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A PRIORI KNOWLEDGE AND INFALLIBILITY

by

Ivette Fred

A dissertation submitted to the Graduate Faculty in Philosophy
in partial fulfillment of the requirements for the degree of
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Date

Chair of Examining Committee

Jun 2, 1997
Date

Executive Officer

Arnold Koslow

Charles Landesman

Richard Mendelsohn

Stefan B. H. Baumrin

William J. Earle

THE CITY UNIVERSITY OF NEW YORK

Abstract

A PRIORI KNOWLEDGE AND INFALLIBILITY

by

Ivette Fred

Advisor: Professor Arnold Koslow

Two extremely important tasks in the epistemology of a priori knowledge occupy me in this dissertation. First, the most urgent: to provide an explicit characterization of the notion of "a priori knowledge" and related epistemological notions. Second, and equally important task though not as urgent as the first task since without the first, the second cannot be settled: I come up with a plausible notion of infallibility which is compatible with the fact that we are fallible knowers and offer an answer to the question whether infallibility, properly understood, has a place only in the realm of a priori knowledge.

Hale's work on a priori knowledge provides a very useful starting point in this project. My dissertation constitutes a response to Hale's views on a priori knowledge. His discussion bears directly on a number of important issues in connection with the notion. For instance, he offers proposals for the characterization of the notion, forcibly argues for the compatibility of a priori knowledge and revision, addresses the issue of the

non-falsifiability of a priori statements, and - given the mere fact that we are fallible creatures prone to make mistakes everywhere - he does not explicitly leave room for any possibility of infallibility, properly conceived, in the a priori realm. Given that Hale's work bears directly on the issues I am interested, it is relevant to discuss his work in detail.

The dissertation consists of two parts. The first part is concerned with my work on Hale. The second part constitutes my positive account of the problems of a priori knowledge that occupy me, that is, to provide an explicit and illuminating characterization of the notion of "a priori knowledge" and to elucidate the relationship between a priori knowledge and infallibility, properly understood.

After an introductory first chapter on Hale's general views regarding a priori knowledge, I examine (in chapter two) Philip Kitcher's views on a priori knowledge. Since Hale consider his work on a priori knowledge to be a response to Kitcher, it is useful to discuss Kitcher's views. Chapter three is concerned with Hale's reactions to Kitcher. Chapter four deals with views that can be attributed to Hale independently of his response to Kitcher. In chapter five I first focus on what I take to be Hale's preferred notion of a priori knowledge in Abstract Objects, and show that this proposal does not work because of circularity. Then I proceed to examine more recent developments where Hale, in seminars, has proposed a new characterization of a priori knowledge. There is an outstanding problem in connection with Hale's new proposal for a priori statements as being those which are non-falsifiable by experience: if Quine's epistemological holism is right, Hale's notion of a priori knowledge would be empty.

In the sixth chapter I offer characterizations of the notions of "a priori knowledge", "warrant" and "method", and proceed to evaluate the

suggestions analyzing whether the truths we usually regard as a priori come out as a priori on my account. In chapter seven I argue that the properties "a prioricity" and "infallibility" are primarily properties of methods. The question to ask concerning a method is whether if I execute it correctly and then apprehend the belief I ought to form according to the result, there is any residual possibility of that belief's being mistaken. I argue that a method is infallible if and only if when it is applied correctly we end up always with true beliefs. I conclude that it does seem that no a posteriori method is absolutely infallible and, it is certainly the case that not all a priori methods are infallible. However, a priori truths are generally known by the correct prosecution of infallible methods.

Acknowledgments

I would like to express my deep gratitude to my advisor Professor Arnold Koslow for his very detailed and very helpful comments on the whole dissertation. It was indeed under Koslow's supervision that I decided to work on a priori knowledge and on Hale.

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Part I

General Introduction:
The problem of a priori knowledge

Two extremely important tasks in the epistemology of a priori knowledge occupy me in this dissertation. First, the most urgent: to provide an explicit characterization of the notion of "a priori knowledge" and related epistemological notions. This task is the most urgent since we have first to clarify what we are talking about before addressing important questions about a priori knowledge. Second, and equally important task though not as urgent as the first task since without the first, the second cannot be settled: I come up with a plausible notion of infallibility which is compatible with the fact that we are fallible knowers (in that compatibility resides precisely its plausibility) and offer an answer to the question whether infallibility, properly understood, has a place only in the realm of a priori knowledge - in other words, whether infallibility is an a priori matter.

The notion of a priori knowledge, as it is discussed today, stems from the epistemology of Kant. [1] Kant distinguished between two kinds of knowledge: a posteriori knowledge and a priori knowledge. A posteriori knowledge is knowledge whose justification must be based upon experience. (It is also called empirical knowledge.) A priori knowledge is knowledge whose justification need not rest on experience. (It is also referred to as knowledge justifiable

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[1] Kant, Immanuel. Critique of Pure Reason. Translated by Norman Kemp Smith. New York: St. Martin's, 1956.

independently of experience.) [2] Now Kant recognized that some empirical knowledge is required in order to obtain even a priori knowledge, since, for him, all knowledge begins with experience. But once we have the experience required to learn the required concepts, experience does not play any further role.

Mathematics and logic are considered paradigms of disciplines constituted by a priori knowledge. In addition many sentences the content of which is neither purely logical nor purely mathematical are said to be known a priori: "All bachelors are unmarried men", "All bodies are extended" and "Nothing is simultaneously red and green all over".

The notion of a priori knowledge is one of those notions that has been widely utilized by philosophers despite the fact that it is terribly vague. In order to clarify the notion of a priori knowledge, one has to explain what the "independence of experience" that is characteristic of it consists in; that is, one has to provide an illuminating account of the kind of independence of experience that is involved. It is a separate and subsequent issue whether some of our knowledge is indeed a priori. In this dissertation, although I do in fact believe that a priori knowledge is possible, I shall give no argument for that. My concern will be rather, on the assumption that there is a priori knowledge, to say what it consists in: what marks it as a priori.

In order to get clear about the property of being a priori we have to settle what "a priori" primarily applies to. I believe that an essential insight here is that the notion of an a priori justification is the primary notion that needs to be characterized in the epistemology of a priori knowledge. My basic proposal is that the distinction between a priori knowledge and empirical knowledge is

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[2] Ibid, Introduction, B2, pp. 42-43.

grounded first and foremost in a distinction between ways in which we can obtain knowledge, and only secondarily in differences in the products.

Furthermore, any adequate account of the way in which a priori knowledge is independent of experience has to allow room for a degree of dependence on experience: certain experiences may be necessary to equip ourselves with the concepts needed if we are to entertain a candidate for a priori knowledge in the first place - or indeed, in the case of inferential a priori knowledge, if we are to understand the premises for the inference in question. [3]

Let me indicate what kind of an account I am seeking. It is necessary for any account of a priori knowledge to be able to satisfy most (if not all) of the following adequacy conditions. In my view, the conditions for such an adequate account are:

(1) it has to make sense;

(2) it has to be an account that involves no notions which are problematic or, at least, the minimal number of notions which are so;

(3) it has to be an account that is not narrowly circular, that is, the notions involved in the account can be related - of course, how could they be useful to characterize the notion in question if they had no connection with it at all - but should not be so close that the account can shed (some) light on the nature of a priori knowledge;

(4) the account should accommodate most, if not all, the truths we usually consider a priori, that is, it should get the extension of a priori truths right; [4]

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[3] Basic a priori knowledge is knowledge which is not obtained by any inference from other premises. For example, elementary arithmetical truths like "2 + 2 = 4" and trivially analytic truths like "All bachelors are unmarried men" are considered items of basic a priori knowledge. In contrast, inferential a priori knowledge is knowledge obtain by inference from premises already known a priori. For example, the conclusion of an argument constitutes inferential a priori knowledge given that the premises in the inference are already known a priori.

[4] It is understood that there is a certain looseness in this condition because of the disagreement among philosophers as to which are the a priori truths.

(5) it has to be an account that explains the possibility of a priori knowledge, and, in so doing, respects its problematic nature; and

(6) the account ought to illuminate the issue of the certainty of a priori knowledge. If a priori knowledge is certain: where does the certainty come from?

Why are these conditions of adequacy desirable? I don't claim that they are exhaustive, but they certainly reflect the salient concerns in the philosophy of a priori knowledge. Furthermore, I do not intend to provide an answer to all of them in this dissertation, but will address them in my account, as best as I possibly can.

Hale's work [5] on a priori knowledge provides a very useful starting point in this project. My dissertation constitutes a response to Hale's views on a priori knowledge. His discussion bears directly on a number of important issues in connection with the notion. For instance, he offers proposals for the characterization of the notion, forcibly argues for the compatibility of a priori knowledge and revision, addresses the issue of the non-falsifiability of a priori statements, and - given the mere fact that we are fallible creatures prone to make mistakes everywhere - he does not explicitly leave room for any possibility of infallibility, properly conceived, in the a priori realm. Given that Hale's work bears directly on the issues I am interested, it is relevant to discuss his work in detail.

The dissertation consists of two parts. The first part is concerned with my work on Hale. The second part constitutes my positive account of the problems of a priori knowledge that occupy me, that is, to provide an explicit and

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[5] Hale, Bob. Abstract Objects. Oxford: Basil Blackwell Ltd, 1987, Chapter Six "Platonism and Knowledge II: Non-Empirical Knowledge", pp. 123-48.

illuminating characterization of the notion of "a priori knowledge" and to elucidate the relationship between a priori knowledge and infallibility, properly understood.

The first part consists of five chapters followed by a conclusion. After an introductory first chapter on Hale's general views regarding a priori knowledge, I proceed to examine (in chapter two) Philip Kitcher's views on a priori knowledge. Since Hale consider his work on a priori knowledge to be a response to Kitcher, it is useful to discuss Kitcher's views. I begin to consider (in chapter three) Hale's work on a priori knowledge which appear in his chapter "Platonism and Knowledge II: Non-Empirical Knowledge" of his book Abstract Objects. This chapter is concerned with Hale's reactions to Kitcher. Chapter four deals with views that can be attributed to Hale independently of his response to Kitcher and that are published in the above-mentioned chapter: with what I call "pure Hale". In chapter five I first focus on what I take to be Hale's preferred notion of a priori knowledge in Abstract Objects, and show that this proposal does not work because of circularity. Then I proceed to examine more recent developments where Hale, in seminars, has proposed a new characterization of a priori knowledge. I argue that there is an outstanding problem in connection with Hale's new proposal for a priori statements as being those which are non-falsifiable by experience. That is, it is in too direct collision with Quine's epistemological holism. The problem is that if Quine's epistemological holism is right, Hale's notion of a priori knowledge would be empty. In the end of this chapter I will closely examine whether it is coherent to talk of revision in connection with a priori knowledge - as some authors have thought, including Hale - sorting out what has to be involved for such a coherence to be possible, and ending up with some remarks on the relationship between the notions of revision and a prioricity.

As far as the issue of infallibility is concerned, Hale's position in his chapter may seem to be that there is no room for infallibility in connection with a priori knowledge - or knowledge in general - given the undeniable fact that we are fallible creatures.

The second part consists of two chapters followed by a conclusion. In the sixth chapter I will offer more illuminating (explicit) characterizations of the notions of "a priori knowledge", "warrant" and "method", and will proceed to evaluate the suggestions analyzing whether the truths we usually regard as a priori come out as a priori on my account. In chapter seven I will try to make sense of the concept of infallibility among others. I shall argue that the properties "a prioricity" and "infallibility" are primarily properties of methods. I will elaborate the thesis that infallibility is a matter of methods alone; and then will address the important issue whether there are empirical methods which are infallible in my sense. In the concluding section I shall recapitulate the most important theses of the chapter and dissertation.

Chapter One
Hale's general epistemological views regarding a priori knowledge

According to Hale, for any adequate general theory of knowledge to require some kind of causal condition is to require, roughly speaking, that for X to know that p, X's belief that p must be causally connected with the fact that p in some adequate way. [6] A good deal of the motivation for accepting a causal requirement as a necessary condition in an adequate general theory of knowledge arises from the perceived inadequacy of the standard account of knowledge in terms of "justified true belief". The Gettier-type counterexamples have motivated the introduction of a causal constraint on knowledge as justified true belief to avoid having to classify as knowledge cases when one does merely possess a lucky guess.

Hale distinguishes between weak and strong causal theories of knowledge. [7] Hale observes that strong causal theories have difficulties for accommodating any sort of a priori knowledge. Strong causal theories are those which require that for knowledge that p, the true conferring fact that p itself must be appropriately causally related to the belief that p. On the other hand, weak causal theories like the one put forward by Philip Kitcher require only that for warranting belief that p, the belief should be caused by the grounds which warrant it. Note that now there is another causal relation: it is no longer required that the true conferring fact that p must cause the belief that p - and in doing so must warrant the belief that p - but rather than it is a necessary condition for

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[6] Ibid, p. 28.

[7] P. 93.

knowledge that the ground for p shall be causally responsible for the belief that p and for warranting p as well. Hale comments on Kitcher's causal proposal:

> "What is further required ... is that the knower's (true) belief shall be caused by the grounds which warrant it. This seems, if not unexceptionable, highly plausible." [8]

> "... whilst he [Kitcher] may be said to be advocating a causal account of knowledge, it is only a weakly, and fairly uncontentiously, causal theory that is on offer. Certainly there is no suggestion here that for X to know that *p*, *the fact that p* must be suitably causally connected with X's belief that *p*. The causal requirement is no more than that *what grounds* X's belief that *p* shall be causally responsible for his belief, if he is to possess knowledge ... Just because the kind of causal theory Kitcher advocates does not require that, when X knows that *p*, the truth-conferring fact that *p* shall itself play a causal role, there is so far no special reason to expect his theory, in contrast with strong causal theories of the sort previously discussed, to confront any insuperable difficulty in accommodating knowledge a priori of necessary truths. But, for the same reason, it could pose no direct threat to platonism based upon the acausality of abstract objects, or states of affairs involving them. Kitcher's endorsement of Benacerraf's argument suggests a failure to appreciate the significance of this difference between the two types of causal account." [9]

That is, merely to require that for warranting belief that p, the belief should be caused by the grounds which warrant it, is quite plausible and that is compatible with the existence of a priori knowledge. As I understand it, this requirement is compatible with a priori knowledge since for a priori knowledge that p it is necessary that the ground is causally responsible for believing that p and, therefore, for knowing that p. Even in the case of inferential a priori knowledge, for example, rightly possessing a proof that p satisfies this weak causal requirement. How is it that rightly possessing a proof that p could cause belief that p? Because when one rightly possesses a proof for p then one justifiably

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8 P. 127.

9 Ibid.

could come to believe that p is true since it is the conclusion of a proof. [10] Kitcher is not aware that this weak causal condition on knowledge is compatible with a priori knowledge. [11]

Hale points out that the fact that a priori knowledge is compatible with a causal theory of knowledge - since it can satisfy a weaker causal condition on knowledge - is not enough to justify the claim that mathematical statements are about abstract objects.

> "a strong causal theory is likely to have troubles making room for any kind of knowledge *a priori*, whether the truths so known are platonistically construed or not. This line of defense was wholly negative, however. And just for that reason, it is unlikely by itself to adequately relieve the general and widely felt anxiety that however natural the platonist's semantics for statements of arithmetic, for example, may be, acceptance of the accompanying ontology leaves us in an epistemologically parlous state." [12]

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[10] It should be specified that the idea that Kitcher and Hale are defending here ought to be that the ground for knowledge has to have the ability to produce the belief in question. It would be implausible to require, on the other hand, that the belief was in fact produced by the ground. This would be too strong because we often can believe that p is true, for example, p being the conclusion of a proof, before following a proof that p. One can follow such a proof later on and know that p is true. As I understand it, the idea that Kitcher and Hale ought to be defending is rather that the ground for knowledge must have the ability to produce the belief, so if I have not thought that p before and then possess a justification for p then I can come to believe that p right then.

[11] Kitcher endorses the Benacerraf epistemological challenge against the platonist understanding of mathematical knowledge as being about abstract objects. (The reference to Benacerraf is: Benacerraf, Paul. "Mathematical Truth", Journal of Philosophy, No. 70, 1973, pp. 661-79.)
Broadly speaking, abstract objects are non-spatio, non-temporal entities which supposedly exist independently of the human mind and language. The Benacerraf epistemological challenge against the platonist consists in the need of explaining how it is that we can obtain knowledge of abstract objects given that we cannot have causal contact with these objects.

[12] Hale, p. 123. The word "parlous" is an archaic form of "perilous", and it is the word that Hale uses.

The epistemological problem for the platonist is how to explain knowledge of abstract objects when there can be no causal route or contact with those objects. Hale asks: 'what sort of positive account is available for the platonist?' [13] The traditional answer is to appeal to a kind of knowledge that does not require causal contact with objects for warranting true belief. Our knowledge of logical and mathematical truths is non-empirical or a priori. Again, this merely constitutes a form of an answer - more elaboration is necessary. It is necessary to analyze the notion of non-empirical knowledge or knowledge a priori.

Hale observes that since the notion of nonempirical knowledge is quite general, it is consistent with different explanations of how we acquire a priori knowledge. Hale thinks that inferential a priori knowledge is obtained by deductive reasoning from basic truths already known a priori. It is in relation to our apprehension of basic truths - truths not known inferentially - that there is most room for conflicting accounts. For instance, Gödel's and Penelope Maddy's views consider our a priori apprehension of basic truths to involve a sort of intuition, which is vaguely constructed following the model of sense perception. On the other hand, Hale considers it to be quite doubtful that the perceptual model can be successful in explaining our knowledge of basic a priori truths. Hale proposes the following alternative explanation of basic a priori knowledge: a priori knowledge is got "by recognition of conceptual connections". Hale adds that such knowledge involves the exercise of "conceptual competence". [14] As a platonist, he is interested in making sure that 'there ought to be no conflict between the view that mathematical statements describe properties and relations

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[13] Ibid.

[14] P. 124.

among mind-independent abstract objects, and the view that their truth is a matter internal to language, that is, their truth is owed to conceptual liaisons'. Hale assumes that this tension can be resolved. [15]

Hale is not explicit about how it is that basic a priori knowledge can be acquired by "recognition of conceptual liaisons" and involves "conceptual competence". It seems to me that Hale's proposal is to understand knowledge a priori of basic truths via the notion of analyticity, vaguely constructed as "truth in virtue of meaning alone". However, Hale does not offer an account of the notion of analyticity, at least not in the chapter with which we are concerned.

According to Hale, the view that some of our knowledge is a priori is quite plausible, for instance, our knowledge of logic and mathematics. And that view is independent of any prior ontological interpretation of logical and mathematical truths. That is why he neither provides an argument for the a prioricity of mathematical and logical knowledge nor for the existence of a priori knowledge in general.

> "The view that some of our knowledge is *a priori*, and specifically that our logical and mathematical knowledge is typically of that character, is perhaps sufficiently plausible for positive supporting argument to be unnecessary, and none will be attempted here." [16]

Nonetheless, Hale recognizes that the appeal to the plausibility of our mathematical and logical knowledge as being a priori does not by itself constitute a decisive consideration against the causal theorist of knowledge.

> "It is worth reminding ourselves that the conception of our knowledge of logic and mathematics ... as *a priori* has considerable initial plausibility, quite independently of any prior attachment to any platonistic

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15 Ibid.

16 P. 125.

> interpretation of say, number theory. It is just because that is so, as Wright has argued, that the apparent incapacity of causal accounts of knowledge to accommodate any kind of knowledge *a priori* affords an independent reason to suspect their claim to embarrass the platonist. But it is, as he grants, well short, of being decisive." [17]

In short, even if one accepts that it is plausible that some of our knowledge is a priori, this alone is not enough to justify the platonist reading of these truths. The reason is this: if the a prioricity of mathematical and logical knowledge is independent of the ontological construction of their truths, then a defense of the former does not constitute a defense of platonism. However, that the a prioricity of mathematical and logical knowledge is independent of the ontological construction of their truths is very embarrassing for strong causal theories. Strong causal theories are in a difficult position if they cannot accommodate a large area of knowledge. And, if they cannot accommodate a large area of knowledge, they cease to be exhaustive epistemological theories, that is, theories which account for all our knowledge; and this is a good reason to reject them (at least as paradigmatic theories of knowledge).

I shall not be concerned with Hale's ontological commitments but will concentrate rather on his epistemology of a priori knowledge. It is possible to clarify the epistemology of a priori knowledge without doing ontology, or at least without depending heavily on a particular ontological position like platonism, let's say. The epistemological points I want to make ought to stand on their own and not be subjected to the fate of any particular ontological theory. However, if it is not possible to isolate completely epistemology and ontology, it seems that to try to do so constitutes a *desideratum*. Hale himself appears to do

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[17] P. 124-5.

the same when he recognizes that the view that there is a priori knowledge is quite plausible independently of platonism.

Of course, there is an initial attraction of platonists for a priori knowledge. Since a priori means of knowing do not require the satisfaction of a strong causal condition for knowledge, they make it possible how to explain our knowledge of mathematical objects, conceived as abstract objects. Since we won't need the satisfaction of a strong causal condition, it is not necessary to have contact with abstract objects in order to acquire knowledge about them. Hale, as a platonist, is interested in responding to the epistemological challenge of how to explain our knowledge of abstract objects when we cannot have causal relations with them.

Moreover, I recognize that there is a connection between the epistemological and ontological issues. For part of what makes a statement a priori or knowable a priori is what the statement is about. Again, the idea is that what a statement is about has an important role in explaining how a priori knowledge is possible. Since I won't be discussing ontological issues in the dissertation, I won't be addressing this interesting issue here either.

Furthermore, I will follow Hale in assuming the existence of a priori knowledge, so I won't be giving an argument for the existence of a priori knowledge either. On the assumption that there is a priori knowledge - for the purpose of clarifying the notion - then the task at hand is to say what it consists in: what makes it a priori? In order to clarify the notion of a priori knowledge, one has to explain, and be explicit about, the "independence of experience" that characterizes a priori knowledge in contrast to empirical knowledge. We can accomplish that without recurring to ontological discussions or at least to keep them to a minimum.

Chapter Two
Kitcher's views on a priori knowledge

Hale's defense of the notion of a priori knowledge takes the form of a response to Philip Kitcher. [18] For Hale, since what can be said about a priori knowledge could as well constitute a response to Kitcher's views, it is relevant to discuss them. Hale affirms:

> "But it is clear that defense [of the notion of a priori knowledge] is much needed, not only because scepticism about the applicability of the notion of *a priori* knowledge has not always, or even typically, been motivated by adherence to a specifically *causal* view of knowledge, but also because at least one writer [Kitcher] has developed a notion of knowledge *a priori* within a framework of a broadly causal epistemology whilst arguing that our mathematical knowledge cannot be of that ilk." [19]

Let me clarify that I will only be briefly discussing the views of Kitcher that Hale reacts to. So I won't be discussing Kitcher's views in any detail but only as much as it is relevant for my subsequent discussion of Hale. I will be depending heavily on Hale's presentation of Kitcher's views for this reason. Kitcher is a (weak) causal theorist who constructs a notion of a priori knowledge on which mathematical knowledge is not a priori. Now let me briefly explain the tradition from where Kitcher is coming from.

Traditionally, mathematical knowledge has been considered one of the paradigmatic instances - besides logical knowledge which is also considered a priori - of knowledge for its exactitude and the special certainty it confers.

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18 Kitcher, Philip. The Nature of Mathematical Knowledge. Oxford: Oxford University Press, 1983. All the quotations from Kitcher are from this book.

19 Hale, p. 125.

Mathematics had been considered "the queen of disciplines" since it is in its domains where our search for certainty was supposed to finally be stopped and be fully realized. As I understand it, mathematics enjoys such a privileged status because of its methods for acquiring mathematical knowledge. [20] Mathematical methods had been considered to be a priori. Since mathematical knowledge is usually obtained by a priori routes, and it was considered a paradigm of knowledge, it followed that a priori methods were better routes for knowledge than empirical ones. Furthermore, it was assumed that a priori routes were not only better routes for knowledge than the empirical ones, but also that they always were secure (infallible) routes for knowledge. (Note that the last step of the argument is not forced upon us, that is, that a priori routes are infallible does not follow from any of the previous premises.) Kitcher's position is very interesting because it is an example of such a conception of an a priori infallible route (though he does not distinguish between methods and warrants as ways of acquiring a priori knowledge) while denying that mathematical knowledge is a priori. [21]

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[20] I consider that, nonetheless, the best available explanation of the certainty of a priori mathematical knowledge is because of its methods. As we will explain in chapter seven, some a priori methods are infallible; and mathematical methods (most of them) are infallible. But things have to be sorted out very carefully, for instance, the sense of infallibility involved, in order to rescue the truth of this assertion.

[21] Kitcher's book constitutes an attack on the thesis that mathematical knowledge is a priori.

The distinction between "methods" and "warrants" is very important. I will be explaining this distinction very carefully in the glossary I provide in chapter seven. I advise the reader to consult pp. 128-136 for the uses I make of all these epistemological terms. But for now let me briefly say that a method is a cognitive routine which can be performed correctly or incorrectly on the occasion, and we get warranted belief (or not) as a result. A warrant is a particular implementation of a method. For example, "constructing proofs" is a method; the particular proof

Section 1: Kitcher's account of the notion of a priori knowledge

For Kitcher, as well as for many other epistemologists, an adequate general account of knowledge has to put some constraint upon how a state of true belief must be produced if it is to amount to knowledge. Kitcher's proposal is that a suitable constraint has to be concerned with the psychological processes which produce beliefs. Since his proposed constraint is on psychological processes, he calls his view "psychologistic". An example of a belief-producing psychological process is the process of "following a proof". [22]

Kitcher's general analysis of the concept of knowledge is:

"(1) X knows that p if and only if X believes that p and X's belief that p was produced by a process which is a warrant for it." (p. 17)

Kitcher specifies that (1) is just the starting point for a general characterization of the concept of knowledge. [23] Kitcher introduces the term "warrant" to refer to

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I construct is my warrant for its conclusion. As we will see in chapters six and seven the notion of infallibility applies primarily to methods, and only derivately to warrants.

[22] Kitcher, pp. 42-3.

[23] But how can the right hand side of (1) be logically equivalent to its left hand side? In the left hand side of (1), truth is implied since knowledge entails truth. In the right hand side, there is no talk of truth. It appears then that in order for the right hand side to be logically equivalent to the left hand side, a warrant has to imply the truth of p.

However, if a warrant implies the truth of p then it has to be an infallible warrant. And if so, then the concept of an infallible warrant won't be distinctive of the concept of a priori knowledge, as Kitcher maintains. (We will discuss this shortly.) Kitcher is trying to provide here a general analysis of the concept of knowledge.

I think this problem may be explained by the fact that (1) is allegedly only providing a starting point in this analysis. Unfortunately, Kitcher does not elaborate on (1) in his book.

those processes which produce belief "in the right way". The notion of what it is for a process to warrant a belief is deliberately vague to leave open which of the various psychologistic accounts is preferable. The notion of "warrant" or "justification" makes possible a distinction between different ways we acquire knowledge, i.e., a priori or a posteriori.

Kitcher's analysis of the notion of a priori knowledge is:

> "(2) X knows a priori that p if and only if X knows that p and X's belief that p was produced by a process which is an a priori warrant for it.
>
> (3) α is an a priori warrant for X's belief that p if and only if α is a process such that, given any life e, sufficient for X for p,
>
> (a) some process of the same type could produce in X a belief that p;
>
> (b) if a process of the same type were to produce in X a belief that p, then it would warrant X in believing that p;
>
> (c) if a process of the same type were to produce in X a belief that p, then p." (p. 24)

How do we get the a priori from this characterization? The a priori is attached to the warrant. Given that X has the necessary concepts to entertain p, an a priori warrant for X for p: could produce belief that p, warrant p, and, given that the warrant produced belief that p, then p is true. 'If someone knows a priori that p then she could know that p whatever sufficiently rich experience she had had (sufficiently rich experience to entertain the proposition to be known).' [24]

Again, given that a subject is equipped with the necessary concepts to entertain a belief to be known, p, a process of the type in question would, were it to produce the belief that p, produce a warranted and true belief. Any belief produced by an a priori warrant is knowledge. It is assumed that the knowers in question are humans and that their intellectual capacities remain fixed across all

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[24] P. 24.

these possible lives. [25] Kitcher's general problem becomes: how can there be warrants that always produce knowledge? Kitcher tries to show that there can be no a priori mathematical knowledge. For Kitcher, a priori grounds for belief must be such that it could not be rational to regard them as insufficient to justify belief under any experiential context. Mathematical grounds allegedly do not satisfy that condition on a priori grounds.

Section 2: On Kitcher's thesis that a priori knowledge is incompatible with revision

For Kitcher, a priori beliefs must be unrevisable because they are produced by a priori warrants. A priori warrants only produce and warrant true beliefs. [26] Mathematical beliefs cannot be warranted by a priori warrants because some of the mathematical beliefs we take ourselves to be justified in believing are false and their justifications faulty.

Kitcher claims that routes to mathematical knowledge can always be upset by unkind experience, [27] they can lead us astray, so they cannot be a priori for that reason. Kitcher's point is twofold: routes to mathematical knowledge are not infallible, so they are not a priori; and they can be upset by unkind experience making things even worse for the defender of mathematical routes as being a priori.

> "Experiences which cast doubt on the accuracy of the book (by appearing to expose errors in many 'theorems', let us say), and in which eminent mathematicians denied the conclusion, would interfere with the ability of the process to warrant the belief. If I check through the proof in a book, thinking I see how the inferences go, and if the proof is very complex,

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25 PP. 26-7

26 P. 24.

27 Unkind evidence is evidence against a warrant or the belief it warrants.

> then, under the circumstances in which there is weighty evidence against both book and theorem, it would be unreasonably arrogant and stubborn of me to form the belief." [28]

Kitcher understands the term "experience" as the knower's total sensory experience; and "independent of experience" is understood counterfactually: knowledge that could have been obtained by a process of the same type that actually produced it, no matter what the course of the subject's sensory experience is, provided it was sufficient to allow him to acquire the relevant concepts required to grasp the proposition in question.

Kitcher claims that the "processes which apriorists take to generate our mathematical beliefs would be unable to warrant those beliefs against the background of a suitably recalcitrant experience". [29] Kitcher explains:

> "If apriorists are to escape this criticism on the grounds that the analysis of a priori is too strong, then they must allow that it is not necessary for an a priori warrant to belong to a type of process members of which could warrant the belief in question given any sufficient experience. To make this concession is to abandon the fundamental idea that a priori knowledge is knowledge which is independent of experience. The apriorist will be saying that one can know a priori that p in a particular way, even though, given appropriate experiences, one would not be able to know that p in the same way. But if alternative experiences could undermine one's knowledge then there are features of one's current experience which are relevant to the knowledge, namely, those features whose absence would change the current experience into subversive experience ... To reject condition (3b) ... would be to strip apriorism of its distinctive claim." [30]

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[28] P. 43; my emphasis.

[29] PP. 88-9.

[30] Ibid; my emphasis.

Kitcher argues that if apriorists try to respond to his criticism alleging that his analysis of a prioricity is too strong, then they have to accept that it is not necessary for an a priori warrant to belong to a type of process which could warrant belief provided there are sufficient experiences for the acquisition of the conceptual repertoire needed for a priori knowledge. For Kitcher, if the apriorist makes this concession, he will be abandoning the crucial claim that a priori knowledge is independent of experience. A priori knowledge would be dependent on experience. The apriorist will find himself in the following awkward position: that one can know a priori that p in a particular way, even though, given unkind experiences, one would not be able to know that p in the same way. Now if there were alternative experiences that could undermine one's a priori knowledge that p, then there are features of one's actual experience which are relevant to the a priori knowledge, that is, those features whose absence would change the current experience into subversive experience. If the apriorist rejects condition (b) - that is, if a process of the same type were to produce in X a belief that p, then it would warrant X in believing that p - of Kitcher's analysis, he would be stripping a priori knowledge of its distinctive character.

Section 3: The issue of long proofs or calculations

Kitcher discusses whether long proofs can be a priori in his sense. He argues that long proofs cannot generate a priori knowledge of their results. [31] Kitcher relies at this point on Descartes' view [32] that long proofs are problematic sources of

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31 PP. 40-3.

32 Descartes, Rene. Rule VII, Philosophical Writings. Edited by E. S. Haldane and G. R. T. Ross. Cambridge University Press, vol. 1, 1967, p. 19. I won't discuss whether Kitcher's interpretation of Descartes's view is correct.

knowledge since we cannot apprehend them all at once. According to Kitcher, Descartes' proposal is to go over the proof several times until we are able to apprehend it as a whole, all at once, and do not have then to rely upon the memory of having justified its earlier steps. Obviously, there is going to be a limit, an upper bound, to what we can achieve by this method. Then, what happens with our knowledge of truths that exceed this upper bound? Kitcher's response is that there can't be any a priori knowledge obtained via long proofs and that there are no rules of inference that preserve a prioricity. That is, there are no rules of inference such that if we start with premises supposedly known a priori, we are assured of ending up with statements known a priori. For Kitcher, the situation is even worse: our a priori knowledge of the premises, our starting points in the proofs, would be lost. The reason is the following: since we cannot apprehend the proof all at once, there is a switch of grounds from an a priori warrant in the beginning for the knowledge of the premises to knowledge based upon the memory of having followed such a warrant. Knowledge based upon memory can provide a reliable warrant, but not an a priori warrant. Kitcher concludes that there could be no rules of inference that could preserve a prioricity since, if there were any, we could construct proofs of arbitrary length whose results, if their premises where known a priori, would be known a priori as well.

Section 4: Knowledge obtained by "non-empirical processes"

Kitcher clearly allows that mathematical knowledge can be obtained by non-empirical processes. But he thinks that it is important to realize that though these processes are considered non-empirical they are not to be considered a priori warrants. These processes fail to come up to his standards for being a priori warrants.

One of the non-empirical processes that Kitcher considers was proposed by Gödel. [33] Gödel's proposal was that mathematical knowledge can be generated by a sort of non-sensory apprehension - Gödel calls it "mathematical intuition" - directed on the platonistically conceived abstract subject matter of mathematics. Kitcher asks: Why might someone who believed that we had such a faculty be led to think that the knowledge which it generated was a priori? Because mathematical intuition is a non-empirical process. Anyone who confuses non-empirical processes which actually warrant belief with a priori warrants will read Gödel as upholding a priorism. Other examples of non-empirical processes which engender belief are following proofs or computations.

Kitcher does not explain what he means by a "nonempirical process". Nonetheless, he seems to be committed to the following theses:

> A. Certain belief-producing processes (for example, following proofs) are non-empirical processes. They may be significantly contrasted with empirical processes which engender belief mainly because they can be available independently of experience.
>
> B. Some of these nonempirical belief-producing processes can generate knowledge (produce true warranted belief).
>
> C. Some mathematical knowledge is generated by such nonempirical processes.

For Kitcher, while the absence of certainty is compatible with knowledge, it rules out a priori knowledge. Kitcher rejects Kripke's claim that a priori knowledge does not necessarily involve being certain. [34] The notion of "certainty"

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[33] Gödel, Kurt. "What is Cantor's Continuum Problem". Reprinted in Benacerraf and Putnam (eds.), The Philosophy of Mathematics, Englewood Cliffs: Prentice Hall, 1984, pp. 470-485.

[34] Kitcher, p. 43. The reference to Kripke is: Kripke, Saul. Naming and Necessity. Cambridge, Mass: Harvard University Press, 1980, p. 39.

in question is left unexplained. Kitcher argues that the process of following a proof may give us knowledge - nonempirical - of its conclusion, but not a priori knowledge, because we can conceive empirical circumstances in which it would be irrational to cling to the proof. In such a situation, our having followed the proof would not be enough for knowledge. In order for the process of "following a proof" to be an a priori warrant, it must always produce knowledge.

Section 5: Some remarks on Kitcher's views

Before proceeding to discuss Hale's criticisms of Kitcher's views, I will be identifying Kitcher's most vulnerable points in this section.

What does Kitcher mean by "belief" when he affirms that unkind experiences "would interfere with the ability of the process to warrant the belief"? Is it the psychological state of believing or the objective belief, the proposition believed? This is important when we try to figure out what unkind experiences are interfering with.

On the one hand, there is the psychological reading of the term "belief". According to this reading, Kitcher's point is a simple one: if there is strong empirical evidence against a warrant for p then one should not believe p (the proposition). Suppose I think have followed a long and complicated proof that p. I find out later on that mathematicians think the purported proof is flawed. Given that I am not a mathematician, I should not believe p. Then since one

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Kripke affirms:

"Something can be known, or at least rationally believed, *a priori* , without being quite certain. You've read a proof in the math book; and, though you think it's correct, maybe you've made a mistake. You often do make mistakes of this kind. You've have made a computation, perhaps with an error." (ibid)

should not believe the belief, one cannot know the belief (the proposition). It is a necessary condition for knowledge that we believe the belief. It is in this qualified sense, that a priori knowledge is not completely independent of experience, and in that Kitcher is right.

However this reading would be a weaker interpretation of Kitcher's position. Since to believe the proposition involved is a necessary condition for knowledge of any sort, it does not involve the distinctive character of a priori knowledge. And Kitcher proposes an account of the latter, so he ought to take into consideration the distinctiveness of a priori knowledge. What I mean is that since any knowledge is dependent on experience in this way, if there is a priori knowledge, as Kitcher thinks there is, its characteristic "independence of experience" has to be accounted for in a way which respects such experience dependence.

Moreover, if experience always has this disturbing role - I should specify that what experience can always primarily disturb is our psychological act of believing and indirectly the proposition believed - even when it is misleading experience, [35] that ought to show to Kitcher that there cannot be independence of experience and, therefore, there can be no a priori knowledge. But Kitcher insists that there is a priori knowledge. Actually I think that Kitcher goes back and forth between the conception of revision as always possible even when it is the wrong thing to do, as in the case of revision, for example, being called for by misleading

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Kitcher is reacting to this passage of Kripke. Neither Kripke nor Kitcher explain what they have in mind when they talk about the notion of certainty.

Kitcher's assertion that the absence of certainty is compatible with knowledge, but not with a priori knowledge is problematic. Knowledge is only compatible with probability of 1. Feelings of sureness is another matter.

35 Kitcher particularly stresses the role of misleading experience on p. 84 of his book.

experience, and revision as the right thing to do on the occasion. But actual knowledge in general is only compatible with revision for the wrong reason and incompatible with revision for the right reason (more below).

On the other hand, Kitcher seems to be talking about the proposition believed as well as the psychological act of believing. Thus, another explanation of "the belief" in the phrase "would interfere with the ability of the process to warrant the belief" is that it is the proposition believed, so that, according to Kitcher, a priori warrants must be "ultra reliable", that is, they must always, in all counterfactual situations in which they are invoked, produce true beliefs (not only beliefs but always beliefs that are true). According to this interpretation, for Kitcher, it is not possible to separate the issue whether a belief has been acquired by an a priori warrant, and that belief's being knowledge. A priori warrants are "ultra-reliable" because they always infallibly produce knowledge by their means. (Note that I use the terms "ultra-reliable" and "infallible" interchangeably for that reason.)

Someone could ask: how "infallible" is involved in (3c)? Remember, "(3c) if a process of the same type were to produce in X a belief that p, then p." [36] Of course, the term "infallible" does not appear in (3c). It is involved in the sense that, for Kitcher, it is a necessary condition for processes to be a priori that they will always end producing true justified beliefs. That is, an a priori process is such that it produces justified belief and it is always the case that that justified belief is true. Isn't such a process infallible given that a priori knowledge is true justified belief a priori? Note that condition (3c) is derivable from (3b). (3b) imposes a condition on warrants to be a priori, namely, that under no

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36 Kitcher, p. 24.

experiential background in which they are available, can we regard them as insufficient to justify belief that p. A priori warrants always warrant belief.

A corresponding distinction can be drawn between the ability of the process to warrant belief, the proposition to be known, and the ability of the process to sustain my act of believing. The first is an epistemological matter; the second a matter of psychology. Kitcher's thought is that not only experience but even misleading experience can affect my ability to believe the proposition (the belief), so a priori knowledge cannot be independent of experience. In this sense, even misleading experience is always capable of interfering with my a priori knowledge, given that to believe (the psychological state of believing) is a necessary condition for knowledge.

The underlying assumption in Kitcher's argument is that if a priori warrants were not ultra-reliable, then "experience" could undermine a priori knowledge, and that would show that the latter is not independent of experience and, therefore, by definition, not a priori at all. It is interesting to note that what Kitcher would have needed is that some a priori warrants must be ultra reliable in order to have a priori knowledge in the first place. It is also assumed that only experience could undermine alleged "a priori" warrants. Kitcher does not consider the epistemic possibility of a priori grounds being undermined by a priori reasons. He does not allow for that possibility because he thinks that a priori warrants cannot be undermined in any way. I think that since Kitcher constructs the term "experience" very broadly, even if he could accept that there could be a priori reasons that could undermine an a priori warrant or its result, he would still characterize this case as the experience of having a priori reasons to reject an a priori warrant or its result.

Kitcher explains that the aforementioned experiences, like those which put into question the accuracy of proofs by appearing to show errors in many of the

theorems, "would interfere with the ability of the process to warrant belief". I propose that a distinction should be drawn here. We want to distinguish between (a) a warrant that is good from (b) a warrant we believe to be good. A warrant, if it is a good one, has to warrant the belief. That is why it is a good warrant. (A warrant warrants belief that p if it gives us a pretty good reason to believe that p.) In the case when our warrant is the possession of a proof that p, that the proof that p warrants p is not a matter of degree; it is an absolute yes or no; it does or it does not warrant p, contrary to what the phrase "would interfere with the ability of the process to warrant belief" appears to convey (my emphasis). A separate matter is our believing that a warrant is a good one. To think that we have a proof, for example, is a matter which admits of degree. We can be more or less confident in believing that our warrant is sound. On the other hand, to think rightly that we have followed a proof that p is surely not a matter of degree. Kitcher is obviously right in the sense that we have to believe the proposition in order to be able to know it. Also, our belief that p would involve the accompanying belief that our warrant for it is a good one; otherwise we won't count ourselves as being justified in believing the proposition in question, let alone knowing it.

It is wrong to think that we have to be very confident about this second-order belief - i.e. the belief that the warrant is good - in order to be justified in believing that p, and in order to know a priori that p, if one knows that p. (Note that the belief that a warrant is good is also accompanied by other collateral beliefs like that I have understood the reasoning in its entirety.) The matter is one of degree here since our necessary belief that the warrant is good, our having confidence in the warrant, admits degrees. Although we ought to attach to the warrant some appropriate degree of confidence (sureness) in order to be able to take ourselves as in possession of pretty good reasons which justify our

believing a proposition. But, again, this is independent from the fact that does not admit of degrees: namely, that the warrant is good or not as a matter of fact.

As already said, Kitcher is right in claiming that empirical considerations can undermine a priori knowledge in the sense that they can undermine our ability to believe that a warrant is sound and, consequently, our ability to believe the proposition which it (presumably) justifies. However, we still can believe, or even know, a proposition without being sure that our warrant for it is good.

Kitcher ignores the fact that a warrant could be good, regardless of our thinking that it is; a warrant could be objectively an a priori warrant in his (strong) sense even if we don't know it. I believe that the reason why Kitcher ignores this fact is because he simply cannot accept it. He understands the notion of a warrant - of any sort - "psychologistically". In the case of an priori warrant, since the a priori warrant is a mental process, like following a proof, and not the proof itself, we have to believe that the warrant is a good one in order for the warrant always to produce belief, that is, to make us believe the belief that p. Now we have seen that we don't have to be totally confident about the second-order belief that our warrant is sound in order for the warrant always to make us believe the belief that p.

But, I think that Kitcher's most vulnerable point is: How can there be belief-producing processes which remain available, warranting, and truth-guaranteeing under such a wide range of experiences a subject can have?

Chapter Three
Hale's reactions to Kitcher's views

In this chapter I will be discussing only the views of Hale which constitute a direct response to Kitcher. (In the next chapter I will focus on Hale's views which are independent of Kitcher.)

Section 1: Hale's claim that a priori knowledge is compatible with revision

In contrast with Kitcher, for Hale, a priori knowledge as well as a priori beliefs do not have to be absolutely unrevisable. It is perfectly reasonable that we can reject mathematical beliefs we thought were true or reasonably justified because subsequent evidence shows or appears to show that they are false or that our grounds for them are not as sound as we previously thought. Consequently, a priori warrants do not have to be infallible - i.e. they do not have always to produce true justified beliefs or (a priori) knowledge - in order to be a priori. Hale explains:

> "any defense of the claim that we can have knowledge *a priori* .... must accommodate the fact that we are liable to error in such matters, not merely in the fairly unexciting sense that an individual may, through inattention, make slips in computation or be taken in by a fallacious proof, but in the more momentous sense that the mathematical community as a whole may find itself obliged to revise confidently held mathematical beliefs, where the demonstration that revision is called for does not merely consist in the exposure of routine error which prior training equipped it, in principle, to avoid, but itself advances mathematical knowledge. Traditionally, the conception of logic and mathematics as bodies of truths known *a priori* went hand in hand with the belief that here at least the ideal of certainty might be realized. This particular liaison can be, as has been, challenged. Kripke (1980, p. 39), in particular, plausibly claims that acquisition of knowledge by following a proof is consistent with absence of certainty that the proof contains no error." [37]

Hale is saying that any defense of the notion of a priori knowledge has to come to terms with the fact that we are fallible creatures. We may not only make mistakes due to lack of attention or enough concentration, but may make mistakes in a more significant way. That is, revision can be called for which does not merely consist of exposure of routine error but itself advances mathematical knowledge. For instance, mathematicians may have to revise beliefs they held to be true because new evidence appears that shows that they were wrong.

Then, as Hale points out, the following question arises:

> "If ... knowledge *a priori* is not to be understood as requiring absolute unrevisability, a pressing question concerns just what scope of revision is consistent with a reasonable construal of what such knowledge amounts to. The central task here is ... to get clear about what should, and what need not, be built into the notion of experience independence in terms of which, at least since Kant, knowledge *a priori* is customarily characterized." [38]

That is, given that a priori knowledge is not conceived as requiring absolute unrevisability [39] of a priori beliefs and a priori warrants, as Kitcher does, then a

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[37] Hale, p. 125; my emphasis. The reference Hale makes in this passage is to Kripke's Naming and Necessity, p. 39.

[38] Hale, p. 125.

[39] Hale does not explain what he means by "absolute unrevisability" (or "absolute indefeasibility"). Hale uses the notions of "absolute unrevisability" and "absolute indefeasibility" interchangeably. As I understand it, absolute unrevisability (indefeasibility) is presumably a property of warrants and the beliefs which they warrant. A warrant and the belief that it warrants are absolutely unrevisable when they cannot be revised (or defeated) either by empirical reasons or by a priori reasons. That is, they cannot be revised at all. Hale thinks that the notion of a priori knowledge does not require for a priori warrants and a priori statements that they have to be absolutely unrevisable (or absolutely indefeasible). As we will discuss in chapter four, Hale argues that the notion of a priori knowledge is compatible with revisions of a priori warrants

pressing issue becomes what scope of revision is consistent with a reasonable construction of a priori knowledge.

As already said, Hale questions Kitcher's assumption that warrants must always in all counterfactual situations produce knowledge in order to be a priori. Hale asks: 'Why do we have to assume that a priori warrants always, in all counterfactual situations in which they are available, must produce true beliefs?' [40]

Hale explains that there is a confusion at this point. It is undeniable that, for X to know a priori that p, it must be true that p. Likewise, p must be true, if X is to know a posteriori that p. It is obvious that from the satisfaction of this necessary condition for knowledge of any sort - that what is known is true - it does not follow that in the case of a priori knowledge, the satisfaction of this condition - that p is true - must be guaranteed by the fact that X has a priori grounds to believe that p. Kitcher's condition (c) demands that a priori warrants are infallible in order to ensure that a priori knowledge is truth-entailing. This condition is gratuitous because truth is already entailed simply by "X knows that p".

According to Hale, the above-mentioned conflation is precisely the mistake Kitcher makes regarding a priori warrants.

> "But why assume that *a priori* warrants must produce true beliefs? On the face of it, there is simply a confusion at this point. Obviously, for X to know *a priori* that p, it must be true that p (just as it must be true that p, if X is to know a posteriori that p). It clearly does not follow that the satisfaction of this condition, in the case of *a priori* knowledge, must be ensured by the fact that X has *a priori* grounds to believe that p. It seems that, by conflating the truth-entailing character of knowledge (of any sort) with the conditions on the way in which a belief is held required for it to

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and a priori statements, but only when such revisions are called for by a priori reasons.

[40] Hale, p.129.

> qualify as knowledge *a priori* , Kitcher gratuitously inflates the concept of knowledge that is independent of experience into that of infallible knowledge." [41]

Hale thinks that Kitcher conflates the truth-entailing character of knowledge (of any sort) with the distinctive character of a priori warrants. For Kitcher, a priori warrants are infallible, that is, by their means we always will end up having knowledge, i.e. true justified beliefs.

But I do not regard this as Hale's major objection to Kitcher. Hale goes out of his way to point out that Kitcher's real reason for insisting on the infallibility of a priori warrants lies in his reasons for insisting on (3b), as Kitcher understands it.

As Hale points out, in order for Kitcher's argument to establish that reasonable uncertainty [42] is compatible with knowledge, but not with a priori knowledge, a proof followed has to be sound. Otherwise, one's having followed a "proof" (a sequence of formulas) is not enough to give us knowledge, even though one may be deceived into believing it does.

> "... none of the envisaged unkind variants upon my kind, actual experience can be ones in which I am confronted with *decisive* grounds for thinking the proof flawed or its conclusion false. That is, Kitcher must rely, as he seems to realize, upon the possibility of unkind variants of my actual experience in which I am presented with non-conclusive, indirect evidence which casts a doubt upon the proof or its conclusion, but which is, in fact, misleading." [43]

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[41] Hale, p.129; my emphasis.

[42] Kitcher talks about reasonable uncertainty being compatible with knowledge but not with knowledge a priori. In note 34 I explained why this assertion is problematic.

[43] Hale, p.136.

It follows that the unkind experiences against the proof or its result do not conclusively show that the proof is flawed or that its result is false. Hale explains that Kitcher has to be thinking of experiences which misleadingly make us doubt a sound proof or its conclusion.

Section 2: The issue of long proofs again and the role of memory

Hale comments that Kitcher is right in not being satisfied with Descartes' remedy, but observes that Kitcher is uncritical about it, and even seems to accept it. The problem with Descartes' solution, as Hale describes it, is that it suggests that grounds for belief that p can only function as such when they are present to the knower's mind. The contention is that, for example, a proof that p can be one's grounds for believing that p only when one is actually reviewing or focused on the proof that p.

According to Kitcher, Descartes proposes to substitute for the actual following of the inferential steps one's memory of having followed the inferential steps. Hale objects that this proposal cannot solve the problem of the inability to have a long proof all present to the mind. We will have the same difficulty all over again: there would be as many memories as inferential steps, so we will have the same difficulty of not having enough room for memories in our heads. [44]

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[44] Hale reacts to Kitcher as follows:

> "Clearly something has gone badly wrong here ... First, while Kitcher is rightly dissatisfied with (what he takes to be) Descartes' remedy, he seems uncritically to accept, and indeed to endorse, the bizarre suggestion that grounds to believe that *p* can only function as such while they are 'present to the mind' - that a proof that *p* from certain other propositions X can only be a man's grounds for belief that *p* while his attention is actually focused upon a deduction of *p* from those premisses, together with his grounds for accepting them. But it is just false that, when you enquire after his grounds for his belief that *p* , and I reply that he believes that *p*

Nonetheless, Hale warns us that he does not deny the real importance of memory in our acquisition of a priori knowledge.

> "None of this should be taken as denying the real importance of memory in the present context. If our man is asked why he believes that p, it will normally be by a straightforward exercise of memory that he comes up with his answer. And should it prove that he has now totally forgotten the grounds upon which he believes that *p*, and can come up with no others, we should conclude that his continued belief is merely a groundless habit - though it is worth noting that it is commonly no easy matter to establish that someone has indeed totally forgotten; if having been temporarily quite enable to recall his reasons for believing that *p*, he subsequently manages, without undue prompting, to come up with some, which he claims, with apparent sincerity, to have recovered rather than invented afresh, we should be unlikely to insist that, during the period within which he was unable to reproduce them his belief that *p* was

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> because it follows from X, which he also believes, I am claiming that he is, right now, contemplating a deduction of *p* from X, along with his grounds for accepting them. I should be quite unabashed to learn that, at the time of my report, our man was sound asleep, or engrossed in a game of chess. It is true that I may, more cautiously, report that he believes that *p* because he *believes* that it follows from X. But the point is thus hedging my report is not to make allowance for the near certainty that he is not currently reviewing any deduction of that conclusion from those premisses, but simply to avoid endorsing the claim that *p* does follow from X. And it is also true that my report may be mistaken because, he has, in the meantime, changed his grounds - but the possibility that is in question here is that he has, for example, come to think that the premisses X are insufficient grounds to believe that *p* , and now, if he still holds that belief at all, would appeal to other grounds; it is not that, no longer having any deduction of *p* from X in his current mental view, he has perforce shifted to believing that *p* on the strength of his memory of having once derived it. Further, if it really were needful to have all before the mind, it would in any case be no solution to substitute memories for attention to the inferential steps themselves and the grounds on which their premisses are accepted - for in that case, the kind of memories required would be acts of current recall, and we should have just as much difficulty finding room in our heads for enough of them as we allegedly do in accommodating the inferential sequence itself. If, on the other hand, remembering in the dispositional sense, which does not require on the spot reviewing of the contents of our memory, is enough, then the problem supposedly alleviated by invoking memory has been misdescribed in the first place." (pp. 139-40)

> groundless. And there is no question that our capacity to follow and understand any but the simplest proofs is dependent upon (short-term) memory - understanding a proof of any complexity is largely a matter of grasping how it hangs together, and we are not in a position to accomplish that unless we can remember the bits that fits into the structure we must discern, if we are to understand the proof. But none of these familiar facts has any tendency to support the peculiar claim that when a man believes that p on the grounds (as we should normally say) that it follows from certain propositions X, which he also has grounds to believe (though not ones he is currently reviewing), properly and strictly speaking, his grounds for belief are provided, not by the *proof* of *p* from X (which he could reproduce, if called upon to do so) but by his memory of having constructed or followed such a proof." [45]

Hale is thinking in this connection of the worry that requiring actual surveyability may be inappropriate since most of the time we only remember that we followed a proof. Rarely can we remember the proof in any detail. So, actual surveyability as a condition for a priori knowledge does not seem to be what is going on in mathematical practice.

Hale does not deny the importance of memory in our acquisition of a priori knowledge. We have to understand the proof in order to know a priori its result, and that largely depends on our memory of its earlier steps; we have to understand how the proof hangs together. The point Hale is making is that our grounds for belief that p, in the case of inferential knowledge a priori that p, is the proof that p, and not our memory of having followed a proof for p. Hale specifies that one should be able to reproduce the proof, if called upon to do so, in order to be entitled to claim to know a priori that p. When such a recall (or reproduction) takes place it is not that a change of grounds - from memory of having followed the proof (an empirical warrant) to actually following the proof - has taken place but rather than all along my grounds for p were the proof for p.

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[45] Hale, p.140; my emphasis.

Hale correctly explains that length of proofs alone may not be particularly problematic but length combined with complicated structure.

> "My claim ... that sheer length raises no special problem, and that, while long proofs may, when very complicated in structure, be epistemologically problematic, it would be an error to suppose that their unsurveyability disqualifies them as routes to *a priori* knowledge, but leaves us able to acquire *empirical* knowledge by their means-applies equally, I think, to big calculations." [46]

If long proofs and calculations are problematic for acquiring knowledge a priori of their results, they will be equally problematic as sources of empirical knowledge. For knowledge, what is required is the same in both cases (a priori and a posteriori): that (long) proofs or calculations are in fact sound, so their results are correct (true). This is against Kitcher who holds, according to Hale, that while long proofs and calculations cannot be sources of a priori knowledge of their results, they are fine to produce empirical knowledge. [47] Hale's point is that if following a proof (long or otherwise), for example, produces knowledge, despite unkind experiential background, it must be a real proof. So why isn't this knowledge a priori?

Section 3: Some remarks on Hale's attack on Kitcher

Note that an important view that Hale shares with Kitcher is that the notion of "a priori warrant" is the basic notion to be characterized in the epistemology of a priori knowledge. [48] The intuition is that in order to distinguish between a

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[46] Hale, p. 260.

[47] Kitcher, p. 43.

[48] Hale, p. 138. Actually, Kitcher's position regarding the thesis that the notion of "a priori warrant" is the basic notion to characterize is not straightforward in the following passage:

priori knowledge and empirical knowledge we have to distinguish between the ways in which we obtain knowledge.

For Hale, given that a priori knowledge does not require absolute unrevisability of a priori beliefs and a priori warrants, then a pressing issue for Hale becomes how much revision is consistent with a priori knowledge.

> " "A priori" is an epistemological predicate. What is *primarily* a priori is an item of knowledge. Of course, we can introduce a derivative use of "a priori" as a predicate of propositions: a priori propositions are those which we could know a priori. In many contemporary discussions, it is common to define the notion of an a priori proposition outright, by taking the class of a priori propositions to consist of the truths of logic and mathematics (for example). But when philosophers allege that truths of logic and mathematics are a priori, they do not intend merely to recapitulate the definition of a priori propositions. Their aim is to advance a thesis about the epistemological status of logic and mathematics." (Kitcher, ibid, p.21)

What does Kitcher have in mind by saying that what is primarily a priori is an "item of knowledge"? Given that he goes on saying that there is a derivative use of a priori which applies to propositions, I take it that an "item of knowledge" refers to "warrant". This is consistent with both the fact that for him a priori warrants always produce knowledge and that the notion of a priori warrant is the basic one.

Then Kitcher goes on affirming that:

> "My present aim is to distinguish a priori knowledge from a posteriori knowledge. We have discovered that the distinction requires us to consider the ways in which what is known is known. Hence I propose to reformulate the problem: let us say that X knows a priori that p just in case X has a true belief that p and that belief that p was produced by a process which is an *a priori warrant* for it. Now the crucial question is that of an a priori warrant, and our task becomes that of specifying the conditions which distinguish a priori warrants from other warrants." (Kitcher, p.23; my emphasis)

In the last quote it is clear that Kitcher considers the notion of an a priori warrant to be the crucial one to characterize.

Let me observe briefly that the claim that revision is consistent with a priori knowledge is incoherent as it stands. Contrary to Hale, Kitcher is right about the incompatibility of a priori knowledge with revision. However Kitcher arrives at this true conclusion by the wrong reasoning, and that is partly why I think Hale thought Kitcher was wrong about the incompatibility in question. According to Kitcher, a priori knowledge cannot be compatible with revision. By contrast, Kitcher thinks that empirical knowledge is so compatible. But this is a serious confusion, again, because knowledge of any sort is incompatible with revision. Revision is compatible with justification and belief (not with knowledge and truth). Only warrants and beliefs are revisable.

I will be fully addressing this confusion in chapter five and will try to resolve it since it is a pervasive problem which creates all sorts of misunderstandings. For now I will just say that the candidates for revision are beliefs, in particular, claims to knowledge (i.e. they are of the form: "I know that p"), and justifications (warrants). We change our minds about beliefs and the warrants which justify them. We can change our minds about what is in fact true. We may erroneously revise our beliefs and its warrants. In a weak sense, knowledge is compatible with revision since we can change our minds about what is in fact true and we are fully justified in believing. In another strong (normative) sense, knowledge is incompatible with revision because if we mistakenly revise, we cease to know since we cease to believe to be justified.

Two cases are to be distinguished: (1) when revision is the right thing to do; and (2) when we mistakenly think that revision is the right thing to do. When we have knowledge then revision in the second sense is the only thing possible and then we end up not knowing or ceasing to know. But then Kitcher is wrong to think that the negation of (1) is only true in the case of a priori knowledge. In

other words, when we have actual knowledge, then (rightly) revising is not possible in any case.

As we will see, for Hale, the grounds for p should be the proof itself. But Kitcher is right that our grounds for believing a proposition p, or knowing p, in the case of inferential a priori knowledge (in Kitcher's view: in the case of inferential "nonempirical" (≠ "a priori") knowledge) that p should be the process of following a proof. A proof becomes a warrant by its being appreciated. And we can only appreciate a proof by following it. What is wrong is to think like Kitcher that the experience independence of a priori knowledge cannot leave room for the experience dependence necessary for the truth that we have followed a proof that p. For Kitcher, since we can make mistakes in following proofs (or in constructing them), then "following a proof" cannot be an a priori ground for believing that p. However, if the process of "following a "proof", I should say, a sequence of formulas, is sufficient for knowledge, (remember that Kitcher considers the process of "following a proof" a non-empirical process which sometimes can engender knowledge) the sequence has to be a proof-token. Then, why "following a proof" cannot be an a priori ground for believing that p if when we know by their means it is implied that we did not make a mistake in following the proof?

Section 4: Hale's remarks on Kant

Confronted with the alleged compatibility between knowledge and revision, Hale proceeds to consider Kant's view of a priori knowledge. Hale makes some interesting comments on Kant's view that are worthy of mention, even briefly.

Hale explains that Kant intended to characterize a priori knowledge as independent of experience in terms of justification or grounds, in contrast with

knowledge based on causal relations. For Hale, though Kant characterizes necessity and strict universality as positive marks of the a priori, his explanation of a priori knowledge is completely negative. According to Hale, Kant does not provide a general positive account of what makes a ground for belief non-empirical. It is rather assumed what empirical grounds for belief are, and non-empirical grounds are simply grounds for belief that are not empirical. [49] Kant only affirms the general claim that they are to be characterized as those grounds which justify belief independently of experience. Hale explains that this point has been ignored, and it has been assumed that Kant distinguished between both sorts of grounds by a positive feature. Then, an easy step (not unavoidable though) is to conclude that this positive feature which makes a ground for belief a priori involves more exacting standards than those involved in the characterization of empirical grounds to be able to produce empirical knowledge. Hale observes that from the claim that a priori grounds have to comply with more exacting standards than empirical grounds, one may easily think that there is no a priori knowledge or much less than we previously thought.

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[49] Hale explains:

> "It seems to me too that, while Kant, after giving that characterization (1963, B1-5), goes on to offer necessity and (strict) universality as positive *marks* of apriority, his official explanation is wholly negative. He offers, that is, no general positive account of what makes a ground for belief non-empirical - it is, rather, taken as understood what empirical grounds for belief are like, and non-empirical grounds are simply grounds for belief that are not of that sort. It is easy to overlook this point, and to suppose, in consequence, that non-empirical grounds must be distinguished from empirical ones by some positive feature. It is then a short, though not inevitable, step to the conclusion that to be *a priori* , knowledge must comply with more exacting standards than have to be met by ordinary empirical knowledge. And it can then readily seem that we have either no such knowledge at all, or vastly less of it than we thought." (p. 125-6)

Then, for Hale, Kant did not succeed in giving an account of what makes a ground for knowledge "a priori". I don't want to commit myself to such a strong claim since it would involve a very careful reading of the first Critique. Perhaps Hale's point is a weaker and correct one: we won't find what we are looking for, namely, an explicit definition of the notion of a priori knowledge. Now from that modest point of view it does not follow that there is no possible (Kantian) account of a priori knowledge that we can reconstruct from his views. Actually what Hale is trying to do is to interpret the Kantian notion of "experience-independence". [50]

Let me speculate a bit more before finishing with this topic. Hale's view is that Kant was not explicit about what makes a ground for knowledge "a priori" and, consequently, that he does not clarify in any detail the "independence of experience in terms of justification" that he claims to be distinctive of a priori knowledge in contrast with empirical knowledge. What can partly explain Hale's view on Kant is that Kant appears to have concentrated primarily on the properties that a priori judgments have. Kant offers his criteria to differentiate judgments a priori from the rest. They are supposed to be strictly universal and necessary.

Moreover, a related point: I think that Kant was not clearly explicit about what is a non-empirical ground simply because Kant did not appear to accept grounds for belief as being more basic than the "judgments" we could possibly know by their means. In any case, this is a much later development. However, I believe that there is an ambiguity in Kant's use of the term "judgment" which leaves open the possibility that he was referring to grounds too, and not to truths, or not only to truths or judgments expressing them.

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50 P. 138.

Section 5: Some remarks on the role of memory in our acquisition of a priori knowledge

Hale does not want to deny that one hardly remembers a proof in any detail, specially so when the proof is long or complicated in structure. However, Hale would say that if one is not able to reproduce the proof, one's continued belief that p, or knowledge that p, would be at best a posteriori: remembering that one followed a proof that p. Note that knowledge based on memory is to be considered a posteriori regardless of the nature of the memory in question. That is, there is no qualification whether it is a memory of knowledge a priori we had before - which is the only possible case which raises the question whether it ought arguably to be considered a priori - or whether it is a memory about either knowledge a posteriori of truths knowable a priori or knowledge a posteriori of a posteriori truths.

However, Hale appeals in the above quote (p. 35) only to what we normally say. I find it quite plausible to say that I know that p, when I am not currently reviewing my a priori knowledge that p - the proof - by my memory of having constructed or followed such a proof. I don't understand why Hale is so worried about a switch of grounds - from memory of having followed the proof (an empirical warrant) to actually following the proof - since such a switch does not interfere with the a priori knowledge that we can have of the same truths. A possible answer which sheds some light on the role of memory in this connection - and that I think is available to Hale - is to appeal to the following distinction: what is actually known a priori (by us) and what is knowable a priori (by us). When I follow a proof of a mathematical statement, following the proof is my ground for belief that p, and I know a priori that p. A statement is knowable a priori (for us) when there is an a priori warrant for it (a proof, for example) that we can possess, apprehend, and then know the statement a priori. When I only

can remember that I followed a proof that p, I know a posteriori that p, since memory only can provide at best an a posteriori warrant. But it does not follow simply from that either that p is not knowable a priori (for us), or even worse, that p is a posteriori.

Chapter Four
"Pure" Hale

In this chapter I will isolate the views that can be attributed to Hale independently of his reactions to Kitcher.

Section 1: Revision and defeasibility of items of a priori knowledge

Hale explains that when someone's ground for a belief he holds is defeasible, there is always the possibility that considerations may be presented which question the ability of the supposed grounds to be enough to warrant his belief. [51] One then has to establish whether the evidence defeats the grounds in question.

> "Whenever a man's justification for some belief he holds is defeasible, considerations may be adduced which cast doubt upon the capacity of his presumed grounds actually to warrant his belief. Such considerations constitute a prima facie defeat; further investigation may reveal that the doubt to which they give rise is well-founded, or that it is not." [52]

Hale proposes to distinguish between two senses in which our grounds for a belief may be defeasible or undermined by subsequent evidence.

> "we may distinguish between two senses in which grounds for a belief may be said to be undermined by new evidence. In one sense, a man's grounds for his belief that *p* are undermined by the citation of new evidence in the light of which, pending a more through investigation, it

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[51] Hale talks about "justifications", "grounds" and "warrants" for beliefs. I take it that these terms have the same reference for Hale. He certainly does not distinguish between them in the chapter. Actually he identifies "a priori grounds for belief" and "a priori warrants" in his chapter, p.128.

[52] Hale, p.137.

> would be unreasonable for him to continue to repose in those grounds the degree of confidence with which he formerly entertained them. But his grounds may, quite differently, be undermined by adducing further considerations which actually show either that they are not in fact grounds for his belief that *p* at all, or that they are not as good as he supposed them to be, and are, perhaps, insufficient to warrant his belief. The experiences to which Kitcher appeals can indeed 'interfere with the ability of the process [i.e. of following the proof] to warrant belief', if that means that can undermine our grounds in the first, weaker sense. But they do not undermine our grounds in the second sense." [53]

That is, there are two senses in which a warrant for believing that p can be defeasible. A warrant is weakly defeasible if new evidence appears to show that the warrant is not in good standing. New evidence weakly undermines one's warrant for believing that p when it calls for a careful examination and in the meantime suggest that it is unreasonable to have in the warrant the same degree of confidence which one formerly had of it. A warrant is strongly defeasible when it has been established that it is not good. A warrant can be undermined by adducing additional considerations which in fact show either that the presumed warrant does not ground one's belief that p at all, or that it is not as good as one previously thought, and is, possibly, not sufficient to justify one's belief. According to Hale, the experiences that Kitcher appeals to can undermine a priori warrants only in the first, weaker sense.

If my sole ground for believing that p (p being some mathematical proposition) is that I have read what I take to be a proof that p in a math book, and then someone produces (a posteriori) evidence against the reliability of the book, I ought - so far - to give up my claim to know that p, at least until I have checked that the proof is fine. According to Hale, when I suspend my claim to know that p, I do just that, that is, I don't back off into a claim to still know that p,

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53 Ibid.

but only a posteriori. (Of course, if I had not followed the approved proof at all, but merely heard that Hardy, for example, say, that it is knowable a priori that p, and I accept this on the basis of my belief that Hardy did not make mistakes, then my knowledge is at most a posteriori in the first place.)

Nevertheless, the envisaged unkind experiences of my kind actual experiences do not necessarily undermine our grounds for belief in the second sense. For instance, if following a proof against kind background experience yields knowledge, the proof must be sound. If considerations of the envisaged unkind experiential background are relevant, we have to assume to be following the same proof. This shows that the unkind experiences Kitcher considers could not show that the proof is unsound. Hale explains:

> "As we have seen, if following the proof against kindly background experience is to yield knowledge, the proof must be good, and if consideration of the envisaged unkind experiential background is to be relevant, we must suppose the same proof to be followed. Consequently, nothing in the unkind setting could show that the proof is flawed. Indeed, however 'weighty' we envisage the indirect evidence against the theorem or its proof as being, it has to be allowed that a sufficiently painstaking examination will reveal the doubt it raises to be unfounded.
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> It is, then, perfectly possible for us to have grounds for belief that *p* , but to think that those grounds less than adequate, when in fact they are. In the presence of sufficiently weighty indirect reasons to doubt their adequacy, such doubt may be quite rational. In the kind of case with which we are concerned, where our grounds are provided by what purports to be a proof, any indirect reason to doubt its soundness will, of course, be a reason to doubt that it affords grounds for belief at all - a flawed proof is useless, and while it may be possible to repair the defect, or adapt it to establish a restricted version of the conclusion, we may ignore these possibilities here, since they make for no essential difference."[54]

To recapitulate: Considerations can be presented which reduce the confidence we have in a warrant or its result. These are perfectly compatible with our in fact

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[54] Ibid.

knowing the proposition, and with the fact that the warrant is a good one. Knowledge is consistent with a certain degree of unsureness and it is only possible with a reasonable degree of sureness (confidence) that our warrant for believing that p is in good standing. This is the weak sense of defeasibility. The strong sense of defeasibility is when the considerations adduced show that we were in fact wrong, that the proof is not sound, or that the result is not true. According to Hale, a priori grounds are indefeasible by experience in this strong sense. Experience cannot show that an a priori warrant is unsound or that an a priori statement is false, though it can have an "incidental" role in our knowledge of these two things.

Hale explains:

> "Any satisfactory elucidation of the notion of experience-independence should respect the foregoing distinction [between the two senses in which grounds are defeasible]. Kitcher's condition (3b) may be read as requiring that *a priori* grounds should not depend for their cogency upon specific features of the experiential setting in which they are invoked. So read, the condition is at least plausibly necessary for knowledge *a priori*. The crucial distinction is, however, obliterated, if it is read, rather, as requiring that there can be no experiential setting in which it would be rational to doubt them." [55]

Hale concludes that a satisfactory clarification of the notion of experience-independence ought to take into account these two senses of defeasibility of warrants (grounds). Also, he continues, an elucidation of the notion of a priori knowledge does not have to exclude the possibility that we can conceive an experiential background in which it would be rational to doubt the adequacy of a priori grounds to warrant belief. That it is always an epistemic possibility that a proof is unsound does not destroy its capacity (the capacity of the proof) to provide a priori grounds for accepting its conclusion.

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[55] Ibid.

Section 2: Empirical indefeasibility as a hallmark of the a priori

Hale endorses a conception of the a priori as that which is not only justifiable independently of experience, but also that which can only be shown unsound or false independently of experience as well. That is, the candidates for a priori knowledge only admit of either a priori justification or revision by a priori reasons. Empirical indefeasibility is one of the hallmarks of the a priori.

Hale does not claim merely that whenever an a priori statement appears to be conflicting with empirical evidence, or it is apparently falsified by empirical evidence, we always will find the corresponding revision by a priori reasoning - analogously as when we can know a posteriori, truths that can be known a priori - but rather that empirical disconfirmation is not possible for a priori statements and that empirical evidence can have only an incidental role in the defeat of a priori warrants.

In an important note, Hale takes up the issues whether a priori warrants could be defeasible by empirical evidence and a priori statements could be in principle falsifiable by experience.

> "Suppose some very large and difficult calculation has been performed, with a certain definite result. We may well have, independently of our calculation, weighty empirical evidence that the result ought to lie within certain limits. Unfortunately, it does not. We may thus, it seems, have *empirical* evidence that our calculation contains some mistake, even if, owing to its size and complexity, it is difficult to locate one, and our best efforts to do so have (so far, anyway) failed to disclose any errors. Does this possibility impugn the claims of calculation to be a potential source of *a priori* knowledge?
>
> A first, and obvious, point to note is that our result, obtained via the big calculation, will, in the kind of case envisaged, be based upon some assignment of particular values to certain empirical, e.g. physical, variables. The significance of this point is twofold. First, since our knowledge or belief in the correctness of those initial value assignments will be at best empirically grounded, there can be no question, in any case,

> of our having knowledge *a priori* of the result of the calculation (e.g. that a certain physical variable has this or that particular value)." [56]

Let me explain. The example is: A long and complex calculation has been performed and its result disagrees with the one we expected according to independent weighty empirical evidence. Because of the complexity of the calculation, we have not been able to locate specifically the mistake in the calculation. The issue Hale wants to illustrate with this example is whether it provides us with empirical reasons to think that this particular calculation is faulty. We possess empirical evidence against its result, and, therefore, empirical (indirect) evidence against the warrant itself. More generally, Hale wants to know whether this example provides empirical evidence against the claim that calculation is a potential source of a priori knowledge.

The result of the calculation - "that a certain physical variable has this or that particular value" - is <u>not</u> known a priori. The reason is that our knowledge of those initial value assignments is at best grounded empirically.

> "Second, the discrepancy between our result and our empirically grounded belief about the range within which it must lie does not point selectively to a mistake *in the calculation* - we might, so far, pin the blame on our assignment of initial values." [57]

Because we don't know where the mistake is in the calculation, we can say that it could be that the original assignment of the physical variables were mistaken, for example, that the measurements were done incorrectly.

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56 Hale, pp. 260-1.

57 P. 261.

> "To get a more threatening looking case, we may suppose that we have, additionally, very strong empirical grounds to believe that our initial values for the physical variables are correct." [58]

Let us now suppose that the initial assignments of the physical variables are correct. It follows that the mistake is in the calculation itself.

> "Now whilst there can be no question of our having *a priori* knowledge of the result as such, it does seem that our calculation should support a claim to such knowledge of a certain *conditional* statement, to the effect that if our initial value assignments, together with the relevant component general statements of the covering physical theory, are correct, then the value of some further physical variable(s) is such and such." [59]

Hale insists that the result of the calculation is obviously not known a priori given that the original value assignments of the physical variables are correct and at best grounded empirically. On the other hand, the calculation should support a priori knowledge of the following conditional: "If our initial value assignments, together with the relevant component general statements of the covering physical theory, are correct, then the value of some further physical variable(s) is such and such." [60]

> "And the claim may now be that we have, in the situation envisaged, good empirical evidence against a statement which we supposedly know *a priori*." [61]

The statement that we supposedly know a priori is the above-mentioned conditional. Now if we look at the conditional, its antecedent is known to be true

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[58] Ibid.

[59] Ibid.

[60] Ibid.

[61] Ibid.

a posteriori (for, by hypothesis, the original value assignments were determined empirically together with the covering physical theory), and the consequent is, in the case envisaged, the result of the calculation. If the result of the calculation is considered false by empirical considerations, then the conditional would be considered likewise false too. That is why Hale affirms that this example, after having made the appropriate clarifications, would appear to constitute empirical evidence against a statement supposedly known a priori. Hale observes:

> "But now... it is important to ask ourselves just how essential it is to the case just sketched that our grounds for belief that our big calculation contains some undetected mistake are *empirical* . Suppose instead that we have carried through some very large and complex purely mathematical calculation, of the correctness of which it is, for obvious reasons, difficult to be fully confident. Suppose further that there is a standard method of obtaining approximate results for the kind of problem in question, and that this method is much simpler than the method employed in our big calculation to get a precise result. Applying this approximation technique, we find that the result of our big calculation is seriously out of line. Since we are prepared to repose greater confidence in the simpler and more straightforward technique, we have now good *a priori* grounds to believe that the big calculation contains some disclosed error. Does this situation differ materially from that envisaged previously? I submit that it does not, and that the possibility that we should be led by empirical considerations to think that a calculation contains a mistake raises no relevant issue that it is not already raised by the possibility of our having *a priori* grounds to think that the result of the calculation must have been done incorrectly. The essential points are these. First, if, in either case, the big calculation does indeed contain a mistake, it is no contingent fact that that is so, regardless of whether the considerations which lead us to think so are empirical or *a priori* ." [62]

Again, Hale's point is: there can be empirical grounds for <u>suspecting</u> that a mathematical statement is false (or that the warrant we use to "prove" it is

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62 Ibid.

unsound). Hale wants to claim that empirical grounds point to an error detectable a priori.

In the case of an empirical generalization like

All swans are white

empirical evidence against it constitutes a direct disconfirmation of the empirical generalization. For example: There is a swan which is not white. (A counterexample of a generalization implies (logically) the negation of the generalization. If A implies not B, then the truth of A disconfirms B.)

Hale's point is that empirical evidence is sufficient to falsify empirical statements. Empirical evidence is not sufficient to falsify a priori statements. We need the a priori evidence that our original a priori statement is false.

For Hale, since empirical evidence against an a priori statement is incidental, empirical evidence cannot falsify an a priori statement. According to Hale, that empirical evidence cannot falsify an a priori statement is obviously true but not because the statement in question is in fact true since in that case the a priori evidence against it would be incorrect, but rather than given that it is in fact false, still empirical evidence could not falsify an a priori false statement.

Section 3: Some remarks about Hale's views on the defeasibility of items of a priori knowledge

For Hale, the case where we have strong empirical evidence against the result of a calculation as well as a priori evidence against the same result is analogous, for example, to the more simple case when we have the experience of receiving a Russellian letter as well as finding the contradiction ourselves. The experience of receiving the letter is incidental. The problem is with what the letter contains, that is, the contradiction. Hale is claiming what can be called "the autonomy of

the a priori": that is, if there is a mistake involved in a route to a priori knowledge and if we are able to locate it, then we could do so by a priori considerations alone.

But the view that our possessing a posteriori considerations for thinking that there is a mistake in the calculation, for example, is incidental is correct only if we always have at hand both kinds of considerations (a priori and a posteriori). I find this implausible. There is no guarantee that we are going to be always in that epistemological privileged position.

Of course, when we have a priori evidence, the a posteriori evidence may be incidental or not necessary; on the other hand, when we yet don't have a priori evidence then the a posteriori evidence is very important, though it may not be necessary in the qualified sense that if we possess a priori evidence at some point later on then we won't need the a posteriori evidence.

Returning to the simple case of the experience of receiving a Russellian letter, I don't see the analogy between this experience and the experience of having strong empirical evidence against the result of a calculation. One thing is the experience of receiving the letter and this experience only containing what really posits the difficulty: the contradiction. The letter is not the ground for questioning the calculation or its result. The letter only contains (provides) the ground, the contradiction, which is what really poses the difficulty and constitutes the ground (a priori) for questioning the calculation or its result. In this case, the incidentality of the experience of receiving the letter is quite uncontroversial. We could just as well know of the contradiction by reading it in a book or by finding the contradiction ourselves, if we are in a position to accomplish that. But the interesting case is when empirical considerations can defeat directly - raise relevant issues concerning the defeasibility of - a priori grounds or its results. That is, it is quite another matter when there is empirical

evidence against a calculation or its result - as Hale's example seems to suggest - that itself constitutes a ground, and may not merely contain (provide) a ground - as in the example of the experience of receiving a Russellian letter. Hale says in the last quote:

> "the possibility that we should be led by empirical considerations to think that a calculation contains a mistake raises no relevant issue that is not already raised by the possibility of our having *a priori* grounds to think that the result of the calculation must have been done incorrectly". (p. 261)

Hale's point is that if we are able to gather a priori evidence against a calculation, empirical evidence would be incidental: that is, it would not point to any distinctive problem with the calculation besides the one a priori considerations alone point out.

The following quote may convey the thought that given that a mistake in a calculation is a matter of necessity, if one is to know that there is a mistake in the calculation, then one is to know it a priori.

> " ... if, in either case, the big calculation does indeed contain a mistake, it is no contingent fact that that is so, regardless of whether the considerations which lead us to think so are empirical or *a priori* ." [63]

At first sight, this implication can appear problematic because the notions of necessity and a prioricity are not coextensive, [64] or at least not necessarily so in all cases. We are talking of mathematics in this context. Hale, of course, is aware of the latter. He is not claiming that the notions of a prioricity and necessity are coextensive. What I think he is implying is that provided that the notions are not coextensive, it does not follow that the possibility that some necessities can only

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63 Ibid; my emphasis.

64 Kripke's Naming and Necessity. Hale endorses this distinction.

be known a priori cannot arise - as, allegedly, some necessities can only be known a posteriori (like the identity statement "Hesperus is Phosphorus"). Then, another reading of that remark is possible: Hale is insisting here again on the same point that the a priori evidence is necessary.

Hale seems to admit that there could be both a priori and a posteriori undermining, but that the a priori undermining has priority (he says it is "paradigmatic" ("essential")). I take it that, for Hale, this situation would be entirely analogous to the case of having both an a posteriori justification and an a priori justification for the same (a priori) statement. But that may be to admit too much. And, if Hale does admit this, then why should only the a priori undermining "count" as it were? The following question arises: How can the incidentality of empirical evidence against an a priori statement, something Hale accepts, guarantee that such empirical evidence cannot falsify the a priori statement in question? I think that Hale would appeal at this point to his distinction between weak defeasibility and strong defeasibility. Empirical evidence can only weakly undermine items of a priori knowledge. That is, new empirical evidence can undermine one's a priori grounds for belief that p when it calls for a careful examination and in the meantime suggest that it is unreasonable to have in those grounds the same degree of confidence which we formerly had in them.

But the point clearly remains: if we accept a priori and a posteriori undermining, why is one superior over the other? This is worrisome since the empirical evidence is pointing to a problem, perhaps that the statement in question is false, so it should be taken seriously.

If Hale says that empirical considerations do not raise any further relevant issue that a priori considerations do raise, how can he say that a priori statements are indefeasible by experience? His two claims that a priori statements are not

falsifiable by experience and that empirical considerations can raise the same relevant considerations - but no more - than a priori considerations in the defeat of warrant or a falsification of its result which we supposedly knew a priori, seem to be incompatible. The second seems to imply that a priori statements can be falsified by experience.

For Hale, empirical evidence is not necessary. I already have explained why this assertion is problematic. Nonetheless, empirical evidence does not seem to be sufficient to obtain knowledge of truths knowable a priori. Note that this assertion does not contradict the possibility of a posteriori knowledge of such truths since a posteriori knowledge of truths knowable a priori appears to be only possible given that there is a priori knowledge of these truths which posteriori knowledge must be based upon. This is surely true in the case of knowledge a posteriori based on testimony of a priori evidence. Such a posteriori knowledge is only possible if there is something to testify about, in the case in question, if there is a priori knowledge to refer to. In a qualified sense then knowledge based on testimony is sufficient for knowledge but only because it depends upon the knowledge a priori obtained already. Testimonial (empirical) evidence of a priori evidence is contrasted with empirical evidence in general. Empirical evidence in good standing is sufficient for knowledge of an empirical statement.

Hale's discussion points to another very interesting question: what is the relationship between an a priori justification and an a posteriori justification for the same proposition or truth? Is one stronger than the other? In the case of testimony we have seen that the a priori evidence is stronger, not because necessarily it is more certain than the other, but rather because it is a necessary condition for the possibility of such a posteriori knowledge. Note that the converse is not the case. It seems that a consequence of Hale's view is that the

only way we can have knowledge a posteriori of truths that are knowable a priori is by testimony of a priori evidence. That is, it seems that testimony is the only a posteriori way to know truths knowable a priori.

Let me conclude this section by observing that the fact that a truth can be known in both ways, a posteriori and a priori, does not make such truth a posteriori. It is interesting to note that we do not call truths that can be known in both ways "mixed truths" but rather continue to considering them "a priori". A possible explanation for the latter is: A priori warrants are more secure than a posteriori warrants, so that we ought always to prefer the a priori warrant for a truth over an a posteriori warrant for the same truth; and for this reason, the truth is "a priori".

I don't think this explanation is needed. The justification needed, in my opinion, for considering "mixed truths", nonetheless, "a priori truths" does not have to depend upon the claim that a priori warrants are more secure than a posteriori warrants. (They may well be; that is a separate issue.) Rather it depends upon two alternative claims: (1) that the notion of (a priori) warrant is more basic than the notions of (a priori) belief and truth; and, more importantly, that (2) a priori warrants only warrant a priori statements (and truths, at best).

Note that (1) and (2) are compatible with the following interpretation of the phrase "more secure" in the statement "A priori warrants are more secure than empirical ones for the same beliefs (or truths)". It is possible to understand the phrase "more secured" here as expressing merely the fact that those a posteriori warrants like testimony, for example, have to depend for their reliability upon the reliability of the a priori warrants which they refer to. For example, I can be told about a mathematician's proof and know its theorem only if there is such a proof. This is a very interesting topic indeed and it has to do with what can sensibly be said about the relationship between a priori warrants

and a posteriori warrants for the same beliefs, in particular, with the issue of the strength of such warrants which I think ought to be considered as the main criterion for determining their relationships.

Section 4: Hale's assertions on a priori knowledge

Hale makes some assertions about a priori knowledge. It is important to get clear about what he means by them, i.e. what their status is.

One of Hale's assertions is the following:

> "It would, obviously, be too much to require, for X's knowledge that *p* to be *a priori*, that X could have known that *p* (or, better, could have come to know that *p* in the way he did), *whatever his experiential background*. We should certainly allow that experiences of certain kinds may be needed for X to acquire the relevant concepts requisite to entertaining the thought that *p* at all, but would not wish that to put the truth that *p* beyond X's *a priori* ken. So, however precisely experience independence is to be cashed, it should be construed liberally enough for *a priori* knowers that *p* to have experiences sufficient for acquisition of the requisite conceptual repertoire. This may involve more than just the concepts explicitly involved in the thought that *p*; knowledge a priori that *p* may ... involve coming to appreciate that *p* by deducing it from other propositions known *a priori* -we need to allow X experiences sufficient for acquisition of concepts involved in his premisses, in cases of mediate knowledge *a priori*." [65]

The idea is that we have to allow room for certain experiences that may be necessary to equip ourselves with the necessary concepts to be able to entertain the thought that p in the first place. The experiences in question may also involve, in the case of inferential a priori knowledge that p, those that may be necessary for the acquisition of the concepts that appear in the premises. That is, experiences that may be necessary for concept acquisition have to be accommodated in an adequate notion of a priori knowledge.

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65 PP. 127-8.

Another assertion Hale makes is:

> "The obvious way to distinguish knowledge *a priori* that *p* from (undifferentiated) knowledge that *p*, where it is knowable *a priori* that *p*, is to require that the *a priori* knower's justification for his belief that *p* should be independent of (all) experience save that which is needed for concept acquisition. We need the idea, that is, of *a priori* grounds for belief, or, ... of an *a priori* warrant." [66]

First, Hale is not providing a definition of a priori knowledge here, but rather a general principle that allegedly any definition of a priori knowledge should satisfy. Hale does not put this principle forward as a definition because he holds that a definition should specify positively what is going to define (i.e. its *definiendum*). For Hale, this principle is not a definition because it does nothing to elucidate the form of "independence of experience" characteristic of a priori knowledge. Another different matter is how to implement this principle. Second, Hale, following other writers like Kitcher, for example, takes the concept of an "a priori warrant" to be the basic concept in the epistemology of a priori knowledge.

Hale agrees with Kitcher in requiring for the notion of "independence of experience":

> "The constraint imposed by condition (3b), of Kitcher's analysis seems simply to be that a priori grounds should not depend for their cogency upon any specific features of the experiential background, and that seems reasonable enough." [67]

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[66] P. 128.

[67] P. 134; my emphasis.

> "a priori grounds should not depend for their cogency upon specific features of the experiential setting in which they are invoked. So read, the condition is at least plausibly necessary for knowledge a priori that p." [68]

I take it that these assertions are consequences of the general requirement Hale imposes on any definition or elucidation of a priori knowledge which makes the latter independent of experience, except in the sense of being dependent on those experiences which are necessary for the acquisition of the conceptual repertoire needed for a priori knowledge.

Section 5: Hale's preferred notion of a priori knowledge in Abstract Objects

Hale makes another assertion - which I call (1) - that I understand as his preferred assertion about a priori knowledge. Hale agreed, in private conversation, [69] that (1) would be the claim to work on if a satisfactory definition of a priori knowledge is to be elicited from the suggestions offered in his book.

> (1) "For knowledge that p to be a priori, our justification for belief that p must not require the truth of any empirical statement." [70]

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68 PP. 137-8.

69 I have profited enormously from private conversations with Hale about a priori knowledge since the Fall of 94 when I first arrived as a visiting graduate to the Department of Logic and Metaphysics at the University of St. Andrews, Scotland to work under the supervision of both Professors Hale and Crispin Wright. Since Fall 95, Hale and I have been meeting in the Department of Philosophy at the University of Glasgow.

In private conversation, Hale explains that his main task in his chapter "Non-Empirical Knowledge" was the issue of revisability in connection with a priori knowledge and to respond to Kitcher who thinks that both are incompatible. Hale did not intend to provide a definition of a priori knowledge in his book. Rather his major task was to rebut Kitcher.

70 Hale, p. 138.

It is quite clear that (1) states at most a necessary condition for a priori knowledge. Condition (1) could be fulfilled without it's being true that p, so it can't possibly be sufficient as it stands. Hale admitted (in private conversation) that (1) could have been converted into a purported definition in the following way:

> (1^1) X knows a priori that p if and only if X knows that p and X's justification for belief that p must not require the truth of any empirical statement.

Certainly the most obvious way to turn (1) into a sufficient condition also would be to augment it along the lines of (1^1). But, as we will see in chapter five, section one, there are problems with (1) and (1^1). Before I go into those, I shall make some remarks.

(1) is intended to respect the distinction between two senses of defeasibility Hale spoke about. Hale says that the requirement formulated in the text (i.e. (1)) "accords with" Bennett's suggestion. Hale expresses Bennett's notion of an a priori statement, "judgment" in Bennett's terminology (I suppose following Kant), as follows:

> "A priori judgments are those which aren't open to falsification by experience." [71]

Hale explains:

> "Certainly it [Hale's conception of a priori knowledge] accords with the suggestion ... that *a priori judgments* are those which aren't open to falsification by experience - clearly, if our grounds to believe that *p* don't

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71 Hale, note 9, p. 259. For the suggestion, see Bennett, Jonathan. Kant's Analytic. Cambridge: Cambridge University Press, 1966, p. 9.

require the truth of any empirical statement, the proposition that *p* itself must not have empirical consequences." [72]

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[72] Hale, note 9, p. 259; my emphasis.

It is not clear whether (1) and Bennett's suggestion are logically equivalent. The problem is that Hale does not explain in his book what he means by "empirical consequences". I think that what he means is that if a statement is a priori then it is not possible to derive from it any empirical statement by deduction. That is, from a priori statements alone one cannot derive any empirical statement. Empirical consequences seem to be empirical consequences obtained by deduction.

According to this interpretation of the notion of "empirical consequence", Bennett's suggestion is rather a consequence of (1). My reason for this belief is the following: Hale thinks that if our grounds for believing that p don't require the truth of any empirical statement, the proposition that p itself must not have empirical consequences. And if p does not have empirical consequences, it is not going to be falsified by experience. The consequent of the last conditional is Bennett's suggestion.

Let's try the other direction:

If a statement is not falsifiable by experience then it does not have empirical consequences.

The problem is that this implication holds only if we accept at this stage that an a priori statement is characterized as not being falsifiable by experience. So we are begging the question. Let's ignore this problem and assume that we can obtain this first conditional. Now another problem arises when we try to derive the second conditional:

If a statement p does not have empirical consequences then our grounds for believing that p don't require the truth of any empirical statement.

The problem is that from a statement having no empirical (deductive) consequences nothing follows about its justification. That is, it does not follow that its justification does not require the truth of any empirical statement. Of course, the latter may characterize its a priori justification. (In any case, that is what Hale thought in the book.) However, even if an a priori justification for p could be characterized as one which does not require the truth of any empirical statement, it is left open at this point the possibility that our justification for p is as a matter of fact a posteriori given that we can obtain knowledge a posteriori of truths knowable a priori.

So, if my interpretation of Hale's notion of "empirical consequence" is correct, it follows that (1) and Bennett's proposal are not logical equivalent.

Hale does not mean that as a matter of fact it is not going to be falsified. Clearly what is important (according to his proposal) is that the statement that p should not be open to empirical falsification at all. It will not be enough if it merely happens not to get falsified.

Is there anything special meant here beyond "falsifiable"? There is a distinction between "open to falsification" and "falsifiable". What is not open to falsification is, perhaps, a maxim (a moral rule). We don't let anything to count against them. They are not open to falsification simply because they are not candidates for truth in the first place. [73]

Do "open to falsification" and "falsifiable" mean the same for Hale? Of course, they do not, but it is very possible that he didn't take into consideration the difference in the book.

Hale did not explicitly make any such distinction between "open to falsification" and "falsifiable". He agrees (in private conversation) that one could and should make a distinction between statements which are not open to falsification of any kind because they are not candidates for truth at all (such as perhaps moral rules, or laws of the land (in contrast with putative laws of nature)) and statements which are. When Hale proposes that statements known a priori should not be open to empirical falsification, he is not saying that they should not be open to falsification at all. On the contrary, Hale meant to allow precisely for the possibility of their being shown to be false (and so falsified) by a

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[73] It can be argued that moral principles don't show the difference. They are not open to falsification, and they are not falsifiable. Of course Hale is thinking only of statements which have truth-values, but there is no harm in allowing the others if one wants the restriction to statements that have truth-value is important for (1) and (1^1). Otherwise one would know a priori that one ought not impose pain unnecessarily (moral rule). But even here, there are people who think those statements have a truth-value.

priori considerations (just as Frege's basic law V is shown to be false by the deduction of Russell's paradox from it).

Bennett affirms:

> "'A priori' and 'a posteriori' are among Kant's hardest worked technical terms. His use of them is complex and many-layered, but all we need at this stage is the division of judgments into a priori and a posteriori on the basis of what risk a judgment runs of being falsified by experience.
> 'Necessity and strict universality', says Kant, 'are ... sure criteria of a priori knowledge'. The context clearly implies that necessity and universality are entailed by apriority as well as entailing it. Thus, if the judgment that all Fs are G is a priori, then experience *cannot* render it false by yielding even a single F which is not G. If it is a posteriori, then it *could* be falsified by experience." [74]

In this quote we can see that Bennett does not use the term "open to falsification" at all. Bennett is talking in this passage about statements which are known (or knowable) a priori, and for a statement to be known a priori, it has to be true, and if it has to be true, then certainly it is a candidate for truth. Furthermore, given Kant's view that the notions of a prioricity and necessity are coextensive, then a priori statements are necessarily true, and if so, certainly they are candidates for truth. Given Hale's clarification of the understanding and acceptance of the distinction between "open to falsification" and "falsifiable", then Hale was simply a bit informal in the passage quoted before and then Bennett and Hale's proposals are both referring to the notion of "non-falsifiability by experience" rather than to the notion of "not open to falsification".

Let me now make a remark about their proposal so understood. It is a point of clarification. It might be argued (incorrectly) that Bennett and Hale's claim that "a priori judgments are those which are not falsifiable by experience" is

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[74] Bennett, ibid; my emphasis. The reference to Kant is the first Critique, B 4.

false. [75] Consider the following statements: "A master criminal leaves no evidence of his crime" and "Bill Clinton is a master criminal". The first is plausibly a priori - and indeed analytic - since a master criminal - a really accomplished criminal - has to be good enough to ensure that he escapes detection. It follows that if the second is true, it will be undetectably true - since there won't be any traces of Clinton's criminality. So if the first is a priori, then the second, if true, will be unknowable. But I see no case for saying that the first is unknowable.

So, the second is not falsifiable (or verifiable) by anything, and it is not an a priori statement. It cannot be an a priori statement because it is not knowable in the first place.

Now this criticism is incorrect. Surely Hale (at least) meant only that if p is an a priori statement it is not falsifiable etc., i.e. he offered a necessary condition. The argument above assumes that non-falsifiability by experience is a sufficient condition for them, i.e., that non-falsifiability by experience is a necessary and sufficient condition for being an a priori statement. But non-falsifiability cannot be a sufficient condition for being an a priori statement because it has to be knowable too. For Hale, non-falsifiability by experience plus knowability of statements does imply their being a priori.

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75 Both Hale and Bennett refer to a priori judgments here. I talk about statements and propositions. Hale talks about judgments, statements and propositions in the same short passage, see note 9 p. 259. I assume that all these terms refer to the same thing.

Chapter Five
Critical examination of Hale's views on a priori knowledge

In this chapter I will critically examine Hale's proposals to define the notion of "a priori knowledge". The chapter consists of three main sections followed by a conclusion. The first section of section one is completely devoted to Hale's published work in contrast with both the rest of section one (i.e., section 1.1) and the two other main sections. In section two I shall be dealing with the most recent developments where Hale (in seminars) proposes a new candidate, (H), for a definition of a priori knowledge. In section three I will closely examine whether it is coherent to talk of revision in connection with a priori knowledge - as Hale and other philosophers have thought - ending up with some remarks on the relationship between the notions of revision and a prioricity. In the concluding section I recapitulate the most important theses of this part of the dissertation.

Section 1: A difficulty involved in Hale's proposal in Abstract Objects

The suggestion I want to consider in this section is prefigured in the following claim of Hale's:

> (1) "For knowledge that p to be a priori, our justification for belief that p must not require the truth of any empirical statement."

Again, we can straightforwardly convert (1) into a purported definition in the following way:

> (1^1) X knows a priori that p if and only if X knows that p and X's justification for her belief that p must not require the truth of any empirical statement. [76]

Is this acceptable? Clearly, some qualification is going to be needed. What is it for a justification not to require the truth of any empirical statement? Consider the case of a purely mathematical or logical proof that P. (I assume that P is the conclusion of some proof). Here there will be, to be sure, no role for any empirical statement as a premise. But my justification for believing P, when so proved, will still require the truth of empirical statements like "I have followed a proof that P", "I am competent to follow proofs of this kind", and so on. Without justification for the truth of empirical statements of this sort, I should simply have no justification for the proved proposition, even in the face of the most straightforward and rigorous proof. [77] This is one of various problems with (1) and (1^1), but I shall begin with perhaps a more immediate difficulty. The principal problem with this kind of proposal is circularity: we can't yet be presumed to know what an empirical statement is, if our business is to explain what an a priori statement is - it is no good giving an account of a prioricity which simply makes an unreconstructed use of the idea of a statement's being empirical. [78] Of course the characterization can still be correct (though I shall

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[76] I am following closely Hale's original formulation of an a priori justification (warrant) in his book in constructing the right hand side of (1^1). Now we can express the same thing by inserting "does" instead of "must" resulting in:

X knows a priori that p if and only if X knows that p and X's justification for her belief that p does not require the truth of any empirical statement.

No modality is needed on the right-hand side of (1^1). What the right-hand side of (1^1) is saying is just that X's justification is independent of experience only if the justification does not require the truth of any empirical statement. There is no point in the "must".

[77] I will elaborate on the distinction between a justification for p and a justification for my belief that p in chapter five, section 2.3.

raise doubts about that too) - the circularity problem has to do with whether or not it is explanatory.

Does Bennett's version escape the circularity criticism? We will see shortly.

Section 1.1: Hale's first reaction to my circularity objection to (1) (and (1^1))

In private conversation (Fall 94) as a reaction to my circularity objection, Hale proposed to give up the attempt to provide a definition of a priori knowledge. He suggested that we have instead to look at the concept in application. When we try to define, we try to provide necessary and sufficient conditions that do not appeal to notions which are close enough to the term to be defined. For example, the notions of analyticity and synonymy are too close. One ought not to appeal to each other in trying to define these notions. According to Hale, the situation with the notion of a prioricity is analogous to the situation with the notion of analyticity. For Hale, being definable is not a condition for making sense. That a notion is not definable does not mean that it is not explainable. Furthermore, being explainable, providing an explanation, does not amount to providing a definition. It is possible that an explanation of the notion would crucially consist of providing examples and comments on the examples.

Hale also suggests that the task is to get somebody to understand what it is for a statement to be subject to empirical revision or a priori revision. The suggestion is to explain the notion of a priori knowledge in terms of the kind of revision to which it is vulnerable, instead of defining the way we positively know

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78 I recognize that, in general, it is possible to know what an a priori statement is (an priori statement being a nonempirical statement) given that we have a clear characterization of what an empirical statement is. But at this point of the discussion of Hale, we cannot know what an a priori statement is in this manner since we don't yet know what an empirical statement is.

a priori. We need an explanation of how there can be a priori knowledge. It may not matter that the a priori cannot be defined, but only explained.

Now the task at hand is to characterize the notion of an a priori warrant in a way that is less circular - it may be that we can only reason in circles - or to get a bigger circle in order to shed more light into the concept of an a priori warrant. The task is to try not get the circle so quickly, as Hale does, such that we don't remain without clarification of the concept of an a priori warrant. To this task I shall return in the next chapter. But, for now, let me discuss Hale's more recent proposals to rebut my circularity objection.

Section 2: Hale's most recent proposals [79]

The worry about circularity in connection with any appeal to the idea of an empirical statement in a purported account of the a priori, is simply that it is not clear how to understand the empirical except as the complement of the a priori. If "empirical proposition" were to mean, roughly, "proposition not knowable a priori", we would have a very small circle indeed.

That suggests that we might improve matters by saying a little more about what qualifies a statement as empirical. And that, indeed, is the direction of the first of Hale's more recent proposals. Hale suggests the following modification of (1) (and (1^1) respectively) to avoid the charge of circularity. First:

(1^2) For knowledge that P to be a priori, our justification for belief that P must not require the truth of any statement Q such that Q is falsifiable by experience.

Second:

(1^3) X knows a priori that P if and only if X knows that P and X's justification for belief that P must not require the truth of any statement Q such that Q is falsifiable by experience.

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[79] This section is entirely based on my conversations with Hale in Fall 95.

That is, the proposal is to substitute "statement falsifiable by experience" (or "experientially falsifiable statement") for "empirical statement" in (1) (and (1^1)). This follows exactly the path suggested by my comment earlier on Bennett's version.

Is Hale's new proposal an advance? It is his view that (1^2) avoids circularity because it implicitly explains what an empirical statement is without appealing to the notion of an a priori statement, characterizing the empirical as what is disconfirmable by experience. This avoids circularity because the bargain was not to explain what experience is (that was taken for granted), but rather what "independence of experience" is. He does not deny that (1^2) (and (1^3)) may be vague - they don't say what "experience" is - but at least they do not involve a vicious circle. We cannot define everything. Sooner or later we must rest upon something undefined.

Can we develop this further? For instance, might (1^2) be simplified to:

(2) X knows a priori that P only if the proposition that P cannot be falsifiable by experience.

One concern with this is that we do not wish to exclude the possibility of a posteriori knowledge of propositions known a priori. (2) might seem to risk violation of this constraint. I suspect that this was what Hale probably had in mind [80] when he did not offer (2) straight away. But it is incorrect to think that (2) excludes knowledge a posteriori of truths knowable a priori. Let me explain.

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80 P. 138.

Assume (2) is correct, and that the categories of knowledge a priori and knowledge a posteriori are exhaustive. Then it might seem that the following principle would hold for knowledge a posteriori that p:

(3) X knows a posteriori that P only if the proposition that P is falsifiable by experience.

From (2) and (3), it would indeed follow that if it is knowable a priori that p, then it is not knowable a posteriori that P. (3) clearly precludes knowledge a posteriori that P when P is a proposition knowable a priori.

Now, it is not clear that one has to accept (3) just because one accepts (2). But, first of all, what would be wrong with just accepting (3)? (3) affirms that a posteriori knowledge is only knowledge of propositions that can be falsified by experience. Then, a priori propositions cannot be known a posteriori. Since it is presumably desirable to leave room for knowledge a posteriori of propositions that can also be known a priori - we don't want to exclude knowledge of a priori propositions based upon testimony, for example - we have to reject (3).

It is clear at any rate that (3) does not follow from (2). The logical form of (2) is: If P then not Q, while that of (3) is: If R then Q.

(1^2) would be paralleled by the following characterization of a posteriori knowledge:

(3^1) X knows a posteriori that P only if X's justification for P requires the truth of some experientially falsifiable statements.

(Note that the consequent of (3^1) is the contradictory of the consequent of (1^2)). (3^1) would, in any case, avoid the undesirable exclusion of the possibility of a posteriori knowledge of truths knowable a priori.

Section 2.1: Problems with (1^2) and (1^3)

First of all, the notion of "require" in the formulations (1^2) and (1^3) is unclear. What does it mean for a justification to require the truth of a statement? What should "require" mean in the formulation? Let's distinguish two cases:

(A) A justification for P requires the truth of Q just in case the justification proceeds by inference to P from a set of premises which include Q.

The evident problem with this is that not all our a priori knowledge is inferential. So no constraint is imposed by (1^2), so interpreted, on non-inferential a priori knowledge; equivalently, no distinction is yet made from non-inferential empirical knowledge. To see this, suppose we interpret (1^3) along the same lines. And let's take a non-inferential belief: say, "I am sitting comfortably" or "I am holding a pen in my hand." Clearly the proposal is too broad. Any non-inferentially known proposition would count as known a priori because - being non-inferential - it does not require the truth of any other empirical statement other than the truth of itself, of course. Any proposition based non-inferentially on my memory, or on my sensory states, would qualify. So, (1^2), so interpreted, could not provide the basis for a biconditional account (it cannot be transformed into (1^3)). It only would provide for a necessary condition for a priori knowledge:

X knows a priori that P only if X's justification for belief that P is not given by inference from any statement Q such that Q is falsifiable by experience.

What is the problem with leaving matters there? It is that if we only offer a necessary condition for a priori knowledge, we leave it open what it would take for there to be any instance of knowledge of this kind. Obviously, the matter is not left entirely open since the necessary condition means that something won't count as a priori knowledge if it does not satisfy the necessary condition. I said

that I intended to offer no argument that there is such knowledge. But it certainly is on our agenda to try to explain what such an argument would have to accomplish. This demands that we offer a sufficient condition for knowledge to count as a priori.

Another possible interpretation of the term "require" in the formulations is:

(B) That I have justification for P is inferred from Q as a premise (possibly among others).

If "requires" is interpreted more liberally - if the requirements of a justification of P are not just any premises which that justification utilizes but include certain collateral beliefs - for instance, in one's own competence - on which justification for P may depend, then (1^2) is surely too demanding a condition to impose on a priori knowledge. For instance, Q might be a statement of the collateral conditions [81] that have to obtain in order for me to be justified in accepting P; say, "I am not dreaming", "I am paying proper attention", and so on.

Any account incorporating (1^2) with "requires" interpreted along these lines will be too strong. It will demand that there are no collateral conditions on a priori knowledge. This would be an unreasonable requirement. Even a priori warrants call for the satisfaction of conditions concerning the appropriate cognitive functioning of the knower.

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[81] Collateral conditions are background empirical conditions that have to obtain in order for a subject to be justified in believing a proposition. I call them also "preconditions". They do not belong to the justification for p, that is, they don't function as premises in an argument for p. Rather they are outside the argument, so to speak, functioning as background empirical assumptions (or conditions) for my being able to respond to the argument by believing that p. As we will see in chapter five, section 2.3, since these background assumptions do play a role in my believing that p, they can be part of the explanation of why I failed to know that p. That is, background conditions expressed in the form of statements can be called for explaining why I didn't manage to know that p, if that were the case.

There is a possible objection to this second reading of the term "require". That is: I don't need to claim that "I am justified in accepting P". The answer is that the preconditions obtain even if one does not state that they are. Moreover, given that these preconditions can always be expressed in the form of statements, then the point vanishes.

It is the concern which has emerged about non-inferential a priori knowledge that motivates introduction of the simplification (2) above. [82]

It is important to realize that in (2), "Q" is gone. So this formulation promises to help avoid the problem of non-inferential beliefs coming out as a priori. For this reason (2) cannot <u>just</u> be a simplification of (1^2).

Could we build the account we need on (2)? No: (2) cannot provide for a suitable biconditional because the result would be to conflate knowledge a priori with a posteriori knowledge of truths that can also be known a priori. Let me explain:

(2) X knows a priori that P only if the proposition that P is not falsifiable by experience.

(2^1) X knows a priori that P iff X knows that P and P is not falsifiable by experience.

The right hand side of (2^1) is satisfiable by X's a posteriori knowledge of truths known a priori by X; and it will then be equivalent to her a priori knowledge that

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[82] Remember that (1^2) does not put any constraint for non-inferential a priori knowledge. So any non-inferential belief comes out as being knowable a priori since any non-inferential belief does not require the truth of any empirical statement (apart from the truth of itself). (2) adds the condition for knowledge a priori that p, that p itself cannot be falsifiable by experience. (2) rules out basic empirical knowledge coming out as a priori since p itself would be falsifiable by experience, so it would not satisfy (2).

P, for example. (2) does not have that consequence if it is understood only as a necessary condition.

Section 2.2: Hale's revised account

In response to the difficulty that (1^2) fails to distinguish between non-inferential knowledge a priori and non-inferential knowledge a posteriori Hale has suggested (in private conversation) the following proposal:

> (H) X knows a priori that P iff X knows that P and neither X's justification for believing that P nor P itself implies (entails) the truth of any experientially falsifiable Q.

This is designed to handle both problems emphasized in our discussion so far. First it leaves room for the possibility of a posteriori knowledge of truths which are also knowable a priori. For while (H) imposes a necessary and sufficient condition on an a priori justification for believing that P, it does not say anything about the range of the a posteriori. Second, (H) distinguishes between non-inferential a priori knowledge and non-inferential a posteriori knowledge since the definition does put a distinctive constraint on non-inferential knowledge that is a priori: viz. P itself cannot imply (entail) the truth of any experientially falsifiable Q. When Q is P, since P implies itself, P also cannot be falsifiable by experience. In contrast, in the case of non-inferential a posteriori knowledge that R, say "I am sitting comfortably", since R implies itself and R is an empirical statement, R is falsifiable by experience. Since R is falsifiable by experience then it won't satisfy the condition that R does not imply (entail) the truth of any experientially falsifiable proposition. So, non-inferential knowledge of R won't count as non-inferential a priori knowledge.

Section 2.3: Some remarks about (H)

When one knows a priori that p, and one's knowledge that p is inferential, then one knows that p because one has a sound proof that p. When one doesn't know that p, something has happened. For instance,

(1) the purported proof is unsound.

(1) would be an a priori reason to reject the alleged proof.

(2) the proof is sound, but we don't understand it.

(2) is a partly empirical reason: one may be not intelligent enough; and a partly a priori reason: the proof is too difficult.

(3) regardless of the soundness of the proof, there is unkind evidence and one doesn't believe that p. So, one doesn't know that p.

In (3) one entirely drops belief that p. (3) is different from the case when one has unkind empirical evidence but still manages to believe that p. It is just that one is not very sure about it. If one manages to believe that p and the alleged proof is in fact sound, then one knows that p, even when one is not sure that one knows that p.

Then, when one knows a priori that p, the empirical evidence concerning the prevailing circumstances, like one's state of mind, is assumed. For example, when I know (a priori or otherwise) it is assumed that I am not too drunk. But that does not mean that it does not come into play in this context. It has a role, and since that role is assumed, it is taken for granted, one doesn't state the prevailing empirical circumstances in the form of empirical statements to which one's a priori knowledge in fact is dependent upon. That is precisely why it is not incoherent to appeal to these empirical reasons or considerations when

things go wrong; they can be part of the explanation why things went wrong. That is, when I don't know that p, it can be due to a priori reasons or empirical ones, or both, as examples (2) and (3) suggest. The empirical prevailing circumstances that were taken for granted when I thought I knew a priori that p would be clearly stated and tested since they could have a role in explaining why I did not succeed in knowing (a priori) that p.

Is the justification for p the same as the justification for our belief that p? I don't think so. I want to distinguish between the conditions for the existence of a justification and the conditions for rightly taking a justification to justify her in believing that p. For example, conditions for the existence of a justification, let's say, a calculation which justified some arithmetical proposition entirely concern what is permitted by the rules of arithmetic, and thus involve no element of contingency. But rightly thinking oneself justified by what is in fact a correct calculation may well involve empirical presuppositions about oneself and the prevailing circumstances (that one is not confused, or too drunk, and so on.)

Accordingly, I distinguish between the role assumptions can have in connection with our a priori knowledge:

(1) In justifying the conclusion of a proof in the reasoning itself

(2) In justifying my belief that I have a proof.

Of course, the statement "I have a proof" is an empirical statement, and depends for its truth on empirical assumptions, for example, that I did not make any mistake in carrying out the proof. My knowledge is empirically defeasible.

Let me illustrate. The statement "I am not good at proofs" can undermine my warrant (my purported proof) for p, but not undermine a proof

for p. If I am not good at proofs, and I in fact made a mistake in carrying out a proof, my warrant is not a token of a proof. So, my warrant is defeated by the experience of having made the mistake, even if I don't realize it. Of course, a proof is not defeasible at all. It is actually indefeasible: since a real proof is a sound argument, it is not possible to defeat it in any way. Likewise truths cannot be falsified. What is always defeasible are the ways we come to believe truths (or propositions).

At this point, I propose to distinguish between two senses in which empirical evidence can defeat items of a priori knowledge. It is one thing to have empirical evidence against an a priori warrant. It is another thing to have empirical evidence against an a priori statement. That is, one sense is when empirical evidence shows that an a priori warrant for a statement is not good. Another is when empirical evidence shows that an a priori statement is false. It seems that it is more plausible that an a priori warrant can be undermined by experience than an a priori statement can be undermined by experience. Again, the fact that I am not good at proofs can undermine my purported proof.

Someone could ask: But how can we know that a statement is false if it is not by showing that we did not have a real warrant for it? Through the warrant, we can know the truth or falsity of the statement. The answer is that even though we can only know that we don't know a statement by learning that we failed to have a good warrant for it, that we failed to have a good warrant is not sufficient to show that the proposition itself is false. There may be another warrant that we still don't possess.

Naturally, Hale could simply restrict the non-falsifiability by experience requirement to a priori statements. That is perfectly in order since in any case the properties of falsifiability / non-falsifiability can only be meaningfully applied

to statements, and not to warrants (rather defeasibility is a property of warrants).[83] But it is implicit in Hale's view also that a priori warrants are themselves indefeasible by experience, though he admits at other times that empirical evidence can be incidental in our knowledge of their defeat.

In my view, making the appropriate clarifications, Hale's position is compatible with (1) accepting the defeasibility of a priori warrants by experience and (2) the non-falsifiability by experience of a priori statements. But then, the first cannot imply the negation of the second: that is, when an a priori warrant is defeated by experience, it does not follow that the statement it justified has been falsified; there may be another (better) a priori warrant for the same statement. (Though if there is an a priori warrant for its denial, and we possess it, then it seems that the original statement has been defeated by experience when the original a priori warrant is defeated by experience. But we only say that the original a priori statement has been falsified when we find the a priori warrant for its denial.)

The real difficulty is that the requirement of non-falsifiability by experience of a priori statements may be too strong. There seems to be a tension between the claim that a priori statements are indefeasible by experience and the possibility of having a posteriori knowledge of some a priori statements. It appears that Hale's view would be more plausible if in the case of a priori knowledge, knowledge as well as refutation (defeat of warrants; falsification of statements) were an entirely a priori matter. But things get complicated because that does not seem to be the case. We have presumably a posteriori knowledge of truths that can also be known a priori.

How can we reconcile Hale's claim that a priori statements are nonfalsifiable by experience when we can have a posteriori knowledge of some

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83 In chapter seven I provide a glossary of all the epistemological terms I use.

of the statements that are knowable a priori too? Again, I think that Hale would say that in the case of alleged a posteriori knowledge of truths known a priori, the empirical considerations could only defeat the warrant (empirical), but not the proposition.

Nevertheless, if Quinean confirmation holism is true, then there can be collateral empirical evidence against an a priori warrant or an a priori statement, for example, against the Parallel Postulate. [84] For Quine, since any statement can be revisable by experience, and it is his view that a priori statements are not supposed to be revisable by experience (even more, they are not supposed to be revisable; they are supposed to be true "come what may") it follows that there can't be any. In contrast, for Hale, an a priori statement may not be true at all. According to Hale, in order for a statement to be a priori: (1) it has to be justifiable by an priori warrant; or (2) it has to be revisable only by a priori reasons.

Quine thinks of epistemological holism and belief in a priori knowledge as incompatible - or seems to do so in "Two Dogmas of Empiricism". Holism challenges a priori knowledge because, as standardly interpreted, it implies that almost any belief whatever can be implicated in one's grounds for almost any other belief, in such a way that grounds for belief may be open to defeat from the

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84 Roughly speaking, confirmation holism, as I understand it, is the thesis that knowledge is a system of interconnected beliefs such that changes in beliefs in one area of knowledge can have repercussions in other areas.

Confirmation holism per se is not a problem, at face value at least. We have to distinguish between confirmation holism and Quinean holism. Confirmation holism is compatible with the view that the a priori is revisable only by a priori reasons. In contrast, Quinean holism is not so compatible. Quine does not specify what sort of revisions statements are vulnerable to.

The reference to Quine here is: Quine, W. V. O. "Two Dogmas of Empiricism" in From a Logical Point Of View. Cambridge, Mass.: Harvard University Press, 1980.

most unexpected sources. For example: my knowledge of a mathematical proposition may be based on what I take to be a proof of it; but my justification for believing that I have a proof may well be defeated if I get some kind of empirical evidence for my, perhaps temporary, unreliability on such matters - for instance, if I learn that a medication I am taking sometimes has a side effect of clouding a patient's judgment. The grounds I have for that belief about the medication may again be potentially very diverse - perhaps I hear a discussion in a medical program on TV. This in turn will implicate collateral beliefs about the nature of the program - that it was a genuine studio discussion, for instance, rather than an excerpt in a drama - and that it represented up-to-date, reliable medical opinion. And so on.

It seems that I need an example of an a priori proposition with the following logical form: A universal proposition: For every x, if F(x) then G(x), where F and G are empirical predicates. How can there be empirical undermining of a universal a priori proposition, all F's are G, where F and G are empirical predicates? I take it that by an "a priori proposition", it's here meant a proposition for which there is some - perhaps defeasible - a priori warrant.

The answer is the following: Such a proposition can be <u>indirectly</u> undermined if it participates in a theory which is running into difficulties - making false predictions, though not necessarily ones which involve either of the concepts, F and G - and when the best way of modifying the theory so as to restore its success is to junk that particular generalization. This of course is the kind of possibility that Quine always has in mind. But also I don't see in general why there shouldn't be the possibility of <u>direct</u> disconfirmation: the Euclidean Postulate of Parallels, for example, is equivalent to the thesis that the interior angles of a triangle always have the sum of 180 degrees. So the counter-evidence might just take the form of our measuring up lots of triangles and finding that

their interior angles tend not to so sum up. Of course there will still be the possibility of saying that what we are measuring are not really triangles - that their sides are not really straight, for example. But they might perfectly well seem to be straight by whatever operational criteria we had. If we say, "still, they are not really straight", then we make a move that is always possible to make if we want (perhaps irrationally, or dogmatically) to protect a hypothesis against disconfirming evidence. One can always deny that one is dealing with genuine F's if some of them seem not to be G, and fall back on the fallibility of one's methods for determining whether or not something is F in order to protect that denial.

Likewise, simple arithmetical generalizations - say, whenever there are exactly seven F's and exactly five G's, then there are twelve F-or-G's - might be disconfirmed by counting. One counts the F's and get seven, one counts the G's and get five, one counts the F-or-G's, and get thirteen! Again, one can protect the original hypothesis by saying that there must have been a miscount, or that the number of things being counted must have changed somewhere along the line, etc. But, again, these are moves that can always be made if one wants to protect a hypothesis against disconfirmation. Nothing special about the a priori case here.

In general, I see no problem about how a hypothesis which can be warranted a priori might be empirically disconfirmed. The point is not that there is no sense to be made of the idea but , rather, that we don't in general allow empirical disconfirmation of such propositions. The difficulty is not in the very idea of a collision between what the a priori warrant suggests and what the empirical data suggest. It is rather that in general we regard the a priori warrants as dominant. But is there any reason why we always have to do so, specially if we are concerned with cases where there is no rigorous explicit proof and the a

priori warrant is rather a matter of appeals to what we think we can conceive, and the like?

Quine urges that no statement is immune to revision. But a defender of a prioricity need not wish to resist the suggestion that logic, or other disciplines conceived to involve a priori statements, are revisable. There should be no interest in maintaining that we cannot be in error in judging a statement to have that status. Can we therefore give Quine the claim that any particular statement which we accept as a priori could, in certain circumstances, reasonably be discarded? To grant the claim need be to grant no more than that our assessment of any particular statement as a priori may always in principle turn out to have been mistaken.

I say "there should be no interest in maintaining that we cannot be in error in judging a statement to have that status". What is the status exactly? Suppose we have an apparent a priori warrant for a particular statement, the Euclidean Parallels Postulate, for instance, which then runs into trouble in the context of empirical theory. Now certainly, a defender of the a priori should not have any interest in maintaining that our recognition of a priori warrants is infallible. And it may be that, in the kind of situation described, when we look again more carefully at the warrant, we find it contains mistakes or oversights. It is important to point out that a defender of the a priori who really wants to leave her position consistent with everything Quine says must do more than accept the fallibility of the epistemology of the a priori: she must allow, in addition, that a claim can be genuinely warranted a priori and yet still defeated by experience. Then there may have been nothing wrong with an a priori warrant except that the statement it warranted was a statement that was destined to be overturned in the light of wider, broadly empirical considerations.

That is the same as saying that a priori warrants can be inconclusive, so that commitment is not just to admitting the fallibility of our appreciation of the a priori, but also to admitting that some a priori warrants provide less than conclusive grounds for what they warrant.

The latter is not implausible when the a priori warrant is based on the mixture of imaginative and visualizational considerations which probably motivated defenders of the Euclidean Parallels Postulate. But it is less plausible in other cases, and I would presumably have to exclude examples of warrants acquired by infallible methods in the sense I discuss in Part Two . So if Quine's thesis is that absolutely any statement may be revised without any implication that an original warrant for it was flawed in its own right, it is tantamount to the contention that there are no infallible a priori methods. It does mean that the account I will offer in Part Two cannot be fully consistent with the view I am ascribing to Quine as long as some a priori methods are infallible. [85]

Though Hale's notion of a priori knowledge is compatible with revision, it does not admit revision of a priori statements by empirical reasons. It is interesting to note that Hale affirms that given that a priori routes are not infallible, agreement in results diminishes the chances of making mistakes. [86] But Hale resists the following idea: that if we made a mistake, better to know it, in whatever way possible, by an a priori or an a posteriori route, it should not

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[85] It is very implausible to suppose that the simplest items of (what we regard as) logical and mathematical knowledge might rationally be dumped in the light of holistic empirical considerations. What about the law of non-contradiction, for instance? Surely any rational empirical methodology, even that of sections five and six of "Two Dogmas", must presuppose certain logical principles and put them in a position of privilege. And surely some principles like modus ponens, non-contradiction, etc., will always be so privileged. It seems a satisfying epistemology of logic should explain this.

[86] Hale, note 12, p. 262.

matter. In my view, there is no need to compromise to a view of a particular kind of defeasibility of a priori warrants and a priori statements. I shall expand on this issue in the next section.

Section 2.4: Problems with Hale's revised account

I shall outline no less than three difficulties with this proposal. Two are due to remediable shortcomings of formulation, but one seems deeper-reaching.

(1) (H) does not say anything about how we might come by non-inferential a priori knowledge. Hale does not say that all justification is inferential. (H) allows for non-inferential a priori knowledge but it does not say anything illuminating about it. (Though, of course, this is a problem for everyone and not specific to Hale's proposal.)

(2) (H) talks about a priori known propositions. It would be desirable to include a priori knowable propositions also. This is desirable because we want to single out a class of truths with a certain feature. The distinction that interests us is not between truths that happen to be already known a priori and those known a posteriori. Rather we need a distinction that does not rely on what we contingently happen to know but draws on a feature that necessarily belongs to this class of truths.

The matter is easily resolved. It is a basic logical point that when one puts a condition for something to be something, for example:

Q iff ..... ,

we can infer from it the following statement:

It is possible that Q iff it is possible that ...

Thus from (H) we can obtain the following as a simple logical consequence:

> (H*) P is knowable a priori iff P itself implies (entails) the truth of no experientially falsifiable Q and it is possible for a thinker X to know that P on the basis of a justification which requires the truth of no experientially falsifiable Q.

Note that (H) and (H*) do not presuppose that there are known a priori truths, or that there is a priori knowledge. (H) and (H*) only put a condition for knowledge to be priori; it may not even be satisfied by any instance of knowledge. Let me stress again that a definition of a priori knowledge merely states the necessary and sufficient conditions for knowledge to be a priori. It does not follow from such a definition that there actually can be a priori knowledge but only that if there is any instance of knowledge that satisfies the conditions stated on the definition, it would count as a priori. When I have insisted that the conditions for a priori knowledge have to be sufficient as well as necessary what I have intended to convey is that we need to come up with conditions which, if satisfied, are able to determine whether instances of knowledge are a priori. The present project is to work towards an explanation of what it would take for there to be a priori knowledge; but the possibility is still open that, when we have such a characterization, we shall see that it applies to nothing.

(3) Perhaps the notion of a priori knowledge can be circumscribed by (H) and (H*), but it may be empty. The problem is that if a holistic view is right, then there won't be any instance of a priori knowledge that could satisfy Hale's proposals. The reason is that there won't be any statement that is not in principle revisable by experience. It is a consequence of Hale's account that an interest in a priori knowledge is incompatible with Quinean holism.

Hale endorses [87] the traditional conception of the a priori as what is not only independent of experience in terms of justification, but also what can only be revised, i.e. shown unsound or false, independently of experience as well.

A serious consequence of Hale's definition of a priori knowledge is that is incompatible with Quinean holism. If Quine's epistemological holism is right, Hale's notion of a priori knowledge would be empty. And that would be problematic because those who are interested in the possibility of having a priori knowledge would have difficulties in accepting any attempted definition of a priori knowledge which cannot render as knowable a priori, or perhaps more immediately important cannot render them as known a priori, the truths we would like to regard as such. Much of the motivation for a search of a definition, or an elucidation, of the concept of a priori knowledge is to get clear about how we can explain our knowledge of many propositions, like mathematical and logical propositions, without looking at the world.

Thus, given that it is an open question whether we are going to have empirical evidence against an priori statement, better to be more cautious and do not hold a position like Hale's which is incompatible with Quine from the start. Now if it is to follow from our best characterization of a priori knowledge that there can be no such thing, it ought at any rate not to follow so immediately or quickly! [88]

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[87] Note 9, p. 259, and p. 148.

[88] Although the tension between Quine's holistic empiricism and a Hale-type account of the a priori only has the consequence too "immediately or quickly" that there is no a priori knowledge if one takes it to be both immediate and quick that Quine is right.
Let me clarify that I don't want to present the motive for looking for something different to Hale as being to avoid a conflict with Quine that is bound to involve

Section 3: Is it coherent to talk of revision in connection with a priori knowledge?

There is an important distinction to be drawn between "a prioricity" and "a priori knowledge". Actually, I find that this basic distinction has been overlooked in the literature, or at least, has not been made explicit. I myself came up with it in my attempt to make sense of the idea that it is possible to talk of revision in connection with a priori knowledge, or what I take to be an equivalent claim in the literature, that of not requiring unrevisable belief in connection with a priori knowledge (or the same thing that a priori knowledge does not require unrevisability). [89] This simple way of talking is indeed quite misleading, and I find that proposing the distinction between "a prioricity" and "a priori knowledge" resolves the tension which many have overlooked.

For example, Paul K. Moser affirms [90] that one of the necessary conditions for an instance of knowledge of a proposition's truth to be a priori (what he calls "minimal" a priori knowledge) is that the evidence (a priori) in question "makes

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defeat for the a priori. Rather, I present the general motive as being to find an account of the a priori which would show that Quine was wrong to dismiss it - if indeed he does (hard to say, since he persistently confuses the a priori with the analytic) - on the grounds that he actually has, even if those grounds are right, i.e., even if Quinean holism is right.

89 Hale, ibid, pp. 125, 143, 148; another example is: Paul K. Moser. A priori Knowledge. Oxford University Press, 1987; introduction, pp. 2-3.

90 Moser, ibid, pp. 2-3.
For Moser, the concept of minimal a priori knowledge consists of an instance of knowledge of a proposition's truth such that the evidence does not rely on sensory experience and that the evidence makes the proposition more likely to be true than its denial. The concept of minimal a priori knowledge, on his view, does not require unrevisable belief, innate knowledge, or self-evidence. That is why the concept is supposed to be minimal: it consists of only the two conditions specified.

the proposition in question more likely to be true than its denial." (my emphasis; p. 2) Moser also affirms that a priori knowledge does not require epistemically unrevisable belief.

> "A justified belief is epistemically irrevisable, let us say, if and only if it would not be epistemically rational to give up that belief under any circumstances, including circumstances where the world is radically different. It is epistemically rational to hold such a belief come what may. Clearly, minimal a priori knowledge does not require epistemic irrevisability, since such knowledge is compatible with the fact that one's a priori justifying evidence might be altered (e.g. expanded) in such a way that what was justified a priori in the original evidence is no longer justified in the altered evidence. Thus, even if all our justified beliefs are epistemically revisable, we may still have minimal a priori knowledge." (my emphasis; ibid, p. 3)

But this is incoherent as it stands. Knowledge entails truth. When I know p at a particular time, the belief that p is unrevisable at that time, since it is true, if not I could not know it. If it were true that all beliefs are epistemically revisable then there won't be any instance of knowledge: neither a priori nor a posteriori knowledge. Why? Because there won't be any true beliefs.

Actually, Hale and Moser are not alone. Albert Casullo [91] starts talking about justified beliefs a priori as being not necessarily unrevisable, but later on collapses into talking about knowledge a priori as being revisable. Hale, [92] on the other hand, is less cautious. Hale talks mainly of knowledge a priori as being revisable. Kitcher [93] talks about the notion of a priori knowledge and the notion of apriority, but he then seems to identify them. Their positions seem incoherent,

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91 Casullo Albert. "Revisability, Reliabilism, and A priori Knowledge", Philosophy and Phenomenological Research, Vol. XLIX, No. 2, December 1988.

92 Hale, ibid, p.125, p.143 and p.148.

93 Kitcher, ibid, p.17.

that is why it is useful to try to make the position more clear to avoid that impression.

The notion of "a prioricity" only refers to "justified belief in an a priori manner" (in short: "justified belief a priori") where the "a priori manner" has been characterized properly. A prioricity is concerned primarily with the notion of an a priori warrant. The notion of an a priori warrant is only concerned with justification independent of experience. Therefore, the notion of a prioricity does not entail that the belief we acquire by an a priori warrant is true and, consequently, that it does constitute knowledge.

Talk of revision makes sense in connection with the notion of a prioricity (i.e. in connection with warrants and the beliefs they form). It is indeed incoherent to talk about revision if we are talking about knowledge. It is a truism that knowledge is not mistaken. Knowledge and truth are inextricably linked in a way in which belief and truth are not. Since if we know p, either a priori or a posteriori, "p" must be true, and, therefore, if true, unrevisable.

Talk about revision on the other hand in relation to a prioricity makes sense because to have an a priori warrant for belief that p amounts to having a pretty good reason to believe that p. That does not amount necessarily to knowledge - it is not enough. Warrants are defeasible, and, therefore, revisable.

I have found it useful to make the distinction between "a prioricity" (i.e. justified belief a priori") and "a priori knowledge" for the following reasons: first, to make it coherent how philosophers defend the possibility of revision in connection with a priori knowledge. Second, because truth is not what is distinctive of a priori knowledge - since it shares this property with any sort of knowledge - but rather the alleged independence of experience that gets attached to the a priori warrant. The distinction between "a prioricity" and "a priori

knowledge" makes it possible to talk about revision in connection with the former and the possibility of infallibility in connection to the latter. It makes it explicit and coherent that a priori knowledge does not have to be infallible by merely being a priori since the a priori is primarily concerned with a way of acquiring a belief and it is not per se knowledge.

However, there is an important sense in which knowledge and revision relate. We have to ask ourselves the following question: Can someone know that p and move into a situation where it is rational to give up p?; here "rationally" to be taken as "justifiably". Revision is a change of mind, of opinion. It is possible that one may change one's mind to what is in fact true. It is possible that there is rational pressure put on someone that is defective and that one gives up belief that p.

Another important question is whether there is a kind of justification where revision as a matter of fact is the wrong thing to do: that is, where revision is in principle always possible but where the new evidence is always defective.

There is an important notion of de facto indefeasibility which applies to warrants. A warrant is de facto indefeasible at t (and later times) if all further evidence against it is in fact misleading evidence. First, we cannot know that a warrant has this property at t, or any subsequent time, because we won't be able to have the relevant evidence at our disposal. Second, we cannot know that a warrant has this property at t and later times because there is no guarantee that we can know about all future evidence against the warrant, provided that we are in a position to gather it all, that it is actually misleading evidence. A priori and a posteriori warrants can be de facto indefeasible. The property of de facto indefeasibility is not the same as the property of infallibility. The first is a weaker notion than the latter. When a warrant is necessarily, not simply de facto,

indefeasible, then it is (indirectly) infallible in the sense of being the result of the correct prosecution of an infallible method.

Knowledge requires that its successful warrants - those which produce knowledge on the occasion - are de facto indefeasible. Because if I know (in particular if I knew) then it is not possible to show by any subsequent evidence that I am (was) wrong.

Of course, that is not to deny that there is always the possibility of defeating evidence but the point is that in the case of a warrant which is de facto indefeasible this new defeating evidence will always be misleading evidence. Though there is no second level guarantee (a priori or otherwise) that warrants are de facto indefeasible. We simply cannot know that. It is always a possibility that we made a mistake in carrying out a warrant.

Section 3.1: Some remarks on revision and a prioricity

I am attracted to the following view: Let's for the sake of argument suppose that the analytic / synthetic distinction can be maintained. The analytic / synthetic distinction among statements is one between contents or subject matter; the epistemological distinction between a priori / empirical warrants is one between ways of acquiring beliefs. The possibility I am proposing is the following:

> A priori warrants only warrant (confirm or disconfirm) analytic statements (or some synthetic). A posteriori warrants can confirm or disconfirm any statement.

This view takes the a priori seriously because if we have a sound a priori warrant, then what we know can only be an a priori statement. An a priori statement is a statement knowable a priori. One can know an a priori statement a posteriori. A posteriori statements are only knowable a posteriori.

Hale thinks that Edidin tries to defend a similar view - that a posteriori warrants can disconfirm a priori statements - distinguishing between defeating and confirming evidence. [94] If Hale's argument against Edidin in the book is any good, then that will be Hale's objection against this view.

Hale's argument against Edidin's view is:

> "If, at some particular time, a state can be affected by conditions of a certain kind, then it can be at most a contingent fact that it is not affected by conditions of that kind at any other time during the period through which it obtains, including earlier times. And the point applies to the kind of state with which we are concerned here, i.e. states of (justified) belief. However I came to believe that p in the first place, if it is true that I can now lose my justification for it by coming to have subversive empirical evidence, then I could have lost my justification in that way at any earlier point. At least, that is so unless it is, in a very strong sense, impossible that such evidence should have been available then; and there is no good reason to suppose that that is ever, much less always the case." (Hale, p.133; my emphasis)

But the experiences that Hale was thinking of could not have been in any relevant sense unavailable at the particular time t, when S (subject) was justified a priori in believing that p. They were what I call "Kitcher's experiences". These

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94 Edidin, Aaron. ""A priori" Knowledge for Fallibilists". Philosophical Studies, Vol. 46, Spring 84, pp. 194-96.
Hale discusses Edidin's view in the following passage:

> "someone's belief will be b-independent [backward-independent] of empirical evidence if such evidence plays no part in producing or sustaining it , and that the notion of apriority is best understood in terms of this kind of independence from empirical evidence, rather than in terms of f-independence [forward-independence], which is best taken as distinctive of the notion of (empirical) incorrigibility. Of crucial importance is the claim that b-independence does not entail f-independence. The thought is that I can know *a priori* that *p*, in virtue of having a justified true belief that *p* which is b-independent of empirical evidence, even though subsequent available evidence of that kind could undermine this belief, i.e. it isn't f-independent of empirical evidence." (Hale, p. 131)

are for example that S may have misread a proof, that S may be too drunk, etc. These experiences are obviously always available. So, if I am still justified in believing that p independently of experience, for example, it is because these experiences did not defeat the claim that I am justified in believing that p. But there could be other experiences unavailable at time t that if I were aware of them could undermine my claim to be justified in believing a priori that p; and these experiences even though they are in principle able to defeat my claim to possess a priori knowledge, were in a relevant sense unavailable at the time t when I thought I was justified in believing a priori that p, so they could not defeat that claim, simply because I could not have had that empirical evidence at time t. Hale thinks that if my argument is sound, then his argument against Edidin is unsound.

Hale thinks that the relativity to time is fatal to Edidin's view.

> "The thought is [Edidin's] that I can know a priori that p, in virtue of having a justified true belief that p which is b-independent of empirical evidence, even though subsequently available evidence of that kind could undermine this belief, i.e. it isn't f-independent of empirical evidence." (Hale, p.131)

Hale is right to complain about this position. Edidin's view is incoherent. It implies that I can know a priori at an earlier time and then at a later time know that I was wrong about knowing a priori at an earlier time as subsequent empirical evidence may show. But if I knew then it is not possible to show by any kind of subsequent evidence that I was wrong. The situation is rather as follows: if at an earlier time t I thought that I knew a priori that p, and at a later time come to know, maybe by empirical evidence, that I was wrong, then I did not know p at t at all. This is my view, not Edidin's. Hale's second criticism (the charge of incoherence) does not apply to the view I am interested in. Hale is

wrong in thinking that this (correct) criticism of Edidin's view applies to my view.

However, as I see it, Edidin's position makes it clear that some clarification about how time and knowledge relate to each other is in order. When I know that p, I know that p at a particular time t. If at a later time I am unable to recall my justification in a decent manner, I don't know that p anymore. I may remember to have known that p before but have forgotten about my justification for p. Analogously, when I know a priori that p, I know a priori that p at t. If at a later time I am unable to recall my a priori justification in a decent manner, I don't know a priori that p anymore. In such a situation, I may remember to have known p before but have forgotten about its (a priori) justification. When my justification was a proof, and I don't remember it at a future occasion then at that later time I don't know a priori that p; if I still know that p, it is because I remember having followed a proof that p at an earlier time, but knowledge based on memory is a posteriori. If I don't even remember to have followed a proof that p, then I don't know that p at all at the later time.

The question that naturally arises is: How does the fact that knowledge and time relate connect with revision? In the case when at a later time I am unable to recall my justification in a decent manner what has been revised is not the statement in question but the warrant which justifies it in the sense not that there was something wrong with the warrant but that one lost the warrant. In the case when I still know that p at a later time because I remember well to have followed a proof for p, for example, then there has been a switch of warrant, from an a priori warrant to an a posteriori one. Though it is true that in a sense if the justification or warrant for p has been switched over time, one can say that it has changed over time, and that is a revision also. Since I have lost my a priori warrant for p, I do not know a priori that p at a later time. When I don't even

remember having followed a proof that p before then I don't know that p at all at a later time.

The point is that one can know at some time and not know the same thing at another time. But the explanation of this situation is not the one Edidin offers.

It may seem that it is incoherent to hold that one can know a proposition at a particular time, and not know the same at a subsequent time. But this is incorrect. There is a difference between the case when I knew at an earlier time and then at a later time when shown to be wrong (Edidin's view) and the case when I knew at an earlier time a proposition and then at a later time I do not know the same proposition any more, not because I was shown to be wrong, but rather because I have lost the warrant for it during the lapse of time between the earlier time t and the later time t_1. The second case is a simple consequence of the fact that our epistemic attitudes towards propositions can change with time, and there is nothing mysterious about that.

I distinguish three basic kinds of defeat in connection with a priori warrants or beliefs:

(a) There is something wrong with the purported proof.
We reject the purported proof by a priori reasons. The reasoning is not reliable. It leads to contradictions. This defeat consists in finding mistakes.

(b) Collateral defeating information: drunkenness, having a headache, etc. The reasoning may be fine but we are in no good condition epistemically to assimilate it.

(c) We are perfectly competent in getting the warrant, justified in accepting p a priori, but the warrant can be defeated by empirical evidence not because there is something intrinsically wrong with the warrant but rather because counter empirical evidence is stronger.

Again, an a priori warrant is complete, nothing is wrong with it, but its conclusion is swamped by bad empirical news.

As I already said, defeasibility is a property of warrants. There are more specific cases to distinguish:

(1) One learns that something is wrong with the warrant that one had. The method has not being properly executed. For example, you may have measured incorrectly because there was something wrong with the apparatus.

(2) There appears new information that tells against the conclusion, but not mis-measurement is involved, for example. In this case the method is insufficient to guarantee the truth of the conclusion.

(3) Additional information such that when one put together with the previous warrant, suspension of judgment occurs.

An example of the third case is the following: P: Smoking causes cancer. Later we know: Q: Common cause for smoking and cancer. Then one doesn't know which one is the cause of cancer. Suspension of judgment occurs as a result.

Which of any of these apply to the a priori case? Certainly (1) applies. (2) is controversial. The Parallel Postulate constitutes an example. There are reasons to doubt the Parallel Postulate. There are collateral reasons to think not P. Case (3) may happen as well in the a priori case.

There are different cases to distinguish when we are dealing with a defeasible warrant whose pedigree is fine. There is nothing wrong with its pedigree but nonetheless the warrant is not sufficient, it is inconclusive, as a way of establishing the proposition which it warrants. Two cases can be distinguish here:

(I) This case is exemplified by Goldbach's conjecture that every even number greater than 2 is the sum of two primes.

We don't have a proof, a conclusive confirmation, of this proposition. It can be envisaged that, for instance, a computer goes through all the integers and doesn't find a counterexample. It works up to a very large number. Then there is good confirmatory evidence that the conjecture is true. This evidence is a posteriori since it relies on assumptions about the functioning of the computer. But let's suppose that in principle, given sufficient time, one could gather the same evidence without sensory input. In this case, the a priori warrant is good because no counterexample has been found in a large number of cases studied. But the warrant is insufficient to conclusively warrant the conjecture.

(II) Geometrical intuitions. The Parallel Postulate constitutes an example of this kind of case.

The warrant (a priori) is fine but it is incompatible with a physical theory. (This is previous case (c) and (2)).

A warrant can be a priori without being conclusive. In the geometrical case in particular, the a priori grounds that e.g. Kant must have thought he had for the Parallel Postulate fall short of conclusiveness. The crucial point is the apparent impossibility of imagining counter-examples - where imagining can mean something very concrete, for instance, drawing. There is prima facie space for empirical defeat of claims based on imagination just because the empirical world does not have to fall in line with our imaginings.

We have to ask an important question: What kind of additions to these warrants may be possible? In the first case, the additional information to defeat the warrant involves finding a counterexample. That is, to defeat Goldbach's conjecture, we need (ideally) to find a priori a counterexample. (Although if a computer finds a counterexample of Goldbach's conjecture, then the evidence would be a posteriori since it involves empirical assumptions concerning the

proper functioning of the computer. The computer finds the counterexample by the a priori method of computation. This case seems to be analogous to the case when we have a posteriori evidence, for example, testimonial evidence, for a claim that is a priori.) In the second case, though, the additional information comes a posteriori.

It can be argued that the Parallel Postulate is not a good example of an a priori statement being disconfirmed by experience since it is a priori true in uninterpreted geometry. But we ought not to forget now that we thought that Euclidean geometry was true of the physical world before non-Euclidean geometries appeared in the picture. Furthermore, we thought we knew that by a priori grounds. So the statement that we arguably thought knew a priori "Euclidean geometry is true of the physical world" has been revised by experience (i.e. the appearance of non-Euclidean geometries) in the sense that if it is true, it only can be shown to be true a posteriori. The Parallel Postulate originally was formulated without any specification about the lines in question, i.e., that the lines in question were only Euclidean lines. The formulation of the postulate had to be restricted. That is a revision as well.

I therefore disagree with the suggestion that insofar as geometry is a priori, it has to be conceived as saying nothing about the physical world. In fact I think this is quite a general mistake that people make. It is crucial to recognize that much of pure mathematics - geometry is actually a special, and difficult case - expresses a priori knowledge of propositions whose application to the physical world involves no special interpretation of them. If one knows that seven times seven is forty-nine, one knows that if one has a square of soldiers on parade, with no gaps and with each row and each column seven women long, then one has

forty-nine soldiers on parade in all. Arithmetic, as Frege stressed, [95] is about all thinkable kinds of objects - so in particular, it is about physical objects.

I also think there's no hope for the idea that insofar as it is a priori, we should regard geometry as uninterpreted - if it's uninterpreted, then it articulates no specific statements, a priori or otherwise. Also, a satisfactory account here has to explain why the Euclidean axioms were initially appealing - why everyone up to Kant thought they had to be true. If they had no interpretation, that's utterly mysterious. The explanation has to concern facts about what people thought they could and couldn't conceive. For example, it seems impossible to imagine a drawing in which we get simultaneously an impression of a pair of lines, each perfectly straight, and yet intersecting more than once. We can't imagine this, and nor can we draw it. So a certain kind of appearance seems impossible - and that's then treated as warranting the claim that counterexamples to the Parallels Postulate are not possible. Obviously this can only be a defeasible warrant, since the most that it can show is that space cannot appear locally non-Euclidean, not that it cannot be (globally) non-Euclidean.

I think my example is good - people thought they could know a priori that Euclidean geometry was true of the physical world, because they thought that the transition from "it seems impossible to imagine a setup which looks like a counterexample to the Parallel Postulate" to "the Parallel Postulate is true" was safe. But although not an unreasonable transition, absent further relevant information (like the astronomical observations that actually persuaded physicists to use non-Euclidean geometries) showed that it is certainly not a proof.

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95 Frege, G. The Foundations of Arithmetic. Oxford: Basil Blackwell, 1953.

Perhaps there's a general point here: when propositions are warranted, a priori, on the basis of imaginings and conceivings, etc., there is a certain sense in which the warrant can't penetrate deeper than the level of appearances - and the inference, from the premise that things cannot appear a certain way - in some sense of "appear" - to the conclusion that they cannot be that way, is of course defeasible.

To conclude: The claim that a priori beliefs can be revisable in light of empirical evidence is very controversial. It goes against almost the whole philosophical tradition on a priori knowledge. Even those who are sympathetic to the notion do not want to accept this possibility. Despite that, I believe that there are cases which tend to support the claim, [96] though I am not committed to

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[96] Other philosophers who share this conviction are: Casullo (ibid) and Edidin (ibid). Burge (in "Content Preservation", The Philosophical Review, Vol. 102, No. 4, October 1993), and Wright (in some of the Hale-Wright correspondence published in Hale's book) believe that the notion of a priori justification ought to be characterized independently of the issue of defeasibility (by a priori reasons or otherwise) that the a priori may be vulnerable to. As I take it, Burge and Wright leave open the possibility that there could be counter-empirical evidence against a priori beliefs or a priori warrants.

Burge thinks that the predicate " a priori" applies primarily to justifications and entitlements (ibid, p. 458). The distinction between justifications and entitlements is irrelevant to our purposes.

Burge affirms:

> "although some a priori justifications ... may be invulnerable to empirical considerations , such invulnerability does not follow from the notion of a priority ... I think that some beliefs with genuine a priori justifications ... are vulnerable to empirical overthrow". (p. 461)

According to Wright, the crucial notion to characterize is the notion of "experience independence" rather than the notions of defeasibility and indefeasibility. The issue of defeasibility / indefeasibility of a priori knowledge is a separate matter. That does not mean, of course, that the issue of defeasibility is not an important one, it is in fact a very important and interesting issue, but Wright's point is that the two issues are separate.

its truth. Even if it turns out that some a priori beliefs can be revisable in light of empirical evidence, that fact won't show that the beliefs in question were not justified a priori.

Conclusion

It is important to realize that the circularity charge in (1) and (1^1) involves two aspects: (a) a vicious circle is involved; and (b) it is not very illuminating. Even if Hale succeeds with (H*) in avoiding the first aspect of the circularity, and his definition is correct, still the second aspect of the circularity remains. An a priori justification is characterized as simply lacking a certain feature.

As far as the issue of infallibility is concerned, it appears that Hale in his chapter leaves no room for infallibility in connection with a priori knowledge - or knowledge in general - given the undeniable fact that we are fallible creatures. Nevertheless, I have to be cautious and refrain from attributing to Hale this position since he simply does not discuss the issue of infallibility at all. The reason why he doesn't in the book may be the one just mentioned, but it is only a possibility among others. On the other hand, that reason may have just been entirely non-philosophical. [97]

I agree, given the qualifications that I have discussed in chapter five, with Hale's claim that the notion of a priori knowledge (more accurately, the notion of a prioricity (i.e. a priori justified belief)) ought to be consistent with the possibility of revision. We are fallible creatures and we can make mistakes. I agree also with Hale that a priori warrants do not have to be infallible. The mistake Kitcher made, and that Hale quite correctly diagnosed as due to conflating the truth entailing character of knowledge and the independence of experience characteristic of a priori knowledge, is to think that an a priori warrant has to be ultra-reliable (infallible) to be a priori. However, Hale does not

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[97] For instance, Hale could just have simply chosen not to discuss the issue for more mundane reasons (i.e. like lacking space and time on the occasion).

consider the idea that some a priori warrants may prove to be infallible, what Kitcher called "ultra-reliable".

The immediate point about infallibility is, of course, not whether what is in fact an item of a priori knowledge can be mistaken - no-one supposes that - but whether the prosecution of the methodology of a priori knowledge can lead to mistaken beliefs. Put like this, the answer, of course, is yes - people can get muddled, make mistakes in inference, miscalculate, etc. Those who allow for infallibility like Kitcher cannot mean to deny this. So what is involved? The question is a substantial one. The idea is that a certain kind of prosecution of the methodology of a priori knowledge cannot steer us false - whereas in the empirical case there are no controls, no safeguards on method such that, if they are complied with, the results are guaranteed to be true. Of course, the substantial question is to say what this ideal prosecution consists in. In the next chapters I shall try to accomplish this task.

To sum up, (i) I doubt if it is coherent to think of human beings as being infallible anywhere. If we have access to substantial truths about the world, whose obtaining is independent of human judgment about the matter, then, however hard it may be for us to understand how this might be, there has to be the bare possibility of misapprehension, or ignorance, of the states of affairs described by such truths. Another way of putting the point would be to suggest that fallibility is implicit in objectivity, though that needs unpacking. (ii) That, however, is consistent with the infallibility of certain methods - with the idea that, at least in certain areas, we have methods of knowledge acquisition at our disposal such that, if we implement these methods properly, we are assured of winding up with true opinions. Of course, this will not guarantee the truth of what we actually come to believe by prosecuting those methods, since we may be

mistaken in thinking that we have done so properly. Ordinary arithmetical calculation is an example of such a method: if one calculates properly, the results one gets are correct. That is a necessary truth. But it does not follow that carefully achieved arithmetical opinions are infallible, since one may be mistaken in thinking that one calculated properly.

As far as a priori knowledge is concerned, then, the interesting questions are: (a) whether a priori truths generally are associated with methods which are infallible in the sense I just have outlined; and (b) whether it is possible to judge with a superior, though not indefeasible, degree of sureness that the appropriate methods have, in particular cases, been correctly executed. Perhaps Hale just goes past these questions because of his assumption that a priori knowledge has to be compatible with revision. As a consequence, Hale does not address the interesting and very difficult question whether a conceivable notion of infallibility has a place only in the realm of a priori knowledge - in other words, whether infallibility, properly understood, is an a priori matter. I shall be discussing this crucial question in chapter seven when I shall turn to the task of disambiguating the notion of "infallibility" - among others - as a necessary step to get some clarification about the matter.

This is what emerges from my discussion of Hale: (1) the task at hand is to illuminate more the notion of a priori warrant; (2) the constraint of "experience independence" ought to be effective only after the acquisition of the conceptual repertoire necessary for a priori knowledge and the obtaining of certain other necessary experiences that underpin the reliability of the knower's state of mind; (3) there is a distinction between defeasibility and infallibility, [98] what Hale did not seem to be aware of; and (4) there is at least one outstanding problem with

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98 I will fully explain this distinction in chapter seven.

Hale's definition (H*) which we must seek to remedy: it is in too direct collision with Quine's epistemological holism.

Part Two

There are two tasks I intend to accomplish. First, to provide more illuminating characterizations of the notion of a priori knowledge and related notions. Second, to disambiguate the notion of infallibility, and to clarify its relationship with the concept of a priori knowledge.

In respect to the first task, what I intend is to provide accounts of these notions that are illuminating even if they do not amount to proper definitions. The idea is to respect and get clear about some of the intuitions we have in relation to a priori knowledge.

In relation to the second task, what I intend to accomplish is to make conceivable the (mere) possibility of infallibility. It is necessary to sort out what would have to be involved for such a possibility to be realizable. It is another matter to argue that the possibility has been realized. I don't have a knock-down argument for the latter, although, of course, that is not to deny that making coherent the possibility of infallibility paves the way for settling the question whether the possibility has been realized (actualized).

This part consists of two chapters followed by a conclusion. In chapter six I attempt to elucidate the concepts of an "a priori method", an "a priori warrant" and "a priori knowledge", and then proceed to evaluate the suggestions. Chapter seven will be wholly devoted to an analysis of the relationship between the notions of a priori knowledge and infallibility. In the concluding section I round out the most important issues in the dissertation.

Chapter Six
What is the a priori?

In this chapter, I attempt to elucidate the concepts of "a priori method", "a priori warrant", and "a priori knowledge". I elucidate these concepts by what I call "the tank insulation argument". We will need these characterizations in order to get clear about the relationship between a priori knowledge and a defensible notion of infallibility.

My proposal constitutes an elaboration of what I take to be a suggestion which Crispin Wright made originally in his Frege's Conception of Numbers as Objects. [99] However, Wright had not developed the suggestion in any detail either in his Frege book or in any of his writings. So, the proposal stayed as a suggestion, very interesting, though hardly developed. Wright's proposal is highly intuitive and simple and, at the same time, fully captures in my opinion the intuition that lies behind Kant's idea that a priori knowledge is independent of experience.

Section 1: Wright's proposal

Wright first introduced what I call "the sensory insulation tank suggestion" in the context of an argument against the causal theory of knowledge, specifically, on how the latter has difficulties in accommodating any kind of a priori knowledge and, in particular, a priori knowledge of necessary truths. The causal theory of knowledge requires that the state of affairs which confers truth upon the statement to be known plays a crucial role. Thus, for the causal theorist, the

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99 Wright, Crispin. Frege's Conception of Numbers as Objects. Aberdeen University Press, 1983, pp. 95-6.

problem is that a priori knowledge does not require the fulfillment of any such causal relation with the world.

> "what is distinctive of any piece of knowledge a priori is precisely that it has no essential causal antecedent save a training in certain relevant concepts. A man can lie suspended in a tank of lukewarm water, blindfolded, ears plugged, etc. - in short, in a state of total sensory insulation - and arrive, if he can concentrate well enough, at the end of elementary, and perhaps some less elementary, arithmetical and geometrical truths which he has never thought before. How, when the events in his consciousness are in this way causally quite unrelated to his present physical environment, is it possible for him to be exposed to the necessary causal influences?" [100]

Wright explains (in private conversation) that the idea is not that a priori knowledge is what will be available to a thinker independently of any causal interaction with the world. The point of the tank idea is one of temporary causal insulation. Something is knowable a priori just in case someone who went into a state of sensory insulation could, while so insulated, come to know something she has never previously thought about.

Section 2: Characterizations of the notions of "a priori method", "a priori warrant" and "a priori knowledge"

First of all, I want to clarify why I suddenly talk about methods when I have been talking about warrants. The reason is that the notion of "method" is crucial for our purposes. I will take the notion of "method" to be the primary bearer of the predicate "a priori". In order to accomplish the task of getting clear about the relationship between a priori knowledge and the notion of infallibility, properly conceived, it is necessary to draw the a priori / a posteriori distinction primarily at the level of methods. This would parallel the situation with the predicate

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[100] Ibid, pp. 95-6; my emphasis.

"infallible" since I shall argue that the interesting notion of infallibility in epistemology is the one that applies primarily at the level of methods.

Methods are ways of getting a result. The methods we are interested in are cognitive methods. Fundamentally, the a priori is a cognitive method by means of which we get into a situation where we are warranted in believing a proposition.

It can be argued that this notion of "method" is vague: For example, are looking, seeing, asking a knowledgeable friend methods? It is not clear to me why it should be necessary to characterize in absolutely general terms what a "method" is. The important thing is that the idea be clear enough in the context in hand - when our interest is in a priori methods. There are certain things which one could in principle do in the tank - calculate, infer, visualize, try to construct a coherent fiction, etc. - and other things (including, incidentally, looking, seeing, asking a knowledgeable friend, etc.) which one could not. According to the proposal, the belief that a priori knowledge is possible is the belief that methods of the former kind can be a source of knowledge. This idea is not compromised by the unavoidable vagueness in the notion of "method", taken in its most general sense.

An priori warrant is the product of an implementation of an a priori method. The property "a priori" ought to be applied derivatively to propositions believed, and to knowledge acquired, by being warranted a priori.

What are warrants? They are cognitive processes which give us reasons (presumably pretty good reasons if they are good) to hold beliefs. Warrants produce belief "in the right way". "The right way" does not entail that a warrant has to be able always to produce true beliefs. Warrants which can warrant belief

given favorable circumstances may be unable to justify belief given unfavorable circumstances.

My definition of the concept of an "a priori method" is as follows: Given that X possesses the concepts necessary for entertaining p,

(Fapm) μ is an a priori method for X's belief that p if and only if it is a routine

(1) that is cognitive and whose implementation by X produces in her a warrant for the belief that p;
(2) that can be implemented by X in a state of sensory deprivation.

In general, a cognitive method is a method that generates belief that p. A method is a general routine which we implement in particular occasions resulting in our being warranted or not for our belief that p. An a priori method is one that satisfies (2), that is, it does not need sensory activity to be implemented and provide a warrant for a belief. Empirical methods need sensory activity.

My definition of the concept of an "a priori warrant" is as follows: Given that X possesses the concepts necessary for entertaining p,

(Fapw) ø is an a priori warrant for X's belief that p if and only if it is a process

(1) that is the result of an implementation of a method,

(2) that can produce in X the belief that p,

(3) that warrants X in believing that p,

(4) that can be accomplished by X in a state of sensory deprivation.

The implementation of a method generates a warrant for believing that p. In (Fapw(3)), the notion of "warranting" does not entail the truth of p, therefore, knowing that p, but rather "warranting" entails that a warrant provides good reasons to believe that p.

My definition of "a priori knowledge" is as follows: Given that X possesses the concepts necessary for entertaining p,

(Fapk) X knows a priori that p if and only if X knows that p and X's belief that p was acquired by an a priori warrant.

Let me clarify the proposal. The position is that we don't have to have sensory information while we carry out an a priori method. Also, an a priori warrant can be accomplished with no sensory input, that is, in a state of sensory deprivation as well.

Let's illustrate these components with the example of calculation. The method consists in the rules of calculation. Warrants are particular calculations, the results of the applications of the method. Our response to the methods is always fallible.

Another example: in cookery, the method is the recipe, the way of getting the meal. The implementation is the way I actually carry out the recipe which can be successful or not. The result is the meal that I generated. In the best case, I get a (delicious) meal. The uptake of the warrant is the meal. The best response to the situation is that one eats the meal. The response to it: I may eat it or I may not. In the epistemological case, if one really generated a warrant, then one

ought to believe the result. In the cookery case, there is no such a pressure; one may not eat the meal.

I would say that methods are like recipes, that the relevant processes (warrants) are like episodes of cooking according to recipe, and that beliefs are like the meals which result. A priori methods - if they are indeed ones that can be implemented in the tank - will in a sense be capable of purely mental implementation; so the process of implementing them will be a psychological routine, but that is not to say that the method itself, or the product, is psychological (in any derogatory sense).

There is a distinction between having a warrant and actually forming the belief that it warrants. It is conceivable that there could be epistemological pressure not to form the belief. To form the belief which a warrant justifies is a matter of psychology. On the other hand, that we ought to form the belief in question given that we possess a good warrant for it is an epistemological matter.

(Fapk) attempts to capture "the independence of experience" characteristic of a priori knowledge in terms of the possibility of an a priori warrant to be constructed in a state of sensory deprivation. Note that (Fapk) is compatible with the possibility of knowledge a posteriori of truths that are knowable a priori. This compatibility constitutes *a desideratum* for any characterization of a priori knowledge since we presumably possess such a posteriori knowledge. (Fapk) only puts a constraint on the a priori justification, namely, that it can be accomplished in a state of sensory deprivation, and leaves it completely open whether experience is capable of providing another sort of justification.

(Fapk) talks about a priori known propositions. It would be desirable, it seems, to include also a priori knowable propositions. This is desirable because we want to single out a class of truths with a certain feature.

(Fapk)* P is knowable a priori iff it is possible for a thinker X to know that P on the basis of a justification which can be accomplished in a state of sensory deprivation (namely, by an a priori warrant).

The distinction that interests us is not only between truths that happen to be already known a priori and those known a posteriori. Rather we need a distinction that does not rely on what we contingently happen to know but draws on a feature that necessarily belongs to this class of truths.

I am proposing an analysis of the notion of a priori knowledge similar to the one Wright has proposed, and that Kitcher also considered, but finally rejected. [101]

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101 Kitcher affirms:

> "Why not define a priori knowledge outright as knowledge which is produced by processes which do not involve perceptual mechanisms? The answer is that ... knowledge which is produced by a process which does not involve perceptual mechanisms need not be independent of experience. For the process may fail to generate warranted belief against a backdrop of misleading experience. (Nor may it generate true belief in all relevant counterfactual situations.) So, for example, certain kinds of thought experiments may generate items of knowledge given a particular type of experience, but may not be able to sustain that knowledge against misleading experiences." (Kitcher, ibid, p. 31.)

> "Let us assume that warrants for items of primary modal knowledge do not involve the processing of perceptual information ... that primary modal knowledge is obtained by some clearly non-perceptual process such as abstract reflection or experimentation in imagination. It does not follow that primary modal knowledge is a priori ... a priori warrants have to be able to discharge their warranting function, no matter what background of disruptive experience we may have. But a fact that a process is non-perceptual does not rule out the possibility that the ability of that process to warrant belief might be undermined by radically different disruptive experiences. I can imagine experiences which would convince me that my own efforts at experimentation in imagination were an extremely unreliable guide to anything at all. Hence, the last step in the popular argument illegitimately conflates non-perceptual sources of

An important observation: By (Fapw), I can pick out a class of beliefs in terms of the way by which they have been justified. There are those justified by a priori routes which ought to be contrasted with those justified by a posteriori routes. It is crucial to realize that one can claim that the class of such a priori beliefs is non-empty while still, if one wishes, denying that there is any a priori knowledge. [102]

To sum up, the question "how is a priori knowledge possible?" can take the form: how can it be that any belief having been reached by an a priori warrant, can be true too? A warrant should aim to presumption of truth. Warranting is what a good warrant does: it entitles us to take a proposition as true. An a priori warrant entitles us to take a proposition as true just by thinking. But how can that be? That a belief is acquired by an a priori warrant does not entail that that belief is knowledge. For it does not entail that the belief is true. The problem of answering the question "how is a priori knowledge possible?" is a serious one for two reasons: (1) a priori justification does not entail truth, as it

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knowledge with sources of a priori knowledge." (Ibid, p. 35; my emphasis)

Kitcher rejected this analysis for one main reason. This analysis allegedly conflates "non-empirical processes" of belief formation - in short, nonempirical warrants - with a priori warrants. Kitcher considers a priori warrants as "ultra-reliable", that is, that they have to able to produce knowledge automatically, so to speak, just by the mere fact of their being a priori. Non-empirical processes cannot be of that sort; they are supposedly unable to sustain their warranting function given unkind experiences. I am not going to react to Kitcher here in any detail since I already did that in chapters two and three. I just wanted to illustrate that Kitcher considered the analysis of a priori knowledge I am proposing here, and to briefly point out his reason for rejecting it.

[102] If there are a priori warrants, then it follows that there are a priori beliefs, since warrants are warrants for beliefs. In other words, given that there are a priori warrants, the class of a priori beliefs is non-empty as a consequence. The point I am making is that this fact is consistent with there being no a priori knowledge at all.

does not in my account, and; (2) it is in tension with something we are at least inclined to think is involved in other cases of knowledge, namely, a satisfaction of a strong causal condition. How can we know anything just by thinking? How can thinking justify conclusions about truth? Any adequate notion of a priori knowledge should fully respect its problematic nature. My proposal captures entirely the problematic nature of a priori knowledge.

What I have intended to accomplish with the proposals I offer in this section is: first, to provide at least a "Carnapian explication" of the notion of a priori knowledge. [103] I have found the notion of a "Carnapian explication" very helpful in this context. As I understand the notion, the idea is that we are supposed to get hold on an intuitive concept, modify and develop it, to make it more sharp. For instance, the notion of a general recursive function is such an explication of the intuitive concept of a mechanical effective procedure. The former constitutes no analysis of the latter. We get something that intuitively we had and then get a concept more sharp that still respects what we had before. We don't get exactly what we had before but we can still recognize what we had previously in an intuitive level of the understanding of the concept.

Second, if (Fapk) (and the others) do not amount to definitions, I hope they can at least explain some of the intuitions regarding a priori knowledge. I think that it may be the case that to clarify the crucial intuitions regarding a priori knowledge we don't need to have a proper (explicit) definition of a priori knowledge, though, of course, that would be very desirable.

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103 Carnap, R. The Logical Foundations of Probability. Chicago: Chicago University Press, 1962, chapter one.

Section 3: What is "experience"?

In order to avoid circularity with (Fapk), I will try to elucidate what "experience" is.

"What is experience?" Experience is what is delivered by our five cognitive sense-organs. Some questions naturally arise: What is a sense-organ? Why are there only five?

Broadly speaking, a subject's experience is the flow of her sensory encounters with the world. A subject's sensory states are caused by stimuli external to the body. Another sense of experience is involved in those sensory states brought about by internal stimuli.

Why are there only 5 senses? I won't be addressing this important question in any detail here. [104] My only concern with the issue is to make it clear that in order to establish whether a particular type of process is an a priori warrant, the existence of worlds in which subjects are endowed with additional senses or faculties is entirely irrelevant. My focus is on the question whether a particular type of process could be carried out in by a subject with senses and abilities we actually have, not on whether the processes we are interested in could be carried out by creatures whose capacities for acquiring knowledge are substantially different from ours, e.g., they are augmented or diminished in a significant way.

To decide whether or not a particular item of knowledge that p is an item of a priori knowledge we consider whether the type of process which produced the belief that p is a process which would have been carried out by the subject,

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104 For an illuminating paper on why there has to be only five senses, see H. P. Grice's paper "Some Remarks About The Senses" in Analytical Philosophy. Edited by R. J. Butler. Oxford, UK: Blackwell, 1962, pp. 133-53.

with the kind of cognitive structure it actually has, if no other experiences - apart from those necessary for concept acquisition - had been fed into it, and whether, under such conditions, such processes would warrant belief that p, and possibly would produce true belief that p, if the subject is to have a priori knowledge that p.

Another question that immediately arises is: what does "sensory deprivation" mean? A first approximate answer is: not seeing, hearing, or smelling, etc., for a period of time. Of course, the notion of experience is more general than that of sensory experience. In any case the kind of "experiences" that are possible for one in the tank include many that would ground claims that are intuitively not a priori knowledge. For instance, claims about what one just dreamt, or how one is feeling, or whether one has a headache, etc. So I need to say something about why a reflection on these matters doesn't constitute an "a priori method". One suggestion: an item of knowledge only counts as a priori if it could be known by any suitably conceptually endowed subject in a state of temporary sensory deprivation.

Is it possible to arrive at knowledge in a state of sensory deprivation? According to the apriorist, the answer is "yes". (Though I think that the claim that it is possible to arrive at new information in a state of sensory deprivation may not be special for the "apriorist". Most people would think it is true.)

Given that an a priori warrant is one that can be carried out in a state of sensory deprivation, the following consequences obtain:

(1) **that it is possible to have no sensory experiences at all,** [105] **of course after those needed for acquisition of the conceptual repertoire for a priori knowledge, and still hold these justifications;**

(2) **one could do them "in the dark", provided one understands the questions.** [106]

Certain experiences are necessary for acquiring a priori knowledge, but they are only those needed to provide the relevant concepts. Nevertheless, we have to reconcile the need for other kind of experiences: the experiences needed to account for the reliability of the knower's state. At this point, it is important to draw the following distinction between conditions for (a priori) processes:

(a) **what are the conditions that enable one to carry out an a priori process? Are there any sensory conditions? The apriorist answer is "no".**

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105 Note that sensory deprivation does not mean "no experiences" (one feels warm, cold, etc. - as I explained on the previous page).

106 Can one read a proof in the insulation tank? Reading a proof involves at least one sense: seeing, so one cannot read a proof in the tank. Is it a consequence then that our a priori knowledge would be restricted? I can read a proof outside the tank but not in the tank. What is then the status of the sensory insulation idea?
This seems to me to be a worry only if it is supposed that whenever we acquire knowledge a priori by means which rely on physical external aids - typically reading and writing marks on paper - it is an essential feature of the methods we use that there is this reliance. But that doesn't seem to be so. It seems plausible to suppose that whenever we do in fact rely on pencil and paper, we could in principle carry out the same process in our heads - if we were lucid and focused enough. It is, for example, surely a contingent fact about human beings that we are not as good at mental arithmetic as we are at written arithmetic, and it certainly seems to be no essential feature of the methods of calculation that they be carried out on paper. Similarly, very good chess players can play without a board. And so on.
In general, the point of the sensory insulation idea is just that it is a graphic way of ensuring that there is no empirical input in the course of implementing the relevant method.

Let me explain. These conditions are those that make the process possible. An a priori warrant is one for which no sensory input is necessary for the subject to carry it out.

The other set of conditions for (a priori) processes:

> (b) **conditions whose obtaining are needed in order for the subject to be able to respond to the process by believing that p.**

What is at issue is: what conditions are necessary for the person who undertakes the process to actually accept p in the end? There is room for empirical considerations here. These are collateral beliefs about the background, conditions that make (possible) a certain response to the (a priori) process. This second set of conditions for processes have to be accommodated in a notion of a priori knowledge too.

Another related distinction has to be made in connection to the role of statements in justifications. A statement plays a justificatory role in a justification for p iff it is playing a role in justifying the truth of p. A statement may play a justificatory for q, related to p in the sense that it expresses a pre-condition for our knowing the truth of p. For example, when p is the conclusion of inferential a priori knowledge, then all of the premises in the proof are playing a justificatory role for the truth that p (in short: for p). However, q, let's say, "I follow a proof that p" (which implies the truth of r: "I am alert enough at the moment I am following the proof that p") is not playing a justificatory role for the truth that p (any more than r); rather it expresses a precondition for my being able to obtain inferential a priori knowledge that p. Actually, the statement "r' is partly a more basic pre-condition since it involves "alertness" which is a precondition for the acquisition of any kind of knowledge.

Section 4: Is the sensory deprivation suggestion any good?

The test for a proposed definition of a priori knowledge rests in its ability to classify appropriately knowledge as a priori. To examine a catalogue of truths supposedly known or knowable a priori [107] will help to know whether my account render them as a priori:

(1) **Analytic truths**:

Let's say that analyticity is construed as "true in virtue of meaning alone". Given that these truths can be known by reflecting on the concepts which figure in them, we can easily see that conceptual analysis is an activity that can be accomplished in a state of sensory deprivation. So, analytic truths trivially come out as a priori. An example of an analytic truth that we know a priori is "Triangles are three-sided figures".

(2) **Logical truths:**

Logic is a purely deductive science. Since one neither verifies logical laws nor obtains logical consequences by perceptual processes, it is possible to obtain logical knowledge in a state of sensory deprivation.

But what if one has to rely on the syntax of an expression (i.e. it has the form of a conjunction, etc.?) That experience is part of the necessary experiences for the acquisition of the conceptual repertoire necessary to obtain a priori knowledge.

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[107] I don't claim that the list is exhaustive. These truths are the ones I have found.

(3) **Mathematical truths:**

If logicism were correct, then the a prioricity of mathematics would follow from the a prioricity of logic only if a prioricity is preserved by logical inference. On the other hand, if mathematics has its distinctive subject matter, for instance, arithmetical statements speak of numbers conceived as abstract objects, then again mathematical knowledge would come out a priori. If mathematical grounds are concerned with entities which are causally inert, as abstract objects are, then a mathematical ground can produce knowledge without having to rely on anything outside the reach of a subject's mind. For instance, to know that the arithmetical statements "The number 2 is the third number in the natural number series" and "The number 2 is an even number" are consistent is something that we can know with our eyes closed, ears plugged, etc..

Note that there is a difference between what we actually do and what we could use in the tank. Of course we are not going to actually get insulated in the tank to know, for example, that these statements are consistent. The point is rather that for the characterization of a priori truths we have the following: it could be done. Our basic proposal is that in order for a proposition to be actually known a priori, it is a necessary and sufficient condition that the method whereby we came to know it could have been used by a lucid subject in the tank, that is, by a lucid subject in a state of total sensory deprivation.

If mathematics were reducible to logic and set theory, since logic is a priori, and sets are abstract objects, then mathematical knowledge comes out as a priori again, provided the required inferences preserve a prioricity. [108]

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108 It appears to some philosophers like Kitcher, for example, that the assumption that logical inference preserves a prioricity is too big an assumption. What I want to say is that for those who are sympathetic to the view that mathematical knowledge is a priori, it is not so. And we are concerned with

(4) **The cogito:**

It comes out as a priori. [109]

(5) **Universally empirical knowledge:**

Examples of propositions which allegedly constitute universally empirical knowledge are "There is an external world", "There are physical objects", "Some objects have shapes", etc. It can be argued that such knowledge should be considered as a priori since it requires no particular kind of experience beyond that needed for the acquisition of the relevant concepts. The traditional conception of a priori knowledge is too vague, it is not articulated enough, to decide alleged cases of universally empirical knowledge. [110]

Universally empirical knowledge does not come out as a priori according to (Fapk) because some experiences are needed for that knowledge, for example, the experience of at least one physical object to know that there are physical objects. It may appear that the contrary is the case because of the fact that any particular experience of any physical object is enough for such knowledge, that is, any particular experience of a physical object would do, the particularity of the object is irrelevant. Nevertheless, we cannot forget what is crucial: that the

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those who are sympathetic to this view. Even more, it would be bad news if according to an account of the a priori, logic turns out not to be a priori. Logic is an a priori discipline if anything is.

In the next section I will try to account for our knowledge of mathematical axioms.

[109] I discuss this case on pp. 151-2.

[110] Contemporarily, Kitcher (ibid, p. 31) has considered the problem of universally empirical knowledge coming out as a priori, but although his analysis points to the right direction, in my opinion, it is not sharp enough to explain why this knowledge is not a priori since he does not elaborate on the notion of the particularity of experiences, for example.

experience in question is not only relevant, but also necessary for that knowledge, and that is what makes it a posteriori instead of a priori. Universally empirical knowledge does not count as a priori according to (Fapk) because beyond the specified experiences, no experiences at all, not particular or otherwise, are needed to obtain a priori knowledge.

Section 5: On basic a priori knowledge

How is it possible to acquire basic a priori knowledge? I shall be modest; I don't have much new to say here. I will discuss some examples and examine what conclusions can be drawn from them.

The picture I endorse is a simple one: given certain concepts that we can learn even empirically, we can have the ability to go from the concepts to the apprehension of truths concerning them just by thinking. [111] Provided we have the necessary concepts, it can occur to us, if we concentrate and are intelligent enough, that some statements are true, or follow, from the combination of, or reflection on, these concepts. For example, having the concept of a triangle, I come to know that "If a figure is a triangle, then it must have three angles", and this follows trivially from the definition of a triangle; or less trivially, if we were a Dedekind or Frege in a state of sensory deprivation, the Dedekind-Peano axioms could occur to us.

First of all, do we just think about the (basic) proposition in question? Possible cases to study that can help us to answer this question are the axioms of

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111 Of course, this is not a theory; it is not controversial. Anyone who accepts that there is basic a priori knowledge will agree with this picture. But my intention is not to provide an original theory but rather try to illuminate what goes on in our acquisition of basic a priori knowledge by studying some examples to establish what general conclusions we can draw from them.

arithmetic, the Parallel Postulate, as a negative case of study, stipulation, and definition. In stipulation and definition, however, not just thinking is involved, but also laying down some principles.

The cases that interest me as examples of propositions which constitute basic a priori knowledge are those which have the following characteristics: (1) the concepts in the statements are already familiar prior to the understanding of the statement, (2) we still have not thought the statements in question. Then, we think the statements in question. What happens in our thinking them? We persuade ourselves that they are true just by thinking. We have to think of the recognition of truth in the light of understanding. For example, if I understand what it is to apply color terms correctly, then I can know that colored objects cannot be colored with two different colors in the same areas. [112]

We have geometrical intuitions. We refine them and get some principles. There are propositions that seem good to us in the light of the concepts we have.

Sometimes a process involved in the study of geometrical figures is visualization. [113] We check a property by visualization and obtain generalizations about cubes, for example. The properties on which we generalize are not accidental properties due to the process of visualization, but they are essential properties of the figures.

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[112] What is interesting about color-exclusion is that even appearances of colors will lead us to the same result; the appearance of looking red or green, for example. We reflect on those appearances. There is a process of reflection involved. Even if we don't have the convention of how to use color terms, it is a fact (empirical) that things look in a certain way, that they don' allow of being of different colors all over.

[113] It can be argued that visualization need not be used. I won't deny that. My point is that given that we use it, to say what goes on in that process. Note that I am not saying that visualization is an infallible method. But, surely, most people will agree that it can be a potential source of knowledge. That is why I consider it.

We have to distinguish between visualizing and making explicit presuppositions. In visualization, we execute processes we use to convince ourselves that certain statements are true. We use diagrams. When we make explicit the presuppositions we are not simply recasting what we did in visualizing, we are trying to be more rigorous about what we visualize. We try to improve what we could understand in an intuitive level.

This simple picture I am suggesting may not be restricted to analytic truths. I leave open the possibility that there could be a priori knowledge of "synthetic" a priori truths.

Here is what happens when we proceed to formalize our informal knowledge or intuitions. Given that we can obtain some statements which state all the relevant relations among the concepts, we compare the statements we get. This comparison establishes the level of "basicness" among the statements. The task of establishing the degree of basicness of a statement is a logical one. We choose the statements that have more deductive power, the ones that by themselves alone can imply the rest. That is, logic helps us to establish which statements (or combination of statements) ought to be our axioms by serving as a tool for testing which statements have more deductive power or have more deductive consequences and at the same time constitute the smallest set (or combination) to do that. This entire procedure of acquiring axioms can be accomplished a priori in my sense.

With the examples of colors, geometry, and basic arithmetic, I have tried to answer the following question: how do I persuade myself that these propositions are true? I think that the role of imagination in a priori knowledge is important. It can be a source for acquiring knowledge. Though, certainly, it is not infallible. Philosophers have the tendency to concentrate on logic, axioms, rules of inferences, logical consequences, and seem to forget the importance of

imagination and ways of knowing that should be taken into serious consideration at this basic level too.

Chapter Seven
A priori knowledge and Infallibility

This chapter is devoted to an analysis of the relationship between a priori knowledge - as partly characterized by the proposed analysis of the notions of an a priori warrant and an a priori method - and the notion of infallibility.

Section 1: Glossary

It strikes me that there are a lot of terminological problems in this area. A variety of predicates - "fallible", "infallible", "defeasible", "indefeasible", "a priori", "empirical", etc. - are jostling with each other to apply to a variety of different kinds of things: truths, methods, warrants, knowledge. One needs to straighten all this out and set things up quite carefully, so that the basic ideas I want to discuss - for instance, the question of the possible infallibility of a priori knowledge - can be clear enough to be discussible at all. I would hope and expect that, when this is done, some of the confusions would be cleared up.

I don't intend to provide proper definitions for all the epistemological terms I discuss. To accomplish that task would take me far from my focus here - which is to get clear about the notion of a priori knowledge and the notion of infallibility, and how they relate to each other. However, I have been using these epistemological terms, and very often they are used in the literature without any adequate (explicit) explanation. What I will offer are clarifications of the ways I use these terms. I shall also offer some reasons why these terms ought to be used in this manner.

I will be saying a lot more about some of the epistemological terms in this part of the dissertation, so in this glossary I intend to give a basic guide for the

reader to be able to follow the discussion. To provide the glossary at this point, though, involves having to avail myself of discussion that comes later on.

As I see the matter, the primary task in clarifying these epistemological terms is to get clear about what their bearers are: what things have these properties.

It is important to bear in mind what I mean in saying that these properties apply objectively to their bearers. One has to distinguish between the bearers having those properties objectively, and our thinking that they possess them (which may not be true at all). For example, if a method is infallible, it is objectively that way. It is so intrinsically and not due to our inability to find something wrong with it. Obviously, if there is nothing wrong with the method intrinsically, then there is no way that we can correctly improve it, though we can incorrectly think we are doing so. In respect to infallible methods, it is not possible that there are ways of improving the method to make it more reliable without showing that it is wrong; that is, no improvement is possible for infallible methods. In contrast, it is possible that there are ways of improving fallible methods without showing that they are wrong.

(1) **Method**

By a "method" I simply understand a general routine which we can perform correctly or incorrectly on particular occasions and wind up warranted (or not) in certain beliefs as a result.

The connection between methods and beliefs is the following: methods are ways for generating beliefs. The methods that are interesting from my point of view are those methods which must generate in a person, rational, reasons for thinking that the belief acquired by X in carrying out the method is likely to be true. In this way, hypnosis can be excluded, for example. A person in knowing

that I have been hypnotized, let's say, is not likely to believe that the belief I acquire by the process of hypnosis is true at all, or if the belief turns out to be true, this rational person would not be likely to think that the belief was justified. Hypnosis gives no warrant, it only causes the belief in the subject. On the other hand, if I believe B by method M, a proof, let's say, then someone else has reasons to believe I am right.

It is methods that are primarily infallible or fallible. An example of an infallible method is calculation. An example of a fallible method is observation.

(2) **A Priori Method**

The predicate "a priori" ought likewise to be applied firstly to methods. A priori methods, I propose as a first approximation, are those which could in principle be implemented in a state of sensory deprivation. Empirical methods require sensory activity.

(3) **Being Warranted**

To be warranted is to be in a state of information which we reasonably take to justify certain beliefs. States of being warranted are the products (results) of the warrants we get by implementing methods.

Talk about "warrants" can create the mistaken impression that they are objects of some sort. But warrants are not objects but processes. For example, when what is warranting is an act of perception, perception (perceiving) is a process. Likewise, one can be warranted by ones' memory, perception, by testimony.

One is inclined to think of a warrant as an object having a proof in mind. An important qualification is in order here: in the case of inferential a priori knowledge, a warrant is not really a proof, but rather rightly possessing a proof.

There is a distinction between having a warrant and being rightly warranted. We may possess a proof without realizing that it is in fact a proof. We may not understand the proof, so we cannot gain knowledge by its means. For inferential knowledge a priori that p, certainly the possession of a proof is a necessary condition but it is not a sufficient one. I may not be even sure that I have a proof, I may not know at all that I possess a proof.

The property of defeasibility applies primarily to warrants, and warrants are typically defeasible. That is, there will usually be no conclusive guarantee either that we have not made a mistake in getting a warrant, or - even if no mistake has occurred - that further information would not compromise the justification we currently have. (More about defeasibility below)

(4) **Being Warranted A priori**

Being warranted a priori is thus a state that can be reached in a state of sensory deprivation. On the face of it, such warrants may be as defeasible as any others, for the same reasons.

(5) **Infallibility**

It is trivially but necessarily true that when we know a proposition p, then we are not mistaken about p. This is the sense of Kitcher's notion of infallibility as Hale understands it.

The interesting notion of infallibility - the nontrivial one - is one that primarily applies to methods. The question to ask concerning a method is whether if I execute it correctly and then apprehend the belief I ought to form according to the result, there is any residual possibility of that belief's being mistaken. A method is infallible if and only if when it is applied correctly we end up always with true beliefs.

Methods are the only candidates, in my opinion, which can make plausible talk of infallibility in an interesting sense despite the fact that we are always fallible. They are the only candidates which can put us in the special epistemological position where we cannot possibly get it wrong.

(6) **A Priori Infallible Method**

An a priori infallible method is one that satisfies (2) and (5). It can be carried out in a state of sensory deprivation and is infallible in the second sense mentioned in (5).

(7) **Defeasibility**

This property applies firstly to warrants and, derivatively, to the beliefs they justify.

A warrant (and its associated justified belief(s)) are defeasible when additional information cannot be ruled out which would either compromise our confidence that the warrant was correctly acquired - that the relevant method(s) were properly executed - or result in a total evidential picture in which the belief(s) in question are no longer justified. It is our essential fallibility - the fact that we are always capable of error - which ensures that defeasibility is so pervasive.

(7*) **De facto indefeasibility**

A warrant is de facto indefeasible at t and later times if all further evidence against it is in fact misleading evidence. First, we cannot know that a warrant has this property at t, or any subsequent time, because we won't be able to have all the relevant evidence at our disposal. Second, we cannot know that a warrant has this property at t and later times because there is no guarantee that we can know about all future evidence against the warrant, provided that we are

in a position to gather it all, that it is actually misleading evidence. Warrants can be de facto indefeasible at t and later times without our knowing that they are. Note that a posteriori and a priori warrants can be de facto indefeasible. Very importantly, de facto indefeasibility is not the same as infallibility. The notion of de facto indefeasibility is weaker than the notion of infallibility. When a warrant is necessarily, not simply de facto, indefeasible, then it is infallible (indirectly) in my sense, that is, it is the result of a correct implementation of an infallible method.

Knowledge requires that its successful warrants - those which produce knowledge on the occasion - are de facto indefeasible. Because if I know (in particular if I knew) then it is not possible to show by any kind of subsequent evidence that I am (was) wrong.

A warrant can be objectively de facto indefeasible (indefeasible as a matter of fact) or objectively necessarily indefeasible (i.e. the warrant is the correct implementation of an infallible method) without our knowing that is objectively indefeasible one way or the other. That is why even these warrants are defeasible in the sense just mentioned in (7).

(8) **Success of Methods**

There is an important distinction to be made between a method of discovery which is in a particular case successful, and a method which in principle could only be successful.

As I take it, an infallible method ought to be understood as a method which in principle could only be successful.

(9) **Revisability**:

The candidates for revision are beliefs, warrants, and claims to knowledge. In the literature, we find two kinds of revisions for the first:

(I) **Revision of Beliefs**

(a) **strong revision**

When subsequent information reveals that the original belief was false.

(b) **weak revision**

When subsequent investigation makes it appropriate to reshape the concepts occurring in the proposition. There can be circumstances in which it would be rational to rethink our concepts or to change them by others which do not guarantee the truth of the original belief.

I propose a corresponding distinction at the level of warrants:

(II) **Revisions of Warrants**

(a) **strong revision**

When the warrant is found to be faulty and rejected completely for that reason. For example, in the case of an alleged proof containing a contradiction and there has been no way of resolving the contradiction. We reject the warrant completely.

(b) **weak revision**

This is the case when the warrant is faulty but repairable. We just have to repair the problem and can still hold the belief under the revised warrant.

(III) **Revision of "claims to knowledge"**

Claims to knowledge are of the form: "I know that p" and they are true iff I know that p, otherwise, they are false.

My claim to know that p can be subverted in the following ways:

(a) **when p is subject to weak revision and I end claiming to know a (slightly perhaps) different belief,**

(b) **when my claim to know p is false because p is false,**

(c) **when I have to reject my claim to know that p because the warrant I had was not a good one.**

Another situation is when I still claim to know that p but I am justified in doing so by a new warrant which can be an improvement of the old one, if that was possible, or a completely new warrant for my belief that p. This is not a revision of the claim to know.

The claim to know that p says something different from the claim that p. I can justify my denial that I know that p without for that reason justify the denial that p. Also not every ground for asserting p is a ground for my claim to know that p.

(10) **Falsifiability**

This property is applied primarily to beliefs or propositions, and secondly, to our claims to know a belief that has been subsequently falsified, shown to be false. Falsifiability of beliefs is the same as strong revision of beliefs. It results in a complete rejection of the warrant which allegedly justified the belief. The claim that "I know that p" is falsified when p itself is falsified.

(11) **Certainty**

Describing necessary propositions as certain doesn't seem to be correct, since - obviously - a proposition could be necessary without even being known (or, some would hold, without being knowable - Goldbach's conjecture, for example).

The best notion of certainty is a normative one: a proposition may be described as certain not because one happens to be immovably convinced of it but because that attitude is rightly regarded as fully justified. Note that if certainty is so characterized, then it is possible for a proposition justifiably to be regarded as necessary <u>without</u> its being certain - in special circumstances where the grounds for regarding it as necessary are not all that strong - and the same may go for a proposition which is grounded a priori. Where the notions - certain,

necessary, a priori - all seem to come together is in the simplest examples of the a priori: fundamental laws of logic, for example, and the simplest elementary arithmetical equalities.

To recapitulate, it was crucial to provide a glossary for two main reasons: First, in order for the reader to get clear about what I have in mind. Second, I have found it extremely important to come up with a notion of infallibility that is workable enough to make it plausible that there is some room for infallibility, properly understood, despite the fact that we are fallible creatures and prone to make mistakes everywhere. It is too easy to disregard the possibility of infallibility given our inescapable fallibility. But I considered that to be a mistake. To postulate infallibility does not have to mean denying our fallibility. So, the important question becomes: what are the reasons that could be offered to postulate infallibility? I think that to take infallibility as a property primarily of methods is a necessary condition to accomplish this task. It was Wright's work which firstly led me to think about this position. [114]

Section 2: Theses on infallibility

I shall proceed to distinguish two theses on infallibility, and to evaluate them. I will not be attributing the first thesis to any particular philosopher but rather will consider it on its own merit as a possibility in logical space. I won't contend though that they exhaust all the possibilities. I just have found them interesting.

(1) **What is primarily infallible are procedures by means of which items of knowledge can be acquired.**

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114 Wright, Crispin. Realism, Meaning and Truth. Basil Blackwell, second edition, year 1987. I will discuss Wright's idea on pp. 137-9.

This thesis of infallibility may be understood as requiring two things: that knowledge must be restricted to cases where we possess infallible ways of knowing as well as incorrigible propositions, that is, risk-free methods of generating knowledge and propositions which in no circumstances could be false.

Does the latter requirement mean that we have to be concerned only with necessary truths? It should not since we presumably should leave room for a priori knowledge of contingent truths. This points to a distinction between senses of the phrase "cannot be false". I distinguish the following:

(I) "cannot be false" in the sense that the proposition in question is a necessary truth;

(II) "cannot be false" in the sense that it may be a contingent truth and nevertheless cannot be false in the world where it is true as long as it is entertained by the subject, for example, the cogito and some simple beliefs about our mental states belong to this group (I call them "cogito propositions"); and:

(III) "cannot be false" in the following sense: when the proposition is generated by the correct prosecution of an infallible method it has to be true - infallible methods only generate true beliefs; the modal status of the proposition known being a separate issue altogether.

Thesis (1) would allegedly exclude the method of observation since it is not a totally reliable method; it is not immune from mistakes. A possible view about risk-free procedures is that only procedures which are self-guaranteeing or self-justifying are accepted. The propositions supposedly known by these "methods" are self-evident truths, for example, the cogito.

Another version of thesis (1):

(2) **It consists of understanding thesis (1) - i.e. that what is primarily infallible are procedures by means of which items of knowledge can be acquired - wholly in terms of identifying methods of knowing which in themselves eliminate the risk of error.**

That is, (2) does not involve an attempt to discover propositions which are supposedly infallible. Again, what is the difference between theses (1) and (2)? The second thesis is entirely concerned with methods which can be infallible.

Wright suggests this interesting thesis of infallibility in his Realism, Meaning and Truth in the context of a discussion about the relationship between an arithmetical computation and its result.

> "A .... prima facie promising aspect of the relation between a feasible computation and any arithmetical statement which it verifies or falsifies is this: that if someone
> i comprehendingly carries out the computation correctly; and
> ii correctly apprehends the outcome; and
> iii possesses a correct understanding of the statement in question, the opinion he forms concerning the truth-value of the statement is bound to be correct; there is no further scope for error." [115]

(2) has to be qualified though. At first sight, the fact that we are always fallible would appear to preclude the possibility of such methods: that is, given that we are always fallible, prone to make mistakes, how can we have infallible methods? First, we could have infallible methods in my sense without being aware of having them given our limited intellectual capacities. Second: Provided that there are infallible methods, their metaphysical status is irrelevant for the epistemological points I am making about them. The latter conditional will obtain irrespectively of: (a) whether we have infallible methods, (b) whether we create them or not, and (c) whether infallible methods exist independently of us, as abstract objects arguably do, and we are just discovering them.

A clarification is in order at this point, and it has to do with something I have insisted upon and, that surprisingly enough, has been almost totally ignored in the literature. There is a distinction between the defeasibility of

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115 Wright, ibid, p. 115.

grounds (warrants) and the possible infallibility of methods, and the fact that given the former, the latter can only be conceivable in conditional form, [116] and in relation primarily to methods instead of ourselves implementing such methods. Infallibility is conditional upon the satisfaction of the appropriate precautions, and lies in the impossibility of no further error or mistake. That is, infallibility entails that there must not be the possibility of further error. Obviously all warrants can be subject to defeat (are defeasible in principle) due to our fallibility. That is, all warrants are defeasible in the sense that it is always an epistemic possibility that we have made a mistake in carrying it out, or even if we did not and, as a matter of fact, the warrant cannot be defeated, we may think otherwise. But given that the methods have been applied correctly on the occasion, and we apprehend correctly the belief we ought to form by an infallible method, then no other logical possibility of error remains.

Another distinction is at work: the distinction between (cognitive) methods and warrants. A (cognitive) method is a general routine we can perform correctly or incorrectly and, consequently, get warranted belief (or not) as a result. The methods can be intrinsically infallible, but our implementation of them - which results in our rightly possessing a warrant - is not infallible because we are not infallible.

Given these qualifications, the second sense is the one I will develop, that is, that infallibility is a matter of certain methods of knowing which in themselves eliminate the possibility of (further) error as described above. The nature of the proposition known will not play a major role in my development of this thesis. (2) is consistent - more precisely, totally analogous at the level of methods, not of warrants directly - with my view, shared by other philosophers,

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116 Again, Wright is aware of the crucial fact that a conditional is involved here.

that the notion of an a priori justification is the fundamental notion to characterize in the epistemology of a priori knowledge. I have extensively argued for this claim in this dissertation, so I won't repeat myself here.

As I already said (p. 107), I intend to make conceivable the (mere) possibility of infallibility. The crucial question, the first task, becomes in my view how to explain that we as fallible knowers can be in a position where we cannot get it wrong, where we cannot be mistaken. Whether the possibility has been realized is to argue for another matter (a second task). I don't have a knock-down argument for the latter, though, of course, that is not to deny that accomplishing the first task paves the way for settling the second question, that is, that the possibility has been realized (actualized). Briefly, an answer to the second question is that if we prosecute an infallible method correctly, and correctly apprehend the result, we are guaranteed both that we are not wrong and that we cannot possibly be wrong.

One could argue that in the basic case, we are infallible when we possess a belief acquired by a self-justifying process. I am inclined to reject basic knowledge of this kind as infallible because something basic can be confused. Also we are not infallible about our own private states. It seems also plausible to suppose that physiological psychology could develop to the point that we would be rightly justified in preferring neural readings to the subject's report when they conflicted.

I am inclined to think that when there is room for ourselves conceivably being infallible about more substantial truths, it appears to me to be the case that there is a method involved, and it involves the correct execution of an infallible method.

I believe that infallibility can be accounted for wholly in terms of the existence of infallible methods and the correct implementation of them,

independently of the status (i.e. analytic or necessary) of the propositions to be known. At least, I find that that would be a desirable result to have. It would fit nicely with taking the term "infallible" as applying primary to methods, and conceivably and derivatively to ourselves carrying such methods, and likewise, in any case, to propositions believed by their means. I don't yet have an argument for this conviction, but will be testing below if it is correct.

Section 3: Some clarifications

My goal in this section is to clarify the relationship between knowledge and infallibility by analyzing statements that we ordinarily accept about them. Some of them are truisms; others are loose ways of trying to convey the same thoughts that these truisms express. The problem is that some of these loose ways of speaking are harmful. They can create the wrong impression that knowledge, by its merely being so, entails infallibility.

The following are some clarifications of the concept of knowledge.

(I) **Necessarily if I know that p, then "p" is true.** [117]

It is a truism that knowledge entails the truth of what is known. (Usually we express the same thought as simply "If I know that p, then "p" is true". Actually, we tend to ignore the modal operator in front of these statements.)

(I^1) **Necessarily if I know that p then I am not wrong about p.**

It is a platitude that one doesn't count as knowing unless one is right. Even more trivial is that:

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117 The kind of modal involved in these statements is conceptual. These statements express conceptual truths about the concept of knowledge. But it is not an issue for me to settle what kind of modal is involved.

(I^{11}) **Necessarily if "p" is true then I am not mistaken in believing that "p" is true (if I believe that p at all).**

It is banal that if p is true then I am not mistaken in believing that p. These are trivial consequences.

It is crucial to note that the modal operator in (I), (I^1), and (I^{11}) governs the whole statement. We usually intend in ordinary English to convey the previous platitudes as follows:

(II) **If I know that p, then I cannot be wrong (or mistaken) about p.**

With (II) we try to capture (I^1): "Necessarily if I know that p I am not wrong about p" (loosely: "If I know that p, I am not wrong about p") by (II) If I know that p, then I cannot be wrong about p.

(I) and (I^1) are satisfied by any sort of knowledge. It may appear to someone that (I) and (I^1) imply that my knowledge that p is infallible. This would constitute a trivialization of the notion of infallibility. This is not the interesting notion of infallibility since it does not point out the problematic nature of infallibility but rather trivializes it conflating it with knowledge per se.

This loose way involved in (II) by which it is usually meant (I^1) is problematic. (II) involves an operator shift and it indeed constitutes a fallacy. We see how the statement "I cannot be wrong about p" can give the wrong impression in this context that because we know there is stronger guarantee that necessarily we know, and that we cannot be mistaken. Because I know that p it does not follow that I cannot be wrong about p since it is not necessary that I know that p. I may not have known that p; I may have not even thought about p, so I might not have been right or wrong about p.

In contrast, the following statement is fine:

(II^2) **If p and I believe that p then I am not mistaken in believing that p. (Logically expressed: Necessarily if p and I believe that p then I am not mistaken in believing that p.)**

There is an important qualification to make at this point: the statement "I cannot be wrong about p" conveys three more possible meanings (at least):

(a) At any time I think that p is true, I can't be wrong in thinking that p is true since p is a necessary truth, that is, it is always true. Why? Because any time I entertain p and believe it to be true I am right because p is always true. This case only involves knowing the truth-value of p and does not involve knowing why it is true or the way it is shown to be true. For example, if p is true and can be proven, and I know it by testimony, I know that it is true, but I don't know its proof, I don't know the reasons why it is true. We may not even know the content of p but just that it is true.

(b) I can't be wrong about the subject matter of p. That is, I will always "track the facts" so to speak - if p is true I will believe that p and if not p is true then I will believe not p instead. [118]

(c) This case does not involve - necessarily - simply thinking that p is true because I will always be right about that given that p is a necessary truth - this is case (a) above - but rather when my warrant for knowing that p is the result of the correct prosecution of an infallible method. That is, given that, on the occasion, the method was properly executed, and that I apprehended the belief I ought to form according to the right prosecution of the infallible method in question, then I cannot be mistaken about p. This sense of "I cannot be wrong" is the one I am interested since it involves the non-trivial notion of infallibility I have in mind.

Infallibility is generated when we form the belief we ought to form. If the method is infallible, the belief that we ought to form on the basis of the correct implementation of the method cannot be false. Note that I am not saying that the belief we actually come to have on the basis of the method (even if infallible) cannot be false because there is a gap, there is a causal chain, from the end of the

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118 I am not endorsing this since it seems an implausible epistemic position. I am only explaining the different meanings the statement "I cannot be wrong about p" can convey.

calculation, for example, to the apprehension of its result. It is possible that we apprehended the result incorrectly, misread the result, something happened, and then we ended up with another belief that the one we ought to form.

The statement "I cannot be mistaken about p" involves not only that I am not mistaken about p (presently) as a matter of fact since, as already said, this is a consequence of my simply knowing that p at any time t, but also three more things: (A) that "I cannot be mistaken about p at t, at the present time I know that p" because I necessarily had to know that p at t when I know that p at t as the result of the correct prosecution of an infallible method; (B) that "I could not possibly have been mistaken about p" whenever in the past I have come to know that p in the same manner; and (C) that "I won't possibly be mistaken about p" in any future time when I will be in the process of coming to know that p again in the same manner as well. These inclusions are obvious consequences of the grammatical fact that the English verb "can" is atemporal. In short: any time I implement correctly an infallible method and correctly apprehend the belief I ought to form by the method, I end up with knowledge.

It is indeed a very strong notion of infallibility that I have in mind. I think that it ought to be so if one is to avoid conflating the possibility of infallibility with the possibility of knowledge. One has to be able to distinguish between three claims:

> **(i) that "if I know then I'm not mistaken",**
> **and the corresponding fact that if I know then the possibility of error is in fact excluded;**
>
> **(ii) the claim that "I know but I can be mistaken"**
> **which entails that the possibility of error is not necessarily excluded;**
> **and**
>
> **(iii) the claim that "I know and I cannot be mistaken" which entails that the possibility of error is necessarily excluded.**

The sentence "I can be mistaken" in (ii) means that I can be mistaken about p at different times: for example, at previous times I could have been mistaken, and at future times I can be mistaken, and also in the present time I might not have known.

To conclude, let me briefly evaluate the following statements:

(1) If I know that p at t, then I am not mistaken about p at t.

(1) is a loose way of expressing "Necessarily If I know that p at t, then I am not mistaken about p at t". It consists of merely dropping the modal operator in front of the statement. Since it does ignore the modal operator, it does not create the wrong impression of infallibility. (It may be harmful for other reasons.)

(2) If I know that p at t then I cannot be mistaken about p at t.

(1) is not logically equivalent to (2). In (2) what we have done is to switch the modal operator inside. It constitutes a fallacy. It leads to confusion and danger since in switching the modal operator inside the statement it creates the wrong impression that knowledge entails infallibility.

The following statements are fine:

(3) If I don't know that p at t then I am not right or wrong about p at t.

(4) If I don't know that p at t and thought I knew p at t then I am wrong about p at t and wrong about my knowing that p at t.

(5) If I know that p at t then it is an open question whether I cannot be wrong about p at t or not. That is, it is left open whether I must have known that p at t or not; the method might be infallible.

(6) If I know that p at t then it is an open question whether I could have been wrong about p when I was coming to know that p, that is, in the process of coming to know that p at t; the method might be fallible.

I have to address two important issues: (1) whether all a priori methods are infallible methods, and, (2) whether some empirical methods are infallible methods or not. I shall shortly return to these questions after section four.

Section 4: Helpful lists

It will be helpful to try to come up with lists in connection with a priori knowledge.

One of them is a list of possible procedures or methods by means of which we could acquire knowledge infallibly: [119]

(1) self-guaranteeing or self-justifying procedures

We saw already the problem with those.

(2) Calculation

When calculating in the abstract realm, it is an infallible procedure otherwise it is "quasi-infallible". [120]

(3) Counting.

When counting in the abstract realm, it is an infallible procedure. Otherwise it is "quasi-infallible".

(4) Construction of proofs, where done correctly.
(5) Some cases of introspection.
(6) Measurement.

List of possible truths or statements that we can know infallibly:

(1) Necessary truths
(2) Contingently true propositions but only a restricted range of factual statements can be accommodated, namely, propositions which describe immediate experiences. For example, the proposition "I am sitting comfortably."
(3) Cogito propositions - example of contingent a priori truths.

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[119] These lists may not be complete. These items are the ones I have found. I am not going to argue for the fallibility/infallibility of all the methods I suggest in the first list.

[120] Briefly, a "quasi" infallible method is an infallible method such that the base to which it is to be applied is unstable, i.e. the objects to which an infallible method is applied do change. More about this notion in section 6.1.

Section 5: The contingent a priori

Traditionally, a priori knowledge was associated exclusively with knowledge of necessary truths. Kant offers in the first Critique [121] necessity as a criterion for distinguishing what is knowable a priori. In recent times, it has been argued by Saul Kripke that the notions of necessity and a prioricity are not coextensive. Kripke argues that there are necessary truths which can only be known a posteriori [122] and also contingent truths which can be known a priori. [123] An example of a necessary truth which can only be known a posteriori is "Hesperus is Phosphorus". Kripke argues that "Hesperus" and "Phosphorus" are rigid designators, that is, in any world in which they refer, they refer to what they refer in the actual world, namely, the planet Venus. The knowledge that these names refer to the same planet was a scientific discovery. An example of a contingent a priori truth is a truth by stipulation: "The standard meter in Paris is one meter long". According to Kripke, for the person that fixes the metric system by reference to stick S at t, let's say, a meter in Paris, the proposition that "The standard meter in Paris is one meter long" is known a priori. Regardless of whether these cases exist, a lesson we obtain from Kripke's arguments is that it cannot be taken for granted that the notions of necessity and a prioricity are coextensive. Argument is needed to prove this thesis.

The issue of the contingent a priori is very complex and I cannot discuss it here in any detail. My goal is modest and clarificatory: namely, to explore how

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121 B 2- B 3, p. 43; B 4, p. 44.

122 Kripke, Saul. "Identity and Necessity" in Identity and Individuation. Edited by M. K. Munitz. New York: New York University Press, 1971.

123 Kripke, Saul. Naming and Necessity. Cambridge, Mass: Harvard University Press, 1980.

the possibility of the contingent a priori may be reconciled with my account of a priori knowledge.

Remember that I proposed to capture the experience-independence characteristic of a priori knowledge as that which is involved in a position where we can know even when we are in a state of total sensory deprivation. It can be argued that the tank insulation suggestion is not good here because it involves sensory deprivation, and this creates a very confused, hallucinatory state. Of course, I am not saying that in such a state there is no possibility of being wrong. But, that sensory deprivation actually leads to confusion and to hallucination seems to me entirely irrelevant - it's just a contingency, and we can easily imagine things otherwise.

It may be that only some contingent a priori statements come out as a priori according to my characterization. What is needed is to find a proposition with genuine clear content and see whether it is contingent and known a priori. [124]

Furthermore, it may be that even if there is such a class of a priori propositions, nothing of general interest arises from studying such a category. Nonetheless, I will discuss some questions I find interesting. I will discuss two kinds of candidates for contingent a priori propositions: (a) those supposedly obtained by stipulation and (b) the cogito propositions (propositions like "I think", "I have some beliefs", etc.)

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[124] Wittgenstein's view in the Philosophical Investigations, for example, is that in the standard meter case there is no proposition involved. There is something non-propositional involved. What is involved is a way of institutionalizing a particular length. There is no proposition involved but rather fixing a length. (The reference here is: Wittgenstein, L. Philosophical Investigations. Translated by G. E. M. Anscombe. Oxford: Basil Blackwell, 1968, p. 25.)

Some of the contingent a priori is concerned with stipulation. When we stipulate we lay down some principles. It is not enough just thinking. Such stipulation can be problematic. First, can we do it in the tank? It does not seem so in some cases. It can involve reference to the external world. In the insulation tank, can we achieve such a reference? [125] For example, one cannot measure physical objects in the tank. That requires contact with the objects. A suggestion is that we can access such reference in the tank perhaps by memory. If so, then I don't only have to have knowledge of concepts prior to the immersion to the tank but also have to be able to recall certain experiences I had previously like when I measured the standard meter. The idea is that we can remember that we achieved such reference prior to the immersion to the tank.

Again, the supposedly knowledge a priori of "The standard meter in Paris is one meter long", for instance, is controversial because it involves reference to physical objects and it appears that we cannot achieve that in the tank. The suggestion is that we can achieve such a reference in the tank by memory. But

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[125] It can appear that Wright's proof of the external world by a priori reasoning and his tank insulation idea as characterizing a priori knowledge are inconsistent. In particular, if one can prove that the external world exist in the insulation tank then one can refer to physical objects in the tank. But to achieve such a reference is problematic.

Wright explains (in private conversation) that what he offers in his paper "On Putnam's Proof that We are not Brains in a Vat" is not so much an a priori proof of the external world as a defense - up to a point - of Putnam's alleged proof that we are not brains-in-vats. (Wright's paper appears in Proc Aris Soc, No. 92, 1992, pp. 67-94.) In any case Wright thinks is wrong to suppose that there would have to be an inconsistency between the characterization of the a priori in terms of what can be known under conditions of sensory deprivation and the possibility of an a priori proof of the external world - the upshot, striking but not incoherent, would be merely that it was possible to know of the existence of an external world in conditions of sensory deprivation. In particular, Wright does not think it would follow that one could refer to physical objects while in the tank.

This all connects with the worry which has vexed a lot of people recently, about whether semantic externalism, so-called, doesn't have some very strange-looking a priori consequences.

this suggestion is problematic for two main reasons: first, if knowledge by stipulation requires stipulating the reference of a term to a physical object by pointing at the physical object then establishing such reference can only be done a posteriori. If we refer by a description rather than pointing at, we will have to remember in the tank not only concepts but, for example, that there is a meter in Paris that can be referred to as the standard meter. We have to know that that object in Paris can be the reference of a definite description. This knowledge is not going to be available in the tank. It is empirical knowledge. Then establishing such reference by a description can only be done a posteriori. In both cases the memory we would need to rehearse in the tank would be a memory of knowledge a posteriori that we obtained prior to the tank. But then how can that be a priori knowledge if it is a memory of knowledge a posteriori? Second, if we could refer to physical objects in the tank (I already explained why this is not possible but let's assume it for the sake of argument) then we could rehearse the a priori knowledge in the tank, so why do we need memory? Each way we see that knowledge obtained by memory cannot be a priori since in some cases the original knowledge was obtained a posteriori; not only that, it appears also that knowledge by memory of knowledge a priori we acquired previously should be considered a posteriori, just like knowledge by testimony of knowledge a priori obtained by someone else. So, some of the "contingent a priori" does not come out as a priori according to my account.

Nevertheless, stipulation is not an infallible method. It seems that talk about fallibility or infallibility in connection with stipulation does not even make sense, it seems incoherent at first sight; or if it were coherent, then one would have to say that stipulation is infallible by stipulation, which makes stipulation come out infallible trivially.

The idea is that the concept and paradigm are very close and do not leave room for the skeptical worry whether something satisfies the paradigm. There is supposedly no space for this question since "meter" and its paradigm are so close to make this question insensible.

But this ignores that stipulation is a fallible method. But if we have stipulate correctly, can we gain truth? Not necessarily so. We may have to change our stipulations because our needs change. So, the standard of correctness can change as well. Another possibility: what about the paradigm being slightly damaged? Something can have the standard meter length, but is the latter still one meter long? The paradigm is subject to contingencies, it can change.

The feature of the (b) cases - the cogito propositions - is that the attempt to doubt the proposition verifies it; that is, the attempt to doubt the proposition necessarily brings about a situation where the proposition in question is true. [126]

With the cogito propositions, there is a process, maybe a method involved. There is a process involved, namely, thinking. But not a method as in the mathematical case, for example, with the following features: (1) suspension of judgment about the truth-value of a proposition, (2) carrying out the method and, consequently (3) then figuring out the truth-value of the proposition we started with. I call a method which has these features a method of assessment. It

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126 What is the status which Descartes conferred to the cogito? It is not clear what metaphysical status the cogito had for Descartes. I doubt if Descartes cleanly confronted the distinction between necessary and contingent a priori. He thought his existence indubitable, (and that it could be shown to be so by the cogito), but he must also have recognized that he did not exist of necessity. In any case if the cogito is indeed cogent, we will see that the thought-routine which it involves is something that can be run in conditions of sensory deprivation, and so whatever is known as a result of it is something that can be known a priori - at least, according to my definition. Equally, it had better be something that isn't necessarily true.

is obvious that if there is a method involved in our knowledge of cogito propositions, it is certainly not a method of assessment. With the cogito propositions we don't suspend judgments about their truth-value, for instance.

The cogito propositions come out true in the tank by certain form of intellectual routine. The routine is not a method in the sense that it is not a method of assessment. A precondition of the doubt is that I exist. As part of the doubt you have to have the very thought in question.

Another example of a contingent a priori truth is: "I have some beliefs". In the tank I can know this proposition. These are psychological contingencies. It constitutes knowledge by introspection. As in the case of cogito propositions, it seems harmless that we can know these things in the tank. The reason is that they only involve understanding.

There are some complications with the cogito that make us rethink whether we can be infallible about it. What am I saying by "I think, therefore, I am"? I may not know who I am. The point is that the cogito propositions may be susceptible to a more sophisticated doubt.

Section 6: Is infallibility exclusively a property of certain methods?

It can be argued that since I am taking infallibility as primarily a property of methods, I should try to argue for infallibility as a matter of methods alone to justify my application, and should reject the view that sees infallibility as in a way connected with the nature of the proposition known also. Is this really a consequence of my view or can I just ignore the matter entirely?

First, I don't think that it is a consequence of my view. Talk of properties applying primarily to certain bearers does not preclude the possibility of intelligibly applying that same property to other bearers. It is the other way

around: in clarifying the primary uses of a property, other derivative uses get clarified as well.

However, I will not ignore the suggestion that infallibility is only a matter of (certain) methods. Actually, I will try to analyze what is involved with that idea in this chapter. To try to answer this issue, one would have to get clear about what is an internal relation, [127] and whether the content and modal status of the proposition possibly known by an infallible method are irrelevant.

I believe that infallibility can be accounted for wholly in terms of the existence of infallible methods and the correct implementation of them, independently of the status (i.e. analytic or necessary) of the propositions to be known. At least, I find that that would be a desirable result to have. It would fit nicely with taking the term "infallible" as applying primary to methods, and conceivably and derivatively to ourselves carrying such methods, and likewise, in any case, to propositions believed by their means. I don't yet have an argument for this conviction, but I will test below if it is correct.

The following question naturally arises: what is there in the method itself that makes it infallible (or fallible)? I want to try to answer that question as follows: that the correct answer is internal to the method (more below).

If infallibility is a matter of methods alone regardless of the nature of the truths known by their means, then it would seem to follow that it is possible that methods which deliver a priori contingent truths can be infallible. If that were not the case, that is, if methods which deliver a priori contingent truths cannot be infallible, then: how can we explain that methods which deliver a priori contingent truths cannot be infallible without appealing precisely to the fact that the proposition in question is contingent as part of such an explanation?

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[127] I will explain the notion of "internal relation" shortly.

In the case of stipulation, we already have seen that it is a fallible method. In the case of an a posteriori method like testimony, say, it is also a fallible method because when we testify to others we can make mistakes, for example, if we report the conversation or a result incorrectly. But can we still be mistaken if we report the conversation correctly ? Yes, because the result we report is false; the authority made a mistake.

Stipulation is not an infallible method; rather it is a fallible method to know some contingent a priori truths. Testimony is a fallible method even when it concerns knowledge of necessary truths.

Observe that the process by which I arrive at the cogito, thinking, is not in general an infallible process. One may wonder whether in cases like these of contingent a priori beliefs, if infallibility were involved, then the process alone would not generate infallibility but rather only in conjunction with the nature of the proposition at issue. Or, alternatively, is it that the possibility of an outcome being internal to a process independent of the modal status of the proposition in question? It may seem that these cases provide an affirmative answer to the first question, and a negative response to the second. But this is a mistake. These cases cannot provide the answers to those questions.

If one were inclined to think otherwise it is because one is ignoring the fact that with the cogito propositions, there is a process, maybe a method involved, but clearly not a method of assessment. With the cogito propositions we don't suspend judgments about their truth-value. In these cases the belief is immediately generated.

To sum up, one might think that the infallibility of methods does not depend on the modal status of the propositions known by their means having the cogito propositions in mind. But this suggestion totally ignores that the case of

the cogito, and similar beliefs, cannot settle this question because there is no method of assessment involved in these cases.

I am inclined to think that the idea of an outcome being internal to a process as generating infallibility is true in the more complicated cases. In these simple cases of knowledge of a priori contingent propositions, since there is no method of assessment involved, if infallibility were involved, it trivially cannot be explained by the fact that the outcome is internal to the process by which we executed the method of assessment. If there is a method involved in our knowing our own existence, it is not a method of assessment.

But what about our knowledge of trivially analytic truths and elementary truths of mathematics and logic? We intuitively expect that in these basic cases there could be more room for infallibility, if at all possible. With proofs we can expect more room for error. But my view is precisely that infallibility takes place in demonstrative knowledge. So it does not capture this intuition.

I think the issues here are complicated and connect with some of the things I said in chapter five about the defeasibility of the kind of intuitions which motivate mathematical axioms. If I stick to the line that I have been taking, that is, that infallibility is always a property of methods, and that we are always fallible in our implementation of them, then I am committed to this stance against the objection. But it is surely right that in the case of very simple and primitive logical mathematical truths, it is specially hard to make sense of the idea that we could be mistaken. But consider: "making sense" of the possibility will mean trying to imagine, in some detail, concrete circumstances in which it would seem to us that the best thing to say was precisely that we had our basic arithmetic, or our basic logic, e.g., wrong in one or other respect. What more is to give the impression of infallibility here than our inability to do this to our

satisfaction, together perhaps with an empirical confidence that no unforeseen circumstances are going to arise - of an unimagined kind - in which we have to admit that we do have basic logic or mathematics wrong? If that is all what is involved here, then it certainly does not seem to merit infallibility. Although it may show that we are never going to accept anything as calling the most basic logical and mathematical intuitions into question.

In any case, infallibility (i. e. a conceptual guarantee against error) is one thing. It is another thing not being able to conceive a situation in which one's fallibility was brought home to one (for example, getting concrete evidence that we made a mistake) together with being empirically certain that that is not going to happen.

Section 6.1: Which methods are infallible?

Are any empirical methods properly regarded as infallible? If so, then infallibility is not distinctive of some a priori methods. Let me try to explain in some detail some of the issues here and the way I am thinking about them.

How is it possible that methods can be infallible while warrants are always in principle defeasible? The distinction between infallibility and defeasibility is very important. The notions come apart and we have to see clearly that and why they do so.

It seems we should say that warrants are always defeasible in principle even when acquired by the prosecution of an infallible method. For it is always an epistemic possibility that a method was not properly applied, however careful we were. But if an infallible method was properly executed and we formed the belief we ought to form on that basis, then the warrant, - the result of the

execution of that method, - cannot be defeated as a matter of fact, (though we might erroneously come to think that it has been defeated as a matter of fact).

Suppose I consult a truthful oracle. Here is where the distinction between methods which are sometimes successful and methods which are always (necessarily) successful play a crucial role. Infallible methods are those which can only be successful. "Consulting a truthful oracle" is not an infallible method because it is not a method which is always necessarily successful. (more below)

Recall that a priori methods, whether infallible or not, are those which can be accomplished in a state of sensory deprivation, and that an infallible method is one such that, if it is done correctly, has a correct outcome. In mathematics, both conditions would seem to hold of necessity. At any rate, much mathematics could presumably in principle be accomplished in a state of sensory deprivation - it would be hard (probably too hard for normal human beings, specially in the case of geometry) but not impossible. And mathematical methods seem necessarily infallible. For with mathematical methods - calculation is the simplest example, but the point is general - the outcome is internal to the method. There is no way the outcome of a correct calculation can vary, and only if it could vary could its correct execution lead to an erroneous outcome.

The last remark has to be qualified though in a very important sense. First of all: why is it true? Consider a method which yields a root to an equation. There may be several.

I respond that the crucial thing, when an outcome is internal to the method, is that a subjunctive conditional along the following lines is necessarily true: had that outcome (or any of a range of outcomes - like, for example, the case of the equation with multiple roots above) not been obtained, the method would not have been correctly applied. That gets rid of the truthful oracle. Because it is not true that had the oracle given a different answer, then

necessarily we would not have asked the truthful oracle. For our question might have concerned a contingency, about which the facts might have been different.

It can be argued that despite the fact that there is a method which yields a root to an equation, and that there may be several, that the root to be found is more specific, so it is not just a root among many, but a specific root among many. In this case there is only one correct answer. We are looking for a specific root. It all depends, I suppose, on the numbers involved: so if certain numbers are involved in an equation, then its root is unique. Analogously, for example: an algebraic law: "a + b = b + a" involves different identities depending on the numbers involved. But given that certain specific numbers are involved then there is only one possible correct identity.

Here is another example. Many different conclusions are possible on the basis of a correct disjunction elimination step on a particular premise. What is characteristic of internal relations generally is, rather, that the obtaining of the relation is essential to the identity of the things related. Even a better example: any statement which may be validly inferred by the disjunction introduction rule from P, say, is such that it would not be the statement it is were it not so inferable. Likewise, if an equation has many roots, each of them will be such that it is an essential feature of them to be a root of that particular equation. Again, what is characteristic of internal relations in general is that the statement that expresses that the internal relation obtains is a necessary truth.

But infallibility may not be distinctive of a priori methods. We have to explore the possibility that there could be empirical methods which are infallible. I will try to explain why people have thought otherwise (of infallibility as distinctive of a priori knowledge in any case). Are their reasons good ones?

There seems to be a confusion between different things:

(a) truth as a necessary condition for propositional knowledge of any kind,

(b) necessary truth, as it is the outcome of the correct implementation of a method in the mathematical case, and

(c) necessarily the outcome is true as in the correct implementation of infallible methods.

If we implemented an infallible method correctly, then necessarily the belief we ought to form by the method is true. [128] That does not mean that the belief is a necessary truth. The modal status of the belief is a separate question.

On the other hand, taking infallibility as primarily applied to methods does not by itself exclude the possibility that the beliefs we can come to know by their means are all necessary truths. The point is that these are two separate issues, though connected in some other respects - as in the strong intuition that if we are dealing with an infallible method and by means of it arrive at a truth that is a necessary truth, then we have the feeling that the infallibility involved has been doubled or it is two-fold. There is a sense in which we feel have reached a double guarantee of infallibility.

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128 Is "get to know that p" an infallible method? For if one knows that p, p has to be true. Why isn't "getting to know that p" a method? One thing that might be said is that the concept of method has a certain generality: if a method is to be something whereby knowledge can be gained, then in general the characterization of a method should not prejudge the truth-value of any particular proposition which might be assessed by the method.
Though I consider that "to get to know that p" is the goal of a method, not the method itself. Now, if in applying a method, one always get to know that p then the method is infallible.

We have to distinguish three components in connection with fallibility and infallibility:

> (1) **we (the subject knowers): who are always fallible**
> (2) **methods: could be infallible**
> (3) **warrants (what the methods deliver): could be objectively indefeasible but given (1) they are always defeasible.**

However, a salient point about infallibility, as we are understanding it, is that it is only possible in conditional form, as it were: if we execute the method correctly, then p, the result, will be true. Cannot this hold in empirical cases too? It does seem so. If I observe correctly that there is a table in front of me, then there has to be a table out there. How can I observe correctly and be wrong about what I observe?

Well, what does it mean to observe "correctly"? Clearly, my organs can be functioning normally, and so in that sense I am observing correctly, but that does not ensure that I will get a correct answer. For I may fall victim to a sensory illusion, for instance, or otherwise misinterpret what I perceive. I can observe correctly - in the sense that my eyes are functioning properly, there is good light, etc. - but given that the table may be a fake one or the result of a visual illusion (like being a hologram), arrive erroneously at the belief that there is a table in front of me. It seems that in the case of the empirical method consisting of simple sensory observation, correctness of implementation - in the sense of proper sensory function, in suitable conditions - is not enough to guarantee the truth of a belief warranted thereby.

The question of whether any empirical methods are infallible in the way that say, calculation is, is I think a very complicated one. Consider counting as a method for determining the answer of an empirical question, say, the number of people in a room. We should reckon this method infallible just in case there is no

way that it can be done right yet we can get the wrong answer. Put like that, isn't such a method infallible? Can you count correctly yet get the wrong answer?

The issue is tricky. We can count abstract entities as well as physical objects. In my view, counting, as applied to physical objects, is not a straightforwardly infallible method. There is no guarantee that in correctly counting physical objects, one gets a correct answer to a particular set question. Counting physical objects is merely a "quasi-infallible" method since, obviously, we can arrive at a wrong answer to the question, how many people are in a room, because the number of people being counted may have changed in the course of my counting.

Now, consider a circle that is decreasing in radius as a function of time? One can do pure mathematics in relation to objects conceived as changing, provided one is given a mathematical characterization of the change (for example of the rate of change, or its extent). Graphs, for example, are typically about such things. Mathematics can be applied to changing things, provided the changes allow of unchanging mathematical representation at some level. So a bit of care is needed in the formulation of the idea that mathematics has an unchanging subject matter.

It might still seem that counting correctly, i.e. correctly following the rules of counting in relation to physical objects that do not move around or otherwise change, necessarily generates knowledge. But even that does not seem right. What if the objects being counted are fakes? In that case, then whether we get a correct result will depend on how we formulate it. In what does "the result" consist? Indeed: if what we had to be right about concerned only the number of objects, genuine or fake, then if the objects, whatever they are, don't move around or change, and we count them correctly, then it seems we must be right about how many of them there are. But such an instruction to count "objects"

would be hardly intelligible. Any intelligible such instruction will concern the F's, for some specific sortal predicate F, like "table". And if "the result" has to reflect the nature of the objects counted, in particular be sensitive to whether they are real or fake F's, then correct counting, as far as it goes, won't guarantee being right about how many F's there are even when the (apparent) F's are stable and unchanging.

Counting can be fallible too when in counting the numbers we rely on some sense-dependent means (for example, "How many chords did she just play?") However counting would seem to be an infallible method when applied to abstract objects defined by their essential characteristics ("Find the number of primes between 12 and 42"), when there is, naturally, no question of fakery.

In any case, it is clear that a distinction is called for between methods whose correct implementation is successful under certain conditions only and methods whose correct implementation is always successful; only the latter should count as infallible, period, and, consequently, counting, applied to physical objects, is not an infallible method. That invites the thought that we might elucidate a range of notions of "quasi" - (i.e. relative) infallibility by adding conditions to the antecedent of the "infallibility-conditional" [129] - conditions such as, for the case of counting, that the objects to be counted should not change in the course of counting.

Again, if a method is infallible provided, properly applied, it is guaranteed to lead to a correct result, then there may be a variety of notions of

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129 An infallibility-conditional expresses in its antecedent the conditions that have to obtain for infallibility to take place: in my view, for example, (1) that an infallible method was properly executed on the occasion and (2) that we formed the belief we ought to form according to the correct implementation of the method, for example. Then the belief generated is true and we end up with knowledge - the latter conjunction is expressed by the consequent of the infallibility-conditional.

infallibility, depending on certain relativities in the guarantee in question: for instance, certain empirical methods may be infallible relative to the assumption that things do not change in relevant respects while the method is being applied.

But there is an obvious risk of trivialization if we are allowed to be too free with such additions. In order to make an interesting issue of the question whether some empirical methods are quasi-infallible, it thus becomes crucial to get clear about which such additional conditions may usefully be contemplated. This is not an easy task and may not be even an attractive one to undertake. But the resulting taxonomy may still be important.

One other major issue is worthy of mention. We noted that correct observation - in the sense of proper sensory function under normal conditions - cannot conclusively settle whether what one is seeing is a table or, say, a hologram. The content of the proposition, "There is a table there" exceeds anything that can be once and for all established by finite episodes of vision or indeed other forms of sensory episode. But that seems to be a function of the content of the concept, "table". Might the situation be different if we considered instead a statement framed in terms of more purely observational concepts? Are there any concepts so intimately related to observation that they don't allow for any analog of the shortfall between observation and the facts that applies in the example of the table?

One range of candidates for such concepts are those of color, sound, and Lockean secondary qualities generally. However something can look red in all respects and not be red - there are phenomena like "red shift" in relation to distant stars, and illusions of color generated by background. But perhaps an interesting "infallibility conditional" can be generated for these concepts; perhaps we can circumscribe the range of ways in which appearances can mislead and

then if none of these applies, observation will be infallible with respect to e.g. color.

A thought is that no method can be absolutely infallible if its outcome is subject to extraneous causal influences. So it is tempting to conjecture that only where there is an internal relation between method, starting point, and outcome is infallibility possible, and hence that it is restricted to a priori knowledge if it exists at all. But this whole issue needs extended treatment. The immediate task is to explore what forms of relaxation/complication of the original simple condition expressed in the antecedent of infallibility conditionals - "If method M is correctly executed on the relevant occasion and the subject S forms the belief she ought to form according to the outcome, then that belief is true" - may be well-motivated on general grounds.

But, why should infallibility require that no causality is involved in the process? - surely there is causality involved in anyone's actually calculating, for example. What is true is that when the relation between bases, process, and outcome is internal, then there are no causal conditions such that, had they varied, the outcome could have been different although the basis and process were the same.

Conclusion

Let's recall that the circularity charge I made in chapter five against Hale's proposal in his book involves two aspects: (1) a vicious circle is involved; and (2) it is not very illuminating. Even if Hale succeeds with (H) - the later development in private conversation: (H) X knows a priori that p iff X knows that p and neither X's justification for believing that p nor p itself implies (entails) the truth of any experientially falsifiable q - in avoiding the first aspect of the circularity, and his definition is correct, still the second aspect of the circularity remains, it is not very illuminating, not positive enough. In his view, an a priori justification is characterized as simply lacking a certain feature. That is why one of my tasks in the second part of this dissertation has been to try to provide more illuminating suggestions than Hale's about the notions of an a priori warrant and a priori knowledge.

I tried to develop a view that is weaker than Hale's since my view doesn't put any constraint on the kind of defeating evidence against an a priori warrant or an a priori statement, and still respects the intuitions regarding a priori warrants and a priori knowledge. What is really relevant in my view for the characterization of an a priori warrant or an a priori statement is not immunity from revisability, or falsification, by empirical reasons, but rather justifiability independently of experience. I have attempted to capture the experience independence of a priori knowledge wholly in terms of the way the justification is carried out.

Given that we don't know if Hale's view can be correct, I prefer to be scrupulous and maintain a weaker view that does not commit me to a position on the nature of defeating evidence against alleged candidates for a priori knowledge. Provided that it is an open question whether we are going to have

empirical evidence against an priori statement, better to be more cautious.

I agree, given the qualifications that I discussed in chapter five, with Hale's claim that the notion of a priori knowledge (more accurately, the notion of a prioricity (i.e. a priori justified belief)) ought to be consistent with the possibility of revision. I agree also with Hale that a priori warrants do not have to be infallible. The mistake Kitcher made, and that Hale quite correctly diagnosed as due to conflating the truth entailing character of knowledge and the independence of experience characteristic of a priori knowledge, is to think that an a priori warrant has to be ultra-reliable (infallible) to be a priori. However, Hale does not consider the idea that some a priori warrants may prove to be infallible, even if indirectly as results of the correct implementation of infallible methods.

Hale does not appear to have distinguished between warrants and methods [130] or at least has not used the distinction at all. As a consequence, Hale does not address the interesting and very difficult question whether the notion of infallibility has a place only in the realm of a priori knowledge - in other words, whether infallibility is an a priori matter - or whether that is not the case.

In general, there is a distinction between the defeasibility of warrants and the possible infallibility of methods. We have to distinguish between the infallibility of certain methods and our fallibility in carrying them out. In his book, Hale accepts the defeasibility of a priori warrants and, for that reason, may

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130 Hale uses the terms "warrants" and "grounds". As already explained (note 51) Hale certainly does not distinguish between them in his book. Actually Hale identifies "a priori grounds for belief" and "a priori warrants" on p. 128.

have thought that a priori routes for knowledge cannot be infallible. But Hale fails to distinguish between defeasibility and infallibility. It is obvious that we are fallible, but it does not follow that there are no infallible methods such that if we execute them properly can get infallible results.

The second main task I have tried to accomplish in this part of the dissertation is to make sense of the concept of infallibility. The crucial issue in my opinion was to explain how given that we are fallible creatures we could nonetheless be in a position where we cannot get it wrong. I have tried to explain the coherence and possibility of such an epistemic position.

I don't have a knock-down argument for the claim that in some areas we have knowledge infallibly as a matter of fact. Though, of course, that is not to deny that making coherent the possibility of infallibility will pave the way for the resolution of the question whether we in fact have knowledge infallibly in certain areas. My point is that I will be satisfied if I have adequately accomplished the task of explaining the coherence and possibility of the notion of infallibility I am attracted to. The question to ask concerning a method is whether if I execute it correctly and then apprehend the belief I ought to form according to the result, there is any residual possibility of that belief's being mistaken. I argue that a method is infallible if and only if when it is applied correctly we end up always with true beliefs.

To settle the issue whether some empirical methods are infallible it becomes crucial to get clear about what we are allowed to add as a condition in the infallibility-conditional such that there could be empirical methods which are infallible. This is not an easy task and may not be even an attractive one to undertake. As soon as we allow ourselves, for example, to "freeze" physical objects so that they don't change as long as we apply any method to them, and to

"assume" that they are real instead of illusory, then how are we going to stop adding more and more conditions? The danger I see with the maneuver of enriching the infallibility conditional in this manner is that it would result in infallibility coming out very cheap indeed. The notion of infallibility I have defended could be in danger of getting trivialized in this way, so infallibility in general would be again trivialized as a consequence. For reasons of space and time I can't provide the full treatment this very interesting issue deserves in this dissertation. But I will say a little more before finishing this topic.

It can be argued that if the notion of infallibility runs the risk of trivialization, that would show that it's not a deep issue. Is that so bad? It will be bad if as a consequence no sense of infallibility can be found. I don't think that would be a consequence. Though what we could find out is that there is some relativity to infallibility. I consider that it won't be so bad if we find that there are degrees of infallibility. That will involve finding more and various assumptions - according to the subject matter - such that depending on whether they have been satisfied the method will deliver infallibly knowledge.

To conclude, it is clear that I have suggested the view that the interesting sense in which infallibility is conceivable as obtaining is in the realm of the a priori. I conclude that it does seem that no a posteriori method is absolutely infallible and, it is certainly the case, that not all a priori methods are infallible. However, a priori truths are generally known by the correct prosecution of infallible methods.

That the interesting sense in which infallibility is conceivable as obtaining is in the realm of the a priori is, in general, what we traditionally have thought, and what we intuitively expected to occur, if at all. But it was crucial to lay down what underlines this basic intuition. And that is precisely what I hope to have

accomplished in this dissertation with a very important qualification. Infallibility is not infallibility any more: that is, we are not infallible about anything, some methods are.

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