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Inflation tax in a post-liberalization environment: Evidence from Turkey (1980–1990)

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INFLATION TAX IN A POST-LIBERALIZATION ENVIRONMENT:

EVIDENCE FROM TURKEY (1980-1990)

by

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CHAPTER I

INTRODUCTION

Following a severe balance of payments problem in the 1970s, Turkey has undergone a structural reform and stabilization program that went into effect in January 1980. Supported by sizeable external financial assistance particularly in its earlier phases, the adjustment program is considered by most to be a story of success: per annum aggregate real GNP growth in the vicinity of 6 percent with the exception of 1980 when a contraction in GNP was observed, total exports grew at an average rate of 16.4 percent per annum during 1980-89 (21.5 percent per annum for the first half of the decade), and a steady reduction in the current account[1]/GNP ratio from -3.6 percent in 1983 to -1.9 percent in 1985 and to -1.2 percent in 1989. Although the growth rate of exports declined in the second half of the decade and did not even display any significant change levelwise, Turkey still managed to pull off a current account surplus in 1988

[1] Current account is defined as net exports of goods and services (including net interest payments) plus private and of icial unrequited transfers with workers' remittances as the major component of these transfers.

and in 1989 and accumulate substantial foreign reserves which enabled the Central Bank to combat effectively the private sector's expectations of devaluation. Furthermore, despite significant increases in its external debt/GNP ratio (less than 40 percent in the first half of the decade, 48.1 percent in 1988 and 51.1 in 1989) Turkey has become a creditworthy country in the international capital markets where it has access to voluntary lending from different sources most presumably due to the increase in its Exports/GNP ratio (below 10 percent before 1985,11.7 percent in 1985 and 16.9 percent in 1988), and the increase in its Debt Service/Exports ratio from 27.1 percent in 1979 to 42.7 percent in 1983 and to a record of 70.7 percent in 1988 (then declining to a still high level of 57.7 percent in 1989).[2]

The "success story" aspect of the adjustment and stabilization program reflected a shift from import substitution and inward orientation to export-led growth. In that effect, the exchange rate has been subject to daily adjustments following a maxi-devaluation of 33 percent in January 1980, current account transactions have been liberalized to an extent by easing some quantity restrictions

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[2] All figures taken and/or calculated from various issues of the The Central Bank of the Republic of Turkey Quarterly Bulletin, The Central Bank of the Republic of Turkey Monthly Statistical Bulletin, and Treasury Monthly Indicators. The time series for the variables in the estimation sections are also obtained from the same sources. Monthly aggregate real income has been simulated from the annual data using a simulation program in RATS, version 3.02.

and exports have been highly subsidized and promoted. Only in 1989 has the government abandoned the daily adjustment of the exchange rate and the Turkish Lira has appreciated against major currencies. Exports have suffered as a result;despite this, the Central Bank has proclaimed early in the year its commitment to present flight from domestic toward foreign currency (i.e., dollarization of the economy). The unprecedented size of the Central Bank's foreign exchange reserves coupled with the expectation of no drastic changes in the current account is likely to render that commitment realizable.

The most disturbing aspect of the adjustment program has been the high inflation which was in the 35 to 50 percent per annum range in the 1980-85 period and in the vicinity of 70 percent recently. The aim of this study is to investigate the inflation tax mechanism and the government's approach to its real revenue from the creation of money in the post-1980 Turkey. The analysis is going to comprise three basic problems in the theory of inflationary finance: firstly, the maximization of inflation tax revenue; secondly, inflationary effects of aggregate real GNP growth rate changes and thirdly a positive theory of government with respect to the collection of the tax revenue. The specification of the money demand function is of pivotal importance in the analysis of the first two problems.

CHAPTER II

FINANCIAL LIBERALIZATION AND INFLATIONARY FINANCE

The liberalization and structural reform program that started in Turkey in 1980 had the deregulation of bank deposit rates as its most crucial domestic component. Increasing savings and redirecting the savings that had previously gone into inflation hedging assets (primarily gold, foreign currency, and real estate) into organized financial institutions were necessary to fight inflation. To serve the purpose nominal deposit rates were increased substantially, resulting in positive real rates after half a decade of severely negative rates.[3]

Further structural changes in the form of lifting the ceilings on the nominal lending rates and reducing the liquidity and reserve requirement ratios were brought about to improve the allocation of financial resources. Most

[3] Real interest rates per annum on demand deposits were around -40% in 1979 but became positive in the second half of 1980. In 1988 the real rate was negative (around an average of -5%) for the first three quarters but was approximately 3% in the last quarter. In 1989 it has mostly remained positive in single digits. Similar fluctuations, sometimes more severe, are observed throughout the decade (an average close to 15% in 1985 and around -15% in 1987).

significantly, the reserve ratios which stood at 35 and 30 percent, respectively, for sight and time deposits were unified at 25 percent in 1983. This unified ratio was further reduced to 15 percent in March 1986.

Having been put into effect in 1984, a partial capital account liberalization has allowed citizens to open foreign currency deposit accounts and such accounts have since become major competitors against other assets.

Although the inherent logic of financial liberalization - enriching the menu of financial assets available to the public and allowing the rates of return on the assets to be determined within the framework of a reasonably deregulated market - is generally desirable, the practice of liberalization may place a heavy burden on the government, particularly on its efforts to finance the budget deficit. Before illustrating this point any further, it should also be noted that in any evaluation of the effects of financial reform on savings, a suspicious approach is warranted for two reasons: the offsetting income and substitution effects of increased interest (deposit) rates imply an ambiguous impact on savings and a number of empirical studies have not established the real deposit rate as a significant determinant of saving in different countries (e.g., Giovannini (1985)).

Bearing in mind that aside from foreign borrowing public sector deficits can be financed within limits by issuing domestic debt or base money, the portfolio shifts which occur as a result of financial liberalization assume a predominant role in deficit financing.

Following financial liberalization the competition coming from the financial assets in the enriched menu forced the government to increase the return on its liabilities in the face of decontrolled deposit rates and foreign currency accounts. The realized real rate of return on the income sharing certificates introduced and issued by the post-reform government was around 30 percent and that of the treasury bonds was around 20 percent in 1988. Both of these rates exceed the corresponding rates on time and foreign currency deposits, thus increasing the cost of deficit financing for the government. The competition of government securities with foreign currency deposits raises the cost of domestic borrowing via the exchange rate policies and devaluation expectations of the private sector as well. To illustrate the point in a most simplistic framework, let i denote the domestic nominal interest (deposit) rate, i^{*} the corresponding foreign (foreign currency deposit) rate, and $\bullet$ the depreciation of Turkish Lira. Then $i = i^{*} + E\,\bullet$, where E stands for the expectations operator, would be the simple portfolio equilibrium requirement. The same equality would preserve its equilibrium characteristic with real rates and real exchange rate depreciation, i.e., $r = r^{*} + E\,g$, where g stands for the expected depreciation of the real exchange rate, could replace the equality above. Hence, keeping the real exchange rate on a declining path would feed into the expectations of the public and raise **r** above **r***. This in turn would put further strains on the government in its endeavors to finance its deficits by resorting to domestic borrowing.

In the theory of public finance, inflation works as a tax which transfers resources from the holders of money to their issuers, thus creating revenue for them.[4] The tax base is the real money balances held by the public and the tax rate is the inflation rate.

Inflation serves as a tax because it induces the holders of money to adjust their real cash balances to the point where the marginal productivity of cash balances equals the expected opportunity cost of holding those balances.[5] The expenditures of the holders of money would be reduced in their endeavors to adjust their cash balances as they constantly spend part of their income in the adjustment process. Those goods and services the purchases of which are foregone by the holders of money can thus be purchased by the issuers of new money.

The proper definition of the tax base in the analysis of

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[4] Two pioneering articles on inflationary finance are Bailey (1956) and Cagan (1956). Other important works on the same topic are Marty (1967 and 1973), Friedman (1971), Phelps (1973) and more recently Calvo and Peel (1983) and Bruno and Fischer (1990).

[5] In the absence of taxes on interest income, the cost is equal to the expected rate of inflation π, or the nominal rate of interest $i = \pi + r$ where r stands for the real rate of interest in this Fisherian equation. This dual definition of the opportunity cost of holding cash balances can be generated by the assumption that the real rate of interest r is adamant to changes in the expected rate of inflation.

inflation tax is crucial. Defining the tax base as narrow money (M1) or broad money would include the transfer of resources from the public to commercial banks rather than to the government. It is only that portion of the transfer of resources which comprises government revenue that is of relevance for deficit financing analysis. Hence, the transfer that occurs from the public and commercial banks to the government through the currency held by the public and the reserves of the commercial banks at the Central Bank suggests that the proper base for inflation tax is high-powered money.

It is often asserted in inflationary finance theory that there is a trade-off between financial liberalization and inflation tax : a reduction in the demand for base money following liberalization requires an increase in the rate of inflation (or for that purpose, the rate of growth of nominal money stock) consistent with financing a given budget deficit. The underlying premise for the asserted trade-off is that the proliferation of financial assets and the liberalization of the rates of return on bank deposits will induce the private sector to substitute away from cash holdings into the assets in the enriched menu, thus eroding the tax base, the high-powered money. This, as mentioned above, would require a corresponding increase in the tax rate, i.e., the rate of inflation, in order to generate a pre-specified revenue from inflation. Contemplating an over-simplified pedagogical framework where currency and time deposits are the competing assets,[6] deregulation of time deposit rates would lead, on the part of the private sector, to a substitution away from inflation tax base thus would be eroded by **(1 - f)** times the amount of funds shifted where **f** stands for the reserve ratio for time deposits.

The reserve requirement ratio is the second key instrument -the first being the rate of inflation - that any government resorting to inflation tax as a source of revenue has access to. It has been demonstrated by Bailey (1956) and Calvo and Fernandez (1982) that financial liberalization in the form of a more competitive banking system with a reduced reserve requirement ratio will push the monetary authorities to inflate at a higher rate to finance a given budget deficit. The argument can be outlined in the light of the simplistic model postulated previously. In a fractional reserve requirement framework that ignores intermediation costs, the assumption of competitive banking will yield

$$i_d = (1 - f)\ i$$

where i_d is the deposit rate and i is the nominal interest

---

[6] McKinnon and Mathieson (1981) use an analytically identical framework where they postulate as the third asset demand deposits yet with a near 100 percent reserve requirement which serves to eliminate abnormal profits after service costs have been deducted.The implication is that the rate of return margin between currency and demand deposits under competitive banking remains intact despite potential changes in the rate of inflation and/or interest rates on other assets.

(loan) rate. The opportunity cost of holding cash balances is i, the nominal interest rate and the opportunity cost of holding deposits can be defined as

$$i - i_d = i - (1 - f)\ i = if\ .$$

Hence any reduction in the reserve ratio increases the spread between the opportunity cost of holding currency and that of holding deposits (note that it is the difference between the two costs, not the ratio of the two that is altered) and raises the deposit rate that can be offered by the commercial banks. The ensuing substitution away from currency into deposits erodes the high-powered money, thus the base of the inflation tax. Thus, a reduction in the reserve ratio will, for any given inflation rate, reduce the inflation tax by reducing the tax base, or equivalently, an increase in the rate of inflation will be necessary to generate a given inflation tax revenue provided that the government was initially operating on the 'correct' side of the Laffer curve.

Yet it is intuitively obvious that the assertion above does not implicate an increase in f to its upper limit, 1 , in order to minimize the inflation rate required to finance a given budget deficit. If the upper limit for f is chosen, the competitive banking assumption would drive down the term deposit rate to zero, thereby inducing the public to hold the more liquid asset currency and diminishing the status of term deposits as a source of government revenue. If the monetary authorities resort to the other extreme of setting the reserve ratio at zero,that in turn would be tantamount on their part to abandoning the term deposit part of the tax base. Hence,the locus of points corresponding to a given budget deficit in the π, f plane would imply a U-shaped curve rather than a one-to-one mapping.

To highlight the immanent dangers of inflationary finance during periods of financial liberalization when large budget deficits are simultaneously observed, we can borrow a simple model developed by Dornbusch and Reynoso (1989). Assume that a fraction β of deficit is financed by issuing high-powered money,i.e.,

$$\dot{H} = \beta \cdot D$$

where D stands for the nominal deficit, H for nominal high-powered money and the dot above indicates the time derivative. Let g represent the nominal deficit/nominal income ratio, μ the rate of growth of nominal high-powered money and define velocity as

$$V = V_0 + {}^{n} \cdot \pi$$

where π is the inflation rate, V_0 and ^{n} are the parameters of the velocity function (V_0 is the velocity at the zero inflation rate and ^{n} represents the sensitivity of velocity to changes in π), it follows that

$$\mu = \beta \cdot g \cdot (V_0 + {}^{n} \cdot \pi) \quad . \tag{i}$$

In the steady state,

$$\pi = y - \mu \tag{ii}$$

where y is the rate of growth of aggregate real income. Substituting (i) into (ii) and solving for π yields

$$\pi = (\beta g V_0 - y)/(1 - \beta g^{n}) \tag{iii}$$

Looking at the first and second derivatives of π with respect to g,

$$d\pi/dg = \beta[V_0 - {}^{n}y]/(1 - \beta g^{n})^2$$

and

$$d^2\pi/dg^2 = 2\beta^{2n}[V_0 - {}^{n}y][1 - \beta g^{n}]^{-3}$$

it can be concluded that for all reasonable values of the parameters V_0 , ^{n} , y , β and g both derivatives will be positive. The conclusion thereby reached is that an increase in the deficit income ratio raises the inflation rate required to finance the deficit and that the higher is the deficit/income ratio, the higher also will be the rate at which inflation must be rising to finance that deficit.

We should finally make note of potential effects of financial liberalization on the velocity of high-powered money. Conventional wisdom in inflationary finance theory, as mentioned before, anticipates a reduction in high-powered money following financial liberalization. That implies a reduction in the velocity of high-powered money and Dornbusch and Reynoso assert that the coefficients of the velocity function, V_0 and ρ, will increase following financial liberalization.[7] Deposit rate deregulation, enriched asset menu, reduced reserve requirement ratios - all crucial institutional aspects of financial liberalization - and the public's enhanced ability to economize on currency holdings are the underlying factors in the above result.

In "financially repressed" economies where inflation hedging assets assume a predominant presence, the deregulation of deposit rates entailed within the liberalization package is apt to bring about a substitution away from such primarily non-monetary assets into bank deposit accounts . In the words of Shaw, "the deepening of finance increases the real size of

[7] Dornbusch and Reynoso (1989), p.19.

the monetary system" and "the velocity of money diminishes."[8] The building block in Shaw's argument is the increase in the real deposit rate: "increases in real deposit rate result in money deepening; they raise the ratio of stock of money demanded to income, and hence of the equilibrium stock of money relative to income..." Thus a reduction in the velocity of money following financial liberalization is predicted in the financial repression paradigm. However, these results should be interpreted as applying to broad rather than high-powered money as the bulk of the shift from inflation- hedging assets and currency is going to be towards deposit accounts.[9]

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[8] Shaw (1973), p.8.

[9] Dornbusch and Reynoso, i.b.i.d., p.10, state that a measure of financial deepening should not even be the M2/GDP ratio but rather the M3/GDP ratio due to potentially significant amounts of deposits in non-bank institutions. They cite South Korea and Brazil as two examples where the M2/GDP ratio remained relatively constant in the former but declined in the latter whereas the M3/GDP ratio has incresed in both. Thus they do indeed concede that a reduction in the velocity of broad money is to be expected while they postulate in their model an increase in the velocity of high-powered money following financial liberalization.

CHAPTER III

INFLATION TAX REVENUE AND THE DEMAND FOR MONEY FUNCTION: THEORETICAL BACKGROUND

Let the demand function for real cash balances be defined by the function

$$(M/P)^d = \theta(Y,\pi) \qquad (1)$$

where $(M/P)^d$ is the demand for real balances, Y is real income and π is the (expected = actual) rate of inflation[10]. Taking the logarithmic time derivative of (1) we obtain

$$d\ln M/dt - d\ln P/dt = 1/\theta(\ )[\partial\theta(\ )/\partial Y\cdot(dY/dt) + \partial\theta(\ )/\partial\pi\cdot(d\pi/dt)]$$

Given $d\pi/dt = 0$ at the new equilibrium,

$$d\ln M/dt = d\ln P/dt + [\partial\theta(\ )/\partial Y\cdot(Y/\theta(\ ))]\{dy/dt\cdot(1/Y)\}$$

---

[10] When real balances fully adjust to a given new inflation rate, $d\pi/dt$ will equal zero at the new equilibrium and the actual inflation rate will be equal to the expected. The analysis can thus be regarded as one pertaining to alternative steady state equilibria.

The term preceding the plus sign is the inflation rate(π). The bracketed term is the elasticity of the demand for real cash balances with respect to real income (denoted from here on as α) and the final term is the rate of growth of real income(g_Y). Thus,

$$g_M = \pi + \alpha \cdot g_Y \qquad (2)$$

where g_M defines the rate of growth of nominal money.

Inflation tax revenue(H) is defined as the rate of growth of (high-powered)[11] money times the quantity of real money ,i.e.,

$$H = (M/P) \cdot g_M = (M/P) \cdot (\pi + \alpha \cdot g_Y) \qquad (3)$$

Prior to selecting a specific functional form for the demand for money one has to note that a per capita demand for real cash balances function and an aggregate one are not by definition interchangeable and can be shown to be equivalent only if the elasticity of the demand for real balances with respect to income is unity[12] and/or the rate of growth of

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[11] From this point on high-powered money which we previously defined as the proper base for inflation tax purposes will simply be referred to as money unless anything to the contrary is stated.

[12] From the Quantity Theory equation MV = PY where all the variables are defined as before and V stands for velocity, solving for V and taking the natural log and the total differential yield

population is zero.

The Quantity Theory equation can be written in per capita terms as

$$M \cdot V = P \cdot X \cdot N \qquad (4)$$

where the new variables X and N stand for output (income) per man and population, respectively. We will assume that population per se does not have a scale effect on output per man and per capita demand for real balances $M/P \cdot N$ (= X/V), i.e., both variables are assumed to be homogeneous of degree zero in population. The elasticity of per capita demand for

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$d\ln V = d\ln Y - d\ln(M/P)$.

Defining

$M/P = m(Y,\pi)$

we obtain

$$\begin{aligned} d\ln V &= d\ln Y - d\ln m(Y,\pi) \\ &= d\ln Y - (1/m(Y,\pi)) \cdot dm(Y,\pi) \\ &= d\ln Y - (1/m(Y,\pi)) \cdot \{\partial m(Y,\pi)/\partial Y \cdot dY + \partial m(Y,\pi)/\partial \pi \cdot d\pi\} \\ &= d\ln y - \{\partial m(Y,\pi)/\partial Y \cdot (Y/m(Y,\pi))\} \cdot (dY/Y) \\ &\qquad - \{\partial m(Y,\pi)/\partial \pi \cdot (\pi/m(Y,\pi))\} \cdot (d\pi/\pi) \\ &= d\ln Y - \alpha \cdot d\ln Y - {}^{n} \cdot d\ln \pi \\ &= [1 - \alpha] \cdot d\ln Y - {}^{n} \cdot d\ln \pi \ . \end{aligned}$$

where ^{n} is the inflation elasticity of the demand for real cash balances. The result indicates that if the elasticity of real cash balances with respect to real income is unity, velocity will be independent of the rate of growth of income but not the inflation rate.

real balances with respect to output per man can be defined as

$$\mathrm{dln}\ (M/P \cdot N)\ /\ \mathrm{dln}\ X = \mathrm{dln}\ (X/V)\ /\ \mathrm{dln}\ X$$

$$= (\mathrm{dln}\ X - \mathrm{dln}\ V)\ /\ \mathrm{dln}\ X = \mu \quad (5)$$

Taking the logarithmic time derivative of (4) we obtain

$$\mathrm{dln}\ M/dt = \mathrm{dln}\ P/dt + \mathrm{dln}\ N/dt + \mathrm{dln}\ X/dt - \mathrm{dln}\ V/dt$$

$$= \mathrm{dln}\ P/dt + \mathrm{dln}\ N/dt + \mu \cdot \mathrm{dln}\ X/dt$$

$$= g_M = \pi + n + \mu \cdot g_X \quad (6)$$

where n is the rate of growth of population and g_X is the rate of growth of per capita output. If n = 0, i.e., population is constant, then $g_X = g_Y$ (as $g_Y = g_X + n$ from $Y = X \cdot N$) and equation (6) will become

$$g_M = \pi + \mu \cdot g_X \quad (6)'$$

which will also equal

$$g_M = \pi + \alpha \cdot g_Y \ .\text{[13]} \quad (2)$$

---

[13] Note that $\mu = \mathrm{dln}\ (M/P \cdot N)\ /\ \mathrm{dln}\ X = \mathrm{dln}\ (M/P \cdot N)\ /\ \mathrm{dln}\ (Y/N)$

Yet if n is different from 0, then (6) will be equivalent to (2) only if $\alpha = \mu = 1$ as then

$$g_M = \pi + g_Y \quad [\text{from}(2)] \qquad \text{and} \quad g_M = \pi + n + g_X \quad [\text{from}(6)]$$

Consequently, in order to justify the utilization of aggregate output (income) in any empirical study of money demand one would have to obtain an estimate of the income elasticity of the demand for aggregate real balances that is statistically not different from unity.

The two most common forms of money demand functions encountered in the literature are the logarithmic and the semi-logarithmic formulations (as functions of the inflation rate). The elasticity of the demand for real balances with respect to the inflation rate is a constant in the former but increases with the rate of inflation in the latter[14]. Furthermore, as will be demonstrated soon under the assumption of a government attempting to keep the ratio of inflation tax revenues to income constant the compensatory change in the

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$$= [d\ln\,(M/P) - d\ln\,N] \div [d\ln\,Y - d\ln\,N]$$

Assuming $n = 0$ (i.e., $d\ln N = 0$), it follows that

$\mu = d\ln\,(M/P) \,/\, d\ln\,Y = \alpha$. Hence the equality of (6)' and (2) in the face of a constant population.

[14] Consider the logarithmic demand function $(M/P)^d = a_0\pi^{-\beta}Y^{\alpha}$. The elasticity of the demand for real balances with respect to the rate of inflation is $-\beta$. For a demand function semi-logarithmic in the rate of inflation, $(M/P)^d = a_0\exp\{-\beta\pi\}Y^{\alpha}$, the same elasticity will equal $-\beta\pi$.

inflation rate in response to a change in the income growth rate is an increasing function of the existing inflation rate in the semi-logarithmic specification but a decreasing one under the logarithmic specification. Thus under the semi-logarithmic formulation which will later be shown to be the appropriate form for the empirical part to follow the theoretical background, a change, say decline, in the rate of growth of income will necessitate a higher increase in the inflation rate the higher is the initial rate of inflation. To formally illustrate the point consider the inflation tax revenue defined in (3) as a percentage of income, i.e.,

$$H/Y = (M/P \cdot Y)\ (\pi + \alpha \cdot g_Y) \qquad (7)$$

Let w stand for $(M/P \cdot Y)$. Assuming that the governments aim is to keep (H/Y) intact, the required compensating change in the rate of inflation can be obtained by setting $d(H/Y) = 0$ and solving for

$d\pi/dg_Y$. Hence, <u>regardless</u> of the specification of the money demand function setting $d(H/Y) = 0$ yields

$$\begin{aligned} 0 &= (\pi \cdot dw + w \cdot d\pi) + \alpha(w \cdot dg_Y + g_Y \cdot dw) \\ &= [\pi(dw/d\pi) + w] \cdot d\pi + \alpha \cdot [w \cdot dg_Y + g_Y \cdot (dw/d\pi) \cdot d\pi] \\ &= w \cdot [(\pi/w)(dw/d\pi) + 1] \cdot d\pi + \alpha \cdot w \cdot dg_Y \\ &\qquad + \alpha \cdot g_Y \cdot d\pi \cdot [(dw/d\pi)(\pi/w)] \cdot (w/\pi) \end{aligned}$$

Collecting terms and defining $\xi = (dw/d\pi)\cdot(\pi/w)$, the elasticity of $(M/P\cdot Y)$ with respect to π, we obtain

$$d\pi\cdot[\xi + 1 + (\alpha\cdot g_Y\cdot \xi/\pi)] = -\alpha\cdot dg_Y$$

Solving for $(d\pi/dg_Y)$ yields

$$d\pi/dg_Y = -\alpha \ / \ [1 + \xi\{1 + (\alpha\cdot g_Y/\pi)\}]^{15} \qquad (8)$$

Note that $\xi = d\ln w/d\ln \pi = d\ln (M/P\cdot Y)/d\ln \pi$ and is different from the inflation elasticity of the demand for real balances which we can define as $^{n} = d\ln (M/P)/d\ln \pi$. To see the relationship between the two consider the general money demand function expressed in (1) rewritten as

$$(M/P\cdot Y) = (1/Y)\cdot\theta(Y,\pi) \qquad (9)$$

Taking the total differential of both sides in natural logs yields

$$\begin{aligned} d\ln (M/P\cdot Y) &= -d\ln Y + d\ln \theta(Y,\pi) \\ &= -d\ln Y + d\theta(Y,\pi)/\theta(Y,\pi) \\ &= -d\ln Y + 1/\theta(Y,\pi)\cdot[(\partial\theta(\ )/\partial\pi)d\pi + (\partial\theta(\ )/\partial Y)dY] \\ &= -d\ln Y + [(\partial\theta(\ )/\partial\pi)\cdot(\pi/\theta(\ ))]\cdot(d\pi/\pi) \end{aligned}$$

---

[15] A necessary and sufficient condition for $d\pi/dg_Y$ to be negative is that $-\xi<[\pi/(\pi + g_Y)]$.

$$+ [(\partial\theta(\ )/\partial Y) \cdot (Y/\theta(\ ))] \cdot (dY/Y) \qquad (10)$$

Given

$^{n} = d\ln\ (M/P)/d\ln\ \pi = d\ln\ \theta(\ )/d\ln\ \pi$

and

$\alpha = d\ln\ (M/P)/d\ln\ Y = d\ln\ \theta(\ )/d\ln\ Y$,

equation (10) can be written as

$d\ln\ (M/P \cdot Y) = (\alpha - 1) \cdot d\ln\ Y + {}^{n} \cdot d\ln\ \pi$.

Solving for $\xi = d\ln\ (M/P \cdot Y)/d\ln\ \pi$ we obtain

$$\xi = (\alpha - 1) \cdot (d\ln\ Y/d\ln\ \pi) + {}^{n} \qquad (11)$$

To further simplify the term, the following relationship can be used:

$[d\ln\ (M/P)/d\ln\ Y] \cdot [d\ln\ Y/d\ln\ \pi] = d\ln\ (M/P)/d\ln\ \pi$

The first bracketed term being equal to α and the right-hand side of the equation denoting ^{n}, $d\ln\ Y/d\ln\ \pi$ can be expressed as $^{n}/\alpha$. Substituting into (11) yields

$$\xi = (\alpha - 1)\cdot(\eta/\alpha) + \eta$$
$$= [(\alpha - 1)/\alpha + 1]\cdot\eta \qquad (12)$$

If the income elasticity of the demand for real balances is equal to unity, then $\xi = \eta$. In an empirical study the increment $(\alpha - 1)/\alpha$ can be used provided the value of α itself is statistically not different from one.

In the case of a semi-logarithmic formulation for the money demand function in the form of

$$(M/P)^d = a_0\cdot \exp\{-\beta\pi\}\cdot Y^{\alpha}$$

the inflation elasticity of the demand for real balances is

$$\eta_{s\cdot l} = d\ln\ (M/P)/d\ln\pi = [d\ln\ (M/P)/d\pi]\cdot\pi = -\beta\pi.$$

From (12) it then follows that

$$\xi_{s\cdot l} = [(\alpha - 1)/\alpha + 1]\cdot(-\beta\pi) \quad \text{and}$$

$$d\pi/dg_y = -\alpha\ /\ [1 + ((\alpha - 1)/\alpha + 1)(-\beta\pi)\{1 + (\alpha\cdot g_y/\pi)\}]$$
$$= -\alpha\ /\ [1 + ((\alpha - 1)/\alpha + 1)(-\beta)(\pi + \alpha\cdot g_y)]\ ^{16} \qquad (13)$$

---

[16] When the income elasticity of real balances (α) is assumed to be unity,

$$d\pi/dg_y = -1\ /\ [1 - \beta(\pi + g_y]$$

which is identical to the compensatory inflation change expression obtained by Melnick and Sokoler(1984, pp.229) and

Taking the derivative of (13) with respect to π we obtain

$$d[d\pi/dg_Y] \; / \; d\pi = -\alpha \cdot (-1) \cdot [\ldots\ldots]^{-2} \cdot (-\beta) \cdot ((\alpha - 1)/\alpha + 1) \; .$$

For all reasonable values of α the final parenthesis term will be positive thus making the entire term negative and $|d[d\pi/dg_Y] \; / \; d\pi|$ positive. The result in plain words and the rationale behind it can be explained as follows: the higher is the existing rate of inflation, the higher would have to be the compensating percentage point (not percentage) change in the inflation rate in the face of a given percentage point change in the rate of growth of real income. That is so because the inflation rate would have to change not only to compensate for the growth rate change (which alters the quantity of taxable balances) but also to compensate for the inflation elasticity of real balances the absolute value of which is an increasing function of the rate of inflation. As the inflation elasticity will be higher at higher rates of inflation, a given percentage point change in the rate of inflation will bring forth subsequently higher percentage changes in the amount of real balances held thus reducing the taxable base by a larger proportion than it would at low rates of inflation.

If the purpose of the government is to maximize the

Bruno and Fischer(1990, pp.360).

inflation tax revenue rather than to keep its ratio to income constant, it would resort to inflating the nominal supply of money at a rate that would accomplish the objective. Considering the semi-logarithmic formulation which will be justified and used in the empirical portion of the study to follow, the inflation revenue maximizing rate is easily calculable. Given

$$(M/P)^d = a_0 \cdot \exp\{-\beta\pi\} \cdot Y^{\alpha}$$

and

$$H = (\pi + \alpha \cdot g_Y) \cdot (M/P)^d$$
$$= (\pi + \alpha \cdot g_Y) \cdot [a_0 \cdot \exp\{-\beta\pi\} \cdot Y^{\alpha}] \; ,$$

differentiating with respect to the rate of inflation and setting $dH/d\pi = 0$ results in

$$dH/d\pi = 0 = [a_0 \cdot \exp\{-\beta\pi\} \cdot Y^{\alpha}] + (\pi + \alpha \cdot g_Y) \cdot [a_0 \cdot \exp\{-\beta\pi\} \cdot Y^{\alpha}] \cdot (-\beta)$$
$$= 1 - \beta(\pi + \alpha \cdot g_Y) \; .$$

Inflation revenue maximizing rate of inflation is thus given as

$$\pi^{*}= 1/\beta - \alpha \cdot g_{Y}$$ [17] (14)

[17] It can be shown analogously that if per capita output and per capita demand for real balances are to be used instead of their aggregate counterparts, the revenue maximizing rate of inflation is
$\pi^{*}= 1/\beta - n - \alpha \cdot g_{x}$.

CHAPTER IV

ESTIMATION TECHNIQUES EMPLOYED IN THE MONEY DEMAND REGRESSION EQUATION

In calculating the regression equation of the money demand for real balances in the post-1980 Turkey, the general formulation

$$(M/P)^d = \theta(Y,\pi)$$

where M stands for nominal money, P for the price level, Y for aggregate real income, and π for the actual (= expected) rate of inflation is used. The crucial step is defining the specific form of the θ in π. The tests performed in this study are limited to finding whether the demand for real balances is logarithmic or semi-logarithmic in π (it is assumed that it is logarithmic in Y). To serve the purpose, the Box-Cox transformation

$$x^{(\lambda)} = (x^{\lambda} - 1)/\lambda \qquad (15)$$

is applied to the π variable.[18] Note that in the following regression

$$\ln y = \alpha + \beta \cdot [(x^{\lambda} - 1)/\lambda] + \varepsilon$$

if $\lambda = 1$, then

$$\ln y = \alpha + \beta \cdot (x - 1) + \varepsilon$$
$$= (\alpha - \beta) + \beta \cdot x + \varepsilon$$

and y becomes semi-logarithmic in x. Although $x^{(\lambda)}$ is undefined at $\lambda = 0$, its limit as $\lambda \to 0$ can be found by applying L'Hôpital's Rule:

$$\lim_{\lambda \to 0} (x^{\lambda} - 1)/\lambda = \lim_{\lambda \to 0} [x^{\lambda} \cdot \ln(x)]/1$$
$$= \ln x.$$

Hence $\lambda = 0$ in the Box-Cox transformation implies

$$\ln y = \alpha + \beta \cdot \ln x + \varepsilon ,$$

i.e., that y is logarithmic in x.

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[18] For a detailed discussion of the several approaches to the estimation of the parameters in Box-Cox models, see Spitzer(1982a and 1982b).

The demand for real balances regression to be estimated in this study is

$$\ln (M/P)_t^d = a_0 + \beta \cdot \pi_t^{(\lambda)} + \alpha \cdot \ln Y_t + \varepsilon_t \qquad (16)$$

where

$$\pi_t^{(\lambda)} = (\pi_t^{\lambda} - 1)/\lambda \ .$$

Analogous to the presentation of the Box-Cox transformation previously, a λ value statistically equal to zero would imply that the demand for real balances is logarithmic in π. A semi-logarithmic Cagan-type demand function would be the proper specification if the value of λ were found to be statistically not different from one.

A standard assumption in money demand models is that short-run actual real balances divert from their long-run desired levels. Transaction costs involved in adjusting money holdings, sluggish adjustment of prices, interest rates or income can all be factors leading to this divergence.[19] The partial adjustment mechanism to be incorporated into the money demand regression equation to account for the lagged

[19] Substantial interest income losses that would be suffered due to pre-mature withdrawal of funds from deposit accounts is another factor causing lagged adjustment of real balances in a high inflation economy like Turkey.

adjustment of real balances is

$$\ln (M/P)_t - \ln (M/P)_{t-1} = h \cdot [\ln (M/P)_t^d - \ln (M/P)_{t-1}] \qquad (17)$$

where actual balances at time t adjust to the gap between desired real balances at time t and the actual balances at the end of period t-1 (and the beginning of period t). In the above formulation h defines the speed of adjustment and a value of one for h implies instantaneous adjustment of real balances, i.e., $\ln (M/P)_t^d = \ln (M/P)_t$. The lower is the frequency of the data used (e.g., quarterly as opposed to monthly) the higher would one expect the value of h to be.[20] Substituting (17) into (16) gives

$$\ln (M/P)_t = (h \cdot a_0) + (h \cdot \beta)\ \pi_t^{(\lambda)} + (h \cdot \alpha)\ \ln Y_t + (1 - h)\ \ln (M/P)_{t-1} + \varepsilon_t \qquad (18)$$

In estimating the above regression equation which is non-linear in π, a simple one dimensional grid search method and a non-linear least squares technique have been employed.

The one dimensional grid search method is basically one of trial and error. In our case, the value of λ is expected to

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[20] The estimate of h in this study using monthly data (1980:1 to 1990:6) was approximately 0.09 whereas quarterly data for the same time interval produced $h \approx 0.2$.

be between 0 and 1.[21] The estimation of λ begins by scanning a selected range of λ in the increments of 0.1 and transforming the variable subject to the Box-Cox transformation using the values of λ in the range. Once a minimum sum of squared residuals is spotted, search will be restricted to the left and right of the current optimum λ in smaller increments of 0.01. The process can be carried on any number of times using smaller increments depending on the degree of precision desired. The optimal value of λ thereby obtained, the least squares estimates can then be estimated by calculating the values of the variable under Box-Cox transformation using the final optimal λ.[22]

Non-linear least squares methods operate as iterative linearization methods.[23] The non-linear equation is first linearized around some initial set coefficient values. Application of linear least squares to the linearized equation

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[21] Greene ((1990), p. 349) states the value of λ in general is expected to be between -1 and 1. Since the dependent variable in this study, ln $(M/P)_t$, is in the natural log form, 0 and its vicinity for λ would imply a formulation logarithmic in π and 1 and its vicinity a semi-logarithmic one. The range to be scanned thus was chosen initially as 0.4 to 1.2.

[22] At the optimal λ, the least squares estimates, the mean squared residual and this value of λ are the non-linear least squares and the maximum likelihood estimates of the parameters (Greene, p.349).

[23] A non-linear regression model can be described as one in which the first order conditions for the minimization of the sum of squared residuals are non-linear functions of the parameters in the model.

yields a new set of coefficient values and these become the next set of initial values around which the non-linear equation is to be re-linearized. The same iterative process continues until convergence in the values of coefficients is attained; i.e., iteration stops when substantial changes (depending on a δ defined at the outset which has a default value in most software packages) are no more observed in the coefficient values.

Underlying the application of non-linear least squares methods is the notion that any non-linear function can be approximated as a linear one through a Taylor series expansion. Consider the regression equation

$y = f(x_i,\ i=1,\ldots\ldots,k\ ;\ \beta_s,\ s=1,\ldots\ldots,r)$[24] $+ \varepsilon$.

Taylor expansion around an initial set of coefficient values $\beta_{10},\ \beta_{20},\ldots\ldots,\beta_{r0}$ will yield

$$y \approx f(x_i,\ i=1,\ldots\ldots,k\ ;\ \beta_{10},\ \beta_{20},\ldots\ldots,\beta_{r0}) + \Sigma(\partial f(\ )/\partial\beta_i)_0\ (\beta_i - \beta_{i0})$$
$$+ (1/2)\cdot\Sigma\Sigma(\partial^2 f(\ )/\partial\beta_i\partial\beta_j)_0\cdot(\beta_i - \beta_{i0})(\beta_j - \beta_{j0}) + \ldots\ldots + \varepsilon \quad (19)$$

where subscript 0 indicates evaluation at the initial

[24] Note that unlike linear regression, the number of parameters(k) is not equal to the number of regressors (k), but r > k.

coefficient values. Linear approximation requires that the terms after the first two be dropped, thus enabling us to write (19) as

$$y - f(x_i,\ i=1,\dots,k\ ;\ \beta_{10},\dots,\beta_{r0}) + \Sigma(\partial f(\ )/\partial\beta_i)_0 \cdot \beta_{i0}$$
$$\approx \Sigma(\partial f(\ )/\partial\beta_i)_0 \cdot \beta_i + \varepsilon \ . \qquad (20)$$

The left-hand side represents the newly constructed variable and the right-hand side entails a set of unknown coefficients. A linear regression with the partial derivatives evaluated at $(\beta_{10},\dots,\beta_{r0})$ as the regressors will generate the new set of initial values β_{11}, $\beta_{21},\dots,\beta_{r1}$. This set consequently becomes the new set of initial values and a re-linearization is performed around these set and

$$y - f(x_i,\ i=1,\dots,k\ ;\ \beta_{11},\beta_{21},\dots,\beta_{r1}) + \Sigma(\partial f(\ )/\partial\beta_i)_1 \cdot \beta_{i1}$$
$$\approx \Sigma(\partial f(\ )/\partial\beta_i)_1 \cdot \beta_i + \varepsilon$$

is obtained where the subscript 1 now denotes evaluation at the new initial values. Ordinary least squares ia applied to this regression, the new set of coefficient values are obtained and utilized in the next linearized regression equation as the new initial values.

Thus given $Y = F(\mathbf{X},\mathbf{B}) + \varepsilon$ as the non-linear regression equation with n observations, linear approximation by means of a Taylor series expansion around an initial coefficient vector B_0 gives

$$Y \approx F(\mathbf{X},\mathbf{B}_0) + \Sigma\{\partial F(\ )/\partial \beta_k\}_0 (\beta_k - \beta_{k0}) + \varepsilon \tag{21}$$

where subscript 0 indicates evaluation at the initial coefficient vector $\mathbf{B}_0$ and k is the number of coefficients in the model. Rearranging we obtain

$$Y - F(\mathbf{X},\mathbf{B}_0) + \Sigma\{\partial F(\ )/\partial \beta_k\}_0 \beta_{k0} \approx \Sigma\{\partial F(\ )/\partial \beta_k\}_0 \beta_k \tag{22}$$

Defining $\tilde{X}_{k0}$ as the k^{th} partial derivative $(\partial F(\ )/\partial \beta_k)_0$, (22) can be written as

$$Y - F(\mathbf{X},\mathbf{B}_0) + \Sigma\, \tilde{X}_{k0}\, \beta_{k0} \approx \Sigma\, \tilde{X}_{k0}\, \beta_k + \varepsilon$$

or

$$Y - F(\mathbf{X},\mathbf{B}_0) + \tilde{\mathbf{X}}_0'\mathbf{B}_0 \approx \tilde{\mathbf{X}}_0'\mathbf{B} + \varepsilon \tag{23}$$

where $\tilde{\mathbf{X}}_0$ is an $(n \times k)$ matrix of partial derivatives. The newly constructed variable on the left-hand side is to be regressed on the right-hand side and the iteration algorithm is as mentioned before.

The non-linear least squares method used in this study is the Gauss-Newton method. It is employed in the minimization of the SSR and can formally be expressed as follows[25] : Suppose for a given function $Z = F(\mathbf{B})$, the first order optimization condition $\partial F(\ )/\partial \mathbf{B} = \mathbf{0}$ is non-linear in $\mathbf{B}$. Expanding the non-linear first order condition around an initial vector $\mathbf{B}_0$ by a Taylor series expansion yields

$$\partial F(\ )/\partial \mathbf{B} \approx (\partial F(\ )/\mathbf{B})_0 + (\partial^2 F(\ )/\partial \mathbf{B}\partial \mathbf{B}')_0\ (\mathbf{B} - \mathbf{B}_0) = \mathbf{0} \qquad (24)$$

as the linear approximation of the first order condition.

Let $(\partial F(\ )/\partial \mathbf{B})_0$ be denoted by $\mathbf{w}_0$ and $(\partial^2 F(\ )/\partial \mathbf{B}\partial \mathbf{B}')_0$ by $\mathbf{W}_0$. Eq.(24) can then be written as

$$\partial F(\ )/\partial \mathbf{B} \approx \mathbf{w}_0 + \mathbf{W}_0(\mathbf{B} - \mathbf{B}_0) = \mathbf{0}$$

It then follows that

$$\mathbf{W}_0\mathbf{B} = \mathbf{W}_0\mathbf{B}_0 - \mathbf{w}_0$$

and solving for $\mathbf{B}$ yields

$$\begin{aligned} \mathbf{B} &= \mathbf{W}_0^{-1}\mathbf{W}_0\mathbf{B}_0 - \mathbf{W}_0^{-1}\mathbf{w}_0 \\ &= \mathbf{B}_0 - \mathbf{W}_0^{-1}\mathbf{w}_0\ . \end{aligned} \qquad (25)$$

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[25] See Greene, p. 367 and Doan, p. 5-21.

Defining $\mathbf{B}_0$ as $\mathbf{B}_t$ and $\mathbf{B}$ as $\mathbf{B}_{t+1}$, we obtain the iteration algorithm

$$\mathbf{B}_{t+1} = \mathbf{B}_t - \mathbf{W}_t^{-1}\mathbf{w}_t \ . \qquad (26)$$

In the non-linear regression case, $\bullet'\bullet$ is to be minimized and the first order condition $\partial(\bullet'\bullet)/\partial\mathbf{B}$ will be nonlinear in the parameters of the model. Thus $F(\mathbf{B}) = -\bullet'\bullet$ and the Gauss-Newton method maximizes $-\bullet'\bullet$ and solves for the first order conditions by the algorithm described above.

As will be shown in the estimation results section, one dimensional grid search method and non-linear least squares result in coefficient estimates that are almost identical. A maximum likelihood approach was also attempted for the non-linear money demand equation in this study but results have been less successful than those achieved by the two above mentioned methods since convergence was not attained despite a large number of iterations. There may be, in Harvey's words " a heavy penalty attached to having a large number of parameters in a non-linear optimization routine" and concentrating the likelihood function by finding ways to reduce the number of parameters in the optimization process is the route recommended by Harvey[26]. As satisfactory results were obtained by the two methods employed, maximum likelihood

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[26] Harvey (1990), p.139.

approach was not explored any further.

CHAPTER V

SPECIFICATION OF MONEY DEMAND REGRESSION EQUATION

The money demand function given in (18) which has the partial adjustment mechanism incorporated into it is estimated for Turkey using monthly data from 1980:1 to 1990:6.[27] The first technique employed is the one dimensional grid-search method. Table (1) on page 39 shows the scanned range of λ from 0.5 to 1.1 in increments of 0.1 and from 1.0 to 1.1 in increments of 0.01 (values of λ smaller than 0.5 and greater than 1.1 do not affect the optimal value of λ suggested in the table and are therefore omitted). Sums of squared residuals (SSR) corresponding to the given value of λ are presented in the second column of Table (2). SSR apparently reaches a minimum at $\lambda = 1.04$ and this value is considered the optimal λ for a precision level of two digits after the decimal point.

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[27] The variables used in the regression have been seasonally adjusted using the Census II method developed by the Bureau of the Census of the U.S. Department of commerce. It is based on computing seasonal indices that try to capture the seasonal fluctuations in the time-series variables and subsequently using those indices to remove the seasonal variations in the variables, thereby deseasonalizing them. For an introductory exposition of seasonal adjustment, see Pindyck and Rubinfeld (1990,p.432-436), Johnston (1984,p.234-239). For a detailed description of the Census II method, see Salzman (1968).

TABLE (1): SSR for λ from 0.5 to 1.1

λ	SSR
0.5	.10362018
0.6	.10208682
0.7	.10090721
0.8	.10004439
0.9	.099486728
1.0	.099226144
1.01	.099215947
1.02	.099208564
1.03	.099203975
1.04	.099202157
1.05	.099203089
1.06	.099206745
1.07	.099213102
1.08	.099222133
1.09	.099233812
1.1	.099248110

The next step is to calculate the values for the π variable that is subject to the Box-Cox transformation using the optimal value of λ and substitute the newly calculated variable as the π regressor in the money demand regression (18) rewritten as

$$\ln (M/P)_t = b_0 + b_1 \cdot ((\pi_t^{1.04} - 1)/1.04) + b_2 \cdot \ln Y_t$$
$$+ b_3 \cdot \ln (M/P)_{t-1} + \varepsilon_t \qquad (27)$$

where b_i coefficients have replaced the coefficients in parentheses in (18) for notational convenience. The following results were obtained (the numbers in parentheses represent t-values for the coefficient estimates):

$$\ln (M/P)_t = \underset{(-5.122514)}{-1.116703} - \underset{(-7.461163)}{0.9872209} ((\pi_t^{1.04} - 1)/1.04)$$

$$+ \underset{(2.730309)}{0.08825156} \ln Y_t + \underset{(28.88483)}{0.9112811} \ln (M/P)_{t-1} + e_t$$

(28)

SSR = .099202157 Durbin-Watson = 2.2659

R^2 = .9667 $\bar{R}^2$ = .9659

As one of the regressors is the lagged value of the dependent variable, the Durbin-Watson statistics is not informative of any autocorrelation in the disturbance term. Under the null hypothesis of no autocorrelation, the Durbin-h statistic

$$h = r \cdot [n/(1 - n \cdot var(\hat{b}_{lag})]^{1/2} \sim \text{Asym.N. } (0,1)$$

where $var(\hat{b}_{lag})$ is the sampling variance of the lagged dependent variable and r is the estimate of the first order autocorrelation coefficient which approximately equals [1 - (D.W)/2]. The value of r being equal to -.13297 and h calculated as -1.5887, the null hypothesis of 0 first order autocorrelation can not be rejected at the 5 percent level of significance.

The money demand regression equation given in (18) was estimated using a non-linear least squares method employing the Gauss-Newton algorithm as well. The following were the estimation results (with the t-values given in parentheses):

$\hat{b}_0$ = -1.118986 (-2.773187)

$\hat{b}_1$ = -0.9915305 (-1.517282)

$\hat{b}_2$ = .08819445 (2.629349)

$\hat{b}_3$ = .9112922 (28.7273)

$\hat{\lambda}$ = 1.041599 (4.396723)

SSR = .0992202122

R^2 = .96669209 $\bar{R}^2$ = .96558183

Note that in comparison to the grid-search estimation results, the only visible change is in the t-value of $\hat{b}_1$. That is not a contradictory result as b_1, unlike the grid-search case, is the coefficient for the π variable which itself is a function of another parameter in the model. The crucial point to observe is that the value of $\hat{\lambda}$ is very close to the value obtained via the grid-search method. There is a slight reduction in the SSR as the degree of precision in the grid-search method, though satisfactory for all practical purposes, can not match that achieved through the use of non-linear least squares. Also pertaining to the comparison of the results from the two estimation techniques is the fact that one should not look at the estimated standard error of the disturbance term in measuring the statistical improvement stemming from the employment of non-linear least squares; SSR is the proper statistic for that purpose. The reasoning is that with $\hat{\sigma}^2 = e'e/(n-k)$ the denominator will always be smaller under non-linear least squares estimation as there is one more parameter, λ , to be estimated. SSR, on the other hand, depends only on n, the number of observations and is independent of k, the number of parameters.

Having derived the estimate of $\hat{\lambda}$ the next step is to test whether the true λ is statistically different from 1. To serve the purpose, two different tests - Likelihood Ratio Test and Lagrange Multiplier Test - were utilized.

a) Likelihood Ratio Test: The likelihood ratio test statistic

$$\delta = -2\ (\ln L_* - \ln L)^{28} \qquad (29)$$

[28] See Greene (p.356) and for a more detailed exposition Harvey (p. 162-166).

is asymptotically distributed as χ^2_j where j is the number of restrictions, subscript * denotes the restricted model and L the likelihood function. Given the standard log-likelihood function

$$\ln L = -n/2 \cdot \ln 2\pi - n/2 \cdot \ln \sigma^2 - (1/2\sigma^2)\ \varepsilon'\varepsilon \qquad (30)$$

under the assumption $\varepsilon \sim N(0,\sigma^2 I)$ where I stands for the identity matrix, the likelihood ratio statistic can be expressed in terms of the estimated error term variances of the restricted and the unrestricted models. As the maximum likelihood estimate of σ^2 is
$\hat{\sigma}_{ML}^2 = e'e/n$, substituting into (30) yields the log-likelihood function evaluated at the least squares estimates:

$$\ln L = -n/2 \cdot \ln 2\pi - n/2 \cdot \ln (e'e/n) - n/2$$
$$= -n/2\ [\ln 2\pi + \ln (e'e/n) + 1]\ .$$

Analogously,

$$\ln L_* = -n/2\ [\ln 2\pi + \ln (e_*'e_*/n) + 1]$$

and using (29) it is straightforward to show that

$$\delta = n \cdot \ln (\hat{\sigma}_*^2 - \hat{\sigma}^2) \qquad (31)$$

(31) is the likelihood ratio test statistic used in this study. The restricted model in both test procedures is obviously the semi-logarithmic function where the restriction $\lambda = 1$ is imposed. Before proceeding any further, on point should be attended to: the unrestricted model is selected to be not the non-linear least squares regression equation but the grid-search equation with the optimal λ substituted into the Box-Cox transformation variable π. The rationale is to provide identical degrees of freedom for the restricted and the unrestricted models which are compared variance-wise in (31). As stated previously, the non-linear least squares equation has a smaller degrees of freedom ($n - k = 125 - 5 = 120$ as opposed to 121 in the restricted model) due to the presence of λ in the estimation process. Thus, despite a smaller SSR achieved through non-linear least squares a larger $\hat{\sigma}^2$ estimate may materialize due to lower degrees of freedom. That in turn may lead to a $(\hat{\sigma}_*^2/\hat{\sigma}^2)$ ratio less than 1 and therefore to a negative χ^2 calculated value provided the SSR's from the restricted and the unrestricted models are "close enough". Such was indeed the case in this study:

$$SSR_* = .099226144 \text{ and } SSR_{NLLS} = .099202122$$

but

$$\hat{\sigma}_*^2 = .00082005 \quad \text{and} \quad \hat{\sigma}^2_{NLLS} = .00082668 \ .$$

To minimize any potential distortions in the testing procedure the value of λ was taken as that estimated via non-linear least squares (i.e., 1.041599) and the Box-Cox variable π was transformed using that value instead of 1.04. SSR/(n-k) from the unrestricted regression model estimated in the above mentioned fashion is .00081985. The likelihood ratio statistic is accordingly calculated as

$$\delta = 125 \cdot \ln (.00082005/.00081985) = .03049$$

which is asymptotically distributed as χ^2 with 1 degree of freedom. The calculated χ^2 value is less than the tabled critical value at the 10% level of significance. The null hypothesis $H_0 : \lambda = 1$ thus can not be rejected at the 10% level of significance.

b) <u>Lagrange Multiplier Test</u>: Asymptotically equal to likelihood ratio test, lagrange multiplier test requires the estimation of the restricted model only and is founded on the reduction that would be observed in the SSR if the restrictions in the restricted model were lifted. For the non-linear regression model, the lagrange multiplier test statistic is:[29]

$$LG = \mathbf{e}_*' \bar{\mathbf{X}}_* [\bar{\mathbf{X}}_*' \bar{\mathbf{X}}_*]^{-1} \bar{\mathbf{X}}_*' \mathbf{e}_* \ / \ (\mathbf{e}_*' \mathbf{e}_*/n) = n \cdot R_*^2$$

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[29] For a general derivation of the test statistic, see Greene, p. 126-131.

where $\bar{X}$ is the $(n \times k)$ matrix of partial derivatives illustrated in (23), $\bar{X}_*$ is the same matrix evaluated at the restricted estimates and e_* is the $(n \times 1)$ vector of the residuals from the restricted equation. The lagrange multiplier statistic, like the likelihood ratio statistic, is asymptotically distributed as χ^2 with degrees of freedom equal to the number of restrictions. What the test reduces to in this study is calculating the residuals from the semi-logarithmic (restricted) money demand equation and regressing those residuals on the partial derivatives of the unrestricted model with respect to the entire parameter set of the model. The partial derivative regressors are to be evaluated at the restricted values of the parameters. Thus, given the unrestricted model

$$\ln (M/P)_t = f\{b_0, b_1, b_2, b_3, \lambda \; ; \; \pi_t, Y_t, (M/P)_{t-1}\} = b_0 + b_1 [(\pi_t^{\lambda} - 1)/\lambda] + b_2 \ln Y_t + b_3 \ln (M/P)_{t-1} + \varepsilon_t$$

the following partial derivative regressors are obtained:

$$z_0 = \partial f(\;)/\partial b_0 = 1$$

$$z_1 = \partial f(\;)/\partial b_1 = (\pi_t^{\lambda} - 1)/\lambda$$

$$z_2 = \partial f(\;)/\partial b_2 = \ln Y_t$$

$z_3 = \partial f(\ )/\partial b_3 = \ln\ (M/P)_{t-1}$

$z_4 = \partial f(\ )/\partial \lambda = b_1 \cdot [\{\lambda(\pi_t^{\lambda} \cdot \ln \pi_t) - \pi_t^{\lambda} + 1\}/\lambda^2]$

Evaluated at the restricted model parameter values $\lambda = 1$ and $b_1 = \hat{b}_1$, the estimate of b_1 in the restricted regression model, the regressors will be defined as follows:

$z_0 = 1$

$z_1 = \pi_t - 1$

$z_2 = \ln Y_t$

$z_3 = \ln\ (M/P)_{t-1}$

$z_4 = \hat{b}_1 \cdot [\pi_t(\ln \pi_t - 1) + 1]$.

Regressing the residuals of the restricted model on these regressors yields $R_*^2 = .00022285$ and the lagrange multiplier statistic $n \cdot R_*^2 = 125 \cdot (.00022285) = .027856$ which is very close to the likelihood ratio statistic obtained before. Thus the conclusion reached previously is intact: the null hypothesis H_0: $\lambda = 1$ can not be rejected at the 10% level of significance.

CHAPTER VI

MONEY DEMAND ESTIMATION RESULTS

The value of the Box-Cox parameter λ not being different from 1 statistically justifies the formulation of the money demand regression as semi-logarithmic in the inflation rate. Thus the estimated regression was

$$\ln (M/P)_t = b_0 + b_1 \cdot \pi_t + b_2 \cdot \ln Y_t + b_3 \cdot \ln (M/P)_{t-1} + \varepsilon_t \qquad (32)$$

where

$b_0 = h \cdot a_0$

$b_1 = h \cdot \beta$

$b_2 = h \cdot \alpha$

$b_3 = 1 - h$

as defined in (18) and (27). The results were as follows(with the t-values in parentheses):

$$\ln (M/P)_t = \underset{(-1.081311)}{-.1773457} - \underset{(-7.4583)}{.8843856\,\pi_t} + \underset{(2.772043)}{.08966587 \ln Y_t}$$

$$+ \underset{(28.87193)}{.9110264 \ln (M/P)_{t-1}} + e_t \qquad (33)$$

SSR = .099226144 Durbin-Watson = 2.26374139

R^2 = .96668403 $\bar{R}^2$ = .96585801

Due to the presence of the lagged value of the dependent variable as a regressor, Durbin-h statistic has to be calculated. Durbin-h was -1.575659 and the null hypothesis of zero autocorrelation in the error term could not be rejected at the 5% level of significance.

Analyses pertaining to inflation revenue maximization and the compensating change in the rate of inflation in response to the change in the rate of growth of real income (under constant inflation tax/GNP assumption) are both developed within the framework of steady state. Consequently, long-run values for the semi-logarithmic function parameters need to be obtained. Using the results in (33),

$$\hat{a}_0 = \hat{b}_0/(1 - \hat{b}_3) = \hat{b}_0/\hat{h} = -1.9932$$

$$\hat{\beta} = \hat{b}_1/(1 - \hat{b}_3) = \hat{b}_1/\hat{h} = 9.9398$$

$$\hat{\alpha} = \hat{b}_2/(1 - \hat{b}_3) = \hat{b}_2/\hat{h} = 1.0078$$

$1 - \hat{h} = \hat{b}_3 = .9110264$

are calculated. As demonstrated early in this study, aggregate real balances and real income are substitutable with their per capita counterparts only if α , the elasticity of the demand for real balances with respect to real income , is unity. The estimated long run elasticity is 1.0078. To test whether the true α is statistically different from 1 , a 90% confidence interval is constructed using the $\hat{\alpha}$ estimate. As $\alpha = \hat{b}_2/(1 - \hat{b}_3)$, it is a function of two estimated parameters and confidence interval construction is somewhat indirect.[30] Consider the linear combination

$$\phi = \hat{b}_2 - z \cdot (1 - \hat{b}_3) = \hat{b}_2 + z \cdot \hat{b}_3 - z$$

where z is to be determined later on. Note that if z is the true ratio $[b_2/(1 - b_3)]$, then $E(\phi) = 0$. Variance of ϕ is equal to

$$var(\phi) = var(\hat{b}_2) + z^2 \cdot var(\hat{b}_3) + 2z \cdot cov(\hat{b}_2, \hat{b}_3) \ .$$

Provided that the degrees of freedom is large, the distribution of ϕ is approximately normal and the sample value of ϕ is within 1.645 standard deviations of its expected value with a 90% probability. Under the assumption that z is

[30] Pindyck and Rubinfeld, p.244-245.

the true ratio, it will follow that

$$p(1.645 \cdot \sigma_\phi \geq \phi \geq -1.645 \cdot \sigma_\phi) = .90$$

implying

$$(\hat{b}_2 + \hat{b}_3 \cdot z - z)^2 \leq (1.645)^2 \cdot [\mathrm{var}(\hat{b}_2) + z^2 \cdot \mathrm{var}(\hat{b}_3) + 2z \cdot \mathrm{cov}(\hat{b}_2, \hat{b}_3)]$$

with a 90% probability. In the regression equation (33),

$$\mathrm{var}(\hat{b}_2) = .0010463$$
$$\mathrm{var}(\hat{b}_3) = .00099566$$
$$\mathrm{cov}(\hat{b}_2, \hat{b}_3) = -.00086689 \ .$$

Using $\hat{b}_2$, $\hat{b}_3$, $\mathrm{var}(\hat{b}_2)$, $\mathrm{var}(\hat{b}_3)$ and $\mathrm{cov}(\hat{b}_2, \hat{b}_3)$, z can be solved for from the quadratic equation by treating it as an equality. The result is of the form $z = d \pm \mu$ where d is the expected value of the long-run elasticity and μ is the range of the confidence interval.

The estimated confidence interval in our case is 1.0785 ± .40713. Hence the null hypothesis $H_0: \alpha = 1$ can not be rejected at the 10% level of significance. Consequently, the steady state framework and the use of aggregate data are justified for the purposes of this study.

Inflation revenue maximizing rate of inflation was given in (14) as

$$\pi^* = 1/\beta - \alpha \cdot g_y \ .$$

Using the estimates of α and β derived from (33) and the average monthly growth rate of aggregate real income in the estimation period, we obtain

$$\pi^* = (1/9.9398) - (1.0078)(.003448653)$$
$$= .09713 \ .$$

Thus the revenue maximizing rate of inflation is approximately 9.7% per month which corresponds to a per annum inflation rate of 219.2%. Monetary authorities apparently were on the correct side of the Laffer curve and could have raised the inflation tax revenue by increasing the rate of growth of the nominal stock of high-powered money. Yet adverse effects of high and unstable inflation should not go unnoticed and the conclusion reached above should not be interpreted as pointing towards sub-optimal policies pursued by the monetary authorities. Apart from the previously illustrated ill-effects of inflationary finance within the framework of the Dornbusch-Reynoso model, high and unstable inflation will inevitably diminish the effectiveness of the price mechanism as a signalling device in the efficient allocation of resources and lead to increased uncertainty among economic agents and a reluctance on their part to accumulate domestic claims. The deregulation of deposit rates under financial liberalization will also become a less effective means of attracting savings into financial institutions as it will become more difficult to maintain a stable and positive real rate of return in the face of high and unstable inflation. The ultimate outcome may well be the "dollarization" of the economy.

To investigate the compensating change in the rate of inflation in response to the change in the rate of growth of real income provided (seignorage/GNP) ratio is constant, two sub-periods in the estimation periods were selected: 1984-87 and 1988-89. The average per annum rate of growth of aggregate real income in the 1st and 2nd sub-periods were 6.59 and 2.64 percent, respectively. The average per annum inflation rate was 41.5 percent for the first sub-period and 65.7 percent for the second one.

The ratio of seignorage to aggregate real income, defined as $H/Y = (\dot{M}/M)(M/PY)$ in (7), was calculated for Turkey from 1980 to 1989. The results are given in Table(2) below:

Table (2): The ratio of Seignorage to Real Income

	(H/Y)
1980	.0332926
1981	.0356307
1982	.0370917
1983	.032330
1984	.0328875
1985	.0551726
1986	.0243388
1987	.0266849
1988	.0368874
1989	.0444482

To test the hypothesis that the ratio remained constant over the estimation period, a simple regression

$$(H/Y)_t = c_0 + c_1 t + \epsilon_t$$

with a linear time trend[31] was used. The t-value for the time variable was .33 and the hypothesis of a constant ratio over the estimation period could not be rejected at the 10% level of significance.

Given the formulation for the compensating change in the inflation rate in (13),

$$d\pi/dg_y \Big|_{1984\text{-}87} = -1.5415$$

and

[31] As the dependent variable is a ratio, a linear time trend implying absolute changes is the appropriate one.

$d\pi/dg_y \big|_{1988\text{-}89} = -1.8232$.

As the rate of growth of real income fell by 3.95 percentage points from the 1st to the 2nd sub-period, $d\pi/dg_y = -1.8232$ dictates that the compensating increase in the rate of inflation from the 1st to the 2nd sub-period is (1.8232)(3.95) = 7.2016 percentage points. The actual change in the inflation rate from the 1st to the 2nd sub-period being 24.2 percentage points, we conclude that the compensating increase in the inflation rate accounts for 29.8% of the actual increase.

The money demand function given in (18) was also estimated under the assumption of adaptive adjustment of inflation expectations. The inflation rate for period t was thus assumed to be forecasted as

$$_{t-1}\pi_t = \pi_{t-1} + \lambda(_{t-2}\pi_{t-1} - \pi_{t-1}) \qquad (34)$$

where the time subscript before a variable indicates expectations formed at the end of that period. Lagging (34) by one period, multiplying by λ and adding the resultant equation to (34) enables us to write the forecasted inflation for period t as

$$_{t-1}\pi_t = (1-\gamma)\pi_{t-1} + \gamma(1-\gamma)\pi_{t-2} + \gamma^2(1-\gamma)\pi_{t-3}\ldots\ldots \qquad (35)$$

, i.e., as a weighted average of all past inflation rates with exponentially declining weights.

The inflation rate variable under the assumption of adaptive expectations framework was estimated as a third degree polynomial distributed lag with no end point restrictions and twelve lags. The resulting inflation rate series was then used in place of the actual inflation rate and the money demand equation to be estimated thus became

$$\ln (M/P)_t = b_0 + b_1 \cdot {}_{t-1}\pi_t^{(\lambda)} + b_2 \ln Y_t + b_3 \ln (M/P)_{t-1} + \varepsilon_t \quad (36)$$

where ${}_{t-1}\pi_t$ was subject to the Box-Cox transformation.

The non-linear least squares method employing the Gauss-Newton algorithm yielded the following estimated values for the parameters in (36):

$\hat{b}_0 = -4.394413$

$\hat{b}_1 = -6.461986$

$\hat{b}_2 = .1248406$

$\hat{b}_3 = .857009$

$\hat{\lambda} = 1.513693$

The estimated value of the Box-Cox parameter was found to be statistically not different from one (the likelihood ratio statistic $\delta = n \cdot \ln \{\hat{\sigma}_*^2/\hat{\sigma}^2\} = .3277517 < \chi^2$ table value at the 10% level of significance).

The semi-logarithmic money demand function justified on grounds that the Box-Cox parameter λ is statistically not different from one yielded the following:

$$\ln (M/P)_t = -.1505109 - 1.15392\ {}_{t-1}\pi_t + .1247557 \ln Y_t$$
$$(-.795887) \quad (-3.70056) \quad (3.094273)$$

$$+ .86238 \ln (M/P)_{t-1} + e_t \qquad (37)$$
$$(21.86851)$$

SSR = .130112 Durbin-Watson = 2.10445

$R^2 = .9563$ $\bar{R}^2 = .9552$

The Durbin-h statistic was found to be -.6506 and the null hypothesis of zero autocorrelation in the error term could not be rejected at the 5% level of significance. The implied long-run values for the semi-logarithmic function parameters obtained from (37) are

$$\hat{a}_0 = \hat{b}_0 \ / \ (1 - \hat{b}_3) = -1.0936369$$
$$\hat{\beta} = \hat{b}_1 \ / \ (1 - \hat{b}_3) = 8.38457$$
$$\hat{\alpha} = \hat{b}_2 \ / \ (1 - \hat{b}_3) = .9065099$$

where $1 - \hat{b}_3 = \hat{h}$, the estimated value for the speed of adjustment in the partial adjustment of real balances.

To test whether the long-run income elasticity of real balances is different from unity, a 90% confidence interval (using the methodology illustrated previously) was constructed for α using the estimated value $\hat{\alpha} = \hat{b}_2 \ / \ (1 - \hat{b}_3)$. The confidence interval thereby estimated for α was 1.2006 ± .3492. Hence we failed to reject that the long-run income elasticity is different from unity.

The revenue maximizing rate of inflation under the adaptive expectations framework was calculated as

$$\pi^* = (1/\beta) - \alpha \cdot g_y = (1/8.38457) - (.9065099)(.003448653)$$
$$= .1161405$$

which corresponds to a per annum inflation rate of 302.96%.

The revenue maximizing rate of inflation is thus found to be higher when the expected inflation rate series is used in place of the actual rate series. This is due to the lower long-run semi-elasticity of real balances with respect to inflation (i.e., the more inelastic demand schedule for real balances) under the adaptive expectations framework. As the revenue maximizing rate of inflation is inversely related to β, the long-run semi-elasticity of real balances with respect to inflation, a lower value for β under adaptive expectations yields the higher revenue maximizing rate.

The compensating change in the rate of inflation in the face of the decline in the rate of growth of aggregate real income is given by

$$d\pi/dg_y\Big|_{1988\text{-}89} = -1.3734616$$

The decline in the rate of growth of aggregate real income from the 1st subperiod (1984-87) to the 2nd being 3.95 percentage points, the compensating increase in the rate of inflation was found to be (1.3734636)(3.95) = 5.425 percentage points. As the actual increase in the rate of inflation from the 1st to the 2nd subperiod was 24.2 percentage points, the compensating increase in the rate of inflation could explain 22.4 percent of the actual increase under the adaptive expectations framework.

CHAPTER VII

A POSITIVE THEORY OF THE GOVERNMENT WITH RESPECT TO THE OPTIMAL COLLECTION OF SEIGNORAGE: THEORETICAL BACKGROUND

Barro (1979) shows that a government minimizing the present value of the social cost of taxation subject to exogenous expenditure and an intertemporal budget constraint would smooth tax rates over time. The intertemporal budget constraint is of pivotal importance in the analysis and is derived from the one period budget constraint

$$G_t + r_{t-1}B_{t-1} = T_t + (B_t - B_{t-1}) \qquad (38)$$

where B_{t-1} is real public debt outstanding at the of period (t-1), r_{t-1} is the real interest rate paid on the end of period (t-1) debt, T_t is real tax revenues at period (t)where taxes are assumed toj be lump-sum, and G_t is real government expenditures at period (t). For each period (t), let

$$d_{t+s} = d_{t+s-1}/(1 + r_{t+s-1})$$

be defined as the present value factor where $d_{t-1} = 1$, thus d_t

=

$1/(1+r_{t-1})$, $d_{t+1} = d_t/(1+r_t) = 1/(1+r_{t-1})(1+r_t)$ and so on.[32]

From (38) the intertemporal budget constraint of the government over a finite horizon (from period (t) to (t+N)) can be obtained as

$$\sum_0^N d_{t+s}T_{t+s} = \sum_0^N d_{t+s}[G_{t+s} - B_{t+s} + (1+r_{t+s-1})B_{t+s-1}]$$

$$= \sum_0^N d_{t+s}G_{t+s} - \sum_0^N d_{t+s}B_{t+s} + \sum_0^N d_{t+s}(1+r_{t+s-1})B_{t+s-1}. \qquad (39)$$

Note that the last summation term in (39) can be written as

$$d_t(1+r_{t-1})B_{t-1} + \sum_1^N d_{t+s}(1+r_{t+s-1})B_{t+s-1}. \qquad (40)$$

As $d_t(1+r_{t-1}) = d_{t-1} = 1$ and $d_{t+s}(1+r_{t+s-1}) = d_{t+s-1}$, (40) reduces to

[32] Barro (1979) initially assumes that the real interest rate on all debts (public and private), r, is constant over time. He later on allows the current rate of return on the one-period debt r_t to differ from the previous rate r_{t-1}. Given the assumed constancy of anticipated rates of return for all future periods at r_t, he states that all the results hold provided all debt variables are measured at market value rather than at par value.

$$B_{t-1}+\sum_{1}^{N} d_{t+s-1}B_{t+s-1}. \qquad (41)$$

Substituting (41) into (39) yields

$$\sum_{0}^{N} d_{t+s}T_{t+s}=\sum_{0}^{N} d_{t+s}G_{t+s}+B_{t-1}+\sum_{1}^{N} d_{t+s-1}B_{t+s-1}-\sum_{0}^{N} d_{t+s}B_{t+s}. \qquad (42)$$

Noting that the last two summation signs reduce to $d_{t+N}B_{t+N}$ and letting $N\to\infty$, the intertemporal budget constraint of the government at period (t) over an infinite horizon is obtained as

$$\sum_{0}^{\infty} d_{t+s}T_{t+s}=B_{t-1}+\sum_{0}^{\infty} d_{t+s}G_{t+s}-\lim_{N\to\infty}d_{t+N}B_{t+N}. \qquad (43)$$

The transversality condition for the government's intertemporal budget constraint is

$$\lim_{N\to\infty}d_{t+N}B_{t+N} = 0. \qquad (44)$$

Equation (44) implies that individuals do not hold as part of their portfolio any public debt that asymptotically has positive present value. In other words, the transversality condition dictates that individuals do not hold government debt that grows at a rate as high as the real interest rate. Hence the condition effectively rules out perpetual debt finance by the government and eliminates Ponzi-games. Thus the final form of the government's intertemporal budget constraint is

$$\sum_{0}^{\infty} d_{t+s}T_{t+s}=B_{t-1}+\sum_{0}^{\infty} d_{t+s}G_{t+s}. \qquad (45)$$

The present discounted value of tax receipts at period (t) is determined by the level of initial debt, B_{t-1}, and the present discounted value of exogenously given government expenditures.

Barro (1979) focused on the determination of the time pattern of taxes under the intertemporal budget constraint (45). Assuming a real cost of taxation function linearly homogeneous in real tax revenues and real income, Barro goes on to show that a government minimizing the present value of taxation costs subject to the intertemporal budget constraint (45) should keep the tax revenue/income ratio constant for all periods,i.e., the tax rates should be smoothed over time. The level of taxes in each period is determined from (45) given the (optimal) constancy of the tax/income ratio and the exogenously set real income and government expenditure values for all periods. The level of taxes thereby determined will in turn determine the budget deficit for each period from the one-period budget constraint (38).

Mankiw (1987) utilizes the tax-smoothing model outlined above for a joint analysis of inflation tax and ordinary taxes related to output. He demonstrates that under the assumption of a government minimizing the expected present value of the taxation costs from these sources, nominal interest rate and inflation rate series are approximately random walks.[33] Furthermore, tax rates move concurrently and in the same direction with nominal interest rates and inflation.

The theory of optimal seignorage is integrated into the optimal fiscal and monetary policy of a government operating under the intertemporal budget constraint (45). Let the exogenous real output level be denoted by Y and the tax rate on output as τ. The revenue from taxes on output is $\tau \cdot Y$ and the deadweight social loss from these taxes $\phi(\tau)Y$ - where $\phi'(\ )>0$ and $\phi''(\ )>0$ - is assumed linearly homogeneous in Y.

The real revenue from seignorage was defined in (3) as

$$H = (\dot{M}/M)(M/P) = (\pi + \alpha \cdot g_y)(M/P).$$

Assuming that the income elasticity of the demand for real balances is unity and the quantity equation

$$M/P = Y/V = k \cdot Y$$

describes the demand for real balances, the real revenue from

[33] Tax rates are also a random walk which is the Barro (1979) result in a stochastic form. The random walk nature of nominal interest rates follows from the assumed constancy of real interest rates over time and the derived random walk nature of inflation rates. The model postulated by Mankiw is classical in the sense that monetary policy has no effect on real output or real interest rates.

seignorage can be written as

$$H = (\pi + g_y) \cdot kY \; . \qquad (46)$$

The social deadweight loss of inflation tax is defined as $\psi(\pi)Y$ where $\psi'(\ )>0$, $\psi''(\ )>0$ and the loss function is homogeneous of first degree in output. Total tax revenue accruing to the government from the two tax sources can be defined as

$$T = \tau \cdot Y + (\pi + g_y) \cdot kY \; . \qquad (47)$$

Using the total tax revenue defined in (47), the intertemporal budget constraint (45) is rewritten as

$$\sum_{0}^{\infty} d_{t+s}(\tau_{t+s}+\pi_{t+s}k+g_{Y_{t+s}}k)\,Y_{t+s}=B_{t-1}+\sum_{0}^{\infty} d_{t+s}G_{t+s}. \qquad (48)$$

The government has to choose the rates of taxation and inflation to minimize the expected present value of total deadweight losses

$$E_t\sum_{0}^{\infty} d_{t+s}[\phi(\tau_{t+s})+\psi(\pi_{t+s})]\,Y_{t+s} \qquad (49)$$

subject to the intertemporal budget constraint (48). The choice variables being π_{t+s} and τ_{t+s} with $s \geq 0$, the following are the first order conditions :

$$E_t\ [\phi'(\tau_{t+s})] = \phi'(\tau_t) \qquad (50)$$

$$E_t\ [\psi'(\pi_{t+s})] = \psi'(\pi_t) \qquad (51)$$

$$\psi'(\pi_t) = k \cdot \phi'(\tau_t) \qquad (52)$$

Intertemporal first order condition (50) equates the marginal social cost of taxes on output today (period (t)) and in any future period (t+s). First order condition (51) equates the marginal social cost of raising tax revenue through seignorage today and in any future period. Finally, (52) equates for period (t) the marginal social cost of raising revenue through taxes on output to the marginal cost of raising revenue through seignorage.[34]

The first order conditions have crucial implications that are empirically testable. (50) and (51) imply that the marginal social cost of taxes on output and the marginal social cost of seignorage follow martingale processes. Quadratic $\phi(\tau)$ and $\psi(\pi)$ functions would imply (through linear

[34] Mankiw shows that when real balances are assumed to be responsive to the inflation rate,i.e., k is not a constant but $k = f(\pi)$, the interpretation of the new first order conditions is basically the same.

marginal social cost functions) that τ and π series themselves would follow martingale processes. In the absence of specific functional forms for the marginal social cost functions, we will assume linear approximations to $\phi'(\tau)$ and $\psi'(\pi)$[35]. With both marginal cost functions increasing in their arguments, first order condition (52) expresses a crucial implication: higher revenue needs necessitate increases in the use of both taxes. Thus the testable hypothesis implied by (52) is that tax rates move in the same direction with the inflation rate (and the nominal interest rate provided the real interest rate is constant).

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[35] A stochastic process is defined as martingale if for increasing set $t-n < t-(n-1) < \ldots < t-1 < t$ where t represents the time parameter

$$E\{X_{t+1} | X_t, X_{t-1}, \ldots X_{t-n}\} = X_t.$$

The testable implication of the first two first order conditions is formulated as the hypothesis that both the tax rates and the inflation rate are approximately random walks.

CHAPTER VIII

REGRESSION RESULTS FOR THE THEORY OF OPTIMAL SEIGNORAGE

To test the implications of the first order conditions (50) and (51) that tax rate and inflation rate approximately follow random walks, unit root tests introduced by Dickey and Fuller are used.[36] The estimation period is from the first quarter of 1980 to the second quarter of 1990. Inflation rate is the quarterly percentage change in the average wholesale price index and the ratio of government revenues to GNP is taken as the tax rate measure.

A non-stationary time series is a trend stationary process if stationarity can be achieved by detrending and a difference stationary process if differencing is required to render the series stationary. A detrended series will still be non-stationary if it can be characterized as a random walk process. The random walk (unit root) test is conducted through the estimation of the regression equation

$$\Delta y_t = \alpha + \beta t + \rho y_{t-1} + \sum_{1}^{p-1} \lambda_i \Delta y_{t-i} + e_t \qquad (53)$$

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[36] Dickey and Fuller (1979 and 1981).

for an autoregressive process of the p^{th} order (the last term would drop for a first order autoregressive process). The unit root test statistics $\hat{\tau}_{\tau}$, Φ_2, Φ_3 supplied by Dickey and Fuller enable us to test the following null hypotheses[37]

(i) H_0: $\rho = 0$

(ii) H_0: $\beta = \rho = 0$

(iii) H_0: $\alpha = \beta = \rho = 0$

where (i) tests for the non-stationarity of a detrended series, (ii) tests for random walk with drift and (iii) for random walk.

The white noise property of ϵ in (53) determines the order of the autoregressive process and therefore the lag length in the summation term in (53). Autocorrelation and partial autocorrelation functions are the standard tools in selecting the proper autoregression order.[38]Among the variables that are subjected to the unit root test in this study, the nominal interest rate (INT) and the tax rate (TAXRAT) have only the first partial autocorrelation coefficient as statistically significant and are therefore

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[37] Fuller (1976, p.373) and Dickey and Fuller (1981, p.1063, tables V and VI).

[38] For the relevant variables in this study, namely the inflation rate, tax rate and the nominal interest rate, the variables themselves and their autocorrelation and partial autocorrelation functions are plotted and shown in figures 1.1 through 3.3.

estimated as an AR(1) processes. The quarterly inflation rate (INF) series was observed to be stationary. For the $(INF_t - \mu) = \varepsilon_t$ series where μ stands for the average quarterly inflation rate for the estimation period, the ε series was white noise: the t-statistic for mean = 0 was .0000015 and the autocorrelation function ρ_k was statistically equal to zero for all k>0. The Dickey-Fuller test, although redundant, thus yielded very large calculated test statistics leading to a firm rejection of the null hypothesis of random walk for the quarterly inflation rate. The calculated and the critical values for the test statistics $\hat{\tau}_\tau$, Φ_2, Φ_3 are given in tables 3 and 4 below:

Table (3): Calculated values of the unit root test statistics

Test Stat.	TAXRAT	INF	INT
$\hat{\tau}_\tau$	-4.434036	-9.619900	-2.249511
Φ_2	9.858460	48.93711	3.397766
Φ_3	6.582462	33.26349	2.757343

Table (4) : Critical values of the unit root test statistics at 0.025, 0.05 and 0.10 levels of significance (n=50)

Test Stat.	0.025	0.05	0.10
$\hat{\tau}_\tau$	-3.80	-3.50	-3.18
Φ_2	7.81	6.73	5.61
Φ_3	5.94	5.13	4.31

The comparison of the calculated statistics to the critical values indicate that we fail to reject the hypothesis of random walk for the interest rate variable but reject it for both (the first order condition variables) the inflation rate and the tax rate.[39] The econometric analysis of the data thus leads to an empirical refutation of the two first order conditions(50 and 51) that the inflation rate and the tax rate follow random walks under the optimal collection of seignorage.

The testable hypothesis implied by the first order condition (52) is that inflation rate covaries positively with the tax rate measure (i.e., the ratio of government revenues to GNP). The regression of the inflation rate on a time trend and the tax rate measure yields the following (with the t-values given in parentheses):

$$INF_t = \underset{(3.144783)}{.3176158} - \underset{(-2.249509)}{.9975346}\ TAXRAT_t + \underset{(.0611298)}{.000045}\ TIME + e_t \qquad (54)$$

SSR = .12161272 Durbin-Watson = 1.175298

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[39] Schwert(1987) indicates that when a time series is generated by an ARIMA rather than a pure AR process, Dickey-Fuller statistics could lead to the conclusion that the series is stationary when they may not be,i.e., the Dickey-Fuller tests could lead to the rejection of the null hypothesis of unit root more often then they ought to. This reservation does not seem to be valid in this study as the variables subject to the unit root test were observed to display pure AR characteristics upon diagnostic checking.

R^2 = .118784 $\bar{R}^2$ = .073594

Q(18)=9.16738

The Durbin-Watson statistic implies positive 1st order serial autocorrelation (though barely; the lower bound d_l = 1.198 at the 1% significance level and d_l = 1.391 at the 5% significance level) and renders any inference from (54) non-valid.[40] The Hildreth-Lu grid search procedure is used to correct for 1st order serial autocorrelation and the results are as follows:[41]

$$INF_t = .2326721 - .7662906\ TAXRAT_t + .0004472\ TIME + e_t \quad (55)$$
$$(4.156219) \quad (-3.144662) \quad (2.659548)$$

[40] Wallis(1972) indicates that the error term may display 4th order autocorrelation in studies that employ quarterly data. Tests of 4th order autocorrelation using the modified Durbin-Watson statistic are also conducted for the regressions in this section.

[41] The Cochrane-Orcutt iteraative method is also employed to correct for serial correlation and the results are almost identical to those obtained by the Hildreth-Lu:

$$INF_t = .2326721 - .7662906\ TAXRAT_t + .00044719\ TIME + e_t$$
$$(4.156219) \quad (-3.144662) \quad (2.659548)$$

rho(ρ) = .097944
(1.134029)

SSR = .032994 Durbin-Watson = 1.865062
R^2 = .402279 $\bar{R}^2$ = .353815
Q(18) = 22.2599

The Hildreth-Lu grid search method is given priority over the Cochrane-Orcutt method as the latter does not guarantee the global optimum for ρ (see Doan p.5-6).

$rho(\rho) = .09794$
(1.134)

SSR = .0329943   Durbin-Watson = 1.865062

$R^2 = .402279$ $\bar{R}^2 = .353815$

Q(18) = 22.1599

Regression equation (54) was suggestive of 4^{th} order autocorrelation as well (the modified Durbin-Watson statistic given by Wallis was d_l=1.288 and d_u=1.469) but the results from (55) indicate the absence of 4^{th} order autocorrelation.

The problem of spurious regression arises when the variables in a regression can be characterized as random walk processes.[42] Although we previously rejected that the tax rate and the inflation rate are random walk processes, we estimate (54) in differenced form to rule out a spurious relation between the tax rate and the inflation rate. The results of the differenced form regression are as follows (with the t-values given in parentheses):

$$DINF_t = \underset{(-.7290599)}{-.00679423} - \underset{(-1.659970)}{.6690588}\ DTAXRAT_t + e_t \qquad (56)$$

SSR = .138794   Durbin-Watson = 1.80096

$R^2 = .066$ $\bar{R}^2 = .042$

Q(18) = 17.5577

where D is the difference operator, i.e., $DY_t = Y_t - Y_{t-1}$.

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[42] Granger and Newbold (1974).

The effect of the tax rate on the inflation rate is negative and statistically significant at the 5% level of significance in (55) and at the 10% level in (56). This result leads to the rejection of the hypothesis entailed in the theory of optimal seignorage that the inflation rate covaries positively with the tax rate.

The first order condition (52) also implies that the inclusion of government expenditures as an additional regressor should not effect the coefficient of the tax rate variable. This result apparently is peculiar to the theory of optimal seignorage where government's revenue requirement is the sole determinant of the inflation rate. On the other hand, if one subscribes to the proposition that deficits induce monetization, the inclusion of an expenditure variable should render the coefficient of the tax rate negative. Holding the expenditure/GNP ratio constant, lower tax rates should lead to higher money growth and inflation rates. An increase in the expenditure/GNP ratio holding the tax rate measure constant should also generate the same conclusion. Using the ratio of expenditures to GNP (EXP) as an additional regressor in (54), the following regression results are obtained (with the t-values in parentheses):

$$INF_t = \underset{(2.326341)}{.222509} - \underset{(-3.61955)}{1.596075}\, TAXRAT_t + \underset{(3.188818)}{.9515395}\, EXP_t$$

$$- \underset{(-.904275)}{.0006145304}\, TIME + e_t \qquad (57)$$

SSR = .0959398 Durbin-Watson = 1.601376

R^2 = .30481242 $\bar{R}^2$ = .24993

Q(18) = 10.4941

The Durbin-Watson statistic is not indicative of serially correlated errors at the 5% level of significance.

To avoid any severe spurious regression problems that may arise when the tax rate and inflation rate can be characterized as random walk processes(again an over-cautious approach as we previously rejected the null hypothesis of unit root for both rates), the regression equation (57) is run in differenced form (t-values given in parentheses):

$$DINF_t = \underset{(-.7014686)}{-.00657485} - \underset{(-1.816559)}{.8408363}\ DTAXRAT_t + \underset{(.7676772)}{.2694016}\ DEXP_t + e_t \qquad (58)$$

SSR = .13667528 Durbin-Watson = 1.87988465

R^2 = .08025534 $\bar{R}^2$ = .03184773

Q(18) = 18.1202

The tax-rate coefficient is still negative and statistically significant at the 5% level of significance . The expenditure variable, despite retaining its positive coefficient, is statistically insignificant in (57) and (58). The results overall still lend no support to the theory of optimal seignorage and suggest a link between the inflation rate and deficit monetization.[43]

[43] Regressions with the interest rate instead of the inflation rate yielded similar results with the tax rate coefficient being negative in all the regressions. As the inflation rate is the variable in the first order conditions for the optimal collection of seignorage, the results pertaining to the interest rate are not mentioned.

CHAPTER IX

CONCLUSIONS

The Turkish economy has been transformed significantly following the implementation of the January 1980 stabilization and liberalization program which focused on export-led growth and on the deregulation and development of financial markets. The program is considered a success story with the one crucial reservation being the persistently high inflation rate. The main domestic component of the program was the deregulation of bank deposit rates. A partial capital account liberalization which allowed citizens to open foreign exchange deposit accounts increased further the competition for the private sector's funds and thus the cost of borrowing for the government in its endeavors to finance the budget deficit.

In the theory of inflationary finance, deregulation of deposit rates induces the private sector to substitute away from cash holdings and may lead to a reduction in the high-powered money, the tax base for inflation tax. Reductions in the reserve requirement ratios have the same effect on the tax base. Hence, for a government relying on inflationary finance containing the budget deficit under control assumes an overwhelming importance during a period of financial liberalization; a reduction in the tax base will force the monetary authorities to inflate the money supply at a higher rate in order to finance a given budget deficit.

We have attempted in this study to analyze the governments' approach to revenue from money creation in Turkey in the post-liberalization era.

First, we calculated the revenue maximizing rate of inflation through a money demand estimation for the 1980-1990 period using first the actual inflation rate and then an expected inflation rate series as regressors in the money demand equations. In both cases, the average actual inflation rate for the estimation period fell considerably short of the revenue maximizing rate indicating that the governments have been on the correct side of the Laffer curve with respect to seignorage.

Secondly, we investigated the compensatory change in the inflation rate in the face of a decline in the rate of growth of real output. The estimation period had two subperiods the first of which (1984-87) could be characterized as a "low inflation, high real output growth" period and the second as one of "high inflation and low real output growth." We concluded that approximately 30% of the actual change in the inflation rate from the first to the second subperiod could be explained by the compensatory inflation rate increase when the actual inflation rate was the regressor in the money demand regression. The corresponding figure was approximately 25% when the expected inflation rate was employed as the regressor.

Finally, we tested the implications of the tax-smoothing model where both output and real balances are sources of tax revenue. The testable implications of the model are that the tax rate on output and the inflation rate follow random walks and that the inflation rate covaries positively with the tax rate. We rejected the hypotheses that the two rates follow random walks and the coefficient of the tax rate regressor in all the inflation rate regressions was negative and statistically significant. Thus, the overall results revealed that the behavior of the post-liberalization governments was not in conformity with the basic tenets of the optimal theory of seignorage.

Figure 1.1 Quarterly Inflation Rate

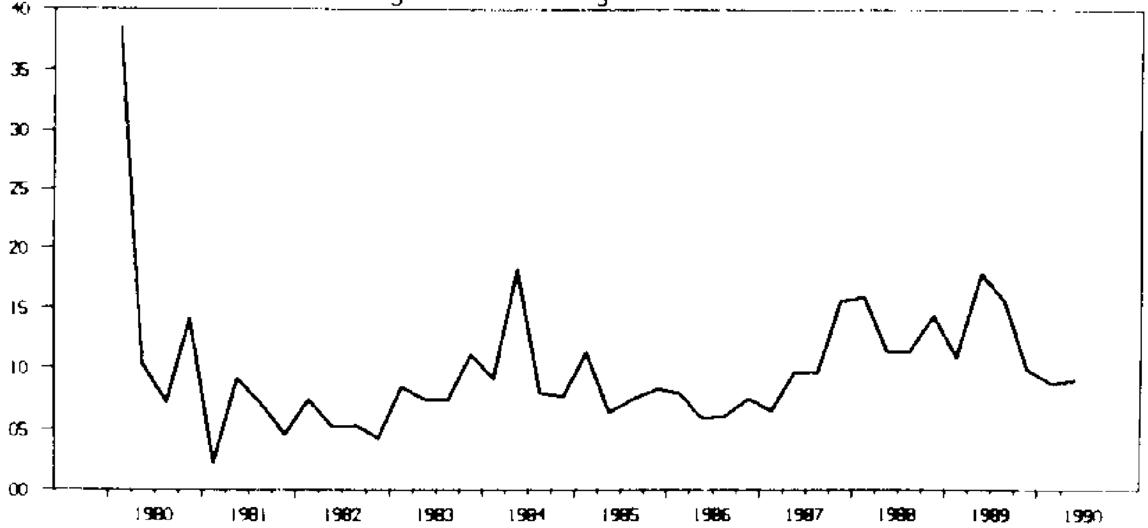


Figure 1.2 Inflation Rate Autocorrelations

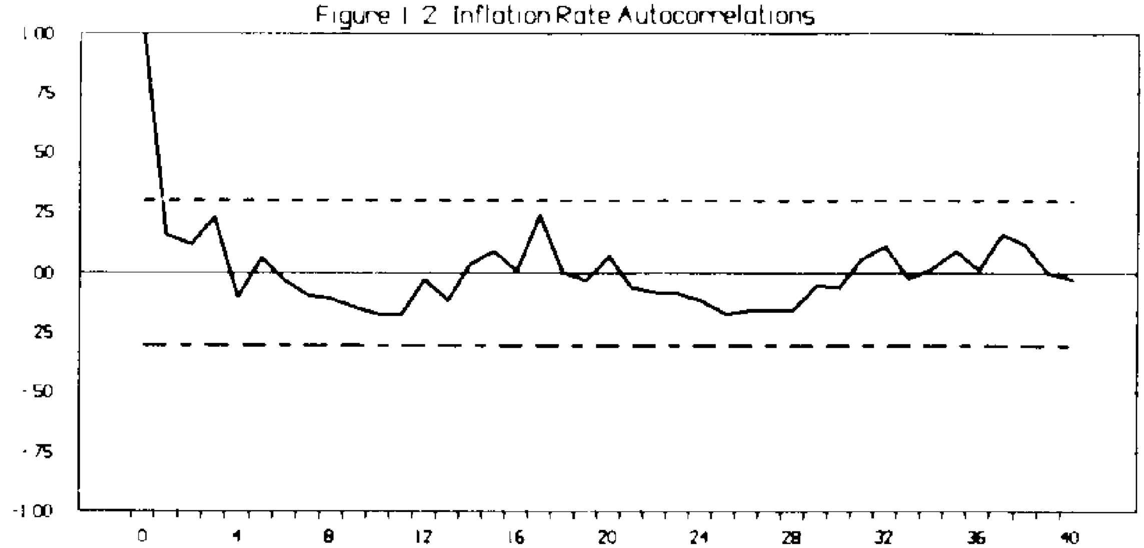


Figure 1.3 Inflation Rate Partial Autocorrelations

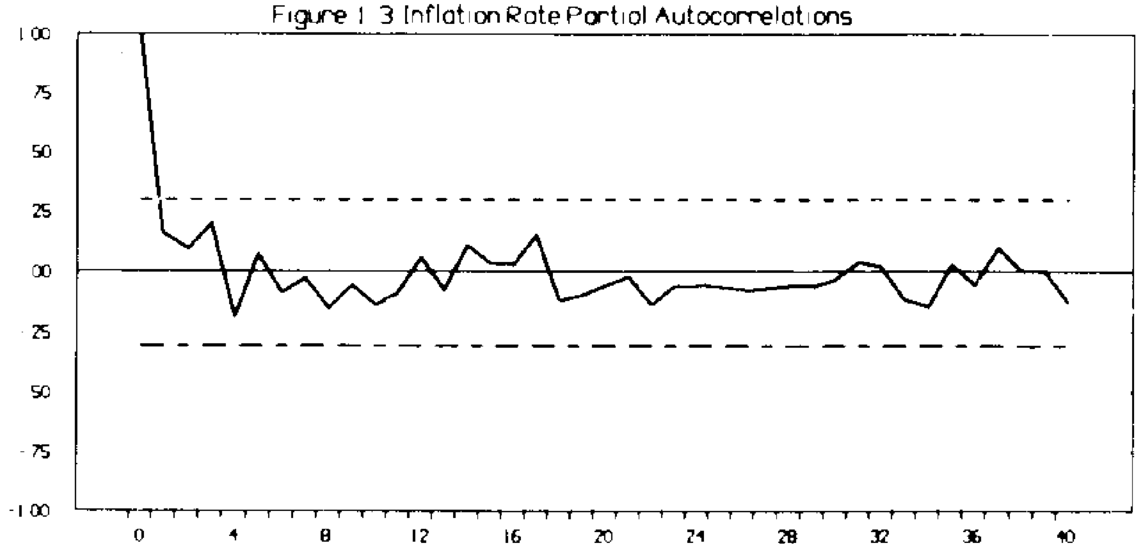


Figure 2 1 Nominal Interest Rate

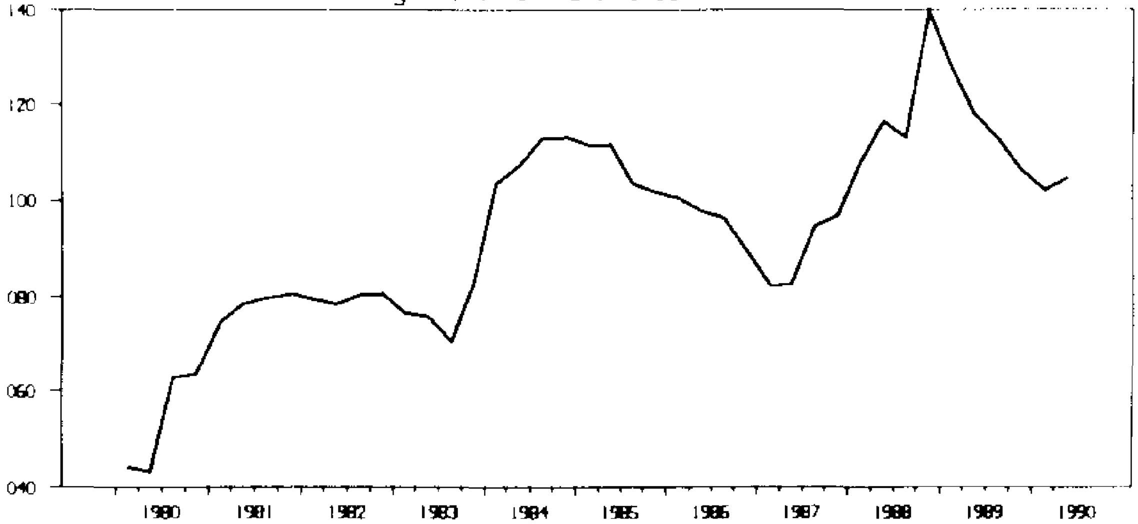


Figure 2 2 Nominal Interest Rate Autocorrelations

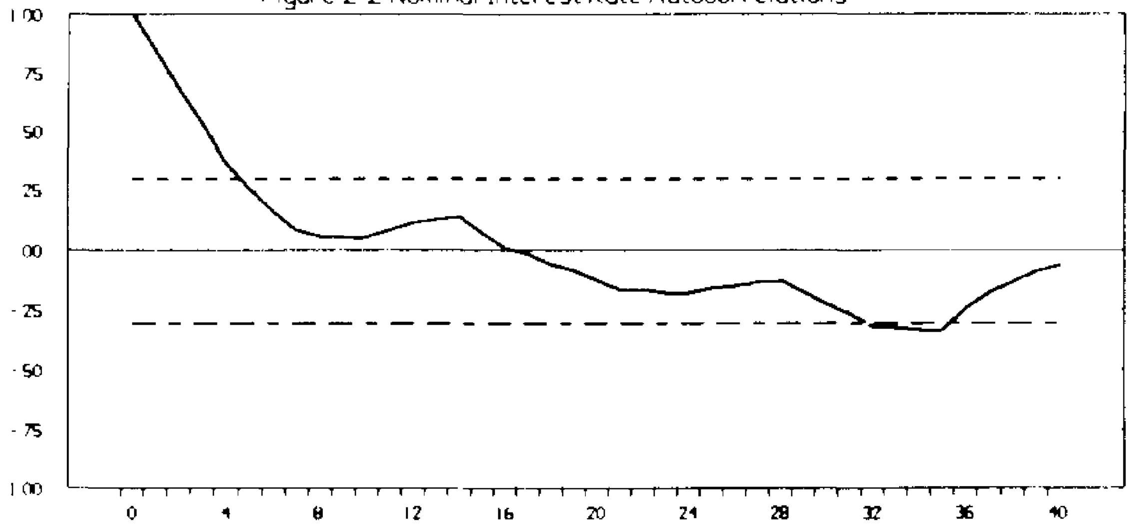


Figure 2 3 Nominal Interest Rate Partial Autocorrelations

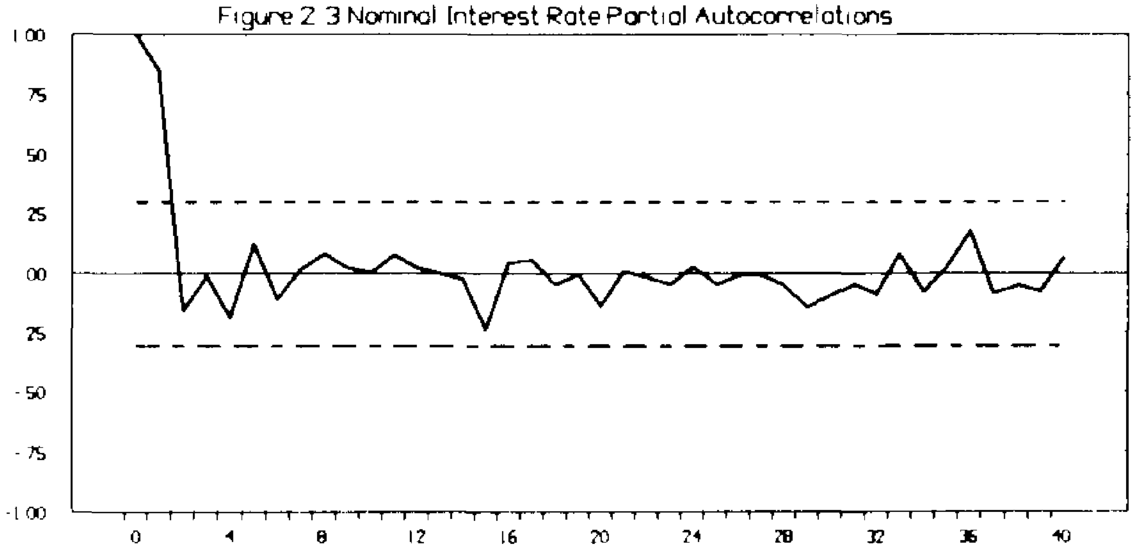


Figure 3.1 Tax Rate

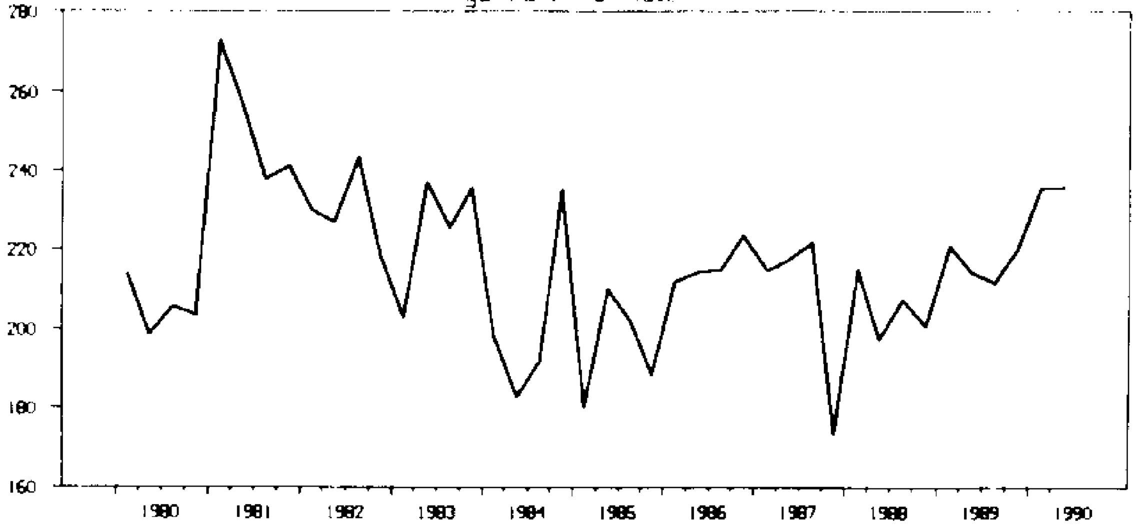


Figure 3.2 Tax Rate Autocorrelations

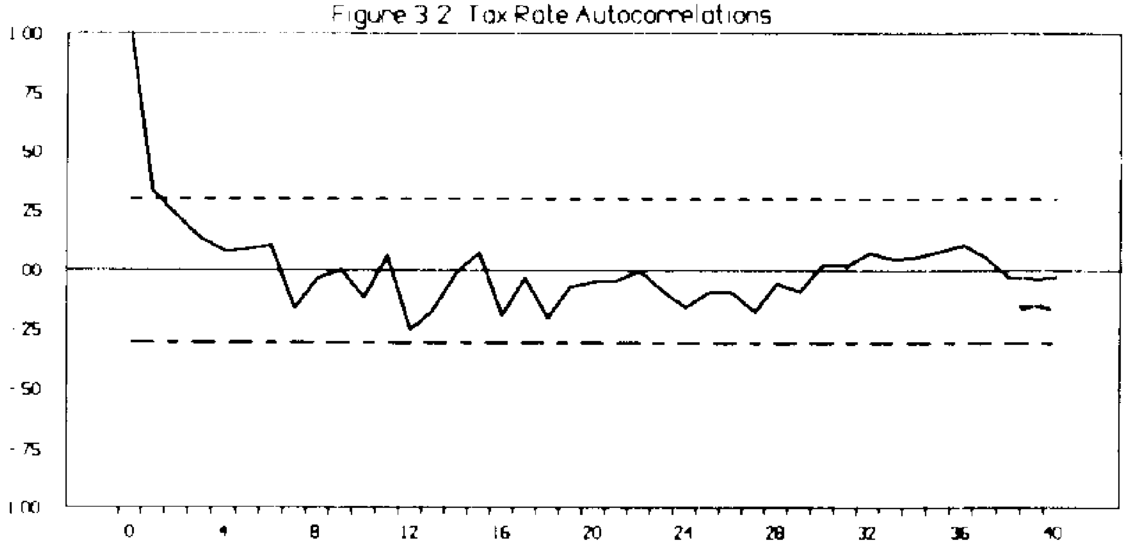
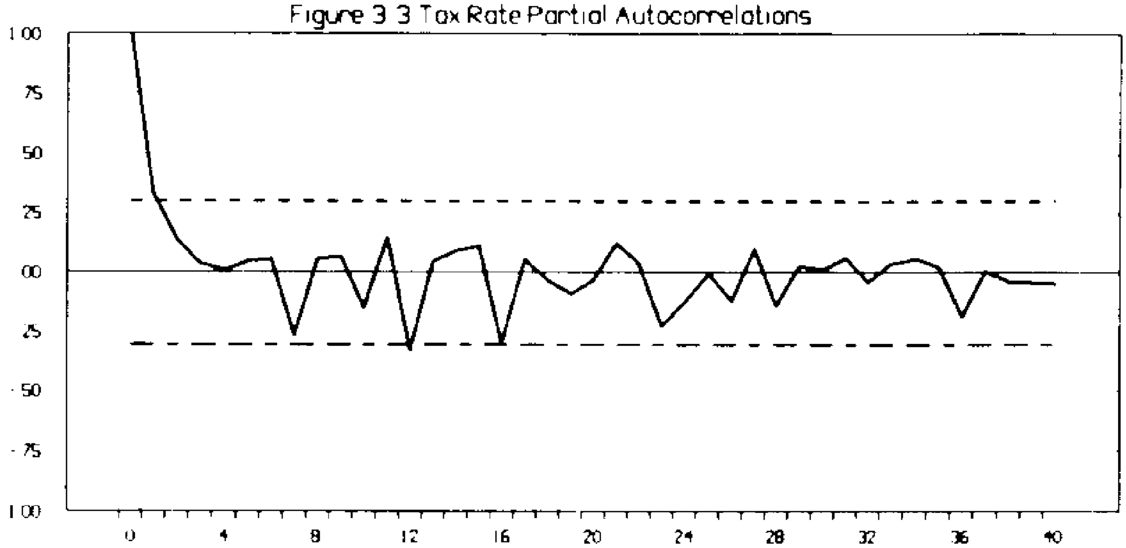


Figure 3.3 Tax Rate Partial Autocorrelations



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