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High speed dynamics of a press feed mechanism: Mathematical simulation and experimental verification

Chassapis, Constantin, Ph.D.

City University of New York, 1988

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HIGH SPEED DYNAMICS OF A PRESS FEED MECHANISM:
MATHEMATICAL SIMULATION AND EXPERIMENTAL VERIFICATION

by

CONSTANTIN CHASSAPIS

A dissertation submitted to the
Graduate Faculty in Engineering
in partial fulfillment of the
requirements for the degree of
Doctor of Philosophy, The City
University of New York.

1988

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This manuscript has been read and accepted for the Graduate Faculty in Engineering in satisfaction of the dissertation requirement for the degree of Doctor of Philosophy.

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Abstract

HIGH SPEED DYNAMICS OF A PRESS FEED MECHANISM:
MATHEMATICAL SIMULATION AND EXPERIMENTAL VERIFICATION

by

Constantin Chassapis

Adviser: Professor Gerard G. Lowen

A mathematical model for a lever-type roll feed mechanism, which furnishes favorable comparisons with simultaneously obtained experimental results, is presented. This device, which is used in all manner of power press work, consists of an RSSR spatial linkage with a bent coupler, an indexing-type sprag clutch, a set of feed and pressure rollers which move the metal strip forward, and a disc brake. Its intermittent motion is characterized by four distinct motion regimes. With the exception of the coupler link which is assumed to be elastic, and the sprag clutch which is modeled by a non-linear torsional spring, all members are considered rigid.

The derivation of the motion equations is based on Hamilton's principle, and the method of Kantorovich is used to reduce the partial differential equations to ordinary ones, which are subsequently solved numerically.

In the experimental set-up, the motion of the mechanism is monitored by three angular encoders. These are attached to the drive shaft, the rocker-link shaft, and the feed-roller shaft, respectively, and their output is stored in a specially designed data acquisition system. Strain gages are attached to the coupler link to determine both in-plane and out-of-plane bending strains.

Experiment and theory, for six different loading conditions, are correlated by way of the following criteria:

In-plane coupler strain traces; the clutch windup angle, which represents the difference between feed roller and rocker rotations, and the total feed length.

January 19, 1988

To my wife Marina,
my parents, and
my aunt Olga.

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I. INTRODUCTION

A. BACKGROUND OF PROJECT

The reliable increase of machine speeds represents an important way of enhancing the productivity of industry. One of the paths towards this desirable goal lies in the greater understanding of the elastic-dynamic behavior of these machines in general and of their linkage components in particular.

A literature survey by G.G. Lowen and the present author [24]^{*}, as well as earlier reviews by several researchers [22,25,10,42]^{**}, attest to the fact that a considerable amount of excellent work has been done in the field of linkage elasticity in recent years by a good number of investigators.

With the exception of a few cases, where external forces were assumed to act on certain links (reference 26 of [24] and references 51 and 52 of [25]) or, where ground and clearance effects were considered (references 13,14,27 of [25]), these efforts assumed the

* Reference [24] discusses the linkage elasticity literature published up to 1984. Section I-E gives an update of related publications that appeared up to the summer of 1987.

** Numbers in brackets designate references.

linkages to stand alone and to run without loads other than the masses of their links. This limitation was necessary in order to make a beginning in a very difficult aspect of machine science.

Some years ago, it became apparent, that the time had come to extend this research to specific industrial machines, in which the elastic linkage represents only one of their components, along with the mass of the product involved, and where the machines have to perform specific functions with a prescribed degree of accuracy. Such a research effort must consist of a proper combination of mathematical modeling and experimentation, with the aim of obtaining the best possible computer simulation of such a machine. Once this tool is available, a parametric study can then point the way to a meaningful optimization approach.

With the above in mind, it was decided to make a lever-type roll feed mechanism, i.e. an automatic press feeding device, the topic of the present dissertation.

Such a mechanism is employed in the stamping of every conceivable sheet metal product from electrical connectors for computers, to building and furniture trim and automotive components of all types, to mention just a few. According to the American Metal Stamping Association, in addition to the large volume manufacturers who do their stamping in-house, there are in the U.S. about 3,000 contract stamping companies which operate about 90,000 mechanical presses with

an annual gross income of approximately \$25 billion. The total number of mechanical presses in the U.S. is between 480,000 and 500,000.

It is of interest to note, that the increase of press speeds has been found much easier to attain than that of economically built and easily adjustable feeding devices.

B. FUNCTION AND DESIGN OF LEVER-TYPE ROLL FEED MECHANISM

As all other press feed mechanisms, the lever-type roll feed must feed strip or coil stock a predetermined distance into a die, which is mounted on a power press. Subsequently, it must cause the stock to stand still during the working portion of the press stroke. The accuracy of all feeds is judged by their ability to intermittently advance the stock in each cycle through the same distance within a tolerance of a few thousandths of an inch. This tolerance depends on the length of feed (feed pitch), the press speed (strokes per minute), and on the mass of the material being fed, in addition to the dynamic properties of the mechanism.

As Figure 1.1 shows, the lever-type roll feed mechanism consists of an RSSR spatial linkage, a sprag clutch, and a set of feed and pressure rollers whose function is to advance the strip or coil stock into a die (not shown). In addition, there is a disc-type friction brake, which retards the motion of the feed-roller shaft as well as prevents its overshooting at the end of the forward stroke. Further,

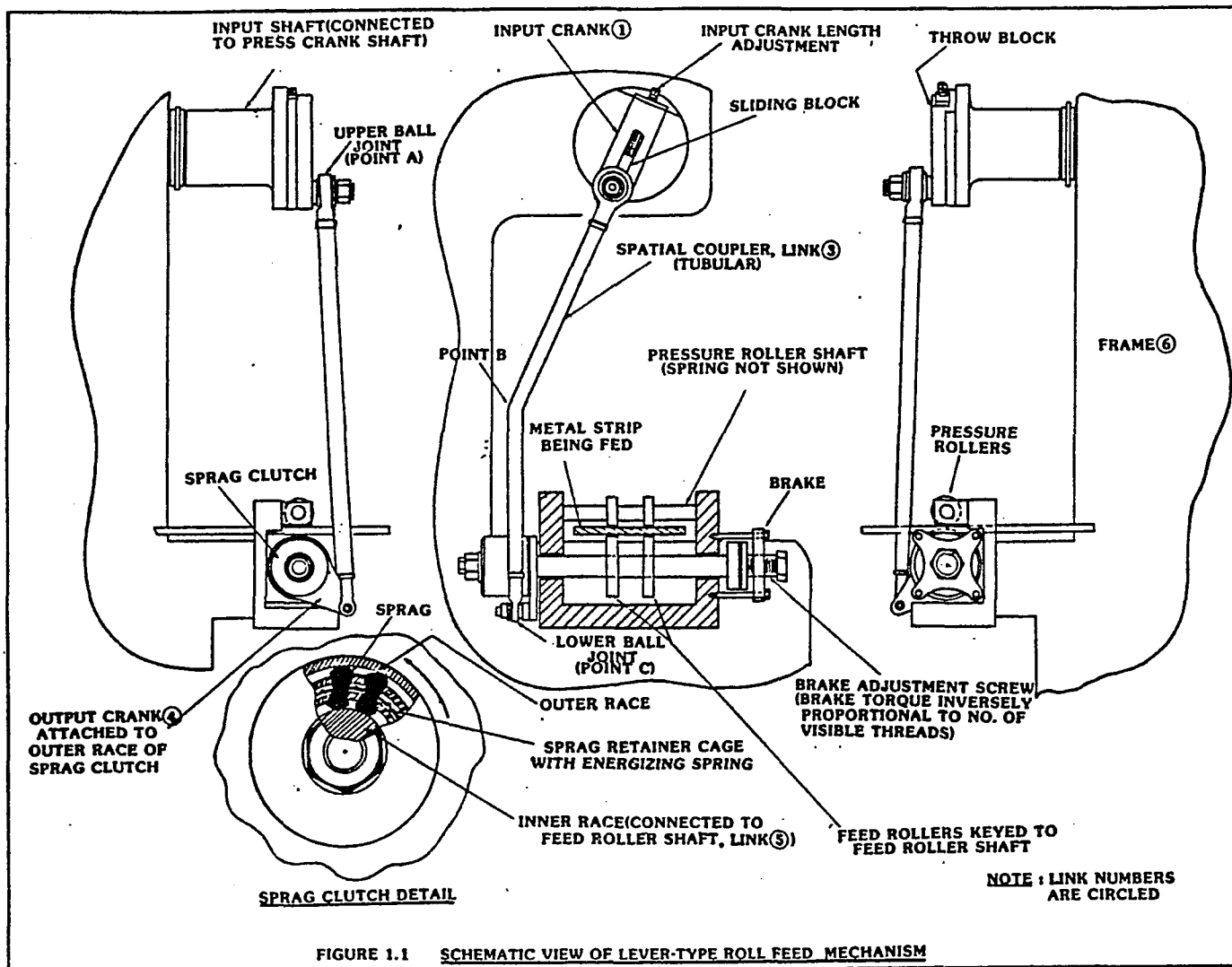


FIGURE 1.1 SCHEMATIC VIEW OF LEVER-TYPE ROLL FEED MECHANISM

it keeps the feed-roller shaft motionless during the return stroke. Its friction torque is adjustable by way of the indicated screw.

The input shaft to the spatial linkage is directly connected to the crank shaft of the power press. It drives the throw block whose angular relationship with respect to the press crank shaft may be varied to adjust the feed cycle with respect to the press cycle. This throw block, which may be counterweighted, together with the sliding block forms the input crank of the linkage. By adjusting the distance of the center of the sliding block from the input axis, i.e. the length of the input crank, the output motion of the mechanism and with that the feed length of the stock is set. The spatial coupler connects the sliding block and the output crank by way of the upper and lower ball joints. The construction details of these commercial ball joints limit the possible rotation of the coupler about the line connecting their centers (this rotation represents the extra degree of freedom of RSSR mechanisms). Thus, when the feed is running one may observe the coupler to rotate through a small angle about the aforementioned line, as well as rebound off the limits of motion imposed by the ball joints. These impacts have been minimized in the test mechanism by appropriate rubber washers.

The coupler has a bend, as shown, in order to be able to clear the feed-roller shaft support assembly as well as the OSHA mandated safety cover (not shown). This link is kept light to prevent the need for balancing.

The rocker type output crank of the spatial linkage is attached to the input, i.e. outer race, of the indexing type sprag clutch (see sprag clutch detail in Figure 1.1). This clutch allows only the forward motion of the rocker to be transmitted to the feed-roller shaft. Its one way action occurs as the outer race causes the variable diameter sprags to roll into a wedging position between it and the inner (output) race*. Once the output crank begins its return motion, the sprags are unlocked and no motion is conveyed to the feed-roller shaft (this makes the metal strip stand still in the press). Depending on the resistance to motion of the feed-roller shaft, due to brake resistance and strip acceleration, smaller or larger elastic deformations of the sprags and the race surfaces will take place as the sprags roll into torque and motion transmitting positions. The resulting difference in rotation between the races is called clutch windup. Because of this phenomenon, the clutch input rotation angle must always be made somewhat larger than the required feed-roller shaft rotation angle.

To assure minimum slip between the feed-rollers and the stock being fed, a set of spring activated pressure rollers is used. Figure 1.1 does not show the springs and the associated pressure adjustment mechanism.

* Rolling is assured by the presence of the retainer cage and the energizing spring, which hold the sprags and push them against the races, respectively.

C. LITERATURE REVIEW ON LEVER-TYPE ROLL FEED MECHANISM

While an investigation of publications on press feed mechanisms, which appeared over the past 25 years (see references [12, 13, 16, 17, 29, 46, 49]), revealed a complete absence of mathematical treatments of lever-type roll feeds, it showed a good deal of descriptive material on this device and its relationship to other types of feed mechanisms.

This information has been found to be very useful since, in order to develop a meaningful mathematical model of such a mechanism, one must fully understand its requirements and how its various components contribute to its successful performance. To this end, the following first gives a brief and necessarily limited overview of the available types of press feed mechanisms and their common performance requirements. It further discusses where each grouping finds its particular application in the overall scheme of press work. Subsequently, a more detailed discussion of the lever-type feed is given.

a. Overview

All press feed mechanisms, whether they are slide feeds or roll feeds, must repetitively feed coil or strip stock a predetermined distance into a die, which is mounted on a power press. Subsequently, they must cause the stock to stand still during the working part of the press stroke. Their accuracy is judged by their ability to

advance the stock in each cycle through the same distance within a few thousandths of an inch, depending on the material being fed, the feed pitch, and the press speed.

Slide feeds advance the stock with reciprocating grippers or clamps which are released during their return motion. They may be activated by a flat cam which is attached to the press slide, an air or hydraulic piston operating between preset stops, or a linkage which is driven by the press crankshaft. They represent the least expensive feeds and are best suited for short-job shop runs on low speed (approximately 200 strokes per minute) and low tonnage presses. Because of their limited pulling power and the vibration difficulties associated with unbalanced reciprocating components, they are generally used in feeding thin, narrow stock requiring short feed lengths. In addition, they can only be applied where possible gripper marks are of no consequence.

Roll feeds advance the stock by pressure between intermittently driven opposed rolls. They are available in several variants, such as lever types, rack and pinion types, cam types, and independently driven roll types. As a group, they are capable of feeding stock in thicknesses of up to 0.25 in., as wide as 60 inches and with feed lengths from less than one eighth of an inch to more than 5 feet. While roll feeds, where comparable, are more expensive than slide feeds, they can be made to feed more accurately and faster.

The relatively simplest of these roll feed mechanisms is the lever type. It consists of a spatial four-bar linkage that is driven off the press crank, an indexing type sprag clutch, a set of feed rollers, and a disc brake which prevents the overshooting of the feed rollers and the stock (see figure 1.1). Its general accuracy is maintained, depending on feed pitch and strip weight, up to press speeds of 400 strokes per minute. The feed pitch of this mechanism is controlled by its adjustable-length input crank. It is limited by the relatively small maximum swing angle of the output link of the four-bar mechanism and the diameter of the feed rollers. This limitation is overcome in the rack and pinion type roll feed mechanism both by a small diameter pinion and by the use of additional step-up change gears. The oscillating input to the sprag clutch is supplied by a modified turning block linkage. Its connecting rod has rack teeth and drives the pinion which is integral with the clutch input race. The 90 degree angle between the clutch output and the feed roller axis is bridged by bevel type gears that are followed by the change gears. As for the lever-type feed, the length of the press driven input crank of the rack and pinion type feed is adjustable. The feed accuracy and speed of this mechanism are comparable to those of the lever type. It finds its application in heavy stamping and drawing work.

Cam-type roll feed mechanisms achieve the advance and dwell of the feed rollers by way of a rotary indexing cam, which is driven from the press crank shaft by way of a timing belt or a Cardan shaft. Feed length flexibility must be provided by change gears, since without them the feed length is fixed for a given cam mechanism and roller

diameter. While this type of feed is considerably more expensive than comparable lever or rack and pinion types, it is capable of providing more accurate feed lengths at higher press speeds (up to 1000 strokes per minute). The later is made possible by the special acceleration and deceleration characteristics, with limited third derivatives, which are designed into the cams and minimize shock loading and resulting vibrations at the beginning and the end of the feed cycle (see the work of J.A. Hrones [19,20] and his successors in cam design for details). These feeds are generally used in conjunction with high-volume items of small thickness and limited-width sheet metal, such as can tops, electrical laminations and electronic connectors.

Even if generally twice as expensive as cam-type roll feeds, independent roll feeds furnish the only practical method of feeding long lengths of wide coils of material. These feeds have separate self-contained drives to power the feed rollers either directly or through gearing. Correlation with the press movement is usually accomplished with rotary-type limit switches. Older models employ independently driven rack and pinion feeds, while the more recent designs use digitally controlled servodrives which regulate position, velocity and acceleration of the stock. Press speeds may be as high as 300 strokes per minute.

b. Lever-Type Roll Feed Detail

The lever-type roll feed consists either of an RSSR or an RRRSR spatial four-bar linkage where the output crank forms the input to the

indexing-type sprag clutch. The output side (usually the inner race) of this one-way clutch is integral with the feed roller shaft and its motion is retarted by a disc brake.

The stock is forced against the feed rollers by spring loaded pressure rollers. In case the final location of the stock with respect to the die depends on pilot pins, the pressure roller is released just before these pins enter their locating holes in the stock. This is generally accomplished by a plunger, which is attached to the press slide. The spring pressure is reapplied as the press slide moves upward and the pilot pins leave the stock. In case the press operation requires no piloting, the pressure roller is in constant contact with the material fed.

If the stock is consumed completely in a given run, or if the stock is fairly rigid, it is only necessary to have one feed mechanism at the input side of the press pushing the stock into the die. In case the skeleton is very fragile, the need for a second mechanism, which pulls the stock out of the die rises. Its motion is correlated with the input feed roller by way of appropriate gearing and a torque shaft.

Lever-type roll feeds find their place in speed and feed accuracy between slide feeds and cam-driven roll feeds. They allow higher speeds than slide feeds and are less noisy. Their feed-pitch is easily adjustable and their relative simplicity of construction makes it easy to build them accurately and maintain them economically. The direct connection to the press shaft assures that they have sufficient

feeding power. They are less expensive than the cam-type roll feeds or the very high priced digitally controlled independent roll feeds.

Lever-type roll feeds are best suited for coil or strip stock of up to 10 inches width and of thickness of up to 0.25 in. As stated earlier, their maximum feed length depends on the diameter of the feed rollers, but usually does not exceed 4 inches. Their feed accuracy is best for short feed lengths and is generally better than 0.005 in., depending on the weight and cross-section of the material being fed, and with press speeds of up to 400 strokes per minute.

The attainment of this accuracy is the result of the successful interaction of such factors as the elastic behavior of both the spatial coupler and the sprag clutch, the combined inertia of the feed roller shaft and the stock being fed, the roller pressure and the feed roller-stock interface friction, as well as the frictional resistance in the bearings, and last but not least, the retarding torque applied by the brake.

Rather than discuss feed accuracy in terms of the above overall system, the existing literature concentrates on the requirements placed on the individual components of the mechanism. With the exception of the sprag clutch publications, which are reviewed in greater detail below, this information is also purely descriptive.

It suggests that, to control overshoot, i.e. the unwanted forward motion of the stock once the forward motion of the clutch has ceased,

the roller moment of inertia should be made as small as possible and that, the brake must act dependably. It points out that the torque of such a friction brake must be the same from cycle to cycle. Since heating causes variations in the resisting moment, brakes are either air or water cooled, and only reach equilibrium after some time has elapsed in a given run.

To obtain a sufficiently high and relatively constant coefficient of friction between the feed rollers and the stock, various surface treatments of the rollers have been devised. They include knurling, grooving, sand blasting, hard chroming and EDM etching. For surface protection of printed, painted or otherwise sensitive stock, rubber and polyurethane-coated rollers are available.

Force balancing of the linkage is only considered if the press speed warrants it, and is generally confined to counterweighting the throwblock.

D. SPRAG CLUTCH LITERATURE

The book by K. Stoezle and S. Hart [34] deals with overrunning clutches and provides a good deal of information on the design and the operation of sprag clutches. It defines the inner and outer strut angles as the angles between the line connecting the inner and outer contact points of the sprag with the races and the normals to these points, respectively. It is indicated that, in order to obtain

reliable locking, the tangent function of these angles must be smaller than the coefficient of friction at the contact points. Since the inner strut angle is somewhat larger than the outer one, one needs only to make sure that the locking condition is satisfied for this angle. Because the clutch components are made of steel, with a coefficient of friction of 0.1, the strut angle should not exceed 6° . It is further shown how to compute these angles if the radii of the circular cam surfaces of the sprag as well as the eccentricity, i.e. the distance between the centers of these circle portions, are known, together with the inner and outer race radii. The book states the need for an energizing spring. Such a garter spring keeps each of the individual sprags pushed against both races and in this way facilitates rapid locking engagement. Using the basic geometric information and contact stress theory, a step by step design procedure for a given torque requirement sprag clutch is developed.

The authors show that if the clutch is interpreted as an equivalent epicyclic gear train, it becomes possible to describe the relative sprag rotation of the planet gear with respect to an imaginary epicyclic arm. To this end, the rotation angles of the outer and inner races, which take the roles of ring and sun gears, respectively, must be known. The discussion on clutch dynamics is confined to automotive and helicopter transmissions, and the clutch elasticity is modeled by way of a linear torsion spring. The associated clutch requirements differ from those in feed mechanisms where indexing takes place.

The first paper of R.L. Daniels [6] discusses the basic geometry of the sprags and defines locking and overrunning conditions. It is pointed out that, in order to get safe locking the strut angle should be as small as possible. In addition, the author shows that for the circular arc-type sprag, the strut angle increases as locking proceeds. Finally, he reports that experimentation has shown the need for a sprag cage to assure simultaneous engagement (full phasing) of all sprags. In a later paper [7], the same author describes various applications of sprag clutches in aircraft and pump power trains which are driven by gas turbines. He is most concerned with the wear of one of the sprag surfaces during overrunning, which causes the strut angle to be increased during subsequent locking. This wear may be overcome by locating the center of mass of the sprag such that a centrifugal force pivots the sprag away from the surface on which it otherwise slides. This paper also shows a design which assures that the individual sprags abut against each other during severe overloads. This prevents their rolling over, which causes loss of torque transmission and the eventual destruction of the clutch.

E.A. Ferris [11] gives much useful information on sprag geometry, material selection and clutch lubrication, as well as the design of energizing springs. He also stresses the need for a sprag retainer ring to assure full phasing. His description of a newly-developed strut angle gage is especially interesting. This device allows the determination of the strut angle using the ratio of the horizontal to the vertical forces necessary to prevent a sprag from falling over as

it is held between two horizontal plane surfaces. While this fixture involves "flat races" as opposed to the circular ones in an actual clutch, it is a valuable comparison tool for inspection purposes. This fixture shows the general increase in strut angle with increasing sprag height (due to rotation in the direction of locking), as well as the abrupt increase of this angle when wear is excessive in the overrunning clutch phase.

V.A. Novikov [27] showed how to determine those sprag radii as well as the associated eccentricity which will keep the increase in the strut angle at a minimum as the sprags rotate from the overrunning to the locking configuration. His formulation of the strut angle includes the contact deformations between the sprag and the races in addition to the various geometric factors. An example indicates that the increase of the strut angle may be kept as low as 4%. In a later paper [81], together with A.K. Radzhpov, the same author extended his earlier optimization to sprags with logarithmic spiral or involute-type cam profiles. While these designs, when optimized, make it theoretically possible to keep the increases of the strut angle small, there is some question concerning the attainable manufacturing accuracy. To this end, the authors determine the magnitudes of the maximum allowable profile tolerances which still would keep the strut angle increase to approximately 4% as full engagement takes place.

In the first of the only set of papers which deal with the dynamics of the indexing sprag clutch, F.C. Williams, D. Tipping and T.A. Henry [47] present a combined analytical and experimental

investigation. The individual sprag is assumed to pivot with respect to a fixed outer race, while its other end is accelerated due to a friction torque exerted by an oscillating inner race. This inner-race motion is due to an externally applied oscillating torque which in their experiment is provided by a torsional pendulum. Under normal circumstances, this external torque locks and unlocks the clutch until the oscillations die out. When the torque was made very large, by increasing the initial deflection of the torsion rod, the authors found that the clutch did not lock properly and that the oscillations of the input became erratic. This instability, which damages the clutch, is explained with the help of a mathematical model that assumes the sprag elasticity to be linear. It is shown that when the acceleration of points on the surface of the inner race, just after motion reversal, is much greater than that which the friction-driven sprag contact point can reach, skidding rather than locking will occur. As might be expected, since locking is a rolling phenomenon, it can only take place when the sprag tip velocity equals the surface velocity of the driving race. In a second paper [48], the same authors demonstrate that skidding may be avoided by a sprag retainer cage which rotates with the velocity of the driving race.

Finally, G.F. Newman [26] is the only author who reported on experimentation on a press feed mechanism which contained a sprag clutch. In this machine the outer race of the clutch is integral with a rack driven pinion. The feed oscillations of the rack are obtained from a planar four-bar linkage whose slotted rocker drives a slider that is pivoted on the rack. While giving no specific details, the author relates clutch windup of up to 2.5 degrees. He further states

that, whatever problems he encountered with the feed accuracy, when running in speeds exceeding 400 RPM, they were due to the linkage and not to the clutch.

E. UPDATE OF THE LITERATURE ON THE ELASTIC BEHAVIOR OF LINKAGES

A. Balasubramonian and E. Raghavacharyulu [1], used Hamilton's principle to derive the equations of motion for the elastic coupler of a planar four-bar linkage. The resulting Hill-type equations were solved numerically using the Runge-Kutta-Merson numerical technique.

W. L. Cleghorn, R. G. Fenton and B. Tabarrok [5], obtained the steady-state solution for the finite element equations which govern the elastic deflections of a four-bar linkage, whose moving links are all flexible, by representing each of the time dependent periodic coefficients by a Fourier series. The results compared favorably to the experimental data of A. Lawrence [ref. 10 in 5].

S. Dubowsky, J.F. Deck and H. Costello [8], extended an existing method of modeling spatial and coplanar mechanisms, developed by W. Sunada and the first author, to include the effects of clearances. The procedure relaxes the kinematic constraints imposed at the connections by ideal joints and replaces them with force or impact constraints. Spherical as well as revolute joints with clearances are considered.

D.R. Flugrad, Jr. [14], considered the motion of the lumped-mass slider of a slider crank mechanism, whose rigid input crank is driven by way of a cam. The massless connecting rod is assumed to have lateral elasticity. The author developed a procedure for synthesizing the cam profile such that a desired slider motion is produced. The equation of motion of this output link was found to be a Hill equation with periodic coefficients and forcing.

A. Garcia-Reynoso and W.P. Seering [15], developed a mathematical model to describe the linear vibrations of a constant velocity four-bar linkage which contains a flexible rocker link as well as flexible input and output shafts, carrying discs. A dimensionless analysis is presented which introduces the concept of families of mechanisms with similar dynamic behavior.

G. N. Sandor and X. Zhuang [30], linearized the equations of motion which resulted from a lumped-mass model of an elastic four-bar linkage that was presented earlier by J. P. Sadler and the first author [ref. 13 of 24]. Comparison of results to those obtained by the earlier non-linear analysis showed that the error introduced by the linearization is small. In addition, the effects of the rotary inertia of the elements and the end masses of the moving links were investigated and were found to be considerable.

Ahmed Shabana [31], used Lagrange's equation to derive the equations of motion governing the thermoelastic response of linkages. A slider-crank mechanism containing an elastic connecting rod was used

to demonstrate how the response of this link increases with increasing temperature. In [32] the same author, along with B. Thomas, used the finite element method to obtain the dynamic response of a flexible machine tool mechanism (crank shaper), which is under the influence of a constant cutting force as well as a chattering force which depends both on the vibratory displacement of the tip of the toolbit as well as its first derivative. All links of the mechanism are considered rigid except the connecting rod and the tool. Results are shown for the forward stroke. The authors conclude that the structural damping of the tool material has a significant effect on the dynamic response of the system.

C.K. Sung, B.S. Thompson [35], used the variational finite element formulation, presented earlier by the second author and M. V. Gandhi (ref. 6 of [24]), to obtain the dynamic response of a four bar linkage whose two flexible links (coupler, rocker) are made either of commercial metals like steel and aluminum, or composite laminates. It is demonstrated that the vibration amplitude of the links is significantly reduced, but not eliminated, when they are fabricated from composites. Based on these analytical results, they suggest that composite materials should be used whenever possible. This claim is supported experimentally in [37], where the same authors, along with P. Crowley and J. Cuccio, present and compare the various strain histories. Experimental responses of the connecting rod of a slider crank are also compared. In [37], the first two authors, used the same analytic methods to demonstrate the advantages of using fiber-reinforced composites to fabricate the moving members of manipulators.

The identical authors, together with J.J. McGrath [36], used a variational formulation to develop the finite element equations of motion governing the response of planar flexible mechanisms subjected to both mechanical and thermal loading.

In a similar manner, B.S. Thompson and C.K. Sung [40], derived the governing differential equations for the nonlinear vibration behavior of linkages with straight slender beam-shaped members. Both axial as well as transverse deflections were considered. Theoretical results are compared with those obtained from an experimental four-bar linkage and a slider crank mechanism whose elastic members are made of steel. Additional favorable experimental comparisons are given by the same authors, along with T. M. Xing and C. H. Wang in [38].

I.G. Tadjbakhsh and C.J. Younis [39], focused on the stability of the flexible connecting rod of a slider-crank mechanism. Application of Galerkin's method (product solution) led to a system of ordinary differential equations with periodic coefficients (Hill's equations). Using Floquet theory, the stability boundaries were obtained.

II. OUTLINE OF INVESTIGATION AND SUMMARY OF RESULTS

The objective of this dissertation was to develop a mathematical model for a lever-type roll-feed mechanism^{*} as used in power presses, which agrees with experimental results for a full range of loading conditions. This task has been accomplished and comparison criteria between computational and experimental results have been developed. These criteria will eventually serve as the basis for a meaningful design optimization.

The mechanism is employed in the stamping of every conceivable sheet metal product. It must feed strip or coil stock a predetermined distance into a die and subsequently it must cause the stock to stand still during the working portion of the press. A typical example of such a device is shown in Figure 1.1. Its basic components are:

- i. An RSSR spatial linkage with a bent coupler.
- ii. A sprag-clutch which engages the feed roller shaft in one direction and overruns it in the opposite one.
- iii. A set of feed rollers which move the stock forward, and a set of pressure rollers which hold the stock against the feed rollers.

* The commercial L-84 feed mechanism used both for analysis and experimentation, was donated to the City College of New York by E. W. Bliss Company of Hastings, Michigan.

- iv. A disk brake which at all times applies a retarding torque on the feed-roller shaft to avoid the overshooting of the stock at the end of its forward motion.

The following outlines the analytical as well as the experimental portions of this effort and gives a summary of the conclusions which may be drawn.

A. THEORY

The theoretical portion of this work begins with the formulation of the mathematical model of this constant input speed press-feed system. While the input and output (rocker) links of the spatial linkage were assumed to be rigid, its coupler link was taken to be elastic*. The latter consists of two straight constant cross-section tubular sections that are rigidly connected to form a corner at point B. A reference plane was established and with that in-plane and out-of-plane lateral deflections were recognized. These deflections, because of the particular safety-mandated 'bent' design of the coupler link, have an effect on the length of the pivot to pivot line AC. The sprag clutch, whose torque-deflection characteristics were obtained experimentally, was modeled by a massless non-linear torsional spring.

* The allowable elastic deformations are those associated with the Euler-Bernoulli beam theory.

The rocker and the outer race are on the clutch input side, while the inner race and the feed-roller shaft are, on the output side.

With the assumption of zero slip between the metal strip and the feed rollers as well as the pressure rollers, respectively, the strip and pressure-roller moment of inertia was referred to the feed-roller shaft. Because of the zero slip, the feed-roller rotation could be related directly to the feed length by way of the roller radius. The retarding brake torque was assumed to be constant, and velocity dependent damping was assumed for both the coupler as well as the sprag clutch.

The intermittent motion of the feed mechanism was characterized by way of four distinct motion regimes. These are represented by their respective motion equations and the transitions between them are controlled by appropriate kinematic as well as dynamic conditions. Since the governing equations for the second motion regime, which describes the forward movement of the stock, are quite general, the remaining regime-equations may be obtained by certain reductions or modifications of terms. The generalized coordinates, of all regimes, are the in-plane coupler deflections $v_1(x_1, t)$ and $v_2(x_2, t)$, the out-of-plane coupler deflections $w_1(x_1, t)$ and $w_2(x_2, t)$ as well as the feed-roller angle $\phi_5(t)$.

The partial differential equations of motion for regime no. 2 are obtained by way of a systems' approach. The associated derivations

are outlined in Section IV and are supported by appendix A, with much of the background material found in appendices F-I. It starts with the formulation of Hamilton's Integral, which is subject to the elastic mechanism constraint equation. This makes it unnecessary to treat each member, as well as the forces acting on it, separately. The constraint equation components which describe the various relationships between the elastic mechanism angles, take the effect of stretching of the coupler pivot-to-pivot line AC into account. In addition, the influence of the foreshortening of the two sections due to bending is also considered. The variations of the elastic angles, which were obtained from the variations of the constraint equation components, are incorporated into Hamilton's integral by direct substitution, rather than by the method of Lagrange multipliers.

Subsequently, Hamilton's integral is linearized leaving only the non-linear term associated with the clutch spring. All D'Alembert loads are approximated by their rigid-body counterparts and only the expression associated with the clutch torque remains in terms of the elastic rocker angle. The elastic angle is later described by a Taylor series expansion about its rigid-body counterpart, with the help of the allowable deflections. This work is shown in appendix C.

The machinery of the calculus of variations then furnishes five coupled partial integro-differential equations, one each for the five generalized coordinates, along with the associated dynamic boundary conditions.

Finally, the above partial differential equations are reduced to ordinary ones by way of the method of Kantorovich. Each of the coupler deflection variables is now represented by an infinite series of products of shape and time functions. The shape functions, which must form a complete set and must satisfy the geometric boundary conditions of the main problem, were obtained for the in-plane and out-of-plane free vibrations of the coupler. The time portions of the in-plane deflections must be identical in both coupler sections. The same must hold true for the ones associated with the out-of-plane deflections. The final number of the generalized coordinates is now reduced to three, i.e., one each for the two types of lateral deflections and one for the feed roller angle. Substitution of the product solution form into the final form of Hamilton's integral leads to two infinite sets of Hill-type equations for each of the coupler time variables and a single expression for the feed-roller angle in this description of regime no. 2. All equations are coupled to each other. This work is found in Section V, with appendices B and J providing most of the background work.

Finally, appropriate reductions of terms furnish the governing expressions for regimes 1, 3, and 4 (see Section VI).

B. APPLICATION AND EXPERIMENT

The theory developed in sections III through VI was applied to a roll-feed mechanism of specific geometry. The computational portion of the application part deals with the following:

Since experimentation indicated that even for the heaviest load runs the out-of-plane strains were insignificant when compared to the in-plane ones, the out-of-plane deflections were eliminated from the differential equations. Further, since computations showed that a first mode description of the remaining in-plane deflections was sufficient to predict all mechanism behavior, all higher modes were disregarded. Subsequent to the enumeration of the remaining governing equations, the various control and transition conditions for the four-regime cycle are presented.

The reduced differential equations, together with the various control conditions are solved numerically, by way of a fourth-order double-precision Runge-Kutta routine, which utilizes four different pairs of external functions to accomodate each of the four regimes.

It was found that the correlation of the theory with the experiment was best accomplished by way of the following criteria:

- i. Comparison of the in-plane coupler strain traces for a complete mechanism cycle.
- ii. Comparison of the traces of the windup angle ϕ_{54} of the clutch during the feed portion of the mechanism cycle. This angle is the difference between the feed-roller angle ϕ_5 and the rocker angle ϕ_4 .
- iii. Comparison of the total feed-roller rotation angle $\phi_{5\text{tot}}$, i.e. of the feed length.

The particular presentation of the results, associated with the first two criteria above, also furnished some basic insights concerning the behavior of the machine:

- i. The windup-angle trace which depicts the behavior of the clutch system reveals that the system vibrates near its linearized natural frequency. It also becomes clear that for proper feed operation this angle must remain (because of its definition) sufficiently negative throughout the feeding cycle.
- ii. The in-plane strain, during the feed portion of the cycle, reflects the action of the clutch on the coupler link. There, the strain frequency is identical to that of the windup angle. During the return portion of the cycle the coupler link is shown to vibrate at a frequency which is somewhat less than its fundamental one.

Further, the computations showed that the elastic rocker angle differs significantly from its rigid body counterpart. This justifies its use in the clutch torque expression. Finally, it is shown that the Taylor series approximation produces results for the elastic rocker angle ϕ_{4e} which are essentially identical to those obtained from the elastic mechanism constraint equation.

The experiment furnished in-plane strain as well as windup-angle traces for a full range of loading conditions between 200 and 400 rpm. Both the windup and the total feed-roller angles were obtained by angular encoders, which are capable of measuring rotation intervals of 0.036 degrees. The associated data acquisition system made it possible to relate all output angles to the mechanism input angle.

Upon comparison of the computed to the experimental results the following conclusions can be drawn (see details in Section VII-H):

- a. The frequencies of the computed windup-angle traces agree well with the experimental ones. The associated experimental amplitudes are somewhat smaller than the computed ones.
- b. The computed in-plane coupler strain traces agree very well with the experimentally obtained ones, both in terms of frequency and amplitude.
- c. The computed total feed lengths are well predicted by the mathematical model. The variations in the experimental results have been shown to be attributable to the commonly encountered inconsistencies in the value of the coefficient of friction in real systems. (In this case it relates to the friction in the brake).
- d. The total rocker angle $\phi_{4\text{tot}}$ is also predicted well by the model. This quantity proved to be dependent both on the load and the input speed. Results have shown that its total rotation decreases with increasing brake torque and strip inertia and increases with input speed.

The above amply proves the validity of the four-regime mathematical model of the roll-feed mechanism.

III. FORMULATION OF THE MATHEMATICAL MODEL

The following first describes the assumptions of the mathematical model which most importantly concern themselves with the spatial coupler and the sprag clutch.

Subsequently, the intermittent motion of the feed mechanism will be characterized by way of four distinct motion regimes. These regimes will eventually be represented by their respective motion equations and the transitions between them will be controlled by the appropriate kinematic as well as dynamic conditions.

A. MODEL ASSUMPTIONS

a. Replacement Mechanism

In order to avoid the extra variable which is needed to express the free, though limited, rotation of the coupler about the axis connecting its two ball joints as well as to obtain a fully defined reference plane for the elastic deflection of the coupler, it is assumed that the RSSR mechanism is replaced by an equivalent RRRSR one [9, 33], which has the same basic dimensions. This is accomplished by substituting the cross link and the coupler clevis shown in figure 3.1, for the upper ball joint. The indicated modification does not change the motion of the pivot to pivot line AC, of the coupler.

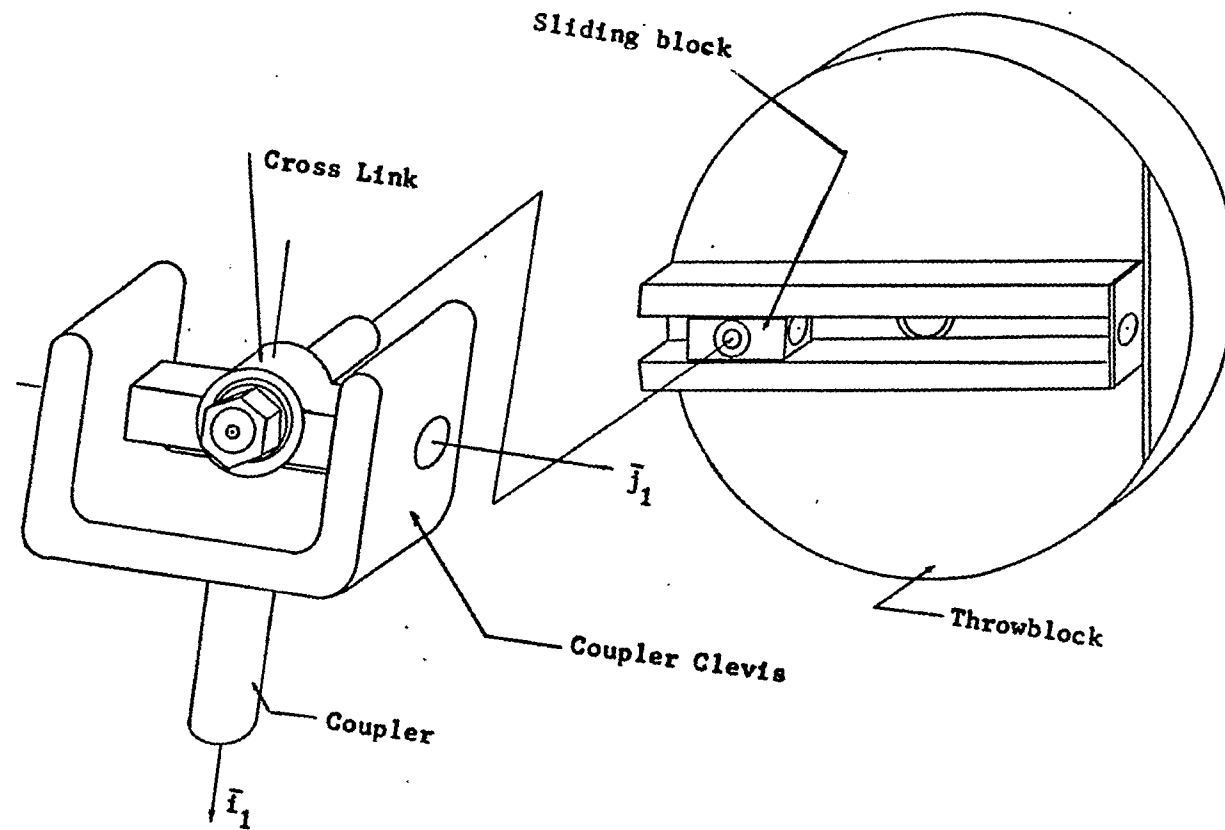


Figure 3.1 Replacement of Top Ball Joint by Cross Link

The above replacement is only applicable to the determination of the kinematic values of the mechanism. It does not extend to the definition of the boundary conditions of the elastic coupler. In this respect, the coupler remains a simply-supported spatial beam, whose endpoints (ball joints) are not subject to moment transmission.

Appendix I furnishes all necessary coordinate systems and resulting kinematic details concerning the rigid-body replacement mechanism (see figure I-1).

Appendix F shows the extension of the above formulation to include the elastic change of the pivot to pivot line AC of the coupler. This results in the recognition of the elastic angles φ_{2e} , φ_{32e} and φ_{4e} .

b. Rigid and Elastic Links. Link Masses and Referred Masses
due to Assumption of Zero Slip

Because of their massive construction, the following links of the feed mechanism will be assumed to be rigid:

1. The input shaft and throwblock
2. The output crank of the linkage and the outer race of the sprag clutch, which form a single component.
3. The inner race of the clutch, together with the integral feed-roller assembly.
4. The ground link, i.e. the frame of the machine.

The two elastic links of the feed mechanism are the spatial coupler and the equivalent torsional clutch spring (see below for details).

With the exception of the cross link which only serves for the purpose of computations, and the equivalent clutch spring, all moving components are assumed to have mass.

In order not to get involved with friction between the feed rollers and the stock to be fed, as well as between the pressure rollers and the stock, zero slip will be assumed*. This makes it possible to refer the inertia of the stock and the pressure rollers to the clutch output assembly. In addition, the feed-roller motion now also represents the output displacement of the mechanism.

c. Treatment of Coupler

1. Coupler Shape

To simplify matters, and because the end sections of the coupler which represent the ball joint retainers are not too dissimilar from the tube portions, it will be assumed that the bent coupler consists entirely of a two-section tube of constant cross-section.

*This slip, as shown by experiment, is generally small.

2. Euler-Bernoulli Beam Model

Because the sections of the coupler are long and slender, Euler-Bernoulli Theory will be used.

3. Neglect of Axial Compression and Tension;

Change of Length of Line AC

Direct axial compression or tension on the individual coupler sections will be neglected, since it has been shown in previous mechanism studies that these effects play no appreciable role. It is to be noted that, the change in length of the pivot-to-pivot distance AC, due to both the foreshortening of the two link sections as well as the change in the angle between these sections, will be accounted for (see appendix F).

4. Reference Axes and Reference Plane

In order to obtain the true elastic deflections of the coupler, it is necessary to define reference axes as well as a reference plane.

As stated in section (a) above, for elastic considerations the link remains a simply-supported beam with two ball joints, and the distance AC between the centers of these spheres depends on its elastic deformation.

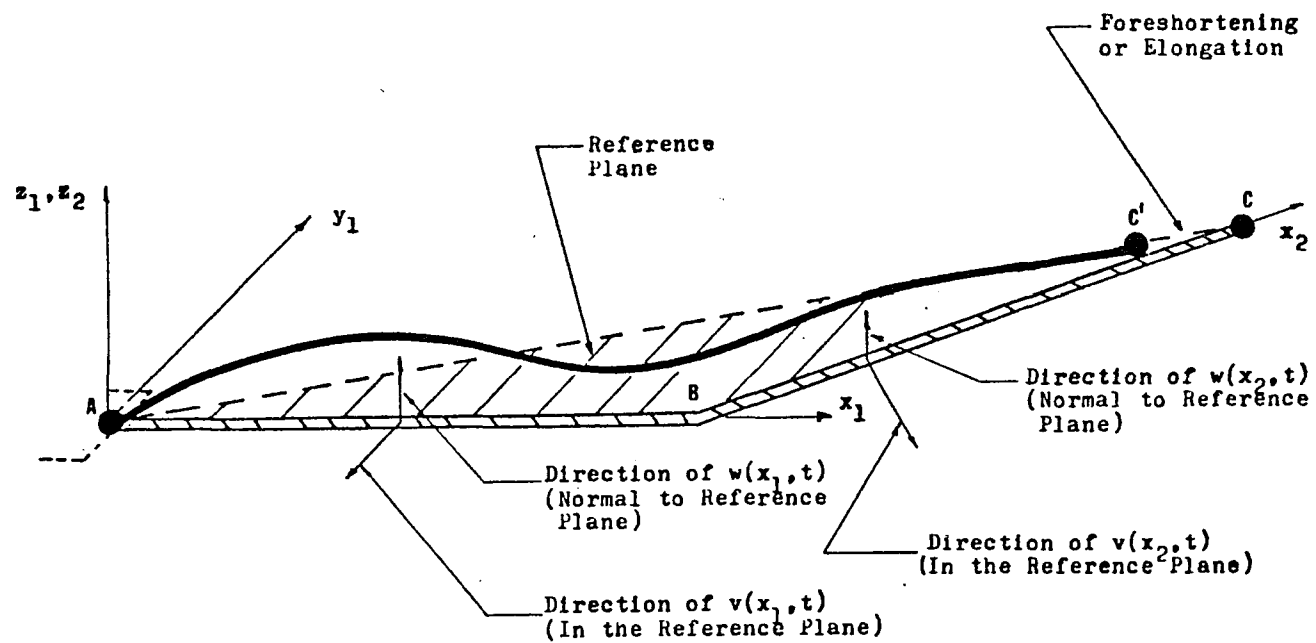


Figure 3.2 : Definition of Reference Plane and Deflected Coupler

Figure 3.2 shows the original length AC and the length AC' , due to deformation, drawn along the same line. This interpretation indicates that the relative position of the above line with respect to the coordinate systems $x_1-y_1-z_1$ and $x_2-y_2-z_2$, associated with the undeformed coupler (and defined in figure I-2 of appendix I), remains the same regardless of the deformation of this link*. Thus the axes x_1 and x_2 each serve as a reference axis, while together they form a reference plane.

The elastic deformations of the coupler will be described in terms of in-plane and out-of-plane deflections. The in-plane displacements of points on section AB are normal to axis x_1 and are expressed by $v_1(x_1, t)$. Similarly, points on section BC have their in-plane displacements, which are normal to axis x_2 , given by $v_2(x_2, t)$, according to [4]. All out-of-plane deflections are normal to the reference plane and are defined by $w_1(x_1, t)$ and $w_2(x_2, t)$, respectively.

5. Velocity Dependent Damping

It will be assumed that the coupler is subject to velocity dependent damping for in-plane and out-of-plane displacements.

* It must be realized that the position of the pivot-to-pivot line in space, as well as the magnitude of the elastic angles φ_{2e} , φ_{32e} , and φ_{4e} (see appendix I) depend on the length AC' .

d. Sprag Clutch Torque Curve and Clutch Damping

With the assumption that the clutch elasticity will be represented by a lumped massless spring, its torque deflection characteristics were obtained experimentally. In this experiment, the inner race of the clutch was held stationary while the outer race could be torque loaded by way of known weights. The angular displacement of the outer race was measured with the help of an angular encoder.

Figure 3.3 shows the experimentally obtained results, together with a $3/2$ power curve which was found to be most representative of the existing torque-deflection relationship. The equation of the curve in Figure 3.3 has the following form:

$$T = K_{SP} (\theta)^{3/2} \quad (3.1)$$

where,

T = Torque (in-lb)

K_{SP} = The torsional spring constant of the clutch
 $= 170,000 \text{ in-lb-rad}^{-3/2}$

θ = Angle of rotation of the outer race (rad)

When equation 3.1 is applied to the nomenclature of the feed mechanism, one obtains according to section 2 of appendix I:

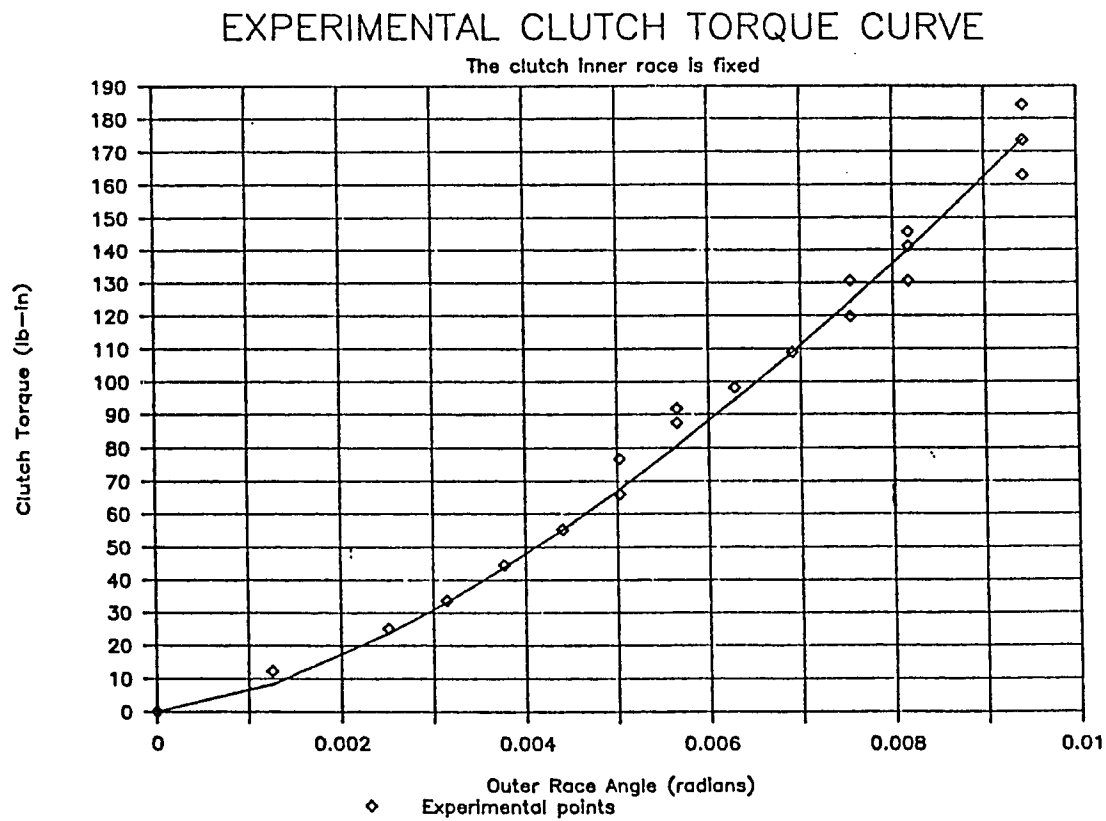


Figure 3.3 : Experimentally Obtained Clutch Torque Curve

$$T = K_{SP} (\phi_5 - \phi_4)^{3/2} \quad (3.2)$$

where

ϕ_5 = Output angle of clutch

ϕ_4 = Input angle of clutch

The clutch damping was assumed to be linear with the following form:

$$T_D = C_D (\dot{\phi}_5 - \dot{\phi}_4)$$

where,

C_D = Clutch damping coefficient

$\dot{\phi}_4$ = angular velocity of the input angle of the clutch

$\dot{\phi}_5$ = angular velocity of the output angle of the clutch

e. Neglect of Bearing Friction

The frictional effects in the various bearings as well as in the clutch will be neglected.

f. Brake Torque

The retarding torque applied by the brake is assumed to be constant throughout a given feed cycle.

B. DEFINITION OF FOUR MOTION REGIMES

a. Forward Motion of the Rocker during Tightening of the Sprag Clutch

Regime no. 1 describes the initial forward motion of the rocker link against the clutch windup, while the feed rollers are held stationary by the brake. This phase of the motion cycle ends when the clutch torque becomes equal to the brake torque and there is incipient motion of the feed rollers. The clutch input motion is continuously monitored.

b. Forward Motion of Strip

Regime no. 2 is concerned with the continued motion of the rocker, which now drives the feed rollers and the metal strip through the sprag clutch. This combined motion is also resisted by the brake. The regime ends when the rocker link reaches its extreme position. Both the clutch input and output motions are monitored.

c. Return Motion of Rocker, Sprag-Clutch Unwinding and Possible Overshoot of Feed Rollers

Regime no. 3 starts with the extreme forward position of the rocker. It deals with the effect of the unwinding of the sprags on the elastic coupler during the initial part of the return stroke and it may entail either of the following two different physical conditions:

- i. In case there is no overshooting of the feed rollers, i.e. their motion ends at rocker reversal, the unwinding of the sprags is only due to the rocker motion.
- ii. If overshooting exists, the unwinding of the sprags is due to both this separate motion of the feed rollers as well as the rocker motion.

d. Return Motion of the Rocker and Possible
Overshooting of the Feed Rollers

Regime no. 4 concerns itself with the portion of the return stroke where the sprag clutch has stopped operating. Thus the coupler experiences no work load. In case the overshooting motion of the feed rollers has not yet come to an end, it must be accounted for separately. This regime ends when the rocker link reaches its extreme return position. Again all motions during this regime are monitored.

IV. DERIVATION OF EQUATIONS OF MOTION FOR REGIME NO. 2

Since the first, third, and fourth motion regimes represent reduced cases of the second one, where the stock is moved forward, only the derivation of this general case will presently be discussed. Its generalized coordinates are the elastic in-plane and out-of-plane coupler deflections v and w , respectively, as well as the feed-roller angle ϕ_5 . The system equations of motion for the other regimes will be obtained by omitting terms related to the clutch spring, the feed roller system, or the brake, either singly or in groups, as applicable.

The following first enumerates the individual energy terms and their variations, which make up Hamilton's integral. Subsequently, a step-by-step outline of the derivation of the associated differential equations and boundary conditions, which is shown in detail in appendix A, is given.

A. HAMILTON'S INTEGRAL

The system equations of motion for the second regime are derived with the help of Hamilton's Principle, i.e.:

$$\delta \int_{t_0}^{t_1} [(T - \Pi) dt + W dt] = 0 \quad (4.1)$$

where:

T = Total kinetic energy of the system

Π = Total potential energy of the system

W = Total work done by non-conservative forces

In addition, an elastic mechanism constraint equation, which relates the elastic angles ϕ_{2e} , ϕ_{32e} and ϕ_{4e} to the coupler link deflections v_1 and w_1 , is introduced (see component equations, F-1 to F-3, in appendix F). This expression originates from the inclusion of the elastic changes in the length of line AC of the coupler, into the loop equation of the replacement mechanism. This change in length is caused by the foreshortening due to bending, of the two link-sections, as well as the accompanying change in the angle between these sections. It could be shown that the x-component of this elongation represents a viable approximation of the total change (see discussion starting with equation F-4 in appendix F).

The kinematic terms used in the determination of the kinetic energy of the system include the effects of the above change of length of the coupler link. Because of this, the work of the forces in the direction of AC is automatically accounted for [22, 23].

The following first shows the various energy and work terms that must be substituted into equation 4.1. Subsequently, the various operations pertaining to this integral are discussed. A complete derivation of the equations of motion for the second regime is shown in appendix A.

B. KINETIC ENERGY OF THE SYSTEM AND ITS VARIATION

a. Kinetic Energy

The kinetic energy term T is equal to the sum of the kinetic energy terms due to all variable speed rigid and elastic links. Since the input link runs at known constant speed, its kinetic energy has no variation and need therefore not be considered. The rigid body links are represented by output crank 4 and clutch output assembly 5.

With both of these links in planar motion, their kinetic energy T_R is given by:

$$T_R = (1/2) * [J_{04}(\dot{\phi}_{4e})^2 + J_{05}(\dot{\phi}_5)^2] \quad (4.2)$$

where

J_{04} = Moment of inertia of link 4 and the outer race of the clutch(both with respect to their common axis of rotation).

J_{05} = Moment of inertia of the clutch output assembly. This includes, as indicated earlier, the moment of inertia of the clutch-output race, the feed roller shaft and the feed-rollers, with respect to their common axis of rotation, as well as the referred moments of inertia of the stock and the pressure rollers.

$\dot{\phi}_{4e}$ = Absolute angular velocity of the rocker, as obtained by taking the elastic deformations of the pivot line AC, of the coupler, into account.

$\dot{\phi}_5$ = Absolute angular velocity of the combined link 5.

The kinetic energy T_{E3} of the two-section elastic spatial coupler is given by:

$$T_{E3} = \frac{1}{2} \rho A \left[\int_{x_1=0}^{L_1} (\dot{\vec{r}}_{P_1} \cdot \dot{\vec{r}}_{P_1}) dx_1 + \int_{x_2=0}^{L_2} (\dot{\vec{r}}_{P_2} \cdot \dot{\vec{r}}_{P_2}) dx_2 \right] \quad (4.3)$$

where

A = Tubular cross-sectional area

ρ = Density of the tube material

L_1, L_2 = Lengths of the individual tube sections

$\dot{\vec{r}}_{P_1}, \dot{\vec{r}}_{P_2}$ = Absolute velocities of arbitrary points P_1 and P_2 on sections AB and BC of the coupler, respectively.

b. Variation of the Kinetic Energy Integral

Because of the complexity of the velocity terms, in the kinetic energy expression, it has been found convenient to employ the following alternate formulation of Hamilton's equation, which uses the acceleration terms [21].

The variation of the integral of the kinetic energy of the elastic member becomes according to equation (4.3):

$$\begin{aligned} \delta \int_{t_0}^{t_1} T_E dt = & \left[\int_{x_1=0}^{L_1} \rho A \ddot{\bar{r}}_{P_1} \delta \dot{\bar{r}}_{P_1} dx_1 \right]_{t_0}^{t_1} - \int_{t_0}^{t_1} \rho A \int_{x_1=0}^{L_1} \ddot{\bar{r}}_{P_1} \cdot \delta \bar{r}_{P_1} dx_1 dt + \left[\int_{x_2=0}^{L_2} \rho A \ddot{\bar{r}}_{P_2} \cdot \delta \dot{\bar{r}}_{P_2} dx_2 \right]_{t_0}^{t_1} \\ & - \int_{t_0}^{t_1} \rho A \int_{x_2=0}^{L_2} \ddot{\bar{r}}_{P_2} \cdot \delta \bar{r}_{P_2} dx_2 dt \end{aligned} \quad (4.4)$$

Similarly, the variation of the integral of the rigid body components is derived from equation (4.2):

$$\delta \int_{t_0}^{t_1} T_R dt = \left[I_{O4} \ddot{\bar{\Phi}}_4 \delta \dot{\bar{\Phi}}_4 \right]_{t_0}^{t_1} - \int_{t_0}^{t_1} I_{O4} \ddot{\bar{\Phi}}_4 \cdot \delta \bar{\Phi}_4 dt + \left[I_{O5} \ddot{\bar{\Phi}}_5 \delta \dot{\bar{\Phi}}_5 \right]_{t_0}^{t_1} - \int_{t_0}^{t_1} I_{O5} \ddot{\bar{\Phi}}_5 \cdot \delta \bar{\Phi}_5 dt \quad (4.5)$$

When one applies the requirement of Hamilton's principle, i.e. that the variations must vanish at the beginning and the end of the time interval, one obtains with equations (4.4) and (4.5):

$$\begin{aligned} \delta \int_{t_0}^{t_1} T dt = & - \left\{ \int_{t_0}^{t_1} I_{O4} \ddot{\bar{\Phi}}_4 \delta \bar{\Phi}_4 dt + \int_{t_0}^{t_1} I_{O5} \ddot{\bar{\Phi}}_5 \delta \bar{\Phi}_5 dt + \int_{t_0}^{t_1} \left[\int_{x_1=0}^{L_1} \rho A \ddot{\bar{r}}_{P_1} \cdot \delta \bar{r}_{P_1} dx_1 \right. \right. \\ & \left. \left. + \int_{x_2=0}^{L_2} \rho A \ddot{\bar{r}}_{P_2} \cdot \delta \bar{r}_{P_2} dx_2 \right] dt \right\} \end{aligned} \quad (4.6)$$

C. POTENTIAL ENERGY OF THE SYSTEM AND ITS VARIATION

a. Potential Energy

The potential energy Π of the system consists of the strain energy of the elastic coupler link as well as the potential energy associated with the spring of the sprag clutch.

The strain energy Π_C of the coupler, due to in-plane and out-of-plane bending is given by:

$$\Pi_C = \frac{1}{2} EI \left\{ \int_{x_1=0}^{L_1} \left[\left(\frac{\partial^2 v_1(x_1, t)}{\partial x_1^2} \right)^2 + \left(\frac{\partial^2 w_1(x_1, t)}{\partial x_1^2} \right)^2 \right] dx_1 + \int_{x_2=0}^{L_2} \left[\left(\frac{\partial^2 v_2(x_2, t)}{\partial x_2^2} \right)^2 + \left(\frac{\partial^2 w_2(x_2, t)}{\partial x_2^2} \right)^2 \right] dx_2 \right\} \quad (4.7)$$

where:

E = The common modulus of elasticity of both sections

I = The area moment of inertia with respect to any centroidal axis of the tubular cross section.

The potential energy Π_{SC} of the non-linear clutch spring becomes, with the torque displacement equation given by equation 3.2:

$$\Pi_{SC} = (2/5) * K_{SP} * [\varphi_5 - \varphi_{4e}]^{5/2} \quad (4.8)$$

b. Variations of the Potential Energy Integral

When the variation of the expression for the strain energy Π_C is integrated by parts twice, equation (4.7) becomes:

$$\begin{aligned} \delta \int_{t_0}^{t_1} \Pi_C dt = \int_{t_0}^{t_1} \left\{ \int_{x_1=0}^{L_1} EI \left[\frac{\partial^4 v_1(x_1, t)}{\partial x_1^4} \delta v_1(x_1, t) + \frac{\partial^4 w_1(x_1, t)}{\partial x_1^4} \delta w_1(x_1, t) \right] dx_1 + \int_{x_2=0}^{L_2} EI \left[\frac{\partial^4 v_2(x_2, t)}{\partial x_2^4} \right. \right. \\ \left. \cdot \delta v_2(x_2, t) + \frac{\partial^4 w_2(x_2, t)}{\partial x_2^4} \delta w_2(x_2, t) \right] dx_2 + \left[EI \frac{\partial^3 v_1(x_1, t)}{\partial x_1^3} \delta \left(\frac{\partial v_1(x_1, t)}{\partial x_1} \right) - EI \frac{\partial^3 v_1(x_1, t)}{\partial x_1^3} \delta v_1(x_1, t) \right]_{x_1=0}^{L_1} \\ + \left[EI \frac{\partial^3 v_2(x_2, t)}{\partial x_2^3} \delta \left(\frac{\partial v_2(x_2, t)}{\partial x_2} \right) - EI \frac{\partial^3 v_2(x_2, t)}{\partial x_2^3} \delta v_2(x_2, t) \right]_{x_2=0}^{L_2} + \left[EI \frac{\partial^3 w_1(x_1, t)}{\partial x_1^3} \delta \left(\frac{\partial w_1(x_1, t)}{\partial x_1} \right) \right. \\ \left. - EI \frac{\partial^3 w_1(x_1, t)}{\partial x_1^3} \delta w_1(x_1, t) \right]_{x_1=0}^{L_1} + \left[EI \frac{\partial^3 w_2(x_2, t)}{\partial x_2^3} \delta \left(\frac{\partial w_2(x_2, t)}{\partial x_2} \right) - EI \frac{\partial^3 w_2(x_2, t)}{\partial x_2^3} \delta w_2(x_2, t) \right]_{x_2=0}^{L_2} \Bigg\} dt \end{aligned} \quad (4.9)$$

Further, the variation of equation (4.8) for the spring potential energy, furnishes:

$$\delta \int_{t_0}^{t_1} \Pi_{sc} dt = \int_{t_0}^{t_1} \left[(k_{sp} \varphi_5 - \varphi_{4e})^{3/2} \delta \varphi_5 - (k_{sp} \varphi_5 - \varphi_{4e})^{3/2} \delta \varphi_{4e} \right] dt \quad (4.10)$$

D. WORK DONE BY NON-CONSERVATIVE FORCES AND ITS VARIATION

The work done by non-conservative forces consists of two parts, i.e. the work associated with the damping in the coupler and the work of the brake on the feed roller shaft, link 5*.

* Damping in the clutch will only be included in the final 'linearized' differential equations.

The variation of the integral of the work done by the velocity dependent damping is:

$$\begin{aligned} \delta \int_{t_0}^{t_1} W_D \cdot dt = & - \int_{t_0}^{t_1} \left[\int_{x_1=0}^{L_1} \left[C_{D1} \frac{\partial v_1(x_1, t)}{\partial t} \delta v_1(x_1, t) + C_{D2} \frac{\partial w_1(x_1, t)}{\partial t} \delta w_1(x_1, t) \right] dx_1 \right. \\ & \left. + \int_{x_2=0}^{L_2} \left[C_{D1} \frac{\partial v_2(x_2, t)}{\partial t} \delta v_2(x_2, t) + C_{D2} \frac{\partial w_2(x_2, t)}{\partial t} \delta w_2(x_2, t) \right] dx_2 \right] dt \end{aligned} \quad (4.12)$$

where

C_{D1} , C_{D2} = Coefficients of damping for the in-plane and out-of-plane deflections, respectively (applicable to both sections of the coupler).

In a similar manner, the variation of the integral of the work W_B of the brake becomes:

$$\delta \int_{t_0}^{t_1} W_B \cdot dt = - \frac{\dot{\Phi}_5}{|\dot{\Phi}_5|} \int_{t_0}^{t_1} T_B \delta \Phi_5 dt \quad (4.12)$$

where

$$T_B = T_{BR} + T_{FR} \quad (4.13)$$

with,

T_{BR} = The retarding torque exerted by the brake. It depends on the brake setting.

T_{FR} = The retarding torque exerted on the feed rollers as a result of the sliding frictional force that is applied

on the metal strip by the feed table (not shown in figure 1.1) . This is proportional to the weight of the metal strip being fed.

The term $\dot{\phi}_5/|\dot{\phi}_5|$ is a signum function which assures that the brake torque always acts in the direction opposite to the rotation of the feed-roller shaft.

E. WORKING FORM OF HAMILTON'S INTEGRAL FOR DERIVATION OF PARTIAL DIFFERENTIAL EQUATIONS (SEE APPENDIX A)

The working form of Hamilton's integral is obtained by the substitution of the variations of the kinetic and potential energies and the non-conservative work terms into the original variational formulation. Its final form is given by equation A-1.

F. SUBSTITUTION OF KINEMATIC TERMS

Expressions for the acceleration of the arbitrary points on the centerlines of both sections of the coupler are substituted into equation A-1', together with the variations of the position vectors of these points. This results in equation A-2 .

The derivations for the acceleration terms are shown in appendix E, while those for the variations of the position vectors are given in appendix G.

The various kinematic terms are functions of the rigid and elastic mechanism displacements, the lateral in-plane and out-of-plane deformations, as well as the axial foreshortening or extension terms which are due to the lateral bending.

Since the deformation of the second section of the coupler depends also on the deformations of the first one, it was found convenient to introduce certain collective terms for that section (see part 1-b of appendix E).

G. FINAL FORM OF HAMILTON'S INTEGRAL

To obtain the final form of Hamilton's integral the following operations are performed on equation A-2:

- i. The variations of the elastic angles, as obtained from the components of the elastic loop equation are substituted (see appendix F). These are functions of the generalized coordinates and their derivatives.
- ii. The expressions for the variations of the aforementioned collective terms for the position vector of a point on the second coupler section are substituted, as per the derivations shown in appendix H.

- iii. Leibnitz's rule is applied to be able to collect terms containing the variations of both in-plane and out-of-plane deformation variables of the coupler.
- iv. With the exception of the output crank angle, which is also the input angle to the sprag clutch, the elastic angles and their derivatives are approximated by their rigid body counterparts (see appendix I). The output crank angle is later expanded in a Taylor series (see appendix C), in terms of the elastic coupler deflections, in order to make the clutch deflection description as realistic as possible. While the influence of the elastic deflections on the angle ϕ_{4e} affects the magnitude of the clutch torque significantly, the D'Alembert forces are affected very little by these deflections. This, on one hand allows the approximation of the aforementioned forces by their rigid-body counterparts, and on the other makes it imperative that the effect of the elastic deflections is accounted for in the clutch torque formulation.

With the above operations completed, the final form of Hamilton's integral, given by equation A-12, consists of six groups of terms, five of which have the variation of one of the generalized coordinates as a factor, while the sixth contains the natural and the dynamic boundary conditions of the problem.

H. PARTIAL LINEARIZATION OF HAMILTON'S INTEGRAL

The partially linearized Hamilton's integral is obtained once the following steps are taken:

- i. The expressions for $U_1(x_1)$, $V_2(x_2, t)$ and $U_2(x_2, t)$, as given by equations E-3, E-6 and E-7, respectively, are substituted.
- ii. All non-linear terms in equation A-12, involving various product combinations of the elastic deflections and their derivatives, are neglected.

This gives rise to equation A-16.

I. DETERMINATION OF PARTIAL DIFFERENTIAL EQUATIONS AND BOUNDARY CONDITIONS

After substitution of the variations of the geometric boundary conditions, i.e. equations A-33 through A-40, into the linearized Hamilton's integral (equation A-16), application of the fundamental lemma of the calculus of variations furnishes a total of five coupled partial differential equations along with their associated dynamic boundary conditions. Two of these expressions specifically describe the in-plane coupler motion, while another set of two equations refers to the out-of-plane motion. The fifth differential equation deals with the feed-roller motion. Thus,

a. In-plane motion

For $0 \leq x_1 \leq L_1$ (Equation A-48)

$$\frac{\partial^2 v_1(x_1, t)}{\partial x_1^2} \left[\cos \alpha \left[\Delta \Phi_{4e} \left(J_{04} \ddot{\Phi}_4 - K_{5p} (\Phi_5 - \Phi_{4e})^{\frac{3}{2}} \right) + G_9(t) \right] + G_{10}(x_1, t) + \cos(\alpha + \beta) G_{11}(t) - \sin(\alpha + \beta) G_4(t) \right] - p A R_1(x_1, t) \frac{\partial v_1(x_1, t)}{\partial x_1} + p A \ddot{\eta}_{1yL} + C_{D1} \frac{\partial v_1(x_1, t)}{\partial t} + EI \frac{\partial^4 v_1(x_1, t)}{\partial x_1^4} = 0 \quad (4.14)$$

For $0 \leq x_2 \leq L_2$ (Equation A-49)

$$\frac{\partial^2 v_2(x_2, t)}{\partial x_2^2} \left[\cos \beta \left[\Delta \Phi_{4e} \left(J_{04} \ddot{\Phi}_4 - K_{5p} (\Phi_5 - \Phi_{4e})^{\frac{3}{2}} \right) + G_9(t) \right] + G_5(x_2, t) \right] - p A R_4(t, x_2) \frac{\partial v_2(x_2, t)}{\partial x_2} + p A \ddot{\eta}_{2yL} + C_{D1} \frac{\partial v_2(x_2, t)}{\partial t} + EI \frac{\partial^4 v_2(x_2, t)}{\partial x_2^4} = 0 \quad (4.15)$$

b. Feed-roller motion (Equation A-50)

$$J_{05} \ddot{\Phi}_5 + K_{5p} (\Phi_5 - \Phi_{4e})^{\frac{3}{2}} + \frac{\dot{\Phi}_5}{|\dot{\Phi}_5|} T_B = 0 \quad (4.16)$$

The boundary conditions that relate the coupled in-plane and feed roller motions are given by (Equations A-51 and A-52):

$$\begin{aligned} & \sec \beta \sin(\alpha + \beta) \left[\Delta \Phi_{4e} (J_{04} \ddot{\Phi}_4 - K_{5p} (\Phi_5 - \Phi_{4e})^{3/2}) + G_6(t) \right] - \left[\Delta \Phi_{4e} (J_{04} \ddot{\Phi}_4 - K_{5p} (\Phi_5 - \Phi_{4e})^{3/2}) \right. \\ & \left. + G_9(t) \right] \left[(\cos \alpha - \cos \beta \cos(\alpha + \beta)) \frac{\partial v_1(L_1, t)}{\partial x_1} - \sin \beta \sin(\alpha + \beta) \frac{\partial v_2(L_2, t)}{\partial x_2} \right] + \int_{x_2=0}^{L_2} \rho A \ddot{\Gamma}_{2xL} \sin(\alpha + \beta) dx_2 \\ & + G_4(t) \sin(\alpha + \beta) \frac{\partial v_1(L_1, t)}{\partial x_1} - EI \frac{\partial^3 v_1(L_1, t)}{\partial x_1^3} + EI \frac{\partial^3 v_2(L_2, t)}{\partial x_2^3} \tan \beta \sin(\alpha + \beta) \\ & + EI \frac{\partial^3 v_2(0, t)}{\partial x_2^3} \cos(\alpha + \beta) = 0 \end{aligned} \quad (4.17)$$

$$\begin{aligned} & \left[\Delta \Phi_{4e} (J_{04} \ddot{\Phi}_4 - K_{5p} (\Phi_5 - \Phi_{4e})^{3/2}) + G_9(t) \right] \cos \beta \cdot L_2 \frac{\partial v_1(L_1, t)}{\partial x_1} + G_8(t) \frac{\partial v_1(L_1, t)}{\partial x_1} \\ & + EI \frac{\partial^3 v_1(L_1, t)}{\partial x_1^3} - EI \frac{\partial^3 v_2(0, t)}{\partial x_2^3} = 0 \end{aligned} \quad (4.18)$$

c. Out-of-plane motion

For $0 \leq x_1 \leq L_1$ (Equation A-53)

$$\begin{aligned} & \frac{\partial^2 w_1(x_1, t)}{\partial x_1^2} \left[\cos \alpha \left[\Delta \Phi_{4e} (J_{04} \ddot{\Phi}_4 - K_{5p} (\Phi_5 - \Phi_{4e})^{3/2}) + G_9(t) \right] + G_{10}(x_1, t) + \cos(\alpha + \beta) G_{11}(t) - \sin(\alpha + \beta) \right. \\ & \left. \cdot G_4(t) \right] - \rho A R_1(x_1, t) \frac{\partial w_1(x_1, t)}{\partial x_1} + \rho A \ddot{\Gamma}_{1zL} + G_{D2} \frac{\partial w_1(x_1, t)}{\partial t} + EI \frac{\partial^4 w_1(x_1, t)}{\partial x_1^4} = 0 \end{aligned} \quad (4.19)$$

For $0 \leq x_2 \leq L_2$ (Equation A-54)

$$\begin{aligned} & \frac{\partial^2 w_2(x_2, t)}{\partial x_2^2} \left[\cos \beta \left[\Delta \Phi_{4e} (J_{04} \ddot{\Phi}_4 - K_{5p} (\Phi_5 - \Phi_{4e})^{3/2}) + G_9(t) \right] + G_5(x_2, t) \right] - \rho A R_4(x_2, t) \frac{\partial w_2(x_2, t)}{\partial x_2} \\ & + \rho A \ddot{\Gamma}_{2zL} + G_{D2} \frac{\partial w_2(x_2, t)}{\partial t} + EI \frac{\partial^4 w_2(x_2, t)}{\partial x_2^4} = 0 \end{aligned} \quad (4.20)$$

The boundary condition for the out-of-plane motion is given by

(Equations A-55):

$$\begin{aligned} & \left[\Delta \Phi_{4e} (J_{04} \ddot{\Phi}_4 - K_{5p} (\Phi_5 - \Phi_{4e})^{3/2}) + G_9(t) \right] \left[\cos \beta \cos(\alpha + \beta) - \cos \alpha \right] \frac{\partial w_1(L_1, t)}{\partial x_1} + G_4(t) \sin(\alpha + \beta) \cdot \\ & \cdot \frac{\partial w_1(L_1, t)}{\partial x_1} - EI \frac{\partial^3 w_1(L_1, t)}{\partial x_1^3} + EI \frac{\partial^3 w_2(0, t)}{\partial x_2^3} = 0 \end{aligned} \quad (4.21)$$

V. COUPLED ORDINARY NON-LINEAR DIFFERENTIAL EQUATIONS FOR THE
ELASTIC COUPLER LINK AND THE FEED-ROLLER SHAFT

A. SOLUTION METHOD

The method of Kantorovich is now used, in conjunction with the final form of Hamilton's integral, to obtain ordinary differential equations for the eventual determination of the coupler deflections v_1 and w_1 , as well as the feed-roller angle ϕ_5 , in terms of the throwblock angle ϕ_1 .

The following outlines how this process is accomplished, drawing on the detailed work of appendices B, C, and J. In addition, it is shown how the Taylor series approximation of the elastic angle ϕ_{4e} , according to appendix C, is being used in the differential equations.

a. Product Solution with Help of Subproblem

Using the basic assumption of the method of Kantorovich, one introduces the following product solutions:

$$v_1(x_1, t) = \sum_n v_{1n}(x_1) q_n(t) \quad (5.1)$$

$$v_2(x_2, t) = \sum_n v_{2n}(x_2) q_n(t) \quad (5.2)$$

$$w_1(x_1, t) = \sum_n W_{1n}(x_1) q_n(t) \quad (5.3)$$

$$w_2(x_2, t) = \sum_n W_{2n}(x_2) q_n(t) \quad (5.4)$$

Where, $V_{1n}(x_1)$, $V_{2n}(x_2)$, $W_{1n}(x_1)$, $W_{2n}(x_2)$ represent the shape functions that are associated with the in-plane and out-of-plane free-vibrations of the coupler link. The geometric boundary conditions of these subproblems must be identical to those of the main problem. The functions $q_n(t)$ and $p_n(t)$ are identical for both sections of the bent beam and correspond to the time portions of the solution. Together with $\varphi_5(t)$, they represent the dependent variables of the ordinary differential equations with t being the independent variable before the change of this variable to φ_1 is made.

Appendix J lists the basic assumptions of the derivations of the subproblems and gives the associated shape functions, characteristic equations, and orthogonality conditions. More specifically, equations J-42 and J-43 give the shape functions for $V_{1n}(x_1)$ and $V_{2n}(x_2)$, respectively, while equation J-53 furnishes the characteristic equation for the eigenvalues of the in-plane deflections. Equation J-60 represents the associated orthogonality condition. Similarly, equations J-91 and J-92 give the shape functions for $W_{1n}(x_1)$ and $W_{2n}(x_2)$, respectively, while equation J-101 furnishes the characteristic equation for the eigenvalues of the out-of-plane deflections. Finally, equation J-108 represents the orthogonality condition for the out-of-plane deflections.

B. SUBSTITUTION OF PRODUCT SOLUTIONS INTO LINEARIZED HAMILTON'S INTEGRAL: ORDINARY DIFFERENTIAL EQUATIONS IN TERMS OF TIME

With the shape functions of the coupler elastic deflections known, one is able to substitute them, together with their derivatives, boundary terms and orthogonality conditions into the linearized Hamilton's integral given by equation A-16 of appendix A.

Appendix B gives details of this substitution. Equation B-11 represents the final integral before the fundamental lemma of the calculus of variations is applied. To better understand the result, it is useful to note the following:

- i. Since the shape functions are prescribed, their variations vanish and with that the variations of the generalized coordinates $v(x_1, t)$, $w(x_1, t)$ reduce to the variations of their respective time functions.
- ii. Whenever the shape functions or their derivatives vanish (for example, for certain boundary terms), the associated variations of equations 5.1 to 5.4 will also vanish.
- iii. The boundary terms $V_1''(L_1)$, $V_1'''(L_1)$, $W_1''(L_1)$, $W_1'''(L_1)$ of the shape functions are expressed according to equations J-36, J-39, J-86 and J-87, of appendix J, respectively.
- iv. The orthogonality conditions, given by equations J-60 and J-107, are used directly, as applicable.

Appendix B shows that the application of the fundamental lemma to equation B-11, furnishes the ordinary equations describing the n th-mode time-portions $q_n(t)$ and $p_n(t)$ of the coupler elastic deflections, as well as the feed-roller angle $\phi_5(t)$. Thus,

a. For the in-plane motion (equation B-58):

$$\begin{aligned} & \rho A W_k \ddot{Q}_k(t) + C_{D1} W_k \dot{Q}_k(t) + \rho A W_k (\omega_{V_k}^2 - J_3^2(t)) Q_k(t) + \sum_{n=1}^{\infty} \left[[-K_{Sp}(\phi_5 - \phi_{4e})^{3/2} J_{4(n,k)} + \right. \\ & \left. + Z_{n,k}(t)] Q_n(t) + \Theta_{n,k}(t) \dot{Q}_n(t) + \Lambda_{n,k}(t) \ddot{Q}_n(t) + \Xi_{n,k}(t) P_n(t) + \Psi_{n,k}(t) \dot{P}_n(t) + \right. \\ & \left. + \Phi_{n,k}(t) \ddot{P}_n(t) \right] + F_{2k}(t) - K_{Sp}(\phi_5 - \phi_{4e})^{3/2} J_{5k}(t) = 0 \end{aligned} \quad (5.5)$$

b. For the out-of-plane motion (equation B-59):

$$\begin{aligned} & \rho A W_k \ddot{P}_k(t) + C_{D2} W_k \dot{P}_k(t) + \rho A W_k (\omega_{W_k}^2 - (J_1^2(t) + J_2^2(t))) P_k(t) + \sum_{n=1}^{\infty} \left[[-K_{Sp}(\phi_5 - \phi_{4e})^{3/2} \right. \\ & \left. J_{6(n,k)} + \dot{\Lambda}_{n,k}(t)] P_n(t) + \Gamma_{n,k}(t) Q_n(t) + \Delta_{n,k}(t) \dot{Q}_n(t) \right] + F_{3k}(t) = 0 \end{aligned} \quad (5.6)$$

a. For the feed-roller motion (equation B-57):

$$J_{05} \ddot{\phi}_5 + K_{Sp}(\phi_5 - \phi_{4e})^{3/2} + \frac{\dot{\phi}_5}{|\dot{\phi}_5|} T_B = 0 \quad (5.7)$$

For $k = 1, 2, \dots, n$. Note that, all equations are mutually coupled and modal coupling exists between the coupler deflection equations.

C. CHANGE OF THE INDEPENDENT VARIABLE AND INTRODUCTION OF CLUTCH DAMPING

Since all the time dependent coefficients of the above differential equations are functions of the input angle ϕ_1 , and the input angular velocity $\dot{\phi}_1$ is constant, the following change of variables is made:

$$\frac{dq_n(t)}{dt} = \dot{\phi}_1 \frac{dq_n(\phi_1)}{d\phi_1} \quad (5.8)$$

$$\frac{d^2q_n(t)}{dt^2} = \dot{\phi}_1^2 \frac{d^2q_n(\phi_1)}{d\phi_1^2}$$

Similarly,

$$\frac{dp_n(t)}{dt} = \dot{\phi}_1 \frac{dp_n(\phi_1)}{d\phi_1} \quad (5.9)$$

$$\frac{d^2p_n(t)}{dt^2} = \dot{\phi}_1^2 \frac{d^2p_n(\phi_1)}{d\phi_1^2}$$

In addition,

$$\frac{d\phi_5(t)}{dt} = \dot{\phi}_1 \frac{d\phi_5(\phi_1)}{d\phi_1} \quad (5.10)$$

$$\frac{d^2\phi_5(t)}{dt^2} = \dot{\phi}_1^2 \frac{d^2\phi_5(\phi_1)}{d\phi_1^2}$$

The above conversion relationships along with the effects of velocity dependent damping (see Section III-A-d), are now introduced

into equations 5.5 to 5.7. Then,

a. For the in-plane motion:

$$\begin{aligned} & \rho A \omega_k^2 \dot{\phi}_1^2 \frac{d^2 q_k(\phi_1)}{d\phi_1^2} + c_{D1} \omega_k \dot{\phi}_1 \frac{dq_k(\phi_1)}{d\phi_1} + \rho A \omega_k (\omega_{vk}^2 - J_3^2(\phi_1)) q_k(\phi_1) + \sum_{n=1}^{\infty} \left[[-K_{SP}(\phi_5 - \phi_{4e})]^{3/2} \right. \\ & \cdot J_{4(n,k)} - c_D (\dot{\phi}_1 \frac{d\phi_5}{d\phi_1} - \dot{\phi}_{4e}) J_{4(n,k)} + Z_{n,k}(\phi_1) \left. \right] q_n(\phi_1) + \dot{\phi}_1 \Theta_{n,k}(\phi_1) \frac{dq_n(\phi_1)}{d\phi_1} \\ & + \dot{\phi}_1^2 \Lambda_{n,k}(\phi_1) \frac{d^2 q_n(\phi_1)}{d\phi_1^2} + \Xi_{n,k}(\phi_1) p_n(\phi_1) + \dot{\phi}_1 \Psi_{n,k}(\phi_1) \frac{dp_n(\phi_1)}{d\phi_1} \\ & + \dot{\phi}_1^2 \Phi_{n,k}(\phi_1) \frac{d^2 p_n(\phi_1)}{d\phi_1^2} \left. \right] + F_{2k}(\phi_1) - [K_{SP}(\phi_5 - \phi_{4e})]^{3/2} + c_D (\dot{\phi}_1 \frac{d\phi_5}{d\phi_1} - \dot{\phi}_{4e}) J_{5k}(\phi_1) = 0 \quad (5.11) \end{aligned}$$

b. For the out-of-plane motion:

$$\begin{aligned} & \rho A \omega_k \dot{\phi}_1^2 \frac{d^2 p_k(\phi_1)}{d\phi_1^2} + c_{D2} \omega_k \dot{\phi}_1 \frac{dp_k(\phi_1)}{d\phi_1} + \rho A \omega_k (\omega_{wk}^2 - [J_1^2(\phi_1) + J_2^2(\phi_1)]) p_k(\phi_1) \\ & + \sum_{n=1}^{\infty} \left[[-K_{SP}(\phi_5 - \phi_{4e})]^{3/2} - c_D (\dot{\phi}_1 \frac{d\phi_5}{d\phi_1} - \dot{\phi}_{4e}) \right] J_{6(n,k)} p_n(\phi_1) + \Lambda_{n,k}(\phi_1) p_n(\phi_1) \\ & + \Gamma_{n,k}(\phi_1) q_n(\phi_1) + \dot{\phi}_1 \Delta_{n,k}(\phi_1) \frac{dq_n(\phi_1)}{d\phi_1} \left. \right] + F_{3k}(\phi_1) = 0 \quad (5.12) \end{aligned}$$

a. For the feed-roller motion:

$$J_{05} \dot{\phi}_1^2 \frac{d^2 \phi_5}{d\phi_1^2} + c_D (\dot{\phi}_1 \frac{d\phi_5}{d\phi_1} - \dot{\phi}_{4e}) + K_{SP}(\phi_5 - \phi_{4e})^{3/2} + \frac{\dot{\phi}_5}{|\phi_5|} T_B = 0 \quad (5.13)$$

With p_n and q_n expressed in terms of the input angle ϕ_1 , the in-plane and out-of-plane total solutions given by equations 5.1 and 5.4 become:

$$v_1(x_1, \phi_1) = \sum_n V_{1n}(x_1) q_n(\phi_1) \quad (5.14)$$

$$v_2(x_2, \phi_1) = \sum_n V_{2n}(x_2) q_n(\phi_1) \quad (5.15)$$

$$w_1(x_1, \phi_1) = \sum_n W_{1n}(x_1) p_n(\phi_1) \quad (5.16)$$

$$w_2(x_2, \phi_1) = \sum_n W_{2n}(x_2) p_n(\phi_1) \quad (5.17)$$

D. DAMPING CONSTANTS

The damping constants C_{D1k} and C_{D2k} associated with each natural frequency ω_{vk} and ω_{wk} of the in-plane and out-of-plane deflections, in equations 5.11 and 5.12 are expressed in terms of the damping ratio by:

$$C_{D1k} = 2\mu\omega_{vk}\zeta \quad (5.18)$$

$$C_{D2k} = 2\mu\omega_{wk}\zeta \quad (5.19)$$

where

μ = mass per unit length of the coupler link

ζ = damping ratio

E. TAYLOR SERIES APPROXIMATION OF THE ELASTIC ANGLE ϕ_{4e}

Equations 5.11 to 5.13 lack an explicit expression for the elastic rocker angle ϕ_{4e} , in terms of the coupler deflections, which serves as the input to the sprag clutch.

While such an expression could be obtained directly from the elastic constraint equation, the resulting form is very unwieldy because the elastic deflections are involved in an inverse tangent function as well as a square root term. This difficulty has been

overcome by developing an approximate expression for this angle by way of a Taylor series expansion around its rigid body counterpart ϕ_4 , in terms of the coupler elastic deflections v_i and w_i (see appendix C for details).

Since the out-of-plane deflections appear only in the form of second order terms, they may be neglected. When the higher order terms associated with the in-plane deflections are also neglected, and the previously introduced change of variables is used, equation C-19 takes the following form:

$$\phi_{4e} = \phi_4 - \frac{2AC}{A\cos\phi_4 - B\sin\phi_4} \sum_{n=1}^{\infty} V_{1n}(L_1) \sec\beta \sin(\alpha+\beta)q_n(\phi_1) \quad (5.20)$$

The above expression is incorporated into equations 5.11-5.13.

F. INITIAL AND FINAL REGIME CONDITIONS

The initial and final regime conditions are discussed in detail in the applications section VII, where it will be shown that the out-of-plane vibrations can be neglected.

VI. GOVERNING EQUATIONS FOR REGIMES 1, 3, 4.

A. DIFFERENTIAL EQUATIONS FOR REGIME No. 1

Regime no. 1 describes the initial motion of the rocker link and with that the spatial mechanism, against the clutch windup, while the feed rollers are stationary. Thus,

a. For the in-plane motion:

$$\begin{aligned} & \rho A \mathcal{W}_K \dot{\varphi}_1^2 \frac{d^2 q_k(\varphi_1)}{d\varphi_1^2} + c_{D1} \mathcal{W}_K \dot{\varphi}_1 \frac{dq_k(\varphi_1)}{d\varphi_1} + \rho A \mathcal{W}_K (\omega_{WK}^2 - J_3^2(\varphi_1)) q_k(\varphi_1) + \sum_{n=1}^{\infty} \left[-K_{SP} (\varphi_5 - \varphi_{4e})^{3/2} \right. \\ & \cdot J_{4(n,k)} + Z_{n,k}(\varphi_1) q_n(\varphi_1) + \dot{\varphi}_1 \Theta_{n,k}(\varphi_1) \frac{dq_n(\varphi_1)}{d\varphi_1} + \dot{\varphi}_1^2 \Lambda_{n,k}(\varphi_1) \frac{d^2 q_n(\varphi_1)}{d\varphi_1^2} \\ & \left. + \Xi_{n,k}(\varphi_1) p_{n,k}(\varphi_1) + \dot{\varphi}_1 \Psi_{n,k}(\varphi_1) \frac{dp_n(\varphi_1)}{d\varphi_1} + \dot{\varphi}_1^2 \Phi_{n,k}(\varphi_1) \frac{d^2 p_n(\varphi_1)}{d\varphi_1^2} \right] \\ & + F_{2K}(\varphi_1) - K_{SP} (\varphi_5 - \varphi_{4e})^{3/2} \cdot J_{5K}(\varphi_1) = 0 \end{aligned} \quad (6.1)$$

b. For the out-of-plane motion:

$$\begin{aligned} & \rho A \mathcal{W}_K \dot{\varphi}_1^2 \frac{d^2 p_k(\varphi_1)}{d\varphi_1^2} + c_{D2} \mathcal{W}_K \dot{\varphi}_1 \frac{dp_k(\varphi_1)}{d\varphi_1} + \rho A \mathcal{W}_K (\omega_{WK}^2 - [J_1^2(\varphi_1) + J_2^2(\varphi_1)]) p_k(\varphi_1) \\ & + \sum_{n=1}^{\infty} \left[(-K_{SP} (\varphi_5 - \varphi_{4e})^{3/2} \cdot J_{6(n,k)} + \bar{A}_{n,k}(\varphi_1)) p_n(\varphi_1) + \Gamma_{n,k}(\varphi_1) q_n(\varphi_1) \right. \\ & \left. + \dot{\varphi}_1 \Delta_{n,k}(\varphi_1) \frac{dq_n(\varphi_1)}{d\varphi_1} \right] + F_{3K}(\varphi_1) = 0 \end{aligned} \quad (6.2)$$

B. DIFFERENTIAL EQUATIONS OF MOTION FOR REGIME No. 3

Regime no. 3 deals with the effects of the unwinding of the clutch in the initial phase of the rocker return motion. With regard to the feed rollers, there may or may not be feed-roller motion due to roller overshoot. Thus,

a. For the in-plane motion:

$$\begin{aligned} & \rho A \mathcal{W}_k \dot{\phi}_1^2 \frac{d^2 q_k(\phi_1)}{d\phi_1^2} + c_{D1} \mathcal{W}_k \dot{\phi}_1 \frac{dq_k(\phi_1)}{d\phi_1} + \rho A \mathcal{W}_k (\omega_{wk}^2 - J_3^2(\phi_1)) q_k(\phi_1) + \sum_{n=1}^{\infty} \left[[-K_{SP}(\phi_5 - \phi_{4e})]^{3/2} \right. \\ & \cdot J_{4(n,k)} + Z_{n,k}(\phi_1) \left. \right] q_n(\phi_1) + \dot{\phi}_1 \Theta_{n,k}(\phi_1) \frac{dq_n(\phi_1)}{d\phi_1} + \dot{\phi}_1^2 \Lambda_{n,k}(\phi_1) \frac{d^2 q_n(\phi_1)}{d\phi_1^2} \\ & + \Xi_{n,k}(\phi_1) p_{n,k}(\phi_1) + \dot{\phi}_1 \Psi_{n,k}(\phi_1) \frac{dp_{n,k}(\phi_1)}{d\phi_1} + \dot{\phi}_1^2 \Phi_{n,k}(\phi_1) \frac{d^2 p_{n,k}(\phi_1)}{d\phi_1^2} \Big] \\ & + F_{2k}(\phi_1) - K_{SP}(\phi_5 - \phi_{4e})^{3/2} \cdot J_{5k}(\phi_1) = 0 \end{aligned} \quad (6.3)$$

b. For the out-of-plane motion:

$$\begin{aligned} & \rho A \mathcal{W}_k \dot{\phi}_1^2 \frac{d^2 p_k(\phi_1)}{d\phi_1^2} + c_{D2} \mathcal{W}_k \dot{\phi}_1 \frac{dp_k(\phi_1)}{d\phi_1} + \rho A \mathcal{W}_k (\omega_{wk}^2 - [J_1^2(\phi_1) + J_2^2(\phi_1)]) p_k(\phi_1) \\ & + \sum_{n=1}^{\infty} \left[[-K_{SP}(\phi_5 - \phi_{4e})]^{3/2} \cdot J_{6(n,k)} + \Lambda_{n,k}(\phi_1) \right] p_n(\phi_1) + \Gamma_{n,k}(\phi_1) q_n(\phi_1) \\ & + \dot{\phi}_1 \Delta_{n,k}(\phi_1) \frac{dq_n(\phi_1)}{d\phi_1} \Big] + F_{3k}(\phi_1) = 0 \end{aligned} \quad (6.4)$$

a. For the feed-roller motion (in case of overshoot):

$$J_{05} \dot{\phi}_1^2 \frac{d^2 \phi_5}{d\phi_1^2} + \frac{\dot{\phi}_5}{|\dot{\phi}_5|} T_B = 0 \quad (6.5)$$

C. DIFFERENTIAL EQUATIONS FOR REGIME NO. 4

This regime concerns itself with that portion of the return cycle where the rocker link has disconnected itself from the clutch. Then,

a. For the in-plane motion:

$$\begin{aligned} & \rho A J_K \dot{\phi}_1^2 \frac{d^2 q_k(\phi_1)}{d\phi_1^2} + c_{D1} J_K \dot{\phi}_1 \frac{dq_k(\phi_1)}{d\phi_1} + \rho A J_K (\omega_{vK}^2 - J_3^2(\phi_1)) q_k(\phi_1) \\ & + \sum_{n=1}^{\infty} \left[Z_{n,k}(\phi_1) q_n(\phi_1) + \dot{\phi}_1 \Theta_{n,k}(\phi_1) \frac{dq_n(\phi_1)}{d\phi_1} + \dot{\phi}_1^2 \Lambda_{n,k}(\phi_1) \frac{d^2 q_n(\phi_1)}{d\phi_1^2} + \Xi_{n,k}(\phi_1) p_n(\phi_1) \right. \\ & \left. + \dot{\phi}_1 \Psi_{n,k}(\phi_1) \frac{dp_n(\phi_1)}{d\phi_1} + \dot{\phi}_1^2 \Phi_{n,k}(\phi_1) \frac{d^2 p_n(\phi_1)}{d\phi_1^2} \right] + F_{2k}(\phi_1) = 0 \end{aligned} \quad (6.6)$$

b. For the out-of-plane motion:

$$\begin{aligned} & \rho A J_K \dot{\phi}_1^2 \frac{d^2 p_k(\phi_1)}{d\phi_1^2} + c_{D2} J_K \dot{\phi}_1 \frac{dp_k(\phi_1)}{d\phi_1} + \rho A J_K (\omega_{wK}^2 - [J_1^2(\phi_1) + J_2^2(\phi_1)]) p_k(\phi_1) \\ & + \sum_{n=1}^{\infty} \left[A_{n,k}(\phi_1) p_n(\phi_1) + \Gamma_{n,k}(\phi_1) q_n(\phi_1) + \dot{\phi}_1 \Delta_{n,k}(\phi_1) \frac{dq_n(\phi_1)}{d\phi_1} \right] + F_{3k}(\phi_1) = 0 \end{aligned} \quad (6.7)$$

In case the feed rollers continue to overshoot, then equation 6-5 is still applicable.

D. INITIAL AND FINAL REGIME CONDITIONS

As for regime no. 2, the initial and final conditions for these regimes will be discussed in detail in the applications section.

VII. APPLICATION AND EXPERIMENT

A. INTRODUCTION

The theory of the dissertation is now applied to the specific example mechanism, and the numerical results are compared with the experimental ones.

The following summarizes the contents of this section and the results of the example:

Section B: Gives the simplified equations of motion which only contain the first mode of the in-plane elastic deflections for all motion regimes and describes the various transition conditions for the four-regime cycle. The elimination of the out-of-plane deflections was based on experimental findings where even for runs with the heaviest load the out-of-plane strain was of insignificant magnitude when compared to the in-plane one. Further, it became apparent from the computations that a first-mode formulation for the in-plane strain was sufficient to adequately describe the behavior of the coupler and with that the motion of the feed rollers and the metal strip.

Section C: Discusses the features of the comparison criteria that were selected to verify the mathematical model and at the same time help gain a thorough understanding of the operation of

this machine. These are, the in-plane coupler strain, the windup angle ϕ_{54} , as well as the total feed-roller angle ϕ_{5tot} .

Section D: Outlines how solutions to the various differential equations of the four-regime cycle are obtained through the use of program FEED which utilizes a fourth-order double-precision Runge-Kutta routine, by way of four different pairs of external functions. Further, details of the numerical solution concerning computation initialization conditions and time step size for the numerical integration, are given.

Section E: Gives the extent of the test runs, both computational and experimental, along with all the physical parameters involved, and presents results of the numerical solution of the simplified equations for an appropriately chosen feed condition. The feed-roller rotation, the windup angle, the difference between the elastic and rigid rocker angle forward rotations as well as the in-plane traces at $x_1 = 11$ inches are obtained for a complete feed cycle. In addition, the elastic angle ϕ_{4e} is obtained both by way of the Taylor series approximation and the exact approach, and is used to demonstrate the validity of the approximation.

Section F: Describes the experimental setup along with the necessary instrumentation.

Section G: Establishes the validity of the mathematical model by comparing various computed runs to their associated experimental ones. The comparison is made principally with

the help of the criteria established in section C. In addition, the forward feed-roller and rocker angles are compared. Finally the effect of loading on the total forward rocker angle is considered. The experiment confirms the four-regime scheme of the mathematical model. A quantitative comparison indicates good agreement throughout the operating range.

Section H: Discusses the results of this investigation and attempts to put analytical and experimental results into proper focus.

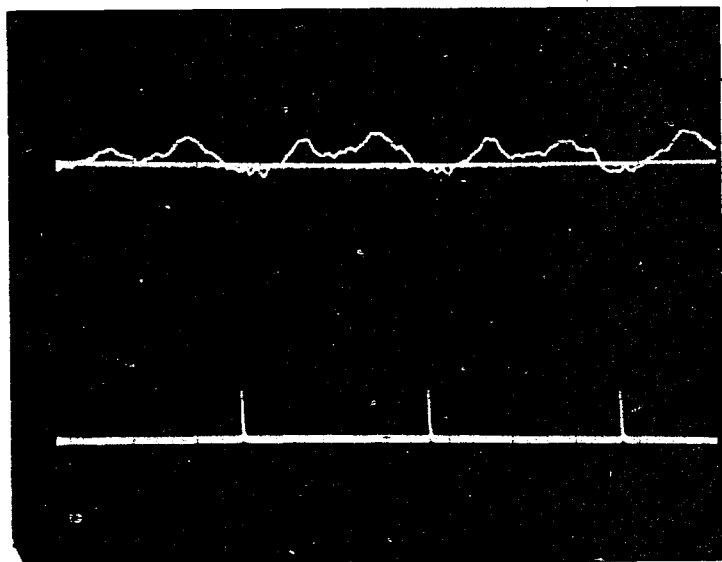
B. SIMPLIFIED REGIME EQUATIONS AND CONTROL CONDITIONS

During the course of the experimental work it was found that even for the runs with the heaviest load, the out-of-plane coupler strains are of insignificant magnitude, when compared to the in-plane ones. Therefore, they may be neglected in all computations as well as the experiment. Figure 7.1 shows out-of-plane as well as in-plane strain traces for such a heavy-load run. The peak-to-peak out-of-plane strain is approximately 5 microstrains, while the peak to peak in-plane one is approximately 185 microstrains. Note that throughout this work all strains are computed and experimentally obtained at $x_1 = 11$ inches.

In addition, it became apparent from the computations that a first mode formulation for the in-plane strain is sufficient to adequately describe the behavior of the coupler and with that the motion of the feed rollers and the metal strip. This is demonstrated by the data of tables 7.1 and 7.2, where both heavy and light load numerical results are given for:

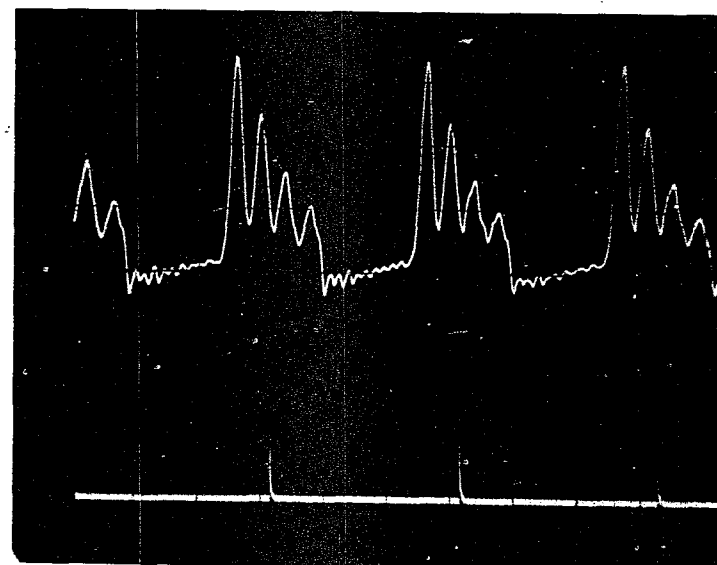
- i. A two-mode formulation which takes modal coupling into account.
- ii. A two mode formulation that disregards modal coupling.
- iii. A first mode formulation

In line with the above experimental and computational findings, the regime equations originally presented in Sections V and VI, are now simplified to include only the first mode of the in-plane coupler



a. Out-of-Plane Strain

Horizontal Scale : 0.1 sec/div
Vertical Scale : 10 microstrains/div



b. In-Plane Strain

Horizontal Scale : 0.1 sec/div
Vertical Scale : 56 microstrains/div

Input Speed : 200 rpm
Feed Length : 0.875 in (nominal)
Strip Size : 4.00 in x 0.250 in
Brake Torque : 141 in-lbs

Figure 7.1 Comparison of In-Plane and Out-of-Plane Strains

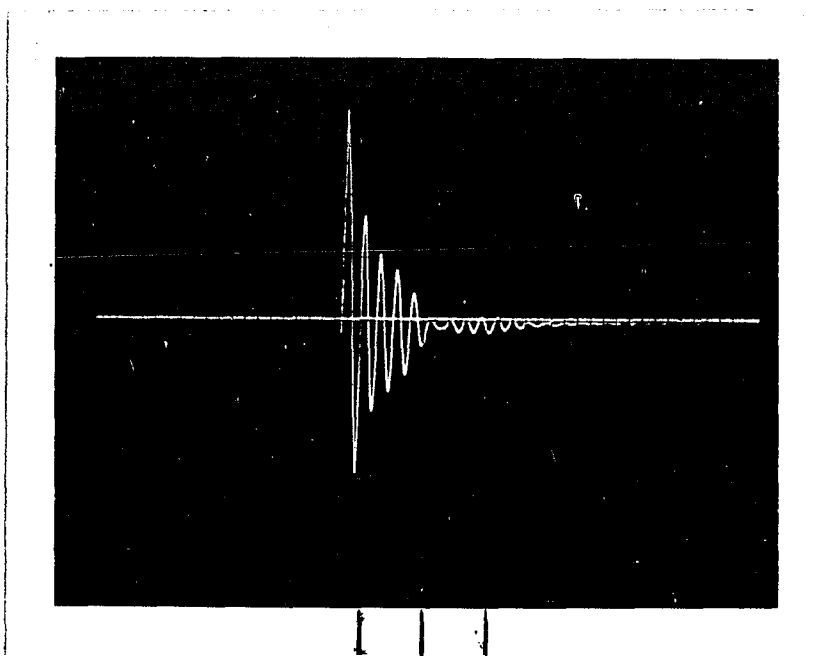


Figure 7.2 : Ringing Response of Coupler

Horizontal Scale: 0.05 sec/div

Input Angle (Degrees)	2 Modes Coupled (microstrains)	2 Modes Uncoupled (microstrains)	Fundamental Mode (microstrains)
0	31.7	31.0	32.6
10	12.5	13.7	13.1
20	20.3	19.5	20.9
30	13.9	14.0	14.4
40	5.1	5.7	5.4
50	9.9	9.2	10.1
60	2.1	2.5	2.2
70	0.1	0.2	0.1
80	0.6	0.7	0.7
90	-22.9	-22.4	-22.5
100	-12.6	-12.4	-12.8
110	-23.1	-23.3	-23.8
120	-12.7	-12.7	-13.0
130	-19.5	-19.5	-19.8
140	-12.0	-12.0	-12.4
150	-13.5	-13.5	-13.0
160	-9.1	-9.1	-9.5
170	-6.3	-6.3	-5.7
180	-3.5	-3.5	-4.0
190	2.1	2.0	1.8
200	2.8	2.7	2.5
210	8.2	8.2	8.6
220	9.0	9.0	9.2
230	13.9	13.9	14.3
240	14.5	14.5	15.0
250	17.6	15.6	18.0
260	34.8	34.6	35.2
270	32.7	32.5	33.1
280	74.8	74.3	76.5
290	15.6	16.7	16.9
300	56.5	55.6	57.8
310	45.9	45.8	47.3
320	23.2	24.3	24.4
330	50.5	49.1	51.7
340	24.4	25.4	25.4
350	26.7	26.7	27.6
360	31.7	31.0	32.6

TABLE 7.1 In-Plane Strain at $x_1 = 11$ in.200 rpm; $T_{BR} = 60$ in-lb; $a_1 = 0.775$ in; Light Strip.

Input Angle (Degrees)	2 Modes Coupled (microstrains)	2 Modes Uncoupled (microstrains)	Fundamental Mode (microstrains)
0	162.1	162.1	162.1
10	122.9	123.1	124.3
20	48.8	48.0	49.2
30	0.2	0.2	-0.2
40	-20.6	-20.7	-20.7
50	-4.0	-4.0	-4.2
60	7.9	7.9	7.9
70	42.2	42.3	42.2
80	37.5	37.7	37.5
90	-3.9	-3.9	-3.9
100	-49.5	-49.5	-49.3
110	-49.8	-49.9	-49.5
120	-24.5	-24.5	-23.8
130	-50.4	-50.5	51.0
140	-29.1	-29.0	-29.2
150	-21.1	-21.1	-20.7
160	-33.4	-33.4	-33.0
170	-5.7	-5.7	-5.8
180	-7.6	-7.6	-7.3
190	-5.0	-5.0	-5.5
200	15.9	15.9	16.9
210	12.2	12.2	12.0
220	20.8	20.7	21.2
230	35.9	35.8	35.5
240	30.2	30.1	30.3
250	39.8	39.8	40.4
260	47.2	47.1	46.6
270	50.4	50.3	50.4
280	123.9	123.7	124.1
290	261.0	260.5	260.9
300	273.8	273.4	275.0
310	179.7	179.6	182.4
320	69.1	69.1	70.4
330	38.9	38.9	39.7
340	68.3	68.3	68.7
350	102.1	102.2	102.3
360	162.1	162.1	162.1

TABLE 7.2 : In-Plane Strain at $x_1 = 11$ in.300 rpm; $T_{BR} = 211$ in-lb; $a_1 = 0.775$ in; Heavy Strip

deflections. In addition, the following gives the appropriate regime transition conditions, necessary for the numerical solution of these equations for the four-regime cycle.

- a. Regime No. 1: Forward Motion of the Rocker during Tightening of the Clutch Spring (Feed Rollers are Stationary).

1. Differential Equations of Motion

With the feed rollers stationary, the reduced differential equation for the elastic deflection of the coupler link takes the following form according to equation 6.1:

$$\begin{aligned} \ddot{\varphi}_1^2 \left[\rho A U_1 + \Lambda_{11}(\varphi_1) \right] \frac{d^2 q_1}{d\varphi_1^2} + \dot{\varphi}_1 \left[c_0 U_1 + \Theta_{11}(\varphi_1) \right] \frac{dq_1}{d\varphi_1} + \left[-k_{sp} (\varphi_5 - \varphi_{4e})^{3/2} \cdot J_{4(1,1)} \right. \\ \left. + Z_{11}(\varphi_1) \right] q_1 + \rho A U_1 (\omega_1^2 - J_3^2(\varphi_1)) q_1 + F_{21}(\varphi_1) - k_{sp} (\varphi_5 - \varphi_{4e})^{3/2} \cdot J_{51}(\varphi_1) = 0 \end{aligned} \quad (7.1)$$

2. Important Conditions for Regime No. 1

i. Initial Conditions for the Coupler

At the beginnig of this regime, the rocker is located at its maximum angle $\varphi_{4e\max}$, and with that its angular velocity

$$\dot{\varphi}_{4e} = 0 \quad (7.2)$$

In the start-up cycle this is a rigid body velocity, while in all subsequent cycles the elastic angular velocity is used.

The initial conditions for the in-plane deflection of the start-up cycle are given by:

$$q_1(\varphi_1 \text{ at } \varphi_{4\max}) = 0 \quad (7.3)$$

and

$$\frac{dq_1}{d\varphi_1}(\varphi_1 \text{ at } \varphi_{4\max}) = 0 \quad (7.4)$$

(See discussion pertaining to computer program in section D below).

Once the start-up cycle has been completed, the above initial conditions are changed to the terminal values of $q_1(\varphi_1)$ and $\frac{dq_1}{d\varphi_1}(\varphi_1)$ of regime no. 4.

ii. Value of Feed-Roller Angle φ_5 during Regime No. 1

The value of the feed-roller angle φ_5 equals that of $\varphi_{4\max}$ until the end of the regime, i.e. until the feed rollers begin to move. This is necessary to describe the buildup of the clutch torque properly.

iii. Termination of Regime No. 1

The motion equation 7.1 is applicable until that time when the torque transmitted by the clutch is equal to or exceeds the brake torque T_B , i.e.

$$K_{SP} (\varphi_5 - \varphi_{4e})^{3/2} \geq T_B \quad (7.5)$$

At this point in the cycle, the values of $q_1(\varphi_1)$ and $\frac{dq_1}{d\varphi_1}(\varphi_1)$ must be stored to serve as initial conditions for the coupler deflections in regime no. 2.

b. Regime No. 2: Forward Motion of Strip (Both the Rocker and the Feed Rollers Advance)

1. Differential Equations of Motion

The coupler deflection differential equation for this part of the feed cycle is simplified to have the following form according to equation 5.11:

$$\begin{aligned} \dot{\phi}_1^2 \left[\rho A \mathcal{U}_1 + \Lambda_{1,1}(\phi_1) \right] \frac{d^2 q_1}{d\phi_1^2} + \dot{\phi}_1 \left[c_{D1} \mathcal{U}_1 + \Theta_{1,1}(\phi_1) \right] \frac{dq_1}{d\phi_1} + \left[K_{SP} (\phi_5 - \phi_{4e})^{3/2} \right. \\ \left. - c_D \left(\dot{\phi}_1 \frac{d\phi_5}{d\phi_1} - \dot{\phi}_{4e} \right) \right] J_{41,1}(\phi_1) + Z_{1,1}(\phi_1) q_1 + \rho A \mathcal{U}_1 \left[\omega_{11}^2 - J_3^2(\phi_1) \right] q_1 \\ + F_{2,1}(\phi_1) - \left[K_{SP} (\phi_5 - \phi_{4e})^{3/2} + c_D \left(\dot{\phi}_1 \frac{d\phi_5}{d\phi_1} - \dot{\phi}_{4e} \right) \right] J_{51}(\phi_1) = 0 \end{aligned} \quad (7.6)$$

Since the feed rollers, together with the metal strip being fed, now move also, the following additional differential equation for the motion of these components becomes applicable (according to 5.13):

$$J_{05} \ddot{\phi}_1 \frac{d^2 \phi_5}{d\phi_1^2} + c_D \left(\dot{\phi}_1 \frac{d\phi_5}{d\phi_1} - \dot{\phi}_{4e} \right) + K_{SP} (\phi_5 - \phi_{4e})^{3/2} + \frac{\dot{\phi}_5}{|\dot{\phi}_5|} T_B = 0 \quad (7.7)$$

Note that the above expression is coupled to the coupler differential equation by way of the elastic rocker angle ϕ_{4e} and its derivative $\dot{\phi}_{4e}$ (see equation 5.20, which expresses ϕ_{4e} in terms of $q(\phi_1)$).

Recall that, J_{05} accounts for the moment of inertia of the feed-roller/pressure-roller assembly together with the referred moment of inertia of the metal strip and the pressure rollers.

T_B accounts for both the retarding torque exerted by the brake as well as the one due to the friction force on the metal strip as it slides on the feed table.

2. Important Conditions for Regime No. 2

i. Initial Conditions for the Coupler

The initial conditions for the coupler deflection, in regime no. 2, are provided by the terminal values of the zeroth and the first derivatives of q_1 of regime no. 1.

ii. Initial Conditions for the Feed Roller Angle φ_5

To accommodate the previously given definition of the feed-roller angle φ_5 in Section VII-B-a-2-ii, the initial conditions for equations 7.6 and 7.7, must be:

$$\varphi_5(\varphi_1) = \varphi_{4\text{emax}} \quad (7.8)$$

$$\frac{d\varphi_5}{d\varphi_1}(\varphi_1) = 0 \quad (7.9)$$

iii. Termination of Regime No. 2

Both coupler and feed-roller motions are terminated at the minimum rocker angle $\varphi_{4\text{emin}}$, which coincides with zero rocker angular velocity:

$$\dot{\varphi}_{4e} = 0$$

At this point in the cycle, the values of the zeroth and the first derivatives of the coupler variable $q_1(\phi_1)$ must be stored to be available as the initial conditions for the coupler deflection of regime no. 3.

In case the feed rollers overshoot, i.e. their angular velocity is not zero at the rocker return point, both their position and velocity must be stored to serve as the initial conditions for the independent feed-roller motion in regime no. 3.

c. Regime No. 3: Return Motion of Rocker, Sprag-Clutch Unwinding and Possible Overshooting of Feed Rollers

1. Differential Equations of Motion

The coupler differential equation for this part of the feed cycle is identical to the one given by equation 7.1. Further, the feed-roller equation becomes independent of the coupler equation and confines itself to the possible overshoot rotation.

i. Feed Roller Overshoot Equation

The independent expression for the overshooting of the feed-roller assembly, together with the metal strip being fed, is also given in terms of the angle ϕ_5 , defined as a function of the throwblock angle ϕ_1 (see change of variable discussion in Section V):

$$J_{05} \ddot{\phi}_1^2 \frac{d^2 \phi_5}{d\phi_1^2} + \frac{\phi_5}{|\phi_5|} \tau_B = 0 \quad (7.10)$$

2. Important Conditions for Regime No. 3

i. Initial Conditions for the Coupler

The initial conditions for the coupler deflection of regime no. 3 are provided by the stored terminal values of the zeroth and first derivatives of the variable $q_1(\phi_1)$ of regime no. 2 (see Section VII-B-b-2-iii).

ii. Initial Conditions for the Feed Roller Motion

In case there is overshooting of the feed rollers, the initial conditions for equation 7.10 are identical to the terminal ones as obtained from the solution of equation 7.7 for the previous regime.

The computation of $\phi_5(\phi_1)$ is terminated once the angular velocity of the feed rollers becomes zero.

iii. Termination of Regime No. 3

This regime terminates once the clutch has completely unwound,
i.e.

$$\phi_5 - \phi_{4e} = 0 \quad (7.11)$$

At this point, the values of the first and zeroth derivatives of q_1 must be stored to be used later as the initial conditions for the coupler deflections of the fourth regime.

In case the feed rollers continue to overshoot, i.e. their angular velocity has not yet become zero, both their position as well as their velocity must be stored to serve as initial conditions for the independent feed roller motion of regime no. 4.

d. Regime No. 4: Return Motion of Rocker and Possible
Overshooting of the Feed Rollers

1. Differential Equations of Motion

While the coupler differential equation for this part of the feed cycle is similar to equation 7.1, it does not contain any clutch torque terms. Thus, simplification of equation 6.6 furnishes:

$$\ddot{\varphi}_1 \left[\rho_A \mathcal{J}_1 + \Lambda_{11}(\varphi_1) \right] \frac{d^2 q_1}{d\varphi_1^2} + \dot{\varphi}_1 \left[c_{D1} \mathcal{J}_1 + \Theta_{11}(\varphi_1) \right] \frac{dq_1}{d\varphi_1} + \rho_A \mathcal{J}_1 \left\{ \omega_{V1}^2 - [\mathcal{J}_3^2(\varphi_1) + \right. \\ \left. + \frac{Z_{n,k}(\varphi_1)}{\rho_A \mathcal{J}_1} \right\} q_1 + F_{21}(\varphi_1) = 0 \quad (7.12)$$

In case the feed rollers continue to overshoot even after the end of the third regime, their motion will be described by an equation identical to 7.10.

2. Important Conditions for Regime No. 4

i. Initial Conditions

The initial conditions for the coupler deflection of regime no. 4 are provided by the stored terminal values of the zeroth and first derivatives of the variable q_1 of regime no. 3 (see Section VII-B-c-2-iii).

In case there is overshooting of the feed rollers, the initial conditions for equation 7.12 are identical to the terminal ones of the previous regime. As mentioned earlier, the computation of $\varphi_5(\varphi_1)$ is terminated once the angular velocity of the feed rollers becomes zero.

iii. Termination of Regime No. 4

With the assumption that the overshooting of the feed rollers has ended before the return of the rocker is completed, the termination conditions for regime no.4 involve only the latter. Thus, regime no. 4 will conclude when:

$$\varphi_{4e}(\varphi_1) = \varphi_{4emax} \quad (7.13)$$

and

$$\dot{\varphi}_{4e} = 0 \quad (7.14)$$

Finally, to provide for the initial values of the variable q_1 for regime no. 1, in the next and subsequent feed cycles, the final values of this variable and its derivative in regime 4 must be stored.

C. SELECTION OF COMPARISON CRITERIA

To best verify the mathematical model and at the same time gain a thorough understanding of the operation of the machine, certain criteria for the comparison between computational and experimental results were introduced. These are the in-plane strain, the windup angle ϕ_{54} , as well as the total feed-roller angle ϕ_{5tot} , and the associated feed length.

a. In-Plane Coupler Strain

In-plane coupler strain histories are obtained experimentally and by computation for a position near the endpoint of section 1 (at $x_1 = 11$ in)^{*} and inside the bend, i.e. on the positive side of the y_1 -axis (see figure 7.10). Thus, an extension of the line AC will cause a positive strain. This coupler strain curve clearly shows how the four motion regimes differ and how each is influenced by the loading conditions (speed, strip size, and brake setting) of the various runs.

This maximum in-plane bending strain is computed according to:

* The particular location has been chosen because it was inappropriate to attach a strain gage at the location of the bend where the maximum strain occurs. It is to be noted that since good feed performance is assured when the elongation of line AC is kept as low as possible, the maximum stress and with that the associated strain must be kept low.

$$\varepsilon_v(x_1, \phi_1) = [1/EI] * M_v(x_1, \phi_1) y_{\max} \quad (7.15)$$

where:

x_1 = point on the first coupler section ($x_1 = 11$ in)

$M_v(x_1, \phi_1)$ = in-plane bending moment at x_1

$y_{\max}$ = outer fiber radius of the coupler cross section

Using elementary stress theory, equation 7.15 becomes:

$$\varepsilon_v(x_1, \phi_1) = -y_{\max} V_{11}''(x_1) q_1(\phi_1) \quad (7.16)$$

With $q_1(\phi_1)$ obtained from the solution of the differential equations, and $V_{11}''(x_1)$ known from the subproblem, equation 7.16 furnishes the maximum bending strain at any coupler location.

b. Windup angle ϕ_{54}

The windup angle is defined as the difference between the net forward clutch-input angle ϕ_{4e}^* and the net forward clutch-output angle ϕ_5^* , during the forward (feeding) motion of the rocker link, i.e.

$$\phi_{54} = \phi_5^* - \phi_{4e}^* \quad (7.17)$$

where: $\phi_5^* = \phi_{4e\max} - \phi_5$

$$\phi_{4e}^* = \phi_{4e\max} - \phi_{4e}$$

During proper operation of the mechanism, the feed-roller angle lags the rocker angle and ϕ_{54} becomes negative by its definition. Pathological clutch behavior, i.e. non-repeatable feeding, occurs when this quantity becomes positive. While this relative angle serves as a

useful means of verifying the model, it is principally a design criterion by which repeatable feed operation can be predicted.

c. Total Angle $\phi_{5\text{tot}}$ and Feed Length

With the assumption of zero slip between the feed rollers and the stock, the angle $\phi_{5\text{tot}}$ is directly related to the feed length. It is important for design purposes to establish which parameters influence the repeatability of this quantity. Its proper value is obtained by taking the difference between the initial value of the feed-roller angle ϕ_5 , at the beginning of regime no. 1, and its final value, at the end of regime no. 2.

D. COMPUTER PROGRAM FEED

The constant changing of regimes and the consequent unavoidable appearance of transients, along with the presence of nonlinear terms in the equations, made the use of a numerical integration scheme very attractive for their solution. Appendix D shows how program FEED adapts an existing double precision fourth-order Runge-Kutta routine* to the four individual regimes. The various parts of this program and their relationship to the regime equations, together with their associated initial and terminal conditions as given in Section VII-B, are thoroughly discussed, and a sample run is reproduced. It shows the manner in which the input and the output of the program appear and it forms the basis for the discussion in section F below.

While the sample run only considers the first mode of the in-plane coupler deflection, the program is capable of simultaneously solving up to four modes of the mutually coupled differential equations for the in-plane coupler deflections v_i , together with the feed-roller angle ϕ_5 . Recall that the coupler differential equations are in-terms of $q_k(\phi_1)$, i.e. the time related portions of the product solutions, which contain both time and space portions. It might be noted that while the present investigation deals with the specific

* DRKGS Routine, IBM SYSTEM/360 Scientific Subroutine Package

bent-coupler design, the program is capable of accomodating any two-section design, including one where the coupler is straight. This could be of importance in a design optimization where other coupler shape criteria are allowed.

The discussion pertaining to the initial conditions of regime no. 1 (shown in Section VII-B), indicated that for the start-up cycle,

$$q_1(\varphi_1 = \varphi_{4\max}) = 0 \quad (7.18)$$

$$\frac{dq_1}{d\varphi_1}(\varphi_1 = \varphi_{4\max}) = 0 \quad (7.19)$$

i.e. the computations are started when the rocker link is at its maximum rigid-body angle. It is further shown that in all subsequent cycles the above initial conditions are changed to the terminal values of $q_1(\varphi_1)$ and its derivative of regime no. 4.

The above choice for the start-up cycle presents a certain difficulty. Since the differential equations are derived with the assumption that the mechanism is already running at constant speed, there must be non-zero values associated with the conditions given by equations 7.18 and 7.19. Of course, it is not possible to know these values before the computations start and therefore the model will only furnish a true response after three to four cycles of computations have passed and the terminal conditions of regime no. 4 have forced out the effects of the faulty initial conditions of the first cycle.

E. TEST RUNS AND SAMPLE COMPUTATIONS

Before the sample computations can be shown, all physical parameters must be enumerated and the extent of the test runs, both computational and experimental, has to be stated.

a. Machine and Material Parameters

Table 7.3 lists the various machine and material parameters. The computed fundamental frequency of the coupler was found to be 94 Hz. When the coupler was struck, its fundamental frequency was found to be 87 Hz (see figure 7.2). Since the concentrated end masses of the ball joints were not considered in the theoretical development, this difference is not surprising. Therefore, the experimentally obtained values will be used in the computations. Figure 7.2 also served to determine the coupler damping ratio. Application of the logarithmic decrement method to this strain trace resulted in a value of $\zeta \approx 0.05$.

With regard to the sprag clutch, its stiffness coefficient was taken to be 170,000 in-lbs-rad^{-3/2} (see figure 3.4 and subsequent discussion), and its damping ratio was chosen to be equal to 0.09, based upon best fit with experimental data. The damping constant C_D was then obtained with the help of the following linearized natural frequency estimate of the clutch:

<u>Link Lengths and Frame Dimensions</u> (see figure I-1 in appendix I)		<u>Coupler Dimensions and Physical Parameters</u> (see figure I-2)	
a_1 = variable		L_1 = 16.562 in	
AC = a_3 = 32.400 in		L_2 = 16.562 in	
a_4 = 4.5 in		α = 12.000 degrees	
s_1 = 1.171 in		β = 12.000 degrees	
s_2 = 7.951 in		Tube O.D. = 1.000 in	
s_3 = 31.478 in		Tube I.D. = 0.750 in	
		E = 30×10^6 psi	
		P = 7.324×10^{-4} lb-sec ² -in ⁻⁴	
		ω_v (computed) = 94 Hz	
		ω_v (experimental) = 87 Hz	
		Damping Ratio ζ : 0.05	
<u>Other Parameters</u>			
Feed Roller Radius	:	2.499 in	
Combined clutch outer race-rocker link moment of inertia J_{04}	:	0.0915 lb-sec ² -in	
Clutch inner race-feed roller shaft moment of inertia J_{05}^I	:	0.0808 lb-sec ² -in	
Feed Table Friction Torque T_{FR}	:	variable	
Brake Torque T_{BR}	:	variable	
Clutch Stiffness Coefficient K_{SP}	:	170,000 in-lbs-rad ^{-3/2}	
Clutch Damping Ratio ζ	:	0.09	

Table 7.3 Machine and Material Parameters

$$C_D = 2 [J_{05} k_{lin}]^{1/2} \quad (7.20)$$

where

- J_{05} : Combined feed roller-pressure roller-metal strip moment of inertia for a given run (see Table 7.4 below)
- $k_{lin} = 16,690$: Stiffness coefficient of the linearized (in-lbs-rad⁻¹) clutch (obtained by straight line regression, not shown)

b. Test Runs and Loading Conditions

The computational and experimental test runs were arranged in such a way as to cover an adequate portion of the permissible loading conditions of the system. A number of factors influence these loading conditions:

- i. input speed
- ii. metal strip size
- iii. feed length (dependent on input crank length a_1)
- iv. brake torque

All test runs were made with two types of strips, each 12 feet long.

Their cross-sections were as follows:

- i. 1.00 in x 0.125 in
- ii. 4.00 in x 0.250 in

Table 7.4 shows for each of these strips the experimentally determined table friction torque T_{FR} and the strip-pressure roller moment of inertia J''_{05} , both referred to the feed-roller shaft. In addition, the combined moment of inertia, $J_{05} = J''_{05} + J'_{05}$, is given. (See table 7.3 for J'_{05}).

The brake torque used in the test runs was taken from associated experiments, where its magnitude was determined by increasing its value until no overshooting was observed and the feed results were reliably repetitive. For design purposes, the brake torque must be chosen such as to definitely keep the windup angle negative.

Table 7.7, in section G below, lists all test runs, both computational and experimental, in ascending order of load severity (where this severity was determined by the magnitude of the product of the individual J_{05} and the maximum angular acceleration of the rocker).

Metal Strip Cross-Section in x in	Table Friction Torque: T_{FR} in-lbs	Referred Strip- Pressure Roller Moment of Inertia J''_{05} lb-sec ² -in	Combined Moment of Inertia $J_{05} = J''_{05} + J'_{05}$ lb-sec ² -in
1.00 x 0.125	4	0.0995	.1803
4.00 x 0.250	40	0.6874	.7682

Table 7.4 : Metal Strips and Strip Dependent Parameters

c. Sample Computations

This section principally deals with the evaluation of the sample computer output, as given by table D-1 of appendix D on page 191. These data are for run no. 5 of table 7.7 on page 117. It is useful to refer to the section on "interpretation of output" in appendix D.

Table D-1 shows the following variables to be available over a complete mechanism cycle as functions of the input angle ϕ_1 :

- i. The in-plane coupler strain at $x_1 = 11$ in (EPSL)
- ii. the rigid body angle ϕ_4 of the rocker (PHI4R)
- iii. the elastic angle ϕ_{4e} of the rocker (PHI4)
- iv. the feed roller angle ϕ_5 (PHI5)

(The velocity shown in table D-1 is presently of no importance).

These data are primarily used to plot the curves associated with the comparison criteria, i.e. the in-plane strain as well as the windup angle ϕ_{54} (see Section VII-C).

In addition, the total angle ϕ_{5tot} will be used to obtain the resulting feed length. Further, it will be shown that the actual elastic rocker angle ϕ_{4e} differs from its rigid-body counterpart. This points up the importance of using the elastic angle in the clutch torque expression.

Certain results for PHI4 from runs 1 and 6 (table 7.7) are subsequently used to justify the Taylor series approximation.

They are compared to results obtained by way of the elastic constraint equation.

1. Windup Angle

For the purpose of logical development, it is best to discuss the behavior of the clutch first. To this end, one considers the history of the windup angle ϕ_{54} which is given in figure 7.3 as a function of the input angle ϕ_1 (for definition see equation 7.17), it extends through regimes 1 (R_1) and 2 (R_2), i.e. the feeding portion of the cycle. Using the clutch tightening data of table D-1, the figure shows that the tightening of the clutch starts at $\phi_1 = 256^\circ$, the beginning of the forward motion of the rocker, and ends at $\phi_1 = 275^\circ$, where the feed rollers start to move (regime 1). The maximum windup angle is reached at $\phi_1 = 290^\circ$ and is equal to 1.023 degrees. Beyond this point, the windup angle oscillates with decreasing amplitude until the end of the forward motion of the rocker is reached, which corresponds to the input angle of 435° (actually 75°). The frequency of this oscillation ranges from 25 to 27 Hz.

While the clutch is clearly non-linear, one may roughly determine its frequency by considering a linearized system whose mass moment of inertia is influenced by the strip size. This was done successfully in all runs. Thus, for the present system, with:

$$J_{05} = 0.768 \text{ lb-sec}^2\text{-in} \quad (\text{see table 7.4}), \text{ and}$$

$$k_{lin} = 16,690 \text{ in-lb-rad}^{-1} \quad (\text{obtained by straight-line regression on the data given in figure 3.3})$$

The natural frequency of the linearized clutch becomes:

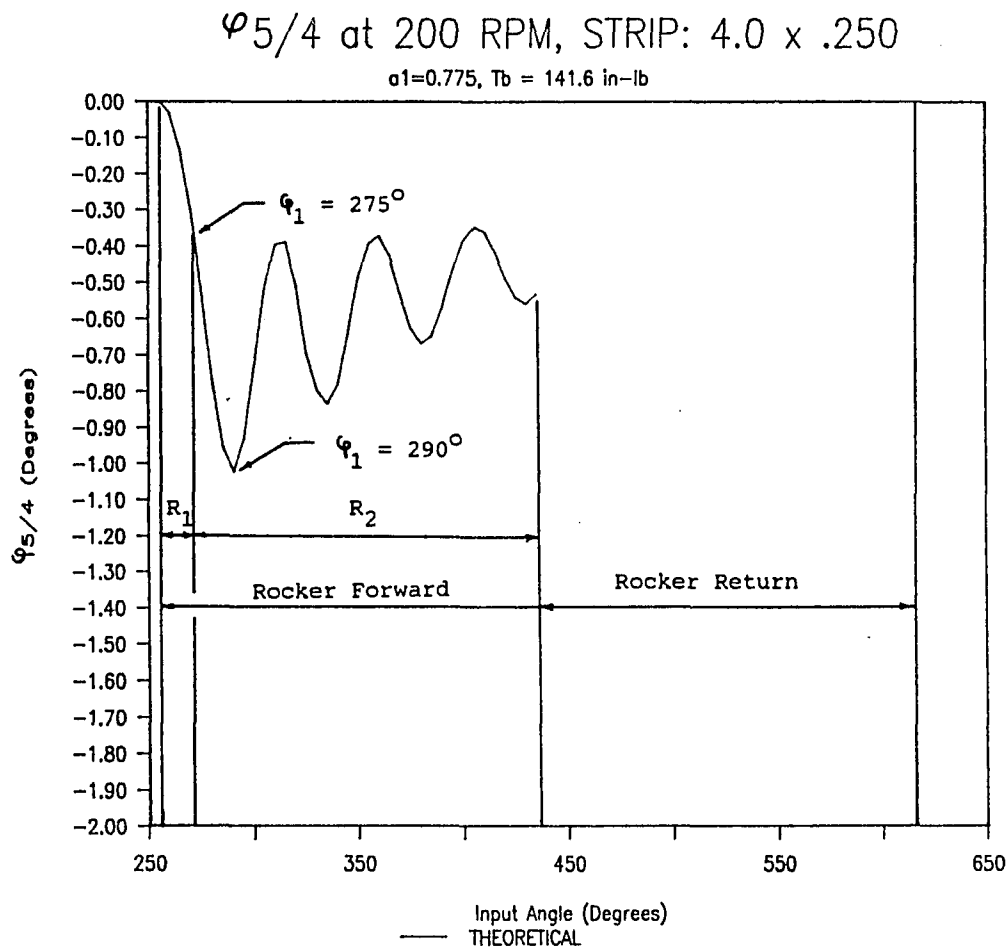


Figure 7.3 : Windup Angle $\varphi_{5/4}$ (theoretical)

Input Speed : 200 rpm
 Feed Length : 0.875 in (nominal)
 Strip Size : 4.000 in x 0.250 in
 Brake Torque : 141 in-lbs

$$f_{lin} = [1/2\pi] \cdot [16,690/0.768]^{1/2} = 23.5 \text{ Hz} \quad (7.21)$$

Finally, note that in order to avoid loss of control, the clutch must be tightened throughout the forward portion of the cycle, i.e. ϕ_{54} must be negative. A positive value would imply that the sprags are in the overrunning mode.

Also note that the figure shows that at the end of regime no. 2 there still exists clutch windup, which finally is released in regime no. 3.

2. In-Plane Strain of Coupler at $x_1 = 11$ in.

Figure 7.4 shows a plot of the maximum bending strain of the coupler link at $x_1 = 11$ in., for a complete cycle, as a function of the input angle ϕ_1 . It indicates that the rocker forward motion, i.e. regimes 1 and 2, extends from $\phi_1 = 256$ degrees to $\phi_1 = 435^\circ$ (actually 75°). The return represents regimes no. 3 and 4, where regime no. 3 extends from $\phi_1 = 435^\circ$ to $\phi_1 = 455^\circ$, and regime no. 4 completes the cycle.

The positive bending strain reaches its maximum at $\phi_1 = 290^\circ$, which also corresponds to the maximum windup angle of the clutch. It continues positive during regimes 1 (R_1) and 2 (R_2) and remains positive for most of regime 3 (R_3). The necessity of introducing regime 3, i.e. a portion in the cycle where the clutch is allowed to unwind, was made clear by the experimental results. Once the clutch

STRAIN AT 200 RPM, STRIP: 4.0 x .250

$\sigma_1=0.775$, $T_b = 141$ in-lb

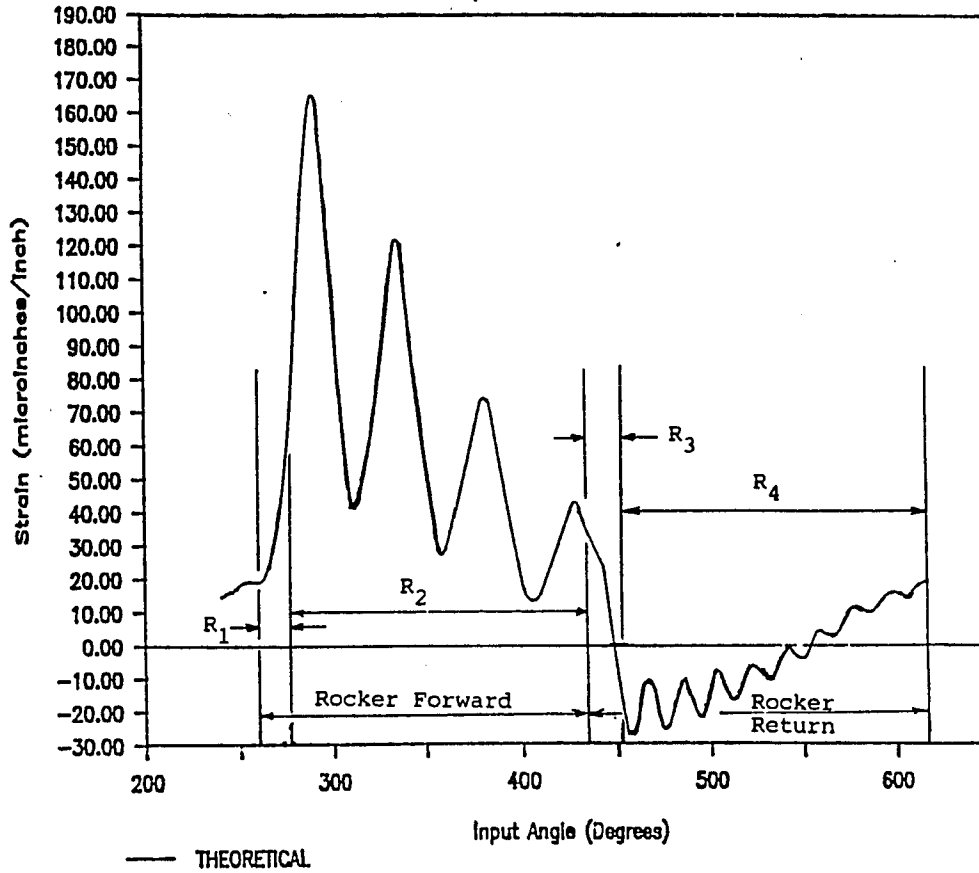


Figure 7.4 : In-Plane Strain (theoretical)

Gage Location : $x_1 = 11$ in

Input Speed : 200 rpm

Feed Length : 0.875 in (nominal)

Strip Size : 4.000 in x 0.250 in

Brake Torque : 141 in-lbs

spring is fully unloaded and regime no. 4 (R_4) starts, the bending strain decreases in magnitude until it reaches its maximum negative value at $\varphi_1 = 455$ (actually 95) degrees. From this point on, it continues in an upward oscillatory manner until the end of the return stroke of the rocker.

An investigation concerning the frequencies of the oscillations revealed that the frequency in the forward stroke is identical to that found for the wind-up angle φ_{54} , i.e. 25-27 Hz. This implies that the clutch torque is acting as a forcing function on the coupler. During the return stroke the frequency of oscillation exhibited a slight variation around an average value of 68.5 Hz. With the coupler beam fundamental frequency at 87 Hz (see figure 7.2), it is apparent that the link does not vibrate at its natural frequency. This may be explained by the following:

If one divides equation 7.12, which describes the elastic deflection of the coupler link during regime no.4, by the coefficient of its second derivative term, one obtains the following form:

$$\frac{d^2 q_1}{d\varphi_1^2} + A(\varphi_1) \frac{dq_1}{d\varphi_1} + [(\omega^*/\dot{\varphi}_1)^2 + D(\varphi_1)] q_1 = -F(\varphi_1) \quad (7.22)$$

where, $\omega^* = \omega_{v1} / [1 + B(\varphi_1)]^{1/2}$, and (7.23)

$A(\varphi_1)$, $B(\varphi_1)$, $D(\varphi_1)$, $F(\varphi_1)$ are functions of φ_1 .

The above is a forced Hill-type equation which represents a system whose natural frequency is equal to ω^* . $B(\varphi_1)$ is a positive

quantity that changes relatively little with the input angle ϕ_1 and depends mostly on the coupler angles α and β . With an average value of 0.6125 for $B(\phi_1)$, and $\omega_{v1} = 87$ Hz, equation 7.23 becomes:

$$\omega^* = 0.7875\omega_{v1} = 68.5 \text{ Hz} \quad (7.24)$$

3. Total Feed Roller Rotation $\phi_{5\text{tot}}$

Figure 7.5 shows the feed-roller angle ϕ_5 as a function of the input angle ϕ_1 . The total feed-roller rotation $\phi_{5\text{tot}}$ corresponds to the value of the feed-roller angle at the end of the forward stroke and is equal to 19.824 degrees, which with a feed-roller radius of 2.499 in. furnishes a feed length of 0.8646 inches ($a_1 = 0.775$ in).

4. Angle $\phi_{4e/R}$, Deviation of The Elastic Rocker Angle from The Rigid Body One

To judge the behavior of the rocker link, due to the pivot to pivot elastic elongation of the coupler and at the same time demonstrate the need for the consideration of the elastic angle in the clutch torque expression, it has been found practical to consider the deviation of the elastic angle ϕ_{4e} from its rigid-body counterpart. Figure 7.6 gives such a plot of,

$$\phi_{4e/R} = \phi_{4e} - \phi_4 \quad (7.20)$$

as a function of the throwblock angle ϕ_1 . The rigid-body angle is obtained from equations I-9 and I-10, in appendix I, where AC

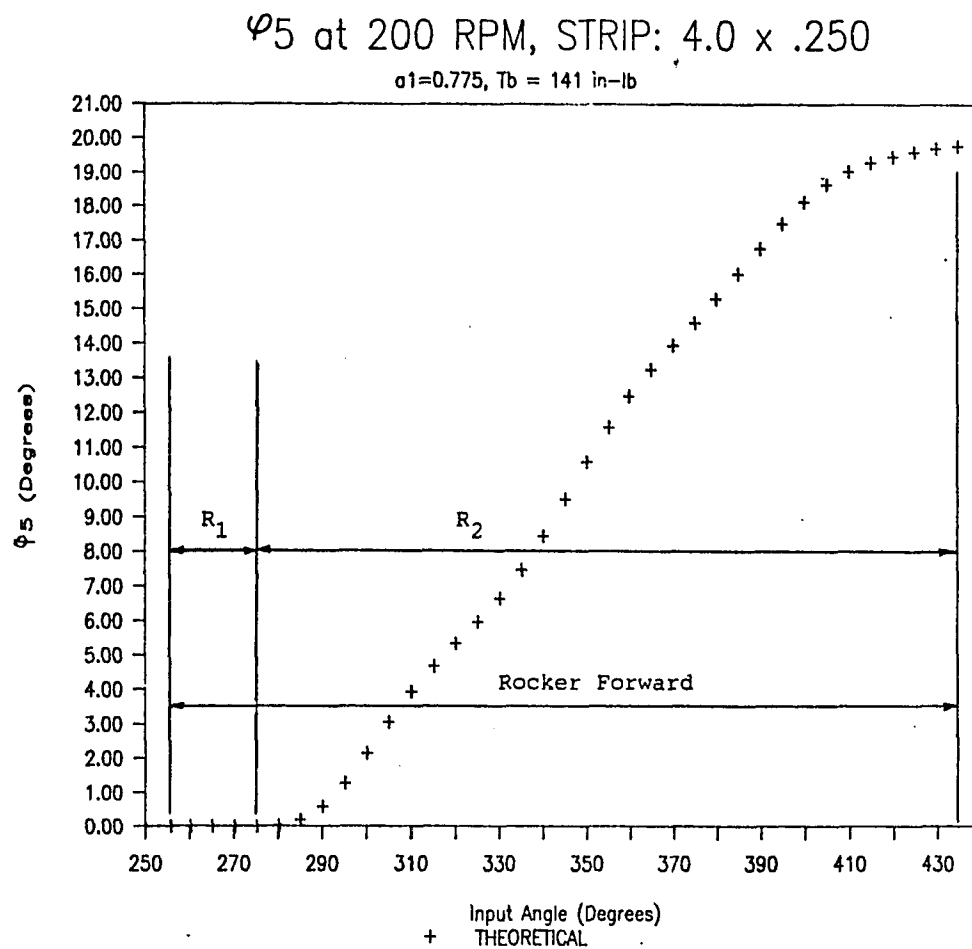


Figure 7.5 : Feed-Roller Angle ϕ_5 (theoretical)

Input Speed : 200 rpm
 Feed Length : 0.875 in (nominal)
 Strip Size : 4.000 in x 0.250 in
 Brake Torque : 141 in-lbs

corresponds to the value of a_3 given in table 7.3. The elastic angle is computed by substituting,

$$AC' = AC + CC'$$

into equations I-9 and I-10. CC' is obtained from equation F-26 of appendix F. The values of the figure are taken from table D-1 in appendix D where $\phi_{4e} = \text{PHI4}$ and $\phi_4 = \text{PHI4R}$.

The figure shows the deviation during both the forward and return motions of the rocker link. Clearly, the elastic angle is larger than the rigid one during the forward stroke when the strip is being fed. The maximum deviation equals 0.248 degrees at $\phi_1 = 290$ degrees. This corresponds to a clutch-torque of 48 in-lbs, according to equation 3.2. The position of this maximum deviation coincides with that of maximum windup angle and the associated maximum bending strain. The overall shape of this curve is identical to that of the bending strain shown in figure 7.4. It should also be noted that here the positive deviation again continues until after the rocker has entered its return stroke. This also indicates that the clutch does not disengage instantly.

5. Taylor Series Approximation of the Angle ϕ_{4e} and the Extent of its Validity

For the range of input speeds and strip sizes considered, the Taylor series approximation of the elastic angle ϕ_{4e} proved to be

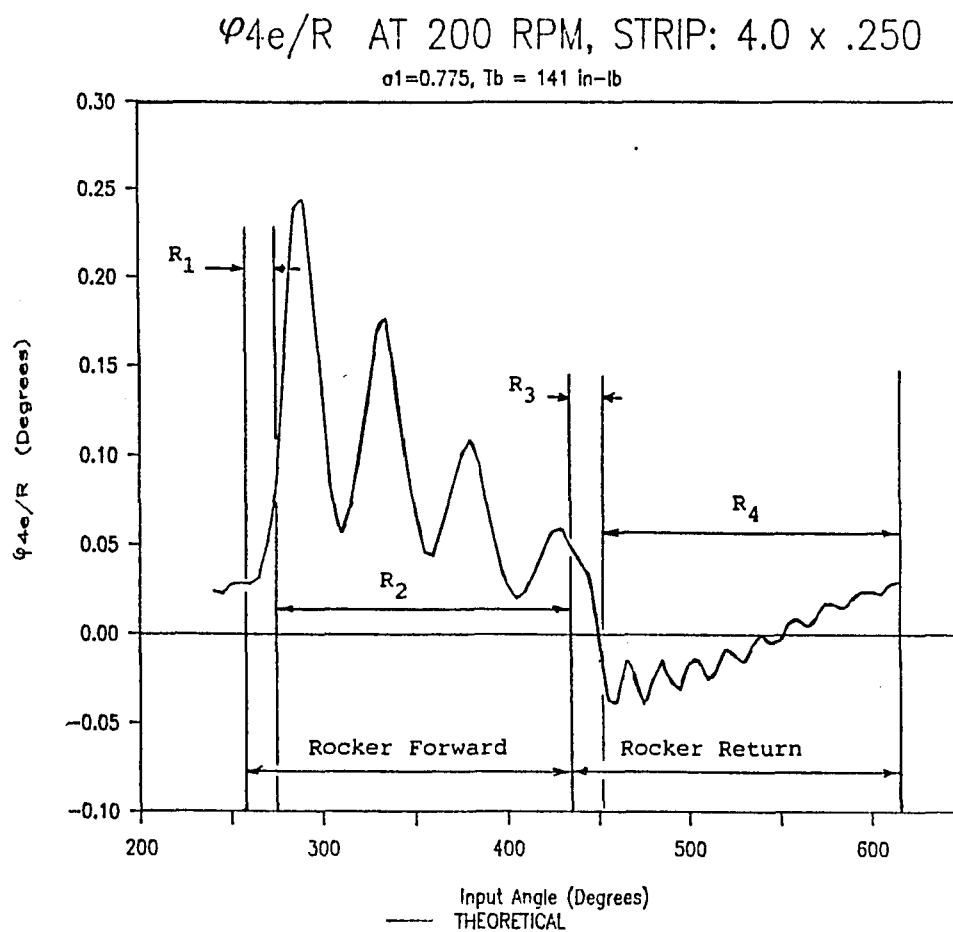


Figure 7.6 : Angle ϕ_{4e}/R , Deviation of Elastic Angle ϕ_{4e}
 from Rigid Body Angle ϕ_4 (theoretical)

Input Speed : 200 rpm
 Feed Length : 0.875 in (nominal)
 Strip Size : 4.000 in x 0.250 in
 Brake Torque : 141 in-lbs

extremely accurate and thus justified its use in the differential equations. This is demonstrated by the results given in tables 7.5 and 7.6, where it is shown that the maximum deviation of the Taylor series approximation of the angle φ_{4e} from that obtained with the help of equations I-9 and I-10, using AC' for AC, is 5×10^{-3} degrees.

Specifically, Table 7.5 shows numerical results for run 1 (see table 7.7 on page 117), where the maximum deviation for this 'light' feeding condition is 3.8×10^{-4} degrees, and occurs at $\varphi_1 = 280$ degrees, i.e. the point of maximum deviation $\varphi_{4e/R}$.

Table 7.6 gives similar results that correspond to test run 6, where the maximum error is 5×10^{-3} degrees. It becomes obvious from the above that this error, although insignificant for the given runs, increases with increased deviation of the elastic angle from its rigid body counterpart. Consequently, in order for the Taylor series to be valid, the coupler must be sufficiently rigid to keep $\varphi_{4e/R}$ as small as possible.

Input Angle (Degrees)	A Taylor Series Approximation	B Elastic Loop Equation	B - A
0	84.28985	84.28092	-0.00007
10	82.59783	82.59781	-0.00002
20	81.0824	81.08237	-0.00003
30	79.72668	79.72666	-0.00002
40	78.58568	78.58568	-0.00000
50	77.71146	77.71144	-0.00001
60	77.09832	77.09832	-0.00000
70	76.78607	76.78607	-0.00000
80	76.77849	76.77849	-0.00000
90	77.03856	77.03853	-0.00003
100	77.63849	77.63848	-0.00001
110	78.48072	77.48068	-0.00004
120	79.60253	79.60252	-0.00001
130	80.91204	80.91201	-0.00003
140	82.41879	82.41878	-0.00001
150	84.04605	84.04603	-0.00002
160	85.76653	85.76652	-0.00001
170	87.52466	87.52466	-0.00000
180	89.26622	89.26622	-0.00000
190	90.95024	90.95024	-0.00000
200	92.51141	92.51141	-0.00000
210	93.91692	93.91691	-0.00001
220	95.10507	95.10506	-0.00001
230	96.05069	96.05068	-0.00001
240	96.70751	96.70750	-0.00001
250	97.06260	97.06258	-0.00002
260	97.09353	97.09351	-0.00002
270	96.82733	96.82725	-0.00008
280	96.28449	96.28411	-0.00038
290	95.30063	95.30061	-0.00002
300	94.20556	94.20533	-0.00023
310	92.81061	92.81047	-0.00014
320	91.22484	91.22479	-0.00005
330	89.58113	89.58096	-0.00017
340	87.78977	87.78972	-0.00005
350	86.01763	86.01757	-0.00006
360	84.28985	84.28092	-0.00007

Table 7.5 : Rocker Angle ϕ_{4e} (Run 1)200 rpm; $T_{BR} = 60$ in-lb; $a_1 = 0.775$ in; Light Strip

Input Angle (Degrees)	A Taylor Series Approximation	B Elastic Loop Equation	B - A
0	84.47385	84.47202	-0.00183
10	82.76047	82.75938	-0.00111
20	81.12499	81.12483	-0.00160
30	79.70528	79.70528	-0.00000
40	78.54551	78.54548	-0.00003
50	77.69105	77.69105	-0.00000
60	77.10623	77.10623	-0.00000
70	76.84779	76.84766	-0.00013
80	76.83346	76.83336	-0.00010
90	77.06618	77.06618	-0.00000
100	77.58456	77.58439	-0.00017
110	78.44294	77.44277	-0.00017
120	79.58648	79.58644	-0.00004
130	80.86653	80.86635	-0.00018
140	82.39414	82.39408	-0.00006
150	84.03501	84.03498	-0.00003
160	85.73210	85.73203	-0.00007
170	87.52436	87.52436	-0.00000
180	89.26156	89.26156	-0.00000
190	90.93942	90.93942	-0.00000
200	92.53264	92.53262	-0.00002
210	93.92206	93.92205	-0.00001
220	95.12296	95.12293	-0.00003
230	96.08225	96.08216	-0.00009
240	96.73062	96.73056	-0.00006
250	97.09564	97.09553	-0.00011
260	97.13528	97.13513	-0.00015
270	96.84938	96.84921	-0.00017
280	96.35661	96.35557	-0.00103
290	95.66660	95.66202	-0.00458
300	94.53063	94.52553	-0.00510
310	92.01327	92.01102	-0.00225
320	91.29149	91.29116	-0.00033
330	89.56477	89.56466	-0.00011
340	87.85290	87.85258	-0.00032
350	86.12751	86.12678	-0.00073
360	84.47385	84.47201	-0.00018

Table 7.6 : Rocker Angle Φ_{4e} (Run 6)300 rpm; $T_{BR} = 211$ in-lb; $a_1 = 0.775$ in; Heavy Strip

F. EXPERIMENT

a. Description of Experimental Set-up

1. Overview

Figure 7.7 gives a schematic view of the experimental setup which was built to test the roll-feed mechanism. Figure 7.8 shows a photograph.

The upper platform with the Varidrive, the right angle drive and the throwblock assembly, as well as the lower mounting box, to which the feed itself is attached, are fixed to the side of a rigid machine frame. The stiffening columns were introduced to make the set-up more rigid. A lathe bed was utilized to build a feed table for the 12-foot long stock strips, of various cross-sections, to slide on. This table is lined up with the feed rollers. The input crank of the spatial linkage is combined with the throwblock assembly (see figure 1.1). All moving parts of the feed mechanism are made of steel. The coupler consists of two tubular sections (look at Table 7.3 for cross-section dimensions), the ends of which are connected to commercially available pre-loaded ball joints. The five HP Varidrive^{*} allows infinite speed variation in the range from 82 to 656 revolutions per minute.

* The unit is manufactured by US Motors. The speed variation is accomplished through a Reeve's Drive.

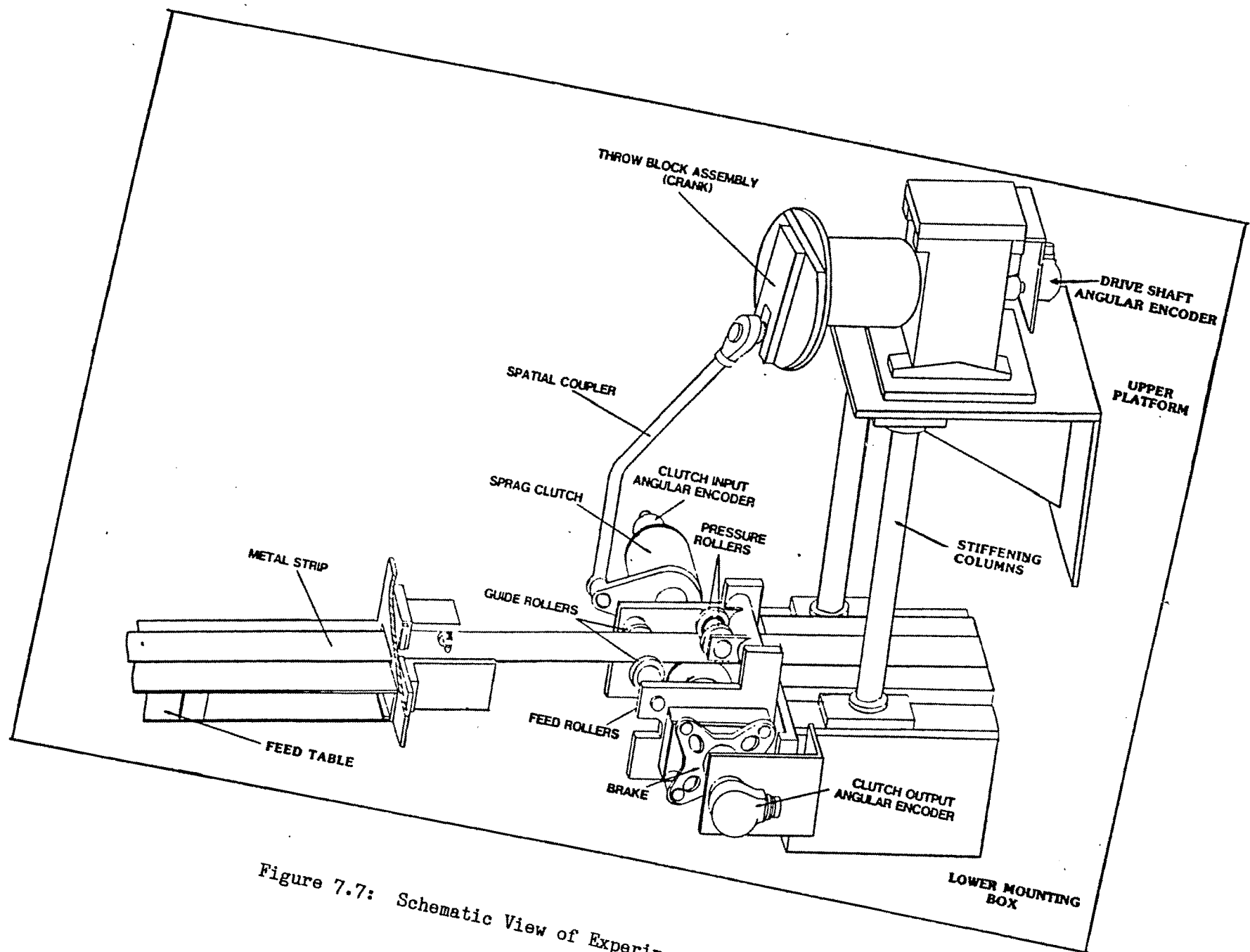


Figure 7.7: Schematic View of Experimental Set-up

Varidrive

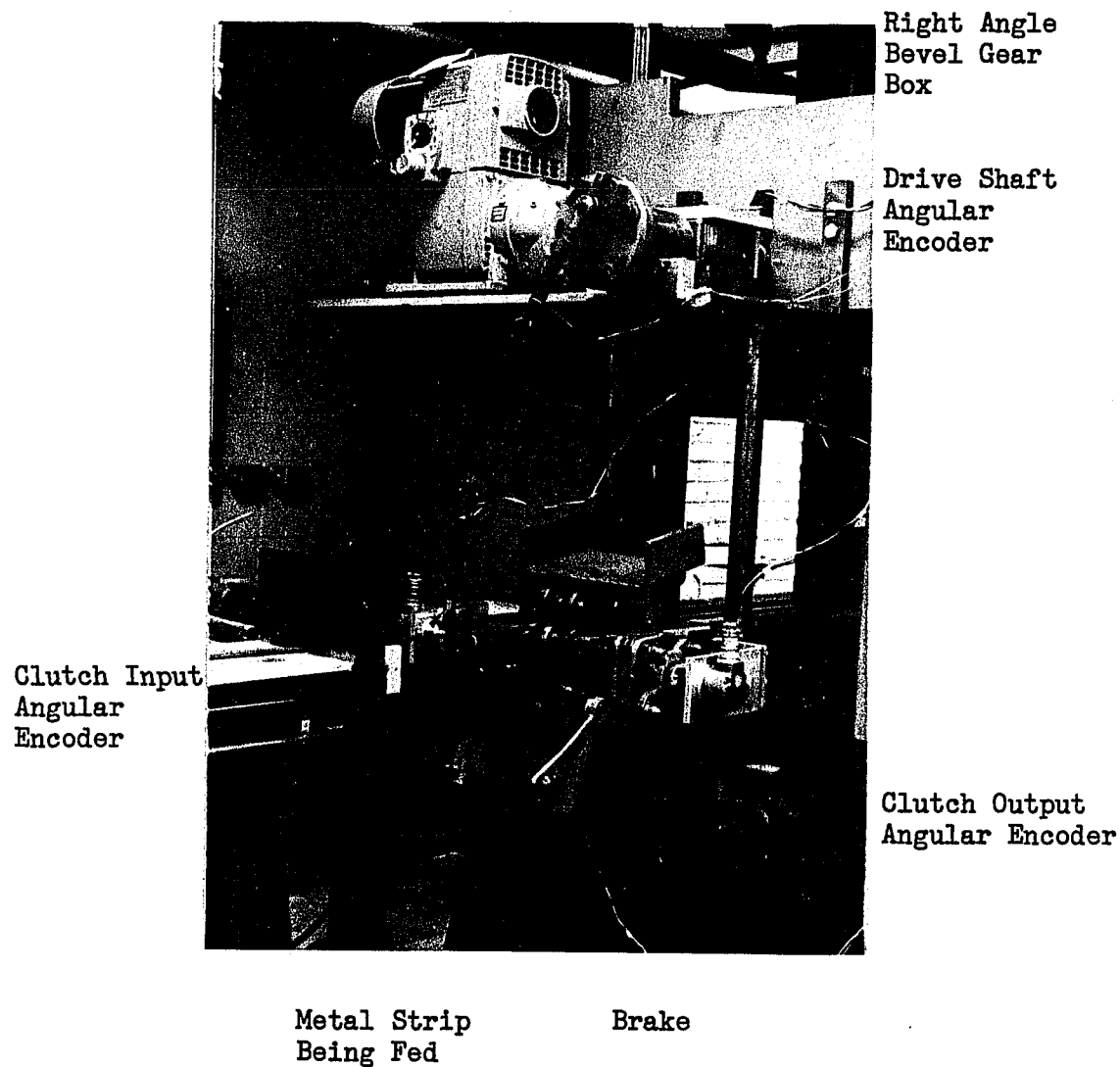


Figure 7.8 : Roller Feed with Variable Speed Drive

Both figures show the three angular encoders*, each of which furnishes 10,000 signals per revolution. They determine the angular displacements at the drive shaft, and the input as well as the output side of the clutch. In addition, strain gages are shown to be attached to the coupler. These are used to monitor the in-plane and out-of-plane deflections. Both encoders and strain gages are discussed in detail below:

i. Drive Shaft Angular Encoder

The drive shaft encoder is mounted in the back of the right angle bevel gear box and provides a pulse every 0.036° of rotation. In addition, it gives a reference signal that is associated with $\varphi_1 = 0$ degrees (see figure I-1 of appendix I). The zero angle reference signal is used to activate the data acquisition system and is fed to the input timer/counter.

ii. Clutch Input Angular Encoder

The clutch input angular encoder is attached to the input side of the sprag clutch and provides a pulse every 0.036° for both the forward and return angular displacements of the rocker link, by way of

* These encoders are all of the same type, i.e. Model No. 29L-11-A14-2500, Dynamics Research Corporation, Wilmington MA [51].

two separate channels. A third channel furnishes a pulse, whenever $\phi_{4e} = 90^\circ$, i.e the rocker is in its horizontal position.

iii. Clutch Output Angular Encoder

The clutch output angular encoder is attached to the feed roller shaft, and also furnishes a pulse every 0.036° of both its forward and return rotations (return rotation has never been observed). Again two channels are involved.

2. Instrumentation

A schematic of the instrumentation is presented in figure 7.9. The input speed of the feed-mechanism is directly measured in revolutions per minute by a timer/counter*. It is activated by the zero input angle reference signal.

The bending strains, both in-plane and out-of-plane, are obtained by way of two sets of two strain gages** each, at 90° to each other, located at $x_1 = 11$ in, and $x_2 = 5$ in (see figure 7.10). Since the

* High resolution timer/counter, Model No. PM 6670, Philips Corp.

** Encapsulated strain gages type EA-06-125D-120, Measurements Group.

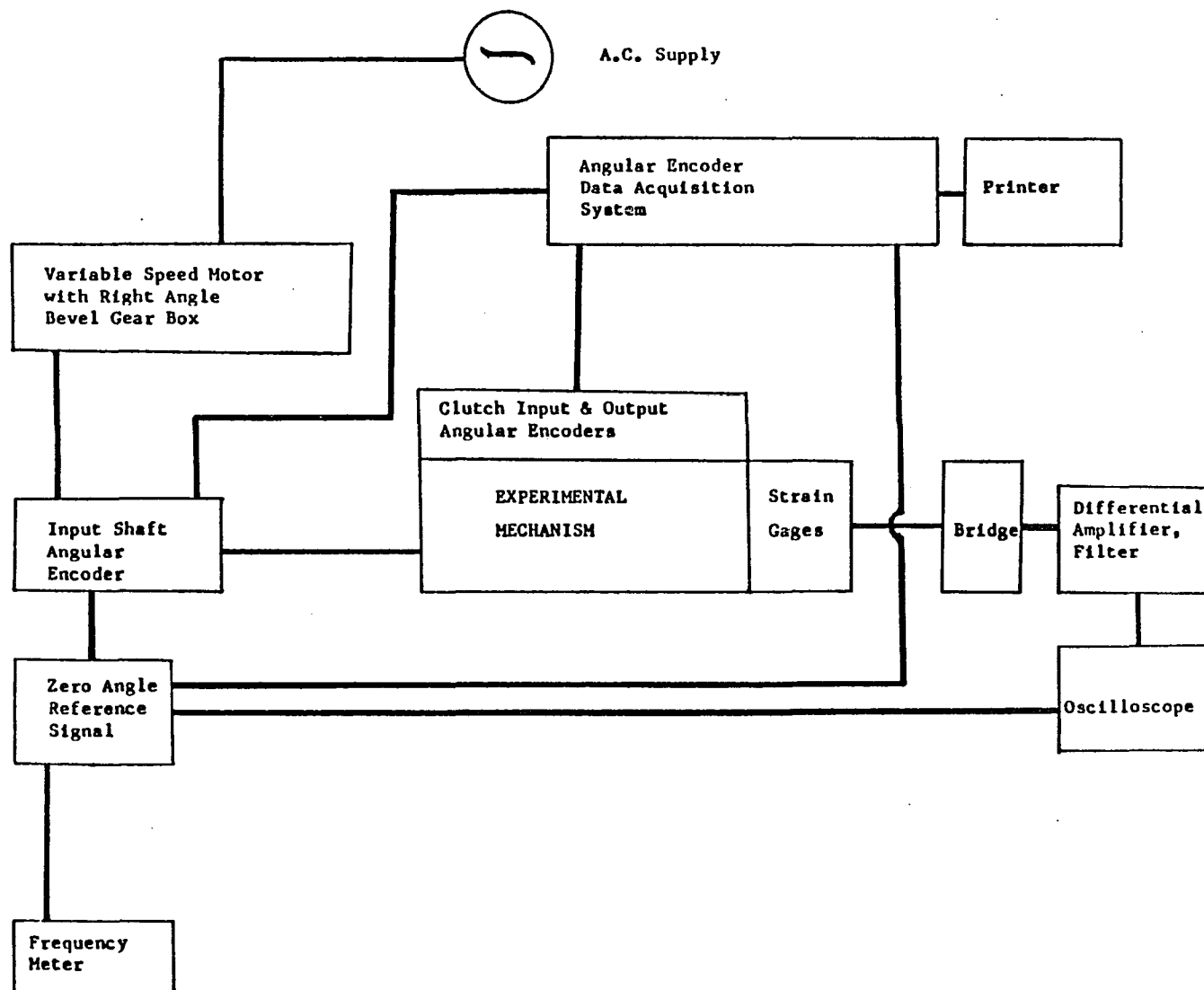


Figure 7.9: Schematic of Instrumentation

form of the strain responses is the same for both locations, only those of the first location are considered in this dissertation.

The strain gage signal is fed into a strain gage conditioner/amplifier* which in turn supplies its output to a dual trace storage oscilloscope** (see figure 7.11). In order to relate the strain gage signal to the instantaneous configuration of the mechanism, the zero input angle reference signal is also fed into the oscilloscope.

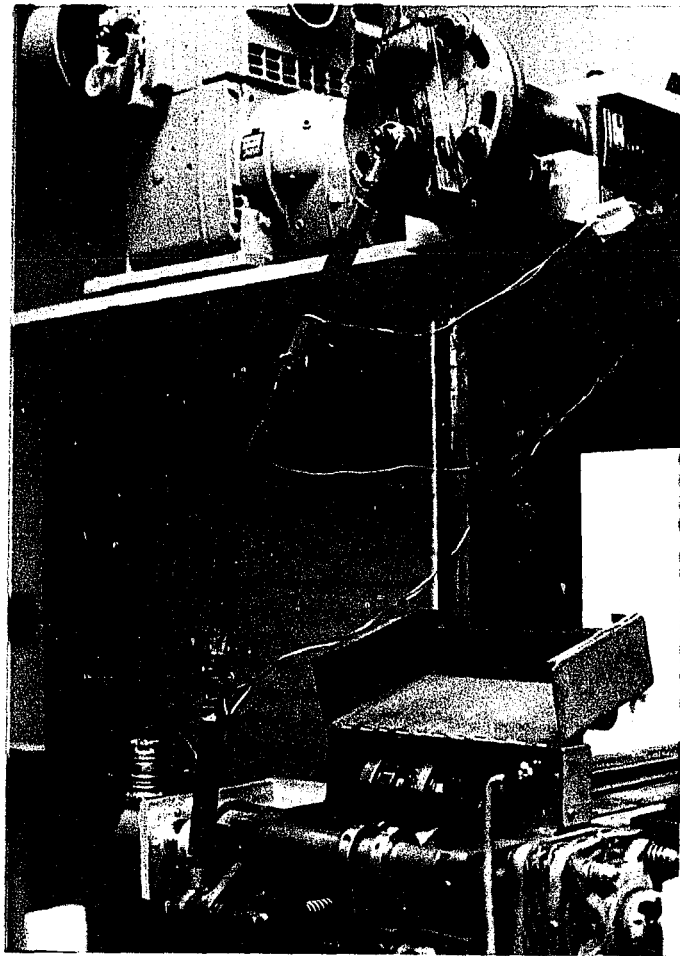
The angular encoder data acquisition system (see figure 7.11), which was designed and built at the City College, is capable of storing and subsequently printing the following data for ten mechanism cycles:

- i. The time associated with each 0.036 degree rotation of the throwblock (input) angle ϕ_1 . The zero input angle reference signal starts the time count when the input angle ϕ_1 is equal to zero. (The two channels of the drive shaft encoder supply these pulses).
- ii. The time associated with each 0.036 degrees of forward and reverse rotation of the rocker link angle ϕ_4 . The aforementioned reference signal indicates the time

* Measurements Group Strain-Gage Conditioner/Amplifier, Model 2100

** Tektronix Dual Trace Storage Oscilloscope, Model 5111.

Out-of-Plane
Strain Gages



In-Plane Strain
Gages

Feed Rollers

Figure 7.10

Coupler with Strain Gages

Timer/Counter	Oscilloscope	Angular Encoder
Power Supply	Differential	Data Acquisition
	Amplifier	System with Printer

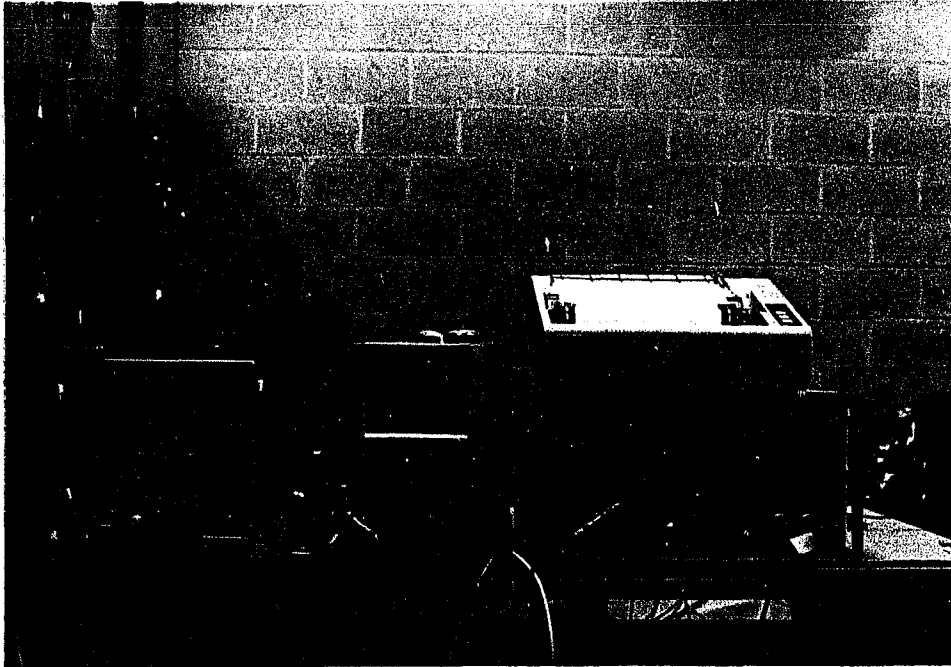


Figure 7.11
Instrumentation System

associated with $\phi_4 = 90$ degrees (three channels of the clutch-input angular encoder supply these signals).

- iii. The time associated with each 0.036 degree of forward and reverse rotation of the feed-roller angle ϕ_5 (two channels of the clutch output angular encoder supply these signals).

The time is incremented by an internal microsecond clock. To be able to register the time in microseconds, an 18-bit wide data bus line is used.

3. Angular Encoder Data Acquisition and Evaluation Procedure

Appendix K outlines the operation of the angular encoder data acquisition system in detail and describes how the data are evaluated, to obtain the rocker angle ϕ_{4e} as well as the feed-roller angle ϕ_5 as functions of the input angle ϕ_1 . To facilitate the above, reference is made to the sample print-out of data given in appendix L. Table K-1 of appendix K shows a typical tabulation of the throwblock, rocker link, and feed-roller angles as functions of time, which was obtained by the indicated evaluation procedure.

4. Brake Setting

The brake torque level of each run is chosen such that the variation of the total clutch output angle ϕ_{5tot} is kept to a minimum from cycle to cycle. (Recall that brake adjustability is provided by the brake adjustment screw shown in figure 1.1).

The magnitude of a specific brake torque setting is obtained by applying an increasing moment (by way of a string and hanging weights) to the clutch outer race, until the feed roller shaft starts moving.

5. Reduction of Impact Disturbances

To prevent bearing impacts during testing, two high precision pre-loaded spherical bearings were installed on the coupler. Each of these requires a no-load rotational breakaway torque of approximately 1.5 in-lbs.

A second type of impact disturbance, which is due to the fact that the coupler center of mass does not lie on the pivot-to-pivot line AC of this member, had to be dealt with. The resulting rotation of the coupler about this line is reversed twice during each cycle, by impacting on the shaft connections with the throwblock and the rocker link. To minimize such impact related contamination of the strain histories, rubber washers were placed at these connections.

G. VERIFICATION OF MATHEMATICAL MODEL: COMPARISON OF COMPUTED TO EXPERIMENTAL RESULTS

The following section establishes the validity of the mathematical model by comparing various computed runs to their associated experimental ones.

Table 7.7 lists these runs in ascending order of their load severity (see discussion on Section VII-E, earlier), giving the rpm, the input crank length a_1 and the associated nominal feed length, the metal strip cross-section, the combined feed roller-pressure roller-metal strip moment of inertia J_{05} , the brake and table-friction torques, as well as the associated load factor.

The comparison will be made principally with the help of the criteria established in Section VII-C, i.e. the in-plane coupler strain history, the windup angle φ_{54} , and the total feed-roller angle φ_{5tot} . In addition, a comparison between computed and experimental values for the angles φ_{4e} and φ_5 will be given. Finally, the effect of loading on the total forward angle φ_{4etot} will be considered.

For the sake of clarity, the strain history coupled with the windup angle φ_{54} are shown in the same figure for each of the six runs. Similarly, the feed-roller angle φ_5 and the rocker forward angle φ_{4e} , are also presented together.

	Input Speed rpm	Input Crank Length* (Feed Length) in	Metal Strip Cross-Section in x in	J_{05} lb-sec ² in	Brake Torque in-lb	Table Friction Torque in-lb	Load Factor
Run 1	200	0.775 (0.875)	1.000 x 0.125	.1803	60	4	1
Run 2	300	0.775 (0.875)	1.000 x 0.125	.1803	87.6	4	1.57
Run 3	200	1.700 (2.000)	1.000 x 0.125	.1803	165	4	2.63
Run 4	400	0.775 (0.875)	1.000 x 0.125	.1803	173.6	4	2.98
Run 5	200	0.775 (0.875)	4.000 x 0.250	.768	141	40	3.13
Run 6	300	0.775 (0.875)	4.000 x 0.250	.768	211	40	5.01

* Nominal

Table 7.7 : Test Runs and Loading Conditions

- a. Comparisons of Windup Angle φ_{54} , In-Plane Strain of Coupler at $x_1 = 11$ in, Rocker Forward Angle φ_{4e} , and Feed-Roller Angle φ_5

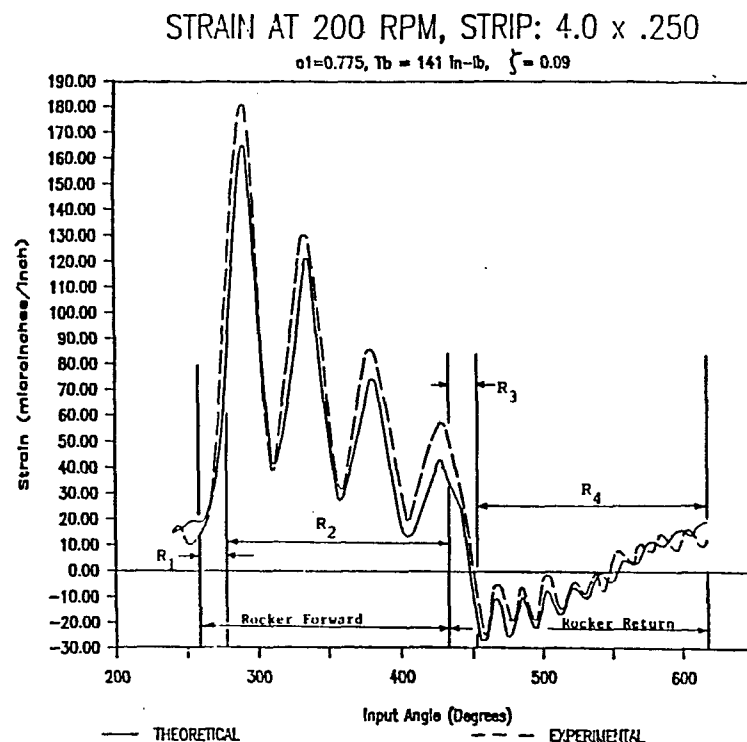
Before considering other runs, certain general observations will be made on hand of run 5 (figures 7.12 and 7.13) which was used for the sample computations in Section VII-E. This run also served for the original refinement of the various regimes as well as for the determination of the damping ratio of the clutch.

In addition to the rocker forward and return motion regimes, the strain histories in figure 7.12 also give the extent of the four regime range, R_1 , R_2 , R_3 , and R_4 . The windup-angle graph is confined, as discussed in Section VII-E, to regimes 1 and 2 (i.e. R_1 and R_2) of the forward stroke. The possible inclusion of regime no. 3, that is the unwinding of the clutch, would not be relevant to the intent of the windup-angle criterion.

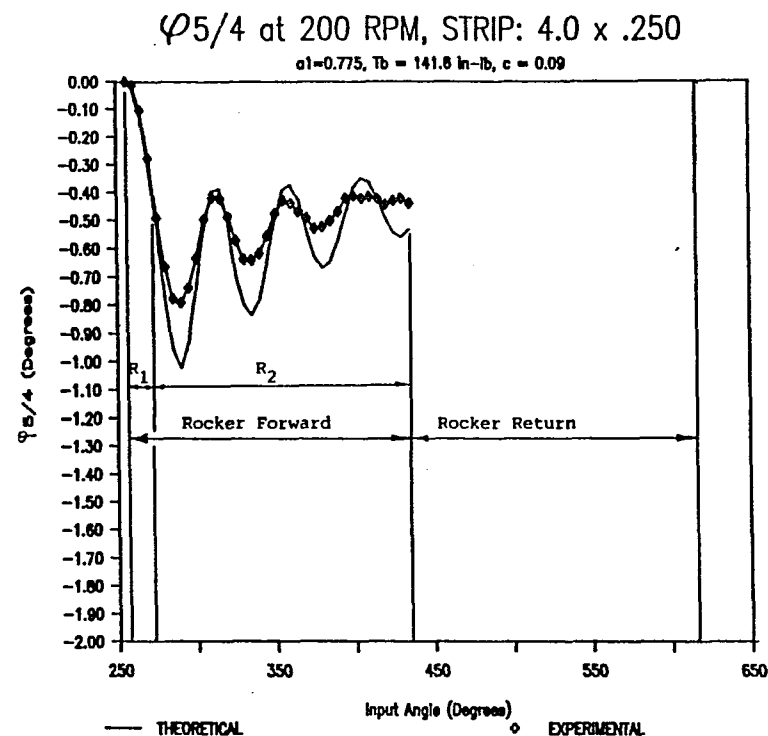
Concerning the comparison between computational and experimental results, the following points can be made:

- i. The general agreement in the overall shape of the computed and the experimental strain histories bears out the four-regime scheme of the mathematical model. Especially, regime no.3 accurately reflects the transition from forward to return stroke.

ii. The relationship between the strain and the windup angle, as discussed earlier in connection with the computational results in Section VII-E, is confirmed by the experiment. The frequencies of oscillation of the in-plane strain and the windup angle are identical during the forward stroke, and are not related to the fundamental frequency of the coupler link. Recall that this transient frequency is roughly that of the unforced linearized clutch system, which includes the referred mass of the specific metal strip being fed (see equation 7.21 which gives 23.5 Hz, when the heavy strip is involved). In addition, the initial peak strain coincides with the maximum windup angle in terms of the input angle ϕ_1 . At this point it is helpful to recall that an extension of line AC will cause a positive strain to be reported by the strain gage, which is located on the inside of the coupler bend. Thus, one would generally expect such a positive strain during regimes 1 and 2 where the strip is moved forward. Closer analysis shows that the coupler strain during the forward stroke originates mainly from both the strip and brake load on the output side of the clutch and from the rocker and outer race on the input side of the clutch. The first component is always positive since the nature of the windup angle always causes the coupler to be elongated. The nature of the rocker acceleration/deceleration may either enhance this positive strain, diminish it or even make it negative. This has its explanation in the fact that at the beginning of the forward



a. In-Plane Strain

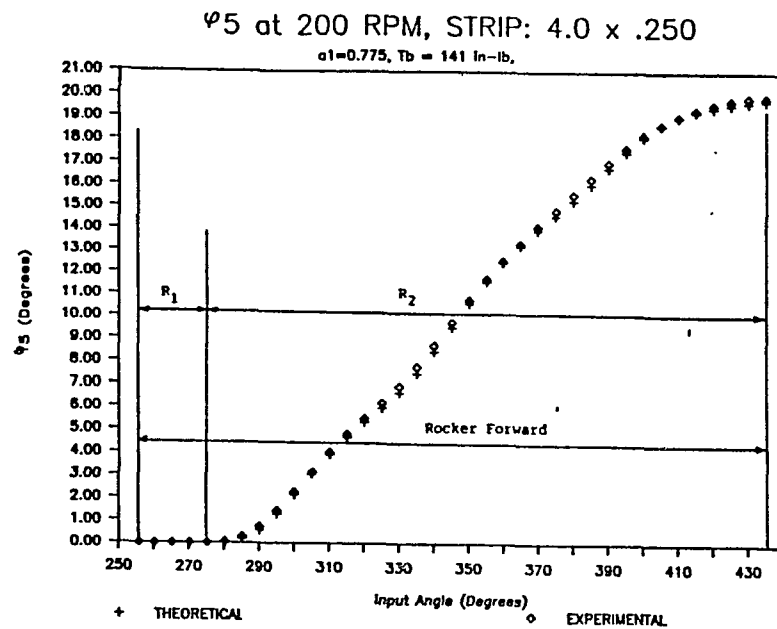


b. Windup Angle ϕ_{54}

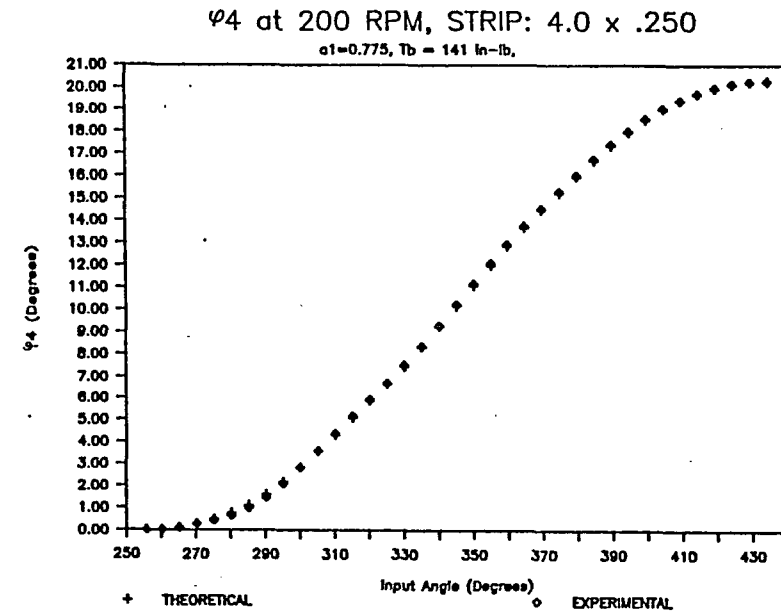
Run No. 5

Input Speed : 200 rpm
 Feed Length : 0.875 in (nominal)
 Strip Size : 4.00 in x 0.250 in
 Brake Torque : 141 in-lb

Figure 7.12: Comparison of In-Plane Strain and Windup Angle for Run 5



a. Feed-Roller Angle ϕ_5



b. Rocker Angle ϕ_{4e}

Run No. 5

Input Speed : 200 rpm
 Feed Length : 0.875 in (nominal)
 Strip Size : 4.00 in x 0.250 in
 Brake Torque : 141 in-lb

Figure 7.13: Comparison of Feed Roller and Rocker Angles for Run 5

stroke the coupler must pull the rocker forward, which will cause an elongation. Towards the end of this motion, the rocker must be decelerated, causing a compression of the coupler. The maximum strain contribution of the combined metal strip and brake load is approximately 85% of the maximum bending strain for run no. 5. However, it is difficult to generalize this, since it depends on the input rpm, the combined feed roller-strip inertia, and the brake setting. Clearly, there may be a case where the total in-plane strain curve is negative at any point during the latter part of regime 2 and /or regime 3, when the work load is lower than the rocker load.

- iii. The frequency of oscillation of the coupler during the return stroke of the rocker link, as indicated by the experimental strain history, is essentially identical with the computed one. This confirms the validity of the results due to the special frequency term ω^* in the Hill's equation which has its origin in the bend of the beam. (See discussion concerning the plucking frequency of 87.5 Hz versus the actual vibration at 68.5 Hz in Section VII-E-c-2). This transient strain oscillation is initiated by the energy content of the coupler, i.e. its deformation, at the end of unloading. Since it represents a free vibration, it must be the same for all runs. In addition to this oscillation, the strain must reflect the rocker-outer race load on the coupler during the return stroke. This load causes compression of the coupler during the speed-up phase of the output

and extension during the slow-down phase. This explains the ascending mean line of the coupler oscillation.

- iv. The predicted value for the maximum bending strain for this run was approximately 9% less than the experimentally obtained one. The windup angle stays negative throughout the forward stroke and with that proper (non-pathological) feeding is assured^{*}. Its computed peak values are overstated. The "minimum" experimental and computed values of the windup angle are (-) 0.42 and (-) 0.38 degrees, respectively. Certain part of these errors may be accounted for by the fact that the value of φ_{54} represents the difference of two quantities, i.e. φ_5 and φ_{4e} , which in themselves contain some error (see discussion in connection with these angles below). Any percentage error associated with either of these quantities will be magnified when the windup angle is determined. Furthermore, the specific choices of the clutch damping ratio and the spring stiffness coefficient of the clutch play a part in this discrepancy. A larger damping factor would have decreased the forward coupler strain, in addition to the peak to peak magnitude of the windup angle. A 10% increase in the spring stiffness coefficient, which would hardly have affected the fit with the clutch characteristics data of figure 3.3, would have given a better amplitude agreement on the windup angle (without

* It would seem desirable that the oscillations of the windup angle are fairly small and that the windup angle "minimum" stays sufficiently far away from the zero line, during regime no. 2.

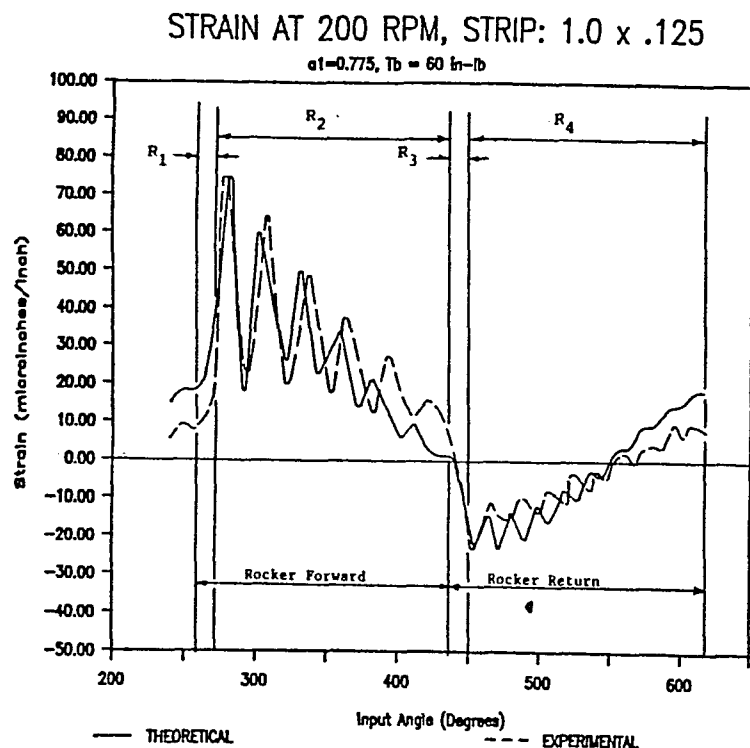
changing the strain amplitude agreement), but it would have caused some changes in the overall phase agreement.

Figure 7.13 shows a comparison between experimental and computed values of the feed-roller angle φ_5 and the rocker forward angle φ_{4e} . In addition to the verification of the validity of the model with respect to these quantities, the plot for φ_5 shows the extent of the regimes 1 and 2 very clearly. Recall that regime no.1 ends as soon as φ_5 acquires a non-zero value, i.e. as soon as the feed rollers start moving. The extent of this regime is proportional to the externally applied brake torque and it will be different from run to run. Regime no. 2 ends with the end of the rocker forward motion. It must be noted here that, both plots should be considered when accounting for the error, between computation and experiment, in the windup angle φ_{54} .

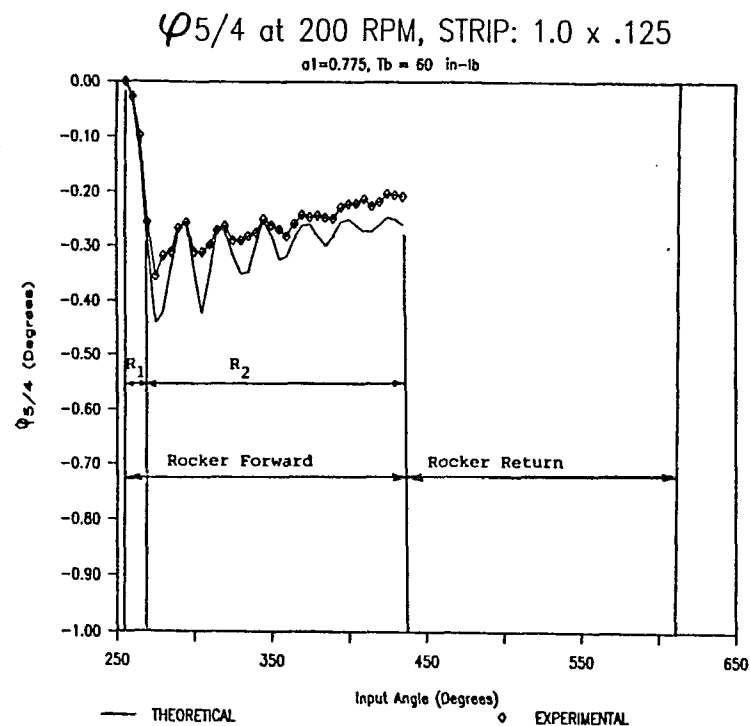
The following discusses the performance of the model as compared to the experimental results for runs 1-4 and 6. It must be kept in mind that runs 5 and 6 deal with the heavier cross-section strip, i.e. 4.000 in x 0.250 in, and that the parameters of choice were attuned to the two more severe load conditions.

Run no. 1 (see figures 7.14 and 7.15)

Run 1, which has a load factor equal to 1, represents a very light load run at 200 rpm, with a light strip and with that a light brake torque, as well as a short feed length (see table 7.7). The



a. In-Plane Strain



b. Windup Angle $\phi_{5/4}$

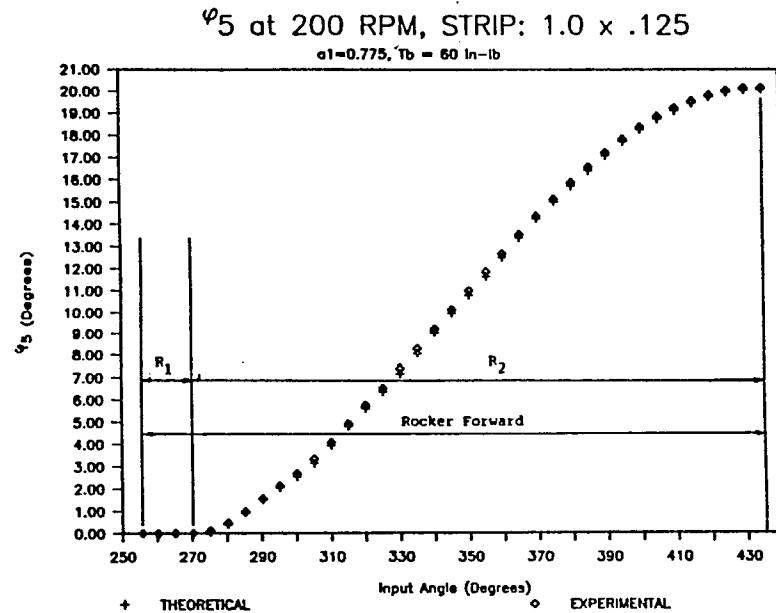
Run No. 1

Input Speed : 200 rpm
 Feed Length : 0.875 in (nominal)
 Strip Size : 1.00 in x 0.125 in
 Brake Torque : 60 in-lb

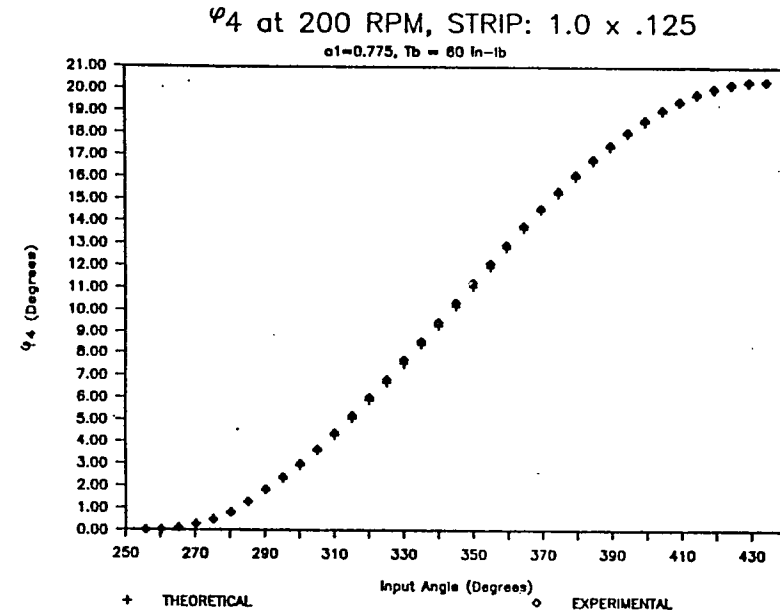
Figure 7.14: Comparison of In-Plane Strain and Windup Angle for Run 1

frequency of oscillation during the forward stroke, both for the strain and the windup angle, agrees at the beginning and grows out of phase towards the end of regime no. 2. This implies that the clutch description is not exactly accurate over the total loading range. It should also be kept in mind that with small differences in frequency, the phase agreement becomes worse as the number of oscillations increases, since the error is cumulative. Note that run 5 had 3 oscillations, while the present run has 6. This experimental transient frequency varies between 40 and 48 Hz, while the linearized spring system frequency is 48.5 Hz. Compare this with the one associated with the heavy strip of run 5 which is equal to 23.5 Hz. The associated computed frequency varies between 40 and 52 Hz. The computed and experimental amplitudes of the bending strain agree quite well during the initial portion of the forward stroke. Throughout, the windup angle stays negative, with its "maximum" experimental amplitude "smaller" than the model prediction. This is most likely due to the errors involved in the prediction of the angles φ_{4e} and φ_5 (see figure 7.15). During regime no. 2 the windup angle has a "minimum" experimental value of (-) 0.21 degrees and a computed one of (-) 0.26 degrees. The extents of regimes 1 and 2 are well predicted.

As expected, the frequency of oscillation in the strain trace during the return stroke of this run is essentially the same as that of run 5. The experimentally observed frequency varies between 60 and 70 Hz, while the computed one ranges between 64 and 72 Hz. The phase agreement is marginal and the experimental peak to peak amplitudes are smaller than the predicted ones. Furthermore, they are smaller than



a. Feed-Roller Angle φ_5



b. Rocker Angle φ_{4e}

Run No. 1

Input Speed : 200 rpm
 Feed Length : 0.875 in (nominal)
 Strip Size : 1.00 in x 0.125 in
 Brake Torque : 60 in-lb

Figure 7.15: Comparison of Feed-Roller and Rocker Angles for Run 1

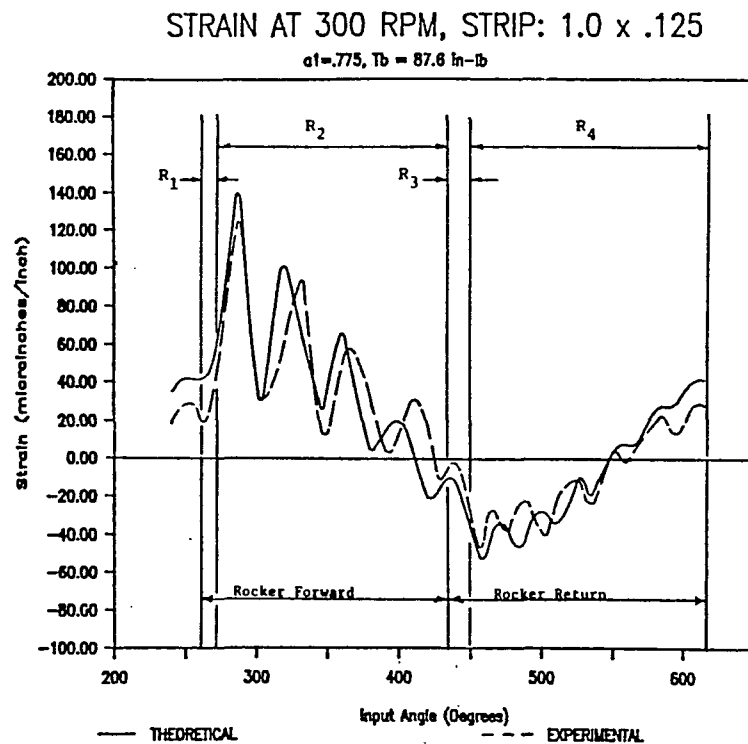
those of run 5 because of the lower energy content of the coupler during the regime transition.

It should be noted that the agreement between the model and the experiment improves for all regimes as the load factor increases.

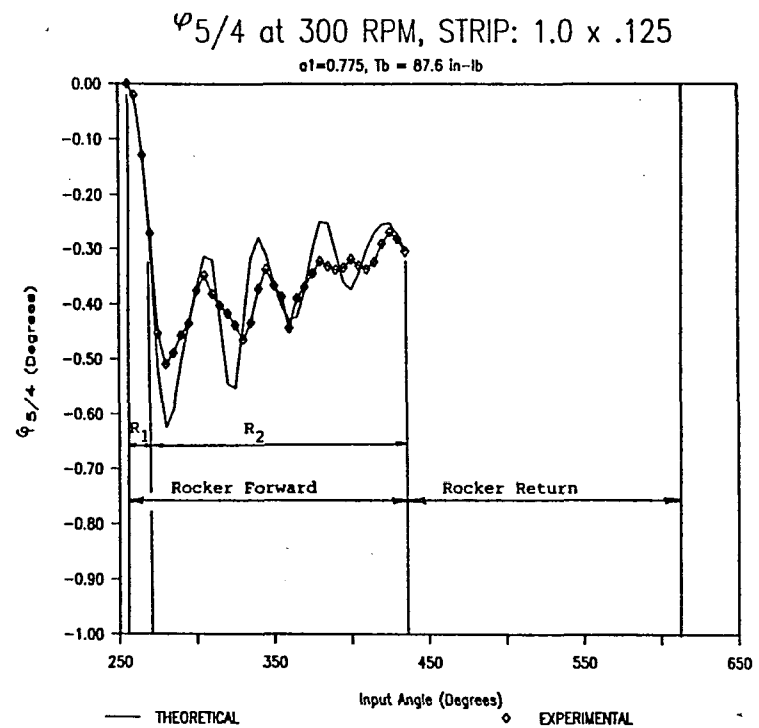
Run 2 (see figures 7.16 and 7.17)

This run pulls the same metal strip through the same feed length as run 1. The input speed is increased to 300 rpm and the brake torque is increased from 60 to 87.6 in-lbs. Thus, the load factor becomes 1.57. The frequency of oscillation of the windup angle and the strain during the forward stroke should be the same as that of run 1, since the same metal strip is involved, which produces a linearized clutch frequency of 48.5 Hz. As it turns out, the experimental frequency varies between 40 and 50 Hz, while the computed one ranges from 42 to 54 Hz. As for run 1, this shows up as a somewhat lower average frequency in the experiment. The computed and experimental amplitudes of the bending strain are within 10% during the initial portion of the forward stroke. Both the computed and the experimental strains become negative towards the end of the forward stroke. As discussed earlier, this is explained by the fact that the work load is smaller than the rocker load, in that region*.

* An increase of the brake torque would have moved the strain into the positive region. This however, is of no significance as long as the associated windup angle does not become positive.



a. In-Plane Strain

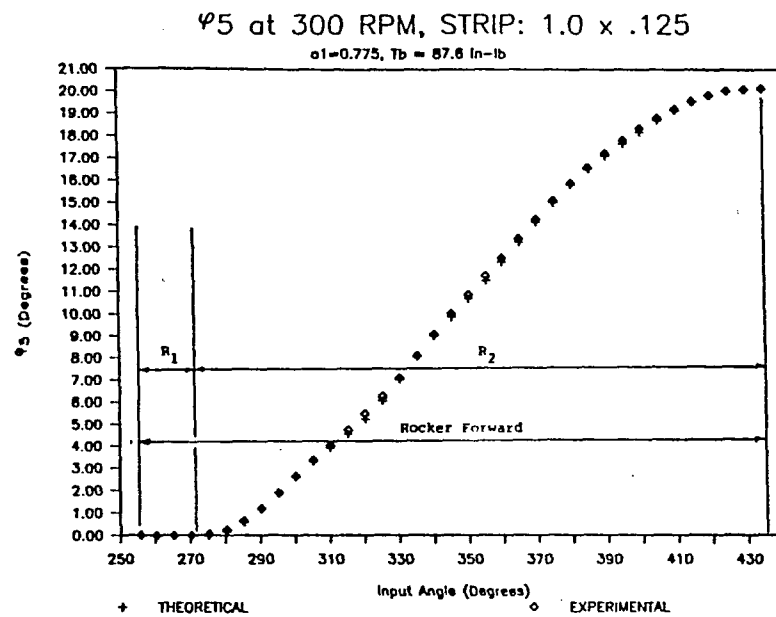


b. Windup Angle $\phi_{5/4}$

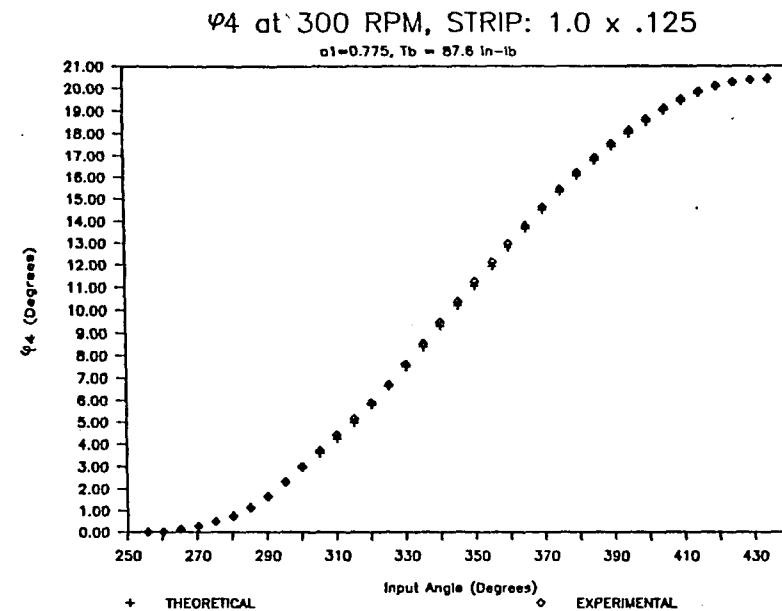
Run No. 2

Input Speed : 300 rpm
 Feed Length : 0.875 in (nominal)
 Strip Size : 1.00 in x 0.125 in
 Brake Torque : 87.6 in-lb

Figure 7.16: Comparison of In-Plane Strain and Windup Angle for Run 2



a. Feed-Roller Angle ϕ_5



b. Rocker Angle ϕ_{4e}

Run No. 2

Input Speed : 300 rpm
Feed Length : 0.875 in (nominal)
Strip Size : 1.00 in x 0.125 in
Brake Torque : 87.6 in-lb

Figure 7.17: Comparison of Feed-Roller and Rocker Angles for Run 2

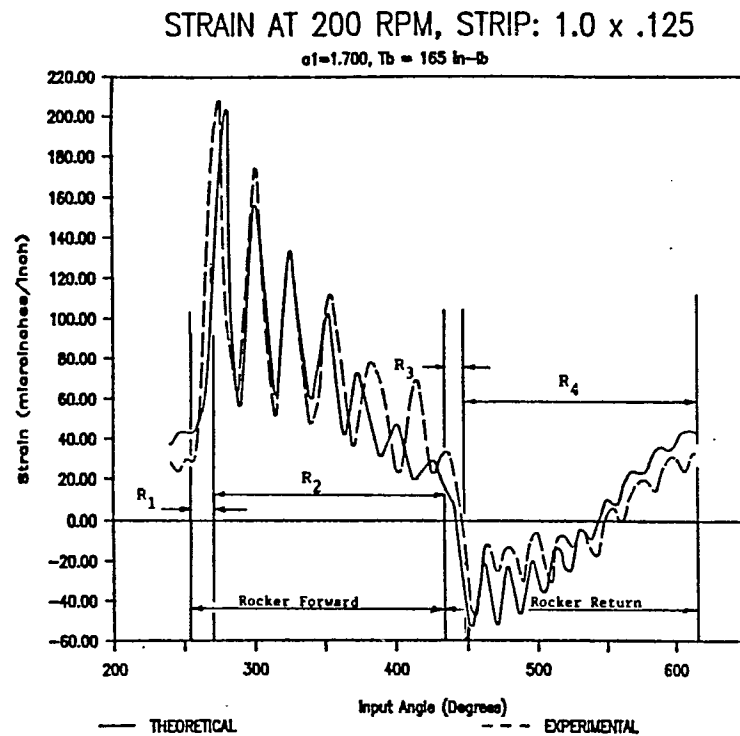
Again, the windup angle stays negative throughout the forward stroke with its "maximum" experimental amplitude "smaller" than that predicted by the model. Again, this is most likely due to the errors involved in the prediction of the angles φ_{4e} and φ_5 (see figure 7.17). During regime no. 2, the windup angle has "minimum" experimental and computed values of $(-) 0.28$ and $(-) 0.27$ degrees, respectively. The extents of regimes 1 and 3 are well predicted.

The strain frequency relationship during the return stroke is the same as in runs 1 and 5. The phase agreement is marginal and the experimental peak to peak amplitudes are slightly smaller than the predicted ones. They are larger, however, than those of run 1 because of the higher energy content of the coupler during the regime transition.

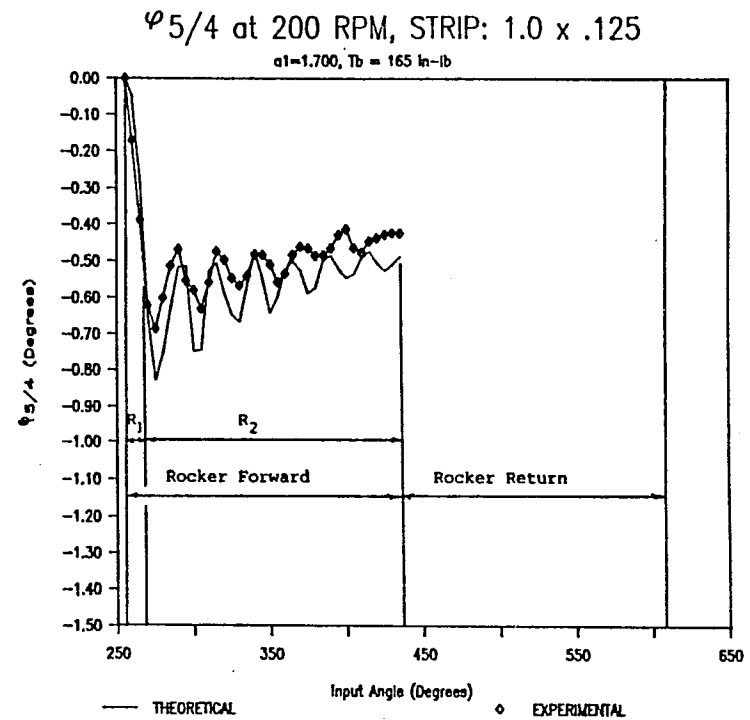
Comparison of the overall agreement with that of run 1 clearly supports the conclusion that the agreement improves as the load factor increases.

Run 3 (see figures 7.18 and 7.19)

Run 3 involves the same metal strip at the same input speed as run 1. The mechanism input crank length is now 1.700 in, and the brake torque has been increased to 165 in-lbs. This gives a load factor of 2.63. Figure 7.18 shows that the computational and experimental frequencies of the windup angle and the strain agree well



a. In-Plane Strain

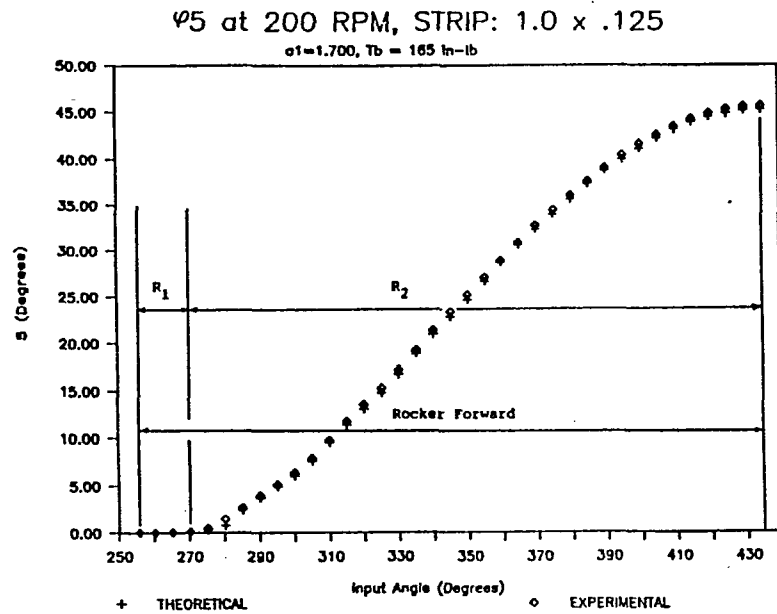


b. Windup Angle $\phi_{5/4}$

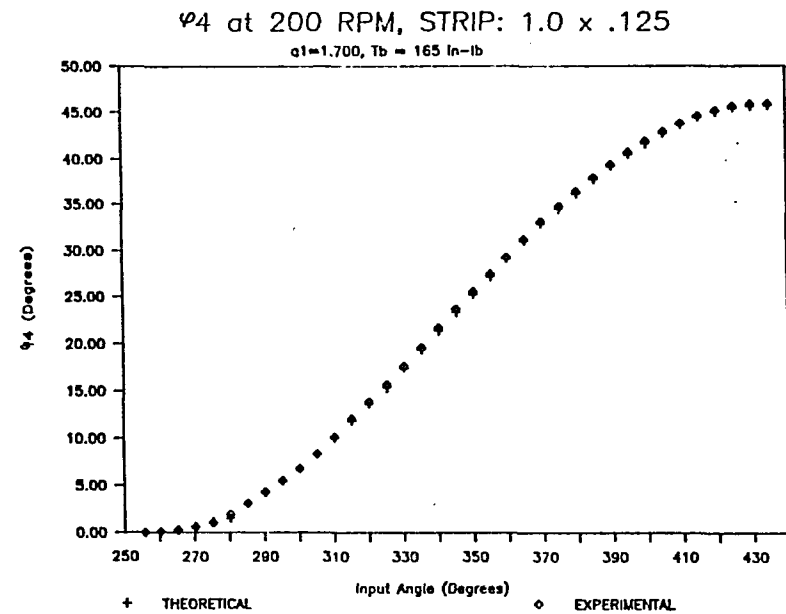
Run No. 3

Input Speed : 200 rpm
 Feed Length : 2.000 in (nominal)
 Strip Size : 1.00 in x 0.125 in
 Brake Torque : 165 in-lb

Figure 7.18: Comparison of In-Plane Strain and Windup Angle for Run 3



a. Feed-Roller Angle ϕ_5



b. Rocker Angle ϕ_{4e}

Run No. 3

Input Speed : 200 rpm
 Feed Length : 2.000 in (nominal)
 Strip Size : 1.00 in x 0.125 in
 Brake Torque : 165 in-lb

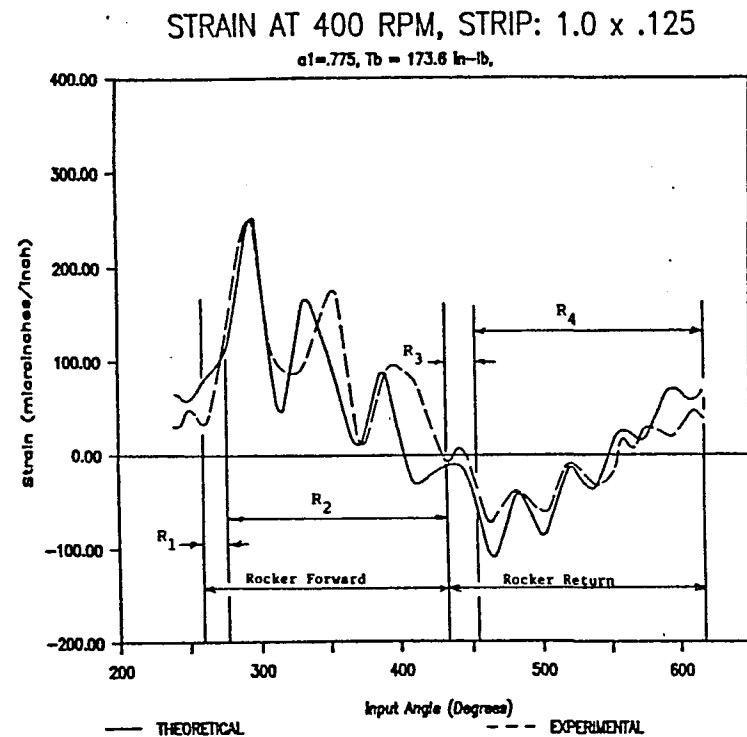
Figure 7.19: Comparison of Feed-Roller and Rocker Angles for Run 3

through most of the forward stroke but grow out of phase towards the end of regime no. 2. This again implies that the clutch description becomes less accurate in the lower load range. The computed and experimental amplitudes of the bending strain agree well during the initial portion of the forward stroke. The windup angle stays negative throughout the forward stroke. Again, its "maximum" experimental amplitude is "smaller" than the computed one. This is most likely due to the errors involved in the prediction of the angles φ_{4e} and φ_5 (see figure 7.19). During regime no. 2, the "minimum" experimental and computed values of the windup angle are (-) 0.42 and (-) 0.485 degrees, respectively. The extents of regimes 1 and 3 are predicted reasonably well.

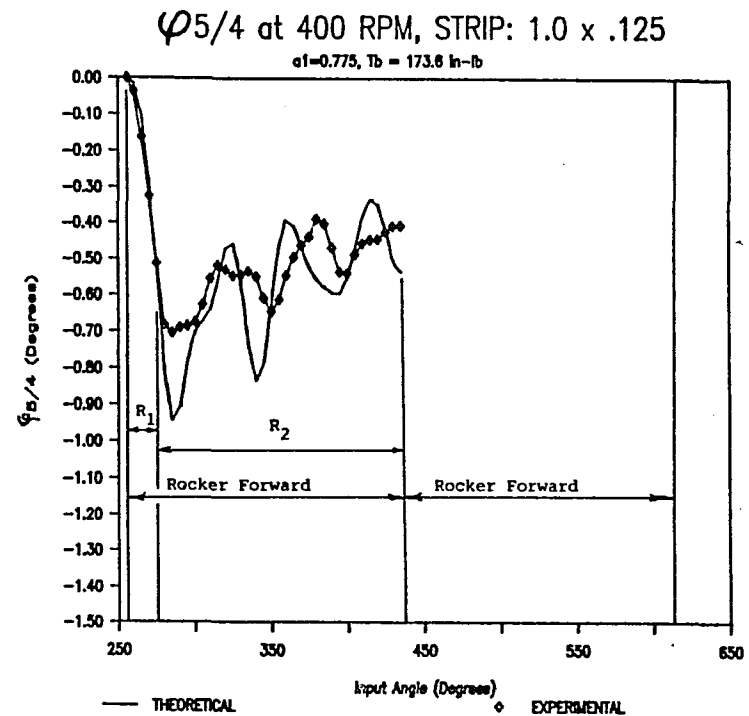
As expected, the strain frequency relationship during the return stroke is essentially the same as that seen in runs 1, 2 and 5. The phase agreement is very good and the experimental peak to peak amplitudes are again smaller than the predicted ones. Both are larger, however, than those of run 1, because of the higher energy content of the coupler during the regime transition.

Run 4 (see figures 7.20 and 7.21)

The metal strip and feed length conditions for run 4 are identical to those of runs 1 and 2. The input speed now is 400 rpm and the brake torque is 173.6 in-lbs. The load factor becomes 2.98. Figure 7.20 shows that the frequency agreement of the windup angle and



a. In-Plane Strain

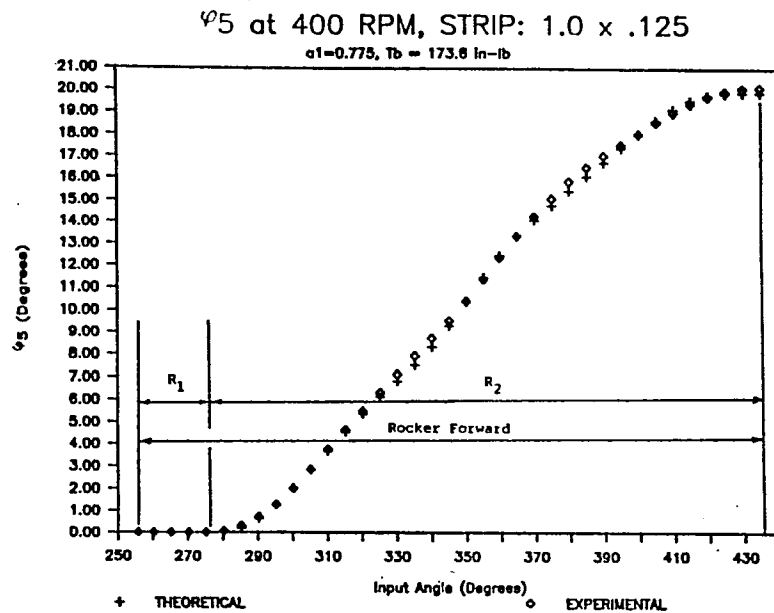


b. Windup Angle $\phi_{5/4}$

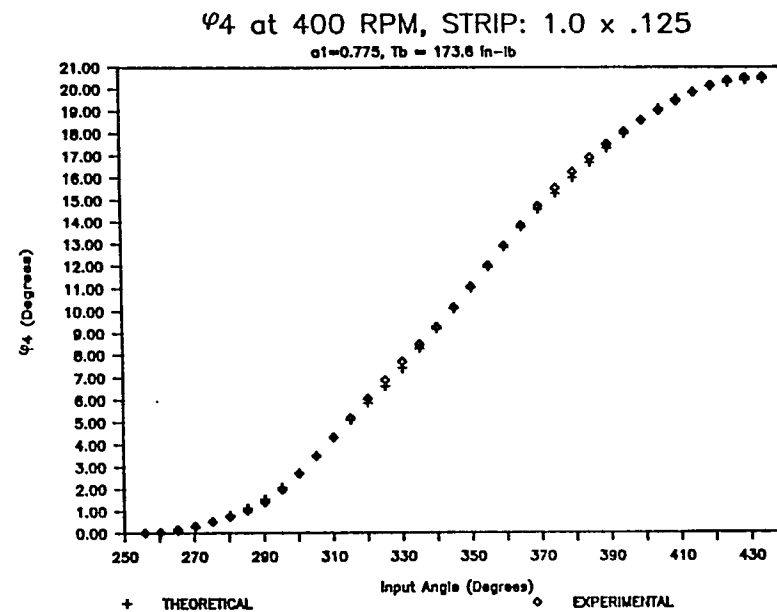
Run No. 4

Input Speed : 400 rpm
 Feed Length : 0.875 in (nominal)
 Strip Size : 1.00 in x 0.125 in
 Brake Torque : 173.6 in-lb

Figure 7.20: Comparison of In-Plane Strain and Windup Angle for Run 4



a. Feed-Roller Angle ϕ_5



b. Rocker Angle ϕ_{4e}

Run No. 4

Input Speed : 400 rpm
Feed Length : 0.875 in (nominal)
Strip Size : 1.00 in x 0.125 in
Brake Torque : 173.6 in-lb

Figure 7.21: Comparison of Feed-Roller and Rocker Angles for Run 4

the coupler strain is marginal during the forward stroke. The computed and experimental amplitudes of the bending strain are very close during the initial portion of the stroke. Both these strains become negative towards the end. Again, this is because the work load is smaller than the rocker load, in that region (see footnote associated with run 2). The windup angle stays negative throughout the forward stroke with its "maximum" experimental amplitude being "smaller" than the computed one. This again, is most likely due to the errors involved in the prediction of the angles ϕ_{4e} and ϕ_5 (see figure 7.21). During regime no. 2, the "minimum" experimental and computed values of the windup angle are $(-) 0.395$ and $(-) 0.35$ degrees, respectively. The extents of regimes 1 and 3 are well predicted.

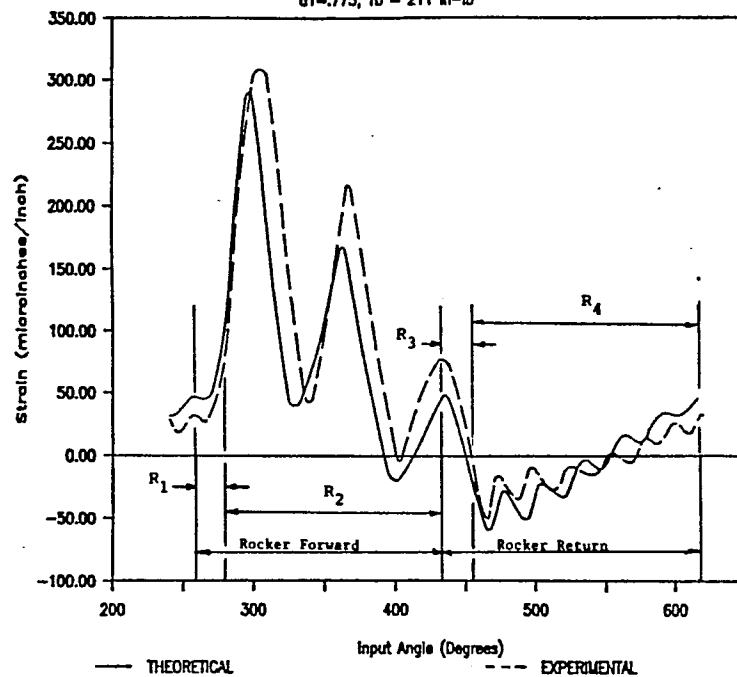
As expected, the strain frequency relationship during the return stroke is essentially the same as that of the previous runs. The phase agreement is good especially during the first portion of the return stroke. The experimental peak-to-peak amplitudes are smaller than the predicted ones. Both are larger, however, than those of run 1 because of the higher energy content of the coupler during the regime transition.

Run 6 (see figures 7.22 and 7.23)

Run 6 deals with the same heavy metal strip and the same feed length as run 5. The input speed is 300 rpm and the brake torque is 211 in-lbs. Thus, while the load factor of run 5 is equal to 3.13 it

STRAIN AT 300 RPM, STRIP: 4.0 x .250

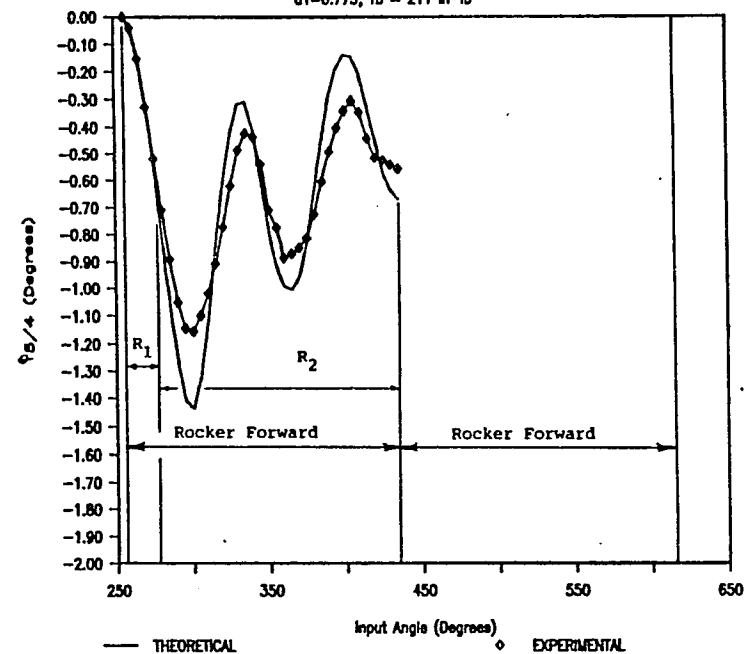
$a_1=0.775$, $T_b = 211 \text{ in-lb}$



a. In-Plane Strain

$\phi_{5/4}$ at 300 RPM, STRIP: 4.0 x .250

$a_1=0.775$, $T_b = 211 \text{ in-lb}$

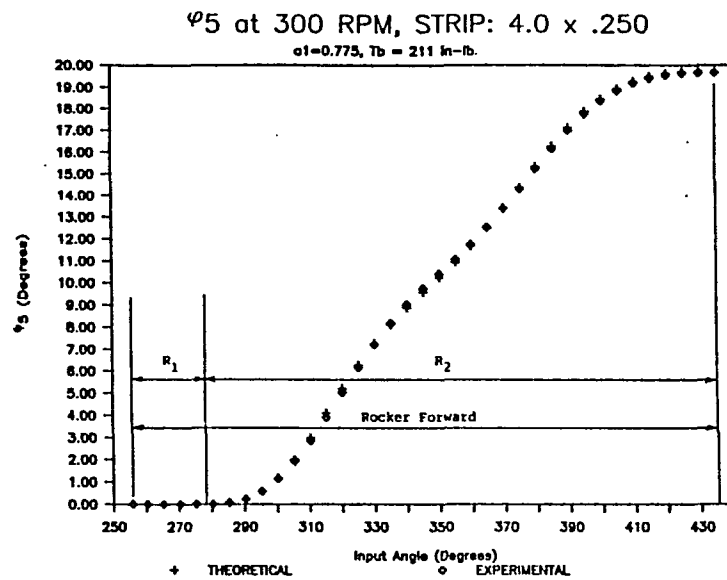


b. Windup Angle $\phi_{5/4}$

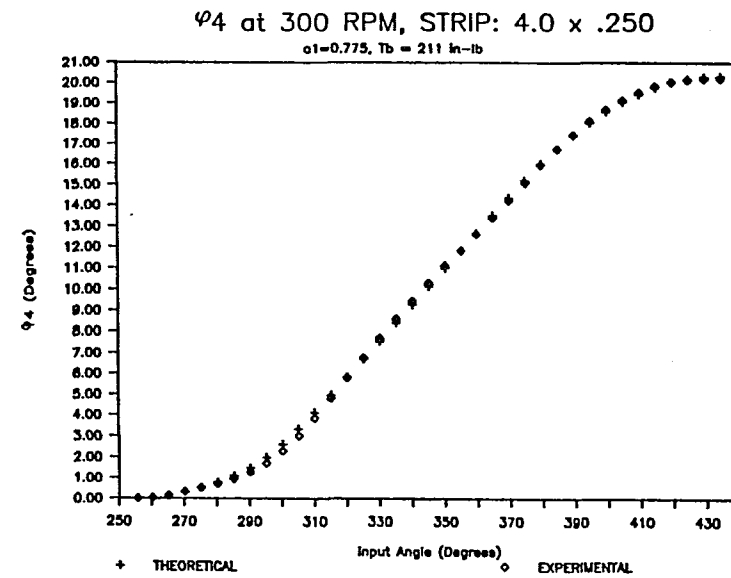
Run No. 6

Input Speed : 300 rpm
 Feed Length : 0.875 in (nominal)
 Strip Size : 4.00 in x 0.250 in
 Brake Torque : 211 in-lb

Figure 7.22: Comparison of In-Plane Strain and Windup Angle for Run 6



a. Feed-Roller Angle φ_5



b. Rocker Angle φ_{4e}

Run No. 6

Input Speed : 300 rpm
 Feed Length : 0.875 in (nominal)
 Strip Size : 4.00 in x 0.250 in
 Brake Torque : 211 in-lb

Figure 7.23: Comparison of Feed-Roller and Rocker Angles for Run 6

becomes 5.01 here. The linearized frequency of oscillation during the forward stroke is the same as that of run 5, i.e. 23.5 Hz. The computed frequency for the strain and the windup angle traces is between 24 and 26 Hz. The associated range of the experimentally obtained frequency is between 21 and 30 Hz. The phase agreement is very good during this portion of the mechanism cycle. The computed and experimental amplitudes of the bending strain are within 10% during the initial portion of the forward stroke. Both the computed and the experimental strains are negative at the mechanism configuration corresponding to the input angle of 400 (actually 40) degrees. Again this is because the work load is smaller than the rocker load, at that point. This could have been changed by an increase in the brake torque. While the windup angle stays negative throughout the forward stroke, its oscillations are larger than in any other run. This shows clearly that, the brake torque must be sufficiently large to keep these oscillations under control and with that prevent a pathological disengagement of the clutch (which would be indicated by a positive windup angle). Note also that during regime no. 2, the "minimum" experimental and computed values of the windup angle are (-) 0.3 and (-) 0.15, respectively. In addition, its "maximum" experimental amplitude is "smaller" than the one predicted by the model. As Figure 7.23 indicates, this may be due to the errors involved in the prediction of the angles φ_{4e} and φ_5 . The extents of regimes 1 and 3 are predicted quite well.

* It was not possible to measure higher brake settings in the experiment under the existing conditions.

The strain frequency relationship during the return stroke is essentially the same as that of the previous runs. The phase agreement is good especially during the first portion of the return stroke. The experimental peak-to-peak amplitudes are smaller than the predicted ones. They are larger, however, than those of run 5 because of the higher energy content of the coupler during the regime transition.

b. Total Forward Angle $\varphi_{5\text{tot}}$

This total forward angle was experimentally obtained by evaluating the angular encoder data output related to the feed-roller angle φ_5 . For each of the six runs 27 data points were obtained, which represented 3 sets of 9 consecutive feed cycles each. It was found that this total angle, and with that the feed length, varied not only from run to run but also from cycle to cycle, for a given input crank dimension. Table 7.8 lists experimentally obtained mean values of feed length and $\varphi_{5\text{tot}}$, as well as the associated maximum tolerances. Table 7.9 compares the computed feed lengths to the above experimental ones.

A plausible explanation for the tolerance in the experimental feed lengths is given by the cycle to cycle variation of the brake torque, which reflects the unpredictable changes in the coefficient of friction. Table 7.10 shows computed feed length tolerances which

result from $\pm 10\%$ changes in the brake torque. These tolerances are of the same order of magnitude as the experimental ones.

Not withstanding the dependence of the computed feed length on the exact magnitude of the brake torque, a comparison of the feed length columns on table 7.9 shows that the model predicts the total feed-roller angle $\phi_{5\text{tot}}$ rather well. A contributory source to the discrepancy that exists between the predicted and the experimental values, may lie in the fact that the rocker angle is very sensitive to the length of the input crank. Since this length was measured before the coupler was tightened on the throwblock, it is quite possible that the tightening changed the length somewhat, and that the computations were made with a slightly smaller than the actual input crank length.

	Mean Feed Length* in	Mean Φ_{5tot} (degrees)	Tolerance ($\pm$)	
			in	degrees
Run 1	0.881	20.196	0.003	0.054
Run 2	0.885	20.304	0.003	0.072
Run 3	1.986	45.576	0.005	0.108
Run 4	0.874	20.052	0.004	0.090
Run 5	0.869	19.933	0.003	0.072
Run 6	0.861	19.750	0.005	0.126

(*) Obtained by multiplying Φ_{5tot} (in radians)
by the 2.499 in, feed-roller radius.

Table 7.8 : Experimentally Obtained Mean Feed-Lengths and
Corresponding Total Feed Roller Angles Φ_{5tot}

	Experimental Mean Feed Length (inches)	Theoretical Feed Length (inches)
Run 1	0.881	0.879
Run 2	0.885	0.880
Run 3	1.986	1.980
Run 4	0.874	0.870
Run 5	0.869	0.865
Run 6	0.861	0.859

Table 7.9 : Comparison of Computed Feed Lengths to Mean
Values of Experimentally Determined Ones

	Nominal Torque in-lb	Variation ($\pm$) in-lb	Computed Tolerance ($\pm$) inches
Run 1	60	6	0.0015
Run 2	87.6	8.76	0.002
Run 3	165	16.5	0.005
Run 4	173.6	17.36	0.0045
Run 5	141	14.1	0.003
Run 6	211	21.1	0.006

Table 7.10 : Influence of plus and minus 10% Brake Load
Variations (Change in the Coefficient of Friction)
on Computed Feed Length Tolerances

c. Effect of Load on Total Forward Rotation ϕ_{4etot} of the Rocker

Table 7.11 gives averaged experimental, as well as computed values for the total forward rotation ϕ_{4etot} of the rocker, for runs 1,2, and 4-6 (for an input crank length equal to 0.775 in). It shows that this angle is different from run to run, although the mechanism dimensions are identical. Thus, the loading influences the extent of the total rocker rotation.

More specifically, if one compares runs 1 and 5, where the speed is the same but the load varies, one can conclude that the higher load causes a reduction in the angle. A similar comparison of runs 2 and 6 leads to the same conclusion. This reduction may be explained by the fact that the coupler link is stretched more at the end of the forward stroke for the higher load. Consequently, the rocker cannot reach its no-load angular minimum position. When, on the other hand, one compares runs of different speeds but approximately equal loading conditions, for example runs 1 and 2, one notices that the forward angle increases as the speed increases. This increase in the total angle is due to the fact that the coupler link is stretched more at the beginning of the forward motion of the rocker. Thus, the rocker exceeds its no-load maximum angular position. The above observation is also verified by runs 1 and 3. It becomes less clear when comparing runs 5 and 6. In this case, the increase in load, which causes a reduction in the angle has a greater effect than the increase in speed which would normally increase this quantity.

While the experimental angles have different values than the computed ones, the above comparisons hold for both sets of data.

a1 = 0.775 in	Total Forward Angle φ_{4etot}	
	Experimental (degrees)	Theoretical (degrees)
Run 1	20.412	20.376
Run 2	20.484	20.431
Run 4	20.520	20.461
Run 5	20.340	20.355
Run 6	20.304	20.352

Table 7.11 : Total Forward Rocker Angle φ_{4etot}

H. CONCLUSIONS

In addition to providing a mathematical model, which predicts the behavior of the lever-type roll feed mechanism quite accurately, this effort has furnished valuable insights concerning the behavior of the various components of this complex system. Meaningful comparison criteria, which will eventually contribute to the development of appropriate design criteria have been developed. These include:

- i. The windup angle, which provides an accurate description of the clutch operation and can be used as a criterion for non-pathological clutch behavior.
- ii. The maximum bending strain and with that the maximum deviation of the elastic rocker angle from its rigid body counterpart.
- iii. The total feed-roller angle.

Finally, the two parameters which contribute most to the performance of the system have been identified as the brake torque, whose proper setting is responsible for satisfying the windup criterion, and the coefficient of friction, which is primarily responsible for feed repeatability.

The results shown in the previous section, indicate that the following assumptions, made in the formulation of the mathematical model, were well justified.

- i. The replacement of the kinematics of the RSSR linkage by that of an RRRSR one.

This led to the elimination of the extra degree of freedom, which represents the small rotation of the coupler about the pivot to pivot line AC.

- ii. The simulation of the sprag clutch by a non-linear torsional spring.

It has been shown that the quantitative prediction of the clutch behavior, although generally good, becomes best when heavy loads are involved. This can be attributed to the fact that the specific curve fit to the clutch deflection data is most accurate in the higher load range.

- iii. The subproblem, which forces the elastically bent beam to slide along line AC.

The resulting shape functions together with the displacement boundary condition at point C, as well as the expression for the change of length of line AC, were responsible for the accurate confirmation of the elastic rocker angle ϕ_{4e} .

- iv. The Taylor series formulation for the elastic angle ϕ_{4e} .

This approximation proved especially valuable for use in the differential equations and with that the correct prediction of the clutch torque and the coupler strain.

- v. The omission of the out-of-plane deflections of the coupler
- vi. The decoupling of the ordinary differential equations and the subsequent fundamental mode formulation.

Naturally, the above assumptions may also have contributed to some of the small discrepancies between the experimental and computed results. The following lists other assumptions which additionally could have contributed to these differences:

- i. Axial tension and compression in the individual coupler sections were neglected.
- ii. The frictional moments of the preloaded bearings were not accounted for. In addition, the effects of the masses of these bearings, at both ends of the coupler link, were ignored.
- iii. The brake torque was assumed constant throughout the cycle, and with that the random nature of friction as well as its speed dependence was neglected.
- iv. The elastic D'Alembert loads were approximated by their rigid-body counterparts.
- v. All non-linear terms, excluding the ones associated with the clutch system, were dropped without regard to their magnitude.
- vi. The method of Kantorovich, which helped to reduce the partial integro-differential equations to ordinary ones, represents an approximation, with the assumption of shape functions belonging to subproblems.

APPENDIX A: DERIVATION OF EQUATIONS OF MOTION FOR REGIME NO. 2

1. WORKING FORM OF HAMILTON'S INTEGRAL

As stated earlier, Hamilton's integral forms the basis for the derivation of the equations of motion. The working form of Hamilton's integral is obtained by the substitution of the variations of the kinetic and potential energies as well as the non-conservative work terms into the variational formulation. Thus, substitution of equations (4.6) and (4.9) to (4.12) into equation (4.1) gives:

$$\begin{aligned}
 & - \int_{t_0}^{t_1} \left\{ I_{04} \ddot{\bar{\Phi}}_{4e} \delta \bar{\Phi}_{4e} + I_{05} \ddot{\bar{\Phi}}_5 \delta \bar{\Phi}_5 + \int_{x_1=0}^{L_1} \rho A \ddot{\bar{v}}_{P_1} \cdot \delta \bar{v}_{P_1} dx_1 + \int_{x_2=0}^{L_2} \rho A \ddot{\bar{v}}_{P_2} \cdot \delta \bar{v}_{P_2} dx_2 \right. \\
 & + \int_{x_1=0}^{L_1} \left[EI \frac{\partial^4 v_1(x_1, t)}{\partial x_1^4} \delta v_1(x_1, t) + EI \frac{\partial^4 w_1(x_1, t)}{\partial x_1^4} \delta w_1(x_1, t) \right] dx_1 + \int_{x_2=0}^{L_2} \left[EI \frac{\partial^4 v_2(x_2, t)}{\partial x_2^4} \delta v_2(x_2, t) \right. \\
 & + EI \frac{\partial^4 w_2(x_2, t)}{\partial x_2^4} \delta w_2(x_2, t) \left. \right] dx_2 + K_{SP} (\Phi_5 - \Phi_{4e})^{3/2} \delta \Phi_5 - K_{SP} (\Phi_5 - \Phi_{4e})^{3/2} \delta \Phi_{4e} + \frac{\dot{\Phi}_5}{|\Phi_5|} T_B \delta \Phi_5 \\
 & + \int_{x_1=0}^{L_1} \left[C_{D_1} \frac{\partial v_1(x_1, t)}{\partial t} \delta v_1(x_1, t) + C_{D_2} \frac{\partial w_1(x_1, t)}{\partial t} \delta w_1(x_1, t) \right] dx_1 + \int_{x_2=0}^{L_2} \left[C_{D_1} \frac{\partial v_2(x_2, t)}{\partial t} \delta v_2(x_2, t) \right. \\
 & + C_{D_2} \frac{\partial w_2(x_2, t)}{\partial t} \delta w_2(x_2, t) \left. \right] dx_2 + \left[EI \frac{\partial^2 v_1(x_1, t)}{\partial x_1^2} \delta \left(\frac{\partial v_1(x_1, t)}{\partial x_1} \right) - EI \frac{\partial^3 v_1(x_1, t)}{\partial x_1^3} \delta v_1(x_1, t) \right]_{x_1=0}^{L_1} \\
 & + \left[EI \frac{\partial^2 w_1(x_1, t)}{\partial x_1^2} \delta \left(\frac{\partial w_1(x_1, t)}{\partial x_1} \right) - EI \frac{\partial^3 w_1(x_1, t)}{\partial x_1^3} \delta w_1(x_1, t) \right]_{x_1=0}^{L_1} + \left[EI \frac{\partial^2 v_2(x_2, t)}{\partial x_2^2} \delta \left(\frac{\partial v_2(x_2, t)}{\partial x_2} \right) \right. \\
 & - EI \frac{\partial^3 v_2(x_2, t)}{\partial x_2^3} \delta v_2(x_2, t) \left. \right]_{x_2=0}^{L_2} + \left[EI \frac{\partial^2 w_2(x_2, t)}{\partial x_2^2} \delta \left(\frac{\partial w_2(x_2, t)}{\partial x_2} \right) - EI \frac{\partial^3 w_2(x_2, t)}{\partial x_2^3} \delta w_2(x_2, t) \right]_{x_2=0}^{L_2} \left. \right\} dt \\
 & = 0
 \end{aligned}
 \tag{A-1}$$

2. SUBSTITUTION OF KINEMATIC TERMS

The following operations are now performed on equation (A-1), in the given order:

- i. The appropriate kinematic terms, as given in appendix E, by equations (E-23) and (E-28), are substituted.
- ii. The required variations of the kinematic terms $\delta \bar{r}_{p_1}$ and $\delta \bar{r}_{p_2}$, as given in appendix H by equations (H-2) and (H-5), are substituted.
- iii. The dot products are computed.

This leads to:

$$\int_{t_0}^{t_1} \left[I_{\alpha\alpha} \ddot{\varphi}_5 + K_{\alpha\alpha} (\varphi_5 - \varphi_{40}) + \frac{\ddot{\varphi}_5}{|\varphi_5|} I_0 \right] \delta \varphi_5 + \left[I_{\alpha\alpha} \ddot{\varphi}_{4e} - K_{\alpha\alpha} (\varphi_5 - \varphi_{4e}) \right] \delta \varphi_{4e} + \int_{x_2=0}^{x_2=L_2} \left[\rho A \ddot{U}_x \delta U(x_1, t) + (\rho A \ddot{U}_y + c_{0y} \frac{\partial U(x_1, t)}{\partial t} + EI \frac{\partial^4 U(x_1, t)}{\partial x_1^4}) \delta V(x_1, t) \right. \\ \left. + (\rho A \ddot{U}_{12} + c_{02} \frac{\partial w_1(x_2, t)}{\partial t} + EI \frac{\partial^4 w_1(x_2, t)}{\partial x_2^4}) \delta w_1(x_1, t) + (\rho A \ddot{U}_{1x} w_1(x_1, t) - \rho A \ddot{U}_{12} w_1(x_2, t)) \delta \varphi_{32e} + (\rho A \ddot{U}_{1y} (x_1, t) \sin \varphi_{32e} - w_1(x_1, t) \cos \varphi_{32e}) \right. \\ \left. - \rho A \ddot{U}_{1x} v_1(x_1, t) \sin \varphi_{32e} + \rho A \ddot{U}_{12} v_1(x_1, t) \cos \varphi_{32e} \right] \delta \varphi_{2e} + \int_{x_2=0}^{x_2=L_2} \left[\rho A \ddot{U}_{2x} \delta U_2(x_2, t) + (\rho A \ddot{U}_{2y} + \frac{\partial w_2(x_2, t)}{\partial t} c_{0y} + EI \frac{\partial^4 w_2(x_2, t)}{\partial x_2^4}) \delta V_2(x_2, t) \right. \\ \left. + (\rho A \ddot{U}_{22} + c_{02} \frac{\partial w_2(x_2, t)}{\partial t} + EI \frac{\partial^4 w_2(x_2, t)}{\partial x_2^4}) \delta w_2(x_2, t) + (\rho A \ddot{U}_{2x} w_2(x_2, t) \cos \varphi - \rho A \ddot{U}_{2y} w_2(x_2, t) \sin \varphi + \rho A \ddot{U}_{22} (V_2(x_2, t) - [x_2 + U_2(x_2, t)] \cos \varphi - L_1)) \delta \varphi_{32e} \right. \\ \left. + (\rho A \ddot{U}_{2y} (L_1 \cos \varphi + x_2 + U_2(x_2, t)) \sin \varphi_{32e} - w_2(x_2, t) \cos \varphi_{32e}) + \rho A \ddot{U}_{2x} (w_2(x_2, t) \sin \varphi_{32e} + [V_2(x_2, t) - L_1 \sin \varphi] \sin \varphi_{32e}) + \rho A \ddot{U}_{22} (V_2(x_2, t) - \right. \\ \left. \cos \varphi_{32e} \varphi_{32e} + [x_2 + U_2(x_2, t)] \sin \varphi_{32e}) \right] \delta \varphi_{2e} + \int_{x_2=0}^{x_2=L_2} \left[EI \frac{\partial^3 w_1(x_2, t)}{\partial x_2^3} \delta \left(\frac{\partial V(x_2, t)}{\partial x_1} \right) - EI \frac{\partial^3 w_1(x_2, t)}{\partial x_2^3} \delta V(x_1, t) \right] + \left[EI \frac{\partial^3 w_2(x_2, t)}{\partial x_2^3} \delta \left(\frac{\partial V_2(x_2, t)}{\partial x_2} \right) - EI \frac{\partial^3 w_2(x_2, t)}{\partial x_2^3} \right. \\ \left. \delta V_2(x_2, t) \right] + \left[EI \frac{\partial^3 w_1(x_2, t)}{\partial x_2^3} \delta \left(\frac{\partial w_1(x_2, t)}{\partial x_1} \right) - EI \frac{\partial^3 w_1(x_2, t)}{\partial x_2^3} \delta w_1(x_1, t) \right] + \left[EI \frac{\partial^3 w_2(x_2, t)}{\partial x_2^3} \delta \left(\frac{\partial w_2(x_2, t)}{\partial x_2} \right) - EI \frac{\partial^3 w_2(x_2, t)}{\partial x_2^3} \delta w_2(x_2, t) \right] \Bigg\} dx_2 = 0 \quad (A-2)$$

where:

$$\ddot{U}_x = R_1(x_1, t) - [I_2^2(t) + I_3^2(t)] U_1(x_1, t) + [I_1(t) I_2(t) - c_{0y}(t) v_1(x_1, t) + (c_{0y}(t) + I_1(t) I_3(t)) w_1(x_1, t) + 2 I_2(t) \frac{\partial w_1(x_1, t)}{\partial t} - 2 I_3(t) \frac{\partial V(x_1, t)}{\partial t}] \quad (A-3)$$

$$\ddot{U}_y = R_2(x_1, t) + (c_{0y}(t) + I_1(t) I_2(t)) U_1(x_1, t) - (I_3^2(t) + I_1^2(t)) v_1(x_1, t) + [I_2(t) I_3(t) - c_{0y}(t) w_1(x_1, t) + 2 I_3(t) \frac{\partial U(x_1, t)}{\partial t} - 2 I_1(t) \frac{\partial w_1(x_1, t)}{\partial t} + \frac{\partial^2 V(x_1, t)}{\partial t^2}] \quad (A-4)$$

$$\ddot{U}_{1x} = R_3(x_1, t) + [I_1(t) I_3(t) - c_{0y}(t) U_1(x_1, t) + (I_2(t) I_3(t) + c_{0y}(t)) v_1(x_1, t) - (I_1^2(t) + I_2^2(t)) w_1(x_1, t) - 2 I_2(t) \frac{\partial U(x_1, t)}{\partial t} + 2 I_1(t) \frac{\partial w_1(x_1, t)}{\partial t} + \frac{\partial^2 w_1(x_1, t)}{\partial t^2}] \quad (A-5)$$

$$\ddot{U}_{12} = R_4(x_2, t) - [I_2^2(t) + I_3^2(t)] U_2(x_2, t) + (I_1(t) I_2(t) - I_3(t)) V_2(x_2, t) + [I_1(t) I_3(t) + I_2(t) w_2(x_2, t) - 2 I_3(t) \frac{\partial V_2(x_2, t)}{\partial t} + 2 I_2(t) \frac{\partial w_2(x_2, t)}{\partial t} + \frac{\partial^2 U_2(x_2, t)}{\partial t^2}] \quad (A-6)$$

$$\ddot{U}_{2x} = R_5(x_2, t) + [I_3(t) + I_1(t) I_2(t)] U_2(x_2, t) + (I_1(t) I_3(t) - I_2(t)) V_2(x_2, t) + [I_1(t) I_3(t) - I_2(t) w_2(x_2, t) - 2 I_3(t) \frac{\partial U_2(x_2, t)}{\partial t} - 2 I_1(t) \frac{\partial V_2(x_2, t)}{\partial t} + \frac{\partial^2 V_2(x_2, t)}{\partial t^2}] \quad (A-7)$$

$$\ddot{U}_{22} = R_6(x_2, t) + (I_1(t) I_3(t) - I_2(t)) U_2(x_2, t) + [I_2(t) V_2(x_2, t) - (I_1^2(t) + I_2^2(t)) w_2(x_2, t) - 2 I_2(t) \frac{\partial U_2(x_2, t)}{\partial t} + 2 I_1(t) \frac{\partial V_2(x_2, t)}{\partial t} + \frac{\partial^2 w_2(x_2, t)}{\partial t^2}] \quad (A-8)$$

3. FINAL FORM OF HAMILTON'S INTEGRAL

Equation (A-2) requires the following modifications:

- i. The expressions for $U_1(x_1, t)$, $U_2(x_2, t)$ and $V_2(x_2, t)$ which are given in terms of the generalized coordinates $v_i(x_i, t)$ and $w_i(x_i, t)$ and their variations, are substituted (see equations H-1 to H-3, of appendix H).
- ii. The variations $\delta\phi_{2e}$, $\delta\phi_{32e}$, $\delta\phi_{4e}$, again expressed in terms of the generalized coordinates and their variations, as given by equations F-36 to F-38 of appendix F, are further substituted.
- iii. In order to be able to collect all terms in v_i and w_i , expressions of the form:

$$\int_{x_i=0}^{L_i} \Lambda(x_i) \int_{\xi_i=0}^{x_i} \frac{\partial^2 \tilde{v}_i}{\partial \xi_i^2} \delta \tilde{v}_i d\xi_i dx_i \quad (A-9)$$

which contain variable limits are transformed according to Leibnitz's rule, into:

$$\int_a^b F(x) \int_a^x G(\xi) d\xi dx = \int_a^b G(x) \int_x^b F(\xi) d\xi dx \quad (A-10)$$

When the limits of integration are constant, one uses:

$$\int_{x_i=0}^{L_i} F(x) \int_{\xi_i=0}^{L_i} G(\xi) d\xi dx = \int_{x_i=0}^{L_i} G(x) \int_{\xi_i=0}^{L_i} F(\xi) d\xi dx \quad (A-11)$$

- iv. Because of the small deflections involved, the elastic angles ϕ_{2e} , ϕ_{32e} , and their derivatives are now approximated by their rigid-body counterparts. The same approximation is applied to all time derivatives of ϕ_{4e} . Finally, the same is done for all terms containing ϕ_{4e} , with the exception of

those that are associated with the term $(\Phi_5 - \Phi_{4e})$, which deals with the clutch elasticity. To account for the coupler elasticity, Φ_{4e} will eventually be described by a Taylor series as a function of v_i and w_i (see appendix C).

Once the above operations are completed, the final form of Hamilton's Integral is obtained. It consists of six groups of terms. Five of these are associated with one of the variations of the generalized coordinates. The sixth group contains all the natural and dynamic boundary conditions of the problem:

$$\begin{aligned}
 & \int_{t_0}^{t_1} \left\{ J_{os} \ddot{\Phi}_5 + K_{sp} (\Phi_5 - \Phi_{4e})^{3/2} + \frac{\dot{\Phi}_5}{|\dot{\Phi}_5|} T_B \right\} \delta \Phi_5 + \int_{x_1=0}^{L_1} \left[\frac{\partial^2 v_1(x_1, t)}{\partial x_1^2} \left[\cos \alpha (H_4(t) + \int_{x_2=0}^{L_2} H_2(x_1, t) dx_2 \right. \right. \\
 & \quad \left. \left. + \int_{x_2=0}^{L_2} H_3(x_2, t) dx_2 \right) + \int_{\xi_1=x_1}^{L_1} \rho A \ddot{\Gamma}_{1x} d\xi_1 + \cos(\alpha + \beta) \int_{x_2=0}^{L_2} \rho A \ddot{\Gamma}_{2x} dx_2 - \sin(\alpha + \beta) \int_{x_2=0}^{L_2} \left(\rho A \ddot{\Gamma}_{2y} + C_{D_1} \frac{\partial v_1(x_1, t)}{\partial t} \right. \right. \\
 & \quad \left. \left. + EI \frac{\partial^4 v_2(x_2, t)}{\partial x_2^4} \right) dx_2 \right] - \rho A \ddot{\Gamma}_{1x} \frac{\partial v_1(x_1, t)}{\partial x_1} + \rho A \ddot{\Gamma}_{1y} + C_{D_1} \frac{\partial v_1(x_1, t)}{\partial t} + EI \frac{\partial^4 v_1(x_1, t)}{\partial x_1^4} \delta v_1(x_1, t) dx_1 \\
 & \quad + \int_{x_1=0}^{L_1} \left[\frac{\partial^2 w_1(x_1, t)}{\partial x_1^2} \left[\cos \alpha (H_4(t) + \int_{x_2=0}^{L_2} H_2(x_1, t) dx_2 + \int_{x_2=0}^{L_2} H_3(x_2, t) dx_2 \right) + \int_{\xi_1=x_1}^{L_1} \rho A \ddot{\Gamma}_{1x} d\xi_1 + \cos(\alpha + \beta) \cdot \right. \right. \\
 & \quad \left. \left. \int_{x_2=0}^{L_2} \rho A \ddot{\Gamma}_{2x} dx_2 - \sin(\alpha + \beta) \int_{x_2=0}^{L_2} \left(\rho A \ddot{\Gamma}_{2y} + C_{D_1} \frac{\partial v_1(x_1, t)}{\partial t} + EI \frac{\partial^4 v_2(x_2, t)}{\partial x_2^4} \right) dx_2 \right] - \rho A \ddot{\Gamma}_{1x} \frac{\partial w_1(x_1, t)}{\partial x_1} \right. \\
 & \quad \left. + \rho A \ddot{\Gamma}_{1a} + C_{D_2} \frac{\partial w_1(x_1, t)}{\partial t} + EI \frac{\partial^4 w_1(x_1, t)}{\partial x_1^4} \right] \delta w_1(x_1, t) dx_1 + \int_{x_2=0}^{L_2} \left[\frac{\partial^2 v_2(x_2, t)}{\partial x_2^2} \left[\cos \beta (H_4(t) \right. \right. \\
 & \quad \left. \left. + \int_{x_1=0}^{L_1} H_2(x_1, t) dx_1 + \int_{x_2=0}^{L_2} H_3(x_2, t) dx_2 \right) + \int_{\xi_2=x_2}^{L_2} \rho A \ddot{\Gamma}_{2x} d\xi_2 - \rho A \ddot{\Gamma}_{2x} \frac{\partial v_2(x_2, t)}{\partial x_2} + \rho A \ddot{\Gamma}_{2y} + C_{D_1} \frac{\partial v_2(x_2, t)}{\partial t} \right. \\
 & \quad \left. + EI \frac{\partial^4 v_2(x_2, t)}{\partial x_2^4} \right] \delta v_2(x_2, t) dx_2 + \int_{x_2=0}^{L_2} \left[\frac{\partial^2 w_2(x_2, t)}{\partial x_2^2} \left[\cos \beta (H_4(t) + \int_{x_1=0}^{L_1} H_2(x_1, t) dx_1 + \int_{x_2=0}^{L_2} H_3(x_2, t) dx_2 \right) \right. \\
 & \quad \left. + \int_{\xi_2=x_2}^{L_2} \rho A \ddot{\Gamma}_{2x} d\xi_2 - \rho A \ddot{\Gamma}_{2x} \frac{\partial w_2(x_2, t)}{\partial x_2} + \rho A \ddot{\Gamma}_{2a} + C_{D_2} \frac{\partial w_2(x_2, t)}{\partial t} + EI \frac{\partial^4 w_2(x_2, t)}{\partial x_2^4} \right] \delta w_2(x_2, t) dx_2 \\
 & \quad + \left[EI \frac{\partial^3 v_1(x_1, t)}{\partial x_1^3} \delta \left(\frac{\partial v_1(x_1, t)}{\partial x_1} \right) - EI \frac{\partial^3 v_1(x_1, t)}{\partial x_1^3} \delta v_1(x_1, t) \right]_{x_1=0}^{L_1} + \left[EI \frac{\partial^3 v_2(x_2, t)}{\partial x_2^3} \delta \left(\frac{\partial v_2(x_2, t)}{\partial x_2} \right) - EI \frac{\partial^3 v_2(x_2, t)}{\partial x_2^3} \delta v_2(x_2, t) \right]_{x_2=0}^{L_2} \\
 & \quad + \left[EI \frac{\partial^3 w_1(x_1, t)}{\partial x_1^3} \delta \left(\frac{\partial w_1(x_1, t)}{\partial x_1} \right) - EI \frac{\partial^3 w_1(x_1, t)}{\partial x_1^3} \delta w_1(x_1, t) \right]_{x_1=0}^{L_1} + \left[EI \frac{\partial^3 w_2(x_2, t)}{\partial x_2^3} \delta \left(\frac{\partial w_2(x_2, t)}{\partial x_2} \right) - EI \frac{\partial^3 w_2(x_2, t)}{\partial x_2^3} \delta w_2(x_2, t) \right]_{x_2=0}^{L_2}
 \end{aligned}$$

$$\begin{aligned}
& + \left(H_4(t) + \int_{x_1=0}^{L_1} H_2(x_1, t) dx_1 + \int_{x_2=0}^{L_2} H_3(x_2, t) dx_2 \right) \left[\sec \beta \cdot \sin(\alpha + \beta) \cdot \delta v_1(L_1, t) + \cos \beta \frac{\partial v_1(L_1, t)}{\partial x_1} \cdot L_2 \delta \left(\frac{\partial v_1(L_1, t)}{\partial x_1} \right) \right. \\
& \left. - \cos \alpha \left[\frac{\partial v_1(x_1, t)}{\partial x_1} \delta v_1(x_1, t) + \frac{\partial w_1(x_1, t)}{\partial x_1} \delta w_1(x_1, t) \right]_0^{L_1} - \cos \beta \left[\frac{\partial v_2(x_2, t)}{\partial x_2} \delta v_2(x_2, t) + \frac{\partial w_2(x_2, t)}{\partial x_2} \delta w_2(x_2, t) \right]_0^{L_2} \right] \\
& + \int_{x_1=0}^{L_1} \rho A \ddot{\Gamma}_{1x} dx_1 \left(\frac{\partial w_1(0, t)}{\partial x_1} \delta w_1(0, t) + \frac{\partial v_1(0, t)}{\partial x_1} \delta v_1(0, t) \right) + \int_{x_2=0}^{L_2} \rho A \ddot{\Gamma}_{2x} \cdot x_2 \frac{\partial v_1(L_1, t)}{\partial x_1} \delta \left(\frac{\partial v_1(L_1, t)}{\partial x_1} \right) dx_2 \\
& + \int_{x_2=0}^{L_2} \rho A \ddot{\Gamma}_{2x} \cdot \left[\sin(\alpha + \beta) \delta v_1(L_1, t) - \cos(\alpha + \beta) \left[\frac{\partial v_1(x_1, t)}{\partial x_1} \delta v_1(x_1, t) + \frac{\partial w_1(x_1, t)}{\partial x_1} \delta w_1(x_1, t) \right]_0^{L_1} + \left[\frac{\partial v_2(0, t)}{\partial x_2} \delta v_2(0, t) \right. \right. \\
& \left. \left. + \frac{\partial w_2(0, t)}{\partial x_2} \delta w_2(0, t) \right] \right] dx_2 + \int_{x_2=0}^{L_2} \left[\rho A \ddot{\Gamma}_{2y} + C_{D1} \frac{\partial v_2(x_2, t)}{\partial t} + EI \frac{\partial^4 v_2(x_2, t)}{\partial x_2^4} \right] \left[\frac{\partial w_1(x_1, t)}{\partial x_1} \delta w_1(x_1, t) \right. \\
& \left. + \frac{\partial v_1(x_1, t)}{\partial x_1} \delta v_1(x_1, t) \right]_0^{L_1} \cdot \sin(\alpha + \beta) \cdot dx_2 \Big\} = 0 \quad (A-12)
\end{aligned}$$

where:

$$H_4(t) = [J_{04} \ddot{\Phi}_{4e} - K_{5p} (\Phi_5 - \Phi_{4e})^{3/2}] \Delta \Phi_{4e} \quad (A-13)$$

$$\begin{aligned}
H_2(x_1, t) = & \rho A \ddot{\Gamma}_{1x} \left[\Delta \Phi_{32e} w_1(x_1, t) - v_1(x_1, t) \sin \Phi_{32e} \Delta \Phi_{2e} \right] + \rho A \ddot{\Gamma}_{1y} \left[(x_1 + U_1(x_1, t)) \sin \Phi_{32e} - w_1(x_1, t) \right. \\
& \left. \cos \Phi_{32e} \right] \Delta \Phi_{2e} + \rho A \ddot{\Gamma}_{1z} \left[v_1(x_1, t) \cos \Phi_{32e} \Delta \Phi_{2e} - (x_1 + U_1(x_1, t)) \Delta \Phi_{32e} \right] \quad (A-14)
\end{aligned}$$

$$\begin{aligned}
H_3(x_2, t) = & \rho A \ddot{\Gamma}_{2x} \left[w_2(x_2, t) (\cos \gamma \Delta \Phi_{32e} - \sin \gamma \cos \Phi_{32e} \Delta \Phi_{2e}) - V_2(x_2, t) \cdot \sin \Phi_{32e} \Delta \Phi_{2e} + L_1 \sin \gamma \right. \\
& \left. \sin \Phi_{32e} \Delta \Phi_{2e} \right] + \rho A \ddot{\Gamma}_{2y} \left[(L_1 \cos \gamma + x_2) \sin \Phi_{32e} \Delta \Phi_{2e} + U_2(x_2, t) \sin \Phi_{32e} \Delta \Phi_{2e} - w_2(x_2, t) \cdot \right. \\
& \left. (\sin \gamma \Delta \Phi_{32e} + \cos \gamma \cos \Phi_{32e} \Delta \Phi_{2e}) \right] + \rho A \ddot{\Gamma}_{2z} \left[x_2 \sin \gamma \cos \Phi_{32e} \Delta \Phi_{2e} - (x_2 \cos \gamma + L_1) \Delta \Phi_{32e} \right. \\
& \left. - U_2(x_2, t) (\Delta \Phi_{32e} \cos \gamma - \sin \gamma \cos \Phi_{32e} \Delta \Phi_{2e}) + V_2(x_2, t) (\sin \gamma \Delta \Phi_{32e} \right. \\
& \left. + \cos \gamma \cos \Phi_{32e} \Delta \Phi_{2e}) \right] \quad (A-15)
\end{aligned}$$

$\Delta \Phi_{32e}$ and $\Delta \Phi_{2e}$ are given by equations F-40 and F-41.

4. PARTIAL LINEARIZATION OF HAMILTON'S INTEGRAL

To obtain a partially linearized Hamilton's integral, the following steps are needed:

- i. The expressions for $U_1(x_1)$, $V_2(x_2, t)$ and $U_2(x_2, t)$, as given by equations E-3, E-6 and E-7, respectively, are substituted and
- ii. all non-linear terms in equation A-12, involving various product combinations of the elastic deflections and their derivatives, are neglected.

Then, equation A-12 becomes:

$$\begin{aligned}
 & \int_{t_0}^{t_1} \left\{ \left[I_{05} \ddot{\Phi}_5 + K_{5P} (\Phi_5 - \Phi_{4e})^{3/2} + \frac{\dot{\Phi}_5}{|\Phi_5|} T_B \right] \delta \Phi_5 + \int_{x_1=0}^{L_1} \left[\frac{\partial^2 v_1(x_1, t)}{\partial x_1^2} \left[\cos \alpha \left[\Delta \Phi_{4e} (I_{04} \ddot{\Phi}_4 - K_{5P} (\Phi_5 - \Phi_{4e})^{3/2}) \right. \right. \right. \right. \\
 & \quad \left. \left. \left. + G_9(t) \right] + G_{10}(x_1, t) + \cos(\alpha + \beta) G_{11}(t) - \sin(\alpha + \beta) G_4(t) \right] - \rho A R_1(x_1, t) \frac{\partial v_1(x_1, t)}{\partial x_1} + \rho A \ddot{\Gamma}_{1YL} \right. \\
 & \quad \left. + C_{D1} \frac{\partial v_1(x_1, t)}{\partial t} + EI \frac{\partial^4 v_1(x_1, t)}{\partial x_1^4} \right] \delta v_1(x_1, t) dx_1 + \int_{x_1=0}^{L_1} \left[\frac{\partial^2 w_1(x_1, t)}{\partial x_1^2} \left[\cos \alpha \left[\Delta \Phi_{4e} (I_{04} \ddot{\Phi}_4 - K_{5P} (\Phi_5 - \Phi_{4e})^{3/2}) \right. \right. \right. \right. \\
 & \quad \left. \left. \left. + G_9(t) \right] + G_{10}(x_1, t) + \cos(\alpha + \beta) G_{11}(t) - \sin(\alpha + \beta) G_4(t) \right] - \rho A R_1(x_1, t) \frac{\partial w_1(x_1, t)}{\partial x_1} + \rho A \ddot{\Gamma}_{1ZL} \right. \\
 & \quad \left. + C_{D2} \frac{\partial w_1(x_1, t)}{\partial t} + EI \frac{\partial^4 w_1(x_1, t)}{\partial x_1^4} \right] \delta w_1(x_1, t) dx_1 + \int_{x_2=0}^{L_2} \left[\frac{\partial^2 v_2(x_2, t)}{\partial x_2^2} \left[\cos \beta \left[\Delta \Phi_{4e} (I_{04} \ddot{\Phi}_4 - K_{5P} (\Phi_5 - \Phi_{4e})^{3/2}) \right. \right. \right. \right. \\
 & \quad \left. \left. \left. + G_9(t) \right] + G_5(t, x_2) - \rho A R_4(t, x_2) \frac{\partial v_2(x_2, t)}{\partial x_2} + \rho A \ddot{\Gamma}_{2YL} + C_{D1} \frac{\partial v_2(x_2, t)}{\partial t} + EI \frac{\partial^4 v_2(x_2, t)}{\partial x_2^4} \right] \delta v_2(x_2, t) dx_2 \right. \\
 & \quad \left. + \int_{x_2=0}^{L_2} \left[\frac{\partial^2 w_2(x_2, t)}{\partial x_2^2} \left[\cos \beta \left[\Delta \Phi_{4e} (I_{04} \ddot{\Phi}_4 - K_{5P} (\Phi_5 - \Phi_{4e})^{3/2}) \right. \right. \right. \right. \\
 & \quad \left. \left. \left. + G_9(t) \right] + G_5(x_2, t) - \rho A R_4(x_2, t) \frac{\partial w_2(x_2, t)}{\partial x_2} \right. \right. \\
 & \quad \left. \left. + \rho A \ddot{\Gamma}_{2ZL} + C_{D2} \frac{\partial w_2(x_2, t)}{\partial x_2} + EI \frac{\partial^4 w_2(x_2, t)}{\partial x_2^4} \right] \delta w_2(x_2, t) dx_2 + \left[\Delta \Phi_{4e} (I_{04} \ddot{\Phi}_4 - K_{5P} (\Phi_5 - \Phi_{4e})^{3/2}) \right. \right. \\
 & \quad \left. \left. + G_6(t) \right] \sec \beta \sin(\alpha + \beta) \delta v_1(L_1, t) + \left[\Delta \Phi_{4e} (I_{04} \ddot{\Phi}_4 - K_{5P} (\Phi_5 - \Phi_{4e})^{3/2}) + G_9(t) \right] \left[\cos \beta \frac{\partial v_1(L_1, t)}{\partial x_1} \right. \right. \\
 & \quad \left. \left. - L_2 \delta \left(\frac{\partial v_1(L_1, t)}{\partial x_1} \right) - \cos \alpha \left[\frac{\partial v_1(x_1, t)}{\partial x_1} \delta v_1(x_1, t) + \frac{\partial w_1(x_1, t)}{\partial x_1} \delta w_1(x_1, t) \right]_0^{L_1} - \cos \beta \left[\frac{\partial v_2(x_2, t)}{\partial x_2} \delta v_2(x_2, t) \right. \right. \right. \\
 & \quad \left. \left. \left. + \frac{\partial w_2(x_2, t)}{\partial x_2} \delta w_2(x_2, t) \right]_0^{L_2} \right] + G_7(t) \left[\frac{\partial w_1(0, t)}{\partial x_1} \delta w_1(0, t) + \frac{\partial v_1(0, t)}{\partial x_1} \delta v_1(0, t) \right] + G_8(t) \frac{\partial v_1(L_1, t)}{\partial x_1} \delta \left(\frac{\partial v_1(L_1, t)}{\partial x_1} \right) \right. \\
 & \quad \left. + \int_{x_2=0}^{L_2} \rho A \ddot{\Gamma}_{2XL} \sin(\alpha + \beta) dx_2 \delta v_1(L_1, t) - G_{11}(t) \cos(\alpha + \beta) \left[\frac{\partial v_1(x_1, t)}{\partial x_1} \delta v_1(x_1, t) + \frac{\partial w_1(x_1, t)}{\partial x_1} \delta w_1(x_1, t) \right]_0^{L_1} \right\}
 \end{aligned}$$

$$\begin{aligned}
& + G_{11}(t) \left[\frac{\partial v_2(0,t)}{\partial x_2} \delta v_2(0,t) + \frac{\partial w_2(0,t)}{\partial x_2} \delta w_2(0,t) \right] + \sin(\alpha + \beta) G_4(t) \left[\frac{\partial w_1(x_1,t)}{\partial x_1} \delta w_1(x_1,t) + \frac{\partial v_1(x_1,t)}{\partial x_1} \delta v_1(x_1,t) \right]_0^{L_1} \\
& + \left[EI \frac{\partial^2 v_1(x_1,t)}{\partial x_1^2} \delta \left(\frac{\partial v_1(x_1,t)}{\partial x_1} \right) - EI \frac{\partial^3 v_1(x_1,t)}{\partial x_1^3} \delta v_1(x_1,t) \right]_0^{L_1} + \left[EI \frac{\partial^2 w_1(x_1,t)}{\partial x_1^2} \delta \left(\frac{\partial w_1(x_1,t)}{\partial x_1} \right) - EI \frac{\partial^3 w_1(x_1,t)}{\partial x_1^3} \delta w_1(x_1,t) \right]_0^{L_1} \\
& + \left[EI \frac{\partial^2 v_2(x_2,t)}{\partial x_2^2} \delta \left(\frac{\partial v_2(x_2,t)}{\partial x_2} \right) - EI \frac{\partial^3 v_2(x_2,t)}{\partial x_2^3} \delta v_2(x_2,t) \right]_0^{L_2} + \left[EI \frac{\partial^2 w_2(x_2,t)}{\partial x_2^2} \delta \left(\frac{\partial w_2(x_2,t)}{\partial x_2} \right) \right. \\
& \left. - EI \frac{\partial^3 w_2(x_2,t)}{\partial x_2^3} \delta w_2(x_2,t) \right]_0^{L_2} \Bigg\} dt = 0 \quad (A-16)
\end{aligned}$$

where:

$$G_4(t) = \int_{x_2=0}^{L_2} \rho A R_5(x_2, t) dx_2 \quad (A-17)$$

$$G_5(x_2, t) = \int_{\xi_2=x_2}^{L_2} \rho A R_4(\xi_2, t) d\xi_2 \quad (A-18)$$

$$G_6(t) = \int_{x_1=0}^{L_1} H_{2L}(x_1, t) dx_1 + \int_{x_2=0}^{L_2} H_{3L}(x_2, t) dx_2 \quad (A-19)$$

$$G_7(t) = \int_{x_1=0}^{L_1} \rho A R_1(x_1, t) dx_1 \quad (A-20)$$

$$G_8(t) = \int_{x_2=0}^{L_2} \rho A R_4(x_2, t) \cdot x_2 dx_2 \quad (A-21)$$

$$\begin{aligned}
G_9(t) = & \int_{x_1=0}^{L_1} \rho A \left(\sin \varphi_{32} \Delta \phi_{2e} R_2(x_1, t) \cdot x_1 - \Delta \phi_{32e} R_3(x_1, t) x_1 \right) dx_1 + \int_{x_2=0}^{L_2} \rho A \left(R_4(x_2, t) L_1 \sin \gamma \sin \varphi_{32} \Delta \phi_{2e} \right. \\
& \left. + R_5(x_2, t) (L_1 \cos \gamma + x_2) \sin \varphi_{32} \Delta \phi_{2e} + R_6(x_2, t) \left(x_2 \sin \gamma \cos \varphi_{32} \Delta \phi_{2e} - (x_2 \cos \gamma + L_1) \Delta \phi_{32e} \right) \right) dx_2 \quad (A-22)
\end{aligned}$$

$$G_{10}(x_1, t) = \int_{\xi_1=x_1}^{L_1} \rho A R_1(\xi_1, t) d\xi_1 \quad (A-23)$$

$$G_{11}(t) = \int_{x_2=0}^{L_2} \rho A R_4(x_2, t) dx_2 \quad (A-24)$$

$$\begin{aligned} \ddot{\Gamma}_{1xL} = & R_1(x_1, t) + (J_1(t)J_2(t) - G_3(t))v_1(x_1, t) + (J_1(t)J_3(t) + G_2(t))w_1(x_1, t) - 2J_3(t)\frac{\partial v_1(x_1, t)}{\partial t} \\ & + 2J_2(t)\frac{\partial w_1(x_1, t)}{\partial t} \end{aligned} \quad (A-25)$$

$$\ddot{\Gamma}_{1yL} = R_2(x_1, t) + (J_2(t)J_3(t) - G_1(t))w_1(x_1, t) - (J_3^2(t) + J_1^2(t))v_1(x_1, t) - 2J_1(t)\frac{\partial w_1(x_1, t)}{\partial t} + \frac{\partial^2 v_1(x_1, t)}{\partial t^2} \quad (A-26)$$

$$\ddot{\Gamma}_{1zL} = R_3(x_1, t) + (J_2(t)J_3(t) + G_1(t))v_1(x_1, t) - (J_1^2(t) + J_2^2(t))w_1(x_1, t) + 2J_1(t)\frac{\partial v_1(x_1, t)}{\partial t} + \frac{\partial^2 w_1(x_1, t)}{\partial t^2} \quad (A-27)$$

$$\begin{aligned} \ddot{\Gamma}_{2xL} = & R_4(x_2, t) + (H_1(t)H_2(t) - I_3(t))v_2(x_2, t) - \sin(\alpha+\beta)(H_2^2(t) + H_3^2(t))v_1(L_1, t) + (H_1(t)H_3(t) + I_2(t)) \\ & w_2(x_2, t) - 2H_3(t)\frac{\partial v_2(x_2, t)}{\partial t} + 2H_2(t)\frac{\partial w_2(x_2, t)}{\partial t} + \sin(\alpha+\beta)\frac{\partial^2 v_1(L_1, t)}{\partial t^2} \end{aligned} \quad (A-28)$$

$$\begin{aligned} \ddot{\Gamma}_{2yL} = & R_5(x_2, t) + \sin(\alpha+\beta)(H_1(t)H_2(t) + I_3(t))v_1(L_1, t) - (H_3^2(t) + H_1^2(t))v_2(x_2, t) + (H_1(t)H_3(t) \\ & - I_1(t))w_2(x_2, t) + 2H_3(t)\sin(\alpha+\beta)\frac{\partial v_1(L_1, t)}{\partial t} - 2H_1(t)\frac{\partial w_2(x_2, t)}{\partial t} + \frac{\partial^2 v_2(x_2, t)}{\partial t^2} \end{aligned} \quad (A-29)$$

$$\begin{aligned} \ddot{\Gamma}_{2zL} = & R_6(x_2, t) + (H_1(t)H_3(t) - I_2(t))\sin(\alpha+\beta)v_1(L_1, t) + (H_2(t)H_3(t) + I_1(t))v_2(x_2, t) \\ & - (H_1^2(t) + H_2^2(t))w_2(x_2, t) + 2H_1(t)\frac{\partial v_2(x_2, t)}{\partial t} - 2\sin(\alpha+\beta)H_2(t)\frac{\partial v_1(L_1, t)}{\partial t} + \frac{\partial^2 w_2(x_2, t)}{\partial t^2} \end{aligned} \quad (A-30)$$

$$\begin{aligned} H_{2L}(x_1, t) = & \rho A \ddot{\Gamma}_{1yL} x_1 \sin \varphi_{32} \Delta \Phi_{2e} - \rho A \ddot{\Gamma}_{1zL} x_1 \Delta \Phi_{32e} + \rho A R_1(x_1, t) [w_1(x_1, t) \\ & \cdot \Delta \Phi_{32e} - v_1(x_1, t) \sin \varphi_{32} \Delta \Phi_{2e}] - \rho A R_2(x_1, t) \cdot \Delta \Phi_{2e} \cos \varphi_{32} w_1(x_1, t) \\ & + \rho A v_1(x_1, t) R_3(x_1, t) \cos \varphi_{32} \Delta \Phi_{2e} \end{aligned} \quad (A-31)$$

$$\begin{aligned}
H_{3L}(x_2, t) = & \rho A R_4(x_2, t) \left[w_2(x_2, t) (\cos \gamma \Delta \phi_{32e} - \sin \gamma \cos \phi_{32} \Delta \phi_{2e}) - v_2(x_2, t) \sin \phi_{32} \Delta \phi_{2e} \right] \\
& + \rho A \ddot{\Gamma}_{2xL} L_1 \sin \gamma \sin \phi_{32} \Delta \phi_{2e} + \rho A \ddot{\Gamma}_{2yL} (L_1 \cos \gamma + x_2) \sin \phi_{32} \Delta \phi_{2e} + \rho A R_5(x_2, t) \cdot \\
& \cdot \left[\sin(\alpha + \beta) v_1(L_1, t) \sin \phi_{32} \Delta \phi_{2e} - w_2(x_2, t) [\sin \gamma \Delta \phi_{32e} + \cos \gamma \cos \phi_{32} \Delta \phi_{2e}] \right] \\
& + \rho A \ddot{\Gamma}_{2zL} \left[x_2 \sin \gamma \cos \phi_{32} \Delta \phi_{2e} - (x_2 \cos \gamma + L_1) \Delta \phi_{32e} \right] - \rho A R_6(x_2, t) \left[\sin(\alpha + \beta) v_1(L_1, t) \right. \\
& \left. (\Delta \phi_{32e} \cos \gamma - \sin \gamma \cos \phi_{32} \Delta \phi_{2e}) + v_2(x_2, t) (\sin \gamma \Delta \phi_{32e} + \cos \gamma \cos \phi_{32} \Delta \phi_{2e}) \right] \quad (A-32)
\end{aligned}$$

5. PARTIAL DIFFERENTIAL EQUATIONS AND BOUNDARY CONDITIONS

The Euler-Lagrange equations are obtained by applying the fundamental lemma of the calculus of variations to the first five groups of equation A-16. The dynamic boundary conditions result from the remaining terms. Before these steps are taken, the variations of the following geometric boundary conditions must be obtained and substituted (see equations J-16, J-18, J-19, J-21, J-70, J-72, J-73, J-6 in appendix J):

$$v_1(0, t) = 0 \quad (A-33)$$

$$w_1(0, t) = 0 \quad (A-34)$$

$$w_2(L_2, t) = 0 \quad (A-35)$$

$$w_1(L_1, t) = w_2(0, t) \quad (A-36)$$

$$v_2(0, t) = \cos(\alpha + \beta) v_1(L_1, t) \quad (A-37)$$

$$\frac{\partial v_1(L_1, t)}{\partial x_1} = \frac{\partial v_2(0, t)}{\partial x_2} \quad (A-38)$$

$$\frac{\partial w_1(L_1, t)}{\partial x_1} = \frac{1}{\cos(\alpha + \beta)} \frac{\partial w_2(0, t)}{\partial x_2} \quad (A-39)$$

$$v_2(L_2, t) = -\tan \beta \cdot \sin(\alpha + \beta) v_1(L_1, t) \quad (A-40)$$

The variations of the above become:

$$\delta v_1(0, t) = 0 \quad (A-41)$$

$$\delta w_1(0, t) = 0 \quad (A-42)$$

$$\delta w_2(L_2, t) = 0 \quad (A-43)$$

$$\delta w_1(L_1, t) = \delta w_2(0, t) \quad (A-44)$$

$$\cos(\alpha + \beta) \delta v_1(L_1, t) = \delta v_2(0, t) \quad (A-45)$$

$$\delta \frac{\partial v_1(L_1, t)}{\partial x_1} = \delta \frac{\partial v_2(0, t)}{\partial x_2} \quad (A-46)$$

$$\delta \frac{\partial w_1(L_1, t)}{\partial x_1} = \frac{1}{\cos(\alpha + \beta)} \delta \frac{\partial w_2(0, t)}{\partial x_2} \quad (A-47)$$

$$\delta v_2(L_2, t) = -\tan \beta \sin(\alpha + \beta) \delta v_1(L_1, t) \quad (A-48)$$

After these terms are substituted into equation A-16, one must let the resulting terms containing zero variations go to zero directly. Further, whenever resulting terms contain non-zero variations their coefficients must vanish.

This leads to the following partial and ordinary differential equations and, where applicable, to the associated dynamic boundary conditions:

a. In-plane motion

For $0 \leq x_1 \leq L_1$

$$\begin{aligned} \frac{\partial^2 v_1(x_1, t)}{\partial x_1^2} & \left[\cos \alpha \left[\Delta \Phi_{4e} \left(J_{\phi 4} \ddot{\Phi}_4 - K_{5p} (\Phi_5 - \Phi_{4e})^{3/2} \right) + G_9(t) \right] + G_{10}(x_1, t) + \cos(\alpha + \beta) G_{11}(t) \right. \\ & \left. - \sin(\alpha + \beta) G_4(t) \right] - \rho A R_1(x_1, t) \frac{\partial v_1(x_1, t)}{\partial x_1} + \rho A \ddot{\Gamma}_{1yL} + C_{D1} \frac{\partial v_1(x_1, t)}{\partial t} + EI \frac{\partial^4 v_1(x_1, t)}{\partial x_1^4} = 0 \quad (A-48) \end{aligned}$$

For $0 \leq x_2 \leq L_2$

$$\begin{aligned} \frac{\partial^2 v_2(x_2, t)}{\partial x_2^2} & \left[\cos \beta \left[\Delta \Phi_{4e} \left(J_{\phi 4} \ddot{\Phi}_4 - K_{5p} (\Phi_5 - \Phi_{4e})^{3/2} \right) + G_9(t) \right] + G_3(x_2, t) \right] - \rho A R_4(t, x_2) \frac{\partial v_2(x_2, t)}{\partial x_2} \\ & + \rho A \ddot{\Gamma}_{2yL} + C_{D1} \frac{\partial v_2(x_2, t)}{\partial t} + EI \frac{\partial^4 v_2(x_2, t)}{\partial x_2^4} = 0 \quad (A-49) \end{aligned}$$

b. Feed roller motion

$$J_{\phi 5} \ddot{\Phi}_5 + K_{5p} (\Phi_5 - \Phi_{4e})^{3/2} + \frac{\dot{\Phi}_5}{|\Phi_5|} T_B = 0 \quad (A-50)$$

The dynamic boundary conditions that relate the coupled in-plane and feed roller motions are given by:

$$\begin{aligned}
 & \sec \beta \sin(\alpha + \beta) \left[\Delta \Phi_{4e} (J_{04} \ddot{\Phi}_4 - K_{5p} (\Phi_5 - \Phi_{4e})^{3/2}) + G_6(t) \right] - \left[\Delta \Phi_{4e} (J_{04} \ddot{\Phi}_4 - K_{5p} (\Phi_5 - \Phi_{4e})^{3/2}) \right. \\
 & \left. + G_9(t) \right] \cdot \left[(\cos \alpha - \cos \beta \cos(\alpha + \beta)) \frac{\partial v_1(L_1, t)}{\partial x_1} - \sin \beta \sin(\alpha + \beta) \frac{\partial v_2(L_2, t)}{\partial x_2} \right] + \int_{x_2=0}^{L_2} \rho A \ddot{\Gamma}_{2xL} \sin(\alpha + \beta) dx_2 \\
 & + G_4(t) \sin(\alpha + \beta) \frac{\partial v_1(L_1, t)}{\partial x_1} - EI \frac{\partial^3 v_1(L_1, t)}{\partial x_1^3} + EI \frac{\partial^3 v_2(L_2, t)}{\partial x_2^3} \tan \beta \sin(\alpha + \beta) \\
 & + EI \frac{\partial^3 v_2(0, t)}{\partial x_2^3} \cos(\alpha + \beta) = 0
 \end{aligned} \tag{A-51}$$

$$\begin{aligned}
 & \left[\Delta \Phi_{4e} (J_{04} \ddot{\Phi}_4 - K_{5p} (\Phi_5 - \Phi_{4e})^{3/2}) + G_9(t) \right] \cos \beta \cdot L_2 \frac{\partial v_1(L_1, t)}{\partial x_1} + G_8(t) \frac{\partial v_1(L_1, t)}{\partial x_1} + EI \frac{\partial^2 v_1(L_1, t)}{\partial x_1^2} \\
 & - EI \frac{\partial^2 v_2(0, t)}{\partial x_2^2} = 0
 \end{aligned} \tag{A-52}$$

c. Out-of-plane motion

For $0 \leq x_1 \leq L_1$

$$\begin{aligned}
 & \frac{\partial^2 w_1(x_1, t)}{\partial x_1^2} \left[\cos \alpha \left[\Delta \Phi_{4e} (J_{04} \ddot{\Phi}_4 - K_{5p} (\Phi_5 - \Phi_{4e})^{3/2}) + G_9(t) \right] + G_{10}(x_1, t) + \cos(\alpha + \beta) G_{11}(t) - \sin(\alpha + \beta) \right. \\
 & \left. \cdot G_4(t) \right] - \rho A R_1(x_1, t) \frac{\partial w_1(x_1, t)}{\partial x_1} + \rho A \ddot{\Gamma}_{1zL} + G_{D2} \frac{\partial w_1(x_1, t)}{\partial t} + EI \frac{\partial^4 w_1(x_1, t)}{\partial x_1^4} = 0
 \end{aligned} \tag{A-53}$$

For $0 \leq x_2 \leq L_2$

$$\begin{aligned}
 & \frac{\partial^2 w_2(x_2, t)}{\partial x_2^2} \left[\cos \beta \left[\Delta \Phi_{4e} (J_{04} \ddot{\Phi}_4 - K_{5p} (\Phi_5 - \Phi_{4e})^{3/2}) + G_9(t) \right] + G_5(x_2, t) \right] - \rho A R_4(x_2, t) \frac{\partial w_2(x_2, t)}{\partial x_2} \\
 & + \rho A \ddot{\Gamma}_{2zL} + G_{D2} \frac{\partial w_2(x_2, t)}{\partial t} + EI \frac{\partial^4 w_2(x_2, t)}{\partial x_2^4} = 0
 \end{aligned} \tag{A-54}$$

The dynamic boundary conditions for the out-of-plane motion are given by:

$$\begin{aligned} & \left[\Delta \Phi_{4e} (J_{04} \ddot{\Phi}_4 - K_{5p} (\Phi_5 - \Phi_{4e})^{3/2}) + G_3(t) \right] \left[\cos \beta \cos(\alpha + \beta) - \cos \alpha \right] \frac{\partial w_1(L_1, t)}{\partial x_1} + G_4(t) \sin(\alpha + \beta) \cdot \\ & \frac{\partial w_1(L_1, t)}{\partial x_1} - EI \frac{\partial^3 w_1(L_1, t)}{\partial x_1^3} + EI \frac{\partial^3 w_2(0, t)}{\partial x_2^3} = 0 \end{aligned} \quad (A-55)$$

$$EI \frac{\partial^2 w_1(L_1, t)}{\partial x_1^2} - EI \cos(\alpha + \beta) \frac{\partial^2 w_2(0, t)}{\partial x_2^2} = 0 \quad (A-56)$$

Note that there is coupling between all the differential equations and all the variables.

APPENDIX B: DETERMINATION OF COUPLED ORDINARY NON-LINEAR DIFFERENTIAL
EQUATIONS BY THE METHOD OF KANTOROVICH

This approximate method assumes the solutions of the coupler variables to be a sum of products of shape and time functions.

To this end one lets:

$$v_1(x_1, t) = \sum_n v_{1n}(x_1) q_n(t) \quad (B-1)$$

$$v_2(x_2, t) = \sum_n v_{2n}(x_2) q_n(t) \quad (B-2)$$

$$w_1(x_1, t) = \sum_n w_{1n}(x_1) p_n(t) \quad (B-3)$$

$$w_2(x_2, t) = \sum_n w_{2n}(x_2) p_n(t) \quad (B-3)$$

The solution of the free vibration subproblems, given in appendix J, furnishes the above space or shape functions, together with the orthogonality conditions. It was kept in mind that the method of Kantorovich requires that the geometric boundary conditions of the subproblem must be identical to those of the main problem.

The following will show what form the linearized integral of equation A-12 will take when the above solution forms, their derivatives, their variations, as well as the variations of their derivatives are substituted in it.

To better understand the result, it is useful to note the following:

- i. Since the shape functions are prescribed, their variations vanish. Thus, the variations of equations B-1 to B-4 take the following form:

$$\delta v_1(x_1, t) = \sum_n v_{1n}(x_1) \delta q_n(t) \quad (B-5)$$

$$\delta v_2(x_2, t) = \sum_n v_{2n}(x_2) \delta q_n(t) \quad (B-6)$$

$$\delta w_1(x_1, t) = \sum_n w_{1n}(x_1) \delta p_n(t) \quad (B-7)$$

$$\delta w_2(x_2, t) = \sum_n w_{2n}(x_2) \delta p_n(t) \quad (B-8)$$

- ii. Whenever the shape functions or their derivatives vanish (for example, for certain boundary terms), equations B-5 to B-8 will also vanish.
- iii. The boundary terms $V_1'''(L_1)$, $V_1''''(L_1)$, $W_1''''(L_1)$, $W_2''(0)$, of the shape functions are expressed according to equations J-36, J-39, J-86, and J-87, of appendix J, respectively.
- iv. The orthogonality conditions, given by equations J-60 and J-107, are used directly, as applicable.
- v. Appropriate manipulation of equations J-29 and J-30 leads to:

$$\int_{x_1=0}^{L_1} v_{1k} v_{1n}^{iv} dx_1 + \int_{x_2=0}^{L_2} v_{2k} v_{2n}^{iv} dx_2 = \lambda_{nv}^4 \left[\int_{x_1=0}^{L_1} v_{1k} v_{1n} dx_1 + \int_{x_2=0}^{L_2} v_{2k} v_{2n} dx_2 \right] \quad (B-9)$$

Similarly, equations J-86 and J-87 furnish:

$$\int_{x_1=0}^{L_1} W_{1k} W_{1n}^{IV} dx_1 + \int_{x_2=0}^{L_2} W_{2k} W_{2n}^{IV} dx_2 = \lambda_{nw}^4 \left[\int_{x_1=0}^{L_1} W_{1k} W_{1n} dx_1 + \int_{x_2=0}^{L_2} W_{2k} W_{2n} dx_2 \right] \quad (B-10)$$

Both expressions allow the replacement of the terms containing fourth derivatives of the shape functions by the orthogonality condition.

Finally, the linearized Hamilton's integral has the following form:

where:

$$M_1(x_1, t) = G_{10}(x_1, t) + \omega \alpha \Delta \Phi_{4e} J_{04} \ddot{\Phi}_4 + G_9(t) + (\cos(\alpha + \beta) G_{11}(t) - \sin(\alpha + \beta) G_4(t)) \quad (B-12)$$

$$M_2(x_1, t) = -\rho A R_1(x_1, t) \quad (B-13)$$

$$M_3(x_1, t) = \rho A \sin(\alpha + \beta) \sec \beta \sin \varphi_{32} \Delta \Phi_{2e} R_1(x_1, t) - \rho A \sin(\alpha + \beta) \sec \beta \Delta \Phi_{2e} R_3(x_1, t) \quad (B-14)$$

$$M_4(x_2, t) = G_5(x_2, t) + \omega \beta \Delta \Phi_{4e} J_{04} \ddot{\Phi}_4 + G_9(t) \quad (B-15)$$

$$M_5(x_2, t) = -\rho A R_4(x_2, t) \quad (B-16)$$

$$M_6(x_2, t) = \sin(\alpha + \beta) \sec \beta [\rho A R_5(x_2, t) \sin(\alpha + \beta) \sin \varphi_{32} \Delta \Phi_{2e} - \rho A R_6(x_2, t) \sin(\alpha + \beta) (\Delta \Phi_{32e} \cos \gamma - \sin \gamma \cos \varphi_{32} \Delta \Phi_{2e})] - \sin(\alpha + \beta) \sec \beta [\rho A L_1 \sin \gamma \sin \varphi_{32} \Delta \Phi_{2e} \sin(\alpha + \beta) (H_2^2(t) + H_3^2(t)) - \rho A (L_1 \cos \gamma + x_2) \sin \varphi_{32} \Delta \Phi_{2e} \sin(\alpha + \beta) (H_1(t) H_2(t) - I_3(t)) - \rho A (x_2 \sin \gamma \cos \varphi_{32} \Delta \Phi_{2e} - (x_2 \cos \gamma + L_1) \Delta \Phi_{32e}) (H_1(t) H_3(t) - I_2(t)) \sin(\alpha + \beta)] - \rho A \sin^2(\alpha + \beta) (H_2^2(t) + H_3^2(t)) \quad (B-17)$$

$$M_7(x_2, t) = -\sec \beta \sin(\alpha + \beta) [\rho A R_4(x_2, t) \sin \varphi_{32} \Delta \Phi_{2e} + (\sin \gamma \Delta \Phi_{32e} + \cos \gamma \cos \varphi_{32e} \Delta \Phi_{2e}) \rho A R_6(t, x_2)] + \sec \beta \sin(\alpha + \beta) [\rho A L_1 \sin \gamma \sin \varphi_{32} \Delta \Phi_{2e} (H_1(t) H_2(t) - I_3(t)) - \rho A (L_1 \cos \gamma + x_2) \sin \varphi_{32} \Delta \Phi_{2e} (H_2^2(t) + H_3^2(t)) + \rho A (x_2 \sin \gamma \cos \varphi_{32} \Delta \Phi_{2e} - (x_2 \cos \gamma + L_1) \Delta \Phi_{32e}) (H_2(t) H_3(t) + I_1(t))] + \rho A \sin(\alpha + \beta) (H_1(t) H_2(t) - I_3(t)) \quad (B-18)$$

$$M_8(x_1, t) = -2\rho A \Delta \Phi_{32} x_1 \sin(\alpha + \beta) \sec \beta J_1(t) \quad (B-19)$$

$$M_9(x_2, t) = -2\rho A \sin(\alpha+\beta) H_3(t) - 2\rho A L_1 \sin\gamma \sin\varphi_{32} \sin(\alpha+\beta) \sec\beta \Delta\phi_{2e} H_3(t) + 2\rho A \sin(\alpha+\beta) \sec\beta (x_2 \sin\gamma \cos\varphi_{32} \Delta\phi_{2e} - (x_2 \cos\gamma + L_1) \Delta\phi_{32e}) H_1(t) \quad (B-20)$$

$$M_{10}(x_2, t) = \sin(\alpha+\beta) \sec\beta \left[2\rho A (L_1 \cos\gamma + x_2) \sin\varphi_{32} \Delta\phi_{2e} \sin(\alpha+\beta) H_3(t) - 2\rho A (x_2 \sin\gamma \cos\varphi_{32} \Delta\phi_{2e} - (x_2 \cos\gamma + L_1) \Delta\phi_{32e}) \sin(\alpha+\beta) H_2(t) \right] \quad (B-21)$$

$$M_{11}(x_1, t) = \sin(\alpha+\beta) \sec\beta \cdot \rho A \sin\varphi_{32} \Delta\phi_{2e} \cdot x_1 \quad (B-22)$$

$$M_{12}(x_2, t) = \sin(\alpha+\beta) \sec\beta \rho A (L_1 \cos\gamma + x_2) \sin\varphi_{32} \Delta\phi_{2e} \quad (B-23)$$

$$M_{13}(x_1, t) = \sin(\alpha+\beta) \sec\beta \left(\rho A R_1(x_1, t) \Delta\phi_{32e} - \rho A R_2(x_1, t) \cos\varphi_{32} \Delta\phi_{2e} \right) + \sin(\alpha+\beta) \sec\beta \left(\rho A \sin\varphi_{32} \Delta\phi_{32e} x_1 (J_2(t) J_3(t) - G_1(t)) + \rho A \Delta\phi_{32e} x_1 (J_1^2(t) + J_2^2(t)) \right) \quad (B-24)$$

$$M_{14}(x_2, t) = \sin(\alpha+\beta) \sec\beta \left[\rho A R_4(x_2, t) (\cos\gamma \Delta\phi_{32e} - \sin\gamma \cos\varphi_{32} \Delta\phi_{2e}) - \rho A R_3(x_2, t) \sin\gamma \cos\varphi_{32} \Delta\phi_{2e} \right] + \sin(\alpha+\beta) \sec\beta \left[\rho A L_1 \sin\gamma \sin\varphi_{32} \Delta\phi_{2e} (H_1(t) H_3(t) - I_2(t)) + \rho A (L_1 \cos\gamma + x_2) \Delta\phi_{2e} (H_1(t) H_3(t) - I_1(t)) - \rho A (x_2 \sin\gamma \cos\varphi_{32} \Delta\phi_{2e} - (x_2 \cos\gamma + L_1) \Delta\phi_{32e}) (H_1^2(t) + H_2^2(t)) \right] + \rho A \sin(\alpha+\beta) (H_1(t) H_3(t) + I_2(t)) \quad (B-25)$$

$$M_{15}(x_1, t) = -2\rho A \sin(\alpha+\beta) \sec\beta \sin\varphi_{32} \Delta\phi_{32e} x_1 J_1(t) \quad (B-26)$$

$$M_{16}(x_2, t) = \sin(\alpha+\beta) \sec\beta \left[2\rho A L_1 \sin\gamma \sin\varphi_{32} \Delta\phi_{2e} H_2(t) - 2\rho A (L_1 \cos\gamma + x_2) \sin\varphi_{32} \Delta\phi_{2e} H_1(t) \right] + 2\rho A \sin(\alpha+\beta) H_2(t) \quad (B-27)$$

$$M_{17}(x_2, t) = \sin(\alpha+\beta) \sec\beta \left[\rho A (x_2 \sin\gamma \cos\varphi_{32} \Delta\phi_{2e} - (x_2 \cos\gamma + L_1) \Delta\phi_{32e}) \right] \quad (B-28)$$

$$M_{10}(x_1, t) = \rho A R_2(x_1, t) \quad (B-29)$$

$$M_{19}(x_1, t) = \sin(\alpha + \beta) \sec \beta \left[\rho A \sin \varphi_{32} \Delta \Phi_{2e} x_1 R_2(x_1, t) - \rho A \Delta \Phi_{32} x_1 R_3(x_1, t) \right] \quad (B-30)$$

$$M_{20}(x_2, t) = \rho A R_5(x_2, t) \quad (B-31)$$

$$\begin{aligned} M_{21}(x_2, t) = \sin(\alpha + \beta) \sec \beta & \left[\rho A L_1 \sin \gamma \sin \varphi_{32} \Delta \Phi_{2e} R_4(x_2, t) + \rho A (L_1 \cos \gamma + x_2) \sin \varphi_{32} \Delta \Phi_{2e} R_5(x_2, t) \right. \\ & \left. + \rho A (x_2 \sin \gamma \cos \varphi_{32} \Delta \Phi_{2e} - (x_2 \cos \gamma + L_1) \Delta \Phi_{32e}) R_6(x_2, t) \right] + \rho A \sin(\alpha + \beta) R_4(x_2, t) \quad (B-32) \end{aligned}$$

$$M_{22}(x_1, t) = \rho A R_3(x_1, t) \quad (B-33)$$

$$M_{23}(x_2, t) = \rho A R_6(x_2, t) \quad (B-34)$$

$$T_1(t) = -\rho A J_1^2(t) \quad (B-35)$$

$$T_2(t) = -\rho A H_1^2(t) \quad (B-36)$$

$$T_3(t) = \rho A (I_3(t) + H_1(t) H_2(t)) \quad (B-37)$$

$$T_4(t) = \Delta \Phi_{4e}(t) L_2 \cos \beta J_{04} \ddot{\Phi}_4 + G_9(t) L_2 \cos \beta + G_8(t) \quad (B-38)$$

$$T_5(t) = (\cos \beta \cos(\alpha + \beta) - \cos \alpha) (\Delta \Phi_{4e} J_{04} \ddot{\Phi}_4 + G_9(t)) + \sin(\alpha + \beta) G_4(t) \quad (B-39)$$

$$T_6(t) = \sin p \sin(\alpha + \beta) (\Delta \Phi_{4e} J_{04} \ddot{\Phi}_4 + G_9(t)) \quad (B-40)$$

$$T_7(t) = 2\rho A H_3(t) \sin(\alpha + \beta) \quad (B-41)$$

$$T_8(t) = \sin(\alpha + \beta) \sec \beta \left[\rho A L_1 \sin \gamma \sin \varphi_{32} \Delta \Phi_{2e} \sin(\alpha + \beta) \right] + \rho A \sin^2(\alpha + \beta) \quad (B-42)$$

$$T_9(t) = \rho A (I_2(t) I_3(t) - G_1(t)) \quad (B-43)$$

$$T_{10}(t) = \rho A (H_1(t) H_3(t) - I_1(t)) \quad (B-44)$$

$$T_{11}(t) = -2\rho A I_1(t) \quad (B-45)$$

$$T_{12}(t) = -2\rho A H_1(t) \quad (B-46)$$

$$T_{13}(t) = \sec \beta \sin(\alpha + \beta) \Delta \Phi_{4e} J_{04} \ddot{\Phi}_4 \quad (B-47)$$

$$T_{14}(t) = \rho A (I_2(t) I_3(t) + G_1(t)) \quad (B-48)$$

$$T_{15}(t) = \rho A \sin(\alpha + \beta) (H_1(t) H_3(t) - I_2(t)) \quad (B-49)$$

$$T_{16}(t) = \rho A (H_2(t) H_3(t) + I_1(t)) \quad (B-50)$$

$$T_{17}(t) = 2\rho A I_1(t) \quad (B-51)$$

$$T_{18}(t) = 2\rho A H_1(t) \quad (B-52)$$

$$T_{1g}(t) = 2\rho A \sin(\alpha+\beta) H_2(t) \quad (B-53)$$

$$\begin{aligned} J_{1(n,k)} = & L_2 \Delta\phi_{4e} \cos\beta V'_{1n}(L_1) V'_{1k}(L_1) + (\cos\beta \cos(\alpha+\beta) - \cos\alpha) \Delta\phi_{4e} V'_{1n}(L_1) V'_{1k}(L_1) \\ & + \sin\beta \sin(\alpha+\beta) \Delta\phi_{4e} V'_{2n}(L_2) V'_{1k}(L_1) \end{aligned} \quad (B-54)$$

$$J_{2(n,k)} = T_4(t) V'_{1n}(L_1) V'_{1k}(L_1) + T_5(t) V'_{1n}(L_1) V'_{1k}(L_1) + T_6(t) V'_{2n}(L_2) V'_{1k}(L_1) \quad (B-55)$$

$$J_{3(n,k)} = (\cos\beta \cos(\alpha+\beta) - \cos\alpha) \Delta\phi_{4e} W'_{1n}(L_1) W'_{1k}(L_1) \quad (B-56)$$

After application of the fundamental lemma to equation (B-11) the following set of coupled differential equations results:

a. For the feed roller motion:

$$J_{05} \ddot{\phi}_5 + K_{SP} (\phi_5 - \phi_{4e})^{3/2} + \frac{\dot{\phi}_5}{|\dot{\phi}_5|} T_B = 0 \quad (B-57)$$

b. For the in-plane motion, there will be one of the following expressions for each of the k modes of the shape functions considered:

$$\begin{aligned} & \rho A J_k \ddot{q}_k(t) + c_{D1} J_k \dot{q}_k(t) + \rho A J_k (\omega_{V_k}^2 - J_3^2(t)) q_k(t) + \sum_{n=1}^{\infty} \left[\left[-K_{SP} \phi_5 - \phi_{4e} \right]^{3/2} J_{4(n,k)} + \right. \\ & \left. + Z_{n,k}(t) \right] q_n(t) + \Theta_{n,k}(t) \dot{q}_n(t) + \Lambda_{n,k}(t) \ddot{q}_n(t) + \Xi_{n,k}(t) p_n(t) + \Psi_{n,k}(t) \dot{p}_n(t) + \\ & \left. + \Phi_{n,k}(t) \ddot{p}_n(t) \right] + F_{2k}(t) - K_{SP} (\phi_5 - \phi_{4e})^{3/2} \cdot J_{3k}(t) = 0 \end{aligned} \quad (B-58)$$

c. For the out-of-plane motion there will also be one of the following expressions for each of the k modes of the shape functions considered:

$$\rho A \mathcal{W}_k \ddot{p}_k(t) + c_{D2} \mathcal{W}_k \dot{p}_k(t) + \rho A \mathcal{W}_k (\omega_{wk}^2 - (J_1^2(t) + J_2^2(t))) p_k(t) + \sum_{n=1}^{\infty} \left[-k_{sp} (\varphi_3 - \varphi_{4e})^{3/2} \right. \\ \left. J_{6(n,k)} + \hat{A}_{n,k}(t) \right] p_n(t) + \Gamma_{n,k}(t) q_n(t) + \Delta_{n,k}(t) \dot{q}_n(t) + F_{3k}(t) = 0 \quad (B-59)$$

where

$$\mathcal{Z}_{n,k}(t) = \int_{x_1=0}^{L_1} \left[M_1(x_1, t) V_{1n}''(x_1) V_{1k}(x_1) + M_2(x_1, t) V_{1n}'(x_1) V_{1k}(x_1) + T_1(t) V_{1n}(x_1) V_{1k}(x_1) \right. \\ \left. - M_3(x_1, t) V_{1n}(x_1) V_{1k}(L_1) \right] dx_1 + \int_{x_2=0}^{L_2} \left[M_4(x_2, t) V_{2n}''(x_2) V_{2k}(x_2) + M_5(x_2, t) V_{2n}'(x_2) V_{2k}(x_2) \right. \\ \left. + T_5(t) V_{2n}(x_2) V_{2k}(x_2) + M_6(x_2, t) V_{1n}(L_1) V_{1k}(L_1) + T_3(t) V_{1n}(L_1) V_{2k}(x_2) + M_7(x_2, t) \right. \\ \left. V_{2n}(x_2) V_{1k}(L_1) \right] dx_2 + J_{2(n,k)} \quad (B-60)$$

$$\Theta_{n,k}(t) = \int_{x_1=0}^{L_1} M_8(x_1, t) V_{1n}(x_1) V_{1k}(L_1) dx_1 + \int_{x_2=0}^{L_2} \left[M_9(x_2, t) V_{2n}(x_2) V_{1k}(L_1) + M_{10}(x_2, t) V_{1n}(L_1) \right. \\ \left. \cdot V_{1k}(L_1) + T_7(t) V_{1n}(L_1) \cdot V_{2k}(x_2) \right] dx_2 \quad (B-61)$$

$$\mathcal{L}_{n,k}(t) = \int_{x_1=0}^{L_1} M_{11}(x_1, t) V_{1n}(x_1) V_{1k}(L_1) dx_1 + \int_{x_2=0}^{L_2} \left[T_8(t) V_{1n}(L_1) V_{1k}(L_1) \right. \\ \left. + M_{12}(x_2, t) V_{2n}(x_2) V_{1k}(L_1) \right] dx_2 \quad (B-62)$$

$$\Xi_{n,k}(t) = \int_{x_1=0}^{L_1} \left[T_9(t) W_{1n}(x_1) V_{1k}(x_1) + M_{13}(x_1, t) W_{1n}(x_1) V_{1k}(L_1) \right] dx_1 + \int_{x_2=0}^{L_2} \left[T_{10}(t) \cdot W_{2n}(x_2) V_{2k}(x_2) \right. \\ \left. + M_{14}(x_2, t) W_{2n}(x_2) V_{1k}(L_1) \right] dx_2 \quad (B-63)$$

$$\Psi_{n,k}(t) = \int_{x_1=0}^{L_1} \left[T_{11}(t) W_{1n}(x_1) V_{1k}(x_1) + M_{15}(x_2, t) W_{1n}(x_1) V_{1k}(L_1) \right] dx_1 + \int_{x_2=0}^{L_2} \left[T_{12}(t) W_{2n}(x_2) V_{2k}(x_2) \right. \\ \left. + M_{16}(x_2, t) W_{2n}(x_2) V_{1k}(L_1) \right] dx_2 \quad (B-64)$$

$$\Phi_{n,k}(t) = \int_{x_2=0}^{L_2} M_{17}(x_2, t) W_{2n}(x_2) V_{1k}(L_1) dx_2 \quad (B-65)$$

$$F_{2k}(t) = \int_{x_1=0}^{L_1} [M_{18}(x_1, t) V_{1k}(x_1) + M_{19}(x_1, t) V_{1k}(L_1)] dx_1 + \int_{x_2=0}^{L_2} [M_{20}(x_2, t) V_{2k}(x_2) + M_{21}(x_2, t) V_{1k}(L_1)] dx_2 + T_{13}(t) V_{1k}(L_1) \quad (B-66)$$

$$A_{n,k}(t) = \int_{x_1=0}^{L_1} [M_1(x_1, t) W_{1n}''(x_1) W_{1k}(x_1) + M_2(x_1, t) W_{1n}'(x_1) W_{1k}(x_1)] dx_1 + \int_{x_2=0}^{L_2} [M_4(x_2, t) W_{2n}''(x_2) W_{2k}(x_2) + M_5(x_2, t) W_{2n}'(x_2) W_{2k}(x_2)] dx_2 + T_5(t) W_{1n}'(L_1) W_{1k}(L_1) \quad (B-67)$$

$$\Gamma_{n,k}(t) = \int_{x_1=0}^{L_1} T_{14}(t) V_{1n}(x_1) W_{1k}(x_1) dx_1 + \int_{x_2=0}^{L_2} [T_{15}(t) V_{1n}(L_1) W_{2k}(x_2) + T_{16}(t) V_{2n}(x_2) W_{2k}(x_2)] dx_2 \quad (B-68)$$

$$\Delta_{n,k}(t) = \int_{x_1=0}^{L_1} T_{17}(t) W_{1k}(x_1) V_{1n}(x_1) dx_1 + \int_{x_2=0}^{L_2} [T_{18}(t) W_{2k}(x_2) V_{2n}(x_2) + T_{19}(t) W_{2k}(x_2) V_{1n}(L_1)] dx_2 \quad (B-69)$$

$$F_{3k}(t) = \int_{x_1=0}^{L_1} M_{22}(x_1, t) W_{1k}(x_1) dx_1 + \int_{x_2=0}^{L_2} M_{23}(x_2, t) W_{2k}(x_2) dx_2 \quad (B-70)$$

$$J_4(n, k) = \int_{x_1=0}^{L_1} \cos \alpha \Delta \Phi_{4e} V_{1n}''(x_1) V_{1k}(x_1) dx_1 + \int_{x_2=0}^{L_2} \cos \beta \Delta \Phi_{4e} V_{2n}''(x_2) V_{2k}(x_2) dx_2 + J_1(n, k) \quad (B-71)$$

$$J_5(k) = \sec \beta \sin(\alpha + \beta) \Delta \Phi_{4e} V_{1k}(L_1) \quad (B-72)$$

$$J_6(n, k) = \int_{x_1=0}^{L_1} \cos \alpha \Delta \Phi_{4e} W_{1n}''(x_1) W_{1k}(x_1) dx_1 + \int_{x_2=0}^{L_2} \cos \beta \Delta \Phi_{4e} W_{2n}''(x_2) W_{2k}(x_2) dx_2 + J_3(n, k) \quad (B-73)$$

APPENDIX C: TAYLOR SERIES APPROXIMATION OF ELASTIC ANGLE ϕ_{4e}

To obtain an expression for the elastic rocker angle ϕ_{4e} (see equations B-57 to B-59), it is necessary to include the deformed coupler length AC'. Its elongation is not only due to the usual foreshortening, which is also experienced in straight bars, but also to the "opening of the legs" under the tensile load.

While an expression for the elastic angle ϕ_{4e} can be obtained in terms of the variables v_i and w_i , by way of the elastic constraint equations (see equations F-1 to F-3). The resulting form is highly non-linear. To avoid the associated difficulties in the subsequent computations, it has been decided to use a simpler expression which results from a Taylor series expansion in v_i and w_i about the rigid body angle ϕ_{4R} . It has the following form:

$$\phi_{4e} = \phi_{4R} + \sum_{i=1}^2 \left[\frac{\partial \phi_{4e}}{\partial v_i(x_i, t)} v_i(x_i, t) + \frac{\partial \phi_{4e}}{\partial w_i(x_i, t)} w_i(x_i, t) \right] \quad (C-1)$$

Where $i=1$ is for section 1, and $i=2$ is for section 2, of the coupler.

With

$$v_i(x_i, t) = \sum_n V_{in}(x_i) q_n(t) \quad (i=1,2)$$

and

$$w_i(x_i, t) = \sum_n W_{in}(x_i) p_n(t) \quad (i=1,2)$$

one obtains, for the above:

$$\dot{\phi}_{4e} = \dot{\phi}_{4R} + \sum_{n=1}^{\infty} \left[\frac{\partial \phi_{4e}}{\partial q_n} \dot{q}_n(t) + \frac{\partial \phi_{4e}}{\partial p_n} \dot{p}_n(t) \right] \quad (C-2)$$

To determine expressions for $\frac{\partial \phi_{4e}}{\partial q_n}$ and $\frac{\partial \phi_{4e}}{\partial p_n}$ one needs a relationship between ϕ_{4e} and the elastic deflections v_i and w_i . This is furnished by the following adaptation of the Freudenstein equation, which relates the output angle ϕ_{4e} to the input angle ϕ_1 :

$$A \sin \phi_{4e} + B \cos \phi_{4e} = C \quad (C-3)$$

where:

$$A = 2s_1 a_4 \quad (C-4)$$

$$B = 2a_4 [s_3 + a_1 \sin \phi_1] \quad (C-5)$$

$$C = a_4^2 + s_1^2 + s_2^2 + s_3^2 + a_1^2 + 2a_1 s_2 \cos \phi_1 + 2a_1 s_3 \sin \phi_1 - AC'^2 \quad (C-6)$$

AC' is given by equation F-26 of appendix F, i.e.

$$AC' = a_3 + u_1(L_1, t) \cos \alpha + u_2(L_2, t) \cos \beta + v_1(L_1, t) \sec \beta \sin(\alpha + \beta), \quad (C-7)$$

with the foreshortening terms:

$$u_1(L_1, t) = -\frac{1}{2} \int_{x_1=0}^{L_1} \left[\left[\frac{\partial v_1}{\partial x_1} \right]^2 + \left[\frac{\partial w_1}{\partial x_1} \right]^2 \right] dx_1 \quad (C-8)$$

$$u_2(L_2, t) = -\frac{1}{2} \int_{x_2=0}^{L_2} \left[\left[\frac{\partial v_2}{\partial x_2} \right]^2 - \left[\frac{\partial v_1(L_1, t)}{\partial x_1} \right]^2 + \left[\frac{\partial w_2}{\partial x_2} \right]^2 \right] dx_2 \quad (C-9)$$

The partial derivatives, $\frac{\partial \phi_{4e}}{\partial q_n}$ and $\frac{\partial \phi_{4e}}{\partial p_n}$, may be obtained by differentiation of equation C-3 with respect to q_n and p_n , respectively. Thus, differentiation of C-3 with respect to q_n , furnishes:

$$A \cos \phi_{4e} \frac{\partial \phi_{4e}}{\partial q_n} - B \sin \phi_{4e} \frac{\partial \phi_{4e}}{\partial q_n} = \frac{\partial C}{\partial q_n} \quad (C-10)$$

While from equation C-6 one obtains:

$$\frac{\partial C}{\partial q_n} = -2AC' \frac{\partial AC'}{\partial q_n} \quad (C-11)$$

When equation C-11 is substituted into C-10, one finds:

$$\frac{\partial \phi_{4e}}{\partial q_n} = - \frac{2AC'}{[A \cos \phi_{4e} - B \sin \phi_{4e}]} \frac{\partial AC'}{\partial q_n} \quad (C-12)$$

A similar sequence of operations, furnishes:

$$\frac{\partial \phi_{4e}}{\partial p_n} = - \frac{2AC'}{[A \cos \phi_{4e} - B \sin \phi_{4e}]} \frac{\partial AC'}{\partial p_n} \quad (C-13)$$

The derivative of AC' with respect to q_n is obtained by direct differentiation of equation C-7, i.e.

$$\begin{aligned} \frac{\partial AC'}{\partial q_n} &= \frac{\partial u_1(L_1, t)}{\partial q_n} \cos \alpha + \frac{\partial u_2(L_2, t)}{\partial q_n} \cos \beta \\ &+ \frac{\partial v_1(L_1, t)}{\partial q_n} \sec \beta \sin(\alpha + \beta) \end{aligned} \quad (C-14)$$

Substitution of equations C-5, C-8 and C-9 into the above and subsequent algebraic manipulation, leads to:

$$\frac{\partial AC'}{\partial Q_n} = V_{1n}(L_1) \sec \beta \sin(\alpha + \beta) - Q_n(t) \left[\cos \alpha \int_{x_1=0}^{L_1} V'_{1n} \cdot V'_{1n} dx_1 + \cos \beta \int_{x_2=0}^{L_2} [V'_{2n} \cdot V'_{2n} + V'_{1n}(L_1) \cdot V'_{1n}(L_1)] dx_2 \right] \quad (C-15)$$

The derivative of AC' with respect to p_n is obtained in a similar manner:

$$\frac{\partial AC'}{\partial P_n} = -P_n(t) \left[\cos \alpha \int_{x_1=0}^{L_1} W'_{1n} \cdot W'_{1n} dx_1 + \cos \beta \int_{x_2=0}^{L_2} W'_{2n} \cdot W'_{2n} dx_2 \right] \quad (C-16)$$

Equation C-2 is now rewritten with the help of equations C-12, C-13, C-15 and C-16:

$$\begin{aligned} \Phi_{4e} = \Phi_{4e} - \frac{2 \cdot AC'}{(A \cos \Phi_{4e} - B \sin \Phi_{4e})} \sum_{n=1}^{\infty} & \left[V_{1n}(L_1) \sec \beta \sin(\alpha + \beta) \cdot Q_n(t) \right. \\ & - \left[\cos \alpha \int_{x_1=0}^{L_1} V'_{1n} V'_{1n} dx_1 + \cos \beta \int_{x_2=0}^{L_2} [V'_{2n} \cdot V'_{2n} + V'_{1n}(L_1) \cdot V'_{1n}(L_1)] dx_2 \right] Q_n^2(t) \\ & \left. - \left[\cos \alpha \int_{x_1=0}^{L_1} W'_{1n} W'_{1n} dx_1 + \cos \beta \int_{x_2=0}^{L_2} W'_{2n} W'_{2n} dx_2 \right] P_n^2(t) \right] \quad (C-17) \end{aligned}$$

When one disregards the non-linear terms, one obtains:

$$\Phi_{4e} = \Phi_{4R} - \frac{2AC'}{(A\cos\Phi_{4e} - B\sin\Phi_{4e})} \sum_{n=1}^{\infty} V_{1n}(L_1) \sec\beta \cdot \sin(\alpha+\beta) \cdot Q_n(t). \quad (C-18)$$

Further linearization of the above involves the replacement of Φ_{4e} and AC' by their rigid body counterparts. This furnishes the final expression:

$$\Phi_{4e} = \Phi_{4R} - \frac{2AC}{(A\cos\Phi_4 - B\sin\Phi_4)} \sum_{n=1}^{\infty} V_{1n}(L_1) \cdot \sec\beta \cdot \sin(\alpha+\beta) \cdot Q_n(t) \quad (C-19)$$

APPENDIX D: PROGRAM FEED

1. INTRODUCTION

The present appendix shows how solutions of the various differential equations of the four-regime cycle with their associated initial and terminal conditons, are obtained by way of program FEED. To clarify the connections between the different regime equations as given in Section VII-B and the program, reference is made, as applicable, to specific line numbers of the program.

In addition, the solution flexibility of the program which utilizes a fourth-order double precision Runge-Kutta routine^{*}, by way of four different pairs of external functions associated with the particular Runge-Kutta scheme, is discussed. Further, the input parameters and output variables of the sample run are listed.

2. FLEXIBILITY OF PROGRAM FEED

While the sample program only considers the first mode of the in-plane coupler deflection, program FEED is capable of simultaneously solving up to four modes of the mutually coupled differential equations for the in-plane coupler deflections v_1 and the associated strain, together with the applicable feed roller expression. Recall

* DRKGS Routine, IBM SYSTEM/360 Scientific Subroutine Package

that the coupler differential equations are in terms of $q_k(\phi_1)$, i.e. the time related portions of the product solutions, which contain both time and space portions.

The above means that there are provisions in the program for the determination of the coefficients of all four coupler deflection equations. These coefficients are dependent on the shape functions of the various modes. The content of the specific external functions (FCT1, FCT2, FCT3, OUTP1, OUTP2, OUTP3, OUTP4) for the Runge-Kutta routine, referred to earlier, depends on the number of modes considered and must be programmed as the need arises.

3. FOUR-REGIME FEED CYCLE (SINGLE-MODE FORMULATION)

The following relates the differential equations of the various regimes, as given in section VII-B, to the specific parts of the program. In each case, the associated initial and terminal conditions are considered.

a. Regime No.1

The points of interest in this regime are the resulting in-plane elastic coupler deflection, which allows the determination of the coupler elongation, the elastic rocker angle, as well as the coupler strain. The above are obtained from the solution of the coupler differential equation 7.1. Note that this expression is solved for

the time portion $q_1(\phi_1)$ of the deflections v_i . This operation is accomplished by way of the external functions FCT1 and OUTP1 (these are given by statement lines 326 to 498).

Subroutines ELAST and PHI4EL (see statement lines 1063 to 1130), furnish the elastic coupler length AC' as well as the elastic rocker angle ϕ_{4e} and $\dot{\phi}_{4e}$.

The various subroutines, which eventually allow the computation of the coefficients of this and other regime equations, are briefly described below:

I. SUBROUTINE KINEM (statement lines 1404 to 1530 of program FEED)	Determines all rigid body kinematic quantities of the spatial four-bar linkage in terms of the input angle ϕ_1 .
II. FUNCTIONS V1(X,N), V2(X,N),	Determine the values of the shape functions for the nth in-plane mode.
III. FUNCTIONS V1P(X,N), V2P(X,N)	Determine the first derivatives of the above functions.
IV. FUNCTIONS V1DP(X,N), V2DP(X,N)	Determine the second derivative of each of the shape functions of section II above.

The subroutines associated with sections II to IV are shown between statement lines 2019 and 2217 of program FEED.

V. FUNCTIONS F1(X) to F66(X) (statement lines 1618 to 2016 of program FEED)	Assemble various expressions containing functions of sections II to IV for subsequent integration.
VI. SUBROUTINE FINTGRL (statement lines 1531 to 1615 of program FEED)	Integrates functions F1 to F66 with the help of subroutine ASQ(G,A,EPS), an adaptive Simpson quadrature method (statement lines 2218 to 2290).
VII. SUBROUTINE COEFF (statement lines 1131 to 1403 of program FEED)	Computes the coefficients of the differential equation 7.1 for every time step.

b. Regime No. 2

The coupler deflection variable $q_1(\varphi_1)$ is solved according to equation 7.6. In addition, the feed roller angle $\varphi_5(\varphi_1)$ is determined by way of equation 7.7, which is coupled to the coupler deflection expressions by way of the elastic rocker angle. Both, the coupler deflection, as well as the relationship between the elastic rocker angle and the feed roller angle are of interest in this part of the feed cycle. The inertia of the metal strip, which is fed by the feed

rolls, is referred to the feed rollers. The regime ends when the rocker link reaches its extreme position.

The above is accomplished by way of a separate Runge-Kutta routine with the external functions FCT2 and OUP2 (shown between statement lines 499 and 721). As for regime no. 1, once the coupler deflection is known, the elastic coupler length AC' and the elastic rocker angle φ_{4e} , together with its derivative, may be obtained with the help of subroutines ELAST and PHI4EL.

The external functions FCT2 and OUP2 also make use of the various subroutines described earlier in section 3a.

c. Regime No. 3

The coupler deflection, the elastic rocker angle and the possible angle of overshoot are of interest. The regime ends when the windup angle is equal to zero.

The solution of equation 7.1 furnishes the variables $q_1(\varphi_1)$. In addition, the uncoupled feed roller equation 7.10 must be solved for the angle φ_5 . This is again accomplished with the Runge-Kutta routine, which is now serviced by the external functions FCT3 and OUP3 (shown by statement lines 887 to 1062).

As for regimes no. 1 and 2, once the coupler deflections are known, AC' as well as the elastic rocker angle and its derivative may be computed with the help of subroutines ELAST and PHI4EL, for the present regime.

It is to be noted that the unrelated differential equations can be solved together by the same Runge-Kutta routine. It has the advantage that the dependent variables of all differential equations involved can be determined for the same value of the independent variable.

The external functions FCT3 and OUTF3 also make use of all the subroutines enumerated in section 3a.

d. Regime No. 4

Again, the coupler deflection, the elastic rocker angle and its relationship to the possible overshoot angle of the feed roller assembly are of importance.

The solution of equation 7.12 furnishes the variable $q_1(\phi_1)$. In addition, the uncoupled feed roller expression 7.9 must be solved for the angle ϕ_5 , if needed. This is accomplished with the Runge-Kutta routine which now is serviced by the external functions FCT4 and OUTF4 (shown between statement lines 722 and 886). The unrelated differential equations are again solved by the same integration routine.

The external functions FCT4 and OUTP4 also make use of all the subroutines enumerated in section 3a.

4. LIST OF INPUT PARAMETERS

The required input parameters which are made part of the program, appear between lines 64 and 117 of program FEED. They are listed below both as computer variables and as symbols. The required eigenvalues λ_{vi}^* are supplied as data.

A1	=	a_1	=	0.775 in	= input crank length
A3	=	a_3 (AC)	=	32.400 in	= length of pivot to pivot line AC
A4	=	a_4	=	4.500 in	= rocker length
S1	=	s_1	=	1.171 in	= X_o -offset (see figure I-1)
S2	=	s_2	=	7.951 in	= Y_o -offset (see figure I-1)
S3	=	s_3	=	31.478 in	= Z_o -offset (see figure I-1)
ALPHA=	α	=	12.000 degrees	= angle that the first coupler section makes with line AC	
BETA =	β	=	12.000 degrees	= angle that the second coupler section makes with line AC	
RHO	=	ρ	=	$0.283/386.4 \text{ lb-sec}^2\text{-in}^{-4}$	= coupler mass density
CD1	=	c_{D1}	=	0.015	= coupler damping coefficient
A	=	A	=	0.3436 in^2	= area of coupler cross-section

* The program uses the IMSL subroutine DEKKER to obtain the in-plane and out-of-plane eigenvalues from the solution of the transcendental equations J-53 and J-101 of appendix J, respectively.

L1	=	L_1	=	16.562 in	= length of first coupler section
L2	=	L_2	=	16.562 in	= length of second coupler section
P1	=	x_1	=	11.000 in	= strain gage location
YMAX	=	$y_{\max}$	=	0.469 in	= coupler cross-section outer fiber
ROUT	=	R_{out}	=	0.500 in	= coupler cross-section outer radius
RIN	=	R_{in}	=	0.375 in	= coupler cross-section inner radius
PHI4L1	=	$\phi_{4\max}$	=	97.045 degrees	= maximum rigid body rocker angle
PHI1IN	=	$\phi_{1\text{in}}$	=	256 degrees	= input angle ϕ_1 at $\phi_{4\max}$
PHI5IN	=	$\phi_{5\text{in}}$	=	97.045 degrees	= feed roller angle ϕ_5 at $\phi_{4\max}$
TBR	=	T_{BR}	=	211 in-lb	= brake torque
FR	=	T_{FR}	=	40 in-lb	= table friction torque
KSP	=	K_{SP}	=	170,000 in-lb-rad ^{-3/2}	= stiffness coefficient of torsional spring
JO4FR	=	J_{O4}	=	0.092 lb-sec ² -in	= clutch outer race-rocker link mass moment of inertia (contains springs)
JO4OV	=	J_{O4}	=	0.090 lb-sec ² -in	= clutch outer race-rocker link mass moment of inertia during return
JO5	=	J_{O5}	=	0.768 lb-sec ² -in	= combined feed roller-metal strip mass moment of inertia
CD2	=	C_D	=	20.381	= torsional spring damping coefficient (gives 0.09 damping ratio)
II	=	I_{yy}	=	0.0336 in ²	= coupler area moment of inertia
E	=	E	=	30 x 10 ⁶ psi	= Young's modulus of elasticity
PHIDT1	=	$\dot{\phi}_1$	=	200 rpm	= mechanism input speed

5. LIST OF OUTPUT VARIABLES

The program provides the output variables in terms of the throwblock angle ϕ_1 . They are listed below both as computer variables and as symbols:

PHI1	:	ϕ_1	degrees	:	Throwblock (input) angle ϕ_1
PHI4R	:	ϕ_4	degrees	:	Rigid body rocker angle ϕ_4
PHI4	:	ϕ_{4e}	degrees	:	Elastic rocker angle ϕ_{4e}
PHIDT4EL:	:	$\dot{\phi}_{4e}$	rad-sec ⁻¹	:	Elastic rocker angular velocity $\dot{\phi}_{4e}$
PHI5	:	ϕ_5	degrees	:	Feed roller angle ϕ_5
EPSL	:	ϵ_v	(in/in)	:	In-plane strain ϵ_v at $x_1 = 11$ in.

In addition, the headings of the various sections of the output given in table D-1 are explained below:

CLUTCH TIGHTENING UP	:	R_1	:	Regime No. 1
STRIP FORWARD MOTION	:	R_2	:	Regime No. 2
SHAFT OVERSHOOT, CLUTCH				
UNLOCKING, LINKAGE RETURN	:	R_3	:	Regime No. 3
SHAFT OVERSHOOT, AND				
LINKAGE RETURN	:	R_4	:	Regime no. 4

6. SAMPLE OUTPUT OF PROGRAM FEED AND INTERPRETATION OF RESULTS

Table D-1 shows a sample output for of a complete mechanism cycle. All values are given in terms of the input angle ϕ_1 , at 10-degree intervals. The four different regimes are separated and their

extent can easily be obtained since both the initial and terminal values of the input angle are given for each regime. The following outlines how each of the parameters discussed in section VII-G, can be obtained with the help of table D-1. For a given mechanism configuration, i.e. $\phi_1 = 10.250$ degrees, one obtains:

$$\phi_4 = 82.551, \quad \phi_{4e} = 82.630, \quad \phi_5 = 83.161, \quad \epsilon_v = 53.6 \times 10^{-6}$$

With the above information, the parameters of interest are obtained in the following manner:

a. In-Plane Strain : This is given directly by the output, i.e.

$$\epsilon_v = 53.6 \times 10^{-6} \text{ strains (in/in)}$$

b. Windup Angle : This is obtained using equation 7.17. To this end, one needs to obtain the value of the elastic rocker angle at the beginning of the rocker forward motion, i.e. the beginning of regime no.1

Table D-1 gives : $\phi_{4\text{emax}} = 97.134$. Then,

$$\phi_{4e}^* = \phi_{4\text{emax}} - \phi_{4e} = 97.134 - 82.630 = 14.504^\circ$$

$$\text{and, } \phi_5^* = \phi_{4\text{emax}} - \phi_5 = 97.134 - 83.161 = 13.973^\circ$$

$$\text{Finally, from 7.17 ; } \phi_{54} = \phi_5^* - \phi_{4e}^* = -0.531^\circ$$

c. Angle $\phi_{4e/R}$: This is obtained by taking the difference between the elastic and the rigid body rocker angle, i.e.

$$\phi_{4e/R} = \phi_{4e} - \phi_4 = 82.630 - 82.551 = 0.079^\circ$$

d. Angle $\phi_{5\text{tot}}$: The total feed-roller rotation $\phi_{5\text{tot}}$ is obtained by taking the difference between the initial feed

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CLUTCH TIGHTENING UP										
PH11=256-500	PH18=	97.104	PH14=	97.134	PH15=97.134	PH10D14=	-0.00033	EPSL=	0.196650479D-04	
PH11=260-230	PH18=	97.076	PH14=	97.105	PH15=97.134	PH10D14=	-0.06253	EPSL=	0.1679035321D-04	
PH11=210-230	PH18=	96.780	PH14=	96.634	PH15=97.134	PH10D14=	-0.06713	EPSL=	0.354930239D-04	
PH11=214-750	PH18=	96.545	PH14=	96.634	PH15=97.134	PH10D14=	-1.24387	EPSL=	0.5937122805D-04	
STRIP FORWARD MOTION										
PH11= 474-750	PH14=	96.545	PH14=	96.634	PH15=	97.134	PH10D14=	-1.24387	EPSL=	0.5937122805D-04
PH11= 280-450	PH14=	96.172	PH14=	96.534	PH15=	97.112	PH10D14=	-1.58835	EPSL=	0.1112861571D-03
PH11= 290-450	PH14=	95.273	PH14=	95.517	PH15=	96.538	PH10D14=	-2.16990	EPSL=	0.1622871795D-03
PH11= 300-450	PH14=	96.112	PH14=	94.429	PH15=	94.972	PH10D14=	-2.67160	EPSL=	0.915150817D-04
PH11= 310-450	PH14=	92.730	PH14=	92.787	PH15=	93.180	PH10D14=	-3.09396	EPSL=	0.3823105568D-04
PH11= 320-450	PH14=	91.172	PH14=	91.268	PH15=	91.774	PH10D14=	-3.41356	EPSL=	0.648564337D-04
PH11= 330-450	PH14=	89.488	PH14=	89.656	PH15=	90.455	PH10D14=	-3.62292	EPSL=	0.113379811D-03
PH11= 340-450	PH14=	87.730	PH14=	87.873	PH15=	88.651	PH10D14=	-3.72042	EPSL=	0.9637481415D-04
PH11= 350-450	PH14=	85.953	PH14=	86.021	PH15=	86.510	PH10D14=	-3.70534	EPSL=	0.4663503119D-04
PH11= 360-450	PH14=	84.209	PH14=	84.253	PH15=	84.625	PH10D14=	-3.58001	EPSL=	0.2993718269D-04
PH11= 370-450	PH14=	82.551	PH14=	82.630	PH15=	83.161	PH10D14=	-3.34920	EPSL=	0.5363130070D-04
PH11= 380-450	PH14=	81.026	PH14=	81.135	PH15=	81.601	PH10D14=	-3.02042	EPSL=	0.7321395269D-04
PH11= 390-450	PH14=	79.681	PH14=	79.749	PH15=	80.320	PH10D14=	-2.60034	EPSL=	0.4664065270D-04
PH11= 400-450	PH14=	78.553	PH14=	78.581	PH15=	78.963	PH10D14=	-2.10953	EPSL=	0.1886498690D-04
PH11= 410-450	PH14=	77.676	PH14=	77.700	PH15=	78.061	PH10D14=	-1.55404	EPSL=	0.1635441290D-04
PH11= 420-450	PH14=	77.076	PH14=	77.124	PH15=	77.613	PH10D14=	-0.95325	EPSL=	0.3235376629D-04
PH11= 430-450	PH14=	76.770	PH14=	76.830	PH15=	77.388	PH10D14=	-0.32536	EPSL=	0.4056706250D-04
PH11= 440-450	PH14=	76.731	PH14=	76.779	PH15=	77.310	PH10D14=	0.02459	EPSL=	0.3252563060D-04
SHAFT OVERSHOOT, CLUTCH UNLOCKING AND LINKAGE RETURN										
PH11= 75-350	PH14=	76.731	PH14=	76.779	PH15=	77.310	PH10D14=	0.02459	EPSL=	0.3252563060D-04
PH11= 80-450	PH14=	76.767	PH14=	76.807	PH15=	77.310	PH10D14=	0.31044	EPSL=	0.269644980D-04
PH11= 90-450	PH14=	77.065	PH14=	77.065	PH15=	77.310	PH10D14=	0.93481	EPSL=	0.195710970D-08
PH11= 95-500	PH14=	77.339	PH14=	77.301	PH15=	77.310	PH10D14=	1.25171	EPSL=	-0.259153815D-04
SHAFT OVERSHOOT AND LINKAGE RETURN MOTION										
PH11= 95-500	PH14=	77.339	PH14=	77.301	PH15=	77.310	PH10D14=	1.25171	EPSL=	-0.259153815D-04
PH11= 100-450	PH14=	77.654	PH14=	77.616	PH15=	77.310	PH10D14=	1.52915	EPSL=	-0.259153815D-04
PH11= 110-450	PH14=	78.517	PH14=	79.455	PH15=	77.310	PH10D14=	2.07663	EPSL=	-0.45938395D-04
PH11= 120-450	PH14=	79.627	PH14=	79.601	PH15=	77.310	PH10D14=	2.50680	EPSL=	-0.177014176D-04
PH11= 130-450	PH14=	80.951	PH14=	80.924	PH15=	77.310	PH10D14=	2.97089	EPSL=	-0.108126929D-04
PH11= 140-450	PH14=	82.454	PH14=	82.435	PH15=	77.310	PH10D14=	3.29687	EPSL=	-0.150429797D-04
PH11= 150-450	PH14=	84.083	PH14=	84.058	PH15=	77.310	PH10D14=	3.56276	EPSL=	-0.167371713D-04
PH11= 160-450	PH14=	85.801	PH14=	85.793	PH15=	77.310	PH10D14=	3.65627	EPSL=	-0.446915093D-05
PH11= 170-450	PH14=	87.555	PH14=	87.540	PH15=	77.310	PH10D14=	3.67902	EPSL=	-0.1021793500D-04
PH11= 180-450	PH14=	89.297	PH14=	89.297	PH15=	77.310	PH10D14=	3.59665	EPSL=	0.329288836D-05
PH11= 190-450	PH14=	90.973	PH14=	90.970	PH15=	77.310	PH10D14=	3.40700	EPSL=	-0.190876190D-05
PH11= 200-450	PH14=	92.533	PH14=	92.542	PH15=	77.310	PH10D14=	3.11252	EPSL=	0.565267232D-05
PH11= 210-450	PH14=	93.929	PH14=	93.939	PH15=	77.310	PH10D14=	2.71875	EPSL=	0.6536415592D-05
PH11= 220-450	PH14=	95.115	PH14=	95.132	PH15=	77.310	PH10D14=	2.23866	EPSL=	0.1099150825D-04
PH11= 230-450	PH14=	96.051	PH14=	96.071	PH15=	77.310	PH10D14=	1.67393	EPSL=	0.1331914240D-04
PH11= 240-450	PH14=	97.050	PH14=	96.728	PH15=	77.310	PH10D14=	1.05303	EPSL=	0.155190647D-04
PH11= 250-450	PH14=	97.050	PH14=	97.078	PH15=	77.310	PH10D14=	0.39271	EPSL=	0.1683504105D-04
PH11= 256-500	PH14=	97.105	PH14=	97.134	PH15=	77.310	PH10D14=	-0.02877	EPSL=	0.196650479D-04
CLUTCH TIGHTENING UP										

roller angle, at the beginning of regime no. 1,
and the final one. Thus,

$$\phi_{5\text{tot}} = 97.134 - 77.310 = 19.824 \text{ (degrees)}$$

e. Feed Length : The feed length can be obtained by multiplying
the total feed roller angle by the feed roller
radius which is equal to 2.499 in, i.e.

$$\begin{aligned} \text{Feed Length} &= (19.824 \times 3.1416 \times 2.499) / 180 \\ &= 0.865 \text{ in.} \end{aligned}$$

7. PROGRAM FEED

CUNYJESJ 12/10/87 13:36:01 G.M.T. WAS THE ORIGIN
 DEST: MEPRT FILE: 8585 OUTPUT RLCs: OUT


```

1.  // JOB
2.  // *MAIN TIME=35,LINES=12
3.  // EXEC WAITFIV,REGION=700K
4.  //SYSIN DD *
5.  $JOB          XX,TIME=2100,PAGES=500
6.  C
7.  C    MAIN PROGRAM
8.  C
9.  C    IMPLICIT REAL*8(A-Z)
10. C    INTEGER I,J,N,K,L,NDIM,NDIM1,NDIM2,IHLF,PLTCNT,NDATA
11. C
12. C    EXTERNAL FUNCTION DESIGNATIONS
13. C    =====
14. C
15. C    EXTERNAL FCT1,OUTP1,FCT2,OUTP2,FCT3,OUTP3,FCT4,OUTP4
16. C
17. C    COMMON STATEMENTS
18. C    =====
19. C
20. C    COMMON/PRTCNT/COUNT
21. C    COMMON/MECHDM/A1,A3,A4,S1,S2,S3
22. C    COMMON/PROPRI/J04,J05,RHO,CD1,TB,KSP
23. C    COMMON/RKDAT/PHIDT1,OMEGAV,PHI4L1,PHI4L2,TIME,PHI5IN
24. C    COMMON/ICLGHT/XIN,XDTIN
25. C    COMMON/ICFRWD/YIN,YDTIN,PH5INF,PH5UNF
26. C    COMMON/ICDVPN/ZIN,ZDTIN,PH5IND,PH5UND
27. C    COMMON LAMDAV,LAMDAW,I,J,ALPHA,BETA,L1,L2,A
28. C    COMMON/INTGRL/FF1,FF2,FF3,FF4,FF5,FF6,FF7,FF8,FF9,FF10,FF11,FF12,
29. C    1FF13,FF14,FF15,FF16,FF17,FF18,FF19,FF20,FF21,
30. C    2FF29,FF30,FF31,FF32,FF33,FF34,FF35,FF36,FF37,
31. C    4FF59
32. C    COMMON/ACINTG/FF64,FF65,FF66,V11L1
33. C    COMMON/ORTH/VV
34. C    COMMON/STRAIN/V11DP,V21DP,YMAX
35. C    COMMON/REGDES/PHI5PR
36. C    COMMON/PLOT/J,PHI,RGMDDES,PLTCNT,NDATA,QMAX
37. C    COMMON/CLTUMP/CD2
38. C
39. C    DIMENSIONAL STATEMENTS
40. C    =====
41. C
42. C    DIMENSION ZINK(4,4),THETNK(4,4),LAMDNK(4,4)
43. C    1F2K(4),J5K(4),
44. C    2J4NK(4,4),J7K(4)
45. C    DIMENSION LAMDAV(4),FF1(4,4),FF2(4,4),FF3(4,4),FF4(4,4)
46. C    1,FF5(4,4),FF6(4,4),FF7(4,4),FF8(4,4),FF9(4,4),FF10(4,4),FF11(4,4)
47. C    2,FF12(4,4),FF13(4,4),FF14(4,4),FF15(4,4),FF16(4,4),FF17(4,4)
48. C    3,FF18(4,4),FF19(4,4),FF20(4,4),FF21(4,4),FF22(4,4)
49. C    4,FF29(4)
50. C    5,FF30(4),FF31(4),FF32(4),FF33(4),FF34(4),FF35(4)
51. C    6,FF36(4),FF37(4)
52. C    9,FF59(4,4)

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53.    DIMENSION FF64(4),FF65(4),FF66(4)
54.    DIMENSION VV(4)
55.    DIMENSION UMEGAV(4)
56.    DIMENSION AUX(8,2),PRMT(5),DERX(2),X(2),Y(4),DERY(4),AUX1(6,4)
57.    I,Z(4),DERZ(4),AUX2(8,4)
58.    DIMENSION Q(10000),PHI(10000),RGMDRES(10000)
59.    C
60.    C    MECHANISM DIMENSIONS
61.    C
62.    C    *****
63.    C
64.    A1=0.77500
65.    A3=32.40000
66.    A4=4.50000
67.    S1=1.17100
68.    S2=7.95100
69.    S3=31.47800
70.    ALPHA=12.000
71.    BETA=12.000
72.    RHO=.28300/386.400
73.    CD1=.01500
74.    A= 0.34361159600
75.    L1=16.56191764
76.    L2=16.56191764
77.    PI=3.14159265358979300
78.    ZI=PI/180.000
79.    ALPHA=ALPHA*ZI
80.    BETA=BETA*ZI
81.    P1=11.000
82.    YMAX=0.4687500
83.    ROUT=.500
84.    RIN=.375
85.    C
86.    C
87.    C    INITIAL VALUES FOR PHI4L AND PHI1IN
88.    C    =====
89.    C
90.    PHI4L1=97.0447200
91.    PHI4L2=PHI4L1
92.    PHI1IN=256.0000
93.    PHI1IN=PHI1IN*ZI
94.    PHI4L1=PHI4L1*ZI
95.    PHI4L2=PHI4L2*ZI
96.    PHI5IN=97.0447200
97.    PHI5PR=PHI5IN
98.    PHI5PR=PHI5PR*ZI
99.    PHI5IN=PHI5IN*ZI
100.   C
101.   C    MOMENTS OF INERTIA AND COUPLER PROPERTIES
102.   C    =====
103.   C
104.   TBR=211.000
105.   FR=40.000
106.   KSP=170000.000
107.   J04OV=.06956601200
108.   J04FR=.09152523500
109.   J05=0.7681558800
110.   LD2=20.381000
111.   I1=0.03355582900
112.   E=30.006
113.   C
114.   C    MECHANISM INPUT SPEED(RAD/SEC)
115.   C    =====
116.   C
117.   PHIDT1=200.000*2.000*PI/60.000
118.   PHUTC=PHIDT1/(2.000*PI)

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119. C
120. C READ AND WRITE NATURAL FREQUENCIES OF COUPLER
121. C =====
122. C
123. READ(5,1) (LAMDAV(I),I=1,4)
124. WRITE(6,1) (LAMDAV(I),I=1,4)
125. 1 FORMAT(4F10.7)
126. C
127. C CALCULATION OF VARIOUS OMEGAS
128. C
129. DO 2 I=1,4
130. OMEGAV(I)=DSQRT(E*I*(LAMDAV(I)**4)/(RHO*A))
131. 2 CONTINUE
132. FRQNT=OMEGAV(1)/(2.000*PI)
133. PRINT, 'OMEGAV(1)=',FRQNT,'CYCLES PER SECOND'
134. COUNT=10.000
135. NDATA=10000
136. PLTCNT=0.000
137. QMAX=0.000
138. REGDST=0.000
139. C
140. C
141. C
142. C
143. CALL FNTGRL(FF1,FF2,FF3,FF4,FF5,FF6,FF7,FF8,
144. 1FF9,FF10,FF11,FF12,FF13,FF14,FF15,FF16,FF17,FF18,FF19,FF20,FF21,
145. 2FF22,FF23,FF24,FF25,FF26,FF27,FF28,FF29,FF30,FF31,FF32,FF33,FF34,
146. 3FF35,FF36,FF37,
147. 4FF38,VV,
148. 5FF39,FF40,FF41,FF42,FF43,FF44,FF45,FF46,FF47,FF48,FF49,FF50,FF51,FF52,FF53,FF54,FF55,FF56,FF57,FF58,FF59,FF60,FF61,FF62,FF63,FF64,FF65,FF66)
149. C
150. C
151. C
152. V11L1=V1(1,L1)
153. V11DP=V1DP(1,P1)
154. V21DP=V2DP(1,P2)
155. AL=ALPHA/Z1
156. BET=BETA/Z1
157. C
158. C PRINT STATEMENTS
159. C
160. WRITE(6,75)
161. 75 FORMAT(' ',3X,'MECHANISM DIMENSIONS')
162. WRITE(6,76) A1,A3,A4,S1,S2,S3
163. 76 FORMAT(' ',3X,'A1=',F7.4,3X,'A3=',F7.3,3X,'A4=',F7.3,3X,'S1=',
164. 1F7.4,3X,'S2=',F7.3,3X,'S3=',F7.3)
165. WRITE(6,77)
166. 77 FORMAT(' ',3X,'COUPLER LINK GEOMETRIC PROPERTIES')
167. WRITE(6,8) A,L1,L2,AL,BET,ROUT,RIN
168. 8 FORMAT(' ',3X,'A=',F10.6,3X,'L1=',F10.6,3X,'L2=',F10.6,3X,'ALPHA=',
169. 1F10.6,3X,'BETA=',F10.6,3X,'ROUT=',F10.6,3X,'RIN=',F10.6)
170. WRITE(6,99)
171. 99 FORMAT(' ',3X,'PARAMETERS USED IN STRAIN EVALUATION')
172. WRITE(6,9) P1,YMAX
173. 9 FORMAT(' ',3X,'THE STRAIN IS EVALUATED AT X1=',F10.6,
174. 1'WITH OUTER FIBER RADIUS =',F10.6)
175. WRITE(6,10) PHOTIC
176. 10 FORMAT(' ',3X,'MECHANISM INPUT SPEED',F12.6,'CYCLES PER SECOND')
177. WRITE(6,11) FRQNT
178. 11 FORMAT(' ',3X,'BEAM NATURAL FREQUENCY',F12.6,'CYCLES PER SECOND')
179. WRITE(6,12) TBR
180. 12 FORMAT(' ',3X,'VALUE OF BRAKE TORQUE',F10.4,'LBS-IN')
181. WRITE(6,13) JO5
182. 13 FORMAT(' ',3X,'VALUE OF COMBINED FEED ROLLER MOMENT OF INERTIA',
183. 1F12.7)
184. WRITE(6,14) KSP

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105. 14 FORMAT(' ',3X,'VALUE OF CLUTCH STIFFNESS',F10.2)
106. WRITE(6,15) CD2,CD1
107. 15 FORMAT(' ',3X,'VALUE OF CLUTCH DAMPING COEF. CD2 IS',F12.6,
108. 13X,'VALUE OF COUPLER DAMPING COEF. CD1 IS',F12.6)
109. WRITE(6,16) FR
110. 16 FORMAT(' ',3X,'MOMENT DUE TO FRICTION ON STRIP IS: FR=',F12.7)
111. C
112. C INITIALIZATION OF COMPUTATIONS
113. C =====
114. C
115. C
116. C
117. C
118. C DATA FOR RUNGE-KUTTA ROUTINES(START-UP VALUES)
119. C
120. XIN=0.000
121. XOTIN=0.000
122. TIME=PHI1IN
123. C
124. C TIGHTENING UP OF CLUTCH
125. C
126. 3 CONTINUE
127. PRMT(1)=TIME
128. PRMT(2)=49.00000
129. PRMT(3)=PI/720.000
130. PRMT(4)=0.000000100
131. TB=TBR
132. JO4=JO4FR
133. UERX(1)=0.500
134. UERX(2)=0.500
135. NDIM=2
136. X(1)=XIN
137. X(2)=XOTIN
138. WRITE(6,*)
139. 4 FORMAT(' ',20X,'CLUTCH TIGHTENING UP')
140. CALL DRKGS(PRMT,X,UERX,NDIM,IHLF,FLT1,OUTP1,AUX)
141. IF (TIME.GT.(PRMT(2)-1.000)) GO TO 7
142. C
143. C STRIP FORWARD MOTION
144. C
145. PRMT(1)=TIME
146. PRMT(3)=PI/720.000
147. PRMT(4)=0.0000001000
148. C
149. JO4=JO4FR
150. TB=TBR+FR
151. UERY(1)=0.2500
152. UERY(2)=0.2500
153. UERY(3)=0.2500
154. UERY(4)=0.2500
155. Y(1)=YIN
156. Y(2)=YOTIN
157. Y(3)=PHI5IN
158. Y(4)=PH5DNF
159. NDIM1=4
160. WRITE(6,5)
161. 5 FORMAT(' ',*STRIP FORWARD MOTION')
162. CALL DRKGS(PRMT,Y,DERY,NDIM1,IHLF,FCT2,OUTP2,AUX1)
163. IF (TIME.GT.(PRMT(2)-1.000)) GO TO 7
164. C
165. C LINKAGE RETURN, CLUTCH UNLOCKING, SHAFT OVERSHOOT
166. C
167. PRMT(1)=TIME
168. PRMT(3)=PI/720.000
169. JO4=JO4OV
170. UERZ(1)=0.2500
171. UERZ(2)=0.2500

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251.      UERZ(3)=0.2500
252.      UERZ(4)=0.2500
253.      NDIM2=4
254.      Z(1)=ZIN
255.      Z(2)=ZUTIN
256.      Z(3)=PH5INU
257.      Z(4)=PH5UNU
258.      IF (Z(4).LT.0.00200) NDIM2=2
259.      WRITE(6,66)
260. 66      FORMAT(' ',20X,'SHAFT OVERSHOOT, CLUTCH UNLOCKING',
261. 1'AND LINKAGE RETURN')
262.      CALL DRKGS(PRMT,Z,UERZ,NDIM2,IHLF,FCI3,OUTP3,AUX2)
263.      IF (TIME.GT.(PRMT(2)-1.000)) GO TO 7
264. C
265. C      SHAFT OVERSHOOT AND LINKAGE RETURN MOTION
266. C
267.      PRMT(1)=TIME
268.      PRMT(3)=PI/720.000
269.      JO4=JO4OV
270.      DERZ(1)=0.2500
271.      UERZ(2)=0.2500
272.      UERZ(3)=0.2500
273.      DERZ(4)=0.2500
274. C      NDIM2=4
275.      Z(1)=ZIN
276.      Z(2)=ZUTIN
277.      Z(3)=PH5INU
278.      Z(4)=PH5UNU
279.      IF (Z(4).LT.0.00200) NDIM2=2
280.      WRITE(6,6)
281. 6      FORMAT(' ',20X,'SHAFT OVERSHOOT AND LINKAGE RETURN MOTION')
282.      CALL DRKGS(PRMT,Z,UERZ,NDIM2,IHLF,FCI4,OUTP4,AUX2)
283.      IF (TIME.GT.(PRMT(2)-1.000)) GO TO 7
284.      IF (TIME.LT.PRMT(2)) GO TO 3
285. 7      CONTINUE
286.      CALL PLOT1(PLTCNT,Q,PHI,RGMD5,QMAX)
287.      STOP
288.      END
289. C
290. C      THE FOLLOWING LISTS THE MAIN PROGRAM SUBROUTINES
291. C      =====
292. C
293.      SUBROUTINE PLOT1(PLTCNT,Q,PHI,RGMD5,QMAX)
294.      IMPLICIT REAL*8(A-Z)
295.      INTEGER I,J,K,PLTCNT,GRAPH(101),COURD
296.      INTEGER SPACE/' '/,CLTGHT/'T'/,STPFWD/'F'/,LNKRT/'R'/,XAXIS/'X'/
297.      DIMENSION Q(PLTCNT),PHI(PLTCNT),RGMD5(PLTCNT)
298.      PI=3.14159265358979300
299.      ZI=PI/180.000
300.      WRITE(6,10)
301. 10      FORMAT(2X,'PHI',16X,'QV')
302.      DO 30 J=1,101
303. 30      GRAPH(J)=SPACE
304.      DO 20 I=1,PLTCNT
305.      PHIIR=PHI(I)
306.      QV=Q(I)
307.      Y=Q(I)/QMAX
308.      VALUE=50.000*Y+51.5
309.      COURD=IFIX(SNGL(VALUE))
310.      IF (COURD.LE.0.000) COURD=1
311.      IF (COURD.GT.101.000) COURD=101
312.      IF (RGMD5(I).EQ.1.000) GRAPH(COURD)=CLTGHT
313.      IF (RGMD5(I).EQ.2.000) GRAPH(COURD)=STPFWD
314.      IF (RGMD5(I).EQ.3.000) GRAPH(COURD)=LNKRT
315.      GRAPH(51)=XAXIS
316.      PHIID=PHIIR/ZI

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317.      WHILE (PHI1D.GT.360.000) DO
318.      PHI1D=PHI1D-360.000
319.      END WHILE
320.      WRITE(0,40) (GRAPH(J),J=1,101),PHI1D,QV
321.      40  FORMAT(' ',2X,101A1,1X,F8.3,1X,D12.5)
322.      GRAPH(COURU)=SPACE
323.      20  CONTINUE
324.      RETURN
325.      END
326.      SUBROUTINE FCT1(T,X,DERX)
327.      IMPLICIT REAL*8(A-Z)
328.      INTEGER I,J,N,K,L,NDIM,IHLF
329.      C
330.      C      DIMENSIONAL STATEMENTS
331.      C
332.      DIMENSION ZINK(4,4),THETNK(4,4),LAMDNK(4,4),
333.      IF2K(4),J5K(4),
334.      ZJ4NK(4,4),J7K(4),JJ7K(4)
335.      DIMENSION VV(4)
336.      DIMENSION OMEGAV(4)
337.      DIMENSION X(1),DERX(1),PRMT(1)
338.      DIMENSION LAMDAV(4)
339.      C
340.      C      COMMON STATEMENTS
341.      C
342.      COMMON/RKNAT/PHI1D1,OMEGAV,PHI4L1,PHI4L2,TIME,PHI5IN
343.      COMMON/COEFF/JJ8,JJ7K
344.      COMMON/ORTH/VV
345.      COMMON/PRUPRT/J04,J05,RHO,CD1,IP,KSP
346.      COMMON LAMDAV,I,J,ALPHA,BETA,L1,L2,A
347.      C
348.      C      INITIALIZATION OF SUBROUTINE
349.      C
350.      PHI1=T
351.      CALL COEF(PHI1,PHI1D1,ZINK,THETNK,LAMDNK,
352.      IF2K,J5K,J4NK,J7K,J8,J3)
353.      C
354.      C      DIFFERENTIAL EQUATIONS OF MOTION
355.      C
356.      TIME=T
357.      PWER=3.000/2.000
358.      JJ8=J8
359.      PHI5 =PHI5IN
360.      JJ7K(1)=J7K(1)
361.      C1=RHO*A
362.      C2=CD1/C1
363.      C3=1.000/(C1*VV(1))
364.      C4=PHI1D1**2
365.      C5=PHI1D1
366.      DEN=(1.000*LAMDNK(1,1)*C3)*C4
367.      COEF1=C2
368.      COEF2=OMEGAV(1)**2-J3**2
369.      TERM1=PHI5 -J8*J7K(1)*X(1)
370.      IF (TERM1.LT.0.000) GO TO 99
371.      TERM1=TERM1**PWER
372.      GO TO 100
373.      99  TERM1=-DABS(TERM1)**PWER
374.      100 CONTINUE
375.      COEF3=-KSP*TERM1*J4NK(1,1)+ZINK(1,1)
376.      COEF4=THETNK(1,1)
377.      DERX(1)=X(2)
378.      TERM=-J8*J7K(1)*X(1)+PHI5
379.      IF (TERM.LT.0.000) GO TO 1
380.      TERM=TERM**PWER
381.      GO TO 2
382.      1  TERM=-DABS(TERM)**PWER

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303.      2      CONTINUE
304.          DERX(2)=(-COEF1*CS*X(2)-COEF2*X(1)-CJ*(CUEF3*X(1)+C5*COEF4*X(2))-
305.          1C3*F2K(1)+KSP*LC3*TERM*J5K(1))/DEN
306.          RETURN
307.          END
308.          SUBROUTINE OUTP1(T,X,DERX,IHLF,NDIM,PRMT)
309.          IMPLICIT REAL*8(A-Z)
310.          INTEGER I,J,N,K,L,NDIM,IHLF,PLTCNT,NDATA
311.      C
312.      C      COMMON STATEMENTS
313.      C
314.          COMMON/PLTCNT/COUNT
315.          COMMON/RKDAT/PHIUT1,UMEGAV,PHI4L1,PHI4L2,TIME,PHI5IN
316.          COMMON/CUEFF/JJ8,JJ7K
317.          COMMON/OKTH/VV
318.          COMMON/STRAIN/V1IDP,V2IDP,YMAX
319.          COMMON/ICFRWD/YIN,YDTIN,PH5INF,PH5UNF
320.          COMMON/PROPR1/J04,J05,RHO,CD1,TB,KSP
321.          COMMON LAMDAV,I,J,ALPHA,BETA,L1,L2,A
322.          COMMON/PLOT/J,PHI,RGMDES,PLTCNT,NDATA,QMAX
323.          COMMON/REGDES/PHI5PR
324.      C
325.      C      DIMENSIONAL STATEMENTS
326.      C
327.          DIMENSION ZINK(4,4),THETNK(4,4),LAMDNK(4,4),
328.          1F2K(4),J5K(4),
329.          2J4IK(4,4),J7K(4),JJ7K(4)
330.          DIMENSION VV(4)
331.          DIMENSION UMEGAV(4)
332.          DIMENSION X(1),DERX(1),PRMT(1)
333.          DIMENSION LAMDAV(4)
334.          DIMENSION W(10000),PHI(10000),RGMDES(10000)
335.      C
336.      C      TEST FOR END OF REGIME(CHECKS THE RIGHT HAND SIDE OF THE
337.      C      THE OUTPUT SHAFT EQUATION.)
338.      C
339.          PI=3.14159265358979300
340.          ZI=PI/180.000
341.          PWER=3.000/2.000
342.          PHI5=PHI5IN
343.          TERM=-JJ8*JJ7K(1)*X(1)+PHI5
344.          PHI4IN=PHI5IN/ZI
345.      C      PRINT,'PHI4IN=',PHI4IN
346.          IF (TERM.LT.0.000) GO TO 111
347.          TERM=TERM*PWER
348.          GO TO 112
349.      111      TERM=-(DABS(TERM)*PWER)
350.      112      CONTINUE
351.          RTIME=(T-(256.000000*ZI))/DAUS(PHIUT1)
352.          TEST=(KSP*TERM-TB)
353.          PRINT,'TEST=',TEST
354.          PHI1=T
355.          Q1=X(1)
356.          QUT1=X(2)
357.          CALL PHI4EL(Q1,QUT1,PHI1,PHIUT1,PHI4E,PHDT4E,ACP,PHIDD4)
358.          PHI4TL=PHI4E
359.          CALL ELAST(PHI1,ACP,PHI4)
360.      C      PRINT,'ACP=',ACP
361.          IF (TEST.GE.0.000) GO TO 10
362.          PHI1=T/ZI
363.          WHILE (PHI1.GE.360.000) DO
364.          PHI1=PHI1-360.000
365.          END WHILE
366.          PHI4TL=PHI4TL/ZI
367.          PHI4=PHI4/ZI
368.          PHI4K=JJ8/ZI

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449.      PHI5=PHISIN/ZI
450.      COUNT=COUNT+10.000
451.      PRPHI1=IFIX(SNGL(PHI1))
452.      UIF=DABS(PKPHI1-PHI1)
453.      DIV=PRPHI1/5
454.      DIFF=SNGL(DIV)-IFIX(SNGL(DIV))
455.      IF ((DIFF.EQ.0.0).AND.(DIFF.LI.0.300)) GO TO 9
456.      IF (COUNT.LT.111.000) GO TO 11
457. 9      CONTINUE
458.      EPSL=-YMAX*V11UP*X(1)
459.      PLTCNT=PLTCNT+1
460.      U(PLTCNT)=EPSL
461.      PHI(PLTCNT)=T
462.      KGMDIS(PLTCNT)=1.000
463.      EP=DABS(EPSL)
464.      UF=QMAX-EP
465.      IF (UF.LI.0.000) QMAX=EP
466.      IF (UF.GE.0.000) QMAX=QMAX
467.      WRITE(6,1) PHI1,PHI4R,PHI4,PHI5,PHI4E,EP
468.      PHI4R=PHI4-PHI4R
469.      PHI54=PHI4-PHI5
470.      WRITE(6,119) PHI4R,PHI54
471. 119  FORMAT(' ',10X,'PHI4R=',F8.5,10X,'PHI54=',F8.5)
472. 1    FORMAT(' ',1X,'PHI1=',F7.3,3X,'PHI4R=',F10.5,2X,'PHI4=',F10.5
473.      1,1X,'PHI5=',F6.3,1X,'PHI4E=',F10.5,1X,'EPSL=',D16.9 )
474.      ELSTDF=PHI4-PHI4TL
475.      WRITE(6,120) ELSTDF,PHI4,PHI4TL
476. 120  FORMAT(' ',3X,'PHI4EL MINUS PHI4TAYLOR = ',F12.8,3X,'PHI4EL=',
477.      IF12.8,3X,'PHI4TAYLOR=',F12.8)
478.      IF (PRMT(5).EQ.1.000) GO TO 11
479.      COUNT=0.000
480.      GO TO 11
481. 10    TIME=T
482.      PHI1=T/ZI
483.      WHILE (PHI1.GE.360.000) DO
484.      PHI1=PHI1-360.000
485.      END WHILE
486.      PHI4=JJ8/ZI
487.      PHI5=PHISIN/ZI
488.      PH5INF=PHISIN
489.      YIN=X(1)
490.      YDIN=X(2)
491.      REGDST=0.000
492.      PH5DNF=0.000
493.      PRMT(5)=1.000
494.      COUNT=110.000
495.      GO TO 9
496. 11    CONTINUE
497.      RETURN
498.      END
499.      SUBROUTINE FCT2(T,Y,DERY)
500.      IMPLICIT REAL*8(A-Z)
501.      INTEGER I,J,N,K,L,NDIM1,IHLF
502.  C
503.  C      DIMENSIONAL STATEMENTS
504.  C
505.      DIMENSION ZINK(4,4),THETNK(4,4),LAMDNK(4,4),
506.      IF2K(4),J5K(4),
507.      ZJ4NK(4,4),J7K(4)
508.      DIMENSION VV(4)
509.      DIMENSION OMEGAV(4)
510.      DIMENSION Y(1),DERY(1),PRMT(1)
511.      DIMENSION LAMDAV(4)
512.  C
513.  C      COMMON STATEMENTS
514.  C

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515.      COMMON/RKDAT/PHIDT1,OMEGAV,PHI4L1,PHI4L2,TIME,PHISIN
516.      COMMON/ORTH/VV
517.      COMMON/CUEFF/JJ8,JJ7K
518.      COMMON/PROPRI/J04,J05,RHO,LD1,IB,KSP
519.      COMMON LAMUAV,1,J,ALPHA,BETA,L1,L2,A
520.      COMMON/CLTDMP/CD2
521.      C
522.      C      INITIALIZATION OF SUBROUTINE
523.      C
524.      PHI1=T
525.      CALL CUEF(PHI1,PHIDT1,ZINK,THEINK,LAMDNK,
526.      IFZK,J5K,J4NK,J7K,J8,J3)
527.      C
528.      C      DIFFERENTIAL EQUATIONS OF MOTION
529.      C
530.      TIME=T
531.      PWER=3.000/2.000
532.      JJ8=J8
533.      C1=RHO*A
534.      C2=CU1/C1
535.      C3=1.000/(C1*VV(1))
536.      C4=PHIDT1**2
537.      C5=PHIDT1
538.      COEF1=C2
539.      COEF2=(OMEGAV(1)**2-J3**2)
540.      TERM=-J8+J7K(1)*Y(1)*Y(3)
541.      IF (TERM.LT.0.000) GO TO 1
542.      TERM=TERM**PWER
543.      GO TO 2
544.      1      TERM=-(DABS(TERM))**PWER
545.      2      CONTINUE
546.      CALL KINEM(PHI1,PHIDT1,PHI2,PHI3Z,PHI4,PHIDT2,PHID3Z,PHIDT4,
547.      IPHI0J2,PHIDU3Z,PHIDU4,J7)
548.      DMPTRM=Y(4)*PHIDT1-PHIDT4
549.      COEF3=-(KSP*TERM+CU2*DMPTRM)*J4NK(1,1)+ZINK(1,1)
550.      COEF4=THEINK(1,1)
551.      IF (Y(4).EQ.0.000) GO TO 89
552.      SIGNUM=(Y(4))/DABS(Y(4))
553.      GO TO 90
554.      89      CONTINUE
555.      SIGNUM=1.000
556.      90      CONTINUE
557.      DEN=(1.000+C3*LAMDNK(1,1))*C4
558.      DERY(1)=Y(2)
559.      DERY(3)=Y(4)
560.      DERY(2)=(-COEF1*C5*Y(2)-COEF2*Y(1)-C3*(COEF3*Y(1)+COEF4*C5*Y(2))
561.      1-C3*FZK(1)+C3*(KSP*TERM+CU2*DMPTRM)*J5K(1))/DEN
562.      DERY(4)=(-KSP*TERM-CU2*DMPTRM-SIGNUM*TB)/(J05*C4)
563.      RETURN
564.      ENU
565.      SUBROUTINE OUTP2(T,Y,DERY,IHLF,NDIMI,PRMT)
566.      IMPLICIT REAL*8(A-Z)
567.      INTEGER 1,J,N,K,L,NDIMI,IHLF,NDATA,PLTCNT
568.      C
569.      C      COMMON STATEMENTS
570.      C
571.      COMMON/PRTCNT/COUNT
572.      COMMON/RKDAT/PHIDT1,OMEGAV,PHI4L1,PHI4L2,TIME,PHISIN
573.      COMMON/ORTH/VV
574.      COMMON/STRAIN/V11DP,V21DP,YMAX
575.      COMMON/CUEFF/JJ8,JJ7K
576.      COMMON/ICTGHT/XIN,XDTIN
577.      COMMON/ICOVRI/ZIN,ZDTIN,PHSINO,PHSINU
578.      COMMON/PROPRI/J04,J05,RHO,LD1,IB,KSP
579.      COMMON LAMUAV,1,J,ALPHA,BETA,L1,L2,A
580.      COMMON/PLOT/Q,PHI,RGMDES,PLTCNT,NDATA,QMAX

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501.      COMMON/REGUES/PHI5PR
502.      C
503.      C      DIMENSIONAL STATEMENTS
504.      C
505.      DIMENSION ZINK(4,4),THETNK(4,4),LAMDNK(4,4),
506.      1F2K(4),J5K(4),
507.      2J4NK(4,4),J7K(4)
508.      DIMENSION VV(4)
509.      DIMENSION OMEGAV(4)
510.      DIMENSION Y(1),DERV(1),PRMT(1)
511.      DIMENSION LAMDAV(4)
512.      DIMENSION Q(10000),PHI(10000),RGMDIS(10000)
513.      C
514.      C      TEST FOR END OF REGIME(CHECKS THE ELASTIC ANGULAR VELOCITY OF
515.      C      THE OUTPUT CRANK)
516.      C
517.      PWER=3.000/2.000
518.      PHI4L=PHI4L1
519.      PI=3.14159265358979300
520.      ZI=PI/180.000
521.      PHI1=T
522.      Q1=Y(1)
523.      QDT1=Y(2)
524.      CALL PHI4EL(Q1,QDT1,PHI1,PHIOT1,PHI4E,PHDT4E,ACP,PHIOT4)
525.      PHI4TL=PHI4E
526.      PHI4TL=PHI4TL/ZI
527.      CALL ELAST(PHI1,ACP,PHI4)
528.      TEST1=PHDT4E
529.      TEST2=PHI4E
530.      IF (TEST1.GE.0.000) GO TO 4
531.      PRMT(4)=0.0000001000
532.      GO TO 5
533.      4 CONTINUE
534.      PRMT(4)=0.000000100
535.      5 CONTINUE
536.      RTIME=(T-(256.000000*ZI))/DABS(PHIOT1)
537.      IF ((TEST1.GE.0.000000).AND.(TEST2.LT.PHI4L)) GO TO 10
538.      UIF=PHI4L-PHI4E-Y(3)
539.      IF (Y(4).GT.0.0000) THEN
540.      PHI5IN=Y(3)
541.      TIME=T
542.      ZIN=Y(1)
543.      ZDTIN=Y(2)
544.      PH5IND=Y(3)
545.      PH5DND=0.000
546.      PRMT(5)=1.000
547.      GO TO 11
548.      END IF
549.      IF ((Y(3)-PHI4E).LT.-0.00261799300) THEN
550.      PHI5IN=Y(3)
551.      TIME=T
552.      ZIN=Y(1)
553.      ZDTIN=Y(2)
554.      PH5IND=Y(3)
555.      PH5DND=0.000
556.      PRMT(5)=1.000
557.      GO TO 11
558.      END IF
559.      PHI1=T/ZI
560.      WHILE (PHI1.GE.360.000) DO
561.      PHI1=PHI1-360.000
562.      END WHILE
563.      PHI4=PHI4/ZI
564.      PHIOT4=PHDT4E
565.      PHI5=Y(3)/ZI
566.      PHIOT5=Y(4)

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647.      COUNT=COUNT+10.000
648.      PRPH11=IFIX(SNGL(PH11))
649.      UIF=DABS(PKPH11-PH11)
650.      DIV=PRPH11/5
651.      DIFF=SNGL(DIV)-IFIX(SNGL(DIV))
652.      IF ((DIFF.EQ.0.0).AND.(DIFF.LT.0.300)) GO TO 9
653.      IF (COUNT.LT.111.000) GO TO 11
654. 9      CONTINUE
655.      PRINT,*,ACP=*,ACP
656.      PLTCNT=PLTCNT+1
657.      EPSL=-YMAX*V11UP*Y(1)
658.      Q(PLTCNT)=EPSL
659.      PH1(PLTCNT)=T
660.      KGMDIS(PLTCNT)=2.000
661.      PH14R=JJB/Z1
662.      EP=DABS(EPSL)
663.      UF=QMAX-EP
664.      IF (UF.LT.0.000) QMAX=EP
665.      IF (UF.GE.0.000) QMAX=QMAX
666.      WRITE(6,1) PH1,PH14R,PH14,PH15,PH1D14,EP
667.      PH14R=PH14-PH14R
668.      PH154=PH14-PH15
669.      WRITE(6,119) PH14R,PH154
670. 119  FORMAT(' ',10X,'PH14R=',F8.5,10X,'PH154=',F8.5)
671. 1    FORMAT(' ',1X,'PH11=',F8.3,1X,'PH14R=',F8.3,2X,'PH14=',F8.3,1X,
672.      1,'PH15=',F8.3,1X,'PH1D14E=',F9.5,1X,'EPSL=',D16.9)
673.      PH1D14=PH1D14
674.      PH15D=DERY(4)
675.      WRITE(6,3) PH1D14,PH15D,Y(4)
676. 3    FORMAT(' ',3X,'PH1D14=',F16.7,3X,'PH15D1=',F16.7,3X,'PH1D15='
677.      1,F16.7)
678.      ELSTUF=PH14-PH14TL
679.      WRITE(6,120) ELSTUF,PH14,PH14TL
680. 120  FORMAT(' ',3X,'PH14EL MINUS PH14TAYLOR = ',F12.8,3X,'PH14EL=',
681.      1,F12.8,3X,'PH14TAYLOR=',F12.8)
682.      IF (PRMT(5).EQ.1.000) GO TO 11
683.      COUNT=0.000
684.      GO TO 11
685. 20   CONTINUE
686.      TIME=T
687.      PH11=T/Z1
688.      WHILE (PH11.GT.360.000) DO
689.      PH11=PH11-360.000
690.      END WHILE
691.      PH14=PH14/Z1
692.      PH1D14=PH1D14E
693.      PH15=Y(3)/Z1
694.      PH1D15=Y(4)
695.      XIN=Y(1)
696.      XD1IN=Y(2)
697.      PH15IN=Y(3)
698.      PRMT(5)=1.000
699.      COUNT=110.000
700.      REGDST=1.000
701.      PH14L2=PH14E
702.      GO TO 9
703. 10   TIME=T
704.      PH11=T/Z1
705.      WHILE (PH11.GT.360.000) DO
706.      PH11=PH11-360.000
707.      END WHILE
708.      PH14=PH14/Z1
709.      PH1D14=PH1D14E
710.      PH15=Y(3)/Z1
711.      PH1D15=Y(4)
712.      ZIN=Y(1)

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713.      ZDTIN=Y(2),
714.      PHSDIN=Y(3)
715.      PHSDND=Y(4)
716.      PRMT(5)=1.000
717.      COUNT=110.000
718.      GO TO 9
719.      11 CONTINUE
720.      RETURN
721.      END
722.      SUBROUTINE FCT4(T,Z,DERZ)
723.      IMPLICIT REAL*8(A-Z)
724.      INTEGER I,J,N,K,L,NDIM2,IHLF
725.      C
726.      C      DIMENSIONAL STATEMENTS
727.      C
728.      DIMENSION ZINK(4,4),THETNK(4,4),LAMDNK(4,4),
729.      1F2K(4),J5K(4),
730.      2J4NK(4,4),J7K(4)
731.      DIMENSION VV(4)
732.      DIMENSION OMEGAV(4)
733.      DIMENSION Z(1),DERZ(1),PRMT(1)
734.      DIMENSION LAMDAV(4)
735.      C
736.      C      COMMON STATEMENTS
737.      C
738.      COMMON/RKDAT/PHIUT1,OMEGAV,PHI4L1,PHI4L2,TIME,PHISIN
739.      COMMON/ORTH/VV
740.      COMMON/CUEFF/JJ8,JJ7K
741.      COMMON/PRNPRT/J04,J05,RHL,C01,TB,K5P
742.      COMMON LAMDAV,LAMDAW,I,J,ALPHA,BETA,L1,L2,A
743.      C
744.      C      INITIALIZATION OF SUBROUTINE
745.      C
746.      PHI1=T
747.      CALL CUEF(PHI1,PHIUT1,ZINK,THETNK,LAMDNK,
748.      1F2K,J5K,J4NK,J7K,J8,J3)
749.      C
750.      C      DIFFERENTIAL EQUATIONS OF MOTION
751.      C
752.      TIME=T
753.      JJ8=J8
754.      C1=RHO*A
755.      C2=C01/C1
756.      C3=1.000/(C1*VV(1))
757.      C4=PHIUT1**2
758.      C5=PHIUT1
759.      COEF1=C2
760.      COEF2=(OMEGAV(1)**2-J3**2)
761.      COEF3=ZINK(1,1)
762.      COEF4=THETNK(1,1)
763.      DEN=(1.000+C3*LAMDNK(1,1))*C4
764.      IF (Z(4).LT.0.00200) NDIM2=2
765.      IF (Z(4).EQ.0.000) GO TO 11
766.      SIGNUM=Z(4)/DABS(Z(4))
767.      GO TO 12
768.      11 SIGNUM=1.000
769.      12 CONTINUE
770.      DERZ(1)=Z(2)
771.      DERZ(3)=Z(4)
772.      DERZ(2)=(-COEF1*C5*Z(2)-COEF2*Z(1)-C3*(CUEF3*Z(1)+COEF4*C5*Z(2))
773.      1-C3*F2K(1))/DEN
774.      DERZ(4)=-SIGNUM*(TB/J05)
775.      RETURN
776.      END
777.      SUBROUTINE OUTP4(T,Z,DERZ,IHLF,NDIM2,PRMT)
778.      IMPLICIT REAL*8(A-Z)

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779.      INTEGER I,J,N,K,L,NDIM2,IHLF,NDATA,PLTCNT
780.      C
781.      C      COMMON STATEMENTS
782.      C
783.      COMMON/PRTCNT/COUNT
784.      COMMON/RKDAT/PHIUT1,OMEGAV,PHI4L1,PHI4L2,TIME,PHI5IN
785.      COMMON/ORTH/VV
786.      COMMON/STRAIN/V11DP,V21DP,YMAX
787.      COMMON/CUEFF/JJ8,JJ7K
788.      COMMON/ICGTHT/XIN,XDTIN
789.      COMMON/PROPR1/JD4,JD5,RHU,CD1,IB,KSP
790.      COMMON LAMUAV,I,J,ALPHA,BETA,L1,L2,A
791.      COMMON/PLDT/J,PHI,KGMDES,PLTCNT,NDATA,QMAX
792.      COMMON/REGDES/PHI5PR
793.      C
794.      C      DIMENSIONAL STATEMENTS
795.      C
796.      DIMENSION ZINK(4,4),THETNK(4,4),LAMDNK(4,4),
797.      IF2K(4),J5K(4),
798.      ZJ4NK(4,4),J7K(4)
799.      DIMENSION VV(4)
800.      DIMENSION OMEGAV(4)
801.      DIMENSION Z(1),DERZ(1),PRMT(1)
802.      DIMENSION LAMUAV(4)
803.      DIMENSION Q(10000),PHI(10000),KGMDES(10000)
804.      C
805.      C      TEST FOR END OF REGIME(CHECKS THE ELASTIC ANGULAR VELOCITY OF
806.      C      THE OUTPUT CRANK)
807.      C
808.      PI=3.14159265358979300
809.      ZI=PI/180.000
810.      PHI1=T
811.      Q1=Z(1)
812.      QDT1=Z(2)
813.      CALL PHI4EL(Q1,QDT1,PHI1,PHIUT1,PHI4L,PHDT4E,ACP,PHIUD4)
814.      PHI4TL=PHI4E
815.      PHI4TL=PHI4TL/ZI
816.      CALL ELAST(PHI1,ACP,PHI4)
817.      C      PRINT, 'ACP=',ACP
818.      TEST1=PHDT4E
819.      TEST2=PHI4E
820.      IF (TEST1.LT.0.1500) GO TO 4
821.      PRMT(4)=0.000000100
822.      GO TO 5
823.      4      CONTINUE
824.      PRMT(4)=0.000000100
825.      5      CONTINUE
826.      PHI4CK=PHI5PR-0.100
827.      IF ((TEST1.LT.0.00000) .AND. (TEST2.GT.PHI4CK)) GO TO 10
828.      PHI1=T/ZI
829.      WHILE (PHI1.GE.360.000) DO
830.      PHI1=PHI1-360.000
831.      END WHILE
832.      PHI4=PHI4/ZI
833.      PHI5=Z(3)/ZI
834.      TIME=T
835.      PHIUT4=PHDT4E
836.      COUNT=COUNT+10.000
837.      PRPHI1=IFIX(SNGL(PHI1))
838.      DIV=UABS(PKPHI1-PHI1)
839.      DIV=PRPHI1/5
840.      DIFF=SNGL(DIV)-IFIX(SNGL(DIV))
841.      IF ((DIFF.EQ.0.0) .AND. (DIV.LI.0.300)) GO TO 9
842.      IF (COUNT.LT.111.000) GO TO 11
843.      9      CONTINUE
844.      EPSL=-YMAX+V11DP+Z(1)

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845.      PLTCNT=PLTCNT+1
846.      U(PLTCNT)=EPSL
847.      PHI(PLTCNT)=T
848.      RGMDEL(PLTCNT)=3.000
849.      PHI4R=JJ8/ZI
850.      LP=OARS(EPSL)
851.      UF=QMAX-EP
852.      IF (UF.LT.0.000) QMAX=EP
853.      IF (UF.GE.0.000) QMAX=QMAX
854.      WRITE(6,1) PHI1,PHI4R,PHI4,PHI5,PHI014,EPSL
855.      PHI4ER=PHI4-PHI4R
856.      PHI54=PHI4-PHI5
857.      WRITE(6,119) PHI4ER,PHI54
858.      119 FORMAT(' ',10X,'PHI4ER=',F8.3,10X,'PHI54=',F8.5)
859.      1   FORMAT(' ',1X,'PHI1=',F8.3,1X,'PHI4R=',F8.3,1X,'PHI4=',F8.3,1X,
860.      1'PHI5=',F8.3,1X,'PHI014=',F9.5,1X,'EPSL=',D16.9)
861.      ELSTOF=PHI4-PHI4TL
862.      WRITE(6,120) ELSTOF,PHI4,PHI4TL
863.      120 FORMAT(' ',3X,'PHI4EL MINUS PHI4TAYLOR = ',F12.8,3X,'PHI4EL=',
864.      1F12.8,3X,'PHI4TAYLOR=',F12.8)
865.      IF (PRMT(5).EQ.1.000) GO TO 11
866.      COUNT=0.000
867.      GO TO 11
868.      10   TIME=T
869.      PHI1=T/ZI
870.      WHILE (PHI1.GE.360.000) DO
871.      PHI1=PHI1-360.000
872.      END WHILE
873.      PHI4=TSI2/ZI
874.      PHI5=Z(3)/ZI
875.      PHIDT4=PHIDT4E
876.      PHI5IN=PHI4E
877.      PHI5PR=PHI4E
878.      XLN=Z(1)
879.      XDTIN=Z(2)
880.      PRMT(5)=1.000
881.      PHI4L1=PHI4E
882.      COUNT=10.000
883.      GO TO 9
884.      11   CONTINUE
885.      RETURN
886.      END
887.      SUBROUTINE FCT3(T,Z,DERZ)
888.      IMPLICIT REAL*8(A-Z)
889.      INTEGER 1,J,N,K,L,NDIM2,IHLF
890.      C
891.      C   DIMENSIONAL STATEMENTS
892.      C
893.      DIMENSION ZINK(4,4),THETNK(4,4),LAMDNK(4,4),
894.      1F2K(4),J5K(4),
895.      2J4NK(4,4),J7K(4)
896.      DIMENSION VV(4)
897.      DIMENSION OMEGAV(4)
898.      DIMENSION Z(1),DERZ(1),PRMT(1)
899.      DIMENSION LAMDAV(4)
900.      C
901.      C   COMMON STATEMENTS
902.      C
903.      COMMON/RKDAT/PHIDT1,OMEGAV,PHI4L1,PHI4L2,TIME,PHI5IN
904.      COMMON/ORTH/VV
905.      COMMON/COEFF/JJ8,JJ7K
906.      COMMON/PROPT/JO4,JO5,RHO,CN1,TB,KSP
907.      COMMON LAMDAV,1,J,ALPHA,BETA,L1,L2,A
908.      C
909.      C   INITIALIZATION OF SUBROUTINE
910.      C

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911.      PHI1=T
912.      CALL CUEF(PHI1,PHIUT1,ZINK,THETNK,LAMDNK,
913.      1F2K,J5K,J4NK,J7K,J8,J3)
914.      C
915.      C      DIFFERENTIAL EQUATIONS OF MOTION
916.      C
917.      PWER=3.000/2.000
918.      TIME=T
919.      JJ8=J8
920.      C1=RH0*A
921.      C2=CU1/C1
922.      C3=1.000/(C1*VV(1))
923.      C4=PHIUT1**2
924.      C5=PHIUT1
925.      COLF1=C2
926.      COEF2=(OMEGAV(1)**2-J3**2)
927.      TERM=-J8+J7K(1)*Z(1)+Z(3)
928.      IF (TERM.LT.0.000) GO TO 1
929.      TERM=TERM**PWER
930.      GO TO 2
931.      1      TERM=-(DABS(TERM))**PWER
932.      2      CONTINUE
933.      COEF3=ZINK(1,1)-KSP*TERM*J4NK(1,1)
934.      COEF4=THETNK(1,1)
935.      DEN=(1.000+C3*LAMDNK(1,1))*C4
936.      IF (Z(4).LT.0.00200) NDIM2=2
937.      IF (Z(4).EQ.0.000) GO TO 11
938.      SIGNUM=Z(4)/DABS(Z(4))
939.      GO TO 12
940.      11      SIGNUM=1.000
941.      12      CONTINUE
942.      DERZ(1)=Z(2)
943.      DERZ(3)=Z(4)
944.      UERZ(2)=(-COEF1*C5*Z(2)-COEF2*Z(1)-C3*(COEF3*Z(1)+COEF4*L5*Z(2))
945.      1-C3*F2K(1)+KSP*C3*TERM*J5K(1))/DEN
946.      UERZ(4)=-SIGNUM*(TB/(J05*C4))
947.      RETURN
948.      END
949.      SUBROUTINE OUTP3(T,Z,DERZ,IHLF,NDIM2,PRMT)
950.      IMPLICIT REAL*8(A-Z)
951.      INTEGER I,J,N,K,L,NDIM2,IHLF,NDATA,PLTENT
952.      C
953.      C      COMMON STATEMENTS
954.      C
955.      COMMON/PRTLNT/COUNT
956.      COMMON/RKDAT/PHIUT1,OMEGAV,PHI4L1,PHI4L2,TIME,PHI5IN
957.      COMMON/URTH/VV
958.      COMMON/STRAIN/V1IDP,V2IDP,YMAX
959.      COMMON/CUEFF/JJ8,JJ7K
960.      COMMON/ICTGHT/XIN,XDTIN
961.      COMMON/ICOVRN/ZIN,ZDTIN,PHSINO,PH5ONU
962.      COMMON/PROPRJ/J04,J05,RHU,CD1,TB,KSP
963.      COMMON LAMDAV,I,J,ALPHA,BETA,L1,L2,A
964.      COMMON/PLDT/U,PHI,RGMDES,PLTENT,NDATA,QMAX
965.      COMMON/REGUES/PHISPR
966.      C
967.      C      DIMENSIONAL STATEMENTS
968.      C
969.      DIMENSION ZINK(4,4),THETNK(4,4),LAMDNK(4,4)
970.      1F2K(4),J5K(4),
971.      2J4NK(4,4),J7K(4)
972.      DIMENSION VV(4)
973.      DIMENSION OMEGAV(4)
974.      DIMENSION Z(1),UERZ(1),PRMT(1)
975.      DIMENSION LAMDAV(4)
976.      DIMENSION U(10000),PHI(10000),RGMDES(10000)

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977.      C
978.      C      TEST FOR END OF REGIME(CHECKS THE ELASTIC ANGULAR VELOCITY OF
979.      C      THE OUTPUT CRANK)
980.      C
981.      PI=3.14159265358979300
982.      ZI=PI/180.000
983.      PHI1=T
984.      Q1=Z(1)
985.      QDT1=Z(2)
986.      CALL PHI4EL(Q1,QDT1,PHI1,PHIUT1,PHI4E,PHDT4E,ACP,PHIDD4)
987.      PHI4TL=PHI4E
988.      PHI4TL=PHI4TL/ZI
989.      CALL ELAST(PHI1,ACP,PHI4)
990.      C      PRINT, 'ACP=',ACP
991.      TEST1=PHDT4E
992.      TEST2=PHI4E
993.      IF (TEST1.LT.0.1500) GO TO 4
994.      PRMT(4)=0.000000100
995.      GO TO 5
996.      4      CONTINUE
997.      PRMT(4)=0.000000100
998.      5      CONTINUE
999.      U=Z(3)-PHI4E
1000.      PHI4LK=PHI5PR-0.100
1001.      IF (U.LE.0.0000) GO TO 10
1002.      PHI1=T/ZI
1003.      WHILE (PHI1.GE.360.000) DO
1004.      PHI1=PHI1-360.000
1005.      END WHILE
1006.      PHI4=PHI4/ZI
1007.      PHI5=Z(3)/ZI
1008.      TIME=T
1009.      PHIDT4=PHDT4E
1010.      COUNT=COUNT+10.000
1011.      PRPHI1=FIX(SNGL(PHI1))
1012.      DIF=JABS(PKPHI1-PHI1)
1013.      DIV=PRPHI1/5
1014.      DIFF=SNGL(DIV)-FIX(SNGL(DIV))
1015.      IF ((DIFF.EQ.0.0).AND.(DIF.LT.0.300)) GO TO 9
1016.      IF (COUNT.LT.111.000) GO TO 11
1017.      9      CONTINUE
1018.      EPSL=-YMAX*V110P*Z(1)
1019.      PLTCNT=PLTCNT+1
1020.      Q(PLTCNT)=EPSL
1021.      PHI(PLTCNT)=T
1022.      RGHDES(PLTCNT)=3.000
1023.      PHI4R=JJB/ZI
1024.      EP=DARS(EPSL)
1025.      UF=QMAX-EP
1026.      IF (UF.LT.0.000) QMAX=EP
1027.      IF (UF.GE.0.000) QMAX=QMAX
1028.      WRITE(6,1) PHI1,PHI4R,PHI4,PHI5,PHIDT4,EP,SL
1029.      PHI4ER=PHI4-PHI4R
1030.      PHI54=PHI4-PHI5
1031.      WRITE(6,19) PHI4ER,PHI54
1032.      119  FORMAT(' ',10X,'PHI4ER=',F8.5,10X,'PHI54=',F8.5)
1033.      1      FORMAT(' ',1X,'PHI1=',F8.3,1X,'PHI4R=',F8.3,1X,'PHI4=',F8.3,1X,
1034.      1*PHI5=',F8.3,1X,'PHIUT4=',F9.5,1X,'EPSL=',D16.9)
1035.      ELSTOF=PHI4-PHI4TL
1036.      WRITE(6,120) ELSTOF,PHI4,PHI4TL
1037.      120  FORMAT(' ',3X,'PHI4EL MINUS PHI4TAYLOR = ',F12.8,3X,'PHI4EL=',
1038.      IF12.8,3X,'PHI4TAYLOR=',F12.8)
1039.      IF (PRMT(5).EQ.1.000) GO TO 11
1040.      COUNT=0.000
1041.      GO TO 11
1042.      10      TIME=T

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1043. PH11=T/21
1044. WHILE (PH11*GE.360.000) UO
1045. PH11=PH11-360.000
1046. END WHILE
1047. PH14=TEST2/21
1048. PH15=2(3)/21
1049. PH1014=PH014E
1050. PH151N=PH14E
1051. PH15PR=PH14E
1052. Z1N=2(1)
1053. Z01IN=2(2)
1054. PH1INO=2(3)
1055. PH5010=2(4)
1056. PRMT(5)=1.000
1057. PH14L1=PH14E
1058. COUNT=110.000
1059. GO TO 9
1060. CONTINUE
1061. RETURN
11
END
1062. SUBROUTINE FLAST(PH11,ACP,PH14)
1063. IMPLICIT REAL*8(A-Z)
1064. COMMON/MECHDM/A1,A3,A4,S1,S2,S3
1065. PI=3.14159265358979300
1066. AA=2*S1*A4
1067. AA=2*A4*(S3+A1*DSIN(PH11))
1068. B=2*A4*(S3+A1*DSIN(PH11))
1069. CC=A4**2+S1**2+S2**2+S3**2+A1**2-ACP**2
1070. C=CC+2*A1*S2*DCOS(PH11)*2*S3*A1*USIN(PH11)
1071. SQURT=DSQRT(AA**2+B**2-C**2)
1072. UEN=C+B
1073. PH141=DA1AN((AA+SQURT)/DEN)
1074. IF (PH141.LT.0) PH141=2*PI-UA05(PH141)
1075. PH142=DA1AN((AA-SQURT)/DEN)
1076. IF (PH142.LT.0) PH142=2*PI-UA05(PH142)
1077. PH141=2*PH141
1078. PH142=2*PH142
1079. IF (PH141.LT.0) PH142) GO TO 10
1080. PH14=PH142
1081. GO TO 11
1082. PH14=PH141
1083. CONTINUE
11
RETURN
END
1084. SUBROUTINE PH14EL(U1,ODT1,PH11,PH10T1,PH14E,PH0T4E,ACP,PH10D4)
1085. IMPLICIT REAL*8(A-Z)
1086. INTEGER I,J,N,K,L,NDIM,NDIM1,NU1N2,INLF
1087. COMMON/ACINTG/FF64,FF65,FF66,V11L1
1088. COMMON/MECHDM/A1,A3,A4,S1,S2,S3
1089. COMMON/PROPRIT/J04,J05,RHU,C01,TB,KSP
1090. COMMON LAMDAV,I,J,ALPHA,BETA,L1,L2,A
1091. DIMENSION FF66(4),FF65(4),FF66(4)
1092. DIMENSION LAMDAV(4)
1093. CALL KINEM(PH11,PH10T1,PH12,PH132,PH14,PH10T2,PH1032,PH10T4,
1094. PH10D2,PH0D32,PH10D4,J7)
1095. PH10D4=PH10D4
1096.
1097.
1098.
1099.
1100. C
1101. C COMPUTATION OF AC*
1102. C
1103. C =====
1104. C
1105. SINAPB=DSIN(ALPHA*BETA)
1106. SECB=1.000/DCOS(BETA)
1107. COSA=DCOS(ALPHA)
1108. COSB=DCOS(BETA)
1109. URUER1=V11L1*SINAPB*SECB*Q1
1110. URUER2=-0.500*(Q1**2)*(COSA*FF64(1)+COSB*(FF65(1)-FF66(1)))
1111. ACP=A3*ORDER1+ORDER2

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1109. AA=2.0000*SI*A4
1110. B=2.0000*A4*(S3*A1*DSIN(PHI1))
1111. BPR=2.0000*A4*A1*DCOS(PHI1)*PHIUT1
1112. COSPH4=DCOS(PHI4)
1113. SINPH4=DSIN(PHI4)
1114. DEN=AA*CUSPH4-B*SINPH4
1115. NUM=AA*SINPH4+B*DCOSPH4
1116. CUEFI=2.0000*A3/DEN
1117.
1118. C
1119. C
1120. C
1121. C
1122. C
1123. C
1124. C
1125. C
1126. C
1127. C
1128. C
1129. C
1130. C
1131. C
1132. C
1133. C
1134. C
1135. C
1136. C
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1162. C
1163. C
1164. C
1165. C
1166. C
1167. C
1168. C
1169. C
1170. C
1171. C
1172. C
1173. C
1174. C

      AA=2.0000*SI*A4
      B=2.0000*A4*(S3*A1*DSIN(PHI1))
      BPR=2.0000*A4*A1*DCOS(PHI1)*PHIUT1
      COSPH4=DCOS(PHI4)
      SINPH4=DSIN(PHI4)
      DEN=AA*CUSPH4-B*SINPH4
      NUM=AA*SINPH4+B*DCOSPH4
      CUEFI=2.0000*A3/DEN

      COMPUTATION OF PHI4E AND PHIUT4E
      =====
      PHI4E=PHI4-CUEFI*VILL1*SEC*SI*APR*Q1
      PHUT4E=PHIUT4-CUEFI*(BPR*SINPH4-NUM*PHIUT4*Q1)/DEN)-QD11)
      FOR THE TIME BEING PHIDT4E IS APPROXIMATED BY PHIDI4 AND PHI4E
      BY PHI4

      PHI4E=PHI4
      PHUT4E=PHIUT4
      RETURN
      END
      SUBROUTINE COEF(PHI1,PHIUT1,ZINK,THETNK,LANDNK,
1F2K,J5K,J6K,J7K,J8,J3)
      IMPLICIT REAL*8(A-Z)
      INTEGER N,K,I,J
      COMMON STATEMENTS
      =====
      COMMON/MECHDW/A1,A3,A4,S1,S2,S3
      COMMON LAMDAV,I,J,ALPHA,BETA,L1,L2,A
      COMMON/PODPR1/J0,J05,RH0,L0D1,TR,K5P
      COMMON/INTGRL/F1,FF2,FF3,FF4,FF5,FF6,FF7,FF8,FF9,FF10,FF11,FF12,
1FF13,FF14,FF15,FF16,FF17,FF18,FF19,FF20,FF21,FF22,
2FF23,FF30,FF31,FF32,FF33,FF34,FF35,FF36,FF37,
4FF59
      COMMON/ORTH/VV
      DIMENSIONAL STATEMENTS
      =====
      DIMENSION LAMDAV(4)
      DIMENSION ZINK(4,4),THETNK(4,4),LANDNK(4,4),
1F2K(4),J5K(4),
2J6K(4,4),J7K(4)
      DIMENSION F1(4,4),FF2(4,4),FF3(4,4),FF4(4,4)
1,FF5(4,4),FF6(4,4),FF7(4,4),FF8(4,4),FF9(4,4),FF10(4,4),FF11(4,4)
2,FF12(4,4),FF13(4,4),FF14(4,4),FF15(4,4),FF16(4,4),FF17(4,4)
3,FF18(4,4),FF19(4,4),FF20(4,4),FF21(4,4),FF22(4,4)
4,FF23(4)
5,FF30(4),FF31(4),FF32(4),FF33(4),FF34(4),FF35(4)
6,FF36(4),FF37(4)
9,FF59(4,4)
      DIMENSION VV(4)
      INITIALIZATION OF SUBROUTINE
      =====
      CALL KMEM(PHI1,PHIUT1,PHI2,PHI32,PHI4,PHIUT2,PHID32,PHIUT4,
1PHIDU2,PHIDU32,PHIDU4,J7)
      GAMMA=ALPHA*BETA
      SECB=1.0000/DCOS(BETA)
      G1=PHIDU2*DCOS(PHI32)-PHIUT2*PHID32*DSIN(PHI32)
      G2=PHIDU32
      G3=PHIDU2*DSIN(PHI32)+PHIUT2*PHID32*DCOS(PHI32)

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1175.      I1=G1*DCOS(GAMMA)+G2*DSIN(GAMMA)
1176.      I2=-G1*DSIN(GAMMA)+G2*UCUS(GAMMA)
1177.      I3=G3
1178.      J1=PHIUT2*DCUS(PHI32)
1179.      J2=PHIUD32
1180.      J3=PHIUT2*DSIN(PHI32)
1181.      H1=DCOS(GAMMA)*J1+DSIN(GAMMA)*J2
1182.      H2=DCUS(GAMMA)*J2-DSIN(GAMMA)*J1
1183.      H3=J3
1184.      AA=2*S1*A4
1185.      B3=2*A4*(S3+A1*DSIN(PHI1))
1186.      DEN=BB*DSIN(PHI4)-AA*DCOS(PHI4)
1187.      DLPH4=2*A3/DEN
1188.      DEN2=A3*UCUS(ALPHA)*USIN(PHI32)
1189.      DLPH32=(UCUS(ALPHA)*DCUS(PHI32)-A4*DCOS(PHI4)*DLPH4)/DEN2
1190.      COEF21=A3*DCUS(ALPHA)*DCOS(PHI2)*DCOS(PHI32)*DLPH32+UCUS(ALPHA)
1191.      I*DCOS(PHI2)*USIN(PHI32)-USIN(ALPHA)*USIN(PHI2)
1192.      DEN11=A3*(USIN(ALPHA)*UCUS(PHI2)+DSIN(PHI2)*DCOS(ALPHA)*DSIN
1193.      I(PHI32))
1194.      DLPH2=COEF21/DEN11
1195.      G4=RHO*A*(A1*(PHIUT1**2)*(UCUS(GAMMA)*USIN(PHI1-PHI2)-USIN(GAMMA)*
1196.      IUSIN(PHI32)*DCUS(PHI1-PHI2))+L2*(I3+H1*H2)*L1*L2*DCOS(GAMMA)+
1197.      2*(I3+H1*H2)*S00*(L2**2)*(H3**2+H1**2)*L2*L1*DSIN(GAMMA))
1198.      G7=RHO*A*(A1*(PHIUT1**2)*DSIN(PHI32)*DCOS(PHI1-PHI2)*L1-U.5DU*
1199.      I(J2**2+J3**2)*(L1**2))
1200.      G8=RHO*A*(U.5*A1*L2*L2*(PHIUT1**2)*(USIN(GAMMA)*DSIN(PHI1-PHI2)
1201.      I*DCOS(GAMMA)*DSIN(PHI32)*DCOS(PHI1-PHI2))+0.5DU*L1*L2*L2*DSIN(
1202.      2GAMMA)*(I3-H1*H2)-(H2**2+H3**2)*(0.5DU*L1*L2*L2*DCOS(GAMMA)+
1203.      3*(1.0DU/3.0DU)*(L2**3)))
1204.      G91=RHO*A*(DSIN(PHI32)*DLPH2*(A1*(PHIUT1**2)*DSIN(PHI1-PHI2)+0.5DU
1205.      I*L1*L1*(G3+J1*J2)*(1.0DU/3.0DU)*(L1**3))-DLPH32*(-A1*(PHIUT1**2)
1206.      2*DCOS(PHI32)*DCOS(PHI1-PHI2)+0.5DU*L1*L1*(J1+J3-G2)*(1.0DU/3.0DU)
1207.      3*(L1**3))+L1*DSIN(GAMMA)*DSIN(PHI32)*DLPH2*(A1*(PHIUT1**2)*(USIN
1208.      4(GAMMA)*USIN(PHI1-PHI2)+UCUS(GAMMA)*USIN(PHI32)*DCOS(PHI1-PHI2))
1209.      5*L2*(I3-H1*H2)*L1*L2*DSIN(GAMMA)-(H2**2+H3**2)*(L1*L2*UCUS(GAMMA)
1210.      6*0.5DU*L2*L2))+L1*DCOS(GAMMA)*USIN(PHI32)*DLPH2*(A1*(PHIUT1**2)*
1211.      7(DCOS(GAMMA)*DSIN(PHI1-PHI2))*L2*(I3+H1*H2)*(L1*L2*DCOS(GAMMA)+
1212.      8U.5DU*L2*L2)*(H3**2+H1**2)*L1*L2*DSIN(GAMMA)))
1213.      G92=RHO*A*(DLPH2*DSIN(PHI32)*
1214.      I(A1*(PHIUT1**2)*(DCOS(GAMMA)*DSIN(PHI1-PHI2)-DSIN(GAMMA)*DSIN
1215.      2(PHI32)*UCUS(PHI1-PHI2))+0.5DU*L2*L2*(I3+H1*H2)*(0.5DU*L1*L2*L2*
1216.      3UCUS(GAMMA)+(L2**3)/3.0DU)*(H3**2+H1**2)*0.5DU*L1*L2*L2*USIN
1217.      4(GAMMA))+(USIN(GAMMA)*DCOS(PHI32)*DLPH2-UCUS(GAMMA)*DLPH32)*(-A1*
1218.      5(PHIUT1**2)*UCUS(PHI32)*DCOS(PHI1-PHI2)+0.5DU*L2*L2-(I1+H2*H3)*
1219.      6U.5DU*L1*L2*L2*DSIN(GAMMA)+(H1+H3-I2)*(0.5DU*L1*L2*L2*UCUS(GAMMA)
1220.      7*(L2**3)/3.0DU))-L1*DLPH32*(-A1*L2*(PHIUT1**2)*DCOS(PHI32)*DCOS
1221.      8(PHI1-PHI2)-(I1+H2*H3)*L1*L2*DSIN(GAMMA)+(H1+H3-I2)*(L1*L2*DCOS(
1222.      9GAMMA)+0.5DU*L2*L2)))
1223.      G9=G91+G92
1224.      G11=RHO*A*(A1*L2*(PHIUT1**2)*(USIN(GAMMA)*DSIN(PHI1-PHI2)
1225.      I*DCOS(GAMMA)*DSIN(PHI32)*DCOS(PHI1-PHI2))+L1*L2*DSIN(
1226.      2GAMMA)*(I3-H1*H2)-(H2**2+H3**2)*(L1*L2*DCOS(GAMMA)+
1227.      3U.5DU*(L2**2)))
1228.      B1=DCOS(ALPHA)*DLPH4*JU4*PHIUD4+UCUS(GAMMA)*G11-USIN(GAM
1229.      MA)*G4+G9
1230.      B2=-RHO*A*DSIN(GAMMA)*SECB*DLPH2*A1*(PHIUT1**2)*DCOS(PHI1-
1231.      IPHI2)
1232.      B3=RHO*A*DSIN(GAMMA)*SECB*DLPH2*((J2**2+J3**2)*DSIN(PHI32)+
1233.      I(J1+J3-G2)*DCOS(PHI32))
1234.      B4=RHO*A*(A1*(PHIUT1**2)*(USIN(GAMMA)*DSIN(PHI1-PHI2)+UCUS(GAMMA)*
1235.      IUSIN(PHI32)*DCUS(PHI1-PHI2))*(I3-H1*H2)*L1*DSIN(GAMMA)-(H2**2+H3
1236.      2**2)*L1*UCUS(GAMMA))
1237.      B5=RHO*A*(H2**2+H3**2)
1238.      B6=DCOS(BETA)*DLPH4*JO4*PHIUD4+G9
1239.      B7=RHO*A*(A1*(PHIUT1**2)*(USIN(GAMMA)*USIN(PHI1-PHI2)+UCUS(GAMMA)*
1240.      IDSIN(PHI32)*UCUS(PHI1-PHI2))*(I3-H1*H2)*L1*DSIN(GAMMA)-(H2**2+

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1241. 2H3**2)*L1*UCUS(GAMMA))
 1242. B9=RHO*A*(H2**2+H3**2)
 1243. U9=RHO*A*(USIN(GAMMA)*SECB*(USIN(GAMMA)*USIN(PHI32)*DLPH2*(A1*
 1244. 1*(PHIDT1**2)*(DCOS(GAMMA)*DSIN(PHI1-PHI2)-DSIN(GAMMA)*DSIN(PHI32)*
 1245. 2*DCOS(PHI1-PHI2))+L1*DCOS(GAMMA)*(I3+H1*H2)*(H3**2+H1**2)*L1*
 1246. 3USIN(GAMMA))-DSIN(GAMMA)*(ULPH32*DCOS(GAMMA)-DSIN(GAMMA)*DCOS(
 1247. 4PHI32)*DLPH2)*(A1*(PHIDT1**2)*DCOS(PHI1-PHI2)-(I1+H2*H3)*L1*
 1248. 5USIN(GAMMA)*(H1*H3-I2)*L1*UCUS(GAMMA))-USIN(GAMMA)*SECB*(L1*
 1249. 6DSIN(GAMMA)*USIN(PHI32)*DLPH2*USIN(GAMMA)*(H2**2+H3**2)-
 1250. 7L1*DCOS(GAMMA)*DSIN(PHI32)*DLPH2*DSIN(GAMMA)*(H1*H2-I3)+L1*DLPH32
 1251. 8*(H1*H3-I2)*USIN(GAMMA))-DSIN(GAMMA)*(H2**2+H3**2)*DSIN(GAMMA))
 1252. B10=RHO*A*(DSIN(GAMMA)*SECB*(DSIN(GAMMA)*DSIN(PHI32)*DLPH2*(
 1253. 1I3+H1*H2)-USIN(GAMMA)*(DLPH32*UCUS(GAMMA)-USIN(GAMMA)*UCUS(PHI32)*
 1254. 2ULPH2*(H1*H3-I2))-DSIN(GAMMA)*SECB*(-DSIN(PHI32)*ULPH2*USIN(GAMMA)
 1255. 3)*(H1*H2-I3)-(USIN(GAMMA)*DCUS(PHI32)*ULPH2-UCUS(GAMMA)*DLPH32)*
 1256. 4*(H1*H3-I2)*DSIN(GAMMA)))
 1257. B11=RHO*A*(-DSIN(GAMMA)*SECB*(USIN(PHI32)*DLPH2*(A1*(PHIDT1**2)*(
 1258. 1USIN(GAMMA)*USIN(PHI1-PHI2)*UCUS(GAMMA)*DSIN(PHI32)*DCOS(PHI1-PHI2
 1259. 2)))+(I3-H1*H2)*L1*DSIN(GAMMA)-(H2**2+H3**2)*L1*DCUS(GAMMA)))+(USIN
 1260. 3(GAMMA)*ULPH32*DCOS(GAMMA)*DCOS(PHI32)*DLPH2)*(A1*(PHIDT1**2)*
 1261. 4UCUS(PHI32)*UCUS(PHI1-PHI2)-(I1+H2*H3)*L1*USIN(GAMMA)*(H1*H3-I2)*
 1262. 5L1*DCOS(GAMMA)))+(DSIN(GAMMA)*SECB*(L1*USIN(GAMMA)*DSIN(PHI32)*
 1263. 6ULPH2*(H1*H2-I3)-USIN(PHI32)*DLPH2*(H3**2+H1**2)*L1*UCUS(GAMMA)-
 1264. 7L1*DLPH32*(H2*H3+I1))*USIN(GAMMA)*(H1*H2-I3))
 1265. B12=RHO*A*(-USIN(GAMMA)*SECB*(USIN(PHI32)*DLPH2*(-(H2**2+H3**2))*
 1266. 1DSIN(GAMMA)*DLPH32*DCUS(GAMMA)*UCUS(PHI32)*DLPH2*(H1*H3-I2))*
 1267. 2USIN(GAMMA)*SECB*(-DSIN(PHI32)*DLPH2*(H3**2+H1**2)*DSIN(GAMMA)*
 1268. 3UCUS(PHI32)*DLPH2*(H2*H3+I1)-DCOS(GAMMA)*DLPH32*(H2*H3+I1)))
 1269. B13=-2*RHO*A*DLPH32*DSIN(GAMMA)*SECB*J1
 1270. B14=-2*RHO*A*DSIN(GAMMA)*(H3+L1*USIN(GAMMA)*DSIN(PHI32)*SECB*
 1271. 1ULPH2*H3+L1*SECB*DLPH32*H1)
 1272. B15=2*RHO*A*USIN(GAMMA)*SECB*(USIN(GAMMA)*UCOS(PHI32)*DLPH2-
 1273. 1UCUS(GAMMA)*DLPH32)*H1
 1274. B16=2*RHO*A*USIN(GAMMA)*SECB*(USIN(PHI32)*DLPH2*DSIN(GAMMA)*H3-
 1275. 1USIN(GAMMA)*H2*(USIN(GAMMA)*UCUS(PHI32)*DLPH2-UCUS(GAMMA)*DLPH32))
 1276. B17=2*RHO*A*USIN(GAMMA)*SECB*(L1*DCOS(GAMMA)*DSIN(PHI32)*DLPH2*
 1277. 2USIN(GAMMA)*H3+L1*USIN(GAMMA)*DLPH32*H2)
 1278. B18=RHO*A*USIN(GAMMA)*SECB*DSIN(PHI32)*DLPH2
 1279. B19=RHO*A*USIN(GAMMA)*SECB*DSIN(PHI32)*DLPH2*L1*UCUS(GAMMA)
 1280. B20=RHO*A*USIN(GAMMA)*SECB*DSIN(PHI32)*DLPH2
 1281. B21=RHO*A*USIN(GAMMA)*SECB*(-DLPH32*(J2**2+J3**2)-UCOS(PHI32)*
 1282. 1ULPH2*(G3+J1*J2)*DSIN(PHI32)*DLPH32*(J2*J3-G1)+DLPH32*(J1**2+
 1283. 2J2**2))
 1284. B22=RHO*A*DSIN(GAMMA)*SECB*(A1*(PHIDT1**2)*(DLPH32*DSIN(PHI32)*
 1285. 1UCUS(PHI1-PHI2)-DLPH2*USIN(PHI1-PHI2)))
 1286. B23=RHO*A*(DSIN(GAMMA)*SECB*(UCUS(GAMMA)*DLPH32-DSIN(GAMMA)*
 1287. 1UCUS(PHI32)*DLPH2)*(A1*(PHIDT1**2)*(USIN(GAMMA)*DSIN(PHI1-PHI2)
 1288. 2*DCOS(GAMMA)*DSIN(PHI32)*DCOS(PHI1-PHI2)))+(I3-H1*H2)*L1*USIN
 1289. 3(GAMMA)-(H2**2+H3**2)*L1*DCOS(GAMMA))-USIN(GAMMA)*DCUS(PHI32)*
 1290. 4DLPH2*(A1*(PHIDT1**2)*(DCOS(GAMMA)*DSIN(PHI1-PHI2)-DSIN(GAMMA)*
 1291. 5USIN(PHI32)*UCUS(PHI1-PHI2)))+(I3+H1*H2)*L1*DCOS(GAMMA)*(H3**2+
 1292. 6H1**2)*L1*USIN(GAMMA))*L1*USIN(GAMMA)*USIN(PHI32)*DLPH2*(H1*H3-I2)
 1293. 7*L1*UCUS(GAMMA)*DLPH2*(H1*H3-I1)+L1*DLPH32*(H1**2+H2**2))*USIN(
 1294. 8GAMMA)*(H1*H3+I2))
 1295. B24=RHO*A*(DSIN(GAMMA)*SECB*(-(DCOS(GAMMA)*DLPH32-DSIN(GAMMA)*
 1296. 1UCUS(PHI32)*DLPH2)*(H2**2+H3**2)-DSIN(GAMMA)*DCOS(PHI32)*DLPH2*
 1297. 2*(I3+H1*H2)+DLPH2*(H1*H3-I1)-(DSIN(GAMMA)*DCOS(PHI32)*DLPH2-
 1298. 3UCUS(GAMMA)*DLPH32)*(H1**2+H2**2)))
 1299. B25=-2*RHO*A*DSIN(GAMMA)*SECB*DSIN(PHI32)*DLPH32*J1
 1300. B26=2*RHO*A*USIN(GAMMA)*SECB*(L1*DSIN(GAMMA)*DSIN(PHI32)*DLPH2*H2
 1301. 1-L1*DCUS(GAMMA)*DLPH2*USIN(PHI32)*H1)
 1302. B27=-2*RHO*A*DSIN(GAMMA)*SECB*USIN(PHI32)*DLPH2*H1
 1303. B28=RHO*A*USIN(GAMMA)*SECB*(USIN(GAMMA)*UCOS(PHI32)*DLPH2-
 1304. 1UCUS(GAMMA)*DLPH32)
 1305. B29=-RHO*A*DSIN(GAMMA)*SECB*L1*DLPH32
 1306. B30=RHO*A*A1*(PHIDT1**2)*DSIN(PHI1-PHI2)

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1307.      B31=RHU*A*(G3+J1*J2)
1308.      B32=RHU*A*USIN(GAMMA)*SECB*A1*(PHIUT1**2)*(DSIN(PHI32)*DLPH2*
1309.      1USIN(PHI1-PHI2)+UCUS(PHI32)*UCUS(PHI1-PHI2)*DLPH32)
1310.      B33=2*RHU*A*USIN(GAMMA)*SECB*((G3+J1*J2)*DSIN(PHI32)*DLPH2-
1311.      1*(J1*J3-G2)*DLPH32)
1312.      B34=RHU*A*(A1*(PHIUT1**2)*(DCOS(GAMMA)*DSIN(PHI1-PHI2)-DSIN(GAMMA)
1313.      1*DSIN(PHI32)*DCOS(PHI1-PHI2))*(I3+H1*H2)*L1*DCOS(GAMMA)+(H3**2*
1314.      2H1**2)*L1*DSIN(GAMMA))
1315.      B35=RHU*A*(I3+H1*H2)
1316.      B361=RHU*A*(USIN(GAMMA)*SECB*(L1*DSIN(GAMMA)*DSIN(PHI32)*DLPH2*
1317.      1*(A1*(PHIUT1**2)*(DSIN(GAMMA)*DSIN(PHI1-PHI2)+DCOS(GAMMA)*DSIN(
1318.      2PHI32)*DCOS(PHI1-PHI2))*(I3-H1*H2)*L1*DSIN(GAMMA)-(H2**2+H3**2)*
1319.      3L1*DCOS(GAMMA))+L1*DCOS(GAMMA)*DSIN(PHI32)*DLPH2*(A1*(PHIUT1**2)
1320.      4*(UCUS(GAMMA)*USIN(PHI1-PHI2)-USIN(GAMMA)*DSIN(PHI32)*UCUS(PHI1-
1321.      5PHI2))*(I3+H1*H2)*L1*DCOS(GAMMA)+(H3**2+H1**2)*L1*DSIN(GAMMA))+
1322.      6ULPH32*L1*(A1*(PHIUT1**2)*UCUS(PHI32)*UCUS(PHI1-PHI2)+(I1*H2*H3)*
1323.      7L1*DSIN(GAMMA)-(H1*H3-12)*L1*DCOS(GAMMA))))
1324.      B362=RHU*A*(USIN(GAMMA)*(A1*
1325.      1*(PHIUT1**2)*(DSIN(GAMMA)*DSIN(PHI1-PHI2)+DCOS(GAMMA)*DSIN(PHI32)*
1326.      2UCUS(PHI1-PHI2))*(I3-H1*H2)*L1*DSIN(GAMMA)-(H2**2+H3**2)*
1327.      3L1*DCOS(GAMMA)))
1328.      B36=B361+B362
1329.      B37=RHU*A*(DSIN(GAMMA)*SECB*(-L1*DSIN(GAMMA)*DSIN(PHI32)*DLPH2*
1330.      1*(H2**2+H3**2)+L1*DCOS(GAMMA)*DSIN(PHI32)*DLPH2*(I3+H1*H2)+USIN(
1331.      2PHI32)*DLPH2*(A1*(PHIUT1**2)*(UCUS(GAMMA)*DSIN(PHI1-PHI2)-USIN(
1332.      3GAMMA)*DSIN(PHI32)*DCOS(PHI1-PHI2))*(I3+H1*H2)*L1*
1333.      4UCUS(GAMMA)+(H3**2+H1**2)*L1*DSIN(GAMMA))*(DSIN(GAMMA)*DCOS(PHI32)
1334.      5*DLPH2-DCOS(GAMMA)*DLPH32)*(-A1*(PHIUT1**2)*UCUS(PHI32)*UCUS(PHI1-
1335.      6PHI2)-(I1*H2*H3)*L1*USIN(GAMMA)+(H1*H3-12)*L1*UCUS(GAMMA))-DLPH32
1336.      7*L1*(H1*H3-12))-USIN(GAMMA)*(H2**2+H3**2)))
1337.      B38=2*RHU*A*USIN(GAMMA)*SECB*(USIN(PHI32)*DLPH2*(I3+H1*H2)+(USIN(
1338.      1GAMMA)*DCOS(PHI32)*DLPH2-DCOS(GAMMA)*DLPH32)*(H1*H3-12))
1339.      B39=-RHU*A*A1*(PHIUT1**2)*UCUS(PHI32)*UCUS(PHI1-PHI2)
1340.      B40=RHU*A*(J1*J3-G2)
1341.      B41=RHU*A*(-A1*(PHIUT1**2)*DCOS(PHI32)*DCOS(PHI1-PHI2)-(I1*H2*H3)
1342.      1*L1*USIN(GAMMA)+(H1*H3-12)*L1*UCUS(GAMMA))
1343.      B42=RHU*A*(H1*H3-12)
1344.      B43=RHU*A*A1*(PHIUT1**2)*DSIN(PHI32)*DCOS(PHI1-PHI2)
1345.      B44=-RHU*A*(J2**2+J3**2)
1346.      B45=-RHU*A*(PHIUT1**2)*DSIN(PHI32)*DCOS(PHI1-PHI2)
1347.      B46=RHU*A*(J2**2+J3**2)
1348.      B47=UCUS(ALPHA)*DLPH4
1349.      B48=UCUS(BETA)*DLPH4
1350.      B49=(DCOS(BETA)*UCOS(GAMMA)-UCUS(ALPHA))*DLPH4
1351.      B50=SECB*DSIN(GAMMA)*DLPH4
1352.      T1=-RHU*A*J1*J1
1353.      T2=-RHU*A*H1*H1
1354.      T3=RHU*A*(I3+H1*H2)
1355.      T4=DLPH4*L2*UCUS(BETA)*J04*PHIUD4+G9*L2*UCOS(BETA)*G8
1356.      T5=(UCUS(BETA)*DCOS(GAMMA)-DCOS(ALPHA))*(DLPH4*J04*PHIUD4+G9)+
1357.      1USIN(GAMMA)*G4
1358.      T6=DSIN(BETA)*USIN(GAMMA)*(DLPH4*J04*PHIUD4+G9)
1359.      T7=2*RHU*A*H3*JSIN(GAMMA)
1360.      T8=RHU*A*(USIN(GAMMA)*SECB*(L1*DSIN(GAMMA)*DSIN(PHI32)*DLPH32*
1361.      1USIN(GAMMA))+DSIN(GAMMA)**2)
1362.      T9=RHU*A*(J2*J3-G1)
1363.      T10=RHU*A*(H1*H3-I1)
1364.      T11=-2*RHU*A*J1
1365.      T12=-2*RHU*A*H1
1366.      T13=SECB*DSIN(GAMMA)*DLPH4*J04*PHIUD4
1367.      T14=RHU*A*(J2*J3+G1)
1368.      T15=RHU*A*USIN(GAMMA)*(H1*H3-12)
1369.      T16=RHU*A*(H2*H3+I1)
1370.      T17=2*RHU*A*J1
1371.      T18=2*RHU*A*H1
1372.      T19=2*RHU*A*USIN(GAMMA)*H2

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1439. C    CALCULATION OF ANGLE PHI32
1440. C
1441. EF=S3+A1*DSIN(PHI1)
1442. H=A3*DCOS(ALPHA)
1443. L=A3*DSIN(ALPHA)
1444. D=S2**2+A4**2+S3**2+A1**2-S1**2-A3**2+2*A1*S3*DSIN(PHI1)
1445. E=EE**2+S1**2
1446. F=(S1/M)*(D-2*(EE**2))
1447. G=(D/(2*M))**2-((A4**2-S1**2)/(M**2))*(EE**2)
1448. SQR2=DSQRT(F**2-4*E*G)
1449. DEN2=2*E
1450. PHI321=DARSIN((F+SQR2)/DEN2)
1451. PHI322=DARSIN((F-SQR2)/DEN2)
1452. IF (PHI321.LT.0) PHI321=2*PI-DARS(PHI321)
1453. IF (PHI322.LT.0) PHI322=2*PI-DARS(PHI322)
1454. IF (PHI321.LT.PHI322) GO TO 20
1455. PHI32=PHI322
1456. GO TO 21
1457. 20 PHI32=PHI321
1458. 21 CONTINUE
1459. C
1460. C
1461. C    CALCULATION OF ANGLE PHI2 (THIS IS OBTAINED FROM AN
1462. C    EXPRESSION WHICH IS A FUNCTION OF PHI1 AND PHI32
1463. C
1464. H2=-DSIN(ALPHA)
1465. H3=(A1/A3)*DCOS(PHI1)+(S2/A3)
1466. H1=DCOS(ALPHA)*DCOS(PHI32)
1467. SQR3=DSQRT(H2**2-H3**2+H1**2)
1468. DEN3=H1+H3
1469. PHI21=DATAN((H2+SQR3)/DEN3)
1470. PHI22=DATAN((H2-SQR3)/DEN3)
1471. IF (PHI21.LT.0) PHI21=2*PI-DARS(PHI21)
1472. IF (PHI22.LT.0) PHI22=2*PI-DARS(PHI22)
1473. PHI21=2*PHI21
1474. PHI22=2*PHI22
1475. IF (PHI22.LT.PHI21) GO TO 30
1476. PHI2=PHI21
1477. GO TO 31
1478. 30 PHI2=PHI22
1479. 31 CONTINUE
1480. C
1481. C    CALCULATION OF ANGULAR VELOCITY PHIDOT4 AND J7
1482. C
1483. NUM4=2*A1*((S3-A4*DCOS(PHI4))*DCOS(PHI1)-S2*DSIN(PHI1))
1484. DEN4=AA*DCOS(PHI4)-B*DSIN(PHI4)
1485. PHIDT4=NUM4*PHIDT1/DEN4
1486. J7=2.000*A3*DSIN(ALPHA+BETA)/(DEN4*DCOS(BETA))
1487. C
1488. C    CALCULATION OF ANGULAR VELOCITY PHIDOT3/2 IN TERMS OF
1489. C    PHI3/2, PHI4 AND PHIDOT4
1490. C
1491. PHID32=(A4*DCOS(PHI4)*PHIDT4)/(A3*DCOS(ALPHA)*DCOS(PHI32))
1492. C
1493. C    CALCULATION OF ANGULAR VELOCITY PHIDOT2 IN TERMS OF
1494. C    PHI1, PHI2, PHI32, PHIDT1, PHID32
1495. C
1496. DEND2=H2*DCOS(PHI2)-H1*DSIN(PHI2)
1497. NUMD2=DCOS(ALPHA)*DSIN(PHI32)*DCOS(PHI2)+PHID32-(A1/A3)
1498. 1*DSIN(PHI1)*PHIDT1
1499. PHIDT2=NUMD2/DEND2
1500. C
1501. C    CALCULATION OF ANGULAR ACCELERATION PHIDDOT4
1502. C
1503. DENDD4=DEN4
1504. NMDD41=AA*DSIN(PHI4)+B*DCOS(PHI4)

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1505. QUOT1=NMDD41/DENDD4
1506. NMDD42=4*A1*A4*DCOS(PHI1)*USIN(PHI4)
1507. QUOT2=NMDD42/DENDD4
1508. NMDD43=2*A1*((S3-A4*UCUS(PHI4))*USIN(PHI1)-S2*UCOS(PHI1))
1509. QUOT3=NMDD43/DENDD4
1510. PHIDU4=QUOT1*(PHIDT4**2)+QUOT2*PHIDT1*PHIDT4-QUOT3*(PHIDT1**2)
1511. C
1512. C CALCULATION OF ANGULAR ACCELERATION PHIDDOT3/2
1513. C
1514. COMP1=A4*DCOS(PHI4)*PHIDU4
1515. COMP2=-A4*USIN(PHI4)*(PHIDT4**2)
1516. COMP3=M*USIN(PHI32)*(PHID32**2)
1517. DENDD32=M*DCOS(PHI32)
1518. PHDD32=(COMP1+COMP2+COMP3)/DENDD32
1519. C
1520. C CALCULATION OF ANGULAR ACCELERATION PHIDDOT2
1521. C
1522. COMP4=DCOS(PHI32)*UCOS(PHI2)*DCOS(ALPHA)*(PHID32**2)
1523. COMP5=(H2*DSIN(PHI2)+H1*UCUS(PHI2))*(PHIDT2**2)
1524. COMP6=(A1/A3)*DCOS(PHI1)*(PHIDT1**2)
1525. COMP7=-2*DSIN(PHI32)*DSIN(PHI2)*UCUS(ALPHA)*PHIDT2*PHID32
1526. COMP8=USIN(PHI32)*UCUS(PHI2)*DCOS(ALPHA)*PHDD32
1527. DENDD2=H2*UCUS(PHI2)-H1*USIN(PHI2)
1528. PHIDU2=(COMP4+COMP5+COMP6+COMP7+COMP8)/DENDD2
1529. RETURN
1530. END
1531. SUBROUTINE FNTGRL(FF1,FF2,FF3,FF4,FF5,FF6,FF7,FF8,
1532. 1FF9,FF10,FF11,FF12,FF13,FF14,FF15,FF16,FF17,FF18,FF19,FF20,FF21,
1533. 2FF22,FF23,FF24,FF25,FF26,FF27,FF28,FF29,FF30,FF31,FF32,FF33,FF34,FF35,
1534. 3FF36,FF37,
1535. 4FF59,VV,NW,
1536. 5FF64,FF65,FF66)
1537. C
1538. C THE PROGRAM CALCULATES VARIOUS QUANTITIES INVOLVING PRODUCTS OF
1539. C THE SHAPE FUNCTIONS AS WELL AS THEIR DERIVATIVES. A SYSTEM'S SUB-
1540. C ROUTINE CALLED OLAURE IS USED TO PERFORM THE REQUIRED INTEGRATIONS.
1541. C THE RESULTS WILL BE SUBSEQUENTLY USED TO EVALUATE THE COEFFICIENTS
1542. C OF THE COMPONENTS OF THE DIFF. EQUATIONS FOR EVERY TIME STEP.
1543. C
1544. IMPLICIT REAL*8(A-Z)
1545. REAL*8 L2,L1
1546. INTEGER I,J,N,K
1547. EXTERNAL F1,F2,F3,F4,F5,F6,F7,F8,F9,F10,F11,F12,F13,F14,F15,F16,F1
1548. 17,F18,F19,F20,F21,F22,F23,F24,F25,F26,F27,F28,F29,F30,F31,F32,F33
1549. 2,F34,F35,F36,F37
1550. 3,F59,F60,F61,F64,F65,
1551. 4F66
1552. COMMON LAMDAV,I,J,ALPHA,BETA,L1,L2,A
1553. DIMENSION LAMDAV(4),FF1(4,4),FF2(4,4),FF3(4,4),FF4(4,4)
1554. 1,FF5(4,4),FF6(4,4),FF7(4,4),FF8(4,4),FF9(4,4),FF10(4,4),FF11(4,4)
1555. 2,FF12(4,4),FF13(4,4),FF14(4,4),FF15(4,4),FF16(4,4),FF17(4,4)
1556. 3,FF18(4,4),FF19(4,4),FF20(4,4),FF21(4,4),FF22(4,4)
1557. 4,FF23(4,4)
1558. 5,FF24(4,4),FF25(4,4),FF26(4,4),FF27(4,4),FF28(4,4),FF29(4,4)
1559. 6,FF30(4,4),FF31(4,4),FF32(4,4),FF33(4,4),FF34(4,4),FF35(4,4)
1560. 7,FF36(4,4),FF37(4,4)
1561. 8,FF59(4,4)
1562. DIMENSION FF64(4),FF65(4),FF66(4)
1563. DIMENSION VV1(4),VV2(4),VV(4)
1564. DO 10 I=1,4
1565. 10 DO 9 J=1,4
1566. C
1567. C COMPUTATION OF INTEGRALS
1568. C
1569. EPS=1.0D-5
1570. AERR=1.0D0

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1571.      CRU=0.0DU
1572.      CB1=L1
1573.      CB2=L2
1574.      FF1(I,J)=ASQ(F1,CBU,CB1,EPS)
1575.      FF2(I,J)=ASQ(F2,CBU,CB1,EPS)
1576.      FF3(I,J)=ASQ(F3,CBU,CB1,EPS)
1577.      FF4(I,J)=ASQ(F4,CBU,CB1,EPS)
1578.      FF5(I,J)=ASQ(F5,CBU,CB1,EPS)
1579.      FF6(I,J)=ASQ(F6,CBU,CB1,EPS)
1580.      FF7(I,J)=ASQ(F7,CBU,CB1,EPS)
1581.      FF8(I,J)=ASQ(F8,CBU,CB2,EPS)
1582.      FF9(I,J)=ASQ(F9,CBU,CB2,EPS)
1583.      FF10(I,J)=ASQ(F10,CBU,CB2,EPS)
1584.      FF11(I,J)=ASQ(F11,CBU,CB2,EPS)
1585.      FF12(I,J)=ASQ(F12,CBU,CB2,EPS)
1586.      FF13(I,J)=ASQ(F13,CBU,CB2,EPS)
1587.      FF14(I,J)=ASQ(F14,CBU,CB2,EPS)
1588.      FF15(I,J)=ASQ(F15,CBU,CB2,EPS)
1589.      FF16(I,J)=ASQ(F16,CBU,CB2,EPS)
1590.      FF17(I,J)=ASQ(F17,CBU,CB1,EPS)
1591.      FF18(I,J)=F18(L1)
1592.      FF19(I,J)=F19(L1)
1593.      FF20(I,J)=F20(L1)
1594.      FF21(I,J)=ASQ(F21,CBU,CB2,EPS)
1595.      FF22(I,J)=ASQ(F22,CBU,CB2,EPS)
1596.      FF29(J)=ASQ(F29,CBU,CB1,EPS)
1597.      FF30(J)=ASQ(F30,CBU,CB1,EPS)
1598.      FF31(J)=ASQ(F31,CBU,CB1,EPS)
1599.      FF32(J)=ASQ(F32,CBU,CB1,EPS)
1600.      FF33(J)=ASQ(F33,CBU,CB2,EPS)
1601.      FF34(J)=ASQ(F34,CBU,CB2,EPS)
1602.      FF35(J)=ASQ(F35,CBU,CB2,EPS)
1603.      FF36(J)=ASQ(F36,CBU,CB2,EPS)
1604.      FF37(J)=ASQ(F37,CBU,CB2,EPS)
1605.      FF59(I,J)=ASQ(F59,CBU,CB1,EPS)
1606.      FF64(I)=ASQ(F64,CBU,CB1,EPS)
1607.      FF65(I)=ASQ(F65,CBU,CB2,EPS)
1608.      FF66(I)=ASQ(F66,CBU,CB2,EPS)
1609.      VV1(J)=ASQ(F60,CBU,CB1,EPS)
1610.      VV2(J)=ASQ(F61,CBU,CB2,EPS)
1611.      VV(J)=VV1(J)+VV2(J)
1612.      9  CONTINUE
1613.      10 CONTINUE
1614.      RETURN
1615.      END
1616.      C
1617.      C
1618.      C  EXTERNAL FUNCTIONS F1(X) THRU F66(X)
1619.      C  =====
1620.      C
1621.      C
1622.      DOUBLE PRECISION FUNCTION F1(X)
1623.      IMPLICIT REAL*8(A-Z)
1624.      REAL*8 L2,L1
1625.      INTEGER I,J
1626.      DIMENSION LAMDAV(4)
1627.      COMMON LAMDAV,I,J,ALPHA,BETA,L1,L2,A
1628.      VIUPN=VIUP(I,X)
1629.      VIK=V1(J,X)
1630.      F1=(L1-X)*VIUPN*VIK
1631.      RETURN
1632.      END
1633.      DOUBLE PRECISION FUNCTION F2(X)
1634.      IMPLICIT REAL*8(A-Z)
1635.      REAL*8 L2,L1
1636.      INTEGER I,J

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1637.    DIMENSION LAMDAV(4)
1638.    COMMON LAMDAV,I,J,ALPHA,BETA,L1,L2,A
1639.    VIUPN=VIUP(I,X)
1640.    VIK=V1(J,X)
1641.    F2=0.500*(L1**2-X**2)*VIUPN*VIK
1642.    RETURN
1643.    END
1644.    DOUBLE PRECISION FUNCTION F3(X)
1645.    IMPLICIT REAL*8(A-Z)
1646.    REAL*8 L2,L1
1647.    INTEGER I,J
1648.    DIMENSION LAMDAV(4)
1649.    COMMON LAMDAV,I,J,ALPHA,BETA,L1,L2,A
1650.    VIUPN=VIUP(I,X)
1651.    VIK=V1(J,X)
1652.    F3=VIUPN*VIK
1653.    RETURN
1654.    END
1655.    DOUBLE PRECISION FUNCTION F4(X)
1656.    IMPLICIT REAL*8(A-Z)
1657.    REAL*8 L2,L1
1658.    INTEGER I,J
1659.    DIMENSION LAMDAV(4)
1660.    COMMON LAMDAV,I,J,ALPHA,BETA,L1,L2,A
1661.    VIPN=VIP(I,X)
1662.    VIK=V1(J,X)
1663.    F4=X*VIPN*VIK
1664.    RETURN
1665.    END
1666.    DOUBLE PRECISION FUNCTION F5(X)
1667.    IMPLICIT REAL*8(A-Z)
1668.    REAL*8 L2,L1
1669.    INTEGER I,J
1670.    DIMENSION LAMDAV(4)
1671.    COMMON LAMDAV,I,J,ALPHA,BETA,L1,L2,A
1672.    VIPN=VIP(I,X)
1673.    VIK=V1(J,X)
1674.    F5=VIPN*VIK
1675.    RETURN
1676.    END
1677.    DOUBLE PRECISION FUNCTION F6(X)
1678.    IMPLICIT REAL*8(A-Z)
1679.    REAL*8 L2,L1
1680.    INTEGER I,J
1681.    DIMENSION LAMDAV(4)
1682.    COMMON LAMDAV,I,J,ALPHA,BETA,L1,L2,A
1683.    VIN=V1(I,X)
1684.    VIK=V1(J,X)
1685.    F6=VIN*VIK
1686.    RETURN
1687.    END
1688.    DOUBLE PRECISION FUNCTION F7(X)
1689.    IMPLICIT REAL*8(A-Z)
1690.    REAL*8 L2,L1
1691.    INTEGER I,J
1692.    DIMENSION LAMDAV(4)
1693.    COMMON LAMDAV,I,J,ALPHA,BETA,L1,L2,A
1694.    VIN=V1(I,X)
1695.    VIK=V1(J,L1)
1696.    F7=X*VIN*VIK
1697.    RETURN
1698.    END
1699.    DOUBLE PRECISION FUNCTION F8(X)
1700.    IMPLICIT REAL*8(A-Z)
1701.    REAL*8 L2,L1
1702.    INTEGER I,J

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1703.    DIMENSION LAMDAV(4)
1704.    COMMON LAMDAV,I,J,ALPHA,BETA,L1,L2,A
1705.    VZDPN=VZDP(I,X)
1706.    VZK=VZ(J,X)
1707.    F8=(L2-X)*VZDPN*VZK
1708.    RETURN
1709.    END
1710.    DOUBLE PRECISION FUNCTION F9(X)
1711.    IMPLICIT REAL*8(A-Z)
1712.    REAL*8 L2,L1
1713.    INTEGER I,J
1714.    DIMENSION LAMDAV(4)
1715.    COMMON LAMDAV,I,J,ALPHA,BETA,L1,L2,A
1716.    VZDPN=VZDP(I,X)
1717.    VZK=VZ(J,X)
1718.    F9=0.500*(L2**2-X**2)*VZDPN*VZK
1719.    RETURN
1720.    END
1721.    DOUBLE PRECISION FUNCTION F10(X)
1722.    IMPLICIT REAL*8(A-Z)
1723.    REAL*8 L2,L1
1724.    INTEGER I,J
1725.    DIMENSION LAMDAV(4)
1726.    COMMON LAMDAV,I,J,ALPHA,BETA,L1,L2,A
1727.    VZDPN=VZDP(I,X)
1728.    VZK=VZ(J,X)
1729.    F10=VZDPN*VZK
1730.    RETURN
1731.    END
1732.    DOUBLE PRECISION FUNCTION F11(X)
1733.    IMPLICIT REAL*8(A-Z)
1734.    REAL*8 L2,L1
1735.    INTEGER I,J
1736.    DIMENSION LAMDAV(4)
1737.    COMMON LAMDAV,I,J,ALPHA,BETA,L1,L2,A
1738.    VZPN=VZP(I,X)
1739.    VZK=VZ(J,X)
1740.    F11=X*VZPN*VZK
1741.    RETURN
1742.    END
1743.    DOUBLE PRECISION FUNCTION F12(X)
1744.    IMPLICIT REAL*8(A-Z)
1745.    REAL*8 L2,L1
1746.    INTEGER I,J
1747.    DIMENSION LAMDAV(4)
1748.    COMMON LAMDAV,I,J,ALPHA,BETA,L1,L2,A
1749.    VZPN=VZP(I,X)
1750.    VZK=VZ(J,X)
1751.    F12=VZPN*VZK
1752.    RETURN
1753.    END
1754.    DOUBLE PRECISION FUNCTION F13(X)
1755.    IMPLICIT REAL*8(A-Z)
1756.    REAL*8 L2,L1
1757.    INTEGER I,J
1758.    DIMENSION LAMDAV(4)
1759.    COMMON LAMDAV,I,J,ALPHA,BETA,L1,L2,A
1760.    VZN=VZ(I,X)
1761.    VZK=VZ(J,X)
1762.    F13=VZN*VZK
1763.    RETURN
1764.    END
1765.    DOUBLE PRECISION FUNCTION F14(X)
1766.    IMPLICIT REAL*8(A-Z)
1767.    REAL*8 L2,L1
1768.    INTEGER I,J

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1769.    DIMENSION LAMDAV(4)
1770.    COMMON LAMDAV,I,J,ALPHA,BETA,L1,L2,A
1771.    VIN=V1(I,L1)
1772.    VIK=V1(J,L1)
1773.    F14=X*VIN*VIK
1774.    RETURN
1775.    END
1776.    DOUBLE PRECISION FUNCTION F15(X)
1777.    IMPLICIT REAL*8(A-Z)
1778.    REAL*8 L2,L1
1779.    INTEGER I,J
1780.    DIMENSION LAMDAV(4)
1781.    COMMON LAMDAV,I,J,ALPHA,BETA,L1,L2,A
1782.    VIN=V1(I,L1)
1783.    VIK=V1(J,L1)
1784.    F15=VIN*VIK
1785.    RETURN
1786.    END
1787.    DOUBLE PRECISION FUNCTION F16(X)
1788.    IMPLICIT REAL*8(A-Z)
1789.    REAL*8 L2,L1
1790.    INTEGER I,J
1791.    DIMENSION LAMDAV(4)
1792.    COMMON LAMDAV,I,J,ALPHA,BETA,L1,L2,A
1793.    VIN=V1(I,L1)
1794.    V2K=V2(J,X)
1795.    F16=VIN*V2K
1796.    RETURN
1797.    END
1798.    DOUBLE PRECISION FUNCTION F17(X)
1799.    IMPLICIT REAL*8(A-Z)
1800.    REAL*8 L2,L1
1801.    INTEGER I,J
1802.    DIMENSION LAMDAV(4)
1803.    COMMON LAMDAV,I,J,ALPHA,BETA,L1,L2,A
1804.    VIN=V1(I,L1)
1805.    V2K=V2(J,X)
1806.    F17=X*VIN*V2K
1807.    RETURN
1808.    END
1809.    DOUBLE PRECISION FUNCTION F18(X)
1810.    IMPLICIT REAL*8(A-Z)
1811.    REAL*8 L2,L1
1812.    INTEGER I,J
1813.    DIMENSION LAMDAV(4)
1814.    COMMON LAMDAV,I,J,ALPHA,BETA,L1,L2,A
1815.    VIPN=VIP(I,L1)
1816.    VIPK=VIP(J,L1)
1817.    F18=VIPN*VIPK
1818.    RETURN
1819.    END
1820.    DOUBLE PRECISION FUNCTION F19(X)
1821.    IMPLICIT REAL*8(A-Z)
1822.    REAL*8 L2,L1
1823.    INTEGER I,J
1824.    DIMENSION LAMDAV(4)
1825.    COMMON LAMDAV,I,J,ALPHA,BETA,L1,L2,A
1826.    VIPN=VIP(I,L1)
1827.    VIK=V1(J,L1)
1828.    F19=VIPN*VIK
1829.    RETURN
1830.    END
1831.    DOUBLE PRECISION FUNCTION F20(X)
1832.    IMPLICIT REAL*8(A-Z)
1833.    REAL*8 L2,L1
1834.    INTEGER I,J

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1835.    DIMENSION LAMDAV(4)
1836.    COMMON LAMDAV,I,J,ALPHA,BETA,L1,L2,A
1837.    V2PN=V2P(I,L2)
1838.    VIK=V1(J,L1)
1839.    F20=V2PN*VIK
1840.    RETURN
1841.    END
1842.    DOUBLE PRECISION FUNCTION F21(X)
1843.    IMPLICIT REAL*8(A-Z)
1844.    REAL*8 L2,L1
1845.    INTEGER I,J
1846.    DIMENSION LAMDAV(4)
1847.    COMMON LAMDAV,I,J,ALPHA,BETA,L1,L2,A
1848.    V2N=V2(I,X)
1849.    VIK=V1(J,L1)
1850.    F21=V2N*VIK
1851.    RETURN
1852.    END
1853.    DOUBLE PRECISION FUNCTION F22(X)
1854.    IMPLICIT REAL*8(A-Z)
1855.    REAL*8 L2,L1
1856.    INTEGER I,J
1857.    DIMENSION LAMDAV(4)
1858.    COMMON LAMDAV,I,J,ALPHA,BETA,L1,L2,A
1859.    V2N=V2(I,X)
1860.    VIK=V1(J,L1)
1861.    F22=X*V2N*VIK
1862.    RETURN
1863.    END
1864.    DOUBLE PRECISION FUNCTION F29(X)
1865.    IMPLICIT REAL*8(A-Z)
1866.    REAL*8 L2,L1
1867.    INTEGER I,J
1868.    DIMENSION LAMDAV(4)
1869.    COMMON LAMDAV,I,J,ALPHA,BETA,L1,L2,A
1870.    VIK=V1(J,X)
1871.    F29=VIK
1872.    RETURN
1873.    END
1874.    DOUBLE PRECISION FUNCTION F30(X)
1875.    IMPLICIT REAL*8(A-Z)
1876.    REAL*8 L2,L1
1877.    INTEGER I,J
1878.    DIMENSION LAMDAV(4)
1879.    COMMON LAMDAV,I,J,ALPHA,BETA,L1,L2,A
1880.    VIK=V1(J,X)
1881.    F30=X*VIK
1882.    RETURN
1883.    END
1884.    DOUBLE PRECISION FUNCTION F31(X)
1885.    IMPLICIT REAL*8(A-Z)
1886.    REAL*8 L2,L1
1887.    INTEGER I,J
1888.    DIMENSION LAMDAV(4)
1889.    COMMON LAMDAV,I,J,ALPHA,BETA,L1,L2,A
1890.    VIK=V1(J,L1)
1891.    F31=X*VIK
1892.    RETURN
1893.    END
1894.    DOUBLE PRECISION FUNCTION F32(X)
1895.    IMPLICIT REAL*8(A-Z)
1896.    REAL*8 L2,L1
1897.    INTEGER I,J
1898.    DIMENSION LAMDAV(4)
1899.    COMMON LAMDAV,I,J,ALPHA,BETA,L1,L2,A
1900.    VIK=V1(J,L1)

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1901.      F32=0.5D0*(X**2)*V1K
1902.      RETURN
1903.      END
1904.      DOUBLE PRECISION FUNCTION F33(X)
1905.      IMPLICIT REAL*8(A-Z)
1906.      REAL*8 L2,L1
1907.      INTEGER I,J
1908.      DIMENSION LAMDAV(4)
1909.      COMMON LAMDAV,I,J,ALPHA,BETA,L1,L2,A
1910.      V2K=V2(J,X)
1911.      F33=V2K
1912.      RETURN
1913.      END
1914.      DOUBLE PRECISION FUNCTION F34(X)
1915.      IMPLICIT REAL*8(A-Z)
1916.      REAL*8 L2,L1
1917.      INTEGER I,J
1918.      DIMENSION LAMDAV(4)
1919.      COMMON LAMDAV,I,J,ALPHA,BETA,L1,L2,A
1920.      V2K=V2(J,X)
1921.      F34=X*V2K
1922.      RETURN
1923.      END
1924.      DOUBLE PRECISION FUNCTION F35(X)
1925.      IMPLICIT REAL*8(A-Z)
1926.      REAL*8 L2,L1
1927.      INTEGER I,J
1928.      DIMENSION LAMDAV(4)
1929.      COMMON LAMDAV,I,J,ALPHA,BETA,L1,L2,A
1930.      V1K=V1(J,L1)
1931.      F35=V1K
1932.      RETURN
1933.      END
1934.      DOUBLE PRECISION FUNCTION F36(X)
1935.      IMPLICIT REAL*8(A-Z)
1936.      REAL*8 L2,L1
1937.      INTEGER I,J
1938.      DIMENSION LAMDAV(4)
1939.      COMMON LAMDAV,I,J,ALPHA,BETA,L1,L2,A
1940.      V1K=V1(J,L1)
1941.      F36=X*V1K
1942.      RETURN
1943.      END
1944.      DOUBLE PRECISION FUNCTION F37(X)
1945.      IMPLICIT REAL*8(A-Z)
1946.      REAL*8 L2,L1
1947.      INTEGER I,J
1948.      DIMENSION LAMDAV(4)
1949.      COMMON LAMDAV,I,J,ALPHA,BETA,L1,L2,A
1950.      V1K=V1(J,L1)
1951.      F37=0.5D0*(X**2)*V1K
1952.      RETURN
1953.      END
1954.      DOUBLE PRECISION FUNCTION F59(X)
1955.      IMPLICIT REAL*8(A-Z)
1956.      REAL*8 L2,L1
1957.      INTEGER I,J
1958.      DIMENSION LAMDAV(4)
1959.      COMMON LAMDAV,I,J,ALPHA,BETA,L1,L2,A
1960.      V1N=V1(I,X)
1961.      V1K=V1(J,L1)
1962.      F59=V1N*V1K
1963.      RETURN
1964.      END
1965.      DOUBLE PRECISION FUNCTION F60(X)
1966.      IMPLICIT REAL*8(A-Z)

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1967.      REAL*8 L2,L1
1968.      INTEGER I,J
1969.      DIMENSION LAMDAV(4)
1970.      COMMON LAMDAV,I,J,ALPHA,BETA,L1,L2,A
1971.      VIN=V1(J,X)
1972.      VIK=V1(J,X)
1973.      F60=VIN*VIK
1974.      RETURN
1975.      END
1976.      DOUBLE PRECISION FUNCTION F61(X)
1977.      IMPLICIT REAL*8(A-Z)
1978.      REAL*8 L2,L1
1979.      INTEGER I,J
1980.      DIMENSION LAMDAV(4)
1981.      COMMON LAMDAV,I,J,ALPHA,BETA,L1,L2,A
1982.      V2N=V2(J,X)
1983.      V2K=V2(J,X)
1984.      F61=V2N*V2K
1985.      RETURN
1986.      END
1987.      DOUBLE PRECISION FUNCTION F64(X)
1988.      IMPLICIT REAL*8(A-Z)
1989.      REAL*8 L2,L1
1990.      INTEGER I,J
1991.      DIMENSION LAMDAV(4)
1992.      COMMON LAMDAV,I,J,ALPHA,BETA,L1,L2,A
1993.      V1PN=V1P(J,X)
1994.      F64=V1PN**2
1995.      RETURN
1996.      END
1997.      DOUBLE PRECISION FUNCTION F65(X)
1998.      IMPLICIT REAL*8(A-Z)
1999.      REAL*8 L2,L1
2000.      INTEGER I,J
2001.      DIMENSION LAMDAV(4)
2002.      COMMON LAMDAV,I,J,ALPHA,BETA,L1,L2,A
2003.      V2PN=V2P(I,X)
2004.      F65=V2PN**2
2005.      RETURN
2006.      END
2007.      DOUBLE PRECISION FUNCTION F66(X)
2008.      IMPLICIT REAL*8(A-Z)
2009.      REAL*8 L2,L1
2010.      INTEGER I,J
2011.      DIMENSION LAMDAV(4)
2012.      COMMON LAMDAV,I,J,ALPHA,BETA,L1,L2,A
2013.      V1PN=V1P(I,L1)
2014.      F66=V1PN**2
2015.      RETURN
2016.      END
2017.      C
2018.      C
2019.      C      EXTERNAL FUNCTIONS  V1(X)  THRU  W2DP(X)
2020.      C      =====
2021.      C
2022.      C
2023.      DOUBLE PRECISION FUNCTION V1(K,X)
2024.      IMPLICIT REAL*8(A-Z)
2025.      INTEGER K,I,J
2026.      DIMENSION LAMDAV(4)
2027.      COMMON LAMDAV,I,J,ALPHA,BETA,L1,L2,A
2028.      ARG1=LAMDAV(K)*L1
2029.      ARG2=LAMDAV(K)*L2
2030.      ARG3=LAMDAV(K)*(L1+L2)
2031.      COEF1=DCOS(ALPHA+BETA)+1
2032.      COEF2=DCOS(ALPHA+BETA)-1

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2033. UENGAM=2.000*(USIN(ARG2)*DSINH(ARG2))
2034. GAMAIN=(UCUS(ALPHA*BETA)*DSIN(ARG1)*DCOS(ARG2)-DCOSH(ARG2))*
2035. USIN(ARG3)*DSIN(ARG2)*DCUS(ARG1)*DCOSH(ARG2)*DSIN(ARG1))/DLNLCAM
2036. GAMAZN=(UCUS(ALPHA*BETA)*DCUS(ARG2)-DCOSH(ARG2))*DSINH(ARG1)-
2037. 1(DCOS(ARG2)*DCUSH(ARG2)*DSINH(ARG1)+2.000*DSIN(ARG2)*DCUSH(ARG1)
2038. 2)/UENGAM
2039. GAMAN=(UCUS(ARG1)-GAMAIN)*DSIN(ARG2)+0.500*DCOS(ARG2)*DSIN(ARG1)*
2040. 1COEF1+GAMAIN*DSINH(ARG2)+0.500*DCOSH(ARG2)*DSIN(ARG1)*COEF2*
2041. 2UTAN(BETA)*DSIN(ALPHA*BETA)*USIN(ARG1)
2042. GAMAN=(UCUSH(ARG1)-GAMAZN)*USIN(ARG2)+0.500*DCOS(ARG2)*
2043. USINH(ARG1)*COEF2+DSINH(ARG2)*GAMAZN+0.500*DCOSH(ARG2)*
2044. 2USINH(ARG1)*COEF1+DTAN(BETA)*DSIN(ALPHA*BETA)*DSINH(ARG1)
2045. GAMAN=GAMAZN/GAMAZN
2046. ARGV1=LAMDV(K)*X
2047. V1=DSIN(ARGV1)-GAMAZN*DSINH(ARGV1)
2048. RETURN
2049. END
2050. DOUBLE PRECISION FUNCTION V2(K,X)
2051. IMPLICIT REAL*8(A-Z)
2052. INTEGER K,I,J
2053. DIMENSION LAMDV(4)
2054. COMMON LAMDV,I,J,ALPHA,BETA,L1,L2,A
2055. ARG1=LAMDV(K)*L1
2056. ARG2=LAMDV(K)*L2
2057. ARG3=LAMDV(K)*(L1+L2)
2058. COEF1=UCUS(ALPHA*BETA)*1
2059. COEF2=UCUS(ALPHA*BETA)-1
2060. UENGAM=2.000*(USIN(ARG2)*DSINH(ARG2)+DCOS(ARG2)-DCOSH(ARG2))*
2061. GAMAIN=(UCUS(ALPHA*BETA)*DSIN(ARG1)*DCOS(ARG2)-DCOSH(ARG2))*
2062. 2UTAN(BETA)*DSIN(ARG2)*DCUS(ARG1)*DCOSH(ARG2)*DSIN(ARG1)/DENLCAM
2063. GAMAZN=(UCUS(ALPHA*BETA)*DCUS(ARG2)-DCOSH(ARG2))*DSINH(ARG1)-
2064. 1(DCOS(ARG2)*DCUSH(ARG2)*DSINH(ARG1)+2.000*DSIN(ARG2)*DCUSH(ARG1)
2065. 2)/UENGAM
2066. GAMAN=(UCUS(ARG1)-GAMAIN)*DSIN(ARG2)+0.500*DCOS(ARG2)*DSIN(ARG1)*
2067. 1COEF1+GAMAIN*DSINH(ARG2)+0.500*DCOSH(ARG2)*DSIN(ARG1)*COEF2*
2068. 2UTAN(BETA)*DSIN(ALPHA*BETA)*USIN(ARG1)
2069. GAMAN=(UCUSH(ARG1)-GAMAZN)*USIN(ARG2)+0.500*DCOS(ARG2)*
2070. USINH(ARG1)*COEF2+DSINH(ARG2)*GAMAZN+0.500*DCOSH(ARG2)*
2071. 2USINH(ARG1)*COEF1+DTAN(BETA)*DSIN(ALPHA*BETA)*DSINH(ARG1)
2072. GAMAN=GAMAZN/GAMAZN
2073.
2074.
2075. COEFFICIENTS OF V2
2076.
2077. ULTA1N=DCOS(ARG1)-GAMAIN-GAMAZN*DCUSH(ARG1)+GAMAZN*GAMAZN
2078. ULTA2N=0.500*(COEF1*USIN(ARG1)-GAMAZN*COEF2*USINH(ARG1))
2079. ULTA3N=GAMAIN-GAMAZN*GAMAZN
2080. ULTA4N=0.500*(COEF2*USIN(ARG1)-GAMAZN*COEF1*USINH(ARG1))
2081.
2082. EXPRESSION FOR V2
2083. ARGV2=LAMDV(K)*X
2084. V2=DLTA1N*DSIN(ARGV2)+DLTA2N*DCOS(ARGV2)+DLTA3N*DSINH(ARGV2)+
2085. 1DLTA4N*DCOSH(ARGV2)
2086. RETURN
2087. END
2088. DOUBLE PRECISION FUNCTION VIP(K,X)
2089. IMPLICIT REAL*8(A-Z)
2090. INTEGER K,I,J
2091. DIMENSION LAMDV(4)
2092. COMMON LAMDV,I,J,ALPHA,BETA,L1,L2,A
2093. ARG1=LAMDV(K)*L1
2094. ARG2=LAMDV(K)*L2
2095. ARG3=LAMDV(K)*(L1+L2)
2096. COEF1=UCUS(ALPHA*BETA)*1
2097. COEF2=UCUS(ALPHA*BETA)-1
2098. UENGAM=2.000*(USIN(ARG2)*DSINH(ARG2)+DCOS(ARG2)-DCOSH(ARG2))*

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2099.      GAMA1N=(UCUS(ALPHA+BETA)*DSIN(ARG1)*(DCOS(ARG2)-DCOSH(ARG2))+
2100.      1USIN(ARG3)*DSIN(ARG2)*UCUS(ARG1)*DCOSH(ARG2)*DSIN(ARG1))/DENGAM
2101.      GAMA2N=(UCUS(ALPHA+BETA)*(UCUS(ARG2)-DCOSH(ARG2))*USINH(ARG1)-
2102.      1(DCOS(ARG2)+UCUSH(ARG2))*DSINH(ARG1)+2.000*DSIN(ARG2)*UCUSH(ARG1)
2103.      2)/DENGAM
2104.      GAMA4N=(UCUS(ARG1)-GAMA1N)*DSIN(ARG2)+0.500*DCOS(ARG2)*DSIN(ARG1)*
2105.      1COEF1+GAMA1N*DSINH(ARG2)+0.500*DCOSH(ARG2)*DSIN(ARG1)*COEF2+
2106.      2DTAN(BETA)*DSIN(ALPHA+BETA)*USIN(ARG1)
2107.      GAMA5N=(UCUSH(ARG1)-GAMA2N)*USIN(ARG2)+0.500*DCOS(ARG2)*
2108.      1USINH(ARG1)*COEF2+USINH(ARG2)*GAMA2N+0.500*DCOSH(ARG2)*
2109.      2USINH(ARG1)*COEF1+DTAN(BETA)*DSIN(ALPHA+BETA)*DSINH(ARG1)
2110.      GAMA3N=GAMA4N/GAMA5N
2111.      ARGV1=LAMDAV(K)*X
2112.      VIP=LAMDAV(K)*(DCOS(ARGV1)-GAMA3N*UCUSH(ARGV1))
2113.      RETURN
2114.      END
2115.      DOUBLE PRECISION FUNCTION VIP(K,X)
2116.      IMPLICIT REAL*8(A-Z)
2117.      INTEGER K,I,J
2118.      DIMENSION LAMDAV(4)
2119.      COMMON LAMDAV,I,J,ALPHA,BETA,L1,L2,A
2120.      ARG1=LAMDAV(K)*L1
2121.      ARG2=LAMDAV(K)*L2
2122.      ARG3=LAMDAV(K)*(L1+L2)
2123.      COEF1=UCUS(ALPHA+BETA)+1
2124.      COEF2=UCUS(ALPHA+BETA)-1
2125.      DENGAM=2.000*(USIN(ARG2)+DSINH(ARG2))
2126.      GAMA1N=(UCUS(ALPHA+BETA)*DSIN(ARG1)*(DCOS(ARG2)-DCOSH(ARG2))+
2127.      1USIN(ARG3)+DSIN(ARG2)*UCUS(ARG1)+DCOSH(ARG2)*DSIN(ARG1))/DENGAM
2128.      GAMA2N=(UCUS(ALPHA+BETA)*(UCUS(ARG2)-DCOSH(ARG2))*USINH(ARG1)-
2129.      1(DCOS(ARG2)+UCUSH(ARG2))*DSINH(ARG1)+2.000*DSIN(ARG2)*UCUSH(ARG1)
2130.      2)/DENGAM
2131.      GAMA4N=(UCUS(ARG1)-GAMA1N)*DSIN(ARG2)+0.500*DCOS(ARG2)*DSIN(ARG1)*
2132.      1COEF1+GAMA1N*DSINH(ARG2)+0.500*DCOSH(ARG2)*DSIN(ARG1)*COEF2+
2133.      2DTAN(BETA)*DSIN(ALPHA+BETA)*USIN(ARG1)
2134.      GAMA5N=(UCUSH(ARG1)-GAMA2N)*USIN(ARG2)+0.500*DCOS(ARG2)*
2135.      1USINH(ARG1)*COEF2+USINH(ARG2)*GAMA2N+0.500*DCOSH(ARG2)*
2136.      2USINH(ARG1)*COEF1+DTAN(BETA)*DSIN(ALPHA+BETA)*DSINH(ARG1)
2137.      GAMA3N=GAMA4N/GAMA5N
2138.      ARGV1=LAMDAV(K)*X
2139.      VIP=(LAMDAV(K)**2)*(-DSIN(ARGV1)-GAMA3N*DSINH(ARGV1))
2140.      RETURN
2141.      END
2142.      DOUBLE PRECISION FUNCTION V2P(K,X)
2143.      IMPLICIT REAL*8(A-Z)
2144.      INTEGER K,I,J
2145.      DIMENSION LAMDAV(4)
2146.      COMMON LAMDAV,I,J,ALPHA,BETA,L1,L2,A
2147.      ARG1=LAMDAV(K)*L1
2148.      ARG2=LAMDAV(K)*L2
2149.      ARG3=LAMDAV(K)*(L1+L2)
2150.      COEF1=UCUS(ALPHA+BETA)+1
2151.      COEF2=UCUS(ALPHA+BETA)-1
2152.      DENGAM=2.000*(USIN(ARG2)+DSINH(ARG2))
2153.      GAMA1N=(UCUS(ALPHA+BETA)*DSIN(ARG1)*(DCOS(ARG2)-DCOSH(ARG2))+
2154.      1USIN(ARG3)+DSIN(ARG2)*UCUS(ARG1)+DCOSH(ARG2)*DSIN(ARG1))/DENGAM
2155.      GAMA2N=(UCUS(ALPHA+BETA)*(UCUS(ARG2)-DCOSH(ARG2))*USINH(ARG1)-
2156.      1(DCOS(ARG2)+UCUSH(ARG2))*DSINH(ARG1)+2.000*DSIN(ARG2)*DCOSH(ARG1)
2157.      2)/DENGAM
2158.      GAMA4N=(UCUS(ARG1)-GAMA1N)*DSIN(ARG2)+0.500*DCOS(ARG2)*DSIN(ARG1)*
2159.      1COEF1+GAMA1N*DSINH(ARG2)+0.500*DCOSH(ARG2)*DSIN(ARG1)*COEF2+
2160.      2DTAN(BETA)*DSIN(ALPHA+BETA)*USIN(ARG1)
2161.      GAMA5N=(UCUSH(ARG1)-GAMA2N)*USIN(ARG2)+0.500*DCOS(ARG2)*
2162.      1USINH(ARG1)*COEF2+DSINH(ARG2)*GAMA2N+0.500*DCOSH(ARG2)*
2163.      2USINH(ARG1)*COEF1+DTAN(BETA)*DSIN(ALPHA+BETA)*DSINH(ARG1)
2164.      GAMA3N=GAMA4N/GAMA5N

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2165. C
2166. C      COEFFICIENTS OF V2P
2167. C
2168.      DLTAIN=DCOS(ARG1)-GAMA1N-GAMA3N*UCOSH(ARG1)*GAMA2N*GAMA3N
2169.      DLTAN=0.500*(COEF1*DSIN(ARG1)-GAMA3N*COEF2*DSINH(ARG1))
2170.      DLTAN=GAMA1N-GAMA2N*GAMA3N
2171.      DLTAN=0.500*(COEF2*DSIN(ARG1)-GAMA3N*COEF1*DSINH(ARG1))
2172. C
2173. C      EXPRESSION FOR V2P
2174. C
2175.      ARGV2=LAMDAV(K)*X
2176.      V2P=(DLTAIN*UCUS(ARGV2)-DLTAN*DSIN(ARGV2)+DLTAN*DCOSH(ARGV2)+
2177.      1DLTAN*DSINH(ARGV2))*LAMDAV(K)
2178.      RETURN
2179.      END
2180.      DOUBLE PRECISION FUNCTION V2UP(K,X)
2181.      IMPLICIT REAL*8(A-Z)
2182.      INTEGER K,I,J
2183.      DIMENSION LAMDAV(4)
2184.      COMMON LAMDAV,I,J,ALPHA,BETA,L1,L2,A
2185.      ARG1=LAMDAV(K)*L1
2186.      ARG2=LAMDAV(K)*L2
2187.      ARG3=LAMDAV(K)*(L1+L2)
2188.      COEF1=UCUS(ALPHA+BETA)+1
2189.      COEF2=UCUS(ALPHA+BETA)-1
2190.      DENGAM=2.000*(DSIN(ARG2)+DSINH(ARG2))
2191.      GAMA1N=(UCUS(ALPHA+BETA)*DSIN(ARG1)+DCOS(ARG2)-UCOSH(ARG2))*
2192.      1DSIN(ARG3)+DSIN(ARG2)*UCUS(ARG1)+DCOSH(ARG2)*DSIN(ARG1)/DENGAM
2193.      GAMA2N=(UCUS(ALPHA+BETA)*(UCUS(ARG2)-DCOSH(ARG2))*DSINH(ARG1)-
2194.      1(DCOS(ARG2)+DCOSH(ARG2))*DSINH(ARG1)+2.000*DSIN(ARG2)*UCOSH(ARG1)
2195.      2)/DENGAM
2196.      GAMA4N=(UCUS(ARG1)-GAMA1N)*DSIN(ARG2)+0.500*UCUS(ARG2)*DSIN(ARG1)*
2197.      1COEF1+GAMA1N*DSINH(ARG2)+0.500*DCOSH(ARG2)*DSIN(ARG1)*COEF2+
2198.      2DTAN(BETA)*DSIN(ALPHA+BETA)*DSIN(ARG1)
2199.      GAMA5N=(UCOSH(ARG1)-GAMA2N)*DSIN(ARG2)+0.500*DCOS(ARG2)*
2200.      1DSINH(ARG1)*COEF2+DSINH(ARG2)*GAMA2N+0.500*DCOSH(ARG2)*
2201.      2DSINH(ARG1)*COEF1+DTAN(BETA)*DSIN(ALPHA+BETA)*DSINH(ARG1)
2202.      GAMA3N=GAMA4N/GAMA5N
2203. C
2204. C      COEFFICIENTS OF V2UP
2205. C
2206.      DLTAIN=DCOS(ARG1)-GAMA1N-GAMA3N*DCOSH(ARG1)+GAMA2N*GAMA3N
2207.      DLTAN=0.500*(COEF1*DSIN(ARG1)-GAMA3N*COEF2*DSINH(ARG1))
2208.      DLTAN=GAMA1N-GAMA2N*GAMA3N
2209.      DLTAN=0.500*(COEF2*DSIN(ARG1)-GAMA3N*COEF1*DSINH(ARG1))
2210. C
2211. C      EXPRESSION FOR V2UP
2212. C
2213.      ARGV2=LAMDAV(K)*X
2214.      V2UP=(-DLTAIN*DSIN(ARGV2)-DLTAN*DCOS(ARGV2)+DLTAN*DSINH(ARGV2)+
2215.      1DLTAN*DCOSH(ARGV2))*(LAMDAV(K)**2)
2216.      RETURN
2217.      END
2218.      DOUBLE PRECISION FUNCTION ASQ(G,A,B,EPS)
2219. C
2220. C *****
2221. C
2222. C TITLE- ADAPTIVE SIMPSON QUADRATURE
2223. C
2224. C PURPOSE- TO EVALUATE A DEFINITE INTEGRAL.
2225. C
2226. C DESCRIPTION OF PARAMETERS
2227. C
2228. C      G(X) = THE INTEGRAND OF THE INTEGRAL TO BE EVALUATED.
2229. C      A = THE LOWER LIMIT OF INTEGRATION.
2230. C      B = THE UPPER LIMIT OF INTEGRATION.

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2165. C
2166. C COEFFICIENTS OF V2P
2167. C
2168. DLTAIN=DCOS(ARG1)-GAMA1N-GAMA3N*DCOSH(ARG1)+GAMA2N*GAMA3N
2169. DLTAN=0.500*(COEF1*DSIN(ARG1)-GAMA3N*COEF2*DSINH(ARG1))
2170. DLTAN=GAMA1N-GAMA2N*GAMA3N
2171. DLTAN=0.500*(COEF2*DSIN(ARG1)-GAMA3N*COEF1*DSINH(ARG1))
2172. C
2173. C EXPRESSION FOR V2P
2174. C
2175. ARGV2=LAMDAV(K)*X
2176. V2P=(-DLTAIN*UCUS(ARGV2)-DLTAN*DSIN(ARGV2)+DLTAN*DCOSH(ARGV2)+
2177. 1DLTAN*DCOSH(ARGV2))*LAMDAV(K)
2178. RETURN
2179. END
2180. DOUBLE PRECISION FUNCTION V2UP(K,X)
2181. IMPLICIT REAL*8(A-Z)
2182. INTEGER K,I,J
2183. DIMENSION LAMDAV(4)
2184. COMMON LAMDAV,I,J,ALPHA,BETA,L1,L2,A
2185. ARG1=LAMDAV(K)*L1
2186. ARG2=LAMDAV(K)*L2
2187. ARG3=LAMDAV(K)*(L1+L2)
2188. COEF1=UCUS(ALPHA+BETA)+1
2189. COEF2=UCUS(ALPHA+BETA)-1
2190. DENOM=2.000*(DSIN(ARG2)*DSINH(ARG2))
2191. GAMA1N=(UCUS(ALPHA+BETA)*DSIN(ARG1)*(DCOS(ARG2)-DCOSH(ARG2))+
2192. 1DSIN(ARG3)+DSIN(ARG2)*UCUS(ARG1)*DCOSH(ARG2)*DSIN(ARG1))/DENOM
2193. GAMA2N=(UCUS(ALPHA+BETA)*(UCUS(ARG2)-DCOSH(ARG2))*DSINH(ARG1)-
2194. 1(DCOS(ARG2)+DCOSH(ARG2))*DSINH(ARG1)+2.000*DSIN(ARG2)*DCOSH(ARG1)
2195. 2)/DENOM
2196. GAMA4N=(UCUS(ARG1)-GAMA1N)*DSIN(ARG2)+0.500*UCUS(ARG2)*DSIN(ARG1)*
2197. 1COEF1+GAMA1N*DSINH(ARG2)+0.500*DCOSH(ARG2)*DSIN(ARG1)*COEF2+
2198. 2UTAN(BETA)*DSIN(ALPHA+BETA)*DSIN(ARG1)
2199. GAMA5N=(DCOSH(ARG1)-GAMA2N)*DSIN(ARG2)+0.500*DCOS(ARG2)*
2200. 1DSINH(ARG1)*COEF2+DSINH(ARG2)*GAMA4N+0.500*DCOSH(ARG2)*
2201. 2DSINH(ARG1)*COEF1+UTAN(BETA)*DSIN(ALPHA+BETA)*DSINH(ARG1)
2202. GAMA3N=GAMA4N/GAMA5N
2203. C
2204. C COEFFICIENTS OF V2UP
2205. C
2206. DLTAIN=DCOS(ARG1)-GAMA1N-GAMA3N*DCOSH(ARG1)+GAMA2N*GAMA3N
2207. DLTAN=0.500*(COEF1*DSIN(ARG1)-GAMA3N*COEF2*DSINH(ARG1))
2208. DLTAN=GAMA1N-GAMA2N*GAMA3N
2209. DLTAN=0.500*(COEF2*DSIN(ARG1)-GAMA3N*COEF1*DSINH(ARG1))
2210. C
2211. C EXPRESSION FOR V2UP
2212. C
2213. ARGV2=LAMDAV(K)*X
2214. V2UP=(-DLTAIN*DSIN(ARGV2)-DLTAN*DCOS(ARGV2)+DLTAN*DSINH(ARGV2)+
2215. 1DLTAN*DCOSH(ARGV2))*LAMDAV(K)**2)
2216. RETURN
2217. END
2218. DOUBLE PRECISION FUNCTION ASJ(G,A,B,EPS)
2219. C
2220. C *****
2221. C
2222. C TITLE- ADAPTIVE SIMPSON QUADRATURE
2223. C
2224. C PURPOSE- TO EVALUATE A DEFINITE INTEGRAL.
2225. C
2226. C DESCRIPTION OF PARAMETERS
2227. C
2228. C G(X) = THE INTEGRAND OF THE INTEGRAL TO BE EVALUATED.
2229. C A = THE LOWER LIMIT OF INTEGRATION.
2230. C B = THE UPPER LIMIT OF INTEGRATION.

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2231. C EPS = DESIRED ACCURACY OF RESULT.
2232. C
2233. C SUBROUTINES AND/OR FUNCTION SUBPROGRAMS REQUIRED- G(X)
2234. C
2235. C *****
2236. C
2237. C IMPLICIT REAL*8 (A-H,O-Z)
2238. C DIMENSION XC(100),FOXC(100),XR(100),FOXR(100),H02(100),S(100)
2239. C SIMP(F0XL,F0XC,F0XR,H)=H*(F0XL+4.00*F0XC+F0XR)/6.00
2240. C H=U-A
2241. C H2=0.500*H
2242. C E=15.00*EPS/H
2243. C N=0
2244. C ASQ=0.00
2245. C X1=A
2246. C WRITE(6,1)X1,ASQ
2247. C F0X1=G(X1)
2248. C X3=A+0.500*H
2249. C F0X3=G(X3)
2250. C X5=B
2251. C F0X5=G(X5)
2252. C 7 SS=SIMP(F0X1,F0X3,F0X5,H)
2253. C 4 X2=0.500*(X1+X3)
2254. C F0X2=G(X2)
2255. C X4=0.500*(X3+X5)
2256. C F0X4=G(X4)
2257. C SS1=SIMP(F0X1,F0X2,F0X3,H2)
2258. C SS2=SIMP(F0X3,F0X4,F0X5,H2)
2259. C SD=SS1+SS2
2260. C IF (N*EPS*(SD-SS).LE.E*H) GO TO 3
2261. C N=N+1
2262. C XC(N)=X4
2263. C F0XC(N)=F0X4
2264. C XR(N)=X5
2265. C F0XR(N)=F0X5
2266. C H02(N)=H2
2267. C S(N)=SS2
2268. C X5=X3
2269. C F0X5=F0X3
2270. C X3=X2
2271. C F0X3=F0X2
2272. C H=0.500*H
2273. C H2=0.500*H2
2274. C GO TO 7
2275. C 3 ASQ=ASQ+15.00*SD/15.00-SS/15.00
2276. C WRITE(6,1)X5,ASQ
2277. C 1 FORMAT(' ',2F10.5)
2278. C IF ((X5-B)*(B-A).GE.0.00) RETURN
2279. C X1=X5
2280. C F0X1=F0X5
2281. C X3=XC(N)
2282. C F0X3=F0XC(N)
2283. C X5=XR(N)
2284. C F0X5=F0XR(N)
2285. C SS=S(N)
2286. C H=H02(N)
2287. C H2=0.500*H
2288. C N=N-1
2289. C GO TO 4
2290. C END
2291. C ENTRY

```

APPENDIX E: SPATIAL KINEMATICS OF ELASTIC COUPLER

1. ELASTIC DEFORMATION OF ARBITRARY POINT ON BEAM ACCORDING TO EULER-BERNOULLI THEORY

Figure E-1 shows a typical section of an elastic link. Its deformations are assumed to be those given by the simplified Euler-Bernoulli theory. Thus, for a point on the centerline of the link the general elastic deformation is given by:

$$\bar{r}_{ei} = U_i(x_i, t) \bar{i}_i + V_i(x_i, t) \bar{j}_i + W_i(x_i, t) \bar{k}_i \quad (E-1)$$

Where $V_i(x_i, t)^*$ represents the sum of all possible in-plane lateral deflections of the arbitrary point ($i = 1$, for section 1, and $i = 2$ for section 2). Similarly, $U_i(x_i, t)$ and $W_i(x_i, t)$ stand for all possible axial and lateral out-of-plane deflections of such a point, respectively.

a. Arbitrary Point P_1 on First Section of Coupler

Figure E-2 shows a point P_1 on the centerline of the first section of the coupler link. Its elastic displacement vector is given, according to equation E-1, by:

$$\bar{r}_{e1} = U_1(x_1, t) \bar{i}_1 + v_1(x_1, t) \bar{j}_1 + w_1(x_1, t) \bar{k}_1 \quad (E-2)$$

* Note introduction of new notation.

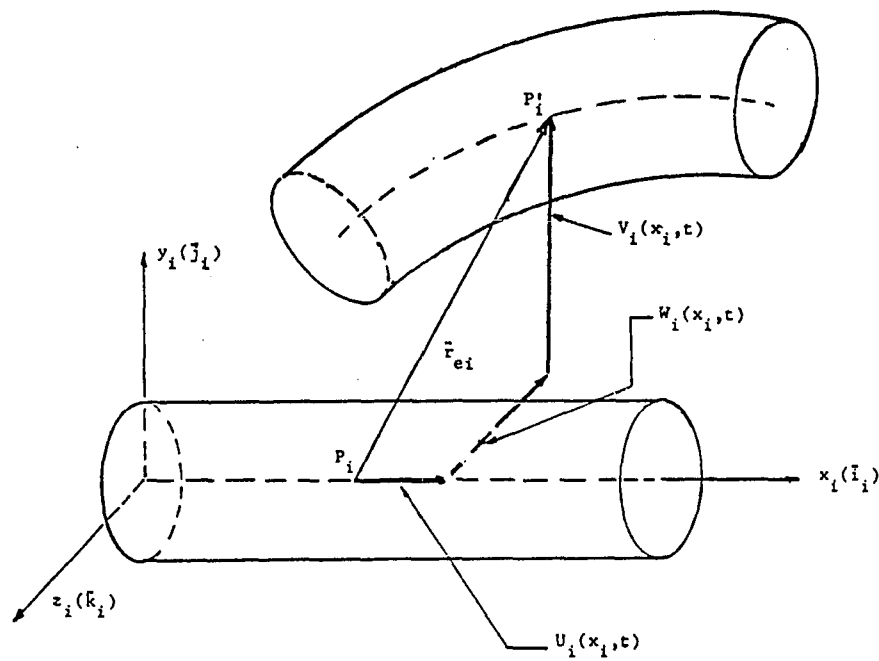


Figure E-1

Displacement of Point on Centerline of Beam

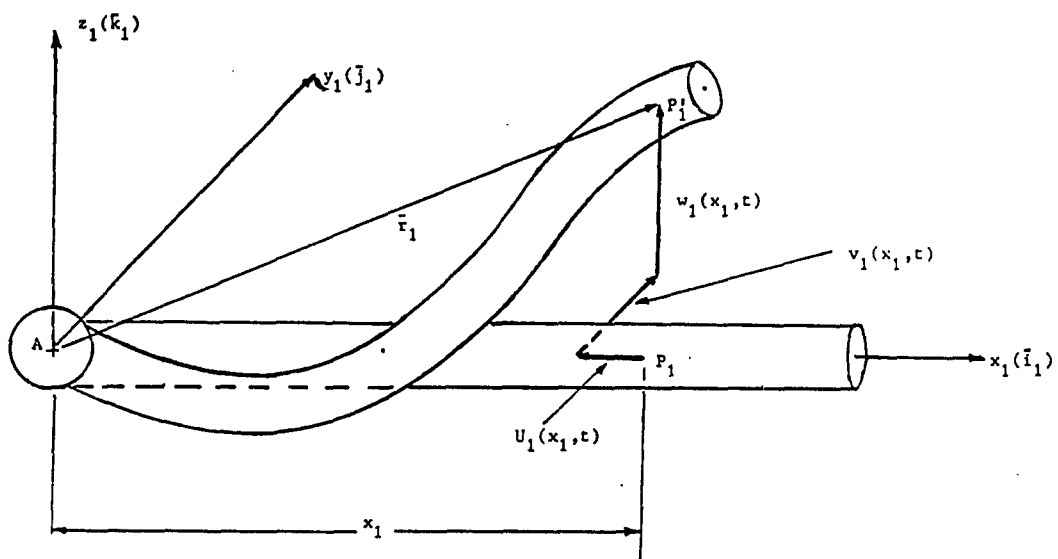


Figure E-2

Displacement of Arbitrary Point P_1 on Section 1 of Coupler

The axial component $U_1(x_1, t)$ is due to the foreshortening of the first section of the beam up to point x_1 , which results from the transverse deflections v_1 and w_1 , i.e.:

$$U_1(x_1, t) = u_1(x_1, t) = -\frac{1}{2} \int_{\xi_1=0}^{x_1} \left[\left[\frac{\partial \tilde{v}_1}{\partial \xi_1} \right]^2 + \left[\frac{\partial \tilde{w}_1}{\partial \xi_1} \right]^2 \right] d\xi_1 \quad (E-3)$$

Where,

$$\tilde{v}_1 = v_1(\xi_1, t)$$

and

$$\tilde{w}_1 = w_1(\xi_1, t) \quad (E-4)$$

b. Arbitrary Point P_2 on Second Section of Coupler

Figure E-3 shows a point P_2 on the centerline of the second section of the coupler link. Its elastic displacement vector is again given, according to equation E-1, by:

$$\bar{r}_{e2} = U_2(x_2, t) \bar{i}_2 + V_2(x_2, t) \bar{j}_2 + w_2(x_2, t) \bar{k}_2 \quad (E-5)$$

The total in-plane transverse deflection $V_2(x_2, t)$ consists of the local lateral deflection $v_2(x_2, t)$ (note lower case) as well as the constant displacement of all points on this section due to the y_2 -component of the foreshortening of section 1. Thus,

$$V_2(x_2, t) = v_2(x_2, t) + \frac{1}{2} \sin(\alpha + \beta) \int_{x_1=0}^{L_1} \left[\left[\frac{\partial v_1(x_1, t)}{\partial x_1} \right]^2 + \left[\frac{\partial w_1(x_1, t)}{\partial x_1} \right]^2 \right] dx_1 \quad (E-6)$$

The total axial deflection $U_2(x_2, t)$ consists of three parts. The first two are due to the x_2 -components both of the lateral deflection as well as the axial foreshortening of the end point of the first section. Both of them are constant throughout section 2. The third part is due to the foreshortening of section 2 itself, and is caused by $v_2(x_2, t)$, $w_2(x_2, t)$ and $\frac{\partial v_1(L_1, t)}{\partial x_1}$. This term is obtained by adapting equation F-23, of appendix F, to any point x_2 .

$$U_2(x_2, t) = \sin(\alpha + \beta) \cdot v_1(L_1, t) - \frac{1}{2} \cos(\alpha + \beta) \int_{x_1=0}^{L_1} \left[\left[\frac{\partial v_1(x_1, t)}{\partial x_1} \right]^2 + \left[\frac{\partial w_1(x_1, t)}{\partial x_1} \right]^2 \right] dx_1 - \frac{1}{2} \int_{\xi_2=0}^{x_2} \left[\left[\frac{\partial \tilde{v}_2}{\partial \xi_2} \right]^2 - \left[\frac{\partial v_1(L_1, t)}{\partial x_1} \right]^2 + \left[\frac{\partial w_2}{\partial \xi_2} \right]^2 \right] \cdot d\xi_2 \quad (E-7)$$

Where,

$$\tilde{v}_2 = v_2(\xi_2, t)$$

and

$$\tilde{w}_2 = w_2(\xi_2, t)$$

(E-8)

2. POSITION VECTORS

The following furnishes expressions for the position vectors of the arbitrary points P_1 and P_2 , on the centerline of sections 1 and 2, respectively. These are given with respect to the origin O_1 of the space fixed coordinate system. They include both rigid body and elastic displacements (refer to appendix I for a discussion on coordinate systems and see figure I-1).

a. Position Vector of Point P_1

The position vector of point P_1 , with respect to O_1 , is given by:

$$\bar{r}_{P1} = \bar{r}_A + \bar{r}_1 \quad (E-9)$$

Where

$$\bar{r}_A = a_1 [\cos\phi_1 \bar{j}_0 + \sin\phi_1 \bar{k}_0] \quad (E-10)$$

and

$$\bar{r}_1 = x_1 \bar{i}_1 + \bar{r}_{e1} \quad (E-11)$$

The above becomes, with equation E-2:

$$\bar{r}_1 = [x_1 + U_1(x_1, t)] \bar{i}_1 + v_1(x_1, t) \bar{j}_1 + w_1(x_1, t) \bar{k}_1 \quad (E-12)$$

Finally, equation E-9 becomes with E-11:

$$\bar{r}_{P1} = \bar{r}_A + [x_1 + U_1(x_1, t)] \bar{i}_1 + v_1(x_1, t) \bar{j}_1 + w_1(x_1, t) \bar{k}_1 \quad (E-13)$$

b. Position Vector of Point P_2

The position vector of point P_2 , with respect to point O_1 , is given by:

$$\bar{r}_{P2} = \bar{r}_A + \bar{r}_2 \quad (E-14)$$

Where,

$$\bar{r}_2 = L_1 \bar{i}_1 + x_2 \bar{i}_2 + \bar{r}_{e2} \quad (E-15)$$

Equation D-15 becomes, with equation E-5:

$$\bar{r}_2 = L_1 \bar{i}_1 + [x_2 + U_2(x_2, t)] \bar{i}_2 + V_2(x_2, t) \bar{j}_2 + w_2(x_2, t) \bar{k}_2 \quad (E-16)$$

Combining equations E-13 and E-16, one obtains:

$$\begin{aligned} \bar{r}_{P2} = \bar{r}_A + [L_1 \cos \gamma + x_2 + U_2(x_2, t)] \bar{i}_2 + [-L_1 \sin \gamma + V_2(x_2, t)] \bar{j}_2 \\ + w_2(x_2, t) \bar{k}_2 \end{aligned} \quad (E-17)$$

3. VELOCITY VECTORS

a. Velocity Vector of Point P_1

The time derivative of equation E-9 has the following form:

$$\dot{\bar{r}}_{P1} = \dot{\bar{r}}_A + \dot{\bar{r}}_1 + \dot{\bar{\phi}}_{3e} \times \bar{r}_1 \quad (E-18)$$

Where,

$\dot{\bar{r}}_1$ = The velocity with respect to the body-fixed coordinate system (x_1, y_1, z_1)

$\dot{\bar{\phi}}_{3e}$ = The absolute angular velocity of the coupler link after elastic deformations

Substitution of equations E-11 and of the adapted forms of equation I-18, to account for the elastic condition, subsequent

differentiation, and applicable coordinate transformation, gives in terms of the x_1 - y_1 - z_1 system:

$$\begin{aligned}\dot{\vec{r}}_{P_1} = & \left[a_1 \dot{\phi}_1 \sin \phi_{32e} \cdot \sin(\phi_1 - \phi_{2e}) + J_2(t) w_1(x_1, t) - J_3(t) \cdot v_1(x_1, t) + \frac{\partial U_1(x_1, t)}{\partial t} \right] \bar{I}_1 \\ & + \left[-a_1 \dot{\phi}_1 \cos(\phi_1 - \phi_{2e}) + J_3(t) \cdot (x_1 + U_1(x_1, t)) - J_1(t) \cdot w_1(x_1, t) + \frac{\partial v_1(x_1, t)}{\partial t} \right] \bar{J}_1 \\ & + \left[-a_1 \dot{\phi}_1 \cos \phi_{32e} \cdot \sin(\phi_1 - \phi_{2e}) + J_1(t) \cdot v_1(x_1, t) - (x_1 + U_1(x_1, t)) \cdot J_2(t) + \frac{\partial w_1(x_1, t)}{\partial t} \right] \bar{K}_1\end{aligned}\quad (E-19)$$

b. Velocity Vector of Point P_2

Appropriate differentiation of equation (E-14) gives:

$$\dot{\vec{r}}_{P2} = \dot{\vec{r}}_A + \dot{\vec{r}}_2 + \dot{\phi}_{3e} \times \vec{r}_2 \quad (E-20)$$

Where,

$\dot{\vec{r}}_2$ = The velocity with respect to the body-fixed coordinate system(x_2, y_2, z_2)

$\dot{\phi}_{3e}$ = The absolute angular velocity of the coupler after elastic deformation

Substitution of equation E-16 and of the adapted forms of equation I-20, to account for the elastic condition, subsequent differentiation, and applicable coordinate transformation results, in terms of the x_2 - y_2 - z_2 system, in:

$$\begin{aligned}\dot{\vec{r}}_{P2} = & \dot{\vec{r}}_A + \left[H_2(t) \cdot w_2(x_2, t) - H_3(t) \cdot (V_2(x_2, t) - L_1 \sin \gamma) + \frac{\partial U_2(x_2, t)}{\partial t} \right] \bar{I}_2 \\ & + \left[H_3(t) \cdot (L_1 \cos \gamma + x_2 + U_2(x_2, t)) - H_1(t) \cdot w_2(x_2, t) + \frac{\partial V_2(x_2, t)}{\partial t} \right] \bar{J}_2 \\ & + \left[H_1(t) \cdot (V_2(x_2, t) - L_1 \sin \gamma) - (L_1 \cos \gamma + x_2 + U_2(x_2, t)) \cdot H_2(t) + \frac{\partial w_2(x_2, t)}{\partial t} \right] \bar{K}_2\end{aligned}\quad (E-21)$$

4. ACCELERATION VECTORS

a. Acceleration Vector of Point P_1

Double differentiation of equation E-9, leads to the following form for $\ddot{\vec{r}}_{P1}$:

$$\ddot{\vec{r}}_{P1} = \ddot{\vec{r}}_A + \ddot{\vec{r}}_1 + \ddot{\vec{\phi}}_{3e} \times \vec{r}_1 + \dot{\vec{\phi}}_{3e} \times (\dot{\vec{\phi}}_{3e} \times \vec{r}_1) + 2\dot{\vec{\phi}}_{3e} \times \dot{\vec{r}}_1 \quad (E-22)$$

Where,

$\ddot{\vec{r}}_1$ = The acceleration with respect to the body-fixed coordinate system (x_1, y_1, z_1)

$\ddot{\vec{\phi}}_{3e}$ = The absolute angular acceleration of the coupler after elastic deformation

Substitution of equation E-11 and of the adapted forms of I-18 and I-26, subsequent differentiation, necessary coordinate transformation and advantageous grouping of terms furnish:

$$\begin{aligned} \ddot{\vec{r}}_{P1} = & \left[R_1(x_1, t) - (I_2^2(t) + I_3^2(t)) \cdot U_1(x_1, t) + (I_1(t) \cdot I_2(t) - G_3(t)) \cdot V_1(x_1, t) + (G_2(t) + I_1(t) \cdot I_3(t)) \cdot W_1(x_1, t) \right. \\ & + 2I_2(t) \frac{\partial W_1(x_1, t)}{\partial t} - 2I_3(t) \frac{\partial V_1(x_1, t)}{\partial t} + \frac{\partial^2 U_1(x_1, t)}{\partial t^2} \Big] \vec{L}_1 + \left[R_2(x_1, t) + (G_3(t) + I_1(t) \cdot I_2(t)) \cdot U_1(x_1, t) \right. \\ & - (I_3^2(t) + I_1^2(t)) \cdot V_1(x_1, t) + (I_2(t) \cdot I_3(t) - G_1(t)) \cdot W_1(x_1, t) + 2I_3(t) \frac{\partial U_1(x_1, t)}{\partial t} - 2I_1(t) \frac{\partial W_1(x_1, t)}{\partial t} + \\ & + \frac{\partial^2 V_1(x_1, t)}{\partial t^2} \Big] \vec{J}_1 + \left[R_3(x_1, t) + (I_1(t) \cdot I_3(t) - G_2(t)) \cdot U_1(x_1, t) + (I_2(t) \cdot I_3(t) + G_1(t)) \cdot V_1(x_1, t) - (I_1^2(t) + \right. \\ & \left. + I_2^2(t)) \cdot W_1(x_1, t) - 2I_2(t) \frac{\partial U_1(x_1, t)}{\partial t} + 2I_1(t) \frac{\partial V_1(x_1, t)}{\partial t} + \frac{\partial^2 W_1(x_1, t)}{\partial t^2} \Big] \vec{K}_1 \end{aligned} \quad (E-23)$$

Where:

$$R_1(x_1, t) = a_1 \dot{\phi}_1^2 \sin \phi_{32e} \cos(\phi_1 - \phi_{2e}) - [J_2^2(t) + J_3^2(t)] x_1 \quad (E-24)$$

$$R_2(x_1, t) = a_1 \dot{\phi}_1^2 \sin(\phi_1 - \phi_{2e}) + [G_3(t) + J_1(t) * J_2(t)] x_1 \quad (E-25)$$

$$R_3(x_1, t) = -a_1 \dot{\phi}_1^2 \cos \phi_{32e} \cos(\phi_1 - \phi_{2e}) + [J_1(t) * J_3(t) - G_2(t)] \quad (E-26)$$

b. Acceleration Vector of Point P₂

Double differentiation of equation E-14, leads to the following form for $\ddot{\mathbf{r}}_{P2}$:

$$\ddot{\mathbf{r}}_{P2} = \ddot{\mathbf{r}}_A + \ddot{\mathbf{r}}_2 + \ddot{\phi}_{3e} \times \bar{\mathbf{r}}_2 + \dot{\phi}_{3e} \times (\dot{\phi}_{3e} \times \bar{\mathbf{r}}_2) + 2\dot{\phi}_{3e} \times \dot{\bar{\mathbf{r}}}_2 \quad (E-27)$$

Where,

$\ddot{\mathbf{r}}_2$ = The acceleration with respect to the body fixed coordinate system (x_2, y_2, z_2)

$\ddot{\phi}_{3e}$ = The absolute angular acceleration of the coupler after elastic deformations

Substitution of equation E-14 and of the adapted form of I-26 and I-20, subsequent differentiation, necessary coordinate transformation and advantageous grouping lead:

$$\begin{aligned}
\ddot{\bar{r}}_{P_2} = & \left[R_4(x_2, t) - (H_2^2(t) + H_3^2(t)) \cdot U_2(x_2, t) + (H_1(t) \cdot H_2(t) - I_3(t)) V_2(x_2, t) + (H_1(t) H_3(t) + I_2(t)) W_2(x_2, t) \right. \\
& - 2H_3(t) \frac{\partial V_2(x_2, t)}{\partial t} + 2H_2(t) \frac{\partial W_2(x_2, t)}{\partial t} + \frac{\partial^2 U_2(x_2, t)}{\partial t^2} \left. \right] \bar{L}_2 + \left[R_5(x_2, t) + (I_3(t) + H_1(t) H_2(t)) U_2(x_2, t) \right. \\
& - (H_3^2(t) + H_1^2(t)) V_2(x_2, t) + (H_1(t) H_3(t) - I_1(t)) W_2(x_2, t) + 2H_3(t) \frac{\partial U_2(x_2, t)}{\partial t} - 2H_1(t) \frac{\partial W_2(x_2, t)}{\partial t} \\
& + \frac{\partial^2 V_2(x_2, t)}{\partial t^2} \left. \right] \bar{J}_2 + \left[R_6(x_2, t) + (H_1(t) H_3(t) - I_2(t)) U_2(x_2, t) + (H_2(t) H_3(t) + I_1(t)) V_2(x_2, t) \right. \\
& - (H_1^2(t) + H_2^2(t)) W_2(x_2, t) + 2H_1(t) \frac{\partial V_2(x_2, t)}{\partial t} - 2H_2(t) \frac{\partial U_2(x_2, t)}{\partial t} + \frac{\partial^2 W_2(x_2, t)}{\partial t^2} \left. \right] \bar{K}_2 \quad (E-28)
\end{aligned}$$

Where,

$$\begin{aligned}
R_4(x_2, t) = & a_1 \dot{\phi}_1^2 [\sin \gamma \sin(\phi_1 - \phi_{2e}) + \cos \gamma \sin \phi_{32e} \cos(\phi_1 - \phi_{2e})] \\
& + [I_3(t) - H_1(t) * H_2(t)] * L_1 \sin \gamma - [H_2^2(t) + H_3^2(t)] * [L_1 \cos \gamma + x_2] \quad (E-29)
\end{aligned}$$

$$\begin{aligned}
R_5(x_2, t) = & a_1 \dot{\phi}_1^2 [\cos \gamma \sin(\phi_1 - \phi_{2e}) - \sin \gamma \sin \phi_{32e} \cos(\phi_1 - \phi_{2e})] + [I_3(t) \\
& + H_1(t) * H_2(t)] * [L_1 \cos \gamma + x_2] + [H_3^2(t) + H_1^2(t)] * L_1 \sin \gamma \quad (E-30)
\end{aligned}$$

$$\begin{aligned}
R_6(x_2, t) = & - a_1 \dot{\phi}_1^2 [\cos \phi_{32e} \cos(\phi_1 - \phi_{2e})] - [I_1(t) + H_2(t) * H_3(t)] * L_1 \sin \gamma \\
& + [H_1(t) * H_3(t) - I_2(t)] * [L_1 \cos \gamma + x_2] \quad (E-31)
\end{aligned}$$

APPENDIX F: DETERMINATION OF VARIATIONS OF ELASTIC MECHANISM ANGLES
IN TERMS OF CHANGES IN COUPLER LENGTH AC

1. ELASTIC CONSTRAINT EQUATIONS

When the coupler AC is considered elastic, its deformed length AC' must be substituted into the loop equation I-2, of appendix I. In addition, the elastic angles ϕ_{2e} , ϕ_{32e} and ϕ_{4e} must be accounted for. Equations I-6 to I-8 then take the following form:

$$s_1 + AC' \cos \alpha \cos \phi_{32e} - a_4 \sin \phi_{4e} = 0 \quad (F-1)$$

$$a_1 \cos \phi_1 + AC' [\sin \alpha \sin \phi_{2e} - \cos \alpha \cos \phi_{2e} \sin \phi_{32e}] + s_2 = 0 \quad (F-2)$$

$$a_1 \cos \phi_1 - AC' [\cos \alpha \sin \phi_{2e} \sin \phi_{32e} + \sin \alpha \cos \phi_{2e}] - a_4 \cos \phi_{4e} + s_3 = 0 \quad (F-3)$$

2. LENGTH AC' OF ELASTICALLY DEFORMED COUPLER

According to the assumptions of the subproblem, which does not include any second order terms, line AC' must coincide with line AC, i.e. any displacement of point C is along the x-axis.

If foreshortening of the individual sections is recognized, AC' does not coincide with line AC, since there will be a component of displacement normal to the line AC, i.e. in the direction of the

y-axis. It can be shown that this displacement represents the difference between two positive second order terms, and is considerably smaller than a sum of similarly positive terms in the x-component of AC' .

All elastic deformations are expressed in the manner of the subproblem (see appendix J) in terms of the in-plane variables $v_1(x_1, t)$, and $v_2(x_2, t)$, as well as the out-of-plane variables $w_1(x_1, t)$ and $w_2(x_2, t)$. As in the subproblem, the reference plane is defined by the undeflected link ABC.

Figure F-1 shows that the vector $\overline{AC'}$ is made up of the following components:

$$\overline{AC'} = \overline{AB} + \overline{BB'} + \overline{B'C_2} + \overline{C_2D} + \overline{DC'} \quad (F-4)$$

where now,

$$\overline{AB} = L_1 \overline{i_1} \quad , \text{ the undeformed length AB}$$

$$\overline{BB'} = v_1(L_1, t) \overline{j_1} + u_1(L_1, t) \overline{i_1} \quad , \text{ with:}$$

$$v_1(L_1, t) = \text{the in-plane elastic deflection at point B}$$

and

$$u_1(L_1, t) = -\frac{1}{2} \int_{x_1=0}^{L_1} \left\{ \left[\frac{\partial v_1(x_1, t)}{\partial x_1} \right]^2 + \left[\frac{\partial w_1(x_1, t)}{\partial x_1} \right]^2 \right\} dx_1 \quad (F-4b),$$

which is a second order term, and represents the foreshortening of section due to both in-plane and

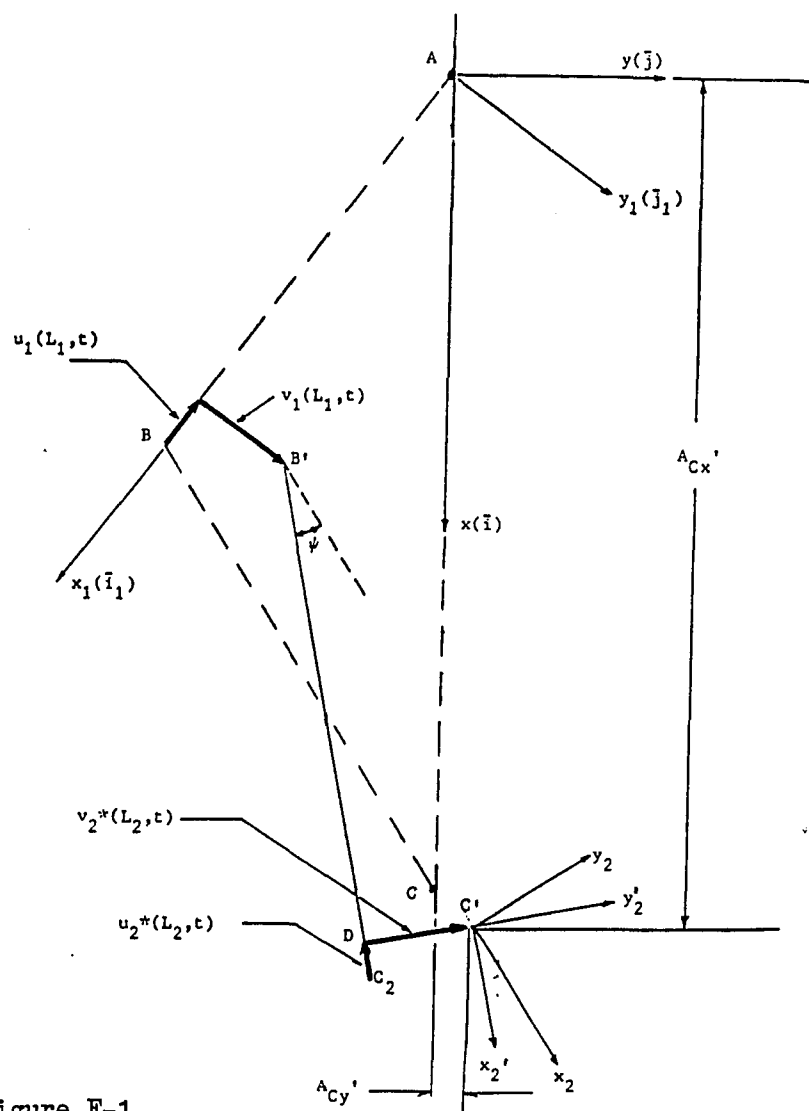


Figure F-1

Determination of AC'

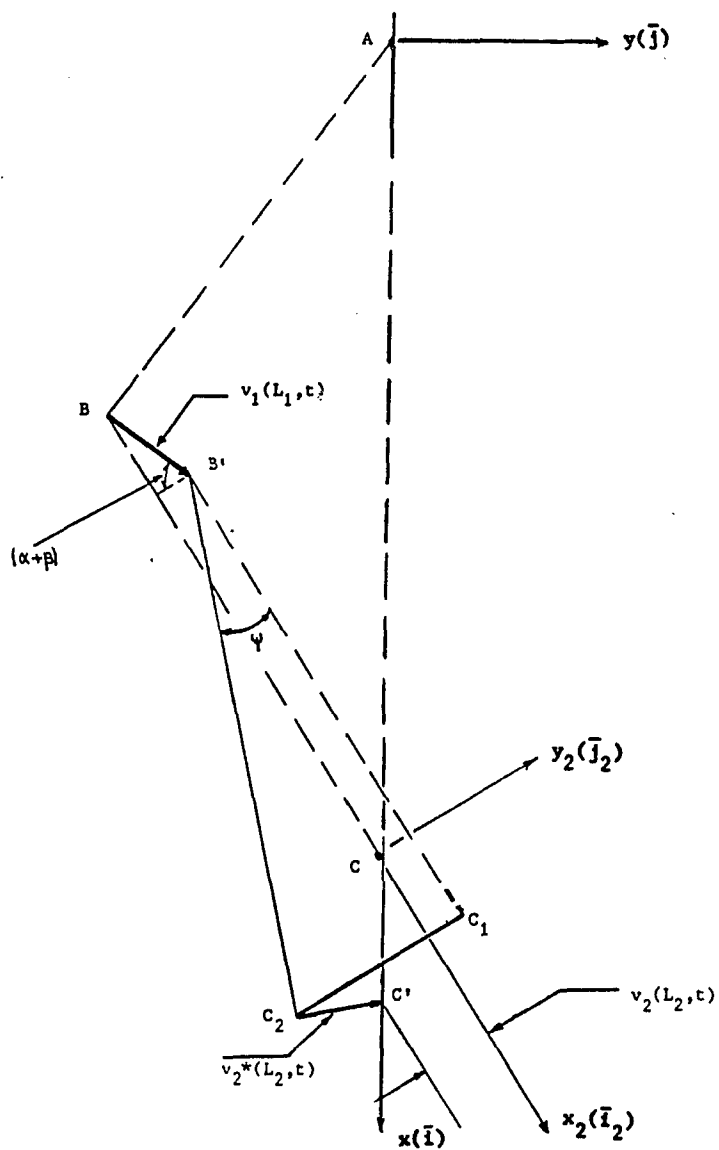


Figure F-2

Determination of Relationship between $v_2(L_2, t)$ and $v_2^*(L_2, t)$

out-of-plane elastic bending.

$\overline{B'C_2} = L_2 \overline{i_2'}$, due to in-plane rigid body rotation of section BC through the angle $\psi = \tan^{-1} \left[\frac{\partial v_1(L_1, t)}{\partial x_1} \right]$

$\overline{DC'} = v_2^*(L_2, t) \overline{j_2'}$, the elastic deflection of section $B'C_2 = BC$

$\overline{C_2D} = u_2^*(L_2, t) \overline{i_2'}$, the foreshortening of section $B'C_2$ along the x_2' axis due to both in-plane and out-of-plane elastic bending.

where

$$u_2^*(L_2, t) = -\frac{1}{2} \int_{x_2'=0}^{L_2} \left\{ \left[\frac{\partial v_2^*(x_2', t)}{\partial x_2'} \right]^2 + \left[\frac{\partial w_2^*(x_2', t)}{\partial x_2'} \right]^2 \right\} dx_2' \quad (F-5)$$

In order to find the form of u_2^* , the length $v_2^*(L_2, t)$ must first be determined with the help of the total deflection $v_2(L_2, t)$ of point C, normal to the x_2 -axis.

Figure F-2 shows the relationship between these two lateral displacements. Then,

$$v_2(L_2, t) = v_1(L_1, t) \cos(\alpha + \beta) + L_2 \sin \psi + v_2^*(L_2, t) \cos \psi, \quad (F-6)$$

or,

$$v_2^*(L_2, t) = v_2(L_2, t) \sec \psi - v_1(L_1, t) \cos(\alpha + \beta) \sec \psi - L_2 \tan \psi \quad (F-7)$$

To evaluate the foreshortening u_2^* , according to equation F-5, equation F-7 must be adapted to an arbitrary position along the x_2' -axis, to furnish the first term of equation F-5, i.e.

$$\frac{\partial v_2^*(x_2', t)}{\partial x_2'} = \frac{\partial}{\partial x_2'} \left[v_2(x_2, t) \sec \psi - v_1(l_1, t) \cos(\alpha + \beta) \sec \psi - x_2 \tan \psi \right]$$

Further,

$$\frac{\partial w_2^*(x_2', t)}{\partial x_2'} = \frac{\partial w_2(x_2, t)}{\partial x_2} \quad , \text{ because the out-of-plane deflection is not influenced by the in-plane slope.}$$

Equation F-5 then becomes:

$$u_2^*(l_2, t) = - \frac{1}{2} \int_{x_2'=0}^{l_2} \left\{ \left[\frac{\partial}{\partial x_2'} \left[v_2(x_2, t) \sec \psi - v_1(l_1, t) \cos(\alpha + \beta) \sec \psi - x_2 \tan \psi \right] \right]^2 + \left[\frac{\partial}{\partial x_2'} [w_2(x_2, t)] \right]^2 \right\} dx_2'$$

To be able to integrate the above, the chain rule is first used to change the variables:

$$u_2^*(l_2, t) = - \frac{1}{2} \int_{x_2'=0}^{l_2} \left\{ \left[\frac{\partial}{\partial x_2} \left[v_2(x_2, t) \sec \psi - v_1(l_1, t) \cos(\alpha + \beta) \sec \psi - x_2 \tan \psi \right] \frac{\partial x_2}{\partial x_2'} \right]^2 + \left[\frac{\partial}{\partial x_2} [w_2(x_2, t)] \frac{\partial x_2}{\partial x_2'} \right]^2 \right\} dx_2'$$

When the differentiation is carried out, one obtains:

$$u_2^*(L_2, t) = -\frac{1}{2} \int_{x_2'=0}^{L_2} \left\{ \left[\left[\frac{\partial v_2(x_2, t)}{\partial x_2} \sec \psi - \tan \psi \right] \frac{\partial x_2}{\partial x_2'} \right]^2 + \left[\frac{\partial w_2(x_2, t)}{\partial x_2} \frac{\partial x_2}{\partial x_2'} \right]^2 \right\} dx_2'$$

Further, since

$$\left. \begin{aligned} x_2 &= x_2' \cos \psi \\ \frac{dx_2}{dx_2'} &= \cos \psi \end{aligned} \right\} \quad \text{one finds:} \quad (F-8)$$

Equation F-8 is now used to replace the limits of integration, i.e.

$$u_2^*(L_2, t) = -\frac{1}{2} \int_{x_2=0}^{L_2 \cos \psi} \left\{ \left[\left[\frac{\partial v_2(x_2, t)}{\partial x_2} \sec \psi - \tan \psi \right]^2 + \left[\frac{\partial w_2(x_2, t)}{\partial x_2} \right]^2 \right\} \cos \psi dx_2 \quad (F-9)$$

Finally, equations F-6 through F-9 are substituted into equation F-4, and one finds:

$$\begin{aligned} \overline{AC'} &= L_1 \bar{i}_1 + u_1(L_1, t) \bar{i}_1 + v_1(L_1, t) \bar{j}_1 + L_2 \bar{i}_2 + u_2^*(L_2, t) \bar{i}_2 \\ &\quad + [v_2(L_2, t) \sec \psi - v_1(L_1, t) \cos(\alpha + \beta) \sec \psi - L_1 \tan \psi] \bar{j}_2 \end{aligned} \quad (F-10)$$

The above expression is now transformed into x and y components of the original reference plane, so that,

$$\overline{AC'} = AC_x' \bar{i} + AC_y' \bar{j} \quad (F-11)$$

To this end, the transformation tables F-1a to F-1c are used.

	$\bar{i}_2$	$\bar{j}_2$
$\overline{i_2'}$	$\cos\psi$	$\sin\psi$
$\overline{j_2'}$	$-\sin\psi$	$\cos\psi$

(a)

	$\bar{i}_1$	$\bar{j}_1$
i_2	$\cos(\alpha+\beta)$	$\sin(\alpha+\beta)$
j_2	$-\sin(\alpha+\beta)$	$\cos(\alpha+\beta)$

(b)

	$\bar{i}$	$\bar{j}$
$\bar{i}_1$	$\cos\alpha$	$-\sin\alpha$
$\bar{j}_1$	$\sin\alpha$	$\cos\alpha$

(c)

Table F-1: Coordinate Transformation Tables

The components of F-11 then become:

$$\begin{aligned}
 AC_x^i &= L_1 \cos\alpha + u_1(L_1, t) \cos\alpha + v_1(L_1, t) \sin\alpha + L_2 \cos(\beta+\psi) \\
 &\quad + u_2^*(L_2, t) \cos(\beta+\psi) + v_1(L_1, t) \cos(\alpha+\beta) \cdot \sin(\beta+\psi) \cdot \sec\psi \\
 &\quad - v_2(L_2, t) \sin(\beta+\psi) \sec\psi + L_2 \sin(\beta+\psi) \cdot \tan\psi
 \end{aligned}
 \tag{F-12}$$

$$\begin{aligned}
AC'_y = & -L_1 \sin \alpha - u_1(L_1, t) \cdot \sin \alpha + v_1(L_1, t) \cos \alpha + L_2 \sin(\beta + \psi) \\
& + u_2^*(L_2, t) \sin(\beta + \psi) + v_2(L_2, t) \cos(\beta + \psi) \cdot \sec \psi \\
& - v_1(L_1, t) \cos(\alpha + \beta) \cdot \sec \psi \cos(\beta + \psi) - L_2 \cos(\beta + \psi) \tan \psi
\end{aligned} \tag{F-13}$$

With the above, the length of AC' is given by:

$$AC' = [(AC'_x)^2 + (AC'_y)^2]^{1/2} \tag{F-14}$$

Because of the square root, the above is in a very cumbersome form for the eventual substitution into Hamilton's integral. Fortunately, it can be shown (see below) that the x-component of CC' , the change in length of AC , is much larger than the corresponding $CC'_y = AC'_y$, and therefore equation F-14 may safely be approximated by:

$$AC' = AC'_x \tag{F-15}$$

To start with, let the following geometric boundary condition, as given by equation J-21, be substituted into equation F-13, for AC'_y .

$$v_2(L_2, t) = -v_1(L_1, t) \tan \beta \sin(\alpha + \beta) \tag{F-16}$$

This furnishes:

$$\begin{aligned}
AC'_y = & -L_1 \sin \alpha - u_1(L_1, t) \cdot \sin \alpha + L_2 \cdot \sin(\beta + \psi) - L_2 \cos(\beta + \psi) \cdot \tan \psi \\
& + u_2^*(L_2, t) \sin(\beta + \psi) + v_1(L_1, t) [\cos \alpha - \cos(\alpha + \beta) \cdot \cos(\beta + \psi) \cdot \sec \psi \\
& - \sin(\alpha + \beta) \cos(\beta + \psi) \cdot \tan \beta \cdot \sec \psi]
\end{aligned} \tag{F-17}$$

Using trigonometric identities, the coefficient of $v_1(L_1, t)$ in the above expression is rewritten:

$$\begin{aligned} AC'_y &= -L_1 \sin \alpha - u_1(L_1, t) \sin \alpha + L_2 [\sin(\beta + \psi) - \tan \psi \cdot \cos(\beta + \psi)] \\ &\quad + u_2^*(L_2, t) \sin(\beta + \psi) + v_1(L_1, t) \cos \alpha \cdot \tan \psi \cdot \tan \psi \end{aligned} \quad (F-18)$$

The coefficient of L_2 , in the above, is simplified by way of:

$$\begin{aligned} \tan \psi &= \frac{v_1(L_1, t)}{x_1} \approx \sin \psi \\ \cos \psi &\approx 1 \approx \sec \psi \\ L_1 \sin \alpha &= L_2 \sin \beta \end{aligned} \quad (F-19)$$

This results in:

$$\begin{aligned} AC'_y &= -u_1(L_1, t) \sin \alpha + u_2^*(L_2, t) \sin \beta + v_1(L_1, t) \frac{\partial v_1(L_1, t)}{\partial x_1} \cos \alpha \cdot \tan \psi \\ &\quad + L_2 \sin \beta \left[\frac{\partial v_1(L_1, t)}{\partial x_1} \right]^2 \end{aligned} \quad (F-20)$$

Now, $u_2^*(L_2, t)$ of equation F-9, is modified with the help of equations F-19, i.e.

$$u_2^*(L_2, t) = -\frac{1}{2} \int_{x_2=0}^{L_2} \left\{ \left[\frac{\partial v_2(x_2, t)}{\partial x_2} - \frac{\partial v_1(L_1, t)}{\partial x_1} \right]^2 + \left[\frac{\partial w_2(x_2, t)}{\partial x_2} \right]^2 \right\} dx_2$$

Expansion of the term in the parenthesis and subsequent integration of the resulting expressions leads to:

$$\begin{aligned} u_2^*(L_2, t) &= -\frac{1}{2} \int_{x_2=0}^{L_2} \left\{ \left[\frac{\partial v_2(x_2, t)}{\partial x_2} \right]^2 + \left[\frac{\partial w_2(x_2, t)}{\partial x_2} \right]^2 \right\} dx_2 \\ &\quad + \frac{\partial v_1(L_1, t)}{\partial x_1} [v_2(L_2, t) - v_2(0, t)] - \frac{1}{2} \left[\frac{\partial v_1(L_1, t)}{\partial x_1} \right]^2 \cdot L_2 \end{aligned}$$

If one further uses the relationship between $v_2(L_2, t)$ and $v_1(L_1, t)$ as well as between $v_2(0, t)$ and $v_1(L_1, t)$, as given by equations J-18 and J-21 in appendix J, one obtains:

$$u_2^*(L_2, t) = -\frac{1}{2} \int_{x_2=0}^{L_2} \left\{ \left[\frac{\partial v_2(x_2, t)}{\partial x_2} \right]^2 + \left[\frac{\partial w_2(x_2, t)}{\partial x_2} \right]^2 \right\} dx_2 \\ - \frac{\cos \alpha}{\cos \beta} \cdot \frac{\partial v_1(L_1, t)}{\partial x_1} \cdot v_1(L_1, t) - \frac{1}{2} \left[\frac{\partial v_1(L_1, t)}{\partial x_1} \right]^2 L_2 \quad (F-21)$$

Substitution of the above into F-20 and subsequent rearrangement and cancelation of terms, gives:

$$AC_y' = CC_y' = -\sin \alpha u_1(L_1, t) + \sin \beta u_2(L_2, t) \quad (F-22)$$

where according to equation F-4b:

$$u_1(L_1, t) = -\frac{1}{2} \int_{x_1=0}^{L_1} \left\{ \left[\frac{\partial v_1(x_1, t)}{\partial x_1} \right]^2 + \left[\frac{\partial w_1(x_1, t)}{\partial x_1} \right]^2 \right\} dx_1$$

and

$$u_2(L_2, t) = -\frac{1}{2} \int_{x_2=0}^{L_2} \left\{ \left[\frac{\partial v_2(x_2, t)}{\partial x_2} \right]^2 - \left[\frac{\partial v_1(L_1, t)}{\partial x_1} \right]^2 + \left[\frac{\partial w_2(x_2, t)}{\partial x_2} \right]^2 \right\} dx_2 \quad (F-23)$$

Note that equation F-22 represents the difference between two second order infinitesimals.

Equation F-12, for AC_x' , will now be rearranged such that the terms which represent CC_x' will appear in the same form as CC_y' , in equation F-22. Thus, when equations F-16, F-19 and F-21 are used and all resulting third order terms are dropped, then:

$$\begin{aligned}
AC'_x = & L_1 \cos \alpha + L_2 \cos \beta + u_1(L_1, t) \cos \alpha - \frac{1}{2} \int_{x_2=0}^{L_2} \left\{ \left[\frac{\partial v_2(x_2, t)}{\partial x_2} \right]^2 + \right. \\
& + \left. \left[\frac{\partial w_2(x_2, t)}{\partial x_2} \right]^2 \right\} dx_2 + v_1(L_1, t) \left[\sin \alpha + \cos(\alpha + \beta) \sin \beta \right. \\
& + \left. \sin \beta \tan \beta \sin(\alpha + \beta) \right] + v_1(L_1, t) \frac{\partial v_1(L_1, t)}{\partial x_1} \left[\cos(\alpha + \beta) \cos \beta \right. \\
& + \left. \sin(\alpha + \beta) \cos \beta \tan \beta \right] + \frac{1}{2} \cos \beta L_2 \left[\frac{\partial v_1(L_1, t)}{\partial x_1} \right]^2 \\
& + \cos \beta \frac{\partial v_1(L_1, t)}{\partial x_1} \int_{x_2=0}^{L_2} \frac{\partial v_2(x_2, t)}{\partial x_2} dx_2
\end{aligned} \tag{F-25}$$

Simplification of the coefficients of $v_1(L_1, t)$ and $v_1(L_1, t) \frac{\partial v_1(L_1, t)}{\partial t}$

by the use of the appropriate trigonometric identities, along with:

$$a_3 = L_1 \cos \alpha + L_2 \cos \beta$$

and equations F-23 to F-24, leads to the final form for AC' :

$$AC'_x = a_3 + CC'_x$$

where:

$$CC'_x = \cos u_1(L_1, t) + \cos u_2(L_2, t) + \sec \beta \sin(\alpha + \beta) v_1(L_1, t) \tag{F-26}$$

Comparison of F-22 with F-26, shows that the magnitude of the difference of the second order infinitesimals in CC'_y , which is small in itself, must be smaller than that made by the sum of these infinitesimals in F-26. In addition F-26 contains a first order term given by $\sec \sin(\alpha + \beta) v_1(L_1, t)$. With the above, the dropping of AC'_y is justified.

3. VARIATION OF LENGTH AC'

Prior to the determination of the variations of the elastic constraint equations, it is necessary to have an expression for the variation of the coupler length AC'. This involves the generalized coordinates $v_1(x_1, t)$, $v_2(x_2, t)$, $w_1(x_1, t)$ and $w_2(x_2, t)$. Therefore, the variation of equation F-26 furnishes:

$$AC' = \cos\alpha \delta u_1(L_1, t) + \cos\beta \delta u_2(L_2, t) + \sec\beta \sin(\alpha+\beta) \delta v_1(L_1, t) \quad (F-27)$$

$\delta u_1(L_1, t)$ and $\delta u_2(L_2, t)$ must also be expressed in terms of the generalized variables. Thus, with $u_1(L_1, t)$ defined by equation F-4b, its variation becomes:

$$\begin{aligned} \delta u_1(L_1, t) = & - \left[\frac{\partial v_1(x_1, t)}{\partial x_1} \delta v_1(x_1, t) + \frac{\partial w_1(x_1, t)}{\partial x_1} \delta w_1(x_1, t) \right]_{x_1=0}^{L_1} \\ & + \int_0^{L_1} \left[\frac{\partial^2 v_1(x_1, t)}{\partial x_1^2} \delta v_1(x_1, t) + \frac{\partial^2 w_1(x_1, t)}{\partial x_1^2} \delta w_1(x_1, t) \right] \cdot dx_1 \end{aligned} \quad (F-28)$$

Further, with $u_2(L_2, t)$ given by equation F-23, one obtains:

$$\begin{aligned} \delta u_2(L_2, t) = & - \left[\frac{\partial v_2(x_2, t)}{\partial x_2} \delta v_2(x_2, t) + \frac{\partial w_2(x_2, t)}{\partial x_2} \delta w_2(x_2, t) \right]_{x_2=0}^{L_2} \\ & + \frac{\partial v_1(L_1, t)}{\partial x_1} \delta \left[\frac{\partial v_1(L_1, t)}{\partial x_1} \right] \cdot L_2 + \int_0^{L_2} \left[\frac{\partial^2 v_2(x_2, t)}{\partial x_2^2} \cdot \right. \\ & \left. \cdot \delta v_2(x_2, t) + \frac{\partial^2 w_2(x_2, t)}{\partial x_2^2} \cdot \delta w_2(x_2, t) \right] \cdot dx_2 \end{aligned} \quad (F-29)$$

4. VARIATION OF ELASTIC CONSTRAINT EQUATIONS AND DETERMINATION OF VARIATIONS OF ELASTIC ANGLES

To obtain the variations of the three elastic angles, $\delta\varphi_{32e}$, $\delta\varphi_{2e}$, and $\delta\varphi_{4e}$ in terms of the aforementioned generalized coordinates, it is necessary to first obtain the variations of the three elastic loop equation components F-1 to F-3. Subsequently, these three expressions are solved for the desired variations of the elastic angles.

Variation of equations F-1 to F-3 furnishes the following set of expressions:

$$AC' \cos\alpha \sin\varphi_{32e} \delta\varphi_{32e} + a_4 \cos\varphi_{4e} \delta\varphi_{4e} = \cos\lambda \cos\varphi_{32e} \delta AC' \quad (F-30)$$

$$AC' [(\sin\alpha \cos\varphi_{2e} + \sin\varphi_{2e} \cos\alpha \sin\varphi_{32e}) \delta\varphi_{2e} - \cos\alpha \cos\varphi_{2e} \cos\varphi_{32e} \delta\varphi_{32e}] \\ = [\cos\alpha \cos\varphi_{2e} \sin\varphi_{32e} - \sin\alpha \sin\varphi_{2e}] \delta AC' \quad (F-31)$$

$$a_4 \sin\varphi_{4e} \delta\varphi_{4e} + AC' [(\sin\alpha \sin\varphi_{2e} - \cos\alpha \cos\varphi_{2e} \sin\varphi_{32e}) \delta\varphi_{2e} - \cos\alpha \sin\varphi_{2e} \cos\varphi_{32e} \delta\varphi_{32e}] \\ = [\cos\alpha \sin\varphi_{2e} \sin\varphi_{32e} + \sin\alpha \cos\varphi_{2e}] \delta AC' \quad (F-32)$$

Simultaneous solution of the above for $\delta\varphi_{4e}$, $\delta\varphi_{32e}$, and $\delta\varphi_{2e}$, results in:

$$\delta\varphi_{4e} = \left[\frac{-2a_3}{[A \cos\varphi_{4e} - B \sin\varphi_{4e}]} \right] \delta AC' \quad (F-33)$$

$$\delta\varphi_{32e} = \left[\frac{\cos\alpha \cos\varphi_{32e} - a_4 \cos\varphi_{4e} \frac{\Delta\Phi}{4e}}{AC' \cos\alpha \sin\varphi_{32e}} \right] \delta AC' \quad (F-34)$$

$$2e = \left[\frac{AC' \cos \lambda \cos \varphi_{2e} \cos \varphi_{32e} \Delta \Phi_{32e} + \cos \lambda \cos \varphi_{2e} \sin \varphi_{32e} \Delta \Phi_{32e}}{AC' [\sin \lambda \cos \varphi_{2e} + \sin \varphi_{2e} \cos \lambda \sin \varphi_{32e}]} - \frac{\sin \lambda \sin \varphi_{2e} \Delta \Phi_{2e}}{AC' [\sin \lambda \cos \varphi_{2e} + \sin \varphi_{2e} \cos \lambda \sin \varphi_{32e}]} \right] \delta_{AC'} \quad (F-35)$$

Equations F-33 to F-35 are now rewritten in the following manner:

$$\delta \varphi_{4e} = \Delta \Phi_{4e} \delta_{AC'} \quad (F-36)$$

$$\delta \varphi_{32e} = \Delta \Phi_{32e} \delta_{AC'} \quad (F-37)$$

$$\delta \varphi_{2e} = \Delta \Phi_{2e} \delta_{AC'} \quad (F-38)$$

Where AC' is given by equation F-27 and

$$\Delta \Phi_{4e} = \frac{-2a_3}{A \cos \varphi_{4e} - B \sin \varphi_{4e}} \quad (F-39)$$

$$\Delta \Phi_{32e} = \frac{\cos \lambda \cos \varphi_{32e} - a_4 \cos \varphi_{4e} \Delta \Phi_{4e}}{AC' \cos \lambda \sin \varphi_{32e}} \quad (F-40)$$

$$\Delta \Phi_{2e} = \left[\frac{AC' \cos \lambda \cos \varphi_{2e} \cos \varphi_{32e} \Delta \Phi_{32e} + \cos \lambda \cos \varphi_{2e} \sin \varphi_{32e} \Delta \Phi_{32e}}{AC' [\sin \lambda \cos \varphi_{2e} + \sin \varphi_{2e} \cos \lambda \sin \varphi_{32e}]} - \frac{\sin \lambda \sin \varphi_{2e} \Delta \Phi_{2e}}{AC' [\sin \lambda \cos \varphi_{2e} + \sin \varphi_{2e} \cos \lambda \sin \varphi_{32e}]} \right] \quad (F-41)$$

APPENDIX G: VIRTUAL DISPLACEMENTS OF ARBITRARY COUPLER POINTS P_1, P_2

1. VIRTUAL DISPLACEMENT OF POINT P_1 ON SECTION AB OF COUPLER

The virtual displacement of point P_1 is obtained by taking the variation of equation E-9, i.e.,

$$\begin{aligned} \delta \bar{r}_{P_1} = & \delta \bar{r}_A + \frac{\partial \bar{r}}{\partial U_1(x_1, t)} \delta U_1(x_1, t) + \frac{\partial \bar{r}}{\partial v_1(x_1, t)} \delta v_1(x_1, t) + \frac{\partial \bar{r}}{\partial w_1(x_1, t)} \delta w_1(x_1, t) \\ & + \delta \bar{\phi}_{3e} \times \bar{r}_1 \end{aligned} \quad (G-1)$$

where,

$\delta \bar{\phi}_{3e}$ represents the virtual rotation of the body fixed coordinate system (x_1, y_1, z_1) .

$\delta \bar{r}_A$ is equal to zero since $\dot{\phi}_1$ is prescribed.

Appropriate substitutions into G-1 using equations E-12 and H-5 furnish:

$$\begin{aligned} \delta \bar{r}_{P_1} = & [\delta U_1(x_1, t) + \Delta_2(t)w_1(x_1, t) - \Delta_3(t)v_1(x_1, t)] \bar{i}_1 \\ & + [\delta v_1(x_1, t) + \Delta_3(t)[x_1 + U_1(x_1, t)] - \Delta_1(t)w_1(x_1, t)] \bar{j}_1 \\ & + [\delta w_1(x_1, t) + \Delta_1(t)v_1(x_1, t) - \Delta_2(t)[x_1 + U_1(x_1, t)]] \bar{k}_1 \end{aligned} \quad (G-2)$$

where:

$$\left. \begin{aligned} \Delta_1(t) &= \cos \phi_{32e} \delta \phi_{2e} \\ \Delta_2(t) &= \delta \phi_{32e} \\ \Delta_3(t) &= \sin \phi_{32e} \delta \phi_{2e} \end{aligned} \right\} \quad (G-3)$$

2. VIRTUAL DISPLACEMENT OF POINT P_2 ON SECTION BC OF COUPLER

The virtual displacement of point P_2 is obtained by taking the variation of equation E-14.

$$\begin{aligned} \delta \bar{r}_{P_2} = & \delta \bar{r}_A + \frac{\partial \bar{r}_2}{\partial U_2(x_2, t)} \delta U_2(x_2, t) + \frac{\partial \bar{r}_2}{\partial V_2(x_2, t)} \delta V_2(x_2, t) + \frac{\partial \bar{r}_2}{\partial W_2(x_2, t)} \delta W_2(x_2, t) \\ & + \delta \bar{\phi}_{3e} \times \bar{r}_2 \end{aligned} \quad (G-4)$$

where

$\delta \bar{\phi}_{3e}$ represents the virtual rotation of the body fixed coordinate system (x_2, y_2, z_2) .

Appropriate substitutions into G-4 using equations E-16 and H-6 yield:

$$\begin{aligned} \delta \bar{r}_{P_2} = & [\delta U_2(x_2, t) + \Delta_5(t) W_2(x_2, t) - \Delta_6(t) [-L_1 \sin \gamma + V_2(x_2, t)]] \bar{i}_2 \\ & + [V_2(x_2, t) + \Delta_6(t) [L_1 \cos \gamma + x_2 + U_2(x_2, t)] - \Delta_4(t) W_2(x_2, t)] \bar{j}_2 \\ & + [W_2(x_2, t) + \Delta_4(t) [-L_1 \sin \gamma + V_2(x_2, t)] - \Delta_5(t) [L_1 \cos \gamma + x_2 \\ & + U_2(x_2, t)]] \bar{k}_2 \end{aligned} \quad (G-5)$$

where:

$$\left. \begin{aligned} \Delta_4(t) &= \Delta_1(t) \cos \gamma + \Delta_2(t) \sin \gamma \\ \Delta_5(t) &= \Delta_2(t) \cos \gamma - \Delta_1(t) \sin \gamma \\ \Delta_6(t) &= \Delta_3(t) \end{aligned} \right\} \quad (G-6)$$

APPENDIX H: DETERMINATION OF VARIATIONS OF $U_1(x_1, t)$, $U_2(x_2, t)$,

$V_2(x_2, t)$ AND ψ_{3e}

1. VARIATIONS OF AXIAL DISPLACEMENT $U_1(x_1, t)$

The variation of equation E-3, of appendix E, leads to:

$$\begin{aligned} \delta U_1(x_1, t) = & - \left[\frac{\partial \tilde{v}_1}{\partial \xi_1} \delta v_1(\xi_1, t) + \frac{\partial \tilde{w}_1}{\partial \xi_1} \delta w_1(\xi_1, t) \right]_{\xi_1=0}^{x_1} \\ & + \int_{\xi_1=0}^{x_1} \left[\frac{\partial^2 \tilde{v}_1}{\partial \xi_1^2} \delta \tilde{v}_1 + \frac{\partial^2 \tilde{w}_1}{\partial \xi_1^2} \delta \tilde{w}_1 \right] d\xi_1 \end{aligned} \quad (H-1)$$

2. VARIATION OF AXIAL DISPLACEMENT $U_2(x_2, t)$

With equation E-7 one obtains:

$$\begin{aligned} \delta U_2(x_2, t) = & \sin(\alpha + \beta) \cdot \delta v_1(L_1, t) - \cos(\alpha + \beta) \left[\frac{\partial v_1(x_1, t)}{\partial x_1} \delta v_1(x_1, t) + \frac{\partial w_1(x_1, t)}{\partial x_1} \delta w_1(x_1, t) \right]_{x_1=0}^{L_1} \\ & + \frac{\partial v_1(L_1, t)}{\partial x_1} \cdot \delta \left[\frac{\partial v_1(L_1, t)}{\partial x_1} \right] x_2 - \left[\frac{\partial \tilde{v}_2}{\partial \xi_2} \delta \tilde{v}_2 + \frac{\partial \tilde{w}_2}{\partial \xi_2} \delta \tilde{w}_2 \right]_{\xi_2=0}^{x_2} + \int_{\xi_2=0}^{L_1} \cos(\alpha + \beta) \cdot \\ & \cdot \left[\frac{\partial^2 v_1(x_1, t)}{\partial x_1^2} \delta v_1(x_1, t) + \frac{\partial^2 w_1(x_1, t)}{\partial x_1^2} \delta w_1(x_1, t) \right] dx_1 + \int_{\xi_2=0}^{x_2} \left[\frac{\partial^2 \tilde{v}_2}{\partial \xi_2^2} + \frac{\partial^2 \tilde{w}_2}{\partial \xi_2^2} \right] d\xi_2 \end{aligned} \quad (H-2)$$

3. VARIATION OF IN-PLANE DISPLACEMENT $V_2(x_2, t)$

The variation of equation E-6 leads to:

$$\delta V_2(x_2, t) = \delta v_2(x_2, t) + \sin(\alpha + \beta) \left[\frac{\partial v_1(x_1, t)}{\partial x_1} \delta v_1(x_1, t) + \frac{\partial w_1(x_1, t)}{\partial x_1} \delta w_1(x_1, t) \right]_0^{L_1} - \int_{x_1=0}^{L_1} \left[\frac{\partial^2 v_1(x_1, t)}{\partial x_1^2} \delta v_1(x_1, t) + \frac{\partial^2 w_1(x_1, t)}{\partial x_1^2} \delta w_1(x_1, t) \right] dx_1 \quad (H-3)$$

4. VARIATION OF ELASTIC ANGLE φ_{3e}

The adaptation of equation I-15 for the rigid body angular velocity $\dot{\bar{\varphi}}_3$ to the elastic velocity $\dot{\bar{\varphi}}_{3e}$ leads to the following form of the variation of the angle φ_{3e} :

$$\delta \bar{\varphi}_{3e} = \delta \varphi_{2e} \bar{i}_0 + \delta \varphi_{32e} \bar{j}_0 \quad (H-4)$$

When the above is expressed in the body fixed system of the first section, one obtains:

$$\delta \bar{\varphi}_{3e} = \cos \varphi_{32e} \delta \varphi_{2e} \bar{i}_1 + \delta \varphi_{32e} \bar{j}_1 + \sin \varphi_{32e} \delta \varphi_{2e} \bar{k}_1 \quad (H-5)$$

Similarly, when expressed in the body fixed system of the second section equation H-4 becomes:

$$\delta \bar{\varphi}_{3e} = [\cos \gamma \cos \varphi_{32e} \delta \varphi_{2e} + \sin \gamma \delta \varphi_{32e}] \bar{i}_2 + [\cos \gamma \delta \varphi_{32e} - \sin \gamma \cos \varphi_{32e} \delta \varphi_{2e}] \bar{j}_2 + \sin \varphi_{32e} \delta \varphi_{2e} \bar{k}_2 \quad (H-6)$$

For complete expressions of equations H-5 and H-6, it is necessary to substitute the expressions for $\delta \varphi_{2e}$ and $\delta \varphi_{32e}$ which are given, in appendix F, by equations F-38 and F-37, respectively.

APPENDIX I: RIGID BODY KINEMATICS OF THE LEVER-TYPE ROLL
FEED MECHANISM WITH REPLACEMENT SPATIAL
LINKAGE

It is the purpose of this appendix to define the variables, the coordinate systems, and the rigid body kinematics of the feed mechanism in which the spatial RSSR linkage has been replaced by an RRRSR one. (see figure I-1, which represents a schematic diagram of the complete machine.)

1. SPACE FIXED COORDINATE SYSTEM

The space fixed coordinate system $O_1-X_0-Y_0-Z_0(\bar{i}_0, \bar{j}_0, \bar{k}_0)$ has its origin on the axis of the input crank in such a manner that points O_1 and A lie in the same plane. The output crank pivot point O_2 , which together with O_1 may also be thought to lie on the ground link 6, is located in the same plane as the center C of the spherical joint of the output crank. The vector $\overline{O_1O_2}$ is defined by:

$$\overline{O_1O_2} = -s_3\bar{k}_0 - s_2\bar{j}_0 - s_1\bar{i}_0 \quad (I-1)$$

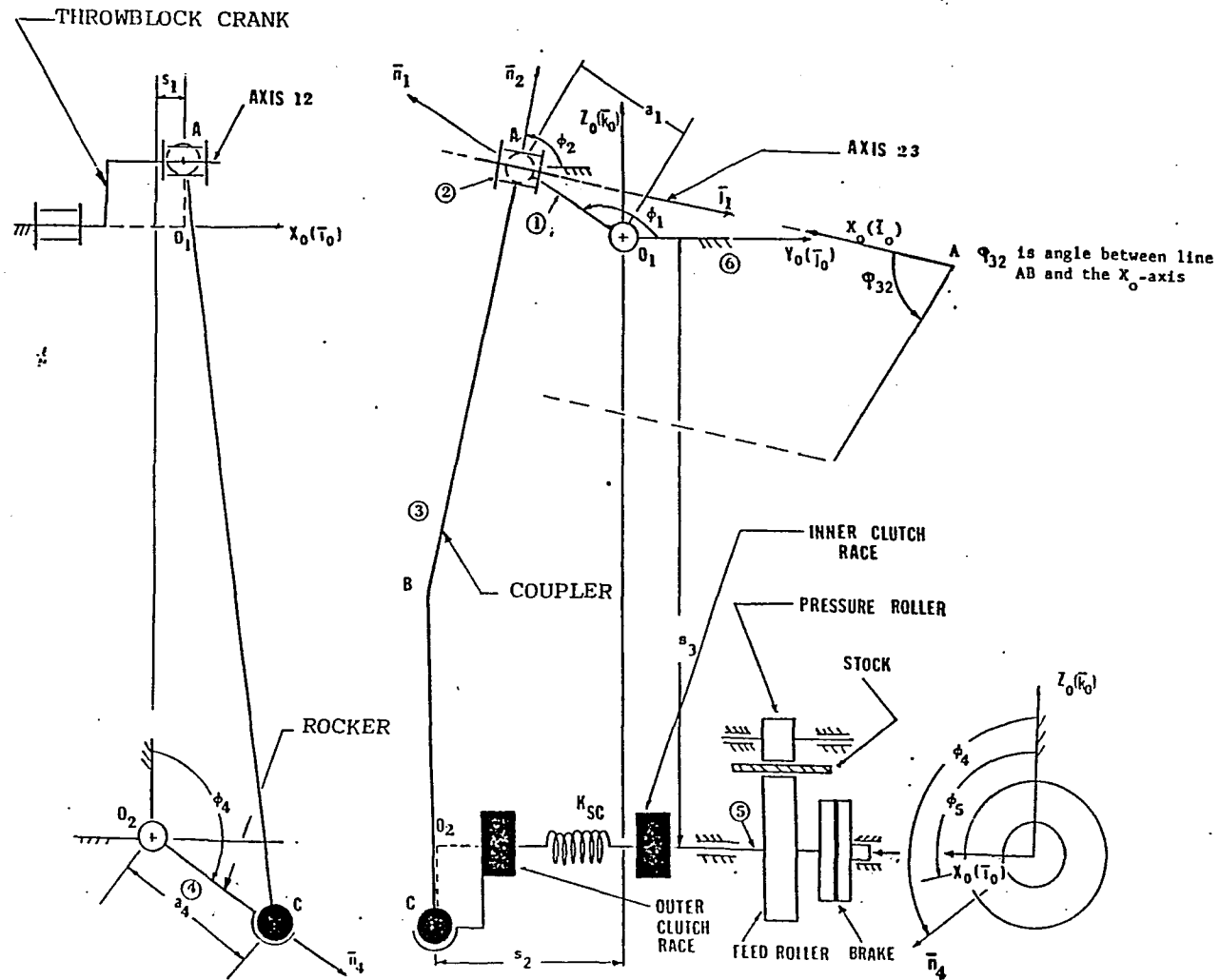
Note that the ground pivot axes are normal to each other and parallel to the X_0 and Y_0 axes, respectively.

2. LINKS AND ASSOCIATED BODY-FIXED COORDINATE SYSTEMS

Link 1 represents the throw block with its adjustable input length a_1 . Unit vector $\bar{n}_1$ which lies in the throw block plane has direction $\overline{O_1A}$ and is defined with the help of angle φ_1 with respect to the Y_0 axis.

Link 2 is the cross link connecting the revolutes of links 1 and 3. The axes 12 and 23 are normal to each other and intersect at point A. Unit vector $\bar{n}_2$ is normal to axis 23, lies in the throw block plane, and is defined by angle φ_2 . Unit vector $\bar{j}_1$, which is fixed in link 3, lies along the axis 23 at an angle $\varphi_2 - (\pi/2)$ with respect to the Y_0 -axis.

Link 3, as shown in greater detail in figure I-2, represents the replacement spatial coupler with pivot to pivot length AC and sections $AB = L_1$ and $BC = L_2$. Lines AB and BC define the link plane in which AB forms the angle α and BC the angle β with the pivot to pivot line AC, respectively. The following body fixed coordinate systems will be found useful: point A is the origin of the x-y-z ($\bar{i}, \bar{j}, \bar{k}$) system whose x-axis coincides with line AC and whose y-axis lies in the link plane. The x_1 - y_1 - z_1 ($\bar{i}_1, \bar{j}_1, \bar{k}_1$) system also has its origin at point A. Its x_1 -axis is along line AB and its x_1 - y_1 plane is identical with the link plane. (The y_1 -axis and axis 23 are identical.) The angle $\varphi_{3/2}$ is



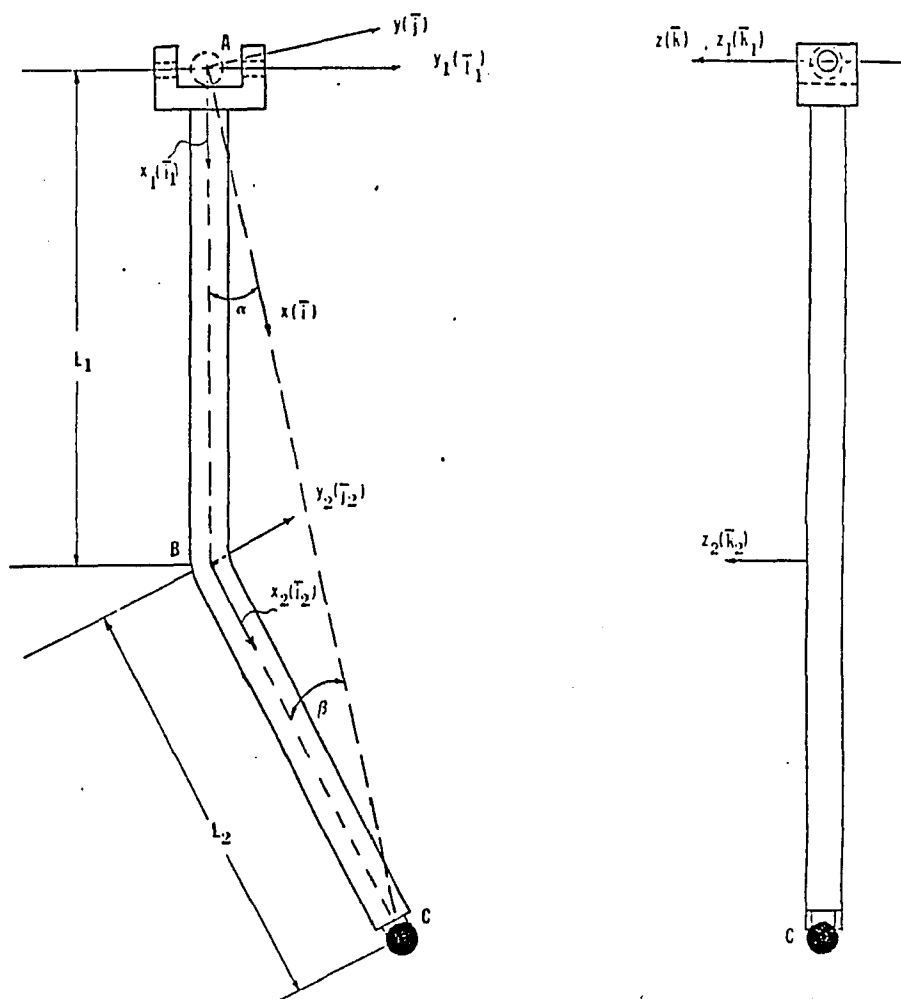


Figure I-2

Replacement Coupler with Body-Fixed Coordinate Systems

formed by this x_1 -axis and the X_0 -axis (see figure I-1).

The x_2 - y_2 - z_2 ($\bar{i}_2, \bar{j}_2, \bar{k}_2$) system originates at point B. Its x_2 -axis coincides with line BC. The x_2 - y_2 plane is again identical with the link plane.

Link 4 is the output crank of the spatial mechanism and has length a_4 . The associated unit vector $\bar{n}_4$ has the direction of $\overline{O_2D}$ at the angle ϕ_4 , with respect to the positive Z_0 axis and in the positive sense about the Y_0 -axis.

Link 5 represents the inner (output) race of the clutch, together with the feed roller shaft and the feed rollers. Its position, as expressed by angle ϕ_5 , is measured in the same manner as angle ϕ_4 (see figure I-1).

3. RIGID BODY KINEMATICS

a. Unit Vector Transformation Tables

The following transformation tables give the relationship between the unit vectors of the space fixed coordinate system and the two body fixed ones, $x_1-y_1-z_1$ and $x_2-y_2-z_2$.

	$\bar{i}_0$	$\bar{j}_0$	$\bar{k}_0$
$\bar{i}_1$	$\cos\phi_{32}$	$-\cos\phi_2 \sin\phi_{32}$	$-\sin\phi_2 \sin\phi_{32}$
$\bar{j}_1$	0	$\sin\phi_2$	$-\cos\phi_2$
$\bar{k}_1$	$\sin\phi_{32}$	$\cos\phi_2 \cos\phi_{32}$	$\sin\phi_2 \cos\phi_{32}$

Table I-1: Relationship Between $X_0-Y_0-Z_0$ and $x_1-y_1-z_1$ Systems

	$\bar{i}_0$	$\bar{j}_0$	$\bar{k}_0$
$\bar{i}_2$	$\cos\alpha\cos\phi_{32}$	$\sin\alpha\sin\phi_2$ $-\cos\alpha\cos\phi_2\sin\phi_{32}$	$-\sin\alpha\cos\phi_2$ $-\cos\alpha\sin\phi_2\sin\phi_{32}$
$\bar{j}_2$	$-\sin\alpha\cos\phi_{32}$	$\cos\alpha\sin\phi_2$ $+\sin\alpha\cos\phi_2\sin\phi_{32}$	$\sin\alpha\sin\phi_2\sin\phi_{32}$ $-\cos\alpha\cos\phi_2$
$\bar{k}_2$	$\sin\phi_{32}$	$\cos\phi_2\cos\phi_{32}$	$\sin\phi_2\cos\phi_{32}$

Table I-2: Relationship Between X_0 - Y_0 - Z_0 and x_2 - y_2 - z_2 Systems

Table I-3 relates the unit vectors of the above body fixed coordinate systems:

	$\bar{i}_1$	$\bar{j}_1$	$\bar{k}_1$
$\bar{i}_2$	$\cos\gamma$	$\sin\gamma$	0
$\bar{j}_2$	$-\sin\gamma$	$\cos\gamma$	0
$\bar{k}_2$	0	0	1

Table I-3: Relationship Between x_1 - y_1 - z_1 and x_2 - y_2 - z_2 Systems

Note that $\gamma = \alpha + \beta$

b. Loop Equation of RRRSR Linkage

The loop equation of the spatial linkage is given by:

$$\overline{O_1A} + \overline{AC} + \overline{CO_2} + \overline{O_2O_1} = 0 \quad (I-2)$$

where

$$\overline{O_1A} = a_1 [\cos\phi_1 \bar{j}_0 + \sin\phi_1 \bar{k}_0] \quad (I-3)$$

$$\begin{aligned} \overline{AC} &= AC \bar{i} = AC [\cos\alpha \bar{i}_1 + \sin\alpha \bar{j}_1] \\ &= AC \cos\alpha \cos\phi_{32} \bar{i}_0 + AC [\sin\alpha \sin\phi_2 - \cos\alpha \cos\phi_2 \sin\phi_{32}] \bar{j}_0 \\ &\quad - AC [\cos\alpha \sin\phi_{32} \sin\phi_2 + \sin\alpha \cos\phi_2] \bar{k}_0 \end{aligned} \quad (I-4)$$

$$\overline{CO_2} = -a_4 [\cos\phi_4 \bar{k}_0 + \sin\phi_4 \bar{i}_0] \quad (I-5)$$

$\overline{O_2O_1}$ is obtained from equation (I-1).

Substitution of equations I-1 and I-3 through I-5 into equation I-2, leads to the following three rigid body constraint equations:

$$s_1 + AC \cos\alpha \cos\phi_{32} - a_4 \sin\phi_4 = 0 \quad (I-6)$$

$$a_1 \cos\phi_1 + AC [\sin\alpha \sin\phi_2 - \cos\alpha \cos\phi_2 \sin\phi_{32}] + s_2 = 0 \quad (I-7)$$

$$a_1 \sin \phi_1 - AC [\cos \alpha \sin \phi_2 \sin \phi_{32} + \sin \alpha \cos \phi_2] - a_4 \cos \phi_4 + s_3 = 0 \quad (\text{I-8})$$

c. Linkage Angles

1. Output Link 4

$$\phi_4 = 2 \cdot \tan^{-1} \left[\frac{A + \sqrt{A^2 + B^2 - C^2}}{B + C} \right] \quad (\text{I-9})$$

where:

$$\left. \begin{aligned} A &= 2s_1 a_4 \\ B &= 2a_4 [s_3 + a_1 \sin \phi_1] \\ C &= a_4^2 + s_1^2 + s_2^2 + s_3^2 + a_1^2 - AC^2 + 2a_1 s_2 \cos \phi_1 \\ &\quad + 2s_3 a_1 \sin \phi_1 \end{aligned} \right\} \quad (\text{I-10})$$

2. Coupler Link 3

The position of the coupler is given by angles ϕ_2 and ϕ_{32} .

$$\phi_{32} = \cos^{-1} \left[\frac{a_4 \sin \phi_4 - s_1}{a_3 \cos \alpha} \right] \quad (\text{I-11})$$

$$\phi_2 = 2 \cdot \tan^{-1} \left[\frac{A^* - \sqrt{A^{*2} + B^{*2} - C^{*2}}}{B^* + C^*} \right] \quad (I-12)$$

where

$$\left. \begin{aligned} A^* &= s_2 + a_1 \cos \phi_1 \\ B^* &= -s_3 - a_1 \sin \phi_1 + a_4 \cos \phi_4 \\ C^* &= -a_3 \sin \alpha \end{aligned} \right\} \quad (I-13)$$

d. Absolute Angular Velocities of Links

1. Output Link 4

$$\dot{\phi}_4 = \frac{2 \cdot a_1 [(s_3 - a_4 \cos \phi_4) \cos \phi_1 - s_2 \sin \phi_1]}{[A \cos \phi_4 - B \sin \phi_4]} \cdot \dot{\phi}_1 \bar{I}_0 \quad (I-14)$$

2. Coupler Link 3

The total angular velocity of the coupler is given by:

$$\dot{\phi}_3 = \dot{\phi}_{32} \bar{I}_1 + \dot{\phi}_2 \bar{I}_0 \quad (I-15)$$

where:

$$\dot{\varphi}_{32} = - \frac{a_4 \cos \varphi_4}{a_3 \cos \alpha \sin \varphi_{32}} \dot{\varphi}_4 \quad (\text{I-16})$$

$$\dot{\varphi}_2 = \frac{a_1 \cos(\varphi_1 - \varphi_2)}{A^* \cos \varphi_2 - B^* \sin \varphi_2} \dot{\varphi}_1 + \frac{a_4 \sin \varphi_4 \cos \varphi_2}{A^* \cos \varphi_2 - B^* \sin \varphi_2} \dot{\varphi}_4 \quad (\text{I-17})$$

When equation I-15 is expressed in terms of the (x_1, y_1, z_1) system, one obtains:

$$\dot{\bar{\varphi}}_3 = J_1(t) \bar{i}_1 + J_2(t) \bar{j}_1 + J_3(t) \bar{k}_1 \quad (\text{I-18})$$

where:

$$\left. \begin{aligned} J_1(t) &= \dot{\varphi}_2 \cos \varphi_{32} \\ J_2(t) &= \dot{\varphi}_{32} \\ J_3(t) &= \dot{\varphi}_2 \sin \varphi_{32} \end{aligned} \right\} \quad (\text{I-19})$$

Further, when equation I-15 is expressed in the (x_2, y_2, z_2) system it becomes:

$$\dot{\bar{\varphi}}_3 = H_1(t) \bar{i}_2 + H_2(t) \bar{j}_2 + H_3(t) \bar{k}_2 \quad (\text{I-20})$$

where

$$\begin{aligned} H_1(t) &= \dot{\varphi}_2 \cos \gamma \cos \varphi_{32} + \dot{\varphi}_{32} \sin \gamma \\ H_2(t) &= \dot{\varphi}_{32} \cos \gamma - \dot{\varphi}_2 \sin \gamma \cos \varphi_{32} \\ H_3(t) &= \dot{\varphi}_2 \sin \varphi_{32} \end{aligned} \quad (\text{I-21})$$

e. Absolute Angular Accelerations of Links

1. Output Link 4

$$\ddot{\Phi}_4 = \left\{ \left[\frac{A \sin \Phi_4 + B \cos \Phi_4}{A \cos \Phi_4 - B \sin \Phi_4} \right] \dot{\Phi}_4^2 + \left[\frac{4Q_1 Q_4 \cos \Phi_1 \sin \Phi_4}{A \cos \Phi_4 - B \sin \Phi_4} \right] \dot{\Phi}_4 \cdot \dot{\Phi}_1 - \left[\frac{2Q_1 [(S_2 - Q_4 \cos \Phi_4) \sin \Phi_1 + S_2 \cos \Phi_1]}{A \cos \Phi_4 - B \sin \Phi_4} \right] \right\} \bar{I}_0 \quad (I-22)$$

2. Coupler Link 3

The absolute angular acceleration of the coupler link is given

by:

$$\ddot{\Phi}_3 = \ddot{\Phi}_2 + \ddot{\Phi}_{32} + \dot{\Phi}_2 \times \dot{\Phi}_{32} \quad (I-23)$$

where

$$\ddot{\Phi}_{32} = \left\{ - \left[\frac{Q_4 \cos \Phi_4}{Q_3 \cos \alpha \sin \Phi_{32}} \right] \ddot{\Phi}_4 - \left[\frac{Q_3 \omega_{31} \omega_{32}}{Q_3 \cos \alpha \sin \Phi_{32}} \right] \dot{\Phi}_{32} + \left[\frac{Q_4 \sin \Phi_4}{Q_3 \cos \alpha \sin \Phi_{32}} \right] \dot{\Phi}_4^2 \right\} \bar{I}_1 \quad (I-24)$$

$$\begin{aligned}
\ddot{\phi}_2 = & \left\{ \frac{a_1 \sin(\phi_1 - \phi_2) [2\dot{\phi}_1 \dot{\phi}_2 - \dot{\phi}_1^2]}{A^* \cos \phi_2 - B^* \sin \phi_2} + \left[\frac{A^* \sin \phi_2 + B^* \cos \phi_2}{A^* \cos \phi_2 - B^* \sin \phi_2} \right] \dot{\phi}_2^2 \right. \\
& + \left[\frac{a_4 \sin \phi_4 \cdot \cos \phi_4}{A^* \cos \phi_2 - B^* \sin \phi_2} \right] \ddot{\phi}_4 + \left[\frac{a_4 \cos \phi_4 \sin \phi_2}{A^* \cos \phi_2 - B^* \sin \phi_2} \right] \dot{\phi}_4^2 \\
& \left. - \left[\frac{2a_4 \sin \phi_4}{A^* \cos \phi_2 - B^* \sin \phi_2} \right] \dot{\phi}_4 \dot{\phi}_2 \right\} \bar{L}_0
\end{aligned} \quad (I-25)$$

When equation I-23 is expressed in the (x_1, y_1, z_1) coordinate system one obtains:

$$\ddot{\phi}_3 = G_1(t) \bar{i}_1 + G_2(t) \bar{j}_1 + G_3(t) \bar{k}_1 \quad (I-26)$$

where:

$$\begin{aligned}
G_1(t) &= \ddot{\phi}_2 \cos \phi_{32} - \dot{\phi}_2 \dot{\phi}_{32} \sin \phi_{32} \\
G_2(t) &= \ddot{\phi}_{32} \\
G_3(t) &= \ddot{\phi}_2 \sin \phi_{32} + \dot{\phi}_2 \dot{\phi}_{32} \cos \phi_{32}
\end{aligned} \quad (I-27)$$

Further, when equation I-23 is expressed in the (x_2, y_2, z_2) coordinate system, it becomes:

$$\ddot{\phi}_3 = I_1(t) \bar{i}_2 + I_2(t) \bar{j}_2 + I_3(t) \bar{k}_2 \quad (I-28)$$

where:

$$\begin{aligned}
I_1(t) &= G_1(t) \cos \chi + G_2(t) \sin \chi \\
I_2(t) &= -G_1(t) \sin \chi + G_2(t) \cos \chi \\
I_3(t) &= G_3(t)
\end{aligned} \quad (I-29)$$

APPENDIX J: DERIVATION OF SUBPROBLEMS

The present appendix gives derivations for both the in-plane and out-of-plane free vibration behavior of the coupler. This includes shape functions, characteristic equations and orthogonality conditions.

1. GENERAL ASSUMPTIONS(See Figure J-1)

Both derivations contain the following assumptions:

- i. The link is a simply supported beam with the ball joint at point A hinged, while the ball joint at point C is free to slide along the original line AC.
- ii. With the reference plane defined by the points A, B, and C, the in-plane deflections $v_1(x_1, t)$ and $v_2(x_2, t)$ are normal to the x_1 and x_2 axes, respectively. The out-of-plane deflections $w_1(x_1, t)$ and $w_2(x_2, t)$ are normal to the above reference plane.
- iii. The two beam sections are rigidly connected at point B.
- iv. While in the derivation of some boundary conditions it becomes necessary to consider axial forces and torsional moments, both direct tension or compression and torsion are neglected.
- v. The two in-plane, as well as the two out-of-plane deflections share common eigenvalues.
- vi. Euler-Bernoulli beam theory is used.

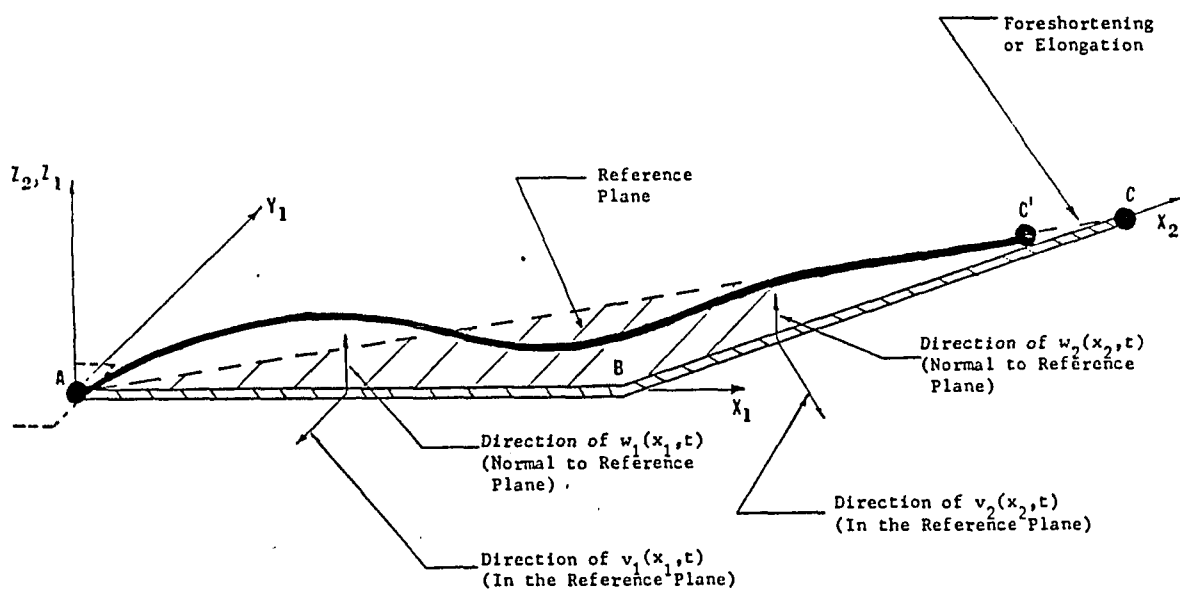


Figure J-1

Definition of Reference Plane and Deflected Coupler

2. IN-PLANE VIBRATIONS

a. Preliminary Work for the Determination of Certain Boundary Conditions.

The following describes certain displacement and force relationships at points B and C, respectively, which are necessary for the subsequent determination of the boundary conditions at these points.

1. Displacement Compatibility at Point B

In line with the assumption of zero axial deformation, figure J-2 shows point B moved to B' due to the in-plane deflection of section 1. Thus,

$$\overline{BB'} = v_1(L_1, t) \overline{j}_1 \quad (J-1)$$

When the above is expressed in the x_2 - y_2 system of section 2, it becomes:

$$\overline{BB'} = u_2(0, t) \overline{i}_2 + v_2(0, t) \overline{j}_2, \quad (J-2)$$

where, $u_2(0, t)$ may be thought of as a "rigid" axial displacement of the origin of the second beam section, and $v_2(x_2, t)$ represents an elastic deflection.

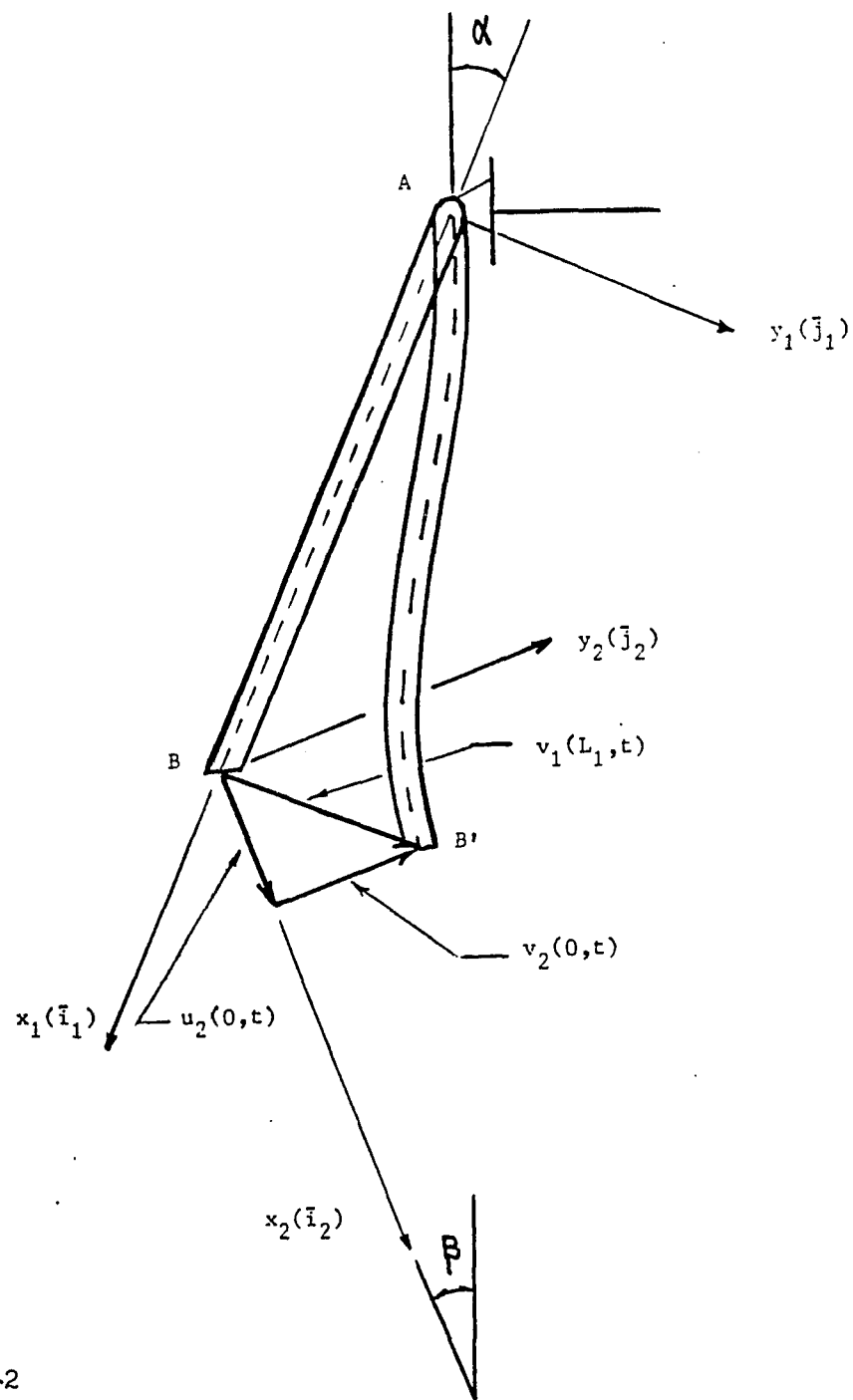


Figure J-2

Displacement Compatibility at Point B

When equation J-1 is transformed into the form of equation J-2, with the help of the angles α and β , one obtains:

$$v_2(0,t) = v_1(L_1,t)\cos(\alpha+\beta) \quad (J-3)$$

$$u_2(0,t) = v_1(L_1,t)\sin(\alpha+\beta) \quad (J-4)$$

2. Displacement Boundary Condition at Point C

The following discusses the permissible elastic motions of point C at the end of the second section.

Normally, one would set $v_2(L_2,t)$ equal to zero, with the implication that point C slides along the x_2 axis. But, in keeping with the first assumption, point C must slide along the x-axis. In order to accomplish the latter, by way of a two step procedure, consider figure J-3.

Since axial elastic deflections are neglected, the displacements along the x_2 -axis are identical at points B and C, and point C is moved to C''. Therefore,

$$u_2(L_2,t) = CC'' = u_2(0,t) \quad (J-5)$$

To bring point C'' back to C' the x-axis, the displacement $v_2(L_2,t)$ is necessary. Thus the total displacement of point C may be expressed by:

$$\overline{CC'} = u_2(L_2,t) \overline{i}_2 + v_2(L_2,t) \overline{j}_2 \quad (J-6)$$

If the above is transformed into the x-y system, one obtains:

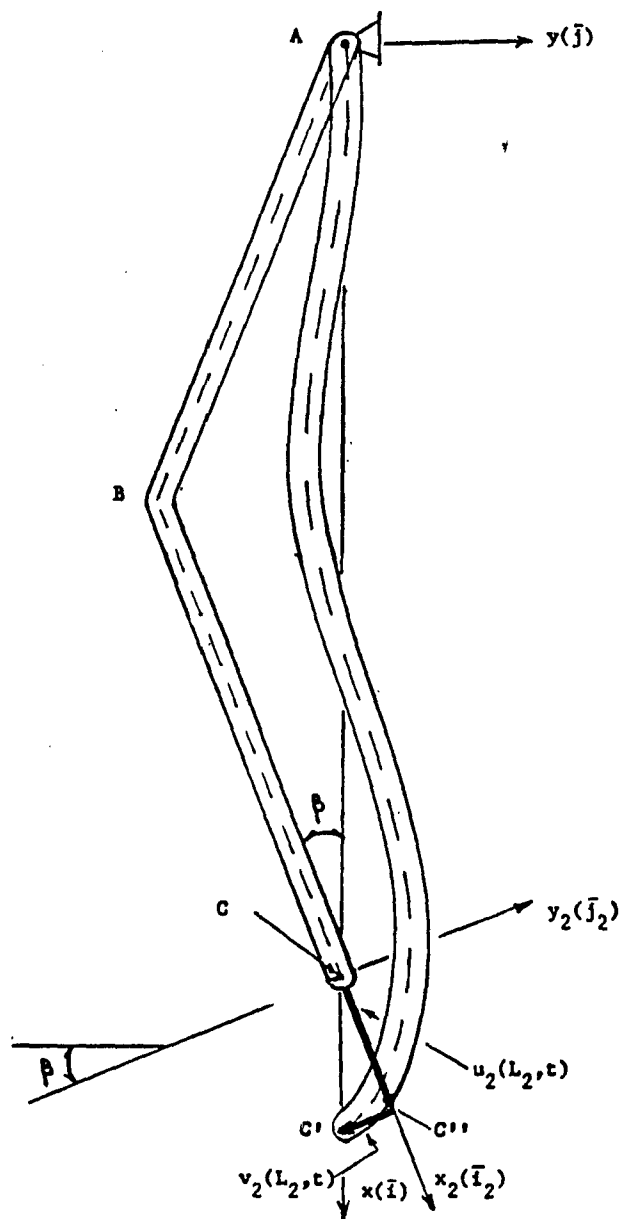


Figure J-3

Displacement Boundary Condition at Point C

$$\begin{aligned}\overline{CC'} = & [u_2(L_2, t)\cos\beta - v_2(L_2, t)\sin\beta] \bar{i} \\ & + [u_2(L_2, t)\sin\beta + v_2(L_2, t)\cos\beta] \bar{j}\end{aligned}\quad (J-7)$$

In order to obtain the displacement boundary condition, at point C, which expresses the fact that point C stays on the x-axis, the y-component of equation J-7 is set equal to zero. This, together with equation J-4 and J-5, gives:

$$v_2(L_2, t) = -\tan\beta \sin(\alpha + \beta) v_1(L_1, t) \quad (J-8)$$

3. Relationships between Shear Forces at Points B and C

In order to develop the shear force boundary condition at point B, it is necessary to express these shear forces in terms of the shear force at point C. Figure J-4 shows the condition of force equilibrium at point B, in terms of the axial and shear force components $F_{x1}(L_1, t)$ and $F_{y1}(L_1, t)$, respectively, acting at the end of section 1, as well as $F_{x2}(0, t)$ and $F_{y2}(0, t)$ which act at the origin of section 2. Thus,

$$-F_{x1}(L_1, t)\bar{i}_1 + F_{y1}(L_1, t)\bar{j}_1 = F_{x2}(0, t)\bar{i}_2 + F_{y2}(0, t)\bar{j}_2 \quad (J-9)$$

By transforming the above into the x-y system, it is possible to express the axial force $F_{x2}(0, t)$ in terms of the shear forces $F_{y1}(L_1, t)$ and $F_{y2}(0, t)$:

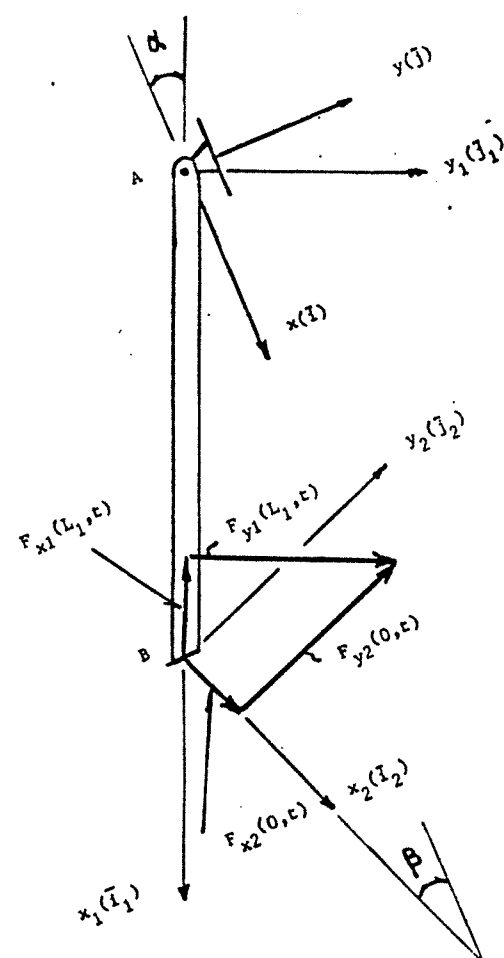


Figure J-4
Force Equilibrium at Point B

$$F_{x2}(0,t) = \frac{\cos\alpha + \sin\alpha \tan\alpha}{\sin\beta + \cos\beta \tan\alpha} F_{y1}(L_1,t) + \frac{\sin\beta \tan\alpha - \cos\beta}{\sin\beta + \cos\beta \tan\alpha} F_{y2}(0,t) \quad (J-10)$$

Note that the above also represents the axial force at point C,
i.e.

$$F_{x2}(L_2,t) = F_{x2}(0,t) \quad (J-11)$$

As has been discussed before, point C must be free to move along the x-axis. Therefore, the reaction force at this point can only be normal to line AC. The force $\overline{F}_C = F_C \overline{j}$, shown in Figure J-5, expressed in the x_2 - y_2 system, becomes:

$$\overline{F}_C = F_{x2}(L_2,t) \overline{i}_2 + F_{y2}(L_2,t) \overline{j}_2 \quad (J-12)$$

The right hand side of this expression is first transformed completely into the x-y coordinate system. Subsequently, equations J-11 with J-10 are substituted. The resulting x-component is then set equal to zero (there is no interest in F_{Cy}). This furnishes the following relationship between the shear forces:

$$\tan\beta F_{y2}(L_1,t) = \frac{\cos\alpha + \sin\alpha \tan\alpha}{\sin\beta + \cos\beta \tan\alpha} F_{y1}(L_1,t) + \frac{\sin\beta \tan\alpha - \cos\beta}{\sin\beta + \cos\beta \tan\alpha} F_{y2}(0,t) \quad (J-13)$$

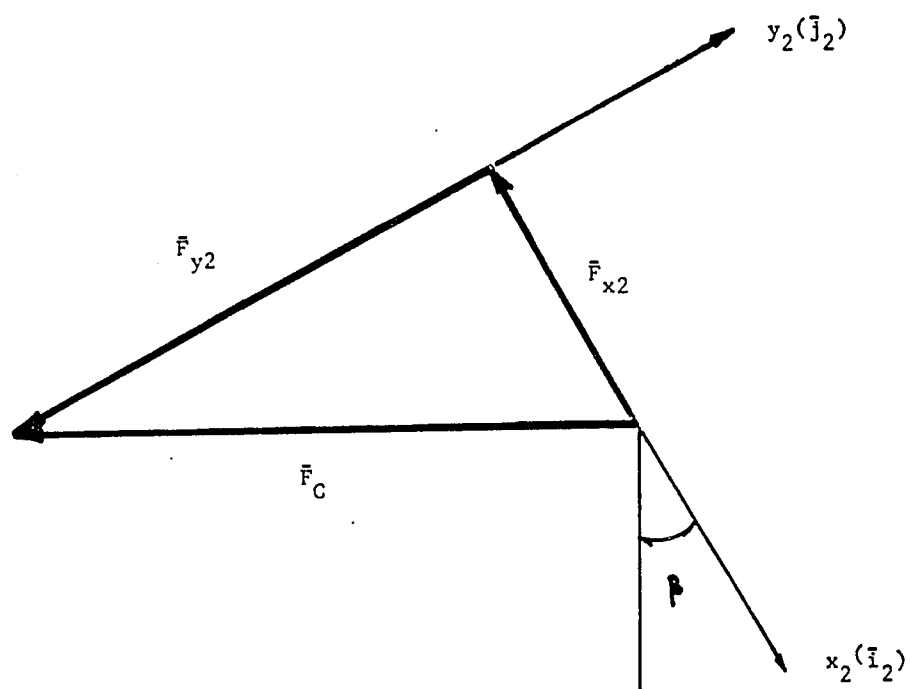


Figure J-5

Force Equilibrium at Point C

b. Equations of Motion and Boundary Conditions

The differential equations of motion for the two straight sections are given by:

For $0 \leq x_1 \leq L_1$

$$\rho A \frac{\partial^2 v_1(x_1, t)}{\partial t^2} + EI \frac{\partial^4 v_1(x_1, t)}{\partial x_1^4} = 0 \quad (J-14)$$

For $0 \leq x_2 \leq L_2$

$$\rho A \frac{\partial^2 v_2(x_2, t)}{\partial t^2} + EI \frac{\partial^4 v_2(x_2, t)}{\partial x_2^4} = 0 \quad (J-15)$$

The associated boundary conditions are:

at $x_1 = 0$

$$v_1(0, t) = 0 \quad (J-16)$$

$$\frac{\partial^2 v_1(0, t)}{\partial x_1^2} = 0 \quad (J-17)$$

at $x_1 = L_1$ and $x_2 = 0$ (point B)

According to equation J-3:

$$v_2(0, t) = \cos(\alpha + \beta) v_1(L_1, t) \quad (J-18)$$

in addition, to satisfy the rigid connection assumption,

$$\frac{\partial v_1(l_1, t)}{\partial x_1} = \frac{\partial v_2(0, t)}{\partial x_2} \quad (J-19)$$

and

$$\frac{\partial^2 v_1(l_1, t)}{\partial x_1^2} = \frac{\partial^2 v_2(0, t)}{\partial x_2^2} \quad (J-20)$$

$$\text{at } x_2 = L_2$$

According to equation J-8:

$$v_2(L_2, t) = -\tan\beta \sin(\alpha + \beta) v_1(L_1, t) \quad (J-21)$$

In addition,

$$\frac{\partial^2 v_2(L_2, t)}{\partial x_2^2} = 0 \quad (J-22)$$

Finally, according to the shear force relationship of equation J-13,

$$b_1 \frac{\partial^3 v_2(L_2, t)}{\partial x_2^3} = b_2 \frac{\partial^3 v_1(L_1, t)}{\partial x_1^3} + b_3 \frac{\partial^3 v_2(0, t)}{\partial x_2^3} \quad (J-23)$$

where

$$b_1 = \tan\beta [\sin\beta + \cos\beta \tan\alpha] \quad (J-24)$$

$$b_2 = \cos\alpha + \sin\alpha \tan\alpha \quad (J-25)$$

$$b_3 = \sin\beta \tan\alpha - \cos\beta \quad (J-26)$$

c. Shape Functions and Characteristic Equation

The partial differential equations J-14 and J-15 are solved by the method of separation of variables, which assumes that:

$$v_1(x_1, t) = V_1(x_1) e^{i\omega_v t} \quad (J-27)$$

$$v_2(x_2, t) = V_2(x_2) e^{i\omega_v t} \quad (J-28)$$

where ω_v denotes the common in-plane vibration frequency of the total beam. Substitution of the above into J-14 and J-15 results in the following:

For $0 \leq x_1 \leq L_1$

$$\frac{d^4 V_1(x_1)}{dx_1^4} - \lambda_v^4 V_1(x_1) = 0 \quad (J-29)$$

For $0 \leq x_2 \leq L_2$

$$\frac{d^4 V_2(x_2)}{dx_2^4} - \lambda_v^4 V_2(x_2) = 0 \quad (J-30)$$

where

$$\lambda_v^4 = \frac{P \cdot A \cdot \omega_v^2}{EI} \quad (J-31)$$

Substitution of the solution form according to equations J-27 and J-28 into equations J-16 through J-23 furnishes the boundary conditions for equations J-29 and J-30.

$$V_1(0) = 0 \quad (J-32)$$

$$V_1''(0) = 0 \quad (J-33)$$

$$V_2(0) = \cos(\alpha+\beta)V_1(L_1) \quad (J-34)$$

$$V_1'(L_1) = V_2'(0) \quad (J-35)$$

$$V_1''(L_1) = V_2''(0) \quad (J-36)$$

$$V_2(L_2) = -\tan\beta\sin(\alpha+\beta)V_1(L_1) \quad (J-37)$$

$$V_2''(0) = 0 \quad (J-38)$$

$$b_1V_2'''(L_2) = b_2V_1'''(L_1) + b_3V_2'''(0) \quad (J-39)$$

Note that the primes represent differentiation with respect to x_1 and or x_2 .

The general solutions of equations J-30 and J-31, i.e. the shape functions, are given by:

$$V_1(x_1) = A_{11}\sin\lambda_V x_1 + A_{12}\cos\lambda_V x_1 + A_{13}\sinh\lambda_V x_1 + A_{14}\cosh\lambda_V x_1 \quad (J-40)$$

and

$$V_2(x_2) = B_{11} \sin \lambda_n x_2 + B_{12} \cos \lambda_n x_2 + B_{13} \sinh \lambda_n x_2 + B_{14} \cosh \lambda_n x_2 \quad (J-41)$$

Application of the boundary conditions of equations J-32 through J-39 to equations J-40 and J-41 furnishes the n th mode shape functions $V_{1n}(x_1)$ and $V_{2n}(x_2)$ (up to the arbitrary constant A_{11n} , which has been chosen to be unity), as well as the characteristic equation:

$$V_{1n}(x_1) = \sin \lambda_{vn} x_1 - \Gamma_{3n} \cdot \sinh \lambda_{vn} x_1 \quad (J-42)$$

$$\begin{aligned} V_{2n}(x_2) = & \Delta_{1n} \sin \lambda_{vn} x_2 + \Delta_{2n} \cos \lambda_{vn} x_2 + \Delta_{3n} \sinh \lambda_{vn} x_2 \\ & + \Delta_{4n} \cosh \lambda_{vn} x_2 \end{aligned} \quad (J-43)$$

where

$$\Delta_{1n} = \cos \lambda_{vn} L_1 - \Gamma_{1n} - \Gamma_{3n} \cdot \cosh \lambda_{vn} L_1 + \Gamma_{2n} \cdot \Gamma_{3n} \quad (J-44)$$

$$\Delta_{2n} = \frac{1}{2} \left[(\cos(\alpha+\beta)+1) \sin \lambda_{vn} L_1 - \Gamma_{3n} \cdot (\cos(\alpha+\beta)-1) \cdot \sinh \lambda_{vn} L_1 \right] \quad (J-45)$$

$$\Delta_{3n} = \Gamma_{1n} - \Gamma_{2n} \cdot \Gamma_{3n} \quad (J-46)$$

$$\Delta 4\tilde{\eta} = \frac{1}{2} \left[(\cos(\alpha+\beta) - 1) \cdot \sin \lambda_{v\eta} L_1 - \Gamma_{3\eta} (\cos(\alpha+\beta) + 1) \cdot \sinh \lambda_{v\eta} L_1 \right] \quad (J-47)$$

$$\Gamma_{1\eta} = \frac{1}{2(\sin \lambda_{v\eta} L_2 + \sinh \lambda_{v\eta} L_2)} \left[\cos(\alpha+\beta) \sin \lambda_{v\eta} L_1 (\cos \lambda_{v\eta} L_2 - \cosh \lambda_{v\eta} L_2) \right. \\ \left. + \sin \lambda_{v\eta} (L_1 + L_2) \pm \sin \lambda_{v\eta} L_1 \cos \lambda_{v\eta} L_1 + \cosh \lambda_{v\eta} L_2 \sin \lambda_{v\eta} L_1 \right] \quad (J-48)$$

$$\Gamma_{2\eta} = \frac{1}{2(\sin \lambda_{v\eta} L_2 + \sinh \lambda_{v\eta} L_2)} \left[\cos(\alpha+\beta) [\cos \lambda_{v\eta} L_2 - \cosh \lambda_{v\eta} L_2] \cdot \sinh \lambda_{v\eta} L_1 \right. \\ \left. - (\cos \lambda_{v\eta} L_2 + \cosh \lambda_{v\eta} L_2) \sinh \lambda_{v\eta} L_1 + 2 \cdot \sin \lambda_{v\eta} L_2 \cdot \cosh \lambda_{v\eta} L_1 \right] \quad (J-49)$$

$$\Gamma_{3\eta} = \frac{\Gamma_{4\eta}}{\Gamma_{5\eta}} \quad (J-50)$$

$$\Gamma_{4\eta} = (\cos \lambda_{v\eta} L_1 - \Gamma_{1\eta}) \cdot \sin \lambda_{v\eta} L_2 + \frac{1}{2} \cos \lambda_{v\eta} L_2 \cdot \sin \lambda_{v\eta} L_1 (\cos(\alpha+\beta) + 1) \\ + \Gamma_{1\eta} \cdot \sinh \lambda_{v\eta} L_2 + \frac{1}{2} \cosh \lambda_{v\eta} L_2 \cdot \sin \lambda_{v\eta} L_1 (\cos(\alpha+\beta) - 1) \\ + \tan \beta \cdot \sin(\alpha+\beta) \cdot \sin \lambda_{v\eta} L_1 \quad (J-51)$$

$$\begin{aligned}
\Gamma_{5n} = & (\cos \lambda_{vn} L_1 - \Gamma_{2n}) \cdot \sin \lambda_{vn} L_2 + \frac{1}{2} \cos \lambda_{vn} L_2 \cdot \sinh \lambda_{vn} L_1 \cdot (\cos(\alpha + \beta) - 1) \\
& + \sinh \lambda_{vn} L_2 \cdot \Gamma_{2n} + \frac{1}{2} \cosh \lambda_{vn} L_2 \cdot \sinh \lambda_{vn} L_1 (\cos(\alpha + \beta) + 1) \\
& + \tan \beta \cdot \sin(\alpha + \beta) \cdot \sinh \lambda_{vn} L_1
\end{aligned} \tag{J-52}$$

Finally the characteristic equation is given by:

$$\begin{aligned}
& \left\{ \left[(b_3 - b_1 \cos \lambda_{vn} L_2) \cdot (\cos \lambda_{vn} L_1 - \Gamma_{1n}) + \frac{b_1}{2} \sin \lambda_{vn} L_2 \cdot \sin \lambda_{vn} L_1 \cdot (\cos(\alpha + \beta) + 1) \right. \right. \\
& \quad + (b_1 \cosh \lambda_{vn} L_2 - b_3) \cdot \Gamma_{1n} + \frac{b_1}{2} \sinh \lambda_{vn} L_2 \cdot \sin \lambda_{vn} L_1 (\cos(\alpha + \beta) - 1) \\
& \quad \left. \left. + b_2 \cos \lambda_{vn} L_1 \right] \right\} \Bigg/ \left[(\cos \lambda_{vn} L_1 - \Gamma_{1n}) \cdot \sin \lambda_{vn} L_2 + \frac{1}{2} \cos \lambda_{vn} L_2 \cdot \sin \lambda_{vn} L_1 \right. \\
& \quad \cdot (\cos(\alpha + \beta) + 1) + \sinh \lambda_{vn} L_2 \cdot \Gamma_{1n} + \frac{1}{2} \cosh \lambda_{vn} L_2 \cdot \sin \lambda_{vn} L_1 \cdot (\cos(\alpha + \beta) - 1) \\
& \quad \left. \left. + \tan \beta \cdot \sin(\alpha + \beta) \cdot \sinh \lambda_{vn} L_1 \right] \right\} = \left\{ \left[(b_3 - b_1 \cos \lambda_{vn} L_2) \cdot (\cosh \lambda_{vn} L_1 \right. \right. \\
& \quad - \Gamma_{2n}) + \frac{b_1}{2} \sin \lambda_{vn} L_2 \cdot \sinh \lambda_{vn} L_1 \cdot (\cos(\alpha + \beta) - 1) + (b_1 \cosh \lambda_{vn} L_2 - b_3) \cdot \Gamma_{2n} \\
& \quad \left. \left. + \frac{b_1}{2} \sinh \lambda_{vn} L_2 \cdot \sinh \lambda_{vn} L_1 \cdot (\cos(\alpha + \beta) + 1) - b_2 \cosh \lambda_{vn} L_1 \right] \right\} \Bigg/ \\
& \left\{ (\cosh \lambda_{vn} L_1 - \Gamma_{2n}) \cdot \sin \lambda_{vn} L_2 + \frac{1}{2} \cos \lambda_{vn} L_2 \cdot \sinh \lambda_{vn} L_1 \cdot (\cos(\alpha + \beta) - 1) \right. \\
& \quad + \sinh \lambda_{vn} L_2 \cdot \Gamma_{2n} + \frac{1}{2} \cosh \lambda_{vn} L_2 \cdot \sinh \lambda_{vn} L_1 \cdot (\cos(\alpha + \beta) + 1) + \tan \beta \cdot \\
& \quad \left. \left. \sin(\alpha + \beta) \cdot \sinh \lambda_{vn} L_1 \right] \right\}
\end{aligned} \tag{J-53}$$

d. Orthogonality Condition

To determine the orthogonality condition, let λ_{vm} and λ_{vn} be two distinct roots of the characteristic equation given by B-53, and let V_{1n} , V_{2n} and V_{1m} , V_{2m} be the associated characteristic functions. Then, using equations J-29 and J-30 one obtains:

$$\frac{d^4 V_m}{dx^4} = \lambda_{vm}^4 V_m \quad (J-54)$$

$$\frac{d^4 V_n}{dx^4} = \lambda_{vn}^4 V_n \quad (J-55)$$

with

$$\left. \begin{aligned} V_m &= \begin{cases} V_{1m} & 0 \leq x_1 \leq L_1 \\ V_{2m} & 0 \leq x_2 \leq L_2 \end{cases} \\ V_n &= \begin{cases} V_{1n} & 0 \leq x_1 \leq L_1 \\ V_{2n} & 0 \leq x_2 \leq L_2 \end{cases} \end{aligned} \right\} \quad (J-56)$$

Equation J-54 is now multiplied by V_n and equation J-55 by V_m .

When the results are subtracted from each other one finds:

$$V_n \frac{d^4 V_m}{dx^4} - V_m \frac{d^4 V_n}{dx^4} = (\lambda_{vm}^4 - \lambda_{vn}^4) V_n V_m \quad (J-57)$$

Now the above expression must be integrated over the total length of the coupler and the subscripts of the variables are adjusted according to equation J-56, i.e.:

$$\begin{aligned}
& \int_{x_1=0}^{L_1} \left[V_{n_1} \frac{d^4 V_{m_1}}{dx_1^4} - V_{m_1} \frac{d^4 V_{n_1}}{dx_1^4} \right] dx_1 + \int_{x_2=0}^{L_2} \left[V_{n_2} \frac{d^4 V_{m_2}}{dx_2^4} - V_{m_2} \frac{d^4 V_{n_2}}{dx_2^4} \right] dx_2 \\
& = (\lambda_{V_m}^4 - \lambda_{V_n}^4) \left\{ \int_{x_1=0}^{L_1} V_{m_1} \cdot V_{n_1} dx_1 + \int_{x_2=0}^{L_2} V_{m_2} \cdot V_{n_2} dx_2 \right\} \quad (J-58)
\end{aligned}$$

Performing each of the left hand integrations twice by parts, and substituting the required boundary terms according to equations J-32 through J-39, leads to:

$$(\lambda_{V_m}^4 - \lambda_{V_n}^4) \left\{ \int_{x_1=0}^{L_1} V_{m_1} \cdot V_{n_1} dx_1 + \int_{x_2=0}^{L_2} V_{m_2} \cdot V_{n_2} dx_2 \right\} = 0 \quad (J-59)$$

Equation J-59 represents the orthogonality condition which may be rewritten in the following form:

$$\int_{x_1=0}^{L_1} V_{m_1} \cdot V_{n_1} dx_1 + \int_{x_2=0}^{L_2} V_{m_2} \cdot V_{n_2} dx_2 = \delta_{n,m} \cdot \mathcal{V}_m \quad (J-60)$$

Where $\delta_{n,m}$ is the Kronecker delta and $\mathcal{V}_m$ represents the value of the expression when $m = n$.

3. OUT-OF-PLANE VIBRATIONS

a. Preliminary Work for the Determination of Certain Boundary Conditions

The following describes certain moment and slope relationships at point B, when the motion is normal to the x-y plane.

1. Moment Compatibility at Point B

Even though at points A and C there are ball joints which cannot transmit moments, in order to establish moment equilibrium at point B, it will be provisionally assumed that section 1 can support a torsional moment. (Section 2 of the beam does not experience any torsional moment). Then,

$$M_{2B} \bar{J}_2 + M_{1B} \bar{J}_1 + M_{1T} \bar{I}_1 = 0 \quad (J-61)$$

where:

M_{1B} and M_{2B} = out-of-plane bending moments acting on section 1
and 2, respectively

M_{1T} = torsional moment acting on section 1

Equation J-61 becomes in component form:

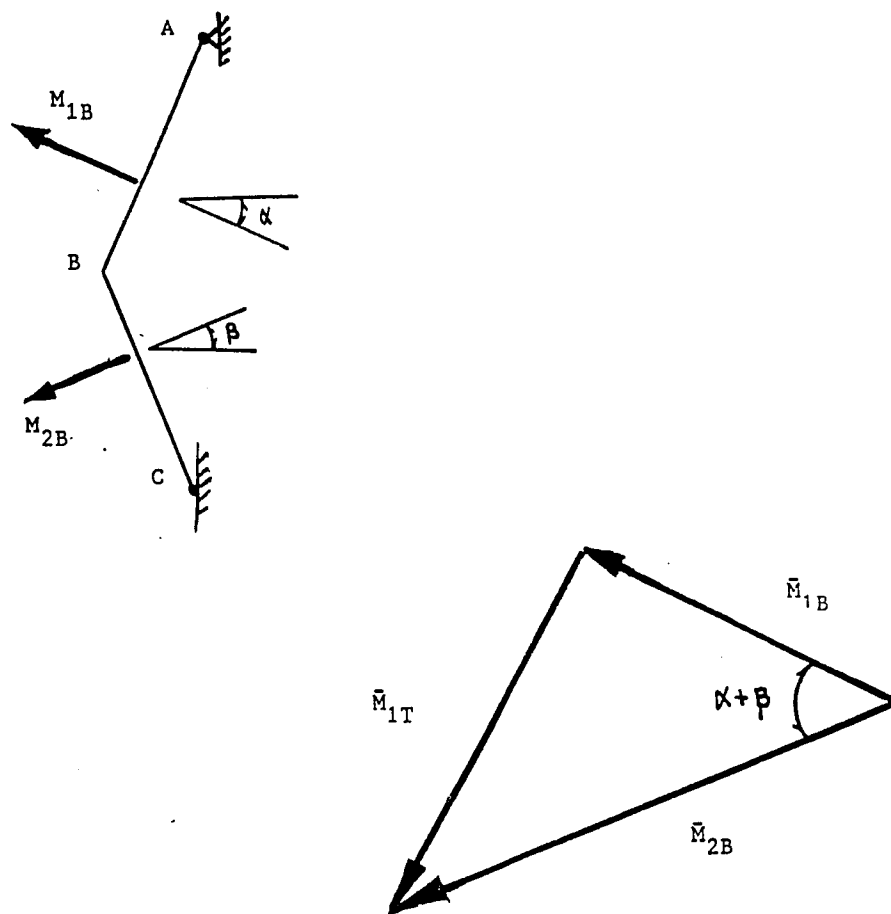


Figure J-6

Moment Equilibrium at Point B

$$M_{1B} = \cos(\alpha + \beta) M_{2B} \quad (J-62)$$

$$M_{1T} = \sin(\alpha + \beta) M_{2B} \quad (J-63)$$

Equation J-62 forms the basis for the moment boundary condition at point B.

Since it is not possible to sustain a torsional moment at point A, equation J-63 will be disregarded. It might be noted that this moment contributes in actuality to the rotation of the coupler about line AC, which has also been disregarded.

2. Slope Compatibility at Point B

The total slope at point B as the endpoint of section 1, must be the same as at point B when it is interpreted as the initial point of section 2.

If one expresses such slopes in terms of their associated infinitesimal rotation vectors, then (see figure J-7)

$$\overline{\frac{dw_1(l_1, t)}{dx_1}} = \frac{dw_1(l_1, t)}{dx_1} \mathbf{j}_1 \quad (J-64)$$

is normal to the x_1 -axis, and

$$\overline{\frac{dw_2(0, t)}{dx_2}} = \frac{dw_2(0, t)}{dx_2} \mathbf{j}_2 \quad (J-65)$$

is normal to the x_2 -axis.

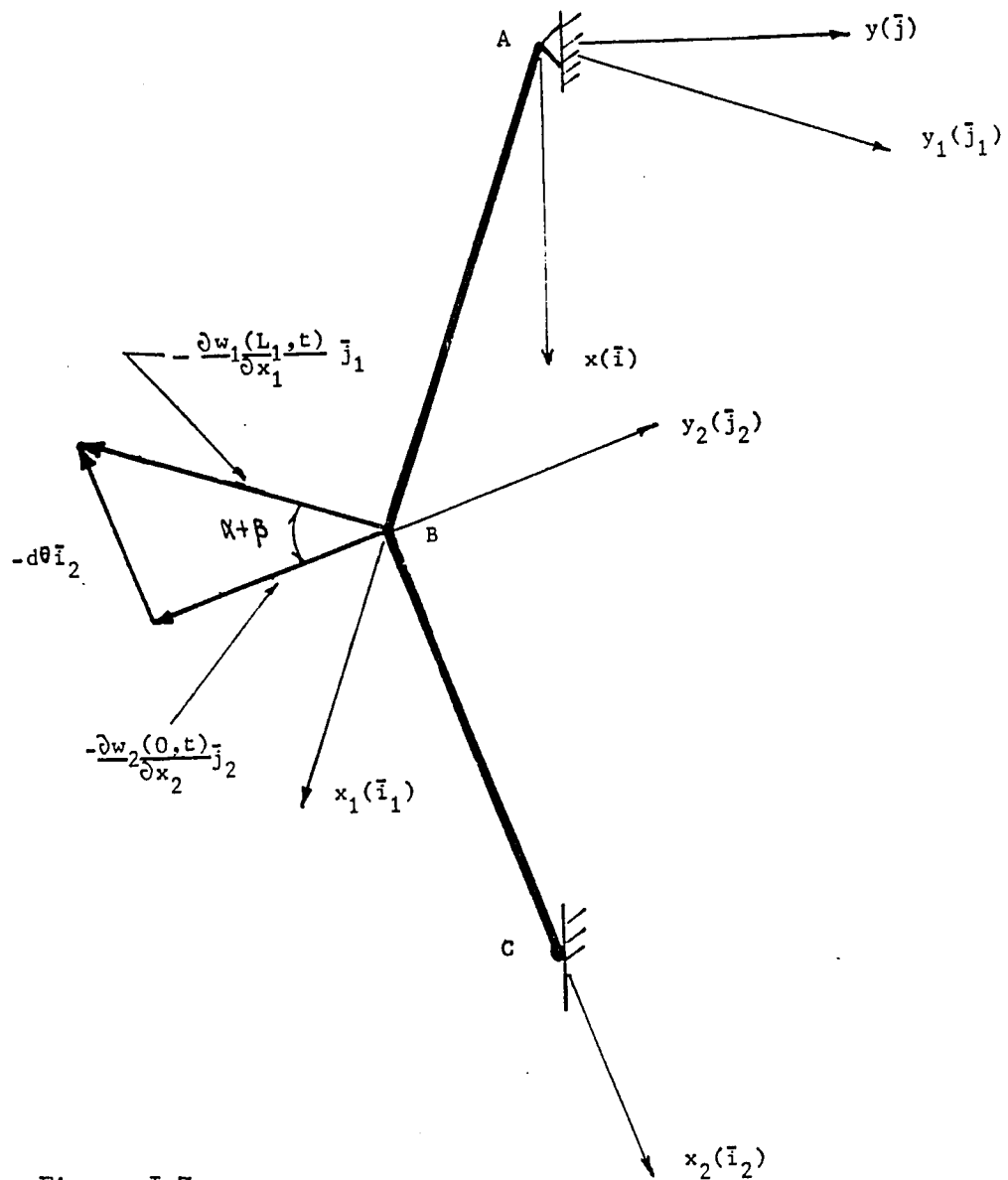


Figure J-7

Slopes of Out-of-Plane Deflections at Point B

A possible infinitesimal torsional rotation about the x_2 -axis is expressed by:

$$\overline{d\theta} = d\theta \overline{i}_2 \quad (J-66)$$

The equality of infinitesimal vectorial rotations at point B may be expressed by:

$$\overline{\frac{dw_1(L_1, t)}{dx_1}} = \overline{\frac{dw_2(0, t)}{dx_2}} + \overline{d\theta}$$

The resulting scalar relationship between $\frac{dw_1(L_1, t)}{dx_1}$ and $\frac{dw_2(0, t)}{dx_2}$, then becomes (according to figure J-7):

$$\frac{dw_2(0, t)}{dx_2} = \frac{dw_1(L_1, t)}{dx_1} \cdot \cos(\alpha + \rho) \quad (J-67)$$

The torsional part $d\theta$ is neglected.

b. Equations of Motion and Boundary Conditions

The differential equations of motion for the two individual straight sections are given by:

For $0 \leq x_1 \leq L_1$

$$\rho A \frac{\partial^2 w_1(x_1, t)}{\partial t^2} + EI \frac{\partial^4 w_1(x_1, t)}{\partial x_1^4} = 0 \quad (J-68)$$

For $0 \leq x_2 \leq L_2$

$$\rho A \frac{\partial^2 w_2(x_2, t)}{\partial t^2} + EI \frac{\partial^4 w_2(x_2, t)}{\partial x_2^4} = 0 \quad (J-69)$$

The associated boundary conditions are:

$$\underline{\text{At } x_1 = 0}$$

$$w_1(0, t) = 0 \quad (J-70)$$

$$\frac{\partial^2 w_1(0, t)}{\partial x_1^2} = 0 \quad (J-71)$$

$$\underline{\text{At } x_1 = L_1 \text{ or } x_2 = 0 \text{ (point B)}}$$

$$w_1(L_1, t) = w_2(0, t) \quad (J-72)$$

In addition, from equation B-67:

$$\frac{\partial w_1(L_1, t)}{\partial x_1} = \frac{1}{\cos(\alpha + \beta)} \frac{\partial w_2(0, t)}{\partial x_2} \quad (J-73)$$

Further, according to equation J-62 the bending moments are related by:

$$\frac{\partial^2 w_1(L_1, t)}{\partial x_1^2} = \cos(\alpha + \beta) \frac{\partial^2 w_2(0, t)}{\partial x_2^2} \quad (J-74)$$

Finally, compatibility of shear forces gives:

$$\frac{\partial^3 w_1(L_1, t)}{\partial x_1^3} = \frac{\partial^3 w_2(0, t)}{\partial x_2^3} \quad (J-75)$$

$$\underline{\text{At } x_2 = L_2}$$

$$w_2(L_2, t) = 0 \quad (J-76)$$

$$\frac{\partial^2 w_2(L_2, t)}{\partial x_2^2} = 0 \quad (J-77)$$

c. Shape Functions and Characteristic Equation

The partial differential equations J-68 and J-69 are also solved by the method of separation of variables, which assumes that:

$$w_1(x_1, t) = W_1(x_1) e^{i\omega_w t} \quad (J-78)$$

$$w_2(x_2, t) = W_2(x_2) e^{i\omega_w t} \quad (J-79)$$

Where ω_w denotes the common out-of-plane vibration frequency of the total beam. Substitution of the above into J-68 and J-69 results in the following:

For $0 \leq x_1 \leq L_1$

$$\frac{d^4 W_1(x_1)}{dx_1^4} - \lambda_w^4 W_1(x_1) = 0 \quad (J-80)$$

For $0 \leq x_2 \leq L_2$

$$\frac{d^4 W_2(x_2)}{dx_2^4} - \lambda_w^4 W_2(x_2) = 0 \quad (J-81)$$

where

$$\lambda_w^4 = \frac{\rho A \omega_w^2}{EI} \quad (J-82)$$

Substitution of the solution forms, according to equations J-78 and J-79 into equations J-70 through J-77 furnishes the boundary conditions for equations J-80 and J-81.

At $x_1 = 0$

$$W_1(0) = 0 \quad (J-83)$$

$$W_1''(0) = 0 \quad (J-84)$$

At $x_1 = L_1$ or $x_2 = 0$

$$W_1(L_1) = W_2(0) \quad (J-85)$$

$$W_1'(L_1) = [1/\cos(\alpha+\beta)] W_2'(0) \quad (J-86)$$

$$W_1''(L_1) = \cos(\alpha+\beta) W_2''(0) \quad (J-87)$$

$$W_1'''(L_1) = W_2'''(0) \quad (J-88)$$

At $x_2 = L_2$

$$W_2(L_2) = 0 \quad (J-89)$$

$$W_2''(L_2) = 0 \quad (J-90)$$

The general solutions of equations J-80 and J-81 are given by:

$$W_1(x_1) = B_{11} \sin \lambda_w x_1 + B_{12} \cos \lambda_w x_1 + B_{13} \sinh \lambda_w x_1 \\ + B_{14} \cosh \lambda_w x_1 \quad (J-91)$$

$$W_2(x_2) = B_{21} \sin \lambda_w x_2 + B_{22} \cos \lambda_w x_2 + B_{23} \sinh \lambda_w x_2 \\ + B_{24} \cosh \lambda_w x_2 \quad (J-92)$$

Applications of the boundary conditions J-83 through J-90, to equations J-91 and J-92 gives the nth shape functions $W_{1n}(x_1)$ and $W_{2n}(x_2)$ (up to the arbitrary constant B_{11n} which has been chosen to be unity), as well as the characteristic equation.

$$W_{1n}(x_1) = \sin \lambda_{wn} x_1 - \frac{\Gamma_{6n}}{\Gamma_{7n}} \sinh \lambda_{wn} x_1 \quad (J-93)$$

$$W_{2n}(x_2) = \Delta_{5n} \sin \lambda_{wn} x_2 + \Delta_{6n} \cos \lambda_{wn} x_2 + \Delta_{7n} \sinh \lambda_{wn} x_2 \\ + \Delta_{8n} \cosh \lambda_{wn} x_2 \quad (J-94)$$

where:

$$\Delta_{5n} = \frac{1}{2} \left[\cos \lambda_{wn} L_1 (1 + \sec(\alpha + \beta)) + \frac{\Gamma_{6n}}{\Gamma_{7n}} \cosh \lambda_{wn} L_1 (1 - \sec(\alpha + \beta)) \right] \quad (J-95)$$

$$\Delta_{6n} = \frac{1}{2} \left[\sin \lambda_{wn} L_1 (1 + \sec(\alpha + \beta)) - \frac{\Gamma_{6n}}{\Gamma_{7n}} \sinh \lambda_{wn} L_1 (1 - \sec(\alpha + \beta)) \right] \quad (J-96)$$

$$\Delta_{7n} = \frac{1}{2} \left[\cos \lambda_{wn} L_1 (\sec(\alpha + \beta) - 1) - \frac{\Gamma_{6n}}{\Gamma_{7n}} \cosh \lambda_{wn} L_1 (1 + \sec(\alpha + \beta)) \right] \quad (J-97)$$

$$\Delta_{\theta\eta} = \frac{1}{2} \left[\sin \lambda_{w\eta} L_1 \cdot (1 - \sec(\alpha + \beta)) - \frac{\Gamma_{6\eta}}{\Gamma_{7\eta}} \sinh \lambda_{w\eta} L_1 \cdot (1 + \sec(\alpha + \beta)) \right] \quad (J-98)$$

$$\Gamma_{6\eta} = -(1 + \sec(\alpha + \beta)) \cdot \sin \lambda_{w\eta} (L_1 + L_2) + (1 - \sec(\alpha + \beta)) \left[\sin \lambda_{w\eta} L_1 \cdot \cosh \lambda_{w\eta} L_2 - \cos \lambda_{w\eta} L_1 \cdot \sinh \lambda_{w\eta} L_2 \right] \quad (J-99)$$

$$\Gamma_{7\eta} = (1 + \sec(\alpha + \beta)) \cdot \sin \lambda_{w\eta} (L_1 + L_2) + (1 - \sec(\alpha + \beta)) \left[\sin \lambda_{w\eta} L_2 \cdot \cosh \lambda_{w\eta} L_1 - \cos \lambda_{w\eta} L_2 \cdot \sinh \lambda_{w\eta} L_1 \right] \quad (J-100)$$

Finally, the characteristic equation is given by:

$$\left\{ \left[(1 + \sec(\alpha + \beta)) \cdot \sin \lambda_{w\eta} (L_1 + L_2) + (1 - \sec(\alpha + \beta)) \cdot \left[\sin \lambda_{w\eta} L_1 \cdot \cosh \lambda_{w\eta} L_2 - \cos \lambda_{w\eta} L_1 \cdot \sinh \lambda_{w\eta} L_2 \right] \right] \right. \\ \left. \left/ \left[(1 + \sec(\alpha + \beta)) \cdot \sinh \lambda_{w\eta} (L_1 + L_2) + (1 - \sec(\alpha + \beta)) \cdot \left[\cos \lambda_{w\eta} L_2 \cdot \sinh \lambda_{w\eta} L_1 - \cosh \lambda_{w\eta} L_1 \cdot \sin \lambda_{w\eta} L_2 \right] \right] \right\} \right. \\ \left. = \left\{ \frac{\Gamma_{6\eta}}{\Gamma_{7\eta}} \right\} \right. \quad (J-101)$$

d. Orthogonality Condition

The orthogonality condition for the out-of-plane vibrations is obtained in a manner similar to the one used for that of the in-plane vibrations. With ω_{wn} and ω_{wm} the two distinct roots of the characteristic equation J-100 as well as W_{1n} , W_{2n} and W_{1m} , W_{2m} , respectively, representing the two sets of characteristic functions, equations J-80 and J-81 become:

$$\frac{d^4 W_m}{dx^4} = \lambda_{wm}^4 W_m \quad (J-102)$$

$$\frac{d^4 W_n}{dx^4} = \lambda_{wn}^4 W_n \quad (J-103)$$

with:

$$\left. \begin{aligned} W_m &= \begin{cases} W_{1m} & 0 \leq x_1 \leq L_1 \\ W_{2m} & 0 \leq x_2 \leq L_2 \end{cases} \\ W_n &= \begin{cases} W_{1n} & 0 \leq x_1 \leq L_1 \\ W_{2n} & 0 \leq x_2 \leq L_2 \end{cases} \end{aligned} \right\} \quad (J-104)$$

Equation J-102 is now multiplied by W_n and equation J-103 by W_m .

When the results are subtracted from each other one finds:

$$W_m \frac{d^4 W_m}{dx^4} - W_m \frac{d^4 W_m}{dx^4} = (\lambda_{W_m}^4 - \lambda_{W_m}^4) \cdot W_m \cdot W_m \quad (J-105)$$

The above expression is now integrated over the total length of the coupler and the subscripts of the equation are adjusted according to equation J-104, i.e.,

$$\begin{aligned} & \int_{x_1=0}^{L_1} \left[W_{m_1} \frac{d^4 W_{m_1}}{dx^4} - W_{m_1} \frac{d^4 W_{m_1}}{dx^4} \right] dx_1 + \int_{x_2=0}^{L_2} \left[W_{m_2} \frac{d^4 W_{m_2}}{dx^4} - W_{m_2} \frac{d^4 W_{m_2}}{dx^4} \right] dx_2 \\ & = (\lambda_{W_m}^4 - \lambda_{W_m}^4) \left\{ \int_{x_1=0}^{L_1} W_{m_1} W_{m_1} dx_1 + \int_{x_2=0}^{L_2} W_{m_2} W_{m_2} dx_2 \right\} \end{aligned} \quad (J-106)$$

Performing each of the integrations of the left hand side twice by parts, and substituting the required boundary terms according to equations J-83 through J-90, gives:

$$(\lambda_{W_m}^4 - \lambda_{W_m}^4) \left\{ \int_{x_1=0}^{L_1} W_{m_1} W_{m_1} dx_1 + \int_{x_2=0}^{L_2} W_{m_2} W_{m_2} dx_2 \right\} = 0 \quad (J-107)$$

Equation J-107 represents the orthogonality condition, which may be rewritten in the following form:

$$\int_{x_1=0}^{L_1} W_{m_1} W_{n_1} dx_1 + \int_{x_2=0}^{L_2} W_{m_2} W_{n_2} dx_2 = \delta_{n,m} W_m \quad (J-108)$$

Where $\delta_{n,m}$ is the Kronecker delta and W_m represents the value of the expression when $m = n$.

APPENDIX K: ANGULAR ENCODER DATA ACQUISITION AND EVALUATION PROCEDURE

1. DATA ACQUISITION SYSTEM

The angular encoder data acquisition system is arranged such that the micro-second clock starts counting, in any cycle, whenever the throwblock angle ϕ_1 is at zero degrees. Counting is initialized by the preset zero-degree signal of the drive shaft encoder (see figures 7.7 and 7.8 in Section VII).

Subsequent time values, associated with angle ϕ_1 , are stored for later printout whenever an additional encoder interval of 0.036 degrees is passed. (All encoders furnish 10,000 signals per revolution, both forward and reverse, in addition to settable zero signals). The rocker angle ϕ_4 is correlated with the angle ϕ_1 by way of the clutch input zero signal, which is preset to $\phi_4 = 90$ degrees. This marker causes the time associated with this angle to be stored. This type of timing occurs twice during each cycle, i.e. during the feed as well as during the return stroke. Two separate channels of the clutch input encoder cause the times associated with each forward or return interval of 0.036 degrees of the rocker to be stored by the system.

The following first describes in which channels of the system memory the various time data are stored. Subsequently, it is shown by way of typical examples how these data are evaluated. Finally, a table giving throwblock, rocker and feed-roller angles as functions of time, as obtained by this evaluation procedure, is presented.

Appendix L shows a complete printout of the encoder data acquisition system for a single cycle*. It provides the data for the examples in the present appendix.

2. DATA STORAGE AND PRINTOUT

The seven channels of memory of the data acquisition system store the time values of the various angles for the subsequent printing as follows (this storage is available for ten successive cycles):

Channel 7: ϕ_1 , forward (ccw) motion of the throwblock; starting with $\phi_1 = 0$ degrees.

Channel 6: ϕ_1 , reverse (cw) motion of the throwblock; starting with $\phi_1 = 0$ degrees.

Channel 1: $\phi_4 = 90$ degree reference for the rocker angle. The first signal occurs during the reverse motion of the rocker, while the second during its feeding motion.

Channel 2: ϕ_4 , forward (feed) motion of the rocker.

Channel 3: ϕ_4 , reverse motion of the rocker.

Channel 4: ϕ_5 , forward (feed) motion of the feed rollers.

Channel 5: ϕ_5 , reverse motion of feed rollers (if it occurs).

* This data was obtained with the mechanism having the following input crank and coupler link dimensions: $a_1 = 0.750$ in. and $AC = 31.965$ in., respectively. Note that these are not the same dimensions as in the rest of the dissertation.

3. EVALUATION OF DATA PRINTOUT

a. Key events during Rocker Cycle

Figure K-1 gives graphical description of the key timing events during the rocker cycle which are used to determine the relationship between the angles ϕ_1 and ϕ_4 . These events are enumerated below with reference to the data printout given in appendix L.

- $T_1 = 0$ represents the start of the time count and corresponds to $\phi_1 = 0$. (First value of channel 7 printout on page L-6)
- T_2 = time near end of forward motion of rocker. (Last value of channel 2 output on page L-2)
- T_3 = time near the beginning of reverse motion of rocker. (First value of channel 3 output on page L-2)
- T_4 = time corresponding to $\phi_4 = 90$ degrees during reverse motion of rocker. (In channel 3 output on page L-3)
- T_5 = time near the end of the reverse motion of the rocker. (Last value of channel 3 output on page L-3)
- T_6 = time near the beginning of the forward motion of the rocker. (First value of channel 2 output on page L-3)
- T_7 = time corresponding to $\phi_4 = 90$ degrees during the forward motion of the rocker. (In channel 2 output on page L-3. See discussion in example)

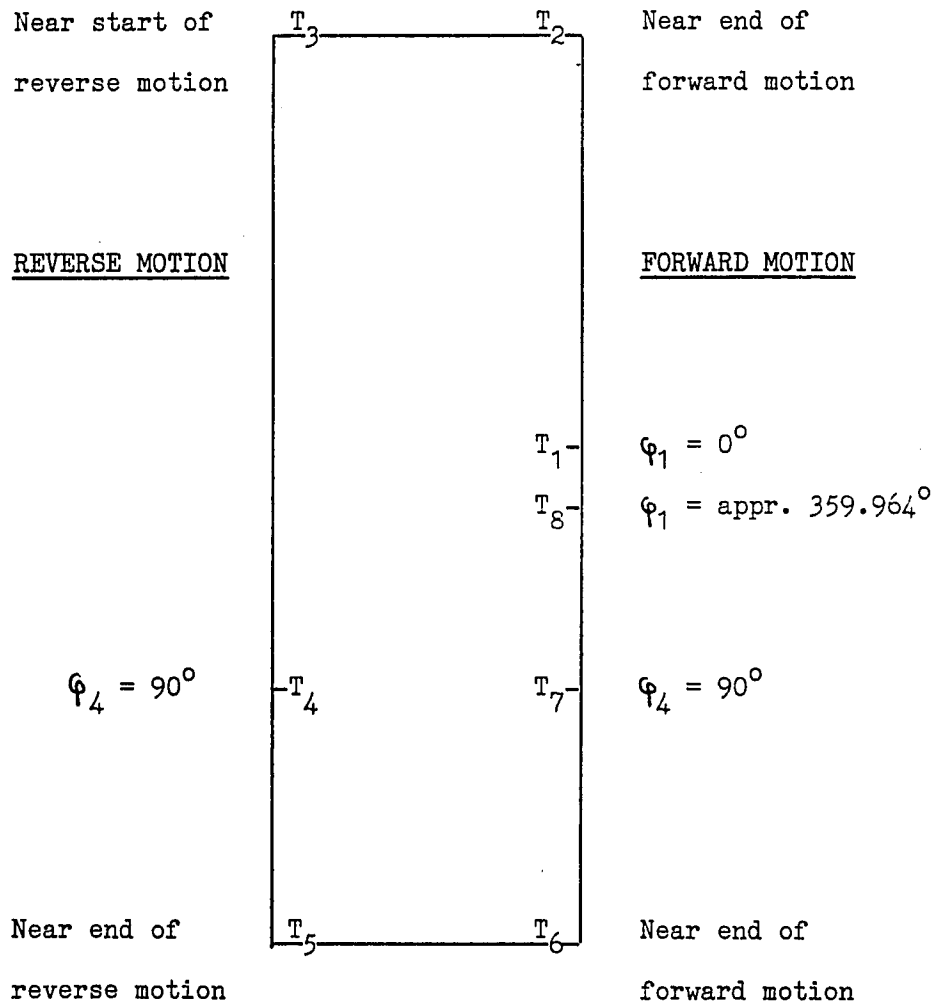


Figure K-1 .

Key Timing Events in Rocker Cycle

b. Determination of Rocker Angle ϕ_4 as a
Function of Throwblock Angle ϕ_1

1. Angle $\phi_4(\phi_1)$ at T_8

Channel 1, as listed on top of page L-1, furnishes T_4 and T_7 , the times associated with the 90-degree rocker reference angle. The first data position and all subsequent odd-numbered ones give the reference times for the reverse motion of the rocker, i.e. T_4 . The second data position and all subsequent even-numbered ones, refer to those times T_7 , when $\phi_4 = 90^\circ$ during the forward stroke of the rocker. Thus, $T_7 = .237412$ (seconds) during the first forward motion.

The channel 2 output on page L-1 lists times associated with 0.036° intervals of angle ϕ_4 between T_7 and T_8 . The last item in this series represents T_8 . It occurs near the time when ϕ_1 is at 359.964° , i.e one encoder interval before $\phi_1 = 360^\circ = 0^\circ$, when the time count starts anew.

To determine the exact angular position of ϕ_4 at T_8 , one must count the number of signal intervals between T_7 and T_8 , multiply these by 0.036 degrees per interval, and subtract the result from 90° . To find the associated angle ϕ_1 at T_8 , the time value of ϕ_1 must be consulted. This is accomplished as follows:

From channels 1 and 2 :	T_7	= .237412
From channel 2 :	T_8	= .298583

The number of signal intervals between these time values is 308.

Thus,

$$\phi_4(T_8) = 90^\circ - 308 \times 0.036^\circ = 78.912^\circ.$$

The channel 2 output on page L-21 indicates that the time at $\phi_1 = 359.964$ is equal to .298584 (seconds). This is essentially identical with T_8 . Therefore:

$$\phi_4(\phi_1 = 359.964^\circ) = 78.912^\circ$$

2. Angle $\phi_4(\phi_1 = 0)$ at T_1

To obtain the time T_1 for $\phi_1 = 360^\circ$ in terms of the earlier cycle, one adds the time corresponding to two previous intervals* to the time, associated with $\phi_1 = 360^\circ - 0.072^\circ = 359.928^\circ$ (see page L-21). Then,

$$T_1 = .298547 + (.298547 - .298487) = .298607 \text{ (seconds).}$$

To find the corresponding value of ϕ_4 , note the channel 2 data on the last line of page L-1 and compare them to the time data associated with the angle ϕ_1 as established above. This comparison yields the table below:

Time	.298221	.298583	.298607
ϕ_4	78.984°	78.912°	?
ϕ_1		359.964°	360° = 0°

* It is best to use two intervals due to the nature of the Schmitt trigger in the encoders [51].

Extrapolation then furnishes:

$$\phi_4(\phi_1 = 0^\circ) = 78.912 + \frac{(78.912^\circ - 78.984^\circ)(.298607 - .298583)}{.298583 - .298221}$$

or,

$$\phi_4(\phi_1 = 0^\circ) = 78.907^\circ$$

3. Angle $\phi_4(\phi_1)$ between T_1 and T_2

As an example for the determination of ϕ_4 as a function of ϕ_1 between T_1 and T_2 , let it be required to find ϕ_4 when ϕ_1 is approximately 10° . Note that because of the size of the encoder intervals one cannot obtain integral values of ϕ_1 without interpolation.

According to channel 7 data on page L-6, the time associated with $\phi_1 = 10.008^\circ$, i.e. 278 intervals past $\phi_1 = 0^\circ$, equals

$$t(\phi_1 = 10.008^\circ) = .008618 \text{ (sec)}$$

To obtain the corresponding angle ϕ_4 , the following channel 2 data of page L-2 are juxtaposed:

Time	Angle ϕ_4
.000216	$78.912^\circ - .036^\circ = 78.876^\circ$
.008495	$78.876^\circ - 44 \times .036^\circ = 77.292^\circ$
.008634	$78.876^\circ - 45 \times .036^\circ = 77.256^\circ$

Interpolation then furnishes:

$$\phi_4(\phi_1 = 10.008^\circ) = 77.292^\circ + \frac{(77.256^\circ - 77.292^\circ)(.008618 - .008495)}{.008634 - .008495}$$

or,

$$\phi_4(\phi_1 = 10.008^\circ) = 77.260^\circ$$

4. Angles $\phi_4(\phi_1)$ at T_2 and T_3

The last value for the forward motion is given by channel 2 on page L-2 as $T_2 = .057684$ (seconds). Its displacement corresponds to 197 intervals past $\phi_4 = 78.876^\circ$. Thus,

$$\phi_4(T_2) = 78.876^\circ - 197 \times .036^\circ = 71.784^\circ$$

The first reverse data point, as given by channel 3 on page L-2, shows $T_3 = .067192$ (sec). Its location must be found by way of $\phi_4 = 90$ degrees at T_4 , together with the number of intervals between T_4 and T_3 . According to the channel 1 data on page L-1, $T_4 = .186634$ (sec). Further, channel 3 shows on pages L-2 and L-3 that the number of intervals from T_4 back to T_3 equals 506. Then,

$$\phi_4(T_3) = 90^\circ - 506 \times .036^\circ = 71.784^\circ$$

This is identical to $\phi_4(T_2)$.

Because of the approximate .0095 (seconds) time difference between T_2 and T_3 , one may assume that the minimum angle of the rocker is $.036/2$ degrees less than 71.784° , i.e.

$$\phi_{4\min} = 71.766^\circ \quad (K-1)$$

The associated angle ϕ_1 is found from an assumed average time value for $\phi_{4\min}$:

$$t(\phi_{4\min}) = T_2 + .5(T_3 - T_2) = .057684 + .5(.067192 - .057684)$$

or,

$$t(\phi_{4\min}) = .062438 \text{ (seconds)} \quad (K-2)$$

The corresponding value of angle ϕ_1 must now be found by way of interpolated channel 7 data of page L-9:

Time	Angle ϕ_1
.062381	$2102 \times .036^\circ = 75.672^\circ$
.062442	$2104 \times .036^\circ = 75.744^\circ$

Interpolation then furnishes:

$$\phi_1(\phi_{4\min}) = 75.672^\circ + \frac{(75.744^\circ - 75.672^\circ)(.062438 - .062381)}{.062442 - .062381}$$

or,

$$\phi_1(\phi_{4\min}) = 75.739^\circ$$

5. Angle $\phi_4(\phi_1)$ between T_3 and T_5

To obtain the angle ϕ_4 as a function of ϕ_1 in the range between T_3 and T_5 , one uses an adaptation of the interpolation procedure described in section K-3-b-3 earlier. Again, one first determines the time associated with an approximately integral value of the angle ϕ_1 with the help of the channel 7 output. Subsequently, the angle ϕ_4 , whose time value coincides with that of angle ϕ_1 , is found by interpolation. The latter time and angle data are supplied by the channel 3 output on pages L-2 and L-3.

6. Angles $\phi_4(\phi_1)$ at T_5 and T_6

The last reverse motion data point on channel 3 on page L-3 gives $T_5 = .208705$ (seconds). It corresponds in position to 37 intervals after $\phi_4 = 90^\circ$ at T_4 . Therefore,

$$\phi_4(T_5) = 90^\circ + 37 \times .036^\circ = 91.332^\circ$$

The first forward motion data point of channel 2 on page L-3 gives $T_6 = .215579$ (sec). Its location is found with the help of the 90° angle of ϕ_4 at T_7 and the number of intervals between T_7 and T_6 .

According to channel 1 data on page L-1, $T_7 = .237408$ sec, in the second cycle. The number of intervals from this T_7 back to T_6 equals 37. Then, as above:

$$\phi_4(T_6) = 90^\circ + 37 \times .036^\circ = 99.332^\circ$$

Because of the approximate .007 second difference between T_5 and T_6 , it is assumed that the maximum rocker angle is $.018^\circ$, i.e. one-half interval, larger than 91.332° . Thus,

$$\phi_{4\max} = 91.350^\circ \quad (K-3)$$

The associated angle ϕ_1 is found from an assumed average time value for $\phi_{4\max}$:

$$t(\phi_{4\max}) = T_5 + .5(T_6 - T_5) = .208705 + .5(.215579 - .208705)$$

or,

$$t(\phi_{4\max}) = .212142 \text{ (seconds)}$$

The corresponding value for ϕ_1 is now found by interpolation of the following channel 7 data on page L-16:

Time	Angle ϕ_1
.212127	$7138 \times .036^\circ = 256.968^\circ$
.212183	$7140 \times .036^\circ = 257.040^\circ$

Then,

$$\phi_1(\phi_{4\max}) = 256.968^\circ + \frac{(257.040^\circ - 256.968^\circ)(.212142 - .212127)}{.212183 - .212127}$$

or,

$$\phi_1(\phi_{4\max}) = 256.987^\circ$$

7. Angle $\phi_4(\phi_1)$ between T_6 and T_8

To obtain the rocker angles ϕ_4 as functions of the throwblock angle ϕ_1 in the range between T_6 and T_8 , one again employs the interpolation procedure first shown in section K-3-b-3: Initially, the time corresponding to an approximately intergal value of ϕ_1 is found from the data of channel 7. Subsequently, the associated angle ϕ_4 is determined by interpolation of the time data of channel 2 on page L-3.

c. Determination of the Feed Roller Angle ϕ_5 as a Function of the Throwblock Angle ϕ_1

The forward time values of the feed roller angle ϕ_5 between T_6 and T_2 are given by the two channel 4 outputs listed on page L-4. As can be seen from the channel 5 data on page L-5, the feed rollers experience no reverse motion during the present run.

1. Initial angle $\phi_1(\phi_{5in})$

The initial forward motion of angle ϕ_5 has occurred at $t = .224517$ (sec). It is assumed that, this data point represents a one-half interval of motion, i.e.

$$\phi_{5in} = .018^\circ$$

The corresponding value of ϕ_1 for this initial value of ϕ_5 is found by interpolation of the following channel 7 data on page L-17:

Time	Angle ϕ_1
.224511	271.908°
.224570	271.980°

Then,

$$\phi_1(\phi_{5in}) = 271.908^\circ + \frac{(271.980^\circ - 271.908^\circ)(.224517 - .224511)}{.224570 - .224511}$$

or,

$$\phi_1(\phi_{5in}) = 271.915^\circ$$

2. Angles $\phi_5(\phi_1)$ during the Remainder
of the Forward Motion of the Rocker.

The remaining angular displacements of the feed roller angle during the forward stroke of the rocker are found for those values of ϕ_1 , for which determinations of angle ϕ_4 have been made. To this end, the interpolation procedure of section K-3-b-3 is adapted. For example,

$$t(\phi_1 = 280.008^\circ) = .2312263 \text{ sec, according to the channel 7}$$

output on page L-17

This is applied to the following channel 4 data, as found on page L-4:

Time	Angle ϕ_1
.231178	.342°
.231958	.414°

Then,

$$\phi_5(\phi_1 = 280.008^\circ) = .342^\circ + \frac{(.414^\circ - .342^\circ)(.231263 - .231178)}{.231958 - .231178}$$

or,

$$\phi_5(\phi_1 = 280.008^\circ) = .352^\circ$$

3. Overshooting of Feed Rollers

The feed rollers are considered to have overshoot, whenever they move beyond the rocker angle $\phi_{4\min}$, as given by equation K-1. This can be judged by noting the last data point for ϕ_5 , as given by the channel 4 output on page L-4, and comparing it to $t(\phi_{4\min})$ which is given by equation K-2.

For the present data, the last time value of ϕ_5 equals .058439 seconds, while $t(\phi_{4\min}) = .062438$ seconds. Therefore, the motion of ϕ_5 has either ceased before or at the time when ϕ_4 reached its minimum value.

d. Determination of Angle ϕ_{54} as a function of Angle ϕ_1

The angle ϕ_{54} , which expresses the difference in the angular displacement of the feed rollers and the rocker is defined as:

$$\phi_{54}(\phi_1) = \phi_5(\phi_1) - \phi_4^*(\phi_1) \quad (K-5)$$

where, with equation K-3:

$$\phi_4^*(\phi_1) = \phi_{4\max} - \phi_4(\phi_1) \quad (K-6)$$

The angles $\phi_5(\phi_1)$ and $\phi_4(\phi_1)$ were originally determined for identical values of the throwblock angle ϕ_1 .

4. TYPICAL SET OF DATA

Table K-1 shows a typical set of encoder data, which were obtained by the evaluation procedures described above. It gives the throwblock, rocker and feed roller angles as functions of time.

Time (microseconds)	ϕ_1 (degrees)	ϕ_{4e} (degrees)	ϕ_5 (degrees)
0	0.0	78.907	12.193
8316	10.008	77.318	13.80
16650	20.088	75.808	15.326
24925	30.168	74.516	16.627
33226	40.248	73.443	17.695
41570	50.328	72.608	18.550
49942	60.408	72.050	19.098
58333	70.488	71.766	19.384
62526	75.528	71.765	19.386
66711	80.568	71.771	0.0
75058	90.648	72.059	0.0
83371	100.728	72.700	0.0
91680	110.808	73.620	0.0
100005	120.888	74.729	0.0
108362	130.968	76.104	0.0
116737	141.048	77.593	0.0
125133	151.128	79.177	0.0
133541	161.208	80.885	0.0
141961	171.288	82.579	0.0
150395	181.368	84.260	0.0
158826	191.448	85.858	0.0
167239	201.528	87.343	0.0
175600	211.608	88.630	0.0
183920	221.688	89.700	0.0
192229	231.768	90.543	0.0
200535	241.848	91.077	0.0
208851	251.928	91.334	0.0
213017	256.968	91.350	0.0
217191	262.008	91.260	0.0
225557	272.088	90.996	0.036
233975	282.168	90.415	0.62
242425	292.248	89.347	1.739
250876	302.328	88.205	2.855
259308	312.408	86.817	4.228
267645	322.488	85.194	5.906
275908	332.568	83.576	7.502
284205	342.648	81.856	9.216
292861	352.728	80.086	11.052

Table K-1

Encoder Data for Feed Run at 200 rpm, with 1.000 x 0.125 inch strip size, crank length $a_1 = 0.750$ in., Coupler Length AC = 31.965 in., $T_B = 60$ lb-in. See footnote on page 305.

APPENDIX L: SAMPLE OUTPUT OF ANGULAR ENCODER DATA ACQUISITION SYSTEM

CHANNEL 1 : Time for $\varphi_4 = 90^\circ$ (1st, 3rd, ... odd: T_4
2nd, 4th, ... even: T_7)

	T_4	T_7								
237121	186668	237412	186634	237408	186587	237402	186593	237268	186398	
237051	186768	237209	186882	237283	186520	237375	186381	237250	186205	
000000	000000	000000	000000	000000	000000	000000	000000	000000	000000	
000000	000000	000000	000000	000000	000000	000000	000000	000000	000000	
000000	000000	000000	000000	000000	000000	000000	000000	000000	000000	
000000	000000	000000	000000	000000	000000	000000	000000	000000	000000	
000000	000000	000000	000000	000000	000000	000000	000000	000000	000000	
000000	000000	000000	000000	000000	000000	000000	000000	000000	000000	
000000	000000	000000	000000	000000	000000	000000	000000	000000	000000	
000000	000000	000000	000000	000000	000000	000000	000000	000000	000000	
000000	000000	000000	000000	000000	000000	000000	000000	000000	000000	

CHANNEL 2 : Time for φ_4 between T_7 and T_8

							T_7			
238337	238512	238873	239067	239431	239605	239968	237412	237776	237971	
241060	241253	241615	241790	242152	242346	242710	240163	240524	240698	
243809	243986	244350	244548	244915	245092	245457	242887	243249	243444	
246558	246754	247115	247291	247649	247841	248194	245654	246018	246195	
249248	249417	249758	249944	250285	250450	250785	248166	248714	248901	
251789	251967	252293	252451	252772	252745	253260	250967	251300	251461	
254191	254338	254635	254796	255087	255228	255514	253412	253721	253889	
256360	256781	000000	256913	257179	257324	257588	255668	255950	256085	
258373	258500	258755	258894	259147	259271	259522	257717	257976	258116	
260282	260418	260665	260787	261033	261167	261411	259659	259910	260033	
262153	262272	262514	262647	262889	263008	263249	261532	261777	261911	
263981	264113	264356	264476	264718	264850	265092	263381	263622	263741	
265830	265951	266194	266328	266570	266690	266934	265212	265455	265588	
267674	267807	268049	268169	268411	268543	268785	267068	267311	267431	
269522	269642	269885	270018	270260	270380	270623	268905	269147	269280	
271364	271498	271741	271863	272106	272241	272485	270757	271000	271121	
273225	273346	273590	273724	273966	274087	274330	272606	272848	272982	
275069	275203	275442	275561	275799	275931	276169	274464	274707	274828	
276890	277007	277242	277371	277604	277721	277954	276288	276524	276655	
278657	278785	279013	279128	279355	279481	279707	278082	278312	278428	
280392	280504	280728	280851	281073	281184	281405	279707	279920	280169	
282078	282200	282419	282529	282747	282868	283085	281528	281748	281859	
283752	284079	000000	284200	284418	284528	284746	283195	283413	283534	
285413	285534	285751	285861	286079	286202	286421	284867	285084	285194	
287094	287206	287428	287551	287773	287885	288109	286532	286751	286874	
288797	288923	289150	289265	289492	289619	289846	288234	288458	288572	
290547	290663	290894	291023	291253	291370	291602	289762	290191	290318	
292311	292440	292673	292791	293023	293153	293386	291731	291962	292079	
294101	294220	294455	294586	294820	294938	295172	293504	293738	293869	
295896	296028	296265	296385	296623	296755	296993	295304	295540	295660	
000000	297847	298087	298221	298461	298583		297114	297354	297726	

T_8

159410	159645	159823	160059	160223	160461	160642	160882	161049	161291
161473	161716	161884	162126	162309	162552	162721	162964	163147	163390
163559	163803	163986	164230	164399	164644	164828	165072	165241	165487
165672	165918	166089	166337	166525	166775	166948	167201	167390	167645
167821	168078	168270	168527	168704	168907	169156	169415	169594	169855
170051	170315	170497	170765	170965	171235	171420	171693	171896	172167
172357	172632	172839	173119	173313	173814	000000	174105	174306	174601
174820	175116	175318	175616	175838	176139	176346	176651	176878	177186
177396	177706	177936	178248	178462	178780	179016	179337	179556	179885
180128	180459	180685	181022	181272	181615	181850	182201	182460	182816
183056	183415	183679	184041	184288	184659	184925	185312	185571	185958
186243	186634	186900	187304	187605	188017	188300	188726	189041	189475
189773	190223	190558	191020	191337	191821	192183	192687	193037	193574
193976	194536	194925	195530	195991	196646	197107	198776	000000	199165
199722	200602	201285	202289	203021	204274	205275	207001	<u>208705</u>	

T_5

CHANNEL 2: Time for Φ_4 between T_6 and T_8

				<u>215579</u>	216779	218689	219551	221017	221656
222902	223529	224665	225207	226292	226855	227831	228267	229104	229520
230235	230551	231167	231481	232035	232292	232797	233064	233537	233758
234205	234437	234863	235064	235467	235682	236078	236264	236648	236852
237228	<u>237408</u>	237778	237974	238340	238517	238879	239073	239437	239612
239973	240169	240534	240710	241071	241268	241633	241808	242172	242369
242733	242910	243275	243470	243835	244012	244375	244569	244932	245106
245462	245653	246009	246181	246533	246724	247079	247250	247601	247791
248140	248309	248656	248843	249187	249354	249697	249882	250221	250386
250721	250900	251228	251387	251709	251883	252202	252357	252672	252841
253152	253303	253607	253771	254071	254218	254513	254672	254964	255106
255393	255548	255832	255972	256252	256403	256677	256812	257084	257232
257499	257630	257895	258038	258427	000000	258685	258824	259080	259204
259456	259593	259842	259965	260214	260349	260596	260717	260962	261097
261339	261459	261701	261834	262076	262195	262437	262570	262812	262932
263176	263309	263551	263671	263913	264045	264287	264407	264648	264780
265022	265141	265383	265515	267757	265877	266119	266251	266492	266610
266851	266982	267223	267341	267581	267712	267952	268070	268310	268441
268681	268799	269039	269170	269410	269529	269770	269902	270143	270263
270506	270639	270883	271003	271247	271381	271624	271745	271989	272123
272366	272487	272731	272865	273108	273229	273472	273606	273848	273968
274209	274342	274583	277702	274942	275074	275313	275432	275670	275801
276037	276156	276392	276523	276758	276876	277111	277241	277475	277592
277826	277955	278188	278304	278536	278665	278896	279011	279242	279369
279597	279712	279940	280066	280294	280407	280633	280759	280984	281096
281321	281445	281668	281780	282004	282127	282349	282460	282681	282804
283025	283136	283357	283480	283701	283812	284033	284157	284378	284489
284711	284834	285056	285168	285390	285736	000000	285848	286072	286197
286420	286533	286757	286882	287106	287219	287444	287569	287794	287907
288132	288258	288483	288597	288822	288947	289172	289295	289510	289636
289861	289975	290201	290327	290552	290667	290894	291021	291249	291364
291593	291721	291949	292065	292295	292423	292654	292771	293003	293132
293364	293482	293715	293846	294079	294197	294432	294563	294797	294915
295150	295282	295517	295637	295873	296005	296242	296362	296599	296732
296969	297090	297329	297463	297701	297822	298063	298197	298438	<u>298560</u>

T_8

CHANNEL 4: Time for ϕ_5 between T_6 and T_8

224517	226629	227395	228256	228817	229387	229823	230363	230723	231178
231528	231958	232314	232706	233007	233402	233723	234077	234354	234724
235016	235344	235603	235954	236225	236530	236776	237106	237362	237653
237887	238199	238442	238723	238946	239243	239479	239753	239971	240258
240488	240758	240972	241253	241478	241745	241958	242240	242467	242734
42949	243235	243466	243738	243958	244251	244488	244769	244996	245299
245545	245837	246073	246386	246640	246943	247511	000000	247773	248081
248330	248658	248922	249233	249484	249811	250072	250377	250621	250936
251186	251478	251711	252010	252248	252523	252743	253022	253245	253502
253707	253969	254179	254421	254617	254865	255063	255294	255482	255722
255919	256147	256324	256550	256738	256960	257138	257365	257549	257763
257938	258165	258346	258552	258720	258937	259114	259321	259490	259704
259876	260077	260242	260352	260621	260817	260978	261181	261346	261540
261700	261904	262068	262258	262413	262613	262776	262967	263122	263320
263480	263667	263822	264020	264182	264369	264523	264721	264883	265072
265227	265426	265588	265776	265931	266131	266294	266484	266640	266840
267005	267195	267352	267554	267719	267910	268069	268272	268438	268631
268792	268997	269362	000000	269524	269732	269903	270101	270266	270476
270649	270848	271314	271226	271400	271602	271769	271982	272157	272359
272526	272740	272915	273118	273286	273499	273675	273878	274045	274257
274432	274633	274800	275010	275183	275382	275546	275754	275926	276124
276287	276493	276662	276858	277021	277226	277394	277588	277748	277950
278116	278307	278465	278663	278827	279016	279172	279368	279530	279716
279871	280064	280224	280408	280561	280751	280909	281091	281243	281432
281588	281768	281920	282107	282263	282443	282594	282782	282940	283122
283272	283456	283609	283788	283940	284127	284283	284463	284614	284800
284956	285138	285292	285479	285633	285812	285963	286150	286305	286486
286639	286825	286980	287160	287312	287499	287654	287833	287985	288171
288326	288505	288656	288844	289000	289179	289330	289518	289674	289854
290006	290195	290353	290533	290685	290876	291036	291218	291372	291564
291725	291910	292065	292260	292423	292609	292765	292962	293127	293315
293474	293672	293838	294028	294188	294390	294557	294749	294911	295113
295282	295475	295638	295842	296012	296206	296370	296576	296746	296942
297107	297314	297486	297683	297848	298057	298230	298428	298595	

CHANNEL 4: Time for ϕ_5 between T_1 and T_8

000096	000300	000463	000677	000850	001056	001219	001434	001607	001812
001974	002185	002357	002560	002721	002934	003106	003312	003473	003685
003856	004059	004220	004431	004602	004806	004966	005177	005349	005553
005713	005924	006095	006298	006458	006671	006843	007046	007207	007420
007592	007797	007958	008172	008345	008551	008713	008928	009107	009309
009471	009688	009863	010071	010235	010454	010630	010841	011006	011226
011404	011615	011782	012004	012184	012398	012567	012792	012977	013197
013361	013588	013770	013988	014160	014389	014572	014791	014965	015195
015380	015601	015776	016008	016195	016420	016598	016834	017023	017252
017435	017675	017868	018101	018287	018532	018728	018965	019155	019405
019606	019848	020041	020292	020495	020745	020944	021203	021412	021668
021872	022138	022350	022610	022819	023094	023315	023586	023802	024086
024316	024596	024820	025115	025351	025640	025870	026411	000000	026707
026943	027251	027498	027799	028039	028352	028604	029151	000000	029467
029720	030028	030274	030597	030856	031171	031423	031753	032019	032743
032603	032945	033221	033556	033825	034179	034463	034810	035089	035457
035753	036117	036408	037792	037102	037480	037785	038190	038516	038716
039239	039671	040021	040454	040804	041271	041651	042117	042498	042906
043422	043936	044360	044935	047413	046014	046514	047204	047779	048507
049111	049942	050629	051494	052213	053231	054116	056433	000000	058439

CHANNEL 5: Time for ϕ_5 between T_3 and T_5

```
000000 000000 000000 000000 000000 000000 000000 000000 000000 000000
000000 000000 000000 000000 000000 000000 000000 000000 000000 000000
000000 000000 000000 000000 000000 000000 000000 000000 000000 000000
000000 000000 000000 000000 000000 000000 000000 000000 000000 000000
000000 000000 000000 000000 000000 000000 000000 000000 000000 000000
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CHANNEL 6: Time for ϕ_1 reverse

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CHANNEL 7: Time for ϕ_1 between T_1 and T_8

T_1

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001469	001193	001229	001252	001289	001312	001348	001372	001409	001433
001772	001493	001530	001554	001590	001614	001651	001675	001711	001735
002070	001796	001832	001856	001892	001916	001952	001975	002012	002035
002367	002094	002130	002153	002189	002212	002248	002271	002307	002330
002666	002390	002426	002449	002486	002509	002546	002569	002607	002630
002970	002690	002728	002751	002788	002811	002849	002872	002909	002932
003268	002993	003029	003053	003090	003113	003149	003172	003209	003232
003566	003291	003327	003350	003386	003409	003446	003469	003505	003529
003867	003588	003625	003648	003686	003709	003745	003769	003807	003830
004169	003890	003928	003951	003988	004012	004049	004072	004109	004132
004466	004192	004229	004252	004289	004312	004348	004371	004408	004430
004763	004489	004526	004548	004584	004607	004644	004666	004702	004726
005062	004785	004822	004845	004882	004905	004942	004965	005003	005026
005364	005086	005123	005146	005183	005207	005244	005267	005304	005327
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006260	005981	006018	006041	006079	006132	006139	006163	006200	006223
006563	006284	006322	006345	006382	006405	006443	006465	006502	006525
006860	006585	006622	006645	006682	006705	006741	006764	006801	006824
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021713	021736	021773	021795	021832	021855	021892	021914	021950	021973
022032	022055	022092	022128	022150	022186	022209	022246	022268	022291
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026145	026168	026204	026227	026263	026286	026322	026346	026382	026405
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027033	027056	027093	027115	027151	027174	027210	027233	027269	027292
027328	027351	027387	027410	027447	027470	027506	027529	027565	027589
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027923	027946	027982	028005	028041	028064	028100	028123	028160	028182
028218	028241	028278	028300	028336	028359	028396	028418	028454	028478
028514	028537	028571	028596	028633	028656	028692	028715	028752	028775
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035344	035368	035404	035427	035463	035487	035523	035546	035582	035606
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037132	037156	037192	037216	037251	037275	037312	037335	037371	037395
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044593	044617	044652	044676	044712	044736	044771	044795	044831	044855
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065551	065577	065611	065637	065669	065697	065731	065758	065792	065818
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AUTOBIOGRAPHICAL STATEMENT

Constantin Chassapis was born on June 5, 1956 in Limnos, Greece, where he completed his elementary and secondary education. In 1975 he arrived in New York to further his education. He attended The City College of New York from which he graduated, Magna Cum Laude, in 1979 with a Bachelor's degree in Mechanical Engineering. He was elected to the City College chapters of Tau Beta Pi and Pi Tau Sigma and served as president and secretary, respectively. He appeared on the City College Dean's List in 1977, 1978, and 1979, and on the National Dean's List in 1979. He is the receipt of the 1979 American Military Engineers Scholarship, and the 1979 Academic Achievement Award of Tau Beta Pi.

In September 1979, he entered the doctoral program in Mechanical Engineering at the City University of New York, receiving a Master's degree in 1981. On January 19, 1988, he successfully defended his dissertation and was awarded the degree of Doctor of Philosophy by the faculty of the University. While pursuing his studies, he was a lecturer in the Department of Mechanical Engineering where he taught courses in mechanics of machines. In 1986, he published an article on the elastic behavior of linkages along with Dr. Gerard Lowen.

He currently is Assistant Professor of Mechanical Engineering at Stevens Institute of Technology, and resides in Queens, New York, with his wife.