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# ANALYSIS, DESIGN, AND RELIABILITY BASED TRIANGULAR ESTIMATION MAINTENANCE FLOAT POLICY FOR LARGE SYSTEMS WITH HIGH OPERATIONS AVAILABILITY REQUIREMENTS

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ANALYSIS, DESIGN, AND RELIABILITY BASED TRIANGULAR
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WITH HIGH OPERATIONS AVAILABILITY REQUIREMENTS

by

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ABSTRACT

ANALYSIS, DESIGN, AND RELIABILITY BASED TRIANGULAR ESTIMATION MAINTENANCE FLOAT POLICY FOR LARGE SYSTEMS WITH HIGH OPERATIONS AVAILABILITY REQUIREMENTS

by

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The equipment failure distributions commonly identified in practice pose great difficulties in the establishment of sound maintenance float policy (MFP). What motivated the present study, however, is a set of analytical reliability based estimation constructs, which have managed to bypass the obstacles of efficient MFP determination. Following their methodology, the MFP estimation model base is extended to include the case of uncertainty due to the lack of data on the underlying failure distribution. A set of reliability based triangular estimation models is formally derived and tested. Guided by these promising constructs, the conceptual schema of the MFP framework is enlarged in order to integrate economic, social, and technological implications of maintenance float systems which had been overlooked in the past.

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Finally, I express my deepest gratitude to my parents Mary and Costas, my brother George, and my lovely wife Arlene. They have been a continuing source of support and encouragement, and have made many sacrifices so that I might accomplish this work. They have helped to provide a balance and sense of perspective throughout the process that might otherwise have been lost.

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# CHAPTER I

# INTRODUCTION AND REVIEW OF THE LITERATURE

## Purpose of the Study

The purpose of this study is to investigate what is appropriate for the establishment of maintenance float policy (MFP), during the initial setup and debugging of a newly planned manufacturing technology, a new factory, or any new production/operations (P/O) facility with a high level of operations availability requirements. The decision makers' knowledge on the equipment failure and repair continuous time distributions, on the capacity requirements in terms of the initial number of units in operations, as well as on the overall system effectiveness and reliability, are at the core of the analysis.

Within the first research module some MFP guidelines for systems planning and design will be analytically derived. That part of the paper will further extend the current model base on MFP to include the case of the reliability based triangular estimation technique. Within a second, but of equal importance experimental module, the theoretical constructs will be tested in terms of their underlying assumptions and factors. The third and final module will explore a set of alternative cost/effectiveness criteria within the context of management planning for maintenance float systems.

Prior to discussing the conceptual framework of the maintenance float policy research at both the single- and multi-facility levels, a brief overview of the developments and applications in the maintenance management literature will be given.

## Statement of the Problem

In the spirit of Barlow, Proschan, & Hunter (1965), the purpose of an MFP is to determine the total number of open service channels, as well as the number of support units which must be held aside in order to maintain a close to continuous flow of operations. The implied management objective is to minimize the total cost of lost production attributable to equipment downtime, maintenance personnel, and standby units, while maintaining operations highly available.

The pursuit of an analytical solution to the MFP problem is not easy. Maintenance float systems pose great modeling difficulties primarily due to the complex equipment failure distributions which occur in practice. Further complications arise from the interdependence between operations and repair facilities, which render the establishment of an effective MFP even more difficult (Gertsbakh & Kordonskiy, 1969).

Based on the above obstacles, the determination of the optimum MFP for a given maintenance float system is considered a classic among production/operations management (P/OM) problems. It has attracted considerable attention and has been solved under specific assumptions regarding both the failure and repair distributions as well as the reliability of equipment.

Levine (1965), for example, theoretically derived an MFP model by utilizing the notion of the repair to failure ratio (r), defined as the ratio of mean time to repair (MTTR) over the mean time between failure (MTBF). His model provides an approximation to the MFP for a maintenance float system with equipment which break down according to an Exponential distribution and their repair times being also Exponentially distributed. Levine's 1965 paper motivated a new research stream. His work was extended by Love & Lewis (1983), who developed an MFP model for the combination of Weibull failure and Lognormal repair distributions. Their analysis was carried further by Madu (1985), to include the cases of Gamma, Lognormal, Normal, and Uniform failure distributions.

These recent studies have employed a number of analytical and experimental simulation techniques, in order to support management decisions on MFP determination. Some of the formulations have reached quite efficient and ready to implement solutions. In addition, they have provided evidence that certain assumptions and factors, deemed important in the maintenance float policy modeling process, should be further studied.

Apart from its purely academic origin the need for further investigation on the maintenance float systems problem also stems from the real world of business. As new technological advances result in the replacement of man-machine systems, organizations rely on more complex and costly automata (e.g.: computers, robotics, flexible manufacturing systems, etc.), in order to compete. Within this process of change they realize higher quasi and/or actual rents for their human and physical assets. This higher cost, coupled with the increased complexity that automation poses on P/OM systems, create the attention that industrial and service, for- and non-profit organizations pay to maintenance management theory and applications (Gilbert & Finch, 1985).

Some real life MFP applications include emergency vehicles, air-force and navy fleet maintenance, life and safety sustaining systems, information systems, and flow- and job- production shops. According to Fersko-Weiss (1986), Fox (1977), Gotvals, Smith, Kruse, & Fortune (1977), Van Eseltine (1974), and Claire (1986), a very high level of operations availability is required for such systems to achieve their purpose. Their performance is determined by management decisions on the number of standby units as well as on the size of their repair crew. In addition, the widely implemented just-in-time (JIT) systems, also depend upon the availability of the production equipment when needed, and on a high level of system reliability during a P/O planning horizon. According to Claire (1986), too many managers regard the maintenance function as a necessary evil, and this is probably the most costly mistake a manager can make. One of the major maintenance resources, plant and production equipment, requires maximum operations availability as well as reliability for a JIT system to be successful. Thus, it becomes imperative to complement management planning for JIT implementation with a sound MFP. Providing decision support to managers of systems with high availability and reliability requirements is precisely the task of the analytically derived and experimentally tested and investigated maintenance float policy construals of this study.

## Contribution of the Study

Within the process of investigation on the above exposed real organizational life problem, this research provides a contribution in a number of ways. First, it adds to the recent normative MFP research by developing a set of flexible, reliability based triangular estimation models. The statistical techniques employed within a set of experimental analysis modules, show that these new models can significantly improve the process of maintenance float policy determination. Second, it enlarges the conceptual schema of the MFP framework, allowing to integrate, study, and increase understanding of the social, economic, and technological implications of maintenance float systems. Some of these implications have been totally overlooked in the past. Maintenance float systems have been considered as the mathematical equivalent to finite queues, limited resource queues, or mixtures of the two, and to dynamic inventory control. This study breaks away from this established plausible but narrow engineering perspective by considering maintenance float systems within their larger P/OM context. Third, by doing so, it evaluates MFP with an enhanced set of physical performance measures whose surfaces allow to recognize and subsequently analyze maintenance float system tradeoffs. Fourth, this study resolves a "mystery" in theoretically explaining why cost optimal maintenance float policies are so sensitive to a firm's underlying cost structure.

## Outline of the Dissertation and Methodology

The remainder of this chapter will present a brief overview of the maintenance and system reliability concepts. A literature review related to the specific area of inquiry will follow, in which the established framework as well as the recently developed MFP models will be presented. Chapter I will conclude with the level of analysis and limitations of the study.

Chapter II begins with the maintenance float policy determination methodology, adopted to provide theoretical consistency with existing MFP models. It explains the underlying structure of the new models that follow and continues with a discussion on the specific arguments that motivated the reliability based triangular estimation models which are built and tested as part of this study. Chapter II also includes the theoretical proofs of the new constructs, and concludes with a summary of results given in a table format.

In Chapter III certain assumptions underlying the employed methodology are revisited for testing on the basis of experimentally generated data. Issues of descriptive and predictive validity are considered as the explanatory power of certain major MFP factors is assessed. A pilot study and a total of three experimental analyses are reported in the chapter. The motivation, the underlying assumptions, the employed methodology, and the significant results are provided for each study under the headings of "experimental design" and "analysis of results." With a separate statistical treatment for theoretical and experimental data, the reliability based triangular estimation models are shown to significantly improve the process of MFP determination.

Chapter IV is primarily concerned with the study of physical performance characteristics of maintenance float systems. Within an integrative effort, a fourth experimental analysis is reported where measures of physical performance are allowed to influence the final determination of maintenance float policy. Three cost optimal MFP examples are provided as an illustration of their sensitivity to a hypothesized firm's underlying cost structure. On the basis of the employed analytical/experimental framework, the sensitivity issue is theoretically explained with the aid of a constrained cost minimization approach.

Chapter V contains a summary of this study's major experimental and theoretical findings, and concludes with a set of propositions for future research. Throughout this paper a total of thirteen figures and sixteen tables are utilized in order to support and better explain a mixture of conservative as well as radical extensions to the existing knowledge base on MFP. It is hoped that the final product stands above the minimum analysis, clarity, methodology, and overall signification standards required for risky research adventures such as this.

## Maintenance and System Reliability

The purpose of an effective maintenance policy is to keep a P/OM system in optimum operating condition so that it can satisfy expected demands at a minimum cost (Hardy & Krajewski, 1975). From a practitioner's point of view, all components of such a system are subject to deterioration and occasional failure in performing their assigned tasks. How fast deterioration occurs and how frequently breakdowns impose idleness on workers, equipment, and perhaps on the entire system, clearly depends on the design of the system, the employed equipment, and the local stress conditions.

In a more narrow sense, maintenance is intended to improve the reliability of such physical assets as machines, material handling of equipment, or computers, and the safety of buildings and facilities. According to Dervitsiotis (1981), the deterioration and failure patterns for highly critical system components can be observed, recorded, and analyzed rather objectively. Thus, a considerable amount of study has led to the development of both practical and theoretical models that can be used to formulate maintenance policies. The result of their successful implementation, however, requires an improvement of the overall reliability of the P/OM system at large.

Although the term "reliability" has many different technical meanings, the following definition by Bazovsky (1961), has become most commonly accepted, according to Mayer (1970).

The reliability of a component (or system) at time t, say R(t), is defined as R(t) = Prob{T>t}, where T is the life length of the component. R is called the reliability function.

The above definition simply says that the reliability of a component equals the probability that the component does not fail during the interval [0, t] (or, equivalently, reliability equals the probability that the component is still functioning at time t). For example, if for a particular item $R(t_1)=0.95$, this means that approximately 95% of such items, used under certain conditions, will still be functioning at time $t_1$.

In terms of the pdf of T, say f,

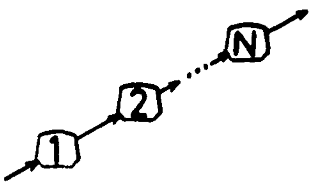
$$R(t) = \int_t^{\infty} f(s)\, ds. \qquad (1)$$

In terms of the cdf of T, say F,

$$R(t) = 1 - Prob(T \leq t) = 1 - F(t). \qquad (2)$$

The concepts of reliability and failure rate are central to maintenance management, and are considered important tools for a thorough study of "failure models." Mayer (1970), poses that the decision on what is a "reasonable" failure law to assume, is equivalent to determining what is a reasonable mathematical model for the description of some observational phenomenon. From a strictly mathematical point of view, one may assume any pdf for T and then simply study the consequences of that assumption. However, if there is interest in having the model represent (as accurately as possible) the actual failure data available, then the choice of the model must take this into account.

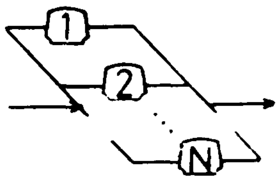
A subsequent and more difficult problem is the evaluation of the reliability of a system when the reliability of all of its components is known. According to Hillier & Lieberman (1986, p. 762), statistical estimation of component reliability is well in hand, but estimation of system reliability from component data is virtually an unsolved problem. One simple (but relatively important) case is the system of N components, connected in series, functioning independently, and the ith component has reliability $R_i(t)$; then, the reliability of the entire system, say R(t), is given by




$$R(t) = R_1(t)R_2(t)\ldots R_N(t), \qquad (3)$$

and the reliability of the system is less than the reliability of any of its parts.

Another important case is a parallel system in which the components are connected in such a way that the system fails to function only if all the components fail to function. For a system of N components operating in parallel, functioning independently, and if all the components have equal reliability, say $R_i(t)=r(t)$, for all i, then the reliability of the system, say R(t), is given by




$$R(t) = 1 - [1-r(t)]^N, \qquad (4)$$

with the system being more reliable than any of the individual components.

Whereas a series operation is often mandatory (that is, a number of components must function in order for the system to function), systems are mostly designed for parallel operation in order to increase their reliability. Despite the form and the design, however, the maintenance system is called upon either in a predictable manner for preventive maintenance, or at arbitrary intervals when failure is signaled by significant deviations from accepted standards in the quality of output or the cost and/or time to produce it. A general description of a maintenance system and its operation (Figure 1), should include a set of decision variables, input, output, constraints, and measures of performance.

Following the prerequisite variety of maintenance issues, a wide spectrum of alternatives has become available in order to keep equipment in satisfactory working order at a minimum cost. The analytical developments in maintenance modeling - other than, and yet related to MFP, can be classified in three major categories (Table I).

(i) Inspection. It is intended to determine the operating status of a component and may be visual or require some tests for measurement with special equipment. If the inspection reveals defective performance, the inspected component may be repaired or replaced. It is a widely employed alternative for maintenance at all levels of P/OM systems. Several authors (Hayre, 1983; Luss, 1976; Sengupta, 1980), have modeled continuous time maintenance problems, where equipment deteriorate according to a Markov or semi-Markov process. Their solutions consist of inspection and replacement schedules that minimize the long run expected average cost per unit of time.

(ii) Corrective Maintenance (CM) or Repair. This is the alternative used after a component breaks down and replacement is not advisable. CM policies take into consideration how critical the component is, and whether or not there is a standby unit that takes over to avoid production stoppage. The literature on CM can be further divided into two major sub-classes.

The first CM sub-class, includes models built to represent minimal repair (MR) policies, which restore a system without affecting its failure rate. The MR concept was introduced by Barlow & Hunter (1960), in their Policy I model that involves maintenance of fairly simple equipment. Brown, Mahoney, & Sivazlian (1983), define MR as the type of repair or replacement which is sufficient to return the equipment promptly to operational status. They combined the MR concept with that of "hysteresis repair" which leads to partial recovery of the system, to extend Eppen's (1965) model, by including a discount factor. As a result, Brown et al. (1983), determined the optimal service age of equipment, which once reached, management must decide whether to replace or completely overhaul. By expressing MR as the failure rate of equipment, that is: $MR(t)=1-R(t)=F(t)$, for $t>0$, Nakagawa & Kowada (1983), derived the two replacement policies that were originally developed by Barlow & Hunter (Policy I, 1960), and Morimura (1970).

The second CM sub-class of models includes constructs concerned with repair limit (RL) policies. The first RL model was developed by Nakagawa & Osaki (1974), and it was extended to its more general form by Nguyen & Murthy (1980), who found it optimal over both deterministic as well as stochastic RL policies. Under the RL formulation, a system is allowed to operate until a failure occurs. A repair is immediately applied, but if it is not completed after a time limit $RL=t*$, the unit undergoing repair is replaced by a new one.

t* minimizes the expected cost per unit of time over an infinite horizon, while a completed repair is assumed to make a component as good as new. Lambe (1974), extended the RL policy model to incorporate past repair cost data, while Beichelt (1982), found it more cost effective than the economic life time policy models (Lamson, Hastings, & Willis, 1983; Lohman, Foster, & Layman, 1982), where replacement decisions are based solely on the age of equipment. Age replacement policies are based on determining the useful age of a unit in operations (Mehrez & Stulman, 1983). After that age is reached, the unit is considered unproductive and thus, costly to maintain. Similarly, the RL policy is used to decide when to repair or replace a unit on the basis of cost or equivalently on the basis of the time required for its repair (Nakagawa & Osaki, 1974).

(iii) Preventive Maintenance (PM). It is generally considered a more comprehensive approach as it relies on both inspection and repair, according to a predetermined plan of action. Such action in PM may range from minor or major repairs to replacing parts or even an entire component. PM may be scheduled at different time intervals, at random or sequentially. An important condition for PM implementation is the small variability in the failure free run time of equipment, which makes an optimal inspection schedule feasible. White (1979), defined PM as a periodically performed operation that prevents failure, increases equipment reliability, and detects components with high failure rates. Policy II of Barlow & Hunter (1960), consists of a PM schedule which once applied, returns a system to a state that is equivalent to that following replacement. Thus, the optimum PM policy maximizes the "limiting efficiency" of a system, defined as the fractional amount of failure-free run time of equipment over long intervals of time. Nguyen & Murthy (1981), studied the PM problem under the assumption that the life distribution of a system changes after each repair, as the failure rate function increases with the number of repairs. For them, the aim of PM is to upgrade the reliability of equipment by reducing the chances of machine breakdown. For Mann & Coates (1980), PM is a control mechanism which provides corrective action according to feedback on equipment performance and maintenance needs. Several authors have utilized optimization techniques to model PM. The assumption underlying their mathematical constructs is that there exists a cut off point where it is more economical to apply PM rather than emergency maintenance. Thus, in order to be more effective, PM must be scheduled just prior to breakdown. Such an intervention may prevent the failure of equipment, and may reduce both the cost of breakdown as well as the cost of frequent inspections. If the equipment breakdown pattern is known, PM becomes much easier to implement, but still there is no certainty with respect to its final effectiveness.

In general, stochastic failure rates necessitate the use of complex mathematical techniques and restrictive assumptions in modeling maintenance policies. Unfortunately, this results in a shift from simple, tractable, and easy to implement models, to more and more complex mathematical construals. On the other hand, as the cost of investing in new machines rises, managers seek more efficient means to predict and control the behavior of the systems they manage.

Bullock (1979), and Chanin & Sphicas (1980), identified and reported on this widening gap between theory and practice in the maintenance management literature. Bullock's survey found that complex mathematical models are confined to large in size and slack resources organizations, while Chanin & Sphicas posed that, although useful for academic purposes, such models are of less practical utility. Most of their prescribed policies claim to minimize the total cost of maintenance by determining an optimum replacement and/or repair schedule. In addition to their complexity these models establish their cost optimum policies by studying maintenance systems on a unit by unit basis. No attempt is made to capture the performance of the system they study as a whole.

In practice, however, there are situations where important equipment must be kept in operation regardless of component failures. In such cases, a system breakdown is likely to have serious consequences due to disruption, downtime cost, or resulting unsafe conditions. That is when a maintenance float system is typically used.

## The Maintenance Float Policy (MFP) Framework

A maintenance float system is typically used when it is important to keep critical operating facilities at a high level of availability. It is a form of redundancy which once introduced improves the reliability of a system as a whole. Thus, the genesis of the literature sampled and classified in the lower part of Table I.

The research domain includes a P/OM system with three major sub-components: an operations facility, a repair station with multiple service channels, and a set of standby float units (Figure 2). The float is a ready (hot) inventory of spare units immediately available to replace the units which fail. Once repaired at the central repair station (CRS), the equipment return to a standby condition. In Figure 2, let N denote productive capacity in terms of a given number of independent, yet identical and simultaneously functioning equipment in their operations system environment. The length of time during which a unit operating under stress conditions occupies an "up" state (viz., the unit is in good working order), can be thought of as the time between two successive "down" states, during which the unit is inoperative, or ceases to function properly under the stress applied. Calling this up time "time between failure," or "life length," say T, a continuous random variable is realized, with some pdf.

Existing empirical evidence indicates that the value of T cannot be predicted from a deterministic model. That is, "identical" components subjected to "identical" stress fail at different and unpredictable times. Some fail quite early in their life and others at later stages. Of course, "the manner of failing" varies with the type of component being considered. A fuse, for example, fails rather suddenly in the sense that one moment is working perfectly and the next moment is not functioning at all. A steel beam under a heavy load, however, weakens over a long period of time. In any event, the use of a probabilistic model, with T considered as a continuous random variable, seems to be the only realistic approach.

When any of the initial N units in the operations facility fails (viz., changes from its up to a down state), it is released for repair. The time required for its repair - to be performed at the CRS, is also a continuous random variable and as such it assumes some probability distribution as well.

Whenever a failure occurs the number of operating units is reduced to N-1. If any reserve units are available, the down unit is replaced by an up, the presence of which immediately restores the number of operating units back to N. It is assumed that any standby units present are not actually in use, and therefore have no probability of failure. Switching in of a redundant unit occurs with perfect reliability and unimpaired performance. In such a system, a fixed number of units (those in operations, plus on reserve, and those undergoing and/or waiting for repair), are allowed to free-float as they circulate within an enlarged, yet closed P/OM system.

The rationale behind maintaining standby units is the following: any time the number of equipment in operations becomes less than N, the operations facility is not fully available for production. As a general management practice, such a reduction in operations availability is recorded as unavailability or "downtime," which can further be translated into cost of lost production.

The above framework can be further extended to model a system of J operations facilities which employ similar but not identical production equipment, they differ in terms of capacity, and they are all serviced by a central repair station (Figure 3). Different stress factors may be realized at each facility, due to different environmental or local conditions, demand load, handling of equipment, etc. Thus, vastly different failure rates may be experienced at each operations facility. Although it would be valuable to model and solve this generalized MFP problem for the J facility system, the challenge is reserved for future research.

The maintenance float system described above has various other names such as "extra equipment," "revolving fund," "maintenance exchange stock," "repairman's problem," or "spare provisioning." Regardless of the name, a unit which fails is replaced by a unit from the float and the old unit is repaired and returned to the float stock. It is this feature of replacement and concurrent repair which is unique to maintenance float systems.

## Maintenance Float Policy (MFP) Models

Several variations of the problem have been discussed in the literature. The most common version involves the determination of the optimal number of spares for operating components which are not repaired once they fail. Related models in the pure replacement spare parts problem had been treated by Geisler & Karr (1956), and Gourary (1958 & 1956). Those models minimized the expected total of weighted shortages, subject to a linear weight or cost restraint, with the demand probability density for spares assumed a priori.

On May 14, 1959, Guy Black and Frank Proschan presented "On Optimal Redundancy" at the Fifteenth National ORSA Meeting in Washington, D. C. (Black & Proschan, 1959). Their model maximized reliability by optimum allocation of redundant units likewise subject to a linear constraint. The demand for redundant units, however, instead of being assumed a priori, it was generated by the failures of operating units following an Exponential distribution. Thus, in order to obtain the composition of the optimal number of redundant units, Black and Prochan were the first to use information on component failure rather than information on component demand distributions.

Proschan (1959), had already shown that an optimal combination of redundant units can be obtained for the case of nonexponential life distributions as well. All that is required for the underlying densities of the component failure times is to possess the monotone likelihood ratio property. This property characterizes the Exponential, Gamma, Normal, and many other distributions. Due to this property a system's reliability takes the form of a concave function which a combination of redundant units seeks to maximize.

Additional models on the pure replacement spare parts problem were developed by Bhattacharyya (1967), Morrison (1961), and Schweitzer (1967). Other approaches consider both provisioning for spares and repair (e.g., Natarajan, 1968; Srinivasan, 1968), but these have been typically concerned with the derivation of the first time to failure, or the interarrival time between failures.

Barlow et al (1965), perceived the maintenance float system in Figure 2 as a repairman's problem. They, likewise Hilliard (1975), and Wahi (1966), formulated and solved it with the aid of the steady state birth and death equations. Such a formulation conveniently allows for analysis which naturally fits into the basic assumptions of queueing theory. It does facilitate the derivation of a cost optimal MFP solution, although White (1967), would rather formulate and solve the problem using dynamic programming. An alternative optimization approach was proposed by Mani & Sarma (1984), who identified MFP determination as a "closed queueing network" problem. Their formulation seems to eliminate the need for explicit arrival and departure assumptions for units which otherwise would be considered extraneous to the maintenance system.

Since a maintenance float policy is so difficult to determine analytically, researchers have frequently employed experimental simulation. Schriber (1974), for example, offered a general description of a maintenance float system and proposed to study the behavior of the expected total cost function on the basis of marginal cost parameters associated with the repair crew size, the number of standby units, and lost production. Additional trial-and-error extensions to the MFP problem via simulation have also been considered by Weir & Tiger (1971), White (1979), Widawsky (1971), and other colleagues. Such an extension yielding a specific solution while simultaneously controlling for the effect of routine maintenance, is the work of Johnson & Fernandes (1978).

The above stream of theoretical and experimental mathematical analyses on maintenance float systems with both replacement and concurrent repair has not explicitly considered the important issue of system reliability. Some of the authors who did incorporate system reliability in their optimization constructs, provided minimum cost solutions to the MFP problem within a spectrum of rather restrictive assumptions, deemed necessary in the development of their mathematical models. Kumagi (1975), Ramanarayanan & Usha (1979), Sabramanian & Venkatakrishnan (1975), Stevenson (1972), and Usha & Ramanarayanan (1980), for example, were forced to confine their formulations to systems with a very small number of units in operations.

Similar difficulties are reported by researchers who developed cost optimal direct search algorithms. The level of complexity increases as a large number of identical equipment which operate simultaneously require a considerable number of service channels and standby units in order to maintain a high level of operations availability and system reliability. Hilliard (1975), and Wahi (1966), for example, pose that for large maintenance float systems the vector grid boundaries get very large. Additionally, likewise other authors on MFP, they both conclude that cost optimal maintenance float policies which are based on the assignment of marginal cost parameters will always be highly sensitive to a firm's underlying cost structure. This last conclusion was also reached by Madu (1985), who followed a cost ratio approach, and Natarajan (1968). None of these authors ever explained why.

Boris Levine (1965), was the first to bypass the difficulties in developing a reliability based, theoretical estimation model without restrictions on the size of the system. He did so by utilizing the notion of a maintenance system's repair to failure ratio (r), defined as the ratio of mean time to repair (MTTR), over the mean time between failure (MTBF). By treating r as a variable, Levine analytically developed an estimation approach to the MFP problem for the case where both the equipment failures and repairs follow an Exponential distribution. Wallace (1985), reports that both the MTBF and the MTTR are well established notions and they are still used today with the MIL-STD-781, under the assumption of constant rates.

Following Levine's methodology, Love & Lewis (1983), developed a maintenance float policy for a public transportation system with Weibull failures and Lognormal repairs. The research domain was further extended by Madu (1985; see also Madu & Chanin, 1986), who showed (in Madu 1985: Corollary 1, p. 49), that the structure of these reliability based models for the estimation of the maintenance float factor (f), are independent of and can be used with any arbitrary repair distribution. Consequently, Madu derived a set of reliability based MFP estimation formuli for the cases of Gamma, Lognormal, Normal, and Uniform equipment failure distributions.

The presentation of the Levine-Love & Levis-Madu research stream developments is simplified in this paper, by presenting essential results in a tabular form, and by referring back to Levine (1965), Love & Levis (1983), and Madu (1985), for a more detailed formal treatment with proofs. Table II contains a summary of the relevant equations for the estimation of the maintenance float factor f, for a given class of equipment failure distributions.

Subsequently, the minimum maintenance total float (F), in terms of both spares and service channels required to achieve a relatively high level of operations availability, say A, can be estimated as a function of the the float factor and the number of units initially in operations. That is

$$F = f N. \qquad (5)$$

Unlike the cases of the Exponential and Weibull equipment failure distributions where the analytical results are relatively easy to implement, the realizations, for example, of a Gamma failure distribution may require the use of a rather unwieldy equation. To deal with the problem, Madu reduced the complexity of his formuli by offering the asymptotic property based alternatives (Table III). From the Gamma distribution the Exponential failure case was obtained and shown to be equivalent to Levine's original formulation. The special cases of Gamma were also obtained, and the f was shown to approach zero at the limit for the Degenerate or Constant failure distribution.

## Level of Analysis and Limitations

The literature sample on maintenance float systems indicates the need for models that decrease the vector grid direct search boundaries for large systems with high availability requirements. This is precisely the need that the reliability based Levine-Love & Lewis-Madu estimation constructs responded to. The experimental analyses reported by Love & Lewis (1983), and Madu (1985), however, indicated that in both the cases of large maintenance float systems and large repair to failure ratio r values, their analytically derived estimates fall short from achieving high levels of operations availability. For such cases, the vector grid boundaries still remain large resulting in inefficient MFP searches. The reliability based triangular estimation models which are built and tested in the chapters to follow, are shown to provide initial float factor estimates that significantly improve the optimal maintenance float policy determination effort, for both large systems and large r values.

Most of the formulations in the MFP literature depend on the a priori assignment of cost parameters. Consequently, the optimal maintenance float policies derived are very sensitive to small changes in the marginal cost parameters of the objective functions they depend on. This study deviates from this explicit a priori marginal cost assignment method. Throughout the theoretical, experimental and statistical analyses which follow in Chapters II, III, and IV, the primary focus is on the physical performance characteristics of maintenance float systems. The improvement of the overall system effectiveness is pursued in this study as new facilities are planned, designed, and built.

The behavior of a MFP total cost function is considered a posteriori in Chapter IV. A steady state cost minimization approach is the final determinant of the optimum maintenance float policy, yet it is restrained by the decision makers' choice on the physical performance of the system they manage.

There are certain limitations inherent in the modeling process of systems. Unfortunately, certain relationships must be excluded from consideration. Little useful information would be generated by a model in which "everything is related to everything else," even if the model were mathematically tractable. For this reason, a set of paths of primary interest was selected for analysis, under the assumption that such selection does not deny the existence of possible relevant others.

Furthermore, additional arguments might be made for the potentially interesting effects of the numerous maintenance issues and attributes not addressed in the present study. Again, the selection process reflects the dimensions of greatest immediate interest, and in no way denies the existence or influence of others.

# CHAPTER II

## RELIABILITY BASED TRIANGULAR ESTIMATION MODELS

### Maintenance Float Policy (MFP) Estimation Methodology

The approach employed in this chapter was adopted to provide theoretical consistency with the reliability based MFP estimation model base as a whole. Each new construct will be defined, developed, and its place in the scheme of relationships will be made explicit. The newly constructed model base will then be ready for the process of experimentation and testing, which involves simulation followed by the appropriate statistical analysis.

Levine (1965), once more, defined the maintenance float factor *f* as the proportion of units which have failed up to time $t_n$. Central to his model was the notion of system reliability as well as the expected time of the nth renewal of a unit, which at steady state is defined as n times the sum of MTBF plus MTTR. That is $T(n)=n(MTBF+MTTR)$. To fix ideas, the following notation and definitions are introduced.

$N$ : number of units initially in operations.

$N_t$ : number of functioning units at time t (at time $t_1$: $N_t = N-1$).

$R(t)$ : system reliability at time t; defined as the fraction of functioning units at time t. Thus, $R(t) = N_t/N$, with $R(t_1) = (N-1)/N$.

MTBF : mean time between failure.

MTTR : mean time to repair.

$r$ : repair to failure ratio; defined as $r$ = MTTR/MTBF.

$t_n$ : the time of the nth failure. It is required that $t_n \geq t_1$+MTTR, so by the time the nth failure occurs, the unit which caused the first failure has already been repaired. With this condition the operations facility is never completely unavailable and an average of N units in operations is achieved.

$f$ : maintenance float factor; defined as the proportion of units that have failed up to time t. Thus, $f = 1-[N_t/N]$ or alternatively, $f = 1-R(t)$.

$F$ : maintenance total float; backup and service channel requirements in support of an average of N units in operations. From (5) defined as $F = fN$.

In order to illustrate the methodology developed and applied by the Levine-Love & Lewis-Madu research stream, the equations yielding $f$ for the Weibull failure distribution (Table II, formulae II-2, and Table III, formulae III-2), are analytically derived. These formuli represent a generalization of the Love &

Levis (1983) model, and they have not appeared elsevhere in the MFP literature. Madu (1985, Theorem 3, p. 49), presented the original Love & Levis result without proof nor any modification.

Weibull Distribution. According to Gredenko, Belyayev, & Solovyev (1969), the distribution function

$$F(t) = \begin{cases} 0, & \text{for } t < 0, \\ 1 - \exp(-(t/b)^a), & \text{for } t \geq 0, \end{cases} \qquad (6)$$

with shape parameter a>0, and scale parameter b>0, has been given the name Weibull distribution. This distribution was used by Weibull in order to describe experimentally observed variations in the fatigue resistance of steel, its elastic limits, the dimensions of particles of soot, etc. It has recently been used to study the variations in the length of service of electronic equipment. Weibull's distribution was known earlier in probability theory as the limiting or Type III extreme value distribution (Type I: Gumbel; Type II: Cauchy-Frechet; Type III: Weibull; see Ledevmann, 1980). Today it is widely used in reliability models for lifetimes of devices and in order to model time requirements to task completion.

<u>Theorem 1</u>.

For a P/OM system with N units initially in operations which fail according to a Weibull(a, b) distribution, the maintenance float factor is given by

$$f = 1 - \exp(-[(\ln[N/(N-1)])^{1/a} + MTTR/b]^{a}).$$

<u>Proof</u>:

assuming time increments small enough so that only one unit can fail at a time, from equation (2),

$$R(t_1) = \exp(-(t_1/b)^{a}). \qquad (7)$$

Since $t_1$, however, signifies the first failure of a unit in operations, the system's reliability becomes

$$R(t_1) = (N-1)/N. \qquad (8)$$

Due to the equal left hand side of (7) and (8),

$$\exp(-(t_1/b)^a) = (N-1)/N, \tag{9}$$

or $$(t_1/b)^a = -\ln[(N-1)/N], \tag{10}$$

or $$t_1 = b(-\ln[(N-1)/N])^{1/a}. \tag{11}$$

Due to the required condition given in the notation and definitions above: $t_n \geq t_1+MTTR$, where MTTR is the mean time to repair of an arbitrary repair distribution, and from (11),

$$t_n \geq b(-\ln[(N-1)/N])^{1/a}+MTTR. \tag{12}$$

Thus, in order to maintain a minimum system reliability, from the equality in (12)

$$R(t_n) = \exp(-([b(-\ln[(N-1)/N])^{1/a}+MTTR]/b)^a), \tag{13}$$

or $$R(t_n) = \exp(-[(-\ln[(N-1)/N])^{1/a}+MTTR/b]^a). \tag{14}$$

Since $f = 1-R(t)$,

$$f = 1-\exp(-[(-\ln[(N-1)/N])^{1/a}+MTTR/b]^a), \tag{15}$$

and due to the relationship

$$-\ln[(N-1)/N] = \ln[N/(N-1)], \tag{16}$$

formulae (15) transforms to

$$f = 1-\exp(-[(\ln[N/(N-1)])^{1/a}+MTTR/b]^a). \tag{17}$$

Q. E. D. □

Theorem 2.

For a P/OM system with a very large number of units initially in operations ($N \longrightarrow \infty$), which fail according to a Weibull(a, b) distribution, the maintenance float factor is given by

$$f = 1-\exp-((MTTR/b)^a).$$

Proof:

in (17), let $N \rightarrow \infty$. Then,

$$\lim_{N\rightarrow\infty}(N/(N-1)) = \lim_{N\rightarrow\infty}(1/[1-(1/N)]) = 1, \qquad (18)$$

and since ln(1) = 0, (17) becomes

$$f = 1-\exp-((MTTR/b)^a). \qquad (19)$$

Q. E. D. □

## Preliminary Arguments and Discussion

Turning back to Table II, in the case of an Exponential distribution (formulae II-1), the value of $f$ depends on two major factors: N and r. In the two parameter failure distribution cases, the number of factors on which $f$ depends is increased. Namely, in the Gamma case (formulae II-3), $f$ is a function of N, r, and s (s signifying the number of phases to final failure). With Lognormal (formulae II-4), and Normal (formulae II-5), failures, the standard deviation becomes an additional factor in the estimation of maintenance float policy.

Table III, which is a summary of the equivalent MFP models in their asymptotic form, shows how the complexity in the equations yielding $f$ is immediately reduced as N is assumed to approach infinity. In all cases, the number of factors which influence the estimation of $f$ is reduced. For example, the maintenance float factor depends only on the repair to failure ratio value in the Exponential case (formulae III-1). It is expressed as a function of r and s in the case of a Gamma failure distribution (formulae III-3).

If equipment fail according to a Gamma distribution, as a becomes large (viz., it approaches infinity), f becomes zero and the Degenerate result is obtained. Otherwise, as the flow of operations reaches a steady state, f becomes Constant and for large values of N, from formulae (5), the maintenance total float F is directly proportional to the initial number of units in operations. The asymptotic results of Table III, have been shown to provide good approximations to the f estimates from Table II, even for N small (Madu & Chanin, 1986).

In spite the paradigmatic elegance, from the inception of the MFP models, several assumptions were postulated. These, although convenient for analytical purposes, can be hardly tenable in practice. The three major ones, which were carried through by the Levine-Love & Lewis-Madu stream, are listed below.

1. A unit is completely rejuvenated after repair, assumed to be as good as new.
2. Waiting time to repair is nonexistent or negligible.
3. The equipment failure distribution has been completely characterized from empirical data.

Due to the modularity concept in manufacturing and maintenance management, the first assumption stands rather close to reality. For the second assumption, however, reality dictates that one should retain a healthy skepticism. The very first experimental analysis which is reported in Chapter III, allows to explicitly test both for the statistical significance of the MFP determination factors N and r, and for the negligibility of the average waiting time to repair, say W.

With respect to the third assumption, when actual observations of times to failure are recorded, it is difficult to distinguish among the various possible underlying probability functions. The differences between certain asymmetrical distributions, such as Gamma, Weibull, and Lognormal (Figure 4), are significant only at the tails. Actual observations, however, can be and in most cases are sparse at the tail even for fairly large samples. Barlow et al (1965), were rather concerned with this important issue of failure distribution characterization. In order to make the problem tractable, Bazovsky (1961), proposed the use of the instantaneous failure rate or "hazard" function, which is associated with the random variable T, and is given by

$$Z(t) = g(t)/[1-G(t)] = g(t)/R(t), \qquad (20)$$

defined for $G(t)<1$. While g, the pdf of T, uniquely determines the hazard function, the converse also holds: Z uniquely determines the pdf g.

In some cases, however, it may not be possible to collect data on equipment failure nor repair times, so the techniques proposed by Barlow et al (1965), and Bazovsky (1961), are not applicable to the maintenance float policy estimation problem. For example, if a newly planned maintenance float system does not currently exist in any physical form other than the blueprint, collecting data from the system is not possible.

## The Triangular Approach

Collecting real life data from a newly planned P/OM facility may indeed be impossible. All that the planner of such a system knows is that the random quantity of interest, time, is a continuous random variable T. The planner may think of T as being the time to perform some task, e.g., the time required to repair a piece of equipment, or as being the duration of an activity, e.g., the time to failure of a new machine, or a new computer component. The first reaction in such cases is to identify a time interval [a, b] (a and b real numbers such that a<b), in which it is "felt" that T will lie with probability close to one; that is, Prob(T<a V T>b) = 0. In order to obtain "subjective" estimates on b (the longest likely time to failure), and a (the shortest likely time to failure), the planner will ask some "experts" for their most optimistic and pessimistic estimates of T, respectively. Once such an interval has been identified, the next step is to place a pdf on [a, b], which is thought to be representative of T.

Inferences for large systems are inevitably based to a great extent on theoretical considerations. Whenever data are available or can be collected, the MFP investigator can affect a considerable improvement in the quality of the input and hence the meaningfulness of the analytical and/or experimental results, by using statistical methods to complement the theoretical considerations and the specification of parameter values.

Data collection and analysis are costly however. In addition, they take time, a rare commodity when answers are needed in a hurry. Yet, the manager and/or planner must give an MFP estimate for a system of operations which might be vital to the organization. Since none of the state-of-the-art maintenance float policy models can explicitly handle the lack of data, this could be a risky decision under uncertainty.

Fortunately, heuristic procedures for choosing a distribution in the absence of data do exist, and practitioners use them. They are based on the Beta[$a_1$, $a_2$], and Triangular[a, b, c], distributions (Britney, 1976; Chase & Aquilano, 1985; Dane, Gray, & Woodworth, 1979; Fishman, 1973; Law & Kelton, 1982).

In the Triangular approach, once an interval [a, b] has been subjectively identified, the next step is to place a pdf on it, which is thought to be representative of the continuous random variable time to failure T. Additionally, the experts are also asked for their subjective estimate on the most likely time to failure. This most likely value, say m, is the mode of the distribution of T. Given a, b, and m, the random variable T is then assumed to follow a Triangular distribution on the interval [a, b] with mode m; that is, Triangular[a, b, c=m].

The practitioners' reliance on the above described approach of "triangulation," may be traced back to the "principle of minimum prejudice," which states that one should be as noncommittal as possible regarding the things that are not known. Jaynes (1957), developed the "minimax entropy estimator" on the basis of this principle. Even if it is not true that one should always try to minimize the maximum entropy within a system, however, another plausible justification for the Triangular heuristic may emerge from the realm of maximum likelihood estimation, which relies upon a particular probabilistic description of data. Goodwin & Payne (1977, p 16-17), for example, formally provide the following:

Definition.

A maximum a posteriori (MAP) estimate is the mode of the posterior distribution $p(h|y)$, where the function $p(y|\cdot)$ is called the likelihood function.

Further, by letting $k = p(h)/p(y)$, where k does not depend upon h, and with

$$p(h|y) = p(y|h)p(h)/p(y) = kp(y|h), \qquad (21)$$

Goodwin & Payne show how the above defined MAP estimator coincides with the maximum likelihood estimator (MLE), for a Uniform[a, b] prior distribution (which is often called the non-informative prior).

Thus, once a failure time interval [a, b] is identified, a non-informative prior or Uniform[a, b] distribution may be assumed. Next, based on the above formal definition and result, a subjective estimate on the most likely time to failure, say m, may serve as an MLE, which coincides with the MAP. Parsimony would pose m as the mode of a posterior Triangular[a, b, c=m].

The following set of analytically derived results is proposed as a class of robust maintenance float policy estimation models, when the only information on the equipment failure pattern available to management, consists of the subjective estimates of experts. In effect, the work of the Levine-Love & Levis-Madu stream is extended, based on all but their third assumption.

Since Madu (1985: Corollary 1, p. 49), showed that the structure of the MFP estimation formuli depends on the equipment failure distribution alone, the distribution of repair times will not be of major concern. Besides, maintenance experts can always provide an interval or point estimate for the MTTR as well as the Triangular parameters a, b, and c. These reliability based constructs are developed for both the Triangular and Left Triangular failure distributions. For maximum uniformity with the Levine-Love & Levis-Madu research results, the asymptotic cases are also derived.

<u>Triangular Distribution</u>. The cdf of T following a Triangular[a, b, c], is given by

$$F(t) = \begin{cases} 0, & \text{if } t < a, \\ ((t-a)^2)/((b-a)(c-a)), & \text{if } a \le t \le c, \\ 1-[((b-t)^2)/((b-a)(b-c))] & \text{if } c < t \le b, \\ 1, & \text{if } b < t, \end{cases} \quad (22)$$

a, b, & c real (a: location, b-a: scale, c: shape parameters).

<u>Theorem 3</u>.

For a P/OM system with N units initially in operations which fail according to a Triangular[a, b, c] distribution, the maintenance float factor is given by

$$f = ([(b-a)(c-a)/N^{1/2} + MTTR)^2/[(b-a)(c-a),$$

for

$$0 < MTTR \le (1-(1/N)^{1/2})([(b-a)(c-a)]^{1/2},$$

$$= 1-(([[(N-1)/N](b-a)(b-c)]-MTTR)^{1/2}/(b-a)(b-c)),$$

for

$$[1-(1/N)^{1/2}][(b-a)(c-a)]^{1/2} < MTTR \le ([(N-1)/N](b-a)(b-c))^{1/2},$$

$$= 1, \qquad \text{for } ([(N-1)/N](b-a)b-c))^{1/2} < MTTR.$$

Proof:

assuming time increments small enough so that only one unit can fail at a time, from the definition of R(t) in (2), and from the first part of (22)

$$R(t) = 1 - [((t-a)^2)/((b-a)(c-a))], \qquad (23)$$

defined for $a \leq t \leq c$. At $t_1$,

$$R(t_1) = 1-[((t_1-a)^2)/((b-a)(c-a))] = (N-1)/N, \qquad (24)$$

or
$$N(b-a)(c-a) - N(t_1-a)^2 = (N-1)(b-a)(c-a), \qquad (25)$$

or
$$t_1 = a + [(1/N)(b-a)(c-a)]^{1/2}. \qquad (26)$$

Due to the condition $t_n \geq t_1 + MTTR$, let

$$t_n \geq a + [(1/N)(b-a)(c-a)]^{1/2} + MTTR. \qquad (27)$$

From (23), and by considering the equality in (27)

$$R(t_n) = 1-(\{[(1/N)(b-a)(c-a)]^{1/2}+MTTR\}^2/[(b-a)(c-a)]) \quad (28)$$

and due to $f = 1 - R(t)$,

$$\left.\begin{array}{l} f = (\{(b-a)(c-a)/N\}^{1/2}+MTTR)^2/[(b-a)(c-a)], \\ \text{for} \quad 0 < MTTR \leq (1-(1/N)^{1/2})([(b-a)(c-a)]^{1/2}), \end{array}\right\} \quad (29)$$

because $0 \leq f \leq 1$. Similarly,

$$\left.\begin{array}{l} f = 1-(\{[[(N-1)/N](b-a)(b-c)]-MTTR\}^{1/2}/(b-a)(b-c)), \\ \text{for} \\ [1-(1/N)^{1/2}][(b-a)(c-a)]^{1/2}<MTTR\leq([(N-1)/N](b-a)(b-c))^{1/2}. \end{array}\right\} \quad (30)$$

Q. E. D. □

Theorem 4.

For a P/OM system with a very large number of units initially in operations ($N \longrightarrow \infty$), which fail according to a Triangular[a, b, c] distribution, the maintenance float factor is ginen by

$$f = (MTTR)^2/(b-a)(c-a),$$

for

$$0 < MTTR \leq [(b-a)(c-a)]^{1/2},$$

$$= 1-([[(b-a)(b-c)]^{1/2}-MTTR]^2/(b-a)(b-c)),$$

for

$$[(b-a)(c-a)]^{1/2} < MTTR \leq [(b-a)(b-c)]^{1/2},$$

$$= 1, \qquad \text{for } [(b-a)(b-c)]^{1/2} < MTTR.$$

Proof:

In (29), let $N \to \infty$. Then,

$\lim_{N \to \infty}(1/N) = 0$. Thus, (31)

$f = (MTTR)^2/(b-a)(c-a)$, for $0 < MTTR \leq [(b-a)(c-a)]^{1/2}$. (32)

Similarly, in (30), let $N \to \infty$. Then,

$$\lim_{N\to\infty}[(N-1)/N] = \lim_{N\to\infty}[1-(1/N)] = 1-0 = 1. \qquad (33)$$

Thus, (30) becomes

$$\left.\begin{array}{l} f = 1 - ([[(b-a)(b-c)]^{1/2}-MTTR]^2/(b-a)(b-c)), \\ \text{for} \\ \quad [(b-a)(c-a)]^{1/2} < MTTR \le [(b-a)(b-c)]^{1/2}. \end{array}\right\} \qquad (34)$$

Q. E. D. □

<u>Left Triangular Distribution</u>. The limiting case as $c \to a$ is called the Left Triangular[a, b] distribution. For a=0 and b=1, the Left Triangular[a, b] is a special case of the Beta[$a_1$, $a_2$] distribution. The cdf of T following a Left Triangular[a, b], is given by

$$\left.\begin{array}{lll} F_L(t) & = 0, & \text{if } t < a, \\ & = 1 - [(b-t)/(b-a)]^2, & \text{if } a \le t \le b, \\ & = 1, & \text{if } b < t. \end{array}\right\} \qquad (35)$$

Theorem 5.

For a P/OM system with N units initially in operations which fail according to a Left Triangular[a, b] distribution, the maintenance float factor is given by

$$f = 1 - \{[(N-1)/N]^{1/2} - [MTTR/(b-a)]\}^2,$$

for

$$0 < MTTR \leq (b-a)[(N-1)/N]^{1/2},$$

$$= 1, \qquad \text{for } (b-a)[(N-1)/N]^{1/2} < MTTR.$$

Proof:

assuming time increments small enough so that only one unit can fail at a time, from the definition of R(t) in (2), and (35)

$$R(t) = [(b-t)/(b-a)]^2, \qquad (36)$$

defined for $a \leq t \leq b$. At time $t_1$,

$$R(t_1) = [(b-t_1)/(b-a)]^2 = (N-1)/N, \qquad (37)$$

or $t_1 = b - (b-a)[(N-1)/N]^{1/2}$. (38)

Since it is required that $t_n \geq t_1 + MTTR$, let

$$t_n \geq b - (b-a)[(N-1)/N]^{1/2} + MTTR. \qquad (39)$$

From (36), and by taking the equality in (39),

$$R(t_n) = (b-[b-(b-a)[(N-1)/N]^{1/2}+MTTR]/(b-a))^2, \qquad (40)$$

or $R(t_n) = ([(N-1)/N]^{1/2}-[MTTR/(b-a)])^2$. (41)

Thus, $f = 1 - ([(N-1)/N]^{1/2}-[MTTR/(b-a)])^2$,

for

$$0 < MTTR \leq (b-a)[(N-1)/N]^{1/2}. \qquad (42)$$

Q. E. D. □

**Theorem 6.**

For a P/OM system with a very large number of units initially in operations ($N \rightarrow \infty$), which fail according to a Left Triangular[a, b] distribution, the maintenance float factor is given by

$$f = 1 - \{1-[MTTR/(b-a)]\}^2, \quad \text{for} \quad 0 < MTTR \leq b-a,$$

$$= 1, \quad \text{for} \quad b-a < MTTR.$$

**Proof:**

In (42) let $N \rightarrow \infty$. Then, due to (33),

$$f = 1 - \{1-[MTTR/(b-a)]\}^2, \quad \text{for} \quad 0 < MTTR \leq b-a, \tag{43}$$

because $0 \leq f \leq 1$.

Q. E. D. □

Table IV contains a summary of the analytically derived results for the reliability based triangular estimation MFP models. Both the Triangular and Left Triangular distribution cases included in the table may be considered as extensions to the Levine-Love & Lewis-Madu research stream, since they are based on all but their third assumption.

Thus, if the underlying equipment failure distribution cannot be characterized for a newly planned maintenance float system due to the lack of data, the models in Table IV can be used to estimate an initial MFP. Once a set of "educated" subjective estimates on the most likely a, b, c, and MTTR, parameter values becomes available from experts, it can be used to estimate the maintenance float factor (f), from which the maintenance total float (F), can be subsequently calculated by using formulae (5). In the case that N is large (viz., $N \rightarrow \infty$), the asymptotic models summarized in Table V may be used.

Again, the above model base is primarily proposed as a set of robust MFP estimation constructs when the only information available to management consists of the subjective estimates of experts. Due to their flexible structure, however, on which some rather desirable properties identified within the pilot study of Chapter III may be attributed, these models can significantly improve the estimation process of maintenance float policy even when the underlying failure distribution has been completely characterized.

# CHAPTER III

## EXPERIMENTAL ANALYSIS AND TESTING

### Estimated versus Actual Need

In order to measure the validity of his model, Levine (1965), performed a set of comparisons with two real life maintenance float systems. In both cases, the theoretically estimated float requirements were found lower than the ones which had been established on the basis of actual need. While Love & Lewis (1983), were more concerned with local failure and repair distribution characterizations, Madu (1985), also found his theoretical f estimates to be low when compared to the float factor requirements which he established through a set of simulation experiments.

Despite the similar findings, the interpretations of Levine and Madu differ. Levine's explanation was that his repair to failure ratio (r) based formulae "... does not determine the effectiveness of the float, only its size" (Levine, 1965, p. 403). Madu (1985), however, attributed the consistency of his results to the second major assumption on the negligibility of the average waiting time to repair, say W. He further posed that although the inclusion of W would make the theoretical estimation of MFP difficult to derive, the explanatory power which the two major maintenance float factors, N and r, have on W should be experimentally assessed.

Levine's theoretical estimation model and its extensions, assume W away by considering it to be a negligible quantity. In real as well as simulated digital life, however, it appears rather difficult to eliminate W. Besides, the cost of such an achievement would be astronomical. In the following section, Levine's model is revisited, and the effects that N and r have on the "negligible" W are statistically tested. Thus, the validity of Levine's second major assumption is experimentally assessed.

## Experimental Analysis #1

### Experimental Design

The maintenance float system studied in this section was modeled with the aid of the following assumptions:

1. The N units initially in operations function independently, and their failure times are identically distributed following an Exponential(b=1) distribution.

2. N was set equal to the fixed values of 5, 20, and 50 units initially in operations.

3. Repair times were assumed to be constant, with the repair to failure ratio (r), set equal to the fixed values of 0.25, 0.50, 0.75, and 1.00.

4. A unit is completely rejuvenated after repair, assumed to be as good as new, functioning within a closed system of operations and maintenance.

A set of float factor (f) estimates was obtained from Levine's model (Table II, formulae II-1). By using equation (5) and rounding off, the maintenance float system requirements were estimated in terms of standby float units (F), as well as service channels (S), at the central repair station (CRS). These estimates were subsequently used in the implementation of a simulation model (Figure 5), on an IBM/370+ series mainframe using GPSS-V (Gordon, 1975). Ten replications were used for each simulation run. The timer was set to 43,200 GPSS time units, an assumed equivalent of 720 hours (90 work days), of operation. Transient state statistics were canceled, in order to avoid contamination of steady state results. Consequently, ten experimental realizations were averaged to obtain the W values, which were indeed realized and recorded in Table VI.

## Analysis of Results

When multiple criterion observations become available and the research goal is to assess the impact of various levels of one or more experimental variables on the criterion values, multivariate analysis-of-variance (ANOVA), is the appropriate data analysis technique. The primary focus in this section was to test for any significant differences on the waiting time to repair "profile," due to changes in the experimentally controlled factors N and r. The statistical tests conducted were based on the variation of W and its decomposition.

The values that the criterion (dependent) variable W took on, clearly constituted an experimental data set for which the random assignment method - rather than random sampling, was used. The two independent variables were treated as a pair of orthogonal metric variates. More specifically, with the provision of a set of fixed values for each of the metric variables N and r, their effect on W was established. The statistical analysis was performed under the assumption of a fixed effects model, using SPSS (Nie, Hill, Jenkins, Steinbrenner, & Bent, 1975).

## Significance of the Effects of W Factors

Within the context of the above described application, the differences in W were assessed for their significance. A set of two hypotheses was formulated and tested (Table VII), at four different confidence levels. In particular, $H_1$ postulated that $E(W|N=5)=\ldots=E(W|N=50)$, or eta=0. The $F_{Computed}$ indicates that the null hypothesis of equal means ($H_1$), should be rejected. The descriptive eta value for N was improved to 0.7543, which signifies that the initial number of units in operations, alone, can explain more than 75% of the variability in the steady state realizations of W. $H_2$ postulated that $E(W|r=0.25)=\ldots=E(W|r=1.00)$, or eta=0. The $F_{Computed}$ indicates that the null hypothesis of equal means ($H_2$), should not be rejected at any conservative (0.01, nor 0.05) alpha level. The descriptive eta value for r was improved only up to 0.5161, which signifies that the repair to failure ratio, alone, explains less than 52% of the variability in the steady state realizations of W.

N was found to be highly correlated to W with a correlation coefficient of 0.9140. Figure 6, which is based on the experimental realizations in Table VI, shows a direct linear relationship among (a) N and W, and (b) r and W, respectively. W increases as N and r get large, but the effect of r is not as significant as that of N. The two positive relationships shown in Figure 6, were both expected since: (a) the arrival rate of units into the queue is directly proportional to the number of units (N) in operations - a property of limited source queueing models; and (b) higher values of the repair to failure ratio (r), imply slower service.

The significant result obtained here is that different rates of change in average waiting time to repair are realized, as the initial number of units in operations and the repair to failure ratio are increased. This result makes it difficult to assume that W is negligible. Since W co-varies with both N and r respectively, it is expected to have a considerable effect on the maintenance float factor (f), and, thus, on the overall effectiveness of the maintenance total float (F).

If the rates of change in W were found to be negligible as N and r vary, then the assumption of no waiting time to repair might have been acceptable. Based on this section's experimental data, however, statistical results in Table VII, and pictorial relationships in Figure 6, waiting time to repair cannot be assumed to be negligible for $N \geq 5$, and $r \geq 0.25$.

## Pilot Study

In order to determine how the reliability based triangular estimation models compare to the Levine-Love & Lewis-Madu formuli, two sets of theoretical float factor ($f$) estimates were generated. Both data sets were obtained through the implementation of the tabulated $f$ equations from Table II and Table IV. The maintenance float factors were calculated for a fixed range of the initial number of units in operations ($N = 5, \ldots, 1000$), and and for a fixed range of the repair to failure ratio ($r = 0.25, \ldots, 1.50$). The data set obtained from the implementation of the formuli in Table II (for the Exponential, Weibull, and Gamma distributions), was plotted against the set of the $f$ values obtained from the implementation of the equations in Table IV. Figure 7 contains these plots for the set of fixed values for $N$; Figure 8, for the set of fixed values for r. A comparison of these preliminary and rather arbitrary theoretical f estimates was conducted and measured on the basis of the absolute percentage error defined as

$$|\%e| = |(f_{Triangular} - f_{Failure\ Dist^n})| / f_{Triangular}, \qquad (44)$$

which is also included in the plots. For most cases, |%e| took on values well below the 10% mark. In all cases, the mode of the Triangular distribution was set equal to the mode of the hypothesized actual failure distribution.

From this pilot study, the reliability based triangular MFP formuli, appear rather promising in estimating the maintenance float factor, when the underlying failure distribution has not yet been characterized by the planner/decision maker. Thus, the third major assumption of the Levine-Love & Lewis-Madu research stream, may be relaxed, at least for certain ranges which the current study purports to identify.

Additionally, the behavior of the derived f equations was observed over the preset wide range of N and r values. In the cases of the Exponential and Gamma failure distributions, and for a wide range in the Weibull case, the reliability based triangular estimation models were found much more sensitive to both large N and large r values than the Levine-Love & Lewis-Madu stream results. This demonstrated sensitivity (Figure 7, and Figure 8), is quite desirable due to the lack of queueing effect considerations in the system reliability based, theoretical estimation process of an initial maintenance float policy (MFP).

Based on the above preliminary observations, and while taking into consideration both the third and second major assumptions of the Levine-Love & Lewis-Madu stream in combination, two more experimental analyses were motivated and are reported below.

## Experimental Analysis #2

### Experimental Design

The results of Experimental Analysis #1 showed that the average waiting time to repair (W), is expected to have a significant impact on the maintenance total float effectiveness. As both N (the number of units initially in operations), and r (the repair to failure ratio), get large, W is proportionally increased. Based on the pilot study, however, the observed behavior of the reliability based triangular estimation constructs, indicated that they can compensate for the non-inclusion of W in the Levine-Love & Lewis-Madu models.

Consequently, the next step was to establish the ranges over which the desirable compensation would be significant. By letting the formuli in Table V be larger or equal to the equations of Table III, a set of five inequalities was formulated. This set was solved for the (b-a) scale parameter of the reliability based triangular estimation models. In order to be consistent with the theoretical functions used by the Levine-Love & Lewis-Madu models, however, the location parameter of the Triangular function was set equal to zero. In addition, the shape parameter (c), was set equal to the theoretical mode of each corresponding distribution function.

Finally, the b values obtained from the solution of each established inequality, completely determined the ranges of the underlying Triangular densities to be tested. The objective was to test if the reliability based triangular estimation models indeed provide significantly higher f values than the Levine-Love & Lewis-Madu MFP constructs. The theoretical experiments of this section were conducted with the aid of the following assumptions:

1. The N units initially in operations function independently, and their failure times are identically distributed, following one of the five theoretical distributions: (1) Exponential, (2) Weibull, (3) Gamma, (4) Normal, and (5) Lognormal.

2. N was set equal to the fixed values of 5, 10, 20, 40, and 80 units initially in operations.

3. Repair times were assumed to be constant with the repair to failure ratio (r) set equal to the fixed values of 0.25, 0.50, 0.75, 1.00, and 1.50.

4. A unit is completely rejuvenated after repair, and it is assumed to be as good as new functioning within a closed system of operations and maintenance.

Two sets of maintenance float factor (f), and maintenance total float (F), theoretical values were obtained, coded, and listed in Table VIII, from the implementation of both the Levine-Love & Lewis-Madu models (method M=1), and the reliability based triangular estimation models (method M=2), on an IBM/370+ series mainframe, using SIMSCRIPT II.5 (Kiviat, Villanueva, & Markowitz, 1968).

## Analysis of Results

The primary focus of this section was to test if the f and F values obtained from the reliability based triangular estimation models, were significantly higher than the theoretical f and F estimates obtained from the Levine-Love & Lewis-Madu constructs. Technically speaking, the research goal was to assess the impact of the two levels of the non-metric experimental variable method (M), on the maintenance float factor (f), and on the maintenance total float (F). The experimental factors N and r had to be considered as well. The statistical tests conducted were based on the variation of f, the variation of F, and their respective decomposition.

Again, the values that the criterion (dependent) variables f and F took on, clearly constituted two experimental data sets for which the random assignment method - rather than random sampling was used. In addition to the dichotomous M, the two independent variables N and r, were treated as a pair of orthogonal metric covariates, with two sets of fixed values (Table VIII). Their effects on f and F were established (and intuitively explained), as well as that of the underlying estimation method M. Finally, the statistical analyses were performed under the verifiable assumption of fixed effect models, using SPSS (Nie et al, 1975).

<u>Significance of the Triangular Approach</u> (Theoretical Data)

The Differences in both the f and F values in Table VIII were examined for M=1 and M=2. Within the context of this specific application, two hypotheses were formulated and tested (Table IX), at four different levels of significance. More specifically, $H_3$ postulated that E(f|M=1)=E(f|M=2), or eta=0. The $F_{Computed}$ indicates that the null hypothesis of equal means should be rejected. The descriptive eta for the estimation method was 0.2917, which signifies that the experimental variable M explains about 30% of the total variability in the theoretical maintenance float factor estimate (f). Similarly, $H_4$ postulated that $E(F|M=1)=E(F|M=2)$, or eta=0, but the $F_{Computed}$ indicates that the null hypothesis of equal means should be rejected even at the conservative alpha level of 0.01 significance. The derivation method, however, explains only about 15% of the variability in the estimated maintenance total float (F).

Within the above specific applications, an orthogonal factorial design with equal cell frequencies was considered. Since the interaction effects were found to be insignificant, only the significance of main effects was reported above. The two way interaction, for example, between estimation method and equipment failure distribution produced an $F_{Computed}$ of 1.246 with df(N1)/df(N2)=4/238, while the $F_{Tabulated}$ gives a higher value of 1.36, even at the non-conservative alpha level of 0.25 significance.

The two way analysis of covariance for (f) gave a multiple descriptive R of 0.924, while the raw regression coefficients for the ratio (r), and the initial number of units in operations (N), were 48.590, and -24.060. The negative sign in the coefficient of N can be attributed to the fact that larger systems require less float - in percentage terms. This is what

Love & Lewis (1983), refer to as "positive returns to scale." The two way analysis of covariance for F gave a multiple descriptive R of 0.855, with the coefficients of r and M being 0.159 and 0.416, respectively. Once the theoretical percentage estimates are converted into backup unit and service channel requirements, it becomes clear that larger systems require more float - in absolute terms.

True Significance of the Triangular Approach (Theoretical Data)

Due to the small explanatory power of the dichotomous experimental factor method (M), testing for the true significance in the differences among f and F values (for M=1, and for M=2, Table VIII), was imperative. The research goal was to test if the means of the maintenance float factor (f) and maintenance total float (F) values, obtained with the reliability based triangular estimation method (M=2), stand significantly above the means of the f and F values which were estimated from the Levine-Love & Lewis-Madu method (M=1).

Within this specific application, a true difference in the results of the two methods was established and signified with the aid of an one-tailed t-test series implementation, using SPSS (Nie et al, 1975). A preliminary set of F-tests on the sample variances indicated that the t-tests in Table X, should be based on the pooled variance estimate. Additionally, since there are more than 150 degrees of freedom involved (Berenson & Levine, 1979: p. 300), the Normal distribution was used interchangeably with the t distribution in obtaining the exact tabulated values. Based on the results of the above statistical tests, the reliability based triangular estimation method indeed provides significantly higher f (within a 99% confidence level), and higher F (within a 95% confidence level), values, than the Levine-Love & Lewis-Madu models.

The above conclusion was based on a comparison of the theoretical f and F estimates provided by the two sets of system reliability based maintenance float policy estimation models. By design, both approaches provide estimates on the maintenance float requirements in terms of backup units and service channels, but not on the effectiveness of the total float. It remains to be seen, therefore, how the two approaches compare once implemented in a real life setting. Such a task, however, would be well beyond the time and resource limits of this research project. Thus, the digital simulation experiments reported in the following section.

## Experimental Analysis #3

### Experimental Design

The maintenance float systems studied in this section were modeled with the aid of the following assumptions:

1. The N units initially in operations function independently, and their failure times are identically distributed. The failure distributions used in the experiments are: (1) Exponential, (2) Weibull, and (3) Erlang-2.

2. N was set equal to the fixed value of 80 units initially in operations.

3. Repair times were also assumed to be independent and identically distributed. The repair distributions used are: (1) Exponential, (2) Erlang-2, and (3) Lognormal.

4. A quite "reasonable" fixed value was chosen for the repair to failure ratio (r=0.25), throughout the experiments.

5. A unit is completely rejuvenated once repaired, assumed to be as good as new, functioning within a closed system of operations and maintenance.

A subset of of the theoretical maintenance total float estimates (F), was extracted from Table VIII, for both methods of estimation (M=1, and M=2), for three failure distributions

(D=1, D=2, and D=3), for r=0.25, and for N=80. This subset of estimates on the maintenance total float requirements was subsequently used in the implementation of the simulation model in Figure 5.

Again, ten replications were used for each simulation run, with the timer set to 43,200 GPSS time units. After cancelling the transient state results - in order to avoid contamination of the steady state results, ten replications were averaged to obtain the operations availability (A), the service channel utilization (P), and the waiting time to repair (W). These system effectiveness values - provided by the GPSS output, were recorded, coded, and tabulated (Table XI).

Table XI also contains the nominal values on the MFP estimation method, and the failure and repair distribution combinations, which were used during the experimental simulation runs. Mathematically exact values for the Exponential, Weibull, Erlang-2, and Lognormal distribution functions, were generated from a Uniform[0, 1], through the "inverse transform" method (Fishman, 1973; Law & Kelton, 1982).

## Analysis of Results

The primary interest of this section's investigation was to test if the implementation of the reliability based triangular estimates (method M=2 in Table XI), can substantially improve the performance of a maintenance float system, in comparison to the performance level achieved through the implementation of the Levine-Love & Lewis-Madu estimates (method M=1 in Table XI). That is, to test if the recorded differences in operations availability (A), service channel utilization (P), and average waiting time to repair (W), due to differences in the estimation method, are statistically significant.

Once more, the research goal was to assess the impact of the non-metric experimental variable method (M), on the criterion values of A, P, and W. Once again, the statistical tests were based on the variation of these three metric depended variables and their respective decomposition, and they were conducted under the assumption of a fixed effects model, using SPSS (Nie et al, 1975).

Significance of the Triangular Approach (Experimental Data)

The differences in A, P, and W values in Table XI, were examined for M=1 and M=2. Within the context of this specific application, three hypotheses were formulated and tested, at four different levels of significance (Table XII). More specifically, $H_7$ postulated that E(A|M=1)=E(A|M=2), or eta=0. The results in Table XII indicate that the null hypothesis of equal means should be rejected. The descriptive eta was improved to 0.8504, which signifies that the estimation method, alone, explains more than 85% of the variability in the operations availability measure.

Similarly, the $F_{Computed}$ statistics indicate that $H_8$ and $H_9$ should be rejected as well. $H_8$ hypothesized that E(P|M=1)=E(P|M=2), or eta=0. The improvement of eta to 0.7499, indicates that the differences in the estimation method, alone, can explain a little less than 75% of the overall variability in the service channel utilization (P). $H_9$ postulated that E(W|M=1)=E(W|M=2), or eta=0. The results in Table XII, however, not only signify that the equal means model does not fit the data well, but over 80% of the variability in W is attributable to the difference in the estimation method alone.

An orthogonal factorial design with equal cell frequencies was considered for the above specific application. Only the significance of main effects is reported here, since all interaction effects were found to be insignificant. The multiple classification analysis (MCA), tables for the operations availability (A), service channel utilization (P), and average waiting time for repair (W), gave multiple R coefficients of 0.950, 0.880, and 0.925, respectively.

True Significance of the Triangular Approach (Experimental Data)

The research goal of this section was to verify that the steady state performance characteristics obtained from the implementation of the reliability based triangular estimates, are significantly different from the Levine-Love & Lewis-Madu implementation results. The true difference in the results of the two methods was established with the aid of an one-tailed t-test series implementation (Table XIII), using, again, SPSS.

A preliminary set of F-tests on the sample variances indicated that the t-tests should be based on the separate variance estimate. According to the statistical test results in Table XIII, the implementation of the reliability based triangular estimates, results in a significantly higher operations availability (A), but in a significantly lower service channel utilization (P), and waiting time to repair (W), in comparison with the Levine-Love & Lewis-Madu estimates implementation.

The above conclusions are reached within a 99% confidence interval, and they are based on the comparison of the steady state experimental realizations on three performance measures, characterizing a set of simulated maintenance float systems. Both the Levine-Love & Lewis-Madu models and the reliability based triangular estimation construals, are built to provide initial MFP requirements in terms of backup units and service channels. The latter, however, can provide initial maintenance float policy estimates that significantly improve the system's effectiveness in terms of both A and W. P was indeed significantly reduced, but it never fell below the 99.40% service channel utilization mark as Table XI indicates.

## Structural Considerations

Through an examination of the structure of his theoretical MFP estimation model, Levine (1965), observed that the time element appears only in the repair to failure ratio (r). Consequently, he concluded that the exponential term (Table II, formulae II-1), can be divided into two factors which become substantially independent of each other, given any specific situation. Similarly, Madu (1985), observed that the structure of his formuli yielding f, depends only on the failure, not the repair distribution (Madu, 1985: Corollary 1, p. 49).

The purpose of this section is to test if indeed the equations developed by the Levine-Love & Lewis-Madu stream, as well as the reliability based triangular estimation models, can be used with any arbitrary repair distribution. This deduction is a major piece of information that has not been adequately justified in the MFP literature. Experimental Analysis #3 was partially designed to achieve this. The section's third assumption (experimental design, p. 74), as well as the structure of Table XI, should reveal the initial intent.

<u>Significance of the Repair Distribution</u> (Experimental Data)

The primary interest in this section was to test if the three arbitrarily chosen repair distributions used during the simulation runs in Experimental Analysis #3, had a significant effect on the system performance measures. That is, if any significant differences can be detected on the operations availability (A), service channel utilization (P), and average waiting time for repair (W), "profiles," due to the realization of three different repair distributions. The statistical tests conducted were based on the random variations of A, P, and W, and their decomposition, respectively. The analysis was performed under the assumption of a fixed effects model due to the equal cell frequencies, using SPSS (Nie et al, 1975).

Within the context of this specific statistical testing application, the differences in A, P, and W, were assessed for their significance. Interaction effects were found to be insignificant, thus, only the significance of main effects is reported. A set of three hypotheses was formulated and tested (Table XIV), at four different levels of significance. More specifically, $H_{13}$ postulated that $E(A|RD=1)=...=E(A|RD=3)$, or eta=0. The $F_{Computed}$ indicates that the null hypothesis of equal means should not be rejected. The descriptive eta for the trichotomous non-metric variable repair distribution (RD), was improved only up to 0.0936. This signifies that the experimental factor RD alone cannot even explain a 10% of the variability in the system performance measure of operations availability (A).

Similarly, $H_1$ postulated that $E(P|RD=1)=...=E(P|RD=3)$, or eta=0; $H_{15}$ that $E(W|RD=1)=...=E(W|RD=3)$, or eta=0. The corresponding $F_{Computed}$ values in Table XIV, indicate that in both cases, the hypothesized model of equal means fits the data very well. Neither of the two hypotheses should be rejected. The descriptive etas for $H_{14}$ and $H_{15}$ improved up to 0.1688 and 0.0846, respectively. These values signify that the repair distribution alone can explain less than 17% of the variability in the service channel utilization (P), and not even a 9% of the variability in the average waiting time for repair (W).

<u>True Significance of the Repair Distribution</u> (Experimental Data)

The research goal of this section was to verify that the steady state realizations on the system performance characteristics A, P, and W, obtained under the Exponential repair distribution (RD=1), were not significantly different from the values obtained with the Erlang-2 repair distribution (RD=2). The lack of any true difference was established with the aid of an one-tailed t-test series implementation (Table XV), using SPSS (Nie et al, 1975).

A preliminary set of F-tests performed on the sample variances, indicated that the t-tests in Table XV should be based on the pooled variance estimate, with ten degrees of freedom. The results of these statistical tests indicate that the differences in operations availability (A), service channel utilization (P), and average waiting time to repair (W), due to the change from the Exponential (RD=1), to the Erlang-2 (RD=2), repair distribution, are insignificant.

The statistical tests conducted in Table XIV (ANOVA-tests), and in Table VI (t-tests), which are based on the steady state experimental realization data recorded in Table XI, do support the initial conclusions by Levine (1965), and Madu (1985: Corollary 1, p. 49). Thus, both the system reliability based Levine-Love & Lewis-Madu, and the triangular MFP estimation models, can be used with any arbitrary repair distribution. The experimentally verified fact that the final effectiveness of a maintenance float policy depends only on the equipment failure and not the repair distribution, is a very important piece of information. The rationale is that only six cases need to be dealt with, from the initial thirty-six possible combinations of Exponential, Weibull, Gamma, Lognormal, Normal, and Uniform distributions, for MTBF and MTTR.

# CHAPTER IV

# SURFACE ANALYSIS OF MFP ECONOMICS

## Motivation

The work in this chapter was motivated, in part, by the serious gap which was identified between theory and practice in maintenance management (Bullock, 1979; Channin & Sphicas, 1980; Gilbert & Finch, 1985). In connection with maintenance float systems, standby equipment can be compared to buffer inventory. Both exist to maintain a smooth flow of operations in the event of undesirable random departures from normal conditions. On the basis of this comparison several reasons for the existing gap between theory and practice can be identified.

One reason is that the marginal downtime, holding, and service channel cost categories typically assumed known in the theory of maintenance float systems (e.g., Hilliard, 1976;

Morrison, 1961; Wahi, 1966; etc.), are difficult if not imposible to measure in practice. These types of cost pose similar measurement problems with the marginal shortage, holding, and ordering cost levels assumed in the theory of inventory management, which Brown (1967), Churchman (1961), Gardner & Dannenbring (1979), Starr & Miller (1962), and Ziegler (1973), have identified. The cost of a service channel, for example, and the cost of down time - for which no accounting methodology exists, are both expressed in terms of unit time since existing cost analyses typically apply to steady state conditions. The holding cost is even more difficult to handle since it requires the depreciation of a backup unit over what may be a rather arbitrary horizon.

The plausibility of another reason rests on the argument of Gardner & Dannenbring (1979), who pointed out that most practitioners are primarily concerned with specific aggregate objectives for customer service, work-load, and investment, rather than with single-item models that inventory theory has traditionally emphasized. Due to difficulties in terms of mathematical tractability, maintenance float systems have also been formally modeled on a unit-by-unit basis. As pointed out by the literature survey in Capter I, existing cost optimal MFP models can only be applied to very small systems.

Finally, the initial assignment of cost parameters make existing maintenance float policy models too sensitive to the underlying cost structure of any particular firm. Although this appears to be the single point of coherence in the MFP literature, none of the authors explained why.

This paper presents an approach to decision making in maintenance float systems that bypasses the initial cost measurement problems. The approach incorporates objectives and constraints set by management within an experimental analysis module. While traditional theory is based on the cost minimization, this section poses that maintenance float decisions be conceived as policy tradeoffs on a three dimensional topology, created by the response surfaces of a system's performance measures. The first two axes of each surface correspond to MFP requirements in terms of standby float units and open service channels. The third axis measures the system's performance characteristics which may be further used in recognizing maintenance float policy existing tradeoffs.

The experimental analysis that follows is close in spirit with the construals of Starr & Miller (1962), and Gardner & Dannenbring (1979), who studied aggregate inventory tradeoffs and developed the "optimal policy curve," and the "optimal policy surface," respectively. The Levine-Love & Lewis-Madu, and the reliability based triangular estimation models that were formally developed and tested in the preceding chapters, are used to guide decision making in MFP estimation, and indicate where a more detailed operational investigation is justified.

## Experimental Analysis #4

Operationally customer service is the term used to describe the availability of items when needed by the customer. The customer may be that of a finished product, a distributor, a plant in the organization, or a department in which the next operation is performed. Seldom if ever can an organization plan or act so that all items are always available in the proper quantity when desired. According to Artes (1977), and Fogarty & Hoffman (1983: p. 159), however, an organization should aim for a target level of customer service and attain a probabilistic result measured in the same terms. Idle time or unavailability of operations is a very useful absolute value type of measure for production activity control.

With respect to the operations/maintenance float systems the present study is concerned with (e.g., life support systems), a high level of availability is required. An obvious cause of the unavailability of operations is machine failure. In order to study how operations managers may design highly available maintenance float systems with the aid of the Levine-Love & Lewis-Madu, and the reliability based triangular estimation models, the "repairman's problem" context of analysis must be enlarged to the "production manager's problem." Production theory is primarily concerned with the way in which inputs are employed to produce outputs. The concept of production is quite broad and encompasses both the manufacture of physical goods and the provision of services. It examines both the technical and economic characteristics of systems used to produce output, with the aim of determining the optimal manner of combining inputs as to minimize cost.

Under this enlarged analytical mode it becomes possible to specify the maximum output that can be produced for a given amount of inputs; alternatively, the minimum quantity of inputs necessary to produce a required level of output. The basic performance characteristics of a maintenance float system can be illustrated by examining a rather simple two-input, single-output system. In the experimental analysis that follows, repairmen become part of a larger system as they assume the role of service channels.

## Experimental Design

A maintenance float system is considered in which various quantities of two inputs, service channels (S), and standby float units (F), are used to produce a required level of operations availability (A). The production function of such a system can be written as the following unspecified relationship:

$$A = f(F, S), \qquad 0 \leq f \leq 1. \tag{45}$$

The system was modeled with the aid of the following assumptions:

1. A high target level of operations availability is aimed for by the management of a hypothesized firm. It is assumed that they need to attain a probabilistic $A \geq 0.95$.

2. A medium size system was selected with N=80 units initially in operations, which function independently, and their failures are identically distributed, following an Erlang-2 distribution.

3. Repair times were also assumed to be independent and identically distributed, following an Erlang-2 distribution, with the repair to failure ratio set equal to the fairly large, yet still "reasonable" fixed value of $r = 0.60$.

4. A unit is completely rejuvenated after repair, assumed to be as good as new, functioning within a closed system of operations and maintenance.

A set of two float factor (f) estimates was obtained from Madu's (Table II, formulae II-3), and the reliability based triangular estimation (Table IV, formulae IV-1), models. The mode of the underlying Triangular was set equal to the theoretical mode of the Erlang-2 distribution. Its range was determined by the process described in Experimental Analysis #2. The maintenance float initial requirements were estimated with equation (5), for both the standby units (F), and the service channels (S). These values were subsequently used with the implementation of the simulation model in Figure 5, using GPSS-V Gordon, 1975). In addition to the theoretical estimates, the values of F and S were increased in increments of ten, until the stochastic realizations of the operations availability reached their upper bound which is equal to 1 (one).

In order to ensure stability for the model, ten replications were used for each simulation run. The timer was set equal to 43,200 GPSS time units, with the transient statistics cancelled in order to avoid contamination of steady state results. The averages of the stochastic realizations were recorded in order to obtain the output values of the system's performance characteristics. These values were subsequently plotted to create a set of three dimensional surfaces that can contribute a profound awareness of MFP economic forces. The shape for each of these grid surfaces was estimated on the basis of thirty-six (6·6), physical system performance steady state realizations, over the bounded (F, S) input surface (Figure 9).

## Analysis of Results

The three dimensional diagram in Figure 9 is a graphic illustration of the operations availability grid surface, as a function of a two-input, single-output system. Following the F axis outwards indicates that an increasing amount of standby float units is being used; going out the S axis represents an increasing usage of service channels; and moving up the A axis means that a higher level of operations availability is achieved. The maximum A that can be realized is that of unity, with maintenance float requirements estimated close to 100% of the existing population (N).

This very first experimental result supports the original argument by Levine who posed that there is "...surprisingly, a maximum size of float..." (Levine, 1965: p. 403). The maximum amount of A that can be produced with each combination of inputs F and S is represented by the height of the operations availability grid surface erected above the input plane.

The three dimensional surface in Figure 9 also demonstrates the property known as the law of diminishing returns. This law states that as the quantity of a variable input is increased, with the quantities of all other factors being held constant, the increases in output eventually diminish. Alternatively stated, the law of diminishing returns holds that the marginal product of a variable input must eventually decline if enough of it is combined with some fixed quantity of one or more other factors in a production system.

According to Douglas (1948), the law of diminishing returns is not a law that can be derived deductively from the laws of physics. Rather, it is a generalization of an empirical relationship that has been observed to be true in every known production system. The basis for this relationship is easily demonstrated for the labor input in a production process where a fixed amount of capital is employed.

From the operations availability surface in Figure 9 it can also be inferred that the factor productivity of the service channels (S), is much higher than the returns to the input factor of standby float units (F). Factor productivity is the key to determining the optimal combination of the maintenance float system inputs that should be used to achieve a required level of operations availability (A). That is, factor productivity could provide the basis for efficient resource employment in MFP determination.

The plausibility of a fourth significant finding from Figure 9 rests on the notion of an operations availability isoquant. According to Pappas & Brigham (1979: p. 215), the term isoquant (derived from iso: meaning equal; and quant: meaning quantity). denotes a curve that presents all the different combinations of inputs which, when combined efficiently, produce a specific quantity of output. Efficiency in this case refers to technological efficiency, under the assumption that the most productive techniques are used in converting inputs to outputs.

An isoquant for the operations availability function displayed in Figure 9 can be located by passing a plane through its grid surface, horizontal to the FS surface. This plane represents a specific availability level and the isoquant can be thought of as a contour or isoelevation line, connecting all points of equal altitude. Such a plane has been passed through the operations availability surface shown in Figure 9 at height A=0.95. Every point on the surface with a height of Q above the input plane, that is, all points along curve Q, represent an equal quantity, or isoquant, that maps out the locus of all input combinations of F and S that result in a 0.95 level of operations availability (A).

This availability isoquant can be transferred to the input surface, as indicated by the dashed curve Q' in Figure 9, and then further transferred to the two dimensional graph shown in Figure 10. This latter curve represents the standard form of an isoquant. Since the shapes of the isoquants generally reveal a great deal about the substitutability of input factors, a local search was performed in the neighborhood of Q.

The "+" symbols in Figure 10 signify some of the actual standby float unit and service channel (F, S) combinations (Table XVI), which are required to attain an operations availability $A \geq 0.95$ for the maintenance float system under consideration. The inputs F and S can be substituted for each other, but the overall substitutability is not perfect.

The experimental simulation results in Figure 10 indicate that below F=42 standby float units, and below S=45 service channels, the two inputs are perfect complements for each other. Outside the above defined region the isoquant Q takes the shape of right angles which indicate that in no way can standby float units be substituted for service channels, nor vice versa. The slope of these right angles, which is also known as the marginal rate of technical substitution (MRTS) of factor inputs, can verify the nonsubstitutability between the inputs F and S, outside the [F=42, S=45] region. Stated algebraically:

$$MRTS = \Delta S/\Delta F = \Delta F/\Delta S = 0, \quad \text{for } A \geq 0.95. \qquad (46)$$

At the other extreme, the alignment of the "+" symbols in Figure 10, signifies that within the above specified region, standby float units and service channels are perfectly substitutable inputs. Inside the bounds one unit of F can be substituted for one unit of S, and vice versa. Algebraically:

$$MRTS = \Delta S/\Delta F = \Delta F/\Delta S = -1. \qquad (47)$$

## Cost Optimal MFP

Based on the empirical law of diminishing returns and the experimental findings in the above section, it would be irrational for the hypothetical firm under investigation to continue the substitution of service channels with standby float units, or vice versa, below the bounds of the [F=42, S=45] range. Such an attempt will not allow the firm to attain an operation availability level on or above the 0.95 mark. On the other hand, any increase in the total float requirements above the [F=51, S=53] range, may improve operations availability, but it costs more. Once the region(s) of perfect substitutability between F and S is (are) identified, the cost optimal among the available combinations can be easily determined.

The three examples in Table XVI show how the optimum (F, S) combination can be identified. In addition these examples verify how sensitive the final solution is to a firm's underlying cost structure. Under cost structure (i), for example, it costs $150 per day to have a standby float unit available ($C_F$=$150/day), and the cost of keeping a service channel open for repairs is $75 per day ($C_S$=$75/day). The optimum (F*, S*) combination is (42, 53), which yields a daily maintenance float policy cost of $10,275. Under cost structure (ii), it is assumed that the daily cost of retaining a standby float unit and the cost of keeping a service channel open for repairs are equal ($C_F = C_S = \$112.50$/day). The optimum among the alternative (F, S) combinations is (46, 48), for the minimum MFP cost being $10,575.00. Finally, under cost structure (iii), it costs 75 per day to have a standby float unit available ($C_F = \$75$/day), and the cost of keeping a service channel open to perform repairs is $150 per day ($C_S = \$150$/day). The optimum (F*, S*) combination is (51, 45), which again yields a daily MFP cost of $10,575.

In all three cases, and for all other factors being held constant, the optimum among the alternative (F, S) combinations, is the one for which the combined cost of backup machines and the cost of maintaining a particular crew size is minimum. For all the three hypothesized underlying cost structures, the set of alternative (F, S) combinations is limited. Its determination is constrained by the managerial choice on the customer service or operations availability objective. Alternatively, this may be considered the case where the cost of downtime or operations unavailability is too high to be included in the cost minimization process.

## Cost Sensitive MFP

Based on the above section's examples in Table XVI, the determination of the cost optimal input proportions (F•, S•), could be viewed as a problem of minimizing the cost of producing a specific level of operations availability (A). This section presents how the Langrangian technique for constrained optimization can be used to develop the optimal (F•, S•) combination rule.

The grid surface of operations availability in Figure 9, and the discrete data points in Figure 10, can be generalized by assuming that the underlying production function, formulae (45), is continuous in nature. This generalization aids the constrained optimization problem which is developed as follows. The constraint states that some specific level of operations availability A• must be realized for the system described by the function A=f(F, S). Written in the standard Langrangian format, the constraint is 0=A•-f(F, S). The combined cost function is given as $TC=C_F \cdot F+C_S \cdot S$. The Langrangian function for the combined cost minimization problem, then, is:

$$L_{TC} = C_F \cdot F + C_S \cdot S + \lambda[A^* - f(F, S)]. \quad (48)$$

The conditions for constrained cost minimization are provided by the partial derivatives of equation (48):

$$dL_{TC}/dF = C_F - \lambda[df(F, S)/dF] = 0, \quad (49)$$

$$dL_{TC}/dS = C_S - \lambda[df(F, S)/dS] = 0, \quad (50)$$

and

$$dL_{TC}/d\lambda = A^* - f(F, S) = 0. \quad (51)$$

Since the last terms on the left hand side in formuli (49) and (50) represent the marginal availability (MA) due to the standby float units (F), and service channels (S), respectively, those expressions can be revritten as:

$$C_F = \lambda MA_F, \qquad (52)$$

and $$C_S = \lambda MA_S. \qquad (53)$$

Taking the ratio of formulae (52) to formulae (53) and cancelling the lambdas produces the basic (F, S) input optimality relationship:

$$C_F/C_S = MA_F/MA_S. \qquad (54)$$

Thus, for a minimum cost (F, S) combination, given the required level of operations availability (A*), the input factors of standby float units and service channels must be combined in such a way that the ratio of their marginal products, in terms of A, is equal to their cost ratio. Alternatively, transposing formulae (54) to derive the expression:

$$MA_F/C_F = MA_S/C_S, \qquad (55)$$

the optimal (F*, S*) input combinations require that their ratio of marginal operations availability to cost of both input factors must be equal.

Formuli (54) and (55) algebraically specify the necessary conditions for optimality in a maintenance float system input combination. The least cost (F*, S*) combination requires input proportions such that an additional dollar's worth of standby float units (F), adds as much to the operations availability (A), as does a dollar's worth of service channels (S), and vice versa. This theoretically derived relationship clearly explains why cost optimal maintenance float policies are so sensitive to a firm's underlying cost structure.

## MFP Performance and Tradeoffs

The foregoing analysis undoubtedly did not exhaust all the possible system performance measures, but it should indicate the myriad of possibilities. The context of analysis was enlarged from the repairman's to the production manager's problem in order to study how the latter can design a highly reliable maintenance float system utilizing system reliability based MFP models. Following Buffa (1984), however, a single measure of performance used across all the parts of a system may be deceptive and may also lead to an adverse bargaining process between the production and maintenance managers. In such an event an "optimal" maintenance float policy will be determined by the basic power structure within which such negotiations take place.

In addition, to establish an MFP based on the probabilistic attainment of a preset level of operations availability (A), without measuring the performance of the employed service channels and equipment, in terms of their utilization (P), is to possess only half a picture. Confucious must have said something about having half a picture.

Assuming that data become available, the performance of the other two major system components (equipment and service channels), can be measured for an established MFP, in addition to the operations availability. The GPSS-V simulation outputs in Experimental Analysis #4, did provide information on the utilization (P) of the employed service channels and equipment for the hypothesized firm which is being used as an example throughout Chapter IV. By taking into consideration the overall MFP economic consequences (physical and cost aspects), it becomes possible to move away from half pictures and adverse bargaining processes, and to recognize existing MFP tradeoffs.

As Gardner & Dannenbring (1979), point out, with stochastic performance measures management decisions become rather complex. In maintenance float systems, operations availability (A), and equipment and service channel utilization (P), are interdependent. In dealing with the real life dramas and complexities of interdependence, it will be helpful to turn back to the three dimensional surface analysis, and establish additional performance measures that an arbitrary MFP seeks to minimize. According to Saaty & Alexander (1981, p. 97), the operations availability at steady state is given by:

$$A = 1 - U, \tag{56}$$

where U is defined as operations unavailability. By combining formulae (56) with formulae (45), operations unavailability can be written as the following unspecified relationship:

$$U = 1 - f(F, S), \quad 0 \leq f \leq 1. \tag{57}$$

Thus, various quantities of the two inputs, service channels (S), and standby float units (F), can be used to minimize the unavailability of operations (U), within a two-input, single-output system. Additionally, the same arbitrary MFP may seek to maximize the equipment and service channel utilization (P), or alternatively, depending on preset management objectives, minimize the service channel and equipment underutilization:

$$U = 1 - P, \quad 0 \leq P \leq 1. \tag{58}$$

Considering matters of signification as secondary for a moment, U can be interpreted or act as a surrogate measure for the maintenance float system physical performance. What is of primary interest, however, is to recognize that a tradeoff exists between operations unavailability and service channel underutilization, as the grid surfaces in Figure 11 illustrate.

The three dimensional diagram in Figure 11, is a graphic illustration of the operations unavailability (OU), and service channel underutilization (SU), grid surfaces, as two functions of a two-input, double-output system. Following the F axis outwards indicates that an increasing number of standby float units is being used; going out the S axis represents an increasing usage of service channels; and moving up the U axis mean that higher levels of operations unavailability and service channel underutilization are realized. The minimum U level that can be achieved is zero. The amount of U that can be achieved with each (F, S) input combination is represented by the height of the operations unavailability (OU), and the height of the service channel underutilization (SU), grid surfaces erected above the FS input level.

The tradeoff displayed between the OU and SU surfaces in Figure 11, is rather straightforward. With a fixed F, increases in S simply reduce operations unavailability, while at the same time increase the service channel underutilization. At a fixed low S, increases in F do not greatly affect neither operations unavailability nor service channel underutilization. At a fixed high S, however, increases in F reduce operations unavailability, but while they reduce service channel underutilization at lower levels, they add to it at a higher F.

The law of diminishing returns becomes evident once more, as it clearly holds true for the marginal product of the service channel input (S), on the operations unavailability (OU), for the whole range of standby float units (F). The inverse of the law also holds true for the marginal product of S on the service channel underutilization (SU), for the entire displayed range of F. The same relationship, however, is not entirely supported by the marginal product of F on SU, although it is still evident in the marginal product of F on OU, for the entire displayed S.

From both the operations unavailability (OU), and the service channel underutilization (SU), surfaces in Figure 11, it can be inferred that the factor contribution of the service channels (S), is much higher than the returns to the input factor of standby float units (F). This contribution may be the key to determining the optimal combination of a maintenance float system inputs that should be used to achieve a balance between operations unavailability and service channel underutilization. It may also be inferred from Figure 11, that such balance, if required, is achieved on (or in the neighborhood of) the line where the OU and SU grid surfaces cross each other.

The data used for the construction of the three dimensional graph in Figure 11, as well as the graphs in Figures 9 and 10, were provided by the GPSS-V simulation results in Experimental Analysis #4. Additional information was also provided by the same outputs, on the average waiting time to repair as well as on the average waiting time of equipment on standby. According to Bhattacharyya (1967), the waiting time for a spare unit is the time elapsed before it is commissioned to replace a failure. On the basis of these two average waiting steady state realizations and the experimental run time, one more physical performance measure was established. Namely, the average equipment underutilization (EU), was recorded and plotted as an additional grid surface in Figure 12, along with the operations unavailability (OU), and the service channel underutilization (SU), grid surfaces.

The three dimensional diagram in Figure 12 is a graphic illustration of the operations unavailability (OU), service channel underutilization (SU), and equipment underutilization (EU), grid surfaces, as functions of a two-input, triple-output system. The minimum U level that can be achieved is still zero. The amount of U that can be produced with each (F, S), input combination of the standby float units and the service channels, is representd by the height of the EU, OU, and SU surfaces, erected above the FS input level.

Among the three surfaces only two reach the value of absolute zero; OU for F=S=80; SU for S≤32 service channels, and for a large number of standby float units (F). The equipment underutilization (EU) surface, remains floating above the FS input level, indicating that equipment cannot be fully utilized. A tradeoff exists between the average waiting time for repair and the average waiting time on standby.

The significant experimental result which the new equipment underutilization (EU), measure and grid surface, contribute to the maintenance float policy tradeoff analysis, becomes evident in Figure 13. This figure is comparable to taking two slices out of Figure 12, each slice being parallel to a plane formed by a unique set of two axes (F and U, and S and U). The exact shape of the EU, OU, and SU curves in Figure 13, depends on the point at which the slice is taken, that is, the point at which one of the variables F or S is held constant.

Thus, by holding F constant at 50 standby float units, for example, it is possible to observe that for low S values, the EU and OU curves cross each other while they are both intercepted by SU. Similarly, by holding S constant at 50 service channels, it becomes clear that while the OU curve remains below the SU curve for the entire displayed range of F, the EU curve crosses them both at different heights above the FS input level. The amazing result is that all of the above line crossing takes place within a fairly small area of the FS input plane as indicated by the shaded triangle in Figure 13.

If an optimal MFP involves balancing operations availability, and equipment and service channel utilization, as it is implied in Figure 12, then, only a small set of alternative (F, S) combinations exists, where this balance can be achieved. This set can be identified by the existing tradeoffs among the system's physical performance measures. The minimum cost (F*, S*) combination can be further determined, depending on a particular firm's underlying cost structure. This last point was made rather explicit by the illustrations in Table XVI, where three different underlying cost structures yielded three different optimal solutions.

Again, the foregoing discussion assumes a balanced maintenance float system with an investment properly proportioned among the P/OM system components: operations, service channels, and equipment. If this is not the case, then some of the the system's components will be under excessive strain and stress, and others inadequately utilized. None of the above relationships will be valid.

# CHAPTER V

## SUMMARY AND FUTURE RESEARCH

The present study represents a piece of exploratory as well as deductive research. It constitutes a first attempt to place a complex array of economic, social, and technological variables into a holistic system for the purpose of analysis. Namely, the classic repairman's problem is extended into a P/OM problem. The present chapter will begin with some discussion of the results reported in the preceding chapters. In analyzing the results of this study in light of its intended purposes, it is important to summarize its major findings. These are deemed important due to their potential implications as they lead to suggestions for future research on maintenance float systems.

The fundamental purpose of this study was to investigate what is appropriate in the establishment of maintenance float policy for production systems from which a high level of operations availability is demanded. The need for this investigation emerged from the review of the developments and applications in the maintenance management literature in general, and in the maintenance float systems literature in particular. The serious gap identified between theory and practice motivated the two-fold research objective reflected in the paper's structure.

First, guided by management practice a set of flexible, reliability based triangular estimation models was developed. Formally derived, these add to the recently established MFP normative theory. Based on both theoretical as well as experimental data, a gauntlet of statistical tests indicated that the triangular approach can significantly improve the process of maintenance float policy determination, for newly planned as well as already existing P/OM systems.

Second, guided by this analytically derived new model base, the conceptual schema of the MFP framework was enlarged in order to integrate, study, and understand the economic, social, and technological implications of maintenance float systems; implications which had been overlooked in the past. In the process, the study departed from the established plausible but narrow engineering perspective and examined maintenance float systems within their larger and closer to real life P/OM context.

## Major Findings and Implications

The MFP literature sample showed how the landmark contribution of Black & Proschan (1959), allows the use of information on component failure rather than information on component demand distributions. Even with this bypass, however, the establishment of MFP for large systems with high availability requirements has not been solved analytically. For such systems researchers and practitioners alike resort to digital simulation. Yet, the vector grid boundaries over which a search must be conducted remain very large.

The Levine-Love & Lewis-Madu stream of models can provide initial MFP estimates which substantially reduce the vector grid boundaries of the search space. For the hypothetical firm in Chapter IV, for example, the search space was reduced from 6,561 to 2,401 alternative (F, S) input combinations, which is a 63.40% reduction over the trial and error approach. One of their major underlying assumptions, however, is that the equipment failure distribution has been completely characterized. Thus, these models cannot be used with newly planned maintenance float systems from which failure data collection is impossible.

The reliability based triangular estimation models developed in Chapter II, can explicitly deal with the lack of empirical data. The pilot study in Chapter III, indicates that these constructs can provide initial MFP estimates which are very close to those that the Levine-Love & Lewis-Madu models would give, had the underlying equipment failure distribution been completely characterized. That is, provided that the subjective estimates of experts are reliable.

The results of Experimental Analysis #1 provided little support for the hypothesized negligibility of W (average waiting time to repair). For large N (number of units initially in operations), and large r (repair to failure ratio), W is substantially increased, having a negative impact on the overall MFP effectiveness. Due to their flexible structure, however, the reliability based triangular estimation constructs, according to this study's results, can compensate for the non-inclusion of W in the Levine-Love & Lewis-Madu models.

Indeed, the results of Experimental Analyses #2 and #3 showed that the reliability based triangular estimation models can significantly improve the process of MFP determination. The reduction of the vector grid, already bounded by the Levine-Love & Lewis-Madu initial MFP estimates, was shown to be statistically significant. The statistical tests were based on both theoretical MFP initial estimates, and experimental system effectiveness data. Back to the hypothesized firm in Chapter IV, once more, the reliability based triangular estimation method reduced the search space down to 1,681 alternative (F, S) input combinations, which is a 29.99% reduction over the Levine-Love & Lewis-Madu models, and a 74.38% (!) reduction over the trial and error approach.

The statistical tests in Experimental Analysis #3 support the propositions by Levine (1965), and Madu (1985), that system reliability based MFP estimation models can be used with any arbitrary repair distribution. In that section it was experimentally verified that the final effectiveness of a maintenance float policy primarily depends on the underlying equipment failure, not the repair distribution.

The exploratory analysis based results, with respect to the size of the maintenance total float required for large systems, support the Love & Lewis (1983), notion on the positive returns to scale. The theoretical MFP percentage estimates, however, must be converted into backup units and service channels requirements, that is, into absolute terms. Then, it becomes clear that larger P/OM systems require more service channels, and more standby float units, in order to attain a high level of operations availability.

Some of the strongest effect observed in this study resulted by shifting from an analysis of the physical performance of inputs to an examination of their economic productivity, or operations availability generating capability. The conversion from physical to economic relationships was accomplished by dividing the marginal product of the input factors F and S by their marginal cost. Within the process, the mystery of why cost optimal maintenance float policies are so sensitive to a firm's underlying cost structure was resolved.

Additionally, three numerical examples were provided in Chapter IV, which show how a restrained cost optimum MFP can be determined. The constraint was imposed by the hypothetical firm's management in order to attain a probabilistic A (operations availability), at a high level. An A isoquant emerged from this physical performance constraint, which was subsequently transferred to the FS input surface. The second example in Table XVI [underlying cost structure (ii)], illustrates the possibility of underlying isocost as well as isoquant curves.

Finally, by turning back to an analysis of physical performance measures for the maintenance float system under consideration, it was shown how existing MFP tradeoffs can be identified. Although it was not realized within the grid surface analysis in Chapter IV, experimental and/or real life cases may emerge where the three surfaces in Figure 12, cross at a single point. If such a case does exist, then, a perfectly balanced maintenance float system will be identified independently of the firm's underlying cost structure. The perpetual difficulties associated with the cost optimal MFP determination will be bypassed.

The overall findings of the very last section in Chapter IV, are based on an exploratory search over a set of experimental steady state realizations. The crudity of their foregoing analysis does not permit to formally list them as results, let alone as theorems at this point in time. Evidently, however, in addition to the interesting results, a set of propositions for future research emerge from the investigation.

## Suggestions for Future Research

The findings and conclusions reported in this study indicate the need for further research in a number of areas. A few are listed here:

1. Validation and stability of the study's results on the basis

of real life and/or other experimental data sets.

2. Extensions from the Triangular Approach to the even more flexible Beta Approach, based on the Beta[$a_1$, $a_2$] equipment failure distribution.

3. A "fine tuning" of the relationships involving MFP tradeoffs, to evaluate further their impact on the overall physical and monetary effectiveness of maintenance float systems.

4. Further development of the definition and measurement of what is collectively referred to as equipment utilization within the MFP context.

5. Research towards the empirical and/or experimental estimation of operations availability (A) functions. With the aid of the statistical methods of regression analysis it may be possible to identify general A concave forms. General in the sense that they may hold for large classes of maintenance float systems, provided that the underlying equipment failure distributions (continuous or discrete), possess the monotone likelihood ratio property.

6. The tradeoff analysis in Chapter IV can be extended to identify and establish an "optimal MFP surface." Once such

a surface is constructed, it can be used to make improvements in any maintenance float system under consideration.

7. Such an "optimal MFP surface," once constructed, can be used to model the more generalized maintenance float system in Figure 3 (p. 22).

The study of maintenance float policy is by definition, dependent upon the understanding of an eclectic array of influences. Economic, physical, and even social issues are involved. The integration of the deductive with an exploratory tradeoff analysis represents a means of capturing some of the complex systems of relationships involved in MFP determination.

It is hoped that this study will at least serve as a basis of reflection, and as a stimulus to research designed to further enhance knowledge on the design and management of maintenance float systems.

# FIGURES

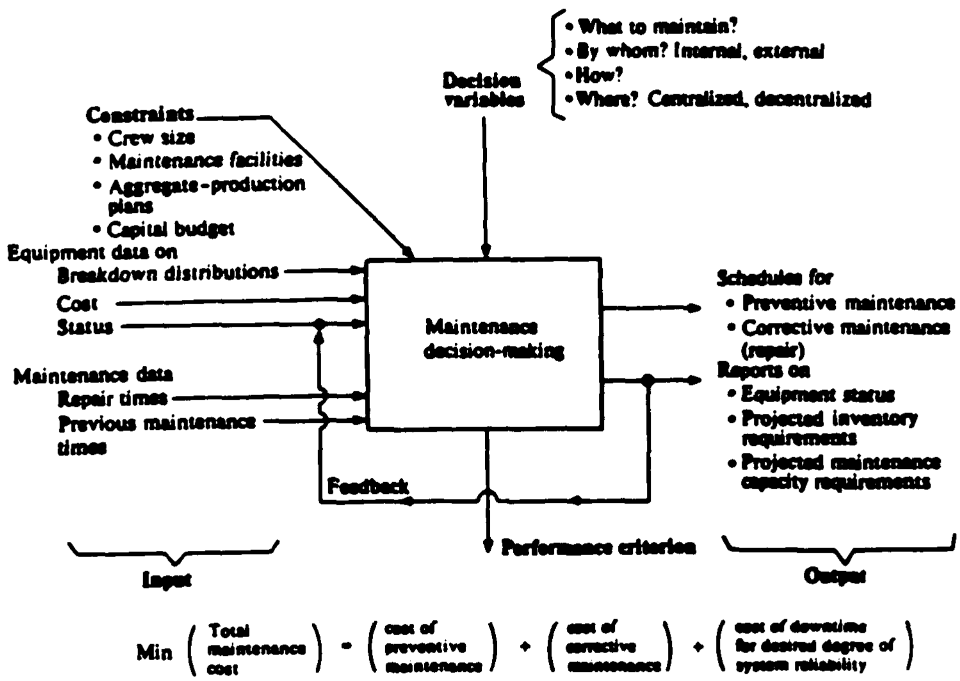



Figure 1. Basic Components of a P/OM System. Adapted from Dervitsiotis, K. N. (1981).

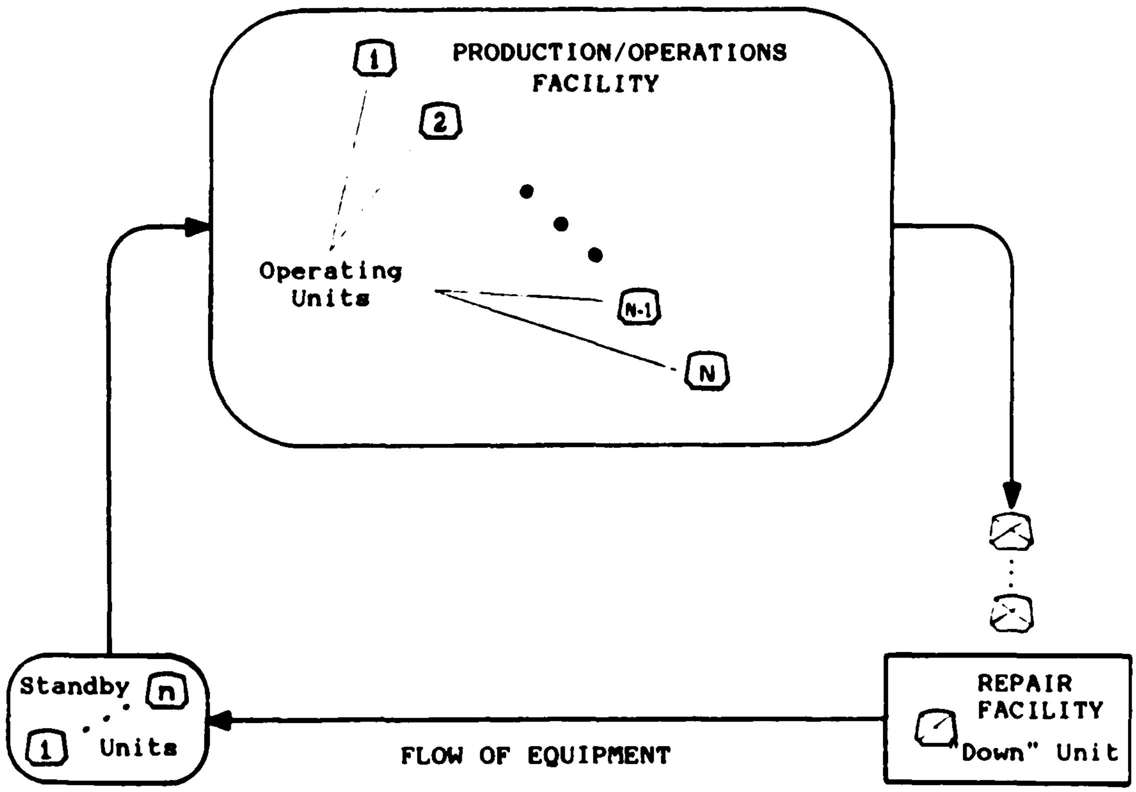



Figure 2. A Maintenance Float Augmented System

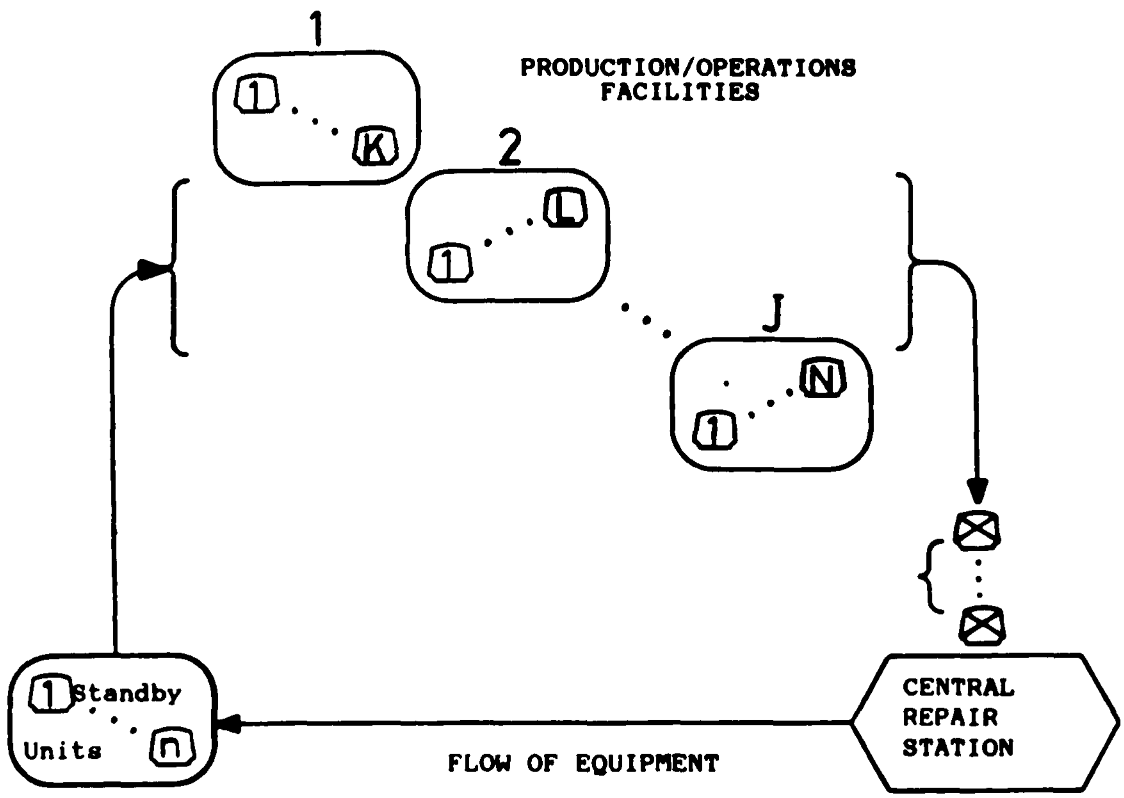



Figure 3. The M F P Problem Generalized

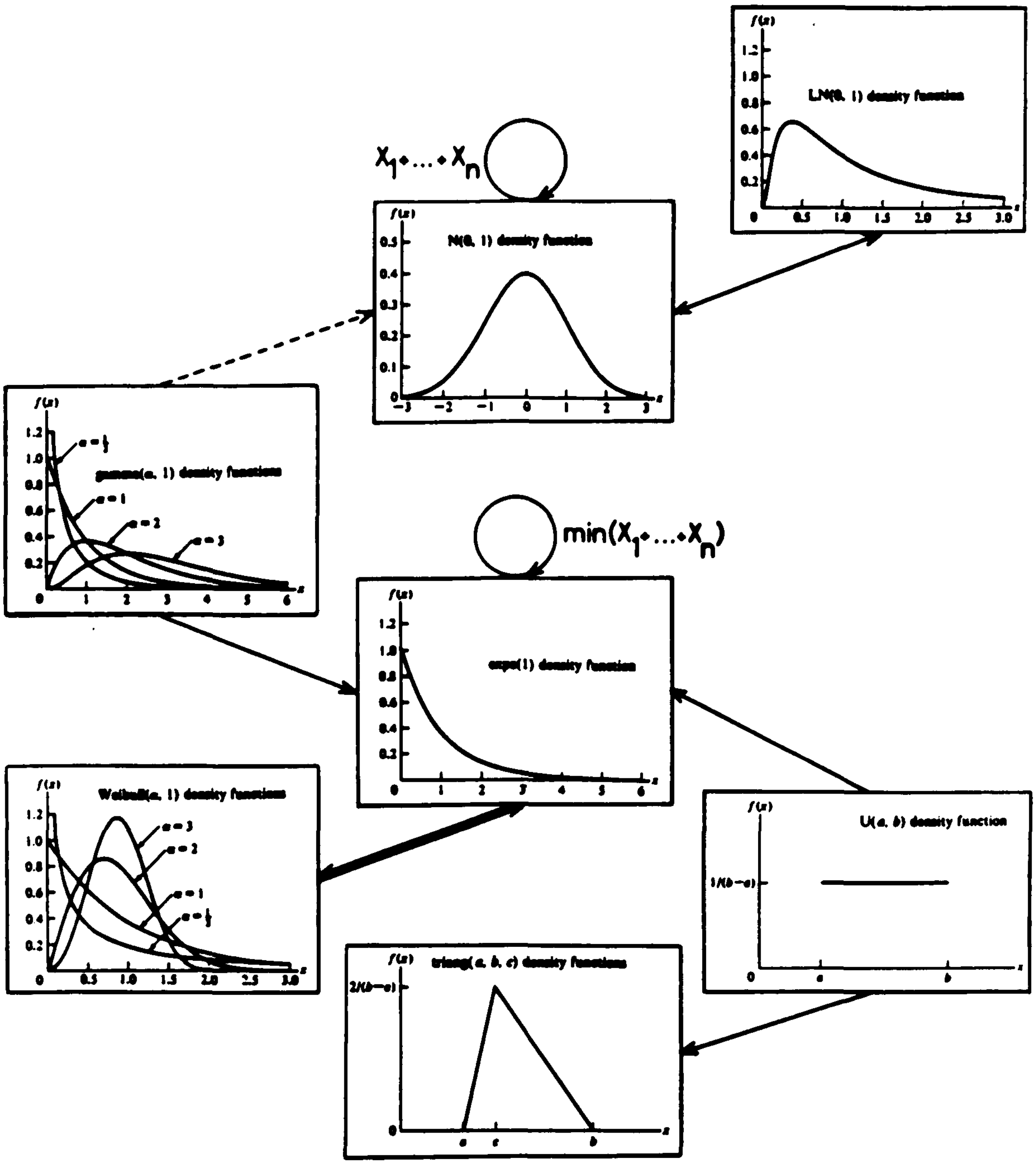



**Figure 4. Density Functions and Relationships among Continuous Distributions used in MFP; --> limiting distribution; ⟶ transformation; ⟷ one-to-one transformation. Adapted from Leemis (1986).**

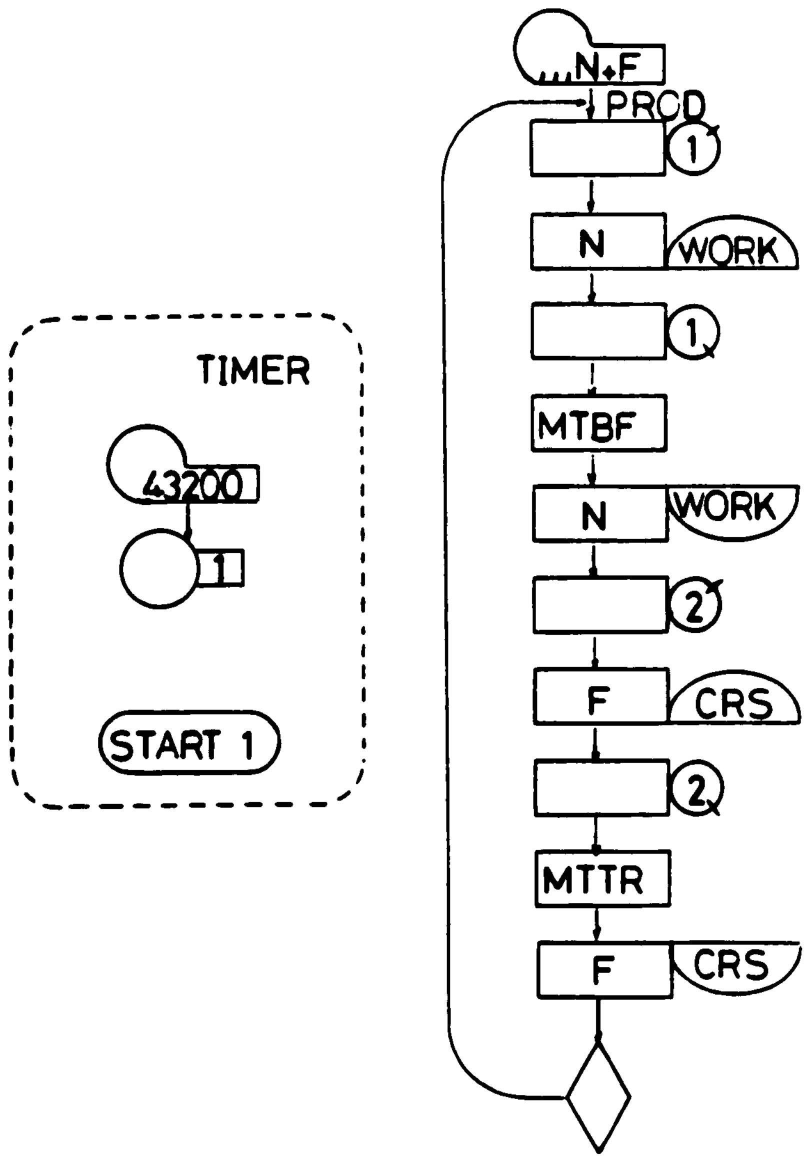
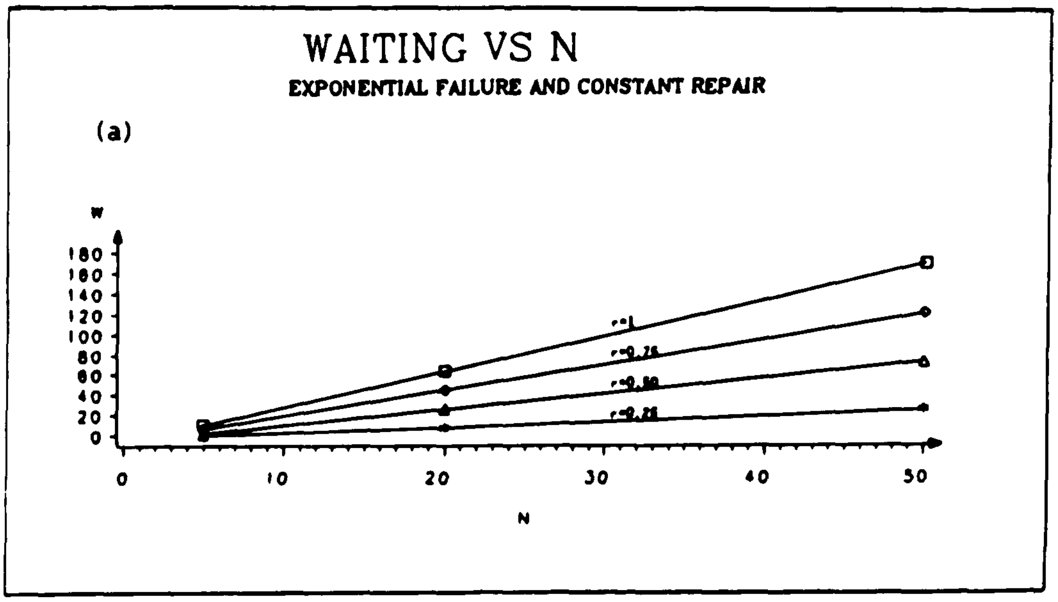



Figure 5. GPSS Flowchart of a Maintenance Float System.



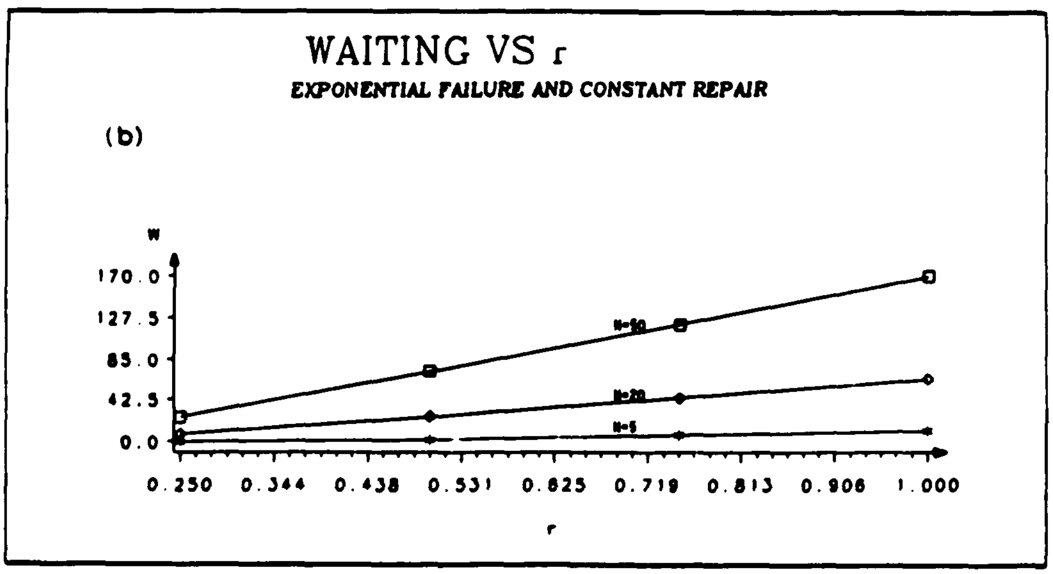




Figure 6. Linear Relationship between: (a) N and W, and (b) r and W.

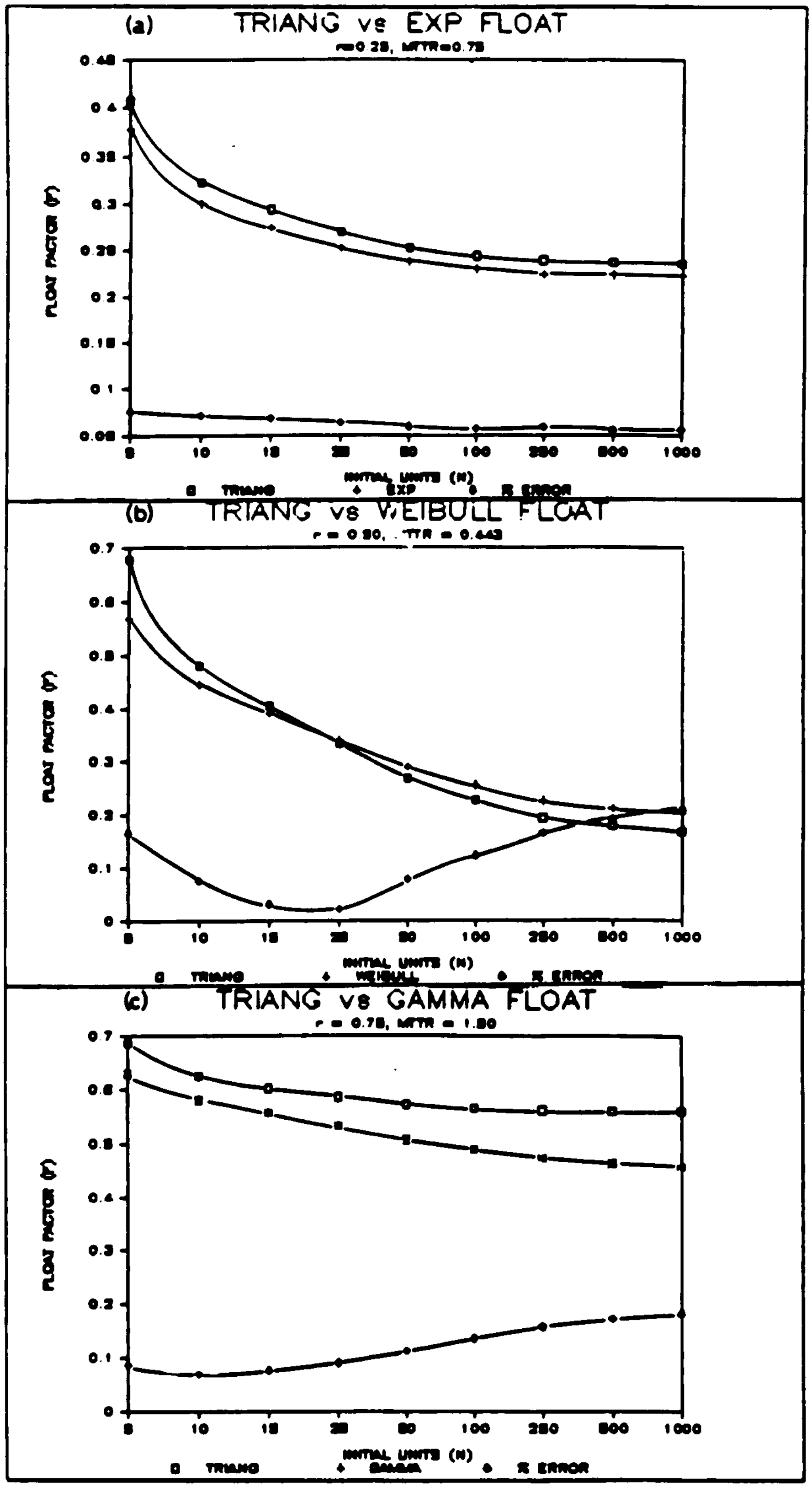



Figure 7. Reliability Based Triangular Estimates on f over N, Compared to: (a) Exponential, (b) Weibull, and (c) Gamma Estimates (Pilot Study Results).

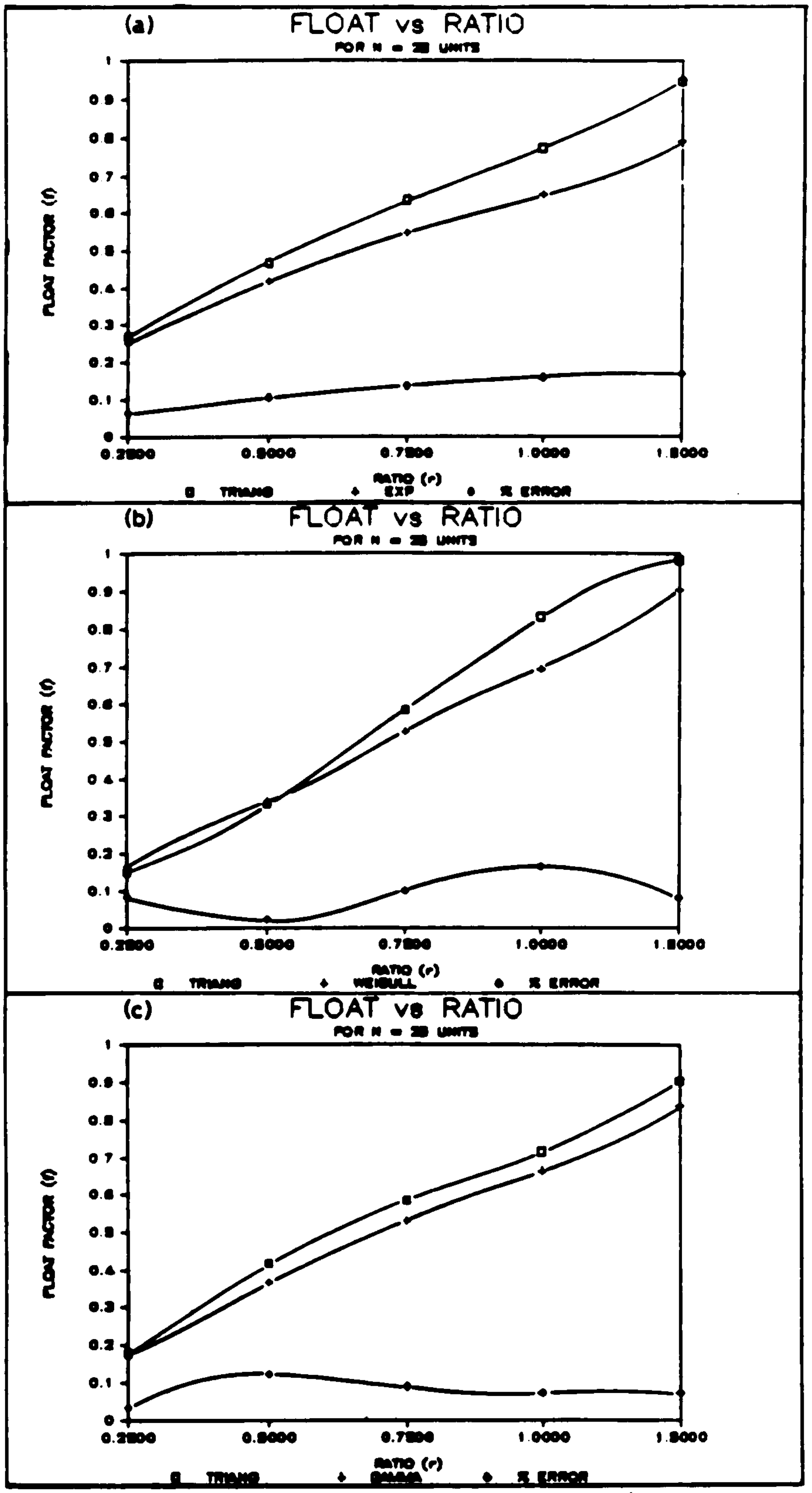



Figure 8. Reliability Based Triangular Estimates on f over r, Compared to: (a) Exponential, (b) Weibull, and (c) Gamma Estimates (Pilot Study Results).

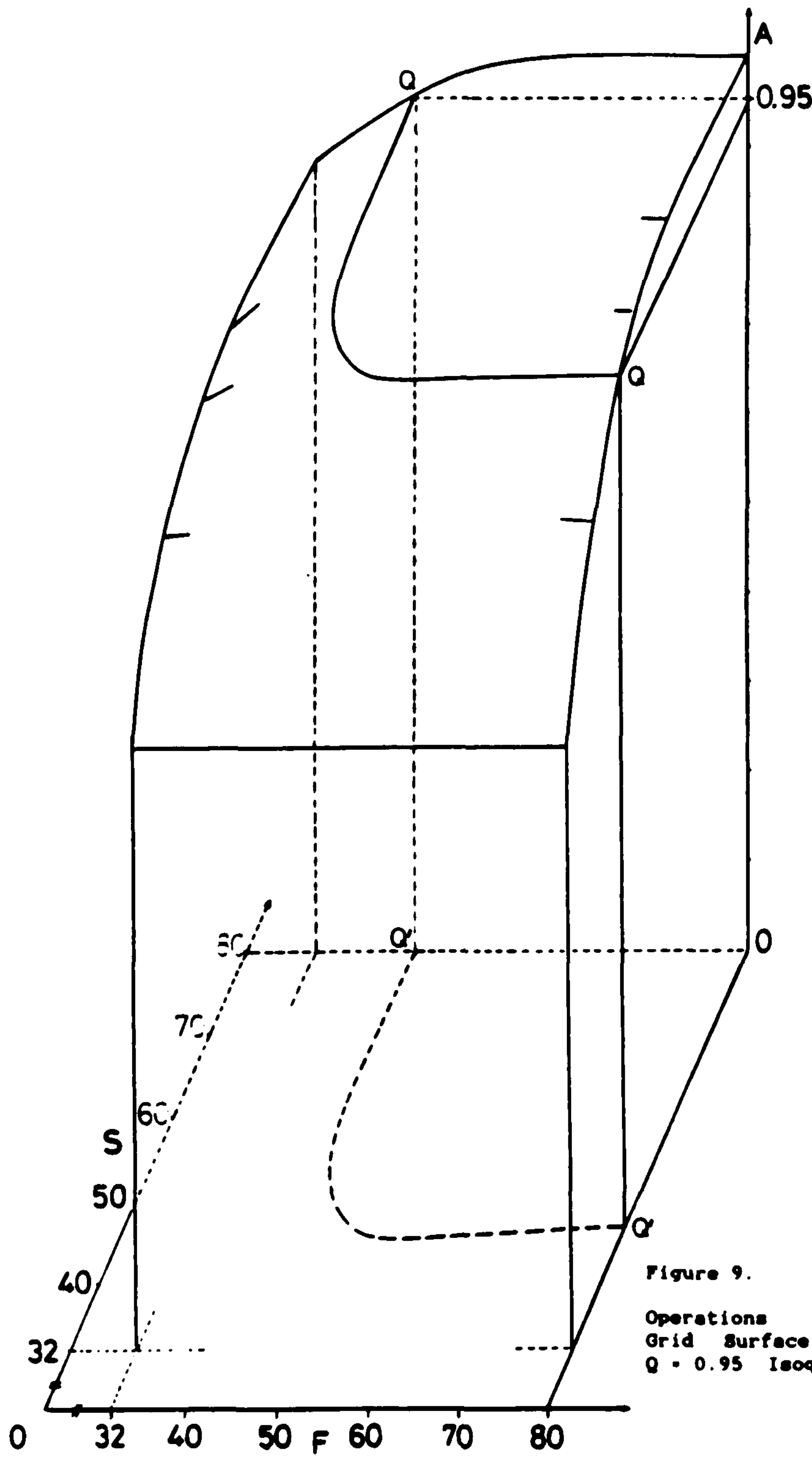



Figure 9.

Operations Availability Grid Surface (A), with Q = 0.95 Isoquant.

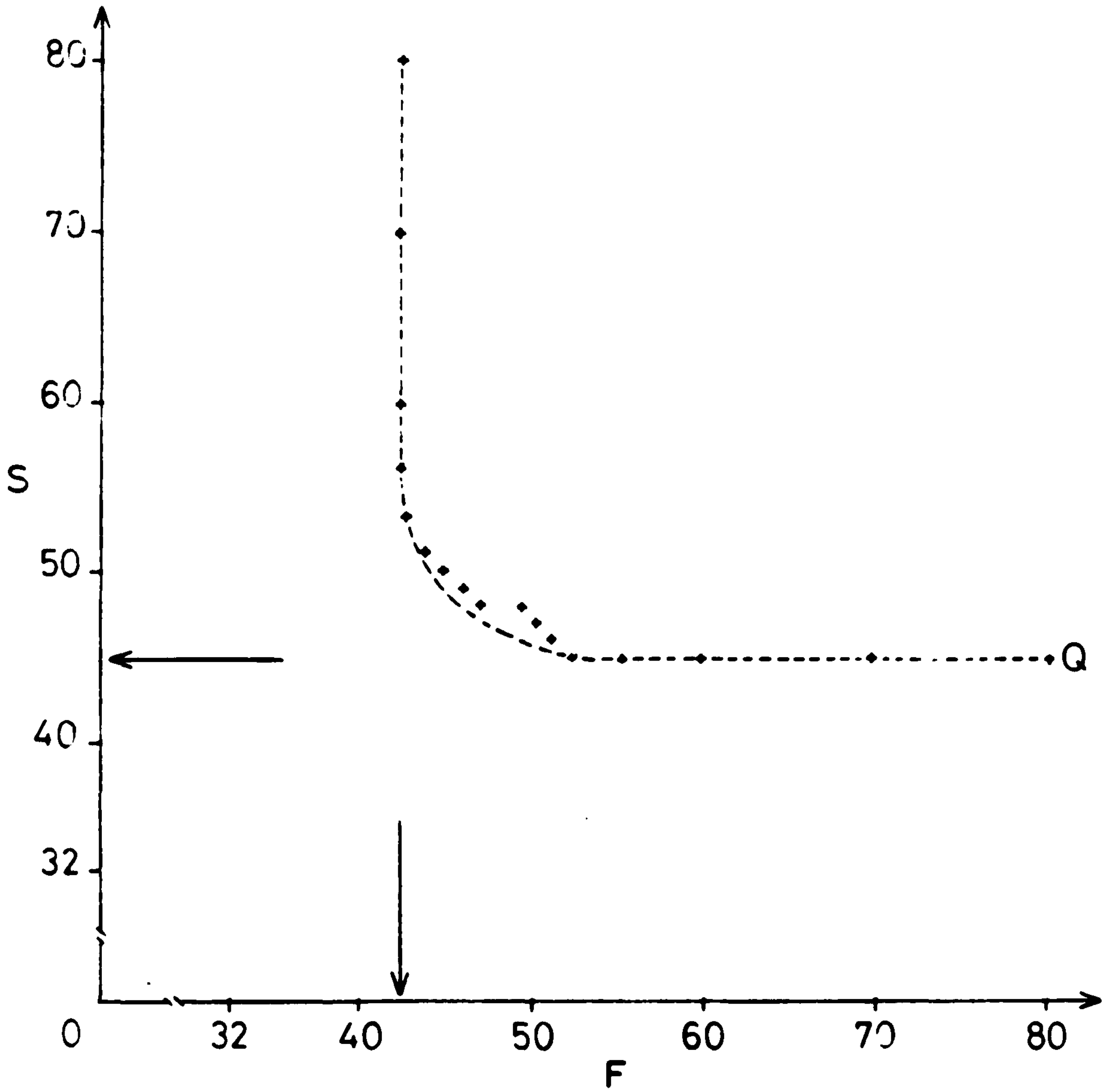



Figure 10. The Q = 0.95 Operations Availability (A) Isoquant, Transferred to the FS Input Surface.

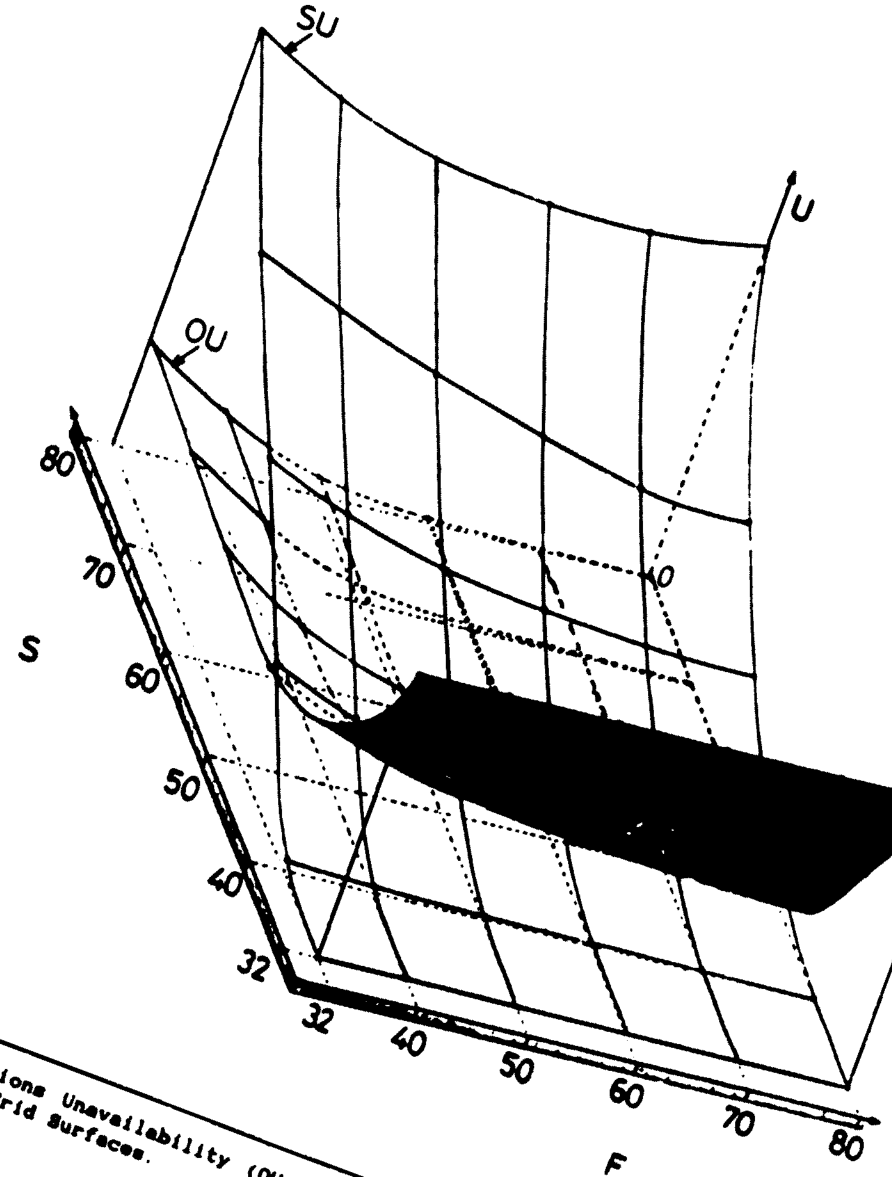



Figure 11. Operations Unavailability (OU), and Service Channel Underutilization (SU), Grid Surfaces.

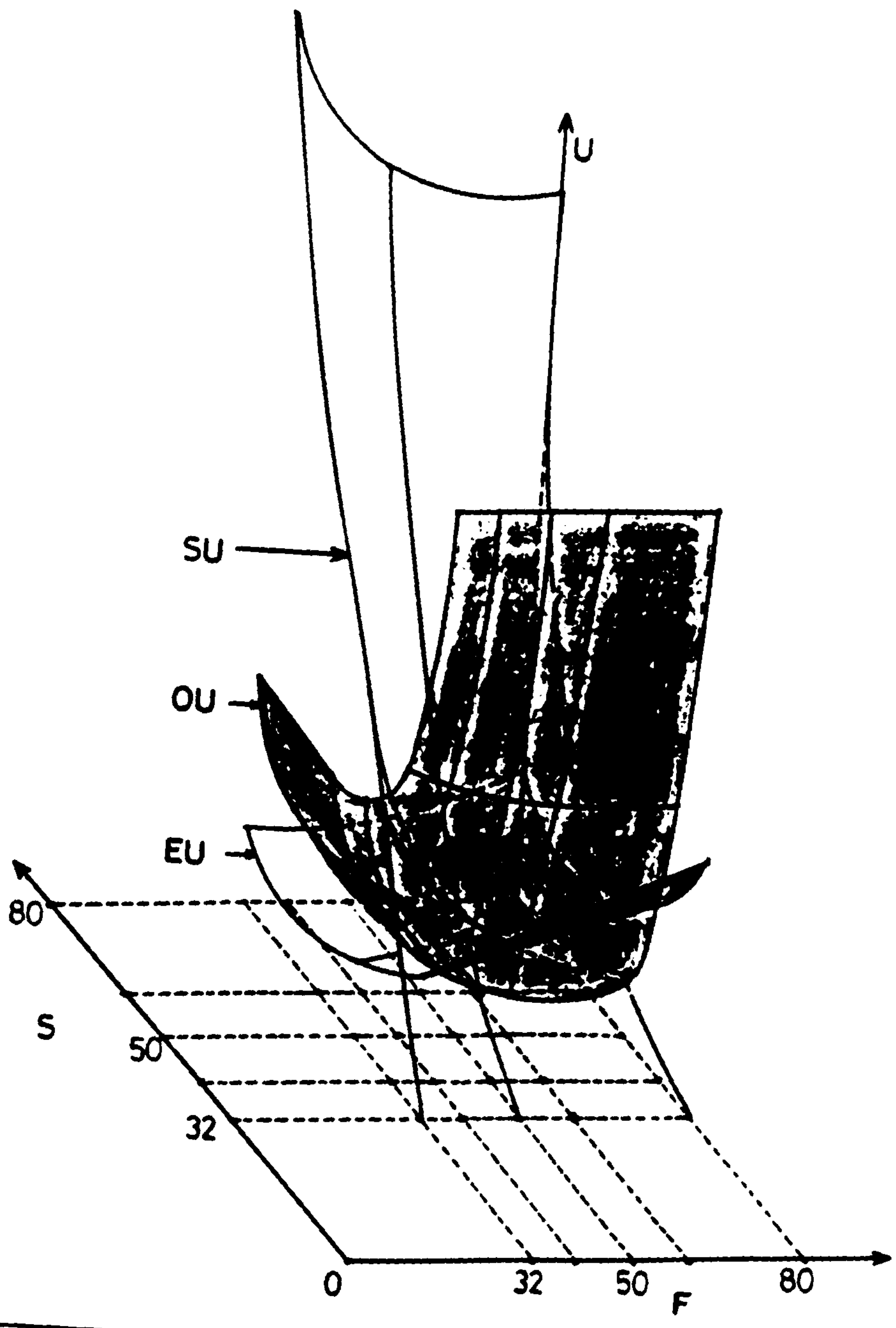



Figure 12. Equipment Underutilisation (EU), Operations Unavailability (OU), and Service Channel Underutilisation (SU), Grid Surfaces.

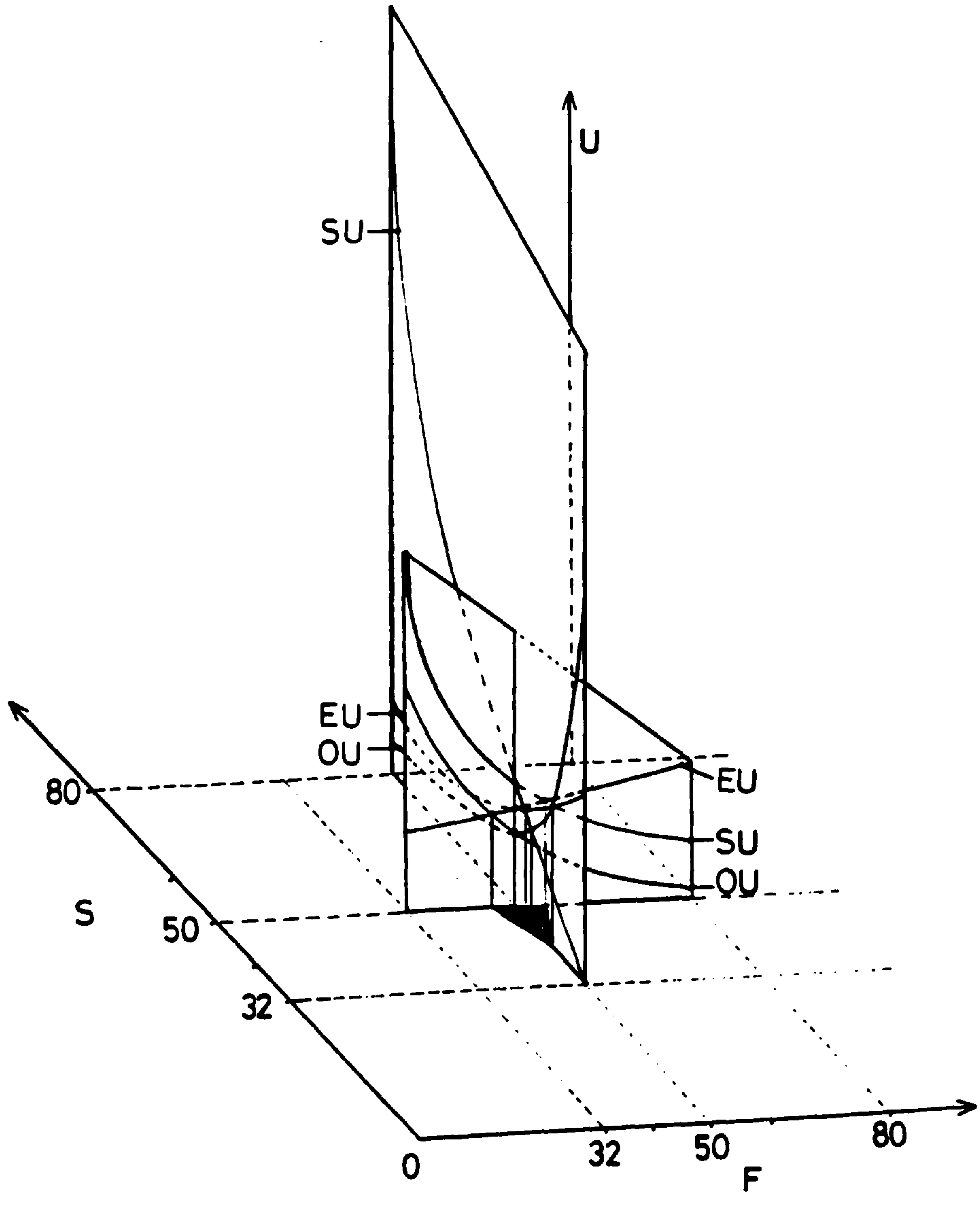



Figure 13. Line Crossing and Tradeoffs between EU, OU, and SU.

# TABLES

TABLE I

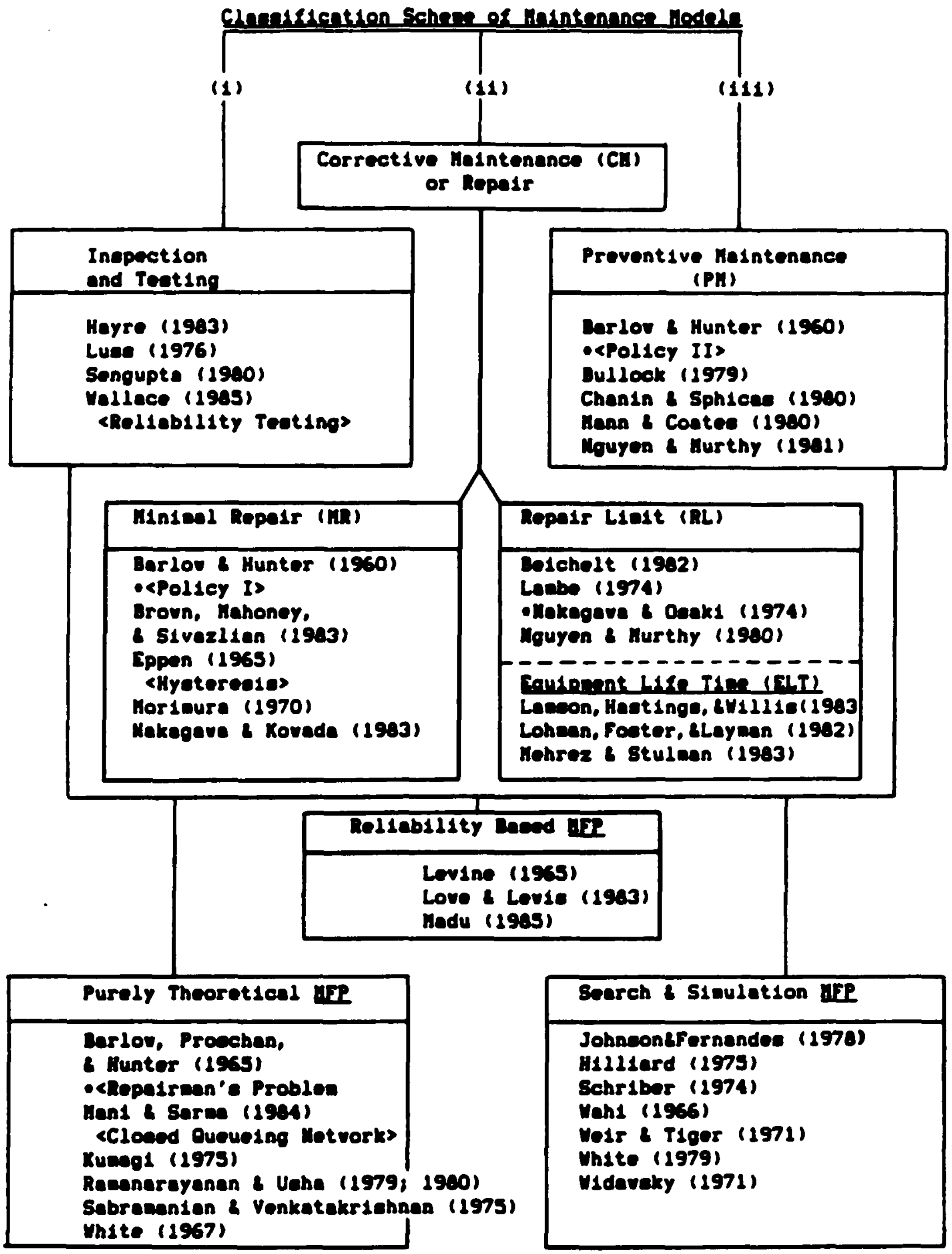

Classification Scheme of Maintenance Models
(i)
(ii)
(iii)
Corrective Maintenance (CM) or Repair
Inspection and Testing
Hayre (1983)
Luss (1976)
Sengupta (1980)
Wallace (1985)
<Reliability Testing>
Preventive Maintenance (PM)
Barlow & Hunter (1960)
•<Policy II>
Bullock (1979)
Chanin & Sphicas (1980)
Mann & Coates (1980)
Nguyen & Murthy (1981)
Minimal Repair (MR)
Barlow & Hunter (1960)
•<Policy I>
Brown, Mahoney, & Sivazlian (1983)
Eppen (1965)
<Hysteresis>
Morimura (1970)
Nakagawa & Kowada (1983)
Repair Limit (RL)
Beichelt (1982)
Lambe (1974)
•Nakagawa & Osaki (1974)
Nguyen & Murthy (1980)
Equipment Life Time (ELT)
Lamson, Hastings, &Willis(1983
Lohman, Foster, &Layman (1982)
Mehrez & Stulman (1983)
Reliability Based MFP
Levine (1965)
Love & Lewis (1983)
Madu (1985)
Purely Theoretical MFP
Barlow, Proschan, & Hunter (1965)
•<Repairman's Problem
Mani & Sarma (1984)
<Closed Queueing Network>
Kumagi (1975)
Ramanarayanan & Usha (1979; 1980)
Sabramanian & Venkatakrishnan (1975)
White (1967)
Search & Simulation MFP
Johnson&Fernandes (1978)
Hilliard (1975)
Schriber (1974)
Wahi (1966)
Weir & Tiger (1971)
White (1979)
Widavsky (1971)

**TABLE II**

**M F P Determination Formuli**

| Failure | f Equation | Number |
|---|---|---|
| Exponential | $f = 1 - \exp - \left\{ \ln\left(\frac{N}{N-1}\right) + \frac{MTTR}{b} \right\}$ | II-1 |
| Weibull | $f = 1 - \exp - \left\{ \sqrt[a]{\ln\left(\frac{N}{N-1}\right)} + \frac{MTTR}{b} \right\}^a$ | II-2 |
| Gamma | $f = 1 - \exp - \left\{ \sqrt[a]{\frac{a!}{N}} + \frac{MTTR}{b} \right\} \cdot \sum_{j=0}^{a-1} \frac{\left\{ \sqrt[a]{\frac{a!}{N}} + \frac{MTTR}{b} \right\}^j}{j!}$ | II-3 |
| Lognormal | $f = \Phi \left\{ \frac{\ln\left[\exp(Z\alpha \cdot \sigma + \mu) + MTTR\right] - \mu}{\sigma} \right\}$ | II-4 |
| Normal | $f = \Phi \left\{ Z\alpha + \frac{MTTR}{\sigma} \right\}$ | II-5 |
| Uniform | $f = \frac{\left(\frac{b-a}{N}\right) + MTTR}{(b-a)}$ , $0 < MTTR < \frac{N-1}{N}(b-a)$ | II-6 |
| | $= 1$ , $\frac{N-1}{N}(b-a) < MTTR$ | |

TABLE III

**Asymptotic M F P Determination**

| Failure | f Equation | | Number |
|---|---|---|---|
| Exponential | $f = 1 - \exp - \left\{\frac{MTTR}{b}\right\}$ | | III-1 |
| Weibull | $f = 1 - \exp - \left\{\frac{MTTR}{b}\right\}^a$ | | III-2 |
| Gamma | $f = 1 - \exp - \left\{\frac{MTTR}{b}\right\} \cdot \sum_{j=0}^{a-1} \frac{\left\{\frac{MTTR}{b}\right\}^j}{j!}$ | | III-3 |
| Lognormal | $f = \Phi \left\{\frac{\ln[\exp(-3.09 \cdot \sigma + \mu) + MTTR] - \mu}{\sigma}\right\}$ | | III-4 |
| Normal | $f = \Phi \left\{-3.09 + \frac{MTTR}{\sigma}\right\}$ | | III-5 |
| Uniform | $f = \frac{MTTR}{b-a}$ | $, \quad 0 < MTTR \leq b - a$ | III-6 |
| | $= 1$ | $, b - a < MTTR$ | |

## TABLE IV

**Triangular M F P Determination**

**Triangular Distribution** (IV-1)

$$f = \frac{\left\{-\sqrt{\frac{(b-a)(c-a)}{N}} + MTTR\right\}^2}{(b-a)(c-a)}, \qquad 0 \le MTTR \le \left\{1-\sqrt{\frac{1}{N}}\right\}\sqrt{(b-a)(c-a)}$$

$$= 1 - \frac{\left\{\sqrt{\left(\frac{N-1}{N}\right)(b-a)(b-c)} - MTTR\right\}^2}{(b-a)(b-c)}, \qquad \left\{1-\sqrt{\frac{1}{N}}\right\}\sqrt{(b-a)(c-a)} \le MTTR \le \sqrt{\left(\frac{N-1}{N}\right)(b-a)(b-c)}$$

$$= 1, \qquad \sqrt{\left(\frac{N-1}{N}\right)(b-a)(b-c)} \le MTTR$$

**Left Triangular Distribution** (IV-2)

$$f = 1 - \left\{\sqrt{\frac{N-1}{N}} - \frac{MTTR}{b-a}\right\}^2, \qquad 0 \le MTTR \le (b-a)\sqrt{\frac{N-1}{N}}$$

$$= 1, \qquad (b-a)\sqrt{\frac{N-1}{N}} \le MTTR$$

**TABLE V**

**Asymptotic Triangular M F P**

**Triangular Distribution** (V-1)

$$f = \frac{(MTTR)^2}{(b-a)(c-a)} , \qquad 0 \le MTTR \le \sqrt{(b-a)(c-a)}$$

$$= 1 - \frac{\left\{\sqrt{(b-a)(b-c)} - MTTR\right\}^2}{(b-a)(b-c)} , \qquad \sqrt{(b-a)(c-a)} \le MTTR \le \sqrt{(b-a)(b-c)}$$

$$= 1 , \qquad \sqrt{(b-a)(b-c)} \le MTTR$$

**Left Triangular Distribution** (V-2)

$$f = 1 - \left\{1 - \frac{MTTR}{b-a}\right\}^2 , \qquad 0 \le MTTR \le b-a$$

$$= 1 , \qquad b-a \le MTTR$$

## TABLE VI

### A SET OF EXPERIMENTAL REALIZATIONS ON THE AVERAGE WAITING TIME FOR REPAIR

| Units Initially in Operations N | Repair to Failure Ratio r | Average Waiting for Repair W (hrs) |
|---|---|---|
| 5 | 0.25 | 0.069 |
| | 0.50 | 0.191 |
| | 0.75 | 0.689 |
| | 1.00 | 1.075 |
| 20 | 0.25 | 0.766 |
| | 0.50 | 2.598 |
| | 0.75 | 4.469 |
| | 1.00 | 6.353 |
| 50 | 0.25 | 2.530 |
| | 0.50 | 7.226 |
| | 0.75 | 12.006 |
| | 1.00 | 16.785 |

## TABLE VII

### HYPOTHESIS TESTING FOR THE SIGNIFICANCE OF DIFFERENCES IN W, DUE TO CHANGES IN THE EXPERIMENTAL FACTORS N AND r

| Postulated Hypothesis | df N1/N2 | Computed F | alpha p<.** | Tabulated F | Descriptive eta |
|---|---|---|---|---|---|
| $H_1$ | 2/11 | 10.363 | .25 | 1.58 | 0.7543 |
| | | | .10 | 2.86 | |
| | | | .05 | 3.98 | |
| <F test for N> | | | .01 | 7.21 | |
| $H_2$ | 3/11 | 3.235 | .25 | 1.58 | 0.5161 |
| | | | .10 | 2.66 | |
| | | | .05 | 3.59 | |
| <F test for r> | | | .01 | 6.22 | |

Rule: at alpha-percent level of significance:
Reject $H_0$ if $F_{Computed} > F_{Tabulated}$;
Do not reject $H_0$ if $F_{Computed} \leq F_{Tabulated}$.

TABLE VIII

THEORETICAL FLOAT FACTOR (f) AND TOTAL FLOAT (F) ESTIMATES

M: Estimation Method; D: Failure Distribution;
N: Units in Operations; r: Repair to Failure Ratio

| No. | M | D | r | N | f | F |
|---|---|---|---|---|---|---|
| 1. | 1 | 1 | 25 | 5 | 3770 | 2 |
| 2. | 1 | 1 | 25 | 10 | 2991 | 3 |
| 3. | 1 | 1 | 25 | 20 | 2601 | 5 |
| 4. | 1 | 1 | 25 | 40 | 2407 | 10 |
| 5. | 1 | 1 | 25 | 80 | 2309 | 18 |
| 6. | 1 | 1 | 50 | 5 | 5140 | 3 |
| 7. | 1 | 1 | 50 | 10 | 5441 | 5 |
| 8. | 1 | 1 | 50 | 20 | 4238 | 8 |
| 9. | 1 | 1 | 50 | 40 | 4086 | 16 |
| 10. | 1 | 1 | 50 | 80 | 4011 | 32 |
| 11. | 1 | 1 | 75 | 5 | 6221 | 3 |
| 12. | 1 | 1 | 75 | 10 | 5749 | 6 |
| 13. | 1 | 1 | 75 | 20 | 5513 | 11 |
| 14. | 1 | 1 | 75 | 40 | 5394 | 22 |
| 15. | 1 | 1 | 75 | 80 | 5335 | 43 |
| 16. | 1 | 1 | 100 | 5 | 7057 | 4 |
| 17. | 1 | 1 | 100 | 10 | 6689 | 7 |
| 18. | 1 | 1 | 100 | 20 | 6595 | 13 |
| 19. | 1 | 1 | 100 | 40 | 6413 | 26 |
| 20. | 1 | 1 | 100 | 80 | 6367 | 51 |
| 21. | 1 | 1 | 150 | 5 | 8215 | 4 |
| 22. | 1 | 1 | 150 | 10 | 7992 | 8 |
| 23. | 1 | 1 | 150 | 20 | 7880 | 16 |
| 24. | 1 | 1 | 150 | 40 | 7824 | 31 |
| 25. | 1 | 1 | 150 | 80 | 7797 | 62 |
| 26. | 1 | 2 | 25 | 5 | 4066 | 2 |
| 27. | 1 | 2 | 25 | 10 | 2812 | 3 |
| 28. | 1 | 2 | 25 | 20 | 2031 | 4 |
| 29. | 1 | 2 | 25 | 40 | 1941 | 6 |
| 30. | 1 | 2 | 25 | 80 | 1229 | 10 |
| 31. | 1 | 2 | 50 | 5 | 6115 | 3 |
| 32. | 1 | 2 | 50 | 10 | 4934 | 5 |
| 33. | 1 | 2 | 50 | 20 | 4101 | 8 |
| 34. | 1 | 2 | 50 | 40 | 3524 | 14 |
| 35. | 1 | 2 | 50 | 80 | 3125 | 25 |
| 36. | 1 | 2 | 75 | 5 | 7756 | 4 |
| 37. | 1 | 2 | 75 | 10 | 6849 | 7 |
| 38. | 1 | 2 | 75 | 20 | 6140 | 12 |
| 39. | 1 | 2 | 75 | 40 | 5624 | 22 |
| 40. | 1 | 2 | 75 | 80 | 5245 | 42 |
| 41. | 1 | 2 | 100 | 5 | 8856 | 4 |
| 42. | 1 | 2 | 100 | 10 | 8270 | 8 |
| 43. | 1 | 2 | 100 | 20 | 7778 | 16 |
| 44. | 1 | 2 | 100 | 40 | 7391 | 30 |
| 45. | 1 | 2 | 100 | 80 | 7097 | 57 |
| 46. | 1 | 2 | 150 | 5 | 9796 | 5 |
| 47. | 1 | 2 | 150 | 10 | 9642 | 10 |
| 48. | 1 | 2 | 150 | 20 | 9492 | 19 |
| 49. | 1 | 2 | 150 | 40 | 9362 | 37 |
| 50. | 1 | 2 | 150 | 80 | 9257 | 74 |
| 51. | 1 | 3 | 25 | 5 | 2213 | 1 |
| 52. | 1 | 3 | 25 | 10 | 1540 | 2 |
| 53. | 1 | 3 | 25 | 20 | 1159 | 2 |
| 54. | 1 | 3 | 25 | 40 | 822 | 3 |
| 55. | 1 | 3 | 25 | 80 | 637 | 5 |
| 56. | 1 | 3 | 50 | 5 | 3129 | 2 |
| 57. | 1 | 3 | 50 | 10 | 2449 | 2 |
| 58. | 1 | 3 | 50 | 20 | 1978 | 4 |
| 59. | 1 | 3 | 50 | 40 | 1641 | 7 |
| 60. | 1 | 3 | 50 | 80 | 1414 | 11 |
| 61. | 1 | 3 | 75 | 5 | 4020 | 2 |
| 62. | 1 | 3 | 75 | 10 | 3364 | 3 |
| 63. | 1 | 3 | 75 | 20 | 2886 | 6 |
| 64. | 1 | 3 | 75 | 40 | 2546 | 10 |
| 65. | 1 | 3 | 75 | 80 | 2305 | 18 |
| 66. | 1 | 3 | 100 | 5 | 4856 | 2 |
| 67. | 1 | 3 | 100 | 10 | 4244 | 4 |
| 68. | 1 | 3 | 100 | 20 | 3790 | 8 |
| 69. | 1 | 3 | 100 | 40 | 3458 | 14 |
| 70. | 1 | 3 | 100 | 80 | 3221 | 26 |
| 71. | 1 | 3 | 150 | 5 | 6288 | 3 |
| 72. | 1 | 3 | 150 | 10 | 5794 | 6 |
| 73. | 1 | 3 | 150 | 20 | 5421 | 11 |
| 74. | 1 | 3 | 150 | 40 | 5141 | 21 |
| 75. | 1 | 3 | 150 | 80 | 4936 | 39 |
| 76. | 1 | 4 | 25 | 5 | 2743 | 1 |
| 77. | 1 | 4 | 25 | 10 | 1492 | 1 |
| 78. | 1 | 4 | 25 | 20 | 888 | 2 |
| 79. | 1 | 4 | 25 | 40 | 436 | 2 |
| 80. | 1 | 4 | 25 | 80 | 233 | 2 |
| 81. | 1 | 4 | 50 | 5 | 3632 | 2 |
| 82. | 1 | 4 | 50 | 10 | 2148 | 2 |
| 83. | 1 | 4 | 50 | 20 | 1251 | 3 |
| 84. | 1 | 4 | 50 | 40 | 722 | 3 |
| 85. | 1 | 4 | 50 | 80 | 409 | 3 |
| 86. | 1 | 4 | 75 | 5 | 4602 | 2 |
| 87. | 1 | 4 | 75 | 10 | 2946 | 3 |
| 88. | 1 | 4 | 75 | 20 | 1841 | 4 |
| 89. | 1 | 4 | 75 | 40 | 1131 | 5 |
| 90. | 1 | 4 | 75 | 80 | 681 | 5 |
| 91. | 1 | 4 | 100 | 5 | 5636 | 3 |
| 92. | 1 | 4 | 100 | 10 | 3859 | 4 |
| 93. | 1 | 4 | 100 | 20 | 2578 | 5 |
| 94. | 1 | 4 | 100 | 40 | 1685 | 7 |
| 95. | 1 | 4 | 100 | 80 | 1075 | 9 |
| 96. | 1 | 4 | 150 | 5 | 7454 | 4 |
| 97. | 1 | 4 | 150 | 10 | 5832 | 6 |
| 98. | 1 | 4 | 150 | 20 | 4404 | 9 |
| 99. | 1 | 4 | 150 | 40 | 3228 | 13 |
| 100. | 1 | 4 | 150 | 80 | 2297 | 18 |
| 101. | 1 | 5 | 25 | 5 | 3403 | 2 |
| 102. | 1 | 5 | 25 | 10 | 2611 | 3 |
| 103. | 1 | 5 | 25 | 20 | 2090 | 4 |
| 104. | 1 | 5 | 25 | 40 | 1736 | 7 |
| 105. | 1 | 5 | 25 | 80 | 1515 | 12 |
| 106. | 1 | 5 | 50 | 5 | 4721 | 2 |
| 107. | 1 | 5 | 50 | 10 | 4013 | 4 |
| 108. | 1 | 5 | 50 | 20 | 3557 | 7 |
| 109. | 1 | 5 | 50 | 40 | 3300 | 13 |
| 110. | 1 | 5 | 50 | 80 | 3085 | 25 |
| 111. | 1 | 5 | 75 | 5 | 5675 | 3 |
| 112. | 1 | 5 | 75 | 10 | 5120 | 5 |
| 113. | 1 | 5 | 75 | 20 | 4761 | 10 |
| 114. | 1 | 5 | 75 | 40 | 4522 | 18 |
| 115. | 1 | 5 | 75 | 80 | 4404 | 35 |
| 116. | 1 | 5 | 100 | 5 | 6406 | 3 |
| 117. | 1 | 5 | 100 | 10 | 5948 | 6 |
| 118. | 1 | 5 | 100 | 20 | 5714 | 11 |
| 119. | 1 | 5 | 100 | 40 | 5517 | 22 |
| 120. | 1 | 5 | 100 | 80 | 5398 | 43 |
| 121. | 1 | 5 | 150 | 5 | 7454 | 4 |
| 122. | 1 | 5 | 150 | 10 | 7157 | 7 |
| 123. | 1 | 5 | 150 | 20 | 7019 | 14 |
| 124. | 1 | 5 | 150 | 40 | 6915 | 28 |
| 125. | 1 | 5 | 150 | 80 | 6808 | 54 |
| 126. | 2 | 1 | 25 | 5 | 4080 | 2 |
| 127. | 2 | 1 | 25 | 10 | 3215 | 4 |
| 128. | 2 | 1 | 25 | 20 | 2780 | 6 |
| 129. | 2 | 1 | 25 | 40 | 2562 | 11 |
| 130. | 2 | 1 | 25 | 80 | 2453 | 20 |
| 131. | 2 | 1 | 50 | 5 | 5847 | 3 |
| 132. | 2 | 1 | 50 | 10 | 5118 | 10 |
| 133. | 2 | 1 | 50 | 20 | 4748 | 10 |
| 134. | 2 | 1 | 50 | 40 | 4562 | 19 |
| 135. | 2 | 1 | 50 | 80 | 4469 | 36 |
| 136. | 2 | 1 | 75 | 5 | 7302 | 4 |
| 137. | 2 | 1 | 75 | 10 | 6709 | 7 |
| 138. | 2 | 1 | 75 | 20 | 6404 | 13 |
| 139. | 2 | 1 | 75 | 40 | 6249 | 25 |
| 140. | 2 | 1 | 75 | 80 | 6172 | 50 |
| 141. | 2 | 1 | 100 | 5 | 9444 | 5 |
| 142. | 2 | 1 | 100 | 10 | 7987 | 8 |
| 143. | 2 | 1 | 100 | 20 | 7747 | 16 |
| 144. | 2 | 1 | 100 | 40 | 7624 | 31 |
| 145. | 2 | 1 | 100 | 80 | 7562 | 61 |
| 146. | 2 | 1 | 150 | 5 | 9791 | 5 |
| 147. | 2 | 1 | 150 | 10 | 9605 | 10 |
| 148. | 2 | 1 | 150 | 20 | 9495 | 19 |
| 149. | 2 | 1 | 150 | 40 | 9436 | 38 |
| 150. | 2 | 1 | 150 | 80 | 9406 | 76 |
| 151. | 2 | 2 | 25 | 5 | 4322 | 2 |
| 152. | 2 | 2 | 25 | 10 | 2772 | 3 |
| 153. | 2 | 2 | 25 | 20 | 1882 | 4 |
| 154. | 2 | 2 | 25 | 40 | 1357 | 6 |
| 155. | 2 | 2 | 25 | 80 | 1037 | 9 |
| 156. | 2 | 2 | 50 | 5 | 7529 | 3 |
| 157. | 2 | 2 | 50 | 10 | 5427 | 5 |
| 158. | 2 | 2 | 50 | 20 | 4140 | 8 |
| 159. | 2 | 2 | 50 | 40 | 3348 | 14 |
| 160. | 2 | 2 | 50 | 80 | 2833 | 23 |
| 161. | 2 | 2 | 75 | 5 | 8967 | 4 |
| 162. | 2 | 2 | 75 | 10 | 8168 | 9 |
| 163. | 2 | 2 | 75 | 20 | 7299 | 15 |
| 164. | 2 | 2 | 75 | 40 | 6223 | 25 |
| 165. | 2 | 2 | 75 | 80 | 5513 | 45 |
| 166. | 2 | 2 | 100 | 5 | 9257 | 5 |
| 167. | 2 | 2 | 100 | 10 | 8932 | 9 |
| 168. | 2 | 2 | 100 | 20 | 8755 | 18 |
| 169. | 2 | 2 | 100 | 40 | 8481 | 34 |
| 170. | 2 | 2 | 100 | 80 | 8278 | 67 |
| 171. | 2 | 2 | 150 | 5 | 9999 | 5 |
| 172. | 2 | 2 | 150 | 10 | 9997 | 10 |
| 173. | 2 | 2 | 150 | 20 | 9982 | 20 |
| 174. | 2 | 2 | 150 | 40 | 9973 | 40 |
| 175. | 2 | 2 | 150 | 80 | 9963 | 80 |
| 176. | 2 | 3 | 25 | 5 | 3894 | 2 |
| 177. | 2 | 3 | 25 | 10 | 2431 | 3 |
| 178. | 2 | 3 | 25 | 20 | 1603 | 4 |
| 179. | 2 | 3 | 25 | 40 | 1122 | 5 |
| 180. | 2 | 3 | 25 | 80 | 833 | 7 |
| 181. | 2 | 3 | 50 | 5 | 6412 | 4 |
| 182. | 2 | 3 | 50 | 10 | 4486 | 5 |
| 183. | 2 | 3 | 50 | 20 | 3331 | 7 |
| 184. | 2 | 3 | 50 | 40 | 2618 | 11 |
| 185. | 2 | 3 | 50 | 80 | 2166 | 18 |
| 186. | 2 | 3 | 75 | 5 | 8112 | 4 |
| 187. | 2 | 3 | 75 | 10 | 7167 | 8 |
| 188. | 2 | 3 | 75 | 20 | 5684 | 12 |
| 189. | 2 | 3 | 75 | 40 | 4740 | 19 |
| 190. | 2 | 3 | 75 | 80 | 4123 | 33 |
| 191. | 2 | 3 | 100 | 5 | 9649 | 5 |
| 192. | 2 | 3 | 100 | 10 | 9416 | 9 |
| 193. | 2 | 3 | 100 | 20 | 8662 | 18 |
| 194. | 2 | 3 | 100 | 40 | 7486 | 30 |
| 195. | 2 | 3 | 100 | 80 | 6776 | 54 |
| 196. | 2 | 3 | 150 | 5 | 9999 | 5 |
| 197. | 2 | 3 | 150 | 10 | 9971 | 10 |
| 198. | 2 | 3 | 150 | 20 | 9936 | 20 |
| 199. | 2 | 3 | 150 | 40 | 9914 | 40 |
| 200. | 2 | 3 | 150 | 80 | 9901 | 80 |
| 201. | 2 | 4 | 25 | 5 | 3193 | 2 |
| 202. | 2 | 4 | 25 | 10 | 1894 | 2 |
| 203. | 2 | 4 | 25 | 20 | 1166 | 3 |
| 204. | 2 | 4 | 25 | 40 | 762 | 4 |
| 205. | 2 | 4 | 25 | 80 | 527 | 5 |
| 206. | 2 | 4 | 50 | 5 | 4664 | 3 |
| 207. | 2 | 4 | 50 | 10 | 3046 | 4 |
| 208. | 2 | 4 | 50 | 20 | 2117 | 5 |
| 209. | 2 | 4 | 50 | 40 | 1551 | 7 |
| 210. | 2 | 4 | 50 | 80 | 1238 | 10 |
| 211. | 2 | 4 | 75 | 5 | 6412 | 4 |
| 212. | 2 | 4 | 75 | 10 | 4486 | 5 |
| 213. | 2 | 4 | 75 | 20 | 3331 | 7 |
| 214. | 2 | 4 | 75 | 40 | 2618 | 11 |
| 215. | 2 | 4 | 75 | 80 | 2166 | 18 |
| 216. | 2 | 4 | 100 | 5 | 8439 | 4 |
| 217. | 2 | 4 | 100 | 10 | 6207 | 7 |
| 218. | 2 | 4 | 100 | 20 | 4830 | 10 |
| 219. | 2 | 4 | 100 | 40 | 3963 | 16 |
| 220. | 2 | 4 | 100 | 80 | 3401 | 28 |
| 221. | 2 | 4 | 150 | 5 | 9649 | 5 |
| 222. | 2 | 4 | 150 | 10 | 9416 | 10 |
| 223. | 2 | 4 | 150 | 20 | 8662 | 18 |
| 224. | 2 | 4 | 150 | 40 | 7486 | 30 |
| 225. | 2 | 4 | 150 | 80 | 6706 | 54 |
| 226. | 2 | 5 | 25 | 5 | 5456 | 2 |
| 227. | 2 | 5 | 25 | 10 | 3692 | 4 |
| 228. | 2 | 5 | 25 | 20 | 2652 | 6 |
| 229. | 2 | 5 | 25 | 40 | 2021 | 9 |
| 230. | 2 | 5 | 25 | 80 | 1626 | 14 |
| 231. | 2 | 5 | 50 | 5 | 6585 | 3 |
| 232. | 2 | 5 | 50 | 10 | 6344 | 7 |
| 233. | 2 | 5 | 50 | 20 | 6103 | 13 |
| 234. | 2 | 5 | 50 | 40 | 5490 | 22 |
| 235. | 2 | 5 | 50 | 80 | 4825 | 39 |
| 236. | 2 | 5 | 75 | 5 | 7703 | 4 |
| 237. | 2 | 5 | 75 | 10 | 7153 | 8 |
| 238. | 2 | 5 | 75 | 20 | 6869 | 14 |
| 239. | 2 | 5 | 75 | 40 | 6725 | 27 |
| 240. | 2 | 5 | 75 | 80 | 6636 | 53 |
| 241. | 2 | 5 | 100 | 5 | 8838 | 4 |
| 242. | 2 | 5 | 100 | 10 | 8438 | 9 |
| 243. | 2 | 5 | 100 | 20 | 8226 | 17 |
| 244. | 2 | 5 | 100 | 40 | 8117 | 33 |
| 245. | 2 | 5 | 100 | 80 | 8062 | 65 |
| 246. | 2 | 5 | 150 | 5 | 9959 | 5 |
| 247. | 2 | 5 | 150 | 10 | 9860 | 10 |
| 248. | 2 | 5 | 150 | 20 | 9791 | 20 |
| 249. | 2 | 5 | 150 | 40 | 9751 | 39 |
| 250. | 2 | 5 | 150 | 80 | 9733 | 78 |

## TABLE IX

### HYPOTHESIS TESTING FOR THE SIGNIFICANCE OF DIFFERENCES IN f AND F DUE TO THE ESTIMATION METHOD (M=1 AND M=2) THEORETICAL DATA: ANOVA-TESTS

| Postulated Hypothesis | df N1/N2 | Computed F | alpha p<.** | Tabulated F | Descriptive eta |
|---|---|---|---|---|---|
| $H_3$ | 1/283 | 23.073 | .25 | 1.32 | 0.2917 |
| | | | .10 | 2.71 | |
| | | | .05 | 3.84 | |
| <F test for f> | | | .01 | 6.63 | |
| $H_4$ | 1/238 | 6.053 | .25 | 1.32 | 0.1544 |
| | | | .10 | 2.71 | |
| | | | .05 | 3.84 | |
| <F test for F> | | | .01 | 6.63 | |

Rule: at alpha-percent level of significance:
Reject $H_\emptyset$ if $F_{Computed} > F_{Tabulated}$;
Do not reject $H_\emptyset$ if $F_{Computed} \leq F_{Tabulated}$.

## TABLE X

### HYPOTHESIS TESTING FOR THE TRUE SIGNIFICANCE OF DIFFERENCES IN f AND F DUE TO THE ESTIMATION METHOD (M=1 AND M=2) THEORETICAL DATA: T-TESTS

| Postulated Hypothesis | df N1+N2-2 | Computed t | alpha p<.•• | Tabulated t |
|---|---|---|---|---|
| $H_5$ | 248 | -4.80 | .25 | -1.156 |
| | | | .10 | -1.645 |
| | | | .05 | -1.960 |
| <Lover Tail Test< | | | .01 | -2.578 |
| $H_6$ | 248 | -2.46 | .25 | -1.156 |
| | | | .10 | -1.645 |
| | | | .05 | -1.960 |
| <Lover Tail Test> | | | .01 | -2.578 |

Lover Tail Rule: at alpha-percent level of significance:
Reject $H_0$ if $t_{Computed} \le t_{Tabulated}$;
Do not reject $H_0$ if $t_{Computed} > t_{Tabulated}$

TABLE XI

CODED EXPERIMENTAL REALIZATIONS ON SYSTEM EFFECTIVENESS
BASED ON THE THEORETICAL TOTAL FLOAT (F) IMPLEMENTATION

| No. | Estimation Method M | Distribution of: Failure F | Repair R | Operations Availabil. A | Channel Utiliz. P | Waiting For Rep. W |
|---|---|---|---|---|---|---|
| 1. | 1 | 1 | 1 | 776 | 999 | 1447 |
| 2. | 2 | 1 | 1 | 886 | 994 | 642 |
| 3. | 1 | 2 | 1 | 581 | 1000 | 3557 |
| 4. | 2 | 2 | 1 | 862 | 994 | 792 |
| 5. | 1 | 3 | 1 | 668 | 1000 | 2461 |
| 6. | 2 | 3 | 1 | 840 | 998 | 952 |
| 7. | 1 | 1 | 2 | 777 | 999 | 1438 |
| 8. | 2 | 1 | 2 | 886 | 987 | 645 |
| 9. | 1 | 2 | 2 | 582 | 1000 | 3546 |
| 10. | 2 | 2 | 2 | 859 | 994 | 817 |
| 11. | 1 | 3 | 2 | 669 | 1000 | 2454 |
| 12. | 2 | 3 | 2 | 844 | 998 | 929 |
| 13. | 1 | 1 | 3 | 749 | 999 | 1670 |
| 14. | 2 | 1 | 3 | 874 | 989 | 722 |
| 15. | 1 | 2 | 3 | 568 | 1000 | 3741 |
| 16. | 2 | 2 | 3 | 830 | 994 | 1014 |
| 17. | 1 | 3 | 3 | 645 | 1000 | 2725 |
| 18. | 2 | 3 | 3 | 818 | 997 | 1107 |

Key to the nominal values of the polytomous variables.

M: (1) Levine-Love & Levis-Madu estimate implementation;
(2) Reliability Based Triangular estimate implementation.

F: (1) Exponential, (2) Weibull, and (3) Erlang-2 failures.

R: (1) Exponential, (2) Erlang-2, and (3) Lognormal repairs.

## TABLE XII

### HYPOTHESIS TESTING FOR THE SIGNIFICANCE OF DIFFERENCES IN A, P, & W, DUE TO THE ESTIMATION METHOD (M=1 AND M=2) EXPERIMENTAL DATA: ANOVA-TESTS

| Postulated Hypothesis | df N1/N2 | Computed F | alpha p<.•• | Tabulated F | Descriptive eta |
|---|---|---|---|---|---|
| $H_7$ | 1/16 | 41.814 | .25 | 1.51 | 0.8504 |
| | | | .10 | 2.67 | |
| | | | .05 | 3.63 | |
| <F test for A> | | | .01 | 6.23 | |
| $H_8$ | 1/16 | 20.563 | .25 | 1.51 | 0.7499 |
| | | | .10 | 2.67 | |
| | | | .05 | 3.63 | |
| <F test for P> | | | .01 | 6.23 | |
| $H_9$ | 1/16 | 30.563 | .25 | 1.51 | 0.8101 |
| | | | .10 | 2.67 | |
| | | | .05 | 3.63 | |
| <F test for W> | | | .01 | 6.23 | |

Rule: at alpha-percent level of significance:
Reject $H_\phi$ if $F_{Computed} > F_{Tabulated}$;
Do not reject $H_\phi$ if $F_{Computed} \leq F_{Tabulated}$.

TABLE XIII

HYPOTHESIS TESTING FOR THE TRUE SIGNIFICANCE OF DIFFERENCES
IN A, P, & W, DUE TO THE ESTIMATION METHOD (M=1 AND M=2)
EXPERIMENTAL DATA: T-TESTS

| Postulated Hypothesis | df N1+N2-2 | Computed t | alpha p<.** | Tabulated t |
|---|---|---|---|---|
| $H_{10}$ | (9.34): 9 | -6.47 | .25 | -0.7027 |
| | | | .10 | -1.3830 |
| | | | .05 | -1.8331 |
| <Lower Tail Test< | | | .01 | -2.8214 |
| $H_{11}$ | (8.28): 8 | 4.53 | .25 | 0.7064 |
| | | | .10 | 1.3967 |
| | | | .05 | 1.8595 |
| >Upper Tail Test> | | | .01 | 2.8965 |
| $H_{12}$ | (8.51): 9 | 5.53 | .25 | 0.7027 |
| | | | .10 | 1.3830 |
| | | | .05 | 1.8331 |
| >Upper Tail Test> | | | .01 | 2.8214 |

Lower Tail Rule: at alpha-percent level of significance:
Reject $H_\emptyset$ if $t_{Computed} \leq t_{Tabulated}$;
Do not reject $H_\emptyset$ if $t_{Computed} > t_{Tabulated}$

Upper Tail Rule: at alpha-percent level of significance:
Reject $H_\emptyset$ if $t_{Computed} \geq t_{Tabulated}$;
Do not reject $H_\emptyset$ if $t_{Computed} < t_{Tabulated}$

## TABLE XIV

### HYPOTHESIS TESTING FOR THE SIGNIFICANCE OF DIFFERENCES IN A, P, & W, DUE TO THE REPAIR DISTRIBUTION [(1) EXPONENTIAL; (2) ERLANG-2; (3) LOGNORMAL] EXPERIMENTAL DATA: ANOVA-TESTS

| Postulated Hypothesis | df N1/N2 | Computed F | alpha p<, ** | Tabulated F | Descriptive eta |
|---|---|---|---|---|---|
| $H_{13}$ | 2/15 | 0.066 | .25 | 1.52 | 0.0936 |
| | | | .10 | 2.70 | |
| | | | .05 | 3.68 | |
| <F test for A> | | | .01 | 6.36 | |
| $H_{14}$ | 2/15 | 0.220 | .25 | 1.52 | 0.1688 |
| | | | .10 | 2.70 | |
| | | | .05 | 3.68 | |
| <F test for P> | | | .01 | 6.36 | |
| $H_{15}$ | 2/15 | 0.054 | .25 | 1.52 | 0.0846 |
| | | | .10 | 2.70 | |
| | | | .05 | 3.68 | |
| <F test for W> | | | .01 | 6.36 | |

Rule: at alpha-percent level of significance:
Reject $H_0$ if $F_{Computed} > F_{Tabulated}$;
Do not reject $H_0$ if $F_{Computed} \leq F_{Tabulated}$.

## TABLE XV

**HYPOTHESIS TESTING FOR THE TRUE SIGNIFICANCE OF DIFFERENCES IN A, P, & W, DUE TO THE REPAIR DISTRIBUTION [(1) EXPONENTIAL; (2) ERLANG-2] EXPERIMENTAL DATA: T-TESTS**

| Postulated Hypothesis | df N1+N2-2 | Computed t | alpha p<,•• | Tabulated t |
|---|---|---|---|---|
| $H_{16}$ | 10 | -0.01 | .25 | -0.6998 |
| | | | .10 | -1.3722 |
| | | | .05 | -1.8125 |
| <Lower Tail Test< | | | .01 | -2.7638 |
| $H_{17}$ | 10 | 0.49 | .25 | 0.6998 |
| | | | .10 | 1.3722 |
| | | | .05 | 1.8125 |
| >Upper Tail Test> | | | .01 | 2.7638 |
| $H_{18}$ | 10 | 0.01 | .25 | 0.6998 |
| | | | .10 | 1.3722 |
| | | | .05 | 1.8125 |
| >Upper Tail Test> | | | .01 | 2.7638 |

Lower Tail Rule: at alpha-percent level of significance:
Reject $H_\emptyset$ if $t_{Computed} \leq t_{Tabulated}$;
Do not reject $H_\emptyset$ if $t_{Computed} > t_{Tabulated}$

Upper Tail Rule: at alpha-percent level of significance:
Reject $H_\emptyset$ if $t_{Computed} \geq t_{Tabulated}$;
Do not reject $H_\emptyset$ if $t_{Computed} < t_{Tabulated}$

## TABLE XVI

**ALTERNATIVE (F, S) COMBINATIONS ALLOWING A FIRM WITH N=80, ERLANG-2 FAILURES AND REPAIRS, AND r=0.60, TO ATTAIN AN $A \geq 0.95$ FOR THREE UNDERLYING COST STRUCTURES**

| Alternative Combination ( F, S) | Underlying Cost Structure (i) $C_F=\$150$/day $C_S=75$/day | (ii) $C_F=\$112.5$/day $C_S=112.5$/day | (iii) $C_F=\$\ 75$/day $C_S=150$/day |
|---|---|---|---|
| | $ | $ | $ |
| 42, 80 | 6,300+6,000=12,300 | 4,725.0+9,000.0=13,725.0 | 3,150+12,000=15,150 |
| ⋮ | ⋮ | ⋮ | ⋮ |
| 42, 56 | 6,300+4,200=10,500 | 4,725.0+6,300.0=11,025.0 | 3,150+ 8,400=11,550 |
| ⋮ | ⋮ | ⋮ | ⋮ |
| 42, 53 | 6,300+3,975=10,275* | 4,725.0+5,962.5=10,687.5 | 3,150+ 7,950=11,100 |
| 43, 52 | 6,450+3,900=10,350 | 4,837.5+5,850.0=10,687.5 | 3,225+ 7,800=11,025 |
| 44, 51 | 6,600+3,825=10,425 | 4,950.0+5,737.5=10,687.5 | 3,300+ 7,650=10,950 |
| 45, 50 | 6,750+3,750=10,500 | 5,062.5+5,625.0=10,687.5 | 3,375+ 7,500=10,875 |
| 46, 48 | 6,900+3,600=10,500 | 5,175.0+5,400.0=10,575.0* | 3,450+ 7,200=10,650 |
| 48, 48 | 7,200+3,600=10,800 | 5,400.0+5,400.0=10,800.0 | 3,600+ 7,200=10,800 |
| 49, 47 | 7,350+3,525=10,875 | 5,512.5+5,287.5=10,800.0 | 3,675+ 7,050=10,725 |
| 50, 46 | 7,500+3,450=10,950 | 5,625.0+5,175.0=10,800.0 | 3,750+ 6,900=10,650 |
| 51, 45 | 7,650+3,375=11,025 | 5,737.5+5,062.5=10,800.0 | 3,825+ 6,750=10,575* |
| ⋮ | ⋮ | ⋮ | ⋮ |
| 55, 45 | 8,250+3,375=11,625 | 6,187.5+5,062.5=11,250.0 | 4,125+ 6,750=10,875 |
| ⋮ | ⋮ | ⋮ | ⋮ |
| 80, 45 | 12,000+3,375=15,375 | 9,000.0+5,062.5=14,062.5 | 6,000+ 6,750=12,750 |

*: minimum combined cost of standby float units (F), and service channels (S), yielding the optimum (F*, S*) combination.

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