \kappa^2 \langle g \rangle = \text{constant} \end{aligned} \quad (5.47)$$

was given by

$$V_L = |\tilde{g}|^2 + m_{3/2}^2 |y_a|^2 + m_{3/2} (A \tilde{g} + \text{h.c.}) + \frac{1}{2} D_\alpha^{\text{light } 2} \quad (5.48)$$

where it was assumed that no mass parameter exists in the cubic superpotential  $g(y_a)$  so that  $g^a y_a = 3g$  (a similar result for a more specific superpotential had been obtained earlier by Barbieri et al)⁽³⁷⁾

Here,

$$\begin{aligned} \tilde{g} &= e^{\frac{1}{2}|b_i|^2} g \\ A &= b_i^* (a_i + b_i) \end{aligned} \quad (5.49)$$

The positive definite quantity

$$|\tilde{g}_a \pm m_{3/2} y_a|^2 = |\tilde{g}_a|^2 + m_{3/2}^2 |y_a|^2 \pm 3m_{3/2} (g + g^*) \quad (5.50)$$

and one can show

$$(A\tilde{g} + h.c.) > \pm 3(\tilde{g} + h.c.)$$

Further  $y=0$  is a trivial solution with  $V_1=0$  so it follows that breakdown of the weak symmetry can occur in this type of scenario only if  $|A| > 3$ . We note in passing that the Polonyi superpotential yields  $|A| = 3 - \sqrt{3}$

Assuming a superpotential  $H$  with  $|A| > 3$  can be found one may ask what type of superpotential leads to a breakdown of weak symmetry. NSW proposed the toy model

$$g = \lambda Y \bar{H} H$$

where  $Y$  is a gauge singlet and  $HH$  are  $SU(2)$  doublets. Then

one can show that while all v.e.v. vanish when  $|A| < 3$ , if  $|A| > 3$

$$\begin{aligned} \langle Y \rangle &= \langle H_0 \rangle = \langle \bar{H}_0 \rangle = U m_{3/2} / \lambda \\ U &= \frac{1}{4} (|A|^2 + \sqrt{|A|^2 - 8}) \\ \langle V_{LE} \rangle &= -\frac{m_{3/2}^4}{\lambda^2} U^2 (U^2 - 1) \end{aligned} \quad (5.51)$$

Unfortunately, the introduction of light Fermion Yukawa couplings in  $g$  such as  $HLE$  ( $L$  and  $E$  are left chiral supermultiplets containing  $(\nu_e)_L$  and  $e_L^c$  respectively) leads to a vacuum that violates charge and other conserved quantum numbers.⁽³⁸⁾ To see this one need merely note that in the minimization of the potential derived from

$$g = \lambda_e \bar{H} L E + \lambda Y \bar{H} H \quad (5.52)$$

the field  $E$  plays a role exactly analogous to  $Y$  and  $L$  to  $H$ . Since

$$\langle H^0 \rangle = \langle A_e \rangle = \langle A e_L^c \rangle \quad (5.53)$$

causes  $\langle D_u \rangle$  to vanish, it follows that another solution to the minimization is that where the above v.e.v. are given by  $m_{3/2} U / \lambda_e$ . The ground state energy is then negative and  $O(m_{3/2}^4 / \lambda_e^2)$ . Now since the  $SU(2)$  breaking v.e.v.

$$\langle H_0 \rangle = m_{3/2} / \lambda$$

while the electron mass  $m_e = m_{3/2} \lambda_e / \lambda$  it follows that

if  $\lambda < \lambda_e$  then  $m_e > m_{3/2}$ . But since  $m_{3/2}$  bounds the light supergap from above this means we get a supergap  $O(m_e)$ . Hence the true vacuum breaks electric charge and lepton number. We have checked that this result cannot be avoided by introducing more generations or special couplings for the matter fermion. However, if  $|\lambda| < 3$  then there is no charge breaking vacuum but neither is there a breaking of weak symmetry. Radiative corrections do not alter this phenomenon.⁽³⁹⁾ Barring ad hoc complication of the superpotential one must accept that the NSW scenario is too simplistic.

To end this section we quote the result of Hall, Lykken and Weinbergs' calculation on the effective low energy potential in SUGRA GUTs. This aspect of SUGRA models has also been studied by ACN⁽³⁵⁾ in their pioneering papers. They worked with a potential separable in to GUT and SUSY breaking sectors

$$\mathcal{V} = \mathcal{V}_{GUT} + \tilde{\mathcal{V}}_{Hidden}(\tilde{\phi}) \quad (5.54)$$

with  $M_{GUT} \sim M_P$  and tuning  $V=0$  at tree level. Expanding around the global solution in powers of  $m_{3/2}$  and solving the extremal equations

$$\partial \mathcal{V} / \partial z^A = 0$$

for the v.e.v. of the heavy fields  $z_A$  as functions of the light fields  $z$  they found that the low energy potential

had the form

$$V_{\text{eff}} = \sum_{\alpha} \left| \frac{\partial g_{\text{eff}}}{\partial z^{\alpha}} \right|^2 + 2 \operatorname{Re} m_1^* g^{(3)} + 4 \operatorname{Re} m_2 g^{(2)} \quad (5.55)$$

$$+ 2 \operatorname{Re} (4m_2 - m_1) g^{(1)} + m_2^2 |z^{\alpha}|^2 + \frac{1}{2} D_{\alpha}^2$$

$$+ V_0$$

$$V_0 \sim O(m_2^4)$$

here

$$g^{\text{eff}} = g_0 + g^{(1)} + g^{(2)} + g^{(3)} \quad E_0 = e^{\kappa^2 |z_i|^2}$$

$$g_0 = \frac{1}{6} \sqrt{E_0} g_{ABC} z_1^A z_1^B z_1^C \quad (5.56)$$

$$g^{(1)} = \frac{1}{2} \sqrt{E_0} g_{\alpha AB} z^{\alpha} z_1^A z_1^B$$

$$g^{(2)} = \frac{1}{2} \sqrt{E_0} g_{\alpha\beta A} z^{\alpha} z^{\beta} z_1^A$$

$$g^{(3)} = \frac{1}{6} \sqrt{E_0} g_{\alpha\beta\gamma} z^{\alpha} z^{\beta} z^{\gamma}$$

here  $z_1^A$  is the first order shift in the heavy fields

$$z_1^A = \sum_B f_{AB}^{-1} z_0^B + m_2 \quad (5.57)$$

$$m_2 = \kappa^2 \sqrt{E_0} \langle \tilde{g} \rangle$$

$$m_1 = \kappa^2 \sqrt{E_0} \left[ \tilde{z} \left( \partial \tilde{g} / \partial \tilde{z}_i \right) + \kappa^2 \langle \tilde{g} \rangle |z_i|^2 \right]$$

and  $\tilde{z}$  are the hidden field v.e.v.

Note that there are no quartic terms from the

$F^2$  contributions of heavy scalars.

### Section 7:

In this section we briefly summarize the ideas of Ibanez⁽⁴⁰⁾ as developed by Alvarez-Gaume, Polchinski and Wise (APW)⁽⁴¹⁾ on the subject of weak scale induction in SUGRA models via renormalization effects.

The basic idea, due to Ibanez, was that the evolution equations for scalar masses obtain opposite sign contributions from Yukawa quartic (cubic) couplings  $h(A)$  and gauge couplings  $\alpha$ . Thus

$$dm_{\phi}^2/dt = a h^2 (\sum m_i^2) + A^2 h^2 m_{3/2}^2 - b \alpha \tilde{m} \quad (5.58)$$

$t = \ln \mu$        $\mu$  - renormalization scale  
 where  $\tilde{m}$  are gaugino masses,  $A$  is the trilinear scalar coupling and  $m_i^2$  are the masses of the scalars at the vertex  $h$ . Thus we see that as  $t = \ln \mu$  decreases the first and second term cause  $m_{\phi}^2$  to decrease and the third term causes  $m_{\phi}^2$  to increase. Since the coefficients for the evolution equations for Higgs scalars are larger than those for s-matters due to colored triplets running in loops the Higgs masses decrease faster than any others. Furthermore, if gluino masses are large they drive colored scalars to larger masses causing additional decreases in  $m_H^2(t)$ . Thus provided a heavy generation (large enough  $h$ ) exists  $m_H^2$  may be driven negative leading

to breakdown of Weak symmetry .

Alvarez-Gaume, Polchinski and Wise have considered a minimal  $G_{123}$  model with soft breakings of the pattern derived by HLW in ref.(39) and implemented this idea. By integrating the renormalization group equations between  $M_P$  and  $M_W$  they find that for  $A_{\text{initial}} = 3 - \sqrt{3}$  and  $m = 0$  the top quark mass lies in the range 140-195 GeV. If the gluino mass is taken to be large the constraints on  $m_{\text{top}}$  can be relaxed to  $m_{\text{top}} \leq 55$  GeV for gluino masses  $m_{3/2}$ . Since the form of the SUGRA Lagrangian leads us to believe that  $m_{\tilde{\chi}} \sim m_{3/2}$  the previous estimate is probably more likely. We refer the reader to Ref.(41) for details.

Chapter 6

Effective Potentials in Different Supergravities

Introduction: In this chapter we digress from the main line of development of this thesis to consider a question which has a bearing on the construction of locally supersymmetric models of particle interactions. We saw in the last chapter that coupling SUSY gauge theories to gravity via the Old Minimal formalism gave rise to a low energy theory (ie. in the limit  $k \rightarrow 0$   $m_{3/2} = \text{const}$ ) which consisted of a supersymmetric Lagrangian of the global type plus soft breaking terms of dimension two and three which were precisely those necessary for compatibility with the known particle spectrum, constraints on Flavor changing neutral current, etc. We mentioned there that the coupling of  $N=1$  SUSY theories to gravity can be accomplished in many ways. Thus one obtains the Old Minimal, New Minimal (Sohnius-West), DeWit-VanNieuwenhuizen, Breitenlohner, etc. supergravities. This plethora of supergravities which have the structure of a gauge theory with gauge group  $G \times OSp(4/1)$  can, however, all be understood as arising from the choice of different 'compensating multiplets' to break the Superconformal group  $SU(2,2/1)$  down to  $OSp(4/1)$  in the context of the superconformal tensor calculus. It is therefore legitimate to wonder whether these alternative  $N=1$  supergravities could be of any relevance to the building of SUSY models. In an attempt to provide at least a partial answer to this question we have derived the form

of the scalar potential in various supergravities to see whether it offers any of the attractive features available in the context of OM SUGRA. We have taken the point of view that the conformal weights (see below) associated with different supermultiplets in superconformal gravity do have a significance in constraining the structure of the theory. If the opposite view point is taken and the fields are redefined via point transformations so that all have the same (zero) conformal weight then an equivalence between (some of) the different SUGRAS can be demonstrated. (28)

Our result, on the other hand, shows that if one retains the original conformally weightful fields as those relevant to a description of the world then the potential in the various alternative supergravities is 'pseudo-global'. That is it depends essentially on the square of the derivative of the superpotential. Thus there is no automatic matter-smatter or GUT vacuum degeneracy splitting available.

This conclusion along with that of the authors of ref. (28) concerning the equivalence of the Lagrangian for the different SUGRAS to that in the OM case under a redefinition of fields lends weight to the use of OM SUGRA alone for the construction of SUSY models.

In section 2 we briefly introduce the notions of the  $SU(2,2/1)$  gauge theory with arbitrary matter

multiplets. In the following sections we consider the potential obtained in the various SUGRAs which follow from using different combinations of compensating multiplets to break the group. In this chapter we shall follow the notation of the review article of Kugo and Uehara.⁽⁴²⁾

## Section 2 Gauge theory of SU(2,2/1).

a) Let us first consider the pure SC theory without matter or gauge multiplets of the usual type. The superalgebra SU(2,2/1) has generators of two types. The 'even' ones generate SU(2,2) x U(1) and are named  $P_m (-\gamma_m P_-)$ ,  $K_m (\gamma_m P_+)$ ,  $M_{mn} (\sigma_{mn})$ ,  $D (-\gamma_5/2)$ ,  $A(\text{diag}-i/4(1_4, -4))$ ,  $(P_{\pm} = (1 \pm \gamma_5)/2)$ . Where we have given 5x5 matrix representation in terms of 4x4 matrices. Of these  $P_m, M_{mn}$  are just the usual generators of the Poincare group while  $K_m, D$  are the 'conformal boost' and dilation generators respectively. Finally A is the generator of an additional U(1) symmetry. In addition to these one has two fermionic generators Q, S which are generators of 'square roots' of P and K respectively. They may also be represented in terms of the 4x4 Clifford algebra. The commutation relations have the generic form

$$\begin{aligned} [E, O]_- &\sim 0 & E: \text{ even} & (6.1) \\ [E, E]_- &\sim E \\ [O, O]_+ &\sim E & O: \text{ odd} \end{aligned}$$

We refer the reader to the review article of Van Nieuwenhuizen⁽⁴²⁾ for details.

Then one can define a gauge theory in exact analogy to the usual gauging of compact groups. Thus one defines gauge potentials  $h_\mu^A : \{e_\mu^m, \omega_\mu^{mn}, \psi_\mu, f_\mu^m, b_\mu, \varphi_\mu, A_\mu\}$  corresponding to the above generators.

$$h_\mu = h_\mu^A X_A = e_\mu^m P_m + \omega_\mu^{mn} M_{mn} + \bar{\psi}_\mu Q + f_\mu^m K_m + b_\mu D + \bar{\varphi}_\mu S + A_\mu A \quad (6.2)$$

Here  $e_\mu^m$  is the usual vierbein (square root of the metric),  $\omega_\mu^{mn}$  the spin connection,  $\psi_\mu$  the gravitino,  $b_\mu$  the dilation gauge field and  $A_\mu$  the chiral gauge field.  $f_\mu^m, \varphi_\mu$  are extra superconformal potentials corresponding to  $K, S$  respectively. As we shall see  $\omega_\mu^{mn}, f_\mu^m, \varphi_\mu$  do not propagate. Note that  $\psi_\mu$  and  $\varphi_\mu$  are fermionic while the others are bosonic.

One can also define a transformation parameter

$$\epsilon = \epsilon^A X_A = \xi^m P_m + \lambda^{mn} M_{mn} + \bar{\epsilon} Q + \xi^m K_m + \lambda_D D + \bar{\xi} S + \lambda_A A \quad (6.3)$$

(where  $\epsilon$  and  $\xi$  are fermionic) and field strengths (curvatures)

$$R_{\mu\nu}^A = [D_\mu, D_\nu]^A = \partial_\mu h_\nu^A - \partial_\nu h_\mu^A + h_\mu^B h_\nu^C f_{BC}^A \quad (6.4)$$

where  $f_{BC}^A$  are the structure constants of  $SU(2,2/1)$ .

$$\delta_{\text{gauge}}(\epsilon) h_\mu^A = (D_\mu \epsilon)^A = \partial_\mu \epsilon^A + h_\mu^B \epsilon^C f_{CB}^A$$

$$\delta_{\text{gauge}}(\epsilon) R_{\mu\nu}^A = R_{\mu\nu}^B \epsilon^C f_{CB}^A$$

just as in the usual gauge theories. After writing down the most general action dependent on curvatures only one finds that the following constraints⁽⁴³⁾ must be imposed in order that the action be invariant under  $SU(2,2/1)$ :

I

$$R_{\mu\nu}(P) = 0 \quad (6.5)$$

$$\text{II} \quad R_{\mu\nu}(Q) \gamma^\nu = 0$$

$$\text{III} \quad R_{\mu\nu}^{mn}(M) e_m^\lambda e_{n\nu} - \frac{1}{2} R_{\lambda\nu}(Q) \gamma_\mu \psi^\lambda - \frac{i}{4} \tilde{R}_{\mu\nu}(A) = 0$$

where  $\sim$  denotes dual

$$\tilde{R}_{\mu\nu}(A) = \epsilon \epsilon_{\mu\nu\rho\sigma} R^{\rho\sigma}(A) \quad (6.6)$$

Although these constraints were originally derived using the invariance of the action they probably have a deeper significance since they allow the interpretation of the (so far) internal transformations generated by  $P_m$  in the tangent space as general coordinate transformations in the base space. It also follows that one may solve for the fields  $\omega_\mu^{mn}$ ,  $\varphi_\mu$ ,  $f_\mu^m$  as functions of  $(e, \psi, b, A)$ . Thus

these connections are dependent and do not propagate. However, since the constraints I-III are not invariant under the  $Q$  transformation (only) the actual  $Q$  transformations of these fields (as functions of the independent ones) are not the same as the original group theoretic ones. The difference being such as to preserve the constraints. This also implies that in the resulting theory invariant under  $Q$  as well as the other transformations additional covariantization terms must be included so as to accord with the modified (actual) transformation rules of the dependent gauge fields under  $Q$  transformations. The (super) algebra and gauging procedure is then redefined in terms of the modified transformations. The matter fields, which we shall shortly introduce, are then defined to obey the modified transformation laws and thus support representations of the (dynamically consistent) SC algebra.

b) Matter multiplets

We denote complex (and Dirac) fields by script letters and real (and Majorana) ones by print letters.

i) Vector multiplets.

The most general multiplet without external indices is the complex vector  $v(\lambda, n)$

$$v = [C, \zeta, \chi, \kappa, B_m, \ell, Q] \quad (6.7)$$

$\lambda, n$  are the so called conformal and chiral weights defined so that the first components of the multiplet transform as

$$S_D(\lambda_D) \epsilon = \lambda \lambda_D \epsilon \quad S_A(\lambda_A) \epsilon = i \frac{n}{2} \lambda_A \epsilon \quad (6.8)$$

the appropriate covariant derivative for  $\epsilon$  is

$$D_m^c \epsilon = \partial_m \epsilon - \frac{1}{2} \bar{\psi}_m i \gamma_5 \not{\partial} - \lambda G_m \epsilon - i \frac{n}{2} A_m \epsilon \quad (6.9)$$

we omit the definitions of  $D^c$  for  $\not{\partial}, B_m, \not{A}$  as well as the other transformation rules for lack of space and since they will not be relevant to the determination of the potential.

The real vector multiplet  $V(\lambda, n=0)$  is obtained from zero chiral weight complex multiplets  $\mathcal{V}$  by decomposing them as  $\mathcal{V} = V_1 + iV_2$ .

The transformation laws are obtained from those of  $\mathcal{V}$  by setting  $n=0$

$$\mathbb{V} = (C, Z, H, K, \Lambda, D) \quad (6.10)$$

ii) The Gauge multiplet  $G(\lambda = n=0)$  is obtained from  $V$  as the sub-multiplet

$$G = G(B'_\mu, \Lambda, D)$$

where  $B'_\mu = e_\mu^m B_m - \frac{1}{2} \bar{\psi}_\mu \not{\partial} z$

This corresponds to the global case of a real vector multiplet in Wess-Xumino gauge ( $C=Z=H=K=0$ ). Then the

variation under Q is

$$\delta_Q (\epsilon) B'_\mu = -\frac{1}{2} \bar{\epsilon} \gamma_\mu \Lambda - \partial_\mu (\frac{1}{2} \bar{\epsilon} Z) \quad (6.12)$$

and the second piece can be gauged away. We note that

$$\begin{aligned} \delta_Q \Lambda &= \frac{1}{2} (\sigma \cdot F^c + i \gamma_5 D) \epsilon \\ F^c &= \partial_\mu B'_\nu - \frac{1}{2} \bar{\psi}_\mu \gamma_\nu \Lambda - (\mu \leftrightarrow \nu) \end{aligned} \quad (6.13)$$

iii) For  $\lambda = \pm n$  one can separate  $\mathcal{V}$  into two 'chiral' multiplets  $(Q, \chi_R, \mathcal{F})$  and  $(Q^*, \chi_L, \mathcal{F}^*)$  where  $\chi$  is Majorana. They are embedded in  $\mathcal{V}$  as

$$\begin{aligned} \mathcal{V}(Z) &= [Q, -i\chi_R, -\mathcal{F}, i\mathcal{F}, iD_m^c Q, 0, 0] \\ \mathcal{V}(Z^*) &= [Q^*, i\chi_L, -\mathcal{F}^*, -i\mathcal{F}^*, iD_m^c Q, 0, 0] \end{aligned} \quad (6.14)$$

and have transformation laws determined by those of  $\mathcal{V}$

$$D_m^c Q = \partial_m Q - \frac{1}{2} \bar{\psi}_m P_R \chi - \lambda G_m Q - i \frac{\lambda}{2} A_m Q \quad (6.15)$$

here

$$Q = \frac{A + iB}{2} \quad \mathcal{F} = \frac{F + iG}{2}$$

A, B, F, G real. We note that

$$\delta_Q \chi_R = P_R (\not{D}^c Q + \mathcal{F}) \epsilon \quad (6.17)$$

iv) One also has the so called 'linear multiplets'  $\mathcal{L}(\lambda = n = 2)$  and  $\mathcal{L}(\lambda = 2, n = 0)$ . These correspond to the global multiplets which obey the constraints

$$\bar{D} \bar{D} \Phi = 0 \quad (6.18)$$

This constraint can be implemented by using a chiral Lagrange multiplier  $\varphi$  ( $\bar{D}\varphi=0$ ) in the action

$$[\varphi \bar{D} \bar{D} \Phi]_F = [\varphi \Phi]_D$$

Super conformal invariance requires that  $\varphi$  and  $\Phi$  have  $\lambda=2, \eta=0$  hence since

$$\lambda_\varphi = \eta_\varphi$$

one finds

$$\lambda_\Phi - \eta_\Phi = 2$$

When such a constraint has been implemented on some  $\Phi = \psi$  one obtains a complex linear multiplet

$$\mathcal{L} = \mathcal{L}(\mathcal{C}, \mathcal{Z}, \mathcal{H}, \mathcal{B}_m, \Lambda) \quad (6.19)$$

( $\Lambda$  is a Majorana field but the others are complex.)

$$v(\mathcal{L}) = [\mathcal{C}, \mathcal{Z}, \mathcal{H}, -i\mathcal{H}^*, \mathcal{B}_m, 2\Lambda_L + \not{D}(\gamma_5 \mathcal{Z}, -i\mathcal{Z}_2), \quad (6.20) \\ -\square^c \mathcal{C} - i\not{D}_m^c \mathcal{B}^m]$$

where  $\square^c$  is the conformal D'Alembertian and  $\mathcal{Z} = \mathcal{Z}_1 + \mathcal{Z}_2 i$

One can also define a real linear multiplet when the chiral weight is zero and hence

$$\mathcal{L}(\lambda=2, \eta=0) = \mathcal{L}(\mathcal{C}, \mathcal{Z}, \mathcal{B}_m)$$

this is embedded in  $v$  as

$$V(\mathcal{L}) = [\mathcal{C}, \mathcal{Z}, \mathcal{C}, 0, \mathcal{B}_m, -\not{D}^c \mathcal{Z}, -\square^c \mathcal{C}] \quad (6.21)$$

but  $\mathcal{B}_m$  must be constrained in order that this structure

be preserved under transformations.

$$D_m^c B^m = 0 \quad (6.22)$$

this constraint may be solved

$$B_m = e^{-1} e_{m\mu} \epsilon^{\mu\nu\rho\sigma} \left( \partial_\nu a_{\rho\sigma} - \frac{i}{4} \bar{\psi}_\nu \gamma_\rho \psi_\sigma - \bar{z} \sigma_{mn} \psi^n \right) \quad (6.23)$$

where  $a_{\mu\nu}$  is an antisymmetric tensor with appropriate transformation laws determined from those of B.

c) Embeddings, multiplication rules and Density formulae: When building actions one needs the following rules

$$i) [\Sigma(V)]_{\lambda_V=2} = \left[ \frac{1}{2} (H - iK), i P_R (\not{D}^c z + \Lambda), -\frac{1}{2} (D + D^c C + i D_m^c B^m) \right] \quad (6.24)$$

has weights  $\lambda = n = 3$ . It corresponds to  $\bar{D} \bar{D} V$  of the global case (D has  $\lambda = 1/2, n=3/2$ ).

$$W_\alpha = \Sigma_\alpha(G) = \left[ \Lambda_{R\alpha}, -\frac{1}{2} (P_R \sigma \cdot F^c - i \gamma_5 D)_{\beta\alpha}, (P_R \not{D}^c \Lambda)_\alpha \right] \quad (6.25)$$

has  $\lambda = n = 3/2$  and corresponds to

$$W_\alpha = [D D (e^{-gV} D_\alpha e^{gV})]_{\text{Wess-Zumino gauge}}$$

but if one tries to embed  $\psi(\mathcal{L})$  in  $\Sigma$  one gets

$$\Sigma(\psi(\mathcal{L})) \equiv 0 \quad (6.21)$$

this will prove important.

ii) One also has the multiplication formulae.

$$\begin{aligned}
 \psi_1 \times \psi_2 &= \psi_3 \quad / \quad n_3 = n_1 + n_2, \quad \lambda_3 = \lambda_1 + \lambda_2 \\
 &= [C_1 C_2, C_1 \beta_2 + C_2 \beta_1, C_1 \mathcal{H}_2 + C_2 \mathcal{H}_1 - \frac{1}{2} \beta_1^T C \beta_2, \\
 &\quad C_1 \mathcal{K}_2 + C_2 \mathcal{K}_1 + \frac{1}{2} \beta_1^T C i \gamma_5 \beta_2, C_1 \beta_{m2} + C_2 \beta_{m1} + \\
 &\quad + \frac{1}{2} \beta_1^T C i \gamma_5 \beta_{m2}, \\
 &\quad \{C_1 \mathcal{L}_2 + \frac{1}{2} (\mathcal{H}_1 - i \gamma_5 \mathcal{K}_1 + i \gamma_5 \beta_1 - \mathcal{D}^C \mathcal{Q}_1) \beta_2 + (1 \leftrightarrow 2)\}, \quad (6.27) \\
 &\quad C_1 \mathcal{D}_2 + C_2 \mathcal{D}_1 + \mathcal{H}_1 \mathcal{H}_2 + \mathcal{K}_1 \mathcal{K}_2 - \beta_{m1} \beta_{m2} - \mathcal{D}_m^C C_1 \mathcal{D}_m^C C_2 \\
 &\quad - \beta_1^T C \mathcal{L}_2 - \beta_2^T C \mathcal{L}_1 - \frac{1}{2} \beta_1^T C \mathcal{D} \beta_2 - \frac{1}{2} \beta_2^T C \mathcal{D} \beta_1]
 \end{aligned}$$

$$g(\mathcal{Z}) = [g(\mathcal{Q}), g^i \chi_{Rj}, g^i \mathcal{F}_i - \frac{1}{4} g^{ij} \chi_{Ri}^T C \chi_{Rj}] \quad (6.28)$$

$$\begin{aligned}
 \varphi(\mathcal{Z}, \mathcal{Z}^*) &= \sum a_{kl}^{ij} \psi(\mathcal{Z}_i^k) \times \psi(\mathcal{Z}_j^{*l}) \quad (6.29) \\
 &= \{ \phi(\mathcal{Q}, \mathcal{Q}^*), -(\phi^i \chi_{Ri} + \phi_i \chi_L^i) + \frac{1}{4} \phi^{ij} \chi_{Ri}^T C \chi_{Rj}, \\
 &\quad i(\phi^i \mathcal{F}_i - \phi_i \mathcal{F}^{*i}) - \frac{i}{4} \phi^{ij} \chi_{Ri}^T C \chi_{Rj} - \phi_{ij} \chi_L^i C \chi_L^j, \\
 &\quad i(\phi^i \mathcal{D}_m^C \mathcal{Q}_i - \phi_i \mathcal{D}_m^C \mathcal{Q}^{*i}) + \frac{1}{4} \phi_j^i \chi_L^j C i \gamma_5 \mathcal{D}_m \chi_{Ri}, \\
 &\quad -i(-\phi_k^i (\mathcal{F}^{*k} + \mathcal{D}^C \mathcal{Q}^{*k}) + \frac{1}{4} \phi_{kl}^i \chi_L^{Tk} C \chi_L^l) \chi_{Ri}, \\
 &\quad +i(-\phi_k^i (\mathcal{F}_i + \mathcal{D}^C \mathcal{Q}_i) + \frac{1}{4} \phi_k^{ij} \chi_L^j C \chi_{Rj}) \chi_L^k, \\
 &\quad -2 \phi_j^i (\mathcal{D}_m^C \mathcal{Q}_i \mathcal{D}_m^C \mathcal{Q}^{*j} - \mathcal{F}_i \mathcal{F}^j + \frac{1}{4} \bar{\chi}^j \mathcal{D} \chi_i) \\
 &\quad + \frac{1}{8} \phi_{kl}^{ij} \bar{\chi}_{Rk} \chi_L^j \chi_L^i \chi_{Rl} - \frac{1}{2} [\phi_{ij}^k \mathcal{F}_k \bar{\chi}_R^i \chi_L^j + \phi_k^{ij} \mathcal{F}^{*k} \bar{\chi}_L^i \chi_{Rj}] \\
 &\quad + \frac{i}{4} \bar{\chi}^j i \gamma_5 \mathcal{D}_m \chi_R (\phi_{ij}^k \mathcal{D}_m^C \mathcal{Q}^i - \phi_k^{ij} \mathcal{D}_m^C \mathcal{Q}_i) \}
 \end{aligned}$$

Finally our list of needed formulae is completed by the density formulae used to construct Superconformally invariant actions. The F type density formula for chiral multiplets of weight 3

$$\begin{aligned} \mathcal{L} &= \int I_F d^4x = \int d^4x [\Sigma_{\lambda=n=3}]_F \\ &= \int d^4x e \left[ F + F^* + \frac{1}{2} \bar{\psi} \cdot \gamma \cdot \chi + \frac{1}{2} \bar{\psi}_m \sigma^{mn} (A - i\gamma_5 B) \psi_n \right] \quad (6.30) \end{aligned}$$

The D type formula for real vectors with  $\lambda=2$  is derivable from this by using the embedding  $\Sigma(V)$ .

$$\begin{aligned} I_D &= e \left[ D - \frac{1}{2} \bar{\psi} \cdot \gamma i \gamma_5 A - \bar{\varphi} \cdot \gamma i \gamma_5 Z \right. \\ &\quad \left. + \frac{1}{3} C \left( R + \frac{1}{e} \bar{\psi}_\mu R^\mu \right) + \frac{2}{3} \epsilon^{mnk\ell} \bar{\psi}_m \gamma_n \psi_k \left( B - A_2 - \frac{1}{2} \bar{\psi}_\ell Z \right) \right] \quad (6.31) \end{aligned}$$

$R = \hat{R}^\mu_\mu - 2 \bar{\psi}_m \sigma^{mn} \varphi_n$  (scalar curvature with torsion)

$$R^\mu = \epsilon^{\mu\nu\rho\sigma} \gamma_5 \gamma_\nu D_\rho^\omega \psi_\sigma$$

### Section 3 Construction of Lagrangians:

In this section we review the construction of super Poincare Lagrangians from the conformal tensor calculus and focus on the derivation of the potential in the OM case as a benchmark for comparison.

Before proceeding further let us note that since we are computing only the potential and not the entire Lagrangian we can ignore fermionic terms. This results in an enormous reduction in the number of terms we must write

down. Thus an ellipsis . . . will stand for omitted or truncated fermionic terms.

In order to go from the superconformal theory to the Poincare one it is necessary to break the extraneous invariances by fixing the components of a compensating multiplet (s) which are variant under the transformations generated by  $K, S, D$  and  $A$  so that the resulting Lagrangian has only the desired residual invariance. The choice of different sets of compensating multiplets then leads to the different supergravities. The nonphysical compensating multiplets must have an odd number of fermionic auxiliary fields. This is because fixing the  $S$  gauge eliminates one of them but the remaining ones can be eliminated only if they occur in pairs in the action:  $\bar{\psi}\psi'$  (see section 6.4,5). For general applicability one must construct an action for the matter sector out of a general gauge invariant function  $g(S)$  of chiral multiplets (the super potential) and an arbitrary vector function  $(S, S_e^G)$  of chiral and gauge multiplets, together with the gauge field action.

$$I = [\phi V_1]_D + [g \Sigma_1]_F + [W_\alpha W_\beta f_{\alpha\beta}]_F \quad (6.32)$$

However, one finds that the gauge field contribution to the potential is generically of the form  $f_{\alpha\beta} D^\alpha D^\beta$  and leads to no interesting structures. Hence we shall omit

the gauge multiplet and focus on the chiral matters alone. In the above formulae  $v_1, \Sigma_1$  are functions of the compensating multiplets which are inserted to compensate for any possible conformal weights that the chiral fields may have in order that the contents of the  $[ ]_F, [ ]_D$  brackets have weights 2 and 3 respectively.

OM supergravity: This is the tensor calculus which uses a single chiral multiplet  $\Sigma_0$  of weight unity as the compensating multiplet.

The S.C. invariance is broken by choosing

$$\left. \begin{aligned} A_0 &= 1 \\ B_0 &= 0 \\ \chi_0 &= 0 \\ h_\mu &= 0 \end{aligned} \right\} a_0 = \frac{1}{2}$$

fixes scale or D invariance  
fixes axial invariance A  
fixes S invariance (6.33)  
fixes K invariance

In this case it is sufficient to take all matter multiplets of zero weight

$$\Sigma_1 = 8 \Sigma_0^3 = [1, 0, 6f] \quad f = f_1 + i f_2$$

$$V_1 = 4 \Sigma_0 \bar{\Sigma}_0 = [1, 0, -2f_1, -2f_2, A_m, 0, 8|f|^2 - \frac{1}{2} A_m^2] \quad (6.34)$$

Substituting (6.34) in (6.33) and expanding gives the following relevant terms

i) Terms with scalars only

$$e (2 \phi^i F_i F^{*j}) + 2 |f|^2 \phi + (2 f^{*j} \phi^i F_i + h.c.) + (6 f g + g^i F_i + h.c.) \quad (6.35)$$

eliminating  $f$  (since there are no kinetic terms for these

fields) we get the derivativeless terms

$$2 \mathcal{F}_i \mathcal{F}^{*j} \phi_j^i - 2 \phi^{-1} / \phi^i \mathcal{F}_i + \frac{3}{2} g^{*12} (g^i \mathcal{F}_i + h.c.) \quad (6.36)$$

and we see that the elimination of  $f$  yields contributions proportional to  $g$  itself.  $\mathcal{F}_i$  may also be eliminated in a similar fashion to give

$$e \left[ - (g_i - 3 g^* \phi_i / \phi) \mathcal{N}_j^i (g^j - 3 g \phi^j) - \frac{9}{2} |g|^2 \phi \right] \quad (6.37)$$

where  $\mathcal{N}_j^i = 2 (\phi_j^i - \phi^i \phi^j / \phi)$

ii) The gravitational action  $e R \phi / 3$

iii) Double derivative terms from the elimination of the auxiliary field  $A_m$  plus the usual kinetic ones

$$e (-2 \phi_j^i \partial_\mu a^{*j}) \partial^\mu a_i - \frac{1}{2\phi} (\phi^i \partial_m a_j - \phi_j \partial_m a^{*i})^2 \quad (6.38)$$

Rescaling the vierbein by

$$e_{m\mu} \rightarrow e^\lambda e_{m\mu} \quad e^{2\lambda} = -3/2 \phi \quad (6.39)$$

so that

$$e \rightarrow e^{4\lambda} e \quad (6.40)$$

one finds that

$$e \phi R / 3 \rightarrow -\frac{1}{2} e R - \frac{3}{4} e (\partial_\mu \ln \phi)^2$$

Now all double derivative terms can be collected into

$$- \frac{3}{2\phi} \mathcal{N}_j^i \partial_\mu a^{*j} \partial^\mu a_i \quad (6.41)$$

if one chooses so that one obtains  
canonical kinetic terms then one finds that the potential is

$$V = e^{2\alpha_i a^{*i}} \left[ \frac{1}{2} |g^i + 2\alpha^{*i} \dot{g}|^2 - 3|g|^2 \right] \quad (6.42)$$

which is the result quoted in chapter 5 apart from a difference in normalization (since in our case  $a=(A+iB)/2$  while there  $Z$  was given in terms of real components as  $A+iB$ ).

#### Section 4 Potentials for One Compensating Multiplet:

We now turn to the other tensor calculi, we shall consider the tensor calculi that are generated by using  $L_0$ ,  $\mathcal{L}_0$  and  $V_0$  respectively as compensating multiplets.

##### i) $L_0$ Tensor calculus

This formalism also known as the Sohnius-West or New Minimal formalism uses  $L_0(C_0, Z_0, B_m^0)$  with conformal weight  $\lambda=2$  to fix the extraneous superconformal symmetries.

We first note that  $\sum(V(L))=0$ . Hence a compensating chiral field cannot be formed out of a linear multiplet and thus our  $F$  formula must read simply  $[g(S)]_F$ . Notice that  $V^k(L_0)$  cannot be embedded in since it does not have  $\lambda=2$ . Thus  $g(S)$  must have weight 3 by itself. This means that some of the  $S_i$  must have non-zero weight.

Thus the combinations of fields that enter the superpotential are restricted. For the D term one can form

$V_1 = V(L_0)$  and take combinations  $[\phi_k V_1^k]$  provided

$\phi_k$  has weight  $2-2k$ .

The conditions  $C_0 = 1$ ,  $Z_0 = 0$ ,  $b_m = 0$  can be used to fix the dilation, supersymmetry and conformal boosts respectively. This leaves the chiral symmetry generated by A as a local gauge invariance of the theory.

In this gauge one then has

$$V(L_0) = [1, 0, 0, B_m^0, 2i\gamma_5 \gamma \cdot \varphi - \frac{1}{2} \gamma^m \beta^0 \psi_m, -\frac{1}{3} \hat{R} - \bar{\psi} \cdot \varphi + \frac{1}{4} \bar{\psi}^m i \gamma_5 \beta^0 \psi_m] \quad (6.43)$$

Since the gauge fixing conditions used above are not Q invariant one must define a new SUSY transformation which is a linear combination of Q, S, K symmetries, as the surviving supersymmetry transformation.

The D term density formula can be written as

$$I_D = [V_\lambda \times V(L_0)^{1-\lambda/2}]_D = [V^P V(L_0)]_D \quad (6.44)$$

where

$$V^P = V_\lambda \times V(L_0)^{1-\lambda/2} \quad (6.45)$$

In terms of this new Poincare supermultiplet

$$I_D = e D^P - \frac{e}{2} \bar{\psi} \cdot \gamma i \gamma_5 \Lambda^P - e^{\mu\nu\rho\sigma} (B_\mu^P - \frac{1}{2} \bar{\psi}_\mu Z^P) \partial_\nu \partial_\rho \epsilon_\sigma \quad (6.46)$$

thus  $C^P, Z^P, H^P, K^P$  do not appear when the density

formula is written in terms of the components of  $v^P$ . Now since

$$\begin{aligned} v^P(\Sigma_{n=0}) &= v(\Sigma_{n=0}) \\ &= (a, -i\chi_R, -\mathcal{F}, -i\mathcal{F}^*, -iD_m^c a^*, 0, 0) \end{aligned} \quad (6.47)$$

thus

$$\begin{aligned} i(v^P(\Sigma_{n=0}) - v^P(\Sigma_{n=0}^*)) & \\ &= i(a - a^*, -i\chi, -i(\mathcal{F} + \mathcal{F}^*), i(\mathcal{F} - \mathcal{F}^*), \\ &\quad i(D_m^c a + D_m^c a^*), 0, 0) \end{aligned} \quad (6.48)$$

hence under

$$V^P \rightarrow V^P + i(v^P(\Sigma_{n=0}) - v^P(\Sigma_{n=0}^*)) \quad (6.49)$$

one finds that the change in the combination

$$\begin{aligned} \Delta(B_\mu^P - \frac{1}{2}\bar{\psi}_\mu Z^P) & \\ &= i(iD_\mu^c a + iD_{c\mu} a^* + \frac{1}{2}\bar{\psi}_\mu \chi) \end{aligned} \quad (6.50)$$

but since

$$\begin{aligned} D_\mu^c a &= \partial_\mu a - \frac{1}{2}\bar{\psi}_\mu \chi_R \\ \Delta B_\mu' &= \Delta(B_\mu^P - \frac{1}{2}\bar{\psi}_\mu Z^P) = -\partial_\mu(a + a^*) \end{aligned} \quad (6.51)$$

an integration by parts and the antisymmetry of then gives zero. Moreover,  $B_m'$  is just the vector component of the gauge submultiplet of  $v^P$  (see Section

6.2). Thus we see that in the general case the action of the New Minimal supergravity has an additional Abelian invariance with chiral gauge transformation parameter . We emphasize that this local gauge invariance is present over and above the two other gauge invariances associated with the (auxiliary) gauge fields  $A_m, a_{mn}$ . The gauge transformation for the latter having a vector parameter in its transformation law

$$\delta a_{\mu\nu} = \partial_\mu \Lambda_\nu - \partial_\nu \Lambda_\mu \quad (6.52)$$

while

$$\delta A_\mu = \partial_\mu \omega \quad (6.53)$$

These two fields have 3 + 3 degrees of freedom so that the number of auxiliary fields is the same as in the Old Minimal formalism where one had  $2(f) + 4(A_m) = 6$ .

This 'spurious' gauge invariance is of some use to us. For if one tries to use the identity multiplet to construct the Poincaré action (just as in the Old Minimal case one might have used  $V_1 = \bar{\Sigma}_0 \Sigma_0$  ) one finds using (6.31), (6.43) that  $[V_0 = (L_0)]_D$  vanishes identically. Thus if one wishes to provide gravity terms even in the absence of matter then the following trick is used. One introduces a supplementary chiral multiplet

of unit weight and writes

$$\mathcal{L}_{SG} = \frac{3}{2} \left[ V(L_0) \ln (V(L_0) / \Sigma_0 \times \Sigma_0^*) \right]_D \quad (6.54)$$

Using the Abelian gauge invariance above it is permissible to fix

$$\Sigma_0 = \left[ \frac{1}{2}, 0, 0 \right] \quad (6.55)$$

since under  $\Sigma_0 \rightarrow e^{-i\Lambda} \Sigma_0$  the argument of the  $[\ ]_D$  operation goes to  $iV_0(\Lambda - \Lambda^*)$ . Thus if  $\Lambda$  is chiral ( and of course of zero weight) then  $\mathcal{L}$  is unchanged.

We now get

$$\Sigma_0 \bar{\Sigma}_0 = \left[ 1, 0, 0, 0, A_m, 0, -\frac{1}{2} A_m^2 \right]. \quad (6.56)$$

and then evaluation of the log multiplet gives

$$\mathcal{L}_{SG} = -\frac{eR}{2} + \dots \quad (6.57)$$

Thus our total action is

$$\mathcal{I} = \sum_k [\phi_k v_1^k] + [g(s)]_F + \mathcal{L}_{SG}. \quad (6.58)$$

Expanding this action yields the following relevant terms

i) Purely scalar terms

$$e \left[ 2\phi_j^i \mathcal{F}_i \mathcal{F}^{*j} + (g^i \mathcal{F}_i + h.c.) \right] \quad (6.59)$$

ii) Gravity terms

$$\frac{eR}{3} \left[ \Phi - \frac{3}{2} - \bar{\Psi} \right]. \quad (6.60)$$

where

$$\bar{\Phi} = \sum_k \phi_k(a, a^*) \quad \bar{\Psi} = \sum_k k \bar{\phi}_k(a, a^*) \quad (6.61)$$

iii) Kinetic terms

$$-2 \bar{\Phi}_j^i \partial_\mu a_i \partial^\mu a^{*j} \quad (6.62)$$

iv) Additional double derivative terms from the rescaling of  $e$  and elimination of the vector auxiliary fields  $A_\mu, a_{\mu\nu}$ . We regard these as  $O(\kappa^2)$  interaction terms since we have not found a way to combine them into a canonical form for the kinetic terms by a choice of  $\phi_k$  alone.

Eliminating  $\mathcal{F}^i$  and rescaling the gravity terms to canonical form  $-\frac{e^2 R}{2}$  via

$$e_{m\mu} \rightarrow e^\lambda e_{m\mu} \quad e^{2\lambda} = -\frac{3}{2[\bar{\Phi} - \frac{3}{2} - \bar{\Psi}]} \quad (6.63)$$

one gets

$$V = [1 - \frac{2}{3} (\bar{\Phi} - \bar{\Psi})]^{-2} g_i^* (2 \bar{\Phi}_j^i)^{-1} g^j \quad (6.64)$$

for the potential. Thus in contrast to the OM case we see that the potential is 'pseudo global' that is it depends on the field derivatives of  $g$  (apart from an inessential scale factor). Thus it offers no improvements over the global case other than a model dependent modification of the metric function  $\bar{\Phi}_j^i$ . To see the similarity to the global case let us consider chiral fields of weight 1 then the

action density is

$$[C_0 SS^* + C_1 (SS^*)^2 V_0^{-1} + \dots]_D + [g(s^3)]_F \quad (6.65)$$

Thus

$$\bar{\Psi} = (-1) C_1 (a a^*)^2 + \dots \quad (6.66)$$

The choice  $C_0 = 1$  gives canonical kinetic terms plus derivative interactions.

$$\bar{\phi}_j^i = \delta_j^i (1 + 2 C_1 a a^* + \dots) \quad (6.67)$$

thus

$$V = \frac{|g|^2}{2} \left( 1 - \frac{2}{3} (a a^* + C_1 (a a^*)^2 + \dots) \right) \quad (6.68)$$

ie. just the global potential times an uninteresting scale factor.

ii)  $\mathcal{L}_0$  tensor calculus: The next example uses the complex linear multiplet  $\mathcal{L}_0$  (which has three fermionic fields - two in the Dirac field  $\mathcal{J}$  and one in the Majorana field  $\Lambda$ ).

The superconformal invariances are broken using

$$\mathcal{C} = 0 \quad \mathcal{Z}_1 = 0 \quad \mathcal{L}_\mu = 0 \quad (6.69)$$

This leaves 14 bosonic ( $H_1, H_2, B_m^1, B_m^2, A_m$ )

and 8 fermionic ( $\mathcal{Z}_2, \Lambda$ ) auxiliary fields to be

eliminated. As before we concentrate only on the result of the elimination of scalar auxiliary fields. One can form

the following multiplets for use in the action

$$\begin{aligned} V_1 &= \mathcal{V}(\mathcal{L}_0) \times \mathcal{V}(\mathcal{L}_0^*) \\ &= [1, 0, 2H_1^0, 2H_2^0, \dots, \dots, 2\mathcal{H}^2 + \frac{1}{3}\hat{R} + \dots] \quad (6.70) \\ V_1^k &= [1, 0, 2kH_1^0, 2kH_2^0, \dots, \dots, \frac{k}{3}\hat{R} + 2k^2|\mathcal{H}^0|^2] \end{aligned}$$

$$\Sigma_1 = \Sigma(V_1) = [\mathcal{H}^*, \dots, -\mathcal{H}^2 + \dots]$$

$$\Sigma_1^\ell = [\mathcal{H}^{*\ell}, \dots, -\ell \mathcal{H}^{*\ell} \mathcal{H} + \dots]$$

Now if all matter fields have zero weight then the F formula can be written as a D type density

$$\begin{aligned} I &= [\varphi_0 V_1]_D + [g_0 \Sigma_1]_F = [\tilde{\phi} V_1]_D \\ \tilde{\phi} &= \phi_0 + V(g_0) \end{aligned} \quad (6.71)$$

since  $[g_0 \Sigma(V_1)]_F = [V(g_0) V_1]_D$ . Then the potential terms are

$$2 \tilde{\phi}_j^i \mathcal{F}_i \mathcal{F}^{*j} + 2 |\mathcal{H}|^2 \tilde{\phi} - 2 (\mathcal{H}^* \phi^i \mathcal{F}_i + h.c.) \quad (6.72)$$

Eliminating  $\mathcal{H}$  gives an expression quadratic in  $\mathcal{F}$  and hence eliminating  $\mathcal{F}$  gives no potential at all! If on the other hand we take chiral fields of different weights then one has in general

$$I = \left[ \sum_k \phi_k V_1^k \right]_D + \left[ \sum_\ell g_\ell \Sigma^\ell \right]_F \quad (6.73)$$

We now get the density

$$I = e [ 2 \bar{\Phi}_j^i \mathcal{F}_i \mathcal{F}^{*j} ] + \frac{1}{3} (\Phi + \bar{\Psi}) \hat{R} \quad (6.74) \\ - 2 (\mathcal{H}^* \bar{\Psi}^i \mathcal{F}_i + h.c.) + (\mathcal{H}^{*k} g_k^i \mathcal{F}_i - k |\mathcal{H}|^2 \mathcal{H}^{*k-1} g_k^i + h.c.)$$

Now for  $k=0$  (only) we get a constraint equation on eliminating  $\mathcal{H}$ . Namely

$$- 2 \bar{\Psi}^i \mathcal{F}_i + g_0^i \mathcal{F}_i = 0 \quad (6.75)$$

Thus  $I$  is left quadratic in  $\mathcal{F}_i$  which kill themselves and leave no potential behind. If however we take  $k=0,1$

we get on eliminating  $\mathcal{H}^*$

$$\mathcal{H} = \frac{1}{g_1^i} (-2 \bar{\Psi}^i \mathcal{F}_i + g_1^i \mathcal{F}_i)$$

so that we are left with

$$2 \bar{\Phi}_j^i \mathcal{F}_i \mathcal{F}^{*j} + g_0^i \mathcal{F}_i - \frac{1}{|g_1|^2} |(g_1^i - 2 \bar{\Psi}^i) \mathcal{F}_i|^2 \quad (6.76)$$

thus eliminating  $\mathcal{F}_i$  again gives apseudo global type of potential proportional to  $|g_0|^2$ . If  $k$  has values other than zero or one then one faces even more severe difficulties because

$$\Sigma_1 = [\mathcal{H}^*, \dots, -i D_m^c B^m - |\mathcal{H}|^2 + \frac{1}{2} B_m'^2 + \dots]$$

$$\Sigma_1^L = [\mathcal{H}^{*L}, \dots, L \mathcal{H}^{*L-1} (-i D_m^c B^m + \frac{1}{2} B_m'^2 - |\mathcal{H}|^2 + \dots)] \quad (6.78)$$

Thus

$$[g_L \Sigma_1^L]_F = g_L(a) L \mathcal{H}^{*L-1} [-i D_m^c B^m + \frac{1}{2} B_m'^2 + \dots] \quad (6.79)$$

so that the elimination of  $B_m$  creates kinetic terms for the auxiliary fields  $\mathcal{H}$ . Clearly the Breitenlohner tensor calculus offers no special possibilities of model building!

iv) The last of the single-compensating multiplet tensor calculi is the one using a vector multiplet  $V_0$  and results in the DeWitt-VanNieuwenhuizen SUGRA.

This tensor calculus introduces the peculiar features associated with using  $V_0$  multiplets. First  $V_0$  has an even number(2) of fermionic fields and hence one of them will actually propagate. This propagating fermion turns out to be the partner of physical gauge field present in the theory. Now if we try to construct a supergravity with  $V_0$  alone the only possible density is (taking, for arguments sake,  $\lambda_{V_0}=2$ ).

$$\mathcal{L}_{S.G} = -\frac{3}{2} [V_0]_D = -\frac{eR}{2} - \frac{1}{2} \bar{\psi} \cdot R + eD + \dots \quad (6.80)$$

where we have fixed  $C_0 = 1$ . But now eliminating  $D$  gives  $e=0$ . To remedy this problem one introduces  $\Sigma_0$  ( $\lambda=1$ ) and defines a transformation

$$\begin{aligned} e^{V_0} &\longrightarrow e^{-i\psi(\Lambda^*)} e^{V_0} e^{i\psi(\Lambda)} \\ \Sigma_0 &\longrightarrow e^{-2i\Lambda} \Sigma_0 \end{aligned} \quad (6.81)$$

Now

$$\begin{aligned} \mathcal{L}_{S.G} &\sim [\psi(\Sigma_0^*) e^{V_0} \psi(\Sigma_0)]_D \\ &\quad + [WW]_F \end{aligned} \quad (6.82)$$

is invariant under this transformation since

$$\psi(\Sigma_0) \rightarrow \psi(e^{-i\Lambda} \Sigma_0) = e^{-i\psi(\Lambda)} \psi(\Sigma_0) \quad (6.83)$$

Here  $G$  is the gauge submultiplet of  $V_0$

(The proof of the invariance of  $[WW]_F$  just consists of the usual demonstration that the field strength of a  $U(1)$  gauge multiplet is invariant under gauge transformations). Now that  $\mathcal{L}_{S.G}$  is no longer linear in  $D, \Lambda$  we fix the new gauge invariance by  $\Sigma_0 = [1/2, 0, 0]$ . The transformation laws for  $Q, S, \Lambda, D$  must then be modified by adding appropriate  $\Lambda$ -Gauge transformations so that they preserve this condition.

One can now fix the superconformal gauges by  $C=1, Z=0, b_m=0$ . The chiral gauge is not fixed since  $V_0$  is  $A$ -inert.

The appropriate compensating multiplet for insertion in  $D$  terms is the gauge invariant combination  $V_1 = \psi(\Sigma_0^*) e^{V_0} \psi(\Sigma_0)$  already used in constructing the pure SUGRA. With  $\Sigma_0 = [1/2, 0, 0]$

$$V_1 = [1, 0, H_0, K_0, B_m^0 + A_m, \Lambda, \quad (6.84)$$

$$D_0 + \frac{1}{2} [|\mathcal{H}_0|^2 - (B_m^0 + A_m)^2] ]$$

$$\mathcal{H}_0 = H_0 + i K_0.$$

Now

$$V_1^k = [1, 0, k H_0, k K_0, k (B_m^0 + A_m), \dots] \quad (6.85)$$

$$\frac{k^2}{2} (|H_0|^2 - (B_m^0 + A_m)^2) + k D_0 + \dots]$$

and

$$\Sigma_1(V_1) = [\frac{1}{2} H^*, i A_0 + \dots, -\frac{1}{2} D_0 - \frac{1}{2} |H_0|^2 + \frac{1}{2} (B_m^0 + A_m)^2] \quad (6.86)$$

$$-(D^c 1) - \frac{1}{2} D_m^c (B_m^0 + A_m)]$$

$$\Sigma_1^l = \frac{1}{2^l} [H^{*l}, \dots, -\frac{l}{2} H^{*l-1} (D_0 + |H_0|^2 + (B_m^0 + A_m)^2$$

$$+ D^c 1 + D_m^c (B_m^0 + A_m)]$$

If we choose zero weights for the matter fields in I then

k=1 and as in the previous case one can write the formula as

$$I = [\tilde{\phi} V_1]_D \quad \tilde{\phi} = \phi_0 + V(g_0) \quad (6.88)$$

Then

$$I = e [\tilde{\phi} D_0 + \frac{1}{2} \tilde{\phi} |H|^2 + 2 \tilde{\phi}_i^j \tilde{F}^{*j} \tilde{F}_i + \frac{1}{2} D_0^2] \quad (6.89)$$

$$+ \frac{\tilde{\phi} R}{3} + \dots]$$

where  $1/2 D_0^2$  comes from  $[WW]_F$ . Eliminating  $\mathcal{H}$

gives  $-\frac{4}{\tilde{\phi}} |\tilde{\phi}^i \tilde{F}_i|^2$  thus leaving a potential quadratic in  $\tilde{F}_i$ .

Hence after eliminating  $D_0$  and  $\tilde{F}_i$  the potential is just  $\frac{1}{2} \tilde{\phi}^2$ .

If  $l=0$  but  $k$  is arbitrary then one finds that

the potential is

$$V = \frac{1}{2} g_i [\tilde{\phi}_i^j - \textcircled{H}^{-1j}_j \tilde{\Psi}_i \tilde{\Psi}^j] \tilde{g}^{*j} + \frac{1}{2} \tilde{\Psi}^2 \quad (6.90)$$

$$\textcircled{H} = \sum_k k^2 \tilde{\phi}_k$$

As in the previous case the structure of  $\Sigma_1^\ell$  implies that for  $\ell \neq 0$  one obtains unacceptable kinetic terms from the elimination of vector fields  $B_\mu$  and  $A_\mu$ .

### Section 5: Potentials for 2 Compensating Multiplets

We now turn to the study of combinations of multiplets. Since we must have an odd number of fermionic auxiliary fields the allowed pairs are just  $(\Sigma_0, V_0)$  (3 fermions),  $(L_0, V_0)$  (3 fermions) and  $(\mathcal{L}_0, V_0)$  (5 fermions). Then one must form  $V_1$  (weight 2) and (weight 3) for use in the density formulae. If  $V_0$  has weight two then  $[\not{D}V_0]_D$  will yield a constraint on matter fields:  $e \not{D}(a)=0$  on eliminating  $D$ . Hence we take  $V_0$  to have unit weight. We also take  $\Sigma_0$  to be of unit weight. For simplicity we consider only the case with all matter fields of zero weight. Since we can form arbitrary functions of them this does not reduce the generality of our results.

Then

$$I = [\not{D}V_1]_D + [g(s)\Sigma_1]_F$$

$$V_1 = V_0^2 + \alpha V_1 (\Sigma_0, L_0 \text{ or } \mathcal{L}_0) \quad (6.91)$$

$$\Sigma_1 = \Sigma_0^3 + \Sigma_1 (V_0^2) \quad (\text{for } (\Sigma_0, V_0))$$

$$\Sigma_1 = \Sigma_1 (V_0^2) + \Sigma(\mathcal{L}_0) = \Sigma_1 (V_0^2)$$

The last equality holds for  $(L_0, V_0)$  and  $(\mathcal{L}_0, V_0)$  since  $\Sigma(L_0) = \Sigma(\mathcal{L}_0) = 0$ .

In order that the reducible conformal pairs  $(\Sigma_0, V_0)$  etc. function as irreducible auxiliary multiplets in the Poincare SUGRA one must mix them by using both to fix conformal gauges. Thus for  $(\Sigma_0, V_0)$  one chooses

$$\begin{aligned} a_0 &= 1/2 && \text{to fix D and A gauges} \\ \mu &= 0 && \text{to fix K gauge} \\ \chi_0 &= -i\gamma_5 \bar{z}_0 && \text{to fix the S gauge} \end{aligned} \quad (6.92)$$

The Poincare transformations which preserve these conditions then mix the auxiliary fields from the two sets. Similarly in the  $[L_0, V_0]$  case one uses

$$c_0^L = 0, z_0^L = 0, b_m = 0 \quad (A \text{ is not fixed}). \quad (6.93)$$

Similar considerations hold for the  $(\mathcal{L}_0, V_0)$  case. The resulting SUGRAs are named after Revielles - Taylor.

However, as we shall immediately see all this arcana is irrelevant since SUGRAs containing  $V_0$  in this linear fashion though not inconsistent (as the naive  $V_0$  construction is) are self quenching. That is all fields in  $V_0$  are automatically eliminated. To see this note that

$$\begin{aligned} V_0^2 = & [c_0^2, c_0 z_0, 2c_0 H_0 - \frac{1}{2} \bar{z}_0 z_0, 2c_0 K_0 + \frac{1}{2} \bar{z}_0 i\gamma_5 z_0, \\ & 2c_0 B_m^0 + \frac{1}{2} c_0 \bar{z}_0 i\gamma_5 \gamma_m z_0, 2c_0 \Lambda + (H_0 - i\gamma_5 K_0 + i\gamma_5 \not{D} c_0 - \not{D} c_0) z_0, \\ & 2c_0 D_0 + [H_0^2 + K_0^2 - B_m^0{}^2 - (D_m c_0)^2 - 2\bar{\Lambda}_0 z_0 - \bar{z}_0 i\not{D} z_0] \end{aligned} \quad (6.94)$$

and

$$\begin{aligned} \Sigma(V_0^2) = [C_0 H_0^*, i P_R (2 C_0 \Lambda + i (H_0 - i \gamma_5 K_0 + i \gamma_5 B_0) Z_0, \\ - \frac{1}{2} (2 C_0 D_0 + |H_0|^2 - B_{m0}^2 - (D_m^c C_0)^2 - 2 \bar{\Lambda} Z_0 \\ - \bar{Z}_0 \not{D} Z_0) ] \end{aligned} \quad (6.95)$$

The D density formula (6.31)

together with

$$\begin{aligned} \Lambda_{\varphi V_0^2} = C_{\varphi} \Lambda_{V_0^2} + C_0^2 \Lambda_{\varphi} + \frac{1}{2} (H_{\varphi} - i \gamma_5 K_{\varphi} + i \gamma_5 B_{\varphi} - \not{D} \varphi) Z_{V_0^2} \\ + \frac{1}{2} (H_{V_0^2} - i \gamma_5 K_{V_0^2} + i \gamma_5 B_{V_0^2} - \not{D} C_0^2) Z_{\varphi} \end{aligned}$$

$$\begin{aligned} D_{\varphi V_0^2} = C_{\varphi} D_{V_0^2} + C_0^2 D_{\varphi} + H_{\varphi} H_{V_0^2} + K_{\varphi} K_{V_0^2} - B_{m\varphi} B_{m V_0^2} \\ - D_m^c \varphi D_m^c C_0^2 - Z_{\varphi}^T C \Lambda_{V_0^2} - Z_{V_0^2}^T C \Lambda_{\varphi} \\ - \frac{1}{2} Z_{\varphi}^T C \not{D} Z_{V_0^2} - \frac{1}{2} Z_{V_0^2}^T C \not{D} Z_{\varphi} \end{aligned} \quad (6.96)$$

$$Z_{\varphi V_0^2} = C_{\varphi} Z_{V_0^2} + C_0^2 Z_{\varphi}$$

$$C_{\varphi V_0^2} = C_{\varphi} C_0^2$$

$$B_{\varphi V_0^2}^m = C_{\varphi} B_{V_0^2}^m + C_{V_0^2} B_{\varphi}^m + \frac{1}{2} Z_{\varphi}^T C i \gamma_5 \gamma_m Z_{V_0^2}$$

now shows that (as expected for  $\theta^3$  and  $\theta^4$  terms)  $D$ ,  $\Lambda$  occur linearly in the action with coefficients proportional to  $C_0$  and  $Z_0$  respectively. Eliminating these kills

$C_0$  and  $Z_0$  everywhere. So that effectively,

$$V_0^2 = [0, 0, 0, 0, 0, 0, |H_0|^2 - B_m^0{}^2] \quad (6.97)$$

so only  $D(\phi V_0^2)$  can contribute. Moreover, including the  $[ ]_F$  term does not help since

$$\begin{aligned} [\Sigma(V_0^2)g]_F &= C_0 H_0^* \mathcal{F}_g(s) - \mathcal{F}_{Z(V_0^2)} g(a) \\ &\quad - \frac{1}{2} \chi_{\Sigma(V_0)}^T C \chi g(s) \end{aligned} \quad (6.98)$$

is still linear in  $C_0$ ,  $D_0$  and  $\bar{A}_0 Z_0$ .

Thus one gets just

$$C_0 [ |H|^2 - |B_m^0|^2 - \frac{1}{2} (|H|^2 - B_m^0{}^2) g(a) ] \quad (6.99)$$

Since  $H_0$  and  $B_m^0$  occur only quadratically they drop out of the Lagrangian leaving no trace of  $V_0$  anywhere!! This is the afore mentioned quenching. Similarly one will find that the  $(L_0, V_0)$  and  $(\mathcal{L}_0, V_0)$  tensor calculi also self quench to the  $L_0$  and  $\mathcal{L}_0$  tensor calculi. The same reduction also occurs for  $3V_0$  multiplets for the same reason: linearity in  $D$  and  $\Lambda$  kills  $C_{0i}$  and  $Z_{0i}$  which then kills everything else. One possibility not investigated so far is of introducing the  $V_0$  via the De Wit-VanNieuwenhuizen mechanism of a weightless  $V_0$  plus an additional  $\Sigma$  allowed by a new gauge invariance. We have not yet treated this case since the consistency of this formulation has not been demonstrated.

Section 5 Potentials for 3 Compensating Multiplets:

Finally we consider sets of 3 compensating multiplets at a time.

1) Our first guess is  $(\Sigma_1, \Sigma_2, \Sigma_3)$  which contains 3 fermionic  $(6 + 6)$  scalar fields. The theory with  $\varphi = 1$   $g = 0$  should reduce to a pure SUGRA so none of the fields in the  $\Sigma_i$  should propagate. Now with unit weight for each  $\Sigma_i$  if we take

$$V_1 = a_{ij} \Sigma_i \Sigma_j^* \quad (6.100)$$

then we will obviously obtain kinetic terms for some of the fields in the  $\Sigma_i$  even after using the S.C. gauge fixings to eliminate some of them. The appropriate choice of  $a_{ij}$  is

$$V_1 = \Sigma_1 \Sigma_1^* + \Sigma_2 \Sigma_2^* \quad (6.101)$$

Fixing A,D,S,K via

$$A_1 = 1 \quad B_2 = 0 \quad \psi_2 = \psi_3 \quad (6.102)$$

still leaves us with  $B_1, \psi_1, \mathcal{F}_1, A_2, \mathcal{F}_2, A_3, \mathcal{F}_3$

Now if we introduce

$$\Sigma_0^{\lambda=3} = \Sigma_3 (\Sigma_1^2 \alpha + \Sigma_2^2 \beta + \gamma \Sigma_1 \Sigma_2) \quad (6.103)$$

$$\begin{aligned}
 [\Sigma_0^{\lambda=3}]_F &= 2 (a_1 \mathcal{F}_1 \alpha + a_2 \mathcal{F}_2 \beta) a_3 \\
 &\quad + \mathcal{F}_3 (\alpha a_1^2 + a_2^2 \beta + \gamma a_1 a_2) \\
 &\quad + \psi_3^T C [2 \psi_1 a_1 \alpha + 2 \psi_2 a_2 \beta + \gamma \psi_1 a_2 + \gamma \psi_2 a_1] \\
 &\quad + a_3 (\psi_1^T C \psi_1 \alpha + \psi_2^T C \psi_2 \beta + \psi_2^T C \psi_1 \gamma)
 \end{aligned}$$

and  $A_3$  are now Lagrange multipliers and eliminate themselves ( 8 fields) and also give 8 equations

$$\begin{aligned}
 a_1^2 \alpha + a_2^2 \beta + \gamma a_1 a_2 &= 0 \\
 2 \psi_1 a_1 \alpha + 2 \psi_2 a_2 \beta + \gamma \psi_1 a_2 + \gamma \psi_2 a_1 &= 0 \quad (6.105) \\
 \psi_1^T C \psi_1 \alpha + \psi_2^T C \psi_2 \beta + \psi_2^T C \psi_1 \gamma &= 0
 \end{aligned}$$

which rid us of 8 more fields. Along with the gauge fixing constraints (6 in all) we have done away with 22 fields leaving two auxiliaries. This is the same as the Old Minimal case.

Similar results hold for other triplets such as  $(\Sigma_1, \Sigma_2, \mathcal{L}_0)$ ,  $(\Sigma_0, \mathcal{L}_0, \mathcal{L}_0)$  and  $(\Sigma_1, \Sigma_2, \mathcal{L}_0)$ . Thus one extends the above trick for preventing auxiliary field propagation and as a result is left with one of the single multiplet tensor calculi. Thus for the  $(\Sigma_1, \Sigma_2, \mathcal{L}_0)$  tensor calculus one writes  $(\Sigma_1, \Sigma_1^*)_D$  but only  $(\Sigma_1, \Sigma_2)_F$  so that  $\Sigma_2$  is a Lagrange multiplier and kills  $\Sigma_1$  leaving the  $\mathcal{L}_0$  tensor calculus. In like fashion, tensor calculi  $(\mathcal{L}_1, \mathcal{L}_2, X \mid X = \mathcal{L}_3, \mathcal{L}_0, \Sigma_0)$  reduce to  $X$  when  $\mathcal{L}_1$  and  $\mathcal{L}_2$  are embedded as

$$V_1 = \mathcal{V}(\mathcal{L}_1) \times \mathcal{V}(\mathcal{L}_2^*) + \text{h.c.}$$

Thus we see that in all known SUGRYs the potential reduces to either the Old Minimal form or a 'pseudo global' one which is of little interest from the point of view of model building. One possible exception to this may be a hybrid DeWit-VanNieuwenhuizen-Old Minimal tensor calculus which, however, has as yet not been shown to be a consistent SUGRY.

In fairness we should mention that the authors of ref.(28) have shown that if one performs a point transformation on all matter fields to render them weightless then the  $L_0, V_0, \mathcal{L}_0$  tensor calculi yield results identical to the Old Minimal case with certain extra invariances imposed upon of the Old Minimal superpotential.

## Chapter 7

### SUGRY and Scale Induction

Introduction: In previous chapters we have seen how an intermediate scale  $O(10^{10}-10^{12}\text{GeV})$  enters naturally into supersymmetric theories of several types. The identification of  $M_R$  and  $M_S$ , cosmological constraints on the Witten mechanism, and on the gravitino and s-matter masses as well as the program of the induction of the weak scale by SUSY breaking effects all require the introduction of such a scale.

Moreover, we saw that the unification scale in the Dimopoulos-Raby⁽²¹⁾ model or in a similar model based on  $SO(10)$  proposed by Kalara and Mohapatra⁽²⁷⁾ necessarily have  $M_U \sim 1/\kappa$ , from constraints on  $\sin^2\theta_W$ ,  $\alpha_3(M_W)$ . Thus it is natural to enquire to the effects of embedding an O'Raifeartaigh superpotential with gauge non-singlets, which we call a Witten superpotential, into a SUGRY theory.

In this chapter we shall explore the consequences of such an embedding. We shall find that SUGRY effects determine the GUT scale at tree level to be  $O(1/\kappa)$  by fixing the undetermined v.e.v's of the global limit (one of which is a gauge non-singlet) to be of that order of magnitude⁽⁴⁴⁾. We derive the low energy effective potential (before scaling) and find that it is essentially that of a  $G_{123}$  supersymmetric gauge theory plus soft

breaking terms. The low energy potential has been previously computed by Ovrut and Raby⁽⁴⁴⁾ under a special assumption. Our demonstration of decoupling does not make this assumption but yields essentially the same result.

In section 2 we discuss the embedding of a gauge singlet superpotential of the OR type in  $N=1$  SUGRA and show how all v.e.v. are determined. In section 3 we extend the analysis to the gauge non-singlet  $SU(5)$  case and demonstrate the induction of the GUT scale rigorously. In section 4 we introduce the electroweak Higgs doublets  $(5 + \bar{5})$  and trace the derivation of the low energy effective potential.

In sections 5-8 we consider the construction of  $SO(10)$  geometric hierarchy models and their embedding in SUGRA theories. In section 9 we indicate how  $\sin^2 \theta_W$  and  $\alpha_3(M_W)$  constrain these models.

## Section II Singlet OR Potentials and OM SUGRA:

The potential in OM SUGRA is given^(29,35,45) by

$$V = E(z) [ |y_i|^2 - 3\kappa^2 |g(z)|^2 ]^2 \quad (7.1)$$

where  $K$  is the Planck length,  $g$  the superpotential

$$\begin{aligned} y^i &= \partial g / \partial z_i + \kappa^2 z^{\dagger i} g \\ E(z) &= e^{\kappa^2 |z_i|^2} \end{aligned} \quad (7.2)$$

and  $i$  runs over all the scalar fields ( $Z_i$ ) of the theory. Our  $\mathcal{L}$  differs from the usual one by a factor of 2.

We take as our superpotential

$$g = \alpha_1 X(A^2 - M^2) + \alpha_2 Z A^2 + C \quad (7.3)$$

where  $X, A, Z$  are fields and  $C$  a constant of mass dimension

3. Then in the global limit the potential is minimized by

$$\langle \alpha X + Z \rangle = 0 \quad ; \quad \alpha = \alpha_1 / \alpha_2 \quad (7.4)$$

$$\langle A^2 \rangle = \frac{\alpha^2 M^2}{1 + \alpha^2} \quad (7.5)$$

On the other hand, if one substitutes eqn. (7.3) into (7.1) and makes the following rescalings

$$x = \kappa X \quad z = \kappa Z \quad a = A M^{-1} \quad v = \frac{C \kappa}{\alpha_2 M^2} \quad (7.6)$$

then one gets

$$V = \alpha_2^2 M^4 E U \quad (7.7)$$

$$U = |\alpha(a^2 - 1) + x^\dagger \tilde{g}|^2 + |a^2 + z^\dagger \tilde{g}|^2 + |2a\delta + a^\dagger \epsilon \tilde{g}|^2 - 3|\tilde{g}|^2 \quad (7.8)$$

where

$$\begin{aligned} \epsilon &= \kappa M \\ \tilde{g} &= (\alpha x + z) a^2 + v - \alpha x \\ E &= \exp(|x|^2 + |z|^2 + \epsilon^2 a^2) \\ \epsilon \delta &= \alpha x + z \end{aligned} \quad (7.9)$$

Our analysis is based on the assumption that

- i) CP is conserved and all vacuum expectations are real ( Actually, most of our conclusions carry through even when complex v.e.v. are allowed as a possibility).
- ii)  $\epsilon \ll 1$
- iii) All the dimensionless scaled quantities are  $O(1)$  or smaller at the minimum of the potential. This assumption can attain its justification only by self-consistency when the v.e.v.'s are actually determined, since the rescalings (7.6) are empty of any necessity other than our expectations of the sizes of the v.e.v. based on the nature of the limit.

Thus  $\epsilon$  is the parameter in which we expand our equations. Substituting into (7.8) one gets, to leading order in  $\epsilon$ ,

$$U = (\alpha^2 |a^2 - 1|^2 + |a|^4 + 4|a|^2 \delta^2 + \ell |x^\dagger \tilde{\nu}|^2 - 2\alpha \operatorname{Re} x^\dagger \tilde{\nu} - 3|\tilde{\nu}|^2) \quad (7.9)$$

where

$$\tilde{\nu} = \nu - \alpha x \quad \ell = (1 + \alpha^2)$$

Note that to leading order 'a' does not couple to 'x'.

Moreover, minimizing with respect to  $\delta$  gives  $\delta = 0$  (if

$a=0$ ). Thus it is permissible to expand around the global solution for  $a$ . As we shall see (Section 5) this is not always the case.

Now,

$$V = [e^{|\chi|^2 \ell} U(x, a) + O(\epsilon)] \alpha_2^2 M^4 \quad (7.11)$$

and thus one immediately recovers, on minimizing  $V$  to leading order in  $\epsilon$  the global values

$$|a|^2 = \frac{\alpha^2}{\ell} \quad S = 0 \quad (7.12)$$

thus

$$U = |\chi^t \bar{\ell} \nu - \alpha / \sqrt{\ell}|^2 - 3|\tilde{\nu}|^2 + O(\epsilon) \quad (7.13)$$

this is a remarkable result for by the redefinitions

$$y = \chi \sqrt{\ell} \quad \bar{\alpha} = \frac{\alpha}{\sqrt{\ell}} \quad \omega = \frac{\nu}{\alpha} \quad \tilde{\omega} = \frac{\tilde{\nu}}{\alpha} = \omega - y \quad (7.14)$$

We get,

$$V = \alpha_2^2 M^4 \bar{\alpha}^2 [e^{|\gamma|^2 \ell} \{ |y^t \tilde{\omega} - 1|^2 - 3|\tilde{\omega}|^2 \} + O(\epsilon)] \quad (7.15)$$

but this is precisely the canonical form of the Polony potential! ⁽⁴⁶⁾*

Thus we see that to leading order in  $\epsilon$  this O'Raifeartaigh potential is equivalent to the Polonyi one.

*Srednicki has shown that this type of reduction occurs quite generally. ⁽⁴⁷⁾

Minimizing with respect to  $y$  one gets**

$$(y\tilde{\omega}-1)(y^3-y^2\omega+2\omega)=0 \quad (7.16)$$

Thus either

$$y = [\omega \pm (\omega^2-4)^{1/2}]/2 \quad (7.17)$$

or  $y^3 - y^2\omega + 2\omega = 0$

hence if  $\omega^2 < 4$  the real solution is

$$y = \left[ -\frac{e}{2} + \left[ \frac{e^2}{4} + \frac{d^3}{27} \right]^{1/2} \right]^{1/3} - \left[ \frac{e}{2} + \left[ \frac{e^2}{4} + \frac{d^3}{27} \right]^{1/2} \right]^{1/3} \quad (7.18)$$

where

$$e = \frac{2\omega}{27} (27 - \omega^2) \quad d = -\frac{\omega^2}{3}$$

We shall now specialize to the case where the vacuum energy is assumed to be tuned to zero order by order in  $\epsilon$  by adjusting  $C$ . While this is clearly an extremely ugly procedure one may consider this to be merely a property that any 'effective' potential must have in order to represent the unknown dynamical mechanism that screens out the vacuum energy with such incredible accuracy.

**The conclusions to  $O(\epsilon)$  are not dependent on assumption(i).

Thus one gets to leading order

$$y = \pm(\sqrt{3}-1) \quad \omega = \mp(2-\sqrt{3}) + O(\epsilon) \\ \tilde{\omega} = \mp 1 \quad (7.19)$$

Furthermore, since

$$U_{aa} = 8\alpha^2 \quad U_{ss} = \frac{8\alpha^2}{\ell} \quad U_{as}; U_{xs}; U_{ax} \sim O(\epsilon) \quad (7.20)$$

so that  $\text{Det}(U_{ij}) > 0$ .

It follows that

$$a^2 = \alpha^2/\ell \quad x = \pm(\sqrt{2}-1)/\sqrt{\ell} \quad v = \mp \frac{(2-\sqrt{3})\alpha}{\sqrt{\ell}} \\ \delta = O(\epsilon) \quad (7.21)$$

indeed represents the minimum which reduces to the global case as  $k \rightarrow 0$ . It remains to fix  $\delta$ . One easily finds that at next to leading order the contributions involving  $\delta$  are

$$U_1 = 2\delta \epsilon (\alpha/\sqrt{\ell})^3 (\tilde{\omega} y^2 - 2y) = \frac{74\alpha^2 \delta \epsilon \alpha}{\sqrt{\ell}} \quad (7.22)$$

thus,

$$\delta = \pm \frac{\epsilon \alpha}{2\sqrt{\ell}} + O(\epsilon) \quad (7.23)$$

It is straightforward to extend the perturbation to higher orders. For example, the only other  $O(\epsilon^2)$  terms are just  $\epsilon^2 \alpha^2 \tilde{v}^2$  and are the first coupling between 'a' and 'x'. We shall not belabor this point here, but will derive the expansion in the gauge case.

Section III Generalization to SU(5):

We now show how the grand unifying scale may be induced by carrying over the previous results to SU(5).

We take as the superpotential

$$g = \alpha_1 X(\text{tr} A^2 - M^2) + \alpha_2 \text{tr} A^2 Z + C \quad (7.24)$$

where  $X$  is a singlet  $A$ ,  $Z$  are complex 24's of SU(5) and  $C$  is a constant. In the global limit one finds that this potential breaks supersymmetry and SU(5) at the scale  $M$ .

Breaking along one of the two maximal sub group directions

$$\begin{aligned} G_{123} &= U(1) \times SU(2) \times SU(3) & : & \langle A \rangle = \alpha M \Lambda_3 \\ G_{14} &= U(1) \times SU(4) & : & \langle A \rangle = \alpha M \Lambda_4 \end{aligned} \quad (7.25)$$

$$\Lambda_n = \text{Diag} (m \mathbb{I}_n, -n \mathbb{I}_m)$$

causes the D term contribution to vanish. The linear combination

$$\alpha_1 x + \alpha_2 (m-n) z = 0$$

where

$$\langle Z \rangle = \Lambda_n z / \kappa \quad \langle X \rangle = x / \kappa \quad (7.26)$$

but 'x', 'z' are not determined. Moreover, the  $G_{123}$  vacuum is chosen since it has a lower (positive) energy than the  $U(1) \times SU(4)$  vacuum. We now rewrite the SU(5) L-SUSY problem so that one can carry over, mutatis mutandis, the results of the previous section for the gauge singlet case. We define

$$\delta C = (\alpha x + e z) \eta \quad (7.27)$$

where

$$\eta = 5nm \quad \rho = m - n \quad \ell = \rho^2 + \alpha^2 \eta$$

$$\nu = C\kappa / \alpha_2 M^2 \quad (7.28)$$

The potential becomes

$$V = \alpha_2^2 M^4 E U \quad (7.29)$$

$$U = \alpha^2 |\eta \alpha^2 - 1|^2 + \eta \alpha^4 \rho^2 + \alpha^2 \tilde{\nu}^2$$

$$- 2\alpha \alpha \tilde{\nu} + \frac{\eta}{\rho^2} \alpha^2 \alpha^2 \tilde{\nu}^2 + 4\alpha^2 \delta^2 \eta$$

$$- 3\tilde{\nu}^2 + O(\delta \epsilon) + O(\epsilon^2)$$

where

$$E = e^{\alpha^2 \ell} + O(\epsilon)$$

the minimum of this potential is, to leading order, and when  $V_{\min}$  is adjusted to zero:

$$a_0^2 = \alpha^2 / \ell \quad \alpha_0 = \pm \frac{1}{\sqrt{\ell}} (\sqrt{3} - 1) + O(\epsilon) \quad (7.30)$$

$$\delta = 0 + O(\epsilon) \quad \nu_0 = \frac{7\alpha}{\sqrt{\ell}} (2 - \sqrt{3}) + O(\epsilon)$$

Using  $a_0^2 = \alpha^2 / 1 + O(\epsilon)$  the potential at the minimum becomes (compare (7.13))

$$V = e^{\alpha^2 \ell} \left[ \left( 6\alpha \tilde{\nu} \sqrt{\ell} - \frac{\alpha}{\sqrt{\ell}} \right)^2 - 3\tilde{\nu}^2 \right] + O(\epsilon) \quad (7.31)$$

furthermore, by including the next order terms

containing  $\delta$  one finds (compare 7.23 !)

$$\delta = \alpha \alpha_0 \eta e^{\frac{(\sqrt{3}+1)}{4}} = \pm \frac{e \alpha \eta}{2\sqrt{2}} \quad (7.32)$$

As in section II), these v.e.v. represent the minimum which reduces to the global one as  $K \rightarrow 0$ . Thus we have shown that under the above stated assumptions, OM SUGRA with an O'Raifeartaigh-Witten Superpotential induces an SU(5) breaking scale at the tree level through the v.e.v.

$$\begin{aligned} \langle Z_0 \rangle &= -\alpha \alpha_0 / K + O(m_{3/2}) \\ \alpha_0 &= \pm (\sqrt{3}-1) / \sqrt{2} + O(e^2) \end{aligned}$$

The mass spectrum in this model is that of the 'geometric hierarchy' model of Dimopoulos and Raby. The presence of a light SU(2) triplet and SU(3)_c octet forces the unification scale to be  $O(M_p)^{(21)}$ . Hence,

$$\alpha = \alpha_1 / \alpha_2 \gtrsim 1$$

On the other hand, one of the chief virtues of naive OM SUGRA models is their induction of uniform masses in the squark-slepton sector. This carries over to the Witten type of model with the common mass given by

$$\begin{aligned} m_{3/2}^2 &= e^{\frac{K^2 \langle |Z_i|^2 \rangle}{K^4 |\langle g \rangle|^2}} = (\alpha^2 / e) (KM^2)^2 e^{K^2 \langle |Z_i|^2 \rangle} \\ &= e^{(\sqrt{3}-1)^2} \frac{\alpha_1^2 \alpha_2^2}{\alpha_2^2 + 30\alpha_1^2} (KM^2)^2 \quad (7.33) \end{aligned}$$

The possibility of GUT scale induction by O'Raifeartaigh models in this manner has been suggested by Srednicki,⁽⁴⁷⁾ who took  $\alpha \sim 10^{-3}$ .

Before going further we can estimate the difference of the  $SU(4) \times U(1)$  vacuum energy from the  $SU(3) \times SU(2) \times U(1)$  case (which we have fixed to be zero). If we assume that the v.e.v. are real then we have, as before,

$$\begin{aligned} a^2 &= \alpha^2 / l_4 + O(\epsilon^2) \\ V &= \alpha_2^2 M^4 \left[ e^{12 l_4^2} \frac{\alpha^2}{l_4} [1 y^t \tilde{\omega}_4 - 1]^2 - 3 |\tilde{\omega}|^2 \right] \end{aligned} \quad (7.34)$$

where  $\omega_4 = \sqrt{l_4} / \alpha$  and we have already fixed  $\nu$ :

$$\omega_3 = \frac{\nu \sqrt{l_3}}{\alpha} = 7(2 - \sqrt{3}) + O(\epsilon) \quad (7.35)$$

so that the vacuum energy in the  $G_{123}$  phase vanishes.

With  $\alpha \sim O(1)$  one finds  $|\omega_4| \sim 10^{-1}$  and then (7.34, 7.35) show that

$$V_4 \approx 10^{-1} \alpha_1^2 M^4$$

so that the  $SU(4) \times U(1)$  vacuum lies higher than the  $SU(3) \times SU(2) \times U(1)$  vacuum, just as in the global case. Thus the scale of vacuum splitting is  $O(10^{-1})$ . Since the next order contributions to the splitting will be  $O(\epsilon V_4)$  it is clear that the choice of vacuum state is stable.

Section IV Decoupling in SU(5) Models:

Let us now turn to the induction of the Electro-weak scale. In order to keep the Electro-weak doublets light, one might attempt to implement the 'sliding-singlet',⁽⁴⁸⁾ mechanism. However, we find that this mechanism does not work.

Let us first see how this comes about. One introduces additional couplings in the superpotential

$$g = \lambda_1 \bar{H} A H + \lambda_2 \bar{H} S H$$

where  $H, \bar{H}, S$  transform as  $5, \bar{5}, 1$  under  $SU(5)$ .

In the global limit  $F_X$  and  $F_Y$  are unchanged, while

$$-F_{\beta}^{\dagger \alpha} = \frac{2\alpha M}{\kappa} (\alpha_1 x + \alpha_2 y) \Lambda_{\beta}^{\alpha} + \lambda_1 (\bar{H}^{\alpha} H_{\beta} - \frac{1}{5} \delta_{\beta}^{\alpha} \bar{H} H) \quad (7.37a)$$

$$-F_H^{\dagger} = (\alpha_1 \alpha M \Lambda + \lambda_2 S) H \quad (7.37b)$$

$$-F_H^{\dagger} = \bar{H} (\lambda_1 A \Lambda + \lambda_2 S) \quad (7.37c)$$

$$-F_S^{\dagger} = \lambda_2 \bar{H} H \quad (7.37d)$$

We see that if  $F_S$  did not force  $\bar{H} H = 0$  then, when

$$\langle \bar{H}_{\alpha} \rangle = \langle H^{\alpha} \rangle = H_0 \delta_{\alpha}^{\alpha} \quad (7.38)$$

and  $\Lambda = \Lambda_3 (\Lambda_4)$ , the singlet  $S$  would slide to a value  $\frac{3\lambda_1 \langle A \rangle}{\lambda_2} \left( \frac{4\lambda_1 \langle A \rangle}{\lambda_2} \right)$  such that  $F_H, F_{\bar{H}}$  vanished,

thus removing a positive contribution to the potential. Furthermore, the color triplets would simultaneously receive masses  $O(\Lambda)$ . Since  $\langle H \rangle = \langle \bar{H} \rangle$  ensures that the contribution to the vacuum energy coming from the gauge auxiliary scalar term  $1/2 D^2$  vanishes it is a plausible ansatz to make. One may remove the constraint that  $H, \bar{H}$  vanish by inserting a term  $-SM_W^2$  into the superpotential. It has been suggested that coupling the theory to supergravity⁽³⁵⁾ obviates the necessity of introducing an additional scale by hand since  $F_S$  may no longer force  $H$  to vanish. However, we find that the common diagonal contribution to the scalar mass matrix has the effect of making it energetically unfavorable for the solution

$$\langle S \rangle = \frac{n\lambda_1}{\lambda_2} \langle A \rangle \quad \langle \bar{H} \rangle_\alpha = \langle H_\alpha \rangle = H_0 \delta_\alpha^5 \neq 0 \quad (7.39)$$

to occur. This is even more true for models where  $\langle A \rangle$  is  $O(M_U)$ . Dine⁽⁵⁰⁾ has shown that a similar suppression of the  $S=0$  pattern takes place due to quantum effects. Realistic models of this type need at least two  $5 + \bar{5}$  pairs in order to avoid ultra fast proton decay via operators of dimension 5⁽²¹⁾. Thus in general the appropriate terms for the low energy superpotential are

$$g = \lambda_3 (\bar{H}_1 A H_1 + M' \bar{H}_1 H_1) + \lambda_4 (\bar{H}_1 A H + M' \bar{H}_1 H) \\ + h_{ij}^U (10)_i (10)_j \bar{H} + h_{ij}^d (10)_i (\bar{5})_j H \quad (7.40)$$

Here  $(10+\bar{5})_c$  are the matter multiplets. Note the absence of the couplings  $\bar{H}H, \bar{H}_1 H_1$  as also the absence of matter couplings for  $\bar{H}_1$  and  $H_1$ . As explained in chapter 5 this is necessary in order to avoid ultrafast proton decay via dimension 5 operators involving  $\bar{H}_3-H_3$  mass insertions of  $O(M)$  (3 denotes color triplet). It can be shown that radiative corrections to such masses are at most  $O(M_S^2/M_U)$ .

We also see that the matter fields do not couple to the SUSY breaking sector  $(A, X, Z)$ . Furthermore, since experimental constraints force their v.e.v if any, ⁽⁴⁹⁾, to be tiny they do not affect the discussion of symmetry breaking patterns. With the addition of these new terms  $F_X$  and  $F_Z$  are unchanged but

$$-F_A^{\dagger\alpha}{}_{\beta} = 2\alpha X A^{\alpha}_{\beta} + 2\alpha_2 (ZA - \frac{1}{5} \bar{U}ZA)^{\alpha}_{\beta} + T^{\alpha}_{\beta} \quad (7.41a)$$

$$T^{\alpha}_{\beta} = \lambda_3 \bar{H}^{\alpha} H_{1\beta} + \lambda_4 \bar{H}_1^{\alpha} H_{\beta} - 1/5 (\lambda_3 \bar{H}H_1 + \lambda_4 \bar{H}_1 H)$$

$$-F_{H_1}^{\dagger} = \lambda_4 (A+M') H \quad -F_{H_1}^{\dagger} = \lambda_3 \bar{H} (A+M') \quad (7.41b)$$

and similar equations for  $F_H$  and  $F_{\bar{H}}$ .

In order to prevent the Electro-weak doublets in  $H$  and  $\bar{H}$  from getting masses  $O(M)$  one fine tunes  $\tilde{m} = \langle A \rangle + M'$  for the  $G_{123}$  breaking pattern of  $\langle A \rangle$  to be  $O(m_{3/2})$ . For simplicity we assume  $\tilde{m}$  to be zero. This

goes with the assumption that  $\langle A \rangle$  has the pattern:  $\text{diag}(2 \mathbb{1}_3, -3 \mathbb{1}_2)$ . Since the SUSY breaking part of the global potential  $(|F_X|^2 + |F_Z|^2)$  has this pattern as its absolute minimum we regard this as a plausible simplifying assumption. However, we expect our results to be essentially unchanged even if we allow  $\langle A \rangle$  to break  $SU(2)$  by small shifts in its  $SU(2)$  sector.

Then it is clear that if  $F_A$  is to vanish (for the general case  $T=0$ )  $\langle Z \rangle$  must shift so as to compensate for the presence of  $T$ . Thus we take

$$\langle Z \rangle = \bar{Z} \text{Diag} (2 \mathbb{1}_3, -3+d\epsilon, -3-d\epsilon) \quad (7.42)$$

Furthermore, assuming that  $\langle H \rangle, \langle \bar{H} \rangle$  do not break color or charge

$$T^\alpha_\beta = \text{Diag} \left( -\frac{T}{5} \mathbb{1}_4, \frac{4T}{5} \right) \quad (7.43)$$

where  $T = \lambda_3 \bar{H} H_1 + \lambda_4 \bar{H}_1 H$

Thus globally the vanishing of  $F_A$  implies that

$$(\alpha_1 X - \alpha_2 \bar{Z}) \bar{A} = T/20 \quad (7.44)$$

and

$$d = T / 12 \alpha_2 \bar{Z} \bar{A} \quad (7.45)$$

This shift in the v.e.v. of the heavy supermultiplets ensures not only their propagating components  $(A, \psi)$  but also their auxiliary fields  $F$  decouple from the low energy theory.

An argument due to Dine⁽⁵⁰⁾ clarifies the

matter. If the low energy theory is to look approximately supersymmetric it should be possible to integrate out the heavy fields so that the low energy potential has the form

$$V = \sum_{\text{light}} |F_\phi|^2 + \sum_\alpha \frac{1}{2} D_{\text{light}}^{\alpha^2} \quad (7.46)$$

Let  $A$  stand generically for the heavy fields. Then after minimization and diagonalization of mass matrices the superpotential has the form

$$g = m_{kl} A_k A_l + \lambda_{ijk} \phi_i \phi_j A_k \quad (7.47)$$

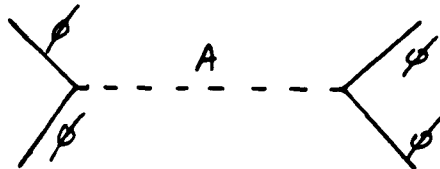
the auxiliary fields

$$-F_{A_k}^\dagger = m_{kl} A_l + \lambda_{ijk} \phi_i \phi_j \quad (7.48)$$

may be expected to give rise to quartic light couplings from

$$|F_{A_k}|^2 = \lambda_{ijk} \lambda_{lmk}^* \phi_i \phi_j \phi_l^* \phi_m^* + m^2 |A_k|^2 + (m_k \lambda_{ijk} A_k^* \phi_i \phi_j + \text{h.c.}) \quad (7.49)$$

However, when one sums up tree level graphs with only light externals and heavy intermediates



one finds that one obtains a contribution which precisely cancels the would-be quartic coupling. An equivalent way of stating the same requirement of decoupling has been emphasized by the authors of refs.35,39 in the context of

SUGRY models. One solves the extremal equations for the heavy fields  $A_k$  for  $A_k$  as a function of the light fields and evaluates the potential obtained by substituting for  $A$ . The heavy fields (including auxiliaries) should then decouple. In the global case it is easy to see how this happens since if the supergap is to be  $O(M_W)$  the matter fields cannot couple directly to fields whose  $F$  components have non-zero v.e.v  $O(M_S^2)$ . The only possible contributions are from fields with  $F = 0$ : but this removes  $F^2$  from the Classical potential! (and Dine's argument shows that there will be no light Quantum quartics associated with the Heavy field  $F$ s). The authors of ref.(39) have shown that this decoupling continues to hold good in SUGRY theories in which SUSY is broken at an intermediate scale by a 'hidden' sector which communicates with the matter sector only via SUGRY effects. In contrast to this the O'Raifeartaigh model considered by us couples  $X, Z$  to  $A$  which couples to the low energy fields. Thus one may ask whether the decoupling still goes through. A partial answer to this question has been previously given by Ovrut and Raby⁽⁴⁴⁾. They set  $T=0$ . But with this substitution along with the fine tuning mentioned earlier the decoupling becomes completely trivial since the moot effects due to the coupling between  $T$  and  $A$  have been removed.

In this section we show that even if one does not

set  $T=0$  one still obtains the expected decoupling.

In what follows we shall assume the breaking patterns discussed above and replace  $(a^2, z^2, \alpha^2, c^2)$  by  $1/30(a^2, z^2, \alpha^2, c^2)$  to remove spurious large numbers from our calculation. We denote

$$\begin{aligned} \beta &= 6M^2\epsilon = [ |H|^2 + |\bar{H}|^2 + |H_1|^2 + |\bar{H}_1|^2 ] \\ t &= T/M^2\epsilon \end{aligned} \quad (7.50)$$

The scalar potential is

$$\begin{aligned} V &= \frac{\alpha^2 M^4}{30} EU, \quad E = e^{\kappa^2 |Z_c|^2} \\ U &= \alpha^2 |a^2 - 1|^2 + a^4 + 4a^2\delta^2 - 12a\delta\epsilon t \\ &\quad + 123a d \epsilon t + \frac{12}{5} a^2 z^2 d^2 \\ &\quad + 2(3a^2\delta\epsilon - 3a\epsilon^2 t - \alpha x)(\nu - \alpha x + a^2\delta\epsilon) \quad (7.51) \\ &\quad + (a^2\epsilon^2 + x^2 + (\alpha x - \delta\epsilon)^2 + (1 + d^2\epsilon^2/15) + \kappa^2 B - 3) \\ &\quad (\nu - \alpha x + a^2\delta\epsilon)^2 \\ &\quad + 24\epsilon^2 t^2 \end{aligned}$$

Note that the natural mass scale for  $T$  is  $O(M^2\epsilon)$  due to the term  $12A(\alpha X - Z)T$ , hence we have scaled  $t = T/M^2\epsilon$ .

One can minimize with respect to  $d$  and obtain

$$d = \frac{-5t}{2\alpha\beta} \epsilon + O(\epsilon^2) \quad (7.52)$$

so that

$$12 a_3 d e t + \frac{12}{5} a^2 z^2 d^2 = -15 e^2 t \quad (7.53)$$

We are left with

$$\begin{aligned} U = & a^2(a^2-1) + a^4 + 4a^2\delta^2 - 12a\delta e t \\ & + 2[3a^2\delta e - a\chi + 3ae^2t][v - a\chi + a^2\delta e] \\ & + [a^2e^2 + \chi^2 + (a^2\chi^2 - 2a\chi\delta e + \delta^2e^2)(1+d^2e^2/15) \\ & \quad + e^3\delta - 3] \quad (7.54) \\ & [v - a\chi + a^2\delta e]^2 + 9t^2e^2 \end{aligned}$$

Notice that at this point one has both quadratic and quartic light Higgs terms. If one sets  $t=0$  one gets just the potential of section (3) (apart from  $m_{3/2}^2 H_1^2$  terms) and the derivation of the low energy potential is trivial. Since there is no condition which enforces this before decoupling we proceed to minimize with respect to 'x', 'a', and solve for them in terms of t and b by expanding

$$\begin{aligned} \chi &= \chi_0 (1 + e\chi_1 + e^2\chi_2 + \dots) \\ \delta &= a\chi_0 (\delta_1 + e\delta_2 + \dots) \\ a &= a_0 (1 + ea_1 + \dots) \\ v &= v_0 (1 + ev_1 + \dots) \end{aligned} \quad (7.55)$$

We also impose

$$\langle v \rangle = 0$$

Then along with the values obtained previously:

$$S_1 = 0 \quad a_0 = \frac{\alpha}{\sqrt{1+\alpha^2}} \quad x_0 = \frac{\sqrt{3}-1}{\sqrt{1+\alpha^2}} \quad v_0 = \frac{\alpha(\sqrt{3}-2)}{\sqrt{1+\alpha^2}} \quad (7.56)$$

One obtains from the solutions to  $O(\epsilon)$

$$a_1 = 0 \quad x_1 = 0 \quad v_1 = 0 \quad S_3 = 0$$

$$a_2 = -\frac{(\sqrt{3}-1)}{4(1+\alpha^2)} \left[ 3t^2 + \frac{\alpha^2}{1+\alpha^2} \right]$$

$$x_2 = +\frac{(\sqrt{3}+1)}{4(1+\alpha^2)} \left[ 3t^2 + \frac{\alpha^2}{1+\alpha^2} \right]$$

$$S_2 = \alpha x_0 \frac{(\sqrt{3}+1)}{4} \left( 1 + \frac{3t^2(1+\alpha^2)}{\alpha^2} \right) \quad (7.57)$$

$$S_4 = \alpha x_0 \left[ -\frac{9}{8} \frac{(1+\alpha^2)}{\alpha^2} t^2 + \frac{(3-3\sqrt{3})t}{16\alpha^2} + \frac{(1-\sqrt{3})v_2}{4(1+\alpha^2)} + \frac{3-\sqrt{3}}{16(1+\alpha^2)} \right]$$

by substituting these values back into  $U$  one obtains

$$U = \frac{4\sqrt{3}-6}{1+\alpha^2} \alpha^2 v_2 \epsilon^2 + O(\epsilon^3) \quad (7.58)$$

So that all  $t^2$  and  $t$  terms decouple to this order. Also

$$v_2 = 0 \quad (7.59)$$

Finally on going to  $O(\epsilon^3)$  one finds that

$$S_5 = \alpha x_0 \left[ \frac{1-\sqrt{3}}{4} v_3 + \frac{(3+\sqrt{3})}{24} b \right] \quad (7.60a)$$

The potential is now

$$U_3 = \left[ \frac{6\alpha^2}{1+\alpha^2} + \frac{6-4\sqrt{3}}{1+\alpha^2} \alpha^2 \nu_3 \right] \epsilon^3 \quad (7.61)$$

Had we kept the term arising from the fine tuning of  $\langle A \rangle + M'$  as being order  $O(m_{3/2})$  we would also have obtained a contribution to  $U_3$  proportional to  $T$  but with an arbitrary coefficient of  $O(1)$ . If large enough this could lead to  $SU(2) \times U(1)$  breaking but then the natural scale for that breaking would be  $O(M)$  or larger since the positive definite quartics due to the gauge terms are zero in the direction where  $\bar{H}_1 H, \bar{H} H_1$  are most negative.

Thus we have demonstrated the decoupling to be expected on general grounds. Note that with  $t=0$  this is certainly not a trivial matter as one can see from the fact that  $x_2, z_2$  etc., all depend on  $t$ .

The low energy potential for the Higgs doublets may now be easily derived since apart from the term

$$m_{3/2}^2 B = e^{\frac{4-2\sqrt{3}}{\alpha_1 \alpha_2}} \frac{(\epsilon M)^2}{\alpha_2^2 + 30\alpha_1^2} \left[ |H|^2 + |\bar{H}|^2 + |H_1|^2 + |\bar{H}_1|^2 \right] \quad (7.62)$$

already shown to survive the other contributions are those exactly computed by Nilles et al (see section 5.6) with the parameter  $|A|$  assuming its Polonyi value  $(3 - \sqrt{3})$  as it must be since we have shown that to leading order the SUSY

breaking effects of the O'Raifeartaigh and Polonyi potentials are identical and since the relation between matter fields and the O'Raifeartaigh sector is the same as between the 'hidden' and 'observed' sectors of section 5.6. Thus the result is that given in eqn. (5.48) with

$$m_{3/2} = e^{2-\sqrt{3}} \left( \alpha_1 \alpha_2 / \alpha_2^2 + 30 \alpha_1^2 \right)^{1/2} (K M^2) \\ \tilde{g} = e^{(2-\sqrt{3})} g(y_a) \quad (7.63)$$

for a homogenous low energy potential  $g(y_a)$ .

The only difference from that case arises due to the presence of a light color octet and a SU(2) triplet supermultiplet in the low energy spectrum. Using the v.e.v. already derived one can determine the low energy potential for the (1,3)(8,1) subreps of Z. Fields with such quantum numbers remain light because of the following phenomenon. When Z breaks the GUT symmetry the superfields in SU(5)/G₁₂₃ get eaten in the course of the Higgs phenomenon. The other multiplets, however, get mass terms of O(M_S) by mixing with A from the term  $\alpha_2 Z A^2$  in the superpotential. Thus for fermions this term gives

$$2 \alpha_2 \psi_2^T C^{-1} \psi_A \langle A \rangle + \alpha_2 \langle Z \rangle \psi_A^T C^{-1} \psi_A \quad (7.64)$$

since  $\langle Z \rangle \sim O(M_P)$  while  $\langle A \rangle \sim O(M_S)$ , and Z gets no self mass we see that the fermionic mass matrix of form  $\begin{pmatrix} M_P & M \\ M & 0 \end{pmatrix}$  has an eigenvalue O(m_{3/2}). A similar phenomenon also takes place for scalars. The dominant mass terms for both

A and Z come from  $|F_A|^2$  and are given by

$$9 A_0^2 Z_3^2 + A_3^2 16 \alpha^2 X_0^2 + 18 A_0 Z_0 A_3 Z_3 \\ + 4 A_0^2 Z_8^2 + 9 A_8^2 X_0^2 + 8 A_0 Z_0 A_8 Z_8 \quad (7.65)$$

thus the mass² matrix has the form  $\begin{pmatrix} 1 & \epsilon \\ \epsilon & \epsilon^2 \end{pmatrix} M_P^2$  and one eigenvalue is therefore again small. A result for the low energy potential (consisting of mass terms) for these light fields has been quoted in the literature⁽⁴⁴⁾, however, since we have not verified this result we desist from quoting it.

We have shown that at tree level all scalars in the low energy theory (with couplings defined at  $M_U$ ) have positive mass²  $O(m_{3/2})$  and the parameter  $|\Lambda|$  which controls tree level breaking (section 5.6) takes its Polonyi value. Thus a radiative breaking mechanism must be invoked to account for the Weak scale. In this respect the Geometric model is at least no worse than the other SUGRA models since it provides the same soft terms and in addition generates the GUT scale very naturally.

The dominant modes of nucleon decay are⁽²¹⁾

$$p \rightarrow K^0 \mu^+, K^+ \nu \\ n \rightarrow K^0 \bar{\nu}$$

and proceed via the light colored triplets present in such models (see section 5.3). The chief defects of this model lie in the fact that the presence of the light triplet and

octet when accounted for in the renormalization group equations give  $M_U \sim 10^{20} \text{ GeV}$ ,  $\sin^2 \theta_W \sim .25$ . Moreover, even with just three families the color coupling constant grows with increasing energy since*

$$b_3 = -9 + 2n_g + 3n_s + \frac{n_t}{2} = 0 \quad (7.66)$$

The other defect is the necessity to fine tune the doublet masses to zero. Although the first problem seems intrinsic to this type of model, it may be possible to avoid fine tunings in the context of an  $SO(10)$  model as we shall discuss in the next sections.

*See section (7.9) for more details

## Section 5:

### SO(10) Geometric Hierarchy

In this and following sections we discuss SO(10) models which induce the GUT scale via SUGRA effects and simultaneously break Left-right symmetry at the SUSY breaking scale. As we shall see the considerations of the previous chapter do not generalize trivially to the SO(10) case due to certain group theoretical constraints. In this section we discuss these constraints on the form of the superpotential and the structure of the resulting potential. In section 6 we discuss the minimization of such potentials and the breaking patterns obtainable thereby. In section 7 we propose a model which dispenses with the fine tuning necessary to make Higgs doublets light in the SU(5) type models. In section 9 we indicate how the Left-Right breaking scale may be associated with  $M_S$ .

As we saw in chapter 3 the only low(er) dimension irreps suitable for breaking SO(10) down to  $G_{123}$  via the left-right symmetric gauge groups contained in SO(10) are the 45, 54 and 210 dimensional representations. The 54(S) can be written as a 10x10 traceless matrix and is thus analogous to the 24 of SU(5) in that respect. However, if we attempt to write a superpotential

$$g = \alpha, X (\bar{t} S^2 - M^2) + \alpha_2 \bar{t} S^2 Z \quad (7.67)$$

we find that this does not break SUSY in the global limit.

The reason for this is easily seen from

$$\begin{aligned} -F_X^\dagger &= t S^2 - M^2 \\ -F_Z^\dagger &= \alpha_2 (S^2 - \frac{1}{10} t S^2) \end{aligned} \quad (7.68)$$

Since  $S_0 \text{Diag}(\underline{1}_5, -\underline{1}_5)$  is a possible v.e.v. for  $S$  which breaks  $SO(10)$  to  $O(5) \times O(5)$  it is clear that both equations can be satisfied with this pattern. Hence, one can only use

$$g = \alpha_1 X (t S^2 - M^2) + M_2 t S Z \quad (7.69a)$$

Similarly, one finds that since the 45 has no cubic coupling the only cubic superpotential which is constructed from 45S and breaks SUSY is

$$g = \alpha_1 X (t A^2 - M^2) + M_2 t A Z \quad (7.69b)$$

Finally in the case of the 210 the same phenomenon takes place. If one constructs a superpotential with only one mass parameter by using a 210(Z) and a 45(A)

$$g = \alpha_1 X (t A^2 - M^2) + \alpha_2 \epsilon_{\alpha\beta\gamma\delta} A_{\alpha\beta} A_{\gamma\delta} \quad (7.70)$$

then

$$-F_{\alpha\beta\gamma\delta}^\dagger = \alpha_2 (A_{\alpha\beta} A_{\gamma\delta} + \text{permutations}) \quad (7.70)$$

Since one can have  $\text{Tr } A^2 = M^2$  with just  $A_{12} = -A_{21} = 0$  we see that since  $F^Z$  must have four indices different it will again be zero.

Thus we must resign ourselves to working with two mass parameters. Although it may be shown⁽²⁷⁾ that global models still induce GUT scales via radiative effects the corresponding demonstration, at tree level, in SUGRA theories is non trivial.

Moreover, the leading order minimum is not the same as the global one. To see how this comes about let us consider a singlet superpotential of this type

$$g = \alpha, X (A^2 - M^2) + M' Z A + C \quad (7.72)$$

after scaling  $Z, X \sim 1/\kappa$  one finds

$$\begin{aligned} U = & (a^2 - 1)^2 a^2 + a^2 + (3 + 2\alpha a x)^2 / \epsilon^2 \\ & + 2(3\alpha a^2 x + 2a_3 - \alpha x)(v + \alpha a^2 x + 3a - \alpha x) \\ & + (x^2 + 3^2 + a^2 \epsilon^2 - 3)(v + \alpha a^2 x + 3a - \alpha x)^2 \end{aligned} \quad (7.73)$$

$$V = M^2 M'^2 e^{\kappa^2 / 2 I^2} U$$

By minimizing with respect to 'z', 'x' at  $O(\epsilon^{-2})$  and  $O(\epsilon^{-1})$  one easily finds

$$\begin{aligned} z_0 &= -2\alpha a_0 x_0 \\ z_1 &= a_1 + x_1 \end{aligned} \quad (7.74)$$

However, unlike the case for the singlet superpotential of chapter 6 or the SU(5) case considered there we see that the substitution of these values into the potential at  $O(\epsilon^0)$  is not sufficient to decouple 'a' from 'x'. Hence

one cannot blindly minimize the global part

$$a^2 + \alpha^2 (a^2 - 1)^2 \quad (7.75)$$

with respect to 'a' and substitute the value obtained in the rest of the potential.

Instead one must actually minimize the global terms plus the terms due to the coupling to SUGRA in order to obtain a solution around which to expand. We have checked that the former procedure in fact leads to inconsistent equations for  $a_2$  etc.

#### Section 6 Extremum Problem for SO(10) Models:

We saw in the previous section that for SO(10) models it is not possible to expand around the global solution. An examination of (7.73) shows that the potential contains terms involving  $a^4 x^4, a^4 x^2 z^2$  thus an exact analytic solution of the simultaneous equations obtained by setting

$$\partial U / \partial a = 0 \quad \frac{\partial U}{\partial x} = 0 \quad \frac{\partial U}{\partial z} = 0 \quad U = 0 \quad (7.76)$$

does not seem feasible even with the aid of a symbolic manipulation program such as MACSYMA.

However, one can easily show that GUT scale induction takes place in such potentials. To see this one need only recognize that the solutions to (7.76) force

Since  $\langle z \rangle$  is used to break the grand unifying symmetry it is clear that the GUT scale will be induced unless either  $a_0=0$  or  $x_0=0$ .

Let us consider the case with  $A(45)$  and  $Z(45)$

$$g = \alpha_1 (\bar{A} A^2 - M^2) + M' \bar{A} A Z \quad (7.77)$$

with

$$\begin{aligned} \langle A \rangle &= (\bar{A} / \sqrt{n}) \text{Diag} (I_n \otimes \eta, 0) \\ \langle Z \rangle &= (\bar{Z} / \sqrt{n}) \text{Diag} (I_n \otimes \eta, 0) \\ \eta &= \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \end{aligned} \quad (7.78)$$

one has the potential

$$\begin{aligned} U &= \alpha^2 (2a^2 - 1)^2 + 4a^2 + \frac{4}{e^2} (z + 2a\alpha x)^2 \\ &+ 2(2az + 2a^2\alpha z - \alpha x + v)(4az + 6a^2\alpha x - \alpha x) \\ &+ (2az + 2a^2\alpha x - \alpha x + v)^2 (z^2 + x^2 + a^2 e^2 - 3) \end{aligned} \quad (7.79)$$

If at the extremum one had  $x_0=0=z_0$  then one finds

$$\begin{aligned} \left. \frac{\partial V}{\partial x} \right|_0 &= 4\alpha v \quad \left. \frac{\partial V}{\partial a} \right|_0 = 8a [\alpha^2 (a^2 - 1) + 1] \\ V_0 &= 4a^2 + (2a^2 - 1)^2 \alpha^2 - 3v^2 \end{aligned} \quad (7.80a)$$

If  $V_0 = 0$  is to be achieved - as is experimentally indicated - then  $v \neq 0$  so  $\partial V / \partial x|_0 \neq 0$ . Hence  $x_0=0$  is not an extremum.

On the other hand, if  $a_0=0=z_0$  (omitting  $M^2 M^{1,2}$ )

$$U_0 = (x_0 v_0 - \alpha)^2 - 3(v - \alpha x)^2$$

$$\partial U / \partial x_0 = 4\alpha^2 x_0^3 - 6\alpha v x_0^2 + 2(v^2 - \alpha^2)x_0 + 4\alpha v \quad (7.80b)$$

we recognize the minimization and zero energy equations for a superpotential  $g = C - M^2 x$ . Thus  $x_0 = \pm(\sqrt{3}-1)$ ,  $v = \pm(\sqrt{3}-2)$

but now

$$\begin{aligned} \left. \frac{\partial^2 U}{\partial a^2} \right)_0 &= -8\alpha^2 x_0^4 + 8\alpha v x_0^3 - 8\alpha^2 x_0^2 - 8\alpha^2 + 8 \\ &= (40 - 32\sqrt{3})\alpha^2 + 8 \end{aligned} \quad (7.81)$$

so that for

$$\alpha^2 > 5 + 4\sqrt{3} / 23$$

we will have a maximum at  $a_0=0, z_0=0$ . Thus it is clear that there exists a range of parameters for which the minimum of the potential has  $z_0 \neq 0$ . Thus scale induction must take place.

### Section 7 Comparison of 54plet and 45plet models:

From the considerations of the last sections we see that when using  $SO(10)$  as the gauge group two

mechanisms for scale induction are available, namely those using 54s and 45s. In this section we discuss their relative advantages.

i) The first problem is that of fine tuning of the Higgs doublet masses. The 54 has a structure similar to that of the 24 plet in the SU(5) models. its coupling

$$S_{\mu\nu} H_{\mu} H_{\nu}$$

to the 10 dim Electroweak Higgses thus tends to give the SU(2) doublets a superheavy mass since in the desired breaking pattern

$$\langle S \rangle \sim (\mathbb{1}_6, -\mathbb{1}_4)$$

Thus just as for the 24 one must introduce(at least)* two different 10plets only one of which couples to the matter fields and a mass parameter M.

Then one writes

$$g = S_{\mu\nu} H_{\mu} H_{\nu}' + M H_{\mu} H_{\nu} \quad (7.82)$$

and fine tunes

$$-3/2 \langle S \rangle + M = O(m_{3/2}) \quad (7.83)$$

so that the electroweak doublets remain massless.

On the other hand in the case of the 45plet the v.e.v breaks  $SO(10)$  to  $SU(2)_L \times SU(2)_R \times SU(3) \times U(1)_{B-L}$ .

* If the standard model's matter spectrum is to be accounted for by using the tree level v.e.v then two sets of 10plets are required.

By coupling it to two 10plets  $H, H$  as

$$\lambda A_{\mu\nu} H_{\mu} H_{\nu}'$$

one can thus give only the color triplet masses  $O(M_S)$  while keeping the  $SU(2)$  doublets light. Hence, provided this v.e.v. is selected by some means no fine tuning is necessary. In this case too one needs to introduce two Higgs doublets so that the 45-10-10' coupling is possible. Also note that in contrast to the 54plet the decoupling in this case is trivial since there is no coupling  $\lambda(\alpha X-Z)T$  at the extremum. This absence of fine tuning must therefore be counted as a definite plus point of the 45plet model.

ii) Next consider the question of vacuum selection.

It is helpful to first consider the global case.

If

$$\langle S_{54} \rangle = S_0 \Lambda_n \quad \Lambda_n = (\mathbb{1}_n, -\frac{n}{10-m} \mathbb{1}_4)$$

let  $\eta = \alpha \Lambda_n^2$

The global potential at the extremum is

$$V = [\eta a^2 + \alpha (\eta a^2 - 1)^2] M^2 M'^2 \quad (7.83)$$

Clearly only  $\eta a^2$  and not  $a^2$  itself is determined by this equation. Thus

$$\eta a^2 = 2\alpha^2 - \frac{1}{2}\alpha^2 \quad \langle V \rangle = M^2 M'^2 \frac{(4\alpha^2 - 1)}{4\alpha^2} \quad (7.84)$$

is independent of  $n$ . Similarly, in the case of the

45plet too all vacua, except the symmetric one, are degenerate. The SUGRA potential continues to be a function of  $a^2 \eta$  or  $\eta a^2$  and the corresponding quantities in the 45plet case. It is therefore apparent that the minimization or  $V = 0$  conditions will not determine the vacuum in either case. For the 54plet model one can hope to circumvent this by introducing a cubic coupling  $\alpha_3 \text{Tr } S^3$ . Then

$$-F_A^T = 2\alpha_1 A S + M' Z + 3\alpha_3 (S^2 - \frac{1}{10} \text{tr } S^2) \quad (7.85)$$

thus if  $\langle A \rangle$  assumes the pattern  $(n \mathbb{1}_m, -m \mathbb{1}_n)$  the potential is no longer a function of  $\eta a^2$  and  $\eta a^2$  only. The square of the last term is  $(\eta a^2)^2 / \eta$  and hence the minimum can recognize the breaking pattern. The presence of this term will affect the v.e.v. of  $Z$  and  $X$  only to  $O(\epsilon)$ . In fact by exactly the same arguments as given above to show that  $\langle Z \rangle$  is induced in the simpler case  $\alpha_3 = 0$  one can show that scale induction still takes place.

No similar mechanism seems available in the case of the 45plet without complicating the model.

iii) We saw in chapter 7 that any sub-multiplets of  $Z$  not eaten by the gauge multiplet remain light. Thus in the breaking

$$SO(10) \xrightarrow{\langle 54 \rangle} G_{PS}$$

45-21=24 fields are absorbed by the gauge particles which

become heavy. These lie in the  $(2,2,6)$  submultiplet of the 54. Under  $G_{PS}$

$$54 = (1,1,1) + (3,3,1) + (1,1,20) + (2,2,6) \quad (7.86)$$

where

$$20 = 6(4/3) + \underline{6}(-4/3) + 8(0) \quad (7.87)$$

under  $SU(3) \times U(1)$

Thus one will have the  $(1,1,20)$  supermultiplet light down to energies  $O(m_g)$ . The color  $6+6+8$  have a total Casimir of 8 and completely dominate the evolution of  $\alpha_3$  even at energies  $O(m_3/2)$ . Including the gauge and matter fields gives

$$b_3 = 5 \quad \text{below } M_{PS} \quad (7.88)$$

$$b_4 = 2 + n_t/2 \quad \text{above } M_{PS}$$

where  $n_t$  is the number of colored triplets. Clearly, this is not acceptable from the point of view of asymptotic freedom or even perturbative unification.

In the case of the 45plet  $Z$  one has

$$45 = (3,1,1) + (1,3,1) + (1,1,15) + (2,2,6) \quad (7.89)$$

thus it can be used to break  $SO(10) \rightarrow G_{LR}$ .

30 supermultiplets, namely  $(2,2,6), (1,1,3(-4/3) + \underline{3}(4/3))$  are eaten. This still leaves the color octet light. Hence at best the situation is no worse than in the  $SU(5)$  case.

We conclude that a model with 45s is favored on

the counts of not needing fine tuning and a less explosive number of light colored particles. It has, however, the drawback that vacuum degeneracy persists even at the local SUSY level.

### Section 8 A 45plet Model:

In this section we write down a concrete model that utilizes the 45plet mechanism and breaks  $SO(10)$  down to  $G_{LR}$  at  $M_U$  and further breaks it to  $G_{123}$  at  $M_S$ . We can achieve the association of  $M_S$  and  $M_R$  via the following simple expedient. One introduces a pair of multiplets. Under  $G_{PS}$  these have subreps

$$16 = (2, 1, 4) + (1, 2, \bar{4})$$

$$4 = 1(1) + 3(-1/3) \quad (\text{under } G_{123}) \quad (7.90)$$

hence the singlet in the right handed 4 (which has the quantum numbers of  $\nu_L^c$ ) can be used to break  $SU(2)_R \times U(1)_{B-L}$  down to  $SU(2)_L \times U(1)_Y$ . We incorporate this into our model by writing

$$g = \alpha_1 X A (A^2 - \bar{\Psi}\Psi) + \alpha_2 M_U Z A + \alpha_3 Y (\bar{\Psi}\Psi - M_R^2) \quad (7.91)$$

where  $Y$  is also a gauge singlet. Now at the global minimum,

$$\langle \bar{\Psi}\Psi \rangle = M_R^2$$

but  $F_X$  and  $F_Z$  cannot be simultaneously zero. As before

$$\langle Z + 2\alpha X A \rangle$$

is not determined. By arguments similar to those given earlier one sees that SUSY and  $SU(2)_R \times U(1)$  are simultaneously broken at  $O(M_S)$  and a GUT scale is induced

at  $O(1/\kappa)$ . The Higgs couplings are introduced as

$$g = \lambda_i H_\mu^i A_{\mu\nu} H_\nu^i + \lambda_j' \chi^T B \not{D}_\mu \chi H_\mu^j$$

The previously discussed mechanism for keeping Higgs light goes through if the correct vacuum be chosen.

Since  $Y = -X$  at the global level we see that although  $Y$  also gets a v.e.v.  $O(M_P)$  the  $\psi, \bar{\psi}$  multiplets get mass  $O(m_{3/2})$  only (from radiative corrections with  $F_X$  insertions).

Although we have now accomplished the desired association between  $M_R$  and  $M_S$ , this model is yet far from being viable as it stands. For instance we have no mechanism to give mass to the right handed neutrino. To do this one could introduce additional  $126 + \underline{126}$  multiplets of which the  $126$  multiplet would couple to the matter multiplets. If it were arranged that

$$\langle 126 \rangle = \langle \underline{126} \rangle \sim M_R$$

for instance by replacing  $16, \underline{16}$  by  $126, \underline{126}$  then the desired neutrino mass could be obtained. Note, however, that since most of the  $126, 126$  would remain light the situation with respect to the coupling constant evolution would be disastrous with  $\alpha_3$  and  $\alpha_2$  growing explosively. This proliferation of  $G_{123}$  variant multiplets of low mass is one of the most characteristic features of the SUGRA-Witten model with canonical kinetic terms. We are presently investigating whether a generalization to the case with non-minimal kinetic terms can be used to induce large

masses for  $z, \psi, \bar{\psi}$ , etc. fields. Pending such a resolution of the problem we do not detail the phenomenological features of the model any further.

Of course, it is evident that one might just as well use a 54plet in the above scheme (with a fine tuning to remove Higgs doublet masses).

### Section 9 Low Energy Constraints:

In this last section we indicate how the coupling constant equations constrain the mass scales in geometric hierarchy models.

We consider the SU(5) case. Then one can derive

$$\begin{aligned} \ln^2 \alpha_W(M_W) &= \frac{3}{8} + \frac{5\alpha(M_W)}{16\pi} \left[ (b_2 - b_1)^B \ln(M_S/M_W) \right. \\ &\quad \left. + (b_2 - b_1)^t \ln(M_U/M_S) \right] \\ \alpha_3(M_W) &= \frac{8/3 \alpha(M_W)}{\left[ 1 + \frac{5\alpha(M_W)}{6\pi} \left[ \left( \frac{8}{5} b_3 - \frac{3}{5} b_2 - b_1 \right)^B \ln \frac{M_S}{M_W} \right. \right. \\ &\quad \left. \left. + \left( \frac{8}{5} b_3 - \frac{3}{5} b_2 - b_1 \right)^t \ln \frac{M_U}{M_S} \right] \right]} \quad (7.92) \\ \frac{1}{\alpha_3(M_U)} &= \frac{1}{\alpha_3(M_W)} - \frac{b_3^B}{2\pi} \ln \frac{M_S}{M_W} - \frac{b_3^t}{2\pi} \ln \frac{M_U}{M_S} \end{aligned}$$

where the superscripts b, t indicate the bottom ( $M_W - M_S$ ) and top ( $M_S - M_U$ ) mass ranges respectively. The  $b_i$  are

the appropriate Casimir operators for supermultiplets (since the light sector is effectively supersymmetric above  $M_W$ ). The general formulae as well as the values for the SU(5) model are given below

$$\begin{aligned}
 b_2^6 &= -6 + 2n_g + 2n_3 + n_H(1/2) = 4 \\
 b_1^6 &= 0 + 2n_g + 0 + n_H(3/10) = 36/5 \\
 b_3^6 &= -9 + 2n_g + 3n_8 + n_H(0) = 0 \\
 b_1^2 &= 2n_g + n_H/2 = 8 \\
 b_2^2 &= 2n_g - 6 + 2n_3 + n_H/2 = 4 \\
 b_3^2 &= -9 + 2n_g + 3n_8 + n_H/2 = 2
 \end{aligned} \tag{7.93}$$

$n_g$  = number of generations = 3.

then

$$\begin{aligned}
 \text{Pen}^2 \theta_W(M_W) &= .285 + 1.78 \times 10^{-3} (.85 - 4u) \\
 s &= \log_{10} M_s(\text{GeV}) \quad u = \log_{10} M_U(\text{GeV})
 \end{aligned} \tag{7.94}$$

with  $U=19$  one gets

$$\text{Pen}^2 \theta_W(M_W) = .25 + .0014s \tag{7.95}$$

while

$$\alpha_3(M_W) = \frac{.019}{[1 - 4.36 \times 10^{-3} (7.2u + 2.4s)]} \tag{7.96}$$

Consistency with low energy data which put  $0.1 < \alpha_3(M_W) < 0.14$  then implies  $21 < U < 20$  for  $s=11$ . Which puts us in a realm where one cannot neglect the unknown physics of the

Planck scale.

For the 45plet  $SO(10)$  model the situation is very similar, apart from the fact that in that case, as explained above, one tends to have at least 5 light 16 plets. Since we do not have a complete  $SO(10)$  model so far it would not be very meaningful to present detailed estimates.

### SUMMARY & CONCLUSIONS

In this thesis we have seen how the problem of preserving the gauge hierarchy of GUT models leads to the introduction of SUSY. The structural constraints imposed by this remarkable symmetry then lead one to consider models with  $M_U \sim M_P$  and  $M_S \sim 10^{11} - 10^{12}$  GeV so that it is natural to generalize to Local SUSY. Not only does this resolve several problems of globally SUSY GUTS but makes a remarkable connection between Gravity and the Weak Interactions via the gravitino mass.

The experimental implications of this class of models are by no means trivial. In model independent manner the whole class of models predicts a rich diversity of new Physics clustered at the Gravitino mass. Since all scalars receive a mass² contribution  $O(m_{3/2}^2)$  we expect that  $m_{3/2} \sim M_W$  in order to avoid a new fine tuning problem. If this is so and  $m_{3/2}$  lies in the range  $10^2 - 10^3$  GeV one expects a great variety of particles with masses in this range. As discussed in the thesis the squarks and sleptons in such theories typically have masses  $O(m_{3/2})$ . Moreover the gauginos are also expected to have masses of this order of magnitude or somewhat less in accordance with the ideas of Appendix II. In addition there will be the (fermionic) partners of the Higgs doublets and possibly light color octet and SU(2) triplet scalars from Geometric mechanisms. These particles will decay into the lighter

ones to which they are connected by the low energy interactions and will have very distinct signals. Moreover unlike the case of Quark-Lepton Spectroscopy all squarks /sLeptons with the same  $G_{123}$  quantum numbers must be nearly degenerate and should therefore show up almost simultaneously. Thus the next generation of accelerators (ie those probing up to about 10TeV) will directly test the predictions of the class of models considered.

In scenarios where Weak breakdown is induced via renormalization group effects a top-like Quark with mass in the range [140,195] GeV is also predicted.

We also saw that the observation of B-L violating phenomena would rule out SUSY SO(10) GUTS with intermediate mass scales.

Finally the values of  $\sin^2 \theta_W$  tend to be larger than those in non SUSY models. This leads one to suspect that the effective value of the parameter is in fact not unity as in the standard model but deviates from it sufficiently to allow the higher values of  $\sin^2 \theta_W$ . This may be due to the cluster of particles with masses  $O(m_{3/2})$ .

We see that SUGRA models with intermediate SUSY breaking offer us the exciting prospect of generating both the GUT scale and the Fermi scale via effects triggered by SUSY breaking at an intermediate mass scale. Structurally, they offer the possibility of solving several vexing

problems simultaneously. However, due to the highly constrained form of the superpotential and the various fine tunings necessary they present features that are aesthetically unappealing and phenomenologically unacceptable.

Of these the problem of what actually screens out the cosmological constant will probably have to await a dynamical resolution, possibly along the lines suggested by Hawking⁽⁵¹⁾ and more recently Rajeev.⁽⁵²⁾ It would be interesting to investigate how such screening mechanisms can be combined with SUSY breaking potentials of the Polonyi or O'Raifeartaigh form.

On the other hand it may be possible to give large masses to the Z-type particles by using non-minimal formalisms. This would then make the theory competitive with respect to the usual calculations of Sin .

Although our investigations have not concluded in a definite completely viable model yet interesting structures have been excavated and mechanisms brought to light. Several lines of investigation, as described above, remain to be pursued in the future. Perhaps it is only by learning what nature will not allow to happen that one may finally stumble through the one door into her inner coherence that she has left open.

Appendix I:

Notation and basic concepts on Superfields

In the sections on global SUSY we use the notation of Salam and Strathdee⁽²⁾.

The metric is

$$\eta_{\mu\nu} = \text{diag} (1, -1_3) \quad (\text{A I.1})$$

The  $\gamma$  matrices satisfy

$$\{\gamma_\mu, \gamma_\nu\} = 2\eta_{\mu\nu} \quad \sigma_{\mu\nu} = \frac{i}{2} [\gamma_\mu, \gamma_\nu] \quad (\text{A I.2})$$

and  $\gamma_5$  is defined by

$$\gamma_5 = \gamma_0 \gamma_1 \gamma_2 \gamma_3$$

The charge conjugation matrix  $C$  satisfies

$$C^{-1} \gamma_\mu C = -\gamma_\mu^T \quad C = -C^T = C^T = C^{-1} \quad (\text{A I.3})$$

The matrices  $(\gamma_0, i\gamma_i, i\gamma_5)$  are Hermitian.

The chiral projectors are

$$P_{L,R} = P_\pm = (1 \mp i\gamma_5)/2 \quad (\text{A I.4})$$

Majorana spinors satisfy

$$\psi^c = C \bar{\psi}^T \quad (\text{A I.5})$$

In the sections on Local SUSY we use the opposite metric and the chiral projectors are

$$P_\pm = P_{R,L} = (1 \pm \gamma_5)/2 \quad (\text{A I.6})$$

since  $\gamma_5$  is taken to be Hermitian rather than  $i\gamma_5$ . Thus we conform to the notation of Kugo and Uehara⁽⁴²⁾.

We introduce the supersymmetry algebra and its representations both in the superfield notation and in that of the 'tensor calculus'. The latter will prove especially

useful when we consider local supersymmetry.

Our conventions are summarized in Appendix I.

A) Following Salam and Strathdee we introduce four anticommuting c-numbers  $\theta^\alpha$  which form a Majorana spinor. We introduce the concept of an anti-commuting derivative by

$$\bar{\phi}(x, \theta + \delta\theta) - \bar{\phi}(x, \theta) = \delta\bar{\theta} \frac{\partial}{\partial\theta} \bar{\phi}(x, \theta) \quad (\text{A I.7})$$

with the property

$$\frac{\partial}{\partial\theta} \bar{\phi}_1 \bar{\phi}_2 = \frac{\partial \bar{\phi}_1}{\partial\theta} \bar{\phi}_2 \pm \bar{\phi}_1 \frac{\partial \bar{\phi}_2}{\partial\theta} \quad (\text{A I.8})$$

where the  $+$ ( $-$ ) sign applies when  $\bar{\phi}_1$  is bosonic(fermionic).

The supersymmetry algebra is defined by

$$\begin{aligned} [\bar{Q}_\alpha, P_\mu]_- &= 0 \\ [\bar{Q}_\alpha, J_{\mu\nu}] &= \frac{1}{2} (\sigma_{\mu\nu})_\alpha{}_\beta \bar{Q}_\beta \\ \{\bar{Q}_\alpha, \bar{Q}_\beta\} &= -(\gamma_\mu C)_{\alpha\beta} P_\mu \end{aligned} \quad (\text{A I.9})$$

where  $P_\mu, J_{\mu\nu}$  are the generators of Lorentz translations and rotations respectively, while  $\bar{Q}_\alpha$  are the supersymmetry generators.

The algebra has a differential realization

$$\begin{aligned} \tilde{P}_\mu &= i \partial_\mu \\ \tilde{Q} &= i \left( \frac{\partial}{\partial\bar{\theta}} + \frac{i}{2} \not{\partial} \theta \right) \\ \tilde{J}_{\mu\nu} &= i(x_\mu \partial_\nu - x_\nu \partial_\mu) - \frac{1}{2} \bar{\theta} \sigma_{\mu\nu} \frac{\partial}{\partial\theta} \end{aligned} \quad (\text{A I.10})$$

If one introduces anti-commuting Majorana spinor parameters the anti-commutator may be written as

$$[\bar{\epsilon}^1 Q, \bar{\epsilon}^2 Q] = \bar{\epsilon}_1 \gamma_\mu \epsilon_2 P_\mu$$

B) A superfield  $\bar{\phi}(x, \theta)$  defined over space-time and the has the expansion

$$\begin{aligned} \bar{\phi}(x, \theta) = & A(x) + \bar{\theta} Z(x) + \frac{1}{4} \bar{\theta} \theta F(x) + \frac{1}{4} \bar{\theta} \gamma_5 \theta G(x) \\ & + \frac{1}{4} \bar{\theta} i \gamma_\nu \gamma_5 \theta V_\nu(x) + \frac{1}{4} \bar{\theta} \theta \bar{\theta} \lambda(x) \\ & + \frac{1}{32} (\bar{\theta} \theta)^2 D(x) \end{aligned} \quad (A I.11)$$

Here  $A, F, G, V_\nu, D$  are Bose fields while  $Z$  and  $\lambda$  are fermi fields.

The expansion terminates at fourth order in  $\theta$  because of the property

$$\theta_\alpha \theta_\beta = - \theta_\beta \theta_\alpha \quad (A I.12)$$

This expansion defines a real vector multiplet

$$V = (A, Z, F, G, V_m, \lambda, D)$$

The product of two vector multiplets is again a vector multiplet

$$\begin{aligned} V_1 \times V_2 = V_3 = & [A_1 A_2, \quad A_1 Z_2 + A_2 Z_1, \quad A_1 F_2 + A_2 F_1, \\ & - \bar{Z}_1^c Z_2 \\ & A_1 G_2 + A_2 G_1 + \bar{Z}_1^c \gamma_5 Z_2, \quad A_1 V_{2\nu} + A_2 V_{1\nu} \\ & + \bar{Z}_1^c i \gamma_\nu \gamma_5 Z_2, \\ & A_1 \lambda_2 + A_2 \lambda_1 + Z_1 F_2 + Z_2 F_1 - \gamma_5 Z_1 G_2 - \gamma_5 Z_2 G_1 \\ & - V_{1\nu} \gamma_5 \gamma_\nu Z_2 - V_{2\nu} \gamma_5 \gamma_\nu Z_1, \\ & 2 V_{1\nu} V_{2\nu} + A_1 D_2 + A_2 D_1 + 2 F_1 F_2 + 2 G_1 G_2 - 2 \bar{\lambda}_1^c Z_2 - 2 \bar{\lambda}_2^c Z_1] \end{aligned} \quad (A I.13)$$

The general vector superfield transforms as

$$\delta \bar{\Phi} = \left[ \alpha_\mu \tilde{P}_\mu + \frac{1}{2} \omega_{\mu\nu} \tilde{J}_{\mu\nu} + \bar{\epsilon} \tilde{S} \right] \quad (\text{A I.14})$$

leading to the componentwise transformations

$$\begin{aligned} \delta A &= \bar{\epsilon} z \\ \delta z &= \frac{1}{2} (F + \gamma_5 G + i \gamma_\mu \gamma_5 V_\mu - i \not{A}) \epsilon \\ \delta F &= \frac{1}{2} \bar{\epsilon} \lambda - \frac{1}{2} \bar{\epsilon} i \not{A} z \\ \delta G &= \frac{1}{2} \bar{\epsilon} \gamma_5 \lambda + \frac{1}{2} \bar{\epsilon} i \not{A} \gamma_5 z \\ \delta V_\nu &= \frac{1}{2} \bar{\epsilon} i \gamma_\nu \gamma_5 z + \frac{1}{2} \bar{\epsilon} i \not{A} i \gamma_5 \gamma_\nu z \end{aligned} \quad (\text{A I.15})$$

By defining the covariant derivatives

$$D \bar{\Phi} = (\not{\partial} \bar{\partial} - \frac{1}{2} \not{A} \bar{\partial}) \bar{\Phi}(x, \theta) \quad (\text{A I.16})$$

one has

$$\begin{aligned} [D, P_\mu] &= \{D, S\} = 0 \\ \{D_\alpha, D_\beta\} &= -(\gamma_\mu C)_{\alpha\beta} i \partial_\mu \end{aligned} \quad (\text{A I.17})$$

One can decompose the real vector multiplet into three smaller multiplets each of which can independently support a representation of the supersymmetry algebra.

Thus one has the two chiral multiplets  $\hat{\phi}_\pm$  which may be embedded in the vector multiplet as

$$\mathcal{V}(\hat{\phi}_\pm) = (A_\pm, \psi_\pm, F_\pm, \pm F_\pm, \pm i \partial_\mu A_\pm, 0, 0) \quad (\text{A I.18})$$

note that

$$\bar{\phi}_{\pm} = e^{\mp \frac{1}{4} \bar{\theta} i \not{\partial} \gamma_5 \theta} \hat{\bar{\phi}}_{\pm} = e^{\mp \frac{1}{4} \bar{\theta} i \not{\partial} \gamma_5 \theta} (A_{\pm} + \bar{\theta} \psi_{\pm} + \frac{1}{2} \bar{\theta} \theta_{\pm} F_{\pm})$$

in superfield notation, and which have the property that

$$\bar{D}_{\pm} \phi_{\mp} = 0 \quad (\text{A I.19})$$

the remaining part of the vector multiplet is the linear multiplet

$$\begin{aligned} v(\phi_i) &= (A, \psi, 0, 0, V_{\mu}, i \not{\partial} \psi, \partial^2 A) \\ \partial_{\mu} V_{\mu} &= 0 \end{aligned} \quad (\text{A I.20})$$

with the property that

$$D D \phi_i = \bar{D} \bar{D} \phi_i = 0$$

The transformation rules are

$$\begin{aligned} \delta A_{\pm} &= \bar{\epsilon}_{\mp} \psi_{\mp} & \delta \psi_{\mp} &= F_{\pm} \epsilon_{\pm} - i \not{\partial} A_{\pm} \epsilon_{\mp} \\ \delta F_{\pm} &= -\bar{\epsilon}_{\pm} i \not{\partial} \psi_{\pm} \\ \delta A_i &= \bar{\epsilon} \psi & \delta \psi_i &= \frac{1}{2} (i \not{\partial}_{\mu} \gamma_5 V_{\mu} - i \not{\partial} A_i) \\ \delta V_{\mu} &= -\bar{\epsilon} \sigma_{\mu\nu} i \gamma_5 \partial_{\nu} \psi \end{aligned} \quad (\text{A I.21})$$

The product of two chiral multiplets of the same chirality is chiral. In fact, we have

$$\begin{aligned} v(\phi_{\pm i}) &= [v(A_{\pm i}), v^2(A_{\pm i}) \psi_{\pm i}, \\ &\quad F_{\pm i} v^i(A_{\pm i}) + \frac{1}{2} \psi_{\pm i}^T C^{-1} \psi_j v^j(A)] \quad (\text{A I.22}) \end{aligned}$$

The product of two chiral multiplets of opposite chirality

is a vector multiplet.

$$\begin{aligned} \bar{\Phi}_{\pm} \bar{\Phi}'_{\mp} = & (A_{\pm} A'_{\mp}, A_{\pm} \psi'_{\mp} + \psi_{\pm} A'_{\mp}, A_{\pm} F'_{\mp} + F_{\pm} A'_{\mp}, \\ & -i A_{\pm} F'_{\mp} + i F_{\pm} A'_{\mp}, (i \partial_{\nu} A_{\pm}) A'_{\mp} + A_{\pm} (-i \partial_{\nu} A'_{\mp}) \\ & + \psi_{\pm}^T C^{-1} \gamma_{\nu} \psi'_{\mp}, \\ & (i \partial_{\nu} A_{\pm}) \gamma_{\nu} \psi'_{\mp} + \gamma_{\nu} \psi_{\pm} (i \partial_{\nu} A'_{\mp}) - i (\not{\partial} \psi_{\pm}) A'_{\mp} \\ & - A_{\pm} (i \not{\partial} \psi'_{\mp}) + 2 \psi_{\pm} F'_{\mp} + 2 F_{\pm} \psi'_{\mp}, \quad (\text{A I.23}) \\ & (-\partial^2 A_{\pm}) A'_{\mp} + 2 (\partial_{\nu} A_{\pm}) (\partial_{\nu} A'_{\mp}) + A_{\pm} (-\partial^2 A'_{\mp}) \\ & + 4 F_{\pm} F'_{\mp} + 2 \psi_{\pm}^T C^{-1} (i \not{\partial} - i \not{\partial}) \psi'_{\mp}) \end{aligned}$$

This can be generalized to an arbitrary function  $d(\bar{\Phi}_{-}, \bar{\Phi}_{+})$

The complex conjugate of a chiral multiplet is chiral but of opposite chirality.

$$\bar{\Phi}_{\mp}^* = e^{\frac{i}{4} \bar{\theta} \not{\partial} \gamma_5 \theta} (A_{\mp}^* + \bar{\theta} \psi_{\mp}^c + \frac{1}{2} \bar{\theta} \theta_{\pm} F_{\mp}^*)$$

or

$$\bar{\Phi}_{\pm}^* = (A_{\pm}^*, \psi_{\mp}^c, F_{\mp}^*) \quad (\text{A I.24})$$

in the 'tensor calculus' notation.

In addition to the chiral and linear multiplets introduced above one also introduces so called gauge and 'field strength' multiplets.

### Gauge multiplet

A vector multiplet in the so called 'Wess-Zumino' gauge is called a gauge multiplet. One sets the  $A, Z, F, G$  fields to zero. The multiplet then obtained can support a

representation of the SUSY algebra - modulo gauge transformations. Thus

$$G = (V_\mu, \lambda, D) \quad G^n = 0 \quad n \geq 3 \quad (\text{A I.25})$$

and by utilizing the gauge freedom of the theory in which  $G$  functions as a gauge multiplet one can define a new SUSY transformation

$$\begin{aligned} \delta V_\nu &= -\frac{i}{\sqrt{2}} \bar{\epsilon} \gamma_\nu \lambda & \delta D &= -\frac{i}{\sqrt{2}} \bar{\epsilon} \not{\gamma}_5 \lambda \\ \delta \lambda &= \frac{i}{\sqrt{2}} (i \not{\gamma}_5 D + \frac{1}{2} \sigma_{\mu\nu} V_{\mu\nu} \epsilon) \end{aligned} \quad (\text{A I.26})$$

while the corresponding transformations of chiral fields become

$$\begin{aligned} \delta A_\pm &= \bar{\epsilon} \psi_\pm & \delta \psi_\pm &= F_\pm \epsilon_\pm - i \not{\gamma} A_\pm \epsilon_\pm \\ \delta F_\pm &= \bar{\epsilon} (-i \not{\gamma} \psi_\pm + i \sqrt{2} \lambda \mp Q^k \pm A_\pm) \end{aligned} \quad (\text{A I.27})$$

Finally one introduces the field strength multiplet which is a chiral multiplet with an external spinor index ( $\alpha$  is a gauge index).

$$\begin{aligned} Q^\alpha W_{\alpha\dot{\beta}} &= \bar{D}_+ D_- (e^{-gV} D_{\dot{\beta}+} e^{gV}) \\ &= Q_\alpha [\lambda^\alpha_+, \frac{i}{\sqrt{2}} (D^\alpha(x) + \frac{1}{2} \sigma_{\mu\nu} F_{\mu\nu}^\alpha), \\ &\quad - i \not{\gamma} \lambda^\alpha_-]_{\dot{\beta}} \end{aligned} \quad (\text{A I.28})$$

The second equality holds when  $V$  is a gauge multiplet, ie. in Wess-Zumino gauge.

Appendix II

Complete Soft Breaking Set for  $G_{123}XSUSY$

In this appendix we discuss certain constraints on the Lagrangian in SUSY theories arising from the assumption of a renormalizable low energy Lagrangian with softly broken SUSY.

Consider the supersymmetrized standard model with two Higgs multiplets. The left-chiral multiplets involved are

Symbol	Field content	SU(3)xS(2)x(1) quantum numbers
$L$	$\begin{pmatrix} \nu \\ e \end{pmatrix}_L \begin{pmatrix} A_\nu \\ A_e \end{pmatrix}$	$(1, 2, -1)$
$E^c$	$e_L^c \quad A_{e^c}$	$(1, 1, 2)$
$Q$	$\begin{pmatrix} u \\ d \end{pmatrix}_L \begin{pmatrix} A_{uL} \\ A_{dL} \end{pmatrix}$	$(3, 2, 1/3)$
$D^c$	$d_L^c \quad A_{d^c}$	$(3^*, 1, 2/3)$
$U^c$	$u_L^c \quad A_{u^c}$	$(3^*, 1, -4/3)$
$H_1$	$\begin{pmatrix} \psi_1^0 \\ \psi_1^- \end{pmatrix} \begin{pmatrix} A_1^0 \\ A_1^- \end{pmatrix}$	$(1, 2^*, -1)$
$H_2$	$\begin{pmatrix} \psi_2^+ \\ \psi_2^0 \end{pmatrix} \begin{pmatrix} A_2^+ \\ A_2^0 \end{pmatrix}$	$(1, 2^*, 1)$

The general superpotential is then

$$g = k_e E^c H_1 L + k_d D^c H_1 Q + k_u U^c H_2 Q + m H_2 \in H_1^T \quad (A II.1)$$

We assume that the Higgs acquire a v.e.v. by some mechanism (either radiative effects of an explicit term  $X(H_2 \in H_1^T - \mu^2)$  (X a singlet) could be used.

Our basic result is that if softbreaking terms other than diagonal soft masses

$$\mu_i^2 A_i^\dagger A_i$$

be introduced, then renormalizability defines a 'self-consistent set' of soft breaking terms.

1) consider first the introduction of soft gaugino masses

$$\sum_i m_{\lambda_i} \bar{\lambda}_i \lambda_i$$

where  $i$  runs over gauge generators. Then the diagram of Fig. 1b shows that at the one loop level there is a logarithmic divergence in the chirality violating mass terms of type  $A_R^\dagger A_L$ . Thus, for instance, if we take  $A_R$  to be associated with  $E^{C*}$  the left vertex of Fig. 1b is  $\bar{\lambda}_L^c A_R$  and via the Yukawa mass for the electron and the soft gaugino mass  $m'$  we get an emergent  $A_e$  at the other vertex  $A_e^\dagger \bar{\lambda}_L$ . Note that  $A_R$  has quantum number -2 while  $A_e$  has -1. Hence this (divergent) term violates gauge invariance (being proportional to  $m_e$ ). Therefore, in order that the Lagrangian be renormalizable and gauge invariant one must

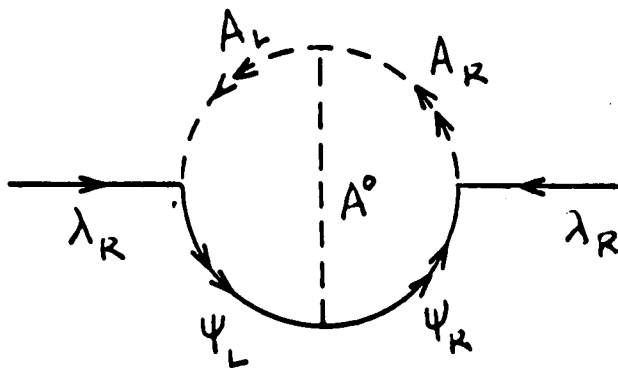


Fig 1a

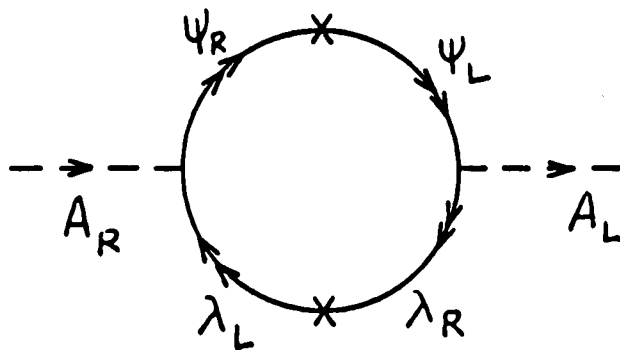


Fig 1b

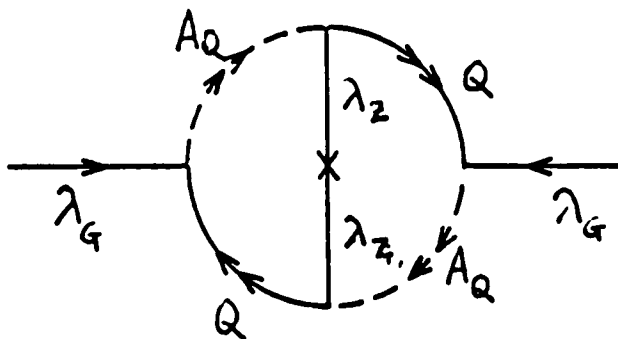


Fig 1c

introduce trilinear couplings at tree level. Namely

$$A_E c (m_1 A_1 + \bar{m}_1 A_2^* e) A_{e_L} + h.c \quad (A II.2)$$

and similar terms for the quarks.

ii) On the other hand if one had soft trilinear couplings, then Fig. 1a shows that gaugino masses, divergent at the two loop level, are induced. Thus conversely to the above one must introduce tree level counter terms for gaugino masses as well.

In chapter 5 (section 4-6) we showed that such trilinear scalar couplings arise naturally in SUGRA models. However, from (5.22) it is apparent that the term,

$$\frac{1}{2\kappa} e^{-g/2} g'^k (g''^{k'}) f'_{\alpha\beta k} \bar{\lambda}^\alpha \lambda^\beta \quad (A II.3)$$

can give rise to gaugino masses only if  $f'_{\alpha\beta k} \neq 0$ . Thus if one takes  $f_{\alpha\beta} = \delta_{\alpha\beta}$  then there are no tree level terms for gaugino masses. If one wishes to evade this inconsistency one may write  $f_{\alpha\beta} = \delta_{\alpha\beta}$  or  $f_{\alpha\beta} = z_\alpha z_\beta$  ( $\alpha, \beta$  are adjoint indices) where  $z, z_\alpha, z_\beta$  are singlet and adjoint fields with v.e.v.  $\sim M_p$ . A gaugino mass  $O(m_{3/2})$  is then generated at tree level. The kinetic terms for the gauge supermultiplet can be put in canonical form by rescaling them as

$$\begin{aligned} \langle f_{\alpha\beta} \rangle \bar{\lambda} i \not{D} \lambda &\rightarrow \bar{\lambda} i \not{D} \lambda \\ \langle f_{\alpha\beta} \rangle F_{\mu\nu}^\alpha F_{\mu\nu}^\beta &\rightarrow F_{\mu\nu}^{\alpha 2} \end{aligned}$$

This generates  $O(\kappa^2)$  corrections but leaves the low

energy effects unchanged.

Also note that Fig. 1c shows that one type of gaugino mass implies divergent masses for other gauginos thus implying chirality breaking masses.

iii) Finally we note that if one introduces soft gauge invariant mass terms for the Higgs multiplet then diagrams such as

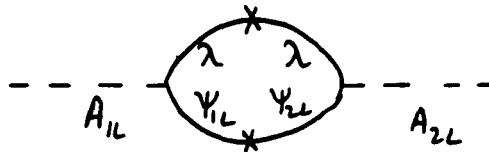


lead to masses

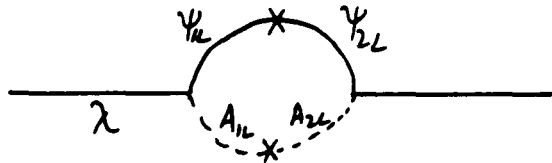
$$\psi_{1L}^T C^{-1} \in \psi_{2L} \quad (\text{A II.4})$$

for their partners. Conversely, such fermionic masses can lead to both gaugino masses and soft bilinear terms above.

1) This occurs via (divergent) diagrams for the



but (non-divergent) diagrams for



However, explicit SUSY breaking chiral fermion masses are not soft (Chapter 2) that is they lead to quadratic divergences. Thus they should not occur in the low energy Lagrangian. They also do not arise in SUGRA models and so

we omit them.

2) Thus although we include masses  $A_1 \in A_2^T$  we omit the corresponding fermionic terms at tree level, secure in the knowledge that they are not induced with divergent coefficients.

Our complete soft breaking set is now

$$\begin{aligned} \mathcal{L}_{\text{soft}} = & -\mu_i^2 |A_i|^2 - A_E c (m A_1 + \bar{m}_1 A_2^* \epsilon) A_L \\ & - A_U c (m_2 A_2 + \bar{m}_2 A_1^* \epsilon) A_Q \\ & - A_D c (m_3 A_1 + \bar{m}_3 A_2^* \epsilon) A_Q \\ & + m_\lambda^i \bar{\lambda}_G^i \lambda_G - m_H^2 A_1 \in A_2^T + h.c \end{aligned} \quad (\text{A II.5})$$

Note that  $\bar{m}_1$  do not arise in SUGRA models.



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